

https://dspace.org/

Evaluating the diversification effects of direct real estate within a multi asset portfolio of shares and bonds

Singh, Vijay

2018

https://hdl.handle.net/10539/26927

Singh, Vijay (2018) Evaluating the diversification effects of direct real estate within a multi asset portfolio of share and bonds, University of the Witwatersrand, Johannesburg, https://hdl.handle.net/10539/26927

Downloaded from a DSpace repository. https://dspace.org/

UNIVERSITY OF THE WITWATESRAND

Evaluating the Diversification Effects of Direct Real Estate within a Multi Asset Portfolio of Shares and Bonds

Research Report

Vijay Singh

0515791H

A research report submitted to the University of the Witwatersrand, Faculty of Engineering and the Built Environment, School of Construction Economics and Management in partial fulfilment of the requirements for a Master of Science degree in Building.

October 2018

Contents

DECLARATION 5
ACKNOWLEDGEMENTS 6
ABSTRACT 7
CHAPTER 1 8
Background to the Study 8
Real Estate as an Asset Class 9
Diversification with Real Estate 9
Multi Asset Portfolios with Real Estate 10
The South African Context for Real Estate Investment. 11
Rationale of Study 12
Aim of Study 13
Objective of Study 13
Statement of the Research Problem 13
Research Questions 14
Scope 14
Delineations 15
Limitations 16
Assumptions 16
CHAPTER 2 18
Literature Review 18
Preamble 18
Theoretical Framework 19
Modern Portfolio Theory 19
Constraints of Modern Portfolio Theory in the Real Estate Sector 21
Liquidity Problems and Modern Portfolio Theory 24
Transaction Costs and Modern Portfolio Theory 24
Allocation, Optimisation and Efficient Frontiers 25
Conceptual Review 29
Real Estate as an Asset Class 29
Direct Real Estate 30
Indirect Real Estate (REITs) 32
Real Estate Return and Risk Characteristics 34
Return Characteristics 34

Risk Characteristics 36
Valuation of Real Estate Assets 38
Appraisal Smoothing 41
Inflation hedging characteristics of Real Estate 43
Portfolio Diversification with Real Estate 48
Real Estate in Multi Asset Portfolios 48
Optimal and Efficient allocation of Real Estate to Portfolios 52
Summary of Literature Review 53
CHAPTER 3 54
Research Design 54
Introduction 54
Data Sources 56
Ethical Considerations 57
Data Analysis and Interpretation 58
Descriptive Statistics 58
CHAPTER 4 65
Data Analysis and Findings 65
Introduction 65
Presentation of Data Analysis 65
Analysis of Data and Presentation of Results 66
Sub Section 1: Portfolio Series 1- 20 Years – 1995 -2014 66
Table 1: Descriptive Statistics 66
Chart 1: Returns of Asset Classes 67
Table 2: Asset Allocation and Performance of Constructed naïve Portfolios 68
Chart 2: Ten Naïve Portfolios 69
Table 3: Optimum Portfolios 70
Chart 3: Efficiency Frontier 70
Findings and Results 71
Summary: Portfolio Series 1- 20 Years – 1995 -2014 74
Sub Section 2: Portfolio Series 2- 10 Years – 1995 -2004 75
Table 4: Descriptive Statistics for 10 years (1995-2004) 75
Chart 4: Returns of Asset Classes 76
Table 5: Asset Allocation and Performance of Constructed naïve Portfolios. 77
Chart 5: Ten Naïve Portfolios 78
Table 6: Optimum Portfolios 79
Chart 6: Efficiency Frontier 79
Findings and Results 80

Summary: Portfolio Series 2- 10 Years – 1995 -2004 83
Sub Section 3: Portfolio Series 3- 10 Years – 2005 -2014 84
Table 7: Descriptive Statistics for 10 years (2005-2014) 84
Chart 7: Returns of Asset Classes 85
Table 8: Asset Allocation and Performance of Constructed naïve Portfolios. 86
Chart 8 – Ten Naïve Portfolios 87
Table 9: Optimum Portfolios 88
Chart 9: Efficiency Frontier 88
Findings and Results 89
Summary: Portfolio Series 3- 10 Years – 2005 -2014 92
Comparative Analysis of Efficient and Optimal Portfolios for the Three Time Series 93
Table 10: Reward and Risk Analysis of the Optimal Portfolios 94
Chapter 5 95
Conclusion 95
Summary of Findings 95
Summary Analysis of the Portfolios 95
Comparison of results to past literature. 97
Future Research Topics 98
Concluding Comments 98
References 99
Annexure A - Portfolio Series 1 – 20 Years – 1995-2014 106
Annexure A1: JSE All Share Data 107
Annexure A2: SA Government Ten Year Bonds 108
Annexure A3: MSCI All Property Index 114
Annexure A4: Portfolio 1 A 115
Annexure A5: Portfolio 1 B 117
Annexure A6: Portfolio 1 C 119
Annexure A7: Portfolio 1 D 121
Annexure A8: Portfolio 1E 123
Annexure A9: Portfolio 1 F 125
Annexure A10: Portfolio 1 G 127
Annexure A11: Portfolio 1 H 129
Annexure A12: Portfolio 1 I 131
Annexure A13: Portfolio 1 J 133
Annexure A14: Portfolio Maximum Returns 135
Annexure A15: Portfolio Minimum Risk 137
Annexure B - Portfolio Series 2 – 10 Years – 1995-2004 139

Annexure B1: JSE All Share Data 140
Annexure B2: SA Government Ten Year Bonds 141
Annexure B3: MSCI All Property Index 144
Annexure B4: Portfolio 2 A 145
Annexure B5: Portfolio 2 B 147
Annexure B6: Portfolio 2 C 149
Annexure B7: Portfolio 2 D 151
Annexure B8: Portfolio 2E 153
Annexure B9: Portfolio 2 F 155
Annexure B10: Portfolio 2 G 157
Annexure B11: Portfolio 2 H 159
Annexure B12: Portfolio 2 I 161
Annexure B13: Portfolio 2 J 163
Annexure B14: Portfolio Maximum Returns 165
Annexure B15: Portfolio Minimum Risk 167
Annexure C - Portfolio Series 3 – 20 Years – 2005-2014 169
Annexure C1: JSE All Share Data 170
Annexure C2: SA Government Ten Year Bonds 171
Annexure C3: MSCI All Property Index 174
Annexure C4: Portfolio 3 A 175
Annexure C5: Portfolio 3 B 177
Annexure C6: Portfolio 3 C 179
Annexure C7: Portfolio 3 D 181
Annexure C8: Portfolio 3 E 183
Annexure C9: Portfolio 3 F 185
Annexure C10: Portfolio 3 G 187
Annexure C11: Portfolio 3 H 189
Annexure C12: Portfolio 3 I 191
Annexure C13: Portfolio 3 J 193
Annexure C14: Portfolio Maximum Returns 195
Annexure C15: Portfolio Minimum Risk 197

DECLARATION

I declare that this research report is my own work, and is submitted in partial fulfilment of the requirements for a Master of Science degree in Building at the School of Construction Economics and Management in the University of the Witwatersrand Johannesburg. It has not been submitted before for any degree or examination at any other university.

Vijay Singh

26 October 2018

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my supervisor, Dr. Omokolade Akinsomi for his understanding, time and enduring patience during the preparation of this research report.

ABSTRACT

This study examined the diversification benefits of including direct real to a multi asset portfolio of stocks and bonds. Modern Portfolio Theory provided the theoretical frame work for construction of thirty multi asset portfolios, over a twenty year time period from 1995 to 2014. The methodology included the calculation of optimal portfolios, efficient barriers, returns, risks and correlation of the asset classes. The results of this study indicate that direct real estate provides significant diversification benefits to multi asset portfolios and furthermore direct real estate proved to be a diversifier of risk and a return enhancer over the time period studied.

CHAPTER 1

Background to the Study

Institutional investors devote substantial amounts of time researching and forecasting the returns of individual assets and asset classes for the inclusion in multi asset investment portfolios. Direct real estate is an asset class that should be considered for inclusion in multi asset investment portfolios. (Smietana, 2014), stated that institutional investors should include property in their investment portfolios, due to the important investment attributes of property such as, satisfactory returns on investment (high gross margin profitability of total capital), low price volatility , high returns in relation to risk, potential for protection against inflation and deflation, stable revenue components, predicable component of operating expenses and low correlation with other assets, which allows for the diversification of the portfolio.

(Adair, McGreal & Webb, 2006), established that direct real estate, possess the attributes of consistency in optimal multi asset portfolios, over time periods ranging from five to twenty five years. Real estate enhances returns and reduces risk of the portfolio and real estate should be considered as a strategic asset especially for investors with long holding periods.

Within the context of institutional portfolios, where diversification, consistency of returns , predictability of returns , risk adjusted returns and inflation hedging are part of the overall investment strategy, direct real estate should be considered as an attractive asset class for investors. Commercial real estate is one of the most important asset classes in institutional investment portfolios, (Fisher & Goetzman, 2005).

Real Estate as an Asset Class

Real Estate, land and buildings is constituted as part of the overall investment universe (Hudson- Wilson, Fabozzi & Gordon, 2003), and as such should be considered an investment option for investors. Real Estate is considered a unique asset class, with specific underlying characteristics, which include, a physical and tangible nature, rights of use, regular income and capital appreciation. Other attributes of real estate as an attractive asset class include, low price volatility, high returns in relation to risk, potential protection against inflation and deflation, predictable components of operating expenses and low correlation with other asset classes, which allow for diversification within multi asset portfolios, (Smietana, 2014).

Real restate provides an additional benefit of diversification, as it provides intra asset diversification, by property type and geographic location.

Real estate, as an asset class can be divided into two sub-asset classes. Firstly as a tangible asset, in the form of land and building, referred to as direct real estate. This asset class provides regular income in the form of rental income and capital appreciation. Secondly real estate as an equity or share, in the form of Real Estate Investment Trust, which trades on established securities exchanges and provides dividend payments and capital appreciation.

Diversification with Real Estate

Real Estate as an asset class attracts substantial attention from both institutional and individual investors. Investment managers include real estate into their portfolios to achieve diversification and superior returns, (Fuerst, Liu & Lizieri, 2016). Portfolios containing multiple assets, have to be constructed in a diversified way in order to avoid risk, which may deprive investors from realizing the return that they are expecting (Bakri, 2014). Modern Portfolio Theory provides the methodology for investors to construct diversified risk adjusted portfolios. It, is a theory of investment finance that provides a methodology to maximize portfolio expected returns for a given amount of portfolio risk or conversely minimizes risk for a given level of expected return.

(Markowitz,1952) pioneering publication on Modern Portfolio Theory, proved mathematically that by diversifying investments the investor can lower the risk of the investment portfolio, or conversely earn a higher return for the same amount of risk as an undiversified portfolio. (Black, 2004).

Multi Asset Portfolios with Real Estate

Asset allocation and portfolio theories show that the inclusion of real estate investments in traditional equity and bond portfolios can provide diversification benefits, (Fuerst, Liu & Lizieri, 2016). Several real estate scholars have investigated the role of real estate as an asset classes within a multi asset portfolio of common shares and bonds. The dominant theme of this research, focuses on the performance and diversifications benefits of real estate with in the portfolio. Researchers on this topic included, (Brueggeman, Chen & Thibodeau, 1984), (Hartzell, Hekman & Mile, 1986), (Adair, McGreal & Web, 2006), (Olaleye, Aluko & Oloyede, 2008), (Akinsomi & Ntuli, 2017).

Other researches, explored specific aspects and perspectives of real estate as an asset class. These included, the optimum allocation of real estate to an efficient multi asset portfolios. (Sivitanides,1997). Investor's time horizons, when including real estates to multi asset portfolios, (Rehring, 2012). Diversification of real estate portfolios by geographic location and property type, (Seiler, Webb & Myer, 1999). (Cheng, 2005), examined the risk and return characteristics of real estate investments.

The investment case for direct real estate, to be included to a multi asset portfolio of common stocks and bond, was validated by many researchers. (Ziering, & McIntosh, 1997), (Black 2004) and (Hudson, Fabozzi & Gordon, 2003) concluded that direct real estate reduced portfolio risk and volatility and improved risk adjusted returns. (Brueggeman, Chen & Thibodeau, 1984), found that direct real estate outperformed stocks and bonds, offered diversification benefits and provided a good hedge against inflation. Direct real estate provided consistency of risk adjusted return and portfolio optimisation benefits over a twenty five year period in multi asset portfolios, (Lee and Stevenson, 2006). (Ziobrowski & Ziobrowski, 1997), concluded that real estate has historically provided rates of return that compared favourably with common stocks and bonds, but at remarkably low levels of risk. Real estate has typically demonstrated very limited positive correlation with both common stocks and bonds. A further advantage of investing in direct real estate, is that this asset class provides a relative predicable income stream, from rental income and lease periods may be extended for long periods with inflation related escalations, (Hoesli & Lekander, 2007).

The South African Context for Real Estate Investment.

(Akinsomi, Mkhabela & Taderera, 2018), stated that the South African commercial market attracts local and international public and private investors such as pension funds, financial institutions, retailers, property developers and syndications. Properties are unique and so are investors but the common goals in real estate investments are to gain profits in a form of returns and diversify risks. It is essential for investors to know the factors that affect the sector's performance to make informed investment selections.

The South African real estate investment environment, consists primarily of direct real estate owned by companies and real estate securities traded on the Johannesburg Securities Exchange.

As at December 2017, there are approximately fifty eight real estate companies listed on the securities exchange and the majority of these companies, (thirty seven) have adopted the Real Estate Investment Trust or REITS, structure to their securities. The market capitalisation of these real estate companies is estimated at around 400 Billion Rand, (33.3 Billion US Dollars), according to (sareits.co.za).

The capital value of direct commercial real estate is estimated at approximately 287 Billion Rand (24,5 Billion US Dollars) in South Africa (MSCI IPD South Africa Property Index Report, 2017).This commercial real estate sector consists, primarily of four sectors, namely retail, office, industrial and others.

The retail sector is the largest at 166 Billion Rand (14.2 Billion US Dollars) or fifty seven percent of the total value of the real estate, followed by the office sector at 70 Billion Rand (6 Billion US Dollars) or twenty five percent. The industrial sector is valued at 38 Billion Rand (3.2 Billion US Dollars) or fourteen percent and the value of the remaining property classified other is valued at 11 Billion Rand (1 Billion US Dollars) or five percent of the total value of real estate in South Africa. (MSCI IPD South Africa Property Index Report, 2017).

Rationale of Study

Substantial academic literature exists on the performance of real estate as an investments asset class. While these studies provide a wealth of knowledge on the topic, they tend to focus on the developed world.

Within the South African context, research has focused on topics on other real estate issues. (Akinsomi & Ntuli (2017). An Overview of the Initial Performance of the South African REIT Market, (Akinsomi, Balcilar, Demirer & Gupta, 2016). The effect of gold market speculation on REIT returns in South Africa: a behavioural perspective. (Akinsomi, Mkhabela, & Taderera, 2018). The role of macro-economic indicators in explaining direct commercial real estate returns: Evidence from South Africa. (Akinsomi, Katlego, Ndlovu & Motloung, 2016). The Performance of Broad-Based Black Economic Empowerment Listed Property Companies in South Africa.

There are limited empirical studies publishes on the topic of the diversification benefits of direct real estate in multi asset portfolios in a developing country such as South Africa. (Olaleys, 2011), researched the effects of adding real estate into mixed-asset portfolios in South Africa. This research provided significant insight into the diversification benefit of adding listed property stocks to a mixed portfolio of shares and bonds. This study therefore aims to fill that gap by examining the performance benefits, if any, of including direct real estate to a multi asset portfolio of shares and bonds.

This research report, has implications for institutional investors, such as pension fund managers, private equity fund managers and asset managers, managing large portfolios of investor funds. The managers of these funds employ various strategies in order to meet their investment objectives. Diversification between asset classes is one of such strategy which can be employed by instructional investors to achieve their desired investment objectives.

A multi asset portfolio can potentially provide diversification benefits as each asset class encompasses specific factors and characteristics which influences the risk and returns dynamics. Combining, asset classes with low correlation with each other may provide enhances performance benefits to multi asset portfolios.

Previous research indicates that direct real estate provides diversification benefits for multi asset portfolios, (Black, 2004), (Brueggeman, Chen & Thibodeau, 1984).

(Hudson- Wilson, Fabozzi, & Gordon, 2003), (Hartzell, Hekman & Miles, 1986), (Adair, McGreal & Webb, 2006).

Diversification of investment portfolios through diversification by asset classes results in dispersion of risk and achieves an appropriate balance between expected return and an acceptable level of investment risk, (Smietana, 2014). The principals of risk and return are fundamental to the investment environment and by applying the theory of diversification investors can achieve the balance between expected returns and risks of the investment.

The issue of allocation or weighting of assets within the portfolio will be addressed to determine the most optimum allocation of the selected asset class within the portfolio. Optimisation in terms of portfolio theory seeks to deliver the risk and reward characteristics preferred by investors.

Aim of Study

The aim of the study is to evaluate the effects of diversifying a multi-asset portfolio of shares and bonds with direct real estate. This is with a view of providing information for investment decisions.

Objective of Study

The objective of this study is to determine the type of research that has been undertaken on the research topic, gain knowledge on the area on interest and understand the framework and methodologies utilised by other researchers.

On completion of this study, the objective is to determine whether direct real estate adds value to a multi the portfolio of shares and bonds in terms of risk reduction and / or return enhancement.

Statement of the Research Problem

Financial investing, generally entails the commitment of a lump of sum of capital, for future streams of income and or capital appreciation. In investment markets, investors are generally faced with numerous alternative investments where capital could be invested. These include shares, government and company bonds, unit trusts, bank deposits, real estate, private equity and commodities. The diversity of investment options eventually creates a problem of choice and is one of the fundamental problems of the investment decision making. This requires investors to choose from a myriad of

investment opportunities and the strategic allocation of capital to these investment opportunities.

The establishment of appropriate criteria and an analytical framework for assessing investment options is fundamental to a successful investing strategy. Success in investing, is mostly dependent upon finding strategically appropriate investment opportunities and being able to accurately forecast their future performance.

Investing in direct real estate, is one asset class that should be considered by investors, as direct real estate returns are determined by factors different from those affecting the returns of other asset classes and hence may produce diversification benefits. Real estate may offer a hedge against inflation and this positive attribute may warrant inclusion in multi asset portfolios. (Hudson- Wilson, Fabozzi, & Gordon, 2003).

(Hartzell, Hekman & Miles, 1986), stated that real estate investments offer higher returns than other financial investments over the studied period. In addition, the real estate returns are less variable than those of other classes. Real estate can offer significant diversification benefits, when added to a portfolio of common stocks and bonds.

Research Questions

1. What are the return and risk characteristics of direct real estate.
2. What are the diversification effects of combining direct real estate with shares and bonds within the South African context.
3. What combination of these assets could offer optimal strategies for the multi asset portfolio.

Scope

This study focused on the investment characterises of direct real estate within a multi asset portfolios, consisting of common stocks and bonds within the South African investment environment.

Common stocks are an equity based investment option whilst bonds are debt based investment option. These two options are traditional investment options and are integral to the investment universe.

Common stocks, were represented by the FTSE/JSE All- Share Index. This index represents ninety percent of the full market capitalization of all ordinary securities or share listed on the main board of the JSE.

The South African Reserve Banks, ten year Government Bonds, were used as a proxy for Bonds.

The Morgan Stanley Capital International, (MSC) I/ International Property Data Bank's South African Property Index, were used as the proxy for investment performance of direct real estate in South Africa.

The investment data of the asset classes were analysed, for a period of twenty years from 1995 to 2014, as this time frame represents the availability of data for direct real estate.

A series of three sets of portfolios were constructed, the first series was based on a twenty year time period and the second and third series was based on two differing ten year periods. The three time periods were selected to determine any substantial changes to the outcomes of the portfolios based on the different time periods.

Delineations

The application of Modern Portfolio Theory, will formed the fundamental theory for this research report as this theoretical framework has been used extensively on this research topic. (Brueggeman, Chen & Thibodeau, 1984), (Hartzell, Hekman & Mile, 1986), (Adair, McGreal & Webb, 2006), (Olaleye, Aluko & Oloyede, 2008), (Olaleye, 2011).

The research report focused on the South African investment environment. All other geographies and countries will be excluded from this report.

Asset management fees, transaction cost, dividends and applicable taxes are excluded from this research report, as these cost vary between financial intuitions, and are generally excluded from this type of analysis.

The asset classes selected for research within multi asset portfolios are direct real estate, common shares and bonds. Common shares and bonds are deemed traditional investment asset classes and form a sizeable portion of the investment landscape held by institutional managers and pension funds. Similar studies have been undertaken by other researches. (Ziering & McIntosh, 1997) and (Black 2004), (Hudson, Fabozzi & Gordon, 2003), (Olaleye, 2011).

Commercial real estate, denotation retail, office and industrial properties were analysed as the asset class representing direct real estate. Residential real estate is excluded from this research report, as the data compiled in the Morgan Stanley Capital International, (MSCI)/ International Property Data Bank's South African Property Index excludes any residential data.

The analysis of Real Estate Investment trusts, within a multi asset portfolio is excluded from this research report as, prior research has been undertaken on this topic (Olaleye, 2011) and (Akinsomi & Ntuli, 2017) an overview of the initial performance of REITs in South Africa.

Limitations

Investment data, was reviewed and analysed for a period of twenty years from 1995 to 2014. This is due to the availability of investment information for direct real estate, which is limited to this time frame.

The annual returns of direct property, was utilized by the following researchers in their work on benefits of diversification with direct real estate within multi asset portfolios.

(Adair, McGreal & Webb, 2006), used annual data of real estate returns for all UK properties, from 1975 to 2003, obtained from the UK, IPD data base. (Sivitanides, 1997), used the annual returns of stocks, bonds and real estate from the period of 1978 to 1995.

(Olaleye, Aluko & Oloyede, 2008), used the annual holding returns of capital value and rental transaction for direct real estate from 1998 to 2003.

Assumptions

The South African All Share Index (ALSI) was used as a proxy for common shares, as this index represents 99% of the full market capital value of all ordinary securities listed on the main board of the Johannesburg Stock Exchange, the regulated securities exchange in South Africa. The FTSE/JSE All Share Index is a major stock market index which tracks the performance of all companies listed on the Johannesburg Stock Exchange in South Africa. It is a free-float, market capitalization weighted index.

The South African ten year bond, represented a proxy for the bonds. The South African Reserve Bank act as an agent for the government in placing government bonds on the market. Therefore, data on the ten year government bond will be sourced from the

South African Reserve Bank for ten year government bonds ,represented by the banks KBP 2003M, ten year bond yield.

The Morgan Stanley Capital International, (MSCI) / International Property Data Bank's South African Property Index, will used as the proxy for returns of direct real estate in South Africa.

The Morgan Stanley Capital International, (MSCI) is an international research organisation, which conducts and publishes real estate investment data in thirty three countries over twenty five years and is an authoritive research organisation which publishes investment data on direct real estate in South Africa.

The Morgan Stanley Capital International,(MSCI) International Property Data Banks, South African Property index is a measure of ungeared total returns on direct held real estate from one open market valuation to another. The index track the performance of 1 635 real estate investments with a total capital value of 287 Billion Rand as at 2017.

CHAPTER 2

Literature Review

Preamble

The literature review will cover three key themes. The theoretical framework of Modern Portfolio Theory. A conceptual review of real estate as an asset class and portfolio diversification with direct real estate. The organisation and arrangement of the literature review follows the following format.

The first section on the theoretical framework will include subsections on modern portfolio theory, allocations and optimisation and efficient frontiers. The second section a, conceptual review of real estate as an asset class will cover sub sections on direct real estate, indirect real estate, returns and risk characteristics of real estate, valuation and appraisal smoothing and inflation hedging and finally a section on portfolio diversification with direct real estate.

Theoretical Framework

Modern Portfolio Theory

(Markowitz, 1952) innovative publication on Modern Portfolio Theory, proved the old adage that “Don’t put all your eggs into one basket.” Investors seeking strategies how best to allocate capital across asset classes and assets can employ the principals of Modern Portfolio Theory as this financial theory can provide a methodology for the prudent allocation of capital across multi asset class. This theory attempts to maximize portfolio expected returns for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets.

Modern Portfolio Theory provides a mathematical framework of the concept of diversification of investments, with the objective of selecting a collection of assets that has collectively lower risks than the weighted average of the individual asset. The primary building blocks of Modern Portfolio Theory or mean – variance portfolio analysis are the measure of historic returns, risks (volatility) and covariance of the asset or asset classes.

Risk or the variance of returns are measured by the standard deviation of the asset. Standard deviation is the amount by which returns vary over time around the average mean. Covariance of the returns of the various assets are determined, in order to discover the relationships between the returns of the assets. Are the returns positively or negatively correlated? A positive correlation implies that the returns of the assets move together and a negative correlation implies that the returns move inversely to each other. By combining assets which are negatively correlated with each other, non-systematic risk or portfolio risk can be reduced. There by providing diversification benefits to the portfolio.

(Black, 2004), stated that Markowitz, essentially proved a mathematically that by diversifying investments , the investor can lower the risk of the investment portfolio, or conversely , earn a higher return for the same amount of risk as an undiversified portfolio and that Markowitz gave us the tools by which we could measure the benefits of diversification.

(Penny, 1982), stated three interesting insights into Markowitz Modern portfolio theory. These include, firstly a better understanding of the relationship between risk and reward, secondly improving portfolio selection by establishing procedures for

combining securities in portfolios so as to optimize the risk – return relationship and finally to perfect performance measurement techniques in order to distinguish between better and weaker portfolios in terms of risk- return relationships.

(Geltner & Miller, 2001), stated the following insights to Modern Portfolio Theory. In this theory the objective of the macro level investment decision maker is taken to be the minimization of portfolio volatility (or variance) subject to an expected return target. In fact, this objective is mathematically equivalent to its dual description: the maximization of portfolio expected return subject to a volatility constraint.

Modern Portfolio Theory deals with risk and returns in a comprehensive and integrated manner, it quantifies the investment decision relevant implications of risk and return and it makes both of these contributions at the portfolio level, the level of the investors' overall wealth.

(Ziering & McIntosh, 1997), suggested that Modern Portfolio Theory addressed the investment rational for portfolios diversification and seeks the best methods for achieving an optimal mix of assets. The goal is to maximize risk- adjusted return by including investments, or investments classes, whose expected returns are negatively correlated. A power fact of investment portfolio performance is that total portfolio risk is lower than the sum of the risks of each individual asset when the correlation between assets fall below 1. The lower the correlation, the greater the risk reduction. Any two assets whose pricing moves in exactly the same way have a correlation coefficient of +1. The combination of these assets in a portfolio does not reduce portfolio risk (volatility). Two assets whose pricing movements bear no relationship with one another display 0.00 correlation, while assets whose prices move exactly the opposite direction displays a correlation of -1.0. In the latter case, portfolio risk is reduced thought diversification.

(Byrne, & Lee, 2004), provided an analysis of the potential weakness of Modern Portfolio Theory. Modern Portfolio Theory established a quantitative framework for assets selection into a portfolio that is now well known. In this, it is assumed that the assets follow a multivariate normal distribution or the investors have quadratic utility function. This approach shows that characteristics of a portfolio of assets can be completely described by the mean and the variance (risk) and is described as the mean – variance (MV) portfolio model.

For a particular universe of assets, the set of portfolios of assets that offer the minimum risk for given level of return from the efficient frontier. The portfolios on the efficient frontier can be found by quadratic programming and such can now be solved easily in

spreadsheets programs. The solutions are optimal and the selection process can be constrained by practical considerations, such as upper and lower bounds, which can be written as linear constraints.

The weakness of the MV approach, however, is that the underlying assumptions of multivariate normality or that investors have a quadratic utility function are not sustainable. This has led researcher to develop portfolio asset allocation models using other measures of risk that have many theoretical and practical advantages over MV. Even so the MV approach remain the most popular approach to the asset allocation problem.

In order to overcome the difficulties associated with the MV model, Markowitz and others proposed semi-variance. The semi – variance concentrates on the returns below the mean (expected return).

Constraints of Modern Portfolio Theory in the Real Estate Sector

(Byrne & Lee, 1995), provided theoretical constraints in the application of Modern Portfolio Theory to portfolios containing real estate.

Firstly the application of the Markowitz model requires “forecast” values of expected returns and volatility for the future period. Although these values could be derived subjectively, or objectively by using forecasting techniques, these measures are more typically estimated by calculating their historic values, and the analysis is thus ex-post. Extreme results may arise when such an optimizing model is applied to a multi – asset portfolio including property, with some assets taking zero weights while other have very large allocations. Although the resulting portfolios are optimal in the statistical sense, the results would be unacceptable to any prudent portfolio manager.

These unrealistic results stem from a number of now well- recognized problems. First, most measures of property return are appraisal rather than market based, and there is a strong view that as a consequence , the volatility of property returns has been considerably underestimated. As a consequence when optimizers consider property as part of the portfolio, these highly attractive features of appraisal- based returns have made property dominant in those studies undertaken using such data. These results sit uncomfortably with most investors and are therefore dismissed out of hand, with the fault firmly placed on the quality of the property data, as not truly representative of investor experience. The case for property therefore needs to be reassessed with more acceptable data in a more representative context.

Secondly potential problem has been identified, in the use of the Markowitz portfolio optimisation tool itself. Optimizers are essentially “estimation –errors maximizes” in that risk and return inputs are invariable subject to estimation error, even using historic data. Optimizers, in searching for those assets which increase portfolio returns and reduce risk, prefer assets with high expected returns and low volatility. Modern Portfolio Theory, optimizers significantly overweight / underweight those assets with large / small estimated returns, negatively / positively correlations and small / large variances. Such assets are more likely to have the largest estimation errors. Moreover, the problem is exacerbated as the number of assets increase. The errors accumulate rather than cancelling one another out.

The above in turns to two difficulties, Firstly the expected return of the efficient portfolio is overstated, while the risk is understated. This problem can be addressed to some extent by adjusting the estimate of risk and return by incorporating as much information as possible about the deviation of the return series in the case of property. This has recently led to the development of a number of methods for desmoothing returns to pick up property’s true risk/return characteristics from an original appraisal- based return series. Previously, the volatility of property return series has been subjectively increased in an attempt to compensate for the downward bias in property returns without any attempt to rationalize the reasons for the level of volatility chosen.

The third problem is more pernicious. Not only are the efficient portfolio’s risk and return mis-estimated, but this will be done for the wrong portfolio. Essentially, a number of assets for which expected returns have been underestimated and risk has been overestimated will be excluded and will be replaced by assets with estimates of risk and return which have erred in the opposite direction. The elimination of some assets, and the over emphasis placed on other, seem contrary to the spirit of diversifying across a large number of assets, and results from the straight application of the unconstrained Markowitz model. The clear implication of this is the representative constraints need to be applied in the optimization process. This can be done by imposing upper and / or lower bounds on the proportion of the portfolio allocated to any single asset, as would be expected in a real world situation.

(Ping, 2005), questioned the two underlying assumptions of Modern Portfolio Theory, that assets are normally distributed and investors view extremely high and extremely low returns as equally undesirable.

Rational investors certainly prefer extremely high returns to extremely low returns, most investors perceive risk as the only chance of earning less than certain target of

return. The potential of earning better than expected returns, on the other hand, is viewed as favourable upside potential. Such an asymmetric view towards risk implies that assets returns need not to be normally distributed. In fact, all things being equal investors will prefer assets distributions with small left tails (negative skewness).

The relevant risk measure, therefore should capture only the downside risk and or the skewness of the asset return distribution. In the context of portfolio management, this means that the inclusion of an asset depends on whether it reduces the downside risk or increases the negative skewness of a portfolio. Standard deviation becomes irrelevant in such an asymmetric world.

(Ping, Zhenguo & Yhang, 2011), studied the nonconformity characteristics of real estate, when applying the principals of Modern Portfolio Theory, to portfolios with real estate, their findings concluded that private real estate does not conform to the high liquidity, informational- efficiency, and frictionless public security market upon which Modern Portfolio Theory was premised, this knowledge is often perceived as perhaps being too basic and encompassing to be practically useful, and it has typically not prevented researchers from treating real estate indifferently from other financial assets in portfolio analysis. But after so much effort for so long, the ever- elusive target of optimal real estate allocation suggests it is perhaps time that this issue be taken more seriously , and investigate the implications of the nonconformity of real estate to the classic finance paradigm in portfolio analysis.

The complex nature of real estate, as well as the complexities of its performance measurements, implies that mixed asset portfolios theory will be more complex that the classical Modern portfolio Theory. In essence, because mixed – assets portfolios have illiquid real estate asset, the overall performance of the portfolios should be also horizontal- dependent and subject to illiquidity risk. Optimal diversification, therefore should be based on multi- period utility maximization, as opposed to the classic single period optimization. Strike an optimal balance between the portfolio's price volatility and its illiquid risk to maximize the portfolio's overall risk – adjusted returns. It is worth noting that illiquidity risk adds another dimension to mixed- asset portfolio decisions. Given that two assets are perfectly correlated, the traditional thinking is that more assets are always better because it reduces the overall portfolio risk. However for a portfolio that includes illiquid real estate assets, more may not be necessary better because the increase illiquidity risk may outweigh the reduction of price volatility. This implies that there may exist an optimal size for the mixed asset portfolio.

Liquidity Problems and Modern Portfolio Theory

(Seiler, Webb, & Myer, 1999), evaluated the constraints of Modern Portfolio Theory. Where the principals of liquidity and transaction costs were reviewed in term of Modern Portfolio Theory.

One of basic Modern Portfolio assumptions is that assets are perfectly divisible. This assumption is made so that exact portfolio allocations can be rendered. Direct real estate is the perfect example of an asset class that thoroughly violates this assumption. The selling of a property involves, on average millions of dollars at a time, as opposed to the infinitely small amount that is assumed in modern portfolio theory.

It is widely known and agreed on that direct real estate also suffers from much lower levels of liquidity than other asset classes. This stems primarily from the venue in which both are traded. Stocks, bonds, treasury securities and REITS are traded on continuous action markets (organized exchanges such as the American Stock Exchange and the New York Stock Exchange) or over the counter where even small investors have enough capital to participate in the buying and selling process, (because REIT shares are much less than the cost of an entire property). This greater number of market participants leads to a greater level of liquidity. Direct real estate on the other hand, is sold individually through negotiations and special arrangements. Because prices of properties are so high due to (indivisibility) and because prices are not publicly communicated, liquidity is much less favourable.

Transaction Costs and Modern Portfolio Theory

The final Modern Portfolio Theory assumption violation concerns the high transaction cost associated with direct estate. This is important when it comes time to rebalance an optimal portfolio. A portfolio must be rebalanced any time a change in expected returns, risks or correlation occurs among any of the assets in the market. Since financial markets are open almost every day, it makes sense that correlations and expectations change every day as well. Hence, optimal portfolios in theory should be rebalanced continuously to reflect continuous changes in risk, returns and inter asset correlations.

From a practical standpoint, however, rebalancing will not occur continuously because of the positive transaction costs associated with rebalancing. Instead, rebalancing should occur only when the cost of rebalancing, is less than the perceived benefit associated with reallocating assets. The greater the cost to rebalance, the less often

the portfolio will be rebalanced, and the greater the amount of time that the portfolio will be out of optimization.

Allocation, Optimisation and Efficient Frontiers

Efficiency barriers or mean variance frontiers are vectors of the expected return of the assets and the standard deviation. This is generally presented in a graphic format, where the Y axis represents the portfolio returns and the X axis represents the standard deviation or volatility of the portfolio, both expressed in percentages. The vector, represents various points of return and risk of the portfolio, (Ziobrowski, Caines & Ziobrowski, 1999).

Optimisation in terms of portfolio theory seeks to identify the best combination and the percentage weighting of allocation of each asset class to hold within the portfolio. This combination of various assets will produce a level of risk and return acceptable or desired by the investor.

(Geltner & Miller, 2001), provided the following explanation on efficiency frontiers. The efficient frontier is a concept within modern portfolio theory. The underlying theory of the efficient frontier is that investment decisions are driven not only by selecting the investment with the greatest return, but the investment that achieves that level of return with the least amount of risk or volatility.

To diversify one's portfolio, investors will combine different assets, such as equities, bonds, property, and other, in different allocations, to form a diversified portfolio of assets that provide the maximum investment return based on one's risk tolerance.

There are endless possible permutations of investments using the various asset classes that can be incorporated into an investment portfolio allocation. The efficient frontier represents the series of "optimal" portfolios derived from the range of combinations available. It ranges from the asset mix which is the least volatile, or demonstrates the least risk, and subsequently provides the lowest returns, to the asset mix which is the most volatile or risky and has the potential for the highest returns.

The efficient frontier uses two measures – expected return of a portfolio of assets and the associated risk or volatility of the specific portfolio. Portfolios situated along the efficient frontier line represent the most "optimal" portfolio and mix of assets based on the expected return of the portfolio for investors when compared to the level of risk the investor will assume.

(Byrne & Lee, 1995), stated that Modern Portfolio Theory remains the standard tool for investigating the importance of an asset in a multi –asset portfolio. It is used to determine the mixed portfolio of assets which achieves the highest level of return for a given level of risk. Such portfolio is described as “efficient” in that a rational investor would choose this portfolio above all others. Applying Modern Portfolio Theory to the asset problem that includes property has been the subject of various studies throughout the world. These which studies have been the subject of criticism, principally for a reliance on appraisal –based data.

The portfolio selection model is based on an optimization of the portfolio mean-variance relationship.

The expected return (or mean) for the portfolio is the weighted average of the returns on the individual assets within the portfolio, weighted by their proportionate share of the portfolio.

Risk is measured by variance. Variance measures the variability of realized returns around an average level. Variance is dependent on the way in which individual assets interact with each other. This interaction is known as covariance. Covariance essentially tells us whether or not the two assets returns are correlated. Covariance measures by themselves do not provide an indication of the degree of correlation between two assets. Covariance is standardized by dividing covariance by the product of the standard deviation of two individual assets. This standardization measure is called correlation coefficient.

For the allocation of assets, two constants are required. The weights must be positive and fractions of the total portfolio must amount to 1.

The problem as classically stated is to minimize the portfolio variance of portfolio return. The two asset or three asset case can be solved by calculus. Beyond this, quadratic programming methods are required.

(Ping & Youguo, 2000), citing (Pagliari, Webb & Del Casino, 1995) state that an efficient portfolio in an ex-post sense may not remain efficient for the future. Further, recent studies have demonstrated that so called “efficient frontier” is not singularly delineated. Instead, the efficiency frontier is a collection of many statistically indistinguishable frontiers. As a result, the efficiency frontier calculated from Modern Portfolio theory is fuzzy and the optimal allocation is even fuzzier. The fuzziness of the results may be caused by, (1) the model’s high sensitivity to key inputs and (2) the

information inefficiency in the real estate industry, which puts Modern Portfolio Theory at risk of being an “error maximizer.

The question of how much of real estate should be allocated to an optimum asset portfolio has been the subject of various researchers. The percentage allocation of weighting vary from zero to seventy six percentage, (Seiler, Webb and Meyer, 1999). The answer of how much real estate to be included into an optimum multi asset portfolio can only be answered empirically, considering the data provided and the time frame of the analysis. Risk returns and correlations are dynamic and the percentage allocation can only be determined based on these factors.

(Ziobrowski, Caines & Ziobrowski, 1999) studied the optimum mixed –asset portfolio composition in the context of longer holding periods (five years) that are more typical of real estate investments. A bootstrap simulation is used to estimate confidence intervals for the real estate component.

The bootstrap, like all nonparametric procedures, is burdened by few restrictive assumptions. Thus the bootstrap is highly effective when analysing a relative small data sample, which is quite common in research involving real estate. The data in this study extends from 1973 to 1994, in order to have a sufficient number of five year holding periods for statistical analysis. In total eighteen five year holding periods were analysed. Returns the common stocks, long term government bonds and Treasury bills were used. Real estate returns were provided by Evaluation Association. The real estate data was all corrected for smoothing using Geltner’s adjustment technique.

Two different opportunities sets were examined. The first contained only common stocks, long –term bonds and Treasury bills. Real estate was then added to create a second opportunity set. Thus, the marginal benefits attributable to the real estate could be determined.

In a departure from earlier studies, the optimum amount of real estate in a mixed asset portfolio, shows that the confidence intervals produced with five year holding periods are quite tight and provide meaningful guidance for portfolio managers along the entire efficiency frontier. Unlike short holding periods (one year or less), when the optimum quantity of real estate could be almost anything depending on the economic conditions, over the long-term holding periods the optimum amount of real estate is amazingly stable. At the lowest levels of risk tolerance, the median optimum real estate component was virtually zero. It is also be stated with a high level of confidence (97,5%) that these low return optimal portfolio contains less than eight percent real

estate , as risk tolerance increases, the median optimal amount of real estate crept upward.

For investors seeking the highest returns and therefore those who were most tolerant to risk, the median optimal real estate composition was approximately nine percent with a (95%) confidence interval ranging from a minimum of four percent to a minimum of eighteen percent.

Overall, the results appear consistent with the historical behaviour of fund managers. Conservative managers seeking low risk and willing to tolerate lower returns should hold little or no real estate. Managers seeking higher returns who are more tolerant to risk should hold some real estate, but not very much, ranging between four and eighteen percent of the portfolio maximum.

Conceptual Review

Real Estate as an Asset Class

The important attributes of real estate, namely its profitability and ability to hold value, makes it a popular object of capital allocation both for individual investors and especially institutions. The latter want to multiply their capital by including real estate in their investment portfolios, which are diversified and as such, offer lower risk and satisfactory returns, particularly to conservative investors. Property as a portfolio component is considered to be an investment that creates future income and ensures the appreciation of the value of the invested capital. Its benefits depend on the usage (utility function), provided standard (quality of the building and leased space), and results of the adopted strategy and actions in terms of value management. Seeking to maximize financial gains related to individual assets, the investor looks for the best possible way in which they can be developed and managed.

The reasons presented to institutional investors for including property in their investment portfolios include, satisfactory return on the investment (high gross margin profitability of total capital), low price volatility, high returns in relation to risk, protection against inflation and deflation, stable rental revenue components, predictable components of operating expenses and low correlation with other assets, which allow for the diversification of the portfolio. (Smietana, 2014).

Property is not only an investment asset, but an asset that has clearly established itself as a strategic role player in the multi- asset portfolio due to its effectiveness as a diversifier of risk in a mixed asset portfolio. Empirical studies have also suggested that one of the principal reasons investors favour property investments, relative to other investments options, is the superior performance of property investments. (Amidu, Aluko, Nuhu & Saibu, 2008).

(Hoesli & Lekander, 2007), state that property as an investment class from an institutional perspective is the long duration of property assets which in combination with the common contractual tie of rents to the rate of inflation offers a match to long term liabilities. The long term asset allocation related benefits of property are, however not the only thing an institutional investor needs to consider. Property offers high risk – adjusted returns, in part due to the liquid premium. The relative predictable income stream from property also offers an attractive opportunity for investors accepting gearing. In low interest rate environment, this can appear as a “free lunch”, as the returns on equity of geared property investments well exceed the perceived risks.

There are also drawbacks to investing in property, such as the lumpiness of the asset, information asymmetries and barriers to entry. Indeed, building values tend to be large and properties are by their very nature indivisible, the seller of the asset generally has an information advantage over the buyer and substantial capital is needed in order to invest in property without being exposed to too much asset specific risk. Another drawback is the management of the asset, which requires an organizational and personnel. This risk puts the investor in a situation of being the manager of an operating company rather than being an investor.

Direct Real Estate

(Geurts & Nolan, 1997), presented a case for investing in direct real estate and indirect real estate. There are several possibilities an investor has when investing in direct real estate, which include undeveloped land, residential rental properties, office buildings, shopping malls, and industrial properties. As with any type of investment, there are many pros and cons pertaining to each investment opportunity.

In general, rental houses provide a good but sometimes unstable cash flow, because of turnover, rent can be increased to keep pace with inflation, depending on the location of the rental property. Furthermore, the value of the property can appreciate significantly over a short period of time, although the opposite is also possible. One of the main disadvantages of residential rental properties is the maintenance problem. Frequently repairs and redecorating can be expensive and are related to the turnover of tenants. Also, the higher the turnover the less stable the cash flow becomes.

Office buildings are generally seem to be more risky than residential properties, since a greater knowledge of property management and leasing is necessary. Office building can be broken into two main types, suburban low- rise buildings that are popular with new investors and high riser buildings in downtown and suburban areas of cities. The length of an office lease is generally longer than residential properties, traditionally for a minimum of two to three years. A lower turnover has obvious advantages and in addition, office buildings have not usually been subject to rent control and many increases in operating expenses can be shifted directly into the tenants. The main disadvantage of direct investment in an office building is the purchase price is substantially greater than for residential property due mainly to the higher cost of land and better quality of construction. Another problem with office buildings is that they are affected by cyclical changes in the economy. Hence there tends to be an over- reaction to produce in a period of demand, thus causing short term surplus that leads to periodic depression of rental rates.

Shopping centres are considered to be the blue chip investment properties despite the fact that there is greater and more skilful competition. The returns on a well located centre are secure and stable. In order to ensure the stability of returns, high credit rating tenants are necessary. For this reason, good property management and lease sophistication is essential for success. Also it is becoming increasingly difficult to find a suitable location for building new shopping centres. This is due to community resistance, which has taken the form of restrictive zoning and environmental controls, thus limiting the number of available sites. The rates of return on shopping centres can be very good. However, outside factors can substantially alter these rates. Some of these factors are competition entering the market, outdated design and layout and changes in trade area income levels and in population density.

Industrial properties refer to warehouses and mini warehouses as well as factories and industrial parks. Factories are considered more risky as they are not easily converted to other use. Warehouses are the most popular type of industrial of industrial property investment, because warehouses have long economic life and require little management effort. Often, a warehouse can be leased to major industrial companies for a long period of time, which provides the investor a steady monthly cash flow. An alternative strategy is to lease the ware house on a square footage basis. The result is a higher return but also greater difficulties since there is a need for more intensive management.

Finally, undeveloped land is seen as the most risky investment. The reason is because it is purchased in a raw state, usually on the outskirts of a city or town, in the hope that someday in the future it will be used for growth. Therefore, before investing in undeveloped land, one must project the path of future growth and estimate whether it is desirable for development, given the risks associated with misjudgement. Raw land is not the ideal investment for a monthly cash flow, in fact most investments yield a negative cash flow. The only hope of getting a return on the investment as if the value of the land risers and the investor can cover the costs of holding the property.

In conclusion, investing in direct real estate requires expertise and high capital expenditure and is only available to well-endowed investors.

Indirect Real Estate (REITs)

A REITs or Real Estate Investment Trust is an investment vehicle similar to shares, which are traded on established stock exchanges. They enable their shareholders to own a portion of real estate, thus bringing diversification possibilities to small and medium investors.

A Real Estate Investment Trust of REIT is a company that owns, and often operates, income producing property. A South African REIT is a listed property investment vehicle that is similar to international recognised REIT structures from around the world. Listed Company REITs or Trust REITs are publically traded on the Johannesburg Stock Exchange REIT board and qualify for REIT tax dispensation.

In order to qualify for the classification of a REIT, the following criteria must be realised.

The company must own at least R 300 million of property, the company's debt must be below sixty percent of its gross asset value, earn seventy five percent of its income from rental property owned or investment income from indirect property ownership, have a committee to monitor risk, not enter into derivative instruments that are not in the ordinary course of business and pay at least seventy five percent of its taxable earnings available for distribution to its investors each year.

REITs were first listed on the Johannesburg Stock Exchange on 1 May 2013. Most SA REITs own several kind of commercial properties like shopping centres, office buildings, factories, warehouses, hotels, hospitals and even, to a lesser extent, residential properties, in cities and towns across the country. Some even invest in properties in other countries.

Twenty seven companies with the REITs structure are listed on the Johannesburg Stock Exchange, with a market capitalization of approximately 380 billion rand or 31 billion US Dollars. These companies own and estimate 1.3 trillion rand worth of real estate.

The benefits of investing in REITs include,

REITs provide a hedge against inflation in two ways, firstly in capital investment growth and secondly the regular income distribution that grows.

By investing in SA REITs you are exposed to the best commercial properties in South Africa, and in some instances, offshore properties.

REITs have equity and bond characteristics. SA REITs offer investors the best of both worlds. A reoccurring cash distribution yield like a bond as well as growth in income like equity.

REITs are accessible to many investors and give everyone the opportunity to invest in income producing commercial property. It is the easiest, quickest and safest way to invest in property.

The entry cost for a REIT investment is the price of a single share. Investors can also buy or sell shares in SA REITs at any time, without the costs and delays involved with physical property ownership.

REITs earn income from property lease income that escalates each year, they have relatively predictable earnings. So you know what to expect from your investment with reasonable certainty.

REITs also provide a lower-risk property investment model because investors are exposed to a diversified portfolio of properties that are actively managed by companies which have strict governance requirements.

REITs earn income from property lease income that escalates each year and they have relatively predictable earnings. So you know what to expect from your investment with reasonable certainty. (SA REIT, www.sareit.co.za, 2017).

Real Estate Return and Risk Characteristics

Direct real estate is the traditional avenue to real estate investing. Owning real estate directly offers the diversification benefits investors seek, grants the investor full control of the strategy and management of the asset.

The accurate measurement of real estate return and risk is of great importance to real estate investors. Real estate is highly heterogeneous, is thinly traded over relatively long holding periods, and is traded through a transactions process that is typically not a simultaneous-bid auction but instead is a sequential bid process which may involve significant transaction costs.

Return Characteristics

(Benjamin, Sirmans & Zietz, 2001), offer some insights into the characteristics of real estate returns. Real estate returns are generally categorised by three factors, initial current yield, growth in net operating income and price movements.

Several issues regarding the general nature of real returns focus on real estate market efficiency, as measured by the distribution of returns, The predictability of real estate returns, the macroeconomic variables that may help explain variation in real estate returns, methods of measuring real estate risk and returns, and the returns of specific types of property.

Macroeconomic variables sometimes are believed to be the keys to understanding variances and as such nominal interest rates explain about sixty percent of the variation in real estate prices. Macroeconomic variables such as growth in employment, do not influence real estate returns in the short run, but actually reduce return betas and volatility. The growth rate in real capita consumption has a significant and consistent positive effect on commercial real estate returns. Another area of study on real estate returns focuses on the performance of different types of property, that being retail, office and warehouse properties. (Akinsomi, Mkhabelab & Taderera, 2018).

The benefits come at a cost, however, some of the most obvious one being large lot sizes, low liquidity, information asymmetries and management intensity.

The large lot size, and also heterogeneity (diversity), of real estate result a large degree of unsystematic risk to the portfolio. Second, direct real estate investments are illiquid, it can easily take three to six months to transact an asset. Third, the direct real estate

market is ripe with information asymmetries, a seller of an asset by definition has more information than the potential buyer. This is further amplified if international investments are taken into account. In this case a foreign buyer is at an information disadvantage with regards to asset level and market information.

Finally real estate is a management intensive asset during the whole cycle of investment, from the technical management of the assets, to leasing activities, refurbishment and transaction and strategy dimensions of the portfolio. (Hoesli & Lekander, 2007),

(Lizieri & Ward, 2000), examined the return dynamics of direct real estate, both the income returns and the capital returns and how these returns are derived.

The estimation of returns in the private real estate market is more complex than in the bond and equity markets. The basic components are, of course income returns and capital growth. Each presents particular problems in calculation.

The income returns come from the rent paid by tenants. The returns must account for the timing of payments (typically quarterly in the UK, monthly in the US), and for the cost of rent collection. The latter is problematic and there is no common standard for calculation of net operating income. Direct costs (maintenance, repairs and insurance not chargeable to the tenant, professional fees, marketing charges for example) are clear, but the indirect costs of managing the property may be obscured. This is particular true for properties held and managed by an institution or property company with an in-house management function.

The estimation of the capital gain component of returns is particular difficult in real estate. Long holdings periods and infrequent transactions mean the capital value is an estimate provided in-house of by external appraisers or valuer. Because the transaction market is so thin, the valuer has to resort to a formalised version of what is, essentially, a dividend discount model of valuation. He or she must consider the current and future income stream, the security of that income (that is, the probability of the tenant defaulting or vacating and the probability of securing a new lease), the investment demand for the property (which will include consensus estimates both of future market rental growth and of the covenant strength of the tenant), the legal terms of the agreement between landlord and tenant and any specific risk factors relating to the building). These risk factors, typically, are incorporated into a single “initial yield” or capitalisation rate. Assets heterogeneity and thin transaction markets mean that the appraisal utilises a very limited information set.

Two consequences of this process are important. First, the appraisal process creates uncertainty as to the true value, and hence about both components of the return. That said, it is the appraisal- based returns that are used to measure fund performance and fund managers added value. Thus this is a case of unadjusted returns. Second, because appraisers are faced with limited information set, it is suggested that they use evidence over a time window around (but generally preceding) the notional date of the valuation and that they adjust prior valuations in the light of new evidence by an intuitive process of Bayesian adjustments. The first gives rise to temporal aggregation effects, the second to an auto-regressive or exponential smoothing effect. The consensus position from published research is that appraisal smoothing reduces the measured volatility of real estate. Further, appraisals may lag turning points and understate both peaks and troughs.

The direct real estate market is hampered by the low frequency of data and uncertainty concerning the validity of appraisal based returns. Nonetheless, concern has been expressed about the distribution characteristics of real estate returns and the possible impact of non-normality. In addition to attempts to “desmoothing” property returns (that is, to attempt to remove serial correlation and aggregation effects to the “true” market signal), a number of authors have tested for normality. The results point both to peaked, fat-tailed distribution and, more tentatively, to skewness.

Risk Characteristics

(Wheaton, Torto, Sivitanides, Southard, Hopkins & Costello, 2001), provided a general overview of risks, when applied to investing. Most economists agree that risk is best conceptualized as uncertainty over likely outcomes. The future outcomes could be thought of as returns over five years and the uncertainty can be thought of as a “deviation” from an expected, or likely outcome. Over the past forty years the exploration of risk has been one of the dominant themes of financial economics and the success of this line of inquiry is evident in the technical advances in financial economics have been applied to nearly all branches of the discipline.

Risk and risk management considerations are, or should be, the heart of every investment decision. Every time an investor puts money into an asset, other than cash, a trade-off is made between risk and return. Cash deposits offer a virtually risk free return at any point in time, government index linked bonds are frequently regarded as the real risk free rate over the long term. No other asset provides the same degree of certainty.

With property investments, tenants can default affecting the rental income stream and there is no certainty over the level of property values in the future. Changes in these and other variables can have a marked effect on the delivered level of returns- hence uncertainty and potential volatility in returns. (Bundell, Fairchild & Goodchild, 2005).

(Bundell, Fairchild & Goodchild, 2005) states that overall portfolio volatility can be broken down into component parts relating to changes in income (default, releasing and rent review) and capital (shifts in capitalisation rate and market rents). These changes are driven in turn by a range of factors partly shared in common. Volatility reflects fluctuation in the two components of return- income and capital plus for some portfolios, the effects of leverage and exchange rate movements. These in turn are driven by a range of factors that can be measured at either a portfolio or a property level.

A range of measures evolved to profile the ex ante risk of a particular portfolio, all based on its current characteristics. They can be divided between fundamental and modulators that either amplify or dampen the volatility emanating from the fundamental factors. The actual risk factors used were refined as our research progressed.

Fundamental characteristics include, Tenant credit worthiness, exposure to volatile sectors, example city offices, speculative development exposure, weight lease length, expected income growths, yield level and vacancy rates.

Modulators include, number of properties / lot sizes and concentration, property type concentration, sector balance, location concentration, leverage, exchange rate ,mismatch, tenant concentrations and lease concentrations.

The fundamental risk can be viewed at a stock level as well as for a portfolio. The modulators, however are relevant for the portfolio risk alone. They determine the scale of volatility in portfolio returns while the volatility itself emanates from the fundamental factors. A portfolio with 50 assets, for example is likely to be less volatile than one comprising 10 properties because the volatility in returns from individual properties is diversified across a larger number of assets.

(Ping, 2005), studied the asymmetric (abnormally high and abnormally low) returns of real estate performance, using a matrix of systematic downside beta (β^-), systematic upside ($\beta+$), skewness and co-skewness based on both conditional and unconditional return distribution and tested their empirical validities to real estate returns. (Beta, β, is the measure of volatility of a security in comparison to the market.

Data was sourced from the NCREIF property database, with twelve sub-types of commercial properties. The analysis calculated (β^-) and (β+), using time series regression. Unconditional and co-skewness.

The major findings were the traditional (β), does not explain the cross –sectional variation very well. In other words “high risk, high return”, is wrong statement if the risk is measured by (β). Second, there is strong evidence indicating that downside risk is associated with a risk premium while the upside potential (unexpected very high returns) are not factored into the valuation. Third, skewness (based on either conditional or unconditional return distributions) provide significant explanatory power to the variation of cross sectional property returns. This remains true in the presence downside risk measure (β^-), suggesting skewness captures some additional aspects of downside risks and commands statistically significant risk premiums. Fourth, (β^-) and skewness are independent of property types, suggesting they have the potential of being valid asymmetric measures for all types of properties. Finally, co-skewness does not explain real estate returns well and therefore is not a good systematic risk measure.

Valuation of Real Estate Assets

The valuation of real estate is a central tenet for all business. Land and property are factors of production and, as with any other asset, the value of the land flows from the use to which it is put, and that in turn is dependent upon the demand and supply for the product that is produced. Valuations, in its simplest form, is the determination of the amount for which the property will transact on a particular date. However, there is a wide range of purposes for which valuations are required. These range from valuations for purchase and sale, transfer, tax assessment, expropriation, investment and finance

The valuation of real estate is required to provide a quantitive measure of the benefit and liabilities accruing from the ownership of the real estate. For any valuation to have validity, it must produce an accurate estimate of the market price of the property. The model should therefore reflect the market culture and conditions at the time of the valuation. It should be remembered that the model should be representative of the underlying fundamentals of the market. Thus, in the property market, what is often called a “valuation”, is the best estimate of the trading price of the building. (Pagourtzi, Assimakopoulos, Hartzichisto & French, 2003).

(Geltner & Miller, 2001), define the term valuation as the process of estimating or determining the value of an asset. The process of estimating or determining value of an asset is fundamental for the rational analysis of investment decisions. The valuation of real estate assets is essential to investors as this process underlies the computation of holding period returns and is therefore basic to the quantification of investment performance.

Determining the value of real estate assets poses a unique challenge to investors, as real estate is traded in private property markets unlike other assets that are traded on public exchanges, where transaction information is easily accessed. The private property market is characterised by four tenets. (1) The market is characterised by trading of whole assets (that is, whole individual properties rather than securities or shares) (2) Each individual property (i.e, traded asset), is unique at least to some extent, (eg, no two locations are exactly identical, not to mention structures, leases and other features important in determining commercial property value). (3) Each asset is typically traded only infrequently and at irregular intervals. (4) Each trade is negotiated privately between only one seller and only one buyer). These four characteristics make it difficult to obtain a clear and precise indication of what the current market value is for any given property at any given time.

When trying to estimate the current market value of a subject property, you can observe the price at which similar properties trade in the past. Traded properties are not identical to the subject property, and, the past is not the same as the present, because of this, buyers and sellers in any transaction go into their price negotiations with imperfect information about the current market value of the subject property. The result is that the transaction price they agreed to will generally contain some unobservable random noise (pure random error) component that causes the price to differ from the conceptual definition of the current “market value” for the subject property. This is also true for appraised values.

Appraisers, like market participants can only estimate market values imperfectly based on incomplete information. So appraised values also will tend to be dispersed randomly around the underlying “true” market value for a given property as of a given time.

When appraisers use transaction price information from (comparative properties) comps, they have to try to adjust the observed transaction prices to reflect cross-sectional idiosyncratic differences between the comps and the subject property. Secondly, the true market value of any given property or portfolio changes over time,

and the relevant transaction prices or appraised values are observed at different points in time , thereby providing value indications as of different points in time.

Investors should distinguish between two sort of differences between any given subject property and its relevant comps, difference that appraisers must consider and try to adjust for the real world: cross –sectional differences and temporal differences. The former are differences in the characteristics of the properties themselves, including physical qualities and size, location the nature of leases and tenants. In contract temporal differences refer to the fact that the comps will all have sold in the past, prior to the time at which the subject property is to be appraised. The true market value of the subject property will normally have changed over the relevant time span.

Temporal Aggression or the value estimation procedure that is optimal at the micro level of individual property appraisal may not be optimal at the macro level for estimating the value of an entire portfolio of many properties.

There are two major types of differences between comps and the subject property that the appraiser must adjust for, so there are two major sources of the bias: cross – sectional and temporal.

The major culprit in the cross- sectional source of bias is what is often called sample – selection bias. This occurs if the comps tend to drawn from a population of properties that is not cross- sectionally different in a systematic way from the subject property. That is, the value of a subject property in one city and all comp properties were in another city, we would likely be susceptible to sample – selection bias. No doubt the appraiser will adjust for the difference between the cities property market as best he could, but this adjustment would probably be imperfect, especially as the appraiser has no comps in in the city of the subject city to compare. In this case, the appraiser might tend to under adjust for the difference between the two cities.

The temporal source of bias is often referred to a temporal aggregation bias. This occurs when the property market has recently turned around. In particular, suppose asset market prices had been rising or holding steady for a couple of years, and then suddenly the market turns sharply down within the past couple of month. The appraiser will no doubt be aware of this change. But the appraiser will have few, if any actual transaction comps from the past couple of months work, with about how much values have fallen in general, or how relevant the general market downturn is for the unique subject property in particular. Almost all the empirical value indications available to the appraiser will be from the period of high prices earlier than two months ago. The appraiser will no doubt try to adjust the estimate value to reflect the new market, but

the adjustments will tend to be imperfect, probably to under adjust. Such a result can also be expressed as a weighted average between the true current market value and a prior appraised value that the same appraiser might have estimated for the same property at a time not too far in the past.

Appraisal Smoothing

(Seiler, Webb, & Myer, 1999), stated that one of the most fundamental differences between direct real estate and all other asset classes stems from the way in which returns are compounded. Direct real estate index (such as NCREIF), returns are based on appraisal data as opposed to transaction based data. REITs, on the other hand, are traded on organized exchanges and are therefore transaction based as are stocks and bonds. Appraisals can occur at any frequency, but most real estate properties are appraised only once a year. Since the direct real estate indexes are given quarterly, while most actual appraisals are performed only annually, there is no way to determine the "true" return for each quarter. Adding to the difficulties associated with appraisal-based data is the fact that annual appraisals are performed at different times of the year for different properties. Thus, these indexes are better described as annual indexes that are updated quarterly rather than quarterly indexes. This phenomenon is known as smoothing.

If the property was sold at any time throughout the year, then the transaction-based return could be used. Most, properties, however, remain under the same ownership for several years at a time, rendering the transaction based index impossible for direct real estate.

Appraisal smoothing causes measured volatility to be much lower than suspected (no one really knows), actual volatility. Because Modern Portfolio Theory allocations are based on risk/ return trade-offs, the understatement of direct real estate volatility results in an overstatement bias of the amount of direct real estate that should be included in optimal portfolios, particularly at the lower levels of risk.

Any time allocations are incorrect this causes the portfolio to be out of optimization.

(Edelstein & Quan , 2006), state that a substantial, respected existing literature, spanning the 1970 and 1980s, conclude that real estate provides a higher risk-adjusted return compared with other investment alternatives. These studies imply that the inclusion of real estate in a portfolio of investment can substantially reduce portfolio risk, and find that real estate is a hedge against inflation. In contact to more recent research, utilizing extended similar methodologies, but subsequent market data, has

found that more modest real estate performance. The differing results, in part may be attributable to cyclical sample specific effects as well as secular real estate market changes. The conclusion of both views in the literature depends upon the construction of one or more of real estate return series which can be compared with similar return indexes for other investments. In the case of real estate, where market transactions are relatively infrequent, professional appraisals are used to represent market values in order to generate rates of return index. Indeed this is the standard practice.

Several difficulties greatly restrict the usefulness of real estate rates of returns series computed from unadjusted appraisal data. The classic criticism alleges that the aggregate real estate rate of return index is smoothed because it employs smoothed individual property appraisals. A smoothed index reduces the variance of returns reported for a sample of appraisals relative to a sample of sales transactions for identical properties. Since the usual metrics of risk and diversification rely crucially upon the measure of dispersion an artificially smoothed series would necessarily underestimate the riskiness of the real estate asset class, and would distort its correlation with the rates of return of other assets.

The notion of smoothing can be viewed as errors resulting from the calculation of a statistic using imperfect appraisal estimates of individual property values. Smoothing is defined as the deviation of an index from one which is never observed, and since it is this deviation between the series which gives to the problem, it is not surprising that smoothing is often demonstrated based on assumptions made about the true series and appraiser methodology and practice. The notion that appraisal –based return indexes are “too smooth “ is so well entrenched in the literature that researchers have proposed methods to reverse engineer to “unsmooth” indexes.

(Marcato & Key, 2007), describe smoothing as the phenomenon causing a lagging effect and reduced volatility in valuation based indices in comparison with the underlying market which would be represented by accurate transaction- based indices. Indicators of smoothing is easily found from time series analysis of valuations – based indices.

There are three main factors which may cause smoothing in a valuation based index; the aggregation process underlying the index constructions itself (i.e. cross –sectional variance of individual returns), valuations spread over time (known as temporal aggregation) and inertia in individual valuations arising from anchoring to prior values in the absence of conclusive market evidence, or thresholds applied by values (of say 1% of capital value) before a change in value is reported.

The issue of smoothing is critical for asset allocation, where the estimation of risk / return profiles of different classes are key to the construction of function – maximising portfolios. Following Markowitz mean – variance model, we would attribute a very high weight to real estate because valuation based index show low levels of risk. However, institutional investors normally show a property weight ranging between 5% and 10% on average. The difference between "ideal" and current property weight is frequently attributed to the understatement of volatility in standard property index such as IPD and NCREIF.

Inflation hedging characteristics of Real Estate

Inflation, is an economic concept that measures the increase in the price of a basket of goods and services that is representative of the economy as a whole from one year to the next. The inflation rate is important within the investment context as it represents the rate at which real value of an investment is eroded and loss in spending power over time. The inflation rate also tells investors exactly how much of returns, in percentage terms, that their investments need to earn to maintain their spending power. http://economics.about.com.

Inflation has long been identified as a major risk for individual investors, especially for those who aim to match their liabilities through the investment process. To counter the inflation risk investors should seek opportunities or asset classes with inflation hedging characteristics to add to their mixed asset portfolios. (Wurstbauer & Schater, 2014). Investors wishing to safeguard the value and purchasing power of their portfolios needs an accurate assessment of the inflation hedging characteristics of the various assets in their portfolio. (Barkham, Ward & Hendry, 1996).

For investors who hold assets for long-term returns, inflation risk is always of great concern because high inflation erodes the real returns accruable to them in the future. On this account, assets that are inflation hedges appear to be more attractive than those that are not. Traditionally, real estate has been regarded as one of the best inflation hedges. (Sing & Low, 2000).

One of the key considerations in understanding the link between inflation and investment returns is to identify the expectations of investor at the time asset prices were set. If investors expect a high level of inflation, they might reasonable be assumed to set prices so as to provide an acceptable real return. One implication of this assumption is that asset prices should provide returns that are sufficient to offset expected rates of inflation. On the other hand, the property of an efficient hedge against

inflation is to provide investors with compensation, not only for expected inflation, but also the inflation that is not foreseen at the time at which prices are set. (Barkham, Ward & Hendry, 1996),

Two arguments are generally given to motivate the decision of including real estate in portfolios of financial assets (example, stocks and bonds). These arguments are the diversification benefits from the less than perfect correlation of real estate with the other assets included in the portfolio and the better protection against inflation provided by real estate. (Hoesli, 1994).

In considering the role of real estate in mixed- asset portfolios many industry commentators point to its supposed inflation hedging qualities. This would be highly desirable characteristic, particularly for those institutional investors, such as pension funds, who need to match real liabilities. Given evidence of a positive relationship between real estate and inflation, this would encourage higher allocation to this asset class in mixed portfolios. (Hoesli, Lizieri & MacGregor, 2007).

(Barkham, Ward & Hendry, 1996), cited (Limmack & Ward, 1988), distinguished between expected and unexpected inflation. Expected inflation is calculated in two ways, the yield on the three- month Treasury bill is taken as one measure and an ARIMA generated is the other. (Autoregressive integrated moving average (ARIMA) model is a generalization of an autoregressive moving average (ARMA) model. These models are fitted to time series data either to better understand the data or to predict future points in the series (forecasting).Unexpected inflation is calculated by deducting the forecast expected rate from the actual rate.

Many researchers have supported the hypothesis that direct real estate provides a good hedge against inflation. (Hoesli, Lizieri & MacGregor, 2007), (Wurstbauer & Schater, 2014), (Sing & Low, 2000), (Hartzell, Hekman & Miles, 1986), (Barkham, Ward & Hendry, 1996), (Hoesli, 1994),

(Hoesli, Lizieri & MacGregor, 2007), explored the relationship between commercial real estate returns from both private and public markets, economic factors and inflation for the United States of America and the United Kingdom markets.

In researching the hedging qualities of commercial (investment) real estate a distinction needs to be made between private and public assets. In both cases, there are conceptual and data- related issues. For private real estate, researchers are forced to rely on appraisal based portfolio indices, such as the NCREIF in the United States or the IPD in the United Kingdom. Appraisal- based indices are influenced by appraiser

behaviour, where an appraiser might adjust a prior value to reflect known inflation and appraisal smoothing may distort capital values. Securitized real estate returns are based on transactions, however the delivered returns depend on the leverage of firms and on management behaviour.

The data set used for both countries ran from 1977 to 2003, using an error correction approach

Results for the inflation hedging characteristics of real estate assets are mixed with differences between the findings for securitized real estate markets and direct markets.

While there is some evidence that directly owned private real estate (direct real estate) provides a partial hedge against some components of inflation, securitized real estate (securities) is often shown to exhibit the same negative relationships found in stock market research particular with respect to unexpected inflation.

(Sing & Low, 2000), tested the inflation –hedging features of a wide range of real estate, all property, shops, office, residential and industrial and financial assets (all share and all property share in Singapore over a twenty one year period from 1978 to 1998. The study tests the assets hedging characteristics against three types of inflation, observed, expected and unexpected inflation. Two sub periods are tested, the first divides samples into four five year periods and examines inter-temporal changes of the inflation hedging characteristics of the assets, the second the sample periods were divided into low and high inflation regimes and tests the responses of assets to inflation. The methodology used in these tests was the standard least square regression techniques.

This study proved empirically that the traditional belief that real estate is a good hedge against inflation. Furthermore, the inflation hedging performance of real estate in general is found to be better than the financial assets of stocks and securitized real estate. Industrial property was found to be the most effective hedge against both expected and unexpected inflation, shops offer a significant hedge only against expected inflation.

The results of the five year sub period analysis showed that except for industrial property that offered consistent positive hedge against both expected and unexpected inflation, all other assets have a mixture of positive and negative results over the different sub periods. It implies that an asset, which is a good hedge against inflation in one period, can become a negative inflation in another period. The findings highlight

the importance of the points of entry and exist of an investment when determining the effectiveness of the inflation hedging characteristics of the asset

(Hartzell, Hekman & Miles, 1986), investigated the inflation hedging benefits of four asset classes, the S&P share index, Treasury bonds, Treasury Bills, and direct real estate, from 1973 to 1983. Using correlation coefficients of the returns of four asset classes. Their finding indicate that, direct real estate has the potential to provide an inflation hedge, the S&P share index and Treasury bonds were negatively correlated to inflation. The Treasury bill and inflation returns were highly correlated to each other.

(Ziering & McIntosh, 1997), stated that when core real estate (direct real estate) is added to a mixed – asset portfolio, the inflation risk of the expanded portfolio is substantially below that of the original and during periods of prolonged inflation, core real estate acted as a hedge by exhibiting a tendency to increase in value as prices and wages climbed. Because core real estate managers were able to pass higher expenses through to tenants and step up rents as inflation rose, it evidenced anti-inflation attributes not seen in other asset performance.

(Hoesli, 1994), tested the inflation hedging abilities of two asset classes. A Swiss real estate mutual fund and stocks to evaluate and compare the two asset classes' ability to hedge against inflation. Real estate mutual funds were selected as a proxy for underlying real estate assets because these mutual fund units' prices should reflect quite accurately the prices of the underlying values of the properties held in the fund. The Bopp ISB Index is an index of real estate mutual funds, which contains the ten largest property funds and accounts for approximately 80 percent of the total worth of the 32 Swiss real estate mutual funds. The Morgan Stanley Capital International index was used as a proxy for stocks. Data from 1943 to 1991 was utilized in this study.

If the asset is a perfect hedge against inflation, the beta should be equal to one. When the beta is larger than one, the hedge is more than complete. Assets that which provide an incomplete hedge will yield a beta between zero and one. A negative beta suggests that the asset class acts as a perverse hedge against inflation.

The results show in the long run real estate seems to provide a better hedge against inflation than common stocks. When the data for five year time increments are used, the beta coefficient for inflation is marginally positive in the case of real estate, whereas it is significant negative for common stocks. Moreover when the inflation rate is broken into its expected and unexpected components, all coefficients are negative for stocks, whereas some coefficients are positive for real estate. This is particular true for unexpected inflation.

(Hudson- Wilson, Fabozzi, & Gordon, 2003), investigated whether real estate is or isn't an effective hedge against unanticipated inflation or deflation. There finding were the following. Inflation elicits different responses in the different property types through divergent impacts on the income and value components o returns, and through variation in the effects and the most recent inflation. But the return- modelling process can generate a clear view of the relationship among all these components. We look first at the response of private equity to periods of inflation and disinflation.

Past inflation is partially embedded in rents set previously, because every seller of every product, including sellers of space, wants to keep prices level or rising in real terms. Thus net operating income (NOI) is partly a function of past inflation, rising if past inflation has been higher, and falling (or rising less) if past inflation has been tame.

How quickly inflation affects NOI, or the time lag necessary to capture inflation's impact on current NOI, depends on the structure of the leases, which in turn varies with property type. Current office NOI reflects the inflation experience of one to ten year ago, while apartment NOI reflects more recent inflation. The impact of past inflation, appropriately lagged, is positive for all four major property types.

Current inflation impacts the level of current rents and expenses. Current inflation raises NOI, by increasing the rental rate on new leases, but reduces NOI by increasing all expenses. The model helps us see which effect dominates. In the office, warehouse and apartment markets, current inflation causes NOI's to fall as the rise in current rents associated with recent leases does not fully offset the increase in expenses, which impacts the entire asset. In the retail sector, however, current inflation raises NOI, as the impact on rents and percentage rents more than offsets the impact on the few expenses that are not passed through. Retail then has two characteristics that render it a very capable transmitter of inflation to asset performance: percentage rents and generous pass- through of expenses.

Inflation impacts the capital value return in two ways. First, it impacts current NOI, as described above, which feeds through to value via capitalization rate. This channel is especially strong for retail assets. Second, inflation affects the cap rate directly by influencing NOI growth expectations and therefore investor's demand for real estate investments. The direct capital value impacted of inflation is significantly positive for apartments and office properties, but not significant different from zero for warehouses. Thus, the empirical assessment shows that private equity real estate is a very useful partial inflation hedge. That said, it is also clear that the degree of inflation – hedging capacity is not uniform across the property types.

Portfolio Diversification with Real Estate

Real Estate in Multi Asset Portfolios

Diversification is a strategy consisting of the distribution of available resources among different types of investments with respect to financial instruments and including sectorial diversification of their issuers. The implementation of this strategy is aimed at reducing risk and achieving the expected returns on investment. The principals of diversification are essentially common to financial instruments and property. In relation to property, diversification is defined as the practice of dispersing and spreading capital by investing in property while accounting for its geographical diversity, with investments concentrated in selected geographic regions which is aimed at reducing risk. The dispersion and spread of investment consist of diversifying asset classes and types within various sectors and regions. (Smietana, 2014).

The notion of diversification is to spread your capital across a wide variety of assets. Since the portfolio contains many assets, it has to be constructed in a diversified way in order to avoid risks, which may deprive investors from realising the return that they have expected. It is important to mention that there are two components of portfolio risk, the first one is non-systematic risk or idiosyncratic risk which can be diversified. The second one is systematic or market risk which cannot be reduced by portfolio diversification. (Bakri, 2014).

Non-systematic risk is associated with random events or specific factors that are unrelated to the factors affecting the whole market, it is the risk that affects a small number of assets. Non – systematic risk can be reduced to zero, through portfolio diversification.

Systematic risk is associated with the factors affecting all assets making up the market portfolio. It is a risk that influences a large number of assets within the investment market. Systematic risk cannot be diversified. (Bakri, 2014).

(Geurts & Nolan, 1997), provide examples of non-systematic risks and systematic risk. Non-systematic risks are events such as lawsuits, strikes and unsuccessful marketing programs. Systematic risks include inflation, recession and high interest rates.

The purpose of diversifying an investment portfolio is to reduce non- systematic risk. The benefits of diversification arise from the less than perfect correlation between the

various market segments. The lower the level of correlation between the assets, the greater the potential for portfolio risk reduction.

Diversification of investments, provides a wider choice of investment opportunities and an improved risk – adjusted returns and reduced volatility with the investment decision. This is primarily due to the low correlation that asset of real estate assets tends to have against main steam capital markets assets classes and the attractive risk and return characteristics of real estate, (Stevenson, 2001).

(Delcoure, 2010), explored the principle of the corrections between asset returns being the common indicator of diversification in a portfolio, as this wide held principle is common in portfolio theory and literature (Black, 2004) and (Ziering & McIntosh, 1997). The study examined the differences between correlation and the return gap as a measure of diversification benefits across asset classes.

The return gap is associated with dispersion – standard deviation of an individual asset return around the mean returns of assets. In the case of two assets, dispersion can be defined as the difference between the return of each asset and the mean return of the two assets.

Return gap and diversification benefits are lower when correlation between assets is higher, however, return gap and diversification benefits are higher when standard deviation is higher between a pair of assets. The return gap should not be confused with covariance, as both of them combine correlation and standard deviation, however, these two functions are different. Covariance and the return gap are higher when the standard deviation is higher. On the other hand, covariance is lower when correlation coefficients is lower but the return gap and diversification benefits are higher.

Correlation is a common indicator of portfolio diversification benefits. However, coefficient of correlation alone does not provide an intuitive indicator of diversification benefits nor do these depend only on the correlation between assets returns, Results of which show that relative rank of assets by diversification benefits is different when measured by correlation coefficient and by return gap. In addition, investors seeking diversification opportunities could go a step further and combine coefficient of correlation and return gap in practical asset allocation to at least partially hedge out market exposure and invest more granularly in regions, countries and asset categories.

(Adair, McGreal & Webb, 2006), analysed the diversification effects of direct real estate versus indirect real estate. Optimal mean – variance portfolios were constructed using four class classes. These asset classes included, UK pooled property funds,

representing direct real estate, pooled real estate funds, representing indirect real estate, UK common stocks, representing equities and gilts, representing government bonds. The data used extended over a period of twenty eight years from 1995 to 2003.

Portfolios were constructed using a combination of these four asset classes. Where the optimum allocation of the asset classes served as the underlying criteria for inclusion into the portfolio. These were further, categorised as, low risk, medium risk and high risk portfolios. The authors did not provide any definition of explanation as to what constitutes the three categories of portfolios.

The conclusion reached were the following. Direct real estate featured significantly in low and medium risk portfolios, with allocations of twenty four and fifty five percent allocations. Similarly, indirect real estate featured in the low and medium risk portfolios, with allocations of thirty three and fifty four percent allocation. Real estate, both direct and indirect do not feature in the high risk portfolios. All high risk portfolios have a hundred percent allocation to common stocks. The results are consistent with higher risk associated with equities and lower risk profile of direct real estate and indirect real estate. The similarity of the allocation between direct and indirect real estate, reflects the high correlation between the returns with common stocks (shares).

(Lee & Stevenson , 2006), examined the attractiveness of direct real estate in the context of a mixed asset portfolio compared with portfolios without real estate, firstly holding the risk constant and second the portfolio return constant. The data used in this study are the quarterly returns of equities, government bonds and direct real estate over the period of 1977 to 2002.

Mean and standard deviations were calculated to provide a relative comparison of the different asset classes on both the risk and return basis. Correlation coefficients were calculated to describe the co-movement between each asset class. To investigate the issue of whether real estate allocations in efficient portfolios are consistent over varying time horizons, efficient frontiers were calculated by minimising portfolio risk for different levels of portfolio returns, where risk is defined as the variance of returns, or more typically the standard deviation of the returns. In other words a portfolio is on the efficient frontier if it has the highest expected return for a given level of risk or the lowest level of risk for a given expected return. The empirical results examined the return enhancements and risk reduction benefits of incorporating direct real estate into a mixed asset framework. The results highlight a number of issues in relation to the role of direct real estate within a mixed asset portfolio. Firstly, the results suggest strongly that real estate has possessed the attributes of consistency in optimised portfolios.

Real estate constantly had positive allocations over time ranging from five to twenty five years, and for most levels of portfolio returns, irrespective of whether real estate is used to enhance returns or reduce risk. Secondly, the benefits from including real estate in the mixed asset portfolio tends to increase as the investment horizon is extended. This implies that direct real estate should be considered as a strategic asset in the mixed asset portfolio especially for those investors with longer holding periods. Third, the position of real estate changes across the efficient frontier from its return enhancing ability to its risk reducing facility. As a consequence, the asset that needs to be replaced depends on whether an investor wishes to increase returns or lower risk of both. Finally, the results show that the gain in returns from adding real estate to the mixed asset portfolio is typically less, compared with the reduction in portfolio risk. This would indicate that as an investor moves along the frontier, the rationale behind the inclusion of real estate alters, with increasing emphasis being placed on the assets risk reduction rather than its return enhancing capabilities. In other words real estate should be considered as diversity rather than a return enhancer in the mixed asset portfolio.

(Hartzell, Hekman & Miles, 1986), examined the appropriateness of real estate in a multi asset portfolio of traditional stocks and bonds and which category of real estate is best within the portfolio.

The data selected for analysis, was derived from a large institutional manager of a pension fund with real estate investments. The data consisted of quarterly operating figures from 1973 to 1983. The total number of properties within the data sets, or held by the portfolio range from 113 to 403, with 78 properties having 40 consecutive quarters, and 220 properties having 20 quarters of consecutive data. The individual properties are diversified both geographically and by property type, with all areas of the country and all major types of constructed commercial space represented.

The mean – variance framework was utilized to analyse portfolio returns, risks and correlation between asset classes. The S&P, 500 was used as a proxy for stocks and Treasury bills as a proxy for Bonds.

The results support the finding of previous work showing that the real estate investments offer higher returns than other financial investments over the sample period. In addition, the real estate returns are less variable than those of other classes. Real estate can offer significant diversification benefits, when added to portfolio of common stocks and bonds. Considering that the major portion of this pension fund portfolio is invested in common stocks and bonds, the zero correlation of real estate returns and common stock and the significant negative correlation of

real estate and bonds implies that pension funds could reduce their overall portfolio risk by combining the three asset types. (Akinsomi & Ntuli, 2017).

Optimal and Efficient allocation of Real Estate to Portfolios

(Hung, Onayev & Tux, 2008), examined the role of real estate in a multi asset portfolio and the optimal asset allocation strategy for intuitional investors, using a wide range of alternative asset classes in the portfolio and, assessed the diversification benefits of real estate in up and down stock markets after considering other alternative vehicles.

The study follows the Modern Portfolio Theory to develop an optimal asset allocation strategy. In the mean variance framework, a vector of expected asset returns and standard deviations, as well as the correlation matrix of the asset classes, were estimated to build efficient frontiers.

The data considers eight asset classes, including large- cap stocks, small –cap stocks, bonds, securitized real estate, direct real estate, commodities, hedge funds and emerging markets. Data for a ten year period from 1996 to 205 was studied.

Efficiency frontiers were created using historical returns/ standard deviations and correlation matrix in the three market states (all states, up market and down market), for the two data sets the first one including monthly REIT index returns and the second one including TBI returns (direct real estate).

The efficiency frontiers in up and down markets dominated the one in all markets state. Furthermore, when direct real estate investments are included, portfolio performance is superior to the one without direct real estate investments. Portfolio performance is enhanced because of the high Sharpe ratio of TBI returns. The optimal asset allocation to direct real estate investment ranges from fifteen to fifty percent for all markets states. On the other hand the optimal weight to direct real estate ranges from nineteen to twenty three percent in up markets and from twenty to thirty eight percent in down markets. Furthermore in down markets, a more risk-adverse investor should allocate a higher weight to direct real estate investments.

Summary of Literature Review

The aim of the literature review was to identify the concepts, theories and methodologies employed by other researchers, when investigating the diversification benefits of direct real estate in multi asset portfolios.

Modern Portfolio Theory or Mean Variance Portfolio analyses, is the dominate theory proposed by researchers when constructing multi asset portfolios. The principal of diversification is central to the theory of Modern Portfolio Theory, and enlightens investors as to combining of asset in order to achieve risk adjusted reward benefits within a portfolio. (Brueggeman, Chen & Thibodeau, 1984), (Hartzell, Hekman & Mile, 1986), (Adair, McGreal & Web, 2006), (Olaleye, Aluko & Oloyede, 2008).

Property both direct and indirect seemed to provide good diversification benefits in low and medium mixed asset portfolios. In portfolios where returns and appreciation were analysed, property provided an enhanced benefits in low risk mixed portfolios. (Newell, Chau, Wong & Mckinnell, 2005).

The attractiveness of the asset class of direct real estate, was reinforced by (Hudson-Wilson, Fabozzi & Gordon, 2003). The uniqueness of this asset class and diversity benefits afforded by real estate, should be considered by investors, seeking risk adjusted returns (Sivitanides, 1997).

Core real estate continues to offer several major benefits that can enhance the performance of a mixed asset portfolio. These include low correlations with stocks and bonds, low volatility. Real estate may lower portfolio risk, provide a hedge against inflation and provides a predictable income stream. (Ziering & McIntosh, 1997).

The allocation or weighting of real estate to a multi asset portfolios, is a common theme amongst researchers with optimal levels of real estate holdings varying from 13% to 75%. A portfolio cannot be constructed like a recipe for a cake where the exactly the same ingredients provide the same results every time. In different time periods, different portfolios will be optimal. (Ziobrowski, Caines & Ziobrowski, 1999).

CHAPTER 3

Research Design

Introduction

The objective of this research report is to investigate the diversification benefits of including direct real estate to a multi asset portfolio of common shares and bonds. This objective can be achieved by using formalized statistical and mathematical methods, therefore the positivist research philosophy applies to this study.

The application of Markowitz Modern Portfolio Theory, is essential to this research report as this financial theory forms the over aching methodology for portfolio construction, and optimizing diversification benefits within multi asset portfolios.

The empirical segment of this research report will follow a quantitative positivist approach. Statistical Analysis, will focus on the returns, risks and relationships of each asset class within the portfolio.

The returns and risks of the selected asset classes, namely common stocks, bonds and direct real estate were analysed within a multi asset portfolio, with the view to identify the asset classes and combinations of assets which offer the optimal risk adjusted returns.

Correlation analyses was used to investigate and identify potential positive and negative correlation relationships of the asset classes, as combining negatively correlated asset classes reduces non- systematic risk within a portfolio and as such provides diversification benefits.

A series of three sets of portfolios were be constructed. The first portfolio based on a twenty year time frame and the second and third portfolios, with differing ten year time frames. Each set of portfolios encompassed ten naïve portfolios, based on a pre-determined selected allocation of the asset classes, (Sivitanides, 1997). The different time frames selected will enable investors to access the outcomes of the portfolios based on their time horizons of the investment strategy.

These three time periods were selected to determine, any substantial changes to the outcomes of the portfolios, based on the different time periods.

Specifically Portfolio Series number 1, was be based on a twenty year time frame from 1995 to 2014.

Portfolio Series number 2, was be based on a ten year time frame from 1995 to 2004.

Portfolio Series number 3, was be based on a ten year time frame from 2005 to 2014.

The differing time periods for the analysis of the portfolios were selected based on two aspects, firstly, the investors preferred investment time frame. The twenty year time frame is deemed a long term investment period and the ten year a medium term investment period. The second aspect is the Global Financial Crisis of 2007 -2008. During the Global Financial Crisis, the contagion effect influenced investments around the world including South Africa. Portfolio Series number 3, from 2005 to 2014, ensues during the Global Financial Crisis.

The individual portfolios were constructed based of the frameworks of Modern Portfolio Theory to determine the return and risk of these portfolios, subject to the allocation of the asset class mix in each portfolio.

Finally efficiency frontiers were be construct for the three series of portfolios.

Research methodology used to solve this problem are of a quantitative experimental method, as exist theory, Modern Portfolio Theory exists in this field. As stated previously Markowitz Modern Portfolio Theory forms the over aching theory for this research report.

Markowitz Modern Portfolio Theory, has been used extensively on research into multi asset portfolio, (Olaleye, 2011), (Hartzell, Hekman, & Miles, 1986) (Brueggeman, Chen & Thibodeau, 1984), (Hekman and Mile, 1986), (Akinsomi, Pahad, Nape & Margolis 2015), and others.

The empirical analysis general follows a pattern of analysing the returns and volatility of each of the asset classes. Statistically determining the optimum risk reward characterises of the asset classes. Quantifying the covariance of the returns of the asset class to discover positively or negative correlation relationships between the asset classes. Thereafter a series of naïve portfolios will be constructed to determine the optimum allocation of the various asset classes. The naïve portfolios assist in determining whether direct real estate provides diversification benefits to a multi asset portfolio. (Olaleye, 2011), and (Sivitanides, 1997)

Data Sources

The MSCI / Investment Property Databank South African Annual All Property Index, were used as a proxy for direct real estate. The data consists of the total return indices for commercial real estate in South Africa from a time period of 1995 to 2014. The data included the index returns for four types of direct property. These are indices for retail, office and industrial real estate and include a category for other real estate types of a more specialised nature, such a hotel and leisure. A further index is provided, which is called the all property index, which combines the four property types listed above.

The indices and benchmarks created from these databases covered both asset and fund-level performance. Asset level reporting encompassed all real estate investment interests that are portfolio structured and professionally managed. It therefore included properties held in insurance and pension funds, sovereign wealth funds, listed property companies including REITs, unlisted pooled funds, charitable trusts, traditional landed estates, and by large private property owners. Fund-level indexes are more narrowly restricted to unlisted pooled structures, since the returns to listed vehicles are covered by equity indexes, and those to segregated private mandates are not normally made public.

Direct real estate performance measurement may be carried out at property, portfolio or market level, or for any other grouping of property assets. The headline market returns reported in the MSCI /IPD indexes are based solely on directly owned standing investments in completed and lettable properties; these returns exclude assets held indirectly through investment funds and the impacts of debt, fund management fees, taxation and cash. Market measures are intended to reflect underlying market trends over the period of analysis. Some properties, such as those occupied by their owners, are screened out of market measures in all periods. Portfolio and benchmark returns include all investment properties within the portfolio, including those bought, sold, and under development or major refurbishment during the measurement period. Performance measures therefore allow the comparison of property and portfolio investment returns relative to an appropriate benchmark, either for the whole investment market or a relevant sub-group of portfolios or properties. The headline measures which are most widely relied upon and used to document the investment performance of commercial real estate are total return and its income and capital components. MSCI calculates these measures monthly and time-weights (chain-links) them over longer periods. They are value-weighted measures for each measurement

period, meaning that the contribution of each asset is in proportion to its monetary weight.

Common Shares or equities are represented by the All Share Index. The South African All Share Index, is designed to reflect the movement of the equity market. The FTSE/JSE All Share Index represents 99% of the full market capital value. The FTSE /JSE All Share Index contains the leading 164 securities listed on the JSE, measured by market capitalisation.

The data on the total returns of the all share index for the period from 1995 to 2014, were be sourced from IRESS data base.

Bonds, shall be represented by the South African Government ten year bond yield (KBP 2003m). Data on the ten year government bond were sourced from the South African Reserve Bank, with a time frame from 1995 to 2014. Government bonds were considered as they are deemed low risk investments because they are backed by the full faith and credit of the government.

Ethical Considerations

This research report is of a quantitative nature and the various data inputs required are available in the public domain.

The data on common shares will be sourced from the IRESS Software Company previously known as the BFA McGregor. IRESS is a private company which provides investment data and research information on financial markets in South Africa. This information is available in the public domain on a subscription basis. The IRESS databases will be accessed via the universities online research portal. Data on government bonds will be sourced from the South African Reserve Bank online research portal.

The data on returns of direct real estate will be sourced from the Morgan Stanley Capital International, (MSCI) / IPD South African Annual Property Index Report. Morgan Stanley Capital International, (MSCI) is an international financial services company, which provide investment data and to the financial community. The performance of direct real estate is one such research report compiled by this research organisation.

Data Analysis and Interpretation

The data analysis provided a quantitative assessment of the returns and risks of each of the asset classes, when combined in a portfolio, with a predetermined allocation to each of the asset classes. The objective of constructing these portfolios to determine the portfolio that provides the lowest risk for a given expected return and the portfolio that provides the highest return for a given level of risk and then by assessing the allocation of each asset class in the portfolio and determining whether direct real estate provides any diversification benefits, if any to a multi asset portfolio.

The time frame of twenty years, from 1994 to 2014 was used for the analysis of the data of the three selected asset classes. This time frame is due to the availability of data on direct real estate sourced from the MSCI /IPD. The data was analysed in two key processes. The first process is that of descriptive statistics to determine the returns and risk of the asset classes and second is the construction ten Naïve Portfolios, based on Modern Portfolio Theory.

Descriptive Statistics

Step 1: Mean Annual Returns of the selected Asset Classes

These are the gains or losses of the asset classes, over a yearly period, as a percentage increase or decrease over the initial investment cost.

For common shares the holding period returns were calculated based on the following formula.

$$\left(\frac{\text{index}_{t} - \text{index}_{t-1}}{\text{index}_{t-1}} \right) \times 100 \quad \text{.................................} (1)$$

Where

Index t **=** returns from the current period

Index t-1 **=** returns from the previous period

Bonds, are represented by the South African Government ten year bond yield (KBP 2003m). The bond yield, represents the return on investment for the holders on these bonds. The yields are expressed as a percentage. The return data on the government ten year bond yields was available monthly. This data are converted to yearly data.

The yearly bond yields were determined based on the average yields for the year. By added the yields for the twelve months and dividing by twelve, there by determining the yield for the year as presented in Annexure A2.

Returns for all property index and retail, industrial and office sectors are available from the MSCI / Investment Property Databank South African Annual Property Index, in an annual format, no further analysis or calculations are required.

Step 2: Standard Deviation of the selected Asset Classes

Standard deviation is a measure of the dispersion of a set of data from its mean. Standard deviation is calculated as the square root of variance.

The standard deviation were calculated from the returns of the various asset classes, to determine the risk or volatility of the asset class. Secondly, the standard deviation of the various asset classes were be used in the calculation of the various naïve multi asset portfolios in terms of modern portfolio theory.

The equation used to measure standard deviation is as follows.

$$s = \sqrt{\frac{\sum(x - \overline{x})^2}{N - 1}} \quad \text{..... (2)}$$

Where

S = the Standard deviation

$\mathcal{X}$ = each value of the population

$\overline{\mathcal{X}}$ = the mean of the values

N = the number of values (the population)

Step 3. The Application of Modern Portfolio Theory to Portfolio Construction

3.1 Allocation of Assets Classes to the Portfolio

A matrix of asset allocations and performance of portfolios were constructed on a naïve allocation basis, to determine diversification benefits of direct real estate in a mixed asset portfolio. A series of portfolios were constructed with ten different asset allocation options of the three asset classes, as constructed by (Sivitanides, 1997).

Step 4. Returns of the Portfolio

Returns of a portfolio is the weighted average of the returns of the individual assets within the portfolio, weighted by their proportionate share of the portfolio. This given as;

$$\mathrm{E}\{r_p\} = w_i \mathrm{E}\{r_i\} + w_i \mathrm{E}\{r_{ii}\} + \mathrm{E}\{r_{iii}\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where

E $\{r_p\}$ = the returns of the portfolio

w_i = the weighting of asset first asset within the portfolio.

E $\{r_i\}$ = the returns of the first asset in the portfolio

E $\{r_{ii}\}$ = the returns of the second asset in the portfolio

E $\{r_{iii}\}$ = the returns of the third asset in the portfolio

Step 5. Risk of the portfolio

There are many different definitions of risk, one widely – used measure is called variance, and others include volatility. Variance measures the variability of realized returns around an average level. The larger the variance the higher the risk of the portfolio.

To calculate the risk or variance of a portfolio based on the Modern Portfolio Theory, four steps are required.

The first step is to calculate the correlation between the various asset classes.

Correlation is a measure that determines the degree to which two variables' movements are associated or in terms of investment terminology the extent to which the returns of two asset classes move together or opposite each other. Modern Portfolio Theory emphasizes that investors can diversify away the risk of investment loss by reducing the correlation between the returns from the asset classes. The range of values for the correlation coefficient is -1.0 to 1.0. If a calculated correlation is greater than 1.0 or less than -1.0. A correlation of -1.0 indicates a perfect negative correlation, while a correlation of 1.0 indicates a perfect positive correlation. The correlation is represented by the following equation.

$$\rho_{xy} = \frac{Cov(r_x, r_y)}{\sigma_x \sigma_y} \quad \text{.....(4)}$$

Where

$P_{(x,y)}$ = correlation of the variables *x* and *y*

$COV(x, y)$ = covariance of the variables *x* and *y*

σx = sample standard deviation of the random variable *x*

σ_y = sample standard deviation of the random variable *y*

The results of the correlation between the various asset classes, are presented in a matrix format.

The second step is to calculate the covariance expectations. This data is presented in a matrix format, representation the three asset classes. The formula to determine the covariance expectation of an asset class is as follows:

$$COV(x,y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots 5$$

Where

x = the independent variable

y = the dependent variable

n = number of data points in the sample

$\bar{x}$ = the mean of the independent variable *x*

$\bar{y}$ = the mean of the dependent variable *y*

The third step in calculating the portfolio's risk or variance. The formula to determine the variance follows:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2x\ covariance\ (1,2) \quad \ldots\ldots\ldots 6$$

Where

$ó^2_p$ = Variance of the portfolio.

W_1 = is the weight of the first asset.

W_2 = is the weight of the second asset.

$ó^2_1$ = is the variance of the first asset.

$ó^2_2$ = is the variance of the second asset.

The Returns of the Portfolio as described in step 4 and the Risk of variance of the portfolio as described in step 5, above are in essence the fundamentals information necessary to determine the performance of the portfolio.

Step 6. The Sharpe Ratio

The Sharpe ratio is a risk adjusted measure of return that is often used to evaluate the performance of a portfolio. The ratio helps to make the performance of one portfolio comparable to that of another portfolio, by making an adjustment for risk. The Sharpe ratio can be explained in just three components namely, asset returns, risk-free rate and standard deviation. After calculating the excess return, which is determined by subtracting the risk free rate return of the portfolio. The resultant figure is divided by the standard deviation of the risky asset to get its Sharpe Ratio. The idea of the ratio is to determine how much additional return you are receiving for the additional volatility of holding the risky asset over a risk – free asset. The higher the better.

The Sharpe ratio, where the applicable risk free rate shall be the South African Government Ten Year Bond Yield.

Sharpe Ratio $S(X) = \left(\mathcal{R}_X - \mathcal{R}_f\right)/StdDev\ (X)$.....7

Where

S(X) = the Sharpe ratio of the investment

Rx = the average rate of return on X

Rf = the rate of return on the risk free security (Ten year bond)

StdDev (X) = the standard deviation of Rx

Step 7. Efficient Frontier and Optimal Portfolio

Constructing the efficiency frontier and determining the optimal portfolio consists of following steps. The first entails the selection of the portfolios constructed previously in portfolio returns and risks.

Secondly finding the lowest standard deviation of the portfolio. This is done with the assistance of Excel Solver.

The next step involves an iterative process, where solver can run a number of times, each time increasing the return of the portfolio slightly. A target cell with the must be

created on the spread sheet, slightly higher than the calculated previously portfolio return. The results of each solver calculation with a slightly higher portfolio return can be set up as illustrated.

The results of each iterative process, should be documented in a table. The slope is determined by dividing the portfolio returns by the standard deviation. This process should be repeated until the value of the slope “peaked”.

The portfolio with the maximum slope value is the optimal portfolio allocation.

CHAPTER 4

Data Analysis and Findings

Introduction

The analysis and presentation of data done follows three primary procedures. The first stage is a process of descriptive statistics to determine the returns, standard deviation, risk to reward and correlation of the asset classes.

The second stage is the application of Modern Portfolio Theory to construct the various naïve portfolios. The application on Modern Portfolio Theory delivers, the returns and risk or volatility of the various portfolios for further analysis. The detail calculations are contained in the annexures section of this research report.

The final stage of the process, entails calculating the return and risks of the various portfolio and determining the Sharpe Ratio of the portfolios.

Presentation of Data Analysis

The analyses are presented in three sub -sections with three tables and three graph for each sub- section.

The three sub sections denote on the various holding periods of the various portfolios. Portfolio set 1 is based on a twenty year time period from 1995 to 2014. Portfolio Set two is based on a ten year holding period from 1995 to 2004 and Portfolio set three is a ten year holding period from 2005 to 2014.

The first table present an analyses of descriptive statistics related to the asset classes under consideration, that being Common Stocks, Bonds and Real Estate. The second presents performance of constructed naïve portfolios and third displays the optimum portfolios. Chart 1 represents the returns of the asset classes, chart 2 reflects the returns and risks of the ten naïve portfolios and the third graph displays the efficient frontier and optimum portfolio.

Analysis of Data and Presentation of Results

Sub Section 1: Portfolio Series 1- 20 Years – 1995 -2014

Table 1: Descriptive Statistics

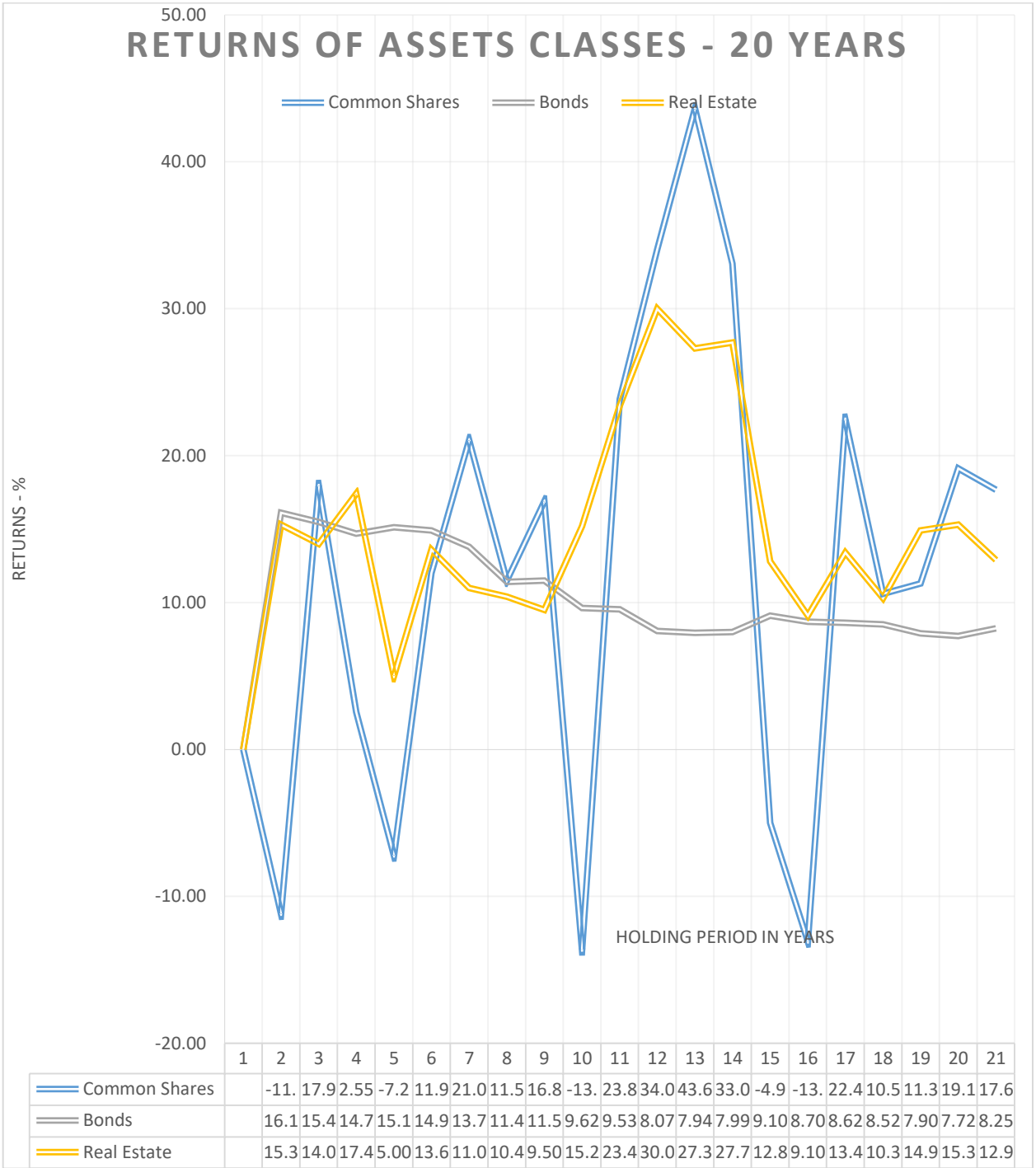
Annual Returns, Standard Deviation and Correlations of Shares, Bonds and Direct Real Estate for 20 years, from 1995 to 2014.

Annual Returns and Standard Deviation Of Asset Classes – 20 Years				
	Date	**Shares**	**Bonds**	**Real Estate**
		%	**%**	**%**
1	31- Dec -95	-11.18	16.11	15.30
2	31- Dec -96	17.94	15.48	14.00
3	31- Dec -97	2.55	14.70	17.40
4	31- Dec -98	-7.22	15.12	5.00
5	31- Dec -99	11.99	14.90	13.60
6	31- Dec -00	21.05	13.79	11.00
7	31- Dec -01	11.50	11.41	10.40
8	31- Dec -02	16.88	11.50	9.50
9	31- Dec -03	-13.64	9.62	15.20
10	31- Dec -04	23.84	9.53	23.40
11	31- Dec -05	34.03	8.07	30.00
12	31- Dec -06	43.63	7.94	27.30
13	31- Dec -07	33.06	7.99	27.70
14	31- Dec -08	-4.99	9.10	12.80
15	31- Dec -09	-13.08	8.70	9.10
16	31- Dec -10	22.46	8.62	13.40
17	31- Dec -11	10.59	8.52	10.30
18	31- Dec -12	11.30	7.90	14.90
19	31- Dec -13	19.13	7.72	15.30
20	31- Dec -14	17.67	8.25	12.90
	Mean Annual Returns	**12.38**	**10.75**	**15.43**
	Standard Deviation	**15.77**	**2.99**	**6.52**

Correlation Matrix Of Asset Classes -20 Years			
	Shares	**Bonds**	**Real Estate**
Shares	**1.000**	**-0.3730**	**0.6516**
Bonds	**-0.3730**	**1.000**	**-0.3811**
Real Estate	**0.6516**	**-0.3811**	**1.000**

Chart 1: Returns of Asset Classes

Annual Returns of Shares, Bonds and Direct Real Estate for 20 years, from 1995 to 2014.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Common Shares		-11.	17.9	2.55	-7.2	11.9	21.0	11.5	16.8	-13.	23.8	34.0	43.6	33.0	-4.9	-13.	22.4	10.5	11.3	19.1	17.6
Bonds		16.1	15.4	14.7	15.1	14.9	13.7	11.4	11.5	9.62	9.53	8.07	7.94	7.99	9.10	8.70	8.62	8.52	7.90	7.72	8.25
Real Estate		15.3	14.0	17.4	5.00	13.6	11.0	10.4	9.50	15.2	23.4	30.0	27.3	27.7	12.8	9.10	13.4	10.3	14.9	15.3	12.9

Table 2: Asset Allocation and Performance of Constructed naïve Portfolios

Allocation of Asset Classes, Portfolio Returns, Risks and Sharpe Ratios for 20 years, from 1995 to 2014.

PORTFOLIO SERIES 1										
1995 – 2014 – 20 YEARS										
			Allocation							
	Portfolio		Stocks %	Bonds %	Real Estate %		Portfolio Returns %	Portfolio Risk %		Sharpe Ratio
1	1A		0.00	36.70	63.30		13.71	3.84		0.77
2	1B		11.20	32.20	56.50		13.57	4.67		0.60
3	1C		22.50	27.30	50.10		13.45	5.90		0.46
4	1D		33.90	22.40	43.70		13.34	7.27		0.36
5	1E		45.20	17.40	37.30		13.22	8.71		0.28
6	1 F		56.60	12.50	31.00		13.13	10.21		0.23
7	1G		67.90	7.50	24.60		13.00	11.73		0.19
8	1H		79.30	2.50	18.20		12.89	13.28		0.16
9	1I		90	0.0	10.00		12.68	14.63		0.13
10	1J		100	0.0	0.00		12.38	15.77		0.10

The risk free rate, was determined as the twenty year average of the government ten bond [KBP 2003m] as 10.71

Chart 2: Ten Naïve Portfolios

Portfolio Returns, Risks for 20 years, from 1995 to 2014.



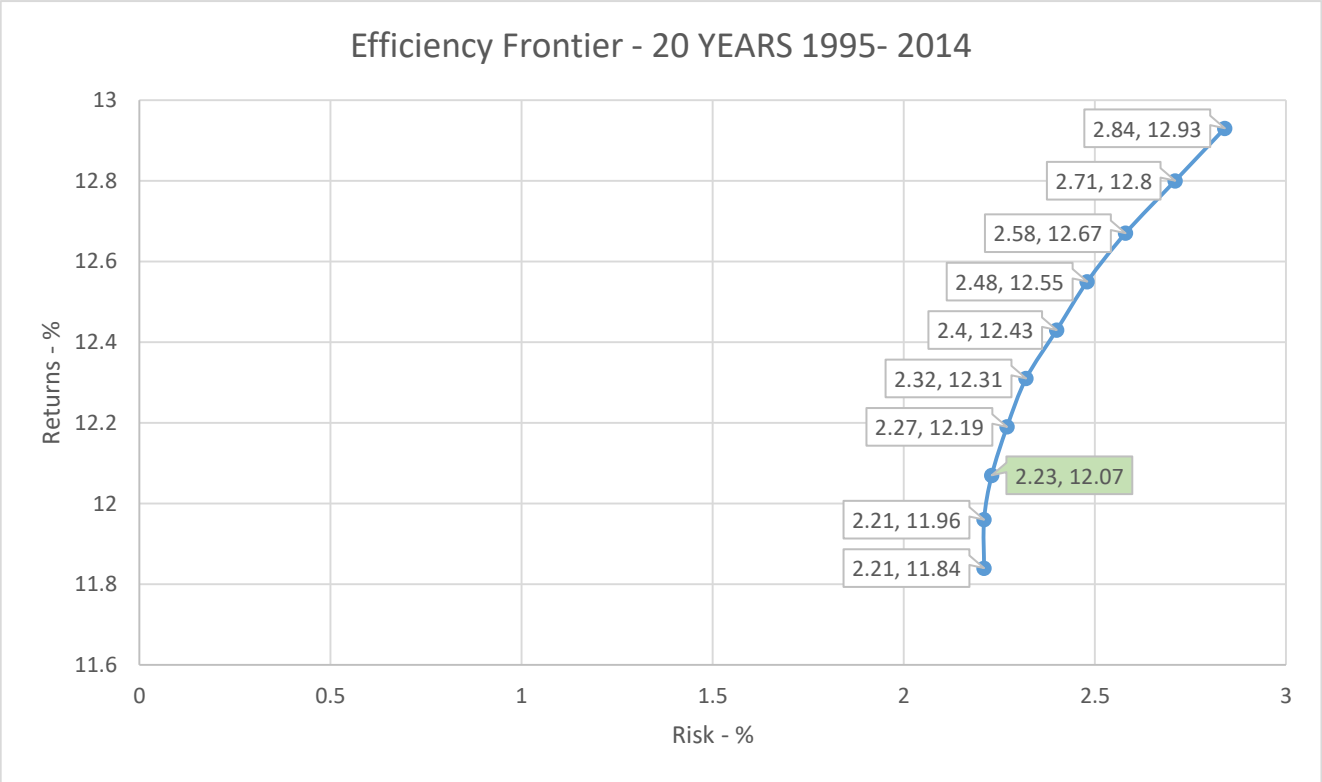
Table 3: Optimum Portfolios

Allocations and returns and risks of efficient portfolios.

DATA - Efficiency Frontier and Optimum Portfolio										
	Min Risk		Optimum Portfolio %							
Portfolio Returns -%	11.84	11.96	12.07	12.19	12.31	12.43	12.55	12.67	12.8	12.93
Portfolio Risk -%	2.21	2.21	2.23	2.27	2.32	2.4	2.48	2.58	2.71	2.84
Slope	5.357	5.412	5.413	5.370	5.306	5.179	5.060	4.911	4.723	4.553
Shares - Allocation %	0	0	0	0	0	0	0	0	0	0
Bond - Allocation -%	77	74	72	69	67	64	61	59	56	53
Real Estate Allocation -%	23	26	28	31	33	36	39	41	44	47

Chart 3: Efficiency Frontier

Efficient portfolios on the efficient frontier



Note: Common Shares do not feature in any of the portfolios on the efficiency frontier.

Findings and Results

Table 1- Descriptive Statistics and Chart 1- Returns of Asset Classes

The mean returns and standard deviations were calculated to provide a relative comparison of the performance of the different asset classes on both risk and return basis from 1995 to 2014.

The performance of the three asset classes from 1995 to 2014 are portrayed in table1 and chart 1. According to the tables and the graph, in the past 20 years, direct real estate provided the highest average returns of 15.43 percent. This is followed by shares with 12.38 percent average return. Government bonds provided the lowest average return of 10.75 percent.

The risk or volatility levels of the asset classes were determined based on the standard deviation of their historical returns, Shares appear to be the most risky of the three asset classes having a standard deviation of 15.77 percent compared to real estate with a standard deviation of 6.52 percent, and expected bonds has the lowest stand deviation of 2.99 percent. Government bonds are generally the least risky of the three asset classes as bonds have the backing of the government.

Correlation of Asset Class – Table 1

The correlation coefficients between shares and bonds was – 0.3730, between shares and real estate +0.6516 and between bonds and real estate – 0.3811.

The highly positive correlation between shares and real estate suggests that having each pair of the assets combined in a portfolio might be expected to bring minimal or no risk reduction benefits to the portfolio. It also shows that they are close substitutes. This finding is similar to the finding of (Akinsomi & Ntuli 2017).

The negative correlation of shares and bonds at -0.3730 and bonds and real estate at, -0.3811, suggest that combining these pairs of assets may provide diversification benefits to a portfolio.

Asset Allocation and Performance of naïve Portfolios – Table 2

Table 2 represents the results of the mean returns, standard deviations and Sharpe ratios of the ten constructed portfolios, based on the principals of Modern Portfolio Theory, with various allocations to the different asset classes.

The results of the analysis are presented in two sub sections. The first on return enchantments and the second on risk reduction.

Return Enhancement

The return enhancement characteristics of real estate in the portfolios, was measured by comparing the Sharpe Ratios of the various portfolios with the differing allocations of real estate to the portfolios.

From Table 2, it is evident that portfolio (1A), with the highest allocation (63.30%) to real estate delivers the highest Sharpe Ratio of 0.77 and the returns of all the portfolio are 13.71 percent. The portfolio with the lowest allocation of real estate, 0 percent (1J) delivered the lowest overall Sharpe Ratio of 0.10 and the lowest returns of 12.38%. There is evidence of a linear relationship between the allocation to real estate of the portfolios and the returns in the portfolios. This is evident from portfolio 1A to portfolio 1J, as the allocation to real estate reduces so do the returns of the portfolio.

The Sharpe ratio for Portfolios 1B to 1I, range from 0.60 to 0.13. The reduction in the Sharpe Ratio is consistent with the reduction with the allocation of real estate to the portfolios.

The returns range of returns from 13.86 to 12.68 percent, these portfolio hold an allocation of real estate between 10.00 percent and 56.50 percent.

Regarding the allocation of shares to the various portfolios and the returns of the portfolios, Portfolio 1A, with the lowest allocation (0.00%) of shares delivered the highest returns of all ten portfolios at 13.71 percent. The portfolio with the highest allocation of shares portfolio, (100%) (1J) delivered the lowest overall returns at 12.38%.

The allocation of bonds to the ten portfolios range from 0.00 percent to 36.90 percent. These percentage allocations were substantially, less that the allocation of shares and real estate to the portfolios, as the maximum allocation of bonds to the portfolio was 36.70 percent. Contrary was shares 100 percent and real estate 63.0 percent. This implies that bonds have a low to minimal impact on the returns of the various portfolios considering the formula used to calculate the returns of the portfolios. The formula,

takes into account the weighting / allocation of the asset class in the portfolio and the returns of the asset class. With bonds the annual mean returns were the lowest at 10.75 percent.

Risk Reduction

Similar to the return enhancement analysis above the risk characteristics of real estate was measured by comparing the risk of the portfolio with the various allocations of real estate to the portfolios.

The interesting observation on the risk values of the ten portfolios, is that as the allocation of real estate to the various portfolios reduces, the risk values of the portfolios increase.

Portfolio 1A, has the highest allocation of real estate to portfolio at 63.30 percent and the portfolio risk was 3.84 percent. As the allocation of real estate was reduced to the various portfolios the risk of the portfolios increased. Portfolio 1J held no real estate and the portfolios consisted of 100 percent allocation of shares had the highest risk of 15.77 percent of all the portfolios

From the analysis above it is evident that a reduction in the percentage allocation of real estate equates to an increase in the risk values of the portfolios.

The converse is applicable to shares as the allocation of shares to the portfolio increase the value of the risk of the portfolios increase.

The Efficiency Frontier and Optimal Portfolio Chart 3 and Table 3.

The efficiency frontier is a graphical representation of all the portfolio combination that maximize returns and minimize risk, with different allocation of the asset classes.

A total of ten efficient portfolios were calculated. These ten portfolios display the following characteristics.

All of the efficient portfolios in Table 3 contain real estate. The allocation of real estate to the ten portfolios range from 23 to 47 percent. An allocation of shares do not feature in any of the efficient portfolios. Portfolio 3, was deemed the optimum portfolio, based on the slope value of 5.413, the highest of all the portfolios. The allocations within portfolio 3 consisted of 72 percent to bonds and 28 percent to real estate, the returns were calculated at 12.07 percent and risk at 2.23 percent.

Summary: Portfolio Series 1- 20 Years – 1995 -2014

Both real estate and bonds provided positive returns of the twenty year holding period. However shares provided negative returns in five years of the 20 year period of analyses. The volatility of shares, as measured by the standard deviation is highest amongst the three asset classes at 15.77 percent, compared to bonds at 2.99 percent and real estate 6.52 percent. The volatility of shares are 58.65 percent higher than real estate and 81 percent higher than bonds.

The negative correlation between shares and bonds (-03730) and bonds and real estate (-0.3811), suggest that combining these pairs of assets may provide diversification benefits to a portfolio. However the benefits of diversification within the portfolio is also depended on the allocation of these assets.

A converse relationship exists between the allocations of shares and real estate to the portfolios and the impact on the returns and risks of the portfolios can be ascribed to the highly positive correlation between shares and real estate (0.6516). The positive correlations limit or minimize the diversification benefits of real estate within a portfolio.

Optimisation in terms of portfolio theory seeks to identify the best combination in terms of and the percentage weighting or allocation of each asset class to hold within the portfolio. This combination of various assets will produce a level of risk and return acceptable or desired by the investor. Portfolio No 3, with a return of 12.07 and risk of 2.23, with an allocation of 72 percent to bonds and 28 percent to real estate was identified as the optimal portfolio.

The allocation of real estate to all ten efficient portfolio, ranging from 23 to 46 percent, further justifies a high allocation of real estate to multi asset portfolios, as real estate provides enhanced diversification benefits as well as enhanced return benefits to the portfolio.

Sub Section 2: Portfolio Series 2- 10 Years – 1995 -2004

Table 4: Descriptive Statistics for 10 years (1995-2004)

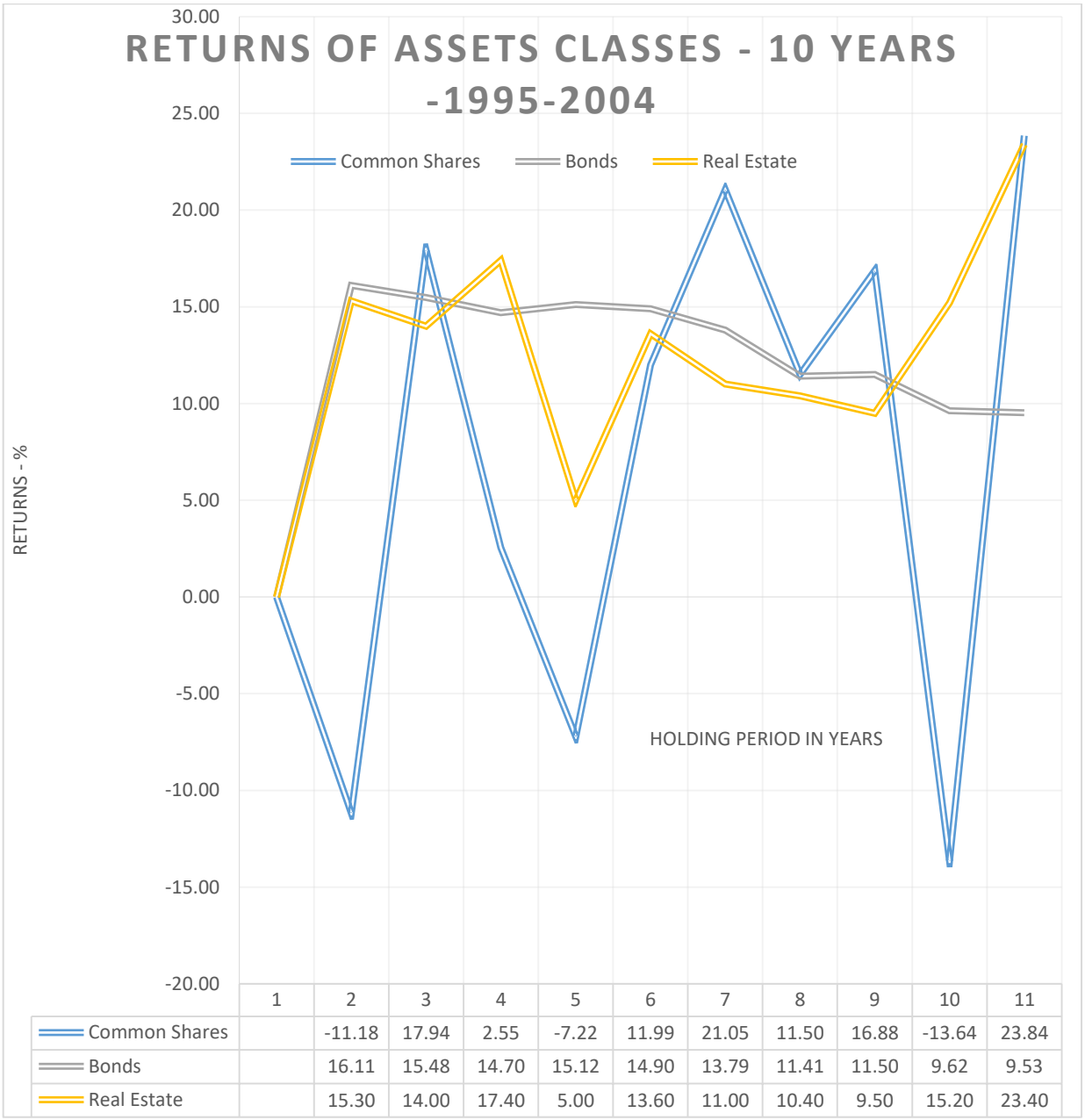
Annual Returns, Standard Deviation and Correlations of Shares, Bonds and Direct Real Estate for 10 years, from 1995 to 2004.

Annual Returns and Standard Deviation Of Asset Classes – 10 Years				
	Date	**Shares**	**Bonds**	**Real Estate**
		%	**%**	**%**
1	31- Dec -95	-11.18	16.11	15.30
2	31- Dec -96	17.94	15.48	14.00
3	31- Dec -97	2.55	14.70	17.40
4	31- Dec -98	-7.22	15.12	5.00
5	31- Dec -99	11.99	14.90	13.60
6	31- Dec -00	21.05	13.79	11.00
7	31- Dec -01	11.50	11.41	10.40
8	31- Dec -02	16.88	11.50	9.50
9	31- Dec -03	-13.64	9.62	15.20
10	31- Dec -04	23.84	9.53	23.40
	Mean Annual Returns	**7.37%**	**13.22%**	**13.48%**
	Standard Deviation	**13.12%**	**2.35%**	**4.73%**

Correlation Matrix Of Asset Classes -10 Years			
	Shares	**Bonds**	**Real Estate**
Shares	**1.000**	**-0.1881**	**0.1932**
Bonds	**-0.1881**	**1.000**	**-0.3191**
Real Estate	**0.1932**	**-0.3191**	**1.000**

Chart 4: Returns of Asset Classes

Annual Returns of Shares, Bonds and Direct Real Estate for 10 years, from 1995 to 2004.



	1	2	3	4	5	6	7	8	9	10	11
Common Shares		-11.18	17.94	2.55	-7.22	11.99	21.05	11.50	16.88	-13.64	23.84
Bonds		16.11	15.48	14.70	15.12	14.90	13.79	11.41	11.50	9.62	9.53
Real Estate		15.30	14.00	17.40	5.00	13.60	11.00	10.40	9.50	15.20	23.40

Table 5: Asset Allocation and Performance of Constructed naïve Portfolios.

Allocation of Asset Classes, Portfolio Returns, Risks and Sharpe Ratios for 10 years, from 1995 to 2004.

PORTFOLIO SERIES 2									
1995 – 2004 – 10 YEARS									
			Allocation						
	Portfolio		Stocks %	Bonds %	Real Estate %		Portfolio Returns %	Portfolio Risk %	Sharpe Ratio
1	2A		0.00	36.70	63.30		13.38	2.84	0.06
2	2B		11.20	32.20	56.50		13.86	4.67	-0.16
3	2C		22.50	27.30	50.10		12.02	3.97	-0.30
4	2D		33.90	22.40	43.70		11.35	5.13	-0.36
5	2E		45.20	17.40	37.30		10.66	6.41	-0.40
6	2F		56.60	12.50	31.00		10.00	7.78	-0.41
7	2G		67.90	7.50	24.60		9.31	9.16	-0.43
8	2H		79.30	2.50	18.20		8.63	10.59	-0.43
9	2I		90	0.0	10.00		7.98	11.91	-0.44
10	2J		100	0.0	0.00		13.12	7.37	-0.01

The risk free rate , was determined as the ten year average of the Government ten year bond [KBP 2003m] as 13.22

Chart 5: Ten Naïve Portfolios

Portfolio Returns, Risks for 10 years, from 1995 to 2004.

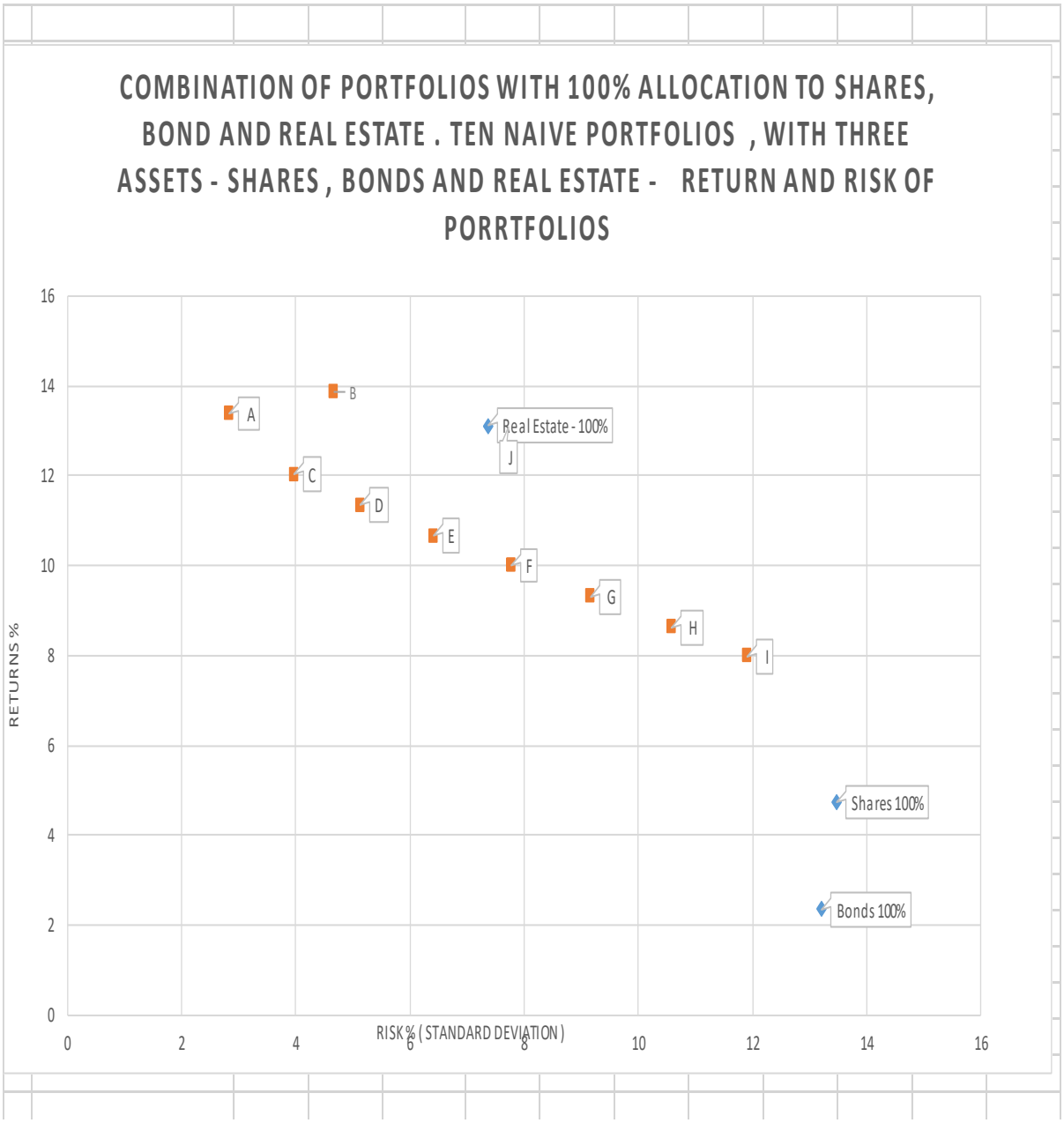
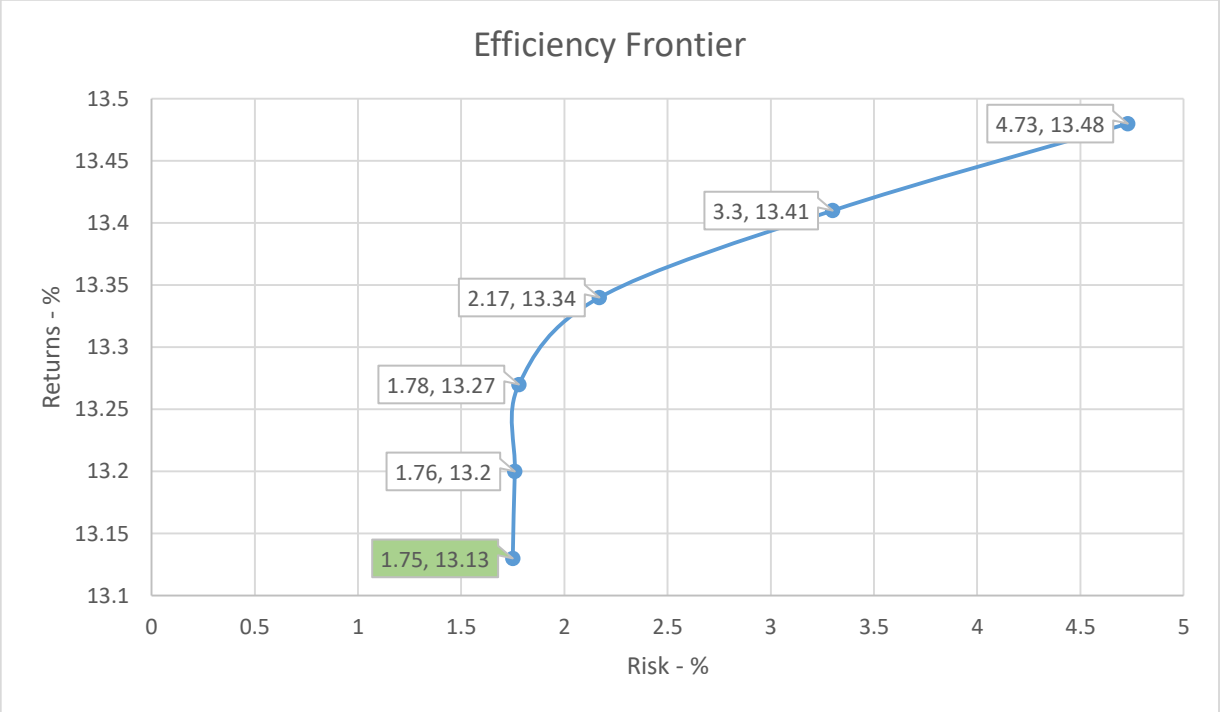


Table 6: Optimum Portfolios
Allocations and returns and risks of efficient portfolios.

DATA - Efficiency Frontier and Optimum Portfolio						
	Optimum Portfolio					
Portfolio Returns %	13.13	13.2	13.27	13.34	13.41	13.48
Portfolio Risk %	1.75	1.76	1.78	2.17	3.3	4.73
Slope %	7.503	7.500	7.455	6.147	4.064	2.850
Shares	3.00	1	0	0	0	0
Bond	73.00	74	75	53	27	0
Real Estate	23.00	25	25	47	73	100

Chart 6: Efficiency Frontier
Efficient portfolios on the efficient frontier



Findings and Results

Descriptive Statistics – Table 4 and Returns of Asset Classes - Chart 4

The mean returns and standard deviations are calculated to provide a relative comparison and performance of the different asset classes on both risk and return basis from 1995 to 2004.

The performance of the three asset classes from 1995 to 2004 are portrayed in Table 4 and Chart 4 According to the tables and the chart, in the past 10 years real estate provided the highest average returns that is 13.48 percent, followed by bonds with 13.22 percent average return. Shares provided the lowest average return that is 7.37 percent.

The risk or volatility levels of the asset classes were determined based on the standard deviation of their historic returns, Shares were to be the most risky of the three asset classes having the highest standard deviation of 13.12 percent compared to real estate with a standard deviation of 4.73 percent, and expectedly bonds has the lowest standard deviation of 2.35 percent. Shares provided negative returns in three of the ten years under consideration. The returns on shares ranged from -13.64 percent to a negative 23.84. This illustrates the risky nature of this asset class. Bonds and real estate did not deliver any negative returns during the ten year holding period. The returns on bonds ranged from 9.53 to 16.11 percent and the range of returns of real estate from to 5.00 to 23.40 percent.

Correlation of Asset Class – Table 4

The correlation coefficients between shares and bonds was – 0.1881; between shares and real estate was +0.1932 and between bonds and real estate – 0.3191.

The positive correlation between shares and real estate suggests that having each pair of the assets combined in a portfolio might be expected to bring minimal or no risk benefits to the portfolios. Contrary the negative correlation between shares and bonds of -0.1881 and bonds and real estate, at -0.3191, suggest that combining these pairs of assets may provide diversification benefits to a portfolio.

Asset Allocation and Performance of naïve Portfolios – Table 5

Table 5 represents the results of the mean returns, standard deviations and Sharpe ratios of the ten constructed portfolios, based on the principals of Modern Portfolio Theory, with various allocations of the different asset classes.

The results of the analysis are presented in two sub sections, the first on return enchantments and the second on risk reduction.

Return Enhancement

The return enhancement characteristics of real estate in the portfolios, was measured by comparing the Sharpe Ratios of the various portfolios with the differing allocations of real estate to the portfolios.

From table 5 it is evident that portfolio 2B, with the second highest allocation (56.50%) to real estate delivered the highest returns of the portfolio at 13.86 percent. Portfolio 2A, with holds the highest allocation (63.30%) of real estate delivered the second highest returns of the portfolio at 13.38 percent. However this portfolio has the highest Sharpe Ratio of 0.06. The portfolios with an allocation of about fifty percent (50.10) to real estate deliver the highest and third highest, returns of all the ten portfolios, thereby confirming real estate's ability to enhance the returns of portfolios.

The allocation of bonds to the ten portfolios range from 0.0 percent to 36.70 percent. These percentage allocations were substantially less that the allocation of shares and real estate to the portfolios. The maximum allocation of bonds to the portfolio was 36.70 percent while that of shares 100 was percent and real estate 63.0 percent. This implies that bonds had a low to minimal impact on the returns of the various portfolios considering the formula used to calculate the returns of the portfolios. The formula, takes into account the weighting / allocation of the asset class in the portfolio and the returns of the asset class. With bonds, the annual mean returns were the lowest at 2.35 percent.

An allocation of 100 percent to shares, in portfolio 2J, produced a return of (13.12) percent return. The third highest returns of all the portfolio. The Sharpe Ratio on this portfolio was -001, the lowest of all ten portfolios.

Risk Reduction

Similar to the return enhancement analysis above the risk characteristics of real estate was measured by comparing the risk of the portfolio with the various allocations of real estate to the portfolios.

The interesting observation on the risk values of the ten portfolios, is that when the allocation of real estate to the various portfolios, was in excess of fifty percent , the allocation produced the lowest risk values of all the ten portfolios.

Portfolio 2A, which had the highest allocation of real estate to portfolio at 63.30 percent produced the lowest risk of all ten portfolios at 2.84 percent. Portfolios 2C with an allocation of 50.10 to real estate produce a risk value of 3.97 percent, similar to portfolio 2B, with an allocation of 56.50 percent produced the third lowest risk value of 4.67 percent.

From the analysis above it is evident that a high allocation of real estate to the portfolios reduces the risk of the portfolios.

The Efficiency Frontier and Optimal Portfolio Chart 6 and Table 6.

The efficiency frontier is a graphical representation of all the portfolio combination that maximize returns and minimize risk, with different allocation of the asset classes.

A total of ten efficient portfolios were calculated. These ten portfolios display the following characteristics.

Six of the ten portfolios were deemed efficient portfolios. Real estate was allocated to all six efficient portfolios, the allocation of real estate to these six portfolios range from 23 to 100 percent. Portfolio number 1, with the highest slope of 7.50 was deemed the optimum portfolio, with a return of 13.13 percent and risk of 1.75 percent.

Summary: Portfolio Series 2- 10 Years – 1995 -2004

Both real estate and bonds provided positive returns during the ten year holding period. However shares provided negative returns in three years. The volatility of shares, as measured by the standard deviation was highest amongst the three asset classes at 13.12 percent, with real estate at 4.73 percent and bonds at 2.35 percent. The risk of shares was 2.77 time higher than real estate and 3.60 times higher than bonds.

The negative correlation of shares and bonds (-0.1881) and bonds and real estate (-0.3191), suggest that combining these pairs of assets may provide diversification benefits to a portfolio. However the benefits of diversification within the portfolio is also depended on the allocation to these assets.

The converse relationship between the allocations of shares and real estate to the portfolios and the impact on the returns and risks of the portfolios can be ascribed to the positive correlation between shares and real estate (+0.1932). Such positive correlation limits or minimizes the diversification benefits within a portfolio.

Optimisation in terms of portfolio theory seeks to identify the best combination in terms of and the percentage weighting or allocation of each asset class to hold within the portfolio. Portfolio No 1, with a return of 13.13 and risk of 1.75, with an allocation of 3 percent to shares, 73 percent to bonds and 23 percent to real estate was identified as the optimal portfolio.

The allocation of real estate to all six the efficient portfolio, ranging from 23 to 100 percent, further justifies a high allocation of real estate to multi asset portfolios. This is because real estate provided enhanced diversification benefits as well as enhanced return benefits to the portfolio.

Sub Section 3: Portfolio Series 3- 10 Years – 2005 -2014

Table 7: Descriptive Statistics for 10 years (2005-2014)

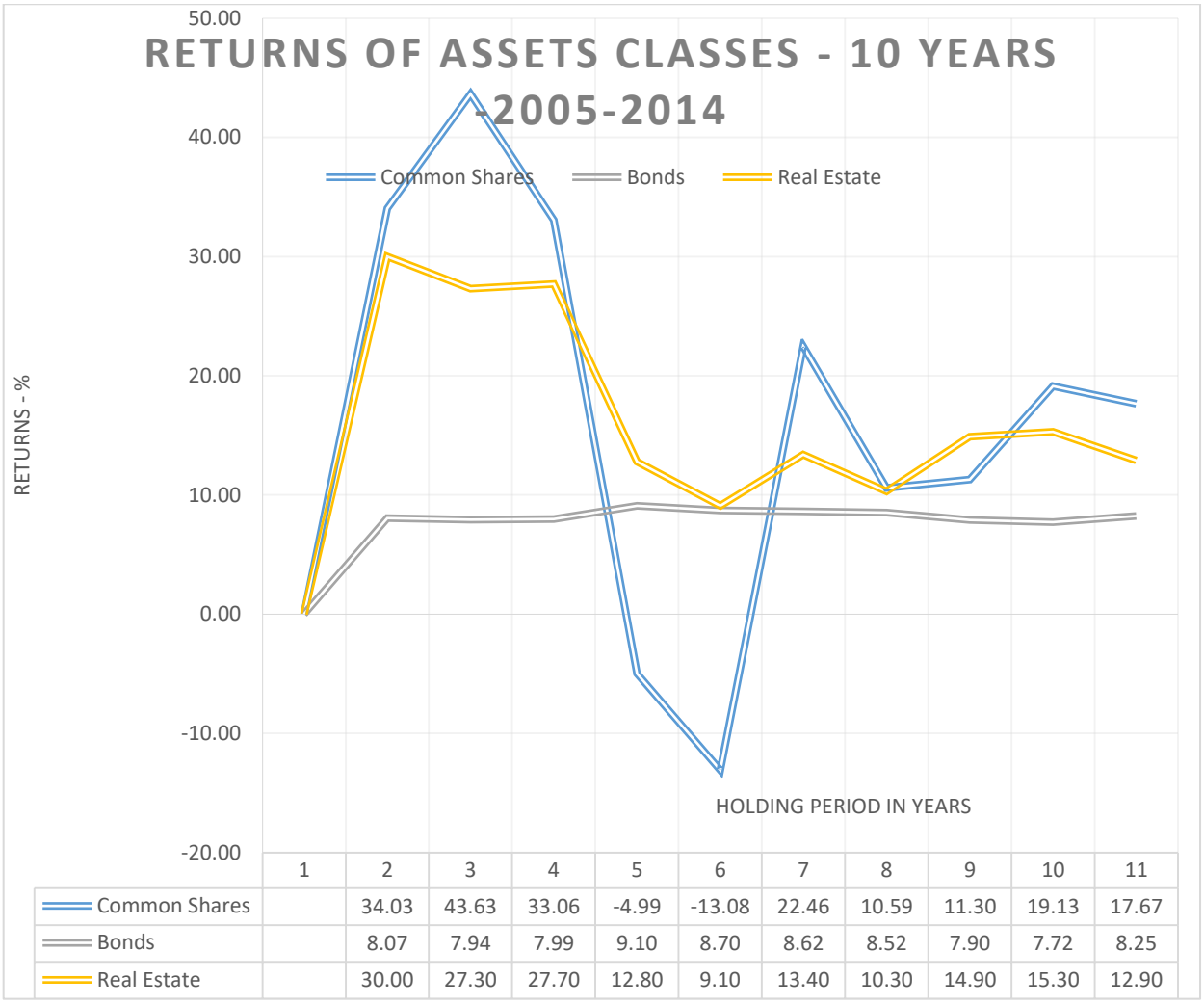
Annual Returns, Standard Deviation and Correlations of Shares, Bonds and Direct Real Estate for 10 years, from 2005 to 2014.

Annual Returns and Standard Deviation Of Asset Classes – 10 Years				
	Date	**Shares**	**Bonds**	**Real Estate**
1	31- Dec -05	34.03	8.07	30.00
2	31- Dec -06	43.63	7.94	27.30
3	31- Dec -07	33.06	7.99	27.70
4	31- Dec -08	-4.99	9.10	12.80
5	31- Dec -09	-13.08	8.70	9.10
6	31- Dec -10	22.46	8.62	13.40
7	31- Dec -11	10.59	8.52	10.30
8	31- Dec -12	11.03	7.90	14.90
9	31- Dec -13	19.13	7.03	15.30
10	31- Dec -14	17.67	8.25	12.90
	Mean Annual Returns	**17.38 %**	**8.28 %**	**17.37 %**
	Standard Deviation	**16.59 %**	**0.42 %**	**7.42%**

Correlation Matrix Of Asset Classes -10 Years			
	Shares	**Bonds**	**Real Estate**
Shares	**1.000**	**-0.6776**	**0.8329**
Bonds	**-0.6776**	**1.000**	**-0.5538**
Real Estate	**0.8329**	**-0.5538**	**1.000**

Chart 7: Returns of Asset Classes

Annual Returns of Shares, Bonds and Direct Real Estate for 10 years, from 2005 to 2014.



	1	2	3	4	5	6	7	8	9	10	11
Common Shares		34.03	43.63	33.06	-4.99	-13.08	22.46	10.59	11.30	19.13	17.67
Bonds		8.07	7.94	7.99	9.10	8.70	8.62	8.52	7.90	7.72	8.25
Real Estate		30.00	27.30	27.70	12.80	9.10	13.40	10.30	14.90	15.30	12.90

Table 8: Asset Allocation and Performance of Constructed naïve Portfolios.

Allocation of Asset Classes, Portfolio Returns, Risks and Sharpe Ratios for 10 years, from 2005 to 2014.

PORTFOLIO SERIES 3										
2005 – 2014 – 10 YEARS										
			Allocation							
	Portfolio		Stocks %	Bonds %	Real Estate %		Portfolio Returns %	Portfolio Risk %		Sharpe Ratio
1	3A		0.00	36.70	63.30		14.03	4.61		1.25
2	3B		11.20	32.30	56.50		13.86	4.67		1.07
3	3C		22.50	27.30	50.10		14.87	7.06		0.93
4	3D		33.90	22.40	43.70		15.34	8.45		0.84
5	3E		45.20	17.40	37.30		15.78	9.87		0.76
6	3F		56.60	12.50	31.00		16.26	11.34		0.70
7	3G		67.90	7.50	24.60		16.70	12.80		0.66
8	3H		79.30	2.50	18.20		17.15	14.29		0.62
9	3I		90.00	0.00	10		17.38	15.55		0.59
10	3J		100	0	0		17.38	16.59		0.55

The risk free rate , was determined as the ten year average of the Government ten year bond [KBP 2003m] 8.28

Chart 8 – Ten Naïve Portfolios

Portfolio Returns, Risks for 10 years, from 2005 to 2014.



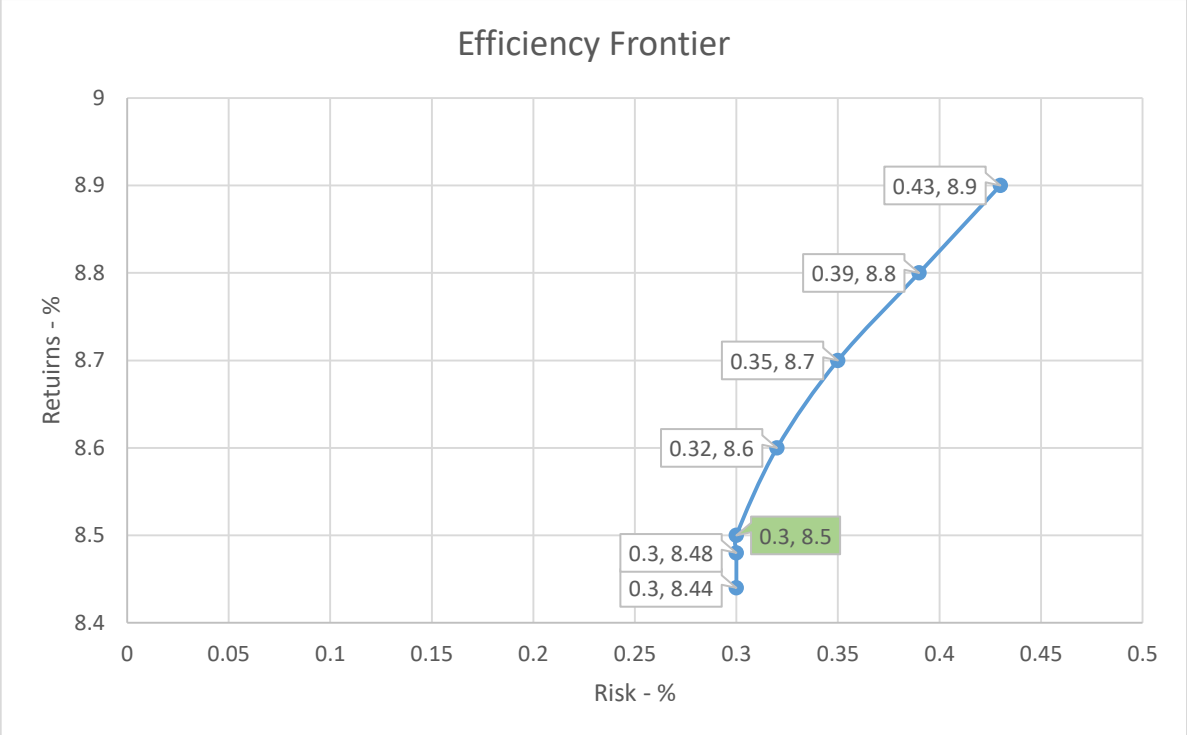
Table 9: Optimum Portfolios

Allocations and returns and risks of efficient portfolios.

DATA - Efficiency Frontier and Optimum Portfolio							
	Min Risk		Optimum Portfolio				
Portfolio Returns %	8.44	8.48	8.5	8.6	8.7	8.8	8.9
Portfolio Risk %	0.3	0.3	0.3	0.32	0.35	0.39	0.43
Slope %	28.133	28.267	28.333	26.875	24.857	22.564	20.698
Shares -Allocation %	2	1	0	1	1	0	0
Bond - Allocation %	98	98	72	96	95	94	93
Real Estate -Allocation	0	1	28	2	4	6	7

Chart 9: Efficiency Frontier

Efficient portfolios on the efficient frontier



Findings and Results

Descriptive Statistics of Asset Classes – Table 7 and Returns of Asset Classes- Chart 7.

The mean returns and standard deviations ratios were calculated to provide a relative comparison and performance of the different asset classes on both risk and return basis from 2005-2014.

The performance of the three asset classes from 2005- 2014 are presented in Table 7 and Chart 7. According to the tables and the chart, in the past 10 years, shares provided the highest average returns that of 17.38 percent; followed by real estate at 17.37 percent. Bonds provided the lowest average return at 8.21 percent.

The risk or volatility levels of the asset classes were determined based on the standard deviation of their historical returns. Shares appear to be the most risky of the three asset classes having the highest standard deviation of 16.59 percent compared to real estate with a standard deviation of 7.42 percent, and expectedly bonds had the lowest standard deviation of 0.42 percent. Shares provided negative returns in two of the ten years under consideration. The returns on shares ranged from -13.08to 43.63 percent. This illustrates the risky nature of this asset class. Bonds and real estate did not deliver any negative returns during the ten year holding period. The returns on bonds ranged from 7.03 to 9.10 percent and real estate from 9.10 to 30 percent.

Correlation of Asset Class – Table 7

The correlation coefficients between shares and bonds was – 0.6776; between shares and real estate +0.8329 and between bonds and real estate – 0.5538.

The positive correlation between shares and real estate suggests that having each pair of the assets combined in a portfolio might be expected to bring minimal or no risk reduction benefits as when one of the pair replaces the other.

The negative correlation of shares and bonds at -0.6776 and bonds and real estate at, -0.5538, suggest that combining these pairs of assets may provide diversification benefits to a portfolio.

Asset Allocation and Performance of naïve Portfolios – Table 8

Table 5 represents the results of the mean returns, standard deviations and Sharpe ratios of the ten constructed portfolios, based on the principals of Modern Portfolio Theory, with various allocations of the different asset classes.

The results of the analysis are presented in two sub sections, the first on return enchantments and the second on risk reduction.

Return Enhancement

The return enhancement characteristics of real estate in the portfolios, was measured by comparing the Sharpe Ratios of the various portfolios with the differing allocations of real estate to the portfolios.

The portfolio with the highest allocation to shares, at 100%, returned the highest portfolio returns at 17.38% and lowest Sharpe Ratio of 0.55. The portfolio with the lowest allocation to shares, at 0%, returned the lowest portfolio returns at 14.03 and the highest Sharpe Ratio of 1.25.

A linear relationship exists between the allocation of shares to the portfolios and the returns in the portfolios. This is evident from portfolio 3A to portfolio 3J, as the allocation of shares reduces so do the returns of the portfolio.

The opposite is applicable when the allocation of real estate to the various portfolios are analysed. Within all the ten portfolios, as the allocation to real estate increases the returns of the portfolios reduces.

The allocation of bonds to the ten portfolios range from 0.00 to 36.70 percent. These percentage allocations were substantially less that the allocation of shares and real estate to the portfolios, as the maximum allocation to bonds of the portfolio was 36.70 percent, shares 100 percent and real estate 63.0 percent. This implies that bonds had a low to minimal impact on the returns of the various portfolios considering the formula used to calculate the returns of the portfolios. The formula takes into account the weighting / allocation of the asset class in the portfolio and the returns of the asset class. With bonds the annual mean returns were the lowest at 8.28 percent.

Risk Reduction

Similar to the return enhancement analysis, the risk characteristics of real estate was measured by comparing the risk of the portfolio with the various allocations of real estate to the portfolios.

The interesting observation on the risk values of the ten portfolios, is that, as the allocation to real estate of the portfolios increase, the value of risk decreases, within the portfolios.

Portfolio 3A, has the highest allocation of real estate to portfolio at 63.30 percent and the portfolio risk was 4.61 percent. As the allocation of real estate was reduced to the various portfolios the risk of the portfolios increased. Portfolio 3J held no real estate and the portfolios which consisted of 100 percent allocation of shares had the highest risk of 16.59 percent of all the portfolios

From the analysis in Table 8 it is evident that a reduction in the percentage allocation of real estate to the portfolio equates to an increase in the risk values of the portfolios.

The converse is applicable to shares as the allocation of shares to the portfolio increase the value of the risk of the portfolios increase.

The Efficiency Frontier and Optimal Portfolio Chart 9 and Table 9.

Seven of the ten portfolios were deemed efficient portfolios. Real estate was allocated to six of the seven efficient portfolios. The allocation of real estate to these six of the seven portfolios range from 1 to 28 percent. Portfolio number 1, was deemed the optimum portfolio, with the highest slope of 28.33 and returns of 8.50 percent and risk of 0.30 percent.

Summary: Portfolio Series 3- 10 Years – 2005 -2014

Both real estate and bonds provided positive returns during the ten year holding period. However shares provided negative returns in two years. The volatility of shares, as measured by the standard deviation was highest amongst the three asset classes at 16.59 percent. Bonds were 0.42 percent and real estate 7.42 percent. Share were 39 times more volatile than bonds and shares were 2.23 times more volatile than real estate.

The negative correlation of shares and bonds and bonds and real estate, suggest that combining these pairs of assets may provide diversification benefits to a portfolio. However, the benefits of diversification within the portfolio is also depended on the allocation of these assets.

The inverse relationship between the allocations of shares to real estate to the portfolios and the impact on the returns and risks of the portfolios can be ascribed to the highly positive correlation between shares and real estate. Such positive correlations limit or minimize the diversification benefits within the portfolio.

This further supports the principles of Modern Portfolio Theory, where risk and returns are treated in a comprehensive and intergraded manner and not as separated aspects, as the portfolio with the lowest overall returns is the most efficient of the ten portfolios analysed.

Optimisation in terms of portfolio theory seeks to identify the best combination in terms of and the percentage weighting or allocation of each asset class to hold within the portfolio. Portfolio No 3, with a return of 8.5 percent and risk of 0.3 percent with an allocation of 72 percent to bonds and 28 percent to real estate was identified as the optimal portfolio.

Real Estate was allocated to six of the seven efficient portfolio, ranging from 1 to 28 percent, further justifies a high allocation of real estate to multi asset portfolios, as real estate provides enhanced diversification benefits as well as enhanced return benefits to the portfolio.

Comparative Analysis of Efficient and Optimal Portfolios for the Three Time Series

Portfolio Series 1 -20 Years -1995-2014, Table 3 and Chart 3, indicate that bonds and real estate are included in all nine portfolios on the efficient frontier. Share do not feature in any of the portfolios on the efficient frontier. The optimum portfolio encompassed an allocation of 72 percent of bonds and 28 percent of real estate to deliver portfolio returns of 12.07 and risk of 2.23 percent.

Portfolio Series 2 -10 Years -1995-2004, Table 6 and Chart 6, indicate that real estate is included in all six portfolios on the efficient frontier. Bonds feature in five portfolios and real share feature in just one of the portfolios on the efficient frontier. The optimum portfolio encompassed an allocation of 73 percent of bonds and 23 percent of real estate to and 3 percent to shares. The portfolio delivered returns of 13.13 and risk of 1.75 percent.

Portfolio Series 3 -10 Years -2005-2014, Table 9 and Chart 9, indicate that real estate and bonds are included in all six portfolios on the efficient frontier. Share feature in three of the portfolios on the efficient frontier. The optimum portfolio encompassed an allocation of 72 percent of bonds and 28 percent of real estate, no share are included in the optimum portfolio. The portfolio delivered returns of 8.5 and risk of 0.30 percent.

In determining the most favourable or best period for investors, over the three time series analysed. The reward to risk methodology was utilized to determine this. This entails dividing the returns (rewards) of the optimum portfolios by the risk of the portfolios and determining a ratio for each of the portfolios. The portfolio ratio with the highest value was he most desirable.

Table 10: Reward and Risk Analysis of the Optimal Portfolios

Portfolio Series									
			Allocation						
	Portfolio		Stocks %	Bonds %	Real Estate %		Portfolio Returns %	Portfolio Risk %		Reward to Risk Ratio
1	20 Year (1995-2014)		0	72	28		12.07	2.23		11.5
2	10 Year (1995-2004)		3	73	23		13.13	1.75		7.50
3	10 Year (2005-2014)		0	72	28		8.5	0.3		28.33

The most favourable time period for investors, over the three time series analysed as the time period from 2005 to 2014, where investors were reward with a factor 28.33 units of reward for every one unit of risk. This was the highest reward to risk ratio of the three portfolio series analysed.

The time period of 1995 to 2014 was the least favourable period analysed.

Chapter 5

Conclusion

Summary of Findings

The objective of this research report was to determine the diversification benefits of including direct real estate in a multi asset portfolio of common shares and bonds. An analysis of the three portfolio sets constructed to determine the stated objective revealed that real estate does provide diversification benefits to multi asset portfolios of common shares and bonds.

The benefits of real estate as a diversifier of risk was determined, over three time frames, a twenty year period from 1995 to 2014, a ten year period from 1995 to 2004 and a ten year period from 2005 to 2014.

Summary Analysis of the Portfolios

Three common trends were evident, in all the three portfolio sets analysed.

The portfolios with the highest allocation of bonds were the optimal portfolios over the three time periods analysed.

Time period 1, (20 years from 1995-2014). Analysis of this time period indicated an optimal portfolio with the allocation of 72 percent to bonds and 28 percent to real estate, with portfolio return of 12.07, and risk of 2.23 percent. Real estate was allocated to all the ten efficient portfolios, with allocation ranging from 23 to 47 percent.

Time period 2, (10 years from 1995-2004). The results indicated an optimal portfolio with the allocation of 73 percent to bonds and 23 percent to real estate and 3 percent to shares. Return of this portfolio was 13.13 and risk 1.75 percent. Real estate featured in all six of the efficient portfolios, with an allocation ranging from 23 to 100%.

Time period 3, (10 years from 2005-2014).The results indicated an optimal portfolio with the allocation of 72 percent to bonds and 28 percent to real estate and 0 percent to shares. Returns of this portfolio were 8.5 and risk 0.30 percent. Real estate featured in all six efficient portfolios and the allocation ranged from 1 to 28 percent.

Real estate delivered the highest returns in two of the three time period analysed.

During the period from 1995 to 2014 (20 years), real estate delivered returns of 15.43, bonds 10.75 and shares 12.38 percent.

During the second period of analysis from 1995 to 2004 (10 years), real estate delivered returns of 13.48, bonds 13.22 and shares 7.37 percent.

During the final period from 2005-2014 (10 years), shares provided the highest returns at 17.38 percent followed by real estate at 17.37 percent. The returns of real estate were 0.01 percent less than shares, in the period.

Real estate provided significant diversification benefits to the series of the three portfolios analysed.

During the period from 1995 to 2014 (20 years) (Table 3, Efficient Portfolios), real estate was included in all the 10 efficient portfolios. Firstly, this strongly suggests that real estate possessed the attributes of consistency in efficient portfolios. Secondly, the allocation of real estate in the efficient portfolios impacted on the returns and risks of the various portfolios. Considering the investors preference for higher returns or lower risk, the allocation of real estate can enhance returns for a given level of risk, or conversely, reduce risk for a given level of returns. The allocation of real estate ranges from 26 to 47 percent. With a maximum allocation of 47 percent, portfolio return was 12.93 and risk, 2.84 percent. With a minimum allocation of 26 percent, portfolio return are 11.96 and risk, 2.21.

During the period from 1995 to 2004 (10 years) (Table 6, Efficient Portfolios), real estate was included in all the six efficient portfolios. Like in the first period Firstly this strongly suggests that real estate possessed the attributes of consistency in optimal portfolios. Secondly, the allocation of real estate in the efficient portfolios impacted on the return and risks of the various portfolios. Considering the investors preference for higher returns or lower risk the allocation of real estate can enhance returns for a given level of risk or conversely reduce risk for a given level of returns. The allocation of real estate ranges from 23 to 100 percent. With a maximum allocation of 100 percent, portfolio return was 13.48 and risk 4.73 percent. With a minimum allocation of 23 percent, portfolio returns was 13.13 and risk 1.75.

During the period from 2005 to 2014 (10 years) (Table 9, Efficient Portfolios), real estate is included in all six efficient portfolios. This also strongly suggests that real estate possessed the attributes of consistency in optimised portfolios. The allocation of real estate ranges from 0 to 28 percent. With a maximum allocation of 28 percent,

portfolio return was 8.5 and risk 0.30 percent. With a minimum allocation of 1 percent, portfolio return was 8.484 and risk, 0.30.

The allocation of real estate to multi asset portfolio, was validated over both the ten year period as well as the twenty year period, thereby justifying the inclusion of real estate in multi asset portfolios in the medium term as well long term.

Comparison of results to past literature.

The results suggest strongly that real estate has possessed the attributes of consistency in optimised portfolios. Real estate constantly had positive allocations over time periods ranging from five to 25 years, and for most levels of portfolio returns, irrespective of whether real estate is used to enhance returns or reduce risks. The benefits of including real estate in a mixed asset portfolio increase as the investment horizon is extended. Direct real estate should be considered as a diversifier rather than return enhancer in a mixed asset portfolio. (Lee & Stevenson, 2006).

Real estate can indeed justify a higher allocation in the mixed asset portfolio, owing to good returns and low covariance within mixed asset portfolios, (Lee, 2005).

Core real estate continues to offer several major benefits that can enhance the performance of a mixed asset portfolio. These include low correlations with stocks and bonds, low volatility. (Ziering & McIntosh, 1997)

Real Estate offers higher returns than other assets and provides attractive diversification opportunities for stocks and bond investors. More existing categories of combining property types, SMSA (Standard Metropolitan Statistical Area, growth rates, and lease maturities offer more efficient procedures for real estate diversification that traditional property type (Hartzell, Hekman & Miles, 1986).

Real Estate provides significant better returns on both shares and bonds on a risk adjusted basis. Real Estate does provide diversifications benefits to a portfolio, however how much of real estate to include in a portfolio is difficult to estimate. (Brueggman, Chen & Thibodeau, 1984).

Real estate commended higher allocations in a mixed asset portfolio. This suggests that even for investors with low risk aversion and high targeted returns it would be optimal to include real estate in their mixed asset portfolios. (Sivitanides, 1997).

The addition of real Estate improves the efficiency of the portfolio, giving either a higher return for the same amount of risk or lower risk for the same return. Private (direct) real

estate provides more efficiency in terms of portfolio benefits because of covariance benefits and lower volatility compared to other asset classes. Public traded REITs have a place in the mixed asset portfolio, when used in combination with private real estate, may provide additional benefits, perhaps the maximum portfolio efficiency. (Black, 2004).

Investors combining property stocks into a mixed portfolio might be expected to obtain diversification benefits in terms of portfolio return enhancements and risk reduction. The gain in return would typically be higher than of the risk reduction and confirmed the fact that listed property stocks tend to offer less diversification benefits as found in previous studies. (Olaleye, 2011). This study found that direct real estate investors could have return enhancement and risk reduction benefits if they include direct real estate in their portfolios. These benefits are however dependent on the percentage allocation to real estate and can best be realized when investing on a medium to long term basis.

Future Research Topics

Further research should be undertaken on the flowing topics.

(1) The diversification benefits of real estate in multi asset portfolios, when direct real estate is represented firstly by property type indices and secondly geographic indices.
(2) Does direct real estate provide a hedge against inflation within the South African investment environment?

Concluding Comments

In this study on the diversification effects of direct real estate within a multi asset portfolio of shares and bonds it can be concluded than investors could have return enhancement and risk reduction benefits if they include direct real estate in their portfolios. These benefits are however dependent on the percentage allocation to real estate and can best be realized when investing on a medium to long term basis.

References

Adair, A, McGreal, S & Webb, JR, 2006," Diversification Effects of Direct versus Indirect Real Estate Investments in the U.K", Journal of Real Estate Portfolio Investment, Vol 12, No 2, pp. 85-90.

Akinsomi, O, Balcilar, M, Demirer, R, & Gupta, R, 2016, "The Effects of Gold Market Speculation on REITS returns in South Africa: a behavioural perspective", Journal of Economics and Finance, Vol 41 Issue 4, pp. 774-793.

Akinsomi, O, Mkhabela, N & Taderera, M, 2018," The role of Macro-economic indicators in explaining direct commercial real estate returns: evidence from South Africa", Journal of Property Research, Vol 35, No1, pp 28-52.

Akinsomi, O & Ntuli, M, 2017, "An Overview of the Initial Performance of the South African REIT Market". The Journal of Real Estate Literature, Vol 25, No. 2, pp 365-388.

Akinsomi, O, Kola, K,Ndlouv, T & Motloung, M, 2016," The performance of the Broad Based Black Economic Empowerment Compliant listed property firms in South Africa", Journal of Property Investment and Finance, Vol 34, Issue 34,pp. 3-26.

Akinsomi, O, Pahad, Nape & Margolis 2015. "Geographic Diversification Issues in Real Estate Markets in Africa". Journal of Real Estate Literature. Volume 23, Issue 2, pp 259-295.

Amidu, A ,Aluko, BT, Nuhu, MB & Saibu, MO, 2008," Real Estate Security and other investments assets: A comparison of investment characteristics in the Nigerian Stock Market", Journal of Property Investment and Finance, Vol. 26, No. 2, pp. 151-161.

Bakri, A, 2014, "Portfolio Diversification Strategy and the Impact on the Middle East Real Estate Investment Decision", International Journal of Economics and Finance, Vol. 6, No.2, pp. 62-74.

Barkham, RJ, Ward, CWR, & Hendry, O.T, 1996, "Inflation Hedging Characteristics of UK Property", Journal of Property Finance, Vol.7, No1, pp.62-66.

Bekkers, N, Doeswijk, RQ & Lam, TW, 2009, "Strategic Asset Allocation: Determine the Optimal Portfolio with in Ten Asset Classes", Social Science Research Network, http:// ssrn.com/abstract = 1368689.

Benjamin, JD, Sirmans, GS & Zietz, EN, 2001, “Returns and Risks on Real Estate and Other Investments: More Evidence”, Journal of Real Estate Portfolio Management, Vol. 7, Issue 3, pp. 183.

Black, RT, 2004, “Real Estate in the Investment Portfolio”, Real Estate Issues, Vol.29, No.3, pp. 1-6.

Bundell, GF, Fairchild, S & Goodchild, RN, 2005, “Managing Portfolio Risk in Real Estate”, Journal of Property Research, Vol. 22, pp. 115-136.

Burgeggeman, WB, Chen, AH & Thibeau, TG, 1984, “Real Estate Investment Funds: Performance and Portfolio Considerations”, AREUEA Journal, Vol.12, No 3.

Byrne, P & Lee, S, 2004, “Different risk measures: different portfolios compositions?” Journal of Property Investment & Finance, Vol. 22, No. 6, pp. 501-511.

Byrne, P & Lee, S, 1995, “Is there a place for property in the Multi -Asset Portfolio”, Journal of Property Finance, Vol.6, No.3, pp. 60-68.

Byrne, P & Lee, S, 1999, “Sector, Region or Function? A MAD reassessment of Real Estate Diversification in Great Britain”, Journal of Property Investment & Finance, Vol. 29, No. 2, pp. 167-189.

Callender, M, Devaney, S & Sheahan, A, 2007, “Risk Reduction and Diversification in UK Commercial Property Portfolios”, Journal of Property research, Vol 24, No 4, pp 355-375.

Cheng, P, 2005, “Asymmetric Risk Measure and Real Estate Returns”, The Journal of Real Estate and Economics, Vol 30 No 1, pp 89-102.

Clayton, J, Ling, DC & Naranjo, A, 2008, “Commercial Real Estate Valuations: Fundamentals versus Investor Sentiment”, Journal of Real Estate Finance and Economics, Vol 38, pp 5-37.

Dapaah, KA, Wee, SG & Ebrahim, MS, 2002,”Real Estate Portfolio Diversification by Source of Return”, Journal of Real Estate Portfolio Management, Vol 8, No1.

Delcoure, N, 2010, “Diversification Benefits: Correlation and Return Gaps”, International Review of Applied Financial Issues and Economics, Vol.2, No.3, pp.597-607.

Edelstein, RH, & Quan, DC, 2006, “How Does Appraisal Smoothing Bias - Real Estate Returns Measurement”. Journal of Real Estate Finance and Economics, Vol. 32, Issue. 1, pp. 41-60.

Eichholtz, PMA, Hoesli, M, MacGregor, BD & Nanthakumaran, N, 1995, “Real Estate Portfolio Diversification by Property Type and Region”, Journal of Property Finance, Vol. 6, No.3, pp.39-59.

Fuerst, F, Lui, X & Lizieri C, 2016, “A Commercial Real Estate Index for an Emerging Market. The case of Beijing”, Cambridge Real Estate Research Centre, University of Cambridge, Working Paper Series, No 2016-02.

Geltner, D & Miller, NG, 2001, “Commercial Real Estate Analysis and Investment, South- Western Publishing.

Geltner, D, 1993, “Estimating Market Values from Appraised Values without Assuming an Efficient Market”, Journal of Real Estate Research, Vol.8, No.3, pp. 326-345.

Geltner, D, 1991, “Smoothing Appraisal based Returns”. Journal of Real Estate Finance and Economics”, Vol.4, No.3, pp. 327-345.

Georgiev, G, 2002, “The Benefits of Real Estate Investment”, Working Paper, PhD Candidate, University of Massachusetts.

Geurts, TG & Nolan, H, 1997,”Does real estate have a place in the investment portfolio of tomorrow,” Review of Business, Vol 18, No 4, pp.19-24.

Giannotti, C & Mattarocci, G, 2008, “Risk Diversification in a Real Estate Portfolio: Evidence from the Italian Market”, Journal of European Real Estate Research, Vol. 1, Issue 3, pp. 214-234.

Giliberto, MS, 1992, “The Allocation of Real Estate to Future Mixed - Asset Institutional Portfolios”, The Journal of Real Estate Research, Vol. 7, No. 4, pp. 423-432.

Hartzell, D, Hekman, J & Miles M, 1986, “Diversification Categories in Investment Real Estate”, AREUEA Journal, Vol. 14, No 2.

Hartzell, DJ, Shulman, DG & Wurtzebach, CH, 1987, “Refining the Analysis of Regional Diversification for Income - Producing Real Estate”, The Journal of Real Estate Research, Vol.2, No.2, pp. 85-95.

Hoesli, M, Lizieri, C & MacGregor, B, 2007,”The Inflation Hedging Characteristics of US and UK Investments: A Multi – Factor Error Correction Approach”, Journal of Real Estate Finance and Economics, Vol 36, pp.183-206.

Hoesli, M & Lekander, J, 2007, “Real Estate Portfolio Strategy and Product innovation in Europe”, Journal of Property Investment and Finance, Vol. 26, No. 2, pp. 162-176.

Hoesli, M, 1994, ”Real Estate as a Hedge against Inflation, Learning from the Swiss Case”, Journal of Property Valuation and Investment, Vol 12, No 3, pp 51-59.

Hudson- Wilson, S, Fabozzi, FJ & Gordon, JN, 2003, “Why Real Estate”, The Journal of Portfolio Management, Special issue, 2003.

Hudson- Wilson, S & Guenther, DP, 1995,” The Four Quadrants: Diversification benefits for investors in Real Estate”, Real Estate Finance, Vol 12, No 2, pp. 82

Hung, K, Onayev Z & Tu , CC, 2008, “Time -Varying Diversification Effects of Real Estate in Institutional Portfolios: When Alternative Assets are Considered”, Journal of Real Estate Management, Vol. 14, No. 4, pp. 241- 261.

Jackson, C & White, M, 2005 ”Challenging Traditional Real Estate Market Classifications for Investment Diversification”, Journal of Real Estate Portfolio Management, Vol 11, No 3, pp.307-321.

Kaiser, RW, 2004, “Real estate as a surrogate for Bonds: A Dynamic Asset Allocation View”, Journal of Real Estate Portfolio Management, Vol.10, No.1, pp. 23-35.

Lee, S & Stevenson, S, 2006,”Real estate in the mixed portfolio: The Question of Consistency”, Journal of Property Investment and Finance, Vol 24, No 2, pp. 123-135

Lee, S & Stevenson, S, 2005,”Testing the Statistical Significance of Sector and Regional Diversification”, Journal of Property Investment and Finance, Vol 23, No 5, pp. 394-411.

Lee, S & Devancey S, 2005, “Real Estate Portfolio Size and Risk Reduction, A paper presented at the annual European Real Estate Society Meeting, Dublin, Ireland , June 2005”.

Lee, S & Stevenson, S, 2004, “Real Estate Portfolio Construction and Estimation Risk”, Journal of Property Investment and Finance, Vol. 23, No. 3, pp. 234-253.

Lee, S, 2012, “The Performance gap in UK Property Returns”, Journal of European Real Estate Research, Vol. 5, No. 2, pp. 121-134.

Lee, L, 2002, “Correlation Shifts and Real Estate Portfolio Management”, A paper presented at the ARES annual meeting April 2002.

Lee, S, 2005, “The Return due to Diversification of Real Estate to the US. Mixed – Asset Portfolio.” Journal of Real Estate Portfolio Management, Vol.11, No.1, pp. 19-28.

Lin, Z & Vandell, KD, 2007,” Liquidity and Pricing Biases in the Real Estate Market”, Real Estate Economics, Vol.35, No. 3, pp. 291-330.

Lizieri, C & Ward C, 2000, “Commercial Real Estate Return Distributions: A review of Literature and Empirical Evidence”, Social Science Research Network, http:// ssrn.com/abstract = 255746.

Marcato, G & Key, T, 2007, “Smoothing and Implications for Asset Allocation Choices”, Journal of Portfolio Management, Vol 33, No 5, pp.85-89.

Malizia, EE, & Simons, RA, 2001, “Comparing Regional Classification for Real Estate Portfolio Diversification”. The Journal of Real Estate Research, Vol. 6, No. 1, pp. 53-77.

Markowitz, H, 1952, “Portfolio Selection: Efficient Diversification of Investments”, The Journal of Finance, Vol. 7, No.1, pp. 77-91.

IPD/ MSCI, (2017), *South African Annual Property Index*.

McGeal, S, Adair, A & Webb, JR, 2009, “Optimal Diversification in US/ UK Private Real Estate only Portfolios: The Good, the Bad and the Uncertain”, Journal of Real Estate Portfolio Management, Vol. 15, No.1, pp. 87-93.

McGeal, S, Adair, A, Berry, JN & Webb, JR, 2006, “Institutional Real Estate Portfolio Diversification in Ireland and the UK”, Journal of Property Investment and Finance, Vol. 24, No.2, pp. 136-149.

Mueller, GR, 2001, “Refining Economic Diversification Strategies for Real Estate Portfolios”, The Journal of Real Estate Research, Vol. 8, No. 1, pp. 55-68.

Neal, M & Naka, A, 2006,” Diversifications Benefits of Japanese Real Estate Over the Last Four Decades”, Journal of Real Estate Financial Economics, Vol 33. Pp. 259-274.

Newell, G, Chau, KW, Wong, SK & Mckinnell, K, 2005, “Dynamics of the Direct and Indirect Real Estate Markets in China”, Journal of Real Estate Portfolio Management, Vol 11, No 3, pp. 263-279.

Olaleys, A, 2011, “The effects of adding real estate into mixed- asset portfolios in South Africa”. Journal of Financial Management of Property and Construction”, Vol.16, No.3, pp. 272-282.

Olaleye, A, Aluko, BT & Oloyede SA, 2008,"Diversification Strategies for Direct Property Investment Portfolios", Journal of Real Estate Portfolio Management, Vol 14, No 3, pp. 223-231.

Pagourtzi, E, Assimakopoulos, V, Hatzichristos, T & French, N, 2003, Journal of Property Investment & Finance, Vol 21, No4, pp.383-401.

Peng, L & Schulz, R, 2013, "Does the Diversification Potential of Securitized Real Estate Vary over Time and should Investors Care", Journal of Real Estate Finance and Economics, Vol. 4, No. 7, pp. 310-340.

Penny, PE, 1982, "Modern Investment Theory and Real Estate Analysis", The South African Journal of Economics, Vol. 48, No. 1.

Ping, C, Zhenguo, L, & Yhang, YY, 2011,"Has Real Estate Come of Age?" Journal of Real Estate Portfolio Management, Vol 17, No 3, pp. 243-254.

Ping, C, 2005, "Asymmetric Risk Measure and Real Estate Returns", The journal of Real Estate Finance and Economics, Vol.30. No. 1, pp. 89-102,

Ping, C & Youguo, L, 2000,"Optimal Diversification: Is it really worthwhile?" Journal of Real Estate Portfolio Management, Vol 6, No 1.

Rehring, C, 2012, "Real Estate in a Mixed- Asset Portfolio: The Role of the Investment Horizon", Journal of Real Estate Economics, Vol 40, No 1, pp. 65-95.

Sa-Aadu, J, Shilling, J & Tiwari, A, 2010, "On the Portfolio Properties of Real estate in Good Times and Bad Times", Real Estate Economics, Vol. 23, No. 3, pp. 529-565.

SAREITS, accessed, 1 March 2017, http://sareits.co.za.

Sing, TF & Low, SY, 2000, "The Inflation –Hedging Characteristics of real Estate and Financial Assets in Singapore. American Real Estate Society, Vol 6, No 4, pp.373-385

Sing, TF, & Zhuang, YT, 2013, "Time – varying correlations between stock and direct real estate returns, Journal of Property Investment and Finance, Vol 31, No 2, pp.179-195

Seiler, MJ, Webb, JR & Neil Myer, FC, 1999, "Diversification Issues in Real Estate Investment", Journal of Real Estate Literature, Vol. 7, pp. 163-179.

Sivitanides, PS, 1997, "Why Invest in Real Estate: An asset allocation perspective", Real Estate Issues, Vol. 22, pp. 30-36

Smietana, K, 2014, “Diversification Principal of Real Estate Portfolios”, Journal of Real Estate Management and Valuation, Vol. 22, No. 1 pp. 54-60.

Stevenson, S, 2001, “The long term advantages to incorporating indirect securities in direct real estate portfolios”, Journal of Real Estate Portfolio Management, Vol. 7,No.1, pp. 5-16.

Stevenson, S, 2002, “Momentum Effects and Mean Reversion in Real Estate Securities”. Journal of Real Estate Economic Research, Vol. 23, Nos. 1-2, pp. 47- 64.

Wheaton, CW, Torto, RG, Sivitanides, PS, Southard, JA, Hopkins, RE & Costello, JM, 2001, “Real Estate Risk: A Forward Looking Approach”, www.tortowheatonresearch.com.

Viezer, TW, 2000, “Evaluating “within real estate” diversification strategies”, Journal of Real Estate Portfolio Management, Vol.6, No.1, pp. 75-95.

Wolverton, ML, Ping, C & Hardin, WG, 1998,” Real Estate Portfolio Risk Reduction through Intracity Diversification”, Journal of Real Estate Portfolio Management, Vol 4, No 1, pp. 35-41.

Worzala, E & Sirmans, CF, 2003, “Investing in International Real Estate Stocks: A Review of the Literature”, Urban Studies, Vol. 40, Nos. 5-6, pp. 1115-1149.

Wurstbauer, D & Wolfgang, S, 2014, “Inflation Hedging and Protection Characteristics of Infrastructure and Real Estate Assets, Journal of Property Investment and Finance, Vol 33, No 1, pp.19-44.

Ziering B & McIntosh, W, 1997, “Revisiting the case for including core Real Estate in a Mixed -Asset Portfolio”, Real estate Finance, Vol. 13, Issue. 4, pp.14-22.

Ziobrowski, A, Caines, RW & Ziobrowski, BJ, 1999, “Mixed - Asset Portfolio Composition with Long- Terms Holding Periods and Uncertainty”, Journal of Real Estate Portfolio Management, Vol. 5, No 2.

Ziobrowski, BJ & Ziobrowski AJ, 1997, “Higher Real Estate Risk and Mixed - Asset Portfolio Performance”, Journal of Real Estate Portfolio Management, Vol. 3, No. 2, pp. 107-115.

Annexure A - Portfolio Series 1 – 20 Years – 1995-2014

Annexure A1: JSE All Share Data

FTSE / JSE ALL SHARE INDEX (Common Stocks)

YEARLY DATA - FROM 1995 TO 2014

20 YEAR PERIOD

	Date	Index Value at Close	% Returns
1	30-Dec-94	5,852.79	
2	31-Dec-95	5,198.17	-11.18%
3	31-Dec-96	6,130.74	17.94%
4	31-Dec-97	6,286.78	2.55%
5	31-Dec-98	5,832.86	-7.22%
6	29-Dec-99	6,531.93	11.99%
7	29-Dec-00	7,907.08	21.05%
8	31-Dec-01	8,816.75	11.50%
9	31-Dec-02	10,305.31	16.88%
10	31-Dec-03	8,899.71	-13.64%
11	31-Dec-04	11,021.29	23.84%
12	30-Dec-05	14,772.16	34.03%
13	29-Dec-06	21,217.15	43.63%
14	31-Dec-07	28,231.84	33.06%
15	31-Dec-08	26,823.29	-4.99%
16	31-Dec-09	23,315.31	-13.08%
17	31-Dec-10	28,552.93	22.46%
18	30-Dec-11	31,577.02	10.59%
19	31-Dec-12	35,058.42	11.03%
20	31-Dec-13	41,764.92	19.13%
21	31-Dec-14	49,144.54	17.67%
	Average Returns		**12.36%**
	Standard Deviation		**15.77%**

Annexure A2: SA Government Ten Year Bonds

SA GOVERNMENT TEN YEAR BOND [KBP 2003M]

YEARLY DATA - FROM 1995 TO 2014

20 YEAR PERIOD

	Date	Monthly - Yield		Annual Yield
1	1995/01	17.02		
2	1995/02	16.82		
3	1995/03	16.72		
4	1995/04	16.82		
5	1995/05	16.95		
6	1995/06	16.78		
7	1995/07	16.62		
8	1995/08	15.96		
9	1995/09	15.49		
10	1995/10	15.15		
11	1995/11	14.39		
12	1995/12	14.56		**16.11**
13	1996/01	13.77		
14	1996/02	14.1		
15	1996/03	15.04		
16	1996/04	15.78		
17	1996/05	16.53		
18	1996/06	15.78		
19	1996/07	15.39		
20	1996/08	15.82		
21	1996/09	15.42		
22	1996/10	15.8		
23	1996/11	16.18		
24	1996/12	16.19		**15.48**
25	1997/01	15.82		
26	1997/02	15.03		
27	1997/03	15.16		
28	1997/04	15.24		
29	1997/05	15.08		
30	1997/06	14.72		

31	1997/07	14.21		
32	1997/08	14.24		
33	1997/09	14.18		
34	1997/10	14.05		
35	1997/11	14.5		
36	1997/12	14.14		**14.70**
37	1998/01	13.61		
38	1998/02	13.49		
39	1998/03	13.33		
40	1998/04	12.92		
41	1998/05	13.46		
42	1998/06	14.6		
43	1998/07	15.89		
44	1998/08	16.95		
45	1998/09	18.3		
46	1998/10	16.51		
47	1998/11	16.05		
48	1998/12	16.36		**15.12**
49	1999/01	15.89		
50	1999/02	14.88		
51	1999/03	14.54		
52	1999/04	14.58		
53	1999/05	15.14		
54	1999/06	14.93		
55	1999/07	14.97		
56	1999/08	15.28		
57	1999/09	15.35		
58	1999/10	15		
59	1999/11	14.3		
60	1999/12	13.96		**14.90**
61	2000/01	13.49		
62	2000/02	13.5		
63	2000/03	13.92		
64	2000/04	14.27		
65	2000/05	14.79		
66	2000/06	14.33		
67	2000/07	13.9		
68	2000/08	13.52		
69	2000/09	13.72		
70	2000/10	13.72		
71	2000/11	13.4		
72	2000/12	12.88		**13.79**
73	2001/01	12.61		
74	2001/02	11.97		
75	2001/03	11.79		
76	2001/04	12.17		
77	2001/05	11.93		

78	2001/06	11.32		
79	2001/07	10.9		
80	2001/08	10.75		
81	2001/09	10.81		
82	2001/10	10.73		
83	2001/11	10.27		
84	2001/12	11.63		**11.41**
85	2002/01	11.61		
86	2002/02	11.89		
87	2002/03	12.63		
88	2002/04	12.47		
89	2002/05	11.66		
90	2002/06	11.6		
91	2002/07	11.15		
92	2002/08	11.08		
93	2002/09	11.35		
94	2002/10	11.27		
95	2002/11	10.86		
96	2002/12	10.44		**11.50**
97	2003/01	10.17		
98	2003/02	9.94		
99	2003/03	10		
100	2003/04	9.93		
101	2003/05	9.76		
102	2003/06	9.26		
103	2003/07	9.57		
104	2003/08	9.59		
105	2003/09	9.61		
106	2003/10	9.13		
107	2003/11	9.28		
108	2003/12	9.15		**9.62**
109	2004/01	9.27		
110	2004/02	9.4		
111	2004/03	9.6		
112	2004/04	9.85		
113	2004/05	10.24		
114	2004/06	10.38		
115	2004/07	10.08		
116	2004/08	9.72		
117	2004/09	9.28		
118	2004/10	9.16		
119	2004/11	8.96		
120	2004/12	8.38		**9.53**
121	2005/01	8.2		
122	2005/02	7.85		
123	2005/03	8.21		
124	2005/04	8.45		

125	2005/05	8.4		
126	2005/06	8.08		
127	2005/07	8.12		
128	2005/08	7.9		
129	2005/09	8.04		
130	2005/10	8.1		
131	2005/11	7.88		
132	2005/12	7.57		**8.07**
133	2006/01	7.37		
134	2006/02	7.26		
135	2006/03	7.36		
136	2006/04	7.46		
137	2006/05	7.63		
138	2006/06	8.29		
139	2006/07	8.58		
140	2006/08	8.59		
141	2006/09	8.64		
142	2006/10	8.39		
143	2006/11	7.87		
144	2006/12	7.81		**7.94**
145	2007/01	7.65		
146	2007/02	7.5		
147	2007/03	7.59		
148	2007/04	7.64		
149	2007/05	7.58		
150	2007/06	8.1		
151	2007/07	8.34		
152	2007/08	8.37		
153	2007/09	8.45		
154	2007/10	8.12		
155	2007/11	8.19		
156	2007/12	8.29		**7.99**
157	2008/01	8.36		
158	2008/02	8.69		
159	2008/03	9.16		
160	2008/04	9.15		
161	2008/05	9.51		
162	2008/06	10.35		
163	2008/07	10.1		
164	2008/08	9.2		
165	2008/09	9.04		
166	2008/10	9.24		
167	2008/11	8.58		
168	2008/12	7.82		**9.10**
169	2009/01	7.85		
170	2009/02	8.26		
171	2009/03	8.43		

172	2009/04	8.57		
173	2009/05	8.7		
174	2009/06	8.88		
175	2009/07	9.11		
176	2009/08	8.8		
177	2009/09	8.7		
178	2009/10	9.11		
179	2009/11	9		
180	2009/12	9.03		**8.70**
181	2010/01	9.24		
182	2010/02	9.11		
183	2010/03	8.92		
184	2010/04	8.75		
185	2010/05	8.97		
186	2010/06	8.99		
187	2010/07	8.78		
188	2010/08	8.17		
189	2010/09	7.92		
190	2010/10	8.03		
191	2010/11	8.12		
192	2010/12	8.38		**8.62**
193	2011/01	8.52		
194	2011/02	8.7		
195	2011/03	8.94		
196	2011/04	8.73		
197	2011/05	8.56		
198	2011/06	8.5		
199	2011/07	8.55		
200	2011/08	8.1		
201	2011/09	8.25		
202	2011/10	8.41		
203	2011/11	8.47		
204	2011/12	8.51		**8.52**
205	2012/01	8.4		
206	2012/02	8.23		
207	2012/03	8.37		
208	2012/04	8.28		
209	2012/05	8.28		
210	2012/06	8.16		
211	2012/07	7.52		
212	2012/08	7.48		
213	2012/09	7.4		
214	2012/10	7.67		
215	2012/11	7.64		
216	2012/12	7.37		**7.90**
217	2013/01	7.26		
218	2013/02	7.29		

219	2013/03	7.39		
220	2013/04	6.97		
221	2013/05	6.96		
222	2013/06	7.94		
223	2013/07	7.98		
224	2013/08	8.37		
225	2013/09	8.15		
226	2013/10	7.87		
227	2013/11	8.18		
228	2013/12	8.32		**7.72**
229	2014/01	8.48		
230	2014/02	8.67		
231	2014/03	8.52		
232	2014/04	8.41		
233	2014/05	8.16		
234	2014/06	8.34		
235	2014/07	8.25		
236	2014/08	8.21		
237	2014/09	8.2		
238	2014/10	8.06		
239	2014/11	7.86		
240	2014/12	7.84		**8.25**

	Average Returns	**10.75%**
	Standard Deviation	**2.99%**

Annexure A3: MSCI All Property Index

MSCI / INVESTMENT PROPERTY DATABANK ALL PROPERTY INDEX

YEARLY DATA - FROM 1995 TO 2014

20 YEAR PERIOD

	Date		Annual Returns
1	31-Dec-95		15.30%
2	31-Dec-96		14.00%
3	31-Dec-97		17.40%
4	31-Dec-98		5.00%
5	29-Dec-99		13.60%
6	29-Dec-00		11.00%
7	31-Dec-01		10.40%
8	31-Dec-02		9.50%
9	31-Dec-03		15.20%
10	31-Dec-04		23.40%
11	30-Dec-05		30.00%
12	29-Dec-06		27.30%
13	31-Dec-07		27.70%
14	31-Dec-08		12.80%
15	31-Dec-09		9.10%
16	31-Dec-10		13.40%
17	30-Dec-11		10.30%
18	31-Dec-12		14.90%
19	31-Dec-13		15.30%
20	31-Dec-14		12.90%

Average Returns	**15.43%**
Standard Deviation	**6.52%**

Annexure A4: Portfolio 1 A

PORTFOLIO - 1. A

	Portfolio Allocation	Stocks - 0%	Bonds - 36.7%	Real Estate - 63.3%
	Data			
		Stocks	Bonds	Real Estate
	Returns	12.38%	10.75%	15.43%
	Volatility	15.77%	2.99%	6.52%
	Allocation	0.00%	36.70%	63.3%
1	**Portfolio Expected Returns**			
	Stocks	Allocation	0.00%	0.000
		Returns	12.38%	
	Bonds	Allocation	36.70%	0.367
		Returns	10.75%	
	Real Estate	Allocation	63.30%	0.633
		Returns	15.43%	
	Expected Returns	(0 .000*12.38)+ (.367*10.75)+ (.633*15.43)		
	Portfolio Expected Returns	**13.71%**		

PORTFOLIO - 1. A

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0000000	0.000000	0.000000
Bonds	0.000000	0.000120	-0.000173
Real Est	0.000000	-0.000173	0.001702
Sub - Total	0.000000	-0.000052	0.001530

Total 0.001477

Portfolio Risk or Variance **3.84%**

Annexure A5: Portfolio 1 B

	PORTFOLIO - 1. B				
	Portfolio Allocation	**Stocks - 11.2%**	**Bonds - 32.3%**	**Real Estate - 56.5%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	12.38%	10.75%	15.43%	
	Volatility	15.77%	2.99%	6.52%	
	Allocation	11.20%	32.30%	56.5%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	11.20%	0.112	
		Returns	12.38%		
	Bonds	Allocation	32.30%	0.323	
		Returns	10.75%		
	Real Estate	Allocation	56.50%	0.565	
		Returns	15.43%		
	Expected Returns	(0.112*12.38)+ (.323*10.75)+ (.565*15.43)			
	Portfolio Expected Returns	**13.57%**			

PORTFOLIO - 1. B

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0003120	-0.000064	0.000424
Bonds	-0.000064	0.000093	-0.000136
Real Est	0.000424	-0.000136	0.001356
Sub - Total	0.000672	-0.000106	0.001644
Total	0.002211		

Portfolio Risk or Variance **4.70%**

Annexure A6: Portfolio 1 C

PORTFOLIO - 1. C

Portfolio Allocation	**Stocks - 22.50%**	**Bonds - 27.30%**	**Real Estate - 50.10%**

Data

	Stocks	Bonds	Real Estate
Returns	12.38%	10.75%	15.43%
Volatility	15.77%	2.99%	6.52%
Allocation	22.50%	27.30%	50.1%

1 Portfolio Expected Returns

Stocks	Allocation	22.50%	0.225
	Returns	12.38%	
Bonds	Allocation	27.30%	0.273
	Returns	10.75%	
Real Estate	Allocation	50.10%	0.501
	Returns	15.43%	
Expected Returns	(0.225*12.38)+ (.273*10.75)+ (.501*15.43)		

Portfolio Expected Returns **13.45%**

PORTFOLIO - 1. C

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0012590	-0.000108	0.000755
Bonds	-0.000108	0.000067	-0.000102
Real Est	0.000755	-0.000102	0.001066
Sub - Total	0.001906	-0.000143	0.001720
Total	0.003483		

Portfolio Risk or Variance **5.90%**

Annexure A7: Portfolio 1 D

	PORTFOLIO - 1. D				
	Portfolio Allocation	**Stocks - 33.90%**	**Bonds - 22.40%**	**Real Estate - 43.70%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	12.38%	10.75%	15.43%	
	Volatility	15.77%	2.99%	6.52%	
	Allocation	33.90%	22.40%	43.7%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	33.90%	0.339	
		Returns	12.38%		
	Bonds	Allocation	22.40%	0.224	
		Returns	10.75%		
	Real Estate	Allocation	43.70%	0.437	
		Returns	15.43%		
	Expected Returns	(0.339*12.38)+ (.224*10.75)+ (.437*15.43)			
	Portfolio Expected Returns	**13.34%**			

PORTFOLIO - 1. D

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0028580	-0.000134	0.000992
Bonds	-0.000134	0.000045	-0.000073
Real Est	0.000992	-0.000073	0.000811
Sub - Total	0.003717	-0.000161	0.001731
Total	0.005286		

Portfolio Risk or Variance **7.27%**

Annexure A8: Portfolio 1E

	PORTFOLIO - 1. E				
	Portfolio Allocation	**Stocks - 45.2%**	**Bonds - 17.40%**	**Real Estate - 37.3%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	12.38%	10.75%	15.43%	
	Volatility	15.77%	2.99%	6.52%	
	Allocation	45.20%	17.40%	37.3%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	45.20%	0.452	
		Returns	12.38%		
	Bonds	Allocation	17.40%	0.174	
		Returns	10.75%		
	Real Estate	Allocation	37.30%	0.373	
		Returns	15.43%		
	Expected Returns	(0.452*12.38)+ (.174*10.75)+ (.373*15.43)			
	Portfolio Expected Returns	**13.22%**			

PORTFOLIO - 1. E

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0050809	-0.000138	0.001129
Bonds	-0.000138	0.000027	-0.000048
Real Est	0.001129	-0.000048	0.000591
Sub - Total	0.006072	-0.000159	0.001672
Total	0.007584		

Portfolio Risk or Variance **8.71%**

Annexure A9: Portfolio 1 F

	PORTFOLIO - 1. F				
	Portfolio Allocation	**Stocks - 56.6**	**Bonds - 12.50**	**Real Estate - 31**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	12.38%	10.75%	15.43%	
	Volatility	15.77%	2.99%	6.52%	
	Allocation	56.60%	12.50%	31.0%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	56.60%	0.566	
		Returns	12.38%		
	Bonds	Allocation	12.50%	0.125	
		Returns	10.75%		
	Real Estate	Allocation	31.00%	0.310	
		Returns	15.43%		
	Expected Returns	(0.566*12.38)+ (.125*10.75)+ (.310*15.43)			
	Portfolio Expected Returns	**13.13%**			

PORTFOLIO - 1. F

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0079671	-0.000124	0.001175
Bonds	-0.000124	0.000014	-0.000029
Real Est	0.001175	-0.000029	0.000408
Sub - Total	0.009018	-0.000139	0.001555
Total	0.010433		

Portfolio Risk or Variance **10.21%**

Annexure A10: Portfolio 1 G

	PORTFOLIO - 1. G				
	Portfolio Allocation	**Stocks - 67.90**	**Bonds - 7.50**	**Real Estate -24.60**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	12.38%	10.75%	15.43%	
	Volatility	15.77%	2.99%	6.52%	
	Allocation	67.90%	7.50%	24.6%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	67.90%	0.679	
		Returns	12.38%		
	Bonds	Allocation	7.50%	0.075	
		Returns	10.75%		
	Real Estate	Allocation	24.60%	0.246	
		Returns	15.43%		
	Expected Returns	(0.566*12.38)+ (.125*10.75)+ (.310*15.43)			
	Portfolio Expected Returns	**13.00%**			

PORTFOLIO - 1. G

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0114658	-0.000090	0.001119
Bonds	-0.000090	0.000005	-0.000014
Real Est	0.001119	-0.000014	0.000257
Sub - Total	0.012495	-0.000098	0.001362
Total	0.013759		

Portfolio Risk or Variance **11.73%**

Annexure A11: Portfolio 1 H

	PORTFOLIO - 1. H				
	Portfolio Allocation	**Stocks - 79.3**	**Bonds - 2.50**	**Real Estate - 18.20**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	12.38%	10.75%	15.43%	
	Volatility	15.77%	2.99%	6.52%	
	Allocation	79.30%	2.50%	18.2%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	79.30%	0.793	
		Returns	12.38%		
	Bonds	Allocation	2.50%	0.025	
		Returns	10.75%		
	Real Estate	Allocation	18.20%	0.182	
		Returns	15.43%		
	Expected Returns	(0.793*12.38)+ (.025*10.75)+ (.182*15.43)			
	Portfolio Expected Returns	**12.89%**			

PORTFOLIO - 1. H

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0156391	-0.000035	0.000967
Bonds	-0.000035	0.000001	-0.000003
Real Est	0.000967	-0.000003	0.000141
Sub - Total	0.016571	-0.000038	0.001104
Total	0.017637		

Portfolio Risk or Variance **13.28%**

Annexure A12: Portfolio 1 I

	PORTFOLIO - 1. I				
	Portfolio Allocation	**Stocks - 90**	**Bonds - 0.0**	**Real Estate - 10**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	12.38%	10.75%	15.43%	
	Volatility	15.77%	2.99%	6.52%	
	Allocation	90.00%	0.00%	10.0%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	90.00%	0.900	
		Returns	12.38%		
	Bonds	Allocation	0.00%	0.000	
		Returns	10.75%		
	Real Estate	Allocation	10.00%	0.100	
		Returns	15.43%		
	Expected Returns	(0.900*12.38)+ (.00*10.75)+ (.1100*15.43)			
	Portfolio Expected Returns	**12.68%**			

PORTFOLIO - 1. I

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0201443	0.000000	0.000603
Bonds	0.000000	0.000000	0.000000
Real Est	0.000603	0.000000	0.000042
Sub - Total	0.020747	0.000000	0.000645
Total	0.021392		

Portfolio Risk or Variance **14.63%**

Annexure A13: Portfolio 1 J

	PORTFOLIO - 1. J				
	Portfolio Allocation	**Stocks - 100**	**Bonds - 0.0**	**Real Estate - 0**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	12.38%	10.75%	15.43%	
	Volatility	15.77%	2.99%	6.52%	
	Allocation	100.00%	0.00%	0.0%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	100.00%	1.000	
		Returns	12.38%		
	Bonds	Allocation	0.00%	0.000	
		Returns	10.75%		
	Real Estate	Allocation	0.00%	0.000	
		Returns	15.43%		
	Expected Returns	(1.0*12.38)+ (.00*10.75)+ (0.0*15.43)			
	Portfolio Expected Returns	**12.38%**			

PORTFOLIO - 1. J

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0248695	0.000000	0.000000
Bonds	0.000000	0.000000	0.000000
Real Est	0.000000	0.000000	0.000000
Sub - Total	0.024869	0.000000	0.000000
Total	0.024869		

Portfolio Risk or Variance **15.77%**

Annexure A14: Portfolio Maximum Returns

	Portfolio - Maximum Return				
	Portfolio Allocation	**Stocks - 0%**	**Bonds - 0%**	**Real Estate - 100%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	12.38%	10.75%	15.43%	
	Volatility	15.77%	2.99%	6.52%	
	Allocation	0.00	0.00	1.00	
		1.00			
1	**Portfolio Expected Returns**				
	Stocks	Allocation	0.00%	0.000	
		Returns	12.38%		
	Bonds	Allocation	0.00%	0.000	
		Returns	10.75%		
	Real Estate	Allocation	100.00%	1.000	
		Returns	15.43%		
	Expected Returns	(0.339*12.38)+ (.224*10.75)+ (.437*15.43)			
	Portfolio Expected Returns	**15.42%**			

Portfolio - Maximum Return

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0000000	0.000000	0.000000
Bonds	0.000000	0.000000	0.000000
Real Est	0.000000	0.000000	0.004248
Sub - Total	0.000000	0.000000	0.004248

Total 0.004248

Portfolio Risk or Variance **6.52%**

Annexure A15: Portfolio Minimum Risk

	Minimum Risk / Variance				
	Portfolio Allocation	**Stocks - 1%**	**Bonds - 76%**	**Real Estate - 23%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	12.38%	10.75%	15.43%	
	Volatility	15.77%	2.99%	6.52%	
	Allocation	0.01	0.76	0.23	
		1.00			
1	**Portfolio Expected Returns**				
	Stocks	Allocation	1.15%	0.012	
		Returns	12.38%		
	Bonds	Allocation	75.91%	0.759	
		Returns	10.75%		
	Real Estate	Allocation	22.94%	0.229	
		Returns	15.43%		
	Expected Returns	(0.339*12.38)+ (.224*10.75)+ (.437*15.43)			
	Portfolio Expected Returns	**11.84%**			

Minimum Risk / Variance

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.3730	0.6516
Bonds	-0.3730	1.0000	-0.3811
Real estate	0.6516	-0.3811	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.024869	-0.001758	0.006698
Bonds	-0.001758	0.000894	-0.000743
Real Est	0.006698	-0.000743	0.004248

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0000033	-0.000015	0.000018
Bonds	-0.000015	0.000515	-0.000129
Real Est	0.000018	-0.000129	0.000223
Sub - Total	0.000006	0.000370	0.000112
Total	0.000488		

Portfolio Risk or Variance **2.21%**

Annexure B - Portfolio Series 2 – 10 Years – 1995-2004

Annexure B1: JSE All Share Data

FTSE / JSE ALL SHARE INDEX (Common Stocks)

YEARLY DATA - FROM 1995 TO 2004

10 YEAR PERIOD

	Date	Index Value at Close		% Returns
1	30-Dec-94	5,852.79		
2	31-Dec-95	5,198.17		-11.18%
3	31-Dec-96	6,130.74		17.94%
4	31-Dec-97	6,286.78		2.55%
5	31-Dec-98	5,832.86		-7.22%
6	29-Dec-99	6,531.93		11.99%
7	29-Dec-00	7,907.08		21.05%
8	31-Dec-01	8,816.75		11.50%
9	31-Dec-02	10,305.31		16.88%
10	31-Dec-03	8,899.71		-13.64%
11	31-Dec-04	11,021.29		23.84%
	Average Returns			**7.37%**
	Standard Deviation			**13.12%**

Annexure B2: SA Government Ten Year Bonds

SA GOVERNMENT TEN YEAR BOND [KBP 2003M]

YEARLY DATA - FROM 1995 TO 2004

10 YEAR PERIOD

	Date	Monthly - Yield		Annual Yield
1	1995/01	17.02		
2	1995/02	16.82		
3	1995/03	16.72		
4	1995/04	16.82		
5	1995/05	16.95		
6	1995/06	16.78		
7	1995/07	16.62		
8	1995/08	15.96		
9	1995/09	15.49		
10	1995/10	15.15		
11	1995/11	14.39		
12	1995/12	14.56		**16.11**
13	1996/01	13.77		
14	1996/02	14.1		
15	1996/03	15.04		
16	1996/04	15.78		
17	1996/05	16.53		
18	1996/06	15.78		
19	1996/07	15.39		
20	1996/08	15.82		
21	1996/09	15.42		
22	1996/10	15.8		
23	1996/11	16.18		
24	1996/12	16.19		**15.48**
25	1997/01	15.82		
26	1997/02	15.03		
27	1997/03	15.16		
28	1997/04	15.24		
29	1997/05	15.08		
30	1997/06	14.72		
31	1997/07	14.21		
32	1997/08	14.24		
33	1997/09	14.18		

34	1997/10	14.05		
35	1997/11	14.5		
36	1997/12	14.14		**14.70**
37	1998/01	13.61		
38	1998/02	13.49		
39	1998/03	13.33		
40	1998/04	12.92		
41	1998/05	13.46		
42	1998/06	14.6		
43	1998/07	15.89		
44	1998/08	16.95		
45	1998/09	18.3		
46	1998/10	16.51		
47	1998/11	16.05		
48	1998/12	16.36		**15.12**
49	1999/01	15.89		
50	1999/02	14.88		
51	1999/03	14.54		
52	1999/04	14.58		
53	1999/05	15.14		
54	1999/06	14.93		
55	1999/07	14.97		
56	1999/08	15.28		
57	1999/09	15.35		
58	1999/10	15		
59	1999/11	14.3		
60	1999/12	13.96		**14.90**
61	2000/01	13.49		
62	2000/02	13.5		
63	2000/03	13.92		
64	2000/04	14.27		
65	2000/05	14.79		
66	2000/06	14.33		
67	2000/07	13.9		
68	2000/08	13.52		
69	2000/09	13.72		
70	2000/10	13.72		
71	2000/11	13.4		
72	2000/12	12.88		**13.79**
73	2001/01	12.61		
74	2001/02	11.97		
75	2001/03	11.79		
76	2001/04	12.17		
77	2001/05	11.93		
78	2001/06	11.32		
79	2001/07	10.9		
80	2001/08	10.75		
81	2001/09	10.81		
82	2001/10	10.73		
83	2001/11	10.27		

84	2001/12	11.63		**11.41**
85	2002/01	11.61		
86	2002/02	11.89		
87	2002/03	12.63		
88	2002/04	12.47		
89	2002/05	11.66		
90	2002/06	11.6		
91	2002/07	11.15		
92	2002/08	11.08		
93	2002/09	11.35		
94	2002/10	11.27		
95	2002/11	10.86		
96	2002/12	10.44		**11.50**
97	2003/01	10.17		
98	2003/02	9.94		
99	2003/03	10		
100	2003/04	9.93		
101	2003/05	9.76		
102	2003/06	9.26		
103	2003/07	9.57		
104	2003/08	9.59		
105	2003/09	9.61		
106	2003/10	9.13		
107	2003/11	9.28		
108	2003/12	9.15		**9.62**
109	2004/01	9.27		
110	2004/02	9.4		
111	2004/03	9.6		
112	2004/04	9.85		
113	2004/05	10.24		
114	2004/06	10.38		
115	2004/07	10.08		
116	2004/08	9.72		
117	2004/09	9.28		
118	2004/10	9.16		
119	2004/11	8.96		
120	2004/12	8.38		**9.53**

Average Returns	**13.21%**
Standard Deviation	**2.35%**

Annexure B3: MSCI All Property Index

MSCI / INVESTMENT PROPERTY DATABANK ALL PROPERTY INDEX

YEARLY DATA - FROM 1995 TO 2004

10 YEAR PERIOD

	Date		Annual Returns
1	31-Dec-95		15.30%
2	31-Dec-96		14.00%
3	31-Dec-97		17.40%
4	31-Dec-98		5.00%
5	29-Dec-99		13.60%
6	29-Dec-00		11.00%
7	31-Dec-01		10.40%
8	31-Dec-02		9.50%
9	31-Dec-03		15.20%
10	31-Dec-04		23.40%

Average Returns	**13.48%**
Standard Deviation	**4.73%**

Annexure B4: Portfolio 2 A

PORTFOLIO - 2. A				
Portfolio Allocation		**Stocks - 0%**	**Bonds - 36.7%**	**Real Estate - 63.3%**
Data				
		Stocks	Bonds	Real Estate
Returns		7.37%	13.22%	13.48%
Volatility		13.12%	2.35%	4.73%
Allocation		0.00%	36.70%	63.3%
1 **Portfolio Expected Returns**				
Stocks	Allocation		0.00%	0.000
	Returns		7.37%	
Bonds	Allocation		36.70%	0.367
	Returns		13.22%	
Real Estate	Allocation		63.30%	0.633
	Returns		13.48%	
Expected Returns	(0 .000*7.37)+ (.367*13.22)+ (.633*13.48)			
Portfolio Expected Returns	**13.38%**			

PORTFOLIO - 2. A

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0000000	0.000000	0.000000
Bonds	0.000000	0.000074	-0.000082
Real Est	0.000000	-0.000082	0.000896
Sub - Total	0.000000	-0.000008	0.000814

Total 0.000806

Portfolio Risk or Variance 2.84%

Annexure B5: Portfolio 2 B

	PORTFOLIO - 2. B				
	Portfolio Allocation	**Stocks - 11.2%**	**Bonds - 32.3%**	**Real Estate - 56.5%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	7.37%	13.22%	13.48%	
	Volatility	13.12%	2.35%	4.73%	
	Allocation	11.20%	32.30%	56.5%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	11.20%	0.112	
		Returns	7.37%		
	Bonds	Allocation	32.30%	0.323	
		Returns	13.22%		
	Real Estate	Allocation	56.50%	0.565	
		Returns	13.48%		
	Expected Returns	(0.112*7.37)+ (.323*13.22)+ (.565*13.48)			
	Portfolio Expected Returns	**12.71%**			

PORTFOLIO - 2. B

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0002158	-0.000021	0.000076
Bonds	-0.000021	0.000058	-0.000065
Real Est	0.000076	-0.000065	0.000714
Sub - Total	0.000271	-0.000028	0.000725
Total	0.000968		

Portfolio Risk or Variance **3.11%**

Annexure B6: Portfolio 2 C

	PORTFOLIO - 2. C				
	Portfolio Allocation	**Stocks - 22.50%**	**Bonds - 27.30%**	**Real Estate - 50.10%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	7.37%	13.22%	13.48%	
	Volatility	13.12%	2.35%	4.73%	
	Allocation	22.50%	27.30%	50.1%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	22.50%	0.225	
		Returns	7.37%		
	Bonds	Allocation	27.30%	0.273	
		Returns	13.22%		
	Real Estate	Allocation	50.10%	0.501	
		Returns	13.48%		
	Expected Returns	(0.225*7.37)+ (.273*13.22)+ (.501*13.48)			
	Portfolio Expected Returns	**12.02%**			

PORTFOLIO - 2. C

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0008711	-0.000036	0.000135
Bonds	-0.000036	0.000041	-0.000049
Real Est	0.000135	-0.000049	0.000562
Sub - Total	0.000971	-0.000043	0.000648
Total	0.001576		

Portfolio Risk or Variance **3.97%**

Annexure B7: Portfolio 2 D

PORTFOLIO - 2. D

	Portfolio Allocation	**Stocks - 33.90%**	**Bonds - 22.40%**	**Real Estate - 43.70%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	7.37%	13.22%	13.48%	
	Volatility	13.12%	2.35%	4.73%	
	Allocation	33.90%	22.40%	43.7%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	33.90%	0.339	
		Returns	7.37%		
	Bonds	Allocation	22.40%	0.224	
		Returns	13.22%		
	Real Estate	Allocation	43.70%	0.437	
		Returns	13.48%		
	Expected Returns	(0.339*7.37)+ (.224*13.22)+ (.437*13.48)			
	Portfolio Expected Returns	**11.35%**			

PORTFOLIO - 2. D

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0019773	-0.000044	0.000178
Bonds	-0.000044	0.000028	-0.000035
Real Est	0.000178	-0.000035	0.000427
Sub - Total	0.002111	-0.000051	0.000570
Total	0.002630		

Portfolio Risk or Variance **5.13%**

Annexure B8: Portfolio 2E

	PORTFOLIO - 2. E				
	Portfolio Allocation	**Stocks - 45.2%**	**Bonds - 17.40%**	**Real Estate - 37.3%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	7.37%	13.22%	13.48%	
	Volatility	13.12%	2.35%	4.73%	
	Allocation	45.20%	17.40%	37.3%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	45.20%	0.452	
		Returns	7.37%		
	Bonds	Allocation	17.40%	0.174	
		Returns	13.22%		
	Real Estate	Allocation	37.30%	0.373	
		Returns	13.48%		
	Expected Returns	(0.452*7.37)+ (.174*13.22)+ (.373*13.48)			
	Portfolio Expected Returns	**10.66%**			

PORTFOLIO - 2. E

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0035152	-0.000046	0.000202
Bonds	-0.000046	0.000017	-0.000023
Real Est	0.000202	-0.000023	0.000311
Sub - Total	0.003672	-0.000052	0.000490
Total	0.004110		

Portfolio Risk or Variance **6.41%**

Annexure B9: Portfolio 2 F

	PORTFOLIO - 2. F				
	Portfolio Allocation	**Stocks - 56.6**	**Bonds - 12.50**	**Real Estate - 31**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	7.37%	13.22%	13.48%	
	Volatility	13.12%	2.35%	4.73%	
	Allocation	56.60%	12.50%	31.0%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	56.60%	0.566	
		Returns	7.37%		
	Bonds	Allocation	12.50%	0.125	
		Returns	13.22%		
	Real Estate	Allocation	31.00%	0.310	
		Returns	13.48%		
	Expected Returns	(0.566*7.37)+ (.125*13.22)+ (.310*13.48)			
	Portfolio Expected Returns	**10.00%**			

PORTFOLIO - 2. F

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0055120	-0.000041	0.000210
Bonds	-0.000041	0.000009	-0.000014
Real Est	0.000210	-0.000014	0.000215
Sub - Total	0.005681	-0.000046	0.000412
Total	0.006047		

Portfolio Risk or Variance **7.78%**

Annexure B10: Portfolio 2 G

	PORTFOLIO - 2. G				
	Portfolio Allocation	**Stocks - 67.90**	**Bonds - 7.50**	**Real Estate -24.60**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	7.37%	13.22%	13.48%	
	Volatility	13.12%	2.35%	4.73%	
	Allocation	67.90%	7.50%	24.6%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	67.90%	0.679	
		Returns	7.37%		
	Bonds	Allocation	7.50%	0.075	
		Returns	13.22%		
	Real Estate	Allocation	24.60%	0.246	
		Returns	13.48%		
	Expected Returns	(0.566*7.37)+ (.125*13.22)+ (.310*13.48)			
	Portfolio Expected Returns	**9.31%**			

PORTFOLIO - 2. G

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0079326	-0.000030	0.000200
Bonds	-0.000030	0.000003	-0.000007
Real Est	0.000200	-0.000007	0.000135
Sub - Total	0.008103	-0.000033	0.000329
Total	0.008399		

Portfolio Risk or Variance **9.16%**

Annexure B11: Portfolio 2 H

	PORTFOLIO - 2. H				
	Portfolio Allocation	**Stocks - 79.3**	**Bonds - 2.50**	**Real Estate - 18.20**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	7.37%	13.22%	13.48%	
	Volatility	13.12%	2.35%	4.73%	
	Allocation	79.30%	2.50%	18.2%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	79.30%	0.793	
		Returns	7.37%		
	Bonds	Allocation	2.50%	0.025	
		Returns	13.22%		
	Real Estate	Allocation	18.20%	0.182	
		Returns	13.48%		
	Expected Returns	(0.793*7.37)+ (.025*13.22)+ (.182*13.48)			
	Portfolio Expected Returns	**8.63%**			

PORTFOLIO - 2. H

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0108199	-0.000011	0.000173
Bonds	-0.000011	0.000000	-0.000002
Real Est	0.000173	-0.000002	0.000074
Sub - Total	0.010981	-0.000013	0.000245
Total	0.011214		

Portfolio Risk or Variance **10.59%**

Annexure B12: Portfolio 2 I

	PORTFOLIO - 2. I				
	Portfolio Allocation	**Stocks - 90**	**Bonds - 0.0**	**Real Estate - 10**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	7.37%	13.22%	13.48%	
	Volatility	13.12%	2.35%	4.73%	
	Allocation	90.00%	0.00%	10.0%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	90.00%	0.900	
		Returns	7.37%		
	Bonds	Allocation	0.00%	0.000	
		Returns	13.22%		
	Real Estate	Allocation	10.00%	0.100	
		Returns	13.48%		
	Expected Returns	(0.900*7.37)+ (.00*13.22)+ (.1100*13.48)			
	Portfolio Expected Returns	**7.98%**			

PORTFOLIO - 2. I

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0139368	0.000000	0.000108
Bonds	0.000000	0.000000	0.000000
Real Est	0.000108	0.000000	0.000022
Sub - Total	0.014045	0.000000	0.000130
Total	0.014175		

Portfolio Risk or Variance **11.91%**

Annexure B13: Portfolio 2 J

	PORTFOLIO - 2. J				
	Portfolio Allocation	**Stocks - 100**	**Bonds - 0.0**	**Real Estate - 0**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	7.37%	13.22%	13.48%	
	Volatility	13.12%	2.35%	4.73%	
	Allocation	100.00%	0.00%	0.0%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	100.00%	1.000	
		Returns	7.37%		
	Bonds	Allocation	0.00%	0.000	
		Returns	13.22%		
	Real Estate	Allocation	0.00%	0.000	
		Returns	13.48%		
	Expected Returns	(1.0*7.37)+ (.00*13.22)+ (0.0*13.4815.43)			
	Portfolio Expected Returns	**7.37%**			

PORTFOLIO - 2. J

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0172059	0.000000	0.000000
Bonds	0.000000	0.000000	0.000000
Real Est	0.000000	0.000000	0.000000
Sub - Total	0.017206	0.000000	0.000000
Total	0.017206		

Portfolio Risk or Variance **13.12%**

Annexure B14: Portfolio Maximum Returns

	Portfolio - Maximum Return				
	Portfolio Allocation	**Stocks - 0%**	**Bonds - 0%**	**Real Estate - 100%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	7.37%	13.22%	13.48%	
	Volatility	13.12%	2.35%	4.73%	
	Allocation	0.00	0.00	1.00	
		1.00			
1	**Portfolio Expected Returns**				
	Stocks	Allocation	0.00%	0.000	
		Returns	7.37%		
	Bonds	Allocation	0.00%	0.000	
		Returns	13.22%		
	Real Estate	Allocation	100.00%	1.000	
		Returns	13.48%		
	Expected Returns	(0.339*7.37)+ (.224*13.22)+ (.437*13.48)			
	Portfolio Expected Returns	**13.48%**			

Portfolio - Maximum Return

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0000000	0.000000	0.000000
Bonds	0.000000	0.000000	0.000000
Real Est	0.000000	0.000000	0.002237
Sub - Total	0.000000	0.000000	0.002237
Total	0.002237		

Portfolio Risk or Variance **4.73%**

Annexure B15: Portfolio Minimum Risk

Minimum Risk / Variance				
	Portfolio Allocation	**Stocks - 1%**	**Bonds - 76%**	**Real Estate - 23%**
	Data			
		Stocks	Bonds	Real Estate
	Returns	7.37%	13.22%	13.48%
	Volatility	13.12%	2.35%	4.73%
	Allocation	0.03	0.73	0.24
		1.00		
1	**Portfolio Expected Returns**			
	Stocks	Allocation	2.59%	0.026
		Returns	7.37%	
	Bonds	Allocation	73.48%	0.735
		Returns	13.22%	
	Real Estate	Allocation	23.93%	0.239
		Returns	13.48%	
	Expected Returns	(0.339*12.38)+ (.224*10.75)+ (.437*15.43)		
	Portfolio Expected Returns	**13.13%**		

Minimum Risk / Variance

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.1881	0.1932
Bonds	-0.1881	1.0000	-0.3191
Real estate	0.1932	-0.3191	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.017206	-0.000580	0.001199
Bonds	-0.000580	0.000552	-0.000355
Real Est	0.001199	-0.000355	0.002237

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0000115	-0.000011	0.000007
Bonds	-0.000011	0.000298	-0.000062
Real Est	0.000007	-0.000062	0.000128
Sub - Total	0.000008	0.000225	0.000073
Total	0.000306		

Portfolio Risk or Variance **1.75%**

Annexure C - Portfolio Series 3 – 20 Years – 2005-2014

Annexure C1: JSE All Share Data

FTSE / JSE ALL SHARE INDEX (Common Stocks)

YEARLY DATA - FROM 2005 TO 2014

10 YEAR PERIOD

	Date	Index Value at Close	% Returns
1	31-Dec-04	11,021.29	
2	30-Dec-05	14,772.16	34.03%
3	29-Dec-06	21,217.15	43.63%
4	31-Dec-07	28,231.84	33.06%
5	31-Dec-08	26,823.29	-4.99%
6	31-Dec-09	23,315.31	-13.08%
7	31-Dec-10	28,552.93	22.46%
8	30-Dec-11	31,577.02	10.59%
9	31-Dec-12	35,058.42	11.03%
10	31-Dec-13	41,764.92	19.13%
11	31-Dec-14	49,144.54	17.67%
	Average Returns		**17.35%**
	Standard Deviation		**16.60%**

Annexure C2: SA Government Ten Year Bonds

SA GOVERNMENT TEN YEAR BOND [KBP 2003M]				
YEARLY DATA - FROM 2005 TO 2014				
10 YEAR PERIOD				
	Date	**Monthly - Yield**		**Annual Yield**
1	2005/01	8.2		
2	2005/02	7.85		
3	2005/03	8.21		
124	2005/04	8.45		
125	2005/05	8.4		
126	2005/06	8.08		
127	2005/07	8.12		
128	2005/08	7.9		
129	2005/09	8.04		
130	2005/10	8.1		
131	2005/11	7.88		
132	2005/12	7.57		**8.07**
133	2006/01	7.37		
134	2006/02	7.26		
135	2006/03	7.36		
136	2006/04	7.46		
137	2006/05	7.63		
138	2006/06	8.29		
139	2006/07	8.58		
140	2006/08	8.59		
141	2006/09	8.64		
142	2006/10	8.39		
143	2006/11	7.87		
144	2006/12	7.81		**7.94**
145	2007/01	7.65		
146	2007/02	7.5		
147	2007/03	7.59		
148	2007/04	7.64		
149	2007/05	7.58		
150	2007/06	8.1		
151	2007/07	8.34		
152	2007/08	8.37		
153	2007/09	8.45		
154	2007/10	8.12		
155	2007/11	8.19		

156	2007/12	8.29		**7.99**
157	2008/01	8.36		
158	2008/02	8.69		
159	2008/03	9.16		
160	2008/04	9.15		
161	2008/05	9.51		
162	2008/06	10.35		
163	2008/07	10.1		
164	2008/08	9.2		
165	2008/09	9.04		
166	2008/10	9.24		
167	2008/11	8.58		
168	2008/12	7.82		**9.10**
169	2009/01	7.85		
170	2009/02	8.26		
171	2009/03	8.43		
172	2009/04	8.57		
173	2009/05	8.7		
174	2009/06	8.88		
175	2009/07	9.11		
176	2009/08	8.8		
177	2009/09	8.7		
178	2009/10	9.11		
179	2009/11	9		
180	2009/12	9.03		**8.70**
181	2010/01	9.24		
182	2010/02	9.11		
183	2010/03	8.92		
184	2010/04	8.75		
185	2010/05	8.97		
186	2010/06	8.99		
187	2010/07	8.78		
188	2010/08	8.17		
189	2010/09	7.92		
190	2010/10	8.03		
191	2010/11	8.12		
192	2010/12	8.38		**8.62**
193	2011/01	8.52		
194	2011/02	8.7		
195	2011/03	8.94		
196	2011/04	8.73		
197	2011/05	8.56		
198	2011/06	8.5		
199	2011/07	8.55		
200	2011/08	8.1		
201	2011/09	8.25		
202	2011/10	8.41		
203	2011/11	8.47		
204	2011/12	8.51		**8.52**
205	2012/01	8.4		

206	2012/02	8.23		
207	2012/03	8.37		
208	2012/04	8.28		
209	2012/05	8.28		
210	2012/06	8.16		
211	2012/07	7.52		
212	2012/08	7.48		
213	2012/09	7.4		
214	2012/10	7.67		
215	2012/11	7.64		
216	2012/12	7.37		**7.90**
217	2013/01	7.26		
218	2013/02	7.29		
219	2013/03	7.39		
220	2013/04	6.97		
221	2013/05	6.96		
222	2013/06	7.94		
223	2013/07	7.98		
224	2013/08	8.37		
225	2013/09	8.15		
226	2013/10	7.87		
227	2013/11	8.18		
228	2013/12	8.32		**7.72**
229	2014/01	8.48		
230	2014/02	8.67		
231	2014/03	8.52		
232	2014/04	8.41		
233	2014/05	8.16		
234	2014/06	8.34		
235	2014/07	8.25		
236	2014/08	8.21		
237	2014/09	8.2		
238	2014/10	8.06		
239	2014/11	7.86		
240	2014/12	7.84		**8.25**

	Average Returns	**8.28%**
	Standard Deviation	**0.42%**

Annexure C3: MSCI All Property Index

MSCI / INVESTMENT PROPERTY
DATABANK ALL PROPERTY INDEX

YEARLY DATA - FROM 2005 TO 2014

10 YEAR PERIOD

	Date		Annual Returns
1	30-Dec-05		30.00%
2	29-Dec-06		27.30%
3	31-Dec-07		27.70%
4	31-Dec-08		12.80%
5	31-Dec-09		9.10%
6	31-Dec-10		13.40%
7	30-Dec-11		10.30%
8	31-Dec-12		14.90%
9	31-Dec-13		15.30%
10	31-Dec-14		12.90%

Average Returns	**17.37%**
Standard Deviation	**7.42%**

Annexure C4: Portfolio 3 A

	PORTFOLIO - 3. A				
	Portfolio Allocation	**Stocks - 0%**	**Bonds - 36.7%**	**Real Estate - 63.3%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	17.38%	8.28%	17.37%	
	Volatility	16.59%	0.42%	7.42%	
	Allocation	0.00%	36.70%	63.3%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	0.00%	0.000	
		Returns	17.38%		
	Bonds	Allocation	36.70%	0.367	
		Returns	8.28%		
	Real Estate	Allocation	63.30%	0.633	
		Returns	17.37%		
	Expected Returns	(0 .000*17.38)+ (.367*8.28)+ (.633*17.37)			
	Portfolio Expected Returns	**14.03%**			

PORTFOLIO - 3. A

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0000000	0.000000	0.000000
Bonds	0.000000	0.000002	-0.000040
Real Est	0.000000	-0.000040	0.002205
Sub - Total	0.000000	-0.000037	0.002165

Total 0.002128

Portfolio Risk or Variance **4.61%**

Annexure C5: Portfolio 3 B

	PORTFOLIO - 3. B				
	Portfolio Allocation	**Stocks - 11.2%**	**Bonds - 32.3%**	**Real Estate - 56.5%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	17.38%	8.28%	17.37%	
	Volatility	16.59%	0.42%	7.42%	
	Allocation	11.20%	32.30%	56.5%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	11.20%	0.112	
		Returns	17.38%		
	Bonds	Allocation	32.30%	0.323	
		Returns	8.28%		
	Real Estate	Allocation	56.50%	0.565	
		Returns	17.37%		
	Expected Returns	(0.112*17.38)+ (.323*8.28)+ (.565*17.37)			
	Portfolio Expected Returns	**14.44%**			

PORTFOLIO - 3. B

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0003453	-0.000017	0.000649
Bonds	-0.000017	0.000002	-0.000031
Real Est	0.000649	-0.000031	0.001756
Sub - Total	0.000977	-0.000046	0.002374
Total	0.003305		

Portfolio Risk or Variance **5.75%**

Annexure C6: Portfolio 3 C

PORTFOLIO - 3. C

Portfolio Allocation	**Stocks - 22.50%**	**Bonds - 27.30%**	**Real Estate - 50.10%**

Data

	Stocks	Bonds	Real Estate
Returns	17.38%	8.28%	17.37%
Volatility	16.59%	0.42%	7.42%
Allocation	22.50%	27.30%	50.1%

1 **Portfolio Expected Returns**

Stocks	Allocation	22.50%	0.225
	Returns	17.38%	
Bonds	Allocation	27.30%	0.273
	Returns	8.28%	
Real Estate	Allocation	50.10%	0.501
	Returns	17.37%	
Expected Returns	(0.225*17.38)+ (.273*8.28)+ (.501*17.37)		

Portfolio Expected Returns **14.87%**

PORTFOLIO - 3. C

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0013934	-0.000029	0.001155
Bonds	-0.000029	0.000001	-0.000023
Real Est	0.001155	-0.000023	0.001381
Sub - Total	0.002520	-0.000051	0.002513
Total	0.004983		

Portfolio Risk or Variance **7.06%**

Annexure C7: Portfolio 3 D

PORTFOLIO - 3. D

Portfolio Allocation **Stocks - 33.90% Bonds - 22.40% Real Estate - 43.70%**

Data

	Stocks	Bonds	Real Estate
Returns	17.38%	8.28%	17.37%
Volatility	16.59%	0.42%	7.42%
Allocation	33.90%	22.40%	43.7%

1 **Portfolio Expected Returns**

Stocks	Allocation	33.90%	0.339
	Returns	17.38%	
Bonds	Allocation	22.40%	0.224
	Returns	8.28%	
Real Estate	Allocation	43.70%	0.437
	Returns	17.37%	
Expected Returns	(0.339*17.38)+ (.224*8.28)+ (.437*17.37)		

Portfolio Expected Returns **15.34%**

PORTFOLIO - 3. D

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0031631	-0.000035	0.001518
Bonds	-0.000035	0.000001	-0.000017
Real Est	0.001518	-0.000017	0.001051
Sub - Total	0.004646	-0.000051	0.002553
Total	0.007147		

Portfolio Risk or Variance **8.45%**

Annexure C8: Portfolio 3 E

	PORTFOLIO - 3. E				
	Portfolio Allocation	**Stocks - 45.2%**	**Bonds - 17.40%**	**Real Estate - 37.3%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	17.38%	8.28%	17.37%	
	Volatility	16.59%	0.42%	7.42%	
	Allocation	45.20%	17.40%	37.3%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	45.20%	0.452	
		Returns	17.38%		
	Bonds	Allocation	17.40%	0.174	
		Returns	8.28%		
	Real Estate	Allocation	37.30%	0.373	
		Returns	17.37%		
	Expected Returns	(0.452*117.38)+ (.174*8.28)+ (.373*17.37)			
	Portfolio Expected Returns	**15.78%**			

PORTFOLIO - 3. E

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0056233	-0.000037	0.001728
Bonds	-0.000037	0.000001	-0.000011
Real Est	0.001728	-0.000011	0.000766
Sub - Total	0.007315	-0.000047	0.002483
Total	0.009750		

Portfolio Risk or Variance **9.87%**

Annexure C9: Portfolio 3 F

PORTFOLIO - 3. F

	Portfolio Allocation	Stocks - 56.6	Bonds - 12.50	Real Estate - 31
	Data			
		Stocks	Bonds	Real Estate
	Returns	17.38%	8.28%	17.37%
	Volatility	16.59%	0.42%	7.42%
	Allocation	56.60%	12.50%	31.0%
1	**Portfolio Expected Returns**			
	Stocks	Allocation	56.60%	0.566
		Returns	17.38%	
	Bonds	Allocation	12.50%	0.125
		Returns	8.28%	
	Real Estate	Allocation	31.00%	0.310
		Returns	17.37%	
	Expected Returns	(0.566*17.38)+ (.125*8.28)+ (.310*17.37)		
	Portfolio Expected Returns	**16.26%**		

PORTFOLIO - 3. F

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0088175	-0.000033	0.001798
Bonds	-0.000033	0.000000	-0.000007
Real Est	0.001798	-0.000007	0.000529
Sub - Total	0.010583	-0.000039	0.002321
Total	0.012864		

Portfolio Risk or Variance **11.34%**

Annexure C10: Portfolio 3 G

	PORTFOLIO - 3. G				
	Portfolio Allocation	**Stocks - 67.90**	**Bonds - 7.50**	**Real Estate -24.60**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	17.38%	8.28%	17.37%	
	Volatility	16.59%	0.42%	7.42%	
	Allocation	67.90%	7.50%	24.6%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	67.90%	0.679	
		Returns	17.38%		
	Bonds	Allocation	7.50%	0.075	
		Returns	8.28%		
	Real Estate	Allocation	24.60%	0.246	
		Returns	17.37%		
	Expected Returns	(0.566*17.38)+ (.125*8.28)+ (.310*17.37)			
	Portfolio Expected Returns	**16.70%**			

PORTFOLIO - 3. G

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0126897	-0.000024	0.001712
Bonds	-0.000024	0.000000	-0.000003
Real Est	0.001712	-0.000003	0.000333
Sub - Total	0.014378	-0.000027	0.002042
Total	0.016393		

Portfolio Risk or Variance **12.80%**

Annexure C11: Portfolio 3 H

	PORTFOLIO - 3. H				
	Portfolio Allocation	**Stocks - 79.3**	**Bonds - 2.50**	**Real Estate - 18.20**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	17.38%	8.28%	17.37%	
	Volatility	16.59%	0.42%	7.42%	
	Allocation	79.30%	2.50%	18.2%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	79.30%	0.793	
		Returns	17.38%		
	Bonds	Allocation	2.50%	0.025	
		Returns	8.28%		
	Real Estate	Allocation	18.20%	0.182	
		Returns	17.37%		
	Expected Returns	(0.793*17.38)+ (.025*8.28)+ (.182*17.37)			
	Portfolio Expected Returns	**17.15%**			

PORTFOLIO - 3. H

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0173084	-0.000009	0.001479
Bonds	-0.000009	0.000000	-0.000001
Real Est	0.001479	-0.000001	0.000182
Sub - Total	0.018779	-0.000010	0.001661
Total	0.020429		

Portfolio Risk or Variance **14.29%**

Annexure C12: Portfolio 3 I

	PORTFOLIO - 3. I				
	Portfolio Allocation	**Stocks - 90**	**Bonds - 0.0**	**Real Estate - 10**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	17.38%	8.28%	17.37%	
	Volatility	16.59%	0.42%	7.42%	
	Allocation	90.00%	0.00%	10.0%	
1	**Portfolio Expected Returns**				
	Stocks	Allocation	90.00%	0.900	
		Returns	17.38%		
	Bonds	Allocation	0.00%	0.000	
		Returns	8.28%		
	Real Estate	Allocation	10.00%	0.100	
		Returns	17.37%		
	Expected Returns	(0.900*17.38)+ (.00*8.28)+ (.1100*17.37)			
	Portfolio Expected Returns	**17.38%**			

PORTFOLIO - 3. I

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0222944	0.000000	0.000922
Bonds	0.000000	0.000000	0.000000
Real Est	0.000922	0.000000	0.000055
Sub - Total	0.023217	0.000000	0.000978
Total	0.024194		

Portfolio Risk or Variance **15.55%**

Annexure C13: Portfolio 3 J

PORTFOLIO - 3. J

Portfolio Allocation	Stocks - 100	Bonds - 0.0	Real Estate - 0

Data

	Stocks	Bonds	Real Estate
Returns	17.38%	8.28%	17.37%
Volatility	16.59%	0.42%	7.42%
Allocation	100.00%	0.00%	0.0%

1 **Portfolio Expected Returns**

Stocks	Allocation	100.00%	1.000
	Returns	17.38%	
Bonds	Allocation	0.00%	0.000
	Returns	8.28%	
Real Estate	Allocation	0.00%	0.000
	Returns	17.37%	
Expected Returns	(1.0*17.38)+ (.00*8.28)+ (0.0*17.37)		

Portfolio Expected Returns **17.38%**

PORTFOLIO - 3. J

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0275240	0.000000	0.000000
Bonds	0.000000	0.000000	0.000000
Real Est	0.000000	0.000000	0.000000
Sub - Total	0.027524	0.000000	0.000000
Total	0.027524		

Portfolio Risk or Variance **16.59%**

Annexure C14: Portfolio Maximum Returns

Portfolio - Maximum Return

Portfolio Allocation	Stocks - 0%	Bonds - 0%	Real Estate - 100%

Data

	Stocks	Bonds	Real Estate
Returns	17.38%	8.28%	17.37%
Volatility	16.59%	0.42%	7.42%
Allocation	0.00	0.00	1.00
	1.00		

1 **Portfolio Expected Returns**

Stocks	Allocation	0.00%	0.000
	Returns	17.38%	
Bonds	Allocation	0.00%	0.000
	Returns	8.28%	
Real Estate	Allocation	100.00%	1.000
	Returns	17.37%	
Expected Returns	(0.339*12.38)+ (.224*10.75)+ (.437*15.43)		

Portfolio Expected Returns **17.37%**

Portfolio - Maximum Return

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0000000	0.000000	0.000000
Bonds	0.000000	0.000000	0.000000
Real Est	0.000000	0.000000	0.005502
Sub - Total	0.000000	0.000000	0.005502
Total	0.005502		

Portfolio Risk or Variance **7.42%**

Annexure C15: Portfolio Minimum Risk

	Minimum Risk / Variance				
	Portfolio Allocation	**Stocks - 1%**	**Bonds - 76%**	**Real Estate - 23%**	
	Data				
		Stocks	Bonds	Real Estate	
	Returns	17.38%	8.28%	17.37%	
	Volatility	16.59%	0.42%	7.42%	
	Allocation	0.02	0.98	0.00	
		1.00			
1	**Portfolio Expected Returns**				
	Stocks	Allocation	1.64%	0.016	
		Returns	17.38%		
	Bonds	Allocation	98.21%	0.982	
		Returns	8.28%		
	Real Estate	Allocation	0.15%	0.001	
		Returns	17.37%		
	Expected Returns	(0.339*12.38)+ (.224*10.75)+ (.437*15.43)			
	Portfolio Expected Returns	**8.44%**			

Minimum Risk / Variance

2 **Portfolio Risk or Variance**

Correlation

	Stocks	Bonds	Real Estate
Stocks	1.0000	-0.6776	0.8329
Bonds	-0.6776	1.0000	-0.5538
Real estate	0.8329	-0.5538	1.0000

Co variance Expectations

	stocks	**bonds**	**real estate**
Stocks	0.027524	-0.000467	0.010250
Bonds	-0.000467	0.000017	-0.000171
Real Est	0.010250	-0.000171	0.005502

Weighted Co variance

	stocks	**bonds**	**real estate**
Stocks	0.0000074	-0.000008	0.000000
Bonds	-0.000008	0.000017	0.000000
Real Est	0.000000	0.000000	0.000000
Sub - Total	0.000000	0.000009	0.000000

Total 0.000009

Portfolio Risk or Variance **0.30%**