



AN ANALYSIS OF THE COGNITIVE DEMANDS OF EXAMPLES PRESENTED IN GRADE 7 TEXTBOOKS IN GEOMETRY

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Submitted by: Dereck Donora

Student number: 1271267

Supervisor: Dr. R. Durandt

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DEDICATION

This research project is dedicated to all those who have been part of my academic journey. To my late mom and dad, Twobouy and Sphelile Donora, for working tirelessly to afford my basic education; I say rest in peace. All the teachers, mentors, colleagues, and lecturers who inspired me to aim for excellence. Education is a product of the people who care and give their best so that students/learners can grow and learn. Without the people who taught me and equipped me with the skills, attitude, and knowledge that I have today, this research report would not have been possible. My special appreciation for all you have done for me, it is you that have allowed me to complete this journey.

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ABSTRACT

The use of textbooks in teaching and learning is very crucial, and in some cases, textbooks are the only resources available for both teachers and learners. Well-constructed textbooks should include examples that support learning, particular in subjects with an abstract nature, like mathematics. This research study is aimed at the analysis of examples in two lower-secondary level (grade 7) mathematics school textbooks in geometry. The selected textbooks were: *Curro Grade 7 Mathematics* and *Mindbourne Grade 7 Mathematics*. The work included an investigation into the levels of cognitive demands of examples, frequency of examples, the types of examples, and the order of thinking required in solving the mathematics problem presented in examples. This was followed by a discussion about real-life examples and how they can be used to enhance the understanding of geometry concepts. This study is important as it contributes to the crucial role of examples in school mathematics textbooks, particularly at the lower-secondary level in a South African context; thereby contributing to the literature and classroom practice as both teachers and learners interact with textbooks in a mathematics classroom.

A constructivism paradigm informed the study, through the use of a qualitative research design. All examples in geometry in the two selected textbooks were analysed using an analysis tool adopted from literature and further developed for the purpose of this study. The researcher has drawn on the framework by Stein, Henningsen and Silver (2000) to analyse the cognitive demands of tasks, and the work from Michener (1978) to classify the types of examples in mathematics. Thus, the overall analytical framework is a merger between the Stein et al. (2000) framework and Michener's (1978) work. The main findings were that there are some similarities and fundamental differences in terms of frequency and presentation of examples in the two textbooks, which may benefit learners as they use both books. There were no counter examples in both textbooks. This might indicate that both textbooks require more examples. One limitation of this study is that only two textbooks formed part of the analysis, due to practical reasons, yet there are so many textbooks being used in South African schools at this level. It is recommended that more research should be done in analysing examples as it is presented in mathematics textbooks; through selecting a larger sample, also focusing on other content areas, and using other theoretical frameworks for analysis. It might provide more insights in how examples in textbooks contribute to teaching and learning in mathematics.

Key words Cognitive demands, Examples, Grade 7, Higher Order Thinking, Lower Order Thinking, Geometry

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GLOSSARY OF TERMS

CG7MT	Curro Grade 7 Mathematics Textbook
MG7MT	Mindbourne Grade 7 Mathematics Textbook
LOT	Lower Order Thinking
HOT	Higher Order Thinking
CAPS	Curriculum Assessment Policy Statement
ANA	Annual National Assessments
TIMSS	Trends in International Mathematics and Science Study

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CHAPTER 1: RESEARCH BACKGROUND

1.1 Introduction and background to the study

This study was undertaken with the aim of providing an analysis of the cognitive demands of examples presented in two lower-level secondary school mathematics textbooks used in South Africa in geometry. This chapter provides the general background to the research problem, the rationale of study, the significance of the study, the research aims, objectives and research questions, the theoretical and analytical frameworks as well data presentation and analysis thereby giving a broader overview of the study.

1.1.1 Mathematics underperformance in South Africa

The problem of underperformance in mathematics is not isolated but rather echoes a national phenomenon in South Africa. Several studies have been undertaken, and researchers agree that South Africa generally faces a crisis in mathematics performance by students (Arends, Winnaar & Mosimege, 2017; Luneta, 2015; Luneta, 2022; Mabena, Mokgosi, & Ramapela, 2021; Mokotjo & Mokhele, 2021; Nkabinde, 2016; Theron, 2025). In their studies, Arends et al. (2017) and Mokotjo & Mokhele (2021) confirmed this notion when they posited that South African students lag significantly behind the international average in international assessments, such as the Annual National Assessments (ANA) and the Trends in International Mathematics and Science Study (TIMSS). Results show that in 2019 about 60% of South African Grade 8 learners failed to reach the low international benchmark in mathematics (Mabena et al. 2021; Mokotjo & Mokhele 2021). Mabena et al. (2021), observed that in the Mpumalanga province learner performance in mathematics is extremely bad resulting in a high failure rate. The same sentiments were echoed by Luneta, (2022), Mokotjo and Mokwele (2021) and Theron, (2025). Causes of poor performance ranges from mathematics teacher practices, learner general dislike of mathematics, shortage of competent and qualified teachers, pedagogical content knowledge (PCK) which the specialised knowledge on how to teach certain content possessed by the teacher, crowded classrooms, and mathematics teaching resources such as textbooks, which may hinder or enable the teaching and learning process. The phenomenon of underperformance in mathematics is however not only isolated to South Africa, but is also experienced outside South Africa. Otieno and Povey (2022) argues that countries in central and Southern Africa face similar problem of underperformance in mathematics.

1.1.2 Textbook related studies

There has been great development and interest in research about mathematics textbooks in recent years (Leshota & Adler 2018; Leshota, 2020; Leshota, 2015; Rezat, Visnovska, Trouche & Fan, 2018). Within the South African context, Leshota (2015) investigated the interplay between textbook affordances and teachers' pedagogical design capacity (PDC). PDC was defined by Brown (2009), highlighting that teachers do not only need to use textbooks as a resource but should rather have in-depth knowledge about textbooks to enhance and improve instructional design. Leshota and Adler (2018) further emphasised the importance of deliberate textbook utilization in demonstrating high levels of pedagogical design capacity (PDC), through an analysis of the Grade 10 South African Classroom Mathematics textbook. Leshota and Adler (2018) investigated mathematics teachers' Classroom Mathematics textbook use in class concluding that a deliberate use of the textbook and an in-depth understanding of its contents and design reflects a high PDC. Leshota (2019), contends that textbooks serve as mediators between authors and teachers, teachers and learners, and the mathematics content itself and the learners. In a quantitative study, Leshota (2020) examined 6 teachers about teacher-textbook relationship in mathematics in context of limited resources. The findings pointed to the general tacit use of textbooks and a need for intervention on textbook use by teachers and also recommended further research on the perceived role of textbooks in teachers' practices. The facts above indicates that there remains a gap to research on textbook analysis pointing towards a need to undertake this study.

From a global perspective, Otieno and Povey (2022) explored the perspectives of learners in Kenya regarding the assessments contained in mathematics textbooks. The study emphasised valued aspects and areas in textbooks that are lacking. This study led to the conclusion that there are some major gaps regarding the value of assessments presented in these textbooks. In a Croatian context, Glasnovic (2018) explored what was termed the "fifth dimension" of textbooks, recommending that authors of textbooks encompass tasks with presentation and modelling, interpretation, argumentation and reasoning activities into mathematics education. The conclusion was that textbook authors should ensure inclusion of tasks and activities that involves various facets of mathematics like modelling and reasoning.

1.1.3 Teaching and learning of geometry at lower secondary level

The mathematics content section to be focused on in this study is geometry. Armah, Cofie and Okpoti (2018) argues that geometry is an essential area of study in school mathematics curriculum throughout history and is of great importance to peoples' lives. The widely accepted

Van Hiele levels of geometric thinking theory posits that learners advance in geometric thinking through five sequential, progressive, and hierarchically discrete levels that characterises learners' thinking and understanding of geometric relationships (Armah et al. 2018; de Villiers, 2010 and Van der Sandt & Nieuwoudt, 2003;). The van Hiele five levels of geometric thinking identify the five levels starting with visualisation or recognition level, advanced description or analysis, ordering or informal deduction, deduction or abstraction, and finally rigor or proof of geometric theories. It was observed by de Villiers (2010) that van Hiele's theory gave the main reason for learners failing to understand traditional geometry was that the curriculum was presented at a higher level than the learners, therefore learners should know geometric concepts of the preceding stage before attempting the next stage. Generally, there is not much research done on examples in geometry. Most research on geometry in South African primary and lower secondary school focuses on other areas of geometric reasoning and not necessarily on textbooks and examples contained in textbooks. Bleeker & Goosen (2009) focused on understanding the barriers to learning senior phase geometry, Sinclair & Bruce (2015) examined new opportunities in geometry education at the primary school through technology, while Bowie (2013) focused on high school geometry textbooks. There is therefore need for more research into textbooks on the section of geometry especially junior high school level in South Africa.

A deeper exploration into the design, content, and instructional potential of textbooks seems necessary. One example is the work from Ratnayake and Pournara (2022) in the content area geometry. They discussed the tasks in Grade 8 and 9 textbooks and argued for an adjustment of the cognitive demand of geometry tasks to better prepare learners for higher grades. In line with these recommendations, this study seeks to analyse the cognitive demand of examples presented in lower secondary level (Grade 7) mathematics textbooks, aiming to contribute to the ongoing discussion on the important role of textbooks as instructional materials in mathematics classrooms.

1.2 Rationale for the study

A reflection of my teaching career involves being a holder of an Honours Degree in Mathematics Education, teaching mathematics in both government and private schools for almost 10 years in the lower and upper secondary level and in the process and during that period attending professional developments programmes like academic conferences and being involved in marking national mathematics examinations. I encountered challenges such as students' poor performance in exercises and tests, students showing low interest in studying

mathematics as a subject and stereotyping mathematics as a subject for a few gifted learners. In my search to deal with the challenges and my efforts to positively change the situation, I applied strategies like individual and small group academic support lessons, use of rewards in class, and the use of real mathematics objects in the classroom to generate interest among students towards the subject. Despite my efforts to enhance students' performance and to make students more interested in the subject and to let them believe that mathematics is for all learners, I was constantly disappointed. Students' performance on assessments consistently fell short of my expectations. This was more prevalent in the geometry section. Initially I expected learners to do well in geometry because they interact daily with shapes and solids in real-life situations.

The reasons above sparked my interest to broaden my knowledge about the teaching and learning of geometry which led me to undertake a study to explore geometry examples in textbooks, particularly at the lower secondary level (grade 7). My focus was specifically on textbooks as they are an important and indispensable resource used by teachers and learners in mathematics classrooms (Jukic & Glasnovic, 2021; Leshota, 2015; Leshota & Adler 2018; Otieno & Povey, 2022). In recent years, a growing body of research has focussed on textbooks in South Africa and beyond (Fleisch, Taylor, Sapire, & Herholdt, 2011; Glasnovic, 2018; Jukic & Glasnovic, 2021; Leshota, 2015; Leshota & Adler 2018; Otieno & Povey, 2022; Schubring & Fan, 2018). Jukic and Glasnovic (2021) and Otieno and Povey (2022) concluded that the role of textbooks in a mathematics classroom, teachers and learners' perspectives on the values and areas that may be lacking in textbooks, textbook presentations and the mediating role played by textbooks, have been and are still very important aspects that underlie the use of textbooks in a mathematics classroom.

1.3 Significance of study

This study is significant mainly because the analysis of examples based on the cognitive demands levels, they address in the mathematics learning and teaching process provides a body of knowledge about the example space in these two textbooks. By conducting this study and communicating the findings, I might be able to provide recommendations to the authors of the textbooks regarding examples presented which may result in possible refinement to the content of the textbooks. The study therefore might assist the authors of the books to reflect on the selection and presentation of examples. Furthermore, the recommendations can assist

curriculum planners and curriculum implementers within the Curro¹ group, in the development of teaching materials for teachers. The curriculum planning department at Curro group may also use the findings for staff professional development which may result in improved use of the textbooks through examples in geometry.

This study is also significant in that its literature and findings might be useful in improving mathematics classroom practices in terms of how learners and teachers can use the textbooks focusing on the example spaces. As such, this study will contribute to literature about mathematics textbooks analysis in terms of examples presented in geometry which will contribute positively towards the improvement textbooks as a major curriculum resource in the South African Grade 7 classrooms. The analysis of the cognitive demands of examples in the textbooks is useful in providing information about learning opportunities availed to learners by the textbooks through various classes and types of examples that addresses learners of different learning capabilities. The broader goal being identifying and analysing the affordances brought about by examples in the process informing in a small way future curriculum development and instructional practices. Finally, this study can also open other avenues and areas of research about textbook analysis and examples.

1.4 Problem statement and research questions

1.4.1 Problem statement

This study is guided by three important themes from the literature, which are: the problems encountered in mathematics performance as experiences through the teaching and learning of mathematics at the lower secondary level, the role of textbooks and examples in teaching and learning of mathematics, and the teaching and learning of geometry. The interaction that mathematics learners encounter in textbooks about mathematics concept is through examples. It is therefore important to examine examples and how they are presented in the two textbooks in the section of geometry. The examination of example space is vital because examples are central in affording learners' ability to understand concepts in the teaching and learning process of mathematics.

The mathematics content section to be focused on in this study is geometry. Armah, Cofie & Okpoti (2018) asserted that geometry is an essential area of study in school mathematics curriculum throughout history and is of great importance to people's lives. The widely accepted

¹ Curro is a provider of private school education in South Africa.

van Hiele levels of geometric thinking theory posits that learners advance in geometric thinking through five sequential, progressive, and hierarchically discrete levels that characterizes learners' thinking and understanding of geometric relationships (Armah et al. 2018; de Villiers, 2010; Van der Sandt & Nieuwoudt, 2003). A conclusion by Van der Sandt and Nieuwoudt (2003) posits that students often find geometrical concepts as being more abstract and difficult to understand especially once above the visual level, which may result in poor performance. This study focuses on the analysis of the *Curro Grade 7 Mathematics Textbook* (CG7MT) and the *Mindbourne Grade 7 Mathematics Textbook* (MG7MT) in terms of examples in the section of geometry. Most research on geometry in South African lower and upper secondary level focuses on other areas of geometric reasoning and not necessarily on textbooks and examples they contain. Bleeker and Goosen (2009) focused on understanding the barriers to learning senior phase geometry, Sinclair and Bruce (2015) examined new opportunities in geometry education at the primary school through technology, while Bowie (2013) focused on high school geometry textbooks. There is therefore need for more research into textbooks with regards to examples on the section of geometry especially lower secondary school level in South Africa. This study, though a small case study, will contribute towards the body of knowledge about mathematics textbooks, the teaching and learning of geometry and examples as contained in textbooks thereby contributing in a small way to the improvements of teaching and learning of mathematics in geometry.

1.4.2 Research questions

This research project is guided by the following main question.

What are the cognitive demands addressed in examples contained in the lower secondary (Grade 7) textbooks in geometry?

The following research sub-questions will guide the study:

1. What are the levels of cognitive demands that are addressed in examples in the textbooks CG7MT and MG7MT in geometry?
2. How do examples in the selected mathematics textbooks support concept development in geometry, and is this sufficient in terms of type of examples, frequency² and quality?

² Frequency refers to how often the examples appear in the textbooks. This study follows a qualitative content analysis approach, and the 'frequency' is used to signify patterns.

3. To what extent are real-life examples included in the selected textbooks (CG7MT and MG7MT) in geometry?

1.5 Research aims and objectives

The aim of this study is to examine the cognitive demands of examples contained in the lower secondary (Grade 7) mathematics textbooks in geometry. The textbooks being analysed are the *Curro Grade 7 Mathematics Textbook* (CG7MT) and the *Mindbourne Grade 7 Mathematics Textbook* (MG7MT) (More information about the textbooks is available in chapter 3).

The following sub-aims (objectives) inform the study.

1. To inspect thoroughly the levels of cognitive demands addressed in examples in CG7MT and MG7MT in geometry.
2. To investigate the type of examples, frequency and quality of examples in geometry in the selected textbooks to support the concepts development.
3. To explore the use of real-life examples (to advance the cognitive demand) in geometry in the selected textbooks.

1.6 Conceptual and analytical frameworks

This study is informed by one conceptual framework and two conceptual areas of study which are interrelated and vital in the teaching and learning of mathematics. The conceptual framework is the teaching and learning of mathematics at the lower secondary level, while the two interrelated areas of the study are the role of textbooks in the mathematics classroom and the teaching and learning of geometry in mathematics.

The main theory contributing to the conceptual idea is the process of **teaching and learning of mathematics**, particularly at the lower secondary level. This involves imparting foundational concepts and problem-solving skills that foster critical thinking and logical reasoning. According to Idris (2005), teaching and learning of mathematics is a complex and dynamic process that involves previously acquired levels of understanding and conceptualising and incorporating new mathematics materials. It means that instruction in mathematics is aimed at cultivating procedural fluency and conceptual understanding which empowers learners to apply mathematical principles in diverse real-world contexts. The idea of ‘scaffolding’ is often associated with this process (see, for example, Ratnayake & Pournara, 2022).

A very important concept central in this study is the **use of textbooks** in a mathematics classroom. Glasnovic (2018, p. 1003) noted that “textbooks play an important role in

mathematics education and are used to a great extent in mathematics classrooms”. The same sentiments were also echoed by Otiono and Povey (2022) concluded that, while textbooks were traditionally designed for teacher-mediated instruction, they now increasingly facilitate independent student use and promote deeper comprehension and understanding of concepts. Glasnovic (2018, p. 1005) observed that “most of the textbooks follow the structure of rule-example-practices”. This approach is referred to as exposition where the author presents subject matter, examples where the author offers students a model to be used, and exercises, where the author gives tasks to check understanding of concepts (Glasnovic 2018). This perspective asserts that in most textbooks there is a presentation involving introduction of a concept through presentation of the mathematical rule, then examples are presented and finally some work to be done by the learner.

Finally, the mathematical content section examined in this study is the **teaching and learning of geometry** and the challenges association with this process. According to de Villiers (2010), the widely renowned van Hiele’s theory observed that the main reason for learners failing to understand traditional geometry was that the curriculum was presented at a higher level than the learners, therefore learners should know geometric concepts of the preceding stage before attempting the next stage. Van der Sandt and Nieuwoudt (2003) observed that students often find geometrical concepts as being more abstract and difficult to understand especially once above the visual level, which may result in poor performance. One way to deal with the cognitive demand of geometry tasks is by following the four strategies presented by Ratnayake and Pournara (2022). These are ‘extending calculation tasks to generalisation’, manipulating the diagram’, scaffolding proof tasks’, and ‘connecting visual and verbal information’.

Since this study is an analysis of content, in this case examples that are presented in two textbooks, it was important to develop an analytical framework. An analytical framework is an illustration of one or more theories or generally accepted schools of thoughts with details of how the theories will be linked and used in a study (Peressini et al. 2004). As such it shows relationships amongst ideas and their relatedness to the research study. The analytical framework is important because it outlines key concepts, variables, and relationships between the variables. It therefore provides a blueprint for the research study which forms the basis of literature, study design and analysis of data. Hence the analytical framework provides an outline of how theories are enabling a more practical process of the research study. In this study the analytical framework is derived from concept and provides a combination of Michener (1978) types of examples fused with Stein et al. (2000) analysis of tasks cognitive demands.

Michener (1978) classified examples into different types or categories based on their use while Sten et al. (2000) developed a framework used to analyse the cognitive demands of tasks. Michener (1978) posited that examples can be classified into start-up examples, reference examples, model examples, and counterexamples. Stein et al. (2000 p. 16) mathematics tasks can be categorized into four levels of cognitive demand being “...memorization tasks, procedures without connections tasks, procedures with connections tasks and doing mathematics tasks”. Although the Stein et al. (2000) framework was primarily developed for analysing tasks, it appears to be applicable in analysing the levels of cognitive demands required when examples are presented in the textbooks. The analytical framework was developed through a merger of the Michener (1978) framework of examples and the Stein et al. (2000) framework of cognitive demands of examples thereby enabling an exploration of the types of examples and the analysis of the cognitive demands they fall under.

Figure 1.1 shows a diagrammatical illustration of the analytical framework which is a combination of the Michener (1978) framework of examples and the Stein et al. (2000) analysis of cognitive demands of tasks. In the centre of this figure (A) is the concept of examples which is at the centre of the textbook analysis in this study and on the left side are the types of examples (1-4). On the right side of the figure are the categories of the cognitive demands of the examples (B-E). The cognitive demands of the tasks are separated into higher order thinking (HOT) examples and lower order thinking (LOT) with two categories (B and C) falling under LOT while the other two categories (D and E) fall under HOT. This framework will form the basis of analysis of the examples contained in the selected textbooks in this study (see section 3.5 and chapter 4).

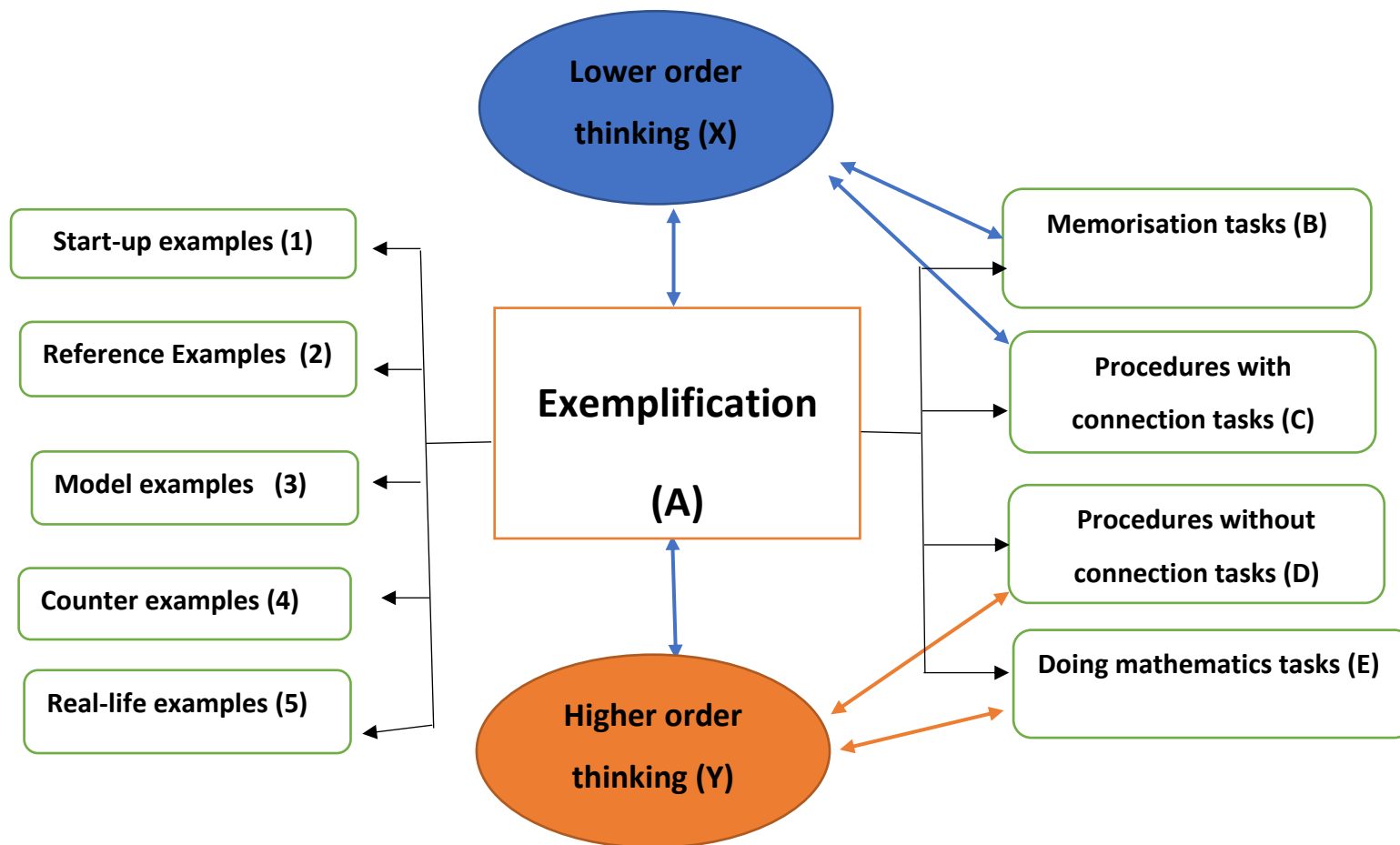


Figure 1.1 Analytical framework combining types of examples (Michener, 1978, p. 368-371) and the cognitive demands of tasks (Stein et al. 2000, p. 128-132)

1.7 Key terminology clarification

Some key concepts – examples, cognitive demands, and geometry – will be used in this study. These are defined to articulate their perceived use in this study.

1.7.1 Examples

In the context of this study, examples are worked solutions in the textbook which are used as illustrations of how mathematical problems are solved. Zodik and Zaslavsky (2008, p. 165) posited that “by examples we mean a particular case of a larger class, from which one can reason and generalise”. As such, in this research study, examples are viewed as questions and worked solutions contained in various teaching and learning resources such as textbooks and used in class to illustrate and communicate concepts between teachers and learners. It means users of teaching and learning resources such as textbooks make use of examples to build generalizations of mathematical concepts. In this study examples were identified from textbooks and analysed in terms of types, frequency of appearance, the cognitive demands and the level of thinking attached to solving the example.

1.7.2 Cognitive demands

Cognitive demands may be viewed as the level and kind of thinking process that students should be engaged with to solve mathematics questions. According to Stein et al. (2000) mathematics tasks be classified into four categories based on the cognitive demands of the task. The categories are memorization tasks, procedures without connections tasks, procedures with connections tasks and doing mathematics tasks. Stein et al. (2000) further observe that in this classification, the first two types (memorization tasks and procedures without connections) are regarded as lower order thinking (LOT) tasks while the last two (procedures with connection tasks and doing mathematics tasks) are regarded as high order thinking (HOT) tasks. In this study, the cognitive demands of examples were central in the development of the analytical framework to be used in the analysis of examples.

1.7.3 Geometry

Geometry is a section of mathematics that predominantly deals with shapes. Breyfogle and Lynch (2010) argued that the van Hiele theory of geometric reasoning identifies the five levels starting with visualisation or recognition level, advanced description or analysis, ordering or informal deduction, deduction or abstraction, and finally rigor or proof of geometric theories.

According to de Villiers (2010) geometry should be seen as a fundamental skill because it has important application to real life problems like measurements, mapwork, critical thinking skills, enables use of models, and provides basic mathematics concepts such as linear programming, area, volume, and essential for understanding calculus and other related fields. In this study, geometry was the section of mathematics whose examples formed the basis of analysis.

1.8 Research design and methodology

The research study is largely guided by the *constructivism* paradigm. According to Ignacio and Paras (2024) constructivism contends that individuals form or construct most of what they learn and understand through their learning experiences. The argument being that what can be perceived to be the truth is a product of personal views based on constructed knowledge through learning experiences. This paradigm will assist in this study to understand the research problem. Further, it will assist in theory generation by analysing the type of examples in the textbook based on Michener (1978) types of examples. Furthermore, the constructivist approach will be used in this study by interpreting and classifying the cognitive demand of the examples contained in the textbooks in the topic of Geometry.

In this study, a *qualitative approach* to research will be followed. The study was aimed at finding qualitative data about the cognitive demands of examples and types of examples from the two textbooks. Qualitative research is useful where the meaning and process may not be examinable through quantifiable amounts of findings (Daniel & Sam 2011). Hendriks and Cruywagen (2024) agrees with this notion when they observed that qualitative research methods offer valuable insights into the experiential and interpretive aspects of educational phenomena thereby providing specific understanding to a phenomenon, in this case analysis of textbook examples. Furthermore, qualitative research is aimed at attaining deep understanding of a specific case with an in-depth exploratory nature to enable finding quality responses throughout the research project. This study follows a *qualitative content analysis* approach (based on the adopted theoretical frameworks), and the ‘frequency’ is used to signify patterns and is presented in the findings as frequency tables and charts (see chapter 4). These visual methods of representing the findings might include elements common to quantitative research, although this study is qualitative in nature. The reason for the adding a visual component is to clearly and accurately convey the message of the data to the intended audience. Qualitative data will be sort through analysis of the examples on the topic geometry contained in the textbooks.

Since this study focussed on only two textbooks out of a variety of textbooks used in lower secondary level in South Africa, it was a case study. Hendriks and Cruywagen (2024) argued that a case study is in-depth research design that primarily uses qualitative research aimed at a particular class or group whose findings can be attributed to a bigger group. In this case document analysis to gather rich, descriptive information was done. In this study the analysis was from only two mathematics textbooks out of several textbook developed and used by lower-level secondary school in South Africa.

1.8.1 Data collection

Data collection is viewed in research as a systematic way of gathering relevant information to answer the research questions. Elaborating on data collection, Sukmawati (2023) stated that data collection is defined as research activities aimed at collecting field data required in an attempt to answer research questions in a research study. This study focused on the types of examples and an analysis of the cognitive demands of the presented examples in geometry section as contained in CG7MT (2015) and MG7MT (2020), through the lens of the analytical framework discussed earlier (see section 1.6) above. As such, purposive sampling was used as these are the only two grade 7 mathematics textbooks used across all Curro schools.

The CG7MT is a specific textbook published for Curro Schools by Brombacher and Associates who are publishers of the Numbersense mathematics programme. In this textbook geometry is presented in such a way that sections of geometry are spread throughout the book. The Numbersense system advocates for covering the major mathematics sections by attempting parts of each section throughout the year but all sections required are covered as per the CAPS curriculum requirements. This is also reflected in the Grade 7 Curro planning used by all teachers which means geometry is done in all the four terms with term 1 for angles, term 2 for properties of lines, term 3 transformation and term 4 unusual containers and revision of all topics. The MG7MT is published by MDP education has a set of two textbooks constituting the grade 7 mathematics textbook. One textbook has all the other mathematics sections and some work on geometry while the second textbook comprises of geometry only. The second textbook is a workbook which has the theory of content, examples and tasks with space to write on (workbook). The outline of the main textbook is aligned to CAPS planning so Curro teachers rely on the planning from CCMD. All areas of geometry are covered in the textbook as per CAPS requirements. The next section will examine how data will be collected through an analysis of examples from these two textbooks.

These two textbooks were chosen because they are the only recommended textbooks in use by Curro schools for Grade 7 throughout its 123 primary schools in South Africa. They are also unique and different in terms of the presentation styles of the content on geometry which gives a better overall view of the content learnt in these schools. This study was an analysis of only two textbooks which may present some limitations to my study in terms of generalisation of findings, although this might be sufficient for the Curro group as they use only these two textbooks.

1.8.2 Data Analysis

Data analysis in the context of qualitative research is the process of organising, analysing, coding, and interpretation of data with the purpose of uncovering patterns, themes, and other insights, in an attempt to have a deeper understanding of the research question as well as addressing the research questions (Creswell 2013). Data analysis is about making meaning and deriving patterns from the data in an attempt to answer the research questions. Data was gathered through the use of an examples analysis grid (see annexure 1). The Creswell and Poth (2016) data analysis spiral which is a simple five-step process advocating for managing and organizing the data (data preparation), reading and noting emerging ideas, describing and classifying ideas into themes, development and assessment of interpretations, and presenting and visualisation of the data was used extensively. This resulted with three components, types of examples, the cognitive demands of the examples and the level of thinking being developed. There was development and use of appropriate tables, figures, and graphs to illustrate key findings.

1.9 Ethical considerations

In addressing ethical issues, this research study was a textbook content analysis which has no human contact, hence it was classified as having no risks according to the University of Witwatersrand ethics policy. Aligned with the standard procedures an ethics waiver application was submitted and granted by the Wits School of Education ethics committee (protocol number: 2024ECE040M, see annexure B).

1.10 Trustworthy of findings

The findings of this study emanate from a relatively rigorous process of analysing the examples in the two textbooks. Further, the process of analysis and findings were checked by an external expert. Thus, all efforts were made to enhance the trustworthiness of findings. Although the

study is a case study of only two textbooks, the results might contribute in a small way to the ongoing discussions about examples and the learning opportunities they afford in the teaching and learning of geometry. The rigour witnessed in the research process makes the research study more credible (particularly through engagement in the content in both textbooks), transferable (particularly through a thick description of the methodology and findings), dependable (through documentation of the raw data), and confirmable (particularly through showing an audit trail).

1.11 Organisation of study

Chapter 1 of this study presented a general background of the study giving an outline of the research project. Chapter 2 comprises of literature that is related to the study. Chapter 3 speaks about the research design and methodology. Chapter 4 presents the findings based on the analysis of examples while Chapter 5 discusses the findings. Finally, Chapter 6 summarises the findings, examines the limitations of study and then concludes the research study.

1.12 Conclusion

This chapter introduces the research study by giving a background of the research problem by highlighting the rationale and the significance of the study. There was a presentation of the problem statement and the research questions. The chapter also presents the research aims and objectives. The chapter also presents the conceptual and analytical framework to be used in the study. There was a discussion of the key terms used in this study. The research design and methodology including leading to data collection and data analysis were addressed. Ethical considerations and trustworthiness of the study were also discussed. The next chapter focuses on the literature review and conceptual framework.

CHAPTER 2 : LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

2.1 Purpose of chapter

A wide range of literature exists about textbooks and examples space in terms of their use, benefits, and choice of such examples in the teaching and learning process. This chapter is aimed at exposing literature related to examples and elucidating on the conceptual framework used in the study. The chapter will start with elaborating on the role of textbooks in learning and teaching mathematics at the lower secondary level, the followed by an examination of the exposition approach to textbook writing. There will be literature about example space in the classroom where the types of examples, the cognitive demands of examples, the role of real-life examples in mathematics, especially use of pictures in geometric shapes, and choice of examples in the classroom. Finally, the chapter will also examine development of geometric reasoning through examples as contained in the textbooks.

It may be important to start by elucidating some background regarding the use of textbooks and the examples as contained in textbooks. Mathematics textbooks which are usually distributed in printed format or, with the advent technology, electronically, exist to serve as comprehensive resources containing explanations and exercises that enables students to engage with mathematical concept (Zeynivandnezhad, Saralar-Aras & Halai, 2024). In an examination of examples, Bills & Watson (2008) noted that examples can be seen as always having played a central role in developing and shaping the teaching and learning of mathematics. It is the responsibility of the mathematics teacher to upgrade the knowledge and upliftment of students' understanding of mathematics, hence the use of examples become an important variable in the teaching and learning of mathematics (Zodik & Zaslavsky 2008). It should be noted that there are many meanings attached to examples in the mathematics education discourse. In this regard, it can be observed that one of the most important instructional strategy that is at the disposal of the teacher which is very useful in helping students construct meaning and understand mathematical concepts is the use of examples (Muir 2007). According to Muir (2007) a good example should be seen as one which is helpful to the learner, transparent its clarification and offer opportunities to acquire new mathematical concepts. In general, examples are choices made from a wide range of possibilities therefore teachers should be able to recognise the best examples. The extent to which examples maybe transparent or useful in

the classroom is subjective, they remain important and fundamental in creating mathematical awareness in the classroom.

2.2 The role of textbooks in learning and teaching mathematics at the lower secondary level

It has been noted earlier in chapter 1 of this research report that in mathematics education, textbooks play an important role and are used to a great extent in the mathematics classroom (Gracin 2018; Zeynivandnezhad et al. 2024). In the same token of advancing the importance of textbooks, in their study, Leshota and Adler (2018) undertook an analysis of Grade 10 Classroom Mathematics textbook and concluded that a deliberate use of a textbook and an intimate relationship with a textbook reflects a high level of pedagogical design capacity which is the unique skill possessed by the teacher that enables perceiving the affordances of a teaching resource. Adler (2000) cited in Leshota (2019, p.3) posits that “textbooks mediate between authors and teachers, between teachers and the learners, and between the mathematics itself and the learners”. Jukic and Glasnovic (2021) posited that textbooks are the most widely used resource in the mathematics classroom at national and international level, which means there is a need to continuously undertake research into textbooks.

On the international scene, Otieno and Povey (2022) examined the perceptions of learners in Kenya on what they value and what they find lacking in core mathematics textbooks. The same study concluded that textbooks were typically designed for teacher-mediation but are now playing a different role of enabling students to use them independently to enable understanding of concepts (Otieno & Povey 2022). Gracin (2018) research was undertaken to embed what was referred to as the fifth dimension of textbooks: mathematical activities, leading to the conclusion that the embedding quality activities in a textbook has a bearing how it affords learning in the classroom. A comparative study undertaken by Mellor, Clark and Essien (2018) aimed at a comparing a South African mathematics textbook *Classroom Mathematics Grade 9* to a Germany textbook *Elemente der Mathematik 3* led to the conclusion that learners from the two streams that used the two different textbooks were exposed to different affordances to learning mathematics by their textbooks in the learning area of functions. It can be observed from the discussions above that textbook analysis remain a critical component of mathematics education research.

In the context of South Africa, Leshota (2015) undertook research on textbooks that focused on investigating the relationship existing between textbook affordances and teachers’ capacity

to design instruction using curriculum resources, in this case textbooks, which was referred to by Brown (2009) as pedagogical design capacity (PDC). This study by Leshota (2015) concluded that there is a strong link between the textbook and the teacher's PDC. Further, in their research that focused on *Mathematics for All* textbook in primary school, Ensor et al (2002) posited that there is extensive use of textbooks in mathematics classrooms and found that textbooks were highly valued as teaching resources. A critique of textbooks uses by Vincent and Stacey (2008) observed that some textbooks provide problems which do not extend learners' thinking beyond the routine use of procedures therefore there is need to augment textbooks with other resources that enables presentation of extending and challenging problems for learners. Regardless of this critique of textbooks and given the central role of textbooks as well as recent development in education where learners are expected to interact independently with the textbook, and reconceptualization of learning options like online learning and self-study, textbook remain a critical resource in terms offering learning opportunities to learners.

The use and purpose of textbooks remain critical in the mathematics teaching and learning discourse. Zeynivandnezhad et al. (2024) concluded that textbooks are strategically designed for the purpose of translating the abstract curriculum policies into more practical operations to be executed by teachers and students by operating as intermediaries between the curriculum objectives as established by curriculum designers and the educators who impart instruction in the classroom. As such mathematics textbooks play an important role in the pedagogical landscape as they serve as vehicles for cultural sensitivity, inclusivity, and the transmission of values in mathematics education through a careful consideration of their visual presentation by adopting innovative teaching materials that can significantly impact students' understanding and engagement of mathematics concepts. It may be important to briefly examine the approach that can be employed in the writing of textbooks in particular the exposition approach.

2.3 The exposition approach to textbook writing

The use of the exposition approach is prevalent in the writing of textbooks. Glasnovic (2018) observed that most textbooks follow a structure involving rule-example-practices. The approach in textbook writing which follows the rule-example-practice is called the exposition approach (Glasnovic 2018). According to Gracin (2018) the exposition approach involves the author presenting content of the topic through explanations and registers, then the author presents examples where learners are offered models to be used or examples, and author gives

exercises or tasks used to check understanding of concepts by learners. The exposition perspective reflects the presentation in the CG7MT and MBG7MT where there is a presentation involving introduction of the mathematics concept by presenting the mathematical rule first. Once mathematical rules are presented, then examples are presented to show illustrations, and finally learners are given some work to do through exercises. In this textbook, there are however exercises which do not follow the exposition perspectives, but questions are just given then learners are expected to figure out the answer and explain their mathematical thinking. The examination of examples and how they are presented in the textbook in terms of learning affordances they present based on the rule-example-practice perspective, is very important in the mathematics education discourse.

The understanding of examples emanating from the exposition rule other aspects of examples (Glasnovic 2018; Gracin 2018) brings to the fore the place, use, benefits, and choice of examples in the mathematics teaching and learning process. According to Bills and Watson (2008) examples plays a critical role in developing and shaping the teaching and learning of mathematics through questions that are worked out after a concept have been explained to learners. It is the responsibility of the mathematics teacher to upgrade the knowledge and upliftment of students' mathematics understanding levels, through a careful selection and sequencing of examples in the process enabling understanding of mathematics concepts by the learners (Zodik & Zaslavsky 2008). In this regard, it can be acknowledged that examples form part of important instructional strategies that is at the disposal of the teacher which enables learners to construct meaning and understand mathematical concepts by providing questions and showing step by step ways of solving the mathematical problem (Muir 2007). Learners can go back to the examples in the textbook to check the logical way of solving mathematical problem which improves understanding of mathematical concepts through practicing the examples. A good example should be helpful to the learner, give clarification and offers opportunities to acquire new mathematical concepts (Muir 2007). In general, the use of examples in the classroom should be seen as an integral part of the mathematics teaching and learning process through creating awareness of how mathematical problems are solved. The next section provides an in-depth examination of examples as used in the classroom.

2.4 Exemplification in the classroom

There are so many variables and aspects of examples that may require intensive discussion in this study. Such variables include the types of examples, the importance of examples, analysis

of example spaces, choice of examples, real life examples, and the cognitive demands of these examples. Although of these variables like choice of examples are outside the confines of this study, they remain crucial and play a critical role in ensuring understanding of mathematical concepts. “Examples can also provide context, while the variation in examples can help learners distinguish essential from incidental features and, if well selected, the range over which that variation is permitted” (Goldenberg & Mason 2008, p. 184). Goldberg and Mason (2008) further posited that examples are very useful as they offer mediation tools that help mediate between learners and mathematical concepts, theorems, and techniques thereby either impeding or enabling the learning process. This shows that examples allow construction of knowledge through the laying out of logical steps to solving mathematical problems hence examples can bridge the gap in the learning process. Examples therefore provide and contain illustrative material that help breakdown and illustrate mathematical concepts. It may be important therefore to examine the types of examples that can be used in a mathematics classroom.

It is important to examine the interrelatedness existing between examples and the cognitive demands of examples. Figure 2.1 provides a graphic illustration of the study overview.

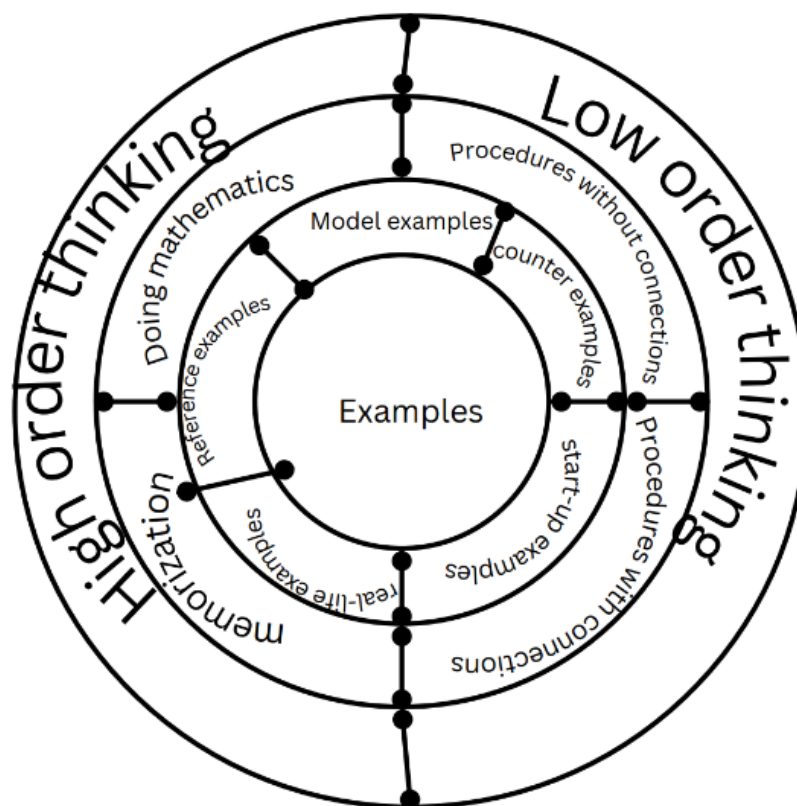


Figure 2.1 An overview of example space according to the research study.

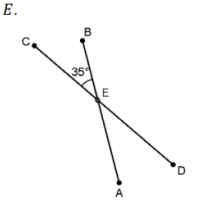
2.4.1 Types of examples by Michener

Since this thesis concerns itself with examples and the learning affordances provided by examples, it is important to discuss the various types of examples used in mathematics. There are several ways of examining examples to come up with the types or classes of examples. Michener (1978) identifies four types of examples being, start-up examples, reference examples, model/generic examples, and counter examples. “The major epistemological classes of examples space are: start-up examples, reference examples, model examples, and counter-examples” (Michener 1978 p. 368). While citing Michener (1978), Cavey and Kinzel (2015) contends that though there are four noteworthy types of examples, these classes are generic and an example can transcend or fall into more than one category of examples. It may be necessary to examine these examples in detail.

Start-up examples help in initiating new subject by motivating by presenting basic information these examples form the basis of delving into a topic, and this usually starts with simple and familiar cases to the learners (Zodik & Zaslavsky 2008). The role of start-up examples is to give basic information from which students can develop deeper understanding of basic fact. In geometry a student may be given several polygons hen the example asks for identifying quadrilaterals. When giving a solution for this example, this starts or initiates knowledge about characteristics of quadrilaterals which learners will use in future.

Reference examples are used to inform and develop the concepts that would have been initiated in the introductory part of the topic through start-up examples. Michener (1978) argues that referent examples are used to build deeper understanding of concepts by combining previously known concepts with new concepts. Figure 2.2 shows a referent example.

Example
Consider the diagram with straight lines AB and CD intersecting at E .
Calculate:
1. $\angle AED$
2. $\angle CEA$
3. $\angle BED$



The diagram shows two intersecting lines, AB and CD, meeting at point E. Line AB is oriented vertically, and line CD is oriented diagonally. The intersection point E is labeled. The angle AED, which is the angle formed by ray EA and ray ED, is labeled with the value 35°.

Method
1. $\angle AED = \angle CEB$
 $= 35^\circ$
 These angles are equal because they lie vertically opposite each other.
2. $\angle CEA = 180^\circ - 35^\circ$
 $= 145^\circ$
 These angles lie on a straight line and must add up to equal 180° .
3. $\angle BED = \angle CEA$
 $= 145^\circ$
 These angles are equal because they lie vertically opposite each other.

Figure 2.2 Referent example from CG7MT

This example from CG7MT is referent example as it combines knowledge of vertically opposite angles with new concept of angles on a straight line.

Model or generic examples can be analysed as examples that are indicative of the general rule or general case of a concept by capturing the essence of the situation or concept. Generic examples makes understanding of concepts by focusing on core mathematics concepts making it easy for students to grasp concepts (Michener, 1978). They enable general application of concepts e.g. the areas of a triangle is calculated by the formula half base multiplied by the perpendicular height. This is applicable to any triangle regardless of the nature and type of triangle.

Counter examples emanate from slight alteration in theorems leading to a conclusion showing that a statement is not true therefore being useful in sharpening distinctions that exist between concepts. (Goldenburg & Mason 2008). Counterexamples are useful and used to show learners what is not true in mathematics. Counter examples are helpful in refining learners' concepts understanding by highlighting exceptions or cases that go against the general rule e.g. drawing a triangle with two obtuse angles. Counter examples are used to disapprove a mathematical statement.

Other classifications and types of examples brings to the fore pre-planned examples, spontaneous examples and non-examples (Zodik & Zaslavsky, 2008). Pre-planned examples there refer to those examples used in the lesson which shows evidence that during the lesson planning process, the teacher would have planned and intended to use them (Zodik & Zaslavsky, 2008). Spontaneous examples are those examples used in the classroom that do not show evidence of them having been planned beforehand hence they are in-the-moment made examples (Zodik & Zaslavsky, 2008). In each case the role of examples remains the provision of something familiar and specific to the learner that provides a basis for exploring new ideas that provides opportunities to learn new concepts. However, the latter may not form part of this discussion as the classification is based on examination of the operational part of examples as used in the classroom which may be outside the confines of this discussion.

2.4.2 The levels of cognitive demands represented in examples

Cognitive demands may be viewed as the level and kind of thinking process that students should be engaged with to solve mathematics questions. According to Stein et al (2000) mathematics tasks be classified into four categories based on the cognitive demands of the task.

Ubuz and Alp (2018) contends that the categories are memorisation tasks, procedures without connections tasks, procedures with connections tasks and doing mathematics tasks. Stein et al. (2000) further observe that in this classification, the first two types (memorisation tasks and procedures without connections) are regarded as lower order thinking (LOT) tasks while the last two (procedures with connection tasks and doing mathematics tasks) are regarded as high order thinking (HOT) tasks. Mainali and Ipek (2025) acknowledged that Smith et al. (2000) developed important levels of cognitive demands taxonomy aimed at categorising mathematics tasks based on the level of thinking required by learners in attempting and solving mathematics tasks or problems. In this study, the cognitive demands of tasks are instrumental in determining the examples that can be used in a mathematics classroom.

One of the widely known lens of analysing mathematical tasks and probably the examples they attract is to examine the levels of cognitive demands of such a tasks. Stein et al. (2000) provides a very useful framework in terms of analysing the level of difficulty of tasks based on the cognitive demands of the question by examining the type and level of thinking elicited by the task. Generally, the framework examines the level and kind of thinking that students engage with to solve mathematical problems. Such processes were classified by Stein et al (2000:16) as “...memorization tasks, procedures without connections tasks, procedures with connections tasks and doing mathematics tasks” The first two are viewed as lower levels cognitive demanding tasks while the last two require higher level cognitive demands. Yu and Huang (2025) and Resnick (2023) argue that higher-order thinking questions or examples provide teaching and learning opportunities that encourages learners to engage in high-quality thinking of mathematics concepts. In this study, the analysis of each example in terms of ‘type’ of example and the cognitive demands expected in answering the question offered by the example, will be presented.

Memorisation Tasks

Stein et al. (2000) posits that memorisation tasks are regarded as demanding low levels of cognitive requirements meaning application of mental ability is relatively low in these tasks. Such tasks usually involve the reproduction of previously learnt content, facts, formulas, and definition which may require simple use of memory to remember the answers. Mainali and Ipek (2025) posited that memorisation tasks usually have no ambiguity hence they involve reproduction of previously learnt concepts, recall of facts, formulas, rules, and definitions without a need to follow procedures or establishing connections in the process. The tasks are

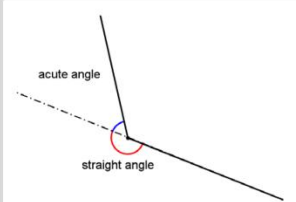
also short, usually one word answer, resulting in less time as there is no need for any connections to solve the mathematical problem. Figure 2.2 (on page 24) shows a memorisation cognitive level example. It can be seen from figure 2.2 that the learner is being given a way to measure an obtuse angle which means if faced with the same question, then the learner should recall how to do it through the use of memorising the process. This then is a LOT in terms of the level of thinking required to solve the problem presented in the example.

Example

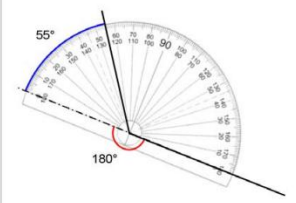
Lebo measures the angle $\hat{F}$ using a protractor. Read through her method shown below. Do you agree with the method she has used? Can you think of another way to measure this angle?

Lebo's method

Lebo draws a straight line which breaks $\hat{F}$ into two angles: $\hat{F} = \text{straight angle} + \text{acute angle}$



She uses her protractor to measure the acute angle.



Lebo's calculation:

$$\hat{F} = 180^\circ + 55^\circ = 235^\circ$$

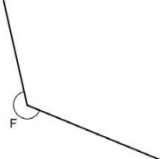


Figure 2.2 Memorisation cognitive level example classified as LOT example CG7MT.

Procedures without connection tasks

This category of tasks according to Stein et al. (2000) falls under lower-level cognitive demand tasks. These tasks require the use of known and predetermined procedures and ways of doing things but there is no connections in terms of mathematical solutions hence they involve the use of algorithms. Such mathematical tasks are also less ambiguous, have no connection to concepts being used and emphasis is deriving the answer rather than the procedure. The concern of such tasks is about deriving the correct answer and not necessarily development of understanding of the mathematical procedure or explanations, but merely description of the procedures used. An example of a procedure without connection maybe... Calculate the value in degrees of the interior angles of a polygon with 9 sides. The formula $(n-2)180^\circ$ can be used to solve this problem. In this case there is no connection but a mere depiction of the procedure being followed using the formula to solve the problem.

Procedures with connection tasks

Stein et al. (2000) classified procedures with connection tasks as higher-level cognitive demand tasks. Procedures with connection tasks focus on bringing out the use and connection of procedures by allowing deeper understanding of mathematics concepts. There is following of certain agreed pathways and there is need for understanding and connections of the pathways to solve mathematical problems. Such tasks also require some high level of cognitive effort and there is need for learners to engage with ideas that underlie those procedures. An example of this kind of task is solving angles or sides of a triangle using the types of angles.

Doing mathematics tasks

According to Stein et al. (2000) ‘doing mathematics tasks’ are higher-level cognitive demand questions which epitomize the study of mathematics as far as the level of difficulty is concerned. Such tasks require complex thinking as there is no predetermined pathways for the task. Exploration, understanding mathematical processes and relationships are very important in these tasks. Students with inner drive to use their cognitive abilities mixed with experiences and background knowledge can complete these tasks. Further, this class of tasks require a high degree of cognitive effort and may even lead to anxiety in learners in terms of their ability to solve the task or not. Such a question maybe as follows; Chelsea football club has a stadium measuring 75 m by 48 m. They want the grounds men to cut the grass into 1000 equilateral triangles. Calculate the base and the height of such a triangle. Figure 2.3 shows a doing mathematics example classified as a HOT example.

Example

Convert 2 km^2 to m^2 .

Method

There are 1 000 m in 1 km.

$$1 \text{ km}^2 = 1\,000 \text{ m} \times 1\,000 \text{ m} = 1\,000\,000 \text{ m}^2$$

So, 2 km^2 is $2 \times 1\,000\,000 \text{ m}^2 = 2\,000\,000 \text{ m}^2$

Figure 2.3 A doing mathematics example being classified as requiring HOT from CG7MT

Table 2.1 The characteristics of mathematical tasks at each of the four levels of cognitive demand (Stein & Smith, 2000 p.128-132)

Table 2.1 Task Analysis Guide	
<i>Lower-Level Demands</i>	<i>Higher-Level Demands</i>
<u>Memorization Tasks</u> • Involve either reproducing previously learnt facts, rules, formulae, or definitions OR committing facts, rules, formulae or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	<u>Procedures With Connections Tasks</u> • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations help to develop meaning. • Require some degree of cognitive effort. Although general procedures maybe followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
<u>Procedures Without Connections Tasks</u> • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	<u>Doing Mathematics Tasks</u> • Require complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Require students to explore and understand the nature of mathematical concepts, processes, or relationships. • Demand self-monitoring and self-regulation of one's cognitive processes. • Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Require students to analyse the task and actively examine task constraints that may limit possible solution strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

2.4.3 The role of real-life examples in mathematics

The use of real-life examples in the teaching and learning of mathematics especially geometry can be viewed as an important approach that fosters understanding of abstract mathematics concepts. Hezron, Deogratias and Lupeja (2024) posited that connecting mathematics concepts being learnt with students' actual life experiences can be possible when teachers use actual teaching aids and objects which mirror students' real life experiences and environments. Since mathematics is seen as a fundamental subject for science related studies, teachers should facilitate the teaching of the subject not as intangible knowledge but as a living part and parcel of student's life. As such, the use of real-life examples helps students to easily understand the mathematical concepts being taught which may have a bearing on performance in the subject (LeMahieu et al., 2017). It means that mathematics teaching and learning should ensure that learners are able to realize and use mathematical knowledge acquired in the classroom to solve their daily life problems as they are reflected in the lesson. The conclusion therefore is that mathematics as a science should be taught in the way reflecting learners' actual living experiences or real-life examples, especially in geometry which involves shapes and figures that learners manipulate on a daily basis. Teachers should develop and improvise teaching aids and examples that connect classroom learning experiences with actual living environment.

2.4.4 Choice of examples in the classroom

The choice of examples used by the teacher regardless of the nature and type of such example has an effect in the teaching and learning process of mathematics in the classroom. Essien (2021) posited that research shows that the type of examples chosen by the teacher as well as how such examples are used in the mathematics teaching and learning process can either constrain or enable students' access to mathematical knowledge. This is because examples give mathematical understanding by demonstrating procedures to be followed and conceptual understanding of relationship and generalizations. It means the use of examples in the classroom is critical because it allows concepts understanding and enhancement of mathematical theory (Alcock & Inglis 2008). The use of examples is seen as a mathematical teaching practice used to explain and afford mathematical understanding in the teaching and learning process. Therefore, the primary actions by the teacher in the classroom also include the choice of examples that are used in interacting with the content being taught as such examples give visuals of the idea in the pedagogical content knowledge. In the confines of this

study, discussion centres on an analysis of the given examples in the textbook. There is still component of choices being made by the teachers from a range of available examples.

Examples are contained in a variety of teaching and learning materials including textbooks which forms the basis of this study. It is therefore important to briefly examine the textbook as a source of the data to be analysed in this study. Leshota (2019) argues that, from a Vygotskian socio-cultural perspective highlights, textbooks are important tools and artefacts used by people with the aim of achieving certain goal-oriented activities in mathematics. Textbooks, therefore, have the potential to enable or hinder the achievement of educational goals especially the way in which examples are presented in the textbook. It means the choice of the textbook must also put into consideration the example spaces in that textbook. Examples in the textbook should also reflect on the teaching approach posited in the textbook being the ability to see and experience mathematics as a human worthwhile activity which involves inculcating problem-solving skills to learners. This approach emphasizes problem solving, building up trajectories through movement from simple to advanced concepts about geometry, interrelatedness of concepts, and regular practice of concepts.

2.5 The development of geometric reasoning through examples

A considerable amount of literature was presented above textbooks and examples, hence the need to examine briefly the third theme of this research project, geometric reasoning, which is content area to be focused on in this research report. Geometry is a section of mathematics that predominantly deals with shapes. Breyfogle and Lynch (2010) posited that the van Hiele theory of geometric reasoning identifies the five levels starting with visualisation or recognition level, advanced description or analysis, ordering or informal deduction, deduction or abstraction, and finally rigor or proof of geometric theories. According to de Villiers (1996) geometry should be seen as a fundamental skill because it has important application to real life problems like measurements, mapwork, critical thinking skills, enables use of models, and provides basic mathematics concepts such as linear programming, area, volume, and essential for understanding calculus and other related fields.

There has been research in geometry both internationally and in South Africa (Breyfogle & Lynch 2010; Mason 2009; de Villiers 2010; Armah et al 2018). Armah, Cofie & Okpoti (2018) posited that geometry is an essential area of study in school mathematics curriculum throughout history and is of great importance to people's lives. Breyfogle and Lynch (2010) posited that the van Hiele theory of geometric reasoning identifies the five levels starting with visualisation

or recognition level, advanced description, or analysis, ordering or informal deduction, deduction or abstraction, and finally rigor or proof of geometric theories. The widely accepted Van Hiele levels of geometric thinking theory agrees with this notion when it posits that learners advance in geometric thinking through five sequential, progressive, and hierarchically discrete levels that characterizes learners' thinking and understanding of geometric relationships (Van der Sandt & Nieuwoudt 2003; Armah et al. 2018 & de Villiers 2010). Van der Sandt and Nieuwoudt (2003) posited that students often find geometrical concepts as being more abstract and difficult to understand especially once above the visual level, which may result in poor performance.

Most research on geometry in South African primary and junior high school focuses on other areas of geometric reasoning and not necessarily on textbooks and examples they contain (Bleek & Goosen 2009; Sinclair & Bruce 2009). Bleeker and Goosen (2009) focused on understanding the barriers to learning senior phase geometry, Sinclair and Bruce (2015) examined new opportunities in geometry education at the primary school through embracing the use of technology in the classroom. Furthermore, research on geometry textbooks was done by Bowie (2013) which focused on the constitution of school geometry in the Mathematics National Curriculum Statement and two Grade 10 geometry textbooks in the context of South Africa. The textbooks researched on were Classroom Mathematics and On Track with Mathematics to examine what was constituted as school geometry in the curriculum documents and the factors and constraints that shaped the production of school geometry in these textbooks. There is therefore need for more research into textbooks on the topic of geometry especially in primary and junior high school level in South Africa.

According to de Villiers (1996) geometry should be seen as a fundamental skill because it has important application to real life problems like measurements, mapwork, critical thinking skills, enables use of models, and provides basic mathematics concepts such as linear programming, area, volume, and essential for understanding calculus and other related fields. Van der Sandt and Nieuwoudt (2003) argued that many learners fail to understand geometrical concepts because these concepts are abstract especially once it goes beyond the visualization level where more abstract geometric concepts are applicable. This position means that competence in geometric thinking is essential in future studies of geometry hence foundation done in Grade 7 must be proper, which brings to the forefront the need to investigate sources of learner struggles in this topic. In the South African education system, grade 7 is part of the

senior phase (lower-level secondary school) covering Grade 7 to Grade 9. The Grade 7 section of geometry forms a firm foundation for senior phase (Grade 7-9) mathematics with 25% of the end of year exam being on geometric reasoning (Department of Basic Education 2011). According to the Curriculum and Assessment Policy Statement (CAPS) document, which determines learning areas South African schools, the weight attached to geometry goes up to 35% in Grade 9 and eventually leads to Analytical Geometry and Euclidean Geometry and Measurement in the Further Education and Training (FET) phase, which is the final phase of high school in South Africa.

2.6 Conclusion

This chapter examined the literature related to this study leading to the conceptual framework to be used in the study. A wealth of literature examined the use of examples in the mathematics classroom, the types of examples, the cognitive demands of examples. There was also a discussion about the use of real-life examples in the mathematics classroom, the choice of examples and the development of geometric reasoning through examples. An analysis of the literature also resulted in the derivation of the conceptual framework based on a combination of Michener (1978) types of examples and Stein et al. (2000) cognitive demands of tasks. This chapter therefore presented literature related to this study creating a broad base of information on the topic of the research topic.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Purpose of chapter

Research design and methodology details the process which researchers design research studies which enables them to achieve their objectives by use of the selected research instruments (Sreekumar & Sreekumar 2023). As such, a research design is a structured plan or a blueprint that having an outline on how to collect, analyse, and interpret data to address a research question(s). All important aspects of research, which includes the research design, methods of collecting data, methods of data analysis, as well as the overall framework used in conducting the research. Ultimately, the research design serves a roadmap that guides researchers to conduct their studies in a systematic way. This chapter is aimed at providing in-depth elaboration of the research design and research methodology through an examination of how the research will be undertaken from research paradigm, the methods of collecting data, the research sample (examples from two textbooks in the topic of geometry) and the instruments used to gather relevant data. This chapter furthermore examines ethical issues related to the study as well as the methods used to analyse the data.

3.2 Research paradigm

The research study is largely guided by the *constructivism* paradigm. According to Ignacio and Paras (2024) constructivism contends that individuals form or construct most of what they learn and understand through their learning experiences. The argument being that what can be perceived to be the truth is a product of personal views based on constructed knowledge through learning experiences. This paradigm will assist in this study to understand the research problem since it is about knowledge construction and knowledge acquisition in the teaching and learning of mathematics through examples in the two textbooks. Further, it will assist in theory generation by analysing the type of examples in two Grade 7 textbooks (CG7MT and MG7MT) textbooks based on Michener (1978) types of examples. Furthermore, the constructivist approach will be used in this study by interpreting and classifying the cognitive demand of the examples contained in the textbooks in the topic of geometry.

3.3 Research approach: Qualitative research

In this study, a *qualitative approach* to research was followed. Qualitative research is useful where the meaning and process may not be examinable through quantifiable amounts of

findings (Daniel & Sam 2011). Hendriks and Cruywagen (2024) agrees with this notion when they observed that qualitative research methods offer valuable insights into the experiential and interpretive aspects of educational phenomena thereby providing specific understanding to a phenomenon, in this case analysis of textbook examples. According to Hendriks and Cruywagen (2024) a qualitative research approach is concerned with establishing the phenomenon in question in the research questions unfolds through descriptive presentations. Furthermore, qualitative research is aimed at attaining deep understanding of a specific case with in-depth exploratory studies to enable finding quality responses throughout the research project. In this study the analysis of qualitative data about and types of examples, the cognitive demands of examples, and the level of thinking required by solving examples presented in the two textbooks. Although the findings included a visual presentation using frequencies and graphs, which might be viewed as elements belonging to quantitative research, the process of collecting data was primarily qualitative and the visual presentation of findings was to enhance the reader's understand of the distribution of the data. Qualitative data will be sort through analysis of the examples on the section of geometry contained in the textbooks to understand the nature of examples and the learning opportunities they afford to learners in mathematics.

Since this study focussed on only two textbooks out of a variety of textbooks used in lower secondary level in South Africa, it was a rather small case study. Hendriks and Cruywagen (2024) argued that a case study is an in-depth research design that primarily uses a qualitative research approach aimed at a particular class or group whose findings can be attributed to a bigger group. In this case document analysis to gather rich, descriptive information was done. In this study the analysis was from only two mathematics textbooks out of several textbook developed and used by lower level secondary school in South Africa. However, the findings might contribute to the ongoing discourse on the teaching and learning of mathematics especially in the textbooks and examples spaces.

3.3.1 Research design and process

The qualitative research process was intended to address the main research question and the sub-questions. Clark & Ivankova (2016) posited that qualitative research methods research process consists of the research purpose and questions, methods, and inferences. In order to ground the inquiry, the researcher analysed examples in CG7MT and MG7MT in the section of geometry. The information emanating from this analysis was presented in frequency charts and descriptive statistics. This was in order to gain greater insights into the cognitive demands

of examples and establish any noteworthy patterns which may be used as inferences for affordances and mathematics teaching and learning opportunities presented by these examples in geometry. This process was meant to gain insights into the learning opportunities presented by examples contained in these two textbooks in geometry through an analysis of examples.

3.4 Research Sample

The sample for this research comprised of examples presented in geometry in CG7MT and MG7MT. These two textbooks were chosen for this study because they are the two textbooks recommended and used by Curro Holdings across all its schools in Southern Africa in Grade 7 mathematics. All examples contained in this textbooks in geometry will be analysed. In the CG7MT textbooks geometry topics are contained in different pages of the textbook while in the MG7MT only the second book contains geometry section hence the examples. The CG7MT is a specific textbook published for Curro Schools by Brombarcher and Associates who are publishers of the Numbersense mathematics programme. In this textbook geometry is presented in such a way that sections of geometry are spread throughout the book. The Numbersense system advocates for covering the major mathematics sections by attempting parts of each section throughout the year but all sections required are covered as per the CAPS curriculum requirements. This is also reflected in the Grade 7 Curro planning used by all teachers which means geometry is done in all the four terms with term 1 for angles, term 2 for properties of lines, term 3 transformation and term 4 unusual containers and revision of all topics.

The MG7MT is published by MDP education has a set of two textbooks constituting the Grade 7 mathematics textbook. One textbook has all the other mathematics sections and some work on geometry while the second textbook comprises of geometry only. The second textbook is a workbook which has the theory of content, examples and tasks with space to write on (workbook). The outline of the main textbook is aligned to CAPS planning so Curro teachers rely on the planning from CCMD. All areas of geometry are covered in the textbook as per CAPS requirements.

In this study data about examples will be collected from CG7MT from sections that deals with geometry. The presentation of textbook is such that all mathematics sections are covered thorough out the year. The following pages will be used to get day from the CG7MT; 12-17, 33-36, 49-69, 91-101, 123-130, 175-190, 235-246, 270-310, 242-370, 426-432, and 488-509. The layout of the MG7MT is more in line with the CAPS document in terms of structure. In

the main textbook analysis will be from geometry topics from pages 71-121, 188-212, and 262-283, while for the geometry workbook (second textbook) analysis will be done on all examples from pages 1-182.

3.5 Data collection and analysis

Data should not only be collected in a systematic manner, but should be analysed to enable effective and meaningful presentation of the data.

3.5.1 Data collection instrument

The data about examples from the two textbooks was collected through a system developed by the researcher. A research instrument is a tool used when collecting, measuring, and analysing data related to the study. The Okeeffe (2013) textbook analysis instrument comprising four key elements; Content, Structure, Expectation and Language was adapted to develop the data collection tool. As such in this study a previously validated instrument was chosen and adapted to respond to the needs of the study. The adaptation was meant to have a better understanding of the nature and frequency of examples to answer the research questions. The adaption was also to suit the analytical and conceptual frameworks as presented in chapter 1 and chapter 2 (see annexure A).

3.5.2 Data analysis

According to Okeeffe (2013) it is important to develop tools to analyse data in qualitative research. Tools enable researchers explore, understand, and make meaning of data. The instrument (annexure A) had sections aimed at identifying the textbook, the section of geometry, the page number, the example reference, the cognitive demand of the examples and the level of difficulty (higher order thinking -HOT or Lower order thinking – LOT). Descriptive statistics were then analysed to show patterns and results as the findings were presented in chapter 4.

3.6 Trustworthiness and findings

The findings of this study emanate from a relatively rigorous process of analysing the examples in the two textbooks. The data collected was collected by use of specific key words to correctly classify the examples. There was also a process of checking the classification done when 2 other teachers who work for Curro Holdings teaching Grade 7 classes at two different schools compared their classification against the classification by the researcher. Teacher 1 had 5 years experience teaching the grade while teacher 2 had 3 years experience. These results can also

be confirmed as the data collection tool has all the information about where the examples are located in the textbooks which allows for verifiability. The rigorous process of documentation and careful considerations makes the findings trustworthy. Although the study is a case study of only two textbooks, the results might contribute in a small way to the ongoing discussions about examples and the learning opportunities they afford in the teaching and learning of geometry. The rigour witnessed in the research process there makes the research study credible, transferable and dependable.

3.7 Conclusion

This chapter laid out the design, procedures and procedure that was undertaken in the process of conducting this research study. In the process of conducting this study, the qualitative research method was utilized. The research sample and research process were also addressed in this chapter. An illustration was made of the research instrument used in collecting data. In the next chapter, all data and analysis from the study will be interpreted in relation to the research question and research objectives to offer findings to the study.

CHAPTER 4: FINDINGS

4.1 Purpose of chapter

The previous Chapter (Chapter 3) presented an outline of how data was to be collected which culminated in the development of a research data collection instrument (annexture A: Data collection instrument). This was a qualitative research aimed at undertaking an analysis of the examples contained in the CG7MT and MG7MT. This chapter is aimed at providing information about the findings of this study as far as collected data about examples from the textbooks is concerned. The data collected through the data collection instrument was collected with the view of responding to the research questions of this study. Each example was examined through highlighting key words to determine the cognitive demands of the example. There was also a table developed to determine the point of topic coverage when the example could be used. The results of the study are divided into four sections: general overview of findings, nature of examples and frequency of appearance, the order of thinking required for the examples and inclusion of real-life examples. These four sections are all aimed at analysing the findings based on an attempt to address the research questions.

4.2 General overview of findings

An exploration of the data about examples from the two textbooks brought out some similarities and fundamental differences. The CG7MT seems to emphasise more on definitions and point to note about a concept before giving questions about that concept than emphasising on examples, whereas MG7MT places emphasis on examples in the presentation of mathematics concepts. It can be seen that CG7MT doesn't align itself with the exposition approach discussed in chapter 2 whereas the MG7MT aligns itself with the exposition approach to mathematics textbook writing. The CG7MT has 19 examples throughout all geometry sections spread throughout the textbook while the MG7MT has 46 examples. This difference shows there is more emphasis of examples in MG7MT than in CG7MT. In the CG7MT every section starts with a reminder of prior knowledge covered pertaining to that section. It is the researcher's conclusion that this view results from the fact that topics are not covered in some sequential order as indicated in Chapter 2. The book is organized in such a way that a section is covered in part e.g. a section of exponents then go to a different section and eventually come back to the previous section. As such this calls for a need to refresh the memory of learners by referring to work previously covered.

Another notable qualitative outcome observed from the analysis of examples is the emphasis on project and discovery approach of the CG7MT. The textbooks is presented in such a way that in some sections the starting point is an investigation which leads to some geometric concepts to be understood by learners before eventually presenting examples or questions. As such, it can be concluded that CG7MT is premised on the problem solving trajectory. On further investigation, the problem solving approach stems from the fact that the textbook attempts to build from the numbersense programme which is used by Curro as the time of the research from Grade R to Grade 6 as far as mathematics content coverage is concerned.

The data collection tool developed (discussed in chapter section 3.5.1 and 3.5.2 – Annexure A) gathered data about the types of examples contained in the two CG7MT and MG7MT in geometry as indicated on the theoretical and analytical frameworks. The tool had 3 sections being types of examples, the cognitive demands of examples and the order of thing required to address the question of the examples. This was aimed at addressing the main research question and research sub-question 3. A snippet of the data collection tool and the 3 sections is presented below. In figure 4.1, S1 (start-up examples), R1 (referent examples), M1 (model examples) C4 (counter examples), MT (Memorisation tasks), PC+ (procedures with connection examples), PC- (procedures without connection examples) and DM (doing mathematics examples),

are

presented.

Textbook		Section of geometry	Page	Types of examples					Cognitive Demands				Thinking	
CG7MT	MG7MT			S1	R2	M3	C4	RL5	MT (B)	PC+(C)	PC-(D)	DM(E)	LOT	HOT
	X	Construction of angles Ex 1	5	X					X				X	
	X	Construction of angles Ex 2	12	X					X				X	

Figure 4.1 Structure of data collection tool

The analysis of examples was a critical component of this research project. Page 38 presents how analysis of examples was done from one of the textbooks using the collection too developed. This example is on page 60 in CG7MT. It was analysed to be a start-up examples as this was the introduction of vertically opposite angles. In terms of the cognitive demands, this example was classified as a memorisation task as solution is derived from memorising a mathematics concept in this case vertically opposite angles. When examining the order of thinking required to solve the problem in this example, the analysis classified this example as a LOT example. As discussed in the literature review HOT thinking examples are seen as providing better and robust learning opportunities in mathematics. Therefore, the ratio of LOT to HOT examples should be central in the examination of the research findings.

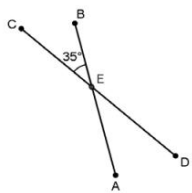
Curro Mathematics Grade 7

Example

Consider the diagram with straight lines AB and CD intersecting at E .

Calculate:

1. $\angle AED$
2. $\angle CEA$
3. $\angle BED$



Method

1. $\angle AED = \angle CEB$
 $= 35^\circ$
 These angles are equal because they lie vertically opposite each other.
2. $\angle CEA = 180^\circ - 35^\circ$
 $= 145^\circ$
 These angles lie on a straight line and must add up to equal 180° .
3. $\angle BED = \angle CEA$
 $= 145^\circ$
 These angles are equal because they lie vertically opposite each other.

Figure 4.2 Sample of example analysis

4.3 Qualitative findings to answer main research question a research sub-question 1.

The main question sought to undertake an analysis of the cognitive demands of examples in the two textbooks. It can be noted that CG7MT has 19 examples in geometry while MG7MT has a total of 46 examples in geometry. As such MG7MT has far more examples that learners can interact with in the teaching and learning process as compared to CG7MT. It can be seen that MG7MT has more examples in each category except in real-life examples where both books have the same number of examples.

Table 4.1 Types of examples per textbook

Ref	Example type	Textbooks	
		CG7MT	MG7MT
A	Start-up examples	5	17
B	Reference examples	8	19
C	Model/generic examples	3	7
D	Counter examples	0	0
E	Real-life examples	3	3
Total amount of examples per textbook		19	46

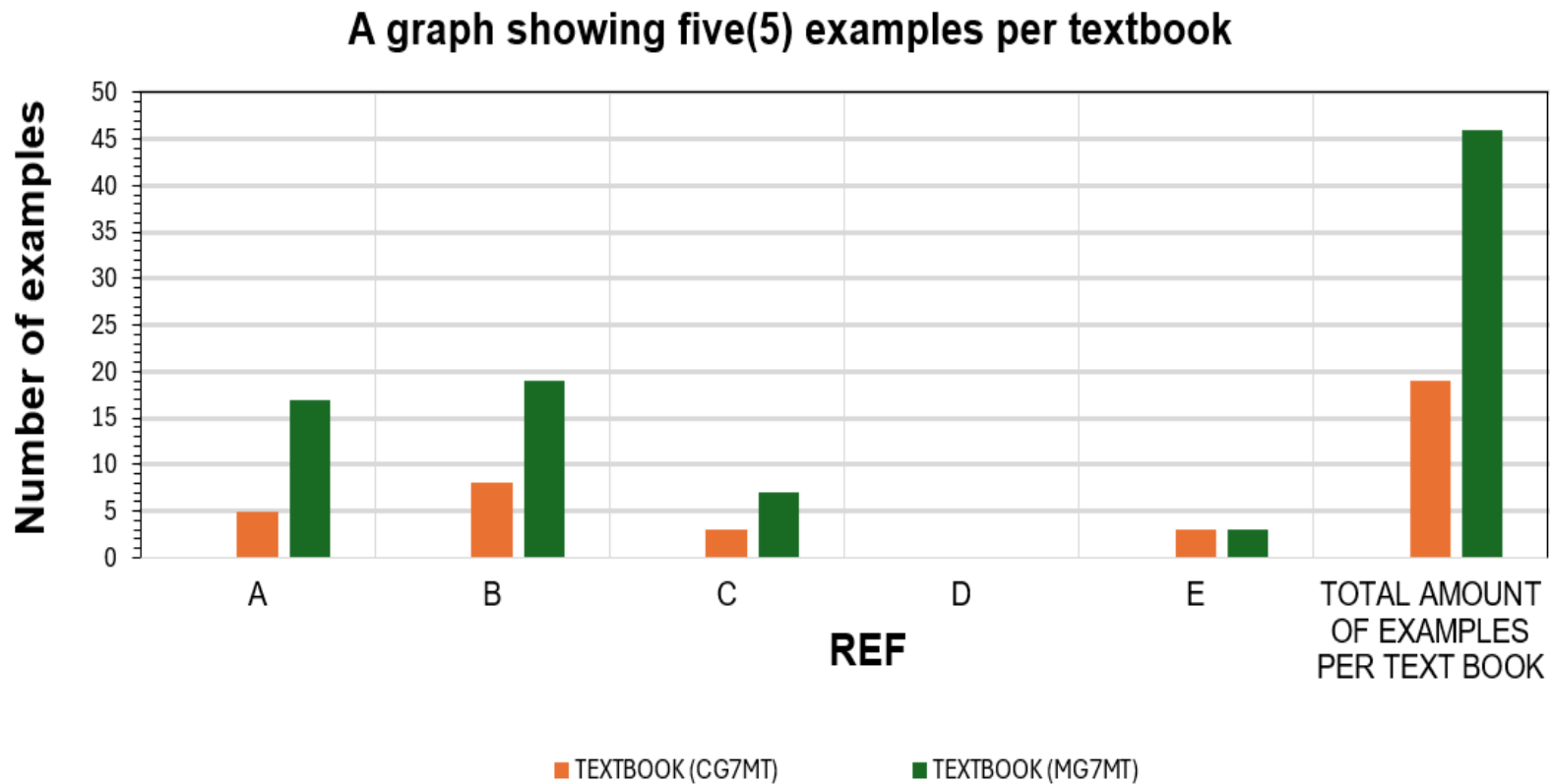


Figure 4.3 shows the graph with types of examples as analysed from the textbooks

Figure 4.3 shows a graphical illustration of the distribution regarding the types of examples per textbook. There was no counter examples used in the two textbooks. Both textbooks have referent examples as the highest quantity. As discussed in the literature review counter examples provide learning opportunities for learners to test deeper understanding of concepts and their absence takes away those learning opportunities.

4.4 Qualitative findings to answer research sub-question 1.

Using the data collection tool, examples presented in these textbooks were analysed in terms frequency of occurrence of the levels cognitive by an examination of cognitive demands of the examples. The analytical framework presented in chapter was used in gathering information about the types of examples. Table 4.2 below shows the results of this analysis. An example of this analysis was done on page 37.

Table 4.2 Cognitive demands of examples per textbook

Ref	Example type	Textbooks	
		CG7MT	MG7MT
A	Memorization tasks examples	12	17
B	Procedures without connections tasks examples	4	19
C	Procedures with connections tasks examples	2	8
D	Doing mathematics tasks examples	1	2

Table 4.2 shows the distribution of examples in terms of the cognitive demands of the examples. It can be observed that the highest number of examples in terms of quantity are procedures without connection from the MG7MT which has a total of 19 examples in this category. However, for CG7MT the highest number of examples fall under memorisation tasks which was 12 examples.

A graph showing cognitive demands of examples as analysed from the textbooks

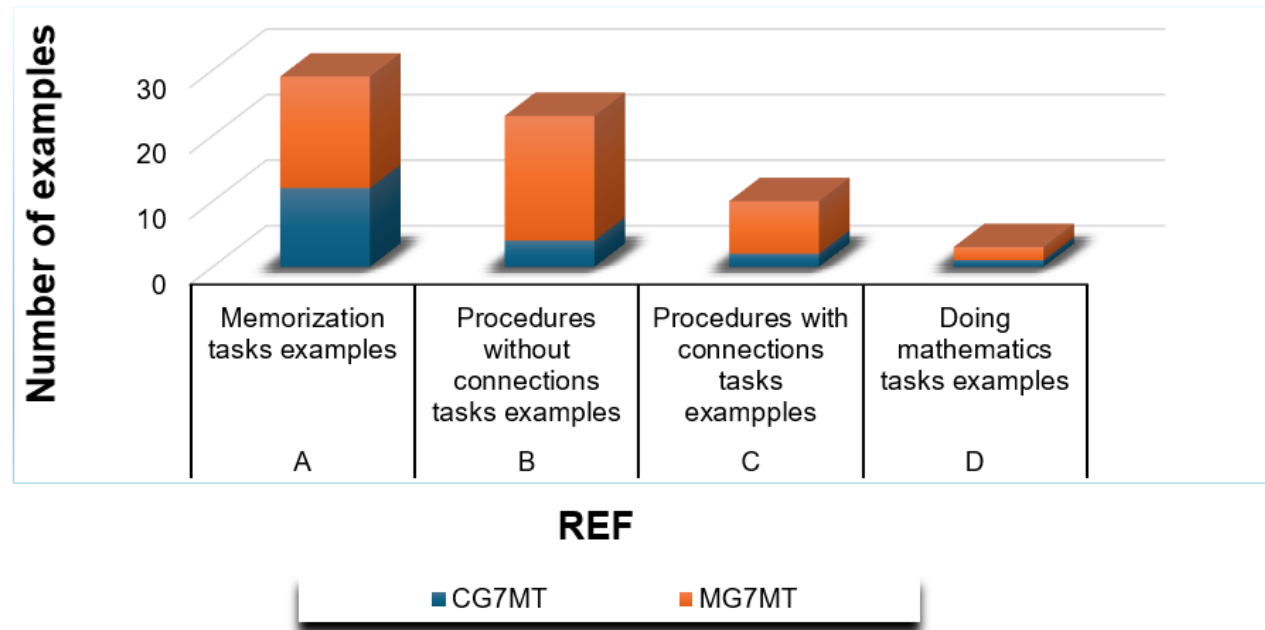


Figure 4.4 shows the graph with cognitive demands of examples as analysed from the textbooks

It can be observed from figure 4.4 that memorisation task examples forms were equally high in terms of frequency of appearance from both textbooks when examining the cognitive demands of examples in geometry. However MG7MT have more examples in each cognitive demands level as compared to CG7MT as shown in figure 6.

4.5 Qualitative findings to answer research sub-question 2.

The final part of the data collection tool had an analysis of the order of thinking addressed by the example. This being either Higher Order Thinking (HOT) and Lower Order Thinking (LOT). As discussed in chapter LOT order thinking examples require lower level of cognitive demands while HOT extends the thinking of learners. The table below shows the findings in terms of the order of thinking required as addressed by the examples. Table 4.3 below shows the distribution of examples in terms of order of thinking required as addressed by the examples.

Table 4.3 Order of thinking of examples per textbook

Textbook	Total examples	Lower Order Thinking		Higher Order Thinking	
		Qty	Percentage	Qty	Percentage
CG7MT	19	16	84%	3	16%
MG7MT	46	35	76%	11	24%

The results show in table 4.3 shows that in both textbooks, lower order thinking (LOT) examples are far more than higher order thinking (HOT) examples. As such 16 out of 19 examples are LOT examples while 3 out of the 19 examples are HOT examples in the CG7MT. In MG7MT 35 out of 46 examples are LOT examples while 11 are HOT examples.

It was also found out that both textbooks have less amount of real-life examples. The MG7MT had 3 out of 46 examples being real-life examples, while CG7MT had 3 out of 19 examples were real-life example. This shows that 6,5% of MG7MT were classified as real-life examples while 15.7% of CG7MT were classified as real-life examples.

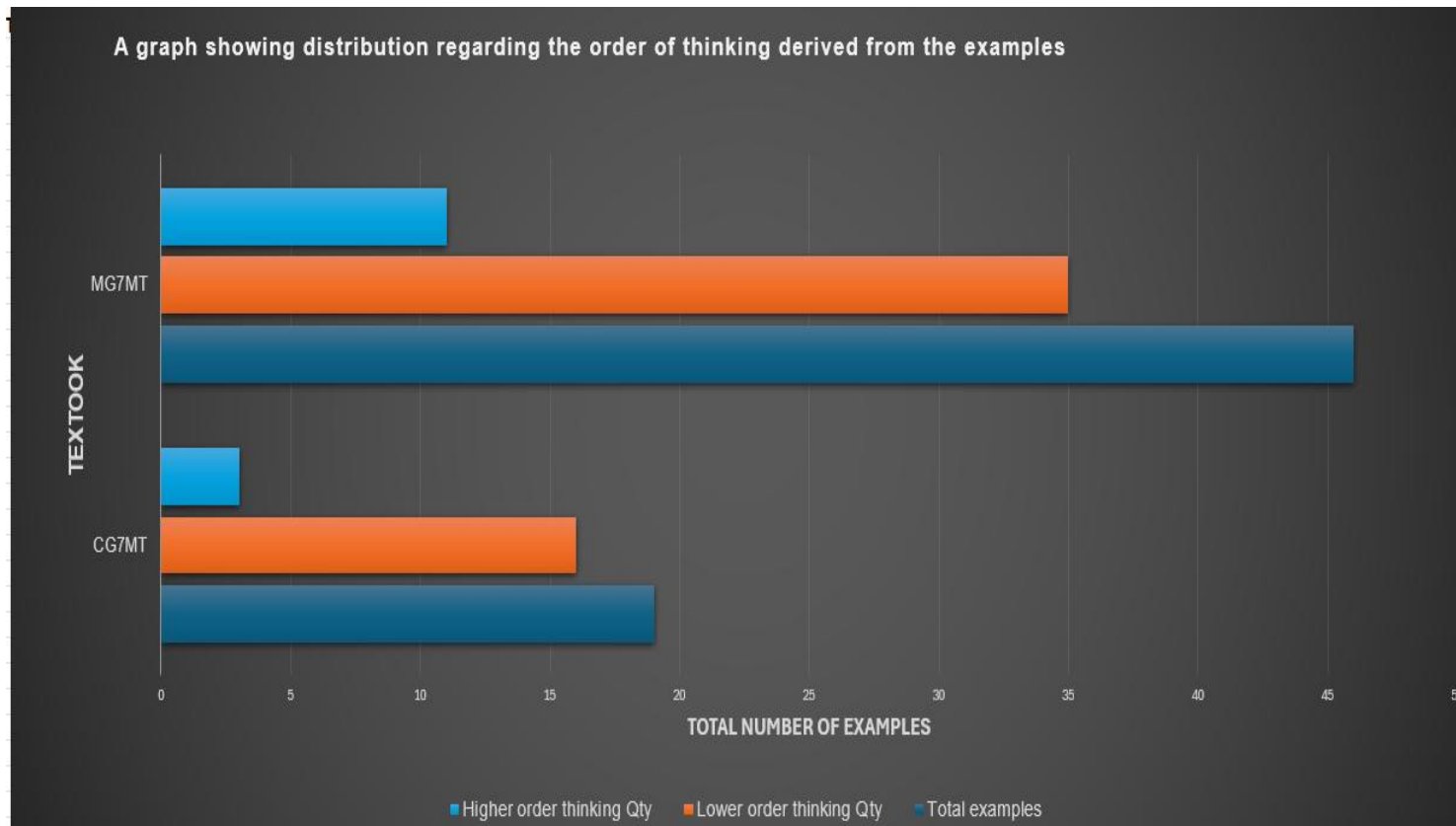


Figure 4.5 shows the horizontal bar graph of distribution regarding the order of thinking derived from the examples.

This graph in figure 4.5 shows the distribution in terms values of examples between the two textbooks in terms of the level of thinking of examples against the total examples in the two textbooks. This graphical representation shows MG7MT have more examples as compared to CG7MT in terms of total number of examples.

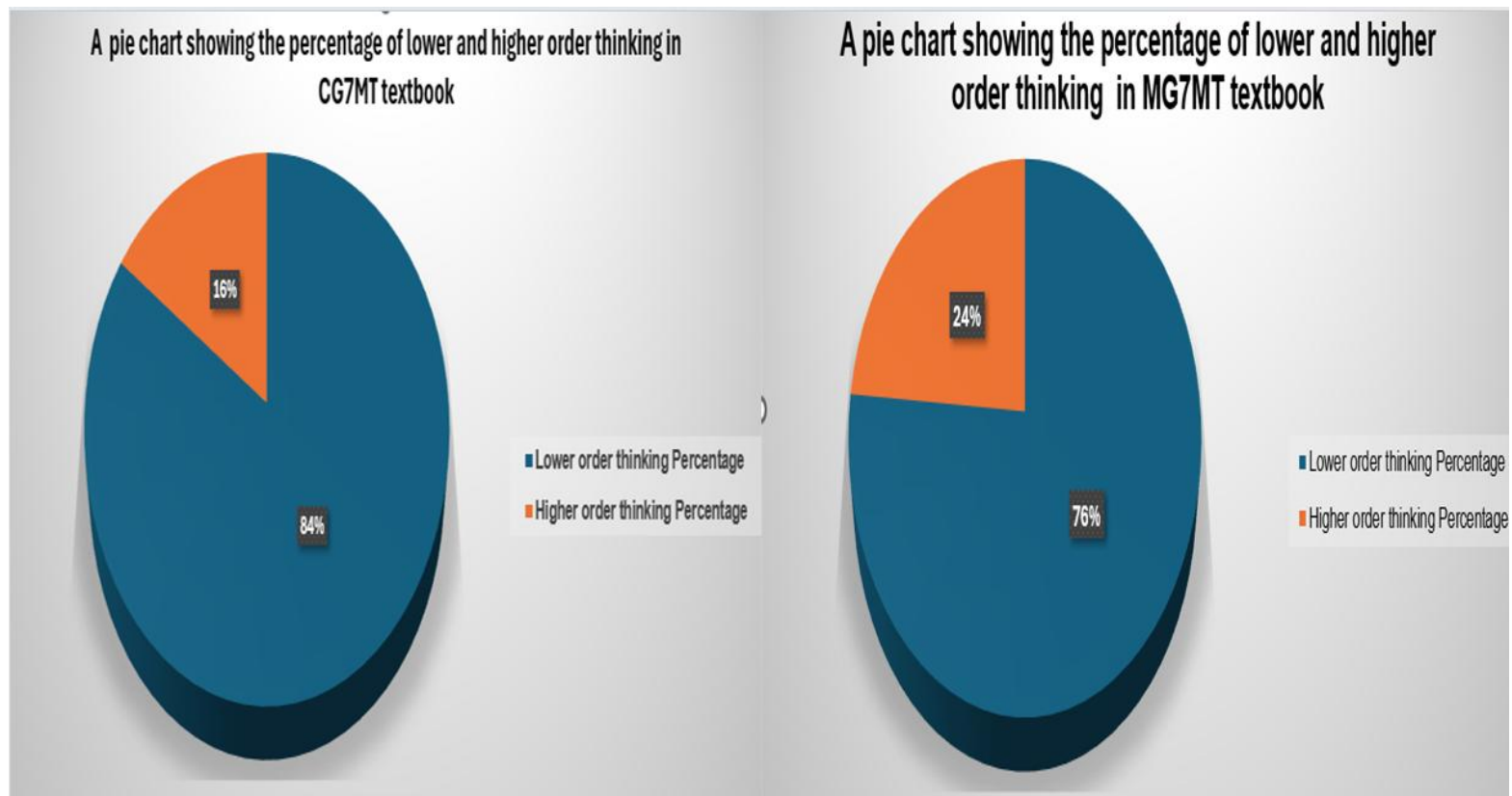


Figure 4.6 shows the pie chart of distribution regarding the order of thinking derived from the examples. The pie chart above gives an illustration of the percentages of examples as far as order of thinking. MG7MT has a higher percentage of HOT examples 24% percent that CG7MT with 16% examples being HOT.

4.6 Qualitative findings to answer research sub-question 3.

The data collection tool had a section examining the types of examples. Table 4.1 on page 38 presents the frequency of each type of example. The findings shows that there are 3 real-life examples in each textbook. As such both textbooks have the same number of real-life examples even though MG7MT has more than double the number of examples contained in CG7MT. Figure 4.3 on page 39 shows the bar graph of examples and how real-life examples are used in the two textbooks as compared to other types of examples. As discussed in the theoretical framework, real-life examples are important in concept development but both textbooks have less amount of real-life examples. There maybe a need to include more of these in textbooks.

4.7 Conclusion

This chapter provided an insight into the research findings in terms through presentation of data gathered through the data collection instrument. The data that was collected through the data collection instrument was presented using tables and graphs. There was also a brief discussion about the qualitative elements of the findings. The next chapter will examine and discuss these findings in an attempt to answer the research questions.

CHAPTER 5: DISCUSSION RELATED TO THE MAIN FINDINGS

5.1 Purpose of chapter

The purpose of this chapter is to discuss the research findings of this study. A discussion in the context is about giving meaning to the data presented and the findings of the study. In Chapter 3 there was presentation of the research design leading to the development and presentation of a research instrument used to collect qualitative data about analysis of examples in geometry. In chapter 4 the data collected was presented and analysed in detail to get some trends and patterns about examples contained in the textbooks. The aim of this process is to expand the scope study by providing in-depth discussion which enables creation of perspectives about examples in the section of geometry as presented in the two textbooks. This also presents opportunities for coming up with value judgement about example spaces based on literature and findings of the research study. The discussion will include findings in an attempt to address the main research question, discussions related to research sub-questions 1 to sub-question 3 as well as some concluding remarks.

5.2 Discussion related to the main findings

The following was this study's main research question. What are the cognitive demands addressed in examples contained in the lower secondary (Grade 7) textbooks in geometry?

As noted in chapter 4,, MG7MT has more examples than CG7MT hence the distribution of examples as per the four cognitive demands have a higher distribution MG7MT. The presentation of sections of geometry is done in a similar chronological order to the CAPS document. It could be noted that of the 46 examples in MG7MT, 16 falls under memorization examples, 19 under procedures without connection examples, 8 fall under procedures with connections examples and 2 fall under doing mathematics examples. On the other hand CG7MT contains 19 examples in the section of geometry, with 12 being memorisation examples, 4 are procedures with connection examples, 2 fall under procedures without connection and 1 is doing mathematics example. It can be concluded therefore that MG7MT provides more examples per each cognitive demand level as compared to CG7MT. In general, both textbooks have more LOT examples being memorization and procedures without connection examples.

5.3 Discussion related to research sub-question 1

The first sub-question was as follows. What are the levels of cognitive demands that are addressed in examples in the textbooks CG7MT and MG7MT in geometry?The process

involved analysing examples contained in CG7MT and MG7MT in the process examining the levels of cognitive demands of these examples based on the theoretical framework derived from a combination of Michener (1979) types of examples and Stein et al.(2000) cognitive demands of tasks. As such the types of examples and the cognitive demands of these examples including the order of thinking required to solve the problem were analysed. It was observed from the findings that MG7MT have more examples, 46 examples as compared to CG7MT with 19 examples in geometry. If example spaces was to be used as a possible measure of affordances and learning opportunities that textbooks present to learners, then MG7MT would be considered as offering better teaching and learning opportunities than CG7MT. The fact that MG7MT has more examples might mean that the textbook enables presentation of a variety examples with different cognitive demands, a wide variety of the types of examples and higher quantity of examples responding to the order of thinking which might benefit learners more as discussed in chapter 2.

5.4 Discussion related to research sub-question 2

This research sub-question 2 sort to find out the quantity and quality of examples in the 2 textbooks. The research sub-question was, Are the examples in the selected mathematics textbooks, that support the concept development in geometry, sufficient in terms of quantity and quality?. In terms of quantity, CG7MT has 19 examples while MG7MT has 46 examples in geometry. This shows that in terms of quantity, MG7MT have more examples that supports concepts development geometry as compared to CG7MT. Furthermore both textbooks have very little amount of doing mathematics examples which are meant to extend learners in terms of mathematical thinking and mathematics problems solving. When examining quality, we can extrapolate the findings of the order of thinking of these examples. The data presented in chapter 4 shows that, on one hand, 84% of CG7MT are LOT examples while 76% of MG7MT are LOT examples. On the other hand 16% of CG7MT examples and 24% of MG7MT examples are HOT examples respectively. It can be noted from these findings that MG7MT has a greater percentage of HOT examples which may mean it provides challenging examples to learners in geometry.

5.5 Discussion related to research sub-question 3

Research sub-question 3 was concerned with the inclusion of real-life examples in these textbooks. The research question was, To what extent are real-life examples included in the selected textbooks (CG7MT and MG7MT) in geometry? The results of examples analysis in

chapter 4 showed that out of the 46 examples from MG7MT, 3 examples (7%) were real-life examples and out of 19 examples contained CG7MT, 3 examples (16%) were real-life examples. This shows that MG7MT seems to place less emphasis on real-life examples than the CG7MT based on these results. In general, the quantity of real-life examples seems to be less given the importance of real-life examples in geometry as discussed in literature review section of this study.

5.6 Concluding remarks

This chapter provided deep insights of the findings as the findings were juxtaposed with the research questions. The chapter explored and attempted to answer the research questions through the analysis of the data presented as per findings in chapter 4. Generally, all research questions were answered using the data presented in chapter 4 giving greater insights into examples spaces in these 2 textbooks in geometry. The next and final chapter will focus on the summary and recommendations.

CHAPTER 6: SUMMARY, LIMITATIONS, RECOMMENDATIONS AND CONCLUSIONS

6.1 Introduction

The purpose of this chapter is to give incites at a glance regarding this research study. It illustrates on the overview of the study an summarises the main findings of the study. The limitations of this study were elaborated. Since this study focused on the analysis of textbooks, some recommendations to authors were presented aimed at improving the process of textbooks presentation in mathematics in geometry. This chapter also observed areas that are related to this study that may require future studies to extend the body of knowledge about mathematics textbooks regarding geometry. Finally, some interesting facts about this study were noted and the chapter was concluded through some final parting word regarding this study.

6.2 Overview of study

The study examined the cognitive demands of examples presented in two Grade 7 mathematics textbooks (CG7MT and MG7MT) in geometry. All examples in geometry from these textbooks were analysed. Sections of analysis of examples focused on the types of examples, the cognitive demands of the examples and the order of thinking required to solve the mathematical problem presented in the examples. The examples were analysed through a data analysis tool developed for the purpose of analysing the data. The data collected was presented through tables and graphs to provide greater insights into the example spaces in geometry as far as these textbooks are concerned.

6.3 Summary of main findings

The main findings showed that the two textbooks have some similarities and fundamental differences in terms of their approach to exemplification in mathematics in geometry. Both textbooks have more start-up and referent examples when examining quantity and frequency of use.. As discussed in Chapter 2 these examples form the basis of the mathematics concept development process. As such they form the highest number of examples from both textbooks. However, both textbooks have more LOT examples than HOT examples. The ability to gain the knowledge through examples required to answer HOT questions may be low as there are fewer examples addressing this order of thinking.

A point to note in this study was that the researcher managed to analyse all examples in geometry from both textbooks. The vast difference between the two textbooks in terms of the quantity of examples was not expected. Further, it was noted that none of the textbooks had

counter examples presented in geometry. It was also noted that CG7MT presented a theoretical section or a brief writeup especially at the beginning of the topic which indicated point(s) to note about the section being covered which gave some insights to the learners regarding the topic being covered.

6.4 Limitations of study

This research study focused on an analysis of only two textbooks, due to practical reasons, yet there are so many textbooks in use in mathematics at Grade 7 level. These two textbooks may not adequately reflect the position of other textbooks. It could have been ideal to analyse more textbooks to give a broader overview, but given time constraints, this could not be possible. The research report was done in a year which made it difficult to engage in analysis of more than one textbook.

The analysis of examples focused on analysis of examples in geometry. As the analysis focuses on examples only, the findings may not be used to a greater extent to generalise the affordances and learning opportunities presented by the textbooks. Furthermore, the analysis focused on one section only, geometry. It would have been ideal to cover other mathematics sections to bring about more convincing generalisations about the textbook content.

6.5 Recommendations

It is important for textbooks to have sufficient quantity of examples of high quality for textbooks to helpful artefacts in the teaching and learning of mathematics. Such examples should cover the various types of examples. Real-life examples should have a higher quantity in mathematics sections like geometry to allow learners to relate mathematics to their lived experiences. This enables learners to grasp abstract concepts as they can relate the examples to their life which fosters better understanding of mathematics concepts. Finally, authors of CG7MT and MG7MT are encouraged to extend the teaching and learning of mathematics by presenting more HOT examples in geometry. Such examples will enable learners to refer to these examples especially in extension questions or more challenging questions which are relatively difficult and require higher order thinking to attempt these questions.

Based on the research process, the data collection and the results of the study, it will be worthwhile to do further study into analysis of examples in geometry contained in other textbooks apart from CG7MT and MG7MT. Further studies can also be done on analysis of examples but focusing on all the other sections done in Grade 7 mathematics.

6.6 Final word

This study provided a glimpse into the example spaces in geometry at lower level secondary schooling through analysis of examples in CG7MT and MG7MT. This study provided great insights into the examples contained in these textbooks giving detail about the affordances they present in the teaching and learning of mathematics. The results of the analysis enabled the researcher to understand the nature of examples based on quantity and the cognitive demands required to solve these examples. There is room for future research on this topic and the researcher aims to explore further on textbook analysis in pursuit of his future studies.

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ANNEXURE A DATA COLLECTION INSTRUMENT

Textbook		Section of geometry	Page	Types of examples					Cognitive Demands				Thinking	
CG7MT	MG7MT			S1	R2	M3	C4	RL5	MT (B)	PC+(C)	PC-(D)	DM(E)	LOT	HOT
	X	Construction of angles Ex 1	5	X					X				X	
	X	Construction of angles Ex 2	12	X					X				X	
	X	Construction of angles Ex 3	15	X						X			X	
	X	Geometry of straight lines Ex 1	22	X					X				X	
	X	Geometry of straight lines Ex 2	38	X					X				X	
	X	Geometry of straight lines Ex 3	38		X					X			X	
	X	Geometry of angles Ex 4	42		X					X			X	
	X	Geometry of triangles Ex 1	47	X					X				X	
	X	Geometry of triangles Ex 2	47		X					X			X	
	X	Geometry of triangles Ex 3	51		X					X			X	
	X	Geometry of triangles Ex 4	55	X					X				X	
	X	Geometry of triangles Ex 5	55		X					X			X	
	X	Geometry of quadrilaterals Ex 6	64	X					X				X	
	X	Geometry of quadrilaterals Ex 7	64	X					X				X	
	X	Geometry of quadrilaterals Ex 8	64		X					X			X	
	X	Geometry of quadrilaterals Ex 9	65		X						X			X
	X	Geometry of quadrilaterals Ex 10	66	X					X				X	
	X	Geometry of circles Ex 11	77					X			X			X
	X	Geometry congruency Ex 12	81		X					X			X	
	19			10	8	0	0	1	9	8	2	0	17	2

Textbook		Section of geometry	Page	Types of examples					Cognitive Demands				Thinking	
CG7MT	MG7MT			S1	R2	M3	C4	RL5	MT (B)	PC+(C)	PC-(D)	DM(E)	LOT	HOT
	X	Geometry similarity Ex 13	81		X					X			X	
	X	Geometry of polygons Ex 14	95	X					X				X	
	X	Perimeter of 2-D shapes Ex 1	98	X					X				X	
	X	Perimeter of 2-D shapes Ex 2	99		X					X			X	
	X	Area of 2-D shapes Ex 3	104		X					X			X	
	X	Area of Triangles Ex 4	107		X					X			X	
	X	Area of Triangles Ex 5	107			X					X			X
	X	Conversion of units Ex 6	111	X					X				X	
	X	Problem solving geometry Ex 7	114			X					X			X
	X	Problem solving geometry Ex 8	114			X						X		X
	X	Problem solving geometry Ex 9	118		X					X				X
	X	Problem solving geometry Ex 10	122		X					X			X	
	X	Problem solving geometry Ex 11	122			X					X			X
	X	Surface Area Ex 1	127	X					X				X	
	X	Surface Area Ex 2	127		X					X			X	
	X	Volume Ex 3	140		X					X			X	
	X	Volume & capacity problems Ex 4	145					X		X			X	
	X	Volume & capacity problems Ex 5	145					X			X			X
	X	Volume & capacity problems Ex 4	146			X						X		X
	19			4	8	5	0	2	4	9	4	2	12	7

Textbook		Section of geometry	Page	Types of examples					Cognitive Demands				Thinking	
CG7MT	MG7MT			S1	R2	M3	C4	RL5	MT (B)	PC+(C)	PC-(D)	DM(E)	LOT	HOT
	X	Translations Ex 1	152	X					X				X	
	X	Lines of symmetry Ex 2	155	X					X				X	
	X	Reflections Ex 3	159		X					X			X	
	X	Rotations Ex 4	162	X						X			X	
	X	Rotational symmetry Ex 5	165		X				X				X	
	X	Rotational symmetry Ex 6	165			X					X			X
	X	Enlargement Ex 7	170		X				X				X	
	X	Enlargement Ex 8	171			X					X			X
	8			3	3	2	0	0	4	2	2	0	2	2
	46			17	19	7	0	3	17	19	8	2	35	11

Textbook		Section of geometry	Page	Types of examples					Cognitive Demands				Thinking	
CG7MT	MG7MT			S1	R2	M3	C4	RL5	MT (B)	PC+(C)	PC-(D)	DM(E)	LOT	HOT
X		Measuring angles	41					X	X				X	
X		Angles on a straight-line	60	X					X				X	
X		Area and perimeter	235					X	X				X	
X		Area and perimeter	243		X				X				X	
X		Conversion	248		X				X				X	
X		Area of triangle	254			X			X				X	
X		Enlargement Ex 7	170		X				X				X	
X		Enlargement Ex 8	171			X					X			X
X		General geometry equations Ex 12	247					X				X		X
X		Surface Area and Volume Ex 1	255	X					X				X	
X		Surface Area and Volume Ex 2	256	X					X				X	
X		Surface Area and Volume Ex 3	258		X					X			X	
X		Volume and capacity Ex 4	262		X					X			X	
X		Volume and capacity Ex 5	264		X					X			X	
X		Scale factor Volume and capacity Ex 6	266			X					X			X
X		Transformations translations Ex 1	311	X					X				X	
X		Transformations translations Ex 2	312		X				X				X	
X		Transformations reflection Ex 3	315	X					X				X	
X		Transformations reflections Ex 4	316		X					X			X	
19				5	8	3	0	3	12	4	2	1	16	3

ANNEXURE B ETHICS CLEARANCE WAIVER



SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE

PROTOCOL NUMBER: 2024ECE040M

PROJECT TITLE

An analysis of the cognitive demands of examples presented in Grade 7 textbooks in geometry.

INVESTIGATOR

Dereck Donora.

SCHOOL/DEPARTMENT OF INVESTIGATOR

Wits School of Education.

DATE CONSIDERED

25 July 2024.

DECISION OF THE COMMITTEE

Approved unconditionally.

RISK LEVEL

No Risk.

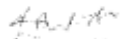
EXPIRY DATE

Date of submission of the Research Report.

ISSUE DATE OF CERTIFICATE

25 July 2024.

CHAIRPERSON


Dr. Lawan Abdulhamid

cc: Dr. Rina Durandt.

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.


Signature

22, 08, 2024
Date