THE SNAP-THROUGH STABILITY OF PLASTICALLY DESIGNED STEEL PITCHED-ROOF PORTAL FRAMES.

John Spencer Bryant

A project report submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

DECLARATION:

I declare that this project report is my own, unaided work. It is being submitted for the Degree of Master of Science in Engineering, in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

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ABSTRACT

Recent proposed rafter slenderness limits, to prevent snap-through of plastically designed pitched-roof portal frames, incorporate the elastic snap-through buckling load of such frames. It has been suggested that the elastic snap-through buckling load used in the proposals is over-estimated making these slenderness limits unconservative. This is supported by a more rigorous elastic analysis. To test the proposals, model frames lying on or close to the slenderness limits were tested to failure in the laboratory. Frame dimensions were chosen so that the frames were only susceptible to snap-through instability. Failure loads far lower than the expected plastic collpase loads were measured, showing that the elastic snap-through buckling load is over-estimated. Since plastic analysis is easily applied to portal frames, these slenderness limits are best replaced by a similar limit incorporating a more accurate elastic snap-through buckling load. A new limit is outlined which must still be tested by further research.

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CONTENTS Page

CONTENTS **Page**

LIST OF FIGURES

Figure **Page**

LIST OF TABLES

LIST OF SYMBOLS

Symbol	Quantity
P_p	plastic collapse load of a bay in a multi-bay frame
P_c	elastic buckling load of a bay in a multi-bay frame
P_F	failure load of a bay in a multi-bay frame
w	uniformly distributed load applied to a rafter
γ_c	load factor corresponding to elastic critical buckling
γ_p	load factor corresponding to plastic collapse
Q	externally applied force, to initiate the desired mode of elastic failure, i.e. snap-through or sway
R_i	axial load in a member
M	moment applied to a member
M_p	plastic moment capacity of a section
A	factor which accounts for the beneficial effects of arching action within a multi-bay frame
Δ, δ	displacement of the frame at the point of application of the load, in the direction of the load, or joint displacement.

Symbol	Quantity
ϕ	rotation of a plastic hinge
β_i	rotation of a member along its length due to the application of load
I_R	moment of inertia of a rafter
I_C	moment of inertia of a column
S	length of a rafter
h	length of a column
d	depth of a rafter section
L	span of a bay in a multi-bay frame
L_i	length of an individual member in a frame
ℓ	effective length of a member
θ	angle of rafters to the horizontal
σ_y	yield stress of the material used in the frame
E	Young's Modulus
Z	elastic section modulus
A_g	gross cross-sectional area of a member

1. INTRODUCTION

Most modern steel design codes permit the use of plastic theory in calculating the strength of structures. Stability problems such as lateral torsional buckling, in-plane member buckling, sway, and snap-through of rafters in portal frames, may result in a failure load lower than that calculated by simple plastic theory.

The British Steel Code (BS 5950)[1] was the first specification to introduce provisions for sway frames to safeguard against snap-through buckling of plastically designed pitched-roof steel frames. These provisions are based on Horne's[2] initial research. In a somewhat modified format these were adopted by the South African Structural Steel Code, SABS 0162-1984.[3]

Some doubt was recently expressed in regard to the validity of these regulations.[4,5] It is therefore the object of this project to investigate the snap-through regulations by way of suitable small-scale laboratory experiments. These experiments were undertaken as a pilot programme to identify whether it would be worthwhile to test full-size frames at a later stage.

Overall frame stability of unbraced pitched-roof frames including sway and snap-through buckling, may be accounted for in two ways:

a) The accurate failure load of the structure can be calculated using a second order elastic-plastic analysis. Alternatively, a number of approximate formulae and techniques have been developed to calculate the failure load. Generally these approximations are easy to apply and take cognisance of stability effects.

b) Stability effects may be limited and simple plastic theory applied. The rigid plastic collapse load will then provide a reasonable estimate of the failure load of such frames.

Some of the techniques which are available to calculate the frame failure load considering overall stability effects are:

* A second-order elastic-plastic analysis. This is often complex and time-consuming.
* The Merchant-Rankine formula.[6]
* The modified Merchant-Rankine formula. This was proposed by Wood and includ the strengthening effect of strain hardening and cladding.[7]
* The use of interaction curves developed by Scholz.[8]

The latter three approximate approaches incorporate the elastic buckling load of the structure. Approximate methods of calculating the elastic buckling load have been developed by Wood, Horne, Scholz and others.

Practical multi-bay pitched-roof portal frames may suffer from overall stability problems. This is exacerbated if use is made of the arching effect in multi-bay pitched-roof portal frames, since very slender internal members result. Two types of stability problems can occur:

* overall sway of frames
* snap-through of rafters.

Since plastic analysis is most easily applied to portal frames, these stability effects must be prevented. Horne[2] proposed equations for both sway and snap-through which result in maximum allowable rafter slendernesses. If these rafter slendernesses are exceeded, plastic theory cannot be used to calculate the strength of a structure. A second-order analysis is then necessary.

Both slenderness equations developed by Horne incorporate the ratio of elastic buckling load to plastic collapse load of the frame, corresponding to the relevant mode of failure, i.e. symmetrical snap-through or side-sway. Therefore if the slenderness limits are to have any significance, the respective elastic buckling loads must be calculated with some accuracy.

In calculating the elastic buckling load, Horne made the following simplifications:

- He used the first order stiffnessess of the members
- The average axial forces in rafters were used.
- Rafter shortening as a result of member deflections under the applied load were ignored.

In making these simplifications the stiffness reducing effect of axial forces, and the effect of changing axial rafter force with length, were not considered. In addition it was also assumed that member forces at the elastic buckling load were proportional to those at the plastic collapse load.

As a result of these simplifications it has been suggested[4.5] that Horne over-estimates the elastic buckling load making the slenderness limits obtained from his equations unconservative. Structures may result which are susceptible to sway or snap-through stability failure, at a load lower than that predicted by simple plastic theory. In research undertaken by Scholz,[4.5] elastic buckling loads eight times smaller than that predicted by Horne were obtained.

In order to test Horne's proposals for snap-through stability, model multi-bay pitched roof portal frames lying on or close to the slenderness limits obtained from his equation, were tested in the laboratory.

This report therefore describes Horne's research work into snap-through stability and discusses its shortfalls. To understand his work more clearly relevant research by Merchant,[6] Wood[7] and Scholz[4,5] is mentioned. A detailed description of the laboratory work is given. Experimental results are compared with Horne's predicted failure loads, to determine whether the elastic snap-through buckling loads are in fact over-estimated. Alternative methods of approaching the snap-through stability problem are outlined and recommendations for further research made.

2 PLASTIC THEORY

Most modern design codes permit the strength of steel structures to be assessed using plastic theory. The failure load of a structure is calculated using the plastic bending capacity of the individual members making up the frame. This provides a sufficiently good estimate of the actual failure load if the stability effects are small.

Member and frame instability may render this method inappropriate, resulting in a frame failure load lower than that estimated. In order to fully understand snap-through instability therefore, plastic theory is briefly described and the effect of deformations on the calculated plastic collapse load illustrated.

2.1 Simple Plastic Theory

When applying simple plastic theory in the analysis of a structure, deflections are assumed to be negligible. The fundamental theorems of plasticity therefore make the assumption that materials have an infinitely high modulus of elasticity. The plastic collapse load - deflection relationship bearing in mind the above is shown in figure 2.1

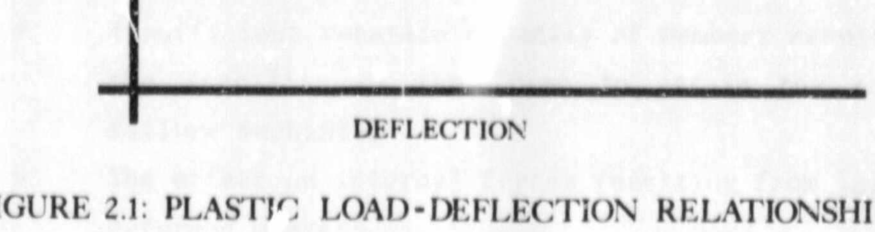


FIGURE 2.1: PLASTIC LOAD-DEFLECTION RELATIONSHIP

The general work equation from which the rigid plastic collapse load may be calculated is:

$$\Sigma(P_p.\Delta) = \Sigma(M_p \phi_i) \qquad (1)$$

The first summation is for all externally applied loads P_p and their associated displacements Δ. The second is for all plastic hinges in the frame with a moment capacity M_p and hinge rotations $\emptyset_i$.

An example showing the calculation of the plastic collapse load of a multi-bay pitched-roof portal frame is given in Appendix A.

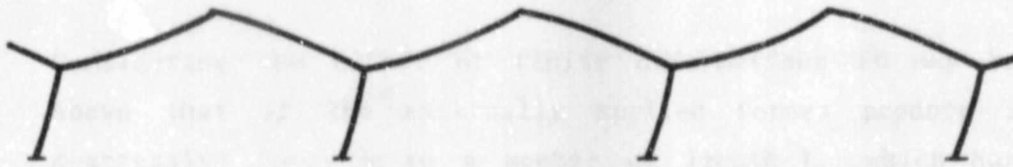
2.2 Stability Problems

If stability effects are negligible, a simple plastic analysis is sufficient to calculate the moments and forces within a frame as well as the load-carrying capacity. Stability problems, however, may result in an actual failure load far below that predicted by simple plastic theory. Stability problems are the direct result of the effect frame, or individual member deformations have on the calculated internal forces.

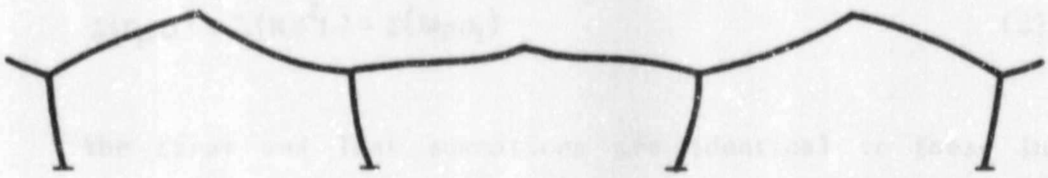
Types of stability problems that occur in steel frames are:

* Instability of individual members by bending about the minor axis
* Lateral-torsional instability of members
* Local buckling of flanges and webs before the plastic moment capacity of the section has been reached
* Insufficient rotation capacity of members resulting in the inability of the frame to attain the expected failure mechanism
* The effect on internal forces resulting from initially deformed members

* The effect of deformations on the calculated internal forces. Deformations may be either within the length of a member or of the frame as a whole. Two types of overall frame instability for a multi-bay pitched-roof portal frame are illustrated in figure 2.2, viz: sway and snap-through instability.



a) SWAY



b) SNAP-THROUGH

FIGURE 2.2: TYPES OF FRAME INSTABILITY

In order to deal with overall instability effects in the design of frames two approaches may be adopted:

a) An analysis may be undertaken in order to determine the failure load of the frame.

b) Overall stability effects may be limited so that a simple plastic analysis will provide a reasonable estimate of the collapse load.

It was with the latter in mind that Horne developed sway and snap-through stability equations that, if satisfied, enabled one to use Equation (1) in calculating the strength of a structure[2]. Since snap-through is the result of deformations within a frame it is best introduced by looking at the effects of deflections on the capacity of a frame as calculated by simple plastic theory.

2.3 The Effect of Finite Deformations on the Plastic Collapse Load

As previously stated a simple plastic analysis involves the assumption that deflections are negligible up to the point of failure. Deformations do, however, occur due to the elastic behaviour of the structure under load.

Considering the effect of finite deformations it may be shown that if the externally applied forces produce a compressive force R_i in a member of length L_i, which has rotated through an angle of β_i, then the work equation becomes:

$$\Sigma(P_p.\Delta) + \Sigma(R\beta^2L) = \Sigma(M_p\phi_i) \qquad (2)$$

The first and last summations are identical to those in equation one. The term $\Sigma(R\beta^2L)$ includes the effect of member deformations.

If equation (2) is applied to frames, a reduction in plastic collapse load with deflection occurs, as shown by line-AB in figure 2.3. From this diagram it is quite clear that finite deformations have a marked effect on the collapse load of a structure. As a result the collapse load as calculated by simple plastic theory is at most an upper bound to the failure load of a frame. The extent to which the failure load falls below the rigid-plastic value gives one an indication of the slenderness or extent of instability of the frame. If the instability of the frame is limited, i.e. deflections are small, a simple plastic analysis in order to calculate frame strength may be adequate.

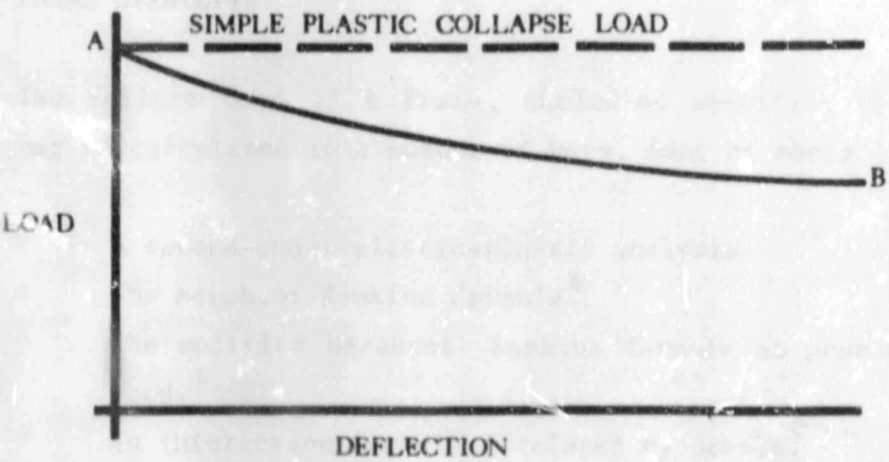


FIGURE 2.3: REDUCTION IN PLASTIC COLLAPSE LOAD WITH DEFLECTION

3 FRAME STABILITY

The failure load of a frame, including stability effects, may be calculated in a number of ways. Some of these are:

* A second-order elastic-plastic analysis.
* The Merchant-Rankine formula.[6]
* The modified Merchant- Rankine formula as proposed by Wood.[7]
* An interaction formula developed by Scholz.[8]

The latter three methods incorporate the elastic critical buckling load. A reasonable approximation to the failure load therefore requires an accurate calculation of the elastic critical buckling load.

The Merchant-Rankine formulae and Scholz' method were both developed to reduce the time spent in calculating the failure load of a frame as a second-order analysis is complex and time-consuming. The above methods are briefly discussed as this will afford a better understanding of Horne's research into snap-through stability.

Alternatively, stability effects can be limited and simple plastic theory applied, to give a reasonable estimate of the collapse load. Since plastic analysis is very easily applied to multi-bay pitched-roof portal frames, Horne proposed limiting slenderness equations, which if satisfied, prevented snap-through instability from occurring.[2] Horne's research into snap-through is looked at in great detail and its shortcomings discussed.

3.1 A Detailed Second-order Elastic Plastic Analysis

In a simple elastic analysis the stiffness reducing effect of axial member forces and the effect of frame deflections are ignored. These effects are introduced in a second-order analysis by including stability functions in the stiffness matrix of the frame. The determinant of the stiffness matrix, if equated to zero, yields the elastic critical buckling load P_c.

However, plastic hinges occur within the frame well before this load is reached. If a similar analysis is undertaken with a plastic hinge at the most highly stressed point in the frame, a reduced elastic critical buckling load P_c^1 is obtained. Successive analyses each time a new hinge forms will eventually lead to the failure load of the structure P_f. This is the load at which the frame becomes unstable, i.e. the applied load no longer increases with increasing frame deformation.

Calculations are tedious and time-consuming. Computer programmes, such as that developed by Kemp[9], can therefore be used. These, however, are often elaborate and expensive. For design purposes, it is best to have approximate methods to calculate the failure load which are sufficiently accurate for engineering purposes, and are not time-consuming. Three such methods have been developed by Merchant, Wood and Scholz.

3.2 Merchant-Rankine Formulae

Merchant[6] suggested that it might be possible to consider the failure load as a function of P_p, the simple plastic collapse load, and P_c, the elastic critical buckling load. With this in mind, he proposed the well-known Merchant-Rankine rule of equation (3)

$$P_F = \left[\frac{1}{1 + P_p/P_c}\right] P_p \qquad (3)$$

This has since been modified by Wood[7] to include the beneficial effects of strain hardening and cladding. He proposed:

$$P_F = \left[\frac{1}{0.9 + P_p/P_c}\right] P_p \quad \text{for } 4 \leq P_c/P_p \leq 10$$

$$P_F = P_p \quad \text{for } P_c/P_p > 10 \qquad (4)$$

Wood also suggested that if $P_c/P_p < 4$ then a second-order analysis is necessary to calculate the failure load.

Both equations require the calculation of the elastic critical buckling load. Approximate procedures to calculate this have been proposed by Horne and Scholz.

3.3 **Horne's Research into Snap-through Stability[2]**

Often practical single-storey frames have sufficiently large elastic critical loads to ensure that $P_c/P_p > 10$, thus eliminating any need to check frame stability. While this is generally true, some frames may have particularly slender internal members and these may need further attention. A case in point are multi-bay pitched-roof portal frames in which full advantage has been taken of the horizontal forces acting at eaves level, as illustrated in figure 3.1. This leads to a reduction in the size of the internal rafters. In fact, in the extreme, theory enables the internal rafters to be designed as fixed-ended beams as shown. This may result in frames becoming susceptible to snap-through instability.

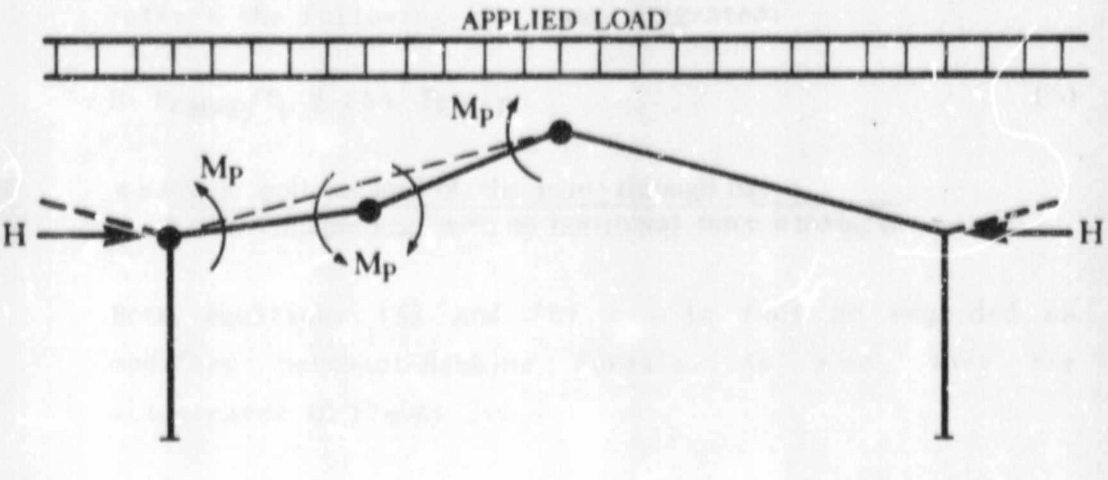


FIGURE 3.1 : DESIGN OF RAFTERS AS FIXED ENDED BEAMS BY UTILISING ARCHING ACTION

The situation is made worse if the internal columns themselves are slender and their bases are close to the pinned condition.

The above discussion leads to the necessity of including in a steel design code, some safeguards against the design of extremely slender multi-bay frames if plastic theory is used. To this end Horne directed his research.

3.3.1 Criteria for Limitations on the Slenderness of Multi-bay Frames

It was proposed that simple plastic theory should be permitted for multi-bay pitched-roof frames, taking $P_f = P_p$, if ranges of slenderness are such that certain minimum values of P_c/P_p are ensured.

For internal sway buckling, Horne proposed the following limit:

$$\text{If } P_{c\,sway}/P_p \geq 5,\ P_F = P_p \tag{5}$$

This is less stringent than the minimum ratio of ten proposed by Wood for multi-storey frames since the beneficial effects of roof cladding are far greater in single-storey frames. As a safeguard against snap-through of rafters the following limit was suggested:

$$\text{If } P_{c\,snap}/P_p \geq 2.5A,\ P_F = P_p \tag{6}$$

$$A = \frac{\text{Plastic collapse load of the snap-through bay}}{\text{Plastic collapse load with no horizontal force} = 16M_p/L}$$

Both equations (5) and (6) can in fact be regarded as modified Merchant-Rankine formula. As such, they are illustrated in figure 3.2.

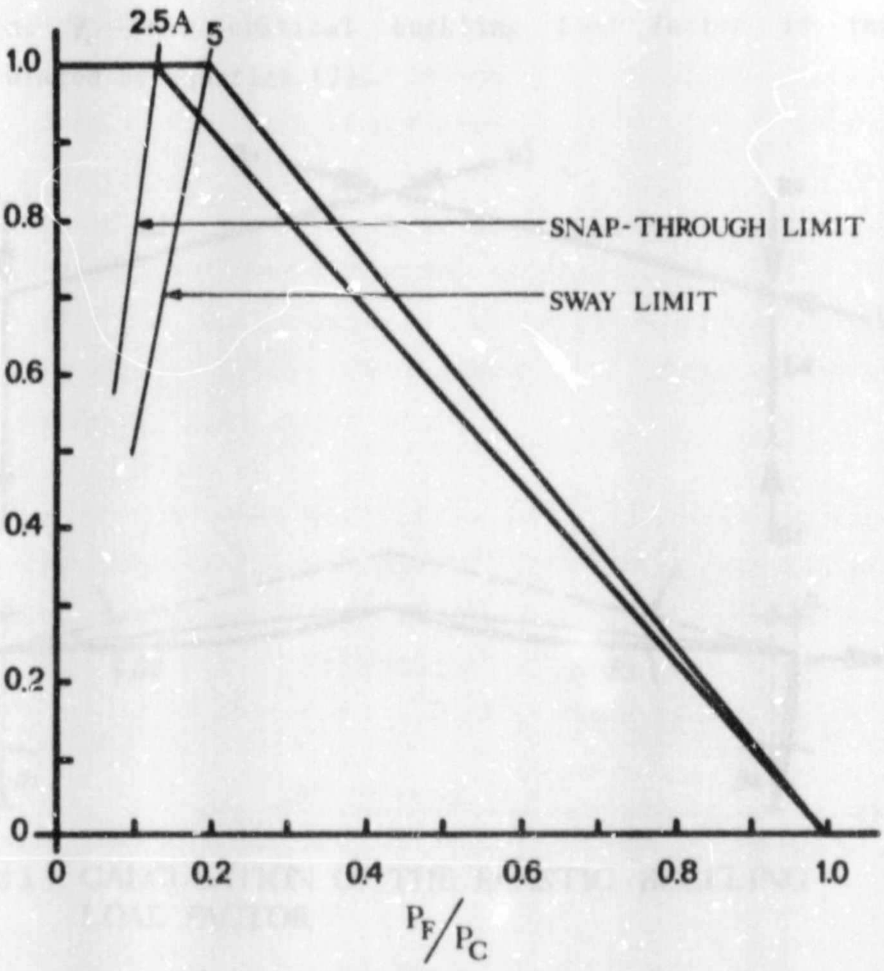


FIGURE 3.2 : HORNE'S PROPOSALS

To calculate the elastic critical buckling load of the multi-bay frames the approximate energy method as described in the next paragraph was used.

Approximate Energy Method

This method is well suited to structures such as multi-bay pitched-roof frames in which snap-through instability may occur.

Suppose it is desired to find the load level at which the axial forces shown in figure 3.3 cause elastic critical behaviour. Let the critical buckling mode be initiated by a disturbing force Q as shown. According to linear elastic analysis this causes a deflection and member rotations ...

given. γ_c the critical buckling load factor is then calculated by equation (7).

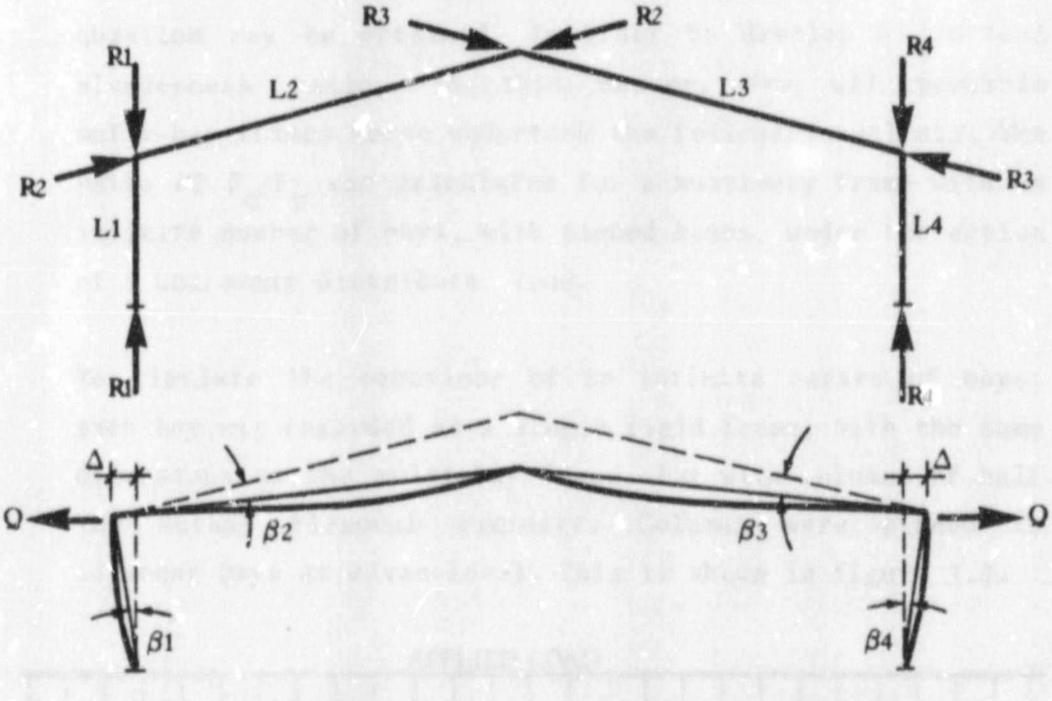


FIGURE 3.3: CALCULATION OF THE ELASTIC BUCKLING LOAD FACTOR

$$\gamma_c = \frac{0.9\ \Sigma(Q\Delta)}{\Sigma(R_i L_i \beta_i^2)} \qquad (7)$$

The summation extends over all members.

In applying this method the assumption was made that the axial forces in the members at the elastic critical load are proportional to the values at rigid-plastic failure. Equation (7) now becomes:

$$P_c/P_p = \frac{0.9\ \Sigma(Q\Delta)}{\Sigma(R_i L_i \beta_i^2)} \qquad (8)$$

The member forces Ri in the above equation are now those at rigid-plastic collapse. The results of equation (8) will be discussed in a later section of this report.

Snap-through Stability

If the ratio of P_c/P_p in equation (8) is given the value 2,5A then a limiting slenderness ratio for the frame in question may be obtained. In order to develop a limiting slenderness ratio, in this manner, for all possible multi-bay frames Horne undertook the following analysis. The ratio of P_c/P_p was calculated for a multi-bay frame with an infinite number of bays, with pinned bases, under the action of a uniformly distributed load.

To simulate the behaviour of an infinite series of bays, each bay was regarded as a single rigid frame, with the same dimensions as the multi-bay frame, but with columns of half the actual flexural rigidity. Columns were pinned to adjacent bays at eaves-level. This is shown in figure 3.4.

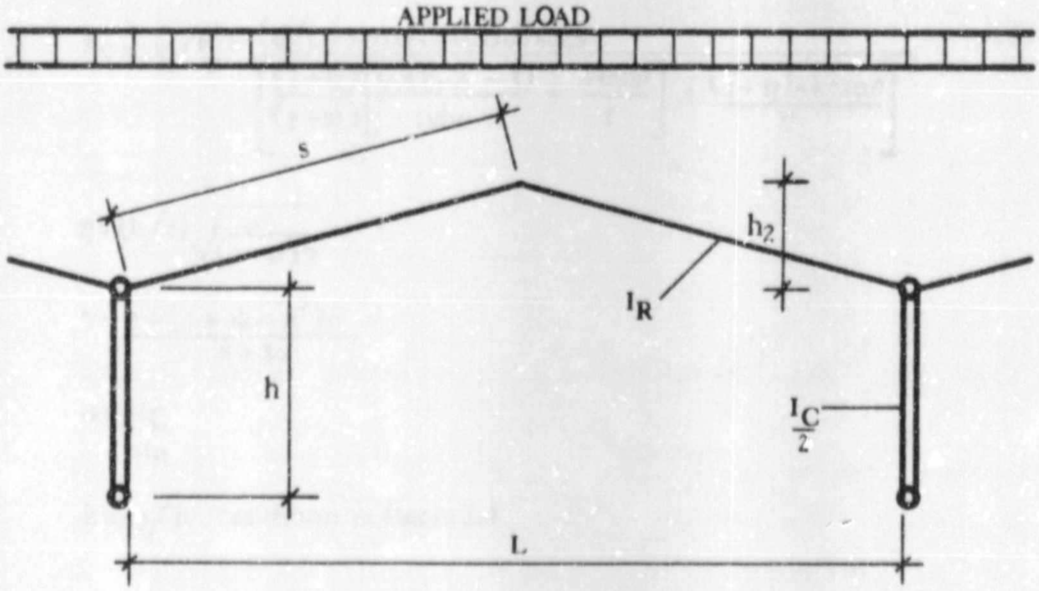


FIGURE 3.4: HORNE'S MODEL FOR MULTI-BAY FRAMES

Horne believed that the above analysis could provide a limiting slenderness ratio for all frames for a number of reasons:

- Horne believed that the distribution of loading, ie. the same uniformly distributed load acting in each bay would give a lower elastic critical load than any other loading configuration.

* The analysis of an infinite number of bays provides one with an under-estimate of the elastic critical snap-through load of the actual multi-bay frame considered.
* Most structures have bases which lie between the pinned and fixed cases. An analysis with pinned bases will give a lower value for the elastic critical load than a detailed analysis using the actual degree of fixity.

Thus a lower bound to the ratio of $P_{c\ snap}/P_p$ for all frames is obtained.

An analysis of the model illustrated in figure 3.4 gives for equation (8):

$$P_{c\ snap}/P_p = \frac{(d/L)\gamma\cos\theta(245/\sigma_y)4025}{\left[\frac{(2+\eta)}{(1+\eta)}\left[\frac{\sqrt{A}(\sqrt{A}-1)}{\tan\theta}+\frac{A\tan\theta}{2}\right]+\frac{(1+\eta)Ak\tan\theta}{\eta}\right]} \quad (9)$$

$$\eta = (k/2)\sqrt{\frac{\alpha}{(2+\alpha)\gamma}}$$

$$\gamma = \frac{2+(3+3k+k^2)\alpha}{8+3\alpha}$$

$$\alpha = \frac{sI_C}{hI_R}$$

$k = h_2/h$ as shown in figure 3.4

If the limiting value of $P_{c\ snap}/P_p = 2{,}5A$ is applied, the following slenderness limit is obtained:

$$(L/d)_{snap} = \frac{1610\gamma\cos\theta(245/\sigma_y)}{\left[\frac{(2+\eta)}{(1+\eta)}\left[\frac{\sqrt{A}(\sqrt{A}-1)}{\tan\theta}+\frac{A\tan\theta}{2}\right]+\frac{(1+\eta)Ak\tan\theta}{\eta}\right]A} \quad (10)$$

Since this equation is too complex to apply directly, the empirical relation for the range $\theta = 0°$ to $20°$ was proposed.

$$(L/d)_{snap} = \frac{25\tan 2\theta (4 + L/h)(1 + I_C/I_R)}{A(A-1)} \cdot \frac{245}{\sigma_y} \quad (11)$$

If slendernesses of the actual frame being analysed are greater than that calculated from equation (11), snap-through instability may be a problem. Failure of the frame in snap-through may therefore occur before the plastic collapse load is reached. If however equation (11) is satisfied simple plastic theory may be used to calculate the collapse load. Of course other effects such as web and flange buckling and individual member stability must also be checked before one can be reasonably sure that the plastic collapse load will be the failure load of the frame.

Equation (11) was developed specifically for I and H sections with a Youngs modulus of 210×10^3 N/mm^2, as these sections are the most commonly used in pitched-roof portal frames. Table 3.1 gives limiting slenderness ratios to prevent snap-through, for various L/h ratios and arching factors. Values were calculated using equation (11) and a yield stress of 350 N/mm^2. The detailed derivation of equation (11) is given in appendix B.

3.4 Failure Load as predicted by Scholz' Interaction Equations[8]

The elastic-plastic failure load of a frame is approximated using an interaction equation which incorporates at the one end a frame failing plastically and at the other, a frame failing elastically. Scholz has shown that frames can be classified into specific frame families and that one such family can be represented as a particular curve in the multi-curve interaction diagram shown in figure 3.5. This figure applies to cases of pure gravity load which are relevant for snap-through failure. A slight modification is introduced to consider cases of combined loading.

TABLE 3.1 : HORNE'S LIMITING SLENDERNESS RATIOS TO PREVENT SNAP-THROUGH $I_C = I_R$

ARCHING FACTOR A	L/h	ANGLE OF RAFTERS			
		5°	10°	20°	30°
1.5	4	66	136	313	647
	8	99	204	470	970
	12	132	272	627	1293
2	4	25	51	117	242
	8	37	76	176	364
	12	49	102	235	485
2.5	4	13	27	63	129
	8	19	41	94	194
	12	26	54	125	259
3	4	8	17	39	81
	8	12	25	59	121
	12	16	34	78	162
4	4	4	8	20	40
	8	6	13	29	61
	12	8	17	39	81

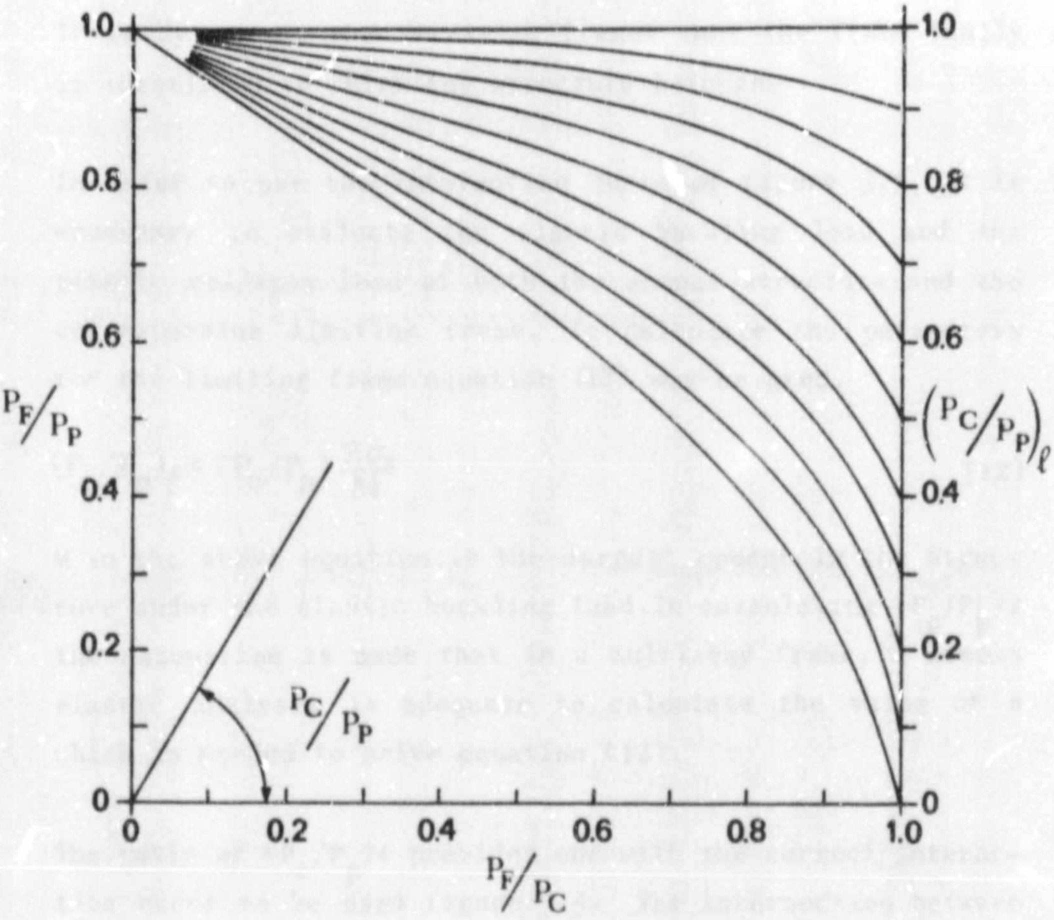


FIGURE 3.5: INTERACTION CURVES FOR GRAVITY LOAD

Characteristics of one of the curves as illustrated are:

- * The intersection on the right vertical axis belongs to frames within the frame family that fail completely elastically.
- * The curve itself represents frames that fail elastic-plastically.
- * The point $P_f/P_p = 1$ on the left vertical axis represents a frame that can be analysed using simple plastic theory. Deflections are negligible and plastic analysis is appropriate.
- * The transition between elastic and inelastic failure is marked by a "limiting frame" of the frame family. This unique structure is identifiable by equating elastic failure with first yield.

It is by way of the "limiting frame" that the frame family is identified to which the structure belongs.

In order to use the interaction curve of figure 3.5, it is necessary to evaluate the elastic buckling load and the plastic collapse load of both the actual structure and the corresponding limiting frame. To calculate the parameters for the limiting frame equation (12) may be used.

$$(P_c/P_p)_\ell = (P_c/P_p)\,\frac{Z\sigma_y}{M} \qquad (12)$$

M in the above equation,is the largest moment in the structure under the elastic buckling load.In calculating $(P_c/P_p)\ell$ the assumption is made that in a multi-bay frame, a simple elastic analysis is adequate to calculate the value of M which is needed to solve equation (12).

The ratio of $(P_c/P_p)\ell$ provides one with the correct interaction curve to be used figure 3.5. The intersection between this interaction curve and the line representing the ratio of P_c/P_p for the actual frame gives the ratio of predicted failure load to plastic collapse load of the structure under analysis. An example of this method is given in appendix C.

This alternative procedure of calculating the failure load can be used for frames subject to gravity or combined loading. Results of frames analysed in this way have proved to be accurate enough for engineering purposes. Also the method is simple, quick to apply and may easily be used for extremely complex structures. As for Horne's proposals, the elastic buckling load has to be calculated. Scholz has developed a computer programme to calculate this which is described in the next paragraph.

3.4.1 Computer Analysis by Scholz[5]

A computer program was written to calculate the elastic snap-through load for multi-bay portal frames with any degree of linear elastic base restraint.

The computer program was based on the frame model illustrated in figure 3.6. The presence of bays next to the snap-through bay is recognised by non-linear lateral springs at eaves level.

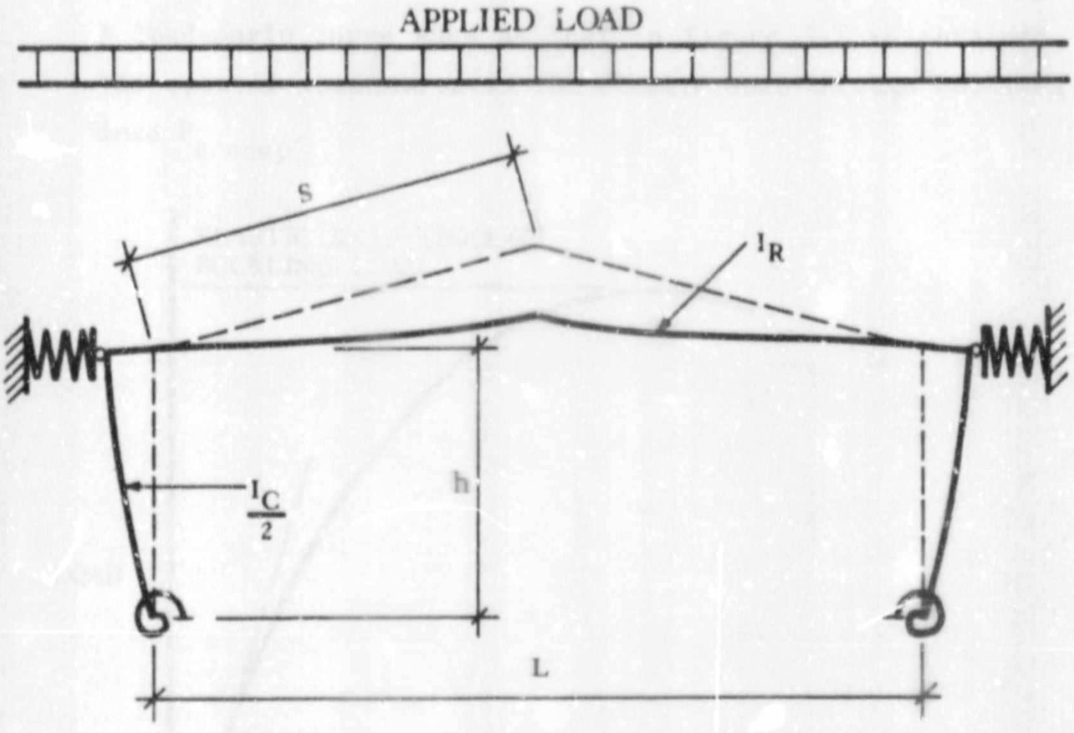


FIGURE 3.6 : SCHOLZ'S MODEL FOR MULTI-BAY FRAMES

In order to increase the accuracy of the analysis the programme includes the following:

* The effects of axial rafter forces due to distributed loading.
* The effect of rafter shortening resulting from the deflected shape of the rafters under load.

The general problem of figure 3.6 is then solved by the programme applying to the deflected shapes of the column and rafter members the differential equation:

$$EI.\frac{\partial^2 y}{\partial x^2} + M + R.y = 0 \quad (13)$$

Here M is defined as the bending moment at a position x