

https://dspace.org/

Investigating challenges that Grade 11 mathematics learners face when translating from word problems to linear algebraic representations

Madzorera, Andrew

2015-05-06
http://hdl.handle.net/10539/17633

Downloaded from a DSpace repository. https://dspace.org/

FACULTY OF SCIENCE
UNIVERSITY OF THE WITWATERSRAND
JOHANNESBURG



Investigating Challenges that Grade 11 Mathematics Learners Face when Translating from Word Problems to Linear Algebraic Representations

By

Madzorera Andrew

Student No: 509288

A Research Report Submitted to the Faculty of Science, University of Witwatersrand, Johannesburg, in Partial Fulfilment of the Requirements for the Degree of Master of Science

Supervisor: Dr Anthony A. Essien, (PhD)

December 2014

Declaration

I declare that this research report is my own, unaided work. It is being submitted for the Degree of Master of Science in the University of Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.



..... Date..... December 2014

Signature.

Abstract

The National Curriculum and Assessment Policy Statement document (CAPS) (Department of Basic Education, 2012: 4) outlines some of the salient skills related to the modelling of word problems that Mathematics learners are expected to acquire during the course of their learning. One such critical skill is the learners' ability to represent and describe situations in algebraic language, formulae and expressions. In line with this broad objective, the present study investigated learners' linguistic knowledge and skills in translating from word to linear algebraic representations. Particularly focussing on errors learners made during the translation process, a class of 40 Grade 11 learners at one school in Gauteng West District were selected and given a written test on word problems. To gain access to learner misunderstandings leading to the observed errors and their sources, five learners were selected for interviews that were audio recorded. A Mixed-method Sequential Explanatory Design was used in the study. The major finding of this present study was that learners committed more semantic errors than syntactic and schematic errors suggesting that they (learners) had challenges in understanding the language used in the word problems. The major sources of semantic errors were found to be learners' lack of vocabulary knowledge, inexperience in interacting with expository text structures and their lack of syntax awareness. In addition to the above finding, learners exhibited limited metacognitive skills essential in word problem solving as well as their inability to reflect on the appropriateness of their written algebraic representations for all categories of word problems. The study concludes by providing recommendations on how errors committed in word problem solving can be attended to.

Acknowledgements

There goes a saying *'It is indeed a long night that will never see the day'*. This briefly summarises the long nights that I spent fruitfully trying to come to terms with the demands associated with writing a research report. However, the above being a reality, I have pleasant words for the following brilliant individuals who stood by my side in this long journey of researching and writing an academic report.

Firstly, my heartfelt thanks and gratitude goes to my supervisor Dr Anthony. A Essien, (PhD). Your persistence and words of encouragement helped me up to the end. To sum it up, I really enjoyed working with you and I hope it is not the last time.

Secondly, to the principal of Thuto-Lehakwe Secondary School Mr D Ngobeni who provided me with moral support and allowed me to carry out the study in his institution, I want to say thank you very much for your unwavering support, patience and tolerance with me.

Thirdly, my deep and heartfelt thanks go to my wife Thandanani, who though a thousand miles away, always reminded me that 'when in a storm, though fearsome, it would not last forever'. I now believe you. Together with our new baby Michelle, I salute you for your love, patience and tolerance.

Lastly, I want to thank all the staff members at the University of Witwatersrand for their moral support.

Table of Contents

Declaration ii

Abstract iii

Acknowledgement iv

Table of Contents v

List of Figures viii

List of Tables ix

CHAPTER ONE 1

INTRODUCTION TO THE STUDY 1

1.1 Problem Statement 1

1.2 Why the Focus is on Word problems? 2

1.3 Significance of the Study 3

1.4 Definition of Terms 4

1.5 Structure of the study 5

1.6 Conclusion 6

CHAPTER TWO 7

CONCEPTUAL FRAMEWORK AND LITERATURE REVIEW 7

2.1 Introduction 7

2.2 Conceptual Framework 7

2.2.1 Problem Representation Stage 8

2.2.2 Problem Solution stage.....9

2.2.3 Connecting Mayer's (1985; 1999) Theoretical Model to the Study 10

2.3 Review of Literature 11

2.3.1 The Nature of Mathematical Language 11

2.3.2 Non-technical and Technical Vocabulary 13

2.3.3 The role of Symbolism in School Mathematics 15

2.3.4 Communication and Mathematical Language 17

2.4 The Role of Reading Comprehension in Answering Word Problems 18
2.4.1 The Role of Prior Knowledge in Word Problem Solving 20
2.4.2 Role of Conceptual Understanding and Procedural Knowledge in Algebra 21
2.4.3 Role of Metacognition in Problem Solving.....22
2.4.4 Errors in Translating Linear Equations 23
2.5 Conclusion 24
CHAPTER THREE 26
RESEARCH DESIGN AND METHODOLOGY 26
3.1 Introduction 26
3.2 Mixed-Method Sequential Explanatory Design 26
3.3 Data Collection Site 26
3.3.1 Participants 26
3.3.2 Data Collection Methods 28
3.3.3 The Test.....28
3.3.4 The Interview.....29
3.3.5 Criteria Used in Selecting Learners for Interviews.....30
3.4 Pilot Study 30
3.4.1 Instructions to Participants 31
3.4.2 Choice of Questions Selected 32
3.4.3 Rationale for Grouping word Problems into Categories 32
3.5 Data Analysis 33
3.5.1 Analysis of Learners' Performance on Multiple-Choice Questions 34
3.5.2 Error Analysis – Open-ended Questions 34
3.6 Definition of Error Descriptors 35
3.6.1 Category E1 - Syntactic Errors.....35
3.6.2 Category E2 - Semantic Errors.....35
3.6.3 Category E3 - Schematic Errors.....36

3.6.4 Procedures Undertaken in Analyzing Open-ended Questions 36
3.7 Phases of Data Analysis Observed in the Study 37
3.8 Rigor 37
3.8.1 Ethical Issues 39
3.8.2 Overview of the Data Collection Plan 39
3.9 Conclusion 40
CHAPTER FOUR 41
DATA ANALYSIS AND FINDINGS 41
4.1 Introduction 41
4.2 Analysis of Errors in Multiple-choice Questions 41
4.2.1 Source of Errors: (1) Learners' Lack of Vocabulary Knowledge.....43
4.2.2 Source of Errors: (2) Learners' Lack of Symbolic Knowledge 44
4.2.3 Source of Errors: (3) Learners' lack of Meta-cognitive Skills 47
4.3 Classification of Incorrectly Written Algebraic Equations into Error Categories.....48
4.3.1 Errors and their Sources [De-contextualised word problems] 50
4.3.1.1 Errors and their Sources in Interpreting Mathematical Texts.....50
4.3.1.2 Learners' understanding of Words *'of', 'less', 'and'* and *'product'*.....51
4.3.1.3 Learners' Use of Word-order-Matching Strategy.....56
4.4 Classification of Incorrectly Written Algebraic Equations into Error Categories.....59
4.4.1 Errors and their Sources ['Additive' compare word problem] 60
4.4.1.1 Learners' Understanding of Words *'more than*' and *'older than'*.....61
4.5 Classification of Incorrectly Written Algebraic Equations into Error Categories.....65
4.5.1 Errors and their Possible Sources ['Multiplicative' compare word problems] 66
4.6 Overall Summary of Results 70
4.7 Conclusion 73
CHAPTER FIVE 74
CONCLUSIONS AND RECOMMENDATIONS 74

5.1 Introduction 74
5.2 Summary of Main Findings.....74
5.3 Recommendations 77
5.3.1 Future Research 79
5.3.2 Reflections 80
5.3.3 Limitations of the Study 80
5.4 Conclusion 81
References 82
APPENDIX A: Research Instrument 94
APPENDIX B: Categories of questions selected for analysis 96
APPENDIX C: Content analysis of each selected question item.....97
APPENDIX D: Student response categories for selected word problems 98
APPENDIX E: Distribution of learners' responses to selected multiple-choice questions.....99
APPENDIX F: Expected responses and (%) of learners with correct responses.....99
APPENDIX G: Interview excerpts 1 - 8 100
APPENDIX H : Interview excerpts 9 - 15 106
APPENDIX I : Interview excerpts 16 - 18.....112
APPENDIX J: Interview excerpts 19 - 20 115
APPENDIX K: Study Consent Documents.....117
GDE Research Approval letter.....117
Letter of consent from the principal.....119
Ethics clearance (University of Witwatersrand).....120
Information Sheet for Parents.....121
Parents' Informed Consent.....122
Consent Form Learners' Interview.....123
Parents' Informed Consent for audio-recording……..…….....….124
Consent Form for Learners Documents.....125

List of Figures

Chapter 2

Fig 2.1: Stages in algebra word problem solving.....8

Chapter 3

Fig 3.2: Two phases of data analysis.....37

Chapter 4

Fig 4.3: Percentage of errors in each category of word problem.....72

List of Tables

Chapter 2

Table 2.1: Summary of knowledge types essential in problem solving stages.....10

Chapter 3

Table 3.2: Research Questions and Sources of Data.....28

Table 3.3: Error Categories Identified from Typologies.....35

Table 3.4: A summary of data collection plan.....40

Chapter 4

Table 4.5: Sample of selected multiple-choice questions under Category A.....42

Table 4.6: Classification of incorrectly written algebraic equations into error categories.....49

Table 4.7: Summary of error subcategories and their possible sources.....59

Table 4.8: Classification of incorrectly written algebraic equations into error categories.....60

Table 4.9: Summary of errors subcategories and their possible sources.....65

Table 4.10: Classification of incorrectly written algebraic equations into error categories.....66

Table 4.11: Summary of error subcategories and their possible sources.....69

Table 4.12: Frequency of errors per category of word problems as a percentage.....70