THE ROLE OF THE GRAPHIC CALCULATOR AS A MEDIATING SIGN IN THE ZONES OF PROXIMAL DEVELOPMENT OF STUDENTS STUDYING A FIRST-YEAR UNIVERSITY MATHEMATICAL COURSE

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DECLARATION

I declare that this research report is my own, unaided work. It is being submitted for the Degree of Master of Science in Education in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

(Signature of candidate)

28th day of May, 1996

ABSTRACT

This study explores the ways in which first-year Mathematics students use the graphic calculator as a tool of semiotic mediation. Twenty students out of a class of one hundred were loaned a graphic calculator for the academic year and encouraged to use these during support tutorials. At year-end, seven students (four with graphic calculator, three without) were audio-taped while solving a mathematical problem aloud in an interview situation. Also statistical data comparing graphic calculator and non-graphic calculator students' performance on a set of five questions was collected.

The qualitative analysis of the interview data suggests that the calculator functioned primarily as a tool which amplified the zones of proximal development of the students, increasing efficiency and speed, rather than as a semiotic system which had been internalised. The quantitative analysis of the statistical data failed to support this notion of amplification. It is suggested that the add-on status of the graphic calculator undermined the possibility for statistical significance on this amplification effect.

In memory of my parents

Boetie and Lily Berger

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CONTENTS

1 INTRODUCTION

1.1 THE CONTEXT

Mathematical courses, even at a tertiary level, are traditionally taught using algebraic representations and algorithmic methods. This can result in a mechanical approach to mathematics in which the student performs the appropriate algorithmic steps without particularly understanding the mathematical concepts behind the algebra. In the wider context this is particularly problematic in that people with problem-solving skills and mathematical flexibility are needed to develop and nurture science and technology in South Africa today.

In recent years a new philosophy stressing the importance of multiple representations (numeric, symbolic and graphical) in the understanding and manipulation of mathematical concepts has become prevalent (Hughes-Hallett, 1991; Confrey and Smith, 1993, Porzio, 1994). In particular many mathematics educators have argued for the greater use of graphs within the multiple representation environment. "Graphical techniques and explorations provide a whole new language with which to impart rich meaning to the central concepts of calculus" (Dick, 1992: 149). Zimmermann and Cunningham (1991) maintain that mathematical visualisation leads to "the kind of intuition which penetrates to the heart of an idea. It gives depth and meaning to understanding, serves as a reliable guide to problem solving, and inspires creative discoveries" (1991:4). They argue that this visualisation should not take place in an isolated environment but rather that

> "visual thinking and graphical representations must be linked to other modes of mathematical thinking and other forms of representation. One must learn how ideas can be represented symbolically, numerically and graphically, and to move back and forth among these modes " (p. 4).

This stress on the pedagogical virtues of the multiple representation environment is to a large extent facilitated and driven by developments in technology. After all, this is the era of computer-generated fractal images and three-dimensional curves, animated or static. But computers are expensive, unwieldy and may be difficult to use.

Fortunately, simpler, smaller and less expensive technological tools such as the graphic calculator have been developed. This calculator facilitates the translation from one representation to another by providing easy to use graphing, numeric and to a lesser extent, symbolic manipulation utilities. Unlike the computer the graphic calculator is affordable, portable, accessible and does not require electricity. It thus represents a potentially appropriate technology for the South African environment.

1.2 THE NEED FOR THE RESEARCH

The evidence for the use of the graphic calculator as a tool for learning is ambivalent. There have been various studies (Ruthven, 1990; Shoaf-Grubbs, 1993) which have pointed to the benefits of the graphic calculator in the learning and development of mathematical concepts, but there is also research which focuses on the problems inherent in the graphical environment. Mathematical educators such as Goldenberg (1988) note that graphs have conventions and ambiguities of their own. These can be a source of confusion rather than clarity if the student is insufficiently skilled in interpreting these conventions. Giamati (1991) found evidence that the use of the graphic calculator over a short-term period, does not necessarily assist in the development of conceptual connections between graphs and equations. In fact, she found that students who had a poorly and partially formed understanding of these conceptual links were cognitively distracted by also having to learn how to use the graphic calculator.

These different perspectives lead to the question: is the graphic calculator an appropriate and possible tool for learning mathematics at a tertiary level in South Africa? How does the use of the calculator effect the individual student? This research will be concerned with these questions.

In addition, a review of the literature shows that the majority of articles concerning the graphic calculator are primarily anecdotal (for example, Demana & Waits, 1993) or principally concerned with quantitative data linking the performance in tests and examinations to the use of the graphic calculator (for example, Quesada, 1993). Kaput, talking about technology in general, refers to a "paucity of publishable research" (1994: 680). Other than Shoaf-Grubbs (1993) and Jones (1993),

> "most studies..[on the graphing calculator]...have been descriptive; they tell us *what* happens in classrooms equipped with graphing calculators. For research effectively to guide curriculum development and instruction, we need to find out *why*" (Dunham & Dick, 1994: 443).

In line with this philosophy, this study will explore the nature of the interaction between the learner and the graphic calculator, locating the exploration within a Vygotskian paradigm.

1.3 RESEARCH OBJECTIVES AND QUESTIONS

The broad objective of this study is to explore and describe the ways in which the graphic calculator acts as a mediating sign in the zones of proximal development of first-year University students in the Mathematics I (major) course at the University of the Witwatersrand.

In order to adequately address this concern and to give direction to the exploration and description, the study focuses on the following issues:

- ♣ The way in which the use of the graphic calculator effects the mathematical conceptions and manipulations of students.
- ♣ Pea's (cited in Salomon, 1990) distinction between amplification and cognitive re-organisation is applied and re-interpreted in Vygotskian terms. The question is asked: Is the graphic calculator primarily a technology which amplifies the zone of proximal development by liberating the student from menial and mundane operations so that he/she is now more free to engage in higher-order thinking? Or does the use of the graphic calculator contribute to the re-organisation of the internal plane of consciousness so that a student with access to this technology shows a change in performance as a *result* of using the graphic calculator rather than just *when* using the tool.

Additionally, and to give depth to this inquiry, the research asks the following questions:

- ♣ What is the difference between the activity of hand-drawing a graph on paper and generating a graph on the graphic calculator?
- ♣ What are the implications of each of these activities for mediation?
- ♣ How does the students' ability to generate graphs on the calculator *before* he/she fully understands how to derive or analyse the function contribute to the power of the graphic calculator as a tool for semiotic mediation?

Furthermore the empirical study aims to contribute to the understanding, within a Vygotskian framework of the use of technology in mathematical courses.

1.4 OUTLINE OF THE RESEARCH REPORT

Since Vygotskian theory underpins and informs this research, Chapter Two of this report gives a broad overview of the relevant Vygotskian principles and themes. Consistent with Vygotsky's approach, the Vygotskian themes are compared and contrasted with other learning theoretic positions in particular those deriving from Piaget. This comparison is used to justify the setting of this study in a Vygotskian framework. In addition, certain extensions and elaborations of Vygotskian theory, such as those developed by Wertsch and Leont'ev, which contribute to a meaningful interpretation of his writings, are described in this chapter. Since many of these neo-Vygotskian concepts became particularly relevant during the development of this research, some of these themes are not mentioned in Chapter Two but rather in the body of the text. This is in accord with the genetic method of Vygotsky which asserts that processes and concepts are best understood in terms of their history.

In Chapter Three I give a broad overview of current research (up until the beginning of 1995) into the use of graphic calculators. In particular I focus on that research which relates to the relationship of the graphic calculator to cognitive processes and its use as a pedagogical tool. Since most of this research is anecdotal or descriptive, I discuss some possible reasons for the non-theoretical nature of so much contemporary research.

In Chapter Four I outline the methodology and method of my study. In particular I justify the predominant use of the qualitative paradigm. I then describe the setting up and execution of the graphic calculator project which forms the core of this research. To this extent I discuss the Mathematics I course at the University of the Witwatersrand within which the project was situated.

Two sets of data, qualitative data from Interviews and quantitative data from a Task were collected during the course of this research. These sets of data are also described in Chapter Four: the process whereby the method for data collection evolved with respect to both types of data as well as the format of each data-collecting activity is outlined. Additionally the framework for the analysis of the qualitative data from the interviews is developed. Since the form and protocol of the interview was suggested by the work of the Soviet psychologist, A.R. Luria, who interviewed peasants in Uzbekistan and Kirghizia in order to demonstrate how the structure of thought relates to the dominant activity within a particular culture, this source of inspiration is acknowledged and discussed. Finally, Chapter Four gives a description of the Task in terms of the content of each of the five questions of which it is comprised.

Chapter Five is devoted to the analyses of the interviews of seven students (four with graphic calculator, three without). Each analysis has two phases: an analysis of the transcript of the interview in tabular form using categories relating to the sign, tool or agent of mediation, and an interpretation of these analytic tables. Since the tables ultimately are used as the data from which the interpretations flow, they are placed in Appendix C. Thus Chapter Five consists of analytic interpretations of each interview where the interpretation is grounded in the empirical data generated by the analytic tables.

In Chapter Six a statistical analysis of the performance data from the task is summarised and presented in tabular form.

Chapter Seven represents the heart of this research report. In this chapter the way in which the graphic calculator acts as a tool for semiotic mediation is critically discussed using Vygotskian and Wertschian concepts. The data from the interviews and the task is used as support and evidence for the theoretical discussion. In particular, attention is paid to the institutional and cultural setting of this study and the influences of the particular context of this research.

Since my conclusions are sometimes implicit in the discussion, a summary of the main points from the discussion is presented in Chapter Eight.

2 THEORETICAL FRAMEWORK

2.1 INTRODUCTORY REMARKS

In order for this study to meaningfully contribute towards an understanding of the role of the graphic calculator in the learning and teaching of mathematics, it is necessary to situate the research within a learning theoretic context. For reasons which are detailed below, I will use ideas and methods which derive mainly from a Vygotskian paradigm. Where relevant, elaborations and extensions of Vygotsky's ideas as developed by his colleagues or more modern theorists are also applied and used. Also, in keeping with the Vygotskian tradition, different learning theories, in particular those deriving from Piaget, are investigated so as to provide a backdrop against which Vygotsky's theories can be assessed.

Many of the ideas and concepts used throughout this research report, albeit they derive from a Vygotskian framework, evolved during the course of the investigation. Consistent with the genetic method of Vygotsky which asserts that processes can be best understood in terms of their history, some of these concepts or their extensions are introduced or interpreted more properly in the body of the text, as they develop or become particularly relevant. Thus this chapter does not represent a comprehensive review of all the Vygotskian and post-Vygotskian ideas used throughout this research; rather it gives a broad overview of the principal Vygotskian themes.

2.2 A SOCIO-CULTURAL APPROACH

The overarching theme in a Vygotskian approach to mind is that human activity is mediated and this mediated activity cannot be separated from the socio-cultural milieu in which it takes place. Wertsch (1991) argues that this does not mean that there are no universals, but rather that a theoretical stress on these universalisms "makes it extremely difficult to deal in a serious, theoretically motivated way with human action in context" (p. 18). In contrast, the socio-cultural approach allows for explanations which recognise a fundamental relationship between the biological constraints on the individual, and society and its tools.

Given the centrality of the idea of socio-cultural situatedness, it is acceptable and necessary to re-locate and re-interpret Vygotsky so that his writings are applicable to mathematics education in South Africa in the late twentieth century. After all, Vygotsky lived and wrote in the pre-technological Soviet Union of the early twentieth century, a very different society and culture to South Africa today. In particular, spectacular advances in technology have spawned an environment in which a multitude of new and exciting tools and symbols flourish. This study is in fact concerned precisely with interpreting the effect of one of these new technological tools, the graphic calculator. Indeed, an important motive of this study is to ascertain the appropriateness and validity of Vygotsky's thesis: "If one changes the tools of thinking available to a child, his mind will have a radically different structure" (Vygotsky, 1978:126).

The decision to use Vygotsky's socio-cultural constructivist theory has to be partly understood in terms of its role as a viable, and as I will argue, more worthy alternative to a radical constructivist position. The radical position, which derives from Piaget, is the predominant influence in the current context of mathematics and science educational research and theory. It "inspires science education reform programmes, infuses textbooks ... and is the foundation of many science-teacher training courses" (Matthews, 1992: 1). Radical constructivism, as expressed by von Glasersfeld, combines two main assumptions: the Piagetian notion of the subject as actively building his/her knowledge and an ontological position which asserts that cognition serves the subject's organisation of the world, not the discovery of an independent ontological reality. Related to this is the notion that the function of cognition is adaptive, tending towards fit and viability.

Thus, unlike the socio-cultural position, radical constructivism focuses on the individual largely ignoring his/her learning context and the social world. As a result, this theory of knowing has been criticised for tending to a view in which the "cognising subject appears to be near-hermetically sealed in a privately constructed experiential world of its own" (Ernest, 1993: 171). Furthermore it may be argued that radical constructivism leads to an overly child-centred romantic position which

> "as part of the progressive teaching ideology, sanctions anything the child does as expressions of its individual creativity, and naively assumes that the child can discover much of conventional school knowledge on its own" (Ernest, 1993: 172).

Such a position is problematic in itself and also in that it can lead to the devaluing of the expert opinion and to a "sharing of ignorance" (Confrey, 1994: 7).

Socio-cultural constructivism with its emphasis on the dialectical relationship between the development of the individual and the social and historical parameters of the environment provides a more satisfactory and useful explanation of the relationship of the individual to the world, as detailed below.

Furthermore, radical constructivism is unable to address the problem of the ontological status of that body of knowledge which we call mathematics. By explaining cognition as an adaptive process whose function is fit and viability, it fails to clarify, in a satisfactory way, how people do communicate and appear to share at least a sense of common meaning, in particular with respect to mathematics. Thus the status of that body of knowledge which we call mathematics is left unresolved by the radical constructivist position. In contrast, socio-cultural constructivism is able to cope with this issue of what constitutes mathematics in a far more consistent and meaningful way. Ernest's (1991) idea that objective knowledge is social, based on convention and the acceptance of linguistic rules, is quite compatible with a Vygotskian position. It is this notion of mathematical knowledge as the body of mathematics which is held by the community of mathematicians and engineers that underlies the mathematics referred to in this research report.

A quick comment on social constructivism, which is currently beginning to find favour among mathematics educators, is required here. Social constructivism derives from radical constructivism but incorporates the external world by allowing meaning to be made by social negotiation (Wood, Cobb and Yackel, 1992). The learning event can be regarded as a process in which the social acts as a major source of perturbation which generates conflict inside the individual. "The utterances of others serve as sources for individuals to generate new meanings as they reflect on these ideas and re-organise their existing structures to assimilate them" (Wood, Cobb, Yackel, 1992:18). Social constructivism, with its inclusion of the social, can and does address some of the problems of radical constructivism such as the learner-in-society and the status of the body of mathematical knowledge, as mentioned above. However, unlike Vygotsky's theory, it does not provide a satisfactory and all-encompassing explanation of the mechanisms which link the external world to the internal world. It treats the social environment as an unanalysed force impinging on the individual rather than considering it in a principled way as part of the process contributing to learning (Newman, Griffin & Cole, 1989: 59).

2.3 LEARNING AND DEVELOPMENT

In contrast, Vygotsky's view of education is both a theory of development and of cultural transmission. He saw education as a socio-historically determined activity in which meaning was mediated by the socially determined environment and by culturally determined signs and tools.

While Piaget described development in terms of universal and biologically related stages, Vygotsky stressed the changing relation between the biological substrata of behaviour and the social conditions.

> "Every function in the child's cultural development appears twice: first, on the social level, and, later, on the individual level; first, between people (interpsychological) and then inside the child (intra- psychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts" (Vygotsky, 1978: 57).

Vygotsky regarded learning as an essential precursor to the development of specific functions whereas Piaget regarded learning as possible only after the appropriate functions were developed. According to Vygotsky, learning takes place when the process whereby the teacher or expert assists the child in developing the maturing functions, is internalised. Thus all higher functions are initially social and shared between people. The transformation of these external activities to create internal processes is called internalisation and it is this internalisation of these processes that constitutes learning. In Leont'ev's (1981) words: "the process of internalisation is not the *transferral* of an external activity to a pre-existing, internal 'plane of consciousness': it is the process in which this plane is *formed*" (cited in Wertsch, 1985 :163).

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Vygotsky called the domain which defines those functions that are maturing rather than those that have already matured the zone of proximal development (ZPD). This zone is

> "the distance between the [person's] actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978: 86).

In keeping with current educational applications of this concept (Moll, 1990; Newman, Griffin and Cole, 1989) the concept of this zone can be generalised to describe a system, defined relative to the learner, the cultural setting and the activity, within which the transformation from the interpsychological to the intrapsychological takes place.

Newman, Griffin & Cole (1989) discuss how Vygotsky's colleague, Leont'ev, provides the key concept, appropriation, for explaining how interaction takes place in the zone of proximal development of the individual. Leont'ev accepts Piaget's notion that children actively construct their knowledge through communicating with the environment but replaces Piaget's biological concept of "assimilation" with the socio-historical concept of "appropriation". The objects in the child's world have a cultural history and meaning that cannot be discovered with unaided explorations alone. Through involvement in culturally organised activities, the child appropriates the meaning and function of culturally devised tools (be it a hammer or language) via the zone of proximal development. Thus the interpsychological gets transformed into the intrapsychological.

Newman, Griffin & Cole (1989) argue that this notion of appropriation attests to the existence of creativity in the zone of proximal development.

> "First, a great variety of systems of cognitive functioning may be appropriate (and appropriatable) entry points into a given zone, such that there is no simple mapping from adult system onto child system. Second, children who may be similar with respect to their entering cognitive systems may enter zones containing very different interpsychological systems, so that there is also no simple mapping between entering child system and resultant child system" (Newman, Griffin & Cole, 1989: 64).

Another useful Vygotskian concept is the division of knowledge into two broad categories, everyday (spontaneous) and scientific (systematic). Scientific concepts originate in the highly structured and systematically organised environment of academic instruction. They are experienced in verbal, rationalised form and the student can consciously and voluntarily regard and manipulate these concepts. In contrast to this are everyday concepts which develop in the context of the person's everyday experience with its rich accessibility of meanings. Vygotsky emphasised the interconnectedness of everyday and scientific concepts. He highlighted the importance of everyday concepts in mediating in the acquisition of scientific concepts. It is through everyday concepts that students make meaning of scientific concepts; but scientific concepts also transform everyday concepts. Ideally, scientific concepts grow down into everyday concepts acquiring meaning and concreteness, and everyday concepts move up towards the scientific to find their place in a rational system wherein they can be consciously regarded and controlled. The extent to which graphical information can be regarded as a link between everyday concepts, accessible and rich with meaning, and scientific concepts, able to be generalised and manipulated, is an important question which Vygotskian theory can address.

Unfortunately the scope of this study does not allow me to investigate this issue explicitly; rather the process whereby the learner uses the graphic calculator to assist in learning mathematics is examined in my research. Specifically the way in which the user interacts with the graphic calculator in the zone of proximal development is explored. Attention is focused on the extent to which the calculator can be regarded as a partner in the learning process and the quality of this partnership. If the mental operations that are activated during the partnership are within the individual's zone of proximal development, the interactions may mirror the zone of proximal development interactions between learner and "more capable peer". Thus some of these operations may become internalised and so contribute to learning. Furthermore, the use of the graphic calculator may provide extra entry points into the zone of proximal development and so help the learner access new information more quickly and easily.

Pea's (cited in Salomon, 1990) distinction between amplification and cognitive re-organisation is applied and re-interpreted within a Vygotskian framework. Amplification refers to the possible ability of the graphic calculator to enlarge or stretch the zone of proximal development of the individual student. Cognitive re-organisation refers to the possible contribution of the graphic calculator to the re-organisation of the internal plane of consciousness of a particular student.

2.4 MEDIATION

Vygotsky saw mediated action as the fundamental mechanism which linked the external social world to the internal human mental processes. Although the lower mental functions are biologically determined, the higher mental functions are products of mediated activity. "[Vygotsky] argued that it is by mastering semiotically mediated process and categories in social interaction that human consciousness is formed in the individual" (Wertsch, 1985: 166). Thus internalisation can be regarded as the mediated process of transforming the external activity to an internal activity.

Vygotsky distinguished between two principle agents of mediation: signs are internally oriented and aimed at mastering oneself whereas tools are externally oriented and aimed at mastering the environment and hence, consciousness. Thus all higher human mental functions are products of mediated activity in which the role of the mediator is played by a psychological tool or sign, such as language, graphs, algebra, or a technical tool, such as a computer or a graphic calculator. These forms of mediation, which are themselves products of the socio-historical context, do not just facilitate activity, rather they define and shape inner processes.

The idea of the mediational means fundamentally *transforming* the action is crucial to Vygotsky's notion of mediation. The incorporation of mediational means does not simply facilitate action that could have occurred without them. "The agent of mediated action is seen as the individual or individuals *acting in conjunction with mediational means*" (Wertsch, 1991; 33). Thus in investigating the role of the graphic calculator in the learning process it is important not to examine either the learner or the graphic calculator in isolation. Rather the quality of the partnership that the student enters into with the graphic calculator is paramount. Additionally the social and institutional context of the study is fundamental. "Mediational means emerge in response to a wide range of social forces. Specifically, forces other than those of localised intermental functioning may be involved" (Wertsch, 1991; 34).

According to Wertsch (1991), one of the central contributions that Vygotsky and his colleagues made to a social theory of mind was the development of the basic unit of analysis of mediated action, and the person(s)-acting-with-mediational-means as the agent involved. "Human action typically employs 'mediational means' such as tools and language, and .. these mediational means shape the action in essential ways"

(Wertsch, 1991: 12). Although Leont'ev is generally credited with elaborating activity as the basic unit of analysis, Wertsch (1991) maintains that Leont'ev's position is flawed due to an insufficient emphasis on semiotic mediation.

In accord with Vygotsky's position as interpreted by Wertsch, this study will look at how the learner uses the graphic calculator and how the signs of the graphic calculator shape the learning process.

3 LITERATURE REVIEW

3.1 GRAPHIC CALCULATORS AND COMPUTERS: DIFFERENT CONTEXTS

It could be argued that any review of the literature of graphic calculators should include a review of literature on computer-based graphical software; superficially, one might hold that most pedagogic or cognitive effects of the use of computer graphics are mirrored in the graphic calculator environment. After all, both provide a means whereby the student can easily represent graphic and tabular and sometimes symbolic information. However even if one assumes that the computer software can only plot graphs and is unable to deal with symbolic representations or manipulations (as is the case with the graphic calculator) I maintain that the computer environment is *fundamentally different* to that of the graphic calculator so that an understanding of the mechanisms of internalisation and interactions in the zone of proximal development cannot be automatically transferred from one technology to the other.

One crucial difference between graphic calculators and computers relates to the respective status of each technology as a cultural artefact. In particular, the way that males and females use and perceive these technologies differ. Studies which indicate a more positive effect for females as compared to males using the graphic calculator (Ruthven, 1990; Shoaf-Grubbs, 1993a) exist. Any possible gender differences which may be attributed to an increased visual-spatial ability amongst females (Blackett and Tall, 1991; Shoaf-Grubbs, 1993a) and which may have developed as a result of using computer graphics or the graphic calculator, are complicated by a very different gender-based attitude to each of these tools. According to Hoyles (cited in Smart, 1993) research has shown that boys tend to monopolise the computer both at home and at school. Boys are more likely to see the computer as a high-status object and thus they tend to compete for its use (Blackett and Tall, 1991; Smart, 1993). Elkjaer (cited in Smart, 1993) explains the difference in gender attitude by expressing the belief that computers are endowed with "male symbolism" and hence that the gender identity of boys depends on dominance in this area. Graphic calculators do not enjoy the same high-status as computers and thus do not reflect the same male need to dominate (Smart, 1993).

The graphic calculator does not have the same interactive capabilities as does the computer. Jones (1993) describes the graphic calculator as an "intelligent technology" which can undertake cognitive processing on behalf of the user. He distinguishes between a positive and a negative partnership. In the positive partnership there is "a complementary division of labour. The user plans and implements the solution, but passes the responsibility over to the calculator ... for example, when there is a need for a graph to be drawn" (p. 213). In the negative relationship "the opportunity also exists for formation of partnership in which the user virtually acts as an input device for the technology and passes over the entire responsibility for cognitive processing to the graphic calculator" (p. 213). I maintain that either of these relationships with a graphic calculator is qualitatively different to most probable or possible relationships with a computer; since the graphic calculator cannot interact meaningfully with a user it is easy for the user to become passive and to unthinkingly use the information presented by the graphic calculator. The graphic calculator acts merely as an algorithmic performing machine and cannot provide feedback as can a lot of computer graphic software. In this important respect the computer and the graphic calculator are fundamentally different.

Although the graphic calculator allows for the easy plotting of 2-dimensional graphs and manipulations which have to do with a stretching or shrinking of scale (such as that permitted by the ZOOM-type function) the user is unable to physically and directly "mess about" with the graph by pulling it or pushing it, as is often possible with computer software, such as *Function Probe* (developed by Confrey and colleagues at Cornell University).

> "Direct graphing actions [with *Function Probe*] allow the user to 'create' the action as a visual experience taking transformations 'back' to their geometric origins. We suggest that many of the difficulties students have with transformations are rooted in the heavy reliance on examining only how changes in algebraic parameters change the graph of the function" (Confrey, 1992: 227).

Thus computer generated graphic images often allow for direct visual action on the representation and for observation of the ensuing changes whereas the user's interaction with the graph on the graphic calculator is mediated by algebraic or numeric symbols. In this fundamental way, the environment of the graphic calculator differs from that of graphical computer software.

Drawing in 3-D on the graphic calculator is particularly difficult and different perspectives of the same object cannot be automatically generated. In contrast to this, 3-D images are dealt with easily by computer software systems such as Maple or Matlab. These computer-generated images can be rotated and manipulated in a manner that is impossible on the graphic calculator.

Even in the most simple of computer environments, such as that described by Shama and Dreyfus (1991), the computer allows for the automatic translation from graphical to symbolic and vice versa; the graphic calculator, however, only allows for the translation of symbolic to graphical and not vice versa. Again this is a fundamental difference between the computer and the graphic calculator.

Finally, there are the socio-economic arguments which are particularly relevant to South Africa. Unlike computers, graphic calculators are relatively cheap, portable and easy to use. They are battery-powered and are thus suitable for using in the electricity-free environments in which many South Africans live and study. Waits and Demana (1993a) although referring to the American environment, expressed it as follows:

> "Frankly, the invention of graphic calculators by Casio is what made C^2PC [the technology-intensive Calculator and Computer Precalculus mathematics course at Ohio University] a viable and implementable course... Students at typical high schools rarely have access to a computer lab during a precalculus class and a computer at home.... We believe that students must use computers on a regular basis for both in-class work and for homework outside of class if there are to be significant changes in the mathematics that students learn in the 1990's" (p. 92).

Notwithstanding the fundamental differences between these two technologies this discussion would be incomplete if some essential features common to both tools were not acknowledged. Like a lot of computer graphic packages, the graphic calculator is a dynamic environment. Graphs can be stretched or squeezed and images can be scaled up or down through algebraic mediations; several graphs can be drawn on one set of axes, highlighting the relationships between them and numerical values are easily extracted from graphs. It is precisely this dynamic aspect which adds another level of pedagogic possibilities to both the computer and the graphic calculator.

> "Dynamic representations of mathematical processes furnish a degree of psychological reality that enables the mind to manipulate [the many mathematical concepts] in a far more fruitful way than could ever be achieved starting from a static text and pictures in a book" (Tall and West, 1986: 107).

In summary, the socio-cultural status of the graphic calculator and the computer differ significantly. In addition, although there are several points of correspondence between the two technologies, the way that each medium operates as a tool for semiotic mediation differs with respect to several fundamental features, in particular the interactive capability of each tool and also the possibility for the user to directly modify the graph through physical acts. Thus this review focuses on graphic calculator research specifically.

3.2 COGNITIVE QUESTIONS : MULTIPLE REPRESENTATIONS

A current view in mathematics educational research is that the use of multiple representations assists in the development of more flexible and deeper understandings of mathematical concepts (Porzio, 1994). A central tenet of the Calculus Reform Movement is the Rule of Three which asserts that mathematical understanding is enhanced by the representation of mathematical information in its graphical, numerical and symbolic form (Hughes-Hallett, 1991). Implicit in most of the research on the use of the graphic calculator is

> "the *assumption* that there will be cognitive residue from learning with graphic calculators [which] is based on the *belief* that presenting the same mathematical material in both algebraic and graphical forms helps students invest meaning in and thereby helps promote an increased depth of learning of the material" (Jones, 215: 93, my emphases).

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However, there is very little practical research with the graphic calculator which attempts to examine the assumption or the belief to which Jones (1993) refers.

In this research report I examine how and whether multiple representations enhance learning. Essentially I argue that learning is promoted by the use of the graphic calculator since the generation of graphical signs provides additional entry points to the zone of proximal development; also that the use of the calculator amplifies the zone.

In the mathematical education literature, there is a large degree of consensus that:

- representations are an inherent part of mathematics. "Certain representations are so closely associated to a concept that it is hard to see how the concept can be conceived without them. Examples: functions and Cartesian graphs" (Duvoir-Janvier, Bednarz, Belanger, 1987: 110);
- representations are multiple concretisations of a concept; by presenting, or discovering different representations, the student should discover the essential and innate properties of the concept (Duvoir-Janvier, Bednarz, Belanger, 1987);
- multiple representations can be used to assist in the understanding of complex concepts by selectively emphasising and de-emphasising different aspects of these concepts (Keller and Hirsch, 1992);
- multiple representations facilitate cognitive linking between presentations (Janvier, 1987; Lesh, Post and Behr, 1987);
- most realistic mathematical problems are multimodal from the start (Lesh, Post and Behr, 1987).

Confrey and Smith (1992) suggest that "building an understanding of functions through multiple representations and contextual problems provides an alternative epistemological approach to functions" (1992: 153). They argue that the formal definition of the function concept restricts both the conceptual and operational understandings that students need to develop and that experience in "doing" functions is more important than static definitions. Since Confrey and Smith use a computer software tool, called *Function Probe,* which they have developed, rather than the graphic calculator in their practical research, I will not examine this research in detail. However, Confrey and Smith (1992, 1993) do express an epistemology of multiple representations which can be applied to the use of the graphic calculator. They articulate a constructivist view of students as learners who need to be actively engaged in the building of their own knowledge and understanding. Confrey and Smith's (1992, 1993) theoretical and pedagogical position, which includes the use of contextualised problems and active participation, underpins, albeit frequently implicitly, much other research that I will describe in this review.

Ruthven (1990) investigated the ability of advanced-level Calculus students to move between graphic and symbolic forms of functions after a year of using the graphic calculator. Ruthven's study indicated a superior performance by graphic calculator students in their ability to algebraically describe a graph. He attributes this to the practice that the students had in moving, albeit in the opposite direction to that tested, from the algebraic to the graphic form of the function. In addition, he posits that the use of the graphic calculator reduces uncertainty and thus diminishes anxiety; as a result, the students perform better.

Shoaf-Grubbs (1993a) also found, in a year-long study, that female students who had been taught with the graphic calculator moved more easily from the graphic to the algebraic representation of the function. In Shoaf-Grubbs' study, two groups of female students were given an identical elementary algebra course except that one group (the experimental group) used the graphic calculator as a matter of normal course; the other group used paper and pencil methods. This study showed that the experimental group exhibited a significant improvement in the level of understanding and spatial skills. Shoaf-Grubbs (1993b) explains this phenomenon within a radical constructivist paradigm. She suggests that by using the graphic calculator, students are more likely to construct their own mathematical understanding through conscious reflection. The student can build connections among representations at her own pace; she is encouraged to focus on understanding concepts rather than on algorithmic procedures and the student, by interacting with the graphic calculator, develops both visualisation and communication skills that extend and reinforce her mathematical skills. Shoaf-Grubbs maintains that the "graphic calculator's visual representation of the mathematical concepts is important as both a heuristic and pedagogic tool.... the student is able to use concrete imagery to move towards a higher level of understanding or abstraction." (1993a: 441).

In a Vygotskian paradigm, one might say that the graphic calculator functions as an effective tool of semiotic mediation in the zone of proximal development of the individual student. In this study, I elaborate on the ways in which the use of the graphic calculator mediates or shapes this learning process.

Rich (1991) studied the C^2PC project administered by Waits and Demana. She found that students who are taught precalculus using a graphic calculator understand the connections between an algebraic representation and its graph better than students exposed to the normal treatment. These students also strengthened their understanding of the global features of graphs, such as asymptotic behaviour.

In contrast to these studies, Giamati (1991) found evidence that the use of the graphic calculator does not necessarily assist in the development of conceptual connections between graphs and equations. In fact, she found that students who had a poorly and partially formed understanding of these conceptual links, were cognitively distracted by also having to learn how to use the graphic calculator; on the other hand, students whose conceptual links were solid, benefited from the use of the graphic calculator. Giamati's findings can partly be attributed to the short-term period (five and a half weeks) of her investigation. Presumably the various functions in the zone of proximal development of the weaker students required more time to mature before they could effectively interact with the tool of semiotic mediation, as indicated by the year-long studies of Ruthven (1990) and Shoaf-Grubbs (1993a, 1993b).

Various researchers have examined the anchor or initial choice of representation by students. Porzio (1994) found that the choice of initial representation for solving mathematical problems was related to the type of technology employed. Students in the *Mathematica* environment, in which access to the different representations was of equal difficulty, did not have particular initial preferences whereas students who had used the graphic calculator or paper and pencil methods of the traditional Calculus course, were most likely to favour a graphical or symbolic approach, respectively. Keller and Hirsch (1992) found a similar result when comparing students in a graphic calculator Calculus course to a traditional course. In this research report I argue similarly that the choice or apparent preference of any student for a particular semiotic system (algebra, calculator-generated graphs etc.) does not necessarily mean that that particular system is more 'basic' to that students' psychological processes. Rather the choice of semiotic system is also determined by the socio-cultural status of the particular symbol system, the extent to which that system is 'privileged' (Wertsch, 1990a) or not.

The identification of the initial cognitive preferences for type of representation has strong pedagogical implications. If a student has a cognitive preference for a particular representation, it is probable that the firmest link between the concept and its representation is constructed using the preferred representation (Keller and Hirsch, 1992). From a constructivist approach, "attempts to enable students to approach mathematics as a sense-making activity may do well to build from students' preferences for representations" (Keller and Hirsch, 1992: 188). Since the apparent preferred use of different representations may relate to the level of privileging of the different representations rather than to cognitive factors alone, careful research which pinpoints actual *cognitive* preferences for representations, if any, is needed.

Unfortunately, the question of cognitive preferences was beyond the scope of this research report. This research focuses on how and whether the graphic calculator enhances mathematical learning *when* it is used rather than *if* it is used.

3.3 COGNITIVE ISSUES: MISCONCEPTIONS AND DIFFICULTIES

In this section I will focus on a review of literature pertaining to misconceptions and difficulties, particularly relating to the use of graphs in a technological environment. Misconceptions are cognitive obstacles which are built firmly, albeit on shaky foundations, in the mind of the student. Given that one would not wish to encourage the use of a tool (the graphic calculator) or sign (the graph) which promotes and encourages misconceptions in the mind of the learner, it is important to be aware of the type and quality of misconceptions which have been reported in the literature.

Goldenberg (1988, 1991) and Dunham and Osborne (1991) discuss the pedagogical challenges inherent in graphical representations. Multi-representational environments in which graphs are accorded a high status are potential sources of much confusion. Graphs have conventions and ambiguities of their own and unless the student is sufficiently skilled in interpreting these conventions, the graph can be a source of confusion rather than clarity (Goldenberg, 1988; Dunham and Osborne, 1991).

> "Students often made significant misinterpretations of what they saw in graphic representations of functions. ... they could induce rules, that were misleading or downright wrong." (Goldenberg, 1988: 137).

Dreyfus and Eisenberg (1991) adopt a broader perspective when dealing with the problems of visualisation and address the difficulties involved in visualising many types of diagrams, not just Cartesian graphs. Like Goldenberg (1988), and Dunham and Osborne (1991), they recognise that there are cognitive difficulties:

"Diagrammatic representations are not immediately intelligible to the uninitiated. It takes cognitive processing to make sense of diagrammatic representations" (p. 33). They also describe difficulties which are socio-cultural in origin. The social explanation rests on the theory of Chevallard who claims that in order to incorporate knowledge into a curriculum, it has to be didactically transposed. It must change from academic, scientific knowledge into instructional knowledge which is necessarily sequential; it is thus best represented sententially, not diagrammatically. Additionally Dreyfus and Eisenberg (1991) present a cultural argument. This argument rests on the traditionally low status of visualisation in the mathematical community. In this research report I argue that the algebraic approach has traditionally been 'privileged' (Wertsch, 1990a) but that with contemporary advances in technology and computer graphics a corresponding shift towards the privileging of visual and graphical representations is taking place.

Goldenberg (1988) distinguishes between two major sources of ambiguities in graphs: perceptual distractions and perceptual illusions. Perceptual distractions are features of the graph which engage the eye (for example the maximum y-value in a quadratic function) but may not, in fact, directly help in interpreting the graph. Perceptual illusions are visual cues which lead to false perceptions. Perceptual illusions arise from the interactions between the position and orientation of the graph and the shape of the graphing window, and the interaction between the scale of the graph and the scale of the window.

Dunham and Osborne (1991) focus on the undeveloped cognitive skills of the student in a graphical environment rather than on phenomena related to visual perception in general. They isolate three major areas of potential confusion. These are the behaviours associated with weak connections between symbols and graphs, with the interaction between the scale of the graph and the size of the viewing window and with the interaction between the scale and shape of the graph. Both Dunham and Osborne

(1991) and Goldenberg (1988) agree that strong algebraic and analytic sophistication allow us to see beyond these perceptual distractions and illusions. Additionally I maintain that a teacher or expert is required to isolate and highlight some of these potential confusions.

Both Dunham and Osborne (1991) and Goldenberg (1988) discuss the inherent problem in software that plots functions as continuous curves which sweep across the domain from left to right. Goldenberg (1988) recommends software that plots the points of a function in a random fashion to emphasise the role of the points as ordered pairs; Dunham suggests the use of paper-and-pencil plotting techniques as a physical exercise with cognitive consequences. Similarly, in this research report I argue that the simultaneous use of hand, eyes and speech when hand-drawing a graph, as opposed to the use of technology to generate a graph, may be a crucial aspect for the internalisation of the symbol system.

Goldenberg (1991) discusses issues particularly related to the computer screen. Those that transfer directly to the graphic calculator are worthy of noting. Firstly, there is the fundamental problem that the mathematical definition of a point gives position, but no size. In this sense, any physical graph fundamentally differs from its abstract conception. Additionally, the computer or calculator screen represents points as lighted pixels. This can cause problems in that curvature is represented by a set of rectangular approximations. Also the extreme magnification of the zoom feature may represent a point as a horizontal line segment.

There are also problems when pixellation and scale interact so that local extrema are missed altogether (Goldenberg, 1988). Minton (1993) suggests a creative solution to this problem. He posits the notion of 'caricature' graphs, which accentuate the important features of the graph by exploiting different scales for small-scale and large-scale features.

Aside from those mathematics educators cited above, and despite the fact that many educators advocate the use of visual stimuli, there seems to be a dearth of literature on possible ways to overcome the cognitive obstacles so prevalent in the graphical environment. Some questions which I have not attempted to address directly in this report but which are worthy of investigation are:

♣ There are conventions attached to the reading of graphs (Goldenberg, 1988). Frequently these are not addressed directly; rather the teacher hopes that the student will learn these codes indirectly through experience. Is this acceptable? In a Vygotskian sense, does the reading of graphs primarily require everyday concepts or scientific concepts?

♣ How strong do algebraic skills need to be, in order to see beyond the perceptual illusions and distractions?

♣ Would more exposure and experimenting in the graphical environment improve the visual ability of the student?

♣ Does a conscious acknowledgement of the problems inherent in visual data in the computer/calculator environment assist in the building of appropriate graphical-reading and manipulating skills? On this point, Vygotsky regarded conscious reflection as essential for higher human thinking processes.

3.4 THE CLASSROOM

The existence of the graphic calculator has led to new and vigorous debates about the content of the curriculum (Waits and Demana, 1993a, 1993b; Demana, Schoen and Waits, 1993; Dick, 1992, Vonder Embse, 1992) and it is in this area that the graphic calculator has had a particularly significant impact. For example, graphic calculators capable of at least numeric differentiation, numeric integration and root finding were required in the 1995 Advanced Placement calculus examination in the United States of America. Additionally, its use in classrooms has encouraged a shift in pedagogical emphasis from procedural, algorithmic type mathematics to problem-solving and investigative mathematics (Waits and Demana, 1993a, 1993b; Smart, 1993; Dick, 1992).

Shoaf-Grubbs (1992) observed that the use of the graphic calculator helped create an interactive learning environment in which students were more likely to construct their own mathematical understanding through conscious reflection; with the use of the graphic calculator the passive student transformed into the active student. She conjectured that through the exploration of the graphic calculator's visual representation, the student is able to use concrete imagery to move to a higher level of understanding or abstraction. The graphic calculator can thus serve as a concrete mechanism that assists the weaker student in making and testing conjectures (Shoaf-Grubbs, 1993a, 1993b).

Ruthven (1992) and Smart (1993) acknowledge and advocate the use of the graphic calculator as a tool for conjecture and investigation. Similarly, Laridon (1994) in describing a project to investigate the effect of introducing graphic calculators into South African schools, notes the capacity of the graphic calculator for changing teaching approaches from the traditional authoritarian approach to a more learner-centred approach. Laridon (1994) however, cautions against the an abrupt and sudden change from rule-oriented mathematics (which is the norm in rural and township schools) to investigative mathematics (which is facilitated by the use of the graphic calculator) since this may lead to confusion. This is an important consideration in any implementation of graphic calculators into the historically conservative South African curriculum.

The role of graphic calculators in problem-solving is acknowledged by many practitioners. Demana, Schoen and Waits (1993) state that the graphic calculator reduces the computational load while computational experience, estimation skills and understanding are enhanced. To substantiate this observation, they quote Rich (1991), who found that precalculus students who had used the graphic calculator were more able to use graphs and functions as a problem-solving tool. Similarly, Dick (1992) notes that students using graphic calculators perceive problem-solving differently: they can now focus on the statement of the problem and analysis of solution rather than on the symbolic manipulation. However as with much of the writings of Waits and Demana, there is little attempt to explain these observations using a learning theoretic approach.

As mentioned previously, I argue that the graphic calculator both opens new doors to the zone of proximal development and amplifies the zone of proximal development. Both these mechanisms contribute to enhancing the problem-solving skills of the student. However since only a small portion (about 5%) of the Mathematics I class had access to a graphic calculator in my study and since its use was inadmissible in examinations and tests, I was morally obliged to limit the use of this technology in the classroom. Essentially the graphic calculator was used as a tool for support and verification rather than as an analytic or problem-solving tool in its own right in this research. In line with a socio-cultural position, I argue that this institutional context was central to this study, blunting and undermining possible cognitive effects of the calculator.

3.5 TECHNOLOGY AND RESEARCH

In the process of reading articles and conference proceedings for this literature review, I was struck by the lack of in-depth research on the role of graphic calculators in the learning of mathematics. There are articles, such as those of Waits and Demana (1993a, 1993b), Dick (1992), Vonder Embse (1992) and several of those found in the 1992 and 1993 International Conference on Technology in College

Mathematics (ICTCM) proceedings (for example, Davis, Moran and Murphy, 1992; Lopez, 1993) which discuss the implications of the introduction of the graphic calculator for the curriculum. Also, there are articles which give useful suggestions for the uses of the graphic calculator (for example, Waits and Demana, 1993a, 1993b; Dick, 1992; Foley, 1992; Huff, 1993; Benson, 1993).

Other articles and references to doctoral dissertations describe experiments in which one group of students is given a graphic calculator, another is not. The two groups of students are then compared on several measures, such as achievement in tests (Ruthven, 1990; Quesada, 1993; Quesada, 1994; Chen, 1992; Jones and Boers, 1992).

The role of multiple representations in the learning and teaching of mathematics is acknowledged by many of the researchers/ practitioners (Waits and Demana, 1993a, 1993b; Ruthven, 1990, 1992; Rich, 1991; Quesada, 1994) but is not examined explicitly by any of these researchers. In fact there are very few articles embedded in an explicit epistemological or psychological framework. Shoaf-Grubbs (1993a, 1993b) and Keller and Hirsch (1992) who locate their research within a constructivist paradigm, and P. Jones (1993) (discussed below) are among the few who do attempt to embed their research in a learning theoretic framework.

Similarly Jere Confrey places her epistemology of multiple representations in the framework of constructivism (Confrey and Smith, 1992) as does Wenzelberger (1992). Likewise Salomon (1990) and Salomon, Globerson and Guterman (1989) focus on the way that technology affects the cognitive processes of the individual in society, frequently using Vygotskian terms and concepts. But Confrey and Wenzelberger are dealing with computer graphing software, not the graphic calculator; Salomon et al are dealing with technology in general. Of course, some educationalists, researching the role of the graphic calculator in understanding, transfer their findings about the graphic calculator directly into these frameworks. For example, Lauten, Graham and Ferrini-Mundy (1994) use Confrey's description of the constructivist clinical interview as a task-oriented interview in which the researcher probes the student's understanding until the researcher understands how the student is doing and thinking about mathematics. Jones (1993) uses the framework developed by Salomon and his colleagues, as discussed later.

Kaput (1994) explains the split between educational research and technological practice in terms of the marginalisation of the technologists in the education community. He suggests that the dearth of technology-oriented articles in the Journal for Research in Mathematics Education in the previous 25 years, can be explained by "the paucity of publishable research" (Kaput, 1994 : 680). Kaput (1994) attributes this state of affairs to two major factors: the mainstream of the mathematics education research community is not involved in technology, and technology is regarded as the province of specialists. Kaput (1994) argues that this state of affairs is borne out by the development of alternate venues, such as Computers in Mathematics and Science Teaching, for the publication of technology-oriented research.

Certainly, my experience in researching this review indicates the existence of two quite separate environments for technology and learning theory in education: there are annual international conferences linked to learning theory, such as the Psychology of Mathematics Education (PME), and there are annual international conferences linked to the practical use of technology in education, such as ICTCM, and there is little overlap between the two research areas.

Kaput (1994) suggests that there are inherent difficulties in doing research in technology in the classroom: the researcher has to continually rethink curricular and pedagogical motives and contexts and there are the logistical and cost problems involved in the introduction of technology. "To exploit the real power of the technology is to transgress most of the boundaries of school mathematics" (Kaput, 1994: 681).

However, since a lot of the graphic calculator research does involve interventions and a rethinking of the curriculum (for example, Waits and Demana, 1993a, 1993b: Demana, Schoen and Waits, 1993; Dick, 1992) and since the logistical and cost problems associated with graphic calculators are relatively small, these reasons are not sufficient. Rather I would agree with Kaput's observation that the mathematical education community, with a few exceptions, has a passive attitude to technology. This is indicated by the historical lack of consultation between educationalists and technologists in the design of different technological tools and by the frequent occurrence in which technology designed for other purposes is introduced into the educational arena (Kaput, 1994).

As mentioned previously, P. L. Jones (1993) is one of the few mathematical educationalists who does go beyond the issues of curriculum and classroom practice to focus attention on the relationship of the user to the graphic calculator. Using the theoretical framework developed by Salomon, Perkins and Globerson (1991), Jones (1993) defines the graphic calculator as an "intelligent technology" which is capable of significant cognitive processing on behalf of the user.

He distinguishes between the two principal effects of working with the graphic calculator. There are the possible changes in performance which occur when the student is working with the technology; these are termed effects *with* the technology. There are also many may be changes that occur as a result of working with the tool; these are termed effects *of* the technology.

Jones (1993) describes the process of working *with* the technology as one in which the user plans, implements and interprets the solution and the graphic calculator performs the techniques. Jones contends that this partnership gives the student the potential to work at a much higher level than may be possible without the graphic calculator. This is a crucial issue and is examined in my report in the Vygotskian terms of the role of the graphic calculator as an amplifier of the zone of proximal development.

According to Jones (1993) the other possible effect occurs as a result *of* the use of the graphic calculator and involves the relationship of the user to the understanding of mathematical concepts. Jones (1993) argues that the graphic calculator does not necessarily improve understanding. He points out that the notion of the graphic calculator as a tool which deepens understanding, stems from the *belief* that learning in a multiple representation environment must give meaning and hence understanding to various mathematical concepts. Jones maintains that there is little evidence for this assumption. In this report I examine this assumption in terms of the capacity of the graphic calculator to re-organise or restructure the internal plane of consciousness.

In order for the graphic calculator to be used and exploited to maximum capacity, it is important that mathematics educators make sense of the interaction between the learner and the graphic calculator. The graphic calculator has the potential to be a potent tool in the learning and teaching of mathematics; for this to happen, an understanding of how it can be used and abused in the development and learning of mathematical concepts and techniques is required. So it is important that analytical research, thoroughly grounded in a theoretical framework, gets done. Thus the situating of my research within a socio-constructivist paradigm is a major concern of my study.

4 METHODOLOGY

4.1 METHODOLOGICAL FRAMEWORK

Traditional approaches to educational research assume an epistemological position which regards certainty as possible and objectivity as attainable. Within this epistemological context, quantitative methods and experimental design which mirror those used in the natural sciences, are the methodology of choice and efficiency. Accepting this epistemology means that the effects of the graphic calculator on Mathematics students could be tested and interpreted using the experimental method, so that claims of cause and effect could be given as statistical probabilities. Inferences about the positive or negative or neutral effects of graphic calculators could be made and these could then be generalised to the population of first-year University students at similar tertiary institutions; in this way, decisions about the value of the graphic calculator could be made in an objective sense. Additionally, given this epistemology, judgements about the validity and reliability of the study, using the traditional sense of these concepts, could be made with reasonable confidence.

But my contention is that such a scenario, based on such an epistemology and hence resulting in such a methodology, is neither possible nor desirable.

Firstly, it is widely accepted that observation is always theory-laden. Due to the work of Hanson, Popper and Wittgenstein,

> "researchers are aware that when they make observations they cannot argue that these are objective in the sense of being 'pure', free from the influence of background theories or hypotheses or personal hopes and desires" (Phillips, 1991: 25).

Secondly, there is the epistemological question: even if I believe that there are general laws (such as those of Freud or Vygotsky) governing the behaviour of people, can I get to these laws in an exact and objective sense? In other words, even if I accept that I am working within a specific paradigm (in a Kuhnian sense) can I achieve objectivity within this paradigm. My answer must be no. Unlike in the natural sciences, where one stands in a subject-object relation to that which is being observed, in the educational arena, one stands in a subject-subject relation. This is crucial, since one's insider status in the educational process is a given, unalterable fact. I have my own beliefs, prejudices and no doubt idiosyncratic ways of approaching technology and any mathematical problem. This must influence not only *how* I measure, but also *what* I measure. Thus I cannot hope to attain objective truth. Kierkegaard (cited in Cohen & Manion, 1994) argued that the ability to consider one's own relationship to the focus of enquiry, to acknowledge the subjective nature of the investigation, is crucial.

Thirdly, there is the problem which Hampden-Turner (cited in Cohen & Manion, 1994) alludes to: in order to use the scientific method, the social scientist "concentrates on the repetitive, predictable and invariant aspects of the person; on 'visible externalities' to the exclusion of the subjective world" (p. 25). This can result in a biased and trivial picture.

Finally, there are practical and ethical concerns:

♣ The graphic calculator was only given to a few students (twenty), the "experimental" group, in the Mathematics I Course. They could not use it for examination purposes since this would have been regarded as unfair to the other students. Because the graphic calculator was not able to be used under test conditions, it is possible that some students resisted and withdrew from its use in order not to become dependent on it.

♣ Since the graphic calculator could not be used in tests and examinations, I was morally bound to encourage the students to use it as a back-up for supporting or verifying their analytic results, rather than as a stand-alone tool in its own right.

These considerations are extremely crucial and must adversely effect the validity of a study looking for statistically significant (and hence objective) effects of the use of the graphic calculator.

Rather than trivialise this study by merely looking for significant results in the limited context described above, I have primarily done an interpretative study in which the ways in which the graphic calculator has effected students' mathematical representations and practices is explored. Furthermore I have chosen (for reasons given in Chapter Two) to work mainly in the Vygotskian paradigm. This I have done through the use of mathematical interviews and a mathematics task. I have collected quantifiable data where possible, but am using this to inform my qualitative inquiry, rather than the other way around. It is hoped that this triangulated approach to data-collection will serve both to corroborate and internally validate the different sets of data.

4.2 THE RESEARCH PROJECT

In order to understand how learners used the graphic calculator as a mediating sign, I needed to work with students who were using the graphic calculator in their mathematics. For this reason I set up a graphic calculator project within the first-year Mathematics course (MATHS 100) at the University of the Witwatersrand in 1995.

4.1 SETTING UP THE PROJECT

The course has a mixed and heterogeneous student population, with students coming from both the economically and educationally advantaged sector of the population as well as from the economically and educationally disadvantaged sector. Normally all students have studied mathematics at the Higher Grade for Matric. But within this student group there are diverse mathematical backgrounds. Some students have also studied Additional Mathematics at school (and obtained A symbols for this subject) whereas others have merely managed a C or D symbol in ordinary Mathematics.

In 1995 there were about 400 first-year mathematics students. These students were allocated, subject to timetable constraints, to one of three parallel courses given at different times by different lecturers. Additionally, the students were assigned to one tutorial period. There were 18 tutorial periods each of which took place on one of three afternoons. The afternoon for the tutorial was chosen by the student, frequently constrained by his/her timetable; the actual division into tutorial groups for the particular afternoon was done by the course co-ordinator.

Twenty students of the Mathematics 100 course were selected for participation in the graphic calculator project. These students constituted the experimental group in this project. (Details of the selection of this group are given below). These students were each loaned a graphic calculator by the Mathematics Department for the academic year. In addition, twenty other students were initially chosen as a comparison group with which the students in the experimental group could be compared. Ultimately the use of the comparison group was limited to their use in a pilot study. (Again, details of the selection of this comparison group are given below).

The students in the comparison and the experimental group attended the same set of lectures (six per week) but were allocated to different tutorial sessions. In addition, there were about sixty other students attending lectures on the same diagonal.

The tutorial period is generally about 45 minutes a week; most tutorials in the MATHS 100 course are unstructured. They provide an opportunity during which students can consult with their mathematics' tutors about problems in the tutorial exercises. These are mathematical exercises which have been set by the course co-ordinator, usually from the prescribed textbook[1], for students to do in their own time.

An important aspect of this project was this prescribed textbook. The textbook is geared towards the multiple representation approach, and as such has many examples of graphs throughout its prose and the exercises. Thus even without a graphic calculator the student was exposed to many graphical and numerical representations in both the prose and the exercises. For this reason it is important to distinguish between graphs as presented a fait accompli and the graphic calculator which can be manipulated by the student to generate graphs of his/her choice.

The textbook was used as the basis for the lectures as well as providing a wealth of exercises from which the tutorial exercises were selected by the course co-ordinator. Some exercises, which I occasionally suggested that the students in my tutorial group did, are specifically designed for use with technology. In addition, the textbook frequently highlights possible misuses of the graphic calculator.

4.2.2 THE STUDENTS

For the purposes of my research project one tutorial group of twenty students constituted the experimental group, with myself as tutor. These students were each loaned a graphic calculator, the TEXAS INSTRUMENT (TI) 82, by the Mathematics

[1] LARSON, R.E, HOSTETLER, R.P. & EDWARDS, B.H. (1994). CALCULUS, Fifth Edition. Lexington, Massachusetts: D.D. Heath and Company.

Department for one year.

Initially the students for the experimental group were chosen arbitrarily from one diagonal by the course co-ordinator. Another tutorial group, the so-called comparison group, was then constituted from the same diagonal. The comparison group was initially structured so that the students in it matched the students in the experimental group in terms of matric mathematics symbols and socio-economic background, as indicated by the last school attended, as far as possible.

The comparison group was tutored by the course co-ordinator in the first-half of the year; in the second half-year, the tutor was a Mathematics 100 lecturer. Both sets of students had the same lectures and lecturers as each other although it may well be that some of these students occasionally attended some lectures on another diagonal.

At the first tutorial of the year, the students in the experimental group were given the choice of remaining in this group or not being part of the project. Students who left the experimental group (three of them) were replaced by students who expressed an interest in the project. Two of these students came from the comparison group, a third came from a different tutorial group but on the same diagonal.

Thus the experimental group consisted of seventeen arbitrarily selected students who indicated a willingness to be part of the project, together with three volunteers. It was not possible to match every student in the experimental group by gender and prior mathematical experience and in the end, ten students were very well matched to the comparison group students (by school, gender, prior mathematical experience and matric mathematics symbol), four were adequately matched (by school and/or matric mathematics symbol) and six were not matched at all.

For this and other methodological reasons, I have moved away from my original intention to only compare the experimental group with the so-called comparison group; rather I have used the rest of the students on the same diagonal as a source for comparisons where relevant although three out of the four non-graphic-calculator students whom I interviewed did come from the comparison group.

4.2.3 USE OF THE GRAPHIC CALCULATOR

As indicated above, the experimental group had the use of graphic calculators for all activities other than tests and examinations for the duration of the academic year. During the tutorial sessions I guided the students in the use of the graphic calculator via written notes, demonstrations and discussions.

An outline of the treatment to which the students in the experimental group were exposed was:

♣ The students in the experimental group were given notes on the basic operation of the graphic calculator. In addition, they were each given the manual which comes with the calculator.

About twenty minutes were set aside at the beginning of the majority of tutorial periods. In this time:

♣ I demonstrated how to use particular functions on the graphic calculator. I had access to a TI 82 specifically designed so that its window could be shown on an overhead projector for demonstration purposes.
♣ The class discussed difficulties that they may have had in interpreting the graphical information in the tutorial exercises and I highlighted possible and less obvious difficulties in the interpretation of the graphical and numerical information on the calculator (for example, that the scaling of the window

may hide a discontinuity in the function).

- I suggested ways of approaching particular tutorial problems which could benefit from the use of the graphic calculator (for example, one can find the points of intersection of two graphs graphically).

For the rest of the tutorial period I walked around the class answering questions about the normal tutorial exercises as well as giving assistance with the graphic calculator.

Throughout this research project, it must be remembered that the students in the experimental group had to write all tests and exams *without* the use of the graphic calculator. Thus I was somewhat limited in what I could ask them to do with this technology. In fact I was morally bound to regard the technology as an add-on, rather than as an integral part of the course. This diminished use of the graphic calculator may have had profound implications for the role that this tool played in the zone of proximal development of students in the experimental group in this project.

4.3 METHODS OF DATA COLLECTION

Two types of data, in the form of answers to a set of mathematical problems (the Task) and in the form of nine distinct mathematical interviews were collected in order to investigate the role of the graphic calculator as a mediating sign in the zone of proximal development.

In this section, the process whereby the method for data collection developed with respect to both the Task and the interviews is outlined and the framework for the analysis of the qualitative data from the interviews is discussed. In addition the format of each data-collecting activity is described.

This investigation necessarily involved the comparison of the graphic calculator as a semiotic mediator with that of other possible mediating signs such as algebra and hand-drawn graphs.

4.3.1 THE INTERVIEWS

Overview

In order to qualitatively examine any possible effects of the use of the graphic calculator on the mathematical conceptions and manipulations of students, nine different students (six who had access to the graphic calculator; three who did not) were given interviews in which they were asked to solve, speaking aloud, a mathematical problem (see Appendix A). These interviews took place in September and October 1995 within a period of three weeks.

The interview was designed so that it was possible to observe the way in which students used different representations and signs in order to solve a mathematical problem. It was hoped that the interview situation would allow me to gain access to the students' conceptions and problem-solving strategies so that I could ascertain whether students with access to the graphic calculator performed differently to non-graphic calculator students as indicated by flexibility, performance and apparent understanding. More details of these terms and categories of analysis are given later.

The mathematical problem was a non-trivial mathematical question which required deep conceptual knowledge and which could be solved using algebraic and/or graphical information. Specifically it involved the relationship between an integral as a polynomial expression (an algebraic representation) and two different graphical representations of this integral: the integral as the graph of a polynomial and the representation of the integral as an area under a curve. Ultimately the student was required to link the different representations. The graphs that were possibly of interest were simple enough to be drawn by hand or with the use of the graphic calculator.

The actual mathematical problem is of a type frequently encountered in the literature of Calculus Reform, such as in the Ostebee and Zorn (1995) textbook. All students had been given a tutorial problem similar to the interview problem (but using a trigonometric function) several months earlier.

The Interview Protocol And Its Genesis

The form of the interview was largely inspired by the cultural-historical work of the Soviet psychologist, A. R. Luria, in the 1930's. Luria's purpose, developed in collaboration with Vygotsky, was to reveal the socio-historical roots of all basic cognitive processes; he wished to demonstrate that "the structure of thought depends upon the structure of the dominant types of activity in different cultures" (Cole, 1976: xiv). For this purpose he went into the villages and remote regions of Uzbekistan and Kirghizia in 1931-1932 in order to observe how the radical changes in Soviet society which followed partly from the programs of mechanisation, collectivisation and literacy training, were contributing to a restructuring of the cognitive processes of the different social groups.

Luria's work has been subsequently criticised for an implicit arrogance and for undervaluing the thought processes of the illiterate peasant. Additionally, the data collected was considered to be insufficient for the theoretical interpretations: Uzbeki culture was represented only on the surface level of objects and vocabulary and Uzbeki modes of interaction were never studied. Thus it was possible only to speculate superficially on the differences in the interviews between different Uzbeki social groups (Cole, 1985).

Nonetheless, Luria's work was innovative and provocative in that it aspired to understand the way in which cultural activity shapes cognitive development. It is this basic objective that has motivated my desire to use relevant aspects and techniques from Luria in the design and conception of my interviews.

Cole (1976) describes Luria as a brilliant craftsman who explored the reasoning process of his subjects with the use of carefully guided probes and hypothetical opponents ("but one man told me..."). I too attempted to use standard probes and questions ("could you explain that in another way..."). More details of these probes and questions are given later in this chapter.

In conducting his interviews, Luria attempted to minimise the artificiality of the situation since this "could yield data that misrepresented the subjects actual capabilities" (Luria, 1976: 16). Thus the experimental sessions usually began with long discussions, often with groups of people. Only gradually did the experimenter introduce the prepared tasks which resembled the "riddles" familiar to the population. This desire to free artificial situations from a stifling contrivedness does not derive only from the tradition of Luria; in fact, it has been adopted fairly generally in qualitative educational research and I have certainly attempted to minimise artificiality in my interviews. However it was not possible to completely normalise the interview situation in my study since the intense nature of the interaction and the need for the student to perform out loud created its own tensions and feelings of artificiality. Of course, I was known to all the students as their lecturer and so the actual giving of the mathematical problem for the student to solve was not unnatural.

Since an interview is not a neutral environment I was conscious of my possible role in the zone of proximal development of the student whom I was interviewing. Rather than pretending that my role was neutral or not affecting the behaviour of the student, I attempted to follow Luria's procedure in which

> "the experimenter... always conducted a clinical conversation or experiment. A subject's response stimulated further questions or debate; as a result the subject came up with a new answer without interrupting the free-flowing interchange" (Luria, 1976: 17).

Within this context, I gave assistance to the student at graduated levels either when the student requested such help or when the student seemed particularly unable to deal with the problem.

Luria's interviews were written down, as they occurred, by an assistant. I, operating in the 1990s, was more fortunate to have a tape-recorder to perform this function.

Following on from Luria, I was concerned to give a problem which could be solved in several ways since this allowed for "a qualitative analysis of the resultant data" (Luria, 1976: 17). The particular mathematical problem given in the interview could be solved in several ways: using the graph of the polynomial or the area under the curve or using an algebraic manipulation.

A contemporary proponent of the mathematical interview in which the researcher listens to a student solve a mathematical problem out loud, is Jere Confrey. Although Confrey conducts most of her research within a radical constructivist paradigm, her description and analysis of an interview (Confrey, 1991) in which a student is asked to represent on a number line a series of historical events given in scientific notation, provides an important example of the way in which an interview can be used to gain access to a students' thinking. My interviews take place within an explicitly Vygotskian framework of form (as described above) and are analysed using Vygotskian theory. Nevertheless, Confrey's example provides an important precedent for the use and importance of the mathematical interview.

The Interviewees

Nine students were selected for interviewing. Five students with different academic performances over the year (as measured by their Mathematics I class record) were selected from the experimental group. Three of these students were matched to three similar students in the comparison group. This matching was done by matric mathematics mark, class record for the Maths 100 course as at August 1995, socio-economic status (in terms of school attended) and gender. A fourth student from the calculator group was matched to another similar student not in the comparison group who it turned out had been using a graphic calculator of his own accord for the previous two years. In addition, I only selected students who were co-operative and reasonably capable of expressing themselves. Details of the students are given in Chapter Five.

In the end, only seven interviews were analysed: one of the interviews was discarded due to the technically poor quality of the recording; the other (my very first interview) was forsaken due to my poor interviewing techniques which resulted in me imposing a particular way of seeing the problem at the beginning of the interview. Thus four interviews with graphic calculator students and three interviews with non graphic calculator students were finally analysed.

When I asked the students if they were willing to be interviewed I explained that it was not compulsory; however no student refused to be interviewed. Students were paid at the rate of R30 per interview.

The Interview Format

Each student was interviewed in my office during September or October 1995 at a time which was mutually suitable to the student and myself. Each interview took about 30 to 60 minutes.

Each interview was audio-taped; in addition, I made notes during and after the interview about the way in which the student used algebra, graphs and/or the graphic calculator in the solution of the problem. Where relevant I recorded any behavioural peculiarities of the student. Any written notes made by the student were kept as raw data.

Recognising that it was impossible for my behaviour in the interview situation to be neutral, I decided to at least strive for consistency. To this extent I attempted to use pre-determined standard responses and graduated levels of assistance where possible.

For example, since I expected that the students would feel awkward if I sat silently throughout the interview and that such a scenario would be counter-productive, I decided to respond to their answers with the use of several set phrases, such as "OK" or "Alright". (Whether I said these phrases in an affirming, neutral or tentative manner was noted in the transcription of the tapes). In order to see whether the students were capable of moving from one type of representation to another, I frequently said: "Can you explain that in another way?". If a student got stuck on an aspect of the problem I gave different levels of assistance where the higher level of assistance was given if the lower level proved unsuccessful. For example, in part c) of the interview problem, I decided that if the student was unable to explain other than by algebraic substitution why **$F(x)= 0$ for $x = 0$,** my graduated responses would be:

ME: (Level 1): Can you explain that in another way?

ME: (Level 2): What is the relationship of g to F?

ME: (Level 3) How would you work out the actual area between the curve y = g(x) and the x-axis?

ME: (Level 4): How does this area relate to $_0\int^0(4t-t^2)dt$*?*

ME: (Level 5): What is F(0) measuring?

ME: (Level 6): What does the graph of g show?

Similarly, throughout the interview, I attempted to use such different levels of assistance.

Some tapes were transcribed immediately after the interviews; others were transcribed in December 1995. The transcriptions were based on 3 sources: the actual taped interview, my field-notes and the writings and notes made by the student during the interview.

In the actual transcriptions, I observe the following conventions:

♣ In transcribing something that the student is writing down as he/she speaks, I indicate that it is **written** and transcribe the passage as written by the student. For example, if the student writes down: $g(x)= 4x - x^2$, then I transcribe this using the student's notation; if the student merely *says* this, then I transcribe this as: g(x) equals 4x minus x squared.

♣ When transcribing the interviewer's responses, I have used the following codes to indicate nuance:

/ neutral

* affirming

> tentative

Development Of Analytic Framework

The framework for the analysis of the interviews developed into two complementary structures: the actual analysis presented in tabular form (the analytic tables) and an interpretation given after each analysis. The development of the categories for the analysis and its interpretation occurred in a dynamic way although the categories serve very different purposes in these two complementary structures. Many of the categories were decided before the actual interviews in terms of the aims of the study and its theoretical underpinnings (particularly for the analytic tables); others were grounded in the actual analysis of the interview (particularly for the interpretations).

For the analytic tables, the categories are directed towards a Vygotskian understanding and primarily involve the tool, sign or agent used for mediation (for example, algebra, graph, interviewer) by the student. Ultimately these tables served to mediate my understanding and interpretation of the role of the graphic calculator. More details of the analytic categories are given in Chapter Five and in Appendix C.

For the interpretation, the categories involve more subjective decisions and are thus less rigorous and uniform and often implicit. To avoid simplification and obfuscation I have largely attempted to let the data generate its own categories which in the interests of accuracy may differ from student to student. Nonetheless, in order to use the interview data constructively to answer the questions posed by the objectives of this study and so as to avoid the tempting distractions of "noise", I have sought, where possible, to comment on the following aspects of problem-solving behaviour as revealed by the analytic table and the raw transcription:

♣ the nature of the mathematical conceptions held by the student. Do they appear to be subjectively meaningful and are they objectively adequate?

- the manipulative skills of the student.
- the performance of the student and his/her ability to answer the question "appropriately".
- the flexibility of the student in terms of his/her ability to move from one representation to another.

In addition, using Pea's distinction (cited in Salomon, 1990) I have attempted to comment on the use of the graphic calculator as a technology of amplification and/or cognitive re-organisation.

- As an instrument for amplification, the use of the graphic calculator increases and improves the ability of the student to accomplish tasks more efficiently and effectively while using the tool. This is similar to what Perkins (cited in Salomon, 1990) calls the "fingertip effect" which involves faster and easier manipulations and computations.

- As an instrument of cognitive re-organisation the graphic calculator promotes effective learning by restructuring and re-organising the internal plane of consciousness. Cognitive re-organisation as a result of interaction with a semiotic system involves the development and internalisation of the symbol system which the learner is then able to use in his/her mathematical conceptions. The technology serves not only as a tool with which to think, but also as a tool which helps thinking to develop (Pea, cited in Salomon, 1990). Specifically, cognitive re-organisation effects are operationalised in terms of whether the student is able to use mathematical conceptions more effectively and meaningfully *as a result* of having used the graphic calculator, rather than just *while* using the technology.

Having completed the analysis and its interpretation for each individual student, I then sought to make links and find patterns across the different interviews. Differences and similarities (if any) between students using the graphic calculator and students without this tool were examined in order to suggest a possible role for the graphic calculator as a tool of semiotic mediation in the zone of proximal development of Mathematics I students.

4.3.2 THE TASK

Description Of The Task

The Task (see Appendix B) was a written mathematical test consisting of five mathematical problems together with questions concerning the method of representation used. Sixteen graphic calculator students and forty-eight other students were required to solve the mathematical problems in whatever way they wished and then to answer, by selecting from a list of possible answers, questions about the method of representation used in the solution. Students with access to graphic calculators were allowed to use these calculators in the Task.

Each item is discussed in detail below but broadly speaking three of the questions on the Task (Questions 1, 3 and 5) could be solved more efficiently and easily if appropriate graphic calculator skills were employed; their purpose was to ascertain whether students with access to the graphic calculator show improved performance when using the tool as compared to non-graphic calculator students. Thus these questions were designed to test whether the graphic calculator functioned as an amplifier in the zone of proximal development by enhancing the powers of the students to accomplish the mathematical tasks more effectively.

The other two questions (Question 2 and 4) could not be solved directly on the graphic calculator. Their solution required deep conceptual knowledge (in particular, Question 2) and a non-algorithmic solution. Their purpose was to determine whether students who had access to the graphic calculator had undergone cognitive changes resulting in increased flexibility and understanding, even when not using the tool. In other words, had the graphic calculator functioned as a cognitive re-organiser?

In addition, students were required to answer questions about the method of representation used (algebra, graphs, numbers or some combination of these) by selecting the appropriate method of representation from a given list of six options, or they could describe an alternate representation by selecting the open-ended seventh option. Students were asked about the type of graph used, if any: calculator-generated, hand-drawn or mental image.

In Chapter Six a statistical analysis of some of this data is presented and interpreted. This analysis compares graphic and non-graphic calculator students in terms of their performance, question by question, and over groups of questions. Although data with respect to choice of different representations was collected and tabulated, in the end I decided not to analyse this data since it did not particularly contribute to the questions posed in the objective of this study.

In Chapter Seven the statistical data is used in conjunction with the qualitative data from the interviews to interpret the role of the graphic calculator in the zone of proximal development.

Development Of The Task

Although I have not found any similar task in the literature, its format was suggested to me by previous studies and current textbooks. The development of the Task from these several sources contributes to its reliability.

- Ruthven's study (1990) in which students are required to move between the symbolic and graphical forms of a function after a year of using the graphic calculator, initially gave me the idea of giving the student an algebraic question and then seeing whether the student used graphs and/or algebra to assist in the solution of the problem (such as in Question 1, 3 and 5).
- The idea of asking questions which required conceptual knowledge but not necessarily the use of the graphic calculator (such as Question 2 and Question 4) came about as a result of reading and browsing through various Calculus Reform textbooks. Examples of questions like Question 2 can be found in many contemporary texts such as that of Ostebee and Zorn (1995), as well as in the Calculus Reform Workshop notes used by Sally Fishbeck at the University of the Witwatersrand in July 1995.
- The research into preferred representations (Porzio, 1994; Keller and Hirsch, 1992)) gave me the idea of asking direct questions about which method of representation was used.

In order to develop an appropriate format and wording for the Task described above, a pilot task was initially developed. This was given in a tutorial period in May 1995 to the experimental group and to the comparison group. This pilot consisted of seven open-ended mathematical questions each with three structured questions concerning the preferred use of representations and generation of graphs. Since the purpose of this pilot was to assist in the wording and format of the actual Task rather than in the choice of actual mathematical problems, I will not discuss the actual questions used in the pilot. During the pilot students were invited to comment on the wording, style and format of the task. Later I discussed the format of the pilot with several students. These comments plus further reflection, enabled me to make some changes to the design of the Task.

- Since the average time it took to solve a problem and then to select answers about preferred methods of representation was about seven minutes (there were seven questions: students took from 35 to 55 minutes to complete the pilot task) I decided to limit the Task to 5 questions.

- Although students did not have particular problems in stating *which* method of representation they had used, they found it difficult to reflect back on the *order* of their methods. For example, several of them struggled to decide whether they had, say, first used algebra, then graphs or vice versa, etc. I thus decided to omit questions about the order of representation.

Using this pilot as a basis, the actual Task developed over several months with me adding or deleting questions from a question bank that I was developing. The final version of the Task was checked by two of my mathematics colleagues whose approval reflected a measure of face validity for the Task.

Question By Question

In Question One a graphical representation of the area helps enormously in simplifying the problem. A graph of the function whether generated on the graphic calculator (an easy mechanical task) or drawn by hand (a task which is of medium difficulty for a first-year Mathematics student, requiring at least some understanding of the behaviour of the exponential function and the ability to deduce a symmetry about the origin of the given function) allows the student to calculate the given area simply as $2 * {}_0\int^1 xe^{x^2}dx$. Without this graphical representation the student may (incorrectly) calculate the area as ${}_{-1}\int^1 xe^{x^2}dx$ which will give an answer of zero. An answer of zero should hopefully alert the student that a reconsideration and re-conceptualisation of the problem is required. Similar problems to Question 1 are littered throughout the regular textbook, Larson & Hostetler and the tutorials.

As mentioned above Question 2 was inspired by the literature of the Calculus Reform

Movement. This problem was non-algorithmic and none of the students in Maths 100 had been required to solve such a problem previously. As such its correct solution required a deep understanding of the relationship between the derivative and the function from which it was generated. The direct use of a graphic calculator has no bearing on the solution to this problem; what is of interest is whether the use of a more graphical approach to the calculus via the use of the graphic calculator over the year has resulted in increased flexibility and understanding of basic Calculus concepts.

Question 3 is very quickly solved with a graphical representation. Since the behaviour of the function $(\tan x)^x$ is complicated and difficult to draw without a graphic calculator, the problem is only easily resolved if a graphic calculator is used appropriately; else the student is forced to use an algebraic algorithm which is rather complicated and long-winded, relying on L'Hopitals Rule. The graphical solution relies on minimal skills in using the graphic calculator; the algebraic solution, although cumbersome is within the capabilities of a technically competent first-year student. Similar problems to this were encountered by all students in the regular tutorials and in the prescribed textbook.

Question 4a) was designed to test the ability of the student to move between a graphical and algebraic representation, without the use of the graphic calculator. Although the concept underlying the question (that of areas) was extensively addressed during the Maths 100 course, students had previously not been asked to move from an algebraic to a graphical representation in this context. The given function is a simple function, thus the ability to correctly answer this question reveals a certain flexibility in the basic conceptual understanding of areas, unencumbered by the need to understand and manipulate sophisticated functions.

After the tests had been printed I detected an error in the wording of Question 4 b). Students were thus told to ignore this question.

Question 5 was a particularly awkward question whether the student used or did not use a graphic calculator. There were three intersection points, two (0 and 1) could be found fairly easily using algebraic methods; an approximation to the third root could be found using the CALC function on the graphic calculator. This question was designed to see how a student approached a tricky problem to which there was no easily found exact solution. A student with confidence and good manipulative skills on the graphic calculator could have been expected to approximate the third root and hence the areas between the two curves; a similar student without the graphic calculator should have been aware of the third root and at least been able to write down the method of finding the exact area, even if he/she did not actually find it.

Administration Of The Task

The Task was given without warning to all students who attended a normal lecture period in September 1995. Unfortunately, and unknown to me, a test in another subject (Computational and Applied Mathematics I) was being written later that day; thus attendance at the lecture was particularly poor (about sixty of the usual one hundred students were present). I thus decided to give the same Task to those students in the experimental group and the comparison group who had missed the lecture, during their tutorial period four days later.

The Task was written under test conditions, although the students were given plenty of time to complete it. Students from the experimental group were allowed to use graphic calculators. Completion of the Task was supposedly for due performance rather than for grading purposes, although in fact no account was taken of whether the student did or did not write the test when it came to refusing due performance certificates.

5 THE ANALYSIS AND INTERPRETATION OF INTERVIEWS

5.1 INTRODUCTORY REMARKS

Broadly speaking, the interviews are considered primarily from a learning perspective, although issues relevant to a teaching perspective are highlighted where necessary. Such an approach is consistent with a Vygotskian framework in which the process of learning is indistinguishable from the process of teaching to the extent that a single word (*obuchenie)* is used in Russian for both.

- From the learning perspective I examine the way in which the student uses algebra and/or graphs as tools and signs of semiotic mediation. A distinction is made between graphic calculator generated graphs and hand-drawn graphs. In addition, the way in which the student uses the interviewer as an agent of mediation in the zone of proximal development is discussed, where relevant.

- From the teaching perspective I examine my role as an expert guide in the zone of proximal development of the student with particular reference to the distinction between assistance which is solicited by the student and assistance which is not.

As discussed in Chapter Four, the transcripts of the interviews were analysed in a two-phase process. In phase one I analysed each interview in tabular form in terms of the tool, sign or agent of mediation (for example, algebra, graphs, interviewer). These analyses constitute the analytic tables.

In phase two I analysed and interpreted each interview using the analytic tables (and the transcripts of the interview where necessary) as data. Throughout the different interpretations I have not aimed to use uniform categories. Rather, in the interests of validity, I have let each interview generate and suggest its own categories, within a learning or teaching perspective.

> "The term interpretive [validity] is appropriate primarily because this aspect of understanding is most central to interpretive research, which seeks to comprehend phenomena not on the basis of the researcher's perspective and categories, but from those of the participants in the situation studied." (Maxwell, 1992: 289).

This need of interpretive validity, for analyses grounded closely in the data, has resulted in quite extensive interpretations for each interview. Thus this chapter, which presents these analytic interpretations, is necessarily long. In order to facilitate the extraction of patterns and links across these rather lengthy and dense interpretations I have highlighted in ***bold italic*** the relevant category or description of activity in my text where possible. Additionally I have summarised the biographical data relating to each interviewee in table form at the end of this chapter.

Since the analytic tables ultimately functioned as mediating tools serving to shape and define my interpretation of each interview, they are presented in Appendix C.

5.2 ANALYTIC INTERPRETATIONS

5.2.1 VT INTERVIEW

The Student

VT is a middle-class white female who has been in the experimental group for the entire year. VT obtained an A-grade for matric Mathematics; she had a class record of 76% for Maths 100 but fared poorly in the end-of-year examination, bringing down her final mark to 69%. This is a good, but not exceptional mark. VT was registered for a B.Sc. degree with her other subjects from the Actuarial Science cluster.

VT is a very responsible, pleasant and diligent student; on one level her desire to be "the good student" means that many of her manners and responses in the interview are determined by her desire to please me rather than by any cognitive considerations. In Chapter Seven, I elaborate, in general, on this aspect of the interview situation.

An Analytic Interpretation

At different points in the interview VT uses ***various tools of mediation*** in solving the sub-problems and establishing the link between the alternate representations (semiotic systems).

The first major problem encountered by the student is resolved when her understanding of the problem of how to deal with the variable x when representing F, is ***mediated by both the interviewer and a change in semiotic system***. In this instance, the interviewer assists in the zone of proximal development at several levels. Firstly, ***the*** *interviewer indicates that it is OK for the student not to use the graphic calculator:*

94 ME: if you don't want to use ... you can use either graphic calculator or paper or whatever... just...

96 VT: **(puts down graphic calculator)** I'll draw the graph **(drawing the graph on paper)**- here we go - here we go now...

Secondly, the ***interviewer guides*** the student into the resolution of the problem by leading VT from the particular x to the variable x (lines 106-121).

Moving from using the graphic calculator as a tool of semiotic mediation to the actual hand-drawing of the graph (lines 131-138) seems to enable the student to deal with the problem of representing F more adequately. Her subjective conception of the problem appears to be more meaningful and her objective explanation is most adequate.

137: VT: That's what I say... it would all this area .. any, any area there **(pointing to correct place on hand-drawn graph of g(x)).**

Here the unity of perception and sensor-motor action of generating the graph herself seems to assist in the resolution of the problem.

The student goes on to solve all the other components of the problem ***most adequately (in the objective sense) giving explanations which use the hand-drawn graph and algebra as mediating signs*** (lines 150- 276). She requires no assistance in the zone of proximal development for these sub-problems and so the execution of the different steps in this mathematical problem may be regarded as an expression of learning which has already been internalised.

The one time she attempts to use the graphic calculator graph in order to explain her thinking (lines 184-210), ***she has difficulty*** verbalising the explanation, although she does imply that she understands the concept.

206 VT: **(giggles)** OK. I don't know how to explain it.

Similarly to the previous episode in which VT moves from the graphic calculator as a tool of mediation to the hand-drawn graph as a semiotic mediator, VT decides to rather use ***the hand-drawn graph in her explanation; and she expresses herself more clearly using this sign*** (lines 212-229).

In lines 279-284, ***the interviewer presents a semiotic challenge*** and asks VT for another explanation of the problem in terms of an alternate representation (a different sign or tool of mediation). Using algebra (lines 289-309) and then the hand-drawn graph (lines 312-321), the student unsuccessfully attempts a different explanation. Finally, however, ***after direct assistance by the interviewer*** in the form of a suggestion, VT draws F as a polynomial on the graphic calculator. Looking at the polynomials F and g ***on the graphic calculator screen, VT suddenly "sees" the link*** between the different graphical representations of F and g and their algebraic expressions:

367 ME: So you are trying to find what now?

368 VT: **(looking at graphic calculator; graphs of F(x) and g(x))**. I'm trying to find.. Oh, you know where it is? **(excitedly)** It is where, um, g(x) cuts the graph, cuts the x-axis, so it will be at x=4.

371 ME: OK.

372 VT: OK. I don't know how.. I just saw it with the calculator here... lets

zoom out...there... it would be a maximum....it would be a maximum value of F(x) would be where g(x) cuts the x-axis.

In this episode, we see VT use ***the graphic calculator as a tool of amplification***. This illustrates the manner in which the tool can be used in the zone of proximal development of a student to assist in the solution of a problem: neither time nor mental energy are wasted in the actual drawing of F or g and so VT is able to concentrate on the link between different semiotic mediators without worrying too much about their generation.

Throughout the interview, VT ***indicates a flexibility*** in mathematical conception: in the first part of the interview, she is able to move easily from an algebraic representation to a representation involving the area under curve (lines 131-278) when explaining all the sub-problems. In the second part of the interview (lines 326-454), having grasped the link between the different signs and their equivalent mathematical meaning, VT moves easily from one representation to another, from F as the graph of a polynomial to the area under g (i.e. the derivative of F).

Also, VT displays particularly ***good manipulative skills*** when using the graphic calculator. Although her use of the graphic calculator as a successful mediating tool is indicated mostly in the second half of the interview, VT displays facility and dexterity when using this tool. She can access the integral function (61-71) and moves easily about the calculator when examining the graphs (lines 187-193, 328-340, 378-431). VT's manipulative skills on the calculator are certainly better than any of the other students interviewed.

5.2.2 RS INTERVIEW

The Student

RS is a white male student who has been a member of the experimental group for the entire year. RS obtained an A grade for Mathematics in his matric year. His marks for Mathematics I have been adequate but not excellent: 53% class record and 58% for the end-of-year exam. RS is registered for a B.Sc. degree and his other subjects in 1995 were Computer Science, Computational and Applied Mathematics and Economics.

At the beginning of the 1995 academic year, RS appeared brash and boorish in his behaviour. However on better acquaintance it seemed that his loud behaviour masked an awkwardness and a lack of confidence.

An Analytic Interpretation

This analysis is characterised by several features which occur regularly during the interview.

Firstly, there is the fact that, unlike the other graphic calculator students, RS ***never attempts to draw any graphs by hand***; ***rather he uses the graphic calculator to generate the graphs*** of the polynomials F and g right at the beginning (lines 115 and lines 164-166) and uses these graphs whenever he requires a graphical representation. Although the motivation for using the graphic calculator so extensively may be to please the interviewer rather than for any particular cognitive preference, this use of calculator generated graphs can be regarded as an instance of the use of the graphic calculator as a ***tool for amplification.***

However, RS displays *poor manipulative skills* when using the graphic calculator and *so limits the role of the calculator as a possible tool for semiotic mediation or amplification* in his zone of proximal development. For example, unlike VT, RS does not use the graphic calculator functions CALC or TRACE to determine key features of the graphs. Rather, he uses calculus, in an algebraic representation, to determine the turning points of F and the intercepts and turning point of g (lines 202-292).

Also, RS's *poor manipulative skills* are demonstrated by several errors or omissions which occur *as he uses the calculator.* For example, he initially attempts to enter F as an integral but is unable to use the appropriate calculator functions (lines 83-115); he then attempts to enter F as a polynomial expression but uses incorrect notation (lines 116-162) and obtains the graph of a discontinuous rational function. At this point RS reveals some internalisation of graphical concepts when he appropriately comments: "that is not what I expect" (line 138). In response to a suggestion by the interviewer (lines 148-149) he re-examines and corrects the error in notation. Yet again RS uses the graphic calculator incompetently when he confuses the data from g with the data from F (lines 412-419) in the graphic calculator table. Again the inter-viewer successfully suggests that he re-consider his reading of the table.

RS's *manipulative skills are relatively more developed in his use of written algebra* (for example, he calculates the critical points of F and g easily and correctly in lines 202-292) although he does make a numerical error (in lines 402-405) which requires several interventions (lines 406, 426-427) by the interviewer before it is corrected.

Notwithstanding RS's poor calculator skills, there are several instances in which *we witness him using the graphic calculator as a tool for semiotic mediation*. Although his initial account of why F(0)=0 is vague and uncertain:

314: RS: I'll go for a bash and say the reason why is because where, um, well I could just substitute x into, into the graph. OK, I'll say because there's no constant term. Because when you integrating with the definite integral there's no constant.

318: Me: So why would that make F(0), zero?

319: RS: Because its only x's.

and a bit later:

328: RS: Um, and there's no constant term, so I would, um, immediately know that F(0) equals zero.

330 Me: OK.>

331 RS: Um. They want you to explain this so I suppose I could.. go back to the original definition. Um.. uuughmm.. **(pause)** well when a question like that comes up I would explain it, that, the reason why is.. No, I can't explain the reason why. Uuumm.. I could say its.. uum.. its where the graph cuts..

336 Me: Alright.>

337 RS: .. the axis, the um.. that's where the graph cuts.. I don't know.

after using the graphic calculator and the interviewer, who acts an agent of mediation by suggesting the use of the integral, ***RS is able to move to a reasonably cogent and lucid explanation***. To witness:

345 RS: ..and um, um. Explain in terms of the integral. Oh well, I suppose its the integral between 0 and 0 which you're substituting into the equation.

348 Me: OK.>

350 RS: Um, which is, nought, I suppose, because the integral has no area. OK?

Similarly *using the graphs on the graphic calculator RS is able to articulate a link between the properties of a graph and its shape and of the link between the graph and its derivative.* In lines 474-493 and again in lines 518-536, RS gives a reasonable explanation of why F(7)<0 in terms of the shape of the graph of F. Again, in lines 622-637, he uses the shape of F to successfully explain why F(2)>0. Later, he uses the shape of the graph of g, which is the derivative of F, to explain why F has a maximum at x=4 (lines 561-576 and 590-620).

Overall, RS's mathematical conceptions are *reasonably flexible* in that he is able to use several different concepts or representations (such as area, derivatives, shape of graph, algebra) to explain most of the different sub-problems. However, this flexibility is sometimes obscured by incoherent speech which may be a manifestation of what Vygotsky calls *"inner speech"*. According to Vygotsky (cited in Wertsch & Stone, 1985: 173) inner speech is "condensed, fragmentary, disconnected, unrecognisable, and incomprehensible in comparison to external speech" and reflects a "transition from external social functioning to external and internal individual functioning " (Wertsch & Stone, 1985: 173). Unfortunately, a detailed analysis of inner speech is beyond the scope of this research report. Suffice to observe that this type of speech, which occurs frequently during RS's interview, may serve to assist RS in internalising various concepts required for an understanding of the mathematical problem.

However, even if one interprets RS's mumblings as manifestations of inner speech, it must be remembered that the sub-questions in the mathematical problem *require* the student to give an objectively adequate account and thus to express him/herself at some point using lucid and cogent external speech. In other words *it is required that the student transforms the internal activity into an external activity with the corresponding use of external speech as opposed to internal speech to answer the* *questions satisfactorily. This RS partly fails to do*, particularly when dealing with the area concept and, also, with the explanations involving algebraic substitutions. In fact, it is only with the relationship between the graph and its derivative or the graph and its shape that RS expresses an objectively adequate account of mathematical concepts.

This inability to make a satisfactory transition from inner to external (or social) speech may indicate an inadequate internalisation of mathematical concepts and a corresponding inability to move beyond inner speech. Or this poor expression of ideas may be partly a function of the social context of the interview in which RS may feel intimidated. RS acknowledges a difficulty with explanation and communication several times (lines 579-583, 621, 665-666, 803-805) and sums it up when he states: "I'm having difficulty in putting this into words" (line 621).

If RS's *general difficulty in moving from apparent inner meaning expressed through inner speech to objectively adequate explanations* expressed through social or external speech is a manifestation of superficial or improperly internalised mathematical concepts then it would be interesting to imagine how RS would have approached or managed the problem without the use of the graphic calculator. If RS's hand curve-sketching techniques are similar in level to either his weak manipulative skills (discussed above) or to his apparently shaky mathematical conceptions, it can be hypothesised that without the assistance of a graphic calculator such a student would struggle enormously in even establishing the framework within which to answer any of the given questions.

5.2.3 RP INTERVIEW

The Student

RP is an Indian male student who has been in the experimental group for the entire year. RP obtained an A-grade for matric mathematics the previous year and performed very well in the Maths 100 course obtaining a class record of 77% and a final mark of 87%. RP is registered for a B.Sc. majoring in Actuarial Science.

RP is a shy student who seems to find it difficult to make eye contact. Although RP is a diligent and pleasant student, his participation in the tutorial period was limited probably as a result of his shyness. During the interview RP was particularly awkward and giggled nervously from time to time.

An Analytic Interpretation

This interview can be split into two broad sections: in the first section (lines 19-181, events II to VIII) the student chooses which tools or signs of mediation to use; in the second section (lines 181-287) the student attempts to respond to the semiotic challenge of the interviewer by using an alternate representation (sign) to interpret and explain the mathematical problem.

In the first section, ***the graphic calculator functions effectively as a tool for amplification***: using this tool, RP is able to draw both the graphs of F and g quickly and efficiently (lines 50-79). This student has ***well-developed algebraic skills*** in that he is able to easily explain the different aspects of the problem using algebraic techniques. He acknowledges that the polynomial graphs can be used for verification but, despite two instances (lines 100-101, lines 155-166) in which the interviewer suggests he give an alternate explanation, the student ***steadfastly sticks to his algebraic explanation***. At most the student ***uses the graphical representation of F as a polynomial to confirm*** that F(6)=0 (lines 95-99) and to explain why F(7)<0 (lines 123-128).

In the second section, and at the request of the interviewer, the student attempts to find a link between the different representations. But, despite a lot of assistance and explicit suggestions by the interviewer (lines 181-190, lines 210-224, lines 225-226, lines 234-263 and lines 271-273) the student ***never does actually establish an adequate link between the area under the graph of g*** (a graphical concept) ***and the function F*** (a graphical or algebraic concept). Clearly the ***interviewer is unable to mediate successfully*** in the zone of proximal development of this student in this particular context.

In fact RP explicitly fails to make the link (lines 214-224 and lines 274-287). He ***acknowledges this unresolved confusion*** and inadequate conception when attempting to sort out the algebraic (and incorrect) expectation that a particular unsigned area would have negative value and the graphical (and correct) expectation that the same area would be positive:

284: Me: Can you explain all of this?

285: RP: (giggle) No.

286: Me: You can't?

287: RP: I just... it just seems...

It seems that RP's strong algebraic skills enable him to execute the different steps in the problem but do not enable him to see the link between the different representations. This ***strong algebraic preference*** is acknowledged by the student himself: " you could have checked on your graph...or by using the table... [but] I just prefer to work it out...algebraically" (lines 110-119).

Despite a year of using the graphic calculator RP does ***not appear to have developed a flexibility*** in terms of moving from one representation to another. A valid question is whether RP's internalisation of one semiotic system (algebra) serves to block the mediating function of an alternate system. If this is the case then RP's well-developed algebraic skills may ultimately serve to discourage the student from using alternate representations and signs and thus developing a more flexible approach.

Alternatively it may be that the ***inability of the interviewer to mediate*** in the zone of proximal development is primarily as a result of the social relationship (RP is shy and nervous with the interviewer) rather than as a result of RP's inability to move away from an algebraic semiotic system.

Another interesting observation is that, despite already having a graphical representation of g on the calculator, the student ***when pressed to use the area concept as a mechanism for explanation, draws the graph of g by hand*** (lines 191-198 onwards). This is a concrete demonstration of the Vygotskian principle that the unity of perception and action generates, by internalisation, the internal visual field in which the student can perform mental experiments. A similar instance can be found in the case of VT who resolves her first major problem by moving from the use of the graphic calculator as a tool of semiotic mediation to an actual hand-drawn graph.

5.2.4 EP INTERVIEW

The Student

EP is a confident and articulate white male student who has not been part of the experimental group but nevertheless has owned his own graphic calculator for the previous two years. EP obtained an A symbol for matric Mathematics and for Additional Mathematics. He is a Law student who is doing mathematics because he enjoys it, not because of any degree requirements. EP was originally chosen as a non-graphic calculator student who had attained a very good class record of 87%. That he had been using a graphic calculator throughout the course was revealed at the interview and was a complete surprise to me. EP's final mark was 78%.

An Analytic Interpretation

EP is a very capable student who finds the above problem "extremely easy" (line 429). Throughout the interview EP reveals mathematical conceptions which are ***objectively adequate*** (he is able to explain and link all the different sub-problems) and which, in terms of the ease and skill with which he performs these explanations, appear to be ***subjectively meaningful.***

What is interesting in this interview is the way in which EP uses the graphic calculator as a ***tool for amplification*** when he uses it to check the orientation of a cubic graph with negative leading coefficient (lines 138-148). This frees EP from the task of drawing the graph of the cubic function from first principles and so leaves him free to devote his intellectual energies to more conceptual problems. This is ***an instance of the significant role that the graphic calculator can play in the zone of proximal development*** of a mathematics student. EP does not use the graphic calculator again throughout the interview but rather *chooses to use hand-sketched graphs and algebra for his explanations.* It is suggested that EP uses the graphic calculator only when other signs and tools are unable to perform the role of semiotic mediators in his zone of proximal development. As was often the case with RP and VT, it is interesting to note that ***EP uses hand-drawn rather than graphic calculator generated graphs for his graphic explanations.***

EP's *apparently deep and flexible understanding of the relationship* between differentiation and integration and the different representations of these relationships is evidenced by his explanations and comments in the first half of the interview (lines 83-284). In this section EP uses the hand-drawn graphs of F as a polynomial or the algebraic representation of F to explain the different sub-problems. The *competent use of different semiotic mediators* in EP's explanations may be regarded as assisting in the externalisation of previously learnt material rather than in mediating in the internalisation of new concepts.

EP *does not spontaneously think of the link* between the area under the graph of g and the polynomial F but once he becomes aware of this alternate representation, as a result of assistance from the interviewer (lines 286-354), he is able to use it very easily to explain all the different sub-problems (lines 356-361, 365-370, 396-401, 403-410, 414-422). Thus, unlike RP or RM, EP is *able to use the interviewer as an agent for mediation* in his zone of proximal development.

Also, EP reveals *flexibility* in his mathematical conceptions, being able to move very easily from one representation to another. The fact that he chooses initially to use the algebraic or graphical representation of F probably indicates a *situation and problem-driven preference for sign rather than a general and context-free preference.* As he himself puts it (lines 457-466):

458: EP: so given no direction I generally would choose the easiest path..

459: Me: Right/.

460: EP: .. the most obvious one that appears to me. Um.. perhaps let's say if I'd run into trouble with the method I had chosen I would have looked for some other method.. um.. or given some direction like using the graph of g ..

463: Me: Right./

464: EP: dah, dah, dah .. answer these questions, then I would have you know, adopted that approach because of instruction. But um.. you know, its not that I avoided it, because oops, I didn't know it or wow, I didn't think of that.. its just that I didn't have to...

EP's interview gives access to the thinking of a student ***who has used the graphic calculator for several years***. An interesting aspect of EP's interview is his ***articulation of his thoughts and ideas*** about how and when he uses the graphic calculator. The initial discussion with this student reveals that the student has previously used the graphic calculator as a ***tool for amplification*** (lines 35-37) in that the graphic calculator "speeded up an about 15 minute problem to about 20 seconds" and as a ***tool for cognitive re-organisation***: (lines 7-8) "it helps me visualise and so on and so forth.. and often if.. if I get stuck um.. or something, I've used a graphic approach to try and help me think of a route to solve a problem at last..".

These comments may be used to infer that, although EP only used the graphic calculator once as a tool within the zone of proximal development in this interview, it has certainly been used as a semiotic mediator in previous mathematics learning.

5.2.5 RM INTERVIEW

The Student

RM is a Chinese male student who has been a member of the comparison group for the entire year. His class record was 72% and his final mark was 71%. RM's matric mathematics mark was an A. RM is studying towards a B.Sc. in Actuarial Science. He has never used a graphic calculator, although he has seen other people use them.

RM is a friendly and pleasant student. Despite appearing quite relaxed at the beginning of the interview, he later became agitated.

An Analytic Interpretation.

RM's interview is characterised by the combination of an ***apparently sophisticated conceptual understanding*** suggesting the existence of previously internalised mathematical ideas together with an ***extreme inflexibility*** indicated by RM's seeming inability to move from one semiotic system to another. RM's representation of F as the area under the curve g is objectively adequate and appears to be subjectively very meaningful. However he seems unable to realise that an alternate representation of F as the graph of a cubic polynomial is desirable and acceptable in the context. Thus he is ***never able to adequately establish the link*** between the polynomial F and the area under the graph, g.

Like VT, RM spontaneously and easily uses the area concept to represent F. However ***RM seems unable to move easily from this representation*** to any other graphical representation. This is evidenced by several episodes in which the interviewer attempts to assist by guiding RM towards the realisation of alternate conceptions of F. These ***interviewer mediations*** do assist the student to formulate an alternate algebraic explanation of the properties of F but are largely unsuccessful in terms of assisting the student to draw F as a polynomial. When, in lines 173-175, the ***interviewer asks directly*** for an alternate explanation:

173 Me: Alright/. Could you, you've explained it.. that's fine.. you've used the area under the graph to explain everything.. is there another way you could explain.. most of those.. facts.

RM responds with an explanation using ***an algebraic representation*** of some of the properties of F.

Again, in lines 195-211, when the ***interviewer encourages*** RM to reformulate the relationship between F and g, RM responds by using ***an algebraic representation*** which refers to the area explanation (lines 221-229, 240-247).

Finally, still seeking an alternate graphical representation, the ***interviewer directly asks*** the student to draw a graph of F to which RM replies:

258 RM: Well, what do you mean?

It is ***as if the possibility of sketching a polynomial as a curve is outside RM's zone of proximal development.*** In reply to several other requests to represent F in another way, he states :

290 RM: I can't see any other way.

291 Me: Can't you?

292 RM: No.

293 Me: Is that..

294 RM: Is there another way? **(giggle).**

Later on in the interview, after ***much suggestion and assistance by the interviewer***, RM finally refers to the area in terms of its polynomial expansion. But he calls the function h and seems almost surprised that it is the same function as F. Again he insists on representing this function h as an area.

382 RM: No just.. you're just showing how your area goes. You're just plotting your area on a graph.

384 Me: Right.>

385 RM: Yeah, which would be a function of x.. let's call it 'h'. **(writes $h(x)=2x^2-x^3/3$)**

387 Me: OK.>

388 RM: h would.. was the same as F(x).

389 Me: OK.>

390 RM: OK. $2x^2-x^3/3$. Alright.. differentiate.. **(writing)** would be $4x - x^2$. OK, so your area's shown as that.

Thus RM is unable to respond satisfactorily to the interviewer's semiotic challenge by expressing an alternate graphical representation of F. Thus he is ***unable to ever establish a satisfactory link between the concept of area under a curve and the polynomial expression of that area.*** This ***inflexibility*** may indicate a poor internalisation of the relationship between a function and its integral or it may indicate weak manipulative skills. Certainly RM's response to the different sub-problems when using the area concept is very clear and precise and so appears to be the expression of well-internalised concepts. For example, RM's explanation of why $F(0) = 0$ is accurate and easy.

132 RM: When.. for c) when F(x) = 0 for x, **(reading)** why F(x) = 0 for x = 0. Its because..**(looking at graph of g)...** you're taking the area under the graph from 0 to 0, which is 0.

Similarly he explains all other sub-problems easily in terms of area (lines 125-172) and has little difficulty, after a request by the interviewer, in explaining the concepts algebraically (lines 173-251).

Thus RM's resoluteness in not drawing F as a polynomial ***may indicate a lack of manipulative skills*** necessary for the easy sketching of curves. He himself hints at this possibility at the beginning of the interview:

73 RM: OK. Um.. I just have to draw it.. so it'll.. basically be.. um.. Graphs, I'm not too good at this **(giggle)**. Um.. ya, you see graphic methods.. its easier with a graphic .. because you can just see the graph and you know where the intercepts are. Like now you have to work out exactly where everything goes and...

An interesting question is whether such a student, given a graphic calculator, would exhibit less rigidity and more flexibility. Perhaps the fact that it is so easy to sketch a polynomial on the graphic calculator would have encouraged RM to give such an alternate representation for F? In this way, the graphic calculator may have served not only as a tool of amplification (in simplifying the task) but also as a tool for cognitive re-organisation. The student would have confronted an alternate representation and this semiotic mediation in the form of F as the graph of a polynomial may have assisted in the development of an adequate linking between a function and the area concept.

5.2.6 MK INTERVIEW

The Student

MK is an Indian student who has been part of the comparison group for the entire year. MK obtained a B symbol for matric mathematics. His class record for Maths 100 was 54% but his November examination brought his mark down to 50%. MK's other subjects are from the Actuarial Science cluster.

Since MK had a friend in the experimental group, he frequently arrived at the end of my tutorial class to fetch his friend and so I had interacted with him, in a limited fashion, previous to the interview. He was always polite and pleasant.

An Analytic Interpretation

MK's interview can be split into three main sections. In the first section (lines 47-275) MK executes the different steps of the given problem mainly ***using algebraic and tabular methods***. In the second section (277-390) MK ***reflects*** on what he has done and, using the information that was given in the problem or, to a lesser extent, generated during his 'explaining' of the sub-problems, redraws F as a polynomial (298-317); corrects the notation in his algebraic representation of g (320-325); confirms the algebraic relationship between F and g (345-358) and declares himself satisfied with his answers to parts a), f) and g) of the problem because they are consistent with "the values of c) and e) that were given" (line 386). In the third section (lines 432-494), the ***interviewer attempts, unsuccessfully, to assist*** MK in explaining the problem using an alternate representation.

Throughout the interview MK reveals ***poor manipulative skills*** which may also indicate a superficial internalisation of basic calculus concepts. This is apparent particularly in his sketching of the graphs of both F and g. The use of ***numerical methods in the form of a table*** (lines 88-111, lines 150-190) for the sketching of polynomial graphs is conventionally regarded as inefficient; more so it is inappropriate for a first-year Mathematics students who has been exposed to so many powerful algebraic methods of curve-sketching using the concepts of calculus.

Furthermore, MK ***does not even use the table method successfully***. Initially, with both F and g, he chooses an inadequate domain. With g, this results in an incorrect graph (lines 88-111). Fortunately MK, appealing to previously internalised graphical concepts, realises that a quadratic should result in a parabola although he also incorrectly maintains (before actually sketching the correct curve) that the turning point is at 0.

112 MK: I don't know, I think it would have given me a parabola, when you, eventually. I'm just, because my turning point is at zero. **(mumbling)**.. turning point...

His method of correcting the graph, by adding one more point to the domain is ***naive*** even if it works in this instance:

118: MK: Oh, ok. I used... the values I've used are too few to plot the graph.
120 Me: Right./
121 MK: So I've just included another one.
122 Me: Ok.>

When he sketches F (lines 150-190), he too uses a limited domain and sketches an incomplete curve which stops before it gets to its interesting features such as its xintercept and the local maximum. His choice of domain and initial sketch of F implies that *he has little expectation of what the graph should look like.* Again he is able to improve the graph later in the interview when he plots more points (lines 228-233) in response to certain questions in the problem.

This practice of returning to incomplete aspects of the solution and improving upon the previous solution, which occurs quite frequently in the second section of the interview (see above for details of instances) indicates that ***some learning or development is taking place*** during the completion of the task. As in the above example, this learning is frequently ***guided by the connection between the actual data contained in the questions of the problem with MK's graphical representation of this data.***

MK's ***manipulative and conceptual weakness*** carries over into his search for the x-value at which F attains a maximum. He finds the two turning points of F by calculus (lines 246-252) but then resorts to the rather clumsy technique of comparing y-values (lines 253-260) rather than using traditional calculus techniques to find the maximum. Again his methods work (since F is a continuous function) but are ***extremely inefficient.*** As with the curve-sketching, MK's algebraic methods are inappropriate for a first-year Mathematics student who has been exposed to so many calculus concepts and techniques. MK himself acknowledges this when he is asked whether he can explain the sub-problems in another way.

364 MK: I can't. I seem to have forgotten most of the, the work we've done in the beginning...

366 Me: Ok./

367 MK: .. like points of inflection and a triple derivative. I don't know what they're used for.

368 Me: Right./

370 MK: So, there's no other way I can explain these three here because I know you need those, those, um, those methods to do these three questions here.

373 Me: Ok./

.

.

383 MK: So I don't see any other way I could explain this.

What MK says in the above statement is in fact quite obvious throughout the interview. To discuss the different sub-problems, MK uses ***algebraic methods of substitution fairly adequately.*** But he never attempts to develop his explanations, nor to elaborate on the underlying concepts, nor to relate his explanations to other representations. For example, his discussion of why F(0)=0 ***confirms rather than explains*** this fact:

193 MK: Ok. Explain why F(x) = 0 for x = 0. F(x).. what is F(x)..F(x) is equal to $2x^2$ minus 1/3 x^3, x equal to zero um.. I'd do this by direct substitution. **(writing)** I'd say that, if you substitute x into the equation you would get zero because, um, when you multiply out, well both terms are going to be zero. Alright. **(mumble)** ..

Similarly, in the other sub-problems MK merely ***seeks to confirm what has been asked of him rather than to express any deep conceptual links*** or alternate semiotic explanations.

MK's admission, and indeed the observation that he has forgotten many of the concepts of calculus may mean that many of the concepts were never adequately internalised. This makes it very difficult for the interviewer to assist in his zone of proximal development or even to expect reasonable answers to questions. This comment is not intended to undermine the previous observation that ***some activity did take place in the zone of proximal development as a result of graphic mediation during the second section of the interview***. In the third section, we witness the ***difficulty of the interviewer*** in her attempt to guide MK into using an alternate semiotic system and of establishing a conceptual link between the different representations of F and g.

For example, we have the following ***confused conversation*** in which MK asserts his ignorance and then having been reminded of the concept, claims that he would have solved the problem in that way (when clearly, a few seconds before, he did not):

442 MK: Ya.. F(x) is.. I don't think, I can't really remember this work.

444 Me: Can't you?

445 MK: Ya, not at all.

.

.

451 Me: So if I said to you: could you find the area under the curve g(x) between 0 and 3 would you not be able to?

453 MK: Of g(x)?

454 Me: Yes. Do you know how you would write that expression?

455 MK: Uh, ugh, would you like to show me. I'd like to know something.

456 Me: Alright, ok, I will, I will. Alright let me switch.. ok I might as well leave that on for a little bit. Um, let's do it here. You want to know how you would find the area under g(x). You would, um, the area would be equal to the integral.. **(Interviewer writes area using integral expression)**. This is the area under g(x), $4x - x^2$.

461 MK: So the function of g(x) is $4x - x^2$?

462 Me: Yes.

463 MK: Oh ok, then I would have done that. If you asked for the area that's what I would have done.

But the interviewer *DID* ask for area and he did *NOT* do that. On further questioning MK maintains that he did not know what g(x) was (lines 471-479) despite the fact that earlier he writes and says: "Um, g(x) is going to equal to $4x - x^2$" (lines 334-335). These contradictions in conversation probably are indications that MK is using inner speech and that he is ***unable to transform this inner speech into external speech***. This inability may reflect the shallowness of his internalisations of basic Calculus concepts. Perhaps under less intense or stressful conditions or given different interventions in the zone of proximal development, MK may have coped better with explaining the different sub-problems.

Of course it would be interesting to see what effect the use of a graphic calculator would have on the conceptions and manipulations of such a student. Possibly, by liberating the student from the actual task and stress of drawing the graphs by hand, this student would have been more mindful of the different concepts and links between the graphs of F and g. Alternatively given MK's poor use of concepts and inadequate manipulative skills perhaps the graphic calculator would not have assisted in the solution and explanation of this problem; conversely it may have served to distract such a student.

5.2.7 JB INTERVIEW

The Student

JB is a white middle-class female student who obtained 66% for her class record and 64% for her final mark. These are good but not brilliant marks. Her matric maths grade was an A. JB's other subjects are Computer Science and Computational and Applied Mathematics.

JB was a student in the comparison group throughout the year. Although I did not often have cause to interact with her, when I did, she was always pleasant and co-operative.

An Analytic Interpretation

JB initially appeared most uncomfortable in the interview situation. She expressed a ***lack of confidence in her ability to perform adequately***: "speaking aloud, its not me" (line 17). Throughout the interview there are instances in which JB's ***speech is confused and uncertain yet her actions are reasonable and mathematically acceptable.*** For example, in lines 251-253 she says: "the small one is to large numbers and then back again... it means that its going to be a slow slope, increasing and then decreasing again...". This is, of itself, a meaningless statement, yet JB is managing, unlike any of the other students interviewed, to infer the graph of F directly from its derivative. (That this inferring of the graph of F from its derivative is a highly inefficient and inappropriate technique for sketching this function is a different issue which I discuss later).

Thus JB's ***incoherent mumblings may be a manifestation of "inner speech".*** This speech, which occurs frequently during JB's interview, may serve to assist JB in internalising various of the concepts required for the explanation of the mathematical problem. Again, an examination of this speech is beyond the scope of the report.

Of particular interest in this interview is JB's ***poor manipulative skills*** as evidenced by her inability to use appropriate and efficient techniques when representing the functions F and g . This is despite the ***frequent use of graphical signs*** as effective mediators in her zone of proximal development. ***Initially*** JB draws g in an extremely ***naive way by plotting points*** (lines 106-131). However she ***soon displays a reasonable conceptual understanding*** and accordingly some measure of internalisation of basic calculus concepts when she realises that since g is the derivative of F the graph of F may be inferred from that of g. Despite being unable, at first, to actually and successfully use this conceptual link in deducing F from g (lines 132-153) or even of plotting F point-by-point (lines 153-180) JB does indicate prior internalisation of some graphical concepts (lines 181-188):

181 JB: That's wrong there..

182 Me: Why do you think that's wrong?

184 JB: .. because of x^3 .. to my knowledge it shouldn't do that.

185 Me: What should it do?

186 JB: It should cut on.. like that one there.. like so..**(draws very rough sketch of general shape of cubic graph).**

187 Me: OK>

188 JB: Sorry, **(laughing)**.. like a cubic graph.

189 Me: That's fine.

190 JB: Like that.

JB then proceeds to successfully infer the shape of the graph of F from the graph of its derivative, g (lines 192-264). The successful inferring of the graph of F from the graph of g (as opposed to using algebraic and calculus-oriented techniques) reveals a ***subjectively meaningful understanding*** of the relationship of the derivative to the function. But as a technique of curve-sketching it is ***extremely inefficient and thus inappropriate.*** The algebraic techniques of calculus which are implicit in JB's approach (for example, in lines 330-334) are a far more efficient and objectively suitable tool for drawing the graph. Overall, JB's ***poor manipulative skills***, as revealed by her graph-sketching performance, serve to retard her performance and progress in this problem. Thus it may be hypothesised that a graphic calculator would greatly benefit such a student by removing the tedium of the actual curve-sketching and in this way, serve as a tool of amplification in the zone of proximal development.

Despite this seeming inflexibility in the context of curve-sketching, JB demonstrates ***a facility in moving between semiotic systems*** when she explains the properties of the function F (lines 265-346) using both algebraic and graphical representations. For instance, in discussing why F(7)<0, JB offers the following graphical and algebraic explanation (lines 304-317):

304 JB: if you join it from .. negative infinity.. you know that's your x cuts .. from, well, I don't know.. that's the way the graph's going, because you know from the derivative that's the way the graph's going to go, so you know its not going to come back up.

307 Me: OK.>

308 JB: Must I explain more?

309 Me: If you can.

310 JB: Um.. (pause).. for um, I don't know, also if you're putting anything into the equation you've got, I don't know.. that's the long way to go about it.. um..

312 Me: Into what equation are you going to put it?

313 JB: Into the F(x) equation.

314 Me: OK./

315 JB: If you put.. say anything greater than 6 in..

316 Me: Right/.

317 JB: .. you know from.. the way that this is that the.. the minus is going to take over, I mean that's a .. .. you can put every point in and prove that to yourself.. you know.

319 Me: OK.>

JB *does not come to explicitly realise the link* between F and the area under g *without a fair amount of graduated assistance from the interviewer* (lines 353-389) in the zone of proximal development. But once the link is acknowledged, the student reveals a *flexibility* in moving from the polynomial-oriented explanations of the sub-problems to explanations using the concept of area. She claims:

491 Me: Does this make sense to you?

492 JB: Ya, it does (giggle) .. completely.

493 Me: Which actually so far makes more sense.. the first..

494 JB: this..

495 Me: .. or ..

496 JB: .. the second way, ya (giggling).. if you look at it like that.

497 Me: OK./

498 JB: At least you can explain it. It makes more sense to try and explain it.

Of course, it may be that she is merely trying to please the interviewer by stating that she prefers this representation. On the other hand, her explanations are much clearer and she is *able to use more social (external) speech*, which may indicate a successful transformation of the external to the internal, ***via the mediation of the area concept***, and a corresponding ability to externalise her thinking more effectively. Compare her explanation of why $F(7)<0$ (lines 487-490) to that given earlier (lines 304-317):

487 JB: Less than 0, because this is carrying on.. so you've cancelled that out.. you've cancelled them out and this is carrying on going in a negative area.. so you get a negative area..

489 Me: OK>

490 JB: .. so this y will be less than 0.

Although the explanation is still not particularly lucid or persuasive, it is far more cogent than that given previously in lines 304-317.

Overall JB reveals a ***strong use of graphical signs as mediators in her mathematical conceptualisation***. Whether the use of the graphic calculator would diminish or enhance the graphical orientation of such a student is difficult to say. Certainly the use of a graphic calculator would speed up the activity of sketching the graphs which took JB so long. And so it may be hypothesised that the use of a graphic calculator would have left JB a lot more time and mental energy to devote to the actual mathematical problem. In this way the tool may have functioned as an amplifier in the zone of proximal development.

5.3 SUMMARY OF INTERVIEWEES' BIOGRAPHICAL CHARACTERISTICS

STUDENT	EXPERIMENTAL (E) or COMPARISON (C) GROUP?	GRAPHIC CALCULATOR (GC) or NOT (N)?	DATE of INTERVIEW	GENDER	RACE	MATRIC MATHS SYMBOL	CLASS RECORD	FINAL MARK
VT	E	GC	19/ 9/ 95	F	W(hite)	A	76%	69%
RS	E	GC	19/ 9/ 95	M	W	A	53%	58%
RP	E	GC	20/ 9/ 95	M	A(sian)	A	77%	87%
EP	N*	GC	21/ 9/ 95	M	W	A	87%	78%
RM	C	N	27/ 9/ 95	M	A	A	72%	71%
MK	C	N	27/ 9/ 95	M	A	B	54%	50%
JB	C	N	5/ 10/ 95	F	W	A	66%	64%

N*: This student was neither in the experimental nor the comparison group.

6 RESULTS FROM TASK

The quantitative aspect of this study consisted of the data from the Task. Sixty-four Mathematics I students wrote this Task. Their answers to the five mathematical questions in the Task were graded and these marks constituted the core of the data.

The general approach was simple. The sixty-four students who wrote the Task were separated into two groups according to whether they had access to the graphic calculator (the experimental group, N=16) or not (the control group, N=48). The data, consisting of their scores on the Task's questions, singly and in various combinations, and also the final Mathematics I mark (see below for details of these variables) were then analysed to test the null hypothesis, H_0, for each variable. The null hypothesis for each variable was that the two groups came from the same population. Also an analysis of these two groups with respect to their matric ratings was performed to see whether they originally came from the same population or not. Since the experimental group had been randomly chosen at the beginning of the year, it was expected that it would not differ significantly from the control group on the matric ratings.

The data was analysed on the computer using the packages SPSS and SAS-LAB. Although it was unnecessary to use both packages (since they both gave the same answers) SAS-LAB also offered advice on interpretation of the results which was useful.

The analysis followed a straightforward procedure. First, it was necessary to decide for each variable whether the two-tailed t-test could be applied meaningfully to the data or whether a non-parametric test should be used. Since the sample sizes were reasonable, in accord with current statistical practice, this decision was based on two considerations: the p-value in the Wilks-Shapiro Test which gave a probability that the data came from a normal population and the look of the data according to the normal probability plot. For example, there was weak statistical evidence ($p = 0,0477$) that the distribution of the matric rating for the experimental group was normal; however given the respectable sample size ($N = 16$) and the general look of the data, it was decided that it was acceptable to apply the t-test to this data. If the data could not be regarded as normally distributed then several non-parametric tests, such as the Wilcoxon-Rank Test and the median 1-way analysis (Chi-Square approximation) were applied.

The results of the statistical analysis are summarised in TABLE I.

In order to understand this table, the way in which several variables have been calculated, needs to be clarified. For more details about these variables, in particular the questions in the Task, see Chapter Four.

The marks for each question in the Task, for example QUESTION 1, are given as a percentage.

YEAR MARK consists of an average of marks from class tests and final examination, all written without the graphic calculator.

PERTOT is the sum of scores for each question in the Task, each weighted equally, given as a percentage.

AMPTOT is the sum of the scores of three questions in the Task (Questions 1, 3, 5) given as a percentage, where each score is weighted equally. These questions were designed to test the amplifying effect of the graphic calculator. This effect is defined in Chapter Four and elaborated on in Chapter Seven.

ORGTOT is the sum of the scores of the other two questions in the Task (Questions 2 and 4) given as a percentage, where each score is weighted equally. These non-algorithmic questions could not be solved directly with the graphic calculator. They were designed to test whether the use of the graphic calculator over the year had resulted in cognitive re-organisation by promoting the internalisation of the graphic calculator semiotic system. Again cognitive re-organisation is defined and elaborated on in Chapter Four and Chapter Seven respectively.

ATTITUDE refers to a variable in which the student rated the Task in terms of its difficulty. The student classified the task into one of five categories ranging from Very Easy (1) to Very Difficult (5).

GROUP:	VARIABLE	SAMPLE SIZE	MEAN	VARIANCE	NORMAL DISTRI-BUTION	STATISTICAL EVIDENCE OF DIFFERENCE BETWEEN MEANS	ACCEPT H_0 AT SIGNI-FICANCE LEVEL: A=0,05.
CONTROL	MATRIC RATING	44	30,3	29,5	YES	NO	YES (TWO-TAILED T-TEST)
EXP		15	32,7	37,4	YES		
CONTROL	YEAR MARK	48	58,8	355,4	YES	NO	YES (TWO-TAILED T-TEST)
EXP		16	61,6	453,5	YES		
CONTROL	QUESTION 1	48	19,6	808,3	NO		YES (WILCOXON RANK-SUM TEST)
EXP		16	32,5	1486,7	NO		

GROUP:	VARIABLE	SAMPLE SIZE	MEAN	VARIANCE	NORMAL DISTRI-BUTION	STATISTICAL EVIDENCE OF DIFFERENCE BETWEEN MEANS	ACCEPT H_0 AT SIGNI-FICANCE LEVEL: A=0,05.
CONTROL	QUESTION 2	48	30,8	1412,1	NO	NO	YES (WILCOXON RANK-SUM TEST)
EXP		16	48,7	1331,7	NO		
CONTROL	QUESTION 3	48	34,2	1074,6	NO	NO	YES (WILCOXON RANK-SUM TEST)
EXP		16	47,3	2033,2	NO		
CONTROL	QUESTION 4	48	47,2	1505,1	NO	NO	YES (WILCOXON SCORE)
EXP		16	62,5	1314,8	NO		

GROUP:	VARIABLE	SAMPLE SIZE	MEAN	VARIANCE	NORMAL DISTRI-BUTION	STATISTICAL EVIDENCE OF DIFFERENCE BETWEEN MEANS	ACCEPT H_0 AT SIGNI-FICANCE LEVEL: A=0,05.
CONTROL	QUESTION 5	48	17,9	1505,1	NO	NO	YES (WILCOXON RANK-SUM TEST)
EXP		16	23,8	425,0			
CONTROL	PERTOT	48	30,0	338,9	YES	YES	NO (TWO-TAILED T-TEST)
EXP		16	43,0	592,2	YES		
CONTROL	AMPTOT	48	23,9	320,0	NO	NO	YES (WILCOXON RANK-SUM TEST)
EXP		16	34,5	622,1	YES		

GROUP:	VARIABLE	SAMPLE SIZE	MEAN	VARIANCE	NORMAL DISTRI-BUTION	STATISTICAL EVIDENCE OF DIFFERENCE BETWEEN MEANS	ACCEPT H_0 AT SIGNI-FICANCE LEVEL: A=0,05.
CONTROL	ORGTOT	48	39,0	1873,3	NO	NO	YES (WILCOXON RANK-SUM TEST)
EXP		16	55,6	970,0	NO		
CONTROL	ATTITUDE	44	3,4	0,5	NO	NO	YES (WILCOXON RANK-SUM TEST)
EXP		15	2,9	0,9	NO		

TABLE I

As expected, the means of the two groups does not differ significantly on MATRIC RATING.

In fact there is a significant difference in means between the two groups of students on only one variable, PERTOT. Thus the statistics indicate that the students with the graphic calculator performed significantly better compared to the students without, when their marks are summed over all the questions in the Task. When each question is taken separately, this difference between means is not significant.

In the sets of questions concerning the amplifying effects (AMPTOT) and in the set of questions concerning the re-organising effect (ORGTOT), the difference between means for the experimental and the control group, although present, is not significantly different for either effect. However it may be noted that the absolute difference in means (10,6% and 16,6% respectively) is fairly large in each case and favours the group with the graphic calculators both times.

The two groups do not differ significantly over the Mathematics I year mark. Here the difference in means, 2,8%, which again favours the experimental group, is relatively small.

In Chapter Seven I will interpret these statistics with reference to my research question. In addition, I will argue that the institutional and cultural setting of this study undermined the effects of the graphic calculator and so diminished the possibility for statistical significance on AMPTOT and, to a lesser extent, ORGTOT and the YEAR MARK.

7 DISCUSSION

7.1 INTRODUCTION

In this chapter I wish to link various themes and motifs which appear in the analyses of the interviews and the Task. Using these links I will move towards a more abstract and generalised description of the role of the graphic calculator in the zones of proximal development of first-year University Mathematics students.

In order to discuss how the graphic calculator functions as a tool for learning, it is necessary to define an appropriate unit of analysis within a socio-cultural framework. Vygotsky saw mediated action as the fundamental mechanism which linked the external social world to the internal human mental processes. Thus, in accord with the Vygotskian tradition, I will use the unit of mediated action, explicitly or implicitly, to discuss the role of the graphic calculator in the zones of proximal development utilising the data from the interviews and the statistics from the task as support and evidence. At times it may be necessary to refer to specific aspects, consequences or the context of this unit of analysis, rather than to regard the unit as an irreducible category only.

This discussion is divided into three inter-linked sections:

First, a description of how and when particular students use different mediational means in their activities, across the interviews, is given. In particular, how and when the learner uses the graphic calculator is highlighted. From this description, an argument for the consideration of hand-drawn graphs and calculator-generated graphs as distinct and separate semiotic systems is developed.

Using the above discussion as well as the data from the interviews and the Task, an attempt is made to explicate the particular role of the graphic calculator as a tool within the zones of proximal development of the students. This role is discussed in terms of a metaphor whereby the graphic calculator amplifies or stretches the zone by minimising clutter and in terms of the way in which the graphic calculator mediates in the transformation of the external activity to the internal plane of consciousness. In addition, a (mainly theoretical) metaphor whereby the graphic calculator provides additional entry points to the zone of proximal development, is developed.

Finally, in order to reflect on the implications and limitations of this study and consistent with a Vygotskian paradigm, the social and institutional milieu within which the research takes place, is discussed. In particular the relationship between the socio-cultural context of the study and the way in which students choose and use semiotic mediators is explored. This leads to a consideration of which sign systems are 'privileged' (Wertsch, 1990a). Also the specificities and peculiarities of the study are discussed in terms of methodological constraints and researcher influences.

Although most of the theoretical concepts which are used in this discussion have been defined previously in this report, it is sometimes necessary to elaborate on some of these concepts and to re-locate them specifically within the actual parameters of this study.

7.2 THE TOOLS OR SIGNS OF MEDIATED ACTION.

The analyses of the interviews show how individual students use different semiotic systems to mediate in the zone of proximal development at diverse points throughout their interviews. This suggests that the use of any specific semiotic system is not uniform but depends on the zone of proximal development of the individual student in relation to the particular task within the interview context in the specific institutional setting.

With respect to the graphic-calculator students it is interesting to compare RS and EP who represent two ends of the spectrum in terms of performance and choice of semiotic mediator. EP uses algebra or hand-drawn graphs entirely except for one instance in which he uses his graphic calculator to decide on the orientation of a general cubic polynomial. As was discussed previously, EP's use of a particular semiotic system may be regarded as assisting in the externalisation of previously learnt material rather than in mediating in the internalisation of new concepts and EP sails through the problem smoothly, easily moving from one semiotic system (algebra or graphs) to another. On the other hand, RS struggles to solve the problem and to express himself. He uses the graphic calculator extensively for generating graphs and tables and supports his graphical assertions with algebra. Paradoxically, RS's mumbled talking (a manifestation of inner speech?) helps to reveal his thinking which appears to be defined and shaped directly by the particular semiotic system he is employing. Thus RS's interview demonstrates clearly how a student uses a semiotic system as a tool of mediation for transforming the external activity into the internal plane of consciousness. A particular example of how RS uses the graphic calculator to shape his thoughts is given later in this discussion.

The other two graphic calculator students, VT and RP, use different semiotic systems at different times throughout their interviews as well, although RP clearly favours an algebraic mediation and VT favours a graphical mediation. Also there are strong contrasts between these two students in terms of flexibility. VT responds fairly comfortably to the semiotic challenges of the interviewer to use alternate semiotic systems to shape and define her internalisations and the expressions of these whereas RP seems quite unable to use certain graphical signs, such as that of area, to mediate in the transformation of the external activity into the internal plane of consciousness.

An interesting motif which emerges from the interviews of the graphic calculator students concerns those instances in which the students choose to use hand-drawn, rather than calculator-generated graphs. Three out of four graphic calculator students (RP, EP and VT) all resort to hand-drawn graphs as opposed to calculator-generated graphs when required to explain or use some concept with which they are grappling during the interview. This demonstrates the Vygotskian principle in which

> "the unity of perception, speech and action which ultimately produces internalisation of the visual field constitutes the central subject matter... of the origin of uniquely human forms of behaviour" (Vygotsky, 1978: 26).

This principle is important in that it enables us to recognise that students use hand-drawn graphs and calculator-generated graphs *differently*. Not only does the hand drawing of the graph enable the student to obtain a unity of action and perception, but additionally the transparency of the procedure of hand drawing the graph may be a crucial aspect for the potential internalisation of the symbol system.

> "Indeed, another important condition is that the technological candidate for internalisation be explicit in its operations. Programs such as spreadsheets... could not be internalised because much of what they accomplish is carried out 'implicitly' *for* users but not *with* them. A tool that is a candidate for internalisation ought to show what it is that has been carried out such that the user could emulate the pro-

cedure and reconstruct it in his or her mind. One cannot reconstruct or emulate procedures that are carried out in hiding." (Salomon, 1990: 198).

Unlike the procedure of hand drawing the graph, the graphic calculator is not explicit in most of its operations. To draw a graph on the calculator the user enters a formula and the tool generates the relevant graph without indicating its derivation. The user is responsible only for defining an appropriate window and scale within which to view the graph.

Thus calculator and hand-drawn graphs may be very different in their effects. For this reason they are treated as separate symbolic systems throughout this discussion.

Like the graphic calculator students, the individual non graphic calculator students reveal different preferences for semiotic mediators in their solution and explanation of the problem. RM, like VT, predominantly uses the graphical representation of area as a mediating sign in the internalisation of the problem and its expression. However, unlike VT, he is not able to use the more conventional sign of the graph of a polynomial as a mediating sign in his zone of proximal development despite a lot of assistance from the interviewer. In contrast, JB, also a non graphic calculator student, uses the graphs of the polynomials most frequently to shape and define the internalisation of the external activity. Although JB struggles with this internalisation as revealed by her extensive use of jumbled and disconnected inner speech, she is ultimately able to move from one representation to another. This indicates a flexibility in that she is able to use mediating signs from the different semiotic systems in her zone of proximal development.

In contrast to all the other students, aspects of this problem seems to lie beyond the zone of proximal development of the other non graphic calculator student, MK, and these aspects are not always satisfactorily addressed during the interview. MK solves the problem using algebra and graphs of polynomial representations but his explanations are never entirely adequate and frequently he has to return to an explanation or solution to fill in the gaps. I would suggest that MK does not, in fact, completely realise the transformation of the external activity to the internal plane of consciousness no matter which mediating sign he uses during the interview although his use of algebra as a mediating sign is objectively more adequate than his use of the graphs.

Overall it is not possible to describe a single pattern of usage of alternate semiotic systems over different students. Which student uses which sign system and when, is determined by a wealth of factors, some psychological and others institutional and social. These factors are discussed later in the chapter.

7.3 THE ROLE OF THE GRAPHIC CALCULATOR IN THE ZONE OF PROXIMAL DEVELOPMENT

In this section I wish to elaborate on the possible role of the graphic calculator as a tool for semiotic mediation. In addition I hope to contribute to the general theoretical debate concerning the place of technology, and in particular the graphic calculator, in a Vygotskian theory of mind.

7.3.1 THE GRAPHIC CALCULATOR AS A TOOL FOR MEDIATION AND AMPLIFICATION.

The role of the graphic calculator as a tool of mediation needs to be examined in terms of the way in which the student uses the technology to shape and transform the external into the internal.

In addition it is worthwhile to consider Pea's (cited in Salomon: 1990) distinction between the amplification effects of technology and the re-organisation effects of technology. These effects are described in Chapter Four and elaborated on here. They are then applied specifically to the graphic calculator using the data from the interview and the Task as evidence or not of their applicability.

The amplification metaphor is a more precise and specific version of the cultural amplification concept of Newman, Griffin and Cole (1989) which describes the existence of intellectual tools from the social world, such as written interpretations in categorical or thematic terms, which assist in the learning of new material. According to the amplification metaphor which I have developed, the graphic calculator amplifies the zone of proximal development by removing cumbersome and time-consuming tasks from this zone. The user thus has more space in his/her zone to perform conceptually demanding tasks with greater effectiveness and ease.

I maintain that the interviews suggest that the graphic calculator may function frequently as an amplifier of the zone of proximal development for an individual learner by "circumventing low-level limitations of human computational ability" (Perkins, 1985). The ease with which the student can gather the graphical information into his/her zone illustrates the amplification power of the tool. The zone of proximal development remains uncluttered by tedious and time-consuming activities and so the student is free to engage in higher-order thinking. Looking at the data, the students with access to the graphic calculator were able to generate the graphs of F and g relatively effortlessly and quickly. This was in marked distinction to the three non graphic calculator students. Two of these (JB, MK) struggled and took a long time and many manipulations before generating the graphs of F and g; the third (RM) did not even get to represent F as a polynomial graph.

The cognitive re-organisation metaphor involves the restructuring and re-organisation of the internal plane of consciousness via the internalisation of a new symbol system.

Since cognitive re-organisation effects are observed more easily when the student is not using the graphic calculator specifically, such effects are more easily inferred from the questions on the Task which did not require the use of the graphic calculator. Nevertheless, an attempt is made to infer these effects in the Interview situation where relevant. However I maintain that the interviews provide little evidence of a cognitive re-organisation role for the graphic calculator. In fact I contend that the role of the graphic calculator as a tool for cognitive re-organisation is limited by the socio-cultural context (discussed later) in this study and also by the hidden nature of the processes of the graphic calculator in general. As discussed previously, the explicit modelling of the actual graph-generating processes may be a crucial attribute for internalisation of the semiotic system.

> "Interactions in which thinking processes are made *explicit*, or modelled, seem to provide important fostering conditions for learning to think well and transfer what one knows to new problem contexts" (Pea, 1987: 655).

Although there are many instances of the graphic calculator being used as a tool with which to think (a tool for semiotic mediation) I suggest that there is little evidence from the interviews or the Task for the role of the graphic calculator as a tool which promotes the internalisation of an alternate semiotic system.

Since the amplification and more general mediation effects of the graphic calculator are inter-related and interdependent, it is not always possible to neatly distinguish between them. For example, the intellectual environment (the zone) frequently needs to be amplified before mediation, in the form of the shaping of thinking, can take place. Thus these two effects are discussed together. Additionally any possible cognitive re-organisation effects are noted where relevant.

One particular episode in VT's interview highlights the way in which the graphic calculator functions as a mediating tool only after it has amplified the zone of proximal development. VT initially represents g on the calculator and attempts to represent F as the area under the curve g. Given that the upper limit of the definite integral (the area) is a variable this proves too difficult and she abandons this attempt. After a suggestion by the interviewer she generates F as a polynomial on the calculator fairly easily. Then using this representation of the graph of F, drawn on the same system of axes as g on the calculator, she establishes the link between the polynomial, F, and the area under g. Using these calculator representations of F and g together with calculator functions like ZOOM, VT proceeds to elaborate most successfully on the link between F and g. This episode illustrates both the amplifying power and the mediating role of the tool since VT uses the calculator directly to establish the link. As she herself says: "I just saw it with the calculator here...". Possibly I could argue that this episode also reveals how the graphic calculator promotes cognitive re-structuring: VT may not have established the link between these mathematical concepts at all (presumably a link which will transfer to other situations even when not using the graphic calculator) if she did not have the use of the graphic calculator. But this is mere speculation with this evidence.

RP's interview also demonstrates how a learner may use the graphic calculator to amplify his/her zone of proximal development. RP sets the stage for the solution of the mathematical problem by using his graphic calculator to successfully generate the graphs of F and g. Although RP seems to prefer an algebraic mediation to a graphic calculator mediation, he uses the calculator graphs to confirm several of his algebraic manipulations. What is important in this interview is not the extent to which RP prefers a graphical or algebraic mediation but rather that RP has the graphs there when he needs them to support his algebraic mediations. RP does not waste time or energy in drawing the graphs from first principles and so he can conserve his intellectual energies for the algebraic thinking which he seems to prefer. He can then use the graphs for verification as he needs. Other than this amplifying role, there is no apparent demonstration of RP using the graphic calculator generally to internalise concepts. Thus the role of the graphic calculator as a mediating sign to shape thinking is limited to an amplification function in this interview.

RS uses the graphic calculator as a mediating tool in general and an amplifier of his zone of proximal development in particular. Initially RS struggles with the manipulation of the graphic calculator (as he does with all computational procedures throughout the interview) but eventually generates the correct graphs. Certainly RS's poor calculator manipulative skills inhibit the amplification potential of this tool although RS's non-calculator manipulations are even more cumbersome and tedious. Given these poor mechanical skills, it is possible that the task of hand-drawing the graphs of F and g would have filled RS's zone of proximal development and left little room for other learning. It is in this comparative non-cluttering of the zone of proximal development, that the graphic calculator shows its effect as an amplifier.

The graphic calculator seems to empower RS: he uses it much more than the other students, moving back and forth between the graphical information and the tabular information. RS's mumblings, despite the incoherencies and contradictions which are all characteristics of inner speech, frequently show how he uses the graphic calculator to mould and define his internalisations. For example, his explanation of why F(7) < 0, shows him using the graph to shape and mediate his thoughts:

> **RS:** And because I've already shown that um, where F, oh dear.. um. OK, I've already got the maxima and minima from the graph, so I know that from the.. that there are only, I know that there are only two turning points.
>
> .
>
> .
>
> **RS:** In other words, there's only two turning points. So from a point..
>
> .
>
> .
>
> **RS:** ..where, where x, where y is equal to zero, where x = 6, its while its increasing, the graph is decreasing. So I could just assume that, but that's what I'm doing at the moment, I suppose. Um, but it's for any x greater than 7 or 7, or any x greater than 6 for that matter.
>
> **Me:** What, what will happen, for any x greater than 6?
>
> **RS:** The graph will be below the x axis.

This episode illustrates how a calculator-generated graph may be used to mediate in the internalisation of a mathematical concept. As described in Chapter Five, there are several other instances in RS's interview which likewise show the graphic calculator functioning as an effective tool of semiotic mediation.

EP uses the graphic calculator just once in his interview and in this instance it functions as an amplifier. By using the graphic calculator to check the orientation of a cubic polynomial, EP frees himself from the time-consuming task of drawing F from first principles. He thus creates more space in his zone of proximal development for the more intellectually challenging tasks. Although we don't witness EP using the graphic calculator as a cognitive re-organiser during this interview, his discussion of how he has used the graphic calculator before:

> "I just found it very useful because um.. it helps me visualise and so on and so forth.. and often if.. if I get stuck um.. or something, I've used a graphic approach to try and help me think of a route to solve a problem at last.."

suggests that the graphic calculator has served to re-organise or restructure his internal plane of consciousness in previous learning episodes.

The interviews with the non graphic calculator students stand in sharp contrast to the interviews with the graphic calculator students. As mentioned previously, both JB and MK spent a lot of time, effort and intellectual energy drawing and correcting the graphs of F and g. For both these students these actions serve to clutter the zone of proximal development so leaving less space in which mathematical conceptions can develop. JB eventually manages to solve the problem using a variety of semiotic systems but it is suggested that her task would have been that much simpler if she had not had to generate the graphs of F and g by hand. MK never really explains any of his reasoning in a satisfactory way and it remains unclear how much of the mathematics he understood. Although RM sketches the parabola fairly effortlessly and is able to discuss the different aspects of the problem fairly easily, he never manages to draw the graph of F as a polynomial and so never forms the link between F and the area representation. In this way, his mathematical experience is diminished and possibly an opportunity for cognitive re-organisation is lost.

The data from the Task supports most of the ideas discussed above, but it undermines others:

- The statistical analysis of the Task data shows that the use of the graphic calculator *significantly effects overall performance* taken over the sum of scores for the mathematical questions in the Task. The significant difference in performance over all the questions in the Task, between those students who used a graphic calculator (who performed better) and those students who did not (who performed worse) may be evidence of several mechanisms: the ability of the calculator to amplify the zone by circumventing cumbersome and tedious tasks and its general role as mediator (described above) or its ability to open additional entry points to the zone (described below).

- In contrast to the qualitative data, there is *no statistically significant difference* between the group with the graphic calculator and the group without, taken over the sum of scores for those questions *related to the amplification powers* of the graphic calculator alone. However there is a fairly large difference between the mean of the group with the graphic calculator and the group without (10,6%), with the graphic calculator students scoring much higher in this category.

- Congruent with the qualitative data, there is *no statistically significant difference* between the group with the graphic calculator and the group without, *with respect to the cognitive re-organisation function* despite a large (16,6%) difference between the mean of the experimental group and the other students over the sum of scores on the cognitive re-organisation questions.

Furthermore the fact that the significant difference in marks taken over all the questions in the Task does not carry over to the ordinary year mark (evaluated when students are not using the graphic calculator) supports the notion that the graphic calculator does not contribute to a cognitive re-organisation role in this study.

I will argue in the next section that the institutional conditions under which the calculator was used blunted its effects and so undermined the possibility for statistical significance with respect to its amplification and to a lesser extent, its cognitive re-organisation consequences.

7.3.2 DOORS TO THE ZONE OF PROXIMAL DEVELOPMENT

An important theoretical proposition which emerged from my study was that the graphic calculator may provide additional entry points to the zone of proximal development by giving the learner easy access to the graphical representation of a function. This access to the zone may have been impossible or difficult without the calculator since the student may not yet have developed the analytic concepts and skills necessary to draw the graph by hand. Thus, analogous to the way in which the child is able to use the word as a reference in social interaction before fully understanding its meaning (Vygotsky, 1962) so the mathematics student may be able to use the graphical information on the calculator before fully understanding its mathematical derivation or analysis.

"[Vygotsky's] argument was that agreement on reference provides an 'entry point'

> for children to participate in social interaction with more experienced members of a culture. ....Although it is tempting to attribute an understanding of meaning to children... the appearance of new words marks the beginning rather than the end point in the development of concepts" (Wertsch, 1985: 169).

In an equivalent sense, it is suggested that the student can use the *graph as a reference* to participate in mathematical activities before fully understanding the full meaning of the graph. Thus the generation of new graphs may mark the beginning rather than the end point in the development of mathematical concepts.

Since the functions in the interview problem were fairly straightforward, it is probable that the three students, VT, RP and RS, all of whom use their graphic calculators to sketch these graphs, probably could have done so without the graphic calculator. Thus the graphs generated by the graphic calculator in the interviews most likely operate as signs with meaning, not just references, for these students. So the interviews do not generate direct evidence for the graph-as-reference analogy.

However there is some quantitative evidence for this analogy in the Task. This evidence, however, needs to be treated with great care as discussed here. Question 3 requires the finding of $\lim_{x \to 0} (\tan x)^x$. This is a graphically easy task (if one already has the graph of $(\tan x)^x$) but an analytically more complicated task requiring an understanding of the application of L'Hopitals Rule. In the Task, 9/16 (56%) of the graphic calculator students used a calculator-generated graph to assist them in finding the limit, whereas 5/48 (10%) of the non graphic calculator students used a hand-drawn graph. Correspondingly the mean mark for the graphic calculator students on this Question was 47,3%, and the mean mark for the non graphic calculator students was 34,2%. Such a difference in mean mark (although not statistically significant) may indicate that the use of the calculator-generated graph shaped a solution to the problem. However this evidence must be treated with caution since getting the right answer does not necessarily mean that the students were developing or extending any mathematical concepts as a result of using the graph as a reference. In this particular problem, both the analytic method, which is essentially algorithmic and the calculator-graph method could be performed fairly mechanically. Thus in general, further research is required in order to obtain a body of grounded evidence for this graph-as-reference proposition and its corresponding metaphor of the graphic calculator as a tool which opens additional doors to the zone of proximal development.

7.4 THE INSTITUTIONAL SETTING OF THE STUDY

> "The fundamental goal of a sociocultural analysis of mind is to explicate the historical, institutional and cultural specificity of human psychological process. Instead of focusing on aspects of human mental functioning that are assumed to be constant across history, institutional settings and cultures, such an analysis seeks to identify ways in which mind reflects and constitutes a specific sociocultural setting" (Wertsch, 1990b: 71).

In this particular study there are two apparently contradictory contexts of interest. On the one hand, there is the interview situation in which students may have felt pressured to use their graphic calculator. On the other hand, there is the wider institutional setting, in which the use of the graphic calculator may have been regarded as unimportant and inconsequential. After all the graphic calculator was used as an add-on rather than as an integral part of the Mathematics I course.

7.4.1 PRIVILEGED SEMIOTIC SYSTEMS

At this point it is useful to consider Wertsch's (1990a) concept of privileging. This notion contextualises the choice of semiotic mediators within the particular socio-cultural setting of the study. It debunks the idea that each student selected the semiotic mediators which were "somehow more basic, more 'cognitive,' or more important than others" (Wertsch, 1990a: 111).

Wertsch (1990b) maintains that the zone of proximal development, which is defined within a particular socio-cultural matrix, can only be understood if one takes into account a 'definition of situation' which includes the social setting and values. It is this setting, rather than just pure psychological factors, which may elevate one form of mental functioning over another and in this way privilege a particular form of mental operation such as algebraic or graphical reasoning.

In accord with this understanding I maintain that the choice and use of semiotic mediators in the study is partly a reflection of the socio-cultural context so that the selection and use of a mediating system is largely determined by the extent of privileging of the corresponding mode of discourse in that specific setting.

7.4.2 THE INTERVIEW SETTING

It is suggested that some students who were part of the graphic calculator group may have felt obliged to use their calculators frequently in the interview partly as a 'sort-of payment' for their use over the year. In this way, the graphic calculator became a privileged tool in the interview.

In witness to this, it is interesting that EP, the only graphic calculator student who was not part of the experimental group (he had his own graphic calculator from school-days) uses his graphic calculator only once in his interview. In contrast RP, although acknowledging that he prefers to work algebraically, frequently attempts to use his graphic calculator to explain various properties of F. VT uses both the graphic calculator and algebra in her interview but she too waits for 'permission' from the interviewer ("you can use either graphic calculator or paper or whatever...") before moving from the graphic calculator to a hand-drawn graph. RS uses his graphic calculator entirely throughout the interview, never using a hand-drawn graph.

It is not suggested that these students did not find the calculator very useful or powerful, particularly in the initial generation of the graphs; what is suggested is that the general elevation of the graphic calculator as a tool for mathematics in this particular setting may have contributed to the definition of situation and so its use may have been a manifestation of the socio-cultural context rather than of psychological factors alone. Of course, since I (the interviewer) actively probed for alternate representations of the mathematical concepts during the interviews, the significance of the above patterns of privileging in the interview situation was lessened.

7.4.3 THE MATHEMATICS I COURSE SETTING

Although I have shown that the use of the graphic calculator may result in qualitative differences between students with and without this tool, in particular as a tool which mediates internalisation of mathematical concepts and as a tool for cognitive amplification, strong statistical evidence of these effects is missing from this account.

In this section I wish to argue that the institutional and cultural setting of this study contributed towards less meaningful encounters with the technology and so undermined the capacity of the graphic calculator to serve as a tool of semiotic mediation.

In the study the graphic calculator was used by a group of twenty students out of about 400 MATHS 100 students. Since it was used by such a small percentage of the total (5%) it was treated as an add-on rather than as an integral component of the course. It was used to support and verify rather than as a stand-alone tool in its own right. Although the students in the experimental group were urged to use their calculators as frequently as possible, its use was never regarded as compulsory. Also since the use of the calculator was inadmissible during tests and examinations, the situation did not intrinsically motivate students to spend a lot of extra time on their own with the graphic calculator. In fact students may have resisted using the calculator as often or as effortfully as they would have, if its use was main-streamed or compulsory.

In addition, for the last two centuries algebra has been privileged as a semiotic system and it is *still privileged* as the most appropriate mode of mathematical discourse as a result of historical practices and traditions many of which still prevail in the contemporary mathematics classroom.

At this junction, a brief description of the rise and fall of the visual in mathematics is summarised. In ancient Greece geometers drew diagrams in the sand. Similarly in the seventeenth century the Calculus developed out of geometric efforts to solve problems of tangents, rates, lengths and areas and Isaac Newton used loose explanations and geometric intuitions rather than analytic arguments when explaining the Calculus. However the intuitive and often visual style of mathematics fell into disrepute in the nineteenth century when it was found that "implicit visual clues had insinuated themselves without logical foundation" into Euclidean geometry. [Tall, 1991: 105]. For example, Euclid's fifth axiom (that parallel lines will never meet) was shown to be logically flawed despite the misleading geometric, visual evidence.

Although visual information is slowly gaining respectability again (Cunningham and Zimmermann, 1991) due largely to the power of computer-generated fractal images and curves and surfaces in three dimensions, the algebraic mode of discourse still maintains its position as the most privileged mode of discourse in the mathematical academic environment.

Within this context I maintain that the graphic calculator may have been regarded as an unimportant tool. As a result students may have interacted with the technology in a less meaningful and conscious way.

Previous studies have shown that when the purpose of learning is socially unimportant, little learning and transfer of skills takes place. For example, Scribner and Cole (cited in Salomon & Globerson, 1987) failed to find transfer from the Vai literacy training to cognitive tasks. Apparently the Vai literates rarely needed or used their literacy skills other than for an occasional letter to a relative. In fact, the only Vai people who did show transfer skills were the teachers for whom the literacy skills had become socially significant. Similarly, many studies on children's programming have failed to show measurable cognitive effects beyond some mastery of programming itself (Pea & Kurland, cited in Salomon, 1990). Presumably when an event or activity is not culturally or socially significant, the student makes less effort to interact or deal with that event in a conscious way.

Vygotsky, in fact, regarded *conscious reflection* or intellectualisation as so fundamental to human thought processes that he deemed it a necessary condition for internalisation and mastery of an activity. "We master a function to the degree that it is intellectualised. The voluntariness in the activity of a function is always the other side of its conscious realisation" (Vygotsky, cited in Wertsch, 1990b: 77). An equivalent concept to this conscious reflection is *mindfulness* as defined by Salomon and Globerson (1987: 623). Mindfulness is "the volitional, metacognitively guided employment of non-automatic, usually effortful processes.. . [Mindfulness] is both a general tendency and a response to situational demands". Thus for internalisation to take place, it is not sufficient that a student is merely exposed to a new technology; rather he/she needs to *mindfully* engage with the technology (Salomon, 1990b). In order to interact in such a mindful way, he/she has to actively and effortfully use the technology in a socially or educationally significant way.

In summary then, the perceived and in fact the actual status of the graphic calculator in the Mathematics I course probably mitigated against strong effects of this technology. In order to fully assess the possible results and modes of interaction of students with this technology, a different institutional or cultural setting is required for the research. In a more ideal institutional setting, meaningful interaction with the technology would be encouraged by allowing the graphic calculator to play a central role in the course content and tutorial exercises; additionally its use would be permitted in tests and examinations.

7.4.4 METHODOLOGICAL AND RESEARCHER SPECIFICITIES

Besides the specific institutional setting of this study (as described above) the interviews need to be understood in relation to the following peculiarities and influences:

♣ Firstly, the extent to which any of the students may have used or not used the graphic calculator, or even graphs, as a tool of mediation may well have been different had I not been the interviewer. Given my role as Mathematics I lecturer and organiser of the graphic calculator research project, many students may have wanted to please me and this may have significantly affected their choice and their use of semiotic mediators in the interview situation as discussed above. Although this phenomenon was minimised by my frequent use of semiotic challenges, it is an important characteristic of the interview data.

♣ Secondly, the amount and type of reassurance which I gave to any student in the interview situation may have unduly influenced the way he/she perceived and executed the problem. Although I did attempt to keep my reassurances as neutral as possible, it is quite likely that I gave subtle but positive reinforcements when I was pleased with a student's performance.

♣ The intensity and personal nature of the interview situation is very artificial in the mathematics context. The normal modus operandi of the Mathematics I course is highly impersonal. Students are not used to one-on-one interaction. Rather the student sees the lecturer in the context of a traditional lecture or at most in the consultative-type tutorial situation. Thus the interview situation may have contributed to a state of nervousness within the interviewee; this may have interfered with his/her ability to solve the problem and see the links.

To enhance the reliability of the analysis of the data, I analysed each interview at least twice, at times one week apart. There were a few minor discrepancies (in particular regarding the extent to which I included or excluded certain statements from my analysis). These inconsistencies were eliminated by reconsidering the relevant parts of the interview.

Despite this care with my application of categories to the analysis of the interview data, the perceived reliability of my analysis may have been enhanced if one of my colleagues had analysed a portion of the interview using my same categories. Unfortunately this implementation of inter-rater reliability was beyond the scope of this research report.

7.5 CONCLUSION

In this chapter I have attempted to develop various metaphors and concepts within a Vygotskian model so as to describe a possible role for the graphic calculator in the zone of proximal development of a Mathematics I University student. I have grounded my theoretical extensions in the data from the interviews and the data from the Task. In addition, and consistent with a Vygotskian approach I have situated the entire study within its socio-cultural context. To avoid replication I will not summarise my discussion here; rather I will summarise this material in the concluding chapter of this report so as to explicate answers to the questions posed in the objectives of this study.

8 CONCLUSION

This study has sought to describe ways in which the graphic calculator acts as a mediating sign in the zones of proximal development of University students in a first-year Mathematics course.

8.1 THE CONTEXT

In terms of a Vygotskian approach, the entire study was located within a socio-cultural context. The broad context is the Mathematics I (Major) course given to students at the University of the Witwatersrand in 1995. Within this course 5% of the class were each loaned a graphic calculator for the year. Since access to this technology was limited to such a small section of the class, its use was confined to support and verification rather than to exploration or problem-solving. In addition the use of this calculator in tests and examinations was prohibited.

I have argued that this limited application of the graphic calculator contributed to its status, or its level of privileging (Wertsch, 1990a) within the course. Analogous to Wertsch's argument about the privileging of rationality in scientific discourse (1991) my contention is that the preference by any student for one semiotic system over another should not be regarded as evidence of a more 'basic' or more 'accessible' semiotic system. Rather it may be a manifestation of the socio-cultural context.

In fact I have maintained that there are two basic contexts of interest in this study: there is the broad context of the first-year course and also the rather narrower context of the interviews. Within the former context, algebra is currently the privileged mode of discourse as a result of historical practices and traditions most of which still prevail in the contemporary mathematics classroom. Within the latter context however, the use of graphs and/or the graphic calculator may have been privileged, since the interviewer (myself) was also the organiser of the graphic calculator research project. Of course, since the interviewer *asks* for alternate representations, the importance of these patterns of privileging in the interview set-up is lessened.

8.2 A TOOL FOR SEMIOTIC MEDIATION

Within these contexts, I have observed that the use of the graphic calculator across different students is not uniform; rather its use appears to be problem-driven and situation- and learner-specific.

From the interview data I have inferred that students use hand-drawn graphs and calculator-generated graphs differently. This inference supports the Vygotskian view that the unity of action, perception and speech (when hand-drawing a graph) contributes to the ability of the student to internalise the visual field. Consequently I argued that hand-drawn graphs and calculator-generated graphs should be regarded as two separate and different symbols in any discussion of tools or signs of mediation in the mathematical context.

I have also described the way in which the calculator functions as a tool which shapes and defines mathematical concepts. In other words the role of the graphic calculator as a tool for semiotic mediation has been demonstrated in this study.

In particular, I have distinguished between the role of the calculator as a tool which promotes mediation *while* it is being used by a student and its role as a tool which promotes internalisation of a mediating symbol system which can then be used by the student even when he/she is *not using* the calculator. Corresponding to this distinction, I have examined the effect of the graphic calculator as an *amplifier* of the zone of proximal development and as a *cognitive re-organiser* of the internal plane of consciousness respectively.

In terms of the amplification concept, I have developed the metaphor of the graphic calculator as a tool which expands the zone by lessening the need of the student to perform laborious and tedious calculations. In other words, the notion of the graphic calculator as a tool which creates extra space in the zone by minimising extraneous clutter has been suggested. This metaphor was qualitatively demonstrated in the interviews but not significantly supported by the quantitative data. My contention is that the patterns of privileging within this study blunted this effect. As previously discussed, the graphic calculator was not an integral part of the course and so may have been regarded as socially inconsequential. I have argued that this status of the tool undermined its relationship with the individual students and so inhibited the possibility for statistical significance with respect to the amplification effect.

I have maintained that the role of the graphic calculator as a tool for cognitive re-organisation has not been shown in this study, neither qualitatively nor quantitatively. As a tool for cognitive re-organisation the graphic calculator should promote internalisation of a new semiotic system. This should impact on the performance of the students even when they are not using the graphic calculator. I have suggested that this lack of demonstration is partly due to the socio-cultural status of the graphic calculator in the study and partly due to the hidden nature of the processes of the graphic calculator. After all, it is not possible to internalise those processes which are invisible. Thus even in a different context I maintain that the graphic calculator has a limited ability to re-organise the internal plane of consciousness of any particular student.

In addition, an important but largely theoretical metaphor which has been developed is that of the graphic calculator as a tool which creates additional entry points to the zone of proximal development of a particular learner. Analogous to Vygotsky's account of the manner in which a word may be used as a tool for social interaction before the child fully understands the meaning of the word, I have proposed that the graphic calculator allows for the generation of graphs which can be used as tools for mathematical interaction before the learner fully understands the meaning of the graph or its function.

> "One of the correlates of the fact that interpsychological semiotic processes require the use of external sign forms is that it is possible to produce such forms without recognising the full significance that is normally attached to them by others" (Wertsch, 1985: 167).

In other words, I have suggested that the graph can be used as a reference from which new mathematical concepts can develop. Since this analogy developed during my exploration of the role of the graphic calculator the research has not really generated sufficient or appropriate data to thoroughly confirm or support this theoretical construct. Thus further research to ground this notion in the empirical world is required.

In order to diminish the undermining effects of the use of a tool whose status in the course is at best ambiguous, and at worst, completely unimportant, it is suggested that any further research about the graph as a reference takes place in a setting in which the use of the graphic calculator is integral to the course. Similarly, the metaphor of the graphic calculator as a tool of amplification and/or cognitive re-organisation needs to be further researched in such an environment.

8.3 A POSSIBLE ROLE

Finally, it needs to be stressed yet again that this entire study was located within a particular socio-cultural context. Within this setting various metaphors which describe the role of the calculator were generated. Since these metaphors developed in a specific context with particular students attempting particular mathematical problems, I do not claim any universal or context-free conclusions for my study. Rather I have explicated a *possible* role for the graphic calculator as opposed to an *absolute* or *certain* role. However, with sufficient acknowledgement of the influences and peculiarities of this particular study, the metaphors and concepts which have evolved can hopefully contribute to the understanding more generally of the possible use of technology in a mathematical course.

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APPENDIX A

EXERCISE

(You may use a graphic calculator for this exercise, if you wish).

Given $F(x) = \int_0^x (4t - t^2)dt.$

Let $g(x) = F'(x)$.

a) Find $g(x)$.

b) Represent or show $F(x)$ and $g(x)$ in any way you want on sets of axes.

c) Explain why $F(x) = 0$ for $x = 0$.

d) Explain why $F(x) = 0$ for $x = 6$.

e) Explain why $F(x) < 0$ for $x = 7$.

f) For what value of x does the maximum value of F occur? Why?

g) Explain why $F(x) > 0$ for $x = 2$.

APPENDIX B

TASK

NAME:____

STUDENT NUMBER:____

INSTRUCTIONS:

1. Answer each mathematical question. Use the back of the previous page for any working. Use any method that you want.
2. In parts A, B, C please try and answer the questions about your method of working. Circle the answer that most approximates the truth.
3. In answering parts B and C please be aware that there are no right or wrong answers. Just **try and be as honest** as possible.
4. This task must be completed for d.p. purposes. It will **not** count towards your class record.

QUESTION 1

Find the area bounded by xe^{x^2} and the x-axis from $x = -1$ to $x = 1$.

A. METHOD OF REPRESENTATION.

Did you solve this problem using:

Algebra only	Algebra and graphs	Algebra and numbers

Graphs only	Graphs and numbers

Numbers only

Other: please specify:

B. GENERATION OF GRAPHS

If you used graphs, did you:

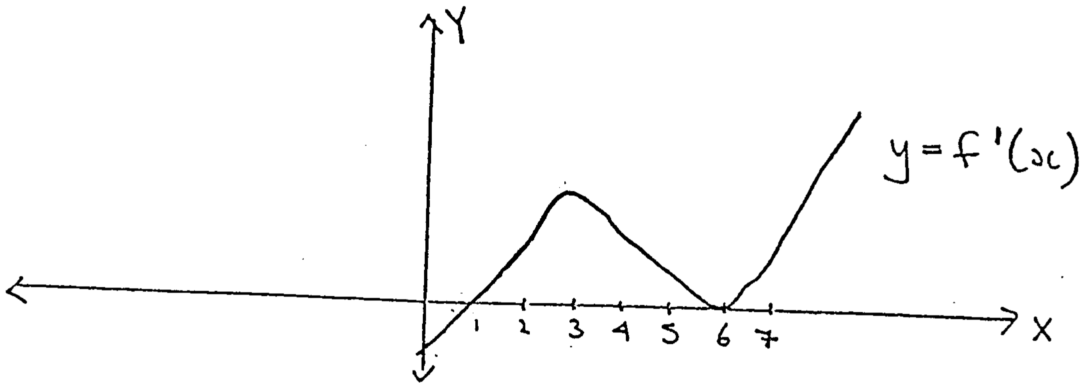
use calculator generated graphs	hand-drawn graphs	a mixture of calculator and hand-drawn graphs	mental image of a graph

C: Any other comments about your working methods?

QUESTION 2

2. Here is the graph of f '(x). Use this to find the local maximum/a and local minimum/a of f(x).

NOTE: The graph of f is not given.



A. METHOD OF REPRESENTATION.

Did you solve this problem using:

Algebra only	Algebra and graphs	Algebra and numbers

Graphs only	Graphs and numbers

Numbers only

Other: please specify:

B. GENERATION OF GRAPHS

If you used graphs, did you:

use calculator generated graphs	hand-drawn graphs	a mixture of calculator and hand-drawn graphs	mental image of a graph

C: Any other comments about your working methods?

QUESTION 3

Find $\lim_{x \to 0+} (\tan x)^x$

A. METHOD OF REPRESENTATION.

Did you solve this problem using:

Algebra only	Algebra and graphs	Algebra and numbers

Graphs only	Graphs and numbers

Numbers only

Other: please specify:

B. GENERATION OF GRAPHS

If you used graphs, did you:

use calculator generated graphs	hand-drawn graphs	a mixture of calculator and hand-drawn graphs	mental image of a graph

C: Any other comments about your working methods?

QUESTION 4

Given $F = \pi\int_0^2 x^2 dx$

a) roughly sketch the region whose area is given by F;

b) roughly sketch the region whose volume is given by F.

A. METHOD OF REPRESENTATION.

Did you solve this problem using:

Algebra only	Algebra and graphs	Algebra and numbers

Graphs only	Graphs and numbers

Numbers only

Other: please specify:

B. GENERATION OF GRAPHS

If you used graphs, did you:

use calculator generated graphs	hand-drawn graphs	a mixture of calculator and hand-drawn graphs	mental image of a graph

C: Any other comments about your working methods?

QUESTION 5

Find area enclosed between $y_1 = x^5 + x^4 - x - 1$ and $y_2 = x^4 - 4x^3 + 4x^2 - 1$.

A. METHOD OF REPRESENTATION.

Did you solve this problem using:

Algebra only	Algebra and graphs	Algebra and numbers

Graphs only	Graphs and numbers

Numbers only

Other: please specify:

B. GENERATION OF GRAPHS

If you used graphs, did you:

use calculator generated graphs	hand-drawn graphs	a mixture of calculator and hand-drawn graphs	mental image of a graph

C: Any other comments about your working methods?

GENERAL:

Please tick the appropriate box.
I found the paper:

Very Easy	Easy	OK	Difficult	Very Difficult.

Approximately how long did you take to do this paper?

Any other comments about this paper?

APPENDIX C: THE ANALYTIC TABLES

CONVENTIONS AND CATEGORIES

In the analytic table, I observe the following conventions:

- Separate activities are grouped by line numbers and categorised according to the most important activity; for example, in the Analysis of VT, lines 61-93 are grouped as one activity in which the use of the graphic calculator generated graph predominates.

- Sometimes a set of activities can be regarded as one super-activity in which the use of one form of mediation dominates, although different signs or tools may be used within this dominant mediation. In such instances, the line numbers of the super-activity are printed in bold-type. For example, in RP's interview, the dominant mediation in lines 234-270 is the Interviewer; within this framework the student uses algebra and hand-drawn graphs (lines 242-270).

- Separate events (for example, each distinct answer to a different part of the mathematical question is a separate event) are distinguished by different Roman numerals and blank lines are left between these events.

- Certain lines are omitted from the analysis. This analysis attempts to investigate broad general themes, rather than every activity and utterance. Thus peripheral activities e.g. pencil falling or superfluous comments of the interviewer or interviewee are not included in the analyses. Similarly, the instructions and comments by the interviewer and interviewee at the beginning and at the end of each interview, are generally omitted from the analyses.

The following categories and codes are used in each analytic table.

1. Algebra	Student uses paper and pencil algebra.
2. C Algebra	Student uses calculator to perform algebraic manipulation.
3. GC Graph	Student uses graphic calculator to draw graph.
4. Hand-drawn graph	Student draws graph with paper and pencil.
5. GC Table	Student uses table of numbers on graphic calculator.
6. Table	Student uses table of numbers written on paper.
7. Speech	Student uses spoken language alone to resolve the problem.
8. Int-as-helper (Solicited)	Student uses the interviewer as an expert assistant.
9. Int-as-helper (Unsolicited)	Interviewer gives unsolicited assistance.
10. Resolution	Student expresses insight into the problem.
11. Reflection	Student discusses his/her attitudes or semiotic preferences.

- ♣ The first nine categories refer to the tool, sign or agent of mediation;
- ♣ the eighth and ninth categories refer to the role of the interviewer in the zone of proximal development of the student;
- ♣ the final two categories refer to the behaviour or apparent experience of the student.

VT: THE ANALYTIC TABLE

Line nos.	Tool, sign or agent for meditation	Description/purpose of event
I		
1-31		I explain to VT the nature of the interview: how she must answer the mathematical question by speaking aloud.
II		
32-37	Speech	Anticipates g(x), but wants to confirm this.
38-41	Algebra	Uses integration to express F as poly.
42-48	Algebra	Uses differentiation to express g as poly.
III		
53-57	GC graph	Represents g(x) as parabola.
61-93	GC Graph	Trying (unsuccessfully) to represent F(x) using ∫ function on graphic calculator; gets stuck on use of x as variable.
94-95	Int-as-helper (solicited)	Suggestion to student to use a different representation.
96-103	Hand-drawn graph	Trying to represent F(x) as area under curve. Gets stuck on where to place x.
106-121	Int-as-helper (solicited)	Interviewer assists by asking questions going from a particular x to the variable x.
113-130	Hand-drawn graph	Trying to represent F(x) as area under curve.
131-138	Resolution Hand-drawn graph.	Successfully represents F as area under g.

IV		
150-157	Algebra	Explains why F(0)=0 using prior knowledge about equal limits in integral notation.
158-165	Hand-drawn graph	Uses area under g to explain that F(0)=0.
168-177	Algebra	Explains why F(0)=0 by substituting x=0 into F as poly.
V		
178-183	Algebra	Explains why F(6)=0 by substituting x=6 into F as poly.
184-210	GC graph	Attempts to use area under g to explain why F(6) = 0. Gets confused.
212-229	Hand-drawn graph	Uses area under g to explain why F(6)=0.
VI		
232-235	Algebra	Explains why F(7)<0 in terms of its polynomial expression.
236-247	Hand-drawn graph	Uses area under g to explain why F(7)<0.
VII		
251-258	Hand-drawn graph	Explaining that F_{max} occurs when area is max (between two x - intercepts of g)
258-260	Algebra	Calculates two x-intercepts.
262-270	Hand-drawn graph	Elaborating on explanation of maximum area.
VIII		
272-274	Algebra	Explains why F(2) > 0 in terms of its polynomial expression.
275-278	Hand-drawn graph	Uses area under g to explain why F(2)>0.

IX		
279-284	Int-as-helper (unsolicited)	Request for student to give another explanation of why F_{max} occurs at x=4.
289-309	Algebra	Explains why of why F_{max} occurs at x=4 using properties of $g = F'$.
310-311	Int-as-helper (unsolicited)	Request for student to clarify explanation.
312-321	Hand-drawn graph	Re-explanation of max. value for F using area concept.
X		
326-327	Int-as-helper (unsolicited)	Suggestion that student draws F as poly.
328-340	GC graph	Draws graph of F (on same system as g).
342-367	Int-as-helper (solicited)	Assisting student to see link between different representations via questioning about F_{max}.
368-374	Resolution GC Graph	"Sees" link between max of F as poly and max area.
375-377	Int-as-helper (unsolicited)	Questioning about where F(x)=0
378-431	GC Graph	Explains link between zero area and F(x)=0
433-444	GC Graph	Explains link between area and why F(7)<0 and why F(2)>0.
XI		
445-454	Reflection	Student confirms that she understands the link between F as poly and area under g.
455-464		Administrative details.

466-483	Reflection	Student states that she likes to use graphs.

RS: THE ANALYTIC TABLE

Line nos.	Tool, sign or agent for meditation	Description/purpose of event
I		
3-31		I explain to student the nature of the interview: how he must answer the mathematical question by speaking aloud.
II		
35-38	Speech	Suspects that integration and differentiation 'cancel' each other out.
47-52	Algebra	Uses integration to express F as a polynomial.
53-65	Algebra	Uses differentiation to express g as a polynomial.
68-74	Reflection	Discusses need to verify algebraically the relationship between differentiation and integration.
III		
83-115	GC Graph	Attempts (unsuccessfully) to represent F as an integral.
115	GC Graph	Realises that he has, in fact, drawn g as poly.
116-133	GC Graph	Attempts to represent F as poly.
135-147	GC Graph	Expresses surprise at shape of F.
148-149	Int-as-helper (unsolicited)	Suggestion that student checks function entered into graphic calculator.

152-162	Resolution	Student finds error with the function format.
164-166	GC Graph	Corrects error and represents F as poly.
202-292	Algebra	Calculates turning points of F and g using calculus.
IV		
314-329	Algebra	Unclear explanation of why F(0)=0 using F as poly.
334-338	GC Graph	Using intercepts of F
339-342	Int-as-helper (solicited)	Suggestion that student uses integral to explain.
345-358	GC Graph	Explanation of why F(0)=0 using area concept.
V		
360-380	Algebra	Explaining why F(6)=0 by substituting x=6 into F as poly.
381-382; 402-405	C. Algebra	Continues calculating F(6).
402-405	Resolution	Express surprise at his (incorrect) answer.
406	Int-as-helper (unsolicited)	Suggestion that student checks his calculations.
412-419	GC Table	Uses GC table (incorrectly) to confirm incorrect result.
420-421	Int-as-helper (unsolicited)	Suggestion that student re-reads table.
422-425	GC Table	Student corrects error in his interpretation of Table.

426-427	Int-as-helper (unsolicited)	Suggestion that student checks his substitution.
428-432	C. Algebra	Corrects error.
435-436	Int-as-helper (unsolicited)	Request for student to explain why F(6)=0 using g.
437-444	GC Table	(Unsuccessful) attempt to use table for explanation.
445-451	GC Graph	(Unsuccessful) attempt to use graph of g for explanation.
VI		
456-473	Algebra	Starts explaining why F(7)<0 by substituting x=7 into F as poly.
474-493	GC Graph	Uses shape of F to explain why F(7)<0.
495-499	Algebra	Explaining why F(7)<0 by substituting x=7 into F as poly.
499-516	C. Algebra	Explaining why F(7)<0 by substituting x=7 into F as poly.
518-536	GC Graph	Uses shape of F to explain why F(7)<0.
VII		
545-559	Algebra	Finds F_{max} using g (=F').
561-576	Algebra; GC Graph	Relates g' (=F'') to graph of g to explain why F_{max} occurs at x=4.
579-583	Reflection	Expresses dissatisfaction with his explanation.
590-620	GC Graph	Re-explains why F_{max} occurs at x=4 using properties of g = F'.

621	Reflection	Expresses frustration at his inability to easily give explanation.
VIII		
622-637	GC Graph	Explains why F(2)>0 using shape of graph of F as poly.
638-639	Int-as-helper (unsolicited)	Request for student to explain why F(2)>0 using g.
643-661	GC Graph	Attempts (unsuccessfully) to explain why F(2)>0 using g=F'. Reveals good (but irrelevant to sub-problem) understanding of relation between graph and its derivative.
665-666	Reflection	Expresses dissatisfaction with his explanation.
667-677	Int-as-helper (unsolicited)	Request for student to consider area under g.
674-692	Resolution	Acknowledges link between F and area under g.
692-699	Algebra	Explains why F(2)> 0 by substituting into F as poly.
701-710	GC Graph	Explains why F(2)>0 using area under g.
X		
733-734	Int-as-helper (unsolicited)	Direct request to student to explain why F_{max} occurs at x=4, using area concept.
736-768	GC Graph	Gives confused explanation which he appears to find meaningful.
770-772	Int-as-helper (unsolicited)	Suggestion that student uses only graph of g in explanation.

773-792	GC Graph	Re-explains why F_{max} occurs at x=4 using properties of $g = F'$.
XI		
795-813; 854-863.	Reflection	States that he found problem easy and understood its different aspects but had difficulty in giving explanation; would have liked more specific guidelines about what was required for explanation.

RP: THE ANALYTIC TABLE

Line nos.	Tool, sign or agent for meditation	Description/purpose of event
4-18		I explain to student the nature of the interview: how he must answer the mathematical question by speaking aloud.
II		
19-29	Algebra	Uses integration to express F(x) as a polynomial.
31-38	Algebra	Uses differentiation to express g(x) as a polynomial.
40-49	Resolution	Acknowledges the algebraic relationship between differentiation and integration.
III		
50-79	GC graph	Represents g(x) as parabola and F(x) as cubic polynomial graph.
IV		
80-83	Algebra	Explains why F(0)=0 by substituting x=0 into F's polynomial expression.
85-88	Algebra	Attempts to give alternate explanation of why F(0)=0. Is unsuccessful.

V

91-93	Algebra	Explains why F(6)=0 by substituting x=6 into F's polynomial expression.
95- 99	GC Graph	Confirms that F(6)=0 by inspection of graph of F.
100-101	Int-as-helper (Unsolicited)	Suggestion that student use integral expression of F or g in explanation.
102-103	GC Graph	Attempts unsuccessfully to link graph of g to above explanation.
106-107	Algebra	Repeats algebraic explanation of why F(6)=0.

VI

110-114	Reflection	Acknowledges that all results could be checked graphically or with a table.
114-121	Reflection	Discusses how he prefers and chooses to use algebra, rather than graph, to find solution.
123-128	GC Graph	Explains why F(7)<0 by looking at poly graph of F.

VII

131-137	Algebra	Uses poly expression of g (i.e. F') to find out where the maximum of F is.
138-148	Algebra	Attempts (unsuccessfully) to 'check' that x=4 does give maximum by finding F(4) using poly expression of F.

VIII

149-152	Algebra	Attempts (unsuccessfully) to use poly expression of g to explain why F(2)>0.
155-166	Int-as-helper (Unsolicited)	Suggestion that area concept be used to explain the answers.
170-181	C Algebra	Does not use suggestion. Uses poly expression for F to explain why F(7)<0. (Previously used GC graph: lines 123-128).

IX

181-190	Int-as-helper (Solicited)	Suggestion that student tries to use concept of area in his explanation of the link.
191-198	Hand-drawn graph	Draws graph of g by hand. (He already has drawn a graph of g on the graphic calculator).
200-209	Hand-drawn graph	Explains that F(7)<0 by looking at signed area under g.
210-224	Int-as-helper (Unsolicited)	Trying to get student to explain the link between area under g and F.
214-224	Algebra	Confuses algebraic expression for real area with that of signed area.
225-226	Int-as-helper (Unsolicited)	Attempt to get student to generalise out his thinking of the link between F and the area under g.
227-231	Hand-drawn graph	Explains why F(0)=0.

X

234-270	Int-as-helper (Unsolicited)	Attempt to get student to see the link between algebraic expression of g and area under g.
242-270	Algebra; Hand-drawn graph	Makes link between algebraic and graphical representation of area with respect to g from x=0 to x=7. Makes no explicit link to F.
271-273	Int-as-helper (Unsolicited)	Attempt to get student to generalise out the link he found above.
274-287	Algebra; Hand-drawn graph	Gets confused about area under g from x=0 to x=4. Is unable to resolve apparent disparity between algebra and graph.

XI

288-305	Reflection	Student states that he could get the answers but that he was unable to explain why.

EP: THE ANALYTIC TABLE

Line nos.	Tool, sign or agent for meditation	Description/purpose of event
I		
4-45	Reflection	Student discusses how and when he uses the graphic calculator.
II		
46-74		I explain to student the nature of the interview: how he must answer the mathematical question by speaking aloud.
III		
83-86	Speech	Student discusses the algebraic relationship between F and g.
88-101	Algebra	Student confirms the algebraic relationship between F and g by integration and differentiation.
103-116	Speech	Explains above result: how differentiation and integration are inverse processes.
IV		
122-124	Algebra	Student uses algebra to calculate intercepts of g.
125	Hand-drawn graph	Student draws g as poly.
127-134	Algebra	Student calculates (erroneously) the intercepts of F.

138-148	GC graph	Student uses graph to remind himself of shape of cubic function with negative leading coefficient.
151-153	Hand-drawn graph	Student sketches rough (correct shape) graph of F as poly.
156-177	Speech	Student discusses how he would use algebra to find turning points of F via F'.
V		
205-212	Algebra	Student explains why F(0) = 0 by substituting x=0 into F as poly.
VI		
214-220	Algebra	Student confirms that F(6)=0 by substituting x=6 into F as poly.
221-232	Resolution	Student realises that he made an algebraic error earlier when he worked out intercepts for F.
VII		
233-235	Hand-drawn graph	Uses graph of F to explain why F(7)<0.
237-241	Algebra	Explains how to substitute x=7 into polynomial expression for F.
243-244	Reflection	Discusses that its easier to just use graph of poly to explain.
VIII		
246-248	Hand-drawn graph	Uses graph of F to (correctly) find maximum of F at infinity.

258-260	Algebra	Uses F' to find where F'=0 i.e. to find x at F_{max}.
260-261	Hand-drawn graph	Uses graph of g to confirm that F has max. at x=4.
268-270	Speech	Repeats explanation that F_{max} occurs at x=4 for above two reasons.
IX		
276-280	Hand-drawn graph	Uses graph of F to explain why F(2)>0.
283-284	Algebra	Explains how to substitute x=2 into polynomial expression for F.
X		
286-292	Int-as-helper (Unsolicited)	Tells student that she wants another explanation.
293-314	Hand-drawn graph	Uses graph of g to explain graph of F in terms of properties of the derivative of F.
322-333	Int-as-helper (Solicited)	Suggests student rereads the question.
335-353	Int-as-helper (Unsolicited)	Interviewer encourages student to think in terms of area.
353-354	Resolution	Beginning to make link between area under g and F.
356-361	Hand-drawn graph	Uses area under g to explain why F(0)=0.
365-370	Hand-drawn graph	Uses area under g to explain why F(6)=0.
372-395	Reflection	States that his preference for explanation is the graph of the polynomials, followed by algebraic explanation, followed by the area explanation.
396-401	Hand-drawn graph	Uses area under g to explain why F(7)<0.

403-410	Hand-drawn graph	Uses area under g to explain why F(2)>0.
414-422	Hand-drawn graph	Uses area under g to explain why F_{max} occurs at x=4.

XII

427-466	Reflection	States that he found problem very easy; that he didn't think of using area as an explanation but that this was because it was not required.

RM: THE ANALYTIC TABLE

Line nos.	Tool, sign or agent for meditation	Description/purpose of event
I		
1-40		I give instructions for the interview.
II		
48-56	Int-as-helper (Solicited)	By direct questioning I help student to remember that the symbol, ', refers to the derivative.
57-59	Algebra	Uses integration to express F as poly; uses differentiation to express g as poly.
61-70	Resolution	Student acknowledges relationship between differentiation and integration.
III		
73-77	Reflection	Expresses an uneasiness about drawing graphs.
78-86	Algebra	Finding turning points of g. Makes error in computation.
87-91	Resolution	Student realises that g is parabola $\therefore$ doesn't need 2nd derivative.
91-95	Hand-drawn graph	Draws g; realises there is an error in turning point.
98-108	Algebra	Recalculates turning point.
109-118	Hand-drawn graph	Draws g as parabola; represents F as area underneath g.

IV		
125-134	Hand-drawn graph	Explains why F(0)=0 in terms of area.
V		
136-152	Hand-drawn graph	Explains why F(6)=0 in terms of area.
VI	Hand-drawn graph	Explains why F(7)<0 in terms of area.
154-162		
VII		
162-168	Hand-drawn graph	Uses area to find max. at x=4.
VIII		
169-172	Hand-drawn graph	Explains why F(2)>0 in terms of area.
IX		
173-251	Int-as-helper (unsolicited)	Trying to get student to give alternate representation of F. First level of assistance: interviewer asks directly for alternate explanation.
192-194	Algebra	Explains why F(0)=0 by substituting x=0 into F as poly.
195-211	Int-as helper (unsolicited)	Attempt to get student to reformulate the representation of F. Using questioning to get him to re-articulate the relationship between F and g.
221-229	C Algebra	Explains why F(6)=0 by substituting x=0 into F as poly.

240-247	Algebra Hand-drawn graph	Explains that $F_{max.}$ occurs at x=4 by substituting x-values, read from area graph, into poly expression of F.
X		
248-260	Int-as helper (unsolicited)	2nd level of assistance: Direct (unsuccessful) request to student to draw graph of F.
262-278	Hand-drawn graph	Student repeats his representation of F as area under g.
279	Int-as helper (unsolicited)	Repeat (unsuccessful) request to student to represent F in another way.
280-290	Hand-drawn graph	Student repeats area representation; suggests a translation of g.
294-315	Int-as helper (solicited)	Attempts (unsuccessfully) to get student to think of F as poly. by focusing on question concerning the max. value of F.
XI		
317-340		Premature and mistaken end of the interview.
XII		
351-421	Int-as helper (solicited)	Encouraging student in his attempt to represent F as poly.
352-356	Reflection	Student expresses concern and confusion.
358-361	Int-as-helper (solicited)	Interviewer assures student that there is more than one correct representation.
362-378	Algebra; Hand-drawn graph.	Acknowledges that area is a function of the poly. F.

379-381	Int-as-helper (solicited)	Direct (unsuccessful) request for student to represent F as poly.
382-392	Algebra	Repeats that area is function of F as poly.
393-421	Reflection	Student indicates confusion.
405-415	Hand-drawn graph	Student re-iterates his area representation.

MK: THE ANALYTIC TABLE

Line nos.	Tool, sign or agent for meditation	Description/purpose of event
I		
1-37		I explain to MK the nature of the interview: how he must answer the mathematical question by speaking aloud.
II		
47-72	Speech	Explains that derivative of integral is function itself; uses this to write g(x) in terms of (incorrect) variable t.
III		
75-76	Reflection	States that he usually draws graphs using a table.
88-111	Table	Sketches g by plotting x-y points with (inadequate) domain: $x \in [-3,2]$; gets incorrect curve.
112	Resolution	Realises that g should be a parabola.
113-133	Table	Adds another point to domain; completes sketch of g correctly.
135-147	Algebra	Finds F as poly. by integrating g.
150-190	Table	Draws graph of F (incompletely) by plotting x-y co-ordinates. Uses inadequate domain: $x \in [-3,3]$.

IV		
193-197	Algebra	Explains why F(0)=0 by substituting x=0 into F's polynomial expression.
V		
203-217	Algebra	Explains why F(6)=0 by substituting x=6 into F's polynomial expression.
VI		
219-228	Table	Substitutes x=7 and x=8 into F to get negative values.
228-233	Hand-drawn graph	Completes sketch of F using values of F(7), F(8). Explains that graph 'dips below axis' after x=6.
VII		
234-260	Algebra	Differentiates F to find turning points. Finds F(0) and F(4) by substitution and compares them to confirm that max. at x =4.
VIII		
260-275	Table	Confirms that F(2), F(3), F(5) >0.
277-282	Reflection	States that algebraic approach would have given more information than the table approach.
285-297	Hand-drawn graph	Looks at his graph of F as poly. to confirm information given in previous sub-problems.

298-317	Hand-drawn graph	Redraws graph of F as poly using information given in parts c), d), e), f) of the question; uses this to confirm that F(2) > 0.
IX		
320-325	Resolution	Corrects the t to an x in his representation of g(x).
327-341	Speech	Repeats that derivative of integral is function itself; uses this to write g(x) (correctly) in terms of variable x.
345-358	Algebra	Confirms algebraic relationship between F and g by integrating and then differentiating.
X		
361-363	Int-as-helper (unsolicited)	Request for student to explain sub-problems in 'another way'.
364-381	Reflection	Student says that he has forgotten relevant work.
384-390	Reflection	Says he's satisfied with answers to a), f) and g) because they are consistent with information given in question.
XI		
391-431		Premature and mistaken end of interview.

XII

432-441	Int-as-helper (solicited)	Graduated assistance: Interviewer suggests student tries to link F to g. Interviewer suggests student thinks of area.
442-450	Reflection	Student maintains that he can't remember this work.
451-455	Int-as-helper (solicited)	Interviewer asks student how to express area under curve between particular points.
456-460	Int-as-helper (solicited)	Interviewer shows student how to express area under curve between particular points.
461-479	Reflection	Student claims he would have done same as interviewer (even though he did not!)
480-489	Int-as-helper (unsolicited)	Interviewer asks questions concerning more general area.
490-494	Hand-drawn graph	Student very confused about area under curve.

JB: THE ANALYTIC TABLE

Line nos.	Tool, sign or agent for meditation	Description/purpose of event
I		
3-28		I discuss the format of the interview; JB expresses anxiety about having to think aloud.
II		
29-43	Algebra	Student discusses the algebraic relationship between F and g; starts differentiating g (Not required).
44	Interruption	Tape is switched off while I deal with knocking at the door. Student has time to reflect on the problem.
48-74	Algebra	Finds F by integration.
74-75	Algebra	Finds g by differentiating F.
80-94	Int-as-helper (unsolicited)	Interviewer questions student about relationship between F and g.
99-104	Resolution	Student 'remembers' the relationship between F and g.
III		
106-131	Table	Draws graph of g by plotting x-y co-ordinates.
132-153	Hand-drawn graph	Attempting to infer graph of F from graph of g. Realises the link (that g represents the slope of F) between the two graphs but is unable to use this link.

153-180	Table	Abandons attempt to infer F from g. Tries to plot F by plotting x-y co-ordinates.
181-190	Resolution; Hand-drawn graph.	Realises that graph of F is wrong; draws (correct) very rough sketch to show the shape it should be.
191	Int-as-helper (Unsolicited)	Suggestion for student to "think of another way".
192-264	Hand-drawn graph	Infers (correctly) shape of F from shape of g; includes point of inflection in graph of F.
IV		
265-279	Algebra	Explains why F(0)=0 using poly expression of F.
V		
280-301	Algebra	Explains why F(6)=0 by finding x where F(x)=0.
298-301	Hand-drawn graph	Relates graph of F to nature of its roots.
VI		
302-306	Hand-drawn graph	Explains why F(7)<0 in terms of shape of F.
310-320	Algebra	Explains why F(x)<0 for any x>7, in terms of poly expression of F.
VII		
323-334	Hand-drawn graph	Uses graph of g = F' to find F_{max}. Distinguishes between local and absolute maximum.

VIII

335-345	Hand-drawn graph	Explains why F(2)>0, using graph of g = F'.
345-346	Algebra	Explains why F(2)>0 for any x=2, in terms of poly expression of F.

IX

353-435	Int-as-helper (Unsolicited)	Interviewer gives graduated assistance to student in finding link between F and the area under the graph of g.
353-363	Int-as-helper (Unsolicited)	Interviewer suggests that student thinks about the relationship of F to g.
360-363	Resolution	Student notes that F is integral of g.
364-375	Int-as-helper (Unsolicited)	Interviewer suggests (unsuccessfully) that student graphically represents F as integral of g.
376-389	Int-as-helper (Unsolicited)	Interviewer suggests that student draws integral function.
390-435	Hand-drawn graph	Student represents F (successfully) as area under curve g.
435-452	Reflection	Student says that this approach "makes sense".

X

453-471	Hand-drawn graph	Student represents F as area under g on graph
472-477	Hand-drawn graph	Student explains why F(0)=0 in terms of area.
467-485	Hand-drawn graph	Student explains why F(6)=0 in terms of area.
486-490	Hand-drawn graph	Student explains why F(7)<0 in terms of area.
491-498	Reflection	Student acknowledges how the new representation makes sense.

500-512	Hand-drawn graph	Student explains why F(4) is max. in terms of area.
514-515	Hand-drawn graph	Student explains why F(2)>0 in terms of area.
XI		
516-538	Reflection	Student discusses her difficulty in thinking aloud; also, she discusses how the new representation makes sense.
XII		
539-562		Thanking the student.