GRADE 8 MATHEMATICS TEACHERS' PROFESSIONAL NOTICING OF LEARNERS' THINKING OF COMMON FRACTIONS

A research report submitted to the Wits School of Education, Faculty of Humanities, University of the Witwatersrand in partial fulfilment of the requirements for the degree of M.Ed in Mathematics Education

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2022

ABSTRACT

This study explores how grade 8 mathematics teachers attends, interprets, and decide how to respond to their learners' mathematical thinking. The study also seeks to understand the extents to which mathematics teachers build on the understanding of particular learners on the basis of insights they (the teachers) have gained from the understanding reflected in those learners' strategies. Potentials for mathematics teachers' development in professional noticing was also explored through development activity involving video stimulated recall (VSR) interviews. Professional noticing is conceptualised in terms of providing a window into learners' mathematical thinking and also in terms of learning opportunities for both teachers and learners. I employed Jacob's et al (2010) framework of professional noticing to analyse the nature and extent of teachers' noticing during their lessons, and I conducted VSR interviews with the teachers thereafter to understand teacher's decision-making process as well as to open up for improvement through reflection-oriented questions.

Two grade 8 mathematics teachers in one non-fees paying secondary school in Johannesburg participated. Data was collected through lesson observation and in-depth semi-structured interview with each case study teacher. Findings from this study indicates that the noticing patterns of the case study teachers in this study were predominantly characterised by low-level consideration of learners' solution action. It also emerged that there is a potential for VSR as a tool for supporting the development of mathematics teachers' professional noticing of their learners' mathematical thinking. The implications of these findings for policy and practice on mathematics teaching in South Africa suggests that teacher education and teacher development programs in South Africa need to do more to equip both pre-service and in-service mathematics teachers with better ways of noticing student understanding. Such support as highlighted by Mason (2011) is necessary for in-service mathematics teachers in South Africa to better notice and build on learners' verbal- and written strategy explanations, particularly.

Keywords

Mathematical thinking, noticing, attending, interpreting, deciding how to respond.

DECLARATION

I declare that this Research Report is my own, unaided work, except as indicated in the text and the references. It is being submitted for the degree of Master of Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

Signature:  **Date: March 15, 2022**

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ACKNOWLEDGEMENTS

I am very thankful to God Almighty who brought me thus far and gave me all the opportunities to study at Wits University and to accomplish this research report. I would like to express my profound appreciation to my supervisor, Dr Lawan Abdulhamid for his constructive criticism, guidance and support throughout this research. It is a great privilege to have you by my side walking me through my journey as a novice researcher.

I would like to express my gratefulness to all my coursework lecturers; Professor Judah Makonye, Professor Tony Essien, and Professor Karin Brodie whose thought-provoking seminars broadened my knowledge in mathematics education. This provides a starting place for me to think about research in mathematics education.

I am equally thankful to my employer at ORT South Africa, Mrs Ariellah Rosenburg, who created a work environment that challenge and encourage my desire for professional growth. I am also grateful to my department manager Mr. Mpho Matlala, and to my former supervisor Mrs. Emily Maphosa for their support motivation. I am also grateful to my wonderful former (Ruth Manala, Percy Nkosi and Serufe Molefe), and present (Debbie Lurie, Amini Murinda, Chamu Mawire, Daphne Mabaso, Thenjiwe Sibiya, Clement Moroke, Fanisa Mkansi, Anele Mkungela, Ora Stollar and Julius Nkoana) colleagues. May the good Lord bless you all for making a career in mathematics education a worthwhile endeavour.

A very special thanks to my wife, Tsakane Eunice Olubodun whose support and understanding gave me courage and assurance to pursue this degree, and for her emotional, moral and financial support throughout this study. To my children AnuOluwaladunmi, IreOluwalayomi and AreOluwaleremi whose innocent faith and expectation motivated my desire for further education. May the good Lord endow you with wisdom, knowledge and understanding abundantly. To my mother Christianah Orubu Olubodun, thank you for your daily prayers. I pray that God will grant you more years of grace with us.

Finally, to the two grade 8 mathematics teachers and who participated in this study. I couldn't have completed this research report without your kind support in sharing with me your space and thoughts relating to your learners' solutions. I say a huge thank you.

Table of Contents

List of Figures

List of Tables

List of Abbreviations

Abbreviation	Meaning
DBE	Department of Basic Education
MKT	Mathematical Knowledge for Teaching
PCK	Pedagogical Content Knowledge
TIMSS	Trends in International Mathematics and Science Study
VSR	Video Stimulated Recall

CHAPTER 1: INTRODUCTION

1.1 Background to the Study

Research evidence shows that mathematics learners at all levels in South Africa demonstrate difficulties in fractions (Basbozkurt, 2010; Makonye & Khanyile, 2015; Maseko, 2015; Brijlall, Maharaj, & Molebale, 2011). Analysis of the "Trends in International Mathematics and Science Study" (TIMSS, 2015) for South African data show that learners' mean performance in fractions stands at 24.6%., and that common fractions and decimals constitute about 50% of the questions on number concept in grade 8 (Diedericks, 2006). Learner's poor performance in South Africa is often attributed to teachers' poor knowledge for teaching mathematics (Venkat & Spaull, 2014) and/or their classroom practices (Arends, Winnaar, & Mosimege, 2017; Muller & Hoadley, 2019). This study therefore focuses on exploring grade 8 teachers 'professional noticing of learners' mathematical thinking.

The focus on professional noticing of learners' mathematical thinking emanate from the fact that teaching that is aligned to reform approach (Kilpatrick, Swafford, & Findell, 2001) seeks to understand students' mathematical thinking and to nurture its development by being responsive to learners' mathematical productions in the learning situations (Wessels, 2018). This suggests that attention to learners' mathematical thinking should extend beyond noticing of errors and misconceptions to also noticing of correct and efficient responses. This can only be attained through teachers' practices of "on-line checking for understanding and misunderstanding" (Shulman, 1987, p. 18) and responding "in the moment" (Rowland, 2013). These pedagogical practices involve teachers' curiosity about, listening to, and employment of professional noticing of the details of the mathematical ideas their learners say, do, or other forms of utterances.

Professional noticing is essential for responsive teaching. Professional noticing suggests that evidence from the learning process inform how the teaching is adapted. At the forefront of professional noticing is the promotion of students' learning in the learning situation. Professional noticing creates greater opportunity for the teacher to provide insightful feedback on the learning activities. Central to the practice of professional noticing is the instructional adaptation that supports student improvement and enhance their mathematical power. Studies show that different levels of opportunities are afforded to learners as a result of the teacher's level of professional noticing (Jacobs, Lamb, & Philipp, 2010).

Davis' (1997) three types of listening; "evaluative listening, interpretive listening, and hermeneutic listening" finds relevance with professional noticing expertise associated with teachers' response to learners' mathematical thinking. In evaluative listening, the teacher use information from learners' offers to make a judgment founded on being right or being wrong. Thereafter the teacher continues with the lesson action as planned. In interpretive listening, the teacher's use of the information demonstrate attempt to understand the learners' mathematical thinking "from the point of view of the learner" (p. 361). Thereafter the teacher modifies the lesson but with no authority given to the learners. In hermeneutic listening, the teacher permits the information from learners' mathematical thinking to influence the direction of instruction. The teacher and learners become co-participants. The hermeneutic listening and action illuminate an effective professional noticing. In this study, I conceptualise the type of listening as one of the social and cultural factors through which teachers engage in professional noticing.

Noticing is a subjective activity, research in noticing points to a range of noticing expertise. For example, Wallach & Even (2005) propose four categories: describing, explaining, assessing, and justifying" (p. 410), suggesting that "the nature of teacher[s'] hearing and interpretation of what students are saying, showing, feeling, and doing, while engaged in mathematics problem solving" (p. 410). Effective teacher response may be influenced by the teacher's type of listening (Davis, 1997). It is noted that teachers 'hear through' social and cultural factors leading to four kinds of hearing, namely: "over-hearing, compatible-hearing, under-hearing, non-hearing, or biased-hearing" (p. 410). Considering types of listening as one of the social and cultural factors through which teachers hear their learners, there is reason to suggest that hearing (Wallach & Even, 2005) is shaped in part by listening (Davis, 1997) and listening is partly shaped by hearing.

The complexity of professional noticing is further heightened by the difficulty to notice everything that happen in the class even if the teacher is willing to. This is due in part to the fact that hearing what children are saying surpasses disposition or hearing capability (Ball, 1993), as noticing is influenced by culture. For example, after watching a videotaped lesson in a comparison study, Chinese teachers were found to focus on the mathematical content, while their American counterparts focused on pedagogical aspects and the personalities of the teacher (Jacobs, Lamb, & Philipp, 2010). Above all, professional noticing requires a "knowledge of what is mathematically significant", so teachers need the "skill in finding those mathematically significant indicators in children's messy, and often incomplete,

strategy or explanations" (Jacobs, Lamb, & Philipp, 2010, p. 195). Three components of professional noticing skill described by Jacobs et al (2010) are: "attending, interpreting, and deciding how to respond on the basis of children's understandings" (Jacobs, Lamb, & Philipp, 2010, p. 195). According to this understanding, these three components precede the eventual cause of pedagogical action (Shulman, 1987) to be taken by the teacher, and there are four distinct expertise in each component. In this study, I therefore conceive professional noticing based on Jacob's et. al's three components.

Jacobs et al (2010) argue that teachers' professional noticing expertise can be developed over time, citing the effectiveness of "lesson-analysis framework as a means for helping prospective teachers gain expertise in observing and reasoning about classroom events" (p. 171). Analysis of classroom videos as an integral part of lesson-analysis framework has been successfully adopted in teacher professional development projects, since the recorded videos make classroom events visible and open for analysis. In video stimulated recall (VSR), teachers watch a video of their own teaching and engage in a collaborative discussion with an expert to reflect on what transpired in the lesson enactment. According to Geiger, Muir, and Lamb (2016), VSR comprise cycles of professional learning involving researchers and teachers. Watching a video of teacher's classroom teaching together by the teacher and researcher, the researcher prompts the teacher, based on a pedagogical situation in the video, who then reflects and scrutinises her/his actions in the episode of teaching. This is followed by discussion in which the researcher probes alternative pedagogies from the teacher. The next cycle follows after the teacher has experimented the pedagogies as discussed in the previous cycle. Contrasting VSR to self-reflection, Geiger, Muir, and Lamb (2016) argues that teachers' efforts to improve their practice are attributed to a process of reflection on self as seen by others. It is because of the power of VSR that I intend to employ in this study as an intervention to support the development of teachers' professional noticing of learners' mathematical thinking.

Against this background, this study therefore aims to explore in-depth, grade 8 mathematics teachers' professional noticing of learners' mathematical thinking and bringing these to teachers' awareness in a context of VSR. This includes noticing of learners' errors and misconceptions as well as emergent strategies and conceptions that learners may present (learner's strategies) in the course of classroom interactions. The aim is to unpack the nature and extent of teachers' professional noticing of learners' mathematical thinking and how they

use learners' conception as a guide to inform next steps of instruction in their teaching of common fractions at grade 8 level.

1.2 Problem statement

Broadened view of mathematics has gained prominence in countries worldwide. One emphasis has been the view of the development of learners' "mathematical power" (Kilpatrick, Swafford, & Findell, 2001). In South Africa, for example, there is emphasis on teaching that support the development of learners' logical and critical thinking (Department of Education, 2011). However the link between this view and the form of mathematical experience offered to learners in the classroom is not a straight forward one. Ernest (1991) noted that social contexts of schooling constitute either a source of support or constrain to productive mathematics instruction in the classroom.

In South Africa, mathematics teachers in previously disadvantaged schools are confronted with pressure to complete the curriculum as evidenced by scripted lesson plans and monitoring tools used by education officials. This pressure has a potential to limit opportunities for effective teacher professional noticing. Recently, Brodie (2017) cautioned against this prescriptiveness, noting that mathematics teachers might not have space to engage meaningfully with their learners in the classroom. Nevertheless, there is a need to support teacher development "conceiving and demonstrating a successful pedagogy that engenders learning in those specific contexts" (Muller & Hoadley, 2019, p. 123).

One of the cornerstones of successful pedagogy is responsive teaching. Responsive teaching seeks to understand students' mathematical thinking and to nurture its development by being responsive to learners' mathematical productions in the learning situations. This way, teachers take up learner's offer (Wessels, 2018) and support the learner to develop his or her understanding further. Teachers need development of three key competencies in professional noticing as part of his or her responsive teaching repertoire namely; "attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (Jacobs, Lamb, & Philipp, 2010, p. 195).

Looking at the competing demands on teachers' instructional practices in the context of responsive teaching, the present study is aimed at exploring grade 8 mathematics teachers' practices of professional noticing of learners' mathematical thinking, and the development of these practices in a context of VSR interviews.

1.3 Purpose of the study

The purpose of this study is to explore how grade 8 mathematics teachers notice their learners mathematical thinking of common fraction. Furthermore, the study also explores the development of noticing expertise of the teachers in a context of VSR interviews.

1.4 Research questions

The central question that framed this study is:

What is the nature and extent of grade 8 mathematics teachers' practices of professional noticing of learners' mathematical thinking, and how does this transform in the context of interim "video-stimulated recall interviews"?

In answering this research question, the following sub-questions were interrogated:

1. How do grade 8 mathematics teachers attend to learners' strategies in the teaching of common fractions?
2. How do grade 8 mathematics teachers interpret learners' strategies and actions in the teaching of common fractions?
3. On what basis do grade 8 mathematics teachers respond to a noticeable learners' strategies and actions in the teaching of common fraction?
4. What shift (if any) in noticing skills is evident in the grade 8 mathematics teachers in the context of an intervention using video-stimulated recall interviews?

1.5 Significance of the Study

Teachers who appreciate their learners as thinkers are more likely to develop their skills in professional noticing and engaging errors and misconception (Brodie, 2014)in mathematics teaching and learning. The potential advantage of this skills is its contribution to highlighting hard to notice classroom elements that are often taken for granted even though they may be more readily engaged. It is hoped that this study will provide valuable trajectories for knowledge growth for teaching, and if the tenets of this concepts are well implemented, responsive teaching will become commonplace in our classrooms.

CHAPTER 2: LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

2.1 Introduction

This chapter locates the present study into the relevant literature and theory on teacher noticing, the concept of common fractions, and video-stimulated recall. The chapter therefore is organised into three distinct bodies of writings. The first part deals with the construct of noticing, with particular emphasis on mathematics teacher noticing in the context of responsive teaching. The second part deals with the concept of common fractions with particular emphasis on learner's mathematical thinking of common fractions. The third part deals with video-stimulated recall interview with a specific focus on reflective awareness practices. The chapter concludes with a conceptual framework that summaries the key concepts that are discussed in the literature and theory on mathematics teacher noticing.

Research interests in listening to learners' utterances and production derives from continual demonstrations by Piaget that children's ways of thinking about the world differ from those that adults' foregrounds (Smith, diSessa, & Roschelle, 1983). Whereas the significance of learners' mathematical ideas as a substance for their construction of mathematical knowledge has been promoted in literature (Borasi, 1994; Ball, 1993), researchers conceptualize the construct of responsive teaching in terms of formative assessment (Wiliam, 2014; Fletcher-Wood, 2018), which requires a combination of teachers' reflection in action, and reflection on action (Wessels, 2018; Schön, 1987). In support of this argument Fletcher-Wood (2018) describes responsive teaching as a blend of planning and teaching, with formative assessment, and consequently adapting teaching in accordance with students learning as revealed in the ideas presented by learners. So, reflection and evaluation are integral parts of responsive teaching.

For some researchers, responsive teaching is about how teachers respond, in the moment, to unplanned contingent events that emerge in the classroom. As an instructional practice in which learners ideas are built into instruction as they emerge in the moment of classroom instruction (Robertson, Scherr, & Hammer, 2015; Fletcher-Wood, 2018; Rowland, 2013), there is a wide gap between research and practice of responsive teaching internationally compared with South Africa (Abdulhamid, 2016). While the concern in international literature is about opportunities offered for learning by teacher's practice of responsive teaching, in South Africa the concern is about professional noticing of learners' offer and of building learners' ideas into instruction.

2.2 Teaching and learning of common fractions

Rational numbers have been identified as the most problematic number system for elementary and middle school mathematics learners (Kilpatrick, Swafford, & Findell, 2001). Data from Diagnosis Report (Department of Basic Education, 2019) of learners' performance in mathematics at the NSC shows that learners have difficulties in solving problems of mathematical concepts such as functions, patterns, calculus, analytical geometry and Euclidean geometry. Whereas the concept of common fraction is not directly identified as a challenge to learners' mathematics performance in the National Senior Certificate (NSC) examination at the end of their Grade 12-year, research shows that these challenges are more prominent when the problem involves rational expressions in these concepts (Makonye & Khanyile, 2015). In support of this argument, Kilpatrick, Swafford, and Findell (2001) noted that learners' proficiency in common fractions foregrounds their proficiency in rational expressions in algebra.

Algebra was identified as one of the common challenges to learners' mathematics performance in the National Senior Certificate (NSC) examination at the end of their Grade 12-year. Research shows that learners' proficiency in common fractions foregrounds their proficiency in rational expressions in algebra (Kilpatrick, Swafford, & Findell, 2001). Hence, a focus on common fraction at grade 8.

2.2.1 Difficulties with teaching and learning of common fraction

Difficulties associated with the teaching and learning of common fraction is well documented in literature (Kilpatrick, Swafford, & Findell, 2001; Cramer, Wyberg, & Leavitt, 2009). Kilpatrick, Swafford, and Findell (2001) linked the difficulties with the teaching and learning of common fractions to two main factors. These are: disparity between learners' informal knowledge of common fraction and school knowledge of common fraction; and the complex relationship between whole number concept and common fraction. The former relates to a recognition that learners' informal knowledge of common fraction does not likely go beyond sharing and partitioning, which is not sufficient for appropriating school knowledge of common fraction. Concerning the latter, it is noted that learners' superficial knowledge of whole numbers has a strong tendency to interfere with their learning of common fractions, since learners tend to apply the same procedures they have invented for working with whole numbers when they transition to common fractions.

Likewise, Cramer, Wyberg, and Leavitt (2009) detailed four sources of this challenges, namely; learners find it challenging to internalise the symbolic representations of common fraction as a single entity, learners' conception of the inverse features of a common fraction is usually inhibited by their whole number ideas, conceptual understanding of common fraction equivalence is not easily developed by learners, learners' development of addition and subtraction of common fraction is constrained by weak conceptual understanding of common fraction.

The aforementioned factors have been found to constrain learners from formulating proper ideas to add, subtract, multiply, divide or order common fractions as is the case with whole numbers. The implication is that learners need more support to extend their limited ideas, developed both from informal knowledge and from whole number concept, in ways that make stable connections with school knowledge of common fraction. To do this, mathematics teachers need to pay attention to, and make sense of, those ideas in order to make instructional decisions that will support intellectually appropriate meaning making of the specific learners based on their ideas. The strength of these difficulties with teaching and learning of common fraction derives from the complex nature of common fractions.

2.2.2 Structure of school knowledge of common fraction

Structurally, a single common fraction is made up of two numbers, that is, a numerator and a denominator, and can be represented with many different written symbols. For example, $\frac{2}{5}$ can be written as $\frac{12}{30}$, or any other equivalence of that, in the same way that $\frac{7}{5}$ can be written as $1\frac{2}{5}$ or as $\frac{5}{5}+\frac{2}{5}$. This composition has been linked to one of the difficulties associated with the teaching and learning of common fraction in that learners do not often see common fraction notation as a single entity.

The relationship between the two numbers is another challenge attributed to the structure of common fraction, since learners often project an additive relationship between numerator and denominator of a common fraction. We see this when learners simply add the same value to both the numerator and denominator in a common fraction in an attempt to generate an equivalence of the initial fraction. In support of this argument, common fractions along with ratios, proportions, decimals, and percents, has been described as constituents of a particular conceptual field connected by multiplicative structures (Vergnaud, 1983 cited in Kilpatrick, Swafford, & Findell, 2001), which has been reported as a central element of the challenge in

the teaching and learning of common fraction. Drawing on Lesh, Post, and Behr (1988) who contend that proportional reasoning is crucial to elementary school arithmetic, and that it also serves as a precursor to further mathematical concepts, namely; algebra, geometry, probability, statistics, and certain aspects of discrete mathematics, Kilpatrick, Swafford, and Findell (2001) describe two dimensions of thinking of common fraction, namely; absolute terms and relative terms. In relative thinking, the numerator and denominator are related through multiplication and division. In contrast, in absolute terms, the numerator and denominator are related through addition. It is noted that learners' thinking of common fractions in absolute terms can be instrumental to their entrance to common fraction, it can become detrimental to their further development of common fraction concepts, especially their development of proportional reasoning. The understanding from this perspective is that learners will find it problematic to develop fraction concept in a context of poor proportional reasoning.

How a common fraction can be interpreted is yet another challenge emanating from its structure. Diverse interpretations for any common fraction say $\frac{2}{5}$, have been reported in literature on rational number structures (Kieren, 1976, 1980, 1988 cited in Kilpatrick, Swafford, & Findell, 2001). These are: (i) a part-whole relation, where $\frac{2}{5}$ is interpreted as 2 out of 5 equal-sized shares; (b) a quotient, where $\frac{2}{5}$ is interpreted as 2 divided by 5; (c) a measure, where $\frac{2}{5}$ is used to describe $\frac{2}{5}$ of the way from the beginning of the unit to the end; (d) a ratio where $\frac{2}{5}$ represents description such as 2 red cars for every 5 green cars; and (e) an operation that enlarges or reduces the size of something such as $\frac{2}{5}$ of 45. This diverse interpretation presents challenge for learners to develop a sound concept of common fraction since some interpretations may be fit for the purpose of analogy as an entrance to common fraction, but not suitable for further development of common fraction concepts. It is therefore essential to pay attention to learners' construction of both the distinctions as well as the relationships among these interpretations.

As with the written notations, spoken words used for common fractions has been identified as another aspect of challenge emanating from the structure of common fraction. For example, $\frac{2}{5}$ is often read in English as two fifths similar to describing order of items or occurrence such as the fifth day, or the fifth from the right. The implication is that interpretation that is based

on these spoken words has the potential to fortify learners' continual use of their incomplete whole number ideas in their development of common fraction concepts.

Considering the complexity of developing common fraction concepts in a context of limited informal knowledge of common fraction, coupled with the abovementioned challenges due to the structure of school knowledge of common, the focus of this study therefore, is to explore how mathematics teachers pay attention to, and act on, the meaning making of their learners as reflected in the strategies employed by those learners when they solve problems on common fractions.

2.3 Professional noticing

2.3.1 Professional noticing in relation to responsive teaching

A key focus in reform-based mathematics teaching is developing mathematics teachers' noticing expertise, particularly expertise in professional noticing of learners' mathematics thinking. Professional noticing enables teachers to put off evaluative judgement of their learners' verbal or written solutions, and to become more attentive to the details in learners' mathematical ideas in the learning situation. In other words, professional noticing underpins teachers' responsiveness to learners' ideas in the moment by being curious about the ideas that learners bring to the learning situation. Being responsive means to actively recognise and work with the knowledge that learners bring to the learning situation, in a context of intellectually demanding mathematical content Lampen and Brodie (2020).

A view of responsive teaching presupposes the existence of a teacher's sound knowledge for teaching including Pedagogical Content Knowledge (PCK) (Shulman, 1987) and their proficiencies in the different domains of mathematics knowledge for teaching (MKT) (Ball, Thames, & Phelps, 2008). In addition to knowledge for teaching, studies with a focus on supporting mathematics teachers' practice of developing learners' mathematical idea identify a range of attributes for responsive teaching, including, listening in order to develop learners' mathematical thinking (Davis, 1997), different types of hearing that occur in the classroom (Wallach & Even, 2005), engaging learners' errors and misconceptions (Borasi, 1994; Brodie, 2014; Makonye & Khanyile , 2015). In responsive teaching, teachers' activity of drawing on their knowledge for teaching to hear, to listen, and engage with learners' ideas, including their errors and misconception in the teaching and learning situation is an embodiment of professional noticing of learners' mathematics thinking.

2.3.2 Professional noticing of learners' mathematical thinking

Research interest on professional noticing of learners' mathematical thinking has been spurred by an understanding that it is neither practicable for a teacher to notice nor respond to everything that happens in a learning situation (Jacobs, Lamb, & Philipp, 2010, p. 195), and that it is reasonable to expect that teachers selective attention is determined by consideration of next instructional steps in that teachers judiciously direct their attention to what is necessary to take action pedagogical commitment – what is most salient to him/her (Erickson, 2011). As a result, the skill to identify and pay attention to noteworthy aspects of a lesson becomes a critical element of noticing (Wessels, 2018). In support of this argument, Jacobs et al. (2010) argues that mathematics teachers must have the "knowledge of what is mathematically significant and [be skilful in finding] mathematically significant indicators in children's messy, and often incomplete, strategy explanations" (p. 194). The complexities and subjective nature of noticing is attributed to several factors namely, teacher's Pedagogical Content Knowledge (PCK) (Shulman, 1987), Mathematics Knowledge for Teaching (MKT) (Thomas, Jong, Fisher, & Schack, 2017), social and cultural factors (Wallach & Even, 2005; Jacobs, Lamb, & Philipp, 2010), mathematical philosophies of the teacher (Ball, 1993; Davis, 1997), the predominant mathematical philosophy of the educational context (Ernest, 1991).

As a result of the aforementioned complexities and subjective nature of noticing, professional noticing resonates well with a diverse range of reflective practices described by several researchers, namely, a process of decision-making that is goal-oriented, "sizing up of students' ideas and responding" and "discipline and an intentional systematic set of practices" (Mason, 2002; Wessels, 2018). These descriptions reveal that noticing is conceptualised severally as a practice, as a process, and as a method and that it is specific to a subject matter and aimed at a specific goal. This is tantamount to Shuman's (1987) notion of pedagogical reasoning and action. In this study, I adopt Shuman's (1987) description which entails a "process [of] on-line checking for understanding and misunderstanding that a teacher must employ while teaching interactively".

Whereas the relevance of professional noticing is in the moment decision making process during the teaching situation, teachers' expertise in noticing is usually developed through reflective practices in retrospect. Rowland's (2013) notion of knowledge quartet, described as "the classification of the situations in which mathematical knowledge surfaces in teaching" (p. 22). This framework recognizes four dimensions of knowledge namely; "foundation,

transformation, connection and contingency" (p. 21). It provides a window into classroom events that are mathematically significant. Described as "the ability to make cogent, reasoned and well-informed responses to unanticipated and unplanned events" (p. 25), contingency embodies significant teacher moves namely; "deviation from agenda, responding to students' ideas, (use of opportunities), teacher insight during instruction, responding to the (un)availability of tools and resources" (p. 25). Contingency finds relevance with this study, not because of hierarchy, but due to the fact that professional noticing plays out "in-the-moment". Of particular interest in contingency is the dimension of responding to students' ideas, described as the cornerstone of a lesson.

Following from reported poor knowledge of teaching and classroom practices of mathematics teachers in South Africa, Abdulhamid (2016) argues that the dimensions of responses to 'in-the-moment' contingencies documented in international literature are far from attainable in the South African mathematics classroom, and proposes the notion of elaboration framework. Elaboration framework identifies three "triggers of elaborations" namely; "breakdown, sophistication, and individuation/collectivisation". According to Abdulhamid, triggers of elaborations are highlights of "important stages of implementation of responsive teaching action" (p. 244), with a potential to advance towards "more responsive teaching in [the South African] context" (Abdulhamid, 2016, p. 236).

2.3.3 Mathematically significant details of common fraction in the senior phase

Knowledge to pick up mathematically significant – "knowledge of what is mathematically significant and skill in finding those mathematically significant indicators in children's messy, and often incomplete, strategy explanations" (Jacobs, Lamb, & Philipp, 2010, p. 194). For the purpose of this study, a lens of theory on teaching and learning as articulated in section 2.2 above on "teaching and learning of common fraction" is used to identify aspects of learner's solution action that is considered mathematically significant. Consequently, the followings are considered mathematically significant: considering that common fraction has multiplicative structure, how learners construct relations between numerator and denominator is noted as mathematically significant; how learners construct relations among different interpretations of common fractions – including for example eighth as a variable because we use 3 eighths, and consequently eighth is not translatable to fourth or to sixteenth; how learners construct relations among different representations of common fractions - including but not limited to mixed fractions, improper fractions, equivalent fractions; learners general computational fluency of fractions (Multiplication, Division, Addition, Subtraction); and

learners' general computational fluency of whole numbers (Multiplication, Division, Addition, Subtraction).

2.4 Video stimulated recall (VSR)

Reflection and evaluation are integral parts of effective teaching (Smith, diSessa, & Roschelle, 1983; Hamza & Wickman, 2008). Video stimulated recall, with a focus on learner reasoning, provide a site "for reflection and the development of teacher professional noticing and sense-making" (Jacobs, Lamb, & Philipp, 2010; Wessels, 2018). Jacobs et al (2010) argue that teachers' professional noticing expertise can be developed over time, citing the effectiveness of "lesson-analysis framework as a means for helping prospective teachers gain expertise in observing and reasoning about classroom events" (p. 171).

Analysis of classroom videos as an integral part of lesson-analysis framework has been successfully adopted in teacher professional development projects, since the recorded videos make classroom events visible and open for analysis. In video stimulated recall (VSR), teachers watch a video of their own teaching and engage in a collaborative discussion with an expert to reflect on what transpired in the lesson enactment. According to Geiger, Muir, and Lamb (2016), VSR comprise cycles of professional learning involving researchers and teachers.

Watching a video of teacher's classroom teaching together by the teacher and researcher, the researcher prompts the teacher, based on a pedagogical situation in the video, who then reflects and scrutinises her/his actions in the episode of teaching. This is followed by discussion in which the researcher probes alternative pedagogies from the teacher. The next cycle follows after the teacher has experimented the pedagogies as discussed in the previous cycle. Contrasting VSR to self-reflection, Geiger, Muir, and Lamb (2016) argues that teachers' efforts to improve their practice are attributed to a process of reflection on self as seen by others.

This study aims to examine mathematics teacher's practice of reflection-in-action demonstrated through their professional noticing of learners during the teaching situation. It also aims to examine in what ways, if any, the process of reflection-on-action through video stimulated recall coupled with prompting, will influence their practice of reflection-in-action in future teaching situation. The aim of the VSR interviews is to gain insight on teachers'

decision-making process and to get teachers to reflect on their practice towards improvement in their professional noticing.

By providing reflection-oriented questions and prompts, researchers are able to obtain responses from participating teachers in order to capture the degree to which the teachers demonstrate evidence of engagement with learners' mathematical thinking. Analysis of the participant's comment on the teaching situation, in response to the video and the prompt will be done by the researcher.

Against the forgoing background, this study therefore aims to explore in-depth, grade 8 mathematics teachers' professional noticing of learners' mathematical thinking and bringing these to teachers' awareness in a context of VSR. This includes noticing of learners' errors and misconceptions as well as emergent strategies and conceptions that learners may present (learner's strategies) in the course of classroom interactions. The aim is to unpack the nature and extent of teachers' professional noticing of learners' mathematical thinking and how they use learners' conception as a guide to inform next steps of instruction in their teaching of common fractions at grade 8 level.

2.5 Conceptual framework

This study is guided by Jacobs, Lamb, and Philipp's (2010) framework of professional noticing of children's mathematical thinking. According to these authors, this framework is grounded by three key principles namely; the need to identify a 'focus for noticing', attention to 'how', and 'extent' of teachers noticing of learners' mathematical thinking.

Jacobs et al (2010) recognizes teachers' expertise in "professional noticing of children's mathematical thinking" as the focus of noticing and describe three components of this expertise namely; "attending to children's [learners'] strategies, interpreting children's [learners'] understandings, and deciding how to respond on the basis of [learners'] understandings" (p. 192).

By attending to learners' strategy, the emphasis is on 'how' teachers attend to noteworthy aspects of complex instructional situations, and the 'extent' to which teachers attend to a particular aspect of instructional situations, in particular the mathematical details in learners' strategies. It is noted that regardless of the degree of intelligibility of learners' strategies, the details of those strategies are essential as they afford opportunities to gain insight into learners' understandings. Jacobs et al (2010) noted that teachers' expertise in children's mathematical thinking is measured by their ability to recall those details of learners'

strategies. Central to this expertise is teachers' development of skills to discriminate traits and synthesize evidence in difficult instructional situations.

Interpreting learners' understandings entails "how teachers interpret" learners' understandings as revealed in the details of their work. Whereas it may not be possible for a teacher to build sufficient depiction of a learner's understandings after encounter with a single situation, however the emphasis is on the degree of consistencies of the teacher's "reasoning with the details of the specific learner's strategies" together with related theories on how learners' "mathematical development".

Deciding how to respond on the basis of learners' understandings is about what informs the teachers the reasoning when deciding how to respond. Whereas the goal is not to judge the teacher's chosen response but the emphasis is on how much the mathematical details in learners' strategies informs the teacher's reasoning and whether the reasoning is associated with theory on learners' "mathematical development". Whereas Jacobs et al (2010) noted that the execution of the response has not been included in their conceptualization of professional noticing, choosing rather to focus on intended responding. However, in this study, because of the situated nature of my study and with a focus on teachers' response in-the-moment of classroom, I have included the execution of the response. The teacher's actual practice of professional noticing is measured through their observed responses demonstrated through their teaching actions subsequent to the solution actions of their learners. The teachers' reflective awareness of their teaching action was also measured to explore possible growth in their future professional noticing actions. The focus on intended responding is equally included in this study because of its significance for the teacher development through VSR interview. I will now turn my attention to the objects of noticing.

My conceptualisation of object of noticing in this study is guided by Abdulhamid's (2016) elaboration framework. As noted by Abdulhamid, the dimensions of responses to 'in-the-moment' contingencies documented in international literature are far from attainable in the South African context. This position is echoed by Muller and Hoadley (2019) who noted that the features of an effective pedagogy in the majority of South African schooling contexts is not yet known. Abdulhamid (2016) proposed three situations of professional noticing within primary mathematics teaching landscape. These are: incorrect learners' mathematical thinking, correct but inefficient learners' mathematical thinking, and correct and efficient learners' mathematical thinking. These three situations form the unit of analysis for

Abdulhamid's (2016) study. I conceived this in the same way as to provide me with the situations for understanding teachers' professional noticing of learners' mathematical thinking. I will now turn my attention to the levels of noticing.

My conceptualisation of levels of noticing in this study is guided by Rowland's (2013) notion of how teachers respond to contingency situations within the knowledge quartet framework. The focus of contingency is on the logical, intelligible and well-informed responses that teachers execute following unanticipated and unplanned events. Rowland's (2013) notion of contingency embodies significant teacher moves namely; "deviation from agenda, responding to students' ideas, (use of opportunities), teacher insight during instruction, responding to the (un)availability of tools and resources" (p. 25). Through this framework teachers noticing will be delineated according to three levels namely; limited noticing, basic noticing and robust noticing.

Jacobs et al (2010) argue that teachers' professional noticing expertise can be developed over time, citing the effectiveness of "lesson-analysis framework as a means for helping prospective teachers gain expertise in observing and reasoning about classroom events" (p. 171).

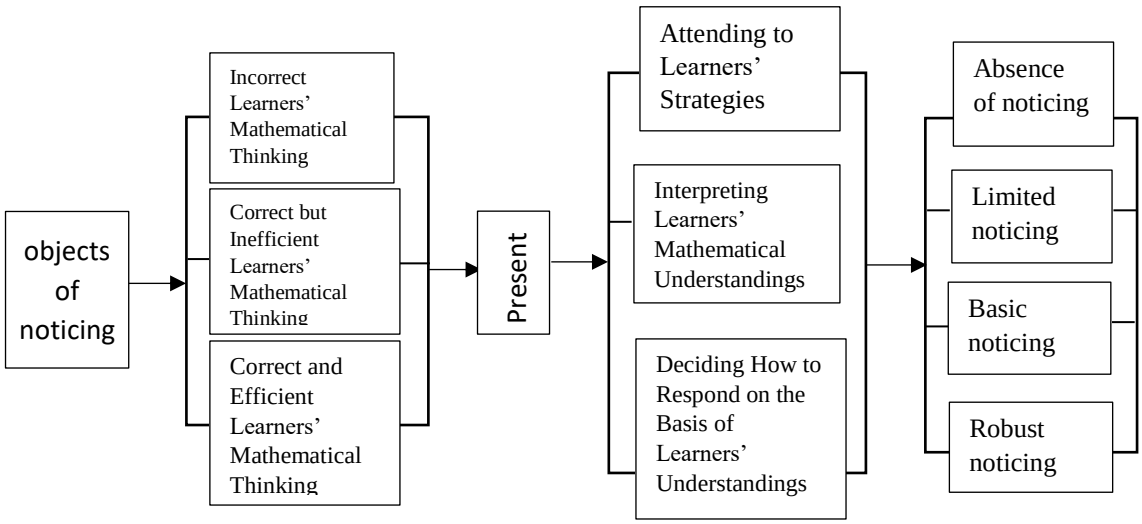

Figure 1: Conceptual framework used in the study – Adapted from Jacobs, Lamb, and Philipp (2010

Figure 1 illustrates how professional noticing of learners' mathematical thinking is conceptualised in this study. This conceptualisation consists of three components namely; object of noticing, skills of noticing, and levels of noticing. 'Skills of noticing' refers to set of skills inherent in professional noticing as a discipline. The skills are: "attending" to

learners' strategies, "interpreting" learners' understandings, and "deciding how to respond on the basis of learners' understandings". 'Object of noticing' refers to what to notice about the learners' mathematical thinking. In essence the mathematical significance of what is notice is located in the object of noticing. Three objects of noticing are considered in this study, namely: incorrect learners' mathematical thinking, correct but inefficient learners' mathematical thinking, and correct and efficient learners' mathematical thinking. Levels of noticing refers to the degrees of expertise that is demonstrated by the teacher in his or her response to the object of noticing.

2.6 Conclusion

From the forgoing, I have highlighted that teachers who appreciate their learners as thinkers are more likely to pay attention to and build on the ideas their learners bring to the learning situation, considering that learners bring both informal knowledge as well as knowledge of other concepts that may or may not conform with the school knowledge of mathematics to be developed in the current situation. develop their skills in professional noticing and engaging with errors and misconceptions. The study is conceptualised by how teachers attend to learners' strategies, interpret learners' understandings, and decide "how to respond on the basis of learners' understandings" as a framework for supporting teacher's development of professional noticing.

The potential advantage of this skills is its contribution to highlighting hard to notice classroom elements that are often taken for granted even though they may be more readily engaged. It can provide valuable trajectories for knowledge growth for teaching, and if the tenets of this concepts are well implemented, responsive teaching that promote learning will become commonplace in our classrooms.

CHAPTER 3: RESEARCH METHODOLOGY

3.1 Introduction

This chapter provides the methodology used in conducting this research. It is composed of the research resign, context and participants, data sources, data analysis, rigour and the ethical considerations

3.2 Research Design

The study employed a qualitative case study approach. A case study permits for a detailed exploration of a phenomenon bounded by a closed system (Merriam, 1998). Through case study, researchers are able to obtain much rich and exhaustive description of human behaviour and experience when compared with quantitative studies (Dayer, 1995). Therefore, a case study is commonly employed for research in social sciences (Yazan, 2015). A core attribute of case study is the case. Merriam (1998) describes a case as "a thing, a single entity, a unit around which there are boundaries" (p.25). In this study, the case is the teacher's competencies for professional noticing of learners' mathematical thinking. The bounded system in this study is the exploration of participants' professional noticing of learners' mathematical thinking.

3.3 Context of the study and participants

This study was conducted in a public non-fees secondary school that was categorised as underperforming. The school is located in Johannesburg. The school was conveniently selected because of my personal relationship with the school and the teachers' willingness to participate in the study. Two grade 8 teachers were purposively selected and participated in this study. In the selection of these two teachers, an information-oriented selection strategy (Flynbjerg, 2006) was employed. This is a strategy for the selection of participants that maximized the utility of information from small samples. Firstly, I consider teachers with mathematics teaching qualification. Secondly, I consider teachers with more than 5 years of teaching experience at grade 8 level. It is assumed that they are much familiar with the content areas of grade 8, and that they may possibly notice and attend to mathematically significant situations in the course of teaching.

3.4 Data sources

Data for this study was gathered from two sources. They are: video recording of observed lessons, and video stimulated recall interviews (VSR) with each teacher. In the first phase, the researcher observed and video recorded three sequences of lessons on common fractions

for each participant. A preliminary analysis of the lessons focusing on how the teachers "attend, interpret and decide how to respond" to their learners "mathematical thinking" was conducted. Critical incidents were identified from this analysis, which were used to engage with each teacher in a video stimulated recall interviews. The critical incidents were identified by the researcher based on the teacher's response or lack of response to learners' mathematical strategies and actions.

The rationales for VSR was to gain insight on teachers' decision-making process and to get the teachers to reflect on their practices with the ultimate goals of improvement in their professional noticing. The VSR also help the researcher to capture the degree to which the participants demonstrate evidence of engagement with learners' mathematical thinking. Analysis of the participant's comment on the teaching situation, in response to the video and the prompt was done by the researcher.

The second phase of lesson observation and video recording was abolished as a result of limited school days due to COVID-19. This impacted the method of analysis, so the result of data analysis from the VSR was compared with the result of data analysis from the first phase involving actual teaching moments. This comparison is used to highlight any possible shifts in participants' skills in each of the three components.

3.5 Procedure for data collection

In gathering data for this study, the researcher met with the principal of the school selected for the study and briefed him about the purpose of the study and obtained permission. The researcher together with the selected teachers examined their Annual Teaching Plan (ATP) and the respective teacher's timetables to ascertain the dates and time when common fractions would be taught. The video recordings were done during normal school teaching schedule. After the video had been analysed, the researcher asked the participating teachers to give convenient time for the next stage of video stimulated recall interview. This was done after teaching periods.

Three video recordings of each teacher were done, constituting three recordings. There was a total of six video recordings of lessons for the two teachers. The interview and prompts were guided by a prepared schedule of questions that were linked with the recorded lessons. Each interview lasted for about ninety minutes with each teacher.

3.6 Data Analysis

As stated previously, there are two sources of data: lesson observations and VSR interviews. Below, I provide how the data were analysed.

3.6.1 Analysis of lesson observation

Data from the lesson observation was analysed qualitatively first with open coding. For each video clip, a unit of analysis was examined to identify the 'how', and 'extent' of teachers' expertise in professional noticing. Table 1 below shows a rubric of the level of engagement demonstrated against each of the three component skills of teacher's professional noticing of learners' mathematical thinking. The rubric was developed by the researcher by bringing together the theoretical field (Jacob's noticing skills) and the empirical field (data gathered for the study). This process provided me with organisational language that give meaning to the data I collected, which allows me to categorised levels of professional noticing within the three skills of noticing that were identified by Jacob et al. Namely; attending, intepreting and deciding to respond. The four levels were: absent (level 0), limited (level 1), basic (level 2), and robust (level 3).

Table 1: Rubric for analysing extent of skills of professional noticing from lesson observation. (Adapted from Jacobs et al., 2010).

	4 – scale level of professional noticing			
Noticing skills	Absence (0)	Limited (1)	Basic (2)	Robust (3)
Attending (What the teacher considers necessary)	No recognition is given to any aspect of the learner's offer or approach. . For example, when an incorrect offer is given, and the teacher moves to another learner without returning to the initial incorrect approach after a correct answer is produced; or if the teacher moves on without affirming a correct answer offered by a learner.	There is evidence of recognition of negligible (Trifling) aspects of learner's strategy in a situation (presence) of salient mathematically significant details. Teacher's response does not demonstrate an understanding of the learner's approach, but focuses on a general evaluation of the outcome of the learner's approach For example, when a teacher's focus is on the tidiness of the learner's work; or the organisation of written symbols.	There is evidence of recognition of general features of learner's strategy. Teacher's response demonstrates an understanding of the learner's approach in relation to overall manipulation of symbols (i.e. computational aspects of the learner's strategy). For example, when the focus of a teacher's response is on a learner's fluency with manipulation of symbols. For example, computations that involves $2\frac{8}{3}$, is seen as $\frac{(2\times3)+8}{3}$ leading to $\frac{14}{3}$, and forms the computational aspects of a learner's strategy to which a basic attending is confined.	There is evidence of recognition of mathematically significant details in the learner's strategy. Teacher's response demonstrates a thorough understanding of the learner's approach in ways that is consistent with research on learner's mathematical development. For example, when the focus of a teacher's response transcends the manipulation of symbols to a consideration of a learner's reasoning with those symbols and why the rules work. For example, $2\frac{8}{3}$ seen first as $2+\frac{8}{3}$ *or* $(2+2+\frac{2}{3})$, next as $\frac{6}{3}+\frac{8}{3}$, then as $\frac{6+8}{3}$, and lastly as, $\frac{14}{3}$. These are some of the different representations of $2\frac{8}{3}$ in its interpretation as thirds. Attending to how a learner constructs relationship among these representations forms part of an attending that is robust.
Interpreting (Teacher's sense making - what was	A complete absence of interpretation of a learner's approach.	There is evidence that the teacher makes sense of a general or alternative issue other than the understanding	There is evidence that the teacher makes sense of a general but superficial aspect of a learner's approach. For example, when the teacher interrogates a learner's manipulation of symbols without any	There is evidence that the teacher makes sense of the details of a learner's strategy and noted how these details reflected what the learner understands, in ways that is coherent with research on children's mathematical development.

	4 – scale level of professional noticing			
Noticing skills	Absence (0)	Limited (1)	Basic (2)	Robust (3)
learned about the children's understandings)		shown in a learner's approach. For example, when the teacher deliberates on the performance of the learners as a group; or the general circumstances/conditioning of the learners such as their learning environment including teaching/learning style (past/present).	knowledge (by the teacher) of that learner's understanding of that symbol and its manipulation. For example, construing that a learner can (or cannot) convert a mixed fraction to an improper fraction, without any knowledge of that learner's understanding of the rules. E.g. Obtaining $\frac{14}{3}$, from $2\frac{8}{3}$ by $\frac{(2\times3)+8}{3}$.	For example, having a knowledge of how the details in a learner's strategy reflects how that learner constructs the relationship of the concepts with other concepts. For example, having a knowledge of how a learner's strategy of computing with $2\frac{8}{3}$ reflects a relationship among the other representations such as $2+\frac{8}{3}$ or $(2+2+\frac{2}{3})$, next as $\frac{6}{3}+\frac{8}{3}$, then as $\frac{6+8}{3}$, and lastly as, $\frac{14}{3}$. These are some of the different representations of $2\frac{8}{3}$ in its interpretation as third.
Deciding how to respond (grounds for responding)	A complete absence of evaluation of learner's offers or approach.	Response transcends evaluative comments on learner's approach but does not draw on insights from the learner's understanding. Response is simply confined to confirmation of a learner's success (or rejection if failure) in the task.	Response is based on the teacher's insights from the learner's understandings coupled with knowledge about next steps that research on children's mathematical development has shown are likely to further that learner's understandings. However, the response is not customised and is characterised by negligible details. For example, when computing $\frac{14}{15}$, Learner A obtains $1\frac{4}{15}$ as the answer, while Learner B obtains $1\frac{1}{15}$ as the answer. A response that interrogates the learner thus, "does fifteen go into fourteen?" Creates some cognitive conflict	Response is based on the teacher's insights from a learner's understandings coupled with knowledge about next steps that research on children's mathematical development has shown are likely to further that learner's understandings. The response is customised and is backed by proper specifics. For example, when computing $\frac{14}{15}$, Learner A obtains $1\frac{4}{15}$ as the answer, while Learner B obtains $1\frac{1}{15}$ as the answer. A response that interrogates Learner A thus, "What will you obtain when you compute $\frac{19}{15}$," and Learner B thus, "What will you obtain when you compute $\frac{16}{15}$," show evidence of responses that are not only based on the understanding of the learner in each case and is consistent with research on children's mathematical development.

3.6.2 Analysis of VSR Interview

The VSR interviews was analysed based on (Muir & Beswick, 2007) two-way framework of reflective response, consisting of object and levels of reflections. The objects of reflection pertain to either self (the teacher), practice, or learners. The levels of reflections are hierarchical based on quality of the teacher's responses. These are: technical description, deliberate reflection, and critical reflection. In technical description, general accounts of classroom practice is given, usually the technical aspects, without the reasoning behind it. Deliberate reflection is characterised by recognition of 'critical incidents' and provision of justifications for action. In critical reflection, a teacher surpasses recognition of 'critical incidents' and its justification but also considers others' perspectives and offering alternatives. The matrix of "levels of reflection" and "object of reflective response" (Geiger, Muir, & Lamb, 2016) is illustrated in Table 2.

Table 2: Rubric for analysing of VSR Interview (Adapted from Geiger et al., 2016).

Levels of reflections	**Object of reflective response**		
	Self	**Practice**	**Students**
Technical	Personal role is described during a teaching event. The description is factual rather than personally insightful	Teaching activity is described in terms of technical aspects. Focus is on consequences or outcomes of their practice	Students' responses to teaching activity are described in terms of technical aspects. Focus is on consequences or outcomes of teaching practice
Deliberate	Personal role is described during a teaching event. A rationale or explanation for the personal behaviour is provided	Critical incidents are related to teaching practice, and a rationale or explanation for the practice is articulated	Students' responses to teaching activity are noted, and a rationale or explanation for the response or behaviour is constructed

Levels of reflections	Object of reflective response		
	Self	**Practice**	**Students**
"Critical"	Personal role is described during a teaching event. The behaviour is critically analysed and alternative behaviours discussed	The purpose of an activity is clearly articulated, and a judgment is made about the success or otherwise of a teaching practice. When unsuccessful, an alternative practice or activity is suggested	Students' responses to teaching activity are noted, and a rationale or explanation for the response or behaviour is constructed. Potential improvements to the activity are related to an anticipated student response

3.7 Ethical considerations

In order to minimize injury to the participants in this study, consistent with all research that involve human participants, attentions were given to ethical issues. It is the responsibility of the researcher to ensure that ethical standards were adhered to. Therefore, in this study, ethical measures were taken, while planning and conducting the study.

For this research, participants were grade 8 mathematics teachers and their learners from one secondary school in Johannesburg. To deal with the ethical issues in my study, I issued information letters to the school and to all the participants, as well as parents of participants (in the case of the learners) in order to seek and obtain their consents before the commencement of the data collection process. The information letters provided the participants with details of the purpose of the study and procedures for the data collection in order to clarify any issues regarding my research. These letters highlighted the beneficial relationship within the research project.

Participants were informed that their involvement in the study was through voluntary participation and that they were free to terminate their involvement at any stage during the course of the study without any consequences. All participants were assured of their confidentiality and anonymity for participating in the study. To guarantee the confidentiality

of the participants, all data collected in the study was kept private. The researcher kept the video recordings of classrooms and audio recordings of the teachers' interviews in a passworded computer. Only myself and my supervisor have access to the data and data will be destroyed after 5 years of completion of the study. Anonymity is guaranteed through the creation of anonymised transcripts for any broader sharing of the data beyond myself and my supervisor. Ethics approval was obtained from both Gauteng Department of Education and the Wits School of Education Ethics committee before the commencement of the data collection.

CHAPTER 4: DATA ANALYSIS AND FINDINGS

4.1 Introduction

The purpose of this chapter is to present findings and discussion of the different patterns of professional noticing of the two teachers in relation to the three component skills of professional noticing of learner's mathematical thinking, namely: attending, interpreting, and deciding how to respond on the basis of learner's mathematical thinking across the six observed lessons. Analysis of the VSR interviews is also presented to explore possibilities of growth in professional noticing by the two teachers.

The chapter begins with the provision of background information on each teacher and followed by a brief description of three sequence of lessons I observed for each teacher. Thereafter, a qualitative analysis of incidents of professional noticing of learners' mathematical thinking that occurred in each of the teachers' lessons was presented. To exemplify the pattern of the observed professional noticing of each teacher, across the three professional noticing skills, I selected excerpts as 'telling cases' that indicated how, and the extent to which, each teacher notice learners' mathematical thinking of common fraction. For each telling case, I provide the background to the case, verbatim evidence in the form of transcript excerpts, followed by interpretation of the noticing event. Additionally, I present findings from the VSR interview based on teachers' reflective awareness to noticing events. Lastly, I provide an overall analysis across the two case study teachers.

4.2 Deborah's case

4.2.1 Deborah's background and overview of the lessons

Deborah teaches grades 8 and 9 mathematics and was in her fourteenth year of teaching at the time of data collection. Deborah is recognised as having a good track record of learner engagement. She is in possession of a Bachelor's Degree, and widely respected by her colleagues and she is the current Head of Department of Mathematics in the school.

Overview of Lesson 1 (57 minutes and 19 seconds***)***

In Deborah's first lesson with grade 8A, she started by telling the class that they will be doing adding, subtracting, multiplying and dividing common fractions. Seven tasks were provided to the learners, and in each case Deborah invited learners to the board to work out the task. This is then followed with classroom discussion on the learner strategy. My focus in each of

these tasks was the presence/absence and the quality of Deborah's noticing of the mathematically significant learner's actions and strategies in the enactment of the task.

In the first task, $6\frac{3}{7}+3\frac{1}{7}$, which requires learners to add two mixed fractions of the same denominator. Three learners' strategies were obtained and recorded in this task. In the second task ($4\frac{6}{9}+5\frac{5}{9}$), one learner's strategy was obtained and recorded, In the third task $2\frac{8}{3}-\frac{9}{3}$, which requires learners to subtract an improper fraction from a mixed fraction of the same denominator. In this task, the denominator of one fraction is a factor of its denominator, and the fractional part of the mixed fraction is an improper fraction as well. Two learners' strategies were obtained and recorded. One learner's strategy was obtained and recorded in the fourth task, $\frac{3}{5}\times\frac{6}{4}$, which requires learners to determine the product of two fractions. In this task, the numerator and the denominator of one fraction have a common factor. Two learners' joint strategy was obtained and recorded in the fifth task, $\frac{9}{7}\times\frac{8}{9}$, which requires learners to determine the product of two fractions. In this task, the numerator of one fraction is the same as the denominator of the other fraction. In the sixth task, $\frac{9}{3}\div\frac{7}{8}$, which requires learners to divide a common fraction by another fraction. In this task, the denominator of the dividend fraction is a factor of its numerator, but neither of the two numerators nor the two denominators (of the dividend fraction and the divisor fraction) have any common factor. One learner's strategy was obtained and recorded in the sixth task. The seventh and the last task in this class, $\frac{3}{7}\div\frac{5}{9}$, which requires learners to divide a common fraction by another fraction. In this task, the numerator of the dividend fraction is a factor of denominator of the divisor fraction, but neither of the two numerators nor the two denominators (of the dividend fraction and the divisor fraction) have any common factor. One learner's strategy was obtained and recorded in the sixth task.

Overview of Lesson 2 (39 minutes and 1 second***)***

In Deborah's second lesson with grade 8B, she started by telling the class that they will be doing adding, subtracting, multiplying and dividing common fractions. In the first task, $\frac{3}{8}+4\frac{2}{8}$, which requires learners to add a proper fraction and a mixed fraction of the same denominator together. In this task, numerator of the fractional part of the mixed fraction is a factor of its denominator. Two learners' strategies were obtained and recorded in this task.

One learner's strategy was obtained and recorded in the second task, $7\frac{2}{5}+3\frac{3}{5}$, which requires learners to add two mixed fractions of the same denominator together. None of the numerator-denominator pairs has a common factor. Deborah introduced the third task, $6+\frac{3}{7}$, to the learners by explaining that six wholes can be expressed as "six over one'. She then proceeded to represent the task as $\frac{6}{1}+\frac{3}{7}$ which requires the learners to add an improper fraction and a proper fraction together. In this task, the denominator of one fraction is 'one'. One learner's strategy was obtained and recorded in this task. In the fourth task $\frac{8}{3}-\frac{5}{3}$, which requires learners to subtract two improper fractions of the same denominator from each other. One learner's strategy was obtained and recorded in this task. One learner's strategy was obtained and recorded in the fifth and final task $5\frac{6}{9}-4\frac{2}{8}$ posed by Deborah in this class. In this task, which requires learners to subtract two mixed fractions of unlike denominators from each other, the numerator and the denominator of one fraction have a common factor, and the numerator of the second fraction is a factor of its denominator.

Overview of Lesson 3 (46 minutes and 14 seconds***)***

In Deborah's third lesson with grade 8C, she started the lesson by telling the class that they will be doing adding, subtracting, multiplying and dividing common fractions. In the first task, $\frac{8}{3}+\frac{17}{3}$, which requires learners to add together two improper fractions of the same denominator. One learner's strategy was obtained and recorded in this task. One learner's strategy was obtained and recorded in the second task, $7\frac{16}{5}+4\frac{3}{5}$, which requires learners to add two mixed fractions of the same denominator together. The fractional part of the first mixed fraction is an improper fraction, and none of the numerator-denominator pairs has a common factor. Similar to the second task, the third task, $3\frac{19}{8}+2\frac{5}{8}$, requires learners to add two mixed fractions of the same denominator together. The fractional part of the first mixed fraction is an improper fraction, and none of the numerator-denominator pairs has a common factor. One learner's strategy was obtained and recorded in this task. In the fourth task $\frac{8}{3}-\frac{17}{3}$, which requires learners to subtract two improper fractions of the same denominator from each other. One learner's strategy was obtained and recorded in this task. In this task, the subtrahend is greater than the minuend. The fifth task, $7\frac{16}{5}-4\frac{3}{5}$, posed by Deborah in this class requires learners to subtract two mixed fractions of like denominator from each other. In this task, the fractional part of the first mixed fraction is an improper fraction. One learner's

strategy was obtained and recorded in the fifth task. Similar to the fifth task, the sixth task, $3\frac{19}{8} - 2\frac{5}{8}$, posed by Deborah requires learners to subtract two mixed fractions of like denominator from each other. In this task, the fractional part of the first mixed fraction is an improper fraction. One learner's strategy was obtained and recorded in this task.

One learner's strategy was obtained and recorded in the seventh task, $\frac{6}{9} \times \frac{5}{3}$, which requires learners to determine the product of two fractions. In this task, the numerator and the denominator of one fraction have a common factor. Seen differently, the denominator of one fraction is a factor of the numerator of the second fraction.

One learner's strategy was obtained and recorded in the eighth and final task posed by Deborah in this class. Deborah introduced the task, $\frac{4}{8} \div \frac{5}{3}$ to the learners by explaining that division by a fraction is done by multiplying the dividend by the multiplicative inverse of the divisor. She then proceeded to represent the task as $\frac{4}{8} \times \frac{3}{5}$ which requires the learners to determine the product of two fractions. In this task, the numerator of one fraction is a factor of its denominator.

The analysis of the lessons as presented in the Table below is organised around four levels across three noticing skills of mathematics teachers in response to learners' strategies in doing mathematics.

I noted fifty-seven incidents of professional noticing of learner's mathematical thinking across all three skills in the three observed lessons. These incidents consist of nineteen incidents of *attending skills*; nineteen incidents of *interpreting skills*; and nineteen incidents of the skill of *Deciding how to respond on the basis of learner's understanding.* For the fifty-seven noticing incidents which emerged, three *Attending* skills were coded *Absence*, four were coded *limited*; and twelve were coded Basic. I coded three *interpreting skills as* a*bsence*, eight as *limited*; and eight as *basic*. For the skill of *Deciding how to respond on the basis of learner's understanding,* three were coded *absence*, thirteen were coded *limited*; and twelve were coded *basic*. In the next section, I provide a qualitative analysis on all fifty-seven incidents.

Table 3: Overview of level of professional noticing of learners' mathematical thinking demonstrated by Deborah

Deborah

4 – SCALE LEVEL OF PROFESSIONAL NOTICING					
NOTICING SKILLS	**Absence (0)**	**Limited (1)**	**Basic (2)**	**Robust (3)**	**Total**
Attending	3	4	12	0	19
Interpreting	3	8	8	0	19
Deciding how to respond	3	13	3	0	19
Total	9	25	23	0	57

Deborah's data overall suggests a notable lack of robust noticing across the three skills of noticing in all three observed lessons. The data also suggests that Deborah's *attending to learner's strategies* is mainly in the form of *basic attending* where 12 instances were noted; followed by *limited attending* where 4 instances were noted; with the least being the *absence of attending* where 3 instances were noted. Next, Within the *noticing* skill of *interpreting learner's mathematical understanding*, Deborah's data indicates an even spread between basic interpreting and limited interpreting where I noted 8 instances each; with the least in this category being 3 instances of absence of interpreting. Lastly, the data shows that Deborah's skill of *deciding how to respond on the basis of learners' understanding* is predominantly in the form of *limited deciding* how to respond where I noted 13 instances; and complemented by 3 instances each of absence of deciding how to respond and *limited deciding how to respond*.

To exemplify the pattern of the observed noticing by Deborah qualitatively across the three lessons, I selected excerpts as 'telling cases' that indicated how and to what extents does Deborah notice learners' mathematical thinking, namely: *absence of attending, limited attending, basic attending, absence of interpreting, limited interpreting, basic interpreting,*

absence of deciding how to respond, limited deciding how to respond, and basic deciding how to respond.

Absence of Attending (Level 0)

In terms of the absence of attending category, Deborah's range of responses in the three lessons were all in response to situations of incorrect learners' mathematical thinking. In particular, responses in this category was characterised by pursuit of correct answer without returning to the initial incorrect one when a correct answer is offered. Below, I exemplify this attending.

In lesson 2, in the context of the task, $\frac{3}{8} + 4\frac{2}{8}$, which requires learners to add a proper fraction and a mixed fraction of the same denominator together. Deborah invited learners to work out the problem on the board. The first learner (L1) gave only the answer which was incorrect (figure 2). In response, Deborah invited a second learner who also gave only the answer which is correct (figure 3.

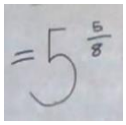

***Figure 2:* L1's incorrect answer of $\frac{3}{8} + 4\frac{2}{8}$**

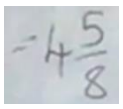

***Figure 3: L2's correct answer of* $\frac{3}{8} + 4\frac{2}{8}$**

The following excerpt then played out:

Excerpt 1

Line	Learner/Teacher accounts	
1	L1:	Learner writes an incorrect answer without the steps (figure 2).
2	Deborah:	(Deborah invites a second learner). "Erase that one" (pointing to L1's answer).
3	L2:	Learner writes a correct answer without the steps (figure 3).
4	Deborah:	"Is it correct?"
5	L (chorus):	"Yes"

6	Deborah:	(Deborah erased the second written answer and continued to show the step by step solution on the board). "Okay. Okay. Eh, except that it is just the answer only. Every time you must show the calculation how you get the answer. I can see you are a bit lost until I can show an example, and everybody will be able to answer that question"

Deborah's utterance in line 2 to erase the incorrect answer of L1 dismisses rather than recognise that learner's strategy. Also, Deborah's action in line 6 of erasing L2's answer is indicative of dismissing learner's input. Furthermore, Deborah's utterance in line 6 'shows the calculation' rather than probes the learners' mathematical thinking of the solution she wants her class to focus on. In the VSR interview, Deborah discloses that her focus was "to show all the calculations so that everyone can understand how to do it". In this singular focus though, learners' ways of thinking about the problem situation are dismissed. This incident exemplifies what Jacobs, Lamb, and Philipp (2010) have described as opportunities missed to open a window into children's understandings, and it is coded within the *'absence of attending'* category. Deborah moved on to work out the problem herself without the resource of the learners' initial strategy. This choice is of interest given that Makonye and Khanyile (2015) have noted teachers can make productive use of learners' errors as a resource that enables them to understand learners' thinking to the benefit of the learner.

Limited Attending (Level 1)

In terms of the limited attending category, Deborah's range of responses in the three lessons were mainly in the form of attention to the way learner organise written symbols. In particular, responses in this category was characterised by a narrow emphasis that certain symbols are not large enough, or that a particular step or parenthesis has been omitted. Below, I exemplify this attending.

In lesson 3, in the context of the task $7\frac{16}{5} - 4\frac{3}{5}$, which requires learners to subtract two mixed fractions of the same denominator from each other. Deborah invited learners to work out the problem on the board. Two learners (L3) worked together to solve the problem. Firstly, they translate both $7\frac{16}{5}$ and $3\frac{3}{5}$ to their improper fraction equivalence, and proceeded to merge the

two fractions into one fraction with two terms in the numerator, which they subtracted from each other to obtain an improper fraction. Finally, the learners converted the improper fraction to a mixed fraction in the final answer which was correct (figure 4). Deborah validated the answer and recounted the solution steps to the class.

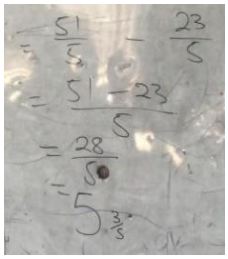

Figure 4: L3's correct solution of $7\frac{16}{5} - 4\frac{3}{5}$

The following excerpt then played out:

Excerpt 2

Line	Learner/Teacher accounts	
1	L3:	Learner works out the solution correctly (figure 4).
2	Deborah:	"Okay. They simplified" (referring to the conversion of an improper fraction to a mixed fraction in the final answer. She then erased the $\frac{3}{5}$ and repositioned it slightly upward because she did not approve of the way the learner wrote it. She proceeded to recount the solution steps to the class).

Deborah's utterance in line 2 that "they simplified" is indicative that the focus of her attention is the mixed fraction ($3\frac{3}{5}$) obtained in the final answer. Also, Deborah's action in line 2 of erasing and repositioning $\frac{3}{5}$ answer is suggestive of attending that focuses on the general but trivial features of the learner's strategy. This incident is not surprising as Jacobs, Lamb, and Philipp (2010) noted similar attending patterns amongst initial participants in their study since some initial participants (and prospective teachers) would provide general description of a learner's strategy, making it difficult to discern if the teacher gained any insight from the learner's strategy. I coded the attending as limited since Deborah recognized that the learners did not leave the answer as improper fraction, however, she paid particular

attention to the positioning of the fraction rather than the relationship between the whole number and the fraction.

Basic Attending (Level 2)

In the category of basic attending, Deborah's array of responses in the three lessons were primarily in the form of attention to incorrect learners' manipulation of common fractions, particularly, failure to translate mixed fractions to improper fractions before they add or subtract; and from improper fraction to a mixed fraction in the final answer. In particular, responses in this category was characterised by drawing learner's attentions to the need (or lack of) for them to translate the fraction under review. Below, I exemplify this attending.

In lesson 1, in the context of the task, $2\frac{8}{3} - \frac{9}{3}$, the learner (L4) who answered the question converted the $2\frac{8}{3}$ by computing 3 (the denominator) minus 2 (the whole) to get 1. Thereafter, the learner subtracted 8 (numerator) from 1 (the previous result) to obtain 7 (instead of negative seven). That learner got $\frac{7}{3}$ from that conversion. Finally, the learner carried out of $\frac{7}{3} - \frac{9}{3}$ to obtain $\frac{2}{3}$ (instead of negative two third) (figure 5. In response, Deborah called on another learner to provide the "correction". The second learner carried out the subtraction correctly (figure 6).

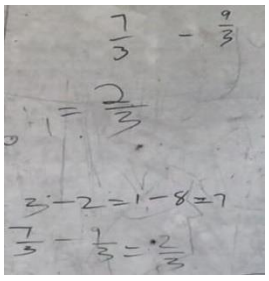

Figure 5: L4's incorrect solution of $2\frac{8}{3} - \frac{9}{3}$

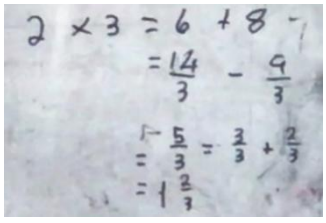

Figure 6: L5's correct solution of $2\frac{8}{3} - \frac{9}{3}$

The following excerpt then played out:

Excerpt 3

Line	Learner/Teacher accounts	
1	L4:	Learner writes an incorrect answer (figure 5).
2	Deborah:	"Come and do the correction" (Deborah invites a second learner).
3	L5:	Learner writes a correct answer (figure 6).
4	Deborah:	"You (the learner who gave the answer in line 1) see how it is done (referring to figure 6)? You got a seven here (pointing to the numerator of the subtrahend in figure 5. Can you see it is wrong"?

Deborah's utterance in line 4 that "you got a seven here" is suggestive that the focus of her attention is the result obtained when the learner converted the mixed fraction $2\frac{8}{3}$ to an improper fraction ($\frac{7}{3}$). Also, Deborah's utterance that "can you see it is wrong?" in line 4 is suggestive of attending that focuses on the result of computation rather than the details in that computation. This incident exemplifies what Jacobs, Lamb, and Philipp's (2010) described as attending that focuses on general features of a strategy but omits the details of how the problem was solved. I coded the *attending* as *basic* since Deborah recognized that the learner attempted to convert a mixed fraction to an improper fraction, however, she paid particular attention to the positioning of the fraction rather than attention to how the learner construct the relationship between the whole number, the numerator and denominator that constitute the mixed fraction. I want to note that I coded the incident as *basic* rather than *limited* since Deborah's attending involves a general but significant feature of that learner's strategy.

Absence of Interpreting (Level 0)

In terms of the absence of interpreting category, Deborah's array of responses in the three lessons were all in response to situations of incorrect learners' mathematical thinking. As with the absence of attending category, responses in this category was characterised by pursuit of correct answer without returning to the initial incorrect one when a correct answer is offered. Below, I exemplify this interpreting.

In lesson 2, in the context of the task, $5\frac{6}{9} - 4\frac{2}{9}$, the learner (L6) who answered the question, wrote a whole number of one (1) (perhaps by subtracting four from five – the whole number parts, followed by $\frac{6}{9} - \frac{2}{9}$ in a bracket. He then proceeded to merge the two fractions into one

fraction with two terms in the numerator, which he subtracted from each other to obtain a proper fraction (Figure 7). The answer was refuted by another learner in the class.

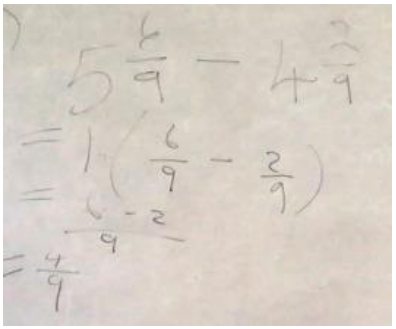

Figure 7: L6's incorrect solution o f $5\frac{6}{9} - 4\frac{2}{9}$

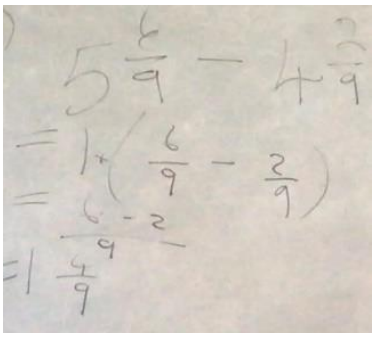

Figure 8: L7's correct solution of $5\frac{6}{9} - 4\frac{2}{9}$

The following excerpt then played out.

Excerpt 4

Line	Learner/Teacher accounts
1	L6: Learner writes an incorrect answer (figure 7).
2	L7: "Add the one that is before the bracket to four over nine" (directing L6).
3	L6: "To make five over nine?
4	Learner (audience): "No. It is one whole. (She proceeded to the board and introduced a plus sign between one whole and the bracket and went on to revise $\frac{4}{9}$ to $1\frac{4}{9}$ (Figure 8).
6	Deborah: "Is it correct?"
7	L (chorus): "Yes. No. Yes. No. Yes. Yes"
8	Deborah: "Go and do it with the proper method".

L6's strategy (Figure 7) is the fourth instance, in this lesson, of learners' departure from the rule established by the teacher that learners must convert mixed fraction to improper fraction before adding or subtracting fractions. Arguably, this learner's understanding of this alternative strategy is not fully developed. Plausibly, this learner also construes a multiplicative relationship between the whole number and the fraction part of the mixed

numbers similar to algebraic expression because the absence of addition sign between *one* and the *bracket* (Figure 7) is indicative that multiplication is implied. Moreover, $1 \times \frac{4}{9}$ is a legitimate operation for L6's answer to remain $\frac{4}{9}$. This was noted by L7 who introduced the plus sign between 1 and the bracket, and then went on to add 1 to the result of the operation from the bracket (Figure 8).

Deborah's dialogue with the learners in lines 6 to 8 is suggestive that Deborah neither attend to, nor interpret any aspect of the learner's strategies. Arguably, Deborah's utterance in line 8 directing the learners to "go and do it with the proper method" suggests that she does not recognise the learner's approach as a proper method. By rejecting the method as improper is indicative that Deborah does not have the mathematical knowledge of the learner's strategy. In the VSR interviews, Deborah reveals that she perceives the learners as a group by stating that "they [the learners] came here [grade 8] very empty". So, she reasons that "the best way is to teach them like grade 4 or whatever in order to move on". This incident is similar to the difficulties experienced by participants in Jacobs, Lamb, and Philipp's (2010) study since the authors noted that it is so challenging for participants who view groups of children only as a group that it becomes almost impossible for them to identify the understandings reflected in specific strategies. I coded the incident as absence of interpreting because Deborah's utterance is dismissive of the method and she did not comment on the strategy.

Limited Interpreting (Level 1)

In terms of the limited interpreting category, Deborah's range of responses in the three lessons were chiefly in the form of interpretation that omission of general but insignificant features of the learner's strategy obscures the validity of the mathematics. In particular, responses in this category was characterised by inconsistencies with the understanding reflected in the learner's strategy. Below, I exemplify this interpreting.

In lesson 2, in the context of the task, $\frac{8}{3} - \frac{5}{3}$. The learner (L8) who answered this question started by converting $\frac{8}{3} - \frac{5}{3}$ (an expression involving two separate fractions - binomial); to $\frac{8-5}{3}$ (an expression involving a merge of two common fractions – monomial) in line 1. However, she could not work out the subtraction with this representation, so she rejected $\frac{8-5}{3}$

and reverted to $\frac{8}{3} - \frac{5}{3}$ in line 2. Remarkably, she obtained $\frac{3}{3}$ and lastly a 1 as the final answer. In response, Deborah revised $\frac{8}{3} - \frac{5}{3}$ to $\frac{8-5}{3}$ in line 5 and validated the solution.

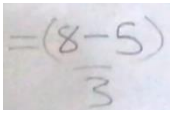

Figure 9: L8's direct solution of $\frac{8}{3} - \frac{5}{3}$

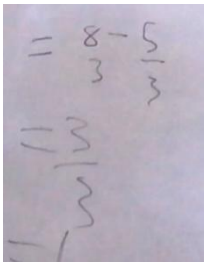

Figure 10: L8's direct solution of $\frac{8}{3} - \frac{5}{3}$

The following excerpt then played out.

Excerpt 5

Line	Learner/Teacher accounts	
1	L8:	Learner writes $\frac{8-5}{3}$ as a correct representation of $\frac{8}{3} - \frac{5}{3}$ but erased it because she could not work out the subtraction with that representation (figure 9).
2	L8:	Learner works out the subtraction, $\frac{8}{3} - \frac{5}{3}$, directly without merging the two fractions (figure 10).
3	Deborah:	"Okay. Good. Is it correct?"
4	L (chorus):	"Yes"
5	Deborah:	"It is just a matter of here (referring to $\frac{8}{3} - \frac{5}{3}$, which she then revised to $\frac{8-5}{3}$. She then indicated with ticks that the solution steps and answer are correct.)

The learner (L8) started with the representation, $\frac{8-5}{3}$ in line 1 but erased it because she found it challenging to work with that representation. However, by working directly with the expression, $\frac{8}{3} - \frac{5}{3}$, she successfully carried out the operation and obtained a correct answer. It is clear that the learner (L8) was able to carry out subtraction of fractions with same

denominator when the fractions are separated (possibly seen as 8 thirds minus 5 thirds), but unable to carry out that operation by combining the fractions. Plausibly, it is likely that this learner's strategy stems from a particular meaning of common fraction that identifies $\frac{8}{3} - \frac{5}{3}$ as eight thirds minus five thirds equal to three thirds. That being the case, the subtraction of the two fractions will be carried out in similar fashion as working with algebraic expression such as: $8x - 5x = 3x$. It is also likely that this learner, wrote one as the answer simply because she construes of three thirds as one whole. Literature on learner's development of common fraction recognises that symbolic representation of common fractions creates a problem of meaning for learners, because each symbol representation means something (Kilpatrick, Swafford, & Findell, 2001).

In the VSR interview, Deborah reveals that her reason for revising $\frac{8}{3} - \frac{5}{3}$ to $\frac{8-5}{3}$ was to let everyone see that denominator 3 in $\frac{3}{3}$ comes from 8 minus 3, by stating that "so that everyone will know where we get that three". Deborah's interpreting in this episode exemplifies the pattern of interpreting by limited-response participants in Jacobs, Lamb, and Philipp's (2010) study since the authors noted that limited-response participants make broad conclusions that are difficult to justify on the basis of the children's approach. I coded Deborah's *interpreting* in this incident as *limited* because her reasoning does not echo the understanding reflected in the learner's approach. Her conclusion is not justifiable, since there is no ambiguity to see that *eight third minus five third* equals *three third*.

Basic Interpreting (Level 2)

In terms of the basic interpreting category, Deborah's range of responses in the three lessons were largely in the form of interpretation that learners have incomplete knowledge of simplifying common fraction. In particular, responses in this category was characterised by reminding learners to simplify, or to find the HCF (Highest Common Factor). Below, I exemplify this interpreting.

In lesson 2, in the context of the task, $\frac{9}{3} \div \frac{7}{8}$, two learners (L9) worked out the problem and obtained $\frac{72}{21}$ but failed to simplify it further.

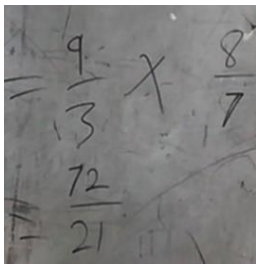

Figure 11: L9's struggle to simplify $\frac{72}{21}$

The following excerpt played out.

Excerpt 6

Line	Learner/Teacher accounts	
1	L9:	The learners translated $\frac{9}{3} \div \frac{7}{8}$ to $\frac{3}{7} \times \frac{9}{5}$ and obtained $\frac{72}{21}$ as a result. However, they struggled to simplify it further (Figure11).
2	Deborah:	"You (L9) can't find the highest common factor? What is the highest common factor of seventy-two and twenty-one? (Deborah directed the question to the whole class).
3	L (class):	No answer
4	Deborah:	"Highest common factor is the number that can divide this (72) and that (21). "What is the highest common factor (of 72 and 21)?
5	L (class):	"Two"
6	Deborah:	"So, two is a factor of twenty-one?
7	L (class):	"No. [It is a factor of] seventy-two".
8	Deborah:	"So, seventy-two and twenty-one, what is the highest common factor?
9	L (class):	No answer
10	Deborah:	"Highest common factor is the number that can divide seventy-two and twenty-one. So, what is the highest common factor?
11	Learner:	"Seven. Seven. Seven."

13	Deborah:	"Okay. Okay. The highest common factor is three. So, do seventy-two divide by three"

Deborah's utterance in line 2 questioning the learners, "you can't find the highest common factor", is indicative that she understands why the learners could not succeed at simplifying the fraction. Corroborating this interpreting in the VSR interview, Deborah stated that "as I said, the kids they don't know how to find the highest common factor". Deborah's interpreting in this episode is similar to what Jacobs, Lamb, and Philipp's (2010) described as interpreting that has specific connections to the learner's strategies but lacked depth. I coded Deborah's *interpreting* in this incident as *basic* since she demonstrates that the learners cannot find highest common factor, but does not show any knowledge of the learners' construal of highest common factor. In this case the learners may not construe of 3 as a factor of 21 that can be considered 'high', even if they know that 3 is a factor of both 21 and 72.

Absence of deciding how to respond (Level 0)

In terms of the absence of deciding how to respond interpreting category, Deborah's array of responses in the three lessons were all in response to situations of incorrect learners' mathematical thinking. As with the absence of attending and absence of interpreting categories, responses in this category was characterised by pursuit of correct answer without returning to the initial incorrect one when a correct answer is offered. Below, I exemplify this responding.

In lesson 3, in the context of the task, $\frac{8}{3} - \frac{17}{3}$, the first learner (L10) who answered the question obtained $\frac{9}{3}$, but did not simplify it further (Figure 12). Deborah did not comment on the work but directed a second learner (L11) to go forward and answer the question.

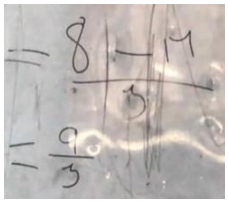

Figure 12: L10 incorrect solution of $\frac{8}{3} - \frac{17}{3}$

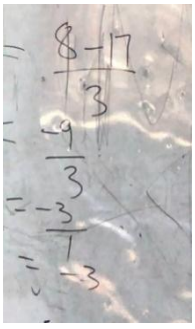

Figure 13: L11's correct solution of $\frac{8}{3} - \frac{17}{3}$

The following excerpt played out.

Excerpt 7

Line	Learner/Teacher accounts	
1	L10:	Learner works out incorrect solution and upholds $\frac{9}{3}$ as the final answer (Figure 12).
2	Deborah:	"Go" (Deborah directed another learner to proceed to the board).
3	L11:	Learner works out correct solution (Figure 13).
4	Deborah:	"Is it correct?"
5	L (chorus):	"Yes"
6	Deborah:	(Deborah narrated the solution step by step and validated the solution with ticks. There was no mention of the original solution being "corrected")

The learner (L10) obtained positive 9 instead of negative 9 from the computation of $(8 - 17)$. Also, the learner (L10) failed to simplify $\frac{9}{3}$ in line 1 (Figure 12), which is suggestive that this learner might be working with a particular meaning of $\frac{9}{3}$ as nine *thirds*. Deborah's directive to the second learner in line 2 did not include any reference to the solution of the first learner (L10). Likewise, her dialogue with the learners in lines 2 to 6 concluded without any mention of L10's solution. This incident is similar to lack of evidence responding by participants in Jacobs, Lamb, and Philipp's (2010) study since the authors noted that lack of evidence responding often involve instructional steps that are applicable in situations that the learner's strategies had not played out. I coded the incident as absence of deciding how to respond because Deborah's subsequent instruction did not mention or address any aspect of the learner's solution.

Limited deciding how to respond (Level 1)

In terms of the limited deciding how to respond interpreting category, Deborah's array of responses in the three lessons were predominantly in the form of stating what the learners were supposed (or not) to do. Below, I exemplify this responding.

In lesson 2, in the context of the task, $7\frac{2}{5}+3\frac{3}{5}$. The learner (L12) who answered this questioned explained the steps verbally and got an incomplete answer of $10\frac{5}{5}$. Initially, Deborah rejected the strategy, however, few learners in the audience disputed that the answer is correct, so Deborah agreed. However, the learner (L12) could not put the answer together as a single whole number.

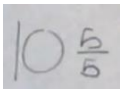

Figure 14: L12's initial solution of $7\frac{2}{5}+3\frac{3}{5}$

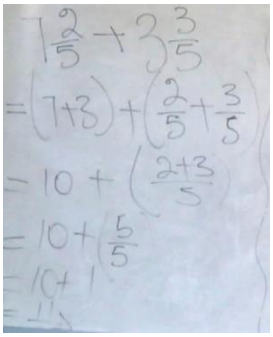

Figure 16: Deborah's response to L12's solution of $7\frac{2}{5}+3\frac{3}{5}$

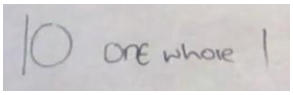

Figure 15: L12's revised solution of $7\frac{2}{5}+3\frac{3}{5}$

The following excerpts played out.

Excerpt 8

Line	Learner/Teacher accounts	
1	L12:	"I am going to add the whole numbers, eight, nine, ten (add three to seven by counting on from 7), it's going to be ten. Then two plus three, it's going to be 5 (writes five above the fraction line).
2	Deborah:	"Eh, what are you doing? Converting mixed fraction?"
3	L12:	(The learner erased the work and listened to the teacher. Thereafter, she started all over again). "First thing you are going to add whole numbers. Seven plus three is equal to ten".
4	Deborah:	"Eh, correction. Correction." (L12 erased the work and listened to the teacher)

5	L(audience):	"She is right". Thereafter, she started all over again). "First thing you are going to add whole numbers. Seven plus three is equal to ten".
1	L12:	(She started all over again). "First thing you are going to add whole numbers. Seven plus three is equal to ten (writes 10). Then two plus three, it's going to be five; over five" (points to the denominator and writes five over five) (Figure 14).
6	Deborah:	"Is that so? Oh! You are adding the whole numbers. Okay"
7	L(audience):	"But five over five is equal to one"
8	L12:	(Learner erased five over five and replaced it with 1 – written in word and in numeral (Figure 15).
9	Deborah:	You are right. (Deborah erased "one whole 1" and created a bracket for the learner to work in). Here (pointing to ten) you have added seven and three together, then you must write the fractions in the bracket".
10	Deborah:	"Okay. She is right but just confused a little bit. This method is grouping like terms together. So, we have two wholes, we can say seven plus three and put them in a bracket, plus another bracket; two over five plus three over five. Seven plus three is equal ten, then you simplify this (pointing to $\frac{2}{5}+\frac{3}{5}$). Then that is two plus three all over five. Then it will be ten; plus, five over five. Which will be ten plus one whole. Which is eleven. So that is why I said She is right but just confused a little bit. So after, grouping your whole, you group your fraction, then start simplifying them the way you simplify others. Eh, is it clear?"
15	Learner:	" Yes".

Conceivably, L12 construes a multiplicative relationship between the whole number and the fraction part of mixed numbers similar to algebraic expression such as $5x$, because she continued to hold the two as separate entities (Figures 14 and Figure 15). In that case it is very likely that L12 is working with the understanding of the fraction as 10 and 5 fifths, which probably just like $10\ and\ 5x$ have to remain apart. As I mentioned earlier, literature on learner's development of common fraction recognises that symbolic representation of common fractions creates a problem of meaning for learners, because each symbol

representation means something (Kilpatrick, Swafford, & Findell, 2001). Deborah's deciding how to respond in this episode exemplifies the pattern of responding by limited-response participants in Jacobs, Lamb, and Philipp's (2010) study since the authors noted that limited-response participants perform follow-up activity without reference to existing understandings or anticipated strategies deductible from the learner's strategies. In the VSR interview, Deborah discloses that her focus was on alternative issue by saying that, "I want to avoid a situation where only a few learners understand the concept and the rest do not". Deborah responded by repeating the whole solution method because "I have to repeat so that even those ones (weaker learners) can understand". I coded Deborah's *deciding how to respond in this incident* as *limited* because Deborah neglected the problem of L12 but focused on showing the class the procedure for organising whole numbers separately from fractions.

Basic deciding how to respond (Level 2)

In terms of the basic deciding how to respond category, Deborah's array of responses in the three lessons were predominantly in the form of probing and learners to complete the task. Below, I exemplify this responding.

In lesson 1, in the context of the task, $\frac{3}{7} \div \frac{5}{9}$, which she translated to $\frac{3}{7} \times \frac{9}{5}$, before inviting learners to solve it. Two learners (L13) worked out the problem and obtained $\frac{27}{35}$. However, they continued to simplify it further, but got stuck.

Figure 17: L13's solution of $\frac{3}{7} \times \frac{9}{5}$

The following excerpt then played out.

Excerpt 9

Line	Learner/Teacher accounts	
1	L13:	Two learners work out the solution of $\frac{3}{7} \times \frac{9}{5}$ and attempt to simplify $\frac{27}{35}$ (Figure 17).

2	Deborah:	(erased the needless division by nine over nine to retain only twenty-seven over thirty-five). "This one ($\frac{27}{35}$) cannot be simplified any further. Can you see it does not have highest common factor? Does any number go into both two numbers?
3	L (chorus):	"No"
4	Deborah:	"We don't have it. So, let us leave it as it is".

Deborah's dialogue with the learners in lines 2 to 4 reveals the reasoning for her response. By stating in line 3 that "this one cannot be simplified any further is indicative of a responding that is informed by the understanding reflected in the learners' strategy. This responding is confined to the general aspect of the computation rather than the details since it did not build on the learner's use of a factor (9) which is applicable to only one aspect of the fraction (the numerator 27) and not the second aspect (the denominator 35). In the VSR interview, Deborah reveals that she finds it necessary to provide additional information to the learners in lines, stating that, "It means, if maybe you cannot divide the denominator by the numerator or find the highest common factor, then this means that it is the end of the road. You cannot simplify it any further". Deborah's *deciding how to respond* in this incident is similar to what Jacobs, Lamb, and Philipp's (2010) described as reasoning that is characterised by consideration for children's strategies and understandings with minimal specificity and lack of customization in deciding how to respond. I coded Deborah's *deciding how to respond* in this incident as *basic* since her reasoning involves an aspect of the learners' understanding, however it lacks the specificity of the learner's use of '9 over 9'.

4.2.2 Findings from the VSR interviews with Deborah

In this section, I present findings from the VSR professional development interview conducted after the three lesson observations. The VSR interview with Deborah focused largely on her responses to learners' solutions of selected tasks in her lesson, and what she might do differently. Preceding the conversation about the learners' solutions, I began the interview by asking Deborah to talk about the objectives for her three lessons.

Excerpt 10

Line	Researcher/Teacher accounts

1	R:	What is your intended goal for this lesson? What do you intend for your learners to be able to do at the end of these lesson?
2	T:	I wanted them to know how to use fractions in four operations. That is, how to use fraction for addition, subtraction, multiplication and division.

The focus of Deborah's utterance in the above excerpt is on the content knowledge that learners will acquire as a result of the lessons without specifying any required strategy nor the significance thereof. This focus is indicative that her reflection was *technical* in relation to *student*. Likewise, when I probed Deborah about her thoughts and decisions in a context of learner's (L4) incorrect solution action (Figure 5, excerpt 3) whereby she focused attention on the learner's incorrect answer of converting mixed fraction to improper fraction, she stated further:

Excerpt 11

Line	Researcher/Teacher accounts	
1	T:	That (converting mixed fraction to improper) is what I wanted to emphasise. Because if you can look at the addition, subtraction, multiplication and division of fraction, we are juggling around this conversion of (to and from) improper and mixed number. This conversion is the key concept of the topic (addition, subtraction, multiplication and division of fractions). I wanted them to know how to use fractions in four operations. That is, how to use fraction for addition, subtraction, multiplication and division.

Thus far, Deborah's utterance echoes what she anticipated that learners would do in their solution to the tasks. Her utterance suggests awareness of content coverage and concepts. There is a remarkable absence of a sense of the need to gain insight into the mathematical thinking of the learners, and to build on their understandings. This absence is indicative of a view of teaching as transmission.

Later in the interview, I probed Deborah's rationale for her thoughts and decisions in a context of two learners' (L14) joint incorrect solution action (Figure 18) in which the teacher emphasized the use of highest common factor.

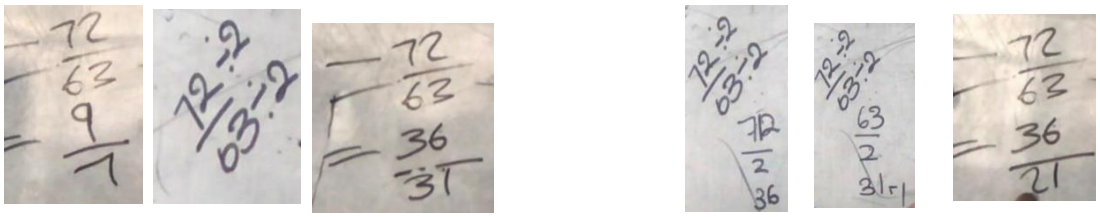

Figure 18: L14's incorrect solutions of $\frac{9}{7} \times \frac{8}{9}$

Excerpt 12

Line	Researcher/Teacher accounts	
1	T:	He divided by two both sides. When he is dividing by two it was going to give him another problem because he didn't find the highest common multiple (factor). Maybe if he uses the highest common multiple the answer was going to be correct on both sides. So that is why I say if he uses any number then it is going to give him another number to simplify again. Then he will find a wrong one again, then he still has to simplify it again.

Deborah's remark showed *deliberate* level of reflection about *students* in relation to the errors and inadequacy of the calculation strategy they employed. Therefore, her comment indicated reflective awareness of the requirements and possibilities of fluency with highest common factor as a competency that can abate such errors committed by her learners. Deborah added on the importance of the highest common factor:

Excerpt 13

Line	Researcher/Teacher accounts	
1	T:	That is why I say, eh, he knows what to do but the problem was the highest common factor. He didn't get the highest common factor. He knew that oh I must find the highest common factor, and divide both numerator and denominator by the highest common factor. So, the only problem is failing to get the highest common factor.

This statement marked a shift in Deborah's level of awareness since she realised the need to consider specific details in learners' strategy. This is indicative of the need for employing her transformation and connection knowledge (Rowlands) within contingent situation to

understand those details in the learners' strategy in relation to theory of how learners develop mathematical knowledge (Jacobs). Deborah's remarks in the excerpt above is therefore construed as a transition from *technical* (how to use fraction for addition, subtraction, multiplication and division) to *deliberate* level of reflection (learners encountered problem because they could not find the highest common factor).

When I probed Deborah on improvements she could make to this incident, Deborah responded as follows:

Excerpt 14

Line	Researcher/Teacher accounts	
1	T:	Because here, [in this lesson], I saw that I just said [asked] highest common factor? Then I gave them the highest common factor. I didn't show them how to find it [the highest common factor], I mean on this topic [this lesson]. I was supposed to show them how to, maybe I will make factors of seventy-two, factors of sixty-three, then take the highest common factor from there. Or else I can use the other method [multiply each fraction by the 'denominator over the denominator' of the other fraction], that is the quickest they can understand.

This statement marked a shift in Deborah's level of awareness since she realised the need to build on learners' understanding rather than transmit knowledge to them in the absence of discernible insights from their strategy details (Jacobs). Deborah's remarks in the excerpt above is therefore construed as a transition from *deliberate* (learners didn't get the highest common factor, so that is why I had to intervene and do it for them) to *critical* level of reflection (I was supposed to show them how to, maybe I will make factors of seventy-two, factors of sixty-three, then take the highest common factor from there. Or else I can use the other method).

Deborah's reflective awareness prompted by the VSR interview showed continuing learning as a result of attentive examination of practice, and open reflection-oriented interrogations about classroom practices. This showed that Deborah had become more cognizant of the limitations of her instructions subsequent to learners' contributions in the classroom. This growing alertness and evidence of critical reflection about *practice* and *students*, showed potentials for advancing classroom practice. In the following section, I link findings from Deborah's shifts in teaching practice with shifts in reflective awareness.

4.2.2 Linking Deborah's practice and reflective awareness

Deborah's professional noticing of learners' mathematical thinking, were mainly in the form of subsequent instructions in which insights from discernible details in learners' strategies are lacking. In the fifty-seven instances of noticing recorded across Deborah's three lessons, about 44% were *limited noticing*; about 40% were *basic noticing*; and about 16% were *absence of noticing*. There was a remarkable lack of instance of *robust noticing*. Deborah's attending pattern were predominantly in the category of *basic attending*, with responses that were characterised by drawing learner's attentions to the need (or lack of) for them to translate fraction from one form to the other. This noticing pattern exemplifies what Jacobs, Lamb, and Philipp's (2010) described as attending that focuses on general features of a strategy but omits the details in the strategy.

Deborah's interpreting pattern was dominated by equal instances of *limited interpreting* and *basic interpreting*, with responses that were characterised either by views that is inconsistent with the understanding reflected in the learner's strategy or by views that have connections with the understanding of the learners but omits the details thereof. While the former noticing pattern exemplifies what Jacobs, Lamb, and Philipp's (2010) described as interpreting that are not consistent with the understanding reflected in learners' strategy, the latter noticing pattern exemplifies what Jacobs, Lamb, and Philipp's (2010) described as interpreting that focuses on general features of a strategy but omits the details in the strategy.

Lastly, Deborah's deciding how to respond pattern was chiefly in the category of *limited deciding how to respond*, with responses that were characterised by instruction which does not build on the understanding reflected in the particular learner's strategy or anticipated next step. This noticing pattern exemplifies what Jacobs, Lamb, and Philipp's (2010) described as decision making that is disconnected with the specifics of children's thinking in a given situation, and inconsistent with the research on children's mathematical development.

Findings from the VSR interviews show that Deborah was at a technical level of reflection when asked to reflect on objectives of her lesson. There was a notable shift to deliberate level of reflection when probed to reflect on the discernible details of her learners' strategies. Likewise, a remarkable shift was noted from deliberate level of reflection to a critical level of reflection when I probed Deborah about possible improvements.

4.3 Thabo's case

4.3.1 Thabo's background and overview of the lessons

Thabo teaches grades 8 to 11 mathematics and was in his seventh year of teaching at the time of data collection. Thabo is recognised as having a good knowledge of mathematics content knowledge, is in possession of a Bachelor's Degree, is widely respected by his colleagues and the Head of Department.

Overview of Lesson 1 (32 minutes and 25 seconds***)***

In Thabo's first lesson with Grade 8D, he started by telling the class that they will be doing addition, subtraction, multiplication and division of common fractions. Six tasks were provided to the learners, and in each case, Thabo invited learners to the board to work out the task. This is then followed with classroom discussion on the learner strategy. My focus in each of these tasks was the presence/absence and the quality of Thabo's noticing of the mathematically significant details of learner's actions and strategies in the enactment of the task.

In the first task, $\frac{3}{5}+\frac{1}{2}$, which requires learners to add two common fractions with unlike denominators, two learners went to the board and demonstrated their strategies one after the other before the teacher proceeded to the second task. The second task, $\frac{3}{5}+\frac{1}{3}$ posed by Thabo, is similar to the first task in that it requires learners to add two common fractions with unlike denominators. Two learners' strategies were also recorded in this task. The third task, $2\frac{1}{3}+\frac{3}{4}$, requires learners to add a mixed fraction to a proper fraction of unlike denominator. Similarly, two learners' strategies were recorded in the third task. Three learners' strategies were recorded in the fourth task, $\frac{3}{4}-\frac{1}{2}$, which requires learners to subtract one common fraction from another in a situation where the denominator of the subtrahend is a factor of the denominator of the minuend. In the fifth task, $\frac{2}{3}of\frac{4}{5}$, which requires the learners to determine the fraction of a fraction, no learner's strategy was obtained until the teacher revised the task, first to $\frac{2}{3}\times\frac{4}{5}$, and lastly to "Just multiply the two by four to get the numerator; and multiply three by five to get the denominator". Thereafter a learner's strategy was obtained and recorded. Similarly, in the sixth and final task, $20\div\frac{1}{2}$, in this class which requires learners to divide a whole number by a fraction, no learner's strategy was obtained until the teacher

revised the task to "Just turn the half upside down to make two and then multiply the twenty by two". Thereafter a learner's strategy was obtained and recorded.

Overview of Lesson 2 (25 minutes and 46 seconds)

In Thabo's second lesson with Grade 8E, he started by telling the class that they will be doing addition, subtraction, multiplication and division of common fractions. The first three tasks were identical to those of 8D. In the first task, $\frac{3}{5}+\frac{1}{2}$, two learners' strategies were obtained and recorded. One learner's strategy was obtained and recorded in the second task, $\frac{3}{5}+\frac{1}{3}$. In the third task, $2\frac{1}{3}+\frac{3}{4}$, three learners' strategies were obtained and recorded. The fourth and last task, $2\frac{1}{5}+5\frac{1}{4}$, posed by Thabo requires learners to add two mixed fractions. One learner's strategy was obtained and recorded for this task.

Overview of Lesson 3 (24 minutes and 17 seconds)

In Thabo's third lesson with grade 8F, he started by telling the class that they will be doing addition, subtraction, multiplication and division of common fractions. The first four tasks were identical to those of class D, although not in the same order. In the first task, $\frac{3}{5}+\frac{1}{2}$, two learners' strategies were obtained and recorded. Similarly, two learners' strategies were obtained and recorded in the second task, $\frac{3}{5}+\frac{1}{3}$. In the third task, $\frac{3}{4}-\frac{1}{2}$, four learners' strategies were obtained and recorded. Three learners' strategies were obtained and recorded for the fourth task, $2\frac{1}{3}+\frac{3}{4}$. The fifth, $\frac{3}{4}\times\frac{2}{5}$ requires multiply two common fractions, and two learners' strategies were obtained and recorded. The sixth and last task, $\frac{-1}{2}\times\frac{3}{4}$, posed by Thabo requires learners to multiply a negative fraction by positive fraction. One learner's strategy was obtained and recorded for this task.

The analysis of the lesson as presented in the Table below is organised around learners' strategies in response to the abovementioned lessons and tasks, and the related teacher's professional noticing of learner's mathematical thinking as detailed in their (learners') responses.

I noted forty-eight incidents of professional noticing of learner's mathematical thinking across all three skills in the three observed lessons. These incidents consist of sixteen incidents of *attending* skills; sixteen incidents of *interpreting* skills; and sixteen incidents of the skill of *Deciding how to respond on the basis of learner's understanding.* For the forty-

eight noticing incidents which emerged, ten *attending* skills were coded *absence*, one was coded *limited*; three were coded *basic*; and two were coded *robust*. I coded ten *interpreting* skills as absence, one as *limited*; and five as basic. For the skill *of deciding how to respond on the basis of learner's understanding*, ten were coded *absence*, one was coded *limited*; and five were coded *basic*. In the next section, I provide a qualitative analysis on all forty-eight incidents.

Table 4: Overview of objects of noticing and level of professional noticing of learners' mathematical thinking demonstrated by Thabo

4 – SCALE LEVEL OF PROFESSIONAL NOTICING					
NOTICING SKILLS	**Absence (0)**	**Limited (1)**	**Basic (2)**	**Robust (3)**	**Total**
Attending	10	1	3	2	16
Interpreting	10	1	5	0	16
Deciding how to respond	10	1	5	0	16
Total	30	3	13	2	48

Thabo's data overall across the three skills of noticing in all three observed lessons suggests that Thabo's *attending to learner's strategies* is largely in the form of *absence of attending* where I noted 10 instances; followed by *basic attending* where 3 instances were noted; with the next being *robust attending* where 2 instances were noted; and the least being *limited attending* where I noted 1 instance. Similarly, within the *noticing* skill of *interpreting learner's mathematical understanding*, Thabo's data shows a prevalent instance of absence of interpreting where I noted 10; followed by basic interpreting where I noted 5 instances; with the least in this category being limited interpreting where I noted only 1 instance. Notably, there is no instance of robust interpreting. Correspondingly, the data shows that Thabo's skill of *deciding how to respond on the basis of learners' understanding* is predominantly in the form of *absence of deciding how to respond* where 10 instances were noted; followed by *basic deciding how to respond* where I noted 5 instances; with the least being *limited deciding how to respond* where 1 instance was noted. There is also a notable paucity of *robust deciding how to respond.* To exemplify the pattern of the observed noticing by Thabo

qualitatively across the three lessons, I selected excerpts as 'telling cases' that indicated how and the extents to which Thabo notice learners' mathematical thinking, namely: *absence of attending, limited attending, basic attending, robust attending, absence of interpreting, limited interpreting, basic interpreting, absence of deciding how to respond, limited deciding how to respond, basic deciding how to respond.*

Absence of Attending (Level 0)

In terms of the absence of attending category, Thabo's range of responses in the three lessons were predominantly in situations of both incorrect and correct learners' mathematical thinking. In particular, responses in this category were characterised by moving on after acknowledging that the answer is correct/incorrect. Remarkably, there was an instance when a learner's offer was correct but judged as incorrect and was cut short as a result. Below, I exemplify this attending.

In lesson 1, in the context of the task, $2\frac{1}{3}+\frac{3}{4}$, and the learner (L14) who answered the question wrote the whole number (2) in a bracket and then proceeded to draw a five by two table to lay out the multiples of 4 and 3 (starting from 1 to 5 multiples). The learner was in the process of populating the table – 4 in cell 4 and 1; 8 in cell 4 and 2; 3 in cell 3 and 1; and 6 in cell 3 and 2 when he was interrupted apparently was doing the wrong thing and another learner was invited to go and answer the question.

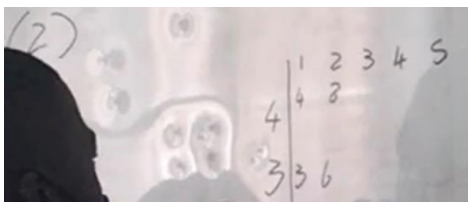

Figure 19: L15's Correct problem-solving strategy of solving $2\frac{1}{3}+\frac{3}{4}$

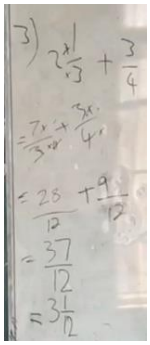

Figure 20: L16's Correct solution of $2\frac{1}{3}+\frac{3}{4}$

The following excerpt played out.

Excerpt 15

Line	Learner/Teacher accounts

1	L15:	Learner correctly draws a table to solve the problem (Figure 19).
2	L16:	"Sir, it is not correct".
3	Thabo:	"Go and take it from him".
4	L16:	Learner erases L14's work and writes own correct answer (Figure 20).
5	Thabo:	"That is correct". (He then moved onto the next task).

Thabo's utterance in line 3 to "go and take it from him", together with his inaction when L16 erased the work of L15 dismisses rather than recognises that strategy. Also, Thabo's action in line 5 of moving to the next task without any reference to L15's strategy is indicative of dismissing that learner's input. In the VSR interview, Thabo reveals that he was confused by that learner's strategy, stating that, "I didn't know what the first learner was doing there. I was just totally confused what the first learner was doing there, using a table or whether it was a tree diagram, or what?", but he made no attempt to probe the learner.

In the VSR interview, Thabo discloses that her focus was "to show all the calculations so that everyone can understand how to do it". In this singular focus though, learners' ways of thinking about the problem situation are dismissed. This incident is similar to the difficulties experienced by prospective teachers and initial participants in Jacobs, Lamb, and Philipp's (2010) study since the authors noted that most initial participants (and prospective teachers) had difficulty to provide evidence of attending to children's strategies, and it is coded within the *'absence of attending'* category. Thabo moved on to the next task after he obtained the solution he wanted from the second learner, without any indication that he recognised L15's strategy as necessary to act upon (Erickson, 2011).

Limited Attending (Level 1)

There is only one instance of Thabo's response in the *limited attending* category, and it emerged in a situation of incorrect solution actions from three consecutive learners. This responding was characterised by a focus on alternative solution method. Below, I provide details about this attending.

In lesson 3, in the context of the task, $2\frac{1}{3} + \frac{3}{4}$, Thabo invited a learner (L17) to solve the problem. L17 added the whole number 2, the numerator 1 and the denominator 3 of the first addend to obtain 6 which he adopted as the numerator of the resulting fraction. Similarly, he

added the numerator 3 and the denominator 4 of the second addend to obtain 7 which he adopted as the denominator of the resulting fraction ($\frac{6}{7}$). Thabo invited two more learners to solve the problem without success in lines 3 to 8, so, he then worked out the solution for the class in lines 9 to 16.

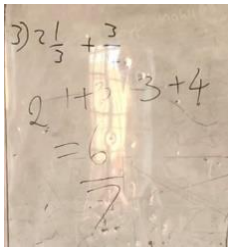

Figure 21: L17's incorrect solution of $2\frac{1}{3}+\frac{3}{4}$

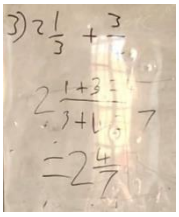

Figure 22: L18's incorrect solution of $\mathbf{2\frac{1}{3}+\frac{3}{4}}$

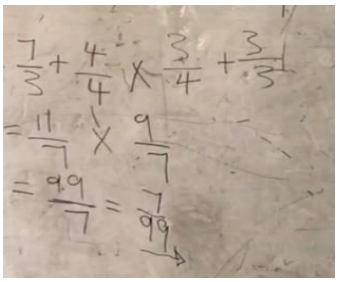

Figure 23: L19's incorrect solution of $2\frac{1}{3}+\frac{3}{4}$

The following excerpt then played out:

Excerpt 16

Line	Learner/Teacher accounts	
1	L17:	Learner works out the solution incorrectly (figure 21).
2	L18:	"It is wrong" (A second learner came forward to provide alternative but incorrect solution Figure 22).
3	Thabo:	"Is it correct?"
4	L19:	"No" (A third learner came forward to provide alternative but incorrect solution Figure 23).
5	Thabo:	"First we must change the mixed fraction to improper fraction".

Thabo's utterance in line 5 that "first, we must change the mixed fraction to improper fraction" is indicative that he is attending to L17's and L18's use of alternative method of solving the problem. This attending also signals inattention to the details in the strategies employed by the learners. In the VSR interview, Thabo reveals that he does not have insight into the learners' solution method, saying, "my thought was that the learners did not understand how to solve the fractions". This incident is similar to the difficulties experienced by limited attending participants in Jacobs, Lamb, and Philipp's (2010) study since the authors noted that limited attending participants focus on alternative issues which suggests

that they do not have a clear understanding of the learner's strategy. I coded the *attending* in this incident as *limited* since Thabo recognized that the L17 and L18 did not convert $2\frac{1}{3}$ to an improper fraction as part of their solution actions, however, he did not demonstrate that he understands the learner's solution method which, if fully developed, is equally a correct and efficient mathematical solution method of the problem.

Basic Attending (Level 2)

In the category of *basic attending*, Thabo's range of responses in the three lessons were primarily in the form of attention to incorrect learners' manipulation of common fractions, particularly, attending to the way learners construct the relationship between corresponding numerators and denominators when they compute with common fractions. In particular, responses in this category were characterised by drawing learner's attentions to the results of their computations. Below, I exemplify this attending.

In lesson 3, in the context of the task, third lesson he posed the question, $\frac{3}{4} \times \frac{2}{5}$, the learner (L20) who answered the question wrote the answer $\frac{6}{20}$ and stopped.

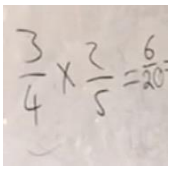

Figure 24: L20's initial solution of $\frac{3}{4} \times \frac{2}{5}$

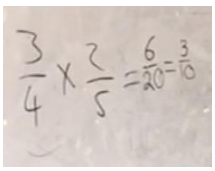

Figure 25: L20's final solution of $\frac{3}{4} \times \frac{2}{5}$

The following excerpt then played out:

Excerpt 17

Line	Learner/Teacher accounts	
1	L20:	Learner works out the solution incompletely (figure 24).
2	Thabo:	"Simplify. Simplify?"
3	L20:	Learner completed the task by simplifying correctly (Figure 25).
5	Thabo:	"Give her a round of applause".

Thabo's directive in line 2 for the learner to simplify is indicative that he is attending to L20's upholding of $\frac{6}{20}$ as a final answer. By stating what L6 should do is further suggestive of a focus on getting the correct answer rather than the mathematical details of the strategy. This incident exemplifies what Jacobs, Lamb, and Philipp's (2010) described as attending that focuses on general features of a strategy but omits the details of how the problem was solved. I coded the *attending* as *basic* since Thabo recognized that the learner had concluded that $\frac{6}{20}$ is the final answer, however, he paid particular attention to getting the correct answer rather than attention to how the learner construct the relationship between the numerator and denominator that constitute the common fraction.

Robust Attending (Level 3)

The two responses that constitute the category of robust attending across the three lessons were largely in the form of probing learners to reflect in details on their solution actions. Precisely, responses in this category were characterised by drawing learner's attentions to their solution action. Below, I exemplify this attending.

In lesson 2, in the context of the task, $\frac{3}{5}+\frac{1}{3}$, Thabo invited learners to the board to answer the question, and the learner (L21) who answered the question obtained $\frac{14}{15}$ as the final answer. In line 2 another learner suggested that $\frac{14}{15}$ has to be simplified, however, L20 ignored the suggestion and upheld $\frac{14}{15}$ as the final answer.

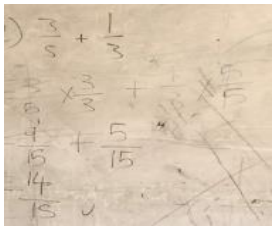

Figure 26: L21's correct solution of $\frac{3}{5}+\frac{1}{3}$

The following excerpt then played out.

Excerpt 18

Line	Learner/Teacher accounts
1	L21: Learner works out the solution correctly (figure 26).
2	L (audience): "Simplify".

3	L21:	Learner ignored the suggestion to simplify.
5	Thabo:	"Is it correct?"
6	Learner:	Silence
7	Thabo:	"Does fifteen go into fourteen?"
8	Learner:	"No".
9	Thabo:	"Great. Give her a round of applause".

In the context of a conflicting view of one learner in line 2 who suggested that L21 should simplify $\frac{14}{15}$, and L20's upholding of $\frac{14}{15}$ as the final answer, Thabo's questioning of the whole class if the answer was correct in line 5 is suggestive of his attending to the learners' thinking of the answer. Likewise, subsequent to the learners' silence in line 6, Thabo probed the learners whether fifteen goes into fourteen, which is suggestive that he is attending to how the learners construct the relationship between the denominator 15 and the numerator 14. This incident exemplifies what Jacobs, Lamb, and Philipp's (2010) described as attending that focuses on the mathematical essence of the strategy. I coded this attending within the *robust attending* category, since Thabo's reflection-oriented question targets the learners' understanding of the relative sizes of the denominator and the numerator as a factor for simplifying a common fraction.

Absence of Interpreting (Level 0)

In terms of the *absence of interpreting* category, Thabo's range of responses in the three lessons were mainly in the form of inattention to response to both incorrect and correct learners' mathematical thinking. As with the absence of attending category, responses in this category were characterised by either moving on to pursue a correct answer without returning to the initially incorrect one; or applauding a correct answer and moving on to the next task. Below, I exemplify this interpreting.

In lesson 1, in the context of the task, $\frac{3}{5}+\frac{1}{2}$, Thabo invited a learner (L22) who obtained $\frac{4}{10}$ as the final answer (Figure 27). In lines 2 to 4, Thabo established that the answer was incorrect and invited another learner (L23) who answered the question correctly (Figure 28). In line 6, Thabo validated the correct answer and sought for alternative solution method, however he could not find any from the learners.

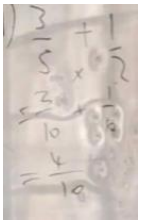

Figure 27: L22's incorrect solution of $\frac{3}{5}+\frac{1}{2}$

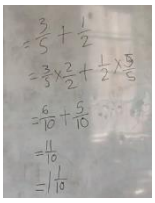

Figure 28: L23's correct solution of $\frac{3}{5}+\frac{1}{2}$

The following excerpt then played out.

Excerpt 19

Line	Learner/Teacher accounts
1	L22: Learner writes an incorrect solution (figure 27).
2	Thabo: "Who can do it correctly?"
3	L23: Learner writes a correct solution (figure 28).
4	Thabo: "That is correct. Another way?"
5	L (audience): "That is the only way".
6	Thabo: "There is another way" (Thabo proceeded to present a different method. He started by writing L.C.D. = 10 on the board – meaning Lowest Common Denominator). "Ten divides by five

In the context of incorrect solution action of L22, Thabo's summons for someone who could do it correctly without any mention of the work to be corrected is suggestive of lack of interpreting. Likewise, subsequent to the correct solution action of L23, Thabo affirmed that L23's solution was correct, but again moved on to seek an alternative solution. Finally, Thabo provided another solution method to the class, without any mention of the solution action being contrasted, before he moved on to the next task. Thabo's action did not indicate that he understands anything from L22's strategy. In the VSR interview, Thabo explained his inattention to L22's strategy stating that "I was very disappointed to find that the learner just multiplied the denominators only". I coded this incident as absence of interpreting because Thabo's actions demonstrate a complete neglect of the strategy.

Limited Interpreting (Level 1)

In terms of the limited interpreting category, there is only one instance which emerged in the same situation of incorrect solution actions from three consecutive learners mentioned in excerpt 16. In particular, the response in this category was characterised by interpreting that is not consistent with the understanding reflected in the learner's strategy. Below, I exemplify this interpreting.

In lesson 3, in the context of the task, $2\frac{1}{3}+\frac{3}{4}$ and incorrect solution actions from three consecutive learners (Figures 21, 22, and 23 in excerpt 16), Thabo worked out the solution for the class in lines 9 to 16. The following excerpt then played out:

Excerpt 20

Line	Learner/Teacher accounts	
1	L17:	Learner works out the solution incorrectly (figure 21).
2	L18:	"It is wrong" (A second learner came forward to provide alternative but incorrect solution Figure 22).
3	Thabo:	"Is it correct?"
4	L19:	"No" (A third learner came forward to provide alternative but incorrect solution Figure 23).
5	Thabo:	"First we must change the mixed fraction to improper fraction".

Thabo's utterance in line 5 that "first, we must change the mixed fraction to improper fraction" is indicative that he considers changing the mixed fraction to an improper fraction as a compulsory first step, and then assumes that the incorrect solution actions of the three learners ensued due to their lack of that knowledge. In the VSR interview, Thabo made this interpreting more discernible by stating that "they (the learners) did not understand anything" before the example he made for them in line 6. This incident exemplifies what Jacobs, Lamb, and Philipp's (2010) categorised as interpreting that is inconsistent with the understanding reflected in learners' strategies because the participants in their study viewed all the three children being observed as having similar understandings. I coded this incident as *limited interpreting* since Thabo's reasoning is not consistent with the understanding reflected in the learners' strategies because L19 correctly changed $2\frac{1}{3}$ to $\frac{7}{3}$. Additionally, Thabo's view that

the learners "did not understand anything" is also not consistent with the understanding reflected in their strategies because L17's and L18's attempt to decompose the mixed fraction into a whole number and a common fraction is a legitimate mathematical idea.

Basic Interpreting (Level 2)

In terms of the basic interpreting category, Thabo's range of responses in the three lessons were fundamentally in the form of interpreting learners' knowledge of simplifying common fraction both in situations of correct and incorrect learners' mathematical thinking. In particular, responses in this category were characterised by calling learners attention to specific aspects of computing common fraction. Below, I exemplify this interpreting.

In lesson 1, in the context of the task, $\frac{3}{5}+\frac{1}{3}$, Thabo invited a learner (L24) who worked out the problem and obtained $\frac{14}{15}$ which he then translated to $1\frac{4}{5}$ but failed to simplify it further.

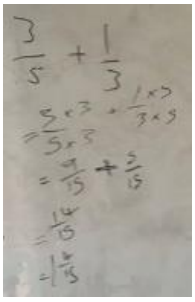

Figure 29: L24's incorrect solution of $\frac{3}{5}+\frac{1}{3}$

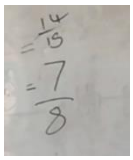

Figure 30: L25's incorrect solution of $\frac{3}{5}+\frac{1}{3}$

The following excerpt played out.

Excerpt 21

Line	Learner/Teacher accounts	
1	L24:	Learner works out the solution incorrectly (Figure 29).
2	Thabo:	"Is it correct?"
3	L(Chorus):	"Yes"
4	Thabo:	"Does fifteen go into fourteen?"
5	L(Chorus):	"No" (A second learner came forward to revise the answer).
6	L25:	Learner works out the solution of $\frac{14}{15}$ incorrectly (Figure 30).
7	Thabo:	"Is it correct?" (Probing L11)

8	L24:	Learner was silence.
9	Thabo:	"Can you divide fifteen by two?"
10	L24:	Learner remains silence.
11	Thabo:	"That answer is here (pointing to $\frac{14}{15}$, and erasing $\frac{7}{8}$)".

In the context of learners' verbal answer affirming an incorrect answer, Thabo's follow-up reflection-oriented question in line 4 is indicative of interpreting that focuses on computational fluency of the learners. Likewise, in the context of L25's answer that $\frac{14}{15} = \frac{7}{8}$, Thabo also responded with a reflection-oriented question that focuses on the computational fluency of L25. In the VSR interview, Thabo articulated his interpreting saying, "the learner (L24) was wrong by saying that fifteen goes once into fourteen", adding further, "he (L24) tried to make that fifteen to go into fourteen. Maybe he did fourteen goes into fifteen by one". This incident echoes what Jacobs, Lamb, and Philipp's (2010) described as interpreting that has specific connections to the learner's strategies but lacked depth. I coded Thabo's *interpreting* in this incident as *basic* since he probed the learners' computational fluency of comparing 14 and 15, as well as the fluency of dividing by two, however, the probing fell short of articulating the multiplicative relationship between numerator and denominator of a fraction.

Absence of deciding how to respond (Level 0)

In terms of the absence of deciding how to respond interpreting category, Thabo's array of responses in the three lessons were mainly in the form of inattention to response to both incorrect and correct learners' mathematical thinking. Like both the absence of attending and absence of interpreting categories, responses in this category were characterised by either moving on to pursue a correct answer without returning to the initially incorrect one; or applauding a correct answer and moving on to the next task. Below, I exemplify this responding.

In lesson 1, in the context of the task, $\frac{3}{4} - \frac{1}{2}$, invited a learner (L26) to answered the question. L25 divided both numerator and denominator of the minuend by the denominator of the subtrahend; and also divided both numerator and denominator of the subtrahend by the denominator of the minuend. However, the solution action of L26 was incorrect. Thabo invited a second learner (L27) who offered a correct solution action.

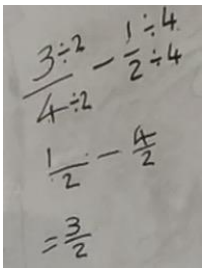

Figure 31: L26's correct solution of $\frac{3}{4} - \frac{1}{2}$

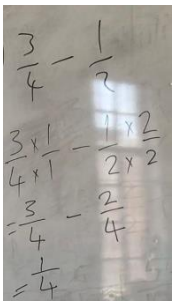

Figure 32: L27's correct solution of $\frac{3}{4} - \frac{1}{2}$

The following excerpt then played out.

Excerpt 22

Line	Learner/Teacher accounts	
1	L26:	Learner works out the solution incorrectly (Figure 31).
2	Thabo:	"Is it correct?"
3	L(Class):	"No"
4	Thabo:	"Go and do it" (Thabo invited a second learner)
5	L27:	Learner works out the solution correctly (Figure 32).
6	Thabo:	"Give him a round of applause" (Thabo then went on to the next task).

In the context of incorrect solution action of L26, Thabo directed a second learner (L27) to "go and do it". However, following the correct solution action of L27 Thabo only applauded L27 and moved on to the next task, leaving open the question what is being rejected in L26's solution and what is being applauded in L26's solution. In the VSR interview, Thabo articulated his responding, saying "I allowed the learner to continue to see what is happening, and fix the mistakes". I coded this incident as *absence of deciding how to respond* because the learner completed the task without "fix[ing] the mistakes", and Thabo's moving to the next task does not constitute a response to L27's strategy.

Limited deciding how to respond (Level 1)

In terms of the limited deciding how to respond category, there is only one instance which emerged within the same task in a situation of incorrect solution actions from three

consecutive learners mentioned in excerpts 16 and 20. In particular, the response in this category was characterised by deciding how to respond that is not consistent with the understanding reflected in the learner's strategy. Below, I exemplify this interpreting.

In lesson 3, in the context of the task, $2\frac{1}{3}+\frac{3}{4}$ and incorrect solution actions from three consecutive learners (Figures 21, 22, and 23), Thabo worked out the solution for the class in lines 9 to 16.

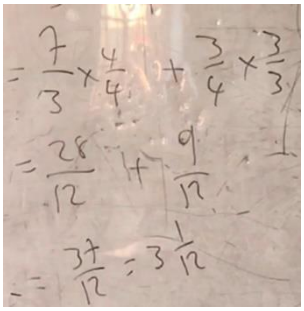

Figure 33: Thabo's solution feedback of $2\frac{1}{3}+\frac{3}{4}$

The following excerpt then played out.

Excerpt 23

Line	Learner/Teacher accounts	
1	Thabo:	"First we must change the mixed fraction to improper fraction. Two times three"
2	L (chorus):	"Six"
3	Thabo:	"Six plus one?"
4	L (chorus):	"Seven"
5	Thabo:	"Over?"
6	L (chorus):	"Over three"
7	Thabo:	"Seven over three plus three over four. Now look at the denominators. We are going to multiply to make them equal. Seven over three multiplied by four over four; plus, three over four multiplied by three over three. Seven times four?"
8	L (chorus):	"Twenty-eight".

9	Thabo:	“Twenty-eight. Over? Four times three”.
10	L (chorus):	“Twelve”.
11	Thabo:	“Plus? Three times three”.
12	L (chorus):	“Nine”.
13	Thabo:	“Over?”
14	L (chorus):	“Twelve”.
15	Thabo:	“Then?”
16	L (chorus):	““Thirty-seven over twelve”.
17	Thabo:	“Mixed fraction? Twelve goes in thirty-seven how many times?”
18	L (chorus):	“Three”.
19	Thabo:	“Remainder?”
20	L (chorus):	“One”.
21	Thabo:	Writes down each step on the board (Figure 32).
22	Thabo:	“Simple, isn’t it?” (Thabo then moved to the next task.

Following incorrect solution actions of three learners in this incident, Thabo proceeded to show the learners the solution in lines 1 to 21. Highlighting the basis of his response in the VSR interview, Thabo stated that “I started to make examples for them so that they can follow”. Moreover, Thabo’s reiteration that the basis of his response is “for the learner that they can solve fractions” suggests that the response was informed by the incorrectness of the solution actions but not the actual understanding reflected in those solutions. This incident exemplifies what Jacobs, Lamb, and Philipp’s (2010) categorised as *deciding how to respond* with little or no reference to building on the children’s understandings or anticipating future strategies. I coded Thabo’s *deciding how to respond in this incident* as *limited* because in showing example for the learners to follow when they solve fractions, Thabo did not address any of the understandings reflected in the strategies of the three learners in this incident.

Basic deciding how to respond (Level 2)

In terms of the *basic deciding how to respond* category, Thabo's range of responses in the three lessons were predominantly in the form of highlighting and emphasising aspects in learner's strategies that contribute to incorrect answer, or opinion. Below, I exemplify this deciding how to respond.

In the first task of Thabo's second lesson he posed the question, $\frac{3}{5}+\frac{1}{2}$, and the learner (L28) who answered the question added the numerators together and obtained 4; added the denominators together and obtained 7; but went on to write $\frac{4}{5}$ as the final answer (Figure 34). In lines 2 and 3, the learners debated the correctness of the denominator, so a second learner (L29) went forward and wrote the answer as $\frac{4}{7}$ (Figure 35). The debate persisted, with majority in support of 7 as the correct denominator. Notably, there was no debate about the addition of the numerators in a context of unlike denominators.

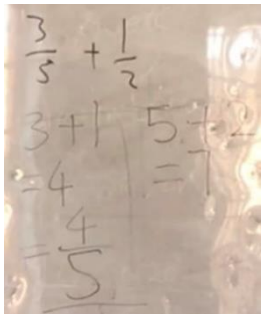

Figure 34: L28's incorrect solution of $\frac{3}{5}+\frac{1}{2}$

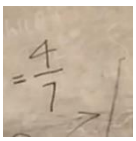

Figure 35: L29's incorrect solution of $\frac{3}{5}+\frac{1}{2}$

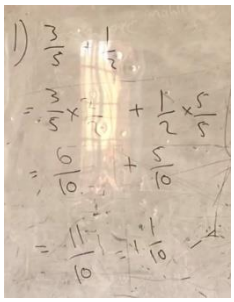

Figure 36: Thabo's feedback solution of $\frac{3}{5}+\frac{1}{2}$

The following excerpt then played out.

Excerpt 24

Line	Learner/Teacher accounts	
1	L28:	Learner works out the solution incorrectly (Figure 34).
2	L (chorus):	"No. not five. It is seven" (Referring to the resulting denominator. Another learner (L29) went forward to revise the answer).
3	L29:	Second learner works out the solution incorrectly (Figure 35).

4	Thabo:	“Is that the answer?”
5	L (chorus):	“Yes. No. Yes. Yes.” (There is argument around which of the two answers is correct)
6	Thabo:	“If you said no, come forward and do it right”
7	Learner:	No one came forward.
8	Thabo:	“I am going to do it, so that you can see how to do it”. You can see there (pointing to denominator 5 of the minuend) is five and here (pointing to denominator 2 of the minuend) is two, we have to make the denominators of the two fractions equal. That means ten. What do we multiply by five to make ten?”
9	L (chorus):	“Two”
10	Thabo:	“What do we multiply by two to make ten?”
11	L (chorus):	“Five”
12	Thabo:	“Three over five multiplied by two over two; one over two multiplied by five over five. Three times two?”
13	L (chorus):	“Six”
14	Thabo:	“Over?”
15	L (chorus):	“Ten”
16	Thabo:	“One times five?”
17	L (chorus):	“Five”
18	Thabo:	“Five over ten. Now we are adding. Six plus five?”
19	L (chorus):	“Eleven”.
20	Thabo:	“Eleven over ten. Mixed fraction? Mixed fraction? Ten goes in eleven, doesn’t it? One remainder one over ten. This is the mixed fraction. You can see.
21	L (chorus):	“Yes”

22	Thabo:	"Ten goes in eleven".
23	L (chorus):	"One"
24	Thabo:	"Remainder?"
25	L (chorus):	"One"
26	Thabo:	"Over ten. This is the mixed fraction (pointing at $1\frac{1}{10}$). Now, you can see". (Thabo writes the steps of the solution simultaneously, Figure 35).

Following incorrect solution actions of two learners in this incident, Thabo questioned the learners, "is that ($\frac{4}{7}$) the answer?" (line4). With that question, two groups of thoughts emerged, one in support of the answer, and the other against the answer. Notably, the debate that ensued was only focused on which of 7 or 5 is correct as the denominator without any mention of how the denominator or the numerator is generated. In response, Thabo highlighted to the learners that the denominators of the two fractions are not the same and that the denominators have to be equal in order to add the two fractions together. Thabo's emphasis on the two unlike denominators echoes what Jacobs, Lamb, and Philipp's (2010) described as clear considerations of children's strategies and understandings in deciding how to respond. I coded Thabo's *deciding how to respond* in this incident as *basic* since he attempted to build on the learner's understanding, albeit lacking specificity and deficient of customization.

4.3.2 Findings from the VSR interviews with Thabo

In this section, I present findings from the VSR professional development interview conducted after the three lesson observations. The VSR interview with Thabo focused largely on his responses to learners' solutions of selected tasks in his lessons, and what he might do differently. Preceding the conversation about the learners' solutions, I began the interview by asking Thabo to talk his objectives for the three lessons.

Excerpt 25

Line	Researcher/Teacher accounts

1	R:	What is your intended goal for this lesson? What do you intend for your learners to be able to do at the end of these lesson?
2	T:	The goal is for them [learners] to be able to make denominators to be the same. So that it can become simple for them to solve fractions.

Thabo's statement in the above excerpt showed *technical level of reflection* about *learners* based on the outcome he expects of his class as a result of the lessons. His utterance shows his awareness of the ease of computation involving common fractions with same denominators. Thabo added on the ease of computation when denominators are the same:

Excerpt 26

Line	Researcher/Teacher accounts	
1	T:	The learner did very well by multiplying the numerator and denominator of the first fraction, and also for multiplying the numerator and denominator of the second fraction to make the denominators to be the same. And to add, and to get the correct answer. I was expecting for the learners to make the denominators to be the same for it to be simple for them to add and get an answer.

Thus far, Thabo's utterance reflects the outcome he anticipated from the learners in their activity during the lessons. His utterance suggests awareness of procedure required for the learners to achieve the desired outcome. There is an absence of a sense of the need to gain insight into the mathematical thinking of the learners, and to build on their understandings. This absence is indicative of a view of mathematics as a finished product (Hersh, 1991). Later in the interview, when I probed Thabo about his thoughts and decisions in a context of three learners' consecutive incorrect manipulations of common fractions involving unlike denominators (excerpt 20 Figures 21, 22 and 23), the following excerpt played out:

Excerpt 27

Line	Researcher/Teacher accounts	
1	R:	Let us look at each of these learners. What is it that the learners did? What mathematics is in these learners' work. Let us start from the first learner.

4	T:	I think the learner is just adding the fractions, and adding the numerators together, and adding the denominators together. Which is wrong because they (denominators) are not the same. He was supposed to make the denominators to be the same first and then add. I think the learner has some misconceptions.

Thabo's statement in line 4 of the above excerpt, shows a shift to a *deliberate level of reflection* on students since Thabo shows awareness that learners bring some misconceptions into the lesson. So, in response to this remark, I drew Thabo's attention to the details in that learner's strategy, as shown in the following excerpt:

Excerpt 28

Line	Researcher/Teacher accounts	
1	R:	What do you think that misconception could have been? If we are going to think deeply into the mathematical misconception that is causing this learner's misconception?
2	T:	It could be that the learner is confusing fractions and numbers (whole numbers). So, the learner is treating the numerators as separate numbers, the denominators as separate numbers.

Again, in response to the reflection-oriented question I asked in this excerpt, Thabo's awareness has deepened to beyond a mere awareness that learners have misconceptions, to include the awareness of the specific misconception that emerged in this particular incident, and in a manner that is consistent with research on learners' mathematical development (Jacobs, Lamb, & Philipp, 2010) of common fraction as shown in the following excerpt:

Excerpt 29

Line	Researcher/Teacher accounts	
9	Researcher:	In that case, how would you respond if you were to repeat this lesson again?
10	Thabo:	To respond to this learner I can say the numerator, one plus three plus what, and the denominator, if we have a denominator, they divide. It's not the same as, maybe, it's not a separate number. They are working together. So, the learner is treating the numerators as separate numbers,

		the denominators as separate numbers, they have to add them or subtract them. So, I have to show the learners that this thing is separate. It's a fraction, a portion of something.

When I probed Thabo on improvements he could make to his lessons, Thabo responded as follows:

Excerpt 30

Line	Researcher/Teacher accounts	
1	T:	Firstly, when you (teacher) are doing exercise to give to the learners, we have seen the learners where there are misconceptions where are the misconceptions, as we have seen a lot of them, there are some misconceptions. So, it's (VSR) adding to me, to my teaching, or to my preparation, so that when I prepare, I explain, you should not do the same mistakes like this. You don't do this, and this, and this. So, if you give them more of activities, it's where you can see their problems, that the learners they face as individual. So, it is important as a teacher when you prepare your lesson to go to the learners, the learners they think differently, allow them to give their thoughts, to give their ideas, to come up with the methods that they know so that you build up on that or you rub that on their mind, or come up with something. So, it is very important that we give them some activities so that we can identify where they are lacking and we fix those mistakes earners that we see.

In the excerpt above, Thabo stated that "it is important as a teacher when you prepare your lesson to go to the learners, the learners they think differently, allow them to give their thoughts, to give their ideas, to come up with the methods that they know so that you build up on that". This statement marked a shift in Thabo's level of awareness since he realised the need to build on learners' understanding rather than transmit knowledge to them. Thabo realised that the absence, in teaching, of discernible insights from learners' strategy details as described by Jacobs et al (2010), inhibits learning. Thabo's remarks in the excerpt above is therefore construed as a transition from *deliberate* (learners have misconceptions so, I have to show them) to *critical* level of reflection (it is important as a teacher when you prepare your lesson to go to the learners, the learners they think differently, allow them to give their

thoughts, to give their ideas, to come up with the methods that they know so that you build up on that) about learners and practice.

Thabo's reflective awareness incited by the VSR interview showed continuing progress as a result of focused view of practice, and focused reflection-oriented questionings about classroom practices. This showed that Thabo had become more conscious of the limitations of his instructions subsequent to learners' contributions in the classroom. This growing alertness and evidence of critical reflection about *practice* and *students,* showed potentials for advancing classroom practice. In the following section, I link findings from Thabo's shifts in teaching practice with shifts in reflective awareness.

4.3.3 Linking Thabo's practice and reflective awareness

Thabo's professional noticing of learners' mathematical thinking, were mainly in the form of subsequent instructions in which insights from discernible details in learners' strategies are lacking. In the forty-eight instances of noticing recorded across Thabo's three lessons, thirty were *absence of noticing*; thirteen were *basic noticing*; three were *limited noticing*; and two were *robust noticing*.

Thabo's *attending pattern*, *interpreting pattern*, and *deciding how to respond pattern* were all dominated by equal instances of *absence of attending, absence of interpreting, and absence of deciding how to respond* respectively. The responses were characterised by moving on after acknowledging that an answer is correct/incorrect. There was a notable instance when a learner's offer was correct but judged as incorrect and was cut short as a result.

Findings from the VSR interviews show that Thabo was at a technical level of reflection when asked to reflect on objectives of his lesson. There was a notable shift to deliberate level of reflection when probed to reflect on the discernible details of his learners' strategies. Likewise, a remarkable shift was noted from deliberate level of reflection to a critical level of reflection when I probed Thabo about possible improvements.

4.4 Synthesis across the two case study teachers

In Table 5 I provide a summary indicating the level of professional noticing by the two teachers across all three component skills of attending, interpreting, and deciding how to respond.

Table 5: Summary of codes used to classify teachers' professional noticing of learners' mathematical thinking

	Absence	Limited	Basic	Robust
Deborah	16% (9/57)	44% (25/57)	40% (23/57)	0% (0/57)
Thabo	63% (30/48)	6% (3/48)	27% (13/48)	4% (2/48)

Overall, all three component skills of professional noticing of learners' mathematical thinking were evident in all of the two case study teachers' observed lessons albeit at very low levels. Remarkably, what this finding suggests is that there is a complete lack of 'robust interpreting' and 'robust deciding how to respond', among the two teachers." These are the highest levels of the component skills of "interpreting" and "deciding how to respond" respectively. However, the quality of professional noticing offered by the two case study teachers differ as illustrated by Table 5. Specifically, the results indicate that there was also a notable absence of robust noticing by Deborah, while Thabo, conversely, demonstrates some amount of robust attending, albeit on two out of 48 instances noticing situations.

Also, the fact that there were more remarks across the two teachers on developing learner's procedures in the VSR interviews, suggests greater focus on the (in)correctness of a learner's strategy rather than the details in those strategies. Intended developments of noticing skills are linked though with any evidence of variation in recognising mathematically significant details of learners' strategies between the teacher's response seen in-the-moment and the reflective awareness remarks during the VSR interviews.

Notably, Thabo's teaching dataset indicates high instances of absence of noticing since he has very strong propensity to ignore incorrect answers and pursue correct solutions; and also, to acknowledge correct answers and move on to the next task. This was attributed to his consciousness about time, as alluded in his VSR interview. He acknowledged in the VSR interviews that modifications to his future teaching will involve shifting priority from time consciousness to gaining insights on learner's understanding through attention to the details in their strategies.

Nevertheless, Thabo, was the only teacher who demonstrated robust noticing albeit in the component skill of attending to learner's strategies by providing probing questions that creates cognitive conflicts. Thabo's responses in these instances built on learners' understanding by making link to integrate/differentiate learner's understandings with related

mathematical concepts. More so, Thabo's data suggests a higher quality of noticing within the instances where noticing was present. Supporting the development of Thabo's professional noticing skills suggests the need to create more awareness of the need for responsive teaching through critical engagement of learners' mathematical thinking.

Limitations in Thabo's patterns of noticing across the three lessons reflects earlier evidence noting that while ability to implement formative assessment (Black, Harrison, Lee, Marshall, & Dylan, 2004; Hodgen & Dylan, 2006) is necessary, they do not offer any assurances for the likelihood of building on learners' understanding, because, his dataset in this study suggests that his knowledge of formative assessment was a necessary influence to a presence of high quality of noticing, this knowledge too may not be sufficient for data informed actions in the classroom.

Conversely, Deborah's data overall suggests a more deliberate attention to professional noticing of learners' mathematical thinking, since only a few instances of absence of noticing emerged in her teaching across the three lessons. Nevertheless, the low quality of noticing within the instances of existing noticing actions, is occasioned by noticing that is inconsistent with the understanding reflected in learner's strategy. Limitations in Deborah's patterns of noticing across the three lessons reflects earlier evidence noting that while ability to identify noteworthy aspects of the class is necessary, it does not offer any assurances for the likelihood of building on learners' understanding, because, her dataset in this study suggests that her knowledge of pedagogically relevant aspects of the class was a necessary contribution to a high presence of noticing, this knowledge too may not be sufficient for data informed actions in the classroom.

What this finding suggests is that the development of the knowledge of both the pedagogically relevant aspects of her teaching together with the knowledge of what is mathematically significant are fundamental for supporting the development of Deborah's professional noticing skills.

4.5 Conclusion

In this chapter I have presented analysis and findings of the pattern of professional noticing of the two case study teachers across three lessons. The findings from this analysis showed that whereas Deborah demonstrated basic noticing skills, which accounts for large instances of the presence of noticing in her teaching, however, the low quality of these noticing was

occasioned by lack of awareness to attend and interpret the mathematically significant details of the learners' strategies. Contrariwise, the analysis also showed that Thabo demonstrated limited noticing skills, which accounts for large instances of the absence of noticing in his teaching, nevertheless, the high quality of the few noticing actions was occasioned by some level of awareness of the mathematically significant details of the learners' strategies. Findings from VSR interviews showed that both teachers have shifted in their awareness of the significance of the details in learner's strategies as a fundamental aspect of their teaching.

CHAPTER 5: SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

Analysing situations of noticing in the two case study teachers' classrooms illuminate how mathematics teachers *attend, interpret* and *decide how to respond* to their learners' mathematical thinking. This provides insight into the extents to which mathematics teachers build on the understanding of particular learners on the basis of insights they (the teachers) have gained from the understanding reflected in those learners' strategies. To bring this research to a close, this chapter begins with a reiteration of the key foci of the research, followed by a summary of key findings which emanated from this research. I then draw attention to the contributions and implications of these findings for South African senior phase mathematics teaching development. Lastly, I present discussion on limitations of the study and suggestions for possible directions for future research.

5.2 Foci of the study

This research had three key foci, which were: firstly, to explore how the two case study teachers *attend, interpret* and *decide how to respond* to their learners' mathematical thinking. Secondly, to understand the extents to which mathematics teachers build on the understanding of particular learners on the basis of insights they (the teachers) have gained from the understanding reflected in those learners' strategies. Professional noticing as discussed in Chapter 2 is conceptualised in the international literature in terms of providing a window into learners' mathematical thinking and also in terms of learning opportunities for both teachers and learners. The third focus is to explore potentials for the two case study mathematics teachers' development in professional noticing through development activity involving VSR interviews.

The central question that guided this study was:

What is the manner and level of grade 8 teachers' practices of professional noticing of learners' mathematical thinking, and how does this transform subsequent to development employing "video-stimulated recall interviews"?

I employed Jacob's et al (2010) framework of professional noticing to analyse the nature and extent of teachers' noticing during their lessons, and I conducted VSR interviews with the teachers thereafter to understand teacher's decision-making process as well is to open up for improvement through reflection-oriented questions. A two-dimensional framework (Geiger's

et al., 2016) which incorporates the notion of level of reflection and object of reflection was used in the analysis of the interviews. In the next section, I present summary of key findings which emanated from my study.

5.3 Key findings of the study

1. Low-level of quality of teacher's noticing of learners mathematical thinking

Findings from this study indicates that the noticing patterns of the case study teachers in this study were predominantly characterised by low-level consideration of learners' solution action. For example, both Deborah and Thabo did not notice and make sense of the discernible details in their learners' strategy, nor noted how these details reflected what each learner understands in the light of research on children's mathematical development. In Deborah's case, she predominantly responds to learner's solution action by repeating the whole solution steps without drawing on any details that emerged in the learner's initial strategy, and her noticing range predominantly between limited and basic. In the case of Thabo, he predominantly pursues correct answer when a learner's solution action is incorrect; or commends and moves on to the next task in response to learner's correct solution actions.

2. VSR interview found to be potentially resourceful for irradiating limitations in teachers' noticing practice that they were not hitherto cognizant of.

Teachers reflective awareness of the limitations of their subsequent response to learners' solution action in the classroom, made possible through the reflection-oriented questions in the VSR interview is indicative of the potential of VSR as a tool for supporting the development of mathematics teachers' professional noticing of their learners' mathematical thinking. For example, by reflecting on the learners' solution actions and making justifications for their actions, both of the two case study teachers demonstrated deliberate level of reflective awareness. Additionally, while envisaging alternatives to his teaching actions subsequent to learners' solution actions, Thabo, came to a critical level of reflective awareness, asserting that it is crucially significant for him (and all teachers) to allow learners to complete their solution actions; to get insights from the learners' strategies; and to build on what the learners understand. These transformations exemplify what Geiger et al. (2016) described as the potential in VSR interview as a professional development tool that can be recruited to support teachers to reflect on aspects of their teaching which they were not

cognizant of beforehand; to discover limitations in their practice; and to envision alternative to unproductive action.

5.4 Implications of findings for policy and practice on mathematics teaching

While assessment for formative (Black & Wiliam, 1998) reason is well-known and promoted in South African literature, findings from this study revealed that grade 8 mathematics teachers do not draw on insights from the understanding reflected in the details of their learners' strategies in the light of research on learners' mathematical development, when responding to solution actions of their learners. Linking a teacher's practice of noticing to his/her pedagogical commitments, Erickson (2011) suggests that what the teacher do notice and emphasize in mathematics teaching is a function of that teacher's pedagogical commitments regarding what counts most in mathematics. The author noted further that mathematics teaching for learner understanding requires learning for teachers. This will help mathematics teachers to become relatively attentive to learner's insight in relation to mathematical knowledge. The implication is that there is potential for mathematics teachers to select and engage learners in more intellectually substantive learning tasks rather than one step tasks in their teaching. Another implication is that there is potential for learners' sense making in a context of engagement in intellectually substantive work, rather than drill, to become salient for the teachers in their instruction. Relatedly, Mason (2011) noted that professional noticing is an intentional activity and suggests that developing the discipline of noticing requires continuing professional development opportunities for mathematics teachers to continually grow from mechanically acting out of habit towards acting freshly. Earlier, in their study, Jacobs et al., (2010) noted an improvement in the three component skills of attending, interpreting and deciding how to respond skills by the practicing K–3 teachers and prospective teachers who participated in their study in a context of two years of professional development.

The difficulty experienced by grade 8 mathematics teachers regarding attending, interpreting and deciding how to respond to their learners' strategies suggests that teacher education and teacher development programs in South Africa need to do more to equip both pre-service and in-service mathematics teachers with better ways of noticing student understanding. In recent years, research by Mason (2011) have proposed that teachers need to be supported to "pre-pare" to notice in the moment, that is, to have come to mind appropriately, and to 'post-pare' by reflecting on the recent past to select what they want to notice". During such program, the

teacher is responsible for using incidents from observed lessons "as stimuli for them to recall and then analyse related incidents from their own teaching" (Mason, 2011, p36), and the implication is that there is potential in this practice for teachers to use "what they saw on the video as a combination of metonymic triggers into, and metaphoric resonances with, their own past experiences" (Mason, 2011, p36). Such support as highlighted by Mason (2011) study is necessary for in-service mathematics teachers in South Africa to better notice and build on learners' verbal- and written strategy explanations, particularly "enabling them to avoid the habitual and to act freshly" (Mason, 2011)

5.5 Limitations of the study

A foremost limitation of this study was the number of teachers in this study. This is attributable to school's strategy of coping with COVID-19 by allocating temporarily employed teachers who have little or no teaching qualification/experience to grade 8 mathematics classes and re-allocating the more experienced and qualified teachers to grades 10 to 12 classes, so there was dearth of mathematics teachers in grades 8 and 9 to be recruited into the study.

The second phase of lesson observation and video recording was abolished as a result of limited school days due to COVID-19. This impacted the method of analysis, so the result of data analysis from the VSR was compared with the result of data analysis from the first phase involving actual teaching moments. This comparison is used to highlight any possible shifts in participants' skills in each of the three components. This deficit of post VSR interview lesson observation is another limitation, which is also attributable to curriculum trimming occasioned by Government's strategy of coping with COVID-19 by reducing school days for learners and I could not conduct the second round of lesson observations for the mathematics teacher's common fraction lessons as envisaged. As a result, I excluded the post VSR interview phase from the study. Consequently, I could not draw any comparisons of the teachers' pre-VSR interview lessons with any post-VSR interview lessons.

5.6 Directions for future research

My suggestion for future research is to explore the development of mathematics teachers' noticing skills by considering the following:

1. Involvement of substantial number of mathematics teachers, involving both inexperienced and experienced teachers.
2. Carried out over a longer period to allow for pre-VSR round of lesson observation, teacher support through VSR interview, and post-VSR round of lesson observation.

5.7 Conclusion

Although reform-based mathematics teaching has been adopted in South Africa, little empirical evidence has hitherto been available in relation to mathematics teachers' practice of noticing. Teacher noticing in mathematics education is conceptualized as a practice that is pivotal to achieving the goals of effective teaching that are advocated in reform documents. The findings in this study showed that low-level of quality of teacher's noticing of learners mathematical thinking is prevalent among mathematics teachers, with VSR interview found to be a potential resource for irradiating limitations in the teachers' noticing practice that they were not hitherto cognizant of.

5.8 Self-reflection

This study was inspired by the difficulties I experienced in my job of supporting the development of in-service mathematics teachers in previously disadvantaged schools around Gauteng province. It is commonplace to find teachers who attribute poor mathematical performance of their learners to forgetfulness by learners. However, in most of these cases there was no evidence of how learners performed during the lesson. Having recognized that it is not feasible for a teacher to check the work of every learner and provide feedback in the moment to each of the learner's work, I began to encourage teachers to take pictures of learners' written work with a phone. The teachers can then peruse these pictures after the lesson to get an idea of learners' understanding of the concept during the teaching. With this experience, I continued to ponder how do expert teachers notice the details in the strategies of their learners in the moment and also build on the understanding reflected in those strategies during the same teaching. This research journey has made significant impacts on my own development as a teacher trainer as well as an apprentice researcher. Focusing on professional noticing of learners' mathematical thinking has made me more discerning of mathematically significant details in learners' solution actions, with particular lens of research on learners' mathematical development. With my engagement in this study, I recognize that my preparation for a lesson has expanded to include exploration of research on learners'

mathematical development of the concepts and identifying the corresponding aspects that are mathematically significant which then becomes units of pedagogically relevant aspects that guide what I consider necessary to act on in the moment.

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APPENDIX A: Sample of VSR interview schedule

Deborah interview questions:

Introduction

I would thank Deborah for allowing me to have video recorded her lesson and making time available for this interview to discuss about her lesson.

The purpose of this interview is to gain insight on how you understand, interpret and respond to learners' mathematical thinking during the lesson

Semi-structured questions

Lesson A:

1. Opening conversation

What was your intended goal for this lesson (lesson A)? In other words, what were you hoping that learners would be able to do, or understand, at the end of the lesson?

2. Discussion

Professional noticing of learner's mathematical thinking. (Selected episodes from the lesson)

Episode 1 (Task1-A)

In this episode, you posed the question, $6\frac{3}{7} + 3\frac{1}{7}$, and in response, and two learners demonstrated their strategies as displayed in the adjoining figure (I will show the printout of the learner's strategy to the teacher).

(a) What were your thoughts at that moment?

   i. Why do you think that inclusion of the denominator at that stage is necessary?

   ii. Why do you think that it is necessary for you to do the conversion of $\frac{67}{7} to\ 9\frac{4}{7}$ (from improper to mixed fraction) for the learners?

   iii. Why do you think that the problem was due to lack of practice by the learners?

(b) **Reflection:** Let us think together.

   i. What is it that these learners did? Let us start with the first learner and then the second learner

   ii. What do you notice?

(c) If you were to repeat this lesson, will you respond to the learners' strategies in task 1 in the same way or differently? If differently, please can you elaborate.

Episode 2 (Task2-A)

In this episode from the second task of this lesson, you posed the question, $4\frac{6}{9}+5\frac{5}{9}$, and in response, a learner demonstrated the strategy displayed in the adjoining figure (I will show the printout of the learner's strategy to the teacher).

(a) What were your thoughts at that moment?

(b) Why do you think that inclusion of $\frac{42+50}{9}$ is necessary?

(c) **Reflection:** Let us think together.

i. What is it that these learners did? Let us start with the first learner and then second learner

ii. What do you notice?

(d) If you were to repeat this lesson, will you respond to the learners' strategies in task 2 in the same way or differently? If differently, please can you elaborate.

Episode 3 (Task3-A)

This episode emanates from the third task of this lesson in which you posed the question, $2\frac{8}{3}-\frac{9}{3}$, and in response, the first learner demonstrated the strategy displayed in the adjoining figure (I will show the printout of the learner's strategy to the teacher).

(a) What were your thoughts at that moment?

(b) Why do you think that a correction is necessary?

(c) **Reflection:** Let us think together.

i. What is it that this learner did?

ii. What do you notice?

(d) If you were to repeat this lesson, will you respond to the learners' strategies in task 3 in the same way or differently? If differently, please can you elaborate.

Episode 4 (Task5-A)

This episode emanates from the fifth task of this lesson in which you posed the question, $\frac{9}{7}\times\frac{8}{9}$ and in response, two learners worked together on this task and demonstrated the

strategy displayed in the adjoining figure (I will show the printout of the learners' strategy to the teacher).

(a) What were your thoughts at that moment?

(b) Why do you think an alternative strategy is necessary?

(c) **Reflection:** Let us think together.

i. What is it that these learners did? Let us start with the first learner and then second learner

ii. What do you notice?

(d) If you were to repeat this lesson, will you respond to the learners' strategies in task 5 in the same way or differently? If differently, please can you elaborate.

Episode 5 (Task6-A)

This episode emanates from the sixth task of this lesson in which you posed the question, $\frac{3}{7} \times \frac{9}{5}$ and in response, two learners worked together on this task and demonstrated the strategy displayed in the adjoining figure (I will show the printout of the learner's strategy to the teacher).

(a) What were your thoughts at that moment?

(b) Why do you think ~~an alternative strategy~~ is necessary?

(c) **Reflection:** Let us think together.

i. What is it that these learners did? Let us start with the first learner and then second learner

ii. What do you notice?

(d) If you were to repeat this lesson, will you respond to the learners' strategies in task 5 in the same way or differently? If differently, please can you elaborate.

Lesson B:

1. Opening conversation

What was your intended goal for this lesson (lesson B)? In other words, what were you hoping that learners would be able to do, or understand, at the end of the lesson?

2. Discussion

Episode 6 (Task1-B)

In this episode, you posed the question, $\frac{3}{8} + 4\frac{2}{8}$, and two learners demonstrated their strategies as displayed in the adjoining figure (I will show the printout of the learners' strategies to the teacher).

(a) What were your thoughts at that moment?

(b) Why do you think that repeating the strategy is necessary?

(c) **Reflection:** Let us think together.

i. What is it that these learners did? Let us start with the first learner and then second learner

ii. What do you notice?

(d) If you were to repeat this lesson, will you respond to the learners' strategies in task 1 in the same way or differently? If differently, please can you elaborate.

Episode 7 (Task2-B)

In this episode from the second task of this lesson, you posed the question, $7\frac{2}{5} + 3\frac{3}{5}$, and two learners demonstrated their strategies as displayed in the adjoining figure (I will show the printout of the learner's strategy to the teacher).

(a) What were your thoughts at that moment?

(b) Why do you think that repeating the strategy is necessary?

(c) **Reflection:** Let us think together.

i. What is it that these learners did? Let us start with the first learner and then second learner

ii. What do you notice?

(d) If you were to repeat this lesson, will you respond to the learners' strategies in task 2 in the same way or differently? If differently, please can you elaborate.

Episode 8 (Task3-B)

In this episode from the second task of this lesson, you posed the question, $\frac{8}{3} - \frac{5}{3}$, and in response, a learner demonstrated the strategy displayed in the adjoining figure (I will show the printout of the learner's strategy to the teacher).

(a) What were your thoughts at that moment?
(b) Why do you think that repeating the strategy is necessary?
(c) **Reflection:** Let us think together.
    i. What is it that these learners did? Let us start with the first learner and then second learner
    ii. What do you notice?
(a) If you were to repeat this lesson, will you respond to the learners' strategies in task 3 in the same way or differently? If differently, please can you elaborate.

Episode 9 (Task5-B)

In this episode from the fifth task of this lesson, you posed the question, $5\frac{6}{9} - 4\frac{2}{8}$, and in response, a learner demonstrated the strategy displayed in the adjoining figure (I will show the printout of the learner's strategy to the teacher).

(a) What were your thoughts at that moment?
(b) Why do you think that repeating the strategy is necessary?
(c) **Reflection:** Let us think together.
    i. What is it that these learners did? Let us start with the first learner and then second learner
    ii. What do you notice?
(d) If you were to repeat this lesson, will you respond to the learners' strategies in task 5 in the same way or differently? If differently, please can you elaborate.

Lesson C:

1. Opening conversation

What was your intended goal for this lesson (lesson 3)? In other words, what were you hoping that learners would be able to do, or understand, at the end of the lesson?

2. Discussion

Episode 10 (Task3-C)

In this episode from the third task of this lesson you posed the question, $3\frac{19}{8} + 2\frac{5}{8}$, and a learner demonstrated the strategies as displayed in the adjoining figures (I will show the printout of the learner's strategy to the teacher).

(a) What were your thoughts at that moment?

(b) Why do you think that repeating the strategy is necessary?

(c) **Reflection:** Let us think together.

i. What is it that these learners did? Let us start with the first learner and then second learner

ii. What do you notice?

(d) If you were to repeat this lesson, will you respond to the learners' strategies in task 3 in the same way or differently? If differently, please can you elaborate.

Conclusion

(a) Is there anything you will like to let me know which I have not asked? Or comment?

(b) Again, thank you very much for your time.

Thabo's interview questions:

Introduction

I would thank Thabo for allowing me to have video recorded his lesson and making time available for this interview to discuss about her lesson.

The purpose of this interview is to gain insight on how you understand, interpret and respond to learners' mathematical thinking during the lesson

Semi-structured questions

Lesson D:

1. Opening conversation

What was your intended goal for this lesson (lesson D)? In other words, what were you hoping that learners would be able to do, or understand, at the end of the lesson?

2. Discussion

Professional noticing of learner's mathematical thinking. (Selected episodes from the lesson)

Episode 11 (Task1-D)

In this episode, you posed the question, $\frac{3}{5} + \frac{1}{2}$, and in response, two learners demonstrated their strategies as displayed in the adjoining figure (I will show the printout of the learners' strategies to the teacher).

(a) What were your thoughts at that moment?

(b) Why do you think an alternative strategy is necessary?

(c) **Reflection:** Let us think together.

i. What is it that these learners did? Let us start with the first learner and then second learner

ii. What do you notice?

(d) If you were to repeat this lesson, will you respond to the learners' strategies in task 1 in the same way or differently? If differently, please can you elaborate.

Episode 12 (Task2-D)

In this episode, you posed the question, $\frac{3}{5}+\frac{1}{3}$, and in response, two learners made contributions as displayed in the adjoining figures (I will show the printout of the learner's strategy to the teacher).

(a) What were your thoughts at that moment?

(b) **Reflection:** Let us think together.

i. What is it that these learners did? Let us start with the first learner and then second learner

ii. What do you notice?

(c) If you were to repeat this lesson, will you respond to the learners' strategies in task 1 in the same way or differently? If differently, please can you elaborate.

Episode 13 (Task3-D)

In this episode, you posed the question, $2\frac{1}{3}+\frac{3}{4}$, and in response, two sets of learners made contributions as displayed in the adjoining figures (I will show the printout of the learners' strategies to the teacher).

(a) What were your thoughts at that moment?

(b) **Reflection:** Let us think together.

i. What is it that these learners did? Let us start with the first learner, the second and then third learner

ii. What do you notice?

(c) If you were to repeat this lesson, will you respond to the learners' strategies in task 1 in the same way or differently? If differently, please can you elaborate.

Episode 14 (Task4-D)

In this episode, you posed the question, $\frac{3}{4}-\frac{1}{2}$, and in response, you received three learners' strategies as displayed in the adjoining figures (I will show the printout of the learners' strategies to the teacher).

(a) What were your thoughts at that moment?

(b) **Reflection:** Let us think together.

i. What is it that these learners did? Let us start with the first learner, the second and then third learner

ii. What do you notice?

If you were to repeat this lesson, will you respond to the learners' strategies in task 1 in the same way or differently? If differently, please can you elaborate.

Episode 15 (Task5-D)

Account of the episode: In this episode, you posed the question, $\frac{2}{3} of \frac{4}{5}$, but none of the learners came forward to answer the question. **In response,** we saw that you revised the question to $\frac{2}{3} \times \frac{4}{5}$.

(a) **Question:** What were your thoughts at that moment?

(b) **Reflection:** Let us think together.

i. What is it that these learners could do that can contribute to their solution to this task:
   - Based on their knowledge of representing multiplication of whole numbers? (Like, x group of y)?
   - Based on their everyday knowledge of fractions? (Like half price of a product)?

ii. What mathematical thinking do you identify in these abilities?

iii. What could be the implications of in/abilities to complete this task when these learners encounter problems involving fraction of a fraction in other domains of mathematics (like, compound interest, depreciation, inflation, in financial mathematics or geometric sequence)?

(c) If you were to repeat this lesson, will you respond to the learners' strategies in task 1 in the same way or differently? If differently, please can you elaborate.

Lesson E:

1. Opening conversation

What was your intended goal for this lesson (lesson E)? In other words, what were you hoping that learners would be able to do, or understand, at the end of the lesson?

2. Discussion

Professional noticing of learner's mathematical thinking. (Selected episodes from the lesson)

Episode 16 (Task1-E)

In this episode, you posed the question, $\frac{3}{5}+\frac{1}{2}$, and in response, two learners made contributions as displayed in the adjoining figure (I will show the printout of the learners' strategies to the teacher).

Account of the episode: The first learner did not get the answer correctly. The second learner revised the answer of the first but the revised answer was also incorrect and the whole class upheld the revised answer as correct. **In response,** we saw that you did not engage with what these learners did but instead proceeded to demonstrate the solution method to the whole class.

(a) **Question:** What were your thoughts at that moment?

(b) **Reflection:** Let us think together.

i. What is it that these learners did:
   - when the first learner:
     - added three and one (the numerators of the two addends) to obtain four for the numerator of the merged fraction?
     - added five and two (the denominators of the two addends) to obtain seven which he rejected, and proceeded to uphold five as the denominator of the merged fraction?
   - when the second learner:
     - retained the four as the numerator, but revised the denominator from five to seven?

ii. What mathematical thinking do you identify in these ideas (presented by: L1? L2?

iii. What could be the implications of these strategies:

- For the first learner if the task involves addends in which one denominator is a multiple of the other (like $\frac{3}{4}+\frac{1}{2}$)?
- For the second learner:
    - if the denominators of the addends are the same (like $\frac{3}{4}+\frac{1}{4}$)?

(c) If you were to repeat this lesson, will you respond to the learners' strategies in task 1 in the same way or differently? If differently, please can you elaborate.

Episode 17 (Task3-E)

In this episode, you posed the question, $2\frac{1}{3}+\frac{3}{4}$, and in response, three learners demonstrated their strategies as displayed in the adjoining figures (I will show the printout of the learners' strategies to the teacher).

Account of the episode: These learners came one after another but none of them got the answer correctly. **In response,** we saw that you contested the result of simplifying $\frac{14}{15}$, but you did not pay attention to this learner's steps in getting to that answer. We also saw that another learner came to the board to revise the simplification of $\frac{14}{15}$.

Similarly, the strategy presented by the second learner was incorrect. **In response,** we also saw that you queried the result, ignored the strategy of that learner, erased the learner's solution and then pronounced that the answer was already obtained.

(d) **Question:** What were your thoughts at that moment?

(e) **Reflection:** Let us think together.

i. What is it that these learners did:
    - when the first learner:
        - translated the first fraction to the numerator (of the merged fraction) by adding the whole number, the numerator and the denominator?
        - translated the second fraction to the denominator (of the merged fraction) by adding the numerator and the denominator?
    - when the second learner:
        - merged the two fractions into one mixed fraction?
    - when the third learner:

- added $\frac{4}{4}$ and $\frac{3}{3}$ respectively to the two fractions to translate the denominators to seven?
- Multiplied the two fractions together.
- Concluded with an inverse of that product.

ii. What mathematical thinking do you identify in these ideas (presented by: L1? L2? L3?

iii. What could be the implications of these strategies:

- For the first learner:
  - if the order of the addends is reversed (like $\frac{3}{4} + 2\frac{1}{3}$)?
  - If there are three addends (like $2\frac{1}{3} + \frac{3}{4} + \frac{1}{2}$)?
- For the second learner:
  - if the two denominators are the same (like $2\frac{1}{3} + \frac{1}{3}$)?
- For the third learner:
  - if the two denominators are the same
  - If the denominator of the product is one (like $\frac{2}{1}$)?

If you were to repeat this lesson, will you respond to the learners' strategies in task 1 in the same way or differently? If differently, please can you elaborate.

Conclusion

(a) Is there anything you will like to let me know which I have not asked? Or comment?
(b) Again, thank you very much for your time.

APPENDIX B: GDE ethics clearance

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	24 November 2020
Validity of Research Approval:	08 February 2021– 30 September 2021 2019/690
Name of Researcher:	Olubodun J
Address of Researcher:	133 Camelthorn Street
	Leondale
	Germiston
Telephone Number:	078 021 8294
Email address:	391357@students.wits.ac.za
Research Topic:	Grade 5 mathematics teachers' professional noticing of learners' thinking of common fractions
Type of qualification	MEd
Number and type of schools:	1 Primary School
District/s/HO	Johannesburg East

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

25/11/2020

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. *Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. ***Because of COVID 19 pandemic researchers can ONLY collect data online, telephonically or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate. The approval letter will then indicate the type of arrangements that have been made with the school.***
4. ***The Researchers are advised to make arrangements with the schools via Fax, email or telephonically with the Principal.***
5. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
6. *A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
7. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
8. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
9. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
10. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
11. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
12. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
13. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
14. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
15. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
16. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

.....

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 24/4/2020

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX C: University of the Witwatersrand ethics clearance

WITS SCHOOL OF EDUCATION

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: 2021ECE069M**
PROJECT TITLE	**Grade 5 mathematics teachers' professional noticing of learners' thinking of common fractions.**
INVESTIGATOR	**JULIUS OLUBODUN**
SCHOOL/DEPARTMENT OF INVESTIGATOR	WITS SCHOOL OF EDUCATION
DATE CONSIDERED	16 August 2021
DECISION OF THE COMMITTEE	Approved unconditionally
EXPIRY DATE	Date of submission of the project report

ISSUE DATE OF CERTIFICATE 25 August 2021 **CHAIRPERSON** (Dr Paul Goldschagg)

cc: Supervisor: Dr. Lawan Abdulhamid

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** emailed to the Ethics Office: Matsie.Mabeta@wits.ac.za .

I fully understand the conditions under which I am authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure be contemplated from the research procedure as approved I/~~we~~ undertake to resubmit the protocol to the Committee.

Signature 26th August 2021 **Date**

PLEASE QUOTE THE PROTOCOL NUMBER ON ALL ENQUIRIES

APPENDIX D: Participants information letters and consent forms

The Principal,

Date

Dear Principal

My name is Julius Olubodun and I am a Part-time Masters student in the School of Education at the University of the Witwatersrand. As part of my studies, I have to undertake a research project, and I am investigating Supporting development of grade 8 mathematics teachers' professional noticing of learners' thinking of common fractions using Video-Stimulated Recall under the supervision of Dr Lawan Abdulhamid. The aim of this research project is to give insight into grade 8 mathematics teachers' practices of professional noticing of learners' mathematical thinking and their use of learners' conception as a guide to inform next steps of instructions during their lessons. Furthermore, the study will also seek to investigate possible development of these practices in a context of a collaborative discussion between myself as a researcher and each teacher while watching the teacher's video recorded lessons. The aim is to use reflection-oriented questions to support teachers to reflect on their practices and possibly alternative and more efficient pedagogies may emerge in this process.

This activity will involve lesson observation of two rounds of three sequential lessons and will take around 30 minutes each. It will also involve interview which will take around 30 minutes. Three grade 8 teachers will be involved in this study. They will be selected based on these criteria: willingness to voluntarily participate in the study; they have been teaching grades 4 to 6 mathematics for over 5 years and are currently teaching grade 8; they are qualified mathematics teachers. As part of this project, I would like to invite your school to take part by allowing me to observe your teachers' lessons and to conduct follow-up interview with them. The reason why I have chosen your school is because the grade 8 mathematics teachers in your school are implementing CAPS (Curriculum and Assessment Policy Statements) and are guided by the Annual Teaching Plan (ATP) prescribed by the Gauteng's Department of Education. I am therefore inviting your school to participate in this research.

I will collaborate with each teacher to decide the days and times of lesson observations in line with their Annual Teaching Plan (ATP). The interview will be done at your school after contact time on a date and time that is suitable for each teacher. This will be arranged with

each teacher beforehand. With the permission of your teachers, I would also like to video record the lessons and audio record the interview using digital devices. These recordings will be stored in a password protected computer and only the researcher and the supervisor will have access to these recordings. It will be deleted after 3 to 5 years.

The consent of your learners' parents/guardians will be obtained and with the cooperation of each teacher, learners whose parents refuse permission will be seated in such a way that they are not included in the video recording, while we ensure that they are not excluded from the class, which would disadvantage them academically. Face-blurring software will be used in the recorded video to preserve anonymity. The research participants will not be advantaged or disadvantaged in any way. Participation in this study is voluntary, so participants will be reassured that they can withdraw their permission at any time during this project without any penalty. The participants will not be paid for this study. Kindly note that there are no financial rewards for participants in this study.

I will ensure privacy of the participants during the interview by making sure that we sit in a closed room where no one can overhear our conversation. The interview will be completely confidential and anonymous as I will not be asking the name or any identifying information of the participants. The information given to me will be held securely and not disclosed to anyone else. I will be using a pseudonym (false name) to represent all their participation in my final research report. The name and identity of the school and all participants will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data which emanates from this study. All research data will be destroyed within 3 – 5 years after completion of this study.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Thank you very much for your assistance.

Yours sincerely,

Researcher's name: Julius Olubodun
Researcher's email: juliusikolubodun@gmail.com or 391357@students.wits.ac.za
Researcher's contact number: 078 021 8294

Supervisor’s name: Dr Lawan Abdulhamid

Supervisor’s email: Lawan.Abdulhamid@wits.ac.za

Supervisor’s telephone number: 011 717 3468Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Information sheet: Teacher

Dear Teacher

My name is Julius Olubodun and I am a Part-time Masters student in the School of Education at the University of the Witwatersrand. As part of my studies, I have to undertake a research project, and I am investigating Supporting development of Grade 8 mathematics teachers' professional noticing of learners' thinking of common fractions using Video-Stimulated Recall under the supervision of Dr Lawan Abdulhamid. The aim of this research project is to give insight into grade 8 mathematics teachers' practices of professional noticing of learners' mathematical thinking and their use of learners' conception as a guide to inform next steps of instructions during their lessons. Furthermore, the study will also seek to investigate possible development of these practices in a context of a collaborative discussion between myself as a researcher and each teacher while watching the teacher's video recorded lessons. The aim is to use reflection-oriented questions to support teachers to reflect on their practices and possibly alternative and more efficient pedagogies may emerge in this process.

This activity will involve lesson observation of two rounds of three sequential lessons and will take around 30 minutes each. It will also involve interview which will take around 30 minutes. As part of this project, I would like to invite you to take part by allowing me to observe your lessons and to conduct follow-up interview with you. I will collaborate with you to decide the days and times of lesson observations in line with your Annual Teaching Plan (ATP). The interview will be done at your school after contact time on a date and time that is suitable for you. This will be arranged with you beforehand. With your permission, I would also like to video record the lessons and audio record the interview using digital devices. These recordings will be stored in a password protected computer and only the researcher and the supervisor will have access to these recordings. It will be deleted after 3 to 5 years.

The consent of your learners' parents/guardians will be obtained and with your cooperation, learners whose parents refuse permission will be seated in such a way that they are not included in the video recording, while we ensure that they are not excluded from the class, which would disadvantage them academically. Face-blurring software will be used in the recorded video to preserve anonymity. Your participation is voluntary, so you are free to withdraw your participation at any time without any prejudice and/or penalty. Kindly note that there are no financial rewards for your participation in this study.

I will ensure your privacy during the interview by making sure that we sit in a closed room where no one can overhear our conversation. The interview will be completely confidential and anonymous as I will not be asking for your name or any identifying information. The information you give to me will be held securely and not disclosed to anyone else. I will be using a pseudonym (false name) to represent your participation in my final research report. Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data which emanates from this study. All research data will be destroyed within 3 – 5 years after completion of this study.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Thank you very much for your assistance.

Yours sincerely,

Researcher's name: Julius Olubodun
Researcher's email: juliusikolubodun@gmail.com or 391357@students.wits.ac.za
Researcher's contact number: 078 021 8294

Supervisor's name: Dr Lawan Abdulhamid
Supervisor's email: Lawan.Abdulhamid@wits.ac.za
Supervisor's telephone number: 011 717 3468

Teacher's Consent Form:

Kindly complete the consent form and return the reply slip below indicating your willingness to be observed/video-recorded and interviewed/audio-recorded for my research project called:

Grade 8 mathematics teachers' professional noticing of learners' thinking of common fractions.

I, _____

Give/do not give (please delete as appropriate)

Permission to be observed in class

I agree to be observed in class. YES/NO

Permission to be video-recorded

I agree to be video-recorded during the lesson observation YES/NO

I know that the video-recordings will be used for this project only YES/NO

Permission to be interviewed

I agree to be interviewed. YES/NO

Permission to be audio-recorded

I agree to be audio-recorded during the interview YES/NO

I know that the audio-recordings will be used for this project only YES/NO

[ ] I know that I don't have to answer all the questions and that I may withdraw from the study at any time without prejudice and/or penalty.

[ ] I am aware that the researcher will keep all information confidential in all academic writing.

[ ] I am aware that the audio and video-recordings will be destroyed within 3—5 years after completion of the project.

Teacher's Signature: _____ Date: _____

Information sheet: Learner

Dear learner

My name is Julius Olubodun and I am a Part-time Masters student in the School of Education at the University of the Witwatersrand. As part of my studies, I have to undertake a research project, and I am investigating Supporting development of Grade 8 mathematics teachers' professional noticing of learners' thinking of common fractions using Video-Stimulated Recall under the supervision of Dr Lawan Abdulhamid. The aim of this research project is to give insight into grade 8 mathematics teachers' practices of professional noticing of learners' mathematical thinking and their use of learners' conception as a guide to inform next steps of instructions during their lessons. Furthermore, the study will also seek to investigate possible development of these practices in a context of a collaborative discussion between myself as a researcher and each teacher while watching the teacher's video recorded lessons. The aim is to use reflection-oriented questions to support teachers to reflect on their practices and possibly alternative and more efficient pedagogies may emerge in this process.

In doing so, I will observe and record video of your teacher as s/he teaches you mathematics. I want you to know that I will be recording using a video camera and I need you to agree that it is alright to record video of your teacher's teaching of mathematics while you are in the class. I also want you to know that you may appear in the video recording. These recordings will be stored in a password protected computer and only the researcher and the supervisor will have access to these recordings. It will be deleted after 3 to 5 years. Should you or your parents/guardians refuse permission, your teacher and I will make a sitting arrangement for you not to be included in the video recording, while we ensure that you are not excluded from the class, which would disadvantage you academically. Face-blurring software will be used in the recorded video to preserve anonymity.

Remember, this is not a test. It is not for marks and it is not compulsory, which means that you don't have to appear in the video. Also, if you decide halfway through that you prefer not to appear in my video recording, this is completely your choice and I will position the camera in a way that will not capture you. Kindly note that there are no financial rewards for your participation in this study.

I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept secret in all my writing about the research. Also, all video recorded will be stored safely and destroyed after 3 – 5 years of completion of this research.

I look forward to working with you.

Please feel free to contact me if you have any questions.

Thank you very much for your assistance.

Yours sincerely,

Researcher's name: Julius Olubodun
Researcher's email: juliusikolubodun@gmail.com or 391357@students.wits.ac.za
Researcher's contact number: 078 021 8294

Supervisor's name: Dr Lawan Abdulhamid
Supervisor's email: Lawan.Abdulhamid@wits.ac.za
Supervisor's telephone number: 011 717 3468

Consent form: Learner

Learner
I am happy to be observed and appear in the mathematics lesson video YES/No
I am happy for the extract from these videos to be viewed by the researchers for research purpose video YES/No
Learner name: _____ Learner signature: _____ Date:_____

Information sheet: Parent/Guardian

Dear Parent/Guardian

My name is Julius Olubodun and I am a Part-time Masters student in the School of Education at the University of the Witwatersrand. As part of my studies, I have to undertake a research project, and I am investigating Supporting development of Grade 8 mathematics teachers' professional noticing of learners' thinking of common fractions using Video-Stimulated Recall under the supervision of Dr Lawan Abdulhamid. The aim of this research project is to give insight into grade 8 mathematics teachers' practices of professional noticing of learners' mathematical thinking and their use of learners' conception as a guide to inform next steps of instructions during their lessons. Furthermore, the study will also seek to investigate possible development of these practices in a context of a collaborative discussion between myself as a researcher and each teacher while watching the teacher's video recorded lessons. The aim is to use reflection-oriented questions to support teachers to reflect on their practices and possibly alternative and more efficient pedagogies may emerge in this process.

My research involves observing and video-recording mathematics lessons at the school where your child is attending with my focus being on the mathematics teacher. I was wondering whether you would mind if I do the observation and video-recording in your child's class while s/he is present. My study is not focusing on the learners in the classroom however since learners will be interacting with the teacher in the classroom, they will be part of the video-recordings and may appear in the video recordings. These recordings will be stored in a password protected computer and only the researcher and the supervisor will have access to these recordings. It will be deleted after 3 to 5 years. Should you or your child refuse permission, the teacher and I will make a sitting arrangement for her/him not to be included in the video recording, while we ensure that s/he is not excluded from the class, which would disadvantage her/him academically. Face-blurring software will be used in the recorded video to preserve anonymity.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name and identity will be kept confidential at all times and in all academic writing about the study. Her/his individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed within 3 – 5 years after completion of the project. Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

Researcher's name: Julius Olubodun
Researcher's email: juliusikolubodun@gmail.com or 391357@students.wits.ac.za
Researcher's contact number: 078 021 8294

Supervisor's name: Dr Lawan Abdulhamid
Supervisor's email: Lawan.Abdulhamid@wits.ac.za
Supervisor's telephone number: 011 717 3468

Parent/Guardian Consent form

Parent/Guardian
I consent/do not consent* for my child to be observed and appear in the mathematics lesson video
I consent/do not consent* for the extract from these videos to be viewed by the researchers for research purpose video YES/No
School name: _____ Parent name: _____ Child name: _____ Parent signature: _____ Date:_____

*Please delete as appropriate