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International Journal of Mining, Reclamation and Environment

ISSN: 1748-0930 (Print) 1748-0949 (Online) Journal homepage: www.tandfonline.com/journals/nsme20

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To cite this article: Joseph Githiria, Cuthbert Musingwini, Milka Madahana & Gift Khangamwa (2025) Incorporating a machine learning approach for commodity price prediction to generate a cut-off grade optimisation policy under grade uncertainty, International Journal of Mining, Reclamation and Environment, 39:6, 437-458, DOI: 10.1080/17480930.2024.2420697

To link to this article: https://doi.org/10.1080/17480930.2024.2420697

Published online: 30 Oct 2024.

INTERNATIONAL JOURNAL OF MINING, RECLAMATION AND ENVIRONMENT
2025, VOL. 39, NO. 6, 437–458
https://doi.org/10.1080/17480930.2024.2420697

Incorporating a machine learning approach for commodity price prediction to generate a cut-off grade optimisation policy under grade uncertainty

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ABSTRACT

The economic importance of cut-off grade (COG) is that it differentiates ore from waste. Therefore, when COG is optimised, the economic value of a mining operation can be maximised. COG is optimised by considering several factors that include economic, geological, and operational parameters. Since uncertainty is inherent in some of these parameters, this paper used grade–tonnage distributions to incorporate grade uncertainty and a machine learning approach for commodity price prediction to account for price volatility when determining COGs. The paper demonstrates how uncertainty in these parameters can be incorporated in developing a dynamic COG policy over the life of mine.

ARTICLE HISTORY
Received 23 May 2024
Accepted 20 October 2024

KEYWORDS
Cut-off grade (COG) optimisation; commodity price; grade–tonnage curve; net present value (NPV); machine learning (ML); deep learning

1. Introduction

A generic open-pit mining operation for a mineral commodity such as gold comprises a mine, ore processing plant, refinery, and a waste dump (Figure 1). The cut-off grade (COG) criterion is used to distinguish ore from waste for the gold mineral deposit. The ore is sent to the processing plant to produce a saleable product, while waste material is dumped on waste dumps. In the processing plant, the ore is initially taken through crushing and grinding processes, before being taken through subsequent processes such as gravity concentration, flotation, and leaching to produce an ore concentrate. At the refinery, the concentrate is then subjected to heating and chemical processes to extract the final gold product.

In practice, a single COG is selected to distinguish the quantity of ore and waste material in the mineral deposit for a certain period set in the operational and tactical plans. The quantity and grade of the ore determine the revenue generated for a specific period during the life of mine (LOM) of the operation. This COG is generally determined at the mine's development phase and reviewed on a yearly basis in business plans to reflect changing market conditions. In this way, the change in COG does not reflect parameter changes on shorter periods such as a month, hence the cumulative quantity of ore and waste material at shorter time periods comprising the longer specified period fail to reflect the overall revenue generated. Therefore, it is important to have a tool that will aid in selecting the appropriate COG when parameters significantly change during the LOM of a mining operation, hence the need to develop a dynamic COG policy. Additionally, since such a tool does

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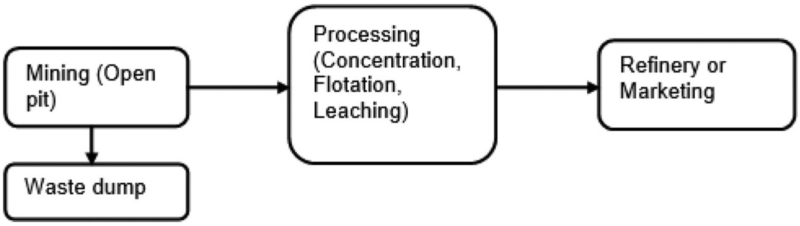

Figure 1. High-level illustration of the flow of material in a generic open-pit mining operation [1].

not generate a static COG but can generate a range of possible COGs, the mine can have the opportunity to stockpile material for later processing when there are changes in parameters.

COG optimisation is often attributed to Lane's seminal work on COG theory, which has been applied with success in most mining projects because it recognises that to create more economic value through increased cash flows or higher net present value (NPV), cut-off grades should be dynamic to account for variable mining conditions over the LOM [2,3]. However, Lane's COG theory has some shortcomings since it does not account for variability in commodity price and uncertainty in the orebody's geological information. This has necessitated heuristic modifications or extensions to the theory [4]. With these modifications, it becomes easier to account for complex mining scenarios that are encountered in actual mining situations.

Most previous studies such as Lane [2,3], Dagdelen and Kawahata [5], Myburgh et al. [6], and Githiria et al. [7] used a static commodity price and grade–tonnage distribution. To overcome the shortcoming of using a static mineral commodity price, this paper applies Lane's COG theory but uses a machine learning (ML) approach to predict and forecast commodity price and Monte Carlo simulation to generate multiple realisations of grade–tonnage distributions to develop a dynamic cut-off grade optimisation policy.

Instead of using a single commodity price for COG determination over the LOM as done in most studies, this paper presents a different perspective by using an ML approach to complement Lane's COG theory. This approach ensures that variable commodity prices combined with grade uncertainty are used to generate more realistic NPVs since uncertainty is inherent to mining operations. In this way, the COG optimisation policy generated using the approach as presented in this paper, closely relates to variable conditions as encountered in actual mining operations.

This paper had two main objectives. Firstly, it sought to show that ML studies which have focused solely on commodity price prediction can be extended to include commodity price prediction in calculating dynamic COGs. Secondly, the paper also sought to demonstrate how commodity price volatility and grade uncertainty affect COG and subsequently, the NPV. Before calculating the COG, grade uncertainty in the mineral deposit was simulated using Monte Carlo simulation to depict a real mining scenario as shown in Section 3.2. The gold price was predicted using ML methods to depict market conditions. The ML approaches were also used to predict and forecast the gold price using a historical price dataset. These two variables were then used in calculating COG and results analysed to illustrate the effect on NPV. The paper, therefore, demonstrated the importance of selecting optimum COGs in a mining operation based on realistic and accurate mining parameters at any specified period on during the LOM.

Machine learning involves the process of utilising data to learn patterns and parameters that help to improve the performance of a given task. Machine learning methods fall into several categories including supervised learning which needs labelled data and unsupervised learning which can use data without labels. The discussion of ML methods shows that supervised learning is the commonly used ML approach for prediction in mining problems. The Autoregressive Integrated Moving Average (ARIMA) model and deep neural network methods are commonly employed in predicting and forecasting commodity prices and operating costs for any specified period during the LOM. Accordingly, ARIMA and the long short-term memory (LSTM) deep neural network algorithm were used in this paper. The literature on the choice of methods employed in variability in the grade and commodity price has been illustrated in Sections 3.2 and 3.3.

The approach developed in this paper was then tested on the McLaughlin gold deposit. The McLaughlin gold deposit case study data presented in this paper is adopted from two studies on COG optimisation [5,6]. The dataset is also publicly available from a mining library which was developed by Espinoza *et al.* [8] for academic research purposes.

2. Input parameters used in the cut-off grade optimisation model

The input parameters for a cut-off grade model typically include economic, geological, and operational parameters. Economic parameters include mineral commodity price, mining cost, processing cost, refining/selling/marketing cost, fixed/period cost, and discount rate. Operational parameters include metallurgical recovery, mining, processing, and refining capacities. Geological input is represented by the grade–tonnage curve distribution inside the ultimate pit limit (UPL) or pushbacks of an orebody model for a mine. Maximum capacities, unit costs, and quantities are important in describing any COG policy.

The block model in this paper and the economic and operational parameters used for COG optimisation are those for the McLaughlin gold mineral deposit as depicted in Table 1. The operation is assumed to have an unrestricted potential to mine and refine or market gold, while the processing capacity is set to be at 1.05 million tonnes of ore which was the capacity of the McLaughlin mineral processing plant.

The mining variables were quantified in United States customary (or imperial) units. These units can be applied equivalently with the metric units in the COG model without errors. When using metric units of measure in the COG calculation, US customary units are then replaced by metric units so that oz/ton is replaced by gram/tonne, US$/ton is replaced by US$/tonne, and US$/oz is replaced by US$/gram. The block model for the McLaughlin gold mineral deposit comprises 2,140,342 blocks with dimensions of 25 ft × 25 ft × 20 ft. In addition, slope profiles of 45° are assumed.

The block model and documentation describing the McLaughlin gold deposit were used to estimate quantities of material that could be extracted from the deposit as indicated in Table 2. The gold grade ranges from 0 to 0.358 oz/ton with an average grade of 0.110 oz/ton and a total tonnage of 125,523,000 tons. Geological input used in the COG optimisation model is the grade–tonnage curve of the deposit as shown in Table 2.

The sheeted vein zone of the deposit, as viewed in a cross-section striking in the north-west direction, is shown in Figure 2 which profiles the average gold concentrations. The block model of the orebody is based on assay results of blast-holes collected during mining. The key geological features of the deposit are indicated by the bold blue line (marked SCF) which refers to the Stony Creek Fault, while the surficial volcanics are marked as Qv.

Table 1. Economic, design, and operational parameters used in the block model for the McLaughlin gold mineral deposit [1,5,6].

Notation	Explanation	Unit	Amount
s	Selling price	US$/oz	1400
m	Mining cost	US$/ton	1.20
c	Milling cost	US$/ton	19
r	Refining or Selling cost	US$/oz	805
f	Annual fixed costs	US$/year	8,350,000
y	Recovery	Decimal	0.90
d	Discount rate	Decimal	0.15
M	Mining capacity	ton/year	Unlimited
C	Milling capacity	ton/year	1,050,000
R	Refining capacity	ton/year	Unlimited
g	Grade	oz/ton	0 to 0.358

Table 2. Grade–tonnage distribution of the McLaughlin gold deposit [1,5,6].

Grade (oz/ton)	Quantity of material in the orebody ('000ton)
0.000-0.020	70,000
0.020-0.025	7,257
0.025-0.030	6,319
0.030-0.035	5,591
0.035-0.040	4,598
0.040-0.045	4,277
0.045-0.050	3,465
0.050-0.055	2,438
0.055-0.060	2,307
0.060-0.065	1,747
0.065-0.070	1,640
0.070-0.075	1,485
0.075-0.080	1,227
0.080-0.100	3,598
0.100-0.358	9,574
Total	**125,523**

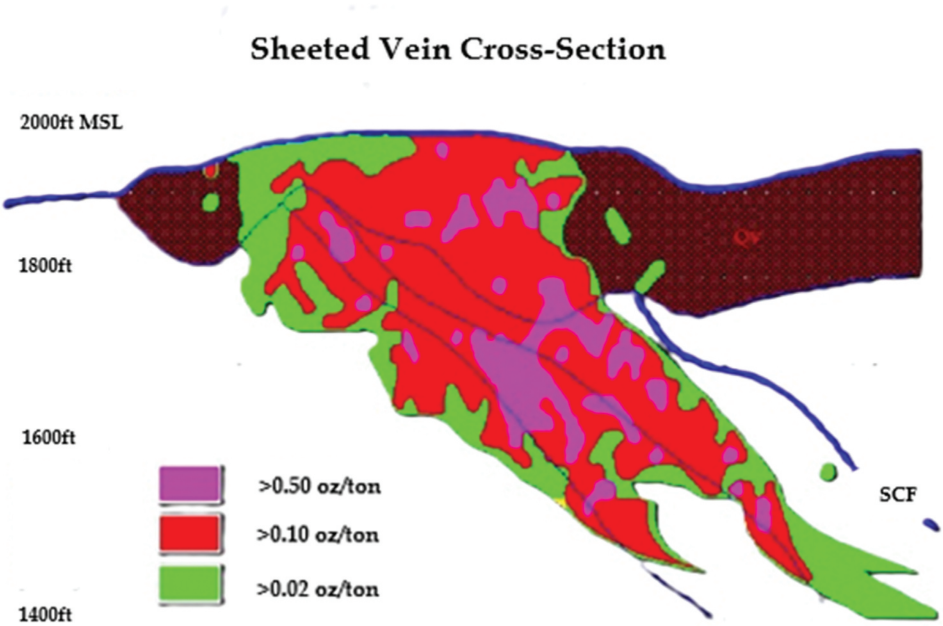

Figure 2. Cross-section of sheeted gold vein in the McLaughlin orebody [9].

3. Stochastic model formulation for COG optimisation

The general flow of the COG optimisation model has several steps that are implemented sequentially and iteratively until an optimum solution is achieved. The process is described in Figure 3, which shows the steps followed in developing the model. The value of the deposit is calculated using this algorithmic process by determining the optimum COG which is then used to maximise the NPV. The methodology involves the development of a stochastic mathematical optimisation model that incorporates a machine learning method to create possible strategies to generate the solution to the COG optimisation model. The COG optimisation model is based on Lane's COG theory with modifications that account for uncertainty in economic, operational, and geological conditions involved in a mining operation.

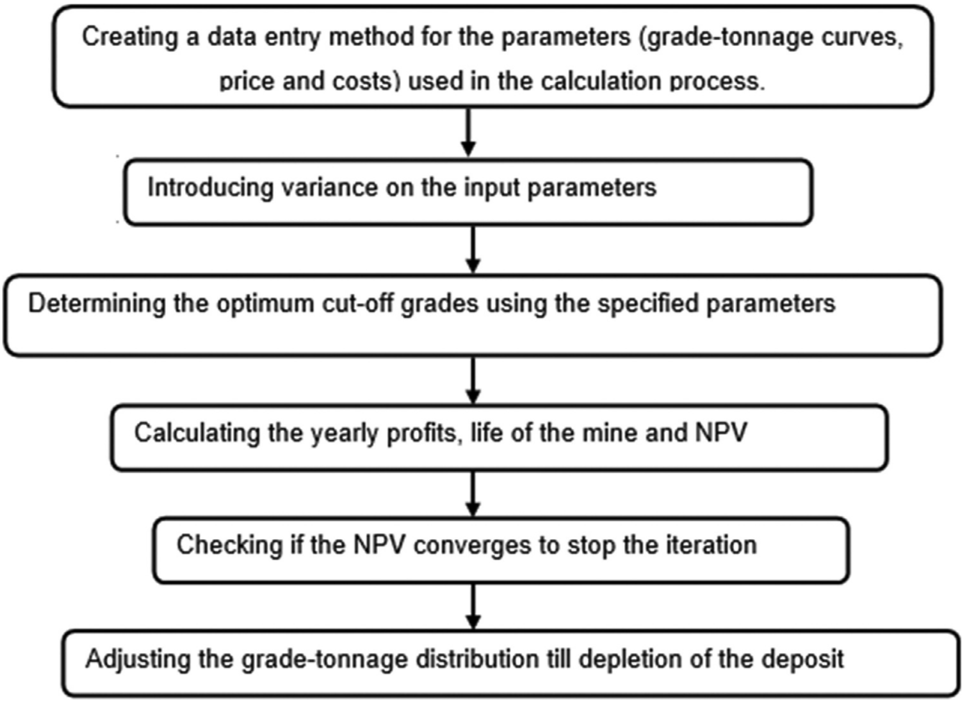

Figure 3. General flow of the stochastic COG optimization model [10].

The objective function used in the COG optimisation model aims to maximise the NPV as represented by Equation 1, where profit (P) or net cash flow per period (i) is given by Equation 2. The COG optimisation model develops a COG policy by determining cut-off grade (G) from periods 1 to N under a set of equally probable grade–tonnage curves (*w*). The NPV of future cash flows is maximised subject to mining, processing, and refining capacity constraints. The objective function of this COG optimisation model is represented as:

$$\text{Max NPV} = \sum\nolimits_{i=1}^{N} \frac{P_{w_i}}{(1+d)^i} \tag{1}$$

The cash flow equation *(*Pw_i*)*, as represented in Equation 2, changes to incorporate the uncertainty of both the mineral commodity price and grade–tonnage distribution.

$$\textit{Cash flow}, P_{wi} = \left(\sum\nolimits_{i=1}^{N} s_i - r_i\right) * Qr_{wi} - c_i * Qc_{wi} - m_i * Qm_{wi} - f_i \tag{2}$$

Subject to ($Qm_{wi} \leqslant M$ for $i = 1 \ldots N$), ($Qc_{wi} \leqslant C$ for $i = 1 \ldots N$), and ($Qr_{wi} \leqslant R$ for $i = 1 \ldots N$)

The notations of these input parameters used in the COG optimisation algorithm are shown in Table 3.

The decision variables are represented by the economic and geological parameters, while the operational parameters are used to construct the constraints that bound decision variables and consist of equalities and inequalities. The constraints are represented as follows: ($Q_{mi} \leqslant M$ for $i = 1 \ldots N$), ($Q_{ci} \leqslant C$ for $i = 1 \ldots N$), and ($Q_{ri} \leqslant R$ for $i = 1 \ldots N$). The quantity of material to be mined, quantity of material mined sent to the concentrator and quantity of the final product are represented by Q_{mi}, Q_{ci} represents Q_{ri} respectively. The variables are governed by a set of non-negativity restrictions: $X_1, X_2 \ldots X_n \geq 0$ where $X_1, X_2 \ldots X_n \in [0, 1]$ [10].

The economic parameters include the selling price (s) of the mineral, mining cost (m_i), processing cost (c_i), refining/selling/marketing cost (r_i), fixed/period cost (f_i), recovery (y), and discount

Table 3. Notation of parameters in the COG algorithm [1,10].

Notation	Explanation	Unit
i	Period (year) indicator	–
N	Mine life	Years
w	Grade–tonnage curve indicator	–
k	Grade–tonnage indicator	–
K	Number of grade–tonnage distribution realizations in a curve	–
s	Selling price	US$/oz
m	Mining cost	US$/ton
c	Milling cost	US$/ton
r	Refining or Selling cost	US$/oz
f	Annual fixed costs	US$/year
y	Recovery	decimal
d	Discount rate	decimal
M	Mining capacity	ton/year
C	Milling capacity	ton/year
R	Refining capacity	ton/year
Qm_{wi}	Quantity of material to be mined for grade–tonnage curve *w* during period *i*	ton/year
Qc_{wi}	Quantity of material mined sent to the concentrator for grade–tonnage curve *w* during period *i*	ton/year
Qr_{wi}	Quantity of the final product for grade–tonnage curve *w* during period *i*	ton/year

rate (d). The geological parameters are defined by the grade–tonnage distribution inside the UPL. These include average grade (g), optimum cut-off grade (G), quantity of material corresponding to a particular grade (q), total quantity of material inside the pit (Q), and quantity of material mined, material processed, and metal refined [10].

The algorithm consists of several stages that are implemented sequentially for a given model of the Mineral Reserves. The first stage in the algorithm is the COG optimisation stage whereby the calculation of cut-off grades is defined using economic, operational, and geological parameters. Variability in these parameters is factored in at this stage before they are used in the calculation of the cut-off grades. These cut-off grades are then used in the second stage (the NPV convergence stage) to calculate the NPV in a specific period. The final stage is the grade–tonnage curve adjustment stage where the grade–tonnage curve is adjusted per the specified period to account for the depletion of the Mineral Reserves.

The optimum cut-off grade policy determines the optimum cut-off grade using two grade categories (limiting economic and balancing cut-off grades) to maximise the NPV of an operation [2,3]. The limiting economic cut-off grades (g_m, g_c, g_r) are based upon economic parameters such as costs and mineral commodity price. They are determined based on either of the three major stages of mining operation (mining, concentrating, and refining). One of these operations is used as the governing constraint which acts as a limiting factor in the system, while the rest of the constraints are varied with respect to the grade-tonnage distribution. The principle of a limiting factor (bottleneck or constraint) in a production system is a concept explained by the Theory of Constraints (TOC) which suggests that at any time only one constraint is limiting the production rate at which the output is generated. By addressing the constraint, the method will generate an improved outcome from the entire production operation in terms of profitability but leads to the identification of the next limiting constraint in the system [11]. In this way, there is an opportunity for continuous improvement of the production system.

The balancing cut-off grades (g_{mc}, g_{rc}, g_{mr}) are dynamic as they vary with the grade variation in the deposit. They do not depend on economic parameters but on the grade distribution of the mined material and capacities where two operations or constraints are to be in balance, i.e. both operating at full capacity. The calculations are based on the grade–tonnage distribution of the mined material by calculating the average grade of the treated material as a function of the selected cut-off grade. In considering the two cut-off grade categories, an optimum cut-off grade will be generated by pairing all the operations. For example, when the mine and the refinery are paired in the calculations, there are three possible options (g_m, g_c, and g_{mc}) of generating an optimum cut-off grade (G_{mc}). Lastly, the optimum cut-off grade (G) is selected as the middle value among the three combinations (G_{mc}, G_{rc}, and G_{mr}) and subsequently, Q_m, Q_c, and Q_r, and the NPV can be computed [2,3].

3.1. Stochastic COG calculation approach

The COG policy is generated using the stochastic COG calculation approach illustrated in Figure 4 and indicates the mine life, annual profits, and NPV. The calculation approach assumes a mineral deposit from which a single mineral commodity is mined, considering uncertainty. The equations for parameters indicated in Figure 4 are listed in the Appendix as Equations A.1 to A.15 [1–3,7,10].

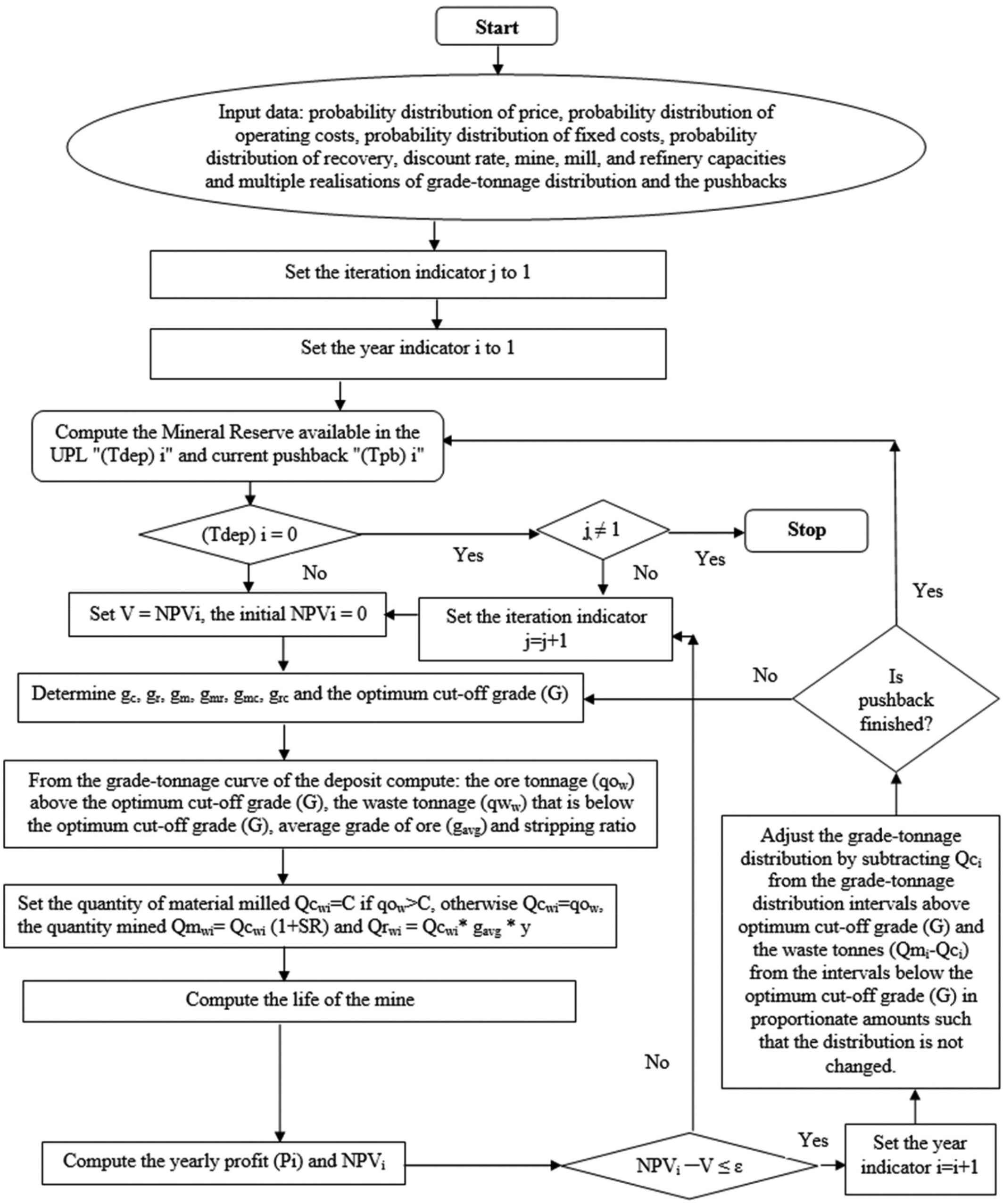

Figure 4. Flow diagram of the cut-off grade calculation algorithm [1,10].

3.1.1. Results: cut-off grade policy generated using baseline data

The COG calculation model determined the optimum COG that maximises NPV in the minimum LOM of 9.12 years. The NPV generated from the model is calculated using the baseline data in Tables 1 and 2. Results from this dataset show that the mining project has a mine life of 9.12 years and the NPV is US$ 467.17million as seen in Table 4.

3.2. Variability in the grade–tonnage distribution

Grade–tonnage curves are used in the determination of balancing cut-off grades. The balancing cut-off grades are best achieved in practice by tabulating the grade distribution data for a mine. Indeed, the balancing cut-off grades are not calculated in the formulae. They can be achieved by tabulating the grade distribution data, but they can also be derived from curves of values versus the respective limiting cut-off grades created for the respective stages that need to be balanced [12,13]. This data is used to calculate the ratios and cumulative values in relation to the different stages in a mining operation. Grade–tonnage distribution in both open pit and underground mining operations is based upon a mineralised orebody. The mineralised material will consist of a range of grades which, in practice, is estimated through a sampling process. The grade distribution is generated from the samples by calculating proportions that exceed a range of grade limits.

In most conventional and deterministic orebody models, over-estimation or under-estimation of grade, volume or tonnage, and other parameters related to the deposit occurs often. It is important to incorporate this when undertaking mine planning to maximise profits [1,10]. Several geostatistical methods used in measuring the geological characteristics of the orebody include sequential indicator simulation, sequential Gaussian simulation, and other simulation methods as described by Vu et al. [14]. Vu et al. [14] stated that the geostatistical simulation is primarily based on the spatial random field and Monte Carlo simulation. The grade and rock type are considered as regionalised variables that are assumed to be the realisation of a spatial random field defined by the spatial distribution of collected samples. Using the generated spatial random field, Monte Carlo simulation is used to generate multiple realisations of the grade–tonnage distribution.

In this study, the Monte Carlo method is used to simulate the grade–tonnage distribution of the orebody (GTC1) to derive six different scenarios (GTC1–GTC6). It uses statistical and graphical techniques to simulate the probable distribution of data. It is mainly used in optimisation, numerical integration and generating draws from a probability distribution. It relies on the possibility of producing a supposedly endless flow of random variables for well-known distributions by modelling and sampling probability density functions (PDFs). The cumulative distribution function (CDF) is used for sampling purposes whereby it is defined as an integral of the PDF, for the uniform distribution. Equation 3 shows the integral of the function of h and f over a range of x to give estimated random values of the whole sample.

$$E_f[h(X)] = \int_x h(x)f(x)dx \tag{3}$$

Table 4. Results as reported from cut-off grade model using constant mining parameters [1].

Economic indicators	Value
Quantity of material mined (Q_m) (tons per year)	125,538,000
Quantity of ore sent to concentrator (Q_c) (tons per year)	9,450,000
Quantity of gold produced (Q_r) (koz)	1931.60
Mine life (years)	9.12
Cash flow or Profits (US$)	887,089,585.49
NPV (US$)	467,172,060.50

Where:

x = rfunction (J),

J = 10^8, represents the whole sample,

rfunction samples x from $f(x)$,

hs = h (X),

(estimate) est = mean (hs),

provided h^2 has finite expectation under f.

The CDF indicates the probability that a number sampled randomly from the PDF will be ⩽ x. With a range of 0 to 1 in the equation, the range used for the grade–tonnage curve (GTC1) to be simulated is 0.7, 0.07257, 0.06319, 0.05591, 0.04598, 0.04277, 0.03465, 0.02438, 0.02307, 0.01747, 0.01640, 0.01485, 0.01227, 0.03598, and 0.09574. The simulated multiple realisations of grade–tonnage distribution developed from this exercise are shown in Table 5. It is evident that the tonnages decrease as the grade limit increases.

A total of six grade–tonnage curves were created using the Monte Carlo simulation method, resulting in six different but equally probable realisations of the orebody. Simulation of several equally probable realisations of the orebody helps in undertaking risk and decision analysis (sensitivity and probabilistic analysis) on the COG policy generated and its relation to the key input parameters. The resultant simulated grade–tonnage curves, as shown in Table 5 have different tonnages and grade distribution across the orebody to depict a stochastic orebody model. The stochastic orebody model reduces over-estimation or under-estimation of grade, volume or tonnage and other parameters related to a mineral deposit.

3.2.1. Results: cut-off grade policy generated using multiple realizations of grade–tonnage distribution

The resultant cut-off grade policies generated by the grade–tonnage curves (GTC1 – GTC6) in Table 5 vary depending on time and quantity of mineable ore in every grade interval. A summary of the calculated annual profit, mine life, and NPV for six equally probable grade–tonnage curves is generated, as shown in Table 6. The results generated are calculated using a gold price of US $1400 per ounce and a refining or selling cost of US$805 per ounce, whereas keeping the other economic and operational parameters constant. The resultant cut-off grade policies from the model

Table 5. Simulated grade–tonnage distribution of the McLaughlin gold deposit [1,10].

	Quantity of material in the simulated orebody realization ('000ton)					
Grade (oz/ton)	GTC1	GTC2	GTC3	GTC4	GTC5	GTC6
0.000-0.020	70,000	70,022	70,081	70,003	69,640	70,900
0.020-0.025	7,257	7,263	7,257	7,364	7,890	7,000
0.025-0.030	6,319	6,325	6,332	6,405	6,860	5,700
0.030-0.035	5,591	5,595	5,616	5,632	6,100	5,200
0.035-0.040	4,598	4,603	4,628	4,608	4,950	4,600
0.040-0.045	4,277	4,279	4,306	4,318	4,570	4,300
0.045-0.050	3,465	3,466	3,488	3,502	3,680	3,000
0.050-0.055	2,438	2,439	2,468	2,438	2,630	2,200
0.055-0.060	2,307	2,309	2,335	2,318	2,520	2,100
0.060-0.065	1,747	1,748	1,754	1,740	1,900	1,500
0.065-0.070	1,640	1,643	1,647	1,637	1,770	1,400
0.070-0.075	1,485	1,488	1,494	1,481	1,560	1,200
0.075-0.080	1,227	1,229	1,237	1,246	1,270	800
0.080-0.100	3,598	3,598	3,625	3,636	3,810	3,300
0.100-0.358	9,574	9,578	9,579	9,641	10,330	9,800
Total	**125,523**	**125,585**	**125,847**	**125,969**	**129,480**	**123,000**

Table 6. Summary of the output generated from six equally probable simulated grade–tonnage curves.

Grade–tonnage curves (GTCs)	Total tonnage	Average annual profit (US$)	Mine life	Net present value (US$)
GTC1	125,523	97,268,594.9	9.12	467,172,060.50
GTC2	125,585	97,268,594.9	9.12	467,172,060.50
GTC3	125,847	97,230,794.9	9.12	466,990,510.60
GTC4	125,969	97,318,994.9	9.18	468,922,686.82
GTC5	129,480	97,999,394.9	9.84	488,180,335.90
GTC6	123,000	97,974,194.9	9.33	475,864,529.70

show a difference of 4.34% between the minimum and maximum NPV generated through a set of six equally probable grade–tonnage curves. The grade–tonnage distribution varies consistently over time in the cut-off grade calculations.

From the summarised results in Table 6, the grade–tonnage distribution affects the cash flow and NPV. GTC5 with a high tonnage of 129,480,000 tons generates the highest NPV (US $ 488,180,335.90), although, this does not happen in the rest of the grade–tonnage curves. For example, GTC6 with the least tonnage (123 million tons) has a higher NPV of US$ 475,864,529.70. The six resultant cut-off grade policies generating NPV in approximately 9 years change with the variable grade–tonnage distribution of the orebody. This indicates that the level of uncertainty in the geological data is high, hence the need to account for it. Therefore, ignoring the geological uncertainty in mining operations may lead to serious economic effects.

3.3. Commodity price volatility and use of machine learning in gold price prediction

Due to mineral commodity prices being characterised by volatility, machine learning methods have sometimes been used to reliably predict mineral commodity prices. This section of this paper, therefore, reviews the application of machine learning methods in the prediction of mineral commodity prices but with a focus on the prediction of gold prices. The focus on gold price prediction is premised on the McLaughlin gold mining operation as a case study. Machine learning involves the use of a wide array of algorithms that are capable of learning meaningful patterns from data that are useful in improving the performance of a given task, for example, prediction of disease outbreaks or prediction of stock prices.

To begin with, various studies have been conducted that have shown that the ML techniques that are applied in the prediction processes can also be used in forecasting the prices of mineral commodities. A distinction must be made between prediction and forecasting. Prediction refers to the use of an ML model to make predictions that can be verified against test data, while forecasting refers to a prediction for a 'future' period where no test data exists.

In general, the models that have been used for prediction tasks can be grouped into statistical or mathematical models, machine learning, and hybrid models which are combined models. Mineral commodities include but are not limited to precious metals, such as gold, silver, platinum, and palladium.

A machine learning method that has often been used for mineral commodity price prediction and forecasting is the ARIMA method. This method has been applied in several fields, for instance in stock problems, economics, energy, social, and engineering [15–18]. Moreover, this model has also been employed as a benchmarking method in the assessment of newer models in the prediction and forecasting of precious commodity price citations.

The second common model is the Linear Regression (LR) model used by Angus et al. [19] to comprehend the price movements of selected recycled materials in the United Kingdom. Also, Artificial Neural Networks (ANNs), with various architectures, have been used extensively in the prediction of commodity prices [18,20,21]. Support Vector Machines (SVM), which was developed in the 1990s has also been used for forecasting future mineral commodity prices and for sales forecasting [22,23]. Various hybrid models have been developed to address the limitations of individual algorithms. Examples of these hybrid models developed are sequential Gaussian simulation and the smoothing spline algorithm. Other documented hybrid models can be found in research works by Chatterjee et al. [24]; Sauvageau and Kumral [25]; Aminul Haque et al. [26].

Literature shows that ANNs have been amalgamated with several other models, for example, ARIMA and Autoregressive Conditional Heteroscedasticity (GARCH) in the prediction of mineral commodity prices [27–32]. ANNs have been previously combined with fuzzy logic systems to form neuro-fuzzy systems that have the advantage of high learning capacity [33]. Current literature shows that the Adaptive Neuro-Fuzzy Inference System (ANFIS) can be used in the prediction of mineral commodity prices [34].

Jianwei et al. [35] applied combined independent component analysis (ICA) and Gate Recurrent Unit Neural Network (GRUNN) to analyse and predict gold prices. The research results show that ICA-GRUNN provides prediction with high accuracy and outperformed the benchmark techniques such as ARIMA, radial basis function neural network (RBFNN), LSTM, GRUNN, and ICA-LSTM. Tripurana et al. [36] used random forest (RF) regression, decision tree (DT), support vector regression (SVR), LR, and ANNs to predict gold prices. The results of the research work showed that the least root mean square error (RMSE) was 0.377 and the least mean absolute percentage error (MAPE) was 0.151 for ANN models in the prediction of mineral commodity prices. Moreover, various performance metrics have been used to compare the performance of these ML models. These metrics include RMSE, absolute error (AE), relative error (RE), Spearman's Rho (SR), and Kendall's Tau (KT).

Cohen and Aiche [37] forecasted gold prices using LR and Darvas boxes (DB), and Bollinger bands (BB). The results of the research work showed that LR forecasted silver and gold prices better, while BB was suited for palladium. Khani et al. [38] used Convolutional Neural Networks (CNN) LSTM, vector sequence output (VSO) LSTM, bidirectional LSTM, and encoder–decoder LSTM to forecast gold prices with respect to pandemics. Sravani et al. [39], predicted gold prices using multiple linear regression (MLR), gradient boosting (GB), and RF, where RF outperformed the MLR and GB. Table 7 below shows some of the machine learning algorithms that have been used in the prediction of gold prices in the main and prices of other minerals often related to gold.

3.3.1. Gold price dataset description

Figure 5 shows historical gold prices from the year 1978 to the current year 2024 as reported by the World Gold Council [45]. The gold prices have gradually changed from approximately US$ 250/oz in 1978 to US$ 2035/oz in 2024. As seen in Figure 5, gold prices started increasing gradually from 2005 due to the metal and energy price boom resulting from the increased global demand. For example, China and India, which jointly account for about 37% of the world's population, have seen their demand for metals grow in this period due to increased development in construction, infrastructure, and manufacturing industries [46]. The gold price increased significantly at approximately 300% from 2005 to date. Table 8 provides the descriptive statistics of the dataset for the monthly gold price data.

Figure 6 shows the gold price distribution for the last 46 years and the price with the highest count is US$250/oz, occurring in 25 years. Historical data, as seen in Figure 6 indicates that the average yearly price of gold has increased by more than 6 times since 2005 [45]. Several factors affect global market conditions which consequently affect the gold price. This paper uses gold price in the year 2019 (US$1400/oz) and a selling cost of US$805/oz as the base case to compare with the works done by Dagdelen and Kawahata [5], and Myburgh et al. [6].

In this paper, the dynamic nature of gold price is explored by using two classes of machine learning algorithms to generate two sets of gold price data; these being the predicted gold prices using observed data as ground truth values and forecasted prices for the future period without any

Table 7. Machine learning approach in the prediction of gold prices.

Author	Commodity	Algorithm	Performance metrics	Key findings
Ufuk and Çağatay [40]	Gold, palladium, silver, platinum, Brent Petrol, natural gas	ANN	RMSE, AE, RE, SR, KT	Error rates are accurate to foresee the market trends
Iftikhar and Nazir [41]	Gold	ANN, LR	RMSE	
Jianwei et al. [35]	Gold	ICA-GRUNN	MAPE, MAD, RMSE	ICA-GRUNN outperforms benchmark methods used in accuracy.
Cohen and Aiche [37]	Gold, silver, copper, platinum, and palladium.	LR, DB, BB		LR is the best technique to forecast silver and gold, while BB is suited for palladium
TTripurana et al. [36]	Gold	ANN, LR, SVR, DT, RF	RMSE, R-Squared, MAPE	The least RMSE is calculated as 0.377 for ANN, and the least MAPE is calculated as 0.151, for ANN as well.
Sravani et al. [39]	Gold	RF, GB, MLR	MAE, RMSE	Random forest model outperformed others
Makala and Li [42]	Gold	SVM, ARIMA	RMSE, SVM	SVM gave RMSE of 0.028 and MAPE of 2.5. ARIMA gave RMSE 36.18 and MAPE of 2897.
Ghule [43]	Gold	RF, LR	R Squared	RF outperformed LR
Khani et al. [38]	Gold	LSTM for CNN, VSO, bidirectional, encoder–decoder	MSE	LSTM achieved an MSE of 6.0e−4, 8.0e−4, and 2.0e−3 on the validation set respectively for 1 day, 2 days, and 30 days predictions in advance.
Sadorsky [44]	Gold	Bagging, stochastic gradient boosting, RF	Accuracy, Kappa, Pos Pred Value, Neg Pred Value	Tree bagging and random forests offer an attractive combination of accuracy and ease of estimation

Figure 5. Historical gold prices (Us$/oz) from 1978 to 2024 [45]).

Table 8. Descriptive statistics for monthly gold price data from 1978 to 2023.

Description	Value
Dataset (Count)	552
Mean Value (Mean)	US$745.07/oz
Standard Deviation (Std)	US$529.80/oz
Minimum Value (Min)	US$207.80/oz
25th percentile	US$352.18/oz
50th percentile	US$422.05/oz
75th percentile	US$1228.18/oz
Maximum Value (Max)	US$2029.30/oz

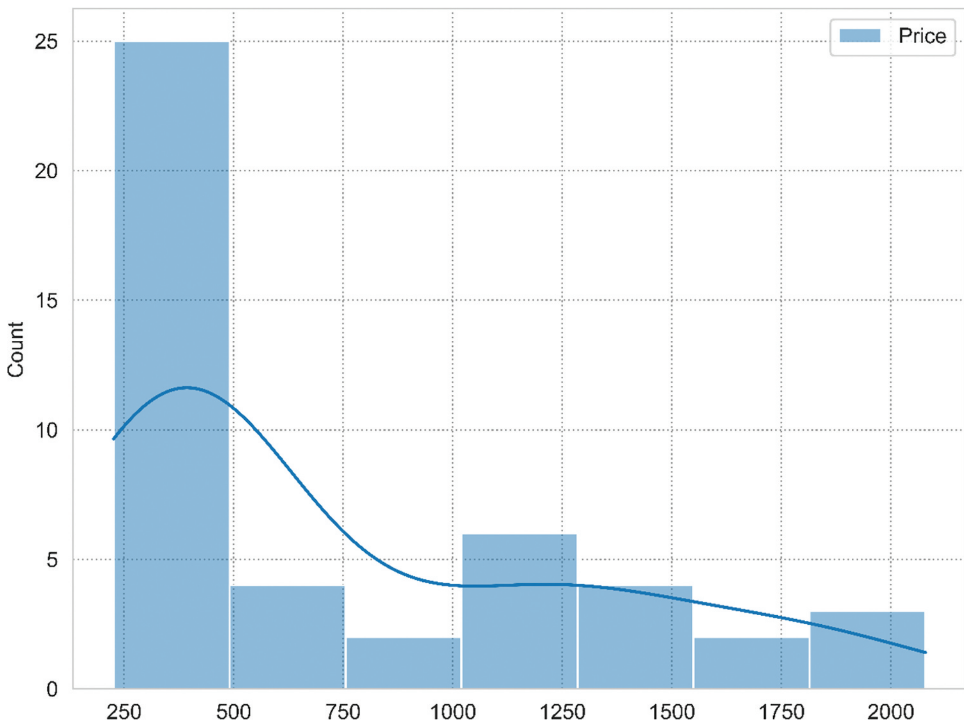

Figure 6. Gold prices distribution showing the number of occurrences from 1978–2024.

observed data. The predicted data was from the period from 2019 to 2024, and the forecasted data started from 2024 to 2033 in the calculation of optimum COG and NPV using the McLaughlin gold mine dataset.

3.3.2. Gold price prediction and forecasting using ML techniques

Pursuant to the earlier discussion on ML approaches to mineral commodity price prediction and forecasting, this paper selected the ARIMA and long short-term memory (LSTM) deep neural network algorithm as the preferred ML approaches for long-term gold price prediction and forecasting. These models are suitable for tasks akin to the one in this study that make use of time-series datasets as they can account for some of the seasonality and cyclic behaviours in the time-series dataset. The study utilised the long-term gold price dataset from 2004 to 2024 with opening and closing prices on a daily, weekly, monthly, and annual periods. Aside from the fact that the gold prices from 2004 have additional features useful in the training of the models, the trajectory of the gold prices from 2004 followed a general upward trend.

The gold price predictions were generated by using the chosen ML techniques, namely, ARIMA and LSTM deep neural network (DNN) models, which were later compared with the actual gold prices. The dataset was split into a training and testing dataset as a way of assessing the effectiveness of the ML models. Figure 7 shows the trend in the predicted and forecasted gold prices as indicated by a blue line, generated using ARIMA with a confidence interval of 95%.

The actual gold price data for the period 2004 to 2019 was used to assess the prediction accuracy of these ML models. The assessment of the predicted results showed that ARIMA was predicting values that were lower than the actual gold prices and hence the choice taken was to focus attention on the LSTM models. Using CNN, Stacked, and Basic LSTM, the predicted gold prices in the period 2019 to 2024 as shown in Table 9 were generated. The predicted gold prices range from US $1406.86/oz to US$1988.93/oz in the years 2019 to 2024.

The mean data was used to represent the actual data as predicted by the machine learning method. The varying gold prices in this period were used in the calculation of cut-off grade to depict real-time market conditions as mining progressed over the LOM. Table 9 shows the gold prices predicted by the ML model for each year indicating the minimum, maximum, and mean values, compared to actual gold prices from that same period as shown in the last column of the table.

Using a linear graph as shown in Figure 8, the predicted gold prices were compared with actual gold prices to show the relationship between them. The correlation coefficient derived from the linear graph was used to measure the linear correlation between the two dependent variables. The correlation coefficient value ranges between +1 and −1, where +1 signifies total positive linear correlation and −1 is total negative linear correlation. If the value is 0, then there is no linear correlation. The coefficient relation between actual and predicted gold prices using LSTM is 0.875, indicating a positive correlation between the two gold price sets, hence the ML model could be used

Figure 7. Figure showing the predicted gold prices from 1978 to 2033 using ARIMA.

Table 9. Gold prices in US$/oz predicted using machine learning methods: CNN, stacked, and basic LSTM.

Year	Model	Mean	Std.	Min.	Max	Actual
2019	CNN LSTM	1406.86	87.77	1259.617	1577.042	1514.8
2019	Stacked LSTM	1434.41	99.91	1266.871	1629.215	1514.8
2019	Basic LSTM	1433.04	98.24	1269.292	1623.689	1514.8
2020	CNN LSTM	1719.77	97.81	1429.625	1928.394	1887.6
2020	Stacked LSTM	1802.23	108.24	1471.348	1951.496	1887.6
2020	Basic LSTM	1795.17	108.04	1471.498	1996.421	1887.6
2021	CNN LSTM	1704.68	38.88	1603.494	1798.503	1805.9
2021	Stacked LSTM	1791.46	44.96	1683.898	1893.643	1805.9
2021	Basic LSTM	1780.39	42.86	1679.18	1876.749	1805.9
2022	CNN LSTM	1714.26	73.83	1565.278	1903.405	1813.8
2022	Stacked LSTM	1797.89	82.79	1618.776	1941.722	1813.8
2022	Basic LSTM	1789.07	82.72	1616.751	1976.259	1813.8
2023	CNN LSTM	1832.96	46.17	1725.622	1940.518	2078.4
2023	Stacked LSTM	1911.83	31.70	1820.355	1955.625	2078.4
2023	Basic LSTM	1913.69	44.40	1803.074	2004.207	2078.4
2024	CNN LSTM	1916.55	18.21	1883.571	1946.09	2160.28
2024	Stacked LSTM	1952.89	3.57	1946.109	1958.546	2160.28
2024	Basic LSTM	1988.93	13.40	1965.416	2010.703	2160.28

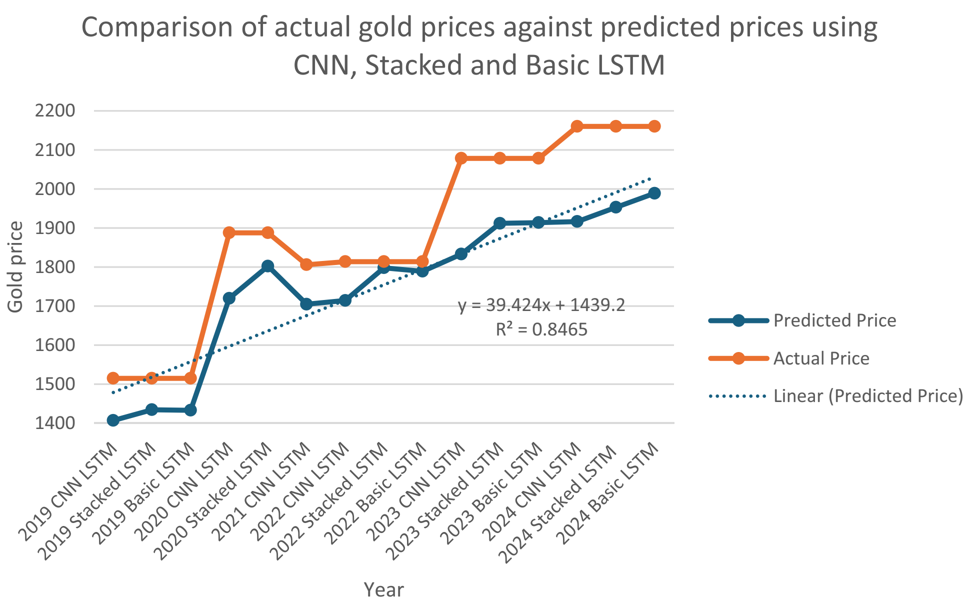

Figure 8. Comparison between predicted and actual gold prices from 2019 to 2024.

to forecast gold prices for the period 2024 to 2033. This is the period for which the cut-off grade optimisation policy had to be developed.

Gold prices from 2004 to 2019 were used to train and test the ML model and with this trend, it predicted gold prices from 2019 to 2024, then later forecasted the gold prices for 10 years from 2024 to 2033. Table 10 shows the forecasted gold prices for the years 2024 to 2033, generated using CNN LSTM deep neural network ranging from US$2191.97/oz to US$2261.19/oz.

3.3.3. *Results: cut-off grade policy generated using varying gold price*

Different gold prices from Table 10 vary depending on time and other market conditions. Using the baseline grade–tonnage distribution (GTC1) and the varying gold prices, cut-off grade calculations generated a policy that can be used to evaluate the mineral project. A summary

Table 10. Forecasted gold prices in US$/oz using machine learning methods: CNN LSTM.

Year	Mean	Std.	Min.	Max
2024	2191.97	51.02	2090.91	2226.86
2025	2326.96	73.50	2202.31	2412.12
2026	2447.62	56.28	2323.95	2497.28
2027	2502.37	51.03	2428.12	2566.83
2028	2534.06	57.11	2461.18	2638.39
2029	2468.62	54.16	2398.96	2556.85
2030	2371.06	42.44	2325.47	2447.53
2031	2363.74	37.61	2317.62	2427.02
2032	2346.63	59.83	2248.01	2449.14
2033	2261.19	42.44	2325.47	2447.53

Table 11. Results generated against different gold price predicted and forecasted using CNN LSTM deep neural network.

Year	Gold price (US$/oz)	Annual profit (US$)	NPV (US$)
2019	1406.86	85,426,913.30	410,297,559.55
2020	1719.77	153,142,201.85	735,527,824.37
2021	1704.68	149,876,650.40	719,843,682.94
2022	1714.26	151,949,810.30	729,800,878.10
2023	1832.96	177,637,083.80	853,174,475.73
2024	2191.97	195,726,377.75	940,055,680.67
2025	2326.96	212,429,708.55	1,020,280,232.83
2026	2447.62	225,511,952.54	1,083,113,040.13
2027	2502.37	237,205,437.68	1,139,275,767.22
2028	2534.06	242,511,407.44	1,164,759,849.01

of the calculated annual profit, mine life and NPV for the 10-year LOM was generated as shown in Table 11. The results generated were calculated using a gold price varying from US$1406.86/oz to US$1988.93/oz, while other parameters were kept constant. The resultant cut-off grade policy from the model generated varied annual profit and NPV increasing significantly over the LOM. The results show a difference of 65.39% between the minimum and maximum NPV generated by varying the gold price from the base value. The base value (US$1400/oz) generated an NPV of US$403,167,459.46, while the gold price in 2028 of US$2534.06/oz had an NPV of US$1,164,759,849.01. This indicates that the level of impact generated from gold price data uncertainty is high, hence the need to account for it. Therefore, incorporating everchanging mineral commodity prices in cut-off grade calculations is important in assessing the overall project value. The cumulative annual profit generated from the varying gold price is US$ 1,831,417,543.61 with an NPV of US$829,394,999.98.

4. Result analysis

The relationship between NPV and the grade–tonnage distribution is modelled using a linear regression approach. It is a linear approach for modelling the relationship between a dependent variable and one or more independent variables. The linear regression models in this section were applied to identify the relationship between NPV and the response variables (grade–tonnage distribution) when all the other variables in the model are held fixed. The linear relationship was generated using values from Table 10. The NPV represents the independent variable as shown in Figures 9 and 10 (on the x–axis), while the dependent variables on the y–axis are gold price and grade–tonnage distribution. The correlation coefficient in the relationship between grade–tonnage distribution and NPV is 0.13 and 0.75 for the relationship between gold price and NPV, respectively.

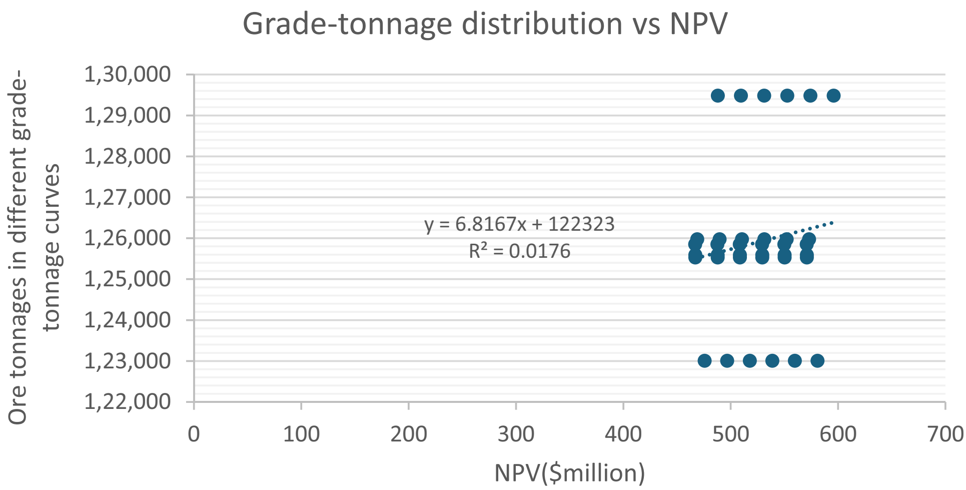

Figure 9. Linear relationship between grade–tonnage distribution and NPV generated.

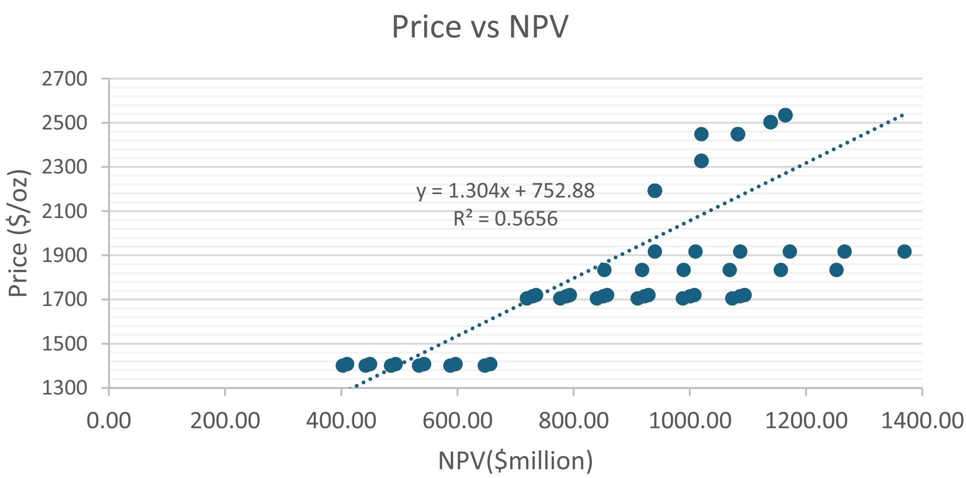

Figure 10. Linear relationship between varying gold price and NPV.

The results indicate that the impact on NPV caused by variations to the grade–tonnage distribution curve is more significant compared to that obtained when commodity price is varied. Any change in mineral commodity price and grade–tonnage distribution affects the NPV significantly, hence the need to accurately determine the cut-off grade considering this variability. Consequently, the resultant COG policy, as presented in Table 12, facilitates a risk-quantified, decision-making process when undertaking major mining investments for sustainable utilisation of Mineral Resources and Mineral Reserves. As seen in Table 12, the mining project value increased to US$829 million due to the increasing gold prices along the 10-year LOM. The actual gold prices during this period (2019–2028) were increasing due to the market conditions at that moment. This increased the quantity of material mined, quantity of ore sent to the plant and quantity of gold produced along the mine life.

Table 12. Results as reported from cut-off grade optimisation model using variable gold price.

Economic indicators	Value
Quantity of material mined (Q_m) (tons per year)	125,538,000
Quantity of ore sent to concentrator (Q_c) (tons per year)	9,450,000
Quantity of gold produced (Q_r) (koz)	1973.61
Mine life (years)	9.12
Cash flow or Profits (US$)	1,831,417,543.61
NPV (US$)	829,394,999.98

5. Conclusion

The cut-off grade policy generated enables the user to sensibly utilise the available data depending on the factors affecting the specific mine such as commodity price and grade–tonnage distribution of the deposit. The commodity price varies with changing market conditions and grade–tonnage distribution varies with the changing geological features of the orebody. The generated results follow a normal distribution where all the key statistics characterise the NPV against the input variables. The results show that the effect of the geological uncertainty in mining and mineral commodity price variations on the project value is significant, hence the need to use mathematical or statistical methods and machine learning models to incorporate the uncertainty. Machine learning methods can be employed in mining to predict and forecast uncertain mining parameters and generate solutions to complex mining problems affecting the mining value chain. Consequently, they optimise the mining operations by minimising costs and maximising the project value. With different commodity prices obtained from using a machine learning approach, different cut-off grades are generated. With different grade–tonnage distributions obtained using Monte Carlo simulation, different cut-off grades are also generated. This paper presented the different perspective that instead of using a static commodity price as has been done in most previous studies, a dynamic cut-off grade policy is generated to mimic variability encountered in actual mining operations. The resultant COG policy generated can be used as a guide to show how mineral commodity price and grade can change NPV. Therefore, monitoring of these two parameters can be done more regularly on shorter time scales such as monthly to cater for any significant changes in economic and operational parameters. In addition, since the proposed approach can generate a range of possible COGs, it can provide the mine the opportunity to stockpile material for later processing when there are significant changes in economic and operational parameters. This then ensures that the mine does not lose out on the opportunity cost brought about by changes in these two parameters.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Appendix

The following equations, inequalities and/or conditions listed below are used in the calculation algorithm in Figure 4 (Lane [1–3,7,10]).

(i) Determine g_c, g_r, g_m, g_{mc}, g_{rc}, g_{mr}, and the optimum cut-off grade G using Equations A.1 to A.10.

$$g_m = \frac{c_i}{\left(\sum_{i=1}^{N} si - ri\right) * yi} \tag{A.1}$$

$$g_c = \frac{ci + \frac{f_i + d * V_{wi}}{C}}{\left(\sum_{i=1}^{N} s_i - r_i\right) * y_i} \tag{A.2}$$

$$g_c = \frac{ci}{\left\{\left(\sum_{i=1}^{N} s_i - r_i\right) - \frac{f_i + d * V_{wi}}{R}\right\} y_i} \tag{A.3}$$

$$g_{mc_w} = \frac{\frac{C}{M} - mc_w(g')}{\frac{mc_w(g'+1) - mc_w(g')}{g_w(k+1) - g_w(k)}} + g_w(k) \tag{A.4}$$

$$g_{rc_w} = \frac{\frac{R}{C} - rc_w(g')}{\frac{rc_w(g'+1) - rc_w(g')}{g_w(k+1) - g_w(k)}} + g_w(k) \tag{A.5}$$

$$g_{mr_w} = \frac{\frac{R}{M} - mr_w(g')}{\frac{mr_w(g'+1) - mr_w(g')}{g_w(k+1) - g_w(k)}} + g_w(k) \tag{A.6}$$

(ii) The optimum cut-off grade G is given by:

$$G_{mc} = \begin{cases} g_m \; if \; g_{mc_w} \le g_m \\ g_c \; if \; g_{mc_w} \ge g_c \\ g_{mc_w} otherwise \end{cases} \tag{A.7}$$

$$G_{rc} = \begin{cases} g_r \; if \; g_{rc_w} \le g_r \\ g_c \; if \; g_{rc_w} \ge g_c \\ g_{rc_w} otherwise \end{cases} \tag{A.8}$$

$$G_{mr} = \begin{cases} g_m \; if \; g_{mr_w} \le g_m \\ g_r \; if \; g_{mr_w} \ge g_r \\ g_{mr_w} otherwise \end{cases} \tag{A.9}$$

$$G = middle\, value(G_{mc}, G_{rc}, G_{mr}) \tag{A.10}$$

(iii) From the grade–tonnage curve of the deposit compute:

(a) The ore tonnage (qo_w) above the optimum cut-off grade (G) is obtained using Equation A.11.

$$q_{o_w}(k*) = \sum_{k=k*}^{K} q_w k \tag{A.11}$$

(b) The waste tonnage (qw_w) that is below the optimum cut-off grade (G) using Equation A.12.

$$q_{w_w}(k*) = \sum_{k=1}^{k*} q_w k \tag{A.12}$$

(c) Average grade of ore (g_{avg}) using Equation A.13.

$$g_{avg_w}(k*) = \frac{\sum_{k=k*}^{K} q_w k \left[\frac{g_w(k)+g_w(k+1)}{2}\right]}{q_{o_w}} \tag{A.13}$$

(d) The stripping ratio (SR) where

$$SR = \frac{q_{w_w}}{q_{o_w}} \tag{A.14}$$

(iv) Set the quantity of material processed as: $Qc_{wi} = C$, if qo_w is greater than the milling capacity (C), otherwise, $Qc_{wi} = q_{o_{wi}}$
(v) The quantity material mined, $Qm_{wi} = Qc_{wi}(1 + SR)$
(vi) The quantity of the final product, $Qr_{wi} = Qc_{wi} * g_{avg} * \sum y_i$
(vii) If the iteration indicator j is equal to 1, compute:
(a) The life of deposit (N) based on the limiting capacity among mining, concentrating and refinery capacity.
(b) The annual profit (P_{wi}) for the life of mine using:

$$P_{wi} = \left(\sum_{i=1}^{N} s_i - r_i\right) * Qr_{wi} - c_i * Qc_{wi} - m_i * Qm_{wi} - f_i \tag{A.15}$$

where:

$$\begin{cases} i = 1 \; if \; N \geq 1 \\ i = N \; if \; N < 1 \end{cases} \tag{A.16}$$