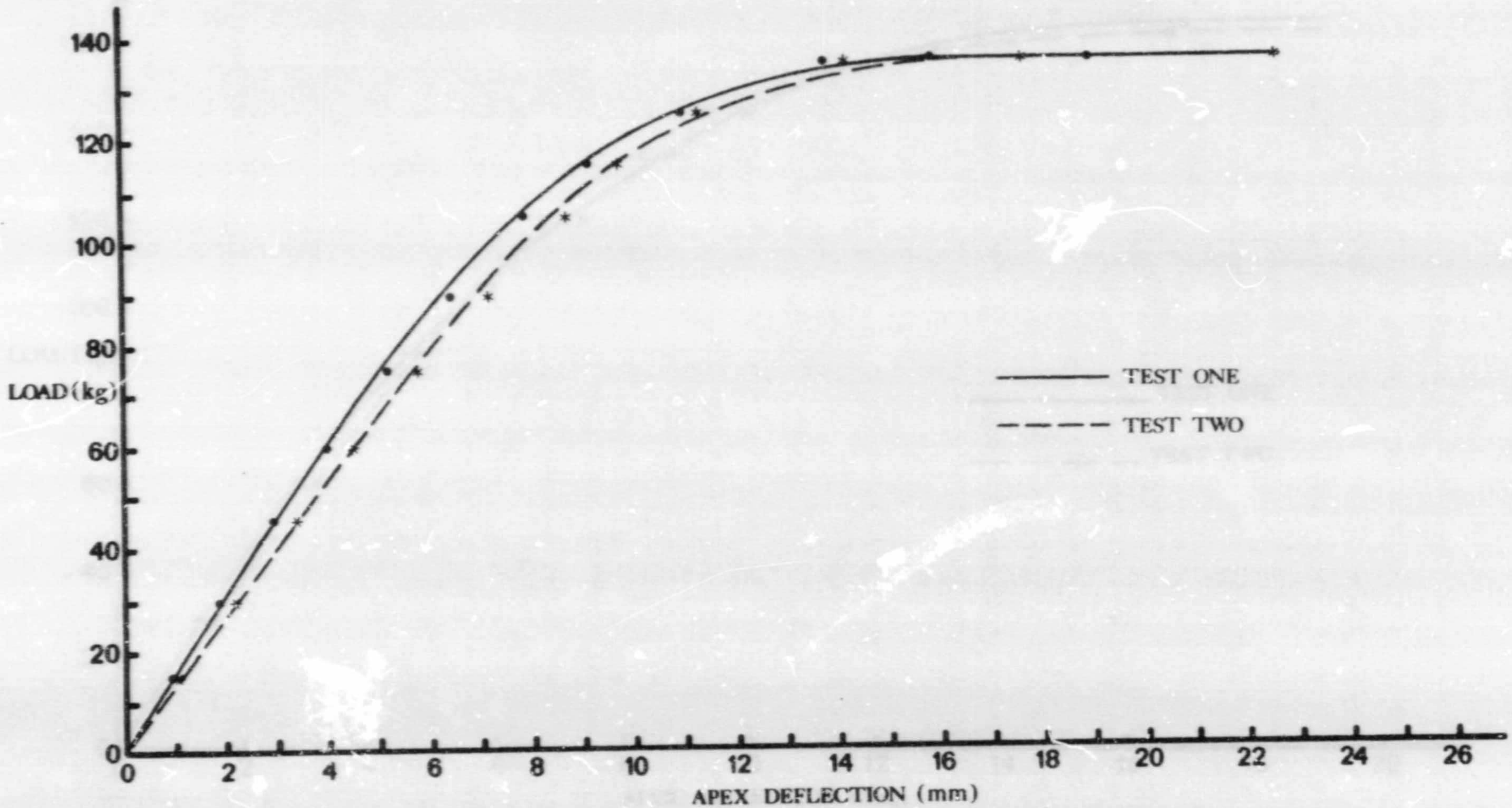


FIGURE 4.11: LOAD-DEFLECTION GRAPH; BAYS UNEQUALLY LOADED, BASES PINNED

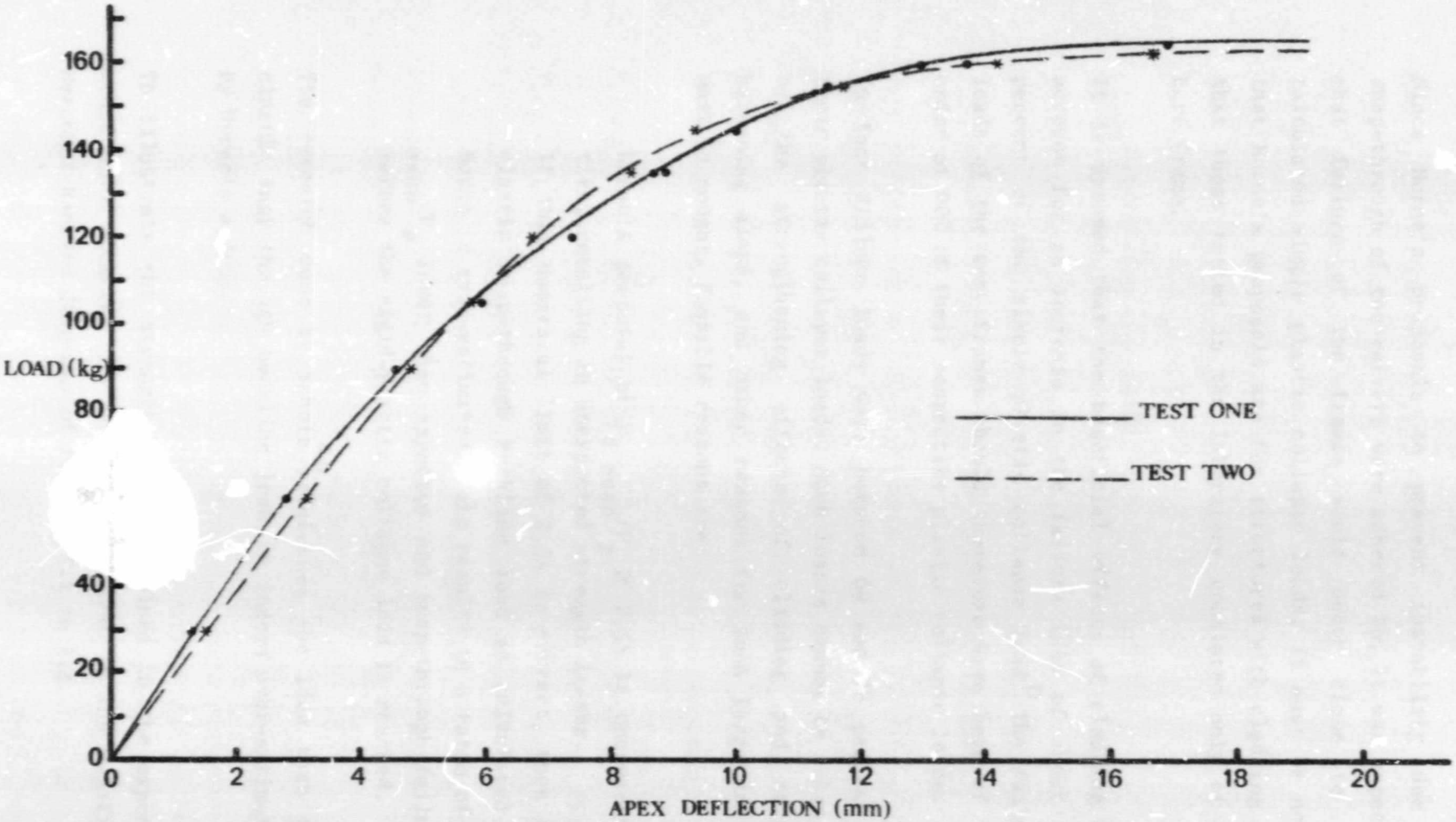


FIGURE 4.12: LOAD-DEFLECTION GRAPH; BAYS UNEQUALLY LOADED, BASES FIXED

5. DISCUSSION OF EXPERIMENTAL RESULTS

Since Horne's proposals to prevent instability due to snap-through of the rafters were adhered to, it was expected that failure of the frames would occur close to the calculated simple plastic collapse loads. It must be noted that Horne's proposals are for structures with cladding and that those tested in the laboratory consisted only of the bare frame.

It is assumed that the beneficial effects of cladding may account for an increase in the failure load of about ten percent of the simple plastic collapse load.[11] The failure loads of the test frames should therefore have been of the order of 90% of their respective plastic collapse loads.

In fact failure loads were between 69 and 78 percent of their plastic collapse loads. Such losses cannot be taken up by the strengthening effects of cladding and strain hardening alone, and other reasons for such large losses must be sought. Possible reasons are:

* Horne's proposal of $P_{c\ snap}/P_p \geq 2{,}5A$ is unconservative, resulting in unexpected strength losses.
* If this numerical limit of 2,5A is correct, then the elastic snap-through buckling load as calculated by Horne is over-estimated. This results in a ratio of $P_{c\ snap}/P_p$ lower than expected and snap-through failure before the rigid-plastic collapse load is reached.

The research done by Scholz[4,5] reinforces the idea that the elastic snap-through buckling load is indeed over-estimated by Horne's approach.

To illustrate the strength losses obtained in the experiments more clearly, the results are shown in the modified Merchant-Rankine diagrams of figures 5.1 to 5.4.

5.1 Modified Merchant-Rankine Diagrams

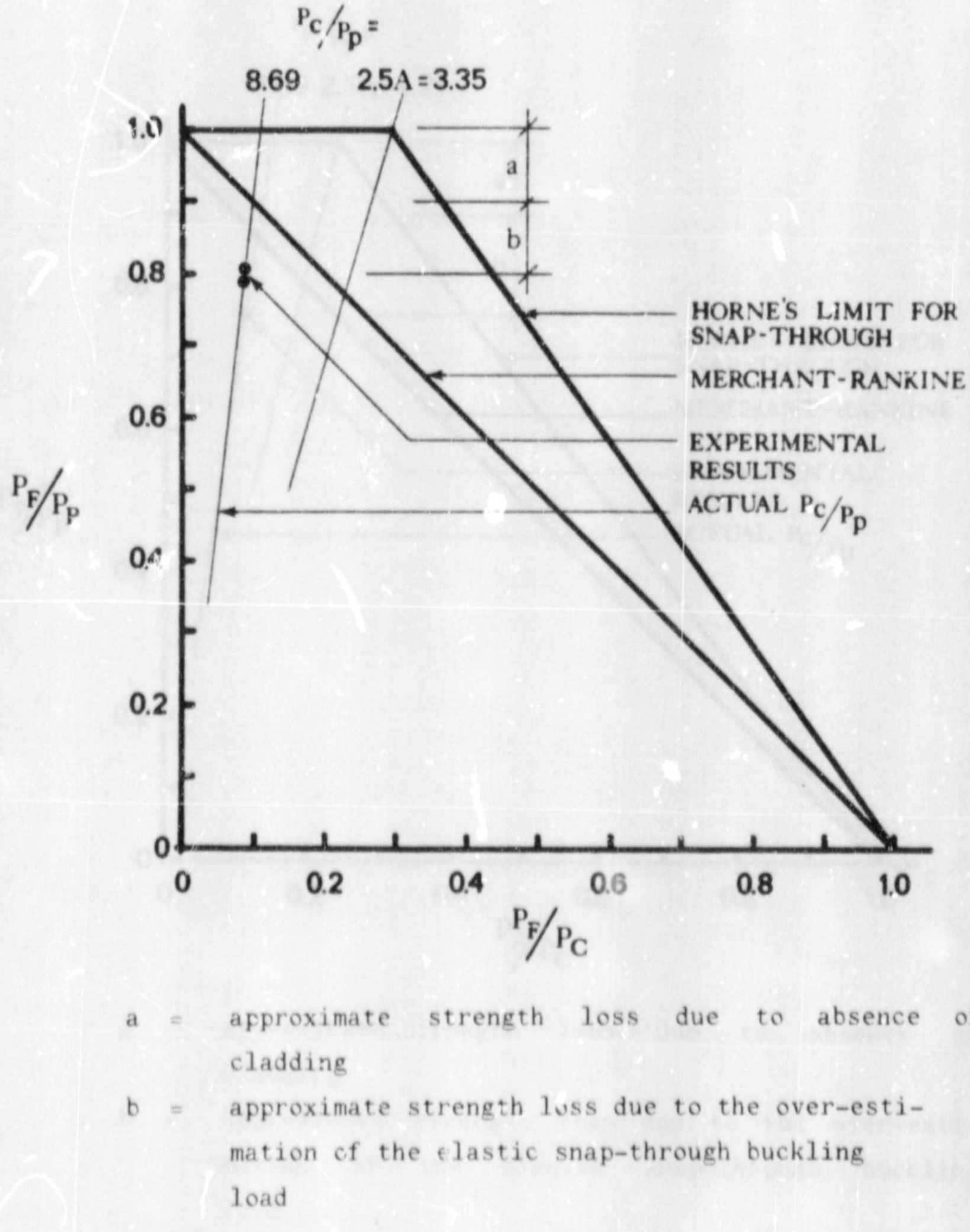
Each figure contains the following:

* The snap-through limit as proposed by Horne, viz: $P_{c\ snap}/P_p = 2{,}5A$.
* The actual ratio of $P_{c\ snap}/P_p$ as calculated by the approximate energy method of equation (8), and using Horne's assumptions.
* The experimental collapse loads.
* The approximate strength loss due to the absence of cladding. This is taken as 10%.
* The approximate strength loss due to the over-estimation of the elastic snap-through buckling load.

Also illustrated in these figures is the fact that the failure loads obtained by experiment are well below those predicted by the Merchant-Rankine formula for a bare frame. (equation 3). Since the Merchant-Rankine failure load is a lower bound for the collapse load of a structure, clearly the elastic buckling load as calculated is over-estimated.

A comparison between the ratios of $P_{c\ snap}/P_p$ as calculated by Horne, and those which result in the experimental failure loads when used in the Merchant-Rankine formula, will give an idea of how much the buckling load is over-estimated. These values are given in table 5.1. Also, simply for comparative purposes, the ratio of $P_{c\ snap}/P_p$ as calculated using Scholz' computer programme is given.

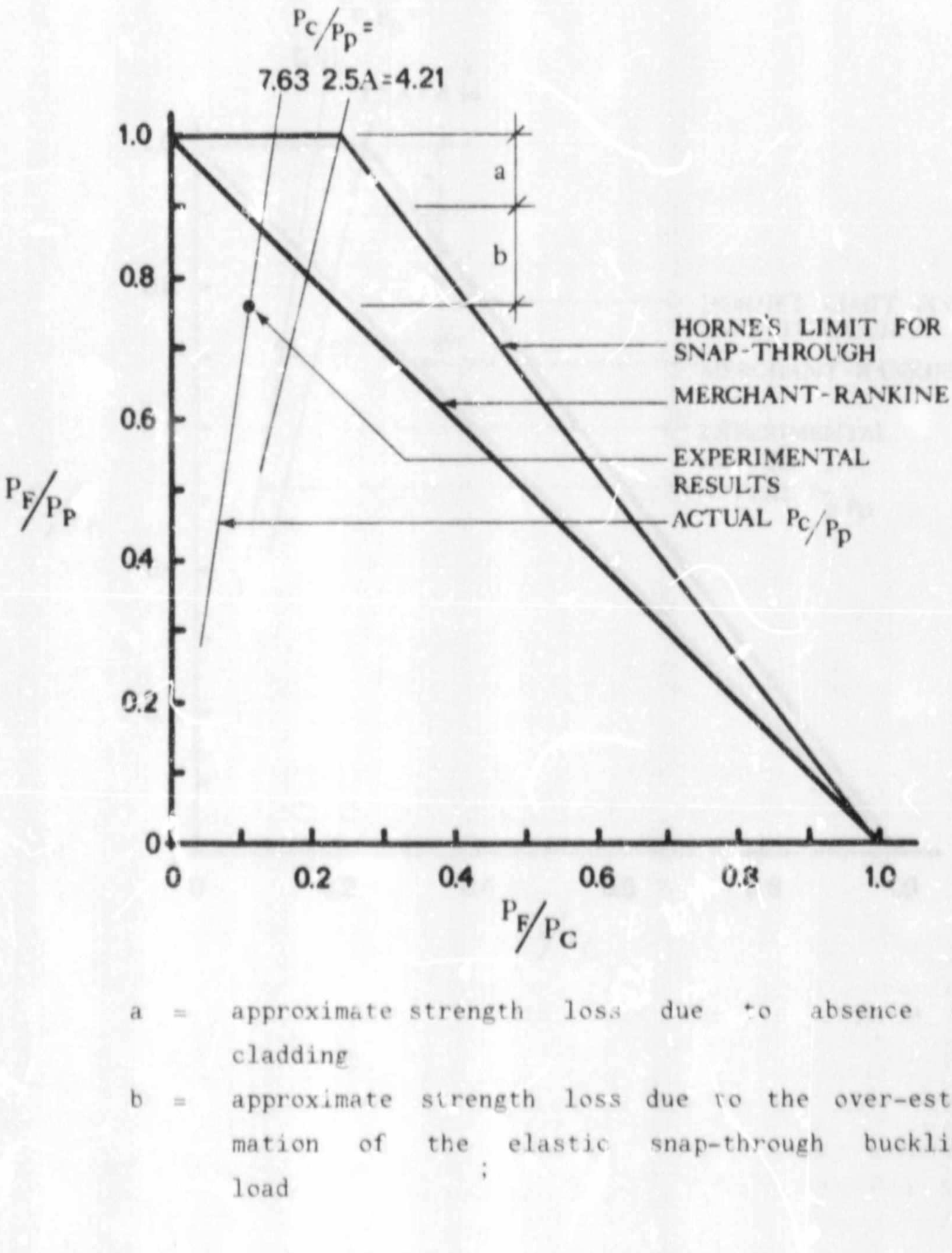
From the limited number of tests done, the elastic snap-through buckling load is over-estimated by between two and three-hundred percent if the straight-line approximation for bare frames by Merchant-Rankine is taken as the basis. Horne's limiting slenderness ratio equation which is designed to prevent snap-through instability is therefore unconservative as it incorporates his approximation of $P_{c\ snap}/P_p$. As a result there is a very real possibility that



a = approximate strength loss due to absence of cladding

b = approximate strength loss due to the over-estimation of the elastic snap-through buckling load

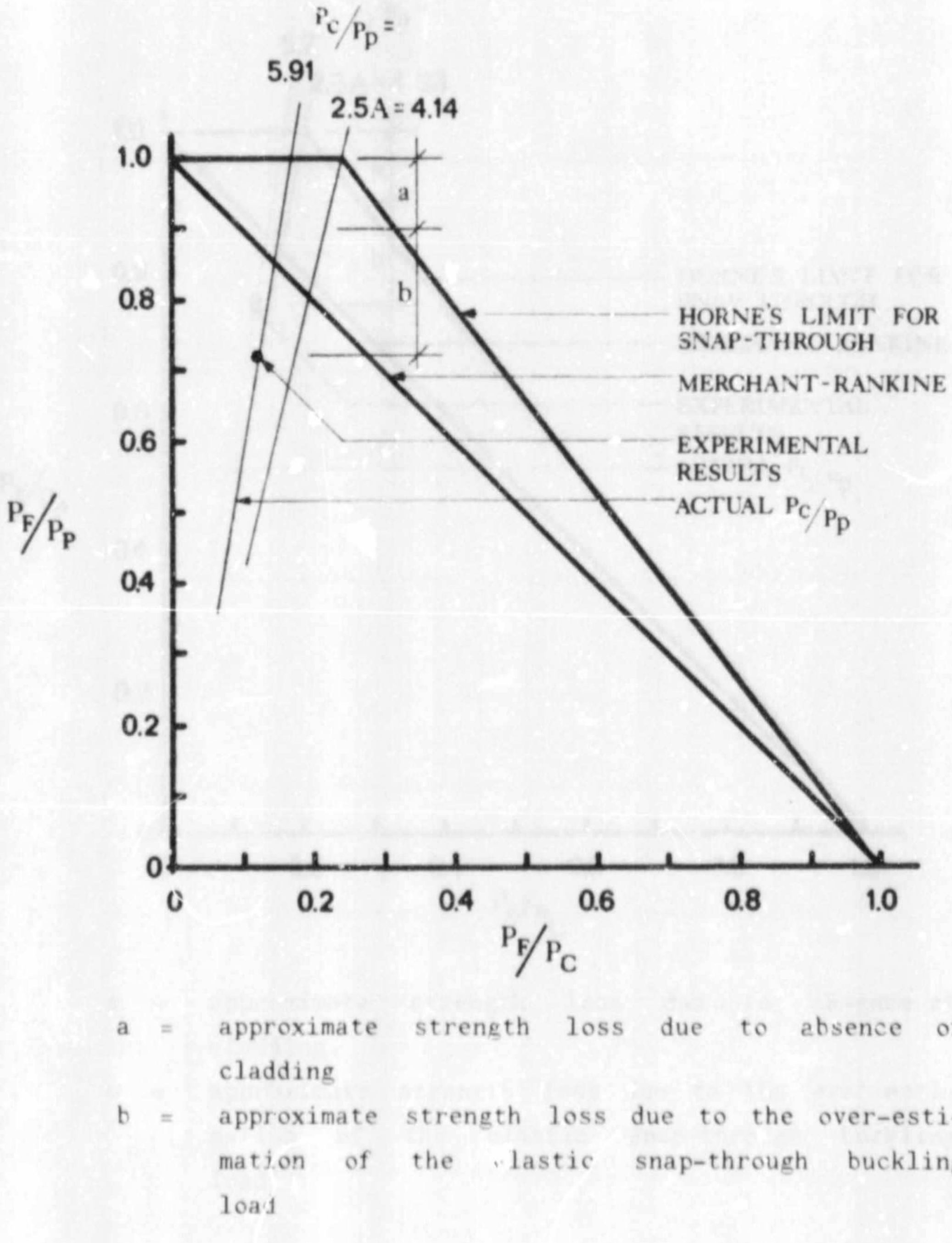
FIGURE 5.1 : BAYS EQUALLY LOADED, BASES PINNED



a = approximate strength loss due to absence of cladding

b = approximate strength loss due to the over-estimation of the elastic snap-through buckling load

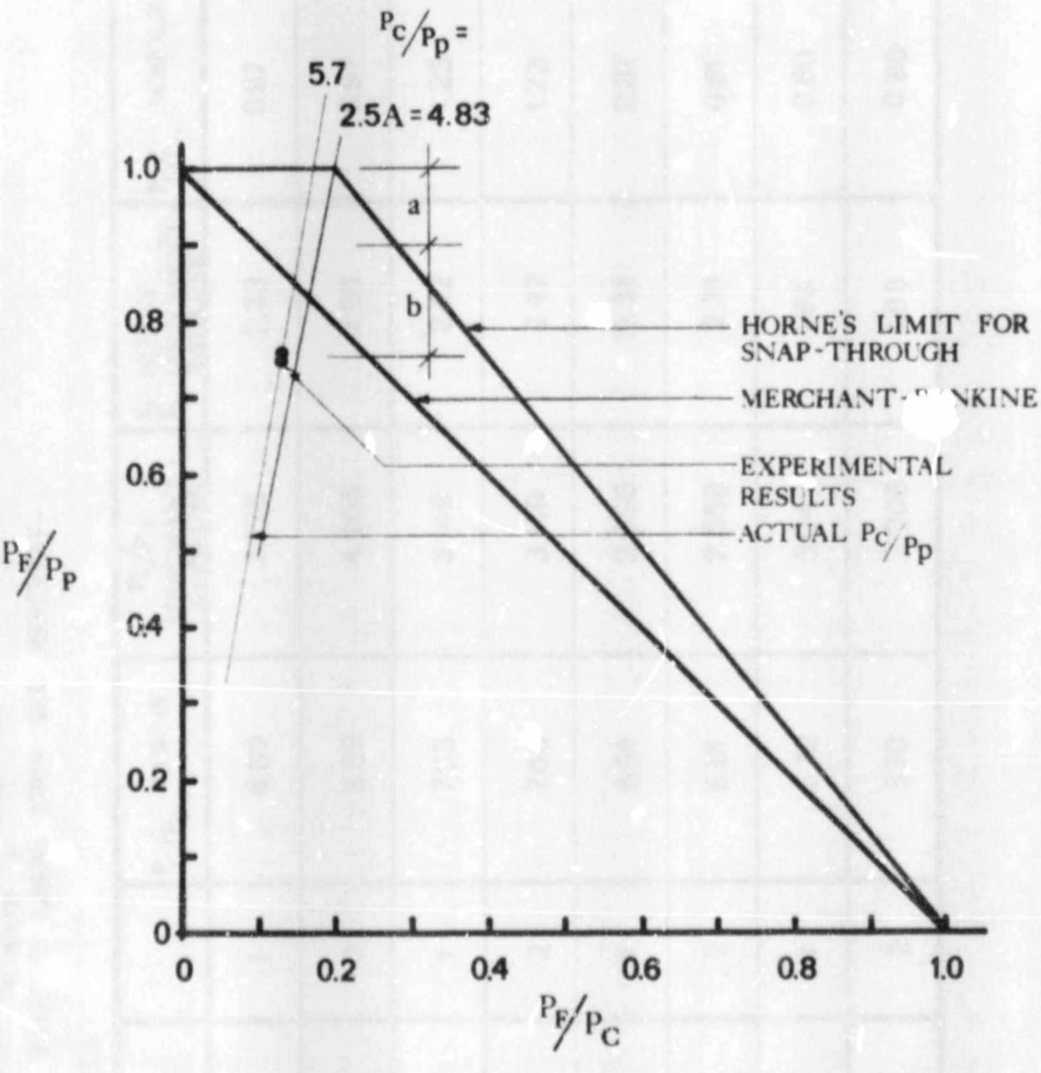
FIGURE 5.2 : BAYS EQUALLY LOADED, BASES FIXED



a = approximate strength loss due to absence of cladding

b = approximate strength loss due to the over-estimation of the elastic snap-through buckling load

FIGURE 5.3 : BAYS UNEQUALLY LOADED, BASES PINNED



a = approximate strength loss due to absence of cladding.

b = approximate strength loss due to the over-estimation of the elastic snap-through buckling load

FIGURE 5.4 : BAYS UNEQUALLY LOADED, BASES FIXED

TABLE 5.1 : COMPARISON OF $P_{c\ snap}/P_p$ CALCULATED BY HORNE, AND THE RATIO OF $P_{c\ snap}/P_p$ REQUIRED TO GIVE THE EXPERIMENTAL FAILURE LOADS FROM THE MERCHANT-RANKINE FORMULA

TYPE OF FRAME		P_c/P_p HORNE	P_c/P_p MERCHANT-RANKINE	$\frac{P_c \text{ HORNE}}{P_c \text{ MERCHANT-RANKINE}}$	P_c/P_p SCHOLZ
BAYS EQUALLY LOADED, ALL BASES PINNED	1	8.69	3.726	2.33	0.97
"	2	8.69	4.208	2.06	0.97
BAYS EQUALLY LOADED, ALL BASES FIXED	1	7.63	3.149	2.42	1.23
"	2	7.63	3.149	2.42	1.23
BAYS UNEQUALLY LOADED, ALL BASES PINNED	1	5.91	2.556	2.31	0.81
"	2	5.91	2.556	2.31	0.81
BAYS UNEQUALLY LOADED, ALL BASES FIXED	1	5.70	3.049	1.86	0.80
"	2	5.70	2.906	1.96	0.80

TABLE 5.1 : COMPARISON OF $P_{c\ snap}/P_p$ CALCULATED BY HORNE, AND THE RATIO OF $P_{c\ snap}/P_p$ REQUIRED TO GIVE THE EXPERIMENTAL FAILURE LOADS FROM THE MERCHANT-RANKINE FORMULA

TYPE OF FRAME		P_c/P_p HORNE	P_c/P_p MERCHANT-RANKINE	$\frac{P_c \text{ HORNE}}{P_c \text{ MERCHANT-RANKINE}}$	P_c/P_p SCHOLZ
BAYS EQUALLY LOADED, ALL BASES PINNED	1	8.69	3.726	2.33	0.97
"	2	8.69	[illegible]208	2.06	0.97
BAYS EQUALLY LOADED, ALL BASES FIXED	1	7.63	3.[illegible]	2.42	1.23
"	2	7.63	3.149	2.42	1.23
BAYS UNEQUALLY LOADED, ALL BASES PINNED	1	5.91	2.556	2.31	0.81
"	2	5.91	2.556	2.31	0.81
BAYS UNEQUALLY LOADED, ALL BASES FIXED	1	5.70	3.049	1.86	0.80
"	2	5.70	2.906	1.96	0.80

structures designed using equation (11) may indeed fail by snap-through instability. In order to prevent this occurring a number of other design methods may be employed as an alternative.

5.2 Alternative Methods to Prevent Snap-through Instability

A number of alternatives exist to prevent the occurrence of snap-through instability in practical frames. These are:

* A detailed second-order elastic-plastic analysis can be undertaken to calculate the exact failure load of the frame.
* An approximate method of calculating the failure load such as that proposed by Scholz can be applied.[8] The use of this procedure was described earlier in section 3.4.
* A more accurate elastic snap-through buckling load may be calculated and Horne's proposal of $P_{c\ snap}/P_p = 2,5A$ applied.
* A new limit can be proposed to replace Horne's limit of 2,5A.

A detailed second-order elastic-plastic analysis is probably the most accurate way of determining the failure load of a structure under applied loading. Calculations are however long and tedious if done by hand and available computer programmes are often expensive. An approximate method needs to be found, which can easily be applied, is not time consuming, and supplies one with sufficiently accurate results. Scholz[8] has developed a method which uses interaction curves and gives results which are accurate to within five percent. This technique may be applied to any type of structure subjected to stability problems.

The object of plastic analysis, of which Horne's proposals form a part, is however to design a structure which will fail at the plastic collapse load.

A detailed second-order analysis of Scholz' interaction method will provide a failure load for any frame, irrespective of the magnitude of the load. If plastic analysis is to be used, stability problems must be limited so that only a marginal reduction in strength below the plastic capacity occurs. It was with this in mind that Horne put forward his proposals and developed the limiting slenderness requirements given as equation (11).[2] This greatly reduces the amount of work necessary in the design of a frame.

Since Horne's slenderness equation for snap-through stability appears to be unconservative, it is best replaced by a similar limit. This would keep the calculations required in the design office to a minimum and allow one to continue using simple plastic analysis. The development of such a limit is outlined in the next paragraph.

Table 5.2 gives a comparison of the following:

* The failure loads as calculated by an elastic-plastic computer analysis developed by Kemp.[9]
* The failure loads as calculated by Scholz' interaction curves.[8]
* The failure loads using Horne's ratio of $P_{c\ snap}/P_p$ in the Merchant-Rankine formula for bare frames.
* The actual ultimate loads obtained by experiment.

From table 5.2 it can be seen that, both Scholz' method and the computer analysis agree with the results obtained by experiment. The values obtained by using Horne's ratio of $P_{c\ snap}/P_p$ in the Merchant-Rankine formula are far greater than the failure loads of the frames. This merely serves to reinforce the author's opinion that the elastic snap-through buckling load is over-estimated, making Horne's proposals unconservative.

TABLE 5.2 : COMPARISON OF FAILURE LOADS (values are percentages of the plastic collapse load)

TYPE OF FRAME	TEST No.	EXPERIMENT	COMPUTER	SCHOLZ	HORNE
BAYS EQUALLY LOADED, ALL BASES PINNED	1	78.84	77.1	73	89.7
"	2	80.80	77.1	73	89.7
BAYS EQUALLY LOADED, ALL BASES FIXED	1	75.90	72.6	74	88.4
"	2	75.90	72.6	74	88.4
BAYS UNEQUALLY LOADED, ALL BASES PINNED	1	71.80	71.0	72	85.5
"	2	71.80	71.0	72	85.5
BAYS UNEQUALLY LOADED, ALL BASES FIXED	1	75.3	73.6	70	85.10
"	2	74.4	73.6	70	85.10

Although this is only a preliminary study of Horne's snap-through stability equations, it appears that they are unconservative. Further research using full-scale frames with cladding is required in order to develop a better slenderness limit, which if satisfied will prevent snap-through stability from occurring.

5.3 Outline of a New Slenderness Limit to prevent Snap-through Instability

As can be seen from Table 5.2, the approximate elastic-plastic method by Scholz gives a good estimate of the frame failure load. This method can be easily processed to give a limiting ratio of $P_{c\ snap}/P_p$ for which the failure load would not fall below $0{,}9P_p$. This loss of 10% of the rigid plastic collapse load has been mentioned previously.[11] This will be compensated for by an equivalent strength gain due to composite action and strain hardening. Hence a minimum ratio of $P_{c\ snap}/P_p$ can be obtained for which the failure load of practical structures could be calculated by a simple plastic analysis.

In Scholz' approximate method to calculate the failure load of a frame, use is made of the following equation:

$$\left(\frac{\gamma_c}{\gamma_p}\right)_\ell = \left(\frac{\gamma_c}{\gamma_p}\right)\frac{Z\sigma_y}{M} \tag{12}$$

γ_c = load factor corresponding to elastic snap-through buckling

γ_p = load factor corresponding to plastic collapse

M = maximum bending moment due to P_C found from a first order elastic analysis

In the above equation $Z.\sigma_y$ represents the yield moment. If M is set to $\gamma_c M_w$, where M_w is the working moment with a load factor of $\gamma = 1{,}0$, it is reasonable to assume that under working load, at most the yield moment is reached, i.e. no yield occurs and $M_w = Z.\sigma_y$. With these assumptions equation (12) reduces to:

$$\left(\frac{\gamma_c}{\gamma_p}\right)_\ell = \frac{1}{\gamma_p} \tag{17}$$

If γ_p is chosen as 1.5, representing an average load factor between dead and live load, the right-hand side of equation (17) becomes 0,67. This value identifies a specific curve on the interaction diagram of figure 3.5. If this curve is intersected with a horizontal line through P_f/P_p = 0,9 a ratio of P_c/P_p of approximately 1.5 is obtained for the actual frame. This is illustrated in figure 5.5.

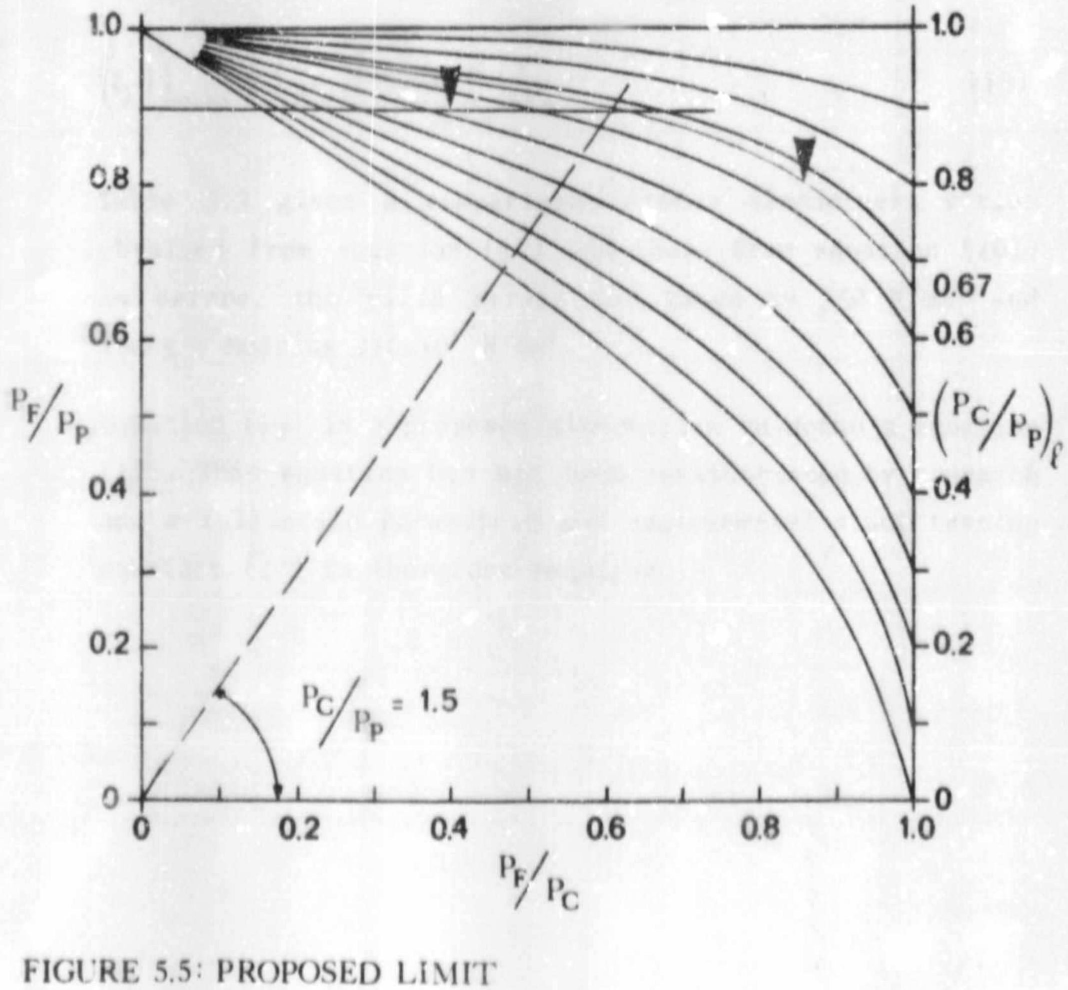


FIGURE 5.5: PROPOSED LIMIT

It is thus proposed to replace Horne's rule of $P_c/P_p \geq 2,5A$ by:

$$P_{c\,snap}/P_p \geq 1.5 \tag{18}$$

In equation (18), the accurate elastic snap-through buckling load such as that obtained from figures 3.8 and 3.9 must be used.

It is also easy to convert equation (18) into a limiting rafter slenderness L/d. If:

$$P_{snap} = \left(\frac{\pi}{\ell/h}\right)^2 \frac{EI_C}{h^2}$$

$$P_p = \frac{16 M_p A}{L}\text{, and}$$

$$M_p = 1.15\, I_R \sigma_y 2/d$$

are substituted into equation (18), this results in:

$$\left(L/d\right)_{snap} \le \left(\frac{\pi}{\ell\ h}\right)^2 \left(\frac{L}{h}\right)^2 \frac{I_C}{I_R} \frac{E}{\sigma_y} \frac{1}{55A} \qquad (19)$$

Table 5.3 gives a comparison between slenderness ratios obtained from equation (11) and those from equation (19). As before, the yield stress was taken as 350 N/mm^2 and Young's Modulus 210x10^3 N/mm^2.

Equation (19) is a proposed alternative to Horne's equation (11). This equation has not been substantiated by research and a full-scale parametric and experimental study testing equation (19) is therefore required.

TABLE 5.3 : COMPARISON OF THE PROPOSED, AND HORNE'S LIMITING SLENDERNESS RATIOS TO PREVENT SNAP-THROUGH $I_C = I_R$

ARCHING FACTOR A	L/h	ANGLE OF RAFTERS						SABS 0162
		10°		20°		30°		10°
		HORNE	NEW LIMIT	HORNE	NEW LIMIT	HORNE	NEW LIMIT	
1.5	4	136	66	313	127	647	217	61
	8	204	83	470	169	970	318	89
	12	272	90	627	194	1293	354	111
2	4	51	50	117	95	242	162	39
	8	76	63	176	127	364	238	60
	12	102	67	235	145	485	265	75
2.5	4	27	40	63	76	129	130	28
	8	41	50	94	102	194	190	46
	12	54	54	125	116	259	212	58
3	4	17	33	39	63	81	108	21
	8	25	42	59	85	121	159	37
	12	34	45	78	97	162	177	48
4	4	8	25	20	47	40	81	13
	8	13	31	29	63	61	119	26
	12	17	33	39	72	81	133	36

6. CONCLUSIONS AND RECOMMENDATIONS

Today most design codes permit the capacity of steel structures to be calculated by using a simple plastic analysis. In low-rise pitched-roof frames stability problems such as:

* In plane-member buckling
* Lateral torsional buckling
* Sway
* Snap-through

may result in a failure load lower than that predicted. To prevent snap-through occurring before the rigid plastic collapse load, Horne proposed the following:[2]

If $P_{c\ snap}/P_p \geq 2,5A$, $P_F = P_p$ (6)

From the limit of P_c snap $/P_p = 2,5A$, Horne developed a limiting slenderness equation, which if satisfied, allowed the use of plastic theory to give a reasonable estimate of the failure load. It was the object of this study to comment on this research by Horne which has become the basis of snap-through stability clauses in various design codes.

In order to test Horne's proposals, eight small-scale model frames were tested in the laboratory. These models were designed with ratios of P_c/P_p, calculated using Horne's assumptions, above the limit of 2,5A. These were tested to failure and collapse loads recorded. Failure loads were of the order of 69% to 78% of the predicted plastic collapse loads.

Although equation (6) was proposed for frames with cladding, in contrast, those tested in the laboratory had no cladding attached to them. A figure of 10% strength gain due to the effects of cladding and strain hardening has been mentioned in literature.[11] Any strength loss below 90% of the rigid plastic collapse load is therefore not acceptable. Even considering the strength gain due to sundry composite action, the failure loads are far lower than expected. This suggests that:

* Horne has over-estimated the value of P_c. This results in actual ratios of P_c/P_p lower than the limit of 2,5A, causing premature failure by snap-through.
* Horne's limit of $P_c/P_p \geq 2,5A$ is unconservative and needs to be revised.

In conjunction with previous research results by Scholz,[4,5] it appears that Horne over-estimates the elastic snap-through buckling load. A more rigorous analysis by Scholz shows that in extreme cases, $P_{c\ snap}$ is over-estimated by as much as eight times. A ratio of P_c/P_p far lower than the expected minimum of 2,5A exists, resulting in snap-through at loads well below that calculated by simple plastic theory.

This strength loss, resulting from Horne's over-estimation of $P_{c\ snap}$ can be prevented by:

* A second-order elastic-plastic analysis. This will provide an accurate estimate of the collapse load.
* Using Scholz' interaction curves to calculate the failure load.[8] Reasonable agreement was obtained between this procedure and experimental results.
* Calculating a revised rafter slenderness limit which incorporates a more appropriate estimate of the elastic snap-through buckling load.

Since Horne's original proposals were introduced so that simple plastic theory can be used, it is obvious that a new rafter slenderness limit, to prevent premature snap-through is required. This will eliminate any need for a tedious second-order analysis as rigid plastic theory may then still be used. A first proposal in this regard has been outlined which incorporates an accurate value of the elastic snap-through buckling load. This approach, however, requires further parametric and experimental confirmation.

The laboratory experiments conducted for this report were only a pilot study. It is recommended that full-scale frames with cladding be tested before new rafter lenderness limits to safeguard against snap-through are formulated.

It can also be concluded that all the findings on snap-through are equally relevant with respect to the South African Steel Code.[3] It is true that other slenderness limits, imposed for a completely different reason will, in this instance, reduce the problem somewhat for practical frames. However these limitations for individual member buckling do not exist in this format in other design codes. It would be wise to subject these slenderness values to a thorough re-examination.

A P P E N D I C E S

THE PLASTIC COLLAPSE LOAD OF A MULTI-BAY PITCHED-ROOF PORTAL FRAME **APPENDIX A**

The plastic collapse load, per bay, of the frame shown in figure A.1 is calculated. This was one of the frames tested in the laboratory.

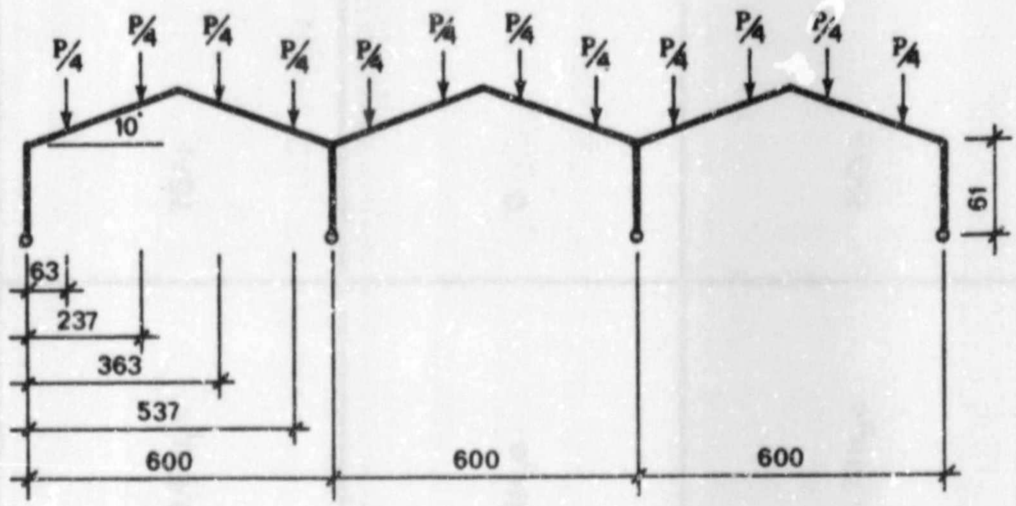


FIGURE A1

The general work equation:

$$\Sigma(M_p\phi_i) = \Sigma(P_p\Delta) \qquad (1)$$

is used to calculate P_p.

From figure A.1

n = the number of possible plastic hinges
 = 25

x = the number of unknown reaction forces
 = 5

N = the number of independent plastic collapse mechanisms
 = n-x
 = 20

These independent mechanisms, and their plastic collapse loads as calculated using equation (1) are illustrated in figure A.2. Combinations of independent mechanisms may also yield the plastic collapse load. Some of these are shown in figure A.2.

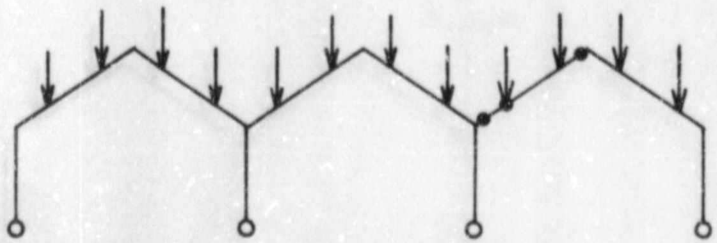
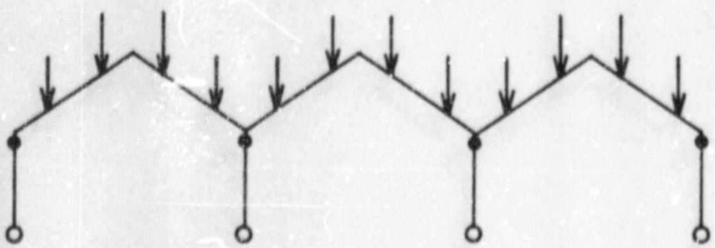
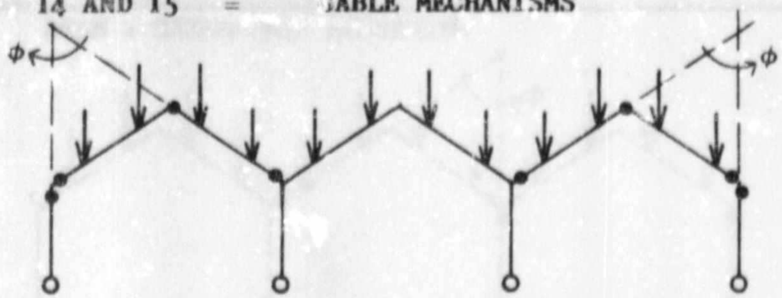
COLLAPSE MECHANISM	$\Sigma M_p \phi_i$	$\Sigma P_p \Delta$	COLLAPSE LOAD
1 TO 12 = BEAM MECHANISMS 	$9.47 M_p \phi$	$75 P\phi$	$0.1264 M_p$
13 = SWAY MECHANISM 	$4 M_p \phi$	0	–
14 AND 15 = GABLE MECHANISMS 	$5.73 M_p \phi$	$150 P\phi$	$0.0382 M_p$

FIGURE A2

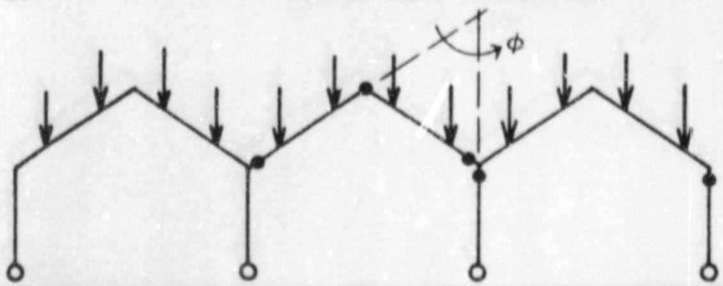
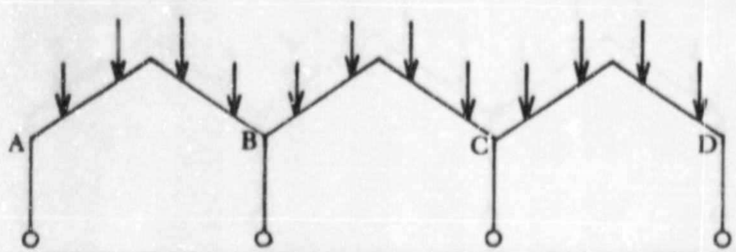
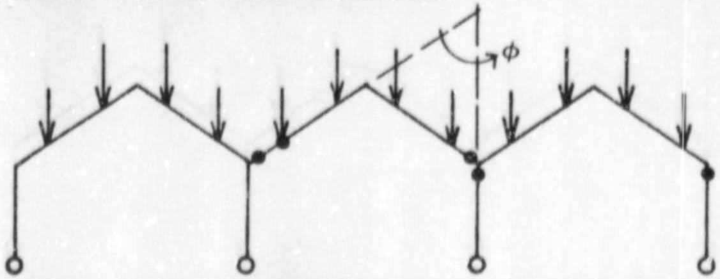
COLLAPSE MECHANISM	$\Sigma M_p\phi_i$	$\Sigma P_p\Delta$	COLLAPSE LOAD
16 = GABLE-SWAY MECHANISM 	$7.468 M_p\phi$	$150 P\phi$	$0.0498 M_p$
17 TO 20 = ROTATION OF JOINTS A TO D 	–	–	–
BEAM + GABLE-SWAY MECHANISM 	$22.42 M_p\phi$	$300 P\phi$	$0.0748 M_p$

FIGURE A2

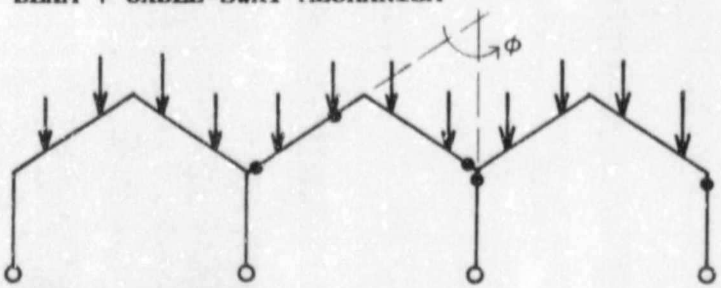
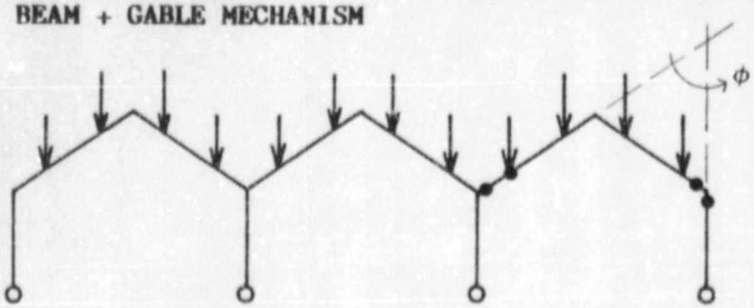
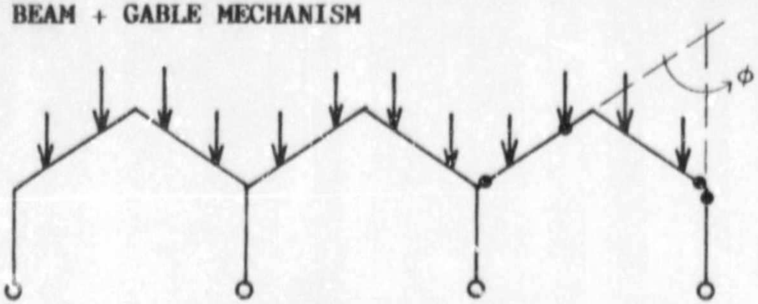
COLLAPSE MECHANISM	$\Sigma M_p \phi_i$	$\Sigma P_p \Delta$	COLLAPSE LOAD
BEAM + GABLE-SWAY MECHANISM 	$8.54 M_p \phi$	$190.1 P\phi$	$0.0449 M_p$
BEAM + GABLE MECHANISM 	$20.69 M_p \phi$	$300 P\phi$	$0.069 M_p$
BEAM + GABLE MECHANISM 	$6.8 M_p \phi$	$190.1 P\phi$	$0.0358 M_p$

FIGURE A2

Since the general work equation only gives an upper bound to the plastic collapse load, the lowest value resulting from the independent mechanisms and all possible combinations of these, will give the plastic collapse load. As a check, no where in the frame is the plastic moment allowed to be exceeded.

From figure A.2, the lowest collapse load per bay is $0{,}0358M_p$. For a 5x20mm section with a yield stress of 334 N/mm² the plastic moment capacity is 41750Nmm. Hence the collapse load per bay is 1494 newtons or 152,2 kg. A bending moment diagram under this load is given in figure A.3. Nowhere is the plastic moment exceeded indicating that 152,2 kg is indeed the plastic collapse load per bay.

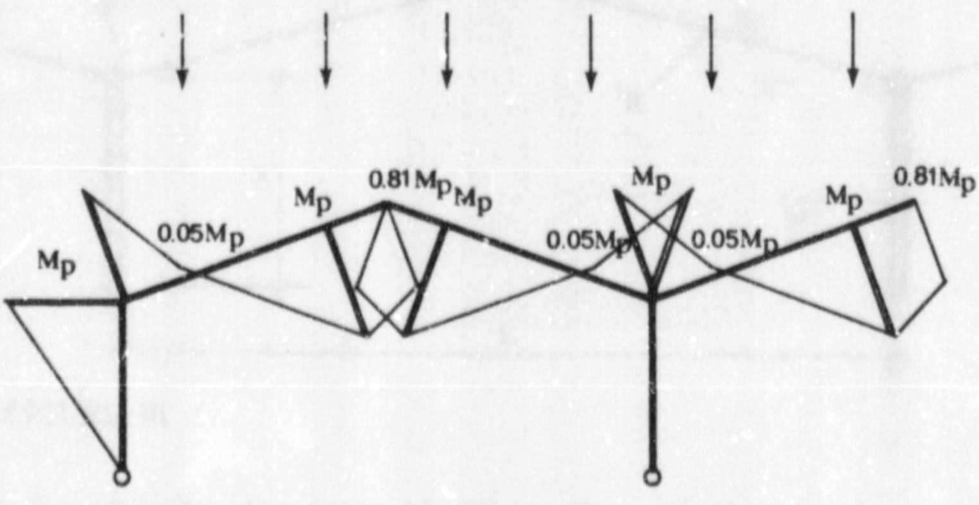


FIGURE A3

The plastic collapse loads for the other frames tested, were calculated in the same way.

A DETAILED DERIVATION OF HORNE'S SLENDERNESS LIMITS TO PREVENT SNAP-THROUGH INSTABILITY[2] **APPENDIX B**

In order to derive a limiting slenderness equation for all multi-bay frames, Horne analysed an infinite series of bays, all with pinned bases. To simulate the behaviour of an infinite series of bays, each bay was regarded as a single rigid frame, but with columns having a flexural rigidity of I/2. This is illustrated in figure B1.

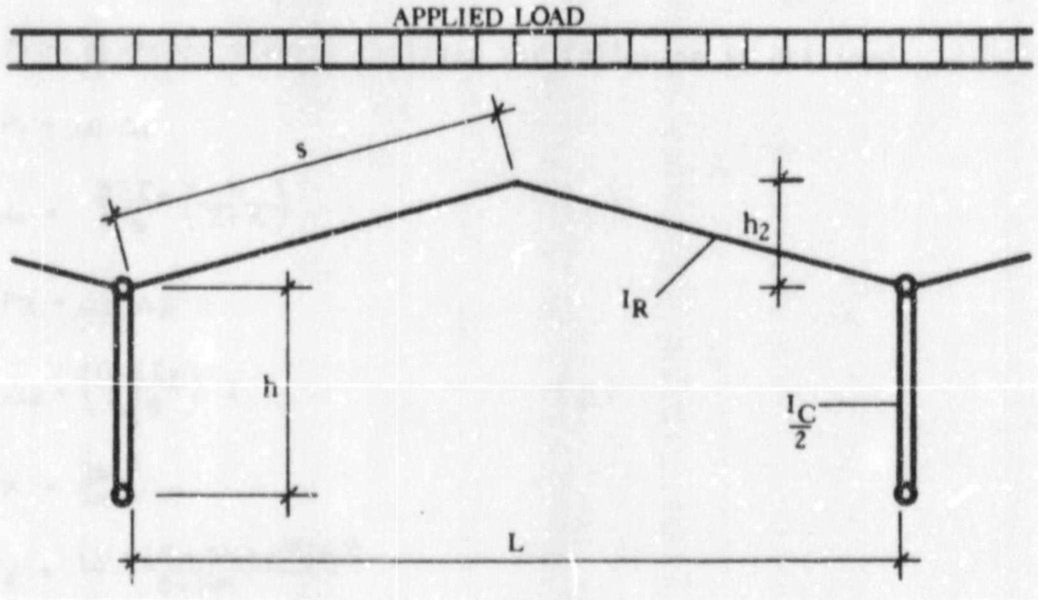


FIGURE B1

In the analysis, two types of deformation are considered

- Symmetrical
- Antisymmetrical

These are shown in figure B2.

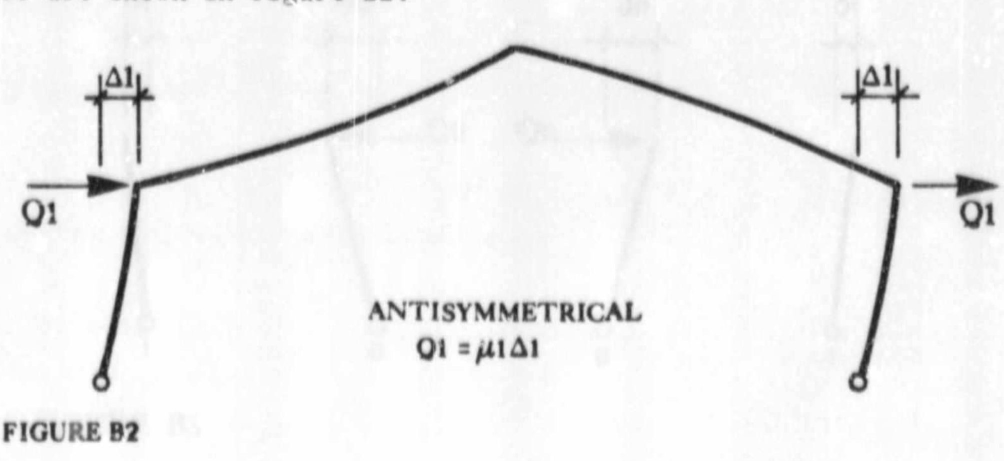


FIGURE B2

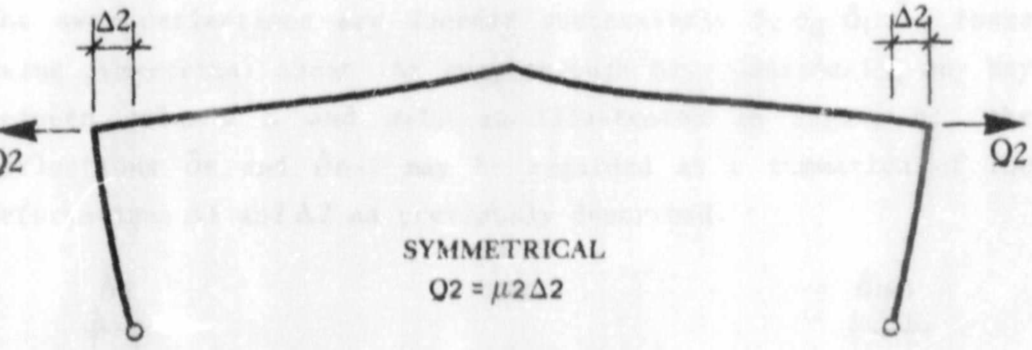


FIGURE B2

From a simple elastic analysis the following is obtained:

$$F_1 = \mu_1 . \Delta 1$$

$$\mu_1 = \frac{3EI_R}{h^2 s}\left(\frac{\kappa}{2+\kappa}\right) \quad \cdots\cdots (1)$$

$$F_2 = \mu_2 . \Delta 2$$

$$\mu_2 = \left(\frac{12EI_R}{h_2^2 s}\right) . \gamma \quad \cdots\cdots (2)$$

$$\kappa = \frac{I_c . s}{I_R . h}$$

$$\gamma = \frac{(2+(3+3k+k^2)\kappa))}{8+3\kappa}$$

$$k = \frac{h_2}{h}$$

Snap-through Deformation

The snap-through deformation of a bay, within an infinite series, is induced by forces Q_o as shown in figure B3.

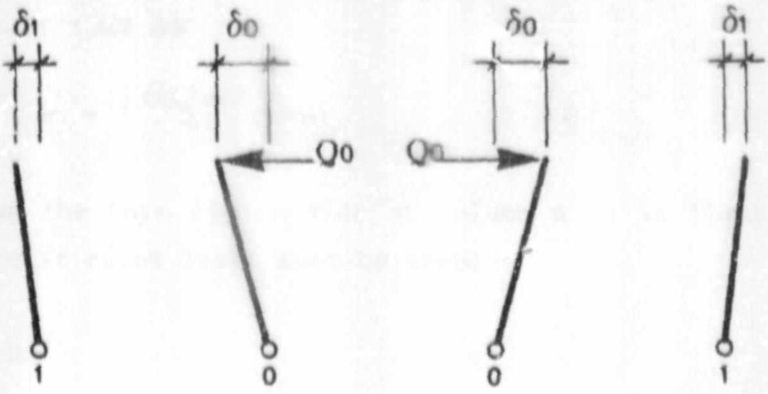


FIGURE B3

The sway deflections are denoted successively by $\delta_0, \delta_1 .. \delta_n$, these being symmetrical about the snap-through bay. Considering any bay between columns n and n+1, as illustrated in figure B4, the deflections δn and $\delta n+1$ may be regarded as a summation of the deformations $\Delta 1$ and $\Delta 2$ as previously described.

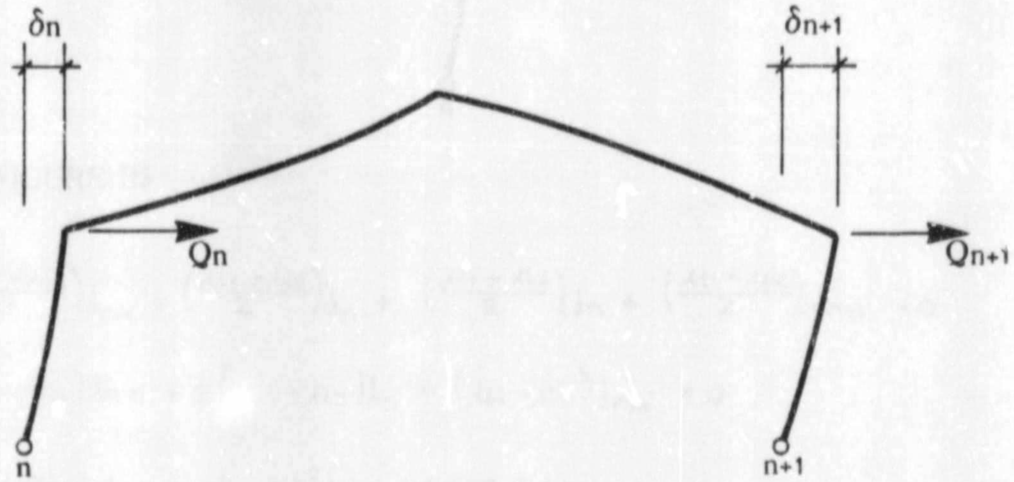


FIGURE B4

$$\Delta_1 = \frac{\delta_n + \delta_{n+1}}{2} \qquad \cdots\cdots (3)$$

$$\Delta_2 = \frac{\delta_{n+1} - \delta_n}{2} \qquad \cdots\cdots (4)$$

The forces Q_n and Q_{n+1} are therefore:

$$Q_n = \mu_1 \Delta_1 - \mu_2 . \Delta_2$$

$$= \left(\frac{\mu_1 + \mu_2}{2}\right) . \delta_n + \left(\frac{\mu_1 - \mu_2}{2}\right) . \delta_{n+1} \qquad \cdots\cdots (5)$$

$$Q_{n+1} = \mu_1 . \Delta_1 + \mu_2 . \Delta_2$$

$$= \left(\frac{\mu_1 - \mu_2}{2}\right) \delta_n + \left(\frac{\mu_1 + \mu_2}{2}\right) . \delta_{n+1} \qquad \cdots\cdots (6)$$

Considering the bays either side of column n as in figure B5 the total force at eaves level must be zero.

$$Q_n^1 + Q_n = 0$$

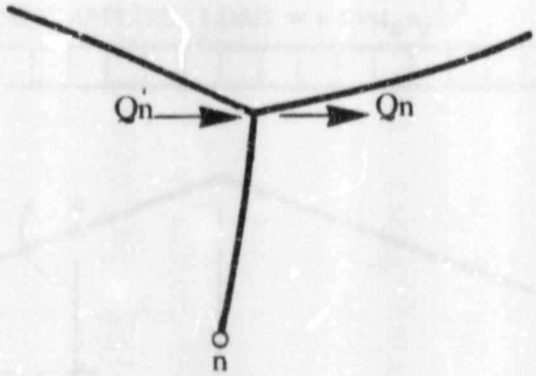


FIGURE B5

$$\therefore \left(\frac{\mu_1-\mu_2}{2}\right)\delta_{n-1} + \left(\frac{\mu_1+\mu_2}{2}\right)\delta_n + \left(\frac{\mu_1+\mu_2}{2}\right)\delta_n + \left(\frac{\mu_1-\mu_2}{2}\right)\delta_{n+1} = 0$$

$$\therefore (\mu_1-\mu_2)\delta_{n-1} + 2(\mu_1+\mu_2)\delta_n + (\mu_1-\mu_2)\delta_{n+1} = 0$$

SUBSTITUTING $\delta_{n+1} = r\delta_n = r^2\delta_{n-1}$ WHERE $r<1$

$$(\mu_1-\mu_2)r^2 + 2(\mu_1+\mu_2)r + (\mu_1-\mu_2) = 0$$

solving for r

$$r = \frac{(\mu_1+\mu_2) - 2\sqrt{\mu_1\mu_2}}{\mu_2-\mu_1} \qquad \cdots (7)$$

Now taking a closer look at the snap-through bay in figure B3, the applied force Q_0 is a combination of forces, viz: the force at eaves level from the snap-through bay, and the force at eaves level from bay 1. Hence:

$$Q_0 = \mu_2\delta_0 + \left(\frac{\mu_1+\mu_2}{2}\right)\delta_0 + \left(\frac{\mu_1-\mu_2}{2}\right)\delta_1$$

Substituting $r\delta_0 = \delta_1$ and the equations for $\mu 1$ and $\mu 2$ into the above yields

$$Q_0 = \frac{12 E I_R \gamma}{h_2^2 s}\left(1 + \frac{K}{2}\sqrt{\frac{\alpha}{\gamma(2+\alpha)}}\right)\delta_0 \qquad \cdots (8)$$

Axial Forces in the Members at collapse

The failure mechanism at plastic collapse, for an infinite series of bays is as shown in figure B6. The uniformly distributed load at failure is magnitude $w = \frac{16M_pA}{L^2}$, where A represents the arching effect.

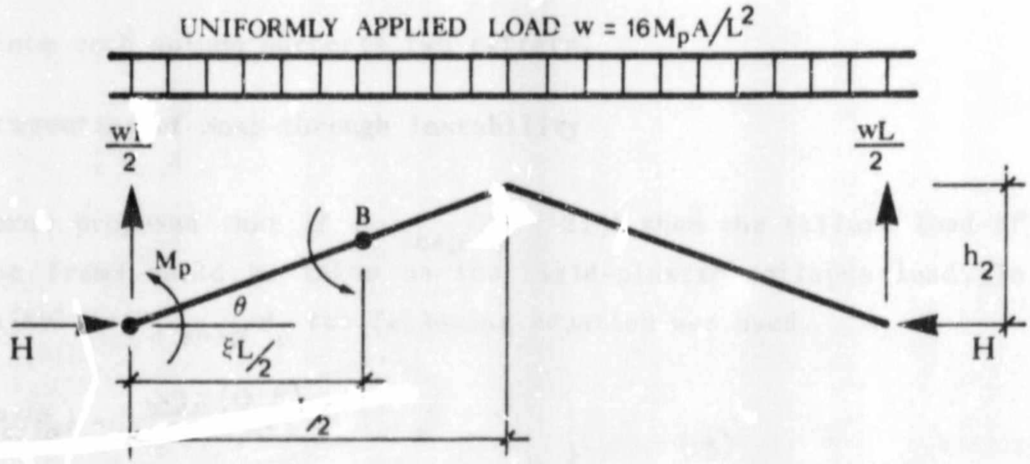


FIGURE B6

Taking moments about point B.

$$M_B + M_p + H.\varepsilon.h_2 + \frac{w}{2}\left(\frac{\varepsilon L}{2}\right)^2 - \frac{wL}{2}.\frac{\varepsilon L}{2} = 0$$

$$\therefore M_B = -M_p - H.\varepsilon.h_2 - \frac{w.L^2.\varepsilon}{4}\left(\frac{\varepsilon}{2} - 1\right)$$

$$= -M_p - H.\varepsilon.h_2 + 4M_p.A\left(\varepsilon - \frac{\varepsilon^2}{2}\right) \quad \ldots\ldots (9)$$

The moment at B reaches a maximum of M_p when

$$\varepsilon = 1 - \frac{H.h_2}{4M_p.A}$$

By substituting the above value of ε into equation (9) one can solve for the horizontal thrust H at eaves level illustrated in figure B6.

$$H = \frac{4M_p.\sqrt{A}(\sqrt{A} - 1)}{h_2} \quad \ldots\ldots (10)$$

The average axial thrust in each rafter at plastic collapse is therefore:

$$R_{RAFTER} = H.\cos\theta + \frac{w.L}{4}.\sin\theta$$

$$\therefore R_R = \frac{4M_p.\sqrt{A}(\sqrt{A} - 1)\cos\theta}{h_2} + \frac{4M_p.A.\sin\theta}{L} \quad \ldots\ldots (11)$$

The axial thrust in each column is:

$$R_c = w.L$$

$$= \frac{16M_p.A}{L} \quad \ldots\ldots (12)$$

Since each column supports two rafters.

Prevention of Snap-through Instability

Horne proposed that if $P_{c\ snap}/P_p$ 2,5A then the failure load of the frame could be taken as the rigid-plastic collapse load. To calculate $P_{c\ snap}/P_p$ the following equation was used.

$$\left.\begin{aligned}(P_c/P_p) &= \frac{0.9\Sigma(Q.\Delta)}{\Sigma(RL\beta^2)}\\ \text{IE: } (P_c/P_p).\Sigma(R.L.\beta^2) &= 0.9\Sigma(Q.\Delta)\end{aligned}\right\}\ldots(13)$$

Q = externally applied force i.e. Q_o in figure B3.
= deflection at the point of application of Q_o
A = axial forces in the members at plastic collapse
L = length of each member
β = rotation of each member as a result of Q

For the above equation:

$$\Sigma(Q.\Delta) = 2.Q_o.\delta_o$$

$$= 2.\frac{12E.I_R.\gamma}{h_2^2.s}\left(1 + \frac{K}{2}\sqrt{\frac{K}{\gamma(2+K)}}\right).\delta_o^2 \quad \ldots(14)$$

The calculation of the left hand side of equation (13) is not as easy.

For the columns:

$$(P_c/P_p).\Sigma(R.L.\beta^2)$$

$$= 2.\frac{P_c}{P_p}.\left(R_c.h.\left(\frac{\delta_o}{h}\right)^2 + R_c.h.\left(\frac{\delta_1}{h}\right)^2 + \cdots\right)$$

$$= 2.\frac{P_c}{P_p}.R_c.h.\left(\frac{\delta_o}{h}\right)^2\left[1 + r^2 + r^4 + \cdots\right]$$

$$= 2.\frac{P_c}{P_p}.R_c.h.\left(\frac{\delta_o}{h}\right)^2\left[\frac{1}{1-r^2}\right]$$

The 2 in the above equation is for columns either side of the snap-through bay. Similarly for the rafters, the left hand side of equation (13) is:

$$(P_c/P_p) . \leq (R.L.\beta^2)$$

$$= \frac{P_c}{P_p} \cdot \left(R_R . S . \frac{\delta_o^2}{h_2^2} . \left(\frac{3+r}{1+r} \right) \right)$$

The left hand side of equation (13) for both columns and rafters is therefore

$$\frac{P_c}{P_p} \cdot \left(R_C \cdot \frac{\delta_o^2}{h} \left(\frac{2}{1-r^2} \right) + R_R . S . \frac{\delta_o^2}{h_2^2} \left(\frac{3+r}{1+r} \right) \right) \qquad \ldots\ldots (15)$$

Substituting equations (11), (12), (14) and (15) into equation (13), an equation for $P_{c\ snap}/P_p$ is obtained.

$$\frac{P_{C\,SNAP}}{P_P} = \frac{10.8 E I_R \gamma \cos\theta}{L M_P \left(A K \tan\theta \left(\frac{1+\Omega}{\Omega} \right) + \left(\frac{2+\Omega}{1+\Omega} \right) \left[\frac{\sqrt{A}(\sqrt{A}-1)}{\tan\theta} + \left(\frac{A}{2} \right) \cdot \tan\theta \right] \right)}$$

$$\Omega = \frac{L}{2} \sqrt{\frac{K}{\gamma(2+K)}}$$

$$\text{IF } E = 210 \times 10^3 \ N/mm^2$$

$$M_P = \frac{1.15 I_R}{d/2} \cdot \left(\frac{\sigma_y}{245} \right) 245$$

This becomes:

$$\frac{P_{C\,SNAP}}{P_P} = \frac{4025 \gamma \cos\theta \left(\frac{245}{\sigma_y} \right) \left(\frac{d}{L} \right)}{\left[\frac{(2+\Omega)}{(1+\Omega)} \left(\frac{\sqrt{A}(\sqrt{A}-1)}{\tan\theta} + \left(\frac{A}{2} \right) . \tan\theta \right) + \left(\frac{1+\Omega}{\Omega} \right) . A K \tan\theta \right]} \qquad \ldots (16)$$

If equation (16) is equated to the proposed limit of $P_{c\ snap}/P_p$ = 2,5A, a slenderness limit to prevent snap-through results

$$(L/d)_{SNAP} = \frac{1610 \gamma \cos\theta (245/\sigma_y)}{A \left[\left(\frac{2+\Omega}{1+\Omega} \right) \left(\frac{\sqrt{A}(\sqrt{A}-1)}{\tan\theta} + \left(\frac{A}{2} \right) . \tan\theta \right) + \left(\frac{1+\Omega}{\Omega} \right) . A K \tan\theta \right]} \qquad \ldots (17)$$

This limit is too difficult to apply directly and the following empirical relation is used.

$$(L/d)_{SNAP} = \frac{25 \tan 2\theta (4 + L/h)(1 + I_C/I_R)}{A(A-1)} \cdot \frac{245}{\sigma_y} \qquad \ldots (18)$$

CALCULATION OF THE FAILURE LOAD OF A FRAME BY SCHOLZ' INTERACTION METHOD **APPENDIX C**

The failure of the frame in figure C.1 will be calculated

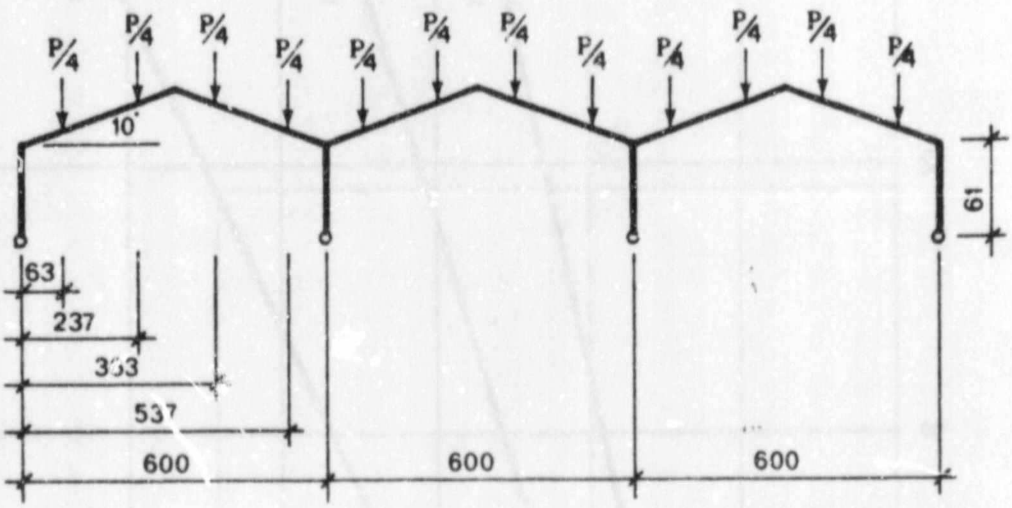


FIGURE C1

From Appendix A the plastic collapse load is 152,2 kg/bay.

From figure C.2 the elastic snap-through buckling load is 146,4 kg/bay.

The ratio of P_c/P_p for the actual frame is therefore

$$= \frac{146,4}{152,2}$$

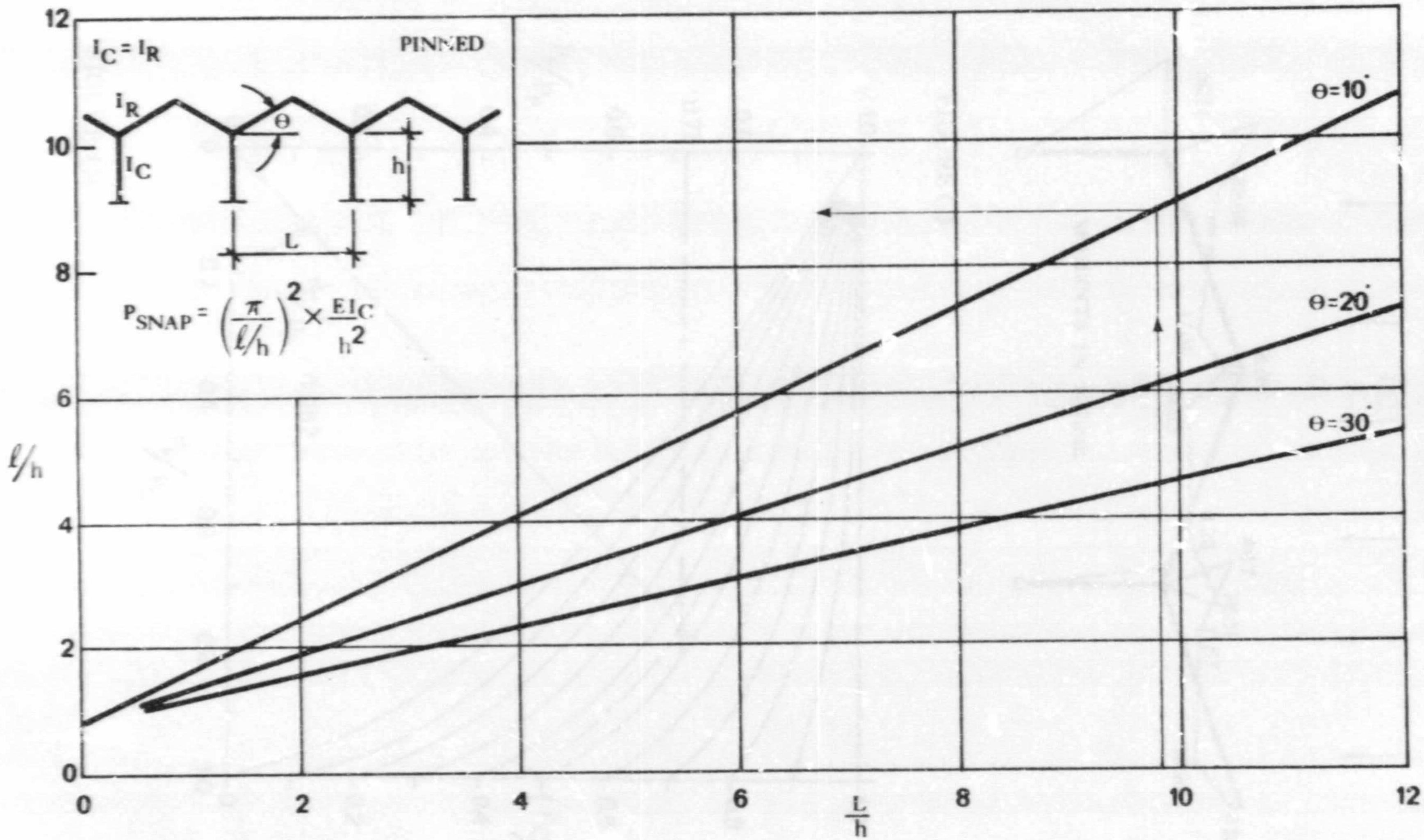
$$= 0,962$$

This parameter must now be calculated for the limiting frame. $(P_c/P_p)\ell$ is found from equation (1)

$$(P_c/P_p)_\ell = (P_c/P_p) \frac{Z.\sigma_y}{M} \qquad (1)$$

M in the above equation, is the largest moment in the structure under the elastic buckling load. An elastic analysis under a load of 146,4 kg/bay yields the moments in figure C.3

The value of M is therefore 59110Nmm. For a 5x20mm section with a yield stress of 334 N/mm², equation (1) gives:



$I_C = I_R$
PINNED
I_R
I_C
Θ
h
L
$P_{SNAP} = \left(\frac{\pi}{\ell/h}\right)^2 \times \frac{EI_C}{h^2}$
$\Theta=10^\circ$
$\Theta=20^\circ$
$\Theta=30^\circ$
ℓ/h
$\frac{L}{h}$
0
2
4
6
8
10
12

FIGURE C2