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Nomenclature

Symbol	Definition	Units
A	Area of a plate acted upon by a force	m ²
A	A breakage parameter that define ore characteristics in the JK dropweight test	-
a _{ij}	a term preferred by the JKMRC for representing a fraction appearing in interval i after breakage in interval j	-
b	radius at the tip of the crack	m
b	an exponential parameter that define ore characteristics in the JK dropweight test	-
b _{ij}	a fraction appearing in interval i after breakage in interval j	-
B _{ij}	a fraction of material that is finer than size X _i after breakage in interval j (B _{ij} = Σb _{ij})	-
C	Damping constant in the dashpot model	Ns/m
C'	Modified Damping constant in the Hunt-Crosley dashpot model	Ns/m ²
CB	Represents breakage where the largest fragment is less than 50% of the original size	-
D	Mill diameter	m
d _i	Fraction representing interval i in the mill discharge	-
d _l , d _u	represents lower and upper screen sizes	mm
d _{max}	described as the volume of pulp presented to the classifier per unit time	
Δ	Overlap distance	m
Δx, Δy, Δz	3D dimensions of a small elastic element	m
ΔF	Force causing elastic extension	N
ΔU	Energy input for elastic extension	J
E _{cs}	Specific comminution energy	Kwh/t
e	Coefficient of restitution	-
E	Energy may be suffixed according to specific application	J
E _{xo}	Minimum energy that should be exceeded before any breakage would be expected to occur	J
ε	strain	-
F _n , F _s	Force in the normal and shear directions	N
γ _s	Surface energy per unit length	J/m
h	height	m
I	Moment of inertia	
K, K _n , K _s	Particle stiffness, normal and shear direction respectively implied when suffixed	N/m
μ	Coefficient of friction	-
M	mass	Kg or g
ω	Angular velocity	Radians/s
P	probability	
P _{CB}	Probability of particle breaking to a level assumed to be complete breakage	-
P _f	Probability of failure of a single component	-
P _i	Chance of survival for a single component	-

p_i	Material in the interval i in the product stream	-
ρ_{ore}	Ore density	kg/m^3
σ	Stress	N/m^2
σ_T	Stress at the tip of a crack	N/m^2
r_i	radius of the first sphere of two neighbouring particles	m
r_j	radius of the second sphere of two neighbouring particles	m
S_i	disappearance rate (by size reduction) per unit time for interval size i	t^{-1}
T	torque	Nm
θ	angular position	radians
t_{10}	represents the fraction of material broken below 1/10th the size of the original particle size after breakage	-
R_f, R_s	Probability of failure and probability of survival for a system comprised of many units	-
S_{20}	A term used by Leung to estimate the geometrical mean size of the top 20% fraction of the mill content	-
u	Strain energy density i.e. strain energy per unit volume	J/m^3
U	refers to various form of energy is subscripted accordingly	J
W	energy supplied to a a body by some external force	J
W_m	Specific elastic energy	
X	particle size	mm
X_m	Mass of a particle of size X	g
Y	Young's modulus	N/m^2
Ψ	shape factor that indicates amount of deviation from spherical shape	-
x_i, y_i, z_i	3D positions of particles	m