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A BAG MODEL FOR KAON-NUCLEON SCATTERING

Etienne Barnard

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ABSTRACT

The success of bag models in hadron physics seems to imply that such models should be taken seriously as an approximation to QCD. In this belief a two-centre harmonic oscillator "bag" is utilised to describe kaon-proton scattering. An adiabatic approximation of the resulting system leads to a coupled channels formulation of the scattering problem.

Because of its complexity a solution of the gluon coupling inside a double-centred bag was not attempted; this turns out to be the most important interaction term and is simply parametrised.

Even though all potentials are approximated by square wells and a variety of other approximations are made, a fair fit to the experimental phase shift data is obtained. It seems that the "bare" masses of the unstable baryons differ appreciably from positions of the experimental resonances.

DECLARATION

I declare that this dissertation is my own, unaided work. It is being submitted for the degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

E C Barnard

E. Barnard.

3 August 1985.

PREFACE

High energy physics envelops an immense diversity of problems and solutions to these. Therefore some aspects necessarily tend to escape attention; gaping holes arise amidst the colourful fabric. One such hole seems to be the dynamic description of baryon-meson scattering. Although the literature abounds with experimental data, theoretical work lags far behind.

This study is an attempt to start rectifying this particular problem.

I would like to express my sincere gratitude to my supervisor, prof. R.H.Lemmer, who has been extremely helpful under circumstances he must have found trying at times. Every satisfied student probably considers his supervisor an uncommonly good educator; I believe that my judgement in this regard carries objective weight.

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I. BAG MODELS AND QUANTUM CHROMODYNAMICS

It is now believed that we have, in quantum chromodynamics (QCD), the correct description of the strong interaction between hadrons. QCD describes the hadrons as compound objects consisting of quarks. Besides the usual symmetries the quark Lagrangian also has local guage invariance under the group SU(3); therefore the quarks are characterised by an additional quantum number which is (whimsically) called "colour". To form a basis of the fundamental representation of SU(3) we require 3 colours. The strong nuclear force is to be seen as a "Van der Waals force" arising from the colour interaction.

Now, since the generators of SU(3) do not commute, QCD is a non-linear theory. This implies that the "carriers" of the QCD interaction - the gluons - can interact with one another. This is to be compared with quantum electrodynamics (QED) which has U(1) symmetry; because U(1) is Abelian (commutative) the photons do not interact amongst themselves.

Because of its non-linearity QCD is not soluble by way of a general perturbative expansion: to utilise QCD we need to make certain phenomenological assumptions, stemming from what we believe QCD to predict, and then calculate by way of this "reduced QCD". The two most important such inputs are

i) asymptotic freedom: at small distances and large momenta the quarks behave as though they were free. This is seen in scattering experiments; it is also a demonstrable consequence of QCD in a certain approximative framework.

ii) colour confinement: collections of quarks that do not belong to a colour singlet representation cannot exist in isolation. Hence free quarks do not exist (as seems to be the experimental

verdict) and the potential to separate a quark from a colour singlet increases monotonically at large separation. Although this has not rigorously been proven to be a result of QCD there are good reasons to expect this to be the case. The fact that QCD on a discrete spacetime lattice [1] gives rise to confinement is particularly suggestive.

Various models which incorporate these ideas into a QCD-like framework exist [2]. One very successful model is the MIT bag model [3]. Here requirement (ii) is satisfied by requiring that the outward flow of current vanishes at a prescribed boundary; (i) is then implemented by modelling the quarks as essentially free within this boundary. The restriction on the current does not imply a unique prescription for obtaining the quark wave functions - the MIT group chose to implement it by enforcing a relation between the wave function and its product with a specific combination of Dirac matrices on the boundary of the bag. This group seems to be very successful. In fig. 1 an example of the fit of hadron masses which has been obtained in this way [4] is shown. Magnetic moments, charge radii and similar properties have also been predicted with a great deal of success using the MIT boundary conditions.

Hence we expect the bag to allow us to carry out perturbative calculations which approximate the QCD result.

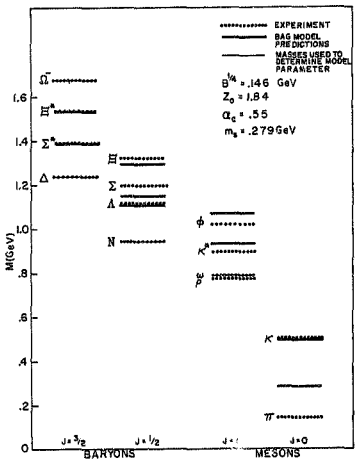
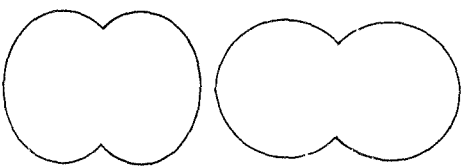


Fig. 1 : Fit to the hadron masses obtained with MIT bag model. The u and d quarks were taken to be massless; other parameters are as indicated. From [4].

II HARMONIC OSCILLATOR BAG

1. Non-spherical MIT-like bags

The success of the MIT bag in describing the static properties of hadrons prompts a scattering description along the same lines. To this end the bag boundary conditions would have to be implemented for other geometries than the conventional spherical one. For instance, the scattering process could be imagined as a simple collision of two bags which maintain their separate identities but exchange quarks across a mutual surface. Then solutions for flattened bags would be required (fig. 1a). On the other hand the bags could be taken to coalesce without deforming, in which case the geometry would be that of two intersecting spheres (fig. 1b).



(a) (b)

Fig. 1 : Examples of non-spherical geometries which may be used in a scattering description.

For "normal" matter (which is nearly incompressible) we know that the first picture is a better representation of the physical process at energies which are of the same order of magnitude as the binding energy of the constuents; however, for the case of interacting bags there is reason to believe that the second

picture is the more likely. This is because the hadronic vacuum is equivalent to a dia-electic medium [2]. The formation of "bubbles" in such a medium is energetically fa ored. Hence we do not expect the merging of bags to be accompanied by the usual resisting force which causes deformation.

Now, solutions of the Dirac equation with the MIT conditions on such a boundary would not be attainable analytically: numerical techniques would have to be employed. Furthermore, there may also be intrinsic difficulties in this: as shown in Appendix 1, a cubic geometry for instance does not admit a solution of the Dirac equation with the MIT boundary conditions. Indeed, a prolate spherical bag is no more successful. (It should be noted that the failure for the cube is hardly less surprising than that for the spheroid. Contrary to a widespread belief, the sharp edges of the cube and the lack of a well defined normal at these edges does not necessarily cause problems - cf. classical electrodynamics [5].)

In view of these difficulties another approach for modelling quark confinement inside interpenetrating bags is presented here. The problem of non-sphherical MIT bags is currently still under consideration.

2. Potential models

As is well known [3], the linear boundary condition of the MIT bag is (in the spherical case) equivalent to the addition of an infinite scalar potential to the mass of the quarks outside the boundary of the bag.

One obvious alternative to such square well confinement is confinement inside a harmonic oscillator potential. In direct analogy to the MIT model one would add an oscillator term to the mass of the quark:

$$m \longrightarrow m+(1/2)Kr^2 \qquad \ldots(1)$$

The Dirac equation with such a scalar potential can be written as

$$(\vec{\sigma}.\vec{p}-m-(1/2)Kr^2)\Psi=0 \qquad \ldots(2)$$

(As noted in Appendix I we use the standard representation of the gamma matrices:

$$\gamma_o = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\vec{\gamma} = \begin{bmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{bmatrix}$$

where 0,1 are the 2x2 null and unit matrices respectively; $\vec{\sigma}$ are the Pauli matrices.)

Writing the quark bispinor as

$$\Psi = \begin{bmatrix} g(r)Y_{KM} \\ if(r)Y_{-KM} \end{bmatrix} \qquad \ldots(3)$$

(for the case of spherical symmetry ; Y being a vector spherical harmonic, K the Dirac quantum number and M the z-projection of the angular momentum [6]) one obtains two coupled equations for f(r) and g(r):

$$\left[\frac{d}{dr} + \frac{1-K}{r}\right]f(r) + (\mathcal{E} - m - V)g(r) = 0 \qquad \ldots(4a)$$

$$\left[\frac{d}{dr} + \frac{1+K}{r}\right]g(r) - (\mathcal{E} + m + V)f(r) = 0 \qquad \ldots(4b)$$

We can now obtain a second order differential equation for each of f(r) and g(r) by differentiating the above equations and solving for the appropriate function:

$$\left\{\frac{d^2}{dr^2}+\frac{2}{r}+\frac{kr}{\mathcal{E}-m-(1/2)kr^2}\cdot\frac{d}{dr}+\mathcal{E}^2-(m+kr^2)^2+\frac{K(1-K)}{r^2}\right.$$

$$\left.+\frac{(1-K)k}{\mathcal{E}-m-(1/2)kr^2}\right\}f(r)=0 \qquad \ldots(5a)$$

The equation for g(r) is

$$\left\{\frac{d^2}{dr^2}+\frac{2}{r}+\frac{kr}{\mathcal{E}-m-(1/2)kr^2}\frac{d}{dr}+\mathcal{E}^2-(m+kr^2)^2-\frac{K(1+K)}{r^2}\right.$$

$$\left.-\frac{k(1+K)}{\mathcal{E}-m-(1/2)kr^2}\right\}g(r)=0 \qquad \ldots(5b)$$

These equations may be useful ; however, their structure lack the attractive simplicity of the MIT solutions. Great simplification can be obtained, however, by employing a slightly more complicated version of the potential . If we add

$$(1+\gamma_0)V(r) \qquad \ldots(6)$$

(that is we use equal amounts of scalar and time like vector potential) to the mass of a particle in the Dirac equation a Schrodinger-like equation for the upper component of the Dirac bispinor arises. The role of the potential in this "Schrodinger equation" is played by V(r).

To see this write the quark bispinor as

$$\Psi=\begin{bmatrix}\phi\\ \chi\end{bmatrix} \qquad \ldots(7)$$

Using the standard representation with the Dirac equation the coupled equations for ϕ and χ are:

$$(\vec{\sigma}.\vec{p})\phi=(\mathcal{E}+m)\chi \qquad \ldots(8a)$$

$$(\vec{\sigma}.\vec{p})\chi=(\mathcal{E}-m-V(r))\phi \qquad \ldots(8b)$$

It can be seen that the potential does not occur in (8a). We can therefore operate with $\vec{\sigma}.\vec{p}$ on that equation without incurring

derivatives of the potential.

This form has been used in conjunction with a harmonic oscillator by Ravndal [7]. He chose a potential of the form (the numerical factors are for convenience)

$$(1/18)(1+\gamma_0)k^3r^2 \qquad \ldots(9)$$

Then the the small component of the quark bispinor is determined by the equation

$$\left[\nabla^2 - (1/9)k^3(\mathcal{E}+m)r^2 + (\mathcal{E}^2-m^2)\right]\phi = 0 \qquad \ldots(10)$$

Making the same separation into radial and angular parts as above we obtain for f(r) an expression very similar to the Schrodinger equation for a harmonic oscillator potential. The only difference is that the energy eigenvalue now occurs both linearly and quadratically, as can be seen in (10).

The solutions of (10) subject to the boundary condition that the wave function be normalizable are confluent hypergeometric functions (see below), and the energies are given by

$$(\mathcal{E}-m)(\mathcal{E}+m)^{\frac{1}{2}} = (2/3)k^{3/2}(2n+l+3/2) \qquad \ldots(11)$$

The symbols have exactly the same significance as in the non-relativistic case : n = 0, 1, ... and l = 0, 1, ... are the radial and angular quantum numbers respectively.

The spin-up spinor with quantum numbers n,l and m is

$$= N\begin{bmatrix} Y_{KM} \\ i/(\mathcal{E}+m)(\frac{d}{dr} + \frac{1+k}{r})Y_{-KM} \end{bmatrix} r^{l+1}\exp(-\alpha^2r^2/2)F(-n,l+3/2;\alpha^2r^2) \qquad \ldots(12)$$

(where $\alpha^4 = (1/9)k^3(\mathcal{E}+m)$ $\qquad \ldots(13)$

and F is the confluent hypergeometric function ; N is a normalisation constant.)

As in the non-relativistic case this has no negative energy solutions. Therefore the antiparticle states have to be found in some other way. For this we note that the zeroth component of the electrodynamic vector potential couples with opposite sign to particle and antiparticle states (since it is accompanied by the electric charge of the particle/ antiparticle in that case). Hence we should make a similar change of sign here, so that the potential for our antiparticle states is

$$(1/18)(1-\gamma_0)k^3r^2 \qquad \ldots(14)$$

We thus get a negative energy spectrum which is the mirror image of the positive energy spectrum. Because this arises from a different potential, however, the particle and antiparticle states will not be orthogonal to one another. As we shall see later on, this might pose a problem. (Extensive use of these negative energy states will be made: we shall assume all of the negative energy states to be populated by "sea" quarks to form a Dirac sea. This implies that the non-vanishing operations on our vacuum are the annhilation of a negative energy quark or the creation of a positive energy quark.)

3. Two-centre harmonic oscillator

We can now extend the above description to include non-spherical geometries too. The picture sketched by the second drawing in fig. 1 will be simulated- i.e. that of two bags coalescing without deforming. This is modelled by using a two-centre harmonic oscillator potential in the Dirac equation. We write - using cylindrical polar coordinates -

$$V(\vec{r},z_0) = (k^3/18)(1+\gamma_0)[(|z|-z_0)^2+\rho^2] \qquad \ldots(15)$$

(See fig. 2)

This potential describes two separate harmonic oscillators in the

asymptotic region (i.e. for the separation between their centres $= 2z_0$ large). On the other hand the confining potential becomes a single harmonic oscillator when the separation vanishes.

To see how solutions with this potential look we again assume a spin-up quark, that is

$$\phi = \begin{bmatrix} F \\ 0 \end{bmatrix} \qquad \ldots(16)$$

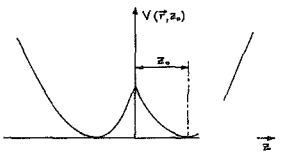


Fig. 2 : Two-centre harmonic oscillator: z-dependence

F is written in the form

$$F = f(\psi)g(\rho)h(z) \qquad \ldots(17)$$

and the Dirac equation separates. Details of the solutions in the three coordinates are given in appendix 2.

It should be noted that the functions $h(z)$ $(\propto D_p[\sqrt{2}\alpha(|z|-z_0)]$ where D_p are parabolic cylinder functions) fall into two distinct categories: since the equation in z is unchanged by the transformation $z \rightarrow -z$ we know that "z-parity" is a good quantum number; hence we get solutions which are either odd or even under this transformation.

The eigenenergies are found to be given by

$$(E-m)(E+m)^{1/2} = (2/3)k^{3/2}(2n_\rho + |m_\varphi| + p + 3/2) \qquad \ldots(18)$$

with $n_\rho = 0, 1, \ldots$; $m_\varphi = \ldots, -1, 0, 1, \ldots$ and $|m_\varphi| \leq n_\rho$. p is restricted by the z-parity conditions (figure 3):

even : $D_p'(z=0)=0$

odd : $D_p(z=0)=0$.

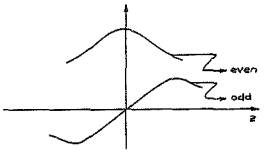


Fig. 3: Continuity of the wave function and its derivative at z=0 restricts the quantum number p.

The centres of the oscillators are situated at $+z_0$ and $-z_0$ respectively. Thus p will change as z_0 changes, and the energy eigenvalues become functions of z_0.

In fig. 4 the lowest few energy levels are shown as a function of z_0 for massless quarks. We note that since the potential becomes two oscillators in the asymptotic region, each level in the spherical limit corresponds to two asymptotic levels: one with positive "z-parity" and the other with negative "z-parity". This fact will be used later on.

Fig. 5 displays the effect of having a single massive quark on the ground state. As in the MIT bag the energy is seen not to be very sensitive to the quark mass. Note that the energies are scaled by the oscillator constant k; this will in turn be inversely proportional to any "radius of confinement", for instance the r.m.s radius. We return to this point below.

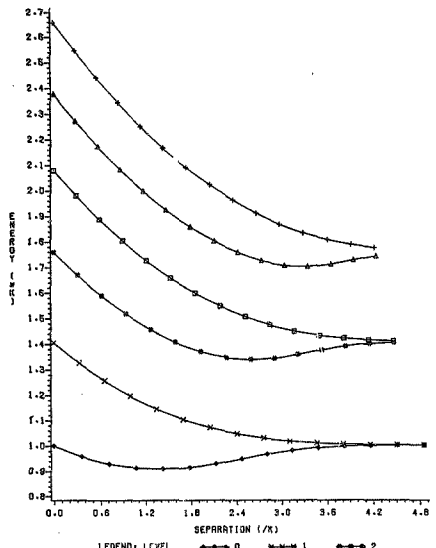


Fig. 4 : Energy levels of massless quarks. Levels are labelled by $p(z_0=0)$; $n_A = m_\psi = 0$.

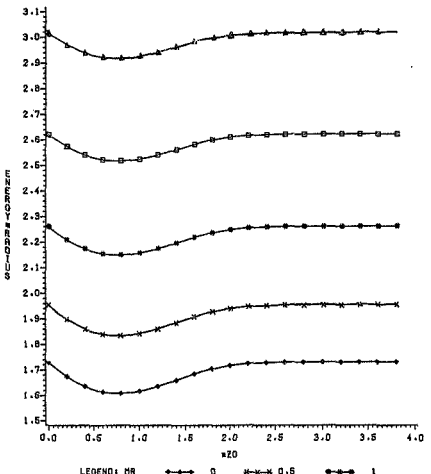


Fig. 5 : Lowest energy level for various values of MR; MR= mass x radius.

4. Adiabatic approximation of particle energies

The energies of the quarks *do not* constitute the only contributions to the mass of hadrons built up out of them. It is well known [3] that contributions from e.g. the volume energy of the bag and the interactions between the quarks will have to be taken into account. At least the *first* of these will depend *strongly* on the "radius" of the hadron, which also scales the energy levels of the quarks.

Hence, whereas the inverse *proportionality of our energy* levels *on* the bag radius implies that the quark energies always decrease as the bag radius increases, the energy of the hadron itself is minimised at a certain finite radius. This is significant for two reasons:

-*in an* adiabatic approximation of the scattering of hadrons we would therefore minimise the energy of the system at each separation ($=2z_0$) to determine the radii of the scattering bags.

-finding this minimum is also equivalent to *mimicking* the non-linear boundary condition of the MIT bag.

We only include the quark energies and the volume energy. At a separation of $2z_0$ the total volume of *two intersecting* bags, *each of* radius R is

$$Vol=(2/3)\pi R^3 + \pi R^2 z_0 + (1/3)\pi {z_0}^3 \qquad ...(19)$$

Furthermore the quark energies are proportional to k from (18); one must therefore find a relation between k and R *to be able to minimise the* energy. In this regard we follow Ravndal [7], who defines the radius as the distance from the centre in the spherical ground state at which the large component of the wave function has fallen to $1/\sqrt{e}$ of its original *value. This determines the* relation between R and k:

$$R=(3/k)[(2/3)(2n_p+1)\pi_{up}+3/2]^{1/6} \qquad \ldots(20)$$

Minimisation of the energies can now be done straightforwardly. As an example each bag was populated with 3 massless quarks in their ground state. The energy was then *minimised* at various values of z_0, keeping z_0 fixed. As can be seen in fig. 6, the energy of the 6-quark system shows a general decrease in energy as z_0 decreases. In a Born-Oppenheimer-like picture we would interpret the total energy of the system as *the potential* between the two particles (since all the quarks remain in the ground state three quarks are on the left hand side asymptotically and three on the right hand side); this would then be seen as an attractive potential. Note that the energies *and separations* are *scaled by* the volume energy parameter B . The way in which the radius of the "bag" changes as a funtion of the separation to give rise to the attractive potential can be seen in the radius-separation plot, fig. 7.

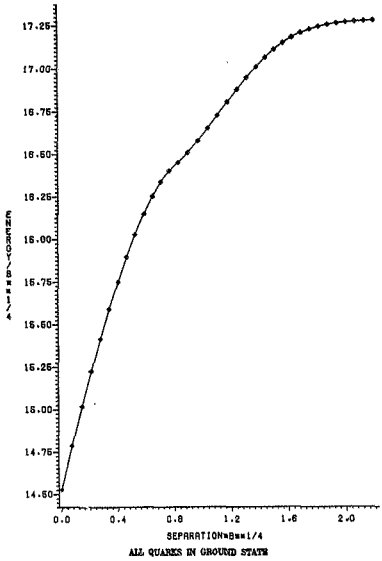


Fig. 6 : Energy of 6-quark system in the adiabatic approximation.

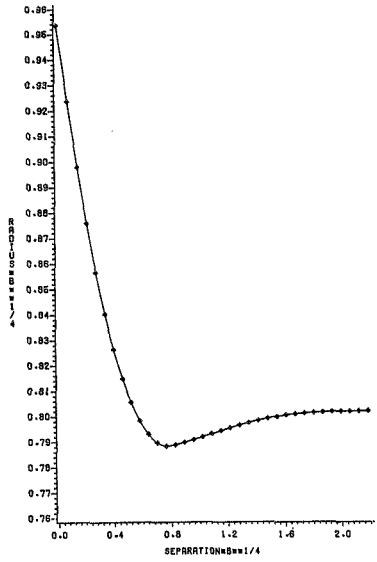


Fig. 7 : Radius of 6-quark system in the adiabatic approximation.

III SCATTERING FORMALISM

1. The Born-Oppenheimer approximation

A useful method of solving scattering problems of compound particles at low energies (that is energies at which the relative motion of the particles is much slower than the internal motion of the constituents) is the so-called Born-Oppenheimer approximation. In this approximation one calculates the total energy of the scattering particles at each separation; the difference between this and the energy in the asymptotic region is taken as the potential for the scattering interaction. This method is commonly used in molecular physics; for instance, the binding energy of the ionized hydrogen molecule is approximated excellently in this manner.

The Born-Oppenheimer approximation is motivated as follows: If the separation between the particles were constant, the force between them would be equal to the derivative of the total energy with respect to the separation. Hence we expect the same to be true if the relative motion is much slower than the internal motion of the particles. Since the force is by definition equal to the derivative of the potential, this implies that in this adiabatic limit the potential is just the total energy (to within an additive constant).

As an excercise in using this approximation we first solved the following simple one dimensional problem. Two coalescing square wells, one containing one particle and the other two, were scattered off one another using this approximation. Appendix 3 contains more details of this calculation.

2. R as collective coordinate

Implicit in the Born-Oppenheimer approximation is the introduction of a *collective* coordinate: the distance between the centres-of-mass of the scattering objects. The Born-Oppenheimer potential then occurs in the Schrodinger equation involving this coordinate.

The quality of an approximation using the centre-of-mass depends on the degree of localization of the constituents of the scattering objects. Since one believes that the quarks *inside* a hadron are *totally confined* to a region of space (inside of which they are essentially free), this requirement should be well satisfied in our case. To introduce this description we shall follow a simplified version of the method of Lemmer and Taepffer [8] that was used to *describe the* scattering of two heavy ions.

It should also be pointed out that we shall follow these authors in using a Schrodinger equation to obtain the wave function for relative motion. As *will be seen, we* shall be working in a region where relativistic effects might just start to be important for this; in the light of the schematic nature of the calculation the complications of a full relativistic treatment was however considered unwarranted.

The Hamiltonian is divided into three parts:

$$H = H_{INTR} + H_{RELATIVE} + H_{GLUON} \qquad \ldots(1)$$

The Hamiltonian *involving relative motion* is

$$H_{RELATIVE} = \frac{-\hbar^2}{2\mu} \frac{1}{R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} \qquad \ldots(2)$$

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(where a term involving collective rotations has been omitted).

The eigenstates of the intrinsic Hamiltonian are now introduced:

$$\langle i|H_{INTR}|i'\rangle = E_i \delta_{ii'} \qquad ...(3)$$

These states normally refer to particles made up of the quarks described by the solutions of the *two-centre Dirac equation* (as in Chapter *II*). *Hence* $|i\rangle$ could be a meson, in which case it consists of a quark in a positive energy eigenstate and an antiquark in a negative energy eigenstate. However, as we shall see later, the particles in the incoming channel are not such eigenstates. In general our "physical" states are therefore *superpositions of the eigenstates*:

$$|\alpha\rangle = \sum_i C_{\alpha i}|i\rangle \qquad ...(4)$$

Using these we can expand the scattering wave function as

$$(1/R)\sum_\alpha w_\alpha(R)|\alpha\rangle \qquad ...(5)$$

To solve the Schrodinger equation with this expansion of the wave function we note that the intrinsic *eigenstates will* depend *parametrically on the* separation R between the two bags. Employing (2) we obtain a set of coupled equations for the amplitudes $w_\alpha(R)$:

$$\left(\frac{-\hbar^2}{2\mu}\left[\frac{d^2}{dR^2} + \langle\alpha|d^2R|\alpha\rangle\right] - (E-E_\alpha) + \langle\alpha|H_{GL}|\alpha\rangle\right)w_\alpha(R) =$$

$$\sum_{\alpha'\neq\alpha}\left(\frac{\hbar^2}{2\mu}\left[2\langle\alpha'|dR|\alpha\rangle\frac{d}{dR} + \langle\alpha'|d^2R|\alpha\rangle\right] - E_{\alpha\alpha'} - \langle\alpha'|H_{GL}|\alpha\rangle\right)w_{\alpha'}(R) \qquad ...(6)$$

with

$$E_\alpha \triangleq \langle\alpha|H_{INTR}|\alpha\rangle = \sum_i |C_{\alpha i}|^2 E_i$$

$$E_{\alpha\alpha'} \triangleq \langle\alpha'|H_{INTR}|\alpha\rangle = \sum_i C_{\alpha' i} C_{\alpha i} E_i$$

The third term in the Hamiltonian, corresponding to gluon exchange, is at this stage still undetermined. We shall return to

this later on.

3. Second quantized form of scattering formalism

We note that

i) the scattering problem will involve the creation *and* destruction of particles/antiparticles and

ii) identical particles will occur, which have to be antisymmetrized properly.

This immediately suggests the use of the method of second quantization. We should therefore write the operators in the coupled equations (6) a in second quantized form. The differential operators d/dR and d^2/dR^2 will be considered here; H_{GLUON} is discussed elsewhere.

To write d/dR and d^2/dR^2 in second quantization we expand the ground state as a Slater determinant, and operate on this with the operators.

i) d/dR in second quantization

The Slater determinant corresponding to a ground state with N particles is

$$\langle x_1\, x_2 \,..\, x_N | \text{ G.S}\rangle = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_a(1) & \phi_b(1) & \dots \\ \phi_a(2) & \phi_b(2) & \dots \\ \vdots & \vdots & \end{vmatrix}$$

By actually expanding the Slater determinant one sees that differentiation with respect to R will affect one column at a time:

$$\langle x_1\, x_2 \,..\, x_N | \text{ G.S}\rangle = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_a'(1) & \phi_b(1) & \dots \\ \phi_a(2) & \phi_b(2) & \dots \\ \vdots & \vdots & \end{vmatrix} + \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_a(1) & \phi_b'(1) & \dots \\ \phi_a(2) & \phi_b'(2) & \dots \\ \vdots & \vdots & \end{vmatrix}$$

Using completeness we write

$$\phi'_a(1) = \sum_\alpha \phi_\alpha(1) \int_{-\infty}^{\infty} dx\ \phi_\alpha(x)\ \phi'_a(x) \quad = \sum_\alpha (\alpha | d/dR | a)\ \phi_\alpha(1)$$

$d/dR\ |\ G.S\ \rangle = \sum_{\alpha\beta} (\alpha | d/dR | \beta) c_\alpha^\dagger c_\beta\ |\ G.S\ \rangle$, and extracting the common

$$d/dR\ |\ G.S\ \rangle = \sum_{\alpha\beta} (\alpha | d/dR | \beta) c_\alpha^\dagger c_\beta\ |\ G.S\ \rangle \qquad \ldots(7)$$

Hence we see that d/dR is a single particle operator; in the above expression it replaces β by α.

ii) d^2/dR^2 in second quantization

This is obtained by differentiating our previous result with respect to R. We must remember that

-the operators c depend on R themselves

-in an expression like

$\sum_c (a|d/dR|c)(c|d/dR|b)$

we cannot use completeness since dR is not a " normal " operator: it operates on the parametric dependence of the states.

We then obtain

$$d^2/dR^2\ |G.S\rangle = \left[\sum_{\alpha\beta} (\alpha | d^2/dR^2 | \beta) c_\alpha^\dagger c_\beta + \sum_{\alpha\beta\gamma\delta} (\alpha | d/dR | \delta)(\beta | d/dR | \gamma) c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta\right] |G.S\rangle \qquad \ldots(8)$$

So this is a sum of one and two body operators.

Combining these results with (6) enables us to actually carry out a calculation with this formalism.

IV. KAON-PROTON SCATTERING

1. Experimental data

Recent years have seen a tremendous increase in the quantity and quality of data on kaon-proton (K+p) scattering. A variety of products result from such scattering: besides elastic scattering, reactions like $K^+p \longrightarrow \pi^+\Sigma$ and $K^+p \longrightarrow \pi^+\Lambda$ are observed. The experiments have outlined interesting structure in the various channels.

Fig. 1 shows the s-wave phase shifts extracted from the elastic channel data for the isospin zero channel (taken from [9]). Two clear peaks, corresponding to lambda resonances at 1680 and 1800 MeV respectively, are discernible. The effect of these resonances can also clearly be seen in the various inelastic channels. Similar resonant structure shows up in the T=1 channel. Here it is the sigma resonances at 1750 and 2000 MeV which are seen.

The significance of these observations becomes clear if the participating hadrons are analyzed in terms of their quark constituents. We shall use u, d and s to denote the up, down and strange quarks respectively, and write $\bar{u}$, $\bar{d}$ and $\bar{s}$ for the respective antiquarks. The proton consists of the three quarks (uud) (the braces indicate that these are to be coupled to the correct spin, isospin and colour - see below), and the kaon of the pair $(\bar{u}s)$. Both lambda and sigma are made up of the three quarks (uds). (They differ in spin and isospin quantum numbers.) Thus we can see that the annihilation of an up quark with an anti-up has to take place for these resonances to occur.

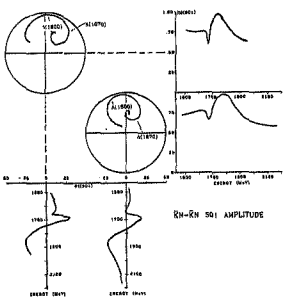


Fig. 1 : Scattering Amplitude in the T=0, I=0 channel extracted from experimental data; from [9].

2. Antisymmetrized operators

An important side issue is the construction of creation and annihilation operators of the hadrons we shall be modelling which have the correct symmetry properties. This is not trivial since i) second quantization will ensure that the operators are totally antisymmetric; however, they also have to be totally antisymmetric with respect to colour exchange to belong to a colour singlet. This antisymmetrization must be done "by hand". ii) in the case of the baryons we shall have to couple 3 quarks, 2 or more of which might be identical, to good spin and isospin.

We shall be interested in 3 types of particles: Kaon, proton, and lambda.

a) Kaon

Because this meson consists of a quark-antiquark pair, it is relatively simple to construct a colour singlet: the quark and antiquark must have the same colour label, and we should sum over all three possible values of this label.

Furthermore the Kaon has spin S=0. Hence the creation operator for a negatively charged Kaon can be written as

$$\langle K^- \rangle^\dagger = N \sum_{\sigma c} (1/2\ \sigma\ 1/2\ -\sigma | 1\ 0\ 0\rangle s^\dagger_{\sigma\kappa}(c) u_{\sigma\beta}(c) (-)^{1/2-\sigma}$$

$$= 1/\sqrt{6} \sum_c [s^\dagger_{\uparrow\kappa}(c) u_{\downarrow\beta}(c) + s^\dagger_{\downarrow\alpha}(c) u_{\uparrow\beta}(c)]$$

$$= 1/\sqrt{6} \sum_c [\bar{s}^\dagger_\alpha(c) u_\beta(c)]_{S=0} \qquad \ldots(1)$$

(where s and u are strange and up quark annihilation operators respectively; c stands for colour; σ ranges over the 2 possible spins and α, β refer to any other quantum numbers. α must be such that in the asymptotic region it refers to the lowest positive energy eigenstate since we know the Kaon to be an unexcited meson. Similarly β must refer to the lowest - least negative - negative energy eigenstate asymptotically.)

At this stage we may address ourselves to the question of the mass of the s quark. This is believed to be more massive than the u and d quarks [4]; since we have seen in chapter II that the energy levels are not very sensitive to quark masses we shall simply set all quark masses equal to zero as a working approximation.

ii) Lambda

As mentioned, this consists of u, d and s quarks arranged in a colour singlet. The u and d are coupled to isospin T=0. So to construct the lambda creation operator we first obtain an operator consisting of the right combination of these two to

couple to T=0 (for specific colours 1 and 2):

$$Q^{\dagger}_{ST}(12) = [q^{\dagger}(1)q^{\dagger}(2)]_{S\ T=0\ M_S=1/2\ M_T=0} \qquad \ldots(2)$$

(q denotes a u or d annihilation operator, depending on the isospin projection τ_i.)

We can now show that the sum of the spin and and isospin of any such two-quark combination of up and down quarks in the same spatial state should be an even number [10]. We have that

$$Q^{\dagger}_{ST}(12) = [q^{\dagger}(1)q^{\dagger}(2)]_{S\ T\ M_S M_T}$$
$$= \sum_{\sigma_i\sigma_2\tau_i\tau_2} (1/2\ \sigma_i\ 1/2\ \sigma_2|S\ M_S)(1/2\ \tau_i\ 1/2\ \tau_2|T\ M_T)$$
$$q^{\dagger}_{\sigma_i\tau_i}(1)q^{\dagger}_{\sigma_2\tau_2}(2)$$

On the other hand, if we swop the colours 1 and 2 we see that

$$Q^{\dagger}_{ST}(21) = -(-)^{S+T}Q^{\dagger}_{ST}(12) \qquad \ldots(3)$$

(where we have made use of

i) $q(1)q(2) = -q(2)q(1)$; the operators anticommute and

ii) $(1/2\ \sigma_i\ 1/2\ \sigma_2|S\ M_S) = -(-)^{1-S}(1/2\ \sigma_2\ 1/2\ \sigma_i|S\ M_S)$).

But we must have antisymmetry in colour alone; this proves that S+T=even.

Therefore the operator Q has S=0 and

$$Q^{\dagger}_{S=0,T=0}(12) = \sum_{\sigma\tau} (1/2\ \sigma\ 1/2\ -\sigma|0\ 0)(1/2\ \tau\ 1/2\ -\tau|0\ 0)q^{\dagger}_{\sigma\tau}(1)q^{\dagger}_{-\sigma,\tau}(2)$$
$$= (1/2)[u^{\dagger}_{\uparrow}(1)d^{\dagger}_{\downarrow}(2) - u^{\dagger}_{\downarrow}(1)d^{\dagger}_{\uparrow}(2) - d^{\dagger}_{\uparrow}(1)u^{\dagger}_{\downarrow}(2) +$$
$$d^{\dagger}_{\downarrow}(1)u^{\dagger}_{\uparrow}(2)] \qquad \ldots(4)$$

To get the full lambda operator (without colour antisymmetrization) this is simply multiplied with an s-quark creation operator with the same spin as the lambda - spin up in our case. Colour antisymmetry is obtained by simply permuting the colours amongst the 3 quarks, picking up a factor of -1 for each permutation. Using the antisymmetry of Q under colour exchange this can be written as

$$(\Lambda^{\circ})^{\dagger}_{\uparrow} = 1/\sqrt{3} \sum_{c} [Q^{\dagger}_{00}(12)s^{\dagger}_{\uparrow}(3)]$$

$$= 1/\sqrt{3}\,[\,Q^{\dagger}_{00}(12)s^{\dagger}(3) + Q^{\dagger}_{00}(31)s^{\dagger}_{1}(2) + Q^{\dagger}_{00}(23)s^{\dagger}_{1}(1)] \quad \ldots(5)$$

C denotes a cyclic permutation of the colour labels.

(iii) Proton

The proton creation operator can now be constructed along the same lines. A different structure arises, however, because there is not any particular pair of operators which is coupled to a definite spin or isospin. Hence we are faced with terms of the form

$$[Q^{\dagger}_{ST}(12)q^{\dagger}_{\sigma\tau}(3)]_{1/2\ \ 1/2\ m_s\ m_t} = (S\ M_S\ 1/2\ \sigma\ 1/2\ m_S)(T\ M_T\ 1/2\ \tau\ 1/2\ m_t)\ \ Q_{S\ M_S\ T\ M_T}(12)q^{\dagger}(3)$$

where Q is again a two-quark operator which must be constructed under the condition that S+T=even.

Now we would expect that, in direct analogy to the above, we should form a linear superposition of such terms with the colour index cyclically permuted:

$$(p_{\uparrow})^{\dagger} = [Q^{\dagger}_{ST}(12)q^{\dagger}(3)] + [Q^{\dagger}_{ST}(23)q^{\dagger}(1)] + [Q^{\dagger}_{ST}(31)q^{\dagger}(2)]$$

(with each term coupled to $m_s=m_t=1/2$ as above.)

The fact that we are now dealing with identical particles in the same spatial state, however, leads to a remarkable simplification. The spin and isospin may be recoupled as follows:

$$[q^{\dagger}(1)Q^{\dagger}_{ST}(23)]^{\frac{1}{2}\frac{1}{2}}_{m_s\ m_t} = +\,[Q^{\dagger}_{ST}(23)q^{\dagger}(1)]_{m_s m_t}$$

$$= \sum_{S'T'} \sqrt{2S+1}\sqrt{2S'+1}\sqrt{2T+1}\sqrt{2T'+1} \begin{Bmatrix} 1/2 & 1/2 & S' \\ 1/2 & 1/2 & S \end{Bmatrix}$$

$$\times \begin{Bmatrix} 1/2 & 1/2 & T' \\ 1/2 & 1/2 & T \end{Bmatrix} [Q_{S'T'}(12)q(3)]_{m_s\ m_t}$$

$$= [Q_{ST}(31)q(2)]_{m_s\ m_t}$$

Hence it follows that all three terms can be written in terms of $[q^{\dagger}(1)Q^{\dagger}_{ST}(23)]_{m_s\ m_t}$:

$$(p_{\uparrow})^{\dagger} = [Q^{\dagger}_{ST}(12)q^{\dagger}(3)]_{m_s\ m_t} + 2\sum_{S'T'} \sqrt{2S+1}\sqrt{2S'+1}\sqrt{2T+1}\sqrt{2T'+1}$$

$$\times \begin{pmatrix} 1/2 & 1/2 & S' \\ 1/2 & 1/2 & S \end{pmatrix} \begin{pmatrix} 1/2 & 1/2 & T' \\ 1/2 & 1/2 & T \end{pmatrix} [Q^{\dagger}_{S'T'}(12) q^{\dagger}(3)]_{m_s\, m_t}$$

$$= 1/\sqrt{6}\ \{-2d^{\dagger}_{\downarrow} u^{\dagger}_{\uparrow} u^{\dagger}_{\downarrow} + d^{\dagger}_{\uparrow} u^{\dagger}_{\downarrow} u^{\dagger}_{\uparrow} + d^{\dagger}_{\uparrow} u^{\dagger}_{\uparrow} u^{\dagger}_{\downarrow}$$
$$+ u^{\dagger}_{\downarrow} d^{\dagger}_{\uparrow} u^{\dagger}_{\uparrow} - 2u^{\dagger}_{\uparrow} d^{\dagger}_{\downarrow} u^{\dagger}_{\uparrow} + u^{\dagger}_{\uparrow} d^{\dagger}_{\uparrow} u^{\dagger}_{\downarrow}$$
$$+ u^{\dagger}_{\downarrow} u^{\dagger}_{\uparrow} d^{\dagger}_{\uparrow} + u^{\dagger}_{\uparrow} u^{\dagger}_{\downarrow} d^{\dagger}_{\uparrow} - 2u^{\dagger}_{\uparrow} u^{\dagger}_{\uparrow} d^{\dagger}_{\downarrow} \} \qquad \ldots(6)$$

3. Identification of the relevant scattering channels

The expansion of scattering wave function (III.5) contains an infinite number of terms; we must now make an appropriate truncation. Because of the strong resonant behaviour of the phase shifts a drastic truncation seems viable. Hence only two (uds)-states of correct isospin (that is two lambda-like states for T=0) will be included in the basis along with the colliding hadron states. The (uds)-states should lie around 1670 and 1800 MeV respectively for the T=0 case, which we shall consider in our calculation.

Constructing the correc. incoming channel intrinsic state requires some care. Obviously we would like all of the quarks in the proton and kaon to be in their ground states in the asymptotic region (i.e. for large R). However, in our two-centre oscillator description this ground state is degenerate (see fig. 2).

This degeneracy is now utilised to get the correct partitioning of quarks to give the kaon on one side of the centre of mass and the proton on the other. We shall lable the energy levels according to the value of the quantum number p at z_0=0, and assume that the other quantum numbers (m_ρ and n_ρ) are zero except when an explicit statement to the contrary is made. As can

be seen in fig. 2, the sum of wave functions 0 and 1 will give a wave function which is localized on the right hand side of the centre asymptotically, while the difference between 0 and 1 is totally localized on the left hand side of the centre. Therefore a product of sums of wave functions will give us the appropriate partitioning; for a Kaon on the left hand side and a proton on the right hand side we would write the total wave function as (suppressing all other lables)

$$\Psi = (\langle s_0+s_1\rangle\langle u_0+u_1\rangle)\ (\langle d_0-d_1\rangle\langle u_0-u_1\rangle\langle u_0-u_1\rangle).$$

u, d and s now represent the relevant wave functions; the subscripts denote the value of p of the wave function at zero separation. If we now replace the wave functions above by their respective second quantized creation and annihilation operators we obtain the partitioned system with full antisymmetry.

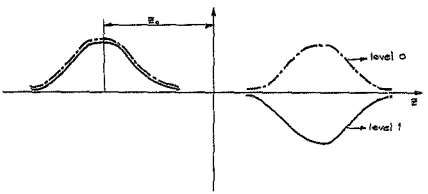


Fig. 2 : The degeneracy of the ground state for large z_0 is utilised to separate the Kaon and proton.

In summary then, we shall include three channels in our calculation:

$|0\rangle = \langle K\rangle_{le}\langle P\rangle_{ri}|D,S\rangle$

$|1\rangle, |2\rangle$ = lambda in appropriate state of excitation

(where the subscripts le and ri denote that the constituent

quarks are asymptotically on the left and right hand sides respectively before antisymmetrization; |D.S⟩ denotes the *filled* Dirac sea.)

4. States included in calculation

The excited states of the lambda which are going to be included in the calculation must now be specified explicitly. The spectrum of I=0, s=1/2 lambdas is indicated in table 1. We can thus see that we are interested in modelling the first and second excited states of appropriate spin.

Each of the first two excited states will be multiply degenerate (any of the three quarks may be excited) if we take the mass of the lambda to stem only from the quark energies. By realising that the inter-quark interaction energy will also be an important contribution to the mass it is, however, possible to argue which arrangement will have the lowest mass and will therefore represent the first excited state. Since the lambda will be restricted to shapes which are almost spherical (to mimic colour confinement) this argument will be based on the spherical MIT [4] results. We note that

-the most important contribution to the interaction energy is the magnetic energy; it is of the form $C\,\vec{\sigma}_1.\vec{\sigma}_2$, with C a positive constant.

-C will depend on the mass of the quarks involved in the interaction- It is maximal for massless quarks. Although we model all the quarks as being massless, it is known that the strange quark in fact has a mass of about 200 MeV [4].

-For the u and d quark in a spin zero state we have $\vec{\sigma}_u.\vec{\sigma}_d=-3$ and $\vec{\sigma}_s.(\vec{\sigma}_u+\vec{\sigma}_d)=0$; if they have spin one we have $\vec{\sigma}_u.\vec{\sigma}_d=1$ and $\vec{\sigma}_s.(\vec{\sigma}_u+\vec{\sigma}_d) = -4$.

Lambdas	Sigmas
1116 P01	1193 P11
1405 S01	1385 P13
1520 D03	1580 D13
1600 P01	1620 S11
1670 S01	1660 P11
1690 D03	1670 D13
1800 S01	1750 S11
1800 P01	1775 D13
1820 F05	1915 F15
1830 D05	

Table 1 : Excited states of the sigma and lambda. 1670 D13 means a resonance at 1670 MeV with angular momentum l=2 (i.e. D-wave), isospin T=1 and total spin J=3/2.

Therefore, if the relation between mass and magnetic energy in the excited levels resembles the relation in the ground state (as one expects it should) the first excited state will have an excited s quark. On the other hand a *more* detailed knowledge of *the relative magnitudes* of the various contributions to the hadron mass will be required to identify the second excited state; we shall simply assume this to have both the u and the d in the first excited state, with the s in the ground state.

Both these states have $n_\rho=0$ (and hence $m_\phi=0$); any excitation of these degrees of freedom will just increase the mass of the resonances.

One further subtlety to note is that the cylindrical eigenfunctions do not necessarily have good angular momentum in the spherical limit. Therefore the coupling to good angular momentum must in general be done explicitly by utilising the degeneracy of the cylindrical eigenfunctions. It turns out, however, that the lowest level and the first excited level have $l=0$ and $l=1$ respectively; since we are only employing these levels we do not have to carry out further coupling to good angular momentum.

5. Calculation of diagrams and matrix elements

A. The operators d/dR and d^2/dR^2

We are now in a position to calculate the matrix elements required for the solution of the coupled equations, (III.6). First we note that the operators (d/dR and d^2/dR^2) conserve

i) spin, colour and flavor, since they do not operate on these quantum numbers.

ii) parity with respect to reflection through the plane z=0. As

will be recalled, this is a good quantum number for the intrinsic states; because the two operators affect only the parametric variables, this will remain unchanged.

Further we remember that d/dR is a single particle operator; d^2/dR^2 is a combination of one and two particle operators. The required matrix elements are now calculated as *follows:*
-*the general form of each* particle (disregarding spin, colour and spatial quantum numbers) is inserted into the appropriate state vector; so for $\alpha = 0$ we write
$|0\rangle = (s^\dagger u)(u^\dagger u^\dagger d^\dagger)|\text{Dirac sea}\rangle$

The operators are then inserted between these state vectors, and the contractions carried out. This will give rise to restrictions on the colours and spins of the particles to obtain *proper* matching.

-the full antisymmetrized form of the operator corresponding to each particle is then considered to calculate the *spectroscopic factor of each such contraction*, i.e. the fraction of the product state vector which has matching colour and spin. Thus we shall also find which spatial states are involved in each diagram.

These steps will now be taken in turn.

Non-vanishing contractions

The diagrams connecting the various channels via the one particle operators are given in table 2; table 3 lists the same for two-particle operators.

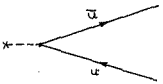


Table 2 : Non-vanishing diagram *for one body* operator. It occurs twice: the $\bar{u}$ can annhilate with either u in the proton.

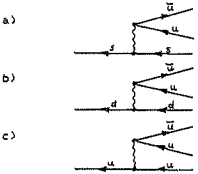


Table 3 : Non-vanishing diagrams *for two body* operator. Each diagram occurs four times: the interaction can involve either u quark in the proton and the second order expansion generates each topologically distinct diagram twice.

a) $1/(2\sqrt{2})(\langle 001|d^2R|\bar{1}1\rangle + \langle 101|d^2R|\bar{1}1\rangle - \langle 001|d^2R|\bar{0}1\rangle - \langle 101|d^2R|\bar{0}1\rangle$
$-\langle 011|d^2R|\bar{1}1\rangle - \langle 111|d^2R|\bar{1}1\rangle + \langle 011|d^2R|\bar{0}1\rangle + \langle 111|d^2R|\bar{0}1\rangle)$

b) and c)
$1/(2\sqrt{2})(\langle 001|d^2R|\bar{1}0\rangle - \langle 101|d^2R|\bar{1}0\rangle - \langle 011|d^2R|\bar{1}0\rangle + \langle 111|d^2R|\bar{1}0\rangle$
$+ \langle 001|d^2R|\bar{0}0\rangle - \langle 101|d^2R|\bar{0}0\rangle - \langle 011|d^2R|\bar{0}0\rangle + \langle 111|d^2R|\bar{0}0\rangle)$

Table 4 : Expansions of diagrams in table 2 for first excited lambda. The expansion *with the second* excited lambda involves similar expansions with $0 \longleftrightarrow 1$ in the lambda.

Matching coeff.cients

The diagram shown in fig. 3 (which occurs in contracting $\langle 0|d^2/dR^2|1\rangle$) will be considered in detail here; results for the other diagrams follow in similar fashion.

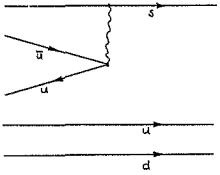


Fig. 3 : Example of diagram connecting the incoming channel to the first excited lambda.

Initially we take the quarks in the kaon to have a particular colour - say 1. In fig. 3 we see that the s-quarks in the kaon and lambda must have the same colour and spin. This means that only the part of the kaon state vector which has the s-quark in the spin-up state contributes (the s quark in the lambda has spin up); hence the anti-up must be spin down. Furthermore, the u and d in the lambda are coupled to spin 0, isospin 0; therefore only the part of the proton wave function which also has these quantum numbers is coupled to the lambda.

Adding up the coefficients of the wave functions which satisfy these conditions we arrive at a weight of -1/3 (where the fact that the anti-up quark can contract with either up in the proton has contributed a factor 2); but this is true for each of the

three possible colours of the s-quark so that the coefficient for this diagram is -1. Furthermore there are two topologically equivalent contractions which can give rise to this diagram. Hence this diagram is eventually accompanied by a factor of -2.

The spatial levels of the participating quarks must now be considered. We let the subscript le denote a quark which is on the left hand side asymptotically (see IV.3) so that e.g.

$$s_{le} = 1/\sqrt{2}\,(s_0 + s_1) \qquad \ldots(8a)$$

(the subscripts 0 and 1 refer to the value of the quantum number p at $z_0=0$; hence 0 represents the lowest symmetric level and 1 the lowest asymmetric). Similarly

$$s_{ri} = 1/\sqrt{2}\,(s_0 - s_1). \qquad \ldots(8b)$$

In this notation

$K^- = (s_{le}\ u_{le})$

$p = (u_{ri}\ u_{ri}\ d_{ri})$

$\Lambda^0 = (s_1\ u_0\ d_0)$

We therefore find that if we contract the operators corresponding to these particles under the assumption that the necessary spin and colour matching has already been done we obtain the following combination of matrix elements:

$$1/(2/\sqrt{2})((001|d^2R|\bar{1}) + (101|d^2R|\bar{1}\bar{1}) - (001|d^2R|\bar{0}1) - (101|d^2R|\bar{0}\bar{1})$$
$$-(011|d^2R|\bar{1}1) - (111|d^2R|\bar{1}\bar{1}) + (011|d^2R|\bar{0}1) + (111|d^2R|\bar{0}\bar{1}))$$

This is now multiplied by the factor -2 derived above.

The expansions of other diagrams are listed in table 4.

The matrix elements indicated by the above calculation can now be calculated. We shall consider particle-particle and particle-hole matrix elements separately.

a)particle-particle matrix elements

Consider the operator d/dR. The single particle matrix element (s.p.m.e.) $F_{12}=\langle 1|d/dR|2\rangle$ can be written as (assuming all particles to be massless)

$$F_{12}=\int d^3x \left[\phi_1^\dagger \quad \phi_1^\dagger \frac{\vec{\sigma}.\vec{p}}{\varepsilon_1}\right] \frac{d}{dR}\begin{bmatrix}\phi_2 \\ \frac{\vec{\sigma}.\vec{p}}{\varepsilon_2}\phi_2\end{bmatrix}$$

$$=\int d^3x \left(\phi_1^\dagger \frac{d}{dR}\phi_2 + \frac{1}{\varepsilon_1}\phi_1^\dagger \frac{d}{dR}\frac{\hat{p}^2}{\varepsilon_2}\phi_2\right) \qquad ...(9)$$

The differential equation for ϕ_2 is :

$$\left(\nabla^2 - \frac{1}{9}k^3[(|z|-z_0)^2+\rho^2] + \varepsilon_2^2\right)\phi_2 = 0 \qquad ...(10)$$

We substitute for $p^2 (= -\nabla^2)_2$ from this. Also, $R=2z_0$. Hence:

$$F_{12}=\frac{1}{2}\int d^3x\left((1+\frac{\varepsilon_2}{\varepsilon_1})\phi_1^\dagger \frac{d\phi_2}{dz_0} + \frac{2k^3}{9\varepsilon_1}\phi_1^\dagger(|z|-z_0)\phi_2 + \frac{1}{\varepsilon_1}\frac{\partial\varepsilon_2}{\partial z_0}\phi_1^\dagger\phi_2\right.$$

$$\left. -\frac{k^3}{9\varepsilon_1}\phi_1^\dagger[(|z|-z_0)^2+\rho^2]\frac{d\phi_2}{dz_0}\right) \qquad ...(11)$$

and

$$\phi_2 = (1/\sqrt{2\pi})C_2 e^{im_2\psi} D_p[\sqrt{2}\alpha_2(|z|-z_0)]h_{n_2m_2}(\alpha_2^2\rho^2)\begin{bmatrix}1\\0\end{bmatrix} \qquad ...(12)$$

Hence this depends on z_0 in 5 ways i.e. in

-the normalisation constant C_2

-the index p_2

-α_2 in the argument of the parabolic cylinder function D_p

-z_0 itself in the argument of this function

-α_2 in the radial function $h_{n_2m_2}$

As shown in appendix 4, the integrals over the radial variable ρ can be done analytically. The integrals over z have to be done numerically, however.

In fig. 4 the results of these calculations are shown . Fig. 5 contains results for the s.p.m.e's of d^2/dR^2, which are calculated in similar fashion.

b) particle-hole matrix elements

The matrix elements $\tilde{F}_{12}=\langle 1|d/dR|\tilde{2}\rangle$ are calculated using the charge conjugation operator $\tilde{C}$: $\tilde{C} = i\gamma_2$ and $\Psi_c = \tilde{C}\Psi^*$ with Ψ_c the charge conjugate bispinor of Ψ . Therefore

$$\tilde{F}_{12} = \int d^3x \, \Psi_1^\dagger (d/dR) \tilde{C} \Psi_2^* = \int d^3x \, [\phi_1^\dagger \quad \phi_1^\dagger \frac{\vec{\sigma}\cdot\vec{p}}{\varepsilon_1}] \, (d/dR) \begin{bmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{bmatrix} \begin{bmatrix} \phi_2^* \\ -\frac{\vec{\sigma}\cdot\vec{p}}{\varepsilon_2}\phi_2^* \end{bmatrix}$$

(where we have used the standard representation of the Dirac matrices.)

$$= \int d^3x \, (-\phi_1^\dagger [\frac{1}{\varepsilon_1}\frac{d}{dR}\frac{\partial\phi_2}{\partial z} + \frac{d}{dR}(\frac{1}{\varepsilon_2}\frac{\partial\phi_2^\dagger}{\partial z})]) \qquad \ldots(13)$$

Figs. 6 and 7 show the values of these matrix elements as well as the particle-antiparticle matrix elements of d^2/dR^2 as a function of the separation variable R.

B. Gluon interaction

The Hamiltonian of the gluon interaction can in general be written as [6]

$$H_{GLUON} = a(\vec{\lambda}_1\cdot\vec{\lambda}_2) + b(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\lambda}_1\cdot\vec{\lambda}_2) + c(\sigma_{2a}\cdot\sigma_{2b})(\vec{\lambda}_1\cdot\vec{\lambda}_2) \ldots(14)$$

Since we have ignored the interaction between quarks in constructing e.g. our intermediate states, and since this interaction is spin-dependent, we must suppress the spin-dependence in (14) too in order to retain a sense of consistency. So this part of the Hamiltonian is simply taken to be

$$H_{GLUON} = A\,\vec{\lambda}_1\cdot\vec{\lambda}_2 \qquad \ldots(15)$$

Now it is easy to prove [4] that the effect of $\vec{\lambda}_1\cdot\vec{\lambda}_2$ on a colour

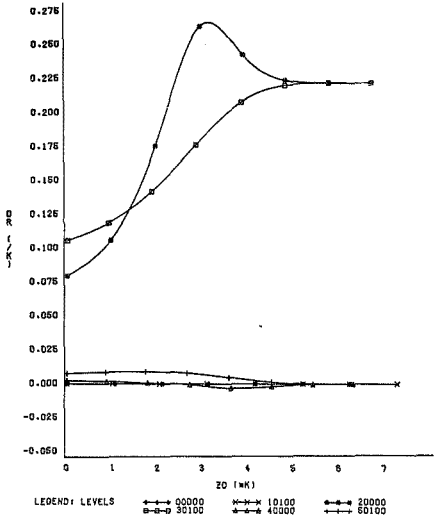


Fig. 4 : Particle-particle matrix elements: (1|dR|2)

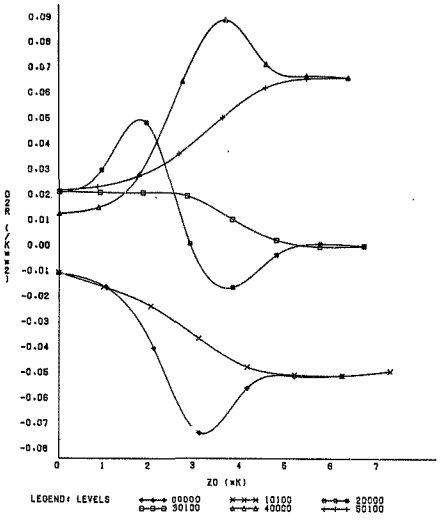
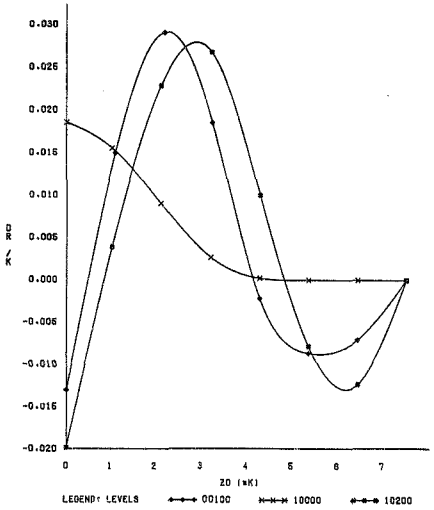


Fig. 5 : Particle-particle matrix elements: $\langle 1|d^2|R12\rangle$



LEVEL CODE: (P1 N1 P2 N2 M)

Fig. 6 : Particle-hole matrix elements: (1|dR|2)

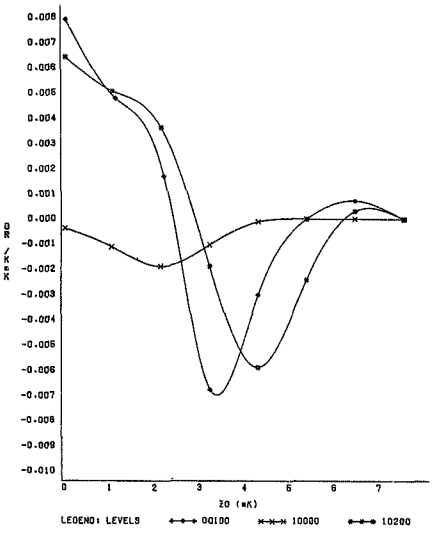


Fig. 7 : Particle-hole matrix elements: $\langle 11|d^2|12\rangle$

singlet baryon is to multiply the wave function by -8/3. We therefore calculate these matrix elements just as we did previously with two body matrix elements, and eventually multiply our result by -8/3 instead of 3.

We did not calculate the coupling between the two lambdas: this represents a higher order effect (that is incoming channel $\longrightarrow$ lambda$_1$ $\longleftrightarrow$ lambda$_2$.)

6. Solution of the coupled equations

At this stage we should again stress the crudeness of our model:
i) our phenomenological bag model is no more than a convenient tool; for instance, the fact that the particle and antiparticle states are not orthogonal shows that this is not a very exact model.
ii) even "better" bag models like the MIT-bag require other corrections - e.g. for spurious centre-of-mass motion. This has been neglected.
iii) we are using a seriously truncated basis to expand the scattering wave function.
iv) the Born-Oppenheimer approximation is being employed.
v) the gluon interaction is introduced in the form of free parameters; even then this is only included to first order.
vi) the hadrons are assumed to be pure three quark or quark-antiquark states.
vii)the relative wave function is treated non-relativistically.

Therefore we should obviously not trust our results in anything but broad outline.

Hence it was decided that solving the set of coupled equations in

their "full splendour" would not be in keeping with the spirit of the rest of the work. The following simplifications were subsequently made:

i) all potentials were set equal to zero outside a joining radius R_a; inside this they were replaced by their average values for $R<R_a$. R_a was chosen to optimise the similarity between the square well approximation and the true potentials.

ii) the Coulomb potential was simply ignored.

iii) colour confinement of the lambda/sigma states was ensured by simply erecting an extremely high square well potential at the joining radius.

Furthermore, a constant imaginary potential was added to the Born-Oppenheimer "potential" E in (III.6) to simulate the effect of background (non-resonant) scattering into channels that are not included in the calculation explicitly.

Now the problem of solving the N coupled equations can be reduced to a matrix eigenvalue problem as follows :

III.6 can be written as

$$\vec{W}+F\vec{W}+G\vec{W}=0 \qquad \ldots(16)$$

where $\vec{W}$ is a column matrix of size N with entries the expansion coefficients w of III.5:

$$\vec{W}=\begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \end{bmatrix} \qquad \ldots(17)$$

By comparing (17) and (III.6) we see that the generic element of F is

$$F_{\alpha'\alpha} = \langle\alpha'|d\mathrm{R}|\alpha\rangle;$$

also

$$G_{\alpha\alpha'} = \frac{2\mu}{\hbar^2}\left(\left(E\,\delta_{\alpha\alpha'} - E_{\alpha\alpha'}\right) + \langle\alpha'|H_{GLUON}|\alpha\rangle\right) + \langle\alpha'|d^2\mathrm{R}|\alpha\rangle.$$

F and G are constant NxN square matrices in the light of our square well approximation.

This is a matrix equation which must be solved subject to the boundary conditions

$\vec{W}(R=0)=\vec{0}$ (N conditions) ...(18a)

and

$$\vec{W}(R>R_a)=\begin{bmatrix} (S_{00}e^{ik_0R}-e^{-ik_0R})/2ik_0 \\ S_{01}e^{-K_1R}/2ik_0 \\ S_{02}e^{-K_2R}/2ik_0 \\ \vdots \end{bmatrix} \quad \text{(N conditions)} \qquad \ldots(18b)$$

where $k_0^2 = \frac{2\mu}{\hbar^2}(E - E_0)$

and $K_\alpha^2 = \frac{2\mu}{\hbar^2}(E_\alpha - E)$, $\alpha \neq 0$

E = centre of mass energy

S_{ij} = S-matrix element for i,j open channels.

In writing decaying exponentials for all channels other than the incoming, we have assumed that these channels are closed ($E_\alpha > E$). This is indeed the case in our calculation - because of colour confinement the lambdas cannot penetrate into the asymptotic region. Hence S_{01}, S_{02} ... are not S-matrix elements; we continue to label them with S, however. These, as well as the S-matrix element S_{00}, are obviously functions of the energy (E).

(16) is now converted into a set of 2N first order equations by writing

$\vec{V}=\dot{\vec{W}}$ ...(19).

Therefore

$$\begin{bmatrix} \dot{\vec{W}} \\ \dot{\vec{V}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0 & -F \end{bmatrix} \begin{bmatrix} \vec{W} \\ \vec{V} \end{bmatrix} \qquad \ldots(20)$$

This system is solved by making an exponential ansatz:

$$\vec{U} = \begin{bmatrix} \vec{w} \\ \vec{v} \end{bmatrix} = e^{\lambda R} \begin{bmatrix} \vec{c} \\ \vec{d} \end{bmatrix} \qquad \ldots(21)$$

This gives us 2N eigenvalues , $\lambda_1, \lambda_2, \ldots \lambda_{2N}$ and 2N eigenvectors - $\vec{c}_1, \vec{c}_2, \ldots \vec{c}_{2N}$. The general solution for W is then

$$\vec{w}(R) = a_1\vec{c}_1 e^{\lambda_1 R} + a_2\vec{c}_2 e^{\lambda_2 R} + \ldots + a_{2N}\vec{c}_{2N} e^{\lambda_{2N} R} \qquad \ldots(22)$$

The a's can now be found by using the 2N boundary conditions (18). This means solving the matrix equation (writing $\vec{c}_i = \begin{bmatrix} c_{1i} \\ c_{2i} \\ \vdots \end{bmatrix}$)

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1(2N)} \\ c_{21} & c_{22} & \cdots & c_{2(2N)} \\ \vdots & \vdots & & \vdots \\ c_{N1} & c_{N2} & \cdots & c_{N(2N)} \\ (\lambda_1 - ik_1)e^{\lambda_1 R} c_{11} & (\lambda_2 - ik_1)e^{\lambda_2 R} c_{12} & \cdots & (\lambda_{2N} - ik_1)e^{\lambda_{2N} R} c_{1(2N)} \\ (\lambda_1 + K_2)e^{\lambda_1 R} c_{21} & (\lambda_2 + K_2)e^{\lambda_2 R} c_{22} & \cdots & (\lambda_{2N} + K_2)e^{\lambda_{2N} R} c_{2(2N)} \\ \vdots & \vdots & & \vdots \\ (\lambda_1 + K_N)e^{\lambda_1 R} c_{N1} & (\lambda_2 + K_N)e^{\lambda_2 R} c_{N2} & \cdots & (\lambda_{2N} + K_N)e^{\lambda_{2N} R} c_{N(2N)} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -\left(e^{-\bar{\gamma}k_0 R}\right)^{\sim} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$\longrightarrow$ N zeros, corresponding to each w being = 0 at the origin

$\longrightarrow$ incoming wave for each of the incoming channels

$\ldots(23)$

The solution of (23) gives us a_1, a_2, $\ldots a_{(2N)}$. From (22) we thus obtain W(R), and hence S_{00}, S_{01} ... from (18b).

7. Results

In the final calculation the following parameters are free:
-the mass of the kaon-proton system when the two bags have fused. We do not expect this to differ too much from the sum of the free kaon and proton masses; on the other hand it will obviously not be identical to this sum because of the gluon interaction.
-the free masses of the two lambdas (that is the mass as it would be measured if the lambda were a stable particle; the word "free" is used to distinguish this from the position of the measured resonance). Because of the strong coupling between the various channels this will not be identical to the position of the observed resonance.
-the magnitudes of the gluon interaction matrix elements.
-an imaginary potential which is required to mimic non-resonant scattering into channels not explicitly included in the calculation.

These parameters were estimated by adjusting them until approximate agreement with the scattering amplitudes extracted from experimental data was obtained. Since the scattering amplitude for the s-wave isospin zero channel has been extracted as such, this can be done straightforwardly.

It turns out that the gluon coupling terms are quite dominant; the matrix elements arising from the operation of the kinetic energy operator on the eigenstates of the intrinsic Hamiltonian are entirely negligible.

As can be seen in fig. 5 the amplitude calculated in this way mimics the main features of the extracted amplitude fairly well. Table 5 lists the values of the parameters which result in this fit.

Because we have chosen to ignore the spin dependence of the gluon interaction, our parametrized coupling elements can not be compared to calculated matrix elements for spherical bags *directly. On the other hand we can compare* the orders of magnitude thus obtained: Viollier [5] found the diagonal ground state gluon matrix element to be around 0.2 GeV; our values are somewhat smaller, as is to be expected from the fact that the matrix elements involved in our calculation couple particle states to antiparticle states.

The free masses of the excited lambdas are interesting: we find the first excited state, which resonates at about 1680 MeV to have a free mass of about 1300 MeV. The higher lying lambda moves even further: the resonance at 1800 MeV is due to a state with free mass 1380 MeV. These results are of course very important if one wants to fit the static properties of the hadrons with some bag model.

Gluon coupling elements:

$\langle 0|H_{GL}|1\rangle = 0.027$ GeV

$\langle 0|H_{GL}|2\rangle = 0.10$ GeV

Free masses of particles:

5-quark system : 1.410 GeV

First excited lambda: 1.300 GeV

Second excited lambda: 1.380 GeV

Imaginary potential well depth: 0.075 GeV

Table 5 : Parameters obtained in fit of scattering amplitudes.

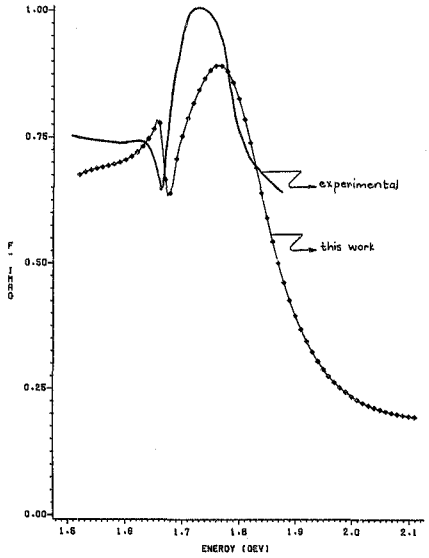


Fig. 5a : Imaginary part of scattering amplitude: calculated and extracted from experimental data.

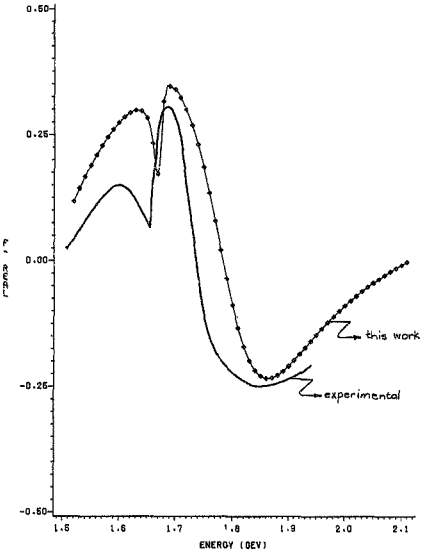


Fig. 5b : Real part of scattering amplitude: calculated and *extracted from experimental* data.

V. CONCLUSION AND OUTLOOK

The work described above is incomplete in various respects. Most important is the lack of knowledge about the gluon matrix elements. To calculate these properly one would specify the restrictions which determine the gluon eigenstates- e.g. that they satisfy the MIT gluon boundary conditions at the "boundary" of the combined bag, where the boundary is determined as in chapter *II*. Then the gluon eigenmodes would be calculated and the values of the required matrix elements ascertained by making a perturbative expansion (as can be done inside the bag [6].)

A further aspect that should be treated more carefully is the calculation of the hadronic state vectors. There is good reason to believe [11] that the mesons, for instance, are not pure quark-antiquark pairs; the gluon component as well as the multi-(quark-antiquark) component of the wave function might in this case not be negligible. Also, the description of the excited lambdas used in this research might well turn out to be incorrect if the spectroscopy is done in more detail by including inter-quark interactions.

It is also obvious that the scheme of confinement utilised here - the sum of equal amounts of scalar and timelike vector potentials - is not to be seen as anything but a convenient model. The effect of the non-orthogonality of the particle and antiparticle states, for instance, should be investigated.

On the positive side a few interesting conclusions can be made:
-there might be a fundamental problem with non-spherical MIT bags.
-an order-of-magnitude estimate of the size of the gluon matrix

elements has been obtained.

-the static masses of the excited lambdas, as opposed to the position of the resonances caused by them, has been estimated. Since this does differ from the static value appreciably one concludes that attention has to be paid to such dynamic aspects if one wants to fit the static properties of the unstable hadrons (which are, after all, measured in scattering experiments.) Thus far this aspect seems to have been ignored [6].

Finally the mere fact that this calculation could be carried out is of some importance; this method could, in a more refined form, be useful in a wide variety of situations involving the strong interaction

APPENDIX 1 : CUBIC MIT BAG

We want to solve the Dirac equation with the linear MIT boundary condition

$$-i\vec{\gamma}.\hat{n}\,\Psi = \Psi \qquad \ldots(1)$$

($\hat{n}$ is the inward normal unit vector)

satisfied on the edges of a cube. To this end we make the usual separation of the quark bispinor into two spinors:

$$\Psi = \begin{bmatrix} \phi \\ \chi \end{bmatrix} \qquad \ldots(2)$$

and obtain from the Dirac equation the coupled equations (using the standard representation)

$$(\vec{\sigma}.\vec{p})\phi = (\varepsilon+m+U)\chi \qquad \ldots(3a)$$

$$(\vec{\sigma}.\vec{p})\chi = (\varepsilon-m-U)\phi \qquad \ldots(3b)$$

Inside the box $U = 0$; hence we obtain (by operating with o.p on (3a) and using (3b))

$$(\nabla^2+k^2)\phi = 0 \qquad \ldots(4a)$$

with $k^2 = \varepsilon^2-m^2$.

Similarly:

$$(\nabla^2+k^2)\chi = 0 \qquad \ldots(4b)$$

Hence both components of both ϕ and χ are superpositions of plane waves with the size of the wave vector equal to k. Therefore, in general:

$$\phi = \begin{bmatrix} \int_{|\vec{k}|=k} d^3k\; A(\vec{k})e^{i\vec{k}.\vec{r}} \\ \int_{|\vec{k}|=k} d^3k\; B(\vec{k})e^{i\vec{k}.\vec{r}} \end{bmatrix} \qquad \ldots(5a)$$

and

$$\chi = (\vec{\sigma}.\vec{p})/(\varepsilon+m)\,\phi = \begin{bmatrix} \frac{\partial}{\partial z} & \frac{\partial}{\partial x}-i\frac{\partial}{\partial y} \\ \frac{\partial}{\partial x}+i\frac{\partial}{\partial y} & -\frac{\partial}{\partial z} \end{bmatrix} \phi/(\varepsilon+m)$$

$$= i/(E+m)\begin{bmatrix} \int_{|k|=k} d^3k \ (A(\vec{k})k_z + B(\vec{k})(k_x - ik_y))e^{i\vec{k}.\vec{r}} \\ \int_{|k|=k} d^3k \ (A(\vec{k})(k_x + ik_y) - B(\vec{k})k_z)e^{i\vec{k}.\vec{r}} \end{bmatrix} \quad \ldots(5b)$$

We now consider a specific edge; say x=a . Here the outward normal is

$\hat{n} = \hat{x}$

and hence

$\hat{n}.\vec{\gamma} = \gamma_x$.

So the linear boundary condition is

$$i \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \Psi = \Psi \quad \ldots(6)$$

for x=a ; all y and z on the face .

This requires that, for all k_x, k_y and k_z having |k|=K

$$-1/(E+m)(Ak_z + B(k_x - ik_y)) = B \quad \ldots(7a)$$

$$-1/(E+m)(A(k_x + ik_y) - Bk_z) = A \quad \ldots(7b)$$

The only solution of this is the trivial solution; there exists no solution to the cubic bag if the boundary condition (1) is adhered to.

APPENDIX 2 : TWO-CENTRE HARMONIC OSCILLATOR BAG

For the two-centre case the scalar part of the potential is (in cylindrical coordinates) :

$$V_{sc}=(1/18)k^3[(|z|-z_0)^2+\rho^2)] \qquad ...(1),$$

and the time part vector potential is, by prescription, equal to it.

Therefore we get for the upper component of the spin up top component of our quark bispinor

$$(\nabla^2-\alpha^4[(|z|-z_0)^2+\rho^2]+P^2)\,\phi=0 \qquad ...(2)$$

where

$$\alpha^4=(1/9)k^3(E+m)$$

and $P^2=E^2-m^2$.

Now, in cylindrical coordinates

$$\nabla^2=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2} \qquad ...(3)$$

The separation is effected by writing

$\phi=f(\rho)g(\varphi)h(z)$.

The separated equations are

$$\frac{1}{f}\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{df}{d\rho}\right)-\alpha^4\rho^2-\frac{m_\varphi^2}{\rho^2}+\mathcal{E}_\zeta=0 \qquad ...(4a)$$

$$\frac{d^2g}{d\varphi^2}+m^2g=0 \qquad ...(4b)$$

$$\frac{1}{h}\frac{d^2h}{dz^2}-\alpha^4(|z|-z_0)^2+(P^2-\mathcal{E}_\zeta)=0 \qquad ...(4c)$$

(with $\mathcal{E}_\zeta$ a separation constant)

Therefore

$$f(\rho)=e^{-(\alpha^2\rho^2)/2}(\alpha^2\rho^2)^{|m_\varphi|/2}L_{n_\rho}^{|m_\varphi|}(\alpha^2\rho^2) \qquad ...(5a)$$

($n_\rho=0,1,2\ldots$)

where L_n^m is a reduced Laguerre polynomial of the indicated order,

$$g(\varphi)=e^{im_\varphi\varphi} \qquad ...(5b)$$

$(m_\psi = \ldots,-1,0,1,\ldots\ldots, |m_\psi| \leq n_\rho)$

$h(z) = D_p[\sqrt{2}\alpha(|z|-z_0)]$ or

$h(z) = \mathrm{sgn}(z) D_p[\sqrt{2}\alpha(|z|-z_0)]$ ...(5c)

with D_p a parabolic cylinder function. p is to be found from the boundary conditions. The equation in z is unaffected by the transformation z -> -z. Therefore parity with respect to this transformation is a good quantum number: the function must be either even or odd under this transformation (these being respectively the first and second solutions in (5c)). For h(z) and its derivative to be continuous at z=0 this requires that

$D_p[-\sqrt{2}\alpha z_0] = 0$ (odd parity) or

$D_p[-\sqrt{2}\alpha z_0] = 0$ (even parity). ...(6)

Finally the energy of of such a solution is obtained from the relation

$(\mathcal{E}-m)(\mathcal{E}+m)^{1/2} = \kappa^{3/2}[(2/3)(2n_\rho + |m_\psi| + p + 3/2)]$ ...(7)

Note that the z_0-dependence of the resulting eigenvalue $\mathcal{E} = \mathcal{E}(z_0)$ resides inthe function $p = p(z_0)$.

APPENDIX 3 : ONE DIMENSIONAL SQUARE WELL SCATTERING

Introduction

The scattering of two infinite 1-dimensional square wells will be considered. Each has width 2d, and each is assumed to contain a single particle; the particles are assumed to have the same mass.

The calculation will be based on the Born-Oppenheimer approximation; calculations are non-relativistic.

Model

During the collision the 2 particles are assumed to co-exist in a combined square well. In the centre of this we insert a δ-function potential to simulate 3-dimensional scattering, as follows:

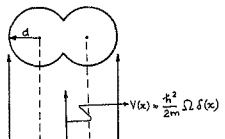


Fig. 1: The scattering potential is assumed to be an infinite square well with a central barrier; the height of the barrier and the width of the square well are functions of the separation between the "bags".

The size of the interior potential was chosen according to

$$\Omega \propto \frac{\text{(maximum cross-sectional area of 3D bag) - (contact area)}}{\text{contact area}}$$

$$\propto [(a-d)^2/d^2(2ad-a^2)] \qquad \ldots(1)$$

(where the factor $1/d^2$ ensures the correct dimensions.)

As can be seen in fig. 1, the length of the combined well - a - is related to the distance between the centres of the "bags", R, by

$$a=d+R/2. \qquad \ldots(2)$$

One particle was chosen to be in the lowest negative parity state (1^-) and the other in the lowest positive parity state, (1^+); no transition from these levels was allowed.

Results

For the indicated potential the energy levels are given in terms of the size of the square well and the interior potential by (cf. Flugge [12])

$$E_n=h^2k_n^2/(2m) \qquad \ldots(3)$$

where

$$-k_n a=n \qquad \text{for even parity states} \qquad \ldots(4a)$$

$$-k_n \cot(k_n a)=-\Omega \qquad \text{for odd parity states} \qquad \ldots(4b)$$

Using this the energy levels of the required states can be calculated as a function of the separation R - see fig. 2.

Then, in the Born-Oppenheimer approximation, the effective scattering potential is given by

$$V(R)=[\,E^{1^-}(R) - E^{1^+}(R)\,] - [\,E^{1^-}(\infty) - E^{1^+}(\infty)\,]. \qquad \ldots(5)$$

The size of the interior potential was chosen according to

$$\Omega \propto \frac{\text{(maximum cross-sectional area of 3D bag)} - \text{(contact area)}}{\text{contact area}}$$

$$= [(a-d)^2/d^2(2ad-a^2)] \qquad \ldots(1)$$

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$$a = d + R/2. \qquad \ldots(2)$$

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$$E_n = h^2 k_n^2/(2m) \qquad \ldots(3)$$

where

$-k_n a = n$ for even parity states ...(4a)

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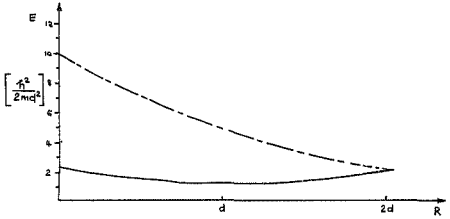


Fig. 2 : Lowest 2 energy levels as a function of the separation.

The resulting potential was approximated by a triangular barrier, as shown in fig. 3. For such a barrier an analytic expression can be found for the transmission coefficient, since the wave function in the region of the potential is a superposition of the two Airy functions .

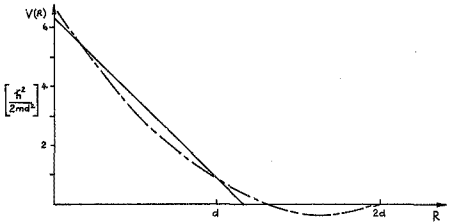


Fig. 3: Born-Oppenheimer potential and triangular [illegible]rier approximation.

The transmission coefficient is plotted as a functi[illegible] energy in fig. 4. (Energies are scaled by the particle mass.) We see

that the transmission function is a smooth, slowly changing function of the energy - the change from T~0 to T~1 takes place in the range E~0,5 x barrier height to E~1,5 x barrier height.

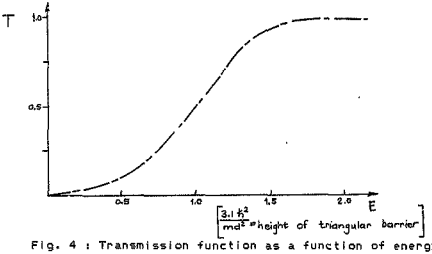


Fig. 4 : Transmission function as a function of energy.

APPENDIX 4 : INTEGRALS OF LAGUERRE FUNCTIONS WITH DIFFERENT ARGUMENTS

We need to calculate integrals of the form

$$Y_{12} \triangleq \int_0^\infty \rho d\rho (\alpha_1^2\rho^2)^{m/2} (\alpha_2^2\rho^2)^{m/2} e^{-(\alpha_1^2+\alpha_2^2)\rho/2} L_n^m(\alpha_1^2\rho^2) L_n^m(\alpha_2^2\rho^2) \quad ...(1)$$

Let

$$\alpha_1^2\rho^2 = y$$

We then obtain

$$Y_{12} = (\alpha^m/2\alpha_1^2) \int_0^\infty dy \; y^m e^{-(1+\alpha^2)y/2} L_n^m(y) L_n^m(\alpha^2 y) \quad ...(2)$$

(with $\alpha = \frac{\alpha_2}{\alpha_1}$)

The generating function of the Laguerre polynomials is now employed :

$$\sum_j s^j L_j^m(y) = e^{-xs/(1-s)}/(1-s)^{m+1} \quad ...(3)$$

This gives

$$\sum_{jk} s^j t^k Y_{jk} = \int_0^\infty dy \; y^m e^{-(1+\alpha^2)y/2 - ys/(1-s) - \alpha^2 yt/(1-t)} / [(1-s)(1-t)]^{m+1} \quad ...(4)$$

This integral is easily calculated: it represents a gamma function. Equating for each order of s and t , and doing the appropriate binomial expansions we finally get

$$Y_{12} = \left(\frac{2}{1+\alpha^2}\right)^{m+1} \frac{\alpha^m}{2\alpha_1^2} \sum_{n=0}^{b} (-)^{a+b+n} \frac{(m+a+n)!}{(b-n)!n!(a-b-n)!} \left(\frac{1+\alpha^2}{1-\alpha^2}\right)^{a-b+2n} \quad ...(5)$$

(for $b \leq a$, which can always be arranged.)

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