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OPTIMISATION OF NO SWAY PLANE RIGID FRAMES AGAINST BUCKLING

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A research report submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

School of Environmental and Civil Engineering

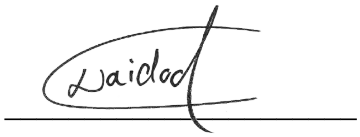
University of the Witwatersrand

Johannesburg, South Africa.

20 September 2019

CANDIDATE'S DECLARATION

I declare that this research report is my own unaided work. It is being submitted to the Masters of Science in Engineering to the University of Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.



Devveena Naidoo (447445)

20 day of September year 2019

ABSTRACT

The report proposes a simple method which optimises the design of plane, rigid no sway frame structures based on the system buckling load. It is centred on either maximising the buckling load or minimising the weight of the structure, or both; and to have all stories buckling at the same time. The method is applied to various frames examples and the results are compared to that obtained from a system buckling analysis performed in the ANSYS Finite Element Analysis (FEA) software. The proposed optimisation procedure proved successful for no sway multi-story rigid frames, as validated by the acceptable percentage differences of below 5% from the FEA analysis. The optimisation method is however limited in its application to multi-story rigid no sway frames only. The methods attempted for the calculation of the system buckling load to account for the influence of adjacent stories of sway frames, was not successful and further development is required.

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LIST OF SYMBOLS

Beam to column restraint parameter	λ
Beam to column stiffness ratio	n
Critical buckling load	P_{cr}
Equivalent column stiffness ratio	ρ
Factor of beam stiffness to column	N
Height of story in a frame	h
Iteration number of optimisation method	k
Modification factor for P-δ effect	β
Modification factor for the upper-bound solution of sway to no sway frames	η
Number of bays in a frame	b
Number of stories in a frame	s
Second moment of area	I
Story load to unit load ratio	γ
Story number	i
Story P_{cr} ratio	α
System buckling load coefficient	μ
System stiffness	K
Young's modulus	E

1 INTRODUCTION

A method is needed in the field of design optimisation of frame structures that is simple to use and realistic in capturing the factors affecting the stability and behaviour of a structure. Current optimisation methods verify the buckling of a frame by only evaluating a column or storey as opposed to the entire system. The report proposes a simple method which optimises the design of plane frame structures based on a stability criterion particularly, the system buckling load. It considers plane, rigid no sway frames only. The method ensures that the stability criterion is satisfied by verifying the buckling load at each subsequent iteration, where the entire structure is verified as opposed to current isolated methods. It is centred on either maximising the buckling load or minimising the weight of the structure, or both.

A simple optimisation method will prove most desirable in a design office environment, which is focused on increasing productivity by spending the least amount of time on analysis or design. With the advent of being more efficient in design, total member capacity utilisation should be achieved to prevent over-design of structures and essentially material wastage. The method proposed will attempt to provide a global approach to instability in terms of buckling, whilst ensuring full member capacity utilisation of multi-story frames.

Before an optimisation procedure could be developed, a system approach to calculate the buckling load of no sway frames needed to be derived. For a structural system, buckling or loss of stability is a system phenomenon. It depends on every element rather than a single member of the system. Buckling is a primary concern for structural design, especially, in the design of steel frame structures. Currently, it is only considered on an isolated basis for a member or column and eventually, the realistic factors affecting the actual behaviour of the structure are missed and is not practical for application.

The stability of the frame during optimisation is verified through a method proposed in the research which incorporates a system buckling load approach, and a load coefficient relating its upper-bound load to the ‘real’ load under non-rigid beam conditions.

Currently, there are methods available for the optimum design of steel frame structures yet some of these methods are not explicitly constrained by a stability criterion but rather by strength and performance requirements outlined in design codes. Those methods which are based on a stability criterion, do not however account for a system approach to the buckling load. In addition, current optimisation methods are laborious and/or complex and require some type of software to calculate the buckling load. A simple hand method calculation will not only add to the efficiency in optimal design, but also provide the design engineer with more control over parameters as the calculation can easily be done by hand.
The current optimisation methods can be expanded upon to make it simpler for efficient design of frame structures and to ensure that they are subject to all or most factors affecting stability.

The outline of the report will first review the current optimisation methods available in the literature for frame structures, in Section 1.1, and will be assessed on its ability to firstly, incorporate a system approach to buckling, and on whether total member capacity is achieved. The optimisation method is then developed in Chapter 2 of the Research Methodology, based on a system buckling load approach that is also derived and presented in the chapter, and applied to various no sway frame examples. The findings from these applications are shown and discussed in Chapter 3. The method was then extended to sway frame applications. The unsuccessful approaches taken to calculate the system buckling load of sway frames are given in Section 3.2 for future research to develop on. The major findings of the research undertaken on the proposed optimisation method are lastly stated in Chapter 4.

1.1 Literature Review

The optimisation of frame structures has been widely explored in the structural engineering field. Generally, optimisation techniques or algorithms are applied with constraints based on the soundness of the structure. The methods address the stability of the structure yet none have accounted for the stability of the entire structure in terms of a system buckling load. The methods check the stability of a single column as opposed to a storey or the entire structure. A story buckling method has been

developed to calculate the system buckling load of plane sway frames which can be utilised to overcome this shortcoming of the current optimisation methods (Li, 2014). In addition, the critical buckling load should be maximised and should be considered as an objective function together with the weight of the structure. The current techniques for optimisation will be discussed on the extent to which the stability criterion is incorporated as a design constraint, the extent at which the design constraints model or encompass the actual behaviour of the structure, and lastly on the simplicity of the method in its use. The literature will be ordered according to its level of contribution towards the research at hand on optimisation based on stability specifically, buckling.

Most optimisation methods for the design of frame structures have design constraints or variables which are based on serviceability and strength requirements outlined in existing codes of practice. An optimisation design method based on the genetic algorithm (Kameshki & Saka, 2001) yielded a frame and bracing system which was of minimal weight while satisfying design criterion on the strength and serviceability requirements outlined in BS 5950. The minimum weight was obtained by section selection from the standard set of steel sections. The only provision made to account for stability was through the lateral torsional buckling of frame members if they are beam-column members.

Another method which was based on the behavioural and performance constraints of the BS 5950, is the harmony search design algorithm (Saka, 2009). The harmony search method is a numerical technique which imitates the musical performance when a musician seeks for a state of harmony. The pitch of each musical instrument determines the quality of the sound in the same way that the value of the objective function was determined by the design variables to which it is constrained. This method determined the optimum section designation, from the list of Universal Beam and Column sections of the British Code, which satisfy the design limitations imposed by the code and resulted in a steel frame of minimum weight. Once again, buckling was only accounted for in an isolated manner on beam-column sections, specifically checking the buckling resistance moment capacity for such members.

A design procedure exists which utilised the genetic algorithm in the discrete optimisation of two-dimensional structures (Camp, et al., 1998). The procedure proposed, has an objective to minimise the weight, and in turn the cost, of the structure which was subject to strength and serviceability requirements. This discrete optimisation method was motivated by the problems associated with the discretisation of a continuous solution; particularly the relationship between the cross-sectional areas and sectional properties of rolled shapes, and that the selected discrete shapes may result in a frame that has a different structural response that may not satisfy performance constraints. The method made use of discrete structural elements as compared to the optimality criteria method which makes use of continuous elements. Thus, the use of discrete elements allows the direct use of rolled shapes or members from a database as opposed to obtaining a cross-section and matching it to the closest available members. Genetic algorithm does not require a relationship between the objective function and the constraints, instead the value of the objective function was adjusted to record the violation of the constraints. When the solution obtained violated a constraint, the value was penalised according to the degree of violation. An optimisation program, FEAPGEN, has been developed based on the genetic algorithm, and the results obtained from this program was compared to those from other optimisation methods. When this design procedure was applied to a frame, which appeals to the focus of the research presented in this report, it was subject to an objective function of minimising the weight with constraints of strength and integrity to meet serviceability. Sway was considered using formulas similar to alignment charts in design codes to get the effective length factors of the columns. Stability was not explicitly accounted for as neither a constraint nor objective function in this design procedure.

After investigating the current optimisation methods based on stability, the one that was most applicable to the current research, was the design method proposed for improving the strength and overall stability of frame structures, where the values were determined by inelastic limit-load behaviour (Pezeshk & Hjelmstad, 1991). It was based on a stability criterion where the designs were constrained to a constant weight. The dominant buckling eigenvalue was maximised to improve stability. It involved linearized buckling eigenvalue and eigenvectors which were applied in an

algorithm based on the optimality-criteria method to solve the optimisation problem. The methodology suggested was driven by the under-design of slender structures and how previous optimisation techniques failed to focus on the stability of the structure. It also further illustrated that by improving the overall stability characteristics of a structure under static loading, it could often improve the dynamic performance.

The second method which enhanced buckling in its optimisation process, made use of finite element analysis, where columns and frames were optimised in order to improve the elastic buckling resistance of structures (Manickarajah, et al., 2000). The method modified each element in the structure with the aim of shifting the material from the strongest part to the weakest part of the structure. The objective function was focused on buckling of the structural elements while keeping the structural weight constant. The hypothesis stated that the optimum design of structures against buckling may be achieved by either finding the minimum weight of the structure that satisfied a buckling load constant, or maximised the buckling load with a given weight or volume. In this design method, the lowest buckling eigenvalue was raised but considered the resulting effect to corresponding eigenvectors and eigenvalues. The structural modifications were limited to cross-sectional changes only. It utilised the eigenvalue problem and resized the elements where the total number of elements resized at each iteration was expressed as the resizing ratio. The final product achieved in this optimisation method, was an optimum shape of the columns in a frame structure. Nevertheless, this method once again considered buckling on a localised level as opposed to a system approach. Furthermore, the optimisation was performed with commercial software, which does not meet the objective of a 'simple' approach proposed in the research at hand.

A method has been proposed on the optimal design of frame structures based on a stability criterion using first-order analysis (Gil-Martin, et al., 2006). Yet, a gap exists in this method in that the system buckling load was not considered. A buckling check was done on one column, as opposed to the whole system which would better account for the performance and stability of the structure. An improvement to this method can be obtained by incorporating the use of system buckling methods, namely the simple story buckling method (Li, 2014). A first-order analysis based method was later proposed by the same authors, which was more suitable for

application in the design office environment. This optimal design method was based on the stability criterion for planar frames and only required data from a first-order analysis of the structure (Gil-Martin, et al., 2006). It aimed to improve the elastic buckling of frame structures, where an instability coefficient was utilised as opposed to a dominant eigenvalue. It was suggested that the overall stability of the structure can be improved by increasing the lateral stiffness of the story with the lowest value of this instability coefficient. Approximate expressions for the second-order effects of the structure identified the structural components that needed to be modified after each iteration. This formed the basis that the resistance to elastic instability can be increased if the properties of the structure can be modified so as to increase the value of the instability coefficient. The approximation, which accounted for second-order effects, enabled a first-order analysis to be used in the iterative modification of the structural components in relation to its elastic buckling strength. This method allowed for a story-by-story modification of the individual components, whereby the critical story was shifted, preferably near the top of structure, upon completion of the optimisation. The proposed method was aimed at either increasing the buckling load, or reducing the weight of the structure.

Nonetheless, buckling of a frame is the behaviour of the whole system (Li, 2014). Many of the current optimisation methods and techniques discussed do not readily account for all or most of the factors affecting stability. System buckling was proposed as the most efficient way to determine the buckling load of a system or the effective length of members in the system. A simple method based on hand calculation of the story buckling was suggested to determine the system buckling load of plane sway rigid frames (Li, 2014). The method was based on the premise that the frame buckled within a story and that the stiffness of the story tends to zero as the system nears buckling. The stiffness of a story was found by first calculating the stiffness without loading and then modifying it with a negative stiffness which represents the effect of the vertical load on the horizontal stiffness. The second-order phenomenon, P-δ effect, was considered by applying a modification factor to the critical load obtained from the first-order analysis. This method proved to be successful when applied to rigid frames with infinitely rigid or ‘stiff’ beams, yet

when applied to frames with non-rigid beams or beams that are not infinitely stiff, it provided the upper-bound value of the system buckling load.

1.1.1 Comparative Analysis

The design methods which utilised optimisation techniques such as the genetic algorithm or harmony search method (Camp, et al., 1998 and Kameshki & Saka, 2001), made use of design constraints based on the strength and serviceability requirements outlined in design codes. The only provision made for buckling in these studies, was the lateral torsional buckling check for beam-column members. The main focus of these design methods was to minimise weight while constrained to design variables for strength and serviceability.

Optimisation methods based on the stability criterion all considered buckling through different approaches. Pezeshk & Hjelmstad (1991), proposed a method which only required the buckling eigenvalues and eigenvectors of the structure. This method also utilised the optimality-criteria method, and its designs were constrained to have a constant weight.

Manickarajah, et al. (2000) also made use of the eigenvalue problem in its method however using finite element analysis. Its objective was also based on improving the fundamental buckling load by local modification of each element after subsequent iterations. It was centred on shifting the material from the strongest part of the structure to the weakest part.

An instability coefficient was introduced as a surrogate for the dominant eigenvalue of the linearized buckling problem in the method proposed by Gil-Martin, et al. (2006). It also used approximate expressions of second-order phenomena to identify those components that need to be modified after each iteration. This method involved a story-by-story optimisation yet the instability coefficient was calculated for each member in a story.

A simple method was developed to account for the system buckling load using story buckling analysis (Li, 2014).

Considering the current optimisation methods discussed, specifically those based on a stability criterion, a gap still exists in this field in that none of the methods account

for buckling of the system in its entirety. Furthermore, these methods did not demonstrate that an optimised frame is achieved where full member capacity was utilised which can address the common practice of over-design and subsequent higher costs.

Subsequently, the research will focus on developing a simple approach on the optimisation of frame structures against buckling based on system stability approach, which will make a significant contribution to the field of structural engineering design. The method will be a systematic approach in that optimisation will attempt to achieve all stories buckling simultaneously as opposed to shifting the critical story or member to a more optimal position in the structure. This will also reduce material wastage by ensuring full member capacity utilisation under buckling. The application of the method to various frame examples will evaluate its ability to achieve these outcomes by comparing it to accepted methods used in the design environment.

2 RESEARCH METHODOLOGY

The methodology applied to develop the optimisation procedure is outlined in the subsequent sections. It is initially developed for a no sway frame. A method is first derived which expands on the current buckling analysis methods and is based on a story approach to buckling as opposed to an isolated member analysis.
The method is then incorporated into an optimisation method designed with the intent to achieve full member utilisation with all stories buckling at the same time. The derivation of this story-based buckling load approach as well the assumptions and objectives of the optimisation method proposed, is presented in this chapter.

2.1 System buckling load of a no sway frame

Before an optimisation technique can be derived, a method is needed to calculate the system buckling load of no sway frames under 'non-rigid' beam conditions.

With the desire of developing a system buckling analysis approach, Li (2014) proposed a simple method for the evaluating of system buckling load of plane sway frame, which can easily be performed by hand. It is a system approach which takes into effect load pattern, stiffness distribution and other aspects in the determination of system buckling load. However, this method is applicable to sway frames only, and not for no sway frames. More importantly, this method is based on the assumptions of 'rigid' beam-column connections and that the beam, compared with the column connected to it, can be considered as "rigid". It is known that most structures in engineering practice do not meet the requirement set by the second assumption, i.e., 'rigid beam'. Essentially, what is obtained with this system buckling analysis method is the upper-bound of the system buckling load. When the beam is not considered as rigid, the real system buckling load would be smaller than this upper-bound obtained. Based on this method, some further developments were made.

Firstly, with the same assumptions outlined previously, an approach for obtaining the upper-bound of the system buckling load of a plane no sway frame is presented by introducing the concept of an accompanying sway frame. The upper-bound system

buckling load is the buckling load when the stiffness of beams in the frame can be considered as infinity when compared with the stiffness of the columns connected to these beams. For a real no sway frame, the stiffness of its beams cannot be considered infinity and accordingly, its system buckling load does not reach the upper-bound. The system buckling load can be evaluated from this upper-bound after it is modified by real parameters of the frame. The suggested system buckling approach developed for such no sway frames under these conditions is outlined in the next section.

2.1.1 Upper-bound of the system buckling load of rigid no sway plane frames

Based on the story buckling method for system buckling load of a plane sway frame (Li, 2014), an approach for obtaining the upper-bound of a no sway plane frame is suggested in this report. As mentioned previously, this method obtains the upper-bound of the system buckling load and it is clear that structures in the engineering practice do not meet this 'rigid beam' assumption. Thus, in addition to further developing on this method to apply it to no sway frames, the method presented in this report also obtains the real system buckling load of the system by taking into account real beams that are not infinitely stiff.

Firstly, the upper-bound system buckling load of a no sway frame is found. The 'bracing' of the no sway frame is first released and a corresponding sway frame, referred to as the 'accompanying sway frame' is obtained. The upper-bound of the system buckling load of the accompanying sway frame can be determined with the story buckling method as suggested by Li (2014). By a comparative study of a single sway column with a single no sway column which is the same as this sway column, a modification factor, η, was derived. The upper-bound system buckling load of the no sway frame can be determined by applying this modification factor to the upper-bound of the system buckling load of its accompanying sway frame.
The procedure will be demonstrated with a single story multi-bay no sway frame, as shown in Figure 2.1, with its loads acting on it. By releasing its bracing, the accompanying sway frame is produced, as shown in Figure 2.2.

The system buckling load of the accompanying sway frame can be found as follows (Li, 2014) under the following assumptions:

a. The connections between columns and beams are rigid, and
b. Compared to the stiffness of the columns, the rigidity of the beams connected to the columns can be considered as infinite.

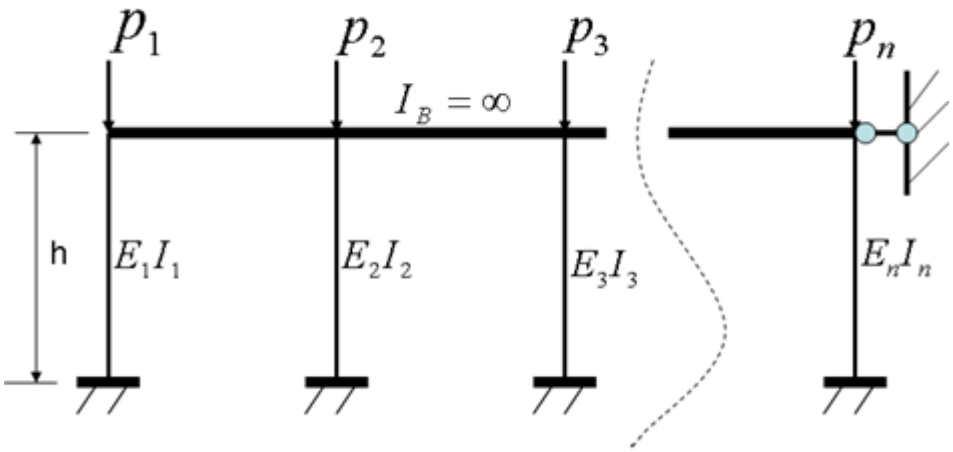


Figure 2.1 One story multi-bay no sway frame with its respective loads.

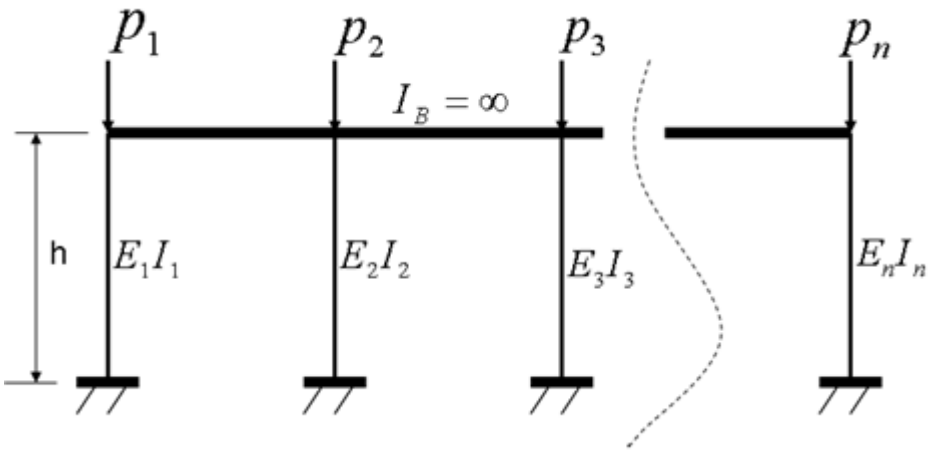


Figure 2.2 Accompanying sway frame of the one story multi-bay frame.

The horizontal stiffness of the system shown in Figure 2.2 before loading (without axial loads) is calculated as:

$$K = \sum_{i=1}^{n} \frac{12E_iI_i}{h^3} \quad (1)$$

where *i* refers to the *i*-th column.

Considering first the P-Δ effect only, the influence of axial loads on horizontal stiffness of the system can be accounted for by introducing the following negative stiffness:

$$K_P = \sum_{i=1}^{n} \frac{P_i}{h} = \frac{P}{h} \sum_{i=1}^{n} \gamma_i \quad (2)$$

where $\gamma_i = \frac{P_i}{P}$.

After loading, the horizontal stiffness of the system becomes:

$$K_{md} = K - K_p \quad (3)$$

As the system approaches buckling, the stiffness of the system, K_{md}, becomes zero;

$$K_{md} = K - K_p = \sum_{i=1}^{n} \frac{12E_iI_i}{h^3} - \frac{P}{h} \sum_{i=1}^{n} \gamma_i = 0 \quad (4)$$

By solving Equation (4), the buckling load of the system, *P*, produced by the first-order analysis above (considering P-Δ effect only), can be obtained.

For a full buckling analysis, the P-δ effect needs to be considered and *P* obtained previously, needs to be modified by a factor, β. The buckling load of the accompanying sway frame can then be obtained as:

$$P_{cr}^{sway} = \beta \times P \quad (5)$$

where $\beta = \frac{\pi^2}{12}$ is the modification factor.

Using the analysis outlined above, the upper-bound buckling load of a sway column with a fixed base and the other end restrained from rotating, as shown in Figure 2.3, can now be determined as $P_{cr}^{sway} = \frac{\pi^2 EI}{h^2}$. If this column is braced and becomes a no sway column, as shown in Figure 2.3, the buckling load then becomes, $P_{cr,upper-bound}^{no\ sway} = \overline{P_{cr}} = \frac{4\pi^2 EI}{h^2}$. Thus, the upper-bound load modification factor, η, for a column with end conditions as per Figure 2.3, can be obtained as the ratio between these upper-bound solutions:

$$\eta = \frac{\overline{P_{cr}}}{P_{cr}^{sway}} = 4 \quad (6)$$

Therefore, the upper-bound of the system buckling load of the original no sway frame can be obtained as:

$$\overline{P_{cr}} = \eta \times P_{cr}^{sway} \quad (7)$$

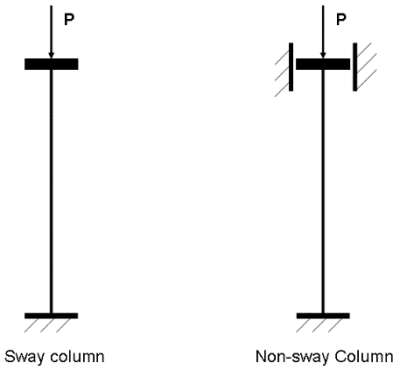


Figure 2.3 A sway column with both ends restrained and the no sway column.

If one end of the column can rotate such as the free end shown in Figure 2.4, or the case of a pinned/hinged base, the system buckling of the sway column becomes, $P_{cr}^{sway} = \frac{\pi^2 EI}{4h^2}$. Once it is braced and cannot sway, the system buckling load becomes $\overline{P_{cr}} = \frac{\pi^2 EI}{0.49h^2}$. In this case, the modification factor η for a column with one end free is found to be $\eta = 8.1633$.

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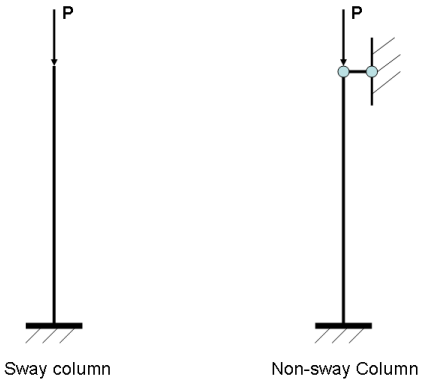


Figure 2.4 A sway column with one free end and the no sway column.

It should be emphasised that the system buckling load produced from the procedures described above, is the upper-bound of the system buckling load of a no sway frame since it was obtained under the assumptions that: i) the connections between columns and beams are rigid and, ii) the rigidity of beams is much higher than that of the columns and can be considered as infinity. For a real frame, where these assumptions are less likely to be valid, such as the infinitely 'rigid' beams, the system buckling load is certainly lower than these upper-bound solutions. When the joints are not rigid or the beams cannot be considered as rigid, the restraint on the columns becomes less and the system buckling load will also reduce accordingly. Thus, the

system buckling load produced before is called the upper-bound system buckling load. This upper-bound solution would need to be modified to account for these conditions of non-rigid joints and non-rigid beams. However, the focus of this research will remain on the latter, namely the evaluation of system buckling load of a no sway rigid frame where the beams are not considered as rigid, or infinitely stiff.

2.1.2 System buckling load of rigid no sway rigid plane frames

In order to derive the modification factor to apply to the upper-bound system buckling loads, it must be noted that the system buckling load of a frame is affected by the load and stiffness distribution. The load pattern and stiffness distribution most often seen in engineering practice is considered in the analysis. The procedure to obtain the real system buckling load which accounts for 'real' beams is outlined as follows.

A single story no sway frame with the number of bays =*b* and the loads acting on it, are presented in Figure 2.5. All columns, except the two columns at the ends of the frame, bear a load of 2*P* and have a stiffness of *I,* and the beams have a stiffness of *nI* ($n=^{I_B}/_{I_C}$).

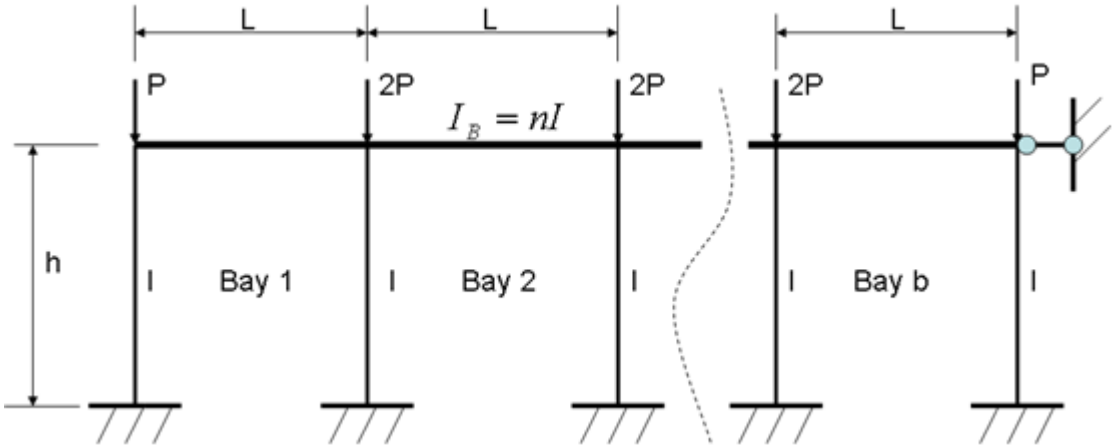


Figure 2.5 A single story b bay no sway frame with its load-stiffness pattern.

If the number of bays, *b*, is singular, the frame shown in Figure 2.5 can be 'folded' into a one bay frame as shown in Figure 2.6 (a); similarly, if the bay number *b* is even, the one bay frame obtained is shown in Figure 2.6 (b). These frames are called

the equivalent one bay frames of the original frames. It must be noted, that when the multi-bay frame is folded, the loads on a column should remain with the column.

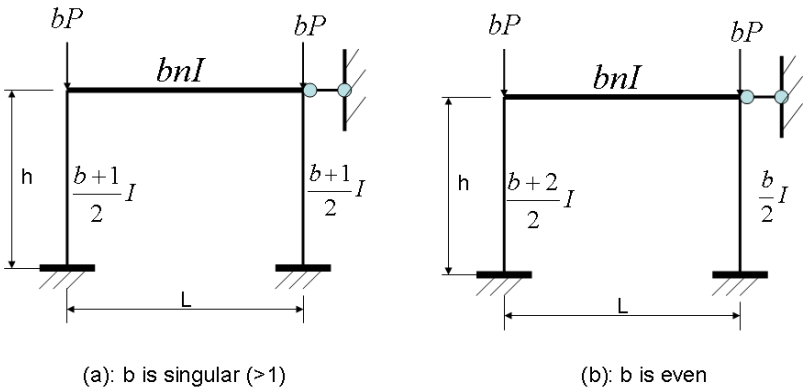


Figure 2.6 Equivalent one bay frame of a frame with b bays.

Two theorems are used in the analysis of the system buckling load:

- Theorem 1: The equivalent one bay no sway frame has the same system buckling load as that of the original no sway frame.
- Theorem 2: The system buckling load of a no sway frame system remains unchanged if the load and stiffness of each member in the system is multiplied by the same constant.

According to these theorems, the system buckling load of a no sway frame is the same as the system buckling load of its normalized equivalent one bay frame as shown in Figure 2.7. Two scenarios are considered; namely, when the number of bays is singular and even.

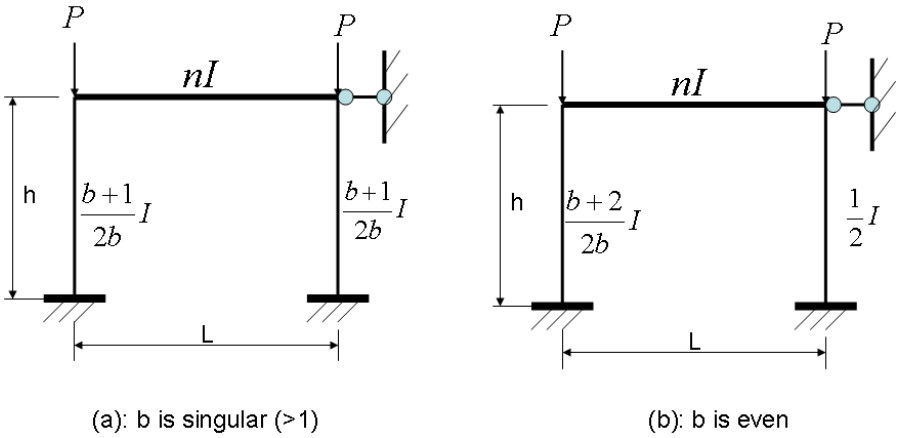


Figure 2.7 Normalised equivalent one bay frame with b number of bays.

Number of bays is singular

If the number of bays, *b*, is singular as shown in Figure 2.7 (a), and *b*>1 with a minimum of 3, the stiffness of the columns $\frac{b+1}{2b}I$ in the normalised equivalent frames when *b*=3, becomes $\frac{b+1}{2b}I=\frac{3+1}{2\times3}=\frac{2}{3}I$. Conversely, as *b* tends to infinity, the limit of the column stiffness $\frac{b+1}{2b}I$ is $\frac{1}{2}I$. The two frames corresponding to *b*=3 and *b*=∞ are shown in Figure 2.8.

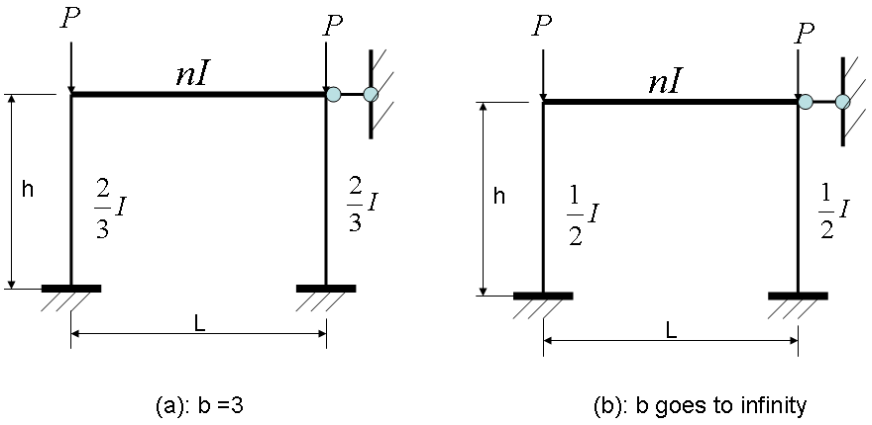


Figure 2.8 Normalised equivalent one bay frames when b is singular.

System buckling analysis using finite element analysis software (FEA) ANSYS, is performed on the frames shown in Figure 2.8 at different beam stiffness' (values of *n*) and their respective system buckling loads obtained is listed in Table 2.1.

Table 2.1 System buckling load of normalised equivalent one bay frames at different values of n.

Beam stiffness ratio, *n*	**System buckling load** $\times\frac{EI}{h^2}$ **(*b*=3)**	**System buckling load** $\times\frac{EI}{h^2}$ **(*b*=infinity)**
0.001	13.4646	10.0852
0.01	13.5019	10.1104
0.1	13.8657	10.2947
0.5	15.3125	11.0474
1	16.7881	11.8745
2	18.9313	13.1968
3	20.3665	14.1878
4	21.3706	14.9483

5	22.1022	15.5510
20	25.0600	18.3535
40	25.3019	19.0090
60	25.4660	19.2352

With the simple approach suggested earlier, the upper-bound of the system buckling load of the frame shown in Figure 2.7 (a) can be obtained as follows:

$$K = 2 \times \frac{12E\frac{b+1}{2b}I}{h^3} = \frac{24EI}{h^3} \times \frac{b+1}{2b} \quad (8)$$

$$K_P = 2 \times \frac{P}{h} \quad (9)$$

$$K - K_P = \frac{24EI}{h^3} \times \frac{b+1}{2b} - 2 \times \frac{P}{h} = 0 \quad (10)$$

Solving Eq. (10) yields:

$$P = \frac{12EI}{h^2} \times \frac{b+1}{2b}$$

Using Eq. (5), the upper-bound system buckling load of the accompanying sway frame of the frame in Figure 2.7 (a) can be obtained:

$$P_{cr}^{sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{12EI}{h^2} \times \frac{b+1}{2b} = \frac{\pi^2 EI}{h^2} \times \frac{b+1}{2b}$$

Therefore, the upper-bound of the system buckling load of the no sway frame, $\overline{P_{cr}}$, corresponding to I_B=nI=∞, can be obtained with Eq. (6) and (7):

$$\overline{P_{cr}} = \eta \times P_{cr}^{sway} = 4 \times \frac{\pi^2 EI}{h^2} \times \frac{b+1}{2b} \quad (11)$$

In this scenario, Eq. (11) indicates that the upper-bound of the system buckling load, $\overline{P_{cr}}$, is related to the number of bays, *b*; as a result:

$$\overline{P}_{cr} = \begin{cases} \frac{8\pi^2}{3} \times \frac{EI}{h^2} & when \quad b = 3 \\ 2\pi^2 \times \frac{EI}{h^2} & when \quad b \to \infty \end{cases} \quad (12)$$

The system buckling load coefficient, μ, can then be defined as the ratio of the system buckling load to its upper-bound:

$$\mu = \frac{P_{cr}}{\overline{P_{cr}}} \quad (13)$$

On the contrary, when the beam is not 'rigid', the length of the beam, L_B, will affect the restraint from the beam on the column. To take this effect into account, the parameter, λ, is introduced:

$$\lambda = \frac{I_B L_C}{I_C L_B} \tag{14}$$

According to this definition, for the frame shown in Figure 2.7 (a), at L_B=1.5h:

$$\lambda = \frac{I_B L_C}{I_C L_B} = \frac{nI \times h}{\frac{b+1}{2b} I \times L} = \frac{2b}{b+1} \times \frac{2}{3} n = \begin{cases} n & b = 3 \\ \frac{4}{3} n & b \to \infty \end{cases} \tag{15}$$

Design graphs can now be produced using the results in Table 2.1, as well as equations Eq. (12), Eq. (13), Eq. (15). The design graphs shown in Figure 2.9 indicate the relationship between the system buckling load coefficient μ and λ for the two frames shown in Figure 2.8, which can now be used in further frame analysis.

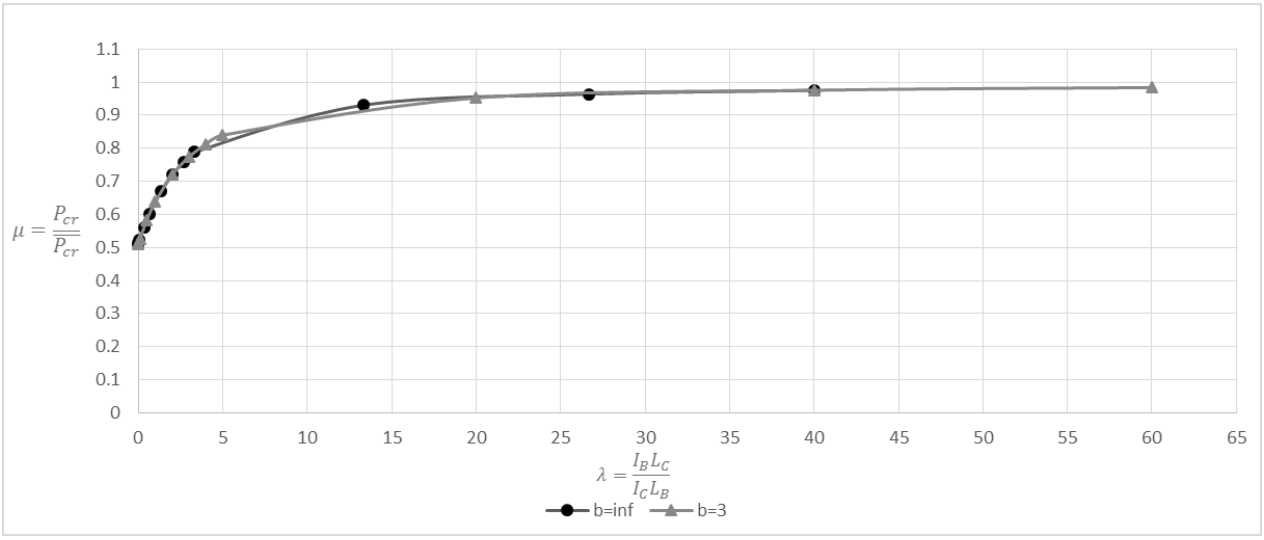


Figure 2.9 Design graph when number of bays is singular.

In Figure 2.9, it is seen that the two graphs plot close to each other and thus it would be reasonable to use the graph corresponding to b=∞ for design examples irrespective of the number of bays. The value of λ for this frame can be easily determined. For a no sway frame where the number of bays, b, is between three and infinity, the corresponding column stiffness $\frac{b+1}{2b}$, should lie between 0.5 and 0.6667 relating to b=3 and b=∞, respectively. Lastly, using the design graph in Figure 2.9, the system buckling load can be easily be evaluated from:

$$P = \mu \times \overline{P}_{cr} \tag{16}$$

Number of bays is even

If the number of bays, *b*, is even, the normalized equivalent one bay frame produced is shown in Figure 2.7 (b). The number of bays can be between two and infinity. At the lower end, *b*=2, $\frac{b+2}{2b} = \frac{2+2}{2\times 2} = 1$, and at the higher end, as b goes to infinity, $\frac{b+2}{2b}$, goes to $^1/_2$. The two frames corresponding to these column stiffness' are presented in Figure 2.10.

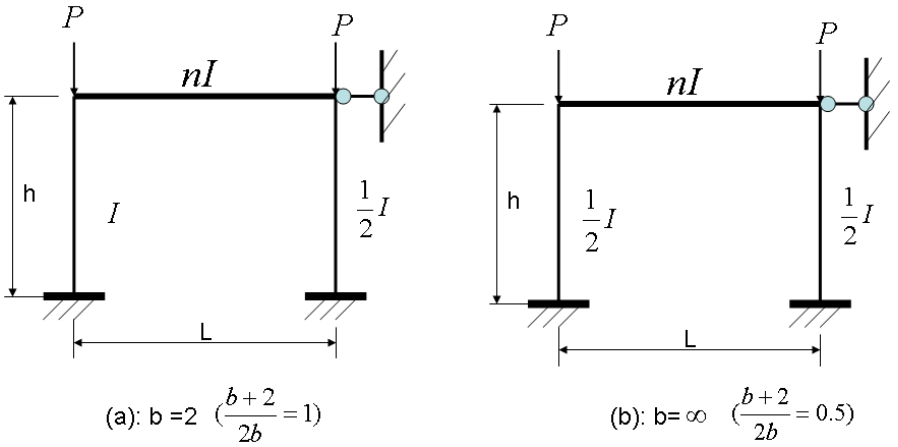


Figure 2.10 Normalised equivalent frames when the number of bays is even.

The design graphs for this case is produced using the same procedure when the number of bays are singular, and is shown in Figure 2.11.

It must be noted however, that the determination of the upper-bound of the system buckling load of the frame shown in Figure 2.10 (a) corresponding to *b*=2, is calculated by taking into consideration the column with the lower stiffness of $\frac{1}{2}I$. For this no sway frame, the buckling of the system is 'local', since the column with stiffness $\frac{1}{2}I$ will reach buckling before the other column. In addition, the upper-bound solution of this frame can easily be determined as $\overline{P}_{cr} = 2\pi^2$.

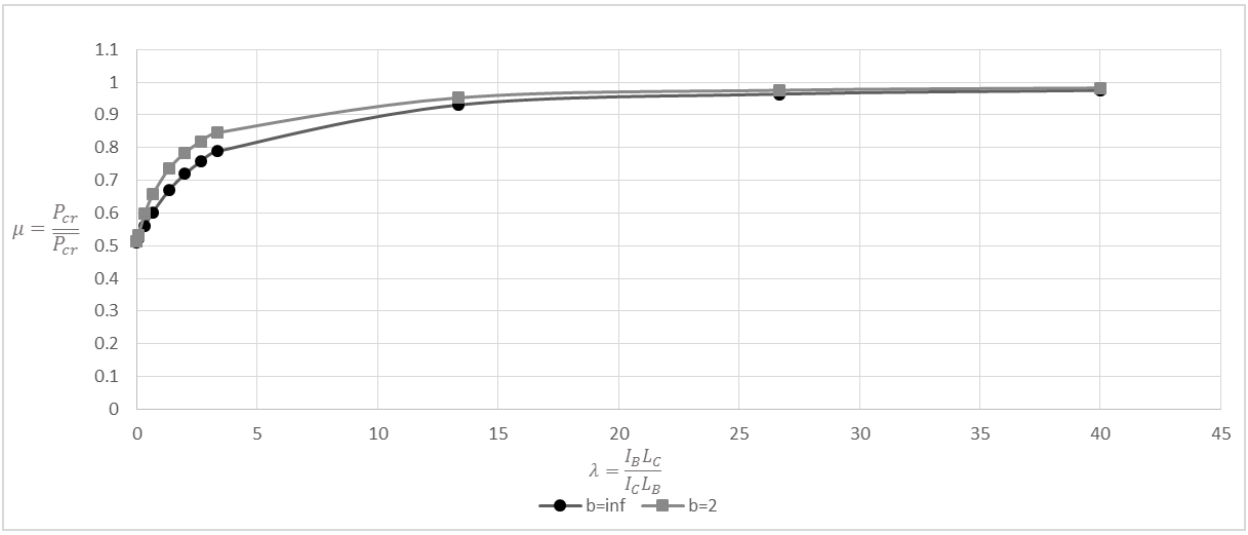


Figure 2.11 Design graph when number of bays is even.

It is seen from Figure 2.11, that the two design graphs deviate it from each other in the lower range of λ and hence the value of μ for a frame where the number of bays lies between two and ∞, will need to be interpolated between these two graphs. From the design graphs produced in Figure 2.9 and 2.11, it is evident that only two design graphs are required in the evaluation of the buckling load of multi-story frames, that being the graph corresponding to *b*=2 and *b*=∞. It was revealed that when the stiffness of the columns is the same within the one bay one story equivalent no sway frame, as in the case of *b*=3 and *b*=∞ (singular bay number), the value of μ did not differ significantly as indicated by the design graphs for these frames plotting close to each other. The graph for *b*=∞ can be used to interpolate the value of μ for these frames. In the case of even bay frames, the value of μ can be interpolated from the *b*=2 and *b*=∞ graphs.

The method proposed in this section was derived for multi-bay one story frames, yet it will be demonstrated that it is also valid for multi-story frames through the application examples presented in Section 3.1. This can be attributed to the fact that there is no beam shear in no sway frames hence the influence of the adjacent stories on the system buckling load is minimal. This was also established by Li et al. (2016) when demonstrating their method for stability analysis of multi-story frames, where it was found that in no sway rigid-base frames, the error produced from their method had no correlation to the number of stories hence no influence on the buckling load.

Furthermore, in the normalised frames presented, the load arrangements on the frames are either 2P or P, but a FEA analysis performed in ANSYS on a single bay single story frame shown in Figure 2.12, indicated that the influence of an increase in loads on the value of μ is minimal. Higher loads are commonly found in the engineering practice especially in multi-story frames.

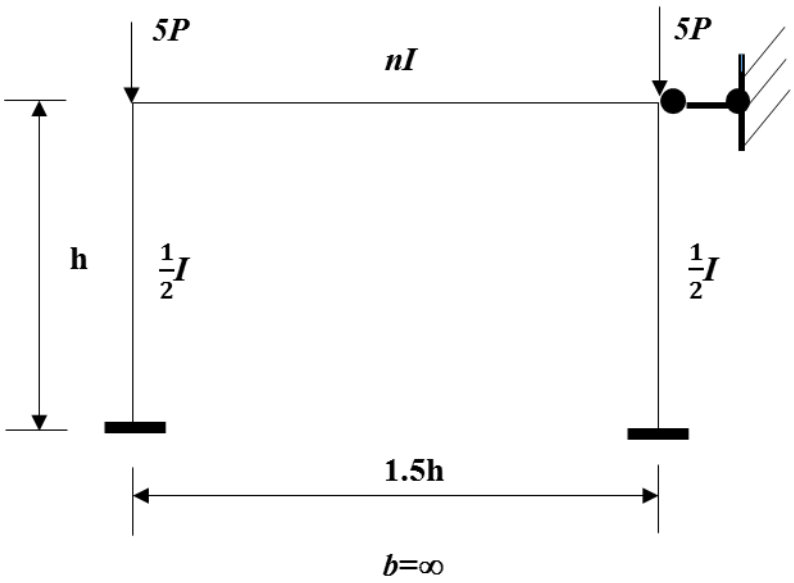


Figure 2.12 Equivalent frame when 'b' is infinity with increased loads.

A higher load on such a one story frame would originate from the fact that the multi-story frames would be folded into a one bay one story equivalent frame, and the loads from the higher stories are transmitted to the first story. Thus, the need to investigate the influence of loads. For the investigation of this effect, the upper-bound system buckling load for the frame in Figure 2.12 was calculated by the method prescribed earlier, the real system buckling load was given by FEA and, Eq. (13) was used to describe the relationship between these two values. The μ values obtained for this frame for various values of *n*, was compared to those obtained from the frame in Figure 2.8 (b) for $b=\infty$ frame. The results are summarised in Table 2.2.

Table 2.2 Comparison of μ values to indicate the influence of loads on equivalent frames when $b=\infty$.

Beam stiffness ratio, *n*	**System load coefficient, μ (P=*P*)**	**System load coefficient, μ (P=*5P*)**
0.001	0.511	0.511
0.01	0.513	0.512
0.1	0.522	0.522
0.5	0.560	0.560
1	0.602	0.602
2	0.669	0.669
3	0.719	0.719

From Table 2.2, it is clear that the influence of loads on the value of μ is minimal and thus the design graphs derived in Figure 2.9 and 2.11 for a single story single bay frame can be applied to multi-story single bay frames.

As previously revealed, when the stiffness of the columns is the same within the one bay one story equivalent no sway frame, as in the case of b=3 and b=∞, the value of μ did not differ regardless of the number of bays. Hence only two design graphs are needed for future frame analysis, namely b=2 and b=∞ in Figure 2.11, which will be used in the frame applications in Section 3.1.

A method to obtain the system buckling load of no sway frame has been outlined which will be used in the development of the optimisation method in Section 2.3. It must be reiterated that the buckling analysis derived in this section, developed on Li's story based buckling approach of sway frames with rigid beams by accounting for 'non-rigid' beams with fixed base support conditions (2014). Once the optimisation procedure can be validated based on these frame conditions, it can be further improved to consider other conditions such as semi-rigid joints or pinned bases. To simplify the evaluation process, the present research undertaken focused on the main objective of optimisation and was limited to the frame conditions as stated.

2.2 Research hypothesis

A simple method to achieve optimisation of plane rigid no sway frame structures against buckling may be achieved through an approximate second order analysis, by either maximising the critical buckling load of the system while fixing the weight of the structure, or minimising the weight of the structure for a given buckling load.

2.3 Optimisation Procedure

2.3.1 Assumptions and Design Criteria

The following assumptions and design criteria were applied in the development of the optimisation method:

i. The connections between the column and beams are rigid;
ii. The support condition of the frames analysed is limited to fixed base only, as the method to obtain the system buckling load of the frame was derived under these support conditions;
iii. Only plane no sway frames are considered. Sway frames were considered in the analysis however the methodology failed to simulate this situation as discussed in Section 3.2;
iv. The same section properties are applied within a story; and
v. The optimised section found will not be selected from a database of existing commercially available sections, but rather expressed as a stiffness. The sectional properties obtained can then be used to select a commercially available section with the closest properties to this optimised stiffness.

2.3.2 Optimisation Method

The method presented develops on the first-order analysis optimal design proposed by Gil-Martin, et al. (2006) by incorporating the story buckling method from Li (2014) and the method to obtain the 'real' system buckling load as outlined in Section 2.1. The optimisation method aims to have all stories buckling at the same time.

The optimisation process is as follows:

1. Select initial section stiffness' for each story for the first iteration, *k*, where its stiffness values are expressed in relation to the stiffness of the first story. The same section stiffness must be used within a story.
2. Calculate the upper-bound system buckling load for each story *i*, $\overline{P_{cr,i}}$ using Eq. (7).
3. Modify $\overline{P_{cr}}$ to account for non-rigid beams and calculate the story buckling load, $P_{cr,i}$, for each story using Eq. (16) and the design graphs in Figure 2.9.
4. Identify the critical story; the one with the minimum buckling load, $P_{cr,\,min}$.
5. The ratio of the each stories' buckling load *Pcr* to the minimum buckling load, $P_{cr,\,min}$, is calculated, namely the story buckling ratio, α_i, for each story, where $\alpha_i = \frac{P_{cr,min}}{P_{cr,i}}$.
6. Determine the new section stiffness for the columns and beams, I_b and I_c, of each story for the next iteration, k=2, based on α_i and the previous stiffness as follows:
   Beam/Column stiffness $_{k=2,i} = \alpha_i \times$ Beam/Column stiffness $_{k=1,i}$
7. Repeat the optimisation procedure (step 2-5) until $\alpha \geq 0.9$ for all stories; at this point the optimisation process is complete.

For ease of computation, the optimisation process was programmed using MATLAB software. The MATLAB code can be found in Appendix A.

The method will be applied to frames with various loading arrangements and geometry and the results will be compared to a system buckling analysis performed on a FEA software. The results are measured against FEA software as the research is aimed at practicing engineers and this approach is commonly used in the design field. The software used in the analysis was ANSYS Mechanical APDL. An Eigenvalue Buckling Analysis (linear behaviour) was performed using a 3D 2-node line with 6 degrees of freedom at each node. All frame elements were modelled with this line element. This type of analysis yields the theoretical buckling strength (bifurcation point) of a structure (ANSYS Inc., 2013). The difference between the results obtained from the method and FEA will then illustrate the validity of the

method and the extent to which the hypothesis is met. The weight of the frames before and after optimisation is compared. Furthermore, the buckled shape obtained from FEA is examined to: i) verify that all stories buckle under the first mode and, ii) evaluate the deviation of the members from a straight line in the deflected shape to indicate that full member capacity is being utilised.

2.3.3 Optimisation Objectives

The proposed optimisation method outlined in Section 2.3.2 attempts to achieve the following:

i. The main objective is to have *all stories buckling at the same time* and to ensure the full member capacity is utilised- this could be demonstrated through the buckled shape of the frame.
ii. To output an optimal design:
    - To minimise the weight of the structure (thus reducing the cost) or maximise the critical buckling load.
    - To meet the stability criterion by verifying the system buckling load after each iteration.
iii. Identify the critical story and modify the remaining stories after each subsequent iteration.
iv. To ensure the method encompasses, to the greatest degree, the factors affecting stability of a structure.
v. To ensure that it is simple to implement.
vi. To apply the method to various load cases and structural design.

An approach has been developed to obtain a story-based buckling analysis of no sway frames, and involves a global approach to buckling thus achieving one goal of the research. The approach is derived based on the relationship of the buckling load between sway and no sway frames with a fixed base and accounts for 'non-rigid' beams. It is limited at this point in the research to no sway frames.

The system buckling load is incorporated into an optimisation procedure which aims to achieve full member capacity utilisation of the frame by resulting in a frame with all stories having the same buckling load. The procedure is simple to perform and

can be validated by various frame examples. In addition, the derivation of the system buckling load of no sway frames, can also be verified through the comparison of the optimisation results with FEA.

3 RESULTS AND DISCUSSION

The optimisation procedure described in Section 2.1 can now be applied to various frame examples to demonstrate its simplicity in providing an estimate to an optimised multi-story no sway rigid frame that can be used in the design practice. The results of the optimisation will be compared to that obtained from a system buckling analysis performed on ANSYS FEA software with the model described in Section 2.3.3 and Section 3.1.1.

The method is then attempted on sway frame applications to determine its effectiveness. As in the case of the no sway frames, the approaches to derive the system buckling load of sway frames, to be incorporated into the proposed optimisation method, is presented. Frame examples are considered in the approaches to compare its results with FEA and evaluate its success in determining the system buckling load of such frames.

3.1 No sway Analysis

The optimisation procedure is applied to various no sway frame examples from literature with its results presented. A comparison of the results with a FE system buckling analysis is discussed to assess its validity and effectiveness in achieving the initial optimisation objectives.

3.1.1 Assumptions

The following assumptions are applicable, as before:

i. The connections between the columns and beams are rigid.
ii. All frames have fixed base support conditions.
iii. All frames are fully braced hence no sway permitted.
iv. Frames are braced out-of-plane.

The weight of the frame for all application examples was calculated using a steel square bar section of density 7800 kg/m^3. This section was used for ease of computation and simplicity sake, however further studies can expand on the sensitivity of the method to different sections. The dimensions of the square bar used

for the column and beam is determined from the second moment of area, *I*, of a square section. For example, if a column has a *I* of $7.5\text{x}10^{-4}\,\text{m}^4$, the size of the square bar is given by: side $= \sqrt[4]{12\times7.5\times10^{-4}} = 0.308\text{m}$. The weight of the frame is then given by: $\rho_{,\text{steel}} \times \sum\text{stories} \times (\sum\text{beams in a story} \times \text{side}^2 \times \text{length of the beam} + \sum\text{columns in a story} \times \text{side}^2 \times \text{height of column})$. The weight of the frames calculated are indicative only. The second moment of area is either initially assigned an arbitrary value if stated as a variable or is given from the literature, as ANSYS software requires numerical values for analysis. When the length of a member is provided in terms of *h*, the value was arbitrarily assigned as 1m. A Young's Modulus, *E*, of 200GPa was used in the FE model to calculate the buckling load.

3.1.2 Application Examples

Example 1: 2 story 1 bay frame

A two story one bay frame and with its loading arrangement as shown in Figure 3.1, was optimised with all columns initially having a stiffness of *I* and all beams having a stiffness of *0.5I*. All stories have the same height.

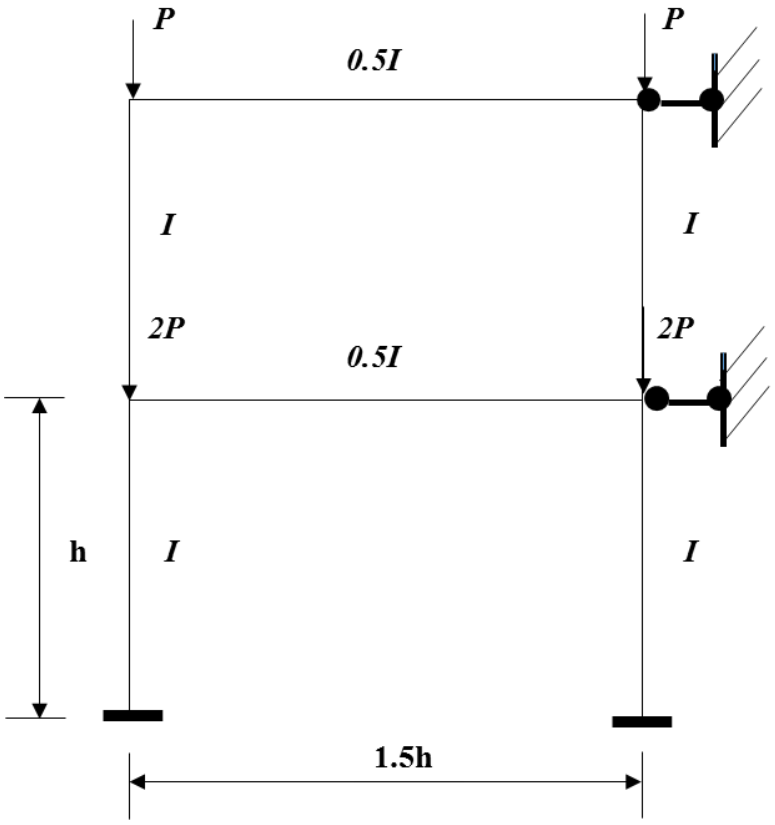


Figure 3.1 A two story one bay frame.

The system buckling load of the original frame is first determined to obtain the story with the minimum story buckling load, from which the other stories can be optimised from.

The accompanying sway frame can be obtained by releasing the bracing of the no sway frame, as described before. For the accompanying sway frame, it is easy to find that the frame will buckle at the bottom story. The upper-bound story buckling load, $\overline{P_{cr}}$, can be found for this frame using the method outlined by Li (2014) for sway frames under 'rigid' beams condition.

The horizontal stiffness of this story before loading:

$$K_1 = 2 \times \frac{12EI}{h^3}$$

The negative stiffness caused by the axial loads on this story, given that the loads from the upper story are transmitted to this story:

$$K_{p1} = 2 \times \frac{3P}{h} = \frac{6P}{h}$$

After loading, the horizontal stiffness of the story becomes:

$$K - K_p = \frac{24EI}{h^3} - \frac{6P}{h} = 0$$

Solving this equation, one can obtain:

$$P = \frac{4EI}{h^2}$$

Taking P-δ effect into account, the system buckling load of the accompanying sway frame then becomes:

$$P_{cr,sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{4EI}{h^2} = \frac{\pi^2 EI}{3h^2}$$

Applying the relationship previously shown between the upper-bound system buckling loads of sway and no sway frames, the upper-bound system buckling load, corresponding to $I_b = nI = \infty$, of the original no sway frame shown in Figure 3.1 can be obtained:

$$\overline{P_{cr,1}} = \eta \times P_{cr,sway} = 4 \times P_{cr,sway} = \frac{4\pi^2 EI}{3h^2}$$

Accounting for the non-rigid beams in the original no sway frame, the real system buckling load can be determined by considering the equivalent normalised one bay frame.

For the frame shown in Figure 3.1, since the number of bays is one; b=1; there is no need for an equivalent frame, thus:

$\rho = \frac{b+1}{2b} = 1$ and $\lambda = \frac{I_b L_c}{I_c L_b} = \frac{0.5I \times h}{I \times 1.5h} = 0.333$.

The design graph when $b = \infty$ was used to obtain the value of μ, as it was determined before that when the column stiffness is the same, the value of μ does not differ significantly. From the graph in Figure 2.9, for a λ value of 0.3333, μ= 0.5588.

Thus, the final story buckling load can be obtained:

$$P_{cr,1} = \mu \times \overline{P_{cr,1}} = 0.5588 \times \frac{4\pi^2 E \times I}{3h^2} = 7.354 \frac{EI}{h^2}$$

The same process is applied to the second story of the frame. For the first iteration of the original un-optimised frame, the story buckling loads and the total weight of the frame is summarised in Table 3.1.

Table 3.1 Iteration 1 of the optimisation procedure of Example 1.

Story number, *i*	$K_i (\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}} (\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$ $(\frac{EI}{h^2})$**	**Story buckling load ratio, α**
1	24	6	4/3	0.333	0.558	**7.354**	1
2	24	2	4	0.333	0.558	22.028	0.33
Weight of frame		4.77kg					
Critical story		**1**					

From Table 3.1, the critical story, the story with the minimum story buckling load, was found to be story 1. The story buckling load ratio α is obtained for each story as follows:

$$Pcr, ratio = \frac{Pcr,1}{Pcr,i}$$

Optimisation is reached when α is greater than 0.9 for all stories hence the frame has not been optimised.

Another iteration is performed where the new column and beam stiffness is reduced by the story load ratio α determined before. For example, the new column stiffness for iteration second iteration of story 2 is given as follows:

Story 2 column stiffness $_{k=2}$ =Column stiffness $_{k=1} \times \alpha = I \times 0.333 = 0.333I$.

The same procedure is applied on the frame using the new column and beam stiffness' with the results summarised in Table 3.2.

Table 3.2 Iteration 2 of the optimisation procedure of Example 1.

Story number, *i*	$K_i(\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}}\ (\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$ $(\frac{EI}{h^2})$**	**Story buckling load ratio, α**
1	24	6	4/3	0.333	0.558	7.354	1
2	7.92	2	4/3	0.333	0.558	7.354	1
Weight of frame		3.76 kg					
Critical story		**All stories**					

From the results of the second iteration shown in Table 3.2, it is found that optimisation has been reached as all story ratios α are equal to one. The weight of the frame has reduced by 21%. Previously the frame would have reached buckling dictated by the story with the lowest buckling load, that being the first story, and after optimisation, all stories have the same buckling load. The final optimised frame is shown in Figure 3.2.

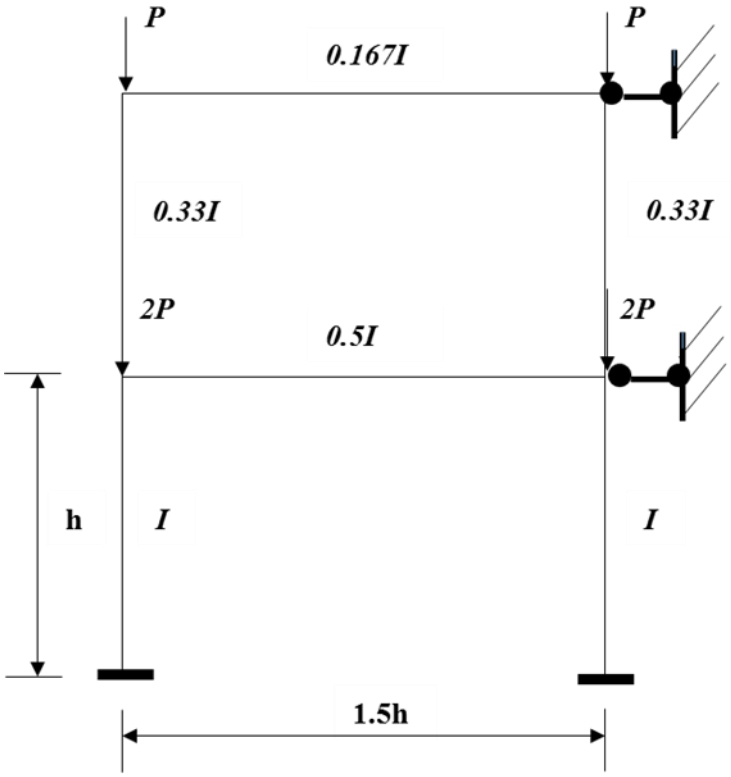


Figure 3.2 Optimised two story one bay frame.

When the optimised frame was analysed in ANSYS, a system buckling load of $5.61\frac{EI}{h^2}$ was obtained, resulting in a 31% smaller difference than the result obtained from the proposed method. The percentage difference could be attributed to the fact that the stiffness of the second story in the optimised frame is considerably lower than the first story- a third of the stiffness. As a result, this story would deflect or buckle at a lower load than calculated and an earlier failure would cause the bottom story to buckle at this lower load as well. The buckled shape of the frame obtained from ANSYS is shown in Figure 3.3. It is noted that when the upper-bound buckling loads, under rigid beam conditions, obtained from FEA and the proposed method was compared, the difference between these values was minimal. Furthermore, it was seen from the buckled shape of the frame, under rigid beams, that only the first story buckled at this load. This could indicate that the upper-bound load used to determine the system buckling load of the frame during optimisation, may have been overestimated as it did not account for the lower stiffness of the second story.

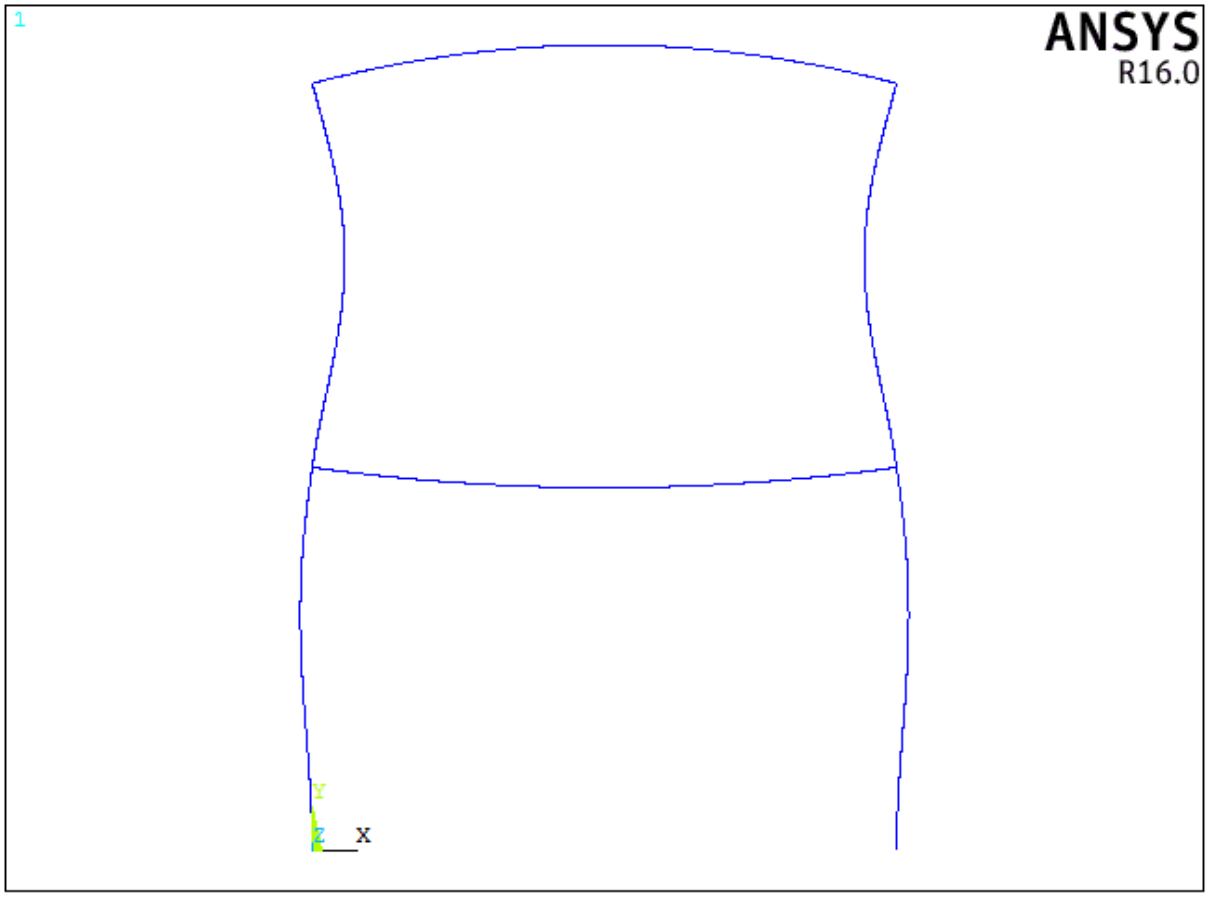


Figure 3.3 Buckled shape of the optimised two story one bay no sway frame.

This percentage difference for the system buckling load, if the FEA result is accepted as correct, is unacceptable and would need to be improved. However, one bay multi-storey frames are not often designed or found in practice and thus it would not deem necessary to improve this result for the purposes of this research at hand.

Example 2: 4 story 3 bay frame

A four story three bay frame and the corresponding loads as shown in Figure 3.4, was optimised with all columns and beams initially having a stiffness as indicated. All stories have the same height.

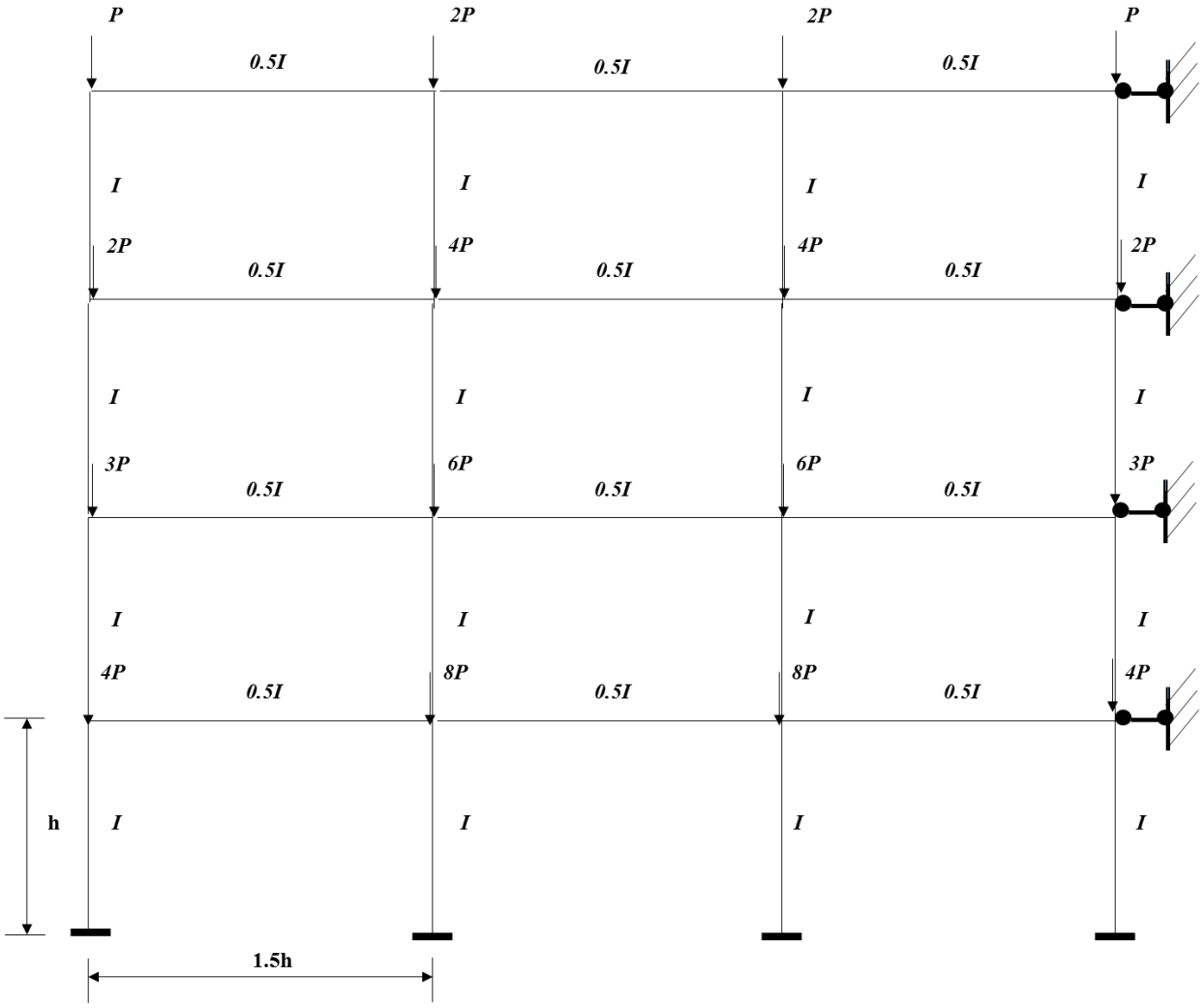


Figure 3.4 A four story three bay no sway frame.

The system buckling load of the original frame is first determined to obtain the story with the minimum story buckling load, from which the other stories can be optimised.

The accompanying sway frame can be obtained by releasing the bracing of the no sway frame, as described before. For the accompanying sway frame, it is easy to find that the frame will buckle at the bottom story. The upper-bound story buckling load, $\overline{P_{cr}}$, can be found using the method outlined by Li (2014) for sway frames under the condition of 'rigid' beams.

The horizontal stiffness of the first story before loading:

$$K_1 = 4 \times \frac{12EI}{h^3}$$

The negative stiffness caused by the axial loads on this story, given that the loads from the upper story are transmitted to this story:

$$K_{p1} = 2 \times \frac{10P}{h} + 2 \times \frac{20P}{h} = \frac{60P}{h}$$

After loading, the horizontal stiffness of the story becomes:

$$K - K_p = \frac{48EI}{h^3} - \frac{60P}{h} = 0$$

Solving this equation, one can obtain:

$$P_1 = \frac{4EI}{5h^2}$$

Taking *P*-δ effect into account, the system buckling load of the accompanying sway frame can be obtained:

$$P_{cr,sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{4EI}{5h^2} = \frac{\pi^2 EI}{15h^2}$$

The upper-bound of the system buckling load, corresponding to $I_b = nI = \infty$, of the original no sway frame shown in Figure 3.4 can be obtained:

$$\overline{P_{cr,1}} = \eta \times P_{cr,sway} = 4 \times P_{cr,sway} = \frac{4\pi^2 EI}{15h^2}$$

Accounting for the non-rigid beams in the original no sway frame, the real system buckling load can be determined using the equivalent one bay frame.

For the frame shown in Figure 3.4, the number of bays is three, b=3, thus the equivalent one bay frame for the first story, when the frame is collapsed or 'folded' into the middle bay, is shown in Figure 3.5:

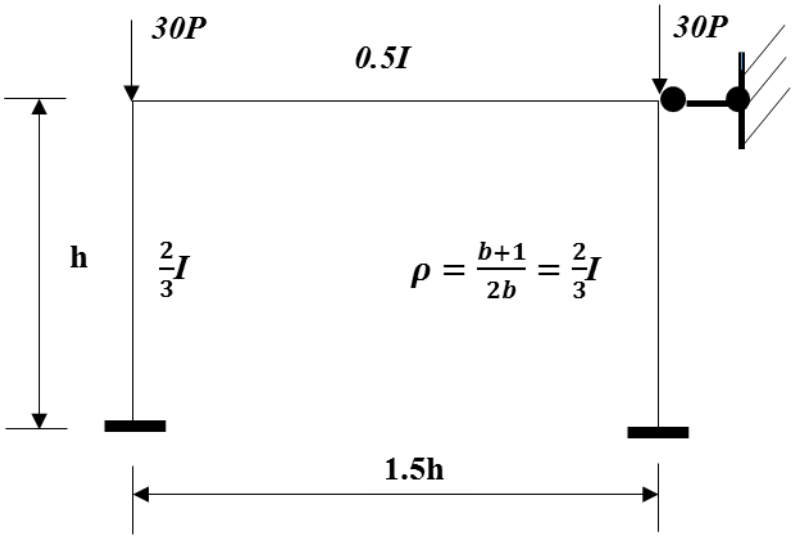


Figure 3.5 Normalised equivalent one bay frame for first story of frame in Example 2.

For the frame in Figure 3.5, the value of $\rho = \frac{2}{3}$ and $\lambda = \frac{I_b L_c}{I_c L_b} = \frac{0.5I \times h}{\frac{2}{3}I \times 1.5h} = 0.5$.

The design graph when $b = \infty$ was used to obtain the value of μ, as it was determined before that when the column stiffness is the same, the value of μ does not differ significantly. From the graph in Figure 2.9, for a λ value of 0.5, $\mu = 0.5818$.
Thus, the final story buckling load can be obtained:

$$P_{cr,1} = \mu \times \overline{P_{cr,1}} = 0.5818 \times \frac{4\pi^2 E \times \frac{1}{2}I}{15h^2} = 0.766 \frac{EI}{h^2}$$

Note that the stiffness of the column was reduced by the value of ρ when $b=\infty$ as the value of μ was taken from the design graph for this equivalent frame.

The same process is applied to the remaining three stories of the frame. For the first iteration, the story buckling loads and the total weight of the frame is summarised in Table 3.3.

Table 3.3 Iteration 1 of the optimisation procedure of Example 2.

Story number, *i*	$K_i\left(\frac{EI}{h^3}\right)$	K_{pi} **(P/h)**	$\overline{P_{cr}}\ \left(\frac{\pi^2 EI}{h^2}\right)$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$ $\left(\frac{EI}{h^2}\right)$**	**Story buckling load ratio, α**
1	48	60	4/15	0.5	0.581	**0.766**	1
2	48	36	4/9	0.5	0.581	1.276	0.6
3	48	18	8/9	0.5	0.581	2.552	0.3
4	48	6	2.67	0.5	0.581	7.656	0.1
Weight of frame		22.4 kg					
Critical story		**1**					

From Table 3.3, it can be seen that the critical story, the story with the minimum story buckling load, was found to be story 1. The story buckling load ratio α is obtained for each story as follows:

$$Pcr, ratio = \frac{Pcr,1}{Pcr,i}$$

From the values of α obtained, the frame has not been optimised as all values are not greater than 0.9.

Another iteration is performed where the previous column and beam stiffness is reduced by the story load ratio α. For example, the new column stiffness for iteration k = 2 of story 2 is given as follows:

Story 2 column stiffness $_{k=2}$ =Column stiffness $_{k=1}$×α=I×0.6=0.6I.

The same procedure is applied with the new column and beam stiffness' with the results summarised in Table 3.4.

Table 3.4 Iteration 2 of the optimisation procedure of Example 3.

Story number, *i*	$K_i\left(\frac{EI}{h^3}\right)$	K_{pi} **(P/h)**	$\overline{P_{cr}}\left(\frac{\pi^2 EI}{h^2}\right)$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $\left(\frac{EI}{h^2}\right)$	**Story buckling load ratio, α**
1	48	60	4/15	0.5	0.581	0.766	1
2	28.8	36	4/15	0.5	0.581	0.766	1
3	14.4	18	4/15	0.5	0.581	0.766	1
4	4.8	6	4/15	0.5	0.581	0.766	1
Weight of frame		14.8 kg					
Critical story		**All**					

From the results of the second iteration shown in Table 3.4, it can be seen that optimisation has been reached as all story ratios α are equal to one. The weight of the frame has reduced by 34%, with all stories having the same buckling load. The final optimised frame is shown in Figure 3.6.

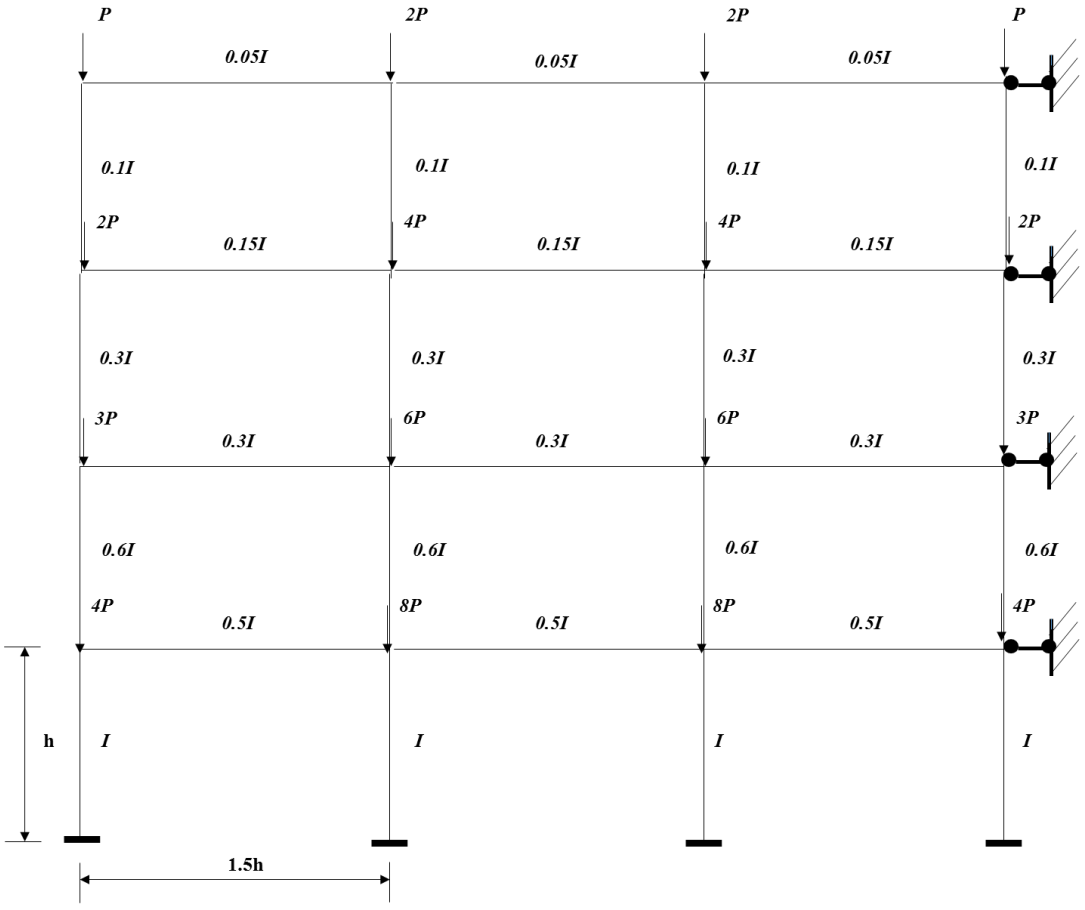


Figure 3.6 Optimised three bay four story no sway frame.

When the optimised frame was analysed in ANSYS, a system buckling load of $0.807\frac{EI}{h^2}$ was obtained, resulting in a 5% larger difference than the result obtained from the proposed method. The buckled shape of the frame obtained from FEA is shown in Figure 3.7. The percentage difference could be attributed to the load arrangement of the frame. The fourth story (top story) has a lower load applied to it and thus the initial system buckling load was considerably higher than the other stories as seen in Table 3.3. As a result, the story buckling load ratio, α, for this story is lower and the final stiffness calculated, contributed to a lower buckling load being obtained. Furthermore, from the buckled shape in Figure 3.7, it is noted that the end columns of the first story did not buckle as extensively as the inner bays. This can be an indication that the full member capacity of these columns had not been reached and hence the buckling load was found to be higher as per the FEA result. However, this percentage difference is an acceptable estimate for the purposes of an estimated simple method to optimise frame structures.

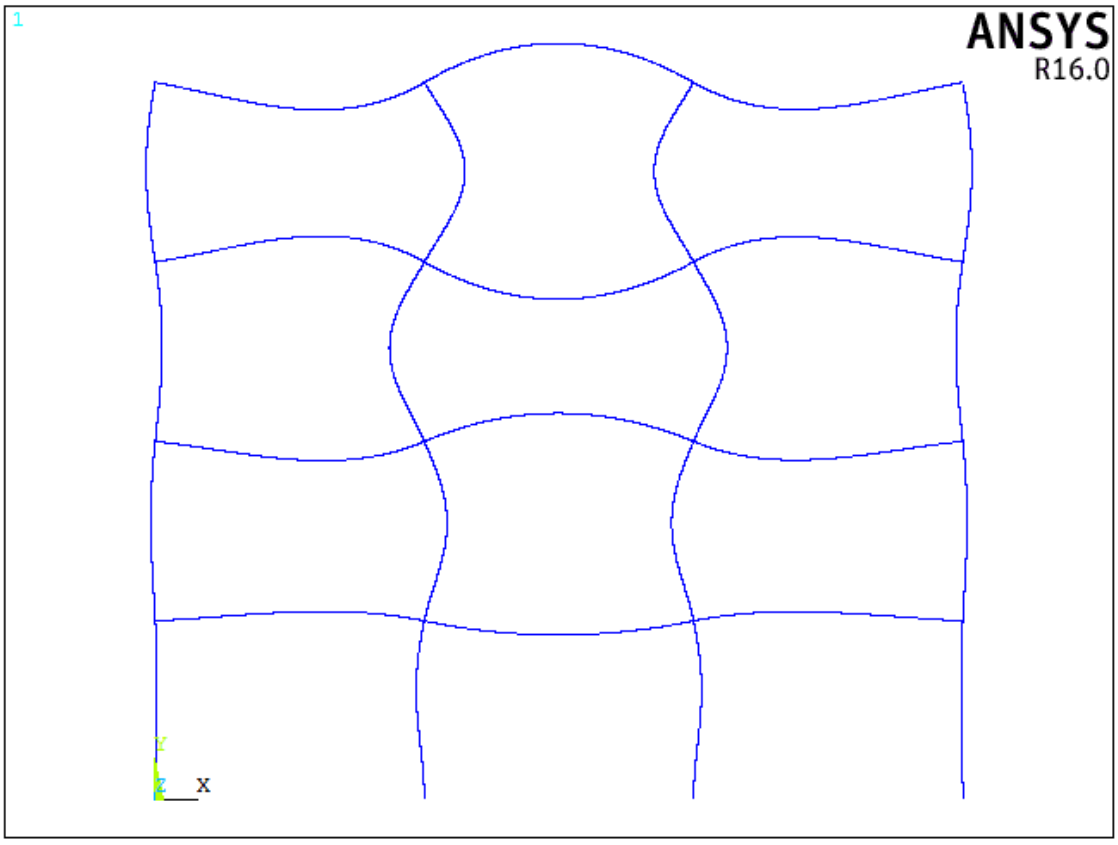


Figure 3.7 Buckled shape of the optimised four story three bay no sway frame.

The optimisation resulted in an optimised frame with a lower weight under the same system buckling load found before optimisation. If the initial objectives that were set out are considered, namely that the buckling load increases whilst the weight of the frame reduces, the frame in Figure 3.4 could be optimised again to attempt to achieve this outcome.

The results of frame buckling loads from the first iteration in Table 3.3 are referred to, but the story buckling ratio, α, is recalculated. If the story with the next lowest buckling load is selected as critical, that being story 2 in this frame, then the new α ratios, shown in Table 3.5, for each story becomes:

Table 3.5 Iteration 1 of the optimisation procedure of Example 2 with story 2 selected as critical.

Story number, *i*	$K_i\left(\frac{EI}{h^3}\right)$	K_{pi} **(P/h)**	$\overline{P_{cr}}\left(\frac{\pi^2 EI}{h^2}\right)$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$ $\left(\frac{EI}{h^2}\right)$**	**Story buckling load ratio, α**
1	48	60	4/15	0.5	0.581	0.766	1.67
2	48	36	4/9	0.5	0.581	**1.276**	1
3	48	18	8/9	0.5	0.581	2.552	0.5
4	48	6	2.67	0.5	0.581	7.656	0.167
Weight of frame		22.4 kg					
Critical story		**2**					

The new column and beam stiffness for the next iteration is calculated as before:

Story 1 column stiffness $_{k=2}$ =Column stiffness $_{k=1}$×α=*I*×1.67=1.67*I*.

The results of the second iteration with the new member stiffness' are summarised in Table 3.6. It is seen that the weight of frame reduced after optimisation and the buckling load of the system, which is determined by the story with the lowest buckling load before optimisation found to be story 1, has now increased.

Table 3.6 Iteration 2 of the optimisation procedure of Example 2 with story 2 selected as critical.

Story number, *i*	$K_i\left(\frac{EI}{h^3}\right)$	K_{pi} **(P/h)**	$\overline{P_{cr}}\left(\frac{\pi^2 EI}{h^2}\right)$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}\left(\frac{EI}{h^2}\right)$	**Story buckling load ratio, α**
1	80	60	4/9	0.5	0.581	1.276	1
2	48	36	4/9	0.5	0.581	1.276	1
3	24	18	4/9	0.5	0.581	1.276	1
4	8	6	4/9	0.5	0.581	1.276	1
Weight of frame		19.1 kg					
Critical story		**All stories**					

The optimised frame is given in Figure 3.8. When the frame was analysed in FEA, a system buckling load of $1.349\,\frac{EI}{h^2}$ was obtained, resulting in a 5.4% larger difference than the result obtained from the proposed method. The same difference was obtained in the first analysis of the optimised frame when the critical story was selected as the first story.

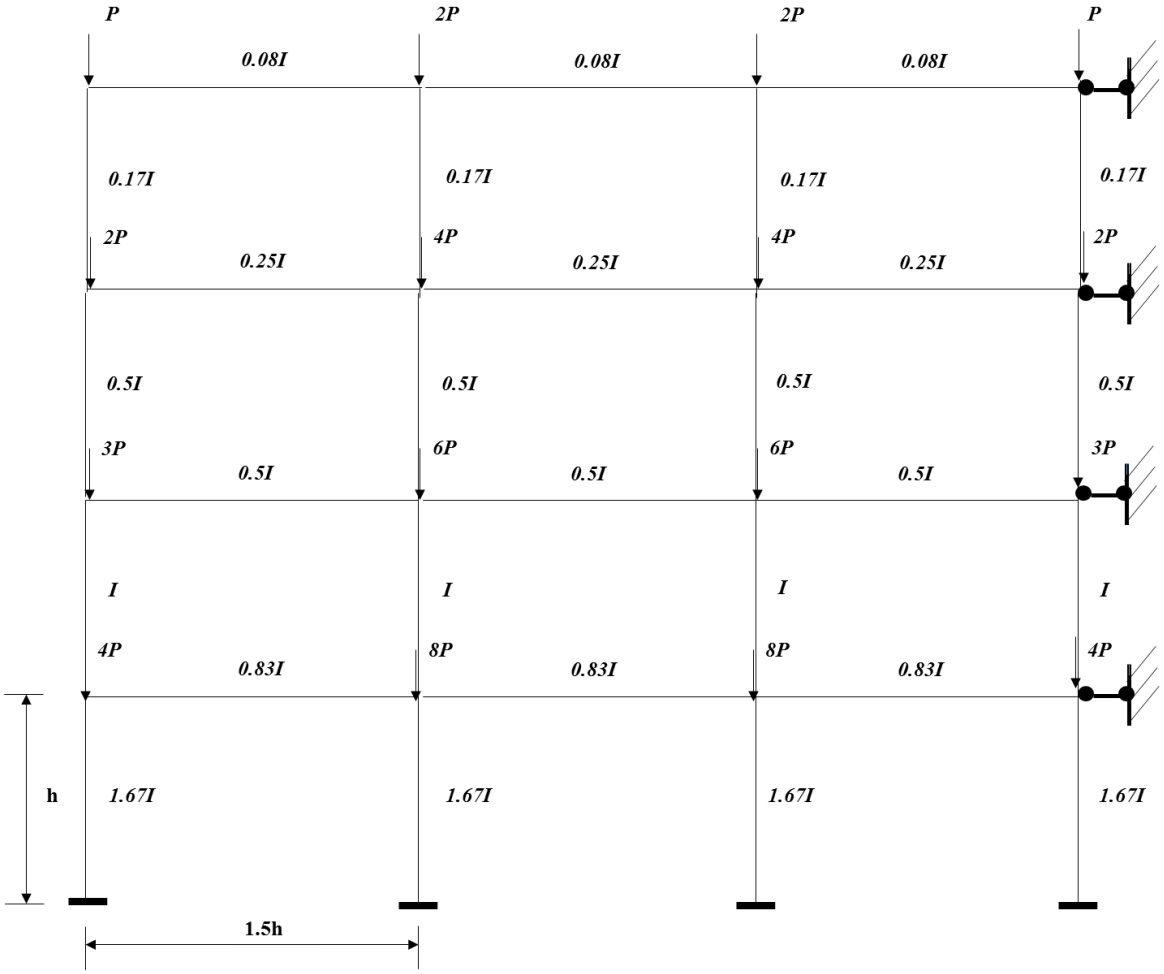


Figure 3.8 Optimised four story three bay no sway frame with increased buckling load.

The results from the two optimisation analyses done, are summarised in Table 3.7. It is noticed that in that latter optimisation analysis, where the second story was selected as critical, the buckling load of the frame increased by 67% and the weight reduced by 15%. The resulting frame is heavier and has stiffer members than that of the first optimised frame. Nevertheless, the objective of increasing the buckling load whilst minimising the weight of the frame was met.

Table 3.7 Optimisation of two frames in Example 2.

	Before optimisation		**After optimisation**		
	System buckling load, P_{cr} $(\frac{EI}{h^2})$= $P_{cr,min}$	**Weight of frame (kg)**	**System buckling load, P_{cr} $(\frac{EI}{h^2})$**	**Weight of frame (kg)**	**FEA result of optimised frame, $(\frac{EI}{h^2})$**
Frame with critical story =1	0.766	22.4	0.766	14.8	0.807
Frame with critical story =2	0.766	22.4	1.276	19.1	1.349

The comparison of the two analyses indicated that a higher buckling load for the given frame configuration under a specific load arrangement, can be achieved whilst minimising the weight of the frame. However, it may not be the optimal solution, as a lighter frame was achieved in the first frame, yet with a lower buckling load. This would be the lower-bound design of the frame seeing that the minimum buckling load possible under this configuration was achieved. This then leads to the fact, that the designer will need to initially outline their objectives that the frame optimisation should achieve.

Example 3: 5 story 5 bay frame

A five story five bay frame and the loads, adapted from an example (Mahfouz, 1999) as shown in Figure 3.9, was optimised with all columns and beams initially having a stiffness as indicated. All stories have the same height. The literature provided starting sections for the frame which were converted into variables in terms of their stiffness values, *I*, with the first story being assigned as *I*.

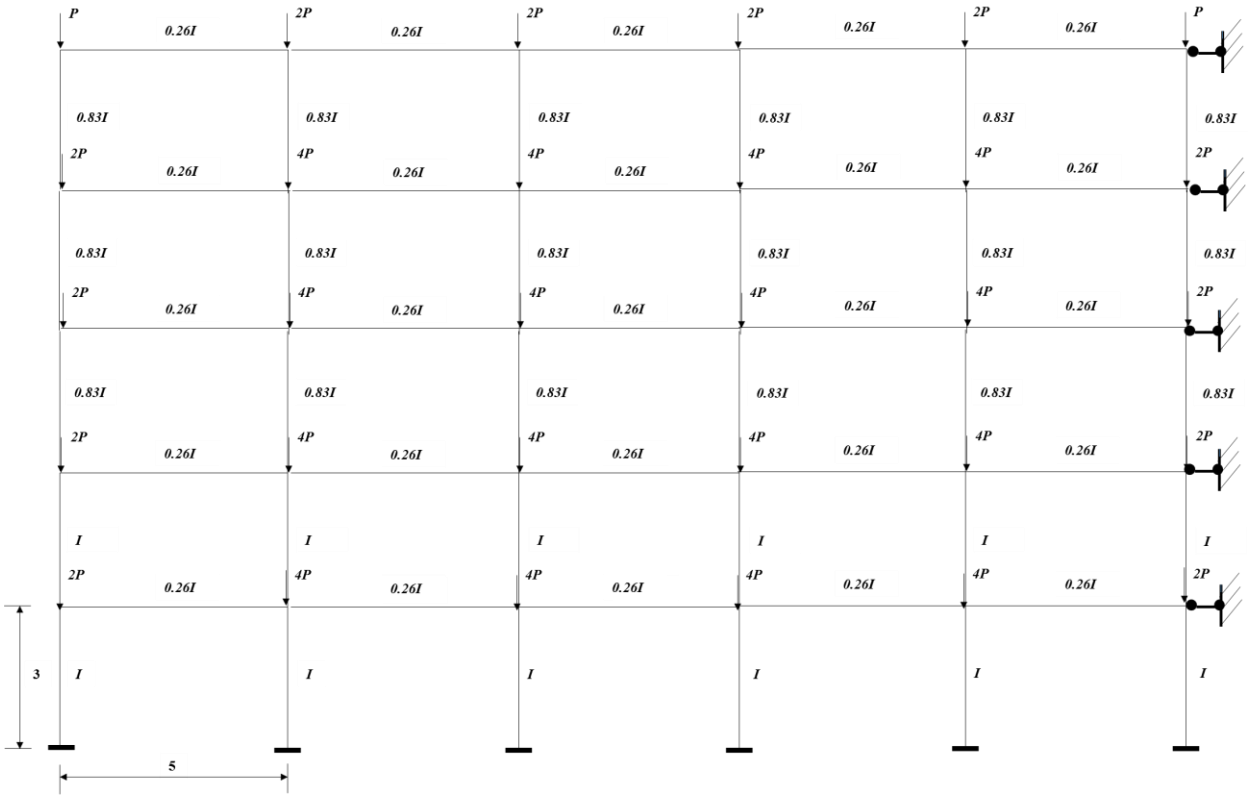


Figure 3.9 A five story five bay no sway frame.

The system buckling load of the original frame is first determined in order to obtain the story with the minimum story buckling load, from which the other stories can be optimised.

The accompanying sway frame can be obtained by releasing the bracing of the no sway frame as before. For the accompanying sway frame, it is easy to find that the frame will buckle at the bottom story. The upper-bound story buckling load, $\overline{P_{cr}}$, can be found using the method previously outlined.

The horizontal stiffness of the first story before loading:

$$K_1 = 6 \times \frac{12EI}{h^3}$$

The negative stiffness caused by the axial loads on this story, given that the loads from the upper story are transmitted to this story:

$$K_{p1} = 2 \times \frac{9P}{h} + 4 \times \frac{18P}{h} = \frac{90P}{h}$$

After loading, the horizontal stiffness of the story becomes:

$$K - K_p = \frac{72EI}{h^3} - \frac{90P}{h} = 0$$

Solving this equation, one can obtain:

$$P_1 = \frac{4EI}{5h^2}$$

Taking *P*-δ effect into account, the system buckling load of the accompanying sway frame can be obtained:

$$P_{cr,sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{4EI}{5h^2} = \frac{\pi^2 EI}{15h^2}$$

The upper-bound of the system buckling load, corresponding to $I_b = nI = \infty$, of the original no sway frame shown in Figure 3.9 can be obtained:

$$\overline{P_{cr,1}} = \eta \times P_{cr,sway} = 4 \times P_{cr,sway} = \frac{4\pi^2 EI}{15h^2}$$

Accounting for the non-rigid beams in the original no sway frame, the real system buckling load can be determined by considering the equivalent normalised one bay frame.

For the frame shown in Figure 3.9, the number of bays is five, b=5, thus the equivalent one bay frame for the first story, when the frame is folded into the middle bay, is shown in Figure 3.10:

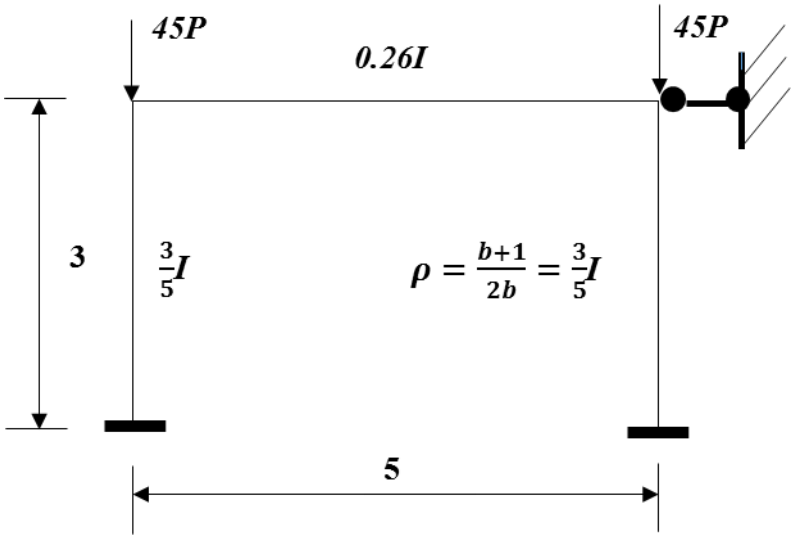


Figure 3.10 Normalised equivalent one bay frame for first story of frame in Example 3.

For the frame in Figure 3.10, the value of $\rho=\frac{3}{5}$ and $\lambda=\frac{I_b L_c}{I_c L_b}=\frac{0.4I\times3}{\frac{3}{5}I\times5}=0.4$.

From the graph in Figure 2.9, the design graph when $b=\infty$ was used to obtain the value of μ, thus for a λ value of 0.4, μ= 0.5482.

Thus, the final story buckling load can be obtained:

$$P_{cr,1} = \mu \times \overline{P_{cr,1}} = 0.5482 \times \frac{4\pi^2 E \times \frac{1}{2}I}{15h^2} = 0.72\frac{EI}{h^2}$$

Note that the stiffness of the column was reduced by the value of ρ when $b=\infty$ as the value of μ was taken from the design graph for this equivalent frame.

The same process is applied to the remaining four stories of the frame. The story buckling loads and the total weight of the frame from the first iteration is summarised in Table 3.8.

Table 3.8 Iteration 1 of the optimisation procedure of Example 3.

Story number, *i*	$K_i(\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}}(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $(\frac{EI}{h^2})$	**Story buckling load ratio, α**
1	72	90	4/15	0.26	0.55	**0.722**	1.00
2	72	70	12/35	0.26	0.55	0.929	0.78
3	60	50	33/83	0.31	0.56	1.091	0.66
4	60	30	55/83	0.31	0.56	1.819	0.40
5	60	10	2	0.31	0.56	5.457	0.13
Weight of frame		88 476 kg/88.4tonnes					
Critical story		**1**					

From Table 3.8, the critical story, the story with the minimum story buckling load, was found to be story 1. The story buckling load ratio α is obtained for each story as before and found to be less than 0.9 for all stories hence the frame has not been optimised.

Another iteration is performed where the column and beam stiffness is reduced as before by the story load ratio α calculated.

The same procedure is applied on the frame with the new column and beam stiffness' and the results are summarised in Table 3.9.

Table 3.9 Iteration 2 of the optimisation procedure of Example 3.

Story number, *i*	$K_i(\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}}(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $(\frac{EI}{h^2})$	**Story buckling load ratio, α**
1	72	90	4/15	0.26	0.55	0.722	1
2	56	70	4/15	0.26	0.55	0.722	1
3	39.48	50	5/19	0.31	0.56	0.722	1
4	23.69	30	5/19	0.31	0.56	0.722	1
5	7.90	10	5/19	0.31	0.56	0.722	1
Weight of frame		65 678 kg/ 65.6tonnes					
Critical story		**All**					

From the results of the second iteration shown in Table 3.9, it can be seen that all story ratios α are equal to one and hence optimisation has been reached. The weight of the frame has reduced by 26% and all stories have the same buckling load. The final optimised frame is shown in Figure 3.11.

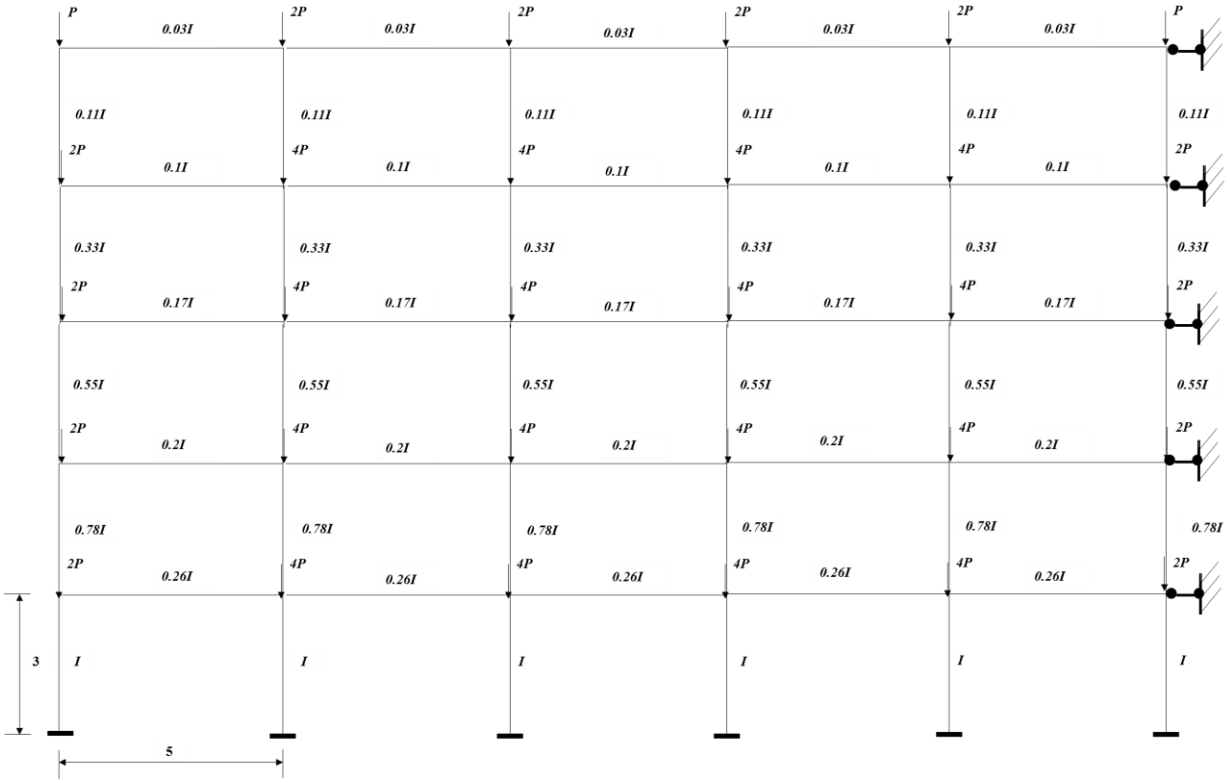


Figure 3.11 Optimised five bay five story no sway frame.

When the optimised frame was analysed in ANSYS, a system buckling load of $0.7\frac{EI}{h^2}$ was obtained, resulting in a 3% smaller difference than the result obtained from the proposed method. The percentage difference is acceptable for the purposes of a simple method to optimise frame structures.

The buckled shape of the frame obtained from the FEA is shown in Figure 3.12. It is observed that all stories buckle under the first mode and all members have deflected, thus leading to the fact that the full member capacity was utilised.

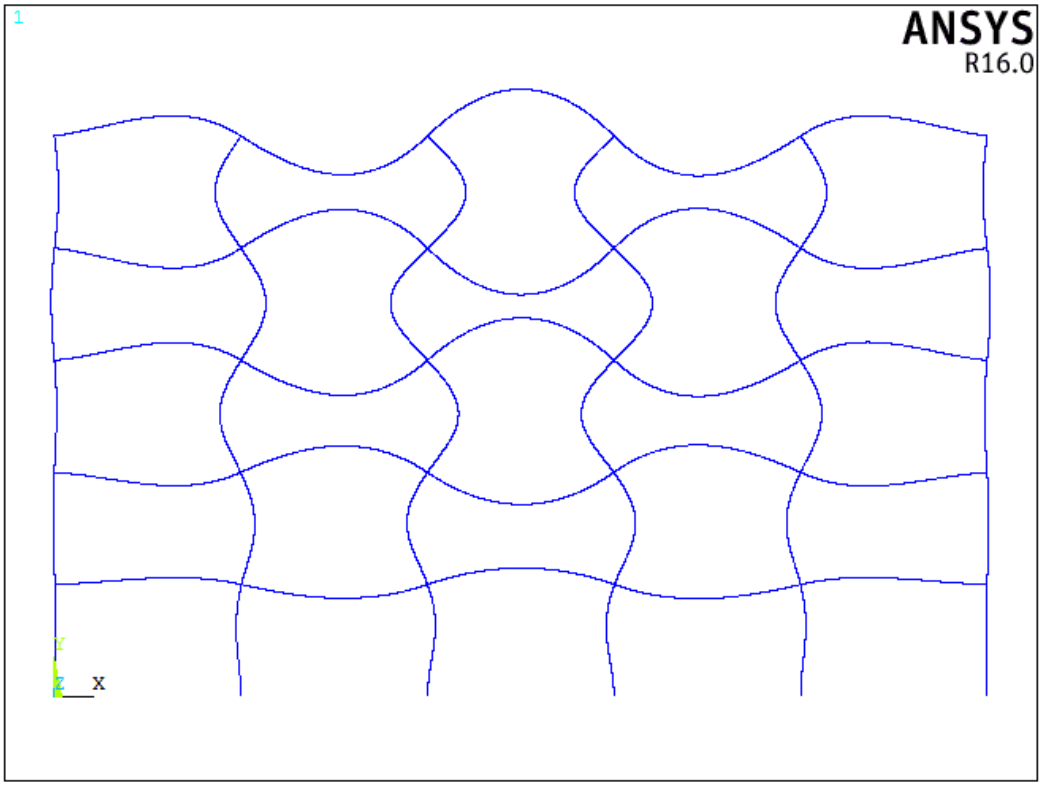


Figure 3.12 Buckled shape of optimised five story five bay no sway frame.

The five story frame was re-optimised in an attempt to increase the buckling load, as done in the frame of Example 2. The frame buckling loads from the first iteration in Table 3.8 are referred to, but the story buckling ratio, α, is recalculated. If the story with the next lowest buckling load is selected as critical, that being story 2 in this frame, then the new α ratios, shown in Table 3.10, for each story becomes:

Table 3.10 Iteration 1 of the optimisation procedure of Example 3 with story 2 selected as critical.

Story number, *i*	$K_i\left(\frac{EI}{h^3}\right)$	K_{pi} **(P/h)**	$\overline{P_{cr}}\left(\frac{\pi^2 EI}{h^2}\right)$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$ $\left(\frac{EI}{h^2}\right)$**	**Story buckling load ratio, α**
1	72	90	4/15	0.26	0.55	0.722	1.29
2	72	70	12/35	0.26	0.55	**0.929**	**1.00**
3	60	50	33/83	0.31	0.56	1.091	0.85
4	60	30	55/83	0.31	0.56	1.819	0.51
5	60	10	2	0.31	0.56	5.457	0.17
Weight of frame		88 476 kg/88.4tonnes					
Critical story		**2**					

The new column and beam stiffness for the next iteration is calculated as before; namely,

Story 1 column stiffness $_{k=2}$ =Column stiffness $_{k=1}$×α=*I*×1.29=1.29*I*.

Optimisation is applied as before on the frame with the new member stiffness' and the results are summarised in Table 3.11. It is seen that the weight of frame reduced after optimisation and the buckling load of the system, given before by the story with the lowest buckling load, has now increased.

Table 3.11 Iteration 2 of the optimisation procedure of Example 2 with story 2 selected as critical.

Story number, *i*	$K_i\left(\frac{EI}{h^3}\right)$	K_{pi} **(P/h)**	$\overline{P_{cr}}\left(\frac{\pi^2 EI}{h^2}\right)$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $\left(\frac{EI}{h^2}\right)$	**Story buckling load ratio, α**
1	92.57	90	12/35	0.26	0.55	0.929	1.00
2	72.00	70	12/35	0.26	0.55	0.929	1.00
3	50.76	50	22/65	0.31	0.56	0.929	1.00
4	30.45	30	22/65	0.31	0.56	0.929	1.00
5	10.15	10	22/65	0.31	0.56	0.929	1.00
Weight of frame		74 472kg/74.4tonnes					
Critical story		**All stories**					

The optimised frame is given in Figure 3.13. When the frame was analysed in FEA, a system buckling load of $0.9\frac{EI}{h^2}$ was obtained, resulting in a 3% smaller difference than the result obtained from the proposed method. The same difference was obtained in the first analysis of the optimised frame when the critical story was selected as the first story.

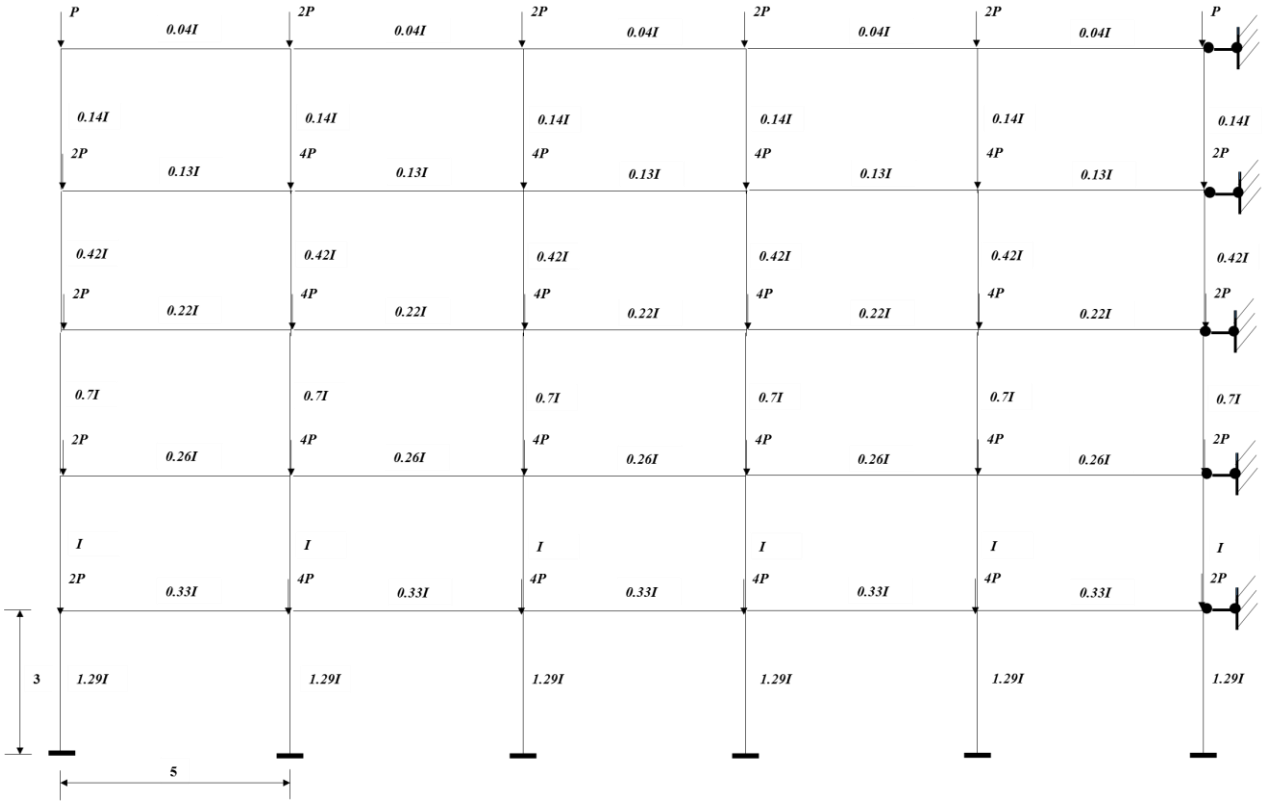


Figure 3.13 Optimised five story five bay no sway frame with increased buckling load.

The results of the two optimisations performed, are summarised in Table 3.12. It is seen that in that latter optimisation analysis, where the second story was selected as critical, the buckling load of the frame increased by 29% and the weight reduced by 17%. The resulting frame is heavier and has stiffer members than that of the first optimised frame. Nevertheless, the objective of increasing the buckling load whilst minimising the weight of the frame was achieved.

Table 3.12 Optimisation of two frames in Example 2.

	Before optimisation		**After optimisation**		
	System buckling load, P_{cr} $(\frac{EI}{h^2})$= $P_{cr,\,min}$	**Weight of frame (kg)**	**System buckling load, P_{cr} $(\frac{EI}{h^2})$**	**Weight of frame (kg)**	**FEA result of optimised frame, $(\frac{EI}{h^2})$**
Frame with critical story =1	0.722	88.4	0.722	65.6	0.807
Frame with critical story =2	0.722	88.4	0.929	74.4	0.900

The comparison of the two analyses reiterates the previous finding that a higher buckling load for the frame configuration and specific load arrangement given, can be achieved whilst minimising the weight of the frame. However, it may not be the

optimal solution, as a lighter frame was achieved as seen by the first frame, yet with a lower buckling load. This would be the lower-bound design of the frame seeing that the minimum buckling load possible under this configuration was achieved. This then leads to the fact, that the designer will need to outline the objectives that the frame optimisation should achieve.

Example 4: 10 storey 4 bay frame

A ten story four bay frame and the loads as adapted from an example in Mahfouz (Mahfouz, 1999), is shown in Figure 3.14. The frame was optimised with all columns and beams initially having a stiffness as indicated. All stories have the same height in the frame example. The literature provided starting sections for the frame which were converted into variables in terms of their stiffness values, *I*, with the first story being assigned as *I*.

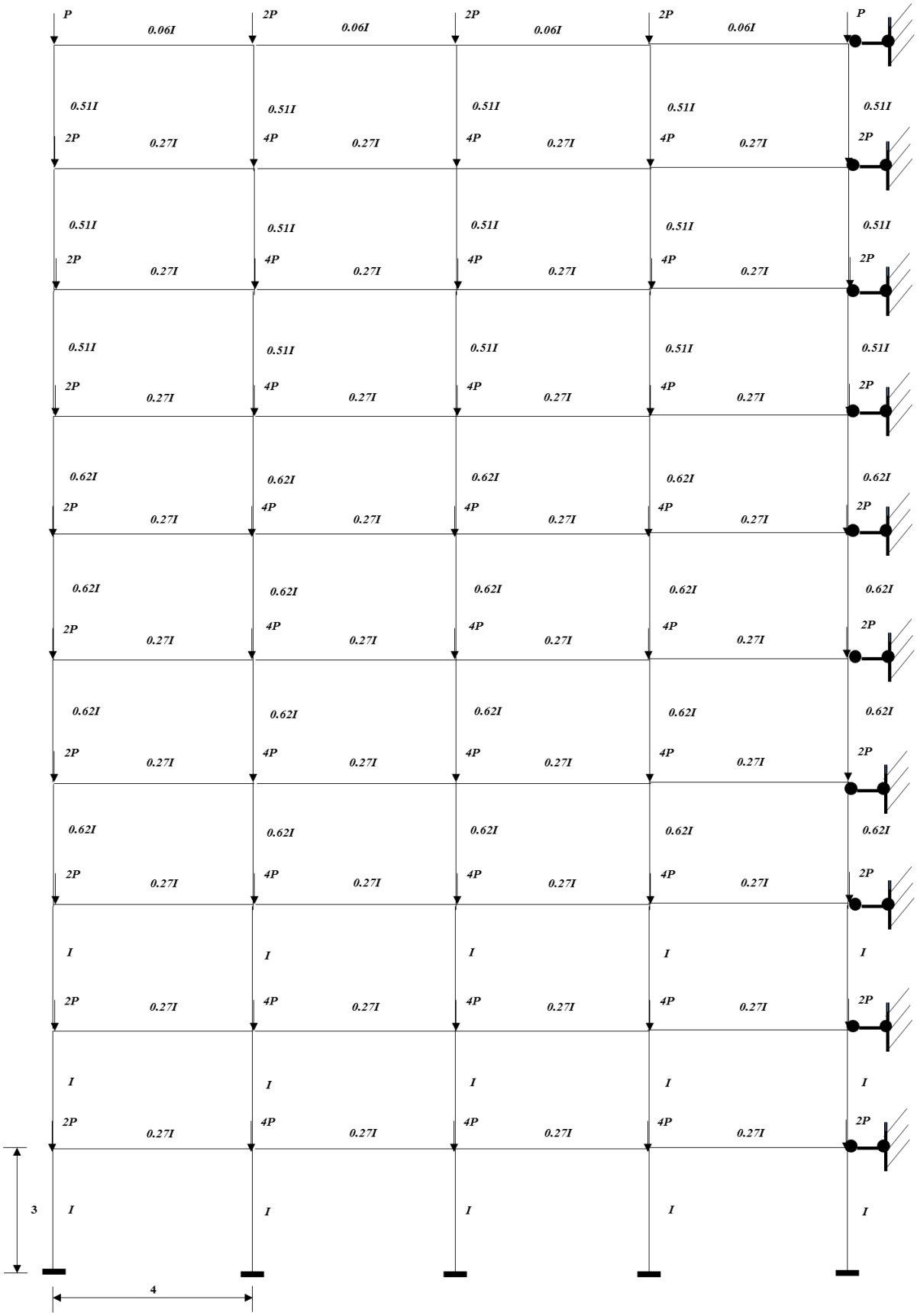


Figure 3.14 A ten story four bay no sway frame.

The system buckling load of the frame is first determined to obtain the story with the minimum buckling load, from which the other stories can be optimised.

As before, the accompanying sway frame can be obtained by releasing the bracing of the no sway frame. For the accompanying sway frame, it is easy to assume that the frame will buckle at the bottom story. The upper-bound story buckling load, $\overline{P_{cr}}$, can be found using the method outlined by Li (2014) for sway frames.

The horizontal stiffness of the first story before loading:

$$K_1 = 5 \times \frac{12EI}{h^3}$$

The negative stiffness caused by the axial loads on this story:

$$K_{p1} = 2 \times \frac{19P}{h} + 3 \times \frac{38P}{h} = \frac{152P}{h}$$

After loading, the horizontal stiffness of the story becomes:

$$K - K_p = \frac{60EI}{h^3} - \frac{152P}{h} = 0$$

Solving this equation, one can obtain:

$$P_1 = \frac{15EI}{38h^2}$$

Taking *P*-δ effect into account, the system buckling load of the accompanying sway frame can be obtained:

$$P_{cr,sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{15EI}{38h^2} = \frac{5\pi^2 EI}{152h^2}$$

The upper-bound of the system buckling load, corresponding to $I_b = nI = \infty$, of the original no sway frame shown in Figure 3.14 can be obtained:

$$\overline{P_{cr,1}} = \eta \times P_{cr,sway} = 4 \times P_{cr,sway} = \frac{5\pi^2 EI}{38h^2}$$

Accounting for the non-rigid beams in the original no sway frame, the real system buckling load can be determined by considering the equivalent normalised one bay frame.

For the frame shown in Figure 3.14, the number of bays is four; $b=4$; thus the equivalent one bay frame for the first story, when the frame is folded into one of the middle bays, is shown in Figure 3.15.

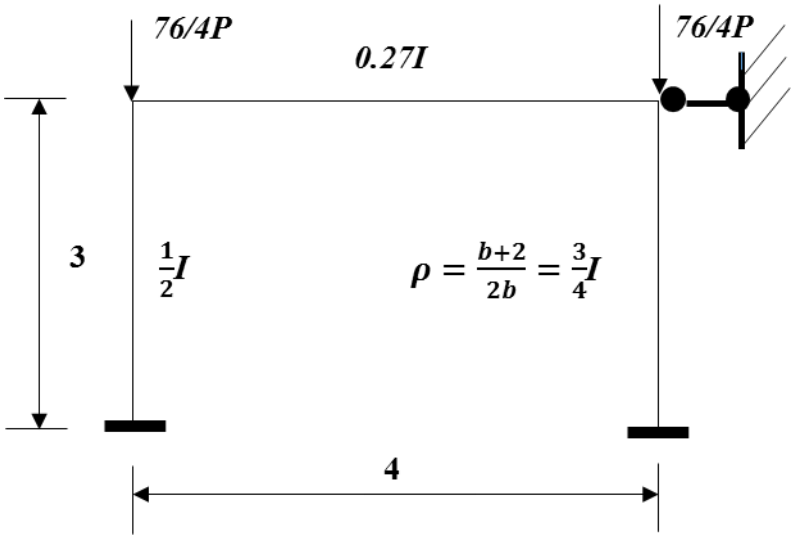


Figure 3.15 Equivalent one bay frame for the first story of frame in Example 4.

For the frame in Figure 3.15, it is reasonable to see that the buckling of the system is localised, since the column with lower stiffness of $\frac{1}{2}I$ will buckle before the other column in the story. Therefore, the upper-bound system buckling load needs to be re-calculated to account for this.

The horizontal stiffness of this column before loading:

$$K_1 = \frac{12E \times \frac{1}{2}I}{h^3}$$

The negative stiffness caused by the axial loads on the column, given that the loads from the upper stories are transmitted to this column:

$$K_{p1} = \frac{\frac{76}{4}P}{h}$$

After loading, the horizontal stiffness of the column becomes:

$$K - K_p = \frac{6EI}{h^3} - \frac{\frac{76}{4}P}{h} = 0$$

Solving this equation, one can obtain:

$$P_1 = \frac{6EI}{19h^2}$$

Taking P-δ effect into account, the buckling load of the column can be obtained:

$$P_{cr,sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{6EI}{19h^2} = \frac{\pi^2 EI}{38h^2}$$

The upper-bound of the system buckling load, corresponding to $I_b = nI = \infty$, of the no sway frame shown in Figure 3.15 can be obtained:

$$\overline{P_{cr,1}} = \eta \times P_{cr,sway} = 4 \times P_{cr,sway} = \frac{2\pi^2 EI}{19h^2}$$

The design graphs in Figure 2.11 when b =2 and b =∞ was used to interpolate the value of μ when b =4. For the less stiff column, $\lambda = \frac{I_b L_c}{I_c L_b} = \frac{0.27I \times 3}{\frac{1}{2}I \times 4} = 0.405$.

From the graph in Figure 2.11, using λ=0.405:

When b =2, μ_1= 0.6372 for ρ_1=1 , and

When b =∞, μ_2= 0.5677 for ρ_2=0.5.

Therefore, the value of μ for b=4 (ρ=0.75) can be interpolated as follows:

$$\mu = \mu_2 + \frac{\mu_1 - \mu_2}{\rho_1 - \rho_2} \times (\rho - \rho_2) = 0.5677 + \frac{0.6372 - 0.5677}{1 - 0.5} \times \left(\frac{3}{4} - 0.5\right) = 0.6024$$

Thus, the final story buckling load for first story can be obtained:

$$P_{cr,1} = \mu \times P_{cr,upperbound,1} = 0.6024 \times \frac{2\pi^2 E \times \frac{1}{2}I}{19h^2} = 0.313\frac{EI}{h^2}$$

The same process is applied to the less stiff columns of the remaining stories of the frame. For iteration 1 of the original un-optimised frame, the story buckling loads and the total weight of the frame is summarised in Table 3.13.

Table 3.13 Iteration 1 of the optimisation procedure of Example 4.

Story number, *i*	$K_i(\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}}$ $(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$** $(\frac{EI}{h^2})$	**Story buckling load ratio, α**
1	6.00	19	2/19	0.41	0.60	0.313	0.966
2	6.00	17	2/17	0.41	0.60	0.350	0.865
3	6.00	15	2/15	0.41	0.60	0.396	0.763
4	3.69	13	7/74	0.66	0.65	**0.302**	1.000
5	3.69	11	1/9	0.66	0.65	0.357	0.846
6	3.69	9	3/22	0.66	0.65	0.437	0.692
7	3.69	7	13/74	0.66	0.65	0.562	0.538
8	3.06	5	11/54	0.80	0.67	0.673	0.450
9	3.06	3	18/53	0.80	0.67	1.121	0.270
10	3.06	1	1.02	0.17	0.55	2.780	0.109
Weight of frame		154 101.5 kg/154t					
Critical story		**4**					

The story buckling load ratio α is obtained for each story as follows:

$$Pcr,ratio = \frac{Pcr,4}{Pcr,i}$$

From Table 3.13, the critical story, the story with the minimum story buckling load and α=1, was found to be story 4, and not the first story as initially assumed. In the original frame shown in Figure 3.14, the story stiffness reduced after the fourth story yet the lower stories still experience higher cumulative loads. Even though the fourth story was found to be critical, the value of α for the first story is certainly close to one as seen in Table 3.13.

Optimisation is reached when α is greater than 0.9 for all stories hence the frame has not been optimised.

Another iteration is performed where the column and beam stiffness is reduced by the story load ratio α as determined. For example, the new column stiffness for iteration k=2 of story 1 is given as: Story 1 column stiffness $_{k=2}$ = Stiffness $_{k=1}$×α=*I*×0.966=0.966*I*. For the second iteration on the frame with new column and beam stiffness', the results are summarised in Table 3.14.

Table 3.14 Iteration 2 of the optimisation procedure of Example 4.

Story number, *i*	$K_i(\frac{EI}{h^3})$	**K_{pi} (P/h)**	$\overline{P_{cr}}(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$ $(\frac{EI}{h^2})$**	**Story buckling load ratio, α**
1	6.00	19	2/19	0.41	0.60	0.313	1.000
2	5.37	17	2/19	0.41	0.60	0.313	1.000
3	4.74	15	2/19	0.41	0.60	0.313	1.000
4	3.82	13	5/51	0.66	0.65	0.313	1.000
5	3.23	11	5/51	0.66	0.65	0.313	1.000
6	2.64	9	5/51	0.66	0.65	0.313	1.000
7	2.06	7	5/51	0.66	0.65	0.313	1.000
8	1.42	5	9/95	0.80	0.67	0.313	1.000
9	0.85	3	9/95	0.80	0.67	0.313	1.000
10	0.34	1	1/9	0.17	0.55	0.313	1.000
Weight of frame		123 380.5 kg/123t					
Critical story		**All stories**					

From the results of the second iteration shown in Table 3.14, it can be seen that optimisation has been reached as all story ratios α are equal to one. The weight of the frame has reduced by 20% and all stories have the same buckling load which has also increased by 4%. The final optimised frame is shown in Figure 3.16. It was noted that the beam stiffness of the optimised frame did not reduce with story number at story four (beam four), as initially this story was identified as the critical story with a load ratio α of 1. As a result, the optimisation was centred on this story and its stiffness of beams and columns was not reduced in the next iteration.

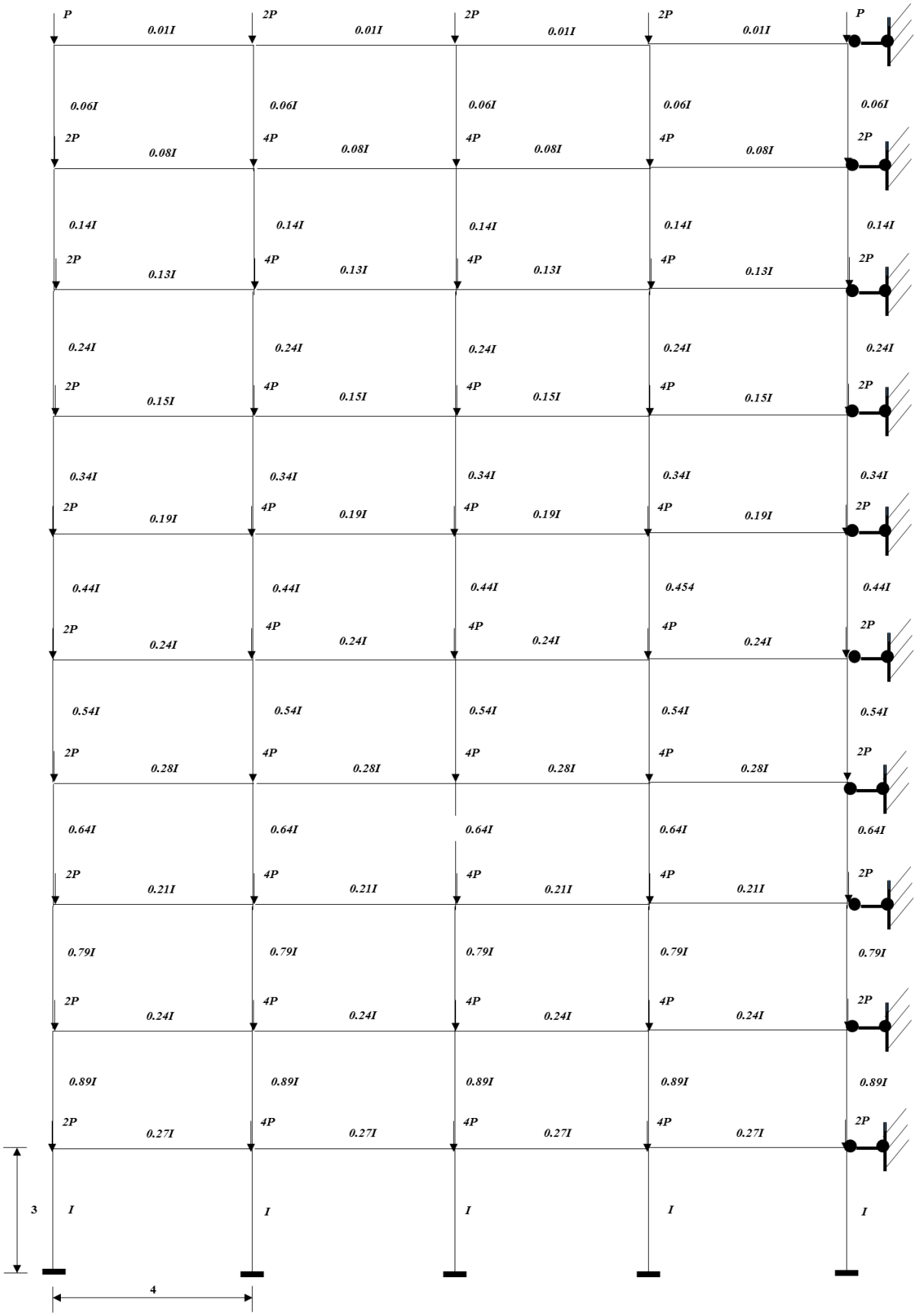


Figure 3.16 Optimised four bay ten story no sway frame.

When the optimised frame in Figure 3.16 was analysed in FEA software ANSYS, a system buckling load of $0.32\frac{EI}{h^2}$ was obtained, resulting in a 2.2% larger difference than the result obtained from the proposed method. The percentage difference could be attributed to the fact that, as previously mentioned, during the optimisation the fourth story was found to be the critical story which resulted in its beam stiffness not being reduced, leading to a localised rigid region or zone within the frame. However, this percentage difference is an acceptable result for the purposes of a simple method to optimise frame structures.

The buckled shape of the optimised frame is shown in Figure 3.17. It is noticed that all stories buckle under the first mode and all members have deflected, thus leading to the fact that the full member capacity was utilised.

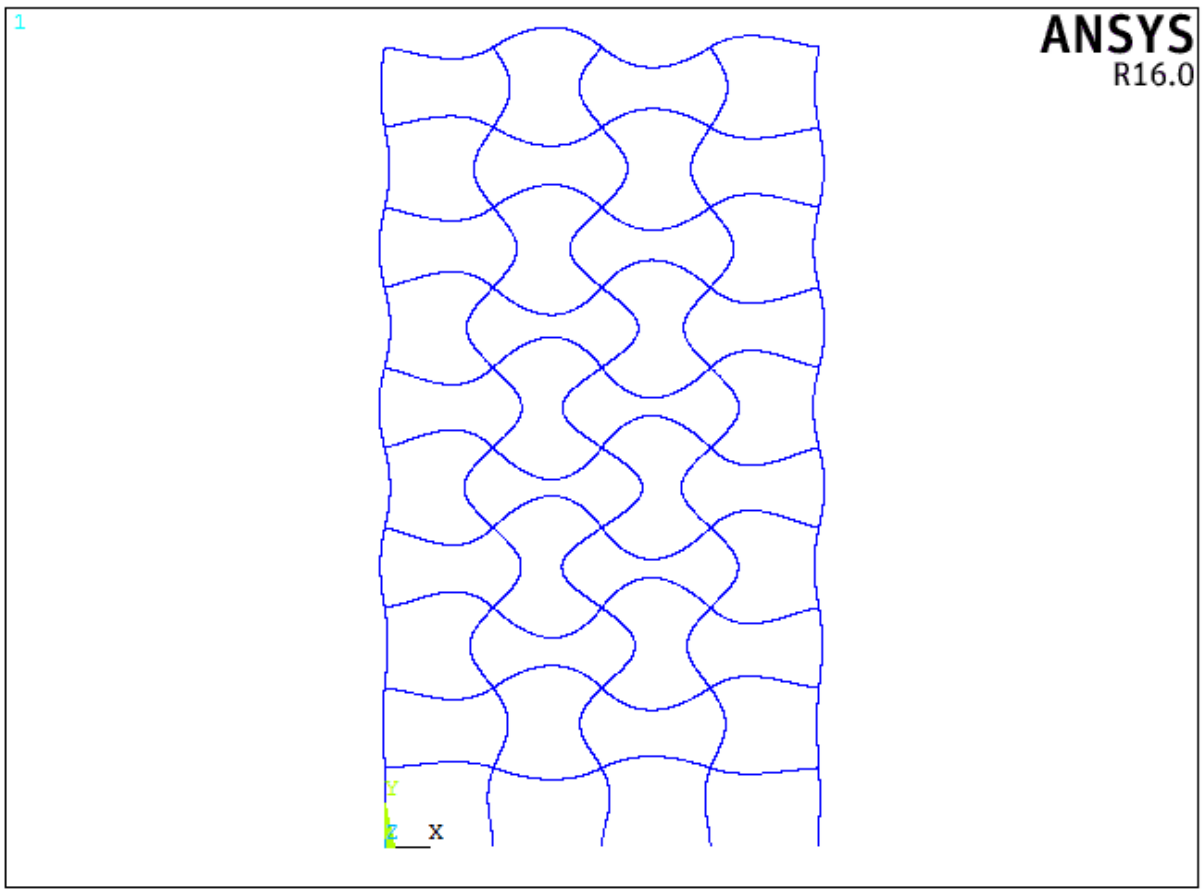


Figure 3.17 Buckled shape of optimised ten story four bay no sway frame under the first mode.

The results presented indicate the norm in design of using stiffer columns for the lower stories of a frame, as was the case of the original frame adapted from the

literature. Yet this may not be the 'optimal' design in terms of member capacity, and thus a scenario was investigated where the design engineer could choose to not select any or know which initial sections to use for the frame and all stories were given the same initial stiffness. The final system buckling load and weight of this frame was examined.

The original ten story four bay frame with its loads as shown in Figure 3.14, is used except all columns have a stiffness of *I* and all beams have a stiffness of *0.27I*. The frame will be referred to as Frame B. The frame is optimised using the same procedure as before and the final results obtained from the first iteration is presented in Table 3.15.

Table 3.15 Iteration 1 of the optimisation procedure of Frame B of Example 4 with all columns having the same initial stiffness.

Story number, *i*	$K_i\left(\frac{EI}{h^3}\right)$	K_{pi} **(P/h)**	$\overline{P_{cr}}$ $\left(\frac{\pi^2 EI}{h^2}\right)$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$** $\left(\frac{EI}{h^2}\right)$	**Story buckling load ratio, α**
1	6.00	19	2/19	0.41	0.60	**0.313**	1.00
2	6.00	17	2/17	0.41	0.60	0.350	0.89
3	6.00	15	2/15	0.41	0.60	0.396	0.79
4	6.00	13	2/13	0.41	0.60	0.457	0.68
5	6.00	11	2/11	0.41	0.60	0.541	0.58
6	6.00	9	2/9	0.41	0.60	0.661	0.47
7	6.00	7	2/7	0.41	0.60	0.849	0.37
8	6.00	5	2/5	0.41	0.60	1.189	0.26
9	6.00	3	2/3	0.41	0.60	1.982	0.16
10	6.00	1	2	0.41	0.60	5.946	0.05
Weight of frame		177160.3 kg/177t					
Critical story		**1**					

From Table 3.15, it can be seen that the critical story, the story with the minimum story buckling load and α=1, was found to be the first story. Optimisation is reached

when α is greater than 0.9 for all stories hence the frame has been not been optimised.

Another iteration is performed where the column and beam stiffness is reduced by the story load ratio α as determined.

The results for the second iteration of the frame with the new column and beam stiffness' are summarised in Table 3.16.

Table 3.16 Iteration 2 of the optimisation procedure of Frame B with all columns having the same initial stiffness.

Story number, *i*	$K_i(\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}}$ $(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $(\frac{EI}{h^2})$	**Story buckling load ratio, α**
1	6.00	19	2/19	0.41	0.60	0.313	1.00
2	6.00	17	2/19	0.41	0.60	0.313	1.00
3	6.00	15	2/19	0.41	0.60	0.313	1.00
4	6.00	13	2/19	0.41	0.60	0.313	1.00
5	6.00	11	2/19	0.41	0.60	0.313	1.00
6	6.00	9	2/19	0.41	0.60	0.313	1.00
7	6.00	7	2/19	0.41	0.60	0.313	1.00
8	6.00	5	2/19	0.41	0.60	0.313	1.00
9	6.00	3	2/19	0.41	0.60	0.313	1.00
10	6.00	1	2/19	0.41	0.60	0.313	1.00
Weight of frame		121 487.2kg/121t					
Critical story		**All stories**					

From the results of the second iteration shown in Table 3.16, it can be seen that optimisation has been reached as all story ratios α are equal to one. The weight of the frame has reduced by 31% and all stories have the same buckling load. The final optimised frame is shown in Figure 3.18.

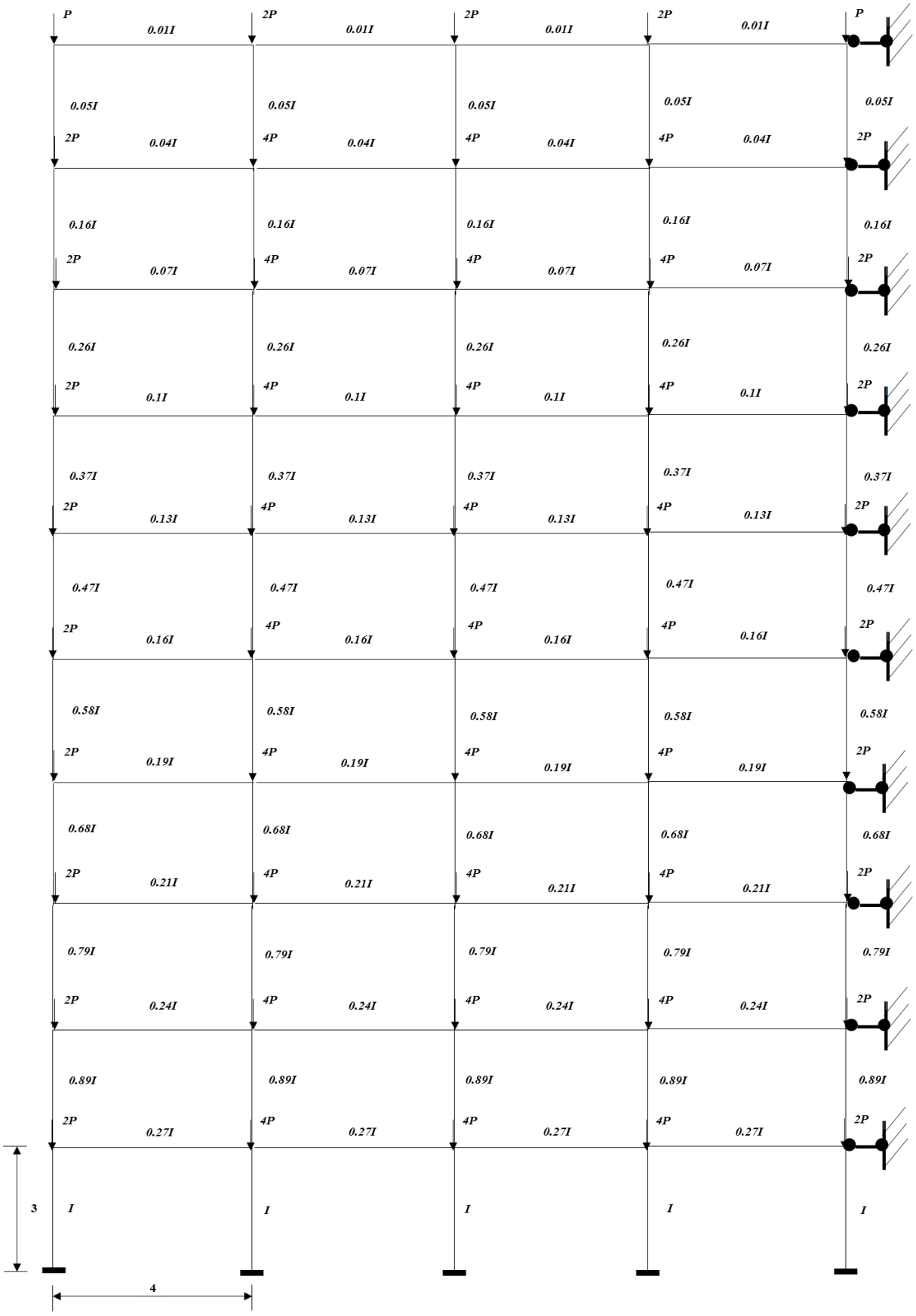


Figure 3.18 Optimised Frame B four bay ten story no sway frame.

When the optimised Frame B in Figure 3.18 was analysed in ANSYS, a system buckling load of $0.32\frac{EI}{h^2}$ was obtained, resulting in a 2.2% larger difference than the result obtained from the proposed method.

The buckled shape of the optimised frame is shown in Figure 3.19. It is noticed that all stories buckle under the first mode and all members have deflected, thus leading to the fact that the full member capacity was utilised.

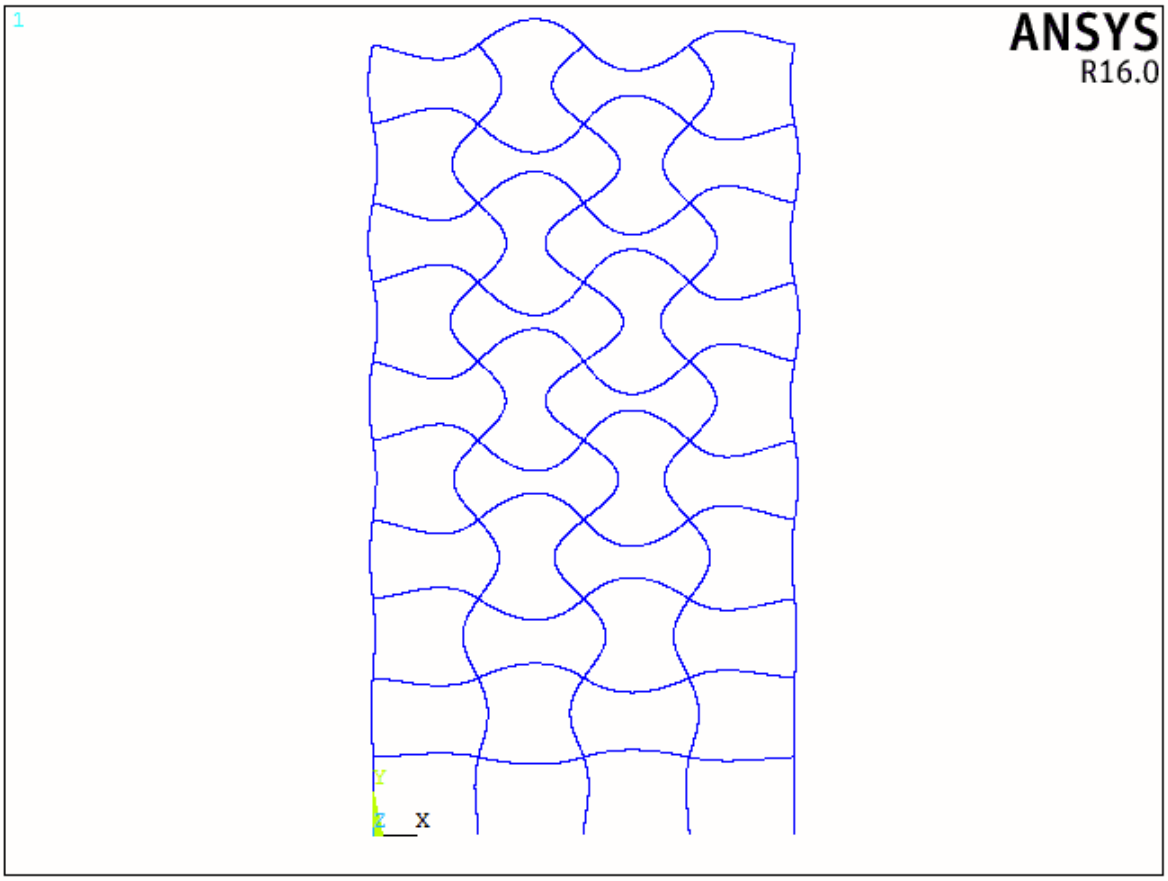


Figure 3.19 Buckled shape of Frame B under the first mode.

If the system buckling loads and weights of the two frames before and after optimisation, as presented in Table 3.17, are compared, it is seen that both frames have the same final system buckling with Frame B only weighing 2% less.

Table 3.17 Final system buckling loads and frame weights of the two ten story four bay frames.

	Before optimisation		**After optimisation**	
	System buckling load, P_{cr} $(\frac{EI}{h^2})$	**Weight of frame (tonnes)**	**System buckling load, P_{cr} $(\frac{EI}{h^2})$**	**Weight of frame (tonnes)**
Frame as shown in Figure 3.14	0.302	154	0.313	123
Frame B	0.313	177	0.313	121
FEA result of optimised frame	0.32	-	0.32	-

For the first frame presented, it was noted that the beams did not reduce in stiffness with story number as was found with the column stiffness'. The optimised frame resulted in the fourth beam having a greater stiffness than its lower stories. This may be attributed to the fact that the original example from the literature had a different story height for the first frame and thus would have resulted in the first story being critical. The optimisation procedure reduces all of the stories stiffness' such that it has the same buckling load as that of the critical story. If the critical storey was initially situated and found in the fourth story of a frame as in this instance, all other storey stiffness' would be reduced by the ratio α premised on this critical story, which led to the critical storey having a higher beam stiffness I_b than the stories below it. Despite this fact, the comparison of the frames indicate that initial sections can be selected before optimisation or the user may have all initial sections with the same stiffness. This will yield an optimised frame with the same final system buckling load as was demonstrated by the two frames.

In consideration of the results found for the two frames, the optimisation procedure was applied to the actual frame with the different story height, as it appears in the example from the literature (Mahfouz, 1999), to investigate the effect on the system buckling load. The frame was referred to as Frame C.

The ten story four bay frame with its member stiffness' and loads as extracted from the literature is shown in Figure 3.20. The first story has a different height whilst stories two to ten have the same height.

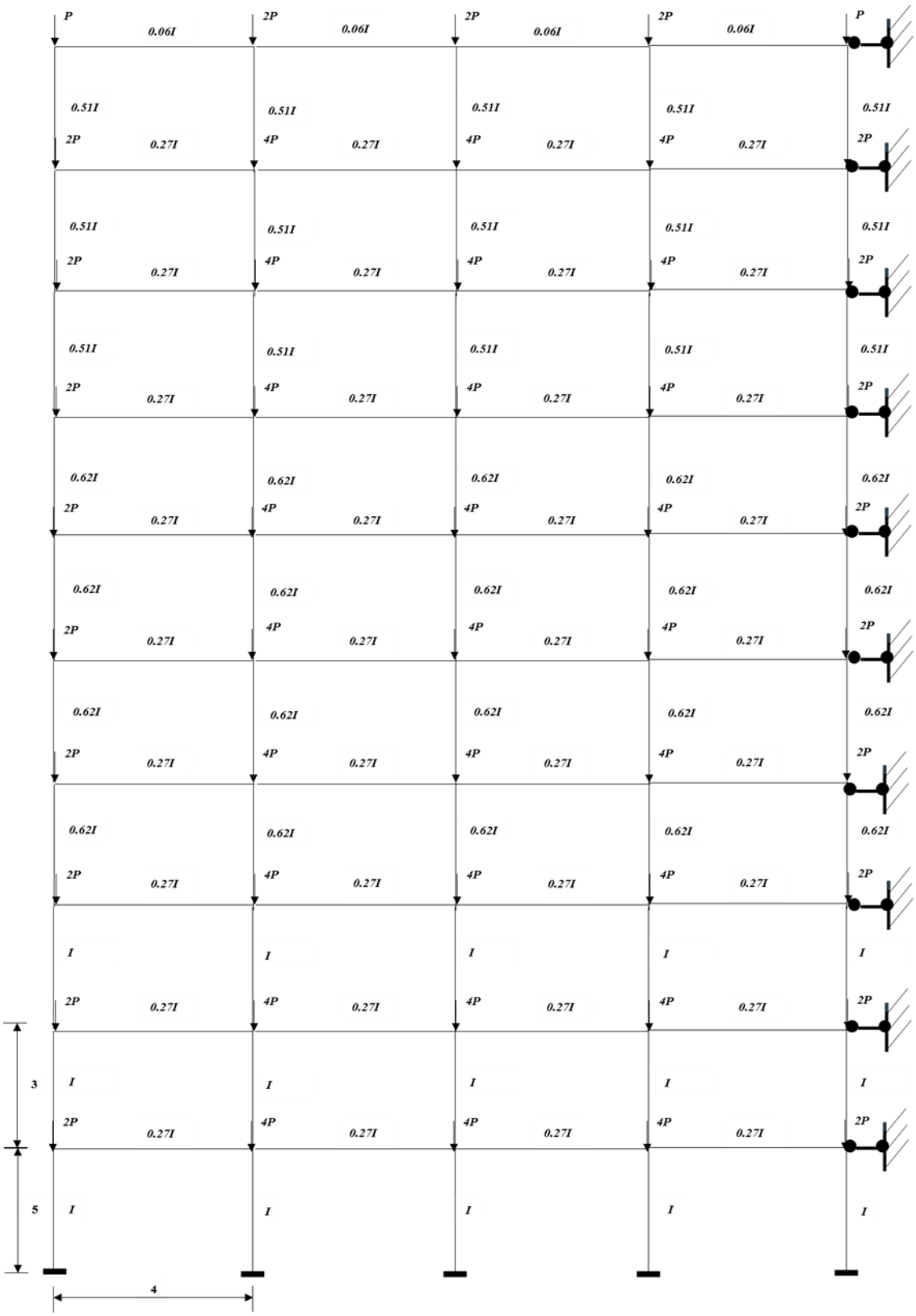


Figure 3.20. Frame C: A ten story four bay no sway frame as per example in Mahfouz (Mahfouz, 1999).

The system buckling load of the frame is first determined to attain the story with the minimum story buckling load, from which the other stories can be optimised.

The accompanying sway frame can be obtained by releasing the bracing of the no sway frame. For the accompanying sway frame, it is easy to assume that the frame will buckle at the bottom story. The upper-bound story buckling load can be found as before.

The horizontal stiffness of the first story before loading:

$$K_1 = 5 \times \frac{12EI}{h^3}$$

The negative stiffness caused by the axial loads on this story, given that the loads from the upper story are transmitted to this story:

$$K_{p1} = 2 \times \frac{19P}{h} + 3 \times \frac{38P}{h} = \frac{152P}{h}$$

After loading, the horizontal stiffness of the story becomes:

$$K - K_p = \frac{60EI}{h^3} - \frac{152P}{h} = 0$$

Solving this equation, one can obtain:

$$P_1 = \frac{15EI}{38h^2}$$

Taking *P*-δ effect into account, the system buckling load of the accompanying sway frame can be obtained:

$$P_{cr,sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{15EI}{38h^2} = \frac{5\pi^2 EI}{152h^2}$$

Applying the relationship previously shown between the upper-bound system buckling loads of sway and no sway frames, the upper-bound system buckling load, corresponding to $I_b = nI = \infty$, of the original no sway frame shown in Figure 3.20 can be obtained:

$$\overline{P_{cr,1}} = \eta \times P_{cr,sway} = 4 \times P_{cr,sway} = \frac{5\pi^2 EI}{38h^2}$$

Accounting for the non-rigid beams in the original no sway frame, the real system buckling load can be determined by considering the equivalent normalised one bay frame.

For the frame shown in Figure 3.20, the number of bays is four; b=4; thus the equivalent one bay frame for the first story, is shown in Figure 3.21.

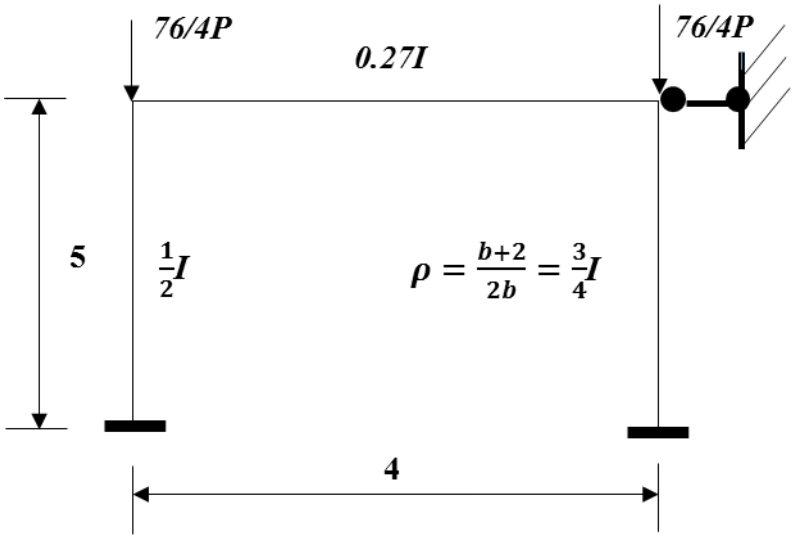


Figure 3.21 Equivalent one bay frame for the first story of Frame C in Example 4.

As stated previously, for the frame in Figure 3.21, the buckling of the system is localised, since the column with lower stiffness of $\frac{1}{2}I$ will buckle first. Therefore, the upper-bound system buckling load is re-calculated.

The horizontal stiffness of this column before loading:

$K_1 = \frac{12E\times\frac{1}{2}I}{h^3}$, where *h* is the height of the first story.

The negative stiffness caused by the axial loads on the column, given that the loads from the upper stories are transmitted to this column:

$K_{p1} = \frac{\frac{76}{4}P}{5}$

After loading, the horizontal stiffness of the column becomes:

$K - K_p = \frac{6EI}{h^3} - \frac{\frac{76}{4}P}{h} = 0$

Solving this equation, one can obtain:

$P_1 = \frac{6EI}{19h^2}$

Taking *P*-δ effect into account, the buckling load of the column can be obtained:

$P_{cr,sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{6EI}{19h^2} = \frac{\pi^2 EI}{38h^2}$

The upper-bound of the system buckling load, corresponding to $I_b = nI = \infty$, of the no sway frame shown in Figure 3.21 can be obtained:

$\overline{P_{cr,1}} = \eta \times P_{cr,sway} = 4 \times P_{cr,sway} = \frac{2\pi^2 EI}{19h^2}$

The design graphs in Figure 2.11 when b =2 and b =∞ was used to interpolate the value of μ when b =4. For the less stiff column, $\lambda = \frac{I_b L_c}{I_c L_b} = \frac{0.27I \times 5}{\frac{1}{2}I \times 4} = 0.677.$

From the graph in Figure 2.11, using λ=0.677:

When b =2, μ_1= 0.6997for ρ_1=1 , and

When b =∞, μ_2= 0.6010 for ρ_2=0.5.

Therefore, the value of μ for b =4(ρ=0.75) can be interpolated as follows:

$$\mu = \mu_2 + \frac{\mu_1 - \mu_2}{\rho_1 - \rho_2} \times (\rho - \rho_2) = 0.6010 + \frac{0.6997 - 0.6010}{1 - 0.5} \times \left(\frac{3}{4} - 0.5\right) = 0.6503$$

Thus, the final story buckling load for first story can be obtained:

$$P_{cr,1} = \mu \times P_{cr,upperbound,1} = 0.6503 \times \frac{2\pi^2 E \times \frac{1}{2}I}{19h^2} = 0.338\frac{EI}{h^2}$$

The same process is applied to the less stiff columns of the remaining stories of the frame. For story number two, the upper-bound system buckling load is calculated;

The horizontal stiffness of the less stiff column before loading:

$K_2 = \frac{12E \times \frac{1}{2}I}{(0.6h)^3}$, where h is the height of the first story.

The negative stiffness caused by the axial loads on the column, given that the loads from the upper stories are transmitted to this column:

$$K_{p2} = \frac{\frac{68}{4}P}{0.6h}$$

After loading, the horizontal stiffness of the column becomes:

$$K - K_p = \frac{6EI}{(0.6h)^3} - \frac{\frac{68}{4}P}{0.6h} = 0$$

Solving this equation, one can obtain:

$$P_2 = \frac{6EI}{17 \times (0.6h)^2}$$

Taking P-δ effect into account, the buckling load of the column can be obtained:

$$P_{cr,sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{6EI}{17 \times (0.6h)^2} = \frac{\pi^2 EI}{34 \times (0.6h)^2}$$

Thus, the upper-bound of the system buckling load, of the no sway frame can be obtained:

$$\overline{P_{cr,2}} = \eta \times P_{cr,sway} = 4 \times P_{cr,sway} = \frac{50\pi^2 EI}{153h^2}$$

The design graphs when b =2 and b =∞ was used to interpolate the value of μ when b =4. For the less stiff column, $\lambda = \frac{I_b L_c}{I_c L_b} = \frac{0.27I \times 3}{\frac{1}{2}I \times 4} = 0.405$.

From the graph in Figure 2.11 for b=2 and b=∞, using λ=0.405, the value of μ was interpolated as 0.6024.

Thus, the final story buckling load for first story can be obtained:

$$P_{cr,1} = \mu \times \overline{P_{cr,2}} = 0.6024 \times \frac{50\pi^2 E \times \frac{1}{2}I}{153h^2} = 0.972\frac{EI}{h^2}$$

For the remaining stories, the results from iteration 1 and the total weight of the frame is summarised in Table 3.18.

Table 3.18 Iteration 1 of the optimisation procedure of the literature example.

Story number, *i*	$K_i(\frac{EI}{h^3})$	**K_{pi} (P/h)**	$\overline{P_{cr}}$ $(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$ $(\frac{EI}{h^2})$**	**Story buckling load ratio, α**
1	6.00	19	2/19	0.68	0.65	**0.338**	1.000
2	6.00	17	50/153	0.41	0.60	0.972	0.348
3	6.00	15	10/27	0.41	0.60	1.101	0.307
4	3.69	13	179/681	0.66	0.65	0.840	0.402
5	3.69	11	73/235	0.66	0.65	0.993	0.340
6	3.69	9	183/482	0.66	0.65	1.213	0.278
7	3.69	7	350/717	0.66	0.65	1.560	0.217
8	3.06	5	167/295	0.80	0.67	1.869	0.181
9	3.06	3	167/177	0.80	0.67	3.115	0.108
10	3.06	1	2.83	0.17	0.55	7.722	0.044
Weight of frame		161 696 kg/162t					
Critical story		**1**					

Note: h is the height of the first story.

The story buckling load ratio α is obtained for each story as follows:

$$Pcr, ratio = \frac{Pcr,1}{Pcr,i}$$

From Table 3.18, it can be seen that the critical story was found to be story 1.

Optimisation is reached when α is greater than 0.9 for all stories hence the frame has not been optimised.

The new column and beam stiffness is calculated using the story load ratio α determined.

The results of the second iteration are summarised in Table 3.19.

Table 3.19 Iteration 2 of the optimisation procedure of the literature example.

Story number, *i*	$K_i(\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}}$ $(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $(\frac{EI}{h^2})$	**Story buckling load ratio, α**
1	6.00	19	2/19	0.68	0.65	0.338	1.000
2	2.09	17	5/44	0.41	0.60	0.338	1.000
3	1.84	15	5/44	0.41	0.60	0.338	1.000
4	1.48	13	89/842	0.66	0.65	0.338	1.000
5	1.26	11	89/842	0.66	0.65	0.338	1.000
6	1.03	9	89/842	0.66	0.65	0.338	1.000
7	0.80	7	89/842	0.66	0.65	0.338	1.000
8	0.55	5	57/557	0.80	0.67	0.338	1.000
9	0.33	3	57/557	0.80	0.67	0.338	1.000
10	0.13	1	93/751	0.17	0.55	0.338	1.000
Weight of frame		92 507 kg/92.5t					
Critical story		**All stories**					

From the results of the second iteration shown in Table 3.19, optimisation has been reached as all story ratios α are equal to one. The weight of the frame has reduced by 43%. The final optimised frame is shown in Figure 3.22. It was noted that the beam stiffness in the optimised frame did not reduce at the fourth story. Even though the first story was found to be critical after the first iteration, the beam stiffness is reduced by the same α value as that calculated for the columns, which is governed by the critical story. Since the column stiffness of the initial frame from the literature in Figure 3.20, reduced from *I* to 0.62*I* in story 4, the story buckling load would reduce and increase the α ratio at this story. Hence the final beam stiffness was higher at this story, as all beams initially had a stiffness of 0.27*I* except for the last story.

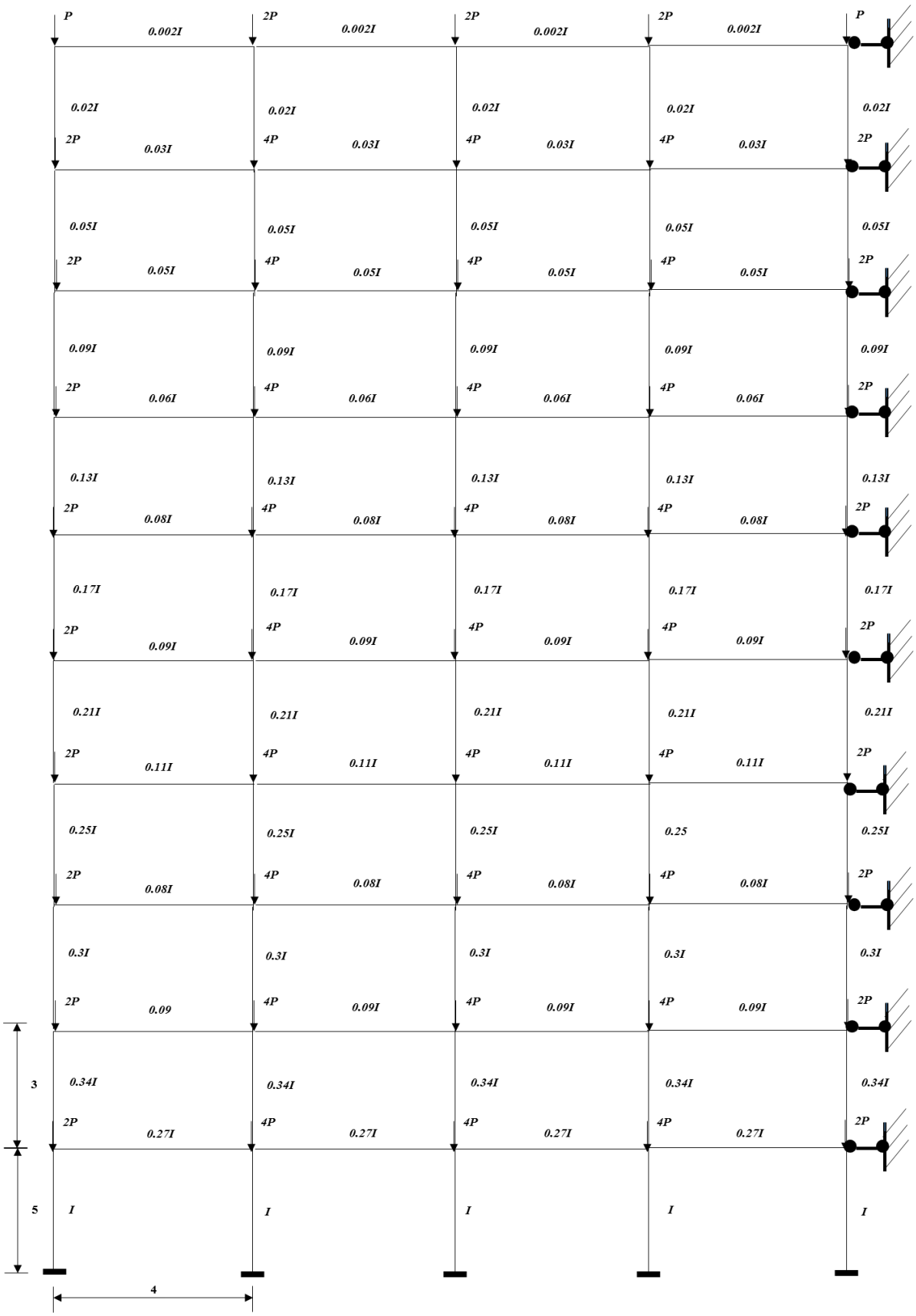


Figure 3.22 Optimised Frame C four bay ten story no sway frame.

When the optimised frame in Figure 3.22 was analysed in ANSYS, a system buckling load of $0.35\frac{EI}{h^2}$ was obtained, resulting in a 3.4% smaller difference than the result obtained from the proposed method. The percentage difference could be attributed to the fact that the fourth story was found to have a beam stiffness that did not reduce with story number, leading to a localised rigid region or zone within the frame. However, this percentage difference leads to an acceptable estimate of the buckling load for the purposes of a simple optimisation method. Nevertheless, from a design aspect, this may not be practical to have one story with a higher beam stiffness and a suggestion can be made to choose initial sections for the beam members that reduce in stiffness with story number as well or to use the same column and beam stiffness across the frame.

The buckled shape of the optimised frame is shown in Figure 3.23. It is seen that all stories buckle under the first mode and all members have deflected thus leading to the fact that the full member capacity was utilised.

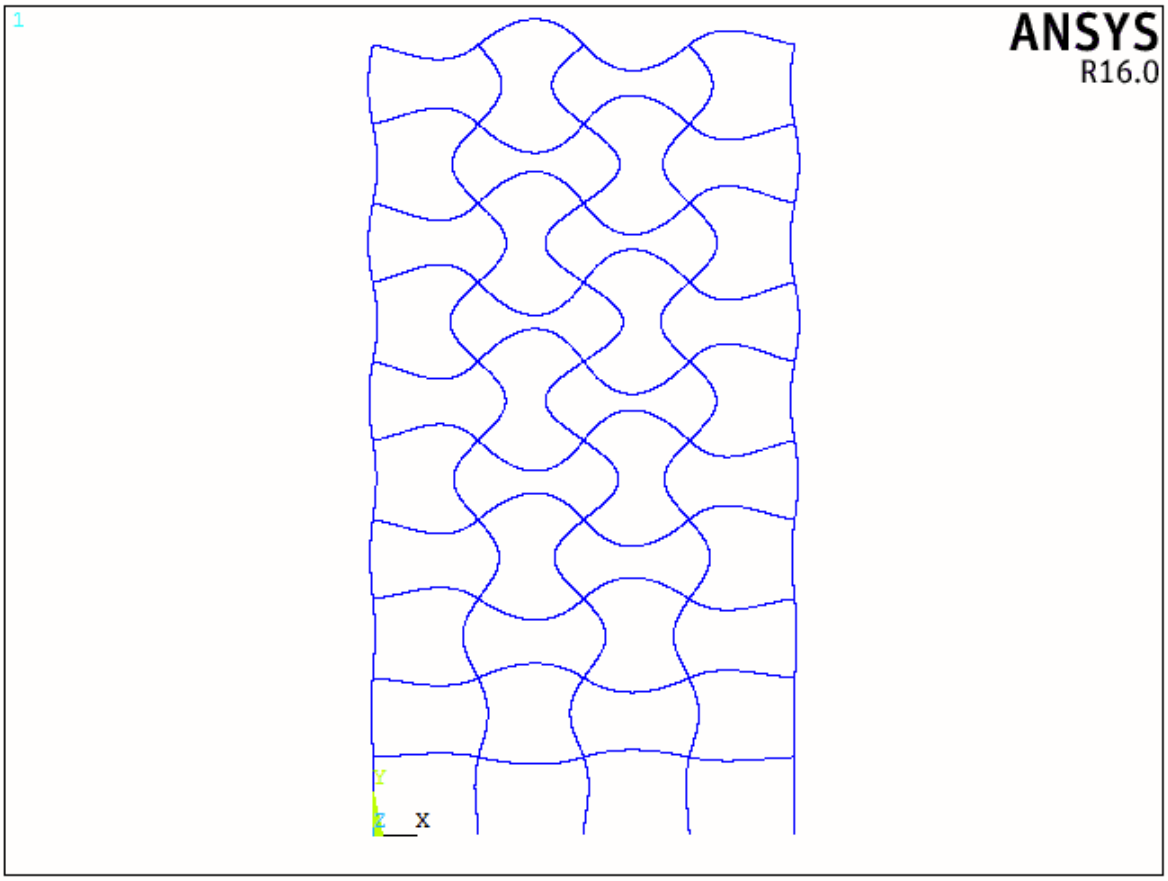


Figure 3.23 Buckled shape of ten story four bay frame as extracted from the literature.

Example 5: 4 storey 2-3 bay frame

A four story frame with different number of bays and its loads is shown in Figure 3.24. The frame was taken from an example in Li (2014) with all columns and beams initially having a stiffness as indicated. All stories have the same height.

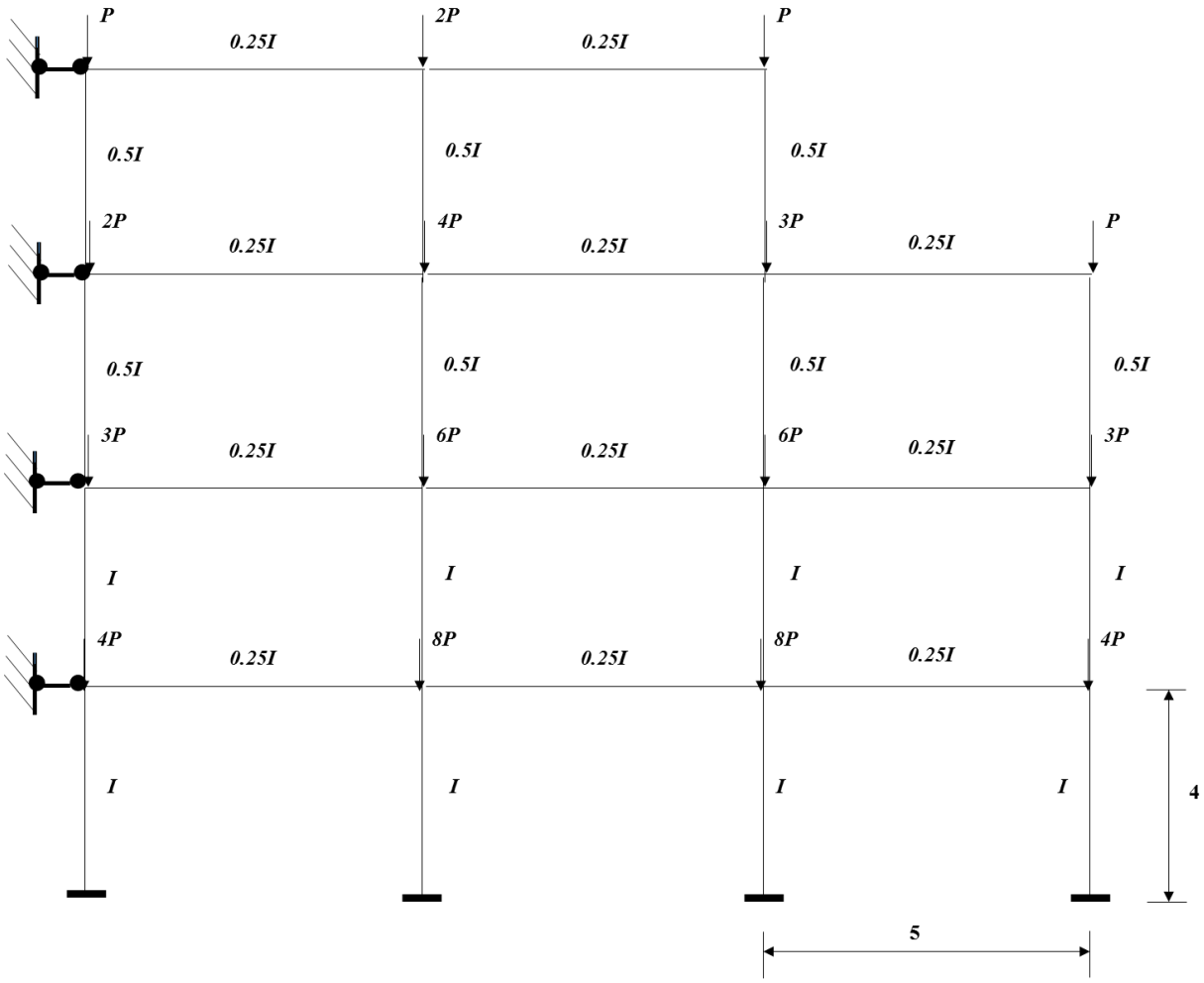


Figure 3.24 A four story no sway frame with different bay numbers.

The system buckling load of the frame is first determined in order to obtain the story with the minimum story buckling load.

The accompanying sway frame can be obtained by releasing the bracing of the no sway frame, as described before. For the accompanying sway frame, it is easy to find that the frame will buckle at the bottom story.

The horizontal stiffness of the first story before loading:

$$K_1 = 4 \times \frac{12EI}{h^3}$$

The negative stiffness caused by the axial loads on this story, given that the loads from the upper story are transmitted to this story:

$$K_{p1} = \frac{10P}{h} + \frac{20P}{h} + \frac{18P}{h} + \frac{8P}{h} = \frac{56P}{h}$$

After loading, the horizontal stiffness of the story becomes:

$$K - K_p = \frac{48EI}{h^3} - \frac{56P}{h} = 0$$

Solving this equation, one can obtain:

$$P_1 = \frac{6EI}{7h^2}$$

Taking P-δ effect into account, the system buckling load of the accompanying sway frame can be obtained:

$$P_{cr,sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{6EI}{7h^2} = \frac{\pi^2 EI}{14h^2}$$

The upper-bound of the system buckling load, corresponding to $I_b = nI = \infty$, of the original no sway frame shown in Figure 3.24 can be obtained:

$$\overline{P_{cr,1}} = \eta \times P_{cr,sway} = 4 \times P_{cr,sway} = \frac{2\pi^2 EI}{7h^2}$$

Accounting for the non-rigid beams in the original no sway frame, the real system buckling load can be determined by considering the equivalent normalised one bay frame for the first story.

For the frame shown in Figure 3.24, the number of bays in the first story is three, b=3, thus the equivalent one bay frame for the first story, when the frame is folded into the middle bay, is shown in Figure 3.25.

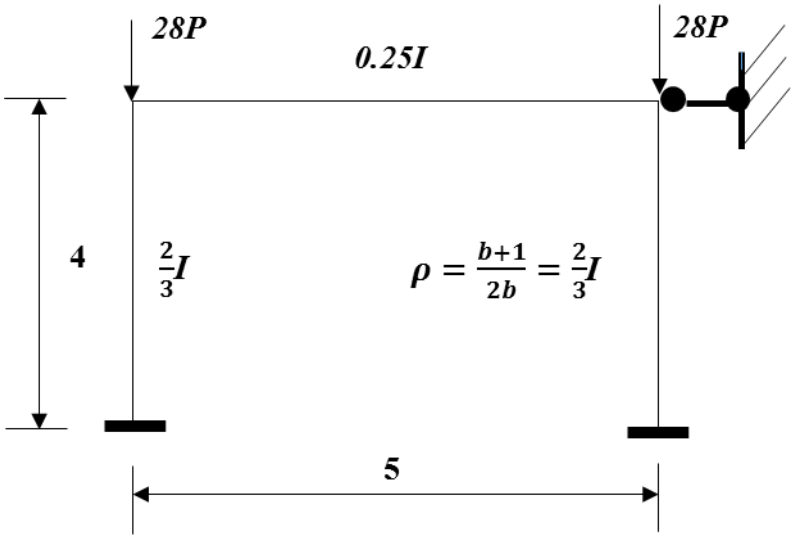


Figure 3.25 Normalised equivalent one bay frame for first story of frame in Example 5.

For the frame in Figure 3.25, the value of $\rho=\frac{2}{3}$ and $\lambda=\frac{I_bL_c}{I_cL_b}=\frac{0.25I\times4}{\frac{2}{3}I\times5}=0.3$.

The design graph when $b=\infty$ was used to obtain the value of μ, thus from the graph in Figure 2.9, for a λ value of 0.3, μ= 0.5539.

Thus, the final story buckling load can be obtained:

$$P_{cr,1}=\mu\times\overline{P_{cr,1}}=0.553857\times\frac{2\pi^2E\times\frac{1}{2}I}{7h^2}=0.781\frac{EI}{h^2}$$

Note that the stiffness of the column was reduced by the value of $\rho=\frac{1}{2}$ as the value of μ was taken from the design graph when $b=\infty$.

The same process is applied to the remaining three stories of the frame.

During the optimisation process, it is noted that for the fourth (top) story, the value of λ is calculated in consideration of it having two bays, whilst the other stories are taken from the graph when bay number b is equal to 3; thus:

Story 1 and 2, $\lambda=\frac{0.25I\times4}{\frac{2}{3}\times I\times5}=0.3\ \rightarrow\ \mu=0.5539$ when b=3

Story 3, $\lambda=\frac{0.12I\times4}{\frac{2}{3}\times0.23I\times5}=0.6\ \rightarrow\ \mu=0.5918$ when b=3

Story 4, $\lambda=\frac{0.05I\times4}{\frac{1}{2}\times0.09I\times5}=0.8\ \rightarrow\ \mu=0.6682$ when b=2 (note $\rho=\frac{1}{2}$)

For the first iteration of optimisation of the original frame, the story buckling loads and the total weight of the frame is summarised in Table 3.20.

Table 3.20 Iteration 1 of the optimisation procedure of Example 5.

Story number, *i*	$K_i\left(\frac{EI}{h^3}\right)$	K_{pi} **(P/h)**	$\overline{P_{cr}}\left(\frac{\pi^2EI}{h^2}\right)$	$\lambda=\frac{I_bL_c}{I_cL_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$ $\left(\frac{EI}{h^2}\right)$**	**Story buckling load ratio, α**
1	48	56	0.286	0.3	0.554	**0.781**	1.000
2	48	33	0.485	0.3	0.554	1.325	0.589
3	24	14	0.571	0.6	0.592	1.669	0.468
4	18	4	1.500	0.8	0.668	4.946	0.158
Weight of frame		8 758 kg/8.7tonnes					
Critical story		**1**					

From Table 3.20, it can be seen that the critical story was found to be story 1. The story buckling load ratio α is obtained for each story as follows:

$$Pcr, ratio = \frac{Pcr,1}{Pcr,i}$$

Optimisation is reached when α is greater than 0.9 for all stories hence the frame has not been optimised.

Another iteration is performed where the column and beam stiffness are reduced by the story load ratio α and the results are summarised in Table 3.21.

Table 3.21 Iteration 2 of the optimisation procedure of Example 5.

Story number, *i*	$K_i (\frac{EI}{h^3})$	**K_{pi} (P/h)**	$\overline{P_{cr}} (\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$ $(\frac{EI}{h^2})$**	**Story buckling load ratio, α**
1	48.000	56	0.286	0.3	0.554	0.781	1.000
2	28.286	33	0.286	0.3	0.554	0.781	1.000
3	11.231	14	0.267	0.6	0.592	0.781	1.000
4	2.842	4	0.237	0.8	0.668	0.781	1.000
Weight of frame		7 479 kg/7.5tonnes					
Critical story		**All stories**					

From the results of the second iteration shown in Table 3.21, it can be seen that optimisation has been reached as all story ratios α are equal to one. The weight of the frame has reduced by 14% and all stories have the same buckling load. The final optimised frame is shown in Figure 3.26.

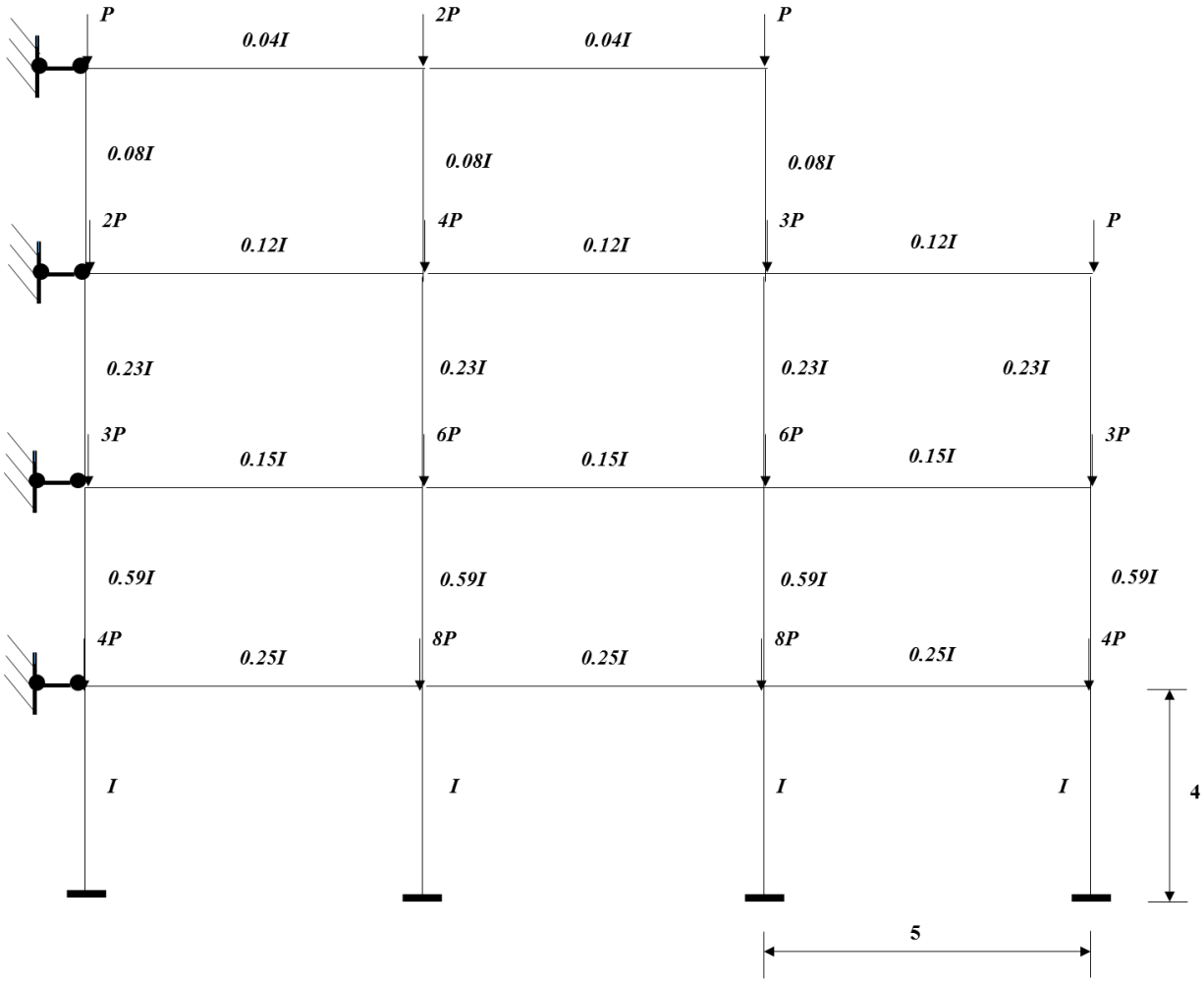


Figure 3.26 Optimised four story 2-3 bay no sway frame.

When the optimised frame was analysed in ANSYS, a system buckling load of $0.719\frac{EI}{h^2}$ was obtained, resulting in a 9% smaller difference than the result obtained from the proposed method. This percentage difference indicates the influence of frame geometry on the system buckling load; the fourth story had one less bay than the rest of the frame. Furthermore, the upper-bound system buckling load, $\overline{P_{cr}}$, calculated from the proposed method, was found to be higher than the result given from FEA.

The first buckling mode shape produced in FEA is shown in Figure 3.27. It can be seen that the entire frame did not 'fully' buckle as not all of the members deflected, as indicated by the straight lines of the end columns in third bay of the frame.

In addition, the higher stories buckled more under this buckling load of $0.719\frac{EI}{h^2}$. Thus, stories with reduced number of bays could be seen to influence the behaviour of the frame.

The fourth story of the frame which has two bays, may have experienced 'localised' buckling which would need to be accounted for. This concept was then explored.

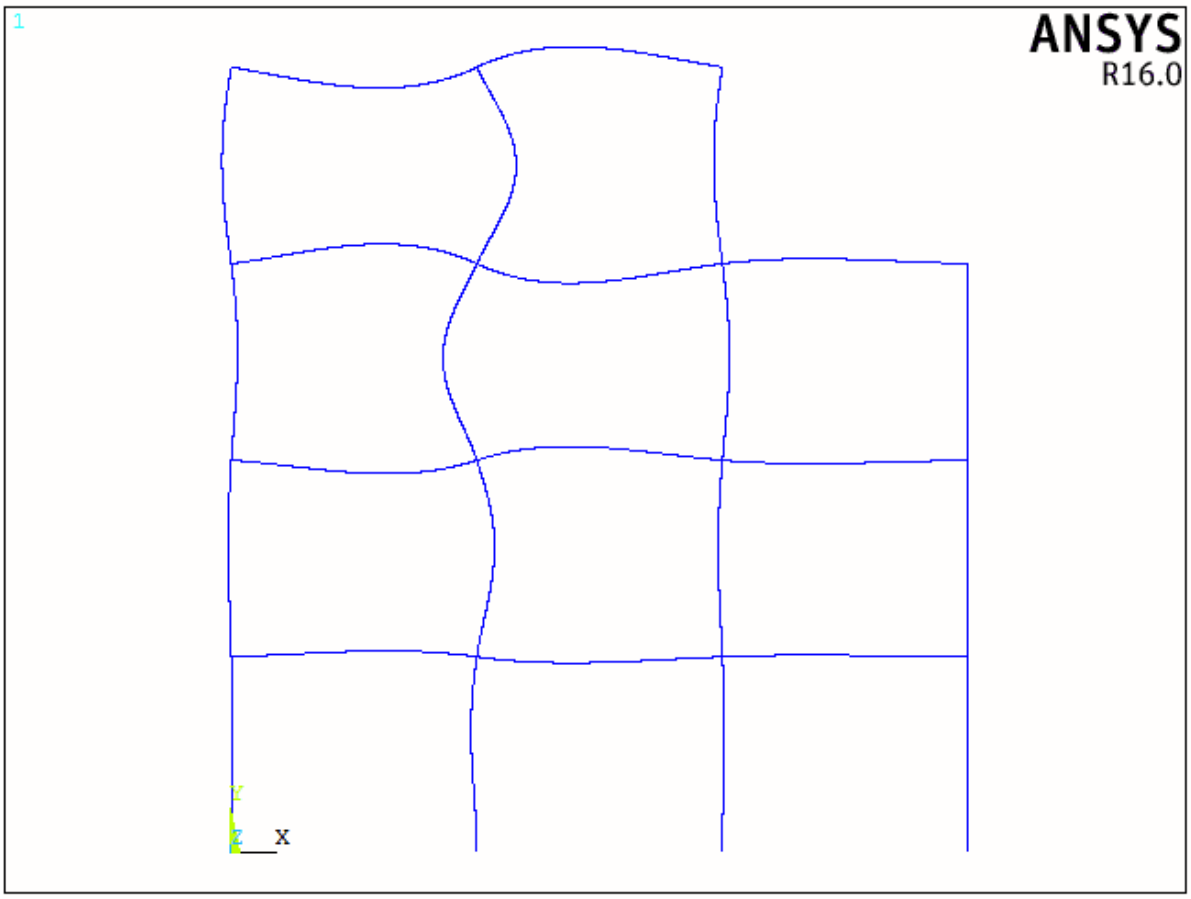


Figure 3.27 Buckled shape of the optimised four story no sway frame.

Using its equivalent one bay frame for the fourth story of the initial frame in Figure 3.24:

$$K_4 = \frac{12E \times \frac{1}{2} \times 0.5I}{h^3}$$

$$K_{p4} = \frac{4P}{h} \times \frac{1}{2} = \frac{2P}{h}$$

$$P_4 = \frac{3EI}{2h^2}$$

$$P_{cr,sway,4} = \beta \times P = \frac{\pi^2}{12} \times \frac{3EI}{2h^2} = \frac{\pi^2 EI}{8h^2}$$

$$\overline{P_{cr,4}} = 4 \times P_{cr,sway} = \frac{\pi^2 EI}{2h^2}$$

$$\lambda = \frac{I_b L_c}{I_c L_b} = \frac{0.25I \times 4}{\frac{1}{2} \times 0.5I \times 5} = 0.8 \rightarrow \mu = 0.6682$$

$$\therefore P_{cr,4} = 0.6682 \times \frac{\pi^2 E \times \frac{1}{2} I}{2h^2} = 2.467 \frac{EI}{h^2}$$

The results for the first iteration of optimisation, taking into account the above, are summarised in Table 3.22.

Table 3.22 Iteration 1 of the optimisation procedure of Example 5 considering the local buckling of story 4.

Story number, *i*	$K_i (\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}} (\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $(\frac{EI}{h^2})$	**Story buckling load ratio, α**
1	48	56	0.286	0.3	0.554	**0.781**	1.000
2	48	33	0.485	0.3	0.554	1.325	0.589
3	24	14	0.571	0.6	0.592	1.669	0.468
4	3	2	0.500	0.8	0.668	2.467	0.474
Weight of frame		8 758 kg/8.7tonnes					
Critical story		**1**					

As before, the load ratio α is used to calculate the stiffness of the members for the next iteration. The results of the second iteration are summarised in Table 3.23.

Table 3.23 Iteration 2 of the optimisation procedure of Example 5 considering the local buckling of story 4.

Story number, *i*	$K_i (\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}} (\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $(\frac{EI}{h^2})$	**Story buckling load ratio, α**
1	48.000	56	0.286	0.3	0.554	0.781	1.000
2	28.286	33	0.286	0.3	0.554	0.781	1.000
3	11.231	14	0.267	0.6	0.592	0.781	1.000
4	1.422	2	0.237	0.8	0.668	0.781	1.000
Weight of frame		7 796 kg/7.8 tonnes					
Critical story		**All stories**					

From the results in Table 3.23, it can be seen that optimisation has been reached as all story ratios α are equal to one. The weight of the frame has reduced by 10.3%. The final optimised frame is shown in Figure 3.28.

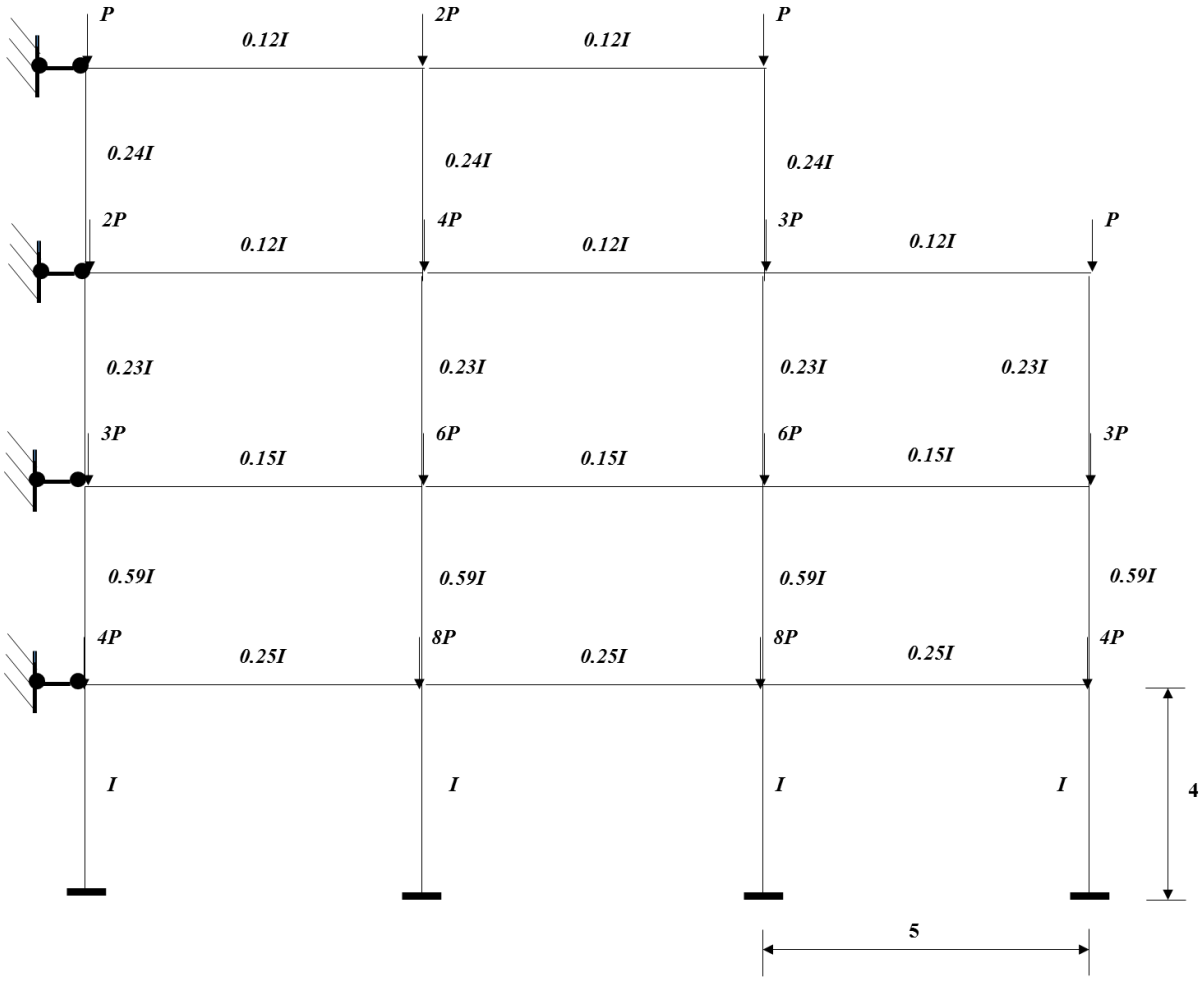


Figure 3.28 Optimised four story frame with different bay numbers considering the local buckling of story 4.

When the optimised frame in Figure 3.28 was analysed in ANSYS, a system buckling load of $0.785\frac{EI}{h^2}$ was obtained, yielding a 0.55% smaller difference than the result from the proposed method. The percentage difference from FEA has reduced significantly in the second approach thus demonstrating the effect of the number of bays on the system buckling load. The 'local' buckling experienced in frames with even bay numbers, should be accounted for in all frames that have different number of bays, as done in the second approach for this frame.

The results of the optimised frame from the two approaches are summarised in Table 3.24.

Table 3.24 Optimised frame from the two approaches on the four story frame with different number of bays.

	System buckling load-proposed method, P_{cr} $(\frac{EI}{h^2})$	**System buckling load-FEA, P_{cr} $(\frac{EI}{h^2})$**	**Difference from FEA (%)**	**Weight of frame (tonnes)**
Frame as shown in Figure 3.26	0.781	0.719	9	7.5
Frame as shown in Figure 3.28	0.781	0.785	0.55	7.8

The second optimised frame in Figure 3.28, has stiffer members in the fourth story and the weight of the frame was 4% heavier when compared to the frame obtained in Figure 3.26. Nevertheless, the latter approach of accounting for the local buckling in the fourth story, resulted in an improved estimate of the system buckling load if the result from FEA is taken as correct.

Example 6: 5 storey 4 bay frame with different story height

A five story four bay frame with its loads, shown in Figure 3.29, was optimised with all columns and beams initially having a stiffness as indicated. The first story has a different height to the rest of the stories within the frame.

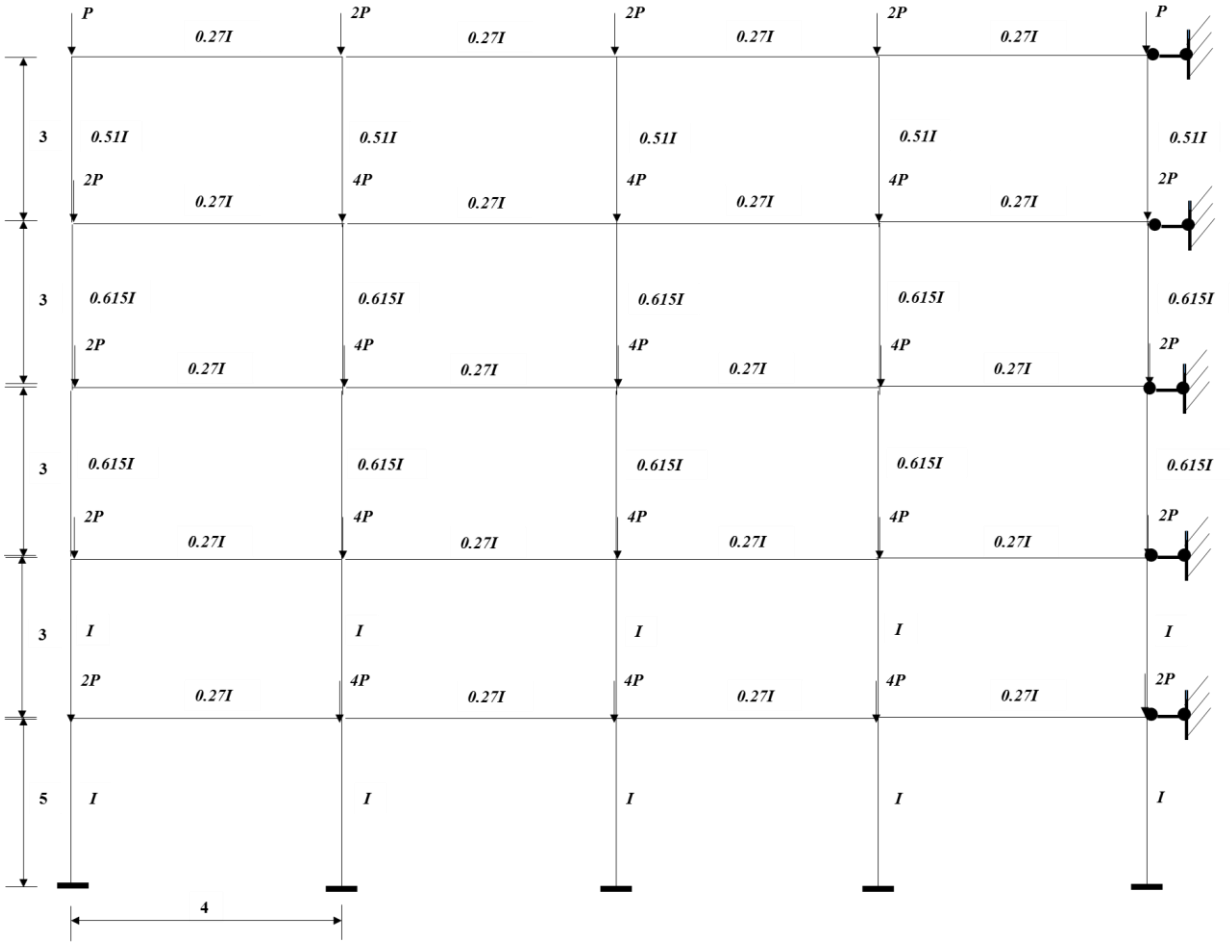


Figure 3.29 A five story four bay no sway frame with different story height.

The system buckling load of the frame is first determined to obtain the story with the minimum story buckling load.

The accompanying sway frame can be obtained by releasing the bracing of the no sway frame, as described before. For the accompanying sway frame, it is easy to assume that the frame will buckle at the bottom story. The upper-bound story buckling load, $\overline{P_{cr}}$, can be found using the method applied previously.

The horizontal stiffness of the first story before loading:

$$K_1 = 5 \times \frac{12EI}{h^3}$$

The negative stiffness caused by the axial loads on this story, given that the loads from the upper story are transmitted to this story:

$$K_{p1} = 2 \times \frac{9P}{h} + 3 \times \frac{18P}{h} = \frac{72P}{h}$$

After loading, the horizontal stiffness of the story becomes:

$$K - K_p = \frac{60EI}{h^3} - \frac{72P}{h} = 0$$

Solving this equation, one can obtain:

$$P_1 = \frac{5EI}{6h^2}$$

Taking *P*-δ effect into account, the system buckling load of the accompanying sway frame can be obtained:

$$P_{cr,sway,1} = \beta \times P = \frac{\pi^2}{12} \times \frac{5EI}{6h^2} = \frac{5\pi^2 EI}{72h^2}$$

The upper-bound of the system buckling load, corresponding to $I_b = nI = \infty$, of the original no sway frame shown in Figure 3.29 can be obtained:

$$\overline{P_{cr,1}} = \eta \times P_{cr,sway} = 4 \times P_{cr,sway} = \frac{5\pi^2 EI}{18h^2}$$

It was determined that for even bay number frames, there is a form of localised buckling as one column has a lower stiffness of $\frac{1}{2}I$, thus the value of λ is calculated accordingly; $\lambda = \frac{I_b L_c}{I_c L_b} = \frac{0.27I \times 5}{\frac{1}{2}I \times 4} = 0.675$.

The design graph in Figure 2.11 when *b* =2 and *b* =∞ was used to interpolate the value of μ when *b* =4 for this column using λ=0.675:

When *b* =2, μ_1= 0.6484 for ρ_1=1 , and

When *b* =∞, μ_2= 0.6008 for ρ_2=0.5.

Therefore the value of μ for *b* =4(ρ=0.75) can be interpolated as follows:

$$\mu = \mu_2 + \frac{\mu_1 - \mu_2}{\rho_1 - \rho_2} \times (\rho - \rho_2) = 0.6008 + \frac{0.6484 - 0.6008}{1 - 0.5} \times \left(\frac{3}{4} - 0.5\right) = 0.6246$$

Thus, the final story buckling load for first story can be obtained:

$$P_{cr,1} = \mu \times \overline{P_{cr,1}} = 0.6246 \times \frac{5\pi^2 E \times \frac{1}{2}I}{18h^2} = 0.856\frac{EI}{h^2}$$

The same process is applied to the columns with the lower stiffness of the remaining stories of the frame. For the first iteration, the story buckling loads and the total weight of the frame is summarised in Table 3.25.

Table 3.25 Iteration 1 of the optimisation procedure of Example 6.

Story number, *i*	$K_i(\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}}$ $(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $(\frac{EI}{h^2})$	**Story buckling load ratio, α**
1	60.00	72.00	2.74	0.68	0.62	**0.856**	1.00
2	277.78	93.33	9.79	0.41	0.58	2.860	0.30
3	170.83	66.67	8.43	0.66	0.62	2.623	0.33
4	170.83	40.00	14.05	0.66	0.62	4.372	0.20
5	141.51	13.33	34.92	0.80	0.64	11.193	0.08
Weight of frame		144t					
Critical story		**1**					

Note: h is the height of the first story.

The story buckling load ratio α is obtained for each story as follows:

$$Pcr, ratio = \frac{Pcr,1}{Pcr,i}$$

From Table 3.25, it can be seen that the first story is critical as it has the lowest buckling load.

Optimisation is reached when α is greater than 0.9 for all stories hence the frame has not been optimised.

The second iteration is performed with the new column and beam stiffness that were reduced by the story load ratio α determined for each story.

The results of the iteration are presented in Table 3.26.

Table 3.26 Iteration 2 of the optimisation procedure of Example 6.

Story number, *i*	$K_i(\frac{EI}{h^3})$	**K_{pi} (P/h)**	$\overline{P_{cr}}$ $(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load, $P_{cr,i}$ $(\frac{EI}{h^2})$**	**Story buckling load ratio, α**
1	60.00	72.00	2.74	0.68	0.62	0.856	1.00
2	83.17	93.33	2.93	0.41	0.58	0.856	1.00
3	55.76	66.67	2.75	0.66	0.62	0.856	1.00
4	33.46	40.00	2.75	0.66	0.62	0.856	1.00
5	10.82	13.33	2.67	0.80	0.64	0.856	1.00
Weight of frame		91t					
Critical story		**All stories**					

From the results of the second iteration shown in Table 3.26, it can be seen that optimisation has been reached as all story ratios, α, are equal to one. The weight of the frame has reduced by 37%. The final optimised frame is shown in Figure 3.30. It was noticed that the beam stiffness of story 3 did not reduce with story number. This may be due to the fact that, since the column stiffness in the initial frame in Figure 3.29, changed from *I* to 0.615*I* in story 3, the story buckling load would reduce and thus increase the α ratio for this story. The initial beam stiffness of 0.27*I* was then reduced by a larger value (α=0.33) than story 2 (α=0.3) and hence the final stiffness was greater at this story as all beam stiffness' was initially set to 0.27*I*.

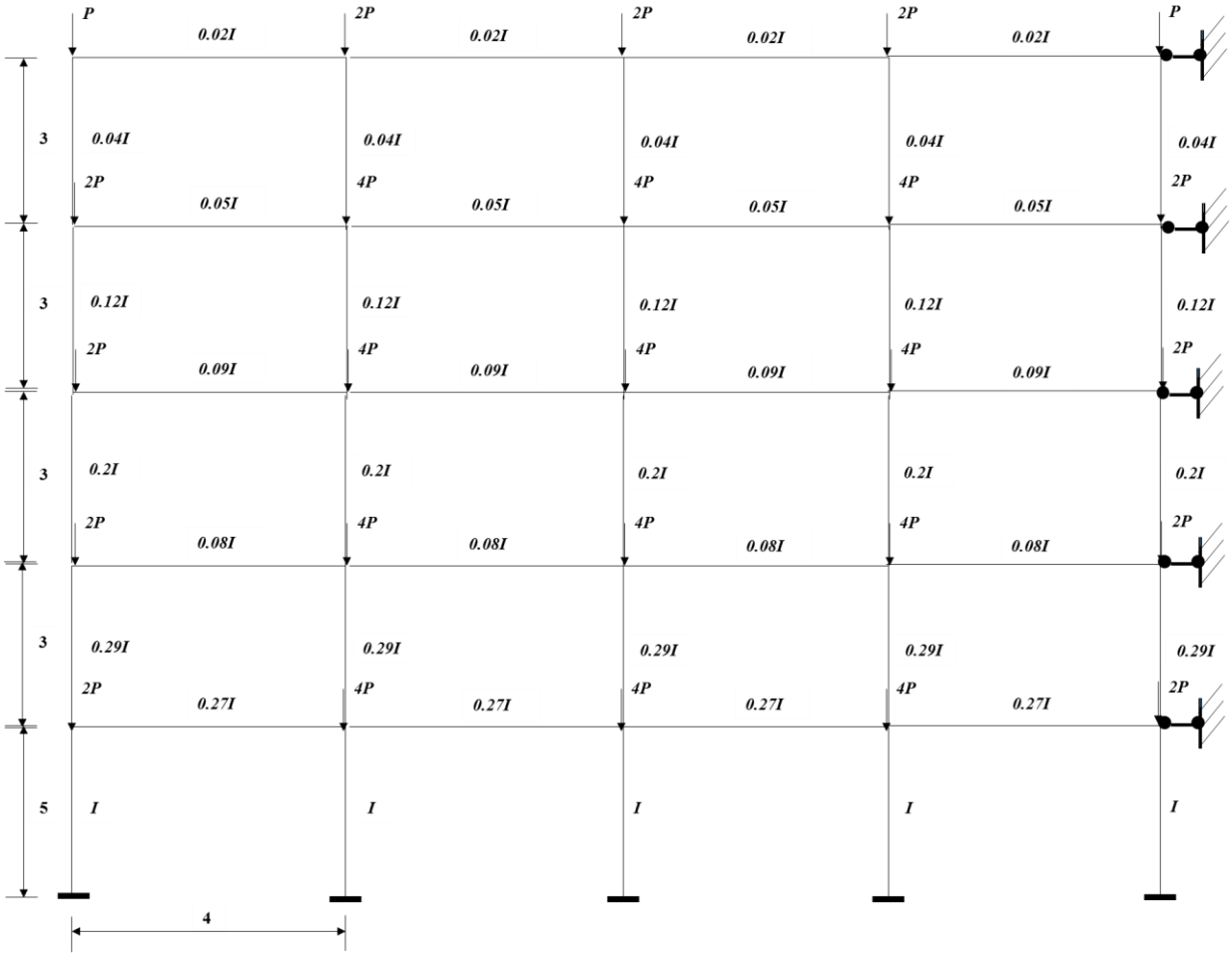


Figure 3.30 Optimised five story four bay no sway frame.

When the optimised frame in Figure 3.30 was analysed in FEA software ANSYS, a system buckling load of $0.82\frac{EI}{h^2}$ was produced, resulting in a 4.4% smaller difference than the result from the proposed method. The percentage difference could be attributed to the fact that as previously mentioned, the third story was found to have a beam stiffness that did not reduce with an increase in story number, leading to a localised rigid region or zone within the frame. However, this percentage difference is acceptable for the purposes of a simple method to optimise frame structures. Nevertheless, this may again not be practical to have an intermediate story with a higher beam stiffness and a suggestion can be made to choose initial sections for the beam members that reduce in stiffness with story number as well or to use the same column and beam stiffness across the frame.

The first buckling mode shape produced in FEA is shown in Figure 3.31.

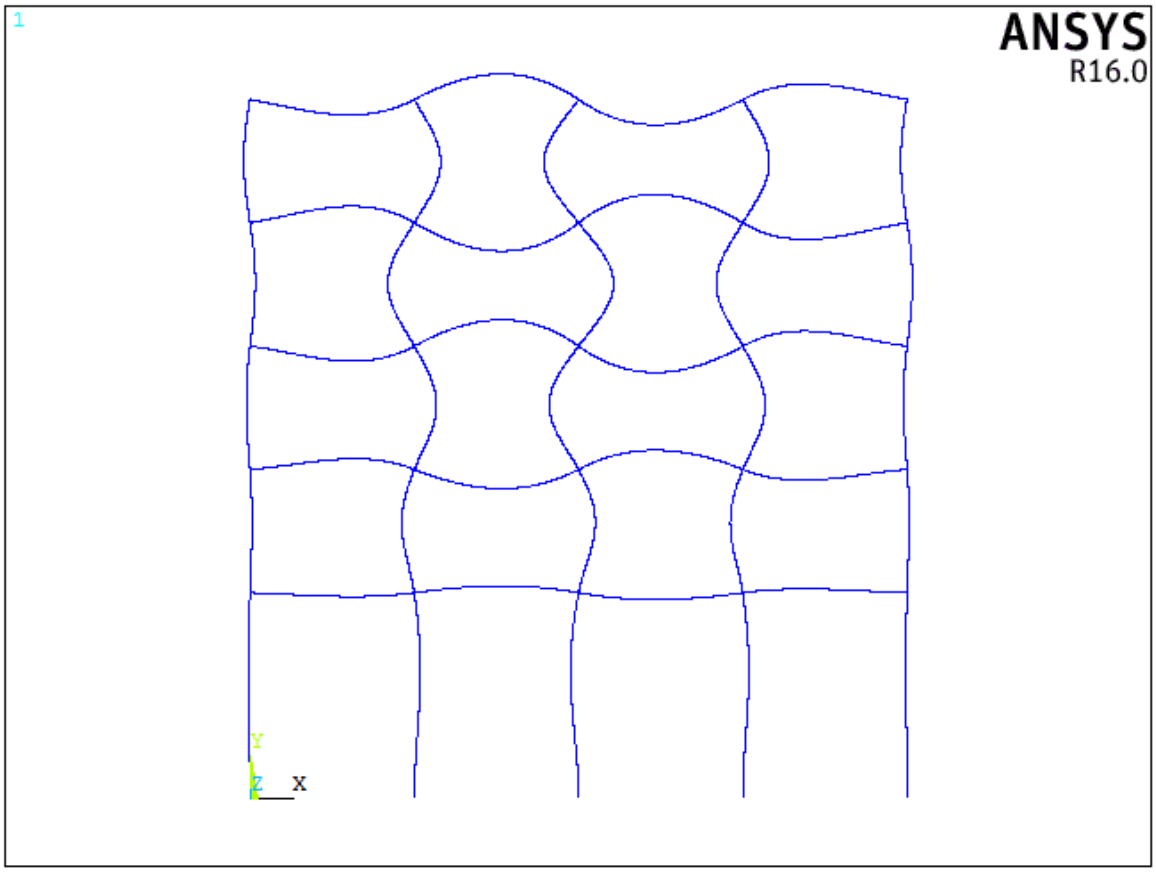


Figure 3.31 Buckled shape of optimised five story four story frame in Example 6.

In light of the previous findings, the option of applying the same column stiffness initially throughout the frame, as seen in Figure 3.32, was explored. The frame will be referred to as Frame C.

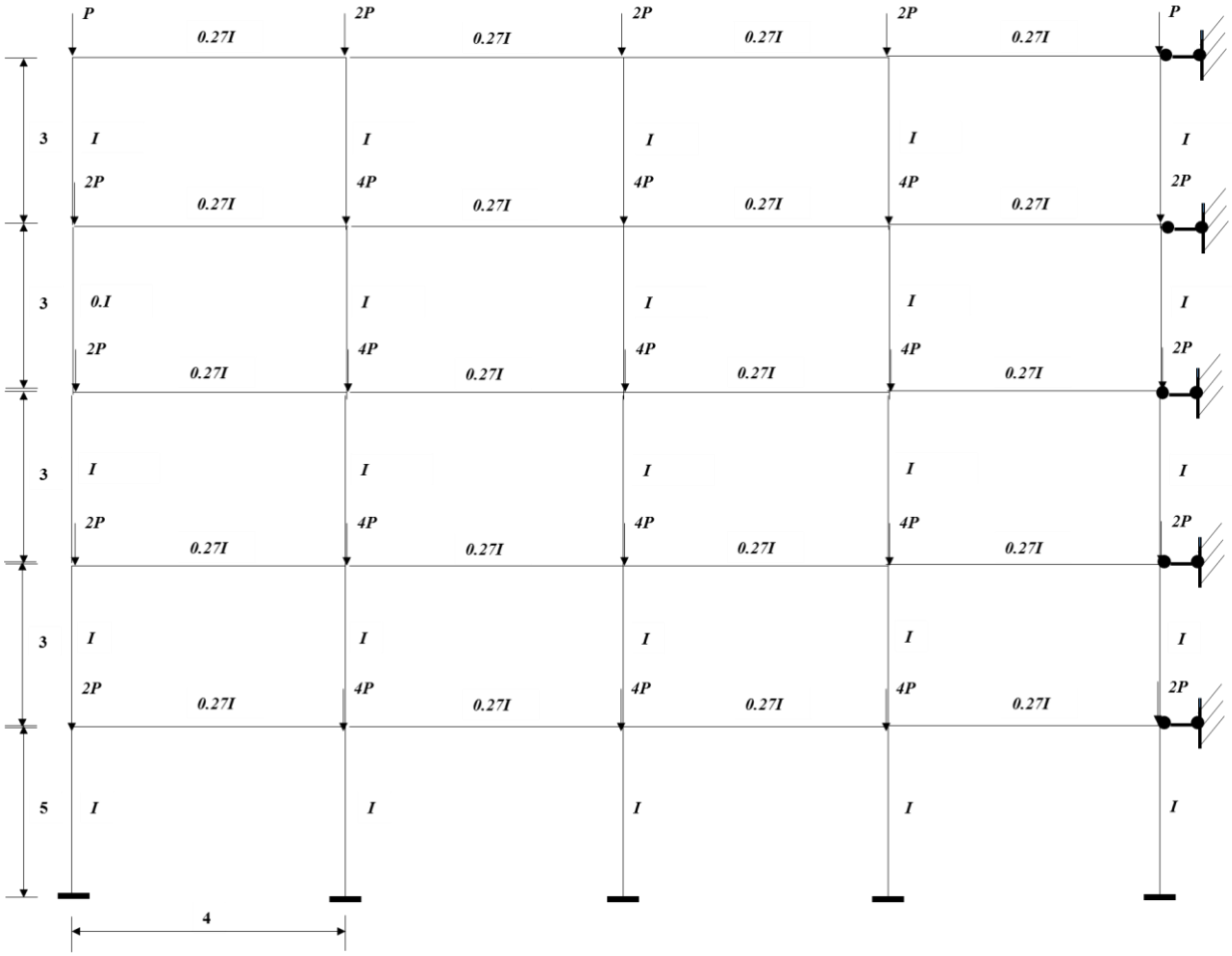


Figure 3.32 Frame C: Five story four bay no sway frame with different story height-same initial column stiffness.

The system buckling load of the original frame is determined as before to obtain the story with the minimum story buckling load, from which the other stories can be optimised. For the first story;

$$K_1 = 5 \times \frac{12EI}{h^3} \qquad K_{p1} = 2 \times \frac{9P}{h} + 3 \times \frac{18P}{h} = \frac{72P}{h}$$

$$P_1 = \frac{5EI}{6h^2} \qquad P_{cr,sway} = \beta \times P = \frac{\pi^2}{12} \times \frac{5EI}{6h^2} = \frac{5\pi^2 EI}{72h^2}$$

$$\overline{P_{cr,1}} = 4 \times P_{cr,sway} = \frac{5\pi^2 EI}{18h^2}$$

$$\lambda = \frac{I_b L_c}{I_c L_b} = \frac{0.27I \times 5}{\frac{1}{2}I \times 4} = 0.675 \quad \rightarrow \quad \mu = 0.6246$$

$$\therefore P_{cr,1} = 0.6246 \times \frac{5\pi^2 E \times \frac{1}{2}I}{18h^2} = 0.856\frac{EI}{h^2}$$

The same process is applied to the remaining stories of the frame. For the first iteration of Frame C, the story buckling loads and the total weight of the frame are summarised in Table 3.27.

Table 3.27 Iteration 1 of the optimisation procedure of Frame C in Example 6.

Story number, *i*	$K_i(\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}}$ $(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $(\frac{EI}{h^2})$	**Story buckling load ratio, α**
1	60.00	72.00	2.74	0.68	0.62	**0.856**	1.00
2	277.78	93.33	9.79	0.41	0.58	2.860	0.30
3	277.78	66.67	13.71	0.66	0.62	4.004	0.21
4	277.78	40.00	22.85	0.66	0.62	6.673	0.13
5	277.78	13.33	68.54	0.80	0.64	20.018	0.04
Weight of frame		96t					
Critical story		**1**					

Note: h is the height of the first story.

The story buckling load ratio α is obtained for each story as follows:

$$Pcr, ratio = \frac{Pcr,1}{Pcr,i}$$

From Table 3.27, it can be seen that the critical story was found to be story 1 and the frame has not reached optimisation.

Another iteration is performed where the column and beam stiffness is reduced by the story load ratio α as determined before.

The results of the second iteration of the frame with the new column and beam stiffness' is shown in Table 3.28.

Table 3.28 Iteration 2 of the optimisation procedure of Frame C in Example 6.

Story number, *i*	$K_i(\frac{EI}{h^3})$	K_{pi} **(P/h)**	$\overline{P_{cr}}$ $(\frac{\pi^2 EI}{h^2})$	$\lambda = \frac{I_b L_c}{I_c L_b}$	**μ from interpolation**	**Story buckling load,** $P_{cr,i}$ $(\frac{EI}{h^2})$	**Story buckling load ratio, α**
1	60.00	72.00	2.74	0.68	0.62	0.856	1.00
2	83.17	93.33	2.93	0.41	0.58	0.856	1.00
3	59.40	66.67	2.93	0.66	0.62	0.856	1.00
4	35.64	40.00	2.93	0.66	0.62	0.856	1.00
5	11.88	13.33	2.93	0.80	0.64	0.856	1.00
Weight of frame		53.2t					
Critical story		**All stories**					

From the results of the second iteration shown in Table 3.28, optimisation has been reached as all story ratios α are equal to one. The weight of the frame has reduced by 45%. The final optimised frame is shown in Figure 3.33.

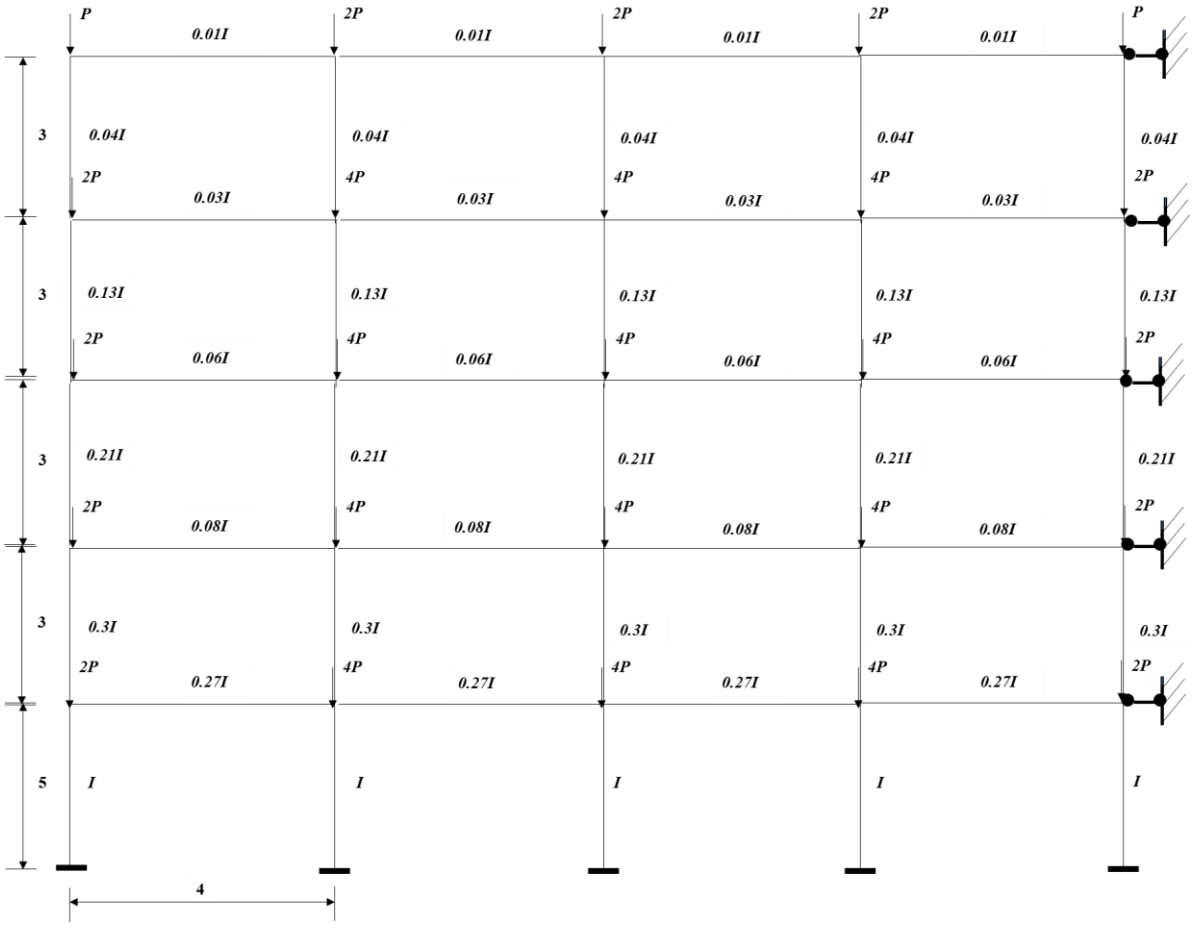


Figure 3.33 Optimised Frame C.

When the optimised frame in Figure 3.33 was analysed in FEA software ANSYS, a system buckling load of $0.8\frac{EI}{h^2}$ was obtained, resulting in a 7% smaller difference than the result obtained from the proposed method. The percentage difference increased from the previous frame analysis.

The first buckling mode shape produced in FEA is shown in Figure 3.34. It is observed that all stories buckle under the first mode and all members have deflected, thus leading to the fact that full member capacity was utilised.

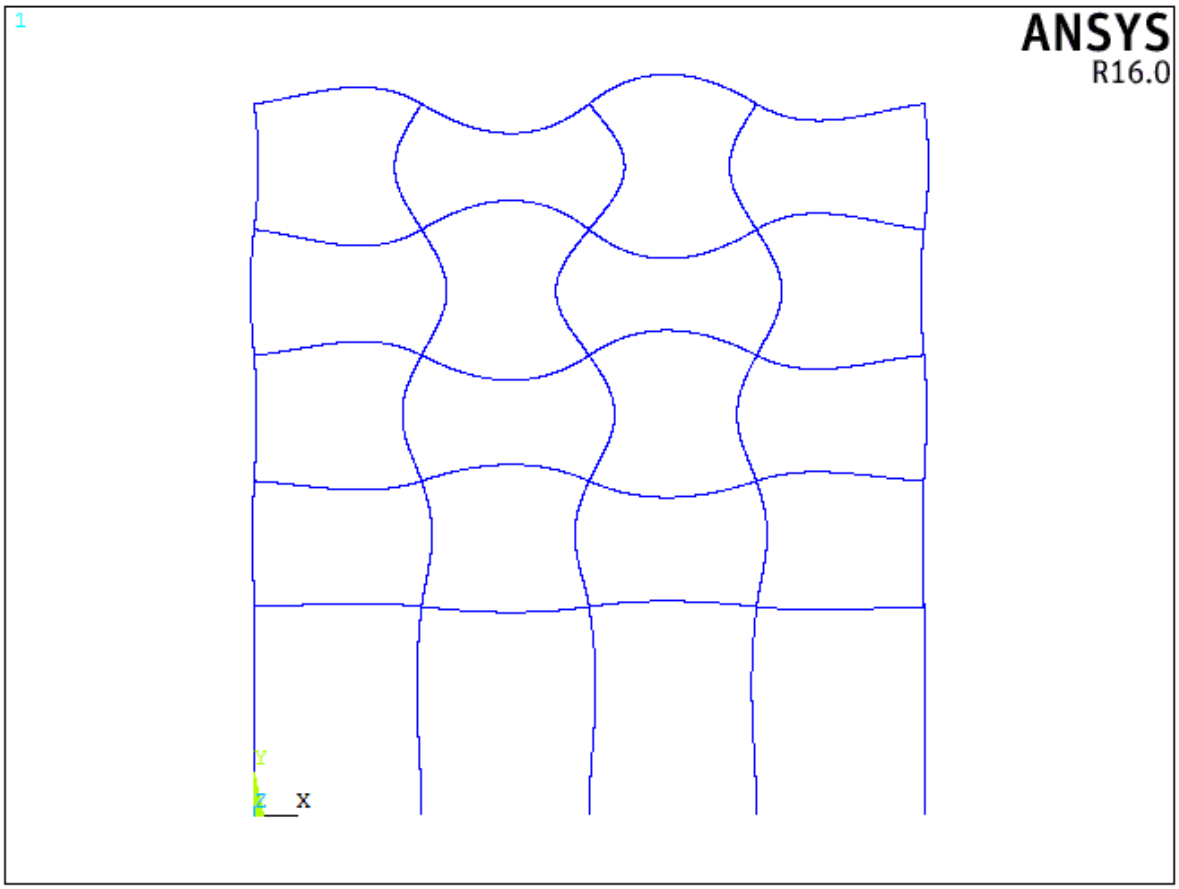


Figure 3.34 Buckled shape of optimised Frame C.

If the system buckling loads and weights of the two frames before and after optimisation presented in Table 3.29 are compared, it is seen that both frames have the same final system buckling, determined by the proposed method, with Frame C weighing 42% less. The first optimised frame resulted in the third beam having a greater stiffness than its lower stories. This may be attributed to the fact that the column stiffness changed in the initial frame at this story. Furthermore, the difference between the system buckling loads for Frame C from FEA is greater than

the first frame. The load obtained in FEA for this frame was found to be lower than that given by the proposed method. In the optimisation analyses of the two frames, the first story was critical in both frames. The frames both had the same initial stiffness of *I* for the first story and as a result, were centred on optimising the frame by the same theoretical system buckling load governed by this stiffness. Yet, the optimisation of Frame C resulted in a 'lighter' with less stiff members, thus leading to the lower buckling load being obtained in FEA for this frame.

Table 3.29 Final system buckling loads and frame weights of the two ten story four bay frames.

	Before optimisation		**After optimisation**		**FEA result of optimised frame**
	System buckling load, P_{cr} ($\frac{EI}{h^2}$)	**Weight of frame (tonnes)**	**System buckling load, P_{cr} ($\frac{EI}{h^2}$)**	**Weight of frame (tonnes)**	
Frame as shown in Figure 3.30	0.856	144	0.856	91	0.82
Frame C	0.856	96	0.856	53	0.8

Despite these differences found, the comparison of the frames demonstrated that whether the designer selects initial sections with same or different stiffness, it will yield the same theoretical system buckling load regardless. This finding was also established in the ten story four bay frames of Example 4.

3.1.3 Conclusion – No sway frames

The application examples presented demonstrate the validity of the proposed optimisation method. The objectives initially set out were achieved, namely; all stories buckled at the same time as indicated by the buckled shape of all stories under the first buckling mode and, the weight of the frame was minimised as shown by the decrease in weight of the optimised frames. These were achieved under a given system buckling load. Furthermore, it is found that the optimisation process outlined in Section 2.3.2, can be modified to end when the buckling load ratio, α, is equal to 1 as opposed to 0.9. The results demonstrated that the same buckling load was achieved for all stories after optimisation with final buckling load ratios of 1.

The buckling load of the frame before optimisation was governed by the critical story with the minimum load. In the optimisation proposed, the stiffness of the stories was reduced in order to obtain the same buckling load of the critical story throughout the frame. As a result, the system buckling load of the frame before and after optimisation remained the same. This indicates that the same theoretical system buckling load, as determined by the proposed method, can be obtained from a 'lighter' frame and material wastage can be reduced, achieving another goal of the optimisation, as all members are engaged in the buckling. Furthermore, the savings in material wastage was demonstrated by the reduction in the stiffness of the stories after optimisation resulting in smaller cross-sectional properties required for members to achieve system buckling. For instance, for Iteration 1 of Example 2 shown in Table 3.3, the fourth story's buckling load was approximately ten times greater than that of the critical story, that being the first story. The frame has globally failed in buckling if it reaches this critical buckling load, regardless of the fourth story still having buckling capacity. In Iteration 2, all stories have buckled under this load and for the fourth story, a stiffness of $0.1I$, as opposed to I initially, was needed to achieve system buckling hence the material reduction for the story was $(I-0.1I)/I\times100 = 90\%$. Varying reductions in story stiffness after optimisation was found across the examples presented in the analysis.

The findings from the analysis of the frames in Example 2 and 3, where the buckling load was increased whilst the weight of the frame reduced, indicated that this objective can be met by the optimisation method suggested. This may not lead to the optimal solution in terms of weight, yet a more 'efficient' frame can be produced with greater structural capacity.

The optimisation procedure was adapted for multi-story frames with even number of bays to account for the effect of localised buckling experienced by the column in the equivalent one bay frames with the lower stiffness. The buckling of the frame was thus governed by this local buckling as opposed to a system buckling. Nevertheless, when this approach was applied, the results confirmed that the system optimisation method suggested still holds for such frames. For tall frame buildings, no sway, the

proposed method can be easily performed by hand for these high-rise buildings, proving its efficiency, as demonstrated by the 10 story frame in Example 4. The method can be further modified to apply to irregular no sway frames or no sway non-rigid frames and shouldn't be applied directly.

The frames presented were based on previous examples given in the literature. Some examples were described in the form of variables but these were converted into numerical values as ANSYS FEA software requires values for analysis. When initial sections were assigned in the literature examples, as in Example 3, the stiffness or *I* value of these were used to find an equivalent square bar section to be used in the FE model. In addition, the values calculated for the weight of the frames are indicative only and was used for comparison after optimisation. However, the reader should not focus on the numerical values of the weight and section properties of the frames but rather on the savings achieved in terms of buckling. In the case of the two story one bay frame, it is stated that it not often designed or found in practice and hence no further improvement of the result was warranted. This frame was included to evaluate the applicability of the method on a range of frames.

The results of the no sway analysis presented, is conclusive that the suggested method to calculate the system buckling load, has been verified through several frame examples and most importantly, that the proposed optimisation procedure has been validated by the acceptable percentage differences of below 5% from the FE analysis. This result however, does not comment on the accuracy of the method. The analysis did not attempt to obtain which method is more accurate between FEA and the proposed method. The results were compared with FEA as the research was aimed at finding a simpler method to be used in the design environment which currently makes use of FE software to design multi-storey frames. Thus, it was reasonable to compare it to current or 'trusted' methods used in the field. To obtain its accuracy would require comparison to other structural buckling methods used or through scale model tests which was not undertaken in the present research.

3.2 Sway Analysis

It is known that majority of engineering structures in practice are not truly no sway and have some form of sway. This section describes the attempts to address the sway frames using the same approach that has been previously validated for no sway frames. Even though the results were found to have been unsuccessful, the approaches are presented for future research to develop on.

3.2.1 Approach 1 – Equivalent multi-bay single story sway frames

The first approach that was explored, was the same as outlined in Section 2.1. The upper-bound system buckling load, $\overline{P_{cr,\,sway}}$, of the rigid beam frames can be easily found by Li's story buckling method (2014). Yet, to account for the real system buckling load for non-rigid beams, P_{cr}, the same methodology that was used on the no sway frames, is applied to obtain this load.

In order to derive the modification factor to apply to the upper-bound system buckling loads of the sway frames, a single story multi-bay sway frame was considered and the same assumptions as before are applied; namely, the connections between the beams and columns are taken as rigid.

A single story sway frame with *b* number of bays and the loads acting on it, are presented in Figure 3.35. It can be seen that all columns, except the two columns at the ends of the frame, bear a load of 2P and a stiffness of *I*, and the beams have a stiffness of *nI* ($n = {}^{I_B}\!/_{I_C}$).

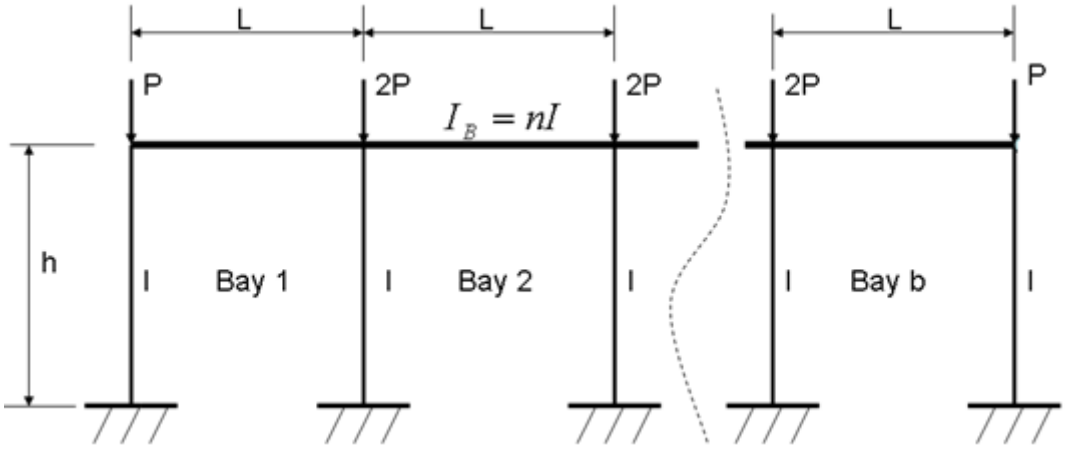


Figure 3.35 A single story b bay sway frame with its load-stiffness pattern.

The frame shown in Figure 3.35 can be folded into a one bay frame as before, to obtain the normalised equivalent one bay frames when the bay number *b* is equal to 1, 2 or infinity, as shown in Figure 3.36 (a) to (c), respectively. It must be noted, that when the multi-bay frame is folded, the loads on a column remains with the column and the same theorems for no sway frames are applied.

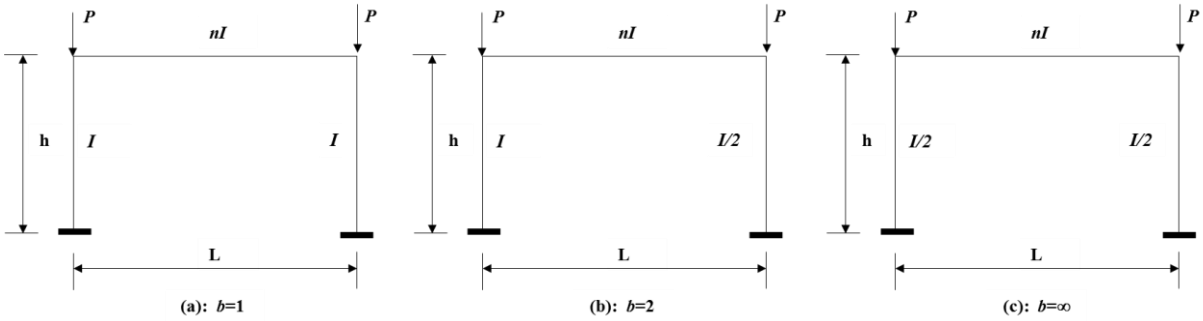


Figure 3.36 Normalised equivalent one bay sway frames when b=1, 2 and infinity.

System buckling analysis is performed in FEA software ANSYS on the frames shown in Figure 3.36 at different beam stiffness' (values of *n*) and their respective system buckling loads are obtained and listed in Table 3.30.

Table 3.30 System buckling load of equivalent sway frames at different values of n.

Beam stiffness ratio, *n* (I_b=*n*I)	**System buckling load, P_{cr} b=1 ($\frac{EI}{h^2}$)**	**System buckling load, P_{cr} b=2 ($\frac{EI}{h^2}$)**	**System buckling load, P_{cr} b=∞ ($\frac{EI}{h^2}$)**
0.001	2.4754	1.8544	1.2417
0.005	2.5072	1.8865	1.2734
0.01	2.5468	1.9261	1.3124
0.05	2.8515	2.2265	1.6030
0.1	3.2057	2.5629	1.9176
0.2	3.8344	3.1267	2.4180
0.3	4.3719	3.5758	2.7917
0.4	4.8336	3.9389	3.0771
0.5	5.2326	4.2370	3.3000
0.6	5.5792	4.4855	3.4779
0.7	5.8822	4.6956	3.6224
0.8	6.1485	4.8755	3.7418
0.9	6.3839	5.0314	3.8419
1.0	6.5930	5.1677	3.9268
1.2	6.9476	5.3951	4.0630
1.4	7.2357	5.5775	4.1672
1.6	7.4737	5.7275	4.2492
1.8	7.6731	5.8532	4.3154
2.0	7.8425	5.9603	4.3700
3.0	8.4079	6.3235	4.5422
4.0	8.7259	6.5351	4.6332
5.0	8.9287	6.6744	4.6894
10.0	9.3626	6.9877	4.8054

With the simple approach outlined by Li (2014), the upper-bound of the system buckling load, $\overline{P_{cr}}$, when I_b=∞, of the sway frames shown in Figure 3.36 can be obtained as follows:

When *b*=1:

$$K = 2 \times \frac{12EI}{h^3} = \frac{24EI}{h^3}$$

$$K_P = 2 \times \frac{P}{h}$$

$$K - K_P = \frac{24EI}{h^3} - 2 \times \frac{P}{h} = 0$$

Solving yields:

$$P = \frac{12EI}{h^2}$$

$$\therefore \overline{P_{cr}} = \beta \times P = \frac{\pi^2}{12} \times \frac{12EI}{h^2} = \frac{\pi^2 EI}{h^2} \quad (17)$$

Similarly;

$$\overline{P}_{cr} = \begin{cases} 0.75 \times \dfrac{\pi^2 EI}{h^2} & when \quad b = 2 \\ 0.5 \times \dfrac{\pi^2 EI}{h^2} & when \quad b \to \infty \end{cases} \tag{18}$$

Note, it was found from the analysis in FEA that when b=2, both columns buckled hence an equivalent column stiffness of 0.75 ((½ + 1)/2=0.75) was used to calculate $\overline{P_{cr}}$, as seen in Eq. (18), as both columns have different stiffness in the equivalent frame shown in Figure 3.36 (b). The system buckling load coefficient, μ, can then be defined as before, seen in Eq. (19), as the ratio of the system buckling load to its upper-bound:

$$\mu = \frac{P_{cr}}{\overline{P_{cr}}} \tag{19}$$

When the beam is not 'rigid', the length of the beam, L_B, will affect the restraint from the beam on the column. To take this effect into account, the parameter, λ, is introduced:

$$\lambda = \frac{I_B L_C}{I_C L_B}$$

According to this definition, for the frames shown in Figure 3.36, with L_B=1.5h:

$$\lambda = \frac{I_B L_C}{I_C L_B} = \frac{nI \times h}{\frac{b+1}{2b} I \times L} = \frac{2b}{b+1} \times \frac{2}{3} n = \begin{cases} \dfrac{2}{3} n & b = 1 \\ \dfrac{8}{9} n & b = 2 \\ \dfrac{4}{3} n & b \to \infty \end{cases} \tag{20}$$

Design graphs can now be created using the results in Table 3.30, as well as equations Eq. (17) to Eq. (20). The design graphs shown in Figure 3.37, indicate the relationship between the system buckling load coefficient μ and λ for the three frames shown in Figure 3.36.

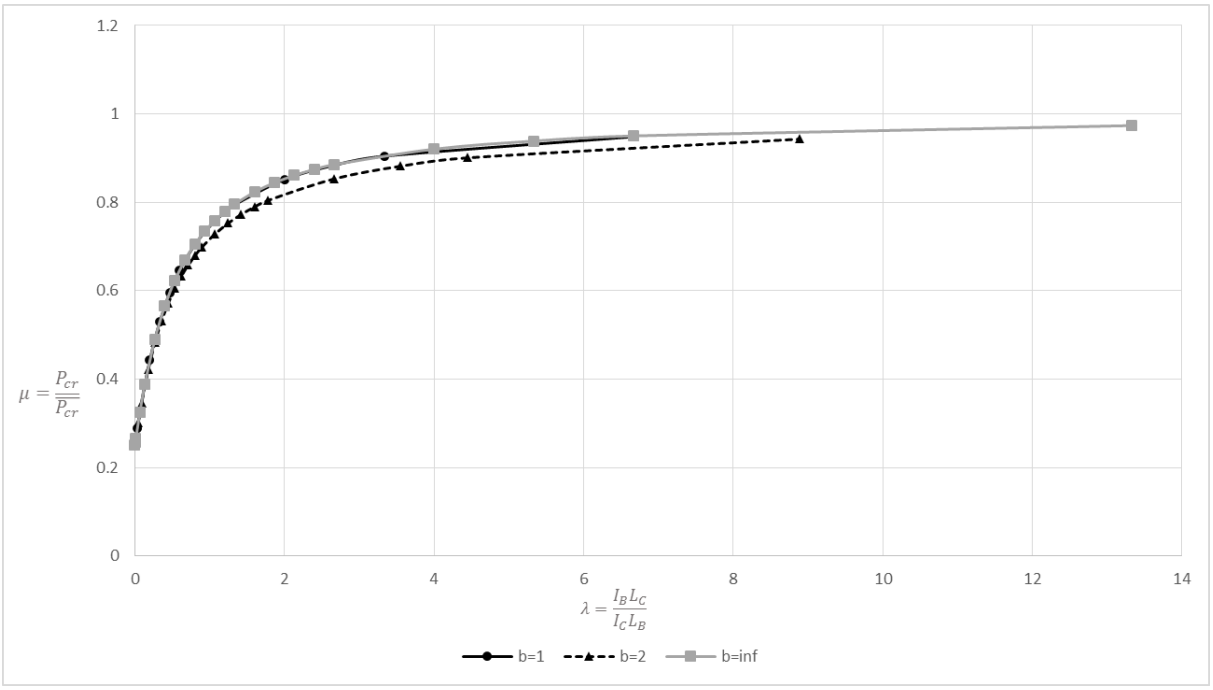


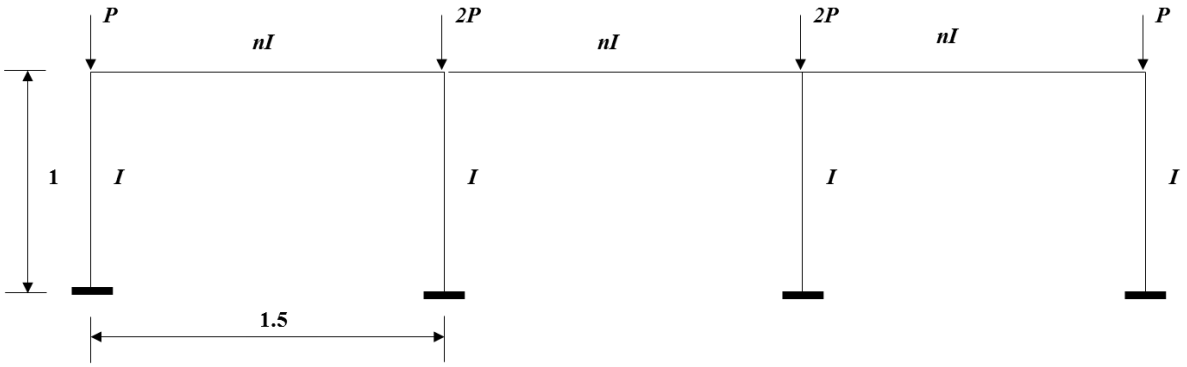
Figure 3.37 Design graph for sway frames when b=1, b=2 and b=∞.

In Figure 3.37, it is seen that the graphs for *b*=1 and *b*=∞ plot close to each other and thus it would be reasonable to only use the graphs corresponding to *b*=∞ and *b*=2 for design examples. Lastly, using the design graph, the system buckling load can be easily evaluated from:

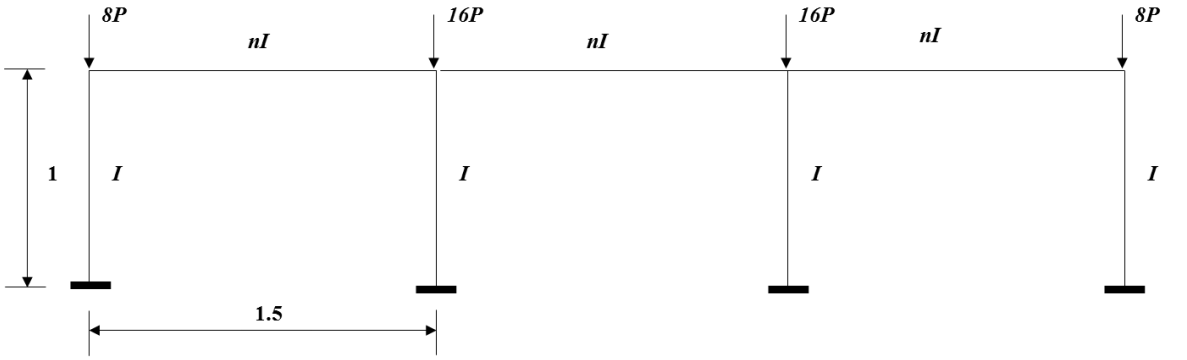
$$P = \mu \times \overline{P}_{cr}$$

The approach outlined above was validated on single story multi-bay sway frames. It was then extended to attain the influence of the adjacent stories and the load arrangement on the value of μ.

A three bay single story frame was first investigated for the influence of loads. Two frames, Frame A and Frame B with different loading magnitudes but same loading patterns applied, are shown in Figure 3.38 (a) and (b).



(a) Frame A – single story three bay sway frame.



(b) Frame B – single story three bay sway frame with increased loads.

Figure 3.38 Two sway frames to demonstrate influence of loads on the system buckling load.

The frames were analysed for its critical load in FEA using values of *n* as 0.1, 0.5, and ∞, for rigid beam conditions, and the system load coefficient, μ, was calculated for the different values. This is summarised in Table 3.31. It was observed that the value of μ did not change significantly between the two frames when the loads increased in Frame B hence there is no influence of loads on the value of μ.

Table 3.31 System buckling loads and load ratios μ calculated for Frame A and B.

	Frame A		**Frame B**	
n	$\boldsymbol{Pcr}\ (\frac{EI}{h^2})$	**μ**	$\boldsymbol{Pcr}\ (\frac{EI}{h^2})$	**μ**
0.1	2.35	0.354	0.28	0.349
0.5	3.93	0.593	0.47	0.588
∞	6.63	-	0.8	-

A second story is then added to Frame A only, producing Frame C in Figure 3.39, since the influence of loads was determined to be minimal. The loads on Frame A were distributed to the second story so that the same cumulative loads would be applied to the first story of Frame C as in Frame A.

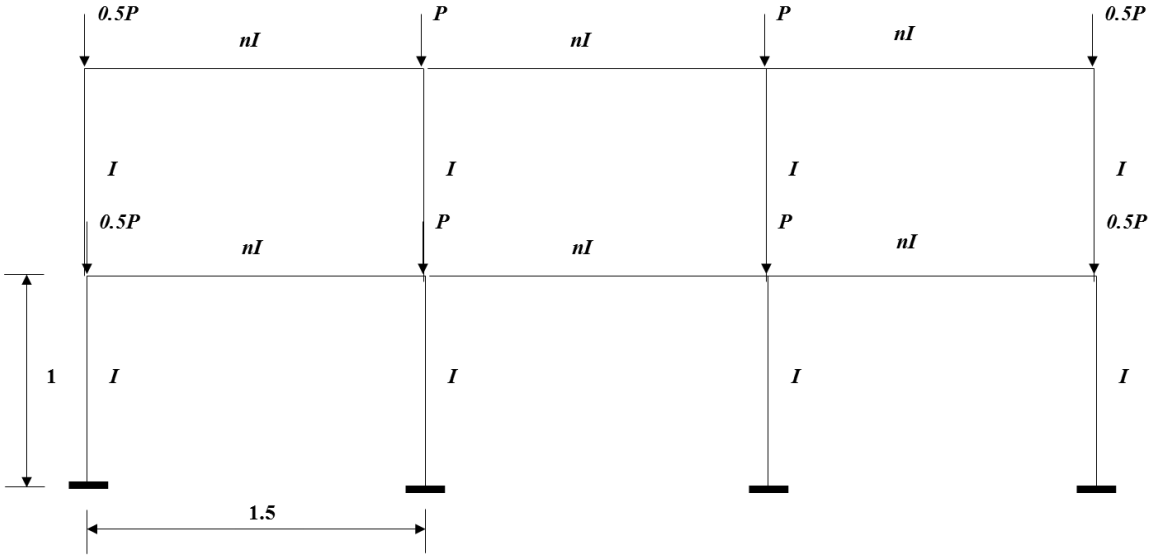


Figure 3.39 Frame C-two story three bay sway frame to demonstrate influence of adjacent stories.

The same analysis was done in FEA to determine the value of μ for Frame C and the results summarised in Table 3.32. The same cumulative loads and column stiffness' of Frame A are used in Frame C, yet the effect of the second story is seen in the decrease in the value of μ and the buckling load, hence causing a destabilising effect of the frame.

Table 3.32 System buckling loads and load ratios μ calculated for Frame A and B.

	Frame C	
n	***Pcr*** $(\frac{EI}{h^2})$	**μ**
0.1	1.54	0.222
0.5	3.41	0.492
∞	6.94	-

The upper-bound system buckling load, $\overline{P_{cr}}$, of Frame A and the first story of Frame C, should theoretically be equal, as calculated below: (Li, 2014)

$$K = 4 \times \frac{12EI}{h^3} \qquad K_p = \frac{6P}{h}$$

$$P = \frac{8EI}{h^2} \qquad \overline{P_{cr,1}} = \frac{2\pi^2 EI}{3h^2}$$

Using the design graph in Figure 3.37 when b=∞, the value of μ for Frame A, single story frame, when n is 0.5:

$$\lambda = \frac{0.5I \times 1}{\frac{2}{3}I \times 1.5} = 0.5 \rightarrow \mu = 0.592$$

$\therefore P_{cr} = 0.592 \times \frac{2\pi^2 EI}{3h^2} = 3.9\frac{EI}{h^2}$ (0.76% smaller than FEA result)

This approach of using design graphs, worked for the one story sway frame of Frame A. The same method is applied to the two story frame of Frame C. The value of λ and μ for the first story is the same as Frame A and hence resulted in the same theoretical system buckling load of $Pcr = 3.9\frac{EI}{h^2}$. However, the FEA result from Table 3.32 for Frame C, was found to be $3.41\frac{EI}{h^2}$, resulting in a 14% smaller difference, hence indicating an influence of additional stories on the buckling load.

To attempt to account for the effect of additional stories, the influence of the beam stiffness was explored first as only the value of vertical stiffness, I_c was reduced by the value of ρ. The results of reducing the beam stiffness, I_b by the same value of ρ was explored, thus for Frame C:

$\lambda = \frac{\frac{2}{3}\times 0.5I \times 1}{\frac{2}{3}I \times 1.5} = 0.33 \rightarrow \mu = 0.503$

$\therefore P_{cr} = 0.503 \times \frac{2\pi^2 EI}{3h^2} = 3.31\frac{EI}{h^2}$ (3% smaller than FEA)

The percentage difference was acceptable and the suggested method of also reducing the beam stiffness, was applied to other multi-story multi-bay frames.

A four story three bay rigid sway frame was optimized using the same procedure as the no sway analysis and the design graphs in Figure 3.37 for sway frames. The final optimized frame is shown in Figure 3.40.

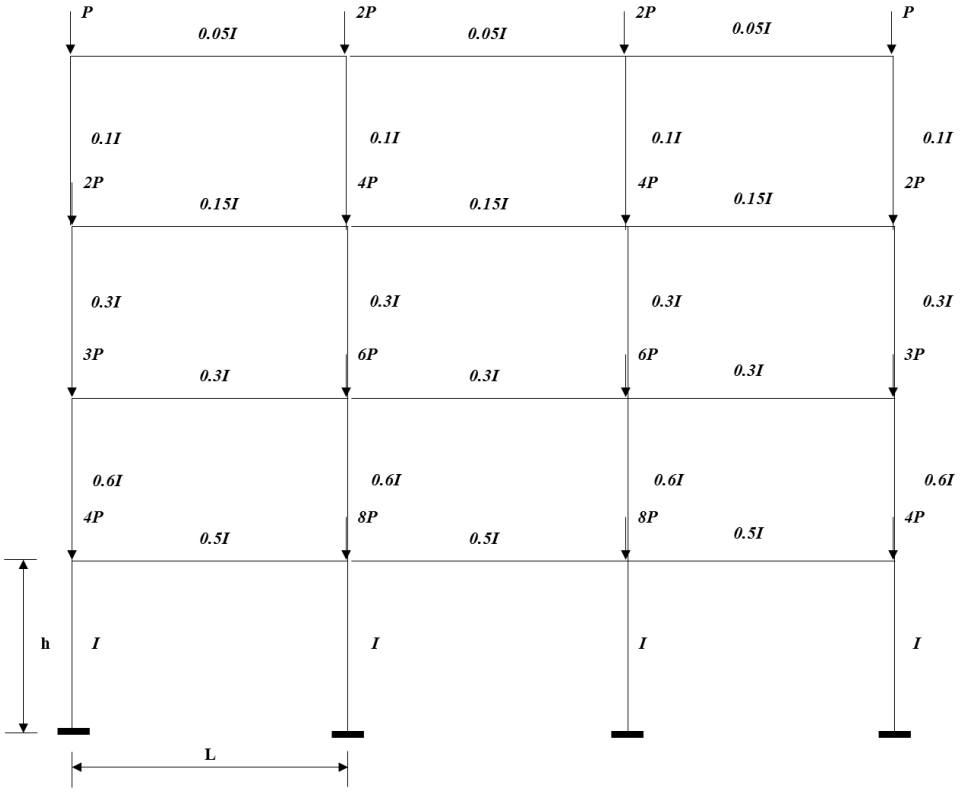


Figure 3.40 Optimised four story three bay sway frame using equivalent single story frames.

Theoretically, the frame was optimized with all stories having the same story buckling load, P_{cr}, thus for story 1 of the frame;

$$K = 4 \times \frac{12EI}{h^3} \qquad K_p = \frac{60P}{h}$$

$$P = \frac{4EI}{5h^2} \qquad \overline{P_{cr}} = \frac{\pi^2 EI}{15h^2}$$

$\lambda = \frac{\frac{2}{3}\times 0.5I\times 1}{\frac{2}{3}I\times 1.5} = 0.33 \rightarrow \mu = 0.503$ (Note: I_b and I_c are both reduced by ρ)

$\therefore P_{cr} = 0.503 \times \frac{\pi^2 E\times\frac{2}{3}I}{15h^2} = 0.22\frac{EI}{h^2}$ (1.44% greater than FEA)

The percentage difference obtained from FEA was low to accept the approach.

The weight of the frames before and after optimisation, initially with all columns having stiffness of I and I_b=*0.5I*, are shown in Table 3.33. The section that was used for comparison of the weights was a steel square bar.

Table 3.33 Weight of initial and optimised four story three bay sway frame.

	Weight of frame (tonnes)
Initial frame	22.403
Optimised frame in Figure 3.40	14.778

The method was applied to another frame example in order to validate its results. A five story five bay rigid sway frame in Figure 3.41, with member stiffness' and loads as shown, was optimized.

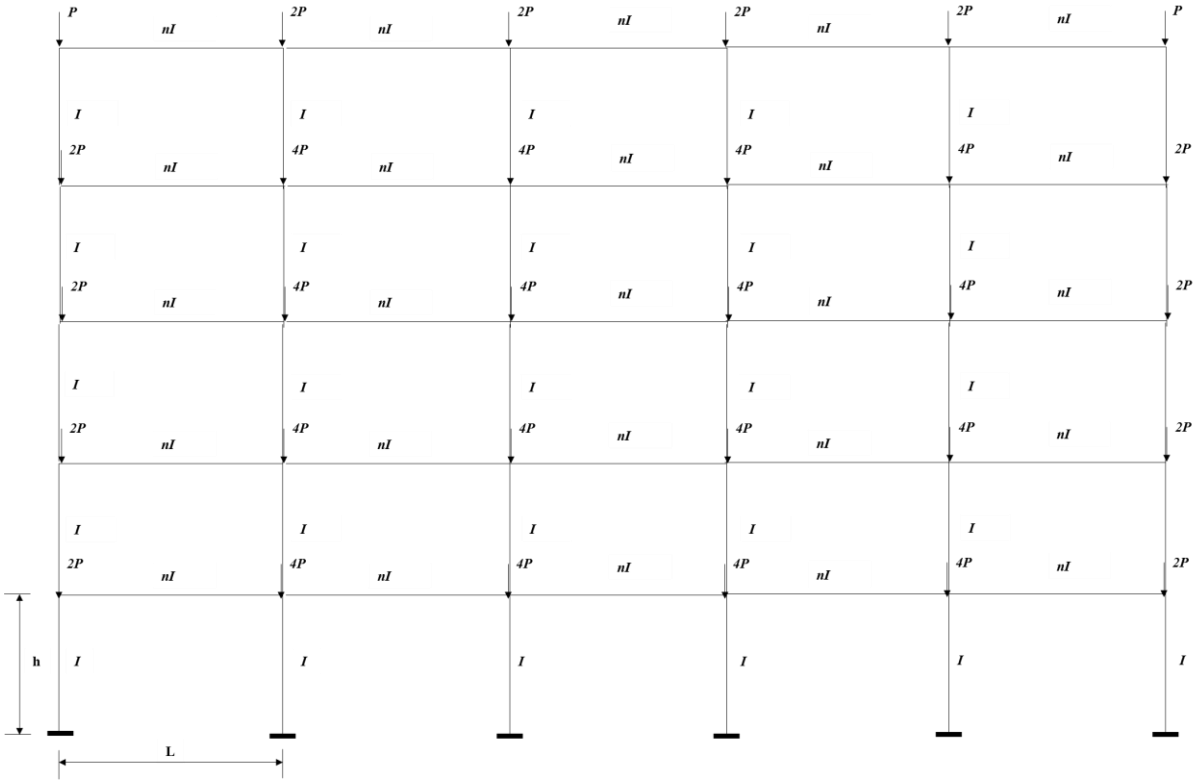


Figure 3.41 A five story five bay sway frame.

The frame was optimised using the same procedure as the no sway analysis where initially n=0.4, and h=3m and L=5m.

For the first story of the frame;

$$K = 6 \times \frac{12EI}{h^3} \qquad K_p = \frac{90P}{h}$$

$$P = \frac{4EI}{5h^2} \qquad \overline{P_{cr}} = \frac{\pi^2 EI}{15h^2}$$

$$\lambda = \frac{\frac{3}{5} \times 0.4I \times 3}{\frac{3}{5} I \times 5} = 0.24 \rightarrow \mu = 0.46446496$$

The value of μ when the bay number was five was interpolated from the design graphs for b=1 and b=∞ in Figure 3.37.

$$\therefore P_{cr} = 0.4645 \times \frac{\pi^2 E \times \frac{3}{5} I}{15h^2} = 0.1834 \frac{EI}{h^2}$$ (6% greater than FEA)

The percentage difference from FEA is acceptable for an estimated optimisation approach. The final optimised frame is given in Figure 3.42.

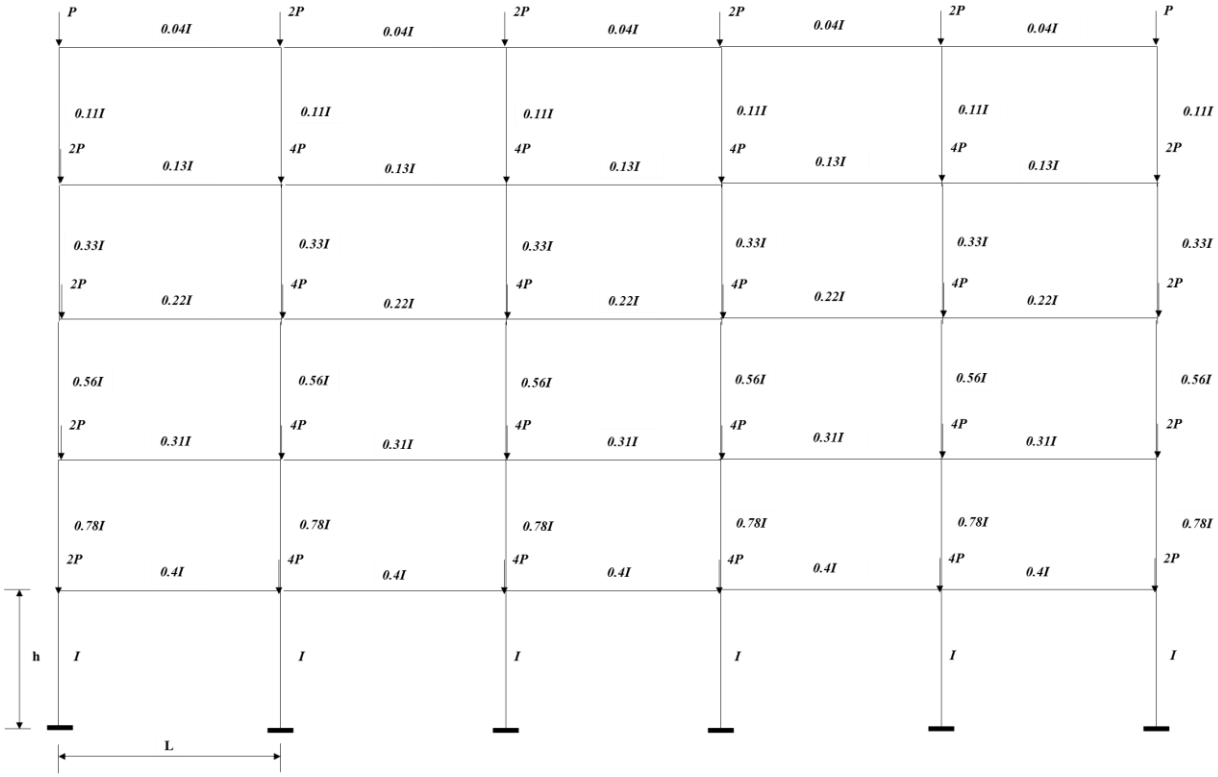


Figure 3.42 Optimised five story five bay sway frame using equivalent single story frames.

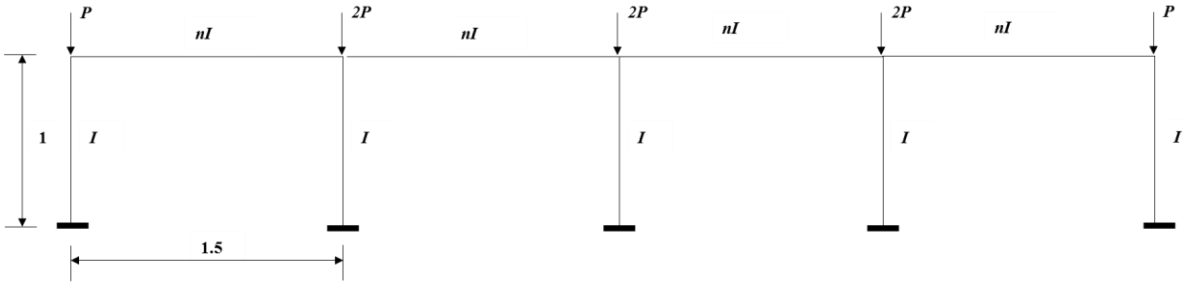
The weight of the frame before and after optimisation, is shown in Table 3.34.

Table 3.34 Weight of initial and optimised five story five bay sway frame.

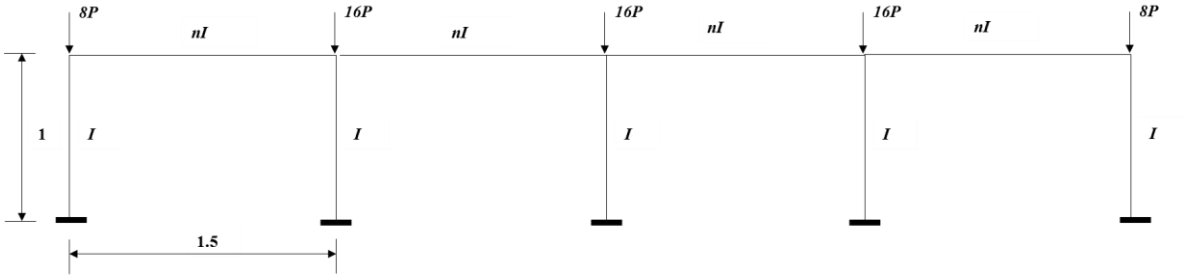
	Weight of frame (tonnes)
Initial frame	91.643
Optimised frame in Figure 3.42	64.846

The suggested method of using design graphs derived from equivalent single story multi-bay sway frames and reducing the beam stiffness as well, was demonstrated on frames where the number of bays was singular. The approach was applied to frames when the number of bays was even.

A four bay single story sway frame was analysed to investigate the results when the bay number was even. The frame was investigated first for the influence of loads. Two frames, Frame D and E with different loading magnitudes but same loading patterns, are presented in Figure 3.43 (a) and (b).



(a) Frame D – single story four bay sway frame.



(b) Frame E – single story four bay sway frame with increased loads.

Figure 3.43 Two sway frames to demonstrate the influence of loads on the system buckling load.

The frames were analysed for its critical load in FEA using values of *n* as 0.1, 0.5 and infinity, and the system load coefficient, μ, was calculated for the different values. This is summarised in Table 3.35. The value of μ did not change significantly between the two frames when the loads increased in Frame E, hence there is no influence of loads on the value of μ.

Table 3.35 System buckling loads and load ratios μ for Frame D and E.

	Frame D		**Frame E**	
n	$\boldsymbol{Pcr\ (\frac{EI}{h^2})}$	**μ**	$\boldsymbol{Pcr\ (\frac{EI}{h^2})}$	**μ**
0.1	2.24	0.349	0.28	0.349
0.5	3.77	0.588	0.47	0.588
∞	6.41	-	0.8	-

A second story is then added to Frame D only, since the influence of loads was determined to be minimal, resulting in Frame F in Figure 3.44. The loads on Frame D were distributed to the second story of Frame F so that the same cumulative loads would be applied to the first story of Frame F as in Frame D.

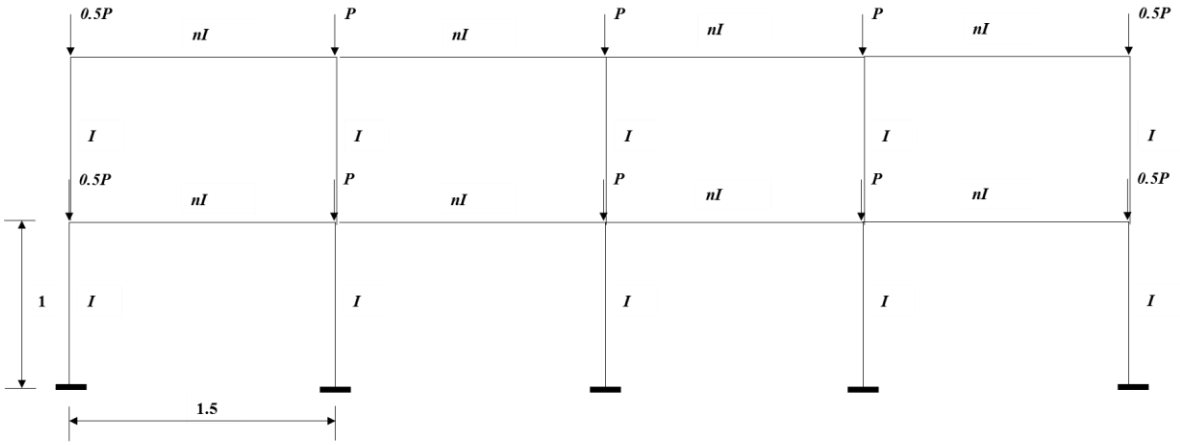


Figure 3.44 Frame F- two story four bay sway frame to demonstrate the influence of adjacent stories.

The same analysis was done in FEA on Frame F to determine the value of μ as summarised in Table 3.36.

Table 3.36 System buckling loads and load ratios μ calculated for Frame F.

	Frame F	
n	**Pcr** $(\frac{EI}{h^2})$	**μ**
0.1	1.48	0.203
0.5	3.33	0.457
∞	6.42	-

The same cumulative loads and column stiffness's of Frame D are used in Frame F. Yet once again, from Table 3.36, the effect of the second story is seen in the decrease in the value of μ and the buckling load, *Pcr*, hence causing a destabilising effect of the frame, as in the case of the singular bay number frames.

The upper-bound system buckling load, $\overline{P_{cr}}$, of Frame D and the first story of Frame F should theoretically be the same, and can be calculated as:

$$K = 5 \times \frac{12EI}{h^3} \qquad K_p = \frac{8P}{h}$$

$$P = \frac{15EI}{2h^2} \qquad \overline{P_{cr}} = \frac{5\pi^2 EI}{8h^2}$$

As mentioned previously, both columns of the equivalent even bay sway frame were found to buckle, thus an equivalent column stiffness (ρ) was used for the equivalent frame in Figure 3.45, when *b*=4, as shown below.

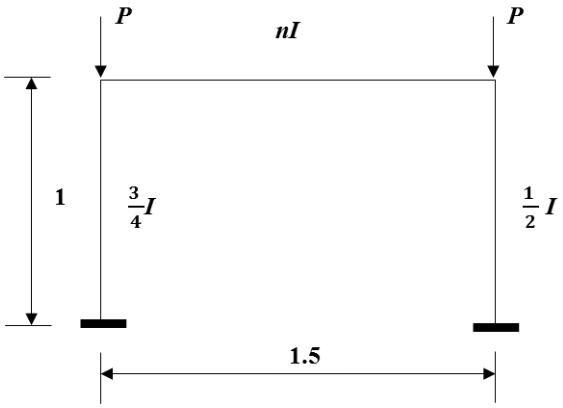


Figure 3.45 Equivalent one bay sway frame when b=4.

From Figure 3.45 when n=0.5, $\lambda = \frac{0.5\text{I}\times 1}{\text{Ic,equiv}\times 1.5}$ where $\text{Ic}_{\text{equiv}} = \frac{\frac{3}{4}\text{I}+\frac{1}{2}\text{I}}{2} = 0.625\text{I}$.

$\therefore \lambda = \frac{0.5\text{I}\times 1}{0.625\text{I}\times 1.5} = 0.5333$

The value of μ for a bay number of four was interpolated from the design graphs for b=2 and b=∞ in Figure 3.37.

For b=2 (ρ=1), μ=0.589 and b=∞ (ρ=1/2), μ=0.6155; thus, the value of μ for b=4 was determined as 0.609.

$\therefore P_{cr,\ Frame\ D} = 0.609 \times \frac{5\pi^2 EI}{8h^2} = 3.757\,\frac{EI}{h^2}$ (0.34% smaller than FEA result from Table 3.35)

The percentage difference indicated that the approach worked for the one story sway frame of Frame D.

The same method is applied to the two story Frame F. The value of λ and μ for the first story is the same as Frame D and thus have the same theoretical system buckling load, *Pcr* of $3.757\frac{\text{EI}}{\text{h}^2}$. However, the FEA result from Table3.36 for Frame F when n=0.5, was found to be $3.332\,\frac{\text{EI}}{\text{h}^2}$ resulting in a 13% smaller difference, hence there was found to be an influence of additional stories on the buckling load.

As explored in the singular bay case, the influence of the beam stiffness, was investigated in the two story frame, Frame F. The beam stiffness, I_b was reduced by the same value of ρ that was applied to the columns:

$\lambda = \frac{0.5\times 0.625I\times 1}{0.625I\times 1.5} = 0.3333$

The value of μ was interpolated from the design graphs corresponding to *b*=2 and *b*=∞.

For b=2 (ρ=1), μ=0.484 and b=∞ (ρ=1/2), μ=0.543; thus the value of μ for b=4 was determined as 0.528.

$\therefore P_{cr,Frame\ F} = 0.528 \times \frac{5\pi^2 EI}{8h^2} = 3.25\frac{EI}{h^2}$ (2.18% smaller than FEA result from Table 3.36).

The percentage difference from FEA is acceptable and the approach was further applied to other multi-story multi-bay frames.

A five story four bay sway frame was optimized using the same procedure as the no sway analysis and the design graphs in Figure 3.37. The final optimized frame is shown in Figure 3.46.

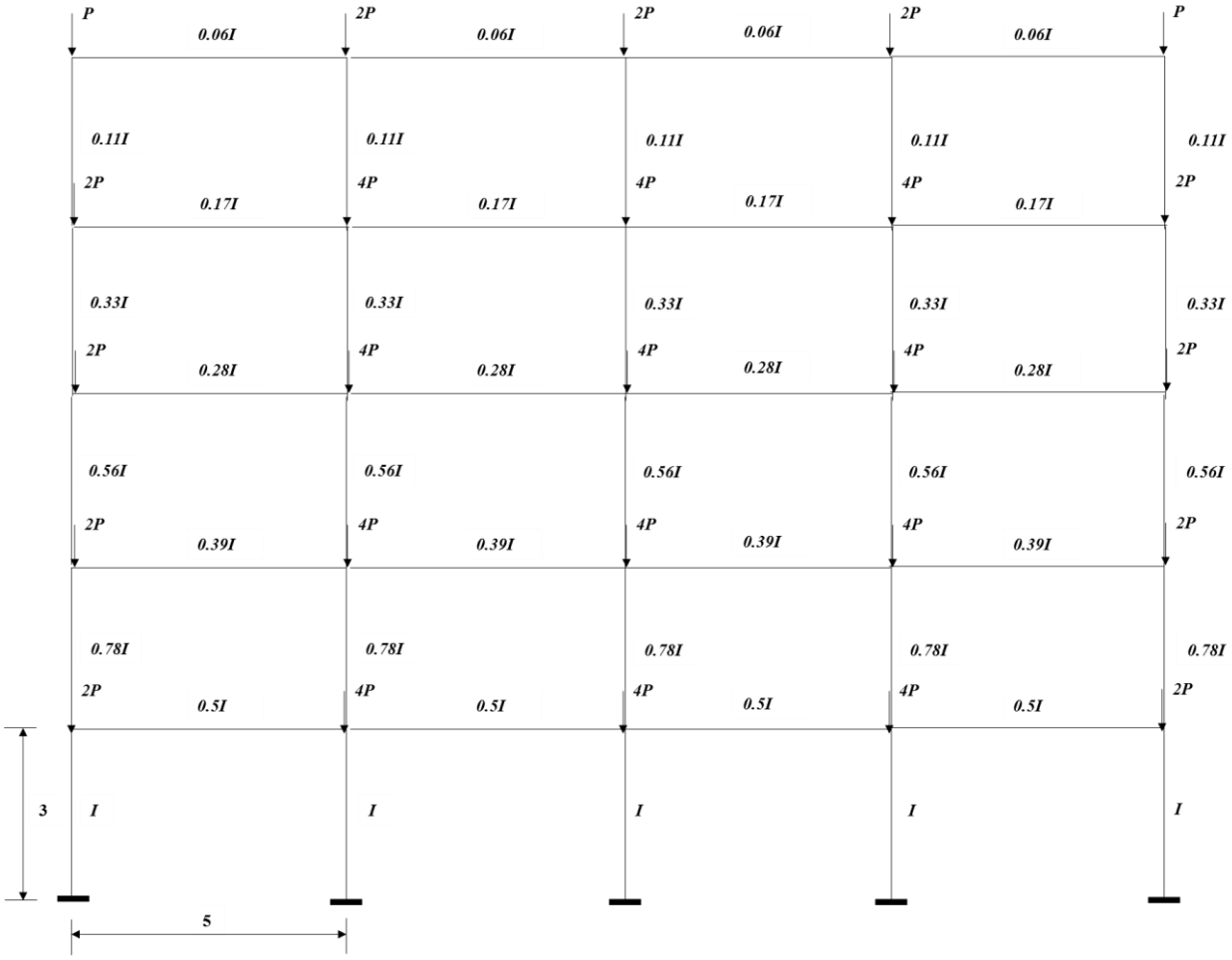


Figure 3.46 Optimised five storey four bay sway frame using equivalent single story frames.

Theoretically, the frame was optimized with all stories having the same story buckling load, $P_{cr,}$ thus for the first story of the frame;

$K = 5 \times \frac{12EI}{h^3}$ $\quad K_p = \frac{72P}{h}$

$P = \frac{5EI}{6h^2}$ $\quad \overline{P_{cr}} = \frac{5\pi^2 EI}{72h^2}$

$\lambda = \frac{0.625 \times 0.5I \times 3}{0.625I \times 5} = 0.3$

The value of μ for a bay number of four was interpolated from b=2 and b=∞ graphs. For b=2 (ρ=1), μ=0.503 and b=∞ (ρ= ½), μ=0.510; thus the value of μ for b=4 was determined as 0.508.

$\therefore P_{cr,1} = 0.508 \times \frac{5\pi^2 E \times 0.625I}{72h^2} = 0.218\frac{EI}{h^2}$ (8.7% greater than FEA result of $0.2\frac{EI}{h^2}$)

The percentage difference obtained from FEA is significant.

The method was applied to another multi-story frame. A two story two bay frame was optimized and the final optimized frame is shown in Figure 3.47.

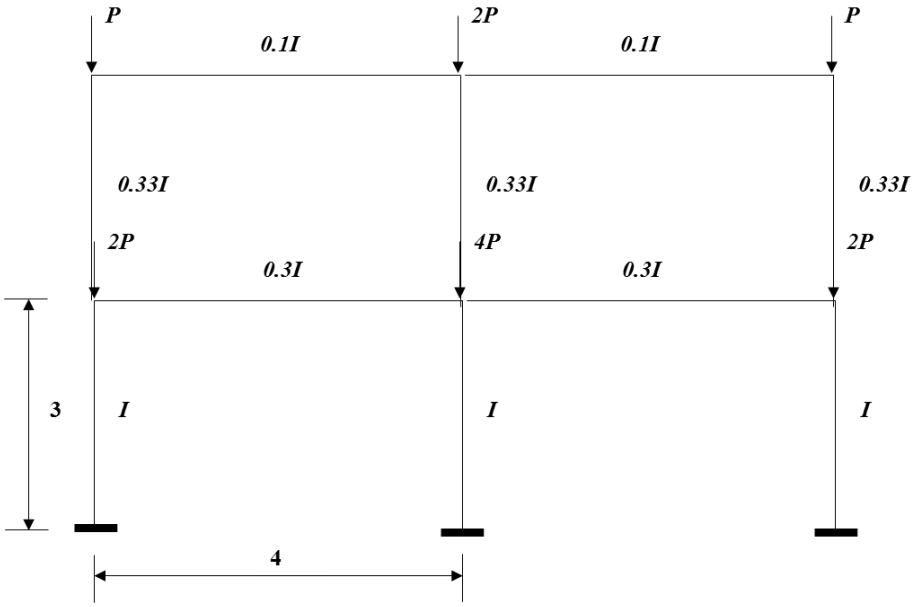


Figure 3.47 Optimised two bay two story sway frame using equivalent single story frames.

Theoretically, the frame was optimized with all stories having the same story buckling load, P_{cr}, thus for the first story;

$K = 2 \times \frac{12EI}{h^3}$ $\quad K_p = \frac{12P}{h}$

$P = \frac{2EI}{h^2}$ $\quad \overline{P_{cr}} = \frac{\pi^2 EI}{6h^2}$

The equivalent one bay frame in Figure 3.48, was used to calculate the equivalent column stiffness (ρ), and thus obtain the value of μ.

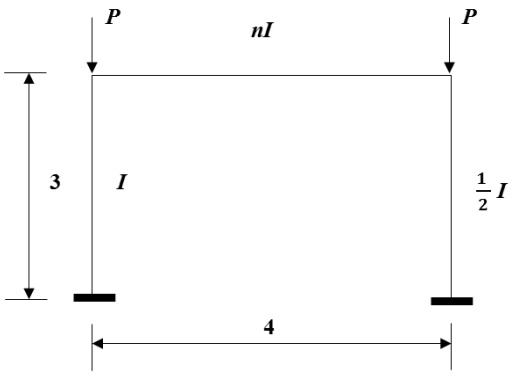


Figure 3.48 Equivalent one bay frame of the two bay two story sway frame.

$$Ic(\rho)_{equiv} = \frac{I+\frac{1}{2}I}{2} = 0.75I$$

$$\lambda = \frac{0.75\times0.3I\times3}{0.75I\times4} = 0.225 \rightarrow \mu = 0.452598$$

$$\therefore P_{cr,1} = 0.452598 \times \frac{\pi^2 E\times0.75I}{6h^2} = 0.838\frac{EI}{h^2}$$ (1.52% smaller than FEA).

The percentage difference from FEA obtained for this frame example is acceptable.

The sway analysis of all cases presented using design graphs produced from equivalent single bay frames, show that there is a story or vertical influence on the critical loads of multi-storey sway frames. The percentage differences between FEA and the proposed method may be acceptable in some of the application examples but needed to be improved in others to make it a 'good enough' estimate of the critical load by simple hand calculations.

Consequently, from the analysis presented, this influence needs to be accounted for when calculating the system buckling load of multi-story sway frames. It is not only reduced by the value of μ to account for non-rigid beams, but an additional effect needs to be considered.

It is known that from the current isolated member buckling analysis, that the effective length of a column is influenced by its end conditions. Various attempts have been made in studies and literature, to capture this vertical interaction of the adjacent stories on the effective length and in turn, the critical buckling load or stability of frame systems. Nevertheless, methods such as those proposed in the

AISC LRFD manual (AISC, 2001) and by Webber et.al (2015) are based on isolated member analysis as opposed to a system approach. Webber et.al (2015) took into account the effect of the adjacent stories as well as the ratio of the loads in the adjoining stories to that of the story of interest. Yet, this approach focused on an isolated column analysis. The method that attempted to incorporate a global system analysis was the Method of Means (Hellesland, 1998). The stability of unbraced frames is considered by finding a mean value of the story stability indices. However, the story stability index was given by the isolated member stability index which contradicted the global approach of the proposed research at hand. All of these methods failed to incorporate a global system approach, nevertheless, they were applied to the application examples in Section 3.2.2 to attain the extent at which they were able to improve the system buckling load estimate of sway frames. It will show that some approaches worked for one application but were unsuccessful in other frames and hence no conclusion could be made on the success of the approach. Furthermore, as noted previously, when the number of bays is even, the equivalent one bay frames yielded a column of lower stiffness which needs to be considered. Literature (Webber, et al., 2015 & Yura, 1971 & Duan & Chen, 1999) have stated that in this instance, the stiffer or stronger column offers some lateral restraint or bracing to the less stiff or weaker column within the same story, hence both columns do not buckle at the same time and there could be a certain degree of 'localised' buckling. This theory or observation was incorporated in the analysis of an even bay frame in Section 3.2.2, however this did not work when applied to another example of such a scenario.

In the end, the attempted approach proposed which considered the adjacent stories in obtaining the system buckling load, using the available methods discussed from literature, is presented in the next section.

3.2.2 Approach 2 – Effect of adjacent stories

It was demonstrated, that for single story multi-bay sway frames, the same method proposed for multi-story multi-bay no sway frames with non-rigid beams can be used; namely using equivalent one bay frames with column stiffness reduced by a factor ρ and obtaining a reduction value μ to be applied to the upper-bound of the

system buckling load. When this was evaluated on a multi-story sway frame as in Approach 1, it resulted in large percentage differences thus leading to the fact that there is another influence on the stability of multi-story sway frames. The different approaches to consider this effect was evaluated.

The first premise that was investigated, was the collapsing of each story of a multi-story multi-bay frame into a one bay frame and accounting for the adjacent story's effect. The results of this investigation showed errors for the application frames, and further research concluded that when the stiffness of columns within the same story differ, as in the case of the equivalent frames for even bay number frames, the stronger columns brace the weaker ones. The AISC commentary (AISC, 2001) which made use of a modified effective length to account for this, was acceptable for a two story two bay frame example but not for a five story four bay frame. The results of this analysis can be found in Appendix B. Another method suggested by Lui (1992) excluded the use of design charts or sway alignment charts and could be used for frames with or without leaning columns, and was found to work for a two story two bay frame example. Yet, this method did not account for the adjoining story's effect in terms of loading pattern or arrangement. Furthermore, it required a more rigorous approach of first calculating the end moments and inter-story deflections, which may be undesirable in a design office environment and is not in line with the focus of this research aimed at proposing a simple optimisation method.

A method proposed by Webber et al. (2015) evaluated the rotational stiffness of columns using the existing effective length charts in the NCCI method (Oppe, et al., 2005) with new distribution coefficients to account for the adjoining stories. It used an effective column stiffness for the equivalent frames assuming that both columns buckled simultaneously within a storey. Yet, as previously mentioned, literature suggests that the columns with a lower stiffness would buckle first when 'braced' by a stronger column within the same story. Thus, the approach by Webber et.al was evaluated on frame applications by considering the weaker column only in the analysis.

The following equations were used:

The distribution factors are given by:

$$\eta_i = \frac{K_{ij}}{K_{ij}+K"_{XY,i}+\sum K"_{b,i}} \quad (21)$$

$$\eta_j = \frac{K_{ij}}{K_{ij}+K"_{XY,j}+\sum K"_{b,j}} \quad (22)$$

where K_{ij} (=I_{ij}/Lij) is the nominal stiffness of column *IJ* being analysed; $K"_{XY,i}$ and $K"_{XY,j}$ are the effective rotational stiffness of the adjoining columns at nodes *I* and *J*; and $\sum K"_{b,i}$ and $\sum K"_{b,j}$ are the effective rotational stiffness of the beams converging at nodes *I* and *J* taken from Table 5 and 4 respectively in the literature.

For the frames being analysed, the far end restraint conditions of the beams apply to double curvature, thus $K"_b = 1.5\frac{I}{L}$; and the frames are sway with free translation hence $K"_{XY} = 0.25\frac{I_{XY}}{L_{XY}}(1 - 0.82\frac{P_{d,XY}}{P_{d,IJ}}\left(\frac{L_{XY}}{L_{IJ}}\right)^2)$ where P_d is the corresponding loads acting on the columns.

The distribution factors are then used to get the effective length factor from the graphs in the NCCI method. The effective length obtained will be used to calculate the value of λ in order to get the corresponding value of μ from the design graphs in Figure 3.37, produced earlier for the single bay single story equivalent sway frames.

With the outlined approach, the five storey four bay sway frame is reanalysed to get the effective length. The final optimized frame is shown Figure 3.49.

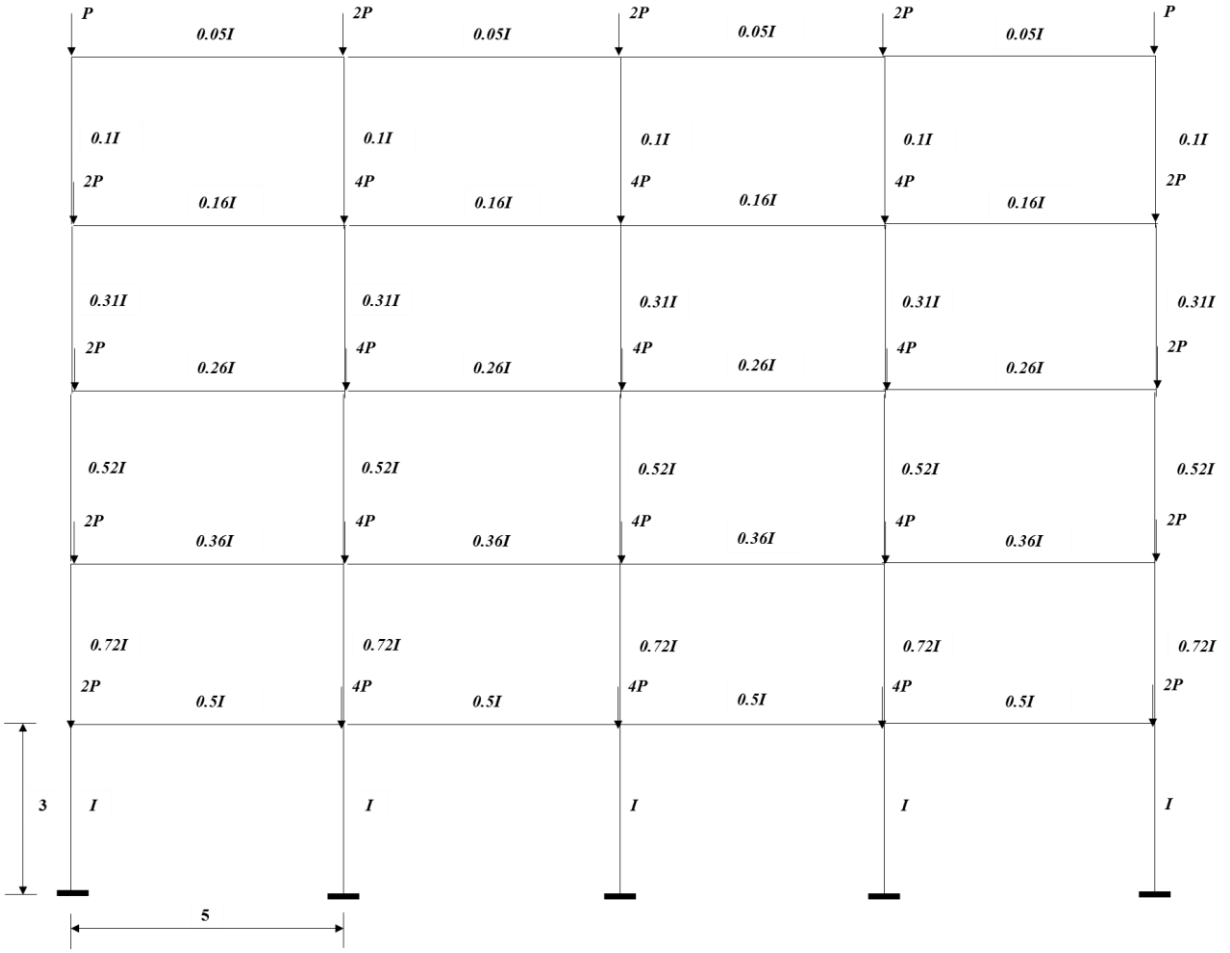


Figure 3.49 A five story four bay sway frame.

The first two stories were folded into an equivalent single bay frame, since all stories were optimised to have the same buckling load thus any story could be used to evaluate the system bucking load. The equivalent two story one bay frame is shown Figure 3.50.

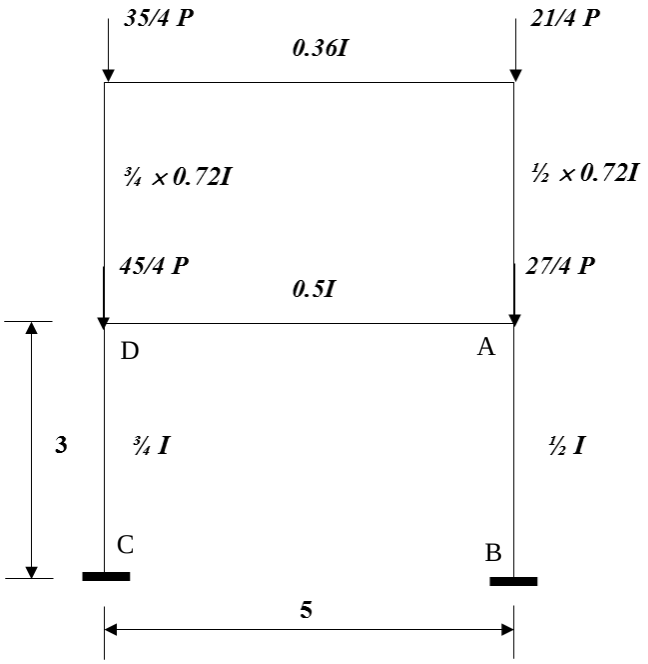


Figure 3.50 Equivalent single bay frame for the first two stories of the five story frame.

The proposed method by Webber et al. (2015) for the calculation of the effective length and the charts in the NCCI method was used.

As established earlier, the stiffer column *CD* offers some lateral restraint to column *AB* of the first storey hence *AB* was the column of interest.

$K_{AB} = \frac{0.5I}{3}$

$K"_{b,A} = 1.5 \times \frac{0.5I}{5} = 0.15I$

$K"_{XY,A} = 0.25 \times \frac{0.5 \times 0.72I}{3}\left(1 - 0.82 \times \frac{\frac{21}{4}P}{\frac{27}{4}P} \times \left(\frac{3}{3}\right)^2\right) = 0.0109$

$\eta_{A(top)} = \frac{\frac{0.5I}{3}}{\frac{0.5I}{3} + 0.15I + 0.0108667} = 0.508$

$\eta_{B(bottom)} = 0$ (fixed support)

$\therefore \frac{L_E}{L}$ from NCCI graphs for sway = 1.23

Applying the above effective length factor to get the critical buckling load of the system and assuming it is governed by buckling of the weaker column;

$K = \frac{12E \times \frac{I}{2}}{h^3}$ $\qquad$ $K_p = \frac{27/4P}{h}$

$P = \frac{8EI}{9h^2}$ $\qquad$ $\overline{P_{cr}} = 0.731\frac{EI}{h^2}$

Using the effective length factor calculated:

$\lambda = \frac{0.5\times0.5I\times\mathbf{1.23}\times3}{0.5I\times5} = 0.369 \rightarrow \mu = 0.547$

$\therefore P_{cr} = 0.547 \times 0.731\frac{E\times0.5I}{h^2} = 0.2\frac{EI}{h^2}$ (5% greater than FEA result =$0.19\frac{EI}{h^2}$). The critical load was reduced by 0.5 as the value of μ was taken from the equivalent frame, not the original frame.

The buckled shape of the frame is presented in Figure 3.51. It was observed that all stories buckled under the first mode.

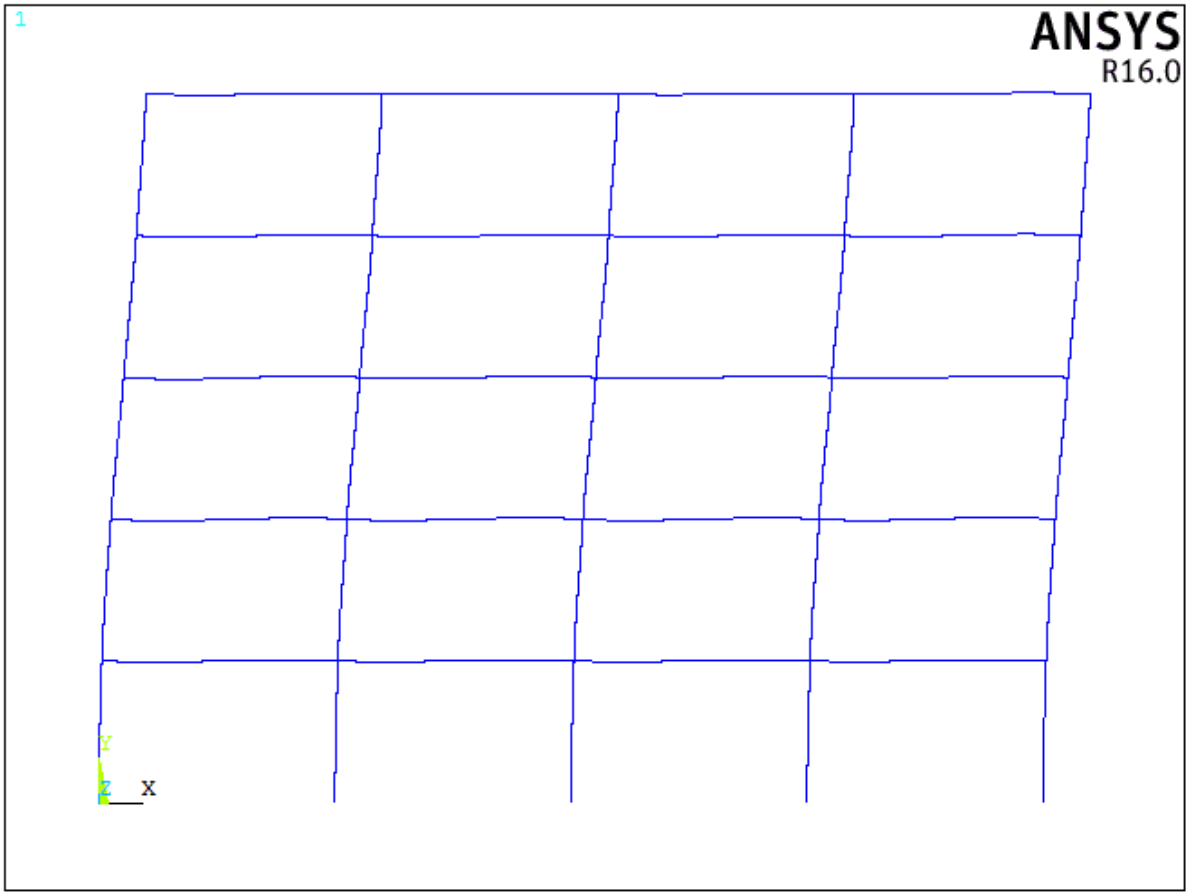


Figure 3.51 Buckled shape of the optimised five story four bay frame.

The approach was applied to another example namely, the previous two story two bay frame in Section 3.2.1.

The two stories of the optimised frame were folded into an equivalent single bay frame, resulting in the equivalent frame in Figure 3.52.

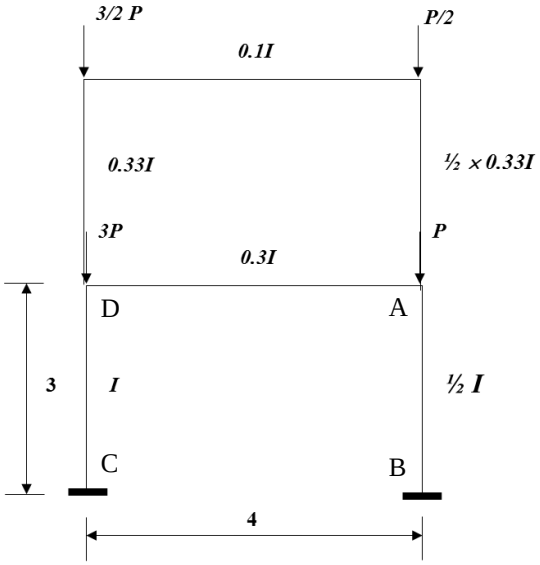


Figure 3.52 Equivalent one bay frame of the optimised two story two bay sway frame.

The stiffer column *CD* offers some lateral restraint to column *AB* of the first storey hence *AB* is the column of interest.

$K_{ij} = \frac{0.5I}{3}$

$K"_{b,A} = 1.5 \times \frac{0.3I}{4} = 0.1125I$

$K"_{XY,A} = 0.25 \times \frac{0.5\times0.33I}{3}\left(1 - 0.82 \times \frac{\frac{1}{2}P}{P} \times \left(\frac{3}{3}\right)^2\right) = 0.0081125I$

$\eta_{A(top)} = \frac{\frac{0.5I}{3}}{\frac{0.5I}{3}+0.1125I+0.0081125I} = 0.58013$

$\eta_{B(bottom)} = 0$ (fixed support)

$\therefore \frac{L_E}{L}$ from NCCI graphs for sway = 1.29

Applying the above effective length factor to get the critical buckling load of the system and assuming it is governed by buckling of the weaker column;

$K = \frac{12E\times\frac{I}{2}}{h^3}$ $\qquad$ $K_p = \frac{1.5P}{h}$

$P = \frac{4EI}{h^2}$ $\qquad$ $\overline{P_{cr}} = \frac{\pi^2 EI}{3h^2}$

Using the effective length factor calculated:

$\lambda = \frac{0.5\times0.3I\times\mathbf{1.29}\times3}{0.5I\times4} = 0.29025 \rightarrow \mu = 0.497$

$\therefore P_{cr} = 0.497 \times \frac{\pi^2 EI}{3h^2} \times 0.5 = 0.818\frac{EI}{h^2}$ (3.82% less than FEA result=0.8505$\frac{EI}{h^2}$).

The percentage difference from FEA is acceptable even though it had increased from the previous approach applied.

The approach was then applied to a four story six bay sway frame as shown in Figure 3.53.

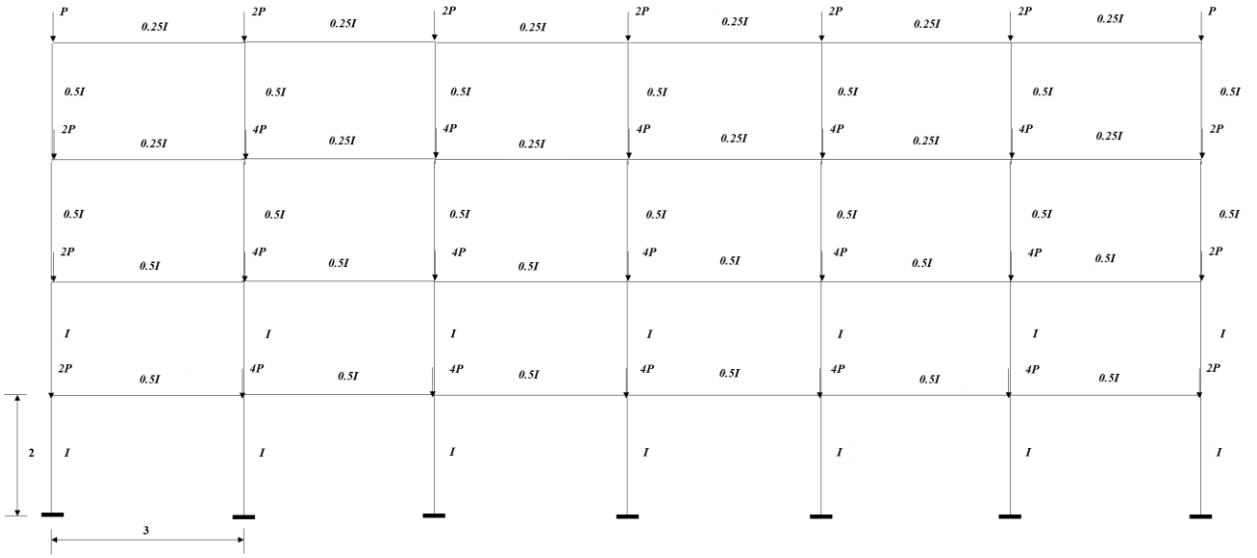


Figure 3.53 A four story six bay sway frame.

Optimisation was applied as before using an equivalent one bay frame and effective lengths so that all stories buckled at the same load.

The final optimised frame obtained is shown in Figure 3.54.

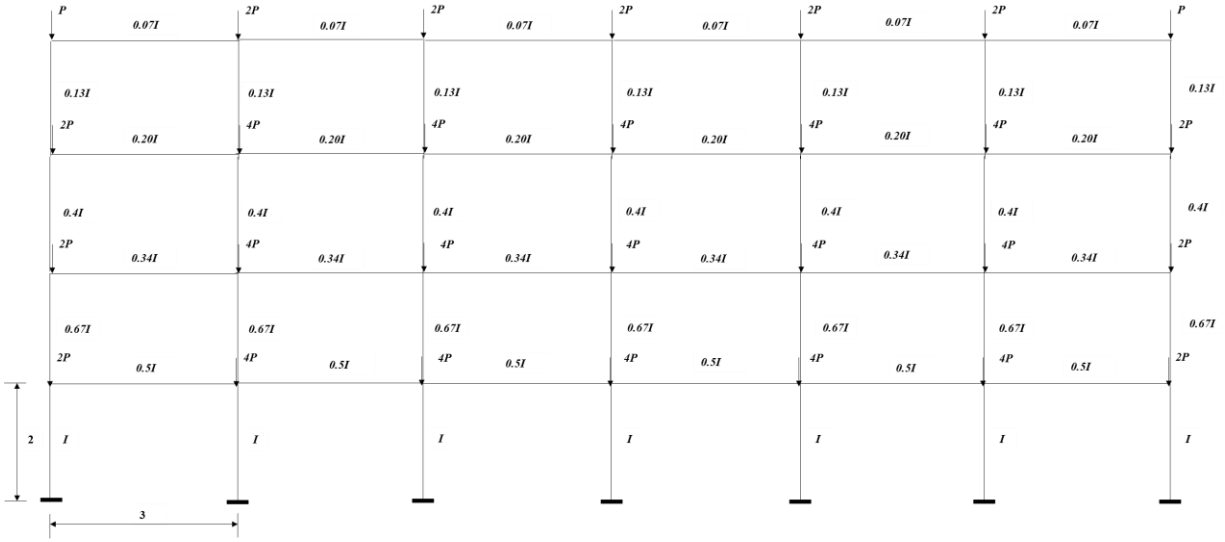


Figure 3.54 Optimised four story six bay frame.

The first two stories were folded into an equivalent single bay frame, as all stories were optimised to have the same buckling load, hence any story can be used to obtain the system bucking load. The equivalent two story one bay frame in Figure 3.55, demonstrates the optimisation procedure.

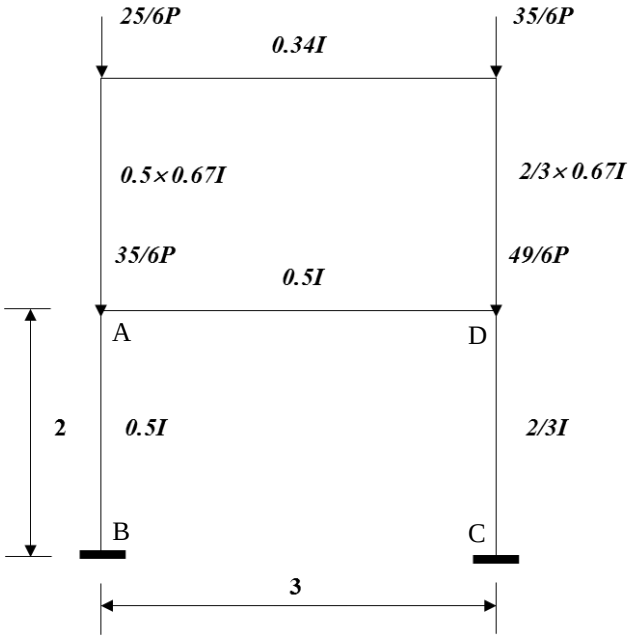


Figure 3.55 Equivalent one bay frame of the first two stories of the frame.

The stiffer column *CD* offers some lateral restraint to column *AB* of the first storey hence *AB* is the column of interest.

$K_{AB} = \frac{0.5I}{2}$

$K''_{b,A} = 1.5 \times \frac{0.5I}{3} = 0.25I$

$K''_{XY,A} = 0.25 \times \frac{0.5 \times 0.67I}{2}\left(1 - 0.82 \times \frac{\frac{25}{6}P}{\frac{35}{6}P} \times \left(\frac{2}{2}\right)^2\right) = 0.017I$

$\eta_{A(top)} = \frac{\frac{0.5I}{2}}{\frac{0.5I}{2} + 0.017I + 0.25I} = 0.48$

$\eta_{B(bottom)} = 0$ (fixed support)

$\therefore \frac{L_E}{L}$ from NCCI graphs for sway = 1.21

Applying the above effective length factor to get the critical buckling load of the system and assuming it is governed by buckling of the weaker column;

$K = \frac{12E \times \frac{I}{2}}{h^3}$ $\qquad$ $K_p = \frac{35/6P}{h}$

$P = \frac{36EI}{35h^2}$ $\overline{P_{cr}} = 0.846\frac{EI}{h^2}$

$\lambda = \frac{0.5\times0.5I\times\mathbf{1.21}\times2}{0.5I\times3} = 0.404 \rightarrow \mu = 0.566$

$\therefore P_{cr,1} = 0.566 \times 0.846\frac{E\times0.5I}{h^2} = 0.24\frac{EI}{h^2}$ (14.5% smaller than FEA result =$0.28\frac{EI}{h^2}$).

The percentage difference from FEA was found to be significant.

The approach was lastly applied to a ten story four bay sway frame as given in Figure 3.56.

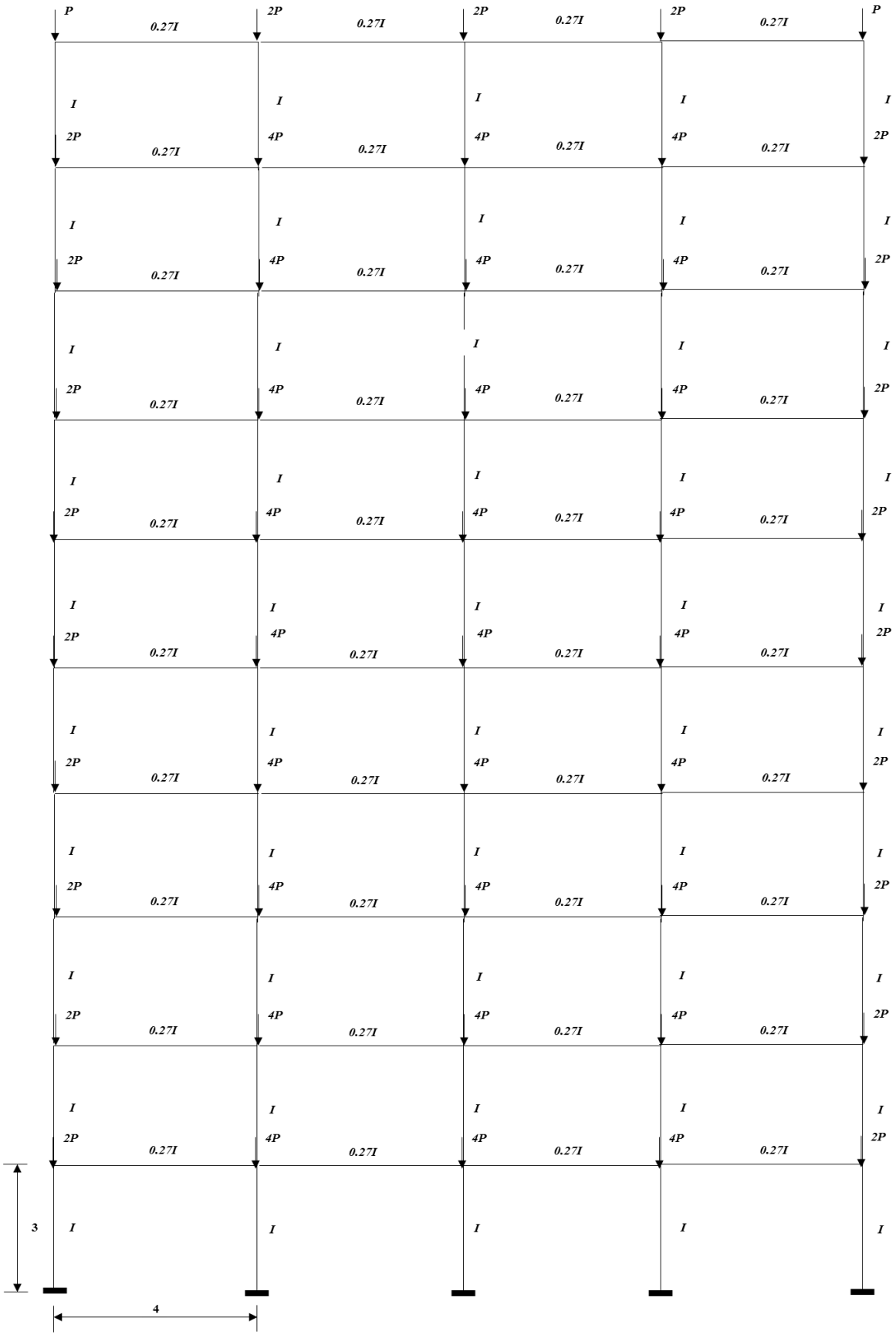


Figure 3.56 Ten story four bay sway frame.

Optimisation was applied as before using an equivalent one bay frame and effective lengths so that all stories buckled at the same load.

The final optimised frame obtained is shown in Figure 3.57.

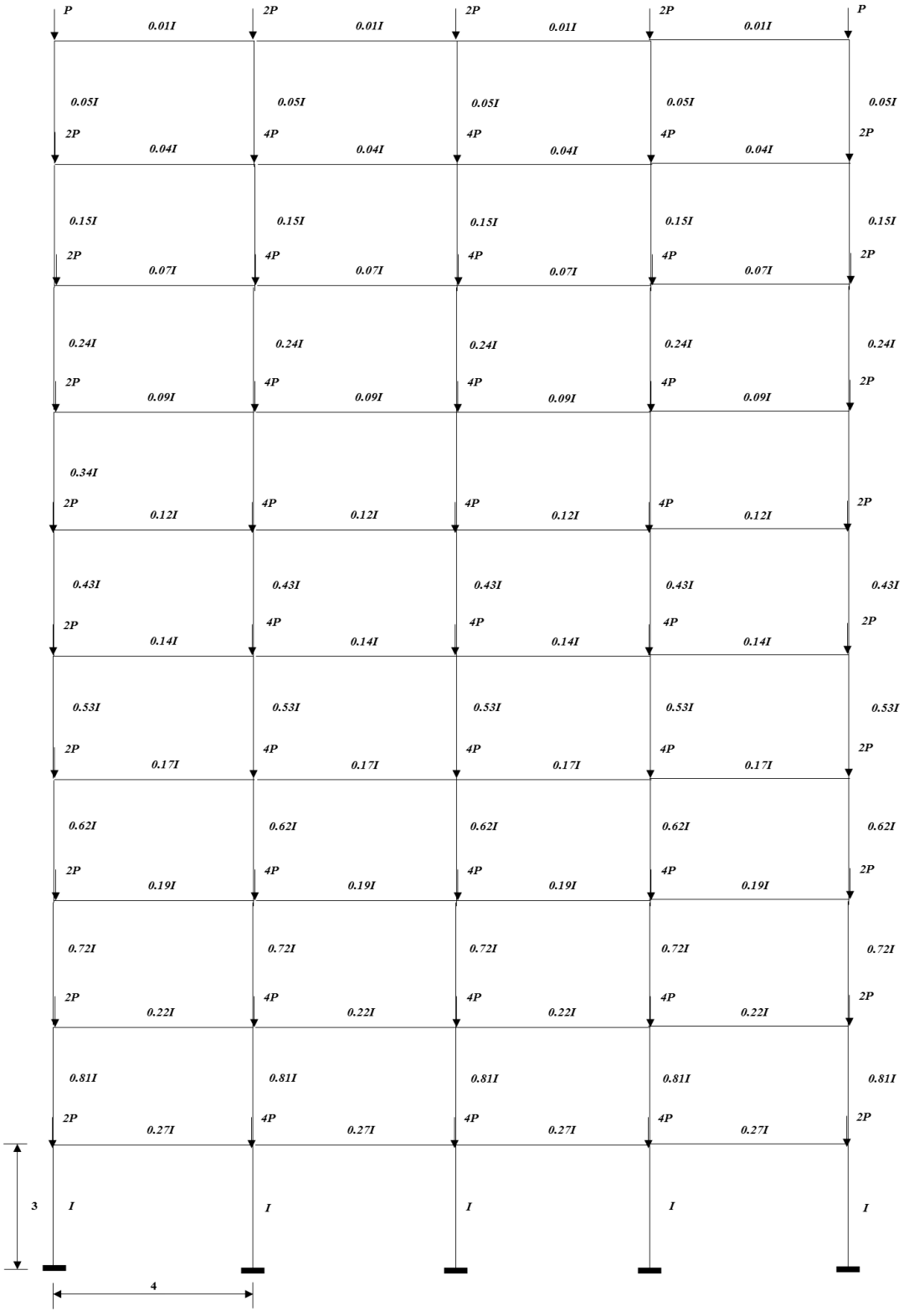


Figure 3.57 Optimised ten story four bay sway frame.

The first two stories were folded into an equivalent single bay frame, as all stories were optimised to have the same buckling load. The equivalent two story one bay frame is shown in Figure 3.58.

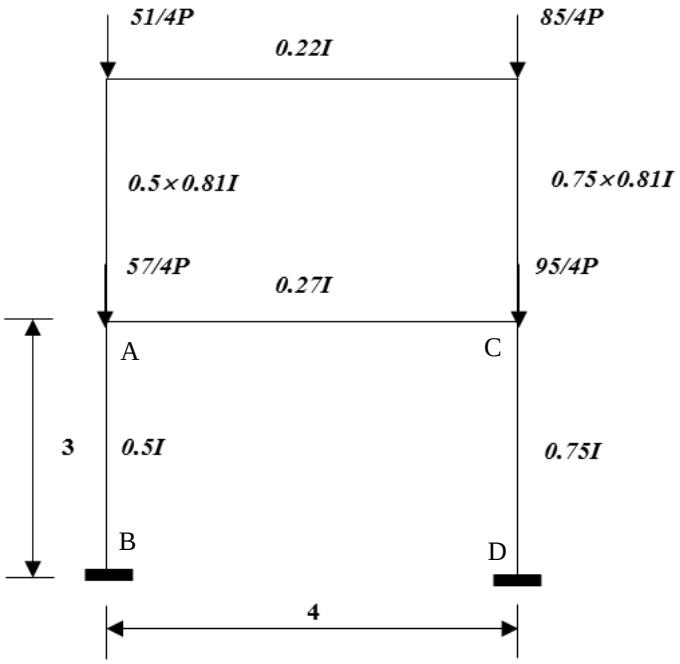


Figure 3.58 Equivalent one bay frame of the first two stories for ten story frame.

The stiffer column *CD* offers some lateral restraint to column *AB* of the first storey hence *AB* is the column of interest.

$K_{AB} = \frac{0.5I}{3}$

$K"_{b,A} = 1.5 \times \frac{0.27I}{4} = 0.10125I$

$K"_{XY,A} = 0.25 \times \frac{0.5\times0.81I}{3}\left(1 - 0.82 \times \frac{\frac{51}{4}P}{\frac{57}{4}P} \times \left(\frac{3}{3}\right)^2\right) = 0.009I$

$\eta_{A(top)} = \frac{\frac{0.5I}{3}}{\frac{0.5I}{3}+0.009I+0.10125I} = 0.6$

$\eta_{B(bottom)} = 0$ (fixed support)

$\therefore \frac{L_E}{L}$ from NCCI graphs for sway = 1.302

Applying the above effective length factor to get the critical buckling load of the system and assuming it is governed by buckling of the weaker column;

$K,1 = \frac{12E\times\frac{I}{2}}{h^3}$ $\qquad$ $K_{p,1} = \frac{57/4P}{h}$

$P,1 = \frac{8EI}{19h^2}$ $\qquad$ $\overline{P_{cr,1}} = 0.346\frac{EI}{h^2}$

$\lambda = \frac{0.5\times0.27I\times\mathbf{1.3}\times3}{0.5I\times4} = 0.2633 \rightarrow \mu = 0.4778$

$\therefore P_{cr,1} = 0.4778 \times 0.346 \times \frac{E\times0.5I}{h^2} = 0.08\frac{EI}{h^2}$ (33% larger than FEA= $0.06\frac{EI}{h^2}$). The percentage difference from FEA was found to be significantly large. When the approach was applied to the previous frame of five stories and four bays, a percentage difference of less than 5% occurred, yet when more stories were added, this difference significantly increased prompting further analysis.

This approach provided useful predictions of the buckling load in some frame examples but in the case of higher order frames, it was found to be unsuccessful. The vertical effect of all stories in the frame needs to be taken into account, as the suggested approach involved a story by story analysis whereby only the adjacent story was considered.

Hellesland (1998) stated that story-based approaches proved to indicate the horizontal interaction between columns and the vertical interaction between stories, provided that inflection points are at mid-height of a column. In other cases, where the inflection points occur elsewhere, these approaches may become inaccurate in determining the system buckling load, which was seen in the higher order or multi-story examples such as the previous ten story frame. The literature suggested that flexible regions may occur in these higher order frames. The 'Method of Means' approach suggested in the literature was applied to the ten story four bay frame, by only considering the stories in this flexible region, in an effort to improve on the previous prediction of the system bucking load.

According to this 'Method of Means' approach, an estimate of the system stability index $\alpha_{,system}$ is usually given by the max story stability index $\alpha_{,storey}$, but in the case of multi-story frames where flexible regions within the frame can occur, this system index was limited to stories within this region; thus

$\alpha_{system} = \frac{1}{n}\sum_{i=1}^{n} \alpha_{story,i}$

where *n* is the number of consecutive stories in the flexible region or within the region of the most flexible storey (one with the largest $\alpha_{,storey}$); and $\alpha_{story} = \frac{1}{m}\sum_{i=1}^{m} \alpha_{,i}$

where m is the total number of columns within a story and α_i is a pseudo member stability index based on isolated free-to-sway *K* factors of the columns: $\alpha_i=\left(K^2\alpha_e\right)_i$, where $\alpha_e=\frac{P}{P_E}$ ('load parameter') and $P_e=\frac{\pi^2 EI}{L^2}$. The *EI* and *P* values are taken at a chosen reference section for members with varying stiffness and axial force along the length. The effective length factor can then be calculated by $K=\sqrt{\frac{\alpha_{system}}{\alpha_E}}$.

The method outlined was applied to the optimised ten story four bay frame and used the equivalent single bay multi-story frame, where the reference column was selected as the column with the stiffness of ½ in story 1, thus:

$$P_e=\frac{\pi^2 EI}{L^2}=\frac{\pi^2 E\times 0.5I}{3^2}=\frac{\pi^2 EI}{18} \quad \text{and} \quad P=\frac{57P}{4} \quad \rightarrow \quad \alpha_e=\frac{P}{P_E}=26EI$$

Thus the story stability index, α_{story}, can be calculated for each story by solving for the member stability indexes given by the K-factors and the load parameter, α_e. The results are summarised in the Table 3.37.

Table 3.37 Story stability index calculated for the ten story frame using the Method of Means.

Story number	K-factor for column with *0.5I* *	K-factor for column with *0.75I**	$\alpha_{story}(\times 10^{-7})$
1	1.302	1.382	2.966
2	1.674	1.895	5.262
3	1.671	1.889	5.233
4	1.670	1.886	5.22
5	1.668	1.883	5.204
6	1.665	1.879	5.185
7	1.662	1.874	5.163
8	1.660	1.870	5.144
9	1.666	1.880	5.19
10	1.698	1.940	5.470

**Calculated using the approach in Webber et al. (2015) and the NCCI method (2005).*

The story with the largest $\alpha_{story,}$ needs to be determined in order to obtain the flexible region of the frame. Thus, from Table 3.37, the tenth story was found to be the most flexible. The flexible region that was used to obtain the system stability index, was from stories six to ten.

$$\alpha_{system}=\frac{1}{4}\sum_6^{10}\alpha_{story,i}=3.3785\alpha_E$$

Thus, the effective length factor *K* is,

$$K = \sqrt{\frac{\alpha_{system}}{\alpha_E}} = \sqrt{\frac{3.3785\alpha_E}{\alpha_E}} = 1.838$$

Since both columns of the equivalent frame were taken into consideration, the upper-bound critical load, $\overline{P_{cr}}$, needs to be re-calculated.

$$K_1 = \frac{12E\times\frac{I}{2}}{h^3} + \frac{12E\times 0.75I}{h^3} = \frac{15EI}{h^3} \qquad K_{p,1} = \frac{57/4P}{h} + \frac{95/4P}{h} = \frac{38P}{h}$$

$$P_1 = \frac{15EI}{38h^2} \qquad \overline{P_{cr,1}} = 0.3247\frac{EI}{h^2}$$

The two columns have different stiffness I_c, thus an equivalent column stiffness was used, $(0.75I + \frac{1}{2} I)/2 = 0.625I$. Hence, using the effective length and equivalent column stiffness:

$$\lambda = \frac{0.625\times 0.27I\times \mathbf{1.838}\times 3}{0.625I\times 4} = 0.3721 \rightarrow \mu = 0.5460$$

$$\therefore P_{cr,1} = 0.5460 \times 0.3247\frac{E\times 0.625I}{h^2} = 0.11\frac{EI}{h^2}$$ (94% larger than FEA result=$0.057\frac{EI}{h^2}$). It was noted that the system buckling load was reduced by the value of 0.625 as μ it was taken by equivalent frame and not the original. The percentage difference when compared to the previous approach applied to the frame, increased greatly and the 'Method of Means' approach cannot be deemed as acceptable to obtain the system buckling load.

The results of the analyses done in Approach 2, indicated that using an effective length factor, which was based on isolated member analysis, to account for the effect of the adjacent stories was not a successful approach. Furthermore, since the effective lengths are determined on an isolated member basis, this is evidently not in line with a system-based approach which is what the research at hand is based on.

The last approach that attempted to address the effect of adjacent stories, was to produce design graphs for equivalent multi-story one bay frames, from which the corresponding μ values could be obtained. This equivalent frame approach was validated for no sway frames and therefore it would lead to exploring this idea on sway frames, particularly multi-story frames. It was demonstrated that the equivalent single story sway frames could not be extended to multi-story frames unlike in the

case of no sway frames, hence new design graphs and equivalent frames were investigated.

3.2.3 Approach 3 – Equivalent multi-story one bay sway frames

Equivalent one bay, multi-story frames, as shown in Figure 3.59, were done for frames where the number of bays, *b*, was equal to 1, 2 and infinity. These frames were used to obtain corresponding values of μ for different values of λ for the first story of the frame. This system analysis was performed using FEA software ANSYS, to get the real system buckling load, *Pcr*. All columns had the same stiffness *I*, whilst the beams had a stiffness of *nI*, and uniform loads of *P* were applied to each story.

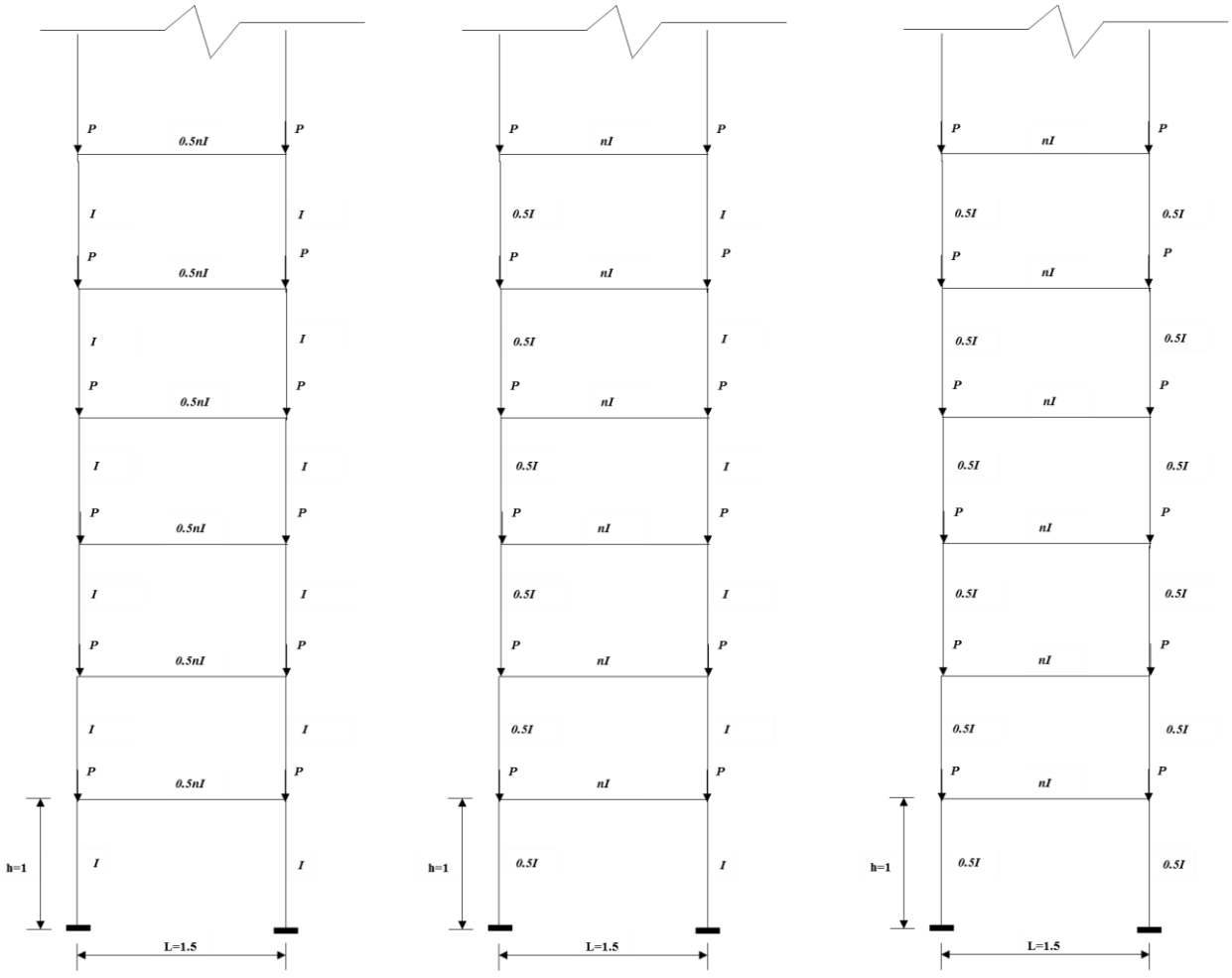


Figure 3.59 Equivalent one bay multi-story sway frames when number of bays 'b' is equal to 1, 2 and infinity.

The upper-bound system buckling load, $\overline{P_{cr}}$, for the first story of each frame shown in Figure 3.59, as well as the parameter λ, can be obtained as before for the equivalent frames (Li, 2014):

When *b*=1,,

$K_1 = 2 \times \frac{12EI}{h^3}$ $K_{p,1} = 2 \times \frac{sP}{h}$

$P_1 = \frac{12EI}{sh^2}$ $\overline{P_{cr,1}} = \frac{\pi^2 EI}{sh^2}$

$\lambda = \frac{0.5nI \times 1}{I \times 1.5} = \frac{n}{3}$

, where *s* is the number of stories.

When *b*=2,

$K_1 = \frac{12E \times \frac{I}{2}}{h^3} + \frac{12EI}{h^3} = \frac{18EI}{h^3}$ $K_{p,1} = 2 \times \frac{sP}{h}$

$P_1 = \frac{9EI}{sh^2}$ $\overline{P_{cr,1}} = \frac{3\pi^2 EI}{4sh^2}$

$\lambda = \frac{nI \times 1}{(\frac{0.5+1}{2})I \times 1.5} = \frac{8}{9}n$

When *b*=∞,

$K_1 = 2 \times \frac{12E \times \frac{I}{2}}{h^3}$ $K_{p,1} = 2 \times \frac{sP}{h}$

$P_1 = \frac{6EI}{sh^2}$ $\overline{P_{cr,1}} = \frac{\pi^2 EI}{2sh^2}$

$\lambda = \frac{nI \times 1}{0.5I \times 1.5} = \frac{n}{0.75}$

Defining the system buckling load coefficient, μ, as the ratio of the system buckling load to its upper-bound as; $\mu = \frac{P_{cr}}{\overline{P_{cr}}}$, the design graphs in Figure 3.60 to 3.62, were obtained from the FEA analysis for the value of *Pcr* and the theoretical analysis of $\overline{P_{cr}}$, to get the value of μ when the number stories, *s*, was equal to 1,2,3,5 and 8.

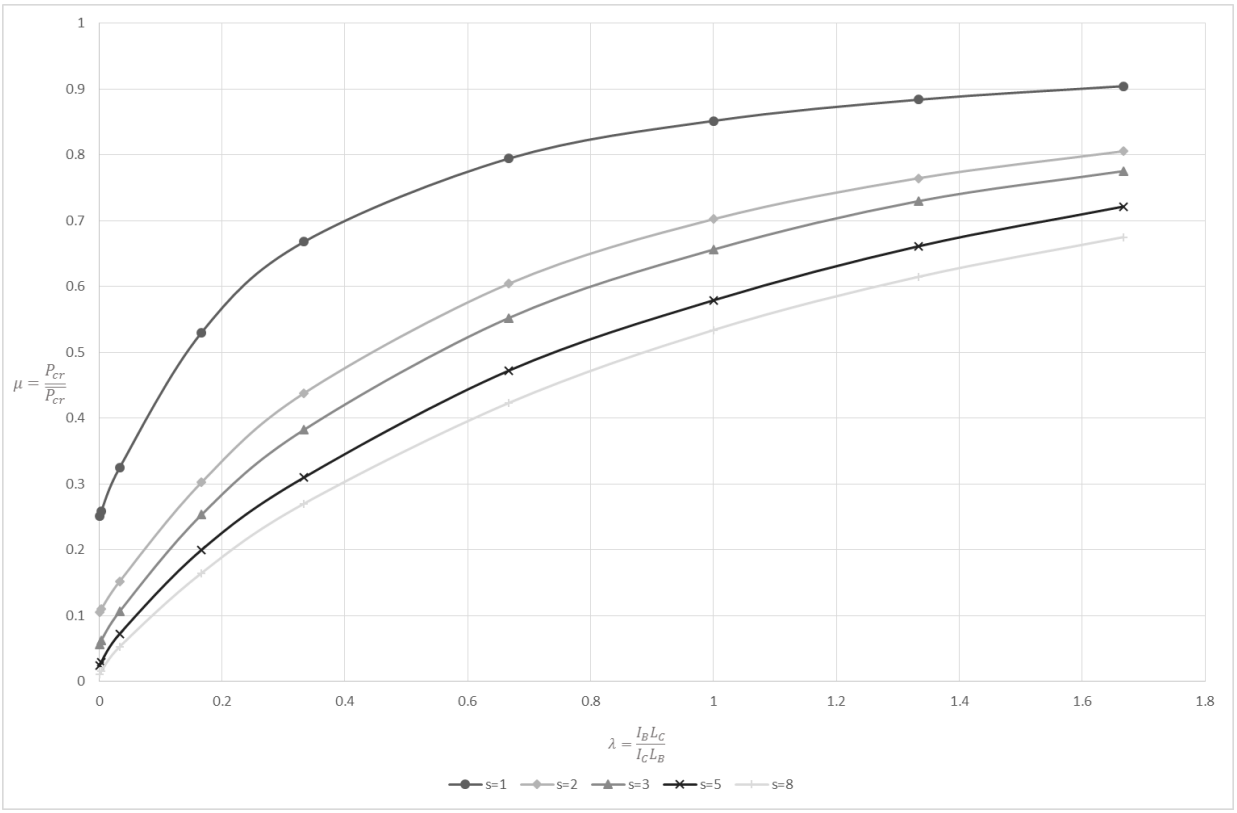


Figure 3.60 Design graph when b=1 for different number of stories of multi-story one bay equivalent sway frame.

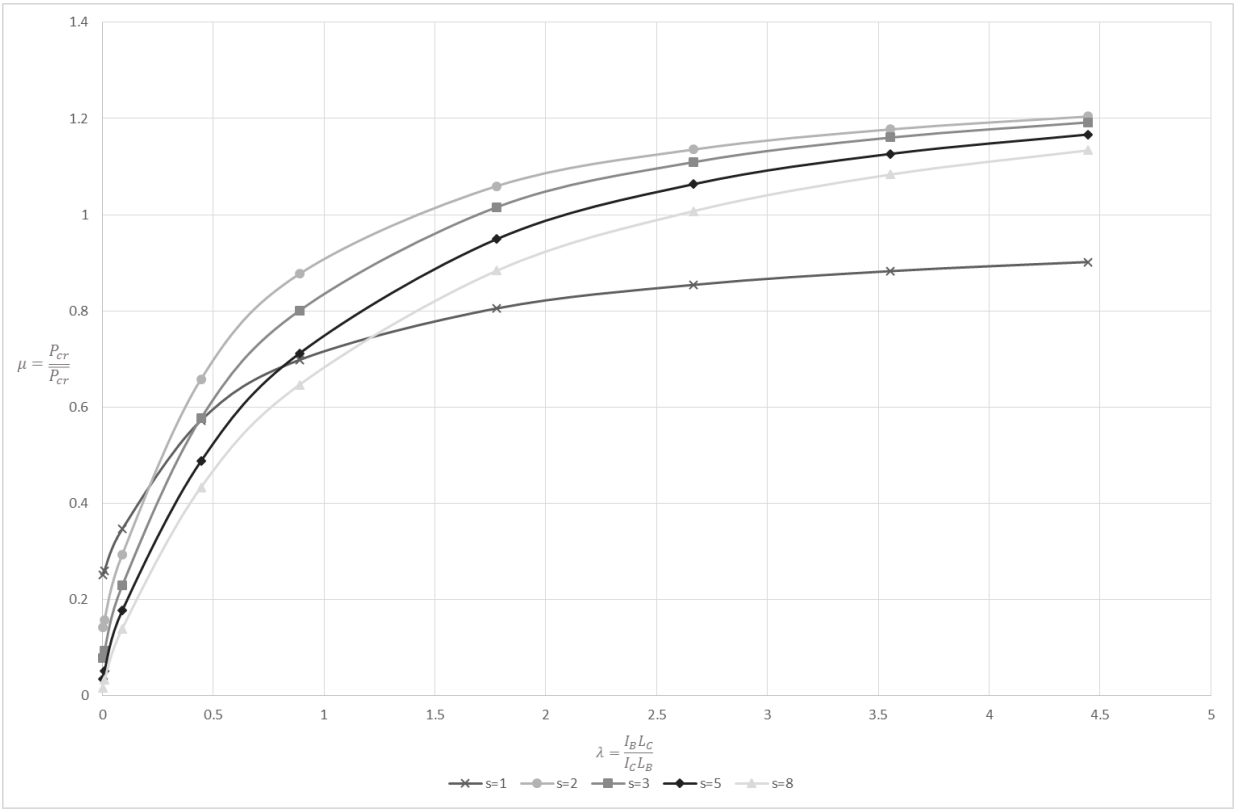


Figure 3.61 Design graph when b=2 for different number of stories of multi-story one bay equivalent sway frame.

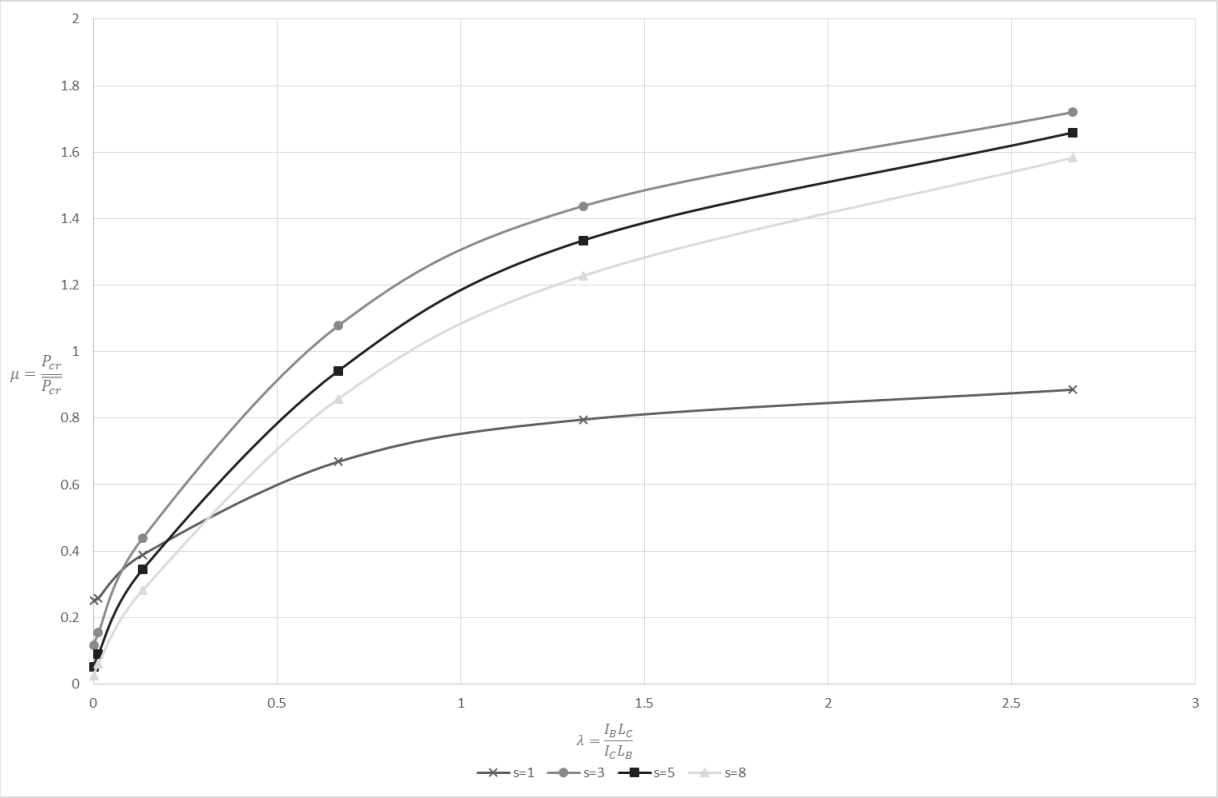


Figure 3.62 Design graph when $b=\infty$ for different number of stories of multi-story one bay equivalent sway frame.

It is seen from the graphs produced for the multi-story one bay frames that no defined trend could be derived between the story number and the value of μ that could be useful in an analysis. Consequently, this approach cannot be used to predict the system buckling load of multi-story sway frames. A value when s=1 and s=∞ for the value of μ for different number of bays, could not be attained to be applied to the upper-bound buckling load determined by Li's method (2014).

3.2.4 Conclusion – Sway frames

The optimisation method for the no sway frames required a derivation of the system buckling load of the frame. The method used to obtain this load for no sway frames was applied to evaluate its effectiveness on sway frames, and was found to be unsuccessful. It was seen that the system buckling load is not only affected by the non-rigid beams as found for the no sway frames, but is also affected by the vertical interaction of the adjacent stories. Even though some results using this method were deemed acceptable, the results were generally inconsistent to incorporate it in the optimisation method.

Methods available in literature to calculate and capture the buckling of multi-story sway frames were applied, yet proved to be unsuccessful when used in a global or system based optimisation approach. The methods are based on isolated member analysis and calculated the buckling load of individual members in sway frames. Even though it accounted for the influence of the adjacent stories, it could not address the story buckling load approach required in the proposed optimisation method.
This led to producing possible design graphs to reduce the upper-bound system buckling load to account for non-rigid beams of multi-story sway frames, as it was known that the upper-bound solution under 'rigid' beams proved to be acceptable (Li, 2014). However, no correlation or relationship between the buckling load and 'non-rigid' beams could be attained from the design graphs produced. As previously stated, this could be due to another influence on the buckling load of a multi-story sway system with non-rigid beams.

The results of all attempts, indicates that there is another influence on the buckling load of sway frames that needs to be investigated. The methods available in the literature which have shown to be adequate in calculating the buckling of individual column members in a multi-story sway frame, could be expanded in further studies to acquire a system or story buckling load. Once a method has been derived, it can be incorporated in the proposed optimisation method to get the system buckling load. This will also study the effectiveness of the optimisation method on sway frames, as the present analysis could not evaluate the optimisation method itself on such frame applications.

The analysis performed on both no sway and sway frames, indicated two significant findings:

- The method derived to obtain the system buckling load rigid no sway frames has been validated by the acceptable percentage differences of the buckling loads to FEA. The approaches to develop a method for system buckling analysis of sway frames was not successful as the results found from the

various frame examples were inconsistent to accept any of the attempts discussed.

- The proposed optimisation procedure can be accepted for no sway frames, as it has achieved its outcome of full member utilisation as all members are engaged in the buckling as demonstrated by the buckled shapes of the optimised no sway frames. The use of the proposed optimisation method on sway frames could not be evaluated. The system buckling load of sway frames, specifically based on a story or global approach, needs to be established to include all buckling influences thereafter, it can be used in the optimisation method. Several approaches were attempted but none were successful.

The various frame applications also highlighted the simplicity of the method that can be easily adopted by design engineers. The accuracy of the solution obtained by either the buckling method presented or from FEA, is however not evaluated in the current research and can be assessed in further studies.

4 CONCLUSION

Presented in this report, is an optimisation method for plane no sway rigid frames against buckling that can be easily implemented in a design environment. The main objectives of the method are to minimise the weight of the frame and reduce material wastage by aiming for all stories within a frame to buckle at the same time. The approach involves a global analysis and its effectiveness and efficiency has been demonstrated through various frame applications. Its difference from other optimisation methods, lies in the fact that it achieves full member capacity utilisation and incorporates a system approach to buckling.

The proposed optimisation procedure proved successful for no sway multi-story rigid frames, as validated by the acceptable percentage differences of below 5% when compared to a FE analysis, which is commonly utilised in the design environment. The accuracy of the solution produced by FEA and the proposed method, was however not evaluated in the present research.

Through the derivation of the optimisation method, a method to calculate the system buckling load of no sway frames under 'real' or 'non-rigid' beams, was also presented and validated. The method developed on the method established by Li (2014) for sway frames under 'rigid' beam conditions. The suggested system buckling analysis method is simple to perform and comprised of two parts; namely:

i. The determination of the upper-bound system buckling load corresponding to rigid beams; and
ii. Producing a system buckling load evaluation from the upper-bound and true system buckling load coefficient, μ, which can be found with the normalized equivalent one bay frames and the design graphs developed.

The no sway frame applications demonstrated how two objectives of optimisation were achieved; namely:

i. All stories buckled at the same time or under the same buckling load whilst minimising the weight of the frame; and

ii. The system buckling load increased whilst the weight of the frame decreased.

The suggested optimisation procedure also indicated that a global approach to optimisation can be performed as opposed to current methods in the literature where the critical story is shifted to higher stories and optimisation is conducted on an isolated level. In addition, the available methods did not address the issue of material wastage or the utilisation of full member capacity in that all members are engaged simultaneously in the buckling of the frame. The optimisation method addresses this, as can be seen from the buckled shapes of the optimised frames, where all members deflected under the first buckling load. This finding is beneficial in the current practice of reducing material wastage in the design environment.

The application of the optimisation method to the no sway frame examples presented, also indicated several findings:

i. The simplicity of the method;
ii. Savings in material wastage was achieved by the reduction in the stiffness of the stories after optimisation resulting in smaller cross-sectional properties required for members to achieve system buckling; and
iii. The comparison of frames indicate that initial sections can be selected before optimisation or the user may designate the same stiffness for all initial sections. This will yield an optimised frame with the same final system buckling load.

The optimisation method is however limited in its application to multi-story sway frames. Specifically, the methods proposed for the calculation of the system buckling load of such frames was not successful. The method adopted for the analysis of no sway single story frames, of using normalised equivalent one bay frames, was found effective when obtaining the buckling load of single story sway frames however, it resulted in large errors when applied to multi-story sway frames as the influence of the adjacent stories became apparent. This could be due to the reduced lateral restraint offered by no sway frames, and possibly introducing secondary effects into

the buckling of the stories. Current techniques in literature to determine the buckling load of these frames have captured this effect however, many are based on isolated member analysis and thus led to the significant differences when applied to a global analysis of frames discussed in the report. The results obtained when implementing these methods were inconsistent and unreliable to accept the approach.
Ultimately, the influence of the adjacent stories and the loss of full lateral restraint on the system buckling load, was found to be prominent in the approaches applied. The attempts to capture this effect were shown to be ineffective and further studies is needed in this area before an optimisation method can be applied. The examples and approaches given in the report, can form a basis for further research to develop on.

At present, the research at hand provides a simple method to optimise multi-story rigid, no sway frames against buckling using a global approach, thus preventing the under-utilisation of members and hence reducing material wastage.

4.1 Future Studies

The proposed optimisation method has been applied to frames under certain conditions such as support conditions, geometry, joint fixity, member sections, load application, and sway conditions. Further studies can extend on the research presented and evaluate its sensitivity to different frame conditions namely;

- Support conditions: fixed base versus pinned base. However, a method may need to be derived first to calculate the system buckling load of the frame as the method presented in Section 2.1.2 was derived using fixed base frames.
- Use of different member sections such as a H, I or hollow sections in the FEA model.
- Different material properties such as different grades of steel, wood or concrete.
- Joint fixity: the analysis currently assumes rigid joints and semi-rigid or pinned joints can be evaluated.

The method derived for the system buckling load of a no sway rigid frame can be expanded to include horizontal loads so as to further its application to a wider range

of frames. Furthermore, the inclusion of horizontal loads will allow for the results to be compared with other optimisation methods in the literature or design codes and standards, which apply these loads. The optimisation method suggested in the research cannot be easily compared to these methods as it is limited to vertical loads only. Even though some of the examples presented in Section 3.1 are adapted from frames available in the literature, the horizontal loads were omitted and hence a true comparison of results could not be made.
The accuracy of the results obtained by the proposed system buckling method was not validated in the research and a comparison to other structural methods and scale model tests could ascertain the accuracy of the solution.

From the approaches presented in the analysis of sway frames, further investigation is needed to firstly, propose a method to calculate the system buckling load of multi-story sway frames and, to then incorporate this in an optimisation method. The effect of adjacent stories and the reduced lateral restraint on the system buckling load is a defining factor which needs to be explored. The optimisation method suggested in this report for the no sway frames, could be used and hence its effectiveness on the optimisation of these frames can be studied. The available methods in the literature which were attempted in the research, could be developed further to obtain a buckling load based on a global analysis.

Despite the number of further developments suggested for the research, it is recommended that future work should begin by expanding on the proposed optimisation method of plane no sway rigid frames to include sway frames before incorporating other frame conditions such as semi-rigid joints or pinned bases.

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APPENDIX A

OPTIMISATION CODE USED IN MATLAB

```
function Optimise_frame_NOSWAY_ReduceColsBeams_FINAL
no_of_stories=xlsread('Optimisation of Plane Frames against Buckling
Input.xls','Frame input','E4')
E=xlsread('Optimisation of Plane Frames against Buckling Input.xls','Frame
input','I5')
Hc=xlsread('Optimisation of Plane Frames against Buckling Input.xls','Frame
input','I6')
Lb=xlsread('Optimisation of Plane Frames against Buckling Input.xls','Frame
input','I4')
no_of_bays=xlsread('Optimisation of Plane Frames against Buckling
Input.xls','Frame input','E7')
extcol_i=xlsread('Optimisation of Plane Frames against Buckling Input.xls','Frame
input',['J35:J' num2str(no_of_stories+34)])
intcol_i=xlsread('Optimisation of Plane Frames against Buckling Input.xls','Frame
input',['F35:F' num2str(no_of_stories+34)])
beam_i=xlsread('Optimisation of Plane Frames against Buckling Input.xls','Frame
input',['N35:N' num2str(no_of_stories+34)])
int_cols=no_of_bays-1
ext_cols=2
weight=weight_of_storey
Storey_stiffness=extcol_i
Pcr_frame=calc_Pcr
find_minPcr

function w=weight_of_storey
intcol_breadth=xlsread('Optimisation of Plane Frames against Buckling
Input.xls','Frame input',['G35:G' num2str(no_of_stories+34)])
```

```
extcol_breadth=xlsread('Optimisation of Plane Frames against Buckling
Input.xls','Frame input',['K35:K' num2str(no_of_stories+34)])
beam_breadth=xlsread('Optimisation of Plane Frames against Buckling
Input.xls','Frame input',['L35:L' num2str(no_of_stories+34)])
w=7800*(((no_of_bays+1).*(Hc).*intcol_breadth.^2) +
(no_of_bays*Lb*beam_breadth.^2))
xlswrite('Optimisation of Plane Frames against Buckling Input.xls',w,'Frame
input',['Q35:Q' num2str(no_of_stories+34)])
end

function Pcr_nosway=calc_Pcr
Loads_ext=xlsread('Optimisation of Plane Frames against Buckling Input.xls','Frame
input',['F23:F' num2str(no_of_stories+22)])
Loads_int=xlsread('Optimisation of Plane Frames against Buckling Input.xls','Frame
input',['E23:E' num2str(no_of_stories+22)])
   %NON-RIGID BEAMS
   disp('Beams are semi-rigid')
   Ki=((int_cols*12*E)./Hc.^3).*intcol_i + ((ext_cols*12*E)./Hc.^3).*extcol_i
%HORIZONTAL/ELASTIC STIFFNESS OF SYSTEM BEFORE LOADING
(UNDEFORMED CONFIGURATION)
   xlswrite('Optimisation of Plane Frames against Buckling Input.xls',Ki,'Output per
Iteration',['C5:C' num2str(no_of_stories+4)])
   Kpi=(Loads_int./Hc)+ (Loads_ext./Hc) %GEOMETRIC STIFFNESS (EFFECT
OF LOADS ON SYSTEM)
   xlswrite('Optimisation of Plane Frames against Buckling Input.xls',Kpi,'Output per
Iteration',['D5:D' num2str(no_of_stories+4)])
   Pi=Ki./Kpi
   Pcr_sway=(pi()^2/12)*Pi
   Pcr_Upperbound=4*Pcr_sway
   xlswrite('Optimisation of Plane Frames against Buckling Input.xls',
Pcr_Upperbound,'Output per Iteration',['E5:E' num2str(no_of_stories+4)])
```

```
    [num,txt,raw]=xlsread('Optimisation of Plane Frames against Buckling
Input.xls','Frame Input',['J34:J' num2str(no_of_stories+34)]);
    xlswrite('Optimisation of Plane Frames against Buckling Input.xls',num,'Output
per Iteration',['F5:F' num2str(no_of_stories+4)])
    if int_cols>0
    [num,txt,raw]=xlsread('Optimisation of Plane Frames against Buckling
Input.xls','Frame Input',['F34:F' num2str(no_of_stories+34)]);
    xlswrite('Optimisation of Plane Frames against Buckling Input.xls',num,'Output
per Iteration',['G5:G' num2str(no_of_stories+4)])
    end
    Ib=beam_i
    Ic=extcol_i
    [num,txt,raw]=xlsread('Optimisation of Plane Frames against Buckling
Input.xls','Frame input','I3');
    load_case='L2S1';
    which_case=strcmp(txt,load_case)
    if which_case==1
    if mod(no_of_bays(1,1),2)==0  %LOAD CASE L2S1
    lamda=(Hc.*Ib)./(Lb*0.5*Ic)%even case for 1/2I col
    else
    lamda=(Hc.*Ib)./(Lb*((no_of_bays(1,1)+1)/(2*no_of_bays(1,1)))*Ic)%odd case
    end
    else
    %LOAD CASE L1S1
    if no_of_bays==1
    lamda=(Hc.*Ib)./(Lb*Ic)%no of bays =1
    else
    lamda=(Hc.*2*Ib)./(Lb*Ic)%odd AND even case
    end
    end
    xlswrite('Optimisation of Plane Frames against Buckling Input.xls',lamda,'Output
per Iteration','H5')
```

```
    mu=input('What is mu from design graph?')
    xlswrite('Optimisation of Plane Frames against Buckling Input.xls',mu,'Output per
Iteration','I5')
    Pcr_nosway=mu.*Pcr_Upperbound
    xlswrite('Optimisation of Plane Frames against Buckling
Input.xls',Pcr_nosway,'Output per Iteration',['J5:J' num2str(no_of_stories+4)])
    end

function min_Pcr=find_minPcr
[min_Pcr,storey_no]=min(Pcr_frame)
crit_storey=[{'Critical story'},storey_no];
xlswrite('Optimisation of Plane Frames against Buckling
Input.xls',crit_storey,'Output per Iteration','B16')
Pcr_ratio=min_Pcr./Pcr_frame
xlswrite('Optimisation of Plane Frames against Buckling Input.xls',Pcr_ratio,'Output
per Iteration',['K5:K' num2str(no_of_stories+4)])
weight_of_frame=sum(weight)
xlswrite('Optimisation of Plane Frames against Buckling
Input.xls',weight_of_frame,'Output per Iteration','C15')

if Pcr_ratio>=0.9
disp('Frame Optimised')
xlswrite('Optimisation of Plane Frames against Buckling Input.xls',{'Frame
optimised'},'Output per Iteration','C17')
[num,txt,raw]=xlsread('Optimisation of Plane Frames against Buckling
Input.xls','Output per Iteration','C5:M17');
xlswrite('Optimisation of Plane Frames against Buckling Input.xls',raw,'Optimised
Frame','C5:M17')
Crit_Storey_stiff=extcol_i(1,1)
Storey_stiff_ratio=Storey_stiffness./Crit_Storey_stiff
xlswrite('Optimisation of Plane Frames against Buckling
Input.xls',Storey_stiff_ratio,'Optimised Frame',['M5:M' num2str(no_of_stories+4)])
```

```
xlswrite('Optimisation of Plane Frames against Buckling Input.xls',{'Storey stiff ratio'},'Optimised Frame','M4')
else
disp('Choose new section based on stiffness ratio')
xlswrite('Optimisation of Plane Frames against Buckling Input.xls',{'Choose new section based on stiffness ratio'},'Output per Iteration','C17')
end
end
end
```

APPENDIX B
SWAY FRAME APPLICATIONS

In Webber et.al (2015), it is stated that there is an oversimplification in the assumption that all columns buckle simultaneously within a story, even if they have different stiffness. If the multi-bay frame with an even number of bays is 'folded' into its equivalent one bay frame, it results in a frame with columns of different stiffness' as seen in previous examples. Thus, using an equivalent column stiffness for both columns may be inaccurate and one may need to account for the fact that the stiffer column offers some translational restraint to the less stiff column. Previously, when the assumption that all columns buckle simultaneously was applied, this translational restraint was taken as zero.

Further literature (Yura, 1971) supported this notion that in such frames, the weaker columns are braced by the stronger columns and the initial assumption that all columns within a storey buckle simultaneously is highly conservative. It found that the more 'rigid' column does not reach its full buckling capacity while the weaker column has buckled; some of its capacity can be offered as bracing hence increasing the lateral stiffness.

Various literature have accounted for this effect by proposing a modified effective length factor. The AISC LFRD method (2001) offers a modified effective length ratio to account for this by Eq. (a):

$$\psi'_i = \sqrt{\frac{\Sigma P_u \, I_i}{P_{ui} \, \Sigma \frac{I}{\psi_0^2}}} \geq \sqrt{\frac{5}{8}\psi_{i0}}; \qquad \text{(a)}$$

where ψ'_i is the modified column effective length ratio; ΣP_u is the required axial compressive strength of all columns in a storey; P_{ui} is the required compressive strength of the column of interest; $\Sigma \frac{I}{\psi_0^2}$ is the ratio of the second moment of area to the effective length factor of each column in the storey based on the sway alignment charts found in AISC manual; I_i is the second moment of area of the column of interest; ψ_{i0} is the unmodified effective length factor for the column.

The approach of using a modified effective length ratio, was applied to the two story two bay frame as its equivalent one bay frame resulted in column of different stiffness' as seen in Figure B.1.

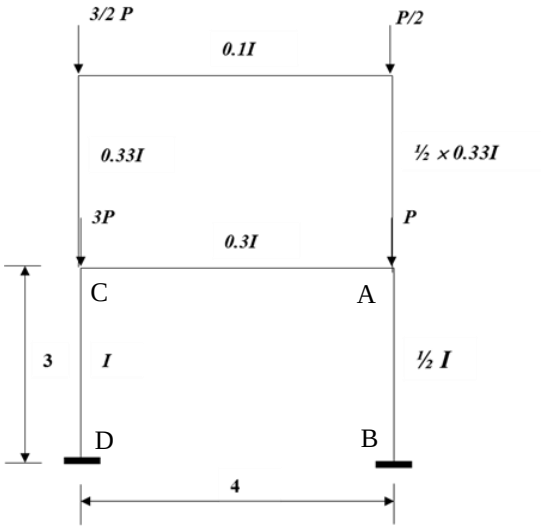


Figure B. 1 Equivalent one bay frame of the two story two bay frame.

The stiffer column *CD* offers some lateral restraint to column *AB* of the first story hence *AB* is the column of interest.

The effective length factors from the sway alignment charts in the AISC manual is obtained by first calculating the ratio of *I/L*, *G*, of all columns to the beams at a rigid connection:

$G_D = G_B = 0$ for a fixed base support.

$$G_A = \frac{\frac{0.5\times 0.33I+\frac{I}{2}}{3}}{\frac{0.3I}{4}} = 2.95 \rightarrow \psi = 1.32$$

$$G_C = \frac{\frac{0.33I+I}{3}}{\frac{0.3I}{4}} = 5.9 \rightarrow \psi = 1.174$$

$$\psi'_{AB} = \sqrt{\frac{6\text{P}}{1.5P} \times \frac{\frac{I}{2}}{(\frac{\frac{I}{2}}{1.32^2})}} = 1.32 \times \sqrt{4} = 2.64$$

Using the effective length factor and the design graphs in Section 3.2.1:

$$\lambda_{AB} = \frac{0.3I\times \mathbf{2.64} \times 3}{0.5I\times 4} = 1.188 \rightarrow \mu = 0.73$$

Assuming that both columns buckle, the upper-bound critical load of the first story is given by:

$$K = \frac{12E\times\frac{I}{2}}{2.64h^3} + \frac{12EI}{1.356h^3} \qquad K_p = \frac{1.5P}{2.64h} + \frac{4.5P}{1.356h}$$

$$P = 2.862\frac{EI}{h^2} \qquad \overline{P_{cr}} = 2.3536\frac{EI}{h^2}$$

$\therefore P_{cr} = 2.3536\frac{EI}{h^2} \times 0.73 \times 0.5 = 0.859\frac{EI}{h^2}$ (1% greater than the FEA result)

The percentage difference from FEA is reduced significantly from the previous approach where an equivalent column stiffness was used.

The approach is applied to another example namely, the five story four bay frame. The first two stories of the optimised frame is shown in Figure B.2.

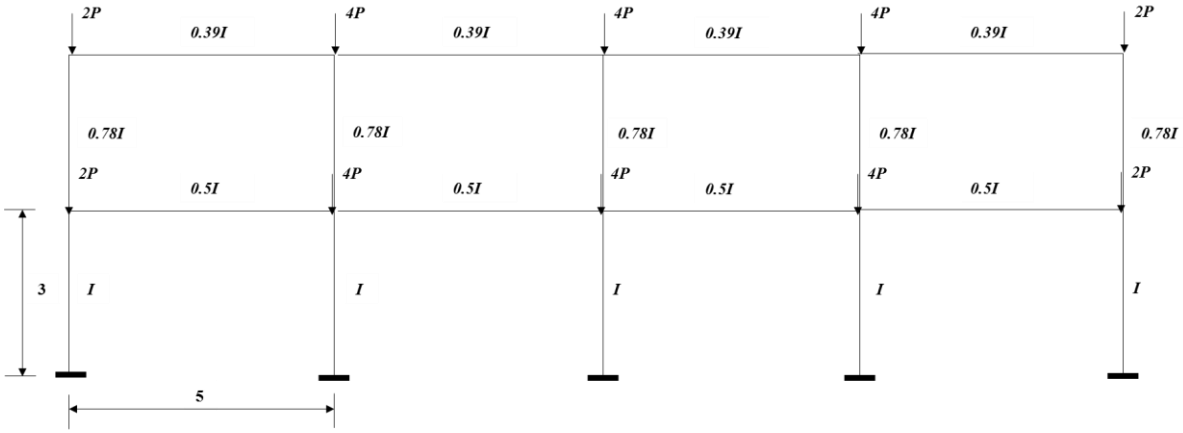


Figure B. 2 First two stories of the optimised five story four bay frame.

These two stories were folded into an equivalent single bay frame, resulting in the frame in Figure B.3.

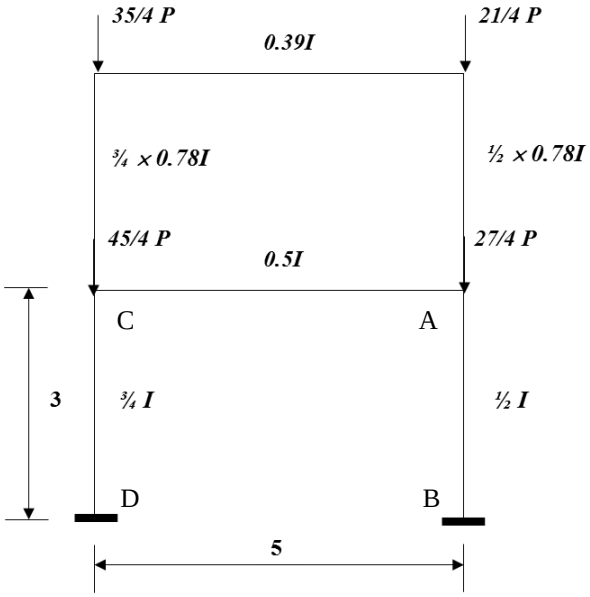


Figure B. 3 Equivalent single bay frame for the first two stories of the five story four bay frame.

The stiffer column *CD* offers some lateral restraint to column *AB* of the first story hence *AB* is the column of interest.

The effective length factors from sway alignment charts in AISC manual is obtained by first calculating the value of *G*:

$G_D = G_B = 0$ for fixed support.

$$G_A = \frac{\frac{0.5\times0.78I+\frac{I}{2}}{3}}{\frac{0.5I}{5}} = 2.96 \rightarrow \psi = 1.325$$

$$G_C = \frac{\frac{0.75\times0.78I+0.75I}{3}}{\frac{0.5I}{5}} = 4.45 \rightarrow \psi = 1.48$$

$$\psi'_{AB} = \sqrt{\frac{\frac{45}{4}\mathrm{P}+\frac{27}{4}P}{\frac{27}{4}P} \times \frac{\frac{I}{2}}{(\frac{I}{1.325^2})}} = 1.325 \times \sqrt{\frac{8}{3}} = 2.1637$$

$$\lambda_{AB} = \frac{0.5I\times\mathbf{2.1637}\times3}{0.5I\times5} = 1.298 \rightarrow \mu = 0.7674$$

Assuming that both columns buckle, the upper-bound critical load of the first story is given by:

$$K = \frac{12E\times\frac{I}{2}}{2.1637h^3} + \frac{12E\times0.75I}{1.872h^3} \qquad K_p = \frac{27/4P}{2.1637h} + \frac{45/4P}{1.872h}$$

$$P = 0.8304\frac{EI}{h^2} \qquad \overline{P_{cr}} = 0.683\frac{EI}{h^2}$$

$\therefore P_{cr} = 0.683\frac{EI}{h^2} \times 0.7674 \times 0.5 = 0.262\frac{EI}{h^2}$ (31% greater than the FEA result)

The percentage difference from FEA has increased significantly and the approach does not hold true for all applications, leading to the need for further investigation into this difference obtained.