

the spin-curvature coupling 'reacts back' on the Lorentz stress only while the stress-torsion couplings 'react back' on the tetrad stress. Equations (6.68), (6.69) and (6.70) give an account of the detailed energy-momentum balance in the system. Note that the inhomogeneous term which is responsible for the non-conservation of the total stress appears in the tetrad stress conservation law.

6.8 Conservation of Spin-Stress and the Symmetry of the Stress Tensors

Using the matter equation of motion in the identity (6.44)^c we have, by definition of the matter spin-stress,

$$\overset{m}{S}{}^{\rho}_{ab;\rho} = - \overset{m}{\Pi}{}^{\rho}_{c} s_{ab}{}^{cd} e_{\rho}{}^d. \quad (6.71)$$

Also, by definition of the matter stress and the Lorentz generator this is

$$\overset{m}{S}{}^{\rho}_{ab;\rho} = \frac{i}{2} (\overset{m}{T}_{ba} - \overset{m}{T}_{ab}). \quad (6.72)$$

This is a result we obtained earlier (Chap. 4) in Special Relativistic form. Similarly, using the definition of the Lorentz stress in the identity (6.49)^c we find

$$\frac{i}{2} (\overset{w}{T}_{ab} - \overset{w}{T}_{ba}) = - \frac{i}{2} \Omega^{\mu\rho}_{cd} C_{ab}{}^{cd} e_{\rho}{}^d R_{\mu\rho}{}^{ef} \quad (6.73)$$

and we see that the inhomogeneous term in the total spin-stress conservation law (6.37) is also responsible for the antisymmetric part of the Lorentz stress tensor. In other words, if the total spin-stress is conserved then the Lorentz stress tensor will be symmetric. Finally, from the identity (6.53)^d we have

$$\frac{i}{2} (\overset{e}{T}_{ab} - \overset{e}{T}_{ba}) = - \frac{i}{2} \Sigma^{\mu\rho}_e s_{ab}{}^{cd} s_{\rho}{}^d{}_{\mu} \quad (6.74)$$

Since the tetrad stress does not occur in the usual theory of Relativity (next chapter) the significance of it being symmetric or not is most unclear. We remark, however, that for most of the simple theories of interest the term on the r.h.s. of (6.74) vanishes identically so that the tetrad stress is symmetric. It is also possible that the symmetry of this tensor may be a restriction which is to be imposed on the theory. The inhomogeneous term in (6.74) appears again when we calculate the conservation of the tetrad spin-stress:

$$s^{\mu}_{ab;\mu} = \frac{1}{2}(T_{ab} - T_{ba}) + \frac{1}{2} \epsilon \Sigma^{\nu\mu}_c S^c_{ab} s^d_{\mu\nu} \quad (6.75)$$

where T_{ab} is the total stress tensor.

6.9 Summary

We collect together all the relevant results from this chapter.

Total Stress: $T_a^{\rho} = \epsilon e_a^{\rho} L + \epsilon \Pi_a^{\rho}$

Total Spin-Stress: $s^{\rho}_{ab} = \epsilon \Sigma^{\rho}_{ab}$

Equations of motion:

$$(\epsilon \psi_a^{\mu})_{;\mu} - \phi_a = 0$$

$$(\epsilon \Sigma_a^{\rho\mu})_{;\mu} - \frac{1}{2} \epsilon s_{\mu\nu}^{\rho} \Sigma_a^{\nu\mu} = T_a^{\rho}$$

$$(\epsilon \Omega^{\rho\mu}_{ab})_{;\mu} - \frac{1}{2} \epsilon s_{\mu\nu}^{\rho} \Omega^{\nu\mu}_{ab} = s^{\rho}_{ab}$$

Total Conservation Laws

$$T_a^{\rho}{}_{;\rho} = \frac{1}{2} \Sigma^{\mu\rho}_b R_{\mu\rho}{}^b{}_a$$

$$s^{\rho}_{ab;\rho} = \frac{1}{2} \Omega^{\mu\rho}_{cd} C^{\rho}_{ef}{}^{cd}_{ab} R_{\mu\rho}{}^{ef}$$

Minimally coupled Lagrangian:

$$L_{\omega} = L_m + L_e + L$$

then
$$T_a^{\rho} = T_a^{\rho} + T_a^{\rho} + T_a^{\rho}$$

where
$$T_a^{\rho} = e (L_m \delta_{\lambda}^{\rho} - \psi_{\alpha}^{\rho} \psi^{\alpha}_{;\lambda}) e_a^{\lambda}$$

$$T_a^{\rho} = e (L_{\omega} \delta_{\lambda}^{\rho} - \Omega^{\mu\rho}_{ab} R_{\mu\lambda}^{ab}) e_a^{\lambda}$$

$$T_a^{\rho} = e (L_e \delta_{\lambda}^{\rho} - \Sigma_b^{\mu\rho} S_{\lambda\mu}^b) e_a^{\lambda}$$

and
$$S_{ab}^{\mu} = S_{ab}^{\mu} + S_{ab}^{\mu} + S_{ab}^{\mu}$$

where
$$S_{ab}^{\mu} = e \psi_{\alpha}^{\mu} S_{ab\beta}^{\alpha} \psi^{\beta}$$

$$S_{ab}^{\mu} = e \Sigma_c^{\rho\mu} S_{ab\,d}^c e_{\rho}^d$$

$$S_{ab}^{\mu} = 0$$

Detailed Conservation:

$$T_{\lambda;\rho}^{\rho} = T_{\sigma}^{\mu} S_{\lambda\mu}^{\sigma} + S_{ab}^{\mu} R_{\mu\lambda}^{ab}$$

$$T_{\lambda;\rho}^{\rho} = T_{\sigma}^{\mu} S_{\lambda\mu}^{\sigma} - S_{ab}^{\mu} R_{\mu\lambda}^{ab}$$

$$T_{\lambda;\rho}^{\rho} = \frac{1}{2} \Sigma_b^{\mu\rho} R_{\mu\rho}^b e_a^a - \left(T_{\sigma}^{\mu} + T_{\sigma}^{\mu} \right) S_{\lambda\mu}^{\sigma} + S_{bc}^{\mu} R_{\mu\lambda}^{bc}$$

Symmetry:

$$(T_{ab} - T_{ba}) = -S_{ab;\rho}^{\rho}$$

$$(T_{ab} - T_{ba}) = -\frac{1}{2} \Omega^{\mu\rho}_{cd} C_{ab}^{cd} R_{\mu\rho}^{ef}$$

$$(T_{ab} - T_{ba}) = -\frac{1}{2} \Sigma_c^{\mu\rho} S_{ab\,d}^c S_{\rho\mu}^d$$

APPENDIX 6A

The Derivation of the Invariance Identities

We will need the following results: from the co-ordinate transformation

$$\bar{x}^\mu = x^\mu + \xi^\mu$$

we get
$$K^\mu_\nu = \frac{\partial \bar{x}^\mu}{\partial x^\nu} = \delta^\mu_\nu + \xi^\mu_{,\nu}$$

and
$$K^\mu_{\nu\rho} = \frac{\partial^2 \bar{x}^\mu}{\partial x^\nu \partial x^\rho} = \frac{1}{2}(\xi^\mu_{,\nu\rho} + \xi^\mu_{,\rho\nu})$$

so
$$\frac{\partial K^\mu_\nu}{\partial \xi^\lambda_{,\tau}} = \delta^\mu_\lambda \delta^\tau_\nu$$

and
$$\frac{\partial K^\mu_{\nu\rho}}{\partial \xi^\lambda_{,\tau\sigma}} = \frac{1}{2}(\delta^\mu_\lambda \delta^\tau_\nu \delta^\sigma_\rho + \delta^\mu_\lambda \delta^\tau_\rho \delta^\sigma_\nu) .$$

Using
$$J^\nu_\mu K^\rho_\nu = \delta^\rho_\mu$$

and
$$J^\eta_{\mu\rho} = - J^\eta_\nu J^\zeta_\mu J^\kappa_\rho K^\nu_{\zeta\kappa}$$

we find
$$\frac{\partial J^\nu_\mu}{\partial \xi^\lambda_{,\tau}} = - J^\tau_\mu J^\nu_\lambda$$

and
$$\frac{\partial J^\eta_{\mu\rho}}{\partial \xi^\lambda_{,\tau\sigma}} = - \frac{1}{2} J^\eta_\nu J^\zeta_\mu J^\kappa_\rho (\delta^\nu_\lambda \delta^\tau_\zeta \delta^\sigma_\kappa + \delta^\nu_\lambda \delta^\tau_\kappa \delta^\sigma_\zeta)$$

These are the only quantities which give a non-zero contribution when the parameter ξ and its derivatives are set to zero.

For the Lorentz transformations we have, to first order

$$D^a_{\beta} = \delta^a_{\beta} + \epsilon^{ab} S_{ab}^a{}_{\beta} + O(2)$$

$$D^a_{\beta, \nu} = \epsilon^{ab}_{, \nu} S_{ab}^a{}_{\beta} + O(2)$$

$$D^a_{\beta, \nu\sigma} = \frac{1}{2} (\epsilon^{ab}_{, \nu\sigma} + \epsilon^{ab}_{, \sigma\nu}) S_{ab}^a{}_{\beta} + O(2)$$

with similar results for $D^a_b = \Lambda^a_b$ except that, in this case, the generators are known explicitly.

The only non-zero quantities are:

$$(D^a_{\beta})_0 = \delta^a_{\beta} ; (D^a_{\beta, \nu})_0 = 0 ; (D^a_{\beta, \nu\sigma})_0 = 0$$

$$\left| \frac{\partial D^a_{\beta}}{\partial \epsilon^{cd}} \right|_0 = S_{cd}^a{}_{\beta}$$

$$\left| \frac{\partial D^a_{\beta, \nu}}{\partial \epsilon^{cd}_{, \lambda}} \right|_0 = \delta^{\lambda}_{\nu} S_{cd}^a{}_{\beta}$$

$$\left| \frac{\partial D^a_{\beta, \nu\sigma}}{\partial \epsilon^{cd}_{, \lambda\tau}} \right|_0 = \frac{1}{2} (\delta^{\lambda}_{\nu} \delta^{\tau}_{\sigma} + \delta^{\lambda}_{\sigma} \delta^{\tau}_{\nu}) S_{cd}^a{}_{\beta}$$

The Lagrangian does not depend explicitly on the co-ordinates hence its total derivative with respect to x^{μ} :

$$L_{, \mu} = \frac{\partial L}{\partial x^{\mu}}$$

is given by

$$\begin{aligned}
L_{,\mu} &= \Psi_a \psi^a_{,\mu} - \Psi_a^{\rho} \psi_{,\rho\mu}^a \\
&= \pi_a^{\lambda} e^a_{\lambda,\mu} - \pi_a^{\lambda\rho} e^a_{\lambda,\rho\mu} \\
&= \Omega^{\lambda}_{ab} \omega^ab_{\lambda,\mu} - \Omega^{\lambda\rho}_{ab} \omega^ab_{\lambda,\rho\mu} = 0
\end{aligned} \tag{6A.1}$$

where the concomitants are defined in the text. (6A.1) is our first invariance identity.

The transformations contain the parametric quantities

$$\xi^{\lambda}_{,\tau} : \xi^{\lambda}_{,\tau\sigma} : e^{ab} : e^{ab}_{,\tau} : e^{ab}_{,\tau\sigma}$$

We differentiate (6.2) with respect to each of these in turn and set them all to zero. We get

$$\begin{aligned}
&\Psi_a^{\mu} \left| \frac{\partial \bar{\psi}^a_{,\mu}}{\partial \xi^{\lambda}_{,\tau}} \right|_0 \\
&+ \pi_a^{\mu} \left| \frac{\partial \bar{e}^a_{,\mu}}{\partial \xi^{\lambda}_{,\tau}} \right|_0 + \pi_a^{\mu\rho} \left| \frac{\partial \bar{e}^a_{\mu\rho}}{\partial \xi^{\lambda}_{,\tau}} \right|_0 \\
&+ \Omega^{\mu}_{ab} \left| \frac{\partial \bar{\omega}^{ab}_{,\mu}}{\partial \xi^{\lambda}_{,\tau}} \right|_0 + \Omega^{\mu\rho}_{ab} \left| \frac{\partial \bar{\omega}^{ab}_{\mu\rho}}{\partial \xi^{\lambda}_{,\tau}} \right|_0 = 0
\end{aligned} \tag{6A.2}^a$$

$$\pi_a^{\mu\rho} \left| \frac{\partial \bar{e}^a_{\mu\rho}}{\partial \xi^{\lambda}_{,\tau\sigma}} \right|_0 + \Omega^{\mu\rho}_{ab} \left| \frac{\partial \bar{\omega}^{ab}_{\mu\rho}}{\partial \xi^{\lambda}_{,\tau\sigma}} \right|_0 = 0 \tag{6A.2}^b$$

$$\begin{aligned}
&\Psi_a^{\mu} \left| \frac{\partial \bar{\psi}^a_{,\mu}}{\partial e^{ab}} \right|_0 + \Psi_a^{\mu} \left| \frac{\partial \bar{\psi}^a_{,\mu}}{\partial e^{ab}} \right|_0 \\
&+ \pi_c^{\mu} \left| \frac{\partial \bar{e}^c_{,\mu}}{\partial e^{ab}} \right|_0 + \pi_c^{\mu\rho} \left| \frac{\partial \bar{e}^c_{\mu\rho}}{\partial e^{ab}} \right|_0 \\
&+ \Omega^{\mu}_{cd} \left| \frac{\partial \bar{\omega}^{cd}_{,\mu}}{\partial e^{ab}} \right|_0 + \Omega^{\mu\rho}_{cd} \left| \frac{\partial \bar{\omega}^{cd}_{\mu\rho}}{\partial e^{ab}} \right|_0 = 0
\end{aligned} \tag{6A.2}^c$$

$$\begin{aligned} \psi_a^\mu \left| \frac{\partial \bar{\psi}^\alpha}{\partial e_{ab}^\mu} \right|_0 + \pi_c^{\mu\rho} \left| \frac{\partial \bar{\theta}^c}{\partial e_{ab}^\mu} \right|_0 \\ + \Omega_c^\mu \left| \frac{\partial \bar{\omega}^{cd}}{\partial e_{ab}^\mu} \right|_0 + \Omega_c^{\mu\rho} \left| \frac{\partial \bar{\omega}^{cd}}{\partial e_{ab}^\mu} \right|_0 = 0 \end{aligned} \quad (6A.2)^d$$

$$\Omega_c^{\mu\rho} \left| \frac{\partial \bar{\omega}^{cd}}{\partial e_{ab}^\mu} \right|_{\lambda\tau} = 0 \quad (6A.2)^e$$

The transformation laws given in the text and the quantities calculated in the beginning of this appendix give:

$$\left| \frac{\partial \bar{\psi}^\alpha}{\partial e_{ab}^\mu} \right|_0 = -\delta_\mu^\tau \psi_{,\lambda}^\alpha$$

$$\left| \frac{\partial \bar{\theta}^a}{\partial e_{ab}^\mu} \right|_0 = -\delta_\mu^\tau \theta_{,\lambda}^a$$

$$\left| \frac{\partial \bar{\theta}^a}{\partial e_{ab}^\mu} \right|_0 = -\delta_\rho^\tau \theta_{,\mu,\lambda}^a - \delta_\mu^\tau \theta_{,\lambda,\rho}^a$$

$$\left| \frac{\partial \bar{\omega}^{ab}}{\partial e_{ab}^\mu} \right|_0 = -\delta_\mu^\tau \omega_{,\lambda}^{ab}$$

$$\left| \frac{\partial \bar{\omega}^{ab}}{\partial e_{ab}^\mu} \right|_0 = -\delta_\rho^\tau \omega_{,\mu,\lambda}^{ab} - \delta_\mu^\tau \omega_{,\lambda,\rho}^{ab}$$

$$\left| \frac{\partial \bar{\theta}^a}{\partial e_{ab}^\mu} \right|_0 = -\frac{1}{2} (\delta_\mu^\tau \delta_\rho^\sigma + \delta_\rho^\tau \delta_\mu^\sigma) \theta_{,\lambda}^a$$

$$\left| \frac{\partial \bar{\omega}^{ab}}{\partial e_{ab}^\mu} \right|_0 = -\frac{1}{2} (\delta_\mu^\tau \delta_\rho^\sigma + \delta_\rho^\tau \delta_\mu^\sigma) \omega_{,\lambda}^{ab}$$

$$\left| \frac{\partial \bar{\psi}^\alpha}{\partial e_{ab}^\mu} \right|_0 = S_{ab}^\alpha{}_\beta \psi^\beta$$

$$\begin{aligned}
\left| \frac{\partial \bar{\psi}^a}{\partial e^{ab}} \right|_0 &= S_{ab}^a \psi^b_\mu \\
\left| \frac{\partial \bar{e}^c}{\partial e^{ab}} \right|_0 &= S_{ab}^c e^d_\mu \\
\left| \frac{\partial \bar{e}^c_{\mu,\rho}}{\partial e^{ab}} \right|_0 &= S_{ab}^c e^d_{\mu,\rho} \\
\left| \frac{\partial \bar{\omega}^{cd}}{\partial e^{ab}} \right|_0 &= \eta^{de} (S_{ab}^c \omega^f_{\mu e} - S_{ab}^f \omega^c_{\mu e}) \\
&= \eta^{de} \omega^{cd}_{\mu e} \\
\left| \frac{\partial \bar{\omega}^{cd}_{\mu,\rho}}{\partial e^{ab}} \right|_0 &= C_{ab}^{cd} \omega^{gh}_{\mu,\rho} \\
\left| \frac{\partial \bar{\psi}^a}{\partial e^{ab}_{,\lambda}} \right|_0 &= \delta^\lambda_\mu S_{ab}^a \psi^b \\
\left| \frac{\partial \bar{e}^c_{\mu,\rho}}{\partial e^{ab}_{,\lambda}} \right|_0 &= \delta^\lambda_\rho S_{ab}^c e^d_\mu \\
\left| \frac{\partial \bar{\omega}^{cd}}{\partial e^{ab}_{,\lambda}} \right|_0 &= \delta^\lambda_\mu S_{ab}^{cd} \\
\left| \frac{\partial \bar{\omega}^{cd}_{\mu,\rho}}{\partial e^{ab}_{,\lambda}} \right|_0 &= \delta^\lambda_\rho C_{ab}^{cd} \omega^{ef}_{\mu} \\
\left| \frac{\partial \bar{\omega}^{cd}_{\mu,\rho}}{\partial e^{ab}_{,\lambda\tau}} \right|_0 &= -\frac{1}{2} (\delta^\lambda_\mu \delta^\tau_\rho + \delta^\lambda_\rho \delta^\tau_\mu) S_{ab}^{cd}
\end{aligned}$$

Substituting these into the identities (6A.2)^{a-e} and writing out the appropriate generators, we find the identities as they are given in the text. In particular the identity (6A.2)^b is:

$$-\frac{1}{2} e^a_\lambda (\mathbb{X}^{\mu\rho}_a + \mathbb{X}^{\rho\mu}_a) - \frac{1}{2} \omega^{ab}_\lambda (\Omega^{\mu\rho}_{ab} + \Omega^{\rho\mu}_{ab}) = 0$$

and (6A.2)^a is

$$- \frac{1}{2} (\Omega^{\mu\rho}_{cd} + \Omega^{\rho\mu}_{cd}) S_{ab}^{cd} = 0$$

from which it follows that

$$\Omega^{\mu\rho}_{cd} + \Omega^{\rho\mu}_{cd} = 0$$

hence (6A.2)^b may be taken as

$$\mathbb{I}_a^{\mu\rho} + \mathbb{I}_a^{\rho\mu} = 0 \quad .$$

APPENDIX 6B

The transformation laws of the Concomitants

If we differentiate both side of (6.2) with respect to the fields in turn and use their transformation laws we get

$$\Psi_a = D_a^\beta \bar{\Psi}_\beta + J_\mu^\nu D_{a,\nu}^\beta \bar{\Psi}_\beta^\mu$$

$$\Psi_a^\rho = J_\mu^\rho D_a^\beta \bar{\Psi}_\beta^\mu$$

$$\bar{\Sigma}_a^\rho = J_\mu^\rho \Lambda_a^b \bar{\Sigma}_b^\mu + J_\nu^\tau (J_\mu^\rho \Lambda_a^b)_{,\tau} \bar{\Sigma}_b^{\mu\nu}$$

$$\bar{\Sigma}_a^{\rho\lambda} = J_\mu^\rho J_\nu^\lambda \Lambda_a^b \bar{\Sigma}_b^{\mu\nu}$$

$$\begin{aligned} \bar{\Omega}_{ab}^\rho &= J_\mu^\rho \Lambda_a^c \Lambda_b^d \bar{\Omega}_{cd}^\mu \\ &\quad + J_\lambda^\sigma (J_\mu^\rho \Lambda_a^c \Lambda_b^d)_{,\lambda} \bar{\Omega}_{cd}^{\mu\lambda} \end{aligned}$$

$$\bar{\Omega}_{ab}^{\rho\sigma} = J_\mu^\rho J_\lambda^\sigma \Lambda_a^c \Lambda_b^d \bar{\Omega}_{cd}^{\mu\lambda}$$

If we now consider the inverse transformation these become:

$$\bar{\Psi}_\beta = D_\beta^{-1a} \Psi_a + D_{\beta,\rho}^{-1a} \Psi_a^\rho$$

$$\bar{\Psi}_\beta^\mu = K_\rho^\mu D_\beta^{-1a} \Psi_a^\rho$$

$$\bar{\Sigma}_b^\mu = K_\rho^\mu (\Lambda_b^{-1a} \Sigma_a^\rho + \Lambda_{b,\lambda}^{-1c} \Sigma_c^{\rho\lambda})$$

$$\bar{\Sigma}_b^{\mu\nu} = K_\rho^\mu K_\lambda^\nu \Lambda_b^{-1c} \Sigma_c^{\rho\lambda}$$

$$\begin{aligned} \bar{\Omega}_{ab}^\rho &= K_\mu^\rho \Lambda_a^{-1c} \Lambda_b^{-1d} \Omega_{cd}^\mu \\ &\quad + K_\mu^\rho (\Lambda_a^{-1c} \Lambda_b^{-1d})_{,\lambda} \Omega_{cd}^{\mu\lambda} \end{aligned}$$

$$\bar{\Omega}_{ab}^{\rho\sigma} = K_\mu^\rho K_\lambda^\sigma \Lambda_a^{-1c} \Lambda_b^{-1d} \Omega_{cd}^{\mu\lambda}$$

If we expand the Lorentz transformation matrices in the last of these and use the fact that, by the properties of the Lorentz generators,

$$\Omega^{\mu\rho}_{cd} = S_{cd}^{ab} \Omega^{\mu\rho}_{ab}$$

we may also use the structure relation to introduce the structure constants then we see that $\Omega^{\mu\rho}_{ab}$ are adjoint tensors transforming as:

$$\Omega^{\mu\rho}_{ab} = K^{\mu}_{\nu} K^{\rho}_{\lambda} \exp(-\epsilon^{ef} C_{ef}^{cd}) \Omega^{\nu\lambda}_{cd} .$$

APPENDIX 6C

The Tensorial Concomitants

To construct tensorial quantities corresponding to the non-tensorial concomitants we use the gauge field transformation laws to eliminate the derivatives of the transformation matrices which occur in the inhomogeneous terms in the non-tensorial transformation laws. We have, for example,

$$D^{-1\alpha}_{\beta,\rho} = K^{\mu}_{\rho} D^{-1\alpha}_{\gamma} \bar{W}^{\gamma}_{\mu\beta} - D^{-1\gamma}_{\beta} W^{\alpha}_{\rho\gamma}$$

The transformation law of the concomitant which corresponds to the matter field is

$$\Psi_{\beta} = D^{-1\alpha}_{\beta} \Psi_{\alpha} + D^{-1\alpha}_{\beta,\rho} \Psi^{\rho}_{\alpha}$$

Using the above expression for the derivative of the transformation matrix we get

$$\Psi_{\beta} - W^{\gamma}_{\mu\beta} \Psi^{\mu}_{\gamma} = D^{-1\alpha}_{\beta} (\Psi_{\alpha} - W^{\gamma}_{\rho\alpha} \Psi^{\rho}_{\gamma})$$

and thus the quantity

$$\Phi_{\alpha} \equiv \Psi_{\alpha} - W^{\beta}_{\rho\alpha} \Psi^{\rho}_{\beta}$$

is a gauge tensor. In a similar manner the other two tensorial quantities are constructed.

CHAPTER 7 Applications: Some Particular Theories of Gravity

7.1 Introduction

In the previous chapter we constructed the general theory which was based on an unknown Lagrangian restricted only by the invariance identities. We will now investigate the properties of theories derived from explicitly known Lagrangians.

It should be clear that the identities cannot be 'solved' to provide us with a unique Lagrangian but it is, in fact, a simple matter to construct Lagrangians which satisfy them. From this point of view the identities merely dictate that the arguments of the Lagrangian be certain tensors, namely the curvature, torsion and covariant derivative. Although we arrived at the structure of these tensors by other means (Chap. 5) they arise within the identities themselves as can be seen in the identities written in covariant form so it is not surprising that a Lagrangian having them as arguments satisfies the identities. We establish this explicitly and then go on to examine a number of simple theories which naturally suggest themselves.

7.2 The General Case

We find that the Lagrangian

$$L = L(\psi^a ; \psi^a_{;a} ; S^a_{bc} ; R_{ab}{}^{cd}) \quad (7.1)$$

where
$$S_{b c}^a = e_b^\mu e_c^\rho S_{\mu \rho}^a \quad (7.2)$$

and
$$R_{ab}^{cd} = e_a^\mu e_b^\rho R_{\mu \rho}^{cd} \quad (7.3)$$

satisfies the identities (6.12)^{a-f} with the exception of the global Lorentz identity (6.12)^d which will also be satisfied once the arguments in (7.1) have been contracted into scalars. Observe that, in (7.1), it is as if we have projected the entire system into local tetrad co-ordinates.

In the minimally coupled case the Lagrangian would be

$$L = L_m(\psi^a; \psi^a_{;a}) + L_e(S_{bc}^a) + L(R_{ab}^{cd}) \quad (7.4)$$

which is the general form of the theories that we shall consider next.

7.3 Einstein's Theory

The Lagrangian is

$$L = L_m(\psi^a; \psi^a_{;a}) + \frac{1}{2} R \quad (7.5)$$

where

$$R = R_{ab}^{ab} \quad (7.6)$$

is the *Ricci scalar*. We impose the additional restriction that the torsion vanishes identically:

$$S_{\mu \rho}^a = 0 \quad (7.7)$$

We have also omitted the torsion Lagrangian entirely which is equivalent, in the general theory, to omitting the derivatives of the tetrad. The concomitants of interest are:

$$\psi_a^\rho = \psi_a^b e_b^\rho \quad (7.8)$$

$$\Sigma_a^{\mu\rho} = 0 \quad (7.9)$$

$$\Omega_{ab}^{\mu\rho} = \frac{1}{2}(e_a^\mu e_b^\rho - e_a^\rho e_b^\mu) \quad (7.10)$$

and so the stress tensors are

$$T_\lambda^\rho = e (L_\lambda \delta^\rho_\lambda - e_a^\rho \psi_a^\alpha \psi_{;\lambda}^\alpha) \quad (7.11)$$

$$T_\lambda^\rho = e (\frac{1}{2} R \delta^\rho_\lambda - R_\lambda^\rho) \quad (7.12)$$

$$T_\lambda^\rho = 0 \quad (7.13)$$

where

$$R = R_{\mu\lambda}^{ab} e_a^\mu e_b^\lambda \quad (7.14)$$

is the *Ricci tensor* while the Lorentz stress tensor is now known as the *Einstein tensor*. The spin-stresses are

$$S_{ab}^\mu = e e_c^\mu \psi_a^c S_{ab}^\alpha \psi_\alpha^B \quad (7.15)$$

$$S_{ab}^\mu = S_{ab}^\mu = 0 \quad (7.16)$$

and in this case the tetrad spin-stress vanishes. Since the torsion is zero the equations of motion are

$$(e \psi_a^\alpha)_{;\alpha} - \Phi_a = 0 \quad (7.17)$$

$$T_\lambda^\rho + T_\lambda^\rho = 0 \quad (7.18)$$

$$S_{ab}^\mu = 0 \quad (7.19)$$

(7.17) shows that the matter behaves as if it is moving in a local lorentz frame. (7.18) are *Einstein's Equations* and (7.19) is a constraint condition on the spin of the matter field. It shows that the Einstein theory cannot support a field with an intrinsic spin.

The conservation equations are

$$\overset{m}{T}_{\lambda}{}^{\rho}{}_{;\rho} = 0 \quad (7.20)$$

$$\overset{\omega}{T}_{\lambda}{}^{\rho}{}_{;\rho} = 0 \quad (7.21)$$

and both stresses are independently conserved. The symmetry relations are

$$\frac{1}{2} (\overset{m}{T}_{ab} - \overset{m}{T}_{ba}) = - \overset{m}{S}{}^{\rho}_{ab;\rho} = 0 \quad (7.22)$$

$$\begin{aligned} \frac{1}{2} (\overset{\omega}{T}_{ab} - \overset{\omega}{T}_{ba}) &= -\frac{1}{2} \Omega^{\mu\rho}_{cd} C_{ab}{}^{cd}{}_{ef} R_{\mu\rho}{}^{ef} \\ &= \frac{1}{2} (R_{ab} - R_{ba}) \\ &= 0 \end{aligned} \quad (7.23)$$

by the second Bianchi Identity and the fact that the torsion is zero.

7.4 Einstein-Cartan Theory

If we wish to formulate a theory which can support a dynamic material spin then we must ensure that the l.h.s. of the Lorentz potential's equation formation does not vanish. The simplest way to do this is simply to relax (7.7) i.e. to allow torsion into the theory.

The Lagrangian is again (7.5), the concomitants are given by (7.8) - (7.10) and the stresses by (7.11) - (7.16). Now, however, the Lorentz potential field equation becomes algebraic; the field equations are:

$$(e \psi_a^{\mu})_{;\mu} - \phi_a = 0 \quad (7.24)$$

$$\overset{m}{T}_{\lambda}{}^{\rho} + \overset{\omega}{T}_{\lambda}{}^{\rho} = 0 \quad (7.25)$$

$$\frac{1}{2} \epsilon (S_{\nu\lambda}^{\nu} \delta_{\sigma}^{\rho} - S_{\nu\sigma}^{\nu} \delta_{\lambda}^{\rho} - S_{\sigma\lambda}^{\rho}) = e_{\lambda}^a e_{\sigma}^b S_{ab}^{\rho} \quad (7.26)$$

It follows that the *torsion vector* is

$$S_{\mu}^{\nu} = S_{\nu\mu}^{\nu} = e_{\mu}^a \psi_{\alpha}^b S_{ab}^{\alpha} \psi^{\beta} \quad (7.27)$$

and that the torsion itself is

$$\begin{aligned} S_{\lambda\sigma}^c &= 2 e_{\lambda}^a e_{\sigma}^b (\psi_{\alpha}^c S_{ab}^{\alpha} \psi^{\beta}) \\ &\quad - (e_{\sigma}^c e_{\lambda}^a - e_{\lambda}^c e_{\sigma}^a) (\psi_{\alpha}^b S_{ab}^{\alpha} \psi^{\beta}) \end{aligned} \quad (7.28)$$

The torsion at a point is therefore generated by the spin of the matter and in empty space, in the absence of matter, the torsion vanishes.

The stress conservation laws are

$$\frac{m}{T}_{\lambda}^{\rho}{}_{;\rho} = \frac{m}{T}_{\sigma}^{\mu} S_{\lambda\mu}^{\sigma} + \frac{m}{S}_{ab}^{\mu} R_{\mu\lambda}^{ab} \quad (7.29)^a$$

$$\frac{\omega}{T}_{\lambda}^{\rho}{}_{;\rho} = \frac{\omega}{T}_{\sigma}^{\mu} S_{\lambda\mu}^{\sigma} - \frac{\omega}{S}_{ab}^{\mu} R_{\mu\lambda}^{ab} \quad (7.29)^b$$

and, by the equation of motion (7.25) the total stress is conserved (in fact it vanishes).

Also,

$$(\frac{m}{T}_{ab} + \frac{m}{T}_{ba}) = - \frac{m}{S}_{ab;\rho}^{\rho} \quad (7.30)$$

and the conservation of total spin-stress gives

$$S_{ab;\mu}^{\mu} = \frac{m}{S}_{ab;\mu}^{\mu} = - \frac{1}{2} \Omega^{\mu\rho}{}_{cd} C_{ab}^{cd}{}_{ef} R_{\mu\rho}^{ef} \neq 0 \quad (7.31)$$

since the torsion is now non-zero. Hence the total spin-stress is not conserved and the matter tensor is not symmetric. It also follows that the Einstein tensor is not symmetric either. On the other hand, in empty space, the torsion vanishes and these conclusions revert to those of the purely Einstein theory.

7.5 The Quadratic Theory

A comparatively simple way to ensure that the term

$$\Omega^{\mu\rho}_{cd} C_{ab}^{cd} \epsilon_{ef} R_{\mu\rho}^{ef} \quad (7.32)$$

vanishes is to take advantage of the symmetry properties of the structure constants. If we take the Lagrangian to be quadratic in the curvature (with non-zero torsion):

$$L = L_m + \frac{1}{2} R_{ab}^{cd} R_{cd}^{ab} \quad (7.33)$$

then,

$$\Omega^{\mu\rho}_{cd} = R^{\mu\rho}_{cd} \quad (7.34)$$

and the term (7.32) vanishes identically.

The Lorentz stress is now

$$T_{\lambda}^{\rho} = e \left(\frac{1}{2} R_{\mu\nu}^{ab} R_{ab}^{\mu\nu} \delta_{\lambda}^{\rho} - R_{ab}^{\mu\rho} R_{\mu\lambda}^{ab} \right) \quad (7.35)$$

which is clearly symmetric. The matter stress is still given by (7.11) and the conservation equations are (7.29)^{a,b}.

In view of the vanishing of the term (7.32) the total spin-stress is conserved and it follows that the matter stress is symmetric.

The field equations are again (7.24) and (7.25) with (7.26) replaced by

$$\begin{aligned} e \left(R^{\rho\mu}_{ab;\mu} - S_{\nu\mu}^{\nu} R^{\rho\mu}_{ab} - \frac{1}{2} S_{\mu\nu}^{\rho} R^{\nu\mu}_{ab} \right) \\ = S_{ab}^{\rho} \end{aligned} \quad (7.36)$$

Which has the appearance of a propagation equation for the curvature which is sourced by the spin-stress of matter.

7.6 Theories which include Torsion in a Non-trivial Way

As we have already remarked there is, in the general theory, a very noticeable similarity between the roles played by the curvature and the torsion. So far the theories we have discussed have only curvature in their Lagrangians, the derivatives of the tetrad, and hence the torsion, being omitted altogether.

Once the torsion is included in a non-trivial way we need to take account of the possible non-conservation of the total stresses embodied in the laws:

$$T_a{}^\rho{}_{;\rho} = \frac{1}{2} \Sigma_b{}^{\mu\rho} R_{\mu\rho}{}^b{}_a \quad (7.37)$$

$$S^{\rho}{}_{ab;\rho} = \frac{1}{2} \Omega^{\mu\rho}{}_{cd} C_{ef}{}^{cd}{}_{ab} R_{\mu\rho}{}^{ef} \quad (7.38)$$

An obvious way of achieving conservation is to reverse the usual roles of the curvature and torsion by taking

$$R_{\mu\rho}{}^{ef} = 0 \quad (7.39)$$

$$S_{\mu\rho}{}^a \neq 0 \quad (7.40)$$

and we make the Lagrangian independent of the derivatives of the Lorentz potentials.

As a first attempt we take the Lagrangian to be quadratic in the torsion:

$$L = L_m + \frac{1}{2} S_a{}^b{}_c S^a{}^c{}_b \quad (7.41)$$

from which we get

$$\psi_a{}^\rho = \psi_a{}^a e_a{}^\rho \quad (7.42)^a$$

$$\Omega^{\mu\rho}{}_{ab} = 0 \quad (7.42)^b$$

$$\mathbb{T}_a^{\mu\rho} = S_a^{\mu\rho} \quad (7.42)^c$$

The stress tensors are

$$\mathbb{T}_\lambda^\rho = e (L_\lambda \delta^\rho_\lambda - e_a^\rho \psi^a \psi_{;\lambda}) \quad (7.43)^a$$

$$\mathbb{T}_\lambda^\rho = e \left(\frac{1}{2} S_{\mu\sigma}^v S_v^\mu \delta^\rho_\lambda - S_{\sigma}^{\mu\rho} S_{\lambda\mu}^\sigma \right) \quad (7.43)^b$$

$$\mathbb{T}_\lambda^\rho = 0 \quad (7.43)^c$$

and the spin-tensors are

$$\mathbb{S}_{ab}^\mu = e e_c^\mu \psi_a^c S_{ab}^\alpha \psi_\beta^B \quad (7.44)^a$$

$$\mathbb{S}_{ab}^\mu = e S_c^{\rho\mu} S_{ab}^c e_d^\rho \quad (7.44)^b$$

$$\mathbb{S}_{ab}^\mu = 0 \quad (7.44)^c$$

(7.44)^b shows that the tetrad spin-stress has now entered the theory.

The conservation laws are

$$\mathbb{T}_{\lambda;\rho}^\rho = \mathbb{T}_\sigma^\rho S_{\lambda\rho}^\sigma \quad (7.45)^a$$

$$\mathbb{T}_{\lambda;\rho}^\rho = - \mathbb{T}_\sigma^\rho S_{\lambda\rho}^\sigma \quad (7.45)^b$$

and the total stress is clearly conserved. For the spin-stress, the total is conserved:

$$(\mathbb{S}_{ab}^\mu + \mathbb{S}_{ab}^\mu)_{;\mu} = 0 \quad (7.46)$$

The equations of motion are

$$(e \psi_a^B)_{;a} - \Phi_a = 0 \quad (7.47)^a$$

$$\begin{aligned} & \bullet (S^{\rho \mu}_{a}{}_{;\mu} - S^{\nu}_{v\mu} S^{\rho \mu}_{a} - \frac{1}{2} S^{\rho}_{ \mu \nu} S^{\nu \mu}_{a}) \\ & = T^{\rho}_{a} + T^{\rho}_{a} \end{aligned} \quad (7.47)^b$$

$$S^{\rho}_{ab} + S^{\rho}_{ab} = 0 \quad (7.47)^c$$

These should be compared to the theory which is quadratic in the curvature. It is also apparent that, at least as far as the structure of the field equations are concerned, the stresses and spin-stresses have also interchanged their roles. Note that the Lorentz potentials play no part in the theory whatever. (7.47)^b has the appearance of a propagation equation for the torsion sourced by the total stress while (7.47)^c shows that the spin-stress of matter and the spin-stress of the tetrad are complementary just as the matter and Lorentz stresses were in the previous cases. Note also that the total spin-stress vanishes by an equation of motion.

Because of the symmetry properties of the Lorentz generators we find that the tetrad stress is symmetric:

$$\begin{aligned} \frac{1}{2} (T^{\rho}_{ab} - T^{\rho}_{ba}) &= -\frac{1}{2} \epsilon^{\nu\rho}_{c} S^c_{ab d} S^d_{ \rho \mu} \\ &= -\frac{1}{2} S^{\mu \rho}_{c} S^c_{ab d} S^d_{ \rho \mu} \\ &= 0 \end{aligned} \quad (7.48)$$

while the matter stress is not:

$$\frac{1}{2} (T^{\rho}_{ab} - T^{\rho}_{ba}) = -\frac{1}{2} S^{\rho}_{ab;0} \quad (7.49)$$

In view of the vanishing of the curvature (7.39) which is an integrability condition for the Lorentz potentials (Chap. 5) we can transform to a special gauge in which they vanish identically. This can have no effect on the theory since, as we have seen, these potentials take no part in it (their stresses vanish).

In such a gauge the manifold connection is simply

$$\Gamma_{\rho\lambda}^{\mu} = e_a^{\mu} e_{\lambda,\rho}^a \quad (7.50)$$

and the torsion is

$$S_{\mu\nu}^a = e_{\nu,\mu}^a - e_{\mu,\nu}^a \quad (7.51)$$

whose non-vanishing means that the tetrad is itself non-integrable. Such a space is called a *Weitzenböck Space* and once we have transformed to such a space we may only perform global Lorentz transformations since local ones will re-introduce the Lorentz potentials.

It is clear that this theory presents an interesting alternative to the usual Einstein theory. In some respects it is simpler since it deals essentially only with gauged translations and thus only has one non-trivial gauge field, namely the tetrad, and yet it encompasses spinning matter. We may refine the Lagrangian somewhat by constructing it out of the irreducible parts of the torsion tensor (see Hayashi and Bregman (1973) for more details). This theory has been called *New General Relativity* (Hayashi and Shirafuji (1979)).

CONCLUSION

We have tried to show something of the power of the gauge concept in its ability to generate interesting theories of interacting fields. Although the theories which emerge are fairly complex, the initial assumptions are not; a circumstance which is very satisfying from the point of view of simplicity. Our approach, throughout, has been to exploit the invariance identities to reveal the structure of the theory without any explicit knowledge of a Lagrangian. This has been particularly fruitful in the case of the theory of gravity where, it would seem, there are many possibilities which remain to be explored.

If the gauge philosophy is correct then the implication is that the forces of nature can all be traced to the existence of certain groups whose representations constitute the naturally occurring fields. We may then also conceive of unifying all these forces into a single gauge framework derived from a single group. It must be emphasised, however, that such a grand unification is still far off and is, in fact, not possible within the relatively simple framework presented here. We do know that it is not possible to unify the Lorentz Group with an arbitrary internal group in any way other than the relatively trivial direct product as we have done (O'Riifeartaigh (1965)). This is why this work is so clearly divided into internal and external parts. It would therefore seem that another approach is called for. However, there is hope that such a unification may yet be achieved within the gauge framework by using groups which are more general than the Lie Groups we have

discussed. Such groups have been found. They are called Graded Lie Groups (their parameters are non-commutative) and the gauge theory they generate is called Supersymmetry (Fayet and Ferrara (1977)). There is also a version which deals with gravity, called Supergravity (Deser (1980)) which may be interpreted crudely as the 'square-root' of ordinary gravity in that binary products of its operators produce the transformations of ordinary gravity. It also turns out that certain of its gauge potentials may be combined into representations of an internal group thus allowing a non-trivial unification of gravity with an internal group (de Witt and Freedman (1977)). Unfortunately, at present, this internal group does not appear to have any physical correspondence and so the theory is somewhat sterile.

There is, however, a ray of hope which springs from an unexpected quarter. We have seen that the gauge fields in our internal theory are massless. At first sight this represents an obstacle to the unified description of the forces which occur in nature because we know that the strong and weak nuclear forces are mediated by massive particles. It turns out that there is a way to break the symmetry of the internal group in such a way that the theory maintains the symmetry in a formal sense only but the gauge fields acquire masses (O'Rai'feartaigh (1979)). This is called a spontaneously broken symmetry and the gauge fields acquire their masses by what is known as the 'Higgs Mechanism' (Higgs (1966)). Furthermore, a quantum mechanical analysis shows that as the energy of the system increases a 'phase transformation' occurs and the original symmetry is restored (Linde (1979)). This means that, at extremely high temperatures, as we expect shortly after the Big Bang, all the symmetries are perfect and the gauge fields are massless. It may be that a similar mechanism will be found to operate in Supergravity and that the perfect, sterile symmetry which it possesses may be broken to produce more interesting subsymmetries which do correspond to reality.

Whatever the final outcome may be it is clear that the gauge concept is extremely useful and will serve our quest for the understanding of the fundamental processes of nature for a long time to come. In the meantime, until the answers are known, we must explore this notion in all its aspects.

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