Application of Derivative Techniques to Improve the Forecasting of Price Volatility of Copper, Gold and Platinum Metals

by

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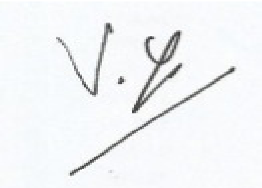
Declaration

I declare that this Ph.D. Thesis is my own, unaided work. It is submitted in fulfilment of the degree of Doctor of Philosophy in the subject area of Mining Engineering at the University of the Witwatersrand, Johannesburg, South Africa. It has not been submitted before for any degree or examination in any other university.

Signed:



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Veriyadi Veriyadi

This 31st________ day of _March_______________ 2023

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Dedication

To my mother (Suryani), father (Lutis Norman), brother (Supandi) and sister (Lusi Oktaviani) with whom I would have liked to share exciting moments such as this one. May Allah bless us all!

List of Publications

Parts of this thesis have been accepted or submitted for publication as follows:

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P3 Veriyadi V (2021). Application of Derivative Techniques to Reduce Volatility of the Future Price Prediction of Copper, Gold and Platinum Metals. *Working Paper*.

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List of Abbreviations

ARCH	Autoregressive Conditional Heteroscedasticity
B-S	Black-Scholes (1973) Model
CAPM	Capital Asset Pricing Model
CRNP	Conditional Risk-Neutral Probability
DCF	Discounted Cash Flow
DJIA	Dow Jones Industrial Average
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GBM	Geometric Brownian Motion
PGMs	Platinum Group Metals
VAR	Vector Autoregression
RNP	Risk Neutral Probability
U.S.	United States of America
ZAR	South African Rand

Abstract

This research investigates the forecasted price volatility of copper, gold and platinum metals based on the selected companies; Palabora Copper Mining Ltd, AngloGold Ashanti Ltd, Gold Fields Ltd, Sibanye-Stillwater, Anglo Platinum Ltd and Impala Platinum Ltd. In responding to the latter sentence, single price volatilities are dual volatilities, where dual volatilities comprise of financial and technical variables. The selected firms either have global operations or they are subsidiaries of global companies. Dual volatility is computed using a Sample Correlation Coefficient and in order to explore the dual volatility, this research introduces three hypotheses. The first hypothesis uses a Decision Tree Analysis to test dual volatility based on financial and technical variables (e.g., mineral commodity price, metal grade, operating cost and production rate) in improving the forecasting of price volatility of copper, gold and platinum metals. For validation, the first hypothesis uses the Markov-Regime Switching Model. The results of this hypothesis illustrate that dual volatilities are more accurate and robust than price only volatilities. Then, the second hypothesis examines dual volatility using a GBM model. This hypothesis tests dual volatility; which is computed based on financial and technical variables (e.g., oil price, copper price, oil production and consumption, copper production and consumption; and the exchange rate from U.S.$ to ZAR and gold and platinum price data). The chosen variables that affect the dual volatility are examined using a Multiple Regression Model and that model confirms that those variables are independent in principle. Finally, the third hypothesis estimates future profits based on a binomial tree, which has risk-neutral probabilities based on dual volatility using mineral commodity price, metal grade, operating cost and production rate. The results of risk-neutral probabilities using dual volatility are less optimal than a mineral commodity price volatility due to not accounting for the mean of logarithmic returns. The robustness test uses the VAR model, which indicates that the profits react differently to different shock stages from revenues, risk-free interest rates and profits. In conclusion, dual volatility can improve future price forecasting performance because duality is underpinned by different variables, which include independent variables from the global commodity markets. The forecasting performance improvement from dual volatility in predicting the future price can be shown by the lower value of the Root Mean Square Error and Mean Absolute Percentage Error results than a mineral commodity price volatility. The findings of this research apply to copper, gold and platinum metals for mining around the globe.

Keywords: Correlations, minerals, multiple regression, profits, risk-neutral probabilities, volatility

1 Introduction

1.1 Chapter overview

The Real Option technique is the valuation method for the real investment which offers management flexibility to respond to the future risks (Black & Scholes, 1973; Merton, 1973; Collan et al, 2009). The selection of options on a mining project is significantly controlled by the future risk prediction, which is derived from the source of uncertainties as illustrated by volatility. This volatility represented in the future risks can reflect the feasibility study assessment to deliver the input to the management in selecting the best option, based on the maximum profits. Some previous studies such as Copeland & Antikarov (2003) recommend employing the Logarithmic Total Ratio for the future net cash flow to the current net cash flow to compute the volatilities (σ). However, it does not consider the technical uncertainties (e.g., metal grades and production rates) when forecasting the future risks. This research intends to illustrate future incorporated risk (i.e., volatility), based on volatility variables using Risk Neutral Probability (hereafter, RNP) methods in optimising the future option. The dual risk in this research is represented by dual volatility, which is made up of market and technical risk volatilities.

1.2. Background of the Study

The Real Options Valuation method in mining projects has mainly employed mineral commodity prices to forecast the future risks (Guj, 2016). Note that risk throughout this thesis means volatility. The mineral commodity prices are an important component in the mining project and play a key role in mining valuation and investments. Haque et al (2014) demonstrated that mineral commodity prices fluctuated randomly following stochastic behaviour. Therefore, one of the most well-known techniques for assessing random movements is the GBM process, which utilises stochastic movements to price the future risk, as illustrated by volatility.

Mun (2002) proposed commodity price volatility to solve real options problems using risk-neutral probabilities. Saliba & Dimitrakopoulos (2019) used annual mean price volatility to simulate the future gold prices risk using GBM; thereafter, they added a Poisson jump diffusion process to improve the original GBM. In term of mining projects, the price volatility in mineral commodity plays a critical role in addressing the issue of risk, since it has a greater risk than other primary commodity products. A greater risk in mineral commodity prices can lead to optimising investment into the mining project (Foo et al, 2018). However, price volatility was commonly overestimated to predict the risks (Haque et al, 2017). Therefore, according to Chandra & Guj (2012), there is a requirement to provide management with more practical ways of investment decision making. The use of historical price and exchange rate, including their volatilities and correlation, has been developed by Haque et al (2017) and Aminrostamkolaee et al (2017). They consolidated their volatilities and correlations into the model using two stochastic processes.

Copeland & Antikarov (2003) proposed the modification of volatility to improve the future option using the Marketed Asset Disclaimer model by aggregated multiple sources of risk. Another major source of risk was measured by aggregated cashflow volatility (Barton & Lawryshyn, 2011). However, the study only considered market and financial perspectives, and no attempt was made by previous authors to quantify the association between market and technical uncertainties.

Although different methods have been developed to combine a myriad of risks to assess the risk in deciding the future option, most of these approaches do not comprehensively incorporate market and technical uncertainties. Therefore, a Sample Correlation Coefficient model is proposed in this research to calculate volatility using a combination of market and technical risks to optimise the future risks. This research proposes to combine market and technical risk volatilities using a Sample Correlation Coefficient model. The Sample Correlation Coefficient model is used across the three hypotheses. The first hypothesis uses a Decision Tree Analysis, the second hypothesis builds on the GBM and the third hypothesis is anchored on the RNP analysis. The reason why those listed techniques are selected will be explained later in Chapter 3.

1.3 Overview of Minerals

The minerals covered in this thesis are (i) copper, (ii) gold and (iii) platinum. According to the World Bank 2019 report, those listed minerals are some of the most highly consumed minerals in the world. Moreover, those minerals are found in abundance across the globe, aside from platinum, and are used in many industries. Therefore, the demand of those listed minerals tends to be high.

1.3.1 Copper

A low average copper grade can play an important role in addressing the issue of mined copper ores in the long term. One of the main concerns for falling average grades is a change to mine the porphyry copper deposit type that can be broadly defined as a low-grade copper mineralisation type. However, different continents reveal different grades that, on average, have higher copper grades than the global average than ones found in Africa and Australia, while North and Latin America tend to have lower graded minerals. Recent evidence suggests that copper grades trend downwards, especially on the American continent, and that the mines are now approaching the end of their lifespan. Prior studies such as Crowson (2012) established that a lower average copper grade can be accepted by the metal price movements and the by-products from the copper mines.

In modern history, copper has been extensively used for electricity. Before 1850, the most interesting aspect of copper was limited to usage as splendid metal for church roofs, bronze bells, doors, and brass hardware. However, another significant demand aspect of copper is the industrial revolution in electricity. First, a telegraph cable performed well across the Atlantic Ocean in 1866. Then, Alexander Graham Bell successfully installed a copper telephone transmission. Next, Thomas Alva Edison needed a copper wire to transfer electricity to an incandescent electric lamp. Finally, the existence of high conductivity industrialisation has led to a massive demand for copper. Recently investigators have examined the effects of copper on industrial and service products, and found that a half of the total consumption for power and telecommunications goes toward building construction, the car industry, aircrafts, and other electronic devices. Studies over the past century have provided important information on the advantages of copper, such as its conductivity transfer, corrosion resistance, and malleability properties (Radetzki, 2009).

1.3.2 Gold

The main challenges faced by gold mines is that gold is a non-renewable resource, ore grades are declining, there are geological and mining limitations, as well as environmental and socio-economic issues (Mudd, 2007). Traditionally, the term “non-renewable resource” has been associated with ore grade sensitivity and mining constraints that can affect the guarantee of future generations’ ability to meet their demands (Cowell *et al*, 1999). With respect to future demands, the operating cost will increase to extract the complex geological deposit with a lower grade. Regarding a lower gold grade, it needs more reagent in order to separate from its impurities, which can cause significant environmental impacts. Taken together, the sustainable development objective for gold mines is to correspond the potential social and environmental risks with the economic risks (Mudd, 2007).

The sustainable developments from gold mines have indicated serious demand trends for gold. Safhiee & Topal (2010) demonstrated that major gold demand is ascribed from jewellery and approximately 2,500 tonnes and 1000 tonnes came from exchange trade funds and retail investors, respectively and the mining industry in the last decade; however, the demand of global gold decreased from 1998 to 2007, while some research such as Safhiee & Topal (2010) has remained to raise for ETFs and retails. Batchelor & Gulley (1995) labelled the jewellery industry as a flexible subset that can be converted from the demand side to the supply side. In broad terms, jewellery can be recycled as renewable resources to enter the central banks as the gold market reserves that can maintain the quality and the new opening gold mines (Batchelor & Gulley, 1995).

1.3.3 Platinum

In the new global economy, the platinum group mining has become a central commodity for the most concentrated among other metal producers. There is evidence that the Platinum Group Metal (PGM, thereafter) plays a crucial role in production which is concentrated over 80 per cent in South Africa and Russia, and has been classified as a critical mineral. In terms of critical minerals, the PGM source has been investigated around the globe, it has been concluded that it can also be found outside South Africa and Russia. Most studies in the field of the PGM producers have only focused on two companies which are Norilsk Nickel in Rusia and Anglo Platinum in South Africa (Humphreys, 2011; Fortescue & Rautio, 2011). However, there is risk in relation to the operations of Norilsk Nickel in the Arctic Ocean's remote shores in the Krasnoyarsk Region because of high operational costs involved in that area.

It is now well established that the PGM is used for auto-catalyst, chemical process, and alloys and data from several studies (Hao et al 2019; Mudd et al 2018) suggest that the PGM can only be dominated by South Africa. There is increasing concerns from South Africa domination that the platinum group elements have accentuated the global supply problem that South Africa experienced, with the main issues being social, economic, and environmental issues. Regarding these issues, a supply side can be distracted that the platinum group elements (from here, PGE) resources and reserves have been reported to have increased 16.4 per cent from 2010 to 2015. Taken together, the platinum group elements labelled these subsets as a critical metal, which is mainly influenced by social issues rather than resource or geological depletion (Mudd *et al*, 2018).

1.4. Problem Statement

One of the greatest challenges in the mining projects is that future risks can be represented by volatility (Guj, 2011). Volatility is an important tool to measure financial risks, however some scholars (e.g., Mun, 2002; Guj, 2011; Saliba & Dimitrakopoulos, 2019) express that the future risk is only based on the historical commodity prices (Guj, 2011). The problem in mining risk, as illustrated by volatility, is not well captured as the risk is modelled purely from mineral commodity prices. Batten et al (2010) addressed the question of mineral commodity prices to represent the future risk as represented by price volatility as it fails to deal with the business cycle. Haque et al (2017) has established that price volatility appears to be overestimated. Another weakness of price volatility is that it cannot draw from the globe financial condition (Muhammad et al, 2019). Studies such as Moel & Tufano (2002) illustrated that the future risk is better captured when one accounts for financial, technical and social variables, among other factors, rather than only from mineral commodity prices. In order to improve the modelling or risk in mining projects, it is suggested to incorporate financial, technical and social variables into a single condensed volatility calculation.

A number of critical experiences have been reported as a consequence of incorporating financial and technical variables to assess volatility in mining projects. According to Brandão et al (2012) and Savolainen (2016), incorporating numerous sources of risks into a single volatility can significantly decrease positive bias. Guj & Chandra (2019) addressed the question of cash flow volatility by incorporating financial and technical variables failing to resolve the future risk. Considering these concerns raised and the well-known fact that incorporating multiple sources of risk into a single volatility is a long-standing problem, a critical study of the main factors causing bias in predicting future risks is required. The main objective of the study is to establish whether incorporating financial and technical variables would reduce a long-standing problem of risks caused by incorporating multiple sources of risks into a single volatility.

1.5 Primary Research Question

Risk is one of the most frequently stated problems with mining projects (Guj, 2011). The understanding of risk from each mining project is important since each of them has different inherent risks on the basis of mineral commodities, economic metal grades, operating costs and production rates. One of the most relevant methods to indicate this risk is to have a better knowledge of the risks in mining globally.

As described above, this research aims to answer the question "what is the most suitable method to quantify volatility risks in mining globally?"

From the primary question, the secondary research questions are identified as:

- What is the best and most efficient way to model volatility using incorporated financial and technical techniques?
- Does efficiency in predicting future volatility improve, when financial and technical variables are modelled together in one model?

1.6 Research Objective

The main objective of this research is to study the future risk of mining projects globally. Risk, as illustrated by volatility, is common in mining projects; however, price volatility would appear overestimated in valuing the mining projects (Haque et al, 2017). Based on the questions raised in section 1.4, then, the objectives are:

- To determine an improved future volatility risk identification by combining financial and technical variables.
- To determine whether forward-looking volatility using GBM processes and volatility is stochastic. This objective should ideally show that the future can mirror the past to some extent.
- To illustrate that dual volatility calculated under a risk-neutral world exemplifies the empirical situations of mining projects, when investigating volatility risk.

1.7 Research Aim

The research aim of this study is to demonstrate that dual volatility, made up of financial and technical variables, is more representative of the risky situation than volatilities made up of only financial prices. This research is focused on identifying the future risks that are represented by volatility. This volatility is firstly computed using the Sample Correlation Coefficient model by incorporating financial (e.g., mineral commodity prices and operating costs) and technical variables (e.g., metal grades and production rates) from Palabora Copper Mining Ltd, AngloGold Ashanti, Gold Fields Ltd and Sibanye-Stillwater Ltd for gold projects, and Anglo American Platinum Ltd and Impala Platinum Holdings Ltd for platinum projects. Then, the Sample Correlation Coefficient model is used to compute a portfolio variance, which can produce a dual volatility. In addition, the volatility is also calculated based on numerous relevant variables such as oil production, oil consumption, copper production and copper consumption as the technical variables plus the financial variables such as oil price, copper price and exchange rate on a monthly basis. Besides using this data, the gold price or platinum price on a monthly basis is included as the historical prices to forecast the future gold price or platinum price, respectively. The dual volatility is then examined using three hypotheses such as a Decision Tree Analysis, GBM model and the Risk-Neutral Probability approaches. For the robustness test, a Decision Tree Analysis is tested using the Markov-Regime Switching Model, which can be used to identify the most sensitive variable to affect a dual volatility. Then, the GBM model, using a dual volatility from numerous relevant variables, is tested using a Multiple Regression model. This model is implemented to identify the correlation between the dependent variables, which can be affected by the independent variables, which are used in the GBM model. The last hypothesis involves the Risk-Neutral Probability approaches being tested using the Vector Autoregression model. This model examines the numerous relevant variables in a single value. This value is used to identify the relationship between the numerous variables into a single commodity to forecast the future commodity prices.

Furthermore, the forecasting performance in predicting the future commodity prices using a dual volatility versus a mineral commodity price volatility with these three hypotheses is tested using the Root Mean Square Error and the Mean Absolute Percentage Error computation results. The forecasting performance can work well since the Root Mean Square Error and the Mean Absolute Percentage Error computation results are close to zero. Therefore, this approach can be selected and recommended to predict the future prices in mining projects.

1.8 Hypothesis

Null hypothesis (H_0): a dual volatility captures risks better than price only volatility,which is tested using three hypothesis such as a Decision Tree Analysis, GBM model and the Risk-Neutral Probabilities approach. Alternative hypothesis (H_1): a dual volatility does not capture risks better than price only volatility. These are examined using three hypotheses such as a Decision Tree Analysis, GBM model and the Risk-Neutral Probabilities approach.

Furthermore, in order to evaluate the future risks in mining projects, volatility was computed by combining the financial and technical variables to optimise the volatility risk, which is called a dual volatility. The optimal future risk by is solved using dual volatility by:

- Collecting the monthly metal prices, mined metal grades, operating costs and production rates;
- Compiling monthly data from gold and platinum prices including oil and copper supply, demand and prices;
- Computing the Sample Correlation Coefficient model using the mined metal grades, operating costs and production rates and data from gold and platinum prices including oil and copper supply, demand and price data;
- Computing a dual volatility based on this data;
- Comparing a dual volatility computation against Samuelson Effect Analysis and Gaussian Autoregressive Conditional Heteroscedasticity (GARCH);
- Examining a dual volatility to Hypothesis One using a Decision Tree Analysis;
- Examining a dual volatility to HypothesisT using the GBM model;
- Examining a dual volatility data to Hypothesis Three using the Risk-Neutral Probabilities approach;
- Comparing the forecasting performance between the forecasted future prices based on a mineral commodity price volatility and dual volatility using the Root Mean Square Error and the Mean Absolute Percentage Error;
- For the robustness test, employing the Markov-Regime Switching Model to test hypothesis one, a Multiple Regression model to test Hypothesis Two and a Vector Autoregression model to test Hypothesis Three with Eviews econometrical commercial software.

1.9 Research Gap

Currently, several methods exist for the measurement of risks by combining the financial and technical risks to calculate volatility. Cortazar et al (2001) studied the effect of incorporating price and geological-technical risks to optimise all exploration phases. However, this research did not take into account the daily operation flexibility during the lifespan of mine. Armstrong et al (2004) measured both components of the financial (e.g., oil price) and technical risks (e.g., geological factor) to value the oilfield project. However, the main weakness of the Armstrong et al (2004) research is that it appeared to focus on the geological variables, which failed to address the oil price fluctuations to predict the future risks. Castillo & Dimitrakapolous (2014) performed a similar series of experiments to show that commodity prices and metal grades risk as a part of financial and geological-technical risks, respectively. These research studies gave sufficient consideration to plan an initial pit limit. Unfortunately, the authors only considered the exploration phase, which offered no explanation for detailed mine scheduling in the production stage to identify the future risks. In a study conducted by Dehghani et al (2014), it was shown that they make an effort to take the production phase into account. They concluded that commodity price and production cost risk can associate to the future forecasting. However, research on this subject has been mostly restricted to assume that commodity price and production cost risk have the same fluctuation behaviour.

In contrast to the authors above, Guj & Chandra (2019) maintain using a single uncertain variable to predict the future risks in the mining projects. They concluded that a single variable produced more accurate prediction results than an incorporated cash flow volatility. The results used by Guj & Chandra (2019) to predict the future risk was similar to that used by Brandão et al (2012) and Savolainen (2016). Both Brandão et al (2012) and Savolainen (2016) shared that a dual volatility can significantly achieve positively biased results.

Although extensive research has been carried out on a dual volatility, no single study has been addressed to deal with a single conclusion. Previously published studies on the effect of a dual volatility on the future risks were only focused on the exploration phase. It was believed that the exploration phase cannot provide the operation flexibility to forecast the future risks. Despite this, little progress has been made in solving the operation flexibility issues in the mining projects. Investigating the operation flexibility is continuing concern with the production variables such as commodity prices, exchange rates, metal grades and processing recoveries in calculating volatility. The probability distributions of these variables were used as inputs to calculate the Discounted Cash Flows Net After-Tax and Operating Projects Cash Flows model (Guj & Chandra, 2019). However, this model also has a number of serious drawbacks, as it fails to specify the risks of each variable in a dual volatility. Therefore, a Sample Correlation Coefficient model is proposed in this research to compute a dual volatility using multiple sources of risk variables (commodity price, metal grade, operating cost and production rate) during the lifespan of mine. In order to forecast the future prices, this research has taken into account the key variables (e.g., oil and copper prices, oil and copper supply and demand, gold price and platinum price) that these variables were also calculated using a Sample Correlation Coefficient model to produce a dual volatility. The accurate forecasting of a dual volatility can be tested based on the Decision Tree Analysis, Geometric Brownian Motion and Risk-Neutral Probabilities. The closest average value between a dual volatility, with a pure price volatility as a result from these tests against historical price and profits, can be concluded as the most accurate forecasting method.

1.10 Assumptions

The following assumption has been made in this study:

- The data as extracted from secondary sources is correct.
    - Data from the companies is correct.
    - Economic information provided is also correct.

1.11 Structure of the Thesis

- Chapter 1 introduces the thesis.
- Chapter 2 is on comparative analysis literature.
- Chapter 3 reviews literature for the three hypotheses.
- Chapter 4 is on research design.
- Chapter 5 presents background on data that will be used for the empirical analysis
- Chapter 6 is on the empirical analysis.
- Chapter 7 concludes the thesis, including listing the contribution and future research areas.

2 Review of Literature on Alternative Valuation Methods

2.1 Conventional Valuation

According to the South African Mineral Code for the Reporting Mineral Asset Valuation in 2016, several mineral asset valuation methods currently exist for the valuation of the mineral and mining industry that have facilitated investigation based on the level of confidence of the minerals and reserves in South Africa. Different conventional valuation methods have been developed to classify the mineral asset valuation methods such as the Market Valuation Approach, the Cost Valuation Approach, and the Income Valuation Approach. Firstly, the term 'Market Approach' has been used to describe the historical market transactions from the similar completed transaction projects, such as the completed comparable transactions, the option agreements, and the gross in-situ value. The completed comparable transactions can broadly represent the similar valuation results from the similar mineral properties that can be extracted from the Deeds Office in Pretoria, South Africa such as the amount of money spent on completed transactions, a real in hectares, the transaction parties and purpose, and the transaction date (Lilford, 2004). The similar series of mineral projects indicate a valuation range that can represent the properties based on economic valuations such as cash flows, risks, costs and technical mineral deposit discoveries, including cut-off grade and mineral resources and reserves (Roscoe, 2002; Lilford & Minnitt, 2005). Investigating a mineral discovery is also continuing concern within geological aspects, mineralisation, neighbouring prospects, and infrastructure that offers some important insight into a premium value for mineral asset properties using the completed transactions (Roscoe, 2002). However, the performance of the completed comparable transactions in a mining project are limited by unavailable real comparison values such as real estate and oil and gas projects. The main challenges faced by the mining industry, such as geological aspects, mineralisation type, geotechnical issues, costs, and infrastructure in unavailable real comparison transactions, have confirmed that each project has unique characteristics even in similar commodities, technical and financial properties (Macfarlane, 2002; Costa Lima & Suslick, 2006). Since the value of real completed transactions do not exist in the market, option agreements can also be considered. Next, the term 'option agreements' is generally understood to mean the agreed price by both parties to take over the mining property. Criteria for selecting the subjects to value the property were best applied in early option periods for active exploration projects in many years. Eligibility criteria required in early periods have offered some real validation processes. A major problem with the method of option agreements is that it can perform well in early years of option. However, the project valuation has the possibility option to increase or decrease based on the most recent exploration results. These results suggest that the project valuation, based on the new results, can be negotiated (Roscoe, 2002).

Both the completed transactions and the option agreement methods that were used in the previous valuation techniques have shown that existing reference values are important to offer the value of a project. Since existing values are unavailable, these cannot demonstrate the inherent value as a basis for real transactions. To increase the reliability of measures, geological engineering databases such as geological occurrences, quality and quantity of metals, geophysical and geochemical instrument results, and ore grade distributions must be considered. In addition, the Kilburn (1990) method has a number of attractive features to include political issues, taxation and royalties, and exploration permits (Kilburn, 1990) while market capitalisation offers an effective way for listed companies to value the mineral assets (Lilford, 2004). Each variable can provide a value factor that can deliver the total valuation stated in U.S. dollar per unit (ha, ton, or oz). To control for bias, measurements were carried out by a competent geologist expert. Unfortunately, the mineral asset valuation is not limited by a strong geological understanding that must include financial, socio-political and legal aspects (Roscoe, 1988; Lilford & Minnitt, 2005).

In summary, one advantage of the Market Approach analysis is that it is simple, internationally comparative, and avoids the problem of data availability. However, there are certain problems with the use of the Market Approach analysis. A major problem with the experimental method is that it ignores technical considerations. Another major problem area in the market approach is in it only being used by listed companies on a stock exchange and the fair value is impeded by illiquidity (Lilford, 2004).

Second, one of the greatest challenges in improving the Market Valuation Approach is to warrant the future costs in valuing mineral assets using the Cost Valuation Approach. The Valuation Cost Approach can be loosely described as an appraised value method, which includes the depreciated replacement costs and historical costs to warrant the previous reasonable and productive exploration expenditures (Gentry & O’Neil, 1984). In broad terms, an Appraised Value Method is sometimes equated with the total previous exploration expenditure costs; while the depreciated replacement costs have been used in the past to value the building and plants that cannot be applied with mineral deposits (Lilford, 2004). Then, the historical costs encompass the current expenditure costs using a multiple of exploration expenditure methods that can improve to understand the geological deposits in valuing the mineral properties (Roscoe, 2002). The existing historical costs have been recognised to play a critical role in the maintenance of the current costs that should be inflated overtime by the time value of money. The time value of money is one of the most common procedures for determining a reasonable transaction price using the multiples of exploration expenditure methods (Lilford & Minnitt, 2005). However, the reasonable transaction price was based upon data from a significant exploration result to deliver an inherent value. Most studies in the field of transaction price have only focused on active exploration projects. Such idle exploration projects for a long period; however, have failed to address an inherent value from a realistic cost, based on incomplete exploration programmes. On the question of incomplete programmes, the exploration expenditures may change to anticipate the ore deposit target, technology improvements, market shifts and interest rates (Lilford, 2004). Before proceeding to examine the exploration costs, it is necessary to comprehensively investigate the minimum exploration standards. One concern expressed regarding the advantage of the multiples of exploration expenditure methods in fulfilling the exploration standards was to adopt the existing comparable exploration expenditure. However, there are certain problems with the use of multiples of exploration expenditure methods that may produce biased valuation results for non-active exploration projects for a long period. In order to identify the probability of future costs from non-active projects, the farm-in analysis can be taken into account to complete the outstanding exploration programmes.

Traditionally, the Farm-In Analysis has been assessed by measuring a mineral property based at a cost of exploration expenditures as proposed by junior mining companies. A case study approach was used to allow a larger mining company to buy the project equity from junior companies based on a cost expenditure or agreed price by both parties. The agreed price can be premium to accept either to control equity and management from the larger company. The design of the exploration expenditures by the larger company was based on the efficient programme to commence production (Lilford, 2004). The main disadvantage of the Farm-In Analysis is the exploration expenditures that cannot identify the probability of future costs. Kruezer et al (2008) described a Future Probabilistic Cost Approach using an Expected Value Method. One of the most well-known tools for assessing an Expected Value Method is a decision tree binomial lattice that can quantify the technical value. The advantages of a decision tree are that it is simple to deliver three main options such as; drilling target, targeting technique, and termination. The drilling target is of interest because it can tailor the target location based on the previous exploration results. Then, the targeting technique is extended towards the down-dip mineralisation zone based on drilling target results. The next option is suggested to terminate the further target that can be perceived as an uneconomic deposit. Kruezer et al (2008) measured both components of the decision trees to indicate the possible expected net present value from the cash flows in corresponding with the future exploration strategy and cost. A major advantage of the Cost Approach is that it offers the available costs and technical data as a basis for mineral asset valuation. However, the main disadvantage of the Cost Approach is to split the historical exploration expenditures with the future exploration costs that have been the subject of many classic studies to value the mineral asset valuation in an exploration phase. Since the performance of the Cost Approach is limited by an exploration phase, the Income Approach should be taken into account to accept the operating project valuation (Roscoe, 1988; Lilford & Minnitt, 2005).

Third, the meaning of this Income Approach has been extended to refer to the operating mineral valuation, which is determined based on the cash flows. The Project Cash Flows-based method provides a means of revenues subtracted by costs, taxation, and royalties (Lilford & Minnitt, 2005). One of the most well-known tools for assessing the mineral asset valuation using the project cash flows is the Discounted Cash Flows (DCFs, from here) (Macfarlane, 2002; Rudenno, 2009). The DCF can play an important role in addressing the issue of a mine plan that includes; (i) available resources and reserves which corresponded with the proposed production rate per annum, (ii) monthly tonnage production and grade, (iii) ore mined and recoveries, (iv) operating costs, (v) capital costs, and (vi) mine rehabilitation cost including other remediation costs (Lilford and Minnitt, 2005). In addition, the conventional DCF based methods provide a means of valuable time, where the current value is higher than the future value which furthermore is compensated using the discount rate. There are three primary aspects of this discount rate in a mining property. Firstly, to investigate the risk-free rate in the long term, which can be guaranteed from a specific bond acquisition of risk-free return from a government bond yield. The term ‘government bond yield’ refers the interest rate that the government borrows at, and is usually based on the current inflation of the same government. These results show an overview of project risk quantification. Secondly, to ascertain mineral project risk perception. Finally, to identify country risks such as political and social risks that can be controlled by civil stability, political conditions, and other factors.

Some of the main factors can be quantified by Moody’s or Fitch as an international standard of credit ratings that can govern to effect an internal rate of return such as a specific operation including general project risks. The use of qualitative case studies for discount rates in a specific operation can be determined by economic and financial knowledge using the capital cost. Another significant capital cost should be taken into account to exploit new reserves during the project life by implementing differential discount rates. The risk solution was then clarified for capital cost using at least one discount rate to generate the DCF. A Case-Study Approach was chosen to adopt the project since the sum of the DCF stated that the net present value (NPV) is positive. All studies described as using some sort of variables above were included in the Sensitivity Analysis. In this analysis there are several sources for risks as a critical value that can be forecasted using a step-wise model. The results were adjusted and then weighted each variable based on its own risk according to valuator perception (Lilford & Minnitt, 2005).

A major problem with the DCF method is that the commodity price of the mineral asset valuators often employs the mean of historical prices to forecast the future prices, known as ‘long-run’. In addition, the synthesis of the high-level accuracy of the DCF remains a major challenge from sufficient technical information in completing the cash flows. In order to assess incomplete technical information in mineral projects that can be observed using a discount rate. In particular, the analysis of the DCF was problematic in alleviating the discount rate to equal with risks that were normalised using the technical inputs. However, another disadvantage of the DCF method should be solved using an equilibrium model to improve the discount rate that comes up with an appropriate value of risk-adjusted discount rate such as the Capital Asset Pricing Model (CAPM). The CAPM approach has a number of attractive features for mining sectors: (i) a main advantage of the CAPM approach in the mining sector is the price undiversifiable financial risks, such as commodity price and exchange rate risks, which can anticipate the unexpected returns by hedging, (ii) in analysis of commodity price risks, even in an unhedged condition was found in mining project that was directly uncorrelated with the market portfolio that was suggested to employ a small risk premium. Lastly, in particular, the analysis of the DCF assumed that once the projects were started, these were necessary to continue to exhaust the total reserves.

However, in the real world, the project managers should have flexible discretion to expand, suspend, or abandon based on DCF calculations using the Option Theory (Moyen et al, 1996). Finally, the mineral asset valuation was then solved using an Option Pricing Approach represented by real option value that offers several options in valuing mineral properties (Macfarlane, 2002). The Option Pricing Model is as an alternative approach to improve the conventional DCF that offers an asset management flexibility by quantifying the additional asset value. In most recent studies, there are two main different ways to value the mineral asset between the real option value and the conventional DCF, (i) the risk of the real option value is adjusted within the cash flow items while the risk of DCF is discounted from at the total net cash flow, (ii) a small discrepancy of the Real Option Value Method allows to differentiate assets based on their characteristics (Samis et al, 2006).

Both the DCF and Real Option Value methods were based on the completed technical financial database (Lilford & Minnitt, 2005). One of the most significant current discussions in incomplete databases using the Lilford Techno Economic Matrix (2005) method is to perform a gold project cash flow using the Rand per hectare method. The Rand per hectare method in valuing the mineral properties was based on four main input variables such as; the mineralisation depth, the resources and reserves classification, the metal grade, the active mining project vicinity or the existing infrastructure. The average depth under the surface is associated with increased risk of capital expenditures to extract the ore. The high capital expenditures impact negatively upon a range risk of returns that can be adjusted using an appropriate risk premium and discount rate. Studies of risk premium and discount rate show the importance of resources and reserves classification of the projects based on the globally recognised reporting codes from the South African Code for Reporting of Mineral Resources and Mineral Reserves in 2016. Throughout this section, the term mineral reserves is used to refer to indicate eventually reasonable and realistic economic ore extraction. An economic ore exploitation has been assessed by known metal prices and exchange rates. The changes experienced by metal prices and exchange rates lead to a renewed reserves classification including cut-off grade. In light of events in reserves classification, it is becoming extremely difficult to ignore the existence of cut-off grade, especially for non-producing projects. Taken together, these results suggest that there is an association between a grade group above cut-off grade with the mineral resources and reserves classification based on the known metal prices using unit price per ha valuation approach (Lilford, 2004).

The term “an average grade for non-producing projects” has been used to refer to situations in which an in-situ grade is stated in unit per ton. An in-situ grade can be loosely described to determine an ultimate prospect value that has a conjunction with other important variables to define economic cut-off grade. Previous studies have based their criteria for selection on cut-off grade greater than 3 g/t for a South African underground gold operation. However, a cut-off grade can adversely be affected under certain conditions such as shallow and wide deposits that can allow an economic factor to operate underground at 1 g/t gold based on the current gold price. It is necessary here to clarify exactly what is meant by cut-off grade in the in-situ grades (Lilford, 2004). A cut-off grade can broadly be defined as the minimum or average metal grade that can economically be extracted based on the current commodity prices (Asanloo & Ataei, 2003). While in-situ grades tend to be used to refer to grades for non-producing projects that do not naturally express the host rock of occurrence such as in oxide, transition or sulphide zone, or placer. A Case Study Approach indicates that no significant reduction in metallurgical recoveries were found compared to these different host rock types. The metallurgical processes will be prepared according to the laboratory testing results to optimise the recovery. In addition, the technology of metallurgical processes was synthesised using the same method that was detailed for the capital cost estimates, using a mineral property proximal analysis. Samples were studied for proximal analysis according to an area of influence. An Area of Influence method is one of the more practical ways of estimating the costs that can depend on the distance between the undeveloped projects with existing infrastructure. On completion of cost estimates, the process of variable analysis was converted using numerical factors. After the numerical values were fitted, the gold projects were valued that should be consistent with the future actual transactions. Lilford (2004) found that most recent completed transactions can improve the variables of cost estimates, which can be accessed from South Africa's Deeds Office in Pretoria. Extensive research has shown that these transactions should represent financial conditions such as metal prices, exchange rates, inflation, and interest rates. These variables are limited by financial conditions that are an urgent need to address the technical issue such as polymetallic economic minerals, metallurgical recoveries, environmental issues, orebody shape and orientation, and geotechnical factors to achieve a better valuation result for the gold mineral projects.

In addition, the main challenge faced by many valuators is to value non-gold commodity, significant dipping and non-tabular or irregular orebodies to perform orebody shape and orientation. The non-gold commodity concept has recently been challenged by gold-equivalence studies demonstrating the matrix modification. One major issue in early gold-equivalence research concerned in ratio between the revenues and the prevailing gold price. These results must be divided by estimated tonnage to come up with a gold grade that will be stated in quantity such as tonnes or ounces and grade. Turning now to significant dipping orebodies rather than the South African mineralisation standard that the matrix must be adjusted to allow the dip increases. Due to practical mining constraints, very steeply dipping deposits must consider the maximum mining depth. The main issues addressed in this deep mine operation are geological risks and economic constraints. The term 'geological uncertainty' will be used to refer to non-consistent dimensions of the deposits, which are challenging in valuing the mineral properties. In an attempt to value the mineral properties with insufficient technical information, the Lilford Techno Economic Matrix method must be taken into account. The Lilford Techno Economic Matrix method offers a number of attractive features such as a simple matriculation system, minimal technical data, updated valuation using actual transactions, and involving economic and political variables. However, this method needs continual matrix updating, experience valuators, and subjective interpretation to solve risks (Lilford, 2004).

Risks become inherent in the mineral projects that were previously simulated using sensitivity analysis and a risk adjusted discount rate. Recent advances in computers have facilitated investigation of risk using Monte Carlo Simulations that associate with the mineral assets valuation. The Monte Carlo Simulations are one of the most common procedures for determining the probability distributions from the random data inputs. The probability distributions provide new insights into the Monte Carlo Simulations using the iterative simulation results. By employing the iterative simulation results, these cannot address the project risks that can be reduced by; (i) hedging for commodity prices (ii) derivatives for currency (iii) secure assets by contract and (iv) insurance for political and commercial risks. Another potential problem for the project risks is that the critical input variables will deliver economic constraints. A number of economic constraints can change the cut-off grade and the resources and reserve reporting that will impact a mine lifespan. Since the cut-off grade moves from uneconomic to being a profit project, the mine plan will change based on the updated economic cut-off grade. It has been demonstrated that the main advantage in using the Monte Carlo Simulations is that the cut-off grade changing can result in the outcome of investors making decisions. Using the DCF results in future decisions being analysed using decision trees. The future analysis should be discounted using discount rates for the future cash flows. However, the main disadvantage of the Monte Carlo Simulations is that it has the same weaknesses as DCF, which can only offer “yes” or “no” options. In order to assess a decision, these simulations often require complicated modelling systems (Lilford, 2004).

2.2 Overall Weaknesses of Conventional Valuation

The literature on conventional valuation methods has highlighted several weaknesses to value mineral assets. In observational studies, there are potential disadvantages from the completed comparable transactions, the option agreements, and the gross in-situ value in the Market Approach. In particular, the analysis of the completed comparable transactions subjectively judges the value of mineral property based on the similar projects. However, these cannot represent the true value of transactions because each mining project is unique. Then, different theories exist in the literature regarding an option agreement, where the mineral asset was prepared according to the exploration report procedure. However, there are certain drawbacks associated with the use of an Option Agreement Approach, which can perform well based on the existing data. Once the database of the mineral projects is incomplete, that the mineral property value probably moved up or down from the option agreement contract price (Macfarlane, 2002). Another existing literature on the Market Approach is concerned with the unit price per tonnage resources and reserves in valuing the mineral assets. Unfortunately, the main criticism of this method is that it does not take into account; metal prices, currency, economic and country issues to value the inherent mineral asset properties (Lillford & Minnitt, 2005).

Much of the literature since the 1990s emphasised the mineral asset valuation using the Cost Approach (Roscoe, 2002). This theme came up for example in discussion of an Appraised Value Method and the Historical Cost Method. Concerns were expressed about the Cost Approach, which employed the past exploration expenditures to value the mineral assets. The majority of these may change over time, where the previous exploration costs have been out-dated. Unfortunately, a major problem with this kind of application is that total previous exploration expenditures which replace costs are subtracted using accrued depreciation from the plant and equipment (Paschall, 1999; Ellis, 2011). One of the most significant current discussions in depreciation is to improve cost analysis by replacing or employing historic cost analysis (Gentry & O'Neil, 1984). However, there are certain drawbacks associated with the use of the Historic Cost Approach, which cannot generally be accepted to mineral deposits for mineral appraisers. The performance of mineral appraisers is limited by long time periods of non-active exploration projects. As a result, the Cost Valuation Approach on the subject has been mostly restricted to limited historical expenditures of the exploration phase, where the Income Valuation Approach is then suggested to value an objective operation project (Roscoe, 1988; Lilford & Minnitt, 2005).

Finally, questions were asked as to the role of the Income Approach in valuing the mineral assets using the DCF. Several weaknesses of the DCF method are that; (i) most companies in the field of mining have only focused on price sensitivity, (ii) the commodity prices were forecasted using a long-run mean, Monte Carlo Simulations, and consensus for the most large mining companies by incorporating with the financial consultants, (iii) the time value of money was often applied by employing the real or constant dollar prices, (iv) the risk rate of the DCF approach was subjectively and poorly understood that was adjusted moving up to compensate the extreme political risk or moving down to attract the investors (Bhappu & Guzman, 1995; Moyen et al, 1996), and (v) in observational studies, there is a potential for bias from the DCF that often implies same risk-adjusted discount rate in the different project risk characteristics (Salahor, 1998). In addition, the DCF does not perform well with high risks and flexibility, where the Real Options Approach is required to prove great risk (Copeland & Keenan, 1998a).

2.3 Unconventional Valuation

2.3.1 Real Options

Much of the current literature on the Real Option analysis pays particular attention to mining sectors that have emphasised the use of flexibility in mining (Shafiee & Topal, 2010). A common view among researchers was that the flexibility in mining significantly appeared to lead to the expansion, suspension, shut down or temporary closure of different mining cycles (Moel & Tufano, 2002). Strong evidence of increased profit using the Real Option analysis was found when the project was selected to shut down or temporarily close, rather than the DCF (Samis et al, 2006; Guj & Garzon, 2007). An increased profit from several operational flexibility studies suggest that the Real Option value revealed to maximise the mining value (Shafiee & Topal, 2010).

Previous studies have explored the relationship between the DCF and the Real Option value. The generalisability of much published on the variable estimation issue of the DCF is problematic. Several systematic reviews of the DCF variable estimation have been undertaken using the Real Option models. Much of the Real Option value models' research has focused on identifying and evaluating a time-varying discount rate to discount the single cash flow items. Detailed examination of mineral price was discounted from a risk revenue stream while labour or fuel price was discounted from a cost stream. In an analysis of the mineral future price, the advantage of the Real Option value models did not take into account the discount rates of price which are embedded with the forward price (Samis et al, 2006). Salahor (1998) forecasted the forward prices underpinned by financial theory, and in that, he contrasted two theories-the CAPM and arbitrate pricing.

However, in Salahor (1998), the future price of projects for the similar characteristics of risk at the same times can come up with the same present value. Another risk can readily be investigated to calculate the future price since in the method such as delivery date and input price are well-identified. The major source of risk is the commodity price, which is 'neutralised'. Following conformational analysis of neutralised, it was necessary to assume the costs are certain, and as such, the result of cash flows are also certain. To enable the subjects to see the certain cash flows clearly, a time-adjusted and risk of discount rate will outperform to identify the risks. The certain time-adjusted and risk of discount rate can configure the risk that must be compensated using the risk-free rate to replace the time value of money from the DCF. The solution of the time value of money from the DCF was then solved using the modified ROV models that entail the volatility neutralisation of the overall project cash flows involving all the sources of risk, besides the prices (Guj & Garzon, 2007).

In an analysis of the project volatility, Davis (1998) found that the Real Option approaches were required by management to determine the flexible option. A great deal of previous research into the Real Option approaches has focused on one of the advantages that offers the managerial flexibility, once the project is started, by responding the production rate changing. Early examples of research into the managerial flexibility proposed by Shafiee & Topal (2010) to include a negative net present value result, where the ROV models have attempted to explain mine closure once the zinc price drops or reopens to increase the zinc price. Concerns were expressed using a Shafiee Topal and Nehring model about the estimated cost to close temporary or permanent scenarios to optimise the risk and flexibility (Shafiee et al, 2009).

2.3.2 Risk-Neutral Probabilities

There is evidence that volatility plays a crucial role in regulating the future risk in optimising the risk and flexibility using the RNP methods. Volatility is an important aspect of the RNP that is established based on the historical transactions from the financial market trading as a proxy. Exposure to the past volatility has been shown to be related to adverse effects in the unavailable historical data of the historical transactions. One of the most significant current discussions in volatility is assumed to equal with the commodity price. Traditionally, the price volatility has been assessed using a GBM approach, where a number of issues identified that the price volatility of gold was lower than that of the project volatility. Characterisation of volatility is important for an increased understanding of Real Options valuation, which indicates high volatility in mining sectors (Guj, 2008). Hall & Nicholls (2007) demonstrated that when high volatility is essentially influenced by the global market, it can come up with high systematic risk. To rule out the possibility of high risk, a project volatility will be taken into consideration. Interestingly, a project valuation was observed to deliver an appropriate valuation result, which can be reached by discounting the risk-free rate. The Appropriate Valuation approach revealed the expected future values probabilities known as the RNP method, which concluded options pricing to discount the expected risk-neutral pay offs and the risk free-rate to provide the risk adjusted capital cost expected present value and the same expected present value. Samis et al (2006) identified that the present value is one of the major causes of future cash flows composing a risk-free and risky asset by replicating a portfolio under free arbitrage conditions.

Jin-suo & Shao-hui (2008) introduced two-factor and mean-reversion models employing risk-free rate as stochastic, which assumed short-term metal price deviations and stochastics long-term prices following a mean reversion. Cortázar & Schwartz (2003) measured dynamic oil price using the risk-neutral (risk-adjusted stochastic processes) that can be obtained for the spot price and the convenience yield by substituting the long-term total price on oil with the risk-free rate. The theory of risk-free rate provides a useful account of how to discount the certainty equivalent value to compensate a timing to calculate the Real Option value. This evidence suggests that the certainty equivalent plays a crucial role in regulating obtaining the RNP that can firstly quantify risk by neutralising an appropriate algorithm. The RNP is fundamental to neutralise the risk, where all input variables have discounted the present value results using the risk-free rate. Much risk still exists in the relationship between all variables of financial risk, which employs a subjective factor to assess the technical risk probability (Guj, 2011). One major issue in early risk probability assessment is assuming that oil price behaviour is stochastic (Cortázar & Schwartz, 2003), where the RNP should be adjusted to follow a commodity price behaviour to forecast option prices.

Dixit & Pindyck (1994) proposed cost and interest rate as a source of risks, rather than a commodity price behaviour to forecast option prices. They believed more information availability, such as cost and interest rate for the projects, plus technical risk might decrease the level of risk. In addition, more information availability can contribute to provide flexibility in making a decision to invest, postpone or relinquish the projects since it can add more variables than commodity price.

The price volatility is computed by a standard deviation of commodity price, since it increases linking with the low operating cost which cannot be applied to long-term mine projects. There were some suggestions that the project volatility can only decrease to respond to the significance of price increasing or operating cost reduction. In summary, these results indicate that the price and cost strongly associate with the project volatility, while the production capacity and taxation are independent (Costa Lima & Suslick, 2006). However, the main challenge faced by many researchers is that the project volatility can be completely different to the commodity price volatility (Dixit & Pindyck, 1994; Trigeorgis, 1996). The project volatility is one of the most frequently stated problems with its implication in the valuation of the mineral assets and the decision making. The difference between the project volatility with the price volatility in the financial assets may be caused by not only the commodity price fluctuation and operating cost, but with their correlation with the cash flow as well. A comparison of the two results in the gold producer reveals that the project volatility responds higher rather than the price volatility to indicate the undervalued assets relating with non-optimal decision making (Costa Lima & Suslick, 2006). Taken together, these results suggest that employing the portfolio volatility incorporates financial variables such as commodity prices and technical variables such as cost, metal grade, and production rate.

3 Literature Review for the Hypotheses

3.1 Introduction

The literature review focuses on three hypotheses to determine an improved future volatility risk identification by combining financial and technical variables, that are; (i) to develop a Decision Tree analysis to determine the best decision, (ii) to determine futuristic volatility using the GBM process, where volatility is stochastic and (iii) to illustrate that dual volatility calculated under a risk-neutral world exemplifies the empirical situations of mining projects, when investigating volatility risk. In order to examine these three hypotheses, a Sample Correlation Coefficient model is proposed in this research to integrate the multiple sources of risk variables that can be represented by a covariance divided by standard deviation product of variables. The crucial point of this calculation result can explain why a Sample Correlation Coefficient model is selected in this research, which can standardise two or more variables (Asuero et al, 2006). Since it can standardise two or more variables and uses them as a benchmark, a Sample Correlation Coefficient model can become a critical tool to compute dual volatility. Then, dual volatility, as a second hypothesis in this research, explores empirical techniques that can be used to optimise risks by incorporating multiple sources of risk in mining projects.

This research in optimising risks by incorporating multiple sources of risk to date has not yet included the production rate to be incorporated into the financial and technical variables. This study seeks to obtain data which can help to address these research gaps in computing volatility in the mining projects by incorporating the financial and technical variables by the extent to which Souza et al (2019) and Akbari's et al (2009) studied. Souza's et al (2019) proposed a mineral commodity and production rate risk as the variables to assess the future risks. In the same view, Akbari et al (2009) obtained a mineral commodity price, metal grades and cost variables to investigate the long-term risks. While, in order to forecast the future price using a GBM model, this research extended Liu's et al (2017) and Alameer's et al (2019) studies by taking the technical variables into account to optimise the future risks. Then, the last hypothesis discussed is to optimise the future profits using the risk-neutral values. This model is developed by the traditional RNP by modifying a mineral commodity price volatility to a dual volatility. A dual volatility as calculated in hypothesis two can accept the future risks since it combines the financial and technical variables to optimise the future profits.

3.2 Basics of Volatility Calculation

Volatility plays a critical role in measuring risks which are associated with the future financial trends, which affect price changes and profits (Brunetti & Gilbert, 1995; Guj, 2011). Lima & Suslick (2006) showed that the effect of risks was price represented by volatility. One of the most well-known tools for assessing the volatility is using the historical prices (Miller & Park, 2002; He, 2007; Guj, 2011) and volatility is computed from the logarithm of the current price divided by the logarithm of the previous price (Ladder & Wu, 2009, Haque et al, 2017). However, the historical prices as determined by volatility, cannot represent appropriately the future volatility risks (Gordon, 1998). The fundamental role of volatility in assessing the future risks was suggested to employ the correct model (Behmiri & Manera, 2015). In order to solve volatility from historical price, Canina & Figlewski (1993) proposed two basic models: (i) to calculate the realised volatility from historical data during the past, and (ii) to compute an "implied volatility" based on current option market prices. As a result, an implied volatility is solved by setting the volatility of pricing model as equal with market prices. Haque et al (2017) proposed the model that the price volatility is computed from the historical price by computing the log returns of commodity prices as a ratio from current day commodity prices to previous day commodity prices. Then, the volatility is determined by the square root of the commodity price variance. Saliba & Dimitrakopoulos (2019) proposed a mean price volatility per annum to simulate the gold prices forecasting using GBM, which incorporates a Poisson Jump Diffusion process. A price risk can also be applied to the production planning. Mokhtarian & Sattavand (2016) considered the price risks to plan the long-term open-pit production planning using an Economic Block model. This proposed model can reduce the future risks of the net present value with an 80% of confident level.

Most studies in the field of volatility have only focused on mineral commodity prices. Canina & Figlewski (1993) argued that the volatility of one mineral commodity prices was purposed to improve historical volatility using a price volatility. A price volatility measures an ex-ante to identify the asset price risk perception. The asset price risk perception is represented by the volatility of market prices expectation, which is assessed by the Standard Fixed-Volatility Approach. The Standard Fixed-Volatility Approach is proposed to examine the efficacy of calculating a price volatility. The Standard Fixed-Volatility Approach provides easy access for a price volatility for trader options. Trader options can decide a price volatility for trading and hedging by forecasting a volatility (Canina & Figlewski, 1993). However, Lamoureux & Lastrapes (1993) concluded that a price volatility was less accurate in forecasting volatility, by using option expiration rather than the models that were developed by the incorporation of historical prices. Therefore, incorporation of historical prices was used to calculate historical volatility. Canina & Figlewski (1993) argued that a price volatility was superior to define historical volatility, since it was purposed to estimate future market volatilities. Therefore, the future market volatility was observed from the market price by computing a price volatility.

Another benefit of price volatility in representing risk was that it can perform well for the project valuation. Al-Aradi et al (2018) proposed a model to assess the future risks using the ore reserves that were determined from a price volatility. This model presented an additional stochastic technique to estimate a price volatility to identify both the best option and project valuation. Using a price volatility, it is possible to significantly determine the mining project valuation from historical data (Haque et al, 2014). Given the challenges in finding historical data from the available sources, Haahtela (2012) proposed a cash flow simulation to account the future risks. Using a cash flow simulation, the future risks, as illustrated by volatility, can be represented by a price volatility. In addition, a study by Copeland & Antikarov (2003) valued the mining project cash flow using the dynamic future price risk. Another scholar employed metal prices to evaluate the project valuation using the Monte Carlo Simulation model.

Smith (2005) criticised Copeland & Antikarov’s (2003) Monte Carlo model to compute a volatility, where the present value of year one simulation was equal to the net present value based on the future cash flows. The major challenge facing the Monte Carlo Simulation, when computing volatility, was that it cannot accept the sensitivity of the project value such as a mineral commodity price. The Monte Carlo Simulation provided a constant volatility throughout the lifespan that can fail to deal with the future risks as reflected by a commodity price sensitivity in the long-term projects (Black & Scholes, 1973). In order to accept commodity price sensitivity, Davis (1998) suggested to compute a volatility using closed-form expression to properly accept the price fluctuations. In closed-form expression, the price fluctuations can forecast the risk including the cash flow effects caused by these fluctuations during the mine lifespan (Davis, 1998).

Another criticism by Ugwegbu (2013) about the Monte Carlo Simulation, as illustrated in Copeland & Antikarov (2003), is that the Monte Carlo Simulation overestimated values of mining projects when compared with traditional valuation methods, such as DCFs. These results were similar to those reported by (Haque et al, 2017). A possible explanation for overestimated results of project valuation using a pure price volatility were caused by a greater risk in mineral commodity prices than other economic variables (Foo et al, 2018).

The findings reported by Vavra (2014) appear to support the assumption that a greater risk in mineral commodity prices has more dynamic countercyclical of volatility in time-varying series than the business cycle. Countercyclical price flexibility was well performed since volatility seemed to have an inverse relationship with productivity. Although, volatility has a direct relationship with productivity, it still cannot perform a perfect countercyclical price flexibility, since the Second Moment Volatility Shocks Model employed aggregate nominal shocks (Vavra, 2014). The Second Moment Volatility Shocks Model responded time-varying with price flexibility. Moreover, this model has a positive correlation between the dispersion of price change and frequency. A positive correlation allowed for time-series movements that were relatively close to the real data. Therefore, time-series movements can associate with the volatility shocks, where these corresponded to price flexibility to forecast the short-term period (Vavra, 2014). A more comprehensive study by Adrian & Rosenberg (2008), confirms that business cycles can be linked to the flexibility of prices. However, a specific economic condition in a different sector should involve different phases of a business cycle to improve macroeconomic factors and historical information datasets from all economic sectors and indices. All economic sectors and all indices cannot be assumed as equal to represent the diversification risks. Therefore, the diversification risks can be responded to by the different phases of a business cycle, since it extracted the statistical factors from sector-based correlations. The sector-based correlations from different phases of a business cycle can improve in identifying the diversification risks. The diversification risks of a specific sector can provide the volatility shocks from the different business cycle phase (Buss & Vilkov, 2012).

The procedures of the study, based on the business cycle phase to identify the volatility shocks, were approved by Day & Lewis (1992) and Lamoureux & Lastrapes (1990). They examined the volatility shocks by employing macroeconomic variables. The assessment of high volatility probability was one of the effective ways in tracing the volatility shocks that linked to the macroeconomic variables, such as the business cycle (Li, 2021). Hall & Nicholls (2007) demonstrated that a high volatility was essentially influenced by the global market. This can provide a high risk of project systematically. However, since the volatility shocks existed, as represented by a high volatility, these ignored the business cycle in the market. Li (2021) concluded that the volatility shocks can be solved using a price volatility since the market is predictable. However, it seemed to poorly perform in the mining sector since it delivered the unpredictable future markets to identify opening and closing costs. The current literature on the unpredictable markets in mining projects, as represented by high volatility, pays particular attention to a hedging strategy in avoiding loss (Haque et al, 2016). There is evidence that a high volatility in mining projects is mainly contributed to by a price volatility from a mineral commodity price. Moel & Tufano (2002) suggested a price volatility in the mining sector from the Real Options model were akin to the opening and closing costs. The opening and closing costs were consistent with the probability of opening and closing that were determined from Monte Carlo model. The Monte Carlo model extracted the sample data to calculate the distribution of independent variables to represent the random variables. As a result, a simple Monte Carlo model was the alternative option to calculate risk, since the opening and closing costs cannot be measured from the cost benchmarks of other mines.

One possible implication of computing the risk of the cost benchmarks of other mines did not exist, where these should be estimated using similar commodity, financial and technical aspects of the mining projects datasets listed on a stock exchange (e.g. Toronto Stock exchange, Johannesburg Stock Exchange, Australian Stock Exchange and other relevant listed mining companies stock exchange globally) (Hull, 1997). Another major source of risks was determined using a mineral commodity price. However, research from Batten et al (2010) examined the four precious metal volatilities from the monthly prices (e.g. gold, silver, platinum and palladium) to investigate the most significant mineral commodity price associated to the macroeconomic variables such as the business cycle. The finding of this study suggested that gold price only can have an effect on the business cycle. In a follow-up study of gold's effect on the business cycle from Batten et al (2010), Muhammad et al (2019) found that gold price has a lower volatility than other precious metals. In summary, gold price is superior to respond the future risks including a project volatility.

A project volatility was one of the most relevant variables with its implication in the valuation of the mineral assets to making decisions. Lima & Suslick (2006) argued that theoretically, a commodity price volatility was equal to a project volatility since the operating costs are not involved. A project volatility has an inverse relationship to a mineral commodity price volatility, such that a project volatility decreased with an increase in a mineral commodity price volatility. A project volatility reflected a direct relationship with the operating costs, in that a decrease in the operating cost volatility was consistent in decreasing a project volatility. Since a project volatility changed consistently to a mineral commodity price volatility and the operating costs volatility, it can be used to identify the existence of high risk in the projects (Hall & Nicholls, 2007). In order to deal with a high risk in the project, they computed a project volatility by discounting the risk-free rate model to obtain an appropriate project valuation result. Davis (1998) determined a project volatility using the production rate variable to deal with the future flexibility option when the mining projects were commenced. However, the main challenge faced by many researchers was a project volatility which can be completely different than a mineral commodity price volatility (Dixit & Pindyck, 1994; Trigeorgis, 1996). A comparison between a project volatility and a mineral commodity price volatility in the gold mines revealed that a project volatility responded to a higher volatility than a price volatility. Therefore, it indicated the undervalued assets relating with non-optimal decision making (Lima & Suslick, 2006). In addition, it has a number of serious drawbacks with non-optimal decision making since it used a single variable such as a mineral commodity price, the operating costs or production rate variables including only their correlations without incorporating them in calculating a volatility to indicate the future risks.

The main problem in investigating the risks as illustrated by a mineral commodity price or other single variables volatility including their relationships failed to capture the multiple risks in the mining projects (Sohrabi et al, 2022). In the follow-up phase of the multiple risks in the mining projects, Copeland & Antikarov (2003) improved the future risks by incorporating the multiple risks in the mining projects using the marketed asset disclaimer model. Barton & Lawryshyn (2011) developed an aggregated cashflow volatility to measure a major source of risks. Dehghani et al (2014) revealed that a price volatility from the multiple sources of risks such as a mineral commodity price, together with operating costs, were the most crucial variables to indicate the future risks. According to Dehghani et al (2014), both operating costs and a mineral commodity price in calculating a project volatility generally have a similar trend that can achieve better results in determining the future risks. Lima & Suslick (2006) argued that the project volatility has a relationship with the price and cost, while it was independent to the production rate and taxation variables. Another scholar such as Liu et al (2017) revealed that the independent variables played a role in determining the effects on the copper price. Liu et al (2017) demonstrated eight external variables (two energy prices such as oil price and natural price, three metal prices such as gold price, silver price and copper price, two commodity prices such as lean hogs price and coffee price and one stock market such as Dow Jones Industrial average). In contrast to Lima & Suslick (2006) and Liu et al (2017), Ramazan & Dimitrakapoulos (2010) extended the Dehghani et al’s (2014) variables to add tax and inflation variables using Lima & Suslick’s (2006) model. Ramazan & Dimitrakapoulos (2010) concluded that operating costs, tax and inflation as economic variables were dependent variables to a mineral commodity price. According to Alameer et al (2019), the main challenge faced by many researchers was the dependent variables to forecast the future gold price risk. They used ten variables of economic variables (three exchange rates from India, China and South Africa, two inflation rates from U.S. and China and five commodity prices from west Texas intermediate crude oil, copper, silver, iron and gold) to assess the future gold price. Another technique to forecast the future risks using economic variables have been proposed by Haque et al (2017). They examined the joint effect and correlation between the iron ore price volatility and the Australian dollar to U.S. dollar exchange rates over a 15 year period. This research concluded that the joint effect and correlation between the iron ore price volatility and the Australian dollar to U.S. dollar exchange rates can accurately predict the future risks.

Different theories exist in literature regarding the correlation between the exchange rates and the mineral price commodity in order to assess the future risks. Ciner (2017) revealed a correlation between the South African currency, platinum and palladium prices. It was consistent that South Africa is the largest platinum and the second largest palladium exporter in the world. Brown & Hardy (2019) demonstrated the Chilean exchange rate as the most relevant predictor variable to copper prices. Copper was a major influence in Chilean exports, which was approximately 23% shares of the global copper supply. While, it has stimulated over 45% of the foreign direct investment with copper which has been over half of total the Chilean exports. Aminrostamkolaee et al (2017) proposed two stochastic process models to consolidate the mineral commodity price and exchange rate volatilities and correlations.

The existing literature on the future risks is extensive and focused particularly on economic variables, where less effort is dedicated to the inclusion of economic policies. Zhu et al (2019) concluded that these were likely the more relevant variables to represent the stock markets coming from macroeconomic and economic policy as reported by the major US newspapers. These theories were similar to those studied by Balcilar et al (2019) in that they employed domestic and global economic policy uncertainties (EPUs) from China, Europe, Japan and the U.S. The domestic and global EPUs databases from China, Europe, Japan, and the U.S. were extracted from the International Financial Statistics database of the International Monetary Fund. These datasets were particularly useful in measuring volatility in the stock returns (Balcilar et al 2019). However, Zhu et al (2019) and Balcilar et al (2019) studied the macroeconomic and economic policy variables using the major global and local published newspapers which ignored an inherent volatility calculated from the real economic conditions (Bonaime et al, 2018). The volatility calculations using the major global and local published newspapers were subjectively determined to contribute to the existence of ambiguity to identify the future risks. New additional information can replace the existence of ambiguity in calculating a volatility (Haahtela, 2012). Studies such as Moel & Tufano (2002) illustrated that the future risks were subject to capturing the financial, technical and social variables to reflect the relevant risks rather than only being assessed using the economic and policy variables. In order to improve the future risks in mining projects, it was suggested to incorporate the financial, technical and social variables into a single volatility calculation. As a result, a dual volatility using the financial, technical and social variables can represent a probability distribution, which can clearly define the major sources of risks in the mining projects (Haahtela, 2012).

Over the past two decades, most research in dual volatility has emphasised the use of a mineral commodity price risk and geological-technical risks. Cortazar et al (2001) optimised all exploration phases by studying an incorporating effect of a mineral commodity price risk and geological-technical risk. However, this study has not dealt with daily mine scheduling flexibility for the lifespan of a mine. Armstrong et al (2004) hold the view that the financial (e.g., oil price) and technical risk (e.g., geological factor) variables were proposed to value the oilfield project. However, they were extensive and focused more on the geological variables, which failed to address the oil price risk to forecast the future risks. Castillo & Dimitrakapolous (2014) identified the financial and geological-technical risks as the major cause of the future risks. In their review of the future risks, the financial (the mineral commodity prices) and geological-technical risk (metal grades) variables can affect an initial pit limit consideration. Savolainen (2016) drew on an extensive range of the financial risks and geological-technical risks to assess the future risks based on a mineral commodity price and orebody using simulation and dynamic programme models. Chatterjee & Dimitrakapoulos (2019) improved an open pit mine schedule under geological variables using an algorithm model. Unfortunately, the authors just considered the exploration phase, which offered no explanation for detailed mine scheduling in the production stage to identify the future risks.

Rimele et al (2020) provided a price and geological risk using an independent scenario for the mine production phase. However, an independent scenario was mainly influenced by price based on the market fluctuations. Other authors such as Maleki et al (2020) have attempted to draw up a mine production and scheduling plan based on the geological-technical variables (rock type, metal grades, magnetic ratio and specific gravity) using the Block Model simulations. The findings of this study can optimise the future risks by providing the appropriate ore quantity and quality at the first stage of the production phase. Both studies from Rimele et al (2020) and Maleki et al (2020) discussed the mine production and scheduling plan based on the geological-technical risks from the exploration stage that they did not pay attention to in the production phase. The exploration phase failed to provide flexibility in the mining operation in predicting the future risks.

Dehghani et al (2014) indicated the future risks in the production phase that used the relationship between a commodity price and production cost risks. However, the research on the future risks, according to Dehghani et al (2014), offered the same trends between commodity price and production cost to investigate the flexibility in the mining operation. In the same vein, Souza et al (2019) developed the relationship between a mineral commodity price and the production rate risks to investigate the future risks. Neingo et al (2018) examined the future risks based on the relationship between the production rates and the actual valuation of the projects using the experimental laws method. They concluded that these appeared to weaken the relationship between the production rates and the actual valuation of the projects based on the platinum mines in South Africa. However, Dehgahni et al (2014), Souza et al (2019) and Neingo et al (2018) only focused on the correlation between two different variables and they did not take into account flexibility in the mining operation.

Guj & Chandra (2019) investigated flexibility in the mining operation, which they related to the mining operation variables such as metal grades, processing recoveries and economic variables such as a mineral commodity price and exchange rate. Then, employed the discounted cash flows net after-tax and operating projects cash flows models to compute the probability distribution of input variables. Similarly, Akbari et al (2009) found that the mining operation variables can be used to assess the future risks. Akbari et al (2009) proposed a mineral commodity price, metal grades and costs to identify the long-term risk.

In contrast to all the authors above, Guj & Chandra (2019) have argued that a single variable contributed more accurate results in predicting the future risks than taking several variables into account. These results were similar to those reported by Brandão et al (2012) and Savolainen (2016), where incorporated variables can provide a positive bias in forecasting the future risks. However, Haahtela (2012) addressed to add the relevant information to reduce the complexity for the future risks to come up with the more accurate results.

By reviewing all the relevant literature systematically, to determine the future risks using the multiple sources or risks, these can be concluded to incorporate the economic and financial variables. Previous writers proposed to incorporate the mineral commodity price, metal grades and operating costs to observe the long-term risk (Akbari et al, 2009). Guj & Chandra (2019) developed a model to incorporate the mineral commodity price, exchange rate, metal grades and processing recoveries in assessing the future risks. However, all previous authors did not take the production rate variables into account to forecast the future risks. Sohrabi et al (2022) argued that the production rates were influenced by the multiple sources of risks in the mining project. Therefore, this research proposed to incorporate an economic variable (e.g. mineral commodity price and operating costs) and technical variable (e.g. metal grade and production rate) during the lifespan of a mine using a Sample Correlation Coefficient model. These results will be examined using the Decision Tree analysis, Geometric Brownian Motion and risk-neutral probabilities results. The closest average value from these tests between a dual volatility versus a pure price volatility against historical price and profits can be concluded as the most accurate forecasting method.

3.3 Decision Tree Analysis to Identify the Future Risks

A dual volatility using the financial and technical variables in this research was firstly examined using a Decision Tree analysis to predict the future risks. The Decision Tree analysis test reflected the future risks based on volatility change. Guj (2016) suggested to investigate the future risks to value a mining project, which should deal with a volatility changing. He revealed that volatility changing can be calculated and interpreted using analytical equations, the partial differential equations and lattice models.

Andersen & Brotherton-Ratcliffe (1997) used the Black-Scholes (1973) (from here, B-S) formula as an analytic equation to interpret the volatility changing. The B-S formula assumed a stock price following a GBM model by employing a constant independent variable process. The constant independent variables from the B-S formula consisted of risk-free rate (r_f), dividend rate (δ), and volatility (σ). Hobson & Rogers (1998) argued volatility as a key variable from the analytical equations to calculate the price that can be represented by the B-S formula. In most scholars, the volatility was calculated from historical data which can readily be found in the financial market. In common practice, the volatility can also be inverted from the B-S formula numerically, so that it is referred to as implied volatility. The implied volatility is consistent with European options of market price to represent the constant independent variables of the B-S formula.

Hull (1997) believed that an implied volatility tended to be higher for the income option than the outcome option. Contrarily, Rubinstein (1994) observed the relationship between the implied volatility with strike from the systematic variation based on the options price analysis. He concluded that the implied volatility decreased since the money was spent in the price options in one period.

However, an implied volatility increased to follow the strike for the next period. Dupire (1993; 1994) expressed that every potential strike for European calls was a circumstance to deliver potential exercise dates to come up with the volatility. Unfortunately, it was difficult to indicate volatility smile or volatility skewness with this model, when it is used as input variables. The existence of volatility smile indicates the stock price deviation from the log-normal probability distribution from the B-S formula (Andersen & Brotherton-Ratcliffe, 1997).

Renault & Touzi (1995) assumed the existence of volatility smile could be found when there was no correlation between a stochastic volatility and the option price. Abdelmalek et al (2009) argued that volatility smile depended on the strike price and maturity. Andersen & Brotherton-Ratcliffe (1997) argued that the volatility smile can sometimes be developed directly from a local volatility parametrisation. However, this approach was not suitable with the volatility smile. It can be concluded that the market smile should be involved as a main input to support the local volatility function using numerical and analytical methods. The analytical method such as the B-S formula unfortunately cannot accept the volatility smile since it cannot deliver the volatility changing over multiple periods as a free arbitrage condition.

Since a commodity future market was recorded as a free arbitrage condition, it can therefore enter a long-term project in investment (Bellalah, 2001; Colwell et al, 2003; Cortazar & Schwartz, 1997; Dixit & Pindyck, 1994). According to Andersen & Brotherton-Ratcliffe (1997), a free arbitrage condition can be developed using the partial differential equations. The partial differential equation can solve the numerical project value from a mining project in a free arbitrage condition using a stochastic differential equation by involving future commodity prices. Similarly, Haque et al (2014) hypothesised the partial differential equation to value a mining project numerically in a gold mining case study, where the commodity future prices were one of the main variables in the partial differential equation.

Another variable for partial differential equation estimation was the hedging strategy. This strategy was employed to hedge the future contract risks over a short period of time in the lifespan of a mine. The short period of mine life, using a hedging strategy, was suggested to earn an investment return that was equal to the total of the risk-free interest rate (r_f) and country risk premium that represents the mining project location (Cortazar & Schwartz, 1997 & Bellalah, 2001). Broadie & Jain (2008) argued that the short period of mining projects employing volatility hedging may be solved using variance swaps. Variance and volatility swaps were believed to be the most effective hedging instruments to capture the asset volatility over a certain period.

Neuberger (1994) and Demeterfi et al (1999) stated that the variance swaps were replicated in put options and European call as a static position. They assumed the variance swaps in the underlying asset were a dynamic trading strategy. Brockhaus & Long (2000) introduced an analytical method to price the volatility swaps. Javaheri et al (2002) suggested to price the volatility swaps by determining the first two moments from the realised variance using the partial differential equation. Little & Pant (2001) extended the B-S formula by developing a finite difference method to value the variance swaps. Broadie & Jain (2008) priced variance and volatility swaps when the variance process fell as a continuous diffusion derived from the Heston Stochastic Volatility (1993) model. According to Broadie & Jain (2008), the Heston Stochastic Volatility (1993) model valued options on realised variance swaps using the PDE. Broadie & Jain (2008) employed the partial differential equation as a numerical method to compute a variance swap hedging. Further, they solved a partial differential equation to identify the risk management variables of variance and volatility swaps. The risk management variables were delivered to hedge volatility derivatives. As a result, volatility derivatives provided delta units to compute dynamic hedging of variance and volatility swaps. Broadie & Jain (2008) argued that dynamic hedging of variance and volatility swaps seemed quite effective to select the best options. However, variance and volatility swaps were difficult in pricing volatility derivatives since the underlying variable and variance were not traded in the market. Therefore, the valuation of volatility derivatives solved by a partial differential equation was computed from the interaction between variance swap plus volatility derivatives in a free arbitrage condition. In practice, a finite number of options are proposed to optimise the hedging of variance swaps.

Another common tool to analyse a volatility changing was the Discrete Lattice models (Guj, 2016). The Discrete Lattice models were readily calculated and interpreted to accommodate the future risks issue by involving numerous sources of risks and variables. The Discrete Lattice models can perfectly accept the stochastic behaviour of datasets per period as represented by a stepping time. A stepping time considered the upward and downward changes to forecast the future risks. This process involved volatility at each stepping time, which assumed a constant risk-free rate during the lifespan of a mine. A constant risk-free rate can affect the future probabilities that were then solved by recombining a binomial lattice to construct the entire lattice. At the end of each lattice point was a new lattice to provide the next future probabilities. Lee (2018) proposed the B-S formula and binomial tree methods to forecast the future commodity price probabilities based on the up and down probabilities. However, a recombining binomial lattice did not correspond to reflect the up and down probabilities perfectly, since it employed a constant volatility for a mine's lifespan. These results were similar to those reported by Andersen & Brotherton-Ratcliffe (1997) that the up and down probabilities cannot achieve accurate results for a high risk-free rate. Therefore, a recombining binomial lattice should take into account a non-constant volatility to deal with the up and down probabilities. The conventional recombining binomial lattice with a non-constant volatility was known as a non-recombining binomial lattice.

Hauge et al (2003) compared a non-recombining binomial lattice and a recombining binomial lattice, which suggested no significant different results over a short-term period. By drawing on the concept of the up and down probabilities, a recombining lattice cannot model the volatility changing (Brandão & Dyer, 2005). The up and down probabilities for a long-term period can impact in very significant results between a non-recombining binomial lattice and a recombining binomial lattice. In order to improve the error in a long-term period, Derman & Kani (1994) introduced the heuristic rules to replace the error nodes that appeared at each stage from the up and down probabilities at the end of each period. However, Derman & Kani (1994)'s method failed to address the accurate future risks since it was insufficient to define the up and down probabilities flexibility in each period (Brandão & Dyer, 2005). Commenting error nodes at each stage, Brandão & Dyer (2005) studied that the large number of samples should employ an algorithm by recombining lattice. These results were similar to those reported by Liu et al (2017), which suggested employing a Decision Tree algorithm to forecast the future copper prices. In the most recent innovation in forecasting the future risks, Dehghani et al (2014) introduced the Multidimensional Binomial Tree method to develop a pyramid technique, which could evaluate the multiple sources of risks in mining such as price and operating costs risk in the mining projects.

Considering all of this evidence, it seems that the Binomial Tree method implemented as a Decision Tree analysis presents a straightforward way to capture the up and down probabilities with a high accuracy of forecasting the future risks (Zheng et al, 2020). Decision Tree analysis offers a visual analysis rather than the statistical regression technique to predict the future risks using the past datasets with similar statistic behaviour. The predicted data comes from the input data that are split into periods of smallest units of the target predictor. Therefore, the target predictor can be estimated with high accuracy using a Decision Tree analysis to examine a dual volatility since the input datasets have a significant positive correlation-Ajak et al (2018). In addition, a highly accurate Decision Tree analysis can control the risk that can be used to forecast the future mineral commodity price. One of the advantages of the Decision Tree analysis is that it can model the GBM behaviour of the future gold price movements (Sohrabi et al 2022).

3.4 GBM to Forecast the Future Price

GBM is one of the popular tools for assessing the future mineral commodity price since it deals with a constant volatility in random walks model (Ardian & Kumral, 2020). Lima & Suslick (2006) suggested to forecast the future mineral commodity price using GBM by extending a closed-form model from Davis (1998). In forecasting the future mineral commodity price, a GBM model can tackle the unavailable historical datasets. Since a GBM model can perform well for the unavailable historical datasets, it should involve numerous variables to estimate the future mineral commodity based on Lima & Suslick (2006):

(i) Current value of a project,
(ii) Present value of investment,
(iii) Time to maturity,
(iv) Future dividend of a project,
(v) Future risk-free interest rate,
(vi) Volatility.

One of the most crucial variables from the six variables of a GBM model in forecasting a mineral commodity price is the volatility (Lima & Suslick, 2006). Volatility can represent the risk including the future price. This volatility is computed from the logarithm of the current price divided by the logarithm of the previous price (Ladder & Wu, 2009, Haque et al, 2017). The logarithm of the current price and the previous price appeared to be associated with the historical price. Shu & Zhang (2006) believed that the historical price can be a proxy to the future mineral commodity prices. However, there are three main drawbacks that were recorded by Lima & Suslick (2006) in using the historical price to calculate the future mineral commodity prices, which differ among different companies, despite the fact that they might be from the same metal commodity. These can be caused by each company as its own financial and technical risks, (ii) the historical price using GBM model cannot accept the long term since it led to allow the random shocks (Schwartz, 1997). He suggested to develop a mean-reverting process to deal with a long-term period, and this was sometimes not always available (for example for unlisted mining companies). Therefore, previous studies have attempted to explain the future mineral commodity price using a GBM model since the historical data was not published or available to the public.

Therefore, Brandão et al (2012) and Hall & Nichols (2007) proposed project volatility to tackle the issue of unpublished or unavailable data from the mining projects for public use, as shown by Lima & Suslick (2006). The equal value between a mineral commodity price and operating costs in project volatility can be achieved since price has a significant positive correlation and a linear function to the operating costs (Lima & Suslick, 2006). However, a mineral commodity price can be different to the operating costs, which can be influenced by fixed costs (Tufano, 1998). Similar with Tufano (1998), Lima & Suslick (2006) recognised that a project volatility was larger than a price volatility in gold mines. A project volatility can provide lower value than a price volatility since it was operating very significant high mineral commodity prices and very low production expenditures (Lima & Suslick, 2006). Since the operating costs cannot follow a mineral commodity price, the future risk prediction using project volatility cannot accurately be determined. In order to address this issue, Godinho (2006) suggested to apply a constant project volatility during the life of a mining project. Therefore, he proposed to employ a constant volatility of each variable to compute a project volatility. However, Bessembinder et al (1995) argued that the constant volatility did not coincide with the traded mineral commodity prices. Therefore, the authors proposed the three alternative variants to solve the dynamic volatility issues. The first model was level-dependent volatility, which assumed the stock prices behavior to follow the debt of a firm's values (Rubinstein, 1994; Bensoussan et al, 1994). Unfortunately, the first model, a Level-Dependent Volatility model, failed to specify a single and simple model to exhibit volatility over time. Dupire (1993; 1994) employed a Specified Level-Dependent Volatility model incorporated with option prices to investigate the volatility calculation. This volatility was a diffusion model that can move consistently according to time and price.

Another criticism about constant volatility is that the most relevant variables in calculating volatility to forecast the future price were difficult to be determined (Alameer et al, 2019). He (2007) has established that oil and copper prices can be used as benchmarks variables against other commodities. Existing research recognised the critical role played by oil price since it was used as a main source of energy (Behmiri & Manera, 2015). It was well established from a variety of studies that oil price fluctuation can be associated with gold prices. Besides oil prices, studies over the past two decades have provided important information on other important variables on copper prices (Buncic & Moretto, 2015). In order to address this question, Buncic & Moretto (2015) involved a number of external variables in determining the effects to copper prices. This research was similar to those reported by Liu et al (2017), which demonstrated eight external variables (two energy prices such as oil price and natural gas price, three metal prices such as gold price, silver price and copper price, two commodity prices such as lean hogs price and coffee price and one stock market such as Dow Jones Industrial average). The findings of this study suggested, firstly, how these variables were related without any assumptions. Secondly, the data reported by Liu et al (2017) appeared to support the assumption that the relationship between input data and the future forecasting plus the most independent variables to predict the future copper prices can be analysed using the Decision Tree Algorithm model.

The comparable studies to compute volatility using the external variables were carried out by Alameer et al (2019). They used ten variables (three exchange rates from; India, China and South Africa, two inflation rates from; U.S. and China and five commodity prices from; west Texas intermediate crude oil, copper, silver, iron and gold) to assess the future gold price. In order to determine the most relevant variables to predict the future price, there were attempts made to investigate the correlation between those variables. In addition, the main weakness of the previous studies in selecting the external variables in assessing volatility was the failure to take the technical variables as one of the main risks in mining projects into account. No attempt has been made to quantify the incorporating between the financial and technical variables. This research makes an attempt to exert eight main variables by incorporating the technical variables (oil supply and demand and copper supply and demand) and the financial variables (oil price, copper price, exchange rate and gold or platinum price) to predict the future short-term and long-term prices. Firstly, oil supply and demand are contributing factors in driving oil prices. There is evidence that oil supply plays a crucial role in regulating oil prices. Oil price risk plays a critical role in the maintenance of demand, which seems to wait after the price tends to be constant. Secondly, another significant aspect of future forecasting variables is oil price. Recent research has revealed that oil price acts as a main indicator on other metal prices. One reason why oil price is a main indicator is that it is used as a main energy source. Thirdly, turning now to the experimental evidence on how copper supply and demand can affect copper prices. A high demand of copper could be associated with industrialisation and urbanisation accelerations. These can significantly affect the copper price. Fourthly, copper price has a positive effect on other metal prices since it is used as alternative investment portfolios besides gold and silver. Finally, previous authors studied the effect of exchange rates in relation to metal prices. They found a significant relationship between metal prices and exchange rates. Ciner (2017) demonstrated that the South African exchange rate has a significant performance to predict the platinum and palladium prices. Brown & Hardy (2019) used the Chilean Peso to predict the future prices of base metals. Results from these studies suggested that exchange rates have a significant and positive correlation to metal prices. Next, investigating the future price is a continuing concern to oil price. There was evidence that oil price played a crucial role in regulating commodity price fluctuations. Oil price was a dominant feature to identify dynamic gold price based on a positive relationship between oil price and gold. Therefore, oil price can be employed to calculate volatility to forecast the future gold prices (Alameer et al, 2019). In summary, by considering the existing literature, the proposed variables in this research compute volatility based on financial aspects such as; oil price, copper price, exchange rate (ZAR to U.S.$), gold price and platinum price and technical aspects such as oil supply and demand and copper supply and demand.

Besides the most relevant variables to compute volatility, different methods have been proposed to forecast the future mineral commodity price using a GBM model. Lima & Suslick (2006) proposed a GBM model to employ the historical volatility in a free arbitrage condition. Since a free arbitrage condition was taken into account, a GBM model tends to be more sensitive to a mineral commodity price fluctuation. Therefore, a sensitive behavior of a GBM model to a mineral price fluctuation can be able to simulate the dynamic future mineral commodity price (Lima & Suslick, 2006). However, since it was sensitive to mineral price fluctuation, traditionally, GBM can only perform well to forecast the short-term period (Ladde & Wu, 2009). The effects of high accuracy over a short-term period on a GBM model were significantly reflected by the same values both tomorrow's price and current price combined with random shocks (Shafiee & Topal, 2010). Therefore, these random shocks in a GBM model were responsible to provide significant accuracy to forecast the short-term mineral commodity price. However, this model still failed to address the long-term trend issue. To answer this issue, Bessembinder et al (1995) found the short-term mineral price fluctuation to revert the long-term trend. Therefore, Hull (1997) developed a stochastic volatility to solve the dynamic mineral commodity price in the long-term period. This model involved a new random classification to employ the second Brownian motion. The second Brownian motion was responsible for an incomplete market to deliver the unique independent prices for stock options by employing the Contingent Claims Approach. The Contingent Claims Approach offered to solve an incomplete market issue by converting to the Complete Market Condition for transaction. The main advantage of the Complete Market Condition is that it can reflect a new asset price which is equal with the replicating portfolio for the value of project, which this represented in free arbitrage conditions. The free arbitrage condition in a project led to deliver the RNP. Therefore, the RNP condition in a project can allow investors to optimise the profits which were used to examine a dual volatility against a price volatility in this research.

3.5 RNP to Optimise the Future Profits

The RNP employed the replicating portfolio that can outperform to value the mining project by optimising the future profits. Commenting on the replicating portfolio, there are multiple steps of definitions that:

(i) Identify and track the replicating and the portfolio to compute its volatility and price.

(ii) Observe the investment size relatively to the replicating portfolio.

(iii) Apply the B-S model as the financial option pricing of a standard financial.

According to Borison (2005), the first step was to observe replicating portfolio characteristics or individual replicating to set up a series of virtual experiments using oil and gas industry cases. These virtual experiments can assess volatility by tracking replicating portfolio of the Western U.S. natural gas valuation. According to Hall & Nicholls (2007) and Guj (2008), replicating portfolio tracking can forecast the future expected profits, which can mainly be influenced by a mineral commodity price. Liu et al (2017) believed a mineral commodity price can refer to the oil price. In the history of global economics, the oil price shock has been thought of as a key factor in driving other sectors, especially in the early to end of the 1980s. The oil price shock associated with the increased price of copper. Understanding the link between the oil price and the copper price can help to investigate the relationship with other metal prices (Liu et al, 2017). Liu et al (2017) provided new insight into these links that can commonly affect the volatility. Characterisation of the volatility was important for an increased understanding of a Real Options valuation that tended towards high volatility in mining the sector (Guj, 2008).

Some authors (Boudt et al, 2011; Areal & Taylor, 2002; Martens et al, 2002; Dominguez & Panthaki, 2006) have been encouraged understanding the future risks based on a high volatility condition. Jackwerth (2000) modified the Jackwerth & Rubinstein’s (1996) methodology to investigate the significant impact of a high volatility on stock price, which was caused by the market crash in 1987. Concerns were expressed around the market crash that could disrupt the long-term stable conditions in the future risk prediction. Another concern expressed regarding a high volatility issue was whether the replicating portfolio could be tracked and valued using a similar price in optimising the future profits. To distinguish between these two possibilities, the replicating portfolio tracking and the valuation, the portfolio has shown no evidence to replicate the risk. These results suggested that the tracking likely seemed to have a negative correlation to the portfolio. Amran & Kulatilaka (1999) suggested that levels of tracking, as represented by the specific project land valuation, were negatively correlated with the project value. This showed that the portfolio provided the tracking error. A recent systematic literature review concluded that the existence of a tracking error represented the unidentified risk (Amran & Kulatilaka, 1999).

To measure the tracking error, many investors have utilised the B-S model to minimise the tracking error volatility that can be computed by the difference between the squared returns deviations and the Mean Square Benchmark model. The smallest difference between the squared returns deviations and the Mean Square Benchmark model can minimise the tracking error volatility. Roll (1992) proposed three broad problems to minimise the tracking error volatility. The first strategy to minimise the tracking error volatility was the Quadratic Model that was commonly employed by financial practitioners. The Quadratic models attempted to provide several suitable statistical products. However, in contrast to the Quadratic Objective model, the tracking error was defined by Clarke et al (1994) as an absolute value of the portfolio return, subtracted by the benchmark portfolio return. One of the biggest challenges, based on the practitioner's fact to minimise tracking error volatility, was to interpret the quadratic objective. Most recent attention has focused on the provision of the tracking error solution, as developed by Konno & Yamazaki (1991) and Speranza (1993). They encountered intense reactions when they tried to optimise absolute deviations portfolio returns using mean absolute deviations, rather than the portfolio returns volatility. However, no model was proposed to minimise the linear deviations results from the portfolio and benchmark returns for the tracking error measurement (Rudolf et al, 1999).

There were several advantages to using the linear tracking error models rather than the quadratic models (Rudolf et al, 1999). The linear functions were one of the most common procedures for determining the tracking error, where the portfolio and benchmark returns from the absolute deviations were taken into account. In observational studies, the portfolio weights from the mean variance models were combined with mean deviations of absolute models in the normal distribution of the portfolio returns. The combination between the mean variance models with mean deviations of absolute models demonstrated an identical shape (Markowitz, 1952; Speranza, 1993). The first question elicited information on the normal distribution of the portfolio returns as the traditional tracking error, can represent a more objective benchmark. In addition, the normal distribution of the portfolio returns can broadly describe and criticise both the positive and negative skewness of the benchmark, which can be a guide in selecting the calculation model. In particular, the use of the positive deviations as the second strategy was a well-established technique to determine the tracking error volatility, since an asymmetric shape appeared to portfolio returns (Franks, 1992; Rudolf et al, 1999). An asymmetric shape of portfolio returns can contribute to the over-performance and the underperformance to assess the tracking error volatility. To optimise the over-performance and the underperformance respectively for the future forecasting, a Tracking Error Model should reflect the weighted average objective function that can be demonstrated by the positive and the negative deviations (Dembo & Rosen, 1999). The term asymmetric shape of portfolio returns was used here to refer to the positive or the negative deviations that corresponded with the Mean Absolute Deviation Approach and Downside Minmax Approach (Rudolf et al, 1999). For Rudolf et al (1999), the mean absolute deviation minimised the difference between the portfolio returns and the benchmark, where the purpose of it was to readily deal with the tracking error for the most investor perceptions, rather than to employ the minimised quadratic tracking error. Using the minimised mean absolute deviation to measure the tracking error, where the mean square objective function was reflected by percentage, referred to as squared percentage, as the third strategy to calculate the tracking error volatility. However, the study of Amran & Kulatilaka (1999) on tracking error in volatility never provided guidelines on appropriate project valuations. Therefore, in the replication portfolio of option pricing, the next question was addressed to the project valuation that can be determined further from the investment size valuation.

The next step to identify the replicating portfolio was addressed to observe the experimental evidence on the investment size valuation. The valuation result was straightforward to equal with a mineral commodity price. The next section of the valuation was concerned with the best strategy to maximise the valuation using a mineral commodity price to acquire the investment. Overall, these results indicated that the maximum valuation from a mineral commodity price was the final prices. Another commented the investment size valuation relative to the replicating portfolio, where Amran & Kulatilaka (1999) applied the B-S model to value a start-up company by duplicating the valuation based on the size of an established company. The main key issue is to duplicate the value based on a similar established company size to compute the price of the company, which has clearly been evaluated according to the relative prices in the market.

The last step to investigate the replicating portfolio, Borison (2005) argued that the price of the company fluctuated to particularly follow a GBM model. Since it followed a GBM model, this suggested to employ the B-S model using the standard financial tool. This view was supported by Copeland & Taylor (1994), who wrote that the price of the company model was determined using the standard financial tool such as the net present value. Then, the net present value was projected to forecast the future company price using the Decision Tree Analysis models. The Net Present Value model appeared to be positively correlated to the risk evaluation, while the Decision Tree Analysis model solved the decision flexibility at each node per period. Further analysis showed that the net present value model can merge the replication portfolio and free arbitrage application by assuming the dynamic mineral commodity price in valuing the price of a company accepted GBM model. A comprehensive study of the dynamic mineral commodity price in valuing the price of company, using the net present value, Luehrman (1997) found very little evidence to support that price followed a GBM model. Therefore, in the same vein, Borison (2005) proposed the B-S model to forecast the dynamic mineral commodity price in valuing the price of a company using a simplified model. The simplified B-S model can assess the probability distribution to characterise the dynamic mineral commodity price based on the regular transactions.

In contrast to Borison (2005), Barndorff-Nielasen & Shepard (2004) argued that the probability distribution of the dynamic mineral commodity price based on the regular transactions can contribute to estimation bias to value the price of company. However, Barndorff-Nielasen & Shepard (2004) suggested that estimation bias caused the probability distribution of the dynamic mineral commodity price can be solved by the spot volatility. Interestingly, the spot volatility was measured using the regular transactions to compute the intraday volatility to test the dynamic mineral commodity price (Lee & Mykland, 2007; Veraart, 2010. However, the intraday volatility cannot optimise the dynamic mineral commodity price since it has no sensitive behaviour to detect the volatility changing (Boudt et al, 2011). To improve the intraday volatility sensitivity to optimise the dynamic mineral commodity price forecasting, Boudt et al (2011) focused on increasing very small jumps when the volatility periodically seemed low, or reducing very high jumps for a high volatility period. In their review of a high volatility period, Boudt et al (2011) identified the main characteristics of intraday volatility that can be more efficient to estimate the dynamic mineral commodity price (Areal & Taylor, 2002; Martens et al, 2002). Dominguez & Panthaki (2006) examined a periodicity price jump using the Monte Carlo Simulation as the robust test to the intraday volatility. According to Dominguez & Panthaki (2006), the Monte Carlo Simulation was one of the most common procedures to detect the significant bias from a periodicity dynamic mineral commodity price. Boudt et al (2011) argued that the dynamic mineral commodity price periodicity was a deterministic function of intraweek for the volatility of the intraday variation rather than intraday volatility. As a result, Boudt et al (2011) concluded to adopt a Case-Study Approach to assess the intraweek to include macroeconomic news that can allow the intraday volatility, such as stochastic volatility.

The stochastic volatility is utilised to optimise the future risks; however, it did not exactly fit with the dynamic mineral commodity price. In order to fit with the dynamic mineral commodity price, Derman & Kani (1998) combined stochastic volatility with the dynamic mineral commodity price model to achieve the best fit to the current prices. This view was supported by Heath et al (1992) who wrote that the stochastic volatility analysis can optimise the dynamic mineral commodity price. However, the Heath-Jarrow-Morton (1992) model failed to straightforwardly implement the stochastic volatility analysis process; therefore, the combined Monte Carlo Simulation and Lattice Method were taken into account by involving an intensive computer program to predict the future dynamic mineral commodity price. Commenting on computer intensive, Britten-Jones & Neuberger (2000) introduced a Markovian Volatility Pricing Algorithm model that developed simpler and more rapid processes to predict the future dynamic mineral commodity price. The major advantage of Markovian assumes that the future dynamic mineral commodity price was deterministic, which assumed that the future probabilities were equal to both the price history and the current price. However, Britten-Jones & Neuberger (2000) can only process the price history and the current price process, which cannot determine a price probability. Therefore, the deterministic volatility should be taken into account to deal with a price probability. The existence of a deterministic volatility process to accept a price probability can be more useful by fitting the historical and current prices using the B-S volatility model. The B-S model can be employed to forecast the future value, since the price history and the current price process have a maturity time in a stochastic process.

Further analysis by Zhao & Ziemba (2008) showed that a stochastic volatility process offered high accuracy to fit the price history and the current price using a drift that was represented by time and a mineral commodity price. Previous research from Britten-Jones & Neuberger (2000) has established to fit the price history and the current price using m up and down factors constantly, where these factors depended on volatility, time, and price. However, the up and down factors in the RNP to forecast the future profits should follow the dynamic behaviour, not constant. Since the dynamic behaviour should be taken to forecast the future profit that the RNP should deal with a stochastic behaviour to accept a mineral commodity price volatility fluctuation. Similar to the two tests above, a single variable cannot deal to accept a mineral commodity price volatility fluctuation, which cannot represent the multiple sources of risks (Sohrabi et al, 2022). Therefore, the multiple sources of risks are proposed by incorporating the financial and technical variables to solve a mineral commodity price volatility fluctuation to optimise the future profits.

3.6 Chapter Summary

The existing literature on volatility changing paid particular attention to the Discrete Lattice Model (Guj, 2016). Andersen & Brotherton-Ratcliffe (1997) demonstrated that the Discrete Lattice Model can follow stochastic behavior of the data. This argument was in line with Derman & Kani (1994) that stochastic behavior associated to a mineral commodity price to allow volatility changing. Then, in order to determine volatility changing, new nodes from the Discrete Lattice Model were added to construct a recombining binomial lattice. Unfortunately, the recombining binomial lattice employed constant volatility, which led to failing to address a non-constant volatility to predict the future risks. Therefore, Andersen & Brotherton-Ratcliffe (1997) conducted a series of improvements in which they suggested a non-recombining binomial lattice to deal with non-constant volatility. However, according to Brandão & Dyer (2005), non-constant volatility has no attempt to solve the flexibility issue in forecasting the future risks that can come up with inaccurate results. Zheng et al (2020) proposed a Decision Tree Analysis that offered flexibility in determining the future risks to come up with the accurate results in forecasting the future risks. To achieve the accurate results in using a Decision Tree Analysis, all of previous authors (Akbari et al, 2009; Guj & Chandra, 2019) have involved the relevant variable of risks that offered to accept the future risks and flexibility. These relevant variables of risk by Akbari et al (2009) and Guj & Chandra (2019) were represented by metal price, exchange rates, metal grades, metal recoveries and operating costs. However, these variables do not offer the production rates as the input data variables to achieve the more accurate results in forecasting the future risks, since these can influence the future risks in the mining projects (Sohrabi et al, 2022).

This thesis develops to quantify the future risks by adding production rates from the existing variables (e.g., metal prices, metal grades and operating costs variables) by Akbari et al (2009) and Guj & Chandra (2019), which are computed using a Sample Correlation Coefficient Model. A Sample Correlation Coefficient Model combines these variables (e.g., metal prices, metal grades, operating costs variables and production rates) to come up with a dual volatility. This dual volatility is examined to a price volatility using a Decision Tree Analysis, which can deal with volatility changing over time.

This research is proposed to solve the flexibility issue to deal with volatility changing using a Decision Tree Analysis in forecasting the future risks. In order to deal with the flexibility issue, the numerous sources of risks including the production rates variable are taken into account to assess a dual volatility. As a result, a dual volatility calculated from metal prices, metal grades, operating costs and production rates variables can improve the accuracy in forecasting the future risks using a Decision Tree Analysis.

Another hypothesis is GBM, which is proposed to examine a dual volatility by forecasting the future mineral commodity price. Ardian & Kumral (2020) studied a GBM model that concluded as the important technique to assess the future mineral commodity price by employing a Random Walks Model with a constant volatility. This argument supported Derman & Kani (1994)'s studies that the future mineral commodity price has a similar trend to the future risks. In order to achieve the most accurate results in forecasting the future mineral commodity price, a GBM model involved numerous components, where the most critical variable was volatility (Lima & Suslick, 2006). According to Shu & Zhang (2006), volatility was computed by a mineral commodity price to represent the future risks, known as a price volatility. However, the main weakness of a price volatility was overestimated than the actual value (Copeland & Antikarov, 2003; Ugwegbu, 2013; Haque et al, 2017). One study by Muhammad et al (2019) examined a price volatility that concluded gold price can only follow the future risks. Another criticism to a price volatility from Lima & Suslick (2006) is that it failed to address the financial and technical risks since these can influence the future risks. Consistent with Lima & Suslick (2006) in suggesting the financial and technical risks as the additional variables, Buncic & Moretto (2015) proposed the external variables to determine the future mineral commodity price (e.g., copper). Liu et al (2017) and Alameer et al (2019) demonstrated the Buncic & Moretto's (2015) studies, which they confirmed to add external variables to forecast the future copper and gold prices. However, both Liu et al (2017) and Alameer et al (2019) have focused on the financial variables that they have no attempt to involve the technical variables as the relevant variables to identify the future risks (Lima & Suslick, 2006).

The GBM model in this thesis is extended by modifying volatility, which is carried out by incorporating the financial and technical variables to forecast the future mineral commodity price. Therefore, this research proposes to incorporate the financial variables (oil price, copper price, exchange rate, gold price and platinum price) and the technical variables (oil supply and demand and copper supply and demand) to predict the future copper, gold and platinum prices using a GBM model. The combination between the financial variables and the technical variables are proposed to improve the future price prediction for the long-term periods since a conventional GBM model can only accept the price shock over a short period. The additional external variables such as the technical variable is able to deal with the long-term future risks to predict the mineral commodity prices.

The last hypothesis is purposed to optimise the future price forecasting using the RNP for future options. In order to optimise the future price forecasting, the RNP can perform well to predict the expected profits using a mineral commodity price volatility (Hall & Nicholls, 2007; Guj, 2008). Another statement from Guj (2008) confirmed that the main issue in mining projects was high risk that can be represented by high volatility. Since it has a high volatility, the RNP seemed to fail in optimising the future prices forecasting (Boudt et al, 2011; Areal & Taylor, 2002; Martens et al, 2002; Dominguez & Panthaki, 2006). Zhao & Ziemba (2008) argued that high volatility in the mining industry can confirm to follow the stochastic behaviour. Britten-Jones & Neuberger (2000) established the relevant method to fit with stochastic behaviour that this should be projected using the constant up and down factor probabilities. However, the constant up and down factor probabilities by Britten-Jones & Neuberger (2000), cannot optimise the future prices forecasting, since stochastic behaviour cannot accept the high volatility. Sohrabi et al (2022) suggested to involve the multiple sources of risks rather than a single variable to optimise the future prices forecasting based on the high volatility datasets.

The RNP methods in this thesis are constructed from multiple sources of risks, such as oil price, copper price, exchange rate and gold or platinum price, and the technical variables (oil supply and demand and copper supply and demand) to calculate a dual volatility using a Sample Correlation Coefficient model to optimise the future prices forecasting. The RNP methods are proposed in this thesis since these can optimise the future profits by valuing the mining projects (Borison, 2005). Therefore, this research attempts to incorporate the multiple sources of risks to improve the optimal future prices in the mining projects. This improvement is carried out by incorporating the financial and technical variables plus adding the production rates variable. The main reason to add the supply variable is that these can influence the future risks in the mining projects (Sohrabi et al, 2022) to optimise the future prices forecasting.

4 Research Design

4.1 Research Philosophy and Approach

This thesis involves analysing and interpreting data from the mining industry in order to understand and investigate challenges that relate to forecasting incorporated risk. Thus, it uses a Positivism Approach, where the quality of the analysis is taken into account. For this thesis, the results are analysed and interpreted from the sample and/or population. According to Harmes (2016), a Positivism Approach is applied in cases where one wants to combine both quantitative and meaning of numbers. Thus, one interprets data while taking into account the quality of the analysis. Therefore, the research approach used in analysing the data is a Positivism Approach.

A Positivism Approach involved the relevant variables that were proposed to minimise the errors. In order to minimise the errors, the relationships between the relevant variables were investigated, where these were averaged to reduce the random variations between two or more variables. To optimise the averaged errors in a Positivism Approach, Irshaidat (2019) proposed to renew the existing variables using different models to achieve more reasonable results.

The Deductive Approach assists in the interpretation of results based on the calculated numbers or ratios (Graneheim et al, 2017); therefore, this thesis uses a Deductive Approach.

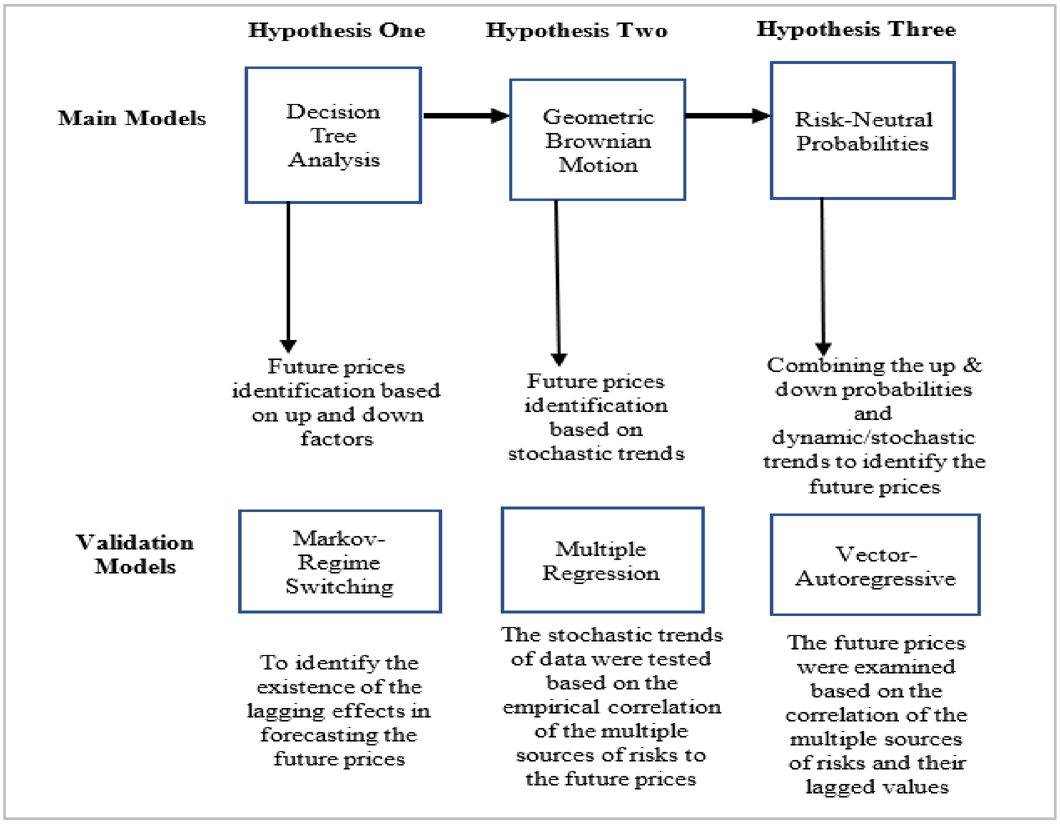
4.2 Methodology

This research uses various mathematical formulae and statistical concepts in order to address the three questions listed in this proposal. According to Hunter & Leahey (2008), where different mathematical and statistical concepts are used to solve certain empirical problems, that methodology is a quantitative one. Hence, this research uses a quantitative methodology.

A quantitative methodology described objectively in solving the problems that it focused on numerical, empirical and statistical approaches. Based on the statistical analysis, it explained the pattern and structural model that can determine the relationship and causal effects between one variable to others (Murshed & Zhang, 2016).

In order to carry the quantitative methodology, this thesis uses the research methods as mentioned in the next subs-section and each research method including supporting validation or robustness method and/or technique are used to first analysis and strengthen the results of the three hypotheses. Figure 4.1 outlines methodological flow chart;

Figure 4. 1: Flow Chart for the Methodology



Source: Author's own conceptualisation

Figure 4.1 presents a flow chart for the methodology used in this thesis. The first hypothesis is a Decision Tree Analysis (DTA) model. This model is selected because the future prices fluctuations can be assessed based on the up and down movements using volatility factors (Guj, 2016). For robustness test of the first hypothesis, the Markov-Regime Switching model is used. The Markov-Regime switching model identifies the lagging effect based on two regimes which are inherent in that model.

However, DTA fails to address the stochastic behaviour of prices. Therefore, the second hypothesis proposes the Geometric Brownian motion (GBM) model that can tackle the stochastic behaviour of prices based on a random walk principle (Ardian & Kumral, 2020). For second robustness test, the multiple regression is used. The multiple regression models different factors that affect the dependent variable. Depending on how independent variables react when group together, in the same model instrumental variables can be used to minimise multicollinearity and heteroscedasticity among econometric challenges. Then, the third hypothesis combines the up and down probabilities including stochastic behaviour of prices using the Risk-Neutral Probabilities

(RNP) to forecast the future prices (Trigeorgis, 1993). In order to test the robustness for RNP results, the Vector-Autoregressive (VAR) model is proposed. The VAR examines the forecasted prices using RNP based on the empirical correlation between the future prices, the multiple sources of risks and their lagged values. Thereafter, this thesis further illustrates the best fit of each model based the empirical findings. The latter responds to, based on forecasted findings, which results have minimal errors?

The most popular technique to measure the forecasting performance is the the Root Mean Square Error (RMSE) and the Mean Absolute Percentage Error (MAPE) (Hollstein et al, 2020a; 2020b). The RMSE measures the difference between the actual prices and the forecasted prices based on the sample size; whilst the MAPE classifies the forecasting performance such as the best performance, good accuracy and sufficient accuracy. According to Lewis (1982), MAPE performs poorly based on Lewis's scale. Fundamentally, the best proposed model in forecasting the future prices is that model that provides the lowest values, for both RMSE and MAPE.

4.3 Research methods

4.3.1 The Sample Correlation Coefficient

In this research, volatility estimation is developed by incorporating financial and technical risks known as dual volatility using the Sample Correlation Coefficient model. Dual volatility is a volatility where it is computed based on incorporating the financial and technical risks. Then, the Sample Correlation Coefficient model is the ratio between covariance and the standard deviation of each variable. The Sample Correlation Coefficient model is employed to integrate several variables that reflect a ratio between a covariance with standard deviation products of variables. This ratio is proposed to standardise between two variables before it is used to measure the Sample Correlation Coefficient model (r_{xy}), as described below in Equation 4.1 (Asuero et al, 2006):

$$r_{xy} = \frac{Cov(x,y)}{S_x\ S_y} = \frac{1}{n-1}\ \Sigma \left(\frac{x_i - \bar{x}}{S_x}\right)\left(\frac{y_i - \bar{y}}{S_y}\right) \tag{4.1}$$

where Cov (x, y) is covariance between random variable *x* and *y*. Then, S_x is the standard deviation of variable x and S_y is the standard deviation of variable y. The n is the number of pairs measurements of variable x_i and variable y_i. The x_i is the random variable of x and y_i is random variable of y. The $\bar{x}$ is the mean of weighted variable x expressed as $\Sigma w_i x_i / \Sigma w_i$ and $\bar{y}$ is the mean of weighted variable y expressed as $\Sigma w_i y_i / \Sigma w_i$. Whilst w_i is the point weight of variable and is denoted as $1/s_i^2$. s_i is the standard deviation of variable.

In order to explore the Sample Correlation Coefficient model deeper to compute a dual volatility, this thesis proposes to construct the portfolio variance (σ_P^2) as expressed in Equation 4.2 (Skintzi & Refenes, 2005).

$$\sigma_P^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N-1} \sum_{j>1} \rho_P \, w_i \, \sigma_i \tag{4.2}$$

where w_i is the weights of the assets i and j in a given time, and σ_i is the variance of asset in a given time. Then, w_i is denoted as:

$$w_i = \frac{V_i}{V_P} \tag{4.3}$$

where V_i expresses the asset value and V_P is the total portfolio value, and ρ_P is the portfolio correlation that represents the implied correlation index to the pairwise correlation weighted average between the portfolio asset returns. ρ_P is expressed as:

$$\rho_P = \sum_{i=1} \sum_{j>i} c_i \, r_{xy} \tag{4.4}$$

where c_i is interpreted as the level of correlation of the stocks from the option prices for out of the money (Linders & Schoutens, 2013). The r_{xy} is the sample coefficient correlation value of the pairwise correlation x and y as shown in Equation 4.1. Then, c_i is expressed as:

$$c_i = \frac{w_i \, \sigma_i}{\sum_{i=1} \sum_{j,k,l>1} w_i \, \sigma_i} \tag{4.5}$$

The standard deviation captures a dual volatility in Equation 4.2. A dual volatility proposed in this thesis is subjected to interact with four variables of uncertainties such as prices, metal grades, costs, and production rates and oil price, copper price, exchange rate and gold or platinum price, oil supply and demand and copper supply and demand. The interaction of those variables enables the improvement of the joint price and exchange rate uncertainties, as proposed by Haque et al (2017). They used stochastic differential equations to calculate the increased volatility (Cortazar et al, 2001). Haque et al (2017) employed the increased volatility by involving the coefficient of correlation that interacts with the gold price and the exchange rate from the American dollar to the Australian dollar.

This thesis develops an implied correlation to interact with four variable uncertainties. An implied correlation involves a pairwise correlation value and the level correlation average of the stocks at a given time. Then, a pairwise correlation value and the level correlation average can provide the portfolio correlation value, the asset portfolio weights, and the asset variance in a given time to produce a dual volatility.

It is proposed that a dual volatility should be used to predict future volatilities within a decision tree setting; thereafter, the GBM prices are presented and finally, the RNP valuation of optimal stochastic profits. Hypothesis One, using a sample correlation coefficient, is purposed to examine a dual volatility such as mineral commodity prices, metal grades, operating costs and production rates against a single volatility.

Note that the sample correlation coefficient is supported by main tests and validation. The first main test for the sample correlation coefficient is the Decision Tree Analysis. A Decision Tree Analysis is a tool that is used to forecast the probability of future risks based on the move up and move down factors (Zheng et al, 2020). The Decision Tree Analysis is set up to predict the future five-year period to investigate the move up and move down future probabilities using a dual volatility. For the Decision Tree Analysis, the move up (u) and move down (d) future probabilities are computed using Equation 4.6 and 4.7 below:

$$u = e^{\sigma\sqrt{t}} \tag{4.6}$$

and

$$d = e^{-\sigma\sqrt{t}} = \frac{1}{u} \tag{4.7}$$

where σ is a volatility, in this case it represents a dual volatility and t is time. Samuelson (1965) exemplified that using the "Samuelson effect" is a better way of forecasting volatilities of commodities. On the other hand, Johnson (2008) works well with other derivatives techniques such as the binomial tree, and Rendleman and Bartter (1979) model. The formula for a generalised axiom of present-discounted expected value, when at each point of time t for future price is;

$$Y(T,t) = \lambda^{-T}\mathrm{E}\left[X_{t,T}|X_t, X_{t-1}, \dots\right] \tag{4.8}$$

According to Samuelsson (1965), Y is may be slightly different for each value of T, and it can be expected a different $(\lambda_1, \lambda_2, \dots, \lambda_T)$ yield for which people hold out if they are to hold for the next period a futures contract with T years to go.

With regards to the "Samuelson effect" in forecasting the volatility, many research studies (Mun, 2002; Saliba & Dimitrakopoulos, 2019) have developed to improve in computing the volatility using the historical price. One of the most well-known models for assessing volatility from historical price is the Gaussian Autoregressive Conditional Heteroscedasticity (GARCH). The benefit of this model is that it simplifies the dynamic historical price of returns, whilst it is used as a benchmark to other heteroscedasticity models in assessing volatility (Pascual et al, 2006). The Gaussian Autoregressive Conditional Heteroscedasticity (GARCH) is expressed as (Bollerslev, 1986):

$$R_t = \sigma_t \varepsilon_t \tag{4.9a}$$

and

GARCH is given by Equation (4.9b);

$$\sigma_t^2 = \omega + \alpha R_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{4.9b}$$

where R_t is returns, σ_t is volatility as a stochastic process, ε_t is the error term which is assumed as a normal distribution. ω, α, β are unknown variables ($\omega > 0, \alpha \geq 0, \beta \geq 0$). R_{t-1}^2 is squared returns of previous time and σ_{t-1}^2 is the conditional variance of previous time.

In order to test the robustness results for validation of Equation 4.6 and 4.7, in this research the Markov-Regime Switching model is used. A robustness test strengthens earlier analysis by offering more insight and/or a different angle of the findings on the earlier analysis. In simple terms, the robustness test is a quality assurance measure. The advantage of the Markov-Regime Switching model is that different regimes of variables are captured by the model and moreover, those regimes can be lagged in order to explore the lagging effect (Fatnassi et al, 2014). The simple Markov-Regime Switching model of the conditional mean is presented when s_t denotes an unobservable state variable assuming the value of one or zero. A simple switching model for the variable z_t involves two AR specifications as shown in Equation 4.10:

$$z_t = \begin{cases} \alpha_0 + \beta z_t + \varepsilon_t, s_t = 0, \\ \alpha_0 + \alpha_1 + \beta z_t + \varepsilon_t, s_t = 1, \end{cases} \tag{4.10}$$

where $|\beta| < 1$ and ε_t are i.i.d. random variables with mean zero and variance σ_ε^2. This is a stationary AR (1) process with the mean $\frac{\alpha_0}{1-\beta}$,when $s_t = 0$, and it switches to another stationary AR(1) process with the mean $\frac{\alpha_{0+\alpha_1}}{1-\beta}$,when $s_t = 1$. If $\alpha_1 \neq 0$,then the model admits two dynamic structures at different levels, depending on the value of the state variable s_t. In this case, z_t are governed by two regimes with distinct means, and s_t determines switching between two different regimes. Kola & Sebehela (2020), used MRS for the same reasons as in this thesis.

4.3.2 Incorporated GBM Volatility

Hypothesis Two is proposed to test a dual volatility in forecasting the future copper gold and platinum prices using a GBM process as the main test. The GBM model is a stochastic model that is employed to forecast the risks price using a Random Walks model with a constant volatility (Ardian & Kumral, 2020). Note that the generated volatilities are based on the financial variables (oil price, copper price, exchange rate and gold or platinum price) and the technical variables (oil supply and demand and copper supply and demand) to predict the future short-term and long-term prices. The future short-term and long-term prices based on GBM is given by Equation 4.11:

$$S_t = S_0 exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right) \quad (4.11)$$

where S_t is the future price, S_0 is the spot price, μ is the mean (return), σ is the volatility, t is the time to maturity and W_t is the Brownian Motion. The log-normally distributed variables with expected value and variance are given by Equations 4.12 and 4.13 below, respectively:

$$\mathrm{E}(S_t) = S_0 e^{\mu t} \quad (4.12)$$

and

$$Var(S_t) = S_0^2 e^{2\mu t}\left(e^{\sigma^2 t} - 1\right) \quad (4.13)$$

The robust test for validation in this GBM is a multiple regression model. In addition, a Multiple Regression model has reliability tests such as Akaike, Durbin-Watson and other tests. Using the same variables in a GBM model, a Multiple Regression model is expressed in Equation 4.14 (Asghar et al, 2019):

$$Y_i = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \varepsilon_i \quad (4.14)$$

where Y_i represents the dependent variable in this case copper price, X_1 is oil price, X_2 is exchange rate (i.e. \$:ZAR), X_3 is copper demand, X_4 is copper supply, X_5 is oil demand, X_6 is oil supply and ε_i is the error term. Note that the price do not follow logarithmic path because prices of mineral commodities are assumed to follow semi-linear movements (See; Bragin et al, 2020). Thus, the price is kept takes its natural form.

4.3.3 Risk-Neutral Probability

Hypothesis Three focuses on the estimated the optimal future price using the risk-neutral probabilities that these proposed as the main test. The risk-neutral probabilities refer to the pricing model as the financial options tool that works based on no arbitrage condition in valuing the real assets (Trigeorgis 1993). Given that the commodity environment is volatile; one proposes a model that will forecast future prices while taking into account the stochastic nature of oil price, copper price, oil production and consumption, copper production and consumption and exchange rate from U.S.$ to ZAR and gold and platinum price variables. One such model to accommodate the stochastic nature of these variables in forecasting the futures prices is shown in Equation 4.15 (Johnson et al, 2008):

$$\mathrm{f}_0 = S_0 r^{nf} - D_t \tag{4.15}$$

Where f_0 is the future prices, S_0 is current prices, r is one plus periodic risk-free rate, nf is the number of expired periods of the futures prices, and D_t is the dividend values at the expired periods. The other parameters of Equation 1 are as follows; $= e^{\sigma_S^A \sqrt{\Delta t} + \mu_S^A \Delta t}$, and $d = e^{-\sigma_S^A \sqrt{\Delta t} + \mu_S^A \Delta t} = \frac{1}{u}$. Where, σ_S^A is the standard deviation of the metal commodity price's logarithmic return, μ_S^A is the mean of the metal commodity price's logarithmic return, Δt is the binomial period duration that is ratio of time to expiration time and number of the option's expiration time.

Equation 4.15 above provided the forecasted futures prices under the Risk-Neutral Probability model. One of the advantages of using a Risk-Neutral Probability model is that once these are calculated, these can value the assets based on their expected payoffs (See; Borison, 2005). The expected payoffs can be represented by strike and exercise price as a specific security tool within a certain period. If the strike value is greater than the exercise price, the expected payoffs are positive or vice versa (Husted, 2005).

In order to test the robustness for validation of Equation 4.15, the thesis uses the VAR model. The advantage of using the VAR model is that it models multiple quantities into one portfolio. In the context, numerous variables are from one commodity; therefore, it is of interest to understand if the relationship between those variables of a single commodity, which explains the profitability of different firms in Equation 4.16 (Kumar et al, 2012).

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + e_t \tag{4.16}$$

Where the l-periods back observation y_{t-1} is called the l-th lag of l-th lag of y, c is a $k * 1$ vector of constants (intercepts), A_j is the time-invariant $k * k$ is the matrix and e_t is a $k * 1$ vector of error terms satisfying $\mathrm{E}(e_t) = 0$, every error term has mean zero. $\mathrm{E}(e_t e_t') = \Omega$, the contemporaneous covariance matrix of error terms is Ω (a $k * k$ positive-semidefinite matrix. A simple switching model for the variable z_t involves two AR specifications are shown in Equation 4.17:

$$z_t = \begin{cases} \alpha_0 + \beta z_t + \varepsilon_t, s_t = 0, \\ \alpha_0 + \alpha_1 + \beta z_t + \varepsilon_t, s_t = 1, \end{cases} \tag{4.17}$$

Where $|\beta| < 1$ and ε_t are i.i.d. random variables with mean zero and variance σ_ε^2. For this study, there will be AR(1) to AR(4). Kola & Sebehala (2020) used VAR for the same reasons as ones that this thesis puts forward to substantiate for the usage of the VAR.

4.4 Population and Sampling

The weekly mining aggregated data is used for the testing the empirical hypothesis and the data includes metal prices, metal grades, operating costs and production rates for Hypothesis One and Hypothesis Three. Then, the Hypothesis Two considers the datasets from oil price, copper price, exchange rate, gold or platinum price, oil supply and demand and copper supply and demand. The data is obtained from 1 January 2004 to 31 December 2019 except for copper, which started from 1 January 2004 to 31 December 2013. The finer details about the data of each company are explained in the data chapter. The companies are focused on copper mining (i.e., Palabora Copper Mining Ltd), gold mining (i.e., AngloGold Ashanti Ltd, Gold Fields Lt and Sibanye Stillwater Ltd) and platinum mining (i.e., Anglo American Platinum Ltd and Impala Platinum Ltd).

4.5 Validity and Reliability

For every empirical analysis carried out in each hypothesis, a supporting robustness test follows. Hypothesis One proposes an empirical thesis of the up and down factors using a Decision

Tree Analysis. Then, the robust test for Hypothesis One is the Markov-Regime Switching model. The empirical Hypothesis Two uses a GBM model to test a dual volatility performance in forecasting the future metal commodity prices and its robustness test for empirical Hypothesis Two is examined using the Multiple Regression Model. Hypothesis Three provided the empirical model of the RNP to optimise the future profits and its robustness test is the VAR model. As a conclusion, this is part of strengthening and validating the findings of each empirical thesis.

Furthermore, as part of validation and reliability tests, this thesis uses the the Root Mean Square Error (RMSE) and the Mean Absolute Percentage Error (MAPE). RMSE and MAPE have been used before by Hollstein et al. (2020a; 2020b). It can be inferred from the authors that RMSE and MAPE measure the difference between sample or population size and the estimated values, and predication accuracy of forecasting processes. Given that this thesis studies volatility risk, the mentioned techniques are appropriate to be used as part of the validation and reliability tests.

4.6 Eviews

For the analysis, this thesis uses Eviews, an econometric analysis software, which is suitable for advanced time series analysis. Among its elegance is its ability to handle the Markov-Regime Switching Model, Multiple Regression Model and the Vector Auto-Regression Model, including being advantageous in forecasting and validating results.

4.7 Chapter Summary

A Sample Correlation Coefficient is used to investigate future risks, i.e., future volatility risk, where the Decision Tree Analysis provides supporting analyses by accounting for up and down probabilities. These probabilities of optimising the future risks are associated with changing volatility (Guj, 2016). The original form of empirical hypothesis developed volatility changing using a mineral commodity price that it was famously known as a price volatility (Shu & Zhang, 2006). However, the drawbacks in using a price volatility is that it contributed to an overestimation in valuing the mining projects (Copeland & Antikarov, 2003; Ugwegbu, 2013; Haque et al, 2017). In order to optimise the overestimated results in valuing the projects, Lima & Suslick (2006) proposed to incorporate a mineral commodity price as the financial variable with the technical variable. Previous scholars (e.g, Akbari et al, 2009; Guj & Chandra, 2019) have incorporated the financial variables (metal price) and the technical variables (metal grades and operating costs). However, they did not consider the production rates as one of the relevant variables to influence the future risks (Sohrabi et al, 2022). This thesis proposes to incorporate metal price, metal grades, operating costs and production rates variables to identify the future risks using a Sample Correlation Coefficient Model.

Another technique proposed to identify the future risks is a GBM model. The original GBM model employed a price volatility to forecast the future mineral commodity price using a Random Walks Model with a constant volatility (Ardian & Kumral, 2020). However, since the GBM model involves a constant volatility, which comes from prices, the GBM failed to predict the long-term period in Ardian & Kumral (2020). In order to optimise the future mineral commodity price forecasting, a GBM should involve the external variables (see Buncic & Moretto, 2015; Liu et al, 2017; Alameer et al, 2019. The work of Liu et al (2017) and Alameer et al (2019) demonstrated that the external variables perform to forecast the future copper and gold prices. However, their approaches did not take into account the other future risks, which can be affected by the technical variables. This research proposes to improve the future mineral price forecasting by incorporating the financial and technical variables using a GBM model.

The RNP methods have recognised that these can optimise the future profits using a price volatility (Hall & Nicholls, 2007; Guj, 2008). However, the mining projects have experience to deal with a high risk, which can be represented by high volatility (Guj, 2008). This has encouraged some authors (Boudt et al, 2011; Areal & Taylor, 2002; Martens et al, 2002; Dominguez & Panthaki, 2006) to develop the volatility that can accept the high risk. The latter comment was suggested by Sohrabi et al (2022) to involve the multiple sources of risk to optimise the future profits. As a result, this research develops a dual volatility, which combines the financial and technical variables to optimise the future profits using the RNP methods.

5 Data Collection

5.1 Introduction

For the empirical analysis, the research used data from the following companies. Palabora Mining Company Ltd for copper, AngloGold Ashanti Ltd, Sibanye-Stillwater Ltd and Gold Fields Ltd for gold, and lastly, Anglo American Platinum Ltd and Impala Platinum Holdings Ltd for platinum. Either of the chosen mining companies have global presence in many jurisdictions around the globe or their holding company does. The companies or their holding companies are listed on the Johannesburg Stock Exchange and have some presence around the globe. The data for forecasting and future risks in this research is collected for the period January 2004 to December 2019 except for Palabora Mining Company Ltd, which is from January 2004 to December 2015 on a monthly basis because of the data availability period. The data for this research contains the technical variables such as ore milled and metal grade in ore and the financial variables such as sold metal price, operating costs and profits. Most studies on forecasting and future risks such as those of Brown & Hardy (2019) and Pincheira & Hardy (2021) used more than 213 monthly datasets. However, this research uses a 192 monthly datasets, except for Palabora Mining Company Ltd, which has a 144 monthly datasets. Therefore, this study is still illustrative, compared to most prior studies. Each individualism and uniqueness of each mining company is discussed in the next sub-sections.

5.2 Copper

Table 5.1 presents an overview of the descriptive statistics for copper based on Palabora Ltd monthly dataset from January 2004 to December 2015. The descriptive statistics can be inferred from Greenland et al (2016) that include variables such as mean, median, minimum, maximum, standard deviation, skewness, kurtosis and the Jarque-Bera test. The descriptive statistics for copper in this research contain the technical variables such as ore milled in tonnes, copper grade in percent and the financial variables such as sold copper price in U.S.$ per tonne, operating costs in U.S.$ per tonne and profits in U.S.$. Furthermore, all these descriptive statistics are calculated using Excel Software.

Table 5. 1: Descriptive Statistics for Copper from January 2004 to December 2015

Description	Technical Dataset		Financial Dataset		
	Ore Milled (Thousand Mt)	% Copper in Ore	Sold Copper Prices (U.S.$/Pound)	Operating Costs (U.S.$/Pound)	Profits (Thousand U.S.$)
Mean	1,477.87	0.5	2.35	1.37	132,529
Median	1,507.23	0.49	2.22	1.24	152,343
Min	916.64	0.34	0.88	0.61	-5,470
Max	2,098.36	0.76	4.36	4.71	271,076
StdDEv	358.59	0.09	0.79	0.62	82,041
Skew	-0.06	0.74	0.53	2.67	-0.37
Kurt	-1.26	0.15	-0.61	11.39	-1.25
No.:	144	144	144	144	144
Jarque-Bera	9.56	13.25	8.93	949.4	12.65

Note; Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB is Jarque-Bera.
Source: Palabora Mining Company Ltd financial statements (2004-2015).

Figure 5.1 shows that the mean of ore milled for Palabora Mining Company Limited from January 2004 to December 2015 is 1.48 million metric tonnes with a minimum and maximum of 0.92 million metric tonnes and 2.10 million metric tonnes respectively. The ore milled data further shows a negative skewness at -0.06 and the kurtosis value at -1.26. Based on the descriptive statistics results of copper from Palabora Mining Company Limited, these suggest that the distributions of these datasets using skewness and kurtosis results are negatively skewed and light tails.

The second variable is percentage copper in ore. The average percentage of copper in ore is 0.50%, with a minimum value of 0.34% of copper and a maximum value of 0.76% of copper. These datasets show skewness and kurtosis that are 0.74 and 0.15 respectively. This data is positively skewed and light tails.

The next variable is sold copper price from of Palabora Mining Company Limited. The mean of sold copper prices is U.S.$ 2.35 per pound. Then, the minimum value of sold copper prices is U.S.$ 0.88 per pound and the maximum value is U.S.$ 4.36 per pound. The skewness and kurtosis of sold copper prices are 0.53 and -0.61. These results demonstrate that the sold copper prices are distributed positively skewness with light tails.

The mean value of operating costs is U.S.$ 1.37 per pound. The minimum and maximum value of operating costs are U.S.$ 0.61 and U.S.$ 4.71 per pound, respectively. Furthermore, the skewness and kurtosis fall at 2.67 and 11.39. These results show that the operating costs distributions are positively skewed with heavy tails.

The last variable is profits that the mean of these is U.S.$ 132.53 million. The minimum and maximum values are U.S.$ -5.47 million and U.S.$ 271.08 million respectively. Then, the skewness and kurtosis are -0.37 and -1.25 that these distributions are negatively skewed and light tails.

5.3 Gold

Table 5.2 below illustrates some of the descriptive statistics of the datasets for gold from AngloGold Ashanti Limited, Gold Fields Limited and Sibanye-Stillwater Limited from January 2004 to December 2019. The datasets from these companies contain the technical datasets such as ore milled in metric tonnes and grade in gram per tonne and the financial datasets such as sold gold prices in U.S.$ per ounce, the all-in costs for AngloGold Ashanti Limited and Gold Fields Limited or the total operating costs for Sibanye-Stillwater Ltd in U.S.$ per ounce and profits in U.S.$. The descriptive statistics from AngloGold Ashanti Limited, Gold Fields Limited and Sibanye-Stillwater Limited are presented in Table 5.2. These descriptive statistics are calculated using Excel Software.

Table 5. 2: Descriptive Statistics for Gold from January 2004 to December 2019

Panel A: AngloGold Ashanti Ltd					
Description	Technical Variable		Financial Variable		
	Ore Milled (Thousand Mt)	Gold Grade in Ore (g/t)	Sold Gold Price (U.S.$/Ounce)	All-in Costs (U.S.$/Ounce)	Profit (Million U.S.$)
Mean	10,572	1.91	1,004.30	740.9	28.91
Median	7,701	1.8	1,194.17	766.35	48.61
Min	5,212	1.07	369.58	286	-370
Max	19,358	4.02	1,779.23	1,190.16	208.1
StdDev	4,630	0.62	432.63	258.29	91.91
Skew	0.52	0.79	-0.03	-0.08	-2.03
Kurt	-1.49	0.09	-1.45	-1.19	6.46
No.	192	192	192	192	192
JB	26.28	19.9	16.79	11.47	465.7
Panel B: Gold Fields Ltd					
Description	Technical Variable		Financial Variable		
	Ore Milled (Thousand Mt)	Gold Grade in Ore (g/t)	Sold Gold Price (U.S.$/Ounce)	All-in Costs (U.S.$/Ounce)	Profit (Million U.S.$)
Mean	3,844	2.13	1,063.55	777.69	27.59
Median	3,776	2.04	1,198.95	821.16	23.02
Min	2,431	0.98	363.06	278.1	-66.9
Max	10,727	3.33	1,772.14	1,697.76	138.9
StdDev	1,147	0.43	396.87	328.5	38.44
Skew	2.43	0.51	-0.34	0.09	0.22
Kurt	11.52	0.23	-0.99	-0.95	1.37
No.	192	192	192	192	192
JB	1,251.12	8.79	11.52	7.51	16.7
Panel C: Sibanye-Stillwater Ltd					
Description	Technical Variable		Financial Variable		
	Ore Milled (Thousand Mt)	Gold Grade in Ore (g/t)	Sold Gold Prices (U.S.$/Ounce)	Total Operating Costs (U.S.$/Ounce)	Profit (Million U.S.$)
Mean	1,156	3.76	1,087.42	1,009.41	35.38
Median	1,163	3.34	1,205.57	886.78	43.13
Min	696	1.12	382.24	521.95	-123
Max	1,539	6.56	1,765.05	6,267.18	226.8
StdDev	162	1.27	380.44	691.97	54.88
Skew	-0.26	0.45	-0.38	6.08	0.02
Kurt	-0.06	-0.88	-0.83	41.03	1.18
No.	192	192	192	192	192
JB	2.17	12.56	10.18	14,651.49	11.16

Note; Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis and No. is sample size.
Source: AngloGold Ashanti Ltd, Gold Fields Ltd and Sibanye Stillwater Ltd financial statements (2004-2019).

Panel A in Table 5.2 presents the results obtained from the descriptive statistics of ore milled in metric tonnes, gold grade in ore in gram per tonne, sold gold prices in U.S.$ per ounce, all-in costs in U.S.$ per ounce and profits in U.S.$ from January 2004 to December 2019, based on AngloGold Ashanti Limited mines globally. The mean of ore milled is 10.57 million metric tonnes. The minimum and maximum values of ore milled are 5.21 million metric tonnes and 19.36 million metric tonnes, respectively. Further descriptive statistics results reveal that skewness and kurtosis are 0.52

and -1.49, respectively. Therefore, the ore milled datasets demonstrate a positive skewness and light tails.

The next variable in Panel A in Table 5.2 is gold grade in ore. The mean of gold grade in ore is 1.91 grams per ton. The minimum value of gold grade in ore is 1.07 grams per tonne and the maximum value is 4.02 grams per ton. Furthermore, skewness and kurtosis fall at 0.79 and 0.09. As a conclusion, gold grade shows a positive skewness and light tails. Furthermore, the descriptive statistics of mean for sold gold prices are U.S.$ 1,004.30 per ounce. Then minimum sold gold prices are U.S$ 369.58 per ounce and the maximum value of sold gold prices reveal at U.S.$ 1,779.23 per ounce. Then, the skewness and kurtosis values are -0.03 and -1.45, respectively. These results suggest a negative skewness with light tails.

The all-in costs for AngloGold Ashanti Limited observe the mean value at U.S.$ 740.90 per ounce. The minimum value of all-in costs falls at U.S.$ 286 per ounce. Then, the maximum value of all-in costs provides at U.S.$ 1,190.16 per ounce. By extension, skewness and kurtosis of all-in costs for AngloGold Ashanti Limited illustrate at -0.08 and -1.19, respectively. These results represent that the all-in costs data for AngloGold Ashanti Limited are negatively skewed with light tails. The last variable in Panel A in Table 5.2 is profits. The mean of profits represents at U.S.$ 28.91 million. Then, the minimum value of profits shows at U.S.$ -369.72 million while the maximum profits are U.S.$ 208.08 million. The skewness and kurtosis indicate at -2.03 and 6.46, respectively. These results imply a negative skewness with heavy tails.

Panel B provides the breakdown of descriptive statistics of technical variables such as ore milled in metric tonnes, gold grade in ore stated in gram per tonne and financial variables such as sold gold prices in U.S.$ per ounce, all-in costs and profits in U.S.$ per ounce and U.S.$, respectively for Gold Fields Limited from January 2004 to December 2019 in monthly basis. The ore milled provides the mean value at 3.84 million metric tonnes. Then minimum value presents at 2.43 million metric tonnes with the maximum at 10.73 million metric tonnes. These calculations from ore milled datasets observe a skewness at 2.43 and kurtosis at 11.52. The ore milled datasets from Gold Fields Ltd demonstrate that the data distributions are a positive skewness with heavy tails. Gold grade in ore for Gold Fields Ltd has 2.13 gram per ton for mean value. The minimum value is 0.98 gram per ton and the maximum value at 3.33. The skewness and kurtosis of datasets are 0.51 and 0.23, respectively. These indicated that these datasets are positively skewed and light tails.

The mean of sold gold price is U.S.$ 1,063.55 per ounce. The minimum and maximum values of sold gold price are U.S.$ 363.06 per ounce and U.S$ 1.772.14 per ounce, respectively. Estimated of skewness and kurtosis are -0.34 and -0.99, respectively. These results are subjected to negative skewness and light tails. The next variable for Gold Fields Ltd is all-in costs, where the mean of these is U.S.$ 777.69 per ounce. The minimum value for all-in costs variable can be seen from Panel B in Table 5.2, which is U.S.$ 278.10 per ounce with a maximum value of U.S.$ 1,697.76 per ounce. The skewness of all-in costs is 0.09, where kurtosis is observed at -0.95. These results indicate that these datasets are positively skewed with light tails.

The last variable provided by Gold Fields is profits. From the datasets, the mean is resulted at U.S.$ 27.59 million. The minimum and maximum values reveal U.S.$ -66.91 million and U.S.$ 138.92 million, respectively. The results of skewness and kurtosis are 0.22 and 1.37, respectively. It is apparent from skewness and kurtosis that the datasets are positively skewed with light tails. The last company for gold in this research is presented in Panel C in Table 5.2. The first variable for Sibanye-Stillwater Ltd is ore milled, where the mean is 1.16 million metric tonnes. The minimum and maximum ore milled are 0.67 million metric tonnes and 1.54 million metric tonnes, respectively. Then, the skewness is -0.26 with a kurtosis at -0.06. These results of skewness and kurtosis can explain that these datasets are negatively skewed with light tails.

The mean of gold grade in ore shows 3.76 grams per tonne. The minimum value of gold in ore is 1.12 grams per ton with a maximum value at 6.56 grams per ton. The skewness of the data observes 0.45 and kurtosis presents at -0.88. From the datasets, these results indicate a positive skewness with light tails. The sold gold price is the next variable for Sibanye-Stillwater Ltd, where the mean of it falls at U.S.$ 1,087.42 per ounce. The minimum value of sold gold price reveals at U.S.$ 382.24 per ounce with a maximum value at U.S.$ 1,765.05 per ounce. The skewness and kurtosis record at -0.38 and -0.83, respectively. These confirm that the datasets are negatively skewed with light tails.

The next variable is the total operating costs, where the mean value falls at U.S.$ 1,009.41 per ounce. The minimum value of the total operating costs is found at U.S.$ 521.95 per ounce. Then, the maximum value shows U.S.$ 6,267.18 per ounce. The skewness and kurtosis results are obtained from the datasets at 6.08 and 41.03, respectively. The results of skewness and kurtosis present a significant positive skewness with heavy tails. The last variables for Sibanye-Stillwater Ltd is profits. It can be seen from the datasets in Panel C that the mean of profits is U.S.$ 35.38 million. As shown in Panel C, the minimum and maximum values are reported at U.S.$ -122.64 million and U.S.$

226.79 million, respectively. Further analysis shows that the skewness and kurtosis are 0.02 and 1.18, respectively. These indicated that the datasets are positively skewed with light tails.

5.4 Platinum

Table 5.3 presents an overview of descriptive statistics for the technical variables such as ore milled in metric tonne, platinum grade in ore stated in gram per ton and the financial variables such as sold platinum price in U.S.$ per ounce, cash costs in U.S.$ per ounce and profits in U.S.$ for Anglo American Platinum Ltd and Impala Platinum Holdings Ltd from January 2004 to December 2019 presented on a monthly basis. The breakdown of descriptive statistics is computed using Excel Software.

Table 5. 3: Descriptive Statistics for Platinum from January 2004 to December 2019

Panel D: Anglo American Platinum Ltd					
Description	Technical Variable		Financial Variable		
	Ore Milled (Thousand Mt)	Platinum Grade in Ore (g/t)	Sold Platinum Price (U.S.$/Ounce)	Cash Costs (U.S.$/Ounce)	Profit (Million U.S.$)
Mean	3,199.58	1.69	1,211.94	1,465.36	25.78
Median	3,281.68	1.64	1,133.11	1,508.87	2.8
Min	1,442.96	1.42	541.23	591.66	-117.13
Max	4,603.05	2.06	2,224.45	2,147.23	2,438.66
StdDev	624.32	0.17	393.48	402.17	179.67
Skew	-0.68	0.74	0.47	-0.51	12.81
Kurt	0.14	-0.15	-0.57	-0.58	172.69
No.	192	192	192	192	192
JB	15.05	17.55	9.6	10.89	243,826.19
Panel E: Impala Platinum Holdings Ltd					
Description	Technical Variable		Financial Variable		
	Ore Milled (Thousand Mt)	Platinum Grade in Ore (g/t)	Sold Platinum Price (U.S.$/Ounce)	Cash Costs (U.S.$/Ounce)	Profit (Million U.S.$)
Mean	1,481.42	3.06	1,206.69	1,349.73	33.26
Median	1,541.10	3.03	1,137.92	1,306.35	37.56
Min	670.56	2.42	459.7	355.89	-91.5
Max	1,999.78	3.61	2,199.88	2,765.49	250.65
StdDev	311.66	0.29	416.36	552.23	60.31
Skew	-0.59	-0.17	0.4	0.31	0.67
Kurt	-0.59	-0.09	-0.66	-0.8	1.54
No.	192	192	192	192	192
JB	14.01	0.96	8.71	8.25	33.17

Note; Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis and No. is sample size.
Source: Anglo American Platinum Ltd and Impala Platinum Holdings Ltd (2004-2019).

Panel D in Table 5.3 illustrates that the mean for ore milled for Anglo American Platinum Ltd is 3.20 million metric tonnes. The minimum value of ore milled is 1.44 million metric tonnes and the maximum of it reveals at 4.60 million metric tonnes. Further analysis shows that skewness is 0.68 and kurtosis is at 0.14. These results suggest that ore milled datasets for Anglo American Platinum Ltd are positively skewed with light tails. The next variable is platinum grade in ore, as shown in Panel D, where the mean is 1.69 gram per ton. The minimum platinum grade in ore reveals 1.42 grams per tonne and a maximum value is at 2.06 grams per ton. The further statistical tests of platinum grade in ore are 0.74 for skewness and -0.15 for kurtosis, respectively. These results indicate that these datasets are positively skewed with light tails.

The sold platinum price mean value can be seen in Panel D, which shows U.S.$ 1,211.94 per ounce. The minimum sold platinum price value falls at U.S.$ 541.23 per ounce and the maximum value of it is U.S.$ 2,147.23 per ounce. Furthermore, skewness value reflects at 0.47 and kurtosis is at -0.57. Therefore, the sold platinum price datasets reveal a positive skewness with light tails. The next variable is cash costs, where the mean value of these falls at U.S.$ 1,465.36 per ounce. Then, the minimum cash cost value is recorded at U.S.$ 591.66 per ounce and the maximum value is at U.S.$ 3,809.35 per ounce. The further statistical analysis reflects that skewness and kurtosis are -0.51 and -0.58, respectively. The skewness and kurtosis analysis results show a negative skewness with light tails.

The final variable for Anglo American Platinum Ltd is profits, where the mean reflects at U.S.$ 25.78 million. The minimum profits value is U.S.$ -117.13 million and the maximum value is U.S.$ 2.44 billion. Furthermore, skewness and kurtosis present at 12.81 and 172.69, respectively. These datasets show a significant positive skewness with heavy tails. Panel E describes the descriptive statistics for Impala Platinum Holdings Ltd. The first variable is ore milled, where the mean of it records at 1.48 million metric tonnes. Then, the minimum ore milled is 0.67 million metric tonnes and a maximum ore milled is 2.00 million metric tonnes. Further statistical tests show skewness at -0.59 and kurtosis at -0.59. Therefore, these datasets indicate a negative skewness with light tails.

Platinum grade in ore as shown in Panel E representing the mean at 3.06 grams per tonne. The minimum value of platinum grade in ore reveals at 2.42 grams per ton. The maximum value records at 3.61 gram per ton. Skewness and kurtosis present at -0.17 and -0.09, respectively. Therefore, these results represent a negative skewness and light tails. The next variable is sold platinum price, where the mean records at U.S.$ 1,206.69 per ounce. This shows that the minimum value at U.S.$ 459.70 per ounce and the maximum value is U.S$ 2,199.88 per ounce. Further analysis reflects skewness at 0.40 and kurtosis at -0.66. It is apparent that these datasets present a positive skewness with light tails.

The cash costs mean value is U.S.$ 1,349.73 per ounce with the minimum value is U.S$ 355.89 per ounce and the maximum value is U.S.$ 2,765.49 per ounce. The skewness and kurtosis present at 0.31 and -0.80, respectively. Therefore, these suggest a positive skewness and light tails. The profits mean of the datasets for Impala Platinum Holdings Ltd are U.S.$ 33.26 million. The minimum profits value present at U.S.$ -91.50 million, where the maximum profits value reflect at U.S.$ 250.65 million. Further statistical tests present that skewness and kurtosis obtain at 0.67 and 1.54, respectively. These results indicate that these datasets are positively skewed with light tails.

In order to examine a dual volatility, it is proposed to test using a GBM model to investigate the accuracy of the forecasting results for the future copper, gold and platinum prices. The dual volatility is computed by a Sample Correlation Coefficient Model using numerous relevant variables such as oil production, oil consumption, copper production and copper consumption as the technical variables plus the financial variables such as oil price, copper price and exchange rate on a monthly basis from January 2004 to December 2019. Besides using these datasets, the gold price or platinum price on a monthly basis are included as the historical prices to forecast the future gold price or platinum price, respectively from January 2004 to December 2019.

The technical datasets consist of monthly oil production and oil consumption stated in barrel and copper production and copper consumption datasets stated in metric tonnes (Mt). The oil production and oil consumption were collected from British Petroleum statistical review of world energy in 2016 and 2020. Then, the copper production and copper production datasets were compiled from the United States Geological Survey in 2020. The descriptive statistics for these technical datasets are represented in Table 5.4 below.

Table 5. 4: Descriptive Statistics for the technical datasets from January 2004 to December 2019

Description	Oil Production (Thousand Barrels)	Oil Consumption (Thousand Barrels)	Copper Production (Thousand Mt)	Copper Consumption (Thousand Mt)
Mean	2,630,858	2,706,716	1,441.31	1,700.32
Median	2,587,650	2,683,619	1,469.01	1,754.69
Min	2,263,435	2,316,611	574.13	678.13
Max	2,943,791	3,025,548	1,882.27	2,275.87
StdDev	164,792	166,861	235.38	307.21
Skew	0.27	0.24	-0.68	-0.62
Kurt	-0.79	-0.71	0.78	0.13
No.	192	192	192	192
JB	7.34	5.81	19.47	12.34

Note; Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis and No. is sample size.
Source: BP (2004-2019); USGS (2004-2019)

The oil production, as shown in Table 5.4, where the mean reveals at 2.63 billion barrels. The minimum oil production records at 2.26 billion barrels and the maximum reveals at 2.94 billion barrels. The skewness and kurtosis, as the results of further statistical analysis, present at 0.27 and -0.79, respectively. These oil production datasets provide a positive skewness with light tails. The oil consumption mean falls at 2.71 billion barrels and the minimum value is at 2.32 billion barrels with the maximum value at 3.03 billion barrels. The skewness is 0.24 and kurtosis is -0.71. Therefore, these datasets present a positive skewness and light tails.

The copper production mean records at 1.44 million metric tonnes with the minimum and maximum values at 0.57 million metric tonnes and 1.88 million metric tonnes, respectively. The skewness and kurtosis reveal -0.68 and 0.78, respectively. These results are subjected to a negative skewness with light tails. The mean of copper consumption can be seen in Table 5.4 that presents 1.70 million metric tonnes. The minimum and maximum values fall at 0.68 million metric tonnes and 2.28 million metric tonnes, respectively. Further statistical tests show that skewness and kurtosis are -0.62 and 0.13, respectively. These results indicate that the copper consumption is negatively skewed with light tails.

The financial datasets indicate monthly oil price in U.S.$ per barrel, copper price stated in U.S.$ per pound, gold price in U.S.$ per ounce, platinum price in (U.S.$ per ounce) and exchange rate in U.S.$ to ZAR from January 2004 to December 2019. The oil price, copper price and gold price were downloaded from Index Mundi in 2022. The platinum price was compiled from Platinum Matthey (Platinum Matthey, 2022). The last variable was exchange rate which was collected from

Nedbank in 2021. The descriptive statistics for these financial datasets are provided in Table 5.5 below.

Table 5. 5: The financial datasets from January 2004 to December 2019

Description	Oil Price (U.S.$/Barrel)	Copper Price (U.S.$/Pound)	Gold Price (U.S.$/Ounce)	Platinum Price (U.S.$/Ounce)	Exchange Rate (U.S.$/ZAR)
Mean	72.02	2.84	1,091.82	1,221.04	9.7
Median	66.53	2.97	1,210.41	1,163.45	8.33
Min	29.78	1.1	383.78	799	5.73
Max	132.83	4.48	1,772.14	2,066.42	16.38
StdDev	24.27	0.77	381.91	330.31	3.02
Skew	0.4	-0.4	-0.38	0.57	0.53
Kurt	-0.92	-0.43	-0.83	-0.72	-1.21
No.	192	192	192	192	192
JB	12.03	6.63	10.18	14.38	20.59

Note; Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis and No. is sample size.
Source: IndexMundi (2004-2019); Platinum Matthey (2004-2019); Nedbank (2004-2019)

Table 5.5 expresses that the mean of oil price is U.S.$ 72.02 per barrel. Then, a minimum value of oil price is U.S.$ 29.78 per barrel with a maximum value at U.S.$ 132.83 per barrel. Furthermore, skewness and kurtosis report at 0.40 and -0.92, respectively. Therefore, these suggest that the oil price datasets are positively skewed with light tails. The copper price mean reveals at U.S.$ 2.84 per pound, where the minimum and maximum values are U.S.$ 1.10 per pound and U.S.$ 4.48 per pound, respectively. The further statistical analysis provide a skewness at -0.40 and kurtosis at -0.43. These results are subjected to a negative skewness and light tails.

The mean of gold price falls at U.S.$ 1,091.82 per ounce and the minimum and maximum values record at U.S.$ 383.78 per ounce and U.S.$ 1,772.14 per ounce, respectively. The skewness and kurtosis obtain at -0.38 and -0.83, respectively. These indicate that the gold price datasets are negative skewness with light tails. The platinum price mean observes at U.S.$ 1,221.04 per ounce and the minimum and maximum values are U.S.$ 799.00 and U.S.$ 2,066.42 per ounce, respectively. The skewness presents at 0.57 and kurtosis is -0.72. These results indicate a positive skewness and light tails.

The mean of exchange rate records at 9.70 and the minimum and maximum values are 5.73 and 16.38, respectively. Skewness and kurtosis observe at 0.53 and -1.21, respectively. These indicate that the exchange rate datasets are positive skewness and light tails.

5.5 Summary on Data

This research selects three commodities (copper, gold and platinum) based on global operations or their groups do include their existing project in South Africa. The copper producer in this research is represented by Palabora Mining Company Limited from January 2004 to December 2015 on a monthly basis. The mean of ore milled per month records 1.48 million metric tonnes. The company extracts copper grade in ore where the mean grade is 0.50%. Then, the company receives the sold copper prices mean at U.S.$ 2.35 per pound. The mean of operating costs reveals at U.S.$ 1.37 per pound. The profits mean of Palabora Mining Company Limited are U.S.$ 132.53 million. The statistical tests confirm that copper in ore, sold copper price and operating costs are positive skewness and ore milled and profits obtained negative skewness. All Palabora Mining Company Limited record light tails, except operating costs which show heavy tails.

The gold commodity is represented by AngloGold Ashanti Limited, Gold Fields Limited and Sibanye-Stillwater Limited datasets from January 2004 to December 2019 on a monthly basis. Based on AngloGold Ashanti Limited datasets, the ore milled mean falls at 10.57 million metric tonnes. Then, gold grade in ore mean records at 1.91 grams per tonne. The mean of sold gold price is U.S.$ 1,004.30 per ounce. The all-in costs obtain at U.S.$ 740.90 per ounce and the profits mean are U.S.$ 28.91 million. Based on the further statistical tests, ore milled and gold grade in ore confirm negative skewness. All datasets for AngloGold Ashanti Limited are light tails, except profits which record heavy tails.

The ore milled mean for Gold Fields Limited reveals 3.84 million metric tonnes. The gold grade in ore mean observes 2.13 grams per tonne. Then, the mean of sold gold price falls at U.S.$ 1,063.55 per ounce. The all-in costs mean record at U.S.$ 777.69 per ounce. The profits mean is U.S.$ 27.59 million. Gold Fields datasets confirm positive skewness except sold gold price is negative skewness and light tails except ore milled is heavy tails.

The mean of ore milled for Sibanye-Stillwater Limited datasets obtains 1.16 million metric tonnes. The gold grade in ore mean falls at 3.76 grams per ton. The sold gold price mean for Sibanye-Stillwater Limited agrees at U.S.$ 1,087.42 per ounce. Then, all-in costs reveal at U.S.$ 1,009.41 per ounce. The mean of profits for Sibanye-Stillwater Limited are U.S.$ 35.38 million. These datasets for Sibanye-Stillwater Limited confirm positive skewness, except ore milled and sold gold price are negative with light tails, except the total operating costs with heavy tails.

The selected companies for platinum are Anglo American Platinum Limited and Impala Platinum Holdings Limited. The ore milled mean for Anglo American Platinum Limited records 3.20 million metric tonnes. The company extracts the platinum grade mean in ore at 1.69 grams per ton. The sold platinum price mean for Anglo American Platinum Limited falls at U.S.$ 1,211.94 per ounce. The mean of cash costs obtains at U.S.$ 1,465.36 per ounce. Therefore, the mean of profits for Anglo American Platinum Limited is U.S.$ 25.78 million. These datasets for Anglo American Platinum Limited indicate positive skewness, except for the cash cost, which is negatively skewed.

Another platinum company is Impala Platinum Holdings Limited, where the ore milled mean records at 1.48 million metric tonnes. Platinum ore in grade mean is 3.06 grams per tonne with sold platinum price mean is U.S.$ 1,206.69 per ounce. The mean of cash costs is U.S.$ 1,349.73 per ounce. Then, the mean of profits is U.S.$ 33.26 million. The datasets for Impala Platinum Holdings Limited indicate negative skewness for ore milled and platinum grade in ore with positive skewness for sold platinum price, cash costs and profits. All of the datasets for Impala Platinum Holdings Limited are light tails.

This research also provides the technical variables such as oil production, oil consumption, copper production and copper consumption and the financial variables such as oil price, copper price, gold price, platinum price and exchange rate. The mean of oil production is 2.63 billion barrels and the oil consumption mean is 2.71 billion barrels. Then, the mean value of copper production is 1.44 million metric tonnes with the copper consumption mean reveals at 1.70 million metric tonnes. The further statistical tests for the technical datasets for this research are subjected to negative skewness except oil production, which shows a positive skewness, where all datasets are light tails.

The financial datasets for this research are started by oil price, where the mean is U.S.$ 72.02 per barrel. Then, the copper price mean shows U.S.$ 2.84 per pound. The mean of the gold price and platinum price is U.S.$ 1,091.82 per ounce and U.S.$ 1,221.04 per ounce, respectively. The exchange rate mean records at U.S.$/ZAR of 9.70. Furthermore, the statistical analysis indicates positive skewness for oil price, platinum price and exchange rate while copper price, gold price reflect negative skewness. All financial variable datasets show light tails.

6 Empirical Analysis

6.1 Sample Correlation Coefficient

6.1.1 Introduction

Hypothesis One is purposed to examine a dual volatility to predict the future probability risk using a Decision Tree Analysis. The up and down factors in a Decision Tree Analysis can address this probability from a dual volatility that is extended from a mineral commodity price volatility by combining metal grade, operating costs and production rates computed by a Sample Correlation Coefficient Model. A metal commodity price volatility is combined by other variables since the most significant risk can come from other variables such as metal grade, operating costs and production rates. That issue can contribute to a mineral commodity price volatility failing to address the future probability risk. The combination of a mineral commodity price volatility with other variables as a dual volatility can optimise the future probability risk.

6.1.1.1 Copper

The results of a Sample Correlation Coefficient of analysis are shown in Table 6.1 in Panel F. As shown in Table 6.1, some of the main variables of a Sample Correlation Coefficient were calculated based on; production, copper grades, the sold copper prices and operating costs from 2004 to 2015.

Table 6. 1: Sample Correlation Coefficient and Difference in Copper Volatility

Panel F: Sample Correlation Coefficient calculation using the Palabora Ltd data from 2004-2015							
Description		Level of Correlation		Portfolio of Correlation		Portfolio Volatility	
Mean		0.01		104		2	
Median		0.01		94		0.30	
Min		0.00		3		0.00	
Max		0.02		350		58	
StdDev		0.00		73		6	
Skew		0.60		0.60		6.20	
Kurt		-0.23		-0.23		51.85	
No.:		144		144		144	
JB		71		71		15.24	
Panel G: Percentage difference of up and down factors between a single volatility and a dual volatility for copper using Palabora Ltd data over five year period							
Description	Production_Down	Cu Grade_Up	Cu Grade_Down	Cu Price_Up	Cu Price_Down	Cost_Up	Cost_Down
Mean	0.14	-0.07	0.08	-0.10	0.12	-0.33	0.56
Median	0.14	-0.07	0.08	-0.10	0.12	-0.35	0.53
Min	0.04	-0.11	0.02	-0.17	0.04	-0.51	0.15
Max	0.25	-0.02	0.13	-0.04	0.20	-0.13	1.03
StdDev	0.08	0.04	0.04	0.05	0.06	0.15	0.35
Skew	0.10	0.05	0.05	0.08	0.08	0.31	0.31
Kurt	-1.19	-1.20	-1.20	-1.19	-1.19	-1.05	-1.05
No.:	5	5	5	5	5	5	5
JB	3.66	3.67	3.67	3.66	3.66	3.51	3.51

Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No. and JB.
Source: Palabora Mining Ltd financial statements (2004-2013).

It can be seen from Panel F in Table 6.1 that the basic statistics for level of correlation the portfolio of correlation and the portfolio volatility have means of 0.01 and 2, respectively. The level of correlation is computed using Equation 4.5, where the ratio between the weights of the asset portfolio is multiplied by the variance of asset with the total of the asset portfolio *l* multiplied by the variance of asset. The weights of the asset portfolio can be computed by the ratio between the asset value and the total portfolio value using Equation 4.3. These computations are based on the relationship between copper prices, copper grade, operating costs and production rates from Palabora Copper Mining Ltd from January 2004 to December 2015. The mean value for the level of correlation is 0.01. Further statistical analysis for the level of correlation shows that the skewness and kurtosis fell at 0.60 and -0.23, respectively. These results indicate that these datasets were negatively skewed. Strong evidence is shown of a negative skewness when the Jarque Bera test is 71. Then, the portfolio correlation is calculated by multiplying the level of correlation with the sample correlation coefficient. There were positive values for portfolio correlation in both skewness and kurtosis at 0.60 and -0.23 respectively. There were significant positive values between the two conditions where the Jarque Bera test is 71. Turning now to portfolio volatility that was computed using Equation 4.2, there is a significant positive correlation between skewness at 6.20 and kurtosis at 51.85, respectively. These results suggest that the Jarque Bera test was significant at 15.24 to indicate non-normal distribution. Based on this data, a dual volatility is computed and come up at 0.05.

In order to verify a dual volatility for copper, the volatility of copper prices from 2004 to 2015 was calculated using the logarithm returns (Copeland & Antikarov, 2003; Sauvageau & Kumral, 2018). This computation comes up at 0.08. Overall, no significant differences were found between price volatility and portfolio volatility. Thus, a copper price volatility is slightly higher than dual volatility.

Panel G in Table 6.1 shows that there is a significant difference from the cost variable, where the up and down factors are -0.33 and 0.56, respectively, for the 5 year lifespan of a project. This implies risk in mining does not only come from price but also from costs. These results are similar to that found in cost volatility, which has a higher value than other variables, at 0.25. Issues related to costs were particularly caused by developing the underground workings in quarter four of 2004, which increased costs from ZAR164 million to ZAR775 million. In summary, these results suggest that a single volatility fails to quantify the real future risks because it is not inclusive of technical variables.

As shown in Panel F, both the level of correlation, portfolio correlation and portfolio variance are positively skewed, based on four main variables from 2004 to 2015. In same to earlier findings, that showed the copper prices from 1995 to 2008, where both skewness and kurtosis of the level of correlation and portfolio volatility were detected as a positive skewness. There was a similarity between the two conditions (2004 to 2015 and 1995 to 2008) in that the end of 2008 coincided with the financial markets' turmoil and the significant peak in copper prices (Khalifa et al, 2011). The financial markets' turmoil at the end of 2008 is a clear trend of greatest low-grade values that is apparent to display a positive histogram shape. The portfolio of correlation did not exist in these hypotheses that a possible explanation for these results may be the lack of other adequate variables in calculation such as; metal prices, production rates, costs and grades.

Panel G presents basic statistics for the difference in up and down factors from main variables to a dual volatility using a Decision Tree Analysis. There are a number of similarities between different main variables to a dual volatility with copper prices from 1995 to 2008 which showed positive skewness with negative kurtosis for the level of correlation and portfolio correlation. It seems possible that these results are due to the largest different values rather than the lowest ones (Khalifa et al, 2011).

To compare the difference between a dual volatility and each volatility of main variables, a basic statistics calculation of each volatility must be taken into account, as shown in Table 6.1. Areas where significant differences have been found include a dual volatility and copper prices volatility from 1995 to 2008. Interestingly, there were also differences in incorporated volatility and copper price volatility that were statistically significant. Concerns were expressed about the lower copper price volatility from 1995 to 2008 that have 0.07 (Khalifa et al, 2011). Note that single volatility is based on prices while a dual volatility is based on technical and financial variables.

6.1.1.2 Gold

The results of a Sample Correlation Coefficient calculation are shown in Table 6.2 for gold from AngloGold Ashanti in Panel H(1), Gold Fields in Panel H(2) and Sibanye-Stillwater in Panel H(3). In order to assess the Sample Correlation Coefficient, four main variables such as productions in tonnes, gold grades stated in g/t, gold prices in U.S.$/Ounce and total production cost or all-in costs stated in U.S.$/Ounce were taken into account. Table 6.2 illustrates measures that are used to explore single and dual volatilities for gold companies. Moreover, variables and/or variables that are typically used to calculate dual volatilities are presented using basic statistics illustrations.

Table 6. 2: Sample Correlation Coefficient and Difference in Gold Volatility

Panel H: Sample Correlation Coefficient for gold using the AngloGold Ashanti, Gold Fields and Sibanye Stillwater 2004-2019 period

Panel H(1): AngloGold Ashanti

Description	Level of Correlation	Portfolio of Correlation	Portfolio Volatility
Mean	0.01	49	0.0043
Median	0.00	23	0.0000
Min	0.00	0	0.0000
Max	1.00	4 514	0.8176
StdDev	0.07	325	0.0590
Skew	13.78	13.76	13.85
Kurt	190.53	190.20	191.96
No.:	192	192	192
JB	287 402	286 410	291 787

Panel H(2): Gold Fields

Description	Level of Correlation	Portfolio of Correlation	Portfolio Volatility
Mean	0.01	111	0.00
Median	0.00	36	0.00
Min	0.00	0.00	0.00
Max	1.00	10 359	0.83
StdDev	0.07	746	0.06
Skew	13.71	13.70	13.86
Kurt	189	189	192
No.:	192	192	192
JB	283 816	283 132	291 885

Panel H(3): Sibanye Stillwater

Description	Level of Correlation	Portfolio of Correlation	Portfolio Volatility
Mean	0.01	51	0.0009
Median	0.01	26	0.0000
Min	0.00	-0.80	0.0000
Max	1.00	4 794	0.1609
StdDev	0.07	345	0.0117
Skew	13.79	13.78	13.66
Kurt	190.74	190.52	188.17
No.:	192	192	192
JB	288 056	287 389	280 272

Panel I: Difference between a single volatility and a dual volatility in percent based on up and down factors calculations from AngloGold Ashanti, Gold Fields and Sibanye Stillwater over a five year period

Panel I(1): AngloGold Ashanti

Variable	Production_Up	Production_Down	Au Grade_Up	Au Grade_Down	Au Price_Up	Au Price_Down	Cost_Up	Cost_Down
Mean	-0.19	0.25	-0.17	0.22	-0.05	0.05	-0.16	0.20
Median	-0.19	0.24	-0.18	0.22	-0.05	0.05	-0.16	0.19
Min	-0.30	0.07	-0.28	0.07	-0.08	0.02	-0.25	0.06
Max	-0.07	0.43	-0.06	0.39	-0.02	0.09	-0.06	0.34
StdDev	0.09	0.14	0.09	0.13	0.03	0.03	0.08	0.11
Skew	0.16	0.16	0.14	0.14	0.04	0.04	0.13	0.13
Kurt	-1.16	-1.16	-1.17	-1.17	-1.20	-1.20	-1.17	-1.17
No.:	5	5	5	5	5	5	5	5
JB	3.63	3.63	3.64	3.64	3.67	3.67	3.64	3.64

Panel I(2): Gold Fields

Variable	Production_Up	Production_Down	Au Grade_Up	Au Grade_Down	Au Price_Up	Au Price_Down	Cost_Up	Cost_Down
Mean	-0.37	0.69	-0.35	0.60	-0.10	0.11	-0.36	0.63
Median	-0.39	0.64	-0.36	0.56	-0.10	0.11	-0.37	0.59
Min	-0.56	0.18	-0.53	0.16	-0.16	0.04	-0.54	0.17
Max	-0.15	1.28	-0.14	1.11	-0.03	0.19	-0.14	1.17
StdDev	0.16	0.44	0.15	0.37	0.05	0.06	0.16	0.40
Skew	0.36	0.36	0.33	0.33	0.08	0.08	0.34	0.34
Kurt	-1.00	-1.00	-1.04	-1.04	-1.19	-1.19	-1.03	-1.03
No.:	5	5	5	5	5	5	5	5
JB	3.45	3.45	3.49	3.49	3.66	3.66	3.47	3.47

Panel I(3): Sibanye Stillwater

Variable	Production_Up	Production_Down	Au Grade_Up	Au Grade_Down	Au Price_Up	Au Price_Down	Cost_Up	Cost_Down
Mean	-0.31	0.49	-0.31	0.50	-0.07	0.08	-0.30	0.48
Median	-0.32	0.47	-0.32	0.47	-0.07	0.08	-0.31	0.46
Min	-0.47	0.14	-0.48	0.14	-0.12	0.03	-0.47	0.13
Max	-0.12	0.90	-0.12	0.91	-0.02	0.13	-0.12	0.88
StdDev	0.14	0.30	0.14	0.31	0.04	0.04	0.14	0.29
Skew	0.28	0.28	0.29	0.29	0.06	0.06	0.28	0.28
Kurt	-1.08	-1.08	-1.08	-1.08	-1.20	-1.20	-1.09	-1.09
No.:	5	5	5	5	5	5	5	5
JB	3.54	3.54	3.53	3.53	3.67	3.67	3.54	3.54

Note: Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera. All variables are in decimals except No. and JB. All variables are in decimals except No. and JB.

The first set of analysis was used to compute simple statistics for the level of correlation, portfolio of correlation and portfolio volatility. On average, level of correlation, portfolio of correlation and portfolio volatility for AngloGold Ashanti were shown to have 0.01, 49 and 0.00, respectively. For Gold Fields, both variables (i.e. level of correlation, portfolio of correlation and portfolio volatility on average) are positively skewed with positive kurtosis. The next step computed the covariance that was calculated from the ratio between total products with the number of samples minus one to come up at 12.91 x 10^9. Then, the basic statistics are used to assess the difference at a given period of time that dual volatility to other volatilities for a 5 year interval, as shown in Table 6.2 in Panel I. The significant risk of AngloGold Ashanti on production is similar to that of Gold Fields, where the up factor fell at -0.37, while the down factor came up at 0.69. Taken together, the most significant volatility of the productions from Gold Fields was indicated by Kloof-Driefontein Complex and Beatrix mines that were closed in South Africa in 2012. Kloof-Driefontein Complex and Beatrix mines had supplied around 40% of Gold Fields gold production in 2012. As a result, the annual production rate dropped from 3,348 to 2,376 ounces from 2012 to 2013.

As shown in Table 6.2, one concern was expressed regarding the level of correlation, portfolio of correlation and portfolio volatility for gold from AngloGold Ashanti in Panel H(1), Gold Fields in Panel H(2) and Sibanye-Stillwater in Panel H(3). These results are similar to those reported by Khalifa et al (2011), where these perform positive values of both skewness and kurtosis. A common view amongst these positive values is that the largest value of both AngloGold Ashanti, Gold Fields and Sibanye-Stillwater main variables, including gold prices from 1995 to 2008, fell in lower values (Khalifa et al, 2011).

Panel H(3) in Table 6.2 shows that level of correlation, portfolio of correlation and portfolio volatility (i.e. dual volatility) are positively skewed with high kurtosis. Normally, high kurtosis leads to a leptokurtic distribution. Leptokurtic distribution increases the margins, especially in a trading environment (See; Marcato et al, 2018; 2019). Interestingly, the dual volatility of Sibanye-Stillwater is quite small (i.e. 0.04) in relation to volatilities of the two gold mining companies (AngloGold Ashanti at 0.07 and Gold Fields at 0.06). That might be due to the fact that when numerous variables are modelled appropriately in a single model, the clustering effect is minimal. The latter phenomenon is not evident in AngloGold Ashanti and Gold Fields. The two companies listed in the latter statement are more global in terms of operations than Sibanye-Stillwater; therefore, there is less volatility in AngloGold Ashanti and Gold Fields.

This study compares the difference between the main variables from AngloGold Ashanti in Panel I(1), Gold Fields in Panel I(2) and Sibanye-Stillwater in Panel I(3) in Table 6.2, and skewness and kurtosis that they are essentially identical. These results indicate that gold volatility was not as high as during the 2014-2017 period than in the 1995-2008 period. For example, all variables of the three companies in Panel I are positively skewed. It can be inferred from Marcato et al (2019) that positive skewness is associated with low investment costs in capital-intensive assets. The latter statement resonates with the mining industry, given that in the last 60 odd years, most of the global mining has been characteristics by deep mining activities. In South Africa, deep mining can be traced back to the 1950s.

A dual volatility from portfolio variance computation equals to 0.07, while the gold price volatility from AngloGold Ashanti contract price was 0.10. These results suggest that no significant differences were found between a dual volatility with the gold prices volatility. A correlation between the main variables from Gold Fields was tested using a Sample Correlation Coefficient, which is presented in Panel I(2) in Table 6.2. The mean score for the level of correlation, portfolio of correlation and portfolio volatility are 0.01, 111 and 0.00, respectively. Further analysis shows that positive values were evident from them that can reveal the skewed distributions. A Sample Correlation Coefficient was then used to compute covariance that was at 12.9 x 10^9. Further calculation revealed that the portfolio of correlation has 3,608. This result was shown to have portfolio variance at 0.07. The main variables of AngloGold Ashanti are significantly different from those of gold prices in several key respects. AngloGold Ashanti main variables tend to have a greater difference than gold prices that were found at 0.04% and 1% on average, respectively (Khalifa et al, 2011). Comparing the two results, it can be seen that AngloGold Ashanti main variables volatility had a higher than average gold prices volatility.

The volatility of gold prices and the portfolio variance (a dual volatility) for Gold Fields were compared in order to analyse the relationship between them. The volatility of gold prices was 0.04, while a dual volatility came up at 0.07. The evidence from this study suggests that the single gold prices volatility fails to draw the future risks. As shown in Table 6.2 for AngloGold Ashanti in Panel I(1), the significant differences between time interval for each volatility during a 5 year period was found from a production variable lower at -0.19 for the up factor and 0.25 for the down factor. It is likely the more extreme volatility in productions compared with a dual volatility is a result of Obuasi mine being closed in Ghana. Obuasi had supplied around 12% of AngloGold Ashanti gold production in 2012.

Gold Fields main variables differ not only in main variable attributes and their correlations, but also in the way in which they have volatility to gold prices during 1995 to 2008 (Batten et al, 2010). The average of gold price differences from 1995 to 2008 is 0.01 on average (Khalifa et al, 2011), while estimates of Gold Fields main variables were 0.06. Key illustrations in Table 6.1 are similar ones forum by Khalifa et al (2011) and Batten et al (2010).

Panels H(3) and I(3) illustrate that the Sample Correlation Coefficient and the difference between single price and dual volatility of Anglo Gold Ashanti. For Panel H(3), in terms of pattern of skewness and kurtosis, the patterns in Sibanye Stillwater are similar to patterns illustrated by AngloGold Ashanti and Golds Fields in Panel H. therefore, findings in Panel H made on AngloGold Ashanti and Gold Fields, equitably apply to Sibanye Stillwater. Similarly, the patterns of Sibanye Stillwater in Panel H3 are similar to patterns illustrated by AngloGold Ashanti and Gold Fields in Panel I. Hence, similar conclusions from Panel on AngloGold Ashanti and Gold Fields can be inferred on Sibanye-Stillwater. Specifically on Gold Fields, the results are similar to those reported by Tholana et al (2013); Neingo and Tholana (2016), where the gold grade declined from approximately 6 grams per ton in 1975 to 2 grams per ton in 2020, while the production has decreased with similar proportions to gold grade.

6.1.1.3 Platinum

It can be seen from Table 6.3, Panels J and K present an overview of a Sample Correlation Coefficient of platinum based on Anglo Platinum and Impala data respectively, from 2004 to 2019. Anglo Platinum may be divided into several variables: (a) produced material in metric tonne, (b) 4E head grade in gram per ton, (c) platinum price in U.S. dollar per ton, and (d) cash operating cost in million ZAR. Then, four variables from Impala from 2004 to 2019 were included for the study such as mined ore in metric tonne, 5PGE+Au in gram per ton, platinum price in concentrate in U.S. dollar per ounce, and cost in ZAR per ounce. Table 6.3 illustrates measures that are used to explore single and dual volatilities for platinum companies. Moreover, variables and/or variables that are typically used to calculate dual volatilities are presented using basic statistics illustrations.

Table 6. 3: Sample Correlation Coefficient and Difference in Platinum Volatility

Panel J: Sample Correlation Coefficient computation for platinum using the Anglo Platinum and Impala data from 2004 to 2019

Panel J(1): Anglo Platinum

Description	Level of Correlation	Portfolio of Correlation	Portfolio Volatility
Mean	0.01	334	197.37
Median	0.00	145	5.87
Min	0.00	0.00	0.00
Max	1.00	31 232	799.98
StdDev	0.07	2 248	798.47
Skew	13.74	13.74	9.63
Kurt	189.92	189.78	111.07
No.:	192	192	192
JB	285 572	285 123	96 400

Panel J(2): Impala

Description	Level of Correlation	Portfolio of Correlation	Portfolio Volatility
Mean	0.01	154.92	0.00
Median	0.00	57.44	0.00
Min	0.00	0.00	0.00
Max	1.00	14446.65	0.44
StdDev	0.07	1039.95	0.03
Skew	13.74	13.73	13.86
Kurt	189.79	189.66	192.00
No.:	192	192	192
JB	285 155	284 781	291 912

Panel K: The different values between a single volatility and a dual volatility in percent based on up and down factors calculations for platinum based on Anglo Platinum and Impala over a 5 year period

Panel K(1): Anglo Platinum

Variable	Production_Up	Production_Down	Pt Grade_Up	Pt Grade_Down	Pt Price_Up	Pt Price_Down	Cost_Up	Cost_Down
Mean	-0.31	0.49	-0.27	0.41	-0.36	0.64	-0.51	1.35
Median	-0.32	0.46	-0.28	0.39	-0.37	0.60	-0.54	1.20
Min	-0.47	0.14	-0.42	0.12	-0.54	0.17	-0.73	0.30
Max	-0.12	0.89	-0.10	0.74	-0.14	1.18	-0.23	2.71
StdDev	0.14	0.30	0.13	0.25	0.16	0.40	0.20	0.96
Skew	0.28	0.28	0.24	0.24	0.34	0.34	0.57	0.57
Kurt	-1.08	-1.08	-1.11	-1.11	-1.02	-1.02	-0.72	-0.72
No.:	5	5	5	5	5	5	5	5
JB	3.54	3.54	3.57	3.57	3.47	3.47	3.15	3.15

Panel K(2): Impala

Variable	Production_Up	Production_Down	Pt Grade_Up	Pt Grade_Down	Pt Price_Up	Pt Price_Down	Cost_Up	Cost_Down
Mean	-0.20	0.26	-0.21	0.28	-0.44	0.97	-0.44	0.94
Median	-0.20	0.25	-0.21	0.27	-0.47	0.88	-0.46	0.86
Min	-0.31	0.08	-0.33	0.08	-0.65	0.23	-0.64	0.23
Max	-0.07	0.45	-0.08	0.49	-0.19	1.87	-0.19	1.81
StdDev	0.10	0.15	0.10	0.16	0.18	0.65	0.18	0.63
Skew	0.17	0.17	0.18	0.18	0.46	0.46	0.45	0.45
Kurt	-1.16	-1.16	-1.15	-1.15	-0.88	-0.88	-0.90	-0.90
No.:	5	5	5	5	5	5	5	5
JB	3.63	3.63	3.62	3.62	3.32	3.32	3.33	3.33

Note: Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera. All variables are in decimals except No. and JB. All variables are in decimals except No. and JB.

Panel J in Table 6.3 summarises a Sample Correlation Coefficient using Anglo Platinum data from 2004 to 2019. The results show that the level of correlation, portfolio of correlation and portfolio volatility are 13.74, 13.74 and 9.63, respectively. The themes identified in these responses of kurtosis fell at; 189.92, 189.78 and 111.07. Then, for the purpose of a Sample Correlation Coefficient calculation, Anglo Platinum data was used to assess covariance that was 11.21 x 10^{12}. In the final part of the computation, covariance was divided by product of SD to deliver a portfolio variance at 70,251.

The salient calculations, which are not included in Table 6.3, illustrate that dual volatility is 0.12, but price volatility for Anglo American Platinum Ltd is 0.19. That latter volatility is computed from the logarithm of the current price divided by the logarithm of the previous price. The results of the difference have been assessed between a dual volatility and production rates, platinum grades and costs as shown in Table 6.1. Another salient point, not in the table, shows that the difference between cost volatility and dual volatility is -0.51 (up node of decision tree) and 1.35 (down node of decision tree), respectively. The highest possible volatility is 0.29 from Mogalakwena mining operations.

In Table 6.3, the positive skewness and kurtosis of level of correlation, portfolio correlation and portfolio variance on their main variables in Panel J(1) and J(2) are similar to those of platinum prices from 1996 to 2006 (Nguyen & Walther, 2020). What is interesting about the data in this table is that the positive skewness and kurtosis of the level of correlation, portfolio correlation and portfolio variance appeared to be unaffected by platinum prices during 1996 to 2006 period.

There are a number of important differences between a dual volatility from Anglo Platinum during the period of 2004 to 2019 and platinum prices from 1996 to 2005. Simple statistical analysis was used to observe the up and down differences for a dual volatility that were found at an average of 0.36. Mean estimated that the up and down differences at platinum prices from 1996 to 2005 was 0% (Nguyen & Walther, 2020). In response to 0% on average differences, this indicated that prices volatility failed to consider the other risks. In the long-run, volatilities converge to their long-term averages and in the long-run, the mean of platinum is zero (Nguyen & Walther, 2020). By extension, the long-term average volatility is zero.

There is a small difference in ratio between a dual volatility and other variable volatilities by employing up and down factors over a 5 year period. A descriptive case study of the up and down factors from Impala differs from an exploratory study in that it uses platinum prices during the period of 1996 to 2005 (Bao, 2020). The average up and down differences for platinum prices were approximately 0% (Nguyen & Walther, 2020). However, with Impala Platinum main variables, caution must be applied, as the findings might discover the up and down differences to be 0.20 on average. As a result, previous studies of platinum prices volatility have not dealt with other variables such as productions, grades and costs. Fundamentally, it seems that the four commodities presented in this study are sensitive to different regimes of commodities. Partly, that can be picked up from skewness and the presence of kurtosis. Therefore, this study proposes using the Markov-Regime Switching Model in order to explore how those regimes affect and/or explain a dual volatility.

In section 4.3.1, the thesis presents and interprets the calculation of the robustness tests for Hypothesis One based on Equation 4.8. Now, in order to ascertain the strength of the Correlation Coefficient technique, this study proceeds to run *Samuelsson effect* on the three commodities-copper, gold and platinum. Volatility inequality 1 illustrates the results of the Samuelsson effect;

$$
\left\{
\begin{array}{ll}
Price\ Volatility & \left\{\begin{array}{ccc} Copper & 0.08 & (a) \\ Gold & 0.04 & (b) \\ Platinum & 0.06 & (c) \end{array}\right\} \\
Dual\ volatility^{**} & \left\{\begin{array}{ccc} Palabora\ Copper\ Mining\ Ltd & 0.05 & (d) \\ AngloGold\ Ashanti\ Ltd & 0.07 & (e) \\ Gold\ Fields\ Ltd & 0.06 & (f) \\ Sibanye - Stiilwater\ Ltd & 0.04 & g \\ Anglo\ American\ Platinum\ Ltd & 0.12 & (h) \\ Impala\ Platinum\ Ltd & 0.09 & (j) \end{array}\right\} \\
Dual\ volatility_{\xi} & \left\{\begin{array}{ccc} Copper & 0.00 & (k) \\ Gold & 0.00 & (l) \\ Platinum & 0.00 & (m) \end{array}\right\} \\
Samuelson\ effect & \left\{\begin{array}{ccc} Copper & 0.18 & (n) \\ Gold & 0.24 & (o) \\ Platinum & 0.24 & (p) \end{array}\right\}
\end{array}
\right\}
$$

Note: $Dual\ volatility^{**}$ is when dual volatility is made up of metal prices, metal grade, operating costs and production rates, and $Dual\ volatility_{\xi}$ is when dual volatility is made up of financial and technical variables.

In this research, the future prices volatility are computed for copper, gold and platinum using the *Samuelson effect*. Data for the implied volatility (VIX) is from the Chicago Board Options Exchange (CBOE). This data represented copper price volatility containing the daily historical volatility that in this research was extracted from 2 January 2004 to 31 December 2019 with a total data of 4,027. Then, for gold and platinum, these extracted from the Chicago Board Options Exchange for the monthly historical volatility for gold volatility exchange trade fund index (GVZ) from July 2008 to December 2019 with a total data of 138 (CBOE, 2022). The futures prices volatility of copper, gold and platinum based on the Samuelson effect are 0.18, 0.24 and 0.24, respectively.

As shown in volatility inequality 1, it is apparent that the Samuelson effect is extremely high in comparison to the futures volatility. This supports prior studies (Chen et al, 1999; Grammatikos & Saunders, 1986; Anderson, 1985; Bessembinder et al, 1986) that, the volatility computed using the Samuelson effect tends to increase when that volatility approaches the expiration point. Therefore, it fails to address time to maturity in computing the future prices volatility for copper, gold and platinum. This result is supported by Duong & Kalev (2005) that the Samuelson effect cannot deal with the future prices volatility dynamism for metals (i.e., copper, gold and platinum), energy and financials.

The Gaussian Autoregressive Conditional Heteroscedasticity (GARCH) model is designed to forecast the future volatility based on the historical prices. In this research, the Gaussian Autoregressive Conditional Heteroscedasticity (GARCH) model is applied for copper, gold and platinum. These results are 0.08 for copper, 0.04 for gold and 0.05 for platinum. These results are similar to those computed by the logarithmic returns that are 0.08, 0.04 and 0.06 for copper, gold and platinum, respectively. However, the Gaussian Autoregressive Conditional Heteroscedasticity (GARCH) model appears to be accurate since it employs over 5,000 datasets. Therefore, it is suggested to use daily data. This research just employs 144 dataset using the monthly data. Then, another weakness of this model is the failure to address the future movement directions (e.g., up or down) since it just focuses on identifying the movement size (Matei, 2009). As a conclusion, this model makes no attempt to incorporate the other risks in computing the volatility that is addressed in this research to deal with the mining risks.

In order to examine Hypothesis One using a Decision Tree Analysis in forecasting future mineral commodity prices, the Markov-Regime Switching model has been proposed to be taken into account. The reason for using the Markov-Regime Switching model for the robustness test for Hypothesis One are provided in section 4.3.1.

6.1.2 Robustness Test for a Decision Tree Analysis

Table 6. 4: Markov-Regime Switching Results

Variables		Palabora Copper Mining Ltd				AngloGold Ashanti				Gold Fields			
		Metal Grades	Operating Costs	Price	Production Rates	Metal Grades	Operating Costs	Price	Production Rates	Metal Grades	Operating Costs	Price	Production Rates
Regime	1	0.053 (0.005)***	-2.609 (0.000)***	0.017 (0.003)***	0.367 (0.000)***	-0.151 (0.000)***	0.081 (0.000)***	0.003 (0.507)	0.106 (0.000)***	0.009 (0.858)	0.79 2(0.000)***	0.078 (0.003)***	1.116 (0.000)***
	2	-0.046 (0.011)***	0.028 (0.593)	-0.616 (0.000)***	-0.355 (0.000)***	0.111 (0.000)***	-0.048 (0.000)***	0.805 (0.000)***	-0.149 (0.000)***	0.009 (0.856)	0.011 (0.006)***	0.00 3(0.546)	0.007 (0.011)***
Transition	1	-22.521 (0.999)	-19.982 (0.998)	4.125 (0.000)***	-3.833 (0.000)***	-14.05 4(0.916)	-0.014 (0.9235)	4.527 (0.000)***	-0.582 (0.025)***	-0.013 (1.000)	-19.403 (0.999)	-2.203 (0.220)	-5.252 (0.000)***
	2	4.316 (0.012)***	-3.593 (0.000)***	19.168 (0.999)	3.417 (0.000)***	0.0659 (0.001)***	-0.012 (0.752)	14.163 (0.987)	24.094 (0.999)	-0.036 (0.923)	-4.536 (0.000)***	-2.654 (0.043)***	-4.897 (0.006)***
AR	1	-0.379 (0.000)***	-0.153 (0.039)**	-0.553 (0.000)***	-1.097 (0.000)***	-0.176 (0.043)***	-1.531 (0.000)***	-0.025 (0.745)	-0.061 (0.599)	-0.448 (0.000)***	-0.736 (0.000)***	0.159 (0.229)	-0.896 (0.000)***
	2	-0.0165 (0.836)	0.024 (0.582)	0.039 (0.640)	-0.366 (0.001)***	-0.022 (0.769)	-1.579 (0.000)***	-0.044 (0.554)	-0.031 (0.681)	0.018 (0.816)	-0.091 (0.357)	0.011 (0.910)	-0.409 (0.000)***
	3	-0.044 (0.571)	0.636 (0.086)*	0.118 (0.163)	-0.367 (0.001)***	0.036 (0.702)	-1.352 (0.000)***	0.048 (0.513)	0.060 (0.504)	-0.249 (0.001)***	-0.064 (0.573)	0.029 (0.753)	-0.262 (0.012)***
	4	-0.162 (0.029)***	0.111 (0.000)***	0.011 (0.884)	-0.1314 (0.071)**	-0.122 (0.103)	-0.749 (0.000)***	-0.035 (0.635)	-0.501 (0.513)	-0.279 (0.000)***	-0.122 (0.159)	0.025 (0.793)	-0.177 (0.024)***
Durban-Watson		1.867	1.935	1.931	1.467	2.402	1.999	1.852	2.436	1.929	1.961	2.072	1.808
Akaike		-1.196	0.488	-1.332	-1.411	-1.895	-1.657	-2.437	-1.818	-1.233	-1.557	3.567	-1.613
Hannan-Quinn		-1.133	0.550	-1.269	-1.348	-1.833	-1.601	-2.374	-1.755	-1.177	-1.494	-3.504	1.557
Schwarz		-1.041	0.643	-1.177	-1.256	-1.740	-1.519	-2.282	-1.663	-1.095	-1.402	-3.412	-1.475
Variables		Sibanye Stillwater				Anglo Platinum				Impala Platinum			
		Metal Grades	Operating Costs	Price	Production Rates	Metal Grades	Operating Costs	Price	Production Rates	Metal Grades	Operating Costs	Price	Production Rates
Regime	1	-0.047 (0.000)***	0.120 (0.181)	-0.002 (0.855)	0.104 (0.000)***	0.077 (0.000)***	0.129 (0.000)***	0.161 (0.000)***	0.571 (0.000)***	0.105 (0.000)***	0.093 (0.000)***	-0.366 (0.000)***	-0.125 (0.000)***
	2	0.068 (0.000)***	0.016 (0.000)***	0.045 (0.119)	-0.154 (0.000)***	-0.058 (0.000)***	-0.199 (0.000)***	-0.188 (0.000)***	0.001 (0.857)	-0.353 (0.000)***	-0.277 (0.000)***	0.099 (0.000)***	0.077 (0.000)***
Transition	1	-23.108 (0.999)	-0.357 (0.709)	2.158 (0.070)*	-0.382 (0.059)*	-14.521 (0.924)	1.1269 (0.000)***	0.269 (0.171)	-3.598 (0.000)***	1.587 (0.000)***	1.560 (0.000)***	-1.755 (0.000)***	-0.745 (0.005)***
	2	22.416 (0.999)	-4.02 5(0.000)***	-0.342 (0.863)	26.591 (0.998)	16.228 (0.963)	0.986 (0.007)***	0.407 (0.088)*	-2.951 (0.051)**	21.661 (0.000)	22.538 (0.999)	-0.977 (0.056)**	-0.491 (0.010)***
AR	1	-0.623 (0.000)***	-0.657 (0.000)***	0.057 (0.784)	-0.182 (0.048)**	-1.042 (0.000)***	-0.551 (0.000)***	-0.352 (0.000)***	-0.504 (0.000)***	0.292 (0.001)***	-0.169 (0.052)**	0.299 (0.000)***	0.022 (0.564)
	2	-0.231 (0.008)***	-0.003 (0.925)	-0.063 (0.688)	0.029 (0.696)	-0.820 (0.000)***	-0.616 (0.000)***	-0.009 (0.905)	-0.043 (0.603)	-0.059 (0.477)	-0.328 (0.000)***	-0.069 (0.392)	-0.896 (0.000)***
	3	-0.103 (0.234)	-0.147 (0.000)***	0.025 (0.832)	0.053 (0.509)	-0.661 (0.000)***	-0.812 (0.000)***	-0.187 (0.008)***	-0.183 (0.035)***	-0.302 (0.001)***	-0.262 (0.000)***	-0.225 (0.009)***	0.013 (0.740)
	4	-0.183 (0.0123)**	-0.142 (0.000)**	0.026 (0.771)	-0.112 (0.119)	-0.505 (0.000)***	-2.384 (0.000)***	0.341 (0.000)***	-0.194 (0.010)***	-0.169 (0.128)	-0.543 (0.000)***	-0.204 (0.019)***	0.860 (0.000)***
Durban-Watson		1.839	1.167	1.996	2.375	1.591	2.302	1.667	1.967	2.257	1.871	2.344	2.066
Akaike		-1.269	-1.337	-3.589	-2.039	-2.203	-0.727	-0.963	-0.856	-0.599	-0.551	0.480	-2.128
Hannan-Quinn		-1.206	-1.267	-3.527	-1.1977	-2.139	-0.678	-0.901	-0.799	-0.536	-0.489	-0.424	-2.066
Schwarz		-1.114	-1.165	-3.435	-1.885	-2.048	-0.606	-0.809	-00718	-0.444	-0.397	-0.342	-1.974

Note: R stands for regime, T for transition, AR stands for Application Markov Switching Regression, MG for metal grades, OC is operating costs, P is price and PR is production rates. ***, ** and * represent significance levels of 1%, 5% and 10%, respectively.
Source: AngloGold Ashanti, Gold Fields, Sibanye Stillwater, Anglo Platinum and Impala Platinum financial statements (2004-2019) and Palabora Ltd financial statements (2004-2013).

Based on Table 6.4, for the six companies, based on the four variables (metal grades, price, production rates and operating costs), almost every regime is statistically significant, except price for AngloGold Ashanti. That might be due to the fact that gold is generally regarded as currency. Therefore, the effects of gold are felt more in currency than in the commodities markets. In terms of regimes being statistically significant, that would be expected as commodities are volatile indeed. Choi & Hammoudeh (2010) illustrated that commodities are more sensitive to low regimes (usually negative) than high regimes (usually positive). That might be due to the fact that negative regimes are more characterised by black swans than positive regimes (Kola & Sebehela, 2020). In terms of transition, regimes transit from the first regime to the second regime. Broadly, based on absolute numbers, first regimes have higher impact than the second regimes. Kola & Sebehela (2020) found similar patterns when they illustrated transatlantic volatility regimes. It can be inferred from Kola &

Sebehela (2020) that the first regime tends to be a trend setter and/or indicator of volatility/returns patterns, including influencing latter regimes.

In terms of Application Markov Switching Regression, the pattern generally starts at a high level and then moves to a lower level. That might be due to declining regimes over time as markets converge to their long-term averages for most parakeets, such as volatilities and returns. The validation measures (Durbin-Watson, Akaike, Hannan-Quinn and Schwarz)-Durbin Watson below 1.3 reflect the presence of autocorrelation. Akaike, Hannan-Quin and Schwarz should be within the 1.6 to 2.7 range in terms of normality. Put it differently, the Akaike, Hannan-Quinn and Schwarz tests are criteria for model selection. Broadly, models with the lowest Akaike, Hannan-Quinn and Schwarz values are preferred. The three criteria also form part of the Likelihood Function Test. That is, the validation measures are more in line for copper and gold firms but not for platinum. That might be due to the fact that platinum is supplied mainly by one country, South Africa-at least 78% of platinum globally is from South Africa. During the 2008/2009 period, commodities were generally low because there was the subprime crisis. During the crisis, commodities movements tend to supersede property prices. And, in 2014, there was a drop in the demand for commodities, largely because of a decline in Chinese economic growth.

Of interest from the Markov-Regime Switching Model is the convergence of the "casual" variables. In the results, where the first variable is metal grades, then operating costs; thereafter, prices and finally, production rates irrespective of the company presented, the iterations to convergence reveal the following. For Palabora Ltd; 34 iterations for metal grades, 28 for operating costs, 56 for prices and 25 for production rates. For AngloGold Ashanti; 8 for metal grades, 24 for operating costs, 15 for prices and 33 for production rates. Gold Fields, 9 for metal grades, 52 for operating costs, 21 for prices and 18 for production rates. Sibanye Stillwater; 28 for metal grades, 6 for operating costs, 25 for prices and 44 for production rates. Anglo Platinum; 14 for metal grades, 11 for operating costs, 14 for prices and 11 for production rates. Impala Platinum; 39 for metal grades, 25 for operating costs, 25 for prices and 11 for production rates. Building on the logic from Marcato et al (2018), independent variables converge quicker when they have a higher impact factor on dependent variables. Therefore, for Palabora Ltd, production rates have a much more causal effect on the value of the company. Most of the causal effect on AngloGold Ashanti is from metal and production grades. For Gold Fields, most causal effect is from metal grades. The Sibanye Stillwater is mostly from operating costs and the same phenomenon is there in Anglo Platinum. For Impala Platinum, the causal effect is mainly from production rates. Fundamentally, each company is driven by a variable, which does not necessarily translate to having a causal effect on another company in the same sector. That is confirmed by causal variables of gold and platinum companies.

6.1.3 Other Tests for Hypothesis One

In this research, the future copper price forecasting was measured using a Decision Tree Analysis in four different scenarios such as up and down sold copper price volatility and up and down a dual volatility from 2016 to 2020. The actual copper price from 2016 to 2020 stated in U.S.$ per pound can be seen in Table 6.5 in Panel K. In 2016, the actual copper price was U.S.$ 2.21 per pound. It increased to be U.S.$ 2.80 per pound in 2017. Then, in 2018, the copper price reached U.S.$ 2.96 per pound. The price declined in 2019 to be U.S.$ 2.73 per pound and bounced to U.S.$ 2.80 per pound in 2020 (IndexMundi, 2022).

The future copper price forecasting using a Decision Tree Analysis with sold copper price volatility for the up factor increased from 2016 to 2018 – it was U.S.$ 2.97 per pound in 2016, U.S.$ 3.19 per pound in 2017, U.S.$ 3.36 per pound in 2018 and U.S.$ 3.52 per pound in 2019. It increased to be U.S.$ 3.67 per pound in 2020. Using a down factor of sold copper price volatility, the future copper price forecasting in 2016 was U.S.$ 2.11 per pound, U.S.$ 1.96 per pound in 2017, U.S.$ 1.86 per pound in 2018, U.S.$ 1.77 per pound in 2019 and U.S.$ 1.70 per pound in 2020. The down factor showed that the price dropped in 2019 and dropped further in 2020.

The future copper price forecasting was recorded based on a dual volatility for the up factor and showed that the price increased from 2016 to 2020. In 2016, the future copper price forecasting was U.S.$ 2.62 per pound, U.S.$ 2.68 per pound in 2017, U.S.$ 2.72 per pound in 2018, U.S.$ 2.75 per pound 2019 and U.S.$ 2.78 per pound in 2020. The dual volatility for the down factor revealed that it consistently dropped from 2016 to 2020. It was U.S.$ 2.38 per pound in 2016, U.S.$ 2.34 per pound in 2017, U.S.$ 2.30 per pound in 2018, U.S.$ 2.27 per pound in 2019 and U.S.$ 2.25 per pound in 2020.

The descriptive statistics for the actual copper price and the future copper price forecasting using sold copper price volatility and dual volatility were presented in Panel L in Table 6.5. The actual copper price mean from 2016 to 2020 revealed at U.S.$ 2.70 per pound. The minimum and maximum values of the actual copper price were U.S.$ 2.21 per pound and U.S.$ 2.96 per pound, respectively. Furthermore, skewness of actual copper price was -1.72 and kurtosis was at 3.51. These results illustrated that the actual copper price from 2016 to 2020 were negatively skewed with heavy tails.

The mean of the future copper price forecasting with a sold copper price volatility for up factor revealed at U.S.$ 3.34 per pound with the minimum value of U.S.$ 2.97 per pound and the maximum value was U.S.$ 3.67 per pound. Further statistical tests showed skewness at -0.31 and kurtosis was -0.94. These results showed negative skewness and light tails.

The mean value of the of the future copper price forecasting with a sold copper price volatility for down factor showed at U.S.$ 1.88 per pound. Then, the minimum and maximum values appeared at U.S.$ 1.70 per pound and U.S.$ 2.11 per pound, respectively. The skewness recorded at 0.55 and kurtosis revealed at -0.61. This skewness and kurtosis suggested a positive skewness and light tails.

The future copper price forecasting with a dual volatility up factor recorded that the mean was U.S.$ 2.71 per pound. The minimum and maximum values were at U.S.$ 2.62 per pound and U.S.$ 2.78 per pound, respectively. Furthermore, the skewness value was -0.39 and kurtosis was -0.84. Therefore, the future copper price forecasting with a dual volatility up factor appeared to be negatively skewed with light tails. The last future copper price forecasting was presented with a dual volatility down factor. The mean value was U.S.$ 2.31 per pound. The minimum value revealed at U.S.$ 2.25 per pound and the maximum value was U.S.$ 2.38 per pound. Further statistical tests showed that skewness recorded at 0.46 and kurtosis was at -0.75. These results indicated that the data was positively skewed with light tails.

Furthermore, the future copper price forecasting results using a single sold copper price volatility and a dual volatility with a Decision Tree Analysis were examined to identify their forecasting performances. These can quantitatively be determined using the Root Mean Square Error. According to Atsalakis (2014), the Root Mean Square Error was one of the most commonly used models to test the future copper price forecasting that was expressed as $RMSE = \sqrt{\frac{\sum_{i=1}^{N} \quad (A_t - F_t)^2}{N}}$, where, A_t is the actual price, F_t is the future price forecasting and N is the number of datasets. The Root Mean Square Error used to assess the forecasting performance can be computed using Excel Software.

The best forecasting performance using the Root Mean Square Error was achieved when the results are close to zero. Panel L in Table 6.5. showed the forecasting performance using the Root Mean Square Error based on four different scenarios, namely, the up and down for sold copper price volatility and up and down for the dual volatility as 1.44, 1.83, 0.02 and 0.88 respectively. Therefore, the best forecasting methodology to predict the future copper price from 2016 to 2020 was provided by a dual volatility with up factor that was 0.02 for a five-year period. A dual volatility with up factor can perform well to forecast the future copper price from 2016 to 2020 because the actual copper price tended to increase from 2016 to 2020.

Table 6. 5: Decision Tree Analysis for Palabora Mining Company Ltd, 2016 to 2019

Panel K: Actual copper price against future copper price forecasting using a Decision Tree Analysis based on copper price volatility and a dual volatility from 2016 – 2020					
Year	Actual Copper Price (U.S.$/Pound)	Forecasted Copper Price using Sold Copper Volatility_Up U.S.$/Pound)	Forecasted Copper Price using Sold Copper Volatility_Down (U.S.$/Pound)	Forecasted Copper Price using Dual Volatility_Up U.S.$/Pound)	Forecasted Copper Price using Dual Volatility_Down U.S.$/Pound)
2016	2.21	2.97	2.11	2.62	2.38
2017	2.8	3.19	1.96	2.68	2.34
2018	2.96	3.36	1.86	2.72	2.3
2019	2.73	3.52	1.77	2.75	2.27
2020	2.8	3.67	1.7	2.78	2.25
Panel L: Descriptive Statistics for actual copper price against forecasting copper price using a Decision Tree Analysis based on copper price volatility and incorporated volatility from 2016 – 2020					
Description	Actual Copper Price (U.S.$/Pound)	Sold Copper Price Volatility_Up (U.S.$/Pound)	Sold Copper Price Volatility_Down	Incorporated Volatility_Up (U.S.$/Pound)	Incorporated Volatility_Down (U.S.$/Pound)
Mean	2.7	3.34	1.88	2.71	2.31
Median	2.8	3.36	1.86	2.72	2.3
Min	2.21	2.97	1.7	2.62	2.25
Max	2.96	3.67	2.11	2.78	2.38
StdDev	0.29	0.28	0.16	0.06	0.05
Skew	-1.72	-0.31	0.55	-0.39	0.46
Kurt	3.51	-0.94	-0.61	-0.84	-0.75
No.	5	5	5	5	5
JB	5.02	0.26	0.33	0.28	0.29
RMSE	0.00	1.44	1.83	0.02	0.88

Note: Mean is the average Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No.
Source: Palabora Mining Company Ltd Annual Report (2004-2015); IndexMundi (2022).

The Root Mean Square Error provides a single value for the five years (2016-2020) and did not provide forecasting for each year. In order to assess the difference between the actual copper price and the future copper price forecasting for each year, one of the most well-known methods for measuring this forecasting performance was the Mean Absolute Percentage Error (MAPE). The Mean Absolute Percentage Error was presented by the mean of absolute percentage error that is expressed as $\text{MAPE} = \frac{1}{N}\sum_{N=1}^{N} \left|\frac{A_t - F_t}{A_t}\right|$, where, A_t represents the actual price, F_t is the future price forecasting and N is the number of datasets. The Mean Absolute Percentage Error was computed using Excel Software.

Table 6.6 presented, in the first column, showed the Mean Absolute Percentage Error of 0.34 in 2016, 0.14 in 2017, 0.14 in 2018, 0.29 and in 2019 and 0.31 in and 2020. In the second column, the Mean Absolute Percentage Error was 0.05 in 2016, 0.30 in 2017, 0.37 in 2018, 0.35 in 2019 and 0.39 in 2020. The third column showed the Mean Absolute Percentage Error 0.19 in 2016, 0.04 in 2017, 0.01 in 2019 and 0.01 in 2020. The last column in Table 6.6, showed a Mean Absolute Percentage Error of 0.08 in 2016 and 0.17 in 2017, 0.22 in 2018, 0.17 in 2019 and 0.20 in 2020.

Previous research has established the forecasting performance using the Mean Absolute Percentage Error that can be classified using Lewis' scale. He suggested that the best forecasting performance was indicated by the Mean Absolute Percentage Error at less than 0.1. In this research (Table 6.6) the Mean Absolute Percentage Error of 0.05 was obtained in 2016 for the sold copper price volatility down factor-suggesting a price volatility and can only perform well in the short-term. However, these results of dual volatility with up factor showed values lower than 0.1 after from Year 2 or 2017 to Year 5 or 2020. The Mean Absolute Percentage Error recorded at 0.04, 0.08, 0.01 and 0.01 respectively for 2017, 2018, 2019 and 2020. On the other hand, the dual volatility with down factor only showed a value of less than 0.1 in 2016 as reflected by a value of 0.08. This concluded that a dual volatility using a Decision Tree Analysis provided the best forecasting performance for copper. In order to forecast the short-term period, the dual volatility with down factor can be taken into account. While the incorporated volatility with up factor can achieve the best forecasting performance to predict the medium to long-term copper price.

The dual volatility for up factor can perform well from 2017 to 2020 however, it failed to forecast the copper price in 2016. A possible reason for a lower copper price in 2016 might be that Chinese demand fell due to low growth from the infrastructure sector. This sector absorbed 30% to 40% in using copper. However, copper price increased in 2017, which was caused by Chinese economic stability from the demand side. In addition, from the supply side, it has a serious impact from the world's largest copper producer, Escondida, which had a strike for 44 days and Indonesian export ban regulation (World Bank, 2017).

Table 6. 6: Mean Absolute Percentage Error 2016-2020 for Palabora Mining Company Ltd

Year	Sold Copper Price Volatility_Up	Sold Copper Price Volatility_Down	Dual Volatility_Up	Dual Volatility_Down
2016	0.34	0.05	0.19	0.08
2017	0.14	0.3	0.04	0.17
2018	0.14	0.37	0.08	0.22
2019	0.29	0.35	0.01	0.17
2020	0.31	0.39	0.01	0.20

This research forecasted the next 5-year period from 2016 to 2020 for copper. The best method to forecast the future copper price using a Decision Tree Analysis was a dual volatility for up factor. However, this model should also accept the extreme global economic condition that was caused by the Covid-19 effects in 2020. In order to test the significant impact from the Covid-19 effects in 2020 for copper price, the best Root Mean Square Error results for 2016 to 2020 were compared to 2016 to 2019. The Root Mean Square Error using a Decision Tree Analysis and a dual volatility with up factor from 2016 to 2020 was 0.02 and 0.04 from 2016 to 2019. These results indicated that these have no significant impact from the Covid-19 effects in 2020 for copper. These results were similar to those reported by Ahmed & Sarkodie (2021), where the copper price was less sensitive to the Covid-19 effects.

Table 6. 7: List of empirical studies for copper price forecasting

Data Type	Period	Observation	Model	Future Prediction Target	RMSE	Author
Yearly	2009-2016	7	Bat Algorithm	1 year	0.13	Dehghani & Bogdanovic (2018)
Monthly	Sept 1987-August 2017	360	GA-ANFIS	36 months	0.08	Alameer et al (2019)
Monthly	Jan 1980-April 2019	472	Mean-reverting	30 days	0.05	Rubaszek et al (2020)
Daily	2 Jan 2006-2 Jan 2018	2971	Support Vector	5 and 10 days	0.02	Astudillo et al (2020)
Monthly	Jan 2004-Dec 2015	144	Decision Tree Analysis with a Dual Volatility up factor	5 years	0.02	Current Study for Palabora Mining Company Ltd
				3 years	0.03	
				1 year	0.17	

Table 6.7 compared the Root Mean Square Error results between previous studies with this study for copper. The study by Dehghani & Bogdanovic (2018), Alameer et al (2019) and Astudillo et al (2020) were comparable to this study based on the Root Mean Square Error results. However, the convergence of Rubaszek et al (2020) is within a range of convergence of this thesis, as shown in Table 6.7.

In Table 6.8, the actual gold price was U.S.$ 1,248.99 per ounce in 2016, U.S.$ 1,257.56 per ounce in 2017, U.S.$ 1,269.23 per ounce in 2018, U.S.$ 1,392.50 per ounce in 2019 and U.S.$ 1,784.94 per ounce in 2020 (IndexMundi, 2022). The future gold price forecasting with sold gold price volatility based on AngloGold Ashanti Ltd data from January 2004 to December 2015 for the up factor was U.S.$ 1,281.18 per ounce in 2016, U.S.$ 1,334.70 per ounce in 2017, U.S.$ 1,377.27 per ounce in 2018, U.S.$ 1,414.22 per ounce in 2019 and U.S.$ 1,447.59 per ounce in 2020.

The future gold price forecasting with sold gold price volatility with the down factor was U.S.$ 1,051.48 per ounce in 2016, U.S.$ 1,009.32 per ounce in 2017, U.S.$ 978.12 per ounce in 2018, U.S.$ 952.57 per ounce in 2019 and U.S.$ 930.61 per ounce in 2020. The future gold price forecasting with a dual volatility for up factor was U.S.$ 1,248.69 per ounce in 2016, U.S.$ 1,287.07 per ounce in 2017, U.S.$ 1,317.33 per ounce in 2018, U.S.$ 1,343.38 per ounce in 2019 and U.S.$ 1,366.77 per ounce in 2020. The future gold price forecasting with a dual volatility for down factor was U.S.$ 1,078.85 per ounce in 2016, U.S.$ 1,046.67 per ounce in 2017, U.S.$ 1,022.63 per ounce in 2018, U.S.$ 1,002.80 per ounce in 2019 and U.S.$ 985.64 per ounce in 2020.

The descriptive statistics were based on the actual and forecasted gold prices and the future gold price forecasting using sold gold price volatility and a dual volatility were shown in Panel N in Table 6.8 based on AngloGold Ashanti Ltd data. The mean of the actual gold price from 2016 to 2020 were U.S.$ 1,390.64 per ounce. The minimum value was U.S.$ 1,248.99 per ounce and the maximum value was U.S.$ 1,784.94 per ounce. The skewness was 1.91 and the kurtosis was 3.61. This data indicated a positive skewness and heavy tails. The mean of future gold price forecasting with sold gold price volatility for up factor was U.S.$ 1,370.99 per ounce. The minimum value was U.S.$ 1,281.18 per ounce and the maximum value was U.S.$ 1,447.59 per ounce. The skewness was -0.36 and the kurtosis was -0.89. This data indicated negative skewness and light tails. The mean of future gold price forecasting with sold gold price volatility for down factor was U.S.$ 984.42 per ounce. The minimum value was U.S.$ 930.61 per ounce and the maximum value was U.S.$ 1,051.48 per ounce. The skewness was 0.50 and kurtosis was -0.70. This data indicated positive skewness and light tails.

The mean of future gold price forecasting with a dual volatility for up factor was U.S.$ 1,312.65 per ounce. The minimum value was U.S.$ 1,248.69 per ounce and the maximum value was U.S.$ 1,366.77 per ounce. The skewness was -0.38 and the kurtosis was -0.86. This indicated a negative skewness and light tails.

The mean of future gold price forecasting with an incorporated volatility for down factor was U.S.$ 1,027.32 per ounce. The minimum value was U.S.$ 985.64 per ounce and the maximum value was U.S.$ 1,078.85 per ounce. The skewness was 0.48 and the kurtosis was -0.72. The data illustrates a positive skewness and light tails. Furthermore, the forecasting performances using a single sold gold price volatility and a dual volatility with a Decision Tree Analysis were tested using the Root Mean Square Error. As shown in Panel N Table 6.8, these were 0.04 for sold gold price volatility for up factor, 0.91 for sold gold price volatility for down factor, 0.17 for a dual volatility for up factor and 0.81 for a dual volatility for down factor. These can be concluded that the best forecasting performance was achieved by sold gold price volatility for up factor over a five-year period.

Table 6. 8: Decision Tree Analysis for AngloGold Ashanti Ltd

Panel M: Actual gold price against future gold price forecasting using a Decision Tree Analysis based on sold gold price volatility and a dual volatility from 2016 – 2020					
Year	Actual Gold Price (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Down (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Down (U.S.$/Ounce)
2016	1,248.99	1,281.18	1,051.48	1,248.69	1,078.85
2017	1,257.56	1,334.70	1,009.32	1,287.07	1,046.67
2018	1,269.23	1,377.27	978.12	1,317.33	1,022.63
2019	1,392.50	1,414.22	952.57	1,343.38	1,002.80
2020	1,784.94	1,447.59	930.61	1,366.77	985.64
Panel N: Descriptive Statistics for actual gold price against forecasting gold price using a Decision Tree Analysis based on sold gold price volatility and dual volatility from 2016 – 2020					
Description	Actual Gold Price (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Down (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Down (U.S.$/Ounce)
Mean	1,390.64	1,370.99	984.42	1,312.65	1,027.32
Median	1,269.23	1,377.27	978.12	1,317.33	1,022.63
Min	1,248.99	1,281.18	930.61	1,248.69	985.64
Max	1,784.94	1,447.59	1,051.48	1,366.77	1,078.85
StdDev	228.03	65.49	47.61	46.48	36.71
Skew	1.91	-0.36	0.5	-0.38	0.48
Kurt	3.61	-0.89	-0.7	-0.86	-0.72
No.	5	5	5	5	5
JB	5.74	0.27	0.31	0.27	0.3
RMSE	0.00	0.04	0.91	0.17	0.81

Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No..
Source: AngloGold Ashanti Ltd Annual Report (2004-2019); IndexMundi (2022).

The future gold price forecasting with sold gold price volatility using up factor based on Gold Field Ltd data from January 2004 to December 2015 was U.S.$ 1,210.23 per ounce in 2016, U.S.$ 1,231.38 per ounce in 2017, U.S.$ 1,247.86 per ounce in 2018, U.S.$ 1,261.92 per ounce in 2019 and U.S.$ 1,274.44 per ounce in 2020. The future gold price forecasting with sold gold price volatility using down factor was U.S.$ 1,113.12 per ounce in 2016, U.S.$ 1,094.01 per ounce in 2017, U.S.$ 1,079.56 per ounce in 2018, U.S.$ 1,067.53 per ounce in 2019 and U.S.$ 1,057.04 per ounce in 2020.

The future gold price forecasting with a dual volatility using up factor was U.S.$ 1,249.68 per ounce in 2016, U.S.$ 1,288.53 per ounce in 2017, U.S.$ 1,319.15 per ounce in 2018, U.S.$ 1,345.53 per ounce in 2019 and U.S.$ 1,369.21 per ounce in 2020. The future gold price forecasting with a dual volatility using down factor was U.S.$ 1,077.98 per ounce in 2016, U.S.$ 1,045.49 per ounce in 2017, U.S.$ 1,021.22 per ounce in 2018, U.S.$ 1,001.19 per ounce in 2019 and U.S.$ 983.88 per ounce in 2020. The descriptive statistics based the actual and forecasted gold prices and the future gold price forecasting using sold gold price volatility and dual volatility were shown in Panel P in Table 6.9 based on Gold Fields Ltd data.

The mean of future gold price forecasting with sold gold price volatility for up factor was U.S.$ 1,245.17 per ounce. The minimum value was U.S.$ 1,210.23 per ounce and the maximum value was U.S.$ 1,274.44 per ounce. The skewness was -0.40 and the kurtosis was -0.84. This indicated a negative skewness and light tails. The mean of future gold price forecasting with sold gold price volatility for down factor was U.S.$ 1,082.25 per ounce. The minimum value was U.S.$ 1,057.04 per ounce and the maximum was U.S.$ 1,113.12 per ounce. The skewness was 0.46 and the kurtosis was -0.76. This indicated a positive skewness and light tails.

The mean of future gold price forecasting with dual volatility for up factor was U.S.$ 1,314.42 per ounce. The minimum value was U.S.$ 1,249.68 per ounce and the maximum value was U.S.$ 1,369.21 per ounce. The skewness was -0.38 and the kurtosis was -0.87. These results indicated a negative skewness and light tails. The mean of future gold price forecasting with dual volatility for down factor was U.S.$ 1,025.95 per ounce. The minimum value was U.S.$ 983.88 per ounce and the maximum value was U.S.$ 1,077.98 per ounce. The skewness was 0.48 and the kurtosis was -0.72. This indicated a positive skewness and light tails. The forecasting performances were tested using the Root Mean Square Error. As shown in Panel P Table 6.9, these were 0.33 for sold gold price volatility for up factor, 0.69 for sold gold price volatility for down factor, 0.17 for a dual volatility for up factor and 0.82 for a dual volatility for down factor. These can be concluded that the best forecasting performance was achieved by a dual volatility for up factor over a five-year period.

Table 6. 9: Decision Tree Analysis for Gold Fields Ltd, 2004-2015

Panel O: Actual gold price against future gold price forecasting using a Decision Tree Analysis based on sold gold price volatility and dual volatility from 2016 – 2020					
Year	Actual Gold Price (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Down (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Down (U.S.$/Ounce)
2016	1,248.99	1,210.23	1,113.12	1,249.68	1,077.98
2017	1,257.56	1,231.38	1,094.01	1,288.53	1,045.49
2018	1,269.23	1,247.86	1,079.56	1,319.15	1,021.22
2019	1,392.50	1,261.92	1,067.53	1,345.53	1,001.19
2020	1,784.94	1,274.44	1,057.04	1,369.21	983.88
Panel P: Descriptive Statistics for actual gold price against forecasting gold price using a Decision Tree Analysis based on sold gold price volatility and dual volatility from 2016 – 2020					
Description	Actual Gold Price (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Down (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Down (U.S.$/Ounce)
Mean	1,390.64	1,245.17	1,082.25	1,314.42	1,025.95
Median	1,269.23	1,247.86	1,079.56	1,319.15	1,021.22
Min	1,248.99	1,210.23	1,057.04	1,249.68	983.88
Max	1,784.94	1,274.44	1,113.12	1,369.21	1,077.98
StdDev	228.03	25.28	22.08	47.05	37.06
Skew	1.91	-0.40	0.46	-0.38	0.48
Kurt	3.61	-0.84	-0.76	-0.87	-0.72
No.	5	5	5	5	5
JB	5.74	0.28	0.29	0.27	0.3
RMSE	0.00	0.33	0.69	0.17	0.82

Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No..
Source: Gold Fields Ltd Annual Report (2004-2019); IndexMundi (2022).

The future gold price forecasting with sold gold price volatility using up factor based on Sibanye-Stillwater Ltd data from January 2004 to December 2015 was U.S.$ 1,209.47 per ounce in 2016, U.S.$ 1,230.28 per ounce in 2017, U.S.$ 1,246.49 per ounce in 2018, U.S.$ 1,260.33 per ounce in 2019, and U.S.$ 1,272.64 per ounce in 2020. The future gold price forecasting with sold gold price volatility using down factor was U.S.$ 1,113.83 per ounce in 2016, U.S.$ 1,094.99 per ounce in 2017, U.S.$ 1,080.74 per ounce in 2018, U.S.$ 1,068.88 per ounce in 2019 and U.S.$ 1,058.54 per ounce in 2020.

The future gold price forecasting with dual volatility using up factor was U.S.$ 1,211.86 per ounce in 2016, U.S.$ 1,233.73 per ounce in 2017, U.S.$ 1,250.77 per ounce in 2018, U.S.$ 1,265.32 per ounce in 2019 and U.S.$ 1,278.28 per ounce in 2020. The future gold price forecasting with dual volatility using down factor was U.S.$ 1,111.63 per ounce in 2016, U.S.$ 1,091.93 per ounce in 2017, U.S.$ 1,077.05 per ounce in 2018, U.S.$ 1,064.66 per ounce in 2019 and U.S.$ 1,053.87 per ounce in 2020.

The descriptive statistics based the actual and forecasted gold prices and the future gold price forecasting using sold gold price volatility and dual volatility were shown in Panel R in Table 6.9

based on Sibanye-Stillwater Ltd data. The mean of future gold price forecasting with sold gold price volatility for up factor was U.S.$ 1,243.84 per ounce. The minimum value was U.S.$ 1,209.47 per ounce and the maximum value was U.S.$ 1,272.64 per ounce. The skewness was -0.40 and the kurtosis was -0.84. This date indicated a negative skewness and light tails.

The mean of future gold price forecasting with sold gold price volatility for down factor was U.S.$ 1,083.40 per ounce. The minimum value was U.S.$ 1,058.54 per ounce and the maximum value was U.S.$ 1,113.83 per ounce. The skewness was 0.46 and the kurtosis was -0.76. This indicted a positive skewness and light tails. The mean of future gold price forecasting with a dual volatility for up factor was U.S.$ 1,247.99 per ounce. The minimum value was U.S.$ 1,211.86 per ounce and the maximum value was U.S.$ 1,278.28 per ounce. The skewness was -0.40 and the kurtosis was -0.84. This data indicated a negative skewness and light tails.

The mean of future gold price forecasting with a dual volatility for down factor was U.S.$ 1,079.83 per ounce. The minimum value was U.S.$ 1,053.87 per ounce and the maximum value was U.S.$ 1,111.63 per ounce. The skewness was 0.46 and the kurtosis was -0.75. This indicated a positive skewness and light tails. The Root Mean Square Error was used to examine the forecasting performances. As shown in Panel R Table 6.10, these were 0.33 for sold gold price volatility for up factor, 0.69 for sold gold price volatility for down factor, 0.32 for a dual volatility for up factor and 0.70 for an incorporated volatility for down factor. It can be concluded that the best forecasting performance was achieved by a dual volatility for up factor over a five-year period.

Table 6. 10: Decision Tree Analysis for Sibanye-Stillwater: 2004-2015

Panel Q: Actual gold price against future gold price forecasting using a Decision Tree Analysis based on sold gold price volatility and dual volatility from 2016 – 2020					
Year	Actual Gold Price (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Down (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Down (U.S.$/Ounce)
2016	1,248.99	1,209.47	1,113.83	1,211.86	1,111.63
2017	1,257.56	1,230.28	1,094.99	1,233.73	1,091.93
2018	1,269.23	1,246.49	1,080.74	1,250.77	1,077.05
2019	1,392.50	1,260.33	1,068.88	1,265.32	1,064.66
2020	1,784.94	1,272.64	1,058.54	1,278.28	1,053.87
Panel R: Descriptive Statistics for actual gold price against forecasting gold price using a Decision Tree Analysis based on sold gold price volatility and incorporated dual volatility from 2016 – 2020					
Description	Actual Gold Price (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Down (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Down (U.S.$/Ounce)
Mean	1,390.64	1,243.84	1,083.40	1,247.99	1,079.83
Median	1,269.23	1,246.49	1,080.74	1,250.77	1,077.05
Min	1,248.99	1,209.47	1,058.54	1,211.86	1,053.87
Max	1,784.94	1,272.64	1,113.83	1,278.28	1,111.63
StdDev	228.03	24.87	21.77	26.15	22.74
Skew	1.91	-0.4	0.46	-0.4	0.46
Kurt	3.61	-0.84	-0.76	-0.84	-0.75
No.	5	5	5	5	5
JB	5.74	0.28	0.29	0.28	0.29
RMSE		0.33	0.69	0.32	0.7

Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No..
Source: Sibanye-Stillwater Ltd Annual Report (2004-2019). IndexMundi (2022).

The Mean Absolute Percentage Error of sold gold price volatility for up factor less than 0.1 was 0.03 in 2016, 0.06 in 2017, 0.09 in 2018 and 0.02 in 2019. Furthermore, a dual volatility for up factor was less than 0.1 that was 0.03, 0.06, 0.09 and 0.02 from Year 1 or 2016 to Year 4 or 2019, respectively for AngloGold Ashanti Ltd. These results suggested that sold gold price volatility for up factor showed the best forecasting technique to forecast the short-to medium term future gold price.

Table 6. 11: Mean Absolute Percentage Error 2016-2020 for AngloGold Ashanti Ltd

Year	Forecasted Gold Price using Sold Gold Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Down (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Down (U.S.$/Ounce)
2016	0.03	0.16	0	0.14
2017	0.06	0.2	0.02	0.17
2018	0.09	0.23	0.04	0.19
2019	0.02	0.32	0.04	0.28
2020	0.19	0.48	0.23	0.45

The Mean Absolute Percentage Error of sold gold price volatility for up factor was less than 0.1 that was 0.03, 0.02, 0.02 and 0.09 from 2016 to 2019 or Year 1 to Year 4 and 0.00, 0.02, 0.04 and 0.03 from 2016 to 2019 for a dual volatility for up factor. These results indicated that a dual volatility for up factor showed the best forecasting technique to forecast the future gold price for short-medium term period.

Table 6. 12: Mean Absolute Percentage Error 2016-2020 for Gold Fields Ltd

Year	Forecasted Gold Price using Sold Gold Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Down (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Down (U.S.$/Ounce)
2016	0.03	0.11	0.00	0.14
2017	0.02	0.13	0.02	0.17
2018	0.02	0.15	0.04	0.2
2019	0.09	0.23	0.03	0.28
2020	0.29	0.41	0.23	0.45

The Mean Absolute Percentage Error was less than 0.1 that was shown by sold gold price volatility for up factor from 2016 to 2019 at 0.03, 0.02, 0.02 and 0.09. The dual volatility for up factor Year 1 in 2016 to Year 4 in 2019 at 0.03, 0.02, 0.01 and 0.09 for Sibanye-Stillwater Ltd. These results showed that a dual volatility for up factor showed the best forecasting technique to forecast the future gold price for a short-medium term period.

Table 6. 13: Mean Absolute Percentage Error 2016-2020 for Sibanye-Stillwater Ltd

Year	Forecasted Gold Price using Sold Gold Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Gold Volatility_Down (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Dual Volatility_Down (U.S.$/Ounce)
2016	0.03	0.11	0.03	0.11
2017	0.02	0.13	0.02	0.13
2018	0.02	0.15	0.01	0.15
2019	0.09	0.23	0.09	0.24
2020	0.29	0.41	0.28	0.41

This research aimed to forecast the future 5-year period from 2016 to 2020. The best method to forecast the future gold price using a Decision Tree Analysis was a dual volatility for up factor based on the Root Mean Square Error results. However, the Root Mean Square Error results in forecasting gold price using a Decision Tree Analysis for up factor have failed to address the accurate forecasting results. These were caused by the significant gold price increasing from 2019 to 2020, where the actual gold price had increased 28.05% from 2019 to 2020 from U.S.$ 1,392 per ounce to U.S.$ 1,784.94 per ounce. One question that needed to be asked was the main source to the significant gold price increasing. A possible explanation for the significant gold price increasing might be that the Covid-19 effects in 2020 might have exacerbated the global tough economic conditions. The extreme global economic effect has pushed to buy more gold as a safe haven that has increased the gold price in the market. Atri et al (2021) demonstrated that Covid-19 has contributed to the extreme global economic effects to impact the short-term period on the gold price. This result was similar to this research based on the future 4-year period from 2016 to 2019 that it has been recognised as the normal condition. The future 4-year period from 2016 to 2019 contributed to a lower Root Mean Square Error value than from 2016 to 2020 for AngloGold Ashanti Ltd of 0.01 from 0.17, 0.02 of Gold Fields Ltd from 0.17 and 0.10 of Sibanye-Stillwater Ltd from 0.32. This can be concluded that gold price was very sensitive to extreme global economic conditions, such as the Covid-19 pandemic.

Table 6. 14: List of empirical studies for gold price forecasting using volatility

Data Type	Period	Observation	Model	Future Prediction Target	RMSE	Author
Monthly	January 1968-December 2008	492	ARIMA	10 years	1.28	Shafiee & Topal (2010)
			Shafiee & Topal Model		1.17	
Daily	3 January 2001-31 January 2017	5,873	GARCH-MIDAS	4 years	3.39	Fang et al (2018)
Yearly	2000-2016	17	ARIMA	17 years	2.49	Madziwa et al (2022)
			Mean Reverting		2.13	
			the Autoregressive Distribution Lag (ARDL)		0.65	
Monthly	January 2004-December 2015	144	Decision Tree Analysis with Dual Volatility up factor	5 years	0.17	Current Study for AngloGold Ashanti Ltd
					0.17	Current Study for Gold Fields Ltd
					0.32	Current Study for Sibanye-Stillwater Ltd

Table 6.14 compared the Root Mean Square Error results between previous studies with this study for gold. The study by Shafiee & Topal (2010), Fang et al (2018) and Madziwa et al (2022) were significant to this study. This study is much better since the Root Mean Square Error results are lower than the previous studies.

In Table 6.15, the actual platinum price in 2016 was U.S.$ 992.72 per ounce and it was U.S.$ 953.35 per ounce in 2017. It was U.S.$ 885.14 per ounce in 2018, U.S.$ 868.88 per ounce in 2019 and U.S.$ 893.39 per ounce in 2020 (Platinum Matthey, 2022). The future platinum price forecasting with sold platinum price volatility using up factor based on Anglo American Platinum Ltd data from January 2004 to December 2015 was U.S.$ 1,287.91 per ounce in 2016, U.S.$ 1,395.05 per ounce in 2017, U.S.$ 1,483.27 per ounce in 2018, U.S.$ 1,561.96 per ounce in 2019 and U.S.$ 1,634.74 per ounce in 2020. The future platinum price forecasting with sold platinum price volatility using down factor was U.S.$ 875.62 per ounce in 2016, U.S.$ 808.37 per ounce in 2017, U.S.$ 760.29 per ounce in 2018, U.S.$ 721.99 per ounce in 2019 and U.S.$ 689.84 per ounce in 2020.

The future platinum price forecasting with a dual volatility using up factor was U.S.$ 1.194.31 per ounce in 2016, U.S.$ 1,253.86 per ounce in 2017, U.S.$ 1,301.55 per ounce in 2018, U.S.$ 1,343.17 per ounce in 2019 and U.S.$ 1,380.94 per ounce in 2020. The future platinum price forecasting with a dual volatility using down factor was U.S.$ 944.25 per ounce in 2016, U.S.$ 899.40 per ounce in 2017, U.S.$ 866.44 per ounce in 2018, U.S.$ 839.59 per ounce in 2019 and U.S.$ 816.63 per ounce in 2020. The descriptive statistics based on the actual and platinum prices forecasting and the future platinum price forecasting using sold gold price volatility and dual volatility were shown in Panel T in Table 6.15 based on Anglo American Platinum Ltd data.

The mean of the actual platinum prices from 2016 to 2020 were U.S.$ 918.70 per ounce. The minimum value was U.S.$ 868.88 per ounce and the maximum value was U.S.$ 992.72 per ounce. The skewness was 0.79 and the kurtosis was -1.42. This data indicated a positive skewness and light tails. The mean of future platinum price forecasting with sold platinum price volatility for up factor was U.S.$ 1,472.59 per ounce. The minimum value was U.S.$ 1,287.91 per ounce and the maximum value was U.S.$ 1,634.74 per ounce. The skewness was -0.29 and the kurtosis was -0.96. This data indicated a negative skewness and light tails. The mean of future platinum price forecasting with sold platinum price volatility for down factor was U.S.$ 771.22 per ounce. The minimum value was U.S.$ 689.84 per ounce. The maximum value was U.S.$ 875.62 per ounce. The skewness was 0.56 and the kurtosis was -0.59. This indicated a positive skewness and light tails.

The mean of future platinum price forecasting with a dual volatility for up factor was U.S.$ 1,294.77 per ounce. The minimum value was U.S.$ 1,194.51 per ounce and the maximum value was U.S.$ 1,380.94 per ounce. The skewness was -0.35 and the kurtosis was -0.90. This data showed a negative skewness and light tails. The mean of future platinum price forecasting with a dual volatility for down factor was U.S.$ 873.26 per ounce. The minimum value was U.S.$ 816.63 per ounce and the maximum value was U.S.$ 944.25 per ounce. The skewness was 0.51 and the kurtosis was -0.68. This showed a positive skewness and light tails.

Furthermore, the forecasting performances have been tested based on a single sold platinum price volatility and a dual volatility with a Decision Tree Analysis being tested using the Root Mean Square Error. As shown in Panel T of Table 6.15, these were 1.24 for sold platinum price volatility for up factor, 0.33 for sold platinum price volatility for down factor, 0.84 for a dual volatility for up factor and 0.10 for a dual volatility for down factor. It can be concluded that the best forecasting performance was achieved by a dual volatility for down factor over a five-year period. The dual volatility for down factor can perform well for platinum since its price tended to decrease from 2016 to 2020.

Table 6. 15: Decision Tree Analysis for Anglo American Platinum Ltd: 2004-2015

Panel S: Actual platinum price against future platinum price forecasting using a Decision Tree Analysis based on platinum price volatility and dual volatility from 2016 – 2020					
Year	Actual Platinum Price (U.S.$/Ounce)	Forecasted Gold Price using Sold Platinum Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Platinum Volatility_Down (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Down (U.S.$/Ounce)
2016	992.72	1,287.91	875.62	1,194.31	944.25
2017	953.35	1,395.05	808.37	1,253.86	899.4
2018	885.14	1,483.27	760.29	1,301.55	866.44
2019	868.88	1,561.96	721.99	1,343.17	839.59
2020	893.39	1,634.74	689.84	1,380.94	816.63
Panel T: Descriptive Statistics for actual platinum price against forecasting platinum price using a Decision Tree Analysis based on platinum price volatility and dual volatility from 2016 – 2020					
Description	Actual Platinum Price (U.S.$/Ounce)	Forecasted Gold Price using Sold Platinum Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Platinum Volatility_Down (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Down (U.S.$/Ounce)
Mean	918.7	1,472.59	771.22	1,294.77	873.26
Median	893.39	1,483.27	760.29	1,301.55	866.44
Min	868.88	1,287.91	689.84	1,194.31	816.63
Max	992.72	1,634.74	875.62	1,380.94	944.25
StdDev	52.27	136.49	73.22	73.45	50.27
Skew	0.79	-0.29	0.56	-0.35	0.51
Kurt	-1.42	-0.96	-0.59	-0.9	-0.68
No.	5	5	5	5	5
JB	0.93	0.26	0.34	0.27	0.31
RMSE		1.24	0.33	0.84	0.1

Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No.
Source: Anglo American Platinum Ltd statements (2004-2019);

The future platinum price forecasting with sold platinum price volatility using up factor based on Impala Platinum Holdings Ltd data from January 2004 to December 2015 started from U.S.$ 1,370.12 per ounce in 2016, U.S.$ 1,522.64 per ounce in 2017, U.S.$ 1,651.08 per ounce in 2018, U.S.$ 1,767.74 per ounce in 2019 and U.S.$ 1,877.34 per ounce in 2020. The future platinum price forecasting with sold platinum price volatility using down factor was U.S.$ 823.08 per ounce in 2016, U.S.$ 740.64 per ounce in 2017, U.S.$ 683.02 per ounce in 2018, U.S.$ 637.94 per ounce in 2019 and U.S.$ 600.70 per ounce in 2020. The future platinum price forecasting with a dual volatility using up factor was U.S.$ 1,167.36 per ounce in 2016, U.S.$ 1,214.04 per ounce in 2017, U.S.$ 1,251.11 per ounce in 2018, U.S.$ 1,283.25 per ounce in 2019 and U.S.$ 1,312.24 per ounce in 2020.

The future platinum price forecasting with a dual volatility using down factor started at U.S.$ 966.04 per ounce in 2016, U.S.$ 928.90 per ounce in 2017, U.S.$ 901.37 per ounce in 2018, U.S.$ 878.80 per ounce in 2019 and U.S.$ 859.38 per ounce in 2020.The descriptive statistics are based on the actual and platinum prices forecasting and the future platinum price forecasting using sold gold price volatility and dual volatility were shown in Panel V in Table 6.16 based on Impala Platinum Holdings Ltd data. The mean of future platinum price forecasting with sold platinum price volatility for up factor was U.S.$ 1,637.79 per ounce. The minimum value was U.S.$ 1,370.12 per ounce and the maximum value was U.S.$ 1,877.34 per ounce. The skewness was -0.25 and the kurtosis was -1.00. This data showed a negative skewness and light tails.

The mean of future platinum price forecasting with sold platinum price volatility for down factor was U.S.$ 697.08 per ounce. The minimum value was U.S.$ 600.70 per ounce and the maximum value was U.S.$ 823.08 per ounce. The skewness was 0.60 and the kurtosis was -0.51. This data showed a positive skewness and light tails. The mean of future platinum price forecasting with a dual volatility for up factor was U.S.$ 1,245.60 per ounce. The minimum value was U.S.$ 1,167.36 per ounce and the maximum value was U.S.$ 1,312.24 per ounce. The skewness was -0.36 and the kurtosis was -0.88. This data showed a negative skewness and light tails.

The mean of future platinum price forecasting with a dual volatility for down factor was U.S.$ 906.90 per ounce. The minimum value was U.S.$ 859.38 per ounce and the maximum value was U.S.$ 966.04 per ounce. The skewness was 0.49 and the kurtosis was -0.70. This data showed a positive skewness and light tails. The Root Mean Square Error was calculated to test the forecasting performances. Table 6.16 in Panel V showed that the Root Mean Square Error was 1.61 for sold platinum price volatility for up factor, 0.50 for sold platinum price volatility for down factor, 0.73 for a dual volatility for up factor and 0.03 for a dual volatility for down factor. It can be concluded that the best forecasting performance was an incorporated volatility for down factor over a five-year period. The dual volatility for down factor was recognised as the best technique to forecast the future platinum price for the next 5-year period that was mainly caused by the platinum price trend relatively decreasing from 2016 to 2020. This research was similar to that reported by Robinson (2016) that the platinum price tended to decrease in the medium term period.

Table 6. 16: Decision Analysis for Impala Platinum Holdings Ltd, 2004-2015

Panel U: Actual platinum price against future platinum price forecasting using a Decision Tree Analysis based on platinum price volatility and dual volatility from 2016 – 2020					
Year	Actual Platinum Price (U.S.$/Ounce)	Forecasted Gold Price using Sold Platinum Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Platinum Volatility_Down (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Down (U.S.$/Ounce)
2016	992.72	1,370.12	823.08	1,167.36	966.04
2017	953.35	1,522.64	740.64	1,214.04	928.9
2018	885.14	1,651.08	683.02	1,251.11	901.37
2019	868.88	1,767.74	637.94	1,283.25	878.8
2020	893.39	1,877.34	600.7	1,312.24	859.38
Panel V: Descriptive Statistics for actual platinum price against forecasting platinum price using a Decision Tree Analysis based on platinum price volatility and dual volatility from 2016 – 2020					
Description	Actual Platinum Price (U.S.$/Ounce)	Forecasted Gold Price using Sold Platinum Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Platinum Volatility_Down (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Down (U.S.$/Ounce)
Mean	918.7	1,637.79	697.08	1,245.60	906.9
Median	893.39	1,651.08	683.02	1,251.11	901.37
Min	868.88	1,370.12	600.7	1,167.36	859.38
Max	992.72	1,877.34	823.08	1,312.24	966.04
StdDev	52.27	199.6	87.69	57.02	42.01
Skew	0.79	-0.25	0.6	-0.36	0.49
Kurt	-1.42	-1	-0.51	-0.88	-0.7
No.	5	5	5	5	5
JB	0.93	0.26	0.36	0.27	0.31
RMSE	0.00	1.61	0.5	0.73	0.03

Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No.
Source: Impala Platinum Holdings Ltd statements (2004-2019);

The Mean Absolute Percentage Error of a dual volatility for down factor less than 0.1 was shown by 2016 to 2020 at 0.05, 0.06, 0.02, 0.03 and 0.09 for Anglo American Platinum Ltd. These results suggest that a dual volatility for down factor was the best forecasting technique to forecast the short to long-term future platinum price.

Table 6. 17: Mean Absolute Percentage Error 2016-2020 for Anglo American Platinum Ltd

Year	Forecasted Gold Price using Sold Platinum Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Platinum Volatility_Down (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Down (U.S.$/Ounce)
2016	0.30	0.12	0.20	0.05
2017	0.46	0.15	0.32	0.06
2018	0.68	0.14	0.47	0.02
2019	0.8	0.17	0.55	0.03
2020	0.83	0.23	0.55	0.09

The Mean Absolute Percentage Error of a dual volatility for down factor less than 0.1 was shown from 2016 to 2020 at 0.03, 0.03, 0.02, 0.01 and 0.04 Impala Platinum Holdings Ltd. These results suggest that a dual volatility for down that was the best forecasting technique to forecast the short to long-term future platinum price.

Table 6. 18: Mean Absolute Percentage Error 2016-2020 for Impala Platinum Holdings Ltd

Year	Forecasted Gold Price using Sold Platinum Volatility_Up (U.S.$/Ounce)	Forecasted Gold Price using Sold Platinum Volatility_Down (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Up (U.S.$/Ounce)	Forecasted Platinum Price using Dual Volatility_Down (U.S.$/Ounce)
2016	0.38	0.17	0.18	0.03
2017	0.6	0.22	0.27	0.03
2018	0.87	0.23	0.41	0.02
2019	1.03	0.27	0.48	0.01
2020	1.1	0.33	0.47	0.04

This study forecasted the next 5-year period for platinum from 2016 to 2020. The best method to forecast the future platinum price using a Decision Tree Analysis was a dual volatility for down factor. This method has also been examined the impact extreme global economic conditions such as Covid-19 effects. The Root Mean Square Error from 2016 to 2020 was 0.10 and decreased to 0.08 from 2016 to 2019 for Anglo American Platinum Ltd. For Impala Platinum Holdings Ltd case, the Root Mean Square Error was 0.03 from 2016 to 2020 and dropped to 0.01 from 2016 to 2019. These results indicated that the platinum price has no significant impact from the Covid-19 effects in 2020. The result in this research was similar to that reported by Yarovaya et al (2022) that platinum price was not affected by the Covid-19 pandemic.

Table 6. 19: List of empirical studies for platinum price forecasting using volatility

Data Type	Period	Observation	Model	Future Prediction Target	RMSE	Previous Author
Daily	4 January 1999-31 December 2009	4,015	FIGARCH	455-day	1.16	Arouri et al (2012)
			GARCH		1.8	
			IGARCH		1.94	
			EGARCH		1.75	
			HYGARCH		1.94	
Daily	1 January 1996-31 December 2015	7,305	GARCH-MIDAS	1-day	0.98	Nguyen & Walther (2020)
				5-day	1.07	
				20-day	1.15	
Daily	1 January 2004-31 December 2019	4,091	Autoregression Distribution Lag (ARDL)	30-day	0.01	Salisu et al (2020)
				60-day	0.01	
				90-day	0.01	
Monthly	January 2004-December 2019	189	Autoregression Distribution Lag (ARDL)	30-day	0.05	
				60-day	0.05	
				90-day	0.05	
Monthly	January 2004-December 2019	144	Decision Tree Analysis with a Dual Volatility Down Factor	5 years	0.10	Current Study for Anglo American Platinum Ltd
					0.03	Current Study for Impala Platinum Holdings Ltd

Table 6.19 compared the Root Mean Square Error results between previous studies with this study for platinum. The study by Arouri et al (2012) and Nguyen & Walther (2020) were significantly different to this study. However, the findings of Salisu et al (2020) are similar to what is found in this research.

Another model that is commonly used to forecast discrete volatility is GARCH(1,1). From Kola (2021), one of the elegances of GARCH(1,1), is the ability to model to measure the conditional variance. The output variables of GARCH(1,1) are presented in inequality 2;

Table 6. 20: GARCH Values

Parameter	Copper	Gold	Platinum
omega	0.0010 (0.03715)**	0.0005 (0.0001)***	0.0101 (0.0016)***
alpha	0.0002 (0.75878)	0.0011 (0.7459)	0.0011 (0.9709)
beta	0.7629 (0.9611)	0.6701 (0.7629)	0.5266 (0.7697)
$alpha + beta$	0.7631	0.6712	0.5376
Historical volatility	0.7362 (0.6322)	0.8343 (0.1593)	0.5471 (0.8411)
GARCH(1,1) volatility	0.8773 (0.0000)***	0.4558 (0.0000)***	0.4484 (0.0000)***

Note: ***, ** and * illustrates statistical significance at 1%, 5% and 10%, respectively.

Table 6.20 illustrates GARCH(1,1) variables of copper, gold and platinum. The omegas of the three mineral commodities are statistically significant, which might be confirmed as instantaneous variation in the commodities (See; Kola 2021). For copper, historical volatility is slower than the GARCH volatility, while for gold and platinum, historical volatilities are higher than GARCH volatilities. Recall, GARCH volatilities are forward looking (See; Kola 2021). Therefore, during the time series period, copper volatility was on an upward trajectory while volatilities of gold and platinum were on a downward trajectory. That is, GARCH calculations confirm slightly different findings in relation to the correlation coefficient calculations. Finally, the sum of alphas and betas add up to a value below one, which confirms the robustness of the GARCH values (see; Kola 2021).

6.2 Dual Volatility of GBM

6.2.1 Introduction

Hypothesis Two examined a dual volatility and a mineral commodity price volatility to forecast the future probability risk for the next 5-year period using the GBM model. The dual volatility was computed based on Equation 4.2 using Excel Software based on oil price, copper price, oil production and consumption, copper production and consumption and exchange rate from U.S.$ to ZAR and gold and platinum price data from January 2004 to December 2015. Based on statistical analysis, these datasets showed non-normal distribution. According to Marathe & Ryan (2005), the GBM model can allow non-normal distribution data by employing the exponential trend. Furthermore, the GBM model, using a dual volatility, was tested to forecast the future mineral commodity price for copper, gold and platinum. The future mineral commodity price forecasting using the GBM model was computed using Excel Software. The future mineral commodity price forecasting using the GBM model, based on a dual volatility, was compared to a mineral commodity price volatility to decide the best forecasting method. The lowest Root Mean Square Error result was determined as the best forecasting method.

6.2.1.1 Copper

The future copper price forecasting was carried out using the GBM model with a copper price volatility and a dual volatility. The copper price volatility was computed from the standard deviation of returns based on the monthly historical copper price from January 2004 to December 2015 using Excel Software. Furthermore, the dual volatility was calculated using Equation 4.2 using Excel Software. The copper price volatility was 0.08 and the dual volatility was 0.00. The future copper price was forecasted with copper price volatility based on monthly historical copper price data from January 2004 to December 2015 using the GBM model. The future copper price results using copper price volatility were U.S.$ 2.05 per pound in 2016, U.S.$ 2.06 per pound in 2017, U.S.$ 2.06 per pound in 2018, U.S.$ 2.11 per pound in 2019 and U.S.$ 2.15 per pound in 2020.

The next section forecasts the future copper price based on the dual GBM volatility model. The future copper price forecasting in 2016 was U.S.$ 2.14 per pound, U.S.$ 2.18 per pound in 2017, U.S.$ 2.22 per pound in 2018, U.S.$ 2.26 per pound in 2019 and U.S.$ 2.30 per pound in 2020. The descriptive statistics, based on the actual copper price and the future copper price forecasting using a copper price volatility and a dual volatility, were shown in Panel Y in Table 6.20. The mean of the actual copper prices was U.S.$ 2.70 per pound. The minimum value was U.S.$ 2.21 per pound and the maximum value was U.S.$ 2.96 per pound. The skewness was -1.74 and kurtosis was 3.56. These indicated a negative skewness and heavy tails.

The mean of future copper price forecasting with copper price volatility was U.S.$ 2.08 per pound. The minimum value was U.S.$ 2.05 per pound and the maximum value was U.S.$ 2.15 per pound. The skewness was 1.05 and kurtosis was -0.41. These indicated a positive skewness and light tails. The mean of future copper price forecasting with a dual volatility was U.S.$ 2.22 per pound. The minimum value was U.S.$ 2.14 per pound and the maximum value was U.S.$ 2.30 per pound. The skewness was 0.04 and kurtosis was -1.20. These indicated a positive skewness and light tails.

The forecasting performances using the GBM model between a copper price volatility and a dual volatility was tested using the Root Mean Square error. As shown in Table 6.20 in Panel X, the Root Mean Square Error was 0.28 of the copper price volatility and 0.03 of a dual volatility over a five-year period. The dual volatility has a lower Root Mean Square Error than a copper price volatility that it indicated more superior to forecast the future copper price using the GBM model.

Table 6. 21: GBM of Copper from 2016 to 2020

Panel W: Actual copper price against future copper price forecasting using the GBM model based on copper price volatility and dual volatility from 2016-2020			
Year	Actual Copper Price (U.S.$/Pound)	Forecasted Copper Price by price volatility (U.S.$/Pound)	Forecasted Copper Price by Dual Volatility (U.S.$/Pound)
2016	2.21	2.05	2.14
2017	2.8	2.06	2.18
2018	2.96	2.06	2.22
2019	2.73	2.11	2.26
2020	2.80	2.15	2.30
Panel X: Descriptive Statistics for actual copper price against forecasting copper price using the GBM model based on copper price volatility and dual volatility from 2016-2020			
Description	Actual Copper Price (U.S.$/Pound)	Forecasted Copper Price by price volatility (U.S.$/Pound)	Forecasted Copper Price by Dual Volatility (U.S.$/Pound)
Mean	2.70	2.08	2.22
Median	2.80	2.06	2.22
Min	2.21	2.05	2.14
Max	2.96	2.15	2.3
StdDev	0.29	0.04	0.06
Skew	-1.74	1.05	0.04
Kurt	3.56	-0.41	-1.2
No.	5	5	5
JB	5.15	0.95	0.30
RMSE	0.00	0.28	0.03

Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No.

Table 6.21 showed the Mean Absolute Percentage Error for copper from 2016 to 2020. The most accurate forecasting was shown by the Mean Absolute Percentage Error value of less than 0.10. The Mean Absolute Percentage Error results less than 0.1 for both the GBM model using a copper price volatility and a dual volatility were shown from 2016 to 2020 that was 0.07, 0.07, 0.07, 0.05 and 0.03 for copper price volatility and 0.03, 0.01, 0.00, 0.02 and 0.04 for a dual volatility. Therefore, the future copper price using this model can perform well to forecast the short to long-term future copper price.

Table 6. 22: Mean Absolute Percentage Error 2016-2020 for copper

Year	Copper Price Volatility (U.S.$/Pound)	Dual Volatility (U.S.$/Pound)
2016	0.07	0.03
2017	0.07	0.01
2018	0.07	0.00
2019	0.05	0.02
2020	0.03	0.04

This research forecasted the next 5-year period from 2016 to 2020 for copper. Based on this research, the best forecasting method was the GBM model using a dual volatility. Since the Covid-19 pandemic in 2020 has had significantly effect on the global economy, then this research also examined the impact of it to the GBM model performance in forecasting the future copper price. This research compared the Root Mean Square Error result of the future 5-year copper price from 2016 to 2020 and the future 4-year copper price from 2016 to 2019. The Root Mean Square Error results of the future 5-year copper price and the future 4-year copper price were 0.03 and 0.02, respectively. These results suggest that copper price has no significant impact from the Covid-19 pandemic in 2020 and similar to this research using a Decision Tree Analysis.

Table 6. 23: List of empirical studies for copper price forecasting using stochastic, time-series and algorithm

Data Type	Period	Observation	Model	Future Prediction Target	RMSE	Author
Daily	1 January 2004-31 December 2007	48	Individual Model	2 years	0.62	Cortazar & Eterovic (2010)
			Multicommodity Model		0.84	
			Modified Multicommodity Model		0.6	
	2009–2016	8	Brownian Motion with Mean Reversion	1 year	0.45	Dehghani & Bogdanovic (2018)
			Particle Swarm Optimization		0.16	
			Differential Evolution		0.21	
			Bat Algorithm		0.13	
Monthly	Jan 2004-Dec 2019	192	GBM with a Dual Volatility	5 years	0.03	Current Study
				4 years	0.02	

Table 6.23 compared the Root Mean Square Error results between previous studies with this study for copper. Cortazar & Eterovic (2010) is different from this research because the convergence of this thesis is much smaller in units than that of Cortazar & Eterovic (2010). Thus, this thesis has a better convergence than Cortazar & Eterovic (2010). On the other hand, Dehghani & Bogdanovic (2018) had similar convergence to this thesis.

6.2.1.2 Gold

The future gold price forecasting using GBM with gold price volatility in 2016 was U.S.$ 1,157.27 per ounce, U.S.$ 1,236.10 per ounce in 2017, U.S.$ 1,342.20 per ounce in 2018, U.S.$ 1,432.49 per ounce in 2019 and U.S.$ 1,530.55 per ounce in 2020. The future gold price forecasting with a dual volatility in 2016 was U.S.$ 1,150.34 per ounce, U.S.$ 1,230.12 per ounce in 2017, U.S.$ 1,315.39 per ounce in 2018, U.S.$ 1,406.59 per ounce in 2019 and U.S.$ 1,504.12 per ounce in 2020. The descriptive statistics based on the actual gold price and the future gold price forecasting using gold price volatility and dual volatility were shown in Panel Z in Table 6.24.

The mean of future gold price forecasting with gold price volatility was U.S.$ 1,339.72 per ounce. The minimum value was U.S.$ 1,157.27 per ounce and the maximum value was U.S.$ 1,530.55 per ounce. The skewness was 0.07 and kurtosis was -1.38. These indicated a positive skewness and light tails. The mean of future gold price forecasting with a dual volatility was U.S.$ 1,321.31 per ounce. The minimum value was U.S.$ 1,150.34 per ounce and the maximum value was U.S.$ 1,504.12 per ounce. The skewness was 0.15 and kurtosis was -1.17. These indicated a positive skewness and light tails. The forecasting performances based on gold price volatility and a dual volatility using the GBM model were examined using the Root Mean Square Error. As shown in Table 6.23 in Panel Z, the Root Mean Square Error was 0.20 of gold price volatility and 0.16 of a dual volatility over a five-year period. These indicated that gold price volatility was superior to forecast the future gold price using the GBM model. This result was similar to that found by Muhammad et al (2019), where the gold price was sensitive to the future risks.

Table 6. 24: GBM of Gold from 2016 to 2020

Panel Y: Actual gold price against future gold price forecasting using the GBM model based on gold price volatility and dual volatility from 2016-2020			
Year	Actual Gold Price (U.S.$/Ounce)	Forecasted Gold Price by price volatility (U.S.$/Ounce)	Forecasted Gold Price by Dual Volatility (U.S.$/Ounce)
2016	1,248.99	1,157.27	1,150.34
2017	1,257.56	1,236.10	1,230.12
2018	1,269.23	1,342.20	1,315.39
2019	1,392.50	1,432.49	1,406.59
2020	1,784.94	1,530.55	1,504.12
Panel Z: Descriptive Statistics for actual gold price against forecasting gold price using the GBM model based on gold price volatility and dual volatility from 2016 – 2020			
Description	Actual Gold Price (U.S.$/Ounce)	Forecasted Gold Price by price volatility (U.S.$/Ounce)	Forecasted Gold Price by Dual Volatility (U.S.$/Ounce)
Mean	1,390.64	1,339.72	1,321.31
Median	1,269.23	1,342.20	1,315.39
Min	1,248.99	1,157.27	1,150.34
Max	1,784.94	1,530.55	1,504.12
StdDev	228.03	149.21	139.89
Skew	1.91	0.07	0.15
Kurt	3.61	-1.38	-1.17
No.	5	5	5
JB	5.74	0.4	0.3
RMSE		0.2	0.16

Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No.

The Mean Absolute Percentage Error value of less than 0.1 using gold price volatility was shown by Year 1 to Year 3 or 2016 to 2018 at 0.07, 0.01 and 0.07, respectively. The Mean Absolute Percentage Error value of less than 0.1 was presented, which was shown by a dual volatility from Year 1 to Year 3 or 2016 to 2018 at 0.08, 0.02 and 0.05, respectively. These results indicated that the best forecasting technique to forecast the future gold price was the GBM model using a dual volatility. The future gold price using this model can perform well to forecast the short to medium-term future gold price.

Table 6. 25: Mean Absolute Percentage Error 2016-2020 for gold

Year	Gold Price Volatility (U.S.$/Ounce)	Dual Volatility (U.S.$/Ounce)
2016	0.07	0.08
2017	0.01	0.02
2018	0.07	0.05
2019	0.15	0.13
2020	0.23	0.20

The Root Mean Square Error result of the next 5-year forecasting period from 2016 to 2020 for the GBM model with a dual volatility was at 0.16, as presented in Table 6.23. In order to examine the Covid-19 effects in 2020, the Root Mean Square Error was tested based on the next 4-year forecasting period from 2016 to 2019 by excluding 2020. The Root Mean Square Error for the next 4-year forecasting period was 0.05 for a dual volatility. This result was significantly impacted by the Covid-19 pandemic.

Table 6.24 compares single-price volatility and dual volatility of gold over a 5-year period, as shown in the table. Fundamentally, the dual volatility on average is smaller than single-price volatility; although, there are exceptions during the 2016-2018 period. During the latter period, the commodities prices were higher than market expectations. The reason pattern as stated in the former statement is that the dual volatility is sensitive to the convergence of volatilities of prices (Moel & Tufano, 2002).

Table 6. 26: List of empirical studies for gold price forecasting using stochastic model

Data Type	Period	Observation	Model	Future Prediction Target	RMSE	Author
Daily	5 January 2005-31 December 2010	1,527	Black-Scholes Model	0.32 years	0.25	Lin et al (2014)
Daily	8 February 1993-8 November 2016	5,820	GBM	3.19 years	0.01	Sauvageau & Kumral (2018)
Yearly	2000-2016	17	GBM	17 years	0.17	Madziwa et al (2022)
Monthly	January 2004-December 2019	192	GBM with a Dual Volatility	5 year	0.03	Current Study

Table 6.26 compared the Root Mean Square Error results between previous studies with this study for gold. Lin et al (2014) and Madziwa et al (2022) are different from this study because this thesis has much smaller convergence than Lin et al (2014) and Madziwa et al (2022), while Sauvageau & Kumral (2018) is similar, fundamentally on results, to what is in this research in terms of convergence.

6.2.1.3 Platinum

The future platinum price forecasting using GBM with platinum price volatility in 2016 was U.S.$ 860.60 per ounce, U.S.$ 822.87 per ounce in 2017, U.S.$ 811.15 per ounce in 2018, U.S.$ 783.51 per ounce in 2019 and U.S.$ 744.27 per ounce in 2020. The future platinum price forecasting with a dual volatility in 2016 was U.S.$ 846.92 per ounce, U.S.$ 828.81 per ounce in 2017, U.S.$ 811.09 per ounce in 2018, U.S.$ 793.76 per ounce in 2019 and U.S.$ 776.81 per ounce in 2020. The descriptive statistics based on the actual platinum price and the future platinum price forecasting using platinum price volatility and dual volatility were shown in Panel BB in Table 6.27.

The mean of future platinum price forecasting with platinum price volatility was U.S.$ 804.48 per ounce. The minimum value was U.S.$ 744.27 per ounce and the maximum value was U.S.$ 860.60 per ounce. The skewness was -0.22 and kurtosis was 0.10. This indicated a negative skewness and light tails. The mean of future platinum price forecasting with a dual volatility was U.S.$ 811.48 per ounce. The minimum value was U.S.$ 776.81 per ounce and the maximum value was U.S.$ 846.92 per ounce. The skewness was 0.05 and kurtosis was -1.20. This indicated a positive skewness and light tails. In order to examine the forecasting performances based on platinum price volatility and a dual volatility using the GBM model, the Root Mean Square Error was taken into account. Table 6.26 in Panel BB showed that the Root Mean Square Error was 0.42 of platinum price volatility and 0.41 of a dual volatility over a five-year period.

Table 6. 27: GBM of Platinum from 2016 to 2020

Panel AA: Actual platinum price against future platinum price forecasting using the GBM model based on platinum price volatility and dual volatility from 2016 – 2020			
Year	Actual Platinum Price (U.S.$/Ounce)	Forecasted Platinum Price by price volatility (U.S.$/Ounce)	Forecasted Platinum Price by Dual Volatility (U.S.$/Ounce)
2016	992.72	860.6	846.92
2017	953.35	822.87	828.81
2018	885.14	811.15	811.09
2019	868.88	783.51	793.76
2020	893.39	744.27	776.81
Panel BB: Descriptive Statistics for actual platinum price against forecasting platinum price using the GBM model based on platinum price volatility and incorporated volatility from 2016 – 2020			
Description	Actual Platinum Price (U.S.$/Ounce)	Forecasted Platinum Price by price volatility (U.S.$/Ounce)	Forecasted Platinum Price by Dual Volatility (U.S.$/Ounce)
Mean	918.7	804.48	811.48
Median	893.39	811.15	811.09
Min	868.88	744.27	776.81
Max	992.72	860.6	846.92
StdDev	52.27	43.58	27.71
Skew	0.79	-0.22	0.05
Kurt	-1.42	0.1	-1.2
No.	5	5	5
JB	0.93	0.04	0.3
RMSE		0.42	0.41

Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No.

The forecasting performance can also be tested using the Mean Absolute Percentage Error. The best forecasting performance was shown by the Mean Absolute Percentage Error value of less than 0.1, 0.11 to 0.2 demonstrated good accuracy of forecasting, 0.21 to 0.50 demonstrated sufficient accuracy of forecasting and over 0.51 demonstrated highly inaccurate forecasting (Lewis, 1982). Based on the Mean Absolute Percentage Error result, the future platinum price forecasting showed good accuracy of forecasting between 2016 to 2018 of platinum price volatility and 2016 to 2019 of a dual volatility. This showed sufficient accuracy of forecasting from 2019 to 2020 for platinum price volatility and 2020 of a dual volatility using the GBM model.

The Root Mean Square Error result over the next 5-year forecasting period from 2016 to 2020 for the GBM model with a dual volatility was at 0.41, as presented in Table 6.36. Due to the Covid-2019, the Root Mean Square Error was computed by excluding 2020 because during that year the pandemic effects were prevalent indeed. The Root Mean Square Error for the next 4-year forecasting period for platinum using the GBM model was 0.35 for a dual volatility. These results indicated that platinum price was not significantly been impacted by the Covid-19 pandemic in 2020.

Table 6. 28: Mean Absolute Percentage Error 2016–2020 for platinum

Year	Platinum Price Volatility (U.S.$/Ounce)	Dual Volatility (U.S.$/Ounce)
2016	0.13	0.15
2017	0.17	0.17
2018	0.18	0.18
2019	0.21	0.20
2020	0.25	0.22

Table 6.28 exemplifies the same pattern as in table 6.25. Therefore, arguments made about calculations in table 6.25 apply to table 6.28.

Table 6. 29: List of empirical studies for platinum price forecasting using stochastic model

Data Type	Period	Observation	Model	Future Prediction Target	RMSE	Author
Weekly	15 January 1999-28 April 2017	954	Univariate Random Forests	1 year	0.98	Pierdzioch C & Risse (2020)
			Multivariate Random Forests		0.73	
Daily	1 January 2005-31 August 2019	505	Heterogeneous Autoregressive	1 day	0.08	Alfeus & Nikitopoulos (2022)
				5 days	0.12	
				22 days	0.13	
			Rough Fractional Stochastic Volatility	1 day	0.12	
				5 days	0.14	
				22 days	0.14	
			Fractional Integrated Generalized Autoregressive Conditional Heteroscedastic	1 day	0.07	
				5 days	0.12	
				22 days	0.13	
Monthly	January 2004-December 2019	192	GBM with a Dual Volatility	5 years	0.41	Current Study

Table 6.29 compared the Root Mean Square Error results between previous studies with this study for platinum. Alfeus & Nikitopoulos (2022) is different to the platinum analysis in this study because Alfeus & Nikitopoulos (2022) had much better convergence than this thesis. While Pierdzioch & Risse (2020) present the same narrative in terms of their findings.

6.2.2 Robustness Test for Dual GBM Volatility

Marcato et al (2019) used a Multiple Regression Model when they determined the relationship between selected macroeconomic variables and exchange options. The authors illustrated that in order to appropriately map numerous independent variables (x-axes) against the dependent variables (y-axis); firstly, they should model each independent variable with the dependent variable. Thereafter, roll over the model by sequentially adding each independent variable until all independent variables are modelled against the dependent variable. In that way, Marcato et al (2019) are able to pick up whether each independent variable is a contributor or hedging variable of the Multiple Regression Model. Secondly, the issues of multicollinearity and/or heteroscedasticity are picked up during the building process of the full model. Most econometricians model macroeconomic variables in an arbitrary way; however, Marcato et al (2019) illustrated that following a specific logic, especially economically grounded theory, makes the results crispy. On the other hand, modelling independent variables against the dependent variable in the same way as suggested by Marcato et al (2019), allows them to account for stepwise regression effects in the model. This study follows a grounded economic heuristic and is presenting a Multiple Regression Model.

The Multiple Regression Model used in this study has the following variables; the independent variable is copper price in U.S.$/ton. The dependent variables are (i) oil price in U.S.$/barrel, exchange rate in U.S.$/ZAR, oil supply in barrel per year, oil demand in barrel per year, copper supply in ton per year, copper demand in ton per year. Kabwe & Yiming (2015) argue that copper price is driven by mainly demand side factors. Therefore, in modelling the variables, they will start by modelling demand factors, followed by supply factors and finally, variables that are indifferent to demand and/or supply side factors. Forbes et al (2018) stated that exchange rates are driven by monetary policy-supply side instruments. It can be inferred from Uddin et al (2018) that oil price is neither driven by demand side factors nor supply side factors. Therefore, oil price will be incorporated in the model last. Note that independent variables used are chosen based on commonly analysed and/or modelled variables against copper based on prior studies.

Table 6.29 presents the results of the Multiple Regression Model, where independent variables are mapped against dependent variables. First, each independent variable is mapped against the dependent variable. The latter process is synonymous with step-wise regression. Thereafter, the model is rolled over until all variables are fitted into one full model. In the latter process, the study is finding whether each stand-alone variable is a causal variable or hedging variable in the given scenario. Finally, in the full model, each independent variable is interpreted in relation to its status as a stand-alone versus in a full model. Typically, when a variable changes its original sign, then it is typically a hedge variable and when it keeps its original sign, it is typically a causal variable. The significance and non-significance changes typically illustrate multicollinearity and heteroscedasticity in the model at different levels. Given that this thesis uses precise variables, then instrumental variables are not used in cases where there is either multicollinearity or heteroscedasticity. An instrumental variable is a variable that represents or is similar to a variable that leads to either multicollinearity or heteroscedasticity in the model but it does not have a relationship including correlation with any variable (be independent or dependent) in the model. If there is any correlation between an instrumental variable and any variable in the set, then the correlation in between those variables is very low.

Table 6. 30: Multiple Regression Analysis

Independent Variable(s)	D_CU	D_Oil	S_CU	S_Oil	ExRate	Oil_P	D_CU and D_Oil	D_CU, D_Oil and S_CU	D_CU, D_Oil, S_CU and S_Oil	D_CU, D_Oil, S_CU, S_Oil and ExRate	D_CU, D_Oil, S_CU, S_Oil, ExRate and Oil_P
Eq.	1	2	3	4	5	6	7	8	9	10	11
Constant	0.003 (0.3684)	0.007 (0.181)	0.003 (0.326)	0.007 (0.181)	0.010 (0.027)***	0.005 (0.253)	0.004 (0.261)	0.003 (0.322)	0.003 (0.339)	0.005(0.079)**	0.004 (0.137)
D_CU	0.587 (0.000)***						0.664 (0.000)***	1.532 (0.000)***	1.717 (0.000)***	1.725 (0.000)***	1.031 (0.000)***
D_Oil		0.061 (0.110)					-0.126 (0.000)***	-0.103 (0.000)***	1.887 (0.024)***	2.147 (0.006)***	0.544 (0.475)
S_CU			0.549 (0.000)***					-0.859 (0.005)***	-1.038 (0.001)***	-1.084 (0.000)***	-0.384 (0.186)
S_Oil				0.061 (0.123)					-1.967 (0.017)***	-2.225 (0.004)***	-0.793 (0.288)
ExRate					-0.698 (0.000)***					-0.415 (0.000)***	-0.308 (0.000)***
Oil_P						0.241 (0.000)***					0.305 (0.000)***
$Adjusted\ R^2$	0.560	0.008	0.521	0.088	0.137	0.079	0.605	0.619	0.628	0.675	0.727
F-Statistic	243.281 (0.000)***	2.573 (0.110)	207.580 (0.000)***	2.537 (0.113)	31.152 (0.000)***	14.249 (0.000)***	146.441 (0.000)***	103.978 (0.000)***	81.416 (0.000)***	79.938 (0.000)***	85.149 (0.000)***
Akaike	-3.328	-2.514	-3.242	-2.514	-2.654	-2.588	-3.429	-3.461	-3.481	-3.609	-3.777
Schwarz	-3.294	-2.480	-3.208	-2.480	-2.619	-2.554	-3.379	-3.393	-3.396	-3.508	-3.658
Hannan-quin	-3.314	-2.501	-3.228	-2.500	-2.639	-2.574	-3.409	-3.434	-3.447	-3.568	-3.729
Durbin-Watson	1.532	1.272	1.501	1.271	1.309	1.555	1.392	1.443	1.481	1.547	1.749

Note: D_CU is for copper demand (tonne/year), D_Oil is for oil demand (barrel/ year), S_CU is for copper supply (tonne/year), S_Oil is for oil supply (barrel/year), ExRate is for exchange rate (U.S.$/ZAR) and Oil_P is oil price (U.S.$/barrel)-all those variables are independent variables. The dependent variable is copper price (U.S.$/tonne). ***, ** and * represent significance levels of 1%, 5% and 10%, respectively. Eq. stands for equation.
Source: AngloGold Ashanti, Gold Fields, Anglo Platinum and Impala Platinum financial statements (2004-2019) and Palabora Ltd financial statements (2004-2013).

In explaining the results regression, it starts with the step wise regressions-Equations (1) to (6) of the robustness test for the GBM. For Equation 1, the demand for copper is positive and statistically significant. This is consistent with prior studies such as Kabwe & Yiming (2015), and Uddin et al (2018) and preliminary expectations because copper is mainly influenced by demand-side factors. Similarly, oil demand is positive and statistically insignificant when modelled against copper price. However, it is common knowledge that oil is regarded as a commodity-same as copper. Therefore, it is not surprising that the supply (demand) of copper is positive and statistically significant (insignificant). Sadorsky (2014) found that oil provides cheaper hedging for countries, while copper is expensive, but copper has variable hedging ratios. That is, the relationship between copper and oil prices is insignificantly positive. For Equation 6, the relationship between copper and oil prices is positive and statistically significant. One possible explanation is that in the latter statement, both variables (copper and oil) are standardised as prices. And, for Equations 1 to 4, the dependent variable is standardised as price, while the independent variables are standardised as quantity. Hence, statistically insignificant when oil quantity is modelled against copper price.

Back to stepwise reasoning, copper (demand and supply) has more of an effect on copper price than oil (demand and supply). This is expected-some commodities versus different commodities. In Equation 5, the relationship between copper price and exchange rate is negative. According to Spilimbergo (2002) the negative relationship can be partly explained by; (i) consumption patterns of both products (copper and currencies), (ii) investment strategies and (iii) wealth creation. Fundamentally, Equations 1 to 6 are consistent with prior studies and pre-modelling expectations in terms of signs and statistically significant patterns. Moreover, the same findings are acceptable when using stepwise reasoning.

Now, one moves to the rolling over "effect". Firstly, Equation 7 shows that copper demand is positive and statistically significant, while oil demand is negative and statically significant. That is, copper demand holds the same effect as before the rolling over process, while oil demand is different. According to Marcato et al (2019), when an independent variable changes sign, irrespective of statistical significance when rolled over, it implies that the variable is a hedge variable. A hedge variable assists in creating an appropriate model, but the same variable does not have any causal effect in the model. Using the same reasoning and/or logic in Equation 8, oil demand and copper supply are hedge variables. On copper supply, it is expected because copper is driven by demand-side factors. In Equation 9, copper and oil supplies are hedge variables, while oil demand is a casual variable-positive and statistically significant. Thus, there might be an issue with multicollinearity because all used variables in the model are considered as commodities. Possible mechanisms of solving multicollinearity include; (i) hybridisation of features (Garg & Tai, 2013), (ii) seemingly unrelated regression (Pan et al, 2020) and instrumental variables usage (Marcato et al, 2019). However, for Hypothesis Two, they cannot use instrumental variables given that they want to model the specified and/or identified variables. On the other hand, controlling for multicollinearity is beyond the scope of Hypothesis Two. For Equations 10 and 11, causal variables stay causal, and the rest is the same as before.

On the other features of models, Durbin-Watson below 1.3 illustrates autocorrelation. Therefore, there is no autocorrelation, except in Equation 4. The latter phenomenon might be due to the close relationship between copper and oil. Other model validation measures (Akaike, Schwarz and Hannan-Quinn) are acceptable within the 1.6-2.7 range. Based on other model validation measures, there is some prediction error. This might be due to the fact the available data is monthly from 2004 to 2019, which implies that there are 180 data points. Most econometric studies recommend at least 500 data points, despite the fact that Central Limit theorem recommends at least 30 data points for normality to occur. Despite that, adjusted R^2 increases with the addition of more independent variables to model. Thus, the more the model is rolled over, the better the rolled over model is. That is partly because of the decrease in error term of the model. Finally, the F-statistic, which measures the goodness of fit of the model, confirms that models are good fitted overall.

6.3 Optimal Risk Neutral Probability

6.3.1 Introduction

In presenting the results, the third hypothesis uses the projected future risk-neutral profits; thereafter, robustness test based on the VAR model. Table 6.30 presents aggregated, forecasted risk-neutral profits based on basic statistics illustrations. The explanations of the presented numbers are below in Table 6.31.

6.3.2 Forecasted Risk-Neutral Prices

Table 6. 31: RNP of Copper, Gold and Platinum based on Prices

Panel CC: Amalgamated Forecasted Future Risk-Neutral Copper Prices					
Variable	Historical Copper Price	Forecasted Future Price using Price Volatility_Up	Forecasted Future Price using Price Volatility_Down	Forecasted Future Price using Dual Volatility_Up	Forecasted Future Price using Dual Volatility_Down
Mean	2.70	2.54	2.42	2.48	2.48
Median	2.80	2.54	2.42	2.48	2.48
Min	2.21	2.49	2.37	2.48	2.48
Max	2.96	2.59	2.46	2.48	2.48
StdDev	0.29	0.04	0.04	0.00	0.00
Skew	-1.72	0.30	-0.26	N/A	N/A
Kurt	3.51	-1.13	-1.18	N/A	N/A
Panel DD: Amalgamated Forecasted Future Risk-Neutral Gold Prices					
Variable	Historical Gold Price	Forecasted Future Price using Price Volatility_Up	Forecasted Future Price using Price Volatility_Down	Forecasted Future Price using Dual Volatility_Up	Forecasted Future Price using Dual Volatility_Down
Mean	1,390.64	1,167.15	1,154.18	1,160.64	1,160.64
Median	1,269.23	1,166.93	1,154.39	1,160.64	1,160.64
Min	1,248.99	1,162.30	1,148.95	1,160.64	1,160.64
Max	1,784.94	1,172.46	1,158.99	1,160.64	1,160.64
StdDev	228.03	4.03	3.99	0.00	0.00
Skew	1.91	0.19	-0.18	N/A	N/A
Kurt	3.61	-1.22	-1.22	N/A	N/A
Panel EE: Amalgamated Forecasted Future Risk-Neutral Platinum Prices					
Variable	Historical Platinum Price	Forecasted Future Price using Price Volatility_Up	Forecasted Future Price using Price Volatility_Down	Forecasted Future Price using Dual Volatility_Up	Forecasted Future Price using Dual Volatility_Down
Mean	918.70	1,079.56	1,044.67	1,061.92	1,061.92
Median	893.39	1,078.42	1,045.67	1,061.92	1,061.92
Min	868.88	1,065.88	1,029.34	1,061.92	1,061.92
Max	992.72	1,095.53	1,057.98	1,061.92	1,061.92
StdDev	52.27	11.79	11.39	0.00	0.00
Skew	0.79	0.33	-0.31	N/A	N/A
Kurt	-1.42	-1.11	-1.14	N/A	N/A

Note: Mean is the average, Min is minimum, Max is maximum, StdDev is standard deviation, Skew is skewness, Kurt is kurtosis, No. is sample size and JB stands for Jarque Bera and it is a joint test of normality of the sample distribution of returns. All variables are in decimals except No. N/A stands for not applicable.

The future copper price forecasting was carried out using the Risk-Neutral Probabilities Model with a copper price volatility and a dual volatility. The copper price volatility was computed based on the standard deviation of the copper price’s logarithmic return from monthly historical copper price data from January 2004 to December 2015 that come up at 0.08. While, dual volatility was calculated based on Equation 4.2 at 0.00.

The descriptive statistics for the actual copper price and the future copper price forecasting using a copper price volatility and a dual volatility are in Panel CC of Table 6.30. The mean of actual copper prices is U.S.$ 2.70 per pound. The minimum value is U.S.$ 2.21 per pound and the maximum value is U.S.$ 2.96 per pound. The skewness and kurtosis are 1.72 and 3.51, respectively. These indicated a negative skewness and heavy tails.

The mean of future copper price forecasting using the Risk-Neutral Probabilities Model for up factor with copper price volatility was U.S.$ 2.54 per pound. The minimum value is U.S.$ 2.49 per pound and the maximum value is U.S.$ 2.59 per pound. The skewness and kurtosis are 0.30 and -1.13, repsctively. These indicated a positive skewness and light tails. The Root Mean Square Error between actual copper price and forecasted copper price using price volatility with up factor is 0.36.

The mean of future copper price forecasting using the Risk-Neutral Probabilities Model for down factor with a dual volatility is U.S.$ 2.42 per pound. The minimum value is U.S.$ 2.37 per pound and the maximum value is U.S.$ 2.46 per pound. The skewness is -0.26 and kurtosis is -1.18. These indicated a negative skewness and light tails. The Root Mean Square Error between actual copper price and forecasted copper price using price volatility with down factor is 0.62.

The mean of future copper price forecasting using the Risk-Neutral Probabilities Model for up factor and down factor with dual volatility were U.S.$ 2.48 per pound. The minimum and maximum values are U.S.$ 2.48 per pound. No skewness and kurtosis values cannot be counted since they have the same data. The Root Mean Square Error between actual copper price and forecasted copper price using dual volatility with up factor and down factor are 0.49.

The descriptive statistics based the actual gold price and the future gold price forecasting using gold price volatility and dual volatility were shown in Table 6.31 in Panel DD. The mean of actual gold price is U.S.$ 1,390.64 per ounce. The minimum value is U.S.$ 1,248.99 per ounce and the maximum value is U.S.$ 1,784.94 per ounce. The skewness and kurtosis are 1.91 and 3.61, respectively. These indicated a negative skewness and heavy tails.

The mean of future gold price forecasting with gold price volatility for up factor is U.S.$ 1,167.15 per ounce. The minimum value is U.S.$ 1,162.30 per ounce and the maximum value is U.S.$ 1,172.46 per ounce. The skewness is 0.19 and kurtosis is -1.22. These indicated a positive skewness and light tails. The Root Mean Square Error between actual gold price and forecasted gold price using price volatility with up factor is 0.50.

The mean of future gold price forecasting with gold price volatility for down factor is U.S.$ 1,154.18 per ounce. The minimum value is U.S.$ 1,148.95 per ounce and the maximum value is

U.S.$ 1,158.99 per ounce. The skewness and kurtosis are -0.18 and -1.22, repsctively. These indicated a negative skewness and light tails. The Root Mean Square Error between actual gold price and forecasted gold price using price volatility with down factor is 0.53.

The mean of future gold price forecasting using the Risk-Neutral Probabilities Model for up factor and down factor with dual volatility were U.S.$ 1,160.64 per ounce. The minimum and maximum values are U.S.$ 1,160.64 per ounce. No skewness and kurtosis values can be counted since they have same data. The Root Mean Square Error between actual gold price and forecasted gold price using dual volatility with up factor and down factor are 0.51.

The descriptive statistics based the actual platinum price and the future platinum price forecasting using platinum price volatility and dual volatility were shown in Table 6.31 in Panel EE. The mean of actual platinum price is U.S.$ 918.70 per ounce. The minimum value was U.S.$ 868.88 per ounce and the maximum value was U.S.$ 992.72 per ounce. The skewness is 0.79 and kurtosis is -1.42. These indicated a positive skewness and light tails.

The mean of future platinum price forecasting with platinum price volatility for up factor is U.S.$ 1,079.56 per ounce. The minimum value is U.S.$ 1,065.88 per ounce and the maximum value is U.S.$ 1,095.53 per ounce. The skewness and kurtosis are 0.33 and -1.11, respectively. These indicated a positive skewness and light tails. The Root Mean Square Error between actual platinum price and forecasted platinum price using price volatility with up factor is 0.36.

The mean of future platinum price forecasting with platinum price volatility for down factor was U.S.$ 1,044.67 per ounce. The minimum value was U.S.$ 1,029.34 per ounce and the maximum value is U.S.$ 1,057.98 per ounce. The skewness is -0.31 and kurtosis is -1.14. These indicated a negative skewness and light tails. The Root Mean Square Error between actual platinum price and forecasted platinum price using price volatility with down factor is 0.28.

The mean of future platinum price forecasting using the Risk-Neutral Probabilities Model for up factor and down factor with dual volatility were U.S.$ 1,061.92 per ounce. The minimum and maximum values are U.S.$ 1,061.92 per ounce. No skewness and kurtosis values canbe counted since they have same data. The Root Mean Square Error between actual platinum price and forecasted platinum price using dual volatility with up factor and down factor are 0.32.

The computation results using dual volatility were compared to the actual price against the forecasted price using price volatility. The best forecasting performance can be achieved by the lowest Root Mean Square Error results. Both of copper, gold and platinum prices indicated the best forecasting performance are shown by price volatility than dual volatility. A possible reason why the price volatility is more superior than dual volatility may be due to dual volatility not computing the mean of logarithmic return. As a result, it fails to address the future price forecasting properly. Based on the results above, the price volatility performs well in forecasting the future prices, better than dual volatility using the risk-eutral probabilities of future contracts. The latter sentence is consistent with the findings of Mun (2002) and Saliba & Dimitrakopoulos (2019). The latter authors pointed out that the price volatility in mineral commodity plays a critical role in addressing the issue of risk.

6.3.2. Robustness Tests for Optimal RNP

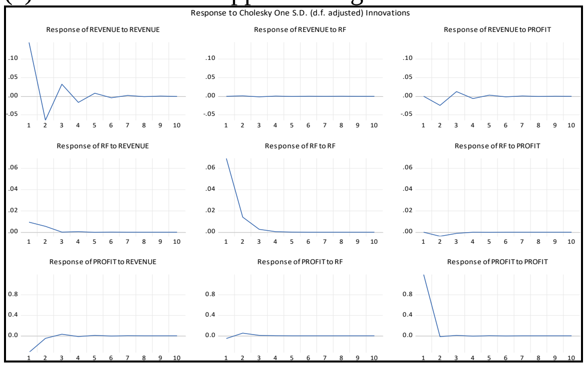
In running the VAR model, Kola & Sebehela (2020) illustrated how they input variables, in terms of numbering order in the model, is important in the type of results one will get. Hypothesis Three uses economic heuristics in assembling the pattern of the variables. Firstly, the analysis is on commodities and, the first thing that determines whether a commodity is affordable or not is its price based on demand and supply. Then, investors buy the commodity leading to mining companies generating revenues. In determining what price the mining company will sell at, they take into account interest rates. However, the interest rates effect is felt throughout the value change of each commodity. Similarly, the operating costs of the mining company. The capital expenditures are funds used to maintain physical operations of the mines. Thus, capital expenditure is incurred before the commodity is even sold. Based on the economic heuristic, then capex will be the first variable, followed by price, then revenue; thereafter, operation costs. Then, interest rates and final profit as profit is the last variable in the economic value chain. However, based on Equation 3.11, the only variables appropriate for Hypothesis Three are revenue, interest rate and profit because of appropriate synchronisation with Equation 3.11-same and/or similar factors. Numerically, the appropriate VAR variables can be represented as follows as an inequality;

$$\begin{Bmatrix} revenue, (i) \\ interest\ rate, (ii) \\ profit, (iii) \end{Bmatrix} \tag{a}$$

In order to decide which lag is appropriate for our studies; firstly, one draws from prior studies-it can be inferred from Bakshi et al (2019) that one lag is appropriate for commodity markets when studying time series because commodities are very much sensitive to immediate changes in the markets. Therefore, “longer” lags will not account effectively for immediate shocks. Secondly, the first and second order tests, and lag-length criterion support using one lag for the time series. The graphs of the Cholesky Decomposition will be interpreted jointly with the VAR (1,1) results in Table 6.31. In order to decide the appropriate lag, the following diagnostic tests were carried out in the orderly manner. First the residuals tests, where the correlation LM 2 lag test is tested for both first and second order correlation, and the results for the former tests for both orders, correlation is significant, which implies any lag up to 2 lags, might be appropriate for the VAR. Thereafter, lag structure test for 1 lag length criterion is tested and results support using 1 lag, when results are based on Aikaike IC and Schwarz SC. Finally, lag structure using AR roots test is carried and based on the AR Roots Table, using the modulus values, all absolute values are less than one; therefore, all roots inside unit are stationary. Fundamentally, results supports using VAR(1,1).

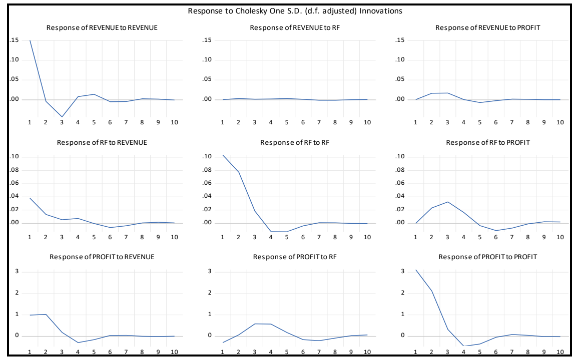
Figure 6. 1: Cholesky Decomposition

(a) Palabora Copper Mining Ltd



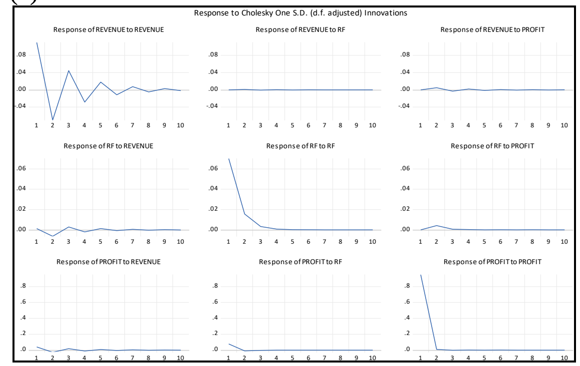
Note: RF stands for interest rates.

(b) AngloGold Ashanti



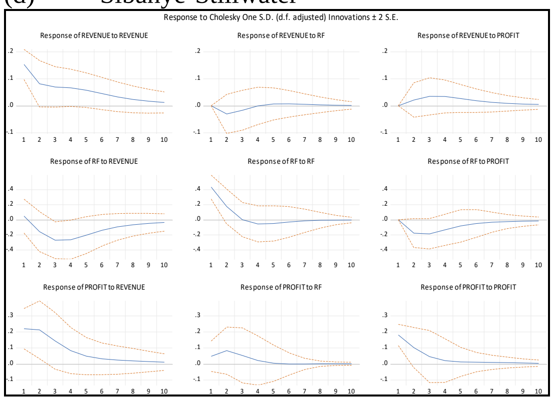
Note: RF stands for interest rates.

(c) Gold Fields Ltd



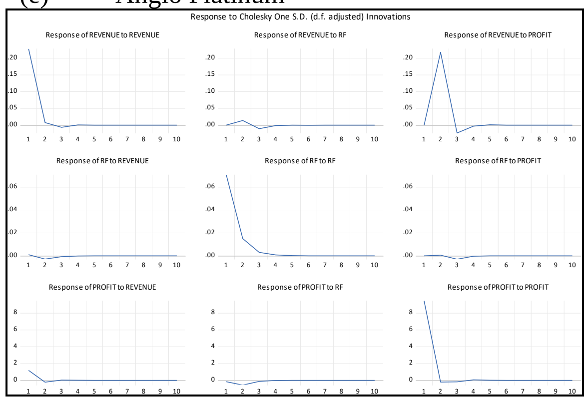
Note: RF stands for interest rates.

(d) Sibanye-Stillwater



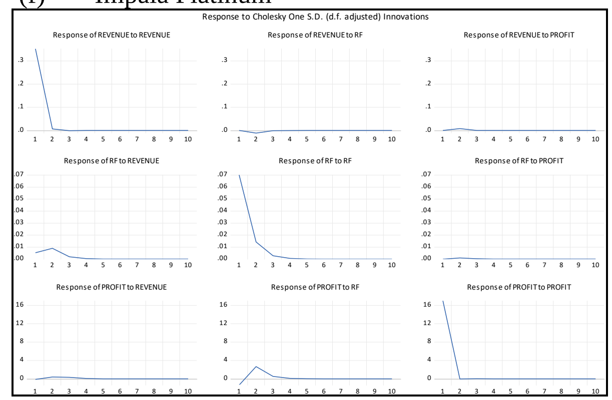
Note: RF stands for interest rates.

(e) Anglo Platinum



Note: RF stands for interest rates.

(f) Impala Platinum



Note: RF stands for interest rates.

Table 6. 32: VAR(1,1)

Panel FF: Copper				GG(3): Sibanye-Stillwater			
FF)1): Palabora Copper Mining Ltd							
Variable	Revenue	Interest Rate	Profit	Variable	Revenue	Interest Rate	Profit
Revenue (-1)	-0.449 (-6.414)##	0.018 (0.486)	-0.424 (-0.651)	Revenue(-1)	0.391 (1.091)	0.211 (0.206)	0.538 (0.793)
Interest Rate (-1)	-0.002 (-0.013)	0.203 (2.429)##	0.733 (0.495)	Interest Rate(-1)	-0.083(- 1.003)	0.513 (2.174)##	0.129 (0.828)
Profit (-1)	0.0201 (-2.031)##	-0.003 (-0.645)	-0.015 (-0.167)	Profit (-1)	0.116 (0.667)	-0.973 (-1.954)	0.564 (1.772)
F-statistic	13.99	2.409	0.209	F-statistic	14.658	19.860	8.139
Log likelihood	74.885	178.569	-229.793	Log likelihood	9.186	19.860	8.319
Akaike	-0.998	-2.459	3.293	Akaike	-0.691	1.406	0.581
Schwarz	-0.915	-2.375	3.376	Schwarz	-0.503	1.594	0.769
GG: Gold				HH: Platinum			
GG(1): AngloGold Ashanti				HH(1): Anglo Platinum			
Variable	Revenue	Interest Rate	Profit	Variable	Revenue	Interest Rate	Profit
Revenue (-1)	-0.055 (-0.631)	-0.158 (-2.464)##	2.709 (1.376)	Revenue (-1)	-0.087 (-1.613)	-0.014 (-0.829)	-0.825 (-0.368)
Interest Rate (-1)	0.054 (0.533)	0.535 (7.160)	0.630 (0.274)	Interest Rate (-1)	0.257 (1.106)	0.213 (2.982)	-8.208 (-0.850)
Profit (-1)	0.003 (1.075)	0.008 (3.523)##	0.471 (7.082)##	Profit (-1)	0.023 (12.727)##	6.4e-05 (0.114)	-0.022 (-0.296)
F-statistic	3.049	16.290	18.492	F-statistic	54.327	3.149	0.323
Log likelihood	92.488	152.100	-489.708	Log likelihood	13.018	235.643	-696.125
Akaike	-0.905	-1.535	5.256	Akaike	-0.095	-2.438	7.369
Schwarz	-0.785	1.415	5.376	Schwarz	-0.027	2.369	7.438
GG(2): Gold Fields				HH(2): Impala Platinum			
Variable	Revenue	Interest Rate	Profit	Variable	Revenue	Interest Rate	Profit
Revenue(-1)	-0.639 (-11.311)##	-0.061 (-1.698)	-0.256 (-0.524)	Revenue (-1)	0.021 (0.288)	0.022 (1.526)	0603 (0.170)
Interest Rate (-1)	0.008 (0.068)	0.217 (3.030)##	-0.1556 (-0.161)	Interest Rate (-1)	-0.150 (-0.419)	0.205 (2.864)##	38.047 (2.199)##
Profit (-1)	0.005 (0.619)	0.004 (0.819)	0.009 (0.127)	Profit (-1)	0.001 (0.309)	6.01e-05 (0.201)	-0.001 (-0.019)
F-statistic	42.849	4.103	0.107	F-statistic	0.116	3.734	1.652
Log likelihood	151.188	237.023	-258.560	Log likelihood	-69.489	236.491	-806.419
Akaike	-1.549	-2.453	2.764	Akaike	0.774	-2.447	8.531
Schwarz	-1.481	-2.385	2.832	Schwarz	0.842	-2.379	8.599

Note: in each cell, the first number is a coefficient and the number in the bracket is a t-test. The t-test is a standard student t-test which is calculated in model from Eviews. All variables that have (##) are statistically significant for VAR values as they are at least 2, irrespective of being negative or positive. Interest rate is based on the 20-year U.S. treasury rate by month.

In order to interpret the VAR results, one should read them in conjunction with the Cholesky Decomposition (Figure 6.1). Based on Figure 6.1(f), starting with the first curve at the top-left corner, the figure shows that changes in revenue reacting to first shocks coming from revenue, then second shocks coming from interest rates and thirdly, shocks coming from profit. That interpretation of the Cholesky Decomposition applies to all figures for Hypothesis Three, Figure 6.1. So, when it is interpreted from the top-left corner graph of Figure 6.1(a), revenue changes negatively to first shocks from revenue. Then, positively to second shocks from interest rate and negatively to shocks from profits. Note that all first shocks are statistically significant. Although, the second and third shocks are statistically insignificant, those shocks are in line with prior expectations and studies such as Kola & Sebehela (2020). For figures starting at zero, it means that they have been imposed by the Cholesky Decomposition. In terms of coefficient signs, interest rate is positive and profit is negative, which is consistent with prior studies (Jarociński & Karadi, 2020). In the second row of Figure 6.1(a), interest rates react positively to the second shocks emanating from interest rate, while interest rates do not react to first and third shocks from revenue and profit, respectively. Interestingly, in row 3 of Figure 6.1(a), profits react negatively to first shocks from revenue. That would be expected as profits include revenues. According to Choi & Prasad (1995), in the U.S.$, revenues can be as high as 50% of profits, depending on exchange rates. Figure 6.1(a) relates to Palabora Copper Mining Ltd.

Figure 6.1(b) is on AngloGold Ashanti. Panel GG(1) illustrates that all three variables (revenue, interest rate and profit) react only to second shocks emanating from revenue, interest rate and profit, respectively. This is probably because AngloGold Ashanti is the third largest gold producer in the world (AngloGold Ashanti, 2019). Therefore, any variable that affects gold directly and/or indirectly, affects the profitability of AngloGold Ashanti. Figure 6.1(e) is on Anglo Platinum and its VAR (1,1) is presented in Panel HH(1). Panel HH1 shows that interest rates react positively to second shocks coming from interest rate. Similarly, profits react positively to first shocks coming from revenue. The broader context of those findings is similar to ones from Palabora Copper Mining Ltd in Figure 6.1(a). Figure 6.1(c) is on Gold Fields. Panel GG(2) illustrates that revenues of Gold Fields react negatively to shocks; firstly, shocks from revenues. For the mining industry that would make sense as gold is regarded both as currency and “safe-haven”, especially during turbulent times. Some years were turbulent during the 2004-2019 period. Similarly, interest rate reacts positively to second shocks coming from interest rates. These are similar findings to ones based in Figure 6.1(e). Generally, copper and gold “control” commodity prices, as they are widely used by different stakeholders throughout the world. Figure 6.1(f) is for Impala Platinum. The only variable that is statistically significant is interest rate, which reacts positively to second shocks coming from interest rates. This finding has been synthesised earlier. It seems that for platinum, interest rate is the cost of owning physical platinum. Some asset managers who hold platinum funds, use interest rate when deciding to either increase or decrease their exposure to platinum stocks. Finally, statistical inference measures (i.e., Akaike Schwarz criteria measures) show that some VAR results are not normally distributed. In Table 6.32, Panel GG(3) supported by Figure 6.1(d) illustrate that the interest rate reacts positively to shocks emanating from one lagged interest rates for Sibanye-Stillwater. One of the reasons why in Sibanye-Stillwater interest rates react positively to interest rates shocks is that Sibanye-Stillwater is on a major growth trajectory both in the U.S. and South Africa (Sibanye-Stillwater, 2019). For gold, its major acquisitions are mainly in South Africa. The conundrum is that gold is regarded as currency and mainly currencies react positively to changes in interest rates (See; Marcato et al, 2019).

6.2 Chapter Summary

This section examined a dual volatility by forecasting the future copper, gold and platinum prices for the next 5-year period from 2016 to 2020 based on the historical data from January 2004 to December 2015. In order to test a dual volatility performance in forecasting the future copper, gold and platinum prices for the next 5-year period from 2016 to 2020, a Decision Tree Analysis, a GBM model and the Risk-Neutral Probabilities Approach for future options were proposed in this research. The main reason for using the Decision Tree Analysis and GBM models to test a dual volatility performance is because both of them can deal with non-normal distribution data (Elnaggar & Noller, 2009; Marathe & Ryan, 2005). Research to date has not yet determined that the risk-neutral probabilities can accept non-normal distribution data to forecast the future probability risk such as the future mineral commodity prices. However, the previous research has suggested that the assumption of normal distribution for the data can lead to underestimation since it has ignored the heavy tails from the data (Samunderu & Murahwa, 2021).

The dual volatility for copper using a Decision Tree Analysis model was computed based on the Palabora Mining Company Ltd data such as ore milled in tonnes, copper grade in ore in percent, operating costs in U.S.$ per pound and copper price in U.S.$ per pound. For gold, these used data from AngloGold Ashanti Ltd, Gold Fields Ltd and Sibanye-Stillwater Ltd such as ore milled in tonnes, gold grade in ore in gram per tonne, all-in costs in U.S.$ per ounce and gold price in U.S.$ per ounce. Platinum used Anglo American Platinum Ltd and Impala Platinum Holdings Ltd such as ore milled in tonnes, platinum grade in ore in gram per tonne, cash costs in U.S.$ per ounce and platinum price in U.S.$ per ounce. For a GBM model to forecast the future copper, gold and platinum prices for 2016 to 2020 that were based on oil price, copper price, oil production and consumption, copper production and consumption and exchange rate data from U.S.$ to ZAR and gold and platinum prices data from January 2004 to December 2015. This dual volatility based on this data was calculated using Equation 4.2 by Excel Software.

The dual volatility performance was tested using a Decision Tree Analysis and GBM model by forecasting the future copper, gold and platinum prices for the next 5-year period from 2016 to 2020. This performance was indicated by the Root Mean Square Error value close to zero. The Root Mean Square Error value for copper based on the Palabora Mining Company Ltd using a Decision Tree Analysis was 0.02, for gold was 0.17 of AngloGold Ashanti Ltd, 0.17 of Gold Fields Ltd and 0.32 of Sibanye-Stillwater Ltd and for platinum was 0.10 of Anglo America Platinum Ltd and 0.03 of Impala Platinum Holdings. The forecasting performance test results using the Root Mean Square Error suggested that a Decision Tree Analysis, using a dual volatility, provided the excellent forecasting performance for copper and platinum. However, it fails to achieve the best forecasting performance for gold using the Root Mean Square Error test. Therefore, for gold case, the Mean Absolute Percentage Error was proposed as an alternative way to examine the forecasting performance for the next 5-year period. For AngloGold Ashanti Ltd, the Mean Absolute Percentage Error mean for the next 5-year period was 0.07, 0.07 of Gold Fields Ltd and 0.09 of Sibanye-Stillwater Ltd. These results suggested that the forecasting performance for gold using a Decision Tree Analysis still provided the excellent performance.

Based on the GBM model with a dual volatility, the Root Mean Square Error value for copper was 0.03, 0.16 of gold and 0.42 of platinum. These results indicated that the GBM model can only perform very well for copper based on the Root Mean Square Error analysis. This has suggested to use another test to examine the forecasting performance for gold and platinum. Using the Mean Absolute Percentage Error, that was 0.10 for gold and 0.18 for platinum. This result indicated that the forecasting performance for gold using the GBM model is excellent and the platinum was still good.

In order to identify a sudden global incident that affects the global economy, such as the Covid-19 pandemic, this research examined the future copper, gold and platinum prices for 4-year period from 2016 to 2019 by excluding to forecast for 2020. Using a Decision Tree Analysis, the Root Mean Square Error result for copper increased from 0.02 to 0.04 and it decreased from 0.03 to 0.02 using the GBM model. For gold, these significantly decreased for AngloGold Ashanti Ltd from 0.17 to 0.01, Gold Fields Ltd from 0.17 to 0.02 and Sibanye-Stillwater Ltd from 0.32 to 0.10. Using the GBM model, it decreased from 0.16 to 0.05. For platinum, it decreased from 0.10 to 0.08 using Anglo America Platinum Ltd and 0.03 to 0.01 for Impala Platinum Holdings Ltd with a Decision Tree

Analysis. Using the GBM model, it decreased from 0.41 to 0.35. As a conclusion, the gold price was very sensitive to sudden global incidents, such as the Covid-19 pandemic, which has been confirmed by the Root Mean Square Error results.

The copper, gold and platinum prices indicated that the best forecasting performance is shown by price volatility rather than dual volatility, using the Risk-Neutral Probabilities approach of future contracts. A possible reason why the price volatility is more superior in this case can be caused by dual volatility not being able to compute the mean of logarithmic return. Therefore, the findings of this research were that a Decision Tree Analysis can perform better than the GBM model and the Risk-Neutral Probabilities approach to forecast the future 5-year period for platinum price.

7 Conclusions and Recommendations

7.1 Conclusions

This research was purposed to improve a mineral price volatility in forecasting the future copper, gold and platinum prices based on RMSE and MAPE results. In order to improve the weakness of a mineral price volatility, this research offered probably a more comprehensive study of a source of risk variables in the mining projects than previous authors synthesised in this thesis. Therefore, this thesis incorporated ore milled, metal grade in ore, metal prices and operating costs from January 2004 to December 2015 to forecast the future metal prices (copper, gold and platinum) from 2016 to 2020. This work also incorporated the multiple sources of risks such as oil price, copper price, oil production and consumption, copper production and consumption and exchange rate from U.S.$ to ZAR and gold and platinum prices data from January 2004 to December 2015. Then, these variables were used to compute a dual volatility using Equation 4.2.

The thesis proposed three methods such as DTA, GBM and RNP in forecasting the future metal prices (copper, gold and platinum). These results suggested that a dual volatility can perform well to forecast the future copper, gold and platinum prices using DTA since RMSE and MAPE results were lower than a mineral price volatility. Furthermore, the up and down movements based on DTA followed the usual price trend, where the increasing price for copper and gold are sensitive to upward factors.

Another test was proposed to examine a dual volatility using a GBM model. The finding for the latter test presents similar results to mineral price volatility when forecasting to forecast the copper price. For platinum, GBM model provided a less satisfied result than copper and gold in forecasting the future prices.

The last model to test the performance of dual volatility in this thesis is RNP. The results based on RNP are somewhat different than when using DTA and GBM, for both three commodities. The results of MAPE suggests that the different results between the actual price and the forecasted price for copper and platinum were consistent for the next-five year from 2016 to 2020. However, the different result between the actual and the forecasted gold prices tended to significantly increase in 2020. The relevant explanation for that is back swans, such as the Covid-19 pandemic in 2020, that have contributed to the extreme global economic turbulence. Possible conclusion is that the gold price is very sensitive to the Covid-19 pandemic than copper and platinum prices.

Overall, dual volatility based on DTA is more appropriate tool to forecast the future copper, gold and platinum prices since it has lower RMSE and MAPE results than when using GBM and RNPs.

7.2 Applied Techniques

In order to understand how a dual volatility regulated the future risks in the Hypothesis One, the relevant variables (ore milled, metal grade in ore, metal prices and operating costs) from January 2004 to December 2015 of mining projects were taken into account. The dual volatility was computed using Equation 4.2. Then, this volatility was used in a Decision Tree Analysis to forecast the future risks. This model was selected because it can accept non-normal distribution data (Elnaggar & Noller, 2009). The Decision Tree Analysis performance using a dual volatility was examined by forecasting the future copper, gold and platinum prices from 2016 to 2020. This result emphasised that a dual volatility can perform better than a mineral commodity price volatility, since it has lower Root Mean Square Error value for copper, gold and platinum prices. This research also improved Haque et al (2017)'s studies, where a mineral commodity price volatility was overestimated based on iron study case. This research found that the overestimation come from copper and platinum prices where gold price was underestimated.

Hypothesis Two was purposed to examine a dual volatility performance by employing the GBM model since a mineral commodity price cannot forecast the future risks for a long-term period (Ardian & Kumral, 2020; Dixit & Pyndick, 1994; Schwartz, 1997; Samis et al, 2006). The GBM model was selected since it agreed with non-normal distribution data (Marathe & Ryan, 2005). Then, in order to solve the long-term risk issue in a GBM model, using a mineral commodity price volatility, Sohrabi et al (2022) suggested to involve other sources of risk variables and Liu et al (2017) recommended to employ the independent variables. In this case, a dual volatility was computed using seven independent variables for copper and eight independent variables for gold and platinum (oil price, copper price, oil production and consumption, copper production and consumption and exchange rate from U.S.$ to ZAR and gold and platinum prices data) from January 2004 to December 2015. The finding of Hypothesis Two was that the GBM model performed better than a mineral price volatility, since it had a lower Root Mean Square Error value. However, the future forecasting price using a GBM model resulted in the underestimation of copper, gold and platinum prices than a mineral price volatility, where this argument was different with Haque et al

(2017) and Hypothesis One using a Decision Tree Analysis in this research. Furthermore, this research indicated that a GBM model cannot provide excellent results for forecasting the platinum prices. Then, this research confirmed that the GBM model can outperform to forecast the long-term period for copper, however it failed to address gold and platinum prices. Overall, this research suggested that a Decision Tree Analysis can perform well to forecast copper, gold and platinum prices better than the GBM model, since it has lower Root Mean Square Error results.

7.3 Contributions of the Hypotheses

7.3.1 Hypothesis One

Hypothesis One discussed forecasting the future price using a Decision Tree Analysis with a dual volatility. Many studies in the field of volatility have only focused on a mineral commodity price however, that has failed to address the optimal future risks variables (Banda, 2019). A reasonable approach to tackle this issue of volatility was optimised by involving the source of risk variables (Sohrabi et al, 2022) which in this case, these were represented by ore milled, metal grade in ore, metal prices and operating costs. This data were extracted from Parabola Mining Company Ltd for copper, AngloGold Ashanti Ltd, Gold Fields Ltd and Sibanye-Stillwater Ltd for gold and Anglo American Platinum Ltd and Impala Platinum Holdings Ltd for platinum from January 2004 to This data was used to calculate a dual volatility, which was used to optimise the future risks. The optimal future risks of a dual volatility were examined by forecasting the future copper, gold and platinum prices from 2016 to 2020, which can be shown by the Root Mean Square Error value. In copper case using Palabora Mining Company Ltd data, the Root Mean Square Error result was 0.02 for a dual volatility up factor and 0.88 for a dual volatility down factor. These results were lower than sold copper price volatility both of up and down factors, which were 1.44 and 1.83, respectively. Therefore, the optimal future risks can be achieved by a dual volatility up factor to forecast the future copper price from 2016 to 2020 since it has the lowest Root Mean Square Error value. For gold, based on AngloGold Ashanti Ltd data, it was 0.01 and 0.51 using a dual volatility for up and down factors, respectively and 0.12 and 0.59 using a mineral price volatility for up and down factors, respectively. Based on Gold Fields Ltd data, it was 0.02 and 0.51 using an incorporated volatility for up and down factors, respectively and 0.11 and 0.41 using a mineral price volatility for up and down factors, respectively. For Sibanye-Stillwater Ltd data, it was 0.10 and 0.41 using a dual volatility for up and down factors, respectively and 0.11 and 0.40 using a mineral price volatility for up and down factors, respectively. For platinum, based on Anglo American Platinum Ltd data, it was 0.84 and 0.10 using a dual volatility for up and down factors, respectively and 1.24 and 0.33 using a mineral price volatility for up and down factors, respectively. Based on Impala Platinum Holdings Ltd data, it was 0.73 and 0.03 using a dual volatility for up and down factors, respectively and 1.61 and 0.50 using a mineral price volatility for up and down factors, respectively.

These results were similar to those reported by Copeland & Antikarov (2003), Ugwegbu (2013), Dehghani et al (2014) and Haque et al (2017), where a mineral commodity price volatility failed to address the future price risk. It was important to bear in mind the possible bias in employing a mineral commodity price volatility that was due to a lack of variables as source of risk variables in the mining projects involved forecasting the future price risk. The study would have been more relevant if the future risks assessment had asked to employ sources of risk variables in the mining projects to represent the future price risk by calculating a dual volatility.

According to Haque et al (2017), another issue with a mineral commodity price volatility was that it contributed to tan overestimation in valuing the mining projects based on iron ore case. However, this research confirmed that copper and platinum prices were subjected to overestimation in forecasting the future price, whilst underestimating the gold price.

7.3.2 Hypothesis Two

Hypothesis Two was purposed to improve the GBM model’s performance in forecasting the future price using a dual volatility since a mineral price volatility failed to forecast accurately for the long-term period (Ardian & Kumral, 2020; Dixit & Pyndick, 1994; Schwartz, 1997; Samis et al, 2006). One of the limitations why a mineral price volatility provided less accurate results to forecast the long-term period was because it had ignored the business cycle to represent the future price fluctuations (Adrian & Rosenberg, 2008). A much more systematic approach would identify how volatility can represent the business cycle, which was believed to be associated to a source of multiple risk variables and the independent variables (Sohrabi et al, 2022; Liu et al, 2017). Different methods using Equation 4.2 have been proposed in this research to incorporate a source of multiple risk variables and the independent variables, where these variables were expected to have a strong link to the future price risk such as oil price, copper price, oil production and consumption, copper production and consumption and exchange rate from U.S.$ to ZAR and gold and platinum prices data from January 2004 to December 2015.

In order to answer the forecasting performance and the long-term period, this result was tested using the Root Mean Square Error (RMSE) and the Mean Absolute Percentage Error (MAPE) for the next 5-year period. For copper, the Root Mean Square Error was 0.28 using a mineral commodity price and 0.03 for a dual volatility. For gold, it was 0.20 using a mineral commodity price and 0.16 for an incorporated volatility. For platinum, it was 0.42 using a mineral commodity price and 0.41 for a dual volatility. This result indicated that a dual volatility performed well to forecast the future copper, gold and copper prices for a 5-year period.

The main purpose using a GBM model in this research examined the forecasting performance for the long-term period that was tested using the Mean Absolute Percentage Error (MAPE) per year. The Mean Absolute Percentage Error suggested that a dual volatility can deal with the future copper and platinum prices for the long-term period however, it failed to address gold price. The main significant changing of the Mean Absolute Percentage Error result for gold was shown in 2020 that was known as the Covid-19 pandemic. This was a sudden incident that contributed to the extreme global economic turbulence. Since gold price has been significantly affected by the Covid-19 pandemic, it can be concluded that gold price was very sensitive in response to sudden incidents.

The Decision Tree Analysis has also confirmed the same issue that the gold price was very sensitive to sudden incidents such as the Covid-19 pandemic rather than copper and platinum. This result was similar to Atri et al (2021)'s study to conclude the gold price was significantly distorted from 2019 to 2020 due to the extreme global economic change in a short-term period.

The GBM model indicated an underestimation in forecasting the future copper, gold and platinum prices. This result was different to a Decision Tree Analysis, which indicated that only the gold price was underestimated. Furthermore, it was reported different to Haque et al (2017) using Partial Differential Equations (PDEs) whilst a mineral commodity price tended to be overestimated.

7.3.3 Hypothesis Three

Metal price plays a critical role in the maintenance of profits since the performance of production rates and operating costs are limited by constant values (Dooley & Lanihan, 2005). However, it is almost certain that production rates, operating costs and other variables in mining projects are generally associated with the future risks contributing to the profits. The research to date has tended to focus on metal prices forecasting, rather than future maximum profits forecasting. The synthesis of maximum future profits probabilities was done according to the Risk-Neutral Probabilities (RNP). A major advantage of the RNP is that it has flexibility to be adjusted with actual data (Le Courtois & Quittard-Pinon, 2006). Investigating the RNP is a continuing concern within volatility that traditionally, has been assessed by measuring a single volatility. Such an approach; however, has failed to address the future risks that are suggested to develop an advanced volatility by incorporating technical and financial uncertainties in mining projects. The results of this study were approved by comparison between historical prices to a single volatility and a dual volatility (Guj, 2016). The average scores of the difference between historical price and forecasted future price using price volatility of copper for up factor and down factor was -5.93% and -10.35%, while a dual volatility was at -8.17%. For gold, that was -17.00% and -16.54% while a dual volatility was -16.54%. For platinum, the difference between historical price and forecasted future price using price volatility of platinum for up factor and down factor was 17.51% and 13.71%, respectively. While, forecasted future price using dual volatility of platinum is 15.59%. These results are different from the previous models since the Risk-Neutral Probabilities approach failed to address the mean of logarithmic return.

Many researchers have utilised the RNP approaches using a price volatility to measure the future probability of prices (Trigeorgis, 1993; Rubinstein, 1994; Dooley & Lanihan, 2005). However, these results therefore need to be interpreted with caution since several important variables in mining projects are not taken into account. Initial observations suggest that there may be a link between these variables and future risks to influence accuracy in forecasting the future prices. A reasonable approach to tackle this issue could be to propose a dual volatility in the RNP approaches in calculating the future prices. Comparing the two results between single volatility and a dual volatility, it can be seen that the RNP approaches, using dual volatility, have no significant different result to forecast the future price to the price volatility since it has a smaller difference mean to the historical prices. RNPs perform in an acceptable manner as illustrated in chapter 6, on Hypothesis Three.

7.4 Future Implications and Future Research Areas

7.4.1 Future Implications

The findings of this study suggested that a dual volatility can improve the future forecasting for copper, gold and platinum prices better than a mineral commodity price volatility. A reasonable approach to explain that a dual volatility can improve the future price forecasting that it incorporated a source of multiple risk variables in mining projects and the independent variables from the global commodity markets. However, the results often exceed the range of each mineral commodity price volatility. Strategies to enhance a dual volatility might divide Equation 4.2 results to ten.

Then, the evidence from this research suggested that a dual volatility can interpret the long-term future risks for copper price. This result emphasised the work of Sverdrup et al (2019) that, one possible implication of this due to copper has the long-term sustainability in the supply side.

As a conclusion, based on this research to forecast the future copper, gold and platinum prices, a Decision Tree Analysis provided more accurate results than a GBM model that was shown by the lower Root Mean Square Error. Furthermore, the GBM model tended to contribute to the underestimation of a mineral price volatility, whilst Haque et al (2017) demonstrated a mineral commodity price being overestimated. Therefore, this result was similar to those resulted by Zhang et al (2015) that a GBM model cannot follow a mineral commodity price.

7.4.2 Future Research Areas

- Selecting the most relevant independent variables

  The most relevant independent variables should be selected to improve a mineral commodity price volatility since it provided unsatisfactory results because it had failed to address the future risks. A possible explanation for this unsatisfactory result might be that a mineral commodity price volatility ignored a source of multiple risks variables and the independent variables in the mining projects and the global commodity markets. The evidence from this research indicated that a source of multiple risk variables and the independent variables were involved in forecasting the future copper, gold and platinum prices have improved more accurate estimation results as shown by lower Root Mean Square Error values than a mineral commodity price volatility. However, this research did not appear to explain quantitatively in selecting a source of multiple risk variables and the independent variables to represent the future risks.

- Examining a dual volatility to other mineral commodities

  Analysis of future price forecasting models using a dual volatility in this research has been done for copper, gold and platinum. The evidence from this research indicated that a dual volatility provided the acceptable forecasting performance for base metal such copper prices and precious metals prices such as gold and platinum. Therefore, this method should be examined for other metals.

- Examining a dual volatility using other models

  The Decision Tree Analysis and GBM model used in this research to examine an incorporated volatility performance were selected because these can deal with non-normal distribution data. The forecasting performance using a Decision Tree Analysis showed to significantly satisfy for copper, gold and platinum. Furthermore, this still performed significantly well for copper and gold using the GBM model, whilst it was still acceptable for platinum. This result suggested to test a dual volatility using other forecasting models.

- Short and long-term forecasting performance investigation

  Previous studies of future copper and platinum prices using volatility were forecasted for less than 2 years. Then, gold price was forecasted for 1 year to 17 years. Then, the Decision Tree Analysis and GBM models used in this research to forecast the future copper, gold and platinum prices for the next 5-year period provided more accurate results than previous studies. However, these models should be examined to forecast the short-term period on a daily basis or more than 5-year period. Therefore, the future research should also make attempts to analyse the short-term period over a period of less than 1 year and more than a 5-year period to test the forecasting performance of a dual volatility.

- Metal price sensitivity to sudden incidents

  This research demonstrated that gold price was very sensitive to the Covid-19 pandemic. However, another test should be carried out to examine the gold price sensitivity to other sudden global incidents such as economic policy changing, war and terrorist attacks and global calamity.

7.5 Recommendations for Future Work

On the basis of the observations, discussions and results obtained so far, the recommendations are as follows:

- It has commonly been assumed that a dual volatility would have been more interesting if it had included a source of multiple risk variables and the independent variables based on quantitative analysis.
- In testing the forecasting performance of a dual volatility, it would have been more useful if it was examined to other metal prices and models.
- These models would have been more relevant if it has been examined over short-term and long-term periods.
- A better comprehensive study to forecast the future mineral commodity prices from the historical data would propose the model(s) to manage non-normal distribution data before using to forecast the future risks.
- In order to test the gold price sensitivity, it might have been more relevant if it examined other sudden global incidents such as economic policy changing, war and terrorist attacks and global calamity.

7.6 Limitations of the Study

Despite the fact that this research contributes numerous contributions to the body of knowledge, in particular volatility analysis of mineral commodities price, there are some limitations. Firstly, the commodities used in this research are; copper, gold and only one of the PGMs, namely, platinum. Therefore, other remaining PGMs; palladium, rhodium, iridium, ruthenium and osmium are not included in the study. Hence, the results of platinum might not apply to other PGMs.

Secondly, a dual volatility includes; (i) ore milled, (ii) metal grade in ore, (iii) metal prices and (iv) total operating costs for Hypothesis One and copper and eight independent variables for gold and platinum such as oil price, copper price, oil production and consumption, copper production and consumption and exchange rate from U.S.$ to ZAR and gold and platinum prices data for Hypothesis Two. Although, the mentioned variables form both financial and technical risks variables in mining, the social, political and natural risk variables are not considered in this research in computing a dual volatility. Hence, the results might be different when social risk, natural risk and political variables are part of dual volatility.

APPENDIX

A Ethical Clearance

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

Registration number: REC-101114-044

10 February 2022

Re: Mr. Veriyadi Veriyadi (1732800)

Waiver letter number: HRECNMW21/02/03

To whom it may concern,

Mr. Veriyadi is currently a registered PhD student at the School of Mining Engineering at the University of the Witwatersrand, Johannesburg. This letter is to confirm that, at the time of writing, Mr. Veriyadi does not need ethical clearance for his PhD study entitled *'Application of Derivative Techniques to Reduce Volatility of the Future Price Prediction of Copper, Gold and PGM Metals'*. This decision has been reached based upon a description of the project supplied by Mr. Veriyadi to the University Human Research Ethics Committee (Non-Medical), which has been evaluated by the Chairs and Deputy Chairs. If, however, Mr. Veriyadi changes the methods of data collection and analysis for this study, this decision may no longer be valid. If such changes take place, this should be communicated to the University Human Research Ethics Committee (Non-Medical) as soon as possible. This waiver letter is valid until 09 February 2024.

Please feel free to contact me should you require any further information.

Thank you.

Yours sincerely,
S Schoeman

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