APPLICATION OF GEOSTATISTICAL ORE RESERVE EVALUATION TECHNIQUES TO OPTIMISE VALUATION OF MINING BLOCKS AT BEATRIX MINE

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A project report submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

Johannesburg, 1998

DECLARATION

I declare that this project report is my own, unaided work. It is submitted for the Degree of Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

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(signature of candidate)

20th day of January 1998

ABSTRACT

This project report describes a geostatistical study undertaken on the Geozone 5 deposit at Beatrix Mine in the Free State. Geostatistical analysis of this deposit is described in considerable detail to illustrate the application of the method to a tabular-type deposit using Geostokos Toolkit, a computer software package developed by Prof Isobel Clark.

Comparison has been made between indicator kriging and lognormal kriging to establish which of the two geostatistical techniques will optimise the valuation of the Geozone 5 deposit. The mean absolute error (MAE) and mean square error (MSE) criteria, and the correlation between kriging estimates and actual values have been used as the basis for this comparison. The results show that lognormal kriging will improve the estimates of resources as a result of lower MAE and MSE values over indicator kriging. This reduction is further confirmed by a higher correlation coefficient for lognormal kriging estimates.

The location of future additional exploratory drilling, particularly in the northern part of the deposit, should be guided by the range of influence of approximately 350 meters as established by the experimental semi-variogram, since samples have no influence beyond this range value from their locations.

This study has demonstrated that geostatistical techniques can be applied at the mine site to improve block estimates and also reduce block estimation variance as new data becomes available.

Dedicated to the Glory of God and to my wife Nana Ekua

and my daughters Naa Lamiley and Naa Lamikor

ACKNOWLEDGEMENTS

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INTRODUCTION

1.1 Location

Beatrix Mine , a division of Gengold, is one of the leading underground gold mines in South Africa. The mine is located some 35 km south of Welkom and 25 km south of Virginia in the Free State (Figure 1.1). The mine has been operating since 1981 and is currently producing about 2.4 million tons of ore annually from two main shafts. An expansion program is currently underway with the excavation of a third shaft to mine the deeper reef in the northern part of the mine.

1.2 General Physical and Geological Settings

Beatrix is the most southerly of the Witwatersrand-type gold mines. The topography of the area is underlain by a thick sequence of flat lying Karoo sediments which overlie the underlying Archaen Witwatersrand and Ventersdrop Supergroups.

Mining operations in the western areas of St Helena Mine during the early 1960's gave a better understanding of the stratigraphic relationship and the structure along the western margin of the goldfields. In the period

1973 to 1980 drilling was concentrated in the Beisa Mine (Oryx Mine) area and also towards the southeast of Beisa . By 1980 an economic auriferous conglomerate at the base of the Eldorado Formation (Figure 1.2) had been proved in the area 14 km to the southeast of Beisa Mine and shaft sinking for Beatrix Mine commenced here in April 1981. The reef mined became colloquially known as the Beatrix Reef, (Genis ,1990).

At Beatrix Mine, the Beatrix reef in the mining sense is taken to be the conglomerate and interbedded arenite deposited on the unconformity surface overlaying the Virginia formation. The upper contact of the reef is taken as the scour surface at the base of the first dark-grey lithic arenite/wacke, or at the base of the first black argillite parting. As such the Beatrix Reef zone, in the definition used in the mine, incorporates the conglomeratic remnants of the Aandenk Formation which occur in the northeastern part of the mine.

The Beatrix reef is characterised by small to medium oligomictic, quartz-pebble conglomerates with a grey quartz-arenite matrix and occurs throughout the mine area. Well packed clast-supported conglomerate and very poorly-packed, matrix-supported pebbly arenite again form two distinct end-member subfacies. The variation between these facies is gradational.

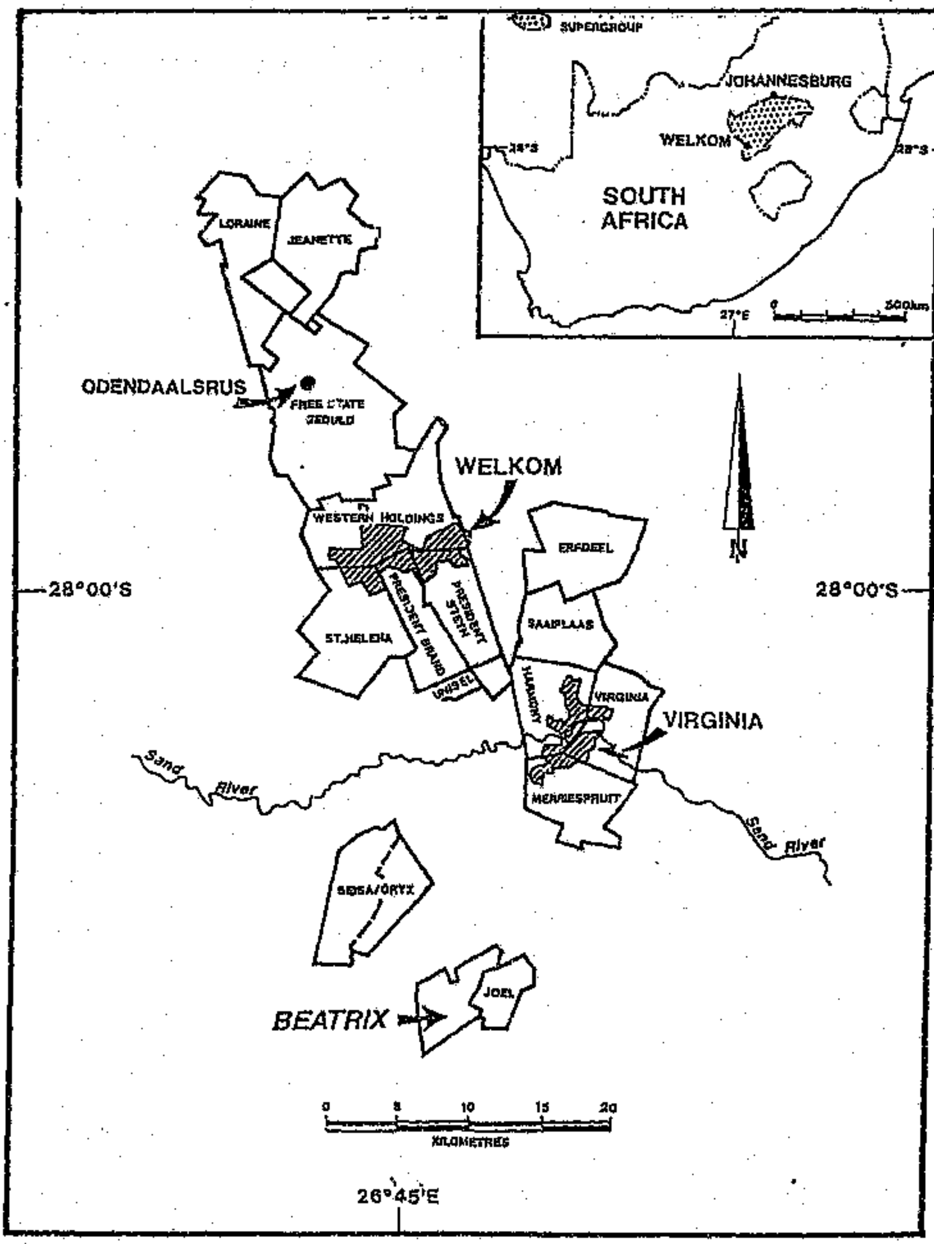

Figure 1.1 Plan showing the location of Bearix Mine (after Genis ,1990)

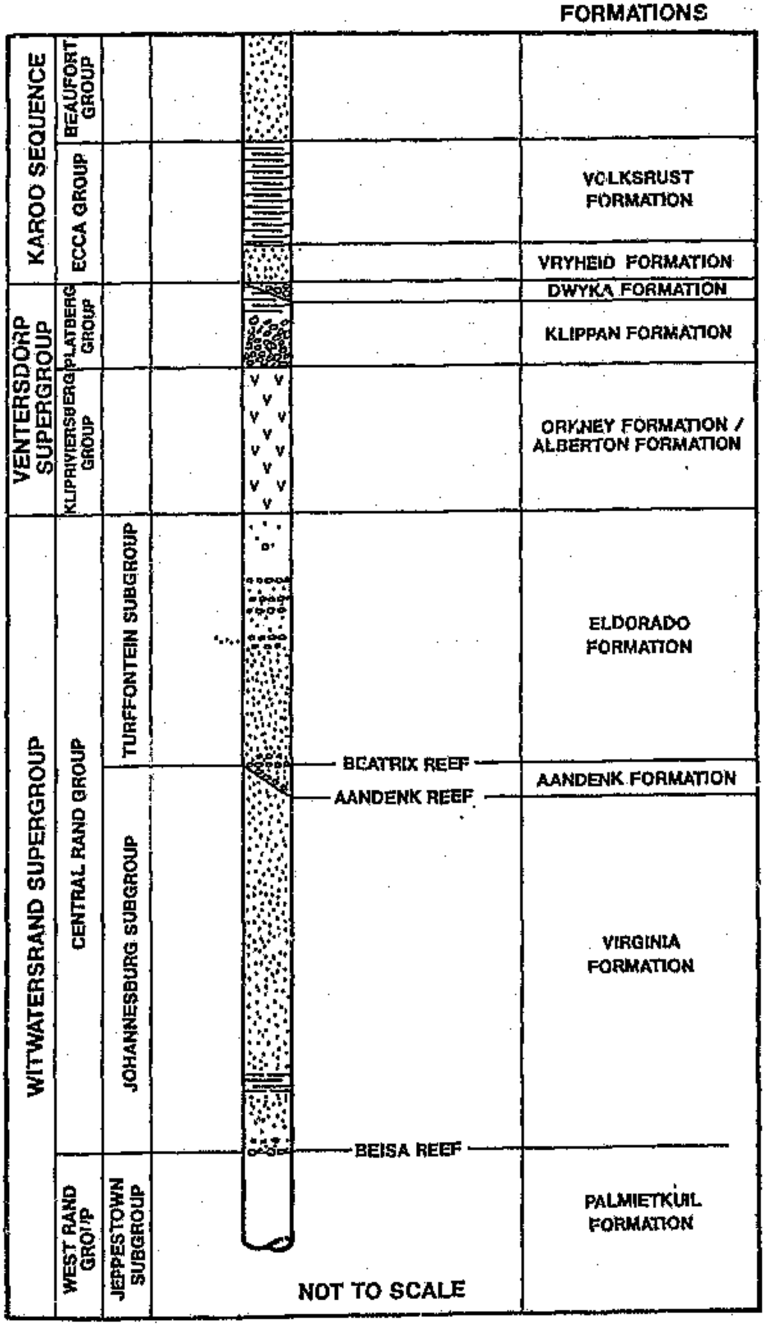

Figure 1.2 Detailed stratigraphic zoning of the Witwatersrand Supergroup in the Beatrix Mine Area (after Genis ,1990)

The Beatrix Reef varies from a thin, single pebble lag to thick sequences of conglomerate and arenite. Much of the variation in thickness can be ascribed to erosional scouring into the unconformity surface at the base of the reef. These variations in thickness are a good indication of channel orientations. An Isopach plan in Figure 1.3 of the Beatrix Reef thickness in the 40 exploration boreholes drilled from surface in the Beatrix Mine and immediate surroundings areas shows that most of the reef intersected is between 20 to 80 cm in thickness. Areas of thicker reef (over 50 cm) define roughly north-south trending zones along the western and central parts of the mine. Areas of reef less than 20 cm thick form irregular elongate areas between these zones of thick reef resulting in most cases into thinner reefs up to 4 cm. This variation is termed Waste on Contact (WOC). The area under consideration in this project work is within this zone of reef formation.

Mineralization within the Beatrix reef occurs as discrete accumulations a few millimetres thick, concentrated along certain bedding and sour surfaces within the arenites and conglomerates. Heavy mineral grains, mainly gold (the most economically significant) and pyrite, occur as the dominant constituent of the matrix in the better packed conglomerate beds.

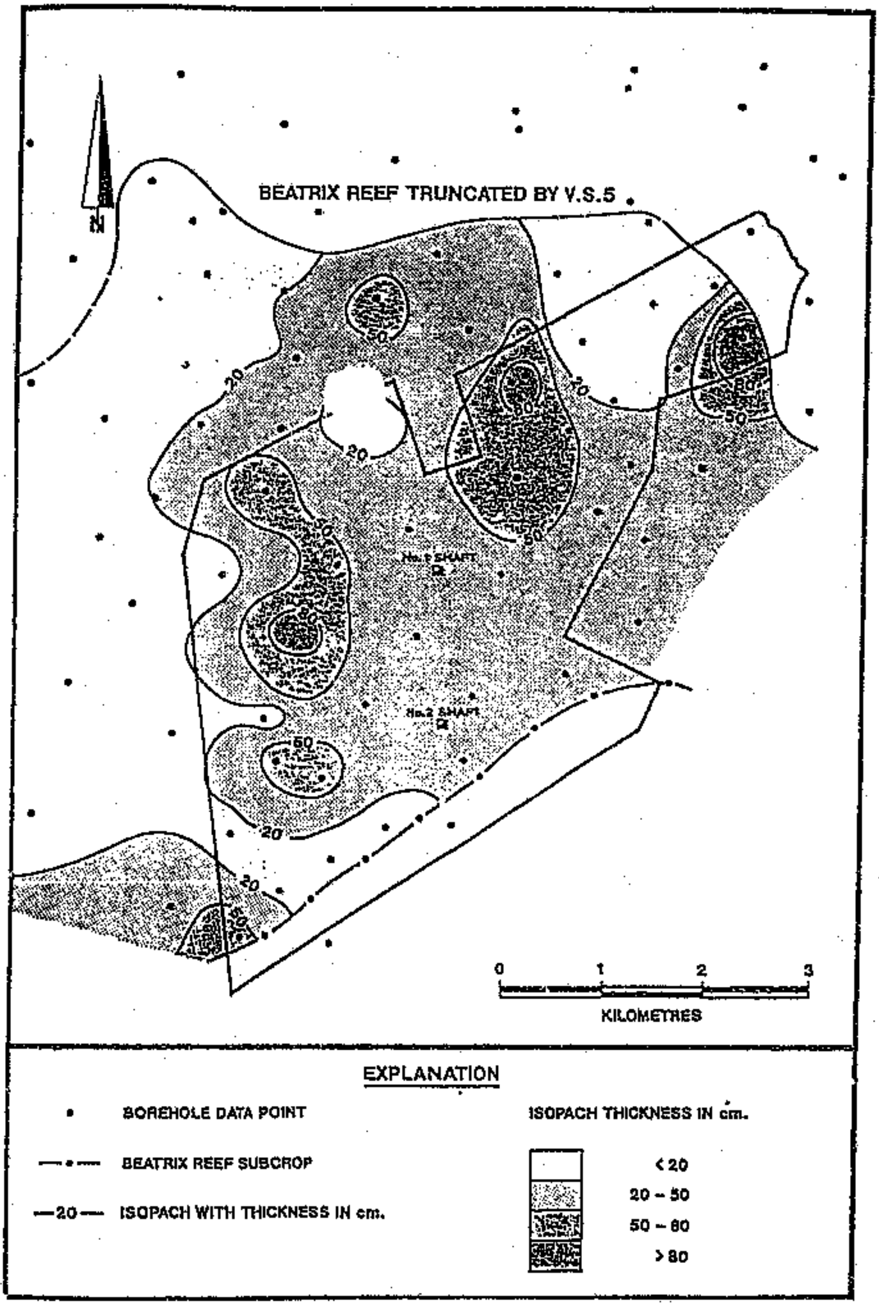

Figure 1.3 Beatrix Reef Isopachs from surface Exploration BoreHoles (after Genis ,1990)

1.3 Problem Definition

The economic viability of mining is subject to many uncertainties, among them political risk, financial risk and project risk. Numerous factors contribute to project risk, but in most cases those relating to ore reserves are the most important. The ore reserve is the principal asset of a mine and one for which variables can be quantified statistically within calculated limits of error. An accurate estimation of reserve base is therefore absolutely necessary for scoping a project and for reliable short and long term planning. In general, it is not sufficient to calculate the average grade of an orebody or parts of it without having some appreciation of the accuracy with which such estimates are made. There are numerous examples of sophisticated ore reserve calculations that led to substantial pre-production expenditures. However, as development progressed, it was realised that ore did not exist in the grades or amounts forecasted. The need for a sound estimation technique which will give as practical as possible an "accurate" valuation of the ore body can not be over emphasised.

Within the Gengold Group, there is a new trend to employ computer aided mineral deposit evaluation packages for block estimations in the currently producing mines. The Valuation department intends to apply geostatistical techniques to estimate reserves within the various delineated mining blocks. In order to establish which techniques could be appropriate, the author as part of his Master of Science degree research programme was requested to investigate the application of geostatistical ore reserve evaluation techniques to optimise the valuation of mining blocks at Beatrix Mine.

1.4 Objectives of the study and Methodology

1.4.1 Objectives

The aims of this study are:

I. to apply a number of geostatistical techniques to the borehole and stope sampled data within a given area and establish an optimal technique that would serve as a tool for reserve estimation.

II. to find global, individual block estimates and associated estimated variance, and to generate grade tonnage curves based on the mine selective mining unit (SMU)

1.4.2 Methodology

The following methods were employed to achieve the aims outlined above:

I. the sample data was subjected to statistical analyses to study the underlying ore value distribution as well as any inherent trends.

II. In the geostatistical studies, lognormal kriging and indicator kriging techniques were employed for the following reasons :

   i. According to Krige (1981), tests have indicated that modified lognormal models (three-parameter lognormal) have been found to eliminate the high skewness associated with Witwatersrand reefs and hence are recommended as a relatively simple and effective model.
   ii. Indicator kriging, according to Fytas (1990), is one of the nonparametric techniques developed to estimate the reserves of highly skewed distributions like gold, uranium, platinum diamonds etc.
   iii. lognormal and indicator kriging techniques are among the most common kriging packages on the software market.

CHAPTER 2

LITERATURE SURVEY

2.1 Introduction

The main objective of ore reserve estimation is to determine the quantity of mineral at a selected cut-off grade present in a given deposit. To obtain reliable ore reserve estimation, exploration, sampling and assays must be carried out thoroughly so that:

i. *geological boundaries within the deposit can be demarcated: the* clearer the boundaries, the more reliable the estimates; and

ii. grades of samples from several locations within the various geological boundaries can be established; the greater the number of samples, the more accurate the estimates.

The usual approach is to create a mineral inventory from the sample value and then employ a cut-off grade to delineate ore reserves based on the relevant S.M.U. (Barnes (1980)).

Ore reserve estimation, according to Davis (1979), involves two general requirements:

i. breaking the property up into mining blocks, and
ii. assigning each block an ore quality and quantity.

The shape of the mining block usually depends on the estimation techniques, and the mining method to be employed. Many methods for ore reserve estimation have evolved over the years and these have been broadly classified under traditional or conventional methods, classical statistics, trend surfaces and geostatistical methods.

2.2 Geostatistical Methods

Geostatistical methods, compared with other methods of ore reserve estimations, have been widely recognised as a superior method for estimating the grade and tonnage of insitu mineralization because they provide a sound theoretical and practical basis for quantifying the geological concept of (i) area of influence (ii) the continuity or lack of continuity of mineralization within the ore body and (iii) the lateral changes in mineralization according to the trend direction of the orebody.

Unlike classical statistics which considers grades to be randomly distributed within a mineral deposit, geostatistics is based on the theory of regionalised variables developed by Matheron (1962). Regionalised variables are those with values which show some relationship to adjacent values - including ore grade, vein thickness, and many others, (Sinclair 1974).

In the 1910s, statistical methods were already used to analyse geological data. However, the origin of geostatistics, as we know it today is best set in the late 1940s, when H.S. Sichel recognised the lognormal distribution of sample values in the South African gold mines. In 1951, Daniel Krige observed that "it can be expected that the gold values in a whole mine will be subjected to a larger relative variation than those in a portion of the mine." In other words, samples taken close together are more likely to have similar values than if taken apart. This observation is the foundation on which spatial statistics, which characterises values defined in a multidimensional space, is built. However, the 1950s were marked by studies based on classical, as opposed to spatial, statistics. It was only in the 1960s that the need was recognised to model the similarity between sample values as a function of the distance between samples and that the semi-variogram was defined. A theoretical framework was developed by Matheron that supplied an elegant mathematical explanation to the empirical observations made by Krige. Matheron coined the term "kriging" in recognition of Krige's pioneering work on the geostatistical evaluation of mineral deposit, (Rendu 1994).

The theory and application of Geostatistics have been outlined by a number of scholars including Krige (1951) and Sichel (1952) in South Africa, de Wijs (1953) in Holland and Hazen (1958), Becker and Hazen (1961) in the United States, Matheron (1962) and Serra (1967) in France, Reedman (1979) and M. David (1979) in Canada and Clark (1979) in England.

2.3 The Semi-variogram

Geostatistics makes use of a semi-variogram, which is a mathematical function derived from the sample data, to give the degree of natural dispersion of assay values. This gives a measure of the expected discrepancies for the estimation method and hence allows the choice of the estimation method with the lowest expected discrepancies. Semi-variograms represent variance between sample pairs as a function of distance (lag) between samples. Experimental variograms are determined for each regionalized variable under consideration by the formula:

$$\gamma(h) = \frac{1}{2n} \sum_{i=1}^{n} \{ Z(X_i) - Z(X_i + h)\}^2$$

Where:

$Z(x_i)$ = the value of the regionalized variable at point x_i,

$Z(x_i + h)$ = the grade of another point at a distance h from the point x_i and

n = the number of sample pairs.

Experimental variograms are commonly prepared for samples aligned in several directions to test for anisotropy (that is, whether the samples have different ranges of influence in different directions), and stationarity - that is whether the samples in a given area came from the same probability distribution. The presence of anisotropy and non-stationarity must be taken into account in obtaining a unifying mathematical model for the variogram that is applicable to the entire deposit, and on which variance estimates will depend.

The study of geostatistics has evolved different types of semi-variogram models. The table in Appendix A shows different types of semi variogram for some of the mineral deposits that are likely to be encountered in nature. Figure 2.1 shows the semi-variogram for a spherical model which is regarded by many as being the most common model (David (1977); Barnes (1979)). In this figure, the range 'a' reflects the classical geologic concept of an area of influence; beyond this distance of separation, sample pairs no longer correlate with one another and therefore become independent.

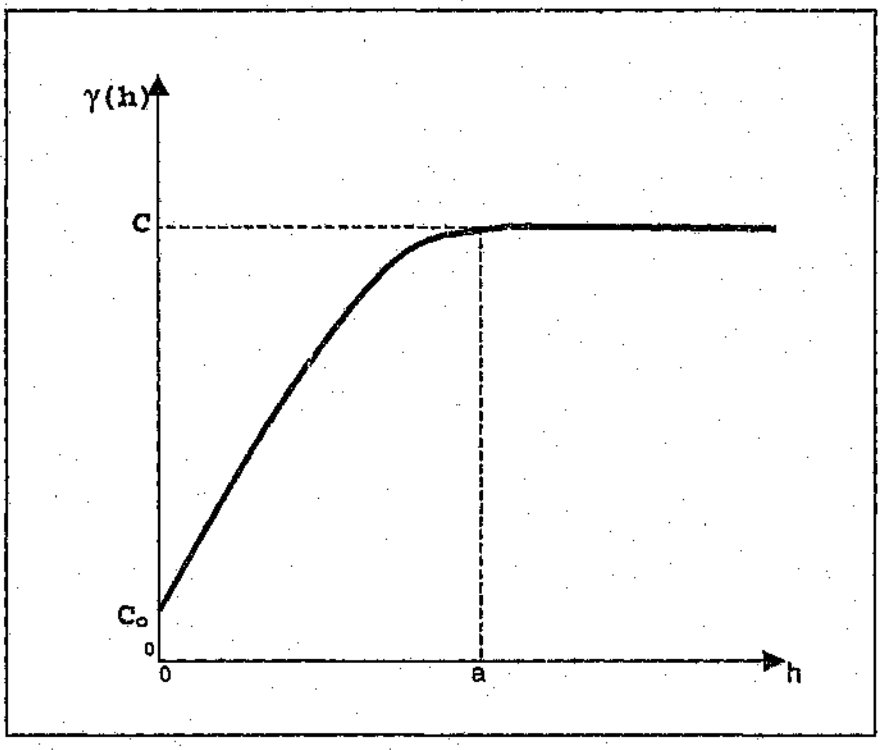

Figure 2.1 The shape for a semi-variogram - the spherical model

The sill (C + Co) is equal to the variance of all samples used to compute the variogram. The nugget effect or variance (Co) is the name given to the semi-variogram value $\gamma(h)$ at a distance of zero. It expresses the local homogeneity (or lack thereof) of the deposit. High nugget effect relative to the sill can indicate that either the mineralization is poorly disseminated or the zone on which the semi-variogram was computed is severely disjointed or that sample preparation and assaying procedures were poorly carried out.

2.4 Cross Validation

Cross validation is one of the main objective techniques for testing a model fit to a semi-variogram. The term "cross validation" according to Clark (1986) is now generally accepted as describing the following procedure:

i. One sample is eliminated from the data set.
ii. The surrounding samples are used to produce an estimate of the value at this (now) unsampled location, using a geostatistical estimation method.
iii. The actual error incurred in this process is measured by: (Actual Value - Estimated Value).
iv. The "expected" or "theoretical" error is measured by the kriging variance calculated during the estimation process or by its square root, that is the kriging standard error.

If the semi-variogram model fits the sample data then the mean or average of the errors should be zero and the ratio of the average kriging variance for all the estimation to the variance of the errors is expected to be one. There are a number of limitations in the application of cross validation techniques on sample data. In the first place there are no objective guide lines as to the acceptable deviation from the ideal figures of zero and one for the mean and standard deviation respectively. Secondly there is the possibility of still getting a mean of zero and a standard deviation one for incorrect semi-variogram model parameters. Cross validation however tests whether the samples in the immediate locality could be reproduced by the samples' values. An unusually high cross validation figure may therefore serve as an indication of some problems with the data set which need to be verified.

2.5 Kriging

Various scientific disciplines require the collection and prediction of data over space. In mining, where the goal is to predict ore concentrations over an entire study area, samples are collected at various locations. To predict concentrations at locations where the samples are not collected, geostatistics uses a technique known as kriging. Kriging was the name given in 1960 by Matheron to the multiple regression procedure for arriving at the best linear unbaised estimator or best linear weighted moving average estimate of the ore grade for an ore block (Krige 1981).
It is one of the most important fundamental methods in geostatistics, with widespread practical applications. In ore reserve calculations, kriging provides the best local estimators of means and variances for a specific panel size. Kriging produces a map of ore concentrations for the entire

site which can be used for planning and operating mining activities (Subhash (1995)). The technique basically involves assigning an optimum set of weights to all the available data in a deposit. It has two main advantages, namely the avoidance of systematic bias errors and the minimisation of the error of estimation, the kriging error

If Z is the unknown grade of a block, then an estimator Z* is determined in the form

$$Z^* = \sum_{i=1}^{n} \lambda_i Z_i$$

where

z_i = the arithmetic means of data within the block to be estimated

λ_i = the corresponding weighting factors or kriging coefficients and

n = the number of samples and

The quality of the estimation is determined by the kriging variance σ_K^2 (that is the variance of Z and Z*) which should take the smallest possible value .

A great variety of kriging methods are now available. Which method should be used depends on the nature of the deposit and on the type of problem that the geologist or the mining engineer wishes to solve. The methods available to model deposits from a large sample base vary from "ordinary kriging" , the original multivariate linear regression method used by Krige to "indicator kriging", "lognormal kriging", "probability kriging", "universal kriging", "disjunctive kriging", and an endless list of other kriging methods (Rendu 1994). For the purposes of this study, lognormal and indicator kriging techniques are discussed further.

2.5.1 Lognormal Kriging

It is found very often that the distribution of ore grades is not even approximately normal, but has a high positive skewness and may be fitted better by a lognormal distribution. The ideal approach according to Krige (1979) is to apply the three-parameter lognormal model. The grade z is transformed by the function log(z+a), where a is an additive constant, that is, the third parameter of the lognormal distribution. The additive constant is added as and when necessary to optimise the fit to a normal distribution. The transformed values are then used to compute semi-variograms and generate the ordinary kriging estimates.

2.5.2. Indicator Kriging

Indicator kriging is one of the nonparametric geostatistical techniques. It discretizes the histogram of the grades in several classes and carries out interpolation separately for every class. The principal difference between ordinary kriging and indicator kriging is that indicator kriging works on transformed data (0,1) according to several cut-off grades. Therefore, the final result of indicator kriging is a cumulative probability distribution for every block (or panel) that gives the probability distribution that the block or panel exceeds a specific cut-off grade (Fytas et al, 1990). The following steps are required to carry out ore reserve estimation:

i. construct the histogram of data ;
ii. choose a few cut-off grades, preferably equi-distant on the histogram scale(e.g. the four quartiles or ten deciles);
iii. transform the drillhole data into 0,1 values for every cut-off grade selected (e.g. 1 if they are below the cut-off and 0 otherwise);
iv. develop and model the indicator variograms separately for every cut-off;
v. perform ordinary kriging on the transformed (0,1) values for each cut-off. By repeating this step for every cut-off grade one gets as an end result a cumulative probability curve as a function of grade for every

block. These probability distributions can then be used for ore reserve estimations.

The advantages of nonparametric geostatistical techniques according to Fytas (1990) are:

i. they are distribution-free and outlier resistant and can be applied to any gold deposit estimation whatever its histogram characteristics;
ii. they provide confidence intervals for the reserves;
iii. they are data value dependent taking into account the outliers.

CHAPTER 3

DATA COLLECTION AND PREPARATION

3.1 Description of Data

Precious metal deposits, particularly gold are typically spatially complex in their geology and ore distribution. The complexity of ore bodies is mainly reflected by two facts: (1) discontinuity in ore grade, and (2) diversity of ore trends. The application of geostatistical techniques could result in erroneous variogram models if the geological domain within the area is not well defined.

The ore deposit used in this study is the "Geozone 5 " deposit - which is one of the eight geological domains defined on the basis of assay and geological information. The area, located in the western part of the Beatrix mine, extends approximately from grid 25755 - 27800 in the east and 21260 - 22780 in the north. The Geozone 5 deposit has reef thicknesses over 50cm in some areas. Areas of reef less than 20 cm thick form irregular elongate patches between these zones of thick reef resulting in most cases into thinner reefs up to 4 cm. This variation is termed Waste on Contact (WOC). The WOC is peculiar to Geozone 5 with no particular channelized orientations. The unpredictability of the WOC formation within the reef has made reserve evaluation in this geological domain very difficult

3.2 Sampling Data

The chip sample values used for the study consist of a total of 4790 samples. These have been categorised into stope samples, primary and secondary developments taken on a 6m x 6m square grid, and underground and surface boreholes. All information related to the sample is stored in a data base and put on a computer disk which includes the following :

i. co-ordinates of the sample points.
ii. centimetres grams per ton (cmg/t)
iii. channel width in centimetres
iv. stope width in centimetres
v. codes or categorisation of sample type

On the mine, the gold accumulation factor known as centimetres grams per ton (cmg/t) is commonly used to express the level of mineralization in the reef. The gold accumulation factor (cmg/t) is derived from the product of the reef thickness (channel width) (cm) and the gold concentration (g/t) over the reef width sampled at any sampling point. The stope width is used for the purposes of estimating tonnages using a relative density of 2.75.

Data for this study was received from the Gengold head office after a further period of attachment on the mine.

3.3 Data Processing and Presentation

Analysis of sample data was carried out using the Geostokos PC Toolkit which has been designed to perform Statistical and Geostatistical analysis of sample data from geological data. The Geostokos Toolkit developed by Prof. Isobel Clark is an interactive package which allows the user complete control of all parameters for the purposes of ore reserve estimation.

CHAPTER 4

DATA ANALYSIS

4.1 Statistical Studies

Statistics is essentially a study of variability, and it involves the use of a suitable mathematical model representative of such variability and the application of this inferred pattern of behaviour to practical problems. Some important parameters of statistics used for the study are the mean (average), variance and standard deviation . To ascertain the di[illegible]ion of data for a particular set of samples, a statistical model is generated in the form of histogram or probability plots (Barnes,1980). The 3-parameter lognormal model was used to describe the underlying ore value distribution.

It is common practise to evaluate tabular deposits such as the Beatrix reef using the accumulation of the samples. It has however been observed that there is the tendency to over estimate the gold produced and / or underestimate the tonnage required to produce it if no relationship exits between the various variables (ore grade, channel width and accumulation) .

Geostatistics is concerned with regionalized variables - those with values, which show some relationship to adjacent values - including ore grade, channel width and accumulation. In order to ascertain the relationship between the gold grade , the channel width and the accumulated grade for the Geozone 5 area, scattergrams were used to investigate the correlation between these regionalized variables.

The histogram and probability plots of the gold grade , channel width and the accumulated grade are shown in Figures 4.1 to 4.6. The variables exhibit a positively skewed distribution with a high coefficient of variation as shown in the statistical summary in Table 4.1. The gold grade data for the study area conforms quite closely to a single population indicating a clearly defined single facies. The channel width on the other hand seems to indicate a number of populations with the majority region consisting of thick channel deposits interspersed with thin channel zones typical of the waste on contact (WOC) formation which is prevalent within the Geozone 5 area. As already explained in section 1.2, this variation in thickness is attributed to erosion of the unconformity surface at the base of the reef .

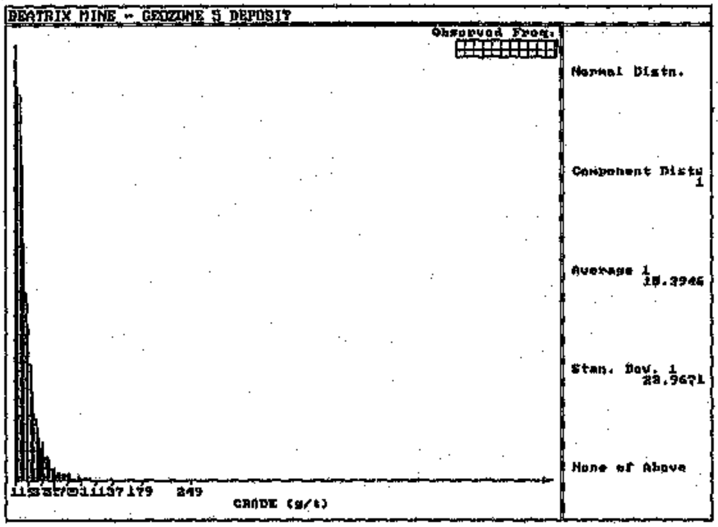

Figure 4.1 Histogram of gold grade

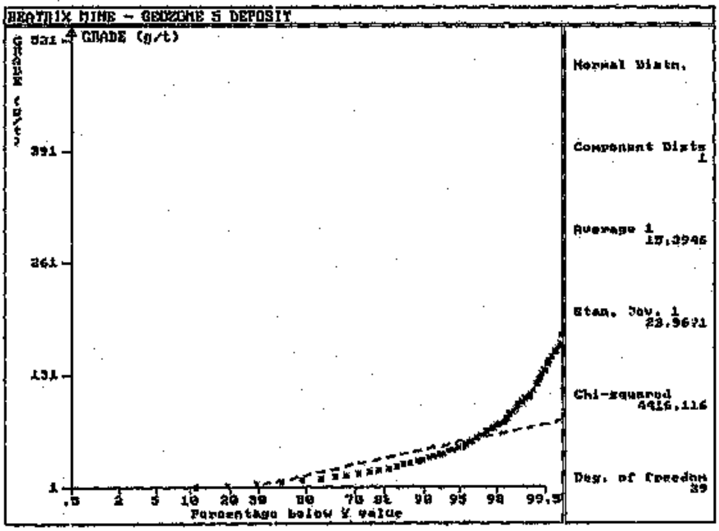

Figure 4.2 Normal probability plot of gold grade

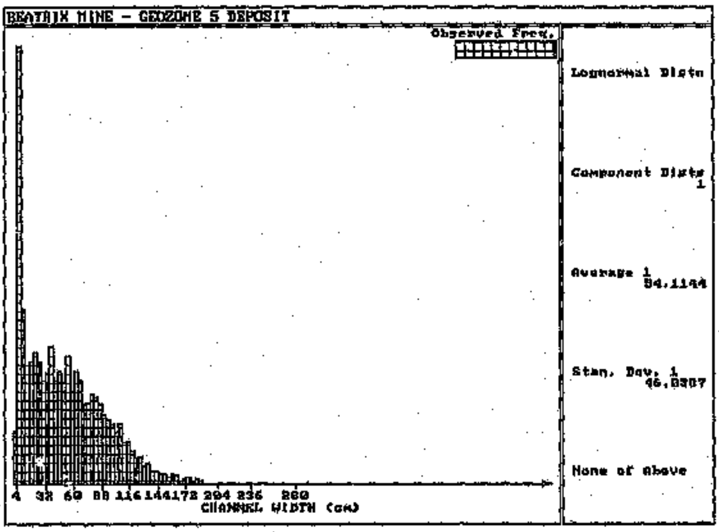

Figure 4.3 Histogram of channel width

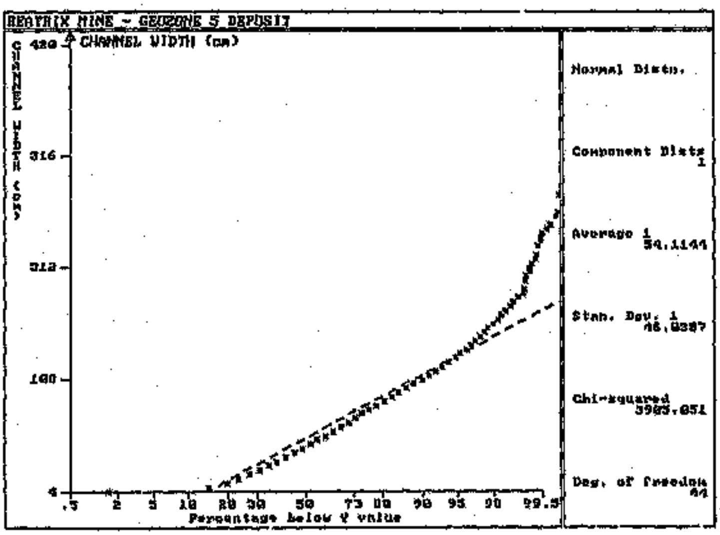

Figure 4.4 Normal probability plot of the channel width

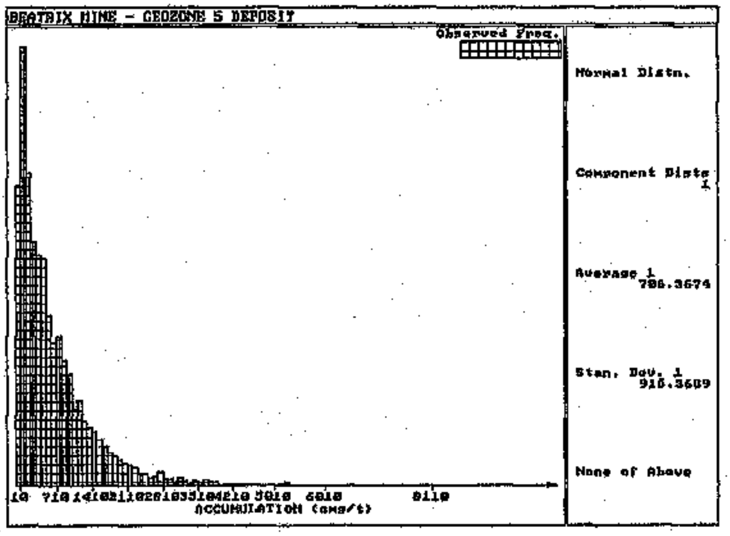

Figure 4.5 Histogram of accumulated grade

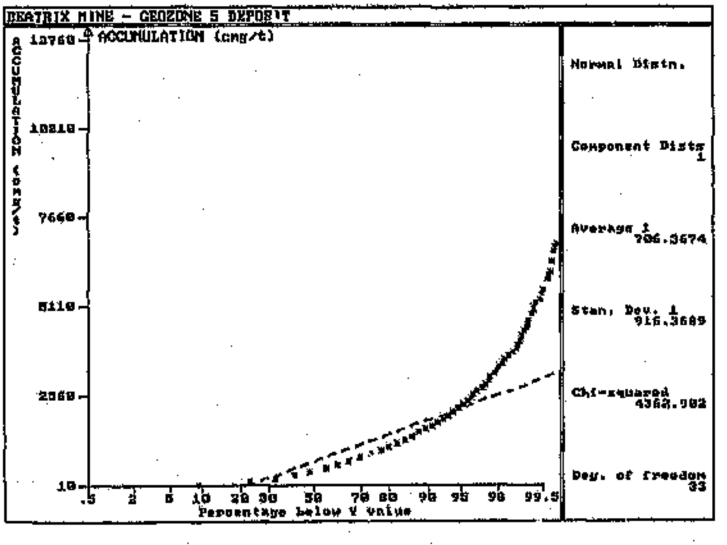

Figure 4.6 Normal probability plot of the accumulated grade

Table 4.1 Summary Statistics of the Geozone 5 deposit

	Gold Grade (g/t)	Channel width (cm)	Accumulations (cm g/t)
No. of Samples	4790	4790	4790
Minimum	0.005	4	1
Maximum	852.22	489	11928.60
Mean	15.3946	54.1144	706.3674
Standard Deviation	22.9671	46.0387	916.3689
Coefficient of Variation	1.4919	0.8508	1.2973

4.1.1 Lognormal Plot and Scattergram of Variables

The lognormal plots shown in Figures 4.7 to 4.9 clearly indicate significant deviations from the two-parameter lognormal model. By introducing an additive constant, the Three-parameter lognormal model was found to be appropriate as shown in Figures 4.10 to 4.12.

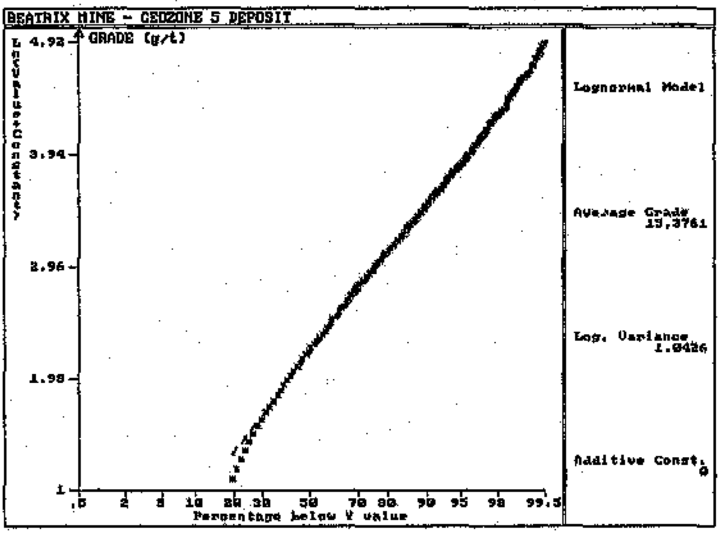

Figure 4.7 Two-parameter lognormal plot of the gold grade

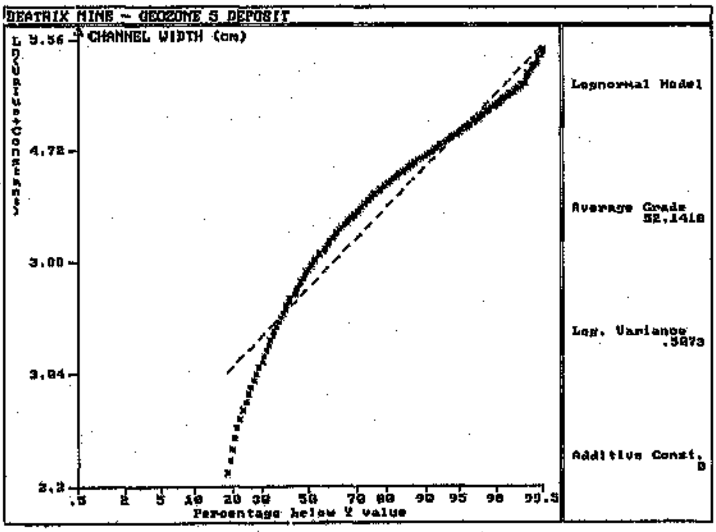

Figure 4.8 Two-parameter lognormal plot of the channel width

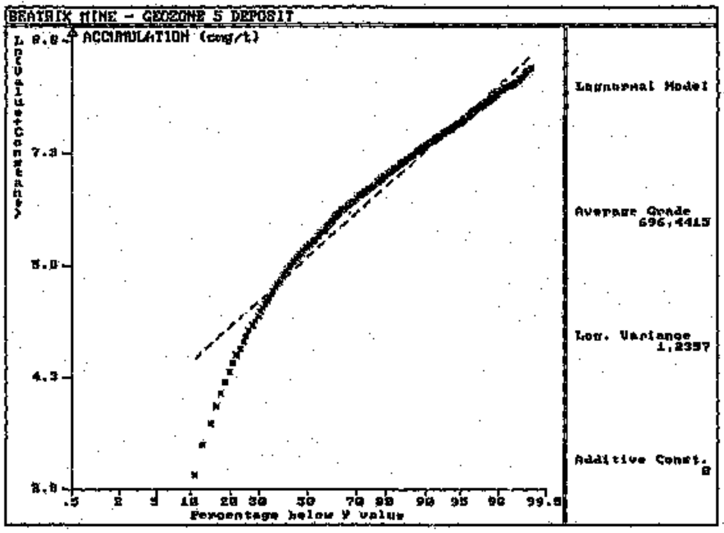

Figure 4.9 Two-parameter lognormal plot of the accumulated grade

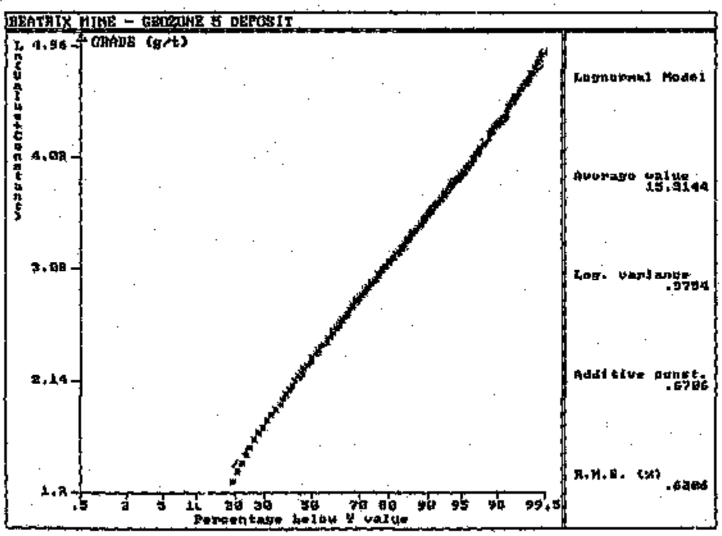

Figure 4.10 Three - parameter Lognormal plot of the gold grade

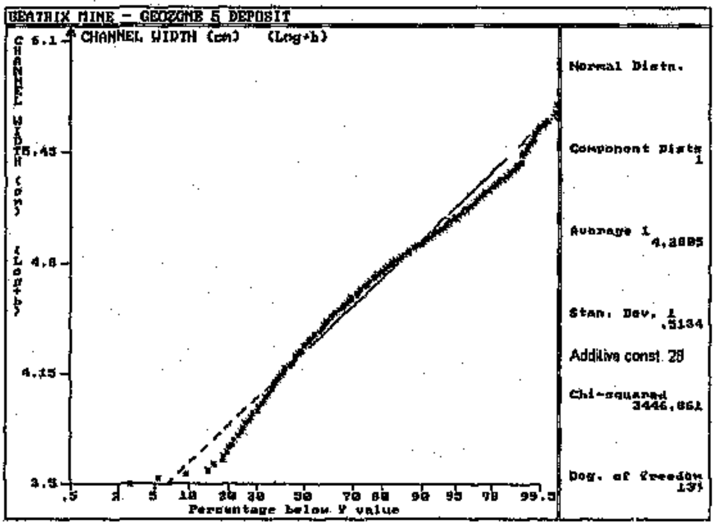

Figure 4.11 Three - parameter Lognormal plot of the channel width

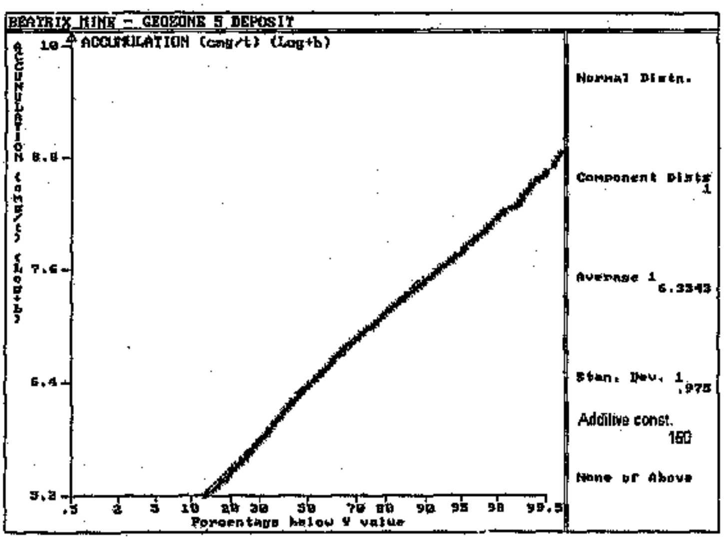

Figure 4.12 Three - parameter Lognormal plot of the accumulated grade

Figures 4.13 to 4.15 show a scattergram of the logarithmic transformation of the three variables . It is clearly evident from the plots that there seems to be no relationship between the gold grade and channel width as a result of a low correlation coefficient of 0.012. The accumulated grade however seems to show a positive linear relationship with the channel width and the gold grade.

4.1.2 Hypothesis Test

To ascertain whether the relationship between the variables is significant, a hypothesis test is set up based on the calculated correlation coefficients in Figures 4.13 to 4.15. Perfect correlation is said to exist if the calculated correlation coefficient *r* is equal to positive or negative one. There is no correlation if *r* is equal to zero. Under the hypothesis that there is no correlation - H_o: $p = 0$ the statistics *s* should follow a specified distribution referred to as the "inverse hyperbolic tangent distribution". From standard statistical tables (Cambridge Statistical Table 13) for values above 130 degrees of freedom *v*, the correlation coefficient *r* is approximately normally distributed with zero mean and variance of **1/(v-1).**

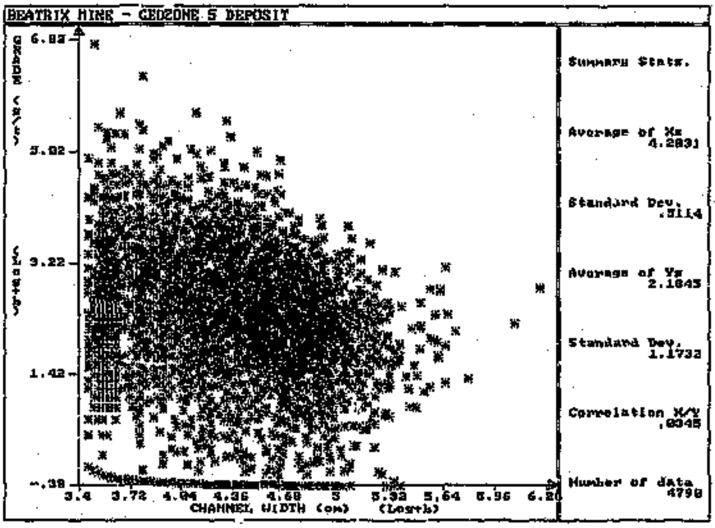

Figure 4.13 Scattergram of the log transformed gold grade and channel width

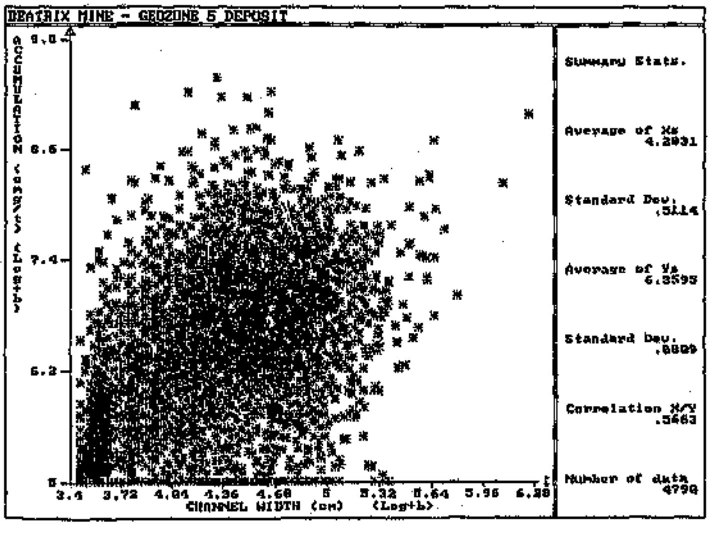

Figure 4.14 Scattergram of the log transformed accumulated grade and channel width

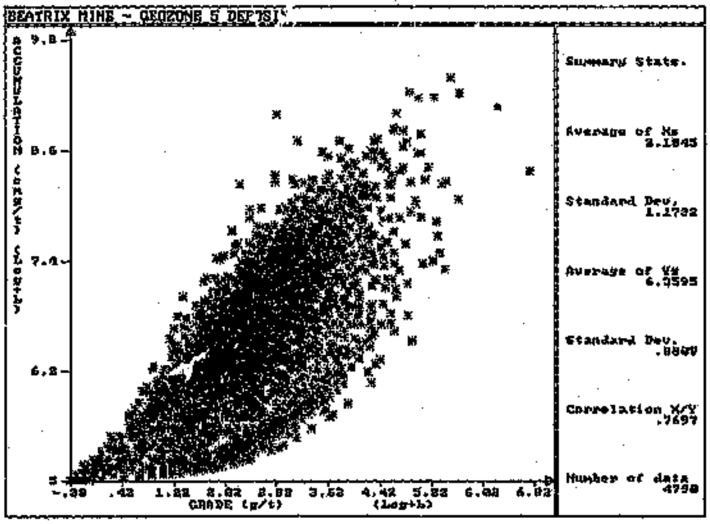

Figure 4.15 Scattergram of the log transformed accumulated grade and gold grade

The sample number of 4790 therefore approximates to a calculated correlation coefficient of 0.0144. The calculated correlation coefficient of 0.0345 between the gold grade and the channel width is rather close to zero. There is therefore no significant relationship between the gold grade and the channel width. In the case of the relationship between the accumulated grade versus the gold grade and the channel width, it can be stated that there is a significant positive linear relationship between the accumulated grade, versus the gold grade (with calculated correlation coefficient of 0.5663) and the channel width, with correlation coefficient of 0.7697 .

4.1.3 Conclusion

A practical consideration in dealing with the Geozone 5 deposit is the use of accumulated grade as is customary for tabular or two-dimensional deposits. This is due to the significant correlation of the accumulated grade with both the gold grade and the channel width. Furthermore the irregular outlines of the deposit, particularly within the waste on contact formations, could be well estimated when accumulation is used for the study.

4.2 Trend Surface Analyses

The distribution of minerals can exhibit very unusual behaviour in terms of rapid increase or decrease in grade over distance as one moves from one point to another. This behaviour of mineralization is known as drift or trend. Since the focus of this study is geostatistically based, and some kriging techniques give erroneous and biased results in the presence of a very strong trend, a trend analysis was carried out for the Geozone 5 deposit.

The Geostokos Toolkit has provision for the analysis of Polynomial trend Surface. This analysis fits three surfaces, namely planar or linear - a constant dip in a single direction, quadratic- a bowl or dome shape, anticline or syncline, and cubic - saddle point, sometimes associated with large scale folding.

Table 4.2 illustrates the trend analysis of the study area. The final column under F-ratio in Table 4.2 is the important parameter for assessing the presence of trend in the deposit. Under statistical assumptions of normality and independence, the statistics shown in this last column would follow the F-distribution, which could be found in any statistical book. The first item under the F-ratio for each of the surfaces compares

Table 4.2 Analysis of Variance

Note: this analysis is based on the assumption of lognormality

Source	Sum of Squares	Degree of freedom	Mean Square	F-ratio
Linear	45.8601	2	22.9301	**46.54**
Residual	2358.3430	4787	0.4927	
Quadratic	46.7993	5	9.3599	**18.99**
Diff	0.9392	3	0.3131	**0.64**
Residual	2357.4040	4784	0.4928	
Cubic	46.6790	9	5.1866	
Diff	-0.1204	4	-0.0301	**10.52**
Residual	2357.5240	4780	0.4932	**-0.06**
Total	2404.2030	4789		

Percentage of Total Sum Of Squares:

Linear Component	1.91
Quadratic Component	1.95
Cubic Component	1.94

the variation on the original set of sample data with that left after fitting the expected sources of possible variation. The second and third items are comparisons between linear / quadratic (18.99), and quadratic /cubic (10.52). These measurements indicate how much more variation remains after the trend has been removed. Comparing these figures in any standard F tables will indicate that the sample data does not show any strong trend, hence no attempt was made to remove trend in the course of the analysis of the data.

4.3 Geostatistical Studies

The accumulated grade within the Geozone 5 area was subjected to various geostatistical studies. The main objective is to assess which of the two kriging methods - Lognormal or indicator kriging - will constitute an appropriate technique for estimating the reserves within the Geozone 5 area. The first stage consisted of construction and interpretation of semi-variograms, and the second the use of the respective kriging method after verification with cross validation.

4.3.1 Regularisation of data set

To compare the two kriging methods, the 4790 chip samples within the Geozone 5 area was divided into two areas and regularised on a 30m by 30m grid or block. The regularisation process was carried for two reasons: firstly because the faces of different stopes are not parallel to each other as shown in the location of stope samples in Appendix H, and secondly because the face advance between sampling varies from stope to stope and within stopes, the overall sample pattern is irregular although stope samples were taken at regular intervals along stope faces. As a result of the above scenario, it became necessary to overlay a regular grid on the surface of the reef and to give each grid a gold value equal to the average value of the chip samples it contains. This yielded an average of approximately 10 samples per grid or block (Table 4.3). Figure 4.16 shows the location of 403 regularised (30m by 30m) block samples for kriging purposes and Figure 4.17 shows the location of 96 (30m by 30m) block samples to be used for performance comparison of local estimates. To mimic extrapolation as observed in practice, the performance comparison samples (actual) were removed and then estimated using the two kriging techniques. Finally, the kriging estimates were compared with the 'actual' values.

Table 4.3 Summary Statistics of regularised samples

	Regularised Samples for kriging	Regularised Samples for performance Comparison
Number of Samples	403	96
Mean	694.48	566.51
Standard Deviation	610.32	582.52
Coefficient of Variation	0.8788	1.0283
Ave. Samples per block	9.66	9.61

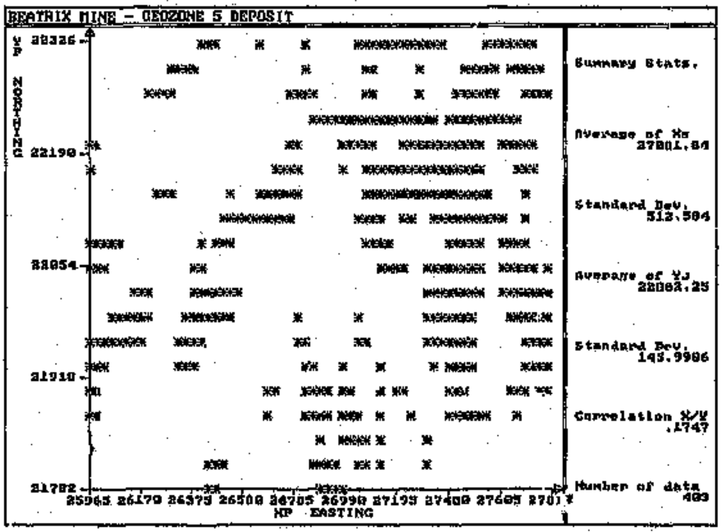

Figure 4.16 Location of regularised samples for kriging

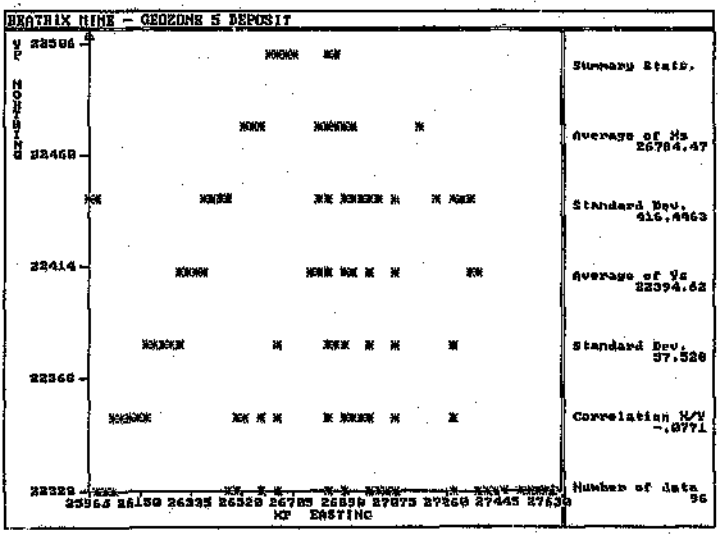

Figure 4.17 Location of regularised samples for performance comparison of local estimates.

4.3.2 Indicator Thresholds

The method of application of indicator kriging, as described in section 2.5.2, requires a number of thresholds or cut-off grades for grade estimation. The selection of thresholds according to Dowd (1996) should be done in such a way as to give an adequate and unbiased representation of the distribution. One recommended way of doing this is to select the thresholds as the values correspond to equal intervals on the probability frequency axis. In mining, very few cut-off values have practical economic significance and in such situations it is necessary to perform indicator at several high cut-offs since the accurate estimation of the upper tail is more important than the estimation of the lower portion of the distribution. For the purpose of this study, two indicator cut-offs were selected from the regularised blocks to give adequate representation of the distribution and also ensure meaningful modelling of variograms. Table 4.4 shows the detailed statistics of the selected cut-off grades .

Table 4.4 Statistics of samples for various cut-off grades

Cut-off grade (cmg/t)	Number of samples	Proportion of blocks above cut-off	Standard Deviation	Mean	Coefficient of variation
400	259	68 %	890.00	953.52	0.9334
800	148	39 %	644.30	1235.06	0.5217

4.3.3 Semi-variograms Study

In order to investigate whether the sample data is exhibiting any form of anisotropy - that is major changes in the range or sill as direction changes - semi-variograms for the log-transformed data and corresponding indicator values for each selected cut-off were calculated in the four main directions, namely SE -NW (azimuth 135) E-W (azimuth 90), NE-SW (azimuth 45), and N-S,(azimuth 0) as shown in Appendices B and C. The over-all mineralization appears to be isotropy, that is not significantly different in the four directions. Subsequent discussions therefore on variogram modelling will deal only with the average variogram in all directions .

4.3.4 Lognormal Variograms

For lognormal kriging purposes, three parameter l· normal distribution was fitted to the samples with a large portion showing a reasonable symmetry of the transformed data (Figure 4.18) The apparent *J-shape* of the transformed distribution is attributed to the large additive constant relative to the lognormal estimator of the mean (*t* estimator), the small sample size and the large variance (Krige 1981). However, recent unpublished research by Sichel (Krige 1981) has shown that provided an additive constant is used which ensures reasonable symmetry of the transformed data, the *t* estimator will have negligible or only very small biases even where the underlying distribution is distinctly non-lognormal or even *J -shaped* (Krige 1981). The resulting omni-directional experimental semi-variogram which shows a reasonably well-behaved variogram was modelled using a spherical model (Figure 4.19) with a nugget effect of 0.24, a sill of 0.188 and a range of influence of 360m.

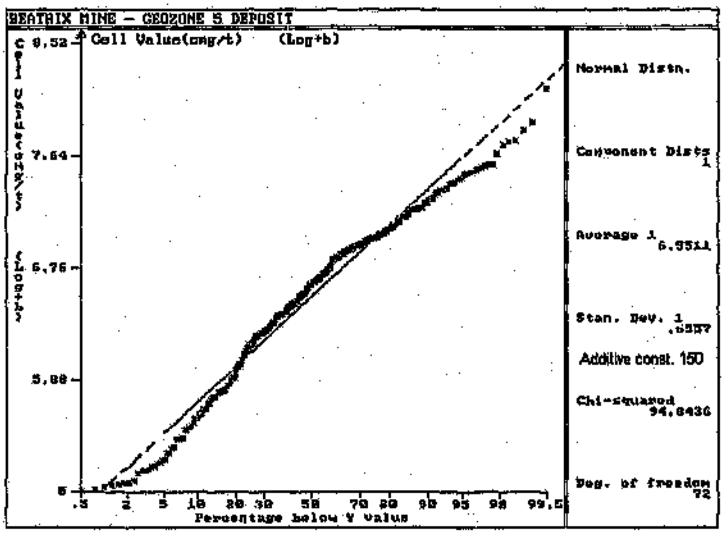

Figure 4.18 Three parameter lognormal distribution of regularised samples

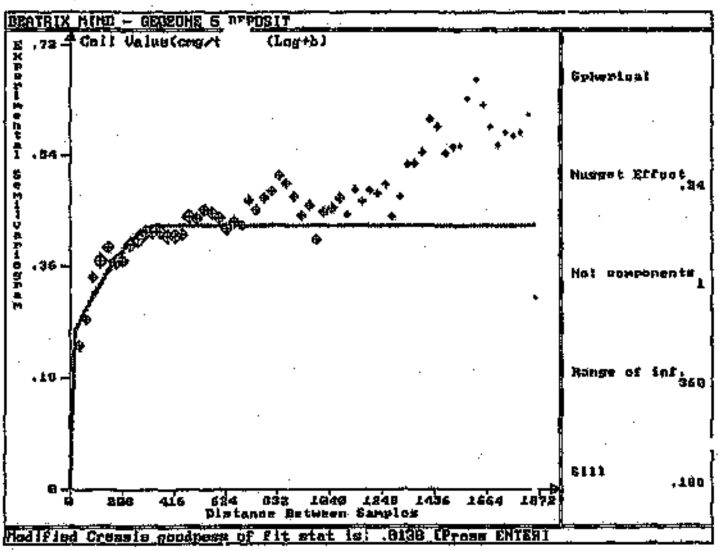

Figure 4.19 Three parameter Lognormal spherical semi-variogram model

4.3.5 Indicator Variograms

Indicator semi-variograms were calculated for each selected cut-off and modelled with a spherical model as shown in Figures 4.20a - 4.20b. Table 4.5 illustrates the indicator semi-variogram parameters for the selected cut-off grade.

Semi-variograms were also calculated for untransformed samples to cover the area under each selected cut-off condition to enable mean grade estimation for each block within the deposit. Appendix D illustrates the semi-variogram and cross validation statistics within each area of the cut-off and Table 4.6 shows a summary of the semi-variogram parameters.

Table 4.5 Indicator Semi-variogram parameters for each cut-off

Cut-off grade	Nugget Effect	Range	Sill
400	0.160	500	0.065
800	0.185	300	0.072

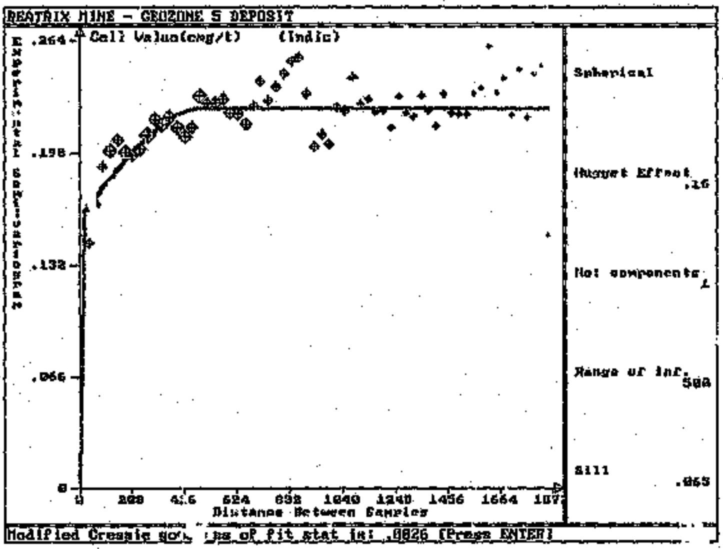

Figure 4.20a Indicator spherical semi-variogram at 400 cmg/t cut-off

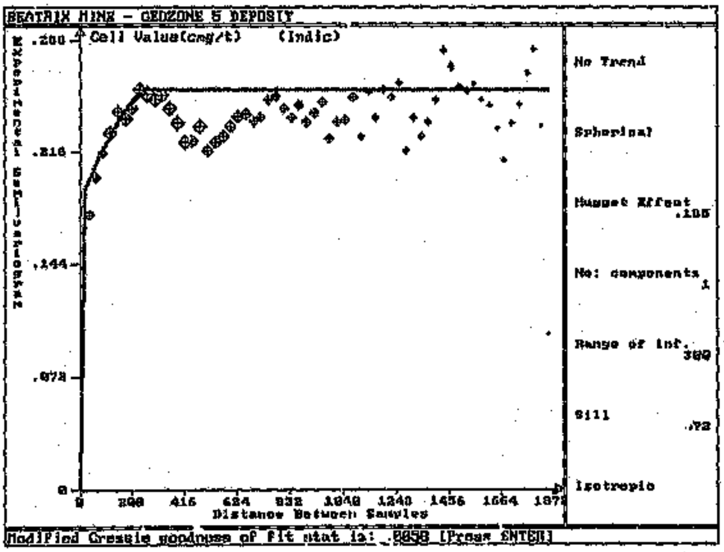

Figure 4.20b Indicator spherical semi-variogram at 800 cmg/t cut-off

Table 4.6 Semi-variogram parameters for gold accumulation for the area based on the selected cut-offs

Values under Cut-off grade	Nugget Effect	Range	Sill
≤400	10000	500	7000
400 - 800	10000	400	4000
≥800	120000	350	180000

4.3.6 Cross Validation

The credibility of the lognormal and indicator semi-variogram parameters was confirmed by the process of cross validation as described in section 2.4. Figures 4.21a to 4.21c illustrate the Z or error statistics at the selected cut-off grade for lognormal and indicator semi-variogram respectively.

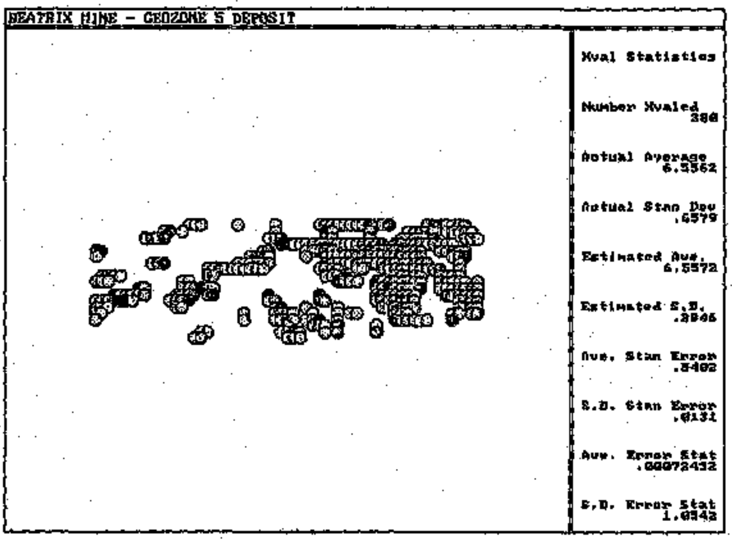

Figure 4.21a Three parameter lognormal cross validation statistics

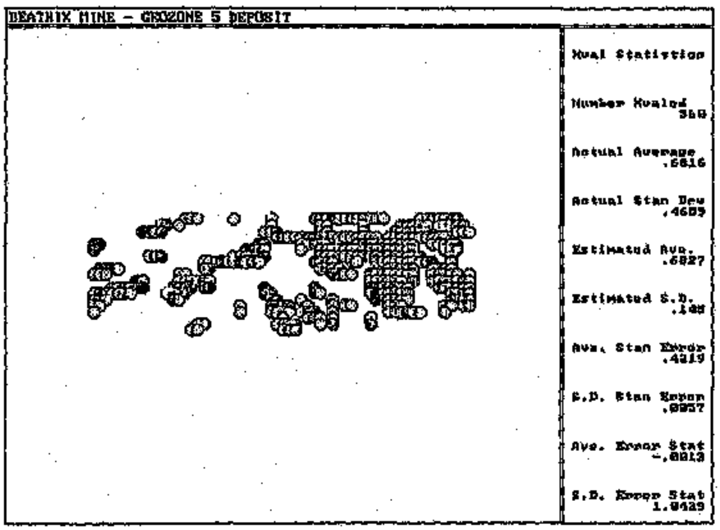

Figure 4.21b Indicator cross validation statistics at 400 cmg/t cut-off

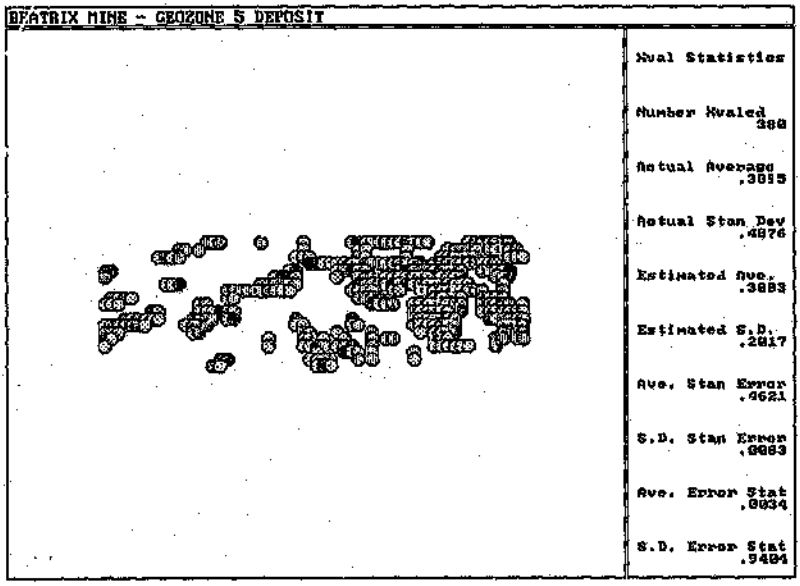

Figure 4.21c Indicator cross validation statistics at 800 cmg/t cut-off

4.3.7 Kriging

Using the lognormal and indicator semi-variogram parameters, kriging was carried out separately for each of the two methods under consideration in this study. Indicator kriging as already discussed in section 2.5 consists of carrying out ordinary kriging on the transformed indicator values (0,1) separately for each of the selected cut-offs. For each cut-off, a series of probabilities is computed from an indicator kriging system. An initial indicator kriging was performed on the transformed indicator samples, to obtain the probabilities of the deposit being mineralised for each indicator value. Kriging was also carried out on the raw data for the mean grade estimates for respective cut-off categories (for data ≤400 cmg/t, between 400 and 800 cmg/t and ≥800 cmg/t). In all cases, a 30 meter grid was used and the search window throughout the kriging procedure has been adjusted to the semi-variogram range. A table of back transformed lognormal kriging estimates, the indicator probabilities at each cut-off and the estimates of the mean grade for each cut-off class are shown in Appendix E. Based on the estimated probabilities, the final mean grade for respective 30m by 30m blocks is computed as follows:

$$g^* = (1-P_{400})\,g_1 + (P_{400} - P_{800})g_2 + (P_{800})g_3$$

where g^* is the mean grade for each 30m by 30m block, P_{400} and P_{800} are the probabilities for each 400 and 800 cmg/t cut-offs respectively and g_1, g_2 and g_3 are the mean grade for the various cut-off classes. An example of block mean grade determination is shown in Appendix F.

One of the difficulties arising in applying indicator kriging, or generally nonparametric geostatistical techniques, is the order relationship problem. Due to the use of a different variogram model for each cut-off grade, the generated probability figures may not be increasing with grade or they may even be negative or greater than 1. During this case study, a few minor order problems were created. However, for the purpose of the study these were eliminated from the data set .

4.3.8 Comparison of lognormal and indicator estimates

Several criteria for comparing estimation methods are described in the literature, such as the correlation between estimates and true values, the degree of smoothing achieved by the interpolation methods or the precision of the methods as measured by mean square error (MSE) or mean absolute error (MAE) (Marcotte & Asli (1995)). The two kriging methods under consideration in this study were compared by the correlation between estimated and true values, the mean absolute error and the mean square error criterion. The MAE and MSE generally incorporate the bias as well as the spread or variance of the error distribution, with MAE being more robust with respect to extreme values (Marcotte & Asli 1995). The MAE and the MSE are given by :

$$MAE = \frac{1}{n}\left[\sum_{i=1}^{n} |Z(x_i) - Z^*(x_i)|\right]$$

$$MSE = \frac{1}{n}\left[\sum_{i=1}^{n} ((Z(x_i) - Z^*(x_i))^2\right]$$

where $Z(x_i)$ is the actual estimate, $Z^*(x_i)$ is the estimator and n is the number of samples.

Table 4.7 Summary Statistics of Actual values versus Estimates of Lognormal and Indicator kriging .

	Lognormal Kriging	Indicator Kriging
Mean Absolute Error (MAE)	278.5341	305.7109
Mean Square Error (MSE)	133,825	145,308
Correlation Coefficient	0.5446	0.4011
Slope of the Linear Regression line	0.8397	0.8593

4.3.9 Discussion and Conclusion

The statistical analysis for estimation error in Table 4.7 indicates that lognormal kriging estimates have lower MAE and MSE compared to those associated with indicator kriging. These criteria sugges , that lognormal kriging will improve the quality of estimation of the gold grades. This is further confirmed by the scattergrams in Figures 4.22 and 4.23 which indicates a better correlation coefficient value between the actual grades and those estimated by lognormal kriging, even though the slope of the linear regression line of actual on estimate (Table 4.7) of the indicator technique shows slightly better results. Figure 4.24 shows a scattergram of the lognormal and indicator estimates. The high correlation coefficient of 0.94 indicates a good linear relationship between the two methods as a result of kriging.

In conclusion, the overall picture suggests that lognormal kriging will produce better estimates over indicator kriging within the Geozone 5 deposit.

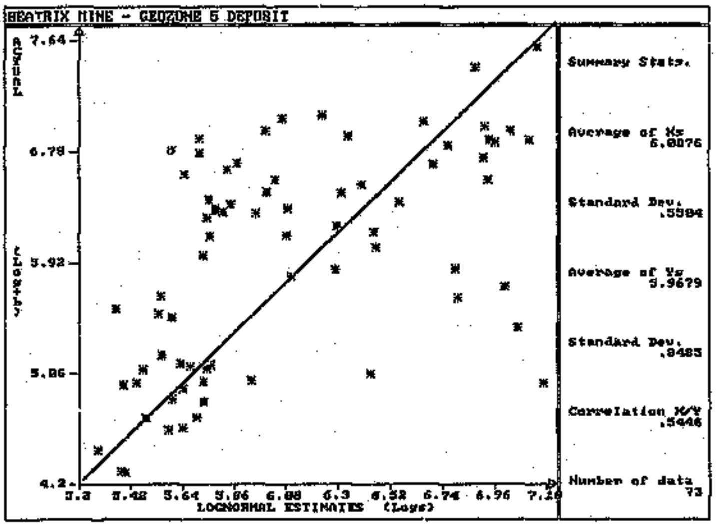

Figure 4.22 Scattergram of the log transformed Actual grades and log-estimates from lognormal kriging

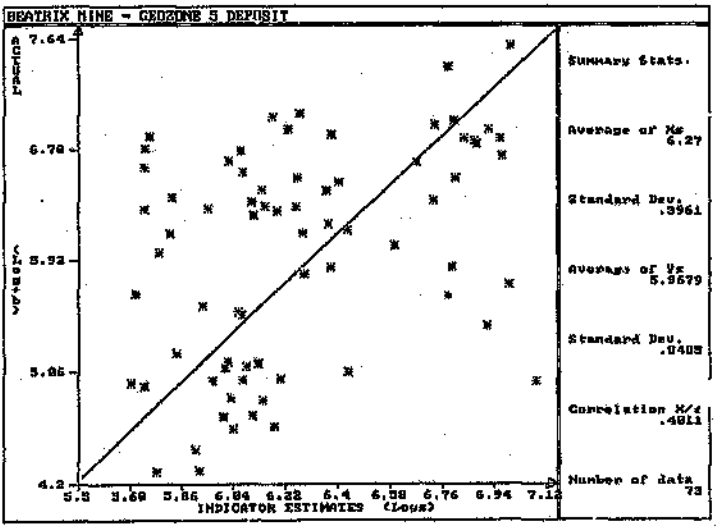

Figure 4.23 Scattergram of the log transformed Actual grades and estimates from indicator kriging

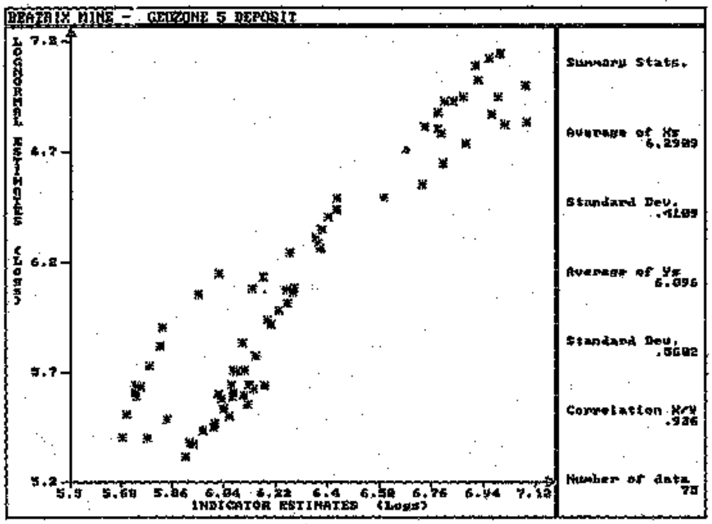

Figure 4.24 Scattergram of the log transformed Lognormal and Indicator estimates

4.4 Global and Local Estimation

Global estimation is commonly used at a very early stage in most studies to obtain some characteristics of the distribution of data values over the whole area of interest. This estimate must also include confidence intervals which will determine the point at which the *in situ* resources are sufficiently well estimated to proceed to the next stage of evaluation.

Global estimates are generally not useful for mine planning purposes; we usually require a complete set of local estimates at particular block sizes to give an idea of the spatial distribution of the *in situ* resources which is necessary for the evaluation of the recoverable reserves. In ore reserve calculations, kriging as described in section 2.5 provides the best estimates of local block means and variances for a specific panel or block size. Using the semi-variogram model parameters in Appendix G, lognormal kriging was carried out on a 30m by 30m block within the area under study in order to conform with block dimensions during mining. The 4790 sample points and the three borehole values (Appendix H) within the northern section of the deposit were used for the kriging process. The backtransformed kriged estimates of grade accumulation in each block and its associated errors are shown in Appendices I and J. It is evident from the maps in Appendices I and J that the northern section of the deposit will require additional drilling to provide realistic block estimates. As a result of the inadequate sample information in the northern section of the deposit, no attempt was made to provide global estimate and grade tonnage curves for the Geozone 5 deposit .

CHAPTER 5

CONCLUSION AND RECOMMENDATION

The main objective of this study within the Geozone 5 area at Beatrix Mine is to apply a number of geostatistical techniques to the available sample data and establish an efficient technique that would serve as a tool for grade estimation purposes.

An initial correlation analysis was carried out to establish whether there is any relationship between accumulation (which is the main regionalized variable for ore value measurements on the mine), the sample grade and the channel width. This was necessary, as there is the possibility of under estimation and / or over estimation of reserves in a situation where these variables do not correlate. Results from the analysis show that accumulation correlates very well with both the sample grade and the channel width.

The performances of two-geostatistical techniques, namely indicator and lognormal kriging, have been investigated and it has been established that lognormal kriging provides a more efficient geostatistical technique necessary for the evaluation of the Geozone 5 area. This was achieved by comparing kriged estimates of the two geostatistical techniques with actual sample values. The mean absolute error (MAE) and mean square error (MSE) criterion, and the correlation coefficient and the slope of regression between the kriged estimates and the actual values were used as the basis for this comparison. The MSE and the MAE criterion were used as they incorporate the bias as well as the spread or variance of the error distribution.

Kriged local estimates of 30m by 30 blocks have been estimated based on the lognormal semi-variogram range of 350 meters. An important significance of the range is that values of the regionalized variables cannot be extended usefully beyond 350 meters from the sample sites. This conclusion is of obvious importance in the estimates of grade in the northern section of the deposit where it is recommended that additional drilling is necessary to improve the grade estimates. No attempt was made to provide global estimates or generate grade tonnage curves for the Geozone 5 deposit as a result of inadequate sample information within the northern section of the deposit.

This study has demonstrated that geostatistical techniques could be employed for evaluation purposes within the Beatrix reef. A potent advantage of geostatistics that can be a useful guide to further development work is the ability to calculate the effects that additional information will have on error estimates. It must be emphasised that the technique of kriging in geostatistics is statistically optimum in the sense that the estimator is unbiased and has the minimum possible uncertainty (error variance) based on the available data. In other words, kriging involves not just the point prediction of an observation at a new location but also, and perhaps more importantly, the uncertainty (i.e., prediction error) associated with it.

The uncertainty or prediction error associated with the distribution can be quantified by calculating confidence limits of the estimated mean grade. It also enables the generation of grade-tonnage curves for economically optimal grades and tonnages based on the prevailing market conditions. This is achieved by applying cut-offs or pay-limits to determine how much of the deposit could be mined and at what average grade of the mineable proportions.

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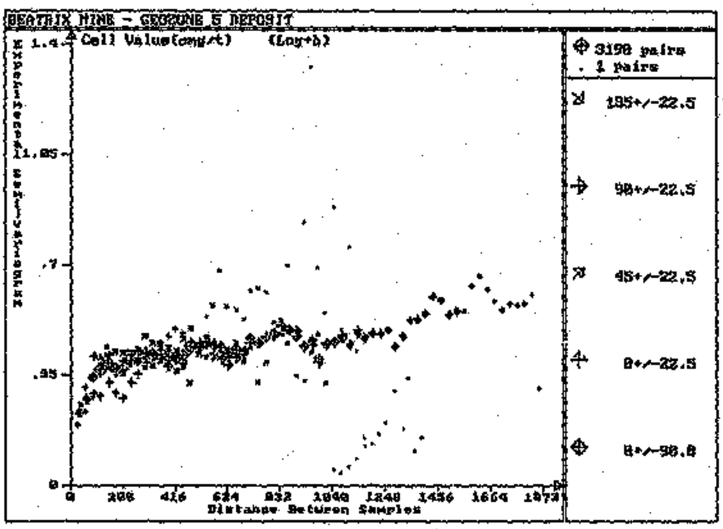
APPENDIX A: TYPES OF SEMI-VARIOGRAM MODELS

MODEL TYPE	EQUATION	COMMENT
Spherical	$\gamma(h) = C_0 + C\left[\frac{3h}{2a} - \frac{h^3}{2a^3}\right]$ @ $0<h<a$ $= C_0 + C$ @ $h > a$	This the most frequent model type encountered in mining practice and it often accompanied by a nugget effect.
Linear	$\gamma(h) = Ah + B$	The simplest model without a range.
De Wijsian	$\gamma(h) = A\ln(h) + B$	An extension of the linear model and its encountered in cases where there is no such thing as a range of dependence.
Exponential	$\gamma(h) = C_0 + C[1 - \exp(-h/a)]$	Almost similar to the spherical model except that it reaches its sill asymptotically and much slower than the spherical model. This model is rare in the mineral deposits.
Guassian	$\gamma(h) = C_0 + C[1 - \exp(-h^2/a^2)]$	The curve is parabolic near the origin and the tangent is horizontal at the origin.
Parabolic	$\gamma(h) = \frac{1}{2}(a^2h^2)$	Observed when there is a linear drift. Its regular behaviour at the origin is seldom found in mining practice.
Hole-Effect	$\gamma(h) = C[1-(\sin(ah)/ah]$	This model has a periodic behaviour and is observed when there is a succession of rich and poor zones.
The symbols stand for the following: C_0 = nugget variance, C = transition variance, h = distance between sample pairs a = range, A and B = constants, and S^2 = statistical variance of sample population		

(Sources ; David (1977), Journel & Huijbregts (1978))

APPENDIX B: LOGNORMAL SEMI-VARIOGRAM FOR FOUR MAIN DIRECTIONS



APPENDIX C: INDICATOR SEMI-VARIOGRAM FOR MAIN DIRECTIONS

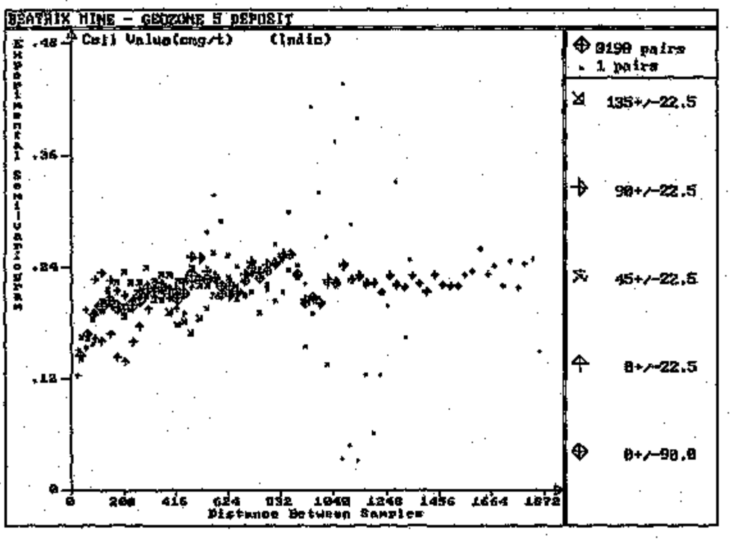


Figure C1 400 cmg/t cut-off

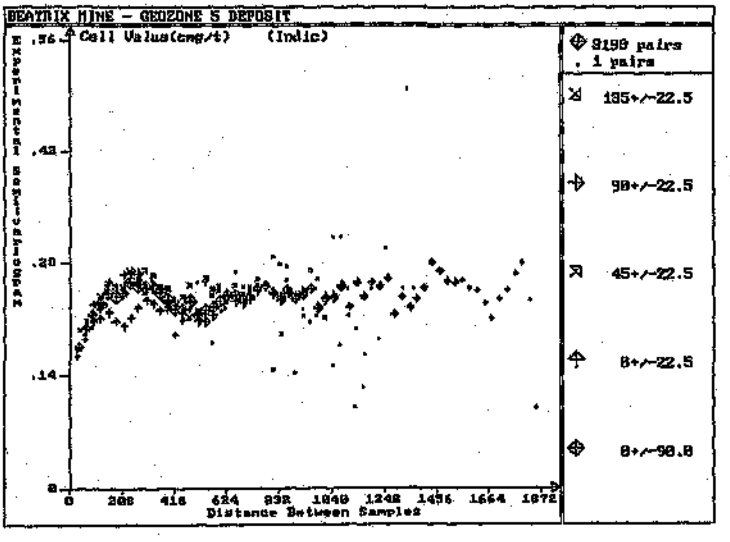


Figure C2 800 cmg/t cut-off

APPENDIX D: SEMI-VARIOGRAM PARAMETERS AND CROSS VALIDATION STATISTICS FOR AREAS UNDER THE SELECTED INDICATOR CUT-OFFS

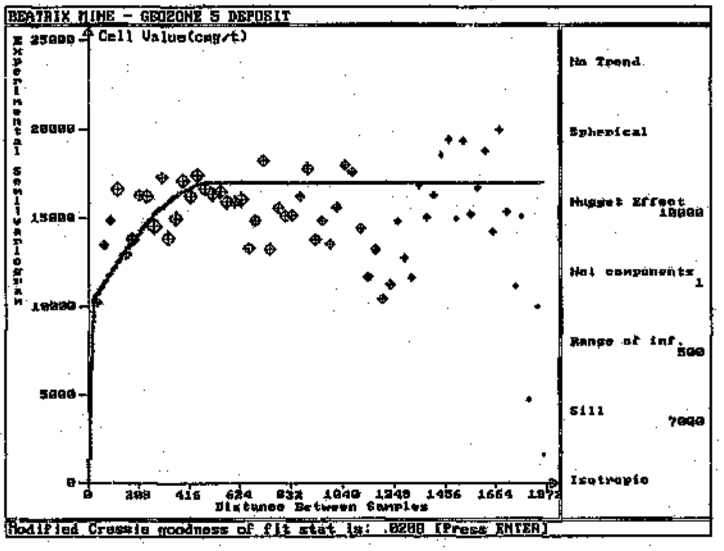


Figure D1 Spherical semi-variogram ≤ 400 cmg/t cut-off

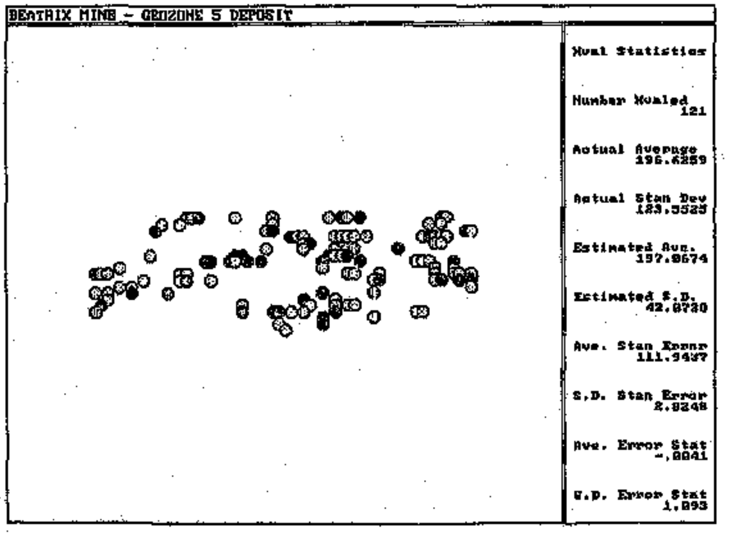


Figure D2 Cross validation statistics ≤ 400 cmg/t cut-off

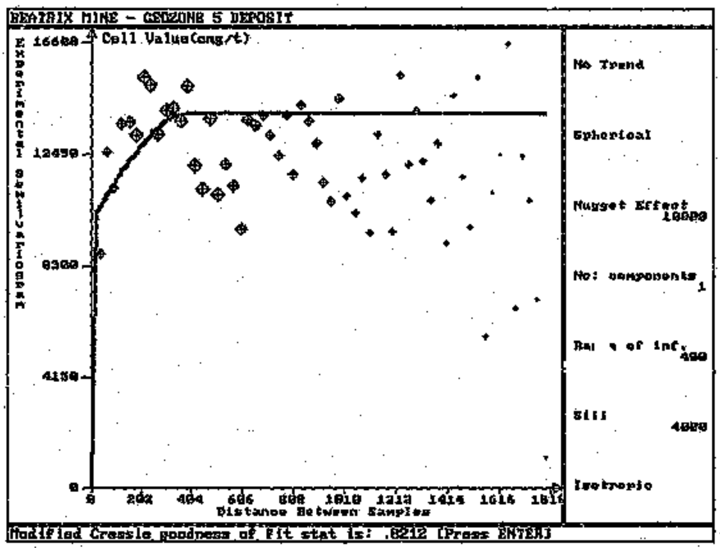


Figure D3 Spherical semi-variogram 400 - 800 cmg/t cut-off

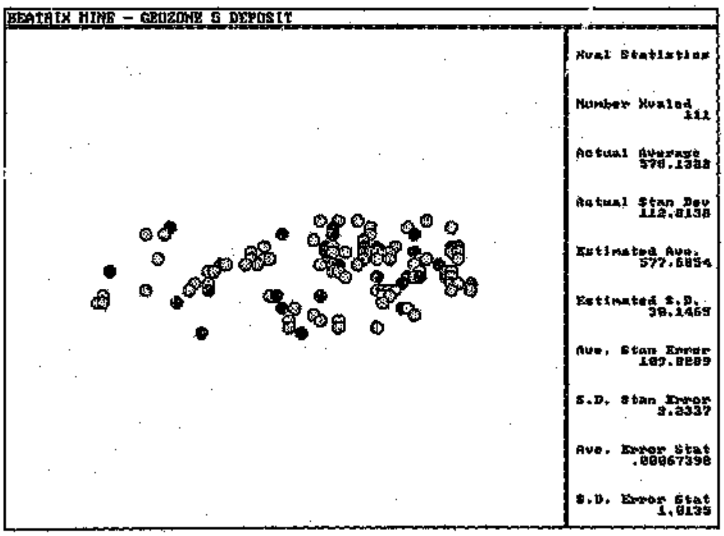


Figure D4 Cross validation statistics 400 - 800 cmg/t cut-off

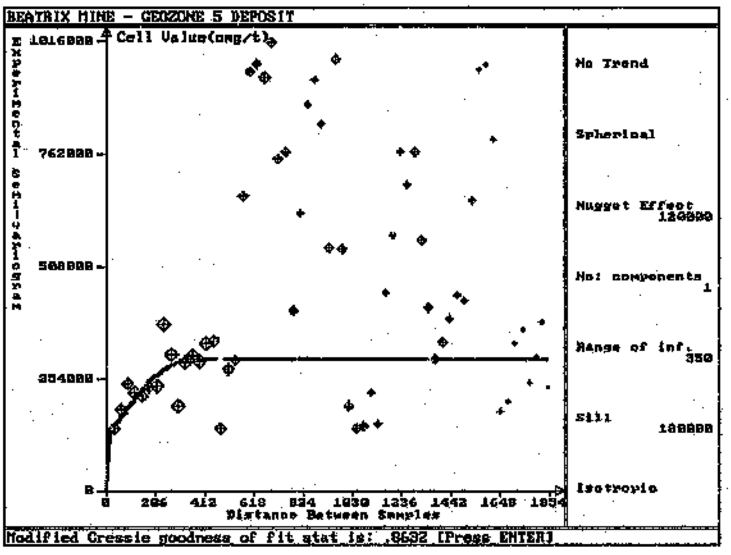


Figure D5 Spherical semi-variogram ≥ 800 cmg/t cut-off

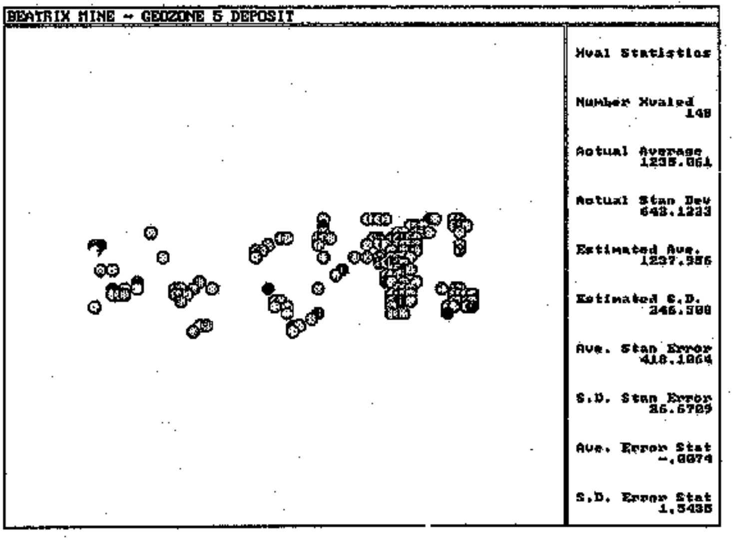


Figure D6 Cross validation statistics ≥ 800 cmg/t cut-off

APPENDIX E: ESTIMATES FOR INDICATOR AND LOGNORMAL KRIGING

EASTING	NORTHING	INDICATOR KRIGING PROBABILITIES AT EACH CUT-OFF		LOCAL MEAN ESTIMATES UNDER EACH CUT-OFF CONDITION			ACTUAL GRADES	INDICATOR KRIGING ESTIMATES	LOGNORMAL KRIGING ESTIMATES
X	Y	400	800	≤400	400-800	≥800			
25995.4	22322.1	0.7305	0.5137	168.869	586.309	1989.719	77.520	1194.741	1300.203
26025.4	22322.1	0.7264	0.4787	167.787	584.724	1870.040	242.125	1085.931	1103.782
26055.4	22322.1	0.7048	0.4922	169.533	582.868	1802.295	775.550	1061.053	1006.247
26055.4	22352.1	0.6767	0.4181	169.960	578.274	1530.983	1006.450	844.594	1012.666
26085.4	22352.1	0.6643	0.4333	172.205	576.679	1809.424	861.980	975.045	866.236
26115.4	22352.1	0.6520	0.4066	172.148	575.155	1723.417	1045.081	901.792	783.240
26145.4	22352.1	0.6289	0.3836	170.049	597.866	1635.266	531.769	8[illegible]7.050	707.095
26175.4	22352.1	0.6169	0.3213	155.123	596.260	1550.097	350.678	733.728	639.264
26175.4	22382.1	0.5910	0.3136	163.681	599.440	1244.422	403.850	623.481	633.664
26205.4	22382.1	0.5606	0.2937	161.419	599.545	1226.979	926.240	591.310	568.382
26235.4	22382.1	0.5399	0.2295	161.148	585.326	1189.415	1102.387	528.800	511.218
26265.4	22382.1	0.5084	0.1342	168.832	581.094	1365.196	1078.713	483.652	432.627
26295.4	22382.1	0.4849	0.1165	173.623	584.336	1375.104	576.350	464.902	403.940
26295.4	22412.1	0.4919	0.1203	179.333	558.177	959.952	739.625	414.020	357.138
26325.4	22412.1	0.4246	0.1067	182.726	554.765	965.999	491.769	384.572	323.937
26355.4	22412.1	0.3433	0.0827	173.295	555.158	972.627	541.793	338.913	316.051
26385.4	22412.1	0.3402	0.0751	179.461	547.132	979.221	390.714	336.993	317.589
26415.4	22442.1	0.2906	0.0581	181.707	550.378	988.071	909.343	314.273	303.285
26445.4	22442.1	0.2654	0.0576	188.080	543.257	993.055	483.640	308.252	334.854
26475.4	22442.1	0.2430	0.0809	184.500	541.379	997.406	698.644	308.114	341.676

26505.4	22322.1	0.2484	0.0541	216.777	559.035	984.020	327.600	324.785	308.258
26535.4	22352.1	0.2161	0.0723	201.169	559.725	980.497	71.780	309.075	281.538
26595.4	22322.1	0.2603	0.0465	193.223	553.880	1029.316	824.467	309.210	268.740
26595.4	22352.1	0.2478	0.0440	184.911	557.398	1055.926	219.420	299.149	257.504
26655.4	22352.1	0.2639	0.0655	158.443	546.353	1058.178	75.530	294.337	219.084
26715.4	22502.1	0.2946	0.1570	142.108	517.371	1100.447	113.433	344.204	258.130
26775.4	22412.1	0.3475	0.1078	110.726	555.143	1080.334	3.400	321.777	220.745
26805.4	22412.1	0.3930	0.1603	115.507	561.628	1078.055	4.500	373.616	217.261
26805.4	22442.1	0.3978	0.2102	122.320	559.692	1080.288	42.300	405.736	241.803
26805.4	22472.1	0.3870	0.2347	136.074	565.389	1083.161	31.990	420.062	265.675
26835.4	22352.1	0.3895	0.1668	103.585	564.921	1073.351	17.289	368.081	196.831
26835.4	22382.1	0.4096	0.1642	111.140	556.130	1068.988	193.735	377.619	212.220
26835.4	22412.1	0.4342	0.1609	120.017	546.092	1081.780	77.430	391.211	231.305
26835.4	22442.1	0.4468	0.2141	130.006	544.019.	1078.740	182.400	429.471	254.183
26835.4	22472.1	0.4594	0.2506	129.337	560.234	1079.037	98.325	457.303	291.388
26835.4	22502.1	0.4751	0.2282	136.079	573.399	1112.712	57.185	466.897	308.318
26865.4	22382.1	0.4508	0.1852	110.688	555.905	1076.772	93.781	407.857	237.924
26865.4	22472.1	0.4597	0.2268	136.647	541.832	1101.774	43.150	449.905	300.486
26865.4	22502.1	0.4554	0.2319	147.119	553.660	1105.609	99.800	460.255	318.942
26895.4	22322.1	0.4652	0.2197	109.143	577.227	1296.758	33.520	484.977	282.318
26895.4	22352.1	0.4676	0.2103	111.023	566.669	1087.465	176.156	434.121	269.641
26895.4	22382.1	0.4570	0.1907	116.105	552.247	1084.371	59.707	416.898	268.930
26895.4	22412.1	0.4551	0.1833	123.263	546.305	1077.843	100.310	413.221	279.448
26895.4	22442.1	0.4428	0.2314	132.447	543.422	1068.228	676.650	435.867	284.962
26895.4	22472.1	0.4428	0.2031	139.198	542.651	1106.300	806.990	432.324	303.516
26925.4	22352.1	0.5038	0.2365	116.743	572.471	1094.599	499.474	469.822	324.907
26925.4	22412.1	0.4647	0.2156	130.031	549.474	1085.347	95.200	440.481	312.282
26925.4	22442.1	0.4651	0.2015	138.966	544.825	1075.495	78.700	434.661	308.842
26925.4	22472.1	0.4630	0.2141	149.115	538.364	1111.367	460.050	452.017	313.171
26955.4	22352.1	0.5145	0.2560	124.004	573.447	1123.878	80.940	496.153	379.467
26955.4	22442.1	0.5048	0.1898	136.157	548.830	1101.907	522.522	449.448	346.743
26985.4	22322.1	0.5609	0.2963	132.570	564.714	1114.627	266.150	537.899	449.007

26985.4	22352.1	0.5592	0.2820	134.312	561.629	1139.876	393.636	536.333	440.452
26985.4	22382.1	0.5462	0.2677	137.387	547.346	1157.445	641.575	524.630	420.769
26985.4	22412.1	0.5207	0.2374	141.900	553.959	1196.924	969.219	509.099	403.103
26985.4	22442.1	0.5040	0.2199	145.973	555.186	1179.946	479.733	489.601	386.342
27015.4	22322.1	0.6200	0.3250	143.234	554.621	1140.842	284.369	588.816	539.698
27015.4	22442.1	0.5294	0.2437	157.898	552.286	1197.161	497.760	523.843	441.598
27075.4	22352.1	0.6481	0.3225	168.554	558.552	1189.974	88.470	624.945	625.861
27075.4	22382.1	0.6378	0.2937	173.025	553.211	1201.113	619.130	605.796	602.096
27075.4	22412.1	0.5979	0.2760	176.189	552.202	1216.134	427.070	584.253	543.338
27075.4	22442.1	0.6178	0.2530	179.856	546.715	1230.417	572.267	579.478	552.410
27225.4	22442.1	0.7207	0.4320	225.502	559.705	1312.236	736.353	791.455	815.388
27285.4	22352.1	0.7317	0.4591	241.385	557.357	1476.671	289.650	894.639	896.640
27285.4	22382.1	0.7395	0.4665	242.016	555.832	1438.356	216.725	885.780	906.179
27315.4	22442.1	0.7807	0.5089	237.211	563.082	1334.104	1631.567	883.992	974.690
27345.4	22412.1	0.8241	0.5721	245.391	576.914	1359.791	885.275	966.483	1055.826
27345.4	22442.1	0.7948	0.5747	242.289	574.761	1319.555	902.600	934.571	1030.289
27375.4	22412.1	0.8479	0.6497	249.653	596.991	1322.109	975.000	1015.270	1130.099
27405.4	22322.1	0.9485	0.7563	251.235	624.817	1275.588	1937.440	1097.756	1259.412
27435.4	22322.1	0.9688	0.7574	251.531	631.770	1207.124	899.450	1055.680	1223.724
27465.4	22322.1	0.9485	0.7474	251.635	640.512	1160.438	196.529	1009.077	1165.612
27525.4	22322.1	0.8262	0.6384	249.397	641.936	1160.302	641.973	904.638	1024.514

APPENDIX F: EXAMPLE OF MEAN ESTIMATE DETERMINATION FOR INDICATOR KRIGING FOR VARIOUS CUT-OFF CLASSES

COORDINATES : Easting - 25995.4 and Northing - 22322.1

Cut-off grade	400	800	
Kriged Probability	0.7305	0.5137	
Actual Probability	0.2695	0.2168	0.5137
Local Class mean	168.869	586.309	1989.719
Class contribution(g_i)	45.5102	127.1118	1022.1187
Mean Grade	**Sum(g_i) = 1194.7407 cmg/t**		

APPENDIX G: LOGNORMAL SPHERICAL SEMI-VARIOGRAM MODEL AND CROSS VALIDATION STATISTICS FOR THE WHOLE DEPOSIT

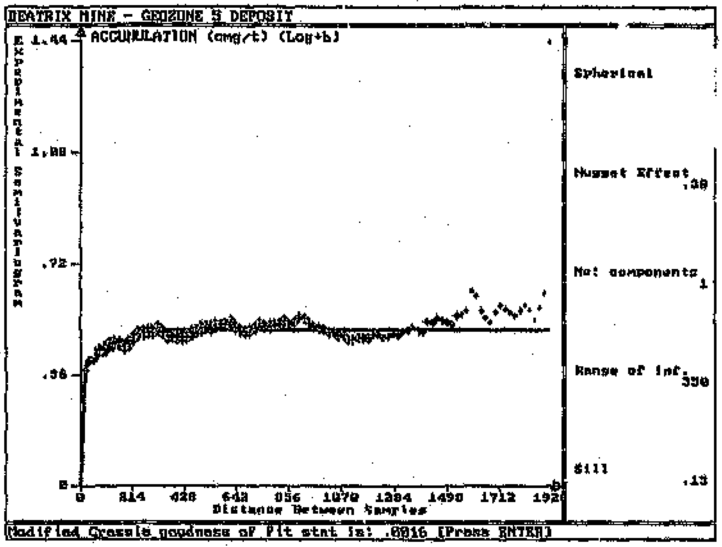


Figure G1 Lognormal semi-variogram model

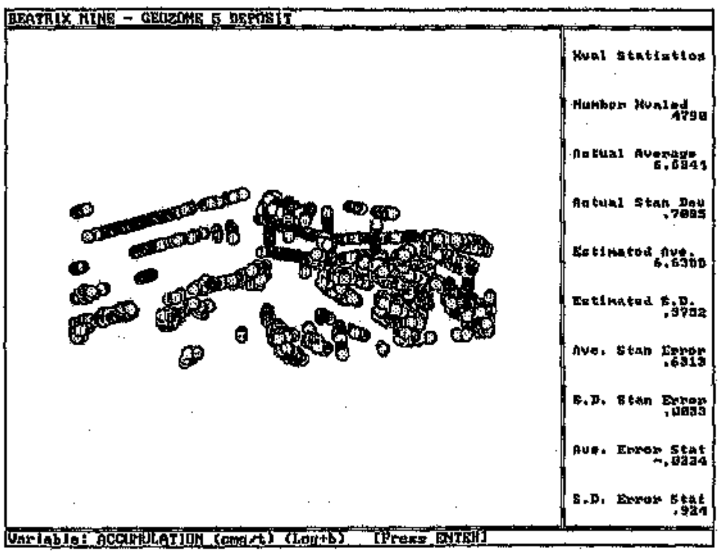
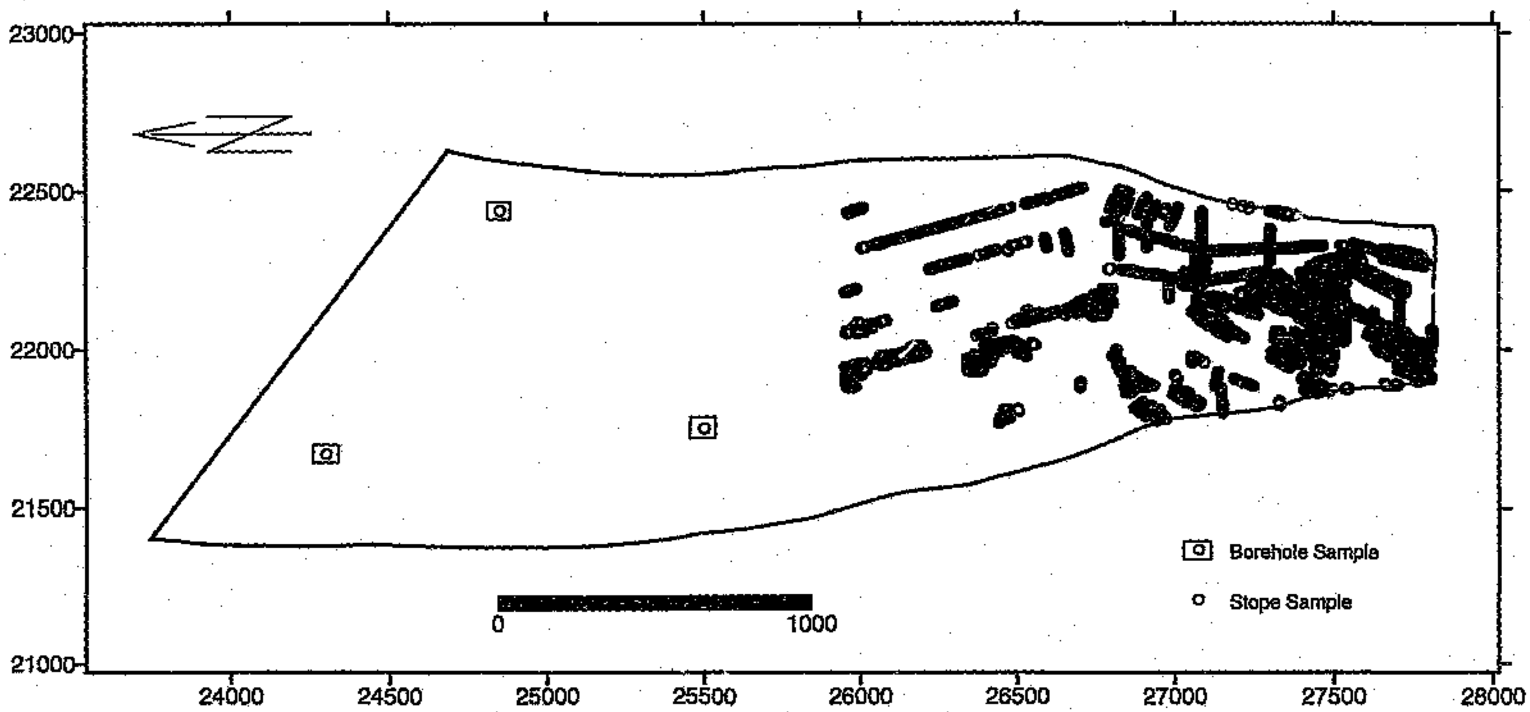
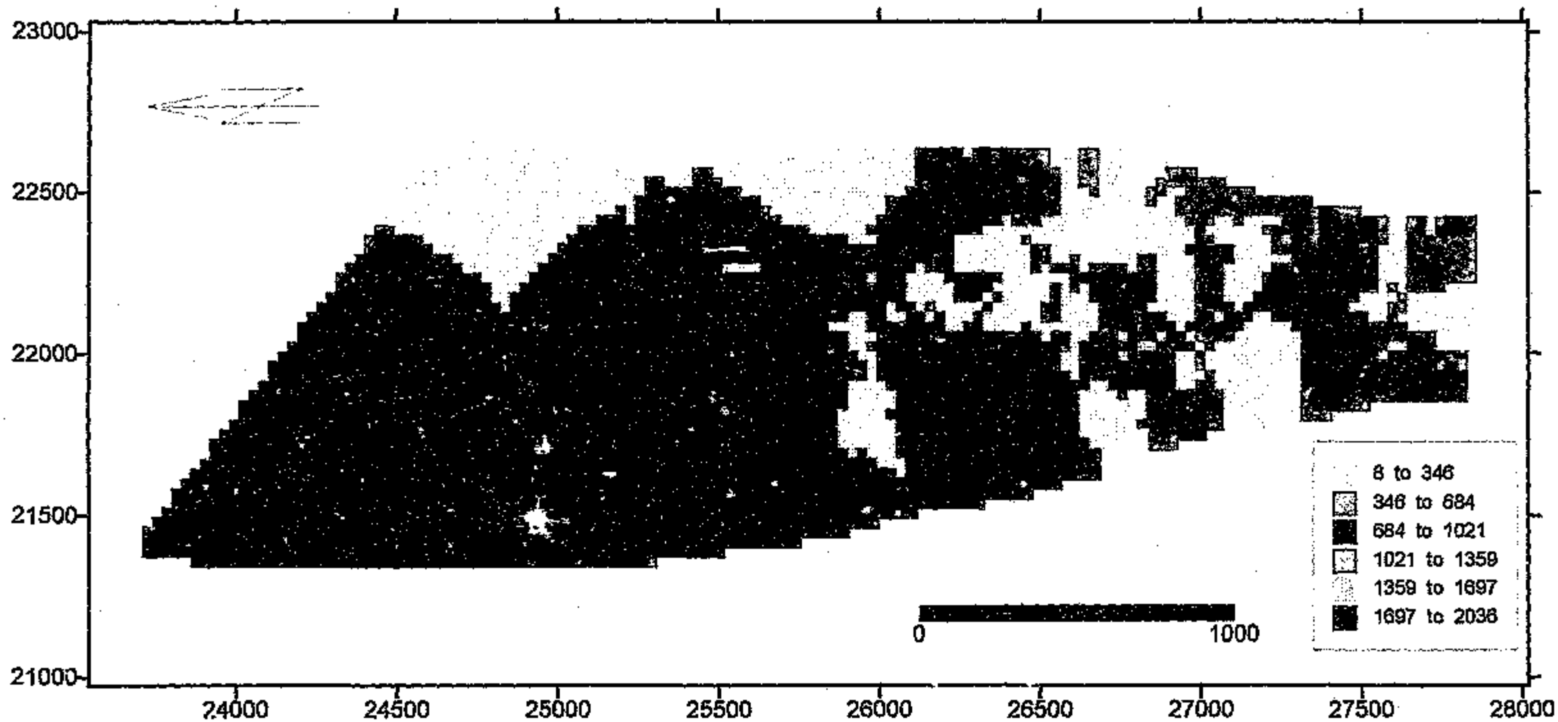


Figure G2 Cross validation statistics for Lognormal semi-variogram model

APPENDIX H LOCATION OF BOREHOLE AND STOPE SAMPLES FOR GEOZONE 5 DEPOSIT



APPENDIX I : BACKTRANSFORMED 30M BY 30M BLOCK ESTIMATES



APPENDIX J   STANDARD ERRORS OF BACKTRANSFORMED 30M BY 30M BLOCK ESTIMATES