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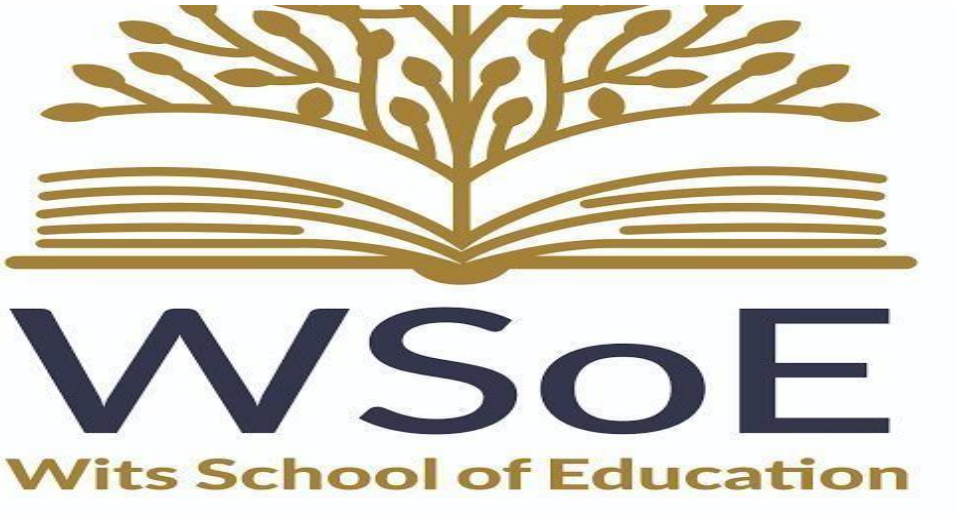

UNIVERSITY OF WITWATERSRAND

THE EFFECT OF USING GRADE 9 LEARNER ERRORS AND MISCONCEPTIONS WHEN EXPLAINING ALGEBRAIC EQUATIONS IN A PROFESSIONAL NOTICING INTERVENTION.

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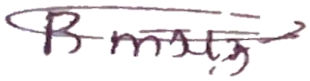
PROTOCOL NUMBER: **2023ECE017M**

15 March 2024

DECLARATION

I take this opportunity to declare that all the content presented in this report is my original work and it was never submitted at any other institution before.

Busisiwe Lerato Msiza



30 November 2023

DEDICATION

This research is dedicated to my late father, Mr Z.Z Ndaba, whom it would have be an honor to be with when I attain this degree. However, his passing has deprived me an opportunity to share this delightful achievement with him although I believe that he is with us in spirit. I will always be grateful for the foundation that you have laid in my studies, daddy, and ensuring that I get the best educational experience from as early as my childhood.

ACKNOWLEDGEMENTS

Firstly, I honor God for granting me this opportunity to successfully complete this report. I have encountered a number of obstacles along the way, but the Almighty has been there with me hence I managed to complete this research. *Ngiyabonga Nkulunkulu wezimanga*.

Secondly, I wish to acknowledge the consistent and profound practical support I have obtained from my supervisor, Prof Judah Makonye from the planning to the execution of this research project. Prof Judah has consistently availed himself with his expertise and knowledge in research. I thank him for sharing relevant articles which were helpful in the preparation and development of this research. I wish him success in the publication of his papers and hope for the best in all his publications.

Thirdly, I honor my mother, Ms Annah Msiza for the invaluable support that she has given me from the proposal phase to the final phase of this study. There are days where I could not perform my household duties because of having so much work to do but my mom made sure that despite her busy schedule, my mom made sure that she covers some of these household chores on my behalf. I pray that God blesses Ms Annah Msiza with all her heart desires.

Fourthly, to my sister, Lindiwe, I thank you for being patient with me throughout the research phase. At times I was not available to give you support in your studies, but you still honored me as your sister and you have worked tirelessly hard with your studies and making sure that you make us proud. I pray that God gives you strength, courage and wisdom in executing your studies, amen.

ABSTRACT

This qualitative error analysis research reveals the effect of using learner errors and misconceptions when explaining algebraic equations in a professional noticing intervention. The study was guided by the research questions: (1) What learner errors and misconceptions are identified by the teacher while attending to the grade 9 learners' work on solving algebraic equations? (2) What learner reasoning and justifications are evident from the learners' work of solving algebraic equations? (3) What possible learning advantages and disadvantages can result from providing explanations that are informed by the learners' errors and misconceptions? (4) To what level can learning and learner achievement in solving algebraic equations be improved if explanations are informed by the learner errors and misconceptions?

To answer the research questions, 40 grade 9 learners from a secondary school in Johannesburg voluntarily took part in the study by, firstly, writing a pre-test. The pre-test served as the instrument for establishing the type of errors that grade 9 learners elicit when solving algebraic equations as well as the misconceptions associated with these errors. These errors, which were categorized using Donaldson's (1963) error classification and Hirst's (2003) categorization framework, were used as a resource to guide the explanations offered as an intervention. Secondly, to understand the learners' intuitive conceptions, the study used a purposive sampling of 8 learners which were interviewed based on their pre-test responses. Thirdly, the participants attended the intervention session where the mathematics teacher from the school offered explanations that were based on addressing the identified errors and misconceptions. The teacher explanations offered in the intervention lesson considered professional noticing intervention as a framework to guide the session. Lastly, the effect of offering these explanations on the performance of learners was comparatively analyzed by considering the pre-test and the post-test responses (which had questions of similar nature as the ones in the pre-test).

It is evident from the findings that indeed offering explanations that are directed at the identified learner errors and misconceptions in a professional noticing context results in a substantial improvement on the learner performance. It is apparent from their responses to the post-test that there were traces of cognitive gains. As a result, this study remains a confirmation that indeed the errors and the misconceptions that learners encounter has the potential to serve as a resource to improve the learner performance in solving the grade 9 algebraic equation. In fact, such

explanations offered in a professional noticing intervention are a general resource that can be considered by mathematical teachers throughout the mathematics discourse.

KEYWORDS

- Algebraic equations
- Constructivism
- Errors
- Misconceptions
- Intervention session

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LIST OF ACRONYMS

- APOS: Activity Process Object Schema
- DBE: Department of Education
- FOIL: First, Out, Inner, Last
- LCD: Lowest Common Denominator
- NSC: National Senior Certificate
- PCK: Pedagogical Content Knowledge

CHAPTER 1: INTRODUCTORY CHAPTER

1.1 INTRODUCTION

This study is introduced by outlining the background, the problem statement as well as the purpose of this research. It also provides research questions guiding the study and also discusses the significance of the study.

1.2 BACKGROUND OF THE STUDY

Mathematics is broadly viewed as a language of discipline embedded with different meanings. Mathematics, as a discipline with abstract structures, is a study of numbers, shape, arrangement, quantity, measure and many other relating concept (Yadav, 2017). In every mathematical subject, there are axioms, properties, theorems, applications and establishment of patterns to formulate conjectures. Murtafiah *et al* (2018) explains mathematics as a language with four fields; arithmetic, analysis, geometry and algebra, which have their own terms and meanings that are understood within the field. Learners are exposed to this body of language from as early as pre-school level to university and college levels. Throughout these levels, it is expected that learners are equipped with the mathematical content and procedural knowledge necessary for their grade. Algebra as an abstract topic in mathematics, involves the use of variables to simplify expressions and to solve equations (Egodawatte, 2011). It is evident even in other mathematical fields such as geometry, trigonometry, functions, etc. It is, therefore, for this reason that learners have to extensively engage with the content, understand the procedural aspects taught in algebra and have an insight understanding of how to use the algebraic knowledge and skills in other fields.

Due to its abstract meanings, learners are mostly challenged with making errors and encountering misconceptions while trying to construct algebraic knowledge and making sense of these abstract meanings during the learning phase. The learner errors are generally the mistakes that learners make while the misconceptions are the incorrect concepts that learners construct which are influenced by the learner's existing knowledge. The errors and misconceptions that the learners encounter will persist until they are addressed. For learners to construct the correct conceptions and avoid making errors, the primary concern should be on examining the learner's written work diagnostically, mainly to attend and to interpret the learner's thinking patterns and ultimately decide or respond to the errors and misconceptions. This strategy, is known as professional

noticing (Jacobs *et al*., 2012). With algebraic equations being the main focus of this study, investigating the effect of using grade 9 learner errors and misconceptions as a tool to provide explanations in a professional noticing intervention is an integral part of the study, on the learners' cognitive growth and in the overall learner academic performance.

1.3 RESEARCH PROBLEM

With all being said, I find it mandatory to highlight that the wrong intuitive conceptions that learners have in algebraic equations as well as the impact of the learners' inability to solve algebraic equations on the overall performance in mathematics serves to be the focal problem in this research study. Algebra as an abstract strand demands an apprehension of the properties of the mathematical operations, and this includes the ability to make meaning from the concept itself and its variables. The elementary properties of the algebraic concepts, its' manipulations, rules that are relevant for simplifying and solving equations are introduced from as early as the lower grades of high school. However, learners in FET phases are still challenged with questions related to simplifying expressions, solving equations and establishing relationships in algebra (Zulfa, Suryadi, Fatimah & Jupri, 2020).

The 2019, 2020 and 2021 National Senior Certificate (NSC) diagnostic reports show a notable trend of the poor algebraic skills portrayed by the grade 12 learners. Using the most recent diagnostic report; part 1 of the 2021 NSC diagnostic report, the overview of the performance in paper one highlights that

"The algebraic skills of the candidates are poor. Most candidates lacked fundamental and basic mathematical competencies which should have been acquired in their lower grades. This becomes an impediment to candidates answering complex questions correctly". (p. 186).

The reports from the Department of Education (DBE) have been and are still indicating the difficulties that learners have in algebra, and most learners are discredited due to their misconceptions that are evident in their responses to the algebra questions. One of the learner's challenges identified on the 2021 grade 12 diagnostic report was factorizing the given equation to find the solution(s) of the equation (p. 187), which was taught from as early as grade 9. Not only

are they struggling with factorization, but they generally show low level of understanding algebra (Marpa, 2019). Subsequently, the poor algebraic skills compromise their achievement in other mathematics concepts that needs algebraic skills.

The learner performance evident from the standardized tests, both locally and internationally, pins out how important it is to ensure that learners develop algebraic understanding conceptually and procedurally (Kilpatrick *et al*., 2001). To promote procedural and conceptual fluency, there is a need to explore the different ways that can be applied in ensuring that the challenges that learners encounter are addressed, the misconceptions that learners have are, by all means, counteracted.

I propose to study the effect of using learner errors and misconceptions as a tool to provide algebraic explanations in a professional noticing intervention, with the aim of examining the cognitive gains which may result from explanations that are informed by their errors and misconceptions in grade 9 solving algebraic equations.

1.4 PURPOSE OF THE STUDY

These misconceptions which learners consistently portray in their exams shows a negative impact on the learners' overall performance. With no intention to disregard all the other academic and contextual factors that might contribute to the problem, this research focuses on the learner errors and misconceptions and the effect on the learners' algebraic understanding when these errors are used as a tool to inform instructional explanations. With an aim of achieving the following research aims, the study was a qualitative research case study. Firstly, the study investigates the errors and misconceptions that the grade 9 learners encounter when solving algebraic equations. Secondly, the study aims to investigate the root cause of these identified errors and misconceptions. Thirdly, the study aims to explore the advantages and/or disadvantages of providing instructional explanations that are informed by the identified errors and misconceptions. Lastly, the study aims to investigate the impact of using these instructional explanations on promoting algebraic understanding thus improving the learners' algebraic skills. Using the identified learner errors and misconception as a tool to provide explanations serve as teaching intervention to address these errors and misconceptions. Ultimately, the findings from the data addressed the subsequent research questions.

1.5 RESEARCH QUESTIONS

1.5.1 What learner errors and misconceptions are identified by the teacher while attending to the grade 9 learners' work on solving algebraic equations?

1.5.2 What learner reasoning and justifications are evident from the learners' work of solving algebraic equations?

1.5.3 What possible learning advantages and disadvantages can result from providing explanations that are informed by the learners' errors and misconceptions?

1.5.4 To what level can learning and learner achievement in solving algebraic equations be improved if teacher explanations are informed by the learner errors and misconceptions?

1.6 SIGNIFICANCE OF THE STUDY

Diagnostic reports and results from standardized tests designed for learners have shown evidence of learner challenges in learning mathematics, not only locally but internationally as well (Reddy, 2004). There are traces of poor algebraic skills evident in the Grade 12 written examinations; these skills which learners were supposed to have from as early as grade 9. Most importantly, algebra is one of the significant fields in mathematics (Egodawatte, 2011) since its' skills are necessary for the manipulations needed in most of the mathematics fields.

Learning is explained as an activity which needs the transformation of prior knowledge present in the leaners' existing schema by assimilating and accommodating (Piaget, 1967) the newly introduced knowledge. It is during this process where learners develop misconceptions, fail to cope and ultimately lose interest in learning due to not being able to meet the cognitive demands. As a teacher, there is a need to consider effective approach that will assist these learners in meeting the demands of the work and developing conceptual understanding the concept taught. To do this, teachers need to first understand the challenges that the learners experience and the root cause of these challenges so that explanations are informed by what the learners need. This study is significant as it reveals the effect of using the learner errors and misconceptions as a resource to providing teacher explanations that will assist learners in developing conceptual and procedural fluency in mathematics, particularly in solving algebraic equations.

The poor algebraic skills that learners portray in their work has motivated me to initiate this study which focuses on the effect of offering explanations that are directed at learner errors and misconceptions in a professional noticing intervention. I believe that this study will add knowledge on to the field of knowledge that already exists in mathematics education. Such contributions will be made by revealing the effect of offering such explanations in promoting cognitive gains and as well as the resulting improvement in the performance of learners. I believe that this study is important as it exposes the causes of poor algebraic skills, particularly in algebraic equations, which are subsequently used to develop trends and understanding the patterns that are then used to make recommendations to improving teacher explanations. Since the outcomes of this study are generalizable, a theory may be generated from the revelation of the study, which is significant for teacher explanations.

Understanding the reasoning behind the identified errors and misconceptions is one way of exploring what makes algebra challenging (Booth 1988), and this strengthens the significance of the study as it reveals the reasoning behind the errors which learners made when solving algebraic equations. The revelation evident in this study will enlighten teachers and brings forth the awareness of such explanations which may subsequently assist them by informing their pedagogical content knowledge (PCK). In Shulman's (1986) terms, PCK is the platform on which the mathematics teachers are challenged with even though it serves as one of the important aspects to consider in ensuring that the teaching of mathematics is well equipped. The outcomes of this study may assist mathematics teachers in informing their PCK and advancing their knowledge on the possible errors which the learners that they teach might have.

The academic researchers and practitioners, in general, will be informed about the errors and misconceptions which act as the barrier to learning, the affordances and constraints of explanations informed by learner errors and misconceptions on the learners' cognitive gains. This study might be an asset in shaping and guiding the academic and practice disciplines as it exposes the researchers and the practitioners to the learners' thinking.

This study exposes me to the different errors and misconceptions that learners have in algebraic equations, being aware of these errors and misconceptions is significant for building a resourceful and informed teaching lessons that will address these errors and misconceptions and deepen their

understanding of solving algebraic equations. Not only will this study deepen my teaching, but will also unfold other research opportunities which I may consider in future.

1.7 CONCLUSION

This chapter discussed the background of the study, outlined the problem statement and the purpose of the study. It also provided the research questions guiding the study and discussed the significance of the study and its contribution in the existing research on algebra.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

This chapter outlines the literature review of the study by first discussing literature on school algebra, algebraic equations and the errors and misconceptions. There are different frameworks adopted in this study, which I wish to discuss in the current chapter.

2.2 LITERATURE REVIEW

2.2.1 School Algebra (algebraic equations)

Algebra is one of the fundamental branches of the mathematics. Algebra, as a form of mathematical language (Guler & Celik, 2021) is embedded with different terms that promotes abstract thinking from learners (Witzel *et al*., 2003). It is a mathematical strand with abstraction and symbolization that allow learners to work with variables and engage in abstract meanings (Star *et al*., 2014). It is viewed as an activity with three broad components as discussed in the following paragraph.

The generational activity involves the formation of objects such expressions as well as equations which are embedded with unknown variables (Kieran, 2007). Comprehending words to devise an equation or an expression, and representing situations using these expressions and equations forms part of the generational activity. The generational activities, therefore, generates algebraic expressions and equations to represent a situation.

The second activity, according to Kieran (2007) is the transformational activity which involves different activities such as simplifying expressions, solving equations, grouping like terms, substituting values or variables and factorizing terms. This activity incorporates the manipulations mainly for simplifying expressions and solving equations respectively.

In the third activity, algebra serves as a modelling resource that is applied in problem-solving situations (Kieran, 2007). With algebraic equations being the focus of this study, activity considers an equation to be the modelling resource necessary for solving problems and make generalizations. Arrangement of elements, also known as *form* according to Ruben's (1992) is influenced by the situation which is modelled, and this forms part of the global/meta-level activity. Vermeulen's (2007) contribution to algebraic literature supports this activity by claiming that *algebra is viewed*

as a problem solving tool. This perspective claims that algebra is applied when modelling different situations, some of which are real-life situations and to represent word problems using expressions, equations and even graphs.

The high school algebra broadly accommodates all three activities, with the first two being greatly witnessed and the last one being rarely used. This is mainly because, firstly, the South African curriculum suggests that learners should engage in activities such as forming algebraic expressions and equations. Secondly, it also needs learners to engage in activities such as collecting like terms with an aim of simplifying and solving (when working with expressions and equations respectively). An act of solving, simplifying and substitutions are, according to Kieran (2007), transformational activities. Thirdly, even though the third activity is the weaker component as it is not exclusive to algebra (Luxomo, 2019), learners do engage in activities such as using algebra as a modelling tool. All these activities are predominant in this study as some of the algebraic misconceptions encountered by learners emanates during the time when learners engage in these activities, for example, the learners encounter errors and misconceptions while they collect the like terms or during substitutions.

The literature on school algebra is has dissimilar views which complement each other. Vermeulen's (2007) contribution to algebraic literature, suggests that there are four different perspectives of school algebra, and one of the four perspectives, namely, *algebra as a problem solving tool*, supports Kieran's (2007) view that algebra is a modelling tool. This perspective claims that algebra is applied when modelling different situations, some which are real-life situations and to represent word problems using expressions, equations and even graphs.

Algebra as a generalized arithmetic which is Vermeulen's (2007) perspective considers algebra as being arithmetic. However, this perspective is critiqued because, firstly, algebra has unknown variables and the equal sign denotes that the value on the left is equal to the value on the right of the equation. This is different from arithmetic because in this case we use known numbers and not variables, instead, variables are used to symbolize the units used, and the equal sign in arithmetic is used to show the answer. Even though learners tend to consider an equal sign as an operation, which is one of the main root cause of misconceptions in algebraic equations (Guler & Celik, 2021). In some instances, learners misconceptualize the idea of an equation showing two equal sides. With the equation $2x + 7 = -x - 5$, for example, comprehending this equation should

evolve around $2x + 7$ being equal to $-x - 3$, where the numerical value of x is the same in both sides of the equation.

The third perspective is *algebra as the study of relations*, such as direct and indirect proportions, exponential growth (Vermeulen (2007). This perspective supports Wilkie's (2016) explanation that algebra is an exploration of functional relationships and making generalizations on relations. Vermeulen's (2007) last perspective, *algebra as the study of structure*, considers that importance of recognizing mathematical structures in algebraic expressions and equations which, according to Kieran (2007) are formed by terms with variables, coefficients and different operations.

With the literature on algebra and algebraic equations discussed thus far, its relevance is still evident in the current CAPS curriculum. Algebraic equations, as one of the strands in the algebra field, involves problem solving and finding the value of the unknown variable. To solve algebraic equations, there are different sub-topics that needs to be understood by the learners, and one of these topics for the ninth graders is factorization. According to the grade 9 curriculum, there are three types of factorization which learners should know which are; taking out the common factor, difference of two squares and factorizing a trinomial method. All these factorization methods will assist learners in solving for an unknown variable in the equation.

The relevance of these different constructs to the study is that the errors and misconceptions that are analyzed in this study are a result of learner misunderstandings and misinterpretation of these constructs. The misconceptions are due to incorrectly constructing these unfamiliar terms, representations and the algebraic characters.

2.2.2 Misconceptions and Errors

Many studies have been undertaken on learner errors and misconceptions in South Africa (e.g Makonye, 2020). Makonye and Fakude (2016) researched on the errors and misconceptions on operations on negative numbers, which is one of the concepts which learners misconceptualizes Moreover, Makonye (2017) added on the existing body of knowledge by researching on the errors and misconceptions on nominal and effective interest rates. The importance of these contributions points out the importance of considering the errors and the misconceptions which learners come to class with as they can impede the process of learning.

Mathematics is naturally embedded with abstract representations with a hierarchical build-up of facts, skills and concepts (Sarwadi & Shahrill, 2014). Successful learning, therefore, needs an understanding of this systemic build-up of this hierarchy of concepts which needs to be introduced to the learners. According to constructivists such as Piaget (1967), the construction of knowledge involves a blend of both experience and the existing mental structures. For this process to occur, the given information should assimilate into the existing schema in the mental structures. Assimilation occurs when the new knowledge confronts and becomes part of the learner's existing knowledge thus becomes accommodated and forms part of the learner conceptions. Once a concept is formed and accommodated, it becomes stable and thus resistant to change and the learners' existing schema determines what will be learned from experience or through instruction (Sarwadi & Shahrill, 2014).

Whenever these conceptions are in conflict with the learners' mathematical meanings, the learner encounters a misconception (Osborne & Wittrock, 1983). Misconceptions occur when there was an unsuccessful confrontation of knowledge resulting to cognitive conflict between the learners' prior knowledge and the newly introduced concepts. This means that manner in which knowledge construction takes place depends on the existing schema, hence it should be mandatory for the teacher to consider the learners' prior knowledge when explanations are offered.

Skemp's (1976) construct supports Piaget's view is a sense that schema is activated in the learner's mind when a learner encounters an example of it. For the development of good concepts, there is a need for the provision of good examples of the concept (Sarwadi & Shahrill, 2014). On the same vein, for learning mathematics as a cumulative discourse, the newly introduced concept should somehow link to the knowledge that already exists in the schema. If the learner is challenged with accepting the newly introduced concepts, a gap in the learning of the concept is created and subsequently, errors and misconceptions results.

While explaining the theoretical meaning of misconception, the theoretical meaning of the term implicitly overarches with errors and both these terms somehow experiences have theoretical underpinnings. The errors that learners make are systematic and results from the misconceptions that learners normally encounter (Radatz, 1980). They reflect on the learner's level of conceptual understanding and procedural development. Moreover, it generally includes the learner's inability to make connections with their prior knowledge and learning cannot take place from these errors.

An error is generally explained as a mistake or lapse of concentration which anyone is bound to encounter. Different from a misconception, an error does not mean the learner has encountered cognitive conflict during knowledge construction, but has rather deviated from the correct solution. This study adopts and views an error as a term that synonymously relates to a mistake that occurs while attempting to algorithmically or procedurally solve a mathematical problem. This study then identifies errors from the incorrectly responded problems.

Learners come to class with these errors and misconceptions which they believe to be true mathematical conceptions. Learners have constructed the mathematical misconceptions chiefly because their existing knowledge was not in conflict with the misconceptions and were therefore accommodated. It is for this reason why learners are able to even justify, or provide reasons, for their mathematical work. If, for example, a learner is tasked to solve for x for the equation: $3x^2 + 6x + 9 = 0$ and the learner writes $x^2 + 3x + 6 = 0$ as his first step. The learners' justification for this step is that he has taken out the common factor by subtracting 3 throughout the equation. It is clear that the learner is aware that he has to make the coefficient of the first term to be 1 as the first step, hence he is able to justify it. Even though he has misconceptualized the concept and decided to subtract instead of dividing. The main point depicted from this explanation is that learners have justifications of the misconceptions evident from their work and it is important that the teacher is aware of these justifications in order to understand the root cause of the misconceptions and to decide upon a construction action to address them.

The learner errors and misconceptions remain persistent unless there are pedagogical actions and interventions implemented. Through diagnostic examination of the learner's written tasks, the teacher is able to find patterns and causes for the learner errors and misconceptions. Using these patterns, the teaching strategies which empower learners to meticulously review their understanding of concepts can be successfully devised. This means that the identified errors and misconceptions shouldn't be viewed as obstacles, instead, they should be considered as an opportunity to reflect on what learners know and use it to generate instructional teaching strategies which will enrich learning.

2.3 THEORETICAL FRAMEWORK

This research study has adopted the theory of constructivism as a theoretical framework. The theoretical framework adopted in this study is the constructivism theory. Learners are viewed as active knowledge constructs who utilize their existing knowledge to construct new information (Cobb & Bauersfeld, 1995). When the learner constructs knowledge, the learner can misconceptualize correct concepts, forming misconceptions. Constructivism is an important theoretical framework as it assists in making predictions of the manner in which learners construct mathematical concepts and how they encounter misconceptions. It is for this reason why the constructivism theory is adopted by the current study.

2.3.1 Constructivism

Constructivism is explained as a philosophical perspective of the nature of knowledge, particularly pointing out the idea that learning is a constructive action (Muengele, 2017). It is a theory which claims that knowledge is individually constructed through cognition that is driven by mental self-regulatory structures (Piaget,1967). In other words, knowledge is not merely propagated from the teacher to the learner, it's rather re-organized by each learner for the purposes of making meaning. The learners' cognition is, therefore, an important aspect for learning (von Glasserfeld, 1989).

The manner in which a learner makes meaning based on the interaction that exists between experience and concepts or ideas is an integral part of constructivism (Muengele, 2017). Experience, in this context, is the prior knowledge which the learner has already constructed, and ideas includes the knowledge which the learner needs to construct. The construction of these ideas is dependent on the existing ones. The knowledge that learner A has is different from the one that learner B has, and this means that the manner in which learner A will uncover and discover the newly introduced concepts is different from that of learner B since they procured dissimilar prior knowledge.

In Piaget (1967)'s view, for knowledge construction to occur, the newly introduced knowledge should assimilate into the existing schema in the mental structures. Assimilation is incorporating knowledge in an existing schema which is the prior or existing knowledge (Piaget, 1967). There is tension developing as a result of the confrontation of the new knowledge into the existing

schema, with the aim of reconciling with and become part of the learner's existing representations. A success in reconciliation results in the accommodation of the new knowledge thus forming part of the learner's conceptions. Accommodation suggests that the new concept was successfully incorporated in with the existing knowledge since the learner reconciled the existing knowledge with the new knowledge (Piaget, 1967).

However, assimilation and accommodation is not always a successful process. When the learner notices the difference between the existing knowledge and the new knowledge, the existing schema experiences a cognitive conflict, which means misconceptions were encountered (Osborne & Wittrock, 1983). Misconceptions, therefore, are a result of an unsuccessful confrontation of knowledge due to cognitive conflict, and in this case, equilibrium is not reached.

Consistent with constructivists' belief, for learning to take place, learners must not be treated as empty vessels or blank slates (Kiong & Yong, 2001). They enter the classroom with different interpretations (or perceptions), some of these interpretations promote learning while others impede the learning process. It is for this reason why learners deserve the opportunity to apply their prior knowledge in engaging with the tasks used in class, either individually, in pairs or in groups to justify the legitimacy of their work and thus take responsibility for their learning.

2.3.2 The concept image and concept definition

On the same vein with the constructivist view of knowledge construction, Tall & Vinner (1981) brings forth the concept image and concept definition theory. Concept image is explained as the bank of ideas in mind that have been developed by the learner regarding the different aspects making up a certain concept (Tall & Vinner, 1981). These cognitive ideas have been personally developed and constructed by the learner due to his or her personal encounters with the specific concept.

A concept image, as the inner picture of a particular concept that is developed based on the individuals' experiences will not always be correct. This means that a concept image is linked to Piagets' (1967) construct of existing knowledge in such a way that the concept image developed by a learner depends on a learners' existing schema. This image is dynamic and is bound to be modified, depending on what the learner thinks about the concept.

The concept image relates to the theory of constructivism in a sense that the learner individually makes meaning of the concept which they are exposed to, forming the inner concept image of the concept. This concept image is modified, critiqued and evaluated when the learner constantly interacts with the mathematical concepts, in this case, the algebraic equations.

Due to the fact that the learners' concept image can be changed, the learner may encounter contradicting ideas about the particular concept which might even lead to developing misconceptions (Tall & Vinner, 1981 as cited in Luneta & Makonye, 2010). This is because since the concept image may be modified, the learner might construct an idea which contradicts with the true mathematical conceptions concerning the concept of taught.

The inherent concept image that learners develop mentally is influenced by the concept definitions exposed by the teachers to the learners. An idea of a concept definition is not the same as that of a concept image in a sense that a concept image is the inner resemblance of the concept constructed by the learner, whereas a concept definition represents the outer resemblance concept presented by the teacher to the learner (Luneta & Makonye, 2010). Limited exposure to the concept definitions by the teacher to the learner may negatively affect learners in a sense that it allows learner to construct partially correct concept images and sometimes even misconceptualize the concept. It is for this reason why teachers should ensure that they maximize the exposure of the concept definitions to the learners. Ultimately, both these conceptions form a model which is necessary for understanding the manner in which the learner views a definition presented to him or her and how the teacher ensures that the concept definition presented to learners is brought even closer to them mainly for promoting a successful construction the concept.

2.3.3 Advantages and disadvantages of learning from the identified errors and misconceptions

Learning from the instructions that are informed by the identified errors and misconceptions may yield a positive outcome, which is an advantage to the learner, or it may impede learning which is a disadvantage. Both these outcomes are influenced by the teachers' formal instruction in the learning space as well as the object of learning. These outcomes are useful in determining the extent to which a correct mathematical conception is made accessible to the learner (Watson, 2007). It is the teachers' responsibility to ensure that the learning space affords learners an opportunity to access the concept hence promoting an advantage to learning through the use of the

identified learner errors and misconceptions. Even though the teacher ensures a "perfect" learning space with the instructions informed by errors and misconceptions, it does not mean that learners will definitely construct the correct conceptions from the object learning. Thus both the advantages and the disadvantages are observed of learning from the identified errors and misconceptions are observed.

2.3.4 The Variation Theory

To be able to construct a concept and understand its features, an experience contributes in making conclusions from the phenomenon given in contrast (Mhlolo, 2013). This theory complements the constructivism theory in such a way that it provides the process of learning and knowledge construction in different patterns; *contrast*, *generalization*, *separation* and *fusion* (Runesson, 2005).

Contrast claims that in order for learning to take place, there should be a contrast between what the object of learning is and what it is not. In this case, the learner makes comparisons and critiques the features making up the object that needs to be learned and what they are not. For a learner to know what algebraic equation $2x + 6 = 0$ is, for example, the learner can be reminded of $2x + 6$, which is an algebraic expression. This expression, which the learner already knows, can be used to compare an expression with the equation in order to learn what an algebraic equation is.

Generalization is a pattern which claims that in order for learning and understanding to take place, generalizations using different activities need to be made (Runesson, 2005). In order for learners to make conclusions, they need to be exposed to different exercises and examples of the same nature. For example, in order for the learner to work out a quadratic equation $t^2 - 6t + 9 = 0$, which needs them to factorize, they should have different examples and activities that will allow to make conjectures that quadratic equations have a two roots and thus be able to construct these generalizations.

Separation pattern suggests that the potential that the learner possess in identifying different features making up a concept and the skill of separating, comparing or differentiating a concept from other concepts promotes learning (Runesson, 2005). For knowledge construction, the learner has to be able to see the features making up a concept and use this distinction to differentiate it from other concepts. For example, working with a quadratic equation as a concept, the learner can

distinguish a quadratic equation from a linear equation once the learner identifies that the degree of a quadratic equation is 2 whereas that of a linear equation is 1.

Fusion, also called synchronic simultaneity, considers the learners' prior knowledge and matches the errors which have the potential of resulting or appearing from the correct concepts (Runesson, 2005). Learners are able to differentiate features of the concept simultaneously.

The space of learning should be created for the purposes of learning and knowledge construction, taking into consideration the learners. This theory is important for this study when explanations which serve as teaching intervention are provided to address the learners' misconception. It will be used in a sense that learners are given space to discern their errors and misconceptions from the correct conceptions, putting them at an advantage of improving their algebraic skill.

2.4 CONCEPTUAL (ANALYTICAL) FRAMEWORK

For a clear and concise understanding of the errors and misconceptions, researchers classify the mathematical errors and misconceptions encountered by leaners. Some of the error classifications developed by scholars are general while others are specific to a particular domain of study (Luneta & Makonye, 2010). For the purposes of this study, I have merged both Donaldon's (1963) and Hirst's (2003) error categorization frameworks mainly to thoroughly investigate the errors that grade 9 learners encounter when solving algebraic equations.

2.4.1 Donaldson's (1963) error classification

This study has adopted the error categories suggested by Donaldson (1963) which proposes three types of which occur during the learning of mathematics. The reason why this study adopted this framework is chiefly because even though the categories are from an old publication, they are still clear and applicable even in the recent years. These errors i.e. *structural*, *executive* and *arbitrary* errors are still evident even in the recent years

The *structural error* category involves the errors that occur due to failure to appreciate the relationships that are part of the concept, or inability to capture the principles which are important for determining the solution to the problem (Donaldson, 1963). These errors are also referred to as conceptual errors (Makonye, 2010), as they are a result of inaccurate perceptions about the nature of mathematical concepts. When a learner fails to understand the relationships that exists between

the parts of a concept, the error which this learner has developed, or will develop, is structural. Structural errors do not only include failure in appreciating relationships, but also due to not being able to hold on or use an important rule for working out an answer (Donaldson, 1963). Given an algebraic equation $(y - 3)^2 = 16$ as a problem which learners have to solve, a learner may choose to associate $(y - 3)^2$ with $y^2 - 3^2$ and justifies this step with the laws of exponents. In this example, the learner has missed the mathematical rule that they multiply out the exponents only when there is a monomial term inside the brackets and not when there is a binomial or any type of polynomial. The learner has shown the structural error of misusing the law of exponents instead of expanding the left hand side to (y – 3) (y – 3). In such instances, this learner completely misunderstood the key concepts that are needed to get to the solution. Therefore, the structural errors are fundamentally conceptual in nature hence they are also referred to as conceptual errors.

The *executive error* category involves failure or inability to manipulate an equation even when the learner understands the concept (Donaldson, 1963). In this case, the learner is enlightened about the concept but does not know how to apply the relevant procedures that are necessary for working out a solution. These errors are independent of the understanding of concepts, meaning the learner may encounter executive errors regardless of whether the leaner understands the concepts of not. They are also referred to as procedural errors since they emanate from an inability to accurately consider an algorithm. When a learner cannot solve the equation $2x^2 - 4x = 0$ by factorizing it mainly to find the product of factors means that the learner has encountered an executive error since the learner fails to recognize factorization as a method of manipulating the equation. Errors such as not being to factorize by taking out a common factor or isolating the *x* variable with an aim of solving for *x* are categorized as being executive.

The third category, *arbitrary errors*, are the errors that result when the learner ignores or neglects some of the information and considers the rest (Donaldson, 1963). These errors can be caused when the learner wants the questions or information to suit their knowledge and understanding and thus the learners are said to act arbitrarily.

2.4.2 Hirst's (2003) error characterizations:

The three generic error categories proposed by Hirst (2003) which I wish to merge with Donaldson's (1963) are *procedural extrapolation, pseudo-linearity* and *equation balancing* errors.

Hirst's (2003) error categorization is employed chiefly because these categories arise out of the learners' tendency to generalize the existing schema to new situations through assimilation and accommodation (Piaget, 1968).

The *procedural extrapolation error*, in Hirst's (2003) terms, occurs when the learner aims to extend a procedure was studied previously and uses it in new situations and contexts that are similar to one learnt in the past. The learner, in this case, extrapolates a method that was learnt before by using it in a different situation where the procedure does not apply. An example of a procedural extrapolation is given below. Please note that this example shows that a learner has wrongly used an algebraic rule in the concept of functions.

The task was to isolate x for the function $f(x) = 2x - 3$. A learner might be tempted to remove the brackets by multiplying f with x to get fx as the answer then the subsequent procedure would be;

$fx = 2x - 3$

$fx - 2x = -3$

$x(f - 2) = -3$

therefore, $x = \frac{-3}{f-2}$

In this example, the learner used the fact that $x(y)$, for example, is equal to xy. The learner in this case has used an algebraic rule in a new context where it does not apply hence a procedural extrapolation error resulted.

The error, Pseudo linearity error, proposed by Hirst (2003) was initially explained by Fischbein & Barash (1993) as erroneous extrapolations. According to Norman and Pritchard (1994 as cited in Luneta &Makonye, 2010), this type of error is characterized as a 'misgenerated distributivity' as they may emanate from simplifying expressions that involve the application of a distributive law. A most common example of pseudo linearity error is $(p + 3)^2 = p^2 + 9$ and $\sqrt{9 + p^2} = \sqrt{9} + \sqrt{p^2}$.

The third Hirst's (2003) error categorization is equation balancing. These types of errors emanate from the basic algebraic principle of equations similar action is presented on the two sides that are separated by an equal sign mainly to maintain the nature of the two sides being equal. An example

would be: $\frac{x}{y} = \frac{6(x)}{6(y)}$, the learner might attempt say write $\frac{x}{y} = \frac{x+6}{y+6}$ and justify this step as a way of forming equivalent fractions. This learner's construction is incorrect.

2.4.3 Other types of errors

Apart from Donaldson's (1963) and Hirst's (2003) error categories, there are other errors which I incorporated in the framework of the study. The justification of this action is because these errors are also relevant to the errors which learners make when they work out solutions to the given equations hence they should be considered and classified as well.

One of these common errors is the incomplete application of rule which is evident when the learner understands the rule and attempts to apply it but fails to complete the application (Luneta &Judah, 2010). These errors are mostly evident in cases where the learner represents a step that shows an understanding of a rule but fails to proceed.

Some errors which are evident in the learners' written work are the unsystematic errors which develop due to factors other than incompetency (Luneta &Judah, 2010). These are the untraceable errors which have no link to concepts from which they may emanate from.

2.4.4 Merging the two error categorizations

Even though I have merged both Donaldson's (1963) & Hirsts' (2003) error categorizations, some of the classifications are not included in the framework used in this research. This is not to disregard the importance of Hirst's (2003) error classifications but it is chiefly because some of the categories are naturally similar. In fact, Donaldson's (1963) structural errors are heuristically similar from Hirst's (2003) pseudo linearity and equation balancing errors (Lutena & Makonye, 2010). Structural errors broadly incorporate both these categories as they are the sub-categories of the structural errors. As a result, the table below briefly tabulates the error categorization that will be used to frame the categorization of errors in this study.

Table 1:Types of errors with descriptions

Error/Misconception	**Description**
Structural	Failure to appreciate the mathematical relationships and principles.
Executive	Inability to apply mathematical manipulations and procedures.

Arbitrary	Ignore some information and considering the rest.
Procedural exploration	Extending a mathematical procedure in situations where it does not apply.
Incomplete application of rule	Applying the rule but unable to proceed.
Unsystematic	Errors that develop due to factors other than incompetency.

Both Donaldson's (1963) and Hirst's (2003) classifications are generalizations, and it is important to note that making generalizations is one of the mathematics qualities which may create errors. These errors may result when, for example, learners carelessly generalize the mathematical processes or procedures which they have learnt and thus develop errors and misconceptions. The merging of the error categorizations proposed by both Donaldson (1963) and Hirst (2003) in this research was also influenced by the constructivist point of view that learners develop errors when they try to interpret an unfamiliar context using their prior knowledge. Bearing the fact that one of the theoretical frameworks used in this research is constructivism, it is, therefore, a thoughtful approach to frame the analysis using these categories. I now wish to turn to the teaching interventions which are necessary in addressing the identified errors and misconceptions.

2.5 TEACHING INTERVENTIONS

This study focused on learners, how they learn as well as their errors and misconceptions. However, for the purposes of considering the aim and the objective of this research, partial attention should be given to the interactions taking place between the teacher and the learners. Although the study identifies the learner errors and misconceptions, there is a need for intervention mainly to examine and refine the identified errors and misconceptions by exposing them to the correct conceptions that will challenge their misconceptions and errors (Smith *et al*., 1994). This section unfolds the teaching intervention in the form of explanations.

2.5.1 Professional noticing

Professional noticing is explained as an act of observing the mathematical thinking and strategies that learners present in their work, and subsequently acting upon their actions (Schack, Fisher, &Wilhelm, 2017). This is a skill that teachers need for the purposes of mediating the learning of

mathematics and promoting high levels of responsiveness (Sherin *et al*., 2011). The application of the noticing skills when offering explanations as an intervention to learners, the intervention encompasses the three interrelated constructs of professional noticing which are *attending*, *interpreting* and *responding* or *deciding* (Jacobs *et al*., 2010).

The different constructs on professional noticing all concur to perceive professional noticing as a noticing resource to teaching (Schack, Fisher, &Wilhelm, 2017; Thomas, 2017). This means that to promote a clear interpretation of the learners' mathematical thinking and subsequently make an instructional decision that will yield better understanding, the teacher should consider the three strands which are, attending, interpreting and deciding.

Firstly, the teacher *attends* to the learners' mathematical thinking by examining their responses on the given tasks. Secondly, the teacher *interprets* the learners' algorithm presented in their responses by understanding their strategies and procedures used to respond to the given tasks. Lastly, the teacher decides upon a constructive response with an aim of addressing what the teacher has identified while attending and interpreting their work (Jacobs *et al*., 2010). During this time, the identified learner errors and misconceptions are addressed. The table below summarizes the strands of professional noticing.

Table 2: Professional noticing strands with descriptions

Professional Noticing strand	**Description**
Attending	Attending to the learner errors and misconceptions in solving algebraic equations.
Interpretation	Interpreting the learner errors and misconceptions by looking at their reasons behind their steps and processes in solving equations.
Deciding/responding	Responding to the errors and misconceptions by looking at the interventions to address the errors and misconceptions. These will be done through offering explanations.

Consistent with the preceding paragraph, the teacher attends to the learners' work by identifying the errors and the misconceptions from which the errors emanate from. The teacher then interprets these errors by finding reasons behind these errors and misconceptions. Lastly, the teacher then decides upon a responsive action that will address the identified errors and misconceptions. A practical example would be examining the learners' response to solving the equation: $2x^2 + 3x -$

$2 = 0$. The teacher attends to the learners' work by noticing the manner in which s/he attempts to solve the equation. The table below shows the learners' response:

Table 3: An example of the learners' interpreted work

Learners' response	Teachers' interpretation
$2x^2 + 3x - 2 = 0$ $x(2x + 3) - 2 = 0$ $x(2x + 3) = 2$ $x = 2\ or\ \ (2x + 3) = 2$ $2x = \ -1$ $x = \frac{-1}{2}$	The learner: ▪ took out the common factor x present in the first two terms. ▪ added 2 on both sides ▪ each factor (both the monomial and the binomial) was equated to zero then solved.

From the table, the teacher has interpreted the learners' manipulations by understanding what the learner was trying to do and the error which resulted from the learners' attempt. It is clear that the learner is aware of the concept of taking out a common factor, but has done it incorrectly by not considering all three terms making up a trinomial. This is an interpretation of the learners' work. The teacher then devices a strategy that would assist in addressing the identified error. The teachers action would then be, for example, offering explanations on what to consider then taking out a common factor, and one of these considerations would be highlighting to the learner that a common factor should be present in all terms of an equation (or expression) and not on selected terms. The teacher would then demonstrate, with a similar example, on how to remove the common factor.

From the demonstrated example, it is clear that the teachers' explanations were based on addressing the identified misconception which was explicit from the learners' response. On the same note of the third strand of professional noticing, I now turn to the teacher explanations as an intervention to address the learners' errors as well as the misconceptions from which the errors emanate from.

2.5.2 Teacher Explanations

According to Mutafiah *et al* (2018), an explanation is a communication which serves as an attempt to promote an understanding of a particular phenomenon to others. There are three categories of explanations i.e. interpretative explanations, descriptive explanations and reason-giving explanations. Interpretative explanations are the explanations that are for the purposes of defining terms and clarifying statements and meanings, descriptive explanations focuses on outlining the procedures and processes that needs to be learnt, and reason-giving explanations are specific to giving the cause-effect relationships that account for a specific phenomenon (Mutafiah et al., 2018) These explanations are typically responding to the *what*, *how* and *why* questions respectively and are equally important to establish learner understanding and successful knowledge construction.

To enforce leaner understanding of solving algebraic equations during intervention, the teacher should pay attention to all the constitutive criteria of explanations. Firstly, *matter* is explained as the elements or the parts that make up the object that is explained (Ruben, 1992). Note that the term object, in this paper, does not refer to a concrete and tangible object, but rather refers to the written and symbolic form of what is taught. For example, the constant term, the coefficients, the operations, equal sign, etc. are the elements that make up an equation hence they are matter. Secondly, *form* describes the arrangement or the structure of matter. This element highlights the pattern established by matter in making an object. Thirdly, *process* (which is also called motion originator) refers to what initiates change or the processes that needs to be applied with an aim of reaching a specific *goal* which is the last constitutive element. Fourthly, *Goa*l refers to the purpose of performing a particular activity, and the goal is to determine the value of the unknown (Ruben, 1992). These four criteria work together as a constitutive criteria of an explanation. Good explanation, according to Ruben (1992), constitutes of these four interrelated constitutive criteria. Hence these should be considered by teachers during teaching interventions when the teacher *responds* (Jacobs *et al*., 2010) to the identified and interpreted errors and misconceptions.

2.6 CONCLUSION

The chapter outlined the notion of algebra and algebraic equations and the notion of errors and misconceptions. It also provided a discussion of constructivism as well as Donaldson's construct of error classification, which serves as the framework of the study. Lastly, the chapter provided a justification of the framework adopted in this study.

CHAPTER 3: METHODOLOGY

3.1 INTRODUCTION

This chapter provides a discussion of the methodological perspectives that was applied for the purposes of data collection in this study. To do this, the chapter first discusses the methodological approach that was considered, followed by the sampling and setting procedures used and the data analysis employed in this study. The chapter then outlines the validity and reliability of these methods as well as the ethical considerations which were adhered to in this study. Lastly, the summary which unites all these methodological aspects is provided.

The main focus of this study was, firstly, to identify the errors and misconceptions which grade 9 learners display as they attempt to solve algebraic equations. Secondly, to interpret the learners' intuitive conceptions and the reasoning that influenced their processes, procedures and steps used to solve the given algebraic equations. Lastly, to establish the effect of offering explanations that are directed at these identified errors and misconceptions on the learners' performance provided that these explanations are offered in a professional noticing intervention.

3.2 METHODOLOGICAL APPROACH

3.2.1 The qualitative research method

For the purposes of meeting the requirements of the study, I have adopted the qualitative research paradigm that is in line with the theory of constructivism in nature. A qualitative research paradigm, in Creswell's (1998)'s terms, particularly focusses on acquiring information that will promote an understanding of a particular phenomenon by eliciting deeper and richer reasoning or ideas about the phenomenon. This study has purposefully adopted the qualitative study methodology mainly because the qualitative methodology, particularly the constructivist qualitative methodology, describes the meanings that human beings construct in their mental capacities (Creswell, 1998). This study focuses on the description of the learners' thinking that is evident in their responses, hence the qualitative research paradigm is purposefully employed in this study as it suits the requirements of this study. Moreover, the qualitative research paradigm was adopted for creating a richer context to better understand phenomenon that is researched. The results obtained from a qualitative paradigm are more descriptive (Cohen & Manion 1994), thus it

assisted me as a researcher to obtain data that is more into explaining and understanding the effect of using the learner errors and misconceptions in informing the explanations. I find it important to point out the fact that a qualitative paradigm was selected as opposed to the quantitative mainly because my intention was to comprehend and understand the effect of using learner errors and misconceptions in a professional noticing intervention and the improvement in their performance when explanations are informed by these errors and misconceptions.

As a researcher, the qualitative research study allowed me to be the main one who is responsible to collect and analyze the data throughout the researching period. This means that I was responsible for ensuring that I carefully decide on the research design applicable for the research, select the sample that will assist in collecting rich raw data, devise strategy useful in collecting and analyzing data.

The data presented in a qualitative paradigm is mostly concrete and represented in the form of descriptions, ways of thinking and stories and less of numerical. In line with the qualitative research paradigm, data was analyzed and represented graphically (both the bar and the line graphs) not to produce quantitative analysis but to rather portray a clear comparison between the pre-test outcomes and the post-test outcomes. Conclusions were then made based on the represented outcomes hence the qualitative research methods have assisted me in ensuring that the research questions are answered.

A qualitative research method allows the researcher to gather a sample that is purposefully selected with an aim of finding patterns and relationships (Creswell, 1998). The one employed in this study is the case study of individual learners to capture the improvement in their performance when explanations are based on their errors and misconceptions. Data is, therefore, collected from a sample of a whole population of grade 9 mathematics learners. Nonetheless, this research approach is influenced by the fact that it is perceived that the qualitative approach has the power to promote rich and valid data with stronger and reliable evidence of knowledge claims (Creswell, 1998). It increases the credibility of the findings (Johnson & Christensen, 2008).

3.2.2 Research design

To address the research questions, this case study incorporated the writing of pre-test and post-test, conducting interviews and intervention session (that was 3 hours long) in which the researcher became the observer. This design was selected mainly because it had the potential of modelling the data collection process in a way that the findings meet the research requirements and I am able to successfully respond to the research questions. Through this design, the validity as well as the reliability of this study was considered.

The data collection process was sequenced in a sense that, during the first stage, data on the errors and misconceptions that learners have in solving algebraic equations was collected using the pre-test instrument. The test instrument is important for answering the first research question:

1.7.1 What learner errors and misconceptions are identified by the teacher while attending to the grade 9 learners' work on solving algebraic equations?

The analysis of these scripts exposed me to the errors and misconceptions which learners are prone to.

The second stage was based on learner interviews where a sample of 7 grade 9 learners who wrote the pre-test were interviewed on their test responses. These interviews are important for eliciting the thinking behind their responses and the information acquired from these interviews was necessary to deepen and enrich the study by answering the second research question:

1.7.2 What learner reasoning and justifications are evident from the learners' work of solving algebraic equations?

From these interviews, I was able to establish the misconceptions from which the learners' errors emanated from. This information, together with the identified errors informed the teacher explanations offered thereafter.

which were offered as an intervention to address the identified errors and misconceptions.

The third stage, the intervention session, served as a session to address the identified errors and misconceptions using teacher explanations in a professional noticing context. The teacher explanations offered were informed by the identified errors and misconceptions as well as the

learners' intuitive conceptions which were evident from the interview sessions. This stage was necessary to answer the research questions:

During the fourth stage, the very same group of learners who wrote the pre-test, including the sampled learners who were interviewed, were given a post test. This stage assisted the researcher in answering the following research questions:

1.7.3 What possible learning advantages and disadvantages can result from providing explanations that are informed by the learners' errors and misconceptions?

1.7.4 To what level can learning and learner achievement in solving algebraic equations be improved if explanations are informed by the learner errors and misconceptions?

The data that was collected using the four above-mentioned four stages were analyzed, findings were discussed and conclusions on the effect of using learner errors and misconceptions in providing instructional explanations in a professional noticing intervention were made (as thoroughly presented in chapters 4, 5 and 6). This research method is employed in the study for the purposes of enriching the results and conclusions of the study (Creswell, 2008).

3.3 SAMPLING AND SETTING

Sampling, as an act or technique used for selecting a representative part of a population (Crawshaw & Chambers, 2008), serves as an integral aspect for data analysis purposes. Setting is devising a platform for the collection of data. To consider the sampling and setting appropriate for this study, there is a need to consider the participants, the instruments as well as the procedure that will be necessary for the study hence this subsection outlines such information.

3.3.1 Participants

(a) Learners:

This research was conducted in a classroom context at a secondary school in Johannesburg, Gauteng province, with a class of 40 grade 9 mathematics learners. For convenience purposes, this group of learners were selected from a school that is very close to my current location, and the selection of participants was on voluntary basis. From this group, a sample of 7 learners, who were

present throughout the sessions, were selected to partake in interviews. These learners were sampled based on their pre-test results where 3 of the 7 learners got below average, 2 were average and 2 of them were the highest.

Since the selection was on voluntary basis, the grade 9 learners at the selected school were informed about the research study. It is important to note that participation to writing the pre-test, post-test and attending the intervention session was on voluntary basis, but the interview session was based on purposive selection. However, even though it was on voluntary basis, participants must have been taught algebraic equations in grade 8 and earlier in grade 9. This decision is influenced by the fact that assessing learners who were never exposed to algebraic equations defeats the purpose of conducting this study and will affect the validity of the outcome. 65 grade 9 mathematics learners who were interested to be participants of the study were given information sheets and consent forms which they, as well as their parents, had to complete. Out of the 65, only 40 were available as their parents agreed for their learners to be part of the research. From the sample, all 40 of them pitched for the pre-test. The sudden decrease in the number of volunteers did not affect the data collection process as we were able to proceed with the number of participants that were present.

(b) Teacher:

During the intervention session, the teacher offers explanations that are directed at the identified learner errors and misconceptions. The mathematics teacher who taught these learners during the intervention session is also from the same school whom I've worked with before. After the discussion I had with her regarding this research, the teacher agreed to be part of the study.

The teacher took part of the study by availing herself in the

- briefing on the objectives of the study
- discussion of the identified learner errors and misconceptions
- Preparations of the intervention lesson
- Teaching by offering explanations.

3.3.2 Professional noticing intervention

I wish to include the professional noticing intervention as it is important for the setting of this study. Professional noticing, as outlined in the preceding chapter, is a skill that mathematics teacher applies with an aim of producing mathematically proficient learners (Jacobs *et al*., 2010). During a professional noticing intervention, the teacher considers the three strands of professional noticing that ensures that the teacher guides learners in ensuring success in knowledge construction.

Firstly, the teacher *attends* to the learners' errors and misconceptions by reviewing their written work. A researcher who serves to be the primary instrument, according to the qualitative research paradigm (Cohen & Manion 1994), I have shared with the teacher these errors and misconceptions which I identified in the learners' responses to the pre-test. I have also ensured that I discuss the learners' reasoning and justifications behind their responses with the teacher. This was to ensure that the teacher is:

- Enlightened about the errors and misconceptions that learner encountered in the pre-test (which I obtained from the analysis of the learners' pre-test responses)
- Aware of the wrong intuitive conceptions which the learner has (which I gathered from the interviews).

To ensure that I am able to meet the objective of the study, which is to investigate the effect of offering explanations that are directed at the learner errors and misconceptions, I notified the teacher about the importance of considering these findings when preparing the teacher's explanations.

Secondly, the professional noticing intervention allows the teacher to *interpret* the identified errors and misconceptions as well as their intuitive conceptions. The teacher does this by linking the learners' misconceptions with the correct concept definition which the learner was supposed to construct (Tall & Vinner, 1981). By trying to understand what the learner was trying to do, it makes it easier to address their misconception ns hence it was important to conduct the interviews. The teacher engages with such interpretation while preparing for the intervention session.

Thirdly, the instructional explanations that the teacher offer serves as the *response* to the learners' mathematical thinking, the third strand of professional noticing. These explanations were informed

by the identified errors and misconceptions. The aim of this intervention was to establish the cognitive gains and the improvement in the performance of the learners as a result of using the errors and misconceptions when teaching.

3.3.3 Procedure:

The participants of the study wrote a pre-test that was 40 minutes long on solving algebraic equations (see annexure A). These tests were marked, mainly for the purposes of noting the errors which learners made.

From their pre-test responses, I have purposefully sampled 7 learners whose papers were used for the analysis that is needed in this study, and were interviewed based on their responses to the pre-test. The purposive sampling of these learners was on non-probability basis where I have considered the below average, average and above average, mainly to ensure that this study yields a generalizable outcome and is not biased in any way.

The interviews were conducted at school, in the very same class where they wrote the test. The interview was in an informal setting I tried by all means to ensure that the learner is comfortable and made aware that s/he will remain anonymous (issues of anonymity and confidentiality are clearly outlined below). The time it took to interview each learner was dependent on the dialogue which took place between the researcher and the learner, with the average time being approximately 9 to 10 minutes per learner.

During the interviews, learners were probed with questions that allowed them to break down the steps which they used to solve the equations. The interrogations were more inclined to understanding their intuitive conceptions and procedural knowledge of solving algebraic equations. These interviews afforded me an opportunity to get a deeper understanding of the misconceptions which yielded the identified errors. I was able to get their algebraic understanding, the reasoning behind their procedures and steps and the strategies that the learners applied when solving algebraic equations in the pre-test Moreover, from their responses in the interviews, I was able to understand their intuitive conceptions on solving algebraic equations. It is for these purposes why the interviews were conducted.

Using the data collected from the pre-test scripts and from the interviews, intervention sessions were conducted by the selected teacher. This was after I had an interaction with the teacher concerning the errors and misconceptions which I identified. During this time, the teacher offered explanations to the learners in a classroom context and I as the researcher was an observer. This session was three hours long and it was conducted immediately after school to avoid disrupting the lessons during the day.

Thereafter, learners were given a post-test (see annexure B) with questions that have the same cognitive difficulty as the pre-test questions. It was administered a day after interventional sessions in order to make analytical comparisons of the learners' responses and performance between the pre-test and the post-test. Applying this procedure assisted me with responding to the research questions of the study.

To avoid spoiling the purpose of the test sessions, both test sessions were conducted under formal examination rules.

3.3.4 Instruments

As it has been mentioned in the preceding sub-sections, the study has used tests, interviews and observation schedules as the instruments of the study. These instruments were meticulously selected on the basis of its function and purpose in data collection.

- A pre-test was used mainly to get the errors and misconceptions which learners have when they attempt to solve algebraic equations.
- A post-test was used, not to observe the learners' errors, but for comparative analysis purposes. Using both the pre-test and the post-test, I was able to trace the cognitive gains and improvement in learner performance which resulted from the use of learner errors and misconceptions when offering explanations.
- Interviews were also used as an instrument mainly for understanding the reasoning behind the learners' misconceptions (Opie, 2004).
- An observation schedule was also used, chiefly to observe the explanations offered by the teacher, the learners' participation in the tasks given to them during the lesson as well as

their expressions throughout the session. Most importantly, the observation afforded me an opportunity to observe the manner in which the teacher applies the professional noticing skills and Ruben's (1992) criteria for offering explanations to address the identified errors and misconceptions.

Ultimately, I was able to collect sufficient data needed to answer the research questions and subsequently meet the objectives of this study using these instruments.

3.4 DATA ANALYSIS

Data analysis is a process of extracting and condensing data to be reviewed it and converting it to determine meaningful information, to formulate an aggregated decision and to draw a conclusion from the collected data (Hatch, 2002). The data analysis process needs different instruments such as interviews, tests, observation notes transcripts, audio and video recordings to collect the raw data and thus using it to draw conclusions. Data analysis involves diagnosis, which is, according to this study, the process of identifying and characterizing the errors and misconceptions that learners encounter when solving algebraic equations (Brueckner & Bond, 1955).

I have adopted a conceptual framework for categorizing the data, however, the data which I will collect is not limited to the framework. The data analysis process accommodates findings that are both within and the possible findings that are outside the framework. For this reason, I have adopted both the inductive approach (with the use of a conceptual framework) and the deductive approach which incorporates the findings outside the framework (Hatch, 2002). Throughout the data analysis phase, I employed Donaldson's error categorization framework to categorize my findings.

3.5 RELIABILITY AND VALIDITY

Reliability is explained as the degree at which the instrument(s) for data collection is trusted to yield consistent results even when it is considered in different research settings by different researchers with similar research conditions (Opie, 2004). Validity is a measure of quality of the collected data, measuring how well the data collected covers the area of investigation (Taherdoost,

2016). The importance of both these constructs are considered hence the methods of collecting data were carefully selected for the aim of this study.

For the objective of the study, I considered the pre-test and post-test which exposed me to the learner's errors and misconceptions in algebraic equations and the extent to which the explanations that are focused at the errors and misconceptions afford cognitive gains on algebraic equations. To maintain reliability of the data, the pre and post tests were of same cognitive difficulty, same marks and equal writing time allocation. The writing of the pre-and post-tests was administered under same conditions where learners adhere to the normal examination rules of learners not being permitted to talk or discuss anything with their fellow classmates, not being permitted to use any notes. Lastly, to maintain reliability of the process and the outcomes, the time between the interventional teaching sessions and the writing of the post-test were minimized as much as possible.

Validity, as a measure of the power of a research instrument in measuring what is supposed to be measured, is embedded with different types. The face validity, for example, is concerned with the degree to which a measure appears to be a specific construct. A test has face validity if its content looks relevant to the person taking the test. It is concerned with the researchers' subjective assessments of the presentation and relevance of the measuring instrument as to whether the items in the instrument appear to be relevant. The content validity is the degree to which the items in an instrument reflect the content universe. Overall, validity considers the research questions of the study, the findings and how these findings are interpreted (Opie, 2004).

To answer the first question, for example, answers provided by learners in the pre-test had errors and misconceptions that learners encountered in algebraic equations. To respond to the second question, conducting an interview session assisted me in determining their cognitive understanding and the reasoning behind their approaches. To respond to the third question on the effectiveness of providing explanations that are focused at the identified learner errors and misconceptions, the interventional sessions together with the post-test results assisted in providing sufficient data.

Important to note, as the researcher, I made sure that I am aware of the personal biases to avoid tempering with the overall validity of the outcomes. To do this, I ensured that the pre-test and

post-test responses were thoroughly and meticulously checked and analyzed. Moreover, to avoid assuming the reasoning behind their steps, I ensured that I interview the sampled learners to get an understanding of their intuitive conceptions thus ensuring that the results of the study are valid in all regards.

3.6 ETHICAL CONSIDERATIONS

Throughout the data collection process of the study, there was an involvement of the selected grade 9 learners as the participants of the study, hence there were ethical considerations which I adhered to. Ethical considerations pertaining the participants is important so as to avoid violating the ethical code of conduct. This study, therefore, highly considered ethics such as the privacy of the learners and the physical or psychological harm of the learners.

Part of adhering to the ethical requirements, I requested permission to conduct this research from the Wits Ethics Committee by applying for ethics clearance. Thereafter, I requested permission from the Principal and the School Management Team of the school which the research was conducted. Once the school approved my application, I, provided parents of the selected learners with consent forms and the learners with information sheets. Both the consent form and the information sheet highlighted the purpose of the study, clearly stating that the study was on voluntary participation basis, and most importantly, guarantee them that the confidentiality and anonymity of the participants are highly considered. To maintain and assure confidentiality and anonymity, I made use of pseudonyms and the collected data is kept safe in a USB flash disc specially bought for the data. This research ultimately contributes to the current literature on the learner errors and misconceptions hence I ensured that ethics are strictly adhered to in all possible ways.

3.7 CONCLUSION

This chapter provided a discussion of the methodology that underpins the research by outlining the methodological approach, the setting and sampling as well as the proposed data analysis process that will suit the data collection process of this study. The chapter also discussed issues of reliability, validity as well as the ethics discussions that needs to be considered.

CHAPTER 4: DATA ANALYSIS

4.1 INTRODUCTION

This chapter provides an analysis of the data that is collected from the learner responses to the written tests, from the interviews and lesson observations. Data collection is explained as a process of collecting data using different methods with an aim of gaining insights of the research (Hamed, 2021). Data analysis is the process of examining and transforming the collected data using the theoretical and conceptual frameworks that are framing a research study. To analyze the data, I have, firstly, framed the data from the test responses using the conceptual framework of identifying and categorizing the learner's errors and misconceptions discussed earlier. Secondly, I have used the learner interviews to understand the learners' intuitive conceptions and make links of the errors and misconceptions to the correct mathematical conceptions. Thirdly, the chapter provides an analysis of the intervention session that was conducted to address the misconceptions and errors made by learners. Even though the explanations offered by the teacher during the intervention session are not the main focus of this study, the discussion of how they proceeded is presented in this chapter mainly to answer the research questions and to ultimately enrich the study. Lastly, the chapter provides a comparison between the pre-test and the post-test responses mainly to understand the effect of using errors and misconceptions during interventions in ensuring learning improvement.

4.2 ANALYSIS OF THE LEARNERS' RESPONSES TO THE PRE-TEST

I now turn to the data analysis process of the responses to written pre-test guided by the conceptual framework which I used to group the errors and misconceptions according to the framework. The discussion of this section incorporates both the pre-test responses and the interview results. The reason why both the pre-test responses and the interview results are incorporated in this section is to bring in an understanding of the learners' reasoning behind the errors which they made and the misconceptions which they had. Ultimately, this segment of the chapter responds to the following questions:

- What learner errors and misconceptions are identified by the teacher while attending to the grade 9 learners' work on solving algebraic equations?

- What learner reasoning and justifications are evident from the learners' work of solving algebraic equations?

4.3 IDENTIFICATION, CLASSIFICATION AND ANALYSIS OF LEARNER ERRORS AND MISCONCEPTIONS

CASE 1:

Question 2: Solve for y: $2^y = 8$

Learner's response:

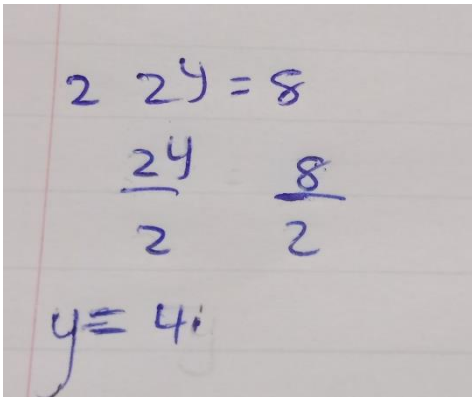


Figure 1: (CASE 1) Learner response to question 2 of pre-test

Error/Misconception identified: Procedural exploration

This response shows that the learner has a background understanding of solving equations, that the value of the unknown has to be isolated hence the learner's aim was to isolate the y variable from 2. The error made by this learner was due to lack of conceptual understanding and failure to appreciate the difference between a coefficient and the base of a term. The procedure used to isolate the unknown from the coefficient of a term is different from when the unknown is an exponent. The learner's response shows that the learner is aware of the procedure of isolating the unknown from the coefficient of a term but the learner extends it by using it in a different situation where it does not apply hence it is a procedural exploration. The learner's procedure of determining the value of y suggests that a coefficient of a term is not different from the base, and this is the root of the learner's error. Given the term 2^y, 2 in this case is the base and y is an exponent and for the term $2y$, 2 is the coefficient of y. Even if the learner's procedure correctly applies in a

situation where the unknown is isolated from the coefficient, this procedure does not apply in a situation where the unknown is the power of a base.

With the equation $5x = 25$, for example, since 5 is the coefficient and we want to find the value of x, dividing both sides by 5 to get x = 5 would apply in this situation. However, to solve for x when $5^x = 25$, the bases both on the left hand side and the right hand side have to be the same. This means that since $5^2 = 25$, then $5^x = 5^2$ and therefore $x = 2$.

CASE 2:

Question 1: Solve for x: $x^2 = 36$

Learners' response:

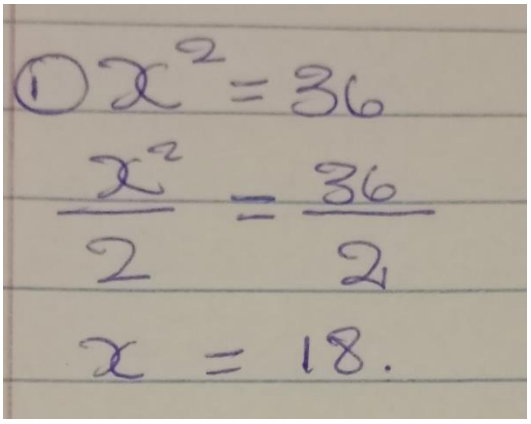


Figure 2: (CASE 2) Learners' response to question 1 of pre-test

Error/Misconception identified: Procedural exploration

The learner is aware that for x to be solved, it needs to be isolated from the known values. The learner divided both sides of the equation by 2 mainly to isolate *x*. The learner, in this case, cannot differentiate an exponent from a coefficient. Similar to case 1 represented above, the learner has encountered a procedural exploration error in a sense that she has failed to appreciate the dissimilarity between the procedure needed to isolate *x* from 2 when 2 is an exponent and *x* from 2 when 2 is a coefficient. The learner knows that both sides have to be divided by 2, but only when it is a coefficient. She has applied this procedure in a situation where the procedure does not apply hence a procedural exploration error was evident.

CASE 3

Question 5: Solve for x: $(x+2)(x+7) = -4$

Learner's response:

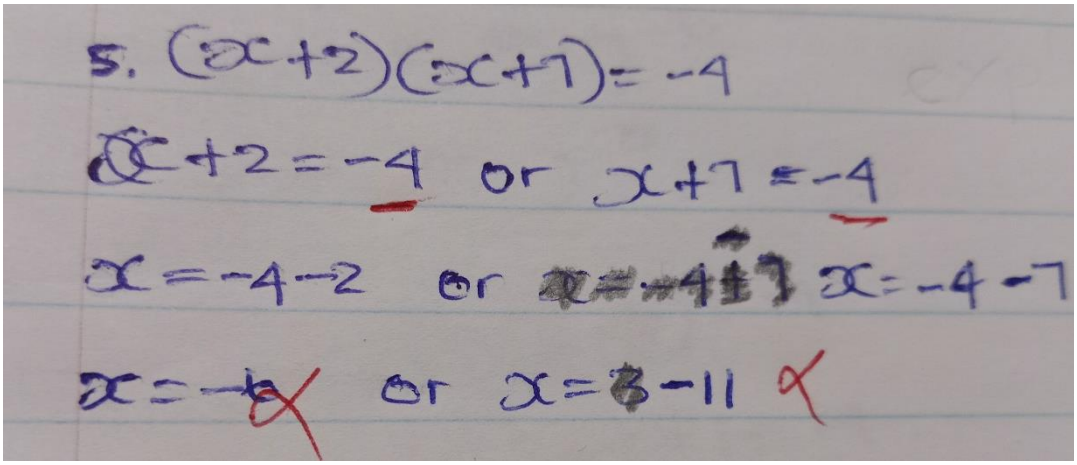


Figure 3: (CASE 3) Learner response to question 5 of pre-test)

Error/Misconception identified: Structural

This response shows a structural error that the learner has made. It emanates from the learner's background understanding of the principle of solving a quadratic equation that has 0 on one side of the equation. The principle of solving quadratic equations which have the product of factors being equal to 0 allow learners to equate each factor to 0. The factors may be monomials, binomials or even trinomials. If $(x+2)(x+7) = 0$ are the two factors that are binomials, then $x + 2 = 0$ and $x + 7 = 0$. In this case, $x = -2$ and $x = -7$ are the solutions. The learner generalized this rule and believed that it is applicable even for equations without a 0 on either side.

To solve the equation$(x+2)(x+7) = -4$, the learner was supposed to first get one side of the equation to be equal to 0 by either transposing -4 to the left hand side or by adding 4 on both sides of the equation thus making the right hand side to be equal to 0. The learner was then going to use the FOIL method to simplify the left hand side then add the like terms to eventually remain with a trinomial which the learner then factorizes to get the product of two binomials equal to 0.

CASE 4

Question 5: Solve for x: $(x+2)(x+7) = -4$

Learner's response:

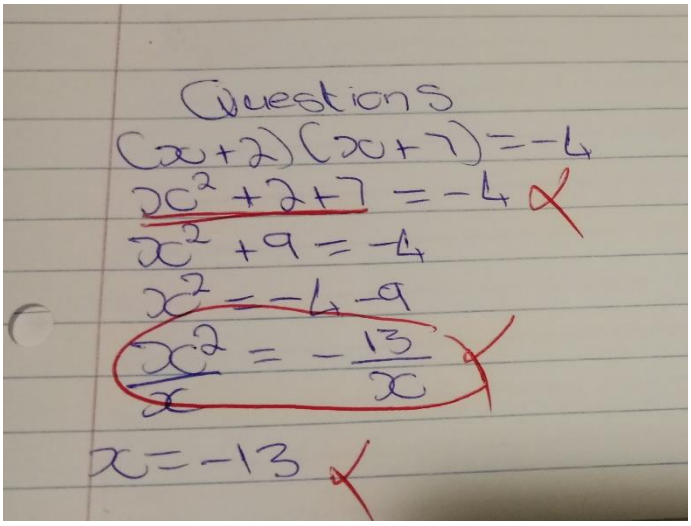


Figure 4: (CASE 4) Learner response to question 5 of pre-test

Error/ misconception identified: Structural and procedural exploration

Looking at the same question but a a response from a different learner, this learner solved for x by attempting to use the FOIL method. To get an understanding on whether the learner has made a mistake or is convinced that this is how the the FOIL method is applied, the learner was interviewed and this is the response:

Researcher: May you explain how you got your answer?

Abe: Okay, so this is what I did; Isn't it to remove the brackets I have to use the, the uhm, the FOIL method? So I used the FOIL , I multiplied x with x so that I get x to the power of 2. Then since 2 and 7 are positive, I said $x^2 + 2 + 7 = -4$

Researcher: Wait, so if you had -2 and -7 instead of +2 and +7, it means you were going to get $x^2 - 2 - 7 = -4$?

Abe: yes ma'am.

Researcher: Do you have an idea of what FOIL stands for?

Abe: First…uhm, I'm actually not sure what the other letters stand for, oh L is for last I think.
Researcher: FOIL stands for First Outer Inner Last. Alright continue.

Abe: Oh, then after removing the brackets, you have to collect the numbers that don't have x . I got -13 then I divided both sides by 2, then my answer is -13.

Researcher: Oh I see. Were you supposed to divide both sides by *x*?

Abe: Yes ma'am because you must remove the 2 that is on top and when you divide both sides by *x*, they will cancel each other and then you will be left with 1*x*. Am I right ma'am?

Researcher: Your answer is incorrect but I understand where and why you went wrong.

It is important to note that these error categories are not mutually independent or distinct from each other, instead, they are somehow in continuum and have some similarities and differences from each other. It is evident that the learner has both structural and procedural exploration errors. Firstly, the learner could not apply the FOIL method correctly. This is a structural error which resulted due to the misunderstanding of how to apply the FOIL method to solve the quadratic equation. Secondly, the Abe decided divide both sides by *x* with an aim of "*removing the 2 on top*" as she said. The learner understands that she has to isolate *x*, however, her step shows that she has procedural exploration error. She knows the quotient of power rule, the laws of exponent, that when you divide and the bases are the same, you subtract their exponents. However, the learner has used this law in a different situation where it does not apply hence the procedural exploration error has resulted.

CASE 5

Question 7: Solve for *x*: $3x^2 + 6x - 9 = 0$

Learner's response:

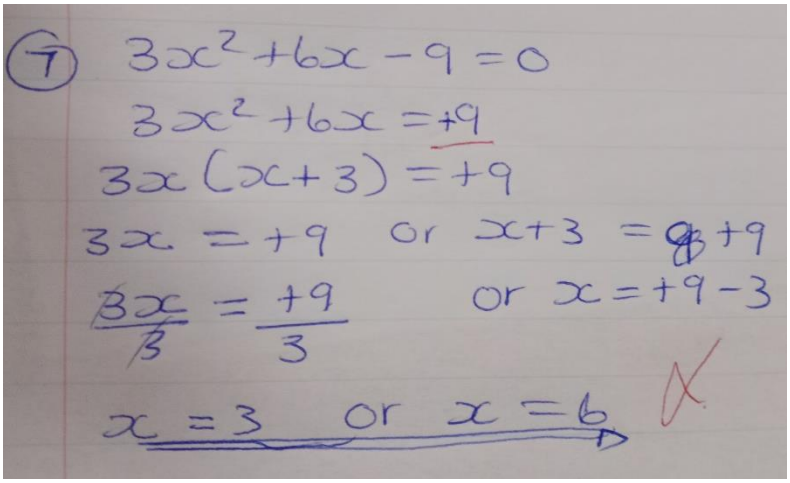


Figure 5: (CASE 5) Learner response to question 7 of the pre-test

Error/ misconception identified: Arbitrary and Executive

Firstly, the second line of the learner's response (i.e. the line after the first line) shows that he attempts to solve the equation by factorizing, particularly taking out a common factor. However, the learner's attempt to factorize seems as if he considered the first two terms and transposed the constant to the right hand side of the equation. This step shows that the learner is aware of the concept of factorizaton but he cannot apply it to manipulate and solve the equation. By considering the first two terms and leaving out the constant term shows that the learner's error is arbitrary in such a way that the equation is rearranged to suit the learner's existing schema and leaving out the rest of the important information. It is also an executive error as the learner is unable to factorize the trinomial; even when he was trying to take out a common factor to satisfy his knowledge, he has done it incorrectly hence it is also an executive error. The learner has $3x$ as the common factor, remaining with $(x+3)$. Instead of dividing $6x$ with the common factor, I am convinced that the learner subtracted 3 from 6. Secondly, the learner decided to equate both the monomial and the binomial factors to 9. I am convinced that the learner was using the rule that if $xy=0$ then $x=0$ and $y=0$. The learner's algorithm and a clarification of why the learner got $(x+3)$ instead of $(x+2)$ was obtained in our dialogue:

Researcher: Zoli, I see that you have transposed the constant to the right hand side, am I correct?

Zoli: Yes ma'am, I wanted to factorize the equation.

Researcher: Explain how you factorized.

Zoli: Okay, ma'am I took out the factor 3*x* because they have 3x.

Researcher: How did you take out 3*x*?

Zoli: I said, uhm, I have $3x^2$ so I said 3 - 3 then I removed 1*x* and I was left with 1*x*. Then I took the second one which is $6x$ then said 6 -3 and I got 3 and cancelled the *x* value. That is why I have $(x + 3)$.

Researcher: Couldn't you factorize with the -9 on that side?

Zoli: No ma'am. -9 does not have *x* so I took it to the side where there is no *x*.

Researcher: Couldn't you factorize the trinomial?

Zoli: how ma'am? Look the first term has 3 so it will be hard to get the factors.

Researcher: You actually can. (I demonstrated how it should be done)

Researcher: Lastly, Zoli, I see you have equated both factors to 9. Can you explain why you did so.

Zoli: I was not sure if I was doing the right thing since I no longer had zero on the other side. I am not sure what to do with the 9 because I can't take it back.

From the learner's explanation, it is clear that the learner has an executive error because he cannot factorize. Firstly, instead of dividing the termsby a common factor, he took out the common factor by subtracting the coefficient of the common factor from the coefficient of the terms. Secondly, he didn't notice that the trinomial itself has a common factor of 3 which he should have taken out before factorizing a trinomial. He is probably more knowledgeable with taking out a common factor rule, making it hard or him to move from his comfort zone of taking out a common factor of a binomial to acquiring new knowledge of factorzing a trinomial. This error could possibly eminate from the conflict that exists between the knowledge that already exists in the learner's cognitive structures and the new knowledge which needs to be assimilated and accommodated into the learners cognitive structures Piaget (1967).

CASE 6:

Question 3: Solve for x: $3^{x-3} = 81$

Learner's response:

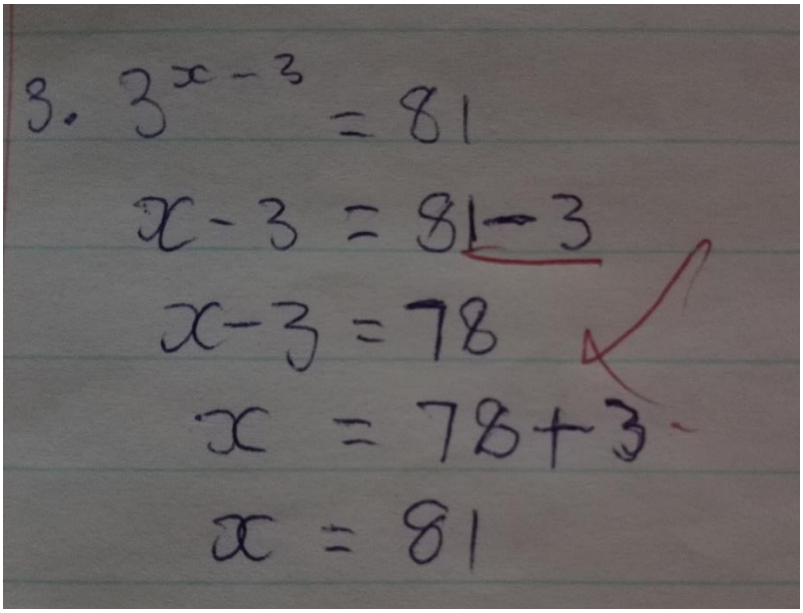


Figure 6: (CASE 6) Learner response to question 3 of the pre-test

Error/ misconception identified by the researcher: Arbitrary and structural

The learner's attempt to solve for x involves transposing base 3 to the right hand side which he subsequently subtracted it from 81. He ended up having $x - 3$, which was initially the exponent, meaning the structure of the equation has changed. The -3 which was an exponent was also transposed to the right hand side. The learners' intention in this regard was to group the like terms and this is where the two errors emanated. However, a clarification of the learner's algorithm is supported by the dialogue below:

Researcher: What happened to base 3 that was on the left hand side?

Samuel: Ma'am, I have taken it to the right because it is like terms with 81.

Researcher: If you decide to transpose the base, what about the exponent?

Samuel: I bring down the exponent because I have to take the 3 also to the other side because I must be left with *x*.

From the dialogue, it is clear that the learner's main aim was to make x the subject of the formula by considering concept of collecting like terms. However, the learner's response as well as the dialogue shows that even though the learner considered the concept of making the unknown the subject of the formula, he failed to appreciate principle of solving exponential equations. As a result of considering one aspect and leaving out the other, the learner encountered an arbitrary error. His steps, particularly the second line when he transposed the base, shows that the learner also lacks conceptual understanding of how to solve for x when x is an exponent and not a base. He has failed to acknowledge the conceptual different that exists between linear equations and exponential equations.

Instead of transposing the base, the learner was supposed to write 81 as 3^4 so as to create the same bases on both sides of an equation. The learner would then work with the exponents to get:

$$x - 3 = 4$$

$$x = 4 + 3$$

$$\therefore x = 7$$

CASE 8

Question 7: Solve for x: $3x^2 + 6x - 9 = 0$

Learner's response:

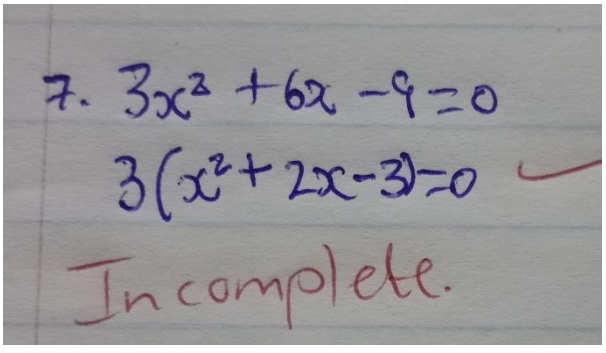


Figure 7: (CASE 7) Learner response to question 7 of the pre-test

Error/misconception identified by the researcher: Incomplete application of rule and executive

The learner is knowledgeable about the concept of factorization and has applied it by taking out 3 as a common factor of all the three terms. Even though the learner recognized the existence of a common factor, her work of determining the value of *x* is incomplete. From the discussion that I had with her, which was to find the reason for an incomplete task, I noticed that she had executive error of not being able to factorize a trinomial. The following snippet of our dialogue below justifies my claim:

Researcher: why didn't you complete the task?

Dudu: I did not know what to do because there is a bracket and there are two numbers that have *x*.

Researcher: So it is not possible to factorize a trinomial that is in the brackets?

Dudu: I have never seen how to factorize the one that is inside the brackets.

From the learner's response, it is evident that the learner does not know how to factorize a trinomial. She is convinced that in order to factorize, there shouldn't be any brackets. I believe that values. where there are no brackets at all. She also believes that to be able to factorize fully, there has to be a common factor, meaning -3 is supposed to be a coefficient of *x* and not be a constant. Her claim that the trinomial has cannot be factorized any further suggests that she is unaware, or has probably forgotten, that a quadratic equation can possibly have up to two roots hence the degree of the equation is 2. To get these roots, she was supposed to factorize the trinomial by writing the factors $(x + 3)(x - 1)$ which she the equate each factor to 0 to get the values of x.

I believe that limiting learners from an exposure to different structures of questioning may also cause errors and misconceptions. Dudu, for example, could not recognize that there is a trinomial inside the brackets which can be factorizes, and this emanated from the fact that she was never exposed to a question of factorization where the expression that needs to be factorized is in the brackets.

CASE 8:

Question 8: Solve for y: $2.4y + 10 = 138$

Learners' response:

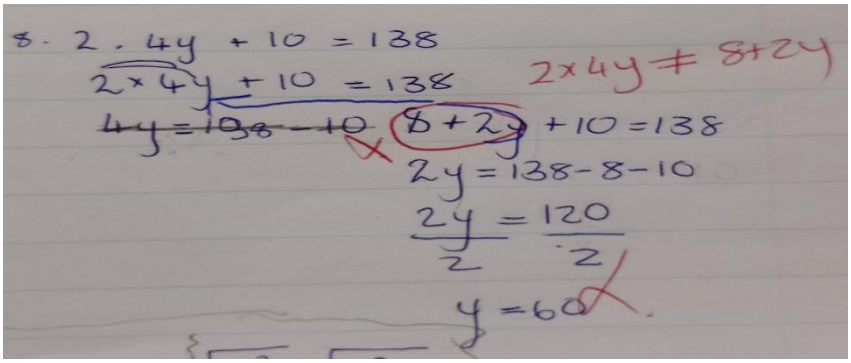


Figure 8: (CASE 8) Learner response to question 8 of the pre-test

Error/Misconception identified by the researcher: procedural exploration

The learner's first step of her response shows that she attempted to apply the distributive law by multiplying 4 with 2 and *y* with 2 respectively. This step shows the learner knows the law, but she has applied it in a wrong situation. It is clear that the learner confused $2(4 + y)$ with $2(4 \times y)$.

CASE 9:

Question 9: Solve for x: $\frac{x+4}{4} - \frac{2x-7}{8} = 9$

Learner's response:

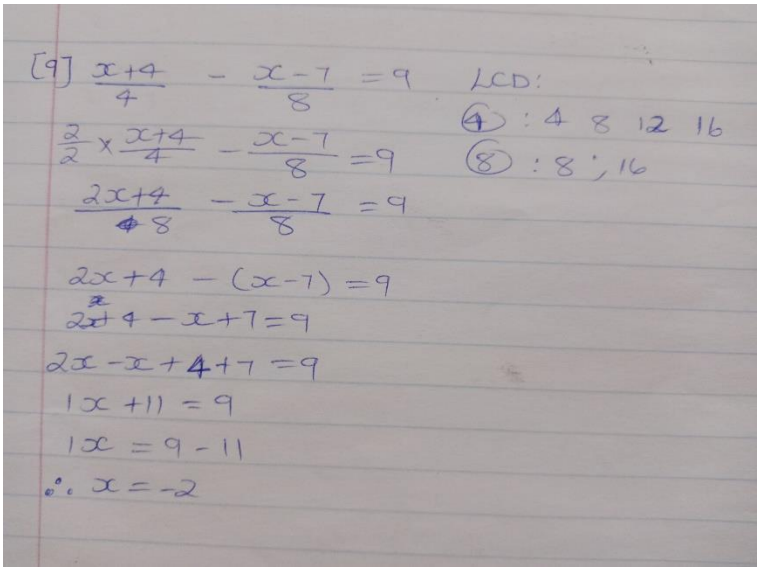


Figure 9: (CASE 9) learner response to question 9 of pre-test

Error/Misconception identified by the researcher: Arbitrary and executive

The learner's error emanates from, firstly, her knowledge and understanding of the concept of simplifying expressions and solving algebraic equations with fractions. She has constructed the rule that algebraic fractions can be simplified only when their denominators are equal. For example, $\frac{2}{3} - \frac{1}{5}$, the learner cannot subtract 1 from 2 because the denominators are unequal not the same hence there is a need to find the lowest common multiple (LCM) between 3 and 5. With 15 being the LCM, the learner needs to then multiply both the numerator and the denominator of $\frac{2}{3}$ by $\frac{5}{5}$ and $\frac{1}{5}$ by $\frac{3}{3}$ to get $\frac{10}{15} - \frac{3}{15} = \frac{7}{15}$. The learner's response shows that she acknowledges the rule ensured that the denominators of the fractions on the left-hand side are equal, however, the learner's arbitrary error emanated when she considered the two terms on the left-hand side but left out the term that is on the right-hand side when ensuring that the denominators are the same. The learner seems to misunderstand the concept of equations and balancing the equations hence the right-hand side was neglected. The snippet of our dialogue clearly outlines the reasoning behind the identified arbitrary error:

Researcher: Dominique, if you were to solve this equation: $2x + 7 = 3$, what would be your first step?

Dominique: I would subtract 7 on both sides of the equation and then…

Researcher? Wait, why would you subtract 7 on both sides? Why would you not only subtract on the left-hand side?

Dominique: Our teacher last year used to say what we do on the left we should also do on the right.

Researcher: Now look at the second and third steps of your answer. I see you found the LCM for the fractions on the left-hand side, what about the right-hand side? Why didn't you consider the right-hand side?

Dominique: you mean 9 ma'am?

Researcher? Yes

Dominique: because 9 is not a fraction ma'am.

Researcher: So you don't balance the equation when you have fractions?

Researcher: uhm, I'm not sure ma'am because 9 is not a fraction and it is on the other side.

From the dialogue, it is clear that Dominique understands the concept of balancing an equation as she mentioned that what is done on the left-hand side should also be done on the right-hand side hence would add -7 on both sides of the equation. However, she did not consider 9 on the right-hand side because it is not a fraction, meaning, if a term is not a fraction, then it will not be considered when finding the LCM. Ultimately, the equality of an equation was not maintained hence her answer of -2 can never be this equation's solution.

The second error identified from Dominique's response emanates from the distributive property of multiplying a monomial with a binomial. If, for example, $3(p + 4)$ is a given expression, the distributive property allows the learner to multiply the monomial with both terms to get $3p + 8$. An executive error was identified from the third line of the learner's response where she suggests that $\frac{2(x+4)}{2\times4} = \frac{2x+4}{8}$. Her justification for this step was that she thought when 2 is multiplied with both x and 4 the whole term would change. Her response shows that she is unable to use the distributive law hence an executive error of not being able to apply the law has resulted.

CASE 10

Question 6: Solve for x: $4(x + 2) - 3 = 2(3 - x)$

Learners' response

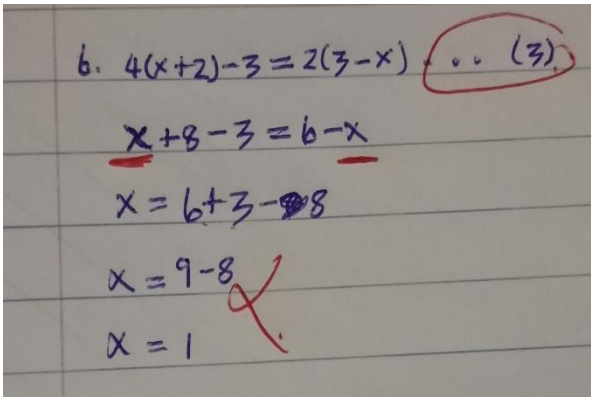


Figure 10: (CASE 10) Learner response to question 6 of pre-test

Error/Misconception identified by the researcher: Structural and executive

The learners' response to question 6 of the pre-test shows a structural error. It seems as if the learner has only multiplied the monomial factor of each side of the equation with the constant terms that are within the brackets. To get a clear understanding of the reasoning behind this approach, an extract of our dialogue is presented below:

Researcher: So Alex, I see that your answer is 1.

Alex: Yes, ma'am.

Researcher: Do you mind explaining how you got to 1?

Alex: You see, I started on the left by multiplying 4 and 2 together and I ended up with 8. Then I multiplied 2 with 3. I took 3 and 8 to the other side of the equation. so I added 6 plus 3 minus 8 and ended up with 1.

Researcher:…but why didn't you multiply 4 with x then 2 with –x as well?

Alex: because they are not the same

Researcher: What do you mean?

Alex: They are not like terms.

Researcher: Oh I see your mistake…but when you collected like terms, what happened to the –x that was on the right:

Alex: Oh no I made a mistake there. I was supposed to take it to the left.

The learners' error emanates from the idea of like terms and removing the brackets by using the distributive law. The learner is aware of the distributive law but does not know how to use it. Moreover, she knows that like terms are added together but she has misconceptualized this idea in this case. She believes that like terms also apply when using the distributive law.

Responses to Question 10 of both the pre-test and the post-test.

Question 10 (Pre-test):

Thando's dad is three times as old as she is now. Ten years ago, he was seven times as she was old then. How old is Thando and her dad now? (3)

Question 10 (Post-test):

Jill runs to the shops to buy a bar of chocolate. It takes her three times as long to get home because she is eating her chocolate. How fast does Jill walk to the shops, if Jill lives 500m from the shops and it took her 25 minutes to go there and back? (3)

Out of all the questions that were given to learners, the responses to question 10 was immensely poor. From both the pre and the post test, a very small quantity of learners answered question 10 and even those few responses were extremely poorly written. The difference in the average performance of question 10 between the pre-test and the post-test shows a significantly small improvement, with percentage pass being 2% and 3.1% respectively. Analyzing the nature of the school and the learners, the following could be the reasons and factors that yielding such results:

- Interpreting word problems and representing them algebraically might be cognitively demanding to most learners as opposed to solving the given algebraic problems.
- Interpreting such word problems and representing them algebraically was introduced to learners from as early as grade 9, if not grade 8. Moreover, the ATP allocated minimal time for the teaching and learning of word problems. As a result, there is a possibility that most learners did not get the opportunity to comprehend such questions and master the rule of representing word problems using algebra.
- English is a learning barrier for most learners, so having to understand, interpret the word problem and to represent the word problem algebraically could be a challenge to most learners.

As a result of the above-mentioned possible reasons, most learners could not even attempt to answer question 10, while a few of them gave one word answers with no steps showing their algorithm.

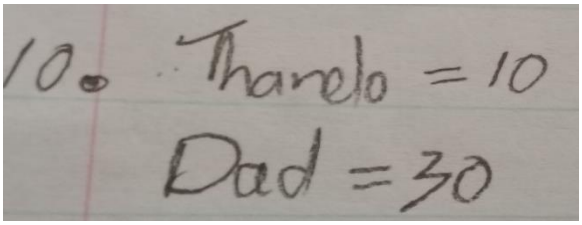


Figure 11: Learner response to question 10 of the pre-test

This learner only wrote 10 and 30 as the solutions, with no steps justifying his line of thinking. The learner might have even guessed the answers due to not knowing what to do. As a result, it

would be practically impossible to analyze their responses to question 10 hence I could not note the errors and misconceptions. Even though I could not use their responses to question 10, their responses to questions 1 to 9 were adequate enough for me to answer the research questions of this study.

4.4 THE TEACHER INTERVENTION LESSON (teacher explanations).

I now turn to the teacher intervention stage of the data analysis which precede prior the writing of the post-test. The objective of the intervention phase is to answer the following research question:

- What possible learning advantages and disadvantages can result from providing explanations that are informed by the learners' errors and misconceptions?

This stage is important as it affords the researcher an opportunity to respond to the question:

- To what level can learning and learner achievement in solving algebraic equations be improved if teacher explanations are informed by the learner errors and misconceptions?

Before the commencement of the intervention session, I had a discussion with the teacher regarding the manner in which the session should be structured. Firstly, I presented the errors which I had identified while analyzing the learner's pre-test papers. This is the first strand of professional noticing, *attending* to the learner errors and misconceptions (Jacobs *et al*., 2010). Secondly, I presented to her the aim of the study, the theories and the frameworks framing this research study. Lastly, we had a discussion on the explanations that she will offer to address the identified errors.

It is important to note that the lesson presented by the teacher is guided by the errors and the misconceptions which learners encounter when solving algebraic equations. The teacher addresses these errors by presenting the equation(s) with errors and misconceptions on the board, breaks it down by ensuring that learners understand the matter (elements making up the equation), the form (arrangement of the elements), the process of solving the equation and ultimately explaining the goal of solving the equation.

After the discussion that I had with the teacher, she needed 3 consecutive days to prepare for the intervention, taking into consideration the identified errors and the possible misconceptions which

learners encountered and the framework which she is bound to adhere to throughout the intervention session.

The teacher started the lesson by welcoming the learners and introduced herself. She then explained the purpose of the session and had a conversation regarding the importance of algebra in the mathematics field. During this introduction, she addressed to the learners that from their responses to the pre-test questions, errors were identified and the session was meant to address these errors and misconceptions. The teacher tried to make them feel comfortable by notifying them that everyone in the class is bound to learn from their mistakes and hence they should comfortably engage throughout the lesson.

She verbally asked learners what an equation is. A couple of learners raised their hands to answer the question. From their response, I noticed that some learners understood what an equation is. However, the teacher saw the need to demonstrate what the concept of equations entails using the following example:

If it is given that $2x + 5 = 9$ and$x = 2$, this means that:

$$2x + 5 = 9$$

$$2(2) + 5 = 9$$

$$9 = 9$$

$$\therefore LHS = RHS$$

The teacher alluded to the learners the purpose of demonstrating this equation, which is because we noted that some of their responses showed that they do not understand the concept of equations and it would be pointless to conduct this session when learners do not know the concept of equations. If $x = 2$ it means that 2 is the solution to the equation, and *x* is replaced with 2 in the equation, the left hand side of the equation has to be equal to the value on the right hand side.

Task 1:

The teacher tasked learners to decide which value of *x* on the second column is a solution to the equation given on the first column. The main aim of this task, as the teacher has explained, is to address the concept of an equation and to ensure that learners are able to check if their solutions

are correct of not. From the identified errors, it is clear that learners did not check whether their solutions were correct or not.

Table 4: Task 1 (matching column A with column B)

EQUATION	SOLUTION
$(x - 7)(x + 8) = 0$	$x = 16$
$3x^2 + 6x - 9 = 0$	$x = 7\ 0r\ x = -8$
$2\times 4y + 10 = 138$	$x = -3\ or\ x = 1$

The teacher demonstrated how to check the correct solution using the first equation. I could see from the learner's facial expressions that most of them did not know and were thus eager to working out the rest. Even when they gave out their answers to the teacher, the excitement on their faces could tell that they are happy to have learnt how to check whether their solutions are correct or not.

Task 2:

The teacher then wrote the following equations on the chalkboard and appointed two learners to explain how these two equations are different from each other:

First equation: $2^x = 16$

Second equation: $x^2 = 16$

The first learner said there is no difference since both equations have 2, *x* and 16. The second learner said the equations are different *because the first one has x on top and the second one has x at the bottom*.

To address he responses, the teacher considered the four criteria for explanations (Ruben, 1992) which we had discussed prior the commencement of the intervention lesson. To explain how these two equations are different from each other, explained the *matter* and the *form* (the elements making up these two equations:

On the first equation, 2 is the coefficient, x is the exponent and on the right hand side we have 16 as the constant term. On the second equation, 2 is the exponent, x is the base and it also has 16 as the constant on the right hand side.

The teacher then considered the *form* by clarifying why *x* on the first equation is called the exponent and why it is called the base on the second equation. This clarification of the arrangement of matter allows learners to notice the functional difference of the value of *x* and thus the procedures used to solve *x* will be different.

She used the contrast pattern of the variation theory for learners to understand the similarity and the difference between these equations and ultimately develop a conceptual understanding of these two equations. The teacher then explained the process (how to solve each of these equations) by working out the solutions on the board. She explained that the goal was to find the value of *x*. This approach allowed the learners to develop procedural understanding and fluency.

As the teacher was explaining, she notified learners of the mistakes that they made when solving for *x* when *x* is a base and when it is an exponent.

She then tasked learners to decide, from the given methods, on which one is a correct method to solve for the given equation.

Task 3:

Equation 1:$5^{-x+4} = 25$

Table 5: Methods for task 2 (part A)

method 1	method 2	method 3
$5^{-x+4} = 25$ $-x + 4 = 25 - 5$ $-x + 4 = 20$ …	$5^{-x+4} = 25$ $5^{-x+4} = 5^2$ $-x + 4 = 2$ …	$5^{-x+4} = 25$ $5^{-x} = 25^{-4}$

More than half of the learners in the class selected method 2, the rest of the learners selected method 1 while none of the learners in the class selected method 3 to be the correct method. Those few learners who selected method 1 strongly agreed that solution 1 is the answer. The teacher then asked the learners to give a reason why they have selected method 1. One of the learner's

justification was that 25 and 5 are like terms so they have to be grouped together. She further explained that the aim is to remain with x on the left hand side hence 5 is transposed and ultimately, 4 will also be transposed. This learner error has emanated from the concept of collecting like terms. To address this error, the teacher explained to the learner that like terms are the terms that have the same base raised to the same power. She then asked the learner whether is 5 and 25 having the same power. From the teacher's question, the learner noticed that 5 and 25 are actually unlike terms. She further demonstrated using the following example which supports what she has explained:

$$5 + 5^{-x+4} = 25$$

$$5^{-x+4} = 25 - 5$$

$$5^{-x+4} = 20$$

The teacher notified the learner that 5 is a constant term hence in this case it has to be transposed. The teacher then asked one of the learners selected method 2 to explain why they have chosen it as a correct solution. The reasoning behind their selection was that the unknown is an exponent and hence the bases should be the same in order to work with the exponents.

Equation 2: $x^2 - 16 = 0$

Table 6: Methods for task 2 (part B)

method 1	method 2	method 3
$x^2 - 16 = 0$ $x^2 = 16$ $x = \mp\sqrt{16}$	$x^2 = 16$ $x^2 = 2^4$ $\therefore x = 4$	$x^2 - 16 = 0$ $x^2 = 16$ $x = \frac{16}{2}$

The teacher wrote equation 2 as well as the three methods on the chalkboard and learners were then tasked to decide on the correct method. Only a few learners selected method 3 with a justification that they *are removing 2 and want to remain with x*. Their justification shows that they know that to solve for x, they need to isolate x, but the challenge is the manner in which they have isolated x. The teacher addressed this misconception using the following demonstration:

$$2x - 16 = 0$$

$$2x = 16$$

$$x = \frac{16}{2}$$

$$x = 8$$

The teacher attended to this misconception by showing them on that they can only divide both sides by 2 when 2 is the coefficient and not an exponent. Method 3 has 2 as an exponent and not as a coefficient. She then explained why method 1 is the correct method and why method 2 is also an incorrect method.

I now turn to the framework which the teacher has used throughout the session from the time she had started presenting the sampled pre-test responses and offering explanations while working out the solutions on the board. Immediately after task 3 was given to the learners, the subsequent actions and tasks used during the intervention sessions were guided by the following:

- *Attending:* As the teacher attended the learners' errors and misconceptions and reasoning behind the learners' steps while preparing for the lesson, the teacher presented the error on the board without mentioning the name of the person who made the error.
- *Interpreting:* The teacher also interpreted the error by highlighting the reason(s) why the presented answer was incorrect. The teacher also mentioned the misconceptions from which these errors emanated from.
- *Deciding:* The then demonstrated how the procedures which the learner was supposed to consider when solving. She worked out the answers on the board while learners were taking notes. As she was working out the solutions, the teacher explained the matter, the form, the process and the goal for each equation which she worked out on the board.

Below is the schematic representation that was used by the teacher during the intervention session:

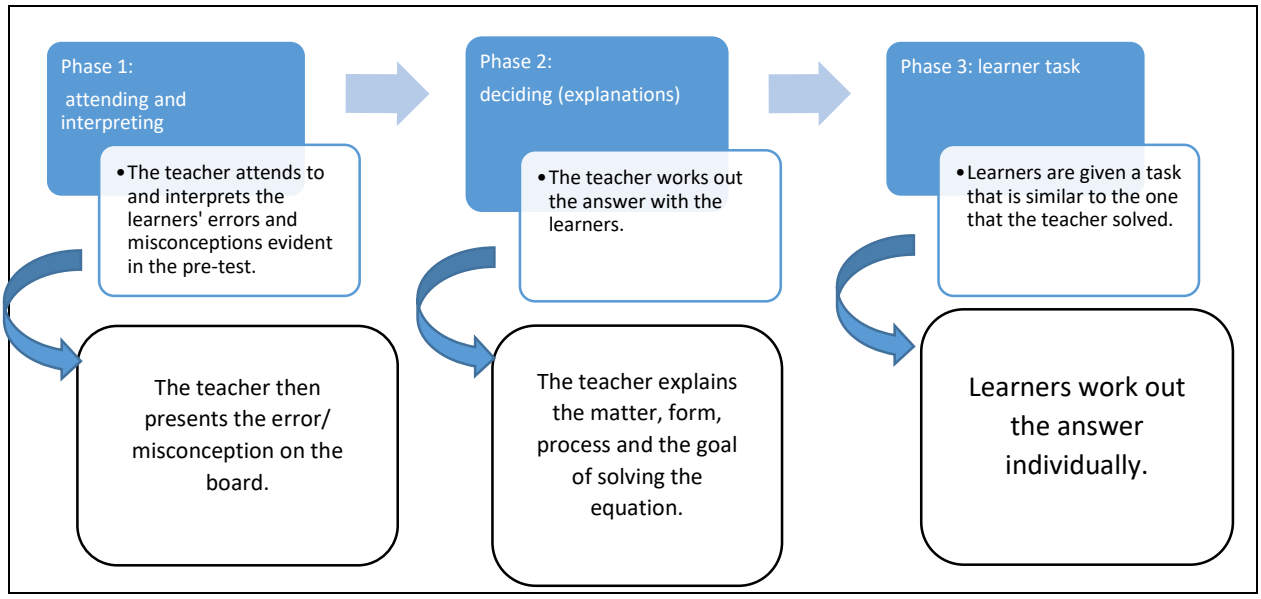


Figure 12: schematic representation of the approach employed for intervention

During the lesson, I also noted how the teacher was using professional noticing even during the lesson. The teacher took note the following:

- The teacher attended to the learner's learning tasks which were given to the learners during the lesson.
- The teacher interpreted their responses to the tasks as well as when they were responding verbally to the pop-up questions.
- The teacher then decided on an explanation that was relevant in responding to the learners' intuitive conception.

Overall, the observations during the intervention session went well as planned were fruitful enough to use, together with the post-test, for comparative analysis which is thoroughly presented in the following subsection.

4.5 COMPARISON BETWEEN THE PRE-TEST AND THE POST-TEST RESPONSES

This section provides an analysis of the pre-test responses in comparison with the post-test responses. This is done by, firstly, providing a graphical representation comparing the performance in both the pre-test and the post-test. Secondly, the analysis between the learner's responses to the pre-test as well as their post-test responses is explained. This analysis affords me an opportunity

to analyze the impact of offering explanations that are guided by the learner's errors and misconceptions on the learner's performance.

The two different graphical representations presented in this section clearly outline the shift in the learners' performance between the pre-test and the post-test. I have presented two different representations chiefly for making the interpretation accessible for the reader. They are important as they assist in answering the research questions:

- To what level can learning and learner achievement in solving algebraic equations be improved if explanations are informed by the learner errors and misconceptions

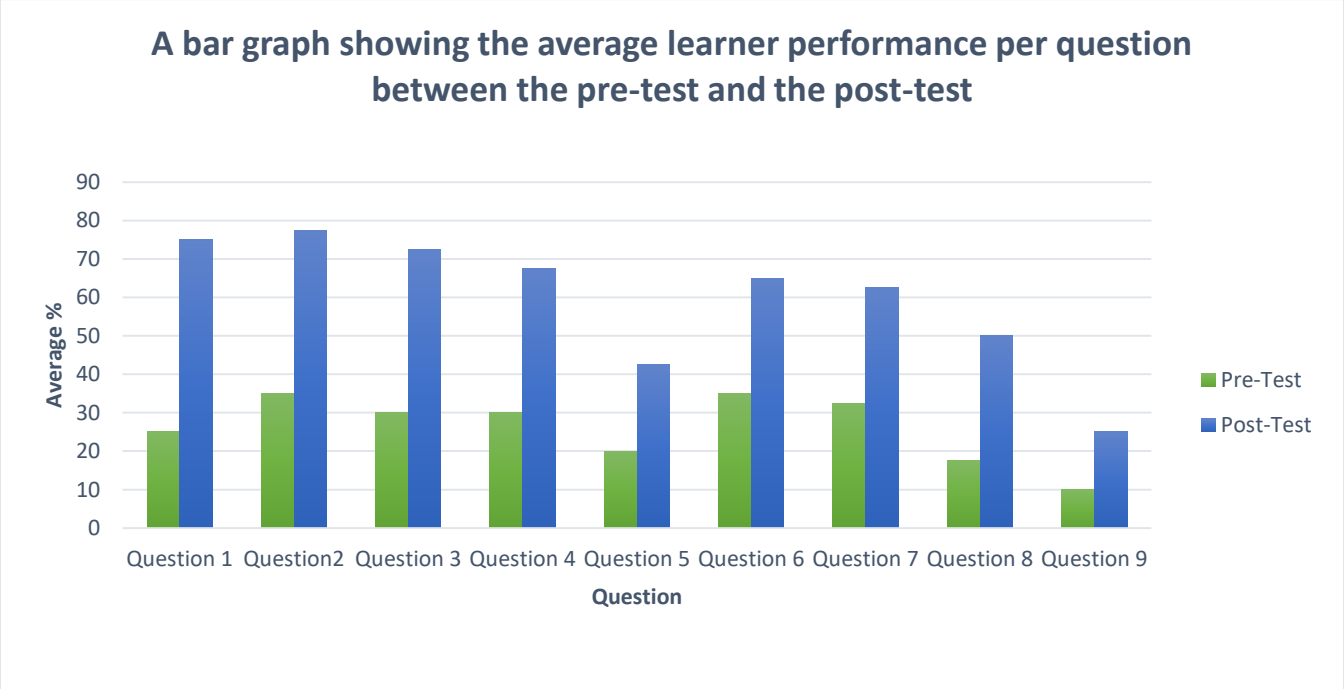


Figure 13: bar graph representing learner performance in pre-test and post-test

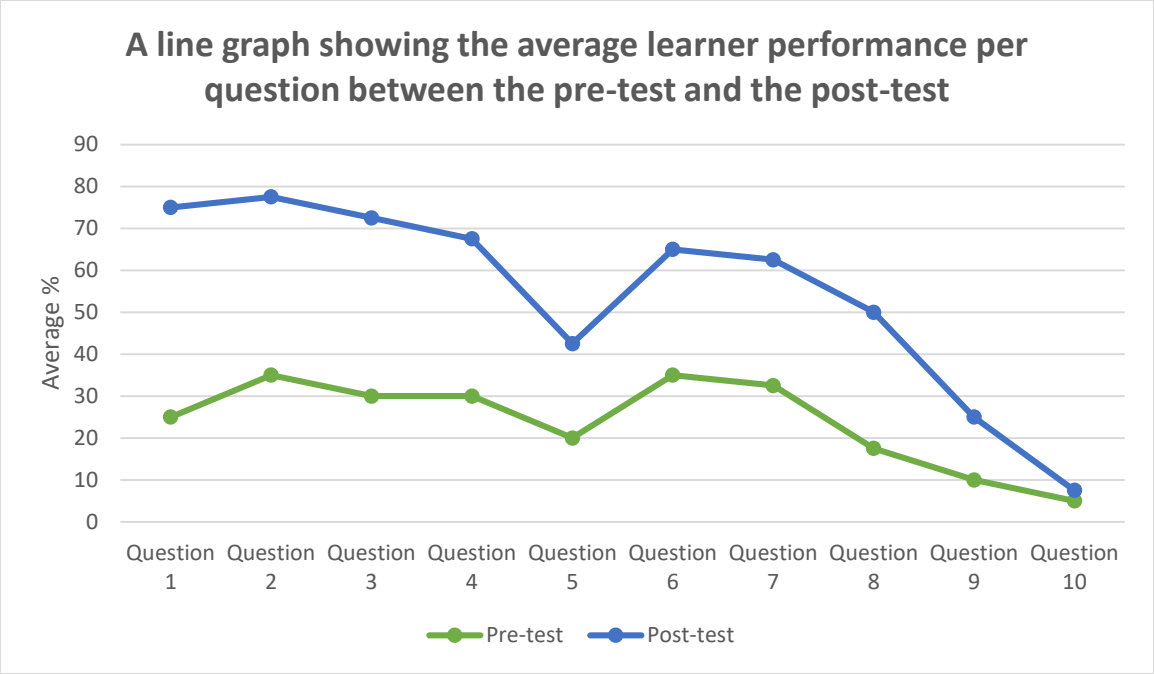


Figure 14: Line graph representing the learner performance in pre-test and post-test

These two graphical representations show an improvement in the learners' performance from the pre-test to the post tests. Even though the last question shows significantly little improvement from the pre-test to the post test, improvement is evident in all the questions given to the learners. The improvement of the performance generally implies a decrease in the number of misconceptions encountered and errors made by learners on the post test. The following analysis specifically shows the comparison between the pre-test and the post-test responses.

Question 1: Pre-test

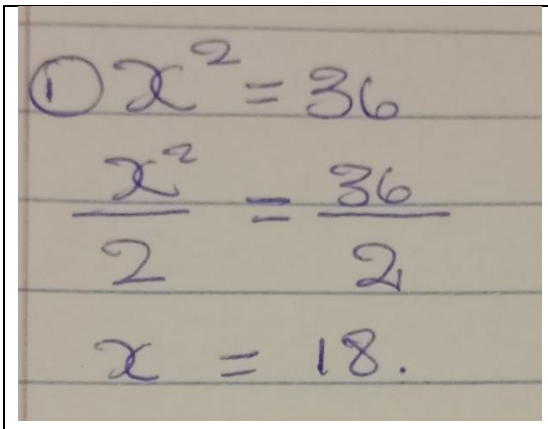
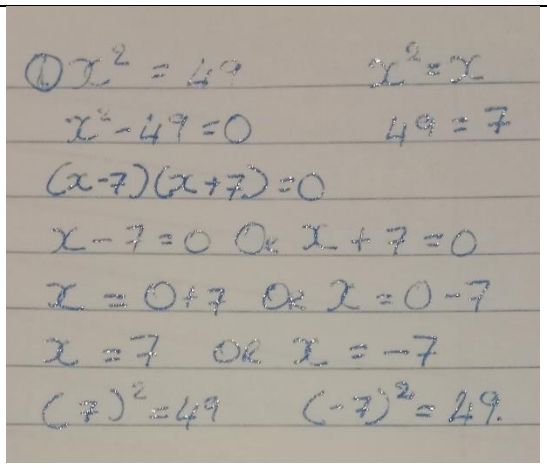
Solve for x: $x^2 = 36$

Question 1: Post-test

Solve for x: $x^2 = 49$

Table 7: Learner responses to question 1 of the pre-test and post-test

Learners' pre-test response	Learners' post-test response

Initially, the learner divided both sides by 2 with an aim of isolating x from the equation. It is clear that the learner divided both sides by 2 as she was convinced that we divide both sides by the numerical value irrespective of whether the value is an exponent or coefficient. However, after the intervention session, the learner was able to correctly solve the equation by transposing 49 then factorized using the difference of two squares method. The comparison of the learners' responses shows that indeed offering explanations by anonymously using the learners' pre-test responses to address the error affords the learner an opportunity to notice the and to successfully apply the correct procedure necessary for that situation.

Question 2: Pre-test

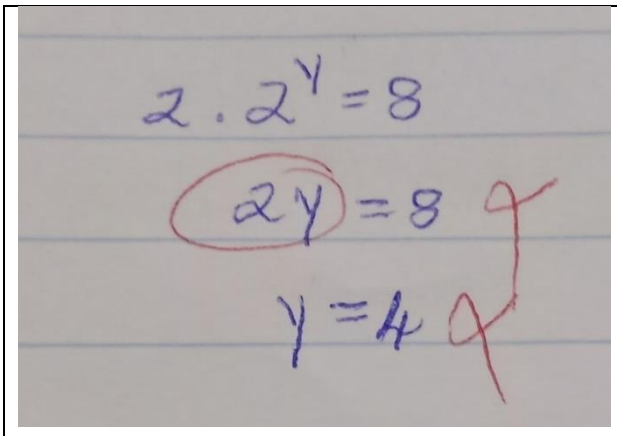
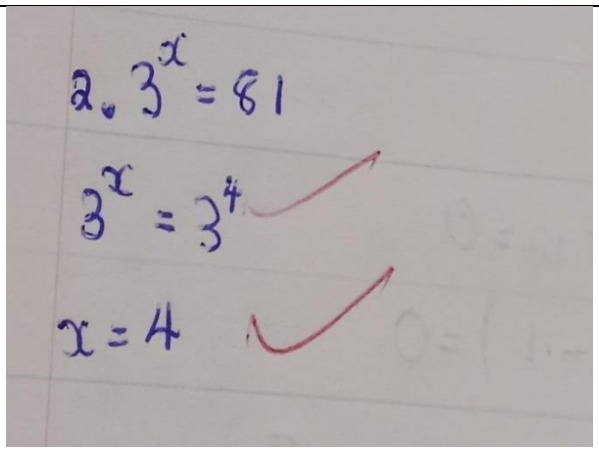
Solve for y: $2^y = 8$ (2)

Question 2: Post-test

Solve for x: $3^x = 81$ (2)

Table 8: Learner responses to question 2 of pre-test and post-test

Learners' pre-test response	Learners' post-test response

From these responses, it is clear that the learner initially could not appreciate the difference between an exponent and a coefficient. Both the pre-test and the post-test questions were testing the learners' ability to solve exponential equations. The learners' pre-test response shows that the learner changed *y* from being an exponent to being a coefficient then divided both sides by 2 to get 4. It is clear That the learner lacked both the conceptual understanding of how an exponent is different from a coefficient and procedural understanding on how to solve for an exponential equation. However, looking at the learners' post-test response, it shows that the learner is now knowledgeable about the procedure that needs to be taken to solve an exponential equation and thus the learner ensured that the bases on both the left hand side and the right hand side are the same.

The main point of this analysis is not to assess whether they have learnt exponential equations or not, but to analyze the learners' improvement in performance after using their errors and misconceptions during the intervention session. The pre-test has made the teacher aware of the fact that some learners could not appreciate the difference between a linear equation and an exponential equation. To foster conceptual understanding, the teacher presented two equations on the board where one of them has *x* as an exponent and the other one had *x* as a coefficient and allowed learners to suggest how the presented equations are different from each other. She attended and responded their suggestions. Thereof, she facilitated the procedural understanding of how to solve each of these equations by using one of the learners' response to question 2 of the pre-test responses learners'. She has done this by showing the learners how the learners' response is incorrect and how the learner was supposed to solve for *y*. From the learners' response presented above and the analysis of the overall drastic improvement on the learners' responses to question

2, it is clear that the intervention as assisted in reducing the number of errors pertaining this concept.

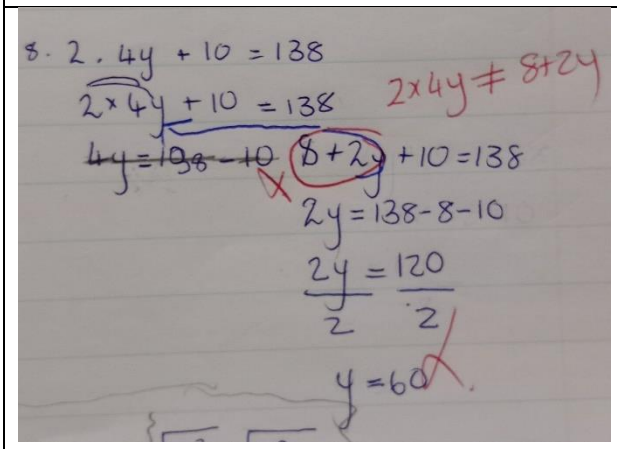
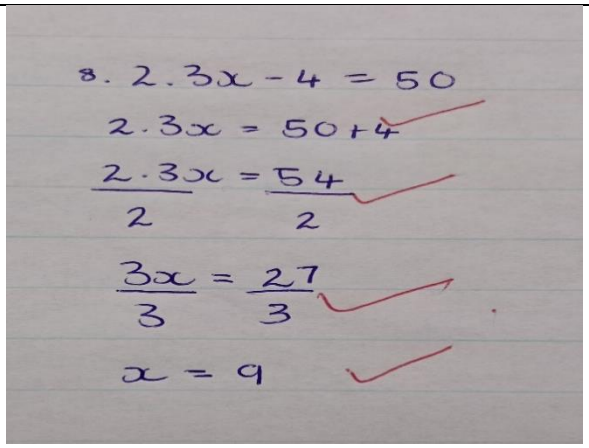
Question 8: Pre-test

Solve for y: $2.4y + 10 = 138$

Question 8: Post-test

Solve for x: $2.3x - 4 + 50$

Table 9: Learner responses to question 8 of pre-test and post-test

Learners' pre-test response	Learners' post-test response
	

From the learners' response to the pre-test question, the learner assumed that 2 had to be multiplied by 4 and y since the learner is enlightened about the distributive law. It is clear that the learner the confused $2(4 \pm y)$ with $2(4 \times y)$ which are two different contexts. Nonetheless, the focal point of this section is not to analyze the error that the learner has made but to bring forth the impact of the explanations guided by the errors on the. It is evident from the learners' post-test response that the learner has successfully constructed the correct conceptions and has a better procedural understanding of solving such equations. This means that by presenting the rest of the class this answer anonymously as an example of one of the identified errors and ultimately demonstrating how the equation could have been manipulated allowed the learners to counteract their errors and challenge the misconceptions associated with such errors. The intervention session has, therefore, afforded learners the opportunity to understand the manipulation of such equations better.

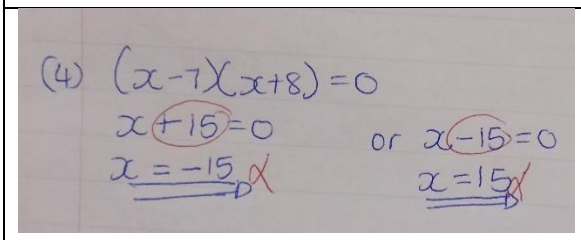
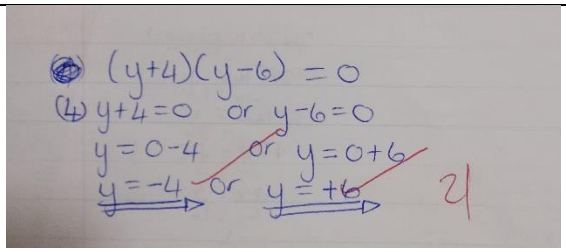
Question 4: Pre-test

Solve for x: $(x-7)(x+8)=0$ (2)

Question 4: Post-test

Solve for y: $(y+4)(y=6)=0$ (2)

Table 10: Learner responses to question 4 of pre-test and post-test

Learners' pre-test response	Learners' post-test response
	

The learners' pre-test and post-test- responses shows improvement in the procedural knowledge in this concept. Initially, the learner believed that since 7 and 8 are constants, they should be added together to get 15 then ultimately transpose the 15 in order to solve for *x*. It is evident from this response that the learner, firstly, was unaware of the application of the FOIL method. Secondly, the learner lacks procedural understanding of how to solve for *x* when two factors of the equation are equated to zero hence her response was incorrect. However, the learners' response to the post-test question shows a cognitive shift as it shows that there is improvement in the procedural understanding of how to solve for the unknown when the equation is represented in such form. This is because the teacher noticed this error by attending to it and interpreted it so as to find a better way to address it using this error. By allowing learners to check whether is the LHS = RHS when their solutions are substituted into the equation, it gave learners an opportunity to notice that their solutions are incorrect and were thus inquisitive to know how they were supposed to work it out. the improvement in their responses to this type of question shows that the teachers' strategy allowed for cognitive gains.

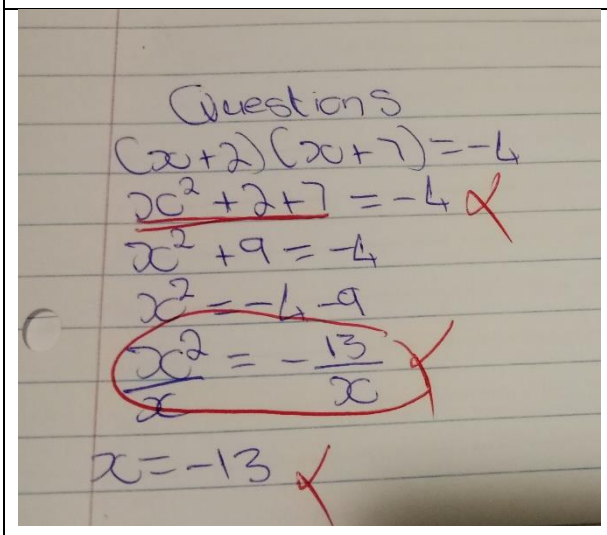
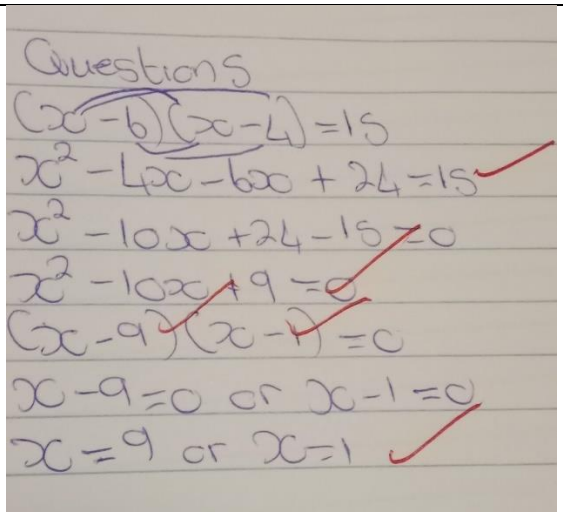
Question 5: pre-test

Solve for *x*: $(x+2)(x+7)=-4$

Question 5: Post-test

Solve for *x*: $(x-6)(x-4)=15$

Table 11: Learner responses to question 5 of the pre-test and post-test

Learners' pre-test response	Learners' post-test response
	

From the learners' response to the pre-test question, it is clear that the learner did not know how to apply the FOIL method. From the interview which I had with the learner, the learner explained FOIL as F for first and L for last, meaning the first terms are multiplied then the last terms are added, and there was no clear justification from the learner as to why are the first terms multiplied and last terms added. The teacher addressed this error by first explaining each element making up the equation then demonstrating the process of applying the FOIL method using the pre-test question. Before explaining the process, the teacher had anonymously displayed the error on the board and explained where the learner went wrong. The teachers' main aim was to ensure that learners know the FOIL method and to demonstrate how it is actually applied. It is evident from the learners' post-test response that the explanations offered during the intervention assisted the learner in addressing the misconception which the learner had hence improvement was evident from the response to the post-test.

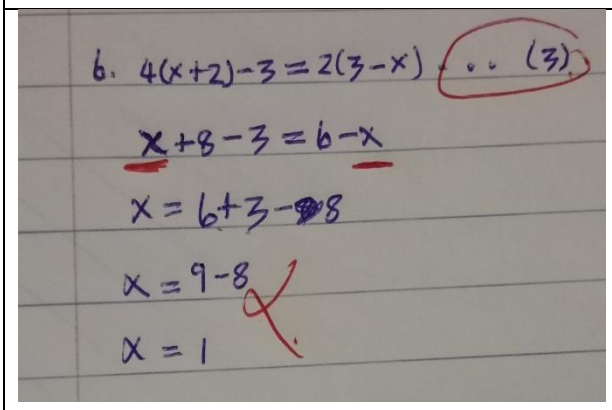
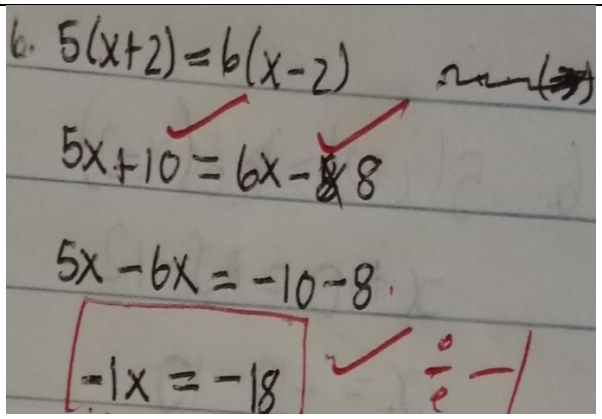
Question 6: pre-test

Solve for *x*: $4(x+2)-3=2(3-x)$ (4)

Question 6: post-test

Solve for *x*: $5(x + 2) = 6(x - 2)$ (4)

Table 12: Learners' responses to question 6 of the pre-test and post-test

Learners' pre-test response	Learners' post-test response
	

From the interpretation of the learners' pre-test response, the learners' error emanated from misconceptualizing the concept of like terms which suggests that like terms are added (or subtracted together). The learner then believed the collection of like terms is also considered when applying the distributive law. Meaning, like terms are also multiplied together whereas it would be incorrect to multiply unlike terms. This interpretation assisted the teacher in deciding upon a constructive approach that helped in addressing this misconception. This approach includes explaining the concept of like and unlike terms and when to consider such terms. she then explained how the anonymous learner was supposed to work out the answer.

Even though the learners' response to the post-test was partially correct, her response is clearly showing that the learner has grasped the concept and her misconception was challenged and addressed by the teachers' explanations. Even though the learner might have forgotten to divide both sides by -1 to remain with x on the left, the learners' response shows traces of cognitive gains which resulted in the improvement in her performance.

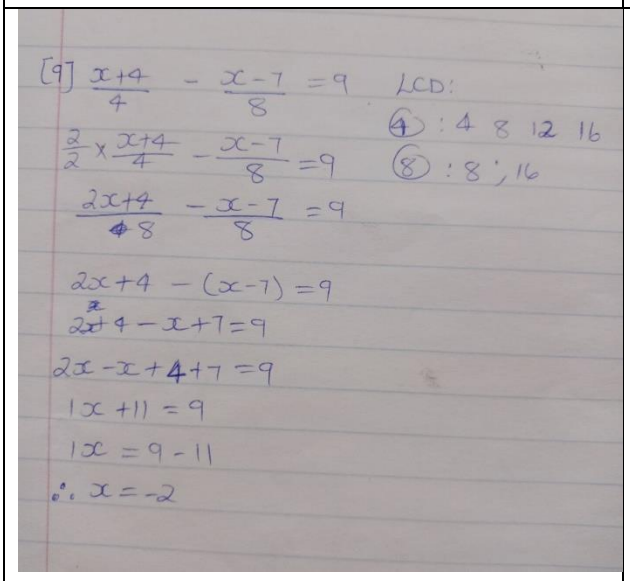
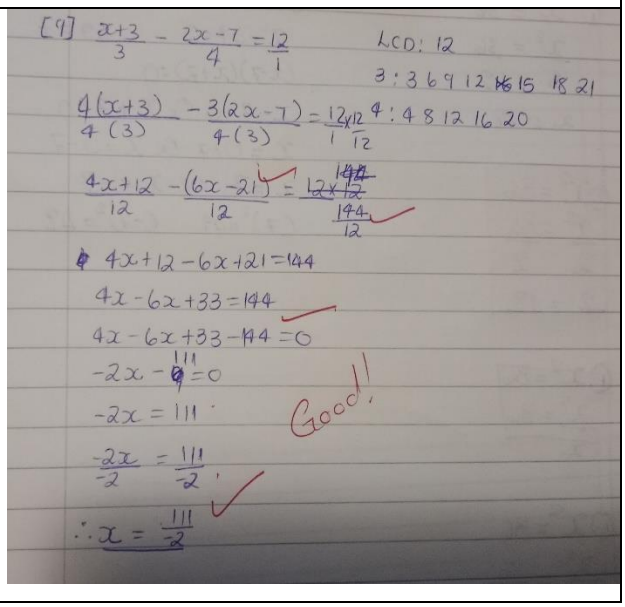
Question 9: pre-test

Solve for *y*: $\frac{y+4}{4} - \frac{2y-7}{8} = 9$ (4)

Question 9: Post-test

Solve for x: $\frac{x+3}{3} - \frac{2x-7}{4} = 12$ (4)

Table 13: Learners' responses to question 9 of the pre-test and post-test

Learners' pre-test response	Learners' post-test response
	

The learners' pre-test response shows that the learner understands the concept of finding the LCD mainly to ensure that the fractions have the same denominator. However, the learners' mistake was to neglect 9 chiefly because it is on a different side of the equation. The learner believed that since 9 was not written in the form of a fraction, it cannot be considered. The teachers' explanation addressed this misconception and demonstrated that 9 can actually be written as a fraction with a denominator of 1. It is clear from the learners' post-test response that the learner benefited from the intervention session as the learners' response presents 12 as a fraction hence there was improvement in her performance.

4.6 CONCLUSION

This chapter has presented a thorough analysis of the results. This was done by, firstly, analyzing the learner errors that were evident in their pre-test responses and were categorized according the

adopted framework of error classifications. Secondly, the chapter has presented an analysis of the classroom observation which took place during the intervention session. From the observations, I noticed that the teacher has offered explanations using the errors that were identified in the pre-tests by presenting these errors on the board for the rest of the learners to see and then explained the process needed to solve the equations. A comparative analysis of the learners' pre-test and post-test responses was presented. From the comparison between their pre-test and post-test, it is clear that the intervention session has assisted the learners in addressing their misconceptions and ultimately improving their performance.

CHAPTER 5: DISCUSSIONS

This chapter discusses the outcomes and findings of this study. This is done by, firstly, discussing the errors which were identified in the learners' pre-test responses. Secondly, the chapter thoroughly discusses the implications of the outcomes of the study, linking it with the frameworks applicable to the findings.

It is worth noting that the main aim of this study was to identify the errors which the grade 9 learners encounter when solving algebraic equations and to determine whether there is improvement in the performance of learners when explanations offered as an intervention are directed at the identified errors and misconceptions. To obtain a clearer understanding of root cause of the identified errors and the misconceptions that these errors emanate from, I scheduled interviews with the sampled learners. Teacher explanations informed by the identified errors and responses from the scheduled interviews were then offered to the learners with an objective of determining whether these remedial sessions could afford improvement in the performance of these learners or not. The intention of identifying the errors and ultimately providing teacher explanations as an intervention was to address the research questions outlined earlier in this study. following research questions:

5.1.1 What learner errors and misconceptions are identified by the teacher while attending to the grade 9 learners' work on solving algebraic equations?

5.1.2 What learner reasoning and justifications are evident from the learners' work of solving algebraic equations?

5.1.3 What possible learning advantages and disadvantages can result from providing explanations that are informed by the learners' errors and misconceptions?

5.1.4 To what level can learning and learner achievement in solving algebraic equations be improved if explanations are informed by the learner errors and misconceptions?

The learner errors and misconceptions were identified using the conceptual framework outlined in chapter 2 of this study. These identified errors were beneficial, firstly, during the lesson preparations. Secondly, it was beneficial to the learners as they were able to shape what they have previously constructed and thus addressing their misconceptions. Lastly, it was beneficial to the teacher who offered explanations because she is enlightened about the errors and misconceptions which she was not aware of. She will subsequently make informed decisions when offering

explanations to her learners. The identified errors are classified using the categories of the conceptual framework adopted in this study.

5.2 THE IDENTIFIED TYPES OF ERRORS:

From the identification of errors in this study, it was evident that the errors that were identified are not mutually independent from each other but are associated with each other. The following errors were evident from the learners' responses:

- Structural errors

Structural errors are conceptual errors. Studies have shown that learners have acquired procedural knowledge but are facing challenges with conceptual understanding (Porter & Masingila, 2000). I believe it is for this reason why structural errors are one of the mostly identified errors. The most commonly identified structural errors were evident when the learners had to remove the brackets by using the FOIL, or distributive, method to solve for the unknown. Case 2 of the learners' response is one example where the learner has shown structural error with an attempt to solve the equation. A learner was required to solve for *x* for the equation: $(x + 2)(x + 7) = -4$

One of the learners responded in the following way:

$(x + 2) = -4$ or $(x + 7) = -4$

$x = -4 - 2$ or $x = -4 - 7$

$x = -6$ or $x = -7$

The learner equated both factors by -4 instead of using the FOIL method. This response shows that the learner understands the principle of solving a quadratic equation that has 0 on one side of the equation. The principle of solving quadratic equations which have the product of factors being equal to 0 allow learners to equate each factor to 0. In this case, the principle does not apply as the right hand side is not equal to zero.

- Executive errors

This error emanates from an inability to apply a mathematical procedure when, in this case, solving for an unknown. They emanate mostly due to an inability to understand how to apply a particular procedure. Most of the executive errors identified in the learner's responses were due to failure to apply factorization methods in order to solve for *x*. Case 6 of the responses is a practical example of an executive error where the learner cannot factorize a trinomial. The learner knows how to take out a common factor, but have failed to factorize the trinomial after taking out the common factor.

- Arbitrary errors

These error emanates from the learner's tendency to change a question to a form that is acceptable to the learner. It is also a result of considering one part of the information that suits the learner's level of understanding and leaving out the rest that seem not to make sense to the learner. With question 7 of some of the learners' responses for example, the learner has behaved arbitrarily by isolating the constant term when taking out the common factor. The learner is aware that to take out a common factor, all terms should have the common factor, but a trinomial has a constant term which does not have *x* and it is for this reason why the learner decided to restructure the equation by transposing the constant term so that it suits the learner's understanding.

Example of Arbitrary and executive errors:

Arbitrary errors result when the learner chooses. Executive errors on the other hand. The analysis of this study has revealed that learners can elicit more than one error in their responses. An example is given below where the learner had to solve for x for the equation: $3x^2 + 6x - 9$, and the response is represented below:

$$3x(x + 3) = 9$$

$$3x = 9 \; or \; x + 3 = 9$$

$$x = 3 \; or \; x = 6$$

From the first step, it is clear that the learner wanted to factorize and noticed that it would not be possible to take out *x* as a common factor when there is 9 hence she transposed it. This step shows that the learner is aware of the concept of factorization but he cannot apply it to manipulate and solve the equation. By considering the first two terms and leaving out the constant term shows that

the learner's error is arbitrary in such a way that the equation is rearranged to suit the learner's existing schema leaving out the rest of the important information. It is also an executive error as the learner is unable to factorize the trinomial; even when he was trying to take out a common factor to satisfy his knowledge, he has done it incorrectly hence it is also an executive error.

- Procedural exploration

This error occurs when the learner over-generalizes a procedure which was previously learnt and uses the procedure in contexts where it does not apply. Cases 1 and 2 of the learner's responses is an example of a procedural exploration where the learner knows that to eradicate the coefficient of a term in an equation, one needs to divide both sides of the equation by the coefficient. The learner uses this rule to eradicate a base where the variable is an exponent. An error was identified because the learner has generalized this rule and applied it when it does not even apply in this situation. The learners' response to solving the equation: $2^y = 8$ was:

$$2^y \div 2 = 8 \div 2$$

$$y = 4$$

The learneer divided both sides by 2 to isolate *y*. However, this procedure does not apply in this situation since 2 is the base and not the exponent.

- Incomplete application

These errors emanate when the learner knows the procedure or the rule but fails to complete the task. There are a number of examples which I could present from the analysis of the learners' responses where they attempted to solve but fails to complete the task. This was evident in most of the questions from the pre-test where learners failed to complete task.

An example is evident when the learner tries to factorize the equation $3x^2 + 6x - 9 = 0$. The learner correctly divides all terms by 3 the ends up with $x^2 + 2x - 3 = 0$. The learner then uses factorization to get$(x - 1)(x + 3) = 0$. Once the learner fails to proceed, it would mean that the learner has failed to answer the question.

- Unsystematic errors

These errors develop due to factors other than incompetency. These are the untraceable errors that have no link to concepts from which they may emanate. They may emanate from factors such as anxiety, tiredness or even due to not feeling well. Learners can correct these errors themselves if they are given an opportunity to correct their work without any assistance from the knowledgeable figure. I have noted these errors from the dialogue which I had with the sampled learners where some of them alluded that they have omitted the equal sign or had omitted the negative sign of a term when manipulating the equation.

5.3 THE IMPLICATIONS OF THE IDENTIFIED TYPES OF ERRORS

The outcomes of this study points out the fact that offering explanations that are guided by the identified errors and misconceptions as an intervention in a professional noticing context affords learners an opportunity to construct the correct conceptions. The professional noticing intervention allows the teacher to notice the learners' responses by attending to their intuitive conceptions evident in their pre-test responses as well as those evident in the tasks given to them during the intervention sessions. These intuitive conceptions were analyzed, interpreted and then explanations were offered as a constructive action that was devised using their errors and misconceptions. The comparison between the learners' responses in the pre-test and the post-test discussed earlier in the chapter confirms that using their errors and misconceptions as a resource in the intervention session allowed learners to unlearn some of their misconceptions and subsequently construct the correct conceptions.

According to Sarwadi and Sharhill (2014), teaching using the learners' errors and misconceptions has the power to serve as a general tool to promote cognitive gains by enhancing successful knowledge construction and ultimately improve learner performance. The outcomes of this study reveals that indeed the use of the learners' errors and misconceptions is an advantage to learning it assists in building on the learners' conceptions and promoting procedural and conceptual understanding of the concepts taught. Hence improvement in the learners' performance was evident their post-test responses. The comparison between the learners' responses in the pre-test and the post-test discussed earlier in the chapter confirms that using their errors and misconceptions as a resource in the intervention session and giving them tasks that maximizes

cognitive conflict between their intuitive conceptions and the correct concepts allowed them to unlearn some of their misconceptions and subsequently construct the correct conceptions.

In Muengele (2017)'s terms, the teacher does not transmit knowledge to learners, instead, the teacher motivates, illuminates or move learners towards the learning of mathematics. It is, therefore, the learner who constructs knowledge. Enlightening learners about their errors and the misconceptions from which these errors emanate from is one of the strategies of incorporating them in the knowledge construction process. They were also timeously engaging in an active dialogue with the teacher. During this time, the teacher also probed learners with questions that allowed learners to:

- discuss the probable causes of the errors and misconceptions
- find ways that can be used to rectify these errors.

When learners are given an opportunity to view the work that is embedded with errors and misconception, to justify the root cause of these misconceptions and to find ways to rectify them, they build confidence in themselves and develop expertise in solving equations. The improvement in the learners' responses between the pre-test and the post-test and their confidence in responding to the given tasks points out the importance of working out and justifying your procedure in promoting conceptual understanding. This action allowed learners to be active participants and not passive receivers of knowledge (Jacobs et al., 2010). An environment for learners to be part of the lesson by contributing their ideas was created, and this environment allowed learners to unlearn the misconceptions which they encountered and learn the correct procedures which they needed in order to solve algebraic equations. Therefore, a successful knowledge construction resulted (Jacobs *et al*., 2010). The reduced number of errors evident in the post–test as opposed to the ones presented by learners in the pre-test proves that offering explanations using their errors enhances learning.

I find it important to discuss what the outcomes reveal about how the approach that the teacher used as a responsive strategy was in agreement with the concept image and concept definition theory of learning (Tall & Vinner, 1981) and the APOS theory of learning (Dubinsky *et al*.,1997). Firstly, the teacher presented the errors and misconceptions which learners have made, and during this time, the concept presented to the learners is considered to be external to them. Secondly, the teacher ensured that learners gain conceptual and procedural understanding by offering

explanations of the matter, the form, the process needed to solve the equation and the goal of solving it. After being exposed to such explanations, the concept then shifts from being external to the learner as the learner starts making meaning to the concept. Thirdly, the teacher gave learners tasks which allowed learners to solve equations of similar nature to the ones given in the pre-test. This approach strengthened the learners' ability to make meaning to the concept. During this time, it is the learners' responsibility to work out the answers, and this approach brings the learners even closer to the concept itself. When learners work out the answers without any assistance from the teacher, they are able to internalize and conceptualize the concept (Dubinsky *et al*.,1997), making it easier for them to accommodate the concept into their existing schema (Piaget, 1967). From comparison between the pre-test and the post-test responses, it is clear that offering explanations that are guided by the learners' errors and misconceptions aids the construction of knowledge by bringing the concept closer to the learners.

Consistent with the discussion in the preceding paragraph, the approach of offering explanations that are guided by the identified errors and misconceptions and exposing learners to tasks of similar nature builds their understanding of the object of learning through generalization pattern of the variation theory (Runesson, 2005). The generalization pattern of the variation theory suggests that learners should be given different exercises of the same nature which will afford them an opportunity to form conjectures and successfully question the misconceptions which they perceived to be true (Mhlolo, 2013). By firstly, showing learners the errors which they have made, secondly, explaining the procedure that needs to followed to solve the equation and thirdly, giving them tasks of similar nature, the teacher allowed learners to challenge their misconceptions. This approach afforded them an opportunity to make conjectures on how to solve a linear, quadratic and exponential equations.

One of the revelations on the outcomes is that learners could not differentiate the procedural knowledge necessary for solving linear, quadratic or exponential equation. Through the contrast pattern of the variation theory, learners were able to create a distinction between these equations. The contrast pattern of the variation theory suggests that for a learner to construct the correct conceptions about the object of learning, there is a need to make comparisons and critiques of the features that make up the object to be learnt (Mhlolo, 2013). Moreover, it is with no doubt that there are learners who do not even know the purpose of an equal sign while others show signs of

misunderstanding the concept of an "equal sign" as "doing something" (Li, 2010). The analysis in this study has also shown, for example, that there are learners who do not even write an equal sign when they manipulate an equation. It is not because they forgot, but because they do not see the purpose of writing it. The danger of such misconception is that it prevents learners from understanding the concept of an equation (Carpenter *et al*., 2003 as cited in Li, 2010). The contrast pattern of variation assisted the learners in creating a distinction between an algebraic expression and an algebraic equation. To foster this distinction, the teacher discussed the similarities and the differences between these two objects of learning by considering the *matter,* the *form,* the *process* and the *goal* of solving an equation and of simplifying an equation. Evident from their post-test responses, learners eventually understood the function of an equal sign and ensured that it is incorporated in the process of solving equations.

Moreover, most of the tasks used during the professional intervention lesson were more inclined to creating and maximizing cognitive conflict that would help learners challenge their intuitive conceptions. This conflict is important because it ensures that learners develop an understanding of the concept (Flavell, 1977). It occurs when learners are enlightened about the two conflicting elements, search for a resolution to the conflict and ultimately achieve a conceptualization that will resolve the conflict (Luneta & Makonye, 2010). Such conflict may seem to be a barrier to learning, but it actually assists learners in counteracting their misconceptions. Nonetheless, the outcomes of the study reveals that learning goes with challenging the learners' intuitive conceptions by showing them the incorrectness of their work and exposing them with the tasks that will maximize cognitive conflict and bring the object of learning closer to them.

It is worth discussing the revelation that some of the errors identified from the learners' responses emanate from the non-algebra concepts. These not-on-task errors (Makonye, 2010) were mostly evident in question 9 of the pre-test where they had to first find the LCD (Lowest Common Denominator) of the given fractions. It is evident from some of their responses that some learners were unaware that the denominators have to be the same while other learners were unable to find the LCD, which was taught from as early as grade 4. To decide upon an action that will overcome this error, the teacher applied the scaffolding technique through analogy (Tirosh *et al*., 2008) where two scenarios were presented to the learners. The first scenario had two fractions of with identical denominators whereas the second scenario had fractions with different values on the denominators,

and learners had to decide whether it would be applicable to add the numerators of the fractions in both scenarios or not. The teacher then presented one of the incorrect responses to the learners and allowed them to provide reasons why the response was incorrect. This approach challenged their thinking and the improvement in their responses in question 9 of the post-test shows that this approach assisted the learners in noticing the incorrectness of their intuitive conception.

I wish to diverge to the importance of scaffolding learners with concept-building tasks mainly because the outcomes have shown the importance of giving learners tasks that will scaffold them with an aim of building conceptual understanding. The teacher started the intervention by giving learners tasks that are not exam type of questions, but were rather meant to:

- Challenge their errors and misconceptions.
- Scaffold learners in building conceptual and procedural understanding.

This outcome does not discredit the importance of giving learners higher order tasks that will promote erratic responses, but to rather emphasize the importance of tasks that enhances conceptual understanding. The subsequent tasks, which are also important as they allow cognitive perturbation (Luneta &Makonye, 2010) may be given to the learners chiefly for the purposes of assessing the learners' maturity in solving algebraic equations.

It is of importance to point out that even though there was a significant improvement in the learners' performance from the pre-test to the post-test, the intervention approach addressed most but not all the identified errors and misconceptions that the learners had. This is evident from the last question where they had to use their algebraic skill to represent and solve a word problem, their post-test responses showed almost no improvement at all. They scribbled meaningless responses which showed lack of understanding on the nature of word problems and lack algebraic maturity of representing word problems algebraically. Moreover, I wish to also highlight that the following, in Moleko and Msimango (2020)'s terms, could be the sources of difficulty for learning and understanding mathematical word problems:

- lack of English proficiency
- limited knowledge of mathematical vocabulary
- the effects of "out of context" meanings
- lack of understanding the mathematical language and structure.

Perhaps this instructional approach did not consider the fact that learners consist of dissimilar perceptions about the world which sometimes contradict with scientific ideas (Baker *et al*. 2007). Some of their false ideas may even be hard to change. This could be the reason why there was very little improvement with the word problems even after they attended the intervention session. For such reason, most learners have presented the same challenge in representing the word problem algebraically.

In agreement with the preceding paragraph, the constructivist view suggests that knowledge, which is driven by mental self-regulatory structures, is individually constructed through cognition (Piaget, 1967). For a learner to re-organize and make meaning out of the newly introduced learning concept, this concept should reconcile with and be accommodated by the existing schema (Osborne & Wittrock, 1983). However, due to the differences in the learners' experiences and prior knowledge, they will not all make meaning in the same way. The way in which the learner accommodates knowledge is dependent on the learner's existing schema and when the learners' existing schema contradicts the new knowledge, it will not be possible for the learner to successfully accommodate the new knowledge. It is worth noting that it won't always be possible to create a link between the learners' existing schema with the new knowledge that needs to be incorporated (Olivier, 1989). As a result, it was hard for the applied approach to address the challenges that learners have with word problems.

5.4 CONCLUSION

This chapter outlined the findings of the study by discussing the errors that were identified in the learners' responses to the pre-test questions. The chapter also provided a thorough discussion of the implications of the identified errors, linking them with the relevant theoretical frameworks that govern this research study. It is clear from the discussion that the reduced number of errors evident in the post-test as opposed to the ones presented by the learners in the pre-test elucidates the fact that indeed providing explanations that are guided by the learner errors and misconceptions is an advantage to learning as promotes improvement in the learner performance, and a disadvantage as it does not cater for all the learners as they have different experiences which may impede the learning process.

CHAPTER 6: OVERVIEW, RECOMMENDATIONS AND LIMITATIONS

6.1 INTRODUCTION

This seeks to conclude the study by providing an overview of the study, offering recommendations and limitations that are pertinent to the study.

6.2 OVERVIEW OF THE RESEARCH

One of the discussions presented in Chapter 2 earlier in this study was on misconceptions, how they develop, and the challenges that the teacher has in addressing them. To mitigate this challenge, this study has presented the effect of using the identified errors and misconceptions when providing explanations in a professional noticing intervention. Below is an overview of the steps which were considered mainly for data collection purposes.

Step 1: The group of 40 grade 9 learners wrote a pre-test on solving algebraic equations. Their responses were marked and analyzed mainly to identify the errors which they have made as well as the misconceptions these errors emanate from. These errors were categorized according to the framework which was adopted in this study.

Step 2: To get clarity on the identified errors and misconceptions, interviews were conducted with the sampled participants. This stage of the data collection process was important as it afforded me an opportunity to understand the reasoning behind the learners' wrong intuitive conceptions. Moreover, the dialogue which I had with the learners also served as a tool in offering explanations that addressed the learners' errors and misconceptions which were evident in their pre-test responses.

Step 3: I conducted a coaching session with the teacher mainly to enlighten her about the errors and the misconceptions which I have identified from the learners' pre-test responses. This is, according to the professional noticing strands (Jacobs *et al*., 2010), attending to the learners' errors and misconceptions. During the coaching session, the teacher was also informed about the learners' reasoning and justifications of their intuitive conceptions. It was important for the teacher to know these justifications and interpret their work as this information was relevant in informing the teacher explanations.

Subsequently, teacher explanations which were directed at addressing the identified errors and misconceptions were offered during the intervention session.

Step 4: A post-test was conducted with the same group of learners. Their post-test responses were also marked and compared with the pre-test responses.

These four stages were important as they offered a platform for collecting sufficient data that assisted me in responding to the following research questions.

- What learner errors and misconceptions are identified by the teacher while attending to the grade 9 learners' work on solving algebraic equations?

The errors that I identified from the learners' pre-test responses were not only based on their inability to answer the question, but were also due to the misconceptions which they had encountered while learning how to solve algebraic equations. Most of the identified errors were influenced by the learners' intuitive conceptions which they initially believed to be true.

- What learner reasoning and justifications are evident from the learners' work of solving algebraic equations?

The dialogue that I had with the learners during interviews had proven that indeed learners come to school with different conceptions that they believe to be true. It was evident from most of the extracts of our dialogue during interviews that their justifications were based on the true mathematical ideas which they have misconceptualized.

- What possible learning advantages and disadvantages can result from providing explanations that are informed by the learners' errors and misconceptions?

From the improvement in the learners' performance in the post-test as opposed to their pre-test has shown that indeed explanations that are informed by learner errors and misconceptions promotes cognitive gains. The analysis of the results and the graphical representation of learner improvement per question (Figure 13) justifies the fact that cognitive shift was evident in most questions. An improvement in the performance means that most of the errors and misconceptions were addressed.

However, looking at the insignificant change that was evident in question 10 of the learners' pre-test and post-test responses, it is evident that this teaching approach did not address the errors that learner had when working with word problems. It is clear that the disadvantage of offering

explanations that are directed at learner errors and misconceptions did not promote cognitive shift. This is due to the fact that word problems demand learners to first understand the scenario before representing it algebraically. There may be other factors affecting the learners' competency in comprehending the question (for example, difficulties in understanding English), and failure to understand the scenario compromises the learners' ability to represent it algebraically. Due to the different scenarios depicted in word problems, addressing their errors using explanations that are informed by these errors would not yield much improvement.

- To what level can learning and learner achievement in solving algebraic equations be improved if explanations are informed by the learner's errors and misconceptions?

The learners' improvement in the post-test confirms that indeed offering explanations directed at the identified errors promoted an improvement in their performance. The graphical representation presented in chapter 4 shows that out of the 10 questions, 9 of them showed an increase in the average performance. These representations, together with the comparative analysis presented in this study has shown that there was a decrease in the number of errors in the post-test as opposed to those that were evident in the pre-test.

6.3 RECOMMENDATIONS AND IMPLICATIONS FOR FUTURE RESEARCH

Based on the findings of this study, I suggest the following recommendations:

a. <u>recommendations for further research:</u>

- The findings of this study has confirmed that teacher explanations that are influenced by the learner error and misconceptions promotes learner academic improvement. Moreover, the findings have shown minimal improvement with the algebraic word problem. Due to this knowledge gap, I recommend that further research on the following should be conducted:

  - getting much deeper into the errors and misconceptions that learners have in solving word problems.
  - acquiring reasons behind the learner challenges in the application of algebraic equations in word problems.
  - researching different instructional practices and approach that will improve learner performance in interpreting and representing word problems algebraically.

This recommendation of further research being conducted will not only enrich the existing research, it will also be beneficial for mathematics educators as it will enlighten them on the errors and the misconceptions that learners encounter.

b. recommendations for the education department:

- As the findings of the study has confirmed that the learners who participated in the study have shown academic improvement after they were taught using the focused instructional approach, I recommend to the department of education that teachers should be advised to use identified learner errors as a resource to improve the learning of algebraic equations.
- I also strongly recommend to the department of education that teachers should be afforded more education training and workshop where teachers are:

  - exposed to the recently identified learner errors and misconceptions, not only in algebra but also in other mathematical topics which seem to challenge learners.
  - given the opportunity to discuss the learner errors which they frequently identify.
  - are collaboratively engaging in discussions on the different instructional approaches which may assist in counteracting the learner errors and misconceptions.
  - are trained on deeper error identification and analysis.

c. recommendations for the teachers:

- The analysis of the study shows that most of the identified learner errors emanate from lack of understanding of algebraic equations. That is why some learners have omitted the equal sign in some of their steps while others while other learners did not know that they can substitute their answer(s) to check if the answer is correct or not. It is for this reason why I recommend that teachers should consider discussing the necessary terminologies such as equation, coefficient, transposing, subject of the formula, etc. as another way of enhancing conceptual understanding.
- To the teachers, I also recommend that they gain the skill to pay careful attention to the learners' contributions as well as their written responses in order to understand the learners' thinking and what they know.

6.4 LIMITATIONS OF THE STUDY

The limitations of the study are outlined in this sub-section. It is important to note that the findings of this study were limited to the sample of a case study of one mathematics educator and a group of 40 grade 9 learners. 7 learners were sampled from this group of 40 learners who participated in the study by writing both the pre-test as well as the post test. Therefore, these learners who participated in the study cannot be generalized over the findings from the sampled learners. It is worth note taking that these learners are sampled from different learning space, are taught by different teachers and have different levels of understanding. As a result of these differences, the findings from this case study of 7 learners cannot be used to generalize over the rest of the learners.

As part of acknowledging the limitations of the study, is considering the limited exposure to word problems during the intervention period. The teacher spent very little time trying to show learners how to represent word problems algebraically. It could be the reason why there was significantly little improvement in solving algebraic word problems.

6.5 CONCLUSION

This chapter has revealed an interesting revelation about the uniqueness in the manner in which individuals, learners in this case, construct knowledge and the strong justifications of their intuitive conceptions. The learners' construction is found to be pleasurable to them irrespective of how their conceptions were discovered or whether it is correct or incorrect.

It is clear from the discussion that the reduced number of errors evident in the post-test as opposed to the ones presented by the learners in the pre-test elucidates the fact that indeed providing explanations that are guided by the learner errors and misconceptions is an advantage to learning as promotes improvement in the learner performance, and a disadvantage as it does not cater for all the learners as they have different experiences which may impede the learning process.

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ANNEXURES

Annexure A: Pre-test

MATHEMATICS	**GRADE 9**
ALGEBRAIC EQUATIONS	**2023**
Mark allocation: 30 marks	
Suggested time:40 minutes	

INSTRUCTIONS and INFORMATION:

1. This paper consists of 1 page with 10 questions on algebraic equations.
2. Answer ALL questions.
3. Number your answers correctly according to the numbering system used in this paper.
4. Clearly show all calculations that you have used to determine your answers.
5. You may use a non-programmable scientific calculator.
6. If necessary, round off your answers to the nearest 2 decimal places.
7. Write neatly and legibly.

Solve for x and y:

1. $x^2 = 36$ (2)
2. $2^y = 8$ (2)
3. $3^{x-3} = 81$ (3)
4. $(x - 7)(x + 8) = 0$ (2)
5. $(x + 2)(x + 7) = -4$ (4)
6. $4(x + 2) - 3 = 2(3 - x)$ (3)
7. $3x^2 + 6x - 9 = 0$ (3)
8. $2.\ 4y + 10 = 138$ (4)
9. $\frac{y+4}{4} - \frac{2y-7}{8} = 9$ (4)
10. Thando's dad is three times as old as she is now. Ten years ago he was seven times as she was old then. How old are Thando and her dad now? (3)

THANK YOU FOR YOUR PARTICIPATION!!!

Annexure B: Post-test

MATHEMATICS	**GRADE 9**
ALGEBRAIC EQUATIONS	**2023**
Mark allocation: 30 marks	
Suggested time:40 minutes	

INSTRUCTIONS and INFORMATION:

1. This paper consists of 1 pages with 10 questions on algebraic equations.
2. Answer ALL questions.
3. Number your answers correctly according to the numbering system used in this paper.
4. Clearly show all calculations that you have used to determine your answers.
5. You may use a non-programmable scientific calculator.
6. If necessary, round off your answers to the nearest 2 decimal places.
7. Write neatly and legibly.

Solve for x and y.

1. $x^2 = 49$ (2)
2. $3^x = 81$ (2)
3. $5^{x+2} - 3 = 2$ (3)
4. $(y + 4)(y - 6) = 0$ (2)
5. $(x - 6)(x - 4) = 15$ (4)
6. $5(x + 2) = 6(x - 2)$ (3)
7. $2x^2 - 14x + 12 = 0$ (3)
8. $2.3^x - 4 = 50$ (4)
9. $\frac{x+3}{3} - \frac{2x-7}{4} = 12$ (4)
10. Jill runs to the shops to buy a bar of chocolate. It takes her three times as long to get home because she is eating her chocolate. How fast does Jill walk to the shops, if Jill lives 500m from the shops and it took her 25 minutes to go there and back? (3)

ANNEXURE C: INTERVIEW SCHEDULE

Learner interview schedule for the study titled:

THE EFFECT OF USING GRADE 9 LEARNER ERRORS AND MISCONCEPTIONS WHEN EXPLAINING ALGEBRAIC EQUATIONS IN A PROFESSIONAL NOTICING INTERVENTION.

Interviewer: Ms Msiza Busisiwe

Interviewee (Grade 9 learner):_____

Date of interview:_____

Note: The suggested length for the interview session is 30 minutes.

General introductory questions:

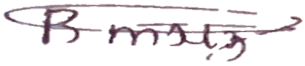
1. How would you rate your level of understanding of algebraic equations? Tick on the relevant box below:

Very poor (level 1)	Elementary (Level 2)	Moderate (level 3)	Adequate (level 4)	Substantial (level 5)	Meritorious (level 6)	Outstanding (level 7)

2. How long did it take you to write and complete the test?
3. Do you feel that you needed more than an hour to complete the test?
4. Which algebraic concepts do you find challenging?
5. Which questions, from the test that you wrote, were easy for you?

Please note that these are not the only questions that will be used to collect data. There will be more learner-individualised questions that are related to the content and the learner's responses. The following is an example of such questions:

QUESTION: Looking at your answer to the question, explain what you have done to get your answer.



ANNEXURE D: LETTER TO THE SCHOOL PRINCIPAL

Dear Mr Burn

I am Busisiwe Msiza. I am a Masters of Mathematics Education student at the University of Witwatersrand.
I am conducting a research study about the effect of using the learners' errors and misconceptions as a tool to provide explanations which serve as an intervention to these errors and misconceptions. The title of my research is: *The effect of using grade 9 learner errors and misconceptions when explaining algebraic equations in a professional noticing intervention*.

To successfully conduct this study, I need approximately 40 grade 9 learners from the school to be the participants of the study. These learners will write a pre- and post-test on solving algebraic equations. They will need 40 minutes per test. I will then sample a few from the learners who wrote the test to be interviewed on the steps which they have taken to solve algebraic equations. I plan to also work together with one mathematics teachers from the school who will conduct intervention lesson where the teacher will offer explanations on algebraic equations which will address these identified errors and misconceptions. During this time, I will observe and take notes on the lesson observations.

Please note that issues of confidentiality and anonymity will be considered and the learners will not be advantaged or disadvantaged in any case. They will also be permitted to withdraw from participating at any time should they decide to do so. I have selected this school chiefly for convenience purposes. All the research data will be terminated between 3 to 5 years after completing the research report. Should you wish to seek more information, do not hesitate to contact me or my supervisor.

Yours sincerely,

Busisiwe Msiza

1424785@students.wits.ac.za

0632458223

Supervisor: Prof. Judah Makonye

Email: Judah.makonye@wits.ac.za

011 717 3086

ANNEXURE E:

PARTICIPATION INFORMATION SHEET FOR LEARNERS AND PARENTS/GUARDIANS

Dear Sir/Madam

I am Busisiwe Msiza, a Master of Mathematics student in the School of Education at the University of the Witwatersrand. I am conducting a research study about the effect of using the learners' errors and misconceptions as a tool to provide explanations which serve as an intervention to these errors and misconceptions. The title of my research is: *The effect of using grade 9 learner errors and misconceptions when explaining algebraic equations in a professional noticing intervention*.

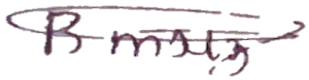
To successfully conduct this study, I invite you as a grade 9 learner to take part of the study. The reason why I invite you as a grade 9 learner is because algebraic equations are part of the grade 9 syllabus and you are, therefore, taught algebraic equations. Should you decide to be the participant of this study, you will engage in activities such as writing assessments (a pre-test and a post-test) which will be conducted for 40 minutes each test. You will also engage in interviews and be observed during lessons which will be approximately two hours. Arrangements for conducting these activities will be discussed and communicated with both the teacher and the learners. Preferably, they will take place at school in the morning before the commencement of classes.

With your permission, I would like to audio record the interview and lesson observation sessions. This data will be kept safe in a USB flash disc and will be terminated within 3 to 5 years after the completion of the research study. Please note that only the researcher will have access to the data. It is important to note that your responses from your tests and from the interviews will be confidential and anonymous. To maintain anonymity throughout this study, I will not write your name in the research report but use pseudonyms instead. Therefore, no one will be able to identify that it's you who produced the responses. With your permission, other researchers may use the data collected from this study, but your name and any personal information about you will never be passed on to them.

Please note that it is not compulsory that you take part of the study, participation is on voluntary basis. You will not lose anything if you choose not to partake in the study, neither will you gain any direct benefits. You are not obliged to answer any question should you feel uncomfortable to respond. If it happens that one of the questions asked during the interview makes you feel sad or uncomfortable, you are permitted to notify the interviewer and you are permitted to stop at any time. There is no participation fee that participants have to pay and there is no money rewarded for participating.

Ultimately, a report will be compiled and will be made available on the university library website. Should you wish to have a summary of this report, I will send it to you. If you need more information, let me know. Thank you very much.

Yours sincerely,



Busisiwe Msiza

1424785@students.wits.ac.za

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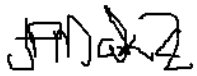
Supervisor: Prof. Judah Makonye



Email: Judah.makonye@wits.ac.za

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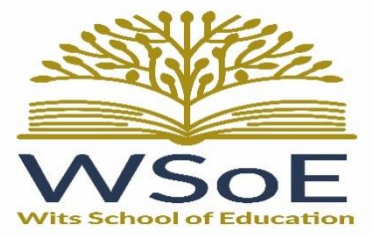
ANNEXURE F: LEARNER CONSENT FORM



LEARNER CONSENT FORM FOR WRITTEN ASSESSMENTS, INTERVIEWS AND OBSERVATIONS OF THE PROJECT CALLED:

THE EFFECT OF USING GRADE 9 LEARNER ERRORS AND MISCONCEPTIONS WHEN EXPLAINING ALGEBRAIC EQUATIONS IN A PROFESSIONAL NOTICING INTERVENTION.

Researcher: Busisiwe Msiza

I,_____, a grade 9 learner at _____ agree to participate in this research project. I am aware and agree to the following:

NB: TICK (✓) on the relevant options.

STATEMENT	**YES**	**NO**
The researcher explained the research study to me in detail and I understand what this study is about.		
The researcher explained that participation is on voluntary basis.		
I agree that I will remain anonymous as a participant of this study and information will be kept confidential throughout the study (My name and surname will not be used by the researcher in their research report).		
I know that I may withdraw from participating (in writing tests, in interviews and observations) in the study at any time.		
I am aware that I am permitted to stop the audio-taping at any time during the study.		
I agree that I will be given a pre-test and post-test on solving algebraic equations.		
I am aware and agree that I do not have to say anything or make use of any study material during the test sessions.		
I agree that I will participate in interview sessions which may be audio-recorded.		
I am aware that I do not have to answer all the interview questions and will not be advantaged or disadvantaged for my decision.		

I agree that I will be observed by the researcher during the intervention lessons of teaching and learning which may be audio-recorded.		
I am aware that I don't have to say anything during the observations and I won't be advantaged or disadvantaged in anyway.		
I agree that direct quotations from my interview may be used by the researcher in their research report.		
I agree that other academic researchers may my test scripts and the information I provide in my interview once they obtain their own ethics clearance, however, my name will not be used in their reports.		
I agree that my written work used in this study will be discarded between 3 to 5 years after the completion of the research report.		

LEARNERS' Signature:_____

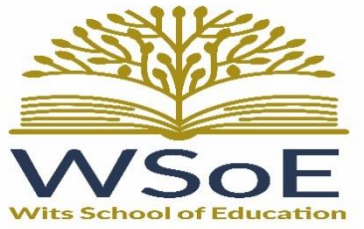
Date (DD/MM/YYYY):_____

Name of researcher:_____

Signature:_____

Date (DD/MM/YYYY):_____

ANNEXURE G: TEACHER CONSENT FORM



TEACHER CONSENT FORM FOR TEACHER'S LESSON OBSERVATIONS and AUDIO-RECORDING DURING THE LESSONS OF THE PROJECT CALLED:

THE EFFECT OF USING GRADE 9 LEARNER ERRORS AND MISCONCEPTIONS WHEN EXPLAINING ALGEBRAIC EQUATIONS IN A PROFESSIONAL NOTICING INTERVENTION.

Researcher: Busisiwe Msiza

I,_____, a mathematics teacher at_____ agree to participate in this research project. I am aware and agree to the following:

NB: TICK (✓) on the relevant options.

STATEMENT	**YES**	**NO**
The researcher explained the research study to me in detail and I understand what this study is about.		
The researcher explained that participation is on voluntary basis.		
I agree that I will remain anonymous as a participant of this study and information will be kept confidential throughout the study (My name and surname will not be used by the researcher in their research report).		
I know that I may withdraw from participating in the study at any time.		
I agree that I may be audio-recorded throughout the study and I am permitted to stop the audio-taping at any time during the study.		
I am aware that the observations will only be used for this research study and the audio will be terminated between 3 to 5 years after the research is completed.		

TEACHERS' SIGNATURE:_____

CONTACT DETAILS:_____

DATE (DD/MM/YYYY):_____

RESEARCHERS' NAME:_____

SIGNATURE:_____

DATE (DD/MM/YYYY:_____

ANNEXURE H: PARENT CONSENT FORM

PARENT CONSENT FORM FOR ASSESSING, INTERVIEWING AND OBSERVING THE CHILD/ LEARNER while participating in the project called:

THE EFFECT OF USING GRADE 9 LEARNER ERRORS AND MISCONCEPTIONS WHEN EXPLAINING ALGEBRAIC EQUATIONS IN A PROFESSIONAL NOTICING INTERVENTION.

Researcher: Busisiwe Msiza

I,_____, a parent/guardian of _____ give consent for my child to participate in the study. I am aware and agree to the following:

NB: TICK (✓) on the relevant options.

STATEMENT	YES	NO
I agree that the researcher will keep all the information gathered during the interview session confidential and my child will remain anonymous during the reporting of the research.		
I agree that my child will write a pre-test and a post-test on solving grade 9 algebraic equations.		
I agree that my child may be interviewed and the interview sessions may be audio-recorded.		
I agree that my child will be observed during lessons.		
I am aware and agree that the researcher will discard the audio-recording between 3 to 5 years after the completion of the study.		

Name of parent/guardian:_____

Signature of parent:_____

Contact Details:_____

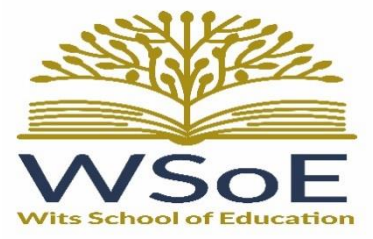
DATE (DD/MM/YYYY:_____

Name of researcher:_____

Signature of researcher:_____

DATE(DD/MM/YYYY) :_____

ANNEXURE I: LETTER TO THE LEARNERS



Dear learner

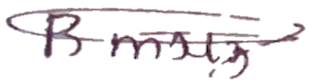
You are invited to participate in the research titled *The effect of using grade 9 learner errors and misconceptions when explaining algebraic equations in a professional noticing intervention*. The research is about the effect of using the learners' errors and misconceptions as a tool to provide explanations which serve as an intervention to these errors and misconceptions. Should you decide to be the participant of this study, you will engage in activities such as writing assessments (a pre-test and a post-test) which will be conducted for 40 minutes each test. You will also engage in interviews and be observed during lessons which will be approximately two hours. Arrangements for conducting these activities will be discussed and communicated with both the teacher and the learners. Preferably, they will take place at school in the morning before the commencement of classes.

With your permission, I would like to audio record the interview and lesson observation sessions. This data will be kept safe in a password protected computer and will be terminated within 3 to 5 years after the completion of the research study. Please note that only the researcher will have access to the data. It is important to note that your responses from your tests and from the interviews will be confidential and anonymous. To maintain confidentiality throughout this study, I will not write your name in the research report but use pseudonyms instead. Therefore, no one will be able to identify that it's you who produced the responses. With your permission, other researchers may use the data collected from this study, but your name and any personal information about you will never be passed on to them.

Please note that it is not compulsory that you take part of the study, participation is on voluntary basis. You will not lose anything if you choose not to partake in the study, neither will you gain any direct benefits. You are not obliged to answer any question should you feel uncomfortable to respond. If it happens that one of the questions asked during the interview makes you feel sad or uncomfortable, you are permitted to notify the interviewer and you are permitted to stop at any time. There is no participation fee that participants have to pay and there is no money rewarded for participating.

Ultimately, a report will be compiled and will be made available on the university library website. Should you wish to have a summary of this report, I will send it to you. If you need more information, let me know. Thank you very much.

Yours sincerely,



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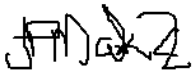

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ANNEXURE J

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: 2023ECE017M**
PROJECT TITLE	The effect of using grade 9 learner errors and misconceptions when explaining algebraic equations in a professional noticing intervention.
INVESTIGATOR	**Msiza Busisiwe**
SCHOOL/DEPARTMENT OF INVESTIGATOR	Wits School of Education
DATE CONSIDERED	06-Mar-23
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	**Minimal risk**
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	20-Mar-23
CHAIRPERSON	Mbokane Dr. Batseba Mofolo-Mbokane

cc: Prof. Judah Makonye

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

B. Msiza
Signature

Date 14/05/2023