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Promoting Mathematical Reasoning in a Multilingual Class of Grade 7 English Second Language Learners

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Abstract

This qualitative study was conducted in one school in an informal settlement, West of Johannesburg. The study explored how a grade 7 teacher promoted mathematical reasoning in multilingual mathematics class of English second language learners. The focus of the research was on how a Grade 7 mathematics teacher interacts with the learners to encourage mathematical reasoning during his teaching in a multilingual class. The study also looked at the kind of tasks the teacher used to promote mathematical reasoning and how he uses language to enable mathematical reasoning. The study was informed by a theory of learning which emphasises the importance of social interaction in the classroom where the teacher encourages learners to interact with each other to explain their thinking and to justify their answers. Data was collected through lesson and teacher interviews. The study shows the teacher focused more on developing the learners' procedural fluency. This focus on procedural fluency was accompanied by the dominance of the use of English by the learners.

DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Education by coursework at the University of the Witwatersrand, Johannesaburg. It has not been submitted before any degree or examination in any other university.

Lindiwe Tshabalala

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TABLE OF CONTENTS

ACKNOWLEDGEMENT 3

CHAPTER 1 INRODUCTION TO THE STUDY 7

WHY MATHEMATICAL REASONING? 7
WHY MATHEMATICAL REASONING IN MULTILINGUAL CLASSROOMS? 8
WHY MATHEMATICAL TASKS? 10
CONCLUSION 11

CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW 12
INTRODUCTION 12
THEORETICAL FRAMEWORK 12
WHAT IS MATHEMATICAL REASONING? 14
THE ROLE OF LANGUAGE IN PROMOTING MATHEMATICAL REASONING 15
THE ROLE OF MATHEMATICAL TASKS WHEN PROMOTING MATHEMATICAL REASONING 18
Lower-level demands 18
Higher-level demands 18
Implementation of tasks 19
CONCLUSION 20

CHAPTER 3 RESEARCH DESIGN AND METHODOLOGY 21
INTRODUCTION 21
SELECTION OF THE CASE 21
WHO IS THIS TEACHER? 21
Description of the class 21
DATA COLLECTION 22
Lesson observation 22
TEACHER INTERVIEWS 23
VALIDITY AND RELIABILITY 23
Descriptive validity 23
Construct validity 24
Interpretive validity 25
PILOTING 25
ETHICAL ISSUES THAT MAY ARISE IN THIS STUDY 25
HOW DATA WAS ANALYSED 26
Lower-level demands tasks 26
Higher-level demands 27
Analysis of classroom interaction 27
CONCLUSION 28

Deleted: 3
Deleted: 3
Deleted: 7
Deleted: 7
Deleted: 7
Deleted: 7
Deleted: 8
Deleted: 8
Deleted: 10
Deleted: 10
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Deleted: 23
Deleted: 23
Deleted: 24
Deleted: 24
Deleted: 24
Deleted: 25

CHAPTER 4 DATA ANALYSIS 29
INTRODUCTION 29
A BRIEF DESCRIPTION OF THE LESSON OBSERVED 29
Analysis of the task 30
Language demands of the task given to learners. 32
THE TEACHER'S ROLE IN PROMOTING MATHEMATICAL REASONING 33
Classroom environment 36
HOW THE TEACHER DEVELOPED MATHEMATICAL PROFICIENCY? 38
Conceptual understanding 38
Procedural fluency 41
Strategic competence 43
Adaptive reasoning 46
Productive disposition 48
LANGUAGE USE 49
Probing the learners to talk about mathematics 49
Interaction amongst the learners and amongst the teacher and the learners 50
CONCLUSION 53

CHAPTER 5 FINDINGS AND RECOMMENDATIONS 54
INTRODUCTION 54
SUMMARY OF THE FINDINGS 54
How did the teacher encourage mathematical reasoning during teaching and how did the way in which he interacted with the learners enable or constrain mathematical reasoning? 54
The kind of task the teacher used to promote mathematical reasoning. 57
How did the teacher use language to enable mathematical reasoning? 57

RECOMMENDATIONS 59

CONCLUSION 62

REFERENCES 63

APPENDICES

APPENDIX A: Research transcript 65
APPENDIX B: Pre-observation schedule 77
APPENDIX C: Post observation interview 78
APPENDIX D: Letter to the teacher 80
APPENDIX E: Letter to the School Principal 82
APPENDIX F: Consent form for the Principal 84
APPENDIX G: Consent form for teachers 85
APPENDIX H: Consent form for learners 86
APPENDIX I: Consent form for parents 87
APPENDIX J: Information letter to parents 88

Deleted: 29
Deleted: 29
Deleted: 29
Deleted: 29
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Deleted: 59
Deleted: 59
Deleted: 62
Deleted: 62
Deleted: 63
Deleted: 63
Deleted: 65
Deleted: 65
Deleted: 77

APPENDIX K: Information letter to learners 89
APPENDIX L: Research observation sheet 90

Deleted: 89
Deleted: 89
Deleted: 90
Deleted: 90

LIST OF TABLES

TABLE 4.1: Teacher talk distributions according to codes and utterances. 33
TABLE 4.2: Analysis of task based on cognitive levels..... 35

Deleted: 33
Deleted: 33
Deleted: 35
Deleted: 35

LIST OF FIGURES

FIGURE 4.1: Task given to learners..... 30

Deleted: 30
Deleted: 30

CHAPTER 1 INRODUCTION TO THE STUDY

INTRODUCTION

Mathematical reasoning is one of the important aspects of the new curriculum in South Africa (DoE, 2002). Learners are expected to actively question, examine, conjecture, and justify their solutions and present arguments. Educators are thus expected to promote these practices in learners when teaching mathematics. Teachers have to ensure that the learners are exposed to mathematical practices that promote mathematical reasoning. Furthermore, the Revised National Curriculum for Mathematics General Education and Training indicates that the mathematics programme should provide opportunities for learners to develop and employ their reasoning skills and be able to evaluate the arguments of others (DoE, 2002). The purpose of this study was to investigate how a Grade 7 teacher in a multilingual class encourages and facilitates mathematical reasoning in a Grade 7 class of English second language learners.

The study was guided by the following questions:

1. How does a Grade 7 mathematics teacher in a multilingual class encourage mathematical reasoning during teaching?
2. How does the way in which the teacher interacts with the learners enable or constrain mathematical reasoning?
3. What kind of tasks does the teacher use to promote mathematical reasoning in a multilingual class?
4. How does the teacher use language to enable mathematical reasoning?

WHY MATHEMATICAL REASONING?

Mathematical reasoning is not a content area like addition; it is a skill that is embedded in the practice of mathematics. It is therefore not easy to teach. It has to be attended to in all the mathematics learning outcomes and concepts that are taught in each grade. Kilpatrick, Swafford and Findell (2001) argue that mathematical proficiency is the evidence of mathematical reasoning. Mathematical proficiency is evident when learners show conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (Kilpatrick et al, 2001). According to Kilpatrick et al (2001), these

strands of mathematical proficiency are not independent of one another; they are interwoven and interdependent in the development of proficiency in mathematics. Mathematical proficiency, and hence mathematical reasoning, enables the learners to cope with the mathematical challenges they come across in their daily lives. As learners are in the process of developing mathematical proficiency, language becomes an important component. Learners require language in order to be able to explain and justify their answers. The study therefore explored how multilingual learners who learn mathematics in a language that is not their home language use language(s) when dealing with mathematical tasks that promote mathematical reasoning. How do the teachers and learners use their home language and/or English when interacting with tasks that encourage mathematical reasoning? Lauf (2004) argues that mathematical reasoning is much wider than just proving results, "it involves firstly the pursuit of what is true, usually leading to the formulation of a conjecture" (p. 23). She cites De Villiers (1990) who suggests that the roles of proof are verification, systematisation, explanation, discovery as means of communication (Lauf, 2004).

Mathematical reasoning is not some activity that is confined within the classroom, but its impact can be seen inside and outside the classroom when learners put into practice the mathematics they have learned in the classroom. Teaching mathematical reasoning requires the teacher to create a mathematical community in which different views are listened to and respected. As Ball and Bass (2003, p. 20) argue, "If students are to engage collectively in substantial mathematical work, they need to learn to listen attentively and critically to views of others." Listening attentively helps learners to critically evaluate mathematical claims, both their own and those of others. This study thus explored how a Grade 7 teacher encourages interaction that promotes mathematical reasoning amongst second language learners.

WHY MATHEMATICAL REASONING IN MULTILINGUAL CLASSROOMS?

In this study a multilingual class is regarded as one in which both the learners and the teacher are multilingual (i.e. they speak more than two languages) and learn and teach mathematics in a language that is not their main language. The fact that most teaching and learning in these classrooms take place in English, which is not the main language of the teachers and learners, makes participation more difficult for learners and the development

of ideas more difficult for teachers (Brodie, 2005). Justification, verification and explanation of answers inevitably requires extensive use of language and this raises questions about learners who learn mathematics in a second language, hence the importance of a study that investigates how a teacher promotes mathematical reasoning in a classroom where learners learn mathematics in a language that is not theirs. As Sfard, Nesher, Streefland, Cobb and Mason (1998) argue language is a major route to the articulation of ideas however natural language is limited in its ability to describe mathematical notions.

Ball & Bass (2003) maintain that while promoting mathematical reasoning; there should be interaction in the classroom. The interaction involves language use, which could enable or constrain mathematical reasoning. In other words, language cannot be avoided in a mathematics class in which mathematical reasoning is encouraged. This is highlighted by Pimm (1981) who argues that mathematics is dependant on language. To ensure effective verbal communication in a mathematics class, Pimm (1981) insists that the role of any mathematics teacher is the encouragement of fluency both in oral and written mathematical language. Hence the importance of this study which explores how the teacher encourages meaningful communication, to understand how the learners use language to construct, express and communicate mathematical meanings especially because they do mathematics in a language that is not their home language. Pimm (1981, p. 2) further argues that "most mathematical classes actually take place in a mixture of ordinary English and mathematical English in which ordinary words are used with a specialised meaning. It is therefore important to ask how second language learners cope with the use of ordinary English and mathematical English especially because mathematics is notorious for attaching specialised meanings to everyday words, words which already have meanings (Pimm, 1981). So, how does the use of language in a multilingual class enable or constrain the use of this specialised language of mathematics?

From my experience as a teacher, I that found English language sometimes can be a barrier in the understanding of mathematics. Pimm (1981) alludes to the same sentiment that many children's difficulty with mathematics may be due more to the complexity of language rather than the mathematical task requested. Educators cannot run away from language involvement in mathematics as Pimm (1981) states that "mathematics when spoken, emerges in a natural language."(p. 16). Mathematical reasoning requires explanation and justification of answers, which poses sophisticated linguistic demand on the learners in terms of communicative process (Pimm, 1981). Backhouse, Haggarty, Pirie and Stratton

(1992) allude to the same sentiment that it is not just technical vocabulary that the teachers need to be careful about but even words that the teacher regards as familiar may cause difficulty to learners. The study thus explores how the teacher copes with such demands where learners have to choose what to say in order to convince the class.

Backhouse et al (1992) also argue that there is potential lack of common understanding of the meanings of words, which leads to failure in communication from learner to teacher and from teacher to learner. This can be due to the use of words which have different meanings in ordinary English which can however cause more trouble and indeed through my experience as a teacher have realised that where language is a resource of learning and teaching it can also be a challenge.

WHY MATHEMATICAL TASKS?

Tasks are important in the teaching and learning of Mathematics. As Stein, Grover and Henningsen (1996) argue, mathematical tasks are important vehicles for building student capacity for mathematical thinking and reasoning. Therefore, in exploring how a teacher in a multilingual classroom promotes mathematical reasoning, a focus on tasks is important. Stein et al (1996) argue that different kinds of tasks make available different kinds of opportunities for the learners. The study explored the kind of tasks the teacher selects to promote mathematical reasoning. Stein et al (1996) emphasise the importance of the cognitive demand of the set task. The research explored whether the tasks given allows the process of mathematical thinking in learners.

The study analysed the different levels of task demands. Stein et al (1996) argue that there are two levels i.e. lower level demand tasks and higher-level demand tasks. Lower-level demand tasks are memorisation tasks and those that require procedures without connections. Higher-level demand tasks include tasks with connections and those that require the doing mathematics tasks. These different kinds of tasks will be explained in more detail in Chapter 2, of importance at this stage is that the kinds of tasks that the teacher selects or develop are critical for promoting mathematical reasoning.

CONCLUSION

The chapter has described the purpose and rationale for the study. The study focuses on mathematical reasoning in a grade 7 multilingual classroom. The work done by Kilpatrick et al (2001) forms the basis of this study.

CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

INTRODUCTION

This chapter discusses Kilpatrick et al's (2001) strands of mathematical proficiency and constructivism which as indicated form the theoretical basis for this study. All five of Kilpatrick's strands for mathematical proficiency will be discussed. Furthermore, review of the literature including literature on the role of language and the use of tasks in mathematics teaching and learning will be discussed.

THEORETICAL FRAMEWORK

The study is broadly informed by the social constructivists' theory of learning (Von Glasersfeld, 1987; Vygotsky, 1987; Taylor and Campbell-Williams, 1993 cited in Jaworski, 1999 which recognises the importance of a knowledgeable other in the construction of knowledge. To understand how the teacher promotes mathematical reasoning in the learners, Kilpatrick et al's (2001) five strands of mathematical proficiency were used to analyse the interaction during the lesson. Kilpatrick et al (2001) argue that when learners are mathematically proficient it means their mathematical reasoning has developed. Mathematical proficiency is the evidence of mathematical reasoning. They describe the five strands of mathematical proficiency as follows:

Conceptual understanding.

Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. Learners understand why a mathematical idea is important and the kinds of contexts in which it is useful.

Procedural fluency.

This strand refers to the knowledge of procedures, knowledge of how and when to use them appropriately and the skill in performing them flexibly, accurately and efficiently. Kilpatrick et al (2001) argue that without sufficient procedural fluency, learners have trouble deepening their understanding of mathematical ideas or solving mathematical problems.

Strategic competence.

This competency means that the learners have acquired problem-solving strategies and have developed the ability to formulate mathematical problems and represent them. Kilpatrick et al (2001) argue that it is crucial for mathematical proficiency that learners should know a variety of solution strategies as well as which strategies might be useful for solving a specific problem.

Adaptive reasoning.

This strand indicates the learners' capacity to think logically about the relationships among concepts and situations. Kilpatrick et al (2001) state, "one manifestation of adaptive reasoning in learners is the ability to justify one's work" p. 130. According to them once the learners' procedures, concepts and solution methods fit together and make sense, then they have developed adaptive reasoning. It is therefore important for learners to develop classroom norms in which students are expected to justify their mathematical claims and make them clear to others. This can be done by allowing students to use new concepts and procedures for some time and to explain and justify them by relating them to procedures that they already understand (Kilpatrick, 2001).

Productive disposition.

According to Kilpatrick et al (2001), learners have developed productive disposition if they have developed a tendency to see sense in mathematics, and perceive it as both useful and worthwhile. They argue that the evidence will be seen when learners believe that steady effort in learning mathematics pays off, and see themselves as effective learners and doers of mathematics. In this strand learners develop confidence and believe that they can make it. Kilpatrick et al (2001) state, "productive disposition develops when the other strands do and helps each of them to develop." (p. 131). They argue that this happens because the learners' attitudes, beliefs, and beliefs about themselves as mathematics learners become more positive.

While the teacher can promote mathematical reasoning using the above-mentioned strands, an issue of social interaction arises especially when learners have to explain and justify their answers. This is supported by Von Glasersfeld (1987); Vygotsky (1987); Taylor and Campbell-Williams (1993) cited in Jaworski, 1999. These theorists emphasise the importance of social interaction in the classroom where the teacher encourages learners to interact with each other to explain their thinking and to justify their answers. Von Glasersfeld (1987) argues that classroom interaction can provide access to the perceptions

and understandings of those involved. From an interactionists perspective it is important to create a teaching environment in which learners are encouraged to talk about their mathematical understandings with each other thereby develop their mathematical thinking (Jaworski,1994). Von Glasersfeld (1987) argues that the teacher uses language to guide the students' construction. He argues that "the student, working on some mathematical task, talks with the teacher and stimulated by the teacher's prompts and responses with the teacher reveals aspects of awareness which provide clues." (cited in Jaworski, 1999, p. 27).

According to Taylor and Campbell-Williams (1993) reality is constructed intersubjectively, it is socially negotiated between the significant others who are able to share meanings. They further highlight the central role of language in learning. The learner is regarded as an interactive co-constructer of knowledge. Vygotsky (1978) placed great emphasis on social and language influences on learning and in particular the role of the teacher in the educative process. Vygotsky (1978) maintains that with appropriate instruction there may be a potential that the child reaches higher conceptual levels than he/she would be able to achieve on their own. According to the constructivism theory, construction of knowledge in the classroom goes beyond interaction between the teacher and students, to the wider interaction between the learners themselves in the social and cultural environment of the classroom.

WHAT IS MATHEMATICAL REASONING?

According to Ball & Bass (2003, p. 2) mathematical reasoning is a basic skill and "mathematical understanding is meaningless without the serious emphasis on reasoning". Mathematical understanding is founded on reasoning and mathematical reasoning is fundamental to using mathematics. Mathematical reasoning enables the learners to tackle problem solving even if they have forgotten the algorithm. As pointed out earlier the biggest challenge with teaching mathematical reasoning is the fact that it is not a content area but a skill that is fundamental to what it means to do mathematics. Ball and Bass (2003) argue that assessing the validity of students' ideas presents a number of challenges for teachers. For teachers to know how and when to probe particular student ideas, they need to be able to identify where there is potential for productive mathematical conversation. (Brodie, 2005).

Teaching is a thinking practice in that teachers do intellectual work in the classroom and the focus of their work is to foster thinking practices among their students (Lampert, 1998 cited in Brodie, 2005). Teachers and learners think alone and together inside and outside classrooms. Their (teachers' and learners') individual thinking shapes and is shaped by their collective thinking, the broader contexts of their thinking and the discourses and systems of activity that constitute teaching and learning mathematics in school (Brodie, 2005). She further argues that listening for sense in students' thinking, even when they seem to be incorrect, is probably the most difficult part that the teachers are facing. It requires that teachers listen to students in ways that draw on teachers' own knowledge of mathematics and of students, and yet allows their ways of knowing to be challenged at the same time. Teachers can never know what learners are thinking; they need to make inferences, and based on these inferences make decisions about their next moves.

Ramnarain (2003) defines mathematical reasoning the same way as mathematical thinking. In his study, he outlines that mathematical thinking entails cognitive processes such as visualisation, abstraction, reflection, integration, synthesis, induction and deduction. Ramnarain (2003) further argues that mathematical thinking is an active dynamic process that includes the abilities to recognise patterns, generalise similar problem situations, identify errors and generate alternate strategies. He regards critical thinking as an important aspect of mathematical thinking. He cites that learners develop critical thinking, which enables them to generate mathematical conjectures as well as to evaluate them (Borasi, 1992).

THE ROLE OF LANGUAGE IN PROMOTING MATHEMATICAL REASONING

Fraser, Murray, Hayward and Erwin (2004) maintain that learners should be given tasks that would encourage them to work co-operatively, discuss, argue and reflect on the methods of others. Talking and hearing others involve language; through talking the learners make their mathematical knowledge public and usable by the collective (Ball and Bass, 2003). This is the reason why Ball & Bass (2003) argue that language is the foundation of mathematical reasoning. Carpenter, Franke and Levi (2003) refer to a teacher in their study, Margaret Jensen, who argues that viewing language and mathematics as separate endeavours limits children's experiences. Hence, she suggests, "many kids might need support when engaging in language-rich mathematics experiences" (Carpenter et al, 2003, p. 51). She maintains that

while language can interfere with learning, it also can nurture learning. Language is fundamental to mathematical reasoning.

Moschkovich argues that mathematics classrooms are shifting from a focus on primarily silent and individual activities as "students are now expected to communicate mathematically, both orally and in writing, and participate in mathematical practices, such as explaining solution processes, describing conjectures, providing conclusions, and presenting arguments" (2002, p. 190). She emphasises the importance of mathematical communication and how classroom instruction can support learners in learning to communicate mathematically. To ensure fruitful interaction in a mathematics classroom; teachers have to have the skill to probe learners to prove their answers (Ball and Bass, 2003). They argue that teachers should be able to help learners justify their claims. Fraser et al (2004) maintain that when teaching mathematics social interaction creates opportunities for learners to talk about their thinking and this encourages reflection in learners. They support this by saying that "to verbalise what one is doing ensures that one is examining it." (p. 27). As learners talk to one another they think about and examine what they are saying if it makes sense or not.

Carpenter et al (2003) in their investigation of mathematical thinking in learners interviewed a teacher, Virginia Koberstein, who highlights the importance of language in making conjectures. Virginia states that the mathematics instruction in her class is heavily dependent on language. She argues that it is very critical to have all kids participate in justifying and explaining their answers in the mathematics class. Language thus becomes one of the tools for participation in this mathematical discourse. Ball & Bass (2003, p. 11) argue, "Language is crucial for mathematical reasoning and communication about mathematical ideas, claims, explanations and proofs. It is a medium in which mathematics is enacted, used and created." Without language, there will be no discussions and arguments.

The role of language in a class of second language learners is thus more than just acquiring vocabulary and word meanings, it is about participating in mathematical discourse practices (Moschkovich, 2002). Mathematics is about not only developing competence in completing procedures, solving word problems, and using mathematical reasoning but it is also about developing socio-mathematical norms, presenting mathematical arguments and participating in mathematical discussions.

Affirming the learners' responses is just as important as encouraging them to justify their answers. Teachers need to develop productive ways of challenging learners to justify their answers. Justification is central to mathematics and mathematics cannot be learnt with understanding without engaging in justification, (Carpenter et al, 2003). Moschkovich (2002, p. 193) alludes to the same sentiment by stating that "in many classrooms teachers are incorporating many forms of mathematical communication and students are expected to participate in a variety of oral and written practices such as explaining solution processes, describing conjectures, proving conclusions and presenting arguments". Sasman; Linchevski; Olivier and Liebenberg's (1998) also argue that learners must be encouraged to justify their answers. These arguments resonate with Carpenter et al's argument that maintains that young children cannot learn mathematics with understanding without engaging in justification (2003).

As learners learn mathematics, they share ideas, which require them to convince others that the procedures they are using to solve problems are valid; they have to use arguments that are convincing to other learners. Carpenter et al (2003) further argue that they have "found that it is productive to ask children whether their conjectures are always true and how they know they are true" (p. 102).

Precision is very important in communicating mathematical ideas (Ball and Bass, 2003, Carpenter et al, 2003) and this can create challenges to second language learners who are still learning the language of teaching and learning. Sasman et al (1998) argue that learners experience major difficulties with mathematical reasoning, as they do not have the tools to address the challenges they come across. These tools include the language of learning and teaching as well as the mathematical language. As Moschkovich (2002) aptly points out sometimes when learning mathematics, multiple meanings can create obstacles in mathematical conversations because students use colloquial meanings of terms whereas teachers may use mathematical meaning of terms. She argues that these multiple meanings make it difficult for second language learners to attain mathematical proficiency because they learn mathematics in a language that is not theirs.

Learners learn to communicate mathematically by using multiple resources and participating in mathematical practices, such as abstracting, generalising, being precise, achieving certainty, explicitly specifying the set of situations for which a claim holds and trying claims to representations" (Moschkovich, 2002, 196). She further states that when

clarifying a description sometimes it becomes an obstacle to second language learners. She therefore suggests that classroom instruction should support second language learners' engagement in conversations about mathematics that go beyond the translation of vocabulary and involve learners in communicating about mathematical concepts. If students carefully articulate, refine and edit conjectures, they confront important mathematical ideas and engage in basic forms of mathematical arguments (Carpenter et al, 2003). In their study they found that as children attempt to articulate conjectures, their language became more precise. The more the learners interact about mathematical ideas and concepts the more their mathematical language improves.

Language has been cited as one of the reasons why some learners perform badly in word problems. Murray (2003) argues that language becomes a hindrance because of poor comprehension skills in second and third language learners.

THE ROLE OF MATHEMATICAL TASKS WHEN PROMOTING MATHEMATICAL REASONING

The types of mathematical tasks that teachers make available to learners and the language used in them is therefore important when teaching in a multilingual class of second language learners. Stein et al (1996) classify the different kinds of tasks into two namely; lower-level demands tasks and higher-level demands tasks.

Lower-level demands

Stein et al (1996) explain lower level demands tasks as memorization tasks and procedures without connections tasks. Memorization tasks require learners to reproduce previously learned facts, rules, formulae or definitions. Procedures without connections tasks are algorithmic; they are focusing on producing correct answers rather than developing mathematical understanding. They require limited cognitive demand.

Higher-level demands

These kinds of tasks refer to procedures with connections tasks and doing mathematics tasks. In procedures with connections tasks, learners use procedures for the purposes of developing deeper levels of understanding of mathematical concepts and ideas. These kinds of tasks require some degree of cognitive effort. In doing mathematics tasks, learners

manifest complex and non algorithmic thinking. These tasks require the learners to explore and understand the nature of mathematical concepts, process or relationships. They also require considerable cognitive effort from the learners. The research will not just focus on task set up but will also focus on the implementation thereof.

Implementation of tasks

Stein, Smith, Henningsen and Silver (2000) argue that if the teacher wants students to learn how to justify or explain their solution processes, he/she should select tasks that are deep and rich enough to afford such opportunities. They contend that tasks should have the potential to engage learners in complex forms of thinking and reasoning. Stein et al (2000) suggest 'doing mathematics tasks' as tasks that can promote mathematical reasoning because they require complex and non algorithmic thinking. With 'doing mathematics tasks' there is not a predictable, well rehearsed approach or pathway explicitly suggested by the task, task instructions or worked out example. These tasks require students to explore and understand the nature of mathematical concepts, processes or relationships.

Stein et al (1996) emphasise that while the learners engage in mathematical tasks that promote reasoning the teacher should provide them with sufficient time and provide encouragement for exploration of mathematical ideas. An exploration of how teachers encourage and develop mathematical reasoning in their learners therefore evokes questions about the nature of the tasks that the teacher uses as well as the interactions that the teacher sets up during teaching. One of the things that this study explored is the kinds of tasks that the teacher uses to promote mathematical reasoning in a Grade 7 multilingual class of second language learners.

The selection and implementation of mathematical tasks is important because tasks can make visible the underlying mathematical reasoning that learners have to engage in (Ball and Bass, 2003). While it is not possible for the learners to know how to reason mathematically, it is the responsibility of the teacher to make reasoning central to their teaching through the tasks they give and by making the mathematical knowledge and language public for the learners. Stein et al (1996, p. 457) argue, "Students must be first provided with opportunities, encouragement and assistance to engage in thinking, reasoning and sense making in the mathematics classroom." In many classrooms teachers set up tasks and group work situations where learners engage with the tasks however, when learners express their thinking, very few teachers are able to support and engage with these ideas to

take them further and to develop them mathematically (Brodie, 2005). Hence, the importance of exploring the tasks that the teacher sets up to encourage mathematical reasoning and the interactions that occur during the implementation of the tasks to support mathematical reasoning.

Wood, Cobb and Yackel (1992) state that when promoting mathematical reasoning teachers create an environment in which children can risk expressing their thinking and also question other learners' ideas. The mathematical community that needs to be created when promoting mathematical reasoning allows learners to explain their methods, for solving mathematical problems and others would listen and decide if the explanation made sense to them or not. The teacher is expected to create opportunities for learning where learners are engaged in discussions in which their ways of solving problems are of central interest. Wood et al (1992) argue that learners should be provided with an opportunity to ask questions or provide information to help clarify the meaning. They further argue that if the teacher expects the learners to express their thinking to the others with the possibility of being questioned or challenged they also expect the teacher to accept and respect their thinking as well. The teacher is not supposed to impose his/her ways of thinking on learners, embarrass or ridicule them in any way or allow other students to do so. (Wood et al, 1992).

CONCLUSION

In this chapter, the theoretical framework that informed this study has been discussed. Furthermore an overview on the role of language in mathematical reasoning has been provided. The chapter that follows describes the research design and methodology used in this study.

CHAPTER 3 RESEARCH DESIGN AND METHODOLOGY

INTRODUCTION

This was a qualitative case study focussing on one multilingual Grade 7 classroom in a township school west of Johannesburg. The purpose of the study was to describe and explain how one grade 7 teacher promoted mathematical reasoning in his multilingual classroom and how the way in which this teacher used language(s) enabled or constrained mathematical reasoning. This chapter discusses the research design and methodology employed in the study.

SELECTION OF THE CASE

For this study, I needed a grade seven mathematics teacher who teaches in a multilingual class where learners learn mathematics in a language that is not their home, first or main language. It was important that this teacher is appropriately qualified and has a reputation as one who promotes mathematical reasoning in his teaching. In this study, the teacher was selected for participation in the study.

WHO IS THIS TEACHER?

He is the Head of Department for mathematics, Science and Technology in a school, West of Johannesburg, where I also teach. He has a three-year diploma majoring in Mathematics and a diploma in computer science. He has been teaching Mathematics for the past nine years. Furthermore, he regularly participates in teacher education initiatives of the Association for Mathematics Educators in South Africa (AMESA). As explained earlier he was selected for this study because he is known to be promoting mathematical reasoning in his class. This teacher was observed teaching in a grade 7 class, which he selected.

Description of the class

This class is one of the three grade 7 classes in an African school that caters for children in an informal settlement area in the West of Johannesburg. The language of learning and teaching in the school is English, however none of the learners and the teachers in the school had English as their main, home or first language. Like the teacher all the learners in

the class are multilingual. They speak the following languages i.e. Setswana, Zulu and Xhosa. They study their home languages as subjects. In addition to these languages, the learners also study English as a subject. The class was heterogeneous in terms of gender and ability. It had a range of learners from those who achieve high marks in mathematics to those who are struggling to pass.

DATA COLLECTION

Data in this study was collected through lesson observation and interviews.

Lesson observation

Three lessons were observed and video recorded. The first two lessons were important for the learners to get used to the presence of two other people in the class as well as the video camera. It was also advantageous to observe three lessons even though the analysis was focussing on one because that would give a broader understanding of the context of the lesson observed for research. Each lesson was 40 minute long.

The lesson was video recorded to ensure that as much as possible of what the teacher and the learners did in class during the lesson presentation was captured. The video camera was able to capture the mathematical conversations that emerged while the teacher interacted with the learners, something that could not be easily done with an observation sheet alone. During the course of the lesson, the teacher was moving towards the learners while they were responding to his questions. The video focused on the interaction between the teacher and the learners and how the teacher communicated with the learners to promote mathematical reasoning. Looking at verbal and non-verbal features I explored if learners comprehend mathematical concepts, operations and relations, strategic competence in which they are able to formulate, represent and solve mathematical problems. During observation the video focused on capturing incidents where learners had capacity for logical thought, reflection, explanation and justification and how they used language to do this (Kilpatrick et al, 2002).

A structured observation schedule was used to ensure a focus on the issues that were related to the research questions as it was easy to get carried away if one was not focused. (Appendix C). The schedule focused on how the teacher helped the learners unpack their

solutions in tasks and opened up opportunities to develop practices of mathematical reasoning. It looked at how the learners used language to deal with these tasks.

TEACHER INTERVIEWS

The teacher was interviewed twice during the study. First before the lesson was observed (pre-observation interview) and second after the lesson observation (reflective interview). These two interviews had different foci and purposes.

A semi-structured pre-observation interview helped to explore the extent to which the teacher knew about mathematical reasoning. The interview also provided information on the types of tasks the teacher believed could promote mathematical reasoning in learners and the role of language. Furthermore, it helped to find out how the teacher saw the importance of his role in promoting mathematical reasoning.

A semi-structured teacher reflective interview was conducted after all the lessons had been observed. The video recording was used during the interview to reflect on the critical incidents that occurred during the lesson.

VALIDITY AND RELIABILITY

Validity refers to a degree to which the method, a test or a research tool measures what it is supposed to measure (Opie, 2004). Maxwell; (1992) argues that validity in qualitative research lies in what is called 'critical realism' which is an account that involves description, inference and explanation i.e. both accuracy and appropriateness. I am aware that in this qualitative research not all possible accounts of the teacher, learners and the classroom are possibly useful, credible or legitimate. I used Maxwell's (1992) categories of validity to ensure rigour and trustworthiness in this qualitative research. Maxwell (1992) argues that qualitative research depends on different kinds of validity namely descriptive validity, interpretive validity, and theoretical validity as well as construct validity. This study used descriptive validity, construct validity, and interpretive validity.

Descriptive validity

According to Maxwell (1992), this type of validity refers to the accurate description of what happened or what was being said during the lesson observation and also in the interview. This is supported by Wolcott (1990b, p. 27) cited in Maxwell (1992) that "description is the foundation upon which qualitative research is built." He further argues that whenever he is engaged in fieldwork he tries to record as accurately as possible in precisely the informants' words. To ensure descriptive validity I used an observation schedule and a video recording. All the description of the lesson analysed in this study are made on the basis of the video recorder. The video recording of the lesson was transcribed. In this study to ensure that the data that had been collected presented an accurate picture of what I claimed to be describing, a video camera, the observation schedule and the tape-recorded interview were used. The video and the tape recorder were used so that I do not distort or make up the things I saw or heard. What the teacher and the learners were doing in the video always remained as it was. The video showed verbal and non-verbal features that could be seen by anyone who could reanalyse the data on this research (Opie, 2004). Using video provided evidence between interpretations and descriptions. Non-verbal features were easily seen in the video than in the observation schedule. The video and the tape recorder helped that the data be presented transparently and in ways that enabled ready re-analysis. The video and the tape recorder were therefore able to give another researcher a true reflection of what happened in the class, so that they are not dependent on what was written on an observation sheet. Construct validity in this study was ensured by transparent data-gathering procedures and the video and the tape recorder ensured the appropriateness and the accuracy of the data collected.

Construct validity

Yin (1994) uses three tactics that can be used to increase construct validity. He mentions the use of multiple sources of evidence, establishing a chain of evidence and lastly has the draft case study report reviewed by key informants. In this study construct validity was ensured by relating the study to the different research that has been done on similar topics. The transcript of the lesson has been coded according to the five strands of mathematical proficiency (Kilpatrick et al, 2001) that inform the theoretical framework of the research. I use the strands exactly as defined and used in the literature by Kilpatrick et. al (2001). After categorising the data, I gave the transcript and the description of the categories to another researcher for validation. The inferences that I draw from the data were guided by the categories I have chosen. The categories would also help different observers to come to agree on their descriptive accuracy if they analyse data in the same perspective and

purposes. The accurate categories guided me with what needed to be included and what needed to be omitted;

Interpretive validity

Maxwell (1992) argues that this type of validity is concerned with the participants engaged in the study. It is concerned with what the participants and behaviours mean to the people engaged in and with the study (Maxwell, 1992). The interpretation in this study was directly linked to the data that was collected through the teacher interview through the use of the tape recorder. The teacher interview helped me to understand why the teacher conducted the lesson the way he did so that my experience may be removed from the interpretation of data.

PILOTING

Piloting was conducted in a different Grade 7 class in the same school. The purpose of the pilot was to check if the observation sheet was useful in collecting appropriate data to explore the research questions. Piloting also checked if Kilpatrick et al's strands of mathematical proficiency were useful to explore how the teacher promoted mathematical reasoning. After doing the piloting, I realised that I could use the observation sheet as well as Kilpatrick et al's strands of mathematical proficiency to explore if the teacher did promote mathematical reasoning.

ETHICAL ISSUES THAT MAY ARISE IN THIS STUDY

Before embarking on the research, I applied for ethical clearance from the Gauteng Department of Education and from the ethics clearance committee at the University of the Witwatersrand to ensure that the rights of all the people who would be participating in this research are respected. The research was done in the school where I was the deputy principal and the informant was my junior. It was therefore very important that I did not manipulate him to be my informant by using my position so that I could get what I wanted. Moral and ethical procedures were followed in order to get information from him. I wrote him a letter requesting his participation in the research (see appendix E) in order to make sure that he was sufficiently protected in written accounts. He was assured anonymity and

made aware that he had the choice to participate. If he chose not to participate, it would not jeopardise his job.

Ethical letters were also written to learners' parents in order to get permission from them too (see appendix J). In the ethical letters, the reasons for the research were spelt out and what it was that the teacher and the learners were going to benefit in it. Consequences of the research were also considered by making sure that the findings did not have a negative impact on the teacher. In my interaction with the teacher, I was very careful how I asked questions. Questions that will embarrass the informant were avoided. The teacher was also given report back after the research.

HOW DATA WAS ANALYSED

As indicated earlier all data collected was transcribed to enable analysis. The data analysis was at two levels: first the type of task that the teacher used was analysed to see how it landed itself to mathematical reasoning. Second, the classroom interactions were analysed. The lesson transcripts were used to enable this analysis. Transcripts of the teacher interviews were used to triangulate the data collected through lesson observations. In the sections that follow, I describe all the steps of data analysis.

The tasks were analyzed according to Stein et al's (2000) analyses of mathematics instructional tasks.

Lower-level demands tasks.

Lower level demand tasks are divided into two, namely memorisation tasks and procedures without connection. Memorization tasks are the kinds of tasks that assess the reproduction of previously learned facts. They assess if the rules, formulas or definitions are known. The demand from the task does not go beyond the memorisation of rules, formulas or definitions. In other words, this kind of task can be tackled without the understanding of rules, formulas or definitions. Procedures without connections tasks are the tasks that are algorithmic, are they focusing on producing correct answers rather than developing mathematical understanding. They require limited cognitive demand.

Higher-level demands

Higher-level demand tasks are also divided into two; namely procedures with connections and doing mathematics tasks. Procedures with connections demand the use procedures for the purposes of developing deeper levels of understanding of mathematical concepts and ideas. These types of tasks demand more than the knowledge of rules, procedures and definitions. The rules and formulas should be used with understanding in order to be able to solve the problem. In other words without the understanding of the underlying reason behind the procedure, rules or the formula it would be difficult to solve the problem. These kinds of tasks are usually presented in word problems. They require some degree of cognitive effort.

Doing Mathematics requires the manifestation of complex and non-algorithmic thinking. The tasks require exploration and understanding the nature of mathematical concepts, processes or relationships. They also require considerable cognitive effort from the learners.

Analysis of classroom interaction

The lesson transcript was categorized according to Kilpatrick's (2001) five strands of mathematical proficiency. These categories were used as described by Kilpatrick et al (2001). I counted utterances in the transcripts based on the categories above. In addition to that I checked the language used for the utterances evidencing the strands. With the help of another researcher, I labelled each category according to Kilpatrick's (2001) strands of mathematical proficiency. I drew a table, and then I counted the each utterance according to the strand and the language that was used. Furthermore, I included additional categories described below:

> **Procedural fluency (PF):** the teacher challenges the learners to explain the procedures that they have used and if they carry out procedures flexibly, accurately, efficiently and appropriately
>
> **Strategic competence (SC):** The teacher challenges learners to carry out procedures flexibly, accurately, efficiently and appropriately. In this strand, the teacher assesses the learners if they have the ability to formulate, represent and solve mathematical problems. In strategic competence, the researcher will also analyze if the teacher challenges the learners to explain the strategies that they have used.
>
> **Conceptual understanding (CU):** the teacher challenges learners comprehension of mathematical concepts, operations and relations.

Adaptive reasoning (AR): the teacher challenges the learners' capacity for logical thought**,** reflection, explanation and justification in learners.

Productive disposition (PD): the learners see mathematics as sensible, useful and worthwhile.

The following categories are the supportive categories that were highlighted from the transcript that supported the promotion of mathematical reasoning.

Classroom environment (CE): the analysis in this category will explore if the teacher creates a classroom environment in which learners respect and value each other's responses.

Giving instructions (GI): the teacher gives clear instructions while encouraging mathematical reasoning to learners.

Affirmation of responses (Aff): the teacher does affirm and reinforce the learners' responses while encouraging mathematical reasoning.

Explaining (E): the teacher explains the questions to the learners when encouraging mathematical reasoning.

These additional categories were included because they supported the promotion of mathematical reasoning. While encouraging mathematical reasoning, the teacher has to ensure that the classroom environment is conducive for all learners so that they can be able to explain and justify their answers without being mocked or intimidated by others. The teacher has also to give instructions in order to give direction of the lesson to the learners. He also has to explain in order to give direction to the learners. Affirming of answers is also important when mathematical reasoning is being promoted. The teacher has to assure the learners that they are on the right tack by affirming their answers.

CONCLUSION

This chapter has presented the research design and methodology. The chapter further explained how ethics, validity, and reliability were ensured. The chapter also explained how data was analysed. The chapter that follows presents analysis of data and findings.

CHAPTER 4 DATA ANALYSIS

INTRODUCTION

This chapter presents analysis of data collected through lesson observation and teacher interview. All data (video recording of the lesson and tape recording of the interview) was transcribed to enable analysis.

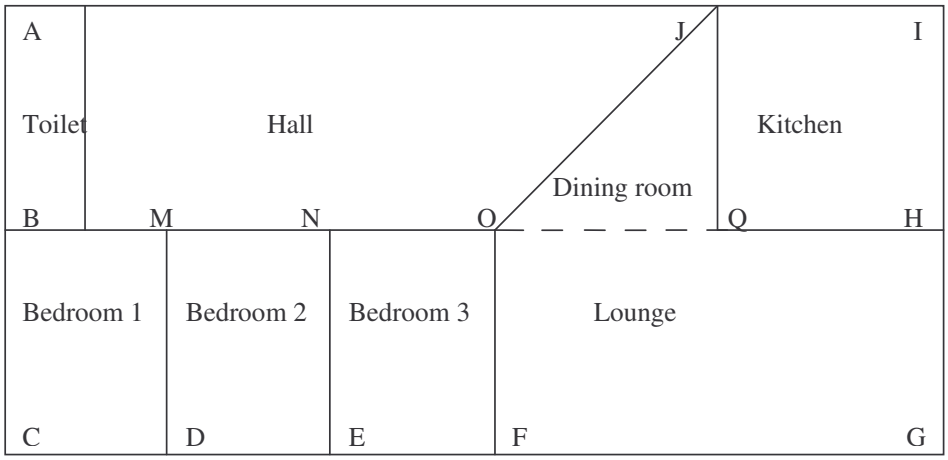
The chapter begins with a description of the lesson that was to be analysed. It also presents the task that was given to learners. Furthermore a brief analysis of the task is presented. A lay out of data analysis table which shows the utterances has also been included to ensure accurate analysis. This is followed by an analysis that focuses on how the teacher promotes mathematical reasoning. I end the analysis by exploring how the language(s) use enabled or constrained mathematical reasoning.

A BRIEF DESCRIPTION OF THE LESSON OBSERVED

The lesson was based on a task that had a real life context that focused on the concept of shape and space as well as money. The task had five questions. The teacher gave learners an opportunity to work on the task in groups and after each question he interacted with them in a whole class discussion about the way they solved the problem. He arranged learners in groups of four. The teacher did not give the entire task to the learners at once. He gave them one question at a time. He read and explained each question to the learners, in some instances he would switch to Setswana to ensure that all the learners understood. He read the questions, explained briefly, what was required and gave hints through probing questions and then gave them an opportunity to work in groups. At the end of each question he requested one member from each group to explain how that particular group got the solution. He did not appoint group leaders to report on behalf of the groups, instead he pointed at learners randomly. When one learner could not explain, he would ask learners either from the same group or other groups to assist. He encouraged learners to share their different solutions. After one learner had given an answer the teacher would ask if there was anybody else with a different solution. After giving a chance to a few learners to explain

their answers the teacher would proceed to the next question. The teacher managed to accomplish the five questions in the task within an 80-minute lesson.

Below is a copy of the task as it was given to the learners:



AB = 3m
BC = 2.5m
CD = 4m
DE = 3m
EF = 3m
PJ = 4m
FG = 8.5m
HI = 3m
OJ = 5m
QH = 4.5m
KP = 7m

1. Find the area of the house.
2. Find the area of the hall.
3. If a square meter box of tiles has five tiles, how many boxes will you use in the kitchen?
4. How much will you pay the person who will put the tiles in the hall if he charges R30 per square meter?
5. What will be the size of the carpet to be used in the bedrooms?

FIGURE 4.1: Task given to learners

Analysis of the task

In this section I draw on Stein et al (2000) to analyse the task that was set up for the lesson. This analysis will focus on the mathematics of the task and what makes it appropriate for use to facilitate mathematical reasoning. As can be seen above the task had five sub-questions. These were a mixture of low and higher-level cognitive demands questions.

Low-level demand questions included ‘memorisation questions’ as well as ‘procedures without connections questions’ (Stein et al, 1996). Questions one and two are memorisation questions as they require the learners to reproduce previously learned facts, rules, formulas or definitions. They require that learners reproduce the formula of an area of a rectangle in order to be able to find the correct answer. These questions also require procedures without connections as learners do not need to connect with the context of the problem in order to solve it. Learners have to display procedural steps to arrive at an answer, which is the area of a house and the area of the hall, not an explanation or justification of why they calculated the area in that manner. The meaning and understanding would be in the process of answering for those learners who would be using rules and procedures with understanding. For those learners who have no understanding this could lead to rote learning. Stein et al (2000) argue that the kinds of questions that require learners to perform a memorized procedure in a routine manner will lead to one type of opportunity for student thinking.

Question three, four and five are higher level demand questions which the teacher used to find out how learners use procedures for the purposes of developing deeper levels of understanding of mathematical concepts and ideas. These three questions require some degree of cognitive effort**.** They can make learners actually engage in high level thinking and reasoning about mathematics. Question three and five requires the learners to be able to come up with the operation signs that have to be used. They need to reason that for them to get the number of boxes or the size of the carpet, they have to calculate the area of a kitchen or the bedrooms first. Question four requires learners to know the formulas for they have to calculate the area of the hall first, get the number of square meters and then multiply it with R30.00. These tasks are classified as ‘doing mathematics tasks’ as well as ‘procedures with connections tasks’ because there is no suggested pathway of solving them. Through these questions learners may be able to explore and understand the nature of mathematical concepts, processes or relationships.

Question 5 requires the learners to find the size of the carpet to be used in three bedrooms. It is possible that learners may follow procedures and make connections between multiple presentations, in other words this task can be solved using different strategies and not just one fixed process. The learners’ responses might show that the facts and the methods were learned with understanding and they are connected or they are learned through memorization of algorithms. Kilpatrick et al (2001) argue that the facts and the methods

learned with understanding are easily constructed when forgotten. Sound understanding of the concept of an area for instance can help learners engage with conceptual ideas that underpin the procedures that are used. These questions require the learners to be able to connect meaning to the formula of an area of a rectangle, in order to successfully complete them. According to Stein et al (2000), the learning of procedures and algorithms with understanding helps learners to be able to link these procedures with higher-level cognitive demands.

In doing mathematics questions learners are required to show complex and non-algorithmic thinking. According to Stein et al (2000) 'doing mathematics tasks' are tasks that can promote mathematical reasoning, as learners have to think and reason about the strategies they need to use in order to solve the problem. These questions are not merely focussing on correct answers and describing procedures but may also enhance the learners' ability to think or reason about important ideas that will help in solving the task. The learners have to be able to understand the diagram so that they can be able to know which points to put together in order to be able to come up with the length and the breadth of the diagram. Through these cognitively challenging questions, the teacher can engage learners in complex forms of thinking. While these questions require learners to retrieve basic facts, definitions and rules they lead to deeper, generative understanding regarding the nature of mathematical processes, concepts and relationships.

Language demands of the task given to learners.

That task required the learners to be able to read and write English. The language used in questions one and two is straight forward, however, the language used in questions three, four and five is more complex. In these questions, the challenge is not just in the meaning of words but also in the interpretation of the question. The word 'if' might be a challenge, as it requires the learners to reason about the words 'supposing that' or 'on the condition that'. The word 'if' has been used without any operational sign or given procedure. It is just followed by the words '…how many?' In addition, mathematical meaning of the word 'square meter' can also be challenging in question three. It is also crucial for 'square meter' to be written symbolically.

The other challenge in question three and four would be to differentiate between the words 'how many' and 'how much.' The adjectives 'many' and 'much' might bring a challenge in the meanings. There is a very thin line when trying to differentiate between the two

adjectives. The challenge will be that these adjectives should be understood within the context of the sentence because 'much' is usually used when referring to something uncountable and money in this case is countable. The other language complexity can be seen in question five where the words 'size of the carpet' should be associated with the word 'area' or 'surface' so that the learners may be able come up with the correct strategy of solving the problem.

THE TEACHER'S ROLE IN PROMOTING MATHEMATICAL REASONING.

In order to be able to analyse data appropriately I counted the utterances of teacher-learner interaction in each of the categories I used to analyse data. This was done before analysing the lesson observed. Below is the table that shows different categories that were used to analyse the lesson observed, teacher-learner interaction, language use, as well as the number of utterances within each category in the transcript. The table displays the utterances that are in the transcript (Appendix A) and have been counted according the categories used to analyse data mentioned in the previous chapter. Using the table to count the utterances helped the researcher not to thumb suck when analysing the lesson. The table also presents the different levels of cognitive demand in the task that was given to learners. See the table below:

TABLE 4.1: Teacher talk distributions according to codes and utterances**.**

Categories	**Teacher-learner interaction**	**Language use: which language did the teacher and the learners use when promoting mathematical reasoning**	**Number of utterances in the transcript.**
1.Procedural fluency (PF)	Does the teacher challenge learners to carry out procedures flexibly, accurately, efficiently and appropriately?	English	91
		Zulu	0

		SeTswana	2
		Xhosa	0
2.Strategic competence (SC)	Does the challenge the learners to explain the strategies and procedures that they have used?	English	12
		Zulu	0
		SeTswana	7
		Xhosa	0
3.Conceptual understanding (CU)	Does the teacher challenge the learners' ability to comprehend mathematical concepts, operations and relations?	English	13
		Zulu	0
		SeTswana	2
		Xhosa	0
4.Adaptive reasoning (AR)	Does the teacher encourage the learners to explain and justify their answers? Is the teacher able to challenge the learners, show capacity for logical thought**,** reflection, explanation and justification?	English	9
		Zulu	0
		SeTswana	0
		Xhosa	0
5.Productive disposition (PD)	Do teachers help learners see mathematics as sensible, useful and worthwhile?	English	
		Zulu	
		SeTswana	
		Xhosa	
6.Classroom environment (CE)	Did the teacher create a classroom environment in which learners respect and value each other's responses when they were talking about their mathematics.	English	8
		Zulu	1
		SeTswana	0
		Xhosa	0
7.Giving instructions (GI)	Did the teacher give clear instructions while encouraging mathematical reasoning?	English	12
		Zulu	2
		SeTswana	0
		Xhosa	0
9.Affirmation of responses	Did the teacher affirm and reinforce the learners' responses while	English	5
		Zulu	1

(AFF)	encouraging mathematical reasoning?	SeTswana	1
		Xhosa	0
10. Explaining (E)	Did the teacher explain the questions to the learners?	English	10
		Zulu	0
		SeTswana	1
		Xhosa	0

The kind of tasks the teacher uses to promote mathematical reasoning according to Stein et al's (2000) analyses of mathematics instructional tasks are described in Table 4.2 below.

TABLE 4.2: Analysis of task based on cognitive levels

Categories	Kinds of tasks the teacher set up to encourage mathematical reasoning.	Number of questions
Lower level demand questions	1. Memorization questions: Did the question require learners to reproduce previously learned facts, rules, formulae or definitions.	Question 1 and question 2
	2. Procedures without connections questions: were the questions algorithmic, were they focusing on producing correct answers rather than developing mathematical understanding. Did they require limited cognitive demand?	Question 1 and question 2
Higher-level demands:	1. Procedures with connections: questions that make learners use procedures for the purposes of developing deeper levels of understanding of mathematical concepts and ideas. Require some degree of cognitive effort.	3, 4 and 5

	2. Doing Mathematics: learners manifest complex and non algorithmic thinking. The questions require the learners to explore and understand the nature of mathematical concepts, process or relationships. They also require considerable cognitive effort from the learners.	

In this section I present an analysis of the lesson with a specific focus on how the teacher promoted mathematical reasoning. The analysis was supported by the number of utterances in each category stated in the table above. A letter 'T' is used to indicate the utterances that were made by the teacher and where the name of the learner was not mentioned, the letter 'L' is used in the utterances made by the learner.

Classroom environment

During the lesson observed the teacher encouraged inquiry, where all the participants had mutual respect and listened to each other's ideas. While he did not formally develop written classroom norms he did establish expectations regarding how academic work gets done. This was evident in his conversation with the learners. He set task conditions and student habits that made the learners feel at ease during teaching. He was monitoring and facilitating the learners' behaviour as they were engaging with the task. The extract below is a typical example of how the teacher set up a classroom environment that ensured that learners were listened to, respected and valued each other's responses.

> 74. T: how did we get that 5.5m. Who can explain? I think we've explained thoroughly now, we want somebody who can stand up, explain it to us in full, how, okay, let's hear. Can we listen to him, can we listen to him, yes.

In line 74 above the teacher urged learners to listen to the student who was explaining. The teacher ensured that there was respect amongst the learners and that others humiliated no learner while explaining his/her answers. The teacher used the words, "can we listen to him, can we listen to him, yes". Using the words "can we listen to him" twice shows that the teacher was urging the whole class to respect the learner who was talking. While the teacher

was creating opportunities for learning by urging the learners to listen to each other, he also urged those who were explaining to others to speak louder. This is a clear indication that the teacher wanted to create a classroom environment mentioned by Kilpatrick et al (2001) who suggest that classroom norms can be established in which students are expected to justify their mathematical claims and make them clear to others.

> 66. T: Okay before we arrive, we haven't been there yet, why did we have to do this? Why were we looking for 5.5m okay khulumela phezulu [speak louder].

The teacher created a community by ensuring that the learners are audible to one another (line 66). In the above extract he emphasises that the learners should speak louder. In other words he was concerned that each learner in class heard clearly what was being said. The teacher also appreciated the learners' attempt by using statements like 'good', 'that's okay' etc. In other words, he was appreciating their thinking. See line 190 in the extract below.

> 190. okay that's good, from there what did you do?

The kind of environment that the teacher created here shows that the teacher wanted everyone to express himself freely. Wood et al (1992) argue that if the teacher expects the learners to express their thinking to the others with the possibility of being questioned or challenged they also expect the teacher to accept and respect their thinking as well. In the extract below the teacher used the word 'please', which is an indication of respect of the learners and their ideas.

> 90. T: **please**, the minute you are finished, raise up your hand so that we can see whether you are right or wrong. Ok there's a hand there's somebody who is finished. ok let's see, now let's see, explain to us what is it that we are doing, what are we multiplying?

In the above extract the teacher says, "explain to us what is it that we are doing?" Wood et al; (1992) stated that when promoting mathematical reasoning the teachers create an environment in which children can risk expressing their thinking and also question other learners' ideas. The mathematical community that needs to be created when promoting mathematical reasoning allows learners to explain their methods. It is imperative that the teacher creates an environment where other learners should listen to the one who is talking. Wood et al (1992) argue that when solving mathematical problems other learners should listen and decide if the explanation made sense to them or not.

Learners were engaged in discussions in which their ways of solving problems were of central interest to the discussions in the lesson. As much as part of creating a community was to let the learners ask questions if they did not understand, there was nowhere in the lesson where the teacher told the learners to ask if they did not understand. Was it because it is in the culture of interactions in this class? Wood et al (1992) argue that learners should be provided with an opportunity to ask questions or provide information to help clarify the meaning. They further insist that the teacher is not supposed to impose his/her ways of thinking on learners, embarrass or ridicule them in any way or allow other students to do so (Wood; 1992). Throughout the lesson there was nowhere where the teacher was found embarrassing the learners when they gave incorrect answers. In line 224, he just swiftly moved to other learners and asked them to explain to those who did not understand.

> 224.T: why did you write the 0? Explain to her. Jeremiah, explain to her, why did you write that zero?

When the teacher realized that Gugu could not explain why she wrote a zero, he then requested Jeremiah to explain to her without embarrassing her. The teacher's attitude suggests that he was instilling the feeling of trust in this class so that the learners may feel free to display their ideas (Schoenfeld; 1996).

HOW THE TEACHER DEVELOPED MATHEMATICAL PROFICIENCY?

In this section I will focus on how the teacher worked with the learners to develop the five strands of mathematical proficiency.

Conceptual understanding

The number of utterances displayed on the analysis table shows that promotion of conceptual understanding was minimal in this lesson. Conceptual understanding in learners was seen when the teacher demanded that they explain how they got 5.5m. Asking how the learners got 5.5m the teacher was assessing the learners' understanding of the concept of measurement. The learners managed to relate the 5.5m to AB and BC. In line 78 below the teacher was promoting sense making and deeper levels of understanding by constantly asking the learners to give the reason why they had to add 3m and 2.5m together. Jeremiah in line 79 linked AB and BC with the measurement of the breadth. The teacher's

encouragement of the learners to justify and explain their answers through questioning helped them to explain how they got 5.5m and why they had to add 3m and 2.5m.

74. T: how did we get that 5.5m. Who can explain? I think we've explained thoroughly now, we want somebody who can stand up, explain it to us in full, how, okay, let's hear. Can we listen to him, can we listen to him, yes.
75. L: AB is 3m, BC, 2,5m
76. T: Then? What do we do with the two measurements?
77. L: you say 3 + 2.5 is 5.5m.
78. T: what is the reason? Why do we do it? Remember in maths all the time ask the reason 'why do we do that'. Why 5.5m, it is correct, the answer is correct but now why, what is it anyone except Percy, Percy, okay Jeremiah, yes.
79. Jeremiah: because we want to get the breadth.

Learners seemed to understand the meaning of breadth and how it is represented in a diagram. In the extract below the teacher also ensures the development of conceptual understanding in learners. In line 48 the teacher used the words 'what do we know about rectangles?' Gomotso came up with one of the properties of a rectangle which is "it has two parallel lines."

48. T: what do we know about rectangles, what is it that we know? Gomotso.
49. Gomotso: It has two opposite parallel sides
50. T: it has two parallel sides akere, so what does this mean? AJ will be equals to what? The distance of AI will be equal to which distance, Sandas
51. Sandas: BG

The learners' conceptual understanding became evident when they tried to explain what a rectangle was. In line 62 in the extract below the teacher asked the learners why they had to substitute the letters with numbers. Percy was able to say 'so that we can find out what is the breadth'. He connected the points that would make the breadth so that he can calculate the area of a house that was a rectangle. This shows that he understood that a rectangle has length and breadth. Kilpatrick et al (2001) maintains that when learners thoroughly understand concepts and procedures, they can extend these concepts and procedures to new areas. In this case, the learners were able to attach the given measurements to the given diagram. In line 63 Percy was able to draw frequent conceptual connections by being able to connect the points that would make the breadth.

56. T: yes what will be the distance how do we get the distance of AI or CG that's it what will be in mm, what will be CG o kgetha [you choose] only the line akere lets find the line the minute le feditse [when you have finished] just raise your hands

okay CD lets see where you… CD it means what, what does it mean can you explain it to us? Ya, ya {*pointing at different learners*}

57. L: {*still thinking*}
58. What are you going to do with these figures? pass it, okay then etlhogufhang [what will it give you?]
59. L: the whole side
60. T: the whole side akere [okay] that's how we should do it , ya thusa babang kamo if uyithotsi.[yes help others if you have got it]
61. L: we say AB is equal to 3m. BC is equal to 2.5 m, ya
62. T: why do we do this? Mh why {*the learner does not respond*}. Percy, help? Why?
63. Percy: so that we find out what is the breadth because when we add from A to C because in this diagram has three points, so can have a point from A to C, we don't have a point from A to C. so that we can add so that we can get to see
64. T: okay, right, I think there is a …..that group has never said anything. Here we are, there we are, Gloria, let's see, who is explaining? Okay let' see.
65. L: we say this one is equal to 5.5m plus .{*educator interrupts*}

When the teacher required the connection to the meaning of area, the learners referred to the surface of the desk in line 21 and 25. This suggests that the teacher managed to challenge the learners' conceptual understanding.

20. Teacher: now what is it that we want to do? We have to look for the area, what do we know about the area? That is the first thing you should think about, we should think about it when we talk about the area, baboa ka area [they are talking about area] when we talk about the area what is the first thing that comes into your mind? Ba bua ka area [they are talking about area], what do you understand, how you understand it lets put it that way, okay lets put it that way,
21. L: the whole space
22. L: the whole space
23. T: can you show us maybe, we are looking for the area of the table there what is it that……? Ya
24. L: The whole space
25. T: the whole space where?
26. L: the surface of the desk

In the above extract, the teacher was assessing learners' understanding of the concept of 'area'. Learners had to find a required rectangle within the diagram of the hall. They were familiar with the isolated diagram of a rectangle but they found themselves having to solve a perimeter of a rectangle within the context of a house (see the hall in a diagram). This resonates with Kilpatrick et al's (2001) argument that "knowledge that has been learned with understanding provides the basis for generating new knowledge and for solving new and unfamiliar problems (p. 119). The unfamiliar knowledge in this case was for the learners to identify the required rectangle in the house and be able to calculate its perimeter. In the extract below in line 196 and 197 the learners' understanding of mathematical

concepts and ideas was also evident when they were able to analyse the formula for an area of a triangle which is 'half base times height'. The learners were able to relate the formula to mathematical concepts. In line 197 Percy was able to explain why half-base multiplied by height is a formula of an area of a triangle. He was able to see that there has to be a half in the formula of a triangle because a triangle is half of a rectangle.

> 196. T: $6m^2$, that's good, okay now let's see, okay I think everybody is finished now, let's finish let's move on let's move on. {*addressing the whole class*}. E e we've got our formula already on the blackboard akere [okay]. ½ Base x height, now somebody who can explain why do we have to use this half all the time when we look for the area of a triangle, why do we use half, besides Percy, besides Percy? Is there anyone who can explain why we use half all the time? Is there any one who can explain to us? Ya. Why do we use half? Okay Percy? Speak aloud so that everyone can hear.
> 197. Percy: because a triangle is a half of a rectangle.

This suggests that the teacher made it possible for the learners to display the capacity to engage conceptually about the concepts that they were learning through the questions he asked. Kilpatrick et al (2001) argues that the teacher cannot ask conceptual questions if he does not teach for conceptual understanding. In other words, there has to be a connection between the kinds of questions the teacher asks and the way he teaches. This suggests that the teacher was aware that conceptual understanding is the bases on which procedural fluency can be developed (Kilpatrick et al, 2001).

Procedural fluency

Kilpatrick et al (2001) describe this strand as the knowledge of procedures, how and when to use them appropriately and the skill in performing them flexibly, accurately and efficiently. Procedural fluency is thus not limited to the ability to use procedures, it also includes an understanding of when and how to use them.

The teacher in the study encouraged mathematical reasoning in learners by ensuring that they were able to use procedures with understanding. The number of utterances displayed in the analysis table and in the transcript show that procedural fluency dominated the entire lesson. In the extract below the teacher wanted the learners to explain what it was they were multiplying and how they have multiplied. The learners were able to put all the points that make the length and the breadth together, i.e. 18.5m for length and 5.5m for breadth. In line 94 and 98 the teacher required that the learners explain the procedures they followed when

multiplying the length and the breadth. He wanted them to explain how they got 101,75 in line 98.

92. T: Ya the length and the breath akere now what are the measurements of that length and the breath
93. L: the length is equal to 18.5m and the breath is equal to 5.5m
94. T: ok now explain to me how did you multiply
95. L: we said 5×5 = 25 carry the two, 5×8 = 40 +2=42, 5×1=5+4=9
96. T: ok
97. L: and then it give us one hundred and one coma seven five meters
98. T: 101.75 m usually numbers after the coma we call them one by one ekere [okay]. Eh, lets see the last group, in this case ok who's explaining here now the first thing what is it we were doing
99. L: 18.5m X 5.5m
100. T: ok?
101. L: after that we said that 5×5 =25 carry the two and then 8 times 5 is 40 plus two is forty two, one times 5 is 5, 5 plus 4 is 9……..

In the above extract, the teacher was asking the learners to come up with efficient and accurate ways of performing basic computations. The calculations required accurate mental arithmetic and a more flexible way of dealing with numbers (Kilpatrick et al, 2001). Algorithms were important in this lesson especially as they were fostering a link between conceptual understanding and procedural fluency. Line 94 shows the teacher was requiring the learners to explain how they multiplied, in other words he was demanding the explanation of the procedure they used in order to get to the answer. In line 95 the learner was able to explain the procedure, doing mental calculations at the same time. This was the evidence that the teacher managed to promote procedural fluency in the learner.

In the extract below the teachers ensured that the learners struggle on their own to get to the correct answer. He used the probing strategy to encourage procedural fluency. Eventually in line 87 the learners were able to say that for them to get an area of a hall they had to multiply the length by the breadth. This suggests that the teacher aligns himself with what Kilpatrick et al (2001) argue about, that without sufficient procedural fluency, learners have trouble deepening their understanding of mathematical ideas or solving mathematical problems. The extract below continues to show how the teacher promoted procedural fluency in learners.

80. T: now from there what do we have? because we've got, from there we have our length which is 18.5m times and what is our breadth?
81. L: 5.5m
82. T: then from there what do we do? Once we have substituted now akere, we say L actually stands for what?

83. Learners and teacher: 18.5m
84. T: so B stands for what stands for what?
85. L: 5.5m
86. T: then what do we do from here? Ya
87. L: we multiply the length by breadth,
88. T: let's multiply the two in your book, let's multiply
89. L: {*learners discuss in their groups*}

The learners did not seem to have forgotten the previously learned rules, definitions, and formulas. From line 80 to 89 the teacher probed learners to recall, reproduce and use formulas and rules with understanding. Some learners managed to explain the formula and rules they used. The following extracts show the learners trying to remember what is meant by area and to find a formula thereof.

29. T: What will be the formulae of the area, what will be the formulae of the area, ja ja
30. L1: height times breadth
31. T: Ya
32. L2: height times breadth
33. T: height times breath ok there's another hand, ja ja ja
34. L: length times breadth
35. T: we have
36. L: length times breadth
37. T: length times breadth akere [okay]
38. Learners: yes

In the above extract, the teacher is trying to persuade the learners to recall the facts about area.. He repeatedly says 'they are talking about area', in other words he seems to be saying they must remember the facts about area. Learners seemed to remember the facts about area. The teacher managed to probe the learners until they came up with a formula of an area of a rectangular diagram. They were also able to recall the formula of an area of a triangle. According to Stein et al (1996), the teacher's implementation of the task led to low-level demand. Stein et al (1996) refer to tasks in which learners are expected to produce previously learned facts as low-level demand tasks. Ball & Bass (2003) argue that learners have to be provided with opportunities, encouragement and assistance to engage in thinking, reasoning and sense making in the mathematics classroom.

Strategic competence

As explained in Chapter 3, strategic competence refers to the ability to formulate, represent, and solve mathematical problems. As much as there is evidence that the teacher demanded that the learners explain how they will solve the problems given to them (see line 230 and

232 below), the number of utterances displayed in the analysis table show that the teacher gave little effort in promoting strategic competence. In line 230 the teacher was provoking the learners to come up with a strategy that they would use in order to find the size of the carpet for the three bedrooms by saying "he wants to put the carpets, what will he do?" Instead of giving the learners the answer, he just gave them the clues so that they could be able to come up with their own strategies. This suggests that the teacher was trying to help the learners develop the ability to formulate mathematical problems and represent them. In line 232 he asked them how they were going to solve the problem, in other words he wanted them to explain the strategy they would use to get the size of the carpet. In line 230 and 232 he used phrases like 'what is it that we should do?', 'what will he do', 'how are we going to determine the size of the carpet?' He also demanded explanation of the strategy by saying 'okay Percy can you explain.' Kilpatrick et al (2001) argue that it is crucial for mathematical proficiency that learners should know a variety of solution strategies as well as which strategies might be useful for solving a specific problem.

> 230. T: okay that's good. Now let's do the last one. {*reads the last question on the chalkboard*} What will be the size of the carpet used in the bedrooms? Now obatla ho Kenya carpet mo di bedrooms tsa haye akere [he wants to put them in his bedroom, okay], now what is it that we should do? Who can tell me? He wants to put the carpets in the bedrooms, otlo yetsang [what will he do]? What is it that we should do? The carpets! Now where are the bedrooms {*pointing at the diagram*}how many bedrooms do we have?
> 231. Learners: three
> 232. T: and we want to put what? Can you tell us without writing, what is it that we should do? Retlotseba joang gore di carpet detshwanetse debe kakang [how are we going to determine the size of the carpet]? Otloyetsang [what will he do]? What is it that we should do, otlo tseba joang [how will he know], okay Percy, can you explain to us?
> 233. Percy: we find area from CF and BO.
> 234. T: simple as that, now what is the area, what is the area? Remember guys, the area, what is the area?

By asking the learners to work in different groups, this suggests that the teacher was provoking learners to represent mathematical situations in different ways. In line 202 below the teacher reminded the learners about the easiest way of multiplying decimal fractions by 10 or 100. This is another way the teacher used to encourage different ways of solving the problem in learners. After getting the response from the learners in line 203, the teacher demanded another different way of calculating. In line 204 he acknowledges that the answer is correct, however he continued to ask; 'somebody else, who wrote it differently.' He was not just satisfied with one strategy; he wanted other learners to come up with different

strategies. This was the reason why he was using words like 'how did others write it?' in line 204.

198. T: okay, let's substitute there, from there we've got height x base. What is our height there? What is our height?
199. Learners: 4 m.
200. T: what is our base?
201. Learners: 3m
202. T: now the easiest way. What is the easiest way of calculating, before we can even start writing what is the easiest way of calculating? Who can tell me? What is the easiest way of calculating? Ya, explain it to us.
203. L: we can say 4 times three is equals to twelve and half of twelve is 6.
204. T: ya, one way of doing that, it is correct. Omong one angweste eng [somebody else, who wrote it differently]? Somebody else ongwetse eng yena [what did others write]? Oyi kwadile jwang yena [how did you write it]? Come, come explain to us then, explain to us, talk to us
205. L1: half times four {*pauses*}

In line 90 in the extract below, the teacher asked the learners to explain what it was that needed to be multiplied and in line 94 he continued to challenge strategic competence in learners by saying "okay now explain to me, how did you multiply."

90. T: please the minute you are finished, raise up your hand so that we can see whether you are right or wrong. Ok there's a hand there's somebody who is finished. ok let's see now let's see, explain to us what is it that we are doing? What are we multiplying?
91. L: the length and the breath
92. T: Ya the length and the breath akere '{okay}, now what are the measurements of that length and the breath?
93. L: the length is equal to 18.5m and the breath is equal to 5.5m
94. T: ok now explain to me how did you multiply.
95. L: we said 5×5 = 25 carry the two 5×8 = 40 +2=42, 5×1=5+4=9
96. T: ok?
97. L: and then it give us one hundred and one coma seven five meters

The teacher was trying to make sure that the questions that were set up to encourage multiple strategies did not end up using a single-solution strategy (Stein et al, 1996). This resonates with Kilpatrick et al's (2001) argument that, of importance in the promotion of strategic competence is how the teacher promotes integrated and functional grasp of mathematical ideas in learners.

Adaptive reasoning

Adaptive reasoning is about capacity for logical thought, reflection, explanation and justification. The teacher's focus on promoting adaptive reasoning was the least of the five strands of Kilpatrick et al (2001) in this lesson. (see analysis table and the transcript). While engaging with the mathematical task the teacher encouraged the learners to explain and justify their answers. He challenged them to explain how and why they used strategies and procedures they were using. Adaptive reasoning was more evident when the teacher was dealing with the question on money. In line 227 below the teacher required Gugu to explain how she got 810.

> 226. T: that row, we want somebody now, so far you have done well. Somebody new, Gugu can you try?, can you explain it to us? Can we…….
> 227. Gugu: 30 x 27, then we said 7×0 is 0, 7×3 it's 21 then we write 21 then we have 20 is 0 under 1 and 2x3 is 6 then we add 210 + 60 then it gave us 810. {*pause*}. I said 0×27, 7x0 is 0, 7×3 is 21 then I have 210. Then I say 2×3 is 6 then I have 60 then I say 210 plus {*pause, looking at 60 and the gap in units*} I think I must put 0 here under the units because I cannot write a number like this, I must write units which is 0. {*she then had 210 + 600 like this*:}

$$\begin{array}{r} 30 \\ \times\ \underline{27} \\ 210 \\ +\ \underline{600} \\ \underline{810} \end{array}$$

> 224. T: why did you write the 0? Explain to her. Jeremiah explain to her, why did you write that zero?
> 225. Jeremiah: if we say 210 plus 60 it won't give us 810
> 226. T: quiet class, ja tell us where did you get the 600?
> 227. Jeremiah: mh…. Eh.. 2 is in 27, 2 is tens then is 20, we say 20 x 30 is 600.
> 228. T: so what will be the total now?
> 229. Jeremiah: total will be 810

The teacher asked Gugu to explain her answer in line 222. In line 224 he continues to say 'explain to her', 'why did you write that zero?' which required the learners to manifest their reasoning. While Gugu was explaining her answer in line 223 she realized that 210 plus 60 cannot give her 810. In line 226 the teacher continued to demand justification from Gugu by saying that 'tell us, where did you get 600?' Kilpatrick et al (2001) argues that for the learners to be mathematically proficient they must have the capacity to think logically about

the relationships among concepts and situations. In this case the teacher expected Gugu to be able to explain why she decided to change from 60 to 600.

In line 224, the teacher demanded the reason 'why' the learners wrote zero. In this way, he wanted justification of the answers. Though Gugu could not explain why she decided to add 0 next to 60 so that she could have 600, she realised that 60 and 210 cannot be equal to 810. Jeremiah's explanation also manifested adaptive reasoning when he was explaining how he got 600 in line 227. Kilpatrick et al (2001, p. 130) state, "one manifestation of adaptive reasoning in learners is the ability to justify one's work". Jeremiah's explanation in line 227 showed that he could make sense of the procedures and concepts that were involved in this question. According to Kilpatrick et al (2001) once the learners' procedures, concepts and solution methods fit together and make sense, then they have developed adaptive reasoning. Stein et al (2000) contend that tasks should have the potential to engage learners in complex forms of thinking and reasoning. This can be done by allowing students to use new concepts and procedures for some time and to explain and justify them by relating them to procedures that they already understand (Kilpatrick et al, 2001).

During the interview, the teacher stated that he preferred the kind of tasks that provoke learners to think. In the interview extract below the teacher mentioned the characteristics he would want to see in the task that promotes mathematical reasoning.

> 1. Researcher: What kinds of tasks or mathematical activities do you use to promote mathematical reasoning in your class?
>
> 2. Teacher: ….. Tasks should require or force them to think, they should talk to one another if necessary, discuss, and the task should drive them to use or recall other maths concepts to find solution.

Despite the fact the teacher mentioned that the task should force the learners to think, to talk to one another, discuss, to recall, to reason, to give explanation with confidence and be able to explain the procedures they have used, but this was very minimal in his lesson. The pre-interview showed that the teacher was aware that there should be a link between his understanding about the tasks that are relevant for promoting mathematical reasoning, the way he teaches and the kinds of questions he asked to assess adaptive reasoning in learners, in this lesson however he does not strongly challenge adaptive reasoning. This suggests that the teacher needed the skill of assessing the mathematical validity of learners' ideas or methods of solution. (Schiffer, 2001). Schiffer (2001) points out that the teachers' attention

need to be continually drawn back to the mathematics in what children are saying and doing and so that he may be able to help them solve mathematical problems on their own.

Promotion of adaptive reasoning can be clearly seen in line 78 below when the teacher tells the learners that it is important to give reasons for one's answers when doing mathematics.

> 78. T: what is the reason? Why do we do it? Remember in maths all the time ask the reason 'why do we do that'. Why 5.5m, it is correct, the answer is correct but now why, what is it anyone except Percy, Percy, okay Jeremiah, yes.

In the above extract the teacher wanted the teacher wanted the learners to explain the reason why they got 5.5m. His mentioning of the words "remember in mathematics all the time ask the reason 'why do we do that" and not fully put it practically in his teaching show that theoretically he knew the importance of promotion of mathematical reasoning but he was still lacking the skills thereof (Schiffer, 2001).

Productive disposition

According to Kilpatrick et al (2001), learners have developed productive disposition if they have developed a tendency to see sense in mathematics, and perceive it as both useful and worthwhile. While learners seemed to be doers of mathematics (especially Percy, Gugu and Jeremiah) there was no evidence of them perceiving mathematics as useful and worthwhile. There was no evidence that learners believed that steady effort in learning mathematics pays off, and see themselves as effective learners and doers of mathematics. However, the way some of the learners were explaining the concepts with confidence showed that they had developed all the first four strands of mathematical proficiency (see extracts in the first four strands above). Kilpatrick et al (2001) state, "productive disposition develops when the other strands do and helps each of them to develop." (p.131). From the analysis presented above it can be argued that the learners displayed procedural fluency, developed strategic competence and had developed adaptive reasoning. Kilpatrick et al argue that this happens because the learners' attitudes, beliefs, and beliefs about themselves as mathematics learners become more positive. However, looking at the analysis table above as well as the transcript (Appendix A) there was a very minimal display of productive disposition as the teacher focused too much focus on encouraging procedural fluency. The teacher gave very little time in attending to learners' reasoning so that they may develop their own powers of mathematical thought (Schiffer; 2001). He rarely challenged the logic behind the procedures. Schiffer (2001) supports this notion by arguing that the teacher should avail the

learners the opportunity to offer mathematical arguments and to assess the validity of those arguments and not just focus on procedures that will lead to the correct answers.

LANGUAGE USE

Probing the learners to talk about mathematics

The teacher intervened consistently throughout the lesson by probing learners to talk. Stein et al (1996, p. 457) argue that, "Students must be first provided with opportunities, encouragement and assistance to engage in thinking, reasoning and sense making in the mathematics classroom." The teacher's talk helped the learners not loose track of the lesson. The teacher set up classroom tasks and group work situations where learners engaged with the task and then gave the learners an opportunity to express their thinking to the whole class. Though he was able to support and engage learners throughout the lesson in order to help them think and talk mathematically but he could not fully engage them around important ideas because of too much focus on procedural fluency. The reason is that advancing the learners' mathematical development is not merely a matter of showing learners that they are wrong or having their classmates demonstrate strategies that work but it is to help them uncover a deep mathematical question for the class to consider and discuss (Schiffer, 2001). Most of the time the teacher was using English language when facilitating the lesson. He rarely switched to Setswana or IsiZulu when explaining questions or scaffolding the question (see appendix A as well as the analysis table). In the extract below the teacher was probing the learners to talk about the procedures they were following. He did not just accept the answer as it was, he used the answer to pose another question. Through the help of the teacher, learners were able to explain why and how they used a particular formula or algorithm.

80. T: now from there what do we have? Because we've got, from there we have our length which is 18.5m times and what is our breadth?
81. L: 5.5m
82. T: then from there what do we do? Once we have substituted now akere, we say L actually stands for what?
83. Learners: 18.5m
84. T: so B stands for what?
85. L: 5.5m
86. T: then what do we do from here? Ya
87. L: we multiply the length by breadth,

In line 80, 82, 84, 86 the teacher accepted the given answer and then probed further. Each response led to the next question. Through this probing process, the teacher was able to engage the learners in the learning process. The teacher seemed to be giving learners procedures while probing them to talk. When he probes the learners to talk in the above extract, he hints a procedure and then let them continue to solve on their own. In doing so the teacher in fact lowers down the mathematical demands of the task to an extent that they all just end up being procedural questions. It is quite clear that the way the teacher used the language to talk about mathematics led the lesson to be predominantly procedural. The above extract clearly shows that the teacher's probing limited the learner-talk as the learners were just responding with one word or short sentence to the teacher's questions. (see appendix A). Short responses when talking about mathematics make it difficult for the teacher to assess the mathematical validity of the learners' ideas as he has to listen for sense in their mathematical thinking (Schiffer, 2001).

Interaction amongst the learners and amongst the teacher and the learners

There was interaction throughout the lesson. Learners' interaction was mostly seen in group work. The teacher gave learners questions to tackle in their groups and then report back to the class. Interaction emerged when the teacher was promoting mathematical reasoning. There is nowhere where we see learners challenging one another's opinions in the whole class discussion. The teacher was playing a facilitating role by using statements like "explain to us", "talk to us". Using the word, "us" which is plural suggests that the teacher wanted the responding learner to address the whole class and not just him (see the extract below). According to Schiffer (2001), this kind of interaction requires the teacher's careful attention and the learners to listen to one another so that they may be able to respond appropriately to the learner who is talking.

There was nowhere in the lesson where learners were found using their home language when explaining and justifying their answers to the teacher or to the whole class. The learners used English throughout the lesson. In line 208 the teacher switches from English to Setswana to find out what the learners wrote, he wanted them to explain their answers to the whole class. He switched to Setswana more than IsiZulu or IsiXhosa.

> 208. T: ya, one way of doing that, it is correct. Omong one angweste eng [somebody else, who wrote it differently]? Somebody else ongwetse eng yena [what did others write]? Oyi kwadile jwang yena [how did you write it]? Come, come explain to us then, explain to us, talk to us.

Also in line 234 to 236, in the extract below, the teacher is the one who switches to Setswana, not the learners. He was explaining English sentences in Setswana in this interaction though other learners were IsiXhosa and IsiZulu speaking and were doing these languages as subjects in the school.

> 234. T: okay that's good. Now let's do the last one. {*reads the last question on the chalkboard*}. What will be the size of the carpet used in the bedrooms? Now obatla ho kenya carpet mo di bedrooms tsa haye akere [he wants to put carpets in his bedrooms], now what is it that we should do? Who can tell me? He wants to put the carpets in the bedrooms, otlo yetsang [what will you do]? What is it that we should do? The carpets! Now where are the bedrooms {pointing at the diagram} how many bedrooms do we have?
> 235. Learners: three
> 236. T: and we want to put what? Can you tell us without writing, what is it that we should do? Retlotseba joang gore di carpet detshwanetse debe kakang [how will you know the size of the carperts]? Otloyetsang [what will you do]? What is it that we should do, otlo tseba joang [how will you know], okay Percy, can you explain to us?

It was only in line 66 and line 228 where the teacher was seen switching to IsiZulu when interacting with the learners. In this case, he was not explaining mathematics to learners. He was asking the learner to speak louder in line 66 and in line 228, he was telling them to look at the chalkboard. There is no indication that the teacher switched to IsiZulu because he wanted to use the learners' home language to enable mathematical reasoning, he used it to create mathematical community.

> 66. T: Okay before we arrive, we haven't been there yet, why did we have to do this? Why were we looking for 5.5m, okay khulumela phezulu [speak louder].

> 228. T: ok lets do it on the board, lets do it on the board asibhekene ngapha [let's look this side] ok lets put down our pens now remember the main thing what you must do is what am I doing why am I doing it akere (okay), so now what is it that we are looking for

As much as the teacher acknowledges in the pre-observation interview in the extract below that language is a barrier but he seldom switched to the learners' home languages. He would just say a few words in the learners' home languages and immediately go back to English. (see appendix A and the analysis table). Though he switched to IsiZulu in line 66, his switching was not going to benefit the learners mathematically (Setati & Adler, 2001). He was just saying they must speak louder. Whether he said it or not; it was not going to constrain or enable mathematical reasoning. However, in the extract below the teacher acknowledged to the researcher in the pre and post-interview questions that language enable

mathematical reasoning and how it can constrain mathematical reasoning. This suggests that the teacher sometimes find it difficult to put his thoughts or knowledge into practice.

Pre-observation interview.

> Researcher: Your learners are all multilingual and they are learning mathematics in a language that is not their home language. How do you factor that in promotion of mathematical reasoning in your class?
> Teacher: Already language is a barrier. It is important to switch to languages represented in class and English. Allow them to express themselves in their mother tongue. This will build their confidence and remove fear of attempting. I let them work in groups, share ideas, present to one another, provide clarity and leads to reasoning, rectify mistakes quickly.

Post-observation interview.

> Researcher: Throughout the lesson the learners are communicating in English instead of their home languages, is it compulsory for them to communicate in English only during the lesson process?
>
> Teacher: Not really, I do not force them to express themselves in English only, though I always encourage them to speak in English. The reason why I am encouraging them is because they learn Maths in English and all their projects and tests are written in English. Maybe they spoke in English because you were in class; they know that you always want them to speak in English.
>
> Researcher: why do you sometimes switch to the learners' home languages?
> Teacher: I wanted to make sure they understood what was going on in the lesson; there is no need to continue with the lesson if some of the learners do not understand what is going on.
> Researcher: why do you think using learners' home language can make them understand? In fact, how do you know that they do not understand because of the language?
> Teacher: Through my experience as a mathematics teacher, I have become aware that most of the time language becomes a hindrance in understanding mathematics. You find that the problem is not always the mathematics but the language. If you can look at their facial expression after I have explained in their home language you can see that they understand better after I have explained in their home language.

Though the teacher is aware that language is a hindrance however, he continues to focus on only two languages in enabling mathematical reasoning instead of all the languages that learners speak in class. In the above extract the teacher acknowledged that code switching helped him to enable mathematical reasoning in class but it was one sided. When the researcher highlighted the issue of why he switched to Setswana more than he did with IsiZulu and IsiXhosa, it was evident that the teacher just assumed that all the learners

understand SeTswana (see the extract below). His assumptions might be untrue because some of the learners are from rural areas where only one African language is dominant, like for instance, KwaZulu-Natal or Eastern Cape.

> Researcher: in our first interview you told me that the learners' home languages in this class includes IsiZulu, Isixhosa and Setswana amongst others, but I hear you switching to Setswana more than other languages, why are you not using all learners' home languages in class?
> Teacher: well, maybe it's because I 'm a Tswana speaking person, well it just happens, I do not plan it, I was not aware.

> Researcher: why did you explain the tasks to learners before they wrote?
> Teacher: For clarity, understanding language and explain context, language problems, explain maths terms, concepts, familiarize them with the context.
> Researcher: Do you think language is a hindrance when promoting mathematical reasoning? Why?
> Teacher: Yes, it is not their mother tongue, they do not understand most English terms, some have poor reading skills, some fear the English language, lack of confidence in the language, feeling embarrassed for not understanding the language.

Though the teacher acknowledged that home language could enable understanding in learners, he however used Setswana language more that other languages though the class was multilingual. This suggests that there was tension between wanting to recognise and acknowledge the learners' main languages and being used to speaking SeTswana as the teacher's language (Setati, 2005). This suggests that this teacher found himself in a dilemma. Adler (2001) argues that dilemmas persist across multilingual contexts in South Africa because of the primary and non-urban contexts our learners come from.

CONCLUSION

In this chapter, I have presented an analysis of data collected in the study. The analysis looked at the kinds of tasks the teacher used, how he promoted mathematical reasoning and how language enabled or constrained mathematical reasoning. The analysis shows that the teacher promoted procedural fluency more than all the five strands of mathematical proficiency presented by Kilpatrick et al (2001). The teacher switched to Setswana more than IsiZulu and IsiXhosa though not all the learners were Setswana speaking. In the following chapter, I will discuss the findings that emerged from the analysis.

CHAPTER 5 FINDINGS AND RECOMMENDATIONS

INTRODUCTION

Promoting mathematical reasoning in a multilingual class of learners who learn in a language that is not their home, main, of first language is not an easy task. While paying attention to what the learners are saying and thinking, the teacher has to at the same time pay attention to how the learners' languages can be used as a resource in their communication of mathematics ideas. This study investigated how a grade 7 teacher promotes mathematical reasoning in a multilingual class. This chapter presents a summary of the findings of the study as well as recommendations for teaching and future research. To present the findings I use the research questions discussed in chapter 1.

SUMMARY OF THE FINDINGS

How did the teacher encourage mathematical reasoning during teaching and how did the way in which he interacted with the learners enable or constrain mathematical reasoning?

The teacher focused more on procedural fluency than the other five strands of mathematical proficiency. As much as he was constantly asking the learners to explain, his probing led to the explanation of strategies and procedures, which suggests that he could not challenge his learners to show the interwoven ness in the strands mentioned by Kilpatrick et al (2001). Now and then he would also ask the learners to explain how they got their answers. The teacher's way of questioning explored the skill of carrying out procedures accurately. This resonates with what Schiffer (2001) aptly points out in his study that he realised that many teachers needed help in order to develop the skills necessary to assess the validity of a mathematical argument because they have gone to school, sometimes very successfully, memorising facts and procedures.

During the process of his teaching, the teacher required explanation and justification of procedures from learners. Carpenter et al (2003) argue that justification is central to mathematics and mathematics cannot be learnt with understanding without engaging in justification. Justification in this class allowed learners to convince others how and why

they came up with particular procedures and solutions; it did not encourage adaptive reasoning at a deeper level. Carpenter et al maintain that young children cannot learn mathematics with understanding without engaging in justification (2003).

While there was evidence of explanation of solutions in this class there was no evidence of conjecturing, and presentation of arguments. Moschkovich (2002, p. 193) indicates that "in many classrooms teachers are incorporating many forms of mathematical communication and students are expected to participate in a variety of oral and written practices such as explaining solution processes, describing conjectures, proving conclusions and presenting arguments". Too much focus on procedural fluency deprived the learners a chance to conjecture, present their arguments and to prove their conclusions. Carpenter et al (2003) also support the importance of conjecturing by stating that they have "found that it is productive to ask children whether their conjectures are always true and how they know they are true" (p102). While the teacher may enter the classroom with stronger mathematics background, there are additional mathematical skills that he needs to call on in order to respond the learners' thinking, skills that are unlikely to be cultivated in explorations of mathematical content (Schifter; 2001).

Learners shared ideas through the teacher's facilitation and convinced others that the procedures they were using to solve, the problems were valid. Given the way in which teacher-learner interactions worked in this class, the teacher could not fruitfully ensure that the interactions move beyond developing procedural fluency. Ball and Bass (2003) insist that to ensure fruitful interaction in a mathematics classroom; teachers have to have the skill to probe learners to prove their answers so that he may understand the reasoning behind their explanations and justifications of answers. The teacher needed the skills necessary to challenge and assess the validity of a mathematical argument or method of solution beyond the level of procedural fluency. Schifter (2001) argues that once the teacher has developed the above-mentioned skills he may easily avoid memorization of facts and procedures, which lead to lack of sense of a mathematical logic. The analysis proves that most of the learners in this class participated in justifying and explaining their answers at procedural level. The teacher seldom asked the learners the reasons behind the procedures they were using.

The teacher did not impose procedures to learners however, his scaffolding led the lesson to end up focussing on procedural fluency more than other strands (see the analysis table on

procedural fluency). He was constantly trying to develop productive ways of challenging learners to justify their answers, but focussing too much on procedures distracted learners from the challenging aspects of the task (Stein et al, 1996). While the teacher was trying to make sure that the implemented task remained consistent with the ways in which they were set up, his challenging strategies made most of the questions to be procedural. As much as he did open up for conversational space for learners to make their ideas public, too much focus on procedural fluency denied learners the opportunity to display adaptive reasoning. The teacher however gave learners sufficient time to explore (not too little, not too much). He gave almost the same amount of time for each question to be appropriately completed. There was appropriate teacher scaffolding of student thinking regardless of the fact that procedural fluency dominated the lesson. Frequent probing made the teacher almost fail to tackle the challenging aspects of the task for the learners. He was sometimes willing to let the learners struggle with a difficult problem; however, he did assist learners who were encountering difficulties. Learners were also willing to persevere in their struggle to solve difficult problems (see appendix A). The teacher created the classroom environment that supported effective learning by explaining questions to the learners. He gave some instructions that were giving guidance to the learners thus creating the classroom environment which made it easy for learners to participate. Throughout the lesson, he was explaining, giving instructions and affirming the learners' responses.

As Kilpatrick et al (2001) argue, the five strands are interdependent, this means that procedural fluency cannot exist without the presence of conceptual understanding. If the teacher focuses more on developing procedural fluency and ignores the other four strands then the learners would not have the kind of procedural fluency that Kilpatrick et al (2001) talks about. This kind of procedural fluency has to be flexible, accurate, efficient and appropriate. It allows the learners to be able to use their knowledge of procedure in different contexts. Hence it's interdependency with the other four strands. It is the holistic focus on all the strands that can enable the promotion of mathematical reasoning. Having been able to promote procedural fluency in learners therefore implies that the teacher had encouraged the development of conceptual understanding as Kilpatrick et al (2001) argue that procedural fluency is not limited to the ability to use procedures, it also includes an understanding of when and how to use them.

The kind of task the teacher used to promote mathematical reasoning.

The teacher was able to select the task that would enable the promotion of mathematical reasoning i.e. lower level demand questions and higher-level demand questions. A weakness of the task was that while the teacher referred to the diagram as rectangular, there was no mathematical indication on it that showed that it was a rectangle. For an example, the teacher could have put 90 degrees signs at the four angles of the shape.

The task that the teacher set up had a potential to develop the five strands of mathematical proficiency (Kilpatrick et al, 2001) i.e. procedural fluency, conceptual understanding, strategic competence, adaptive reasoning and productive disposition. The implementation however, focused mainly on procedural fluency more that the other strands. This indicates that a good task in and of itself does not guarantee that the interactions that will occur during the implementation of the task will promote mathematical reasoning. The probing questions that the teacher asks will ensure that the learners engage in a range of discourses and thus develop mathematical proficiency.

How did the teacher use language to enable mathematical reasoning?

Language is a medium in which mathematics is enacted, used and created. The teacher used language to explain, affirm responses, reprimand, give instructions and to probe learners to talk about mathematics. He explained each question before letting the learners engage with it. He gave instructions in order to give direction to the learning process.

The teacher interacted with the learners in English. None of the learners who spoke during the lesson showed that they were experiencing language problems. They might not have experienced severe problems because the teacher was explaining each question thoroughly before the learners could deal with it. Explanation of the procedures involved language. Most of the time the teacher used English which was not his or the learners' home language as stated in Chapter 3. While he acknowledged that the English language is problematic because it is not the learners' home language however, he could not reach out to all of them by using their home languages to promote mathematical reasoning.

One of the ways the teacher used language to probe was through code switching. He even indicated in the post interview (appendix C) with the researcher that because sometimes language becomes a barrier he therefore switches to the learners' home language to ensure

understanding. This is supported by Adler (2001) who argues that the teacher ensures the establishment of the intersubjectivity in a multilingual class through the use of mediation and code-switching. There were few instances though, where the teacher was seen code-switching. Though the teacher believed that it was his responsibility to ensure that the learners understood the language thoroughly before tackling the mathematical problems, he however focused on switching to Setswana much more than Isixhosa or IsiZulu. The teacher's support on language could not embrace all the learners in class though he managed to engage the learners in the learning process through the language of learning and teaching as well as the mathematical language. Setati's work suggests that mathematics discourses in multilingual classrooms where the teacher uses only English remain procedural because the learners' level of fluency in English does not enable them to engage in conceptual discourse.

Throughout the lesson, the teacher was the one who initiated conversations. He asked the questions, made selections as to who answers those questions. In essence, he controlled the form and the function of the discourse in his class. This reflected evaluative listening on the teacher's side (Davies, 1997). There was nowhere where the learners just decided to ask questions from the teacher. While the learners did not manifest language problems, there was no clear indication that all of them had no language problem as some of the learners never responded to the teacher's questions as individuals. The learners might have been quiet because of what Moschkovich (2002) aptly points out that sometimes multiple meanings make it difficult for second language learners to navigate to attain mathematical proficiency because they learn mathematics in a language that is not theirs.

In my view, this teacher displayed the form of teaching for mathematical reasoning but not the function. He managed to choose the appropriate task with a mix of low and high cognitive level-demand. He gave learners opportunity to talk and he asked questions. He was able to create a classroom environment in which there was mutual respect amongst the learners. He did not humiliate the learners who could not give him the answers he required. He explained questions to learners and affirmed correct responses. His lack of function is evident when his probing focused on procedures. This implies that one can have all the ingredients of baking a sponge cake but miss the method. For an example, when baking a sponge cake you need to be careful not to knead the dough too much as this will disturb the function of the baking powder and thus causing the cake not to raise. The teacher almost

had all the ingredients of promoting mathematical reasoning but his probing led the lesson to focus more on procedural fluency than other strands.

RECOMMENDATIONS

This study points to the importance of learner talk in promoting mathematical reasoning and how the teacher facilitates the process. It is not sufficient for a teacher to select an appropriate task. The manner in which the task is implemented is also critical. As Schiffer (2001) argues, before the teacher gives the task to the learners he should consider the following:

- How he can attend to the mathematics in what the learners will be saying and doing?
- How he will assess the mathematical validity of learners' ideas?
- Have the skill to be able to listen to the sense in learners' mathematical thinking even when something is amiss?
- Have ability to identify the conceptual issues the learners are working on?

Schiffer (2001) argues that though teachers may have stronger mathematics background they still need the skills in order to be able to attend to learners' mathematical thinking. They need to be able to listen to learners with sharpened curiosity and interest and even be able to know which questions to ask when learners respond and how to mediate the learning process. (Schiffer, 2001).

The above-mentioned points by (Schiffer, 2001) also require a teacher who will have developed good listening skills mentioned by Davies (1997). Listening is an important aspect of engaging with learners' thinking. It is important to the teacher to consider how he listens. Davies (1997) talks about three different kinds of listening i.e. evaluative listening, interpretive listening and hermeneutic listening. Evaluative listening is displayed when the teacher have "correct" answers in mind. Davies (1997) argues that the teacher whose listening is evaluative strives for a structured lesson and this is what this teacher displayed. Evaluative listening limits the learners' contributions as either correct or incorrect. Again, as much as interpretive evaluation seeks for information from learners than just responses, that is not enough. Elaborate answers with some sort of demonstration or explanation are also not enough when promoting mathematical reasoning. Listening for the promotion of

mathematical reasoning should be more than attending to answers in different ways as this still leads to listening for a particular response. When promoting mathematical reasoning the teacher has to develop hermeneutic listening skill. He should be able to listen in order encourage participation and interaction in learners. The teacher's listening should not make him direct the learners to some pre-given understanding but must show the willingness to interrogate the learners in order to get reasoning behind their responses.

As much as procedural fluency is one of the most important strands of mathematical proficiency, focus should not be given to it more that the other strands. The teacher should also let the learners give details when using and explaining their strategies as Kilpatrick et al (2001) argue that the more the learners interact about mathematical ideas and concepts, the more their mathematical proficiency will develop. The use of clues should not lead to procedural routes. For instance, the teacher was quick to tell the learners to think about the length and the breadth when answering question 5 of the given task. He could have just challenged the learners further; he could have let them come up with the concepts of 'length and breadth' instead of telling them (see line 143 in appendix A). The teacher should be cautious of how and when the strands of mathematical proficiency can be interactively engaged. Kilpatrick et al (2001) argues that the strands of mathematical proficiency are connected and coordinated and problem solving that makes them pit one another misconstrues the nature of mathematical proficiency.

The teacher also needs to pay attention when learners respond so that he does not end up accepting incorrect answers as he accepted an incorrect answer in line 51 (see appendix A). Accepting incorrect answers could lead to misconceptions. This is supported by Schiffer; (2001) who argues that listening plays an important role in promoting mathematical thinking in learners.

What a teacher in a multilingual classroom has to ensure is that the learners understand the mathematics concepts i.e. the what, how and why. He needs to ensure that the learners are able to talk about the concepts, in explaining their understanding and justifying their answers. The teacher also needs to ensure that the learners are able to respond to questions about those concepts in English because English is the language of learning and teaching.

The other point that the teacher needs to consider is that language is the foundation of mathematical reasoning (Ball & Bass, 2003). As the teacher regards home language as a tool that enables mathematical reasoning, learners should then be openly told to use the language of their choice at the beginning of the lesson. This will help the teacher to know that the learners who are not participating are not quiet because they are scared to express themselves in English. Setati (2005) supports this by pointing out that for second language learners in a multilingual classroom to be successful in their learning of mathematics, it is important that their home languages be regarded as legitimate language(s) of interaction and that they be used in a range of mathematical Discourse. If the learners were openly given an opportunity to use their home languages, the teacher could have been able to speculate that they were quiet because of mathematical problems and not English language and thus help them. Setati (1996) further points out that the extent to which the teacher creates opportunities for learners to articulate their own thinking in their first language facilitates the use of language as an interpersonal tool because sometimes second language learners do not possess enough vocabulary in the language of learning to convey those meanings. This is supported by teacher, Margaret cited in Carpenter et al (2003, p. 51) who argues that "many kids might need support when engaging in language-rich mathematics experiences". She maintains that while language can interfere with learning, it also can nurture learning.

The teacher should create an environment that will encourage learners to challenge one another. He should also conduct a lesson in such a way that the learners become bold enough to ask him or challenge his explanation so that he may be able to understand the reasoning behind their questioning.

REFLECTIONS

What does it mean to do research in a school where the researcher works and hold a senior management position. This is a situation I found myself in as a researcher in this study. The fact that I played multiple roles in this study might have affected the findings about the research. As explained in chapter 3, this research was done in a school where I was principal; this could mean that my presence in the classroom was not benign (Setati, 2000). Setati (2000) in her research found out after the research was conducted that one of the teachers in her study told the learners to speak English only in the presence of the researcher because she thought the researcher was there to spy on her. My presence in the class might have caused the learners not to be free to speak in their home language. If I had asked

somebody the learners did not know, they might have been free to use any language. As their principal, they knew my expectations therefore; they might have spoken in English just to impress me. Again, the teacher might have tried to impress me by encouraging them to speak in English during my visit, thinking that I would be impressed because I always encourage learners to speak English at school.

CONCLUSION

Promoting mathematical reasoning on its own is a challenge in teaching and learning of mathematics. This research has shown that teacher's knowledge of the content on its own cannot develop mathematical proficiency in learners. Teachers need more than knowledge of the content. They need skills that will help them to be able to attend to the mathematics in what the learners are saying, to assess the mathematical validity of the learners' ideas, to be able to listen for the sense in the learners' mathematical thinking even when something is missing and be able to identify the conceptual issues the learners are working on (Schifter, 2001). The other challenge that faces mathematics teachers is the ability to set up tasks and be able to implement them such that mathematical reasoning becomes developed in learners. The challenge becomes even more intense when promoting mathematical reasoning in a multilingual class of English second language learners. The challenge is both the ability to promote mathematical reasoning and to make sure that the language does not become a hindrance in the promotion of mathematical reasoning.

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APPENDIX A: Research transcript

1. T: we ‘ve got our plan akere[okay], it’s a house plan it’s a typical house plan, this house plan has T for toilet, H for what, for the whole space sena [this space], K for kitchen, D for dining room, B1 is for bedroom 1 , bedroom 2, bedroom 3 {*pointing at B1, B2 and B3 and L for lounge. now there’s a broken line*}, At the broken line ya, what is the meaning of that broken line the way according to wena the way uyibonang kateng [the way you perceive it], what do you think is the meaning of this broken line? (E)
2. L: {*silent*}
3. T: (before learners could respond) Ya, it simply means that line ena [this line]is sort of , it can mean there are steps, they differentiate between the lounge and the dining room, you can see that I separate it can be ukare [it seems like] dining room yarona inyane [our dining room is small], its not small its just that we are using a small scale. (E)
4. T: Now this is our plan what is it that we are concerned about? Already we have different, we different measurements for instance this is AB, AB yarona kebokae [how much is our AB]? (PF)
5. L:3m
6. T: what about our BC? (PF)
7. L:4m
8. T: where is KP, where is KP? What about DE? (PF)
9. L: 3m
10. T: What about EF? (PF)
11. L:3m
12. T: What about FG? (PF)
13. L:8,5m
14. T: What about HI? (PF)
15. L: 3m
16. T; What about OJ? here is OJ (PF)
17. L: 5m
18. T: What about QH? Where is QH?: What about KP? yes, where is KP? KP, KP where is KP? KP,KP,KP okay, lets carry on. What about PJ? (PF)
19. Learners: 4m

20. T: now what is it that we want to do? We have to look for the area, what do we know about the area? That's the first thing you should think about, we should think about it when we talk about the area baboa ka area. [they are talking about area]. (E) When we talk about the area what is the first thing that comes into your mind? Baoba ka area [they are talking about area], what do you understand, how you understand it lets put it that way, okay lets put it that way, (CU)

21. L: the whole space

22. T: can you show us maybe, we are looking for the area of the table there what is it that……? Ya (CU)

23. L: The whole space

24. T: the whole space where? (CU)

25. L: the surface of the desk

26. T: the surface of the desk akere [okay], now, coming to our diagram. Through our diagram, our plan, we want what, the area of the whole space kaufela akere [the whole space, okay] (E)

27. L: yes

28. T: What will be the formulae of the area, What will be the formulae of the area, ja ja (PF)

29. L1: height times breath

30. T: Ya

31. L2: height times breath

32. T: height times breath ok there's another hand, ja ja ja (Aff)

33. L: length times breath

34. T: we have? (Aff)

35. L: length times breath

36. T: length times breath akere (okay) (Aff)

37. Learners: yes

38. T: usually the length, it's stands for which side huh the length ja (PF)

39. L: for the longer side

40. T: for the…., what about the breath (PF)

41. L: for the shorter side

42. T: ja ok now now lets look for the where is our length in our diagram, which points makes up the length in our whole diagram ja (PF)

43. L: AI

44. T: A ,ok for what? length ne (okay) A I (Aff), now what will be the distance from A to I now before we can talk about it, what kind figure is this ,ja? (CU)

45. L: a rectangle

46. T: a……

47. L: rectangle

48. T :what do we know about rectangles, what is it that we know? Gomotso. (CU)

49. L: It has two opposite parallel sides

50. T: it has two parallel sides akere (okay) (Aff) , so what does this mean? AI will be equals to what? The distance of AI will be equal to which distance, Sandas? (PF)

51. L: C to G

52. T: Yes the distance of , now let's calculate hore (that) what will be either the distance AI and CG, areye [let's go] in your papers work it out, remember we have quite a number of broken, we are going to use this table e akere, now rebatla hotseba hore [we want to know that] distance of A to I how much is it A to I or C to G, how much will it be in meters? (PF)

53. L: {*silent*}

54. Yes areye [let's go], lets find out, lets work it out, the minute you finish just raise up your hand, mara [but] only that distance (GI)

55. Learner: {*discuss.}*

56. T: yes what will be the distance, how do we get the distance of AI or CG? that's it what will be in m, what will be CG o kgetha [you choose] only the line akere lets find the line the minute le feditse [when you have finished] just raise your hands okay C D lets see where you… CD it means what, what does it mean, can you explain it to us? Ya, ya {*pointing at different learners*} (CU)

57. L: {*still thinking*}

58. What are you going to do with these figures? (SC) Pass it, okay then elhogufhang [what will it give you?]

59. L: the whole side

60. T: the hole side akere [okay], (Aff) that's how we should do it , ya, thusa babang kamo if uyithotsi [yes help others if you have got it] (GI)

61. L: we say AB is equal to 3m. BC is equal to 2.5 m, ya

62. T: why do we do this? Mh why {*the learner does not respond*}. Percy, help? Why? (AR)

63. Percy: so that we find out what is the breadth because when we add from A to C because in this diagram has three points, so can have a point fro A to C, we don't have a point from A to C. so that we can add so that we can get to see

64. T: okay, right, I think there is a …..that group has never said anything. Here we are, there we are, Gloria, let's see, who is explaining? Okay let' see. (AR)

65. L: we say this one is equal to 5.5m plus .{*educator interrupts}*

66. T: Okay before we arrive, we haven't been there yet, why did we have to do this? Why were we looking for 5.5m. (AR). Okay khulumela phezulu. [speak louder] (CE).

67. L: A and B equals to 3m and BC is equals to 2.5m

68. T: and then

69. L: and then we plus, we add them and they give us 5.5

70. T: which is what, the 5.5? (PF)

71. L: our breadth

72. T: okay I think that's okay, I think almost everybody has finished akere, now what will be our breadth in this case? Ya (PF)

73. L: 5.5m

74. T: how did we get that 5.5m. Who can explain? I think we've explained thoroughly now, we want somebody who can stand up, explain it to us in full, how, okay, let's hear. Can we listen to him, can we listen to him, yes. (PF) and (CE)

75. L: AB is 3m, BC, 2,5m

76. T: Then? What do we do with the two measurements? (PF)

77. L: you say 3 + 2.5 is 5.5m.

78. T: what is the reason? Why do we do it? Remember in maths all the time ask the reason 'why do we do that'. Why 5.5m, it is correct, the answer is correct but now why, what is it anyone except Percy, Percy, okay Jeremiah, yes. (AR)

79. Jeremiah: because we want to get the breadth.

80. T: now from there what do we have? Because we've got, from there we have our length which is 18.5m times and what is our breadth? (PF)

81. L: 5.5m

82. T: then from there what do we do? Once we have substituted now akere, we say L actually stands for what? (PF)

83. Learners and teacher: 18.5m

84. T: so B stands for what stands for what? (PF)

85. L: 5.5m

86. T: then what do we do from here? Ya (PF)

87. L: we multiply the length by breadth,

88. T: let's multiply the two in your book, let's multiply. (GI)

89. L: {*discuss}*

90. T: please, the minute you are finished, raise up your hand so that we can see were you are right or wrong (GI). Ok there's a hand there's somebody who is finished. ok let's see now let's see explain to use what is it that we are doing what are we multiplying? (SC)

91. L: the length and the breath

92. T: Ya the length and the breath akere now what are the measurements of that length and the breath (PF)

93. L: the length is equal to 18.5m and the breath is equal to 5.5m

94. T: ok now explain to me how did you multiply (SC)

95. L: we said 5x5 = 25 carry the two 5x8 = 40 +2=42 by 2 5x1=5+4=9

96. T: ok

97. L: and then it give us one hundred and one coma seven five meters

98. T: 101.75 m usually numbers after the coma we call them one by one ekere. (E) Eh lets see the last group, in this case ok who's explaining here now the first thing what is it we were doing. (PF)

99. L: 18.5m X 5.5m

100. T: ok

101. L: after that we said that 5X5 =25 carry the two and then 8 times 5 is 40 plus two is forty two one times 5 is 5, 5 plus 4 is 9……..

102. T: ok I think so far we've got two groups, already they have found it right, (Aff) the correct answer akere. Now what is the correct answer? (PF)

103. Learners: 101,75

104. T: 101.75, now which units are we going to use? (PF)

105. L: Meters

106. T: Square Meters {*writing it on the chalkboard*}, why do we use square meters, who can tell me? why do we use square meters? Yes. (CU)

107. Percy: because we multiply.

108. T: what is it that we multiply? (PF)

109. Percy : the metre and the breadth

110. T: what? what are you saying? A metre ?

111. L: and the breadth.

112. T: a meter times a meter, it gives a what {*pointing at the units*}? (PF)

113. L: square.

114. T: I think that's it with number 1, now let's go to number 2. What is number 2? Find the area of the hall, now let' see, where is the hall? Let's have a different colour, here is the hall {*shading it in yellow}* can you see? That is our hall. (E)

115. L: yes

116. T: now what kind of figure is our hall? What kind of figure is our hall? {*no response}* ya, how many sides, let's start, our hall has got how many sides? (PF)

117. L: It has four sides ,

118. T: By the way a figure with four sides, what do we call a figure with four sides? (PF)

119. L: A quadrilateral

120. T: Quadri means what? (CU)

121. L: 4

122. T: for us to get the area of a square what is it that we notice about this figure? This figure here, we can actually say out of one figure we have? {*separating a rectangle and a triangle*} (PF)

123. L: two figures

124. T: mainly this figure will be what? (CU)

125. L: rectangle

126. T: and this one? (CU)

127. L: triangle

128. T: now for us to get the whole area what is it that we must do? Before we go writing what is it that we have to do for us to get the whole area here? Okay (SC).

129. Thabang: we must find the area of a triangle and the area of a rectangle and we must add them together.

130. T: can someone repeat that for me oh yes. Anyone, Maunye. (GI)

131. Maunye: we must find the area of a rectangle and a triangle then we must add them together.

132. T: why do we do that? By the way why do we do that? Okay ya Thusi why? (AR)

133. Thusi: because we want the area of the hall.

134. T: yes, Tunisia:

135. Tunisia: the area of the hall

136. L: an area of the hall

137. T: yes

138. Nkuna: so that we can get an area of the hall.

139. T: so that we can get an area of the hall. (Aff). Okay why did we have to split the area of this quadrilateral? (AR)

140. Nkuna: because we cannot measure it because the parts are not equal.

141. T: okay, we can still measure it, let's hear Percy. (E)

142. Percy: because it is different from other quadrilaterals, because it has two figures, in one figure you can make two figures, we must divide them into two and we measure both of them and we add them.

143. T: okay, ya ya, remember we have never measured an area of this figure but what do we notice? (CU). If we split this figure we have the area of a rectangle, that we know akere? And we have an area of triangle, do we know? Yes we know and the two figures what do they make, they make the area of the whole quadrilateral so it means for us to get the whole area we have to do them separately and then we add them know let start with the area of the rectangle where is our rectangle here is our rectangle akere now were is our length and our breath (E), what will be our length this side, what will be our length this side, there's a number here (PF)

144. L:K

145. T: this is K akere yes thank you, now what will be, what is this?

146. L: P

147. T:P, ok now what will be our length here? What will be the size of our length, what will be the size of our length ok, here tell us (PF)

148. L: 7m

149. T: now the size of our, by the way this is the area of what area of a

150. L: Rectangle

151. T: ok now lets calculate it in our books we have got the area of we have got our length what will be our breath? (PF)

152. L: 3m

151. T: ok lets calculate it, lets look for the area of the rectangle lets start, Lets start, area of the rectangle the minute you are finished raise up your hand areye (let's go). (GI)

L:(discuss)

152. T: lets see girls, identify the length and the breath and substitute, instead of the length write the measurements, instead of the breath write the measurement calculate, lets see where are you, where are you working at. (GI). Ok, let's see first group has finished here. Who is explaining here? I think you are, there areyeng (let's go) kids explain? (CU)

153. L: child explain (can't hear)

154. T: Ya we add or we multiply ok which one do we do, do we add, do we add the area. What is it we should do, ok correct yourselves, we are coming, we are still coming ok come this side m'am ok ya (CU)

155. L: we multiply

156. T: then and then what do we do, and then, ya, square m. Ok 21 square m. Ok by the way why is it that we do this, why did we do this why did we do that what is that we are doing? What ya ok lets see this group, I think this group has not said anything lets see what they say. (AR)

157. L: child explaining {*can't hear*}

158. T: taking {*can't hear}*

159. T: ok lets do it on the board lets do it on the board a si bhekene ngapa (GI) [let's look at this side] ok lets put down our pens now remember the main thing what you must do is what am I doing, why am I doing it akere (E), so now what is it that we are looking for? (PF)

160. L: the area

161. T: the area of what? the length, what is the area of the rectangle? (PF)

162. L: length times breath

163. T: length times breath, what is our length? (PF)

164. L:7m

165. T:7m (Aff)

166. T: and what is our breath? (PF)

167. L: 3m

168. T: it is important to talk note gore [that] when we multiply we don't add akere. (E)

169. L: yes

170. T: length times breath, area is length times breath not length plus breath (E), now three by seven is how much? (PF)

171. L: twenty one m^2

172. T: ay! 21 square metres (Aff). Are we finished with the whole diagram?

173. L: no

174. T: now what are we left with? The area of the rectangle akere. Now when we look at the area of the rectangle what is it we should look for? The area of a triangle? what is it we should look for ya (PF)

175. L: the height and the base

176. T: the height and the base (Aff), then is that all?

177. L: no

178. T: there's something that is missing ya there is something missing (PF)

179. L: half

180. T: yes we have to half the size of the height and the base (Aff), let's calculate our base.

(learners work in groups.)

181. T: {*after a pause}* okay who has finished? Who has finished? Arieng, arieng [let's go, let's go], let's see {*moving around*} are you finished? Let's see. Can you explain? Can you explain? From here, who is explaining? Jeremiah. Okay can you explain, how did you get it? (SC)

Jeremiah: I said the are of a triangle is 0ne and half times height times base. I said PJ is one and half times four meters times 3metre is equal to 12 square metres,

182. T: now who can tell me why we use the half? That half stands for what? Why do we say a half, every time half times base times height, why that number? ya. (AR)

183. L: that figure is divided into two parts,

184. T: what are the names of those parts? (CU)

185. L: a rectangle and a triangle

186. T: ya, a rectangle and a triangle (Aff), okay something that you are missing, ya let's hear laphaya emuva [there at the back], Percy did you do it?

187. Percy: yes, it's half base times height.

188. T: why do we use that half? (AR)

189. Percy: because a half, a triangle is a half of a rectangle.

190. T: okay that's good (CE), from there what did you do. (PF)

191. Percy: then we said the height which is there which is mostly on the side of the 90 degrees of a triangle which in our diagram is 3 m

192. T: ok, and then? (PF)

193. Percy: then we said the base is 4m and then we said what is a half of base 4, it gave us 2m.

194. T: ya?

195. Percy: then we said these 2 m we multiply them by 3 m then it gives us 6 square meters ($6m^2$)

196. T: $6m^2$, that's good, okay now let's see, okay I think everybody is finished now, let's finish let's move on let's move on. {*addressing the whole class now}*. E e we've got our formula already on the blackboard akere. (½ base x height) now somebody who can explain why do we have to use this half all the time when we look for the area of a

triangle why do we use half, besides Percy besides Percy? Is there anyone who can explain why we use half all the time? Is there any one who can explain to us? Ya. Why do we use half? Okay Percy? Speak aloud so that everyone can hear.(CE) and (AR)

197. Percy: because a triangle is a half of a rectangle.

198. T: okay, let's substitute there, from there we've got height x base. What is our height there? What is our height? (PF)

199. Learners: 4 m.

200. T: what is our base? (PF)

201. Learners: 3m

202. T: now the easiest way. What is the easiest way of calculating, before we can even start writing what is the easiest way of calculating? Who can tell me? What is the easiest way of calculating? Ya, explain it to us. (PF)

203. L: we can say 4 times three is equals to twelve and half of twelve is 6.

204. T: ya, one way of doing that, it is correct (Aff). Omong one angweste eng [what did the other write]? Somebody else ongwetse eng yena [what did the other write]? Oyi kwadile jwang yena [how did he write it]? Come, come explain to us then, explain to us, talk to us (PF)

205. L1: half times four {*pauses*}

206. T : aha it gives us how much, {*pause*}? What is half of four? You've got four, (PF)

207. L: it gives us two.

208. T: okay show us the four, Okay raise up your hands, show us four, come on show us the four, look at the four, {*learners raise up their hands showing four fingers*}. Now show me of the four, half of that.{*the learners shows half of four fingers*} (GI)

209. T: you are left with how much? (PF)

210. L2: two

211. T: so it means what? (PF)

212. L2: equals to two

213. T: okay can you see now, so it means what? (PF)

214. L2: half times four equals to two.

215. T: okay from there what do you do with the other number? ya Thusi. (PF)

216. Thusi: you say two times three.

217. T: it gives us? (PF)

218. Learners: 6

219. T: then from there it means the area is what? (PF)

220. Learners: 6 square meters

221. T: $6m^2$, now what will be the total of the whole space? What will be the total of the whole surface area, meaning this diagram and that diagram {*pointing at a rectangle and a triangle ie hall and lounge*} who can explain to us, start writing. (PF)

222. T: that row, we want somebody now, so far you have done well. Somebody new, Gugu can you try?, can you explain it to us? Can we, (PF)

223. Gugu: 30 x 27, then we said 7x0 is 0, 7x3 it's 21then we write 21then we have 20 is 0 under 1 and 2x3 is 6 then we add 210 + 60 then it gave us 810. {*pause*}. I said 30x27, 7x0 is 0, 7x3 is 21 then I have 210. Then I say 2x3 is 6 then I have 60 then I say 210 plus (pause, looking at 60 and the gap in units) I think I must put 0 here under the units because I cannot write a number like this, I must write units which is 0. {*she then had 210 + 600 like this*:}

$$\begin{array}{r} 30 \\ \times\ \underline{27} \\ 210 \\ +\ \underline{600} \\ \underline{810} \end{array}$$

224. T: why did you write the 0? Explain to her. Jeremiah, explain to her, why did you write that zero? (CU)

225. Jeremiah: if we say 210 plus 60 it won't give us 810

226. T: quiet class, ja tell us where did you get the 600? (CU)

227. Jeremiah: mh…. Eh.. 2 is in 27, 2 is tens then is 20, we say 20 x 30 is 600.

228. T: so what will be the total now? (PF)

229. Jer: total will be 810

230. T: okay that's good. Now let's do the last one.{*reads the last question on the chalkboard*}. What will be the size of the carpet used in the bedrooms? (PF)Now obatla ho Kenya carpet mo di bedrooms tsa haye akere [put carpets in his bedrooms okay], now what is it that we should do? Who can tell me? He wants to put the carpets in the bedrooms, otlo yetsang [what will he do]? What is it that we should do? The carpets! Now where are the bedrooms {*pointing at the diagram*} how many bedrooms do we have?(SC)

231. Learners: three

232. T: and we want to put what? Can you tell us without writing, what is it that we should do? Retlotseba joang gore di carpet detshwanetse debe kakang [how will we know the size of the carpets]? Otloyetsang [what will he do]? What is it that we should do, otlo tseba joang [how will he know]?, okay Percy, can you explain to us? (SC)

233. Percy: we find area from CF and BO.

234. T: simple as that, now what is the area, what is the area? Remember guys, the area, what is the area? (CU)

235. Learners: is the surface.

236. T: the surface of how many bedrooms from here { *pointing at B to E.*} (PF)

237. Learners: three bedrooms

238. T: three bedrooms (Aff), so from three bedrooms what will be our length and our breadth? From other people, other people. Yes tell us, tell us, what will be the length? (PF)

239. Dikolomela: the length will be 10m.

240. T: sure, sure, the length will be 10m (Aff), how do we get the length, yes? How do we get the length? {*pause, giving them a chance to think and respond*} Anyone? (PF)

241. L: we add mh…CD, DE and EF

242. T: yes we add them and how much do we get? (PF)

243. Learners: 10m

244. T: what will be the breadth, ya Gloria, you can actually multiply without using a pen, what will be the answer? What will be the answer? (PF)

245. Gloria: 2.5m

246. T: yes 2.5m, you remember when we multiply the decimals by ten, remember we used to shift whatever {*he writes 2.5 x 10 on the chalkboard*} (E)

247. Obed: 12.5

248. T: what? 12.5, how did you get 12.5? how did you do that? Explain to us. Explain. (PF) and (AR)

249. Obed: I count 2.2 plus 10 equals to 12 then we have 12.5.

250. T: ya, {*after giving him a chance he points at other learners.*} let's hear from others, I can see many hands. (PF)

251. Learner: 20.5

252. T: 20.5? how did you get that, why did you get that? (PF)

253. L: I said 2 x 10 is 20.

254. T: then what happened to the 5 (PF)

255. L: {*could not respond*}

256. T: I can see another hand , Gloria, there is a hand

257. Gloria: it will be 12.5

258. T: how did you get the 12.5? (PF)

259. Gloria: because I add 2 and 10 then I did not add comma five because it zero comma something. When I add together 10 and 2 with that 5 it gives me12.5

260. T: right who can explain this to us? What is the answer? Who can explain it once more? If you multiply ten and hundred, Percy. (PF)

261. Percy: you add the zeros.

262. T: so what will be our answer? We have a comma, 2.5, what will be the answer? (PF)

263. Nkuna: 25.0

264. T: how did you get it? (SC)

265. Nkuna: because there is one number after the comma.

266. okay, that's it for today.

APPENDIX B: Pre-observation schedule

1. Researcher: What kinds of tasks or mathematical activities do you use to promote mathematical reasoning in your class?

2. Teacher: Tasks should be contextualised. They should be related to their daily lives, tasks should be in simple English language and mathematics language. Tasks should require or force them to think, they should talk to one another if necessary, discuss, the task should drive them to use or recall other maths concepts to find solution.

3. Researcher: Is there any reason you use the kinds of tasks you do in your teaching?

4. Teacher: Yes, contextualised tasks improve their understanding, thinking, reasoning, strategies on how to tackle tasks, improves knowledge of maths concepts and language.

5. Researcher: During the lessons that I will be observing, what kinds of tasks or activities have you prepared to use?

6. Teacher: Contextualised tasks, tasks that will engage learners mentally and physically, tasks that will engage them with other maths concepts in order to find solution, tasks that will make them or force them to reason or give explanation with confidence and understanding tasks that apply to their daily lives.

7. Researcher: Is there any reason you have selected those tasks/activities?

APPENDIX C: Post observation interview

1. Researcher: When giving these mathematical tasks to your learners what are your main expectations from them?
2. Teacher: To read, understand, interpret and solve them. Apply own knowledge and understanding.
3. Researcher: When giving these mathematical tasks to learners what is it that you want to achieve?
4. Teacher: Note their level of understanding, to know what method or technique they apply, to implement them in their daily lives.
5. Researcher: 3. why did you explain the tasks to learners before they wrote?
6. Teacher: For clarity, understanding language and explain context, language problems, explain maths terms, concepts, familiarize them with the context.
7. Researcher: I heard you say to one of the learners 'when doing maths you must always give a reason why you are doing that particular thing'. Why were you saying that?
8. Teacher: To avoid guess work, provide or show knowledge and understanding in in tackling problems, they should get used to expressing themselves in maths language, rectify misunderstandings and engage them in maths language.
9. Researcher: how would you view the role of home language when promoting mathematical reasoning? Why?
10. Teacher: well English is not their mother tongue, they do not understand most English terms, some have poor reading skills, some fear the English language, lack of confidence in the language, feeling embarrassed for not understanding the language, I think it is important that I sometimes use their mother tongue to explain to them.
11. Teacher: Yes, I prefer contextualised tasks that will require the learners to come up with their own procedure that needs to be used. They will have to think and reason about the operational signs that are supposed to be used; I do not have to tell them, I love the tasks where they will have to explain why they have chosen that particular method. I want tasks that will provoke or challenge them to think, I think this is the reason why I also like group work, because that is where they discuss and explain their answers to one another.
12. Researcher: in line 51 you accepted an incorrect answer from the learners, why?

13. Teacher: oh really, ah! I was not aware. Which one is that, let me see (looks at the transcript). Okay I see. Yo, Next time I will be careful sis Lindi. I hope I did not mislead these learners.

APPENDIX D: Letter to the teacher

UNIVERSITY OF THE WITWATERSRAND
MATHEMATICS RESEARCH PROJECT

My name is Lindiwe Tshabalala. I am currently doing my Med degree in Mathematics Education. As part of my studies I am doing a study investigating whether and how the use of learners' home language can be a support for learning and teaching mathematics.

Your Principal has given me permission to send you this letter to invite you to participate in this research project. Once you have read the letter you can decide whether you want to take part or not. Should you agree to participate, you would be asked to contribute in two ways. First, allow me to observe your teaching in one of your mathematics classrooms for a week at agreed upon times. Second, I would ask you to participate in a reflective interview focusing on your observed lessons. With your permission, the lessons will be video-recorded and interviews tape-recorded so that I can ensure that I make an accurate record of what you say and do. When the tape has been transcribed, you would be provided with a copy of the transcript, so that you can verify that the information is correct.

I intend to protect your anonymity and the confidentiality of your responses to the fullest possible extent. Your name and contact details will be kept in a separate file from any data that you supply. This will only be able to be linked to your data by me. In any publication emerging from this research, you will be referred to by a pseudonym. I will remove any references to personal information that might allow someone to guess your identity, however, you should note that as the number of people involved in the research is very small, it is possible that someone may still be able to identify you. If, however, for any reason you would like your real name to be used in the publications you will need to make written request to me.

Once the research has been completed, a brief summary of the findings will be available to you. It is also possible that findings will be presented at academic conferences and published in national and international academic journals.

Please be advised that your participation in this research project is completely voluntary. Should you wish to withdraw at any stage, or withdraw any unprocessed data you have supplied, you are free to do so without prejudice. Your decision to participate or not, or to withdraw, will be completely independent of your dealings with the University of the Witwatersrand.

If you would like to participate, please indicate that you have read and understood this information by signing the accompanying consent form and returning it to me.

Should you require any further information do not hesitate to contact me – my telephone numbers are below.

Ms Lindiwe Tshabalala
(011)765 8421

APPENDIX E: Letter to the School Principal

UNIVERSITY OF THE WITWATERSRAND
MATHEMATICS RESEARCH PROJECT

My name is Lindiwe Tshabalala. I am currently doing my M.Ed degree in Mathematics Education. As part of my studies I am doing a study investigating whether and how the use of learners' home language can be a support for learning and teaching mathematics.

I am requesting you as the Principal of the school to give me permission to work with one of the educators and grade 7 learners in your school in this research project. Should you allow them to participate, they would be asked to contribute in two ways. First, allow me to observe his teaching in one of his mathematics classrooms for a week at agreed upon times. Second, I would ask him to participate in a reflective interview focusing on his observed lessons. With your permission, the lessons will be video-recorded and interviews tape-recorded so that I can ensure that I make an accurate record of what he says and does. When the tape has been transcribed, he would be provided with a copy of the transcript, so that he can verify that the information is correct.

I intend to protect their anonymity and the confidentiality of their responses to the fullest possible extent. Their names and contact details will be kept in a separate file from any data that they supply. This will only be able to be linked to their data by me. In any publication emerging from this research, they will be referred to by pseudonyms. I will remove any references to personal information that might allow someone to guess their identity, however, they should note that as the number of people involved in the research is very small, it is possible that someone may still be able to identify them. If, however, for any reason they would like their real names to be used in the publications they will need to make written request to me.

Once the research has been completed, a brief summary of the findings will be available to the educator. It is also possible that findings will be presented at academic conferences and published in national and international academic journals.

Please be advised that the participation of your school in this research project is completely voluntary. Should you wish to withdraw at any stage, or withdraw any unprocessed data you have supplied, you are free to do so without prejudice. Your decision to participate or not, or to withdraw, will be completely independent of your dealings with the University of the Witwatersrand.

If you would like to participate, please indicate that you have read and understood this information by signing the accompanying consent form and returning it to me.

Should you require any further information do not hesitate to contact me – my telephone numbers are below.

Ms Lindiwe Tshabalala
(011)765 8421

APPENDIX F: Consent form for the Principal

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

MATHEMATICS REASONING & LANGUAGE PROJECT

Researcher: Ms. Faith Lindiwe Tshabalala

Supervisor: Prof M. Setati

Consent form for the principal.

I, ……. agree that the school can participate in the project named above, particulars of which (i.e. details of lesson observations and interviews) have been explained to me. A written information letter has been given to me to keep.

I give consent to the following:

- Video recording of the lessons in which the teacher and the learners might appear as part of the videotext.

  Yes ☐ No ☐

- The possible future use of the videotext for teaching purposes

  Yes ☐ No ☐

- The participants being interviewed at some point during the research

  Yes ☐ No ☐

- Tape recording of the participants' interview sessions with the researcher.

  Yes ☐ No ☐

…….. ……..
Signature of the principal Date

…….. ……..
Signature of witness Date

…….. ……..
Signature of researcher Date

APPENDIX G: Consent form for teachers

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

MATHEMATICS REASONING & LANGUAGE PROJECT

Researcher: Mrs. F. L. Tshabalala

Supervisor: Prof M. Setati

Consent form for teachers in the study.

I, ……. agree to participate in the project named above, particulars of which (i.e. details of lesson observations and interviews) have been explained to me. A written information letter has been given to me to keep.

I give consent to the following:

- Video recording of my lessons in which I might appear as part of the videotext.

  Yes ☐ No ☐

- The possible future use of the videotext for teaching purposes

  Yes ☐ No ☐

- Being interviewed at some point during the research

  Yes ☐ No ☐

- Tape recording of my interview sessions with the researcher.

  Yes ☐ No ☐

…….. ……..
Signature of participant Date

…….. ……..
Signature of witness Date

…….. ……..
Signature of researcher
Date

APPENDIX H: Consent form for learners

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

MATHEMATICS REASONING & LANGUAGE PROJECT

Researcher: Mrs. F. L. Tshabalala

Supervisor: Prof M. Setati

Consent form for learners in the study.

I, ……. agree to participate in the project named above, particulars of which (i.e. details of lesson observations and interviews) have been explained to me. A written information letter has been given to me to keep.

I give consent to the following:

- Video recording of the lesson in which I might appear as part of the videotext.

  Yes ☐ No ☐

- The possible future use of the videotext for teaching purposes

  Yes ☐ No ☐

…….. ……..
Signature of participant Date

…….. ……..
Signature of witness Date

…….. ……..
Signature of researcher
Date

APPENDIX I: Consent form for parents

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

MATHEMATICS REASONING & LANGUAGE PROJECT

Researcher: Mrs. F. L. Tshabalala

Supervisor: Prof M. Setati

Consent form for learners' parents in the study.

I, ……. agree that my child participate in the project named above, particulars of which (i.e. details of lesson observations and interviews) have been explained to me. A written information letter has been given to me to keep.

I give consent to the following:

- Video recording of the lesson in which my child might appear as part of the videotext.

  Yes ☐ No ☐

- The possible future use of the videotext for teaching purposes

  Yes ☐ No ☐

…….. ……..
Signature of participant Date

…….. ……..
Signature of witness Date

…….. ……..
Signature of researcher
Date

APPENDIX J: Information letter to parents

UNIVERSITY OF THE WITSWATERSRAND
MATHEMATICS RESEARCH PROJECT

My name is Lindiwe Tshabalala. I am currently doing my Med degree in Mathematics Education. As part of my studies I am doing a study investigating whether and how the use of learners' home language can be a support for learning and teaching mathematics.

Your child's mathematics teacher and headmaster have given me permission to send you this letter to invite your child to participate in this research project.

Children whose parents agree that they participate in this study will be video recorded for at least a week during mathematics lessons in the month of August. The focus in these video-recordings and lesson observations will be on how the teachers use the learners' home language(s) to facilitate learning.

I intend to protect learners' anonymity and confidentiality. Their real names will not be used in the final report. I will remove any reference to personal information that might allow someone to guess the identity of the learners and teachers.

Remember that your child is not obliged to participate. Should you require any further information do not hesitate to contact me – my telephone number is below.

If you agree that your child be part of this research project please complete the consent form on the next page and sign in the space provided.

Ms Lindiwe Tshabalala
(011)765-8421

APPENDIX K: Information letter to learners

UNIVERSITY OF THE WITSWATERSRAND
MATHEMATICS RESEARCH PROJECT

My name is Lindiwe Tshabalala. I am currently doing my Med degree in Mathematics Education. As part of my studies I am doing a study investigating whether and how the use of learners' home language can be a support for learning and teaching mathematics.

Your mathematics teacher and headmaster have given me permission to send you this letter to invite you to participate in this research project.

Children who agree that they participate in this study will be video recorded for at least a week during mathematics lessons in the month of August. The focus in these video-recordings and lesson observations will be on how the teacher promote mathematical reasoning to facilitate learning.

I intend to protect your anonymity and confidentiality. Your real name will not be used in the final report. I will remove any reference to personal information that might allow someone to guess your identity and of your teacher.

Remember that you not obliged to participate. Should you require any further information do not hesitate to contact me – my telephone number is below.

If you agree to be part of this research project please complete the consent form on the next page and sign in the space provided.

Ms Lindiwe Tshabalala
(011)765-8421

APPENDIX L: Research observation sheet

1. TASKS IMPLEMENTATION.

Memorisation tasks

1	2	3	4
Learners have forgotten the previously learned rules, definitions, formulae etc.	Learners reproduce formulae and rules without understanding	Learners use recall and use formulae and rules with understanding	Learners can recall, use and explain formulae, rules and definitions.

Comments: _____

Procedures without connections tasks.

1	2	3	4
Learners cannot explain the procedures they have used.	Learners slightly understand the procedures they have used.	Learners can explain the procedures they are using.	Learners can explain and justify the procedures and algorithms they are using.

Comments: _____

Procedures with connection tasks

1	2	3	4
Learners can't follow procedures, can't make connections among multiple representations.	Learners follow procedures mindlessly with no understanding of underlying concepts.	Learners follow procedures, can make some connection among multiple representations. Slight understanding of underlying concepts.	Learners engage with conceptual ideas that underlie the procedures, show cognitive effort, successfully complete the task and develop understanding.

Comments: _____

Doing Mathematics Tasks.

1	2	3	4
No understanding of mathematical concepts, processes and relationships	Algorithmic thinking, poor analysis of the task, and still requires a predictable approach	Show understanding of concepts and relationships.	Thorough analysis of the tasks, good understanding of mathematical concepts, processes and relationships. Shows non algorithmic thinking.

Comments:_____

THE ROLE OF THE TEACHER.

1	2	3
The teacher does not probe the learners to explain their answers.	The teacher accepts one word answers from learners	Encourages the learners to explain and justify their answers.

Comment:_____

MATHEMATICAL PROFICIENCY. (Kilpatrick;2001).

1	2	3	4	5
There is evidence of conceptual understanding in learners.	There is evidence of procedural fluency in learners.	Learners show strategic competence.	There is evidence of adaptive reasoning in learners.	There is evidence of productive disposition in learners.

Comments:_____

LANGUAGE USE

Learners-learner interaction without the teacher

1	2	3	4
Learners do not question each other or probe for details,	Learners question each other and discuss privately.	Learners only question or help other learners when	Learners freely enter into discussions with

they do not have discussions with each other.		prompted to do so by the teacher.	each other.

Comment: _____

_____

_____

_____

Teacher-learner interaction.

1	2	3
Totally controlled by the teacher	Predominantly controlled by the teacher.	Control of interaction shifts between learners and teacher.

Comment: _____

_____

_____

_____

Use of languages in public domain (both English and home language).

1	2	3	4
Predominantly main language is used by both teacher and learners.	Predominantly English spoken by learners and a teacher	Only the teacher code-switches for a range of purposes	Teacher and learners code-switch for range of purposes.

Comments: _____

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