The Effect of Operating Conditions on the Kinetics of Density Stratification in a Batch Jig

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In fulfilment of the requirements for the degree of Master of Science in Engineering

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Declaration

I declare that this dissertation is my own work aided only by the guidance of my supervisors. It is being submitted for the Degree of Master of Science in Engineering to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.

Anil Kumar Tripathy

28th October 2021

Abstract

King's stratification model is a prominent and elegant model of stratification in a jig once an equilibrium condition has been reached. It has been well validated for synthetic systems of mono sized particles. However, it does not account for (1) kinetic effects, (2) the effect of operating conditions on jigging performance, or (3) differences in particle size and shape. Very little work has been done to investigate how the King model can account for the first two of these shortcomings. The focus of the investigation reported in this thesis was therefore to enhance the predictive ability of the King model with regard to accounting for the effect of kinetics and operating conditions.

Jigging performance has been found to be a function of both (1) jig operating conditions and (2) feed characteristics such as feed composition, density distribution, the size and shape of the particles. Hence, tests were conducted in a batch jig using 8mm artificial particles (glass beads) of different colour and density. The effect of key operating conditions such as pulsion time, hold time, bed depth, and water displacement (stroke) on density stratification and stratification kinetics has been investigated.

The results showed that, provided the bed depth was 100mm or less and the stroke was not excessive, the operating conditions did not affect the stratification pattern at equilibrium but did affect the kinetics of stratification. This has the important implication that kinetic and equilibrium effects can be decoupled. The key operating variables that were found to influence stratification kinetics were the density difference between the particles being stratified, the stroke, and the hold time after the pulsion stroke.

It was found that the effect of operating conditions on stratification kinetics could be modelled by means of an 'approach-to-equilibrium' metric. The approach of the stratification pattern asa whole

to the equilibrium was essentially first order and could be described in terms of a time constant, $\boldsymbol{\Theta}$, and a delay time of $\boldsymbol{t_o}$. A multi-linear regression model was then developed to describe the effect of density difference, stroke, pulsion time, and pulsion hold time on the time constant $\boldsymbol{\Theta}$. From this a kinetic model for stratification was developed in which the equilibrium stratification profile is described by the King model, and the kinetics are described by the regression model. The quality of fit of this model to the kinetic data generated in the study is variable and more work is required to improve its predictive ability.

Dedication

To God Almighty

And to my family;

My parents Mr. Nabin. C Tripathy and Haripriya Tripathy

My family members

My friend Abhishek Pradhan

Anil Kumar Tripathy

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Above all I would like to give thanks and praise to the Lord almighty for granting me the strength and the opportunity to do a masters project. It is by His grace that I was able to endure through every step of this project.

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Publications

The following contributions from this investigation has been published:

1. L.C. Woollacott, A.K. Tripathy and J.H. Potgieter, The effect of operating conditions on the stratification kinetics in a batch jig, Proceedings of the XXX International Minerals Processing Conference, 18-20 October 2020, Cape Town, South Africa, pp. 886-897

2. L. Woollacott, A.Tripathy and H. Potgieter, The Effect of Operating Conditions on Density Stratification in a Batch Jig Part 1: The influence on the Equilibrium Stratification Profile, Minerals Engineering, 2021, 170, 106838

3. L.C. Woollacott , A.K. Tripathy , J.H. Potgieter, The effect of operating conditions on density stratification in a batch jigII: The influence on stratification kinetics, Minerals Engineering, 2021, 164, 106846

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Chapter 1

Introduction

1.1 Background

Jigging is one of the oldest gravity separation methods used in mineral processing for the separation of gangue particles from an ore or secondary materials (**Burt, 1984**). It shows several positive aspects such as cost effectiveness, high separation precision, high throughput and easy maintenance (**Mehrotra, *et al* 1997**). A variety of minerals can be processed ranging from coal to diamonds, andalusite to zirconia, mineral sands to metal oxides and from industrial minerals to precious metals (**Lin, *et al* 1997, Mutibura, 2015**). It is particularily effective for the separation of minerals that have a large difference in density.

Although jigging is an old method for mineral beneficiation and it has been the focus of closer investigation for many years, the dynamics that affects the performance of a jig still not adequately understood. This represents that the current ability to efficiently model the jig performance in mineral processing circuits can be upgraded **(Mutibura, 2015)**.

A number of models have been developed (as discussed in the literature review) to understand the dynamics of the jigging process and to predict jig performance. Among these, King's stratification model is one of the most prominent, effective and easy to use **(Crespo, 2016)** but has some limitations.This study aims to enhance the predictive ability of the King model by addressing some of these limitations.

The King model has been formulated on the assumption that all particles in a jig bed are mono-sized spheres. Another limitation is that it is an equilibrium model and does not take into account the effect of kinetics on jig performance. A further limitation is that the model does not provide an adequate phenomenological link between the operating parameters of a jig and its performance. This study will focus on the second and third of these limitations but not the first.

The King model describes the nature of density stratification of particles in a jig bed by predicting how the concentrations of particles of different density vary in that bed – i.e. it predicts the ‘concentration profile’ in the bed. A jig achieves a mineral separation by splitting the bed into two products –the top layer and the bottom layer. The performance of the jig can be expressed in several ways. One is by means of a partition curve which indicates the extent to which a particle of a given density will be recovered to either the top or bottom layer. A problem with using this approach for jig performance is that the partition curve depends on where the bed is split into two layers – i.e. the height at which the bed is split. A better means of describing jig performance is therefore to indicate the recovery of density components as a function of the height at which the bed split. The concentration of the density component in the top or bottom layer is also an important indicator of performance. All of these performance indicators can be calculated from the concentration profile.

One of the reasons for the popularity of the King model is that only one experimentally determined parameter is needed in order to predict the concentration profile at equilibrium. This is the ‘King stratification index’, alpha. The King model has been well validated by different authors **(King, 1987, Tavares and King, 1995, Woollacott et al., 2015).** In order for the King model to predict the effect of operating parameters on jig performance, it is necessary to model the effect of these operating parameters on the stratification index, alpha. However, no work of this kind has been reported in the literature. Some authors have studied the effect of operating conditions such as frequency and amplitude on the jig performance **(Jinnouchi et al., 1984, Rong and Lyman, 1993)** but with variable success and without using the King model**(Mukhrjee and Mishra, 2006).**

Density stratification in jigging is a kinetically oriented process – i.e. the concentration profile changes with time from a homogeneous bed to a stratified bed at equilibrium. Some authors (**Lin et al., 1997**, **Mehrotra et al., 1997** and **Rong et al., 1993)** developed empirical models for jigging kinetics, for example by using a power function equation to relate stratification indices to the jigging time. Other authors have studied the kinetics of the jigging process and tried to correlate the stratification rate with jigging parameters empirically. None of these authors have attempted to describe jigging kinetics relative to the equilibrium concentration profile for which a well validated model exists.

1.2 Problem Statement

Although much work has been done on modelling jig performance, inadequate advantage has been taken of the King model and its proven ability to model the equilibrium concentration profile in a stratified bed. In order to extend the predictive ability of the King jig model, it is necessary to address its current limitations with regard to its ability to describe kinetic effects and the effect of operating conditions on jig performance. To do this, the study addresses the following research questions.

- How do the operating conditions of a batch jig affect density stratification at equilibrium?
- How do the operating conditions affect jigging process kinetics?
- How can these effects be modelled effectively?

1.3 Structure of this thesis

The introductory chapter provides a background to the problem to be tackled for this study and the research questions to be addressed.

In the second chapter a review of jigging technology, jigging parameters, relevant theories and models are presented. Emphasis is placed on theories and models that describe the mechanism by which a bed of particles for specific particle sizes is stratified in a jig.

Chapter three provides a detailed description of the experimental work and equipment used to achieve the set objectives. Presented there are the experimental design, procedures followed, the laboratory work performed and how the data was analyzed.

The fourth chapter presents the experimental results relating to research question 1. It presents the concentration profiles at equilibrium achieved in the batch jig under the operating conditions tested. The results are analyzed to show how those conditions affected stratification and the King stratification index, alpha.

The fifth chapter presents the experimental results relating to research question 2. It presents the findings of the kinetics study into how the concentration profiles changed with time under different operating conditions. The results are analyzed to show how those conditions affect the stratification rates achieved.

The sixth chapter presents a quantitative analysis relevant to research question 3and develops a kinetic modelbased on the data generated in chapter 5.

The final chapter discusses and summaries the major findings, their relevance and provides some recommendations for future work.

Chapter 2

Literature Review

2.1. The jigging process

Among the separation processes available in mineral processing industries, jigging is considered to be one of the oldest among them. In this separation process a bed of particles is stratified in a fluid medium primarily according to density. Vertical pulsation of the fluid generates hydrodynamic and gravity forces that cause the bed to be repeatedly fluidized and the particles to move relative to one another. Figure 2.1 shows a typical jigging process. It is used for the beneficiation of minerals as well as for coal washing.

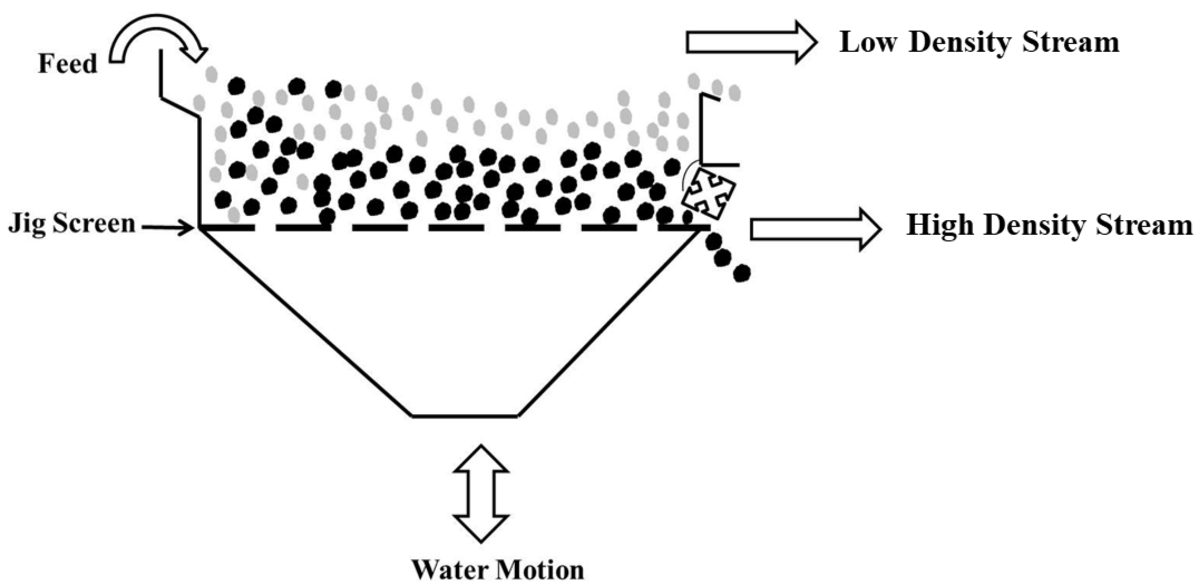


Figure 2.1 Schematic diagram of the jigging process

(Roux, 2017)

Jigging was described as a cyclic process by **Burt (1984)** and (**Mehrotra et al.,1997)**and consisting of three sometimes four distinct stages. The first stage is termed pulsion or the upstroke and takes place at the initial stage of the jig cycle when the fluid moves upwards and lifts and loosens the bed expanding its volume. A second stage can sometimes be recognized towards the end of the up-stroke especially when the fluid level is held for a short period at its maximum level. During this stage the bed transitions from expansion to consolidation. The two final stages – the exhaust and compaction stages - occur during the suction or down-stroke part of the jig cycle when the fluid moves downwards. In the exhaust stage the particles settle in the partially fluidized bed and the bed volume returns to its original condition. During the compaction stage at the end of the down stroke smaller particles are able to trickle downwards to some degree through voids in the resettled bed.

The stratification of the bed is brought about by differences in the density, size and shape of the particles in the bed and by the cyclic repetition of these four stages. Layers of different density are formed due to the stratification process such that the particles having less density (and smaller size) drift towards the upper part of the bed whereas higher density (and larger) particles drift towards the bottom of the bed. The extent of stratification is influenced by the residence time of the particles within the jig as well as the thickness of the bed and the nature of the jig cycle (**Myburgh, 2010;Myburgh et al.,2014**).

2.2. Type of Jigs

Many different types of jigs have been used in industry through the years to improve efficiency, to reduce extraction costs and to fit specific applications. Figure 2.2 provides an overview of different types of jigs with their applications. It distinguishes between fixed vs moveable screen jigs; over- and through-screen jigging; and by different fluid pulsing mechanisms. Today most jigs are of the fixed screen type where the fluid pulsation is generated by some form of piston, diaphragm or air pulsed system. In moveable-screen jigs these mechanisms are not necessary because pulsation is caused by the screen itself moving up and down. Figure 2.1 is a typical over-screen jig where both the denser and less dense products move through the jig over the screen – the lower density product flowing over the dam at the end of the jig and the denser product withdrawn by some form of extraction system as suggested in the diagram. Through-the-screen jigging is used to extract fine heavy particles from the jig feed by allowing these particles to percolate through the bed and through the screen where it is removed from the hutch chamber below the screen.

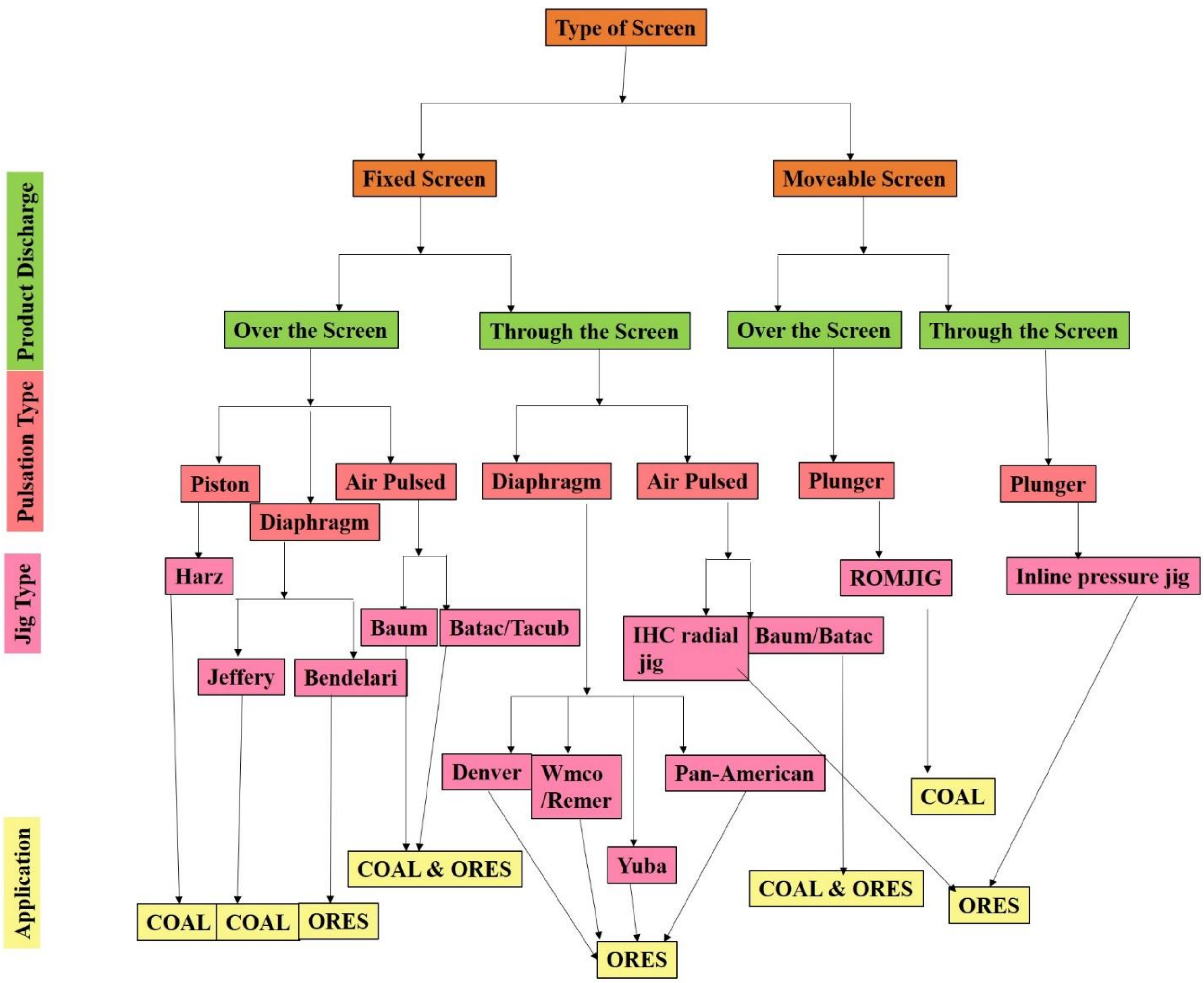
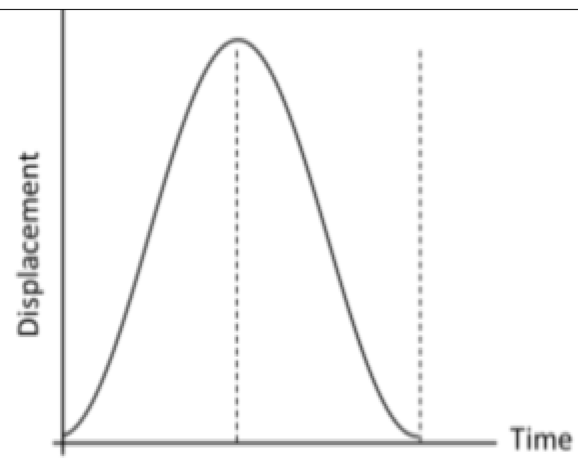


Figure 2.2 Type of Jigs as per the classification proposed by Sampaio and Tavares and reported by Ambrós,2020

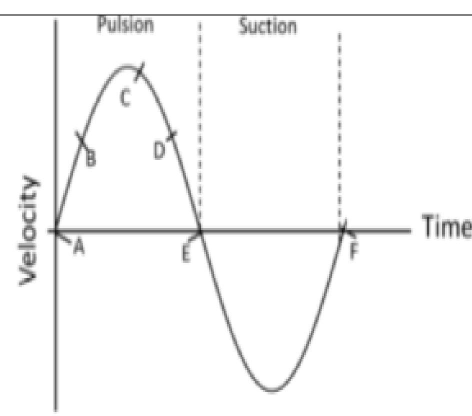
2.3. Operating Parameters

2.3.1 Jigging Cycle

One of the most important variables of the jigging process is the jigging cycle. It determines both the displacement and water velocity profile inside the jig. For instance, the reciprocating movement of the piston generating the fluid movement in a jig produces a harmonic or sinusoidal waveform as shown in Figure 2.3A explains the fluid displacement and Figure2.3B shows the fluid velocity with sinusoidal fluid motion. Points A to F in figure B represents the behaviour of jig bed at different times during a jig cycle Figure 2.4a shows the simplest pulse shape showing harmonic motion where the duration and intensity of the pulsion and suction stroke are similar. This waveform typically occurs in piston type jigs and in some diaphragm type jigs such as the Harz, Denver and the Belendari jigs.



A: Fluid displacement



B: Fluid velocity

Figure 2.3 Harmonic motion cycle of a jig

(Roux, 2017)

Figure 2.4b shows the waveform of air-pulsated jigs like the Baum and Batac jigs. Here the wide variation of the cycle can be achieved by controlling the opening and closing of the air valves controlling the air flow to air chambers that control the fluid level in the jig chamber. These waveforms are commonly used for coal processing.

Figures 2.4c and d show examples of the saw-tooth waveform commonly used in through-the-screen jigging. Here the waveform selected depends on the relative proportion of fine particles in the jig feed and of fine heavy and fine light particles in the jig feed. In the In-Line pressure jig (Figure 2.4c), the fast upward stroke minimizes the loss of fine products to tailings and the slow downward stroke makes for smoother percolation trickling of fines through the bed and provides more time for recovery of the fine heavy product [**Ambrós,2020**]. On the other hand, the ROMJIG (Figure 2.4d) shows a saw-tooth pulse with a rapid downward stroke and slow upward stroke. This may be beneficial in minimizing the contamination of the fine heavy particles with fine light particles.

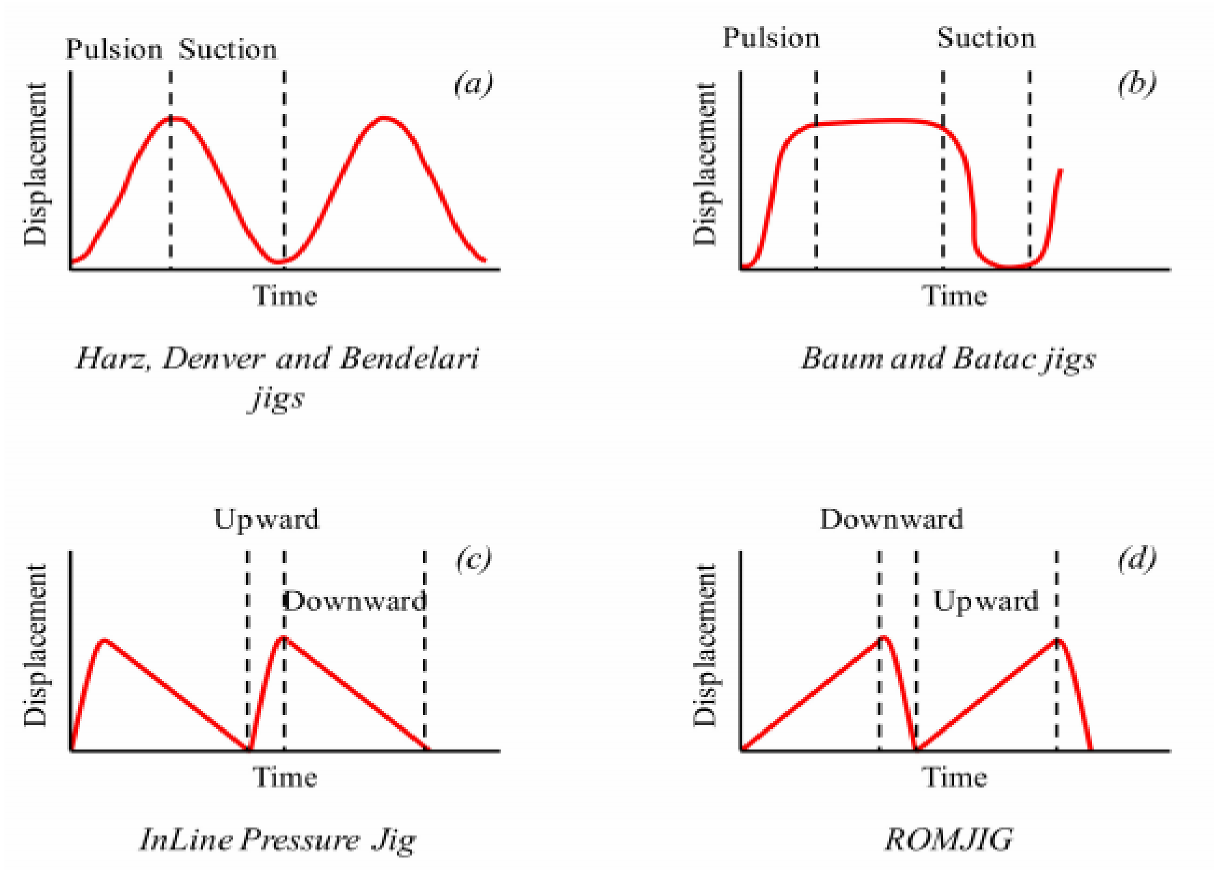


Figure 2.4Typical pulsation diagrams of different jigs.

(a) sinusoidal, (b) trapezoidal, (c) "saw-tooth" with rapid upward, and (d) "saw-tooth" with rapid downward movement.(Ambrós,2020)

2.3.2 Stroke frequency and amplitude:

The optimum pulse cycle frequency for a specific application depends on the rate, size and density of the feed to a jig as well as the jig design. The length of the jig cycle should be sufficient that adequate time for stratification is provided and affects the throughput capacity of the jig. The capacity will be affected negatively if the cycle time is longer than the minimum time required, since little stratification takes place during when the bed is consolidated **(Gupta and Yan, 2006).**

In high capacity jigs that pulse enormous volumes of water, account needs to be taken of the natural pulsation motion that is described by:

$$T = 2t = 2\pi\sqrt{\frac{L}{g}} \qquad [2.1]$$

where T= Period of pulsation (s), g= acceleration due to grvity and L= "Distance between the centres of the water mass at the two extremes of the oscillation (m)"**(Roux,2017).**

From the above equation, it can be concluded that the amplitude as well as the frequency of pulsation is having equivalent significance in jigging process. (Mishra and Mukherjee,2006) reported the significance of frequency on jig efficiency. Pulsated fluidized bed was observed by them to be a mass damper system which was tuned with the help of multi components. This can be explained by an instance where a mass damper system that is tuned by two-components have two natural frequencies. The one with lower frequency resembles to a larger effective mass and visa vis. In this case particles with two varying sizes are considered where respective sizes have two varying density particles. Combining all the jig system is represented with particles consisting of four different masses. A mechanical system that corresponds to a jig is shown in Fig. 2.5a, where, the particles as well as water are epitomized by an independent pair of springs and dashpots. The particles were observed to initiate resonate, eventually providing higher mobility at a while when the pulse frequency tend towards the particle's natural frequency.

Particle was observed to start resonating, consequently giving higher mobility when pulse frequency approached natural frequency of the particle as expressed by

$$\omega_n = \sqrt{\frac{K}{m}} \qquad [2.2]$$

K: spring stiffness

m: particle mass

Mishra and Mukharjee(2006)also reported how the maximum velocity of water affects jigging efficiency. They suggested maximum velocity of water was important factor that needed control as it was used for determination of fluidisation of particles in jig also water velocity does not remain constant in the overall jigging cycle. Also it was observed by them, for varying particle size, the maximum water velocity also varied in order to accomplish maximum separation efficiency as shown in Figure 2.5b.

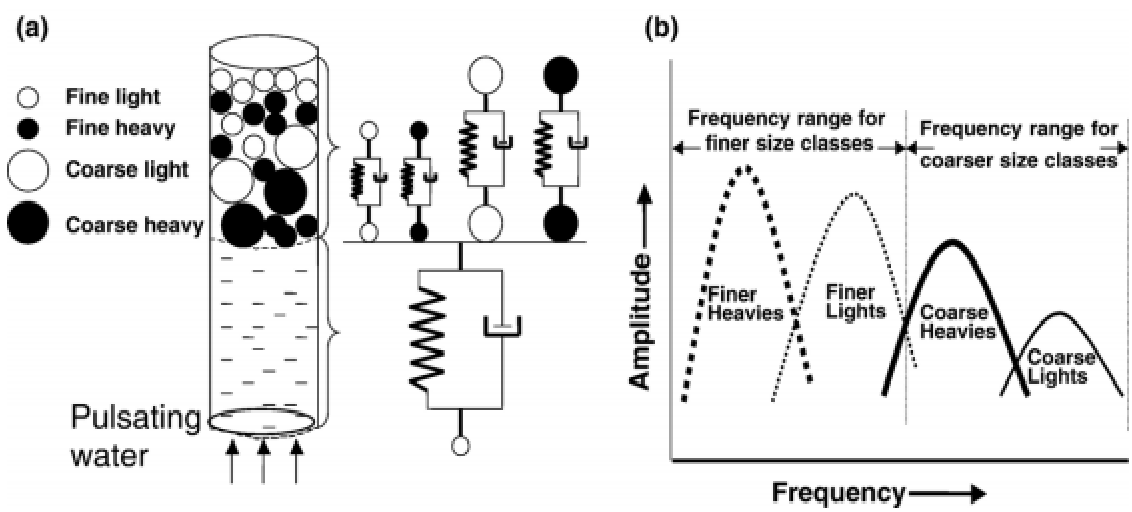


Figure 2.5 Frequency regimes of different group of particles

"(a) four different types of particles in the jig bed are shown as independent tuned mass dampers and water in the jig acts as a separate vibratory system; (b) frequency regimes of particles." (Mishra and Mukharjee, 2006)

The amplitude and the frequency can be expressed by equation 2.3 (**Gupta and Yan, 2006**):

$$V = \frac{Na\pi}{60} \qquad [2.3]$$

Where

V: velocity required to suspended the mineral bed

a: amplitude of the stroke

N: Number of strokes/minute

The minimum water velocity needed for bed lifting is represented in terms of terminal velocity for the largest particle to be lifted. It can be expressed as equation 2.4 and 2.5.

$$V_t = \frac{\sqrt{\left(4g(\rho-\rho')\right)D_p}}{\sqrt{3C_D}\rho} \text{(for Stoke's law region)} \qquad [2.4]$$

$$V_t = \frac{gd^2(\rho-\rho')}{18\mu} \qquad \text{(for Newton's law region)} \qquad [2.5]$$

Where

V_t= terminal velocity $\qquad$ C_D= Drag coeficient

µ= Viscosity $\qquad$ g= acceleration due to gravity

d= Diameter of the particle

ρ *and* ρ' are the densities of particle and fluid respectively

2.4 Jigging Theories and Models

There are many theories developed to understand the dynamics of the jigging process since decades by several researchers. Wills **(1992)** claimed that there is no established theory that can well explain about a jig performance although it has been a primitive process used in the mineral processing industries. As reviewed by **Mehrotra et al.(1997) and (Mutibura, 2015)** the important theories related to the jigging process are classified as follows:

- Classic theory based on single particle behaviour
- Potential energy theory
- Dispersion models of particle suspension
- Energy dispersion theories
- Stochastic analysis
- Empirical models
- CFD and DEM models
- The King stratification model

2.4.1. Classic theory based on single particle behaviour

This theory explains jigging to take place by following the mechanism of differential acceleration, hindered settling as well as interstitial trickling (**Gaudin, 1934**).It proposed a jig function by combining the following mechanisms:

2.4.1.1 Differential acceleration

The particle acceleration in a fluidis expressed as function of the relative density of only the particle as well as fluid, and can be given in equation 2.6.

$$\frac{dv}{dt} = \left(1 - \frac{\rho_f}{\rho_s}\right) g \qquad [2.6]$$

Where, v= velocity of a settling particle

ρ_f andρ_s denotes the densities of the fluid and the solid particle respectively

t= time

and g= acceleration due to gravity

This states that particles having the same densities will have the same acceleration at the beginning. After that, provided the time of fall is short and effective, and the particles are permitted to fall frequently, the relative distance travelled by the given particles should be dependent on their initial acceleration. (**Burt, 1984**). This implies that the corresponding distance covered by various particles should depend further on the acceleration of the particles under original conditions, when they are allowed to fall through the medium a significant number of times within a short period of time. Under such conditions, the specific gravity is responsible for the stratification taking place (**Mutibura, 2015**).

2.4.1.2 Hindered settling

When particles are allowed enough time for settling in a fluid medium, they attain their terminal settling velocity. The settling velocity of such particles follows either Stokes' law (when it follows laminar flow) or Newton`s, law depending on whether it's a small (less than 0.1mm) or a large particle (larger than 2 mm),as described by equations 2.4 and 2.5. This behaviour most likely does not occur as the number of particles becomes large and crowding of particles takes place. In this situation,hindered settling occurs as particles are not able to settle freely and are hindered. At this stage the entire bed performs as if it has a uniform density of the slurry.Consequently, the terminal velocity becomes a function of the weight of the particle instead of the particle density (**Burt, 1984**).

2.4.1.3 Interstitial trickling

The settling down ofthe bed particles towards the end of a particular jig cycle, leads to consolidation and forcing the larger particles to intertwinewith the effect of the suction force. The intertwining results in the formation of channels to the base of the jig bed, which cause the sufficiently smaller size heavier particles to trickle through the channels and reach the base. The process is highly efficient to recover smaller heavy minerals **(Burt, 1984).**It is evident that stratification takes place by depending on the characteristics of the feed to and the jig cycle. If there is a marginal difference in size among the smallest and the largest particles and the time duration of down stroke is sufficiently long, then there is a possibility of interstitial trickling to take place.

There are, however, two major drawbacks with this model. The first one is that it applies to an idealised case to explain the water behaviour in the jig.**Lin et al.(1997), de Jong et al.(1996)** and **Viduka et al.(2013)** all examined the complex behaviour of water in the jig,which is the reverse from the classic theory.Secondly, it only focused on 2 dimensional perspectives, by which the process becomes difficult for the modelling of interstitial trickling **(Mehrotra et al., 1997).**

2.4.2 Potential energy theory

Mayer (1964) established that the stratification occurring in a jig bed,is due to the reduction in potential energy of the system.It states that a stratified bed has a lower potential energy compared to an un-stratified bed. Under gravity potential, the un-stratified bed of particles seems to be unstable. As every system strive to attain its equilibrium by curtailing the Gibbs free energy, an un-stratified particle bed will attain to its equilibrium by reducing the difference between the stratified and un-stratified states of the bed. This theory also has several drawbacks. The important one is that this theory was unable to provide a fundamental explanation of the dynamics associated with a system having a stable configuration. Hence it cannot describe the separation rate. His theory suggests that stratification can be described by equation 2.7, which is a first order equation.. The flaw in this approach is that it fails to take into account the dispersive processes that occur in the bed during jigging.

$$S = J \exp(-kt) \qquad [2.7]$$

Where,

S: Stratification rate

J: the jiggability index

K: rate constant, which is a function of amplitude and frequency of the jig stroke

This is an empirical equation to evaluate the rate constant and to find out the order of stratification, but when size distribution taken into account, it fails to implement (**Mehrotra et al.(1997)**

2.4.3 Dispersion models of particle suspension

Many authors reported extensively on the approach of the potential energy theory that describes the effect caused by intra particle collisions. However, **Vetter et al. (1987)** opposed this philosophy, stating that stratification of minerals in a jig bed takes place as a result of potential energy drop. They developed a mathematical model by assuming that the mixing in the uniformly distributed fluidized bed was diffusion controlled. The model assumes automatic rearrangement by varying density, uniform shape and size particulate materials in the bed. A differential equation representing the mathematical model for motion of particles within the jig bed was created by construing conventional force balance on a particle where a noise term was included for intra particle interactions. The proposed model is shown in equation 2.8

$$m\frac{dv(t)}{dt} + Bv(t) = A(t) \qquad [2.8]$$

Where

m: mass of particle

V(t):velocity

A(t): collision force.

B:Prticle drag term

When an external force term k(t) was introduced to the above equation, it becomes

$$m\frac{dv(t)}{dt} + Bv(t) + mk(t) = A(t) \quad [2.9]$$

The above equation was modified by ignoring the inertial term and expressed it as a stochastic differential equation:

$$dy(t) = -\frac{mk(t)}{B}dt + dw(t) \qquad [2.10]$$

w(t): Wiener process

Wiener process is expressed as

$$w(t) = \int \frac{A(t)}{B}dt \qquad [2.11]$$

Finally, the dispersion equationis expressed as follows

$$\frac{\partial C_i}{\partial t} = -\frac{\partial[k_\mu(\rho_i - \bar{\rho}_i)C_i]}{\partial y} + k_D\frac{\partial^2 C_i}{\partial y^2} \qquad [2.12]$$

[C_i is the conditional probability]

Where,

k_μ: drag coefficient

k_D: coefficient measuring dispersive mixing of jig bed particles.

The model was limited mainly to particles having an uniform shape and size only. Hence, this model failed in application in real life scenarios.

2.4.4 Energy Dissipation Theories

The theory proposed by **Rong (1990)** and **Rong et al. (1993)** worked on air pulsated jigs. They targeted mainly the identification of a single parameter which has a key influence on the stratification of the bed, which in turn was reliant on the operating parameters, size of the jig and characteristics of the feed. They found that this parameter was nothing but the total dissipated energy of a single jig cycle. The theory suggested that the combination of the jigging parameters within the similar dissipation energy in jig bed, resulted in similar stratification. They found that the dissipated energy was greatly affected by pulsation frequency, as well as operating air pressure.

This approach was combined with the mechanism for bed stratification, operating parameters, and air-water interaction in the jig. The primary shortcoming of this theory is that it is unable todetermine the concentration profiles in the jig bed.

2.4.5 Stochastic Analysis

This theoretical approach includes jig behaviour analysis by focussing on single particles **(Vinogradov et.al., 1968)**.The fact that single body behaviour of particles in overlooked critical processes occurring during single particle analysis, is a drawback of this approach. Essentially this analysis relies on applying the theoretical laws of statistical mechanics to describe the behaviour ofa particle in the bed **(Mehrotra et al., 1997)**. It concluded that a statistically unstable bed resulted in effective bed stratificationin case of a complete jig cycle, based on the density of the material in the bed. Statistical stability can be explained as a condition when bed volume remains unchanged in comparison to a mean value at the relaxed bed state. The reported theory explains the fluid movement regime and stratification degree reasonably well. However, the dependence of the stratification performance on operating parameters was not well explained with this theory **(Lin et al., 1997)**.

2.4.6 Empirical Models

This approach used empirical models to report the kinetics of jigging process, based on the assumption that the bed stratification is strongly affected by jigging time. When all the assumptions and the empirical model are applied, stratification is found to be a function of jigging time. Using two empirical parameters, the Weibull distribution can be proposed and a relation between jig performance and number of jigging parameters expressed as follows:

$$Y(t) = 1 - exp\left(-\frac{t}{\theta}\right)^{\beta} \qquad [2.13]$$

Where,

Y(t): Yield at time t

θ and β: Empirical parameters from experimental results

β: A measure of the relative delay of the process

θ: Jiggability

Both θ and β are dependent on different parameters such as, frequency of pulsation, stroke length, water level, bed thickness, particle size, specific gravities of particles, and feed grade. Considering that jig stratification of a jig bed is effectively affected by various operating conditions that affect the water behaviour in a jig bed. , **(Rong and Lyman,1991)** established a power law to establish the relationship between jiging time and stratification indices. **Mehrotra et al.(1997).** After conducting a number of experiments the stratification parameters were determined that yield to be affected by jigging time and they modelled this with a power law equation. The relation between the water behaviour parameters and the jig stratification are as follows:

"(a) The maximum and average water pressure above the bed plate within a cycle

(b) The average water pressure above the bed plate during the time of pulsation

(c) Water oscillation amplitude

(d) Duration of pulsation" **Mehrotra et al.(1997)**

2.4.7 Discrete Element Method (DEM) and Computational Fluid Dynamics (CFD)

With the rapid growth of super computers, the application of the discrete element method (DEM) and computational fluid dynamics (CFD) are now often applied as effective tools for the simulation of jigging theories.DEM is an numerical method effectively used for computation and simulate the motion of a large number of particles interacting in a given boundary.DEM simulations are able to track the individual motion of a number of particles in a short time period. **Mishra and Mehrotra(1998)** have used a 2 Dimensional DEM model and assumed idealised fluid behaviour. Due to the superior computational convenience, theEuler-Lagrange DEM model is superior to other DEM models (**Viduka et al.,2013**). It uses Navier-Stokes equations, as well as continuity equations, to solve liquid flow behaviour. Newton's second law of motion can be used to calculate the motion of individual particles and Newton`s third law for liquid-particle coupling.

The Newton's second law of motion equations are presented in equation 2.14 and 2.15.

$$m_{i\frac{dV_i}{dt}} = \mathrm{f_{f,i}} + \sum_{j=1}^{k_i} \left(\mathrm{f_{c,ij}} + \mathrm{f_{d,ij}}\right) + \mathrm{f_{g,i}} \qquad [2.14]$$

And $I_i \frac{dW_i}{dt} = \sum_{j=1}^{k_i} T_{ij}$ [2.15]

And the Navier-Stokes equations are presented in 2.16 and 2.17.

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla.(\mathcal{E}u) = 0 \qquad [2.16]$$

And $\frac{\partial(\rho_f \mathcal{E}u)}{\partial t} + \nabla.\left(\rho_f \mathcal{E}uu\right) = -\nabla P - \frac{\sum_{i=1}^{k_c} f_{f,i}}{\nabla V} + \nabla(\mathcal{E}\tau) + \rho_f \mathcal{E}\mathrm{g}$ [2.17]

Where, V=volume in m^3, ΔV =volume of computational cell, m^3, Ɛ =porosity, dimensionless ρ=density, kg/m^3 τ=continuum phase viscous stress tensor, $kgm^{-1}s^{-2}$, ω=rotational velocity of particle, s-1

f= drag force, N, g= acceleration due to gravity, ms^{-2}, m= mass, kg, u= gas velocity, ms^{-1}, v=solid velocity, ms^{-1} k_c =number of particles in a computational cell, dimensionless, k_i =number of particles in contact with i, dimensionless

Subscripts: c= contact, d= damping, f= fluid phase, i= particle i, j= particle j

2.4.8 The King stratification Model

According to Mehrotra and Mishra **(Mehrotra and Mishra, 1997),** a number of researchers reported potential energy to be inclusive of dispersive forces, based on considering certain factors such as inter-particle collisions. The most substantial contribution towards quantitative analysis of the phenomenon of stratification was probably initially done by **King (1987)**. Later Tavares and King (**Tavares and King, 1995)** made further contributions. They assume dispersive dynamics Fickian in nature in their approach. Strong empirical endorsement of the model was based on further work by other researchers in the field **(Tavares et al., 1995; Venkoba Rao et al., 2003 and 2007; Woollacott et al., 2014)**. Based on the large volume of work carried out, it is generally accepted that the model can be applied without limits. For an instance, there is no limit upto which the model can be applied to determine stratification patterns under conditions of particle size variations.

This approach**(King, 1987; Tavares and King, 1995; King, 2001**) is based on uniform particle size of the bed with varying density. The reduction in potential energy of this system that results due to particles switching their position can be modelled by equation 2.18.

$$\frac{dE}{dH} = -gV_p(\rho - \rho') \quad [2.18]$$

Where,

V_p: Particle volume

E: Potential energy of the system

H: Bed height

The jig bed stratification can be evaluated based on volumetric considerations. The concentration profile is expressed as $C\rho$,the volumetric concentration for component p, and its variation with bed height.

The upward and downward movement of the particles depend on the stratification potential. The particles will move downwards when there is a positive difference and vis-a-vis. The particle flux occurring due to the potential gradient for a density ρ can be expressed as follows

$$\emptyset_{st}^{v} = -C_p\, u\, \frac{dE}{dH} \quad [2.19]$$

Where

$\emptyset_{st}^{v}$: Stratification flux (A volumetric basis is applied for perceiving this quantity and that is expressed by the superscript v)

u:Migration or penetration velocity

dE/dH: Stratification potential gradient.

When the stratification potential is 1 and the dispersive forces are negligible, the penetration velocity of a particle in the system is obtained.

The stratification potential dE/dH was substituted in the above equation, to obtain

$$\emptyset_{st}^{v} = -C_p\, u\, g\, V_p\, (\rho - \rho') \quad [2.20]$$

King (1987) noticed that in a real system the assumption made by Mayer, i.e. the achievement of perfect stratification, was not possible. This feature differentiated King's model from the potential energy minimization method proposed by Mayer. This happens due to the continuous de-stratification of the bed caused by the Fickian dispersive forces. The de-stratifying or dispersive flux, $\emptyset_{disp}^{v}$ is expressed as

$$\emptyset_{disp}^{v} = -D\frac{dC_p}{dH} \qquad [2.21]$$

Where, the diffusion coefficient, D is a function of both particle shape and size, as well as the bed expansion mechanism **(Tavares and King, 1995)**. The bed of particles achieved dynamic equilibrium when allowed for anadequate time. This consequently equated the stratifying flux in opposite directions as given in equation 2.22.

$$\emptyset_{disp}^{v} = -\emptyset_{disp}^{v} \qquad [2.22]$$

Therefore,

$$\frac{dC_p}{dH} = -\frac{C_p ugV_p}{D}\,(\rho - \rho') \qquad [2.23]$$

The relative bed height, h, is expressed as,

$h = H/H_{bed}$,

Where the H_{bed} is the bed height. The quantities giving the volume of particles, penetration velocity, diffusion constant and acceleration due to gravity, are all constant entities that can together be collectively expressed as a single parameter naming stratification coefficient (α):

$$\alpha = \frac{ugV_pH_{bed}}{D} \qquad [2.24]$$

The jigging action can be described by this coefficient, which furthermore also does not depend on the particle density. **Woollacott et al. (2014)** reported similar values of α for mono-sized particles of similar shapes. The King model is expressed as:

$$\frac{dC_p}{dh} = -\alpha C_p(\rho - \rho'(h)) \qquad [2.25]$$

Upon integration of the equation, the concentration profile of the jig bed is obtained as:

$$C_p = C_p^0 \exp(-\alpha\rho_p h + \alpha \int_0^h \rho'(K)dk) \qquad [2.26]$$

Here, k represents the relative height inside the integrals as an alternative to h, whereas $\boldsymbol{\rho}$ is used to represent the particles' volumetric concentration within the density class *p*. The concentration of component *p* is represented as$\boldsymbol{C_\rho}^{\mathbf{0}}$ at the bottom of the jig bed. The relative height, i.e., is 0.

The relationship between concentration for a component at the bottom of the jig bed and concentration of the same component present in the feed can be expressed as:

$$C_p^0 = \frac{C_f}{\left[\int_0^1 \left\{\exp(-\alpha\rho_p h + \left|\alpha \int_0^h \rho' du\right) dh\right\}\right]} \qquad [2.27]$$

King **(King, 1987, 2001)** suggested an iterative procedure to obtain a numerical solution of the equation. He proposed an iterative procedure starting with an assumption of the initial density profile. Later numerical integration can be implemented and successive estimates normalized for $C_\rho{}^0$ to determine the constraint to calculate $\sum_{all\rho} C_p^0$=1**.**

2.5 Summary

From this review, it is clear that although jigging is one of the oldest technologies for minerals beneficiation, the dynamics of a jigging process is still inadequately understood at the fundamental level. it is also evident that various mechanisms affect density stratification in jigging and that operating variables influence jigging performance, but that these need to be understood better. The operating variables that affect density stratification and the performance of jigs include variables that affect the water displacement – the amplitude, frequency and shape of the jig cycle; and external variables that affect the stratification profile – the bed depth, and the difference in the density of the particles.

There are several modelling approaches for describing the jigging process which can be used. Some authors have developed models to describe the kinetics of stratification, but most are empirically based and refer to a time constant and some kind of 'jiggability' index. Several authors have developed models to describe stratification at equilibrium. Among them the King stratification model stands out not only as an elegant model, but as one that is able to describe stratification in a number of different contexts using only one empirical parameter.The validation of this model has largely been limited to mono size particles and it does not say describe the kinetics of the stratification process.

Not much work has been reported that take jig operating variables into consideration. In conclusion, it is clear that more research is required to understand the influence of operating variables on density stratification. This is necessary from two perspectives – (1) their influence on the performance of jigs and (2) the ability to model that influence. Work in this dissertation describes further efforts in this regard.

Chapter 3

Experimental Design

This chapter presents the methodology, experimental equipment and procedures used to perform the experimental work needed to address the research objectives of this study.Each aspect of the equipment, procedures used and associated data are detailed in this chapter.

To address research question 1, "How do the operating conditions of a batch jig affect density stratification at equilibrium?" a batch jigging system is required that can stratify a bed of particles of different density (and suitable size and shape) under a variety of jigging conditions. The jig bed in the system mustbe able to be split into different layers so that the stratification profile achieved by the jig can be determined. For research question 1 the test must be run to equilibrium. For research question 2, "How do the operating conditions affect jigging process kinetics?" the tests need to be run for different times so that the way the stratification profile changes with time can be determined. Research question 3, "How can these effects be modeled effectively?" is addressed by analyzing the results from the equilibrium and kinetic tests.

The equipment and procedures used and the basis for selection of the particles used for the tests is explained first. After that the experimental design and the rationale for analyzing the results and developing a stratification model are explained.

3.1 The Jigging System Used

As explained in Chapter 1, batch jigging was selected as the most convenient way to establish and measure the stratificationpatterns in a density-stratified bed.In continuous jigs, factors other than stratification dynamics affect these patterns – factors such as fluid transport patterns and remixing. Also, batch jigs are simpler to operate and to conduct the stratification tests needed.

Figure 3.1 shows a photograph of the ‘Wits batch jig’ – the batch jig from the School of Chemical and Metallurgical Engineering laboratory at the University of the Witwatersrand.The jig chamber is made up of 15 mm and 20 mm thick rings stacked one on top of the other as shown in Figure3.1a. The chamber is 200 mm in diameter. This design allows the particle bed inside the chamber to be split into discrete layers by pushing a slide plate between the rings.

A pneumatic drive system causes the bellows below the chamber to move up and down according to the type of jig cycle wanted. This causes the water and particle bed in the chamber to move up and down – the jigging action. The larger or denser particles in the bed move towards the bottom because of the segregation that occurs as the particles are momentarily suspended in water. The smaller or less dense particles move towards the top, while, the medium density particles tend to concentrate in the middle of the jig.

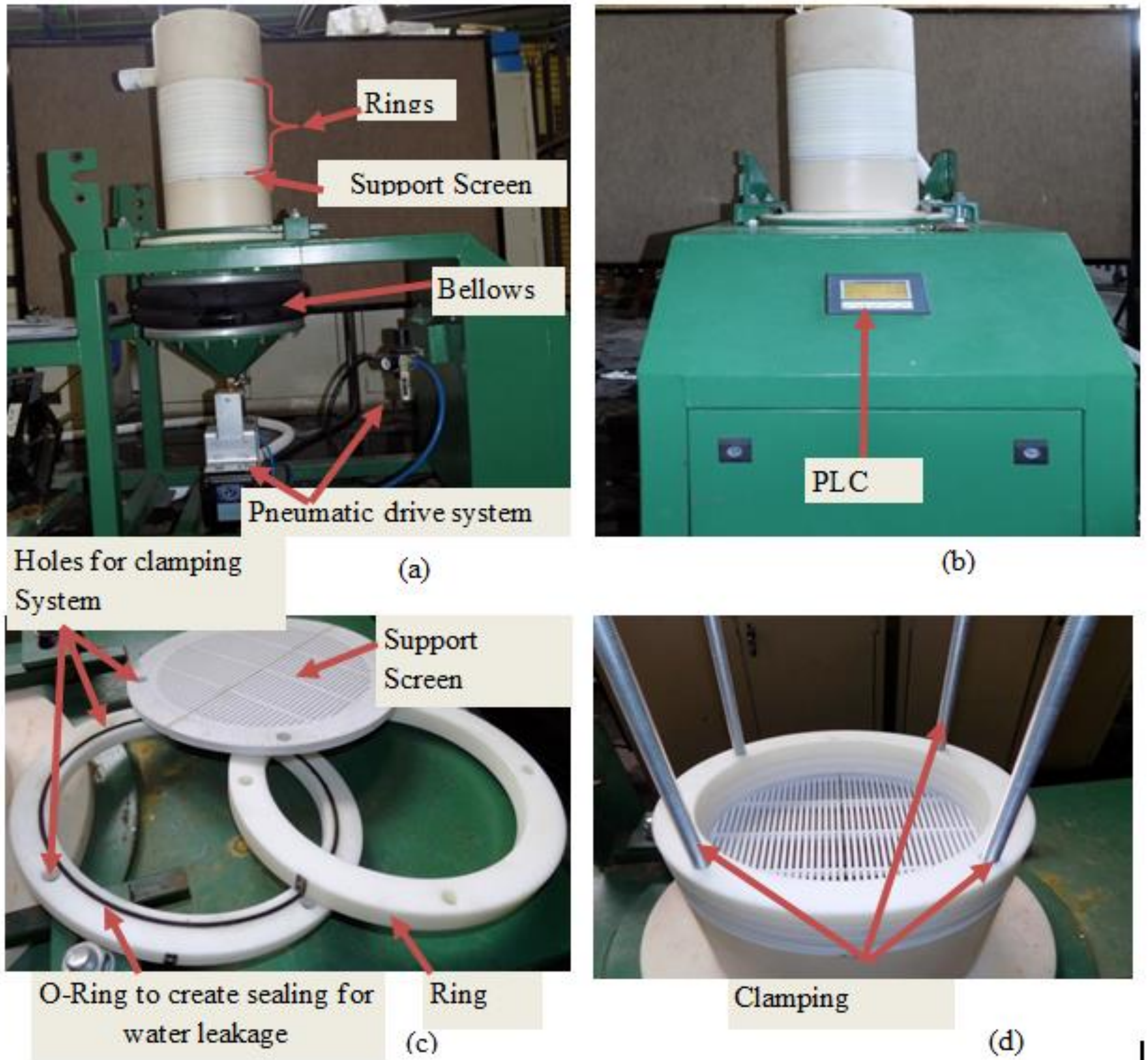


Figure 3.1The jigchamber set up

(a) Front view, (b) Rear view,

(c) Rings and bed support screen, (d) Clamping system

The bed of the chamber is supported by a screen and an O-ring to create sealing for water (see Figure 3.1c and Figure 3.2). Clamping rods are used as a support system for the chamber as shown in Figure 3.1d. In order to maintain the operating variables of the jig chamber, a PLC system is utilised as shown in Figure 3.1b.

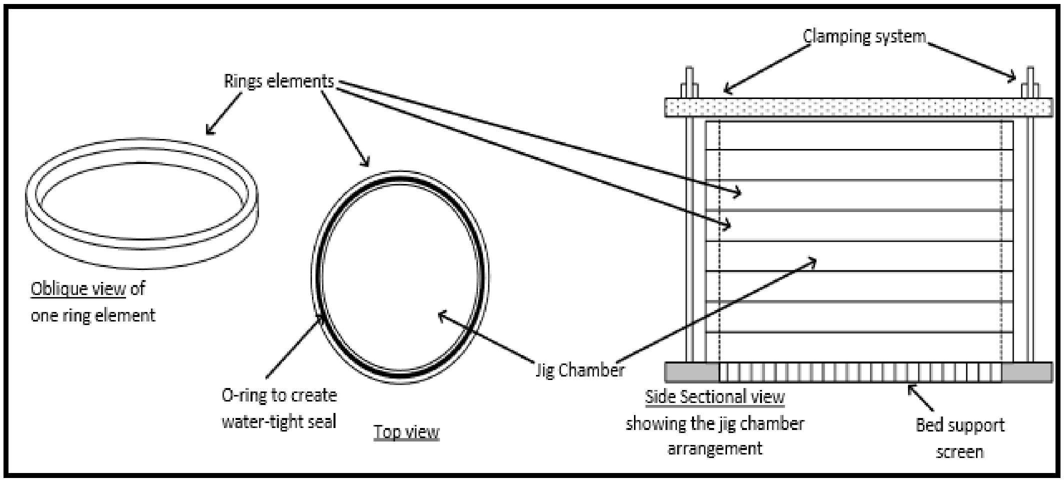


Figure 3.2 The jigchamber set up

3.1.1 Selection of particles for the study

Glass beads of different properties were used to carry out the tests needed to achieve the aims of the investigation. These properties are indicated in Table 3.1. The beads of different density were of different colour or appearance so that they could be separated manually. This allowed the concentration of the different density components in a sample to be determined easily.

Table 3.1 Properties of the glass beads used in the study

Colour of the beads	Density (g/cc)	Size (mm)
Boro (matte)	2.226	8.021
Green	2.463	8.190
Red	2.554	7.800
Blue	2.577	7.960

3.1.2 Test procedure

The jig chamber was set up by assembling the rings in an appropriate order so that the thickness of each layer split from the bed was always the same. After the rings had been clamped, water and particles for the test were added to the chamber. The water level was then adjusted so that the bed was always flooded during jigging. The desired jigging conditions were then set up by means of the PLC and the jig was run. After conducting an experiment, the height of the bed was measured. The bed was then sliced into layers by pushing the slide plate between each ring and removing the layers as separate samples. The samples were dried using a hair dryer and the different density components in each sample were separated manually. The mass of each component was then measured. The volumetric concentration of beads in each layer was then calculated on the mass and density of each component in the layer.

3.1.3 Tests Variables

Because the intention of the study was to investigate the effect of operating conditions on stratification, it was not necessary to vary the composition of the particles of the beads in the bed from test to test. Therefore the same sample of beads was used for all tests except those involving a change in the depth of the bed. In those tests, a greater or lesser volume of beads were used, but in each case the composition was kept the same and the total volume of beads was adjusted according to the height of the particle bed desired.

The literature survey in Chapter 2 showed that that the common operating variables that influence stratification in jigging are the amplitude and frequency of the jig cycle, the shape of the jig cycle, and height of the bed. The Wits batch jig controlled these variables in the following way.

- ***Bed height:*** The bed height, which is based on the jig capacity, is a critical factor in jigging operations as it independently affects all the other operating variables. Here in this study 70mm to 150mm of bed height have been used. This was controlled by manipulating the volume of beads added to the jig chamber as already described.
- ***Shape of the jig cycle:*** The pulsion unit consisted of a set of bellows driven below by a pneumatic piston. Figure 3. shows how the Wits batch jig controlled the jig cycle in the jig chamber. The piston moved up for a period of time T1, the pulsion stroke; was held in the up position for a period T2, the pulsion hold time; then moved down for a period T3, the suction stroke; and then was held in the down position for a period T4. All the units of the time periods were in centi seconds.

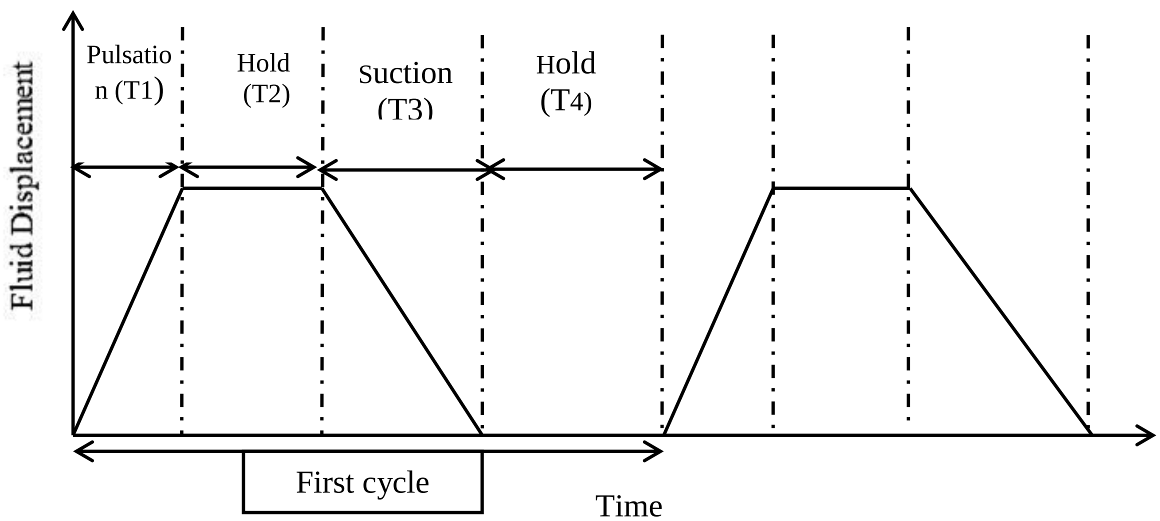


Figure 3.3 The pattern and settings of the pulsion unit

- ***Amplitude of the jig cycle***: The amplitude of the jig cycle is the maximum water displacement in the jig chamber. This is not the same as the maximum movement of the piston driving the bellow movement and, consequently the movement of water in the jig. However, the two are obviously strongly related. The Wits batch jig provided two control variables that affected the movement of the piston: the pressure applied to the piston; and the percentage of the maximum piston movement possible, referred to from here as the % Stroke.

- ***Frequency of the jig cycle:*** The frequency of the jig cycle is the number of cycles per unit time. This is controlled by the cycle time T5, which is the sum of the time periods T1 to T4. The frequency is therefore 1/T5.
- ***Run time:*** The Wits batch jig had the facility to stop jigging after a pre-set period of time.
- ***Hutch water flowrate:*** Hutch water is water that is added into the space below the bed-support screen and flows continually up through the screen and the bed and out of the jig chamber through an overflow pipe. This facility was not used during the tests instead, sufficient water was added to ensure that the bed was always flooded during jigging.

3.2 Experimental Design

To address the research questions requires stratification tests to be done under equilibrium conditions (question 1) and dynamic conditions (question 2) under an appropriate range of operating conditions. Prior to these tests, however, the appropriate range of operating conditions need to be established. The experimental program needed for the study therefore required the following three stages.

- Stage 1: Selection of the range of operating conditions for the tests.

  Section 3.1.3 identified the operating variables that affect stratification in a batch jig and how these are controlled in the jigging equipment used in the study. There are 7 variables if hutch flow rate is not included: bed depth; percentage stroke and applied pressure (determining the amplitude of the jig cycle); the cycle time (determining the cycle frequency); and three time periods (among T1, T2, T3, and T4) for controlling the shape of the jig cycle; the fourth time period is set by the cycle time, i.e. cycle time minus the three time periods selected.

Experience with the Wits batch jig has shown that percentage stroke and applied pressure are not the only factors affecting the jig cycle amplitude; the time periods T1 to T4 also have an influence. Therefore, an investigation into the effect of these six variables on the cycle amplitude is required over an appropriate range of the six operating variables as suggested by previous experience with the Wits batch jig. To conserve the number of tests to be undertaken in order to establish this relationship a partial, 2^{6-1} factorial screening design is required. A statistical analysis of the data generated would indicate if further tests are required to refine the relationship established by the initial experimental design.To set up a partial factorial screening experimental design requires the selection of a centre point for each of the six variables and then an appropriate value above and below the centre point value. These points were suggested from previous experience with the Wits batch jig and the experimental design was then developed using Design Expert 7.0. The results of this investigation were analysed using the same program.Analysis of the relationship established by this experimental program would indicate the most significant operating variables affecting the cycle amplitude and the appropriate range of their values. To conserve the number of tests to be done in the next two stages of the investigation, the number of variables should be reduced if it is appropriate to do so.

- Stage 2: Equilibrium test program

  To investigate the effect of operating conditions on stratification at equilibrium, a series of equilibrium stratification tests must be conducted over the range of the operating conditions established from the results of the Stage 1 program. Again a partial factorial screening experimental design is necessary to conserve the number of tests that are required to establish how these variables affect stratification. Analysis of the results from the experimental program will indicate if and how the program needs to be expanded or refined.

  The experimental design requires appropriate response variables, preferably just one. The response variable 'quality of stratification' is not adequate for this purpose. Chapter 2 has shown that the King stratification index, alpha, has the capability of describing the stratification profile achieved in a batch jig when the particles differ only with respect to density. As such it can function as a parameter describing the quality of stratification. A sharply stratified bed will have a high value of alpha and a poorly stratified bed will have a low value of alpha. Accordingly, the King stratification index found for each stratification test was used as the response variable in the experimental design.The method used to calculate the stratification index for a stratification test is described in Section 3.5.1.

- Stage 3: Kinetic test program

  The design of the investigation into the effect of operating conditions on the kinetics of stratification is similar to the design of equilibrium investigation but with a number of differences.

The first difference is that a series of from about 8 to 13 tests was required to establish the kinetics of stratification for each set of operating conditions investigated. Only one test (possibly with replicates) is necessary for an equilibrium test. Accordingly, practical considerations limit the number of sets of operating conditions that could be tested.

The second difference stems from the previous point. In order to maximize the usefulness of the information derived from the kinetic test program, it is desirable to situate the experimental design around the set of conditions that give the optimum stratification at equilibrium. Accordingly, the Stage 2 test program was designed not only to establish the relationship between operating conditions and the quality of stratification (as indicated by the stratification index), but also to establish the optimum operating condition – the one giving the sharpest stratification, i.e. the one with the highest stratification index. In view of the large number of operating variables in consideration, even only a rough indication of the optimum set of operating conditions would be invaluable.

The third design difference from the Stage 2 test program is the lack of a simple descriptor of the kinetics of stratification. To use a partial factorial design for the kinetic test program requires, ideally, a single parametric descriptor of the kinetics of stratification. The literature shows that descriptors with this property have been used but none of these were considered adequate. Firstly, they are entirely empirical. Secondly, they do not relate kinetics to equilibrium stratification or to the stratification index at equilibrium, alpha. Theoretically, it is reasonable to expect that the factors affecting stratification at equilibrium would also affect the kinetics of stratification. Both, after all, are driven by the same set of stratifying and diffusional dynamics. Accordingly, the kinetic program was designed from the perspective of providing information that would be useful for developing a kinetic model that was based on the assumption that the stratification pattern at equilibrium was either known or could be predicted reliably from an equilibrium stratification model such as the King Model.

There were several design consequences from adopting this approach to the kinetic testwork program. Firstly, the nature of the kinetics of stratification is not well understood, and certainly not well understood from the perspective of the equilibrium stratification condition or the equilibrium stratification index. Secondly, this lack of adequate understanding means that there is no experimentally established way of representing the kinetics of stratification by means of a single (or perhaps two) parameters that could be used as a response variable in any factorial experimental design. Accordingly, the investigation into the effect of operating conditions on stratification kinetics was based on the simply multi-linear approach of varying one operating variable at a time around a centre point. The variations tested in the values of each of the operating variables investigated were based on the findings of the Stage 1 and 2 testwork programs.

3.3 Modification of the Planned Experimental Design

In addition to the uncertainties just described for the design of the kinetic testwork program, additional factors emerged during the Stage 1 and 2 programs that required the original experimental design to be modified. The most significant is that the equilibrium stratification profiles obtained in the equilibrium test program – the Stage 2 program – were unusual and were poorly described by the King Model. This meant that the King stratification index from the equilibrium stratification tests were not sufficiently reliable to be used as a response variable for the planned partial factorial testwork program. One of the reasons appeared to be an unfortunate combination of size, shape and density differences in the four component particle system selected for the tests. Accordingly, a second investigation was undertaken with a three component particle system and the use of a partial factorial experimental design had to be abandoned. Accordingly, the investigation of the effect of operating variables on stratification at equilibrium was undertaken using the simpler approach of varying the values of operating variables one by one around a centre point value. The details of the modifications made and the reasons for them are given in Chapters 4 and 5.

3.4 Validity Issues

The validity and reproducibility issues associated with the test data generated depended on the nature of the test programs undertaken. They are discussed below for each of the three testwork programs.

- Stage 1 program: Partial factorial design to establish the relationship between operating variables and the jig cycle amplitude. Taking into account the variance associated with the experimental results in a factorial design is inherent in the design and in the analysis of the results obtained. Tests were done on the centre point in triplicate as recommended in the experimental design.
- Stage 2 program: The first investigation in this program – a partial factorial design for tests conducted on the four component particle system – was effectively abandoned because of the unreliability of the stratification indices obtained. See Chapter 4 for details. Investigation II – the multi-linear variation in the values of individual operating conditions around a centre point – did not have the sophistication of the experimental design to establish the reliability of the test data obtained. Accordingly, replicate tests were done for each set of operating conditions tested until it was clear that the associated stratification profile was reliably represented. Typical examples of the reliability of the tests are shown

in Figures 3.4a to c. These indicate that a very satisfactory level of test reproducibility was obtained. Where ‘hv’ is heiht from the bottom of the bed and volumetric concentration calculated as $\frac{Volume\ of\ each\ component\ in\ the\ layer}{Total\ volume\ of\ all\ components\ in\ the\ layer} * 100$.

- Stage 3 program: Kinetic tests. In these tests, each kinetic curve was established from multiple tests. Inspection of those curves – see Chapter 5 – shows that a fair degree of variability is evident in each kinetic curve. However, the trend of each curve is established by a collection of points (typically from 8 to 13) and so replicate tests are unnecessary. The trend of a kinetic curve was checked after each data point on the curve had been measured. When the variability was such that the trend was not clearly established, additional tests at times other than those already tested were undertaken. The reliability of the kinetic curves was therefore evident by the extent to which the data points deviated from the clearly established trend.

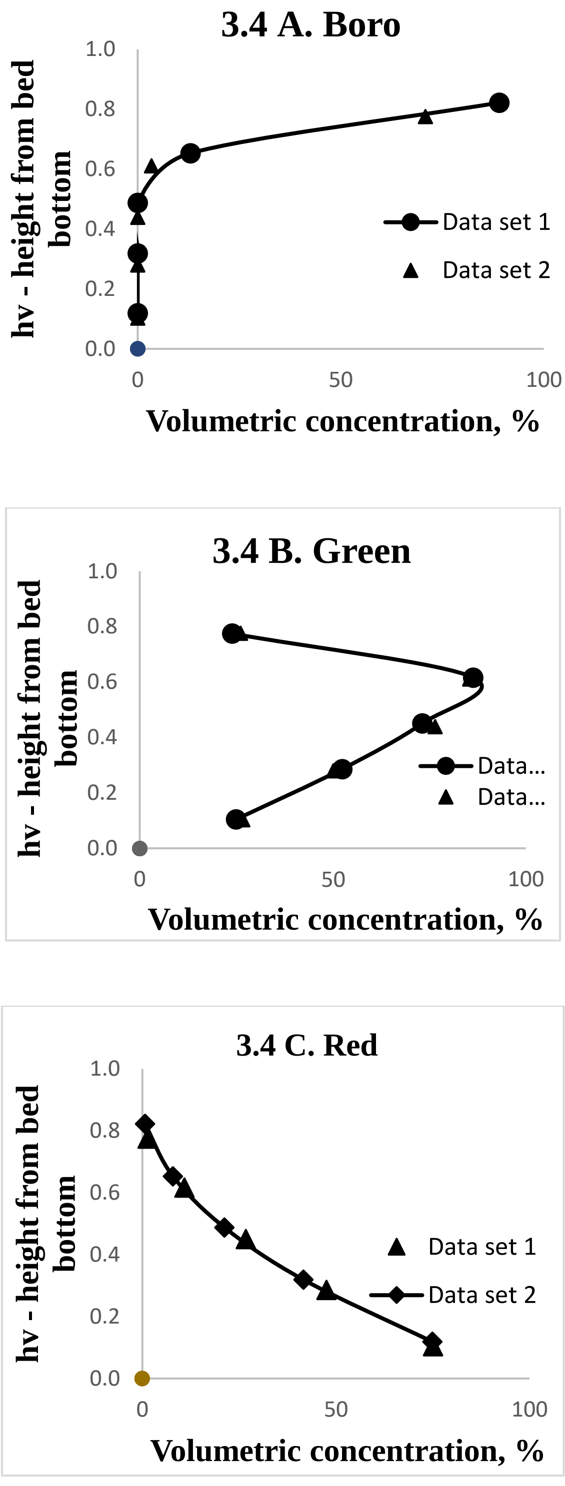


Figure 3.4 An example of the reproducibility of the stratification profiles

3.5 Analysis of Experimental Results

3.5.1 Analysis of the Equilibrium Data

The King Model has been well validated as a way of describing and predicting stratification in systems of particles in which particles differ only with respect to density. As discussed in Chapter 2, the model is expressed as follows:

$$C_i = C_i^0 \exp(-\alpha\rho_i h + \alpha \int_0^h \rho'(k)\, dk) \qquad \textbf{[3.1]}$$

where C_i is the volumetric concentration of component *i* in the thin layer located at *h* or *k* (i.e. from *k* to *k*+*dk*), for *i*=1, N, the number of components.C_i^0 is the concentration of that component at the bottom of the bed (h=0) and can be evaluated from the concentration of component *i* in the feed. ρ_i is the density of component *i*; ρ' is the average density of the particles in the thin layer at *h*; andα is the King stratification index.

The nature of the stratification pattern in a stratified bed can be described in several ways: as the concentration profile, C_i; as the cumulative concentration profile, $\overline{C_\iota}$, Equation 5.2; or as the cumulative recovery profile, $\overline{R_\iota}$, Equation 5.3. In Equations 5.2 and 5.3, *t* and *b* are the top and bottom of the slice being considered. If the concentration or recovery are being cumulated from the bottom of the bed upwards, then b= 0 and t= h; if they are being cumulated from top of the bed downwards, b= h and t= 1. $R_i(h)$ is the recovery of component *i* to the thin layer at *h*.

$$\overline{C_\iota} = \frac{1}{t_i - b_i}\int_{b_i}^{t_i} C_i(h)dh \qquad [3.2]$$

$$\overline{R_\iota} = \frac{1}{t_i - b_i}\int_{b_i}^{t_i} R_i(h)dh \qquad [3.3]$$

The value of the King stratification index for a given set of stratification can be determined by a parameter estimation routine based on a minimization of the squared differences between the predicted and measured values of $\overline{C_\iota}$ or $\overline{R_\iota}$**(King, 2001)**. A numerical routine to do this was developed by Prof Woollacott at Wits, and that routine was used to estimate the stratification indices of the equilibrium data sets generated in the study.

3.5.2 Analysis of the Kinetic Data

From the literature (Chapter 2), a number of different approaches to describing and modelling the kinetics of stratification have been developed or used. They can be grouped into two categories: entirely empirical; or phenomenological. Empirical models are statistically based and attempt to relate stratification to relevant operating variables by means of empirical measurements on stratification equipment. The King model is an example of a phenomenological model. As can be seen from the derivation in Chapter 2, it develops the model from a mathematical conception of the phenomena driving stratification. In the King model the phenomena are a stratification force and a diffusive force in opposition. Consequently, phenomenological models are generic to the process being modelled. In contrast, empirical models are generally not very generic, because they relate to data generated from specific equipment or specific groups of equipment.

The phenomenological kinetic model developed by **Vetter et al., 1987** is not usable in this study because it assumes the same set of phenomena on which the King model is based – a model which, as discussed earlier, does not fit very well the data generated in this study. Accordingly, a semi empirical approach was used to investigate and model the kinetics of stratification.

The approach used was based on the assumption that the equilibrium stratification profile could be modeled phenomenologically. The King Model came reasonably close to being able to do that in this study, and it is anticipated that once it has been adapted to take into account the effect of particle size (and shape) on stratification, it will be better able to reliably predict the equilibrium stratification profile. Accordingly, in this study, when data for equilibrium stratification profiles was needed to test the model developed, measured rather than predicted data was used.

Therefore, this modelling approach meant that kinetic effects would need to be modelled with reference to equilibrium stratification profiles and the rate at which a stratification profile changed from being homogeneous before stratification to the stratification profile at equilibrium. This is illustrated in Figure 3.5.

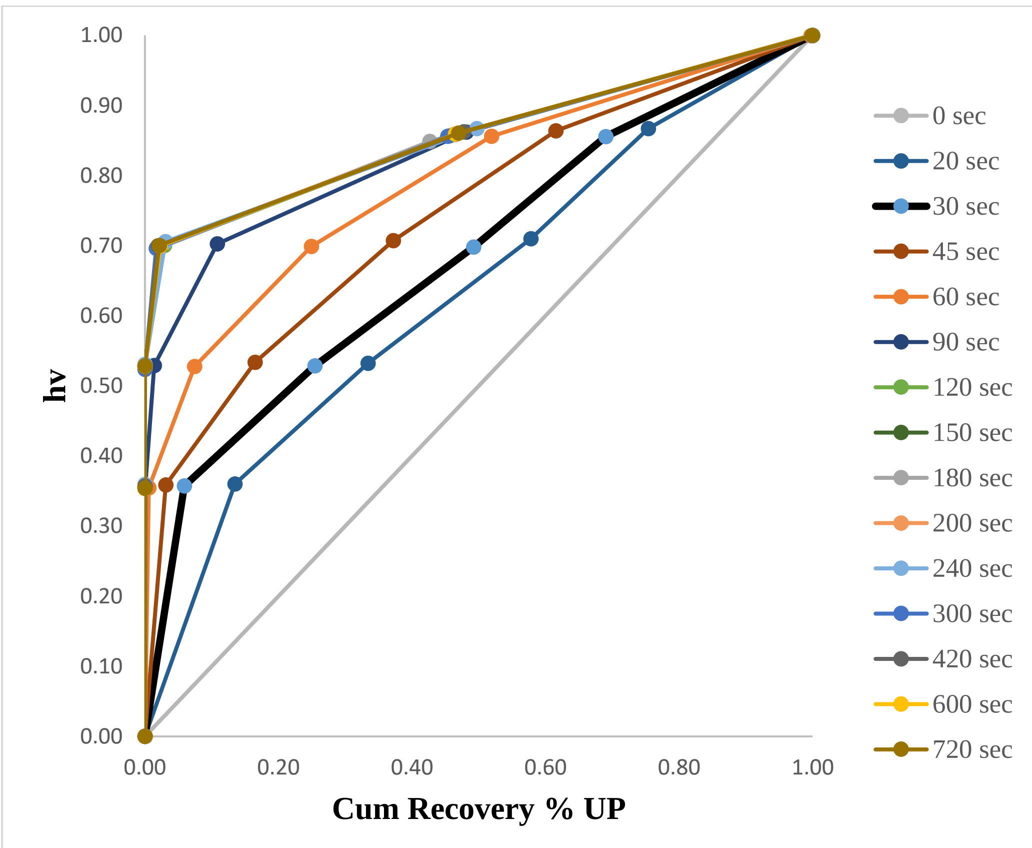


Figure 3.5A typical set of stratification profiles showing how the profiles changed

In the figure the stratification profile is expressed in terms of the cumulative recovery up (i.e. from the bottom of the bed) vs height in the bed. Before stratification starts the bed is homogeneous and the stratification profile will be a straight line as indicated. This is because the recovery of the component to a slice of thickness *h* (measured from the bottom of the bed) will be equal to the fraction of the bed removed. As stratification progresses with time, the stratification profile will shift towards the equilibrium profile as indicated in the diagram. The extent to which the stratification profile has shifted towards – 'approached' – equilibrium can be therefore be estimated by comparing the areas on the plot between the stratification profile at time *t* , i.e. *At*, and the profile at equilibrium, *Ae*. On this basis, the ratio of these two areas was taken as a 'Kinetic Metric', $X_F(t)$, Equation 5.4, to evaluate the extent to which a stratification profile had 'approached equilibrium'.

$$X_f(t) = \frac{At}{Ae} = \frac{Area\ relative\ to\ homogeneity\ at\ time\ t}{Area\ realtive\ to\ homogeneity\ at\ equilibrium} * 100 \qquad [3.4]$$

X_f will have a value of 0 at zero time, and 1 at equilibrium. The kinetics of stratification was therefore studied in terms of how the 'Approach Metric', X_f, varied with time during stratification.

3.6 Modelling Kinetics

The basis for modelling how the operating conditions affect stratification in the batch jig has already been explained. The equilibrium stratification profile is determined by a phenomenological model such as the King Model. The effect of operating conditions on equilibrium is modelled in terms of their effect on the stratification index. The effect of operating conditions on the kinetics of stratification is modelled in terms of their effect on the Kinetic Metric (or Approach Metric)X_f

.

The model will therefore take the form shown in Equation 5.5.

$$\overline{C}_i(h,t) = C_i^{feed} + \{C_i^{feed} - C_i^{equil}(h)\}\ X_f(t) \qquad [3.5]$$

for $i = 1$ to number of particle components.

where C_i^{equil} is the stratification profile at equilibrium and is a function of h and is dependent on operating conditions; and C_i^{feed} is the concentration of component i in the homogeneous feed to the jig.

Chapter 4

Results I: The Effect of Operating Conditions on Stratification at Equilibrium

Introduction

The previous chapter described the equipment used in the study and the experimental design behind the study. This chapter addresses the results of the tests relating to the first research question – "How do the operating conditions of a batch jig affect density stratification at equilibrium?". To address this question, two sub questions must be considered as follows:

Question 1.1: What are the operating conditions that are of practical relevance to an understanding of the performance of a batch jig?

Question 1.2: How do these operating conditions affect density stratification at equilibrium?

Therefore, this chapter is divided into three parts. The first two addresses the two sub questions and presents the experimental results obtained, and the third analyses the results and discusses their relevance.

4.1 Identifying the specific operating conditions to be tested

The literature search in Chapter 2 found that jig performance is affected primarily by the shape of the jigging cycle; its amplitude and frequency; and by the depth of the particle bed. Other factors also can influence it such as hutch water flow rate, but these effects are less relevant and have not been investigated in this study. Hutch water is most relevant to continuous jigs. Many shapes of a jig cycle could be investigated but the popular one described in Chapter 3 (Section 3.1.3, Figure 3.3) was investigated. It is characterized by the pulsion time, T1, the pulsion hold time, T2, the suction time, T3, and the suction hold time T4. The overall cycle time, T5, is the sum of the other four times. It determines the frequency of the cycle, i.e. the number of jig cycles per unit time. This frequency = 1/T5. The amplitude of the cycle is the maximum water displacement during a jig cycle. This is determined by the stroke length of the pulsion device – the drive piston in this case – and by the pressure driving the pulsion device.

Accordingly, seven independent operating variables affect stratification in the batch jig used in the study: T1, T2, T3, T4 (or T5 = T1+T2+T3+T4), stroke length measured as a percentage of a full stroke length (i.e. %Stroke), applied pressure, and bed depth. The first six variables each can affect the amplitude of the jig cycle – i.e. the maximum water displacement in the jig.These six variables are therefore mutually interdependent, at least potentially. Therefore, a preliminary investigation was undertaken to investigate the relationship between these variables and the maximum water displacement in the jig. To do this a partial factorial 2^{6-1} experimental design was used with the cycle amplitude (stroke in mm) as the response variable. As explained in Chapter 3, the tests were conducted using a transparent jig chamber so that the bed motion could be videoed. A ruler was fitted to the chamber to measure the vertical water displacement. The video editor, VideoPad, was used to isolate frames showing the maximum and minimum position of the water in the chamber so that the maximum water displacement could be measured by reference to the ruler.

The design of the series of tests and the analysis of the results were done using Design Expert 7.0. Tables 4.1 shows the ranges selected for each variable based on prior experience of the operation of the Wits batch jig used. Table 4.2 summarizes the overall design and results. As can be seen the conditions for the 2^{6-1} design were randomized with three tests on the centre point being conducted near the beginning, middle and end of the series of tests – 35 tests in all (i.e. 2^{6-1}+3). The regression model developed from the results is shown in Table 4.3 along with the statistics related to the regression.

Table 4.1The ranges of operating variables tested in the partial factorial design

(The bed depth for all tests was 80mm. The beads used were spherical, 8mm in diameter and had a density of 2.567 g/cc.)

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
	A:T1	**B:T2**	**C:T3**	**D:T5***	**E:% Stroke**	**F:Pressure**
	cSec	cSec	cSec	cSec	%	psi
lower value (-1)	**14**	**20**	**20**	**133**	**30**	**59**
centre point (0)	**18**	**25**	**25**	**100**	**35**	**67**
upper value (+1)	**22**	**28**	**28**	**78**	**40**	**75**

* In the analysis, Factor 4 was taken either as T5 or as the cycle frequency 1/T5. Accordingly, the low and high values are inverted in this table.

Table 4.2Design and results of the partial factorial test series

(Refer to Table 4.1 for the actual values for each factor)

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	
	A:T1	B:T2	C:T3	D:T5	E:% Stroke	F:Pressure	Response
Run	cS	cS	cS	cS	%	psi	mm stroke
1	-1	1	-1	-1	-1	1	43
2	1	-1	-1	1	1	1	42
3	0	0	0	0	0	0	34
4	-1	1	1	-1	1	1	48
5	1	-1	1	1	-1	1	23
6	-1	1	-1	-1	1	-1	46
7	-1	-1	-1	-1	-1	-1	19
8	1	-1	1	-1	-1	-1	25
9	1	1	-1	1	-1	1	22
10	-1	-1	1	1	-1	-1	9
11	-1	1	-1	1	-1	-1	18
12	1	-1	-1	-1	-1	1	29
13	1	1	-1	1	1	-1	36
14	-1	-1	-1	1	-1	1	18
15	1	-1	1	1	1	-1	40
16	-1	1	1	-1	-1	-1	27
17	1	1	1	-1	1	-1	55
18	-1	-1	-1	1	1	-1	30
19	1	1	-1	-1	-1	-1	35
20	-1	-1	1	-1	1	-1	30
21	0	0	0	0	0	0	40
22	1	-1	-1	-1	1	-1	44
23	-1	1	1	1	-1	1	31
24	-1	-1	-1	-1	1	1	40
25	1	1	1	1	-1	-1	26
26	1	-1	-1	1	-1	-1	17
27	1	-1	1	-1	1	1	51
28	-1	-1	1	-1	-1	1	28
29	1	1	1	-1	-1	1	41
30	-1	-1	1	1	1	1	39
31	1	1	1	1	1	1	40
32	1	1	-1	-1	1	1	68
33	-1	1	-1	1	1	1	47
34	0	0	0	0	0	0	43
35	-1	1	1	1	1	-1	40

Table 4.3 Model and related statistics for the relationship between cycle amplitude (stroke in mm) and six jig operating variables

(See the left hand column or Tables 4.2 or 4.3 for the identity of factors A to F)

Model relating Amplitude (Stroke in mm) to the six operating variables (Factors A to F)

(Factor D is T3. The other factors are indicated below)

Stroke (mm)	=
94.903	
-10.318	* T1 (Factor A)
2.725	* T2 (Factor B)
-158.065	* 1/T5 (Factor C)
-5.060	* % Stroke (Factor E)
0.437	* Pressure (Factor (F)
9.174	* T1 * 1/T5
0.359	* T1 * % Stroke
-1.590	* T2 * 1/T5
5.922	* 1/T5 * % Stroke
-0.308	* T1 * 1/T5 * % Stroke

Statistics for the Model Regression

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	4810.2	10	481.0	32.2	< 0.0001
A-T1	203.8	1	203.8	13.6	0.0011
B-T2	650.0	1	650.0	43.5	< 0.0001
D-1/T5	722.9	1	722.9	48.4	< 0.0001
E-% Stroke	2620.7	1	2620.7	175.5	< 0.0001
F-Pressure	400.0	1	400.0	26.8	< 0.0001
AD	94.0	1	94.0	6.3	0.019
AE	28.4	1	28.4	1.9	0.18
BD	93.0	1	93.0	6.2	0.012
DE	8.4	1	8.4	0.6	0.46
ADE	87.2	1	87.2	5.8	0.023
Residual	373.3	25	14.9		
Lack of Fit	331.3	23	14.4	0.7	0.75
Pure Error	42.0	2	21.0		
Cor Total	5183.6	35			

The variables T1, T2, 1/T5 (frequency), percentage stroke, and applied pressure were found to be significant along with two factor interactions (T1 and frequency; and T2 and frequency) and one three factor interaction (T1, frequency and percentage Stroke). T3 was not found to be significant, i.e. it has no significant influence on the cycle amplitude (Stroke in mm). Higher order interactions were not needed in the model because there was no significant lack of fit. The model explains 93% of the influence of the six variables on the cycle amplitude; i.e. the adjusted regression coefficient, R^2, was 0.93. Figure 4.1 compares the measured values of the cycle amplitude with the values predicted by the model.

Reference to the F values for the various factors and factor groupings in Table 4.3 indicates that the variables exerting the most important influence on the amplitude are percentage Stroke, T2 and frequency (1/T5) in that order.

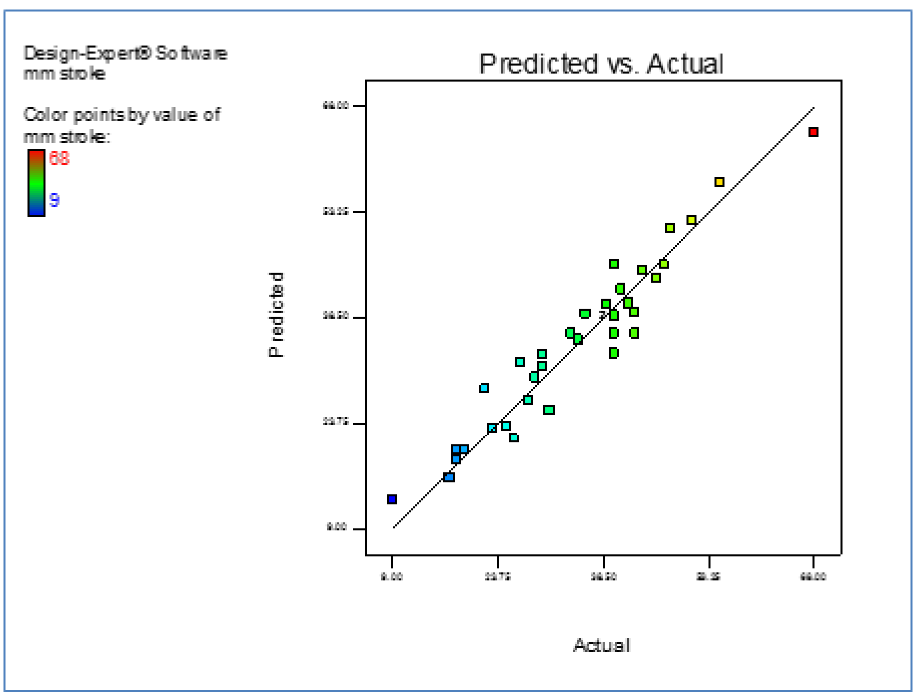


Figure 4.1 Comparison of measured and predicted values of the cycle amplitude

4.2 Selection of Operating Conditions for the Equilibrium Tests

The investigation into the relationship between the cycle amplitude and the operating variables that affected it showed that the suction time, T3, could be ignored. It was apparent from the many videos taken of the water and bed movement for the 35 tests, that the particle bed was completely settled and stationary well before the end of each suction stage of the jig cycle. This was a second reason for ignoring T3 as an operating variable that could affect stratification performance. It did not affect the amplitude of the cycle and the bed always settles well before the end of the suction period.

Another consequence of the observation that the bed is settled well before the end of the suction stage of each jig cycle is that T5 is also not an operating variable relevant to stratification performance. In the first place, stratification in beds consisting of the same sized particles occurs only when the bed is in a dilated condition. The time when the bed is stationary is dead time. In this regard, therefore, the influence of the frequency of the jig cycle is that it controls the number of times when stratification processes are active in the bed – i.e. it affects the jigging capacity not the jigging quality. The only way it can affect the quality of jigging – i.e. the nature of stratification achieved – is through its influence on the cycle amplitude and shape of the jig cycle. This influence was demonstrated in the previous section. It is therefore apparent that T5 can be ignored as a relevant operating variable if a long cycle time of fixed magnitude (a long T5) is used for all tests. The argument is that the critical part of the jig cycle is T1 and T2. T3 is not critical provided it is long enough for the bed to settle. T4 is dead time.

The sixth operating variable – applied pressure – influences the jig cycle amplitude (stroke in mm) as was shown in the previous section. The purpose of adjusting the applied pressure and percentage stroke is to adjust the extent of water displacement (stroke in mm, i.e. the cycle amplitude). Given that percentage stroke is statistically the most significant operating variable influencing the cycle amplitude, it makes sense to use the same applied pressure in all the tests and use only percentage stroke to adjust the cycle amplitude.

On this basis, the operating variables relevant to a study of density stratification in the jig were reduced from seven to four: T1 and T2 control the critical aspect of the shape of the jig cycle, percentage stroke controls the cycle amplitude, and the bed depth is the fourth relevant operating variable. The ranges over which these variables were varied are shown in Table 4.4. The values of the other three operating variables – T3, T5 (1/frequency), and applied pressure – were help at 25 cS, 133 cS and 67 psi respectively. The value of T4 – the suction hold (which was irrelevant dead time) – was adjusted in each test to a value = 133 – T1 – T2 – T3(=25). The range of bed depths tested, as shown in Table 4.4, were selected on the basis of prior experience with the Wits jig.

Table 4.4The Operating Conditions Selected for the Equilibrium Stratification Tests

	T1 (cS)	T2 (cS)	T4 (cS)*	Stroke (mm)	Hbed (mm)
High value (+1)	22	28	58	54	150
Mid point (1)	18	24	66	47	115
Low Value (-1)	14	20	74	40	80

4.3 The Effect of Operating Conditions on Stratification at Equilibrium: Investigation I

A partial 2^4-1 factorial design was used to investigate how the jig operating conditions affected density stratification with the King stratification index as the response variable. As discussed in Chapter 3, the rationale behind this approach had three aspects. The first was that the experimentally determined King stratification index provided a single parameter that is entirely adequate to describe density stratification in a bed of particles with the same size and shape but with different densities. The second was that such an experimental design would establish how the operating conditions tested affected density stratification. The third aspect was that the design would also provide a rough estimate of the operating conditions that would yield optimum density stratification. This estimate would be useful as a center point for the subsequent investigation into how operating conditions affected the kinetics of stratification.

Chapter 3 described the jigging equipment and procedures used in the tests. Table 4.4 above indicates the range of conditions tested. Table 4.5 shows the percentage stroke settings that were used to obtain the desired values of the jig cycle amplitude (stroke in mm) for the conditions specified in Table 4.4. The bed used in the tests was made up of a four component system of 8mm, spherical glass beads with the density and composition shown in Table 3.1 in the previous chapter. A four component system was selected because it would generate more data per test than binary or ternary systems would and therefore it would provide a more precise measure of the King stratification index. The experimental design structure and the experimental results are presented in Table 4.6. The alpha values were determined from the stratification profiles by the procedure described in Section 3.5. Examples of the stratification profiles measured for the tests are shown in Figures 4.6 to 4.8. The curves in the plots represent therecoveries calculated using the model while the points represent the experimental data. The data points and plots for all tests are shown in Appendix A.

Table 4.5 The water displacement at different set of conditions as per the design on table 4.1

T1, cS	T2, cS	Stroke, mm	Stroke, %
14	20	40	50
14	20	54	58
14	28	40	36
14	28	54	44
18	24	47	38
22	20	40	38
22	20	54	43
22	28	40	31
22	28	54	38

Table 4.6Design and results of the equilibrium stratification tests

(Refer to Table 4.4 for the specific factor values)

	Factor 1	Factor 2	Factor 3	Factor 4	Response 1	Response 2
Tern	A:T1	B:T2	C:Stroke	D:H bed	Stratification index	Sum of squared errors
	cS	cS	mm	mm	Alpha, cc/g	
1	1	1	1	-1	100.4	0.110
2	0	0	0	0	80.0	0.421
3	-1	-1	-1	1	57.3	0.775
4	-1	-1	1	1	28.5	0.789
5	-1	1	-1	-1	87.9	0.136
6	-1	1	-1	1	45.6	0.970
7	1	1	-1	1	11.3	0.515
8	-1	-1	-1	-1	86.6	0.131
9	0	0	0	0	86.2	0.398
10	-1	1	1	-1	88.9	0.0378
11	1	-1	-1	1	34.7	0.434
12	1	-1	1	1	60.6	0.716
13	1	-1	1	-1	97.1	0.148
14	1	1	-1	-1	93.8	0.095
15	-1	-1	1	-1	81.9	0.122
16	-1	1	1	1	67.9	0.765
17	0	0	0	0	89.6	0.262
18	1	-1	-1	-1	97.2	0.108
19	1	1	1	1	67.9	0.827

It is evident from Table 4.6, that the King Stratification Model did not always fit the test data very well. This can be seen from the wide variation and magnitudes of the sum of squared errors (SOS) in the table. The sometimes poor fit is illustrated by the selection of the stratificationprofiles plotted in Figures 4.2 to 4.4. The profiles are expressed in terms of the cumulative recovery of each component to the top slice when the bed is split at different heights from its base. (The full set of plots is found in Appendix A) For example, the best fit obtained was for run Test 14 as shown in

Figure 4.6. The alpha value for this test is 93.9 cc/g and the sum of squared errors (SOS) was 0.0956. The worst fit obtained was for Test 19 as shown in Figure 4.3 with an alpha value of 67.9 cc/g and an SOS value of 0.827. The profile for Test 4 (alpha = 28.5 cc/g; SOS = 0.7893) – see in Figure 4.4 – shows not only a poor fit but also how the shape of the profile deviates very significantly from the shape predicted by the King Model. This is a surprising result given the strong validation of the model found in the literature as discussed in Section 2.5.

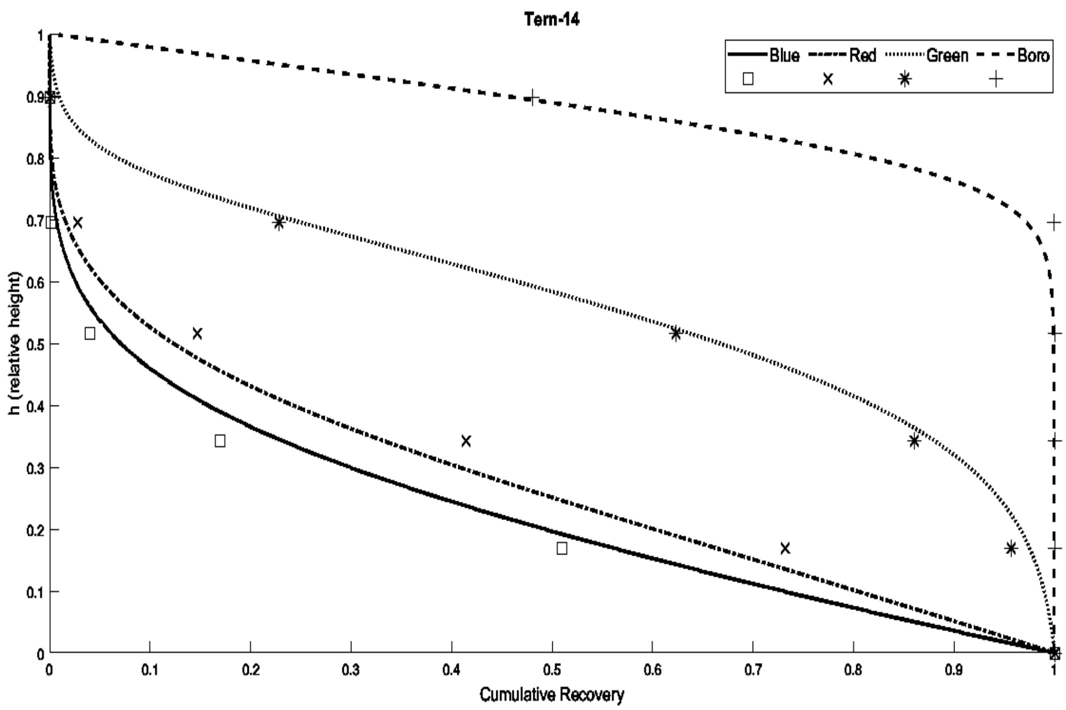


Figure 4.2 Cumulative recovery profile for 80mm bed height and 40mm stroke length

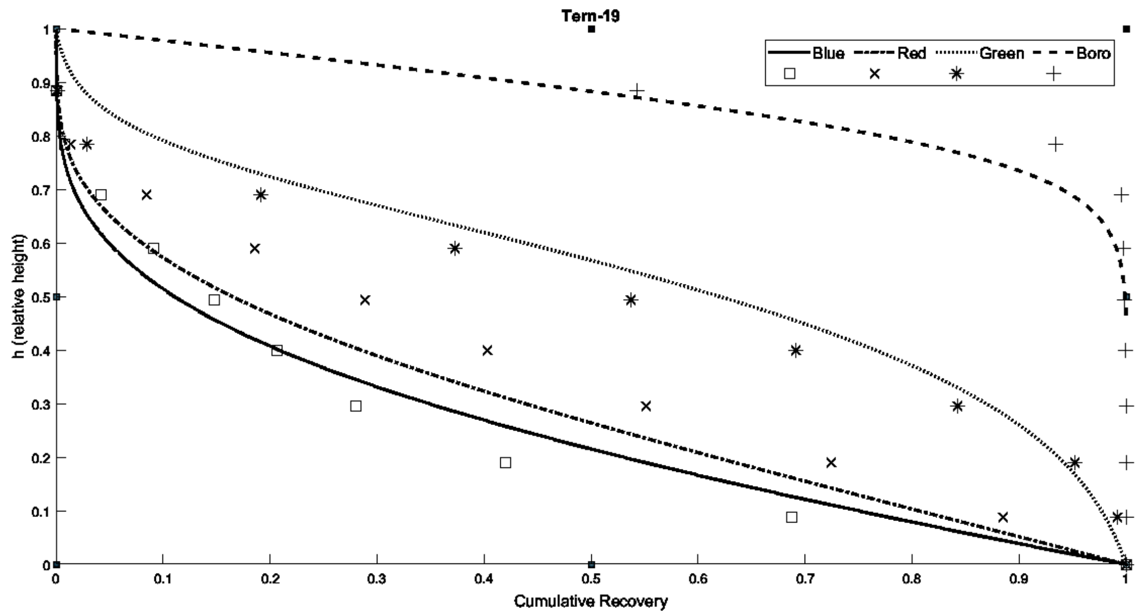


Figure 4.3Cumulative recovery profile for bed height of150mm and 40mm stroke length

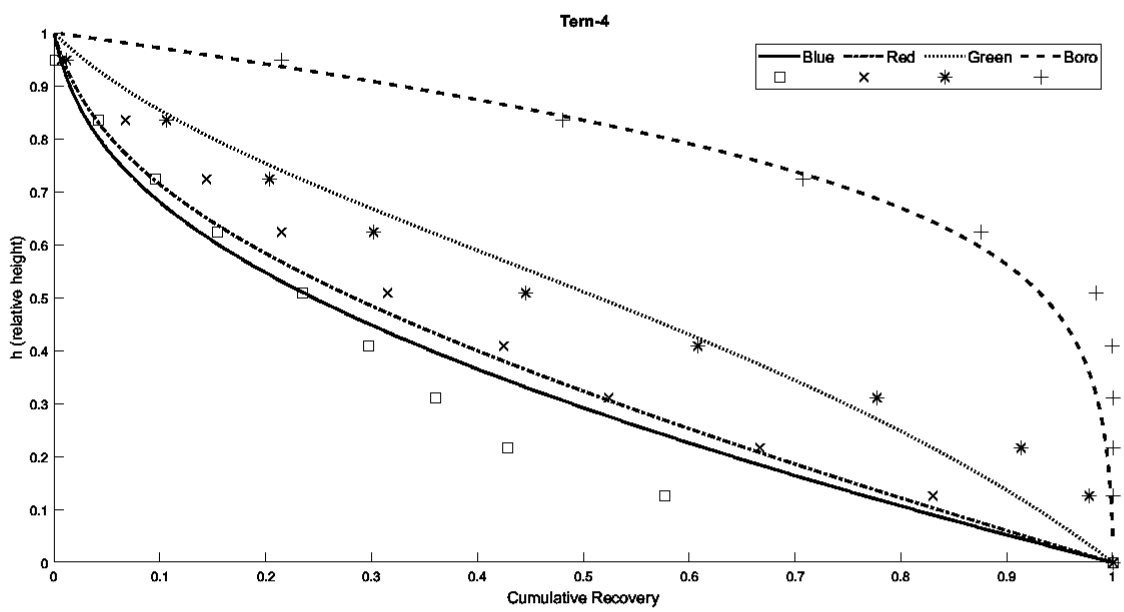


Figure 4.4 Cumulative recovery profile for bed height 150mm and 54mm stroke length

There are two possible reasons for the poor agreement between the experimental data and the King Model. The first is that there was a slight difference in shape among the beads; the fourth component in the bed – the blue beads with a density of 2.567 g/cc – were not strictly spherical.

They each had a slight ridge around their equators. The second possible reason is that there were size differences between the beads used. The blue and boro beads were very close in size – 7.96 and 8.02 mm respectively. However, the green beads were about 2.4% larger and the red beads about 2.4% smaller than these, with a difference of about 4.8% between the smallest and largest beads.(See Table 3.1 for the relevant data.) These differences are not very large and it is hard to imagine how they could cause such odd stratification behavior. Tests on the Wits Jig involving beads similar to the boro, red and green beads did not show such discrepancies. It may be that it is the combined effect of the differences in size, shape and density that is the cause of the poor fits**(Woollacott, 2018)**.

There are several consequences of these observations. First, it is evident that the alpha values obtained are virtually meaningless from the point of view of the intention behind the experimental design. Alpha is not an adequate response variable for the kind of optimization work that was originally designed. Consequently, the results of these tests were not analyzed further and the use of the King stratification index was abandoned as a meaningful response variable in subsequent tests. Second, it seemed prudent not to use the blue beads in subsequent tests and therefore to continue the investigation with a ternary system of beads. This would eliminate the major difference in shape among the beads although the relatively small difference in bead sizes remains. The third consequence was that a different experimental design was required to investigate the effect of operating conditions on density stratification in the jig at equilibrium. This is explained in the next section.

4.4 The Effect of Operating Conditions on Stratification at Equilibrium: Investigation II

The expected ability of the King stratification index to be a single parametric descriptor of a complex stratification profile was foundational to the previous experimental design just described. Without this simple means of evaluating and describing a stratification profile, the only way to investigate the effect of operating variables on stratification was by means of direct comparisons of stratification profiles obtained under different operating conditions. This meant that tests would need to be done varying only one operating variable at a time with all the other variables being held constant at some standard condition. That standard condition would ideally need to be the set of operating conditions that achieves the optimum density stratification. A rough idea of what that standard set of optimum conditions might be was obtained by a subjective examination of the data and plots that had already been generated. The standard set of conditions – or 'base case' – selected is indicated Table 4.7 investigation II. The 'center point' conditions in Investigation I are also shown in the table to highlight where the new 'base case' conditions are different.

Table 4.7Comparison of the Base Case ConditionsComparison of the Base Case Conditions with the Centre Point Conditions in Investigation I

T1, cS	T2, cs	% stroke	Hbed , mm	
22	28	38	80	Estimated Optimum
18	24	38	115	Centre point (Quaternary system)

Table 4.8 shows the full set of conditions tested in Investigation II. Each set of conditions in the table differs from the base case with respect to only one variable. These variations are highlighted in the table to show the specific tests that investigated the effect of each of the six operating variables relating to an investigation test.It can be seen from Table 4.8 that emphasis has been placed on those operating variables which the previous investigation had identified as being the most relevant to stratification – i.e. T1, T2, %Stroke, and bed height. (T3 and T5 are kept constant

at 25 and 133 cSeconds respectively, with T4 varying to maintain the constancy of T5 as before.)Other operating conditions were also included to investigate (1) the effect of pressure, and (2) the effect of %stroke at different bed heights.

Table 4.8 The Operating Conditions Tested in Investigation II
(In all these tests, T3 was 25 cSeconds – as it was in Investigation 1)

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
Tern	T1	T2	T4	Pressure(psi)	Stroke	H bed
	cS	cS	cS	psi	%	mm
1	22	28	58	64	38	80
2	22	28	58	64	45	80
3	22	28	58	64	45	90
4	22	28	58	64	50	90
5	22	28	58	64	38	70
6	22	28	58	64	38	90
7	22	28	58	64	45	70
8	22	28	58	64	34	80
9	22	28	58	64	38	100
10	18	28	62	64	38	100
11	26	28	54	64	38	100
12	22	28	58	64	38	120
13	22	28	58	64	38	150
14	22	24	62	64	38	100
15	22	32	54	64	38	100
16	22	28	58	70	38	100
17	22	28	58	59	38	100
18	22	28	58	64	34	90
19	22	28	58	64	30	90

4.5 The Results and Their Implications

The results of the tests in Investigation II are presented and discussed with respect to the effect of each of the operating variables investigated. The evaluation is based on an examination of the stratification profiles expressed as before in the form of cumulative recovery vs bed height. The plots show R_j^i, the component recoveries to the top (lighter) fraction when the bed is split ata relative height h. The points on the plots show the experimental data obtained. Also shown is the best fit of the King Model to the data. Note that the King Model does not take any of these operating variables into account. It only takes account of the particle densities and their proportions.

To evaluate the effect of an operating variable on stratification, the data from tests with different values of that variable are plotted on the same diagram. To show the stratification profile that most data points follow – i.e. the 'common' stratification profile – a thick red line is shown on the plots.An examination of that diagram reveals the extent to which the stratification profiles obtained under different conditions differ. Deviation of the shape of the experimental profile from that predicted by the King Model is noted and commented on. The slope of the profiles gives a qualitative indication of the quality of stratification; the flatter the curve as presented is, the better is the stratification of that component. On the basis of these observations, the effect which each of the operating variables has on stratification is now evaluated in turn.

4.5.1 The Effect of Bed Height

Figure 4.5 compares the stratification profiles from tests done at different bed heights. The variation in bed height was tested at six different bed heights: 70, 80, 90,100,120 and 150mm, respectively tests Tern-1, Tern-5, Tern-6, Tern-9, Tern-12 and Tern-13.

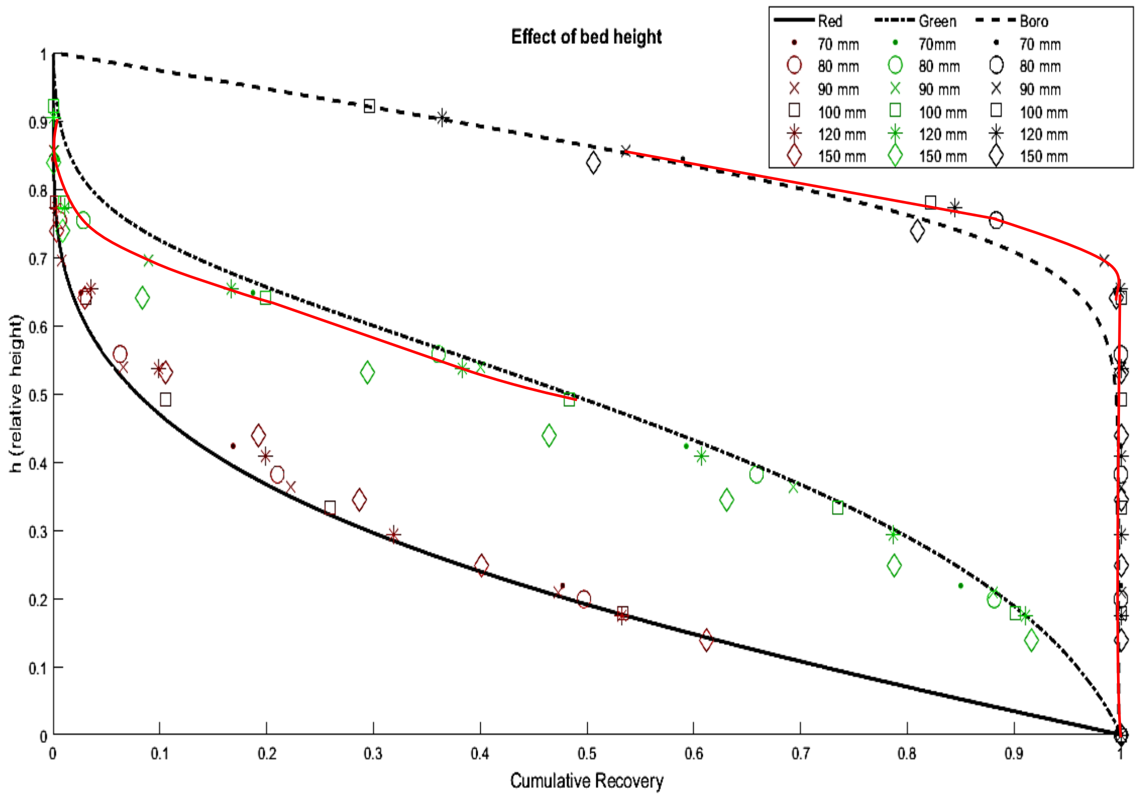


Figure 4.5 The effect of bed height on the equilibrium profile

(Red line shows the common stratification profile)

Examining the curve for the lighter component (boro), it is evident that all the profiles are superimposed except for the profile for the largest bed depth, 150mm. All except for the 150mm curve deviate somewhat from the model curve for recoveries greater than around 0.8 (80%). Much the same can be said about the curves for the intermediate component (green). They are all superimposed except for the 150mm curve. There is a slightly greater deviation when it comes to the heavy component (red). Here,both the 150 and 120 mm curves deviate from the others at recoveries below about 0.35 (35%). Otherwise the profiles are superimposed on each other and follow the King Model quite closely.

Figure 4.6 shows the profiles individually. It can be seen that the shape of the profiles for the 120 and 150mm curves deviate significantly from the shape predicted by the King Model, with the deviation being worse for the 150mm curve. The deviation is in the direction of poorer stratification. The shape of the other curves is similar to that predicted by the King Model although the King Model does not fit the data very well. It appears that the same factors that gave anomalous results in Investigation I are at play here as well, although their effect is not as great. The more significant conclusion, however, is that up to a bed depth of between 100 and 120mm the stratification pattern achieved in the jig at equilibrium appears to be independent of bed height.

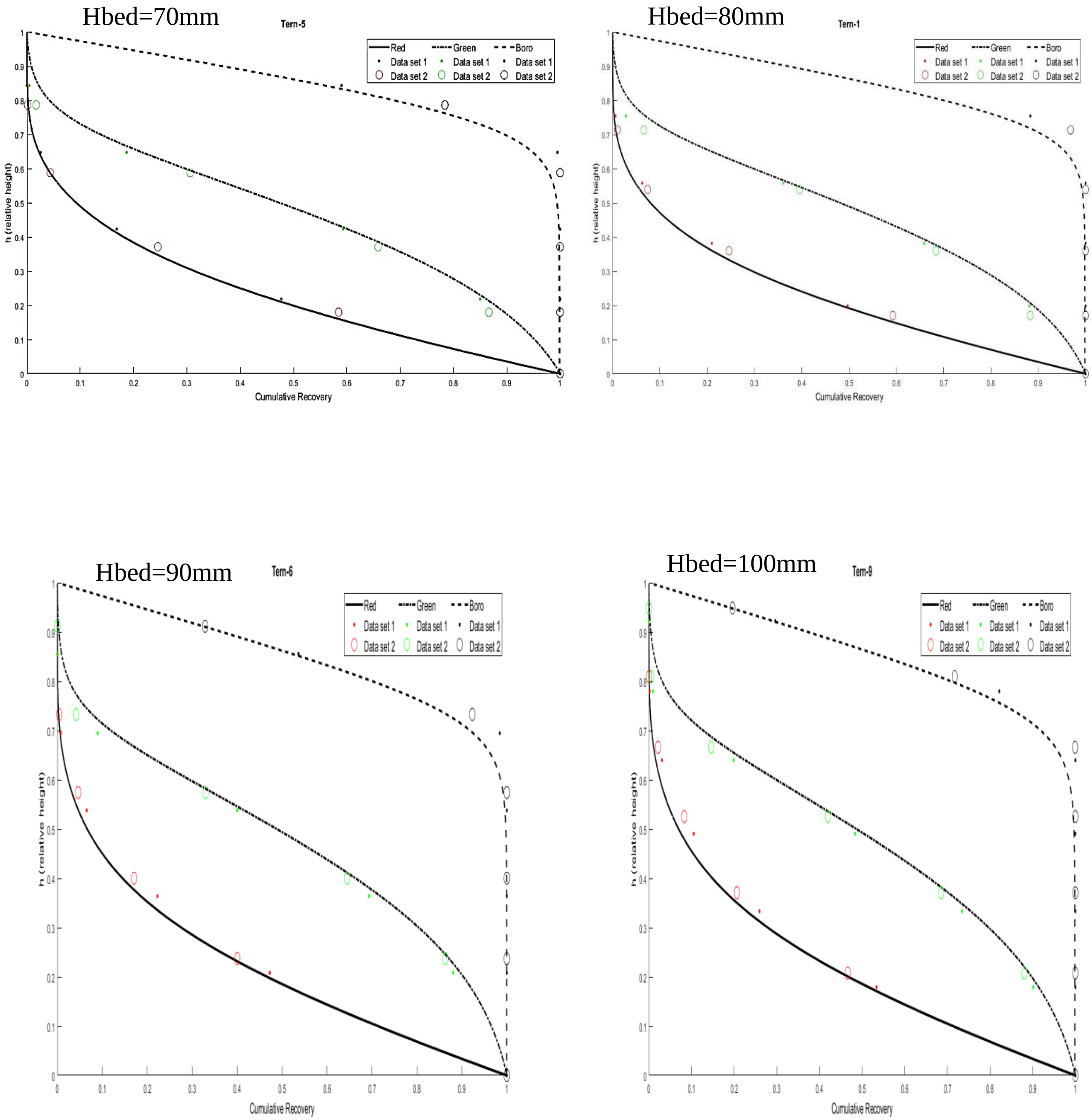


Figure 4.6 The stratification profiles for individual tests at different bed heights

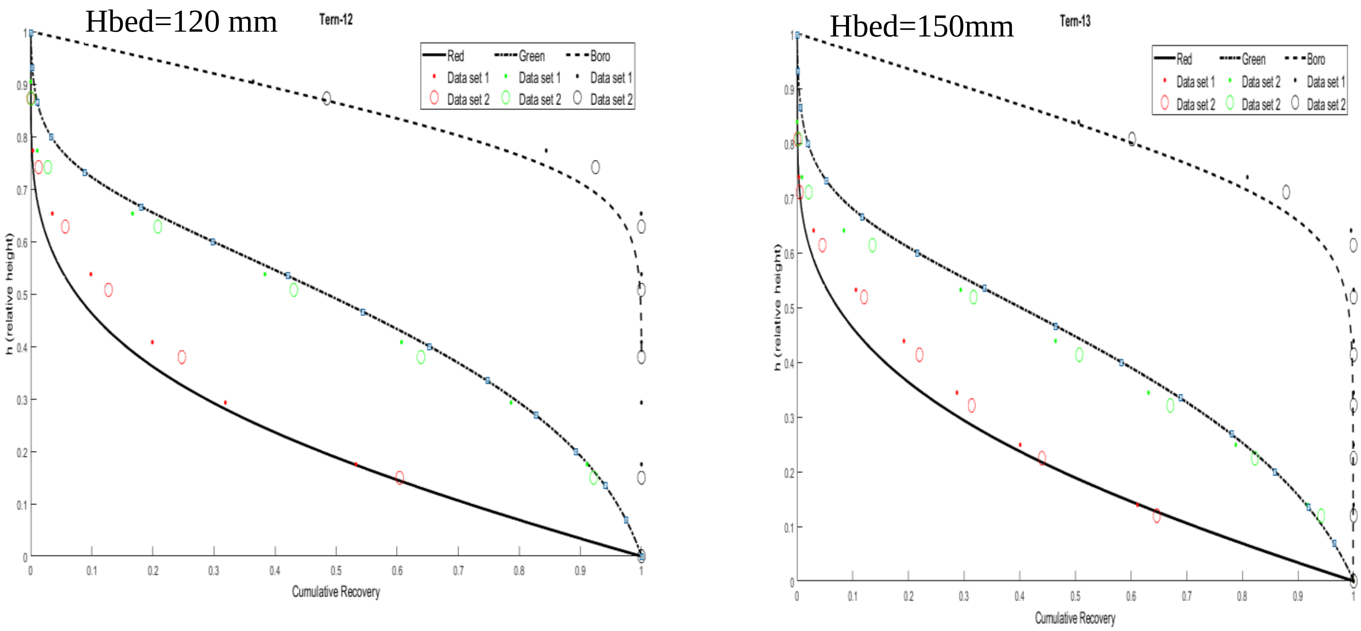


Figure 4.7 The stratification profiles for individual tests at different bed heights

4.5.2 The Effect of Percentage Stroke

Figure 4.7 compares the recovery profiles when the percentage stroke is varied between 30% and 50%. Tests at 30%, 34%, 38%, 45% and 50% %Stroke were conducted (tests Tern-19, Tern-18, Tern-9, Tern-3 and Tern-4 respectively). As before, the equivalent predictions of the King Model are also shown.

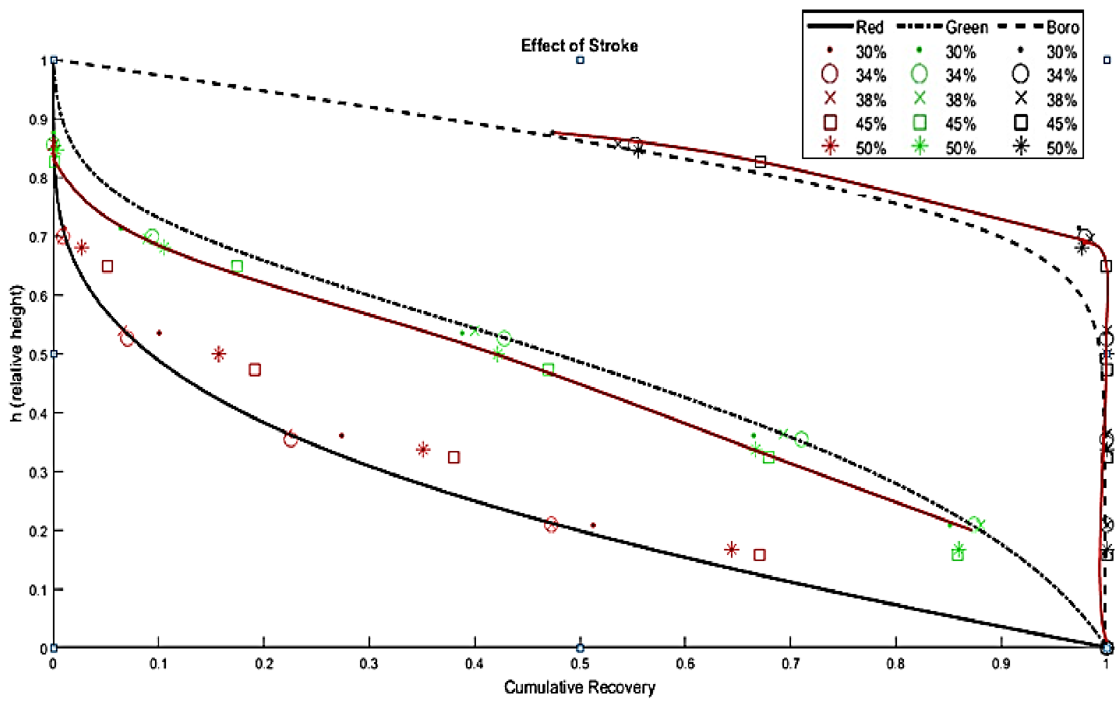


Figure 4.8 The effect of Percentage Stroke on the equilibrium profile

(Red line shows the common stratification profile)

In this Figure 4.7 it can be seen that the stratification of the lighter component (boro) is essentially identical for the entire percentage strokes tested. The sharpest stratification of the intermediate component (green) is achieved with percentage strokes from 34 to 38%. Above this range, i.e. when the stroke is 45 or 50%, stratification is poorer, (the curves are not as flat). It is also somewhat poorer below this range, i.e. at 30%Stroke.

It can be seen from Figure 4.8 that King's Model fits the data better than was the case for different bed heights, but the distortion in the shape of the curves relative to the model predictions is absent. The conclusion from this analysis is that if the stroke is not vigorous enough or is too vigorous, it has a detrimental effect on stratification. Between these two extremes the quality of stratification appears to be independent of percentage stroke.

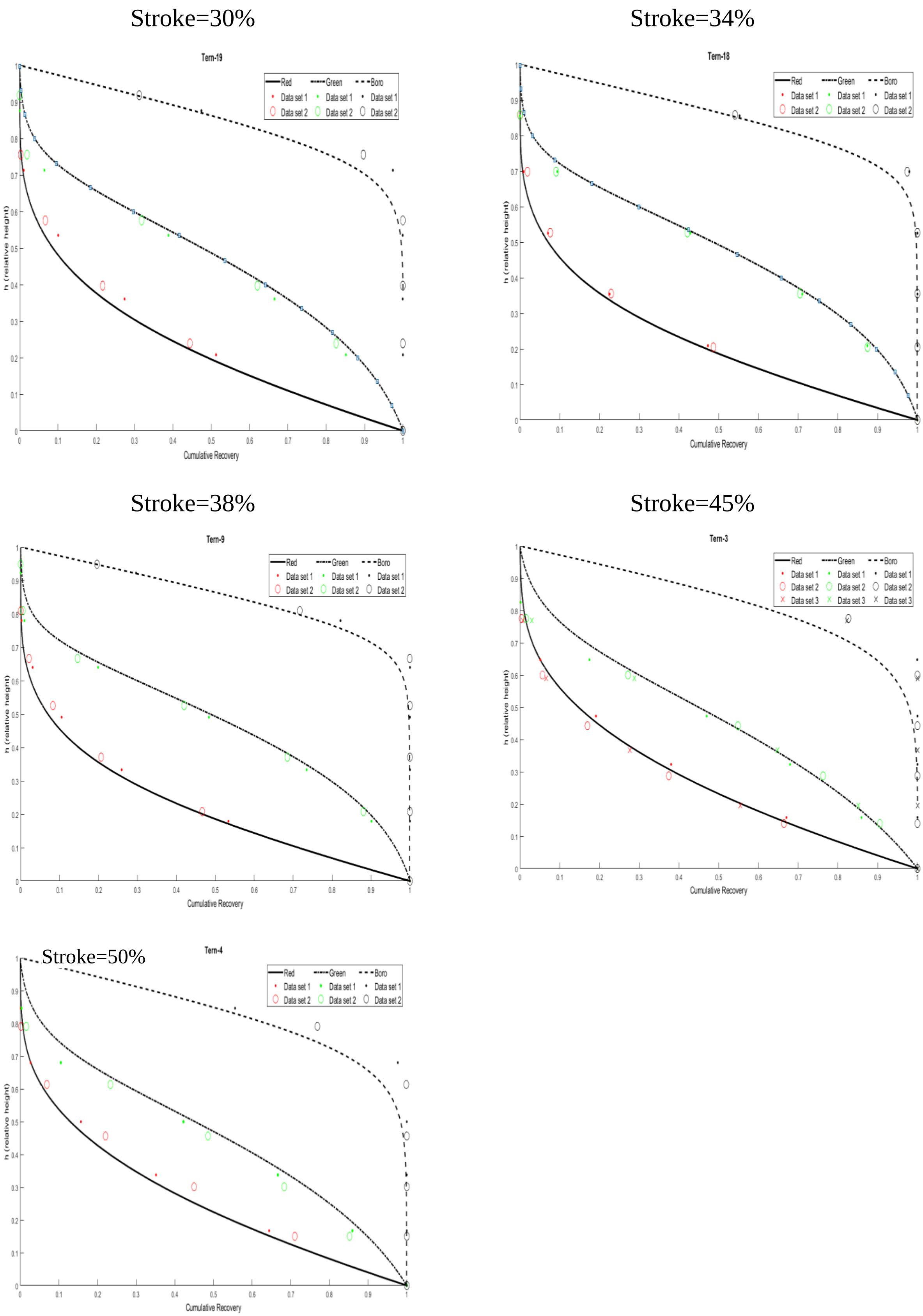


Figure 4.9 The stratification profiles for individual tests at different %Stroke

4.5.3 Effect of pulsion time (T1)

Figure 4.9 compares the recovery profiles achieved at pulsion times, T1, from 18cS to26cS.All the curves superimpose on one another. Figure 4.10 shows the profiles for individual tests. King's Model fits the data reasonably well. There are no distortions in the recovery profiles. The conclusion reached here is that stratification at equilibrium appears to be independent of the pulsion time, T1.

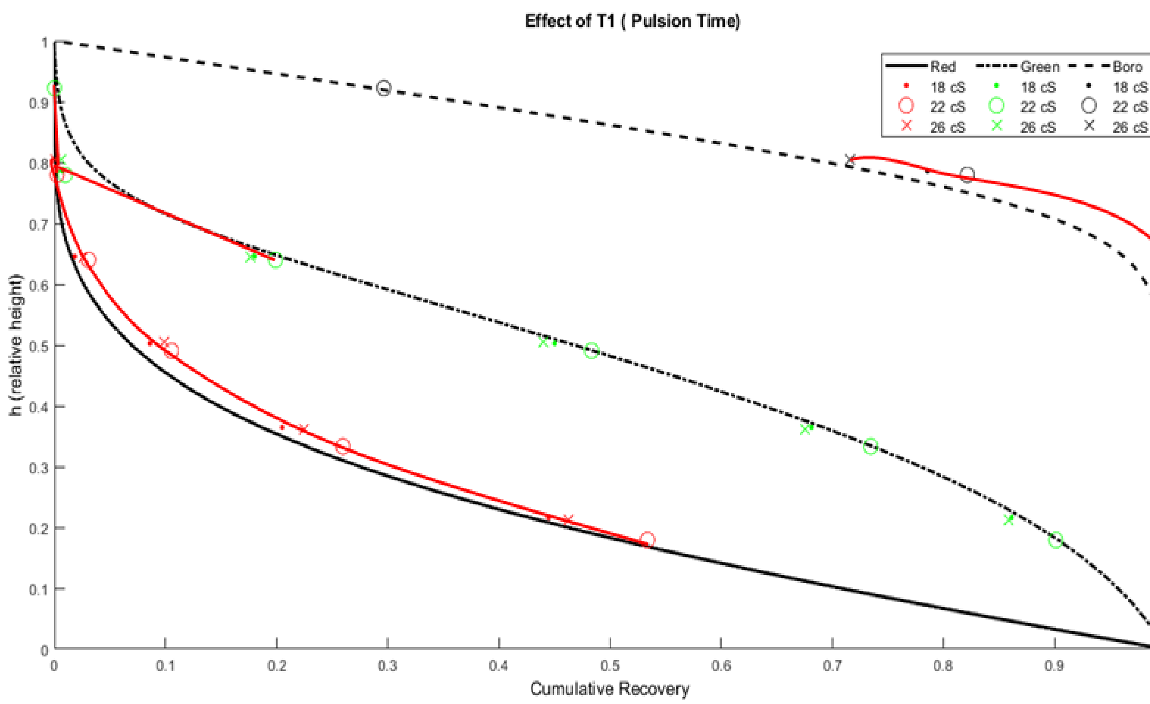
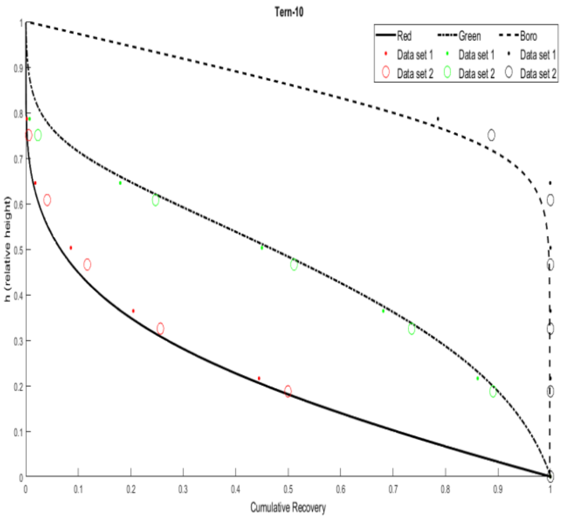
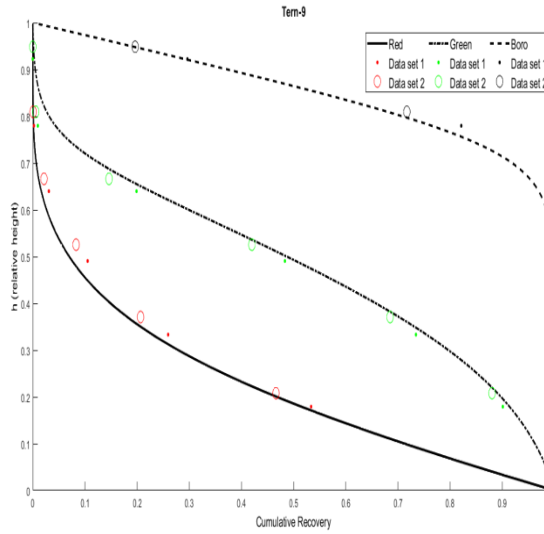


Figure 4.10 The effect of Pulsion time, T1, on the equilibrium profile (Red line shows the common stratification profile)

T1=18cS



T1=22cS



T1=26cS

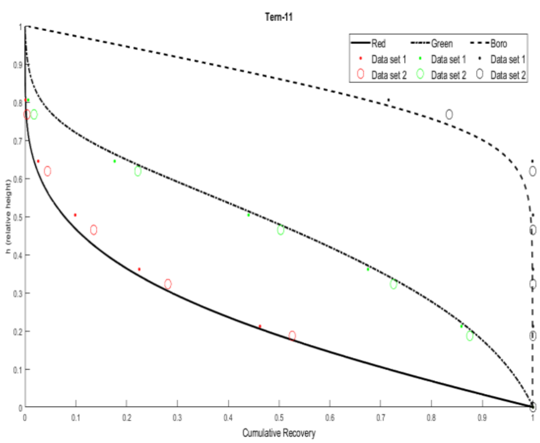


Figure 4.11 The stratification profiles for individual tests at different Pulsion Times, T1

4.5.4 Effect of Pulsion Hold Time, T2

Figure 4.11 compares the stratification profiles achieved for different pulsion hold times, T2. Figure 4.11b shows the profiles for the individual tests. The plots are very similar to those observed for the tests at different pulsion times, although the King Model does not fit the data quite as well as is the case when T1 is varied. Again the conclusion is that the quality of stratification at equilibrium appears to be independent of the pulsion hold time, T2.

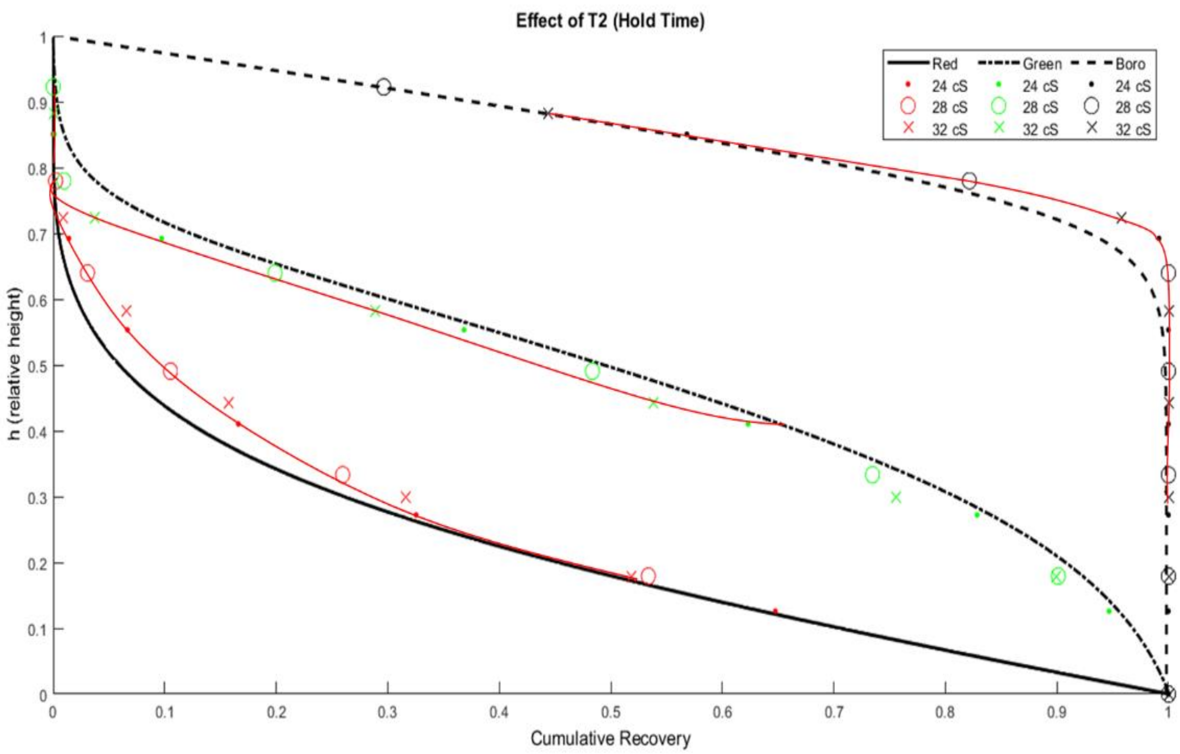
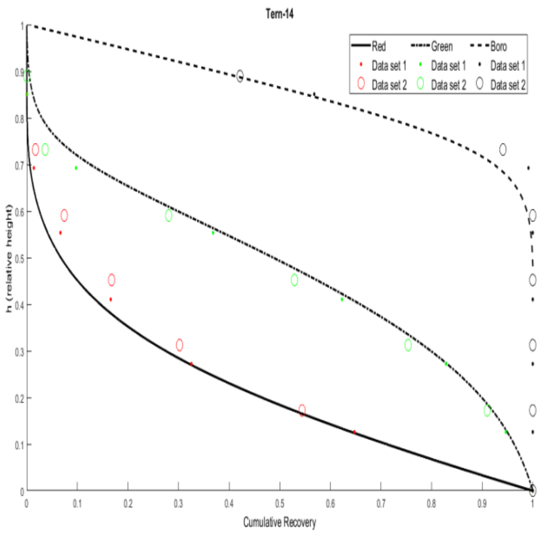
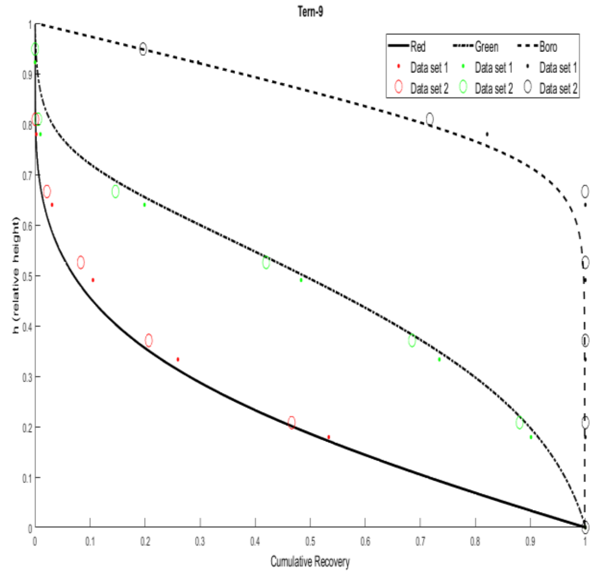


Figure 4.12 The effect of Pulsion hold Time, T2, on the equilibrium profile (Red line shows the common stratification profile)

T2=24cS



T2=28cS



T2=32cS

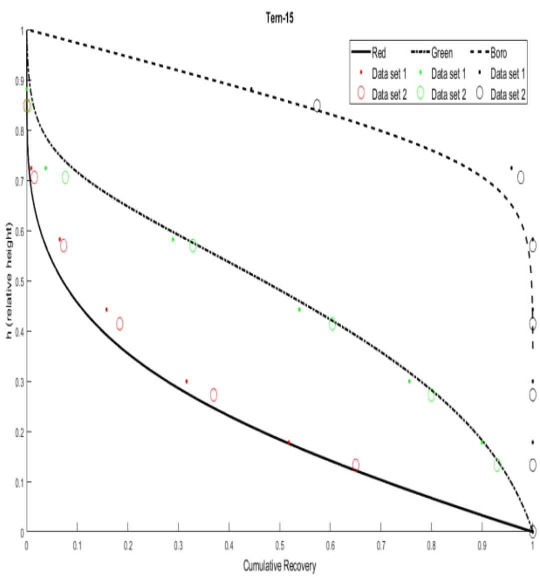


Figure 4.11 bStratification profiles for individual tests at different Pulsion Hold Times, T2

4.5.5 Effect of pressure

Figure 4.12 compares the stratification profiles achieved when different pressures are applied to the pulsion device. Figure 4.13 show the profiles for individual tests. The same observations made for the tests with different T2 values applies in this case as well. The conclusion is that variation in the applied pressure had no effect on the quality of stratification achieved at equilibrium.

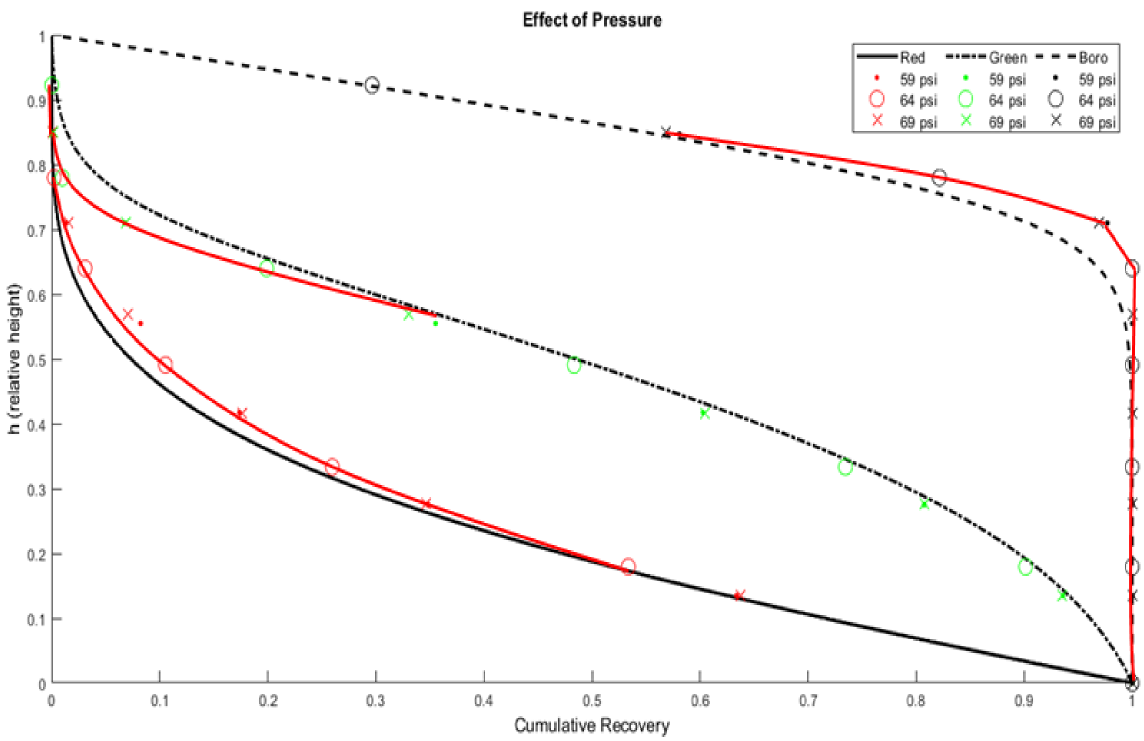
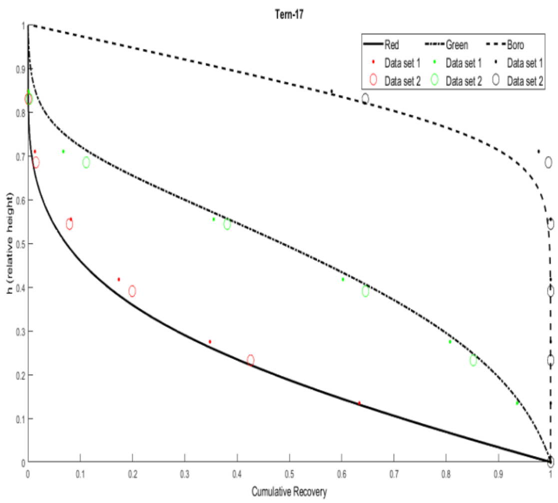


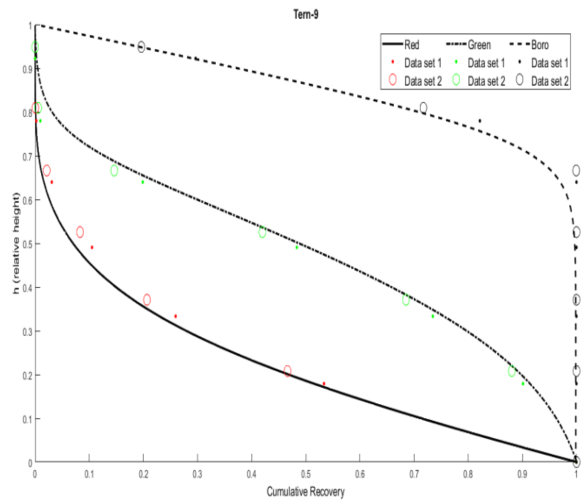
Figure 4.13The effect of pressure on the equilibrium profile

(Red line shows the common stratification profile)

59psi



64 psi



69 psi

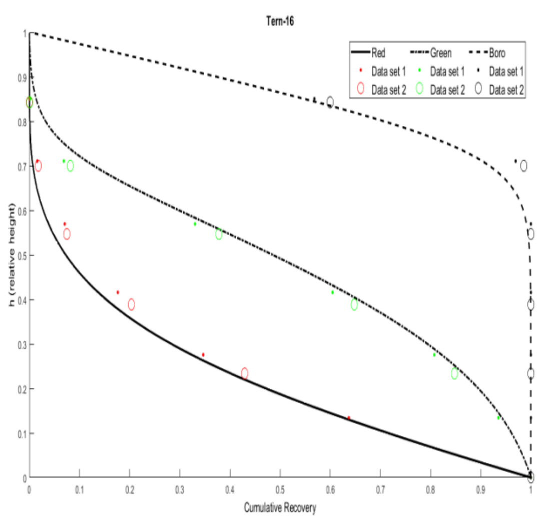


Figure 4.14 The stratification profiles for individual tests at different applied pressures

4.6 The Effect of Operating Conditions on the King Stratification Index

Overall the King Model does not fit the data from the equilibrium tests in Investigation II sufficiently well to be used as a reliable means for predicting the equilibrium stratification profiles for all the conditions tested. The quality of the fits varies from reasonably good in some tests to bad in others. Mention has already been made of possible reasons for this unreliability. However, to conclude the presentation of the results, the values of the stratification indices are reported and discussed here. This is done to show how the operating variables appear to affect the stratification index and the goodness of fit evaluated qualitatively.

4.6.1 The Influence of bed height on the stratification index

As can be seen from Table4.9 the value of alpha value for higher bed height is comparatively small than the lower bed height and provides a bad fit. It can also be seen that for a good and reasonable fit the sum of square difference value are below 0.2.

Table 4.9The influence of bed height on stratification index and quality of fit

Tern	H bed(mm)	SOS	Stratification index	Quality of fit
5	70	0.122	65.0	reasonable
1	80	0.182	71.9	resonable
6	90	0.144	81.5	resonable
9	100	0.153	78.3	resonable
12	120	0.344	75.2	bad
13	150	0.610	66.9	bad

4.6.2 Influence of % stroke on stratification parameter

Table 4.10 shows that the fits for the low and higher % strokes were poor – with sum of square difference significantly above 0.2.

Table 4.10 The influence of % stroke on stratification index and quality of fit

Tern	% stroke	SOS	Stratification index	Quality of fit
19	30	0.329	68.2	bad
18	34	0.137	71.9	resonable
6	38	0.144	81.5	resonable
3	45	0.784	43.1	bad
4	50	0.412	65.2	bad

4.6.3 Influence of T1 on stratification parameter

Table 4.11 shows the fits when T1 was varied were better. The sum of square difference value are all below 0.2.

Table 4.11The influence of T1 on stratification index and quality of fit

Tern	T1, cS	SOS	Stratification index	Quality of fit
10	18	0.127	77.7	reasonable
9	22	0.153	75.3	reasonable
11	26	0.133	70.7	reasonable

4.6.4 Influence of T2 on stratification parameter

Table 4.12 shows the value of alpha and the value of the sum of square differences for different T2 settings. The sum of square difference values are all below 0.3.

Table 4.12The influence of T2 on stratification index and quality of fit

Tern	T2, cS	SOS	Stratification index	Quality of fit
14	24	0.23004	79.8504	reasonable
9	28	0.15339	75.3546	reasonable
15	32	0.30202	74.6304	reasonable

4.6.5 Influence of pressure on stratification parameter

As can be seen from Table 4.13 that the alpha values are essentially the same for all the pressure variation and the fits are reasonable fit, with the sum of square differencesless than 0.25.

Table 4.13 The influence of pressure on stratification index and quality of fit

Tern	Pressure, psi	SSQ	Stratification index	Quality of fit
17	59	0.216	76.0	reasonable
9	64	0.153	75.3	reasonable
16	69	0.220	76.0	reasonable

4.7 Summary and conclusion

This chapter has established which operating conditions affect stratification and has investigated their effect on the equilibrium stratification profile. Because the cycle amplitude – i.e. themaximum water displacement in the jig – could not be measured directly, a preliminary investigation was undertaken to establish the relationship between the controllable variables which affect that amplitude. To do this a partial factorial 2^{6-1} experimental design was used with the cycle amplitude (stroke in mm) as the response variable. Out of the six controllable variables that influence the amplitude,only three were found to have a significant effect. T3, pressure and cycle frequency did not show any significant effect. It was therefore concluded that the remaining three controllable variables – T1, T2 and %Stroke – and bed height were the only operating conditions that needed to be investigated with regard to their effect on the equilibrium stratification profile. Accordingly, the effect of operating conditions on equilibrium stratification was investigated using a 2^4-1 factorial experimental design. The sample selected for the tests was a four component particle system.

The results from this investigation showed that the agreement between the model profiles and the experimental points were poor, so the values of the King Stratification Index could not be used as a reliable response variable for the evaluation of the test results. Therefore a second investigation was undertaken based on a multi-linear experimental design and a three component particle system.

The results from this second investigation showed that the operating conditions had no discernible effect on the stratification equilibrium provided the bed depth and % stroke were within an optimum range – an ‘optimum stratification zone’. Provided the bed depth was 100mm or less and the percent stroke was not too little (less than 36%) or too vigorous (greater than about 41%), none of the four operating variables affected the stratification pattern at equilibrium.In addition, outside the limits of this ‘optimum zone for stratification’, the stratification was poorer than within these limits.

This result is surprsing and unexpected because the literature clearly indicates that operating conditions have a profound effect on stratification. What the finding of the work reported in this chapter therefore indicates is that whatever effect operating conditions have on stratification, they must have on the kinetics of stratification and not on the final stratification profile achieved at equilibrium.

The findings also helped to establish an appropriate range of operating conditions and an appropriate base case for the kinetic tests that needed to be conducted.

Chapter 5

Result II: Effect of operating Conditions on Stratification Kinetics

5.1 Introduction

This chapter addresses research question 2 – "How do the operating conditions affect the kinetics of stratification in the batch jig?". The previous chapter investigated theeffect of operating conditions on stratification in the jig at equilibrium and found that within an optimum operating range of bed depth and %stroke, the operating conditions did not affect the equilibrium stratification achieved. It also provided information on the range of operating conditions that needed to be tested to establish how operating conditions affect the kinetics of stratification. These tests are presented and analyzed in this chapter.

Chapter 3 (Section 3.5.2) describes the experimental method used for the kinetic tests. For practical reasons, it was necessary to limit the number of kinteic tests conducted. Whereas the equilibrium stratification profile at one set of operating conditions could be established experimentally by a single test (with perhaps one or two replicates), very many more tests – from 8 to 15 or so – are required to establish the kinetics at that one set of conditions. Accordingly, the original experimental design (as explained in Chapter 3) aimed to establish the set of operating conditions that would achieve optimum stratification at equilibrium after which a kinetic study would be conducted for conditions at and around that optimum. Unfortunately it was not possible to establish that optimum set of conditions by means of a partial factorial design. However, it was possible to estimate qualitatively from the equilibrium testwork program (as described in Chapter 4) where that optimum was. On this basis, a base case set of conditions for the testwork program was established as shown in the first row of Table 5.1 with the 'optimum' bed depth being 90mm.

Table 5.1 The operating conditions for the kinetic tests

Set number	T1	T2	% stroke	
1	**22**	**28**	**38**	**Base case**
2	22	28	34	Variation in Stroke
3	22	28	36	
4	22	28	41	
5	22	28	45	
6	22	28	48	
7	18	28	38	Variations in T1
8	26	28	38	
9	30	28	38	
10	22	24	38	Variations in T2
11	22	32	38	
12	22	35	38	
13	22	38	38	

Other operating conditions were fixed as follows: T3 25cS; pressure = 64psi;
T4 was varied as per the variation of T1 and T2 to make the cycle time=133cS.
Sample composition (volume %): Red =28.9%, Green=44.8%, Boro= 26.3%.

Chapter 4 identified 7 operating variables that could potentially influence stratification in a batch jig. Of these, three were eliminated as not having any significant influence on stratification under normal conditions. For the kinetic testwork, the number of operating conditions investigated was then reduced from 4 to 3 by keeping the bed depth constant at the estimated optimum bed depth of 90mm. Tests were then conducted by varying these three variables – T1, T2 and percentage stroke – one at a time above and below the base case set of conditions. Additional tests were undertaken to investigate the extent to which stratification kinetics changed around the boundaries of this ‘optimum zone’. The full range of conditions tested is shown in Table 5.1. Table 5.2 indicates the height of each of the 6 slices (layers) taken rom the jig bed after each kinetic test. Care was taken to make sure that these heights remained the same in all the kinetic tests. All tests were done using the same set of particles used for the equilibrium tests.

Table 5.2 Height of Each Layer

Layer no	Sliced height (mm)
1	18.5
2	32.5
3	46
4	60.5
5	76.5
6	90 (top of bed)

5.2 Base Case: Results

Figures 5.1, 5.2 and 5.3 show how the concentrations profiles for the light, intermediate and heavy particles change with time in each of the six layers. Each point on a kinetic curves is derived from a separate experiment. The trend of each kinetic curve is seen to be clearly indicated, with only a few minor anomalies. These are the result of experimental errors such as some segregation during initial mixing of the sample before a test or when the sample was poured into the jig chamber.

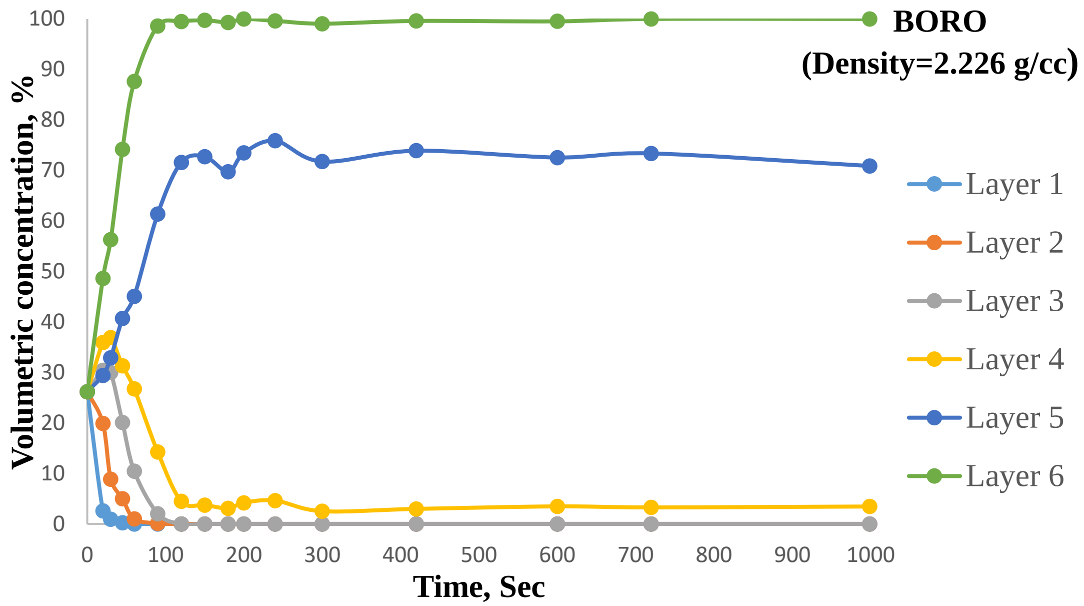


Figure 5.1 Changes in the concentrations of the light(boro) particles in each layer with time

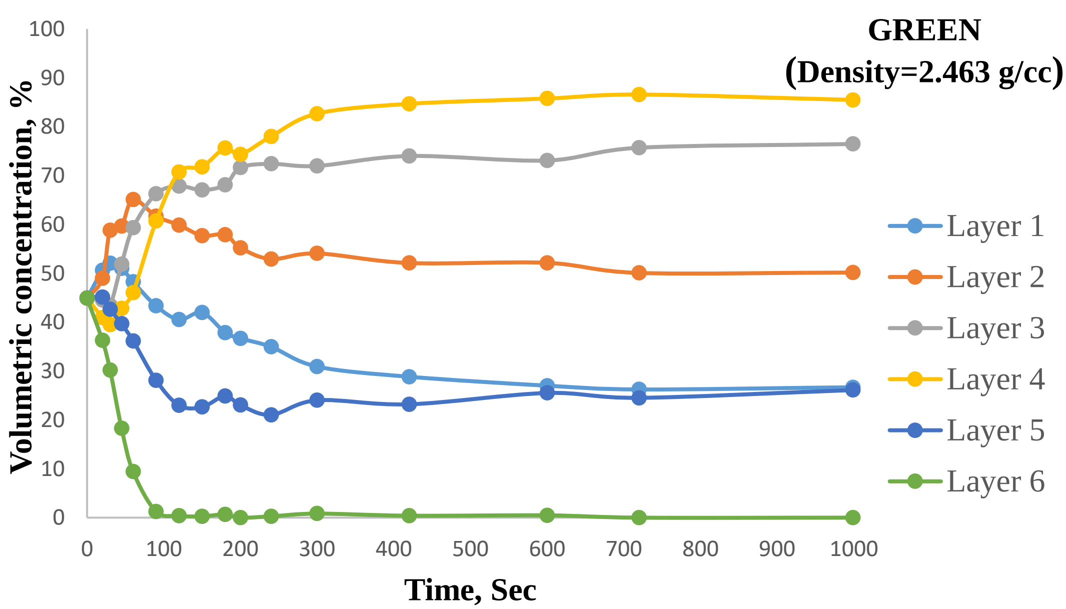


Figure 5.2 Changes in the concentrations of the intermediate (green) particles in each layer with time

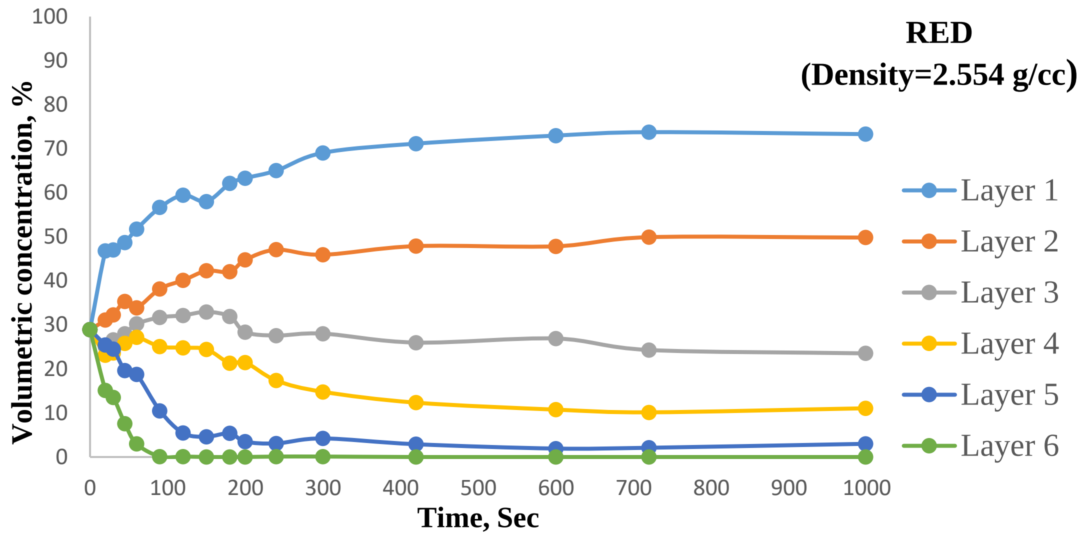


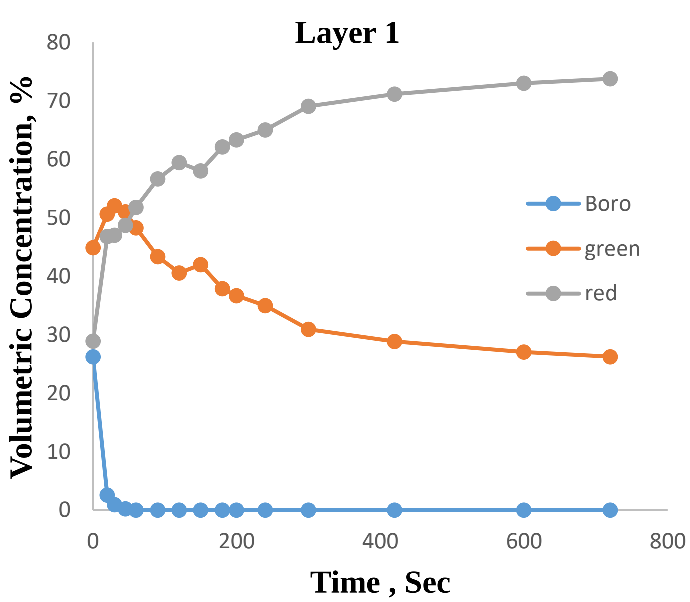
Figure 5.3 Changes in the concentrations of the heavy (red) particles in each layer with time

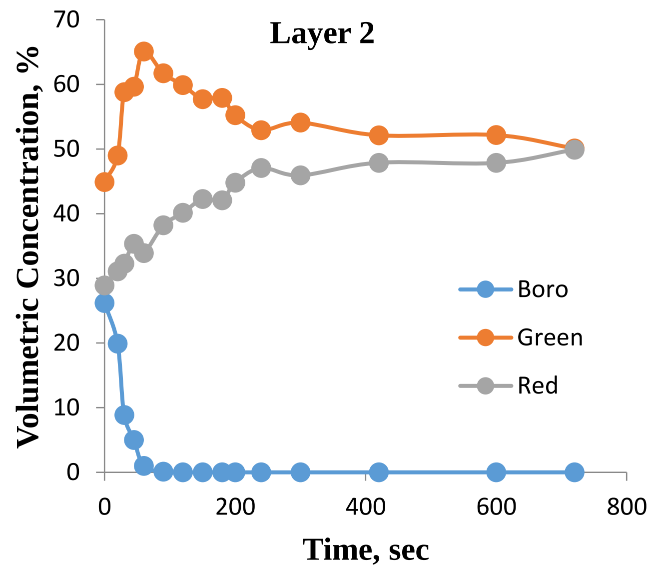
Figure 5.1 shows that the concentration of light particles in all layers reach the equilibrium after 120 to 150 secs. In layers 1 to 2, the concentrations decrease rapidly to zero. In these layers, the shape of the kinetic curves look exponential. The pattern in layer 3 and 4 are different; the concentration increases slightly before decreasing rapidly to zero or a low value.

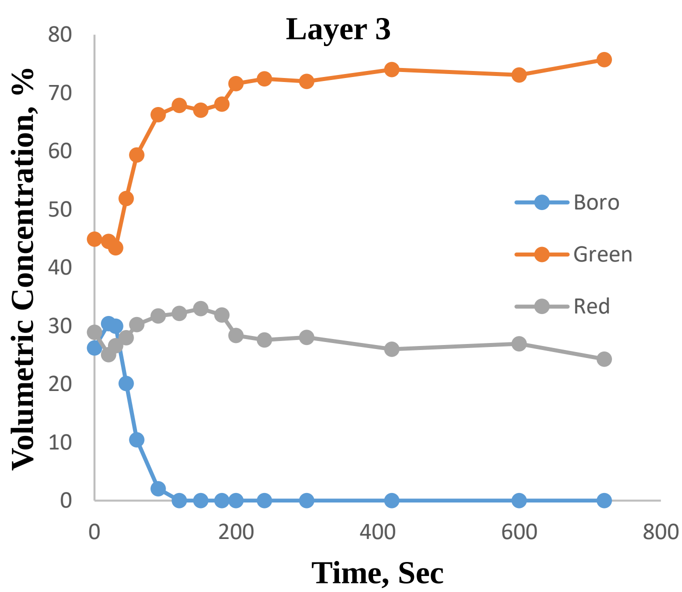
In Figure 5.2 (the intermediate component), the same kind of reversal of direction is observed in all layers except in layer 6 which rapidly reaches the equilibrium value of zero. The reversal of direction is most noticeable in layer 2.

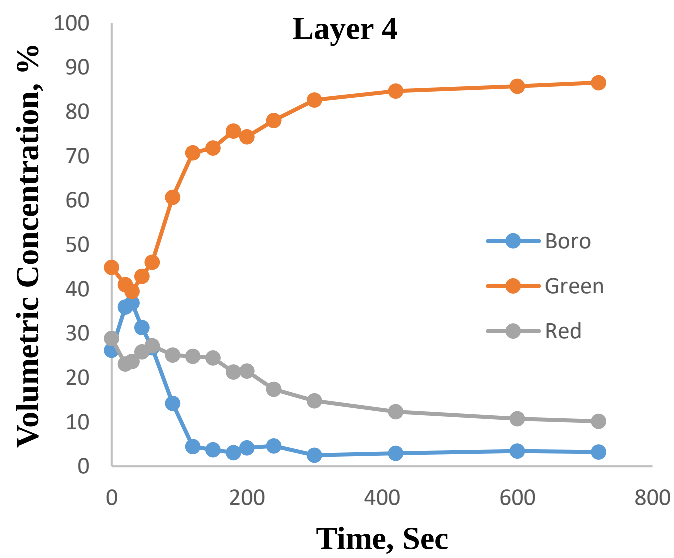
In Figure 5.3 (the heavy component), the changes in concentration are slower and only in layer 4 is there evidence of a slight reversal in the direction of change of the concentration.

Figure 5.4, compares the kinetics of each component in each layer. It shows the approach towards the equilibrium of each layer and the segregation in the bed. This shows that in layer 1 the lighter particles reachthe equilibrium value of zero concentration very rapidly – within 20 secs, where green took 240secs and red 300 secs. In layer 2, the time taken was 60 secs for boro, 240secs for green and the same for red. In layer 2 the concentration profile for green initially increased then decreased. In layer 3, boro took 120 secs to reach equilibrium, where green and red both took 200 secs. In layer 4, boro and green both reached equilibrium in 180secs, whereas red took longer – 240 secs. In layers 5 and 6, all three components reached equilibrium in about the same time - 90 secs.









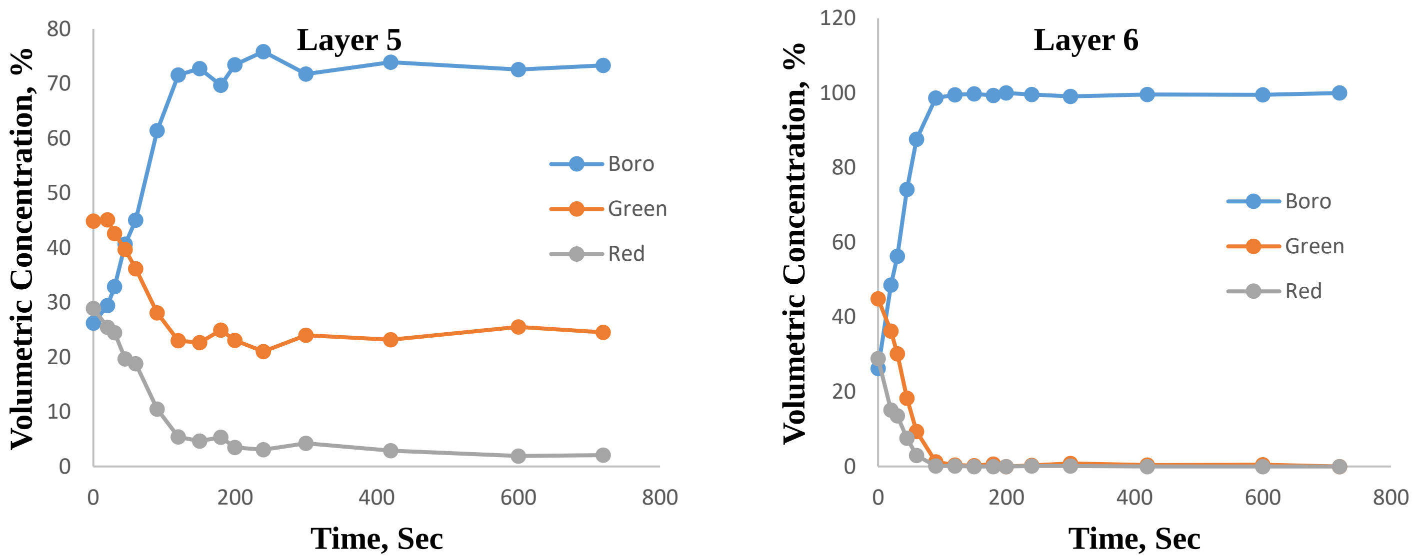


Figure 5.4 Comparison of the kinetic curves for each component in each layer

5.3 Base Case: Preliminary Implications

The time taken to reach equilibrium is different for each of the components. 150 sec for boro, 300 for green and 400 or so for red. The reason is the difference in density –i.e. the larger the density difference the faster the kinetics of stratification. This can be seen very clearly in fig 5.2 – where the time taken for green to reach the equilibrium in layers 5 and 6 is considerably shorter (respectively about 90 and 120 seconds) which is about the same time as the time taken for the boro particles to reach an equilibrium in these layers. Essentially, what is happening in the top two layers then, is that the red particles fall out of these layers quickly and it is essentially the segregation of boro from green that happens in the top layers during jigging. Similarly, the lighter particles quickly migrate out of the bottom 3 layers so that the kinetics in these layers is dominated by the segregation of green from red particles. The density difference between these two is 0.091g/cc which is much smaller than the density difference between boro and green 0.237g/cc.

In layers 3 and 4, the concentrations of the green particles increase slightly then decrease. This can be understood as the result of the migration rates of the different components through and into these layers being different. The effect is small for the light and heavy components but much more pronounced for the intermediate component (Figure 5.4). The sharp rise and then gradual falling off of the concentration of the intermediate components in this layer can be explained as the result of the lighter components in layer 1 and 2 rapidly migrating to the top layers, and then the slower migration of red (heavy) particles from the upper layers through layer 2

5.4 Base Case: Analysis with respect to Approach to Equilibrium

Figures 5.1 to 5.4 are interesting in that they reveal in detail how the different components migrate over time and how the concentrations of the different components change with time in each of the six layers. However, the picture portrayed is complex and non-generic – for example, the figures would be somewhat different if the bed had been sliced in a different way, or the composition of the bed was different. A more generic way of understanding the kinetics of stratification is therefore desirable.

Chapter 3 discussed the various approaches that have or could be used to analyse and model stratification kinetics. It developed an approach which aimed to analyse and model kinetics with reference to the equilibrium stratification profile. This meant that kinetics would be analysed and modelled in terms of the rate at which the stratification profilemoved towards the equilibrium stratification profile.An 'Approach to Equilibrium Metric', $X_f(t)$, was developed based on the areas between a homogeneous stratification profile and the stratification profile achieved after a time t or at equilibrium – Equation 5.1.

$$X_f(t) = \frac{Area\ at\ time\ t\ (relative\ to\ homogeneity)}{Area\ at\ equilibrium\ (relative\ to\ homogeneity)} * 100 \qquad [5.1]$$

The Approach Metric was calculated from the stratification profiles expressed in terms of cumulative recovery up as shown in Figure 5.5 to 5.6, i.e. the recovery of a component to the bottom fraction when the bed is split at a height *h*. A homogeneous stratification profile – the profile that exists before jigging (i.e. before segregation and stratification) – is depicted as the straight line in the diagram as explained in Chapter 3. The areas between that line and the stratification profile at time *t* was estimated using the trapezoid method.

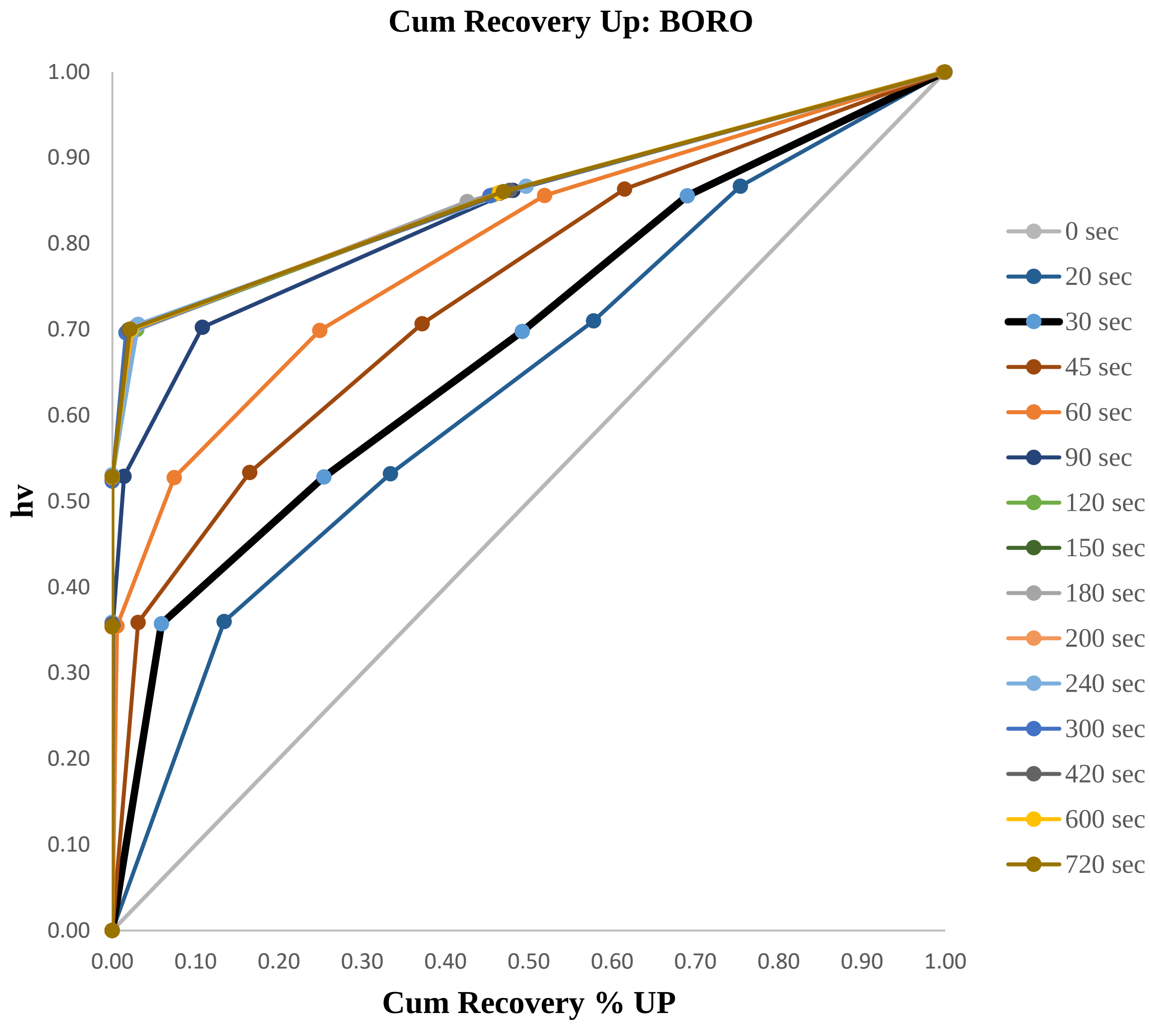


Figure 5.5 Cumulativerecovery profiles for the light particles.

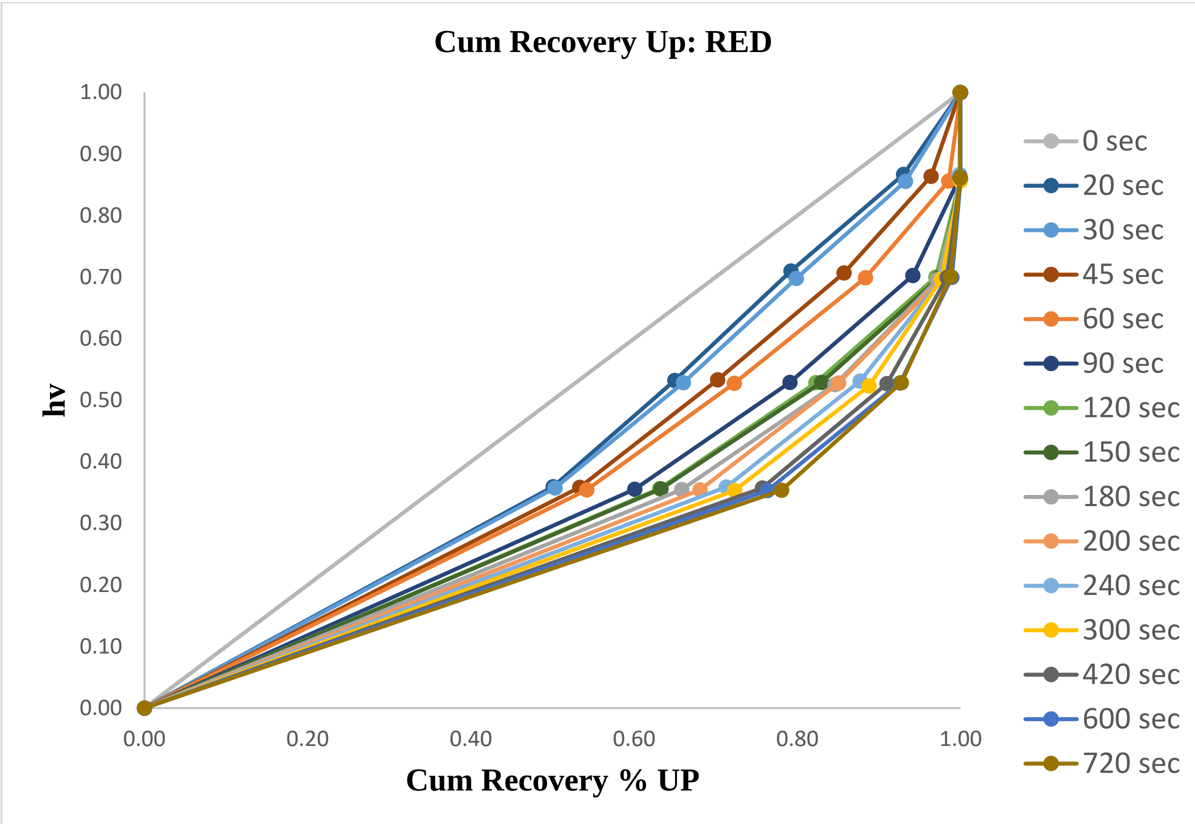


Figure 5.6 Cumulativerecovery profiles for the heavy particles

In a ternary system, the Approach Metric for the light component, $X_f(boro)$, is different from the Approach Metric, $X_f(red)$, for the heavy component. As Figure 5.7 shows, the kinetics of the intermediate component is complex because its approach to equilibrium is different in the upper and lower parts of the bed. Therefore, the concept behind the Approach Metric does not apply as easily to this component. However, the kinetics associated with the intermediate component can be estimated by difference noting that the concentrations of the three components must sum to one in any layer and at any time.

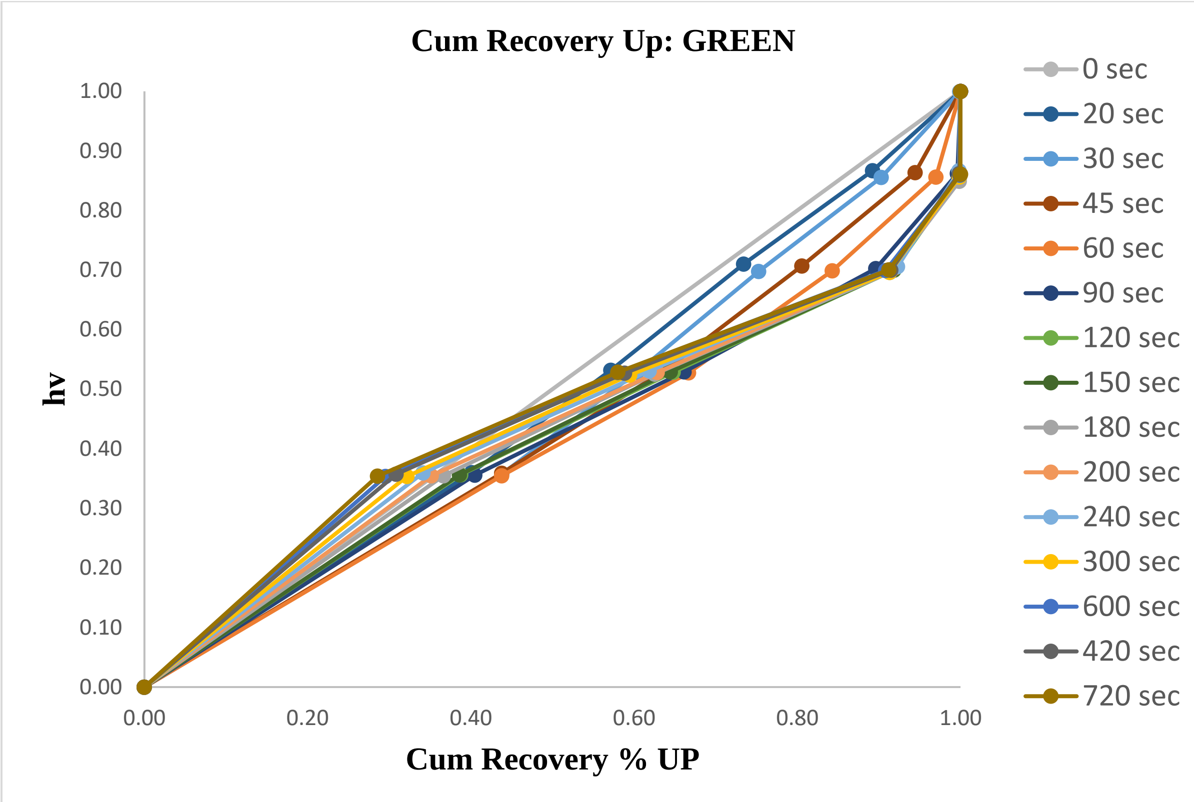


Figure 5.7 Cumulative recovery profiles for the intermediateparticles.

Figure.5.8 shows how the Approach Metrics for the heavy and light particles vary with time. It is clearly seen that light particles reaching the equilibrium much faster than the heavy particles–720 sec (6minutes), compared to 120 sec. In both cases, the trend towards an equilibrium conditions is clearly established and seems to be exponential in nature. The change in Approach Metric with time is seen to be a convenient way to simplify the description of the stratification kinetics; the whole system can be described in a single plot which appears to be exponential in form.

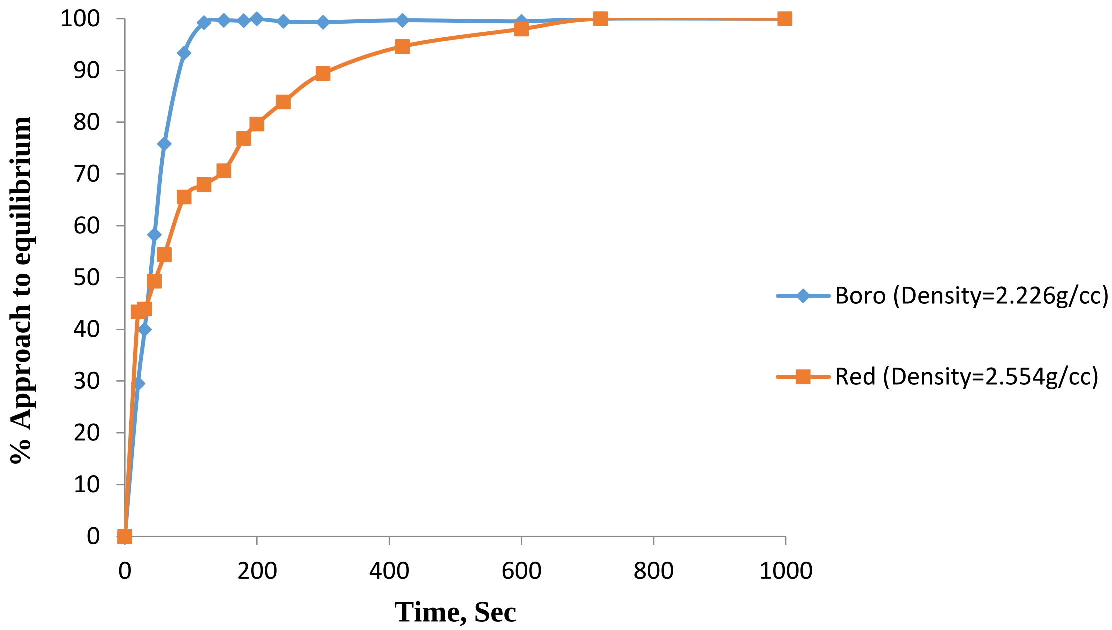


Figure 5.8 The kinetic effect comparison between heavy and light particles

5.5 Effect of operating variables on stratification kinetics

Table 5.1 summarizes the kinetic tests that were conducted to investigate the effect of operating conditions on the kinetics of stratification. The results and kinetic plots are presented in Appendix B The plots of the associated Approach Metric vs time are presented and discussed in this section to show how the operating conditions affected stratification kinetics. The raw results are presented in this chapter and discussed qualitatively. The effects of the operating conditions are analysed in more depth and quantitatively in Chapter 6.

5.5.1 Effect of stroke

To investigate the effect of stroke on the stratificationkinetics, the stroke was varied from 34% to 48%.The other operating conditions were kept the same as for the base case, i.e. T1=22cS, T2=28cS, T3=25cS, T4=58cS and pressure=64 psi.

Figure 5.9 and Figure 5.10 shows the effect of stroke on the kinetics for light and heavy particles respectively expressed in terms of the Approach to Equilibrium Metric. It is clearly seen from the figures that the stroke has a strong influence on the kinetics of both light and heavy particles. The greater the stroke, the faster the stratification approaches the equilibrium profile. Figure 5.11 illustrates this more dramatically by comparing the time it takes for each component to reach 90% of the equilibrium condition when the stroke varies over the range from 34% and 48%(equivalent to a water displace from 36 mm to 54 mm, i.e. from 40 to 60 % of the depth of the bed). The figure shows that when the stroke is increased from 34% to 48%, the time taken for the light particles (boro) to reach 90% of equilibrium decreased from 115 to 70 seconds. The time taken for the heavy particles (red) decreased from 420 to 150 seconds.

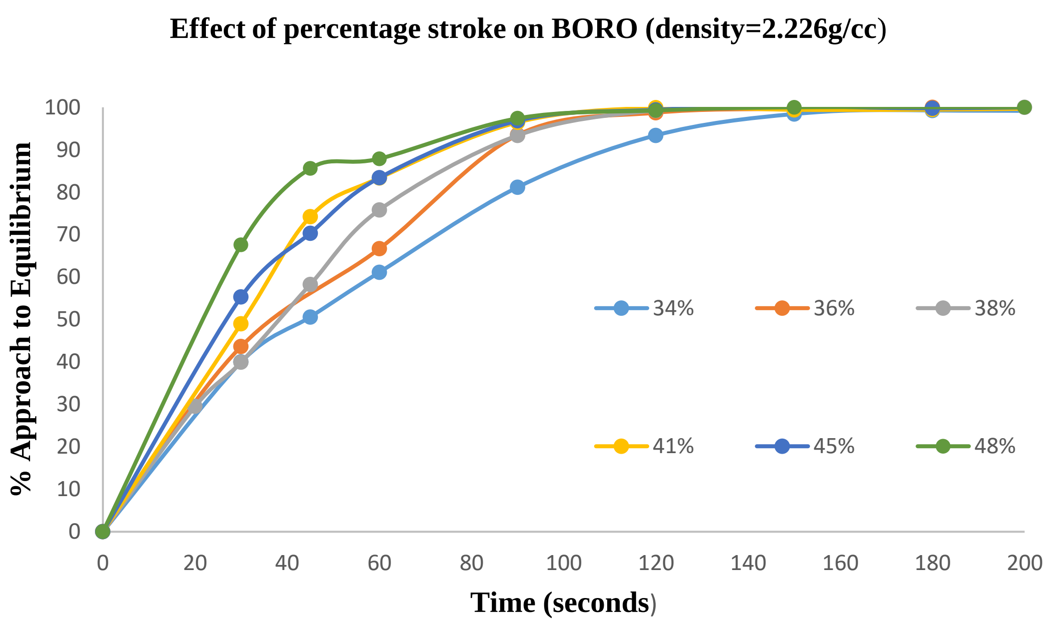


Figure 5.9 Effect of stroke on light particles

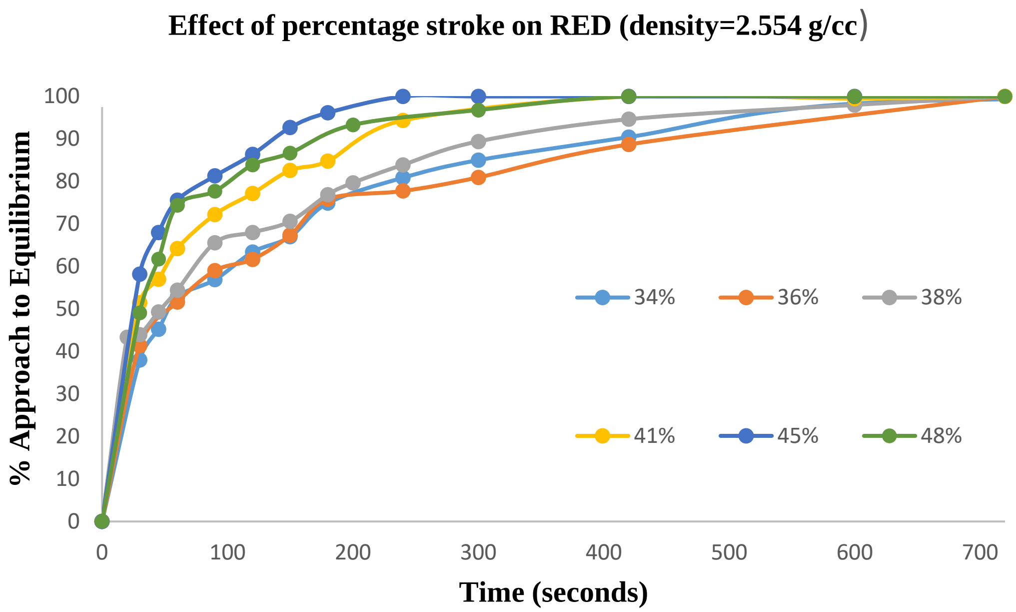


Figure 5.10 effect of stroke on heavy particles

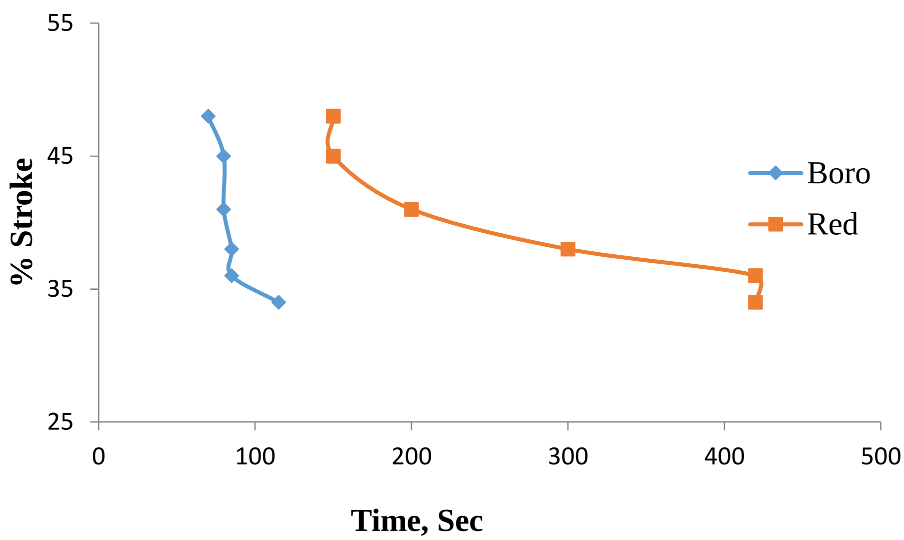


Figure 5.11 Effect of stroke on 90% approach to equilibrium

Looking more closely at Figure 5.9 and 5.10, it can be seen that although the general trend of faster kinetics with increasing percentage stroke is clear, some curves cross over each in places, especially in Figure 5.9 – for the lighter component. This is undoubtedly the effect of experimental errors and suggests that there is a degree of uncertainty in the exact path of each curve that must be taken into account when comparing them. Unfortunately, there is insufficient data available to examine the variance statistically.

Another factor that must be taken into consideration when comparing these kinetic curves is the conclusion reached in Chapter 4 regarding the quality of stratification at equilibrium. A ‘zone of optimum stratification’ was identified for a range of stroke from just below 34% to just above 38%; stratification dynamics with a 30% stroke were not sufficiently vigorous, and a 45% stroke and above was too vigorous for optimum stratification. Over the optimum zone, therefore, it appears, firstly, that increasing the stroke improves the kinetics of stratification of the light component (boro) in a steady and marked manner (Figure 5.9) but, secondly, the effect is less marked for the heavy component (Figure 5.10).

5.5.2 Effect of Pulsion time (T1)

The effectof T1 on the jigging stratification kineticswas studied by varying T1 from 18cS to30 cS while keeping the other operating conditions the same as for base case. (T4 was varied as the T1 values were varied so that the whole cycle time remained constant at 133 cS.) The operating conditions were:T1=varied, T2=28cS, T3=25cS, Pressure=64 psi, Stroke=38%. T4 was varied to maintain T5 at 133cS. Figures5.12and 5.13 shows the effect of T1 on the kinetics for light and heavy particles respectively expressed in terms of the Approach to Equilibrium Metric.

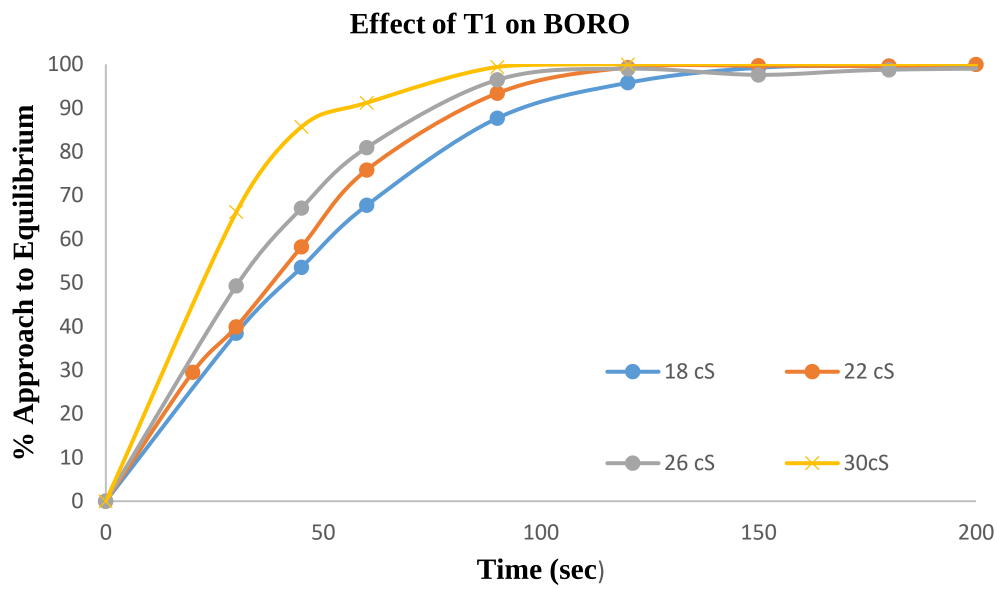


Figure 5.12 The effect of pulsion time on jigging kinetics of the light particles

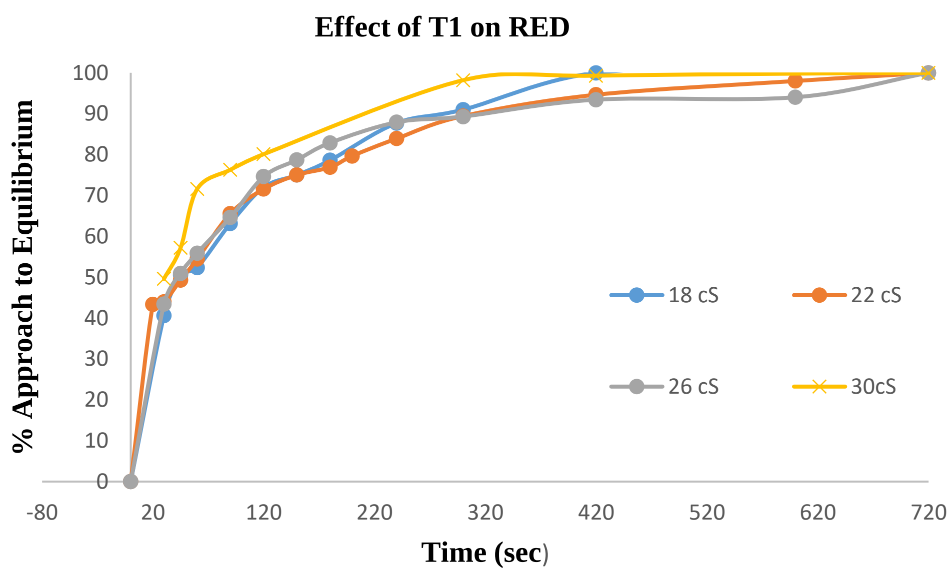


Figure 5.13 The effect of pulsion time on jigging kinetics of the heavy particles

Figure 5.12 and 5.13 shows the effect of pulsion time on the stratification kinetics for the light and heavy particles respectively. A clear trend of increasing kinetics of stratification of the light component (boro) with increase in the pulsion time can be seen in Figure 5.12 – although the curve for the longest pulsion time seems anomalous. The trend, if it exists at all, is very much smaller for the heavy component (red) –and again the 30cS curve appears tobe anomalous. These effects are highlighted in Figure 5.13 which compares the times taken for each component to reach 90% of equilibrium for different pulsion times.

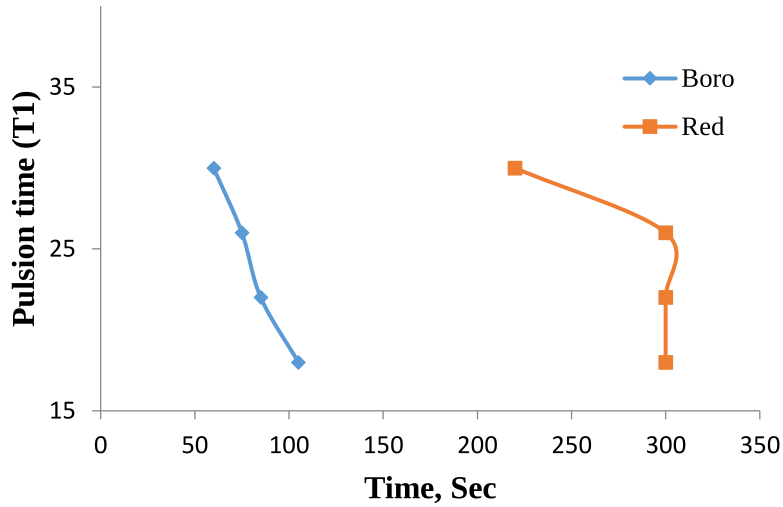


Figure 5.14 Effect of T1 on time taken to reach 90% of equilibrium

5.5.3 Effect of hold time after pulsion (T2)

The effectof T2 on stratification kinetics was studied by varying T2 from 24cS to 38cS with the other operating conditions kept the same as at their base casevalues.The operating conditions were:T1=22cS, T2=Varied, T3=25cS, Pressure=64 psi, Stroke=38%. T4 was varied to maintain T5 at 133cS.Figures 5.15 and 5.16 shows the effect of T2 on the kinetics for light and heavy particles respectively expressed in terms of the Approach to Equilibrium Metric.It can be seen that there is a clear effect of T2 on the kinetics of both the light and heavy particles – though the effect is stronger and more clearly evident with the lighter particles (Figure 5.15). The trend is that if the value of T2 is increased, stratification kinetics are faster.

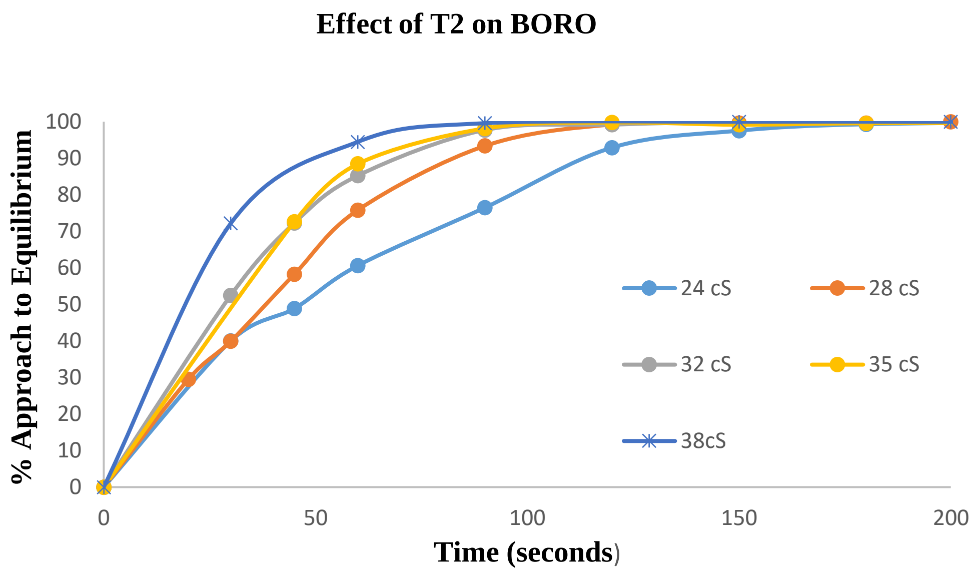


Figure 5.15 The effect of hold time on jigging kinetics of the light particles

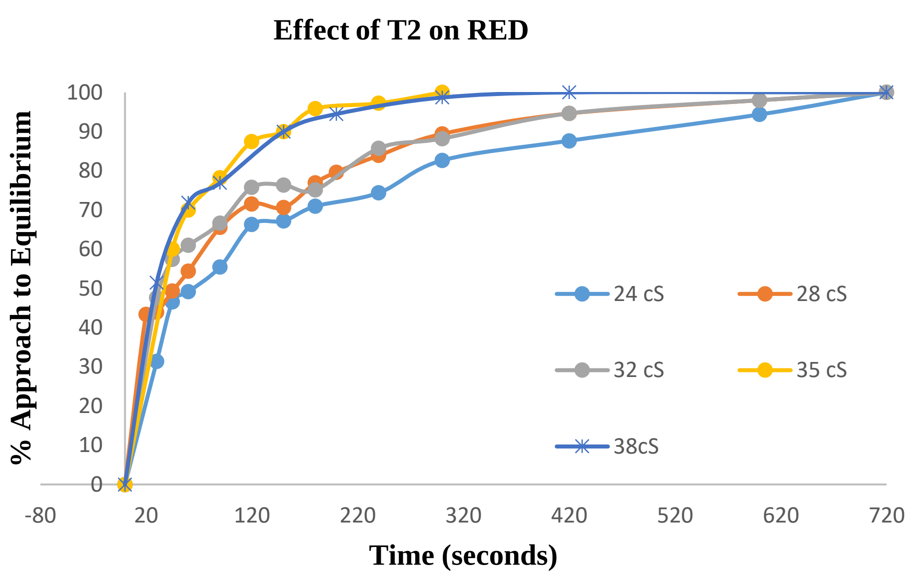


Figure 5.16 The effect of hold time on jigging kinetics of the heavy particles

The trend for the lighter component (boro) appears to be a bit confusing for T2 greater than 32cS because the path of the 35cS curve seems uncertain. Figure 5.15 seems to suggest that the kinetics plateaus off over the range 32 to 35 cS and that 38cS curve is anomalous. Alternatively, if the 35cS curve is intermediate between 35 and 38 cs curves, then there is a steady increase in the kinetics over the whole range of T2 values tested.

With regard to the kinetics of the heavy component (red), stratification does appear to increase with T2, but the effect is smaller and the trend is less well established by the test results. These effects are highlighted in Figure 5.17 which compares the times taken for each component to reach 90% of equilibrium for different T2 times.

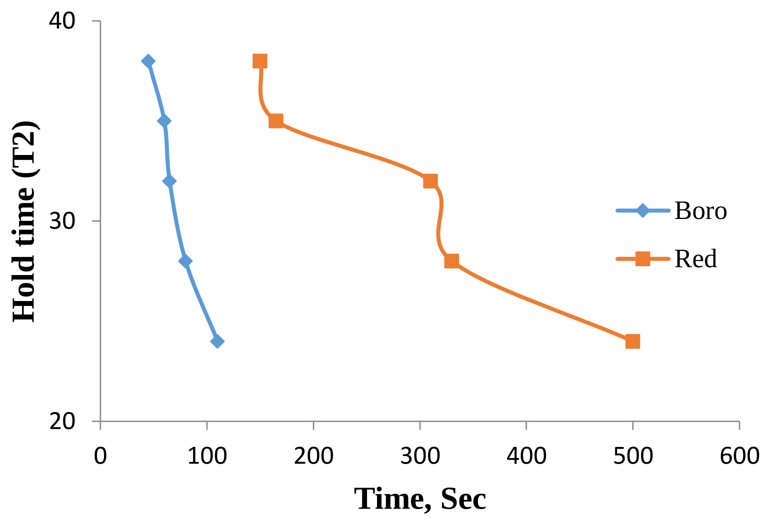


Figure 5.17 Effect of T2 on 90% approach to equilibrium

5.5.4 Effect of Density on the Stratification Kinetics: Binary tests

The effect of density on stratification kinetics was investigated by conducting tests on binary mixtures. This was done for several reasons. First, the effect of density on kinetics is clearly evident in the ternary tests that have been discussed up to this point; stratification kinetics for the lighter component were always faster than for the heavy component because of the relative difference between the density of those components and the density of the intermediate component. The density difference between boro and green beads was 0.237 g/cc and between green and red beads it was 0.091.

The other reasons for using binary systems to test the effect of density on stratification kinetics is that a wider range of densities could be investigated and also that the potentially confusing influence of an intermediate density component on the results could be avoided. Accordingly, tests on five different binary systems were conducted using the four different components as indicated in Table 5.3.

Table 5.3 The Binary Systems Tested

Sample number	Composition of mixtures by colour	Composition by density (g/cc)	Density difference
1	Blue-Boro	2.567-2.226	0.341
2	Red-Boro	2.554-2.226	0.328
3	Green-Boro	2.463-2.226	0.237
4	Blue-Green	2.572-2.463	0.109
5	Red-Green	2.554-2.463	0.091

Figure 5.18a compares the stratification kinetics for these five systems in terms of the Approach to Equilibrium Metric. The very strong dependence of the kinetics on density difference is clearly seen. When the density difference is relatively large, the kinetics are fast and the approach to equilibrium appears to be exponential in form. The kinetics is much slower when the density difference is small and the form of the kinetic curve is more erratic, probably because slower kinetics are more sensitive to experimental error. In addition, the kinetic curve for the system with the largest density difference – 0.341 g/cc (blue-boro) – appears to be anomalous; its kinetics are

slower than expected when compared to the other two kinetic curves in Figure 5.18b. The most probably reason is that the blue beads used are not strictly spherical and each has a small ridge around its equator. (This feature appears to be one of the reasons for the problems with the tests in Investigation I in Chapter 4.)

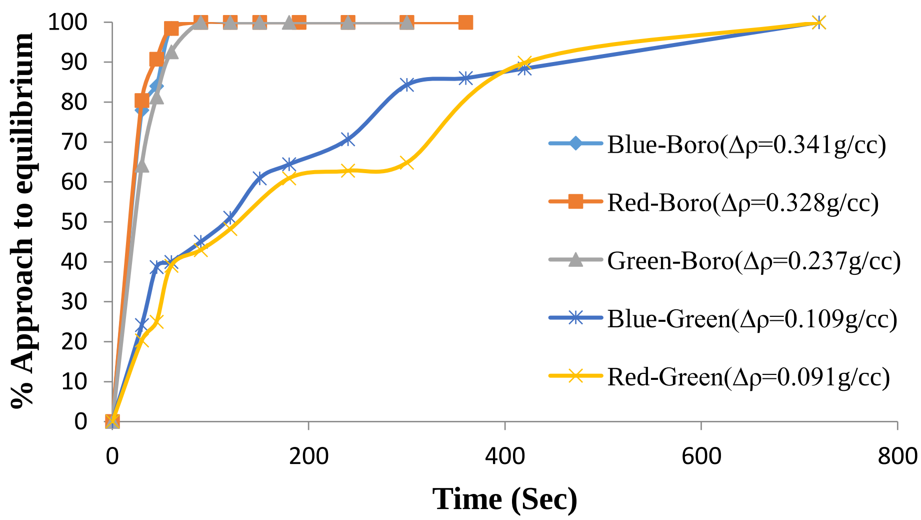


Figure 5.18a The effect of density difference on stratification kinetics

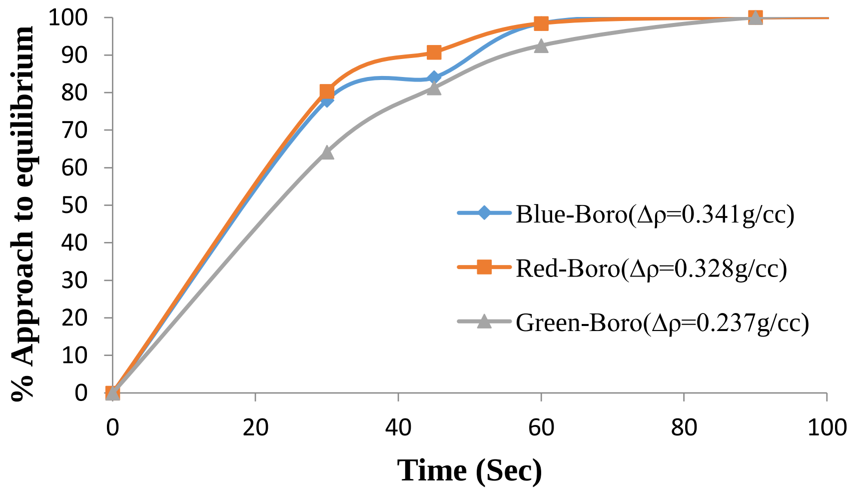


Figure 5.18b The effect of density difference on the stratification kinetics for larger density differences

5.6 Summary: Qualitative Conclusions from the Results of the Kinetic Tests

The kinetic study began with investigating the base case set of operating conditions – i.e. the estimated 'optimum' condition for stratification. The raw kinetic data showed how the concentration of each density component in each of six layer changed with time. To understand the kinetic processes more generically, metric was required that was based on how stratification profiles as a whole changed with time and how that profile shifted from a homogeneous condition to the stratified profile at equilibrium. An 'approach to equilibrium' metric was developed based on area measurements from plots of cumulative recovery of a component to the lower layer when the bed is split at a given height. The change in this metric with time gives an indication of the rate at which stratification approaches the equilibrium condition.

By adopting this area metric approach, the effect of three critical operating conditions on stratification kinetics was investigated. In the ternary system tested, the kinetics of the light components were very significantly faster than the kinetics of the heavy particles. This was because of the density differences of the three components in the jig bed. At the top of the jig bed, stratification involved the segregation of light from intermediate density particles with a relatively large density difference, 0.237 g/cc. However, at the bottom of the jig bed, stratification involved the segregation of intermediate and heavy particles with a much lower density difference – 0.091 g/cc. To explore the obvious effect of density on the stratification kinetics, further tests were conducted on binary systems having a range of density differences. The result of these tests emphasized the very strong effect that density difference has on the kinetics of stratification.

A more quantitative analysis of the kinetic data generated is undertaken in the next chapter.

Chapter 6

6.0 Modeling Stratification Kinetics

6.1 Introduction

The previous chapter presented the results of the kinetic tests carried out and analysed them qualitatively. This chapter extends that work by conducting a quantitative analysis of the results and building a kinetic model of stratification. It therefore addresses research questions 3 (i.e, how can the stratification kinetics be modelled effectively?) as well as completing the analysis related to question 2.

6.2 Quantitative Analysis of the Kinetic Results

To study the rate at which the stratification profile in a jig bed approaches the equilibrium condition, an 'Approach to Equilibrium Metric' was developed by referring to the cumulative recovery vs height h curve after a jigging time of *t*. The metric is defined as the ratio of the area under the stratification curve at time *t* and the area under the equilibrium stratification curve (see Equation 3.5.2). The kinetics of stratification could then be represented by the way the Approach to Equilibrium Metric changed with jigging time. The kinetic curves seemed to be exponential in nature. The way the operating conditions affected stratification kinetics was therefore examined (qualitatively) by comparing the kinetic curves associated with different operating conditions. In this section, those kinetic curves are examined more quantitatively in order to obtain appropriate kinetic metrics for describing the rate at which the stratification profile changes with time.The way in which jigging operating conditions affects stratification kinetics can then be represented and modelled by establishing the relationship between those metrics and the values of the relevant operating variables.

6.2.1 Options for Describing Stratification Kinetics

Figure 6.1 shows typical kinetic curves expressed in terms of the 'Approach to Equilibrium Metric'(it is a copy of Figure 5.8). At time *t*=0 the metric is zero and at equilibrium (t=∞) it is 1 or, expressed as a percentage, it is 100. As can be seen, the general form of both kinetic curves is exponential suggesting the kinetics may be first order or close to first order in nature. If it is first order, the kinetic curve can be described in terms of a time constant, θ, as in Equation 6.1a and b.

$$X_f \;=\; 1 - \exp[-(^t/_\theta)] \qquad [6.1a]$$

$$-\ln(1 - X_f) \;=\; (^t/_\theta) \qquad [6.1b]$$

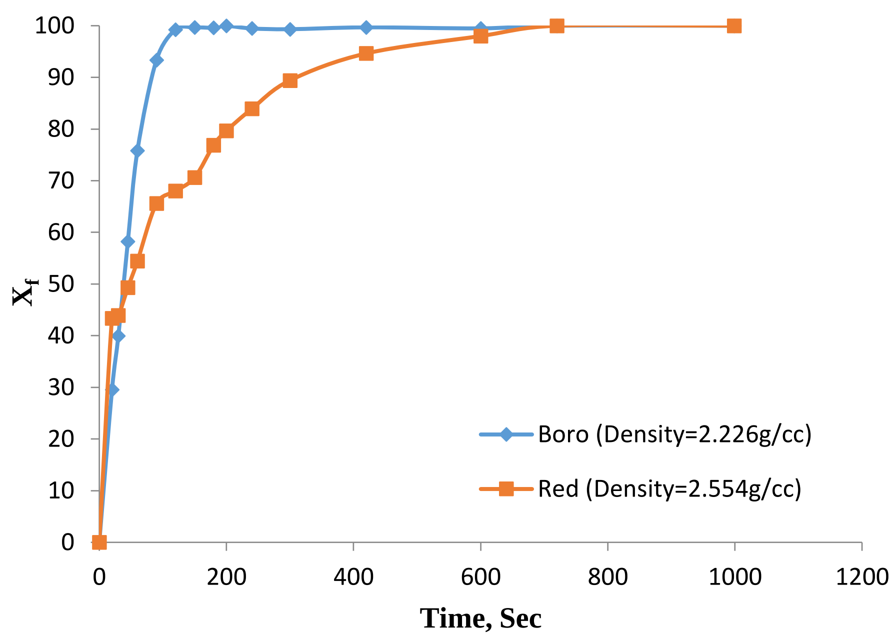


Figure 6.1A typical kinetic curve expressed in terms of the approach to equilibrium metric

In cases such as this one, factors in addition to a first order rate process may influence the shape of the kinetic curve. There may be an initial 'delay time', t_o , that offsets the curve from the starting time *t*=0. This can be described by Equation 6.2a and b.

$$X_f \;=\; 1 - \exp[-\left(\frac{t+t_0}{\theta}\right)] \qquad [6.2a]$$

$$-\ln\left(1 - X_f\right) \;=\; \left(\frac{t_0}{\theta}\right) + \left(\frac{t}{\theta}\right) \qquad [6.2b]$$

Figures 6.2 a and b illustrate the curve shapes associated with the first two of these options.

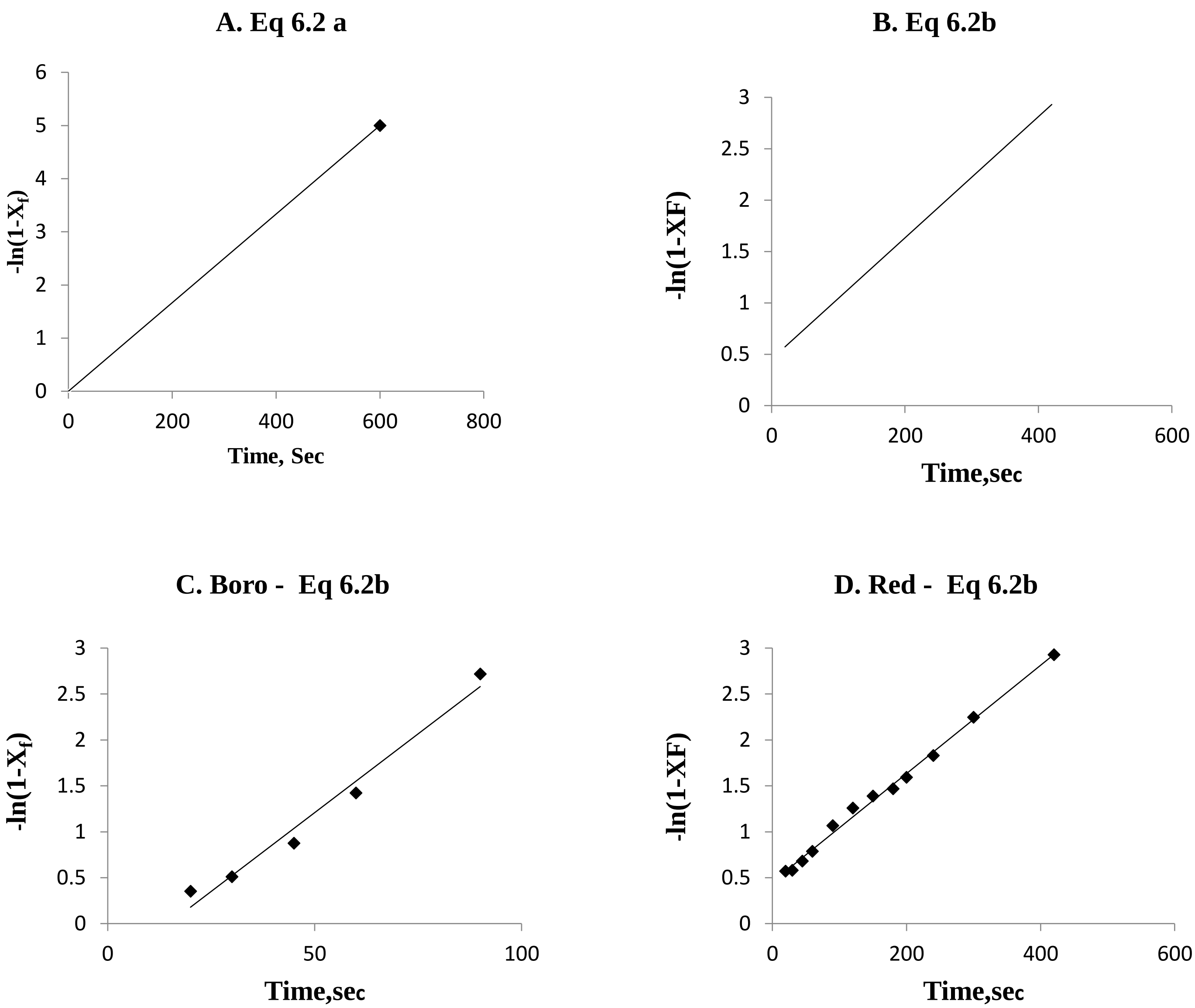


Figure 6.2 Typical kinetic plots for Equations 1b and 2b

The kinetic curve may also have a 'tail' – i.e. its final rate of approach to equilibrium may be slower than is suggested by the initial rate at which the curve changes. This can be described by means of two concurrent rate processes – a fast rate process described in terms of time constant θ_1, and a slower rate process characterized by time constant θ_2 as indicated in Equation 6.3. The parameter *a*in the equation indicates the relative contribution of the faster rate process.

$$X_f = 1 - a \exp\left[-\left({}^{t}/_{\theta_1}\right)\right] - (1 - a) \exp[-\left({}^{t}/_{\theta_2}\right)] \qquad [6.3]$$

6.2.2 Selecting a Model to Describe the Stratification Kinetic Curves

A convenient method for determining which model is most appropriate for describing a kinetic process is to plot the kinetic data in the form of (- ln [1-Xf]) vs time *t*. If the plot is a straight line through the origin, then kinetics is well described by Equation 6.1 and the time constant, θ, is equal to the inverse slope. If the plot is a straight line but offset, then kinetics is described by Equation 6.2a, and the time constant is the inverse slope and the intercept on the Y axis is t_o/θ . If the plot approximates two straight lines, then the kinetics can be described by Equation 6.3, and the two time constants are the inverse of the slopes of the two lines.

Figures 6.2C and D show two typical kinetic curves plotted in the way just described. These illustrate what is found in all the curves from the kinetic tests. It is apparent that Equation 6.2b describe the kinetics quite well. It is interesting to note that the offset, t_o, for the boro curves is positive but it is negative for the heavy component (red). This is found in all the plots, as will be seen. No explanation is offered for this effect.

6.3 The Kinetic Parameters for the Kinetic Data

The values of the kinetic parameters – the time constant Θ and the delay time t_o in Equation 6.2 – for the kinetic data were determined from the log normal kinetic curves of the kind described in the previous section. Two time constants and two delay times were determined for each set of operating conditions – one for the light component (boro) and one for the heavy component (red). The results are presented in Table 6.1. The plots from which the time constants are derived are presented in Figures 6.3 to 6.9. The linearity of all the plots is clear and the associated regression coefficients are all above 0.93, with most well above 0.95.

Table 6.1 Parameters describing the stratification kineticsfor different operating conditions

(The parameters are Θ and t_o in Equation 6.2)

Operating variables*	Θ, Sec(Boro)	Θ,Sec(Red)	t_0, Sec(Boro)	t_0, Sec(Red)
T1=18 cS	37.03	166.66	-15.18	59.49
T1=22cS	29.41	166.66	-14.91	75.49
T1=26cS	22.72	166.66	-18.88	78.49
T1=30cS	22.72	83.33	-4.49	28.33
T2=24cS	45.45	250	-16.22	108.5
T2=28cS	29.41	166.66	-14.91	75.49
T2=32cS	22.72	166.66	8.11	101.82
T2=35cS	15.62	76.92	-26.28	27.46
T2=38cS	14.28	76.92	-14.37	30.99
Stroke=34%	41.66	250	-16.08	104.75
Stroke=36%	29.41	250	-20.26	124.75
Stroke=38%	29.41	166.66	-14.91	75.49
Stroke=41%	22.72	111.1	-15.54	44.44
Stroke=45%	22.22	76.92	-15.78	39.22
Stroke=48%	23.25	76.92	-1.62	37.92

From Binary Tests

	Θ	**t_0**
Density diff=0.341g/cc	12.5	-13.25
Density diff=0.328g/cc	12.98	-10.87
Density diff=0.237g/cc	19.23	11.44
Density diff=0.109g/cc	200	31.8
Density diff=0.091g/cc	250	30.5

* The operating variables for each test were those for the base case except for the variable indicated in the table.

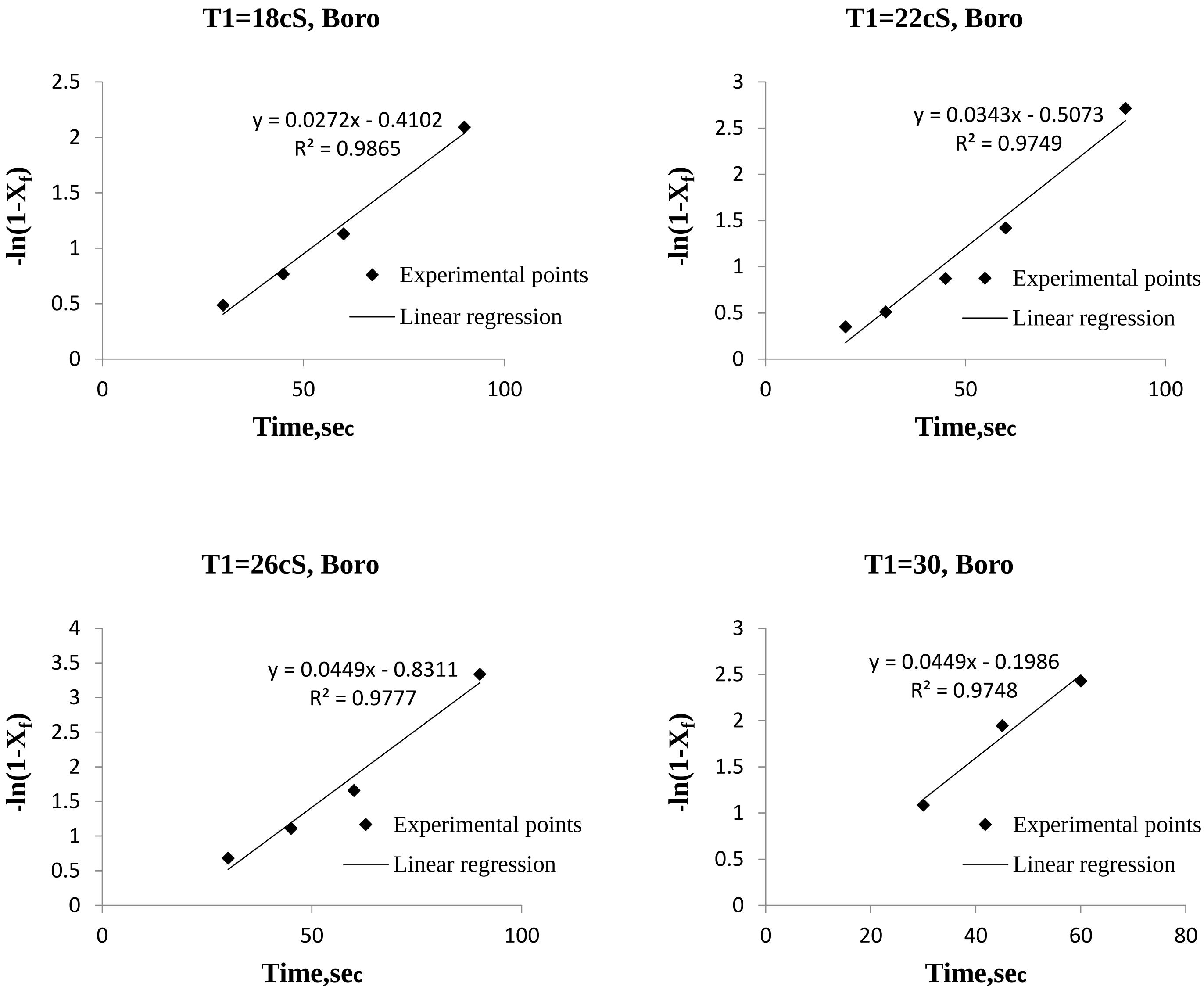


Figure 6.3 Log normal kinetic plots for different hold times (T1):light component (boro)

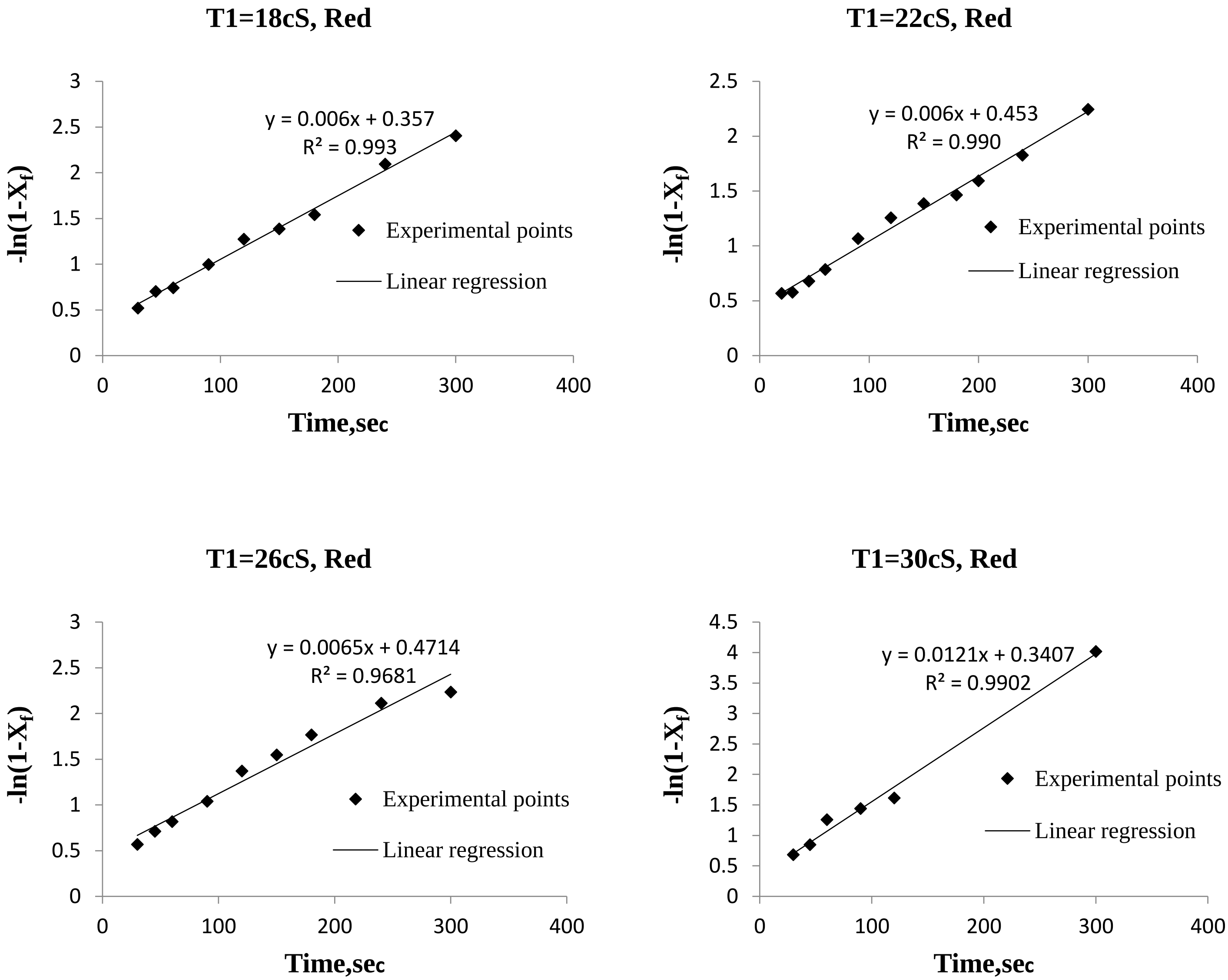


Figure 6.4 Log normal kinetic plots for different hold times (T1): heavy component (red)

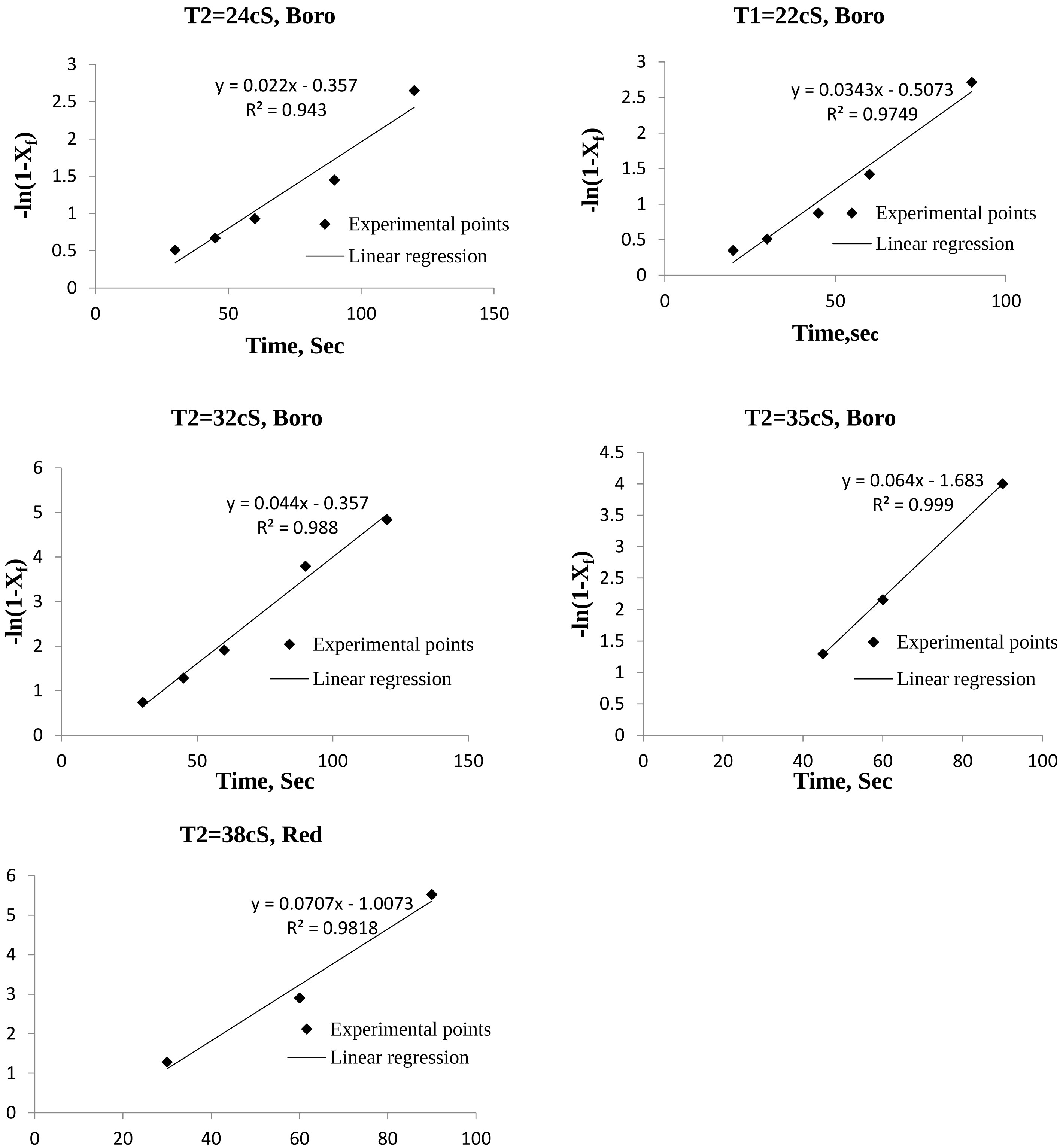


Figure 6.5 Log normal kinetic plots for different hold times (T2):light component (boro)

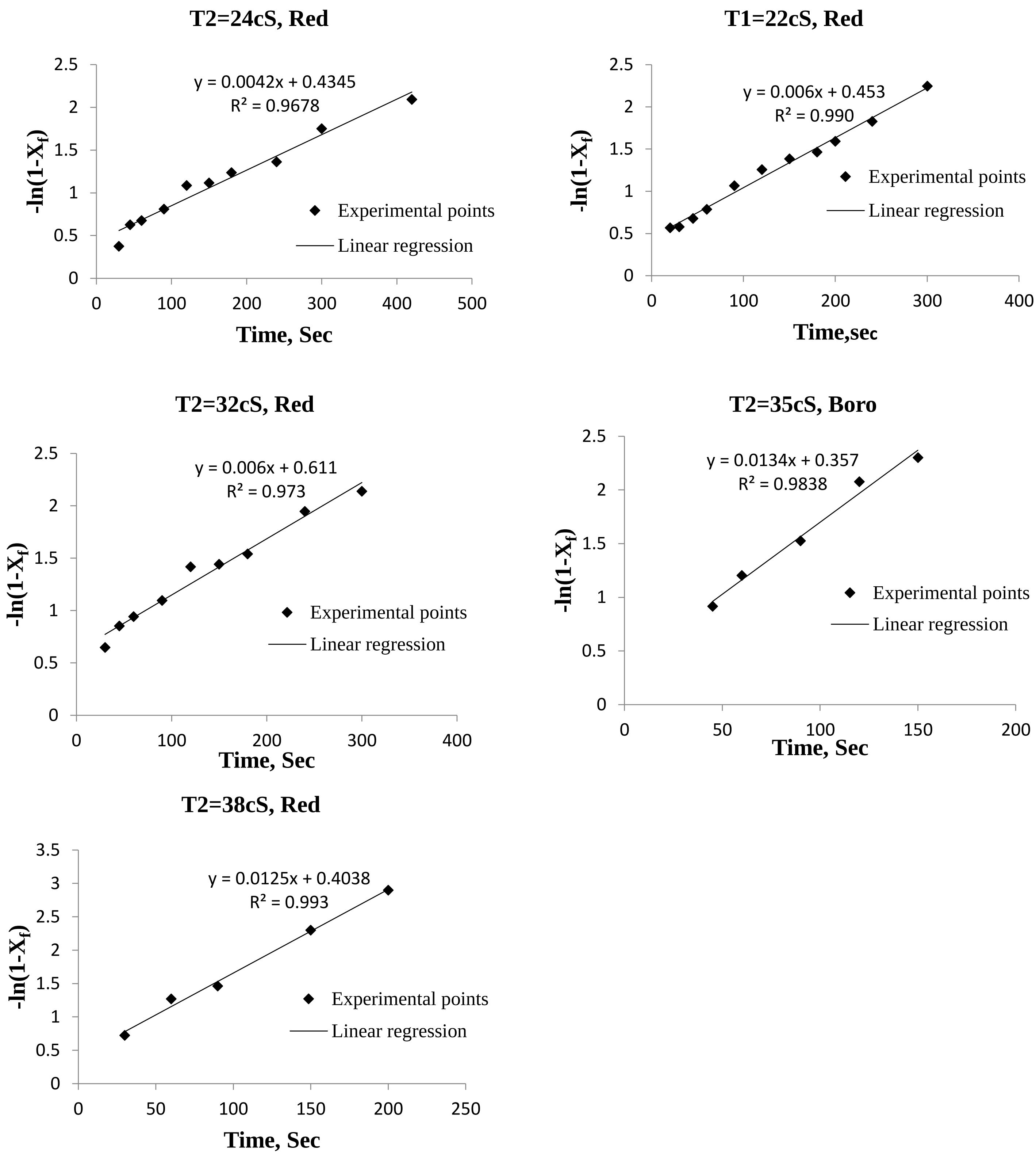


Figure 6.6 Log normal kinetic plots for different hold times (T2):heavy component (red).

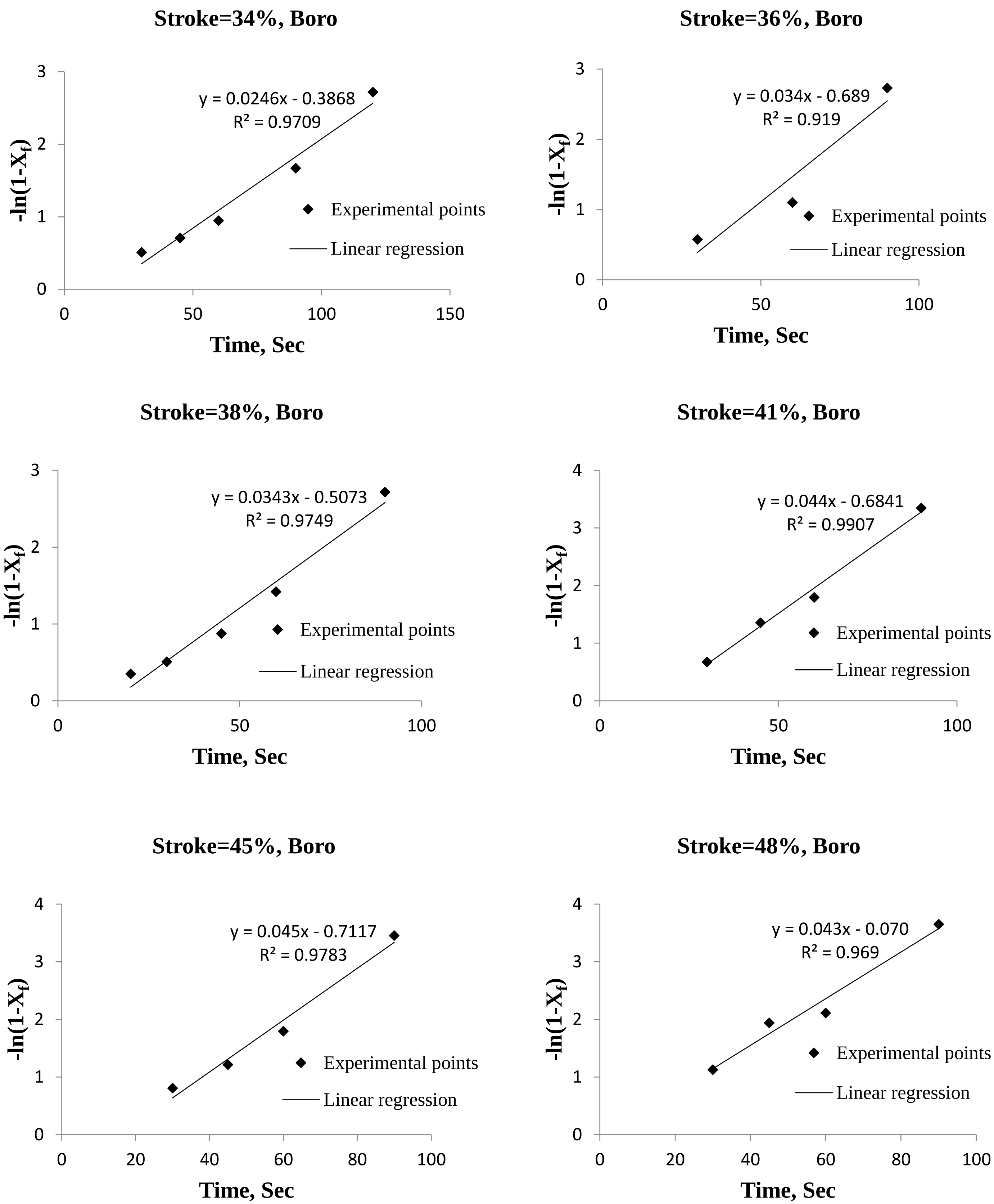


Figure 6.7 Log normal kinetic plots for different %stroke: light component (boro).

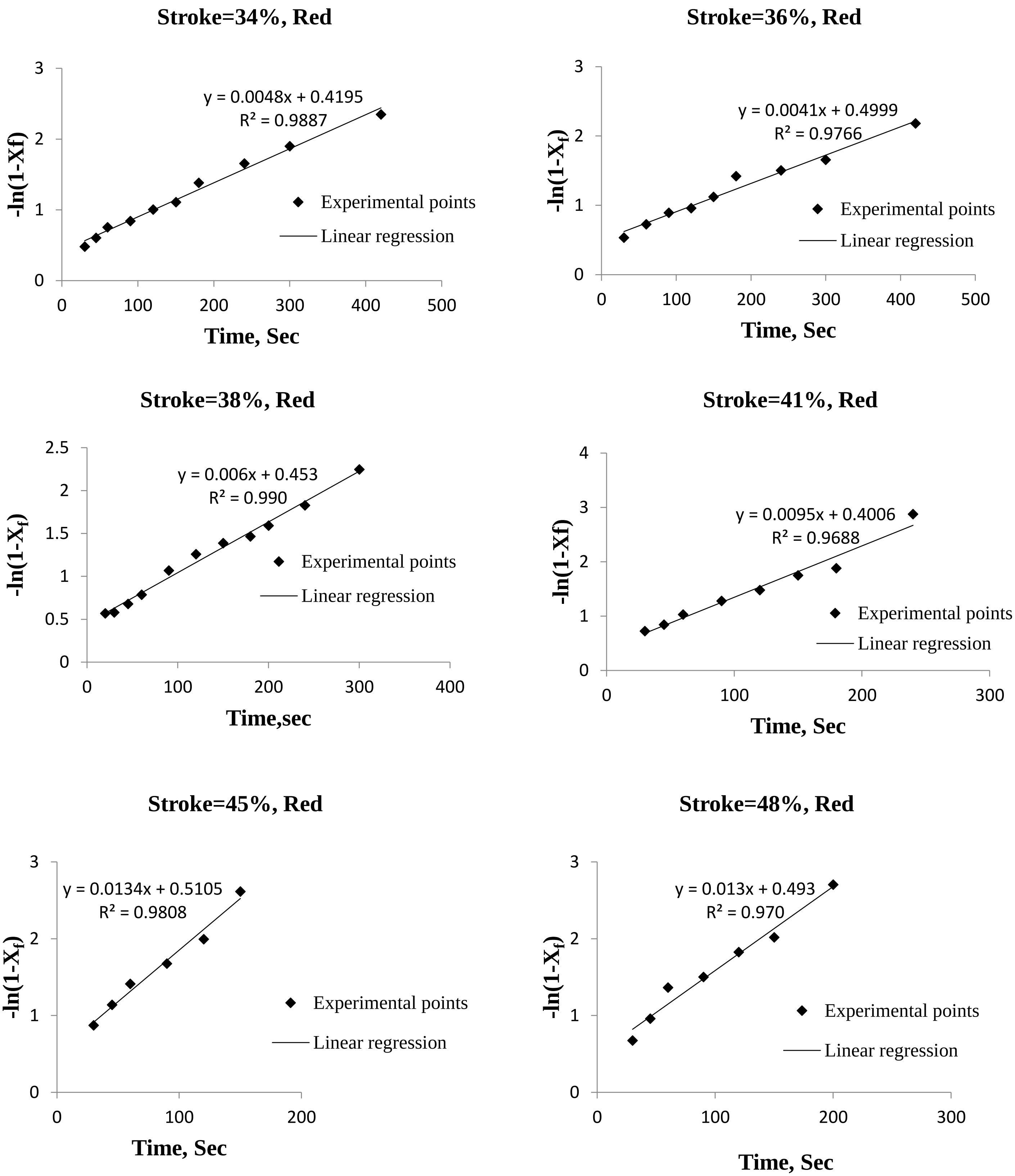


Figure 6.8 Log normal kinetic plots for different %stroke: heavy component (red)

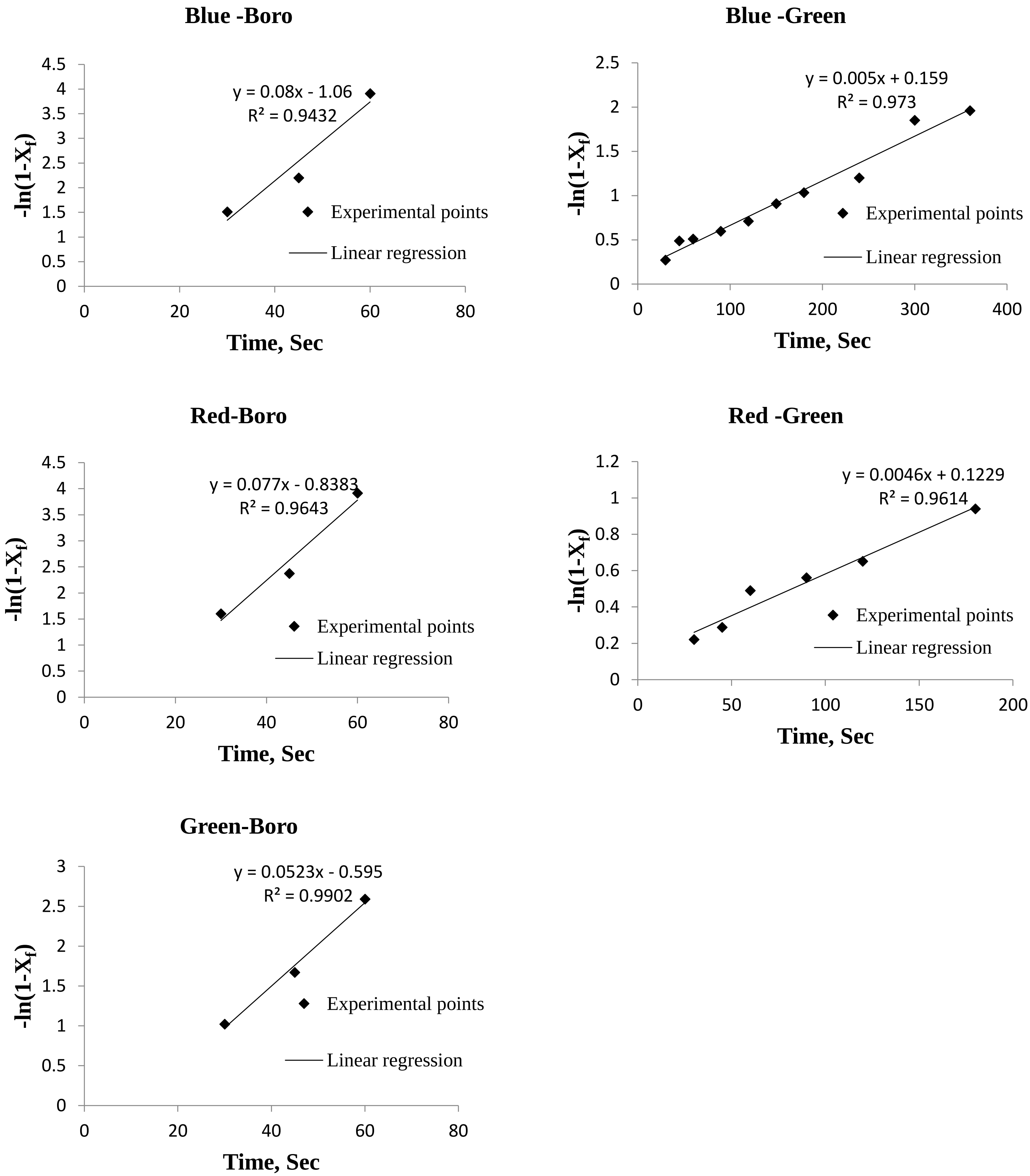


Figure 6.9 Log normal kinetic plots for binary tests with different density differentials

6.4 The Effect of Operating Conditions on Stratification Kinetics

Figures 6.10 to 6.12 show the effect of operating conditions on the stratification kinetics in the kinetic tests. Each figure shows how the variation in one of the operating variables affects the time constant ϴ for the light and heavy density components in the ternarysystems tested. Figure 6.13 does the same for the binary systems tested to investigate the effect of density differences. The actual values have already been presented in Table 6.1.

6.4.1 Effect of T1

Figure 6.10 shows the effect of the pulsion time, T1, on ϴ for both the light and heavy component in the ternary system. It can be seen from the figure that it shows a clear trend of rate of approach to equilibrium for both light and heavy particles. T1 has a strong effect on the kinetics of boro particles i.e, the greater the value of T1, the smaller the value of theta, and the faster the kinetics. But it does not have much effect on the heavy particles.

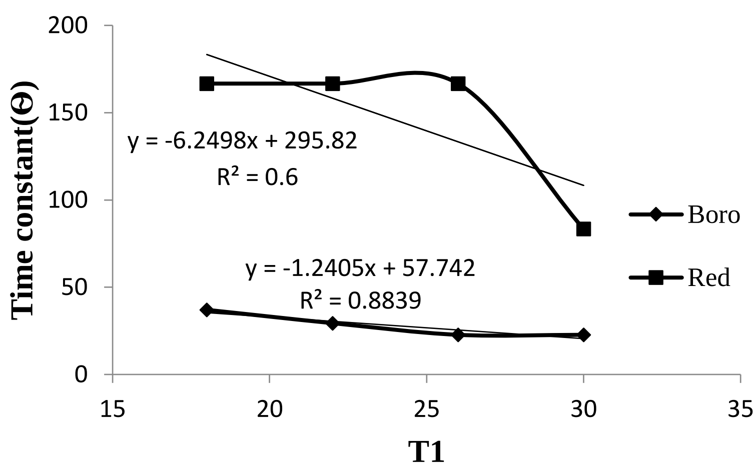


Figure 6.10The effect of T1 on the time constant ϴ.

6.4.2 Effect of T2

Figure 6.11 shows the effect of the pulsion hold time, T2, on Ө for both the light and heavy component in the ternary system. It shows the rate of approach to equilibrium was faster for light particles than for the heavy particles and shows a clear trend with the increase in T2 value. The greater the value of T2, the smaller theta and the faster the kinetics for boro particles. Whereas for red particles the effect is similar but less marked.

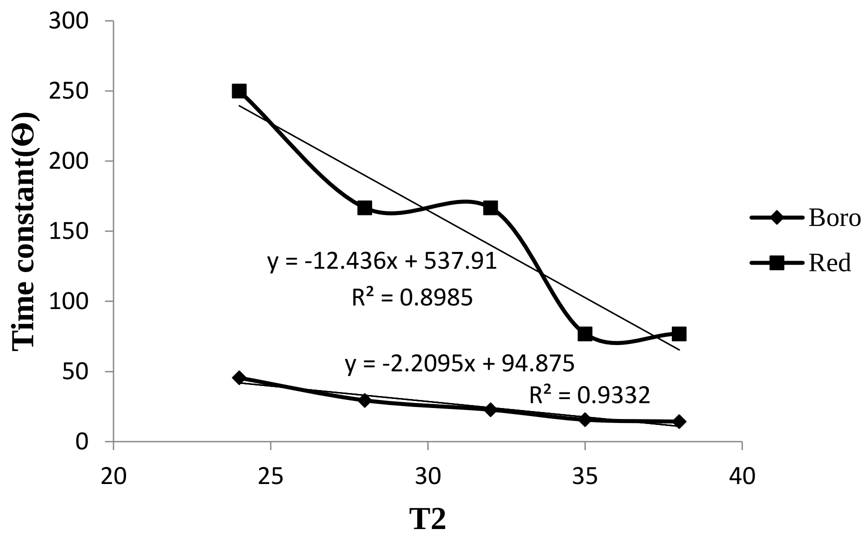


Figure 6.11Effect of T2 on the time constant Ө

6.4.3 Effect of %Stroke

Figure 6.12 shows the effect of the %Stroke on Ө for both the light and heavy component in the ternary system. It can be seen from the figure that the greater the % stroke, the smaller the time constant and the faster the approach to equilibrium for both light and heavy particles. Again the light particles approach equilibrium much faster than the heavy particles.

The optimum zone shown in the diagram was originally defined in Chapter 4 where it was evident that below 34% stroke and at 45% stroke and above, the quality of the equilibrium stratification deteriorated. Figure 6.12 endorses that conclusion but also refines the definition of the upper boundary of the zone to be in the region of a 41% stroke. This conclusion should be tested further.

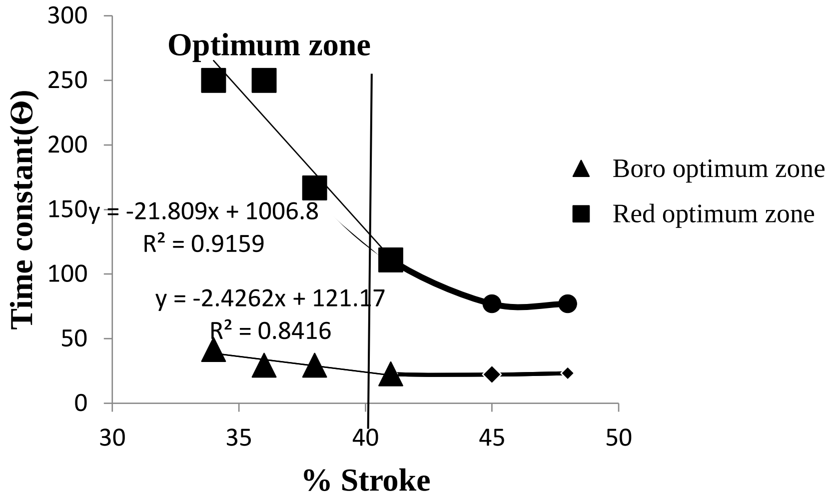


Figure 6.12Effect of %stroke on time constant Θ

6.4.3 Effect of Density Difference

Figure 6.13 shows the effect of the difference in density between the stratifying particles on Θ in binary systems. It can be seen that the greater the difference in density, the smaller theta is and the faster is the approach to equilibrium. The effect is very strong over the range tested.

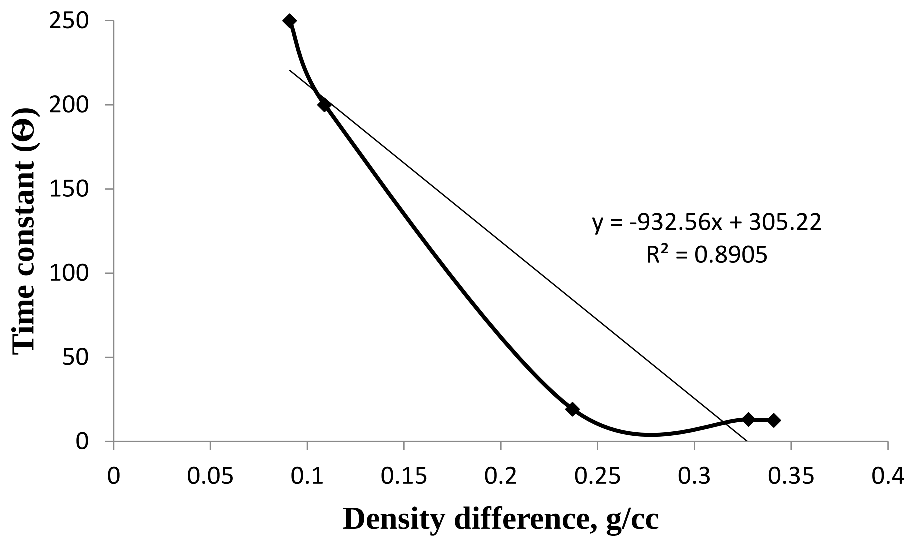


Figure 6.13Effect of density difference on the time constant Θ

6.5 Modelling the Effect of Operating Conditions on the Approach to Equilibrium

The previous section has shown that the effects of operating conditions can be described reasonably well by linear relationships. These relationships are summarized below.

Operating variable	Component	Linear Equation
T1	Boro	Θ= -1.240T1 + 57.74[6.4]
T1	Red	Θ= -6.249T1 + 295.8[6.5]
T2	Boro	Θ= -2.209T2 + 94.87 [6.6]
T2	Red	Θ= -12.43T2 + 537.9[6.7]
%Stroke	Boro	Θ=-2.4%stroke+121.1 [6.8]
% Stroke	Red	Θ=-21.8%stroke+1006 [6.9]
Density difference, $\Delta\rho$		Θ= -932 $\Delta\rho$+305.2 [6.10]

The kinetic testwork program was originally designed on the assumption that the kinetics could not be described in terms of a simple parametric response variable. This meant that, in the tests done, all operating variables were held constant except for the one being investigated. The advantage of this is that the effect of each variable could be investigated and evaluated separately. The disadvantage is that interactive effects could not be investigated. Therefore,a reliable model of the overall, combined influence of the operating variables cannot be developed from the results of the testwork that has been conducted. Further tests would need to be done to investigate these interactive effects. However, it is possible to conduct a multi-variable regression on the data that has been produced to give at least some indication of the relative effects of the operating variables investigated. The regression model that emerges is Equation 6.4. Table 6.2 shows the full results of the regression analysis. Table 6.3 shows the results of the same regression analysis but now in relative terms (i.e. each variable was analysed in the range from +1 to -1).The adjusted R^2 value is 0.86.

$$\Theta = 1323.05 - 5.39\,T1 - 7.57\,T2 - 19.44\%Stroke - 3513\,\Delta\rho + 70.1\,\Delta\rho*\%Stroke \qquad [6.4]$$

(The unit of the terms in the model are as follows: T1 and T2 are in cSeconds, $\Delta\rho$ is in g/cc, and Θ is in seconds.)

Table 6.2Results of the Multi-Variable Regression Analysis

Effect	Model Coefficient	SE	df	t	p-value
Intercept	1323.05	199.61	25.00	6.63	<.0001
T1	-5.39	2.50	25.00	-2.16	0.041
T2	-7.57	1.89	25.00	-4.01	0.0005
Stroke	-19.44	4.55	25.00	-4.27	0.0002
Density_Diff	-3513.47	978.49	25.00	-3.59	0.0014
Stroke*Density_Diff	70.10	25.20	25.00	2.78	0.010

Table 6.3Results of the multi-variable regression analysis showing the relative effect of each term

(The units of all terms in this table are dimensionless and relate to a change from +1 to -1.)

Effect	Model Coefficient	t	p-value
Intercept	56.8	6.14	<.0001
T1	-32.4	-2.15	0.041
T2	-53	-4.01	0.0005
Stroke	-55.6	-4.25	0.0003
Density_Diff	-46.7	-6.80	<.0001
Stroke*Density_Diff	35.8	2.77	0.010

The relative impact of each of the operating conditions on theta can be assessed by referring to the magnitude of the t statistic in Table 6.3 (noting that any negative signs derive from the sign on the coefficient and are irrelevant to this aspect of the analysis). The regression results derive from a variation in the value of each variable in the range from +1 to -1. It is clearly evident that density-difference has the most influence on stratification kinetics. It has a strong individual influence and is also influential in its interaction with %stroke. %stroke is the next most influential variable followed by T2. T1 has a smaller effect and, as with T2, has no discernible interactive influence with either stroke or density difference.

6.6 Modelling the Effect of Operating Conditions on Stratification Kinetics

As discussed in Chapter 3, the original approach to modelling the kinetics of stratification in a batch jig was to decouple the kinetics from the equilibrium. The reason for this approach was that a very adequate, validated model – the King Model – was available for describing and predicting the stratification of particles at equilibrium if those particles differed only with respect to their density. The general form of the model envisaged was

$$\overline{C_i}(h,t) = C_i^{feed} + \{C_i^{equil}(h) - C_i^{feed}\} X_f(t) \qquad [6.12]$$

for i = 1 to number of particle components.

Here, C_i^{feed} is the uniform concentration of density component i in the bed before jigging begins. $\overline{C_i}(h,t)$ is the concentration of that component in either the upper or lower part of the bed when it is split at a height h. This is the cumulative concentration profile in the bed after a jigging time of t. This profile changes with time until the cumulative concentration reaches an equilibrium condition, C_i^{equil}. The kinetics of the change in the concentration profile is represented by the 'Approach to Equilibrium Metric', X_f, which indicates how close to equilibrium the concentration profile is after jigging time t.

The kinetic model that was found to describe the approach to equilibrium reasonably well was expressed in two forms – either Equation 6.2a or 6b and was based on the Approach to Equilibrium Metric, X_f, defined in Equation 5.1 on the basis of the cumulative recovery of component i, $\overline{R_i}(h,t)$, to either the upper or lower layer when the bed was split at a height h. The Approach metric was originally defined to describe the stratification profile in the bed as a whole, i.e. in Equation 5.1 as follows

$$X_f(t) = \frac{Area\ at\ time\ t\ (relative\ to\ homogeneity)}{Area\ at\ equilibrium\ (relative\ to\ homogeneity)} * 100 \quad .$$

To develop this metric into a more convenient form, it is first assumed that the approach to equilibrium in any layer up to (or down to) h is the same. If so, the approach to equilibrium in the layer will be

$$X_f = \frac{R_{i(t,h)} - h}{R_{i(\infty,h)} - h} = 1 - e^{\frac{-(t+t_0)}{\theta}} [6.13]$$

$$R_{i(t,h)} = h + (R_{i(\infty,h)} - h)(1 - e^{-\frac{t+t_0}{\theta}}) \qquad [6.14]$$

Ri is related to Ci as follows

$$R_{i(t,h)} = \frac{Volume\ of\ i\ in\ layer\ h}{Volume\ of\ i\ in\ bed}$$

$$= \frac{h}{C_i^{feed}} C_{i(t,h)} \quad [6.15]$$

Putting equation 6.15 into equation 6.13 and rearranging, the model becomes as follow,

$$C_{i(t,h)} = C_i^{feed} + \left(C_{i(\infty,h)} - C_i^{feed}\right)(1 - e^{-\frac{t+t_0}{\theta}}) \quad [6.16]$$

The raw kinetic data was reworked to calculate the stratification kinetics in terms of C_i – seeappendix B. To inspect the degree to which the model predicted the change in $C_i$with time at different heights, a comparison was made between the data for the base case and the predictionsof the model taking the experimentally determined values of theta and t_o . This is shown in Figure 6.14 and 6.15 for light and heavy particles respectively.

As can be seen, the fit is not very good; it under predicts in some places and over predicts in others. However, the assumption that the kinetics in each layer are the same is clearly not valid. Accordingly, the model developed – Equation 6.2 – applies well to the stratification profile as a whole, but less reliably as Equations 6.14 or 6.16 to the kinetics in any particular layer *h*.

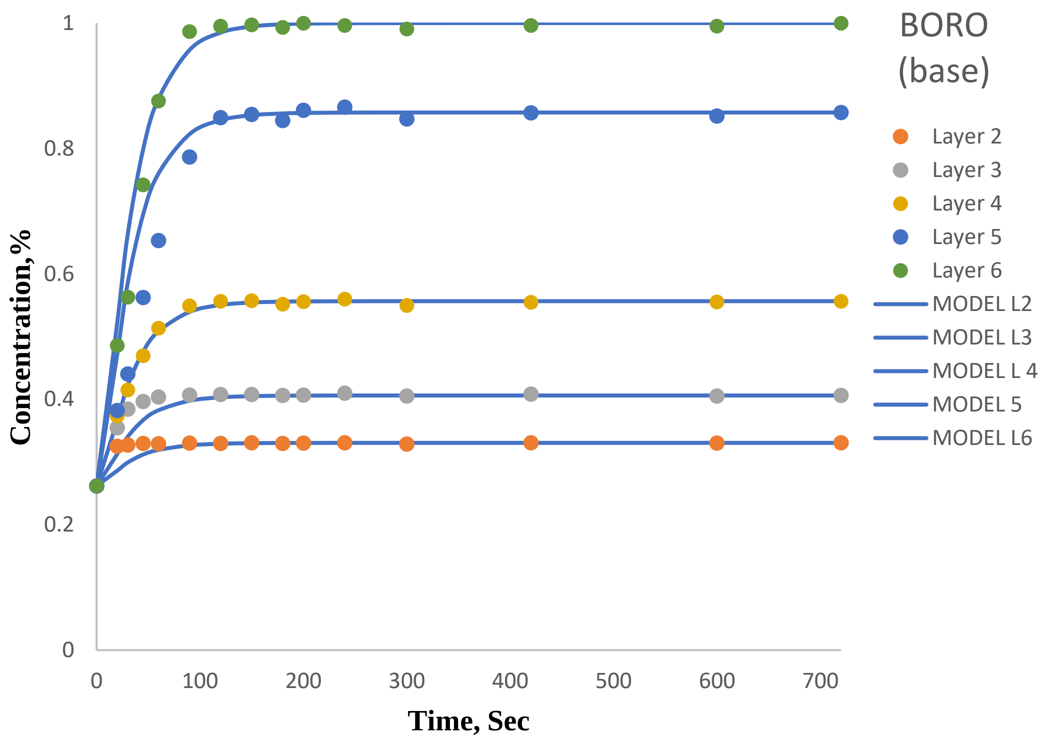


Figure 6.14 Changes in the concentrations of the light(boro) particles in each layer with time

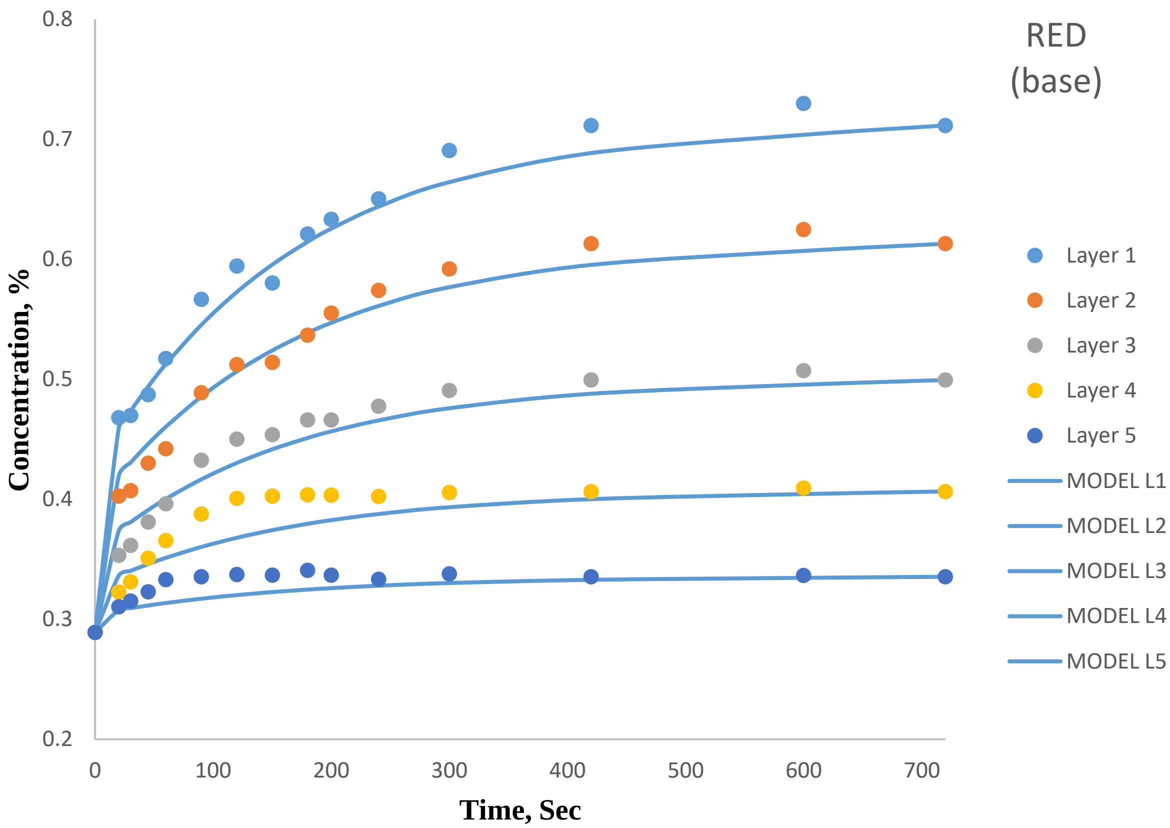


Figure 6.15 Changes in the concentrations of the heavy(red) particlesin each layer with time

6.7 Discussion and Summary

Chapter 4 showed that the equilibrium stratification profile was essentially independent of the operating conditions provided the depth of the particle bed was below 100mm or so, and the stroke (the amplitude of the jig cycle) was in an optimum range – i.e. not too mild and not too vigorous. In the batch jig tested, this range was found to be about 34 to 41% of the full stroke of the drive piston. (The range indicated in Chapter 4 was just below 34% to somewhere above 38%. The kinetic work in Chapter 5 suggested that the upper bound of the 'optimum zone for stratification' was just above 41% Stroke.) This meant that the main influence of operating conditions on stratification was on the kinetics.

Chapter 5 investigated the kinetics of stratification with respect to the metric X_f. The results of the investigation were analysed quantitatively in this chapter. It was found that X_f can be adequately described as a first order rate process in terms of a time constant Θ and a delay time of t_o.Equations 6.13, 6.14 and 6.16 express the final stratification model that has been developed.

$$\overline{C_\iota}(h,t) = C_i^F + \left(\overline{C_\iota^E} - C_i^F\right) e^{\frac{-t}{\theta}} \qquad [6.17]$$

for i = 1 to

Ө =1323.05 -5.39 T1 -7.57 T2-19.44%Stroke-3513 Δρ +70.1 Δρ*%Stroke [6.18]

Here, Ө is the time constant in seconds, and T1, T2, are respectively the pulsion and pulsion hold times (in cSec), Δρ is the density difference (in g/cc).

$$\overline{C_N}(h,t) = 1 - \sum_{j=1}^{N-1} \overline{C_J} \ ; \ \ N \leq 3 \qquad [6.19]$$

The model focuses on the kinetics of the light and heavy density components. The kinetics of a third component can be determined by difference (Equation 6.20). However, this does not apply if there are two or more intermediate components. Therefore the model applies only to binary and ternary systems.

Chapter 7

Discussion and Conclusion

7.1 Introduction

The aim of this study was to investigate the effect of operating conditions on the kinetics of density stratification in a batch jig. Two questions framed the core of the study: how do operating conditions affect (1) stratification at equilibrium and (2) the kinetics of stratification? An initial investigation focused on the question of which operating conditions should be studied. The study concluded with an attempt to develop a model from the findings that described quantitatively the effect of operating conditions on stratification. All tests were conducted in the Wits batch jig under different operating conditions using multi component systems of 8mm spherical glass beads of different density and colour. As the study progressed, the experimental design was modified to adapt to the findings that had emerged.

7.2 Discussion of Findings I: What operating variables affect stratification?

From the literature survey in Chapter 2 it is clear that many variables affect the performance of a jig. **Burt (1984)** and **Mehrotra*et al.* (1997)** summarize the key variables as those that affect the amplitude, frequency and shape of the jig cycle.Several workers reported on the effect of different jig cycle patterns (**Burt, 1984; Viduka et al., 2012 and 2013; Jinnouchi, 1984**). However, many of these do not relate to the trapezoid pattern commonly employed in large industrial jigs that process coal and iron ore. That pattern—consisting of a pulsion and suction stroke with a holding time between each—has been described by **Myburgh (2010 and 2013)** and this is the pattern that was investigated in this study. However, it appears that little detailed investigation has been conducted with regard to how the relative lengths of these different phases in the jig cycle affect jigging performance. The findings in this study therefore help to fill that gap.

In the Wits jig used in this study, the amplitude of the jig cycle—the maximum water displacement in the jig chamber—was manipulated by two variables: the %stroke which controlled the extent of movement of the drive piston, and the applied pressure which affected the rate at which the piston moved. Accordingly, there were 6control variables that potentially could affect stratification in the jig: %stroke and pressure; the four time settings T1, T2, T3 and either T4 or T5 that controlled the shape of the jig cycle and the frequency of the jig cycle. Of these, all but T1, T2, and %stroke were eliminated from the study on the basis that they either did not have a significant effect on stratification (T3); or their values could be fixed so that they did not exert any significant influence (frequency and pressure). Frequency was eliminated as a factor because, as argued and shown by **Gupta and Yan (2006)**,any cycle time longer than a minimum time negatively affects the capacity of the jig, since no segregation takes place when the jig beds in a consolidated state.Accordingly, the cycle frequency was fixed at an abnormally low value—0.45 cycles per minute (133 sec per cycle)—so that the values of the pulsion time, T1, and pulsion hold time, T2, could be varied without influencing the cycle frequency. (The value of the suction hold time, T4, was adjusted to keep the overall cycle frequency at that value.)Pressure was eliminated on the basis that the variation in the cycle amplitude—the maximum water displacement—could be achieved by varying the %stroke at a fixed applied pressure.

Two external variables that affect stratification were also included in the study: the depth (or height) of the particle bed in the jig; and the differences in the density of the particles. Other variables that could affect stratification were not included in the study included the maximum velocity of the water displacement (**Mukharjee and Mishra, 2006)**, the maximum bed displacement, and the composition of the material being jigged. These were not considered in the investigation because they were either response variables (water velocity) or variables external to the jig operating variables. However, their influences on both the kinetic and equilibrium aspects of stratification are interesting and should be investigated in the future.

7.3 Discussion of Findings II: What effect do the operating variables have on stratification at equilibrium?

The primary motivation for the study was to enhance the predictive ability of King's stratification model. Its ability to predict or describe density stratification has been well validated (**King, 1987, 2001; Tavares et al., 1995; and Woollacott et al., 2015**) for a variety of contexts in which all the particles in a jig bed had the same properties except for density. As indicated in the review of the model in Chapter 2, the model is an equilibrium model – it describes the stratification profile in the bed only after it has reached an equilibrium state – and it does so in terms of the King stratification index, $\boldsymbol{\alpha}$, Equation [7.1].

$$\alpha = \frac{ugV_pH_{bed}}{D} \qquad 7.1$$

According to this formulation, the stratification index is a function of the bed depth, $\boldsymbol{H_{bed}}$, particle volume, $\boldsymbol{V_p}$, (which is proportional to the cube of particle size), and the ratio of the specific penetration velocity, $\boldsymbol{u}$, and the diffusion coefficient, $\boldsymbol{D}$. According to King, the first of these terms, $\boldsymbol{u}$, has to do with the mobility of particles under the forces that drive stratification, while the second term, $\boldsymbol{D}$, has to do with the diffusive mobility of particles driven by differences in the concentrations of particles that result from stratification. Both terms, according to King, should depend on particle size and shape, and also on the nature of the jigging conditions in the jig bed. However, almost no work has been done to establish the effect of all these variables on the stratification index, and hence on the ability of the King model to describe or predict the equilibrium stratification profile under a typical range of operating conditions. Apart from some experimental work on the size dependence of $\boldsymbol{\alpha}$ (**Woollacott, 2018)**, no experimental work has been undertaken to investigate how any of these operating variables affect $\boldsymbol{\alpha}$ or stratification at equilibrium in a batch jig. (In addition, as discussed in the next section, virtually no work has been done to relate the King model to stratification kinetics.)

In regard to the factors affecting alpha and equilibrium stratification, the results of this study are surprising. Firstly, the King model did not always describe the equilibrium stratification as well as previous validation work suggested it should, particularly in the preliminary testwork with a four component system. The fits to experimental data were better after one particle component – which consisted of off-shape beads – were removed from the sample tested. However, the fits were still not good enough to conduct a reliable experimental program to determine the effect of the key operating variables on the value of $\boldsymbol{\alpha}$.

The second surprise, was that in the subsequent testwork the operating conditions appeared to have very little influence on the equilibrium stratification profile.(That testwork examined the effect of operating conditions on the equilibrium stratification profiles instead of their effect on $\boldsymbol{\alpha}$). It was found that neither T1, T2, cycle amplitude (%stroke) or bed depth had any effect at all on the nature of the stratification profile, provided the bed depth was not excessive, and the stroke was in an appropriate range – neither too vigorous not too small. Provided the operating conditions remained within this ‘optimum range’, the equilibrium stratification profile was always the same, whatever the operating conditions were. Outside of that ‘optimum range’, the quality of stratification achieved at equilibrium deteriorated – i.e. the equilibrium stratification profile was not as sharp, so that lower recoveries and poorer concentrations of the different particle components were obtained when the bed was split at any given height.

The practical implications of these findings are significant. Firstly, the effect of operating conditions on jigging performance must be a kinetic effect – they must effect stratification kinetics because their effect on equilibrium profiles is negligible if the jig is set up in the 'optimum range' of operating conditions so that the best equilibrium stratification profile is obtained. In effect, different operating conditions may influence the rate at which stratification approaches the equilibrium condition, but ultimately the same stratification profile is achieved at equilibrium whatever the kinetics of stratification are. If the operating conditions are outside the 'optimum range', then poorer jig performance is to be expected whatever beneficial effect those conditions may have on improving the kinetics of stratification.

With regard to the definition of the 'optimum range' of operating conditions, the following remarks can be made. Firstly, that range must be established for the specific jigging context. In the context of the 200mm diameter Wits jig with its specific dynamics when jigging 8mm glass spheres, the conditions were found to be a bed depth of about 100mm or less, and a stroke setting of between 34 and 41%stroke. This is equivalent to a maximum water displacement of between about 40 and 50mm, or between about 40 and 50% of a bed depth of 100mm. This is compatible with typical plant practice which recommends an amplitude of at least three times the top size of the particles being treated (**Myburgh, 2010 and 2013)** – i.e. a 24mm amplitude in this case. The maximum bed depth for optimum equilibrium stratification, i.e. it was found to 100mm for the 8mm beads, is somewhat lower than typical plant practice.Myburgh indicates that bed depths of 250 to 500mm are more typical of large iron ore jigs. The findings of this study suggest that that practice may result in poorer stratification than it is possible to achieve, although this conclusion must be qualified by the fact that the findings of this study derive from a small batch jig as compared to practice in large continuous jigs.

The second remark to make concerns the findings with regard to the shape of the equilibrium stratification profile when the bed depth starts to exceed the optimum. It is apparent from the profiles in Figures 4.8 that with an excessive bed depth (150 mm in this case) the profiles for the lower layers deteriorate significantly – i.e. they 'lose their shape' and deviate significantly from the sharper stratification that can be achieved with shallower beds.

A third remark about the findings of this study relates to the implications for modelling jig performance. If the stratification index $\boldsymbol{\alpha}$ is relatively independent of operating conditions within the 'optimum range', then the King's formulation of the factors affecting the index are misleading. Aside from the size dependence of $\boldsymbol{\alpha}$, about which the results from this testwork can say little, the relationship to bed depth seems wrong – $\boldsymbol{\alpha}$ appears to be independent of bed depth in the 'optimum zone'. In addition, $\boldsymbol{\alpha}$ seems to be independent of the ratio $\boldsymbol{u/D}$. This implies that the influence of operating conditions on $\boldsymbol{u}$ and$\boldsymbol{D}$ is identical.

7.4 Discussion of Findings III: What effect do the operating variables have on stratification kinetics?

Several authors have reported on stratification kinetics in jigs based on the assumption or observation that the jigging time strongly affects bed stratification. These authors therefore attempted to express stratification as a function of jigging time. **Lin et al. (1997)**, **Mehrotra et al. (1997)** and **Rong et al. (1993)** proposed a power function equation to relate stratification indices to the jigging time because they argued that stratification is strongly affected by the parameters that affect water behavior in a jig.None of these workers attempted to decouple kinetic effects from equilibrium effects as was done in this study.

The kinetic data generated in this study showed how the concentration of each density component in each of six layer in the bed changed with time. It also showed that, in the ternary system tested, light particles reached equilibrium faster, whereas heavy particles took longer times to attain equilibrium. In order to express the results in a more generic manner, and also to relate kinetics to the equilibrium stratification profile, an 'Approach to Equilibrium' metric, X_f, was developed. The metric aimed to take into account how the stratification in the bed as a whole changed from that associated with a homogeneous bed to that associated with the stratified bed at equilibrium. This approach was novel in studies on stratification kinetics, and was a point of departure from the more usual approach of developing a dynamic mass or volume balance around a differentially small element in the kinetic system being studied.

The decision to take this approach proved to be justified by the subsequent analysis of the results. More specifically, it was justified by (1) the finding discussed in the previous section that the equilibrium stratification was not dependent on the operating conditions so that (2) the effect of operating conditions on stratification reduced to their effect on stratification kineticsalone; and (3) the Approach-to-Equilibrium metric could be modeled as a first order rate process with a delay time, t_o, and a time constant, Θ.

The significance of the third of these findings is that the time constant has a useful physical meaning; it is the time taken for the kinetic system to reach 63.2% of the equilibrium condition. Further, the system reaches 86.5% of equilibrium in a time of 2Θ, 95% of equilibrium in a time 3Θ, and 98.2% of equilibrium in a time 4Θ.On this basis the effect of operating conditions on stratification kinetics can be evaluated in terms of their effect on Θ; the smaller the value of $\Theta$the faster are the stratification kinetics.

The analysis of the kinetic data generated by the study showed that the relationship between $\boldsymbol{\Theta}$ and 4 of the 5 key variables (the effect of bed depth was not investigated) was essentially linear in the 'optimum zone' for stratification (see Table 6.2). Also, a multi-variable regression analysis showed that $\boldsymbol{\Theta}$was significantly related to these four variables as follows:

$$\Theta = 1323.05 - 5.39\ T1 - 7.57\ T2 - 19.44\%Stroke - 3513\ \Delta\rho + 70.1\ \Delta\rho * \%Stroke$$

(The units of the terms in the model are as follows: T1 and T2 are in cSeconds, $\Delta\rho$ is in g/cc, and Θ is in seconds.)

The adjusted R^2 value was 0.86. The related statistical data (see Table 6.3) showed that the variables that exerted the greatestrelative influence on the time constant – and hence on stratification kinetics – were the difference in density between the stratifying particles, the amplitude of the jig cycle (%stroke) and the pulsion hold time, T2. The pulsion time, T1, exerted a somewhat smaller influence.

The relative importance of density difference on stratification kinetics is as expected. The greater the density difference, the greater is the driving force for stratification, and, therefore, the faster the kinetics would be expected to be. The relative importance of cycle amplitude is also to be expected. As long as the amplitude (stroke) is not increased too much (in which case, excessive turbulence and poorer stratification would be expected), an increase in the amplitude should increase the degree of bed dilation and enhance the rate of segregation.

The relative importance of the pulsion hold time, T2, has not previously been fully tested, although **Jennouchi (1984)** and **Burt (1984)** have argued that this hold time affects the length of time available for segregation by hindered settling and that extending this hold time should therefore improve stratification. Given that the pulsion time, T1, does not have as much impact on the kinetics as the hold time, T2, it appears that the length of time available for hindered settling to take place is more important than the degree to which the bed is dilated. This conclusion is argued on the following basis. The longer pulsion is applied, the greater should the cycle amplitude (stroke) be, and the greater should be the degree of dilation in the bed, other things being equal. The hold time, T2, effectively delays the onset of the suction stroke which would tend to accelerate the re-consolidation of the bed. Therefore, the longer T2 is, the longer the bed remains in a dilated condition. There are obviously limits to such arguments, and also, as many authors comment, the effect of the variables controlling cycle shape and amplitude are inter-dependent. The suggested conclusion should also apply to the jigging of particles using different jig cycle patterns. However, such a conclusion would require further testing.

The very significant effect of differences in particle density on jigging kinetics has an important practical implication for jigging practice. When there are few locked particles in the feed to a jig, the difference between the density of the lightest and heaviest particles is a maximum and stratification kinetics should be relatively fast. However, if there is a significant degree of locking of the mineral components, then the density differences between particles with different degrees of locking will be significantly smaller, and the rate at which these particles stratify will be very much slower than for unlocked particles.This means that the mixed zone between the upper layer of light particles and lower layer of heavy particles will not only be extensive (depending on the degree of locking) but will stratify and reach towards an equilibrium condition only slowly. It is therefore a very real possibility, in such a context, that the residence time in the jig may be far too short for the mixed zone to stratify as it could if given more time. This means that better grades and recoveries of products extracted from the jig could be achieved if the mixed zone were to be removed from the jig as a middlings product and re-jigged for a longer period of time, possibly with some prior crushing to improve the degree of liberation of the minerals.

7.5 Discussion of Findings IV: How can stratification kinetics be modeled effectively?

Taking into account the results of the effects of the operating conditions on the kinetics, an attempt was made to develop an empirical model, as explained in Chapter6.The model derived from the earlier finding that the Approach-to-equilibrium metric could be modeled as a first order rate process with a time delay. However, that model relates to how the stratification profile as a whole changes with time and interesting implications could be drawn from that model, as discussed in the previous section. However, to develop a more detailed model that relates to the kinetics of specific layers – i.e. how the concentration or recovery of components to a layer split from the bed at height ***h*** – required the assumption that the approach-to-equilibrium metric applied to any layer split from the bed. As shown in Chapter 6, the model that emerged from this assumption did not always fit the experimental data well; the assumption was problematic. While the attempt to develop this more detailed model failed, it did highlight the extent to which the kinetics associated with different layers in the bed were different. Given the earlier finding that stratification is strongly influenced by the difference in density between stratifying particles, and the fact that different layers in the jig bed consist of different proportions of particles of different density, the failure of the detailed kinetic model is not surprising. What is required, therefore, is the more fundamental approach of a dynamics balance over an appropriate element in the bed.

7.6 General Implications of the Study

The findings of the study relate to the specific context of the Wits batch jig processing systems of 8mm glass beads of different densities. However, the findings can be generalized to some extent with regard to trends and qualitative conclusions. These have already been discussed in the previous sections of this chapter, but can be summarized as follows.

- Provided the operating conditions remain within an 'optimum zone' for stratification, the equilibrium stratification profile is independent of the operating conditions. The influence of operating conditions on stratification is therefore the result of their effect on the stratification kinetics. In the modelling of stratification kinetics, it is therefore feasible to decouple kinetic effects from equilibrium effects.
- As the King model shows, the factors determining the nature of the equilibrium stratification profile in a system of mono-sized particles, is the proportion and density of each density component in the feed, and an experimentally determined stratification index that accounts for the nature of particle movement during the stratification process. Reliable models that also account for the effect of particle size and shape on the equilibrium stratification profile have still to be developed.
- Some of the control variables available on a batch jig do not have a significant effect on stratification kinetics and so can be ignored when stratification kinetics are being investigated or modelled.
- With the jig cycle typically used in the kind of large, pneumatic jigs employed in coal and iron ore beneficiation, the key control variables affecting stratification kinetics are the cycle

amplitude (stroke) and the pulsion hold time, T2. The pulsion time, T1, influences the kinetics to a smaller degree. The cycle frequency does not influence the kinetics during each jig cycle provided the frequency is small enough that it does not affect the amplitude and shape of the water displacement pattern. The cycle frequency then only affects jigging capacity – i.e. the number of occasions per unit time that the bed is subjected to stratifying dynamics.

- It appears that the influence of all these variables on the rate of approach of the stratification profile to an equilibrium condition can be modelled as a first order rate process characterized by a time constant, $\boldsymbol{\Theta}$, and a time delay, $\boldsymbol{t_o}$. The significance of this finding is that the overall stratification kinetics in the jig – the rate at which the stratification pattern as a whole within the jig bed approaches an equilibrium condition – can be assessed in terms of a generic, well understood parameter, the time constant. So, for example, the time taken to reach 63%, 86%, 95% or 98% of the attainable stratification profile at equilibrium is $\boldsymbol{\Theta}$, $\boldsymbol{2\Theta}$, $\boldsymbol{3\Theta}$ and $\boldsymbol{4\Theta}$ respectively.
- In the 'optimum zone', the relationship between the time constant and any of the variables tested appears to be reasonably linear. The most influential variables, in order of decreasing influence, are the differences in particle density, cycle amplitude (stroke), pulsion hold time, T2, and pulsion time, T1. The effect of the other variables on kinetics was not investigated. A regression model has been developed to encapsulate these effects quantitatively, but further work is required to enhance the reliability and range of the model and to account for any interactive effects between the variables. The extent to which the regression model is generic to large jigs or to continuous jigs or to jigs with different jig cycle patterns would need further investigation.

- The regression model that has been developed applies to the jig bed as a whole. It is less successful in predicting the kinetics of stratification in specific layers within the bed – for example the layer removed as a product by splitting the bed at a height $\boldsymbol{h}$. The reason is that the model assumes the kinetics in all layers is the same which is clearly not so. For one thing, the kinetics of segregation are strongly influenced by the differences in the densities of particles in a given layer, and these differences change with time until the equilibrium condition is reached. Accordingly, amore detailed kinetic model must be developed if a reliable prediction of this aspect of stratification is required.

7.7 Limitations of the study

The primary limitation of this study is that it was conducted in a small (200mm diameter) batch jig on essentially mono-sized, 8mm spherical particles. The effect of size, shape and the density, size and shape distributions of particles on kinetics was not investigated. Nor were the effects of bed depth or bed composition on kinetics investigated. The kinetic model developed is empirical rather than phenomenological in nature. It refers to the rate of change of the stratification profile as a whole rather than to kinetics in the layer that is ultimately split off the bed as a product. In addition, it is applicable only to ternary or binary particle systems.

7.8 Recommendation for future work

A more phenomenological model of stratification kinetics should be developed and validated using the data generated in this study. Further testwork to investigate the effect of bed depth and of bed composition on kinetics should be undertaken. It would also be interesting to investigate how the displacement of the bed responds to the water displacement in the jig for different operating conditions and to relate this information to the findings that have emerged from this study.

7.9 Significance of the study

The findings of this study have significant implications for both jigging practice and for the modelling of jigging kinetics. With regard to practice, three findings are particularly significant. The first is that the operating conditions do not affect the equilibrium stratification profile within the 'optimum zone' for stratification but do affect stratification kinetics. The second is that the stratification kinetics associated with locked particles are slow and that the performance of jigs processing feeds with high proportions of locked particles could be significantly improved by strategies that extend residence times in the jig. The third significant finding is that the extraction and reprocessing of a middlings fraction with longer residence times may significantly improve the performance of an overall jigging circuit.

With regard to the modelling of stratification kinetics, it is apparent that kinetic effects can be decoupled from equilibrium effects. Modelling stratification kinetics using something like an approach-to-equilibrium metric has merit in describing how stratification in the bed as a whole progresses from a homogeneous state to an equilibrium condition but does not evolve very successfully into a model describing the kinetics associated with the layer that is split from jig bed as a product. For that kind of model, a more traditional approach to modelling kinetics is required. In addition, the study has provided useful kinetic and equilibrium data that can be used to test models developed in that way.

7.10 Conclusions

In conclusion it is found that, within a broad range of operating conditions, the control that can be exerted on a jigging operation affects the kinetics of stratification but not the ultimate quality of stratification that can be achieved in that jig. Provided sufficient time is allowed for stratification, and provided the control variables remain within this range of conditions, the same stratification pattern is attained at equilibrium. Put another way, the study has found that, within a broad range of operating conditions, the equilibrium stratification profile is independent of control variables and it is the control of stratification kinetics that has the primary influence on the quality of density separations in a jig.

This study has produced findings that have significance for both jigging practice and for the modelling of jigging. It has provided insights about the fundamentals of jigging that have potentially useful practical outworking. It has developed a novel approach to the modelling of jigging kinetics and provided insights that can inform the modelling efforts of others in the field. It has also highlighted a number of areas for further investigation that are worth pursuing.

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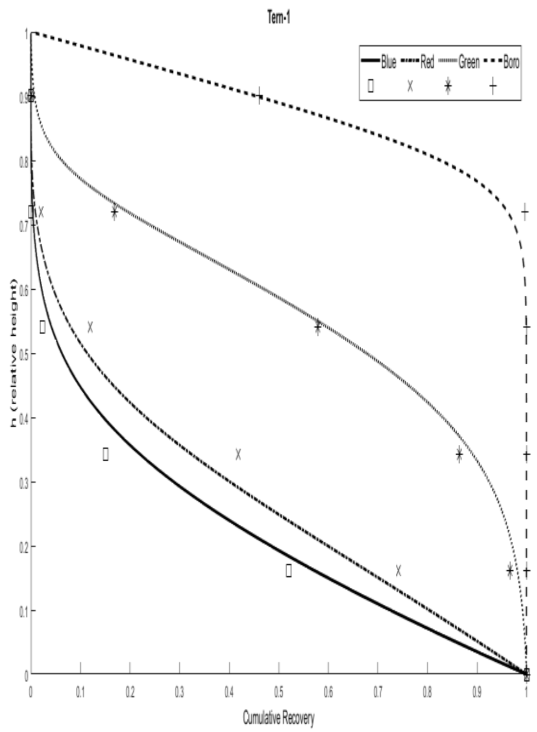
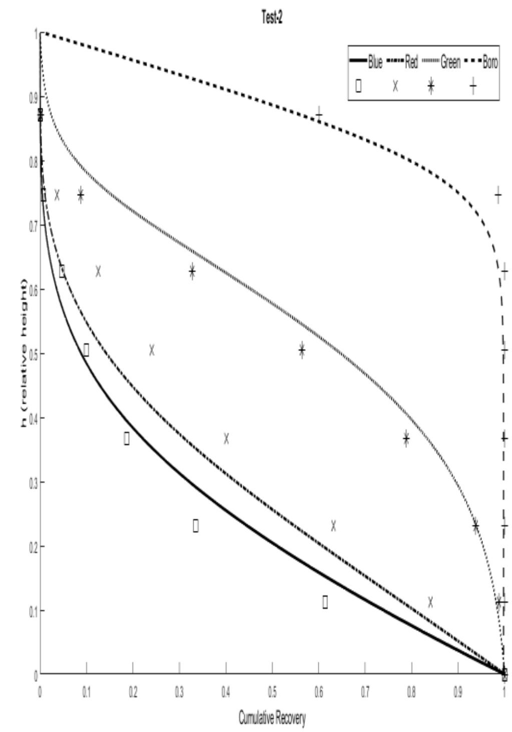
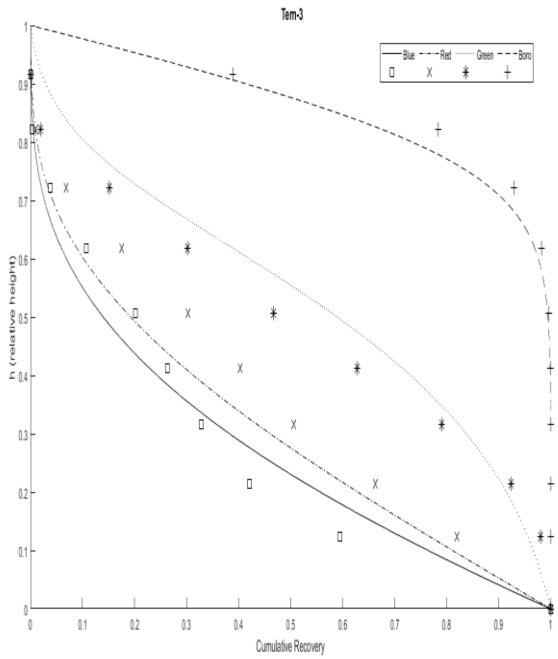
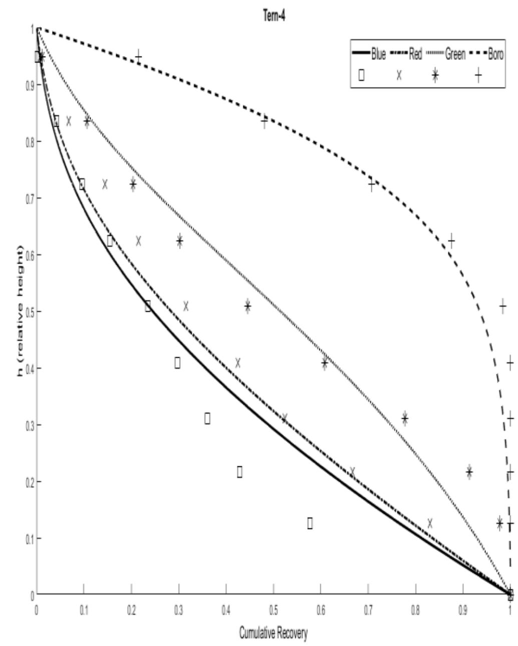
Appendices

Appendics A: Equilibrium plots for quartenary system






Tem-3
Blue
Red
Green
Boro
h (relative height)
Cumulative Recovery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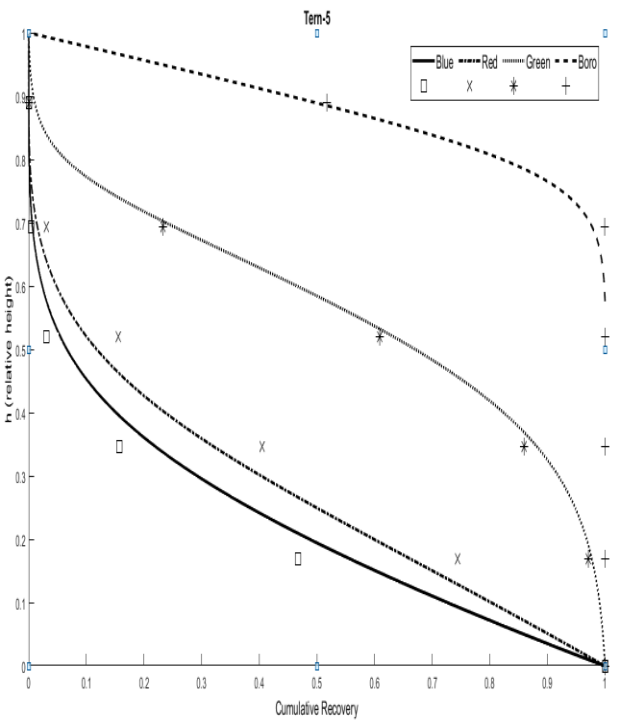
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Red
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Boro
h (relative height)
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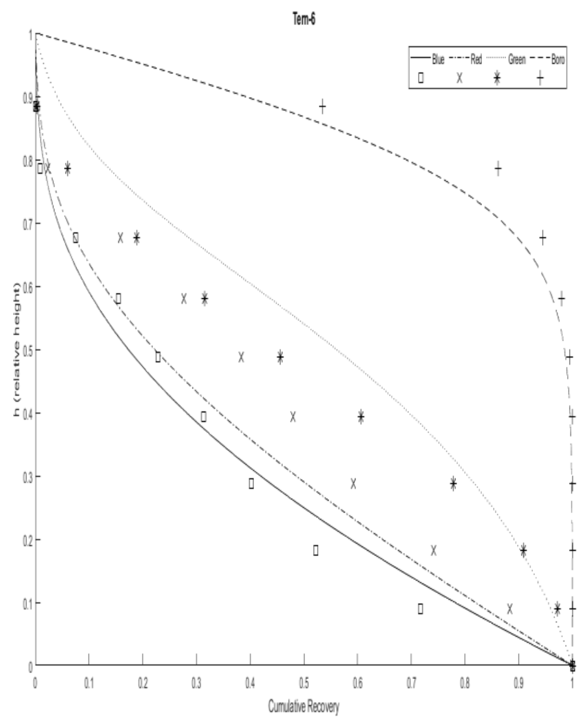
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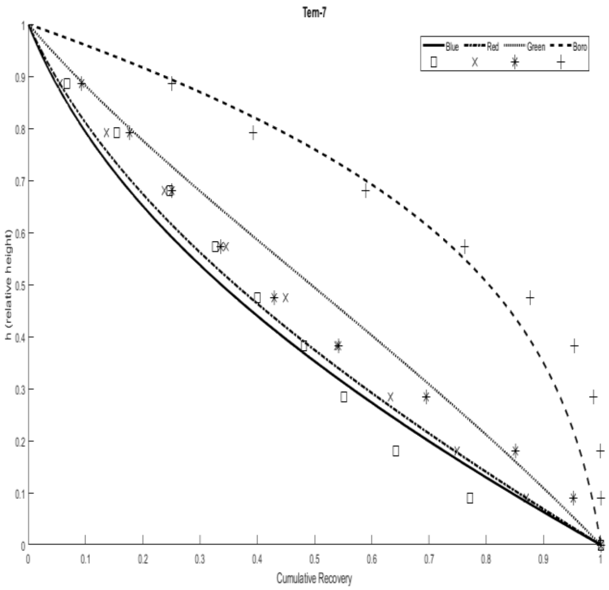
Boro
h (relative height)
Cumulative Recovery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Boro
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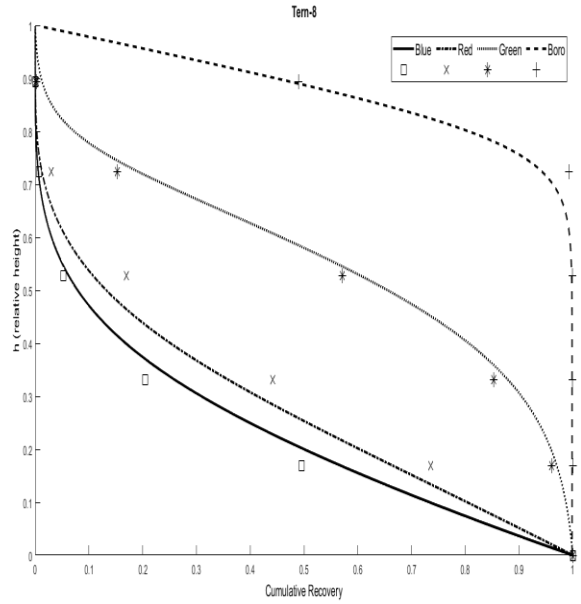
Cumulative Recovery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Blue
Red
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Boro
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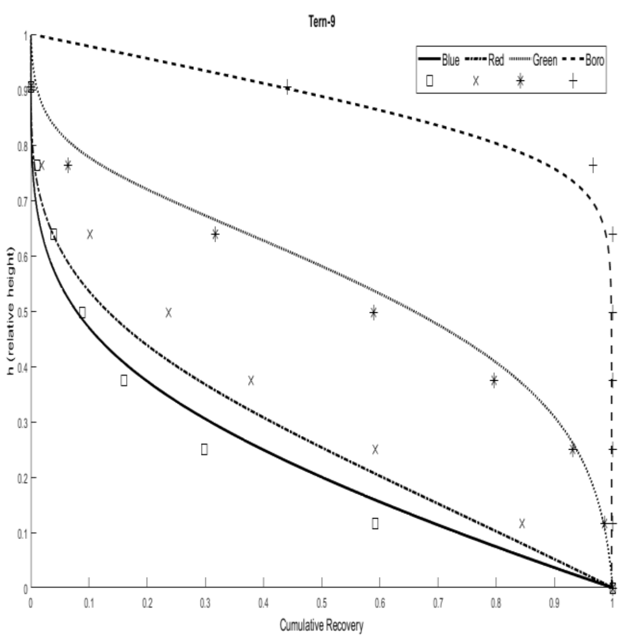
Cumulative Recovery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Red
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Boro
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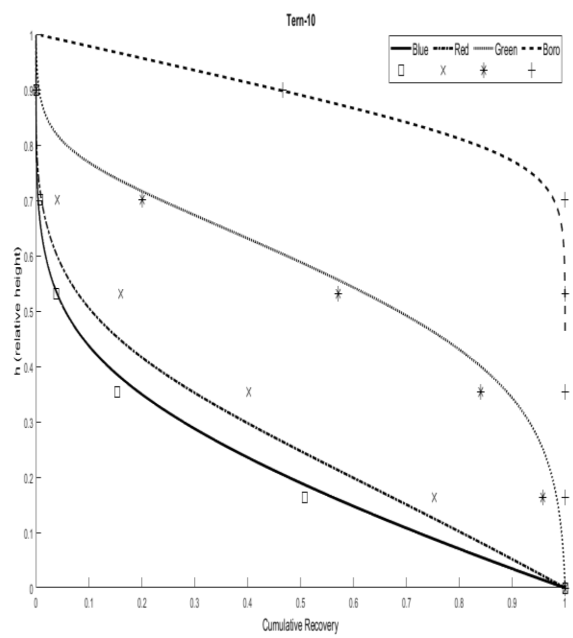
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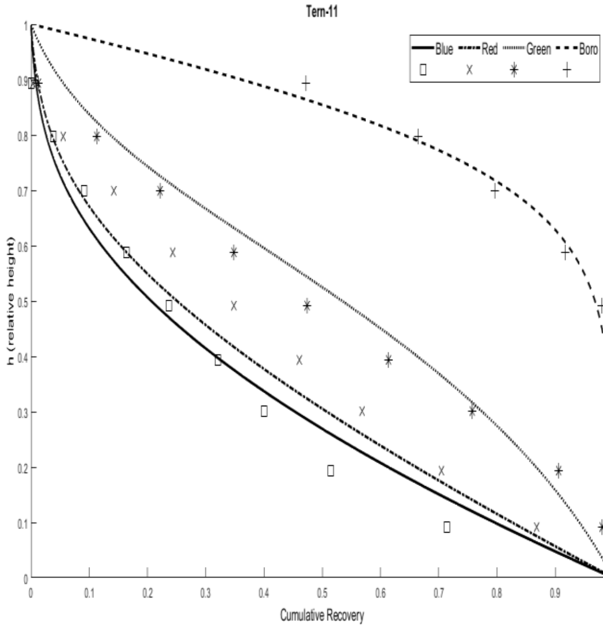
Green
Boro
h (relative height)
Cumulative Recovery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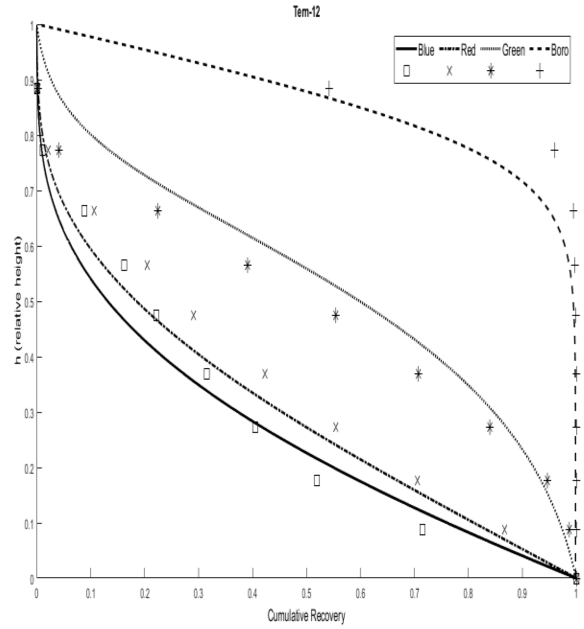
Green
Boro
h (relative height)
Cumulative Recovery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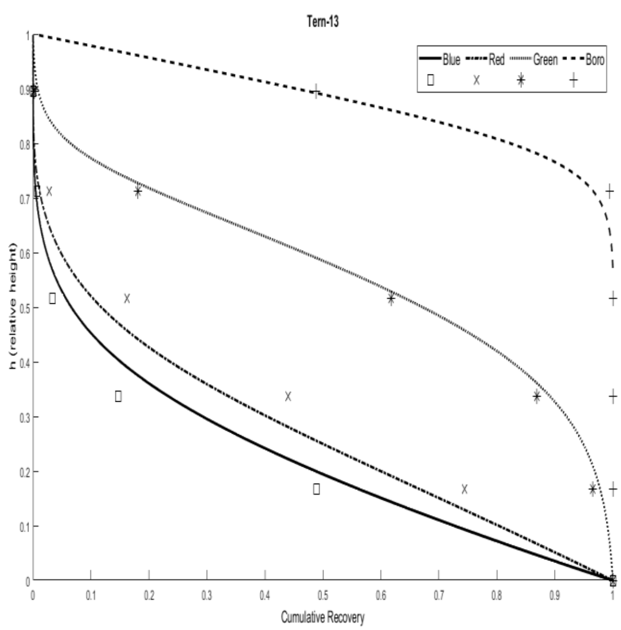
Red
Green
Boro
h (relative height)
Cumulative Recovery


Tern-13
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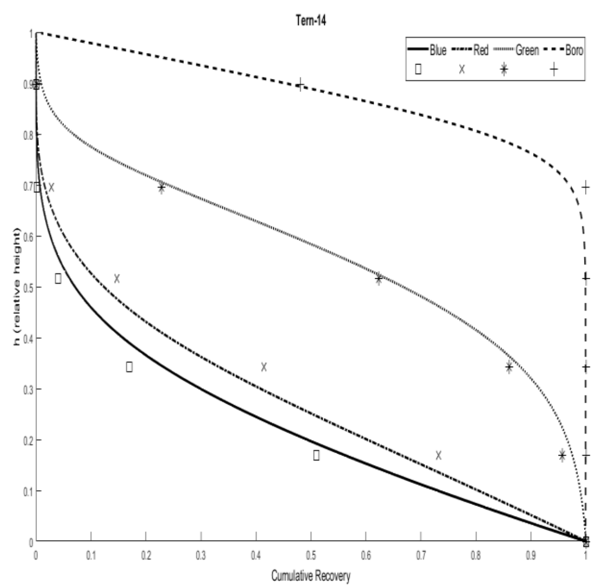
Red
Green
Boro
h (relative height)
Cumulative Recovery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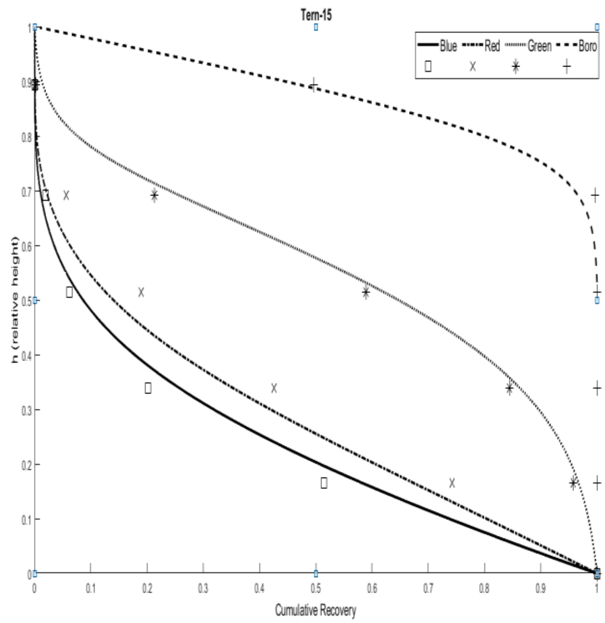
Red
Green
Boro
h (relative height)
Cumulative Recovery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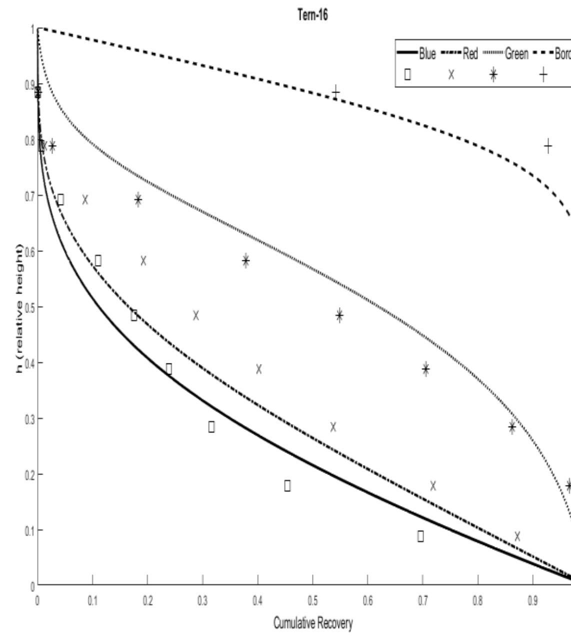
Red
Green
Boro
h (relative height)
Cumulative Recovery


Tern-16
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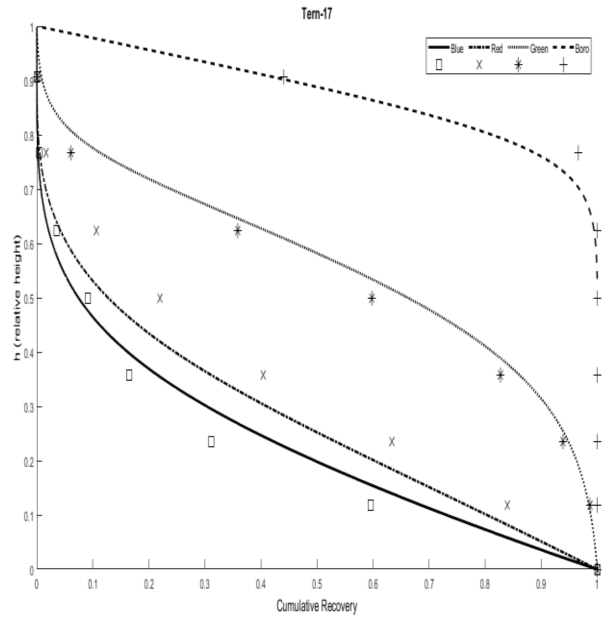
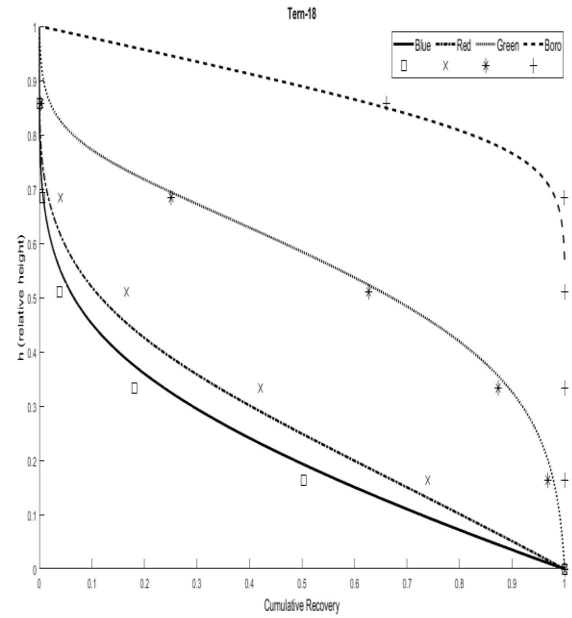
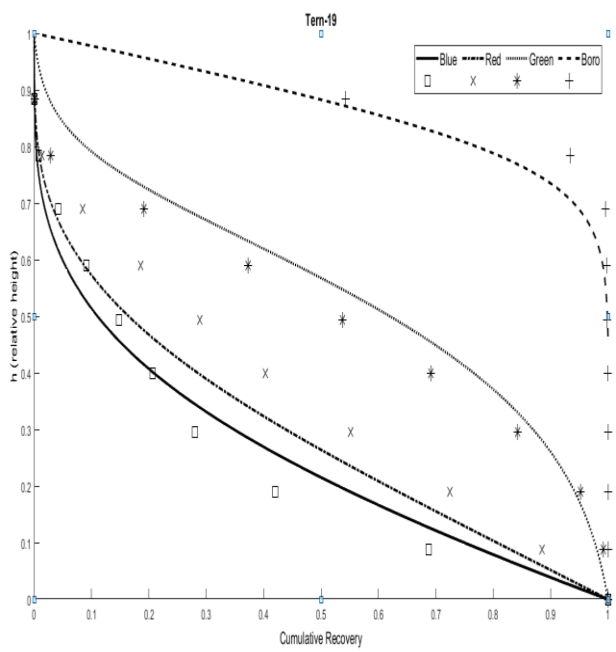
Red
Green
Boro
h (relative height)
Cumulative Recovery


Tern-17
Blue
Red
Green
Boro
h (relative height)
Cumulative Recovery

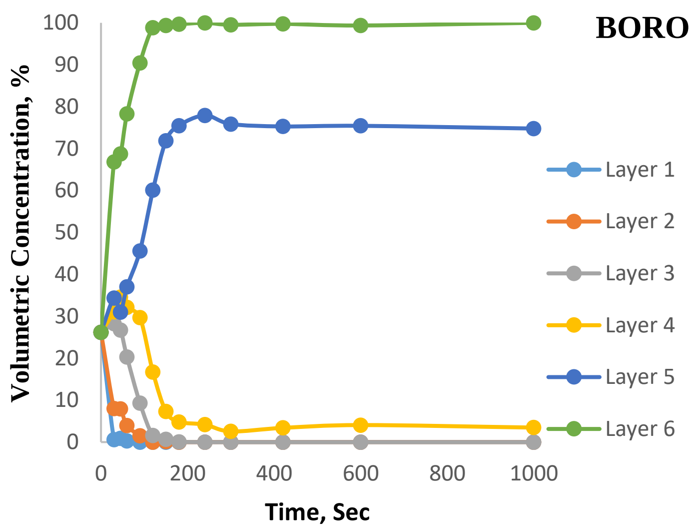

Tern-18
Blue
Red
Green
Boro
h (relative height)
Cumulative Recovery

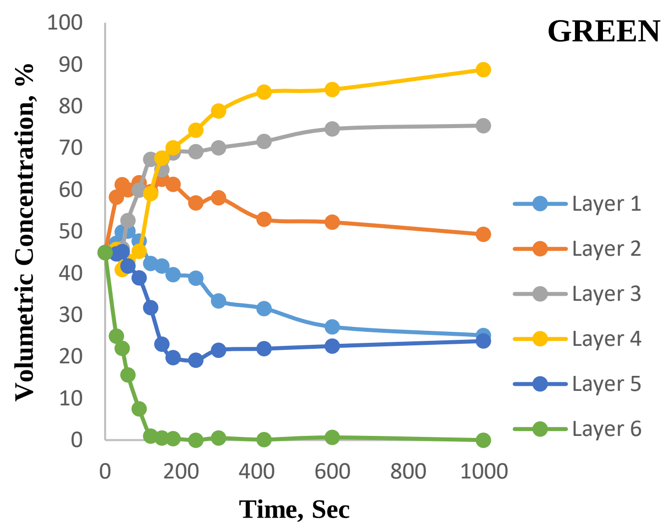

Tern-19
Blue
Red
Green
Boro
h (relative height)
Cumulative Recovery

Appendics B: Kinetics plots and raw data

Figure. B1changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=22cS, T2=28cS, Stroke=34%





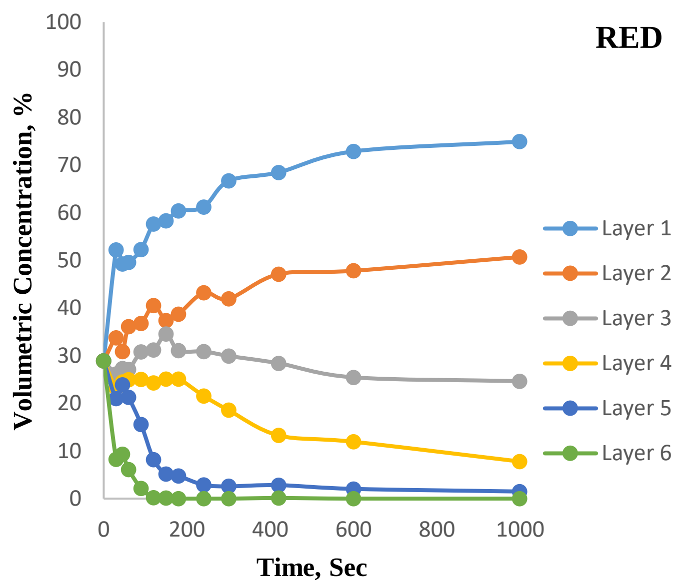
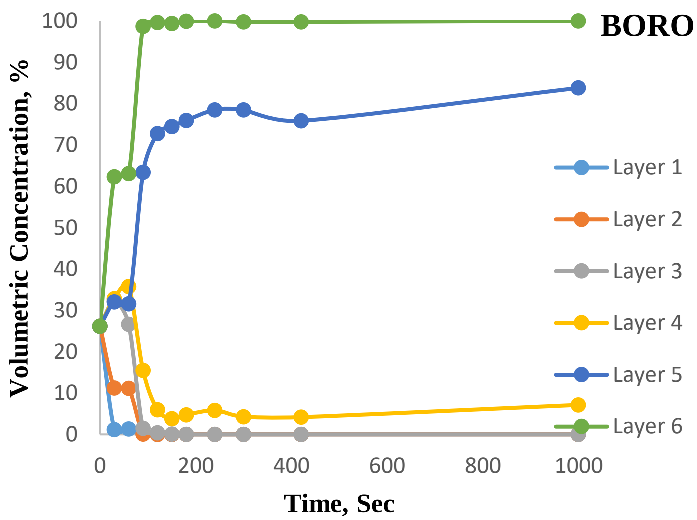
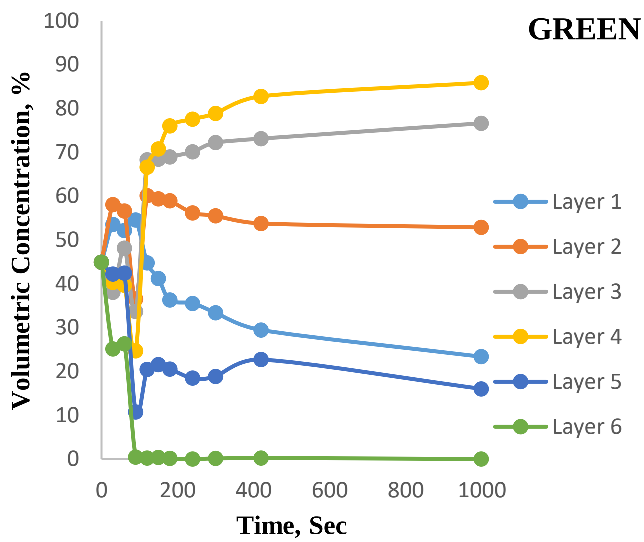


Figure. B2changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=22cS, T2=28cS, Stroke=36%





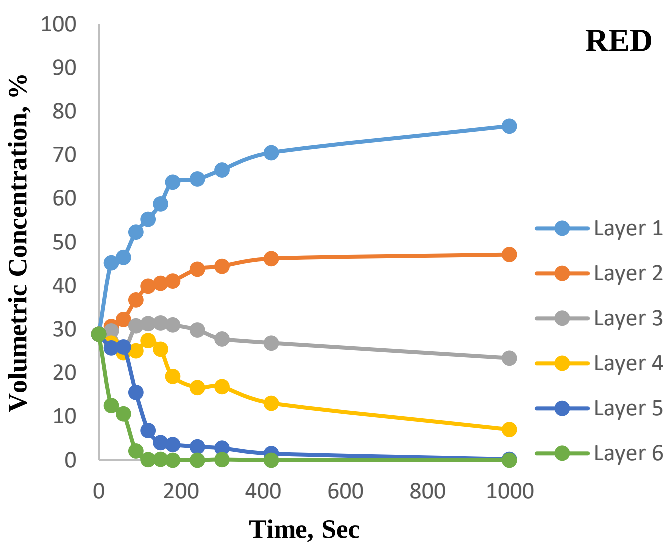


Figure. B3changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=22cS, T2=28cS, Stroke=41%

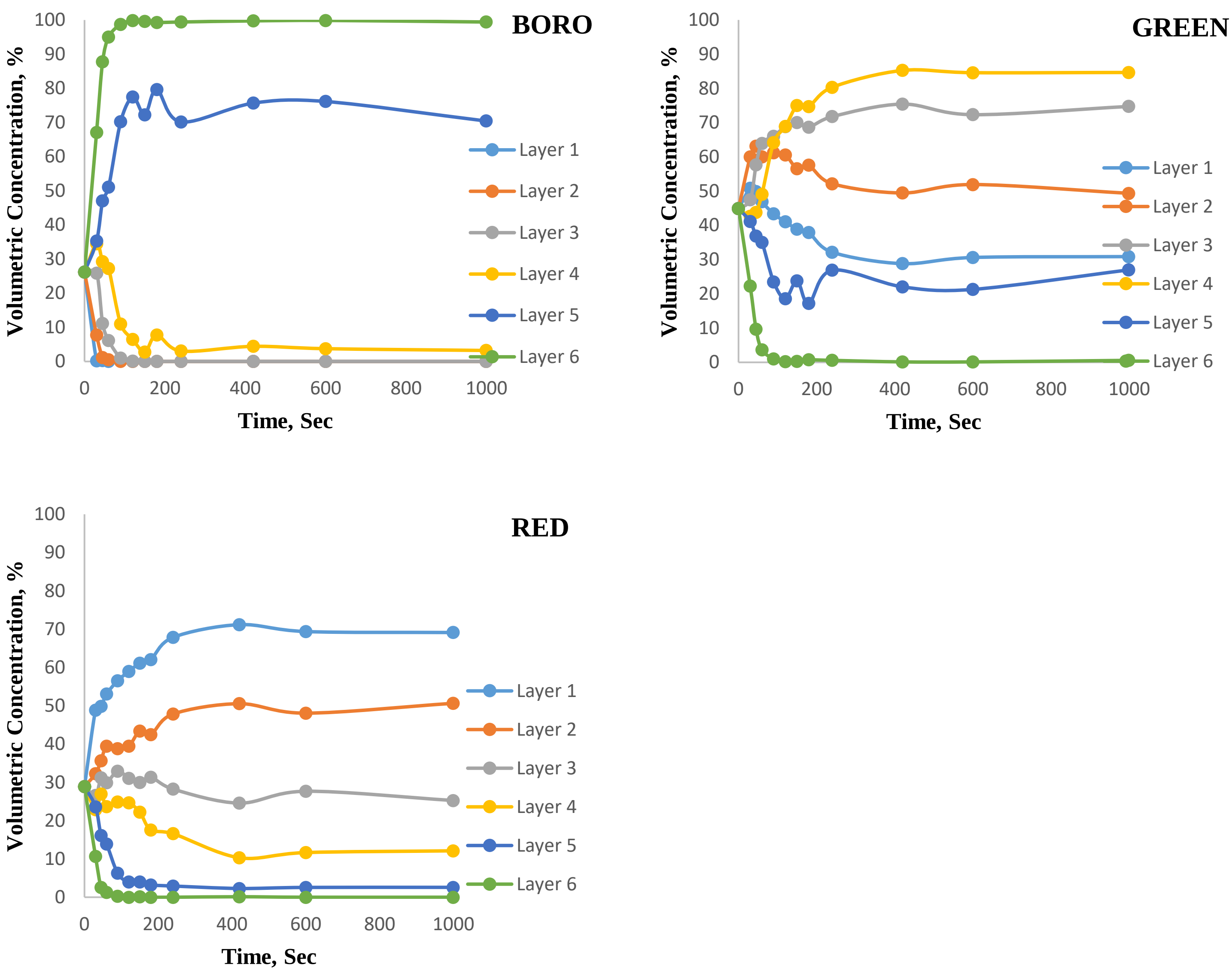
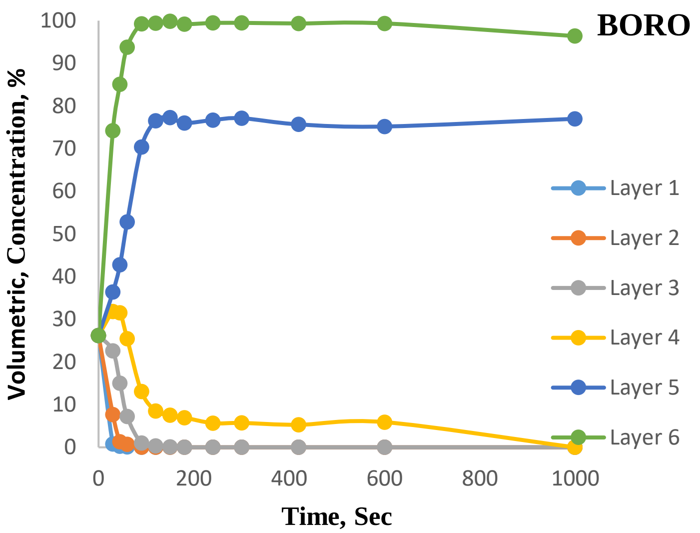
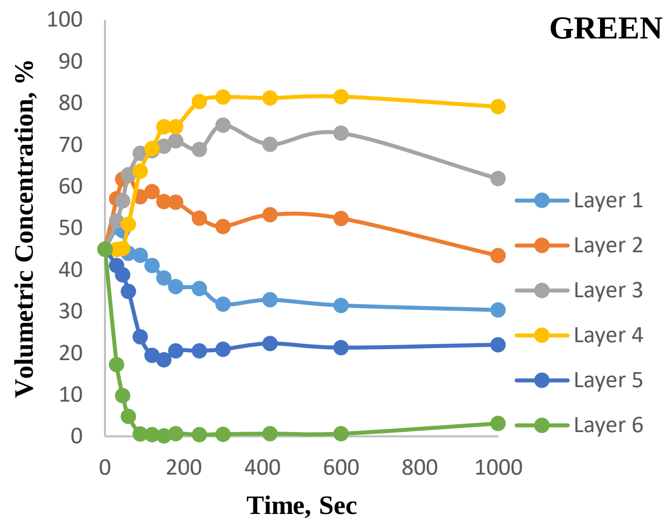


Figure. B4changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=22cS, T2=28cS, Stroke=45%





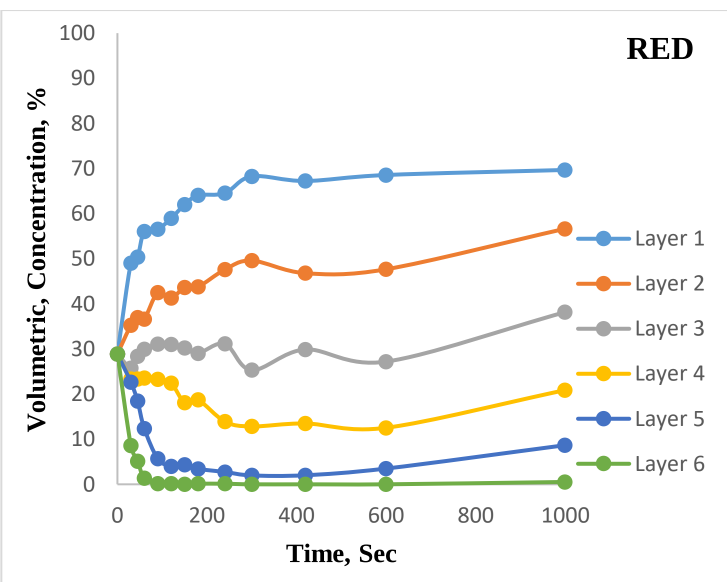
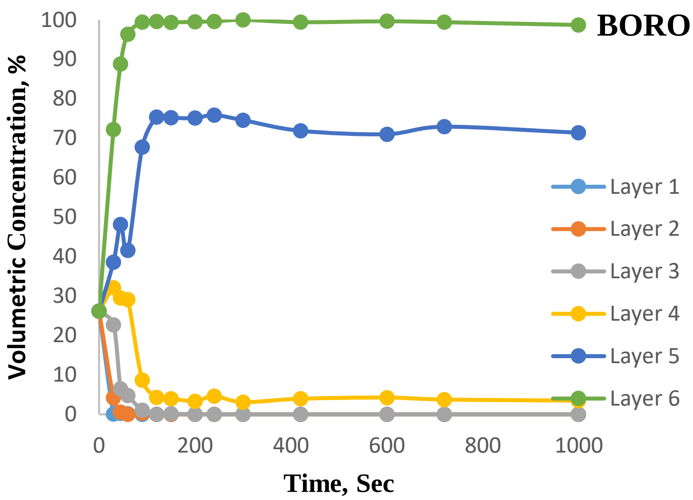
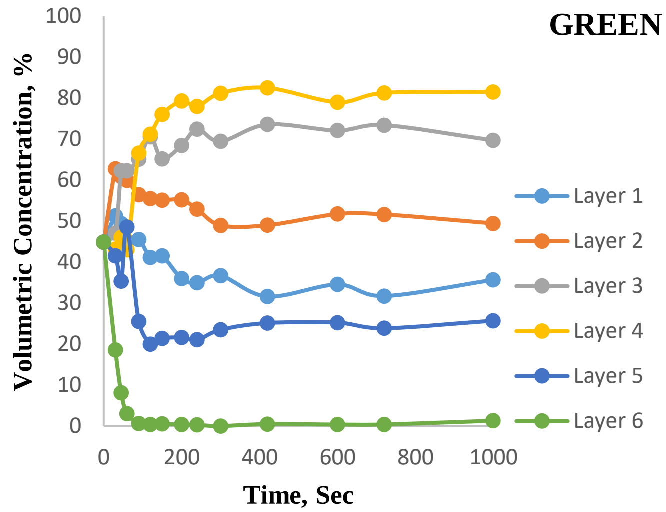


Figure. B5changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=22cS, T2=28cS, Stroke=48%





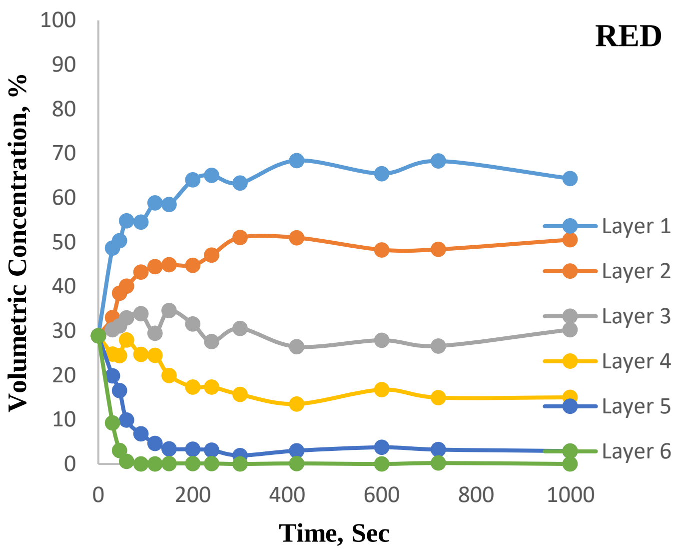
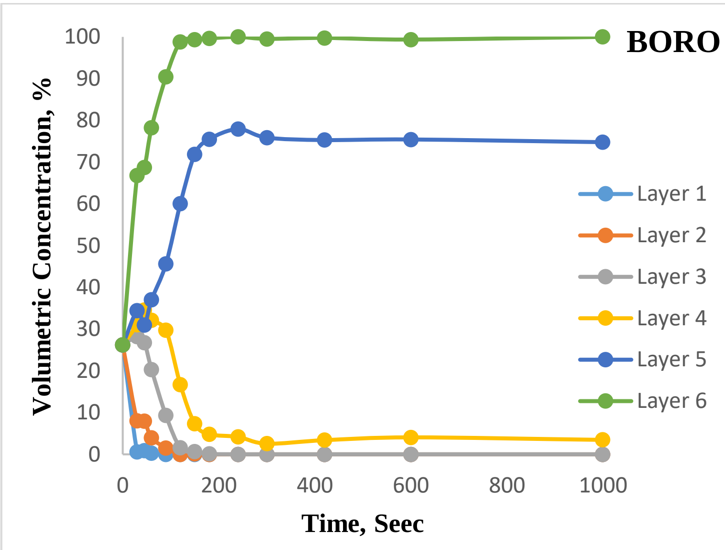
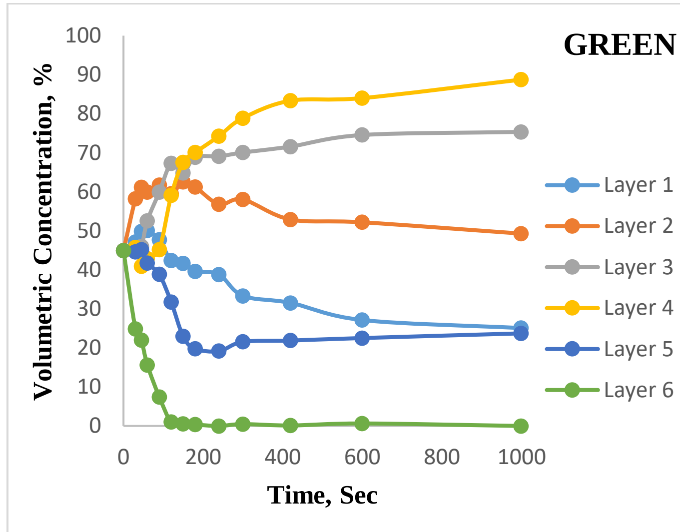


Figure. B6changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=22cS, T2=24cS, Stroke=38%





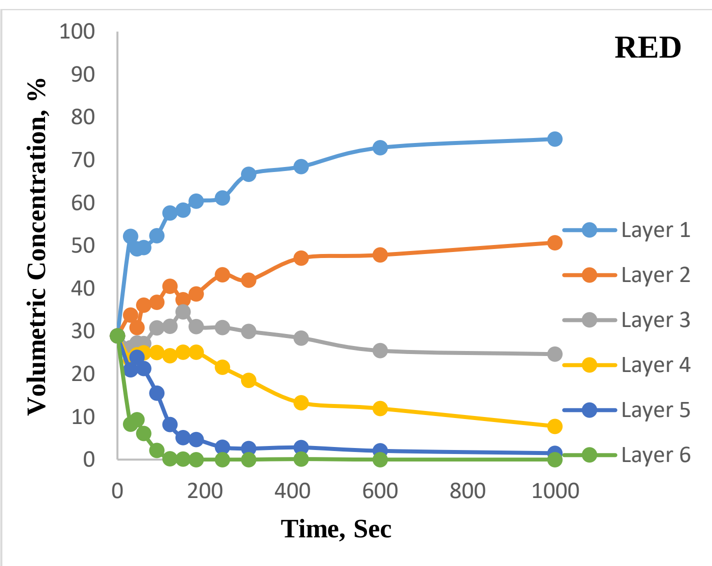
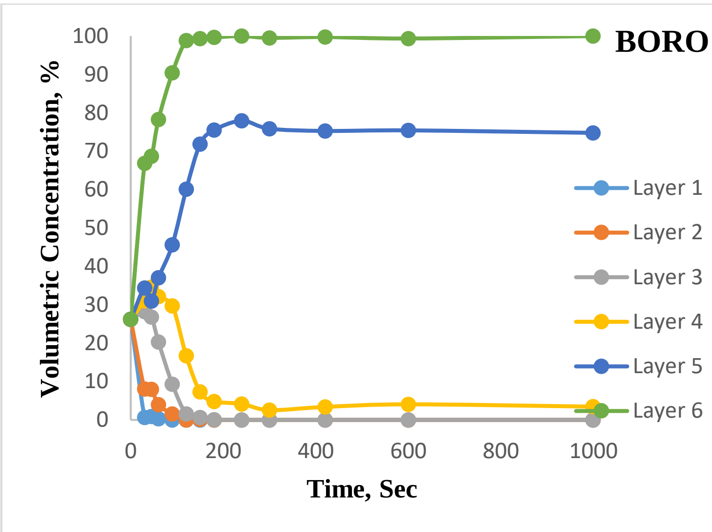
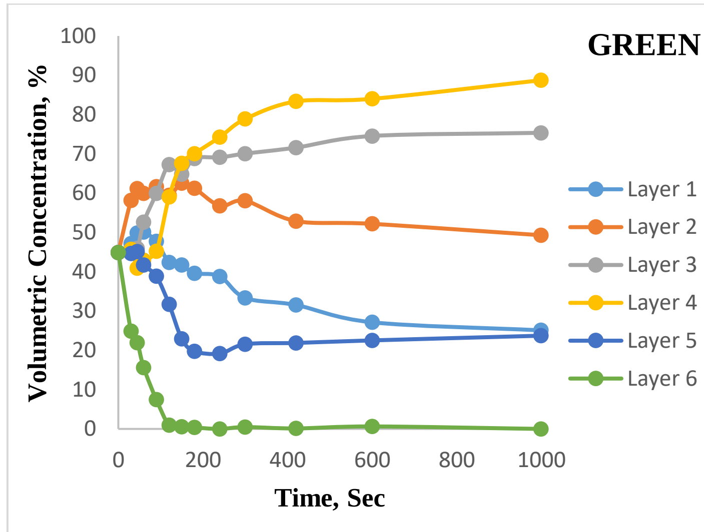


Figure. B 7changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=22cS, T2=32cS, Stroke=38%





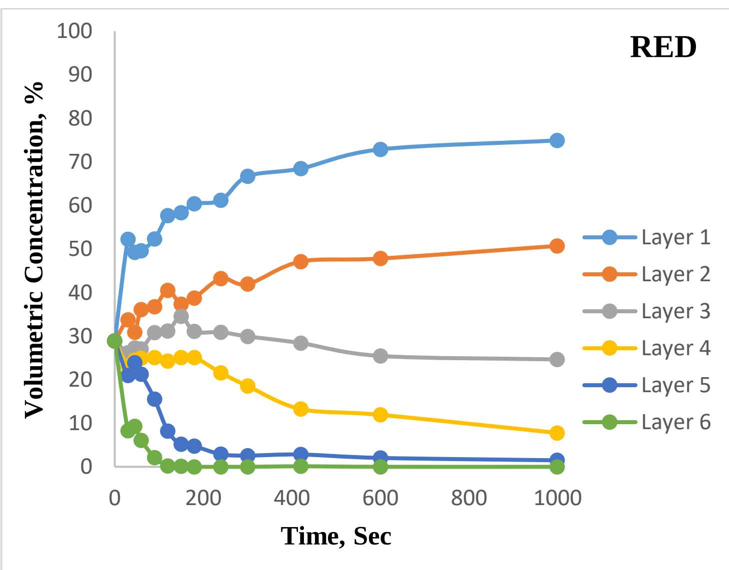
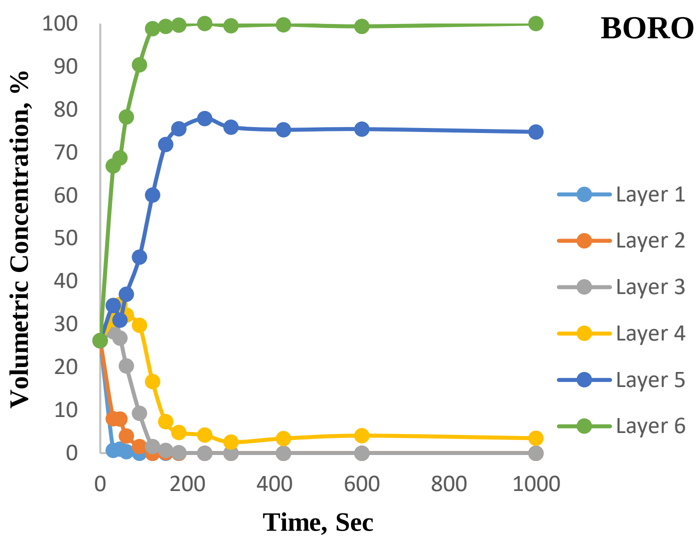
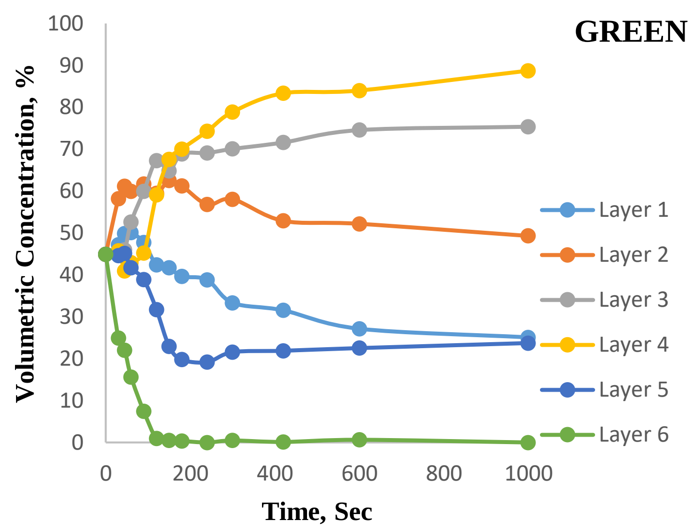


Figure. B8changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=22cS, T2=35cS, Stroke=38%





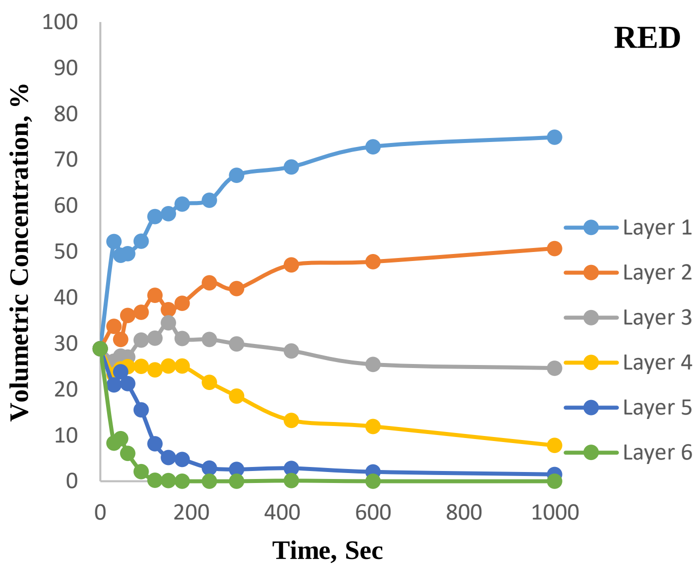


Figure. B 9changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=22cS, T2=38cS, Stroke=38%

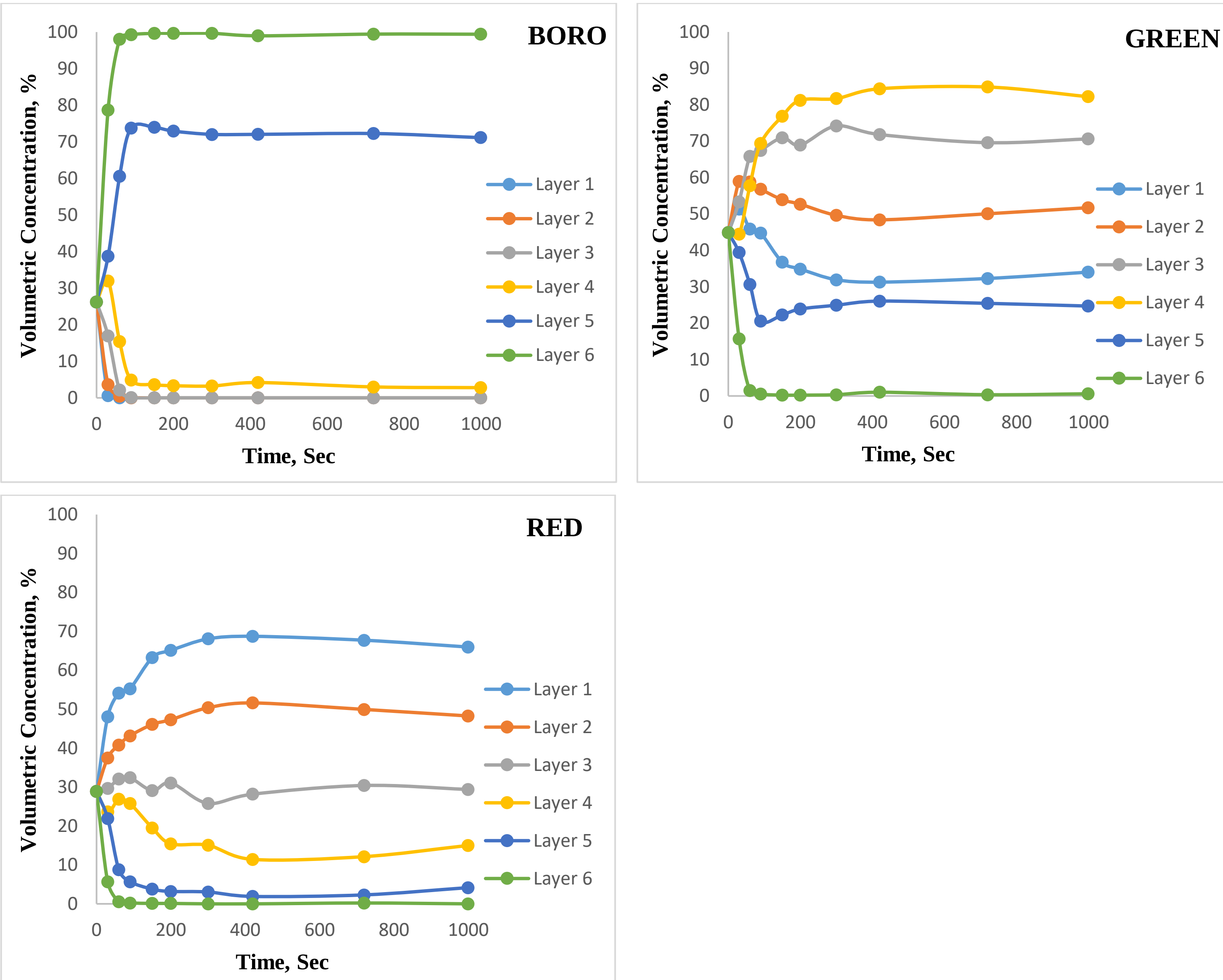
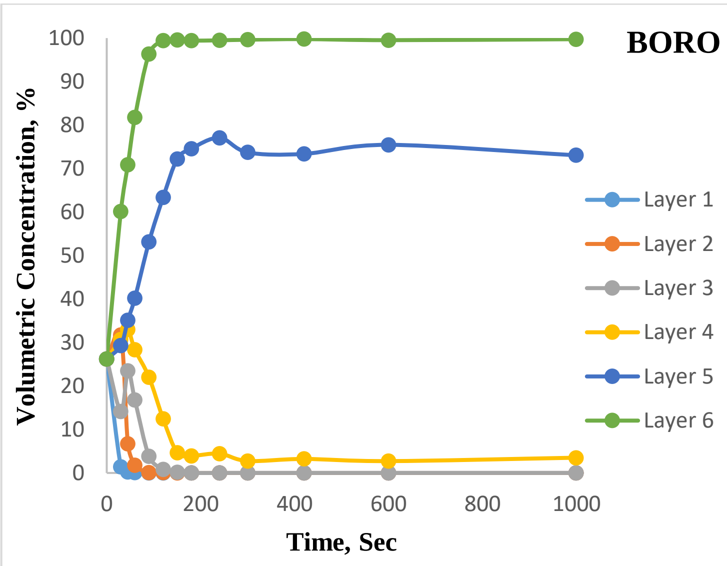
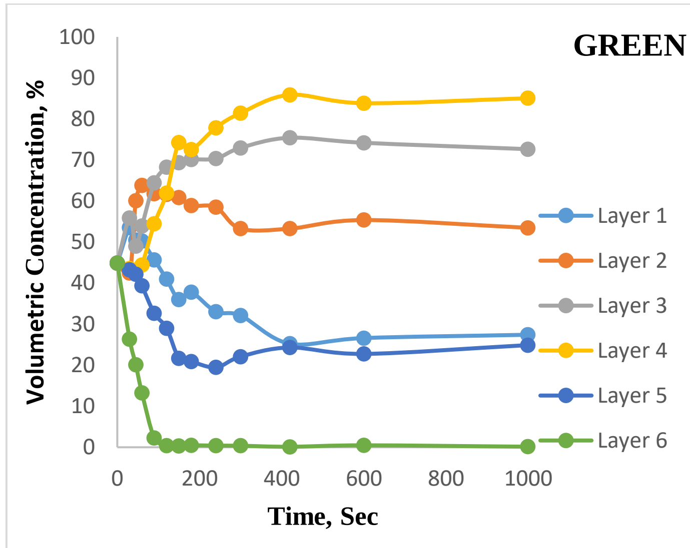


Figure. B 10changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=18cS, T2=28cS, Stroke=38%





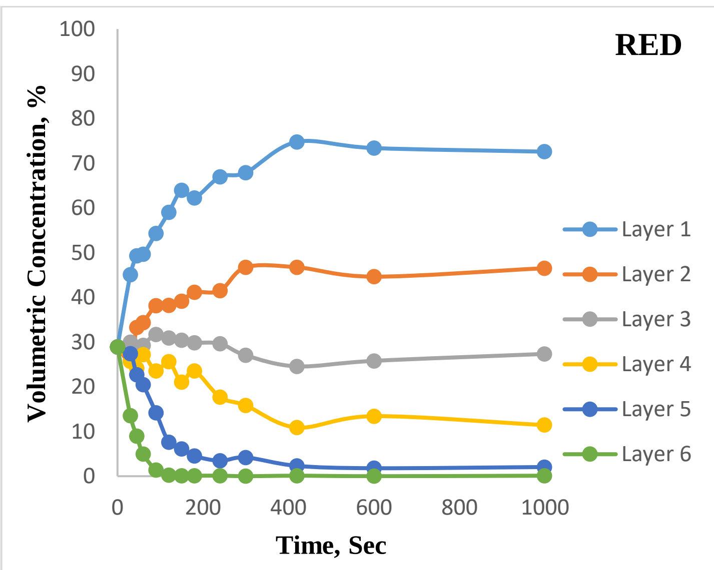


Figure. B 11changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=26cS, T2=28cS, Stroke=38%

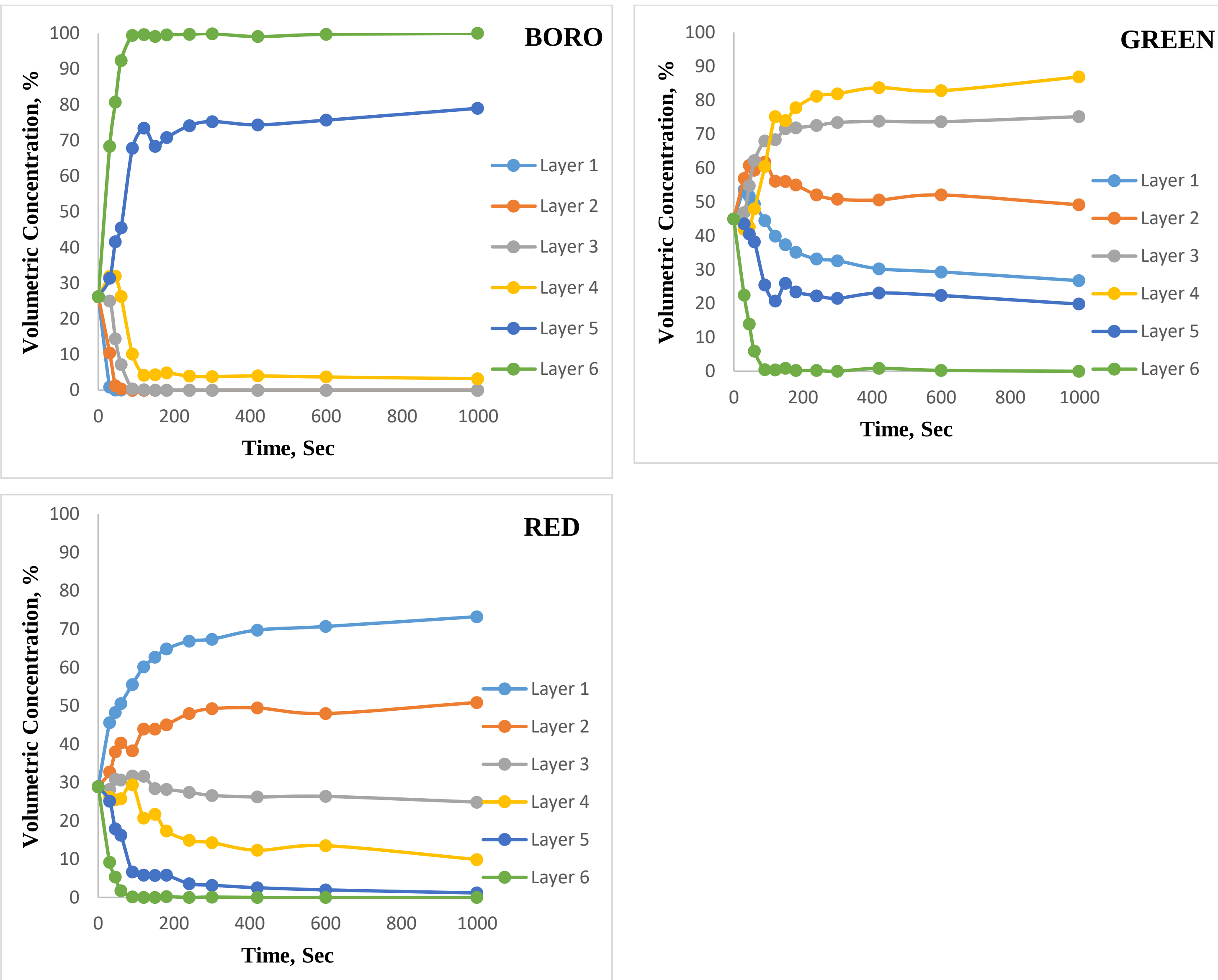


Figure. B 11changes in concentration of light (Boro), intermediate (Green) and heavy(Red) particles in each layer with time

Operating conditions : T1=30cS, T2=28cS, Stroke=38%

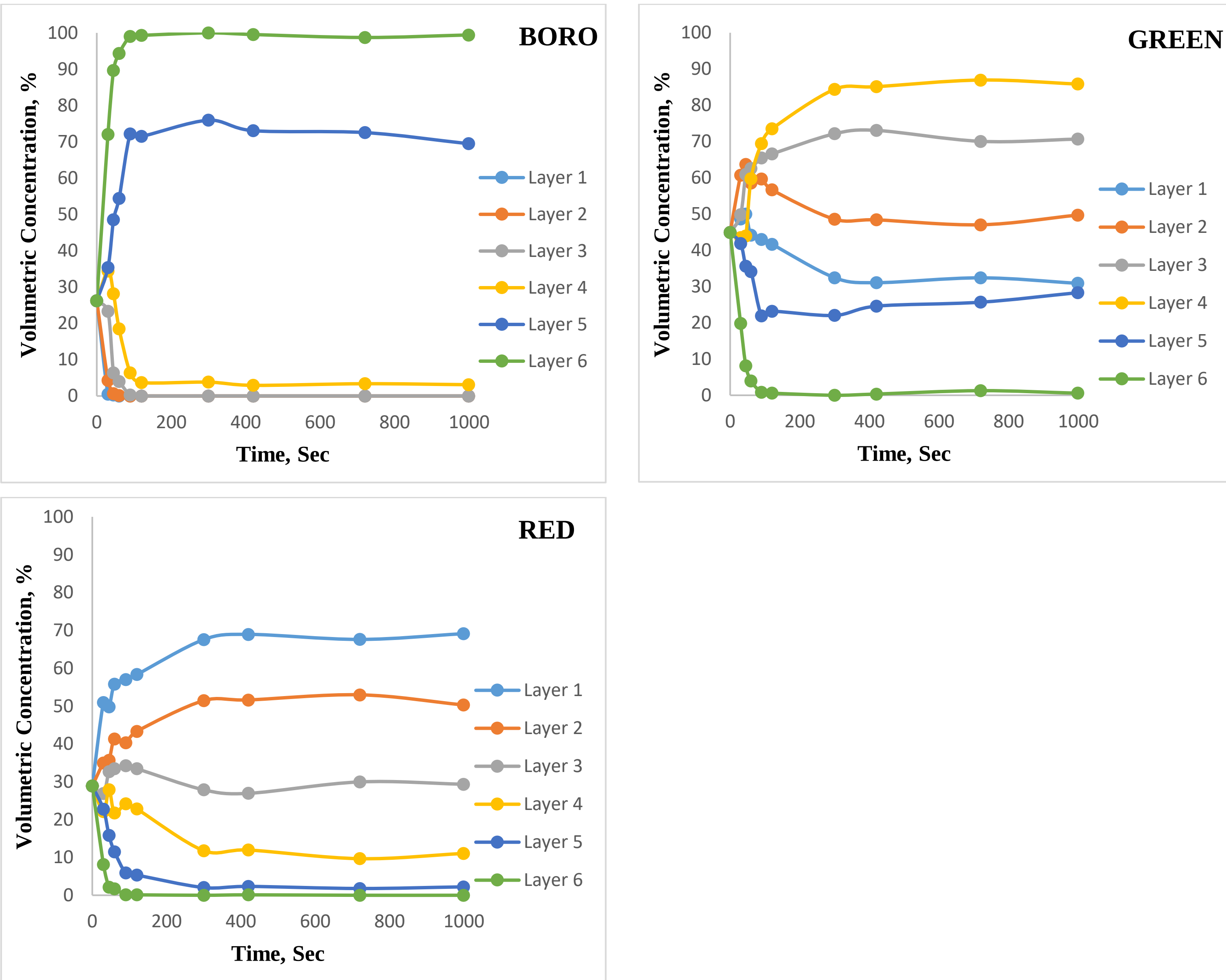


Table. B1 Raw kinetics data for the base case

30 Sec	VOLUME %		
Layer	red	green	boro
1	46.8	50.6	2.6
2	31.1	49.0	19.9
3	25.1	44.5	30.4
4	23.1	41.0	35.9
5	25.4	45.1	29.4
6	15.1	36.3	48.6

45 sec VOLUME %		
red	green	boro
47.0	52.1	0.9
32.3	58.8	8.9
26.6	43.4	29.9
23.6	39.5	36.9
24.5	42.6	32.9
13.5	30.2	56.3

60 sec VOLUME %		
red	green	boro
51.7	48.3	0.0
33.9	65.1	1.0
30.2	59.4	10.4
27.2	46.1	26.8
18.8	36.2	45.0
3.0	9.4	87.6

	90 Sec VOLUME %		
Layer	red	green	boro
1	56.7	43.3	0.0
2	38.2	61.7	0.1
3	31.7	66.3	2.0
4	25.1	60.7	14.2
5	10.5	28.1	61.4
6	0.1	1.2	98.6

120 sec VOLUME %		
red	green	boro
59.5	40.5	0.0
40.1	59.9	0.0
32.2	67.8	0.0
24.8	70.7	4.5
5.4	23.0	71.6
0.1	0.4	99.5

150 sec VOLUME %		
red	green	boro
58.0	42.0	0.0
42.3	57.7	0.0
33.0	67.0	0.0
24.5	71.8	3.7
4.6	22.6	72.7
0	0.2	99.7

	180 sec VOLUME %		
Layer	red	green	boro
1	62.1	37.9	0.0
2	42.1	57.9	0.0
3	31.9	68.1	0.0
4	21.3	75.6	3.1
5	5.4	24.9	69.7
6	0	0.6	99.31

200 sec VOLUME %		
red	green	boro
63.3	36.7	0.0
44.8	55.2	0.0
28.4	71.6	0.0
21.5	74.3	4.2
3.5	23.0	73.5
0	0	100

240 sec VOLUME %		
red	green	boro
65.0	35.0	0.0
47.1	52.9	0.0
27.6	72.4	0.0
17.4	78.0	4.6
3.1	21.0	75.9
0.1	0.2	99.6

	300 sec VOLUME %		
Layer	red	green	boro
1	69.1	30.9	0.0
2	45.9	54.1	0.0
3	28.0	72.0	0.0
4	14.8	82.7	2.5
5	4.2	24.0	71.8
6	0.1	0.8	99.0

420 sec VOLUME %		
red	green	boro
71.2	28.8	0.0
47.9	52.1	0.0
26.0	74.0	0.0
12.3	84.7	3.0
2.9	23.2	73.9
0	0.3	99.6

720 sec VOLUME %		
red	green	boro
73.0	27.0	0.0
47.8	52.2	0.0
26.9	73.1	0.0
10.8	85.8	3.5
1.9	25.5	72.6
0	0.4	99.5

Table B2 raw kinetic data for stroke=34%

	30 sec VOLUME %			45 sec VOLUME %			60 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	48.7	50.3	1.0	46.0	53.0	1.0	49.6	50.3	0.1
2	33.8	56.4	9.8	31.8	60.1	8.1	36.2	59.9	3.9
3	23.9	47.6	28.4	28.2	47.8	23.9	27.2	52.7	20.0
4	25.5	42.4	32.1	26.3	40.5	33.2	24.4	42.1	33.5
5	23.9	46.3	29.8	22.6	42.1	35.3	21.0	43.0	36.0
6	9.1	22.0	68.9	10.2	20.4	69.5	5.1	14.6	80.2

	90 sec VOLUME %			120 sec VOLUME %			150 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	50.8	49.0	0.2	53.3	46.7	0.0	55.4	44.6	0.0
2	36.6	62.8	0.6	39.1	60.4	0.4	38.9	61.1	0.0
3	30.0	61.7	8.3	33.7	64.5	1.9	32.9	66.9	0.2
4	27.3	46.5	26.2	26.5	59.3	14.2	27.3	65.2	7.5
5	15.0	35.7	49.2	8.1	26.9	65.0	5.4	19.8	74.8
6	1.8	3.9	94.1	0.2	0.7	99.0	0.0	0.1	99.9

	180 sec VOLUME %			240 sec VOLUME %			300 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	59.0	41.0	0.0	62.9	37.1	0.0	57.7	42.2	0.1
2	41.3	58.6	0.1	41.4	58.6	0.0	40.1	59.3	0.6
3	31.9	67.8	0.3	29.4	70.5	0.1	28.8	62.8	8.4
4	23.3	71.6	5.1	22.0	75.9	2.1	19.1	51.7	29.2
5	3.5	19.1	77.4	3.4	17.0	79.7	3.1	40.8	56.2
6	0.0	0.3	99.7	0	0	100	0	4.0	95.97414

	420 sec VOLUME %			600 sec VOLUME %			999 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	67.3	32.7	0.0	72.1	27.9	0.0	71.6	28.4	0.0
2	45.5	54.5	0.0	44.3	55.7	0.0	49.4	50.6	0.0
3	27.8	72.2	0.0	26.5	73.5	0.0	25.0	75.0	0.0
4	15.2	80.4	4.5	13.4	82.5	4.2	10.9	85.9	3.2
5	2.4	19.2	78.4	1.0	20.4	78.6	1.2	21.4	77.4
6	0	0.1	99.8	0	0.1	99.8	0	0	100

Table. B3 Raw kinetic data for stroke=36%

	30 sec VOLUME %			60 sec VOLUME %			90 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	45.3	53.5	1.2	46.5	52.2	1.3	54.6	45.3	0.2
2	30.7	58.1	11.2	32.3	56.6	11.1	36.5	63.4	0.1
3	29.7	38.1	32.3	25.3	48.1	26.6	33.7	64.8	1.5
4	26.8	40.3	32.8	24.6	39.6	35.7	24.7	59.9	15.5
5	25.8	42.2	32.0	26.0	42.4	31.6	10.7	25.9	63.4
6	12.5	25.2	62.3	10.6	26.3	63.0	0.4	0.8	98.6

	120 sec VOLUME %			150 sec VOLUME %			180 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	55.2	44.8	0.0	58.8	41.2	0.0	63.7	36.3	0.0
2	39.9	60.1	0.0	40.6	59.4	0.0	41.1	58.9	0.0
3	31.3	68.3	0.4	31.5	68.4	0.1	31.0	69.0	0.0
4	27.5	66.6	6.0	25.5	70.8	3.8	19.2	76.0	4.8
5	6.8	20.4	72.7	4.0	21.6	74.4	3.5	20.5	75.9
6	0.1	0.2	99.6	0.2	0.4	99.4	0.0	0.1	99.9

	240 sec VOLUME %			300 sec VOLUME %			420 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	64.5	35.5	0.0	66.6	33.4	0.0	70.6	29.4	0.0
2	43.8	56.2	0.0	44.5	55.5	0.0	46.2	53.8	0.0
3	29.9	70.1	0.0	27.8	72.2	0.0	26.9	73.1	0.0
4	16.6	77.6	5.8	16.9	78.9	4.3	13.1	82.8	4.2
5	3.1	18.5	78.5	2.7	18.8	78.4	1.5	22.7	75.8
6	0	0	100	0.1	0.1	99.7	0	0.2	99.7

	999 sec VOLUME %		
Layer	red	green	boro
1	76.6	23.4	0.0
2	47.2	52.8	0.0
3	23.4	76.6	0.0
4	7.0	85.9	7.1
5	0.2	16.0	83.8
6	0	0	100

Table.B4 Raw kinetic data for stroke=41%

	30 sec VOLUME %			45 sec VOLUME %			60 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	48.9	50.9	0.2	49.9	49.9	0.2	53.1	46.9	0.0
2	32.3	60.0	7.7	35.7	63.2	1.2	39.4	60.0	0.5
3	26.6	47.5	25.9	31.2	57.7	11.1	29.9	64.0	6.1
4	22.9	42.6	34.5	27.0	43.8	29.2	23.7	49.1	27.3
5	23.6	41.1	35.3	16.1	36.9	47.0	13.9	35.1	51.0
6	10.6	22.3	67.1	2.5	9.7	87.8	1.2	3.6	95.0

	90 sec VOLUME %			120 sec VOLUME %			150 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	56.6	43.4	0.1	58.9	41.1	0.0	61.1	38.9	0.0
2	38.8	61.2	0.0	39.4	60.6	0.0	43.4	56.6	0.0
3	32.9	66.0	1.0	31.1	68.8	0.1	29.9	70.1	0.0
4	24.9	64.2	10.9	24.6	68.9	6.5	22.2	75.0	2.8
5	6.2	23.5	70.2	4.0	18.6	77.4	4.0	23.8	72.2
6	0.2	1.0	98.7	0	0.1	99.8	0.1	0.3	99.6

	180 sec VOLUME %			240 sec VOLUME %			420 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	62.1	37.9	0.0	67.8	32.2	0.0	71.2	28.8	0.0
2	42.4	57.6	0.0	47.8	52.2	0.0	50.5	49.5	0.0
3	31.3	68.7	0.0	28.2	71.8	0.0	24.6	75.4	0.0
4	17.5	74.7	7.8	16.6	80.3	3.1	10.3	85.3	4.4
5	3.2	17.2	79.6	2.9	26.9	70.2	2.3	22.1	75.7
6	0.0	0.7	99.3	0	0.5	99.4	0.1	0.1	99.7

	600 sec VOLUME %			999 sec VOLUME %		
Layer	red	green	boro	red	green	boro
1	69.4	30.6	0.0	69.1	30.9	0.0
2	48.1	51.9	0.0	50.6	49.4	0.0
3	27.7	72.3	0.0	25.3	74.7	0.0
4	11.7	84.6	3.7	12.1	84.7	3.2
5	2.6	21.3	76.1	2.6	27.0	70.4
6	0	0.1	99.8	0	0.5	99.4

Table B5 Raw kinetic data for stroke=45%

	30 sec VOLUME %		
Layer	red	green	boro
1	49.0	50.2	0.8
2	35.2	57.1	7.7
3	25.8	51.7	22.6
4	23.3	44.9	31.8
5	22.6	41.0	36.4
6	8.6	17.2	74.2

45 sec VOLUME %		
red	green	boro
50.4	49.4	0.2
37.0	61.7	1.3
28.4	56.5	15.0
23.4	45.1	31.5
18.5	38.8	42.8
5.1	9.8	85.1

60 sec VOLUME %		
red	green	boro
56.0	43.9	0.1
36.6	62.7	0.7
30.0	62.8	7.2
23.6	50.9	25.5
12.4	34.8	52.8
1.3	4.7	93.8

	90 sec VOLUME %		
Layer	red	green	boro
1	56.5	43.5	0.0
2	42.5	57.5	0.0
3	31.0	68.0	1.0
4	23.3	63.7	13.1
5	5.7	23.9	70.4
6	0.1	0.5	99.2

120 sec VOLUME %		
red	green	boro
59.0	41.0	0.0
41.3	58.7	0.0
31.0	68.6	0.4
22.4	69.1	8.5
4.0	19.4	76.6
0.1	0.4	99.4

150 sec VOLUME %		
red	green	boro
62.0	38.0	0.0
43.6	56.4	0.0
30.2	69.7	0.1
18.1	74.4	7.5
4.4	18.4	77.3
0.0	0.1	99.9

	180 sec VOLUME %		
Layer	red	green	boro
1	64.0	36.0	0.0
2	43.8	56.2	0.0
3	29.1	70.9	0.0
4	18.7	74.3	7.0
5	3.4	20.6	76.0
6	0.1	0.7	99.2

240 sec VOLUME %		
red	green	boro
64.5	35.5	0.0
47.6	52.4	0.0
31.1	68.9	0.0
13.9	80.4	5.7
2.7	20.6	76.7
0.1	0.4	99.4

300 sec VOLUME %		
red	green	boro
68.3	31.7	0.0
49.6	50.4	0.0
25.3	74.7	0.0
12.8	81.4	5.7
2.0	20.9	77.1
0	0.5	99.4

	420 sec VOLUME %		
Layer	red	green	boro
1	67.2	32.8	0.0
2	46.8	53.2	0.0
3	29.9	70.1	0.0
4	13.5	81.2	5.3
5	2.0	22.3	75.7
6	0	0.6	99.3

600 sec VOLUME %		
red	green	boro
68.6	31.4	0.0
47.7	52.3	0.0
27.2	72.8	0.0
12.5	81.6	5.9
3.5	21.3	75.2
0	0.6	99.3

999 sec VOLUME %		
red	green	boro
69.7	30.3	0.0
56.6	43.4	0.0
38.1	61.9	0.0
20.9	79.1	0.0
8.7	65.5	25.9
0.5	3.0	96.3

Table B6 Raw kinetic data for stroke=48%

	30 sec VOLUME %			45 sec VOLUME %			60 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	48.7	51.3	0.1	50.4	49.4	0.2	54.8	45.0	0.2
2	33.0	62.8	4.2	38.5	60.9	0.6	40.1	59.9	0.0
3	30.3	47.1	22.7	31.2	62.3	6.5	33.0	62.3	4.8
4	24.8	43.2	32.1	24.4	46.1	29.5	27.9	43.0	29.1
5	19.9	41.5	38.6	16.5	35.3	48.1	9.9	48.6	41.5
6	9.3	18.5	72.2	3.0	8.0	88.8	0.6	3.0	96.4

	90 sec VOLUME %			120 sec VOLUME %			150 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	54.5	45.5	0.0	58.8	41.2	0.0	58.5	41.5	0.0
2	43.2	56.5	0.3	44.5	55.5	0.0	44.9	55.1	0.0
3	33.9	65.1	1.0	29.5	70.5	0.0	34.6	65.2	0.2
4	24.7	66.6	8.7	24.5	71.2	4.3	19.9	76.1	4.0
5	6.8	25.5	67.7	4.6	20.0	75.3	3.4	21.4	75.2
6	0	0.5	99.4	0.0	0.4	99.6	0.1	0.4	99.3

	200 sec VOLUME %			300 sec VOLUME %			420 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	64.0	36.0	0.0	63.3	36.7	0.0	68.4	31.6	0.0
2	44.8	55.2	0.0	51.0	49.0	0.0	51.0	49.0	0.0
3	31.5	68.5	0.0	30.6	69.4	0.0	26.4	73.6	0.0
4	17.4	79.3	3.4	15.7	81.2	3.1	13.5	82.5	4.0
5	3.3	21.6	75.1	1.9	23.5	74.6	3.0	25.1	71.9
6	0.1	0.3	99.5	0	0	100	0.1	0.4	99.4

	600 sec VOLUME %			720 sec VOLUME %			999 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	65.5	34.5	0.0	68.3	31.7	0.0	64.4	35.6	0.0
2	48.3	51.7	0.0	48.4	51.6	0.0	50.6	49.4	0.0
3	27.9	72.1	0.0	26.6	73.4	0.0	30.3	69.7	0.0
4	16.7	79.0	4.3	14.9	81.3	3.8	15.0	81.5	3.5
5	3.8	25.2	71.0	3.2	23.9	72.9	2.9	25.7	71.4
6	0	0.3	99.6	0.2	0.3	99.4	0	1.2	98.7

Table B7 Raw kinetic data for T2=24cS

	30 sec VOLUME %			45 sec VOLUME %			60 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	52.2	47.2	0.6	49.2	49.9	0.9	49.6	50.1	0.3
2	33.8	58.2	8.0	30.9	61.2	8.0	36.1	59.9	4.0
3	26.2	45.6	28.2	27.3	46.0	26.8	27.1	52.6	20.3
4	23.2	45.7	31.1	24.5	40.9	34.6	24.9	42.9	32.1
5	21.0	44.6	34.4	23.9	45.1	31.0	21.3	41.7	37.0
6	8.3	24.9	66.8	9.3	22.0	68.7	6.0	15.6	78.2

	90 sec VOLUME %			120 sec VOLUME %			150 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	52.3	47.7	0.0	57.6	42.4	0.0	58.3	41.7	0.0
2	36.8	61.7	1.5	40.5	59.5	0.0	37.4	62.5	0.1
3	30.8	59.9	9.3	31.2	67.3	1.6	34.5	64.8	0.6
4	25.1	45.2	29.7	24.3	59.1	16.7	25.1	67.6	7.3
5	15.5	38.8	45.6	8.2	31.7	60.1	5.2	23.0	71.9
6	2.1	7.4	90.4	0.2	0.9	98.8	0.1	0.5	99.4

	180 sec VOLUME %			240 sec VOLUME %			300 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	60.4	39.6	0.0	61.2	38.8	0.0	66.7	33.3	0.0
2	38.7	61.3	0.0	43.2	56.8	0.0	42.0	58.0	0.0
3	31.1	68.8	0.1	30.9	69.1	0.0	29.9	70.1	0.0
4	25.1	70.0	4.8	21.6	74.3	4.2	18.6	78.9	2.6
5	4.7	19.8	75.5	2.9	19.2	77.9	2.6	21.6	75.9
6	0.0	0.4	99.6	0	0	100	0	0.4	99.5

	420 sec VOLUME %			600 sec VOLUME %			999 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	68.5	31.5	0.0	72.9	27.1	0.0	74.9	25.1	0.0
2	47.1	52.9	0.0	47.8	52.2	0.0	50.7	49.3	0.0
3	28.4	71.6	0.0	25.4	74.6	0.0	24.6	75.4	0.0
4	13.3	83.4	3.4	11.9	84.0	4.1	7.8	88.7	3.5
5	2.8	21.9	75.3	2.0	22.5	75.4	1.5	23.7	74.8
6	0.1	0.1	99.7	0	0.6	99.3	0	0	100

Table. B8 Raw kinetics data for T2=32cS

	30 sec VOLUME %			45 sec VOLUME %			60 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	48.6	50.9	0.5	53.5	46.2	0.2	54.6	45.4	0.0
2	34.0	60.8	5.1	35.6	63.0	1.4	38.5	61.3	0.2
3	26.8	48.1	25.1	30.7	57.6	11.8	30.5	63.8	5.8
4	25.6	41.4	32.9	23.4	44.4	32.2	23.4	51.8	24.8
5	23.3	40.9	35.8	17.3	40.9	41.8	13.8	35.7	50.5
6	7.4	23.4	69.2	2.1	10.9	87.0	1.5	3.2	95.1

	90 sec VOLUME %			120 sec VOLUME %			150 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	56.9	43.1	0.0	62.7	37.3	0.0	62.7	37.3	0.0
2	40.6	59.4	0.0	39.6	60.4	0.0	40.2	59.8	0.0
3	32.9	66.4	0.6	30.2	69.8	0.0	31.3	68.7	0.0
4	24.6	67.2	8.2	21.3	72.4	6.3	21.0	74.3	4.7
5	6.0	22.9	71.1	5.4	19.5	75.2	4.2	20.1	75.7
6	0.3	0.3	99.3	0.2	0.5	99.2	0.1	0.2	99.6

	180 sec VOLUME %			240 sec VOLUME %			300 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	58.7	41.3	0.0	66.7	33.3	0.0	68.6	31.4	0.0
2	44.1	55.9	0.0	47.0	53.0	0.0	45.8	54.2	0.0
3	28.9	71.1	0.0	27.3	72.7	0.0	27.9	72.1	0.0
4	20.3	74.6	5.1	15.5	80.8	3.7	15.1	80.9	4.0
5	3.8	20.0	76.2	2.7	18.5	78.8	2.9	23.1	74.0
6	0.0	0.5	99.5	0	0	100	0	0.6	99.3

	420 sec VOLUME %			600 sec VOLUME %			999 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	71.4	28.6	0.0	71.4	28.6	0.0	74.6	25.4	0.0
2	48.9	51.1	0.0	50.1	49.9	0.0	50.1	49.9	0.0
3	25.8	74.2	0.0	26.6	73.4	0.0	23.2	76.8	0.0
4	10.6	84.9	4.5	11.4	85.9	2.7	9.5	86.4	4.1
5	2.6	22.8	74.6	2.2	26.5	71.3	1.8	24.9	73.3
6	0	0.1	99.81	0.1	0.2	99.6	0.09	0.3	99.5

Table B9 raw kietic data for T2=35 cS

	45 sec VOLUME %			60 sec VOLUME %			90 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	54.2	45.7	0.2	51.3	48.6	0.1	56.0	43.9	0.1
2	35.3	63.8	0.9	39.5	60.5	0.0	42.2	57.8	0.0
3	28.1	59.4	12.4	31.3	64.7	4.0	30.1	69.7	0.2
4	23.4	48.1	28.5	27.4	53.5	19.1	26.2	66.4	7.4
5	17.2	36.5	46.2	10.8	29.3	59.9	5.7	21.0	73.3
6	4.2	10.0	85.8	1.0	2.9	96.0	0	0.6	99.36

	120 sec VOLUME %			150 sec VOLUME %			180 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	60.9	39.1	0.0	58.6	41.4	0.0	63.5	36.5	0.0
2	41.4	58.6	0.0	46.7	53.3	0.0	47.6	52.4	0.0
3	30.9	69.1	0.0	33.8	66.2	0.0	29.5	70.5	0.0
4	22.0	73.9	4.2	18.2	76.3	5.4	15.8	79.1	5.2
5	4.2	19.1	76.6	2.3	22.2	75.5	3.1	21.9	75.1
6	0.12	0.24	99.6	0.4	0.1	99.4	0.0	0.0	100.0

	240 sec VOLUME %			300 sec VOLUME %		
Layer	red	green	boro	red	green	boro
1	64.1	35.9	0.0	64.6	35.4	0.0
2	49.0	51.0	0.0	51.4	48.6	0.0
3	28.8	71.2	0.0	31.3	68.7	0.0
4	15.1	80.0	4.9	11.3	85.0	3.7
5	2.6	20.2	77.1	2.1	22.3	75.6
6	0	0	100	0	0.1	99.8

Table. B10 Raw kinetic data for T2=38cS

	30 sec VOLUME %			60 sec VOLUME %			90 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	48.1	51.3	0.6	54.2	45.8	0.0	55.3	44.7	0.0
2	37.5	58.9	3.6	40.8	58.8	0.4	43.2	56.8	0.0
3	29.7	53.4	17.0	32.1	65.8	2.1	32.4	67.5	0.1
4	23.6	44.4	31.9	26.9	57.7	15.4	25.8	69.4	4.9
5	21.9	39.4	38.7	8.8	30.6	60.6	5.7	20.6	73.8
6	5.7	15.7	78.7	0.5	1.5	98.0	0.2	0.5	99.2

	150 sec VOLUME %			200 sec VOLUME %			300 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	63.2	36.8	0.0	65.1	34.9	0.0	68.1	31.9	0.0
2	46.1	53.9	0.0	47.3	52.7	0.0	50.4	49.6	0.0
3	29.1	70.9	0.0	31.1	68.9	0.0	25.8	74.2	0.0
4	19.5	76.9	3.6	15.4	81.2	3.3	15.1	81.7	3.2
5	3.8	22.3	73.9	3.2	23.9	72.9	3.1	24.9	72.0
6	0.1	0.2	99.6	0.1	0.2	99.6	0	0.3	99.6

	420 sec VOLUME %			720 sec VOLUME %			999 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	68.7	31.3	0.0	67.7	32.3	0.0	66.0	34.0	0.0
2	51.6	48.4	0.0	49.9	50.1	0.0	48.3	51.7	0.0
3	28.2	71.8	0.0	30.4	69.6	0.0	29.4	70.6	0.0
4	11.4	84.4	4.2	12.1	84.9	3.0	15.0	82.2	2.8
5	1.9	26.1	72.0	2.3	25.4	72.3	4.2	24.7	71.1
6	0	1.0	98.97	0.2	0.3	99.4	0	0.5	99.4

Table. B11 Raw kinetic data for T1=18cS

	30 sec VOLUME %			45 sec VOLUME %			60 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	45.1	53.6	1.4	49.3	50.5	0.2	49.6	50.3	0.1
2	25.8	42.5	31.7	33.3	60.1	6.7	34.4	63.8	1.8
3	30.0	55.9	14.1	27.5	49.0	23.5	29.3	53.9	16.8
4	25.7	43.6	30.7	24.1	42.9	33.0	27.2	44.5	28.3
5	27.4	43.2	29.3	22.7	42.2	35.1	20.5	39.3	40.2
6	13.6	26.3	60.1	9.0	20.1	70.9	4.9	13.2	81.7

	90 sec VOLUME %			120 sec VOLUME %			150 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	54.3	45.7	0.0	59.0	41.0	0.0	63.9	36.0	0.1
2	38.1	61.8	0.1	38.2	61.7	0.1	39.1	60.9	0.0
3	31.7	64.4	3.8	30.9	68.2	0.8	30.4	69.4	0.2
4	23.5	54.4	22.0	25.6	61.9	12.4	21.0	74.3	4.7
5	14.2	32.7	53.1	7.6	29.0	63.3	6.1	21.7	72.2
6	1.3	2.2	96.3	0.2	0.3	99.4	0.1	0.3	99.5

	180 sec VOLUME %			240 sec VOLUME %			300 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	62.2	37.8	0.0	67.0	33.0	0.0	67.9	32.1	0.0
2	41.1	58.9	0.0	41.5	58.5	0.0	46.7	53.3	0.0
3	29.9	70.1	0.0	29.6	70.4	0.0	27.0	73.0	0.0
4	23.5	72.5	3.9	17.7	77.9	4.4	15.8	81.4	2.8
5	4.5	20.9	74.6	3.4	19.5	77.1	4.2	22.1	73.7
6	0.1	0.5	99.4	0.1	0.3	99.4	0	0.3	99.6

	420 sec VOLUME %			600 sec VOLUME %			999 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	74.8	25.2	0.0	73.4	26.6	0.0	72.6	27.4	0.0
2	46.7	53.3	0.0	44.6	55.4	0.0	46.5	53.5	0.0
3	24.6	75.4	0.0	25.8	74.2	0.0	27.3	72.7	0.0
4	10.9	85.9	3.2	13.4	83.8	2.7	11.4	85.1	3.5
5	2.3	24.3	73.4	1.8	22.8	75.5	2.1	24.9	73.1
6	0.1	0.1	99.7	0	0.4	99.5	0.1	0.1	99.7

Table. B12 Raw kinetic data for T1=26cS

	30 sec VOLUME %			45 sec VOLUME %			60 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	45.6	53.5	0.9	48.3	51.7	0.1	50.6	49.4	0.1
2	32.7	56.9	10.4	38.0	60.8	1.3	40.3	59.4	0.3
3	28.2	46.8	25.0	30.8	54.8	14.4	30.6	62.2	7.2
4	26.2	41.9	31.9	25.4	42.6	32.0	25.8	48.0	26.3
5	25.1	43.5	31.4	17.9	40.5	41.6	16.3	38.2	45.5
6	9.2	22.5	68.4	5.3	13.9	80.7	1.7	5.9	92.3

	90 sec VOLUME %			120 sec VOLUME %			150 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	55.6	44.4	0.0	60.1	39.9	0.0	62.6	37.4	0.0
2	38.3	61.7	0.0	43.9	56.1	0.0	44.0	56.0	0.0
3	31.7	67.9	0.4	31.6	68.3	0.1	28.4	71.6	0.0
4	29.4	60.5	10.1	20.7	75.1	4.1	22.1	75.4	2.5
5	6.7	25.5	67.8	5.8	20.7	73.5	5.7	25.8	68.5
6	0.1	0.4	99.3	0	0.3	99.6	0.0	0.8	99.2

	180 sec VOLUME %			240 sec VOLUME %			300 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	64.8	35.2	0.0	66.9	33.1	0.0	67.4	32.6	0.0
2	45.0	55.0	0.0	48.0	52.0	0.0	49.2	50.8	0.0
3	28.2	71.8	0.0	27.5	72.5	0.0	26.6	73.4	0.0
4	17.4	77.8	4.9	14.9	81.2	3.9	14.3	81.9	3.8
5	5.8	23.4	70.8	3.6	22.3	74.1	3.2	21.5	75.3
6	0.2	0.2	99.6	0	0.2	99.7	0.1	0	99.8

	420 sec VOLUME %			600 sec VOLUME %			999 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	69.8	30.2	0.0	70.7	29.3	0.0	73.3	26.7	0.0
2	49.4	50.6	0.0	48.0	52.0	0.0	50.9	49.1	0.0
3	26.2	73.8	0.0	26.4	73.6	0.0	24.9	75.1	0.0
4	12.3	83.7	4.0	13.5	82.8	3.7	9.9	86.9	3.2
5	2.5	23.1	74.4	2.0	22.4	75.7	1.2	19.8	79.0
6	0	0.8	99.1	0	0.2	99.7	0	0	100

Table. B13 Raw kinetic data for T1=30cS

	30 sec VOLUME %			45 sec VOLUME %			60 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	50.9	48.6	0.5	49.8	50.0	0.2	55.8	44.2	0.0
2	34.9	60.7	4.3	35.7	63.7	0.6	41.3	58.6	0.1
3	26.9	49.8	23.3	32.7	60.9	6.4	33.4	62.6	4.0
4	22.1	43.5	34.4	27.9	43.9	28.2	21.8	59.7	18.5
5	22.8	41.9	35.4	15.8	35.6	48.5	11.5	34.1	54.4
6	8.2	19.8	72.0	2.1	8.1	89.6	1.7	4.0	94.3

	90 sec VOLUME %			120 sec VOLUME %			300 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	57.0	43.0	0.0	58.3	41.7	0.0	67.6	32.4	0.0
2	40.3	59.7	0.0	43.3	56.7	0.0	51.4	48.6	0.0
3	34.3	65.5	0.3	33.4	66.6	0.0	27.9	72.1	0.0
4	24.2	69.4	6.4	22.8	73.5	3.7	11.8	84.4	3.8
5	5.9	21.9	72.2	5.3	23.2	71.5	2.0	22.0	75.9
6	0.1	0.8	99.0	0.1	0.6	99.3	0	0	100

	420 sec VOLUME %			720 sec VOLUME %			999 sec VOLUME %		
Layer	red	green	boro	red	green	boro	red	green	boro
1	68.9	31.1	0.0	67.6	32.4	0.0	69.1	30.9	0.0
2	51.6	48.4	0.0	53.0	47.0	0.0	50.3	49.7	0.0
3	26.9	73.1	0.0	30.0	70.0	0.0	29.3	70.7	0.0
4	12.0	85.1	2.9	9.7	87.0	3.4	11.0	85.8	3.1
5	2.3	24.6	73.1	1.8	25.7	72.5	2.2	28.3	69.5
6	0.1	0.3	99.5	0	1.2	98.7	0	0.5	99.4