Balance sheet management solution: Integrating credit, interest and liquidity cost

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Declaration

I declare that this Thesis is my own, unaided work. It is being submitted for the Degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

Abstract

This thesis main contribution is the model framework based on the projection of the banks' balance sheet taking all major financial risks into account. Real bank data is used to support this analysis, covering a large range of customer and product level data across both retail and corporate products. This thesis explicitly explores the link between retail customers from a deposit, lending activity and performance and other market driven factors such as liquidity risk premium of cost of funding. Various optimization techniques are tested, confirming the value of a strategy that dynamically re-balance the funding profile of the bank versus a more static approach.

In this thesis, we apply two optimisation frameworks to determine the optimal wholesale funding mix of a bank, given uncertainty in both credit and liquidity risk. A stochastic linear programming method is used to find the optimal strategy to be maintained across all scenarios. A recursive learning method is developed to provide the bank with a trading signal to dynamically adjust the wholesale funding mix as the macroeconomic environment changes. The performance of the two methodologies is compared in chapter 3. The optimisation target is the net interest income of the bank. The on-line recursive learning method provides superior results as this allows the bank to dynamically adjust the funding profile.

This thesis integrates the sub-components underlying the bank's balance sheet to facilitate the projection of the net interest income allowing for both liquidity, interest and credit risk. The sub-components include retail and wholesale loans, retail and wholesale deposits and bank issued debt instruments. Actual historical data was obtained from a South African bank to calibrate a model for each of these sub-components (discussed in chapters 4-7).

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Chapter 1

Introduction

1.1 Background

Traditionally banks provide loans to both retail and corporate counterparties. These loans yield a certain interest income for the bank. The bank requires funding to support these lending activities. The bank incurs a cost to raise the required funding (i.e. the interest paid on the funding). The main types of funding available to a bank are:

- Deposits, to both retail and corporate customers.
- Debt instruments of varying term, issued directly to the market.

The bank needs to ensure the interest income on its loans exceeds the rate paid on the funding to ensure profitability.

The description of the operation above is based on the traditional lending and deposit taking activities of a bank, and excludes the investment banking division and non-interest revenue related activities. Optimising these components is beyond the scope of this thesis.

The above expose the bank to the risk of counterparties failing to repay the loans, which is termed credit events. The deposit and debt instruments used to fund the loans are usually short term in nature. This creates a mismatch compared to the long-term nature of the loans (i.e. 20-year mortgage loans). This mismatch exposes the bank to interest rate risk (loans and funding re-price at different durations) and liquidity cost (the uncertainty of the cost of funding at future dates). The extreme and novel macroeconomic realities observed over the last couple of years exposed a number of weaknesses in the risk management methodologies used by banks. This includes much higher credit losses than expected, higher liquidity premiums on wholesale funding during times of distress and the volatility of the deposit base during a flight to safety. A major weakness in the current risk management methodologies is the understanding of the relationship between credit, liquidity and interest rate risk.

A large body of research exists across the various banking risk types focusing on credit risk strategies, capital management or liquidity risk assessment in isolation. Limited research on integrating the various risk types into one model exists. Empirically results to test the importance of the interdependencies between risk types is also limited.

1.2 Research problem

This thesis aims to develop an model to represent the main components underlying the bank's balance sheet: retail and corporate loans, retail and corporate deposits and bank issued debt instruments. The interest income and interest cost from each of these components is simulated via a set of macroeconomic scenarios over the projection period. The integration of credit, interest and liquidity risk is achieved from the balance sheet model of the bank. The generation of the macroeconomic scenarios are outside the scope of the thesis.

The wholesale funding instruments available to the bank range from overnight to 5 year instruments. The bank needs to decide on the mix of these wholesale funding instruments. This thesis aims to use the various sub models of the interest income, credit risk and interest cost to feed the model office, and allows us to find the optimal mix of wholesale funding instruments for the bank, given uncertainty in interest, liquidity and credit risk.

The optimisation target of this thesis is the net interest margin (after taking credit losses into account). A representation of the monthly net interest income ("NII") is shown below:

$$NII = (Interest_{income} - credit_{loss}) - Interest_{cost}. \tag{1.1}$$

Non-interest income is excluded, as this more a business strategy component as opposed to a interest income and credit cost view.

The empirical study is based on actual data from a South African bank. At least 5 years of historic portfolio and account level data and observed market data is used to develop the various sub models. The model that integrates the various components to construct the balance sheet is termed the model office and facilitates the projection of the interest income and cost to support the optimisation. This requires the following components:

- Interest earned and credit losses on a portfolio of retail mortgages and personal loans.
- Interest earned and credit losses on a portfolio of corporate loans.
- The interest cost and corresponding supply of deposit funding through retail and corporate channels.
- The remaining wholesale funding required and its corresponding interest cost.

Each of the components listed above is a dedicated research field in its own right. For this thesis, in-depth complex sub models were chosen for some of the components and standard sub models for others. This thesis aims to add to current literature by:

- Proposing a new balance sheet management approach, taking all major risks into account and calibrating this to actual bank data, to analyse the optimal balance sheet structure given a set of macroeconomic outcomes.
- In-depth sub models: Calibrating a credit model to two retail portfolios.
- In-depth sub models: Decomposing and calibrating the wholesale funding mechanism.
- In-depth sub models: Exploring the link between retail customers from a deposit base and lending activity basis.
- Simplified sub models: The sub models for corporate loan portfolios and the available deposit pool are based on simple existing models.

1.3 Conclusion

The section below provides an overview of the current research with regard to the stated research problem and a methodical breakdown of the overall structure of the thesis. The thesis shows that an on line recursive learning optimisation approach will assist a bank in better managing its balance sheet compared to the static approach that tries to maintain an optimal strategy across all scenarios. The on line learning approach has the benefit of adjusting the wholesale funding as markets start to change. The link between credit and liquidity risk is essential to determine any empirical optimisation structure given the overlap of the performance of the bank's assets to this price it pays for funding in the wholesale market, this relationships was tested as part of this thesis. A simple linear buildup of the banks' balance sheet and corresponding profit function is used, with the complexity of credit losses and funding costs changes captured in a series of complex sub models. The output of the calibration of the various sub models further supports the choice of the sub models and provides a solid reference for the successful application of complex models on empirical data. The output from these sub models are both logical and robust.

1.4 Current research

The focus of this section is to provide a review of the literature on optimising the bank's balance sheet under credit risk and liquidity cost. The literature study of the sub models are addressed in subsequent chapters.

Bank asset liability modelling

Asset liability management is an extensive topic of study, especially for insurance companies and pension schemes. A thorough review of banking asset liability management can be found in Kosmidou and Zopounidis (2004) [67]. We will not attempt a separate survey here.

In a paper, Hahm, Shin and Shin (2011) [51] investigated the relationship between the credit risk underlying a portfolio over a cycle and the availability of non-core deposits (liabilities). The focus of the thesis was to investigate the ability of banks to increase the size of their balance sheet, taking both credit and liquidity risk into account.

Birge and Judice (2013) [13] extended the idea by combining key risks underlying the balance sheet, such as credit, liquidity and interest rate risk into an asset liability management framework. The aim of the research is to develop a balance sheet management framework that integrate all the major risks faced by a bank.

The approach underlying this thesis is similar to Birge and Judice (2013). Birge and Judice (2013) considered a theoretical asset and liability profile. In this thesis, we construct a realistic balance sheet including multiple asset portfolios and a diverse funding mix. The calibration of the model is based on actual historical performance allowing us to empirically test the research question. This thesis builds on current research by explicitly incorporating credit and liquidity into an asset liability management framework to analyse an optimal balance sheet structure. The research extends the balance sheet to a more realistic representation. A large number of numerical intensive sub models calibrated using actual performance data supports this thesis.

Optimisation

There are a number of approaches supporting portfolio optimisation. Two of the approaches considered as part of this thesis are; the first is based on the Markowitz approach to optimise the expected return, given a level of risk measured by the variance of the return. Pyle (1971) [87] developed a methodology under which a financial intermediary is willing to sell deposits at an uncertain rate in return for assets at an uncertain rate. This methodology is based on the maximisation of the concave preference function (utility function). This methodology is also used by Birge and Judice (2013).

The second approach optimises the expected return underlying a portfolio, given a set of constraints. This approach was chosen for this thesis as it ties in with the empirical approach chosen to address the research question.

Deterministic models use a linear programming methodology and assume a certain deterministic realisation of the random variables such as asset returns and interest cost. An example of a deterministic model is Chambers and Charnes (1961) [24] which determined the optimal portfolio for an individual bank given a set of limits and restrictions. Tayi and Leonard (1988) [101] applied a multi-objective constraint model, where the decision maker was presented with a choice of alternative strategies to derive an implicit utility function. This implicit function is used to optimise the portfolio by allowing for the multiple objectives provided.

Stochastic versions of the optimisation methodology allow for parameter uncertainty and include change constraint programming, dynamic programming, sequential decision theory, stochastic decision trees and linear programming under uncertainty.

An approach to allow for uncertainty of the random variables is to investigate this as part of a simulation study by generating a series of economic scenarios using their probability density function. The optimal portfolio under each scenario is determined deterministically. Such an approach was used by and Kosmidou and Zopounidis (2004) [67], with a goal programming model to optimise the portfolio under each simulated scenario. However, this does not guarantee an optimal portfolio overall, as this is dependent on the completeness of the individual scenarios generated. Stochastic Programming is an example of this this approach and is covered in model detail in chapter 3.

Kosmidou and Zopounidis (2004) [67] utilised a goal programming systems using a stochastic interest rate process for determining the interest rates. The interest rate income and cost are assumed to follow a normal distribution, and 2500 scenarios are drawn from these distributions to allow for parameter uncertainty. This approach does not explicitly allow for the credit loss, and does not link the 2500 interest rate scenarios to a macroeconomic environment.

Birge and Judice (2013) [13] developed a dynamic model of the balance sheet, explicitly taking the interest earned and interest cost uncertainty into account, as well as explicitly allowing for credit risk in the model of the balance sheet. Their work forms the basis for an optimisation model to determine the balance sheet management strategy. Birge and Judice (2013) provided a framework to integrate funding uncertainty and credit risk into a balance sheet optimisation framework; their work did not extent to the actual optimisation of the utility function.

The approach proposed for this thesis will use a similar approach to Kosmidou and Zopounidis (2004) to simulate various outcomes of the uncertain variables and deterministically optimise the choice of liabilities under each outcome. The optimal liability strategy for each simulation will be determined deterministically, using an integrated credit risk and liquidity cost approach as put forward by Birge and Judice (2013).

Taking the above into accounts this thesis proposes a methodology that satisfies the following criteria:

- Integrates interest rate risk, liquidity cost and credit risk in the construction of the balance sheet and net interest income function.
- The optimisation methodology should allow for the uncertainty in the following parameters; interest earned on assets, interest cost and credit loss.

- The model should allow for the liquidity cost in the balance sheet framework. Liquidity cost should be incorporated in the framework through the availability and cost of future funding.

- The approach should allow for the dynamic interaction between macroeconomic factors and the above parameters.

- The model of the balance sheet and profit function should allow for the contractual agreement of assets and liability, including any re-pricing agreements due to changes in interest rates.

1.5 Structure of thesis

Chapter 2: Model office

A model office is a mathematical process to replicate the maturing profile of the asset portfolio and corresponding new business written and corresponding interest income. The model office allows for the projection of the deposit funding and corresponding interest cost. This is used to determine the funding gap to be funded via issuing wholesale funding.

Chapter 3: Optimisation

Discussion of the two optimisation methodologies and corresponding results of the optimal wholesale funding mix.

Chapter 4: Retail credit risk

Decompose the observed credit risk experience into an age, vintage and economic cycle effect. This methodology is used for the two retail credit risk portfolios considered in this thesis.

Chapter 5: Corporate credit risk

The model used to project the credit risk underlying corporate assets. The Cox regression model is applied to determine the impact of macroeconomic scenarios on the observed rating migration of the corporate loan portfolio.

Chapter 6-7: Funding

The funding available to a bank consists of retail deposits, corporate deposits and wholesale funding that each varies in term, risk profile and cost. The balance sheet optimisation framework needs to incorporate the characteristics of these funding options to determine the optimal funding mix. A sub model of the cost and risk profile of each funding option is required for the optimisation model.

Chapter 6 sets out the framework applied to capture the volatility in the cost of funding of the wholesale funding. Chapter 7 sets out the framework applied to capture the volatility in both the level and interest cost of the retail and corporate deposits. The behaviour of deposit holdings under certain economic scenarios is required to allow a consistent evaluation with the level of wholesale funding.

Chapter 2

Model office

The model office facilitates the projection of the banks income statement given the loans and funding mechanisms.

2.1 Methodology

The specification of the NII function in equation 1.1 is a simplification of the complexity of the mathematical system required to project the NII. To support this thesis, a detailed model office was developed to simulate the banks' balance sheet and corresponding NII over the 60-month period based on a set of macroeconomic scenarios.

The model office projects the bank's balance sheet over a 60-month projection period. The balance sheet is defined per the following structure $Assets = Liabilities + Equity$.

The assets are defined at month t as:

$$A_t = X_t^1 + X_t^2 + X_t^3 + X_t^4 \tag{2.1}$$

where X^1 represents the retail mortgages, X^2 the personal loans, X^3 the corporate loans and X^4 the risk-free asset portfolio (i.e. government bonds). At the end of each period t, the asset portfolio reduces due to the monthly capital repayment, maturing loans and incurred credit losses. New loans make up for this natural reduction in the asset portfolio. New loans allow the bank to reprice the asset portfolio. We assume the asset portfolio stay constant over the projection period. For the purposes of this thesis we set $X^1 = 240$ billion Rand, $X^2 = 30$ billion Rand, $X^3 = 107$ billion Rand

and $X^4 = 63$ billion Rand.

The liabilities are defined at month t as:

$$L_t = X_t^5 + X_t^6 + X_t^7 + X_t^8 + X_t^9 + X_t^{10} \tag{2.2}$$

where X^5 represents the retail deposits, X^6 the corporate deposits and X^7,...,X^{10} the wholesale funding portfolio of duration 6,12,18 and 24 months. This funding mix was chosen, given data available, to calibrate the sub model in chapter 6 and it represents the range of funding options available to South African banks. For the purposes of this thesis we set $X^5 = 100$ billion Rand and $X^6 = 100$ billion Rand.

For the purpose of this thesis, we will keep the equity level constant over the projection period.

A representation of the monthly **net interest income margin** ("NII") from equation 1.1 using the model office notation is:

$$\begin{aligned} NII = {} & X^1 \times (x^1 - CL^1) + X^2 \times (x^2 - CL^2) + X^3 \times (x^3 - CL^3) + X^4 \times (x^4) \\ & - X^5 \times x^5 - X^6 \times x^6 - X^7 \times x^7 - X^8 \times x^8 - X^9 \times x^9 - X^{10} \times x^{10} \end{aligned} \tag{2.3}$$

where x^1,x^2,x^3,x^4 is the interest rate received on the assets above.
CL^1,CL^2,CL^3 is the credit loss on the assets above.
x^5,x^6 is the interest paid on retail and corporate deposits.
x^i, for $i = 7, 8, 9, 10$ represents the interest rate paid on each debt instrument.

For the purposes of this thesis, we used 6,12,18 and 24 months debt instruments for X^i, for $i = 7, 8, 9, 10$. The interest earned on the asset portfolios (x^1,x^2,x^3) is net of the credit loss (CL^i) for the remainder of this thesis.

The funding costs on the retail and corporate deposits (X^5 and X^6) are in general cheaper compared to wholesale funding instruments. For the purposes of this thesis, we assumed the bank will always use deposit funding before accessing wholesale funding. In addition to this, the size of available retail and corporate deposit funding fluctuate over the 60-month projection period, influencing the gap or size of wholesale funding required.

The balance sheet structure is defined as

$$X_t^1 + X_t^2 + X_t^3 + X_t^4 = X_t^5 + X_t^6 + X_t^7 + X_t^8 + X_t^9 + X_t^{10} + E, t \in [1, 60] \quad (2.4)$$

where E is fixed over the projection period.

A portion of the wholesale funding base will mature each month based on previous funding decisions. For example, the entire portfolio will matures if only funded via monthly instruments. Let Xm_t^i indicate the portion of the wholesale funding portfolio that mature in month t for each $i = 7, 8, 9, 10$. Define Xm_t^7, Xm_t^8, Xm_t^9 and Xm_t^{10} as the wholesale funding instruments maturing in month t.

Define G_t as the size of debt instruments the bank needs to issue in month t to ensure equation 2.4 holds. This is termed the size of the funding gap in month t for the remainder of the thesis. We assume both the equity level (E_t) and the asset portfolio remain constant. The funding gap G_t is thus a function of a change in the size of the deposit portfolios and the sum of all the maturing wholesale instruments (Xm_t^i), where $i = 7, 8, 9, 10$. G_t is defined as:

$$G_t = -(X_t^5 - X_{t-1}^5) - (X_t^6 - X_{t-1}^6) + Xm_t^7 + Xm_t^8 + Xm_t^9 + Xm_t^{10}. \quad (2.5)$$

Each month, the bank needs to choose between the various wholesale funding instruments to fill the funding gap. The optimisation problem tries to identify the best funding mix by optimising the NII function.

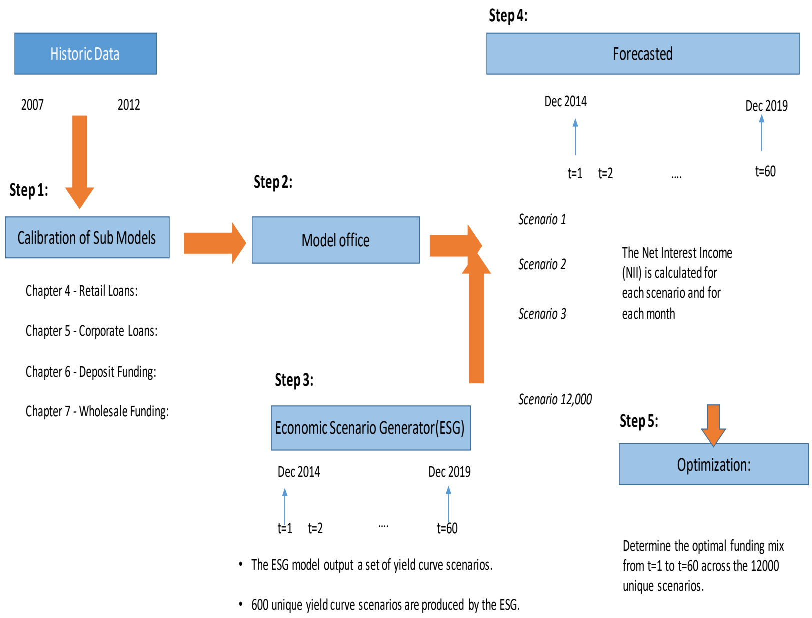
Let F_t be a vector of the funding decision, $\overline{F_t} = \left\langle F_t^7, F_t^8, F_t^9, F_t^{10} \right\rangle$ such that F_t^7 represents portion of the funding gap (G_t) to be filled by wholesale instruments X_t^7.

2.2 Projection

Figure 2.1 highlights the steps followed to apply the two optimisation methodologies to optimise the NII as set out in equation 2.3.

Step 1: A sub model is required for each of the components in equation 2.3. Each of the sub models are calibrated to historical performance data of the bank. The output of the sub model is a mathematical expression to facilitate the projection of each of the components in equation 2.3.

Figure 2.1: Diagram of the model framework to apply the optimisation methods.



Step 2: The model office combines the asset and liability behavior from the sub models to facilitate the projection of the balance sheet.

Step 3: An economic scenarios generator (”ESG”) is used to generate a monthly view of prevailing risk-free rates for a 60-month projection period. A propriety scenario generator using the methodology set out by Sheldon and Smith (2004) [93] is used to generate 600 unique interest rate scenarios. 600 scenarios is based on the size required to ensure convergence for the valuation of certain insurance liabilities, this existing data was used as it was readily available for the purpose of this thesis, rather than building a ESG simulator from first principles. The starting point for the projection is December 2014. The ESG outputs a 60-month projection horizon of prevailing risk-free yield curves for each month from December 2014 to December 2019. Twenty liquidity risk scenarios was calibrated for each of the 600 interest rate scenarios to arrive at the 12000 unique scenarios.

Step 4: The model office and mathematical expressions for the various sub components is used to project the NII per equation 2.3. This is calculated for each of the 600 scenarios, from December 2014 to December 2019.

Step 5: The optimisation methodology is applied to the simulated scenarios from December 2014 to December 2019.

2.3 Sub models

2.3.1 Chapter 4: Retail assets

The capital and interest repayment (after allowing for the credit loss) for the existing retail asset portfolios are required in the model office projection. The data as at the calculation date (December 2014) is used to create model points based on the origination vintage. We assume a 20-year term for mortgage loans and 5-years for personal loans to calculate the amortisation schedule. The personal loans are fixed rate loans with the average interest rate per model point available to support the projection. The mortgage loans are all floating rate loans with the average spread above the prime rate available for each model point. The credit loss is derived based on the decomposition by vintage, age and economic cycle effects per the methodology sets out in chapter 4, where the economic cycle effect is, in turn, derived from the simulated macroeconomic scenarios.

The size and mix of the retail asset portfolio over the 60-month projection period is assumed to stay fixed for this thesis. The monthly new business volume required to support this is based on the capital repayment and credit loss experience on the existing portfolios. The projection of the probability of default ("PD") for the new business requires the vintage effect that is not available per chapter 4. The correlation of the vintage effect calibrated in chapter 4 and the Business Confidence Index ("BCI") macroeconomic variables is the highest. Based on this, the vintage effect required to calculate the PD on new business is based on the projected BCI index. The economic cycle and age effect is the same as for the existing business.

The credit loss in each month is calculate as the monthly PD times a fixed loss given default ("LGD") (20% for mortgages and 65% for personal loans). This amount is deducted from the interest earned to calculate the net cash flow position each month. X_t^1 and X_t^2 represent the size of the retail mortgage and personal loans portfolio based on the new business decision. The sub model discussed in chapter 4 produce the net interest rate, after allowance for credit losses (Cl_t^1 and Cl_t^2) for the mortgage and personal loan portfolios x_t^1 and x_t^2 .

2.3.2 Chapter 5: Corporate assets

The interest repayment (after allowing for the credit loss) for the existing corporate asset portfolio is required in the model office projection. The type of loan products offered to corporate clients are more diverse and complex compared to the straight forward amortisation mortgage and personal loan products. For the purpose of this thesis, all corporate loan facilities are assumed to be overdraft facilities paying a floating rate interest only (thus no capital repayments). This simplification is justified as the majority of the large corporate loans are a derivative of this simplified structure.

A Cox-hazard rate model is fitted to the corporate loan portfolio to calibrate the PD, see a detailed discussion in chapter 5. Chapter 5 discuss the covariates required for the projection of the PD over the 60-month period. As there is no capital repayment, the new business are only in response to credit losses.

The interest rate charged is based on the risk-free rate plus a spread, where the spread is based on the internal rating of the client. The internal rating is obtained in the bank data provided. A better credit rating will result in a lower spread being charged. The spread information was obtained from the bank.

The credit loss in each month is calculate as the monthly PD times a fixed LGD (45 %). This amount is deducted from the interest earned to calculate the net cash flow position each month. X_t^3 represents the size of the corporate loans portfolio based on the new business decision. The sub model discussed in chapter 5 produces the net interest rate, after allowance for credit losses (Cl_t^3) for the corporate portfolio x_t^3 .

2.3.3 Chapter 6: Wholesale funding

The cost of the wholesale funding instruments (duration 6, 12, 18 and 24 months) at time t is based on the appropriate forward rate at time t plus the calibrated credit and liquidity spread. Chapter 6 discusses the data supporting the calibration and the detailed methodologies to allow for the simulation of future credit and liquidity spreads over the 60-month projection period. The forward rates are obtained from the ESG.

Liquidity cost is introduced to the NII optimisation problem via the uncertainty of the liquidity spread at time t for each macroeconomic scenario. A number of simulated liquidity cost paths is calibrated for each macroeconomic scenario. For example, if we consider 50 unique macroeconomic scenarios, and 100 unique liquidity cost paths for each macroeconomic path then the NII optimisation problem is extended to 5000 unique scenarios. The credit risk premium is unique for each macroeconomic scenario, but the same for each of the liquidity cost paths within a scenario.

The size of the various wholesale instruments for duration 6,12,18 and 24 are shown as X_t^7, X_t^8, X_t^9 and X_t^{10}, this is derived as part of the optimisation discussed below. The interest cost on each of these instruments is obtained from the sub model discussed in chapter 6 and are represent as x_t^7, x_t^8, x_t^9 and x_t^{10} below.

2.3.4 Chapter 7: Retail and corporate deposits

A portfolio replicating methodology is used to calibrate both the size and interest rate models for the corporate and retail deposit portfolios. Chapter 7 details this methodology. Each macroeconomic scenario has a unique set of spot and forward rates of varying term that form the basis for the input into the corporate and retail size and interest rate models.

Using this information, the level (X_t^6 and X_t^5) of the retail and corporate deposit portfolio and the corresponding interest rate (x_t^6 and x_t^5) payable each month are projected over the 60-month period.

2.3.5 Risk-free assets

A bank has a portfolio of risk-free assets, usually in the form of long-term and short-term government bonds or treasury bills. An explicit sub model is not required to project the interest earned on these instruments, as this is obtained directly from the risk-free spot and forward rates obtained as part of the macroeconomic scenarios. For the purposes of this thesis, the banks published balance sheet was inspected to obtain the split between short-term (3-month instruments) and long-term (10-year instruments) risk-free instruments. X_t^4 and x_t^4 represent the size and monthly interest income on this portfolio.

Chapter 3

Optimisation

Chapter 2 defines the mathematical model or model office developed to facilitate the projection of the bank's income statement and balance sheet from December 2014 to December 2019. The uncertainty is introduced to the research problem as a set of economic scenarios over the projection. Appendix 1 explains the process used to generate the scenarios supporting the optimisation models.

Both a stochastic linear programming and recurrent reinforcement learning approach is considered. The results of these two methodologies are compared in the final results section. The model office and output from the optimisation models are used to empirically answer the research questions focusing on the optimal funding mix and the impact of certain factors on the funding mix. The results of this is shown in the final results section of this chapter.

3.1 Literature study

The inherent uncertainty of the cash flows and cost of funds for assets and liabilities has prompted banks to seek greater efficiency in the management of their assets and liabilities. This has led to studies concerned with the structure of the bank's assets and liabilities to achieve some optimal trade-off among the various risks. Chambers and Charnes (1961) [24] wrote one of the first papers based on maximising profitability within capital and liquidity constraints. The asset liability management problem deals with the strategic decisions management make, where the performance and outcome of assets and liabilities are uncertain. This uncertainty is reflected in the credit, liquidity and interest rate risk embedded in the performance of both assets and liabilities. Mathematical programming models that incorporate this uncertainty into the solution are known as stochastic program.

Available stochastic program methodologies include; change constraint programming, dynamic programming, sequential decision theory, stochastic decision trees and linear programming under uncertainty (or stochastic linear programming (SLP)).

The textbook by Zenios and Ziemba (2007) [116] discusses the practical application of stochastic programming, focused on finance. Kusy and Ziemba (1986) [69] were one of the first practitioners to advocate the use of stochastic linear programming with simple recourse for an asset liability framework, identifying challenges with available computer power to solve these large problems. Guven and Persentili (1997) [50] also put forward the SLP approach to solve the stochastic program presented by the asset liability problem. The evolution of both computational power and more refined search algorithms have promoted this methodology for dynamic asset liability modelling. The linear programming under uncertainty method is widely used to support financial decision making, see Kouwenberg and Zenios (2001) [68], Carino *et al.* (1994) [21], Edirisinghe and Patterson (2007) [37], Hill *et al.* (2007) [55] and Ying-jie and Cheng-iin (2000) [115]. This methodology allows for a traceable solution when extended over a multi-period and support the path dependency. A multi objective approach is not considered as part of this thesis, however the current methodology can be extended to include this, see Aouni, Colapinto and La Torre (2014) [3] and Kosmidou and Zopounidis (2004) [67].

The setup of the NII optimisation problem, in particular the path dependency and dynamic nature of the decision process aligns well with a dynamic programming methodology. This methodology is chosen to compare the performance with the linear programming under uncertainty. The reward function underlying the reinforcement learning methodology does not need to be linear as with the SLP model. This flexibility allows for the risk in the form of earnings volatility to be included in the optimisation criteria. The main difference between the two methodologies is the dynamic nature of portfolio adjustment. The stochastic multi period linear model provide the optimal liability mix the bank should target given the scenarios considered, where the recurring reinforcement learning algorithm provides trading rules to adjust the portfolio dynamically. Dynamic programming solves the optimal value function by for which prior knowledge of the Markov decision process is required, recursive learning algorithms are model-free and us the reward data from the process to learn.

3.1.1 Stochastic linear programming ("SLP")

The solution to address the challenge in solving stochastic linear programs, including the various forms of recourse, rests on the pioneering work by Benders (1962) [10], Dantzig (1963) [27] and Dantzig and Wolfe (1960) [28]. These authors developed various methodologies to decompose a problem using either an inner or outer linearisation to solve a large and complex problem often with non-linear functions included. Benders decomposition breaks a large problem into a number of smaller problems that can be solved individually, while mining for a global solution through an iterative process. The Dantzig - Wolfe decomposition focus on the duel of the linear problem.

The properties of the linear problem and, in particular, the properties of the recourse function is key to determine the convergence and feasibility. Van Slyke and Wets (1969) [104] extended the decomposition above into a stochastic program solution termed the L-Shape method. This will be the method used to solve the stochastic linear problem in this thesis. The textbooks by Birge and Louveaux (1997) [14] and Kall (1976) [64] provide a good overview of developments in linear programming, including the L-Shape methodology and the various important theoretical consideration to ensure feasibility and convergence. Murphy (2013) [81], Wets *et al.* (1990) [111] and Dempster (1980) [30] provide a good review on the L-Shaped methodology. Ruszczynski and Shapiro (2003) [92] further advanced the work by Van Slykes and Wets. There has been a number of enhancement to the original L-Shape method, such as more robust feasibility cuts using a multi cut approach to speed up convergence, and methods such as bunching and realisations, see Birge and Louveaux (1997) [14] for a good discussion on these approaches. A parallel computing methodology that speeds up convergence has been studied by a number of authors, see Wolf and Koberstein (2013) [113] and Shiina (2000) [94], for a discussion. In some cases, these methods greatly improved the speed of convergence. Investigating the performance with regard to convergence is outside the scope of this thesis.

The extension of the two-stage recourse problem to a multi-stage version and the corresponding extension on the L-shaped method is key to the application in this thesis. Work by Ho and Manne (1974) [56] and Glassey (1973) [45] extended the decomposition from a two-stage to a multi-stage framework. The framework presented by Bridge (1985) [12] used an outer linearisation similar to the L-shape algorithm. Additional work by Louveaux (1980) [73] considered other types of algorithms such as quadratic problems.

3.1.2 Recurring reinforcement learning (”RRL”)

Dynamic programming, and in particular, reinforcement learning is used in financial decision models.The setup of the optimisation problem, in particular the path dependency and dynamic nature of the decision process aligns well with a dynamic programming methodology. This technique also ensure the result is comparable with the stochastic linear programming as this is part of the aim of the thesis to test the performance of two alternative (static versus dynamic or on line learning techniques) and the scenario based view of uncertainty. The reward function underlying the reinforcement learning methodology can be non linear providing more flexibility than the SLP method. This flexibility allows for the risk in the form of earnings volatility to be included in the optimisation criteria.

The optimisation problem shares similarities with a Markov decision process (”MDP”). Formulating the optimisation problem in this way opens up the field of reinforcement learning. As discussed in Marsland (2009) [76], Goldberg (1989) [46], Busoniu *et al.* (2009) [20] and Sutton (1992) [98], a MDP is a mathematical formulation partitioned over various statuses or time intervals, with a transition function to measure the movement across the various statuses and a corresponding reward function to measure the impact of the decision. A MDP has an agent (or multiple agents) that makes policy decisions affecting the transition function. The aim is to train the agent or policy function to optimise the reward, usually based on historic data or real time on-line learning.

An important consideration in specifying the MDP is the path dependency of the reward function. Optimizing the policy decision at time t is dependent on the output of the reward function from time $t = 0$ to time $t - 1$. Dynamic programming is a method used to find an optimal policy for the MDP. Busoniu *et al.* (2009) [20] constructed a Q-function as the cumulative discounted rewards from time 0 to time t to find the optimal policy. A common methodology used to find the optimal solution is based on the Bellman optimal equations based on the Q-function. The Q-function requires each possible state and action pair to be identified to specify an iterative policy search across all these pairs to optimise the cumulative returns.

The action space underlying the optimisation problem in this thesis is multidimensional and continuous, or even if a more simplified discrete option is constructed, consists of a very large number of possible action states. The Q-function optimisation requires the evaluation across all or a large portion of possible states. This, together with curse of dimensionality, requires a fairly large training dataset to support the optimisation.

Reinforcement learning differs from supervised learning in that no target outcome is provided. In supervised learning, the MDP is trained to historic or on-line data by minimising the difference of the target and model outcome. For reinforcement learning, the system takes actions based on some policy and receives feedback on the performance based on these actions. The parameters driving the policy are adjusted to increase the reward function. There is no target return or outcome for the optimisation.

A number of reinforcement learning methodologies have been applied in the context of automated trading decisions and active portfolio management. Neuneier (1996) [82] developed a Q-learning approach to support a portfolio management approach using on-line reinforcement learning.

A recurrent learning algorithm is a recognised methodology applied to train a MDB that is path dependent. Examples of these algorithms are backpropogation through time, see Werbos (1990) [108], and an on-line learning algorithm called real-time recurrent learning ("RTRL") set out in Rumelhart *et al.* (1985) [91].

Moody *et al.* (1998) [79] and Moody and Saffel (2001) [78] developed a recursive learning algorithm called recursive reinforcement learning ("RRL") based on the recursive methodologies from Werbos (1990) [108] and Rumelhart *et al.* (1985) [91] using the Sharpe ratio (defined as the average return divided by the standard deviation of the return) or differential Sharpe ratio as the reward function. This methodology was developed to optimise the return of the portfolio selection framework.

The RRL methodology developed has been used in a number of portfolio selection and rule-based trading systems. See Dempster and Leemans (2006) [31], Li,Dagli and Enke (2004) [72], Maringer and Ramtohul (2012) [75], Gorse (2011) [47] and Bertoluzzo and Corazza (2014) [11] for application in automated trading rules. The papers extended the RRL to allow for either uncertainty through a stochastic process, an alternative iterative process compared to the gradient rule or more granularity, such as transaction costs and non-stationary data.

3.2 Stochastic linear programming (”SLP”)

The computing resources required to solve certain algorithms operating in the higher dimensional input space grows exponentially, causing intractable problems (curse of dimensionality). In multistage stochastic programming the curse of dimensionality is present when the number of decision stages increases. Therefore, methods to approximate the stochastic continuous process will attempt to cover only the realisations of the random process, which are truly needed to obtain the near-optimal decision. In the case of the stochastic linear optimisation problem discussed is achieved by breaking down the problem to a finite approximation using an event tree (See Appendix 2).

3.2.1 Methodology

The section below discuss the L-Shape method in more detail, including some of the theoretical consideration supporting this method. Consider the following standard linear optimisation problem.

The objective function: Minimise $z = c^T x$.

Subject to the constraints:

$$Ax \leq b$$
$$x \geq 0 \tag{3.1}$$

where $c \in \Re^{n_1 * 1}$ is the coefficient of the objective function.
$x \in \Re^{n_1 * 1}$ is the vector of n_1 decision variables.
$A \in \Re^{m_1 * n_1}$ is the matrix of constraints.
$b \in \Re^{m_1 * 1}$ is the vector of constraint levels.

In the above formulation, the optimal decision x can easily be determined as c, A and b are known. In the two stage recourse problem, the information available to determine the optimal choice of x is segmented into a deterministic portion that is known prior to time t, and some random information that effect the optimal decision, but is unknown prior to time t. The initial decision based on the deterministic information is called the first-stage decision and is denoted by x. At time t full information is received on the realisation of some random vector ξ, where after the second-stage of corrective action y is taken.

A stochastic recourse program has some input data that are uncertain and a structure where a recourse action can be taken after the uncertainty is disclosed. The decision operation is split into a first-stage decision taken before the experiment, and a second-stage decision after the outcome of the experiment is known. A two-stage recourse linear program is set out below:

The objective function: Minimise $z = c^T x + E_\xi(\min\ q_\xi^T y)$.

Subject to the constraints:

$$\begin{aligned} Ax &\leq b \\ T_\xi x + Wy &= h_\xi \\ \text{and } x \geq 0&, y \geq 0 \end{aligned} \tag{3.2}$$

In the second stage, a random event $\xi \in \Omega$ is realised. For a given realisation ξ the second-stage problem data, q_ξ, h_ξ and T_ξ become known and the second stage decision, y can be determined. Note the relationship between the first-stage decision and the subsequent corrective action in the second stage is defined as $Tx + Wy = h$. The second-stage term is more difficult as for each ξ, the value y is a solution of a linear program. The deterministic equivalent program is often written as follows:

$$Q(x, \xi) = \min\ (q^T y | Wy = h - Tx, y \geq 0). \tag{3.3}$$

Define the expected second-stage value function as $Q(x) = E_\xi(Q(x, \xi))$. A deterministic equivalent program (DEP) follows as:

The objective function: Minimise $z = c^T x + Q(x)$.

Subject to the constraints:

$$\begin{aligned} Ax &\leq b \\ x &\geq 0 \end{aligned} \tag{3.4}$$

The computational burden for the DEP is apparent, as a solution of all the second-stage recourse linear programs are required. The computational effort is significantly reduced if the random vector ξ has a finite number, say K, of realisations, with probabilities p_k, $1 <= k <= K$. An appropriate sampling technique can be followed. The extensive form of this formulation summarises to:

Minimise $z = c^T x + \sum_{i=1}^{K} p_i y_i q_i$. Where y_i and q_i are the i-th realisation of the random vector with probability p_i.

Feasibility

A challenge in solving a two-stage recourse linear problem is the feasibility of the second-stage problem, especially for consideration across all $\xi \in \Omega$. This section summarises the key definitions and theorems for feasibility, and is based on discussion in Wets (1974) [110], Walkup and Wets (1967) [106] and a summary in Murphy (2013) [81].

Define the following two sets:

$$\begin{aligned} K_1 &= [x | Ax \leq b, x \geq 0] \\ K_2 &= [x | Q(x) \leq \infty]. \end{aligned} \tag{3.5}$$

The DEP follows as: min $z = c^T x + Q(x)$ subject to $x \in K_1 \bigcap K_2$.

The difficulty with computing K_2 is that we need to evaluate $Q(x)$, which is computationally expensive. The check for a feasible second-stage solution for a particular first-stage decision x without computing $Q(x)$ is set out below.

$$\begin{aligned} K_2(\xi) &= [x | Q(x, \xi) < \infty] \\ K_2^p &= [x | \forall \xi \in \Xi] \end{aligned} \tag{3.6}$$

where Ξ is a finite countable set.

If Ω is continuous and ξ has a finite second moment, then K_2 and K_2^p coincide, and for W fixed, the following important properties hold:

Properties: K_2 is a closed convex set.
If T is fixed, K_2 is polyhedral.

These properties are important in defining the feasibility of the problem. If $x \in K_2$, then there exists a y such that $x \in [x|\forall\xi, \exists y \geq 0, Wy \geq h_\xi - T_\xi x]$, or $h_\xi - T_\xi x \in$ pos W for all ξ, where pos $W = [t|Wy = t; y \leq 0]$. This definition follows directly from the definition of K_2 as the feasible set for recourse decision y such that $Q(y, \xi) < \infty$.

Now consider some x and ξ such that $h_\xi - T_\xi x \notin$ pos W. For this to hold there must exist a hyperplane $[x|\sigma^T x = 0]$ that separates $h_\xi - T_\xi x$ and pos W. This hyperplane must satisfy $\sigma^T t \leq 0$ for t pos W and $\sigma^T(h_\xi - T_\xi x) > 0$. The L-Shape search algorithm has been developed using Benders decomposition, Bender (1962) [10], where the inequality $\sigma^T(h_\xi - T_\xi x) > 0$ is used to define an additional constraint to be added to the original $Ax \leq b$ first stage constraints, to ensure a feasible set in K_2. See the discussion on the feasibility cuts in the L-shaped method below for more detail.

Optimally

The feasibility cut mentioned above ensure, that the constraints added to the first-stage problem has a feasible solution for the second-stage decision. Optimally cuts are required for the search algorithm to approach an optimal solution across both stages.

As per Benders decomposition (1962) [10], the problem stated in equation 3.4 relates to a master and sub problem. In the current form, the objective function is complex, non-linear and expensive to evaluate due to the function $Q(x)$. Using Benders decomposition, replace $Q(x)$ with a variable θ and define the master problem as min $z = c^T x + \theta$. The feasibility and optimally cuts, as well as the evaluation of $Q(x)$ based on the realisation of ξ, is defined as the sub problems. This method forms the basis for the L-Shape search algorithm discussed in more detail later.

The idea of the optimally cuts are to use the sub problem defined as min $z = q^T y$, subject to $Wy <= h_\xi - T_\xi x$ after the realisation of ξ to set the boundary for θ from the master problem defined above. Through an iterative process, we aim to find an optimal solution instead of evaluating the complex function $Q(x)$ directly. Important properties of $Q(x)$ are to be piece-wise linear and convex as shown in

Kall (1976) [64], Wets (1974) [110] and Dempster (1980) [30].

Consider the sub problem with a given realisation of ξ:

$Q(x, \xi) = \min q^T y$ subject to $Wy \leq h_\xi - T_\xi x$. Where $Q(x)$ is defined as the probability weights sum across all ξ based on the DEP. Taking the duel of this problem we get:

$Q(x, \xi) = \max_\pi \pi[h_\xi - T_\xi x]$ subject to $\pi W \leq q$

where π is defined as the simplex multipliers of the original problem. As $Q(x, \xi)$ is piece-wise linear and convex, we can infer a number of key observations. In the same linear piece of x, the value of $Q(x, \xi)$ varies linearly with x, i.e. $Q(x + \Delta, \xi) = max_\pi \pi[h_\xi - T_\xi(x + \Delta)]$. This argument follows from the duality theorem. Moving off the piece-wise linear application to x_2, the following holds due to the convexity of Q: $Q(x_2, \xi) \geq max_\pi \Pi[h_\xi - T_\xi(x_2)]$. This becomes the lower boundary on $Q(x, \xi)$ and thus ultimately $Q(x)$. Using this optimal cut, calculat the θ defined in the masters problem is bounded by $\theta \geq \sum_{\xi \in \Omega} \pi_\xi [h_\xi - T_\xi(x_2)] * p_\xi$.

The L-Shape method

The L-shaped method is an approximation of the non-linear $Q(x)$ objection function. To make this approach possible, we need to assume that the random vector ξ has a finite support, where $k = 1, 2, 3, ..., K$ is the index of all possible realisations of ξ, each with an associated probability p_k. The DEP can be written in the following extensive form.

The objective function: Minimise $z = c^T x + \sum_{k=1}^{K} p_k q_k^T y_k$.

Subject to the constraints:

$$
\begin{gathered}
Ax \leq b \\
T_k x + W y_k = h_k, k = 1, 2, ...K \\
x \geq 0, y_k \geq 0, k = 1, 2, ...K
\end{gathered}
\tag{3.7}
$$

The block structure of the problem above can be exploited with either a decomposition of the primal problem using an outer linearization (Benders (1962) [10]), or and inner linearization of the duel (Dantzig and Wolfe (1960) [28]).

The L-Shaped method segments the problem set out in equation 3.7 into a master and sub problem. The feasibility of the sub problem is tested for each realisation of the random vector ξ, and the corresponding x decision obtained from the master problem. A feasibility cut is included for each random realisation, $k = 1, 2, ..., K$, to the master problem if the solution to the sub problem is infeasible.

The L-Shaped method steps into the optimisation cuts once all sub problems for $k = 1, 2, ...K$ have a feasible solution. There are a finite number of cuts required to obtain an optimal solution based on the convexity of the non-linear objective function $Q(x)$.

Step 0: Set $r = s = v = 0$.

Step 1: Set $v = v + 1$. Solve the linear program defined below.

The objective function: Minimise $z = c^T x + \theta$.

Subject to the constraints:

$$Ax \leq b \tag{3.8}$$

$$D_l x \geq d_l, l = 1, 2, ...r \tag{3.9}$$

$$E_l x + \theta \geq e_l, l = 1, 2, ...s \tag{3.10}$$

$$\text{and } x \geq 0, \theta \in \Re$$

Let (x^v, θ^v) be an optimal solution. For the first iteration, no constraints exits for 3.10 and $s = 0$, in which case θ is not considered in the calculation of x.

Step 2: For $k = 1, 2, ...K$, solve the following linear problem to determine if a feasible solution exisits for each k.

The objective function: Minimise $\omega = e^T \nu^+ + e^T \nu^-$

Subject to the constraints:

$$Wy + I\nu^+ - I\nu^- = h_k - T_k x^v \tag{3.11}$$

$y \geq 0, \nu^+ \geq 0, \nu^- \geq 0$.

where $e^T = (1, ...1)$. Repeat this from $k = 1, 2, ...K$, until, for some k, the optimal value is $\omega \neq 0$, in which case a scenario exist, k, where the constraint defined by $T_k x + W y_k = h_k$ is not feasible, with associated simplex multiplies (σ^v) based on problem 3.11. A feasibility constraint is added to the master problem by setting $r = r + 1$ and defining

$$D_{r+1} = (\sigma^v)^T T_k \tag{3.12}$$

$$d_{r+1} = (\sigma^v)^T h_k \tag{3.13}$$

to generate the constraint to be added to 3.9. Return to step 1 and repeat until no further feasibility cuts are added.

Step 3: For $k = 1, 2, ...K$, solve the following linear problem to determine if an optimal cut is required for each k.

The objective function: Minimise $\omega = q_k^T y$

Subject to the constraints:

$$Wy = h_k + T_k x^v \tag{3.14}$$

$y \geq 0$.

Let π^v be the simplex multipliers associated with the optimal solution of the problem 3.14. Define:

$$E_{s+1} = \sum_{k=1}^{K} p_k (\pi^v)^T T_k. \tag{3.15}$$

$$e_{s+1} = \sum_{k=1}^{K} p_k (\pi^v)^T h_k. \tag{3.16}$$

Let $\omega^v = e_{s+1} - E_{s+1} x^v$. If $\theta^v \geq \omega^v$, stop and x^v is optimal. Or else set $s = s + 1$ and add constraint to 3.10 and return to step 1.

The nested L-Shape method

The ability of the two-stage recourse function to incorporate uncertainty in the decision making process as the random vector ξ is observed has strong appeal for the application of the funding problem presented in the model office discussion above.

This methodology needs to be extended to cover the full projection horizon. A well accepted methodology building on the L-Shape method is the nested L-Shape method discussed below. This methodology allow for the recourse decision to be extended across multiple periods, while still maintaining the path dependency of previous decisions and including parameter uncertainty.

In addition to the scenarios $k = 1, 2, ...K$, let t represent the time period across the projection period. Let $t = 1, 2, 3...H$ present the full projection period.

The objective function: Minimise $z = (c^t)^T x_k^t + \theta_k^t$.

Subject to the constraints:

$$W_k^t x_k^t = h_k^t - T_k^t x_{a(k)}^{t-1} \tag{3.17}$$

$$D_{k,j}^t x_k^t \geq d_{k,j}^t, j = 1, 2, ...r_{k,j}^t \tag{3.18}$$

$$E_{k,j}^t x_k^t + \theta_k^t \geq e_{k,j}^t, l = 1, 2, ...s_{k,j}^t \tag{3.19}$$

And $x_k^t \geq 0$. Where $a(k)$ is the ancestor scenario of k at stage $t-1$ and $x_{a(k)}^{t-1}$ is the current solution from the ancestor scenario. Let Υ_j^t denote the period t decendents of a scenario j at time $t-1$.

Abbreviate the problem stated above with constraints shown in equations 3.17,3.18 and 3.19 with $NDLP(k,t)$. Note in the above, problem there is no constraint $Ax = b$ as equation 3.8, this constraint has been incorporated into equation 3.17.

Step 0: Set $t = k = 1, r_k^t = s_k^t = 0$. Add constraint $\theta_k^t = 0$. And set the direction $DIR = Forward$, go to step 1.

Step 1: Solve $NDLP(k,t)$. There are three possible outcomes from solving this problem:

1. if $t = 1$ and infeasible, then STOP as problem stated is not feasible.
2. if $t > 1$ and infeasible, then follow the steps below to add a feasibility cut to $t-1$.
    - $r_{a(k)}^{t-1} = r_{a(k)}^{t-1} + 1$.
    - $DIR = BACK$.
    - Using a similar setup to 3.11, calculate the simplex multipliers π_k^t.

- Compute $D^{t-1}_{a(k),r^{t-1}_{a(k)}} = (\pi^t_k)^T T^{t-1}_k$.
- Compute $d^{t-1}_{a(k),r^{t-1}_{a(k)}} = \pi^t_k h^t_k$.
- Add constraint $D^{t-1}_{k,r^{t-1}_{a(k)}} x^{t-1}_k \geq d^{t-1}_{k,r^{t-1}_{a(k)}}$.
- Let $t = t - 1$ and return to step 1.

3. if $t > 1$ and feasible, update x^t_k, θ^t_k and store the duel multipliers from constraints 3.17,3.18 and 3.19 as $(\pi^t_k, \rho^t_k, \sigma^t_k)$.

   - Let $k = k + 1$ and return to step 1.
   - If $k = K$, let $t = t + 1$ and return to step 1.
   - If $t = H$, set $DIR = BACK$ and move to step 2.

Step 2: For all scenarios $j = 1, 2, ...K^{t-1}$, at $t - 1$, compute the following optimally cuts:

$$E^{t-1}_j = \sum_{k \in \Upsilon^t_j} \frac{p^t_k}{p^{t-1}_j} (\pi^t_k)^T T^{t-1}_k \tag{3.20}$$

$$e^{t-1}_j = \sum_{k \in \Upsilon^t_j} \frac{p^t_k}{p^{t-1}_j} \pi^t_k h^t_k + \sum_{i=1}^{r_k} (\rho^t_{k,i})^T d^t_{k,i} + \sum_{i=1}^{s_k} (\sigma^t_{k,i})^T e^t_{k,i}. \tag{3.21}$$

Let $\overline{\theta^{t-1}_j} = e^{t-1}_j - E^{t-1}_j x^{t-1}_j$, where x^{t-1}_j was calculated in step 1 above. If $\theta^{t-1}_j = 0$ appears in $NDLP(j, t - 1)$ remove it, set $s^{t-1}_j = 1$ and add the constraint to 3.19.

If $\overline{\theta^{t-1}_j} > \theta^{t-1}_j$, then let $s^{t-1}_j = s^{t-1}_j + 1$ and add the constraint to 3.19 from $NDLP(j, t - 1)$. If $t = 2$ and no constraints are added to $NDLP(j, t - 1)$, then stop with x^1_1 as optimal. Or else let $t = t - 1$, $k = 1$ and go to Step 2. If $t = 1$, let $DIR = FORWARD$ and go to Step 1.

3.2.2 Calibration

The stochastic linear program (”SLP”) is used to optimise the NII function per equation 2.3. The optimisation decision is focused on the duration mix of funding issued to fill the monthly funding gap G^k_t (see equation 2.5) at time t for scenario k. The subscript notation for the remainder of this section is t for time period and k for the scenario.

The objective is to minimise the funding cost to the bank. The cost impact of the new funding is a function of the current interest rates and the size of the funding gap, where the previous funding decisions drive the size of the funding gap. Choosing mostly long-term funding will lock in historic interest rates and reduce the exposure of jumps in funding costs as the funding gap will be smaller. However, longer term funding is generally more expensive.

$\overline{F_t^k}$ is the decision vector representing the funding mix $< F_t^{7,k}, F_t^{8,k}, F_t^{9,k}, F_t^{10,k} >$ to fill the gap G_t^k such that $G_t^k = F_t^{7,k} + F_t^{8,k} + F_t^{9,k} + F_t^{10,k}$. The setup needs to be expanded to explicitly allow decisions made in time $t-1$ to influence the optimal decision in time t. To achieve this, add $F_t^{11,k}$ to vector $\overline{F_t}$ and to the NII function, where F_t^{11} is the sum of all the wholesale funding not maturing in month t. Thus, F_t^{11} is known based on previous funding decisions. $F_t^{11,k}$ introduces the path dependency of previous decisions. Note $F_t^{7,k} \neq X_t^{7,k}$ as $F_t^{7,k}$ is only the portion of the funding gap filled by the 6-month instruments, where $X_t^{7,k}$ will also include 6-month instruments issued over the last 5 months. The interest rate paid on an instrument relates to the rate as at issue date, thus the rate $x_t^{7,k}$ will only apply to $F_t^{7,k}$. Let the vector $x_t^k : < x_t^{7,k}, x_t^{8,k}, x_t^{9,k}, x_t^{10,k}, x_t^{11,k} >$ represent the interest rate paid on the various instruments under scenario k.

Let d_t^k be the outcome at time t for scenario k, where d_t^k represents the change in the deposit funding from month $t-1$ to month t. Thus, $d_t^k = X_{t-1}^{5,k} - X_t^{5,k} + X_{t-1}^{6,k} - X_t^{6,k}$. If the level of the deposit portfolios reduce then $d_t^k > 0$ and thus the size of the wholesale funding will increase.

Per chapter 2 $Xm_t^{i,k}$ is the level of the wholesale funding $i = 7, 8, 9, 10$ to mature in month t, for scenario k. A 6-month instrument issued in month $t-6$ will mature in month t, thus $Xm_t^{i,k} = F_{t-Mi}^{i,k}$, where Mi is the term of the instrument i. Based on the above definition the gap G_t defined in equation 2.5, summarise as follows:

$$G_t^k = \sum_{i=7}^{10} Xm_t^{i,k} + d_t^k. \tag{3.22}$$

Per the model setup, the bank needs to fill the funding gap G_t by the funding choice such that:

$$G_t^k = F_t^{7,k} + F_t^{8,k} + F_t^{9,k} + F_t^{10,k}. \tag{3.23}$$

From the path dependency discussion above, $F_t^{11,k}$ is defined as follows:

$$F_t^{11,k} = \sum_{i=7}^{11} F_{t-1}^{i,k} - \sum_{i=7}^{10} Xm_t^{i,k}. \tag{3.24}$$

Let $x_t^{11,k}$ be the interest rate paid on the remaining wholesale liabilities prior to funding the gap in month t. This interest rate is a function of the previous funding decisions and corresponding interest rates that are applied, thus it is fully computable using information from the previous known outcomes at $t = 1, 2...t-1$.

$$x_t^{11,k} = \frac{\sum_{i=7}^{10}[F_{t-1}^{i,k} x_{t-1}^{i,k}] - [\sum_{i=7}^{10} Xm_t^{i,k} x_{t-Mi}^{i,k}]}{F_t^{11,k}}. \tag{3.25}$$

Define $F_t^{i,k} = X_t^{i,k}$ for $i = 1, 2, 3, 4$ to be the size of the asset portfolio. The deposit portfolio in terms of F is $F_t^{5,k} = X_t^{5,k}$ and $F_t^{6,k} = X_t^{6,k}$. This notation is used to support the linear model formulation in F rather than X. The only change in the size of $F_t^{5,k}$ and $F_t^{6,k}$ is due to the change in the deposit portfolio, where $F_t^{i,k}$ for $i = 1, 2, 3, 4$ is assumed to be constant over time. Thus, the following equality holds $F_t^{5,k} + F_t^{6,k} = F_{t-1}^{5,k} + F_{t-1}^{6,k} + d_t^k$.

Formulating the linear model

The NII is formulated in F using equation 2.3, this is formulated in terms of the SLP optimisation methodology as:

$$Max(x_t)^T F_t. \tag{3.26}$$

Equation 3.26 is the same as minimising the cost of funding $\sum_{i=7}^{11} -x_t^i F_t^i$. The expanded form of the linear program can be written as per the L-Shape method:

Maximise $(x_t)^T F_t + E_\xi[(x_{t+1})^T F_{t+1} + E_\xi[(x_{t+2})^T F_{t+2}] + ...]$. Where the realisation of the random event in stage $t+1$, $t+2$, .. is $\xi \in \Omega$. Applying the master and sub problem per the L-Shape, the problem simplifies to Maximise $(x_t)^T F_t + \theta_t$, where θ_t is iteratively expanded.

The constraints applicable to this linear problem are:

$$F_t^{1,k} = F_{t-1}^{1,k} = X_1 \tag{3.27}$$

$$F_t^{2,k} = F_{t-1}^{2,k} = X_2 \tag{3.28}$$

$$F_t^{3,k} = F_{t-1}^{3,k} = X_3 \tag{3.29}$$

$$F_t^{4,k} = F_{t-1}^{4,k} = X_4 \tag{3.30}$$

$$F_t^{5,k} + F_t^{6,k} = F_{t-1}^{5,k} + F_{t-1}^{6,k} + d_t^k \tag{3.31}$$

$$F_t^{7,k} + F_t^{8,k} + F_t^{9,k} + F_t^{10,k} = \sum_{i=7}^{10} Xm_t^{i,k} + d_t^k \tag{3.32}$$

$$F_t^{11,k} = F_{t-1}^{7,k} + F_{t-1}^{8,k} + F_{t-1}^{9,k} + F_{t-1}^{10,k} + F_{t-1}^{11,k} - \sum_{i=7}^{10} Xm_t^{i,k}. \tag{3.33}$$

$$\tag{3.34}$$

The constraints can be written in the form of equation $Wx_t^k = h_t^k - T_t^k x_{t-1}^{a(k)}$. The multi period nested L-Shape algorithm is used to determine the optimal strategy, if feasible.

3.2.3 Results

Table 3.1 show three trading strategies where F_7 represent the 6-month instruments, F_8 the 12-month instruments, F_9 the 18-month instruments and F_{10} the 24-month instruments. The % represents the portion of the funding gap to be filled by the various instruments. Trading strategy 1 is more weighted towards longer dated instruments (mainly 24-month instruments) where strategy 3 focuses on short-dated instruments. Trading strategy 2 is a mix of the above, however still more weighted towards the longer-dated funding. These three strategies represent a general long, short and medium term funding profile of a bank, this can easily be extended to more scenarios. Only three scenarios are shown here to assist in interpreting the results split acorss the three (short, medium or long term) funding strategies only.

The SLP optimisation methodology is used to select the optimal trading strategy for the bank. The SLP optimisation is designed to maximise return only. Other performance metrics such as the Sharpe Ratio (average return divided by the standard deviation), Value at Risk and Conditional Value at Risk are not considered as part of the SLP optimisation. Equation 3.26 can be extended to target other

Table 3.1: The three funding strategies with corresponding weights to the various instruments.

Trading strategy	F_7	F_8	F_9	F_{10}
Strategy 1	0	0	12.5%	87.5%
Strategy 2	0	12.5%	25%	62.5%
Strategy 3	87.5%	12.5%	0%	0%

performance metrics, however a more complex optimisation methodology will apply due to the non-linearity of the optimisation criteria.

The SLP optimisation method selected trading strategy 1 as optimal in terms of maximising the return. The performance of strategy 2 and 3 is shown for comparison purposes only. Short-dated debt is cheaper compared to long-dated debt per the model setup. Funding the bank with short-dated debt exposes the bank to funding at a very high cost during periods to distress. The SLP optimisation methodology selected a longer funding approach to cushion the bank from these liquidity events.

Strategy 1 maximises the average return over a 60-month projection period and across the 12000 scenarios. The preference to fund the bank with longer dated instruments mitigates the liquidity risk introduced by continuously rolling funding at shorter durations. Table 3.2 shows the return distribution for each of the strategies split into 4 buckets for simplicity. Strategy 1 has the biggest portion in the high return bucket, this is the driving force of the superior returns for strategy 1. This coincides with periods of higher interest rates, where the return on assets reprice faster than the cost of funding due to the longer funding duration, confirming the importance of funding for longer durations.

Table 3.2: Strategy 1 has a higher incident in the high-return category.

Return category	Strategy 1	Strategy 2	Strategy 3
Loss	8.1%	7.1%	7.3%
Low return	23.4%	24.4%	24.3%
Medium return	57.9%	66.4%	65.8%
High return	10.6%	2.2%	2.6%

8% of the outcomes under strategy 1 results in a loss compared to 7% for strategies 2 and 3. The 95% VAR and CVAR is based on the return of assets instead of the nominal loss. This return should be multiplied with the size of the asset portfolio to obtain an absolute level. This confirms the slightly worst 95% VAR and CVAR for Strategy 1 as shown in table 3.3. The positive skewness in the NII distribution results in a higher standard deviation of the return under strategy 1, resulting in the lower Sharpe ratio per table 3.3. A summary of the performance of the three trading strategies across a number of performance metrics are shown in the table 3.3.

Table 3.3: Comparison of the performance metric across the three strategies.

Trading strategy	Average return	Sharpe Ratio	95% VAR	CVAR
Strategy 1	3.1%	5.65	-0.2%	-0.64%
Strategy 2	3.0%	6.56	-0.2%	-0.61%
Strategy 3	3.0%	6.77	-0.1%	-0.52%

The optimal solution is a function of both the scenarios considered and the assumptions on the sub-components, such as the credit loss, deposit portfolio behaviour and cost of wholesale funding. The impact of choosing a different starting date for the projection and lower liquidity risk in the cost of funding is tested. This resulted in a shorter optimal funding compared to strategy 1 above.

The strengh of the above methodology is to isolate specific impacts to facilitate the bank to determine the optimal wholesale funding mix given specific outcomes. We investigated the impact of reducing the liquidity risk via the liquidity premium projection, using a Poisson jump process with fewer jumps. The optimal strategy approaches the short strategy from table 3.1 as the frequency of the jumps is reduced. This is intuitive as the bank will seek shorter dated instruments, which are cheaper if liquidity risk diminishes. This confirms the importance of this tool to assist the bank with scenario planning. A further research topic from this thesis is determining the optimal funding strategy under various scenarios and assumptions, isolating the key drivers of specific funding strategies.

3.3 Recurrent reinforcement learning

The recurring reinforcement learning algorithm calibrates a training function across the same scenarios as the SLP. The training function provides management with trading rules for the duration of the wholesale funding required to fill the funding gap based on observed data, such as a change in the risk-free rate, credit rate and the change in the yield curve. This will allow the bank to dynamically adjust the wholesale portfolio based on actual experience, where the SLP method will target a set strategy to ensure optimisation of the expected return across all scenarios.

3.3.1 Methodology

The SLP optimisation methodology considers 4 wholesale funding instruments. For the purpose of the RRL methodology, we simplify this to two instruments, namely a 6-month and 12-month instrument. The same projection of the NII as per the SLP is used, including the projection period, ESG scenarios and sub models. As per the SLP optimisation, the trading decision is made every 6 months. This setup simplifies the complexity of the trading decision, the return function and the algebra required to support the RRL optimisation methodology. The methodology can be extended to more instruments and monthly trading rules with an increase in the complexity of the solutions; this will also require more data to train the trading function.

The funding gap each month is defined as G_t. Let $\bar{F}_t =< F_t^7, F_t^8 >$ represent the decision vector at time t, where F_t^7 represent, the portion of the gap G_t to be filled by issuing 6-month instruments.

The policy is a function with explicit weights to be trained during the reinforcement learning process. For the purposes of this thesis, the policy function is a trading function shown below:

$$F_t^7 = \tanh(e^{\theta*(x_t^8 - x_{t-1}^8 - 0.005)}) \tag{3.35}$$

where θ is the parameter to be solved, and controls the speed of change in the trading rule. See Moody and Saffel (2001) [78] for a discussion on the choice of this trading signal. The choice of the trading function seems fairly arbitrary, however the properties of this function have intuitive appeal. The month-on-month change in the 12-month interest rate is the main driver of credit losses on the asset portfolio, which in turn, drives the probability and the size of the liquidity jumps in the liquidity premium calibration. Due to this relationship, we expect the trading strategy to move to a longer duration to protect the bank from liquidity risk that increases during an interest raising cycle. The *tanh* function ensures that F_t^7 is bounded between $[0, 1]$, where the *exp* function allows for a fairly steep change in the trading strategy as Δx_t^8 changes. The θ parameter controls the speed of this change. Per this setup, $F_t^8 = 1 - F_t^7$.

The above policy function is chosen as the change in both the risk-free rate and credit risk components will highlight changes in the economic outlook and thus assist to position the balance sheet accordingly.

The NII equation 2.3 presents the initial setup of the net interest rate margin, or return function used in the RRL optimisation. This equation simplifies for the RRL application as only 2 types of wholesale funding instruments are used:

$$R_t = x_t^1 * X_t^1 + x_t^2 * X_t^2 + x_t^3 * X_t^3 + x_t^4 * X_t^4 - x_t^5 * X_t^5 - x_t^6 * X_t^6 - x_t^7 * X_t^7 - x_t^8 * X_t^8. \quad (3.36)$$

The return in month t is a function of the previous funding decision F_{t-1}^7 and the current funding decision F_t^7 and F_t^8. This is because X_{t-1}^7 matures by t where X_{t-1}^8 only matures by $t+1$, where t relates to the 6 monthly intervals. Based on this R_t follows as:

$$R_t = F_{t-1}^7 * [x_t^7 * F_t^7 + x_t^8 * (1 - F_t^7)] + x_{t-1}^8 * (1 - F_{t-1}^7). \quad (3.37)$$

The Sharpe ratio is used as the optimisation function for the purposes of the RRL optimisation. The Sharpe ratio is a well-known performance function used in portfolio management as this uses both average returns and the standard deviation of these returns. The Sharpe ratio as time t is defined below.

$$\begin{aligned} S_t &= \frac{Average(R_t)}{Std(R_t)} \\ S_t &= \frac{A_t}{K_t(B_t - A_t^2)^{0.5}} \end{aligned} \quad (3.38)$$

where $A_t = 1/t \sum R_t$, $B_t = 1/t \sum R_t^2$ and $K_t = (\frac{t}{t-1})^{0.5}$.

The differential Sharpe ratio is key if an on-line learning algorithm is required. We use the differential Sharpe ratio as the reward signal for the RRL problem. For the differential Sharpe ratio A_t and B_t are defined below.

$$\begin{aligned} A_t &= A_{t-1} + \eta(R_t - A_{t-1}) \\ B_t &= B_{t-1} + \eta(R_t^2 - B_{t-1}) \end{aligned} \quad (3.39)$$

where η is the adaption rate.

The recurrent reinforcement leaning algorithm aims to maximise S_t, using an online learning approach via the differential Sharpe ratio. This is done, by adjusting the policy function via the θ from F_t^7 with each time step across all simulations. The weight is updated using the gradient method as discussed in detail in Williams (1992) [112].

$$\Delta\theta = \alpha \frac{dS_t}{\theta} \tag{3.40}$$

where α is the learning rate of the RRL process. The equation for $\Delta\theta$ can be broken down into $\frac{dS_T}{d\theta} = \frac{dS_T}{dR_T} * \frac{dR_T}{d\theta}$. Consider the components in two steps.

First consider: $\frac{dS_T}{dR_T}$

As S_t is a function of both B_t and A_t, the derivative above can be written as $\frac{dS_T}{dR_T} = \frac{dS_T}{dA_T} * \frac{dA_T}{dR_T} + \frac{dS_T}{dB_T} * \frac{dB_T}{dR_T}$. Using equation 3.39 to define B_t and A_t the derivation follows from algebra.

$$\frac{dS_T}{dR_T} = \eta * \frac{B_{T-1} - A_{T-1} * R_T}{(B_{T-1} - A_{T-1}^2)^{3/2}}. \tag{3.41}$$

Next consider: $\frac{dR_T}{d\theta}$

The real-time recurrent learning (”RTRL”) set out in Rumelhart *et al.* (1985) [91] is used for the derivation of the recursive learning algorithm. As per Moody and Saffel (2001) [78], the RRL algorithm is given as $\sum_{t=1}^{T}[\frac{dR_t}{dF_t^7} * \frac{dF_t^7}{d\theta} + \frac{dR_t}{dF_{t-1}^7} * \frac{dF_{t-1}^7}{d\theta}]$. The second term in this equation is required as the return function R_t is a function of the incremental decision, thus both F_{t-1}^7 and F_t^7 directly affect the calculation of the R_t.

Note that the quantities $\frac{dF_t^7}{d\theta}$ are total derivatives that depend upon the entire sequence of previous trades from time t=0 to T.

The derivation of the first derivative from equation 3.37, $\frac{dR_t}{dF_t^7} = F_{t-1}^7 * x_t^7 - F_{t-1}^7 * x_t^8$ and $\frac{dR_t}{dF_{t-1}^7} = F_t^7 * x_t^7 + (1 - F_t^7) * x_t^8 - x_{t-1}^8$. The derivation of the second element is obtained using the recurrent learning algorithm RTRL.

$$\frac{dF_t^7}{d\theta} = \frac{\partial F_t^7}{\partial\theta} * [\theta * \frac{dF_{t-1}^7}{d\theta} + R_{t-1}] \tag{3.42}$$

where $\frac{dF_0^7}{d\theta} = 0$ and thus the above equation is solved recursively.

The derivative of $\frac{\partial F_t^7}{\partial\theta}$ is shown below:

$$\frac{\partial F_t^7}{\partial\theta} = sech^2(\exp(\theta*(\Delta x_t^8 - 0.005)))*\exp(\theta*(\Delta x_t^8 - 0.005))*(\Delta x_t^8 - 0.005). \quad (3.43)$$

3.3.2 Calibration

Figure 3.1 sets out the real-time recurrent learning framework. The optimisation framework is initiated with a predefined θ per the trading rule per equation 3.35 in step 0. This trading rule is applied across the 12000 unique scenarios to calculate the return at time $t = 1$. The recurrent learning algorithm per equation 3.40 is applied to update θ, to obtain the new trading rule updated with the information up to time $t = 1$ (Step 2 per figure 3.1). The new trading rule is applied across the 12000 unique scenarios from time $t = 0$ to obtain the return at time $t = 2$. The recurrent learning algorithm per equation 3.40 is applied to update θ to obtain the new trading rule updated with the information up to time $t = 2$. This process repeats till time $t = 60$. It is important to note that the new trading rule will be applied from time $t = 0$ for every step.

3.3.3 Results

Figure 3.2 shows, the trading function, tagged with the "optimal" data label, calibrated per the RRL methodology. Per this trading rule, the bank would issue 70% short-dated and 30% long-dated instruments when there is no change in Δx_t^8. The bank would increase the portion of short-dated instruments if Δx_t^8 is negative, while increasing the long-dated instruments if Δx_t^8 is positive.

Similar to the SLP methodology, we tested the impact on the trading rules as we reduce the impact of liquidity risk via the probability and size of the jump parameters in the cost of wholesale funding. This trading rule is shown as "Sensitivity 1" in figure 3.2. The reduced impact of liquidity risk will result in the bank continuing to issue short-dated instruments as credit losses change.

Figure 3.1: Steps in the RRL optimisation methodology.

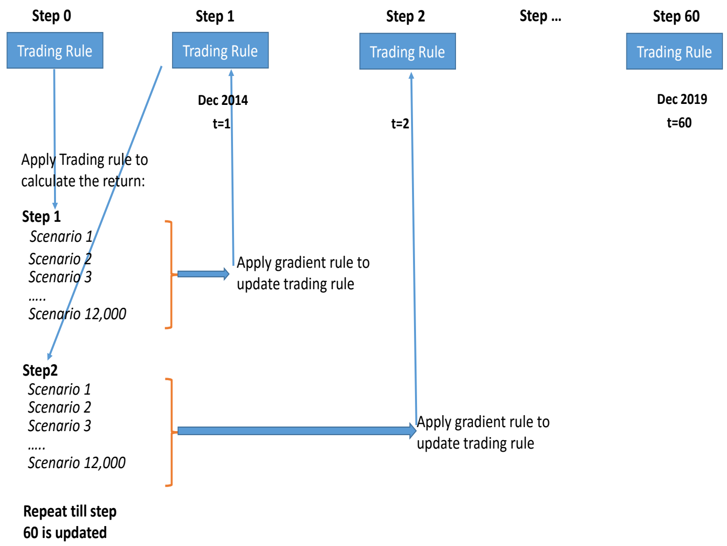
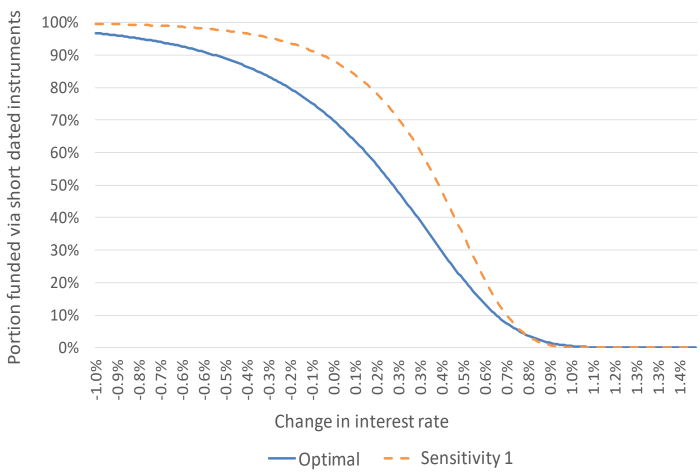


Figure 3.2: Portion of funding gap filled with short-dated debt as interest rates change.



3.4 Results

The SLP optimisation aims to define the trading strategy to follow over the entire projection period. The trading strategy is chosen to target the optimal return. The SLP optimisation method selected strategy 1 as optimal in terms of maximising the return, while taking liquidity risk into account. Strategy 1 utilise, mainly longer-dated instruments to fund the bank as this strategy manages liquidity risk by locking in funding rates due to longer dated instruments. This confirmed that the introduction of liquidity risk via jumps in the cost of funding of the bank requires the bank to switch funding to longer-term instruments.

The RRL method dynamically adjusts the trading strategy over the projection period. The credit and liquidity premium paid by banks to issue debt increase as credit losses increase in the underlying bank portfolios. The RRL methodology attempts to capture this dynamic by calibrating the trading rule based on changes in interest rates that drives credit losses. This allows the bank to maintain cheaper funding via short-dated instruments when credit losses are low, switching to longer-dated instruments to protect against liquidity risk as credit losses start to deteriorate. The RRL methodology provides a higher average return compared to the SLP method.

The trading rule supporting the RRL method is based on a change in interest rates. The calibration of the trading rule resulted in funding with shorter duration instruments when the month-on-month change in interest rates is very small. This switches to longer-dated instruments when the interest rates start to increase. The switch is fairly aggressive once it goes beyond a certain point.

Table 3.4 compares the return distribution for the SLP and RRL methodologies, split into 4 buckets for simplicity. The RRL method has a higher portion in the high-return bucket with a similar portion in the loss-making bucket. Strategy 1 from the SLP method provides superior returns compared to other static funding strategies, when liquidity risk is high due to the longer-dated funding. The RRL also benefits from this as the trading rules drive longer-dated funding as liquidity risk builds up, while focusing on shorted-dated instruments during benign periods.

Table 3.4: The RRL method has a higher portion in the high-return category.

Return category	SLP:Strategy 1	RRL
Loss	8.1%	8.3%
Low return	23.4%	18.7%
Medium return	57.9%	31.8%
High return	10.6%	41.2%

Table 3.5 compares the average return, Sharpe ratio, 95% value at risk and CVAR measure for two methods.

Table 3.5: Metric to compare performance of the two methods

Trading strategy	Average return	Sharpe Ratio	95% VAR	CVAR
RRL	3.32%	4.33	-0.4%	-0.9%
SLP: Strategy 1	3.07%	5.65	-0.2%	-0.6%

The average NII improved significantly when using the RRL method with the dynamic trading rule. Most notable is the shift in the NII distribution towards higher profits. The positive skewness of the RRL method results in a higher standard deviation and thus lower Sharpe ratio. Although the loss distribution has a fatter tail, it indicates a higher level of large losses than under the SLP optimisation (supported by the higher 95% VAR and CVAR).

The scenarios and assumptions supporting the optimisation does impact the optimal strategy under both the RRL and SLP methodologies. Choosing a different starting position for the projection and higher liquidity risk assumptions did results in a different SLP optimal strategy and a dynamic trading rule more weighted towards short-dated funding, due to the lower liquidity risk. A further research topic is to empirically test the optimal funding strategy under various scenarios and assumptions, isolating the key drivers of specific funding strategies.

Chapter 4

Retail credit model

A methodology is required to project the expected credit loss on an underlying loan portfolio given a selected economic outcome. A retail banking portfolio is categorized by a large number of counterparties and default events, and thus there exists a number of both statistical and mathematical methodologies available to calculate a default rate for each loan.

A credit loss is realized through the income statement via an impairment methodology largely driven by the accounting rules. A simplified calculation of the impairment process is the outstanding balance $\times$ probability of default $\times$ loss in the event of a default, where the probability coincide with the time increment of the calculation, i.e. monthly or annual.

For the purpose of this thesis a model relating the probability of default ("PD") to the macroeconomic scenarios will be developed. A fixed input of the loss given default ("LGD") will be assumed.

The focus of this thesis is not a model to calculate the PD per customer but to rather understand how the portfolio level PD will change as the economic environment changes. The aim of this section is to use historical default rates to calibrate the macroeconomic effect from the portfolio level time series of observed default rates. The relationship between the macroeconomic effect calibrated and the historic economic time series is used to forecast the credit outcome based on the simulated macroeconomic scenario. The observed portfolio level default rate is dependent on the economic environment, quality of the loans portfolio and the mix of the portfolio. A change in the portfolio level default rate is due to a change in the mix and quality of the portfolio as well as a change in the economic cycle. For example the impact on a personal loans portfolio of a 200 basis point increase in the base interest rate will be less severe if management applied very strict origination criteria in the 12 months prior to the increase. The proposed methodology supporting this thesis decompose the time series of crude default rates into three components: age of the loan, quality of the vintage and the exogenous economic environment that applied. This is termed the maturity-exogenous-vintage ("MEV") decomposition as discussed in Breeden 2007 [17] and Zhang 2009 [117].

The crude default rate is calculated as the portion of the portfolio that default over the next 12 months, this is used as a proxy for the credit risk outcome.

The exogenous component explains the impact of a change in the economic environment on the observed default rates. This is used to facilitate the forecast of the credit risk outcome given a macroeconomic scenario. A linear regression model is developed to project the exogenous component based on a set of macroeconomic variables. This linear regression model is used to forecast the economic cycle effect of the credit loss on the retail book under the simulated macroeconomic scenarios.

4.1 Literature review

Credit risk as categorized in the Basel II capital framework consist of the probability of default ("PD"') and the loss given default ("LGD"), as defined in The New Basel Capital Accord (2009). This thesis will focus on the PD, while keeping the LGD constant. The two main interpretations of the PD are the real world or observed default rates and the risk neutral default rates. The second is the market implied default rates, see Hull (1990) [57]. The risk neutral default rates are mainly applied in the valuation of assets where real world default rates are used internally by the bank for capital management and risk reporting.

The real world PDs are mostly used in the retail credit environment due to the high volume of observed defaults. The observed default rates forms the basis for the development of the statistical or parametric models of default risk. This thesis use actual observed default rates to develop a statistical model for credit risk.

There are a number of approaches to credit risk modeling. Examples of approaches based on the implied default rates are structural and reduced form models. The structural approach was proposed by Merton (1974) [77] and assumes that a default occur if the asset value of the company at maturity is below this value of the debt. Black and Cox (1976) [15] extended the above framework such that a customer can default anytime prior to time t. This occurs when the asset value falls below a prespecified level.

An alternative to structural credit risk models are reduced from or statistical credit risk models. This approach assumes that a customers default time is unpredictable, and is driven by some default intensity process that is dependent on some latent variable. Jarrow and Turnbull (1995) [62] and Hull and White (2005) [59] provides examples of reduced form models. Reduced form credit risk models have the flexibility to define the intensity process. This intensity process may be related to the asset value of the company, or even a set of macroeconomic factors such as in the Cox regression model (1972)[26]. This thesis used the reduced form or statistical approach given as this was supported by the available data.

Corporate loan portfolio are characterized by very few defaults, thus structural and reduced form models plays an integral part to assess the credit risk in the absence of observed defaults. The Cox regression model applied in chapter 5 is an example of a reduced form application. Retail credit portfolios are characterized by a large number of defaults, allowing us to calculate crude observed default rates without relying on structural or statistical models.

Thomas, Oliver and Hand (2005) [102] provides a summary of the development in retail credit risk management including an overview of the current issues. The original focus of credit risk modelling was to support the initial loan origination decision. This resulted in the development of various credit scoring approached based on score cut offs. The development of the Basel I and Basel II regulatory requirements forced banks to shift the focus to estimate the probability of default and loss given default. Popular methods used by banks to calibrate the PD at a customer level are linear regression, logistic regression or mathematical programming methods. See Baesens (2003) [4] for a comparison of 17 different type of methods and Altman and Saunders (1998) [2] for a summary of existing methodologies.

The aggregated portfolio level view of credit risk is used to forecast the performance of the loan portfolio. Bellottie and Crook (2009) [9], Bucay and Rosen (2001) [19] and Rosh and Scheule (2004) [89] developed various hazard rate or correlation focused methodologies where the default rate is linked to the macroeconomic variables. Malik and Thomas (2007) [74] extended this by considering the behavioural score, age of the loan and macroeconomic variables in the default rate estimate by either using a hazard rate or Markov chain model.

Credit risk forecasting methodologies that ignore the credit score and age effect and only consider the macroeconomic variables will exclude and important component in the forecasting methodology, see Breeden, Thomas and McDonald (2007) [18] for a discussion. This thesis will separate the age, quality of origination vintage and exogenous or macroeconomic cycle impacts in forecasting the PD. The quality of the origination vintage is used a proxy for the average credit score or quality of the origination cohort.

Maturation- exogenous- vintage (MEV) decomposition

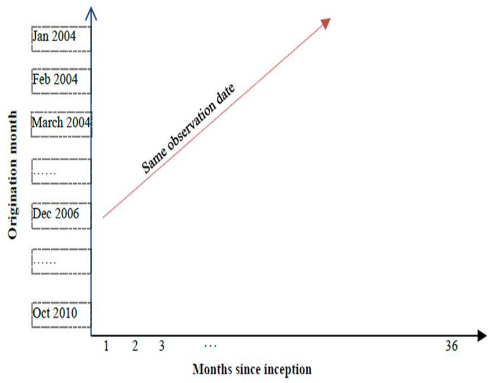
Zhang (2009) [117] developed a detailed methodology to decompose the default rate of various credit risk portfolios that share the duel time dynamic of origination vintage, age of contract and the external environment. In this paper Zhang proposed a Gaussian process with an adaptive smoothing kernel function to decompose the observed credit risk into a maturation-exogenous-vintage ("MEV") effect. This allows the practitioner to understand the observed credit experience in terms of a time since origination (m), exogenous influenced by macroeconomic conditions (t) and heterogeneity introduced from the various vintages (v). The aim of this methodology is to develop a model to predict the default rate for a particular cohort by considering the three impacts. This thesis is based on the MEV decomposition proposed by Zhang.

Breeden (2007) [17] considered a similar problem as Zhang. The multiple time dimensions observed was discussed in terms of both a dendrochronology and credit vintage analysis. The modeling approach described in Breeden is referred to as duel-time dynamics (DtD) because of the two time scales involved. This thesis is based on the MEV decomposition proposed by Zhang.

4.2 Stating the duel dynamics problem

A vintage in the context of this thesis consists of all loans originated in the same month. All loans from the same origination vintage have the same maturity or age development over the the observation and projection period.

Figure 4.1: Overview of panel data



The origination month of the loan is shown on the y-axis of figure 4.1 where the duration (age of account) of the cohort is shown on the x-axis. The diagonal development represents the outcome across different origination vintages. Note the performance measurement on the diagonal relate to the outcome under the same economic conditions, thus under the same time period t.

For example loans written in Dec 2006 will be age 1 in Dec 2006, age 2 in Jan 2007 and ... age 36 in Nov 2009. The diagonal relate to the performance of the portfolio at the same time period. For example this will include loans originated in December 2006 at age 1, loans originated in November 2006 at age 2, ...

Vintage data as presented above forms part of a class of panel data with a duel-time characteristic. The data shares the following cross sectional time series features:

- Origination month (v_j).
- Calendar observation month (t_{j+l}).
- And time since origination (m_l).

The specification of the cross sectional data follows as: There are J cohorts and the credit performance of each of these cohorts are observed for L_j months. Where the Lj is different for each cohort. Each of the J cohorts are observed at monthly intervals t_{j+l}, where $l = 1, 2, 3, ...L_j$.

In our case each of the J cohorts relate to a specific origination cohort v_j such that $v_1 < v_2 < ... < v_J$. The duration in force for each cohort is specified by m_l, this allows us to derived $t_{j+l} = v_j + m_l$.

Using the notation above the vintage data is presented in a panel format as

$$(m_l, t_{j+l}, v_j) \tag{4.1}$$

for $l = 1, 2, ..L_j, j = 1, 2, ...J$.

4.3 Discussion of Gaussian Process and Kernel trick

The discussion of the Gaussian Process is based on the work by Rasmussen (2006)[88].

Starting with the following model framework:

$$q = r(s) + \epsilon \tag{4.2}$$

where q is the dependent variable, and r is a function that maps the input vector to predict the level of the dependent variable, and ϵ the Gaussian error.

If the function $r(s)$ is linear, then a linear regression technique can be applied to calibrate the function $r(s)$. Gaussian process (GP) require less structure w.r.t the format of the function $r(s)$. A GP represent the $r(s)$ by letting the data speak for itself. The GP methodology does not try and define the function $r(s)$, but rather consider the distribution function of all possible functions $r*$, that satisfied the mapping observed in the training data, i.e the sample of data points $q* : s*$. It is not possible to test all possible formats of function $r(s)$ that satisfies the mapping observed in the training dataset $q* : s*$. The GP estimation overcomes this problem by replacing the focus on defining the format of the function $r(s)$, by applying the Bayes theorem and focusing on the replicating kernel. The estimation suffers from the computational complexity especially as we extend this to cubic or quadratic time.

Definition 4.1. Definition of a Gaussian process: A Gaussian process is a collection of random variables, any finite number of which has a joint Gaussian distribution.

Starting with the standard linear model:

$$r(s) = s^T w, q = r(s) + \epsilon \tag{4.3}$$

where s in the input vector.
w is the vector of weights of the linear model.
q is the observed values.

For the purposes of this model we assume the ϵ term follows an independent, identically distributed Gaussian distribution with mean zero and variance σ_n^2.

In the Bayesian approach we need to specify the prior distribution over the parameters or coefficients w from the model above. The assumption underlying the Gaussian Process is that the prior distribution is Gaussian, with a mean of zero and a covariance matrix Σ_p.

Thus: $w \sim N(0, \Sigma_p)$.

Applying the Bayesian model the posterior distribution over the parameter w is:

Posterior $= \frac{likelihood*prior}{marginallikelihood}$, $p(w|q,S) = \frac{p(q|S,w)*p(w)}{p(q|S)}$

where $p(w) \sim N(0, \Sigma_p)$.
$p(q|S,w) \sim N(S^T w, \sigma_n^2 I)$.
$p(q|S) = \int p(q|S,w)p(w)dw$.

It follows from algebra that:

$$p(w|S,q) \sim N(\frac{1}{\sigma_n^2} A^{-1} Sq, A^{-1}) \tag{4.4}$$

where:

$$A = \frac{1}{\sigma_n^2} SS^T + \Sigma_p^{-1}. \tag{4.5}$$

To make a prediction for the test case $s*$ we need to average over all possible parameters w, weighted by the posterior distribution of w ($p(w|S,q)$).

Thus the distribution of the predictive value given s* is r(s*):

$$p(r*|s*,S,q) \sim N(\frac{1}{\sigma_n^2} s*^T A^{-1} Sq, s*^T A^{-1} s*) \tag{4.6}$$

where $s*$ is the input vector, multiplied with a measure of all possible parameters values weighted by the probability function $p(w|S,q)$.

The above model suffers from limited expressions due to the assumed linear structure proposed by the model structure. A method to overcome this problem is to first project the input vector s into some higher dimensional space using a set of basis functions. The solution derived for the linear space above can then be applied to the higher dimensional space, instead of applying the linear model to the inputs directly. For example $\phi(s) = (1, s, s^2, s^3, ...)$ is applied to implement a polynomial regression. For example if $s = (2, 3)$, then s^2 represent an input vector where each of the individual components are raised to the power two, $s^2 = (4, 9)$.

Now the function $r(s)$ is:

$$r(s) = \phi(s)^T w \tag{4.7}$$

where $\phi(s)$ maps the D-dimensional input vector s into a N dimensional space. If the degrees of the polynomial regression is p, then $N = pD$.

Let $\Phi(S)$ be the matrix aggregation of columns $\phi(s)$ for all cases in the training set.

The algebra from 4.4 above holds and we can replace s with $\Phi(S)$ everywhere and $s*$ with $\phi(s*)$. Then:

$$p(r * |s*, S, q) \sim N(\frac{1}{\sigma_n^2}\phi^T A^{-1}\Phi q, \phi *^T A^{-1}\phi*) \tag{4.8}$$

where $\phi* = \phi(s*)$ and $\Phi = \Phi(S)$.

If we define $K = \Phi^T \Sigma_p \Phi$, we can rewrite the equation 4.8 as:

$$\begin{aligned} p(r * |s*, S, q) \sim N(&\frac{1}{\sigma_n^2}\phi *^T \Sigma_p \Phi(K + \sigma_n^2 I)^{-1} q, \\ &\phi *^T \Sigma_p \phi * - \phi *^T \Sigma_p \Phi(K + \sigma_n^2 I)^{-1}\Phi^T \Sigma_p \phi*). \end{aligned} \tag{4.9}$$

This is referred to as the Kernal trick discussed in Rasmussen (2006)[88]. We do not need to estimate the basis function or the Σ_p as this has been replaced with the estimation of the kernel function $K(s, s*)$.

Note from the initial discussion that we are estimating the mean of the function $r(s*)$ at the observed input vector $s*$ instead of trying the estimate the format of the function $r(s)$.

4.4 Application to the MEV model framework

Observed default rates per the panel data

Each of the J origination cohorts are observed for L_j months, this results in n observations, where $n = \sum^{J} \sum^{L_j} 1$. The crude default rate is observed for each of the n observations. Let $x_{j,l} \equiv (m_l, t_{j+l}, v_j)$ be the panel data of vintage v_j observed m_l months since origination which will translate to calender month $t_{j+l} = v_j + m_l$. Let $y(m_l, t_{j+l}, v_j)$ be the observed default rate for the observation per the panel data $x_{j,l}$.

Let $\eta(x_{j,l})$ be a transformation function that maps the input data (m_l, t_{j+l}, v_j) to the default rate $y(m_l, t_{j+l}, v_j)$.

Formulation of $\eta(x_{j,l})$

The η function is broken down into a linear and non linear portion

$$y(m_l, t_{j+l}, v_j) \approx \eta(x_{j,l}) = \mu(x_{j,l}) + Z(x_{j,l}) + \epsilon \tag{4.10}$$

where $\mu(x_{j,l})$ is a linear function of the panel data (included to capture the base or intercept impact or general linear trends), $\epsilon \approx N(0, \sigma^2)$ the random error, and $Z(x_{j,l})$ a zero-mean Gaussian processes.

Definition 1 : Covariance structure of $Z(x_{j,l})$

Let $\mathbf{x}$ be a vector of the n panel observations $x_{j,l}$. Define the covariance between $Z(x_{j,1})$ -$\mu(x_{j,1})$ and $Z(x_{j,2})$ -$\mu(x_{j,2})$ as the $(j+1, j+2)^{th}$ element of the $n*n$ matrix K. K is a matrix of the covariance of each of the n inputs with each other. Let $\sigma^2 K(\mathbf{x}, \mathbf{x}')$ be the $n*n$ matrix, where $\mathbf{x}$' is the transpose of vector $\mathbf{x}$. The covariance of a specific panel data observation $x_{j,l}$ with the n observation is defined as $\sigma^2 K(\mathbf{x}, x_{j,l})$.

The Identification Lemma per Zhang (2009) require us to break up the linear dependency of the panel data (m, t, v) in the specification of the linear portion of η. This is required to ensure the model is identifiable. This is achieved by specifying the linear portion of η with reference to m and t only as

$$\mu(x_{j,l}) = \mu_0 + \mu_1 m_l + \mu_2 t_{j+l}. \tag{4.11}$$

The linear portion of the model cannot include all three inputs per the panel data (m, t, v). We excluded v_j from equation 4.11 to satisfy this requirement. However the formulation of $\mu(x_{j,l})$ can be simplified to $\mu(x_{j,l}) = \mu_0$ without any loss to the MEV decomposition. The linear portion of the model is very useful if there is a general trend in exogenous and maturity effect to be captured by a linear formulation.

Estimation of η

The solution requires a formulation of the expected default rate for each input data point $x_{j,l}$. This is achieved by estimating:

$$E[\eta(x_{j,l})] = \mu(\hat{x_{j,l}}) + E[Z(x_{j,l}) + \epsilon|\tilde{z}] \tag{4.12}$$

where $\mu(\hat{x_{j,l}})$ is the linear estimation and $\tilde{z} = y(m_l, t_{j+l}, v_j) - \mu(\hat{x_{j,l}})$.

From equation (4.9) the expected value of the Gaussian process is

$$E[Z|\tilde{z}] = K(\mathbf{x}, x_{j,l})[K(\mathbf{x}, \mathbf{x}') + I]^{-1}(y - \hat{\mu}) \tag{4.13}$$

where $K(\mathbf{x}, x_{j,l})$ and $K(\mathbf{x}, \mathbf{x}')$ is per Definition 1.

The MEV decomposition

Zhang postulated $Z(x_{j,l})$ as a Gaussian process to allow the expected value to be defined in terms of the covariance functions with an estimation based on the spline estimation technique. A Gaussian process was chosen as this require less structure w.r.t the format of the function that maps the input data $(x_{j,l})$ to the observed default rates, but rather consider the distribution function of all possible functions that satisfy the mapping of the observed data in the training data. The GP estimation focus on the replicating kernel rather than specify the structure of the mapping function.

Let $Z(x_{j,l}) = Z_f(m) + Z_g(t) + Z_h(v)$ be the sum of three independent Gaussian processes. Zhang assumed the three Gaussian processes are independent for simplicity, this model can be postulated to allow for the some dependency between the maturity, exogenous and vintage impacts. This require a more complex structure as it is difficult to ensure a positive definite cross-covariance function between the Gaussian processes, see Boyle and Frean (2005) [16]. The definition of $Z(x_{j,l})$ can be simplified by replacing one of the Gaussian processes (for example $Z_h(v)$) by a constant term or simple deterministic formula, apply the same methodology to decompose the remaining two Gaussian processes and apply the back-fitting algorithm adjusted to estimate the deterministic portion.

The three Gaussian processes have the following practical explanations:

- $Z_f(m)$ represents the impact of the duration in force on the default rate.
- $Z_g(t)$ represents the exogenous influence of the macroeconomic condition on the default rate.
- $Z_h(v)$ represents the vintage heterogeneity measuring the origination quality.

Where $Z_f(m)$ is a zero-mean Gaussian processes. Let $\mathbf{m}$, $\mathbf{t}$ and $\mathbf{v}$ be a vector of the individual elements of the input panel data. For example $\mathbf{v}$ is a vector representing the J input cohorts. Following the same covariance structure as per Definition 1 let $\sigma_f^2 K_h(\mathbf{v}, \mathbf{v}')$ be an $J * J$ matrix of the covariance of each of the J input cohorts with each other. The same definition apply for $\sigma_f^2 K_h(\mathbf{m}, \mathbf{m}')$ and $\sigma_g^2 K_h(\mathbf{t}, \mathbf{t}')$.

By the independence assumption

$$K(x_{j,l}, x_{j,l+1}) = \frac{\sigma_f^2}{\sigma^2} K_f(m_l, m_{l+1}) + \frac{\sigma_g^2}{\sigma^2} K_g(t_{j+l}, t_{j+l+1}) + \frac{\sigma_h^2}{\sigma^2} K_h(v_j, v_j) \tag{4.14}$$

with $\lambda_f = \frac{\sigma_f^2}{\sigma^2}$,$\lambda_g = \frac{\sigma_g^2}{\sigma^2}$ and $\lambda_v = \frac{\sigma_h^2}{\sigma^2}$. From proposition 3.2 and 3.3 in Zhang (2009) the additive separability properties of the spline estimator can be expresses as

$$\eta(x_{j,l}) = \mu(x_{j,l}) + \sum_{i=1}^{L} \alpha_i K_f(m_l, m_i) + \sum_{i=1}^{J+L} \beta_l K_g(t_{j+l}, t_i) + \sum_{i=1}^{J} \gamma_j K_h(v_j, v_i) \tag{4.15}$$

with coefficients determined by,

$$\min[||y - \mu(\hat{x_{j,l}}) - \tilde{\Sigma}_f \alpha - \tilde{\Sigma}_g \beta - \tilde{\Sigma}_h \gamma||^2 + \lambda_f ||\alpha||^2_{\Sigma f} + \lambda_g ||\beta||^2_{\Sigma g} + \lambda_h ||\gamma||^2_{\Sigma h}]$$

where $\tilde{\Sigma}_f = K_f(\mathbf{m}, m_i)_{n*1}$ and $\Sigma_f = K_f(m_i, m_i)_{1*1}$.

Definition 2 : The spline solution

The MEV decomposition define the Gaussian process formulation per equation 4.10

as the sum of three Gaussian processes. The expected value of Gaussian process is defined per equation 4.13. A spline formulation of equation 4.13 is used as an estimation technique. The spline solution is used to define equation 4.15 as the spline estimator of the expected value of the Gaussian process. Equation 4.15 formulates the expected default rate for each input from the panel data $x_{j,l}$. The isolated impact for panel data $x_{j,l}$ of the duration in force (m_i), exogenous influence (t_{j+l}) and the vintage impact (v_j) is shown in equation 4.15 as $\sum_{i=1}^{L} \alpha_i K_f(m_l, m_i)$, $\sum_{i=1}^{J+L} \beta_l K_g(t_{j+l}, t_i)$ and $\sum_{i=1}^{J} \gamma_j K_h(v_j, v_i)$.

Evaluating the three impacts for all the input data points ($j \in J$ and $l \in L_j$) allows a graphical representation of the duration, exogenous influence and vintage impacts. Based on the above this is a graphical representation of the expected value of the Gaussian process, not the actual process. This graphical representation will be termed $Z'_f(m)$, $Z'_g(t)$ and $Z'h(v)$. See figures 4.2, 4.3 and 4.4.

Zhang (2009) proposed a back fitting procedure to estimate the above parameters.

The work by Zhang (2009) established the link between the Gaussian process $Z_g(t)$ and the adaptive smoothing spline function. Where $E[Z_g(t_{j+l})] = \sum_{i=1}^{j+l} \beta_i K_g(t_{j+l}, t_i)$. This allows us to estimate the $Z_g(t_{j+l})$ for each of the n panel data inputs. The same formulation holds to estimate $E[Z_h(v_j)]$ and $E[Z_f(m_l)]$.

4.5 Back fitting algorithm

The dependency of the panel data results in a collinearity problem in estimating the individual Gaussian processes. The back fitting procedure approach follows a steps by step approach where each of the Gaussian processes are estimated in isolation this deals better with the collinearity problem compared to a maximum likelihood estimation.

Assume a pre-specified structure for the three covariance functions $K_f(\mathbf{m}, \mathbf{m}')$, $K_g(\mathbf{t}, \mathbf{t}')$ and $K_h(\mathbf{v}, \mathbf{v}')$. Assume either an exponential of Matern function completely described by θ_f, θ_g and θ_v. Matern is a class of covariance functions $u(k, \theta, r)$ with parameters k, $p = k + 0.5$, θ and with $r = x_i - x_j$ the elements in the covariance matrix. See Guttorp and Gneiting (2006) [49] for a discussion on Matern functions. The Matern function is defined as:

$$u(k,\theta,r) = \exp(-\frac{\sqrt{2kr}}{\theta})(\frac{\Gamma(p+1)}{\Gamma(2p+1)})\sum_{i=1}^{p}\frac{(p+1)!}{(p-1)!}(\frac{\sqrt{8kr}}{\theta})^{p-i}. \tag{4.16}$$

The covariance functions $K_f(), K_g()$ and $K_h()$ are defined by $(\theta_f, \theta_g, \theta_v)$, and given $\lambda_f, \lambda_g, \lambda_f$ equation 4.15 is explicitly defined. An optimal solution may be found by setting the partial derivative of equation 4.15 to zero to solve the remaining parameters. However this require a process to estimate θ and λ first. The back fitting algorithm overcomes this problem.

Cross-validation is a methodology used in spare data statistics where one input dataset is used to both fit and test a specific model. A leave-1-out method train the function on both the complete input dataset as well as on the dataset less 1 data point. The cross-validation score is measured as the difference in the actual versus expected using the various leave-1-out fits. Generalized cross validation (”GCV”) is a well-established method used to fit smoothing splines as the smoothing matrix supporting the spline has a direct link to the generalized cross validation measure see Gelub, Heath and Wahba (1979) [43].

The back fitting algorithm is based on the principle of generalized cross validation (GCV) as described in Bates, Lindstorm, Wahba and Yandell (1986) [8] and Gelub, Heath and Wahba (1979) [43]. A ridge regression methodology is used to apply the GCV methodology to determine the parameters θ,λ_f, λ_g and λ_v.

Define the complete set of parameters by $[\mu, \alpha, \theta_f, \lambda_f, \beta, \theta_g, \lambda_g, \gamma, \theta_h, \lambda_h, \sigma^2]$. Where λ are the smoothing parameters under the adaptive smoothing spline structure; θ_i the parameters specifying the covariance function; α, β, γ the reproducing kernel function and μ, σ the mean and variance.

The 6 steps in the back fitting algorithm are:

Step 1: Set the initial value of μ by the ordinary least squares.

Step 2: Create pseudo data $\tilde{y} = y - \hat{\mu(x)} - \Sigma_g\beta - \Sigma_h\gamma$. Estimate α by ridge regression and determine (θ_f, λ_f) by minimizing the GCV(θ_f, λ_f).

Step 3: Create pseudo data $\tilde{y} = y - \hat{\mu(x)} - \Sigma_f\alpha - \Sigma_h\gamma$. Estimate β by ridge regression and determine (θ_g, λ_g) by minimizing the GCV(θ_g, λ_g).

Step 4: Create pseudo data $\tilde{y} = y - \hat{\mu(x)} - \Sigma_g \beta - \Sigma_f \alpha$. Estimate γ by ridge regression and determine (θ_v, λ_v) by minimizing the GCV(θ_v, λ_v).

Step 5: Re-estimate μ for $Z(x)$ given $[\theta_f, \lambda_f, \theta_g, \lambda_g, \theta_v, \lambda_v]$.

Step 6: Repeat steps 2-5 until convergence. Once convergence is reached obtain σ as derived from the multivariate setup for Z(x).

This methodology is set out in Zhang (2009).

4.6 Results of MEV decomposition

The above methodology was fitted to the retail mortgages and retail personal loans portfolios of the bank. The section below provides an overview of the data underlying the calibration and the results.

4.6.1 Retail mortgages

An account level monthly snapshot of all mortgage accounts were provided from 2006/12/01 - 2011/12/01. All loans originated from 2004/01/01 to 2010/12/01 were included for the calibration. The performance status of the loans are tracked on a monthly basis. An account is classified as in default if the account is in the "Legal" status in the particular month. The Legal status is a system driven indicator assigned by the bank. Over 500 000 individual accounts were tracked over the observation period. There were over 40 000 defaults identified over the observation period. The crude default rate for origination month i at duration j is calculated as the number of defaults from origination month i between duration j and $j + 12$ divided by the number performing accounts in origination month i at duration j.

The vintage data is denoted by $x_{j,l} = m_l, t_{j+l}, v_j$ with $l = 1, 2, ...L_j$, $j = 1, 2, ...J$. In this example j is the origination vintage from 2004/01/01 (1) to 2010/12/01 (49) and L_j refers to the number of monthly observations available for each origination vintage j. L_1 consist of loans originated in 2004/01/01. The observation period started in 2006/12/01 thus the duration available for cohort L_1 was from 36 to 50 months. For the purpose of this model L_j was capped at 36 months.

A number of kernel functions were tested as part of this thesis, the best fit is defined by the minimum deviation of the actual versus expected across the calibration set. The detail of the model calibration is shown below and in table 4.1.

$K_f(m)$ - Maturity curve, based on exponential kernel with $k = 1$. With $m = 1$ to 36.
$K_g(t)$ - Exogenous influence curve, based on Mater family kernel with $k = 2/3$. With $t = 1$ to 49.
$K_h(v)$ - Origination vintage curve, based on Mater family kernel with $k = 2/3$. With $v = 1$ to 72.

Table 4.1: Parameter calibration for the Gaussian process for mortgages.

Parameter	value
μ_0	0.02605
μ_1	0.02605
μ_2	0.02605
θ_g	2.6
θ_f	6
θ_h	3.5

The θ_g, θ_f and θ_h parameters are required to calibrate the covariance functions K_g, K_f and K_h per equation 4.15. The parameters α, β and γ from equation 4.15 are vectors estimated in step 1-2 and are the spline formulation of the Kriging. Per definition 2 the spline formulation provides an estimate of the expected value of the Gaussian processes, with a graphical representation per $Z'_f(m)$, $Z'_g(t)$ and $Z'h(v)$ shown in figures 4.2,4.3 and 4.4.

Figure 4.3 show a graphical representation of the exogenous impact on the observed default rates. A positive value is an add-on to the estimate default rate and thus indicates a deterioration in the credit quality of the portfolio. The figure show large positive values during the economic crises from April 2008 to June 2009, indicating the deterioration of the credit quality of all loans during this period.

$Z'_g(t)$ represent the impact on the crude default rate due to the changes in the macroeconomic environment. This will facilitate the projection of the default rate

Figure 4.2: Mortgages: Age impact $Z'_f(m)$

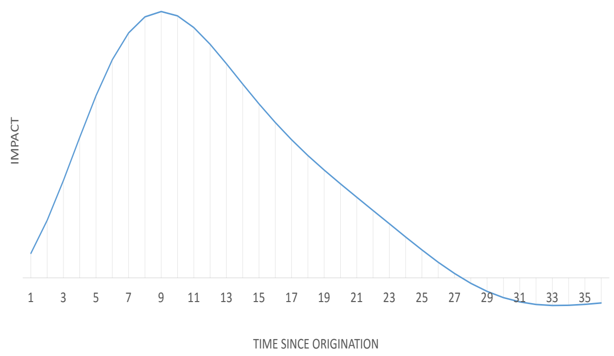


Figure 4.3: Mortgages: Economic cycle impact $Z'_g(t)$

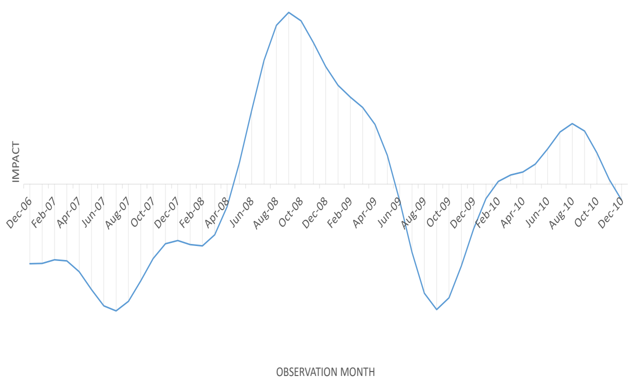
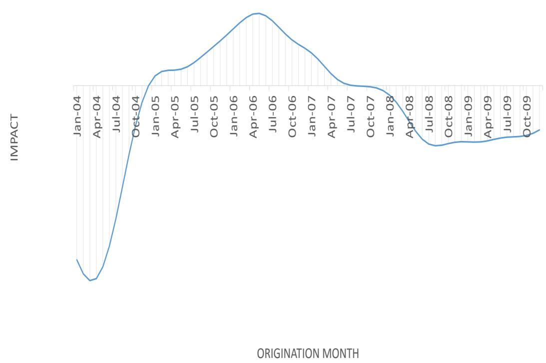


Figure 4.4: Mortgages: Vintage impact $Z'_h(v)$



under various macroeconomic scenario.

4.6.2 Retail personal loan

An account level monthly snapshot of all personal loans were provided from 2004/10/01 - 2011/12/01. All loans originated from 2004/01/01 to 2010/12/01 were included for the calibration. The performance status of the loans are tracked on a monthly basis. An account is classified as in default if the account is in the "Legal" status , or more than 3 installments in arrears in the particular month. Over 2.4 million individual accounts were tracked over the observation period. There were over 350 000 defaults identified over the observation period. The crude default rate for origination month i at duration j is calculated as the number of defaults from origination month i between duration j and $j + 12$ divided by the number performing accounts in origination month i at duration j.

A number of kernel functions were tested as part of this thesis, the best fit is defined by the minimum deviation of the actual versus expected across the calibration set. The detail of the model calibration is shown below and in table 4.2.

$Z_{f(m)}$ - Maturity curve, we assumed an exponential family kernel with $k = 1$. With $m = 1$ to 36.
$Z_{g(t)}$ - Exogenous influence curve, we assumed an Mater family kernel with $k = 2/3$. With $t = 1$ to 75.
$Z_{h(v)}$ - Origination vintage curve, we assumed an Mater family kernel with $k = 2/3$. With $v = 1$ to 72.

Table 4.2: Parameter calibration for the Gaussian process for personal loans.

Parameter	value
μ_0	0.03994
μ_1	-0.00189
μ_2	0.00151
θ_g	2
θ_f	5.5
θ_h	1.5

The θ_g, θ_f and θ_h parameters are required to calibrate the covariance functions

K_g, K_f and K_h per equation 4.15. The parameters α, β and γ from equation 4.15 are vectors estimated in step 1-2 and are the spline formulation of the Kriging. Per definition 2 the spline formulation provides an estimate of the expected value of the Gaussian processes, with a graphical representation per $Z'_f(m)$, $Z'_g(t)$ and $Z'h(v)$ shown in figures 4.5,4.6 and 4.7.

Figure 4.5: Personal Loans: Age impact $Z'_f(m)$

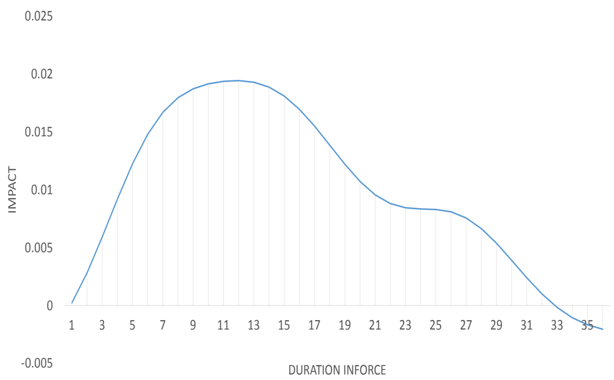


Figure 4.6: Personal Loans: Economic cycle impact $Z'_g(t)$

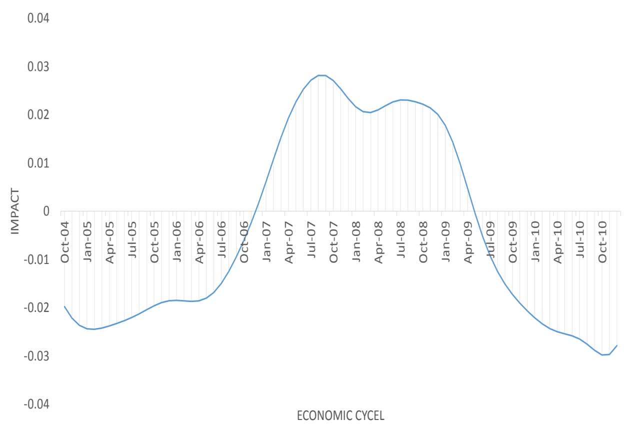
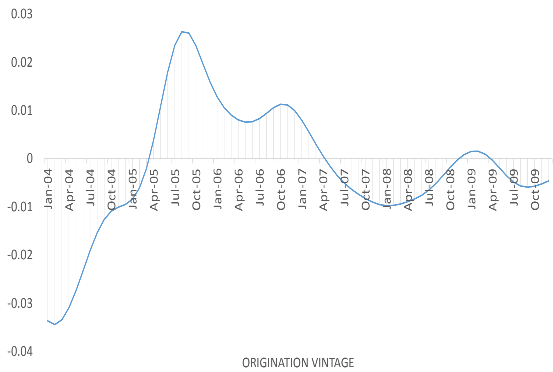


Figure 4.7: Personal Loans: Vintage impact $Z_h'(v)$



4.7 Credit forecasting model

A forecasting methodology is required to project the level of default rates over the projection period. Per the MEV methodology above (equation 4.10) the projected default rate per cohort is the sum of linear ($\mu(x_{j,l})$) and non-linear portion ($Z_f(m) + Z_g(t) + Z_h(v)$). The vintage effect ($Z_h(v)$) is constant per cohort, the age effect ($Z_f(m)$) is the same across all vintages. Need to projection the exogenous impact ($Z_g(t)$) where t expands beyond the calibrated period. The MEV methodology is used to determine the spline formulation of the exogenous impact. The spline formulation is used to calculate the impact over the observation period ($j \in J$ and $l \in L_j$) as shown by $Z_g'(t)$. This section set out the methodology used to forecast $Z_g'(t)$, and compare the results against traditional forecasting methodologies that does not apply the decomposition method before forecasting.

Havrylchyk (2010) [53] developed a regression type model to empirically test the impact on the credit loss due to a change in a set of macroeconomic variables. Breeden (2007) [17] argued that the change in credit risk is not only due to the change in the macroeconomic environment but also due to the vintage or portfolio quality and age effect of the portfolio. Breeden, Thomas and McDonald (2007) [18] argued that a macroeconomic model that links the evolution of credit risk to macroeconomic variables to facilitate the projection should not use a portfolio level credit risk measure without segmenting the age and vintage impacts.

We use the MEV decomposition discussed to first isolate the exogenous impact ($Z_g'(t)$) from the observed default rate time series before applying a similar approach as Havrylchyk (2010) to develop a regression model with a series of macroeconomic independent variables to estimate $Z'_g(t)$. The estimate of $Z'_g(t)$ is $Z *_g (t)$

The linear regression model has the following form

$$Z *_g (t) = Factor_1 * \beta_1 + Factor_2 * \beta_2 + Factor_3 * \beta_3 + Factor_4 * \beta_4 + \varepsilon. \quad (4.17)$$

The economic cycle impact on the portfolio default rate is obtained by using the simulated macroeconomic variables (Factor 1 - 4) in equation 4.17. A liner regression method is used to estimate $\beta_1, \beta_2, \beta_3$ and β_4.

Identify possible factors:

The following categories are used to define the dimensions of the macroeconomic environment:

- Business
- Interest rates
- Price Stability
- Household

The above categories was based on the analyses by Havrylchyk discussed above for the South African market. A number of factors within each of these categories are considered to develop the regression model. The data underlying this investigation was obtained from the South African Reserve Bank ("SARB") website. Figure 4.8 show the factors considered.

The mortgage loans exogenous curve is available from 2006/12/01. The macroeconomic data from the SARB website is available from 2006/01/01.

Multivariate Analysis:

A stepwise regression methodology was used to select a linear regression model for the mortgages loans. The first step is to include variables into the linear model using a tolerance level of 10%. The initial selection criteria does not consider the correlation of the macroeconomic variables. It is expected that a number of the macroeconomic factors will be highly correlated due to the interdependencies observed in the economy.

Figure 4.8: Macroeconomic variables considered for the forecasting regression model.

	Variable	Indicator	Increments	Mean	Std Deviation
	Business				
A	GDP at Market price	Stats SA	Quarterly	3.2	3.31
	Interest Rates				
D	Prime overdraft rate	KBP1403M	Monthly	12.5	2.14
	Prices				
E	M1: Money Supply	KBP1370A	Monthly	9.5	6.56
F	M2: Money Supply	KBP1373A	Monthly	10.9	7.94
G	M3: Money Supply	KBP1374A	Monthly	13.1	8.68
	Household sector				
B	House hold expenditure to GDP	KBP6280L	Quarterly	61.1	1.78
C	Consumption by households	KBP6007S	Quarterly	3.7	3.72
H	Credit extension to private sector	KBP1347A	Monthly	12.4	9.67
I	Total Credit extension	KBP1368A	Monthly	13.4	9.83
J	Credit to Private sector: Loans and Advances	KBP1369A	Monthly	12.8	10.70
k	Ratio of debt-service cost to disposable income	KBP6289L	Monthly	10.9	1.82
L	ABSA Small House Price Index (Inept)	ASAHPI (CL	Monthly	1.005	0.012
M	ABSA House Price Index (Inept)	ASASHI (CL	Monthly	1.004	0.007
N	Household debt to disposable income of households	KBP6525L	Monthly	80.5	1.85

A cluster and correlation analysis is performed as part of the final model selection. The cluster analysis identified 3 main clusters, and confirmed that a number of the macroeconomic variables are highly correlated.

Only 1 factor per cluster is selected for the final regression model. The result of the cluster and correlation analysis is used to remove factors selected by the stepwise model in step 1.

4.7.1 Retail mortgages :$Z'_g(t)$ projection

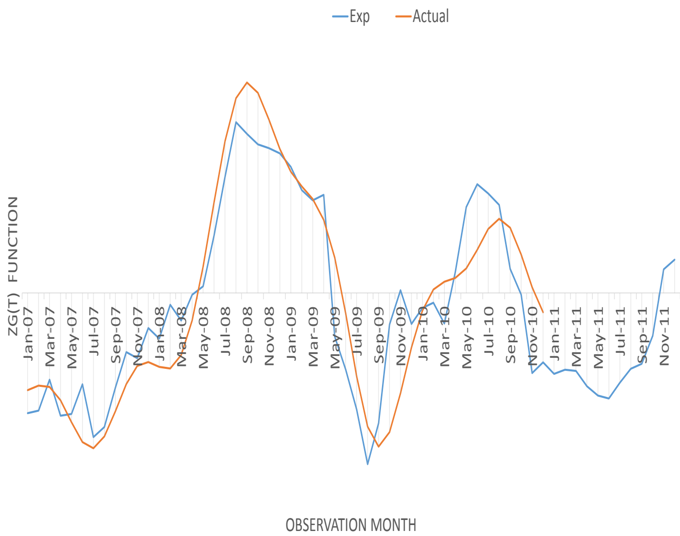
The final model is shown in table 4.3. This model has an R^2 of 84% over the calibration period.

The calibrated model was fitted to historic data from 2006/12 to compare the actual versus fitted $Z_g(t)$. This is shown in figure 4.9. The actual is shown as "Actual" on the graph and model fitted as "Exp". The $Z_g(t)$ function was only calibrated to Dec 2010. As the macroeconomic factors is available till Dec 2011 the fitted $Z'_g(t)$ function for 2011 is also shown.

Table 4.3: Regression parameters for the mortgage regression model.

Intercept	μ	0.15531
D	β_2	0.0009262
F	β_4	-0.0002027
M	β_1	-0.13976
N	β_3	-0.0003046

Figure 4.9: Comparison of the actual and fitted $Z'_g(t)$ function for mortages over calibration period.



4.7.2 Retail personal loans :$Z'_g(t)$ projection

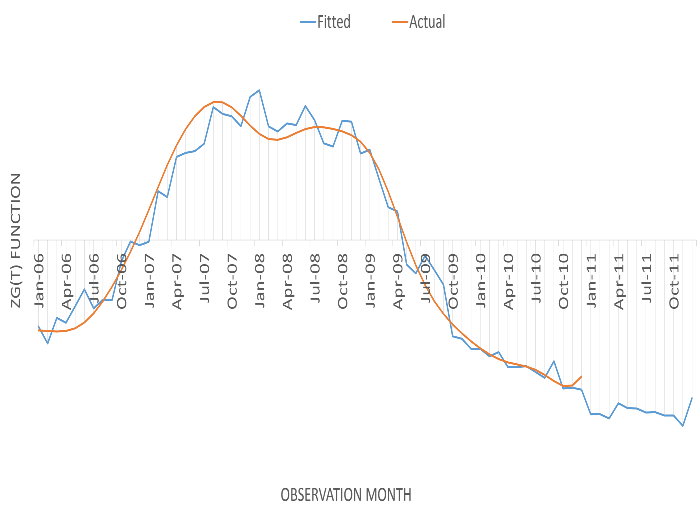
The final model is shown in table 4.4. The model has a R^2 of 97% over the calibration period.

Table 4.4: Regression parameters for the personal loans regression model.

Intercept	μ	-0.44288
D	β_1	0.00537
E	β_2	0.000998
N	β_3	0.00237
B	β_4	0.00294
C	β_5	-0.0004896

The calibrated model was fitted to historic data from 2006/12 to compare the actual versus fitted $Z_g(t)$. This is shown in figure 4.10. The actual is shown as "Actual" on the graph and model fitted as "Exp". The $Z_g(t)$ function was only calibrated to Dec 2010. As the macroeconomic factors is available till Dec 2011 the fitted $Z'_g(t)$ function for 2011 is also shown.

Figure 4.10: Comparison of the actual and fitted $Z'_g(t)$ function for personal loans over calibration period.



Chapter 5

Corporate credit model

A model relating the probability of default ("PD") of the corporate loan portfolio to the macroeconomic scenarios is developed in this thesis to support the NII calculation. A fixed input of the loss given default ("LGD") is assumed. A simpler methodology chosen for the corporate portfolio, compared to the retail portfolio as the corporate portfolio is not the focus of the thesis. This choice is further supported by the idiosyncratic view of corporate loan portfolios in South Africa and the relative few defaults to apply the MEV framework. Thus an approach focused on rating migrations was chosen.

5.1 Literature review

Chapter 4 sets out both the structural and reduced from credit risk models applied in the literature. Using the structural form model requires a projection of the underlying asset values of the client. The client level data required to utilise the structural model is extensive. The reduced form credit model approach assumes a client's default time is unpredictable, and is driven by some default intensity process, or hazard rate formulation. The hazard rate corresponds to the first jump time of a Poisson process. A constant hazard rate formulation will treat all clients in a given group as homogeneous, for example clients in the same rating band.

A challenge to the constant hazard rate formulation is the empirical evidence that the assessment of the credit risk is affected by conditions in the macro-economy. See Diebold *et al.* (2002) [5] and Duffie *et al.* (2007) [35]. The factors driving the observed credit risk are examples of covariates that impact the hazard rate of a specific client in a rating band. Cox (1972) [26] proposed a semi-parametric model to allow for the calibration of certain covariates in the specification of the hazard rate, removing the requirement of a constant hazard rate assumption.

Empirical work confirmed that a positive correlation in rating migrations exists, known as rating drift. Cross-sectional differences in the credit risk within a particular rating band also exist. See Altman and Kao (1992) [1] for a discussion.

The above factors are examples of covariates that impact on the hazard rate of a specific client in a rating band. The Cox (1972) model is a semi-parametric model. The parametric portion of the model specifies the hazard rate of the client with reference to the chosen covariates in an exponential regression form. The non-parametric portion of the model specifies the baseline impact on the hazard rate, without specifying any detail for the baseline.

The calibration of the Cox (1972) model focuses on the proportional representation of the client's hazard rate with reference to the portfolio. The non-parametric portion is removed by the proportional reference, and the focus is on the calibration of the coefficients of the covariates. For example, the relative observed rating migration of clients in a particular rating category for a long term period versus recent downgrades are used to calibrate the coefficients of the covariate time in the current rating. Figlewski, Frydmanand Liang (2012) [39] calibrated the Cox regression model based on the observed rating migration for a portfolio of corporate bonds, using both macroeconomic and client specific covariates. A similar methodology is used in this thesis.

5.2 Cox proportional hazard rate

5.2.1 Methodology

Consider a population of clients each with a corresponding monthly rating. The rating in this case is based on the internal credit rating applied by the bank, as not all clients have an external rating. Define a credit event as a downgrade of a particular client in a certain month. This is called a failure of the particular client. Assume there are N clients in our data sample. We have a monthly time series of ratings for each client. Using this data, identify all the failures and time to failure or censored clients. A client is censored if the failure occurs after the end of the observation period or after the client is removed from the dataset. Censored accounts are important, as we know that the time to failure for censored accounts are greater than the censored time.

Let T denote the random variable representing the failure time of an client. Let $F(t)$ be the survivor function, such that:

$F(t) = P(T \geq t)$, and let $\lambda(t)$ be the hazard rate defined as:

$$\lambda(t) = lim_{\Delta t \to 0} \frac{P(t \leq T \leq t + \Delta t | t \leq T)}{\Delta t}. \tag{5.1}$$

The Cox hazard regression model specifies $\lambda(t)$ as

$$\lambda(t) = \lambda_0 e^{\beta Z(t)}. \tag{5.2}$$

The Cox model is semi-parametric, as it consists of an unspecified baseline function λ_0 and a parametric part $e^{\beta Z(t)}$. The parametric part $e^{\beta Z(t)}$ is a logistic regression model with β, a vector of the coefficients to be calibrated and $Z(t)$ a vector of the covariates considered as part of the model. An example of the covariates are: macroeconomic information, the company credit rating and industry. The calibration of the parametric part requires a dataset of corporate clients, each with individual covariates and the recorded time to failure. This dataset is used to calibrate which covariates are predictive of the failure rate and calibrates their corresponding coefficients for each covariate. Table 5.2, show the covariates considered for this model. The focus is on the macroeconomic covariates, as the overall optimisation approach requires the forecast of the corporate portfolio default rate over a range of macroeconomic scenarios.

An important formulation of the Cox model is the proportional hazard rate property, where the ratio of the hazard rate for two clients does not involve the baseline hazard rate function. Using this proportional hazard rate property, Cox derived the partial likelihood function for the parameter/covariate estimation in the original paper. The likelihood function is presented as:

$$L(\beta) = \prod_{n=1}^{N} \frac{e^{\beta Z_n(T_n)}}{\sum_{i \epsilon R_n} e^{\beta Z_i(T_n)}} \tag{5.3}$$

where: n is the label of the client who experiences a failure event at time T_n
R_n is the population of clients at risk to experience the failure at time t.

The log partial likelihood function is:

$$l(\beta) = \sum_{n=1}^{N} [\beta Z_n(T_n) - log(\sum_{i \epsilon R_n} e^{\beta Z_i(T_n)})] \tag{5.4}$$

where the score function is: $U(\beta) = \frac{dl(\beta)}{d\beta}$.

The coefficient β can be determined by maximum likelihood, by setting $U(\beta) = 0$.

The above derivation is based on the assumption that only one failure event can occur in each time period. In our case, we have a monthly snapshot view of each client although it is possible for multiple clients to experience a failure event in the same month.
Without knowledge of the true ordering of the survival times, we need to consider all possible orderings of failures. For example, if we observe two tied failures at time t=1, then the likelihood function as at time t=1 needs to take into account that either client 1 failed before client 2 (event A1), or client 2 failed before client 1 (event A2). Thus $L_{t=1}(\beta) = P[A1] + P[A2]$. The above is relative straight forward if there are only 2 ties, however if there are d tied survival times then d! different orderings have to be considered and $L(\beta)$ is the sum of d! terms, each with is a product of d terms. This becomes computationally intensive. Breslows approximation is proposed to address the computational challenge with a large number of tied agents.

5.2.2 Calibration

The bank assigned an internal rating to each client on a monthly basis on the data provided. The focus of this thesis is not to model the PD per client, but rather to understand how the portfolio level PD will change given certain economic scenarios. A mapping table to assign a PD to each internal rating is available. This is a point-in-time PD, reflecting the likelihood of the default over the next 12 months, given the current economic conditions. The focus of this chapter is the migration of the internal calibrated PD of the corporate loan portfolio, given certain economic scenarios. This will enable the evaluation of the credit risk component in the corporate loan portfolio, given a simulated economic scenario.

The bank provided us with a monthly snapshot of the large corporate loan portfolio from June 2008 to November 2012. This dataset includes a monthly snapshot of 8045 clients, each with an internal rating based on the banks internal rating system. The monthly rating migration of 8045 clients is tracked from June 2008 to November 2012. These mainly relate to large corporate clients with an annual turn over exceeding R100 million per client and credit facilities ranging between R20 million to R3 billion.

For the purpose of this investigations all the positive and negative (+ and -) outlook ratings were grouped together, i.e. BBB+, BBB, BBB- into a BBB category. Using this information, all the credit downgrades were identified. There were a 7382 downgrades observed over the 52 observation months of the 8045 clients. It is possible for n client to have multiple downgrades over the observation period. If a client had 3 consecutive downgrades over the observation period, only the final downgrade was captured. This results in 6924 unique downgrade sequences identified in the data.

In the Cox hazard rate framework, a failure either relates to a death (in life insurance or medical research) or an event such as a positive diagnosis. In this application an outcome event or failure is derived from the downgrades.

The corporate portfolio consists of global clients from 190 countries, with 87% being South African clients and the remaining 13% mainly focused in Africa. We have not excluded non-South African clients from the investigation.

The client's business sector is one of the covariates tested as part of the Cox model. The corporate portfolio consists of clients across 56 sectors. For the purpose of this investigation, these sectors were grouped into the high level categories shown in table 5.1.

Table 5.1: Summary of portfolio by industry.

Sector	**Distribution**
Financial	2.9%
Retail	2.7%
Leisure	1.4%
Manufacturing	17.2%
Agriculture	12.8%
Services	17.6%
Property	27.8%
Government	2.6%
Other	15%

5.2.3 Results

From equation 5.2, the hazard rate relating to a downgrade on a client is given by:

$$\lambda(t) = \lambda_0 e^{\beta Z(t)}. \tag{5.5}$$

The focus of the Cox hazard rate model is on calibrating the parametric portion of the hazard rate only, namely $\exp(\beta Z(t))$. The covariates $Z(t)$ in the equation above relates to client-specific factors that explain a portion of the downgrade probability of a client, in relation to the population. As per Figlewski, Frydmanand Liang (2012) [39], both macroeconomic and client-specific covariates are tested. Table 5.2 shows the covariates that are tested for inclusion in the final calibration.

Table 5.2: Covariates tested in the Cox hazard rate model including source of information.

Variable	**Source**	**Increments**
Internal credit rating	Internal	Quarterly
GDP	Stats SA	Quarterly
BCI (business confidence index)	SA Reserve Bank	Monthly
Prime overdraft rate	SA Reserve Bank	Monthly
M2: Money supply	SA Reserve Bank	Monthly
M3: Money supply	SA Reserve Bank	Monthly
Change in total bank deposits	SA Reserve Bank	Monthly
Industry	Table 5.1	Quarterly

Table 5.3: Result of the Cox Hazard rate model fitted and supporting p-values.

Parameter	**Coefficient**	**P-value**
Industry 1	-0.08749	0.1261
Industry 2	-0.30477	<.0001
Industry 3	-0.34555	0.0015
Industry 4	-0.04487	0.4235
Industry 5	-0.56651	<.0001
Industry 6	-0.27311	<.0001
Industry 7	-0.69952	<.0001
Industry 8	-0.20359	0.0511
Rating AA	1.81615	<.0001
Rating A	2.07453	<.0001
Rating B	1.3223	<.0001
Rating BB	1.60866	<.0001
Rating BBB	1.91488	<.0001
GDP	0.19956	<.0001
BCI	-0.05782	<.0001
Prime	0.61467	<.0001
Deposit	-4.40085	0.0094
M3	-0.12687	<.0001

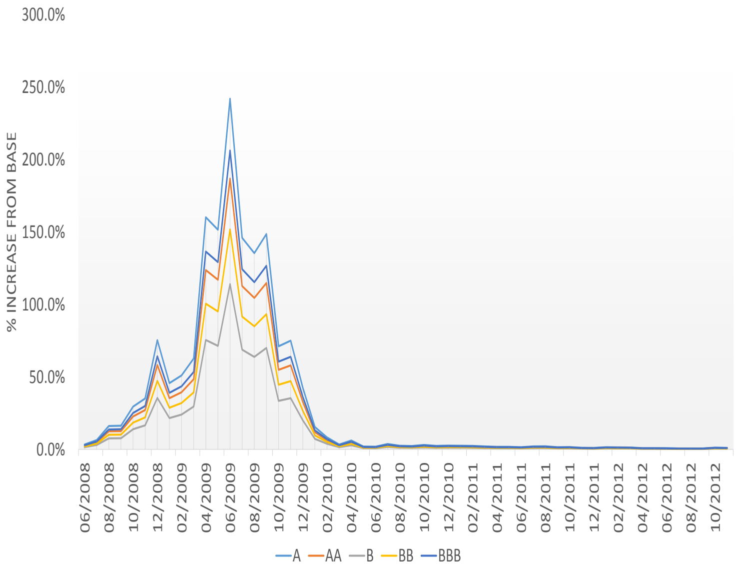
A stepwise selection criterion is used to calibrate the Cox regression model with a 5% selection criterion. The result of this calibration is shown in table 5.3.

All the risk factors considered have a p-value of less than 5%. The M2 money supply is excluded due to the high correlation with the M3 money supply. The hazard rate of a downgrade a specific client is given by equation 5.2, where the calibration above only focuses on the parametric portion $\exp(\beta Z(t))$. However, the ratio of the downgrade rate of the same client at two different time periods is given by:

ratio $(t, t+k) = \frac{e^{\beta Z(t)}}{e^{\beta Z(t+k)}}$, which is independent of λ_0.

The Cox regression model is used to identify the macroeconomic factors that are statistically significant to predict the change in the probability of default on a corporate client. The output of the Cox regression model is a set of macroeconomic factors Z that is statistically significant in predicting the probability of a credit event. The calibration of the coefficients β allows for the estimation of the probability of default

Figure 5.1: Increase in the hazard rate from the base over the historic data period per credit rating.



due to changes in the identified macroeconomic factors.

The proportional hazard rate property is used to estimate the change of the current PD assigned by the bank under the various simulated macroeconomic scenarios, using the identified covariates and estimated coefficients. Figure 5.1 shows the proportional increase in the default rates based on the calibrated Cox regression model from 2008 to 2012 for a number of rating bands. The figure only focuses on the effect of the macroeconomic factors on the hazard rate by keeping all the other covariates the same. The 7382 downgrades observed on the historic data was mainly focused around 2008 and 2009, with only limited downgrade information from 2010 to 2012. Thus, the hazard rate return to the base line from 2010 onwards. Changes in the corporate portfolio downgrade experience post 2010 was mainly due to portfolio management action, such as a switch to higher quality customers and more defensive industries as opposed to the macroeconomic impacts.

Chapter 6

Wholesale funding

The bank attracts retail and corporate deposits from investors and uses these funds to provide loans to customers. The bank also obtains funding directly from the market by issuing various debt instruments to the wholesale market, this is termed wholesale funding. Wholesale funding is important as it diversifies the funding mix available to a bank.

Funding long-term assets with short-term funding will require the bank to roll the funding at uncertain rates on future dates. This exposes the bank to both interest and liquidity risk due to the mismatch with the asset profile. A sub model is required to calculate the risk premium above the risk-free rate for the wholesale funding, this is the cost of funding, which is uncertain on future dates. The risk premium depends on the credit quality of the bank and the market view of liquidity risk (driven by the duration of the funding). This chapter builds on the existing literature and proposes a new approach to include the liquidity risk premium into the risk premium.

The spread above the risk-free rate represents the credit and liquidity risk premiums compared to government instruments of a similar term. The two components of the spread (i.e. the credit and liquidity risk premiums) are projected separately over the projection period (December 2014 to December 2019) in this thesis. These models are calibrated based on the observed spreads from April 2007 to March 2012. The credit-risk premium model is assumed to be a function of the implied PD, derived from the credit rating of the bank. The liquidity risk premium model is assumed to follow a diffusion process with Poisson jumps.

The credit rating of the bank is related to its financial strength. The current and historic credit ratings of the bank is known, based on the public ratings performed by Moody's and S&P. We need the future credit rating of the bank over the projection period based on the macroeconomic scenarios. Chapters 4,5 and 7 provide a number of key parameters required to assess the financial strength of the bank under the various economic scenarios. The outcome from these chapters are used to adjust the current credit rating of the banks and thus the credit risk premium over the projection period.

The projected liquidity risk premium is based on a diffusion process with Poisson jumps. The diffusion model is calibrated using historical liquidity risk premiums. The probability and size of the Poisson jump in the liquidity risk premium is linked to a simulated credit rating of the bank. Following this approach, the probability and size of jumps in the liquidity risk premiums will increase when the credit risk of the underlying asset portfolios increase.

6.1 Characteristics of funding

For the purpose of this thesis, we assume the bank issue, the following wholesale instruments:

- 6-Month fixed zero coupon instruments.
- 12-Month fixed zero coupon instruments.
- 18-Month fixed quarterly coupon instruments.
- 24-Month fixed quarterly coupon instruments.

The quarterly coupon rate is determined such that the note is issued at par. The above funding structure is chosen to represent a range of the main funding options available in the South African market.

6.2 Literature overview

The literature review is segmented into a section covering the decomposition of the credit and liquidity premium from observed data, and a section setting out the use of stochastic jump models in valuing economic time series data.

6.2.1 Decomposing the credit and liquidity risk premium

Chrum and Panigirtzoglou (2005) [25] use the Leland and Toft (1996) [71] model to calculate the credit risk component of observed spreads of UK corporate debt securities, using a structural credit risk model as proposed by Merton (1974) [77]. This is based on the principle where a default is defined if the asset value for the firm falls below the debt. The main factors that drive the default process are the value of the firms assets, the uncertainty of the asset value and the level of leverage. Chrum and Panigirtzoglou (2005) [25] calculated a spread on investment grade bonds of 132 basis points, of which 72 basis points (55 %) is explained by the credit risk component as introduced in the Leland and Toft model. The portion of the observed spread not explained by the credit spread is termed the liquidity spread. Webber and Churm (2010) [107] also applied a structural credit risk model approach to decompose the observed spreads on corporate bonds into a credit and liquidity component. They confirmed that the non-credit component was an important driver of high yields observed during the recent market turmoil.

The above research confirms the spread on corporate bonds compensate investors for credit risk, liquidity risk and parameter uncertainty. Banks that are highly leveraged firms and thus the Leland and Toft model is not ideal for banks with high leverage ratios (above 90 % as is the case in this thesis). Pagratis and Stringa (2009) [84] developed a rating replication model specifically calibrated for banks.

Fons (2005) [40] used the marginal default rates obtained from Moody's to analyse the relationship between credit risk premium, default probabilities, recovery rates and the term structure of corporate bonds. Elton *et al.* (2001) [38] extended this concept by using the S&P and Moody's transition probabilities to derive the marginal default rates per rating category. The default rates are applied in a risk-neutral bond pricing algorithm to calculate the implied credit risk premiums. Using the implied or calibrated default rates to calibrate the credit risk premium is widely used in the industry. This approach is used in this thesis to decompose the historically observed spread into a credit and liquidity risk premium.

6.2.2 Stochastic jump process

The literature overview will discuss the application of a diffusion process with Poisson jumps in general financial time series models.

Jump diffusion models have been proposed in financial literature for a number of time series analysis. Oldfield , Rogalski and Jarrow (1977) [83] investigated the impact by allowing jumps in an auto regressive framework for equity prices. Bates (1996) [7] proposed a diffusion process with Poisson jumps to value the volatility in foreign currency options. The exchange rate was assumed to follow a geometric jump diffusion process. Tauchen and Zhou (2006) [100] proposed a methodology to identify jumps in time series data, using a bi-power variation and realised variance on the time series. This methodology is used to determine the distribution of the jumps for the S&P equity index, 10-year US Treasury bond rate and Dollar:Yen exchange rate.

Zhou (2001) [118] and Pedersen and Krogsgaard (2008) [86] proposed a model that builds on both the structural and reduced credit risk models. The asset return is assumed to follow a pure diffusion process as per Merton's (1974) [77] structural model, but also includes the arrival of unexpected information via the Poisson process, in line with the reduced form models. Zhou assumed the jump size from the Poisson process follows a log-normal distribution.

This thesis uses the compound Poisson process as per Bates (1996) [7]. This model is applied to the historic time series of liquidity premiums observed in figure 6.2, where the large discontinuous jumps are defined as the outcome from the Poisson counting process.

The development of the liquidity risk premium over time has both a diffusion process and a number of discontinuous jumps. A similar time series exists in the commodities and electricity price market, see Suarez and Fernandez (2009) [96] and Cartea and Figueroa (2005) [22] for examples.

Suarez and Fernandez (2009) applied a mean reversion jump diffusion process to model the copper price. The jumps are introduced to the stochastic process from a Poisson process, where the size of the jumps is exponentially distributed. The formulation of the discrete model is required to allow for simulations and is based on Dais and Rocha (2001) [32] and Dixit and Pindyck (1994) [33]. Dixit and Pindyck (1994) used an approach to calibrate the mean reversion parameters in the partial differential equation. A similar mean-reversion jump diffusion model was proposed by Cartea and Figueroa (2005), where the size of the jump was assumed to be normally distributed.

A similar mean reversion calibration, as set out by Cartea and Figueroa, is applied in this thesis. Due to the limited data, a simplified discrete distribution of the jump size was assumed. This could be improved if daily liquidity premium data is available.

6.3 Methodology: Overview

Figure 6.1 provides an overview of the process used to forecast the total risk premium to obtain the wholesale funding cost to support the optimisation in chapters 2 and 3. The historical total risk premiums for the bank is available from April 2007 to March 2012. A methodology is required to decompose this total risk premium into a credit and liquidity risk premiums. A model is applied to decompose the credit risk premium and the remainder is termed the liquidity risk premium.

The forecast of the credit risk premium is driven by the credit rating of the bank in this thesis. An established rating model developed by Pagratis and Stringa (2009) is used to predict a rating change of the bank over the projection period (December 2014 to December 2019). The credit risk premium forecast is a function of the credit ratings and thus implied probability of default of the bank. The historical credit risk premium is thus only required to calculate the historical liquidity risk premium.

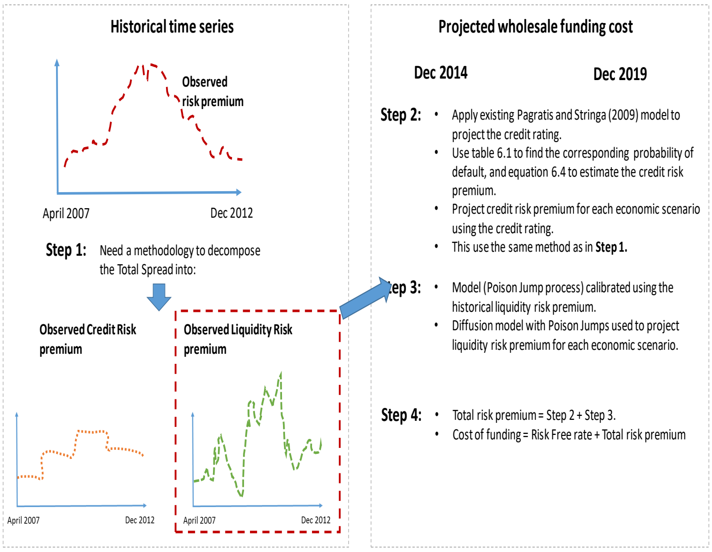
The historical time series of the liquidity risk premium is used to develop a Poisson jump diffusion model of the liquidity risk premium. This jump diffusion model is used to forecast various liquidity risk premium paths from December 2014 to December 2019 to support the optimisation in chapters 2 and 3.

6.4 Methodology: Cost of wholesale funding

Let S_m be the base overnight rate published by the South African Reserve Bank. Let $P_{d,m}$ be the total risk premium above the base overnight rate required by an investor to invest funds for longer than overnight. Where d is the duration 6,12,18 and 24 months and m the monthly time stamp.

The cost of funding paid by the bank is $C_{d,m} = P_{d,m} + S_m$.

Figure 6.1: Diagram of the modelling framework supporting the projection of the wholesale funding costs.



Let $R_{d,m}$ be the rate from the risk-free yield curve observed at time m and duration d.

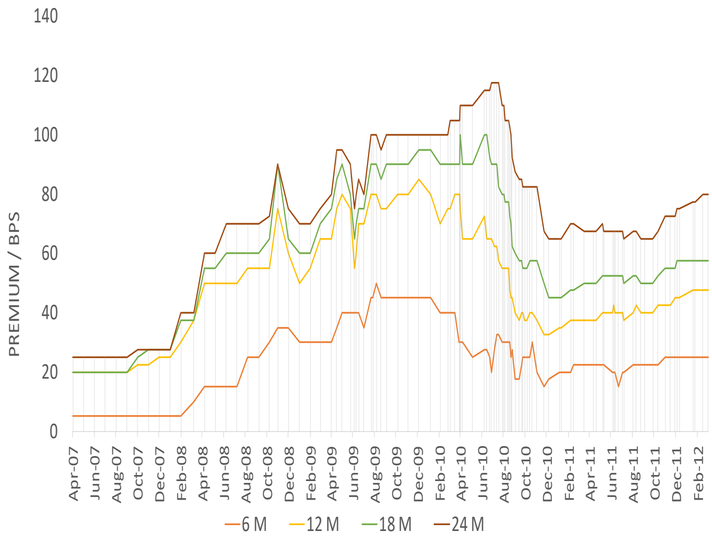
The decomposition of the observed total risk premium follows: $OS_{d,m} = C_{d,m} - R_{d,m}$.

The observed total risk premium $OS_{d,m}$ paid by the bank includes both a credit and liquidity risk premiums where d and m are the same as above. For the purposes of this thesis, we will decompose $OS_{d,m}$ into a credit component $Pc_{d,m}$ and a liquidity component $Pl_{d,m}$. Both these risk premiums will be projected over the observation period for each economic scenario.

The available data represents the historical total risk premium paid by the bank on 6-,12-,18- and 24-months instruments. The total risk premium is quoted over the overnight rate published by the South African reserve bank. Figure 6.2 shows the total risk premium ($OS_{d,m}$) from April 2007 to March 2012.

A time series of historic liquidity risk premiums is required to calibrate the diffusion process with Poisson jumps used to simulate various liquidity risk premium paths for each macroeconomic scenario. The time series of historical liquidity risk premiums is obtained by first calculating the credit risk premium $Pc_{d,m}$ and defining the

Figure 6.2: Total risk premium ($OS_{d,m}$) from April 2007 to March 2012 for d = 6-,12-,18- and a 24-month instrument.



liquidity risk premium as $Pl_{d,m} = OS_{d,m}$-$Pc_{d,m}$.

6.5 The model: Credit risk premium

6.5.1 Methodology

This thesis requires a model to forecast the credit risk premium over the observation period. A methodology was chosen, based on the credit rating of the bank. Alternative models are available, however this methodology was chosen based on the available data. The components supporting this methodology is well established in the banking industry, with a large number of concepts derived from the Basel Capital Accord (2009). More complex and detailed models are available to model credit risk premiums, however the credit risk premium is not the focus of this thesis.

For the purposes of this thesis, $Pc_{d,m}$ is a function of the rating of the individual bank. A methodology derived from risk-neutral pricing based on the probability of default and loss given default is used to derive the credit spread based on the credit rating (see Elton *et al.* (2001) [38]). A projection of $Pc_{d,m}$ over the observation period requires a projection of the bank's rating. The model calibrated by Pagratis and Stringa (2009) is used to project future rating upgrades or downgrades of the individual bank. A rating upgrade or downgrade is based on the projection of key financial ratios, using results from chapters 4, 5 and 7. This ensures that the projection of the credit risk premium ties in with the projection of the other components of the bank's balance sheet over the projection period.

Historical credit risk premium

In a risk-neutral environment the expected cash flows from a bond can be discounted at the risk-free rate to determine the value of the bond. This principle is used to derive the credit risk premium per the following:

The value of the bond $V_M = 1$ at time M.

Let $V_{m,M}$ represent the value of the bond at time m given maturity at time M.

If $M = 2$ the value of the bond at time $m = 1$

$$V_{1,2} = [C(1 - P_2) + P_2 \alpha V_2 + (1 - P_2)V_2] \exp(r^G_{1,2}) \tag{6.1}$$

where: C is the coupon payable at time m = 2.
P_2 is the probability of default between times 1 to 2.
α is the recovery rate if the bond defaults.
$V_2 = 1$ is the value of the bond at time m =2.
$r^G_{1,2}$ is the risk-free rate (forward rate between time m = 1 and 2).

An alternative expression for the value of the bond $V_{1,2}$ is based on the return an investors will require r^C_{12} such that

$$V_{1,2} = [C + V_2] \exp(r^C_{1,2}). \tag{6.2}$$

From equating 6.1 and 6.2 above the credit risk premium is calculated as

$$\exp(-(r^C_{1,2} - r^G_{1,2}) = (1 - P_2) + \frac{\alpha P_1}{(1 + C)}. \tag{6.3}$$

The above formula can be expanded by first considering the formula to value a bond at time 0 maturing at time 2. Expanding this equation to derive the credit risk premium between time m and m+1 as:

$$\exp(-(r^C_{m,m+1} - r^G_{m,m+1}) = (1 - P_{m+1}) + \frac{\alpha P_{m+1}}{(V_{m+1} + C)} \tag{6.4}$$

where $\exp(r^G_{m,m+1})$ is the risk-free rate obtained from the swap curve. And C is the coupon payable at time $m + 1$. $Pc_{d,m}$ is obtained by aggregating equation 6.4 from time m for the required duration d.

Projection of the credit risk premium

Pagratis and Stringa (2009) developed a model to upgrade or downgrade the credit rating of a bank based on its financial statements and country information. The Pagratis and Stringa (2009) model is a rating replicating model, as it targets the specific Moody's rating band during calibration. The rating process follows five areas of fundamental analysis; capital adequacy, asset quality, management, earning and liquidity risk. This model was calibrated on the rating history of 293 banks across the US, Europe and Asia. Pagratis and Stringa (2009) calculated a threshold trigger for each of the rating buckets. A rating change is triggered if the *Rating* calculated, using the banks specific published accounts and country specific information, breaches the ratings triggers as calculated by Pagratis and Stringa (2009). An ordered probit model was used by Pagratis and Stringa (2009) to calibrate both the model for *Rating* and the rating thresholds simultaneously. The same model was used to effect any rating changes for the bank over the projection period for each economic scenario.

The Pagratis and Stringa (2009) model follows a two-step process. The first step calculates a targeted equity-to asset-ratio, based on the banks financial position, the targeted equity-to-asset ratio is compared to the actual equity-to-asset ratio of the bank. This is used in the second model, together with a range of other factors. See Appendix 3 for more detail on the covariates per the Pagratis and Stringa (2009) model.

Table 10.2 in Appendix 3 details the final covariates selected by Pagratis and Stringa (2009) to project the target $Rating_m$ at time period m. Table 6.1 show the trigger threshold for each Moody's rating bands based on the $Rating_m$ at time period m per the Pagratis and Stringa (2009) model. The financial statements of the bank in this thesis is used together with the projected balance sheet and income statement items from chapters 4,5 and 7 to project the $Rating_m$ at m over the observation period. The projected $Rating_m$ is used to identify any credit rating changes per the lookup from table 6.1.

Table 6.1: Thresholds used to map $Rating_m$ to the Moody's rating.

Moody's rating	**Lower bound for** *Rating*	**Upper bound for** *Rating*
Aaa-Aa1	6.929	.
Aa2	6.402	6.929
Aa3	5.489	6.402
A1	4.725	5.489
A2	3.746	4.725
A3	2.746	3.746
Baa1	1.188	2.746
Baa2-Baa3	-0.851	1.188
Ba1-Ba3	-3.061	-0.851
B1 below	< -3.061	.

6.5.2 Calibration

Historical credit risk premium

The credit risk premium $Pc_{d,m}$, where d is the duration and m the observation month, is calculated for m = April 2007 to March 2012 using equation 6.4.

P_{t+1} is the probability of default between time t and t+1 from equation 6.4. The default rates are obtained from the Moody's transition rates. Moody's publish the observed rating migrations over the period 1981 to 2014. The transition rates are updated each year based on observed experience. This is presented in the format of a migration matrix representing the probability of migrating over 1 year to the various rating categories, including default. The bank in this thesis was rated A in Nov 2005 and downgraded to BBB in Dec 2011. Based on the A and BBB rating the probability of default over 12,24,36 and 48 months is calculated (see table 6.2). The probability of default for other rating bands were also calculated to support the credit premium calibration over the simulated time period. Allowance is made for the bank to migrate to a lower grade within the period and subsequently default from these lower grades.

The default rate over a shorter period is estimated using the hazard rate approximation. For example, the probability of default over 6 months ($P_{0,6}$) will be estimated as $1 - (1 - P_{0,12})^{0.5}$.

The credit risk component $Pc_{d,m}$, where d is the duration and m the observation

Table 6.2: Default rate implied by the Moody's rating for 2014.

Duration	Rating: A	Rating: BBB
$P_{0,12months}$	0.08%	0.21%
$P_{0,24months}$	0.17%	0.46%
$P_{0,36months}$	0.28%	0.76%
$P_{0,48months}$	0.39%	1.08%
$P_{0,60months}$	0.52%	1.42%

month is obtained by applying equation 6.4 based on the risk-free rate $\exp(r^G_{m,m+1})$ and default rate $P_{0,m}$, for m from April 2007 to March 2012.

Projection of the credit risk premium

The public financial statements of the bank underlying this thesis were available to calculate the *Rating* per the Pagratis and Stringa (2009) model as at December 2014. Based on this, the bank should be rated BBB, which was the public rating of the bank. The output from chapters 4-7 is used to forecast the covariates underlying the Pagratis and Stringa (2009) model from December 2014 to December 2019. This is used to calculate $Rating_m$ for month m over the projection period for each scenario, to determine if any rating adjustment is required. The projected rating is used to determine the probability of default per table 6.2. $Pc_{d,m}$ is calibrated using equation 6.4, based on the simulated risk-free rate and the probability of default based on table 6.2.

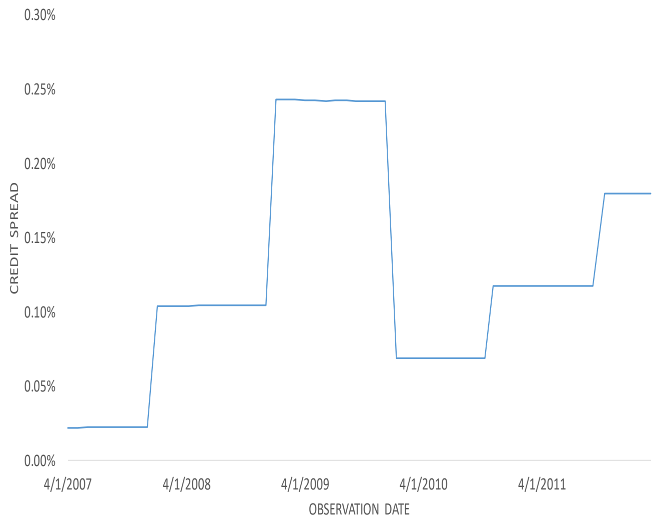
6.5.3 Results

This section sets out the results of the credit risk premium decomposition over the historic data period.

Historical credit risk premium

Figure 6.3 shows the calibrated credit risk premium for the 6-month instrument over the observation period. The jumps in the credit risk premium is based on the annual update of the Moody's migration matrix and the ratings downgrade in November 2011.

Figure 6.3: Credit risk premium ($Pc_{6,m}$) for the 6-month bank instrument.



6.6 The model: Liquidity risk premium

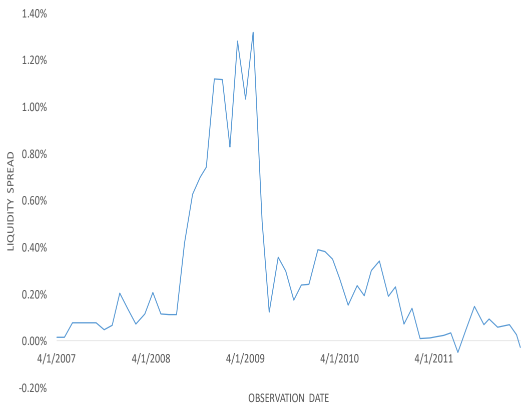
The remaining component of the observed spread is the liquidity risk premium. Figure 6.4 shows the historical liquidity risk premium after deducting the credit risk premium per figure 6.3 for the 6-month instrument. A characteristic of the historic liquidity risk premium is the large jumps observed. A diffusion process with Poisson jumps is fitted to the historical data. This choice is based on the limited liquidity premium data (i.e. monthly) and that the choice of model fitted the requirement of the simulation approach. This model is used to simulate $Pl_{d,m}$.

6.6.1 Methodology

Liquidity risk premium

The liquidity risk premium, including the jumps, is highly correlated across instruments of different duration. The correlation over the observation period (April 2007 to March 2012) is between 80% and 97%. The large jumps in the liquidity premium is due to new information observed by investors. This information affects the liquidity premium across all durations. Based on this, a diffusion process with Poisson jumps is fitted to the six-month instrument only. The liquidity premium for the other instruments is derived using the follows linear regression models:

Figure 6.4: Liquidity risk premium ($Pl_{6,m}$) for the 6-month instrument.



$$Pl_{12,m} = \beta_{1,6} * Pl_{6,m} + \epsilon_m$$
$$Pl_{18,m} = \beta_{2,6} * Pl_{6,m} + \beta_{2,12} * Pl_{12,m} + \epsilon_m$$
$$Pl_{24,m} = \beta_{3,6} * Pl_{6,m} + \beta_{3,12} * Pl_{12,m} + \beta_{3,18} * Pl_{18,m} + \epsilon_m \tag{6.5}$$

where β is estimated from the historical time series, from April 2007 to March 2012 and ϵ_m is the error term. In the above notation $\beta_{1,6}$ explain the linear relationship between $Pl_{12,m}$ and $Pl_{6,m}$, where $\beta_{2,6}$ explain the linear relationship between $Pl_{18,m}$ and $Pl_{6,m}$ which will not be the same.

Mean reversion process

The notation for the diffusion process is expressed in terms of x_m, where m is the monthly time interval and $x_m = log(Pl_{6,m})$.

Start by assuming x_m is a mean reversion process with the following stochastic differential equation:

$$dx = \eta(\bar{x} - x)dm + \sigma dz \tag{6.6}$$

where $\bar{x}$ is the expected value of x, η, the speed of the mean reversion process and σ the diffusion parameter, with dz a Wiener stochastic process drawn from a $N(0, 1)$ distribution. Per Suarez and Fernandez (2009) [96], the solution to this stochastic differential equation in terms of Ito's stochastic integral is:

$$x_M = x_0 e^{-\eta M} + \bar{x}(1 - e^{-\eta M}) + \sigma e^{-\eta M} \sum_{m=0}^{M} e^{\eta m} dz \tag{6.7}$$

where $E(x_M) = x_0 e^{-\eta M} + \bar{x}(1 - e^{-\eta M})$ and the $Var(x_M) = (1 - e^{-2\eta})\frac{\sigma^2}{2\eta}$ as per Dixit and Pindyck (1994) [33].

Since $x_m = log(Pl_{6,m})$ the approach to simulated $Pl_{6,m}$ from $Pl_{6,m-1}$ is obtained via:

$$\begin{aligned} Pl_{6,m} = e^{[}log(Pl_{6,m-1})e^{-\eta\Delta m} &+ log(\bar{Pl_6})(1 - e^{-\eta\Delta m}) \\ - (1 - e^{-2\eta\Delta m})\frac{\sigma^2}{4\eta} &+ \sigma\sqrt{\frac{1 - e^{2\eta\Delta m}}{2\eta}}dz] \end{aligned} \tag{6.8}$$

where dz is a Wiener stochastic process drawn from a $N(0, 1)$ distribution.

Mean reversion process with Poisson jumps

A jump is added to the mean reversion process per equation 6.6 as follows:

$$dx = \eta(\bar{x} - x)dm + \sigma dz + Jdq. \tag{6.9}$$

The dq term is a Poisson term where the value is zero most of the time, yet in some cases, dq takes on a value of 1. J is the proportional size of the jump if dq is 1. The Poisson process dq is defined as:

$$dq = \begin{cases} 1 & \text{with probability } \lambda \; dm. \\ 0 & \text{with probability } 1 - \lambda \; dm. \end{cases} \tag{6.10}$$

For this thesis, assume J, dq and dz are independent. A generic representation of the Poisson process assumes the probability of a jump (λ) has a distribution ω and the size of the jump (J) has a distribution Φ. In the most simplistic case, the distribution ω might be equal to λ with probability 1 and the size of the jump (J) equal to k with probability 1. This is the assumption in this thesis, given the few data points available to calibrate the model. Thus $E[J] = k$ and $E[\omega] = \lambda$.

It is important for equation 6.9 to be a martingale. This is achieved by subtracting λk in equation 6.9 to define the new stochastic differential equation.

$$dx = \eta[(\bar{x} - x) - \lambda k]dm + \sigma dz + Jdq. \tag{6.11}$$

Dais and Rocha (2001) [32] and Dixit and Pindyck (1994) [33] showed that the discrete model for simulation is given by the equation:

$$\begin{aligned} Pl_{6,m} = e^{[}log(Pl_{6,m-1})e^{-\eta\Delta m} + log(\bar{Pl}_6 + \lambda k\eta)(1 - e^{-\eta\Delta m}) \\ + \sigma\sqrt{\frac{1 - e^{-2\eta\Delta m}}{2\eta}}dz + dq - (1 - e^{-2\eta\Delta m})\frac{\sigma^2 + \lambda var(\Phi)}{4\eta}] \end{aligned} \tag{6.12}$$

where $var(\Phi)$ is the variance of the jump process and is estimated directly from the historical market date.

6.6.2 Calibration

Calibrating the mean reversion parameters

The approach described by Dixit and Pindyck (1994) is followed to calibrate the mean reversion parameters. Dixit and Pindyck (1994) showed that 6.9 is the same as an $AR(1)$ discrete time model:

$$x_m - x_{m-1} = \bar{x}(1 - e^{-\eta}) + (e^{-\eta} - 1)x_{m-1} + \epsilon_m \tag{6.13}$$

where ϵ_m follows a normal distribution with standard deviation σ_ϵ as:

$$\sigma_\epsilon^2 = \frac{\sigma^2}{2\eta}(1 - e^{-2\eta}). \tag{6.14}$$

The parameter of equation 6.13 can be expressed in the format:

$$x_m - x_{m-1} = a + bx_{m-1} + \epsilon_m. \tag{6.15}$$

The mean reversion parameters are obtained as:

$$\bar{x} = \frac{-\hat{a}}{\hat{b}}. \tag{6.16}$$

$$\hat{\eta} = -log(1 + \hat{b}). \tag{6.17}$$

The volatility of the mean reversion process is calculated as:

$$\sigma^2 = \frac{\sigma_x^2}{2\eta}(1 - e^{-2\eta}) \tag{6.18}$$

where σ_x is the variance of x_m.

Table 6.3: Summary of the parameters to calibrate to project $Pl_{6,m}$ per equation 6.12.

Parameters	Description	Value
η	Mean reversion parameters of the x_m time series	0.078
$\overline{x}$	The long run estimate of the x_m time series	1.0023
σ_x	The volatility of the time series x_m	0.674
σ	Derived from equation 6.18	0.625
λ	Frequency of jumps	23%
Jump	Definition of a jump	8%
$var(\Phi)$	Variance of the jump size	0.14240
k	The expected value of the jump size	8%

Calibrating the Jump distribution

A number of methodologies exists to calibrate the Poisson jump parameters; the probability of the jump (λ) and the size of the jump Φ. It is required to define what is classified as a jump from the time series data. Tauchen and Zhou (2006) [100] calibrated the jump size using a realised variance and Bi-power analysis, based on their research, they identified a jump as a 13%-20% change in asset values. Suarez and Fernandez (2009) [96] assumed 3 standard deviation of the asset returns to identify jumps.

Suarez and Fernandez (2009) assumed a fixed probability for an up jump and a fixed probability for a down jump. These two parameters were calibrated from the historical data. The size of the jump was assumed to follow an exponential distribution.

For the purpose of this thesis, we assume a simplified discrete distribution calibrated from the actual observed jumps, with a fixed probability for an up and a down jump of 23 %. The jump size was set to 8% for an up and down jump. The main reason for this is the lack of data, as only monthly data for a 5 year period was available for the liquidity premium calibration. The jump size can be calibrated if daily of weekly data is available, and should be refined in future research.

Table 6.3 show the parameters required to implement the jump diffusion model for the liquidity premium.

The calibration of the above parameters is done for the $Pl_{6,m}$ time series $d = 6$ only. The equations in 6.12 are used to derive the liquidity risk premium for the other instruments.

The dataset available for calibration consists of 59 data points (April 2007 to February 2012), with 13 observed jumps as defined with the 8% jump above. This fairly small dataset restricts the calibration of the distribution of the actual jump.

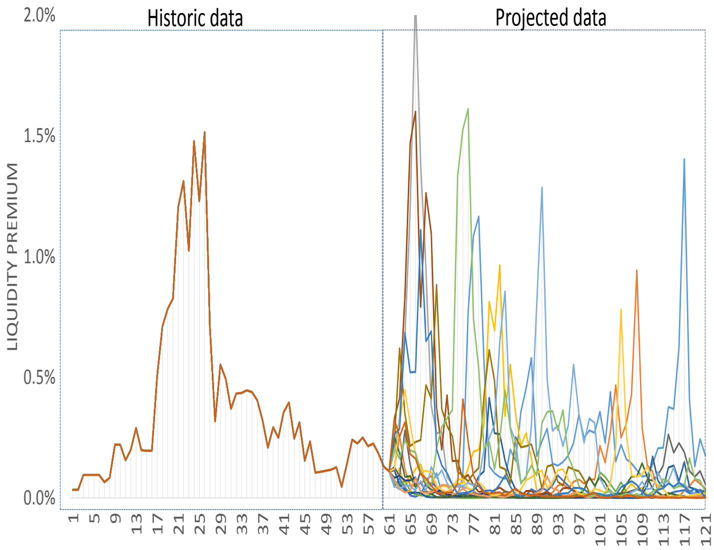
6.6.3 Results

The observed spread is based on the average monthly spread based on a large number of individually traded instruments issued by the bank. Using the daily instrument level price data instead of the monthly average rate will allow for a more robust calibration of the jump size distribution. This calibration is for future research, as the focus of this thesis is on the methodology to incorporate liquidity cost into an optimisation framework, and not the detailed calibration of the liquidity premium model.

Equation 6.5 are used to estimate the $Pl_{d,m}$, where d = 12,18 and 24 months. A linear regression model was fitted to the time series data to calibrate the required parameters.

Figure 6.5 show 10 simulated paths of the the liquidity premium from the model calibrated above.

Figure 6.5: Historical and corresponding projected liquidity premium for a 6-month instrument.



Chapter 7

Deposit base

Traditionally, banks relied on a large deposit base to fund the various loan portfolios. The development of modern financial markets provided investors (both retail and corporate) with an alternative to depositing money with banks. This increased competition for deposits has changed the behaviour of both customers and banks.

The model office underlying the NII function in chapter 3 requires an estimate of the interest rate paid on deposits and the corresponding size of deposits. Both the rate offered on deposits and the size will change under macroeconomic outcomes. The size of the deposit portfolios is an important input into the overall optimisation framework, as a sudden drop in deposit levels needs to be funded by wholesale funding.

Banks offer mainly fixed term and call deposits to both corporate and retail clients. The bank provides a fixed interest rate for a specified period for fixed term products, irrespective of the prevailing market rates. A large number of fixed rate products roll over on maturity based on the new market rates. Call deposits are termed non-maturing accounts ("NMA"), where the bank has the option to change the deposit rate with no advance notice. The client can also call the deposit at a moments notice. The modelling of the duration of NMA is of particular interest in the literature.

The deposit rate set by the bank is at its own discretion. The basic economic principle of supply and demand applies in determining the client rate offered by banks. The client rate offered depends on alternative investments in the market and the client rate provided by competitors. It is expected that the client rate offered on deposits should react with some delay to changes in the prime rate. Duecker (2000) [38] and Rosen (2002) [29] argued that the information available to customers and cost involved in switching investments will cause the speed of changing the deposit rate to react differently to changes in market rates for loans.

A model of both the deposit rate and the size of deposit portfolios is required to model the deposit portfolio of the bank under various economic scenarios, and in particular how the client rates are reset when prevailing market rates change.

7.1 Literature review

The interest rate offered on deposits has been a research topic from the mid 1950's following large scale removal of banking regulations by moving to free market operations. The various literature studies over the last 50 years have found the following important characteristics in deposit rates.

- Interest rates are sticky, and thus the speed of changes in both lending and deposit rates differ from changes in market rates.
- The change in deposit interest rates depends on the market conditions, and in particular if market interest rates are increasing of falling.

Weth (2002) [109] and Gambacorta (2004) [42] argued that the stickiness of deposit and lending rates is a result of the balance sheet structure of the bank. Their research confirmed that banks with a larger deposit base, and thus less reliance on wholesale funding, have more sticky deposit and lending rates.

A number of different models has been proposed to model the behaviour of client interest rates offered on deposits by banks. Paraschiv (2011) [85] proposed various methodologies to calibrate the deposit rate of NMA for accounts published by the Swiss National Bank. These included simple and multiple regression models (such as replicating portfolios), cointegration, error correction models, allowance for asymmetries in the deposit rate, a threshold model for asymmetry and a friction (rigidity) model. For the Swiss Nation Bank deposit rates, the friction model was the best at predicting the client rate offered by banks based on both in- and out-of-sample testing.

Simple and multiple regression models are used to explain the movement in the deposit rate as a function of changes in the market rate of various maturities. This methodology is called a replicating portfolio, as a portfolio of fixed interest securities is obtained to replicate the deposit rate behaviour. Bardenhewer (2007) [6] proposed a model where the client rate is a function of the moving average of the market rate at various maturities, explaining in part the stickiness of deposit rates. This incorporates both the size and rate behaviour of deposits into one model. Von Feilitzen (2011) [105] applied the Bardenhewer (2007) model to calibrate the client rate model for NMA for the Handelsbanken. A similar multiple regression model was developed by Paraschiv (2011) [85]. The empirical evidence from the multiple regression model fitted by Paraschive (2011) resulted in residual terms with very high autocorrelations. Paraschive argues that the client rate and market rate time series are non-stationary and thus a cointegration model should rather be used. Paraschiv (2011) applied a similar cointergration vector as Hansen and Seo (2002) [52].

An alternative to the cointegration model is an error correction model as proposed by Weth (2002) [109]. Applying the regression model above assumes that there is a stationary relationship between the deposit rate and the market rate. The error correction model explains the change in the deposit rate as a function of the change in the 3 month lag deposit rate and change in the 3 month lag market rates, with corresponding restrictions on coefficients to ensure a stationary process.

Hutchinson and Pennachi (1996) [60] assumed an imperfect market for deposit rates offered by banks and applied the concept of the deposit rate rent achieved by the bank. The deposit rent is derived by assuming the bank has a downward sloping demand curve, which is a function of deposit interest rates and competitive market rates and other local market variables, used to derive the bank's optimal deposit interest rate. This provides a framework to calculate the optimal deposit rate, given changes in competitive market rates.

Based on Dueker (2000) [34], banks adjust their deposit and prime lending rate on an asymmetrical basis. Dueker (2000) proposed a dynamic ordered probit model to estimate the threshold above which the prime rate will impact the deposit rate. The coefficients of the corresponding changes given a breach in the threshold, are also calibrated. The probit model requires the development of the latent level of the prime rate that depends on a set of explanatory variables (in this case, 3 month T-bills). The change in the prime rate is based on the build up of the latent level of prime. Dueker (2000) confirmed that the coefficient for an increase and reduction in prime is different, confirming the asymmetric movement in bank lending and deposit rates.

Rosett (1959) [90] proposed a friction model revolves on a mass point to estimate a dependent model based on a certain outcome of the world. The dependent variable will not change for small changes in the independent variables. Forbes and Mayne (1989) [41] applied the likelihood function derived by Rosett (1959) to calibrate a model for the change of the prime lending rate to changes in observed market rates. One of the methodologies applied in Paraschiv (2011) to model the client rate on non-maturing deposits was a friction model based on the methodology followed by Forbes and Mayne (1989).

A model of the size of the deposit portfolio is required. The investor has the right to withdraw funds from the bank at a short notice and thus it is expected that the size of deposit portfolios should fluctuate from month to month. Jarrow and Van Deventer (1998) [61] modelled the size of deposits as a function of the client or market rates. Paraschiv (2011) proposed a model that takes the size of the market rate, size of the client rate, spread between client rates and markets rates and macroeconomic factors into account. Bardenhewer (2007) [6] included a model of the portfolio size directly in the replicating portfolio, where the main factor in the portfolio size model is the spread of the client rate and market rates above its long-run average.

The deposit portfolio size and rate model is not the focus of this thesis, thus a relatively simple method compared to the above discussions was chosen. A portfolio replicating approach was selected for this thesis. The model for the deposit portfolio size follows the same format at per Paraschiv (2011).

7.2 Methodology

The client rate model is required to determine the interest rate the bank needs to pay on the portfolio of deposits, where the portfolio size model is required to calculate the size of the deposit portfolio. Both the client rate and portfolio size models are driven by the risk-free forward rates. A time series of the average portfolio size and rate on various deposit products are available at a portfolio level (as opposed to a client level). The portfolio size and rate models developed are based on a linear regression model, using the historical time series, the portfolio size and rates per product as a dependent variable and the risk-free forward rates as independent variables. Separate models are developed for the various deposit product types.

7.2.1 Portfolio rate

The replicating portfolio is an approach to transform the non-maturity liabilities into a portfolio of fixed-rate products with known maturities. The principle of a replicating portfolio is to match the historical rate adjustments of the non-maturing liabilities with the historical rate adjustments of a portfolio of fixed interest instruments.

The proposed model follows the principle as per Paraschiv (2011), where the replicating portfolio is chosen to match the movement in the customer rate.

Let CR_t^j be the client rate at month t for product j. $\Delta CR_t^j = CR_t^j - CR_{t-1}^j$ represents the change in the client rate between month $t-1$ and t for product j. The list of products considered is shown in table 7.1.

Let $R_{i,t}$ be the risk-free forward rate of duration i at month t. $\Delta R_{i,t} = R_{i,t} - R_{i,t-1}$ represents the change in the risk-free forward rate of duration i between month $t-1$ and t.

The specification of the model follows:

$$\Delta CR_t^j = \alpha^j + \sum_{i=1}^{n} \beta_i^j * \Delta R_{i,t} + \epsilon_t^j. \tag{7.1}$$

We will focus on a model of the change in the deposit rates rather than the absolute level of the rates. This follows the argument by Paraschiv (2011) that the test statistics (t-statistic) for α^j, β_i^j obtained as OLS estimators will be misleading if the dependent and independent variables are non-stationary. Paraschiv (2011) proposed a cointegration approach to resolve this estimation difficulty.

An extension to the above model specification is to use a moving average of the change in the risk-free forward rate. This is based on the fact that the client rate will adjust with a lag to changes in risk-free forward rates due to the cost required for customers to switch and the asymmetry of information.

Define:

$$\bar{R}_{i,t}^k = \frac{1}{k} \sum_{n=0}^{k-1} R_{i,t-n} \tag{7.2}$$

where $\bar{R}_{i,t}^k$ represents the average of the risk-free forward rate of duration i, calculated over the time period $t-k$ to t.

The specification of the model follows as:

$$\Delta CR_t^j = \alpha^j + \sum_{i=1}^{n} \beta_i^j * \Delta \bar{R}_{i,t}^k + \epsilon_t \qquad (7.3)$$

where n represents the set of risk-free forward rates at various durations included as independent variables. The parameters α^j, β_i^j are obtained using a ordinary least square approach.

7.2.2 Deposit portfolio size

Various independent variables have been suggested to explain the change in deposit portfolio size. At a high level, the independent factors considered to explain the change in deposit portfolio size are:

- The risk-free forward rate.
- The spread between the client rate and risk-free forward rates.
- An additional factor such as income growth, inflation, a house price index and aggregate money supply (M1 or M2).

The above is based on the argument that customers adjust their savings habits in the form of deposits based on the returns on alternative investments and other macroeconomic factors that affect their wealth. Bardenhewer (2007) [6] proposed a deposit portfolio size model by considering the client rate relative to a long-term trend and the forward rates relative to its long-term trend.

We adopt an approach where the deposit portfolio size is a function of the absolute level of risk-free forward rates at various durations, the spread between these rates and the client rate. A stepwise regression model approach is applied to test the significance of each of the independent variables. Additional macroeconomic factors can be included to improve the model, such as GDP and business confidence.

Let V_t^j represent the deposit portfolio size as at month t for product j. The list of products considered is shown in table 7.1.

The specification of the model is as follows:

$$\Delta V_t^j = V_t^j - V_{t-1}^j = \mu^j + \sum_{i=1}^{n} \beta_i^j \Delta R_{i,t} + \sum_{i=1}^{n} \delta_i^j \left(R_{i,t} - CR_t^j \right) + \epsilon_t \tag{7.4}$$

where: ϵ_t is a random error term with a zero mean and variance σ^2.
$R_{i,t}$ is the risk-free forward rate of duration i at time period t.
CR_t^j is the client rate offered at time t for product j.
j is the product number per table 7.1.
μ^j, δ_i^j and β_i^j and are the coefficients of the model to be estimated.
n represents the set of risk-free forward rates at various durations.

7.3 Calibration

The methodologies used for the deposit rate and portfolio size models are at a portfolio level rather than an per account level.

Based on this, the bank provided us with a summary of the corporate and retail deposit portfolios at a product level at month end. The bank provided data from Jan 2007 to Dec 2012 for both retail and corporate portfolios. Table 7.1 show the product classification used by the banks including the product code and a short description of the portfolios.

Table 7.1: Deposit products considered as part of this thesis.

Product	Product code	Description
1	*Call*	Immediately callable deposits
2	*NCD*	Negotiable certificates of deposits
3	*CD*	Certificates of deposits
4	*FD*	Fixed deposits
5	*FRN*	Forward rate instruments
6	*Fix*	Retail fixed deposits
7	*Cheque*	Retail cheque accounts

Figure 7.1 shows the historical time series of the deposit portfolio size from April 2007 to Dec 2012 for some of the products, where as figure 7.2 shows the historical time series of interest rates paid on the same products over the same period. This is the input data used to calibrate the regression model.

Figure 7.1: Historic time series of the deposit portfolio size split by product.

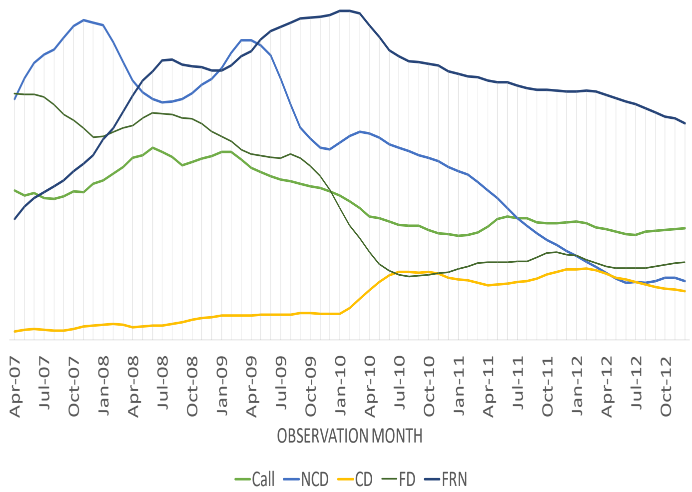
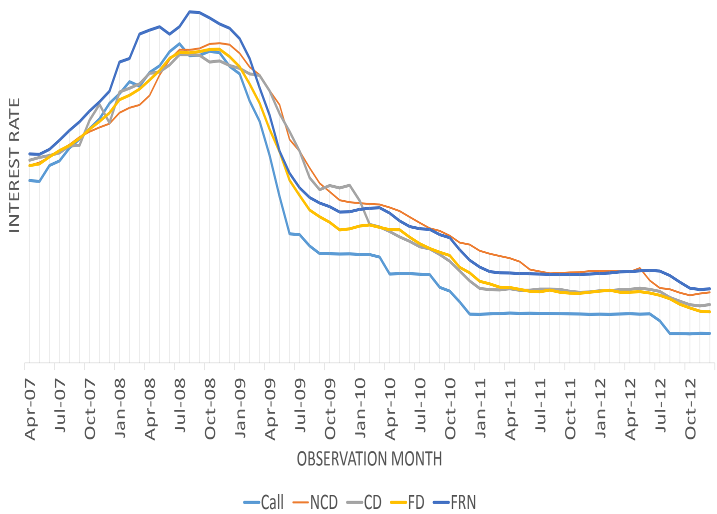


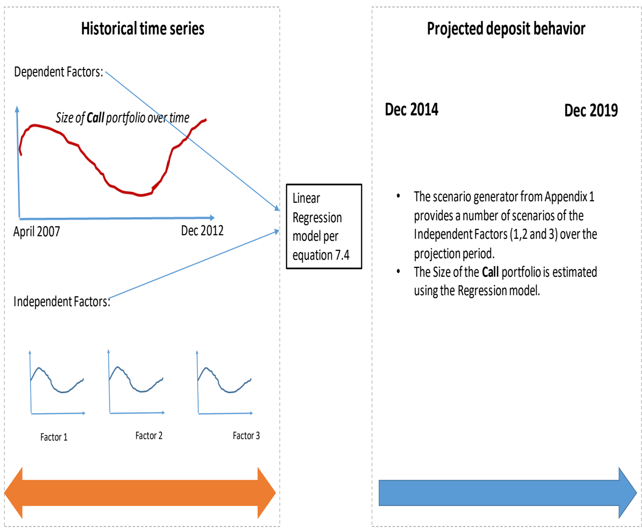
Figure 7.2: Historic time series of the average interest paid on deposit portfolios split by product.



7.4 Results

This section contains the results of the portfolio rate and size models calibrated as discussed above. Diagram 7.3 provides an illustrative example of how the history of the size of the call product is used to calibrate a regression model to relate the change in the size of the portfolio to a set of independent risk factors. This regression model is used to forecast the size call deposit product in chapters 2 and 3 as part of the portfolio optimisation framework. The focus of the thesis is the optimisation model, where the deposit portfolio rate and size models are only sub models used to add to the dynamic nature of the model office projection in chapter 2. Thus, we only focused on the high-level performance of the various models over the calibration period as appose to detailed out-of-sample and out-of-time testing.

Figure 7.3: Regression models calibrated on historical data to support forecasting



7.4.1 Portfolio rate

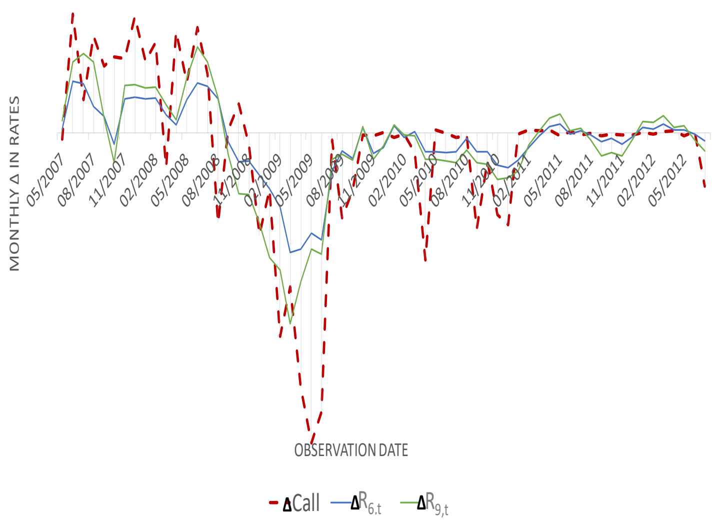
A portfolio replication approach is used to model the interest rate at product level. The approach requires a set of risk-free forward rates to replicate the interest rate behaviour of the selected products.

The same risk-free forward rates as in chapter 6 are used. This consists of forward rates across various durations at each month end. The durations included for this exercise ranged from overnight up to 243 months.

Per equation 7.3, the aim of the model is to explain the observed month on month change in the interest rate for each product. This is done via the observed changes in the forward rates. The 3-month moving average of the forward rates was used for this calibration.

Figure 7.4 shows the monthly change in the interest rate paid on call deposits within the corporate portfolio, together with a change in the 3-month moving forward rate for durations 6 and 9 for illustrative purposes (shown as R_6 and R_9).

Figure 7.4: Change in interest rate on the call product versus the change in the 6 and 9 month forward rate.



A stepwise regression model is used to calibrate the replicating portfolio for each product. Table 7.2 shows the regression model calibrated for the rate of the call product based on the historical rates (April 2007 to December 2012). A separate model is calibrated for each of the products listed in table 7.1.

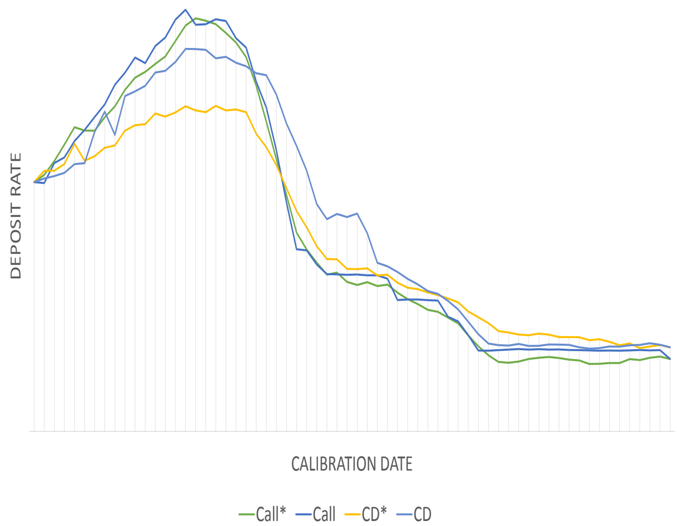
In general the modelling approach yielded acceptable results with an R^2 ranging from 50% - 85% across the various products.

Table 7.2: Coefficients for the call deposit linear regression rate model.

Covariate	Coefficient	P-Value
Intercept	0.0000094	
ΔR_6	3.36018	0.0022
ΔR_9	1.49994	≤ 0.0001

Figure 7.5 shows the actual deposit rate paid over the observation period compared to the fitted deposit rate using the regression model for the call and CD products. The fitted model is shown with an *.

Figure 7.5: Compare the actual to modelled rate on call and CD products over the observation period.



As discussed above, the deposit portfolio rate models are only a sub-model used to support the optimisation in chapters 2 and 3. The focus of the thesis is not on the results of the deposit models and is thus limited to the R^2 over the observation period. Out-of-sample and out-of-time testing of the performance of these models should support future research.

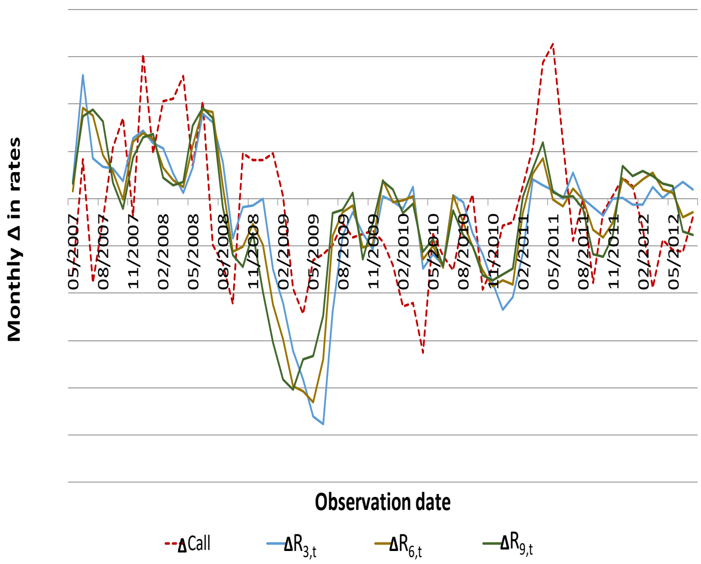
7.4.2 Deposit portfolio size

The size of the deposit portfolio is based on the absolute level of a set of forward rates and the differential compared to the portfolio rate. The same forward rates as for the portfolio rate calibration are used.

Per equation 7.4, the aim of the model is to explain the observed month-on-month change in the portfolio size for each product, by the changes in the forward rate and differential compared to the portfolio rate. The 3 month moving average of the forward is used for this calibration.

Figure 7.6 shows the monthly change in the size of the call portfolio together with a change in the 3 month moving forward rate at durations 3,6 and 9 (shown as R_3,R_6 and R_9).

Figure 7.6: Change in interest rate on the call product versus the change in the 3,6 and 9 month forward rates.



A stepwise regression model is used to calibrate the replicating portfolio for each corporate products. The independent variables considered in the stepwise regression model included the risk-free forward rate at various durations and the difference between the portfolio rate and the risk-free forward rate at various durations. Only the differential between the client and risk-free forward rates is significant for some products, where both the risk-free forward rate and differential is significant for other products. A moving average of 3 months is used as this maximises the adjusted R^2 measure for each model. Table 7.3 shows the final parameters selected for the call product and table 7.4 the parameters for the FRN portfolio.

Table 7.3: Coefficients of the linear regression model for the size of the call portfolio.

Covariate	Coefficient	p-value
Intercept	0.0075	≤ 0.001
CR^1- R_3	-6.4724	≤ 0.001

Table 7.4: Coefficients of the linear regression model for the size of the FRN portfolio.

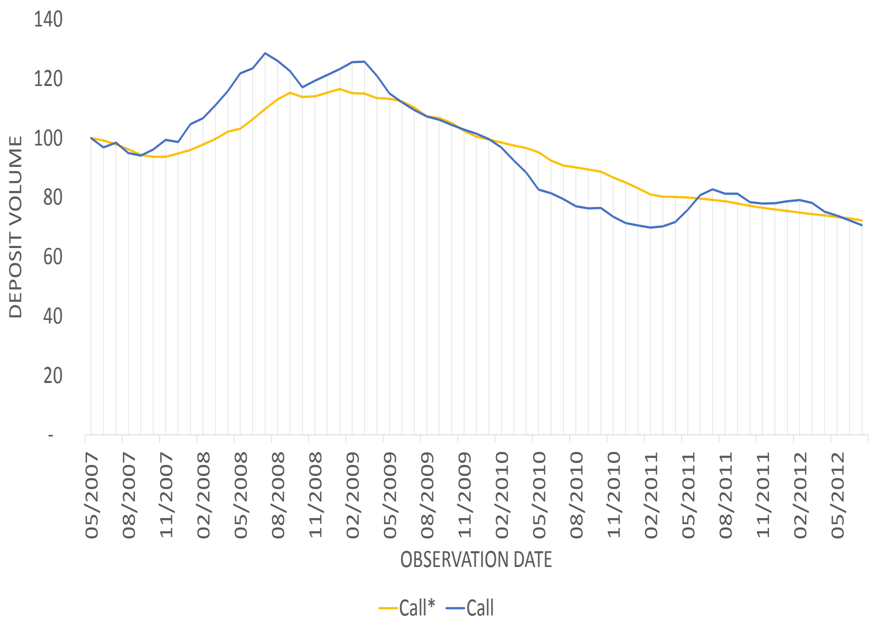
Covariate	Coefficient	p-value
Intercept	0.021	≤ 0.001
ΔR_1	0.351	≤ 0.001
ΔR_{73}	0.298	≤ 0.001
CR^j-R_1	6.628	≤ 0.001
CR^5-R_3	-3.110	≤ 0.001
CR^5-R_{73}	-0.376	≤ 0.001

In general the modelling approach yielded acceptable results for the products with an R^2 ranging from 45% - 80%. The absolute level of the risk-free forward rate is in general significant in the model for the retail-type portfolios, where the marginal level to the client rate is more important for the corporate deposit products.

Figure 7.7 shows the actual deposit levels over the observation period compared to the fitted deposit level using the regression model for the call product. The fitted model is shown with an *.

As discussed above, the deposit portfolio size models are only a sub model used to support the optimisation in chapters 2 and 3. The focus of the thesis is not on the results of the deposit models and is thus limited to the R^2 over the observation period. Out-of-sample and out-of-time testing of the performance of these models should support future research.

Figure 7.7: Compare the actual to modelled portfolio size on the call product over the observation period.



Chapter 8

Appendix 1

8.1 Scenario generator

The macroeconomic scenarios are used to support the projection of each of the components from equation 2.3, using the sub models discussed in chapters 4 - 7. The mathematical expressions underlying the optimisation methodologies are not dependent on the methodology used to generate the macroeconomic scenarios. As a result, existing scenario generator and simplified regression models are used to generate the scenarios as apposed to developing a new framework as part of this thesis.

Each of the models constructed in chapters 4 -7 requires at least one of the macroeconomic variables shown in table 8.1 to facilitate the projection of the outcome over the 60-month projection period. The most significant variable that is fundamental to each of the sub models is the risk-free yield curve. The non-interest rate factors are only required to support the projection of the credit loss rates for the retail and corporate portfolios.

An economic scenario generator is used to generate the risk-free yield curve scenarios. A simple linear regression methodology is used to derive the other macroeconomic variables based on the risk-free yield curve scenario. The literature and methodology sections below set out the methodology for both the risk-free yield curve generator and other macroeconomic variable projections.

Table 8.1: Variables use in the projection of components in chapters 4-7.

Variable	Short Description	Impact following chapters
Risk-free curve		4,5,6,7
Prime lending rate	Prime	4
M2 money supply	M2	4,5
Gross domestic product	GDP	5
Business confidence index	BCI	5
ABSA House Price Index	ABSA Index	4
Household debt to income ratio	DtI	4
Household expenditure to GDP	EtG	4
Consumption by households	CtG	4

8.1.1 Literature study

Risk-free yield curve scenarios

Equilibrium theory in economics use the supply and demand of a large set of interactions to explain the equilibrium of the entire economy. Dynamic stochastic general equilibrium (DSGE) models are an example of this theory and is an industry standard, as these explicitly use the objectives and constraints faced by households and firms. The equilibrium position is used to calculate the impact of changes in an isolated driver such as interest rates on all the interactions. This type of economic equilibrium model is very useful if the main focus is items such as GDP, debt levels, consumption and real wages. They are widely used in the central banking community, see Tovar (2008) [103]. However, a number of important considerations underlying the interest rate process such as market efficiency and non-arbitrage conditions are not addressed.

Economic scenario generators (ESG) with the required properties to satisfy the efficient market conditions and non-arbitrage was propagated by Hull and White (1990, 1993)[57] and [58] to price interest rate-based financial instruments using Monte Carlo type approaches. The evolution in the financial regulatory environment encouraged research into formulating market consistent views of both assets and liabilities to improve the understanding of both inherent risks and the asset liability matching. See Lee and Wilkie (2000) [70] for a discussion on popular ESG models applied in the insurance industry. See Smith and Speed (1998) [95] for a discussion of the Smith model. See CEIOPS (2009) [23] for a discussion on the approach used to incorporate multiple asset types in the ESG models, supporting the calculation of regulatory capital for an insurance company.

A propriety scenario generator, using the methodology set out by Sheldon and Smith (2004) [93] and Smith and Speed (1998) [95] is used in this thesis. The starting point for this exercise is December 2014. The ESG outputs a 60-month projection horizon of prevailing risk-free rates for each month from December 2014 to December 2019. The ESG model provided 600 unique scenarios, each projected from December 2014 to December 2019. The model framework is arbitrage-free, with calibrations based on the observed or quoted market prices of various instruments. The model satisfies the efficient market hypothesis, and for most asset classes assumes some type of Ornstein-Uhlenbeck process that is a mean reverting random walk process. The application of the risk-neutral and real-world prices are based on the deflators, see Smith and Speed (1998) [95] for a discussion on the use of deflators. The average 1 year risk free rate acorss the 600 scenarios is 6.61% with a standard deviation of 2.01%.

Other macroeconomic scenarios

The other macroeconomic variables required per table 8.1 are generated based on the relationship to the risk-free yield curve scenarios.

Forecasting macroeconomic variables is a very important tool to support the fiscal and monetary policy decisions. These projections assist the policy makers to better understand the impact of policy decisions, such as monetary easing on the future state of the economy. See Giovanni, McCrary and Watcher (2009) [44] for a review of econometric models used in this regard.

The impact of risk-free yield curve scenarios on macroeconomic variables such as GDP, consumption and debt levels is required to formulate the macroeconomic scenarios. Khabo and Harmse (2005) [66] tested the relationship between monetary policy, especially through the money supply mechanism on the real economy, and in particular the GDP, using South Africa as a test case. They confirmed the hypothesis that money supply and inflation are significantly related to the economic growth. The impact of interest rate on South Africa's output over different monetary policy periods was assessed by Mualbauer (2002) [80]. Related work by Kanyama and Thobejane (2013) [65] tested the impact of using parametric and non-parametric methods to calibrate the relationship between monetary policy and the real economy.

Jordaan (2013) [63] focused more narrowly on the impact of interest rate changes on the macroeconomic factors, such as household consumptions and GDP. This research reaffirmed the hypothesis that interest rates and GDP are related, the research further quantified the impact of a 100 bp increase in interest rates on GDP.

The methodologies used to assess the impact of monetary policy on the real economy varies from time series analysis (Kanyama and Thobejane (2013) [65]) to more complex stochastic approaches (Mualbauer (2002) [80]). A very simplistic linear regression model is used to identify the relationship between macroeconomic variables and risk-free rates in this thesis as the macroeconomic simulations are only an enabler to the optimisation algorithm.

8.1.2 Methodology

Macroeconomic scenarios

A linear regression approach is used to determine the relationship between changes in the risk-free yield curve and the macroeconomic variables listed in table 8.1. This relationship is calibrated based on historic data (see below). The following explanatory variables was considered to support the regression analysis:

- The annual spot rate.
- The annual forward rate at duration 12 month.
- The annual forward rate at duration 60 month.
- The 12 month lag in each of the variables above.
- The month on month change in the variables above.

The linear regression models are used to forecast the level of the macroeconomic variables given the simulated risk-free yield curve per the ESG.

The first step in the regression analysis is to find the most significant interest rate related variable listed above to predict each of the dependent or target macroeconomic variables listed in table 8.1.

To enhance the predictability of the regression model, consider the inclusion of the other macroeconomic variables as explanatory variables. For example, using the M3 and BCI as explanatory variables in specifying the GDP regression model. This approach require a systematic approach to estimate the series of the regression models to address the circular reference that may arise. Table 8.2 shows the order applied to develop the various regression models. Note Set 0 is only depended on the risk-free curve obtained from the ESG model. Set 1 is depended on all the Set 0 variables, etc.

Table 8.2: Calibration set

Set 0	Set 1	Set 2	Set 3	Set 4
Interest rate related	Index	M3	GDP	CtG
BCI	EtG	M1		M2
Prime		DtI		

The criteria for inclusion of an explanatory variable is based on the significance assessed at a 1 % P-Value.

8.1.3 Calibration

Macroeconomic scenarios

Monthly macroeconomic data from Jan 1986 to Dec 2013 is used to calibrate the regression models listed above. Table 8.3 shows the source of the data, including the source code from the South African Reserve Bank used as reference. Basic properties, such as the mean and standard deviation over the data period, is also shown in the table. Quarterly variables such as GDP was transformed into monthly data points, using a linear interpolation method to enrich the available dataset.

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8.1.4 Results

Macroeconomic scenarios

Table 8.4 shows the performance measured as the R^2 for each model measured over the calibration period (Jan 1986 to Dec 2013). The R^2 for the Prime, BCI, ABSA

Table 8.3: Macroeconomic variables considered

Short description	Source	Code	Mean	Standard deviation
Prime	Reserve Bank	KBP1403M	15	4.0
M2	Reserve Bank	KBP1373A	14	7.7
GDP	Reserve Bank		3.2	2.2
BCI	Reserve Bank		105	16
ABSA Index	Inet		162	134
DtI	Reserve Bank	KBP6007S	62	11
EtG	Reserve Bank	KBP6280L	62	2.0
CtG	Reserve Bank	KBP6525L	3.3	2.8

Index and EtG models shows a strong relationship to risk-free rates with R^2 exceeding 65%.

Table 8.4: R^2 of the regression models to predict each macroeconomic variable.

Short description	R^2
Prime	97%
M2	56.4%
GDP	47%
BCI	67%
ABSA Index	85%
DtI	45%
EtG	83.6%
CtG	45%

Tables 8.5 and 8.6 show both the explanatory variables included in each of the regression models, as well as the regression coefficients. The following definition applied to the table, I is the intercept, $S0$ is the annual spot rate, $F12$ is the 12-month forward rate, $12F12$ is the 12 month lag in the 12-month forward rate and delta is the change in the preceding rates.

Using tables 8.5 and 8.6 the regression model to estimate M3 at month t would be:

$$M3_t = -225.9 + 1.1 \times S0_t + 3 \times EtG_t + 0.4 \times BCI_t \tag{8.1}$$

where $S0_t$ is the spot rate at month t. The variables EtG_t and BCI_t are derived from the regression models.

Table 8.5: Regression coefficients: Interest rate inputs

Var	I	S0	Δ_{S0}	F12	Δ_{F12}	12F12
M3	-225.9	1.1				
M2	-229.2	1.1				
M1	-110.7				2.8	0.8
Prime	3.1	1.0	-0.4	0.1		
CtG	3.4	-0.2				
BCI	143.4	-3.4	3.6			
ABSA Index	240.8			-14.5		
EtG	62.9	0.1				
DtI	59.9					1.0
GDP	1,2	-0.4		0.3		

Table 8.6: Regression coefficients: Macroeconomic inputs

Var	GDP	EtG	BCI	Index	CtG	M3
M3		3.0	0.4			
M2		2.8	0.6	-0.01	-0.5	
M1	1.2	1.8				
Prime						
CtG	0.7					
BCI						
ABSA Index			5.3			
EtG				-0.01		
DtI			-0.3	0.1		
GDP			13.8			0.2

Figures 8.1,8.2,8.3 and 8.4 show the observed level of the macroeconomic variables (M1, M2, Prime, CtG, ABSA Index, EtG, BCI, GDP and DtI) from 2000 to 2013. These are tagged as **Actual** in the figures. This is compared to the output obtained from the application of the regression models from tables 8.5 and 8.6. These are tagged as **Model** in the figures. These graphs provides a visual representation of the R^2 per table 8.4.

Figure 8.1: Compare the actual M1, M2 and Prime time series with the result from the regression model.

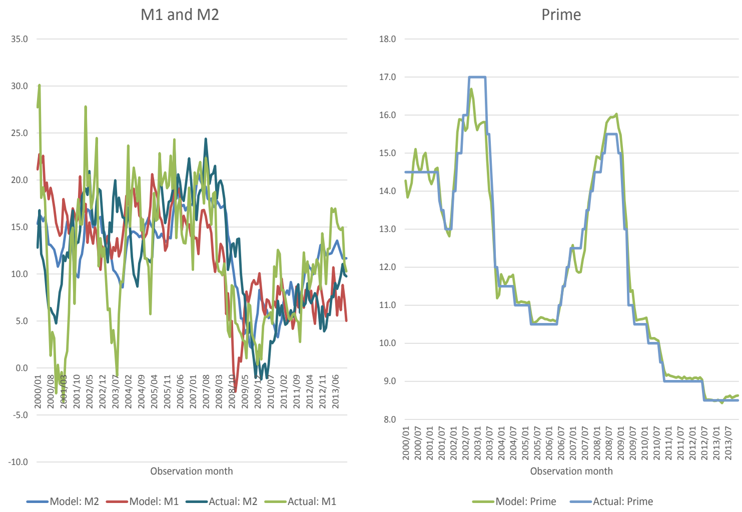


Figure 8.2: Compare the actual CtG and ABSA Index series with the result from the regression model.

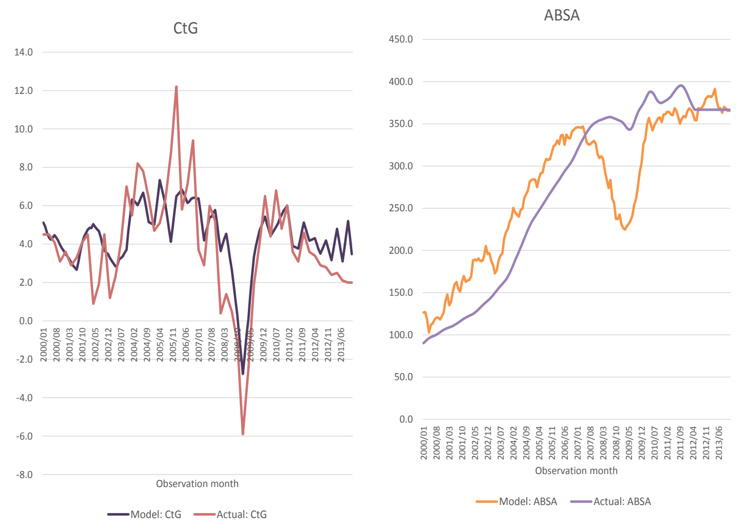


Figure 8.3: Compare the actual EtG and BCI time series with the result from the regression model.

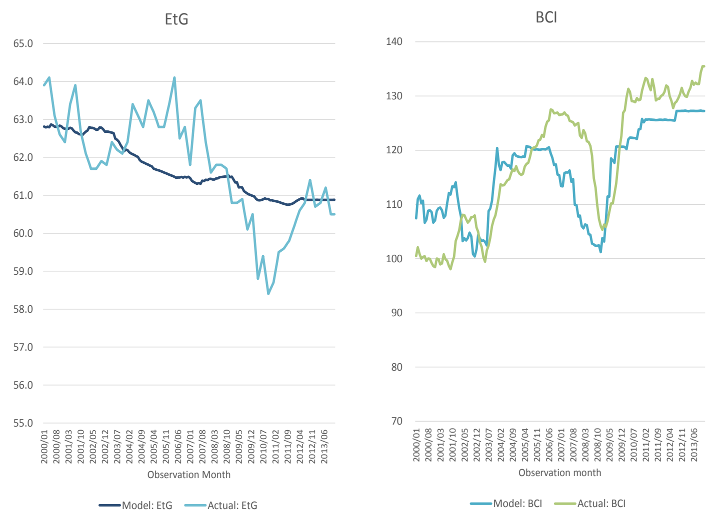
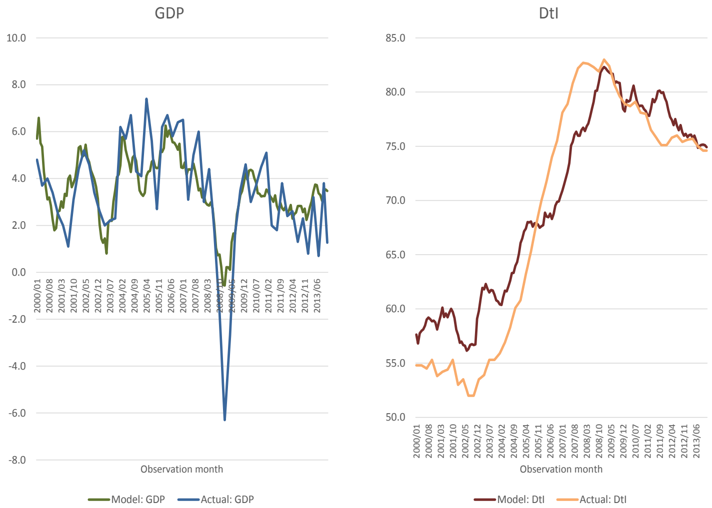


Figure 8.4: Compare the actual GDP and DtI time series with the result from the regression model.



Chapter 9

Appendix 2

9.1 Event tree

9.1.1 Literature study

The event tree is a tool to express a continuous process with a simple discrete approximation via a set of nodes and branches (see Dupacova, Consigli and Wallace (2000) [36]). It is important to recognise that the event tree is an approximation of the continuous process only. A binomial tree, with two branches at node is used in a number of option pricing models, see Hull and White (1990) [57].

There are a number of methods available to construct an event tree from a set of observed scenarios. A method proposed by Hayland and Wallace (2001) [54] and Xu, Chen and Yang (2012) [114] is to construct an event tree that will match a series of statistical properties of the underlying process. The idea supporting this method is to consider the statistical properties of either the random scenarios or the actual stochastic process used to generate the scenarios. The event tree is calibrated to match the selected statistical properties. This is achieved by minimising the difference between the measure and the estimate per the event tree. The moment-matching method set out in Hayland and Wallace (2001) [54] is used to solve these problems.

For this thesis, the set of scenarios is an input into the process, rather than generated as part of thesis. Thus, the statistical properties or multivariate distributions of the process supporting the scenarios are not available to facilitate a statistical method. The approach discussed in Gulpnar, Rustern and Settergren (2004) [48] is used to calibrate the event tree. This procedure is based on a simulated and randomized clustering approach. This approach was chosen given the limited data available to calibrate the event tree. The other more complex methods discussed below require a richer set of data to calibrate the event tree.

Sutiene and Pranevicious (2007) [97] and Tapas *et al.* (2002) [99] discuss alternative methods such as K-clustering to improve the efficiency of the convergence. The event tree is generated via a clustering approach combined with the matching of the statistical properties of the process. The clustering algorithm is used to identify the node structure, where the optimisation considers the probabilities of the nodes. The randomised clustering approach is straight forward to implement, however the computational cost is high as the centroids of the nodes are assigned randomly and enhanced via iterations.

9.1.2 Methodology

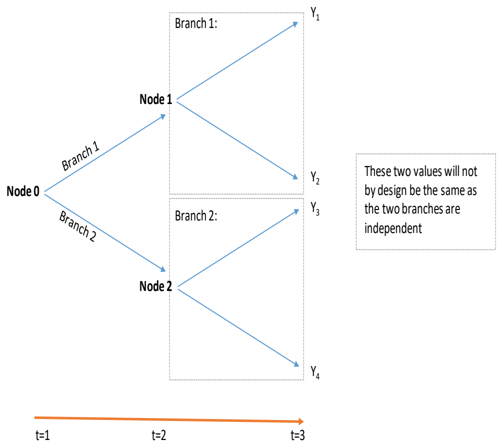
The methodology proposed by Gulpnar, Rustern and Settergren (2004) [48] is chosen to fit the event tree for the purposes of this thesis. The structure of the event tree is shown in figure 9.1. The event tree consists of decision nodes and branches originating from the same base. The event tree has two event branches originating at each node. The sub set of branches created under this structure is independent. Thus, from figure 9.1 moving down from node 1 and up from node 2 will not end in the same position, thus $Y_2 \neq Y_3$.

9.1.3 Calibration

The starting position of the event tree is node 0 at time $t = 1$. Extending this to time $t = 2$ result in 2 nodes. Extending this to time $t = 3$ with two unique branches each result in 4 nodes at time $t = 3$. The projection horizon supporting this thesis is 60 months. A monthly extension of the event tree will result in $1.152 * 10^{15}$ unique nodes at $t = 60$. This dimension exceeds the number of scenarios to calibrate the event tree and is thus unrealistic. To overcome this challenges we partition the 60-month time period into annual time intervals. The time interval used in months are as follows $[1, 7, 13, 19, 25, 30, 35, 40, 45, 50, 55, 60]$.

The following iterative method is used to calibrate the event tree to the scenarios:

Figure 9.1: The event tree structure used.



Start at time $t = 2$, this will be month 7 per the interval above.

- There are two unique nodes at time $t = 2$.
- Select two random scenarios from the calibration dataset and assign as seed values (at time $t = 7$) for each of the two nodes.
- For each scenario, calculate the distance (at time $t = 7$) to the seed value and assign each scenario to either node 1 or 2 based on the smallest distance.
- Replace the seed value with the average calculated across all scenarios assigned to node 1 and 2.
- Repeat from step 1 until the average converge.

Next move to time $t = 3$, this will be month 13 per the interval above.

- Start with node 1 per the previous step.
- Select only the scenarios assigned to node 1 as the calibration dataset.
- There are only two nodes at time $t = 3$ that scenarios from node 1 at time $t = 2$ can move to.

- Select two random scenarios from the calibration dataset and assign as seed values (at time $t = 13$) for each of the two nodes.
- For each scenario, calculate the distance (at time $t = 13$) to the seed value and assign each scenario to either node 1 or 2 based on the smallest distance.
- Replace the seed value with the average calculate across all scenarios assigned to node 1 and 2.
- Repeat from step 1 until the average converge.
- Next move to node 2 per the previous step and repeat steps above.

Next move to time $t = 4$, this will be month 19 per the interval above.

Repeat the above process with the 4 nodes from time $t = 3$.

The output of the above process will be an event tree with 12 time periods with 2 unique branches for each node.

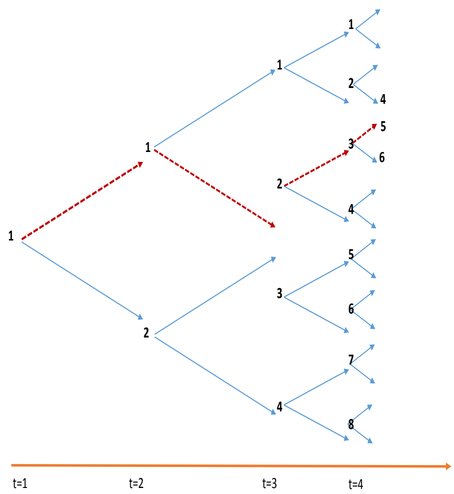
9.1.4 Results

Per Appendix 1, the main macroeconomic component underlying the scenarios is the risk-free interest rate. The ESG model discussed in Appendix 1 is used to generate 600 monthly risk-free interest rate scenarios over a 5-year period. For the purposes of the event tree calibration, we used the annual interest cost on the 6 month debt instrument. The annual interest cost on the 6-month debt instrument is determined as the risk-free rate (obtained from the ESG simulation), plus the simulated credit and liquidity spread as discussed in chapter 6. For the purposes of this thesis, we calculated 20 liquidity spread simulations for each risk-free interest rate simulation obtained from the ESG model. This resulted in 12000 scenarios supporting the calibration of the event tree.

The event tree is only used in the stochastic linear program (SLP). The aim of the event tree is to assign each scenario to a node an an annual basis to simplify the dimensionality supporting the SLP calibration. Each scenario will have a number of outputs that impacts the optimisation problem, these include the credit loss from the retail and corporate models, the deposit funding and the wholesale funding at various durations. The wholesale funding cost for the 6 month debt instrument was used to assign the entire scenario (across all variables) to the constructed nodes per the event tree.

Figure 9.2 is an example of the output from the event tree module. For example, if the node structure for scenario 1 is $[1, 2, 3, 5]$, then the path shown in red shows the node allocation for scenario 1.

Figure 9.2: Example of event tree structure with one sample path.



Chapter 10

Appendix 3

10.1 Bank rating model

This section set out the detail of the Pagratis and Stringa (2009) model used to map the credit rating of the bank based on a series of balance sheet and income statement items. The first step is to calculate the target equity to asset ratio of the bank. The target equity ratio is presented as a linear regression model as per table 10.1, where the coefficients are per the calculation from Pagratis and Stringa (2009) and the values of the covariates per the individual banks financial statements as at December 2014. The ratio of the individual banks actual equity-to-asset ratio and the target ratio calculated per the linear regression model are an input into the next section of the model.

Table 10.2 show the linear regression model proposed by Pagratis and Stringa (2009) to determine the credit rating of the bank. C_1, C_2 and C_3 represent the covariates based on the target equity ratio calculation in table 10.1. If the target to actual equity-to-asset ratio is less than -15% then use C_1, if between -15% and 0% use C_2, else apply C_3. The covariate values are based on the actual financial statement of the bank as at December 2014. To project the credit rating of the particular bank over the 60-month time horizon, we need to project the balance sheet and income statement items affecting the model as per tables 10.1 and 10.2. For the purpose of this thesis, only the line items in the financial statement affected by the sub models from chapters 4-7 are varied, these are highlighted with an $**$ per tables 10.1 and 10.2. The $Rating_m$ is used with table 6.1 to identify the credit rating of the bank in month m.

Table 10.1: The actual equity-to-asset ratio as at December 2014 compared to the target ratio per Pagratis and Stringa (2009).

Description	**Coefficient**	**Covariate**	**Calculation**
Constant	3.243	1.0000	3.2430
Loan loss reserve/Total assets**	-2.044	1.7002	-3.4752
Net Interest Income/Total assets**	1.674	2.8771	4.8162
Other Income/Total assets	0.472	2.3375	1.1033
Off balance sheet/Total assets	0.004	29.475	0.1179
IFRS Bank? (yes/no)	-0.704	1	-0.7040
Domestic slowdown	0.428	0	0.0000
Region	0.168	1	0.168
A: Target: Equity/Assets			**5.27**
B: Actual: Equity/Assets			**7.97**
1- B/A			**-51.25%**

Table 10.2: The $Rating_m$ value per the Pagratis and Stringa model as at $m =$ December 2014.

Description	**Coefficient**	**Covariate**	**Calculation**
C1	0.0070	5.07	0.0355
C2	0.0170		
C3	-0.0020		
Loan loss reserve/Net Interest income**	-0.0050	59.0952	-0.2955
Above × GDP Gap**	-0.0560	-0.7891	0.0442
Operating cost/Total assets	-0.0840	2.8734	-0.2414
Pre tax, pre provision profit/Total asset**	0.1940	1.5474	0.3002
Above × GDP gap**	-0.0270	-0.0207	0.0006
(Loans - Deposit)/Loans**	-0.0020	7.8272	-0.0157
Log(Loans/Total credit extension)	0.2130	1.3248	0.2822
Use IFRS accounting?	-0.6130	1	-0.6130
Large bank indicator	1.8150	1	1.8150
10 -3 month bond curve**	0.0510	0.5	0.0026
GDP Gap**	0.0200	-0.0134	-0.0003
Market credit extension gap	-0.0010	-1.6525	0.0017
Above × GDP gap**	0.1690	0.0221	0.0037
Stock market gap	-0.0010	0.0200	0
Sovereign rating	1.8230	1	1.8230
Country	-0.3910	1	-0.3910
$Rating_m$			**2.7418**

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