Appendix Q

SAIDE Open Educational Resources Project

Teaching and Learning Mathematics in Diverse Classrooms

Unit Five

Building assessment into teaching and learning

PILOT VERSION

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Draft pilot edition, 2007: Teaching and Learning Mathematics in Diverse Classrooms

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Introduction to the module

This is the fifth unit of a six unit module entitled *Teaching and Learning Mathematics in Diverse Classrooms.*

The module is intended as a guide to teaching mathematics for in-service teachers in primary schools. It is informed by the inclusive education policy (Education White Paper 6 Special Needs Education, 2001) and supports teachers in dealing with the diversity of learners in South African classrooms.

In order to teach mathematics in South Africa today, teachers need an awareness of where we (the teachers and the learners) have come from as well as where we are going. Key questions are:

Where will the journey of mathematics education take our learners? How can we help them?

To help learners, we need to be able to answer a few key questions:

- What is mathematics? What is mathematics learning and teaching in South Africa about today?
- How does mathematical learning take place?
- How can we teach mathematics effectively, particularly in diverse classrooms?
- What is 'basic' in mathematics? What is the fundamental mathematical knowledge that all learners need, irrespective of the level of mathematics learning they will ultimately achieve?
- How do we assess mathematics learning most effectively?

These questions are important for all learning and teaching, but particularly for learning and teaching mathematics in diverse classrooms. In terms of the policy on inclusive education, all learners – whatever their barriers to learning or their particular circumstances in life – must learn mathematics.

The module is divided into six units, each of which addresses the above questions, from a different perspective. Although the units can be studied separately, they should be read together to provide comprehensive guidance in answering the above questions.

Unit 1: Exploring what it means to 'do' mathematics

This unit gives a historical background to mathematics education in South Africa, to outcomes-based education and to the national curriculum statement for mathematics. The traditional approach to teaching mathematics is then contrasted with an approach to teaching mathematics that focuses on 'doing' mathematics, and mathematics as a science of pattern and order, in which learners actively explore mathematical ideas in a conducive classroom environment.

Unit 2: Developing understanding in mathematics

In this unit, the theoretical basis for teaching mathematics – constructivism – is explored. A variety of teaching strategies based on constructivist understandings of how learning best takes place are described.

Unit 3: Teaching through problem solving

In this unit, the shift from the rule-based, teaching by telling approach to a problem-solving approach to mathematics teaching is explained and illustrated with numerous mathematics examples.

Unit 4: Planning in the problem-based classroom

In addition to outlining a step-by-step approach for a problem-based lesson, this unit looks at the role of group work and co-operative learning in the mathematics class, as well as the role of practice in problem-based mathematics classes.

Unit 5: Building assessment into teaching and learning

This unit explores outcomes-based assessment of mathematics in terms of five main questions – Why assess? (the purposes of assessment); What to assess? (achievement of outcomes, but also understanding, reasoning and problem-solving ability); How to assess? (methods, tools and techniques); How to interpret the results of assessment? (the importance of criteria and rubics for outcomes-based assessment) ; and How to report on assessment? (developing meaningful report cards).

Unit 6: Teaching all children mathematics

This unit explores the implications of the fundamental assumption in this module – that ALL children can learn mathematics, whatever their background or language or sex, and regardless of learning disabilities they may have. It gives practical guidance on how teachers can adapt their lessons according to the specific needs of their learners.

Process of developing the module

The units in this module were adapted from a module entitled *Learning and Teaching of Intermediate and Senior Mathematics*, produced in 2006 as one of the study guide for UNISA's Advanced Certificate in Education programme. The original guide was based on the following textbook:

Van de Walle, JA (2004). *Elementary and middle school mathematics: teaching developmentally.* New Jersey: Pearson Education.

A team of mathematics educators collaborated in the adaptation of the module so that issues related to inclusive education (the teaching of diverse learners), as well as a more representative selection of 'basic' mathematical knowledge could be included. In addition, to avoid the need to purchase the van der Walle textbook, the adapted version summarises relevant excerpts, rather than simply referring to them.

The team of mathematics educators consisted of the following:

- Constance Babane (University of Limpopo)
- Sam Kaheru / Nicholas Muthambi (University of Venda)
- Norman Khwanda (Central University of Technology)
- Marinda van Zyl / Lonnie King (Nelson Mandela Metropolitan University)
- Sharon Mc Auliffe / Edward Chantler / Esmee Schmitt (Cape Peninsula University of Technology)
- Ronel Paulsen / Barbara Posthuma (University of South Africa)
- Tom Penlington (Rumep at Rhodes University)
- Thelma Rosenberg (University of KwaZulu-Natal)
- Ingrid Sapire (Radmaste at University of the Witwatersrand)

Permissions

Permission has been granted from UNISA to adapt the following study guide for this module:

UNISA. (2006). *Learning and teaching of Intermediate and Senior Mathematics (ACE ME1-C)* (Pretoria, UNISA)

Permission has also been sought for the additional materials included in the various units specified below.

Unit 1

RADMASTE Centre, University of the Witwatersrand (2006). Chapters 1 and 2, *Mathematical Reasoning* (EDUC 263).

UNISA (2006). Study Units 1 and 2 of *Learning and Teaching of Intermediate and Senior Phase Mathematics.*

RADMASTE Centre, University of the Witwatersrand (2006). *Number Algebra and Pattern* (EDUC 264).

Stoker, J. (2001). *Patterns and Functions*. ACE Lecture Notes. RUMEP, Rhodes University, Grahamstown.

Unit 2

UNISA (2006). Study Unit 3: *Learning and Teaching of Intermediate and Senior Phase Mathematics.*

Penlington, T. (2000). *The four basic operations.* ACE Lecture Notes. RUMEP, Rhodes University, Grahamstown.

RADMASTE Centre, University of the Witwatersrand (2006). *Number Algebra and Pattern* (EDUC 264).

Unit 3

UNISA (2006). Study Unit 4: *Learning and Teaching of Intermediate and Senior Phase Mathematics.*

Malati (1999). Geometry activities: Geometry Module 3: Representations (nets, models and cross sections), Grades 4 to 7 Learner Materials.

RADMASTE Centre, University of the Witwatersrand. (2006). Mathematical Reasoning (EDUC 263) Chapter 7.

Unit 4

UNISA (2006). Study Unit 5: *Learning and Teaching of Intermediate and Senior Phase Mathematics.*

RADMASTE Centre, University of the Witwatersrand (2006). *Mathematical Reasoning* (EDUC 263) Chapter 6.

Malati (1999). Geometry Module 3: Representations (nets, models and cross sections). Grades 4 to 7 Learner Materials.

Unit 5

UNISA (2006). Study Units 7 to 10: *Learning and Teaching of Intermediate and Senior Phase Mathematics.*

MM French (1979). Tutorials for Teachers in Training Book 7, SIZE, Oxford University Press, Cape Town.

RADMASTE Centre, University of the Witwatersrand (2005). *Data Handling and Probability* (EDUC 187) Chapters 3, 8 and 9.

Unit 6

UNISA (2006). Study Unit 6: Learning and Teaching of Intermediate and Senior Phase Mathematics.

University of the Witwatersrand (2006). Module 3 of the Advanced Certificate for Learner with Special Education Needs: Understanding Cognitive, Emotional and Motivational Differences in Development.

Department of Education (2005). Guidelines for Inclusive Learning Programmes. http://curriculum.wcape.school.za/resource_files/20091831_Guidelines_for_Curriculum_June_2005.doc.

Contents

Unit Five: Building assessment into teaching and learning

After working through this unit you should be able to:

- explain the term assessment
- identify the four purposes of assessment
- explain the principles of outcomes-based assessment (OBA)
- describe the role and purpose of assessment in mathematics
- implement a variety of types of assessment in assessing your learners' performance in mathematics
- identify and explain the aspects of mathematics learning you ought to consider when assessing learners
- reflect on the assessment potential of mathematical tasks used in the teaching of basic data handling concepts
- select appropriate methods, techniques and tools for assessing a learner's performance in mathematics
- draw up or design your own assessment tasks and rubrics to be used when assessing a learner's work
- compare various methods of recording a learner's performance

5.1 Introduction

In our discussion of the term '**assessment**', we will take into account the perspectives embodied in the principles of outcomes-based education (OBE), as implemented in South Africa. In this unit we analyse the purposes of assessment and give an overview of the main types of assessment and their use or function in classroom practice within the framework of OBE.

Assessment occupies a central place in education and especially in the mathematics curriculum. When assessment is done well, it empowers everyone because it:

- informs learners about what they have learned, what they have still to learn and how best to learn it;
- informs teachers about how to instruct or teach more effectively;
- informs parents about how best to support their child's learning.

When done poorly, however, assessment can lead to a misrepresentation of learning outcomes and thereby result in superficial teaching and learning. Thus, **assessment should be an integral part of teaching and learning** which functions as a quality assurance mechanism to ensure good teaching and learning practice.

The idea that assessment can and should contribute constructively to the curriculum has led to some debate and controversy about the nature, role, importance and the place of assessment in education. One view is that there is a need for new assessment practices to complement more traditional, widely used techniques. These new assessment practices ought to

- take into account the current curriculum, content and goals
- inform teaching initiatives in terms of achieving outcomes
- comply with national and institutional policies.

Pegg (2002:227) adds an additional component to assessment by arguing that an empirically-based theoretical perspective must be taken into consideration when planning assessment. He further states that assessment issues that are always sensitive to the learner's cognitive development should underpin assessment initiatives. To realise the positive potential of assessment in our classrooms, we need a clear idea of

- why we are doing assessment in the first place
- what it is we are assessing
- how best to go about it.

After reading this unit you will be aware that assessment is more than a set of tests or assignments. Assessment has a purpose and we need to establish the purpose of assessment in order to design an appropriate assessment programme that will enable us to achieve our goals. This unit will elaborate on:

- how the purpose of assessment has changed in the new curriculum
- the four main purposes of assessment in an educational system.

We will give an illustration of baseline assessment tests used to establish the readiness of learners to measure items using standard units of measurement.

All of the material in the earlier units of this guide has suggested that teaching in accordance with the NCS will result in learner centred teaching. This style of teaching will assist learners to develop their reasoning skills and their ability to solve mathematical problems both in and out of real contexts. The diverse classes that many teachers have to face will also benefit greatly from learner centred teaching, which will be able to address individual needs where appropriate. Assessment which is not in line with good teaching methods could undermine the value and benefits of that teaching. It is thus essential that the assessment approach you use should support your teaching methods.

Much of the mathematical content used to illustrate and work with the assessment ideas put forward in this unit will come from LO5 (Data Handling). This will give you the opportunity to think about setting tasks that support sound mathematical

teaching. We will look at the difference between assessment methods, techniques, and skills. Most importantly we will show that you must relate the purpose of the assessment with what is being assessed. You need to think about what, how and why you assess, how you interpret the results of the assessment and how you will respond to the learners and engage stakeholders in the process.

The following quotation from *Assessing Students: How shall we know them?* (Derek Rowntree, 1997, Kogan Page, p.11) will serve as a framework for this unit.

1. *Why assess?*
   Deciding why assessment is to be carried out; what effects or outcomes it is expected to produce.

2. *What to assess?*
   Deciding, realising, or otherwise coming to an awareness of what one is looking for, or remarking upon, in the people one is assessing.

3. *How to assess?*
   Selecting, from among all the means we have at our disposal for learning about people, those that we regard as being most truthful and fair for various sorts of valued knowledge.

4. *How to interpret?*
   Making sense of the outcomes of whatever observations or measurements of impressions we gather through whatever means we employ; explaining, appreciating, and attaching meaning to the raw 'events' of assessment.

5. *How to respond?*
   Finding appropriate ways of expressing our response to whatever has been assessed and of communicating it to the person concerned (and other people).

Rowntree's points are very much in line with the RNCS for Mathematics for grades R-9, (DOE 2002:93), which states that

> Assessment is a continuous, planned process of gathering information about the performance of learners measured against the Assessment Standards (ASs) of the Learning Outcomes (LOs). It requires clearly defined criteria and a variety of appropriate strategies to enable teachers to give constructive feedback to learners and to report to parents and other interested people.

As this unit unfolds, it will assist you in meaningfully interpreting and applying this RNCS definition of assessment.

5.2 Why assess?

Assessment is an integral part of our daily lives. Every time we have to make a decision, we have to, for example, assess, **value, judge, estimate, appraise**, the situation first before we can go any further. To assess means to measure.

Activity 1

For this activity you need not think of assessment in the context of a mathematics class. Think of it broadly as you experience it or have seen others experience it in everyday life.

1) What ideas about assessment come to mind in your personal experience? Write down some of the ideas.
2) Reflect on the uses of assessment in everyday life. Write down some of the uses that you think of and the impact they have on the individuals being assessed.
3) Discuss what each of the following ideas signify to you in the context of assessment:

evaluate	appraise	estimate	competition
tests	examinations	fail	pass
study	poor marks	stress	worried
motor-vehicle test	doctor's report	sports coaching success	

The purposes of assessment in outcomes-based education

Your own experience of assessment will have an influence on the way in which you set about assessing your learners. This experience may or may not have features common to those we would like to see emerging in OBA (outcomes-based assessment). The key features of OBA are the following:

- **Outcomes-based assessment** (OBA) should assist learners to reach their full potential - it should be developmental rather than judgmental.
- Teachers should assess learners to determine what they **know, understand, can apply** and **can do**, and to provide learning experiences that lead to higher levels of performance by learners.
- OBA should involve learners actively in using the relevant knowledge in real-life contexts to make sense of the world and to construct meaningful links between the skills, knowledge, concepts, processes, attitudes and values (outcomes) covered. This principle is embodied in the NCS in its **critical outcomes**, from which the learning outcomes and assessment standards were developed.
- OBA should be **integrated** into the **teaching and learning process**. Effective assessment and recording comes from integrating assessment planning into curriculum planning, phase and programme planning and classroom activity plans (lesson plans).

The four purposes of assessment given below call for assessment methods and techniques that are varied to suit the purpose of the assessment.

Baseline assessment is used to measure learners' existing ideas, knowledge, experiences and skills. It is used at the beginning of a new set of learning activities to determine what the learner already knows (i.e. recognition of prior learning [RPL]). It is used to assist in determining what levels of demands to build into the learning plan. Baseline assessment is not always formally recorded, but occasionally takes the form of standardised tests such as 'readiness tests' for school beginners. It is used to enable teachers to identify learners with special needs early on in the learning programme. This assessment can be based on teacher judgment or objective assessment, for example the teacher develops tests to determine the range of ability amongst learners. Another aspect of baseline assessment is that it can involve the interface between teacher and parents when collecting information or when providing feedback.

Formative assessment is assessment of learning which results in process evaluation. It allows teachers to inform the learner and to plan future learning. Formative assessment should involve a developmental approach. It can be formal or informal. It is designed to monitor and support learning progress, and should occur throughout the learning process. Formative assessment involves both the teacher and learner in a process of continual reflection and reviewing of progress, and helps the teacher to determine the learners' strengths and needs. It provides quality feedback to empower learners to take appropriate action and allows teachers to adjust the learning/instruction process according to the response to formative assessment. As such has it has the potential to have the greatest impact on learning. It can be done by the teacher, learner, peer group and parents. The quality of formative assessment is determined by evaluative feedback in order to achieve improvement. An accumulation of formative assessment tasks can be used for summative purposes.

Your experience of assessment may have been very much test-based. Tests can be formative, if the teacher can use the tests to analyse where learners are and provide specific, focused feedback to the learners based on the tests. The teacher should set frequent short tests rather than infrequent long tests, and guidance can be provided. If more serious or extensive learning problems are diagnosed (through formative assessment) these should be addressed. Formative assessment should test new learning soon after the lesson/theme/topic has been completed.

Diagnostic assessment is specifically focused on uncovering the nature and cause of a learning problem and providing appropriate support and guidance. It can guide the teacher so that appropriate support can be given and would determine whether specialist advice and support should be requested.

Summative assessment is assessment of learning in a holistic context. It is carried out at the end of a learning programme, unit, theme, term, year or grade. Summative assessment provides an overview of the learner's overall progress. It allows the teacher to make a judgement about the learner's performance in relation to a particular standard. It provides valuable data records that can be used for various management purposes (e.g. transfer to other school, scholastic progress for further studies, information for selection and certification purposes etc). Summative assessment also determines how well a learner has progressed towards the achievement of specific outcomes and in this way facilitates feedback to the learner.

Summative assessment should encompass a series of assessment activities which result in an overall report on the performance of the learner. It is usually done by the teacher or specialist. It is reasonably formal, for example tests and examinations are often the means for performing summative assessment.

Assessment in OBE quite naturally takes the form of what is known as continuous assessment (CASS).

Continuous assessment

Activity 2

1) Why do you think that assessment for OBE 'quite naturally' takes the form of continuous assessment?
2) Is continuous assessment the same as formative assessment?
3) Can continuous assessment be used summatively? Can the results of various tests and assignments and investigations in the course of the year be added to create a global assessment of whether or not the learner has achieved the required set of outcomes at the required level?

Continuous assessment is 'natural' for OBE, because the philosophy of OBE is that learners are on a journey of lifelong learning. As they journey, they will learn the values, attitudes, skills and knowledge that they need to achieve the goals they set for themselves in life. Learners will not only learn things at school. The outcomes, set out in the curriculum documents, speak about certain values, attitudes, skills and knowledge that learners can achieve through the schooling they will receive. It is the responsibility of teachers to ensure that they do indeed achieve the outcomes, as they have been expanded on, grade by grade, as they progress through school.

It is in the nature of outcomes that they will be achieved at different rates by different learners according to factors such as the situation they find themselves in and their individual ability. This achievement will be facilitated by ongoing – **formative** – feedback, which can be given if assessment is carried out on a continuous basis, using a variety of methods. Learners should be given several opportunities to show that they are progressing in the achievement of the learning outcomes. A policy of continuous assessment facilitates the formative use of assessment.

To rely on a final high stakes test/examination at the end of a learning process may not allow all learners to demonstrate the range of skills they have developed, and the stages of their progression. Continuous assessment gives you the opportunity to vary the kind of assessment you are using because you assess the learners a number of times and in different ways. The results of all these ways can be used in the final (summative) assessment of learners' achievement.

In summary, continuous assessment embraces all four of the purposes of assessment described above – baseline, diagnostic, formative and summative – and supports the philosophy of OBE.

When to assess

The simple answer to the question, 'When should assessment take place?' is that assessment should be ongoing and continuous. But the purpose of the assessment at different points in teaching and learning will be different.

Assessment before instruction

Before beginning a unit of study one has to determine what the learners already know about what is to be taught, and inform them of how they will benefit from what they are about to learn, and what it is that will be of benefit to them. Secondly one has to determine the prior knowledge that the learner has already acquired in relation to the current topic.

Assessment during instruction

During the lesson, one should do 'in-process assessment'' (also known as 'feedback'' or 'formative assessment''). This is important as it provides information on the learner's progress on an ongoing basis. It also indicates to teachers and learners what outcomes have or have not been attained and is used in order to plan follow-up instruction. Performance indicators or assessment standards provide criteria for making a judgment about learning and are used as benchmarks in assessing achievement. The following three levels of achievement may be evaluated during this process:

- Learners show no evidence of having an understanding or skill.
- Learners are developing an understanding or skill.
- Learners have developed an understanding or skill.

Assessment after instruction

During this phase, assessment is undertaken at specified times after instruction has taken place. The learners' achievements are then communicated to them, their parents and the school personnel. This type of assessment can be classified as **summative assessment** since it provides information about the achievements of outcomes that can be compared with the assessment standards. Sources of obtaining summative data include in-process assessment, portfolios, assignments individual or group projects, tests, etc. New directions in teaching mathematics (as in OBE) rely less on norm-referenced evaluation and more on outcomes that focus on all learners attaining important mathematical knowledge.

An example of assessment before instruction

An interesting example of the way assessment can be used **before instruction** is found in the conservation tests of the Swiss educational psychologist, Jean Piaget. In this section we refer to the tests he devised to assess a learner's progress in acquiring fundamental measurement concepts. He stated that unless a learner has a clear concept of conservation (constancy or invariance) of length, area, or volume, it is no use teaching him about how to measure these things. The tests he developed (called

conservation tests) were diagnostic tests to be carried out before the teacher proceeds with teaching learners how to use measuring instruments.

Measurement is important because of its power to help learners see that mathematics is useful in everyday life. Learners need to be able to differentiate between perimeter and area. They should also be able to compute the area and perimeter of polygons and other shapes. The topic of measurement can be used extensively for problem solving through multiple entry points to accommodate the diversity of learners in a classroom. Multiple entry measurement activities will encourage learners to investigate and to use their own level of reasoning and understanding. They also allow for differentiation, both in the completion of the activity and in the assessment of the activity. This is very useful to teachers working in diverse classrooms.

Even an adult's concept of amount can be faulty. For example, sometimes as adults we are surprised to find that our coffee mug does not hold more coffee than an ordinary tea-cup. We think that, simply because it is taller, it must hold more. Piaget found that concepts are attained in a particular order, and gave approximate ages for their attainment. However, some learners are very late in forming concepts, so you should not assume that an 11-year old has attained the concept of conservation of area, for example. You have to test for the achievement of the concept, which can be done very quickly as you will see below.

Before we teach learners to measure an amount, they should have attained the concept of conservation (invariance) of that amount, even when the appearance has been distorted. The attainment of the concept is a matter of development and a learner is not ready to measure an amount he or she believes it to be bigger at one time and smaller at another.

The tests proceed as follows, and the teacher must be very careful not to ask leading questions which could prompt the correct answer.

1. The learner looks at equal amounts, A and B. He is asked which is more, A or B. The learner states that A and B are the same.
2. The appearance of the thing with amount B is distorted. The learner is asked if amounts A and B are the same, or different; (and not the suggestive question 'Which is more?')
3. If the learner says that the amounts A and B are still the same, distort further.

   If the learner continues to say A and B are the same, the learner has attained the conservation concept.

   If the learner can explain that amount B is the same as amount A because you can restore the thing with amount B to its original appearance, he or she has the concept of reversibility.

4. If the learner says amounts A and B are not the same, ask which is more.

   The learner believes what he or she says; it is the learner's truth at this stage of his or her development. Reasoning will not persuade the learner differently if he or she has not attained the concept of conservation of this quantity. If this is the

case, the learner needs to do more practical work comparing quantities, until the concept is understood.

Following are illustrations of the conservation tests for length, area and volume (French: 1979).

Conservation of length

Test 1

Materials: 2 unsharpened pencils of the same colour.

Give these to the learner and ask if the pencils are the same length. If the learner says yes, place one pencil to the left of the other and ask again if the pencils are the same length.If he or she says yes, change the position of one of the pencils

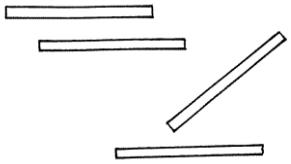
and repeat the question.

Try this with the pencils in several positions.

Ask the learner why he or she is sure.

If the learner says no, ask him or her to point to the longer one.

Repeat the test with other objects of equal length.



Test 2

Materials: 2 pieces of string of equal length.

Follow the procedure in Test 1.

One piece of string can be curved, or cut.

Conservation of area

Test 1

Materials: 2 postcards of the same size, on the table.

Ask the learner if the postcards cover exactly the same area of the table surface. If the learner says yes, tear one postcard in two, and place both pieces, moved apart from each other, on the table. Ask the learner whether the two pieces cover the same area of the table as the whole card covered.

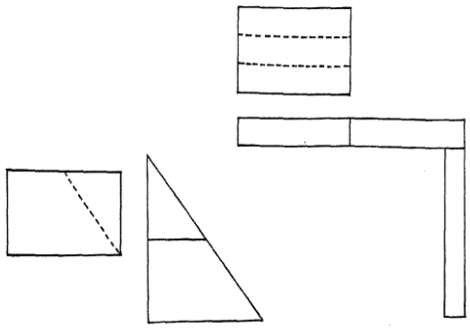
If the learner says yes, tear the each of the two pieces of card in half and place all four pieces, apart from each other, on the table and repeat the question.

If the answer is yes, tear the four pieces in half, so that there are eight pieces, and repeat the question. Ask the learner why she or she is sure of the answer.

Test 2

Follow the procedure in Test 1.

Keep one postcard for comparison, while other postcards are cut up and the pieces placed together to form new shapes.



Test 3

Materials: 2 congruent sheets of paper (green if possible); at least a dozen congruent cubes; 2 toy horses.

Place one horse on each sheet of paper, which represents a field of grass. Ask: Has each horse the same amount of grazing?

Each farmer begins to build. (Place one cube on each sheet of paper.) Ask: Has each horse the same amount of grazing now?

Ask this question as another building, and another, and another … is put up. (Place the cubes close together on one sheet of paper, and scatter them over the second.)

If the learner states each time that the horses have the same amount of grazing, ask why he or she is sure. He or she may be able to explain that each learner started with the same amount of grass, that they have built the same number of buildings, and therefore have the same grass left. If this is the case, the learner has attained the concept of conservation of area.

If the learner says no at any stage, provide further experience to develop the concept.

Conservation of volume (and capacity)

Test 1

Materials: 2 congruent balls of plasticine (equal volumes).

Let the learner see that these have the same volume.

Roll one ball into another shape, such as a long 'snake'.

Ask the learner if the two shapes have the same amount of plasticine.

If the learner says yes, break up the 'snake' into several small balls and repeat the question.

If the learner says yes, ask why he or she is sure.

If the learner says no, further experience is necessary to develop the concept.

(This experiment can also be used to test for conservation of mass. Compare the original masses on a balance.)

Test 2

Materials: 2 similar glasses containing equal volumes of cold-drink, several other containers, wide and shallow, tall and thin.

The learner agrees that the two glasses contain the same amount of cold-drink. Pour the contents of one glass into another container and ask if it contains the same amount of cold-drink as the other glass. Follow the usual questioning procedure.

Test 3

Materials: At least 24 congruent cubes.

Make a shape using a certain number of cubes. Ask the learner to make a copy of the shape. Ask weather your shape or the learner's shape takes up (or fills) the same amount of space. If the learner says yes, rearrange your cubes to form a different shape, and repeat the question.

Continue questioning, making other shapes, and using more cubes.

Activity 3

How would you use the conservation tests in the box above in your teaching of measurement in the intermediate phase?

Describe a lesson where you use one (or more) of the conservation tests.

1) Write down your observations on learners who have achieved an understanding of conservation of the concept you chose.
2) Write down your observations on learners who have NOT achieved an understanding of conservation of the concept you chose.
3) What will you do to help the learners who have NOT achieved an understanding of conservation of the concept you chose?

As we have said above, 'it is in the nature of outcomes that they will be achieved at different rates by different learners according to factors such as the situation they find themselves in and their individual ability.' It is easy to say this in theory, but more difficult in practice.

When it comes to teaching measurement, there are five stages that are usually followed and there needs to be assessment at each stage – to establish readiness to move to the next stage. It is important to know the purpose of each stage, so that you assess the correct thing.

Stage 1: Readiness

You won't be able to teach measurement at all if your learners do not have a grasp of the basic conservation concept. Piaget's conservation tests will help you assess readiness.

Stage 2: 'How much'

At this stage, learners practice measuring the size of a 'thing' using an arbitrary 'standard' (a piece of the same kind of 'thing' having an arbitrary unit size.) Learners should compare directly — they should have a 'standard' object, and they should count how many 'standard' objects make up the object whose size is being measured. They should be trained to give the measurement correct to the nearest unit.

Through incidental learning, the principle of measurement is established and the activity of measurement practised. At this stage, the result of the measurement is unimportant; it is the activity which counts.

Stage 3: The necessity for a widely accepted standard and unit is established

At this stage, a mental image of the unit size is established by estimation and then by measurement.

Learners love measuring, especially 'themselves' — their heights, their masses, length of feet, circumference of waists! You can play around with the idea of a standard and unit by, for example, using their (small) hands to measure something, and your (larger) hand to measure something – and then seeing how different the measurement results are.

Stage 4: Measurement, followed by computation

Here you get to down to the business of measurement and calculation by, for example, finding the mass of one brick and then calculating the mass of 10 bricks. Unless you have gone through the previous stages, learners will simply do calculations mechanically, without understanding.

Stage 5: Computation with physical quantities

Finally, all units and conversions between units can be used.

5.3 What to assess?

We have already established in the earlier units of this guide that outcomes-based education (OBE) is an approach to teaching and learning which stresses what learners are expected to achieve. In OBE, the teacher states in the beginning the performance expected of the learners - this is an outcome. The teacher's task is to teach in order to support or help the learners to achieve the outcomes stated in the curriculum, and the learners' task is to achieve the expected outcomes. Assessment is essential to OBE because

- The teacher is able to measure to what extent a learner has achieved each outcome;
- Curriculum-development requires assessment to control whether the curriculum is being implemented successfully.

Assessing outcomes

We now need to think more specifically about what to assess in our mathematics classes. Within the framework of the RNCS, the Critical Outcomes (COs), Developmental Outcomes (DOs), Learning Outcomes (LOs) and Assessment Standards (ASs) serve as the basis for assessment as follows:

- **The Critical Outcomes (COs)** are a list of outcomes that are derived from the Constitution and are contained in the South African Qualifications Act of 1995. They describe the kind of citizen that the education and training system should aim to create. There are seven critical outcomes. The COs give the goals (outcomes) we are all in the process of working towards in the process of life-long learning.
- **The Developmental Outcomes (DOs)** are a list of outcomes that are derived from the Critical Outcomes. There are five Developmental Outcomes. The DOs give more clarity on the steps towards the achievement of the COs in the learning process.
- **A Learning Outcome (LO)** is derived from a particular CO and is a description of what (knowledge, skills and values) learners should know, demonstrate and be able to do at the end of the General Education and Training (GET) Band. A set of LOs should ensure integration and progression

in the development of concepts, skills and values through the ASs. LOs do not prescribe content or method as stated in the overview to the RNCS (2002:14).

- **Assessment Standards (ASs)** describe the level at which learners should demonstrate their achievement of the particular LO and the ways (depth and breadth) of demonstrating their achievement. An AS is grade specific and shows how conceptual progression will occur. It embodies the knowledge, skills and values required to achieve a particular LO. An AS does not prescribe a specific method. According to Chisholm (2000:90), learning outcomes and assessment standards should be seen as covering the minimum of the core concepts, content and values that should be covered in each grade in each learning programme.

The Critical Outcomes and Developmental Outcomes

(The Overview, National Curriculum Statement Grades 10 – 12 (GENERAL), page 7)

OUTCOMES-BASED EDUCATION

Outcomes-based education (OBE) forms the foundation for the curriculum in South Africa. It strives to enable all learners to reach their maximum learning potential by setting the Learning Outcomes to be achieved by the end of the education process. OBE encourages a learner-centred and activity-based approach to education. The National Curriculum Statement builds its Learning Outcomes for Grades 10 – 12 on the Critical and Developmental Outcomes that were inspired by the Constitution and developed through a democratic process.

The CRITICAL OUTCOMES require learners to be able to:

- Identify and solve problems and make decisions using critical and creative thinking;
- Work effectively with others as members of a team, group, organisation and community;
- Organise and manage themselves and their activities responsibly and effectively;
- Collect , analyse, organise and critically evaluate information;
- Communicate effectively using visual, symbolic and/or language skills in various modes;
- Use science and technology effectively and critically showing responsibility towards the environment and the health of others; and
- Demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

The DEVELOPMENTAL OUTCOMES require learners to be able to:

- Reflect on and explore a variety of strategies to learn more effectively;
- Participate as responsible citizens in the life of local, national and global communities;
- Be culturally and aesthetically sensitive across a range of social contexts;
- Explore education and career opportunities; and
- Develop entrepreneurial opportunities.

The outcomes given and referred to above may seem vague, but you will notice that they become more specific as you go down the levels. By expressing outcomes as actions and performances that embody and reflect learner competence in using ideas, information, content and skills, the curriculum emphasises that learners are able to do important things with what they know and are able to link it with real life experiences. This implies that you cannot assess learners unless they do something to reveal their understanding (or lack of it).

Thinking about outcomes can provide a teacher with a clear focus and structure for his or her lessons. Teachers should communicate those desired outcomes to learners - if they know what the teacher wants them to achieve, they will have a better chance of achieving it.

According to the RNCS for Mathematics for grades R-9, (DOE 2002:93)

> Assessment is a continuous, planned process of gathering information about the performance of learners measured against the Assessment Standards (ASs) of the Learning Outcomes (LOs). It requires clearly defined criteria and a variety of appropriate strategies to enable teachers to give constructive feedback to learners and to report to parents and other interested people.

Activity 4

The paragraph above from the RNCS gives concise but detailed guidance to the teacher about OBA. Give explanations of the following words or phrases taken from this definition. The explanations should expand on the meaning of the word/phrase in its context above.

Your explanations can draw on your experience as well as the information you have gathered from the material in this guide. For some of the ideas you may wish to look ahead to the end of this chapter.

1) continuous
2) planned
3) process of gathering information about the performance of learners
4) measured against the Assessment Standards (ASs) of the Learning Outcomes (LOs)
5) requires clearly defined criteria
6) variety of appropriate strategies
7) enable teachers to give constructive feedback to learners
8) enable teachers to report to parents and other interested people.

Assessing for reasoning, understanding, and problem-solving

The broad view of assessment that underpins the RNCS is seldom part of the assessment process in mathematics. Instead, assessment is often dominated by

- a focus on content (in the form of facts)
- a focus on skills (associated with computational techniques)
- the ability of learners to reproduce these on demand.

This narrow focus has had a sterile effect upon innovation and development in mathematics curricula and even on what thinking mathematically means. Learning programmes that provide for limited developmental work, that place an emphasis on

symbol manipulation and computational rules, and that rely heavily on paper-and-pencil worksheets do not fit in with the natural learning patterns of learners and do not contribute to some important aspects of learners' mathematical development.

Take the following worksheet on basic operations as an example.

Operations Worksheet

Use your slate/whiteboard/rough paper to do any working you need to do.

1) Calculate the following. Then give your answers:
   a) 34 + 56 =
   b) 27 + 18 =
   c) 40 + 26 =
   d) 15 + 69 =
   e) 18 + 72 =
   f) 38 + 43 =
   g) 29 + 39 =
   h) 59 + 46 =
   i) 37 + 27 =
   j) 69 + 13 =
2) Give answers to the following questions:
   a) 67 – 36 =
   b) 28 – 16 =
   c) 45 – 19 =
   d) 78 – 58 =
   e) 73 – 47 =
   f) 62 – 45 =
   g) 89 – 50 =
   h) 48 – 39 =
   i) 87 – 68 =
   j) 33 – 18 =

This worksheet simply has twenty repetitive addition and subtraction questions. There is no discussion on different methods that could be used, and no indication that working is worth recording. The ultimate purpose of doing this worksheet is to 'give answers'. This does not convey the message to the learners that their reasoning process is important to the teacher, only their answers are important. This goes against the ethos of teaching for meaningful understanding. Using a worksheet such as this one would undermine good teaching because it gives a message that contradicts what we have said about good teaching.

- It does not allow for the development of reasoning skills (there is not even one little question which could call on reasoning skills).
- It does not allow learners to show what they have understood (or more importantly NOT understood). If learners get the wrong answer, that is that. There can be no investigation as to where they went wrong, because only answers are being considered.

- Problem solving is not present here. The questions are not grouped (as they potentially could be) so that patterns could be identified as an aid to doing the calculations. Identifying patterns is an important problem solving technique that can be introduced at this level, since it is effective and appropriate as a strategy for learners to use when they move from simple bonds to addition and subtraction of bigger numbers.

The activities that you set should allow learners to develop the knowledge and skills set out in the Assessment Standards for their grade. The main lesson about good mathematics teaching is that it should develop reasoning, understanding, and problem solving.

Activity 5

Comment on the task below, by answering the questions that follow after it.

Pizza Problem

1. Complete the next two rows in the following pattern, which is known as Pascal's triangle.

				1						
			1		1					
		1		2		1				
	1		3		3		1			
1		4		6		4		1		

2. Pizza Palace has asked you to design a form to help them to keep track of certain pizza choices. The basic pizza, which everyone has, is cheese and tomato. Patrons can select extra toppings which they like. The toppings they can choose are: Peppers, Olives, Sausage, Salami, and/or Mushroom. They can choose as many of the extras as they like.
   - List all of the possible choices they have, given the above selection.
   - Find a way to convince somebody else (without doubt) that you have accounted for ALL possible choices.
   - How many different choices for pizza at the Pizza Palace does a customer have?

1) In what way does 'the pizza problem' allow learners to develop their reasoning skills?
   a) Write out an example of a question (or a part of a question) from the task that allows for the development of reasoning skills.
   b) What evidence of reasoning skills (or lack thereof) will the teacher be able to find in the learners' work that they submit having completed this task?
2) In what way does it allow learners to develop their understanding?
   a) Write out an example of a question (or a part of a question) that allows for the development of understanding.
   b) What evidence of understanding (or lack of understanding) will the teacher be able to find in the learners' work that they submit having completed this task?
3) In what way does it allow learners to develop their problem solving skills?
   a) Write out an example of a question (or a part of a question) that allows for the development of problem solving skills.
   b) What evidence of problem solving skills (or lack thereof) will the teacher be able to find in the learners' work that they submit having completed this task?

Although the quantitative (such as content and computational skills) aspects of mathematics learning have dominated the school scene for a long time, it is the qualitative considerations that have greater significance. Thus, it is far more important that learners come to understand mathematical ideas than that they possess a wide array of mathematical skills. Mathematical learning is not only dependent on learners' attitudes towards and decisions about learning mathematics but also depends on the intellectual, social and affective dimensions of learning, and these must all influence curriculum and instruction.

Reflecting on what to assess through the teaching of data-handling

We will now examine the use of assessment for different purposes in the context of LO 5: Data Handling. The material in Appendix A emphasises some of the fundamentals of data handling. It comes from the RADMASTE (2006) ACE materials.

Activity 6

Read and work through the mathematical information and five exercises on data handling in Appendix A. As you work through the examples, think about the different mathematical competences that are being developed and assessed in the activities.

Once you have worked through the main skills of data handling – collecting data, representing data, and interpreting data – and looked at the sample exercises that you could use with learners, consider what this means for assessment.

Activity 7

1) Reflect on each of exercises 1 to 5 by answering the following questions.
   Refer to a copy of the RNCS to find the appropriate curriculum references where necessary. Explain all of your answers.
   a) Which AS (from LO 5 Data Handling) do you think this exercise could address, and for which grades do you think it would be appropriate? [Some exercises could be used for several grades, while others could not.]
   b) Which of the questions in the exercise simply ask for memorisation?
   c) For the purposes of OBE/OBA are simple memorisation tasks useful, and if so where?
   d) Which of the questions in the exercise call for routine procedures that require no explanation or ambiguity?
   e) Which of the questions in the exercise call for procedures in order to develop deeper levels of understanding?
   f) Which of the questions in the exercise require the learner to analyse the question and think about the solution before proceeding?
2) Which exercises call for group activity or could be used for group work, even if this is not indicated?

Look back over all of the exercises and decide which of them deal with concepts and procedures, which of them deal with processes and which of them deal with the learners' disposition.

As a teacher, you need to be able to decide what aspects are being addressed in the different activities, exercises and assessment tasks that you set. When you plan your teaching programme, you need to ensure that you provide a good mix of activities for your learners. You will need to examine the text book that you use in your mathematics classes (especially if you make regular use of a text book) to satisfy yourself that it provides a balanced set of activities that address all of the things you need to teach and assess your learners on.

5.4 How to assess?

In this section we deal with the question of how to assess. Assessment is the process of collecting and interpreting evidence in order to make a judgment on a learner's achievement and competencies. Evidence can be collected at different times and places, by using various methods, tools and techniques.

Teachers select the methods, tools and techniques used on the basis of

- the purpose of assessment
- the specific learning area
- what the teacher wants to assess: knowledge, skills, values and attitudes

Assessment should make a meaningful contribution to learning and teaching. When we assess learners, we should convey to our learners that what matters most is not so much passing (success) or failing, but what can we learn from the past to improve future learning. Constructive assessment therefore requires a commitment to quality information and quality communication. Assessment must incorporate a sufficient range of methods, tools and techniques to meet the teacher's obligations. In particular, such assessment must attend to language, tools, level of sophistication, task type, context and communication mode. No single task or practice can adequately cover all these dimensions.

Let us now turn our attention to educational assessment. In educational assessment our objective is not simply to measure what learners have achieved, but to help them learn and achieve more. Educational assessment is part of the process of learning, not a separate process. If you restrict your view of assessment to tests, quizzes, projects, etc, you are missing the point that assessment can help learners to grow and that it can inform instruction.

Let us now systematically analyse the question: 'What is assessment?''

The definition of assessment as stated in the *Assessment standards for school mathematics* by the National Council of Teachers of Mathematics in the United States (NCTM) (1995:3) is: 'Assessment is defined as the process of gathering evidence about a student's knowledge of, ability to use, and disposition towards, mathematics and of making inferences from that evidence for a variety of purposes.'

The focus on gathering evidence and making inferences indicates that assessment is a process of uncovering what mathematics learners know and can do.

The phases of assessment

The assessment process can be explained in terms of four interrelated phases that highlight the principal points at which critical decisions need to be made in the assessment process.

The following are the four phases

1) Plan the assessment.
2) Gather the evidence.
3) Interpret the evidence.
4) Use the results.

In practice, the four phases outlined above are interrelated and the distinctions between them are blurred. Assessment does not always proceed in a neat, linear fashion. Each phase can be characterised by the decisions and actions taken within that particular phase.

Plan the assessment

The following are important during this phase:

- Assessment must be planned on the basis of the outcomes expected of the learner.
- The teacher must decide on the purpose served by the assessment.
- The teacher must decide on the methods that are going to be used for gathering and interpreting evidence.
- The teacher must decide on the criteria to be used for evaluating performance.

Gather the evidence

The teacher must decide on how

- activities and tasks are to be created or selected
- procedures are to be selected to engage learners in the activities

Interpret the evidence

The teacher must decide on how

- to determine quality of the evidence
- to infer an understanding of the performance from the evidence
- to apply the criteria appropriately
- to summarise the evaluation in terms of results

Use the results

The teacher must decide on how

- to report the results

- to make inferences from the results
- what action to take based on the inferences made

In this section we are looking at how the assessment will be done in the planning and gathering of evidence. When we begin to talk about assessment tools, we are also starting to answer the question of how to interpret the evidence, which will lead us into the discussion of criteria.

Assessment methods, tools and techniques

There is a wide range of assessment methods, tools and techniques available to teachers. In order to assist you to decide on which methods, tools and techniques are most appropriate and useful in a particular learning situation, you need to understand the various criteria in terms of best practice.

Most curriculum packages and teachers organise their lessons around mathematical content. When the mathematics curriculum is organised into content-specific topics, assessment is similarly structured to document appropriate performances within the range of appropriate performances for the different content categories. This can be restrictive and there is a need for mathematics curricula and assessment to represent a model of valued mathematical activity.

It must be remembered that most assessment questions in mathematics are content-specific and must therefore be open enough to allow most learners to show what they know and have grasped (e.g. problem-solving, reasoning, understanding, etc).

The choice of what assessment methods, tools and techniques to use is a subjective one, that is, it is

- unique to each teacher, grade and school
- dependent on the teacher's professional judgment
- dependent on the specific learning area.

The assessment methods chosen must take into account

- the learning outcomes (LOs) and the assessment standards (ASs) of the learning area
- the purpose of assessment.

Therefore, the teacher should consider using a variety of methods to allow the learners to demonstrate their abilities. The chosen methods, tools and techniques must provide a range of opportunities for learners to demonstrate their achievement of knowledge, skills, values and attitudes. In selecting and setting appropriate assessment activities, the teacher should ask the following questions:

- What concept, skill or knowledge am I trying to assess?
- What type of knowledge is being assessed - reasoning, memory or process?
- At what level should the learners be performing?

The chosen methods, tools and techniques must provide a range of opportunities for learners to demonstrate their achievement of knowledge, skills, values and attitudes. The following table gives a list of possibilities, but although it looks long, it is not fully inclusive of all of the possible methods, tools and techniques you may use as a teacher of mathematics.

METHODS

A method is a procedure you will follow to assess the learner.

(Who does the assessing? How?)

- Self-assessment
- Peer assessment
- Group assessment
- Interviews
- Conferencing
- Observation
- Oral questions and answers
- Textual questions and written answers
- Performance of prior learning (RPL)

TOOLS

A tool is any instrument that you use in your assessment method, for example an observation sheet.

(What records does the teacher keep?)

- Portfolios
- Observation sheets
- Worksheets
- Journals
- Questionnaires
- Cassettes
- Assessment grids/rubrics
- Exhibitions
- Photographs/videos
- Class lists
- Profiles
- Tests
- Examinations
- Written assignments

TECHNIQUES

A technique (skill) is a special way in which you use a method and tool.

(What evidence does the learner produce?)

- Project work
- Collage
- Research project
- Assignment

- Survey
- Debate/argument
- Role-play
- Interview
- Drama
- Presentation
- Panel discussion
- Practical demonstration
- Scenario
- Constructions
- Music/songs
- Poetry/rhymes
- Story telling/oral presentation
- Model making/plans
- Designs, e.g. toys
- Sculptures/paintings
- Drawing/graphs
- Mind mapping
- Game design
- Physical activities
- Maps
- Posters
- Charts
- Tables
- Descriptions
- Written presentations e.g. reports, essays
- Posing questions

Methods of assessment

The most important thing to consider when choosing an assessment method is to link the method with what you are intending to assess and why you want to assess it. In this section, we give examples of a number of methods of assessment and ask you to reflect on whether they are 'fit for purpose'.

Self assessment:

Here is an example of a self-assessment form.

Self assessment form

Name: ..... Class: .....

Teacher: ..... Date: .....

Learning area: .....

Write down the two most important things you have learned in mathematics during the past month. .....

What would you like more help with?.....

How would you best describe how you feel in mathematics classes at the moment? (Circle the words that apply to you.)

(a) interested	(b) relaxed	(c) worried
(d) successful	(e) confused	(f) clever
(g) happy	(h) bored	(i) rushed

What is the biggest problem that you are currently facing in mathematics?

.....

(adapted from Clarke: 1997)

Activity 8

1) Which of the following **purposes** are appropriate for self assessment? Circle the ones that you think are important. You should be able to explain your choices.
    a) Learners are guided to assess their own performance or work.
    b) Self-assessment encourages learners to assume more responsibility for their own learning and work.
    c) Self-assessment helps learners to think critically about their own work.
    d) Self-assessment gives learners a good idea of their progress.
    e) Teachers find out what learners value in their own work.
    f) Self assessment promotes the development of independent learners who will be well placed to access learning opportunities throughout their lives.
    g) Learners come to appreciate and know their own work.
    h) Self-assessment develops learners' confidence.
    i) Learners become aware of their strengths and weaknesses.
2) When would you use self assessment in your mathematics class?
3) How would you use the results of self assessment from a mathematics lesson?

Peer or group assessment

Here is an example of a peer assessment form.

Peer or group assessment form

Name of peer/ group members: .....

Date: .....

Name of assessor: .....

Grade: .....

Task: .....

Competence (criteria)	Yes	No	Uncertain
Did the learner work in the team? Did the learner listen to the peer group discussion? Did the learner work with confidence? Did the learner in the group show competence in his or her logical thought processes to formulate, test and justify? Does the learner know and use mathematical language? Can the learner read the data accurately?			

Activity 9

1) Who does this form of assessment benefit?
2) In what way does the assessment benefit them?
3) What different types of group assessment have you used in your mathematics classes? Which of these were the most successful and why do you think this was the case?

Observation

Here is an example of an observation sheet.

Observation sheet for Grade 4 lesson on data handling			
Name of pupil:	Date:		
Class:	Teacher:		
Criteria	*Not yet*	*Fine*	*Comment*
1) Takes time to read and understand the work before beginning to complete the task. 2) Able to read the data from the table and draw up the tally table. 3) Able to find the frequencies using the tally table. 4) Able to draw the bar graph using tabled information. 5) Bar graph completed with the axes correctly marked and labelled. 6) Bar graph given the correct title. 7) Able to summarise the information presented and interpret the findings correctly. 8) Able to justify or explain work when questioned.			

Activity 10

1) Which of the criteria in the observation sheet above relate to content issues?
2) Which of the criteria in the observation sheet above relate to problem solving skills?
3) What are some of the valuable contributions that observation can make to assessment?
4) How could you adapt the observation sheet above to make it possible to use the sheet for a whole class? Draw up the sheet with the names of the learners in your class.
5) Compare the usefulness of an individual observation sheet with a whole class observation sheet indicating strengths and weaknesses of both types of sheet.

Performance based tasks

Performance based tasks should provide the opportunity for all learners to demonstrate at least some knowledge or skill, though this might be at different levels (Van de Walle, 2003).

Performance based assessment looks at the learners' ability to use their knowledge and skills to produce things like:

- presentations
- research projects

- investigations
- demonstrations or exhibitions
- drawings or graphs
- games or designs
- models.

This type of assessment requires learners to demonstrate a skill or proficiency in creating or doing something, often in a setting that involves real-life applications. The teacher should not only assess the end product but also the process used by the learners to complete the task. This method of assessment is very useful in diverse classrooms, since it takes into consideration the process as well as the product. The teacher needs to keep careful records of observations made during the completion of the task, so that the assessment is not simply reduced to the assessment of the final product.

Performance based task

You are going to find out about the different forms of transport learners use to come to school.

Draw up a list of questions that you can use to find out which different forms of transport are used by the learners in your class. (You should include at least four different means of transport on your list.)

1) Ask all of the learners in your class which form of transport they use, and draw up a tally table of their responses.
2) Find the frequencies from your tally table.
3) Represent your data using a bar graph or a pictogram.
4) What does your graph tell you about the forms of transport used by the learners in your class?
5) Could you use this information to decide about the forms of transport used by the learners in your school? Explain your answer.

Activity 11

Use the following check list to evaluate the design of the performance based task above.. In each case explain your answer and suggest improvements where necessary.

1) Does the task enquire an integration of knowledge, skills and values?
2) Does it require a variety of outcomes?
3) Is the task based on a real-life context?
4) Is the task practical enough to be done? (doable)
5) Are multiple solutions possible?
6) Is the nature of the task clear?
7) Is the task challenging and stimulating to the learners?
8) Are criteria for scoring included?

(DOE: Curriculum 2005 Assessment Guidelines)

Tools for assessment

A tool is any instrument that you use in your assessment, such as

- a portfolio
- an observation sheet
- a journal
- a questionnaire
- a test.

In this section we will only discuss portfolios, though you may wish to research some of the other tools mentioned in the list above.

Portfolios

A portfolio can be described as a collection of samples of a learner's work that shows how the learner has developed over time through the learning process. In mathematics, portfolios offer learners the opportunity to demonstrate the evolution of their mathematical knowledge and performance over particular period of time. During this process of collecting items, learners make decisions about what items to put in their portfolios, and it is this decision-making process that builds learner involvement.

Portfolio collections may include input by

- learners
- teachers
- parent/s
- peers
- the school
- possibly, the departmental official.

Portfolios should reflect the learner's actual day-to-day learning activities. They should be a measure of where the learners were and what they have accomplished. Portfolios should be ongoing to show-case the learner's efforts, progress and achievements over time. The selected works may be in a variety of media and multidimensional.

In their document on assessment standards, the National Council of Teachers of Mathematics suggests that mathematics portfolios be organised in exhibits that have focused purposes and clear criteria for judgement.

Here is an example (Clarke:1997):

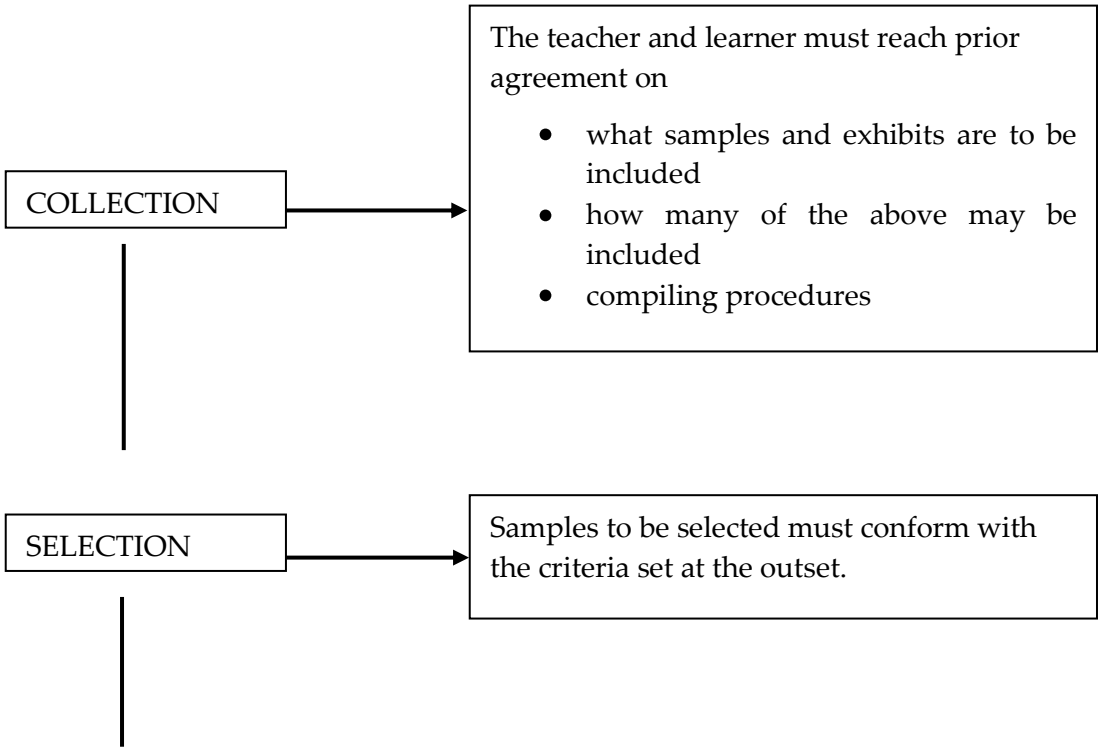
Exhibit	Purpose and criteria
Conceptual understanding	Use conceptual understanding to solve problems and represent the concept in multiple ways (through numbers, graphs, symbols, diagrams or words).
Problem solving	Using mathematical concepts and skills to solve non-routine problems that do not lay out specific and detailed steps to follow.
Putting mathematics to work	Type of investigations: data study design a physical structure pure mathematics investigation
Skills and communication	Lists of skills and communications are presented on the entry slips.

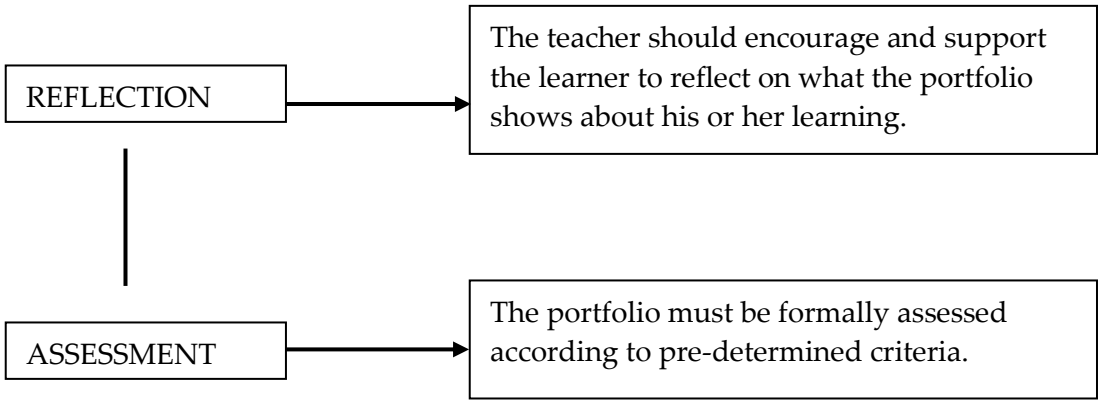
Assembling a portfolio

Learners must be guided on how to review all the work done so far and then select a few examples of the best work for inclusion in the portfolio. Before including a particular document (example), the learner should ask the following questions:

- What is this piece about?
- What makes this piece representative of my best work?
- What mathematics did I learn or apply in this piece?

Stages in the creation and design of a portfolio





The benefits and value of using portfolios for assessment

The portfolio offers

- a broader, more in-depth look at what learners know and can do
- the opportunity to asses more 'authentic' work
- a supplement or alternative to report cards and formal tests
- a better way to communicate learner progress to parents.

How do you assess a portfolio?

The following questions can be used to guide you in assessing a portfolio:

- Does the portfolio show the learner's growth and development over time?
- Does the portfolio show evidence of the process of planning and creating, as well as the final product?
- Does the portfolio show evidence of thoughtful reflection by the learner on his or her learning and achievement?
- Does the portfolio show collected pieces of work of an acceptable quality?
- Is the portfolio varied to show achievement through differing pieces of work?
- Does the portfolio communicate the learner's achievements clearly, effectively and convincingly?

Sample of a portfolio assessment sheet *(Adapted from Kramer: 1999)*				
Name: ..... Class: ..... Due Date: ..... Teacher: ..... Portfolio outcome:				
Assessment criteria	**Very good**	**Good**	**Satis-factory**	**Poor**
Evidence of mastery of the concepts				
Evidence of process: Shows the thinking, planning and process that lead to the final product				
Presentation of the portfolio: Neatness, visual appeal, creative design and layout				
Management: Sufficient number of samples, each dated and annotated, presented in sequence				
Writing skill: Appropriateness of language, vocabulary, style				
Variety and quality of contents: various work samples, clear evidence of competence				
Self-reflection: Honest personal commentary, highlighting of areas of excellence and issues for improvement				
Explanation: Justifies and explains each sample included				

The assessment sheet above is a simple rubric with criteria in the first column, and then four columns in which to indicate the standard of the work achieved by the learner in relation to each of the criteria.

Activity 12

1) Take some time to reflect on the purpose of a portfolio in assessment.
2) The rubric above would be even more useful to the learners if the criteria were explained in more detail in each of the columns of the table. Copy the rubric onto a sheet of paper and write in the detail so that a learner would know exactly what was expected of him/her in order to achieve at each of the levels.

Rubrics

The rubric given for the portfolio assessment above is an appropriate assessment tool, since it lays out clear criteria on which learners' portfolios will be assessed. The fully expanded rubric (which you had to complete for the activity) would be even more successful, since this would make it completely clear to a learner why he/she achieved the given ratings.

Assessment tasks cannot be effectively evaluated simply by adding up all the correct answers, or awarding a quantitative mark (say 60%). Such a mark gives no information to the learner on what, in particular, was good or bad about his or her work. We need to find ways to manage this information and make it useful. One very useful tool with which to do so is a rubric. Rubrics are rating scales that are used in the assessment of performance. They are formally defined as scoring guides, consisting of pre-established performance criteria, and are used in evaluating or assessing a learner's performance. The expansions of the criteria at the different levels are called level descriptors.

On the next page is an example of a learner friendly mathematics rubric.

Activity 13

1) For this question, refer to the **six point analytic rubric** on the next page.
    a) Which column/row in the rubric above sets out the criteria which are being assessed?
    b) Which column/row in the rubric above sets out the levels of the criteria which are being assessed?
2) Set a task (such as the performance task given in activity 11 in this unit) and allow your learners to work through the task. Write up the statement of the task in full.
    a) Design a rubric, using all or some of the criteria and level descriptors that you will use to assess the learners' work.
    b) Assess your learners' work using the rubric.
    c) Comment on the success/failure of the assessment, and on the learners' responses to their work being evaluated in this way.
3) Draw up a project assessment task on any topic from space and shape (LO3) for a grade of your choice.
    a) Indicate the grade on the task. Write out the task in full.
    b) Draw up a rubric that you will use to assess your learners' projects.
    c) Discuss the value of using a rubric as opposed to a memorandum for a task such as a project, compared to a task such as an activity worksheet.

Six-point analytic rubric for assessing mathematics problem solving			
	Understanding	**Support for Thinking**	**Communication**
Outstanding **6** (Exceptional – goes beyond what was asked)	• Finds *all* important parts of the problem • Has *full* understanding of mathematics needed • Uses unusual, creative thinking	• Finds more than one way to solve the problem • Uses many ways to show thinking like diagrams, charts, graphs, etc • Student experiments, designs, analyses • Does *more* than what the problem asks	• Writes a clear, convincing, thoughtful answer • Writes to an audience • Diagrams are very clear
Meritorious **5** (Very good, clear, strong)	• Finds *most* of the important parts of the problem • Has good understanding of mathematics needed	• Finds one or more ways to solve problem • Uses several ways to show thinking like diagrams, charts, graphs, etc • May experiment, design, analyse • May compare the problem to another, predict	• Writes clearly • Makes sense • Writes to an audience • Diagrams clear
Satisfactory **4** (Pretty good, gets the job done)	• Finds *most* of the important parts of the problem – some less important are missing • Understands *most* of the mathematics needed	• Uses one way to solve problem • Some ways to show thinking may be missing • May experiment, design or analyse	• Addresses all parts of the problem • Writes to an audience • Writing may be unclear
Adequate **3** (OK, good try, unclear)	• Finds a *few* of the important parts of the problem • Understands *some* of the mathematics needed • Thinking gets mixed up • Might miss the big idea	• May or may not solve the problem • Mathematical thinking is unclear or limited • Chooses wrong ways to solve problem	• Has trouble writing ideas • May or may not write to an audience • Diagrams or charts not clear
Partial **2** (Incomplete, confusing)	• Has little understanding of the problem • Finds less important parts of the problem • Understands bits and pieces of the mathematics needed	• Doesn't explain thinking • Uses ways to solve problem which don't fit the problem	• Writes in a confusing way • May or may not write to an audience
Inadequate **1** (May make an effort, no understanding)	• Doesn't understand the problem	• Answer is difficult to understand • Makes little or no attempt to explain results	• Writes in a way that is very hard to understand

5.5 How to interpret assessment? How to report?

Rubrics are one of the forms of interpretation for an assessment task. Rubrics are time consuming to draw up, but facilitate the reporting process on an assessment task since they present a good overview of the learners' achievement on the completed task. All assessment, ultimately, needs to be interpreted and reported on. The reports will not only go to the learner who completed the assessment but potentially to other interested parties, and thus the interpretations on which these reports are based are important and need to be carefully considered so that the reporting can be fair and representative of the work done.

Three points of reference that can be used in assessment

The following are the three points of reference that can be used in educational assessment.

Self-referencing

Self-referencing could be used by the teacher to help learners plot their own development without this being downgraded (or over-rated) in comparison to the achievements of other learners. In the case of self-referencing, the learner

- is the reference point for his or her own achievement
- compares his or her achievement to what he or she has done before
- sets targets for the tasks he or she has to do, depending upon his or her previous achievements

Example: The parent or teacher may criticise the learner for getting poor marks for a test. However, the learner may draw their attention to the fact that he or she has improved on his or her previous test. In this case, the learner is self-referencing.

Criterion-referencing

Criteria are used as reference points in OBE. The criteria are the learning outcomes (LOs) and the assessment standards (ASs). These criteria are used to

- ascertain the learner's progress in terms of learning outcomes and assessment standards, which are independent of other learners' achievements
- give every learner a fair and equal opportunity to achieve or master the outcomes
- bring learners into the assessment process because the learners might take part in the process of drawing up the criteria, or will at least be aware of the criteria before attempting to do the task, so they will know what to spend time on when completing the task
- the assessment criteria are explicit, and so the learners will understand how their work is being assessed

- the explicit nature of the criteria will also enable the teacher to justify the assessment more easily and clearly to the learners and other interested parties
- move away from an assessment system which is primarily designed to create a comparison between all learners' achievements to one designed to credit achievement at different levels

Example: When learners are given a task to do, they are also given the criteria on the basis of which the task will be assessed. From the beginning, they are aware of what criteria they have to meet and how they will be assessed.

Norm-referencing

Norm-referencing is useful as a tool for things such as university entrance rating. The norm-referenced results can be used to decide whether or not a learner has achieved sufficiently well (against the given norm) to be allowed into the different faculties of the University. In terms of norm referencing

- learners' progress is described in relation to standards set for a group (such as a class average)
- learners' progress is also defined in terms of other individuals in the class and then inferences are made about how much a learner has learnt in comparison with others
- learner achievement is assessed in a is competitive way
- results are often given as a mark or a symbol which give a picture of the learners' overall achievement, for example by means of matric examinations all grade 12 learners are rated in comparison to a national standard and in relation to individual achievement against this standard.
- standardised tests are often used to measure learner achievement

Pegg (2002:235) writes that traditional assessment can mostly be referred to as 'norm-referenced', while outcomes-based education is more 'criterion-referenced'. While the main focus of these two approaches is different, they are not incompatible or in direct conflict. In fact, when determining appropriate criteria to evaluate levels of achievement in OBE, criteria are being defined within a norm-referenced context.

Activity 14

Read the following statements made by teachers and then state which statements illustrate norm-referencing, which illustrate criterion-referencing and which illustrate self-referencing, giving reasons for your response in each case.

Teacher A

Sipho has done well this term. He can calculate the area of flat surfaces, which he had difficulty with last term. He has made good progress.

Teacher B

I am worried about Mary. She doesn't seem to be able to work as fast as the other learners in my mathematics class. I think she is unable to do computation involving two or three digits.

Teacher C

My class seems to be doing okay. They're about as far into division as last year's class was at this time of year.

Teacher D

Ernest is a born mathematician. His spatial perception is outstanding. He is able to convert 2D drawings into simple 3D working models. He has a good grasp of perspective.

How to report?

The ultimate success of a continuous assessment model rests on sound and meticulous methods of recording learner achievement over an ongoing period of time. Cumulative evidence of learner achievement must be recorded and these records should accompany all learners throughout their learning careers. Cumulative records should include information on the holistic development of the learner, such as his or her social development and the development of his or her values and attitudes.

Each school should develop an assessment programme based on national and provincial guidelines. To ensure a professional approach to assessment, the school assessment programme should clearly outline:

- the way continuous assessment is to be planned and implemented
- how record books are to be kept, their accessibility and security
- internal verification of assessment
- how moderation takes place in the school

- the frequency and method of reporting; and
- the monitoring of all assessment processes.

Activity 15

Study the two different report cards given below.

1) Discuss the differences and similarities between them.
2) Which report card would you prefer to use? Explain your answer.
3) Which report card do you think the learners would prefer to receive? Explain your answer.
4) Design a report card for the second term for the work you have covered in your mathematics class. (It need not look anything like the two examples below).
   a) Write it up and show it to a colleague.
   b) Make changes to the report card according to your discussion with your colleague.
   c) Show the report card to your learners. Write down their responses to its layout and information it offers.
   d) Show the report card to some parents of your learners. Write down their responses to its layout and information it offers.
   e) Draw up a final version of the report card that you think would satisfy all of the stakeholders in the assessment process. Comment on how different it is from the first version you drew up.

Report Card A for Mathematics	*Out-standing*	*Achieved*	*Partially achieved*	*Not achieved*
Project: Conducting a survey				
Working with Multiples and Factors				
Aids statistics: Graph work				
Class Test				
TEACHER				
Strengths/ **support needed**				

Report Card B for Mathematics	*Term Mark*	*Comment*
Data Handling		
Number and Algebra		
Space and Shape		
Overall Average		
TEACHER:		

The control and ownership of assessment was traditionally in the hands of teachers, but since the implementation of outcomes-based education (OBE), the teacher and learner have shared responsibility to assess learning and knowledge.

Although in the first place it is the teacher's responsibility to assess and report on the progress of the learner, this does not imply that it is his or her sole responsibility. There are other stakeholders in the assessment process. These include the learner, other schools, district level managers and support services within the department, parents of the learner and the public at large. The teacher is accountable to all of these stakeholders and must ensure they he/she has effective means for reporting on and communicating to these stakeholders.

Self-assessment

Tick the boxes to assess whether you have achieved the outcomes for this unit. If you cannot tick the boxes, you should go back and work through the relevant part in unit again.

I am able to:

explain the term assessment	
identify the four purposes of assessment and am able to implement these in my classroom instructional activities	
explain the principles of outcomes-based assessment (OBA)	
describe the role and purpose of assessment in mathematics	
identify and explain the aspects of mathematics learning I ought to consider when assessing learners	
reflect on the assessment potential of mathematical tasks used in the teaching of basic data handling concepts	
select appropriate methods, techniques and tools for assessing a learner's performance in mathematics	
draw up or design my own assessment tasks and rubrics to be used when assessing a learner's work	
compare various methods of recording a learner's performance.	

Appendix A: Exercises on teaching data-handling

Learners will gain the skills to make sense of data by gathering data, organising and interpreting data and drawing conclusions from the data collected. These processes are appropriate for learners because they can be used to solve problems that are interesting to them. They can also represent significant applications of mathematics to practical questions. There are many graphs used to represent statistical data. Learners have to work with these graphs to gain an understanding of how to interpret and communicate the information represented in graphical form.

Collecting Data

The data that you collect in a survey or questionnaire may be very varied – it may be about the colour of peoples eyes, their mode of transport to work, an opinion (which chocolate do you prefer?) or a like or dislike. It may also be numerical, such as how many cars come through the school gate in the morning?

There are two forms of numerical data:

- Information that is collected by counting is called discrete data. The data is collected by counting exact amounts, e.g. the number of children in a family; the number of children with birthdays in January; the number of goals scored at a soccer match.
- Continuous data is collected by measurement and the values form part of a continuous scale, e.g. the height of learners in a Grade 8 measured in centimetres and fractions of a centimetre; temperature measured in degrees and fractions of a degree.

The mass of a baby at birth is continuous data, as there is no reason why a baby should not have a mass of 3,25167312 kg – even if there is no scale that could measure so many decimal places. However, the number of children born to a mother is discrete data, as decimals make no sense here.

Tables, lists and tallies

When you first look at data, all you may see is a jumble of information. You need to sort the data and record it in a way that puts order into it so that it makes more sense.

Some data is easy to sort into lists that are either numerical or alphabetical.

Other data can be sorted into tables. Some tables can be used to keep count of the number of times a particular piece of data occurs. Keeping count like this is called keeping a tally. There is an example of a tally table in the activity below. If you need to learn more about tally tables you'll be able to find the information in an intermediate phase mathematics text book. Another name for such a tally table is a frequency table. The frequency of something happening is the number of times it happens.

The content in the exercise that follows relates to the assessment standards on collecting (using a survey and by experimenting) and organising data. There are also some interpretive questions based on the data.

Exercise 1

1) A survey was conducted to find the ten most spoken languages in the world and the number of people speaking them. The results were written out as follows:

Chinese:	700 million	German:	119 million
English:	400 million	Spanish:	240 million
Russian:	265 million	Japanese;	116 million
Bengali:	144 million	Arabic:	146 million

Organise the information into an ordered list in two different ways.

2) In an experiment I toss a dice 50 times and keep a record of the number that appears each time. The numbers are shown below:

2 ; 4 ; 3 ; 3 ; 1 ; 5 ; 6 ; 3 ; 2 ; 2

2 ; 2 ; 6 ; 1 ; 5 ; 5 ; 3 ; 3 ; 4 ; 2

2 ; 3 ; 4 ; 3 ; 6 ; 5 ; 1 ; 1 ; 2 ; 1

3 ; 5 ; 6 ; 3 ; 1 ; 2 ; 2 ; 5 ; 5 ; 1

6 ; 2 ; 2 ; 4 ; 1 ; 6 ; 2 ; 3 ; 3 ; 5

Complete the tally table and then answer the questions.

Number	Tally	Frequency
1.		
2.		
3.		
4.		
5.		
6.		
Total		

Notice:

- The tally total is the same as the frequency total
- The data can go across or down the page.
- The 'items' come first in the table.
- Do not confuse the frequency with the number on the dice

a) How many threes were tossed?
b) What number was tossed the most times?
c) Why do you think more sixes were not tossed?
d) How many more times was a two tossed compared to a five?
e) Do you think this dice is a fair dice? What does fair mean in this question?

3) Conduct a simple survey of the learners in your class to ask about the month of their birthday.
   a) Record the information in a frequency table:

Month	Tally	Frequency
January		
February		
March		
April		
……		

   b) In which month do most birthdays occur?
   c) In which month do the least birthdays occur?

Representing data

Once you have collected the data, you have to be able to display it in a way which effectively communicates the information that you have found. This can be done by means of picture diagrams and several different forms of graphs. Presenting data visually means that it is easier to read and make sense of.

Pictograms

Suppose you collected the birthday months of all the learners in your class. You could organise this information into a table like this:

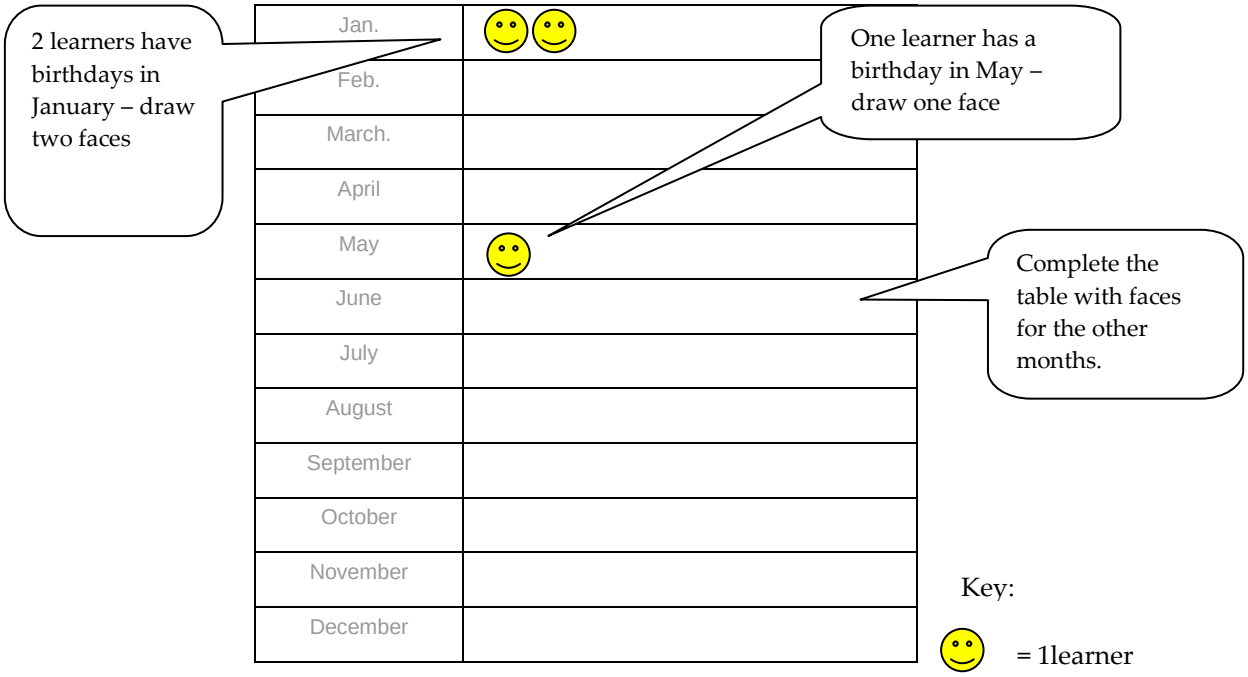
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Mmatsie	Sipho	Abdul	Thandi	Jonas	Mary	Cita	Anna	Alix	Farah	Rachel	Adit
Jama	Zeta		Michael		Jabu	Jo	Makhosi	Sandep	Peg		Devy
	Beth		Zoe			Ahmed	Puleng	Jane	Pumlani		
							Chandra	Zula			
								Fatima			

If you are primarily interested in how many learners have birthdays each month their names do not matter. You can represent each child with a symbol –

 = 1learner

You can rearrange the list of names in this way:

Birthdays in Grade 8



Redraw this table and complete the pictogram.

This method of displaying the data is called a 'pictogram'. You can clearly see how many children have birthdays in each month. Pictograms are useful as they provide a quick visual impression of the data.

If you had colleted a lot of data – say birthday months of everyone in the school – there might 30 or more in one month. It would be tiresome to draw so many little faces so you could choose a scale – say 1 face represents 10 faces. This explains why all pictograms need a key to say what the symbol or 'picture' represents.

When drawing pictograms there are a number of things to remember:

All pictograms must have a title and a key.

Choose a simple 'picture' or symbol that is easy to draw.

Always give a key and say clearly what each symbol stands for. If

= 10 people, you will need to draw

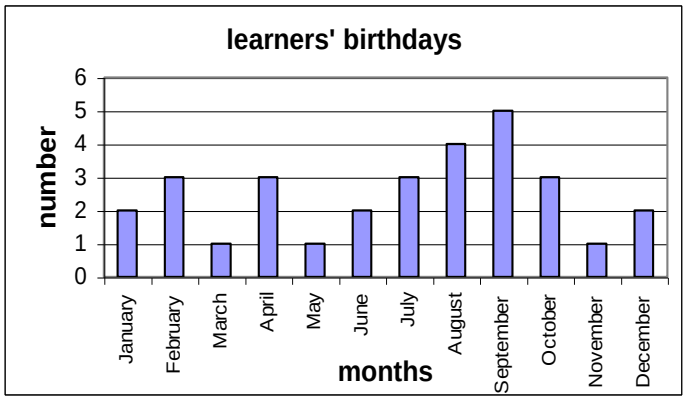
= 5 people

Work out how many symbols you need for each data column or row carefully.

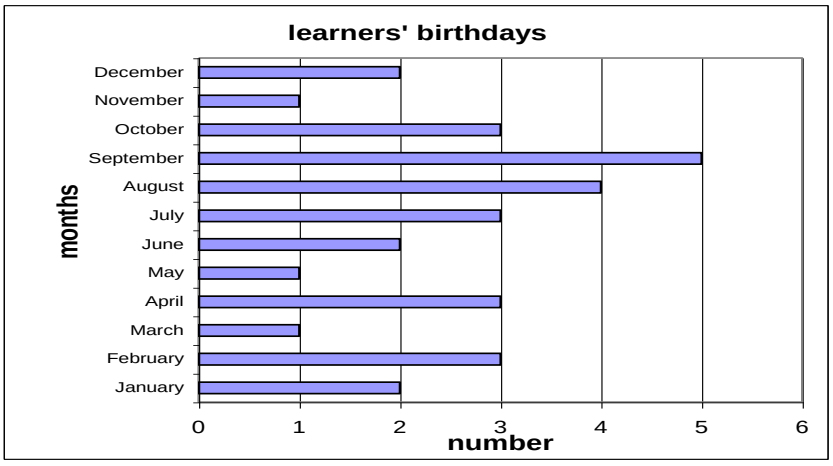
Draw on squared paper as this helps keep the symbols neatly in line.

Bar graphs

Another way of displaying the birthdays would be to put them in a vertical chart like this:



Or a horizontal chart like this:

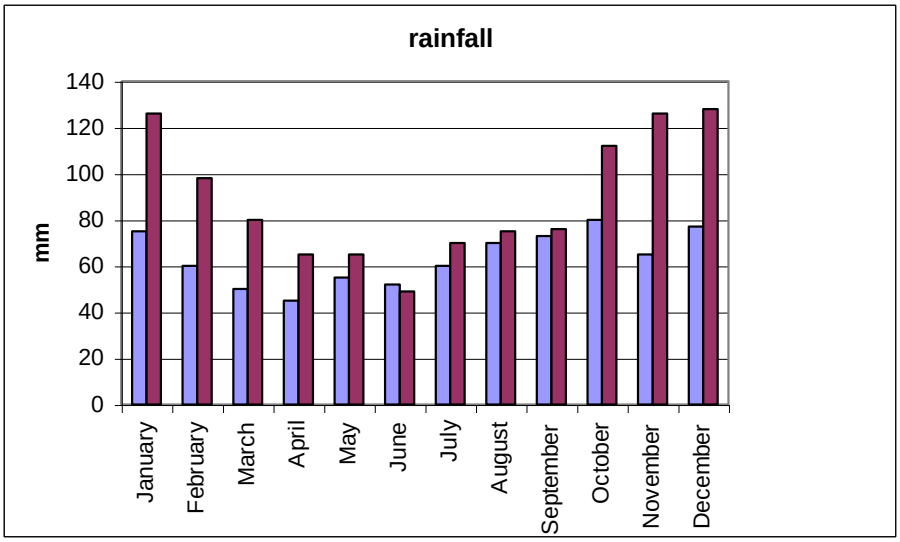


This method of displaying the data is called a 'bar chart' or 'bar graph'. This is a very popular way of displaying information, as it is easy to read accurately and gives a very good visual impression of the data. A bar graph uses bars, side by side, to display information. A bar graph shows clearly how data items compare – you can see at a glance which bar is longer – however it is difficult to compare one item of data to the whole data set. A bar graph can show frequencies – numbers of things, as in the birthdays above; or amounts of things such as heights of mountains, or hours spent watching TV.

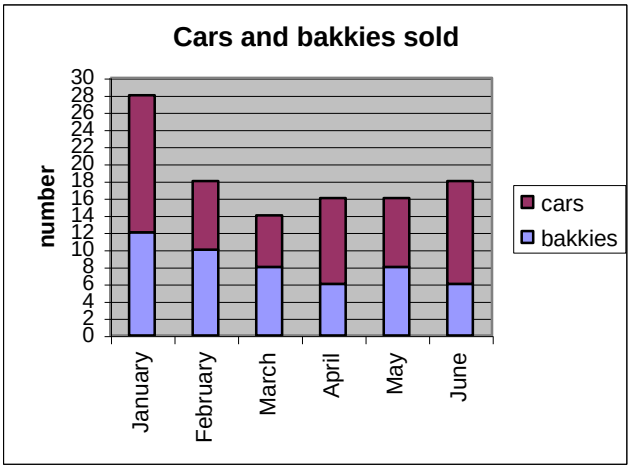
Notice:

1. The bars can be horizontal or vertical.
2. The length of the bar stands for the frequency of the data.
3. A bar graph has two axes – the scales of the axes must be accurate.
4. All bars are the same width.
5. All bar graphs have a title
6. Bar graphs can be used for discrete and continuous data

7. Bar graphs can also be used to illustrate grouped data
8. Sometimes a bar graph has two sets of bars - representing different data side by side. This allows you to compare two sets of data on one graph rather than on two graphs. This bar graph shows the rainfall at two different places on one graph.



9. Sometimes a bar graph has different sets of data on the same topic shown as different sections on a bar. This is called a sectional bar graph.



This bar graph shows the number of bakkies and cars sold by a garage in 6 months

Exercise 2

1) The table below shows the estimated percentage HIV prevalence per province in South Africa in 1998.

Province	%
Eastern Cape	16
Free State	22
Gauteng	22
KwaZulu-Natal	33
Mpumalanga	30
Northern Cape	10
Limpopo	12
North West	21
Western Cape	5
South Africa	22

This means that 16% of the population of the Eastern Cape is estimated to be HIV positive.

1) Show this information in a pictogram. Use  as a symbol that represents 5 %

2) Draw a vertical bar chart to show the information. Put the provinces on the horizontal axis and percentages on the vertical axis.
From what you know about the HIV AIDS pandemic do you think the graph would look the same today? Discuss this with your group.

The table on the below shows the percentage of households in South Africa that have 2 or fewer rooms. The data is listed by province and is an estimate taken in 1996

Province	%
Eastern Cape	39
Free State	37
Gauteng	36
KwaZulu-Natal	35
Mpumalanga	33
Northern Cape	39
Limpopo	29
North West	28
Western Cape	23
South Africa	33

This means that 33% of households in Mpumalanga were living in 2 or fewer rooms.

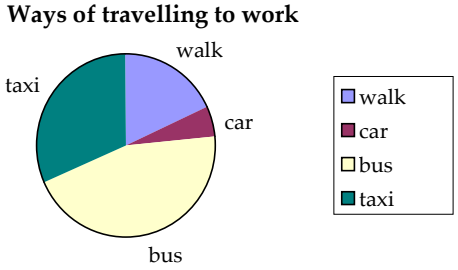
1) Draw a bar chart to represent this information.

2) Discuss with your group why you think the percentage is lower in the Western Cape. Write down the main ideas from your discussion.

Pie charts

A pie chart is another way of representing data. A pie chart is a circular diagram divided up into 'slices' like a pie. It is particularly useful if you want to illustrate a whole population divided into parts and show what portion of the whole each part represents. The whole circle represents the whole population. Each slice represents a part of the whole. The size of the slice shows the size of that part.

This pie chart shows the way a group of people travel to work. It is easy to see that most people go by bus.

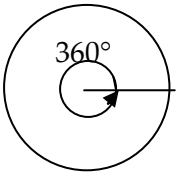


Pie charts are very tedious to draw by hand. If you have a computer and know how to use Excel, click on the Chart Wizard and select the pie chart once you have entered your data on a spreadsheet and the pie chart will be drawn for you almost instantaneously.

There are two main steps in drawing a pie graph by hand.

Steps in drawing a pie chart by hand

1. Calculating the angle in the centre. This is relatively straightforward if you remember that there are 360° around a circle.



2. Drawing the slices of the pie.
   For this you need a protractor.

   To find the angle in the centre:

   Find the total amount to be shown - i.e. add up the frequencies
   This total represents the whole circle – i.e. 360°. So each part is a fraction or percentage of 360°

Let's look at an example to illustrate this. Suppose you did a simple count of the colour of the eyes of the learners in your class.

The table shows this

Colour	Number
Brown	32
Grey	6
Blue	22
Total	60

Look at brown eyes:

32 out of 60 learners have brown eyes;

the fraction of learners with brown eyes is $\frac{32}{60}$;

so the fraction of the circle for brown eyes must be $\frac{32}{60}$;

so the angle of the slice for brown eyes must be $\frac{32}{60}$ x 360° = 192°;

Work out the angles for blue and grey eyes:

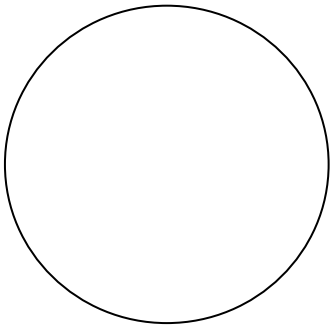
Angle for blue = $\frac{22}{60}$ x 360° = ……

Angle for grey = $\frac{6}{60}$ x 360° = ……

Check that the total of the angles is 360°

Exercise 3

1) Copy and complete this pie chart to represent the data about eye colour given above.
2) Draw a radius in the circle. This is where you start measuring the angles.
3) Measure the angles at the centre.
4) Give a title and a key for the pie chart.
5) Use Excel to draw the pie chart if you are able to.
6) Compare your hand drawn graph with the computer generated graph if possible.

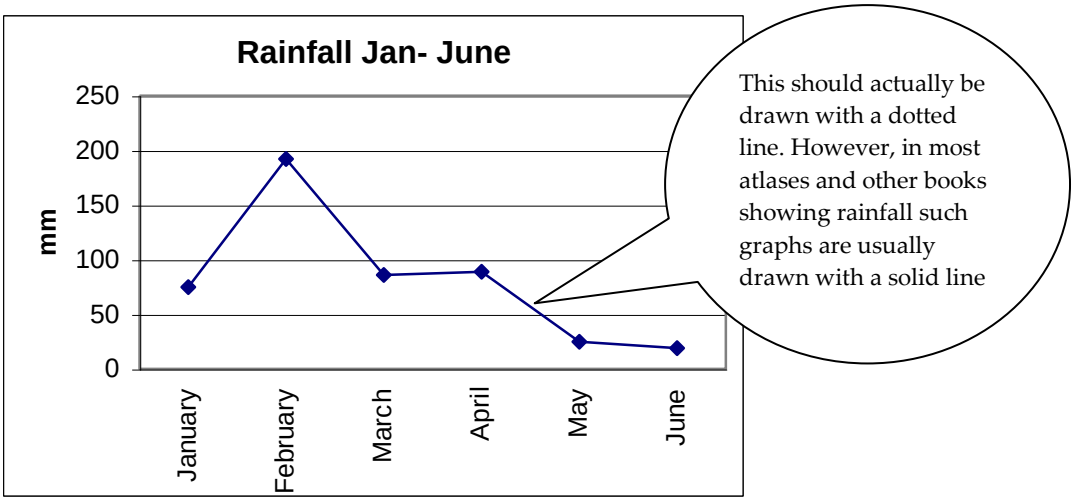


Line graphs and broken line graphs

You can replace a bar graph by a line graph if the data on the horizontal axis is continuous such as time, temperature or age. In this case the data is plotted as a series of points that are joined by straight lines. Businesses often use line graphs to show information about profits or periods of production. Geographers use line graphs to show monthly rainfall or crop yields over time.

Line graphs are useful as they show trends and can easily be extended. This means that with some line graphs it might be possible to continue the line to show what might happen in the future.

The line graph below shows rainfall measured over a period of 6 months.



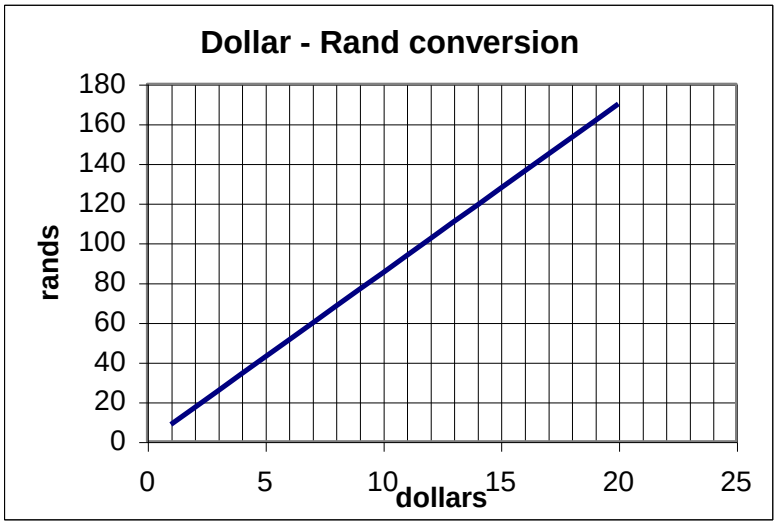
Look at the graph carefully.

i. Which month has the most rainfall?

ii. Which month has the least rainfall?

NB: The line graph above shows the rainfall at a certain place. Because the points are joined it suggests that the rainfall changes as shown by the lines. This is not so. You are just joining isolated rainfall readings. A line graph like this is sometimes called a broken line graph.

Line graphs can also be drawn for conversions – i.e. a change from one unit of measure to another.



Conversion graphs can be used to change form metric to imperial units and vice versa or from one currency to another.

Notice in this line graph the scale on the two axes is different. The vertical axis goes up in twenties, the horizontal axis goes up in fives.

In the following exercises, you will represent data in the four ways shown in this chapter – pictogram, bar graph, pie chart and line graph.

Exercise 4 a

Thembi kept a record of the hours she spent on different activities during the day. This information is shown below.

1) Complete the table to show the degrees needed for each activity when drawing a pie chart.
2) Draw the pie chart.
3) Represent the information using a line graph.

Activity	Number of hours	Number of degrees
School	5	$\frac{5}{24}$ x 360° = …….
Meals	1	
Homework	3	
TV	2	
Travel	1	
Sleep	8	
Other	4	
Total	**24**	

Exercise 4 b

Joe did a survey of the colours of cars parked at the local sports club. The observation sheet is given below.

1) Complete the frequency table
2) Display the data as a pictogram, a bar graph and a pie chart.
3) Which representation do you think is best? Explain your response.

Joe's observation sheet		
Colours	Tallies	Frequency
Red	𝍸 𝍸	
Blue	||	
Green	𝍸 𝍸 𝍸	
Black	𝍸	
Orange	𝍸 𝍸 ||	

Interpreting data

You have read about some ways to collect, organise and present information or data. Remember that the whole point of collecting data is to help you understand more about the world you live in. Information can be collected through questionnaires or surveys that ask people questions about their lives. You can develop questionnaires if you want to find out what different people think about products or opinions or aspects of life. You can do surveys if you want to know people's opinions about things. A government will conduct a census if it wants to know how many people live in a place, who they are, what they do, how big their families are, where they live etc. Now we will look at what the representations of the data you have collected can tell us about our world.

You have seen that drawing graphs or pie charts helps give a picture of data collected. Rather than having long lists of numbers or facts, a graph can help you understand the data. Now we look at how to interpret the graphs you drew to illustrate the data; how to analyse and draw conclusions about the data, how to answer the questions you asked in your survey or questionnaire.

When interpreting data it is a good idea to discuss your interpretation with the other members of your study group. You may 'see' things that the others do not. By sharing your ideas you can get a fuller idea of what the data is saying. It is a well-known fact that statistics can be misleading. They are often used to prove a point, and can easily be twisted in favour of that point!

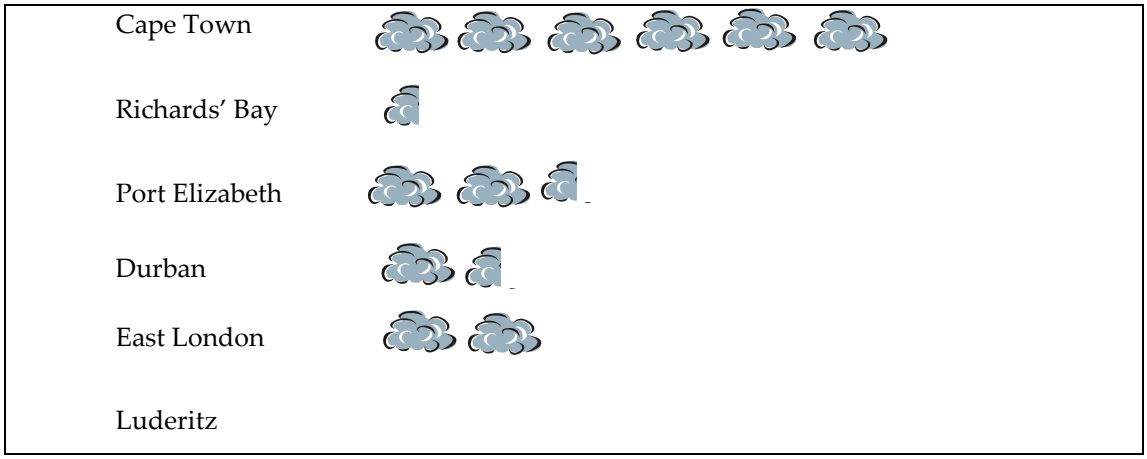
Data displayed in graphs and on bar and pie charts can sometimes be distorted to give false impressions. It is important to look very carefully at the graphs and diagrams so that the visual aspects of the charts, graphs and diagrams do not deceive you. The way you interpret the data usually depends on the reason for collecting it. Your initial question or hypothesis is important in determining the emphasis of your interpretation.

When analysing data you can make general conclusions: more people like chocolate ice cream; and/or mathematical conclusions: the mean of the marks is 54%. Again your initial question or hypothesis will determine your approach.

Interpreting pictograms

Remember that a pictogram shows data as little symbols or pictures. A pictogram gives you a quick impression of the information. You must always look at the key to see how many items of data the symbol or picture represents. Part of a picture or symbol represents a fraction of the number of items.

The pictogram below shows the number of rainy days during the month of June at six towns on the coast of South Africa. You can immediately see which town had the most rainy days and which town had the least. If you look at the key you can tell that Richards Bay had 2 rainy days, Port Elizabeth had 10 rainy days and Luderitz had none.

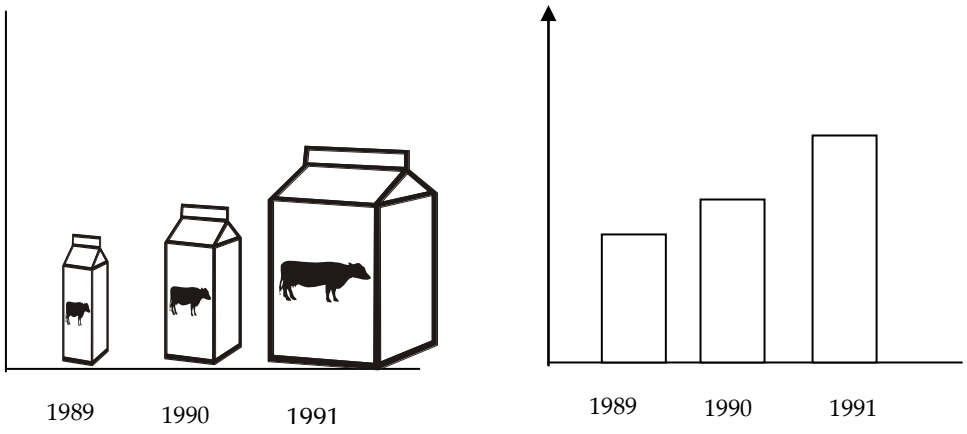


Misleading pictograms

Sometimes pictograms are designed especially to give misleading information. Sometimes pictograms are drawn badly and the information that they portray is misleading. You must be aware of this when drawing your own pictograms and when making interpretations from pictograms in magazines and newspapers.

The ambiguity arises when symbols are incorrectly drawn or when spacing is unequal. Sometimes this is intentional – specifically to give information that is unclear – and sometimes it results from poorly set out work. Look at the examples below:

The two diagrams below show information about the sales in a milk depot during 3 consecutive years. The first diagram is a pictogram with milk cartons used to show the sales of milk. The second is a bar graph showing the sales of milk. Which diagram do you think shows misleading data?



The pictogram shows misleading data. This is because, although the height of the milk bottles is the same as the bars on the bar graph, the differing width of the cartons makes the 1991 sales appear far more than the 1989 sales.

Interpreting bar graphs

In the same way that pictograms are easy to read, bar graphs are also very easy to read. The lengths of the bars stand for the size of the data. Important points to look for when reading information off a bar graph are:

The title – what is the bar graph about?

The axes – check the labels of the axes. One axis gives the labels of the bars, the other tells you how many items in each bar.

The scale – the scale on the number axis tells you how many. You might have to work out how many items are between numbers shown on the axes.

The data about the rainy days in coastal towns on the previous page could have been shown in a bar graph like this:

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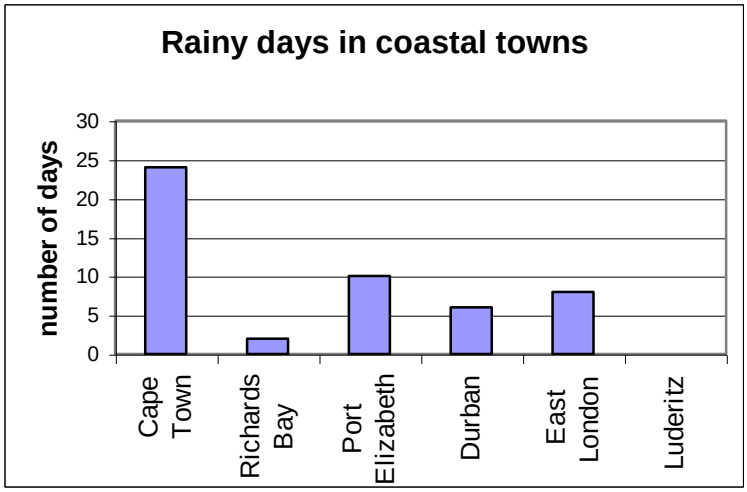

With the scale as shown you need to judge whether Cape Town has 23 or 24 rainy days.

However, it is easy to compare the number of rainy days at the different towns. It is easy to see that Cape Town has the most rainy days and that Luderitz has the least.

Misleading bar graphs

Bar graphs can also be misleading. An important thing to look at is the scale on the axes:

- Does the scale start at zero
- Is the scale distorted – too squashed up or too spread out. This is a trick that some companies use to enhance the look of their sales.

Also check the width of the bars. These can be distorted as in the milk cartons on the pictogram.

The bar charts on the next page show a comparison between two different brands of cereal.

A company that wants to show that Wonder Cereal is better than Loopy Cereal could use any of these bar graphs. They all show misleading information. Look at them carefully to see the different ways in which they are misleading.

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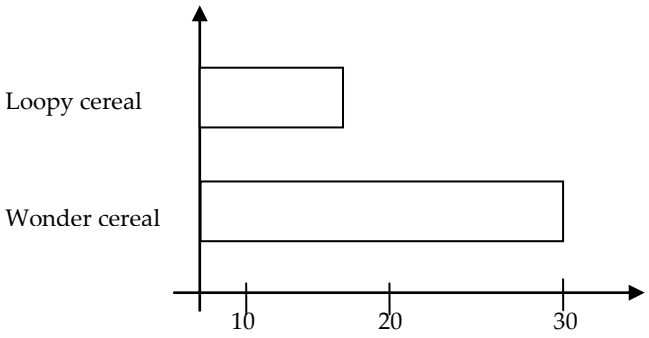
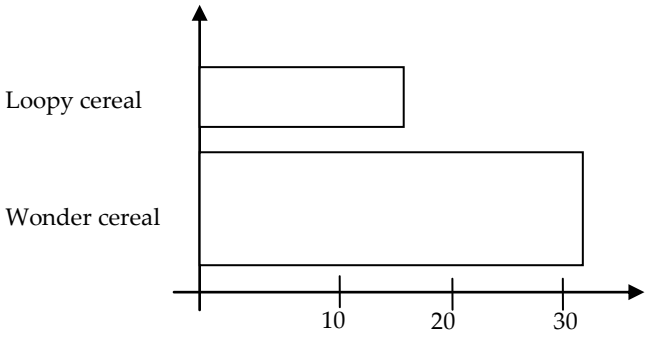
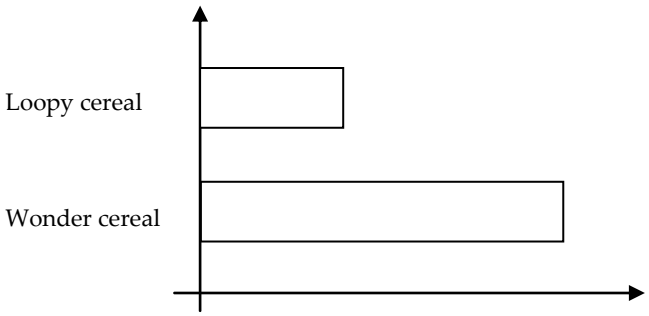
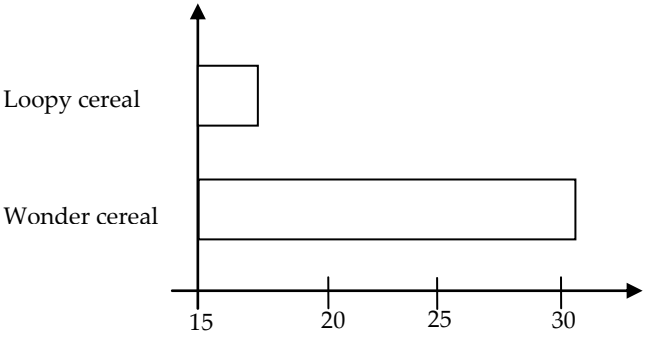




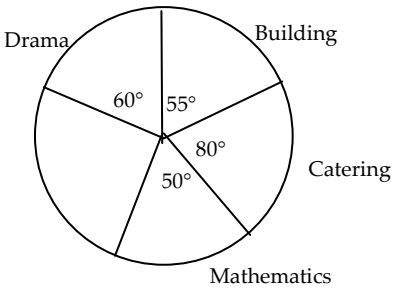




Interpreting pie charts

The interpretation of pie chart is based on the fact that the largest 'slice of pie' relates to the largest item of data and the smallest 'slice' to the smallest item. It is therefore easy to make comparisons between the relative sizes of data items.

Sometimes the size of the respective slices is easy to estimate by eye. 90°at the centre of a circle is a quarter of a circle. Sometimes it is not so easy to see the relative slices of the sizes of the pie. This could be frustrating if you want to make comparisons.



The pie chart shows the number of learners in different sections of a college. 220 learners are in the building department. How many learners are taking mathematics?

You know that 55 ° represents 220 learners.

You can work out that 1° represents $\frac{220}{55}$ = …….learners.

And that 50° represents 50 x ……. learners

So the mathematics department has ……learners.

If badly drawn, pie charts can also display misleading information. Remember that one of the main advantages of pie charts is that you can see part of the data as a fraction of the whole data.

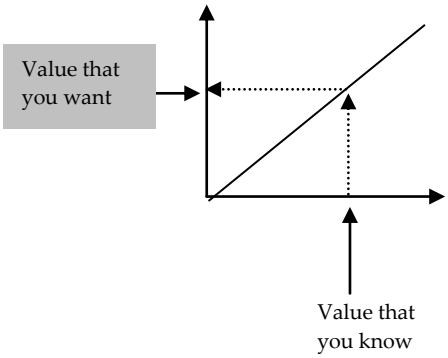
Interpreting line graphs

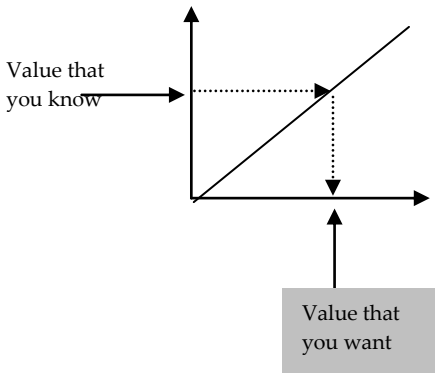
Line graphs are an important feature of much of mathematics. You might have looked at some straight-line graphs with the learners in your class. Line graphs are most often used for representing continuous data.

Some important things to help you interpret line graphs are:

- The title – what is the line graph about?
- The axes – check the labels of the axes.
- The scales on the axes – do the scales start at zero? What else do the scales tell you?

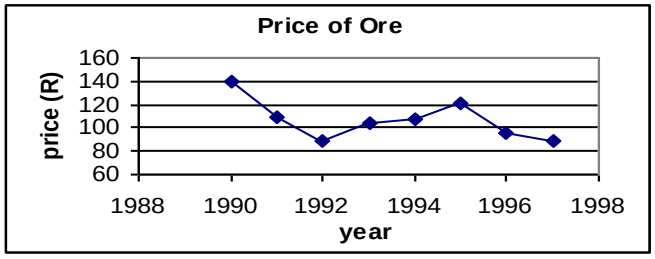
Reading a line graph that illustrates data is like reading any other sort of straight-line graph. Look carefully at the labels of the axes and then read off the values.

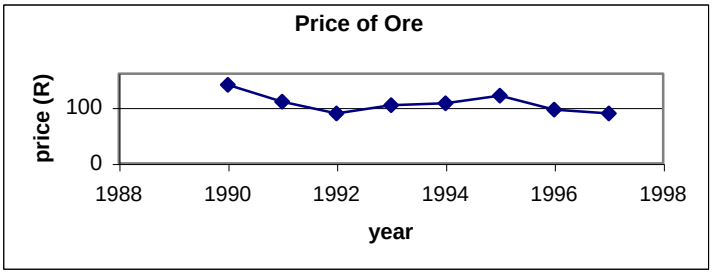




Misleading line graphs

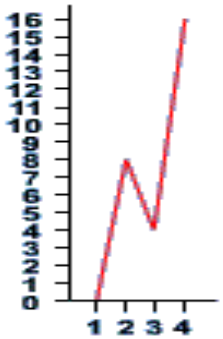
Some line graphs are purposely drawn to convey misleading information. Look carefully at the two graphs below. They both show the same information but the 'look' of each is completely different because the scales are different. They appear to tell a different story.





Look at this graph:

Although the vertical scale starts at 0, it does not go up in even steps. This has the effect of distorting the graph, and making it look as though the biggest jump is between 1 and 2 rather than 3 and 4. Also, there are no labels on the axes. We have no idea what this graph represents!



Here is the same graph with a correct vertical scale.

Does it tell the same story?

In what way is it the same?

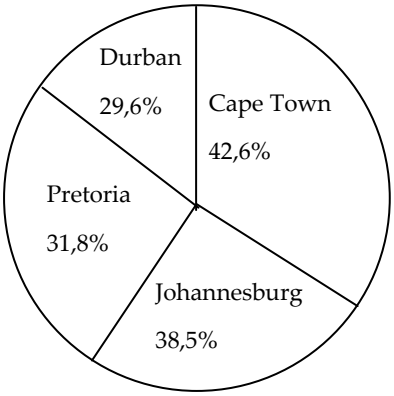
In what way is it different?

Graphs taken from:

http://www.bbc.co.uk/schools/gcsebitesize/mathematics/datahandlingfi/representingdatarev5.shtml

Exercise 5

Below are a graph and a newspaper article taken from The Star newspaper (1999) about impulse buying. Read the article carefully. Think about things like the assumptions that are made by the writer/researcher and whether the article and the graph tell the same story.



Who are the impulse buyers?

South Africa is a nation of shoppers with increasing numbers defined as impulse buyers who respond to glossy adverts and come-ons such as 'never to be beaten bargains' and 'buy one and get one free'.

This, in part, has emerged from one of the most comprehensive surveys of consumer shopping behaviour which has just been compiled by Media & Marketing Research (MMR).

MMR's research provides answers to a host of questions about South Africa and how its people buy.

Capetonians were the most likely to respond to bargains, good buys as well as advertising come-ons on TV and in newspapers (42,6%). Johannesburg shoppers came in second at 38,5%, Pretoria notched up 31,8% and Durbanites were rated at 29,6%.

The ranks of impulse shoppers were most likely to come from these 'bargain hunting' groups.

a) What is wrong with the pie graph?
b) Who do you think 'impulse buyers' are?
c) Who do you think 'bargain hunter' shoppers are?
d) Redraw the given data in a more suitable and correct graph.
e) According to the researchers, what does the given data represent?
f) What does the newspaper headline suggest the data represents?
g) What assumptions are made about the meaning of results of research on 'bargain-hunting' shoppers?
h) What does the data not tell us about impulse buying?

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