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Andrew Forbes,[1,a)] Mostafa Youssef,[2] Sachleen Singh,[1] Isaac Nape,[1] and Bora Ung[2,a)]

AFFILIATIONS

[1]School of Physics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg, South Africa
[2]Department of Electrical Engineering, Ecole de Technologie Superieure, Montreal, Quebec H3C 1K3, Canada

[a)]**Authors to whom correspondence should be addressed:** andrew.forbes@wits.ac.za and bora.ung@etsmtl.ca

ABSTRACT

Quantum photonic platforms have proven to be essential in realizing fundamentally secure quantum transfer of information, with commercially ready systems already deployed in municipal and terrestrial links. The drive toward higher bit rates and robustness to eavesdropping and noisy channels has focused attention on moving from the present two-dimensional quantum states of polarization, to harnessing all of light's degrees of freedom for multi-dimensional quantum coding with structured photons. In this Perspective, we outline the present state-of-the-art in achieving this control with spatial modes of light, both as single photon and entangled states, highlight the open challenges that remain, and consider the roadmap that might see its full potential realized.

INTRODUCTION

Exchanging information fast and securely is core to modern society, wrapped in global networks of fiber and terrestrial links. Classical technologies allow this to be done very fast, but the security hinges on computational complexity, typically number theory problems such as the prime factorization and the discrete logarithm problem: easy to state but difficult to solve. The underlying assumption is that your adversary does not possess tools to decrypt the message in real-time, already proven inadequate by the Enigma machine example of World War II and the birth of classical computing. Quantum computing poses a perceived risk in that such problems can, in principle, be solved in "polynomial" time—fast enough to become a potential threat. Although this can be circumvented by increasing the key length and adapting asymmetric encryption schemes to be resilient to phase estimation transformations and periodic findings (the key techniques used in Shor's Algorithm), these are generally considered temporary solutions, which fail to address the problem that computational encryption is inherently insecure. In contrast, quantum cryptography allows the establishment of a key between parties in a manner that is fundamentally secure,[1] exploiting the randomness, superpositions, and the inability to perfectly clone quantum states for securing information, thus fueling the nascent quantum global network.[2] In such a network, quantum information can be physically sent across a link by exchanging keys from one peer to another through quantum key distribution (QKD),[3] splitting the key among many nodes in the form of quantum secret sharing[4] and quantum secure direct communication without a key.[5] It is also possible to transfer quantum information without physically sending it using protocols such as remote state preparation[6,7] and teleportation.[8] This list is not exhaustive, and for completeness, we point out that many other cryptography approaches exist.[9] To date, most of these protocols have been developed using two-dimensional (2D) quantum states, qubits, with polarization qubits having seen tremendous progress over the past decades.[10]

A recent drive is to move beyond two-dimensional (2D) quantum states to higher-dimensional quantum states: from qubits to qudits with dimension, $d > 2$. The benefits to quantum cryptography include: (1) a higher information capacity[11] of $I(d) = \log_2(d)$ bits per photon of information, so that less photons need to be sent as qudits in order to transmit the same information as a qubit channel. This is important because in entanglement-based protocols, the minimum coincidence time window of Δt sets the maximum rate at which one can generate secure keys, i.e., one cannot simply send photons faster for ever higher key rates. (2) An upper bound to the cloning Fidelity of $F = \frac{1}{2} + \frac{1}{1+d}$. For qubits, $F = \frac{5}{6} \approx 83\%$, whereas for a very high-dimensional state, we find that $F \to 50\%$. Similarly, qudit based QKD schemes allow larger thresholds (as high as 50% for qudits, compared to 13% for qubits[12]) of quantum bit error rate (QBER), owing to either environmental noise or eavesdropping attacks. This means that it is far more difficult for an eavesdropper to hack a high-dimensional state than it would be to hack a qubit state. (3) Larger violations of Bell-type inequalities, limited to $S \leq 2\sqrt{2}$, i.e., the Tsirelson bound for qubits ($d = 2$) but increasing with dimensions and reaching up to $S \leq 3\frac{1}{5}$, crucial in certain QKD protocols to indicate the presence of an eavesdropper. (4) Better resilience to noise, with the cutoff purity at which a

quantum state becomes separable due to isotropic noise scaling as $p_c = \frac{1}{1+d}$, while the higher dimensionality of the Hilbert space allows many pathways to "find" the embedded entanglement.[13]

Here, structured light[14] has come to the fore, where the many degrees of freedom (DoFs)[15] of photons can be exploited for multi-degree-of-freedom and high-dimensional quantum states,[16,17] illustrated artistically in Fig. 1. To unpack the notion, consider a biphoton polarization qubit written in the following form:

$$|\Psi\rangle = c_1|H\rangle_1|H\rangle_2 + c_2|V\rangle_1|H\rangle_2 + c_3|H\rangle_1|V\rangle_2 + c_4|V\rangle_1|V\rangle_2, \quad (1)$$

where $H(V)$ refers to the horizontal (vertical) basis states and the subscripts refer to photons 1 and 2. The four terms are derived from the tensor product of the $d = 2$ states, $\{|H\rangle, |V\rangle\}$, of each individual photon. The degree of entanglement can be deduced from the Concurrence given by $C = 2|c_1c_4 - c_2c_3|$, assuming that the coefficients are normalized such that $\sum_i |c_i|^2 = 1$. We see that depending on the weightings, the biphoton state's entanglement could vary from 0 (no entanglement) to 1 (maximally entangled). One example of the latter is given by

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2\right). \quad (2)$$

Now let us alter the basis to orbital angular momentum (OAM),[18] a property of light that is conserved down to single photons.[19] This alphabet is very large, and we can identify the dimensions by observing that the entangled biphoton state can be written as

$$|\psi\rangle = \underbrace{|\ell_1\rangle|\ell_p - \ell_1\rangle}_{\text{dimension 1}} + \underbrace{|\ell_2\rangle|\ell_p - \ell_2\rangle}_{\text{dimension 2}} + \underbrace{\overbrace{|\ell_3\rangle}^{\text{photon 1}}\ \overbrace{|\ell_p - \ell_3\rangle}^{\text{photon 2}}}_{\text{dimension 3}} + \cdots. \quad (3)$$

Note that each photon's state lives in a Hilbert space of d dimension, while the biphoton entangled state lives in a Hilbert space of d^2 dimensions.

Here, the basis states are $|\ell\rangle \equiv \exp(i\ell\phi)$, where ϕ is the azimuthal angle, referring to photons with a wavefunction characterized by a twirling azimuthal phase, and we assume that the entanglement is created by a spontaneous parametric downconversion process (SPDC) excited by a pump source with OAM state $|\ell_p\rangle$. If this pump light has no OAM ($\ell_p = 0$), then we find the vanilla version of the state as $|\psi\rangle = \sum_{\ell=-\infty}^{\infty} c_\ell|\ell\rangle| - \ell\rangle$. We see that the large alphabet of OAM allows access to high-dimensional Hilbert spaces spanned by spatially structured modes and can be extended past just two photons to multi-photon entanglement too.[20]

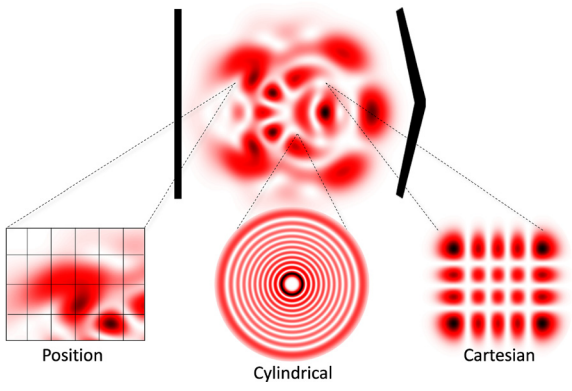


FIG. 1. The spatial degrees of freedom of light such as position and spatial mode can be used to construct high-dimensional quantum structured states.

The question is how to use these spatially structured photons, entangled or not, for cryptography? As this is a short perspective, we will answer this question through the lens of two common protocols, quantum key distribution, and quantum secret sharing.

QUANTUM KEY DISTRIBUTION (QKD)

In QKD, two parties exchange information across a quantum channel. In a standard version of the protocol, Alice chooses a state to prepare from a selection of two bases, often termed the computational and mutually unbiased basis (MUBs), and sends it to Bob. Within a basis, the states are orthogonal, while between them, they are mutually unbiased, so that a projection between bases has the same probability of an outcome regardless of the choice of state in each. Bob then chooses a basis in which to measure. Crucially, they both make random selections for their choices. Alice and Bob then use a classical channel to reconcile their prepared and measured states by revealing their choice of basis, discarding results that do not correlate, which, if truly random, would mean about 50% of the time. What remains is a secret key unique to both Alice and Bob. To check the security of their transmission, both Alice and Bob publicly announce a subset of their secret keys accepting it as secure if the error (perhaps due to an eavesdropper) is below the QBER threshold. In high dimensions, the procedure is the same, but one may use the computational basis comprised of d orthogonal states and at least one MUB from the set of all $d+1$ MUBs (known to exist when d is an integer power of a prime number[21]) that must satisfy $|\langle\Psi_j|\Phi_k\rangle|^2 = 1/d$ for $(j, k = 1, \ldots, d)$, where $|\Psi_j\rangle$ and $|\Phi_k\rangle$ are the computational and MUBs basis sets, respectively. For example, while selecting one MUB will suffice, using $d+1$ ensures more robust and secure QKD. However, in cases where not all $d+1$ MUBs can be found (e.g., $d = 6$), at least three MUBs will exist[21] and can, therefore, be used. The computational $|\Psi_j\rangle$ basis elements are immediate via the choice of structured mode, e.g., $|\Psi_\ell\rangle = |\ell\rangle$ for OAM modes. The basis $|\Phi_k\rangle$ that is mutually unbiased to $|\Psi_j\rangle$ can easily be constructed from

$$|\Phi_k\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1} \exp(i2\pi jk/d)|\Psi_j\rangle. \quad (4)$$

An example of these MUB sets is shown for OAM in $d = 3$ in Fig. 2(a). In Fig. 2(b), the probabilities computed from the inner products between the states confirm that each basis is orthogonal but mutually unbiased with respect to every other basis—a key trait for selecting MUBs in QKD.

To illustrate how the extra dimensions from the structure of light can add value, consider the use of a hybrid state, i.e., a state that stems from two DoFs (polarization and OAM), in a prepare-and-measure QKD scheme. Suppose Alice and Bob wish to share a secure key. For a given OAM subspace, there exist four states in the computational basis,

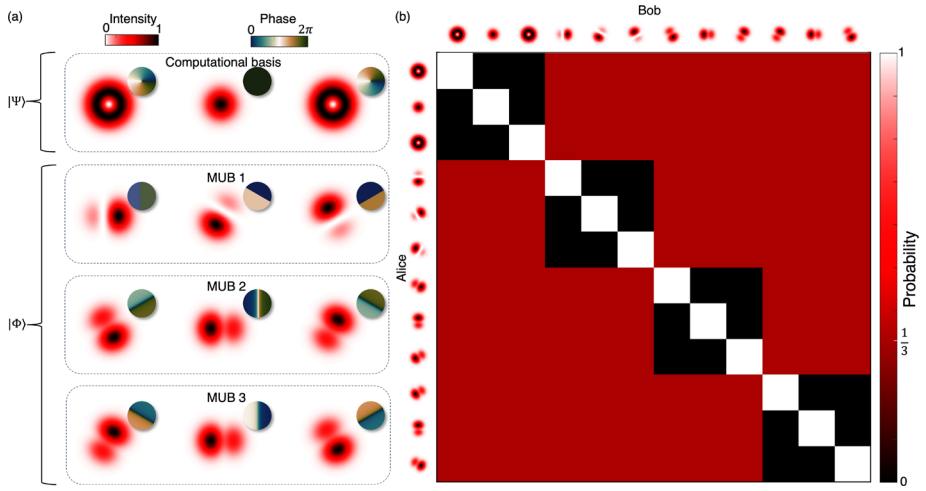


FIG. 2. (a) Exemplary qutrit $d = 3$ dimensional secure encoding basis for QKD. Here, the computational basis, $\{|\Psi\rangle\}$, is made up of three orthogonal spatial modes. From the computational basis, three MUBs can be constructed from superpositions of the computational basis where each basis ($\{|\Phi\rangle\}$) is mutually orthogonal. (b) Detection probabilities for the four basis sets, showing that each set is mutually orthogonal (diagonal) but mutually unbiased (off-diagonal) with respect to the other basis states. In QKD, the computational basis is often selected along with one of the MUBs to ensure security over the quantum channel.

$$
\begin{aligned}
|\Psi_1\rangle = |00\rangle &= \frac{1}{\sqrt{2}}(|R\rangle|\ell\rangle + |L\rangle|-\ell\rangle),\\
|\Psi_2\rangle = |01\rangle &= \frac{1}{\sqrt{2}}(|R\rangle|\ell\rangle - |L\rangle|-\ell\rangle),\\
|\Psi_3\rangle = |10\rangle &= \frac{1}{\sqrt{2}}(|L\rangle|\ell\rangle + |R\rangle|-\ell\rangle),\\
|\Psi_4\rangle = |11\rangle &= \frac{1}{\sqrt{2}}(|L\rangle|\ell\rangle - |R\rangle|-\ell\rangle),
\end{aligned}
\tag{5}
$$

and its mutually unbiased counterpart

$$
\begin{aligned}
|\Phi_1\rangle = |00\rangle &= |D\rangle|-\ell\rangle,\\
|\Phi_2\rangle = |01\rangle &= |D\rangle|\ell\rangle,\\
|\Phi_3\rangle = |10\rangle &= |A\rangle|-\ell\rangle,\\
|\Phi_4\rangle = |11\rangle &= |A\rangle|\ell\rangle,
\end{aligned}
\tag{6}
$$

for creating a key, shown in Fig. 3. Here, $R, L, D,$ and A refer to the right-circular, left-circular, diagonal, and anti-diagonal polarization states, respectively. As before, Alice randomly prepares her photon in a given state, while Bob randomly measures the photon in one of the bases. At the end of the transmission, Alice and Bob reconcile the prepare and measure bases and discard measurements in complementary bases. The challenge is to prepare and measure such states

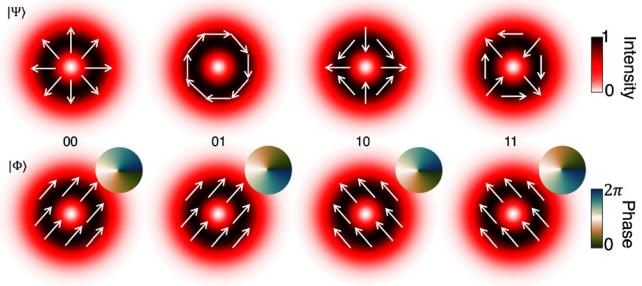


FIG. 3. The computational (top) and mutually unbiased basis (bottom) for hybrid OAM-polarization states. The $d = 4$ space can be used to encoded information as binary strings 00, 01, 10, and 11.

deterministically. Without deterministic detection, the final sifting will result in very low key rates with the benefit of the dimensionality completely erased.

Quantum key distribution with structured photons was first shown with OAM up to $d = 3$ using an entanglement based protocol[22] and then up to $d = 5$ with an OAM prepare-and-measure protocol,[23] later extended to $d = 7$ for a secret key rate of 2.05 bits per sifted photon,[24] twice that of qubit QKD. In the latter experiment, the measurement was performed deterministically with mode sorters, avoiding the filtering problem. The hybrid polarization-OAM entangled states shown in Eqs. (5) and (6) have been exploited for $d = 4$ QKD up to subspaces of $\ell = \pm 10$, with a key rate of 1.52 and a Fidelity of $> 97\%$, enabled by using a deterministic detection scheme and used to encrypt and decrypt an image.[25] Spatial modes give access not only to higher dimensions but also to properties of the modes themselves. For instance, Bessel beams are known to be non-diffracting and self-healing after obstacles, which, in the quantum realm, could be exploited for QKD through obstacles in free-space using Bessel mode photons.[26] This approach has been extended to $d = 21$ with excellent performance.[27] By coupling different degrees of freedom, QKD using polarization, radial, and OAM encoding in $d = 8$ has realized a secret key rate of 2.15 bits per sifted photon.[28] Beyond the standard QKD protocols, the ability to digitally control spatial light modulators for modal detection has seen a myriad of protocols tested with simple test beds,[29] including a new Round-Robin Differential-Phase-Shift protocol up to $d = 64$ with OAM.[30]

While we have seen profound developments in the generation and manipulation of structured light, its reliable transmission remains a challenge. In free-space, atmospheric turbulence mangles the spatial structure, degrading the quality of entanglement upon propagation.[31] In principle, the modal coupling does not change the overall entanglement as the process can be considered unitary, but cryptography becomes adversely affected due to crosstalk across the bases. This has limited quantum experiments using spatial modes to $d = 2$ qubits, initially over a 210 m free-space link with prepare-and-measure OAM[32] followed by entanglement distribution over an intra-city link of 3 km with OAM entangled states,[33] later a $d = 4$ QKD protocol over a 300 m intra-city free-space link using hybrid states.[34] A summary of what has been achieved is shown in Fig. 4.

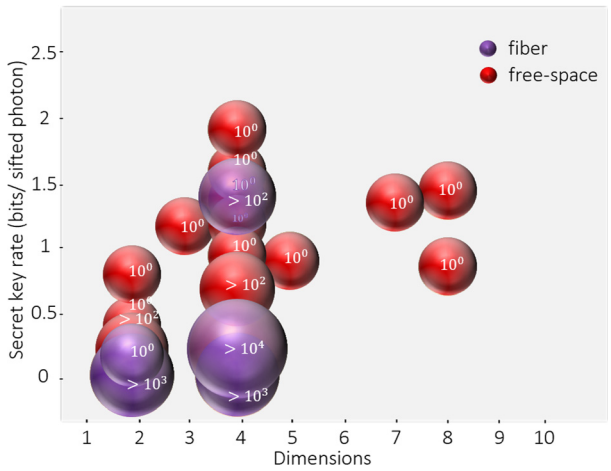


FIG. 4. Advances in free-space and fiber QKD, with the bubble size and inset value indicating the distance (m) reached. Long distances have only been reached with $d = 2$ and hybrid states, with $d > 6$ only demonstrated in the laboratory at meter scale distances.

Progress with high-dimensional spatial states in optical fiber has fared slightly better of late,[35] often requiring custom fiber to prevent state decoherence and intermodal dispersion. Qutrit OAM entanglement has been transported over 1 km of few mode fiber[36] with good state fidelity but without QKD, while prepare-and-measure $d = 4$ QKD has been demonstrated over 1.2 km in air core fiber with high fidelity (>90%) and 37.85 kbit/s secret key-rate (SKR).[37] Hybrid states have reached 25 km in ring core fibers,[38] with entanglement distribution reaching fidelities up to 98% distributed over 5 m of an air core fiber.[39] These high fidelities of transporting the structured light through optical fibers allow exchanging information, in a prepare and measure protocol, with SKRs of 0.15 and 0.515 bits per sifted photon in 60 m and 4 km of a ring core fiber, respectively.[38,40] Some more examples are shown graphically in Fig. 4. All of these require custom fiber, where applications choose the required fiber giving the importance of the fiber index design (e.g., graded index), core shape (e.g., ring core), and cutoff frequency (e.g., number of fiber modes). However, present deployed fiber networks make use of conventional single mode fiber (SMF), which, by definition, does not support any spatial modes beyond a simple $|\ell = 0\rangle$ state along two orthogonal polarizations. Recently, it was shown that even such a medium can be used for quantum transport, but multi-dimensional rather than high-dimensional. A hybrid entangled state, with one photon expressed in the polarization basis and the other in the OAM basis, gives rise to $N \times (d = 2)$, i.e., N different qubit options. Crucially, the polarization encoded photon that is sent to Bob is in a Gaussian spatial mode, so can pass through SMF, while the OAM encoded photon remains in the laboratory of Alice. The OAM state of Alice's photon is prepared by spin–orbit optics, selecting superpositions of ± 1, ± 2, and so on, each by its own unique optic. This subspace preparation allows many different qubits to be prepared at will and can be used to distribute qubits across the network to many parties, a quantum version of multiplexing. To demonstrate the approach, a hybrid polarization-OAM state was used to transport multiple two-dimensional states over 250 m of single mode fiber with high fidelity across multiple OAM subspaces, offering an alternative approach to the transmission of high-dimensional spatial modes.[41]

QUANTUM SECRET SHARING (QSS)

Modern networks are typically not peer-to-peer and require a network of nodes to be connected, invalidating many QKD protocols. Indeed, the expeditious increase in collaborative tasks has resulted in growing interest in multi-party communication protocols such as QSS. In QSS, no party in the node should have the full key so that some number of parties must collaborate to produce the key, essential in applications such as large money transfers and missile launch codes. Instead, a secret is divided into unique shares sent to N nodes (participants) and then securely reconstructed by the collaborating parties. In an entanglement based QSS protocol, the N parties share a N-photon entangled state, with each party performing a measurement from a set of MUBs. If any of the parties uses a different measurement basis than Alice, then their measurement results will be uncorrelated with Alice's result, and the round is discarded. If their measurement bases are correlated, then the parties can collaborate by sharing their measurement bases and measurement outcomes to infer the secret shared by Alice. In this way, Alice can use her key, constructed from her secret measurement results, to encrypt a message, while the other parties can use the inferred key to decipher the message. In this protocol, at no point, does Alice reveal her measurement results. Unfortunately, preparing multi-photon high-dimensional states is very challenging, and no demonstration of entanglement based high-dimensional QSS has been made thus far.

A prepare-and-measure equivalent with single photons sees Alice prepares a photon in an initial state and sequentially passes it from party to party, each acting on it with a phase operator chosen from a basis set. The last participant then performs a measurement on the photon. If the bases are correlated, then the collaborating parties can share a secret key. To unpack this for high-dimensional structured photons, imagine that Alice prepares an initial single photon state from a set of MUBs, say $|\Phi_1\rangle$ following Eq. (4) and then modulates it with operators $X_d = \sum \omega^{\ell} |\ell\rangle\langle\ell|$ and $Y_d = \sum \omega^{\ell^2} |\ell\rangle\langle\ell|$, where $\omega = \exp(i2\pi/d)$. Each is applied x_1 and y_1 times, respectively, with the values selected randomly in the range $x_1, y_1 \in [0, d-1]$ for a final operation of $X_d^{x_1} Y_d^{y_1}$. The X operator is analogous to a change of state within a basis, while the Y operator corresponds to a change of basis. The photon is then sent to the next participant (node) who, upon receiving the single photon, randomly chooses x_2 and y_2, thus applying the operation $X_d^{x_2} Y_d^{y_2}$ and so on until the final N participant (node), who, after applying the operation, passes the photon back to Alice, the distributor. Alice asks that all parties broadcast their choice of y_n, keeping x_n a secret. Alice can ensure a deterministic outcome (with 100% efficiency) by applying a local unitary Y_d^J, where $J = \sum_{n=1}^{N} y_n \bmod d$ for a final measurement outcome of a. Alice now creates a secret key $x_1^{\text{secret}} = (a - x_1) \mod d$ by resetting her value of x_1. If the participants collaborate, they can reconstruct Alice's secret value $x_1^{\text{secret}} = \sum_{n=2}^{N} x_n \mod d$.

The challenge in QSS is how to translate the protocols to high-dimensions and find the necessary phase operator for spatial modes of light. As an example, consider the $d = 3$ protocol using the hybrid state $|R, \ell = 0\rangle + |R, \ell = 1\rangle + |L, \ell = -1\rangle$. How to create this state and transform it to $|R, \ell = 0\rangle + \exp(i\alpha)|R, \ell = 1\rangle + \exp(-i\alpha)\exp(i\theta)|L, \ell = -1\rangle$ after each phase operation? The experimental set-up to do so is illustrated in Fig. 5 for $N = 3$ participants, where a series of Dove prisms and half-waveplates perform the necessary unitaries,[42] while

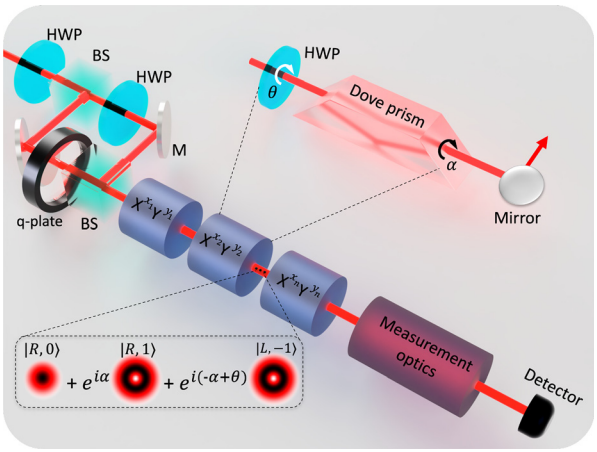


FIG. 5. Experimental realization of quantum secret sharing with a three dimensional hybrid state, following the approach outlined in Ref. 42. The challenge is the translation of the protocol to physical laboratory elements for execution, e.g., the phase operations (X and Y) require Dove prisms and half-waveplates (HWP).

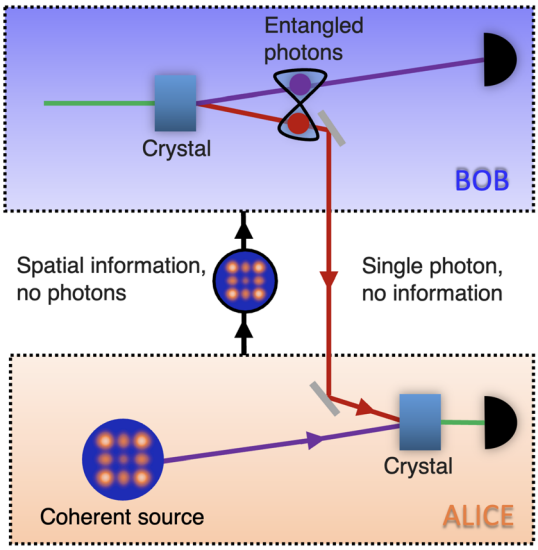
the preparation and detection requires a combination of spatial and polarization control using geometric phase and spatial light modulators. The translation of the protocol to the laboratory is a non-trivial task. Nevertheless, networks have been extended to $d = 3$ and $N = 3$ nodes,[43] pushed up to $N = 10$ nodes with hybrid states[42] and a state-of-the-art of $d = 11$ and $N = 10$ nodes for a secret key rate of 2.89 bits per sifted photon.[44]

DISCUSSION AND CONCLUSION

Despite the advances, several significant challenges remain that if addressed would unlock exciting opportunities. For instance, deterministic detection in d dimensions requires d detectors and the ability to sort both orthogonal and MUBs, presently not possible except in very special cases. To circumvent this, one could consider not encoding information directly into the DoFs of the quantum state but rather the topology that emerges as a consequence of entanglement between the DoFs, e.g., quantum skyrmions.[56] Using topological invariants may make quantum information more robust to noise but would require significant work to advance the toolkit, e.g., to encode/decode information by topology. Furthermore, present quantum cryptography

An Aside: Sending encrypted images

Is it possible to encrypt an image and securely transport it to some distant party? Standard quantum protocols such as QKD and QSS assume that the image is sent classically, with the encryption and decryption by a shared quantum key. Classical image encryption based on computational imaging is decades old,[45] but only very recently have quantum versions surfaced. Prepare-and-measure schemes have used OAM and speckle as dual keys to encrypt an image, which is subsequently sent to Bob. The OAM makes the outcome helicity specific, forming different images for different OAM. The scheme is a combination of OAM holography with ghost imaging for image encryption.[46] High-dimensional entanglement based holography[47] uses OAM specific holograms that can only be decoded by correlated measurements by Alice and Bob, revealing images out of noise. The spatial and temporal correlations embedded in quantum ghost imaging also allows one to hide images in noise.[48] Here, without the correlations of both photons, only noise is detected, while with the joint measurements, the image is deciphered. This builds on the quantum ghost imaging property that neither photon has information of the object, but together an image of the object can be inferred.



In the aforementioned schemes, the photons with the information travel from Alice to Bob. Could they be teleported, so that parties can share information that is not physically sent across a connecting link? Presently, the answer is no. Teleportation of spatial information has been limited to $d = 2$ using OAM[49–51] and $d = 3$ using the path degree of freedom;[52] in other words, we can teleport images with up to three pixels! Attempting to do more without ancillary photons (you need more photons per dimension for teleportation) results in images with low contrast, quickly becoming background noise as the number of pixels is increased.[53] Very recently, a new approach to securely transporting images without actually sending the information across the channel has been demonstrated,[54,55] which we term teleportation-inspired quantum transport. As shown in the adjacent illustration, Alice has sensitive spatial information she wishes to share (perhaps a fingerprint), which she encodes into a bright laser beam (Coherent source). To transport this information to Bob, she requests that Bob send her one photon from an entangled pair, which Bob creates using spontaneous parameteric downconversion (SPDC). Alice has in her possession a nonlinear crystal for a detector, which she uses to overlap both her information and Bob's photon. The crystal performs an operation analogous to the inverse of SPDC: two photons are upconverted to one rather than one downconverted to two. A result of this operation is that Bob will have Alice's image information transported to him when they measure their outcomes in coincidence. This protocol overcomes the ancillary photon challenge by using a nonlinear detector, allowing transport of a $d = 15$ dimensional state with only two entangled photons as a resource.[54] The scheme is neither teleportation nor remote state preparation in that Alice *could know* what she wishes to send but does not *need to know*. It would become teleportation if Alice could use a single photon for the image information. Crucially, the single photon from Bob to Alice carries no information, while the image information from Alice to Bob flows "photon-free" in the opposite direction, making the scheme fundamentally secure. Significant work will be needed to advance this to true teleportation of images.

approaches all make use of linear optics in the measurement stage, which has known limitations in the context of high-dimensional states.[57] Nonlinear optics has emerged as an exciting alternative for spatially structured light[58] and has been proposed to enhance high-dimensional protocols,[59] with the potential for deterministic detection in any dimension. The challenge is to engineer new materials (perhaps as artificial atoms in resonant metasurfaces) with many orders of magnitude enhancement in nonlinear efficiencies. Measurement efficiencies also include the inability to perform full state reconstruction without a quantum state tomography, a very time intensive task, although witnesses[60] and fast quantitative measures[61] are steadily becoming available.

The distance reached with high-dimensional spatial modes has been limited due to loss and modal coupling that degrades the subspaces in which the information is encoded. Extending the reach will require quantum error correction and active compensation for disturbances, perhaps using adaptive optics in the case of phase errors, which has been shown theoretically to yield benefit but not yet experimentally implemented.[62] A mixture of classically entangled light with quantum light has shown promise to allow error analysis by probing with the equivalent classical state to allow real-time quantum error correction,[63] or to use the high-dimensional states themselves as problems for unscrambling noisy states.[64] Alternatively, it may be possible to exploit the symmetry of the state for entanglement distillation, an approach that has shown to work[65] but not yet used in cryptography through noisy channels.

One important challenge for the fiber-medium distribution of high-dimensional quantum states, which is absent in free-space, pertains to intermodal dispersion that results in the different group velocities of the propagating structured photonic modes. The ensuing time delays at the receiver between the different hybrid states have a detrimental effect on the achievable secret key rates, lead to decoherence of superposition states, and, therefore, must be compensated, e.g., via an unbalanced Mach–Zehnder interferometer.[36] Generally, in the design of FMF, there is a trade-off between achieving low intermodal dispersion and large intermodal separation (to prevent mode mixing and state decoherence with near-degenerate modes). In the last decade,[66,67] ring-core fibers have been widely used for transmitting OAM modes up to a few km distances owing to their strong intermodal separation, albeit with significant intermodal dispersion and increased losses (>1 dB/km). In circular symmetric optical fibers, OAM structured photons are fundamentally described as hybrid states with a given (R/L) polarization state. Therefore, the ability of a fiber to preserve the purity of structured photons depends not only on its modal properties but also on its polarization dispersion. Recent advances in fiber manufacturing methods have demonstrated promising results in the development of ultralow-perturbed-polarization hollow-core fibers[68] and circular polarization-maintaining helical fibers.[69] However, these recent reports are mostly concerned with fundamental mode fiber guiding that only supports two-dimensional quantum states. Again, more control over structured photons guiding in fibers can be gained through the development of new optical materials (e.g., metamaterials) that would, for example, enable to engineer radially anisotropic fibers.[70,71] Moreover, the investigation of novel broadband fibers offers a promising route to enabling the co-propagation of both classical and (structured) quantum channels in the same fiber with low crosstalk.[72,73] The latter would accelerate real-world deployment of quantum cryptographic applications. For example, classical signaling may be transmitted over the telecom C-band (e.g., 1550 nm wavelength), while the quantum channel would benefit from operating at the zero chromatic dispersion point (near 1310 nm wavelength in fused silica fibers).[74] Hence, there are several challenges for the quantum communication with structured photons in optical fiber. Going forward, new advances in the field would benefit from the development of novel low intermodal/polarization dispersion and ultralow loss fibers.

The opportunity is to exploit the full benefits of the spatial modes beyond just their dimensionality. Bessel modes as entangled states[75] have shown promise to control the OAM spectrum,[76] realizing non-diffracting[77] and self-healing states,[78] inspiring studies into other spatial modes including Ince–Gaussian modes,[79] Hermite–Gaussian modes,[80] and radial Laguerre–Gaussian modes.[81] Rather than the free-space and fiber modes, why not to use the eigenmodes of complex media to make the medium appear distortion free[82] for error-free communication? This is only recently gaining traction in the quantum realm[83] but yet to be deployed for cryptography. Another exciting and fast developing area is the integration of multi-plane light converters as high-dimensional operational gates[84] and detectors,[85] bringing functionality from classical communication into quantum optics.

In conclusion, we have offered a short perspective on modern quantum cryptography with spatially structured photons, highlighting the exciting prospects that this direction brings but with many open challenges too. The promise is not only advances in fundamental quantum mechanics in high-dimensions but also high-performance quantum technological applications from cryptography to beyond.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Andrew Forbes: Conceptualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Mostafa Youssef:** Writing – original draft (equal); Writing – review & editing (equal). **Sachleen Singh:** Writing – original draft (equal); Writing – review & editing (equal). **Isaac Nape:** Writing – original draft (equal); Writing – review & editing (equal). **Bora Ung:** Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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