DIGITAL COMPUTER SOLUTION OF SPECIAL ELECTROSTATIC AND ELASTOSTATIC PROBLEMS WITH APPLICATIONS TO THE MINING OF TABULAR DEPOSITS

BRIAN COHEN

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I hereby declare that this dissertation entitled "Digital Computer Solution of Special Electrostatic and Elastostatic Problems with Applications to the Mining of Tabular Deposits" is my own work and has not been submitted to any other university.

Brian Cohen

BRIAN COHEN

ABSTRACT

Problems arising with established methods of solution for the stress and displacement fields surrounding mining excavation are reviewed, and the feasibility of a novel digital computer solution to the electrostatic analogy of the tabular mining situation is investigated.

This technique has proved reliable in terms of convergence, and accurate. However, applying the basic method to a full-scale practical problem becomes too time-consuming even on today's computers. Various remedies have been investigated and employed and, at the expense of a small loss in accuracy, have reduced computing times to acceptable limits. These investigations have resulted in the development of the MINSIM I system which has proved itself as a valuable tool to the South African gold mining industry.

The simplifications inherent in MINSIM I restrict its application to relatively deep mines in which surface effects are negligible and reef inclinations of less than 25^{o}. As a result the situation on the Zambian copper-belt cannot be studied with any degree of accuracy. The electric analogues described in the literature suffer from the same limitation. There is, therefore, scope for a system similar to MINSIM I which can cater for arbitrarily steep and deep tabular excavations.

Analysis shows that such a system requires from three to five times as much computational effort as MINSIM I. Additional time-saving techniques have been developed to cope with the situation.

MINSIM II - the latest version of the system - incorporates all these results. Many user oriented features have been added in order to facilitate the solution of practical problems.

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CHAPTER 1

INTRODUCTION

1.1 Background and History

In a mining based country such as South Africa, one would expect there to be a great deal of research into Mining Rock Mechanics, a subject dealing with the study of the stresses and displacements induced in rock by mining. However, this has only been the case in recent years.

In the early days, mines were merely mined. During the first decade of this century, when mining had reached a few hundred feet in depth, it was thought that earth tremors were associated with the mining. In 1908, the Ophirton Earth Tremors Committee [1] recommended that pillars be replaced by waste packs. In 1915, the Witwatersrand Earth Tremors Committee was appointed. This committee found that rockbursts due to crushing of pillars were the major cause of tremors. There was also settling of overlying strata. They also recommended artificial support. Dangerous underground collapses were becoming a problem even at such relatively shallow depths as 2 000 ft. In 1924, the Witwatersrand Rockburst Committee was appointed. It recommended many specific techniques to reduce the hazard such as leaving large protective pillars around vertical shafts.

All these suggestions were largely intuitive guesses or judgements made by mining engineers experienced in the "art of mining" and inevitably many mistakes were made. The excavation plan was drawn up almost entirely on the basis of easiest and maximum production with the minimum of precautions. Even to this day, many problems associated with rockbursts still require solution.

However, it is established that rockbursts originate in highly stressed areas releasing the potential and strain energy of the rock. On the assumption that one is dealing with a large elastic body, and if the excavated configurations are simple in shape, for example, a long slit or a flat circular area, analysis of the regions of high stress can be achieved in closed form and, with a little experience, can be identified by inspection. But, the excavation is usually complex, there being as a rule, many interconnected irregular shapes and "feelers" progressing into the intact regions.

The relevant description of gold-mining conditions is as follows :

(a) The gold-bearing reefs are usually tabular;

(b) The mine openings used for the extraction of the reef are also tabular, i.e. their dimensions in the plane of the reef are orders of magnitude greater than that perpendicular to it.

(c) In practice, the mine lay-outs usually consist of several excavations of complex shape in one plane.

Conditions (a) and (b) simplify the geometry of the problem and condition (c) indicates that a practical method of solution must allow analysis of irregular mining geometries.

It is desirable then that some form of analytic solution should be sought or model be constructed and used to solve these stress problems. It is the aim of this dissertation to show that such problems are computable to a satisfactory degree of accuracy.

1.2 The Problem

Beginning in 1949, Rand Mines Limited, and subsequently the mining industry as a whole, sponsored rock mechanics research. It had been shown by Ryder and Officer [(2)], Leeman [(3)], Ortlepp & Cook [(4)], among others, that the rocks of the Wutwatersrand System could be considered as an elastic continuum. Thus, their behaviour under load could be adequately determined by calculations based on elastic theory. This basis was a good first approach. What is ignored is the fact that in the vicinity of abutments, the rocks are usually fractured. This "fractured zone" is clearly not an elastic phenomenon. Also the possible time dependency of displacement and stress distributions is neglected. These effects have been observed to be small by comparison with the elastic response.

The fundamental assumptions, implicit in the applications of linear elasticity are: [(5)]

(a) rock behaves according to linear stress-strain relationships;

(b) the strains produced by mining are infinitesimal;

(c) the rock bodies, within the requirements of individual models, are continuous, which in practice means that the dimensions of distances between discontinuities of the material are much smaller than the smallest representative linear dimension occurring in the problem under consideration.

In describing the situation, it is convenient to divide the rock mass into two parts, separated by the reef plane. These parts are referred to as the hanging wall (roof) and footwall (floor), depending on whether they are above or below the reef.

For ease of reference, the regions of mining are collectively referred to as A. (see figure 1.1)

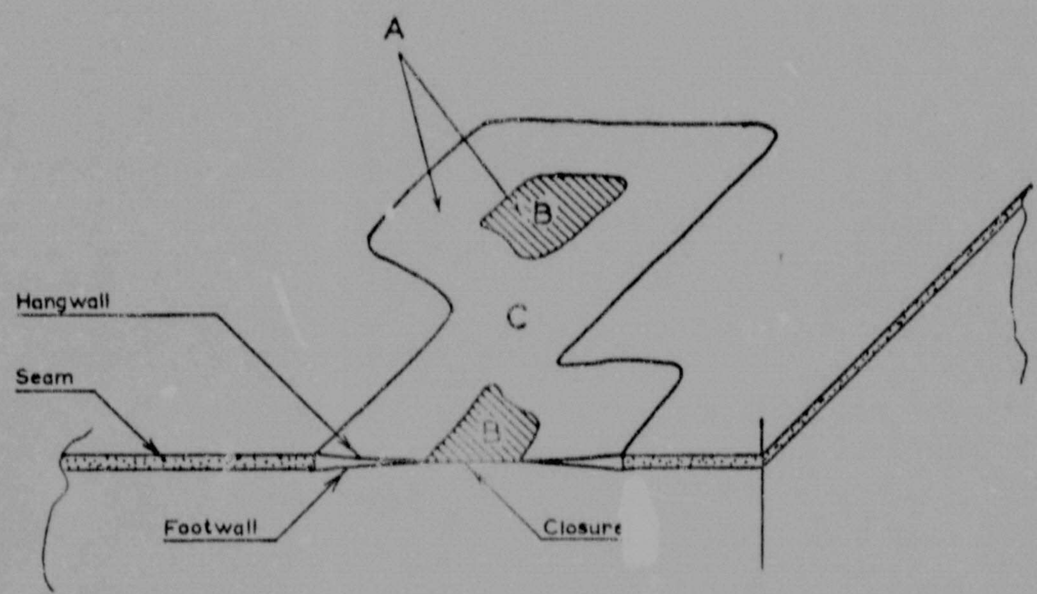

Figure 1.1 Model of a Tabular Excavation

This may consist of several excavations spread over a finite area. Areas in zone A might be sufficiently large to establish contact between the hanging - and footwall. The zone of contact is referred to as B and the remainder of A as C. The determination of the boundary between zones B and C is part of the problem.

The boundary conditions must be specified on four planes - the surface plane, the plane at infinite depth and the planes of the hanging - and footwall.

Thus, this is a boundary value problem and, with the assumption of an elastic medium, this effectively resolves itself as the solution of Laplace's equations. The solution can be expressed in terms of the distribution of relative displacements in the mined area A.

In 1963, Salamon [5] showed that the induced stress and displacements at any point in the three-dimensional medium could be calculated by an integration method, if the elastic convergence in the stoping excavations were known. Again, because of the limitation that irregular shapes were necessarily being created, numerical techniques of integration had to be used. The aim of this analysis is the calculation of stresses and displacements anywhere in the medium in the circumstances involved.

1.3 Solution of the Problem

1.3.1 Analogues

1.3.1.1 The Electrolytic Tank

Salamon [6,7] showed that an electrical analogue can be constructed which determines the distribution of displacements on A in terms of a potential distribution V, but the analogue cannot be used conveniently to measure the distribution outside this plane. However, as mentioned before, a function describing the full solution can be generated in the form of an integral over the solution on the boundary.

It is well known that the potential V, associated with steady state current flow in a homogeneous, continuous medium satisfies the Laplace equation [22]. The continuous medium analogue or electrolytic tank was thus built and is described in detail by Salamon, Ryder and Ortlepp [8].

The tank consists basically of two electrodes forming the upper and lower surfaces of an electrolyte (tap-water) in a non-conductive tank. The lower electrodes, which may be flat or curved, is energised by a voltage, V_I = 10V, 1000Hz relative to the upper electrode which is earthed. The latter electrode consists of a sheet of "Perspex" which has been sprayed with a conductive paint over the area representing the unmined portion of the reef leaving the area of mining, A, non-conductive. A is then punctured with a grid or matrix of holes through which an electrode may be pushed in order to measure the potential at that grid point. If a potential in A exceeds a certain V_m, corresponding to total closure in the mined portion - Zone B, an estimated portion of the upper electrode is also sprayed with conducting paint and that area is maintained at a constant voltage, V_m.

From the voltage measurements in region C, the corresponding convergence can be found. Some experimentation is required to attain the correct boundary shape between B and C. Finally, a set of surface integrals over the reef plane distribution will yield the stresses and displacements at any point in the medium. Exhaustive tests were carried out with this analogue proving its effectiveness.

1.3.1.2 The Electrical Resistance Analogue

The potentials at nodal points of a resistance network satisfy the finite-difference equivalent of the Laplace equation. The conductivity of the electrolyte is replaced by that of an orthogonal net of resistors. This form of analogue is described fully by Cook and Schumann[10]. Their first analogue consisted of a resistance network occupying a rectangular prism equivalent to 62 x 42 x 38 cubes. One of the larger faces represents the reef plane. Nodes of an area of 50 x 30 cubes can be short-circuited or open-circuited individually by means of plugs in holes at the nodes. Open-circuit potentials and closed-circuit currents can be individually measured. A D.C. potential of 10V is used. In order to represent region B of closure, corresponding plugs are connected to a fixed potential.

Potentials and currents thus measured are converted via suitable scale factors to relative displacement components of the hanging-and footwall (convergences) and the stress components on the plane of the reef.

The rate at which energy is released - another quantity found to be useful in the study of rockbursts, is approximately half the product of the reef plane stress before mining a particular 'node' and the relative displacement across the same node immediately after mining.

1.3.2 Digital Computer Solution

Based on a numerical method proposed by Deist[11], a computer system to handle the problem was implemented in 1968. A description of this system and its underlying numerical methods forms the bulk of this dissertation. Qualitatively this system may be introduced as follows :

In terms of the solution to Laplace's differential equation, the boundary conditions for the South African gold mining situation, i.e. tabular excavations, are :-

(a) The potential in the intact areas is zero;

(b) The normal derivative of the potential on the reef plane in the open mined areas is proportional to normal virgin stresses caused by the weight of the overlying strata. The derivative of the potential is usually referred to as the field.

(c) The potential over the closed mined areas is constant and proportional to the stoping width. This condition involves a complication in that the extent of "total closure" forms part of the solution.

As will be shown later, employing the classical Green functions for a half space, the three-dimensional problem is reduced to a quasi-two-dimensional one. A Green function has the characteristic that, while satisfying Laplace's equation, it allows one to specify an arbitrary potential over an infinitesimal region of the boundary without having any influence over the potential over the rest of the boundary. This function, henceforth referred to as the V-function exists for an infinite half-space.

A similar function, the E-function, has the same properties with regard to the normal field on the reef plane.

By imposing a mesh of squares on the reef plane such that each square represents an infinitesimal area, then a Green function is to be associated with each square so that the overall boundary conditions are met.

Suppose one satisfies the boundary condition (b), above, by associating an E-function with every square in the mined areas. This superposition then results in zero fields but finite potentials on the reef plane in the intact and totally closed areas. But boundary conditions (a) and (c) impose certain definite potentials over these areas and so a compensating set of V-functions is imposed on the intact and closed areas to restore those potentials to the real boundary conditions.

While the V-functions have no effect on the potentials in the open mined areas, they do affect the initially imposed fields. Thus it becomes necessary to adjust the E-functions to restore the correct field boundary conditions .... and so on.

Thus an iterative method inherent in the use of the above function evolves which, if it converges, gives a solution that can be made as accurate as computing time permits.

Complications arise in that the closure boundary has to be adjusted during the solution, thus leading to possible instability. The problem is thus no longer linear.

Once convergence is obtained, the solution is in terms of E-functions in the mined areas and V-functions in the intact areas. To obtain the desired potentials in the mined area, a further integration of the superimposed effect of all E-functions at every point in the mined area, and, in a similar manner the V-function is made to form the normal field which corresponds to the normal reef plane stresses.

For an n x n grid of squares, this is equivalent to the solution of n^2 linear equations (ignoring closure adjustments). For example, for n = 64, the solution involves 4 096 equations. It will be shown that an accurate enough (4 percent accuracy) solution can be made to converge in approximately 10 iterations. This would take up to 6 hours on an IBM 360/50 and require 128 megabytes of core storage using ordinary matrix techniques. However, by making use of the $\frac{1}{r}$ and $\frac{1}{r^3}$ character of the V- and E-functions and other numerical techniques, the system used solves a problem in 15 minutes using only 350 kilobytes of core.

1.3.3 Choice of the Solution

Consider the relative merits of the three methods of solution described above.

One of the most important differences between the analogue and digital systems is that the analogue results are not complete, giving a solution only on the reef plane; these must be read and encoded for use by a digital computer in order to find stresses and displacements off the reef plane; whereas the digital system has these results available and can apply them to a subroutine for off-reef calculations without the intermediate step.

The electrolytic tank, at the time of publication of the Salamon, Ryder and Ortlepp paper [8] had other draw-backs;

(i) Total closure was difficult to implement, requiring the painting of areas dependent on readings and the iterative adjustment of the painted boundaries.

(ii) Setting up a problem can be described as difficult. The tank must be levelled and floating dirt removed and bubbles wiped away from the top electrode.

(iii) The height of the water in the test-holes was found to be fairly critical.

(iv) Both the frequency response pattern and the voltage levels changed by as much as ± 10% over a period of only months, leading to non-repeatability of results.

Many of the difficulties could be overcome in principle and an extension to the system is a possible measurement of the currents in the upper electrode, thus allowing the ascertaining of stresses on the reef plane. The resistance analogue has overcome most of these difficulties.

The digital system has a number of advantages in addition to that mentioned earlier;

(i) A digital system is inherently more reliable than an analogue system in that results are always repeatable.

(ii) The closure boundary is an estimation built into the program without the need for manual adjustments.

(iii) Updating the mining configuration in a small region leads to almost immediate results thus allowing a remote terminal interactive operation with a reasonable time for computation and a virtually instantaneous reply after calling for the displacements and stresses of any point in the medium.

(iv) Configurations can be easily stored, as a solution, ready for any future inquiry or update.

(v) There is a facility for adjusting the scale of the problem. This allows for the solution of a problem which will take in the details of a wide area.

Through successive scaling by 2, solutions are found without losing the coarse more distant effects. The program handles the problem by computing modified boundary values for any specified zone of interest. This facility is particularly useful in studying the details around a shaft pillar, where, on the first scale of interest, the edges of the problem appear unsupported by intact areas.

(vi) Not being tied down to physical constraints of models, the digital computer system can, and has been adapted to, solve problems of steeply inclined reefs and reefs close to the surface. The analogue models are restricted to the "infinite depth" situation.

In general, the digital system shows greater versatility and possibilities of development as more advanced analytical techniques become available. It is expected, that these new methods would be more detailed refinements to the simplified theory and computing time.

CHAPTER 2

BASIC THEORY LEADING TO A NUMERICAL METHOD

2.1 Elastic Model

In the present discussion, the model will be developed assuming horizontal reefs at infinite depth. In later chapters, the model will be extended to treating small angles of dip, $<25^{\circ}$, (ch. 5) and finally, in general, to any angle of dip and the finite depth problem (ch. 6).

As stated earlier, §1.3, the assumptions underlying the basic solution are that the rock body as a whole, as opposed to the fracture zone and localised discontinuities, may be treated as an elastic and isotropic continuum[5]. Stratified behaviour is ignored in formulating the basic model.

The rock mass, prior to mining, is in equilibrium. The stresses maintaining this equilibrium are called primitive stresses. Underground excavations upset this state of balance, resulting in induced stresses. Because of linear elasticity, the total or resultant stresses may be formed by the superposition of the primitive and induced stresses. Similarly one can refer to induced displacements but, since there are no other types of displacements, the term "induced" may be dropped.

The problem is one of satisfying the basic equations of elastic theory subject to the boundary conditions prevailing in the particular situation.

2.1.1 Boundary Conditions

Considering here the model where the reef plane is at "infinite depth", the implication is that the effects of the stress-free surface can be ignored. As a result, only a half-space solution need be considered as the plane of the seam or reef is a plane of symmetry.

The boundary conditions, stated generally, are as follows :-

(i) The ground surface must be stress-free.

(ii) At great depth, induced stresses and displacements must be zero.

(iii) At the seam, since the lateral dimensions of the excavations are much larger than the vertical ones (i.e. tabular excavations), only conditions of the roof and floor (hanging- and footwalls) are considered. The assumption is that the stresses normally opposing each other in the roof and floor are equal over the whole plane of the seam. This simplification will introduce an error in the close vicinity of the solid stope faces.

In order to aid the mathematical description, it is convenient to set up the following co-ordinate systems, shown in figure 2.1.

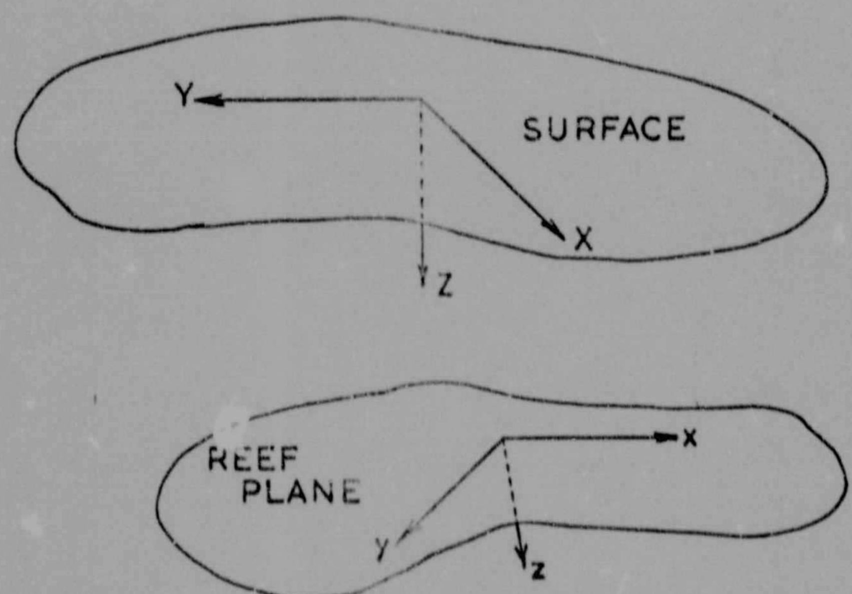

Figure 2.1 Co-Ordinate Systems

They are :

(i) A general co-ordinate system (X,Y,Z) with the X-Y plane in the surface and the Z axis pointing vertically downward.

(ii) A similar co-ordinate system (x,y,z) with x and y axes in the plane of the reef and the z axis normal to the reef pointing downward.

The convenience of operating with two co-ordinate systems will become apparent. Considering the problem as a whole, the surface co-ordinate system will be used, whereas, when considering the reef plane, especially as regards the solution on that plane, the reef co-ordinate system is more convenient.

Using the index "r" (roof) and "f" (floor) for strata immediately above and below the seam, the above conditions for horizontal reefs may be described mathematically in the following form :-

$$Z = o \qquad \sigma_{z_r} = \tau_{xz_r} = \tau_{yz_r} = o$$

$$Z = \infty \qquad w_f = u_f = v_f = o$$

$$Z = H \qquad \sigma_{z_r} = \sigma_{z_f} = \tau_{xz_r} = \tau_{xz_f}, \ \tau_{yz_r} = \tau_{yz_f} \qquad \ldots\ldots\ldots\ldots\ldots(2.1)$$

where H is the depth of the seam

In the seam itself, consider initially that the lateral dimensions of the excavations are so small that no contact has been established between the roof and the floor by "total closure", and that no artificial support, such as waste-packing, is used.

Since the resultant normal and shear stresses are zero throughout the floor and the roof of the excavations, the induced boundary stresses must assume the values of the primitive stresses with opposite signs.

Denoting the regions of mining by A, and the vertical primitive stress component by q_z, the remaining boundary conditions to consider are as follows :-

$$Z = H \qquad \sigma_{z_f} = -q_z, \ \tau_{xz_f} = \tau_{yz_f} = o \qquad \text{inside A}$$

$$\left.\begin{aligned} S_z(x,y) &= w_r - w_f = o \\ S_x(x,y) &= u_r - u_f = o \\ S_y(x,y) &= v_r - v_f = o \end{aligned}\right\} \qquad \text{outside A}$$

$$\ldots\ldots\ldots\ldots\ldots(2.2)$$