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ALIGNMENT BETWEEN THE COGNITIVE DEMANDS OF THE WRITTEN AND ASSESED TRIGONOMETRY CURRICULA IN SOUTH AFRICA

A research report submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Masters of Science

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Professor Margot Berger
Doctor Benita Nel

JOHANNESBURG, MARCH 2017

DECLARATION

I declare that **ALIGNMENT BETWEEN THE COGNITIVE DEMANDS OF THE WRITTEN AND ASSESED TRIGONOMETRY CURRICULA IN SOUTH AFRICA** is my own unaided work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references. It is being submitted for the first time for the degree of Masters of Science at the University of Witwatersrand, Johannesburg. It has not been submitted before for any other degree or examination in any other University.

NATHANIEL SAMUEL MOLEPO **MARCH 2017**

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DEDICATION

This work is dedicated to memory of my beloved mother Mankeng Agnes Molepo for bringing me up as a single parent and providing me with education under very difficult circumstances. It was through her sacrifices that I was able to reach heights in my life. I also dedicate this work to my wife Kgomotso, for all her support and understanding during the course of my study. My two daughters Olebogeng and Oageng and my son Tshiamo, particularly my little ones, Oageng and Tshiamo who understood the sacrifice and gave me space needed to complete this report. GOD blesses you all exceedingly and abundantly.

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- My study partner Sony Dlamini for allowing me to use his learner's resource material (textbooks) for analysing the tasks used in this study.

ABSTRACT

This study investigated the degree of alignment of cognitive demand between the written and the assessed trigonometry curriculum in South Africa. The written curriculum describes the content as it is depicted in the textbook. The assessed curriculum on the other hand refers to the content knowledge upon which student learning is measured. The issue that the study sought to understand is: what type of cognitive demands are implied in the instructional tasks around the general solutions of trigonometric equations in two Grade 12 NCS (CAPS) prescribed mathematics textbooks and in the assessment tasks in the NSC examinations. The study also examined the extent to which the cognitive demands of tasks in each of these curriculum documents align. Anderson (2002) curriculum alignment framework was used as a conceptual framework for the study. The study was a qualitative case study, the methodology of which was framed by Survey of Enacted Curriculum by Porter (2002). The SEC instrument was used to assign judgements of the cognitive demand of the tasks in the textbooks and the NSC examinations. Findings indicate that the degree of alignment between tasks of textbooks and NSC examinations varied. Most of the tasks presented in the most used textbook (Platinum Mathematics Grade 12) corresponded to the categories that exhibited lower order cognitive demands. These were mainly associated with 'memorisation' and 'perform procedures' of Porter's (2002) cognitive demand framework. However tasks in the NSC examinations were basically at the higher order level of Porter's (2002) framework. A reduced level of alignment was noted between the cognitive demands of tasks in Platinum Mathematics Grade 12 textbook and the tasks in the NSC examinations. In terms of tasks from Classroom Mathematics Grade 12 textbook and tasks in the NSC examinations, there was a strong alignment of cognitive demands between each of the these curriculum documents. The study recommended that those charged with the responsibilities of curricula management should be mindful of the fact that if students are expected to learn skills associated with higher order cognitive demands, then it is essential that all forms of curricular documents, particularly prescribed textbooks that guides teaching and learning on a daily basis should make such skills available, If not, then the students may be deprived from learning the envisaged skills.

Keywords:
written curriculum, assessed curriculum, cognitive demand, curricular alignment.

ABBREVIATIONS

ABET	- Adult Basic Education and Training
ANA	- Annual National Assessment
C2005	- Curriculum 2005
CAPS	- Curriculum Assessment Policy Statement
CCSSO	- Council of Chief State School Offices
ECD	- Early Childhood and Development
FET	- Further Education and Training
GDE	- Gauteng Department of Education
GET	- General Education and Training
NCS	- National Curriculum Statement
NSC	- National Senior Certificate
OBE	- Outcomes Based Education
RNCS	- Revised National Curriculum Statement
SEC	- Survey of Enacted Curriculum
TIMSS	- Trends in International Mathematics and Science Study
USA	- United States of America

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CHAPTER 1
BACKGROUND TO THE STUDY

1.1 INTRODUCTION

The latest curriculum reforms in South Africa, following the recommendations of the Curriculum Review Committee and the Chisholm report (Chisholm, 2000), resulted in the revised and repackaged curriculum referred to as the National Curriculum Statements for Grade R-12 (Department of Education, 2011). This new curriculum was formulated out of new policies and directions. The revised curriculum is commonly known as the Curriculum and Assessment Policy Statement (Department of Education, 2011). CAPS 2012 is framed within a standard-based (Näsström, 2008) system of education and places strong emphasis on content to be taught learnt and examined.

Näsström (2008) point out that in a standard-based schooling system, standards are seen as descriptors in policy documents. These descriptors articulate the content that the students are supposed to know as well as how they are expected to achieve standards for that content. Fuhrman (2001) point out that teaching with all its variations is a vehicle that is supposed to equip students with opportunities to achieve standards. To measure whether the students have attained the standards (both content and performance), assessment should measure the standards and provide feedback about how well the students have attained the standards (Näsström, 2008). Finally, when all the three components (standards, teaching and assessments) are in agreement, they are said to be aligned (Webb, 1997; Anderson, 2002; Biggs, 2003; Näsström, 2008).

This study does not constitute a judgment about the quality of the tasks of the Grade 12 Mathematics textbooks and NSC examinations. Instead, this study intends to start conversations about the alignment levels of cognitive demands of tasks between the various components of the Further Education and Training Mathematics curriculum with respect to the content topic of trigonometric equations and the general solutions. The interest of the study is to understand the extent to which cognitive demands of tasks as documented in the curriculum statement (CAPS) are addressed in the different types of the South African FET

Mathematics curriculum (i.e. prescribed grade 12 mathematics textbooks as well as the National Senior Certificate Examination Mathematics Paper 2).

1.2. HISTORY OF THE SOUTH AFRICAN CURRICULUM

The South African education system preceding the democratic dispensation was characterised by large inequalities which enforced segregation along racial lines, ethnic groups and created "class inequalities in the distribution of education resources and teaching" (NEPI, 1992a, p.9.). Within this context of segregation, the then school curriculum known as NATED 550 also organised school subjects including mathematics into grades i.e. Higher and Standard Grades. The major difference between these grades across all subjects was in the way they were conceptualised, interpreted and perceived especially in terms of cognitive demands levels of content. These assumptions further aggravated segregation along racial abilities by associating certain grades with certain sections of the population (i.e. black South African were perceived as being less capable academically and mainly expected to take their subjects on Standard Grades, while on the other hand white South Africans were believed to be highly capable and perceived to be Higher Grade material). Jansen (1999) characterised the South African curriculum of the time as racist, euro-centred, sexist, authoritarian, prescriptive and discriminatory.

With the dawn of democracy and radical political transition in South Africa, the education system underwent a major overhaul in order to redress the legacy inherited from the apartheid education system. The Outcomes-Based Education (OBE) was adopted in 1997 as an approach for restructuring and overcoming curricular division of the past. OBE was based on a learner-centred and outcomes based approach to education. OBE also known as Curriculum 2005 (C2005 for Grades 0 – 9) was underpinned by principles behind integration, holistic development, relevance, participation and ownership, accountability, learner orientation, flexibility, critical and creative thinking, progression and inclusion (NDE, 1997).

The implementation of C2005 was characterised by and showed evidence of weak classification between school subjects (weak internal classification), (Dowling, 1998), school and everyday knowledge (weak external classification), (Dowling, 1998). This weak classification among other challenges was cited as inhibiting the learning of conceptual knowledge for disciplines such as mathematics. Weak classification and emphasis on

integration were also cited as obscuring mathematical learning and that mathematics was dominated by non-mathematical considerations. Challenges such as the above prompted a review of C2005 by the Curriculum Review Committee in 2000. The Curriculum Review Committee (2000) and the Chisholm report (Chisholm, 2000), raised further concerns that C2005 was technically over-designed and neglected conceptual coherence and progression.

In an attempt to address the cited problems and other problems related to the complexity of C2005 design and terminologies, the Curriculum Review Committee proposed that a revised, streamlined National Curriculum Statement be produced for ECD, GET, FET and ABET. This led to the first post democratic dispensation curriculum revision: the Revised National Curriculum Statement Grades R-9 and the National Curriculum Statement Grades 10-12 (2002). Ongoing implementation and challenges of the revised curricula prompted a further review in 2009.

The latest curriculum reforms, following the 2009 review resulted in the revision and repackaging of the 2002 Curriculum Statement (RNCS Grades R–9 and NCS Grades 10-12) into a single curriculum referred to as the National Curriculum Statements for Grade R-12 (Department of Education, 2011). This new curriculum is formulated out of new policies and directions. The revised curriculum is commonly known as the Curriculum and Assessment Policy Statement (Department of Education, 2011). CAPS 2012 specifies the combination of content topics and cognitive demand levels that Mathematics students are expected to master in every grade, term and assessment.

1.3. THE STATEMENT OF PROBLEM

The South African education system is made up of four phases: The foundation phase (Grades R-3), intermediate phase (Grades 4-6), senior phase (Grades 7-9) and the Further education and Training phase (Grade 10-12). Grade 12 is the final and the exit grade of the schooling system in South Africa. This grade is marked off by means of nationally administered assessments (for all subjects making the FET band) known as the National Senior Certificate examinations and the certification processes endorsed by the quality assurance body UMALUSI. The quality of data derived from such assessments is of outmost importance since it is used to certify that candidates were competent in the acquisition of defined subject content and the mastery of competencies measured by such examinations.

The first phase of the new curriculum (CAPS) was implemented as policy in 2012 and climaxed to final phase with a senior certificate examination in 2014. Just like the previous curriculum, CAPS explicitly documents cognitive demand taxonomy to guide teaching, learning and assessment at all phases of the schooling system. The cognitive demands documented in CAPS for FET Mathematics are based on those suggested in the Trends in International Mathematics and Science Study (TIMSS) of 1999. The four cognitive levels in CAPS with their guidelines percentages are listed as knowledge (20%), procedures (35%), complex procedures (30%) and problem solving (15%). The four cognitive levels are organised by the levels of difficulty required to complete the task successfully.

However, according to the 2014 and 2015 National Senior Certificate (NSC) diagnostic reports, the 2015 mathematics performance showed a general decline compared to 2014, and a specific decline in respect to mathematics paper 2. The report further indicates that student's achievement levels remained on memorization, factual recall and executing well known procedures. It is also emphasised in the report that examination questions categorised as cognitively demanding, challenged students and students also struggled with concepts in the curriculum that required conceptual understanding and interpretation of information.

It is on that note, that it seems appropriate to be concerned about: How different are the cognitive demands of examinations tasks as opposed to tasks in the textbooks that support student learning on a daily basis?

1.4. THEORETICAL ASSUMPTIONS OF THE STUDY

The study is guided by the theoretical assumption that textbooks with their variations ought to depict the intended curriculum. At the centre of this assumption stands a believe system that textbooks as a form of written curriculum should systematically mirror the intended curriculum for operational purposes. It is assumed that the content and other performance measures of the textbooks should be congruent with content and other performance measures as articulated in the curriculum statement (intended curriculum). In this study specifically, the assumption is that the cognitive demands of textbook tasks and NSC assessments tasks should be in agreement and mirror the cognitive demands documented in the CAPS cognitive demands taxonomy of the curriculum statement.

1.5. OBJECTIVES AND THE RESEARCH QUESTIONS

The primary objective of this study is to examine the alignment of cognitive demand of textbooks instructional tasks and the tasks in the National Senior Certificate examinations with a particular focus on the topic trigonometric equations and the general solutions.

The study is asking the following questions:

1. What kinds of cognitive demands are implicit in the instructional tasks around the general solutions of trigonometric equations in two Grade 12 NCS (CAPS) prescribed mathematics textbooks and in the assessment tasks in the NSC examinations?

2. To what extent do the cognitive demands of tasks around the general solutions of trigonometric equations in two Grade 12 NCS (CAPS) prescribed textbooks align with the cognitive demands of the assessment tasks around the general solution of trigonometric equations in the NSC examinations?

1.6. RATIONALE AND SIGNIFICANCE

In other countries there have been studies on alignment between various mathematical content and assessment (Bhola, Impara & Buckendahl, 2003) and various mathematical content and instruction (Porter, 2002). However, little attention has been given to trigonometry content and currently there is no study that I know of in South Africa that has examined how cognitive demands of instructional tasks in the Grade 12 textbooks aligns with cognitive demands of task in the National Senior Certificate assessment items particularly with reference to trigonometry.

The FET Mathematics content in South Africa is divided into two sections making up Paper 1 and 2. Mathematics Paper 2 content includes topics such as Statistics, Analytical Geometry, Trigonometry, Euclidean Geometry and Measurement. As mentioned earlier this study pays attention to the content of Trigonometry. Trigonometry is a mathematical content area that combines algebraic, graphical and geometric reasoning (Thompson, 2008). Trigonometry is also a significant portion of the mathematics FET Paper 2 curriculum. According to the weighting of content areas as documented in CAPS, trigonometry weighs approximately 28% of the Mathematics Paper 2 curriculum. This translates to about 42

marks out of 150 marks in each of the final Mathematics Paper 2 Senior Certificate examination. The hope is that an understanding of how cognitive demands are addressed in the textbooks trigonometry tasks will give an indication as to why students' achievement levels are dominated by responses from the lower level of the cognitive demand continuum.

The data from the 2015 Mathematics Paper 2 examination report (figure 1.1) indicates consistently declining and low marks for questions relating to trigonometry (question 5, 6 and 7). The low marks may be attributed to a host of factors, including the inability of students to engage with the demands posed by trigonometry tasks. Orhun (2002) studied students' responses in solving problems in trigonometry in Turkey. She found that the students have a fragmented understanding of the concepts of trigonometry and partially answered the questions at the lower levels of cognitive demands and inadequately attempted questions at the higher levels of cognitive demands. This notion resonates with the findings of Thompson, Carlson, and Silverman (2007) and Webb (2005) that students faces difficulties reasoning about and engaging with trigonometry.

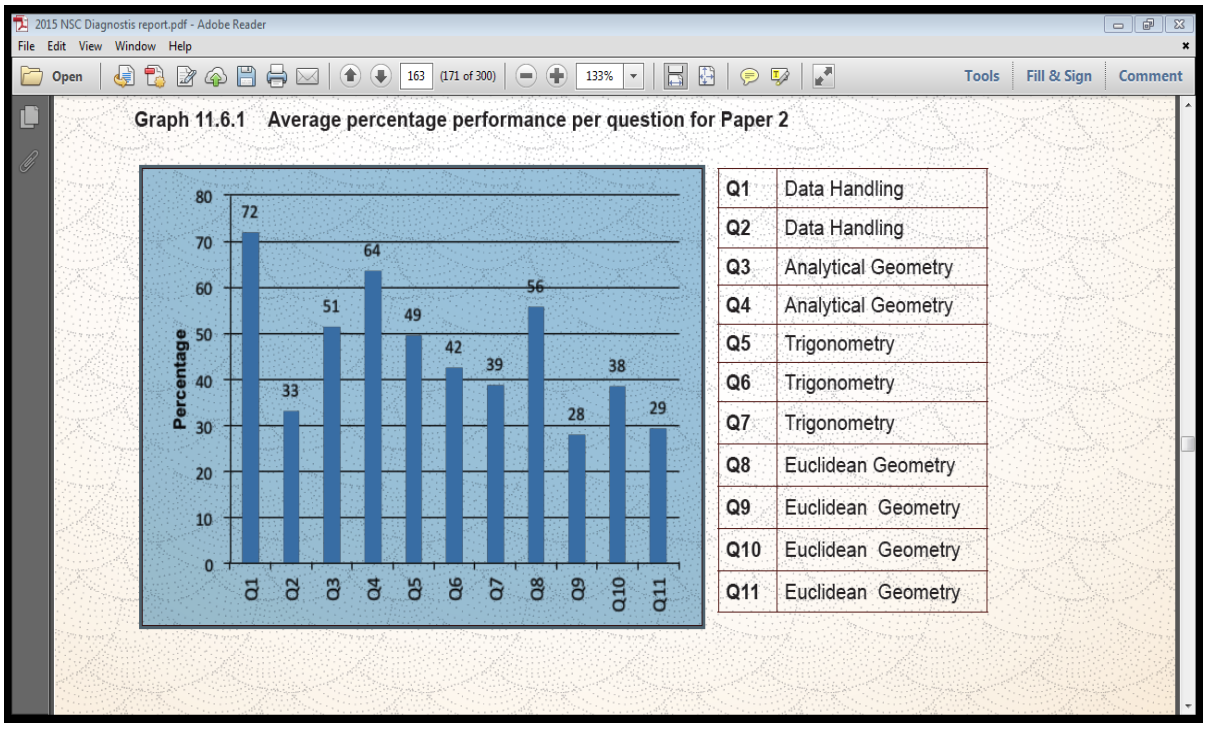


Figure 1.1: Average percentage performance per question – extracted from 2015 NSC Diagnostic report for Mathematics Paper 2.

This study aims to make a contribution to knowledge within mathematics education into how cognitive demands of textbooks trigonometry tasks align with the cognitive demands of

examination tasks. Additionally this study will provide insights into developing and improving mathematics textbooks, particularly with focus on cognitive demands. The assumption is that strong alignment within curriculum components will assist students to eliminate the cognitive gaps that are evident in trigonometry and increase achievement in Paper 2.

1.7. THE PURPOSE OF THE STUDY

The purpose of the study is to evaluate the extent to which tasks (in the prescribed textbooks and the senior certificate examination papers) provide opportunities for students to engage with a variety of cognitive demands, as proposed in the curriculum statement. The term cognitive demand of a task is used in this study to characterise the hypothetical operations required to engage with the tasks. The aim will be achieved by examining the cognitive demands of the tasks comprising exercises in the topic of trigonometric equations and the general solution of the two prescribed and commonly used textbooks in Grade 12 and the trigonometry questions in the 2014, 2015 and 2016 Mathematics Paper 2 National Senior Certificate Examinations.

1.8. OUTLINE OF THE REPORT

A total of five chapters build up this report.

Chapter ONE - Background to the study

Chapter 1 provides the background of the research and describes the statement of problem, theoretical assumptions underpinning the study, research questions and the rationale of the study.

Chapter TWO – Conceptual Framework and Literature Review

The Conceptual Framework was guided by (Anderson, 2002) triangular metaphor of curricular alignment. The contribution and benefits of curriculum alignment to learning are discussed within this chapter, followed by a discussion on different types of curriculum and the three models of alignment. A discussion on the mathematical tasks is followed by the literature on cognitive demands' taxonomies.

Chapter THREE – Research Design and Methodology

Chapter three presents the description of the research design and methodology used in the study. The chapter also describes the credibility of the study and the inter-rater reliability agreement between the raters.

Chapter FOUR – Data Analysis and Discussions

Chapter four documents the analysis of the data collected from the tasks of the two prescribed textbooks under review and the 2014, 2015 and 2016 set of NCS Mathematics paper 2 examinations. This chapter also attempts to make a comparison of the two textbooks and the NSC examinations in terms of their cognitive demands.

Chapter FIVE – Summary of Findings, Implications, Limitations and Recommendations

Chapter five describes the findings and discusses the limitations of the study. It also provides the study's implications for practice and research as well as suggestions for future research and the closing arguments.

CHAPTER 2
CONCEPTUAL FRAMEWORK AND LITERATURE REVIEW

2.1. INTRODUCTION

In an effort to explore the alignment of cognitive demands between various components of the education system, several questions immediately arise. These are questions like: Are the cognitive demands of tasks in the prescribed textbooks the same as the cognitive demands of tasks assessed in the NSC examinations? This study is guided by frameworks that address these questions.

This chapter presents a conceptual framework and the literature that guided the study. The chapter starts by discussing Anderson (2002) conceptual framework of curricular alignment. Curricular alignment is relevant to this study because it emphasises the importance of curriculum components that must cohere in order to improve student achievement. The chapter further discusses types of curricula in research and practice, theories of alignment and different alignment model used to evaluate the degree of alignment between various curricular components. Mathematical tasks and their importance to learning are also documented in this chapter. The chapter ends with the discussion of the various cognitive demand taxonomies.

2.2. CONCEPTUAL FRAMEWORK.

The study is guided by the Curricular Alignment framework of Anderson (2002).

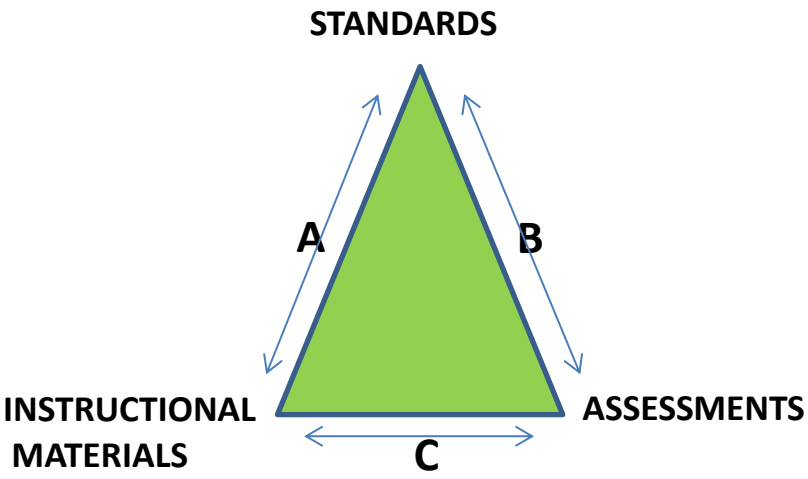


Figure 2.1: Alignment Framework – Anderson 2002

In recent years, one of the most important strategies in education reform and for improving student achievement (Cawelti, & Protheroe, 2003) has been cited as curriculum alignment (Anderson, 2002). This phenomenon is based on the notion that the components of the education system must be directed toward the same end, send a consistent message and support each other rather than pulling towards opposite ends and aiming at cross-purposes (Pellegrino, 2006). Anderson (2002) compares the process of curriculum alignment within a triangular metaphor that links standards, instructional materials and assessments (See Fig 2.1).

In Anderson (2002) triangular metaphor, the sides of the triangle represents the relationships between components of an education system: Side A represents the relationship between standards and instructional materials, Side B represents the relationship between standards with assessment and Side C represents the relationship between assessments with instructional materials. Paraphrasing Anderson (2002), curriculum alignment happens when curriculum components work together in order to have a balanced, consistent and coherent curriculum. As Porter (2002) states, "when a system is aligned, all the messages emanating from policy are consistent with each other, such that policy (curriculum statements) drive the system, and assessments together with learning materials are tightly aligned to the policy (curriculum statements)" (p. 11).

Every system has inputs, processes and outputs as the three basics that must be synchronised both internally and externally for the system to operate as intended. Within this conceptual framework, 'standards' act as the inputs, 'instruction materials' (learning support materials and teachers) are the processes and 'assessments' are determines the outputs. These components are the elements in which content and other performance criteria must be in agreement (aligned). According to Squires (2009) alignment should not only relate to the match of content but extend to the match of other features such as task coverage, variety of skills and level of difficulty.

In this study, the measure of alignment is extended to that of cognitive demands and Anderson's (2002) original triangular model of the relationship between components of the education system has been modified to replace standards by curriculum statement and assessments by NSC examinations.

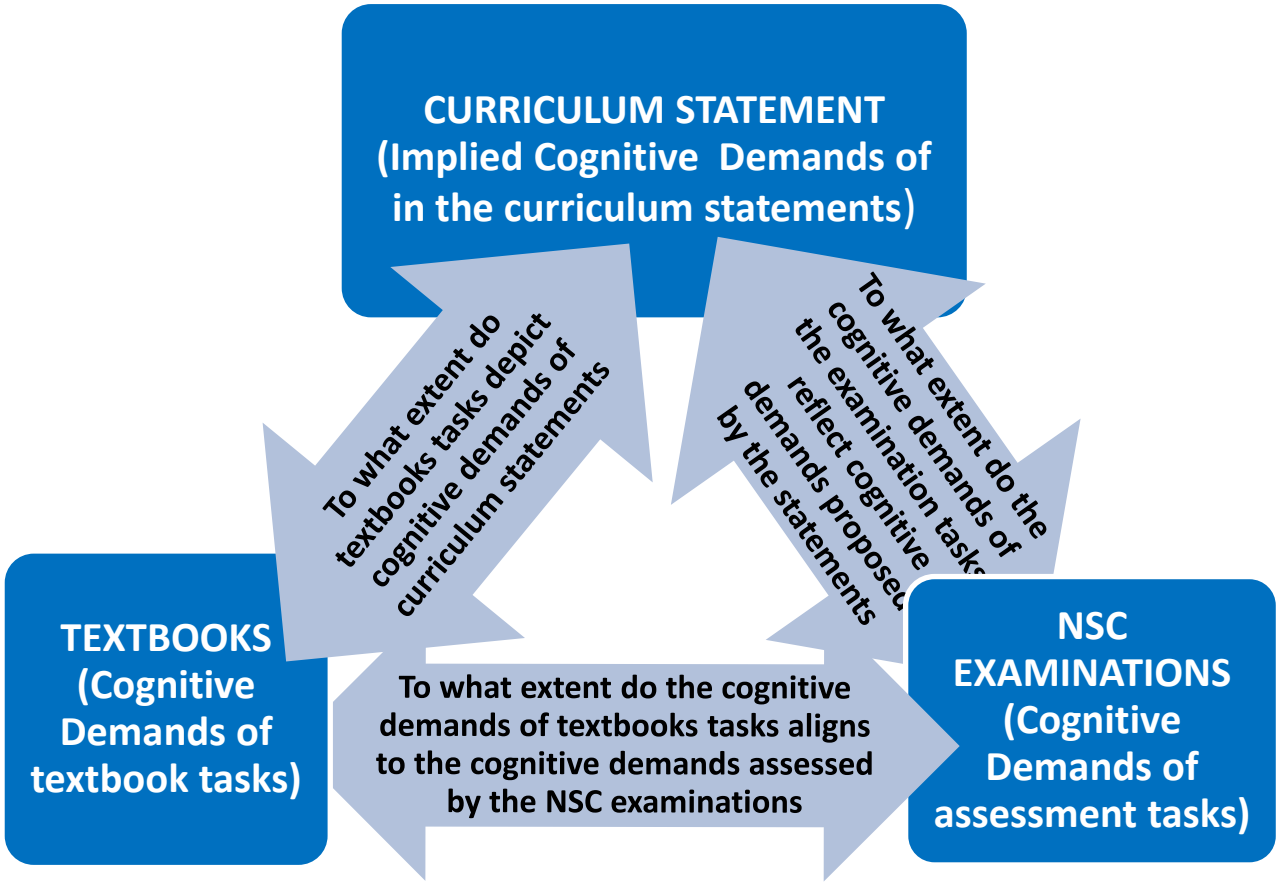


Figure 2.2: Conceptual Framework – Modified from Anderson (2002)

This modified Anderson (2002) (Fig 2.2) triangular model is used in this study to compare alignment of the vertices and sides of the triangle. The vertices of a triangle represent the components of the curriculum and the sides represent a solid link from one vertex to the other. In a triangle, the vertices join with each side to form a concrete figure. If one of the vertices is not tightly joined with the other sides, then the triangle may fall apart and even break. Similarly, if all the sides and vertices are tightly joined, then the triangle will resist a lot of turbulences and still keep together.

Similarly, in an education system, if the three curricular components (statements, textbooks and NSC examinations) are tightly joined, the system will be aligned and create 'educational opportunities' that are consistent and similar for all students. In defining an aligned educational system, La Marca, Redfield, Winter, Bailey, & Hansche, (2000) put it that:

> "Alignment is the degree to which assessments yield results that provide accurate information about student performance regarding academic standards at the desired level of detail. The assessment must adequately cover the standards with the appropriate depth,

> stresses the importance of the standards and allocate marks that covers a variety of standards," (p. 24).

Ideally, what is tested on the examinations must be derived from what is advocated in the curriculum statement, as well as what is taught in the classroom. The triangular model investigates the above assumption in terms of the cognitive demands of tasks in all the curriculum components of the education system. The model further evaluates if the cognitive demands of tasks in the prescribed textbooks depict the espoused cognitive demands of the curriculum statement; if the cognitive demands of the textbooks tasks are consistent with the cognitive demands assessed in the NSC examinations and if the cognitive demands of the NSC examinations are congruent to the cognitive demands of the curriculum statement. It is true that not all that is documented in the curriculum statement or taught to the student in class can or should be assessed. However, a theory underpinning alignment is that there must be a consistent message from all components of the curriculum. Baker (2004) points to a dilemma of alignment by saying:

> "Without a semblance of alignment, nothing hangs together. Goals may or may not be exemplified in practice, children may or may not learn what is expected, and test scores could represent standards or miss their mark entirely. Inferences about results could rarely be tightly justified, and subsequent findings may not respond to actions by students and educators" (p.5).

The above concerns remain central to issues of curricular alignment. The central issue is that, if any of the cognitive demands of any components is not well synchronised with the others, it will disturb and skew the balance of the educational system (Mhlolo, 2011) and impact student performance. To act on against such concerns, the curricula components must demonstrate that what is covered in the national examinations and what transpires in the instructional materials (textbooks) or vice versa, aligns with what the curriculum statement advocates, both in terms of content and other performance criteria. This conceptual framework brings together different types of curriculum, theories of alignment and cognitive demands taxonomies.

2.3. DEFINING CURRICULUM

The term curriculum has been used in many different ways both in practice and in literature to describe different components of a country's education system. Travers, Crosswhite, Dossey, Swafford, McKnight and Cooney (1985) described the curriculum as being tripartite in nature. According to Travers et al., (1985), the three components that makes curriculum tripartite are the 'intended' curriculum (the intent of education system), the 'implemented' curriculum (teachers and students classroom instruction and practices), and the 'attained' or 'learned' curriculum (what students have learned). Similarly, different researchers such as Venezky (1992); Valverde, Bianchi, Wolfe, Schmidt, Houang (2002); Robitaille (1993); Porter (2004) make a distinction between types of curriculum: 'intended', 'written', 'enacted' and the 'assessed' curriculum as well as the steps that each curricular undergoes before being delivered to the student. In this research, the curricular sequence will consist of the 'intended', 'written' and the 'assessed' curriculum.

Glatthorn (1999) defines the intended curriculum as the documents produced by official government bodies that specify what should be taught. Intended curriculum describes the mathematical content that students should learn at a specific grade. According to Porter (2002), the intended curriculum is defined as "statements of what every student must know and be able to do by some specified point in time" (p. 1). Travers et al. (1985) describes the intended curriculum as curriculum "reflected in curriculum guides, course outlines and syllabi" (p.3) that documents the curriculum.

The curriculum presented to students by their teachers is referred to as the enacted curriculum. Glatthorn (1999) also called this curriculum the taught curriculum and Valverde et al (2002), Tarr, Reys, Reys, Chavez, Shih and Osterlind (2008) refer to it as the implemented curriculum and the delivered curriculum (Venezky 1992). The enacted curriculum encompasses the classroom tasks, workbook exercises, lecture notes or activities that the teacher utilizes to organise teaching and to assist students in structuring their understanding of the content or topic under study.

The 'written curriculum' describes the content as it is portrayed in the textbook, instructional materials and other teaching and learning resources. This form of curriculum, is also denoted as the 'textbook curriculum' (Tarr, et al., 2008); the prescribed curriculum

(Venezky 1992), the implemented curriculum (Valverde et al, 2002); and does “not only define(s) the content to be taught but also the sequence of topics and quite often the instructional strategies to employ in teaching” (p. 439).

The assessed curriculum refers to the disciplinary knowledge upon which student learning will be measured. Forms of this type of a curriculum include national examinations such as the national senior certificate (NSC) examinations, state-mandated assessments (Annual National Assessments), provincial common examinations, district common examinations or teacher developed tests (Porter, 2002). Glatthorn (1999) also refers to the assessed curriculum as the achieved curriculum (Hirsch, Lappan, Reys, & Reys, 2005) or the tested curriculum. The outcomes of this curriculum measures student achievement against teaching and learning programmes and it is used as an accountability measure to districts and schools.

Tasks as appearing in the prescribed textbooks and the NSC examinations were selected as the unit of analysis in this study. Tasks are essential in learning because they are the means to afford the opportunities to students to demonstrate what they have learnt and to engage in thinking and reasoning. This study is attending to the cognitive demand of the tasks as presented in the textbooks and the NSC examinations without the involvement of teacher’s instruction and the learner’s practices and as such the enacted curriculum, even though very crucial form of curriculum, will be excluded in the chain since it will make the scope of the study too large.

2.4. WHY MATHEMATICAL TASKS

Mathematical tasks are an important part of learning mathematics because they foster student’s engagement with the content taught and promote the advancement of knowledge as well as the mastery of concepts taught. Tasks also serve as means of assessing student progress against the envisaged mathematical knowledge. In the words of Stein and Lane (1996) doing a task can be viewed as the “act of engaging in an activity in order to build or develop particular concepts and/or skills” (p. 52).

Hiebert, et al., (2003) defined tasks as units of learning in which individuals engage with the ideas in a problem. Stein, Grover and Henningsen (1996) modified Doyle's (1983) notion of

academic task for use in mathematics teaching and learning. They defined a mathematical task as “a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea” (p. 460). According to Hiebert and Wearne (1993), instructional tasks carries with them a potential to influence student learning and performance gains.

Stein, Grover and Henningsen (1996) support the notion that the nature of the tasks exposed to the students determines what students learn. Also, advocates of reform practices such as Stein and Lane (1996); Brodie (2008) call for the type of mathematical tasks that promote mathematical reasoning and problem solving. They argue that student engagement with the task not only determine what they learn but also how they think and draw sense out of mathematics. Tasks can take form of activities, problems to be solved, tests, etc. It is imperative that tasks should cover content taught and also include what was learned previously.

Tasks may also ask content taught in a different context for supporting the learning experiences. Horoks and Robert (2007) argue that “exposing students to a variety of tasks in the classroom enhances the learning environment because it allows for an assortment of solution methods and provides an opportunity for students to use their knowledge in a real manner” (p.281). Tasks have the ability to restrict or expand students’ understandings about the subject matter presented. It is in this light that the nature of tasks presented to students be taken into account in order to ascertain their impact to stimulate and construct the way students thinks.

In addition, Stein et al. (1996) highlight that it is imperative to evaluate the cognitive demand of mathematical tasks because of their potential influence on learning. In the video study of the TIMSS 1999, the mathematical tasks used during the lesson from participating countries were analysed for the levels cognitive demand. The study revealed that the top performing countries, of which Japan is one of, had only about 17% of their tasks at low cognitive demand levels and about 39% of tasks at higher order cognitive demand continuum. Similarly, lower performing countries such as Australia had 77% of their tasks addressing content at lower cognitive demand level while only 8% of their tasks addressed content at higher cognitive demand level (Hiebert, Gallimore, Garnier, Givvin Bogard,

Hollingsworth & Jacobs 2003). The central feature that separated mathematical performance in countries where students performed higher was the ability to retain the higher order demands of tasks during instruction (Silver, 2009).

Stein and Lane (1996) "indicate that the greatest student gains on mathematics performance assessment are related to the use of instructional tasks that engaged students in high levels of cognitive processing" (p.23). The cognitive demand of the task depends on what the task is intended for. For example, if mathematical tasks present mathematics as a collection of definitions, algorithms and procedures, then students will not need to think about mathematics or reason through the concepts (Stein et al., 1996). Similarly, if tasks that students work with require engagement with ideas through thinking and reasoning, students will have opportunities to learn mathematics through reasoning, adapting strategies, conjecturing, justifying solutions and defending each another's ideas (Stein et al., 1996).

2.5. WHY TRIGONOMETRY

Trigonometry is one of the topics of senior secondary school mathematics in South African and elsewhere in the world. According to Delice, & Roper (2006) trigonometry has an important place in the mathematics curriculum and in other disciplines. It is one of the fundamental topics in the transition to FET mathematics within the South African context. A solid understanding of trigonometric functions is also essential for the study of calculus.

Trigonometry is a mathematical content area that combines algebraic, graphical and geometric reasoning (Thompson, 2008). This multifaceted nature of trigonometry makes it somehow challenging for students to understand it. Also, the multifaceted nature of trigonometry fails to assist students to form connections between different representations or see trigonometry as an integrated part of the mathematics discipline. Even (1998); Moschkovich Schoenfield and Arcavi (1993) point out that many students show difficulty linking representations or lack the ability to move flexibly across representations and perspectives.

Weber (2005) found that most approaches to trigonometry teach only procedural skills and do not allow students to fully understand trigonometric functions. Weber (2008) also found

that much of trigonometry instruction focused on procedures and computations without an emphasis on applying the process. This greatly hinders the ability of students to make connections between representations. Moschkovich et al. (1993) emphasised proficiency instead of procedural mastery in the teaching of trigonometry. It is imperative that students must understand concepts even if they understand a procedure; otherwise the students do not gain much from the procedure (Van Dyke & White, 2004).

To address problems like this, Hirsch, Weinhold and Nichols (1991) stated that trigonometry programs needed to shift from memorization to conceptual understanding, multiple representations, mathematical modelling and problem solving. If procedural and conceptual knowledge are not linked together then students may generate an answer but not understand what they are doing (Hiebert & Lefevre, 1986). These effects are also seen during the learning of trigonometry in many classroom in South Africa, hence this study wanted to explore how the cognitive demands of the trigonometric tasks are attended to in the textbooks and NSC examinations.

2.6. ALIGNMENT

The term alignment holds many different meanings in different settings and in social sciences in general. Macmillan English dictionary defines alignment as “the position that something is in when it is straight or in the correct position in relation to other things; or the systems so that things match or fit well together” (p.34). Within a schoolwide system Tyler (1969) points to alignment as the degree to which curriculum across different grades reinforces what is learned in the preceding grades. In the context of classroom instruction, (Tyler, 1969) refers to instructional alignment as the agreement between the objectives, activities and assessments that are jointly supportive in a classroom setting.

The term ‘alignment’ as used in this study moves a mile away and looks at curricular alignment (Porter, 2002). The term curricular alignment is perceived as a phenomenon that is used to describe the link and compatibility between components of the education system. Anderson (2002); Biggs (2003); Näsström (2008a) & Webb (1997) have pointed to curricular alignment as the most effective way of promoting student achievement. According to Bhola, Impara and Buckendahl (2003) and Näsström (2008) curricular alignment is defined as the extent of match or measure of consistency between standards and assessment to measure

achievement of those standards. Webb (1997) refers to 'alignment' as occurring when policy and assessments are in agreement with one another to direct the system towards students learning what they are supposed to learn. Curriculum is considered aligned if the policy, instruction and assessment are all in agreement with each other; aims at the same goals for learning and work together to support student achievement (Mhlolo & Venkat, 2009). The term alignment in this study is used to refer to the degree of agreement of the cognitive demands between the curriculum statements (CAPS), written curriculum (in the form of textbooks) and the assessed curriculum.

Alignment between policy, instruction and assessment is important for the efficiency of an educational system (Webb, 1997) , students learning (Anderson, 2002), and evaluation of educational reforms effort (Ensor, 2001). Curriculum systems with high degree of agreement ensure that most of the advocated standards are included in the assessment. This will reduce imbalances between the anticipated knowledge in the standards and the assessed knowledge in the assessment items. Another added advantage of curriculum with high degree of agreement is that it has high chances of influencing instruction (teaching and textbooks) to cover all standards. As such the instruction (teaching and textbooks) will be in more agreement with the standards and assessment. If a curriculum is aligned, the education system will be well-organised and the outcomes from the assessment will give a true reflection of how adequately students attained the anticipated knowledge and skills. This will provide a perfect measure for accountability purposes.

On the other hand, a curriculum system with low degree of agreements results in curriculum components that are operating at opposite ends. The link between these curriculum components becomes weak and may break. The components maybe insulated from each other thereby emphasising different knowledge and skills. This may result in conflicting messages sent to the students about what they are supposed to learn. As such, it may imply that the fragmented pieces of the standards might be included in the assessment. The result of this may have dire consequences for the entire educational system, resulting in reduced opportunities for the students to attain all expected standards.

Edwards (2010) also investigated the levels of cognitive demand and coherence between the South African Physical Sciences Curriculum for Grade 12 and the 2008 Senior Certificate Physical Sciences exemplar papers and the 2008 and 2009 Physical Sciences Senior Certificate Examination papers. The results of the study revealed that the focus in each curriculum was on lower order cognitive and process skills.

TIMSS bench mark assessments are conducted on a four yearly bases for the Grade 4 and 8. South Africa previously participated in this assessments and its performance has always been a major concern. Ndlovu and Mji (2012) conducted a study investigating degree of alignment between the South Africa's RNCS assessments standards for Grade 8 Mathematics and the TIMSS 2003 Grade 8 Mathematics assessment framework. The study used Porter alignment index to calculate the level of match between the Grade 8 RNCS assessment standards and the TIMSS assessment objectives. The study reported an alignment of 0.751, which was interpreted to be low. This low alignment index suggested a gap and misalignment between the Grade 8 mathematics curriculum and the TIMSS assessment framework in terms of cognitive demands level descriptors.

In alignment evaluations, cognitive demand of an item should be at the same level with the cognitive demand required by the standard for which the item is designed to assess. In previous research on assessment of cognitive demand and alignment of standards, Chisholm (2000); Muller (2004; 2005); Umalusi (2009) raised concerns about the level of cognitive demand that were deteriorating in the examination papers while the pass rates were increasing. Misalignment of the curriculum was pointed to as a key issue within this problem and this necessitated that levels of cognitive demand needed to be investigated between various components of the South African curriculum.

In another study, Alcazar (2007) investigated the degree of alignment of cognitive demand between the Peruvian official curriculum, the national assessments, teaching and the approved textbook. The study found lower cognitive demands among the official curriculum and the approved textbooks, while the cognitive demands of the national assessment tasks were more aligned to categories of Problem Solving and Comprehension. This investigation revealed that there was relative alignment between curriculum components within the Peruvian education system.

2.7. ALIGNMENT MODELS

Alignment studies between standards and assessment focus on three models to evaluate the degree of alignment. The models were revised by Bhola et al. (2003) in terms of their level of difficulty. These models evaluate the degree to which the components of education systems send an unambiguous and consistent message about what to teach and what to assess. The most commonly used alignment models are the 'Webb Model', the 'Achieve Model' and the 'Surveys of Enacted Curriculum (SEC) Model'.

2.7.1. Webb Model

In his alignment study of four USA states, Webb (1999) developed a comprehensive model to evaluate the degree of agreement between assessments and standards. The Webb Model explores five dimensions to understand the degree of agreement and alignment. The five dimensions are 'content focus', 'articulation', 'equity and fairness', 'pedagogical implications', and 'system applicability' (Webb, 1997). Of these five dimensions, 'content focus' has been used in alignment studies to compare content between different curriculum components. The 'content focus' dimension uses four indicators to explore the connections between the assessment and policy in a varied way. The four indicators are 'categorical concurrence', 'depth of knowledge consistency', 'range of knowledge correspondence', and 'balance of representation'. These indicators are discussed in depth below.

- Categorical concurrence measures the similarity of content in the standards and assessments. The indicator of categorical concurrence is achieved when similar or constant sets of content appear in both the assessment and the standards. To expand on this notion, if factorisation is one of the content standards in mathematics, then does the assessment item have questions that target factorisation?
- Depth of knowledge (DOK) compares the uniformity between the cognitive demands of the standards and cognitive demands of assessments. The levels associated with DOK are: Recall - Level 1 - which is associated with the recall of a fact, definition, term, or a simple procedure, Skill or Concept - Level 2 - comprises the use of information or conceptual knowledge in solving a task. Strategic Thinking - Level 3 - requires thinking; constructing a plan or steps in solving a task. Extended Thinking -

Level 4 requires exploration of the problem and time to reflect and formulate numerous conditions of the problem.

- Range of knowledge examines the number of objectives within a standard assessed by at least one assessment item. The primary criterion is that what is assessed should be at the same cognitive level or above the same cognitive level as what is espoused in the standards. To achieve this alignment criterion, about 50% of the items matched to the standard must be at the same cognitive level or above the cognitive level of that standard (Webb, 2002). This assumption is based on the view that students should be assessed on at least half of the knowledge contained in the standard. This part of the "alignment process also assumes that all of the objectives have equal weighting and all of the objectives accurately cover the skills needed to complete that standard" (Martone & Sireci, 2009, p. 1339).
- Balance of representation concentrates on the extent to which items are evenly distributed within a standard and compares the degree to which certain standards have been given more emphasis on an assessment than others.

2.7.2. Achieve Model

The Achieve model has been developed to compare alignment of state's standards to those of other states or nations. The Achieve model applies five criteria for alignment: 'content centrality', 'performance centrality', 'challenge', 'balance', and 'range' (CCSSO, 2002).

- Content centrality matches how well the content of each assessment item corresponds to the content of the standard.
- Performance centrality looks at how well the cognitive demand of each assessment item matches the cognitive demand of the standard.
- Challenge compares the extent to which the set of items have varied level of difficulty that is both matched to the level of difficulty of the standards and appropriateness (age, grade) for the target students.
- Balance and range outline a quantitative and qualitative emphasis of topics in the assessment compared to the emphasis placed on the same topics in the standards.

2.7.3. Surveys of Enacted Curriculum (SEC) Model

Porter and Smithson (2001) developed the SEC alignment model to categorise the standards and assessments according to content topics and cognitive demand. In simple terms, the

SEC alignment models assist those involved in education to understand the relationship between the standards developed by the state, the instruction in the classroom and the content tested in the examinations. SEC model of alignment encompasses two alignment pointers. The two alignment pointers are content match and expectations for student performance (cognitive demand) (Porter & Smithson, 2001). The SEC model uses a content matrix of two dimensions: content topic and expectations for student performance (CCSSO, 2002). Content match pointer lists topics of content that are specific to each subject area, while the expectations for student performance describes five levels of cognitive demand or expectations for student performance. The five levels are "memorize," "perform procedures," "communicate understanding," "generalize/prove," and "solve non-routine problems." (Porter, 2002)

This categorization of content and cognitive demands yields a matrix that permits a comparison of the standards and assessments of different curriculum components. To evaluate alignment using the SEC model, reviewers (four in a usual case) categorise the content of the standards and assessments according to content topic and cognitive demand. Once the categorisation has been accomplished, the degree of alignment between the standards and assessments can be calculated using statistical calculations. The SEC alignment model can also be modified to evaluate other elements of an education system, including classroom instruction.

2.8. COGNITIVE DEMANDS AND TAXONOMIES

Mathematical tasks engage students in a mathematical activity. A distinguishable variation between tasks is the degree of cognitive effort that must be exerted in order to successfully engage with the task. This implies that each task require different levels and kinds of thinking (Stein et al., 2009) to engage successfully with. To expand what is meant by this notion, several interpretations found in the mathematics education literature that characterise the variation in the types of thinking necessary to engage with a mathematical task are presented .

This variation in the types of thinking needed to engage with a task is referred to as cognitive demand. The term cognitive demand of a task refers to "the cognitive processes students are required to use in accomplishing the task" (Doyle, 1988, p. 170). Stein et al

(2000) defined cognitive demand as “the kind and level of thinking required of students in order to successfully engage with and solve the task” (p. 11).

Doyle (1983) and Stein et al., (1996) use the term cognitive demand to characterise the thinking that students need to solve a particular mathematical task. Over the last many years, refined and complex methods and tools to measure cognitive demands have emerged within and across various educational disciplines. However, despite all the efforts, cognitive demand still remains difficult to consistently measure in practice (Schraw & Robinson, 2011). Furst (1981) also highlight that to date there is no single “all-inclusive, all-purpose tool” to precisely measure cognitive demand (p. 451).

Brändström (2005) defines taxonomies as classification schemes according to a predetermined system and discuss their frequent use in education to measure the difficulty levels of mathematical tasks. A review of literature on cognitive demand models reveals a number of taxonomies that have been useful in categorising both content and cognitive demand. Their usefulness may-be associated with guiding teaching and assessments (CAPS, 2012), classroom tasks (Stein et al., 1996), framework against which tests may be constructed (TIMSS, 1999, 2003) or alignment of curriculum with assessment (Porter, 2002). According to Crowe (2012), taxonomies of cognitive demands vary and all taxonomies tell their own story. Berger, Bowie, & Nyaumwe (2010) also point out that taxonomies have different purposes and some purposes may-be associated with the cognitive demands of mathematical tasks.

In the educational literature there are a number of different taxonomies. Among these are the Bloom revised taxonomy (Anderson & Krathwohl, 2001); A New Taxonomy of Educational Objectives (Marzano, 2001); Webb’s Depth of Knowledge categories (Webb, 1997); Surveys of Enacted Curriculum (SEC) model (Porter, 2002). The discussions on standards indicate that standards comprise content and performance standards (Hambleton, 2001).

Anderson & Krathwohl (2001) revised Bloom’s (1956) taxonomy to reflect the cognitive demands reflected in the task in general. In the taxonomy Anderson & Krathwohl (2001), content is defined as different kinds of knowledge: i.e. as factual, conceptual, procedural

and metacognitive. These categories lie in a scale from “concrete in factual knowledge to abstract in metacognitive knowledge” (p.182). The cognitive demand dimensions categorises the cognitive demands of the task into remembering, understanding, applying, analysing, evaluating and creating. The underlying phenomenon in the cognitive demand dimension is the cognitive process required to solve the task, ranging from little cognitive process in remember to the most cognitive process in creating.

Marzano (2001) taxonomy categorises cognitive demand from assessment questions. The taxonomy comprised six levels of mental processing and thinking: ‘retrieval process’ ‘comprehension’, ‘analysis’, ‘knowledge utilization’, ‘metacognition’, and ‘self-system thinking’. The basis of Marzano’s taxonomy is the realisation of how to process thinking and applies to educational discourse rather than identification of cognitive level of assessment questions.

Stein et al (2000) cognitive demand taxonomy defined four levels of cognitive demand of mathematical tasks. The four levels of cognitive demands are memorization, procedures without connections, procedures with connections, and doing mathematics. Memorization and procedures without connections are both categorized as lower level cognitive demand, while procedures with connections and doing mathematics are categorized as higher-level cognitive demand. The findings of the study revealed that student who performed well in problem solving and reasoning tasks were more likely those using tasks at high levels of cognitive demands.

The selection of suitable cognitive demand taxonomy depends on how the result of the study will be used for. In the study concerned with determining the alignment between Mathematics and English high school assessments and higher education expectations in USA, Brown and Conley (2007) uses data gathered from (Marzano, 2001) cognitive demand taxonomy to characterise cognitive processes crucial for higher education studies. Paterson (2003) used Anderson and Krathwohl's (2001) two-dimensional framework of cognitive demand to determine whether the verbs used during the learning and teaching of mathematics could categorise levels of cognitive demand necessary to engage with online assessment system.

Larson (2003) her study of the alignment between science tests and standards used a modification of Webb's Depth of Knowledge categories (Webb, 1997) and the Bloom's Taxonomy of Educational Objectives (Bloom et al., 1956) to highlight the cognitive demand of science tests and science standards developed for USA students. The present study adopts Porter's (2002) cognitive demand taxonomy because it is a strong alignment model for instruction to standards, instruction to assessment, and standards to assessment (Blank, 2004) and because this study measure the alignment of cognitive demands of instruction (textbooks) to assessments.

Porter (2002) developed a two-dimensional framework for aligning standards, instruction and assessments. One part of the framework categorises the content based upon the topics presented. The second part of the framework reports the level of cognitive demand of the topics presented. Cognitive demands as described earlier distinguish "memorization; perform procedures; communicate understanding of concepts; solve non-routine problems; and conjecture, generalise, and prove." (p. 7).

Memorization involves reproducing previously learned facts, rules, formulae, or definitions. Memorization tasks do not require any explanation; they are straightforward rules and learners use well-rehearsed facts to solve the tasks. Perform procedures require reproduction of a procedure and following of procedures or instructions to solve familiar and routine problems but without connection to underlying concepts. Such tasks are focused on producing correct solutions rather than developing mathematical understanding. Communicate understanding of concepts focus attention on the use of math ideas to develop relationships between concepts. Solve non-routine problems deals with mathematics for the purpose of developing deeper levels of understanding of mathematics. Such tasks focus on applying and adapting a variety of strategies to solve non-routine problems in a meaningful way. Conjecture, generalise, and prove does not require any procedure to be followed. There is no rehearsed way of solving the tasks. Tasks of such nature employ more than one way of engaging with them. The foci of these tasks are on analysing the task in order to generate a pattern or a general rule for engaging with them.

In addition to producing an alignment analysis of different components of the education system (standards, assessments and instruction), alignment statistics can also be computed

from the matrices to demonstrate the differences and similarities between standards, assessments, and instruction. This approach allows raters to create matrices of proportions for standards, instruction and assessments. These matrices are then compared, cell by cell, to examine the relationships between standards, instruction and assessments. The alignment index is then calculated to indicate the percentage of content in common between the matrices compared. Porter's model, and the way that it is applied in my study, is further discussed and demonstrated in Chapter 4.

2.9. CONCLUSION

The literature reviewed in this chapter provides the foundation for this study. The review illustrates the importance of the role of alignment between standards, textbooks and examinations using the conceptual framework by Anderson (2002). To summarize, the review discussed the types of curriculum explored in this study. The review also included information about the role of curricular alignment and the primary models of alignment as well as the techniques used by each model for evaluating the degree of alignment between standards and assessments. Additionally, some of the cognitive demand taxonomies that guide the allocation of cognitive demands to task were also reviewed.

CHAPTER 3
RESEARCH DESIGN AND METHODOLOGY

3.1. INTRODUCTION

This chapter presents the research design and the methods that underpinned this study. The data for this study was obtained from different documentary sources. The documents that were analysed in detail with foci on the topic of trigonometric equations and the general solution are the two prescribed Grade 12 mathematics textbook and the 2014, 2015 and 2016 Mathematics NSC paper 2 examinations. The FET mathematics curriculum statement (NCS Grade R – 12, CAPS) that is used to guide teaching, learning and assessment at all levels within the South Africa context is also referred to in the data analysis and the discussion of results. This study is interested in the alignment of cognitive demands as a specific phenomenon (alignment of cognitive demands). It aims at understanding how policy translates into practice within South African mathematics curriculum documents

3.2 RESEARCH PARADIGM

Wahyuni (2012) defines a research paradigm as a "set of fundamental assumptions and beliefs as to how the world is perceived which then serves as a thinking framework that guides the behaviour of the researcher" (p.69). Joubish (2011), reports that a research paradigm is "essentially a worldview, a whole framework of beliefs, values and methods with which research takes place" (p. 2083). Paraphrasing this definition, Terre Blanche and Durrkein (2006) refer to the research paradigm as "perspectives that provide a rationale for the research and commit the researcher to particular methods of data collection, observation and interpretation" (p.40).

The main three types of paradigms are positivism, interpretivist and critical. This study is located within the interpretivist paradigm since it entails gaining deeper insight into the situation (alignment of cognitive demands of task between different curriculum documents) and making sense of the situation by interpreting how different curriculum documents mediate and advance the cognitive demands prescribed by policy statements.

3.3. RESEARCH APPROACH

The specific interest of the study was the investigation of the alignment of cognitive demands between textbooks tasks and NSC examination questions with respect to the topic of trigonometric equations and the general solution within the South African FET Mathematics Curriculum. Porter's (2002) framework of cognitive demands, as previously discussed, was employed to interpret and to give voice and meaning to the cognitive demands of the tasks examined.

The study employed a qualitative approach to give judgements to the quantitative statistics (Alignment Index) of the data counts of cognitive demands of trigonometric tasks within the different types of the curriculum documents. Strauss and Corbin (1990) broadly define qualitative research as "any kind of research that produces findings not arrived at by means of statistical procedures or other means of quantification" (p. 17)

3.4. WHY CASE STUDY

The method of investigation of this study is a case study. Stake (1995) noted that a case study is intensive research in which interpretations are given "based on observable concrete properties and subjects within an actual setting" (p. 95). According to Opie (2004) a case study can be "viewed as an in depth study of interactions of a single instance in a closed system" (p.74). Gay and Airasian (2000) define case study research "as research in which the researcher focuses on a unit of study known as a bounded system" (p. 426). Merriam (1988) upholds the idea that the "single most defining characteristic of case study research lies in demarcating the object of study: the case" (p. 27). The case may be a unit, entity, or phenomenon that the researcher can demarcate.

These explanations resonate with the research methods of this study. The focus of the study is to explore the alignment of cognitive demands of tasks within the topic of trigonometric equations and the general solution between the various components of the South African mathematics FET curriculum. My main interest as the researcher was to explore if the cognitive demands documented in the CAPS document are expressed consistently across the other curriculum documents (textbooks and NSC examination) that actualise the mathematics curriculum. One section of trigonometry is a case in this study, observed at

micro level (Rowley, 2002), which is Grade 12 , in a closed and bounded system which is a South African.

Stake (2000) distinguishes three types of case studies: the intrinsic, instrumental and the collective. An intrinsic case study is carried out when a researcher wants to examine a specific problem of an individual case. In an instrumental case study, the researcher examines a small group of subjects in order to get insight into a certain phenomenon. Collective case study involves the researcher synchronising data from several different sources, such as schools or individuals to allow for some generalisation of findings. The type of case study used in this study is an instrumental case study (Bertram & Christiansen, 2004); its purpose is to explore a specific issue (i.e. the alignment of cognitive demands) in order to gain insight into how cognitive demands of tasks aligns across different South African FET mathematics curricula.

Yin (1984) notes three categories of case studies: exploratory, descriptive and explanatory case studies. Paraphrasing Yin (1984), the exploratory case studies aims to explore a phenomenon in the data which the researcher has interest in. The descriptive case study describes the natural phenomena which ensue within the data in question. On the other hand, the explanatory case studies examine the data closely both at a surface and deep level in order to explain the phenomena in the data. The research design for this study is best characterised as explorative since the data analysis explored one phenomenon (alignment of cognitive demands), which was the point of interest to the researcher. As the researcher, I wanted to explore and understand in depth, how cognitive demands of tasks in the topic of trigonometric equations and the general solution aligns across the various components of the South African mathematics curricula representing the teaching and learning of the topic under review.

Case study has always been condemned for its lack of rigour and the inclination of a researcher to have a biased interpretation of the data. Yin (1984) notes that “too many times the case study investigator has allowed equivocal evidence or biased views to influence the direction of the findings and conclusions” (p.21). Due to the small number of subjects or small sample size, case studies often provide very little basis for scientific generalisation. Cresswell (2003) indicate that the dependency of case study on a single case

exploration makes it difficult to reach a generalising conclusion. Similarly, the results obtained in this case study cannot be generalised since the two textbooks analysed are not representative of the entire content of Grade 12 mathematics textbooks in South Africa nor of all prescribed textbooks. However, of importance is that the depth of description in the study suggests that similar scenario with other prescribed textbooks and other topics are worth exploring. That is, the study may be 'transferable' to other similar scenarios within the South African context. But this possibility of transferability requires further research.

3.5. DATA SOURCES

The 2014, 2015 and 2016 NSC mathematics examinations paper 2 (assessed curriculum) and the two Grade 12 Mathematics prescribed textbooks (written curriculum) are the sources of data that were analysed. The intended curriculum (Mathematic s FET NCS Grade R – 12 CAPS) comprises the Curriculum and Assessment Policy Statements (CAPS) for all approved subjects; the National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R-12 and National Protocol for Assessment Grades R-12. In the context of the NCS these three policy documents complement each other in articulating the content, knowledge and skills to be transmitted and how they must be transmitted (Morais & Neves, 2001). The National policy pertaining to the programme and promotion requirements of the NCS Grades R-12 and the National protocol for assessment Grades R-12 are not considered in this study as they focus more on strategies pertaining to assessment and promotion whereas the foci of my analysis is on cognitive demand levels of the task.

The textbook (written curriculum) is an interpretation of the curriculum statement (intended curriculum) and textbooks tasks were analysed to evaluate whether the intended and the written curriculum were coherent with respect to the cognitive demand levels. The two textbooks analysed were selected from the list of approved textbooks for Grade 12 Mathematics from the Department of Education. This list consists of 11 leaner's books and teacher's guide which have been published by 10 different publishers0. The study was conducted in Gauteng and the two textbooks that were chosen are mathematics textbooks that Gauteng Department of Education recommend to schools as a form of Learner and Teaching Support Materials (LTSM) and one of these textbooks in particular (Platinum

Mathematics) is distributed to most public and no-fee paying schools within the province. The two books are described in chapters 4. They are:

> Platinum Mathematics Grade 12 (M. Bradley; J. Campbell; S. McPetrie, 2012), pages (209-237).
>
> Classroom Mathematics (M. Pike; A. Jawurek; A. Kitto; P. Laridon; M. Myburgh; R. Rhodes-Houghton; M. Sasman; J. Scheiber; S. Tebeila and H. Wilson. 2012) pages (151-169).

As mentioned earlier, the specific interest of the study is the investigation of the alignment of cognitive demands between textbooks tasks and NSC examination questions with respect to the topic of trigonometry within the South African FET Mathematics Curriculum. However, due to the practical concerns relating to the unit of analysis in this study (i.e. each relevant trigonometry task in each of the textbooks and examination question papers) which would result in a very large scope, the study was adjusted to focus only on one topic of trigonometry: **trigonometric equations and the general solutions.** Therefore only the general solutions tasks from each of the textbooks were examined in this study together with relevant questions from the 2014, 2015 and the 2016 National Senior Certificate Examinations.

Tasks in the NCS examination papers were also analysed per question relating to trigonometric equations and the general solution to validate adherence of the assessed curriculum to the intended curriculum in terms of the cognitive demands levels prescribed.

3.6. DATA COLLECTION METHODS

Webb (1997) suggested three major methods of evaluating the degree of alignment between the curriculum documents. The methods are ‘sequential development’, ‘expert review’, and ‘documents analyses’. Sequential development is associated with the construction and approval of curriculum documents, which successively serve as a design for the formation of successive curriculum materials. The method of expert review involves the panel of content experts to analyse the curriculum documents and evaluate the extent of their agreement. Document analysis comprises the coding and analysis of existing

curriculum documents that characterise the various types of curricular materials. Document analysis was adopted as the most appropriate and useful approach to answer the research questions that were conceptualised and posed.

Caulley (1983) defines a document as any archived source of information that can be official, unofficial or informal or as any type of source for the purpose of gathering facts. According to Roach, Niebling and Kurz (2008), document analysis entails the analysis and coding of documents that represent the various policy documents. Witkin and Altschuld (1995) define document analysis as content analysis that involves the processes of tracing and examining facts or trends in already existing documents. This kind of analysis was used in this study to elicit interpretations about the degree to which curriculum documents are likely to be in agreement with the objectives set in the curriculum. In the words of Bryman (1989), document analysis is viewed as another valuable way of highlighting the gap between official policy and practice.

3.7. ANALYTIC TOOLS

In studies of alignment, researchers Webb (1997), Porter & Smithson (2001), Porter (2002) and Council of Chief State School Officers (2002) developed alignment models which use data collection methods discussed earlier to enable more refined alignment analysis. The most commonly used models are the Webb Model, Achieve Model and the Surveys of Enacted Curriculum (SEC) Model.

The SEC model by Porter (2002) was chosen as a suitable analysis tool for my study for three reasons. Firstly, the SEC tool has been successfully used in at least 17 states internationally, including major projects in at least 7 school districts in the USA (Roach, et al., 2008). This international usage also contributes to the credibility of the SEC model and the Porter (2002) tool. Secondly, Porter's (2002) SEC model uses understandable and relevant to mathematics descriptors to code the cognitive demands of the mathematics tasks. Thirdly, the SEC methodology employs a common language framework for examining the degree of coherence between curriculum documents. The key feature of a common language framework is that content within different curriculum components is described with a common language. This allows for direct comparisons between the curriculum, instruction and assessments elements (Porter, 2002).

The original framework by Porter (2002) does not require any modifications to fit my study, since its use has been thoroughly described by the author, providing both theoretical information and examples of analysing mathematical tasks. The Porter (2002) cognitive demand framework is outlined below.

Table 3.1: Porter's taxonomy – Adapted from Porter (2002)

	Cognitive Demand Dimension				
	Lower Order Skills		**Higher Order Skills**		
	A	**B**	**C**	**D**	**E**
	Memorization	**Perform procedures**	**Communicate understanding of concepts**	**Solve non-routine problems**	**Conjecture, generalize, prove**
Topic Dimension	Recite basic math facts. Recall math terms and definitions or formulae and computational procedures.	Use numbers to count, order, denote. Computational procedures or algorithms. Follow procedures/instructions. Solve equations/formulae/ routine word problems. Organise or display data. Read or produce graphs or tables. Execute geometric constructions	Communicate math ideas. Use representations to model math ideas Explain finding and results from data analysis strategies. Develop relationships between concepts. Show or explain relationships between models, diagrams or other representations	Apply and adapt a variety of appropriate strategies to solve non-routine problems. Apply mathematics in contexts outside of maths. Analyse data, recognise patterns. Synthesise content and ideas from several sources.	Determine the truth of a math pattern or proposition. Write formal or informal proofs. Recognise, generate or create patterns. Find a mathematical rule to generate a pattern or number Identify faulty arguments. Reason inductively or deductively.

As can be seen in Table 3.1, the levels of cognitive demands of task in the framework distinguish the following categories: memorization, perform procedures, communicate understanding, solve non-routine problems and conjecture, generalise and prove. As

outlined in the literature review section, Porter's (2002) does not provide distinctions between lower and higher order cognitive demands. However using descriptors from the literature on lower and higher order cognitive demands, it was not difficult to associate Porter's (2002) descriptors given in his taxonomy of cognitive demands with those proposed for lower and higher order.

In the analysis, a record was created in the form of codes with descriptions as mentioned earlier to illustrate cognitive demands judged necessary to engage with each task. Using the cognitive demand descriptors in the Porter (2002) tool, each of the trigonometric equations and the general solution tasks in the prescribed Grade 12 textbooks and NSC examination papers were analysed and placed into corresponding categories (A, B, C, D and E) in terms of the cognitive demand they purport. This is illustrated in Chapter 4.

The analysis then moved on to find out how aligned the two prescribed Grade 12 textbooks and NSC examinations tasks are with respect to cognitive demand. The alignment was based on a highly specialised technique to compute alignment. The details of the technique are described in the next chapter (Chapter 4).

3.8. TRUSTWORTHINESS OF THE STUDY

Guba and Lincoln (1994) emphasise that the trustworthiness of any study depends on issues of reliability and validity. However, qualitative researchers such as Guba and Lincoln (1994), and Golafshani (2003) agree that reliability and validity as are described in quantitative research are different and inadequate in qualitative research. Similarly, Golafshani (2003) highlights that although reliability and validity are essential tools for quality in quantitative research; these concepts are viewed differently in qualitative research. In qualitative paradigms, terminology such as credibility, transferability and confirmability provides alternative descriptions to the terms reliability and validity.

3.8.1. Credibility

Qualitative research is interpretive in nature and to increase credibility, the researcher must ensure that the results of the study are believable. To enhance credibility and quality in this study, triangulation was used as a strategy. According to Knafl and Breitmeyer (1989), a

number of triangulation exists: triangulation of data methods, triangulation of data sources, theoretical triangulation and triangulation of investigators.

The data collected in this study was subjected to triangulation of researchers as a reliability measure to enhance credibility. Triangulation of researchers in this study involved the primary researcher (myself) discussing the concepts under investigation and the data coding process (coding symbols, and the coding instrument) with other raters who have experience with this qualitative approach. One of the experts serves as an external moderator for National Senior Certificate mathematics paper 2 and another reviewer holds a PhD in Mathematics Education. With the researcher this made a total of three raters. According to Lombardi, Seburn, Conley and Snow (2010), on average alignment studies use between three and ten expert raters to rate and code content and the error ascribed to these rating should be as small as possible

In the triangulation of researchers' process, we adopted the deductive approach, in which the cognitive demands taxonomy and framework by Porter (2002) was used as base knowledge to classify the cognitive demands represented in the textbook tasks and the NSC examination questions. In the words of Elo and Kyngäs (2008), deductive analysis is used when the structure of analysis is operationalised on the basis of previous knowledge. All the three raters independently conducted a pilot coding of cognitive demands in a randomly selected set of tasks in the textbooks and all the tasks in the assessment items of the NSC examination within the topic under investigation. In the pilot coding process, each rater recorded the cognitive demands (Annexure L) present in the tasks so that we could find discrepancies easily. It was found there were some discrepancies regarding which specific cognitive demands some questions and tasks could be placed in. Such discrepancies were settled through discussion in which, inter alia, we agreed that tasks involving two cognitive demand levels will be coded with the highest cognitive demand.

After the discrepancies were settled through discussion, each rater then independently categorised all the questions and tasks within the NSC examination papers and chapters in the textbooks. In order to limit 'chance agreement' and to reach acceptable reliability levels of coding we agreed on using the inter-rater reliability measure. Inter-coder reliability evaluates the degree to which coding's of content by multiple raters are similar. Zorin

(2011) point out that inter-rater reliability is associated with the extent to which the coding for the content is consistent across different coders. According to Miles & Huberman (1994); Boyatzis (1998) with inter-coder reliability, the more coders agree on the coding of a text, the more we can consider the codebook a reliable instrument.

After coding the entire tasks in the NSC examinations and the textbooks, we then met again to compare and reconcile coding (using Annexure M), code definitions and to evaluate inter-coder reliability. This study used Holsti's (1969) method among the many reliability measures (Krippendorff, 1980); (Carey, Morgan & Oxtoby, 1996) listed in the literature to assess to what degree a set of texts were consistently coded by different raters. Holsti's (1969) method uses the following reliability formula:

$$\text{Reliability} = \frac{\mathbf{2M}}{\mathbf{Na+Nb}} \text{ where,}$$

2M is the number of coding decisions agreed upon, and **Na** and **Nb** represent the total number of coding decisions made by the raters. The Holsti (1969) measure yield coefficient measures between 0.00 (no agreement) and 1.00 (total agreement). Landis and Koch, (1977) proposed the following conventions of agreement:

Table 3.2.: - Adapted from Landis and Koch (1977) conventions of agreement:

co-efficient range	level of agreement between raters
0.81 – 1.00	almost perfect agreement
0.61 – 0.80	substantial agreement
0.41 – 0.60	moderate agreement
0.21 – 0.40	fair agreement
0.00 – 0.20	slight agreement
< 0.00	poor or no agreement

Researchers, Lombard, Snyder-Duch and Bracken, (2002) list coefficient measure of 0.80 as an acceptable level of agreement and 0.90 as perfect level of agreement. Lombard et al (2002) inter-coder reliability measure of 0.80 or higher was deemed acceptable for this study since it indicates agreements levels that are almost perfect. A breakdown of the reliability measures per each curriculum document analysed is presented in Table 3.3 showing the level of agreement between raters per curriculum document.

Table 3.3.: - Reliability Measures per curriculum document

Curriculum Document	Total Tasks	Total Coding Decisions	Total Agreed Decisions	Reliability Measures per Document
Platinum Mathematics	**38**	**114**	**102**	**0.89**
Classroom Mathematics	**60**	**180**	**162**	**0.9**
NSC Examinations	**7**	**21**	**20**	**0.95**
TOTAL	**105**	**315**	**284**	**0.90**

A total of 105 tasks from the textbooks and the NSC examinations, containing 315 coding's by the raters were used for check-coding in the inter-coder reliability process. 284 of these total tasks represent the number of coding decisions agreed upon. Using Holsti's formula, the overall reliability measure of 0.90 was obtained in this study. This represents a perfect agreement level of inter-coder reliability according to Lombard et al. (2008) and Landis et al. (1977).

The coding of the cognitive demands of textbook tasks was relatively easy considering the fact that the stated descriptors in the task such as solve, prove, etc. had an almost perfect match with the descriptors in Porter's cognitive demand tool. However, coding of cognitive demands of the examination question was not as straight forward as textbooks tasks since examination questions uses questions in a particular context e.g. determine the general solution of **f(x) = g(x**). This question is embedded in the context of the functions and some processes must unfold first before the general solution can be determined**.** Other questions used representations that needed to be manipulated to equivalent representation first e.g. Given that $\boldsymbol{sin\ (x + 60°)}$ **+ 2** $\boldsymbol{cos\ x}$ **= 0**, show that the equation can be written as $\mathbf{\tan x}$ **= - 4 -** $\sqrt{\mathbf{3}}$ and determine the general solution of $sin\ (x + 60°)$ **+ 2** $\boldsymbol{cos\ x}$ **= 0**, in the specified interval. This task is typical of a non-routine task that involves routine procedures of

expanding and rewriting the compound formula for $\boldsymbol{sin}(\boldsymbol{A}+\boldsymbol{B})$ and some level of interpreting solutions in the specified intervals. This is consistent with Newman's (1990) point that some tasks need integrated processes in order to appropriately assign judgement about lower order and higher order cognitive demands.

3.8.2. Transferability

Merriam (1998) points out that external validity is concerned with the extent to which the findings of one study can be applied to other situations. The concern in issues of transferability often lies in demonstrating that the results of the study at hand can be applied to a wider population. Since the findings of this study are specific to only two Grade 12 textbooks, it is impossible to demonstrate that the findings and conclusions can be transferable to all populations of all Grade 12 textbooks. Erlandson, Harris, Skipper and Allen (1993) note that many inquirers believe that even in practice, conventional generalisability is never possible as all observations are defined by the specific contexts in which they occur.

According to Li (2000) “to allow judgements about how well the research fits with other contexts, thick descriptive data of details regarding procedure and background of analysis should be included in the research report” (p. 305). The thick description helps other researchers to replicate the study using similar conditions in other contexts or settings. Guba and Lincoln (1984) assert that a rich description of findings and conclusions makes it easier for the reader to determine if the study is applicable to their site. In this study, thick descriptors of analysis and judgements of cognitive demands were included to enable other researchers to locate this study to their similar sites.

3.8.3. Confirmability

Guba and Lincoln (1994) argue that confirmability refers to whether the researcher can be neutral or non-judgemental when interpreting and reporting the data. Confirmability denotes the degree to which the results can be confirmed or corroborated by others. Triangulation of researchers served as a measure of confirmability in this study to reduce effects of researcher’s biasness. Tasks from the textbooks, examination tasks and analysis tools that assisted in the analysis of data in the study are attached to the report as audit trails so that the course of the research may be traced.

3.9. ETHICAL CONSIDERATIONS

This study examined material in the public domain such as the curriculum statements, textbooks and NSC examination question papers and these required no ethical considerations. The textbooks used in the study are on the department of education's catalogue and in keeping with the requirements of research and the guidelines provided by the University of the Witwatersrand, the books forming the sample are acknowledged in the reference section of this report.

I also stated in the discussions section that the intention of the study is to analyse the cognitive demands of tasks concerning the topic under review, with neither the intentions to measure one textbook against the other nor to tarnish or harm the reputation and image of any book, publisher or publication. The selection of the textbook used in the study was also guided by the highest level of ethics, purely on the basis of their popularity and use in schools within Gauteng province.

3.10. CONCLUSION

In this chapter I have located and justified the research paradigm that underpinned the study. The study was conducted making use of a case study approach. Document analysis was offered for the two curricula documents under investigation. The tasks contained in the section of trigonometry and the general solution of the two Grade 12 mathematics prescribed textbooks and NSC examination papers for mathematics paper 2 constituted the unit of analysis.

CHAPTER 4
DATA ANALYSIS AND DISCUSSIONS

4.1. INTRODUCTION

The interest of this study is in measuring the level of consistency of cognitive between the various components of the South African NCS FET mathematics curriculum. This focus is informed by conceptions of curriculum alignment which assumes that alignment of standards; instruction and assessment between various components of the education system are beneficial to student's achievement.

The analysis of data regarding the two research questions about the alignment of cognitive demands of instructional tasks within the South African FET mathematics context is presented in this chapter. To examine the alignment levels of cognitive demand implied in trigonometry instructional tasks of the prescribed textbooks and the assessment questions of the NCS examinations of the South African FET Mathematics curricula, the following research questions are posed:

1. What kinds of cognitive demands are implicit in the instructional tasks around the general solutions of trigonometric equations in two Grade 12 NCS (CAPS) prescribed mathematics textbooks and in the assessment tasks in the NSC examinations?

2. To what extent do the cognitive demands of tasks around the general solutions of trigonometric equations in two Grade 12 NCS (CAPS) prescribed textbooks align with the cognitive demands of the assessment tasks around the general solution of trigonometric equations in the NSC examinations?

This chapter is divided into two major sections to address the two individual research questions. In the first section, the analysis of the implied cognitive demands of the Grade 12 prescribed textbooks trigonometry instructional tasks and of the trigonometry tasks in the National Senior Certificate (NSC) examinations are presented. In the second section, the alignment of cognitive demands between the instructional tasks of the prescribed mathematics textbooks and the national senior certificate examinations are discussed.

4.2. THE DESCRIPTION OF THE FIRST TEXTBOOK ANALYSED:

Table 4.1.: - Detail of the first textbook

Series	Platinum Mathematics
Grade	12
Authors	M. Bradley; J. Campbell; S. McPetrie

In Platinum Grade 12 learners' mathematics textbook, trigonometry is covered by topic five (5) and six (6). Each topic is further divided into units as in the order below:

Topic 5: Trigonometry: Compound and double angle identities:

Unit 1: Revision: Grade 11 Trigonometry

Unit 2: Derive the compound and double angle identities

Unit 3: Prove identities using compound and double angle identities

Unit 4: Solve equations and determine the general solution

Topic 6: Trigonometry: Problem solving in two and three dimensions:

Unit 1: Problems in two dimensions

Unit 2: Problems in three dimensions

Each unit is organised into three parts: the introductory sections which begin with an acknowledgement of prior knowledge that is assumed to be present from the preceding grade or topic. Additionally, in the introductory section, a short summary of what will be learned within each new unit is also presented. After the explanation of the key concepts to be learned, worked examples are presented step by step to support student understanding of the new concepts. The intentions of the worked examples are to expose students to the methods of engaging with the new concept and similar tasks. The unit ends with a set of exercises where students are expected to practice what they have learned throughout the unit.

In this report tasks that students are supposed to work with are termed exercises. Exercises within the topic follow the chronological natural numbering system irrespective of the unit (e.g. in Unit 1, if exercises are numbered exercise 1 and exercise 2, then in Unit 2 they will continue from exercise 3, exercise 4 and so on). Summing up the topic, there is the revision

test which contains mixed problems that require the application of the acquired knowledge and skills about the main topic or units.

The analysis in this study only focuses on tasks headed as exercise and revision exercises in the textbooks; these tasks are designed for students to work with. These tasks exclude introductory, explanatory and worked examples. Attention was given only to what is explicit in the tasks as presented in the textbooks. This was without either the involvement of a student or a consideration of what the teacher actually does. This study used numerals during the analysis to quantify cognitive demands that the task demanded and used percentages to quantify the skewness towards lower or higher order cognitive demands levels.

4.3. ANALYTIC TOOL FOR RESEARCH QUESTION 1

To gain insight into the levels of cognitive demand of each task and to address the first question, the cognitive demand of each instructional task as presented in the two prescribed Grade 12 mathematics textbooks is analysed using descriptors of Porter (2002) framework of cognitive demands. Each task is then assigned one of the five levels of Porter (2002) cognitive demand: **(A)** memorisation, **(B)** perform procedures, **(C)** communicate understanding of concepts, **(D)** solve non-routine problems, and **(E)** conjecture, generalise and prove as discussed earlier.

A procedure that was used to review each task is explained as follows. If a task demands memorisation then **(A),** was selected as the category for the cognitive demands of the task. Similarly, if the cognitive demands of the task are weighted more towards performing procedures, then **(B)** was used as a code to categorise the demands of the task. A similar approach was followed to categorise the cognitive demands and place demands of the task into other cognitive demands categories i.e. **(C), (D) or (E)** as discussed earlier**.** However, according to Stein et al (2000) placement of a task in an appropriate category using any taxonomy can be a difficult one since the same task can belong to several categories of cognitive demands. Berger et al (2010) also highlight that conflation of “levels” and “kinds” of thinking required by the task makes placement of task into appropriate category less straightforward (p. 33).

However, the discussion on cognitive demand hierarchical order by Anderson & Krathwohl (2001), highlight that the underlying phenomenon in the cognitive demand dimension is the cognitive demand required to solve the task, ranging from little cognitive demand in memorization to the most cognitive demand in conjecture, generalise and prove. Therefore in cases where the task required more than one demand, the highest demand was used to categorise the task.

4.4. ANALYTIC REVIEWS FROM PLATINUM MATHEMATICS GRADE 12 MATHEMATICS, *Topic 5: Trigonometry: Compound and double angle identities: Unit 4*: *solve equations and determine the general solution*: Exercises 6, 7, 8 and Revision Test.

The unit (unit 4) has three exercises: exercise 6, exercise 7 and exercise 8 and all the tasks in the three exercises will be used as a source of data required to answer the research questions.

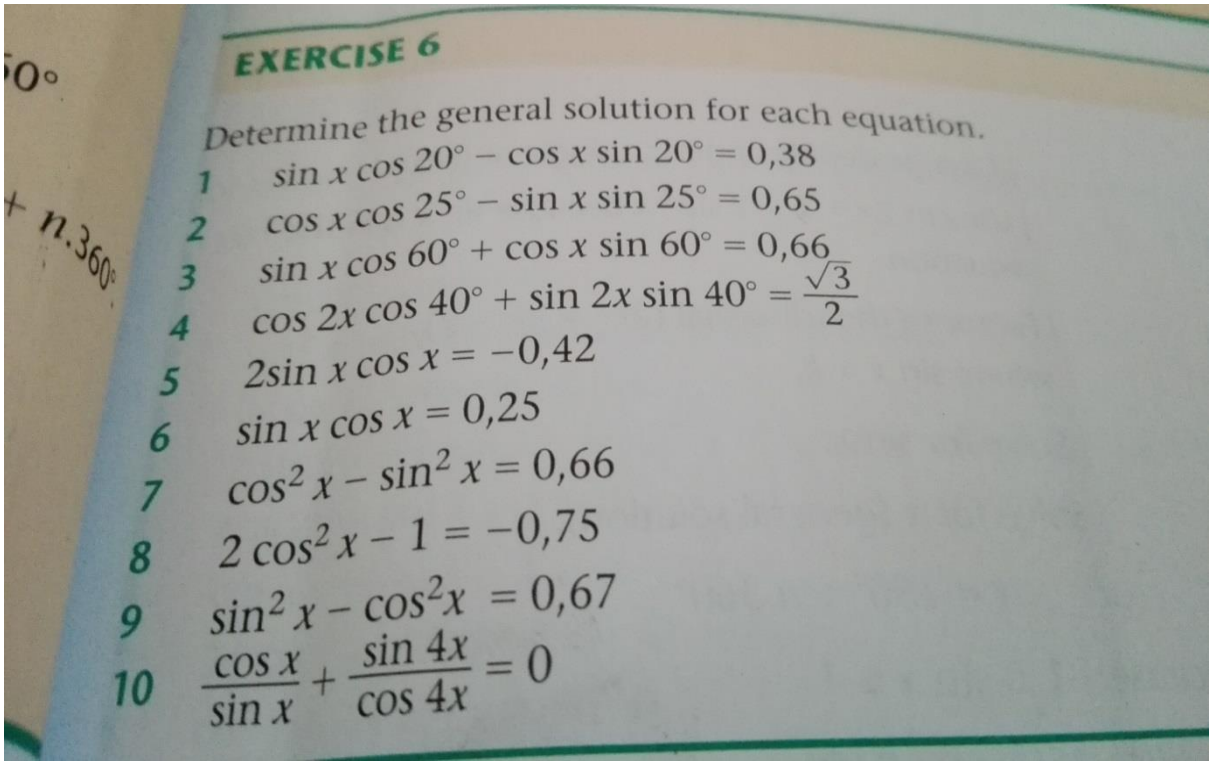
Table 4.2.: describes the number of instructional tasks relating to the content area (*trigonometric equations and the general solution*) addressed in the Platinum Grade 12 learner's textbook.

Table 4.2.: Number of Instructional Tasks by Topic and Content Domain in Topic 5, unit 4, exercise 6 of Platinum Grade 12

Topic	Content Domain	Unit	Exercise	Number of tasks
Trigonometry	*Trigonometric equations and the general solution*	4	6	10

As illustrated in Table 4.2., exercise 6 of unit 4 consists of 10 tasks. Each of the 10 tasks was reviewed to make judgement about and assign the levels of cognitive demands implied by each of the task.

4.4.1. Analytic Reviews of Task in Topic 5, Unit 4, Exercise 6 of Platinum Grade 12:



EXERCISE 6

Determine the general solution for each equation.

1. $\sin x \cos 20° - \cos x \sin 20° = 0{,}38$
2. $\cos x \cos 25° - \sin x \sin 25° = 0{,}65$
3. $\sin x \cos 60° + \cos x \sin 60° = 0{,}66$
4. $\cos 2x \cos 40° + \sin 2x \sin 40° = \frac{\sqrt{3}}{2}$
5. $2\sin x \cos x = -0{,}42$
6. $\sin x \cos x = 0{,}25$
7. $\cos^2 x - \sin^2 x = 0{,}66$
8. $2\cos^2 x - 1 = -0{,}75$
9. $\sin^2 x - \cos^2 x = 0{,}67$
10. $\frac{\cos x}{\sin x} + \frac{\sin 4x}{\cos 4x} = 0$

Figure 4.1.: Tasks from Platinum Grade 12 Mathematics, Topic 5 - Trigonometry, Unit 4 – Solve equations and determine the general solution, Exercise 6, page 105

The procedure that was used to review each tasks in ***Platinum Grade 12 Mathematics, Topic 5, Unit 4, Exercise 6, page 105*** is demonstrated through explanations below. For each task, Porter's (2002) framework of cognitive demand is used.

Exercise 6: Determine the general solution for each equation:

Task 1: $\boldsymbol{\sin x \cos 20° - \cos x \sin 20° = 0{,}38}$

As defined earlier, cognitive demands refers to the "kind of thinking processes entailed in solving the task and the thinking processes in which students engage with the task" (Henningsen & Stein, 1997, p.529). In this study, the interpretations and the judgements of demands of the tasks neither involved the analysis of student's solution nor teacher's explanations to the task. The level and kind of thinking involved in solving the mathematical tasks dictated the demands of the task. Therefore, in order to assign a judgement in terms of the cognitive demands in each case, a decision had to be made as to whether a process leading to the solution of the task requires a routine procedure or it would be non-routine.

By looking at the demands of the process leading to the solution, it was then possible to deduce what was expected of a Grade 12 student in engaging with the task. This was instrumental in determining whether or not a task was a lower order or higher order for a Grade 12 student.

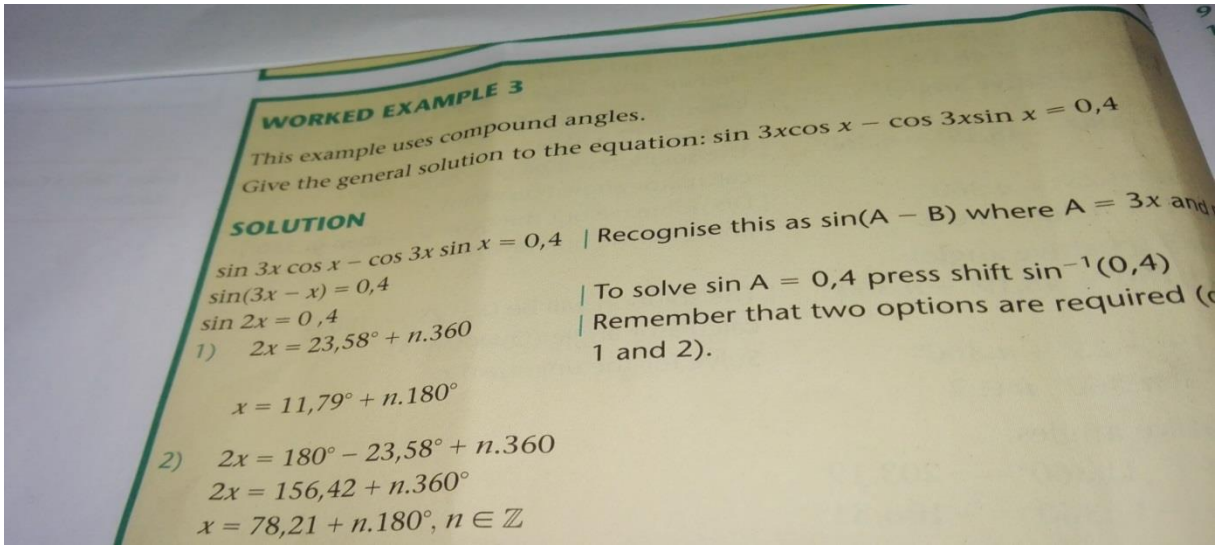
In this task, the cognitive demands are assigned in accordance with Porter (2002) descriptors of 'memorization' and 'perform procedures'. In Porter's view, the cognitive demands 'memorization' and 'perform procedures' include reproducing previously learned facts, rules, formulae and definitions, using algorithms, following instructions and committing facts, rules, formulae, or definitions to memory. Consistent with these characteristics, this task requires the recall of basic formula: specifically, the compound angle identity formula for $\boldsymbol{sin\,(A-B)}$. The rationale behind this coding is based on the context that students at Grade 12 level are familiar with the compound angle for $\boldsymbol{sin(A-B)}$. The task is explicitly of the form $\boldsymbol{sin(A-B)}$. The procedures necessary to solve this task should immediately be recognisable to the student because of recurrent exposure to similar task during preceding text and worked examples. In attempting the task, all that a student may need to do is executing similar procedures for working with $\boldsymbol{sin(A-B)}$, in this case only replacing $\boldsymbol{A}$ by $\boldsymbol{x}$ and $\boldsymbol{B}$ by $\boldsymbol{20°}$.

Furthermore the task follows learned pathways for finding the critical points of a trigonometric equation i.e.

- Recognise the compound or double angle identities, then
- Rewrite the compound or double angle into a single ratio.
- Follow procedures to compute a reference angle e.g. $\boldsymbol{\sin^{-1}(0.42)}$ and
- Apply the two options i.e. relevant reduction formulae relative to quadrants where the ratio is negative or positive. In this case $\boldsymbol{2sin\,x\,cos\,x = -0,42}$, will require recognising that $\boldsymbol{sin\,2x}$ is negative in the third and fourth quadrant and that $\boldsymbol{sin\,2x}$ = $\boldsymbol{sin(180° + ref\angle)}$ in the third quadrant and $\boldsymbol{sin(360° - ref\angle)}$ in the fourth quadrant.

This is a process which according to Hiebert et al (2003) is a repetition where students are expected to continue with the same procedure. A task is classified as repetition if it is "the

same or mostly the same as the preceding task or requires the same operations to solve although the numerical or algebraic expression might be different" (p. 76). The worked example 3 in (Figure 4.2 below): $\boldsymbol{\sin 3x \cos x - \cos 3x \sin x = 0,4}$ and task 1: $\boldsymbol{\sin x \cos 20° - \cos x \sin 20° = 0,38}$ are fundamentally the same and solving $\boldsymbol{\sin x \cos 20° - \cos x \sin 20° = 0,38}$ is observed as the repetition of solving $\boldsymbol{\sin 3x \cos x - \cos 3x \sin x = 0,4}$. Maier (1933) regard a task as repetition if it follows patterns which are typical of a learned behaviour or follows patterns that comes from experiences with previous repetitions.



WORKED EXAMPLE 3

This example uses compound angles.

Give the general solution to the equation: $\sin 3x\cos x - \cos 3x\sin x = 0,4$

SOLUTION

$\sin 3x \cos x - \cos 3x \sin x = 0,4$ | Recognise this as $\sin(A - B)$ where $A = 3x$ and

$\sin(3x - x) = 0,4$

$\sin 2x = 0,4$ | To solve $\sin A = 0,4$ press shift $\sin^{-1}(0,4)$

1) $2x = 23,58° + n.360$ | Remember that two options are required (

1 and 2).

$x = 11,79° + n.180°$

2) $2x = 180° - 23,58° + n.360$

$2x = 156,42 + n.360°$

$x = 78,21 + n.180°, n \in \mathbb{Z}$

Figure 4.2.: from Platinum Grade 12 Mathematics, Topic 5 - Trigonometry, Unit 4 –Give the general solution, page 104 - Worked Example 3

According to Porter (2002), just as with routine problems, performing procedures is not connected to the mathematical meaning that underlies the concept and the directions are straightforward. All that is demanded of lower order cognitive demands tasks is to follow a similar procedure and reproduce processes.

Following the above interpretations, the judgement is that the cognitive demands of this task are associated with:

Table 4.3.: judgements of cognitive demands for task 1 of exercise 6, unit 4, Platinum Mathematics – Grade 12

A	- *recall formulas and computational processes for compound angle for* $\boldsymbol{\sin(A - B)}$,

B	- *computational procedures and follow instructions to solve trig equations i.e. obtain the reference angle, CAST rule and procedures for the general solution*
C	- *none*
D	- *none*
E	- *none*

This task is categorised with more than one demand (memorisation and perform procedures) and the highest demand (perform procedures) is used to categorise the task. The task is accordingly judged as associated with 'perform procedures' (B).

Exercise 6: Determine the general solution for each equation: see figure…on page…

Drawing from the above interpretations for task 1, also in tasks 2, 3, 4, 5, 6 and task 10 there is no ambiguity in terms of what students need to do to answer the questions. These tasks follow similar patterns and require repetition of previously learned procedures for working with compound angles and solving the general solutions of trig equations. This implies that these tasks are also algorithmic, and a process leading to their solutions will require students to only draw from the preceding text and worked examples and replicate the processes. These tasks are thus of a lower order cognitive demand and classified according to the cognitive demand category 'perform procedures' (B).

Table 4.4.: judgements of cognitive demands for task 2, 3, 4, 5, 6 and 10 of exercise 6, unit 4, Platinum Mathematics – Grade 12

A	- *recall procedures for compound angles and double angle identities*
B	- *computational procedures and follow procedures/instructions to obtain the reference angle, rules for CAST and the general solution*
C	- *none*
D	- *none*

E	- *none*

In contrast, tasks 7, 8 and 9 are categorised according to descriptors in the literature that are associated with higher order cognitive demands. As mentioned earlier in the study, Porter does not explicitly refer to lower order and higher order, but using descriptors from the literature on higher order cognitive demands it is possible to associate the descriptors given in Porter's cognitive demands 'communicate understanding of concepts' with those proposed for higher order demand tasks. According to Porter (2002) the cognitive demand 'communicate understanding of concepts' require students to communicate math ideas, use representations to model math ideas, explain findings and results, develop relationships between concepts and engage in metacognition.

The rationale for assigning task 7, 8 and 9 to higher order cognitive demands despite explicit pathways and the predominance of procedures, is that they also incorporate a higher cognitive demand. For engaging with them a capacity to regulate cognitive processes and to choose an appropriate alternate representation ($\boldsymbol{cos^2 x - sin^2 x}$, $\boldsymbol{2\,cos^2 x - 1}$ or $\boldsymbol{1 - 2sin^2 x}$) of expressing the cosine of a double angle **($\boldsymbol{cos\,2x}$)** or vice versa is required. Hiebert et al (2003) point out that "applications that require students to make decisions about how to adjust approaches are conceptually (more) demanding than routine exercises" (p. 90).

This suggests that task 7, task 8 and task 9 requires student to go beyond procedural fluency and to make connections between mathematical concepts. Kilpatrick et al (2001) describe conceptual understanding as an integrated and functional grasp of mathematical ideas that enables connections amongst concept and procedures. Hiebert and Lefevre (1986) stated that for one to make connections between representations, one must possess conceptual knowledge about how pieces of information relate together. As such**,** in these tasks, the interpretations are that the levels of cognitive demands are associated with:

Table 4.5: judgements of cognitive demands for task 7, 8, and 9 of exercise 6, unit 4, Platinum Mathematics – Grade 12

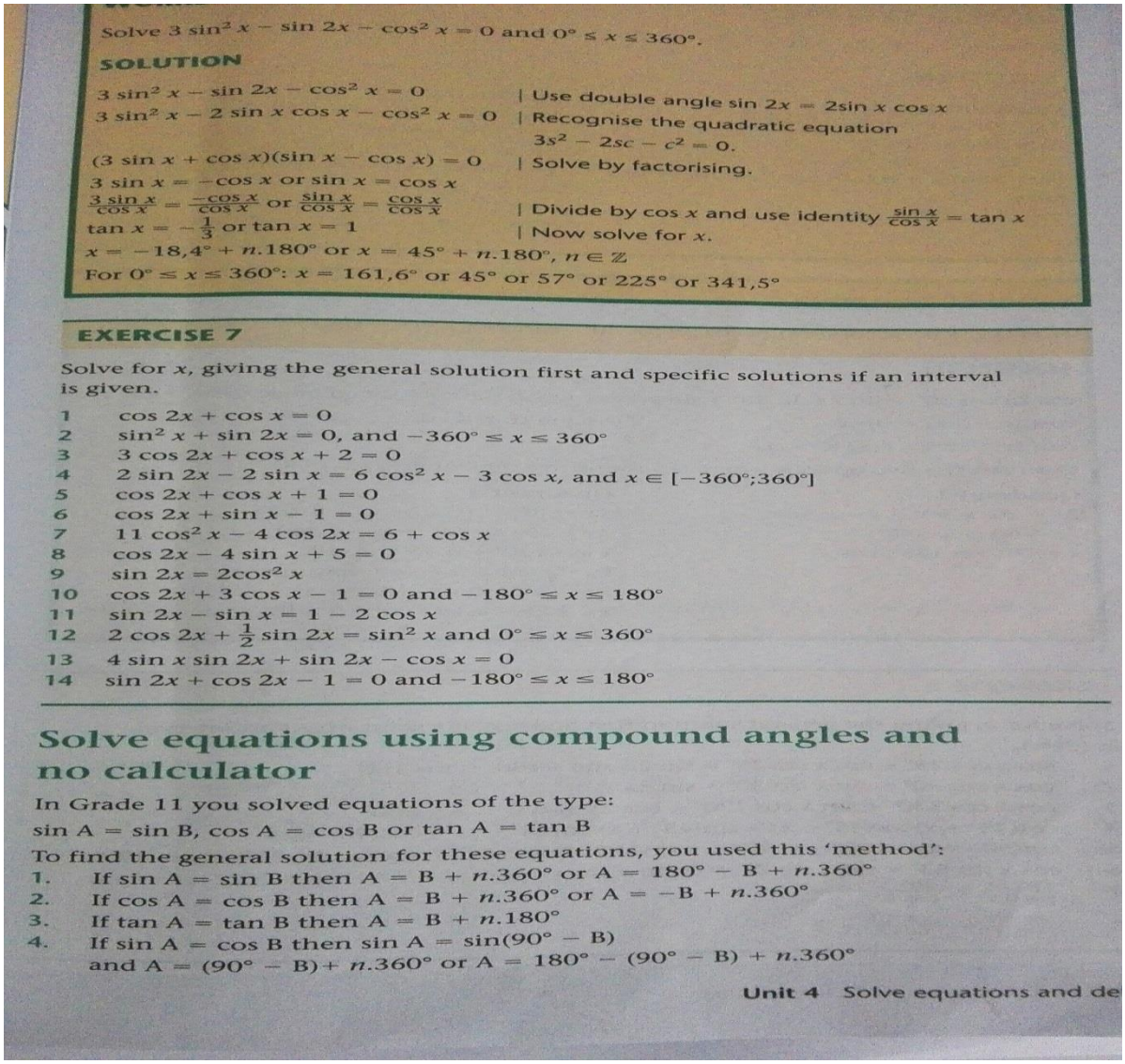
A	- *recall definitions for double angle identities*

B	- *computational procedures and follow procedures/instructions to obtain the reference angle, rules for CAST and the general solution*
C	- *develop relationships between procedures and concepts*
D	- *none*
E	- *none*

The highest cognitive demand from the interpretation in the above table is that task 7, 8 and 9 requires deeper levels of thinking and thus categorised under 'communicate understanding of concepts' (C).

In total 10 instructional tasks were analysed across exercise 6. Most of the cognitive demands of the tasks are concentrated on cognitive demands 'perform procedures' and 'communicate understanding of concepts' with primary prominences on lower order demand 'perform procedures'. In terms of (Kilpatrick, et., al (2001) strands of mathematical proficiency, most of these tasks are associated with knowledge of procedures/procedural fluency and require very little cognitive effort.

4.4.2. Analytic Reviews of Task in Topic 5, Unit 4, Exercise 7 of Platinum Grade 12:



Solve $3\sin^2 x - \sin 2x - \cos^2 x = 0$ and $0° \le x \le 360°$.

SOLUTION

$3\sin^2 x - \sin 2x - \cos^2 x = 0$
$3\sin^2 x - 2\sin x \cos x - \cos^2 x = 0$ | Use double angle $\sin 2x = 2\sin x \cos x$
| Recognise the quadratic equation $3s^2 - 2sc - c^2 = 0$.
$(3\sin x + \cos x)(\sin x - \cos x) = 0$ | Solve by factorising.
$3\sin x = -\cos x$ or $\sin x = \cos x$
$\frac{3\sin x}{\cos x} = \frac{-\cos x}{\cos x}$ or $\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$ | Divide by $\cos x$ and use identity $\frac{\sin x}{\cos x} = \tan x$
$\tan x = -\frac{1}{3}$ or $\tan x = 1$ | Now solve for x.
$x = -18{,}4° + n.180°$ or $x = 45° + n.180°$, $n \in \mathbb{Z}$
For $0° \le x \le 360°$: $x = 161{,}6°$ or $45°$ or $57°$ or $225°$ or $341{,}5°$

EXERCISE 7

Solve for x, giving the general solution first and specific solutions if an interval is given.

1 $\cos 2x + \cos x = 0$
2 $\sin^2 x + \sin 2x = 0$, and $-360° \le x \le 360°$
3 $3\cos 2x + \cos x + 2 = 0$
4 $2\sin 2x - 2\sin x = 6\cos^2 x - 3\cos x$, and $x \in [-360°;360°]$
5 $\cos 2x + \cos x + 1 = 0$
6 $\cos 2x + \sin x - 1 = 0$
7 $11\cos^2 x - 4\cos 2x = 6 + \cos x$
8 $\cos 2x - 4\sin x + 5 = 0$
9 $\sin 2x = 2\cos^2 x$
10 $\cos 2x + 3\cos x - 1 = 0$ and $-180° \le x \le 180°$
11 $\sin 2x - \sin x = 1 - 2\cos x$
12 $2\cos 2x + \frac{1}{2}\sin 2x = \sin^2 x$ and $0° \le x \le 360°$
13 $4\sin x \sin 2x + \sin 2x - \cos x = 0$
14 $\sin 2x + \cos 2x - 1 = 0$ and $-180° \le x \le 180°$

Solve equations using compound angles and no calculator

In Grade 11 you solved equations of the type:
$\sin A = \sin B$, $\cos A = \cos B$ or $\tan A = \tan B$
To find the general solution for these equations, you used this 'method':
1. If $\sin A = \sin B$ then $A = B + n.360°$ or $A = 180° - B + n.360°$
2. If $\cos A = \cos B$ then $A = B + n.360°$ or $A = -B + n.360°$
3. If $\tan A = \tan B$ then $A = B + n.180°$
4. If $\sin A = \cos B$ then $\sin A = \sin(90° - B)$ and $A = (90° - B) + n.360°$ or $A = 180° - (90° - B) + n.360°$

Figure 4.3: Tasks from Platinum Grade 12 Mathematics, Topic 5 - Trigonometry, Unit 4 – Solve equations and determine the general solution, Exercise 7, page 107

Table 4.6.: Number of Instructional Tasks by Topic and Content Domain in Topic 5, unit 4, exercise 7 of Platinum Grade 12

Topic	Content Domain	Unit	Exercise	Number of tasks
Trigonometry	*Trigonometric equations and the general solution*	4	7	14

As illustrated in Table 4.6, exercise 7 of unit 4 consists of 14 tasks. Each of the 14 tasks was reviewed to make judgement about and to assign the levels of cognitive demands implied by each of the task.

The procedure that is used to review each tasks in ***Platinum Grade 12 Mathematics, Topic 5, Unit 4, Exercise 7, page 107*** is explained below:

Exercise 7: Solve for x, giving the general solution first and specific solutions if an interval is given:

Task 1**,** 3, 5, 6, 7, 8, 9, 11 and 13 are accounted for in the following interpretations. The cognitive demand required to solve these tasks is based on reproducing well known procedures for solving routine problems. These tasks requires the recall and recognition of definitions for double angle identities and computational fluency in changing double angles into single angles, factorisation practices, techniques for solving equations and understanding of concepts. According to Van De Walle (2004) mathematical knowledge can be learned by balancing procedural knowledge and conceptual knowledge. When procedural knowledge is balanced with conceptual knowledge, students can explain not only how procedures are performed but also why they are performed. In most of these tasks a conceptual understanding is required to enable connections and relational understanding (Skemp, 1976) that angle $\boldsymbol{2x}$ is double angle $\boldsymbol{x}$ and that the trigonometric functions are supposed to be operating on the same angle.

Most of the tasks are essentially recognised as quadratic equations which then invite the use of algebraic strategies and routine procedures for solving quadratic equations. These algebraic strategies will follow a standard operating procedure for solving quadratics that is similar to the following:

- If equations have 2 terms, then you look for a common factor.
- You may have to form the $\tan$ ratio if each side has a cosine and sine of the same angle.
- If equations have three terms, it is usually a quadratic trinomial
- If there are 4 terms, you have to group them in pairs

In contrast, it should be noted that the level of demand of a task in not necessarily determined by the contents of the task in isolation. The relationship of the task to prior experience of the students also influences the level of cognitive demand of the task. For example, if students had exposure with similar tasks on a multiple occasions, the high

demand task might become procedural and lower order. Using Porter (2002), we[1] characterized these tasks as solving tasks where routine procedure is required. The cognitive demands are judged as:

Table 4.7.: judgements of cognitive demands for task 1, 3, 5, 7, 8, 9, 10, 11 and 13 of exercise 7, unit 4, Platinum Mathematics – Grade 12

A	- *Recall definitions for double angles for* $\cos 2x$ *and/or* $\sin 2x$.
B	- *Algorithms and instructions to obtain the reference angle, CAST rules and associated reduction formulae for two quadrants and the procedures for solving quadratics.*
C	- *none*
D	- *none*
E	- *none*

The needed procedures to solve these tasks are apparent because of previous experience with the preceding examples. According to Porter (2002) descriptors, these tasks are examples of lower level order demand that are based on reproducing procedures. The processes of recognising quadratic equation outlined in the steps of the worked examples are repeated in a similar way in most tasks. Similarly, the procedure for factoring trinomials and/or solve quadratic equations are also explicit and needs to be followed consistently. Accordingly, these tasks are categorised in terms of Porter (2002) descriptors of 'perform procedures' (B) since they follow a step by step algorithms modelled from preceding texts.

Solve for x, giving the general solution first and specific solutions if an interval is given:

Task 2: $\boldsymbol{sin^2 x + \sin 2\, x = 0}$, and $\boldsymbol{-360° \leq x \leq 360°}$.

In this task, the interpretations are that the cognitive demands are associated with:

[1] We – the use of we refers to the primary researcher and the two experts who assisted in the coding of the cognitive demands of the task.

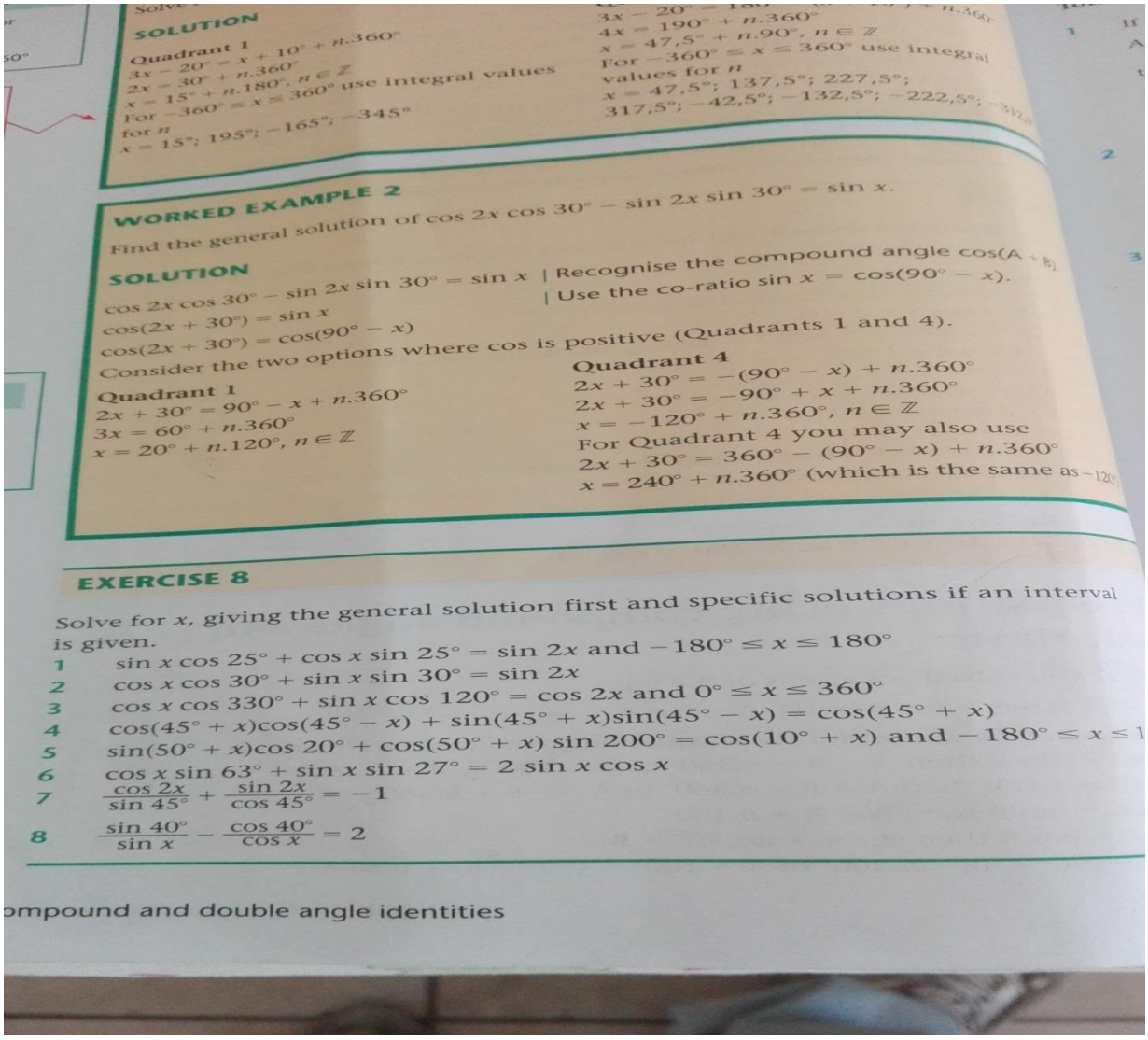
Table 4.8.: judgements of cognitive demands for task 2, 10, 12 and 14 of exercise 7, unit 4, Platinum Mathematics – Grade 12

A	- *recall formulas and computational processes for double angles identities*
B	- *procedural fluency for obtaining the reference angle, procedures for CAST rules and reduction formulae for two quadrants where the ratios are positive and negative, procedures for the general solution and organising data in the given intervals*
C	- *Understanding the relationships between double angle and single angles i.e. that trigonometry functions are supposed to operate on single angles and why to change* $\sin 2x = 2\sin x \cos x$ *and* $\cos 2x$ *to either* $cos^2x - sin^2x$ or $1 - 2sin^2x$ or $2cos^2x - 1$, *together with the multiple representation of expressing* $\cos 2x$ - *demonstrate an understanding of the specific solutions that satisfy an equation in a restricted interval*
D	- *none*
E	- *none*

Manipulating double angles identities formulae should be a familiar procedure; that is associated with recall and as such can be classified as lower order cognitive demand task. However, the focus of this task beyond computational fluency and formulating general solutions is to find the fixed solution in the required intervals:$-360° \leq x \leq 360°$.

According to Porter (2002), computing procedures do not require explanations for why a particular procedure was performed, but interpreting the solution in a specified interval demonstrates an understanding of differences between infinite and finite solutions and what the restricted solution means. As stated, this task could be more accurately described as higher order cognitive demand (Porter 2002) and is classified according to the highest demand: 'demonstrate understanding' (C). Task 2, 10, 12 and 14 (see figure 4.3, page 50) are also interpreted as demonstrating understanding of concepts.

4.4.3. Analytic Reviews from Task in Topic 5, Unit 4, Exercise 8:



SOLUTION

Quadrant 1
$3x - 20° = x + 10° + n.360°$
$2x = 30° + n.360°$
$x = 15° + n.180°, n \in \mathbb{Z}$
For $-360° \leq x \leq 360°$ use integral values for n
$x = 15°; 195°; -165°; -345°$

$3x - 20° = 180° - (x + 10°) + n.360°$
$4x = 190° + n.360°$
$x = 47,5° + n.90°, n \in \mathbb{Z}$
For $-360° \leq x \leq 360°$ use integral values for n
$x = 47,5°; 137,5°; 227,5°; 317,5°; -42,5°; -132,5°; -222,5°; -312,5°$

WORKED EXAMPLE 2

Find the general solution of $\cos 2x \cos 30° - \sin 2x \sin 30° = \sin x$.

SOLUTION

$\cos 2x \cos 30° - \sin 2x \sin 30° = \sin x$ | Recognise the compound angle $\cos(A + B)$.
$\cos(2x + 30°) = \sin x$ | Use the co-ratio $\sin x = \cos(90° - x)$.
$\cos(2x + 30°) = \cos(90° - x)$
Consider the two options where cos is positive (Quadrants 1 and 4).

Quadrant 1
$2x + 30° = 90° - x + n.360°$
$3x = 60° + n.360°$
$x = 20° + n.120°, n \in \mathbb{Z}$

Quadrant 4
$2x + 30° = -(90° - x) + n.360°$
$2x + 30° = -90° + x + n.360°$
$x = -120° + n.360°, n \in \mathbb{Z}$
For Quadrant 4 you may also use
$2x + 30° = 360° - (90° - x) + n.360°$
$x = 240° + n.360°$ (which is the same as $-120°$

EXERCISE 8

Solve for x, giving the general solution first and specific solutions if an interval is given.

1 $\sin x \cos 25° + \cos x \sin 25° = \sin 2x$ and $-180° \leq x \leq 180°$
2 $\cos x \cos 30° + \sin x \sin 30° = \sin 2x$
3 $\cos x \cos 330° + \sin x \cos 120° = \cos 2x$ and $0° \leq x \leq 360°$
4 $\cos(45° + x)\cos(45° - x) + \sin(45° + x)\sin(45° - x) = \cos(45° + x)$
5 $\sin(50° + x)\cos 20° + \cos(50° + x) \sin 200° = \cos(10° + x)$ and $-180° \leq x \leq 1$
6 $\cos x \sin 63° + \sin x \sin 27° = 2 \sin x \cos x$
7 $\frac{\cos 2x}{\sin 45°} + \frac{\sin 2x}{\cos 45°} = -1$
8 $\frac{\sin 40°}{\sin x} - \frac{\cos 40°}{\cos x} = 2$

ompound and double angle identities

Figure 4.4.: Tasks from Platinum Grade 12 Mathematics, Topic 5 - Trigonometry, Unit 4 – Solve for x, giving the general solution first and specific solutions if an interval is given, Exercise 8, page 108

Table 4.9.: Number of Instructional Tasks by Topic and Content Domain in Topic 5, unit 4, exercise 8 of Platinum Grade 12

Topic	Content Domain	Unit	Exercise	Number of tasks
Trigonometry	*Trigonometric equations and the general solution*	4	8	8

As illustrated in Table 4.9, exercise 8 of unit 4 consists of 8 tasks. Each of the 8 tasks was reviewed to make judgement about and assign the levels of cognitive demands implied by each task.

The procedure that reviewed each tasks in ***Platinum Grade 12 Mathematics, Topic 5, Unit 4, Exercise 8, page 108*** is explained below:

Exercise 8: Solve for x, giving the general solution first and specific solutions if an interval is given

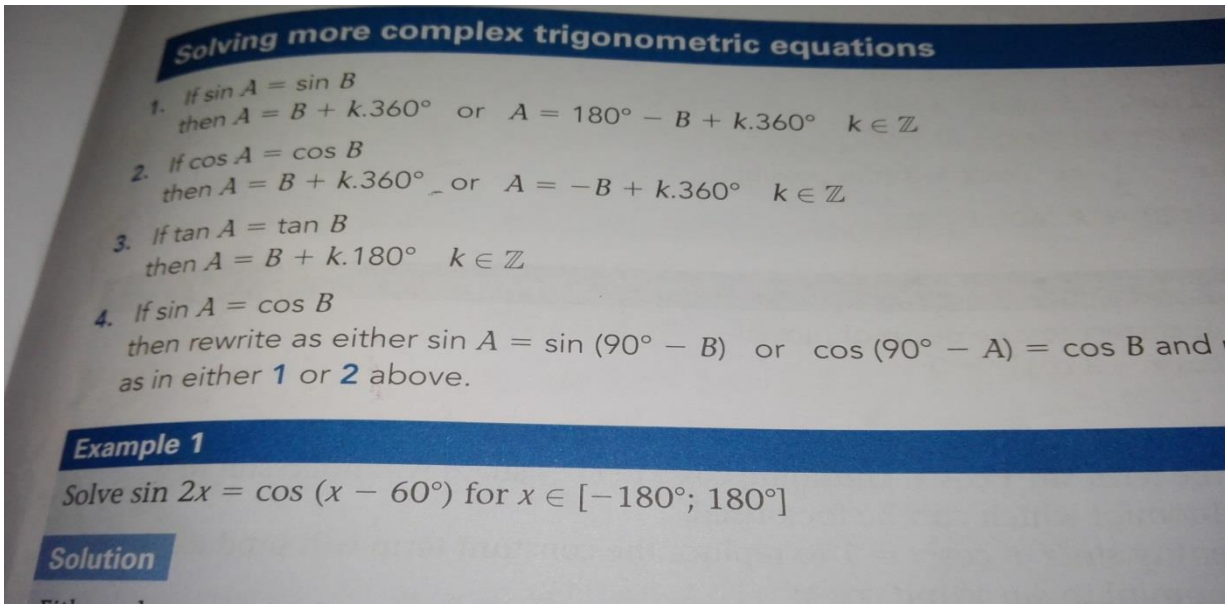
Task 1: $\boldsymbol{\sin x \cos 25° + \cos x \sin 25° = \sin 2x}$ and $\boldsymbol{-180° \leq x \leq 180°}$.

The demands of this task are explained in the steps and solution below:

Step 1: recognise and recall the compound angle for $\boldsymbol{\sin(A + B)}$, where **A** is $\boldsymbol{x}$ and $\boldsymbol{B}$ **is** $\boldsymbol{25°}$ and rewrite $\boldsymbol{\sin x \cos 25° + \cos x \sin 25°}$ as a single ratio of $\boldsymbol{\sin}$ of the repeated angles: $\boldsymbol{\sin\ (x + 25°)}$

Step 2: $\boldsymbol{\sin\ (x + 25°) = \sin 2x}$**,** in this case the ratios are already balanced and the focus is on the angles. The ratio are dropped based on the rule if $\boldsymbol{\sin A = \sin B}$, then $\boldsymbol{A = B + \text{n}.360°}$ or $\boldsymbol{A = 180° - B + \text{n}.360°}$, $\text{n} \in \boldsymbol{Z}$ (see figure below)

Step 3: using the general solution for sin**:** (see figure below)



Solving more complex trigonometric equations

1. If $\sin A = \sin B$ then $A = B + k.360°$ or $A = 180° - B + k.360°$ $k \in \mathbb{Z}$
2. If $\cos A = \cos B$ then $A = B + k.360°$ or $A = -B + k.360°$ $k \in \mathbb{Z}$
3. If $\tan A = \tan B$ then $A = B + k.180°$ $k \in \mathbb{Z}$
4. If $\sin A = \cos B$ then rewrite as either $\sin A = \sin(90° - B)$ or $\cos(90° - A) = \cos B$ and as in either **1** or **2** above.

Example 1

Solve $\sin 2x = \cos(x - 60°)$ for $x \in [-180°; 180°]$

Solution

Figure 4.5.: General Solution rule in Classroom Mathematics, Chapter 5, and page 129

If $\boldsymbol{sin(x + 25°) = sin(2x)}$, then:

$$\boldsymbol{(x + 25°) = 2x + \text{n}.\,360\,°} \textbf{ or } \boldsymbol{(x + 25°) = (180° - 2\,x) + \text{n}.\,360°,}\ n \in \boldsymbol{Z}$$

Step 4: manipulation, recalling reduction formulae and procedural fluency in the general solution results in the solutions below:

Solution 1	Other Solution
$\boldsymbol{x = 25° + n.\,360\,°,}\ n \in \boldsymbol{Z}$	$\boldsymbol{x = 51.7° + n.\,120\,°,}\ n \in \boldsymbol{Z}$

Step 5: for specific solution in $\boldsymbol{-180° \leq x \leq 180°}$ **,** use integral values for n to organise data in the required interval.

This task involves interpreting the solution in a specified interval. As mentioned earlier, demonstrating an understanding of differences between infinite and finite solutions could be more accurately described as higher order cognitive demand (Porter 2002) and classified as cognitive demand (C) 'demonstrate understanding of concepts'. The solution involves adapting the solution to a specified interval to satisfy the given equation. Similarly Task 3 and 5 (figure 4.4, page 54) are also interpreted as 'demonstrate understanding of concepts' (C)

Table 4.10.: judgements of cognitive demands for task 1, 3, and 5 of exercise 8, unit 4, Platinum Mathematics – Grade 12

A	- *processes for compound and double angles identities*
B	- *procedural fluency for CAST rules and reduction formulae for two quadrants where the ratios are positive and negative, procedures for the general solution and organising data in the given intervals*
C	- *interpretations of specific solution in the specified interval*
D	- *none*
E	- *none*

Task 2, 4, 6, 7 and 8 (see figure 4.4, page 54) display similar patterns and places greater emphasis on procedural fluency and a similar approach will be repeated to engage with

each task. In each of these tasks, the interpretations are that the cognitive demands are associated with:

Table 4.11.: judgements of cognitive demands for task 2, 4, 6, 7 and 8 of exercise 8, unit 4, Platinum Mathematics – Grade 12

A	- *Recall formulas and computational processes for compound and double angles identities*
B	- *procedural fluency for CAST rules and reduction formulae for two quadrants where the ratios are positive and negative, procedures for the general solution and organising data in the given intervals*
C	- *none*
D	- *none*
E	- *none*

The tasks are therefore classified as 'perform procedures' (B) because they explicitly calls the use of procedures previously learned or explicitly taught and emphasises a routine to be followed to solve them.

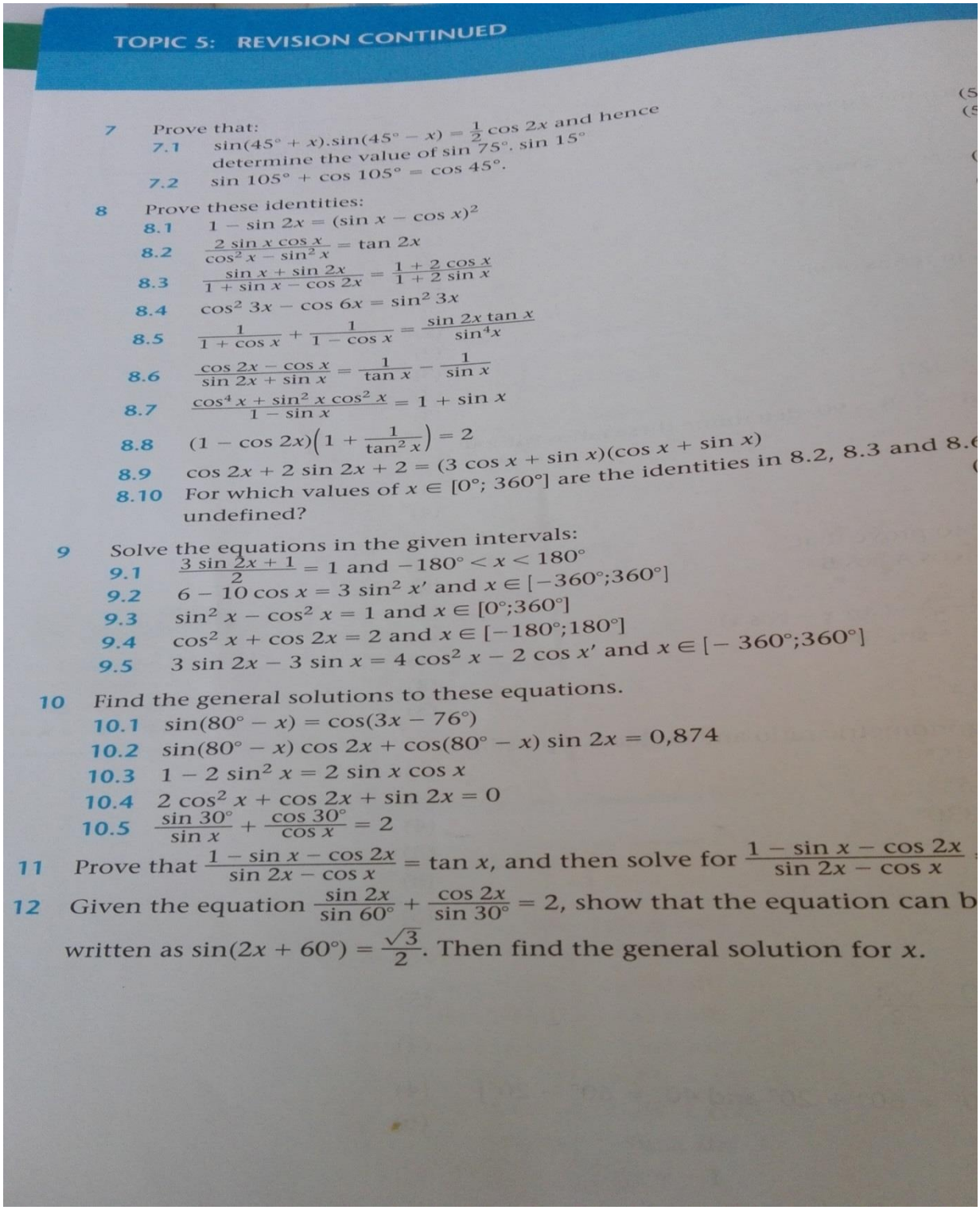
Pólya (1973) made a distinction among the levels of difficulty found in problems. He defined routine problems as those that "can be solved either by substituting special data into a formerly solved general problem or by following step by step, with some well-worn conspicuous procedures" (p. 171). The steps above are dominated by recall of basic mathematic facts, formulae and conspicuous procedures relating to compound angles and double angles identities. The process of solving the equation and determining the general solution described in the steps of worked examples will be repeated in a similar way across the exercise. The directions for these tasks are explicit as to what procedures are to be used with the expectation that procedure must be tailored to a particular task context.

4.4.4. Analytic Reviews from Task in Topic 5, Unit 4, Revision Test:

Table 4.12.: Number of Instructional Tasks by Topic and Content Domain in Topic 5, unit 4, revision test of Platinum Grade 12

Topic	Content Domain	Unit	Exercise	Number of tasks
Trigonometry	*Trigonometric equations and the general solution*	4	Revision test	6 (10.1 – 10.5) and 12.

As illustrated in Table 4.12, the Revision test of unit 4 consists of 6 tasks that address the general solution. Each of the 6 tasks was reviewed to make judgement about and assign the levels of cognitive demands implied by the task.



TOPIC 5: REVISION CONTINUED

7 Prove that:
7.1 $\sin(45° + x).\sin(45° - x) = \frac{1}{2}\cos 2x$ and hence determine the value of $\sin 75°.\sin 15°$
7.2 $\sin 105° + \cos 105° = \cos 45°$.

8 Prove these identities:
8.1 $1 - \sin 2x = (\sin x - \cos x)^2$
8.2 $\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \tan 2x$
8.3 $\frac{\sin x + \sin 2x}{1 + \sin x - \cos 2x} = \frac{1 + 2\cos x}{1 + 2\sin x}$
8.4 $\cos^2 3x - \cos 6x = \sin^2 3x$
8.5 $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} = \frac{\sin 2x \tan x}{\sin^4 x}$
8.6 $\frac{\cos 2x - \cos x}{\sin 2x + \sin x} = \frac{1}{\tan x} - \frac{1}{\sin x}$
8.7 $\frac{\cos^4 x + \sin^2 x \cos^2 x}{1 - \sin x} = 1 + \sin x$
8.8 $(1 - \cos 2x)\left(1 + \frac{1}{\tan^2 x}\right) = 2$
8.9 $\cos 2x + 2\sin 2x + 2 = (3\cos x + \sin x)(\cos x + \sin x)$
8.10 For which values of $x \in [0°; 360°]$ are the identities in 8.2, 8.3 and 8.6 undefined?

9 Solve the equations in the given intervals:
9.1 $\frac{3\sin 2x + 1}{2} = 1$ and $-180° < x < 180°$
9.2 $6 - 10\cos x = 3\sin^2 x'$ and $x \in [-360°;360°]$
9.3 $\sin^2 x - \cos^2 x = 1$ and $x \in [0°;360°]$
9.4 $\cos^2 x + \cos 2x = 2$ and $x \in [-180°;180°]$
9.5 $3\sin 2x - 3\sin x = 4\cos^2 x - 2\cos x'$ and $x \in [-360°;360°]$

10 Find the general solutions to these equations.
10.1 $\sin(80° - x) = \cos(3x - 76°)$
10.2 $\sin(80° - x)\cos 2x + \cos(80° - x)\sin 2x = 0{,}874$
10.3 $1 - 2\sin^2 x = 2\sin x \cos x$
10.4 $2\cos^2 x + \cos 2x + \sin 2x = 0$
10.5 $\frac{\sin 30°}{\sin x} + \frac{\cos 30°}{\cos x} = 2$

11 Prove that $\frac{1 - \sin x - \cos 2x}{\sin 2x - \cos x} = \tan x$, and then solve for $\frac{1 - \sin x - \cos 2x}{\sin 2x - \cos x}$

12 Given the equation $\frac{\sin 2x}{\sin 60°} + \frac{\cos 2x}{\sin 30°} = 2$, show that the equation can b written as $\sin(2x + 60°) = \frac{\sqrt{3}}{2}$. Then find the general solution for x.

Figure 4.6.: Tasks from Platinum Grade 12 Mathematics, Topic 5, Unit 4, Revision Test, page 110

Revision Test: Find the general solution to these equations: Task 10.1; 10.2; 10.3; 10.4 and 10.5.

Task 10.1: This task requires some degree of cognitive effort and the connection that to solve trig equations the ratios must be balanced and or operating on the same ratio. Students will be required to utilize their understanding that sine becomes cosine in vertical

reduction and to use the co-ratio to change either $\boldsymbol{sin}(\mathbf{80}° - \boldsymbol{x})$ $\boldsymbol{to}$ $\boldsymbol{cos}[\mathbf{90}° - (\mathbf{80}° - \boldsymbol{x})]$ or vice versa.

Step 1: recognise that sine becomes cosine in vertical reduction and recall co-ratio that $\boldsymbol{sin}(\mathbf{80}° - \boldsymbol{x}) = \boldsymbol{cos}[\mathbf{90}° - (\mathbf{80}° - \boldsymbol{x})]$

Step 2: in this case the ratios are balanced $\boldsymbol{cos}(\mathbf{10}° + \boldsymbol{x}) = \boldsymbol{cos}(\mathbf{3x} - \mathbf{76}°)$ and the focus is on the angles, so the ratios are dropped. Using the rule (see figure…below) if $\boldsymbol{cos\ A = cosB}$, then $\boldsymbol{A = B} + \mathrm{n}.\mathbf{360}°, \mathrm{n} \in \boldsymbol{Z}$ or $\boldsymbol{A = -B} + \mathrm{n}.\mathbf{360}°, \mathrm{n} \in \boldsymbol{Z}$

Step 3: using the general solution for cos**: (see Figure….in page…..)**

If $\boldsymbol{cos}(\mathbf{10}° + \boldsymbol{x}) = \boldsymbol{cos}(\mathbf{10}° + \boldsymbol{x})$, **then**

$(\mathbf{10}° + \boldsymbol{x}) = \pm(\mathbf{3x} - \mathbf{76}°) + \mathrm{n}.\mathbf{360}°, \mathrm{n} \in \boldsymbol{Z}$

$\therefore (\mathbf{10}° + \boldsymbol{x}) = \mathbf{3x} - \mathbf{76}° + \mathrm{n}.\mathbf{360}°, \mathrm{n} \in \boldsymbol{Z}$ **or** $(\mathbf{10}° + \boldsymbol{x}) = \mathbf{-}\ (\mathbf{3x} - \mathbf{76}°) + \mathrm{n}.\mathbf{360}, \mathrm{n} \in \boldsymbol{Z}$

Step 4: manipulation, recalling reduction formulae and procedural fluency in the general solution results in the solutions below:

Solution 1	Other Solution
$\boldsymbol{x} = \mathbf{43°} + \mathrm{n}.\mathbf{180}°, \mathrm{n} \in \boldsymbol{Z}$	$\boldsymbol{x} = \mathbf{16.5°} + \mathrm{n}.\mathbf{90}°, \mathrm{n} \in \boldsymbol{Z}$

According to Stein et al. (2000), two distinguishing thoughts are necessary when evaluating the cognitive demand of a task. Firstly, the superficial features of a task should be recognised. The argument here is that the superficial features of a task often do not designate the level of mathematical difficulty found in the task. In their example, they argue that tasks presented in a complex manner, necessitating use of manipulatives, diagrams, or real-world contexts may often require students to apply simple, well-rehearsed procedures whereas some tasks that are stated simply may require mathematical thought. They also argue on one hand that the level of cognitive demand of a task is partly dependent upon the experience of students who engage with the task.

This task is algorithmic with little ambiguity on how to engage with the task. As such, this task is classified as low level because distinctive well-rehearsed procedures for balancing the

ratios by changing sine to its relative co-functions, determining and using reduction formulae where cos is positive are strongly implied. The processes of solving this task replicate a standard textbook problem that will be solved by observing the preceding examples. The task is therefore classified as 'perform procedures' (B).

Task 10.2: this task requires the recall of basic formula: compound angle identity formula for $\boldsymbol{sin}\ (\boldsymbol{A}+\boldsymbol{B})$**,** to replace A and B by $(\mathbf{80}^\circ - \boldsymbol{x})$ and $\mathbf{2}\boldsymbol{x}$ respectively. This task is a routine task following similar processes for engaging with $\boldsymbol{sin}\ (\boldsymbol{A}+\boldsymbol{B})$, obtaining the reference angle, procedures for using reduction formulae where a function is positive or negative according to the CAST rule. According to Artut and Tarim (2009), routine exercises are generally similar instances of a previously solved task or they require applying a learned formula to the new task. A similar approach will be repeated to task 10.3 to 10.5. In each of these tasks, the judgements are that the cognitive demands are associated with:

Table 4.13.: judgements of cognitive demands for task 10.2 – 10.5 of revision exercise, unit 4, Platinum Mathematics – Grade 12

A	- *recall formulas and computational processes for compound and double angles identities*
B	- *procedural fluency for recognising quadratic equation and use of algebraic strategies for factoring trinomials and solving quadratic trig equations*
C	- *none*
D	- *none*
E	- *none*

The tasks are therefore associated with 'perform procedures' (B) because they explicitly calls for the use of procedures previously learned and the pathways are explicit and require a routine to be followed to solve them.

Task 12: Given the equation $\frac{\boldsymbol{sin}\ \mathbf{2}\boldsymbol{x}}{\boldsymbol{sin}\ \mathbf{60}^\circ} + \frac{\boldsymbol{cos}\mathbf{2}\boldsymbol{x}}{\boldsymbol{sin}\mathbf{30}^\circ} = \mathbf{2}$**,** show that the equation can be written as $\boldsymbol{sin}\ (\ \mathbf{2}\boldsymbol{x} + \mathbf{60}^\circ) = \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$**.** Then find the general solution.

Determining the level of cognitive demands of a task can be tricky due to misleading superficial features of the task and at times high level tasks can sometimes appear to be low level (NCTM, 1991; Stein et al., 1996). The rationales for coding this task as 'communicate understanding of concepts' (C)**,** is based on the premise that it addresses relationships between several mathematical concepts. There are no preceding texts with similar features to recall or memorise from in order to mimic an algorithm to solve this task. This task employs integration of mathematical content and practices (find the LCD, using special angles, and employing reduction formulae) that requires some degree of cognitive effort to addresses the tasks of this form.

Within and across the revision test, most of the tasks are associated with low levels of cognitive demands; primarily 'perform procedures' (B). The procedure for solving most task and pathways are explicit and there is less ambiguity about procedures to be followed in order to engage with the task. The rationale behind this coding is based on the following interpretations:

4.5. SUMMARY OF ANALYSIS FROM PLATINUM MATHEMATICS GRADE 12

A summary of the analytic review for Topic 5, Unit 4, exercises 6, 7, 8 and the Revision Test is given in the table 4.14 below. The data counts shows how the tasks in Topic 5, Unit 4, of Platinum Grade 12 learner's textbook were coded with respect to the different categories of cognitive demands.

Table 4.14.: Allocation of cognitive demands for Topic 5, Unit 4, Exercises 6, 7, 8 and Revision Test in Platinum Mathematics Grade 12.

	Cognitive Demands					
Unit 4, Exercise 6	**A**	**B**	**C**	**D**	**E**	TOTAL
1.		1				1
2.		1				1
3.		1				1
4.		1				1
5.		1				1
6.		1				1
7.		1				1
8.			1			
9.			1			1
10.			1			1
Unit 4, Exercise 7						
1		1				1

2.			1			1
3.		1				1
4.			1			1
5.		1				1
6.		1				1
7.		1				1
8.		1				1
9.		1				1
10.			1			1
11.		1				1
12.			1			1
13.		1				1
14.			1			1
Unit 4, Exercise 8						
1.			1			1
2.		1				1
3.			1			1
4.		1				1
5.			1			1
6.		1				1
7.		1				1
8.		1				
Revision Test Topic						
10.1		1				
10.2		1				
10.3		1				
10.4		1				
10.5		1				
12			1			
TOTALS		**26**	**12**			**38**

A major area of interest in table 4.14 above is the data counts in relation to lower order cognitive demand levels (A & B) and higher order cognitive demand levels (C, D & E). Comparing the totals of columns B and C it becomes apparent that the cognitive demands of the majority of tasks in exercise 6, 7, 8 and the revision exercise corresponds to a lower level cognitive demand category by Porter (2002). In details and to be more specific, the analysis reveals that there are 26 data counts against column B with 12 data counts against column C. The review suggests that 68.4% (26 of 38) of tasks in unit 4 require knowledge of procedures (perform procedures), while 31.6% of the tasks require understanding of concepts (communicate understanding of concepts).

These results are also shown graphically in the figure 4.7 below, to emphasize the cognitive demands being emphasized, or not emphasized, by the tasks under review.

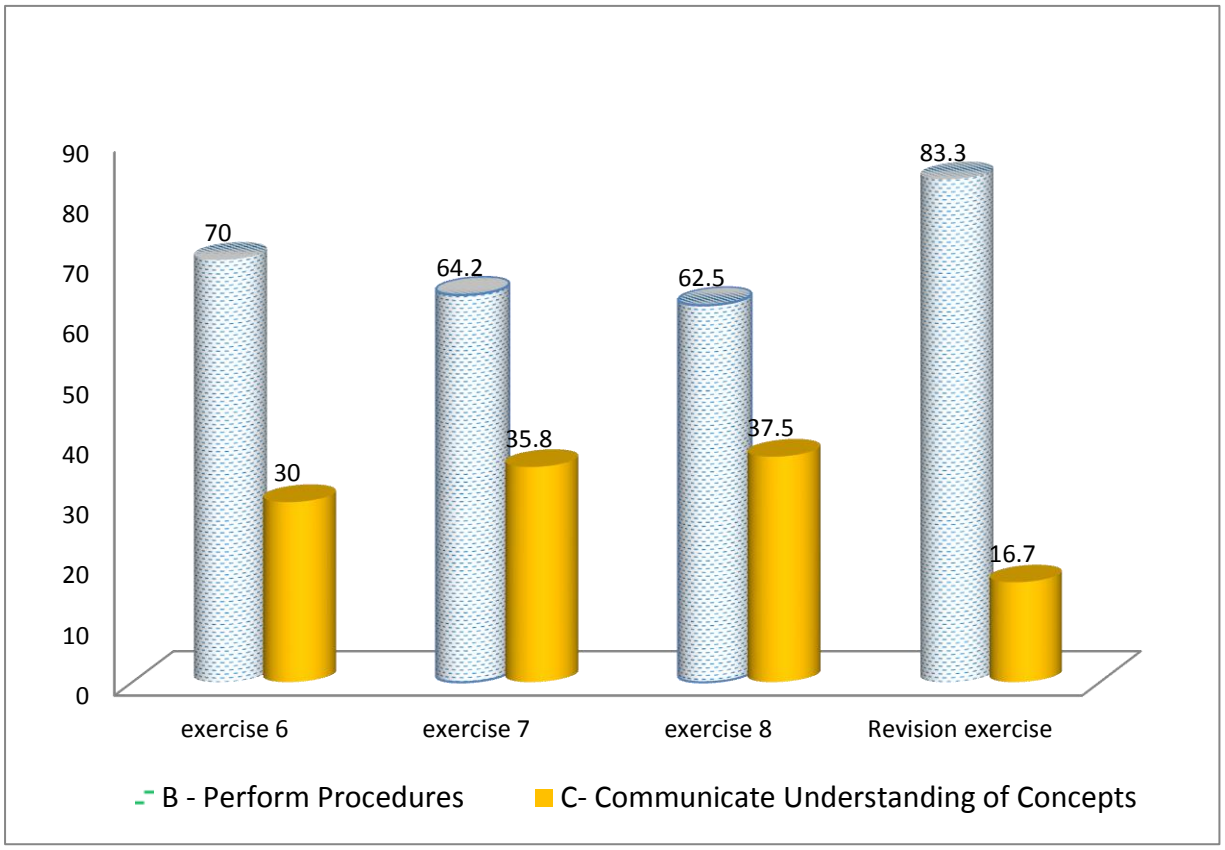


Figure 4.7.: Cognitive Demands of tasks in Platinum Grade 12 Mathematics, Topic 5 and Unit 4

Figure 4.7., depicts that the cognitive demands of most of the tasks in the exercises 6, 7, 8 and revision exercise draws more on recall, definitions, formulas, and procedures. In detail, exercise 6 has 7 out of 10 tasks (70%) classified at cognitive demand B, with the remaining 3

tasks (30%) at cognitive demand C. Figure 4.7 further indicate that 9 out of 14 tasks (64.2%) in exercise 7 are associated with cognitive demand B while only 5 tasks (35.8%) are at cognitive demand C. The same pattern is also evident in exercise 8 where 5 tasks (62.5%) are classified relative to cognitive demand B with only 3 tasks (37.5%) relative to cognitive demand C. In the revision test, 5 tasks (83.3%) are also classified at cognitive demand B while only 1 (16.7%) task is associated with cognitive demand C, with neither of the tasks in the entire unit 4 at cognitive demand level D nor cognitive demand level E of Porter's (2002) descriptors of cognitive demand framework.

Porter (2002) developed a two-dimensional framework for aligning standards, curricula, and assessments. The two dimensional framework employs the use of a single language for measuring content and ensuring the **"**description at a consistent level of depth and specificity" (p. 3) when comparing cognitive demands of various tasks. The two-dimensional framework has the topic dimension arranged as rows on one end and levels of cognitive demands arranged as columns at the other end. In the words of Porter (2002), content of instruction is described as the intersection between topics and cognitive demand, based on the data gathered. Gamoran, Porter, Smithson and White (1997) assert that the intersection between topics and cognitive demand gives a better understanding than when topic or cognitive demands are used alone. This view is in agreement with Marzano and Costa's (1988) assertions that cognitive skills only have meaning when viewed together with content. According to Porter (2002) conceptualizing content in this manner denotes what students should know and be able to do with what they know and much would be lost in alignment studies if studies reduce research instruments to topics, performance or cognitive demand only.

In line with previous studies on alignment (Porter, 2002; Webb, 2005, 2006; Polikoff, Porter, & Smithson, 2012), this study also classifies content of instruction (Porter, 2002) at the intersection of the topics and the cognitive demand. The data counts in Table 4.14 above will now be shown in the Matrix A in Table 4.15 to try and answer the first research question. This will be done by placing demands of each task into cognitive demand categories based on Porter's descriptors discussed earlier and to determine the relative emphasis of the cognitive demands of each exercise.

Table 4.15.: Matrix A with Data Counts for Unit 4, Exercises 6, 7, 8 and revision test in Platinum – Adapted from Mhlolo (2011)

Topic Dimension (Trig Equations and the General Solution)	**Cognitive Demand Categories**							
	Lower Order Demand			**Higher Order Demand**				
	A	**B**	**Sub Totals**	**C**	**D**	**E**	**Sub Totals**	**Total**
Exercise 6 : Determine the general solution for each equation		7	**7**	**3**			**3**	**10**
Exercise 7 : Solve for x, giving the general solution first and the specific solutions in the interval is given		9	**9**	**5**			**5**	**14**
Exercise 8 : Solve for x, giving the general solution first and the specific solutions in the interval is given		5	**5**	**3**			**3**	**8**
Revision Test Topic 5		5	**6**	**1**			**1**	
TOTAL		**26**	**26**	**12**			**12**	**38**

Looking through the columns of sub totals, it would appear that the tasks for unit 4 (Solve equations and determine the general solution) in Platinum Grade 12 places more emphasis on lower order skills. The small number of tasks that make high level demands indicates that there are very limited opportunities for learners to be challenged and to develop sophisticated mathematical skills.

Apparent in these exercises is that tasks used to introduce the topic incline to be lower level based on memorisation and performing procedures. Similarly, tasks used for practice by students also tends to have a lower level of cognitive demand, (memorisation, perform procedures and solve routine problems) and only serves as a review of previously learned concepts and procedures. Pólya (1973) accepted that routine problems are necessary and suitable for some educational goals such as to develop students' fluency in performing

routine mathematical operations quickly and correctly. However, he stressed that routine problem should by no means constitute the entire curriculum.

4.6. THE DESCRIPTION OF THE SECOND TEXTBOOK ANALYSED:

Table 4.16.: - Details of the second textbook

Series	Classroom Mathematics
Grade	12
Authors	M. Pike; A. Jawurek; A. Kitto; P. Laridon; M. Myburgh; R. Rhodes-Houghton; M. Sasman; J. Scheiber; S. Tebeila and H. Wilson.

Classroom Mathematics covers trigonometry in Chapters 5 and 6 of the textbook as follows: ***Chapter 5 is divided into two main parts.*** The first part deals with trigonometric identities and the general solutions of the trigonometric equations and the second part cover solving trigonometric equations using compound and double angle identities.

As mentioned earlier in the study, tasks that students are supposed to work with and practice are termed exercises. In total there are twelve (12) exercises across chapter 5. Each exercise has between five and thirteen tasks/questions/problems. The numbering of exercises within the chapter follows a progressive order starting with the chapter in which the exercise is located and progresses according to the natural numbering system (e.g. in Chapter 5, exercises are exercise 5.1, exercise 5.2 and so on). Summing up, the chapter contains exercises for students to check and extend their skills. These exercises comprise mixed problems that require the application of the learned knowledge and skills about the main topic.

4.7. ANALYTIC REVIEWS FROM CLASSROOM MATHEMATICS GRADE 12 MATHEMATICS, *Chapter 5: Trigonometry: General solution of trigonometric equations and Solving trigonometric equations using compound and double angle identities:* **Exercises 5.1, 5.2 and 5.9.**

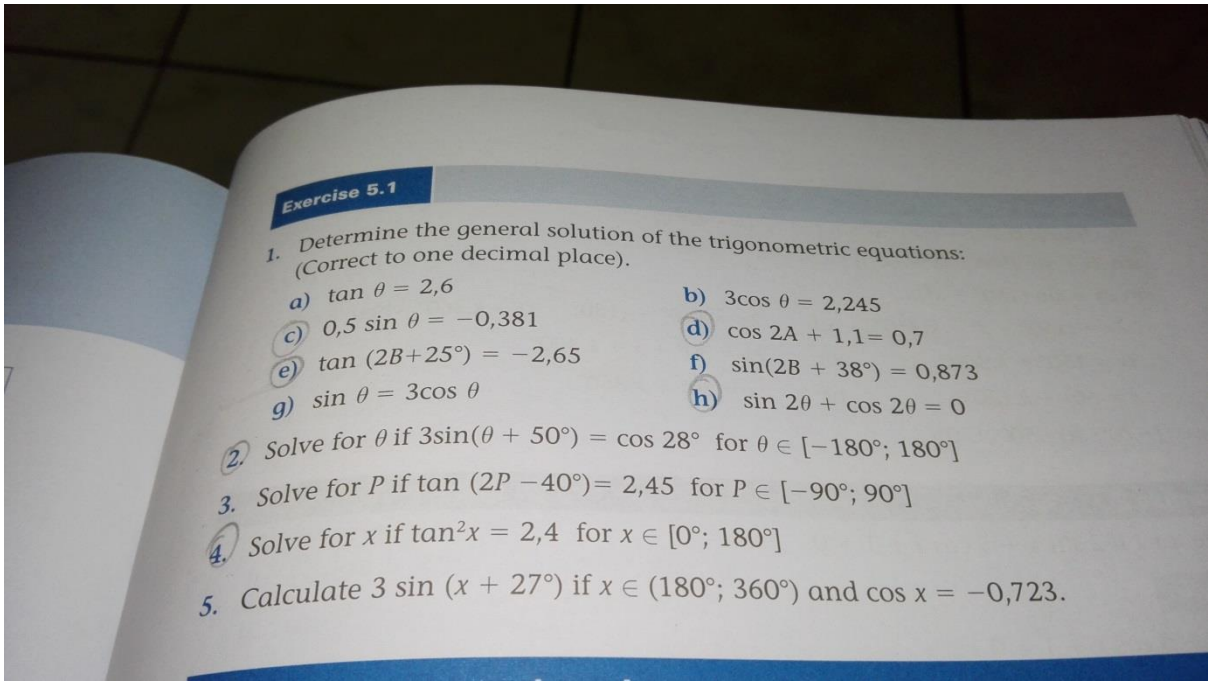
Table 4.17 describes the number of instructional tasks relating to the content area (*general solution of trigonometric equations*) covered in Classroom Mathematics Grade 12 learner's textbook.

Table 4.17.: Number of Instructional Tasks by Topic and Content Domain in Chapter 5, exercise 5.1 of Classroom Mathematics Grade 12

Topic	Content Domain	Chapter	Exercise	Number of tasks
Trigonometry	*General solution of trigonometric equations*	5	5.1	12

As illustrated in Table 4.17, exercise 5.1 consists of 12 tasks. Each of the 12 tasks in Classroom Mathematics Grade 12 were analysed and coded to judge the cognitive demand placed on each task using Porter's (2000) cognitive demand framework.

4.7.1. Analytic Reviews of Task in Chapter 5, Exercise 5.1 in Classroom Mathematics Grade 12:



Exercise 5.1

1. Determine the general solution of the trigonometric equations: (Correct to one decimal place).
   a) $\tan\theta = 2{,}6$
   b) $3\cos\theta = 2{,}245$
   c) $0{,}5\sin\theta = -0{,}381$
   d) $\cos 2A + 1{,}1 = 0{,}7$
   e) $\tan(2B+25°) = -2{,}65$
   f) $\sin(2B + 38°) = 0{,}873$
   g) $\sin\theta = 3\cos\theta$
   h) $\sin 2\theta + \cos 2\theta = 0$
2. Solve for θ if $3\sin(\theta + 50°) = \cos 28°$ for $\theta \in [-180°; 180°]$
3. Solve for P if $\tan(2P - 40°) = 2{,}45$ for $P \in [-90°; 90°]$
4. Solve for x if $\tan^2 x = 2{,}4$ for $x \in [0°; 180°]$
5. Calculate $3\sin(x + 27°)$ if $x \in (180°; 360°)$ and $\cos x = -0{,}723$.

Figure 4.8.: Tasks from Classroom Grade 12 Mathematics, Chapter 5, Exercise 5.1, page 129

Task 1: *Determine the general solution of the trigonometric equations (Correct to one decimal place)*

(a): $\boldsymbol{\tan\theta = 2.6}$. As mentioned earlier in the analysis, a decision had to be made as to whether a process leading to the solution of each task would require a routine procedure or it would be non-routine. A process leading to the solution is shown below:

Table 4.18.: Possible solution to task 1 (a), Classroom Mathematics, Chapter 5, Exercise 5.1, page 129

General Solution Method	***Positive ratio, acute angle and quadrant method***
$\boldsymbol{\tan\theta = 2.6}$ - obtain $tan^{-1}\ 2.6$ (the reference angle ≈ 69.0°) - then for tan General Solution: $\therefore \theta = tan^{-1}\ 2.6 + k.180°, k \in Z$ $= 69.0° + k.180°, k \in Z$	$\boldsymbol{\tan\theta = 2.6}$ - obtain $tan^{-1}\ 2.6$ (the reference angle ≈ 69.0°) - using cast rule, select options where ratio is $\pm$ (in this case tan $is\ a\ positive$ function - tan is(+) in the 1st and 3rd quadrant. The general solution takes into account the period of a trigonometric function. For tan the period is 180°, which then suggest that after every 180° turns, the same numerical values will be obtained. From this understanding we then confine our solution to 1st quadrant, since the 3rd quadrant will give the same numerical values lying in excess of 180°. $\therefore \theta = tan^{-1}\ 2.6\ (\boldsymbol{ref}\angle) + k.180°, k \in Z$ $= 69.0° + k.180°, k \in Z$

The process leading to the possible solution above suggest that the cognitive demand required to solve this tasks is based on recall and recognition of computational procedure for obtaining the reference angle and well known procedures for the general solution or the positive ratio, acute angle quadrant method. This implies that a limited cognitive effort is required to engage with this task. The cognitive demands of the task are therefore associated with lower order demand as accounted for in the Table 4.19 below. Task 1 (b) to

1(h) will follow similar process to obtain the solutions to the task. In light of the afore mentioned interpretations, the cognitive demands of these tasks (1b -1h) are similarly associated with lower level cognitive demands and coded B – 'perform procedures' according to Porter (2002) cognitive demand framework.

Table 4.19.: judgements of cognitive demands for task 1(a) of exercise 5.1, chapter 5, Classroom Mathematics

A	- *recall formulas and computational processes leading to obtaining the reference angle*
B	- *procedural fluency for applying the general solution and/or using the positive ratio, acute angle and quadrant method*
C	- *none*
D	- *none*
E	- *none*

Task 2: Solve for θ if $3\,sin(\theta + 50°) = cos28°$ for $\theta \in [-180°; 180°]$. The possible solution to this task follows the steps as shown below. The process leading to the possible solution for this task uses explicit pathways and familiar procedures for solving the trigonometric equations.

Table 4.20.: Possible solution to task 2, Classroom Mathematics, Chapter 5, Exercise 5.1, page 129

- $3\,sin(\theta + 50°) = cos28°$
- $3\,sin(\theta + 50°) = 0.8829472592$
- $sin(\theta + 50°) = 0.294315864$
- $sin^{-1}0.294315864 \approx 17.1°$

then for the General Solution of sin :

$\therefore (\theta + 50°) = 17.1° + k.360°, k \in Z$ or $(\theta + 50°) = 180° - 17.1° + k.360°, k \in Z$

$\boldsymbol{\theta = -32.9^\circ + k.360^\circ, k \in Z}$ **or** $\boldsymbol{\theta = 112.9^\circ + k.360^\circ, k \in Z}$

Senk, Beckmann & Thompson (1997) characterised high order demand task as solving tasks where justification or explanation are required. This task is coded as higher order demand – 'communicate understanding of concepts' (Porter, 2002). The rationale is based on the view that the process transcends procedures for finding the general solution and extends to the interpretation of the solutions that satisfy the restricted interval and the justification for why other solutions are not acceptable. Task 3, 4 and 5 are also accounted for within the same interpretations and their cognitive demands are judged as 'communicate understanding of concepts' – (C)

Table 4.21.: continuation of possible solution to task 2, Classroom Mathematics, Chapter 5, Exercise 5.1, page 129

From the general solution: $\boldsymbol{\theta = -32.9^\circ + k.360^\circ, k \in Z}$ **or** $\boldsymbol{\theta = 112.9^\circ + k.360^\circ, k \in Z}$
Interpretation of the solution has to be made in the specified interval., using the integral values of k i.e. if $k = -1$, then $\boldsymbol{\theta = -392.9^\circ}$ **or** $\boldsymbol{\theta = -247.1^\circ}$

$k = 0$, then $\boldsymbol{\theta = -32.9^\circ}$ **or** $\boldsymbol{\theta = 112.9^\circ}$

$k = 1$, then $\boldsymbol{\theta = 327.1^\circ}$ **or** $\boldsymbol{\theta = 472.9^\circ}$

for $\theta \in$ **[-180°; 180°],** then it means θ has to be ≥ -180° and θ has to be ≤ 180°, then

$\theta = -392.9^\circ$ or $\theta = -247.1^\circ$	$\theta = -392.9^\circ$ - outside the specified interval $\theta = -247.1^\circ$ -outside the specified interval
$\theta = -32.9^\circ$ or $\theta = 112.9^\circ$	$\theta = -32.9^\circ$ - within the specified interval $\theta = 112.9^\circ$ - within the specified interval
$\theta = 327.1^\circ$ or $\theta = 472.9^\circ$	$\theta = 327.1^\circ$ - outside the specified interval $\theta = 472.9^\circ$ - outside the specified interval
$\therefore \boldsymbol{\theta = -32.9^\circ, 112.9^\circ}$ within the interval **[-180°; 180°],**	

Table 4.22.: judgements of cognitive demands for task 2, 3, 4 and 5 of exercise 5.1, chapter 5, Classroom Mathematics

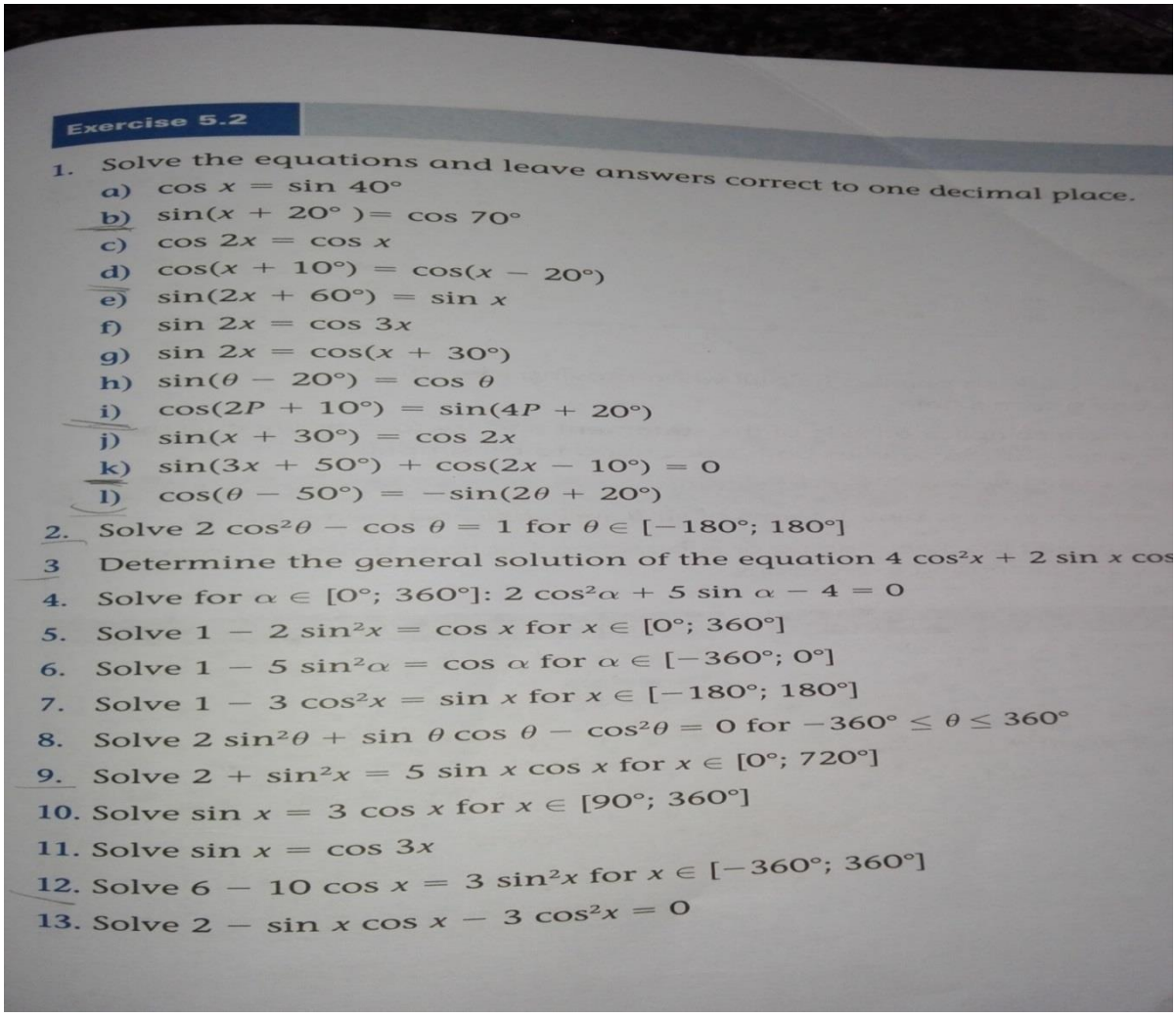
A	- *recall formulas and computational processes leading to obtaining the reference angle*
B	- *procedural fluency for applying the general solution and/or using the positive ratio, acute angle and quadrant method*
C	- *interpretation of solutions in the specified interval*
D	- *none*
E	- *none*

4.7.2. Analytic Reviews of Task in Chapter 5, Exercise 5.2 in Classroom Mathematics Grade 12:

Table 4.23.: Number of Instructional Tasks by Topic and Content Domain in Chapter 5, exercise 5.2 of Classroom Mathematics Grade 12

Topic	Content Domain	Chapter	Exercise	Number of tasks
Trigonometry	*General solution of trigonometric equations*	5	5.2	24

As illustrated in Table 4.23, exercise 5.2 consists of 24 tasks. Each of the 24 tasks in Classroom Mathematics Grade 12 were analysed and coded to judge the cognitive demand placed on each task using Porter's (2002) cognitive demand framework.



Exercise 5.2

1. Solve the equations and leave answers correct to one decimal place.
   a) $\cos x = \sin 40°$
   b) $\sin(x + 20°) = \cos 70°$
   c) $\cos 2x = \cos x$
   d) $\cos(x + 10°) = \cos(x - 20°)$
   e) $\sin(2x + 60°) = \sin x$
   f) $\sin 2x = \cos 3x$
   g) $\sin 2x = \cos(x + 30°)$
   h) $\sin(\theta - 20°) = \cos \theta$
   i) $\cos(2P + 10°) = \sin(4P + 20°)$
   j) $\sin(x + 30°) = \cos 2x$
   k) $\sin(3x + 50°) + \cos(2x - 10°) = 0$
   l) $\cos(\theta - 50°) = -\sin(2\theta + 20°)$
2. Solve $2\cos^2\theta - \cos\theta = 1$ for $\theta \in [-180°; 180°]$
3. Determine the general solution of the equation $4\cos^2 x + 2\sin x \cos$
4. Solve for $\alpha \in [0°; 360°]$: $2\cos^2\alpha + 5\sin\alpha - 4 = 0$
5. Solve $1 - 2\sin^2 x = \cos x$ for $x \in [0°; 360°]$
6. Solve $1 - 5\sin^2\alpha = \cos\alpha$ for $\alpha \in [-360°; 0°]$
7. Solve $1 - 3\cos^2 x = \sin x$ for $x \in [-180°; 180°]$
8. Solve $2\sin^2\theta + \sin\theta\cos\theta - \cos^2\theta = 0$ for $-360° \leq \theta \leq 360°$
9. Solve $2 + \sin^2 x = 5\sin x \cos x$ for $x \in [0°; 720°]$
10. Solve $\sin x = 3\cos x$ for $x \in [90°; 360°]$
11. Solve $\sin x = \cos 3x$
12. Solve $6 - 10\cos x = 3\sin^2 x$ for $x \in [-360°; 360°]$
13. Solve $2 - \sin x \cos x - 3\cos^2 x = 0$

Figure 4.9.: Tasks from Classroom Grade 12 Mathematics, Chapter 5, Exercise 5.2, page 131

Task 1(a): ***Solve the equations and leave answers correct to one decimal place.***

(a): $\boldsymbol{\cos x = \sin 40°}$. At Grade 12 level, students are expected to recognise that when solving trigonometric equations, the trigonometric functions must operate on the same function or operate on the same ratio. In this case, the functions on each side of the equation are not the same. To make the functions the same, they should be converted into an equation involving either sine or cosine using the co-functions identities:

$$\boldsymbol{\cos(90° - x) = \sin x} \text{ or } \boldsymbol{\sin(90° - x) = \cos x}$$

Using the co-functions identities, then:

$\boldsymbol{\cos x = \sin 40°}$, then $\boldsymbol{\cos x = \sin(90° - 50°)}$

$$\boldsymbol{\cos x = \cos 50°}$$

Using the rule in Figure 4.5, then $\boldsymbol{cos\, x = \pm\, 50° + k.360°, k \in Z}$ **.** These tasks do not require the students to work outside familiar context and well known algorithm. The process leading to the solution is a familiar procedure for working with co-functions. Also the process executes familiar and well-rehearsed procedures for solving equations with functions that are not balanced. Within the cognitive demand literature, familiar and well-rehearsed procedures are considered lower order cognitive demands and in this case it will be 'perform procedures' (B). Similar interpretations will also apply to tasks 1b to 1L, task 3, task 11 and task 13.

Table 4.24.: judgements of cognitive demands for task 1(b) – task 1(l), task 3, task 11 and task 13 of exercise 5.2, chapter 5, Classroom Mathematics

A	- *recall formulas and computational processes for compound and double angles identities*
B	- *procedural fluency for recognising quadratic equation and use of algebraic strategies for factoring trinomials and solving quadratic trig equations*
C	- *none*
D	- *none*
E	- *none*

Task 2, 4, 5, 6, 7, 8, 9, 10 and 12 are placed in the cognitive demand category 'communicate understanding of concepts' – C as in table 4.25 below.

Table 4.25.: judgements of cognitive demands for task 2, task 4 – task 10, task 12 of exercise 5.2, chapter 5, Classroom Mathematics

A	- *recall formulas and computational processes for compound and double angles identities*
B	- *procedural fluency for recognising quadratic equation and use of algebraic strategies for solving quadratic trig equations*
C	- *interpretation and placement of intervals within specified intervals*

D	- *none*
E	- *none*

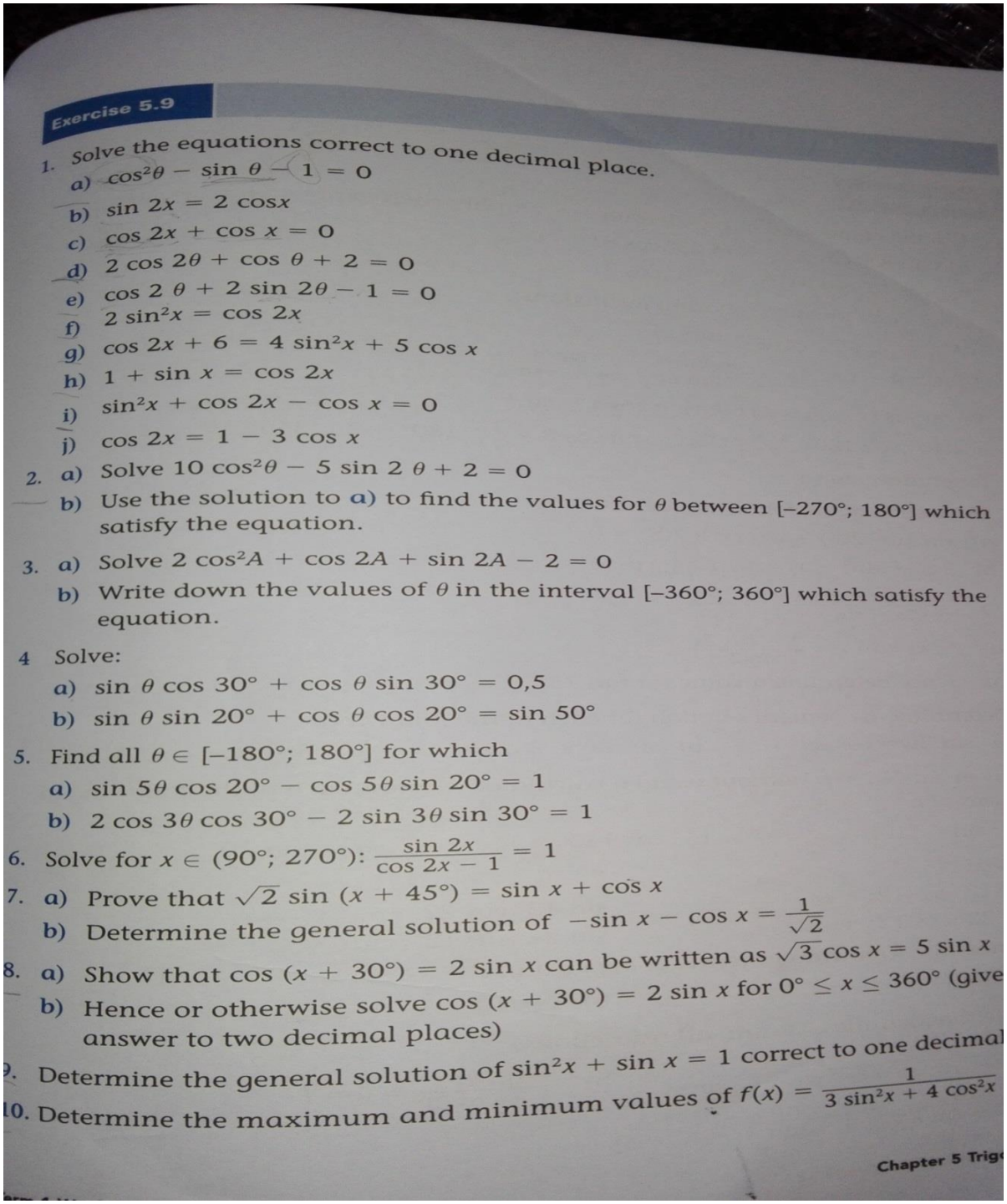
The cognitive demands of these task are considered more demanding since they do not only call for following procedures to solve the equations but also to extend thinking to the interpretations of the intervals {the differences between $\boldsymbol{\theta} \in (0°, 360°)$; $\boldsymbol{\theta} \in [0°, 360°]$ and to places the values of the unknown angle within the specified intervals.

4.7.3. Analytic Reviews of Task in Chapter 5, Exercise 5.9 in Classroom Mathematics Grade 12:

Table 4.26.: Number of Instructional Tasks by Topic and Content Domain in Chapter 5, exercise 5.9 of Classroom Mathematics Grade 12

Topic	Content Domain	Chapter	Exercise	Number of tasks
Trigonometry	*General solution of trigonometric equations*	5	5.9	24

Table 4.26, illustrates the number of exercises in exercise 5.9. Each of the 24 tasks in Classroom Mathematics Grade 12 were analysed and coded to judge the cognitive demand placed on each task using Porter's (2002) cognitive demand framework.



Exercise 5.9

1. Solve the equations correct to one decimal place.
   a) $\cos^2\theta - \sin\theta - 1 = 0$
   b) $\sin 2x = 2\cos x$
   c) $\cos 2x + \cos x = 0$
   d) $2\cos 2\theta + \cos\theta + 2 = 0$
   e) $\cos 2\theta + 2\sin 2\theta - 1 = 0$
   f) $2\sin^2 x = \cos 2x$
   g) $\cos 2x + 6 = 4\sin^2 x + 5\cos x$
   h) $1 + \sin x = \cos 2x$
   i) $\sin^2 x + \cos 2x - \cos x = 0$
   j) $\cos 2x = 1 - 3\cos x$
2. a) Solve $10\cos^2\theta - 5\sin 2\theta + 2 = 0$
   b) Use the solution to a) to find the values for θ between $[-270°; 180°]$ which satisfy the equation.
3. a) Solve $2\cos^2 A + \cos 2A + \sin 2A - 2 = 0$
   b) Write down the values of θ in the interval $[-360°; 360°]$ which satisfy the equation.
4. Solve:
   a) $\sin\theta\cos 30° + \cos\theta\sin 30° = 0{,}5$
   b) $\sin\theta\sin 20° + \cos\theta\cos 20° = \sin 50°$
5. Find all $\theta \in [-180°; 180°]$ for which
   a) $\sin 5\theta\cos 20° - \cos 5\theta\sin 20° = 1$
   b) $2\cos 3\theta\cos 30° - 2\sin 3\theta\sin 30° = 1$
6. Solve for $x \in (90°; 270°)$: $\frac{\sin 2x}{\cos 2x - 1} = 1$
7. a) Prove that $\sqrt{2}\sin(x + 45°) = \sin x + \cos x$
   b) Determine the general solution of $-\sin x - \cos x = \frac{1}{\sqrt{2}}$
8. a) Show that $\cos(x + 30°) = 2\sin x$ can be written as $\sqrt{3}\cos x = 5\sin x$
   b) Hence or otherwise solve $\cos(x + 30°) = 2\sin x$ for $0° \leq x \leq 360°$ (give answer to two decimal places)
9. Determine the general solution of $\sin^2 x + \sin x = 1$ correct to one decimal
10. Determine the maximum and minimum values of $f(x) = \frac{1}{3\sin^2 x + 4\cos^2 x}$

Chapter 5 Trig

Figure 4.10: Tasks from Classroom Grade 12 Mathematics, Chapter 5, Exercise 5.9, page 147

A procedure that reviewed each task is explained below**:**

The cognitive demands of task 1(a), (b), 2(a), 4(a), (b), 5(a), 5(b) are judged in accordance with lower order cognitive demand. Consistent with descriptors of cognitive demands 'perform procedures' by Porter (2002), these tasks require the application of procedural knowledge and previously acquired operation that are familiar to students. The worked examples 1, 2 and 3 in page 146 covered similar types of tasks and students will follow

similar procedures in the examples to solve the tasks, hence they are considered lower order demands that require remembering the procedure.

Task 1c, 1d, 1e, 1f, 1g, 1h, 1i, 1j, 2b, 3a, 3b and 6 are interpreted in accordance with higher order cognitive demands. These tasks require suitable choices to be made between alternate representations of the cosine double angle and integrated processes of using knowledge from quadratics to solve these equations.

Task 7a, 7b, 8a, 8b and 9 are examples of tasks which require students to adapt a variety of strategies to solve non-routine problems, which is therefore a higher order cognitive demand.

4.8. SUMMARY OF ANALYSIS FROM CLASSROOM MATHEMATICS

A summary of the analytic review for Chapter 5, Exercise 5.1, 5.2 and 5.9 is given in the table below. The data counts shows how the tasks in Chapter 5, Exercise 5.1, 5.2 and 5.9 of Classroom Mathematics Grade 12 learner's textbook were coded with respect to the different categories of cognitive demands.

Table 4.27.: Allocation of cognitive demands for Chapter 5, Exercise 5.1; 5.2 and 5.9 of Classroom Mathematics Grade 12:

	Cognitive Demands					
Chapter 5, Exercise 5.1	**A**	**B**	**C**	**D**	**E**	**TOTAL**
1.a		1				1
b.		1				1
c.		1				1
d.		1				1
e.		1				1
f.		1				1
g.		1				1
h.		1				1
2.		1				1
3.			1			1
4.			1			1
5.			1			1
Chapter 5, Exercise 5.2						
1a.		1				1
b.		1				1
c.		1				1
d.		1				1
e.		1				1
f.		1				1

g.		1				1
h.		1				1
i.		1				1
j.		1				
k.		1				1
l.		1				1
2.			1			1
3.		1				1
4.			1			1
5.			1			1
6.			1			1
7.			1			1
8.			1			1
9.			1			1
10.			1			1
11.		1				1
12.			1			1
13.		1				1
Chapter 5, Exercise 5.9						
1a.		1				1
b.		1				1
c.			1			1
d.			1			1
e.			1			1
f.			1			1
g.			1			1
h.			1			1
i.			1			1
j.			1			1
2a.		1				1
b.			1			1
3a.			1			1
b.			1			1
4a.		1				1
b.		1				1
5a.		1				1
b.		1				1
6.			1			1
7a.				1		1
b.				1		1
8a.				1		1
b.				1		1
9.				1		1
TOTALS		**31**	**24**	**5**		**60**

The results for the analysis of cognitive demands of each task in exercise 5.1, 5.2 and 5.9 are summarised in Table 4.28 below.

Table 4.28.: Summary of categories of cognitive demands per exercise

	Exercise 5.1	Exercise 5.2	Exercise 5.9
Perform procedures - B	9 (75%)	15 (62.5%)	7 (29.2%)
Communicate understanding of concepts - C	3 (25%)	9 (37.5 %)	12 (50%)
Solve non-routine problems - D			5 (20.8%)

Based on the analysis, the highest numbers of tasks in exercise 5.1 are at lower order at cognitive demand B – 'perform procedures'**.** In total, 9 (75%) of the tasks are associated with lower order cognitive demand – 'perform procedures'. These tasks often involved the replication of basic information from the examples or preceding text and do not foster more thinking, strategies and the developments associated with higher order thinking processes. It is further observed from the analysis that only 3 (25%) of the tasks are associated with higher cognitive demand C – 'communicate understanding of concepts', with none of the tasks at cognitive demand levels D - 'solve non-routine problems' and cognitive demand level E of Porter (2002) cognitive demand framework.

Whilst there is an increase in the number of tasks in exercise 5.2, however, the cognitive demand of most of the tasks seems to cover lower levels of the cognitive demand framework. 62.5% (15 out of 24) of the tasks are consistent with reproducing material that has been seen before (Smith & Stein, 1998) and using familiar procedures for solving trig equations. Tasks associated with cognitive demand C – 'communicate understanding of concepts' only constitute about 37.5% (9) of the tasks.

According to Smith and Stein (1998) higher order level tasks involve complex thinking where the approach to the problem is not immediately obvious and not detailed in the preceding text or in the worked example. The analysis further shows that descriptors associated with cognitive demand D – 'solve non-routine problems' and cognitive demand E – 'conjecture, generalize and prove', are non-existent across tasks in exercise 5.2. This state of affairs indicates that the students may have fewer opportunities to engage in higher order thinking processes.

A noticeable observation in exercise 5.9 is that about 70.8 % of the tasks are categorised as higher order cognitive demands. The focus of these tasks are on exploring and linking

mathematical concepts and processes necessary to complete the tasks. Although some of the procedures to engage with these tasks have been provided beforehand, they require more cognitive efforts to explore the links between mathematical ideas. These tasks include making choices as to which alternate representations to choose particularly of the three cosine double angles identities. A student have to consider how and which identities to use to produce equations with quadratic trinomials in terms of sine and cosine.

Of the 70.8% (24) of tasks categorised as having higher order cognitive demands in exercise 5.9, 20.8% (5) of tasks are associated with cognitive demand D - 'solve non-routine problems'. Non-routine problems require mathematical thinking and the capacity to find an alternative strategy other than the one previously learned and to adapt the strategy to engage with the task. According to Altun (2005), non-routine problems require more thinking compared to routine problems given that the method to solve the task is not obvious. An advantage associated with non-routine tasks is that it assists students in structuring mathematical thoughts by themselves instead of just following procedures. For this reason, there is a conviction that exercise 5.9 provides the students with opportunities that are associated with higher order cognitive levels.

Earlier in the study it was detailed that Kilpatrick et al (2001) argued that conceptual understanding is concerned with the relationships between knowledge and the ability to link different aspects of knowledge. The tasks that are associated with higher order cognitive demands in exercise 5.9 require the use of procedures for the purpose of developing deeper levels of understanding. The crucial factor within these tasks is how alternate representations become connected to one another.

The allocation of the data counts for exercise 5.1, 5.2 and 5.9 of Classroom Mathematics is now shown in the data matrix Table 4.29 below. The allocation is done according to Porter's descriptors discussed earlier and it allows for the comparison of the relative emphasis of the cognitive demands by each exercise.

Table 4.29.: Matrix B with Data Counts for Chapter 5, Exercises 5.1, 5.2 and 5.9 in Classroom Mathematics Grade 12 - Adapted from Mhlolo (2011)

Topic Dimension (Trig Equations and the General Solution)	**Cognitive Demand Categories**							
	Lower Order Demand			**Higher Order Demand**				
	A	**B**	**Sub Totals**	**C**	**D**	**E**	**Sub Totals**	**Total**
Exercise 5.1 : Determine the general solution of the trigonometric equation (correct to one decimal places)		9	**9**	**3**			**3**	**12**
Exercise 5.2 : Solve the equations and leave answers correct to one decimal places		15	**15**	**9**			**9**	**24**
Exercise 5.9 : Solve the equations correct to one decimal places		7	**7**	**12**	**5**		**17**	**24**
TOTAL		**31**	**31**	**24**	**5**		**29**	**60**

As seen in the Table 4.29, the numbers of tasks in the section trig equations and the general solution in Classroom Mathematics Grade 12 that associates with lower order cognitive demands (31) is greater than the number of task that provides for higher order cognitive demands (29). Although the difference (31 versus 29) between the two categories of cognitive demands is small, in general it appears that the demand of most of the tasks is that of lower order cognitive demand B – 'perform procedures' of Porter's (2002) cognitive demand framework.

The percentage of each cognitive demand in exercise 5.1, 5.2 and 5.9 of chapter 5 in Classroom Mathematics is shown graphically in Figure 4.11 below to emphasize the cognitive demands being emphasized, or not emphasized by the tasks under review.

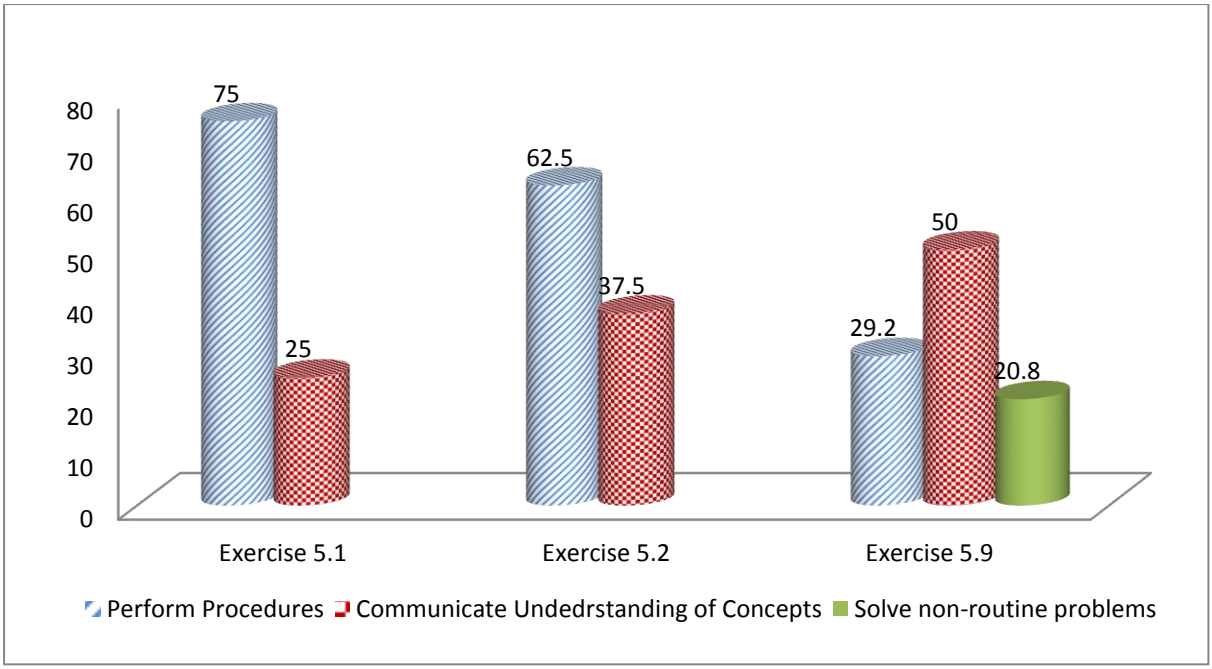


Figure 4.11.: Cognitive Demands of tasks in Grade 12 Classroom Mathematics, Chapter 5, exercises 5.1; 5.2 and 5.9.

4.9. COMPARATIVE ANALYSIS OF LEVELS OF COGNITIVE DEMANDS IN BOTH TEXTBOOKS

This study was not interested in the number of tasks per exercise covering the topic of trigonometric equations and the general solution, but more in measuring the levels of cognitive demands of tasks involving trigonometric equations and the general solution. In total 98 tasks from both textbooks (38 from Platinum Grade 12 and 60 from Classroom Mathematics Grade 12) were reviewed using Porter's (2000) frameworks of cognitive demands to assign judgement about the level of cognitive demands of each task. Each textbook was examined and analysed independently and the analysis of the cognitive demands of each textbook are shown graphically below.

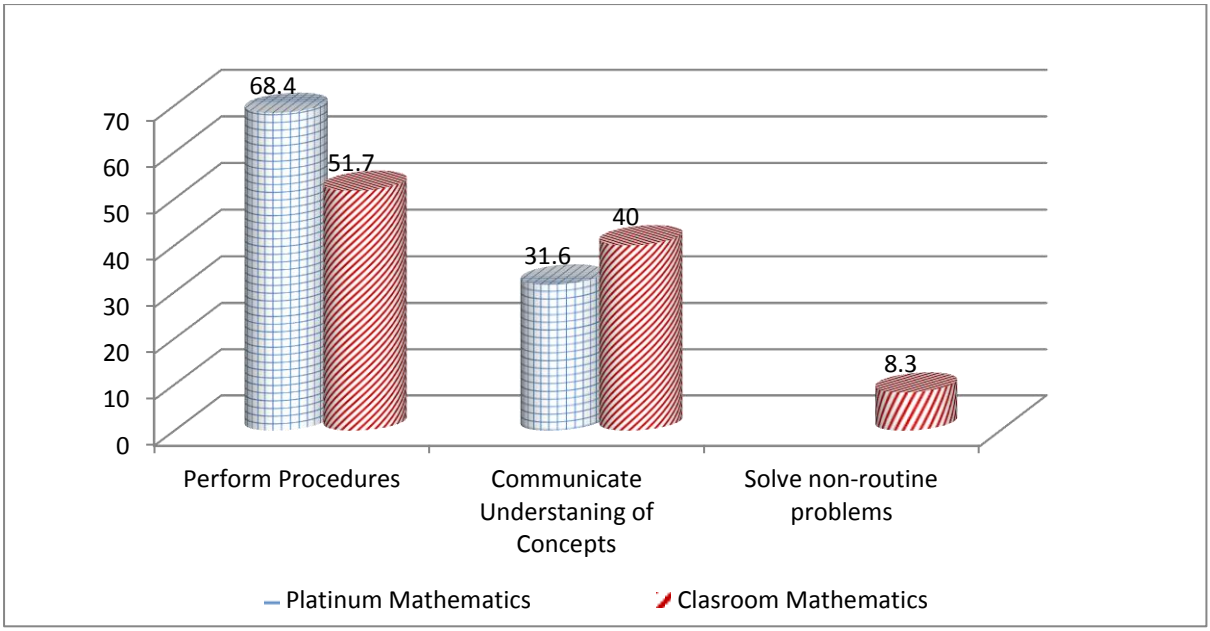


Figure 4.12.: comparative spread of cognitive demands in Platinum Grade 12 and Classroom Mathematics Grade 12.

The analysis revealed that Platinum Mathematics Grade 12 includes a greater proportion of lower order cognitive demand tasks than Classroom Mathematics Grade 12. A total of 26 out of 38 (68.4%) tasks in Platinum Mathematics Grade 12 showed the dominance of lower order cognitive demands at the level of 'performing procedures' – cognitive demand B. In contrast, only 12 (31.6%) of the tasks in Platinum Mathematics are in accordance with higher order cognitive demands. An assumption based on this analysis might be that Platinum Mathematics Grade 12 prioritises the approaches that merely repeat procedures without applying knowledge, insight and understanding to the tasks. Smith and Moore (1991) point out that much of what students actually learn from these sorts of tasks is a set of coping skills for getting past the next assessment.

CAPS suggests that at least 55% of assessments tasks must incorporate lower order type of questions (knowledge and routine procedures) and 45% should be tasks associated with higher order cognitive demands (complex procedures and problem solving). Judging by the data above and in line with the prescripts and expectation of CAPS, it can be argued that 68.4% is too extensive for lower order cognitive demands. The margin (13.4%) between the recommended CAPS percentage of higher order assessment tasks (45%) and the actual percentage of higher order assessment tasks (31.6%) is off the mark as far as the policy expectations are concerned. This observation suggests that students using these textbook

(majority in the public schooling sector) may have limited opportunities to engage in higher order thinking processes and to develop critical and analytical thinking in mathematics. These implications "undermines the goal of helping students with lesser socio economic status to close the gaps, thereby denying them equal educational opportunities" (Zohar & Dori, 2003, p. 146) to their peers who might have access to books promoting higher order tasks.

Results from the coding of the tasks in Classroom Mathematics textbook show that 31 of the 60 (51.7%) tasks are classified according to the cognitive demand that corresponds to the lower order cognitive demand category. Further analysis reveals that there are a noticeable number of the tasks that associate with higher order cognitive demand tasks in Classroom Mathematics. Cumulatively, a total 29 of the 60 tasks (48.3%) are classified as higher order cognitive demands in Classroom Mathematics. From this analysis it can be argued that Classroom Mathematics attempts a balance between lower and higher order tasks more in keeping with CAPS recommendations.

The table below shows the comparative data counts between the two textbooks; it summarises how tasks in each textbook under the study led to the results.

Table 4.30.: Matrix C with data counts for Platinum Grade 12 and Classroom Mathematics Grade 12

Topic Dimension (Trig Equations and the General Solution)		**Cognitive Demand Categories**					
		Lower Order Demand		**Higher Order Demand**			
		A	**B**	**C**	**D**	**E**	**Total**
Platinum Mathematics Grade 12	**Occurrence**		26	12			**38**
	Percentage		(68.4%)	(31.6%)			**100**
Classroom Mathematics Grade 12	**Occurrence**		31	24	5		**60**
	Percentage		(51.7%)	(40%)	(8.3%)		**100**

The results show that the majority of the tasks in each textbook are located towards the lower levels of Porter's (2002) cognitive demand framework. Generally, in Platinum Mathematics Grade 12 it is shown that a total of 68.4% (26 out of 38) of the tasks are

weighted towards lower order cognitive demands - 'perform procedures' as opposed to a total of 31.6% (12 of 38) devoted towards higher order cognitive demand. The analysis highlights that the majority of tasks in Platinum Mathematics Grade 12 often merely involve the generic information from the worked examples and do little to enhance higher order thinking. The cluster of most tasks towards lower order cognitive demands implies that the majority of the tasks in the textbooks do not fully develop students' ability to engage with more cognitively challenging tasks that foster thinking and the adapting of strategies to solve complex procedures.

The findings of this analysis with respect to Platinum Mathematics Grade 12 are consistent with earlier research that indicates that most textbooks place a dominant emphasis on tasks that promote lower order cognitive demands. While higher order cognitive demands are treated as a critical in the curriculum statement (CAPS) and endorsed in the examination papers, Platinum Mathematics Grade 12 seems to tone down this position. The predominance of the lower order cognitive demands in the Platinum Mathematics Grade 12 indicates that the textbooks does not comply with the provisions set in the CAPS FET Mathematics document with respect to the suggested cognitive level or lower and higher order prescriptions. A direct implication of this scenario is that students who use the Platinum Mathematics Grade 12 exclusively have limited opportunities as per policy prescripts to acquire skills associated with higher order demands.

On one hand, most public (state) schools, particular in Gauteng uses Platinum Mathematics Grade 12 as a form of learners support material (textbook) supplied by the state. Taking this into account, it may also be argued that the lack of emphasis on higher order cognitive demands in the Platinum Mathematics Grade 12 textbook possibly contributes to most Grade 12 students not being adequately prepared for NSC examination. This argument is in part supported by the quotes below from the DBE, 2015 examination diagnostic report. This report relates to learner performance in mathematics paper 2. It highlights that:

- Item-by-item analysis revealed that many candidates were mostly exposed to knowledge and routine type questions.
- Candidates showed confidence in dealing with work that they had seen previously.

- Candidates struggled with concepts in the curriculum that required deeper conceptual understanding.
- Questions where candidates had to interpret information or provide justification, presented the most challenges.

4.10. ANALYSIS OF THE SOUTH AFRICAN NATIONAL SENIOR CERTIFIATE MATHEMATICS PAPER 2 EXAMINATION QUESTION PAPERS

4.10.1. Background and Context of the NSC Examinations.

An analysis of the South African National Senior Certificate (Grade 12) Examination for Mathematics Paper 2 was made using the 2014 November final examination paper, 2015 November final examination paper and the 2016 Feb/March supplementary and 2016 November final examination papers. The 2015 Feb/March supplementary examination papers did not included questions on trigonometric equations and the general solutions and as such it is not included in the analysis. These examinations are the only set of CAPS National Senior Certificate Examinations available at the present moment. CAPS became policy in 2012 when it was rolled out to the Grade 10 cohort, with systematic progression that climaxed into the first Senior Certificate Examinations (Grade 12) in 2014. The same methodology (Porter's cognitive demands framework) used to analyse the NSC - CAPS textbooks was applied to the NSC - CAPS examination papers. The examination questions dealing with trigonometric equations and the general solution are analysed to judge the level of the cognitive demands posed by the examination questions.

The examination papers (2014, 2015 and 2016) had various questions covering the entire mathematics papers 2 grade 12 curriculum. However, in this study only questions relating to trigonometry equations and the general solution are reviewed. The questions are reviewed with references to the cognitive demands that the examinations questions demanded. The reviews are informed by Porter (2002) framework of cognitive demands and other associated literature on cognitive demands.

4.10.2. Analytical Review of the 2014 November Mathematics Paper 2 Examinations Question Paper.

As mentioned earlier in the study, according to the weighting of content areas as documented in CAPS, trigonometry weighs approximately 25% to 30% of the Mathematics

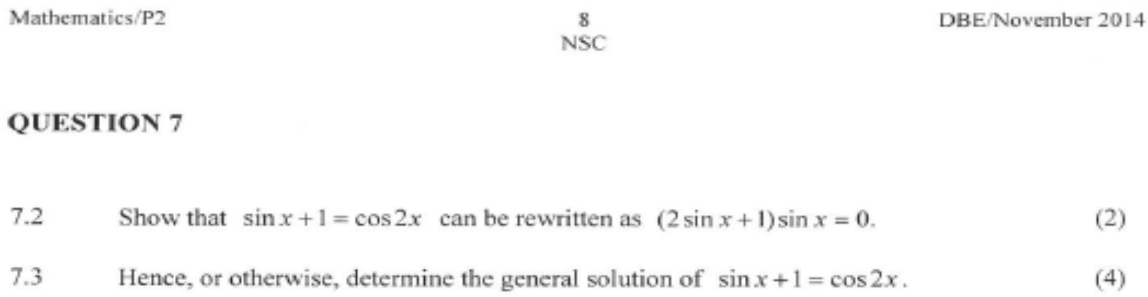
Paper 2 curriculum. This translates to approximately 40 ± 3 marks. This indicates that trigonometry marks must lie between the ranges of 37 to 43 out of 150 marks in each Mathematics Paper 2 Senior Certificate examination. The structure of the 2014 November NSC final mathematics paper 2 examination showed the distribution below relating to the trigonometry content.

Table 4.30.: Structure of the 2014 NSC Mathematics Paper 2 Trigonometry questions

QUESTIONS	CONTENT AREA	CONTENT WEIGHTING ACCORDING TO CAPS	MARKS
Q5	TRIGONOMETRY	40 ± 3	**9**
Q6			**12**
Q7			**19**
			TOTAL MARKS: 40

In the 2014 Senior Certificate examination, the question that relate to trigonometry and the general solution were found in Question 7, sub-question 7.3. The total trigonometry marks for this paper equal 40 marks and is in accordance with the recommended CAPS weighting of 40 ± 3 .

A procedure that reviews each examination question associated with trig equations and the general solution based on Porter (2000) framework is shown below:



Mathematics/P2 8 NSC DBE/November 2014

QUESTION 7

7.2 Show that $\sin x + 1 = \cos 2x$ can be rewritten as $(2\sin x + 1)\sin x = 0$. (2)

7.3 Hence, or otherwise, determine the general solution of $\sin x + 1 = \cos 2x$. (4)

Figure 4.13.: extract of questions from 2014 November NSC mathematics paper 2 Examinations.

Q.7.2. Show that $\boldsymbol{sin\, x + 1 = \ cos\, 2x}$ can be written as ($\boldsymbol{2\, sinx + 1)sin\, x = 0}$.

Table 4.32.: - suggested solution to Q.7.2. Nov 2014 Mathematics P 2

$\boldsymbol{sin\,x + 1 = cos\,2x}$ - (choose and change the cosine double angle to a suitable alternate cosine identity representation that will simplify the equation) $\boldsymbol{sin\,x + 1 = 1 - 2sin^2\,x}$ $\boldsymbol{2sin^2\,x + sin\,x = 0}$ $\boldsymbol{sin\,x(2sin\,x + 1) = 0}$ rearrange to $\boldsymbol{(2sin\,x + 1)sin\,x = 0}$

According to Porter (2002), lower order tasks require students to recall a fact, perform a simple operation and solve familiar types of problems. In contrast, the procedure to solve this task is not immediately recalled and explicit. Students may have to think and adapt strategies to engage with this task. As such it follows that this task is not one of the routine tasks and requires students to employ more cognitive efforts to deal with it. Additionally, in the context of mathematics to show means to illustrate, demonstrate or prove which are characteristics associated with some level of reasoning from the cognitive demand literature. To prove that $\boldsymbol{sin\,x + 1 = cos\,2x}$ can be rewritten as $\boldsymbol{(2\,sinx + 1)sin\,x = 0}$, students will need to be aware of the relationship between alternate representations of the cosine double ($\boldsymbol{cos\,2x}$) angle and to choose a suitable representation that will reduce the double angle into a single trigonometric function that will simplify the equation into a factorisable equation.

In line with the analysis performed earlier that encompasses $\boldsymbol{cos\,2x}$, it was highlighted that in working $\boldsymbol{cos\,2x}$ (double angle for cosine) there is an element of choice and the associated justification to choose a particular representation. Associated with similar and earlier interpretations that classify tasks where there is choice with the ones that are associated with high order cognitive demands, this task is therefore associated with higher order cognitive demand 'communicate understanding of concepts' – C and 'solve non-routine problems' – D. Consistent with earlier explanations regarding assigning judgement where there are two cognitive demands, the highest cognitive demand will therefore be used to categorise this task.

Q.7.3. Hence, or otherwise, determine the general solution of $\boldsymbol{sin\,x + 1 = cos\,2x}$. The word hence is explicitly mentioned, so students should be able to recognise that

information required to solve this task flows and is dependent on the preceding task. The suggested solution below shows that to engage with this task, the form $\boldsymbol{\sin x(2\sin x + 1)}$ **= 0,** obtained in the preceding question will be used to find the general solution.

Table 4.33.: - suggested solution to Q.7.3. Nov 2014 Mathematics P 2

$\boldsymbol{\sin x(2\sin x + 1) = 0}$
$\boldsymbol{\sin x = 0}$ or $\boldsymbol{2\sin x + 1 = 0}$
$\boldsymbol{\sin x = 0}$ or $\boldsymbol{\sin x = -\frac{1}{2}}$
Then $\boldsymbol{\sin^{-1}0 = 0}$ **° or** $\boldsymbol{\sin^{-1} -\frac{1}{2}}$ **= - 30° - reference angles**
Using the General solution for sine:
Solution 1
$\boldsymbol{x = 0° + k.360°, k \in Z}$ or $\boldsymbol{x = 180° + k.360°, k \in Z}$
Other Solution
$\boldsymbol{x = 210° + k.360°, k \in Z}$ or $\boldsymbol{x = 330° + k.360°, k \in Z}$

In solving the above task and for finding the general solution, the process leading to the solution will follow straightforward and familiar procedures for finding the general solution. This task is therefore associated with descriptors associated with cognitive demand B - 'perform procedures'

Table 4.34.: judgements of cognitive demands for question7.3 of 2014 November NSC Mathematics Paper 2 Examination.

Q.7.2	**D**	- Adapt strategies to Solve non-routine tasks
Q.7.3	**B**	- Follow routine procedures for finding the general solution

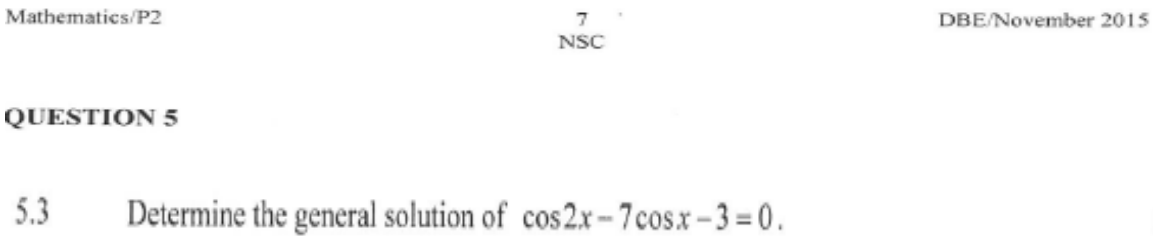
4.10.3. Analytical Review of the 2015 November Mathematics Paper 2 Examinations Question Paper.

The trigonometry questions dealing with the general solution were found within question 5 in the 2015 November Senior Certificate examination. Question 5 is sub divided into four sub question: 5.1; 5.2; 5.3 and 5.4. Sub question 5.1 was further divided into 5.1.1, 5.1.2 and 5.1.3. The distribution of marks for each question is shown in Table 4.35.

Table 4.35.: Structure of the 2015 NSC Mathematics Paper 2 Trigonometry questions

QUESTIONS	CONTENT AREA	CONTENT WEIGHTING ACCORDING TO CAPS	MARKS
Q5	TRIGONOMETRY	40 ± 3	**24**
Q6			**8**
Q7			**10**
			TOTAL MARKS: 42

The total trigonometry marks for this paper is 42 and is in agreement with the recommended CAPS weighting of the range between 37 and 43 marks.



Mathematics/P2 7 NSC DBE/November 2015

QUESTION 5

5.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$. (6)

Figure 4.14.: extract of question from 2015 November NSC Mathematics Paper 2 Examination.

As Figure 4.14 displays, only question 5.3 was analysed as it is the only question that relates to a trigonometry equation and the general solution. To assign judgement about the cognitive demands demanded by the question, the interpretations as in Table 4.36 below are provided.

Table 4.36.: judgements of cognitive demands for question 5.3 of 2015 November NSC Mathematics Paper 2 Examination.

Q.5.3	**C**	- Recognising $\boldsymbol{cos\ 2x}$ as a double angle and link it to other representations of cosine double angle identity. - Recognising the equation as quadratic and follow routine procedures for solving quadratic equations, however, the highest demand of the question is cognitive demand – C – link relationships, therefore the category of the task.

The question required the linking and the relationship between concepts and as such categorised as a higher order cognitive demand C – 'communicate understanding of concepts' using descriptors in the literature.

4.10.4. Analytical Review of the 2016 Feb/Mar November Mathematics Paper 2 Examinations Question Paper.

Table 4.37.: Structure of the 2016 Feb/Mar NSC Mathematics Paper 2 Trigonometry questions

QUESTIONS	CONTENT AREA	CONTENT WEIGHTING ACCORDING TO CAPS	MARKS
Q5	TRIGONOMETRY	40 ± 3	**13**
Q6			**13**
Q7			**15**
			TOTAL MARKS: 41

The three trigonometry questions in this paper make up a total of 41 marks. This range is consistent with the recommendation of weighting of content according to CAPS.

Mathematics/P2 8 NSC DBE/Feb.–Mar. 2016

QUESTION 6

Given the equation: $\sin(x + 60°) + 2\cos x = 0$

6.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)

6.2 Determine the solutions of the equation $\sin(x + 60°) + 2\cos x = 0$ in the interval $-180° \leq x \leq 180°$. (3)

Figure 4.15.: extract of question from 2016 Feb/Mar NSC Mathematics Paper 2 Examination.

Manipulation of $\boldsymbol{\sin(x + 60°) + 2\cos x = 0}$ will result in the equivalent equation: $\boldsymbol{\tan x = -4 - \sqrt{3}}$. This equation will be used to calculate the general solution in 6.1. So Question 6.2 is dependent on Question 6.1 and it means that the questions are interrelated and both questions will be analysed to judge the associated cognitive demands. The suggested solution is shown below.

Table 4.38.: suggested solution to Q.6.1. Feb/Mar 2016 Mathematics P 2

$\boldsymbol{s}in(\ x + 60°) + 2cos\ x$ = 0
$sin\ x\ cos\ 60° + cos\ x\ sin\ 60° + 2cos\ x$ = 0 expanding and rewriting in a format for $sin(\ A + B)$
$\frac{1}{2}\ sin\ x\ \ \frac{\sqrt{3}}{2}\ cos\ x + 2\ cos\ x\ $ = 0 special angles values
$\frac{1}{2} sin\ x\ = -2cos\ x - \frac{\sqrt{3}}{2} cos\ x$
$sin\ x\ = -4cos\ x - \sqrt{3} cos\ x$ simplification
$sin\ x\ = cosx\ (-4 - \sqrt{3})$ simplification
$\frac{sin\ x}{cos\ x} = \frac{cos\ x\ (-4 - \sqrt{3})}{cos\ x}$ simplification
$\therefore \boldsymbol{tan\ x}\ = \boldsymbol{-4 - \sqrt{3}}$ tan identities is $\frac{sin\ x}{cos\ x}$

Table 4.39.: suggested solution to Q. 6.2. Feb/Mar 2016 Mathematics Paper 2

$tan\ x\ = -4 - \sqrt{3}$
$tan\ x\ = -(4 + \sqrt{3})$
$tan\ x\ = -(5.732050808..)$
$tan^{-1} - (5.732050808..)$ =
Using the General solution for tan:
x = $tan^{-1} - (5.732050808..) + k.180°, k \in Z$
x = $-80.1° + k.180°, k \in Z$
From the general solution: $\boldsymbol{x}$ = $\boldsymbol{-80.1° + k.180°, k \in Z}$. Interpretation of the solution has to be made in the specified interval. To find the solutions in the specified interval, we use the integral values of *k*
i.e. if k=-1, then $\boldsymbol{x = -260.1°}$, outside the specified interval

$k = 0$, then $\boldsymbol{x = -80.1°}$, within the specified interval $k = 1$, then $\boldsymbol{x = 99.9°}$, within the specified interval For solution in the interval -180°$\leq x \leq 180°$, $\therefore x$ has to be -80.1° or 99.9°

Table 4.40.:- judgements of cognitive demands for question 6.1 – 6.2 of 2016 Feb/Mar NSC Mathematics Paper 2 Examination.

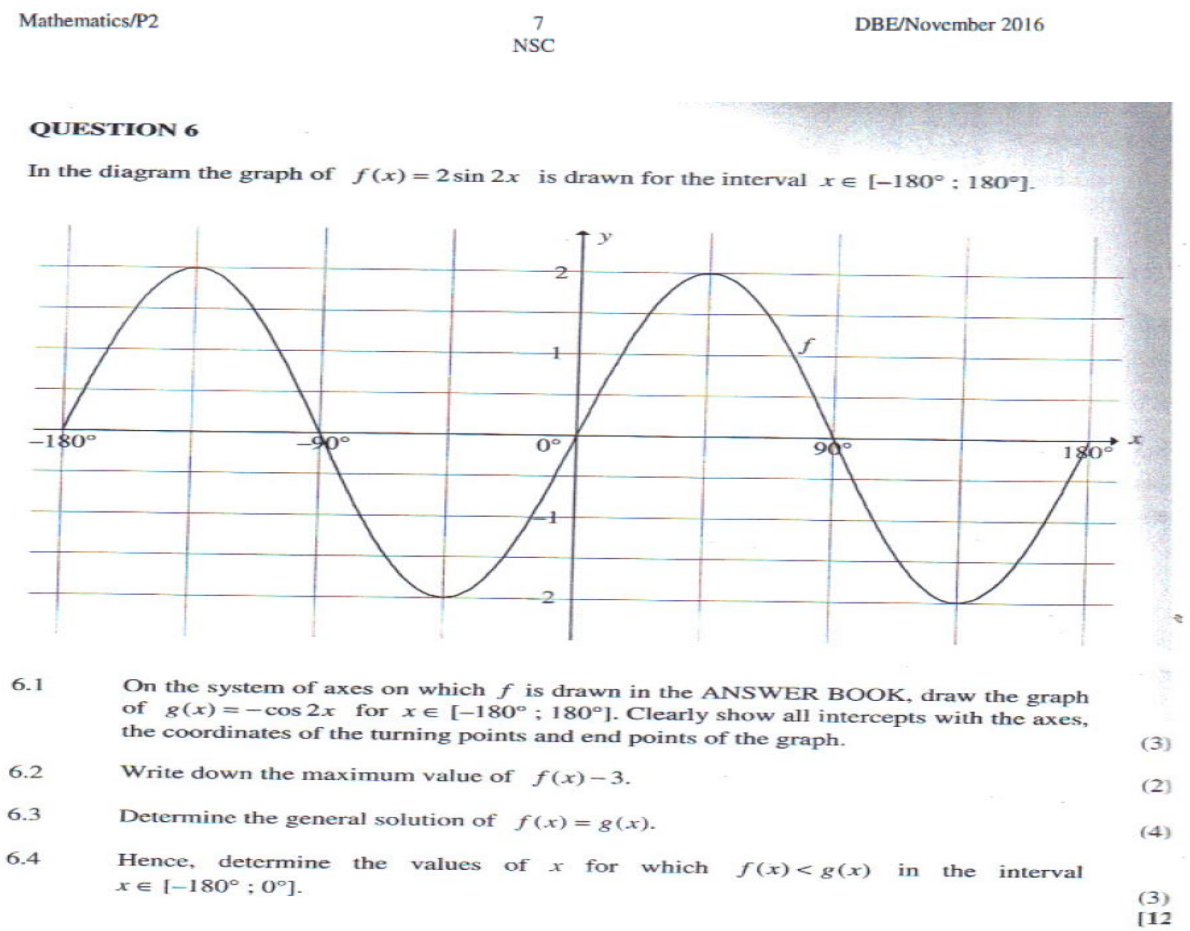
Q.6.2	**B**	- Recognising $\boldsymbol{sin\,(x + 60°)}$ as a compound angle identity. Students are exposed to compound angles of such nature $sin\,(A + B)$, and will just need to insert $x\ and\ 60°$ into the previously learned formula and follow familiar procedures for working with compound angles. The cognitive demand will therefore be 'perform procedures' – B.
Q.6.3	**C**	- The possible solution follows rehearsed procedures for obtaining the general solution of $tan\ x$**.** The question does not explicitly say find the general solution but it is always encouraged that the general solution must be find first and then solve for specific solutions in a given interval**.** - As mentioned elsewhere in the analysis, interpreting the solution in the specified interval requires a justification for why other solutions are excluded and is associated with the descriptor of a higher order cognitive demand 'C' in the literature that require "explain finding and results from data analysis strategies" (Porter, 2002).

4.10.5. Analytical Review of the 2016 November Mathematics Paper 2 Examinations Question Paper.

Table 4.41.: Structure of the 2016 Nov NSC Mathematics Paper 2 Trigonometry questions

QUESTIONS	CONTENT AREA	CONTENT WEIGHTING ACCORDING TO CAPS	MARKS
Q5	TRIGONOMETRY	40 ± 3	**21**
Q6			**12**
Q7			**12**
			TOTAL MARKS: 45

In this examination paper, the total trigonometry marks is 45 and slightly outside the range of between 37 and 43 (± 40) by 2 marks. However it is consistent with 30% weighting of trigonometry curriculum within CAPS curriculum FET mathematics paper 2.



Mathematics/P2 7 NSC DBE/November 2016

QUESTION 6

In the diagram the graph of $f(x) = 2\sin 2x$ is drawn for the interval $x \in [-180° ; 180°]$.

6.1 On the system of axes on which f is drawn in the ANSWER BOOK, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$. Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)

6.2 Write down the maximum value of $f(x) - 3$. (2)

6.3 Determine the general solution of $f(x) = g(x)$. (4)

6.4 Hence, determine the values of x for which $f(x) < g(x)$ in the interval $x \in [-180° ; 0°]$. (3)

[12

Figure 4.16.: extract of question from 2016 Nov NSC Mathematics Paper 2 Examination.

Question 6.3 and 6.4 are interdependent since the general solution obtained in 6.3 will be instrumental in answering question 6.4. The suggested solution is shown below.

Table 4.42.: suggested solution to Q.6.3. Nov 2016 Mathematics P 2

$\mathbf{2}\ \boldsymbol{sin}\ \mathbf{2}\boldsymbol{x}\ = -\ \boldsymbol{cos}\ \mathbf{2}\boldsymbol{x}$

$\frac{\mathbf{2}\ \boldsymbol{sin}\ \mathbf{2}\boldsymbol{x}}{\boldsymbol{cos}\ \mathbf{2}\boldsymbol{x}} = \frac{-\ \boldsymbol{cos}\ \mathbf{2}\boldsymbol{x}}{\boldsymbol{cos}\ \mathbf{2}\boldsymbol{x}}$ $\frac{\boldsymbol{sin}\ \mathbf{2}\boldsymbol{x}}{\boldsymbol{cos}\ \mathbf{2}\boldsymbol{x}} \approx \boldsymbol{tan}\ \mathbf{2}\boldsymbol{x}$ identities

$\therefore \boldsymbol{tan}\ \mathbf{2}\boldsymbol{x}\ =\ -\frac{\mathbf{1}}{\mathbf{2}}$

Using the General solution for tan function:

$\mathbf{2}\boldsymbol{x} = \boldsymbol{tan}^{-1}(-\frac{\mathbf{1}}{\mathbf{2}}) + \boldsymbol{k}.\mathbf{180}°, \boldsymbol{k} \in\ \boldsymbol{Z}$

$\mathbf{2}\boldsymbol{x} = -\mathbf{26}.\mathbf{6}° + \boldsymbol{k}.\mathbf{180}°, \boldsymbol{k} \in\ \boldsymbol{Z}$

$\boldsymbol{x} = -\mathbf{13}.\mathbf{3}° + \boldsymbol{k}.\mathbf{90}°, \boldsymbol{k} \in\ \boldsymbol{Z}$

Q.6.3. Determine the general solution of $f(x) = g\ (x)$. The equation: $\mathbf{2}\ \mathbf{sin}\ \mathbf{2}\boldsymbol{x} = -\ \mathbf{cos}\ \mathbf{2}\boldsymbol{x}$ contains double angle on both sides of the equation. However, students will need to be pre-emptive and recognise that applying double angles will not simplify the equation but complicate it. Although the task is more procedural, students will need to think about other equivalent identities to engage with this task.

Table 4.43.: suggested solution to Q. 6.4. Nov 2016 Mathematics Paper 2

Using the General solution obtained in Q.6.3 above: $\boldsymbol{x} = -\mathbf{13}.\mathbf{3}° + \boldsymbol{k}.\mathbf{90}°, \boldsymbol{k} \in\ \boldsymbol{Z}$, to find the solutions in the specified interval $\boldsymbol{x}\ \in [-\mathbf{180}°;\ \mathbf{0}°]$. Using the integral values of *k*, we get

if k=-1, then $\boldsymbol{x} = -\mathbf{103}.\mathbf{3}°$, within the specified interval

$k = 0$, then $\boldsymbol{x} = -\mathbf{13}.\mathbf{3}°$, within the specified interval

$k = 1$, then $\boldsymbol{x} = \mathbf{76}.\mathbf{7}°$, outside the specified interval

For solution in the interval $\in [-\mathbf{180}°;\ \mathbf{0}°]$, x has to be - 103.3° or - 13.3°

The interpretations of the judged cognitive demands are illustrated in Table 4.43 below.

Table 4.44.: judgements of cognitive demands for question 6.3 – 6.4 of 2016 Nov NSC Mathematics Paper 2 Examination.

Q.6.3	**B**	- The process leading to the solution of this task requires recognising that the use of the tan quotient identity will

		simplify the equation, i.e. $\tan 2x = \frac{\sin 2x}{\cos 2x}$. - The process will follow rehearsed procedures for solving trigonometric equation, obtaining $\mathbf{tan^{-1}}\left(-\frac{\mathbf{1}}{\mathbf{2}}\right)$ and following familiar processes for obtaining the general solution for tan function.
Q.6.4	**C**	- Similar interpretation earlier placed demands of similar tasks where intervals are specified into cognitive demand C- 'Communicate understanding of concepts'

The interpretations of the NSC examinations papers will now be shown in Matrix C below to place the data counts of each question into the appropriate columns of cognitive demand using the Porter (2000) cognitive demand framework.

4.11. SUMMARY OF ANALYSIS FROM EXAMINATION PAPERS

A summary of the analytic review for the 2014, 2015 and 2016 NSC examination papers is presented in the table below. The data counts shows how the tasks in the NSC examination papers were coded with respect to the different categories of cognitive demands.

Table 4.45.: Matrix D for data counts of the cognitive demands in the NSC examination questions - Adapted from Mhlolo (2011)

Topic Dimension (Trig Equations and the General Solution in NSC Examination)	**Cognitive Demand Categories**					
	Lower Order Demand		**Higher Order Demand**			
	A	**B**	**C**	**D**	**E**	**Total**
NSC Examinations Mathematics Paper 2		3	**3**	**1**		**7**
TOTAL		**3**	**3**	**1**		**7**

In the NSC examinations tasks higher order cognitive demands and processes appears to be emphasised. The findings are supported by 57.1% (4 out of 7) of the tasks that lies against higher order cognitive demands.

4.12. DISCUSSIONS and FINDINGS

The first research question is:

"What kinds of cognitive demands are implicit in the instructional tasks around the general solutions of trigonometric equations in two Grade 12 NCS (CAPS) prescribed mathematics textbooks and in the assessment tasks in the NSC examinations"?

In order to offer an answer to this question, the percentages of tasks by different cognitive demands in each textbook and the examination papers are summarised in Table 4.46.

Table 4.46.: Matrix E for data counts of the cognitive demands across all documents analysed - Adapted from Mhlolo (2011)

Trig Equations and the General Solution (in all documents analysed)	Cognitive Demand Categories							
	Lower Order Demand			Higher Order Demand				
	A	B	Sub Totals	C	D	E	Sub Totals	Total
Platinum Mathematics Grade 12		26 (68.4%)	**26 (68.4%)**	12 (31.6%)			**12 (31.6%)**	**38**
Classroom Mathematics Grade 12		31 (51.7%)	**31 (51.7%)**	24 (40%)	5 (8.3%)		**29 (48.3%)**	**60**
NSC Examinations Mathematics Paper 2		3 (42.9%)	**3 (42.9%)**	3 (42.9%)	1 (14.2%)		**4 (57.1%)**	**7**
TOTAL		**60**	**60**	**39**	**6**		**45**	**105**

Comparatively and as indicated earlier in the analysis, an increase is noticed in the percentage of the higher order cognitive demands in Classroom Mathematics Grade 12. This interpretation does not necessarily suggest that Classroom Mathematics is a better textbook than Platinum Mathematics. The submissions here primarily highlight that based on the analysis Classroom Mathematics Grade 12 textbook is more likely to provide opportunities for students to engage with higher order cognitive demand tasks as compared to Platinum Mathematics Grade 12 textbook in the section reviewed. These beliefs blends with Haggarty and Pepin's (2002) inferences that students have varying opportunities to learn depending on the textbook they use. Similarly, Stein et al., (2000) highlight that

student learning gains are greatest in instructional tasks which consistently encouraged higher order thinking and reasoning and reduced in classrooms in which instructional tasks are consistently dominated by procedures.

Important issues around the above discussion are that: although Classroom Mathematics Grade 12 appears in the Department of Basic education (DBE) catalogue for approved textbooks, the majority of students in no fee public schools (quintile 1 to 3 schools, which is the biggest portion of the public school domain) are provided with Platinum Mathematics Grade 12 textbook without charge and only students with additional means procure Classroom Mathematics Grade 12 as an extra resource to supplement their learning.

Since this study does not take the enacted curriculum (teachers and their teaching styles, methods, etc. and students responses) into account, the study is unable to confirm with certainty that the students using Classroom Mathematics as a primary textbook are indeed provided with opportunities to engage with higher order tasks or whether the teachers still continue to choose tasks that promote lower order demands for teaching and learning during. This is area for further research.

On the other hand, the analysis of the NSC examinations for mathematics paper 2 demonstrates that the examinations papers are weighted towards higher order cognitive demands. The 57.1% share of tasks classified as higher order cognitive demands in the NSC examination papers could be an indication that there is a shift away from simple recall to more demanding skills. The shift towards higher cognitive levels tasks is in line with Boaler and Staples (2008.); Gutiérrez (2000) thinking that mathematical achievement improves and gaps diminishes when students experienced instruction focused on problem solving, conjecturing, and explanation and justification of ideas.

The similarity in the NSC examination papers however indicates that the majority of tasks with higher order cognitive demand levels are still clustered towards cognitive demand C – ‘communicate understanding of concepts’ and that other categories of higher order cognitive demand such as ‘solve non-routine problems and conjecture, generalise and prove’ are still in the minority or completely absent in most tasks.

The graph below shows the comparison of the spread of cognitive demands across all documents analysed.

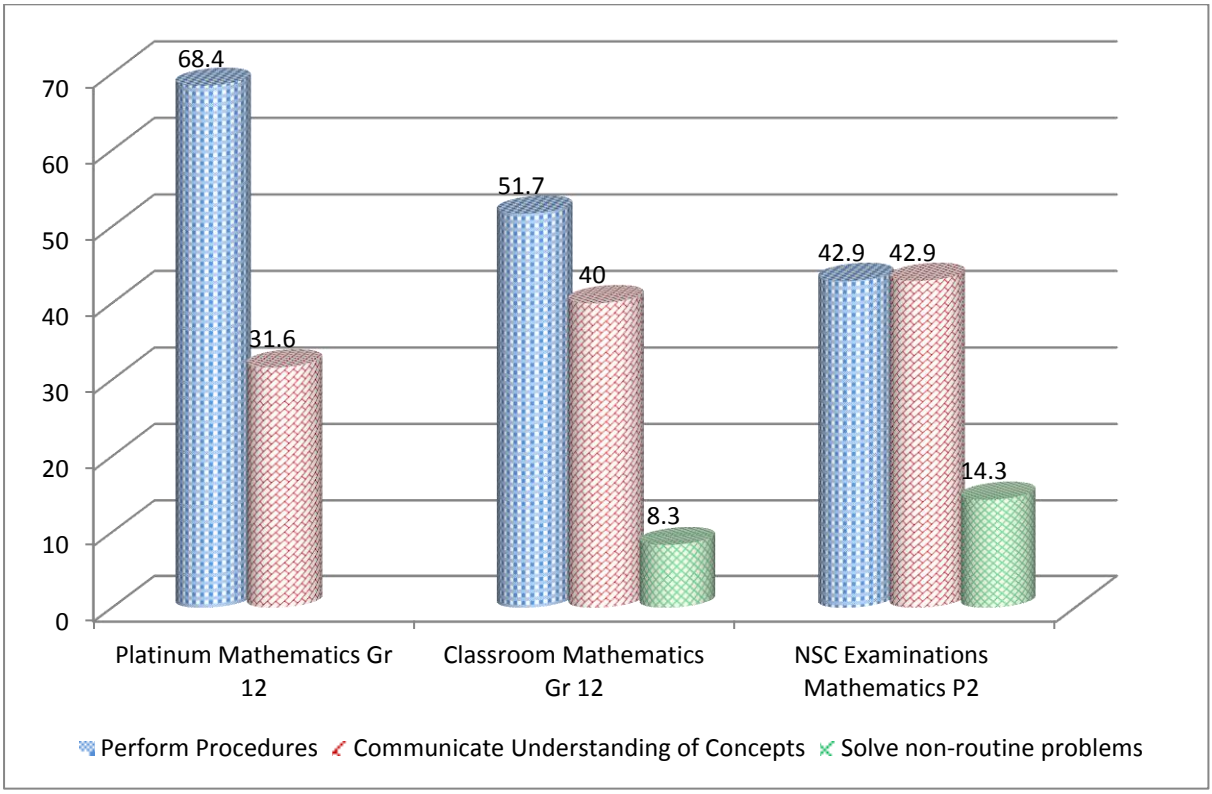


Figure 4.17.: Summary of the cognitive demands across all documents analysed

The findings of this study in relation to Platinum Mathematics Grade 12 are in agreement with the study by Alcazar (2007) who investigated the degree of alignment of cognitive demand between the Peruvian official curriculum, the national assessments, teaching and the approved textbook. Her study found significantly lower cognitive demands tasks among the official curriculum and the approved textbooks, while the cognitive demands of the national assessment tasks were more aligned towards higher order categories such as Problem Solving and Comprehension. The finding of Riazi and Mosallanejad (2010) also found that the lower order cognitive processes were more frequent in English as First Language and English as Language of Teaching textbooks in Iran.

Edwards (2010) also investigated the levels of cognitive demand and coherence between the South African Physical Sciences Curriculum for Grade 12 and the 2008 Senior Certificate Physical Sciences exemplar papers and the 2008 and 2009 Physical Sciences Senior Certificate Examination papers. The investigation revealed that the focus in each curriculum was on lower order cognitive and process skills.

This study also finds that lower order cognitive demands are over represented in the instructional tasks around the general solutions of trig equations in the most popular prescribed mathematics textbooks for Grade 12 (Platinum Mathematics) but the assessment tasks in the NSC assessments tasks tends to shift towards higher order demands.

In view of the critical role played by the textbook as a bridge between the official declaration of content standards and the actual tasks with which students engage (Schmidt, McKnight, & Raizen, 1997), the findings of this study raise concerns about the extent to which curriculum prescripts are being translated into practice. However it must be noted that while the present study was attempting to evaluate the cognitive demands of tasks in two popular Grade 12 textbooks, it did not include all the trigonometry tasks in the mentioned textbooks nor did it examine all the mathematics Grade 12 prescribed textbooks in use in South Africa. Therefore, further investigation is needed to track the cognitive demands of all tasks and to examine if these results are consistent across the content of trigonometry in a larger sample of the prescribed and approved textbooks.

4.13. ANALYTIC TOOL FOR RESEARCH QUESTION 2

The next step after having completed the placement of data counts into appropriate categories of cognitive demands (see Matrix A, B, C and D), was to determine the match or mismatch between curricular documents (textbooks and NSC examination question in this case). Näsström, G. (2008), argue that when components of the education system work together, have matching expectations, are in agreement and serve in conjunction with one another, they are said to be aligned.

Porter (2002) developed an alignment model using an alignment index to describe the match or mismatch between various curricular components. According to Liu and Fulmer, (2008), Porter's alignment model has two advantages over other models: it adopts a common language to describe curriculum, instruction and assessment; and it produces a single number as alignment index. An alignment index can be determined in two ways, each of which complements the other and which yield related mathematical results. It must be highlighted that the two alignment methods do not necessarily yield the same results but approximations that are similar and comparable. The possible values of the alignment index

range from 0 to 1 with 0 indicating no alignment and 1 indicating perfect alignment (Liang & Yuan, 2008).

To measure the level of alignment between two sets of curricular documents (textbooks and NSC examination question) in this study, an alignment index is produced. An alignment measure is produced by comparing the level of agreement of proportion in each corresponding cells of the two matrices. The proportions are obtained by dividing each data count in each cell (cell values) by the grand total of data counts in that matrix. This process converts and standardises each cell value into proportions of the grand total (Liang & Yuan, 2008). The proportional values across all cells of a matrix for any given document should add up to 1. The proportions are then used to determine the alignment index and to make comparisons between different curricular components. It should be noted that the analysis of alignment does not rate the quality of tasks but rather the relationship in terms of the cognitive demand levels.

The first procedure to determine alignment index is easier to compute (Roach, et al., 2008) and readily understood. The process as explained below is repeated for each pair of corresponding cells in the two compared matrices. Each cell value in the matrix is divided by the grand total value of that matrix to produce proportions in each cell of each matrix. Proportions in the corresponding cells of the two matrices are then compared and the smaller proportion is identified for alignment computational purposes. Finally all the smaller values are added together to obtain the final alignment value. It is on this alignment value that the interpretations of the measure of match or mismatch are made.

The second procedure calculates the alignment index using the following formula:

$$\mathbf{AI} = 1 - \frac{\sum|\boldsymbol{x}-\boldsymbol{y}|}{\mathbf{2}}$$

where ***x*** denotes cell proportion in one matrix and ***y*** denotes cell proportion in another matrix. The argument in favour of these types of alignment analyses are that they provides a relatively precise mathematical procedure for calculating the degree of similarity between any two curricular components that are using the same language framework (Edwards, 2010; Squires, 2009).

The second research question asks "***To what extent do the cognitive demands of tasks around the general solutions of trigonometric equations in two Grade 12 NCS (CAPS) prescribed textbooks align with the cognitive demands of the assessment tasks around the general solution of trigonometric equations in the NSC Examination*"?**

To answer this question, each cell value in the total row of the data counts for each of the two textbooks in Tables 4.30, Matrix C, (in page 84) was used to produce proportional quantification as in Matrix F and G in Table 4.47 and 4.48 below.

Table 4.47.: Matrix F with proportional values for Platinum Mathematics Grade 12 - Adapted from Mhlolo (2011)

Topic Dimension (Trig Equations and the General Solution)	**Cognitive Demand Categories**				
	Lower Order Demand		**Higher Order Demand**		
	A	**B**	**C**	**D**	**E**
Determine the general solution for each equation in Platinum Mathematics Grade 12	0	0.7	0.3	0	0

The proportional quantification for each cell of the Platinum Mathematics textbooks matrix is computed as shown in the following example which involves the cognitive demand B cell: $\frac{26}{38}$ = 0.684210526, rounded to 1 decimal place ≈ 0.7. The proportions are obtained by taking data count in each cell of the total row (e.g. wherein the total in cell B is 26) and divide by the total data count of all the cells (wherein the grand total of lower and higher order cognitive demands totals is 38) to obtain the proportional value of 0.7. The procedure was repeated for every cell of the totals row column to give proportional values in Table 4.47 Matrix F above.

The next process was to compute the proportional values for the Classroom Mathematics Grade 12 textbook. A similar process of computing the data counts for the content of the Platinum Mathematics textbook is used for the Classroom Mathematics from Table 4.29 Matrix B to produce Table 4.48 Matrix G with proportional values for Classroom Mathematics Grade 12 as shown below.

Table 4.48.: Matrix G with proportional values for Classroom Mathematics Grade 12 - Adapted from Mhlolo (2011)

Topic Dimension (Trig Equations and the General Solution)	**Cognitive Demand Categories**				
	Lower Order Demand		**Higher Order Demand**		
	A	**B**	**C**	**D**	**E**
Determine the general solution for each equation in Classroom Mathematics Grade 12	0	0.5	0.4	0.1	0

After computing the proportional values for each textbook, the next step was to compute the proportional values for the NSC assessment tasks. A similar process of computing the data counts for the content of the textbooks is used for the NCS examinations from Table 4.45 Matrix D to produce Table 4.49 Matrix H with proportional values for the NSC examinations as shown below.

Table 4.49: Matrix H with proportional values for NSC Examinations - Adapted from Mhlolo (2011)

Topic Dimension (Trig Equations and the General Solution)	**Cognitive Demand Categories**				
	Lower Order Demand		**Higher Order Demand**		
	A	**B**	**C**	**D**	**E**
NSC Examinations Mathematics Paper 2	0	0.4	0.4	0.2	0

From the proportional values, it is now possible to attempt to answer the second research question of this study. To determine the level of alignment between these two sets of curricular components (textbooks and examinations), the two earlier explained procedures are used. It can be either procedure 1 or procedure 2 but it is not compulsory to use both procedures simultaneously. In this study both procedures are used to establish if they yield comparable results considering the limited data obtained from the NSC examination questions.

4.14. Calculation of Alignment Index for proportional values between Platinum Mathematics and the NCS examinations:

Using the first procedure, the proportion between corresponding cells (cognitive demand B) in Matrix F (Platinum Mathematics Grade 12 textbook) and Matrix H (NSC Examinations) is 0.7 and 0.4 respectively. The smaller proportion is 0.4 and will be used in the alignment calculation process. The smaller proportion of cognitive demand C in Matrix F and Matrix H is 0.3. For cognitive demand D the smaller proportion is 0. The final alignment index will be obtained by summing up the smallest proportions between each pair of the corresponding cells. The smallest proportion between the corresponding cells in this study is 0.3, 0.4 and 0. The smallest proportion will then be summed up to obtain the alignment value of 0.7.

The second procedure uses Porter alignment index formula: **AI = 1** $- \frac{\sum|x-y|}{2}$

Porter alignment index totals the absolute value of the difference between each pair of corresponding cells across Matrix F in Table 4.47 and Matrix G in Table 4.48. The total is then divided by 2 and the result is subtracted from 1 to end up with the alignment index.

The proportion values between the Platinum Mathematics and the NSC examinations questions resulted in Matrix F and H above (with abridged versions below)

Platinum Mathematics

Proportional Values				
A	**B**	**C**	**D**	**E**
0	0.7	0.3	0	0

NSC Examinations

Proportional Values				
A	**B**	**C**	**D**	**E**
0	0.4	0.4	0.2	0

Using the alignment index formula, then:

$$\text{AI} = 1 - \frac{\sum|x-y|}{2}$$

$$= 1 - \frac{|0.7-0.4|+|0.3-0.4|+|0-0.2|}{2}$$

$$= 1 - \frac{|0.3|+|-0.1|+|-0.2|}{2} = 1 - \frac{(0.3+0.1+0.2)}{2} = 1 - \frac{(0.6)}{2} = 1 - 0.3 = 0.7$$

Both procedures yield alignment indices of 0.7. As mentioned earlier a typical alignment score lies between 0 and 1, with 0 indicating no match or alignment at all and 1 signalling perfect alignment (Liang & Yuan, 2008). Using a natural number scale from 0 to 1 to describe alignment criterion, it follows that alignment of 0.5 is characterised as moderate alignment, whereas indices less than 0.5 signals weaker alignment and anything above 0.5 indicates a strong alignment. Office of Superintendent of Public Instruction (2005 & 2006) in the Washington State Institute for Public Policy used the 4-point scale below to describe the degree of alignment:

- Full or Strong alignment - both content and cognitive demand is aligned.
- Moderate alignment - all content and some cognitive demands are aligned.
- Weaker or Partial alignment - content but not cognitive demand is aligned.
- No alignment - neither content nor cognitive demand is aligned.

The alignment indices of 0.7 obtained in the analyses between Platinum Mathematics textbook and the NCS assessment tasks reflects that almost 70% of the cognitive demands of the tasks in the Platinum Mathematics and NSC examinations are in agreement, but the degree of the agreement is not as strong as suggested by the curriculum statement. This can be traced to the 13.4% variance between the recommended CAPS percentage of 45% for higher order and the actual percentage of 31.6 % for higher order tasks in Platinum Mathematics Grade 12. Also, as highlighted earlier, Platinum Mathematics includes more tasks (68.4%) at lower level cognitive demand and the NSC examinations include about (57.1%) of tasks at higher level. This difference in cognitive demand levels seems to be the main contributing factor for the lack of a stronger relationship or perfect alignment.

4.15. Calculation of Alignment Index for proportional values between Classroom Mathematics and the NCS examinations:

Following similar procedure as for Platinum Mathematics, the proportion between corresponding cells (cognitive demand B) in Matrix G (Classroom Mathematics Grade 12 textbook) and Matrix H (NSC Examinations) is 0.5 and 0.4 respectively. The smaller proportion is 0.4 and will be used in the alignment calculation process. The proportion

between cognitive demand C of Matrix G and Matrix H is the same (0.4) and it will be used for alignment calculation. For cognitive demand D the smallest proportion is 0.1. The final alignment index will be obtained by summing up the smallest proportions between each pair of the corresponding cells. The smallest proportion between the corresponding cells in this study is 0.4, 0.4 and 0.1. These smallest proportions will then be summed up to obtain the alignment value of 0.9.

The second procedure uses Porter alignment index formula: **AI = 1** $- \frac{\sum|x-y|}{2}$

Porter alignment index totals the absolute value of the difference between each pair of corresponding cells across Matrix G in Table 4.48 and Matrix H in Table 4.49. The total is then divided by 2 and the result is subtracted from 1 to end up with the alignment index.

The proportion values between Classroom Mathematics and the NSC examinations questions in Matrix G and H are shown below with abridged versions.

Classroom Mathematics

Proportional Values				
A	**B**	**C**	**D**	**E**
0	0.5	0.4	0.1	0

NSC Examinations

Proportional Values				
A	**B**	**C**	**D**	**E**
0	0.4	0.4	0.2	0

Using the alignment index formula, then:

$$\text{AI} = 1 - \frac{\sum|x-y|}{2}$$

$$= 1 - \frac{|0.5-0.4|+|0.4-0.4|+|0.1-0.2|}{2}$$

$$= 1 - \frac{|0.1|+|0|+|-0.1|}{2} \quad = 1 - \frac{(0.1+0+0.1)}{2} \quad = 1 - \frac{(0.2)}{2} = 1 - 0.1 = 0.9$$

For Classroom Mathematics Grade 12 and the NSC examinations, calculations yield an alignment index of 0.9. Interpreting these using percentages, then it means that about 90%

of cognitive demands of tasks between Classroom Mathematics and the NSC examination are in agreement. This signals a strong alignment index with about 90% of the cognitive demands of tasks in Classroom Mathematics in alignment with the cognitive demands of tasks in the NSC examinations. In relating back to the analysis it is highlighted that Classroom Mathematics Grade 12 maintained a more balanced distribution of cognitive demands as recommended by CAPS. The analysis of Classroom Mathematics shows that about 51.7% of instructional tasks are weighted towards lower order and 48.3% towards higher order cognitive demands. These weightings are not off the mark as far as policy prescripts. The high level of agreement between the recommended CAPS percentages and the actual percentages in the textbook in terms of cognitive demand seems to be the key factor for the stronger alignment shown by the alignment indices of 0.9 or 90%. Hosts of scholars who developed the alignment index associate the larger value of the index, with better the alignment (Porter, et al., 2002). In light of the fact that alignment index can best be described as a measure of relative emphasis (Mhlolo, 2011), it can be argued that there is some level of parity between what the two textbooks emphasizes and what is being emphasized in the NSC examination papers with regard to cognitive demand.

According to Webb (2005), a significant aspect of alignment between standards and assessment is whether both address the same categories. Earlier in the analysis it was highlighted that both textbooks prioritises lower order cognitive demands while NSC examinations prioritises higher order cognitive demands. In light of this consideration, concerns may be raised as to what kind of disparities exists between these curricular documents (textbooks and examinations). Roach et al., (2008) point out that usually reviewers have to make qualitative judgments to attempt to account for the low or high alignment index and to see where the differences in emphasis could be.

In his study which investigated coherence of the levels of cognitive demand between the Grade 12 South African Physical Sciences Curriculum and the 2008 & 2009 Senior Certificate examination papers for Physical Sciences, Edwards (2010), used the concept of discrepancies to highlight the emphasis that the curriculum statement or examination papers place on a particular cognitive demand for a particular science topic. The discrepancies are reported as the difference between the proportional values of a particular cognitive demand in the curriculum statement and in the examination papers. Negative

discrepancies indicate less emphasis of a particular curriculum document towards a particular cognitive demand while positive discrepancies indicate more emphasis of a particular curriculum document towards a particular cognitive demand. A discrepancy of 0 indicates equal emphasis on curriculum documents for a particular cognitive demand (Edwards, 2010).

The following table present the discrepancies by cognitive demands between the textbooks analysed and the NSC examinations. The discrepancies attempt to address the concern about the kinds of disparities that exists between the cognitive demands of tasks in the textbooks and the NSC examination question papers and where the emphasis lies.

Table 4.50.: - Discrepancies between proportional values of Platinum Mathematics Grade 12 and NSC examination questions - Adapted from Mhlolo (2011)

	A	**B**	**C**	**D**	**E**
Platinum Mathematics	0	0.7	0.3	0	0
NSC Examinations Mathematics Paper 2	0	0.4	0.4	0.2	0
Discrepancy	**0**	**0.3**	**- 0 .1**	**- 0 .2**	**0**

The cells for the textbooks analysis come first in each case and to calculate the discrepancies, the proportional values of the NSC examinations are subtracted from the proportional values of the textbooks. The discrepancy values indicate the emphasis of a particular curricular document (e.g. textbook) towards a particular cognitive demand (e.g. perform procedures). As mentioned earlier and according to Edwards (2010), negative discrepancies indicate less emphasis on a particular curricula document at a particular cognitive demand while positive discrepancies indicate more emphasis of a particular curricula document at a particular cognitive demand. A discrepancy of 0 indicates equal emphasis on a particular curricular document at a particular cognitive demand.

Using the same thinking, in this study, negative discrepancies indicate that the textbooks place less emphasis on that particular cognitive demand while the NSC examination papers place more emphasis on the same cognitive demand. Positive discrepancies similarly

indicate that the textbook places more emphasis on that particular cognitive demand while the NSC Examination papers place less emphasis on the same cognitive demand.

In Table 4.50 above, the discrepancies shown in the last row indicate that the Platinum Mathematics and the NSC Examination papers placed equal emphasis (of zero) on cognitive demand A and E. For cognitive demand B – the Platinum Mathematics placed more emphasis on 'performing procedures'. Discrepancies of -0.1 and -0.2 respectively in cognitive demand C and D indicates that Platinum Mathematics textbook placed less emphasis on higher order cognitive demand while the NSC examinations placed more emphasis on higher order cognitive demand. These observations strengthen the argument that the disparities in each cognitive demand levels of tasks, considered separately, between Platinum Mathematics and the NSC examinations is the main contributing factor for a less strong alignment.

These findings in part confirm and support the claim that poor performance in examination relating to the performance of students might be because the materials (Platinum Mathematics Grade 12, shown in the analysis in this study) that guides learning of mathematics on a daily basis does not balances the cognitive demand categories as per policy prescripts, and places less emphases on higher order cognitive demands.

Table 4.51.: - Discrepancies between proportional values of Classroom Mathematics Grade 12 and NSC examination questions - Adapted from Mhlolo (2011)

	A	**B**	**C**	**D**	**E**
Classroom Mathematics	0	0.5	0.4	0.1	0
NSC Examinations Mathematics Paper 2	0	0.4	0.4	0.1	0
Discrepancy	**0**	**0.1**	**0**	**0**	**0**

The discrepancies between Classroom Mathematics Grade 12 and the NSC examinations shown in the last row point to some parity between what Classroom Mathematics appear to emphasize and what the NSC examinations seem to test in terms of cognitive demands. Judged by the results from the table, it can be pointed out that Classroom Mathematics and the NSC examination papers placed equal emphasis at cognitive demand A, C, D and E. The

disagreements are pronounced only in cognitive demand B where Classroom Mathematics placed more emphasis on 'performing procedures' while the NSC examinations placed less emphasis in that same cognitive demand.

4.16. CONCLUSION

In this chapter the analysis of data attempting to offer answers to the research questions were discussed. The cognitive demands of instructional tasks in the textbooks and the assessment tasks in the NSC examinations relating to trigonometric equations and the general solutions of the Grade 12 Mathematics curriculum revealed differences between cognitive demands of instructional tasks in the predominantly used textbook (Platinum Mathematics Grade 12) and assessment tasks in the NSC examinations and led to the following conclusions.

Most of the instructional tasks of the Platinum Mathematics fell mainly under the lower order cognitive demands of Porter's (2002) cognitive demand framework. The reason is that most of the instructional tasks in Platinum Mathematics Grade 12 explicitly resembled the ones in the worked examples and ask the same knowledge that corresponds to the preceding text. In most of the tasks analysed, students were required to recall, memorise and reproduce similar procedures and definitions previously learned with limited room for adapting procedures to slightly different situations.

On the other hand, after averaging the data analysis of the NSC examinations for the past three years, the NSC exams showed a bias towards the attainment of higher order cognitive demand categories at 57.2%, with about 42.8% of tasks devoted to the attainment of lower order cognitive demands.

The level of coherence between the cognitive demands of instructional tasks in the textbooks and assessments tasks in the NSC examination was determined using alignment indices. The two alignment methods used in the study to measure the level of coherence and agreement between the cognitive demands of instructional tasks in the textbooks and assessment tasks in the NSC examinations revealed alignment indices of 0.7 and 0.9 respectively for Platinum Mathematics Grade 12 and Classroom Mathematics Grade 12.

Focusing on the predominately used textbook (Platinum Mathematics) and judging by the alignment index of 0.7, it can be concluded that about 70% of the cognitive demands of the written component represented by Platinum Mathematics are moderately aligned with the assessed component represented by the examination papers. This is due to Platinum Mathematics testing mainly lower order cognitive demands as opposed to the advocated higher order cognitive demands in the intended curriculum (CAPS document).

Perfect alignment (i.e. where the cognitive demand of all the tasks in the compared curricula documents being at the same level) is indicated by the index of 1. Any indices apart from 1 point to some degree of disagreement between the curricula documents compared. Measures of discrepancies between the textbooks and the examination papers were used to analyse which curricular document could have possibly contributed to the suggested disparities.

The results showed that there was an almost equal emphasis in terms of cognitive demands of tasks in Classroom Mathematics and tasks in the NSC examinations. However the disparities were more pronounced in the tasks of Platinum Mathematics and tasks in the NSC examinations in terms of cognitive demands, where the highest number of task in Platinum Mathematics were weighted towards lower order cognitive demands. Discrepancies of such nature are imperative and help to highlight and channel efforts to areas that need to be reviewed when textbooks are being revised or selected in the future.

CHAPTER 5
SUMMARY OF FINDINGS, IMPLICATIONS, LIMITATIONS AND CONCLUSIONS

5.1. INTRODUCTION

The previous chapter analysed tasks to determine the levels of agreement in terms of cognitive demands of instructional tasks in the textbooks (written curricula) and assessment tasks in the NSC examinations (assessed curricula). This chapter draws up some conclusions obtained from the analysis, discuss limitations and makes some recommendations for future research around the cognitive demands of instructional tasks in the textbooks. The research study as described in the preceding chapters aimed at answering the following research questions.

1. What kinds of cognitive demands are implicit in the instructional tasks around the general solutions of trigonometric equations in two Grade 12 NCS (CAPS) prescribed mathematics textbooks and in the assessment tasks in the NSC examinations?

2. To what extent do the cognitive demands of tasks around the general solutions of trigonometric equations in two Grade 12 NCS (CAPS) prescribed textbooks align with the cognitive demands of the assessment tasks around the general solution of trigonometric equations in the NSC Examination"?

To answer the research question, a task by task analysis in the textbooks and in the NSC examinations relating to trigonometric equations and the general solution was done. The focus was on the levels of cognitive demands required by each tasks in the textbooks and in the NSC examinations.

5.2. SUMMARY OF FINDINGS.

The first research question that guided the study is. ***"What kinds of cognitive demands are implicit in the instructional tasks around the general solutions of trig equations in two NCS (CAPS) FET prescribed mathematics textbooks for Grade 12 and in the assessment tasks of the NSC examinations?"***

In attempting to answer this question, instructional tasks within the content of trigonometric equations and the general solution from the two prescribed Grade 12 mathematics textbooks were analysed and placed into categories of cognitive demands using Porter's (2002) cognitive demand framework. The first two levels of Porter's (2002) cognitive demands framework (memorisation and perform procedures) are associated with lower order levels of cognitive demands while 'communicate understanding of concepts' 'solve non-routine problems' and 'conjecture, generalise and prove' are classified as higher order cognitive demands levels.

Since the enacted curriculum (particularly in terms of students' responses and their solutions) was not taken into account, the process leading to the solution of the task (as suggested by worked examples in the textbook) dictated what was expected of a Grade 12 student in engaging with the task. The demands of the process leading to the solution, was used to tell whether the task required a routine procedure or non-routine procedure. This was instrumental in determining whether or not a task was of a lower order or higher order task for a Grade 12 student.

Data from the analysis indicated that in the Platinum Mathematics Grade 12 tasks that mainly addressed lower order cognitive demands stood out. The number of instructional tasks labelled as lower order was much higher than the number of tasks characterised as higher order cognitive demands. On the other hand, Classroom Mathematics Grade 12 recorded a slightly higher percentage of instructional tasks at higher order cognitive demand compared to the Platinum Mathematics Grade 12. Overall, more tasks in Platinum Mathematics Grade 12 were of the lower order cognitive demand level of 'perform procedures' of Porter (2002) cognitive demand framework with far fewer higher order cognitive demand tasks appearing in cognitive demand C and no task in cognitive demand D and E of Porter's (2002) cognitive demand framework.

Judging by this finding it can be argued that the implied cognitive demands in the predominantly used textbook (Platinum Mathematics Grade 12) among the non-fee public ordinary schools in Gauteng (majority of schools) are of the lower order levels. Research elsewhere also evidences the bias of textbooks towards lower order cognitive demands. Similarly, the 16.4% margin difference between the CAPS recommended percentage for

lower order cognitive demands levels and the tasks in Platinum Mathematics classified as lower order lead to a conclusion that the types of cognitive demands implicit in correspond to lower order cognitive demands.

The assessment tasks in the NSC examinations painted a different picture and espoused mainly higher order cognitive demands. The is evidenced by a high number of tasks that displayed a greater focus on higher order cognitive demands prioritizing cognitive demand C – 'communicate understanding of concepts' and cognitive demand D – 'solve non-routine problems' of Porter's (2002) cognitive demand framework. In total 57.2% of task were dedicated to higher order cognitive demands in the assessment tasks of the NSC examinations and only 42.8% of the task to lower order cognitive demands. Based on these interpretations it was then possible to conclude that the types of cognitive demands emphasised in the NCS examinations are those with higher order cognitive demands.

The second question that guided the study was phrased as ***"To what extent do the cognitive demands of two NCS (CAPS) FET grade 12 prescribed textbooks tasks around the general solutions of trigonometry equations align with the cognitive demands of the trigonometry around the general solution in the NSC Examination?"***

Anderson (2002) defined curricular alignment as a strong link between standards and assessment, between standards and textbooks, and between assessments and textbooks. As discussed earlier, curriculum documents are considered aligned when they are consistent (Biggs, 2003) and in agreement (Bhola et al., 2003).

In view of the fact that alignment indices can best be described as a measure of relative emphasis, it can be argued that there is some level of agreement between the cognitive demands of the curricula documents analysed. The two alignment indices of 0.7 and 0.9 indicates that the written component represented by textbooks (Platinum and Classroom Mathematics respectively) and the assessed component represented by the examination papers are respectively moderately and fully aligned. The prominent difference in terms of cognitive demands that appears in Platinum Mathematics are the key factors contributing to lack of a very strong or perfect alignment (Liang & Yuan, 2008); (Porter, 2002).

Another indicator of coherence was concerned with interpreting discrepancies (i.e. level of emphasis of a particular curricular component towards a particular cognitive demand) (Edwards, 2010). Using proportional values, the result shows that Classroom Mathematics and the NSC examinations reflects a more balanced emphasis in terms of cognitive demand at almost all cognitive demand levels, while between Platinum Mathematics and the NSC examinations there are pronounced differences in terms of emphasis of cognitive demands. In the words of Mhlolo (2011) it could be argued that the curriculum statements, textbooks and examinations are "presenting a coherent message internally but a splintered vision externally" (pg. 249). In other words the espoused cognitive demands seem to be articulated consistently through the policy documents but differently between the textbooks and examinations.

5.3. IMPLICATIONS FOR PRACTICE

Textbooks as a main resource for teaching and learning constitute a crucial component of mathematics instruction. Niss (2003); Stein, Grover, & Henningsen (1996) highlight that learning mathematics includes developing the capacity to engage in the processes of mathematical thinking that include building models, looking for patterns, generalizing methods, challenging chains of arguments, proving conjectures and so on. If this perspective is assumed as the anticipated outcome of mathematics, then curriculum documents, particularly the written curriculum (textbooks) should include tasks that are more cognitively demanding. This is not only to facilitate higher order thinking processes but also to assist in raising the number of students interested in mathematics and giving them the opportunities to develop mathematical knowledge, skills and practice that are flexible and adaptable.

Kulm, Wilson and Kitchen (2005) point out that curriculum documents must be continuously assessed to ascertain their effectiveness in assisting students to accomplish the envisaged mathematical outcomes. Therefore to make a good decision in the preparation and selection of mathematics textbooks; textbook authors, curriculum planners and teachers should take into account the prospects of such textbooks, particularly their cognitive demands and the opportunities they afford to the students to learn the mathematics (higher order skills) which there is a general consensus about. After all, textbook is for the

majority of students in the non-fee paying schools (majority of schools) the only source of the learning support material over and above the teacher.

5.4. IMPLICATIONS FOR RESEARCH

This study was designed to analyse the cognitive demands of the written curriculum in the form of the two Grade 12 textbooks tasks to which students have direct exposure and the NSC examinations questions. The findings are also restricted to the results of the analysis based on one section of trigonometry from the perspectives of only two prescribed mathematics Grade 12 textbooks tasks to which students have direct exposure. Future research might consider the analysis of a larger sample of Grade 12 textbook to expand this focus. Also the research might be broadened to include other additional resources (e.g. study guides, teachers guide and so on) that accompany and support the textbooks. The use of a larger sample size of textbooks would provide wider coverage on trigonometric equations and the general solution treatment and provide a complete picture of the cognitive demands that task in the textbooks advances and promote. A larger sample size might also allow for generalization of results.

The level of cognitive demand as envisaged by the textbook writer of a tasks may or may not be the level at which the student engages with the tasks in an actual classroom setting. In their work, Stein and Kaufman (2010) commented on the numerous phase that a task takes. They argue that instructional task goes from its initial appearance on the textbook to its actual enactment in the classroom by students and teachers. It is in this light that further research is also needed to investigate the curriculum that is enacted in the classroom to examine the levels of cognitive demand of tasks in student's written responses and how the student interacts and enacts the tasks.

Additional research might also analyse the levels of cognitive demand of examples presented in the textbooks and that the teacher uses in their instructions to make firm judgements about how they compare with the levels of cognitive demand of all exercises across the unit or chapter of the textbooks.

5.5. LIMITATIONS

There are several limitations of this study. These affect the findings and, hence call for a careful interpretation of the findings. First, the findings of this study cannot be generalised to all Grades 12 mathematics textbooks presently in use in South Africa since the study was conducted on a reduced range (only two) of Grade 12 textbooks used in Gauteng Province. However, the selected section of the trigonometry curriculum – trigonometric equations and the general solution was analysed in depth. In addition, the analyses conducted were restricted to the perspectives of cognitive demands of tasks whereas more features of tasks may have been included.

Secondly, the mathematics textbooks analysed were written only for Grade 12 students but the Grade 12 mathematics curriculum, in particular trigonometry includes the content of two-year curricula (Grade 11 and 12). The premise for this study was to examine the textbooks to which the grade 12 student is directly exposed in their exit year. Therefore it was not possible to account for grade 11 textbooks and other materials that influence grade 12 students including the content of classroom instruction.

Another limitation of the study is in comparing the cognitive demands of tasks in the textbooks and the NSC because of the different expectations which affect the type of tasks in the textbooks and in the examinations. Within the textbooks, tasks are used as a component of the lesson or unit and might be confined to a single or lower order cognitive demand levels at the initial stages of the lesson or unit with the advancement to different or higher cognitive demands levels in the subsequent exercises. In contrast, tasks in the examinations serve as a component of national assessment encompassing different content and levels of cognitive demands in one task. Doyle (1988) noted that academic tasks in the context of test items exist at several different levels at once.

A fourth limitation relates to the use of Porter's (2002) cognitive demand framework for determining the level of cognitive demand in the tasks. Porter's (2002) cognitive demand framework has evolved over time and has been used successfully for a variety of purposes; however, the cognitive demands may be insufficient or too fine-grained to capture the distinctions among cognitive demands. It might be possible that the results would have

been different if an alternate framework for investigating cognitive demand of task had been used.

5.6 RECOMMENDATIONS

Curricular alignment in every single curricular document is vital and should be an ongoing activity to identify discrepancies between the intended, the enacted, the written and the assessed curriculum developed by the state. This analysis could help to identify areas of the intended curriculum that are not being addressed or addressed with only limited emphasis amongst other curricular documents. As such, those charged with stewardship of curriculum with all its variations should be cognizant of the fact that if students are expected to learn higher order cognitive skills then it is imperative that all forms of curricular documents, particularly prescribed textbooks made available to the majority of students within the public schooling sector should include such skills. If not, then the majority of students may be deprived and the public could be misled by the results of examinations, particularly performance in mathematics.

5.7. CONCLUDING REMARKS

In conclusion, I refer back to Anderson (2002) who defined curricular alignment as a strong link between standards (intended curriculum), instructional materials (written curriculum) and assessments (assessed curriculum). In this study alignment appears to highlight a moderate link in terms of cognitive demands espoused by the intended curriculum (CAPS) and promoted by the written curriculum (textbook). The intended curriculum espoused higher order skills while on the other hand the written curriculum is sending a different message (i.e. prioritising lower order cognitive demands). This is consistent with Bernstein's (2000b) assertions that disciplinary knowledge does not equal the didactic knowledge of that discipline because the process of production (intended curriculum) and transmission (written and enacted curriculum) of knowledge may have contradictions.

It is in this thinking that I highlight that the textbooks (written curriculum) is one curricular document in the curricular link that plays a significant role. They play a significant role as a primary source of learning guiding teaching and learning for the majority of learners within the public schooling sector. They also play a role in providing support for students to

achieve the desired outcomes and the opportunities to do well in the national examinations. Reys, Reys, & Chávez (2004) suggests that textbooks have been identified as potential agents of change to transform curricula (Collopy, 2003 ; Remillard, 2000) but such potential depends upon the extent to which the textbooks align to other official curriculum documents.

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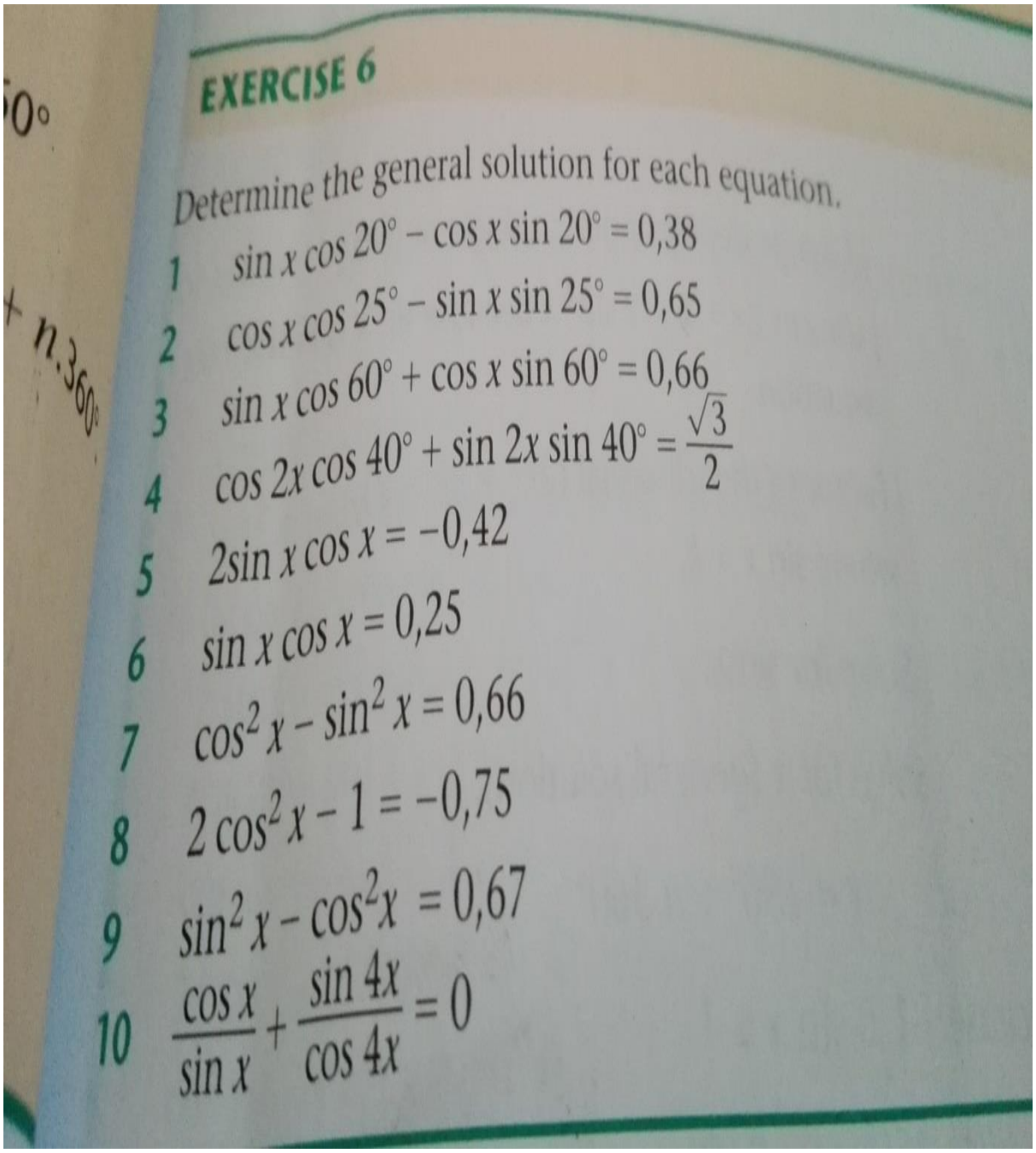
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APPENDIX A



EXERCISE 6

Determine the general solution for each equation.

1 $\sin x \cos 20^\circ - \cos x \sin 20^\circ = 0{,}38$

2 $\cos x \cos 25^\circ - \sin x \sin 25^\circ = 0{,}65$

3 $\sin x \cos 60^\circ + \cos x \sin 60^\circ = 0{,}66$

4 $\cos 2x \cos 40^\circ + \sin 2x \sin 40^\circ = \frac{\sqrt{3}}{2}$

5 $2\sin x \cos x = -0{,}42$

6 $\sin x \cos x = 0{,}25$

7 $\cos^2 x - \sin^2 x = 0{,}66$

8 $2\cos^2 x - 1 = -0{,}75$

9 $\sin^2 x - \cos^2 x = 0{,}67$

10 $\frac{\cos x}{\sin x} + \frac{\sin 4x}{\cos 4x} = 0$

Tasks from Platinum Grade 12 Mathematics, Topic 5 - Trigonometry, Unit 4 – Solve equations and determine the general solution, Exercise 6, page 105

APPENDIX B

Solve $3\sin^2 x - \sin 2x - \cos^2 x = 0$ and $0° \leq x \leq 360°$.

SOLUTION

$3\sin^2 x - \sin 2x - \cos^2 x = 0$ | Use double angle $\sin 2x = 2\sin x \cos x$

$3\sin^2 x - 2\sin x \cos x - \cos^2 x = 0$ | Recognise the quadratic equation $3s^2 - 2sc - c^2 = 0$.

$(3\sin x + \cos x)(\sin x - \cos x) = 0$ | Solve by factorising.

$3\sin x = -\cos x$ or $\sin x = \cos x$

$\frac{3\sin x}{\cos x} = \frac{-\cos x}{\cos x}$ or $\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$ | Divide by $\cos x$ and use identity $\frac{\sin x}{\cos x} = \tan x$

$\tan x = -\frac{1}{3}$ or $\tan x = 1$ | Now solve for x.

$x = -18{,}4° + n.180°$ or $x = 45° + n.180°$, $n \in \mathbb{Z}$

For $0° \leq x \leq 360°$: $x = 161{,}6°$ or $45°$ or $57°$ or $225°$ or $341{,}5°$

EXERCISE 7

Solve for x, giving the general solution first and specific solutions if an interval is given.

1 $\cos 2x + \cos x = 0$
2 $\sin^2 x + \sin 2x = 0$, and $-360° \leq x \leq 360°$
3 $3\cos 2x + \cos x + 2 = 0$
4 $2\sin 2x - 2\sin x = 6\cos^2 x - 3\cos x$, and $x \in [-360°;360°]$
5 $\cos 2x + \cos x + 1 = 0$
6 $\cos 2x + \sin x - 1 = 0$
7 $11\cos^2 x - 4\cos 2x = 6 + \cos x$
8 $\cos 2x - 4\sin x + 5 = 0$
9 $\sin 2x = 2\cos^2 x$
10 $\cos 2x + 3\cos x - 1 = 0$ and $-180° \leq x \leq 180°$
11 $\sin 2x - \sin x = 1 - 2\cos x$
12 $2\cos 2x + \frac{1}{2}\sin 2x = \sin^2 x$ and $0° \leq x \leq 360°$
13 $4\sin x \sin 2x + \sin 2x - \cos x = 0$
14 $\sin 2x + \cos 2x - 1 = 0$ and $-180° \leq x \leq 180°$

Solve equations using compound angles and no calculator

In Grade 11 you solved equations of the type:

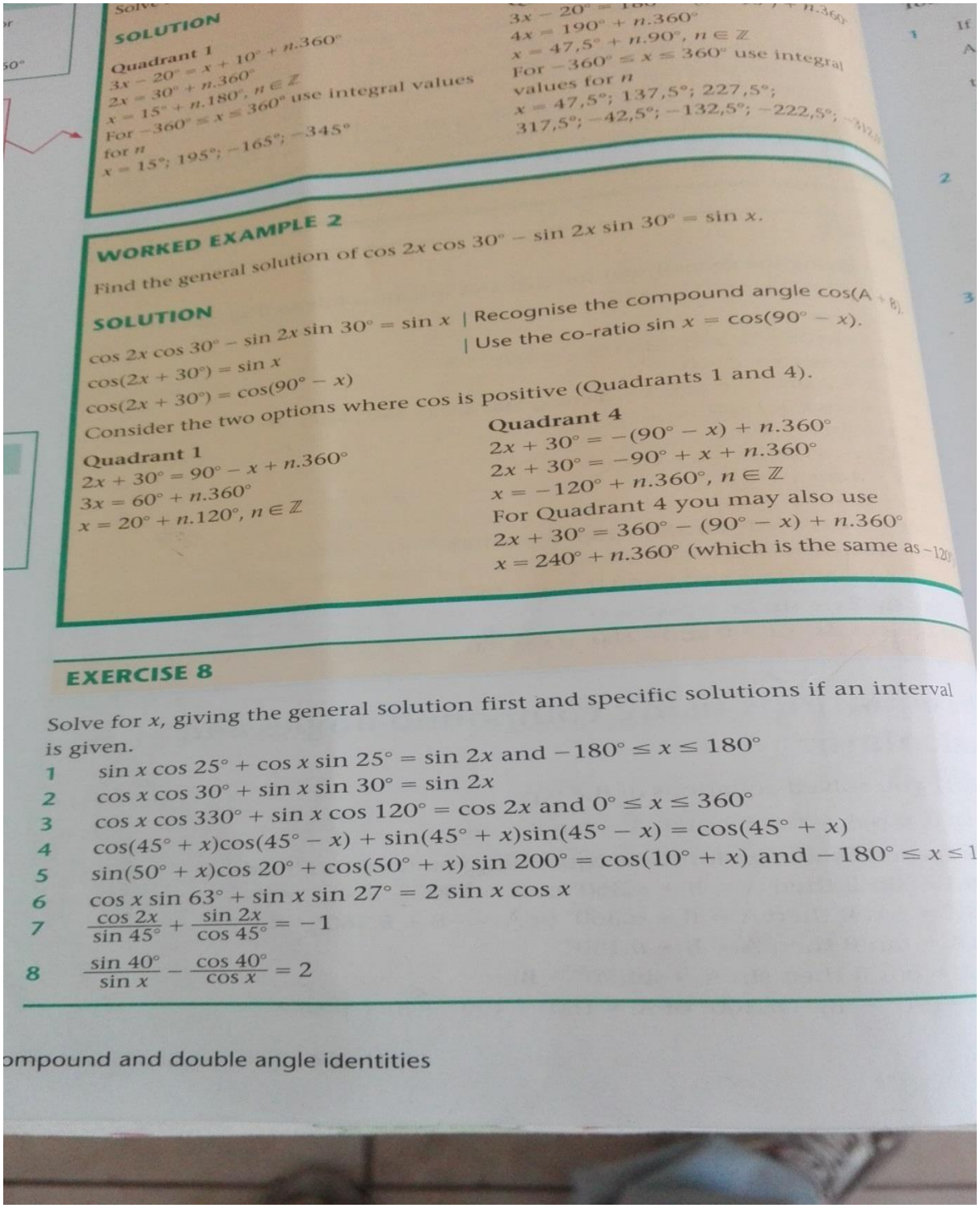
$\sin A = \sin B$, $\cos A = \cos B$ or $\tan A = \tan B$

To find the general solution for these equations, you used this 'method':

1. If $\sin A = \sin B$ then $A = B + n.360°$ or $A = 180° - B + n.360°$
2. If $\cos A = \cos B$ then $A = B + n.360°$ or $A = -B + n.360°$
3. If $\tan A = \tan B$ then $A = B + n.180°$
4. If $\sin A = \cos B$ then $\sin A = \sin(90° - B)$ and $A = (90° - B) + n.360°$ or $A = 180° - (90° - B) + n.360°$

Tasks from Platinum Grade 12 Mathematics, Topic 5 - Trigonometry, Unit 4 – Solve equations and determine the general solution, Exercise 7, page 107

APPENDIX C



SOLUTION

Quadrant 1

$3x - 20° = x + 10° + n.360°$
$2x = 30° + n.360°$
$x = 15° + n.180°, n \in \mathbb{Z}$
For $-360° \leq x \leq 360°$ use integral values for n
$x = 15°; 195°; -165°; -345°$

$3x - 20° = 180° - (x + 10°) + n.360°$
$4x = 190° + n.360°$
$x = 47,5° + n.90°, n \in \mathbb{Z}$
For $-360° \leq x \leq 360°$ use integral values for n
$x = 47,5°; 137,5°; 227,5°; 317,5°; -42,5°; -132,5°; -222,5°; -312,5°$

WORKED EXAMPLE 2

Find the general solution of $\cos 2x \cos 30° - \sin 2x \sin 30° = \sin x$.

SOLUTION

$\cos 2x \cos 30° - \sin 2x \sin 30° = \sin x$ | Recognise the compound angle $\cos(A + B)$.
$\cos(2x + 30°) = \sin x$ | Use the co-ratio $\sin x = \cos(90° - x)$.
$\cos(2x + 30°) = \cos(90° - x)$

Consider the two options where cos is positive (Quadrants 1 and 4).

Quadrant 1

$2x + 30° = 90° - x + n.360°$
$3x = 60° + n.360°$
$x = 20° + n.120°, n \in \mathbb{Z}$

Quadrant 4

$2x + 30° = -(90° - x) + n.360°$
$2x + 30° = -90° + x + n.360°$
$x = -120° + n.360°, n \in \mathbb{Z}$

For Quadrant 4 you may also use
$2x + 30° = 360° - (90° - x) + n.360°$
$x = 240° + n.360°$ (which is the same as $-120°$)

EXERCISE 8

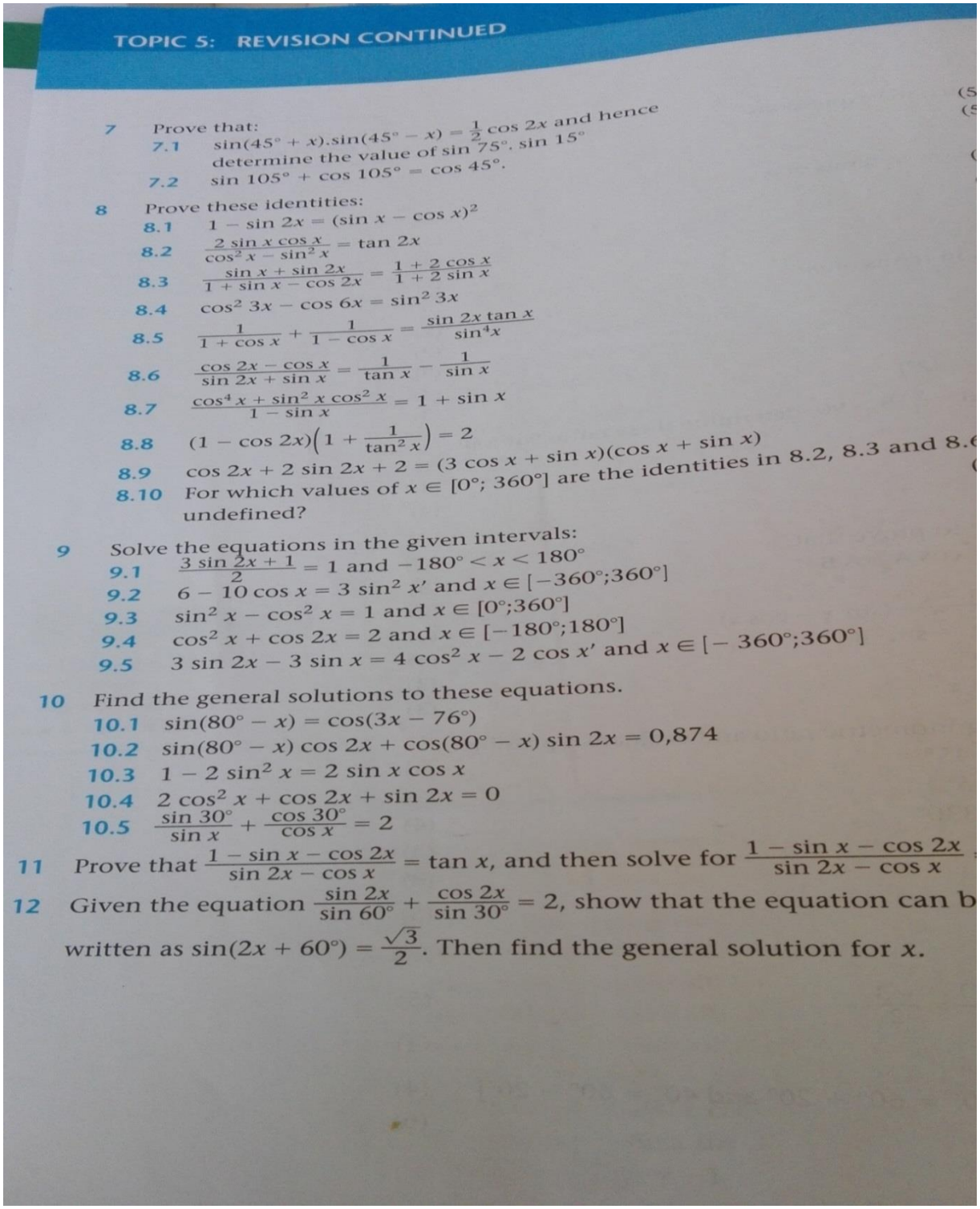
Solve for x, giving the general solution first and specific solutions if an interval is given.

1 $\sin x \cos 25° + \cos x \sin 25° = \sin 2x$ and $-180° \leq x \leq 180°$
2 $\cos x \cos 30° + \sin x \sin 30° = \sin 2x$
3 $\cos x \cos 330° + \sin x \cos 120° = \cos 2x$ and $0° \leq x \leq 360°$
4 $\cos(45° + x)\cos(45° - x) + \sin(45° + x)\sin(45° - x) = \cos(45° + x)$
5 $\sin(50° + x)\cos 20° + \cos(50° + x) \sin 200° = \cos(10° + x)$ and $-180° \leq x \leq 1$
6 $\cos x \sin 63° + \sin x \sin 27° = 2 \sin x \cos x$
7 $\frac{\cos 2x}{\sin 45°} + \frac{\sin 2x}{\cos 45°} = -1$
8 $\frac{\sin 40°}{\sin x} - \frac{\cos 40°}{\cos x} = 2$

ompound and double angle identities

Tasks from Platinum Grade 12 Mathematics, Topic 5 - Trigonometry, Unit 4 – Solve for x, giving the general solution first and specific solutions if an interval is given, Exercise 8, page 108

APPENDIX D



TOPIC 5: REVISION CONTINUED

7 Prove that:

7.1 $\sin(45° + x).\sin(45° - x) = \frac{1}{2}\cos 2x$ and hence determine the value of $\sin 75°.\sin 15°$

7.2 $\sin 105° + \cos 105° = \cos 45°$.

8 Prove these identities:

8.1 $1 - \sin 2x = (\sin x - \cos x)^2$

8.2 $\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \tan 2x$

8.3 $\frac{\sin x + \sin 2x}{1 + \sin x - \cos 2x} = \frac{1 + 2\cos x}{1 + 2\sin x}$

8.4 $\cos^2 3x - \cos 6x = \sin^2 3x$

8.5 $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} = \frac{\sin 2x \tan x}{\sin^4 x}$

8.6 $\frac{\cos 2x - \cos x}{\sin 2x + \sin x} = \frac{1}{\tan x} - \frac{1}{\sin x}$

8.7 $\frac{\cos^4 x + \sin^2 x \cos^2 x}{1 - \sin x} = 1 + \sin x$

8.8 $(1 - \cos 2x)\left(1 + \frac{1}{\tan^2 x}\right) = 2$

8.9 $\cos 2x + 2\sin 2x + 2 = (3\cos x + \sin x)(\cos x + \sin x)$

8.10 For which values of $x \in [0°; 360°]$ are the identities in 8.2, 8.3 and 8.6 undefined?

9 Solve the equations in the given intervals:

9.1 $\frac{3\sin 2x + 1}{2} = 1$ and $-180° < x < 180°$

9.2 $6 - 10\cos x = 3\sin^2 x'$ and $x \in [-360°; 360°]$

9.3 $\sin^2 x - \cos^2 x = 1$ and $x \in [0°; 360°]$

9.4 $\cos^2 x + \cos 2x = 2$ and $x \in [-180°; 180°]$

9.5 $3\sin 2x - 3\sin x = 4\cos^2 x - 2\cos x'$ and $x \in [-360°; 360°]$

10 Find the general solutions to these equations.

10.1 $\sin(80° - x) = \cos(3x - 76°)$

10.2 $\sin(80° - x)\cos 2x + \cos(80° - x)\sin 2x = 0{,}874$

10.3 $1 - 2\sin^2 x = 2\sin x \cos x$

10.4 $2\cos^2 x + \cos 2x + \sin 2x = 0$

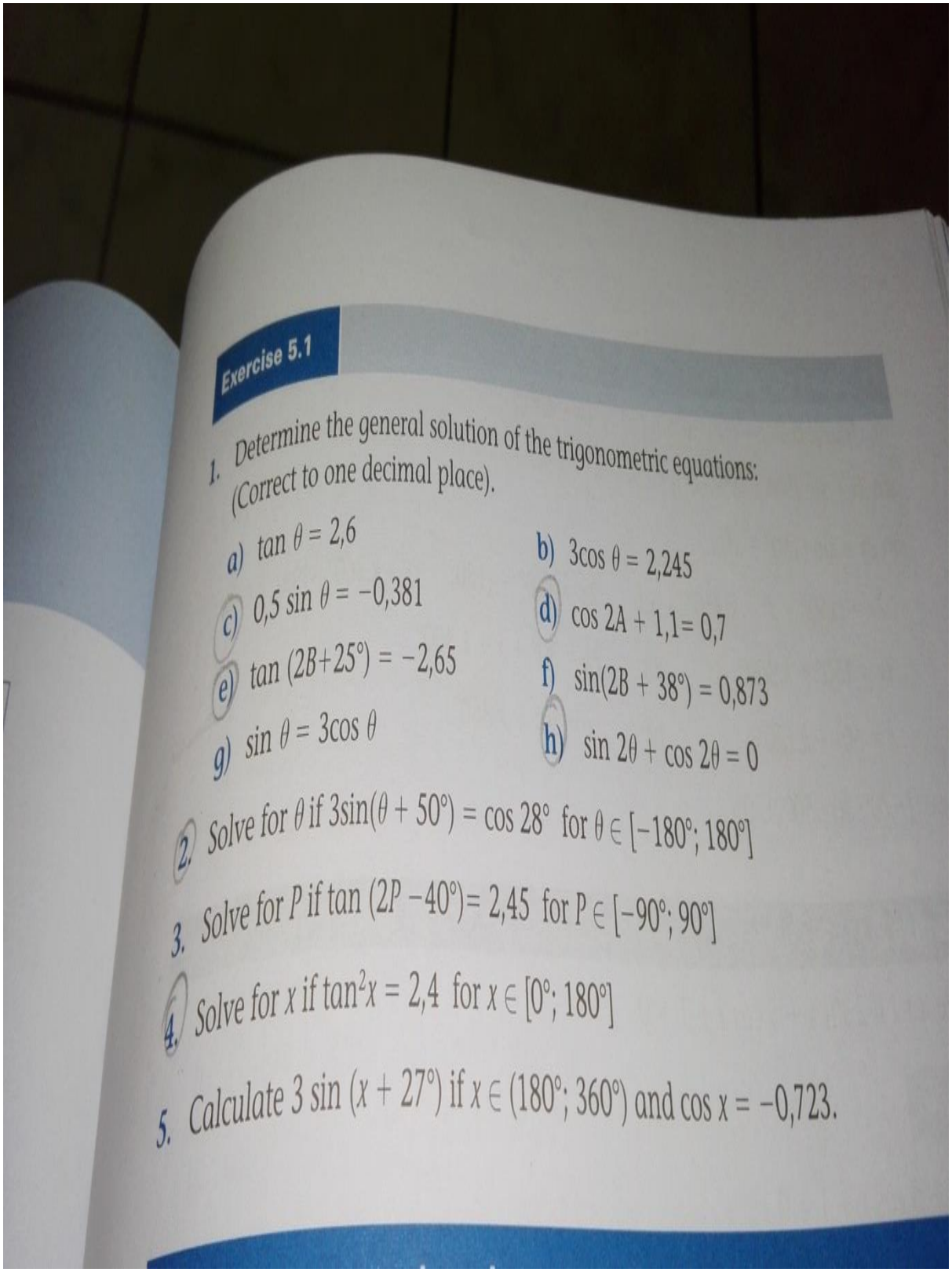
10.5 $\frac{\sin 30°}{\sin x} + \frac{\cos 30°}{\cos x} = 2$

11 Prove that $\frac{1 - \sin x - \cos 2x}{\sin 2x - \cos x} = \tan x$, and then solve for $\frac{1 - \sin x - \cos 2x}{\sin 2x - \cos x}$

12 Given the equation $\frac{\sin 2x}{\sin 60°} + \frac{\cos 2x}{\sin 30°} = 2$, show that the equation can b written as $\sin(2x + 60°) = \frac{\sqrt{3}}{2}$. Then find the general solution for x.

Tasks from Platinum Grade 12 Mathematics, Topic 5, Unit 4, Revision Test, page 110

APPENDIX E



Exercise 5.1

1. Determine the general solution of the trigonometric equations: (Correct to one decimal place).

   a) $\tan\theta = 2{,}6$

   b) $3\cos\theta = 2{,}245$

   c) $0{,}5\sin\theta = -0{,}381$

   d) $\cos 2A + 1{,}1 = 0{,}7$

   e) $\tan(2B + 25°) = -2{,}65$

   f) $\sin(2B + 38°) = 0{,}873$

   g) $\sin\theta = 3\cos\theta$

   h) $\sin 2\theta + \cos 2\theta = 0$

2. Solve for θ if $3\sin(\theta + 50°) = \cos 28°$ for $\theta \in [-180°; 180°]$

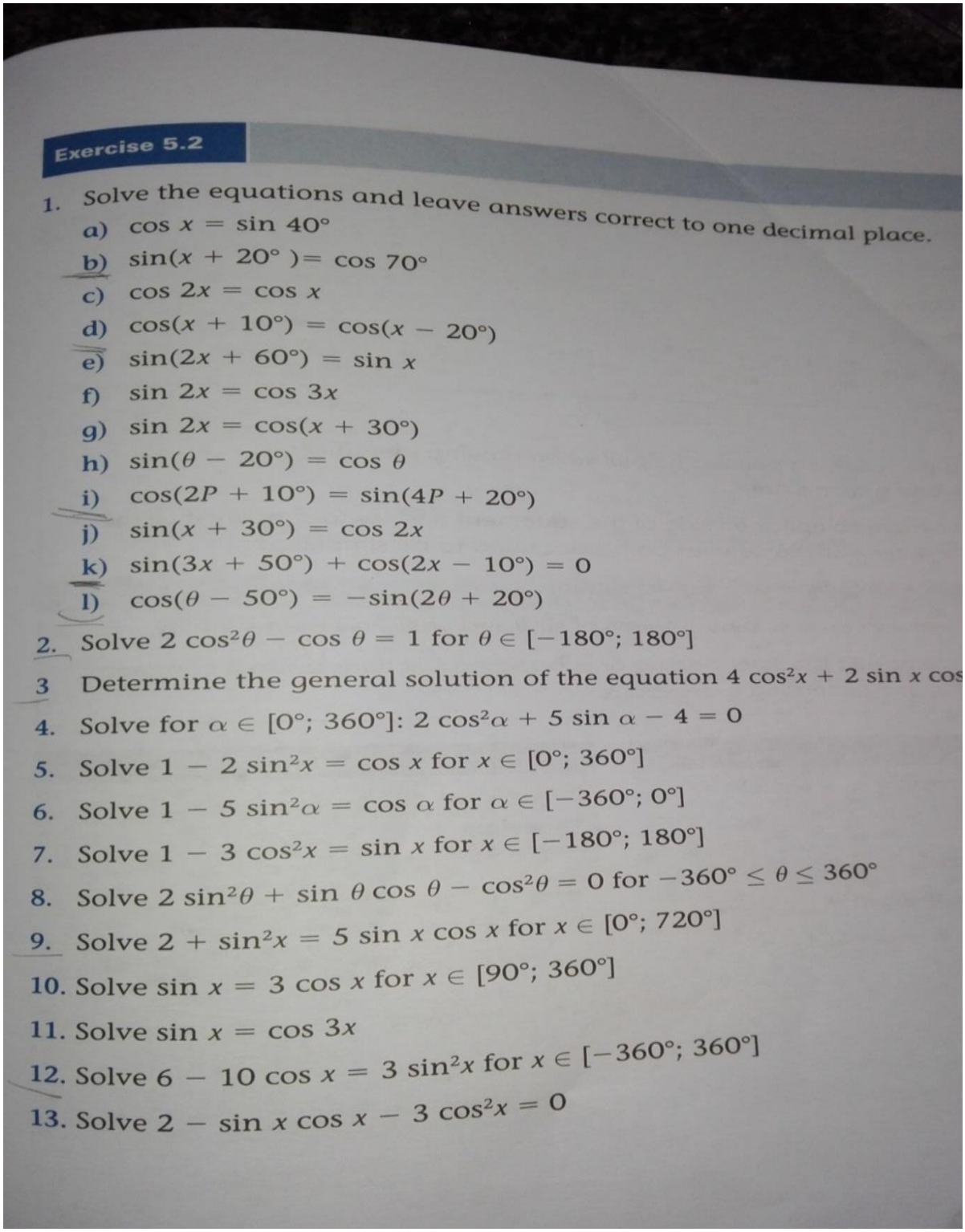
3. Solve for P if $\tan(2P - 40°) = 2{,}45$ for $P \in [-90°; 90°]$

4. Solve for x if $\tan^2 x = 2{,}4$ for $x \in [0°; 180°]$

5. Calculate $3\sin(x + 27°)$ if $x \in (180°; 360°)$ and $\cos x = -0{,}723$.

Tasks from Classroom Mathematics Grade 12, Chapter 5, Exercise 5.1, page 129

APPENDIX F

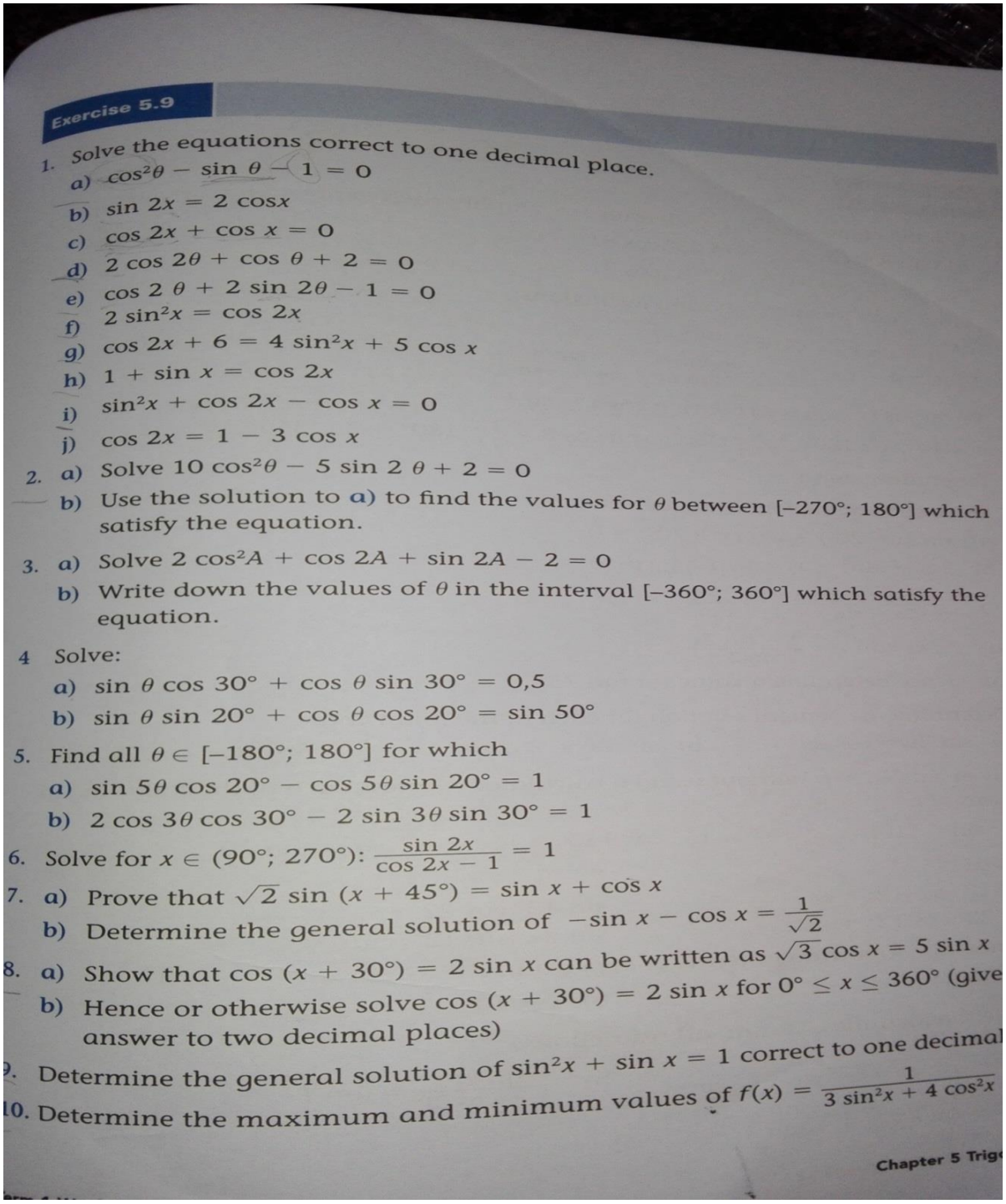


Exercise 5.2

1. Solve the equations and leave answers correct to one decimal place.
   a) $\cos x = \sin 40°$
   b) $\sin(x + 20°) = \cos 70°$
   c) $\cos 2x = \cos x$
   d) $\cos(x + 10°) = \cos(x - 20°)$
   e) $\sin(2x + 60°) = \sin x$
   f) $\sin 2x = \cos 3x$
   g) $\sin 2x = \cos(x + 30°)$
   h) $\sin(\theta - 20°) = \cos \theta$
   i) $\cos(2P + 10°) = \sin(4P + 20°)$
   j) $\sin(x + 30°) = \cos 2x$
   k) $\sin(3x + 50°) + \cos(2x - 10°) = 0$
   l) $\cos(\theta - 50°) = -\sin(2\theta + 20°)$
2. Solve $2\cos^2\theta - \cos\theta = 1$ for $\theta \in [-180°; 180°]$
3. Determine the general solution of the equation $4\cos^2 x + 2\sin x \cos$
4. Solve for $\alpha \in [0°; 360°]$: $2\cos^2\alpha + 5\sin\alpha - 4 = 0$
5. Solve $1 - 2\sin^2 x = \cos x$ for $x \in [0°; 360°]$
6. Solve $1 - 5\sin^2\alpha = \cos\alpha$ for $\alpha \in [-360°; 0°]$
7. Solve $1 - 3\cos^2 x = \sin x$ for $x \in [-180°; 180°]$
8. Solve $2\sin^2\theta + \sin\theta\cos\theta - \cos^2\theta = 0$ for $-360° \leq \theta \leq 360°$
9. Solve $2 + \sin^2 x = 5\sin x \cos x$ for $x \in [0°; 720°]$
10. Solve $\sin x = 3\cos x$ for $x \in [90°; 360°]$
11. Solve $\sin x = \cos 3x$
12. Solve $6 - 10\cos x = 3\sin^2 x$ for $x \in [-360°; 360°]$
13. Solve $2 - \sin x \cos x - 3\cos^2 x = 0$

Tasks from Classroom Mathematics Grade 12, Chapter 5, Exercise 5.2, page 131

APPENDIX G



Exercise 5.9

1. Solve the equations correct to one decimal place.
   a) $\cos^2\theta - \sin\theta - 1 = 0$
   b) $\sin 2x = 2\cos x$
   c) $\cos 2x + \cos x = 0$
   d) $2\cos 2\theta + \cos\theta + 2 = 0$
   e) $\cos 2\theta + 2\sin 2\theta - 1 = 0$
   f) $2\sin^2 x = \cos 2x$
   g) $\cos 2x + 6 = 4\sin^2 x + 5\cos x$
   h) $1 + \sin x = \cos 2x$
   i) $\sin^2 x + \cos 2x - \cos x = 0$
   j) $\cos 2x = 1 - 3\cos x$
2. a) Solve $10\cos^2\theta - 5\sin 2\theta + 2 = 0$
   b) Use the solution to a) to find the values for θ between $[-270°; 180°]$ which satisfy the equation.
3. a) Solve $2\cos^2 A + \cos 2A + \sin 2A - 2 = 0$
   b) Write down the values of θ in the interval $[-360°; 360°]$ which satisfy the equation.
4. Solve:
   a) $\sin\theta\cos 30° + \cos\theta\sin 30° = 0{,}5$
   b) $\sin\theta\sin 20° + \cos\theta\cos 20° = \sin 50°$
5. Find all $\theta \in [-180°; 180°]$ for which
   a) $\sin 5\theta\cos 20° - \cos 5\theta\sin 20° = 1$
   b) $2\cos 3\theta\cos 30° - 2\sin 3\theta\sin 30° = 1$
6. Solve for $x \in (90°; 270°)$: $\frac{\sin 2x}{\cos 2x - 1} = 1$
7. a) Prove that $\sqrt{2}\sin(x + 45°) = \sin x + \cos x$
   b) Determine the general solution of $-\sin x - \cos x = \frac{1}{\sqrt{2}}$
8. a) Show that $\cos(x + 30°) = 2\sin x$ can be written as $\sqrt{3}\cos x = 5\sin x$
   b) Hence or otherwise solve $\cos(x + 30°) = 2\sin x$ for $0° \leq x \leq 360°$ (give answer to two decimal places)
9. Determine the general solution of $\sin^2 x + \sin x = 1$ correct to one decimal
10. Determine the maximum and minimum values of $f(x) = \frac{1}{3\sin^2 x + 4\cos^2 x}$

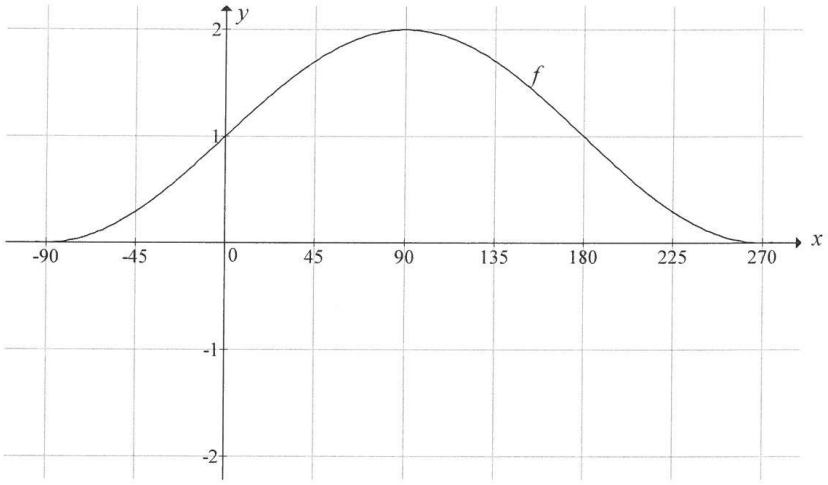
Chapter 5 Trig

Tasks from Classroom Mathematics Grade 12, Chapter 5, Exercise 5.9, page 147

APPENDIX H

QUESTION 7

In the diagram below, the graph of $f(x) = \sin x + 1$ is drawn for $-90° \leq x \leq 270°$.



7.1 Write down the range of f. (2)

7.2 Show that $\sin x + 1 = \cos 2x$ can be rewritten as $(2\sin x + 1)\sin x = 0$. (2)

7.3 Hence, or otherwise, determine the general solution of $\sin x + 1 = \cos 2x$. (4)

7.4 Use the grid on DIAGRAM SHEET 2 to draw the graph of $g(x) = \cos 2x$ for $-90° \leq x \leq 270°$. (3)

7.5 Determine the value(s) of x for which $f(x + 30°) = g(x + 30°)$ in the interval $-90° \leq x \leq 270°$. (3)

7.6 Consider the following geometric series:

$1 + 2\cos 2x + 4\cos^2 2x + \ldots$

Use the graph of g to determine the value(s) of x in the interval $0° \leq x \leq 90°$ for which this series will converge. (5)
[19]

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APPENDIX I

QUESTION 5

5.1 Given that $\sin 23° = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator:

5.1.1 $\sin 203°$ (2)

5.1.2 $\cos 23°$ (3)

5.1.3 $\tan(-23°)$ (2)

5.2 Simplify the following expression to a single trigonometric function:

$$\frac{4\cos(-x).\cos(90° + x)}{\sin(30° - x).\cos x + \cos(30° - x).\sin x}$$ (6)

5.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$. (6)

5.4 Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5)

[24]

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APPENDIX J

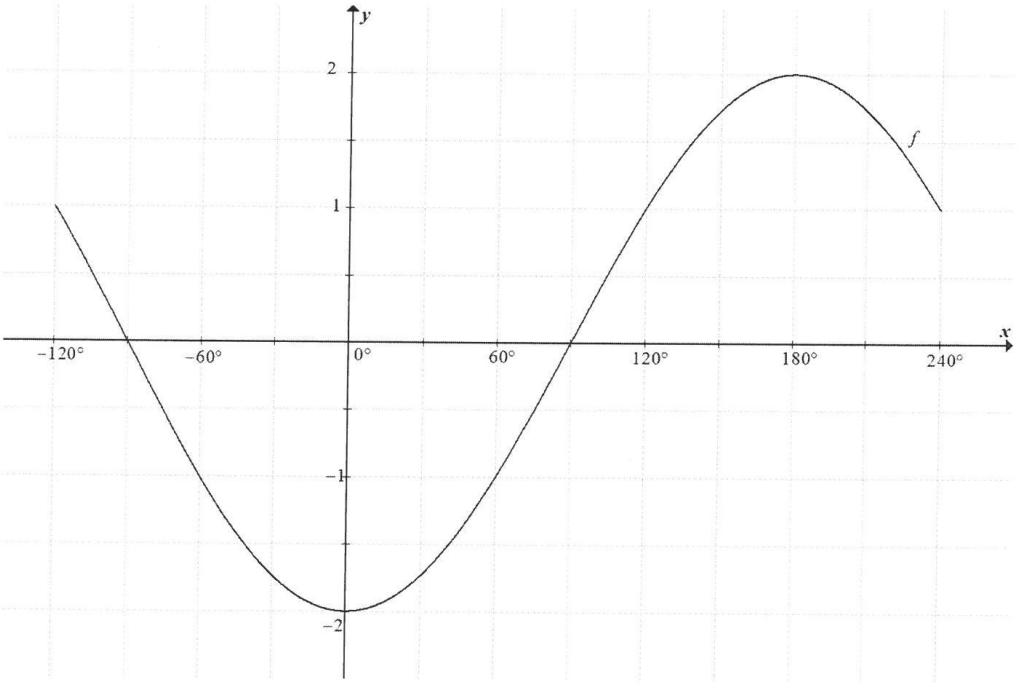
QUESTION 6

Given the equation: $\sin(x + 60°) + 2\cos x = 0$

6.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)

6.2 Determine the solutions of the equation $\sin(x + 60°) + 2\cos x = 0$ in the interval $-180° \leq x \leq 180°$. (3)

6.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120° \leq x \leq 240°$.



6.3.1 Draw the graph of $g(x) = \sin(x + 60°)$ for $-120° \leq x \leq 240°$ on the grid provided in the ANSWER BOOK. (3)

6.3.2 Determine the values of x in the interval $-120° \leq x \leq 240°$ for which $\sin(x + 60°) + 2\cos x > 0$. (3)

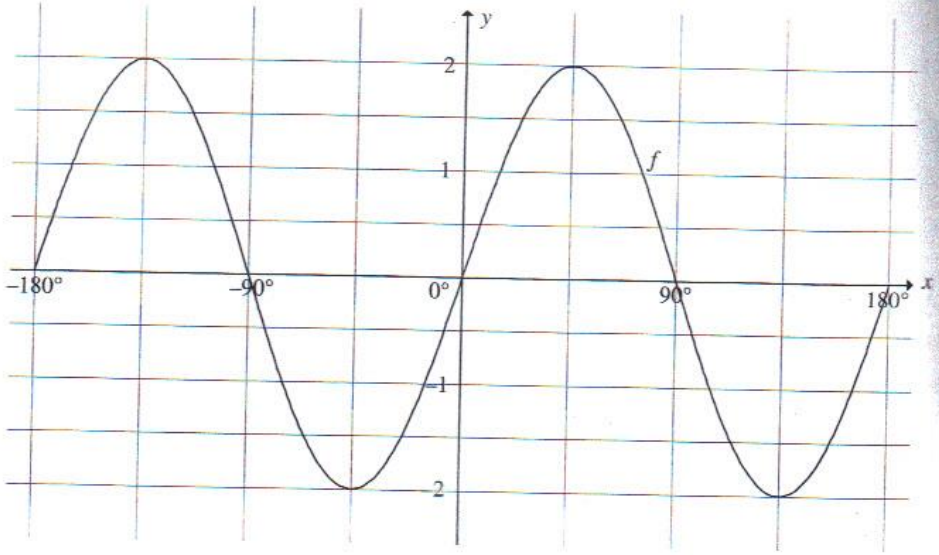
[13]

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APPENDIX K

QUESTION 6

In the diagram the graph of $f(x) = 2\sin 2x$ is drawn for the interval $x \in [-180° ; 180°]$.



6.1 On the system of axes on which f is drawn in the ANSWER BOOK, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$. Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)

6.2 Write down the maximum value of $f(x) - 3$. (2)

6.3 Determine the general solution of $f(x) = g(x)$. (4)

6.4 Hence, determine the values of x for which $f(x) < g(x)$ in the interval $x \in [-180° ; 0°]$. (3)
[12

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APPENDIX L

FINAL RECONCILED TASK CODING LIST

EXERCISE 5.6 – PLATINUM MATHEMATICS GRADE 12

	Final Agreed Coding Agreement	Coding settled through discussion
1.	B	
2.	B	
3.	B	
4.	B	
5.	B	
6.	B	
7.		B
8.		C
9.	C	
10.	C	

EXERCISE 5.7 – PLATINUM MATHEMATICS GRADE 12

	Agreed Coding Agreement	Coding settled through discussion
1.	B	
2.		C
3.	B	
4.		C
5.	B	
6.	B	
7.	B	
8.	B	
9.	B	

10.		C
11.	B	
12.		C
13.	B	
14.		C

EXERCISE 5.8 – PLATINUM MATHEMATICS GRADE 12

	Agreed Coding Agreement	Coding settled through discussion
1.		C
2.	B	
3.	C	
4.	B	
5.		C
6.	B	
7.	B	
8.	B	

REVISION – PLATINUM MATHEMATICS GRADE 12

	Agreed Coding Agreement	Coding settled through discussion
10.1	B	
10.2	B	
10.3	B	
10.4	B	
10.5	B	
12.	C	

EXERCISE 5.1 – CLASSROOM MATHEMATICS GRADE 12

	Agreed Coding Agreement	Coding settled through discussion
1a.	B	
1b.	B	
1c.	B	
1d.	B	
1e.	B	
1f.	B	
1g.	B	
1h.		B
2.		B
3.	C	
4.	C	
5	C	

EXERCISE 5.2 – CLASSROOM MATHEMATICS GRADE 12

	Agreed Coding Agreement	Coding settled through discussion
1a.	B	
1b.		B
1c	B	
1d	B	
1e.	B	
1f.		B
1g.		B
1h.	B	
1i.	B	
1J		B

1k.	B	
1l.	B	
2.		C
3.	B	
4.		C
5.	C	
6.	C	
7.	C	
8.	C	
9.	C	
10.		C
11.	B	
12.	C	
13.		B

EXERCISE 5.9 – CLASSROOM MATHEMATICS GRADE 12

	Agreed Coding Agreement	Coding settled through discussion
1a.	B	
1b.		B
1c.		C
1d.		C
1e.		C
1f.	C	
1g.	C	
1h.	C	
1i.	C	
1j.	C	
2a.		B
2b.	C	
3a.		C

3b.	**C**	
4a.		**B**
4b.		**B**
5a.		**C**
5b.		**C**
6.	**C**	
7a.	**D**	
7b.		**D**
8a.	**D**	
8b.	**D**	
9.	**D**	
10.	**D**	

2014 NOVEMBER NSC MATHEMATICS PAPER 2

	Agreed Coding Agreement	Coding settled through discussion t
QUESTION 7.2.		**D**
QUESTION 7.3	**B**	

2015 NOVEMBER NSC MATHEMATICS PAPER 2

	Agreed Coding Agreement	Coding settled through discussion
QUESTION 5.3	**C**	

2016 FEB/MARCH NSC MATHEMATICS PAPER 2

	Agreed Coding Agreement	Coding settled through discussion
QUESTION 6.1		B
QUESTION 6.2	C	

2016 NOVEMBER NSC MATHEMATICS PAPER 2

	Agreed Coding Agreement	Coding settled through discussion
QUESTION 6.3	B	
QUESTION 6.4	C	