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# Creation and analysis of structured light fields for application in optical tweezers

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*A thesis submitted to the Faculty of Science in fulfillment of the  
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the degree of Doctor of Philosophy*

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February 21, 2019

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## Declaration

I, Nkosiphile Andile Bhebe, declare that this thesis titled, '*Creation and analysis of structured light fields for application in optical tweezers*' and the work presented in it are my own. I confirm that:

This work was done wholly or mainly while in candidature for a research degree at this University. Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated. Where I have consulted the published work of others, this is always clearly attributed. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work. I have acknowledged all main sources of help. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.



Signed:

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Date: February 21, 2019

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## Abstract

The work presented in this thesis focuses on the creation of customised structured light beams, their analysis, characterisation and application in optical trapping and tweezers.

In the first Chapter, we start by presenting an over-view of the thesis as well as a review of literature on laser beam shaping and its applications in optical tweezers. A theoretical description and definitions of customised laser modes utilised in this thesis such as LG and HG beams are presented in Chapter 2. Concepts on digital laser beam shaping techniques to realise customised structured light fields are described in Chapter 3. Two methods of generating customized laser modes namely complex amplitude modulation and phase only modulation are considered. Based on the ability of the SLM to create multiple beams simultaneously, the multiplexing concept is also discussed. Following the same line, a new approach to obtain multiple vector beams on a single hologram is presented. In addition, an approach to create shape invariant vector flat-top beams is discussed. Since we are also interested in the application of these custom light fields in optical tweezers, we discuss the fundamentals of optical trapping in Chapter 4.

Experimental realisation of LG and HG beams is then presented in Chapter 5. With the use of the beam shaping methods in Chapter 3, we demonstrate light beam shaping and multiplexing. In addition, a quantitative analysis to determine the multiplexing properties of SLMs as well as an investigation on the maximum number of beams that can be multiplexed is presented.

A novel experimental method that enables the simultaneous generation of many vector beams using a single digital hologram is described in Chapter 6. This method is interferometric in nature and relies on the multiplexing concept. We demonstrate the simultaneous generation of multiple vector vortex beams each with various polarization distributions. The flexibility of our approach is further confirmed through the creation of multiple vector Bessel beams.

Finally in Chapter 7, we present the application of our structured light fields in an optical trapping and tweezers system. The vector flat-top beam is firstly considered where a new holistic classical and quantum toolkit to analyse this beam during propagation is presented. The experimental realisation of such beams which exploits the polarisation dependent efficiency of spatial light modulators is described. We then demonstrate the versatility of our vector flat-top beam in an optical trapping

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and tweezing application. Following the experimental generation of multiple vector beams in Chapter 6, a novel vector holographic optical trap with arrays of digitally controlled Higher-Order Poincaré Sphere (HOPS) beams is also presented. We employ a simple set-up using a spatial light modulator and show that each beam in the array can be manipulated independently and set to an arbitrary HOPS state, including replicating traditional scalar beam HOTs. We demonstrate trapping and tweezing with customized arrays of HOPS beams comprising scalar orbital angular momentum and cylindrical vector beams, including radially and azimuthally polarized beams simultaneously in the same trap. Our approach is general enough to be easily extended to arbitrary vector beams.

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## Abbreviations

**BG:** Bessel Gaussian

**CVB:** Cylindrical Vector Beams

**DMD:** Digital Micro-mirror Device

**FT:** Flat Top

**HG:** Hermite Gaussian

**HOT:** Holographic Optical Trap

**HOPS:** High Order Poincaré Sphere

**LCP:** Left Circular Polarization

**LG:** Laguerre Gaussian

**OAM:** Orbital Angular Momentum

**RCP:** Right Circular Polarization

**SLM:** Spatial Light Modulator

**VQF:** Vector Quality Factor

**VVB:** Vector Vortex Beams

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## List of Publications

### Peer-reviewed Journal papers:

**BHEBHE N A**, WILLIAMS P A C, ROSALES-GUZMAN C, RODRIGUEZ-FAJARDO V and FORBES A “A vector holographic optical trap” Scientific Reports 8 (2018), 17387.

**BHEBHE N A**, ROSALES-GUZMAN C, and FORBES A “Classical and quantum analysis of propagation invariant vector flat-top beams” Applied Optics 57 19 (2018), 5451.

ROSALES-GUZMAN C, **BHEBHE N A**, MAHONISI N and FORBES A “Multiplexing 200 spatial modes with a single hologram” Journal of Optics 19 (2017), 113501.

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*To mum and dad*

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# Chapter 1

## Introduction

### 1.1 Motivation and thesis overview

The structuring, tailoring or shaping of light in its incoherent or coherent form is a process which involves the redistribution of the intensity of a beam of optical radiation [1]. This phenomena dates back at least to the third century. Archimedes equipped an army of soldiers with flat mirrors (polished metal surfaces) which they used to redirect the sun's rays towards a common target. This was reportedly the first beam shaping method that divided a single input beam (the sun) into multiple light sources before redistribution to a common target [1]. In addition to being the first multi-faceted structuring of light, it is an example of the tailoring of an incoherent source of light. The concept of shaping incoherent light source carried on into the 1700s where Georges-Louis Leclerc de Buffon (1748) developed the idea of apportioning a lens to reduce its mass forming concentric rings on its surface [1]. Fresnel then used this idea in 1820 to construct what is known today as the Fresnel lens for use in light houses. The Fresnel lens would redistribute the illuminating light source to follow a narrow path allowing it to propagate over long distances. This resulted in the first record of a physical beam shaping optic[1].

Undoubtedly, the invention of the laser, first predicted by Einstein in 1917 [2] and demonstrated by Maiman in 1960 [3] then led to modern day beam shaping of coherent light sources. In 1965, Frieden demonstrated lossless structuring of coherent light beams [4]. He developed this through geometric means by employing aspheric lenses that transformed an input Gaussian mode into a beam with a uniform peak intensity distribution (flat-top) [4]. More physical beam shaping optics have since been developed ranging from spiral phase plates, axicons and q-plates which

modulate the phase, intensity and polarization of some input optical field creating customized structured light fields [5–7]. Further technological advances resulted in the development of additional and relatively flexible laser beam shaping elements. Coupled with the fabrication of these modern diffractive and/or refractive optical elements, novel techniques have since been elucidated to generate arbitrary spatial modes of light, internal [8] as well as external to resonator cavities [9–13]. Among the several beam shaping methods, digital holographic beam shaping stands out as one of the most prevailing methods, stimulated by the introduction of modern digital devices, such as Spatial Light Modulators (SLMs) and Digital Micromirror Devices (DMDs) [14–16].

In general, the resulting engineered light fields have been found useful in a myriad of research applications in Physics, Biology and Engineering [6]. At present, implementation techniques to realise arbitrary complex custom light fields together with the tools to analyse the resultant beams have not been extensively investigated and as such continue to be a topic of research interest. Following the same line, an interesting class of light beams are those structured to have orbital angular momentum (OAM) and polarisation coupled in a non-separable manner such that they present some quantum-like properties. For this reason, the aim of this thesis is to explore different approaches to shape the spatial and polarisation degrees of freedom of light waves as well as to develop novel experimental methods to implement these concepts. Furthermore, we wish to describe a new approach that utilises classical and quantum tools to characterise and analyse the properties of the resultant tailored light fields. Finally, we describe their applications in optical trapping and tweezers system although these structured light fields will be beneficial to other laser based applications too.

The content of this thesis is arranged as follows:

Chapter 2 presents an introduction to the theory of scalar and vector light beams. We firstly consider the wave equation in its scalar form taking into account the paraxial approximation. Solutions to the scalar Helmholtz equation in cylindrical and rectangular coordinates obtaining Laguerre (LG) and Hermite Gaussian (HG) modes, respectively, are discussed. Further more, we will consider the non-diffracting Bessel-Gauss beams as well as flat-top beams. We then consider the vectorial wave equation and the solution thereof, with particular interest in cylindrically symmetric vector vortex beams. The geometrical representation of the polarisation states of

these cylindrical vector vortex beams on a sphere called the Higher order Poincaré sphere (HOPS) is then presented and discussed.

In Chapter 3, we present the theoretical concepts of laser beam shaping using digital holography. We began by introducing the LG and HG infinite basis sets and consider these as our programmable modes. Thereafter, we give a detailed explanation of beam shaping using two normally used methods: phase-only and complex amplitude modulation, followed by an in depth guide on the creation of the digital holograms. We will then consider the theory of generating flat-top beams in particular vector flat-top beams which are shape invariant during propagation. We show that our field may be well represented by a super-Gaussian intensity profile and derive expressions for the beam size and divergence. Finally, we present the concept related to the creation of multiple vector light fields using a single hologram and how this translates to realising multiple HOPS beams simultaneously.

The theory on the fundamentals of optical trapping and tweezers is briefly presented in Chapter 4. We describe the respective forces associated with an optical trapping and tweezers system. Two common regimes used to describe the trapping force with respect to the particle size relative to the laser beam wavelength are presented. It is important to note that this thesis's main focus is on the creation of structured light fields and optical trapping was chosen as an immediate application of these customized light beams. As such we will not go into detail on the optical trapping dynamics.

In Chapter 5, we experimentally investigate the multiplexing capabilities of the phase only HoloEye SLM using Laguerre-Gaussian (LG) and Hermite-Gaussian (HG) modes as our beams of choice. We employ the multiplexing concept described in Chapter 3 to simultaneously generate approximately 200 spatial modes encoded on a single hologram. We describe the experimental implementation of this method as well as present a quantitative analysis of the highest possible number of modes that can be multiplexed on a SLM. The quality of the generated beams is determined by performing 2D correlation measurements of the experimentally obtained images against theoretical. The correlation factor ranges from 1 to 0 for identical and non-identical images respectively.

Chapter 6, introduces a novel experimental technique that enables the rapid generation of any cylindrical vector beam (CVB) and allows for the simultaneous generation of multiple vector beams (vector-beam multiplexing) using a single holo-

gram. Our proposed method is interferometric in nature but differs from the rest as it relies on dividing the wavefront of the initial beam and not its amplitude. This principle allow us to split the original beam into many that can be later recombined in pairs to generate multiple vector beams simultaneously. The phase, amplitude and shape of each beam can then be digitally manipulated in an independent way to generate any vector beam without the need to manipulate any external optical element. That is, any combination of multiple vector beams can be generated by simply changing the digital hologram displayed on the SLM.

In Chapter 7, we demonstrate the application of the presented structured light beams in an optical trapping and tweezers system. Using the multiplexing concept discussed in Chapter 3 we firstly show the flexibility and dynamics of our holographic optical trap (HOT) by arranging multiple trapped particles into different 2D configurations. We then describe the experimental realisation of propagation invariant vector flat-top beams. In our generation method, we simply tune the initial polarisation and show a graduate evolution from Gaussian to vortex beam, with a flat-top as the superposition. Since both beams are eigen modes of free-space, so is the vector superposition. We calculate and measure the Stokes parameters of this beam, and introduce a new approach to analyse such beams where we apply a quantum toolkit to study the non-separability and vector nature (vector quality factor) of the vector flat-top beam, showing that all properties are conserved and invariant during propagation. We then use our vector flat-top beam in an optical trapping and tweezing experiment, demonstrating the control of micrometer sized particles.

Furthermore, we present a new holographic optical trap (HOT) for arbitrary higher order Poincaré sphere (HOPS) beams that allows for multiple HOPS beams to be delivered into a trap in some desired array. Each HOPS beam in the array is independently controlled by an SLM and may be switched from scalar states (on the poles of the HOPS) to cylindrical vector vortex beams (on the equator of the HOPS). By this approach, we are able to tailor on-demand the 3D shape of the optical forces at specific locations inside the optical trap. To show the potential of this technique, we simultaneously compare the trapping strength of cylindrical vector vortex and scalar vortex beams in the same trap.

Finally in Chapter 8, we conclude the thesis by summarising our contributions to the field of structured light fields and possible future work within this field.

## 1.2 Laser beam shaping

Structured light, refers to the ability of engineering or shaping light's properties creating custom light fields that can be employed in laser-based applications. In this particular topic, the intensity, phase and polarization degrees of freedom are tailored to suit a given application. Previously, structured light only considered the shaping of light's intensity creating different spatial patterns [6]. However, technological advances in the last 40 years have seen the development of optical devices with the ability to control the phase, amplitude and polarization of light[6, 16, 5]. This has since opened alternative avenues towards the realisation of many more applications in the fields of classical and quantum optical communication, high resolution microscopy, optical metrology and optical micro and nano-manipulation to mention a few [6, 17–21].

Most of the refractive, adaptive or diffractive optics were hard coded physical devices designed to generate a specific mode once fabricated. Due to the restrictive nature of physical optics, other laser beam shaping devices with some degree of flexibility such as SLM were then later developed. SLMs are digitally controlled devices with a miniaturized screen (approximately  $15.36 \times 8.64$  mm for HoloEye SLMs) consisting of hundreds of pixels each containing thousands of liquid crystal molecules. The orientation of the molecules in each pixel prescribes how the SLM will modulate the impinging beam (set to some linear polarisation state) i.e. phase-only, amplitude only or both amplitude and phase. The device is controlled by means of a regulated voltage that rotates the liquid crystal molecules by a predefined angle, resulting in a birefringent material for the impinging linearly polarized light. The voltage is adjusted for each individual pixel using digital holograms which are basically grayscale images with 255 shades of gray (example images presented in Chapter 3). These images are of the same resolution as the SLM screen such that a pixel by pixel mapping of the hologram (gray level image) and screen is realised. Once the hologram is encoded on to the SLM a local phase shift is induced by the rotation of the molecules in each pixel of the SLM, thereby engineering the incident field. Apart from manipulating the phase and amplitude of an input field, SLMs can also be used to modulate the polarization.

SLMs demonstrate a very high degree of accuracy in generating custom light fields [22, 23]. This was demonstrated by the high mode purity values (0.97) obtained with experimentally generated Laguerre Gaussian modes using a phase only SLM

[22, 23]. In addition, unlike hard coded optical elements, digital holographic devices such as the SLM are very flexible. Since these devices are computer controlled, they can be linked to other interfaces in the form of touch screens [24, 25]. This feature has afforded, the development of user-friendly multi-touch screens for optical tweezers [26, 24, 25]. Using multi-touch screens, rapid reconfiguration of the digital hologram can be achieved [27, 18], allowing one to change the shape and/or position of the output field. This is an additional property of the SLM that has shown many advantages in optical tweezers [28], particle tracking [20, 18] and the characterization of complex light fields [29–35] among many others.

Remarkably, SLMs afford the simultaneous generation of multiple light beams (multiplexing), a feature that has had a great impact in optical communication, optical microscopy and optical tweezers [36–42]. In optical communication, the simultaneous creation of multiple spatial modes has been identified as a means to increase data transmission rates [36]. This was demonstrated using over 100 modes, where each spatial mode represented a data transmission channel in free space optical communication [36, 37]. With respect to optical microscopy, depth of field multiplexed imaging using a high resolution SLM has been realised [40, 41]. The implementation of this technique involved the use of an SLM to achieve simultaneous imaging at various focal planes within the sample under investigation [40]. By encoding a superimposition of multiple Fresnel lens holograms on the SLM, an array of multiple beams was created with each adjacent beam focusing at a specific plane as dictated by the SLM [40]. Importantly in optical tweezers, multiplexing has been exploited to generate multiple optical traps through the incorporation of an SLM forming what is commonly known as holographic optical tweezers (HOTs) [43–45]. By programming on the SLM a phase function that transforms the input light field into an array of multiple high intensity beams in the trapping plane simultaneously creating multiple optical traps [43–45]. As a result of the dynamic properties of the SLM, the arrays of optical traps can be rearranged to form any shape on demand [44, 45]. With the use of multi-touch devices coupled with a network system intuitive remote monitoring and multi-particle manipulation has been demonstrated [25]. Furthermore, this has opened doors to a non-invasive means of micro and/or nano-structure assembly [44, 45]. Generally, multiplexing is granted due to the superposition principle in optics [46] which, by virtue of digital holography, allows the addition (multiplexing) of several holograms written into a single one. Each holo-

gram of the superposition can be programmed to generate specific beam shapes at particular locations in the observation plane.

As mentioned previously, the SLM is typically designed to modulate the phase but it can be used to modulate amplitude as well by complex amplitude modulation [9, 10]. For this work, we employed a phase only HoloEye Pluto VIS SLM as our primary beam shaping element. A more detailed discussion on how to generate custom light fields by modulating the phase of a reference beam will be presented in Chapter 3. We will further describe an example technique of modulating the amplitude of the input beam using a phase only SLM via complex amplitude modulation.

### 1.3 Polarization shaping in light waves

The ability to tailor the spatial properties of light thereby generating custom scalar fields has significantly changed the landscape of photonic-based applications as previously discussed. This is also the case for complex light fields classically entangled in polarization and phase, a topic which has gained research interest of late due to their various applications. In particular, classically entangled fields with cylindrical symmetry, commonly known as Cylindrical Vector Beams (CVB), are nowadays routinely used in fields such as laser material processing, optical tweezers, high-resolution microscopy, optical metrology, and classical and quantum communication, among many others [47, 21, 6, 17, 48]. For example in super resolution imaging techniques such as stimulated emission depletion microscopy (STED), vector beams are employed as they enable the manipulation of the depletion region allowing imaging beyond the diffraction limit [19]. By controlling the polarisation states of the input beam the excitation point spread function can be manipulated on demand [49, 19]. However for this work, we are mainly interested in their application in optical tweezers and the benefits thereof which will be discussed later. Prior to considering the application of vector beams in optical tweezers, one needs to consider the generation schemes of such vector light fields. The generation of CVB has been achieved internal or external to laser cavities, using geometrical phase elements [50, 7, 51, 52] or optical interferometers [53, 54]. With geometric phase elements such as the q-plate, only a specific vector beam can be generated at a time [5] for which the output phase, amplitude and polarization cannot be manipulated further. For interferometric generation, two vortex beams with opposite topological charge and orthogonal polarizations are recombined interferometrically to generate

CVB with various polarization distributions and spatial shapes. Remarkably, interferometric techniques have been fueled by the advent of SLMs, which have provided with one of the most flexible and versatile methods to shape light [15, 16, 55]. For this case, the amplitude, phase and polarization can be modulated on demand [29]. Importantly, SLMs also enable the simultaneous generation of multiple scalar beams (multiplexing), a feature that has found applications in optical tweezers and optical communications [44, 56, 57, 36, 58].

Other methods utilize an interferometric approach where two scalar beams with orthogonal polarization states carrying opposite orbital angular momenta (OAM) (i.e. opposite topological charges) are coaxially superimposed. In this case the OAM and polarization of each beam is independently altered by a spiral phase plate and a set of half and quarter wave plates, respectively. This approach, in general, can only produce one vector beam at a time. To overcome this limitation, one of the aims of this work is to investigate an SLM based and novel approach of implementing the simultaneous generation of multiple vector beams.

## 1.4 Gaussian to flat-top laser beam shaping

The need to transform the fundamental Gaussian beam (continuously varying intensity profile with a peak at the centre) to a flat-top beam (uniform peak intensity profile with steep roll off edges) [59, 60, 14] continues to be a topic of research. This has been driven by the many laser-based applications where a beam with uniform irradiance is preferred, for example in laser material processing. The creation of such beams is in general a difficult task as it requires an infinite spatial frequency spectrum which ideally cannot be realised in the laboratory. As such, only close approximations to this problem can be achieved in reality, some of which are super-Gaussian, super-Lorentzian, Fermi-Dirac, and flattened Gaussian beams. [14, 61–64]. Nonetheless, several methods to generate flat-top beams external of the laser cavity have been presented ranging from refractive intensity mapping [60], to diffractive interference-based designs [59, 65] which are implemented using digital micro mirrors [66] or SLMs [15]. In some cases, intra-cavity transformation from Gaussian to flat-top has been demonstrated (i.e. flat-top beam creation from the source) [67–72].

Although many generation methods have been demonstrated, the resultant flat-top beams have some limitations. These include their inability to maintain their

uniform peak intensity profile during propagation in other words the desired flat-top only occurs at a specific plane [73–75]. This configuration requires complicated relay imaging systems to ensure that the desired uniform irradiance occurs at the target plane even though it is expected that the beam profile should remain invariant within the Rayleigh range equivalent. A solution to this problem in the form of vector flat-top beams has been demonstrated recently [76]. This has seen a wide range of generation techniques emerge [77–81, 71] yet the properties of these vector flat-top beams have not been extensively investigated.

Here we demonstrate a holographic means of creating vector flat-top beams by exploiting the polarisation dependency of SLMs. This approach involves the coaxial superposition of a Gaussian beam and a vortex beam, resulting in a flat-top beam [82, 83]. Along this line, our aim is to present a novel tool that employs classical and quantum principles to characterise and analyse such structured light fields. This work will advance that of others on this topic by providing a holistic toolkit comprising both classical and quantum approaches for the analysis of such structured light beams, which we believe will be invaluable for their application, for example, in optical traps and laser materials processing.

## 1.5 Structured light fields and optical tweezers

Optical trapping and/or optical tweezing was first experimentally realised in 1970 [84], and have since become a versatile non-invasive tool to manipulate micro and nano matter. Important to their discovery was the proposed idea that light carries momenta: linear momentum (dating back to at least as far back as Kepler), spin angular momentum [85, 86] and OAM [87], such that a change in momentum results in a force being transferred to matter. In the field of physics, these light-matter interactions in particular optical forces exerted on atomic ensembles led to the invention of powerful techniques for laser cooling [88–90], which in turn paved the way to the realization of Bose-Einstein condensates [91, 92]. From a biological point of view, optical tweezers provided a non-mechanical and relatively non-invasive tool (if low laser intensities are employed) for the study of biological matter. This made it possible to study and enable the understanding of some of the complex properties transpiring inside a cell or to investigate the physical properties of DNA all of this done with the aid of additional spectroscopic or imaging tools [93–98]. There are numerous applications that have been facilitated by optical tweezers and these can

be found in the following Refs. [99–103] and references therein.

An important and major advancement in optical manipulation techniques came about from the field of structured light[6], particularly the utilisation of Spatial Light Modulators (SLMs) [15, 16] to create digitally controlled holographic optical traps (HOTs) [104, 43, 105]. SLMs as previously mentioned are digitally controlled devices which operate on liquid crystal technology that allow, amongst other things, a fast and precise generation and control of almost any beam shape, their digital focusing and propagation, as well as the simultaneous generation and detection of high numbers of individual beams [106, 36]. This computer controlled device has been used to control multiple particles in 2D and 3D configurations enabling the assembling of micro structures [28, 107–110] as well as transportation of trapped particles along exotic trajectories [111–119].

The above mentioned work was performed using scalar light fields, while recently vector states of light have become a topic of research interest and continues to be investigated [47, 120]. Of particular attention are the implementation methods in creating these vector states of light. These have since progressed from simple interferometric techniques, with the ability to generate only one vector beam, to more complex but advanced arrangements relying on computer controlled devices such as SLMs [55, 121, 47, 42, 122]. Importantly, there is great interest in using such vector light fields in optical trapping and manipulation due to the improved focusing ability of certain vector states of light [123–131] thereby enhancing the trapping efficiency [132, 123]. An example of the benefits of these beams was demonstrated by Michihata *et al.* where they pointed out that radially polarized beams improve the axial trapping strength by approximately  $2\times$  compared to linearly polarized beams. This however was at the expense of a decrease in the transverse strength by about a half [123]. Other research work has shown that radially and azimuthally polarized beams have higher axial trapping forces on core-shell magnetic microparticles as compared to the fundamental Gaussian beam [124]. On comparing azimuthally and radially polarization states in vector beams, it has been demonstrated that the azimuthally polarized beam exhibits a stronger lateral trapping force compared to the radially polarized beam [125, 126]. These topical beams are geometrically described by states on a generalized Poincaré Sphere [133, 134], the so-called Higher-Order Poincaré Sphere (HOPS), to account for the total angular momentum of light, that is spin angular momentum (SAM) and OAM [133]. A detailed description and

discussion of this geometric representation of polarization states on a sphere will be presented in Chapter 2.

## 1.6 Conclusion

We began the Chapter with a motivation and an overview of the thesis. This was followed by a literature review, as guided by the thesis summary, where we considered the different approaches and means of achieving the creation of structured light fields that have been demonstrated. We then considered the application of these custom light fields in optical tweezers as this is the immediate application that we identified as part of the scope of this thesis. The next Chapter focuses on the theory of these structured light waves which we will consider in this work as well as some mathematical concepts which will be useful in the analysis of these custom modes.

## Chapter 2

# Structured scalar and vector light fields

All propagating electromagnetic waves are governed by Maxwell's equations. In this Chapter we revisit the wave equation starting from Maxwell's equations and their respective relations. We then consider the solutions to the wave equation within the paraxial regime taking into account the scalar and vector cases. For the scalar case we pay particular interest to the solutions given in cylindrical and rectangular coordinates as well as consider Bessel-Gauss and flat-top beams. Thereafter, we consider a general solution of the vector Helmholtz wave equation, which includes the polarization component of the wave equation, a property of light waves normally neglected in the scalar case. Finally, a discussion on light fields whose polarisation and spatial distribution are coupled in a non-separable manner is presented focusing on the geometric representation of the polarization states of such light fields on a sphere.

### 2.1 Scalar light fields

Scalar light fields are light waves characterised by intensity profiles with a uniform polarization distribution. These, like any electromagnetic wave, can be described by the paraxial Helmholtz wave equation. The Helmholtz wave equation is derived from Maxwell's equations which were a result of experimentally observed phenomena in light waves[135]. For an electromagnetic field propagating in free space, Maxwell's equations are expressed as follows,

$$\nabla \cdot \mathbf{D} = 0, \tag{2.1}$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2.2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2.3)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}. \quad (2.4)$$

$\mathbf{D}$ ,  $\mathbf{B}$ ,  $\mathbf{E}$  and  $\mathbf{H}$  are the electric displacement, magnetic induction, electric field vector and magnetic field vector, respectively, and  $\nabla$  is the differential operator.  $\mathbf{D}$  is related to the electric field vector by  $\mathbf{D} = \epsilon_0 \mathbf{E}$ , while  $\mathbf{B}$  is related to the magnetic field vector by  $\mathbf{H} = (1/\mu_0) \mathbf{B}$ . With the permeability and permittivity given by  $\mu_0$  and  $\epsilon_0$ , respectively, and relate to the speed of a propagating electromagnetic wave  $c = 1/\sqrt{\epsilon_0 \mu_0}$ . The wave equation can be derived from Faraday's induction law (Eqn. (2.3)), by taking the curl to obtain,

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial(\nabla \times \mathbf{B})}{\partial t}, \quad (2.5)$$

and employing the vector identity,  $\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$ , Eqn. (2.5) becomes,

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{\partial(\nabla \times \mathbf{B})}{\partial t}. \quad (2.6)$$

The wave equation is then obtained by substituting Eqn. (2.1) and Eqn. (2.4) into Eqn. (2.6) and using the magnetic and electric field relations to the magnetic induction and electric displacement, respectively, the following equation is obtained

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 (\mathbf{E})}{\partial t^2} = 0. \quad (2.7)$$

Applying separation of variables to separate the spatial component from the temporal component in the above equation we obtain the time-independent Helmholtz wave equation [136, 135],

$$(\nabla^2 + k^2)u(x, y, z) = 0 \quad (2.8)$$

where  $\nabla^2$  is the Laplacian operator ( $\frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 y} + \frac{\partial^2}{\partial^2 z}$ ),  $k$  is the wave vector defined by  $k = 2\pi/\lambda$  and  $u(x, y, z)$  defines the amplitude of related field.

Considering the conditions for the paraxial approximation, given by Eqn. (2.9), for an electromagnetic field primarily propagating in the  $z$  direction. The condition states that the amplitude,  $u(x, y, z)$ , has very small variations along the axial direction  $z$  compared to the transverse  $x$  and  $y$  amplitude and phase variations.

$$|\frac{\partial^2}{\partial^2 z} u(x, y, z)| \ll k |\frac{\partial}{\partial z} u(x, y, z)| \quad (2.9)$$

Such that by neglecting the  $z$  component in the Laplacian of Eqn. (2.8), the Helmholtz equation can then be approximated by

$$(\nabla_{\perp}^2 + 2ik\frac{\partial}{\partial z})u(x, y, z) = 0, \quad (2.10)$$

where  $\nabla_{\perp}^2$  is the transverse,  $x$ - $y$ , component of the Laplacian. Equation (2.10) therefore forms the paraxial approximation of the Helmholtz equation.

The paraxial approximation provides a good description of laser beam propagation where the divergence angle is small. There are many solutions to the paraxial Helmholtz wave equation and these can be obtained in cylindrical or rectangular coordinates. Some of these solutions include Gaussian, Laguerre-Gaussian, Hermite-Gaussian, Bessel-Gaussian and Super-Gaussian functions which we implemented in this thesis and are discussed in the following sections

## 2.2 Laguerre-Gaussian beams

Solutions to the Helmholtz wave equation in the paraxial regime can be given in cylindrical and Cartesian coordinates. The former is given in terms of the generalized Laguerre polynomials  $L_p^{\ell}$  modulated by a Gaussian envelope, which forms an infinite orthogonal set of solutions, known as the Laguerre-Gaussian,  $LG_p^{\ell}$ , modes [137]:

$$LG_p^{\ell}(\rho, \varphi, z) = \sqrt{\frac{2p!}{\pi(\ell+p)!}} \left[ \frac{\sqrt{2}\rho}{\omega(z)} \right]^{\ell} L_p^{\ell} \left[ \frac{2\rho^2}{\omega^2(z)} \right] \frac{\exp[i(2p+\ell+1)\zeta(z)]}{\omega(z)} \exp \left[ -\frac{\rho^2}{\omega^2(z)} \right] \exp \left[ \frac{-ik\rho^2}{2R(z)} \right] \exp[-i\ell\varphi], \quad (2.11)$$

where,  $(\rho, \varphi, z)$  is the position vector of the cylindrical coordinates,  $\ell$  is an integer number that accounts for the number of times the phase wraps around the optical axis,  $p$  is a positive integer that accounts for the number of maxima ( $p+1$ ) along the transverse direction. Each solution represents a paraboloidal wave with radius of curvature  $R(z) = z[1 + (z_R/z)^2]$ , beam waist  $\omega_0$  and Rayleigh range  $z_R$ ;  $\omega(z) = \omega_0\sqrt{1 + (z/z_R)^2}$  is the beam width as function of  $z$  and  $\zeta(z) = \arctan(z/z_R)$  is the Gouy phase. Figure 2.1(a) shows the intensity distributions of the first few  $LG_p^{\ell}(x)$  modes with combinations of  $p = [0, 1, 3]$  and  $\ell = [0, 1, 2, 4]$ .

## 2.3 Hermite-Gaussian beams

In Cartesian coordinates, the paraxial wave equation allows as solutions the Hermite-Gaussian modes,  $HG_{nm}$ , a product of Hermite polynomials  $H(x)$  and a Gaussian envelope, with  $n$  and  $m$  positive integers:

$$HG_{nm}(x, y, z) = \frac{1}{\omega(z)} \sqrt{\frac{2^{-(n+m-1)}}{\pi n! m!}} \exp[i(n+m+1)\zeta(z)] \\ H_n \left[ \frac{\sqrt{2}x}{\omega(z)} \right] H_m \left[ \frac{\sqrt{2}y}{\omega(z)} \right] \exp \left[ -\frac{x^2 + y^2}{\omega^2(z)} \right] \exp \left[ \frac{-ik(x^2 + y^2)}{2R(z)} \right] \exp[-ikz]. \quad (2.12)$$

The parameters  $\omega$ ,  $R(z)$ ,  $\omega_0$ ,  $z_R$  and  $\zeta(z)$  have the same definitions as for the  $LG_p^\ell$  modes. The positive integer subindexes  $n$  and  $m$  are related to the number of bright lobes of the transverse intensity profile along  $x$  and  $y$ , respectively. Figure 2.1 (b) shows the intensity profile of the modes described by combinations of  $n = [0, 1, 2, 3]$  and  $m = [0, 1, 2]$ . A detailed discussion on the generation of digital holograms to

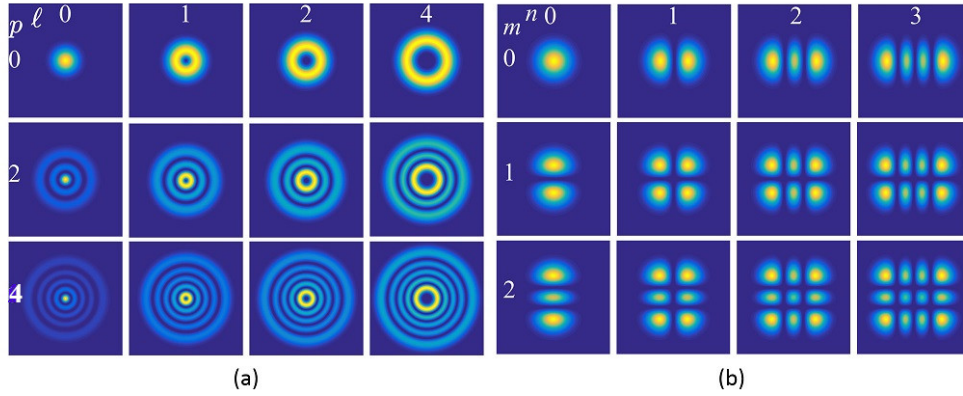


Figure 2.1: Theoretical 2D intensity profiles of  $LG_p^\ell$  and  $HG_{nm}$  modes. Intensity distribution of (a) LG modes with  $\ell = [0, 1, 2, 4]$ ,  $p = [0, 2, 4]$  and (b) HG modes with  $n = [0, 1, 2, 3]$   $m = [0, 1, 2]$ .

create these structured light fields is presented in Chapter 3 while their experimental realisation is presented in Chapter 5.

## 2.4 Bessel-Gauss beams

Bessel-Gauss beams form another solution set to the Helmholtz wave equation. This particular solution set was first realised by Durnin [138] who found these beams to be non-diffracting and demonstrate invariance during propagation. These beams are mathematically described by Bessel functions and are characterised by a central on

axis intensity and an infinite number of concentric rings as shown by the first image in Fig. (2.2). In theory, these beams are expected to carry an infinite amount of energy as they have an infinite number of rings extended over an infinite space. However, this is not experimentally possible as such one can only generate an approximation in the form of a Bessel-Gauss beam described as

$$BG(\rho, \varphi, z) = \frac{\omega_0}{\omega(z)} \exp \left[ i \left( k - \frac{k_r^2}{2k} \right) z - i\zeta(z) + i\ell\varphi \right] J_\ell(k_r r / (iz/z_R)) \times \exp \left[ \left( \frac{-1}{\omega^2(z)} + \frac{ik}{2R(z)} \right) \left( r^2 + \frac{k_r^2 z^2}{k^2} \right) \right], \quad (2.13)$$

where  $J_\ell$  is the  $\ell^{th}$ -order Bessel function,  $k_r$  is the radial wave-vector and  $\omega_0$  is the initial beam waist. The parameters  $\omega(z)$ ,  $z_R$ ,  $R(z)$  and  $\zeta(z)$  have the same definitions as for those presented in section 2.2. When  $z = 0$ , the above equation (2.13) reduces to

$$BG(\rho, \varphi, 0) = J_\ell(k_r r) \exp \left[ - \left( \frac{r^2}{\omega_0} \right)^2 + i\ell\varphi \right], \quad (2.14)$$

which is the Bessel function with a Gaussian envelope. The zero-order ( $\ell=0$ ) and higher-order ( $\ell=\dots -3, -2, -1, 0, 1, 2, 3, \dots$ ) Bessel beams both exhibit non-diffracting properties over relatively long propagation distances. Figure 2.2 shows theoretical 2D intensity distributions of zero-order and higher order Bessel beams. This set of beams is of particular interest because of their non-diffracting and self-healing properties as well as their ability to carry OAM a characteristic we will consider in section 6.5 of Chapter 6.

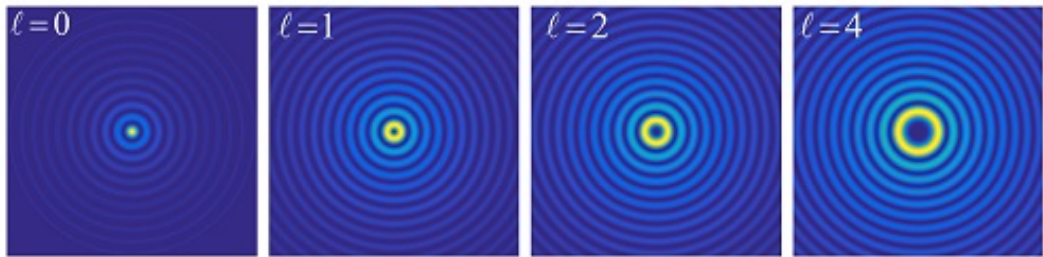


Figure 2.2: Theoretical 2D intensity profiles of the zeroth order Bessel-Gauss beam (first image) and higher order Bessel-Gauss modes with OAM  $\ell=1,2,4$  modes

## 2.5 Flat-top beams

Flat-top beams are modes of light characterised by uniform peak intensity distribution with very steep roll off edges. These can be classified as the Fermi-Dirac beam,

the super-Lorentzian beam, the super-Gaussian and the flattened-Gaussian [14]. As an example lets consider the super-Gaussian which is mathematically described by,

$$I_{FT}(r) = \frac{4^{1/N} N}{2\pi\omega_{FT}^2 \Gamma(2/N)} \exp \left[ -2 \left( \frac{r}{\omega_{FT}} \right)^N \right], \quad (2.15)$$

where  $r$  is the radial coordinate,  $\omega_{FT}$  is the flat-top beam size in the super-Gaussian approximation and  $N$  defines the order of the beam. By setting  $N = 2$  the beam will have a Gaussian profile. As the order,  $N$ , is increased the intensity profile of the beam evolves from a Gaussian to a flat-top as illustrated in Fig. (2.3).

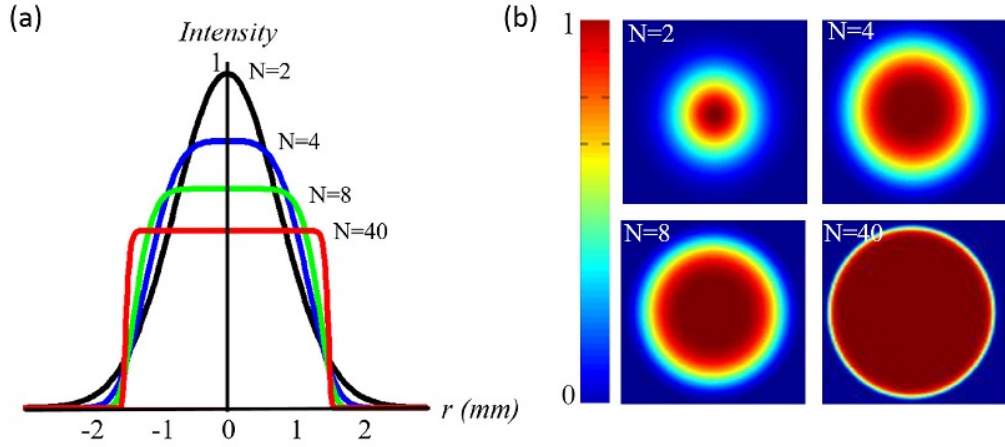


Figure 2.3: (a) Shows the transverse profiles of a Gaussian ( $N=2$ ) evolving to a Super-Gaussian ( $N=40$ ) while (b) are the corresponding 2D intensity distributions.

## 2.6 Vector light fields

In the above sections we have considered structured light fields in their scalar form where the polarization component in the generation process is neglected [139]. The polarization component is associated with the electric ( $\mathbf{E}$ ) field of a light wave where,  $\mathbf{E}$ , has some magnitude (amplitude of the field) in some defined orientation (direction) hence a vector quantity. As such, it then follows that light waves have a vector nature. The vector nature can be described by the polarization structure associated with the light wave. Similar to customized structured scalar light fields, one can tailor the polarization degree of freedom by modulating the orientation of the electric field ( $\mathbf{E}$ ) creating structured vector light fields commonly known as vector beams.

Considering the transverse  $x - y$  plane, the electric field,  $\mathbf{E}$ , can be independently chosen to orient along the  $x$  ( $E_x$ ) or  $y$  ( $E_y$ ) directions thereby defining the

polarisation of the electromagnetic wave as polarized in the  $x$  direction for  $(E_x)$  and polarised in the  $y$  direction for  $(E_y)$ . A coaxial superposition of the  $(E_x)$  and  $(E_y)$  polarization components where each component can have a unique amplitude and phase results in a beam with custom polarization states. The individual amplitude components will thus contribute to the transverse intensity profile of the resulting beam given by  $|E|^2$ . In this way by performing a coaxial addition of beams with orthogonal polarization states and various amplitudes and phases vector light fields or vector beams can be created. These are generally characterised by an inhomogeneous polarization distribution across their spatial intensity profile. The orientation of the electric field for an incident light wave can be altered on demand using optical elements such as wave plates. This change can be mathematically traced using Jones vector representation of polarized light presented in the next section.

A common example of vector light fields are cylindrical vector vortex beams which are a solution to the vector Helmholtz wave equation in cylindrical coordinates. These can be generally defined as linear combinations of scalar vortex beams with opposite OAM indices and orthogonal polarization states (usually circular) [47]. Such modes can be mathematically represented as,

$$U(r, \theta) = A_R(r)e^{i\ell_1\theta_1}\hat{\mathbf{R}} + [A_L(r)e^{-i\ell_2\theta_2}]e^{i\delta}\hat{\mathbf{L}} \quad (2.16)$$

where,  $\hat{\mathbf{R}}$  and  $\hat{\mathbf{L}}$  are the right and left circular polarization vectors while  $A(r)_R$  and  $A(r)_L$  are the respective radial profiles of the beams. The azimuthal angle is defined by  $\theta$  and the OAM is given by  $\ell$  for which  $\ell \in \mathbf{Z}$ . The term  $e^{i\delta}$  introduces an additional phase term where  $\delta = \{0, \pi\}$ . By setting  $\ell_1 = \ell_2 = +1$  or  $-1$ ,  $\theta = \theta_1 = \theta_2$  and  $\delta = 0$  or  $\pi$ , a common set of cylindrical vector vortex beams given by the following equations,

$$\begin{aligned} TM &= \frac{1}{\sqrt{2}} \left( e^{i\theta}\hat{\mathbf{R}} + e^{-i\theta}\hat{\mathbf{L}} \right), \\ TE &= \frac{1}{\sqrt{2}} \left( e^{i\theta}\hat{\mathbf{R}} - e^{-i\theta}\hat{\mathbf{L}} \right), \\ HE^e &= \frac{1}{\sqrt{2}} \left( e^{i\theta}\hat{\mathbf{L}} + e^{-i\theta}\hat{\mathbf{R}} \right), \\ HE^o &= \frac{1}{\sqrt{2}} \left( e^{i\theta}\hat{\mathbf{L}} - e^{-i\theta}\hat{\mathbf{R}} \right) \end{aligned} \quad (2.17)$$

is obtained. The above equations mathematically describe the four fundamental low order cylindrical vector vortex beams whose theoretical intensity profile and polarisation maps are as shown in Fig. (2.4). The above structured vector light fields demonstrate non-separability between the spatial and polarisation degrees of

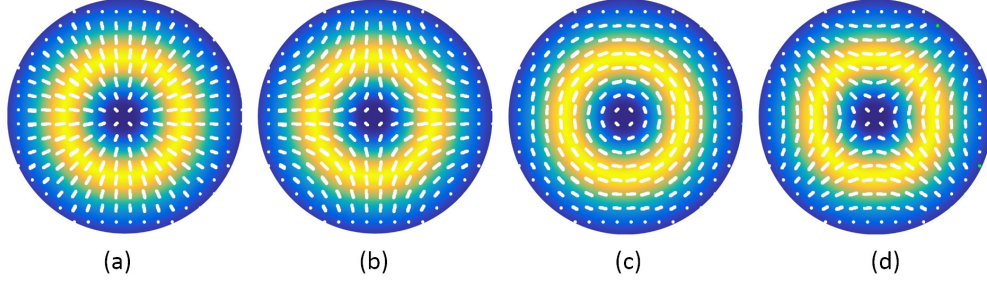


Figure 2.4: Theoretical intensity and polarization states of common low order cylindrical vector vortex beams with (a) radial, (b) hybrid even, (c) azimuthal and (d) hybrid odd polarisations

freedom. The spatial profile of these modes is governed by the selected OAM define by  $\ell$ .

## 2.7 Jones polarisation vectors

Any arbitrary polarization state associated with a light wave can be represented by the sum of two orthogonal and linearly polarized components as

$$\mathbf{E}(z, t) = E_x \cos(\omega t - kz + \delta_x) \mathbf{x} + E_y \cos(\omega t - kz + \delta_y) \mathbf{y} \quad (2.18)$$

where the angular frequency is given by  $\omega = 2\pi f$ ,  $k$  is the wave number,  $\delta_x$  and  $\delta_y$  are the corresponding phases and  $E_x$  and  $E_y$  are the amplitudes. Using Jones vector notation the above equation can be expressed as

$$\mathbf{E}(z, t) = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_x \cos(\omega t - kz + \delta_x) \\ E_y \cos(\omega t - kz + \delta_y) \end{bmatrix}. \quad (2.19)$$

Equation (2.19) can also be represented in complex notation as shown in the following,

$$\mathbf{E}(z, t) = \begin{bmatrix} E_x \exp i(\omega t - kz + \delta_x) \\ E_y \exp i(\omega t - kz + \delta_y) \end{bmatrix} = \exp i(\omega t - kz + \delta_x) \begin{bmatrix} E_x \\ E_y \exp i\delta \end{bmatrix} \quad (2.20)$$

where  $\delta$  is the phase difference between the  $E_x$  and  $E_y$  components given by  $\delta = \delta_y - \delta_x$ . For simplicity the common phase factor can be neglected such that Eqn. 2.20 becomes

$$\mathbf{E}(z, t) = \begin{bmatrix} E_x \\ E_y \exp i\delta \end{bmatrix} \quad (2.21)$$

which is the Jones vector representation of an electromagnetic wave. The normalized Jones vector is then expressed as

$$\mathbf{E}(z, t) = \frac{1}{\sqrt{E_x^2 + E_y^2}} \begin{bmatrix} E_x \\ E_y \exp i\delta \end{bmatrix}. \quad (2.22)$$

The above Jones vector can describe any arbitrary polarization state, for example linear polarization can be realised when  $E_x = \cos \theta$ ,  $E_y = \sin \theta$  and  $\delta = 0$ . In the case of horizontal and vertical linear polarization states,  $\theta$  must be equal to 0 and  $\pi/2$ , respectively, while any other linear orientation can be obtained by varying  $\theta$  provided  $\delta = 0$ . Circular polarisation states can be obtained by equally superimposing  $E_x$  and  $E_y$  but a relative phase difference between the two of  $\delta = \pi/2$  is required for right circular polarisation (RCP) and  $\delta = -\pi/2$  for left circular polarisation (LCP). The four described polarization states (vertical-V, horizontal-H, RCP and LCP) can be mathematically represented using Jones vector notation by

$$V = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, H = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, LCP = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ -i \end{bmatrix}, RCP = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ +i \end{bmatrix}. \quad (2.23)$$

Using the equations in 2.23, diagonal (D), anti-diagonal (A), RCP and LCP can be expressed by linear combinations of vertical and horizontal polarization as shown below,

$$A = \frac{H - V}{\sqrt{2}}, D = \frac{H + V}{\sqrt{2}}, LCP = \frac{H - iV}{\sqrt{2}}, RCP = \frac{H + iV}{\sqrt{2}}. \quad (2.24)$$

For polarisation states which lie between linear and circular, i.e. elliptically polarised, the Jones vector can be generalised to,

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \psi \pm i \sin \theta \sin \psi \\ \sin \theta \cos \psi \mp i \cos \theta \sin \psi \end{bmatrix} \quad (2.25)$$

where  $\psi = \arctan (E_x/E_y)$  describes the ellipticity of the polarization and  $\theta$  is the angle between the major axis of the ellipse and the horizontal axis (Fig. 2.5 (a)).

Another way of representing polarization employed a geometric approach in which any polarization state could be described is on a sphere. This approach was first introduced by Poincaré in 1892 giving birth to what is now known as the Poincaré sphere (Fig. 2.5 (b)). In this representation, cartesian coordinates  $(x, y, z)$  using the conversions,  $x = r \cos 2\theta \cos 2\psi$ ,  $y = r \cos 2\theta \sin 2\psi$  and  $z = r \sin 2\psi$ , are transformed to spherical coordinates  $(r, \theta, \psi)$ . In the spherical coordinate system,  $r$  is the radius,  $\theta$  is the azimuthal angle and  $\psi$  is the polar angle. It can be seen from

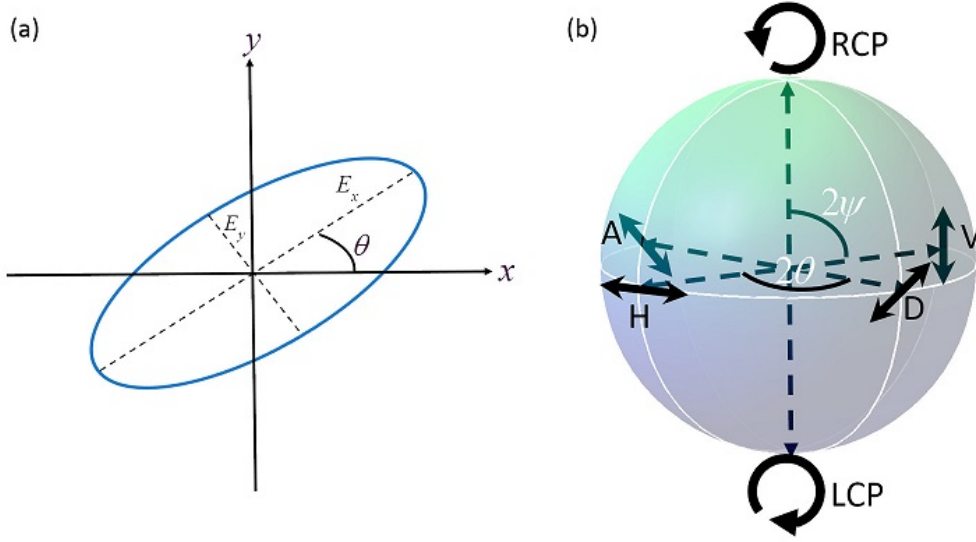


Figure 2.5: (a) Shows a geometric representation of the polarisation ellipse while (b) illustrates the different polarisation states as represented on the Poincaré sphere.

this representation that the angles  $\theta$  and  $\psi$  can be related to polarisation ellipse making the Poincaré sphere an alternative representation of any polarization state. On the sphere, all possible linear states lie on the equator, the circular polarisation states are located on the poles and everywhere else are elliptical states. Basically, any polarization state on the sphere (Fig. 2.5 (b)) can be realised for  $\theta \in [0, \pi]$  and  $\psi \in [-\pi/4, \pi/4]$ .

So far we have considered polarization representations in which it is assumed that the initial polarisation state of the light wave is known. However, in some cases the polarisation of the initial beam is not known. As such, for one to determine the polarisation state of any light field we need to consider Stokes parameters which we discuss in the next section.

## 2.8 Stokes parameters

In section 2.7, we note that Jones vectors are only applicable to polarized light. In 1852 George Gabriel Stokes proposed a new approach for determining the polarization state of any beam [135]. The approach involves four experimental measurements coupled with some theoretical calculations on the light wave of unknown polarization from which one can then mathematically determine the light's polarization.

The measurable parameter are known as the Stokes parameters. These parameters are defined by a set of six intensity measurements which are

$$\begin{aligned} S_0 &= I_0 = |E_x|^2 + |E_y|^2, \\ S_1 &= I_H - I_V = |E_x|^2 - |E_y|^2, \\ S_2 &= I_D - I_A = E_x^* E_y + E_x E_y^*, \\ S_3 &= I_R - I_L = i(E_x^* E_y - E_x E_y^*). \end{aligned} \quad (2.26)$$

where  $I_0$  is the intensity of the initial beam while  $I_H$ ,  $I_V$ ,  $I_D$ ,  $I_A$ ,  $I_R$  and  $I_L$  are the transmitted intensities through a linear polarizer set at horizontal, vertical, diagonal, anti-diagonal, right circular and left circular polarization transmission, respectively. Stokes parameters generally indicate which state of polarization is dominant in a given light field, for example if  $S_1 < 0$  then the polarization state of field is closer to vertical linear polarization. The same applies to parameters  $S_2$  and  $S_3$ . From Eqn. 2.26, it can be deduced that  $I_0 = I_H + I_V = I_D + I_A = I_R + I_L$ . It then follows that the Stokes parameters can be expressed as,

$$\begin{aligned} S_0 &= I_0, \\ S_1 &= 2I_H - I_0, \\ S_2 &= 2I_D - I_0, \\ S_3 &= 2I_R - I_0. \end{aligned} \quad (2.27)$$

Similar to Jones vectors, one can use Stokes parameters to represent various polarisation states and for linearly polarized light the Stokes parameters are expressed as [135],

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} |E_x|^2 + |E_y|^2 \\ |E_x|^2 - |E_y|^2 \\ 2E_x E_y \cos \delta \\ 2E_x E_y \sin \delta \end{bmatrix} = \begin{bmatrix} |E|^2 \\ |E|^2 \cos 2\theta \\ |E|^2 \sin 2\theta \\ 0 \end{bmatrix} \quad (2.28)$$

where  $E$  is the amplitude,  $E_x = E \cos \theta$ ,  $E_y = E \sin \theta$ ,  $\theta$  is the angle between the electric field and the  $x$  axis while  $\delta$  is the relative phase difference. As such the corresponding Linear horizontal (H), vertical (V), diagonal (D), anti-diagonal (A), Left (L) and right (R) circular polarization states are given by

$$H = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, V = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, D = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, L = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, R = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad (2.29)$$

where  $\theta = 0$  for (H),  $\theta = \pi/2$  for (V),  $\theta = \pi/4$  for (D),  $\theta = -\pi/4$  for (A) with respect to the linear polarization states and for the circular states  $S_1$  and  $S_2$  are both 0. Equation (2.29) forms the normalized Stokes parameters from which the following relation can be deduced

$$\begin{aligned}\frac{S_1}{S_0} &= \cos(2\psi) \cos(2\theta), \\ \frac{S_2}{S_0} &= \cos(2\psi) \sin(2\theta), \\ \frac{S_3}{S_0} &= \sin(2\psi).\end{aligned}\tag{2.30}$$

The above  $\psi$  and  $\theta$  have the same definitions of ellipticity and orientation angle, respectively, as mentioned in the previous sections and the associated equations in (2.30) also relate to the Poincaré sphere in section 2.7.

$$\psi = \frac{1}{2} \arctan\left(\frac{S_3}{S_0}\right), \theta = \frac{1}{2} \arctan\left(\frac{S_2}{S_1}\right).\tag{2.31}$$

As such any arbitrary polarisation state can be obtain as defined by  $\psi$  and  $\theta$  given in Eqn. (2.31).

Similar to the Poincaré sphere which is used to describe polarization states, OAM states can also be represented on a sphere known as the Bloch sphere. This geometric representation of OAM relies on the fact that Laguerre-Gaussian ( $LG_p^\ell$ ) modes can be given by linear superpositions of Hermite-Gaussian ( $HG_{nm}$ ) modes [139]. To help us describe this sphere, we consider spatial OAM modes in the form of Laguerre-Gaussian beams such that the various OAM states on the sphere can be mathematically expresses as,

$$U(r, \theta) = \cos\left(\frac{\psi}{2}\right) LG_0^{\ell_1} + \sin\left(\frac{\psi}{2}\right) LG_0^{\ell_2} e^{i\theta}\tag{2.32}$$

where the terms  $\cos(\psi/2)$  and  $\sin(\psi/2)$  define the weightings of each mode and  $\theta$  is the inter-modal phase. The geometric representation of OAM states on a sphere with respect to  $LG_p^\ell$  modes is as illustrated in Fig. 2.6. In this representation, each  $LG_p^\ell$  mode is associated with a unique point on the sphere such that opposite OAM states are situated at the poles (i.e.  $\ell_1 = +1$  and  $\ell_2 = -1$ ). On the equator are equally weighted superpositions of  $LG_p^{\ell_1}$  and  $LG_p^{\ell_2}$  modes resulting in ( $HG_{nm}$ ) modes. Thus far, the polarization and OAM spheres have been described separately yet vector light fields take into account the total angular momentum of light waves, that is, spin angular momentum (SAM-polarization) and OAM. Following the two geometric representations discussed above, the total angular momentum can also be

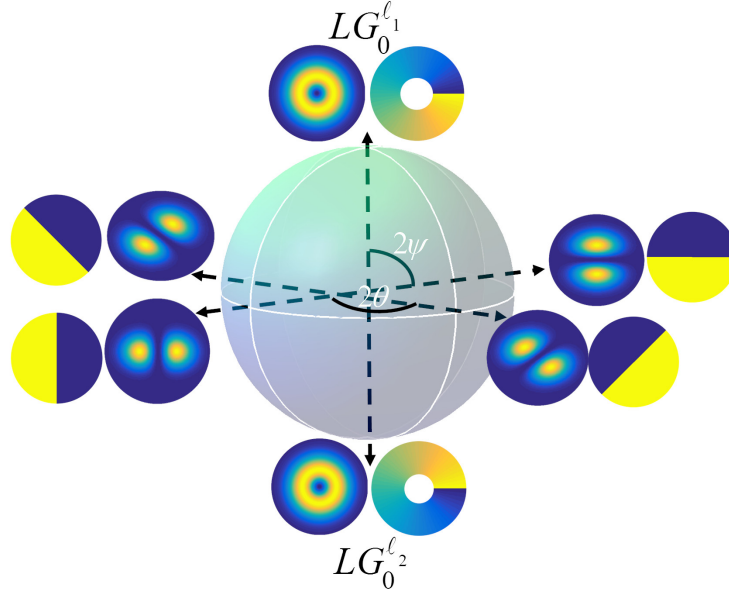


Figure 2.6: A geometric representation of OAM states on the Bloch sphere with pure OAM states  $\ell_1 = +1$  and  $\ell_2 = -1$  located at the poles.

described on a sphere called the higher order Poincaré sphere which is presented in the next section.

## 2.9 Higher Order Poincaré Sphere (HOPS) beams

As mentioned in the previous section, the combined polarisation and OAM state of each cylindrical vector vortex beam can be geometrical represented by a point on a sphere called a Higher Order Poincaré Sphere (HOPS) shown in Fig. (2.7). In this geometric representation, the scalar beams consisting of the left and right circularly polarisation (separable states) are located on the poles, while the cylindrical vector vortex beams with a spatially inhomogeneous polarization distribution (non-separable states) lie along the equator and the elliptical polarizations (partially non separable states) are situated in between the poles and the equator.

$$U(r, \theta) = \cos\left(\frac{\psi}{2}\right) LG_0^\ell e^{i\theta/2} \hat{\mathbf{R}} + \sin\left(\frac{\psi}{2}\right) LG_0^{-\ell} e^{-i\theta/2} \hat{\mathbf{L}} \quad (2.33)$$

The HOPS can be mathematical described by the above equation which is derived from Eqn. (2.17), where the amplitudes  $A_R(r)$  and  $A_L(r)$  are chosen to be  $\cos(\psi/2)$  and  $\sin(\psi/2)$ , respectively, while  $\theta \in [0, 2\pi]$  and  $\psi \in [0, \pi]$ . In this case the Stokes parameters can then be expressed such that they take into account both OAM and

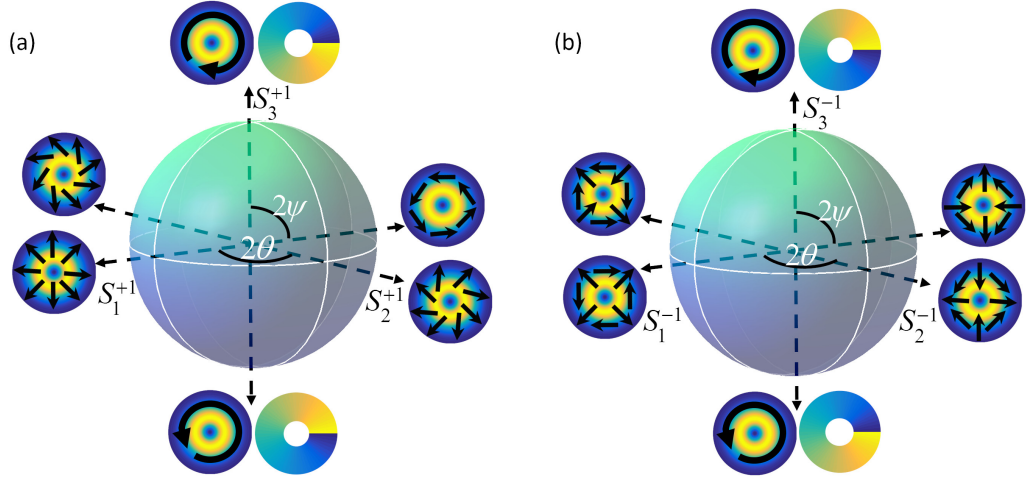


Figure 2.7: The positions of the polarisation states on the high order Poincaré sphere are as shown in (a) and (b). The sphere on the left defined by OAM  $\ell = +1$  shows the radial ( $S_1^{+1}$ ) and spiral ( $S_2^{+1}$ ) polarization states while the sphere on the right defined by OAM  $\ell = -1$  shows the hybrid states ( $S_1^{-1}$  and  $S_2^{-1}$ ).

polarization as represented on the higher order Poincaré sphere in the following manner,

$$\begin{aligned}
 S_0^\ell &= |\Psi_R^\ell|^2 + |\Psi_L^\ell|^2, \\
 S_1^\ell &= 2|\Psi_R^\ell||\Psi_L^\ell|\cos(\delta_2 - \delta_1), \\
 S_2^\ell &= 2|\Psi_R^\ell||\Psi_L^\ell|\sin(\delta_2 - \delta_1), \\
 S_3^\ell &= |\Psi_R^\ell|^2 - |\Psi_L^\ell|^2.
 \end{aligned} \tag{2.34}$$

## 2.10 Conclusion

In this Chapter we presented scalar light fields by taking into account the general Helmholtz wave equation within the paraxial approximation for a propagating light wave. Analytical solutions to the wave equation in cylindrical coordinates (Laguerre-Gaussian beams), rectangular coordinates (Hermite-Gaussian beams) were considered as well as Bessel-Gaussian and super-Gaussian (flat-top) beams. We introduced vector light fields with particular interest in cylindrical vector vortex beams. Mathematical representations of polarization using Jones vectors and Stokes parameters were presented. Furthermore, geometric representations of polarization (SAM) and OAM states on spheres called the Poincaré and Bloch spheres, respectively, were

discussed. Along this line, a geometric representation of the total angular momentum of a light wave in the form of the higher order Poincaré sphere was discussed. We were particularly interested in the aforementioned modes as the following Chapters of this thesis focus on the experimental realisation as well as the analysis and characterization of such structured scalar and vector light fields.

## Chapter 3

# Laser beam shaping using digital holographic techniques

### 3.1 Introduction

The tailoring of light's spatial profile can be achieved by modulating its amplitude or phase and in some cases, both by complex amplitude modulation. This can be realised by employing a transfer function capable of modulating either the amplitude or the phase or both. Since the HoloEye Pluto SLM used in this work only allows the modulation of only the phase, there are several other techniques that have demonstrated the indirect modulation of the amplitude using these phase only devices [10, 11, 22, 23, 140–143]. In addition, all the structured light beams presented in the following Chapters are realised with an SLM. As such this Chapter will be limited to digital beam shaping methods although such customized light fields can also be obtained from physical hard coded optics such as phase plates, q-plates and meta materials to mention a few [6].

In this Chapter, we explain laser beam shaping using computer controlled devices such as SLMs since these digital devices provide on demand a very accurate, flexible and relatively fast holographic means to tailor light's spatial profile. Using the customised laser modes discussed in Chapter 2 as examples, we present a detailed guide on how to generate the necessary holograms to create these structured light beams. Our guide only considers two major beam shaping methods which are (i) phase only modulation and (ii) complex amplitude modulation where the latter method requires the modulation of both amplitude and phase. We also describe the general basics through which multiple beams can be generated using a single

hologram (multiplexing). From the multiplexing concept, we then present a new approach of realizing multiple vector beams and effectively arrays of higher order Poincaré sphere beams.

## 3.2 Phase Modulation

Firstly, let us consider an input light beam which can be mathematically described by,  $U_A = U_a e^{i\ell\phi_a}$  where  $U_a$  is the amplitude and  $\phi_a$  defines the phase. This beam can be transformed into a new light field,  $U_B = U_b e^{i\ell\phi_b}$ , where  $U_b$  and  $\phi_b$  represent the amplitude and phase of the new optical field. This transformation is governed by a transfer function given by [16]

$$t = \frac{U_A}{U_B} = \frac{U_a}{U_b} e^{i\ell(\phi_a - \phi_b)} \quad (3.1)$$

and is associated with the physical optical element used to achieve the desired modulation.

In the case of phase only modulation where  $U_A = U_B$ , that is the amplitude of the respective transmission function is unity, confirming that a phase-only response is required. The light is only redirected to specific regions according to the encoded phase function. Since light is only being redirected, one would expect the input amplitude to be the same as the amplitude of the new desired light field hence a unity amplitude term in the transmission function. Experimentally, this is not possible due to losses within the optical system which cannot be avoided. In general, phase only modulation has a higher conversion efficiency compared to complex amplitude modulation which is discussed later in this Chapter. As an example of phase only modulation, a beam can be propagated to some position in the transverse plane using a phase-only blazed grating. To generate a beam with some OAM, an azimuthal phase profile is encoded on the SLM as shown in Fig (3.1). The phase hologram encoded on the SLM is given by  $\text{mod}[\phi_a - \phi_b, 2\pi]$ . Normally, the initial light field is a plane wave characterised by a flat phase, such that the programmed hologram is just the modulus of the desired phase,  $\text{mod}[\phi_b, 2\pi]$ . It is important to note that for this type of modulation the desired intensity pattern forms in the far field [16].

### 3.2.1 Multiplexing scalar modes using phase holograms

In order to generate a phase-only hologram capable of reproducing the amplitude changes in  $LG_p^\ell$  modes, it is sufficient, in a first approximation, to encode the changes

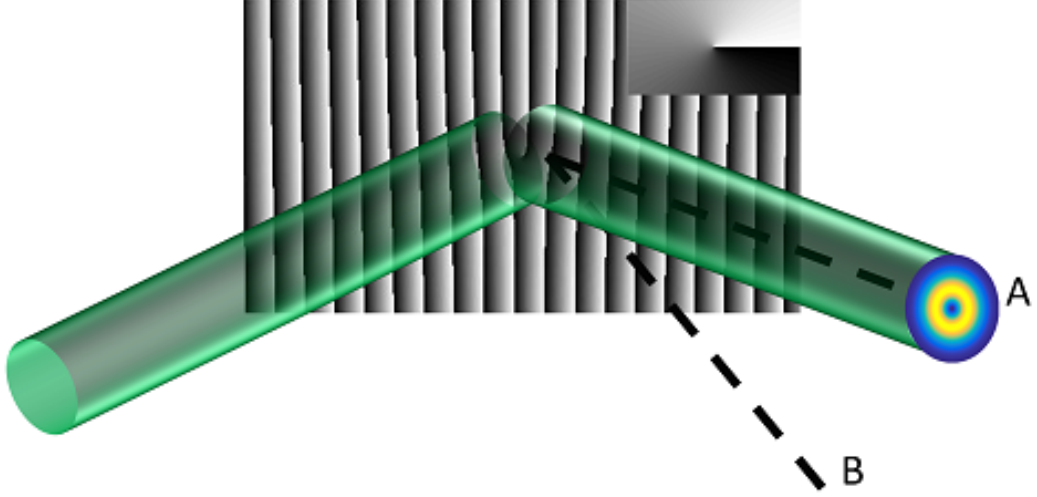


Figure 3.1: A schematic showing a phase only fork hologram used to generate a vortex mode. The required azimuthal phase is shown by the inset. By altering the period blazed grating, the generated vortex mode can be redirected to any position in this case from A to B.

of sign given by the Laguerre polynomial  $L_p^\ell$  in the radial direction. This, in combination with an azimuthal phase variation, generates the transfer function[22],

$$t(\rho, \varphi) = \exp \{ i [\ell \varphi + \pi \Theta(\rho)] \}, \quad (3.2)$$

where,  $\Theta(\rho)$  is the Heaviside function that takes into account the sign variations of the Laguerre-Gauss function and is given by,

$$\Theta(\rho) = \Theta \left[ L_p^{|\ell|} \left( \frac{2\rho^2}{\omega_0^2} \right) \right]. \quad (3.3)$$

The above transfer function can be encoded on the SLM along with a linear phase grating, to separate the different diffraction orders. The transfer function of a linear phase grating is given by,

$$t_g(x, y) = \exp [i2\pi(ux + vy)], \quad (3.4)$$

where  $u$  and  $v$  are the spatial frequencies of the grating along the horizontal and vertical direction respectively. A lens of focal length  $f$  is placed at a distance  $f$  from the SLM to separate the diffraction orders in the focal plane of the lens. The final coordinates  $U$  and  $V$  of the first diffraction order are related to the grating frequencies, the focal length  $f$  and the wavelength of the incident field, by the relations  $U = u\lambda f$  and  $V = v\lambda f$ , respectively. Hence, the transfer function in terms

of  $U$  and  $V$  takes the form,

$$t_g(x, y) = \exp \left[ i2\pi \left( x \frac{U}{\lambda f} + y \frac{V}{\lambda f} \right) \right]. \quad (3.5)$$

Hence, the hologram to generate the  $LG_p^\ell$  mode combined with the diffraction grating takes the final form,

$$\Phi = \text{mod} \left\{ \ell\varphi + \pi\Theta \left[ L_p^{|\ell|} \left( \frac{2\rho^2}{\omega_0^2} \right) \right] + 2\pi \left( x \frac{U}{\lambda f} + y \frac{V}{\lambda f} \right), 2\pi \right\}. \quad (3.6)$$

In the above equation,  $\text{mod}[x]$  represents the modulus function that wraps the phase around  $2\pi$ . A similar approach can be followed to generate  $HG_{nm}$  beams [143]. Figure 3.2 shows the amplitude, phase and holograms for  $LG_p^\ell$  3.2(a) and  $HG_{nm}$  3.2(b) modes.

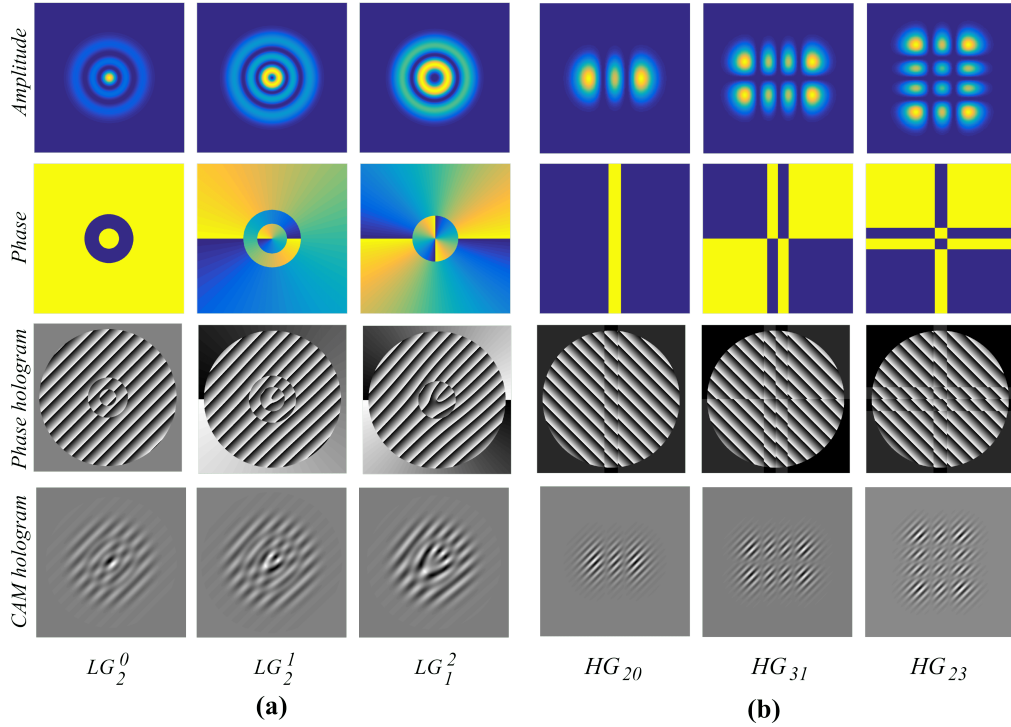


Figure 3.2: Digital holograms, phase and amplitude for  $LG_p^\ell$  and  $HG_{nm}$  modes. The first row shows the amplitude while the corresponding phase profiles are given in the second row. Phase only and complex-amplitude modulation (CAM) holograms shown in the third and fourth row, respectively were encoded on the SLM to generate (a)  $LG_p^\ell$  and (b)  $HG_{mn}$  modes.

To generate multiple beams simultaneously, we combined on a single hologram a superposition of multiple holograms, each with a unique spatial carrier frequency, as illustrated in Fig. 3.3(a). Clearly, each spatial frequency  $(u, v)$  directs the generated beam to a particular location in the Fourier plane, as illustrated in Fig 3.3(b).

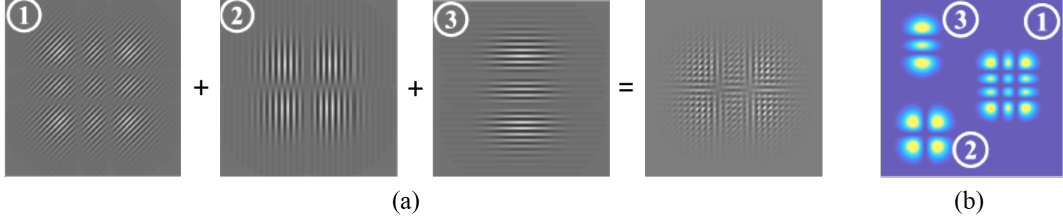


Figure 3.3: (a) A superposition of holograms with unique carrier frequencies (grating) forming a single hologram encoded on to the SLM. The unique carrier frequencies correspond to a specific spatial position, on the Fourier plane, of the encoded modes as shown in (b).

Overlapping between the multiplexed modes can be avoided by taking into account the spatial bandwidth of the beams to be created. The spatial bandwidth in the Fourier plane, as a function of the second moment beam size and beam quality factor  $M^2$  for each mode is given by,

$$\omega_f = \frac{M^2 \lambda f}{\pi \omega_0} \quad (3.7)$$

where  $\omega_0$  is the initial programmed size on the SLM. The spatial bandwidth  $\Delta u$  of the beam is then  $\Delta u = \omega_f / \lambda f$ . Given that the position in the Fourier plane of each beams along both coordinates is defined by  $U = u \lambda f$  and  $V = v \lambda f$ , the bandwidth will be given by  $\Delta U = \Delta u \lambda f$  and  $\Delta V = \Delta v \lambda f$ . This relation provides an optimum distance by which to separate the modes ensuring no overlap. If for example, we choose  $\Delta U = \omega_f$ , then the separation in grating spatial frequencies between neighboring modes would be

$$\Delta u = \frac{M_i^2}{\pi \omega_i} + \frac{M_{i+1}^2}{\pi \omega_{i+1}} \quad (3.8)$$

where  $\omega_i$  and  $\omega_{i+1}$  is the second moment beams size of each mode. The final hologram  $\Phi_M$  for M multiplexed beams takes the form,

$$\Phi_M = \sum_{j=1}^M \text{mod} \left[ \ell_j \varphi + \pi \Theta_j(\rho) + 2\pi \left( x \frac{U_j}{\lambda f} + y \frac{V_j}{\lambda f} \right), 2\pi \right]. \quad (3.9)$$

Specific modes and their positions in the Fourier plane can be selected by properly choosing  $\ell_j$ ,  $\Theta_j(\rho)$  and  $(U_j, V_j)$ .

### 3.3 Complex Amplitude modulation

As was mentioned in the introduction, it is possible to change the amplitude and phase using complex amplitude modulation on a phase-only device. This can be

achieved by incorporating the amplitude term into the phase term such that by modulating the phase, to obtain the desired optical field, the amplitude is also changed. There are several approaches to complex amplitude modulation that have been reported in the literature [141, 10]. In this holographic technique, beam shaping is achieved by redistributing light such that the desired beam is generated in the first diffraction order. The unwanted light is therefore redirected to either higher diffraction orders or to the undiffracted zero order. In the latter case, the phase depth of the encoded hologram is decreased while for the former the spatial frequencies are increased resulting in higher diffraction angles [16]. As expected, this technique unfortunately has a much lower conversion efficiency but presents better modulation as control of both phase and amplitude is enabled. For this case the desired beam shape or pattern forms in the near field as such relay optics are required to propagate the beam to the detector [16].

### 3.3.1 Multiplexing scalar modes using complex amplitude modulation

Complex amplitude modulation can be achieved in many ways, however in this thesis we will only focus on the method proposed by Arrizon *et al.* [10] that produces the highest quality mode compared to other methods [142, 141]. To start with, let's consider a field of amplitude  $A(\mathbf{r})$  and phase  $\phi(\mathbf{r})$  of the form,

$$U(\mathbf{r}) = A(\mathbf{r}) \exp[i\phi(\mathbf{r})] \quad (3.10)$$

with  $A(\mathbf{r})$  and  $\phi(\mathbf{r})$  in the intervals  $[0,1]$  and  $[-\pi, \pi]$ , respectively. One way to indirectly modulate the amplitude from phase modulation can be done by writing Eq. 3.10 as

$$h(\mathbf{r}) = \exp\{i\Phi[A(\mathbf{r}), \phi(\mathbf{r})]\} \quad (3.11)$$

Everything reduces to finding the function  $\Phi[A(\mathbf{r}), \phi(\mathbf{r})]$ . This can be done by expanding Eq. 3.11 as a Fourier series of the form

$$h(\mathbf{r}) = \sum_{q=-\infty}^{\infty} c_q^A \exp[iq\phi(\mathbf{r})], \quad (3.12)$$

where,

$$c_q^A = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\{i\Phi[A(\mathbf{r}), \phi(\mathbf{r})]\} \exp[-iq\phi(\mathbf{r})] d\phi. \quad (3.13)$$

The field  $U(\mathbf{r})$ , can be recovered from the first term of the above equation provided,

$$c_1^A = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\{i\Phi[A(\mathbf{r}), \phi(\mathbf{r})]\} \exp[-i\phi(\mathbf{r})] d\phi = A(\mathbf{r})a, \quad (3.14)$$

with  $a$  a positive constant. Equation 3.14 can be fulfilled if,

$$\int_{-\pi}^{\pi} \sin\{i\Phi[A(\mathbf{r}), \phi(\mathbf{r})]\} d\phi = 0, \quad (3.15)$$

$$\int_{-\pi}^{\pi} \cos\{i\Phi[A(\mathbf{r}), \phi(\mathbf{r})]\} d\phi = 2\pi a, \quad (3.16)$$

which provides an appropriate basis to determine  $\Phi[A(\mathbf{r}), \phi(\mathbf{r})]$ . In Eqn. (3.16) the value of the constant  $a$  is in the interval  $[0,1]$ , which limits the efficiency of the generated hologram. While different solutions to Eqns. (3.15) and (3.16) can be found, here we will only focus on a solution with odd symmetry, namely,

$$\Phi[A(\mathbf{r}), \phi(\mathbf{r})] = f[A(\mathbf{r})] \sin[\phi(\mathbf{r})], \quad (3.17)$$

which corresponds to type 3 hologram in reference [10], according to which  $h(\mathbf{r}) = \exp\{if[A(\mathbf{r})] \sin[\phi(\mathbf{r})]\}$  can be expanded, using the Jacobi-Anger identity as,

$$\exp\{if[A(\mathbf{r})] \sin[\phi(\mathbf{r})]\} = \sum_{s=-\infty}^{\infty} J_s\{f[A(\mathbf{r})]\} \exp[is\phi(\mathbf{r})], \quad (3.18)$$

where,  $J_s$  are the Bessel function of order  $s$ . Direct comparison of Eqns. (3.18) and (3.12) yields,

$$c_q^A(\mathbf{r}) = J_q\{f[A(\mathbf{r})]\}. \quad (3.19)$$

Equation 3.14 yields the final expression

$$c_1^A(\mathbf{r}) = J_1\{f[A(\mathbf{r})]\} = A(\mathbf{r})a, \quad (3.20)$$

where  $J_1(x)$  is the first order Bessel function. Finally,  $f[A(\mathbf{r})]$  can be obtained by numerical inversion of Eqn. (3.20) as,

$$f[A(\mathbf{r})] = J_1^{-1}[A(\mathbf{r})a] \quad (3.21)$$

The inversion of  $J_1(x)$  is guaranteed in the interval  $[0, x_m]$ , being  $x_m = 1.84$  the value where  $J_1(x)$  reaches its first maximum  $J_1(x_m) = 0.5819$ . This restriction limits the values of  $\phi[A(\mathbf{r}), \phi(\mathbf{r})]$  to the reduced domain  $[-0.586\pi, 0.586\pi]$ , which implies that the maximum phase shift that can be obtained with this method is  $\Delta\phi = 1.17\pi$ .

To generate the required holograms to transform an input light field into  $LG_p^\ell$  or  $HG_{nm}$ , we can consider the SLM to be located at  $z = 0$  and simplify Eqns. (2.11) and (2.12) in Chapter 2 as,

$$LG_p^\ell(\rho, \varphi, 0) = \frac{1}{\omega_0} \sqrt{\frac{2p!}{\pi(\ell+p)!}} \left[ \frac{\sqrt{2}\rho}{\omega_0} \right]^\ell L_p^\ell \left[ \frac{2\rho^2}{\omega_0^2} \right] \exp \left[ -\frac{\rho^2}{\omega_0^2} \right] \exp[-i\ell\varphi] \quad (3.22)$$

and

$$HG_{nm}(x, y, 0) = \frac{1}{\omega_0} \sqrt{\frac{2^{-(n+m-1)}}{\pi n! m!}} H_n \left[ \frac{\sqrt{2}x}{\omega_0} \right] H_m \left[ \frac{\sqrt{2}y}{\omega_0} \right] \exp \left[ -\frac{x^2 + y^2}{\omega_0^2} \right], \quad (3.23)$$

respectively. The phase  $\phi(\mathbf{r})$  and amplitude  $A(\mathbf{r})$  can be easily obtained from the above equations and inserted into Eqn. (3.21) and finally in Eqn. (3.17) to obtain the desired hologram, which will take the form

$$\Phi = f[A(\mathbf{r})] \sin \left[ \phi(\mathbf{r}) + 1.17\pi \left( x \frac{U}{\lambda f} + y \frac{V}{\lambda f} \right) \right]. \quad (3.24)$$

Figure 3.2 (fourth row) shows examples of the holograms obtained as described above to generate  $LG_p^\ell$  and  $HG_{mn}$  modes. Multiplexing of  $LG_p^\ell$  and  $HG_{mn}$  can be achieved by encoding a hologram of the form,

$$\Phi_M = \sum_{i=1}^M f[A(\mathbf{r})] \sin \left[ \phi_i(\mathbf{r}) + 1.17\pi \left( x \frac{U_i}{\lambda f} + y \frac{V_i}{\lambda f} \right) \right], \quad (3.25)$$

where,  $A(\mathbf{r})_i$  and  $\phi(\mathbf{r})_i$  represents the amplitude and phase of the encoded mode, respectively,  $U_i$  and  $V_i$  are the final positions of each mode in the vertical and horizontal direction, respectively. Figure (3.3) shows multiplexing of three  $HG_{mn}$  modes  $HG_{01}$ ,  $HG_{11}$  and  $HG_{22}$  with different grating frequencies.

### 3.4 Flat-top generation using phase modulation

One of the many techniques of realising a flat-top beam from an input Gaussian is through modulating phase. This method relies on the size of the input Gaussian which is governed by a dimensionless parameter  $\beta$  given by [1, 14],

$$\beta = \frac{2\sqrt{2\pi}\omega_G\omega_{FT}}{\lambda f} \quad (3.26)$$

where  $\omega_G$  and  $\omega_{FT}$  are the Gaussian and flat-top beam sizes while  $f$  is the focal length of the fourier lens which is placed between the modulating element and the focal plane where the flat-top forms. In this case, the phase function required to obtained a flat-top is as follows,

$$\phi(r) = \beta \frac{\sqrt{\pi}}{2} \int_0^{\frac{\sqrt{\pi}}{\omega_{FT}}} \sqrt{1 - \exp(-\rho^2)} \quad (3.27)$$

This function is radially symmetric and transforms a circular Gaussian beam of size  $\omega_G$  into a circular flat-top (Fig. 3.4 (b)) of width  $\omega_{FT}$ .

A major drawback to this technique is the degradation of the created flat-top upon propagation as previously mention in Chapter 1. A solution to this problem is presented in the following section.

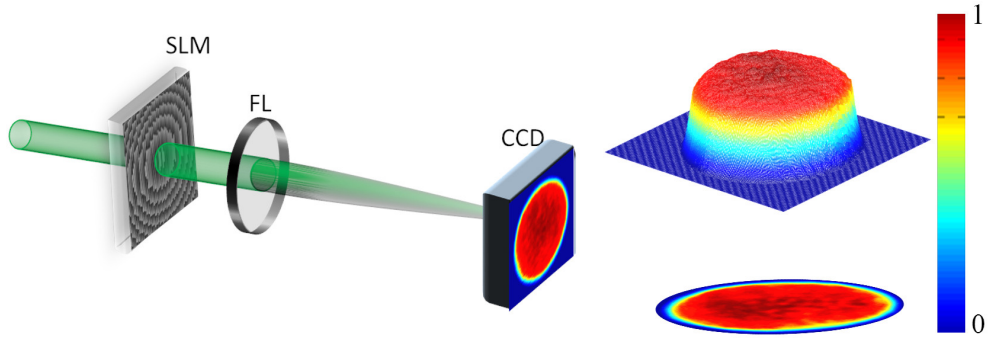


Figure 3.4: Experimental circular flat-top generation using an SLM. By encoding a phase function, the desired flat-top forms at the focal plane of a fourier lens (FL) as illustrated.

### 3.5 Shape invariant vector flat-top beams

In this section, we begin by illustrating the propagation dynamics of Super Gaussian beams. As was explained in the previous section, these beams can be digitally generated using SLMs where an input Gaussian beam is transformed into a flat-top at the focal plane of a focusing lens [14]. However, the resulting uniform peak intensity distribution degrades while the beam propagates in space as illustrated in Fig. (3.5). Shape invariant vector flat-top beams which are described in the subsequent sections present a solution to this problem. As such, we present analytical simulations of propagation invariant vector flat-top beams which can be obtained from a coaxial superposition of a Gaussian and a donut beam with orthogonal polarization states.

#### 3.5.1 Vector flat-top beams

Without any loss of generality, imagine that a Gaussian beam in some linear polarisation state is incident on a SLM and consider that the SLM is orientated to diffract only the horizontal component, passing the vertical component undiffracted. If no grating is added to the SLM phase pattern then the undiffracted beam (vertical component) will be collinear with the diffracted (horizontal) component of the initial beam. One can see that by careful choice of the initial polarisation and programmed phase pattern, an engineered beam can be created (one could also of course alter the initial beam profile too, but this would be akin to altering the phase profile on

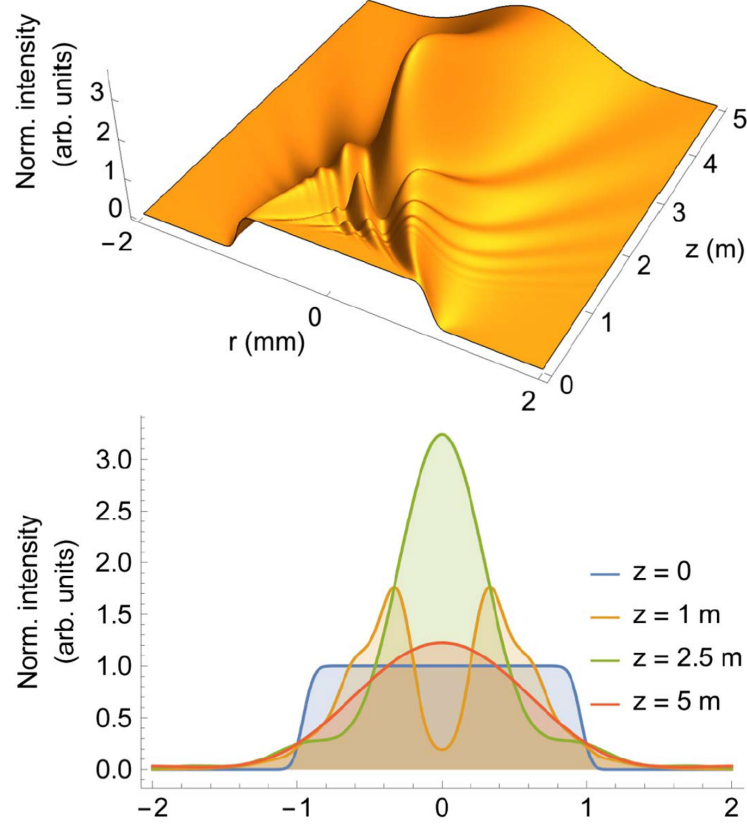


Figure 3.5: Propagation of a flat-top beam (radius of 1 mm and wavelength of  $\lambda = 532$  nm) using the flatten Gaussian beam approximation, clearly showing that the advantage of the initial uniform intensity is quickly negated. Some example profiles during this propagation are shown in the bottom panel, with the initial flat-top beam shown in blue and the final beam after 5 m shown in red, with complex shapes in-between.

the SLM). We can formulate this mathematically by writing the initial beam in the Jones matrix form (in the horizontal/vertical basis) as

$$u_0(r) = \exp(-(r/w)^2) \begin{pmatrix} \sqrt{1-\alpha} \\ \sqrt{\alpha} \end{pmatrix}, \quad (3.28)$$

so that the output beam after an SLM phase of  $\exp(i\psi(r, \phi))$  is

$$u(r, \phi) = \exp(-(r/w)^2) \begin{pmatrix} \sqrt{\alpha} \exp(i\psi(r, \phi)) \\ \sqrt{1-\alpha} \end{pmatrix}. \quad (3.29)$$

Now if the SLM phase screen is set to create a vortex mode of topological charge  $\ell = 1$  (or  $-1$ ), then the output beam after some filtering (see experimental discussion) yields

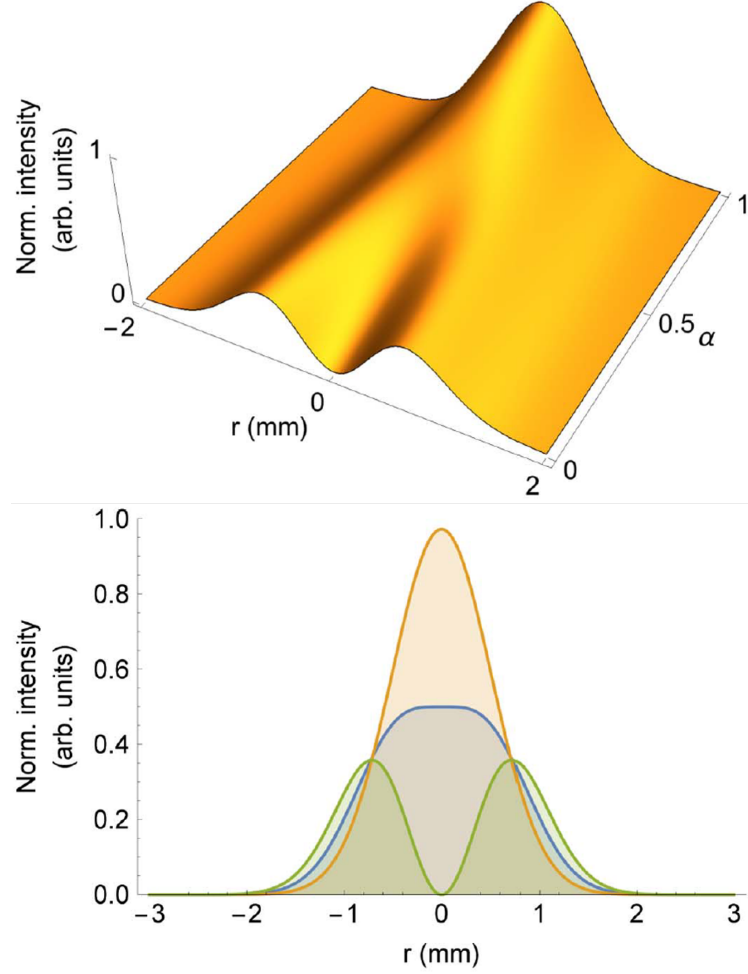


Figure 3.6: In the top frame we show example plots of the vector intensity profile as  $\alpha$  is changed, evolving from a vortex to a Gaussian with a near flat-top beam at a critical superposition state of an equal weighting. Cross-sections of this evolution are shown in the bottom panel for the three cases of interest, with a flat-top beam clearly shown at the intermediate value of  $\alpha$ .

$$U_{FT}(r, \phi, z) = \sqrt{\alpha} \text{LG}_{00}(r, \phi, z) \hat{\mathbf{e}}_{\mathbf{H}} + \sqrt{1 - \alpha} \text{LG}_{01}(r, \phi, z) \hat{\mathbf{e}}_{\mathbf{V}}, \quad (3.30)$$

where  $\text{LG}_{p\ell}(r, \phi, z)$  refers to the Laguerre-Gaussian modes of radial order  $p$  and azimuthal order  $\ell$ , i.e., a Gaussian and vortex mode in the above with  $p = 0$ , and the vectors denote the horizontal and vertical components of the Jones matrix. Such an  $\text{LG}_{0\ell}(r, \phi, z)$  mode may be expressed as

$$\begin{aligned} \text{LG}_{0\ell}(r, \phi, z) &= \sqrt{\frac{2}{\pi w^2(z) |\ell|!}} \left( \frac{\sqrt{2}r}{w(z)} \right)^{|\ell|} \\ &\times \exp\left(-\frac{r^2}{w^2(z)} + ik \frac{r^2}{2R(z)}\right) \exp(i(1 + |\ell|)\zeta(z)), \end{aligned} \quad (3.31)$$

where  $\zeta(z) = \arctan(z/z_R)$ ,  $w(z) = w_0\sqrt{1 + (z/z_R)^2}$ ,  $R(z) = z(1 + (z_R/z)^2)$ ,  $z_R = \pi w_0^2/\lambda$  and have the same physical meanings defined in Chapter 2.  $w_0$  is the Gaussian beam width and  $\lambda$  is the wavelength to give a Gaussian divergence of  $\theta_0 = \lambda/(\pi w_0)$ . The intensity of such a beam is dependent on the initial polarisation of the Gaussian beam, which is easily changed with wave plates and a polariser, but which in general is given by

$$I(r, \phi, z) = \alpha |\text{LG}_{00}(r, \phi, z)|^2 + (1 - \alpha) |\text{LG}_{01}(r, \phi, z)|^2, \quad (3.32)$$

with some example plots shown in Fig. 3.6. Our desired flat-top beam appears near the equal weighting superposition state. We note from Eq. (3.32) that the intensity profile as a function of propagation distance is known analytically since each orthogonal polarisation component is analytic. Intriguingly, the two modes making up  $I(r, \phi, z)$  have exactly the same Rayleigh distance, and hence the contribution of each to the desired beam remains invariant during propagation. This is because the beam sizes after the SLM are not the same: the vortex mode is a factor  $\sqrt{|\ell| + 1}$  bigger, just the right amount to counter the increase in divergence due to the larger mode number. To see this, if the Rayleigh distance of the Gaussian mode is  $Z_{00}$  and that of the vortex mode  $Z_{01}$  then

$$Z_{01} = \frac{\pi w_{01}^2}{M^2 \lambda} = \frac{\pi (w_{00} \sqrt{|\ell| + 1})^2}{(|\ell| + 1) \lambda} = \frac{\pi w_{00}^2}{\lambda} = Z_{00}$$

A consequence of this is that the intensity profile of the vector superposition maintains its intensity profile during propagation, albeit with a change in size. This is also true for the special case of  $\alpha = 0.5$  where we find a propagation invariant flat-top beam, as shown in Fig. 3.7. Because the final intensity appears as an incoherent mix of the two components, the second moment beam width ( $w_{FT}$ ), divergence ( $\theta_{FT}$ ) and resulting scalar beam quality factor,  $M_{FT}^2$ , can be analytically calculated from the weighted sum of the modal values [144], which simply gives

$$w_{FT} = w_0 \left( \alpha + (1 - \alpha) \sqrt{1 + |\ell|} \right),$$

$$\theta_{FT} = \frac{M_{FT}^2 \lambda}{\pi w_{FT}},$$

and

$$M_{FT}^2 = \alpha + (1 - \alpha)(1 + |\ell|).$$

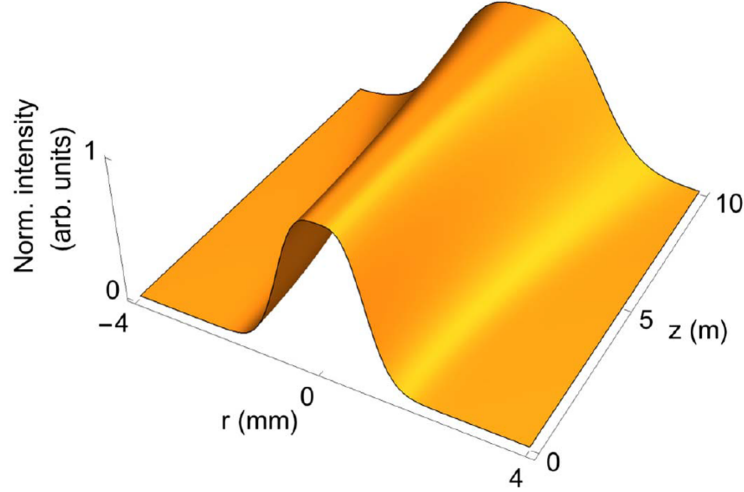


Figure 3.7: Propagation of a vector flat-top beam with  $w_0 = 1$  mm over an extended distance of 10 m for a wavelength of  $\lambda = 532$  nm. The peak intensity has been normalised to unity in order to visualise the intensity shape evolution. It is clearly seen that the profile is propagation invariant.

It is not only the intensity profile that remains invariant but the vector nature too, for which we demonstrate experimentally and outline an analysis toolkit in section (7.2).

### 3.6 Cylindrical vector vortex beams

In this section we consider the concept of digital generation of cylindrical vector vortex beams. We present a novel approach which employs interferometric means to simultaneously generate multiple vector beams using a single hologram (vector beam multiplexing). Furthermore, we extend this concept to realise arrays of higher order Poincaré sphere beams. This approach follows the multiplexing concept previously described in section (3.2.1).

#### 3.6.1 Interferometric generation of vector vortex beams

In general, CVB can be generated as linear combinations of optical vortices of opposite topological charge  $\ell$  (with  $\ell \in \mathbf{Z}$ ) and orthogonal circular polarization as [133, 145, 146],

$$\Psi = \Psi_R^\ell e^{i\ell\phi} e^{i\alpha_1} \hat{e}_R + \Psi_L^{-\ell} e^{-i\ell\phi} e^{i\alpha_2} \hat{e}_L \quad (3.33)$$

where, the unitary vectors  $\hat{e}_R$  and  $\hat{e}_L$  represents the right and left circular polarization with corresponding amplitudes  $\Psi_R^\ell$  and  $\Psi_L^\ell$ .  $\phi$  is the azimuthal angle of the

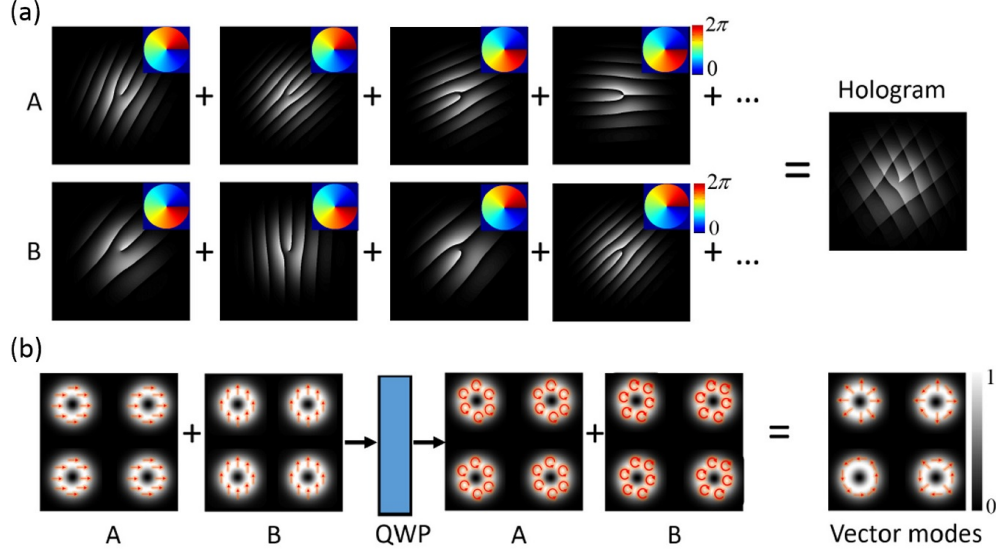


Figure 3.8: (a) Several holograms are multiplexed with appropriate gratings to generate two sets of beams, one traveling along path **A** and another along **B**. The topological charges of each set should be selected carefully to generate the desired CVB. (b) By recombining the beams on paths **A** and **B** with appropriate orthogonal polarizations and phase delays any vector beam can be generated. This figure only depicts the principle behind our generation method, the holograms shown here do not correspond to the intensity profiles shown. The same applies for the topological charges, they have to be carefully chosen to generate the desired vector beam.

cylindrical coordinates. As mentioned before, in order to generate multiple CVB simultaneously, two independent sets of scalar beams traveling along different paths are first generated. This can be easily achieved on the SLM by multiplexing each beam's hologram with different spatial gratings into a single hologram, as illustrated in Fig. 3.8(a). The spatial shape, amplitude and phase of each beam can be manipulated accordingly via the digital hologram. The purpose of sending two sets of beams along different paths is to rotate the polarization of each group to orthogonal circular polarization  $\hat{e}_L$  and  $\hat{e}_R$ , as illustrated in Fig. 3.8(b). Coaxial superposition of these two sets of beams enables the generation of the desired CVB. Manipulation of the spatial shape and phase difference between both beams, via the digital hologram, makes possible to generate CVB with various polarization states as conceptually illustrated in Fig. 3.8, where four CVB are shown. It should be stressed that the gratings shown in this figure should only be taken as illustration of our concept. Analogously the intensity profiles and CVB shown only illustrates the con-

cept, as the intensity, topological charge and phase shift of the beam within each group has to be properly selected.

To multiplex various CVB spatially separated in the far field, each must be encoded with a unique carrier frequency. This can be achieved by adding a linear phase grating to the hologram of each beam. Linear gratings in combination with spatial filters are commonly used to isolate the first diffraction order from undesired zero and higher diffraction orders. The transfer function of a linear phase grating is given by [106],

$$t_g(x, y) = \exp \left[ i2\pi \left( x \frac{U}{\lambda f} + y \frac{V}{\lambda f} \right) \right], \quad (3.34)$$

where  $U$  and  $V$  are the spatial coordinates of the generated beam in the far field, achieved with a Fourier lens of focal length  $f$ .  $U$  and  $V$  are related to the grating frequencies  $(u, v)$  as  $U = u\lambda f$  and  $V = v\lambda f$ . Here,  $\lambda$  is the wavelength of the input light beam. The final expression for the multiplexed hologram that we display on the SLM to generate a number  $M$  of CVB takes the final form,

$$\Phi_M = \text{mod} \left\{ \sum_{j=1}^M \left[ h_j + 2\pi \left( x \frac{U_j}{\lambda f} + y \frac{V_j}{\lambda f} \right) \right] + \sum_{k=1}^M \left[ h_k + 2\pi \left( x \frac{U_k}{\lambda f} + y \frac{V_k}{\lambda f} \right) \right], 2\pi \right\} \quad (3.35)$$

where,  $\text{mod}\{\cdot\}$  represents the modulus function that wraps the phase around  $2\pi$ . The first sum represents the scalar modes traveling along path **A** whereas the second one, those traveling along path **B**.  $h_j$  and  $h_k$  represent the holograms required to generate each scalar beam. In order to generate each CVB, the coordinates  $(U_j, V_j)$  of each beam generated by the hologram  $h_j$  is carefully selected to match the coordinates  $(U_k, V_k)$  of the beam generated by the hologram  $h_k$ .

### 3.6.2 Creating multiple higher order Poincaré sphere beams

Again to explain the concept of generating multiple higher order Poicaré beams, we use the Laguerre Gaussian basis as an example mathematically given as

$$U(r) = LG_{p_1}^{\ell_1} e^{i\alpha_1} \hat{\mathbf{R}} + LG_{p_2}^{\ell_2} e^{-i\alpha_2} \hat{\mathbf{L}}. \quad (3.36)$$

Here  $\hat{\mathbf{R}}$  and  $\hat{\mathbf{L}}$  represent the unitary vectors of the right and left circular polarization, respectively, and  $LG_{p_1}^{\ell_1}$  and  $LG_{p_2}^{\ell_2}$  are the amplitudes of each orthogonal component and represent the well-known Laguerre-Gaussian modes with azimuthal and radial indexes  $\ell$  and  $p$ , respectively, while  $\alpha_1$  and  $\alpha_2$  define modal phases. In this study, we will restrict ourselves to modes with  $\ell_1 = -\ell_2 = \pm\ell$ ,  $\alpha_1 = 0$  and  $\alpha_2 = \alpha$  in order

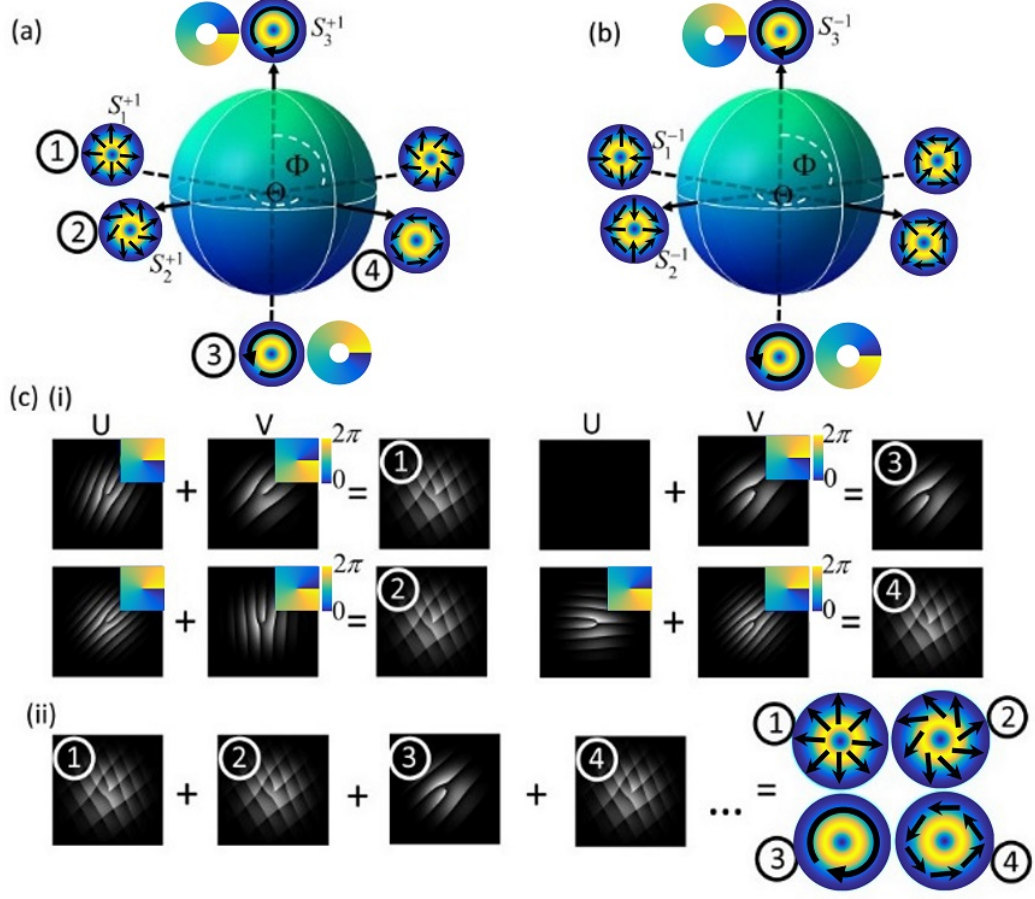


Figure 3.9: Geometric representation of vector beams with arbitrary polarization states on the higher-order Poincaré sphere, for (a)  $\ell = 1$  and (b)  $\ell = -1$ . Each vector beam is digitally generated by encoding a multiplexed hologram consisting of two beams with unique carrier frequencies, as shown in (c) (i), propagating along different paths, that upon coaxial superposition, can create a specific higher-order Poincaré beam (position 1 in (a)). Manipulation of the programmed holograms allows digital creation of any arbitrary polarization state on the sphere (examples denoted by positions 2, 3 and 4 in (a) and the corresponding hologram pairs are given in (c) (i)). Superposition of these multiplexed hologram pairs results in the simultaneous generation of multiple HOPS beams at unique spatial positions as shown in (c) (ii).

to create HOPS beams

$$U(r, \Phi, \Theta) = \cos\left(\frac{\Phi}{2}\right) LG_0^\ell e^{-i\Theta/2} \hat{\mathbf{R}} + \sin\left(\frac{\Phi}{2}\right) LG_0^\ell e^{i\Theta/2} \hat{\mathbf{L}}, \quad (3.37)$$

where  $\Phi \in [0, \pi]$  and  $\Theta \in [0, 2\pi]$  are the coordinates on the HOPS, as shown in Fig. 3.9 (a) and (b). In this geometric representation, the left and right circularly

polarized OAM states are located on the poles (our scalar vortex beams); the cylindrical vector vortex beams lie along the equator; and intermediate states occupy the rest of the sphere. Such beams have been created by a myriad of techniques [51, 42] and found many applications [147, 120, 17].

To generate a single vector beam, we follow the same approach presented in section (3.6.1). To realize HOPS beams is necessary to superimpose appropriate scalar beams with orthogonal polarizations. The beauty of this approach lies in the fact that all properties of each individual beam can be manipulated digitally, without any mechanical component, allowing the generation of any beam on the HOPS. For example, to generate all cases shown in Fig. 3.9 (a), labeled as **1**, **2**, **3**, and **4**, one needs only to adjust the relative phase and amplitude of each scalar beam, associated to the parameters  $\Theta$  and  $\Phi$ , respectively. For the case of the radially polarized beam, labeled as **1**, the corresponding parameters are:  $\ell = 1$ ,  $\Phi = \pi/2$  and  $\Theta = 0$  (see Fig.3.9 (c, i) inset **1**). By superimposing each of the multiplexed pairs of holograms into a single hologram, as shown in Fig. 3.9 (c) (ii), the simultaneous generation of multiple vector beams can be realized.

### 3.7 Conclusion

In this Chapter, digital beam shaping techniques were presented with particular interest in phase only modulation and complex amplitude modulation. A detailed explanation on the generation of the phase only and complex amplitude holograms, using *LG* and *HG* modes as examples was presented. Along the same line the concept of creating multiple light beams simultaneously (multiplexing) was described. We introduced the concept of creating shape invariant flat-top beams which was confirmed by analytical simulations. Finally, a novel approach of creating multiple vector beams which employs the multiplexing concept was presented. Furthermore, the theoretical aspects of generating arrays of HOPS beams was also presented. The experimental realization as well as the analysis and characterization of these structured light fields is presented in Chapters 5, 6 and 7.

## Chapter 4

# Fundamentals of optical trapping

### 4.1 Introduction

In this Chapter we briefly discuss the fundamentals of optical tweezers and consider the theoretical aspects used to model the forces acting on trapped particles of different sizes with respect to the wavelength of the trapping laser. Since the focus of this thesis is on the creation of custom structured light fields, we do not go into detail regarding the optical trap dynamics but however utilize optical tweezing as an immediate application for our structured light fields.

The concept of optical tweezers can be described as a process in which a micron sized particle is confined in 3 dimensional space using a single laser beam that has been tightly focused by a high numerical aperture objective lens [148–150]. The dynamics of trapping a transparent micro particle using a single beam is governed by two forces, the gradient and scattering forces due to the refraction and reflection of the incident light rays, respectively. In the case of the scattering force, the impinging irradiation reflects off the particle, exerting a force per unit area on the particle (radiation pressure) which pushes the particle in the direction of propagation. The gradient force, however, acts in the direction of the spatial intensity gradient (i.e. towards the most intense region of the beam), both in the transverse and propagation planes (Fig. 4.1). In order to realise a stable three-dimensional trap, the gradient force in the axial direction must be greater than the scattering force so as to overcome the radiation pressure. This requires a very steep intensity gradient, which is realised by tightly focusing the laser beam on to a diffraction-limited spot by using a high

numerical aperture objective. The trapping force is typically in the femto to pico-

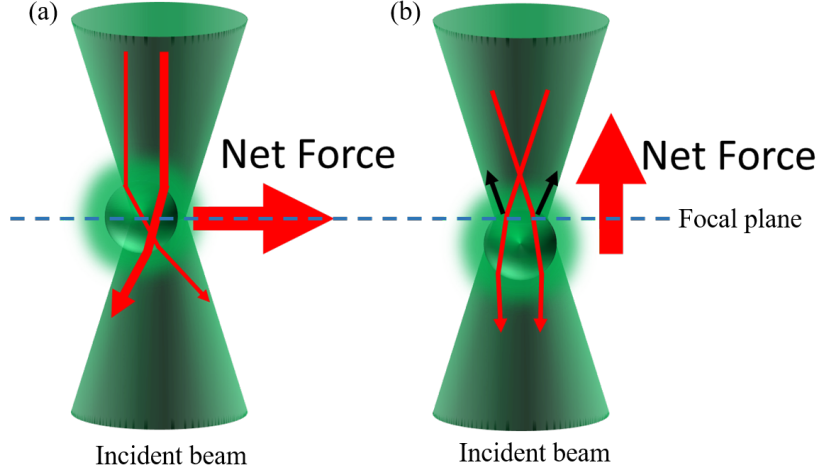


Figure 4.1: Schematic illustrating a single beam optical trap. (a) Shows the net force in the lateral direction towards the most intense region of the focused incident beam while (b) shows an equilibrium state between the scattering forces (in red) and the gradient force (in black) in the axial direction.

newtown range. These optical forces associated with an optical tweezer can be explained using two theoretical concepts: ray optics and Rayleigh theory which depend on the particle size. The following sections will give a brief description of these two theoretical concepts.

## 4.2 The ray optics regime

Ray optics theory is used to model the trapping forces when the dimension of the particle  $a$  is much larger than the wavelength  $\lambda$  [151]. We can calculate from ray optics the optical forces and it is worth noting that as the size of the trapped object becomes bigger, the forces acting on it decrease. The expression of the force caused by a single ray of a power  $P$  can be written as follows [151]:

$$\mathbf{F} = \frac{n_m P}{c} (1 + R \cos(2\theta)) - \left( \frac{T^2 [\cos(2\theta - 2\phi) + R \cos(2\theta)]}{1 + R^2 + 2R \cos(2\phi)} \right) \mathbf{k} + \frac{n_m P}{c} \left( R \sin(2\theta) - \frac{T^2 [\sin(2\theta - 2\phi) + R \sin(2\theta)]}{1 + R^2 + 2R \cos(2\phi)} \right) \mathbf{i} \quad (4.1)$$

where  $\theta$  and  $\phi$  are the angle of incidence and refraction respectively.  $R$  and  $T$  are the Fresnel reflection and refraction coefficients, and  $\mathbf{i}$  and  $\mathbf{k}$  are a perpendicular

unit vectors. The trap can be strengthened by maximizing the force in the  $z$ -direction. This is better if an objective with a high numerical aperture (NA) is used. These optical forces are based on the third law of Newton, due to an exchange of momentum happening between the photon and the bead, a force proportional to the light intensity arising from refraction. This force can be in the direction of the gradient intensity if the refraction index of the sphere  $n_p$  is higher than that of the medium  $n_m$ . In the case where the refraction index of the sphere is lower than the refraction index of the surrounding medium the opposite effect is observed. For the ray optics case we are able to control and manipulate some micron-sized particles such as biological cells however it's difficult to trap some viruses which are smaller than 100 nm. Therefore, for particles much smaller than the wavelength the Rayleigh regime is considered and is described in the following section.

### 4.3 The Rayleigh regime

In this regime, the trapped particle is much smaller than the wavelength of the incident laser beam such that it is considered a dipole [152]. The electric field of the incident light acts uniformly on the particle at a given instant [148, 152]. Similarly, for this regime, the forces acting on the nano particle are divided in two; the force due to radiation pressure (scattering force) and the gradient force related to the intensity distribution of the laser beam. The scattering force is proportional to the scattering cross section and therefore increases according to the following equation [152];

$$\vec{F}_{scat}(\vec{r}) = \frac{8\pi n_m}{3c} (ka)^4 a^2 \left( \frac{m^2 - 1}{m^2 + 2} \right) I(\vec{r}) \quad (4.2)$$

where  $k = \frac{2\pi}{\lambda}$  is the wave number in the medium and  $I(r)$  is the intensity profile of the irradiating laser beam. When  $m - 1 \ll 1$  the scattering force scales to  $(n_p - n_m)^2 a^6$ . However, the gradient force has a linear relation to the polarisability given by

$$\vec{F}_{grad}(\vec{r}) = \frac{2\pi n_m}{c} a^3 \left( \frac{m^2 - 1}{m^2 + 2} \right) \vec{\nabla} I(\vec{r}) \quad (4.3)$$

and scales to  $(n_p - n_m)a^3$  for a small difference in the particle and medium indices of refraction. Considering the paraxial regime of a propagating Gaussian beam, the intensity profile of the the beam can be expressed as

$$I(\vec{r}) = \frac{2P}{\pi w(z)^2} \exp\left(\frac{-2r^2}{w(z)^2}\right) \quad (4.4)$$

at some propagation distance  $z$  from the focus and a radial distance  $r$  from the propagation axis. Where  $w(z)^2 = w_0^2(1 + (\frac{z}{z_0})^2)$ ,  $z_0 = \frac{\pi w_0^2}{\lambda}$ ,  $w_0$  is the waist of the beam and  $P$  is the power of the incident laser beam. The radial trapping stiffness  $k_r$  and the axial trapping stiffness  $k_z$  can, respectively, be obtained using the following equations

$$k_r = \frac{32P}{3w_0^4 c} a^3 (n_p - n_m) \quad (4.5)$$

$$k_z = \frac{64P}{3k^2 w_0^6 c} a^3 (n_p - n_m) \quad (4.6)$$

Equations 4.5 and 4.6 hold if the conditions  $m-1 \ll 1$  and  $\vec{F}_{grad} \vec{z} \gg \vec{F}_{scatt}$  are assumed, respectively. Thus far we have considered two case where the particle is much greater and much smaller than the trapping laser beam wavelength. The next section takes into account when the particle diameter is approximately equal to the laser beam wavelength.

## 4.4 The Mie regime

The Rayleigh approximation described in the previous section is valid for particles much smaller than the wavelength and does not apply for a particle with a diameter comparable to the trapping laser beam wavelength ( $a \approx \lambda$ ). As such, a more general approach referred to as the Generalized Lorenz-Mie Theory (GLMT) is employed. This approach enables the modelling of the scattering fields and radiation forces that act on a particle of any size being trapped by a Gaussian beam or other general beam shapes [153]. The optical trapping forces in this case are expressed with respect to 3 cross-section in the transverse  $x$ ,  $y$  planes and the axial  $z$  plane. This force is mathematically described by equation (4.7) [153].

$$\vec{F}(\vec{r}) = \frac{n_2}{c} I_0 [\hat{x} C_{pr,x}(\vec{r}) + \hat{y} C_{pr,y}(\vec{r}) + \hat{z} C_{pr,z}(\vec{r})] \quad (4.7)$$

where  $I_0 = 2P/\pi\omega_0^2$  is the intensity of the beam irradiating the particle with a power  $P$  while  $C_{pr,x}$ ,  $C_{pr,y}$  and  $C_{pr,z}$  are the respective cross sections [154, 155]. The necessary forces required to trap particles with respect to their size and trapping wave length using a plane wave have been discussed. The next section will consider the concept of holographic optical trapping.

## 4.5 Holographic optical traps

Light forces in optical tweezers provide adequate forces to manipulate micro and nano particles and arrange them into various configurations in 2D and 3D [117, 28, 21, 6, 43]. The ease at which these micro particles are assembled has been made possible by the dynamic properties of SLMs being integrated into optical tweezers forming what is conventionally known as "Holographic optical traps". A schematic illustration of miniaturized particles arranged to spell the word "WITS" using a dynamic holographic optical trap is shown in Fig. (4.2).

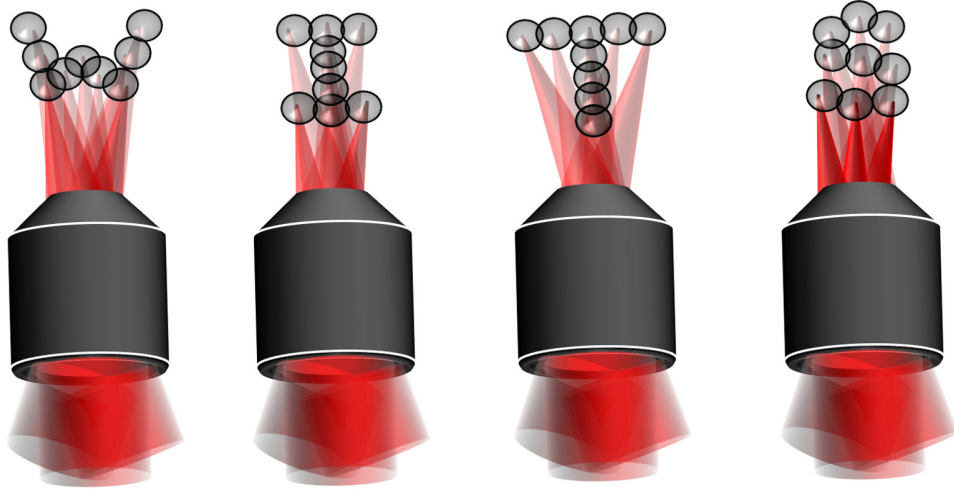


Figure 4.2: Schematic illustrating multiple optical traps arranged into unique configurations as dictated by a holographic optical trap.

The introduction of holographic optical tweezers has enabled the generation of multiple traps in a desired array or arrangement (Fig. (4.2)) each manipulated independently of its nearest neighbour [6, 21, 28, 117]. HOTs also provided a means to real-time manipulation of trapped particles by simply encoding arbitrary phase patterns on the SLM demonstrating its flexibility. In addition, more complex laser beam structures have since enabled other features such as trapping of low refractive index particles. The ability to trap with structured light fields also resulted in an improved trapping efficiency[125, 126, 123].

Generally, HOTs are designed from the convectional optical tweezers system but instead one of the mirrors directing the laser into the trap is replaced with an SLM as shown in Fig 4.3. The effect of the dynamics of the SLM, i.e. the ability to manipulate particles by altering the encoded hologram, is realised when the SLM is at the conjugate plane of the objective lens. Normally this is achieved

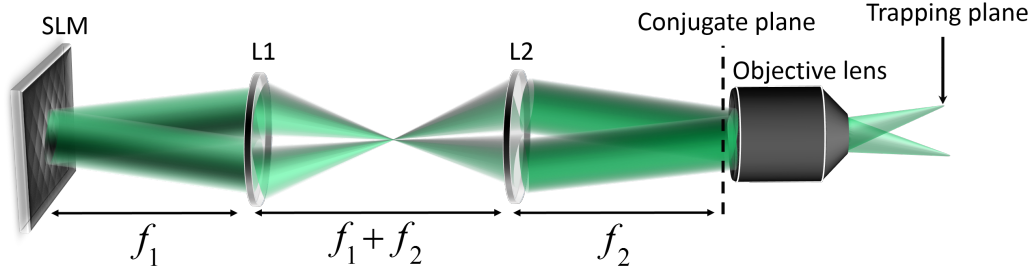


Figure 4.3: Schematic illustrating an SLM controlled optical trap where the SLM is imaged onto the back entrance of the objective lens (conjugate plane) using a telescope ( $f_1$  and  $f_2$ ). Using this configuration, a change in the encoded hologram causes a change in the beam position in the trapping plane.

using relay optics to image the SLM plane to the conjugate plane of the objective. The versatility of the structured light fields discussed in Chapters 2 and 3 will be demonstrated in a holographic trap presented in Chapter 7 as such it was worthwhile to briefly introduce this concept.

## 4.6 Conclusion

To summarize this Chapter, the fundamental forces responsible for optical trapping were briefly discussed. We then consider the three common theoretical concepts used to model the trapping force in an optical tweezers system with respect to the size of the particle being trapped. Finally, we briefly described the basics of holographic optical traps and its benefits thereof.

## Chapter 5

# Generation and manipulation of scalar beams using an SLM

### 5.1 Introduction

Laser beam shaping using computer controlled devices such as spatial light modulators has become a relevant tool in modern day optics laboratories. Importantly, these digital devices provide on demand a very accurate, flexible and fast holographic means to tailor light's spatial profile. In addition, this digital holographic approach allows the simultaneous shaping and generation of several beams (multiplexing), a feature which has been found very useful in numerous laser based applications. In this Chapter, we demonstrate the experimental realisation of light beam multiplexing using Laguerre-Gaussian and Hermite-Gaussian modes as our chosen scalar light fields. The experiments presented here are based on the hologram generation and multiplexing concepts discussed in Chapter 3 section (3.2.1). We qualitatively compare the beam shaping techniques used by performing 2D image correlation of the experimentally generated modes against theoretical images. Additionally, we present a quantitative analysis on the multiplexing capabilities of SLMs to assess the maximum number of beams that can be multiplexed on a single SLM, in which we show multiplexing of approximately 200 modes. Finally, theoretical simulations which take into account different SLM resolutions are presented.

## 5.2 Experimental implementation of mode multiplexing

A schematic of the multiplexing setup is shown in Fig. 5.1. The laser source, linearly polarized, was expanded using a telescope constituting a  $10\times$  microscope objective and a lens  $L_1$  of focal length  $f = 100$  mm to approximate a plane wave. The expanded beam impinged onto a reflective SLM, Holoeye Pluto, with a resolution of  $1920 \times 1080$  pixels and a pixel pitch of  $8 \mu\text{m}$ . The SLM screen was split into three sections which enabled independent hologram addressing on each third [36]. A multiplexed hologram to generate  $LG_p^\ell$ ,  $HG_{nm}$  or a combination of modes was encoded on each third of the SLM. Each mode was encoded with a unique carrier frequency, as explained in section 3.2, to separate all modes in the far field. The far field was obtained using a  $2f$  configuration system, in which a lens of focal length  $f = 250\text{mm}$  ( $L_2$ ) was inserted halfway between the SLM and the detector, located at  $2f$  from the SLM. All the multiplexed modes were generated in the  $1^{st}$  diffraction order which was separated from the rest using a spatial filter. The mode quality of the generated modes was tracked using 2D image correlation, as will be explained in the next section.

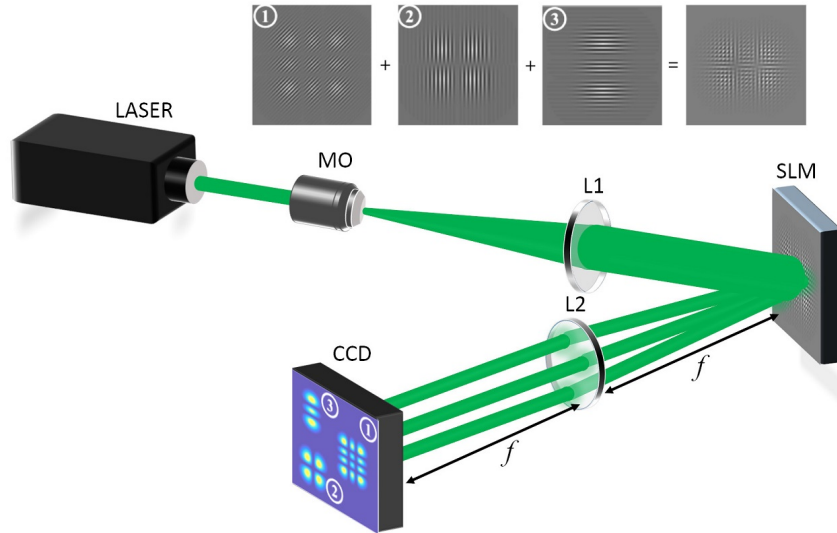


Figure 5.1: Schematic representation of a mode multiplexing setup. A laser beam ( $\lambda = 532$  nm) is expanded using a  $10\times$  microscope objective (MO) and a collimating lens  $L_1$  on to a reflective SLM where a hologram of multiplexed  $LG_p^\ell$  and  $HG_{mn}$  modes is displayed. The desired modes are generated in the focal plane of the Fourier lens  $L_2$  (Screen) with unique spatial positions.

### 5.3 Analysis of mode quality

The quality of the experimentally generated modes was quantified by correlating their intensity profile against their theoretical counterpart, represented by images  $A$  (experimental) and  $B$  (theoretical) respectively. The 2D images correlation was computed as,

$$C = \frac{\sum_i \sum_j (A_{ij} - \bar{A})(B_{ij} - \bar{B})}{\sqrt{\sum_i \sum_j (A_{ij} - \bar{A})^2 \sum_i \sum_j (B_{ij} - \bar{B})^2}}, \quad (5.1)$$

where  $C$ , the correlation coefficient, is a dimensionless parameter that measures the similarity between two images, 0 for nonidentical images and 1 for identical images.  $A_{ij}$  and  $B_{ij}$  are the intensity values per pixel.  $\bar{A}$  and  $\bar{B}$  are the mean intensity values of  $A$  and  $B$  respectively. In this approach, an overlap of the experimentally generated and theoretical images both with identical  $x \times x$  pixel dimensions is performed. This way a pixel to pixel comparison of the experimental intensity profile and beam structure to that of the theoretical is performed. In this case,  $C=0$  means the experimental mode is not identical to the theoretical and  $C=1$  the modes are identical. In these experimental measurements, external filters were employed to filter out any noise resulting in an improved signal to noise ratio on the detector with no additional post image processing required.

A straightforward correlation measurement of the generated modes provides information about their quality as a function of their structure complexity. Figure 5.2 (top) shows the transverse intensity profile of the modes  $LG_3^2$ ,  $LG_4^4$ ,  $HG_{44}$  and  $HG_{73}$  generated through complex amplitude modulation. Their corresponding correlation coefficients are  $C = 0.96$ ,  $C = 0.93$ ,  $C = 0.97$  and  $C = 0.96$ . A cross-sectional plot of the mode's intensity profile, illustrating the intensity variations across the beam, is also shown in Fig. 5.2 (bottom). From the chosen modes in figure 5.2 the  $HG_{nm}$  modes demonstrate higher correlation values than  $LG_p^\ell$ . A comparison of the mode quality between the two beam shaping techniques, phase-only and complex amplitude modulation (CAM), was also performed. Figure 5.3 shows the theoretical (top) intensity profile of the modes  $LG_1^3$ ,  $LG_2^1$ ,  $LG_3^2$ ,  $LG_4^4$ ,  $HG_{12}$ ,  $HG_{12}$ ,  $HG_{31}$ ,  $HG_{23}$  and  $HG_{44}$  along with their corresponding experimental profiles generated through phase-only (Fig. 5.3 center row) and CAM (Fig. 5.3 bottom row). As expected, the correlation values decreases as the complexity of the modes increases. Moreover, the correlation values are always higher for modes generated through CAM since this method provides control of both phase and amplitude while in phase only

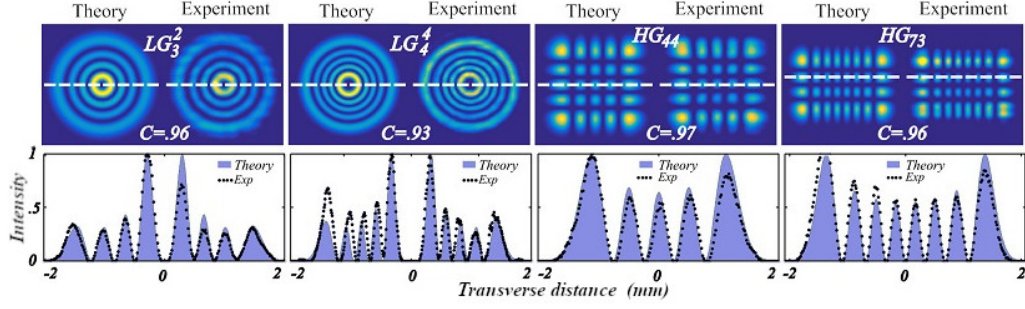


Figure 5.2: Experimental and theoretical intensity profiles of  $LG_p^\ell$  and  $HG_{nm}$  modes with corresponding correlations values  $C$  (top row). Cross-sectional intensity profile along the dashed line comparing theory and experiment (bottom row) where the horizontal axis is the radial profile in mm and the vertical axis is the normalized beam intensity. Notice how the correlation values decrease as the mode complexity increases.

modulation only the phase of the incident beam can be manipulated.

## 5.4 Beam quality of multiplexed modes

In order to quantify the maximum number of modes an SLM can support, we tracked the correlation for a specific mode while increasing the number of multiplexed modes. Our threshold for a good quality mode was set at a correlation  $C = 0.8$ , which is the maximum correlation value that a theoretical mode can achieve when compared to its own binary version, horizontal line in Fig. 5.4. In our experiment, we split the SLM's screen in three independent sections, each of which was addressed with its own multiplexed hologram. In this way, each third of the SLM behaves as an independent SLM and can be illuminated with the same source or with different sources and/or wavelengths, increasing the number of multiplexed modes by a factor of three [36]. Next, we encoded arbitrary modes on each third of the SLM and tracked the correlation of one of them, while increasing the number of multiplexed modes. This procedure was carried out for both multiplexing schemes and compared afterwards. Figure 5.4 (a) shows the correlation as a function of the number of modes for phase-only holograms whereas Fig. 5.4(b) shows the same measurements for CAM holograms. Notice how the correlation values decay very rapidly for phase-only holograms, enabling multiplexing of only 75 modes with correlations higher than  $C = 0.8$ . On the contrary, for CAM, the correlation coefficient remains higher than 0.8 for up to 192 modes. The obtained correlation values and number of modes are for

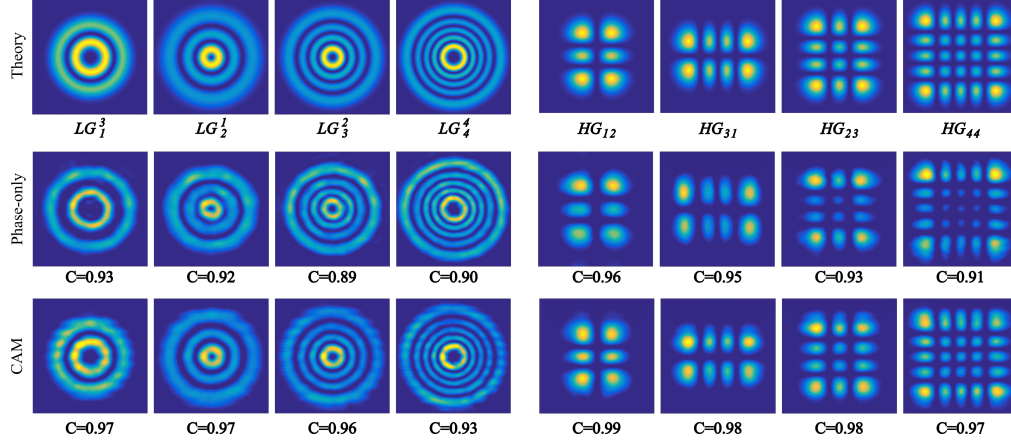


Figure 5.3: The top row shows the theoretical intensity profile of  $LG_p^\ell$  (left) and  $HG_{nm}$  (right) modes. Second and third row show experimental intensity profiles of phase only and complex amplitude modulation (CAM), respectively. Notice that CAM allows for higher correlation values.

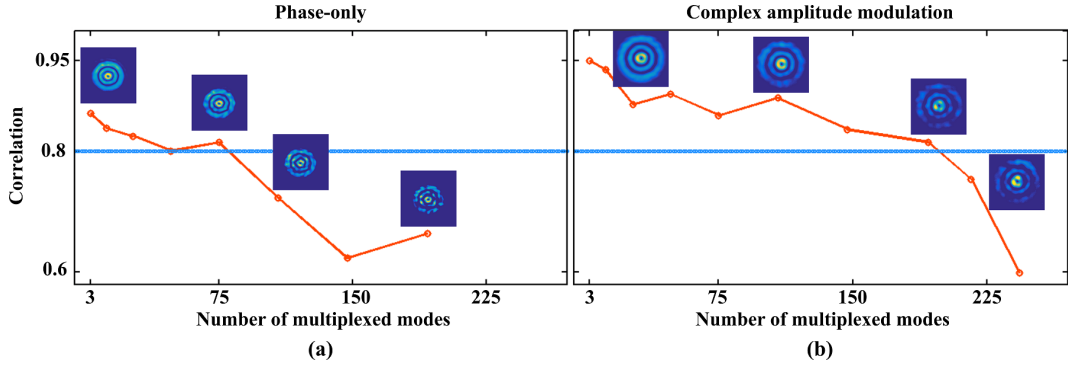


Figure 5.4: Correlation of an  $LG_2^1$  while increasing the number of multiplexed modes for phase-only (a) and CAM (b) modulation. Notice that only 75 modes with  $C > 0.8$  were achieved with phase-only modulation while CAM allows multiplexing of 192 modes.

this specific case and exhibit slight variation according to the selected modes when multiplexing. In other words, the observed variations emanate from the addition of more spatial modes which is done in a random manner, i.e. a different mode is multiplexed each time, thereby resulting in the random variations in our plots. If the same mode is used for multiplexing a more smooth decay in the correlation is expected with increase in the number of modes. In this case we are mainly concerned with the general trend and importantly that CAM enables the generation of much larger mode sets compared to phase only modulation.

Figure 5.5 shows six sets of multiplexed modes, captured with a CCD camera

(Pointgrey,  $1288 \times 964$  pixels,  $3.75 \mu\text{m}$  pixel size). By properly choosing the grating period of each multiplexed mode, we generated 25, 36, 42 and 49 modes, as shown in Fig. 5.5(a), (b), (c) and (d) respectively. These figures also illustrates that we can generate, modes from the same set Fig. 5.5 (a),(b) and (d) or from different sets Fig. 5.5(c) and (f).

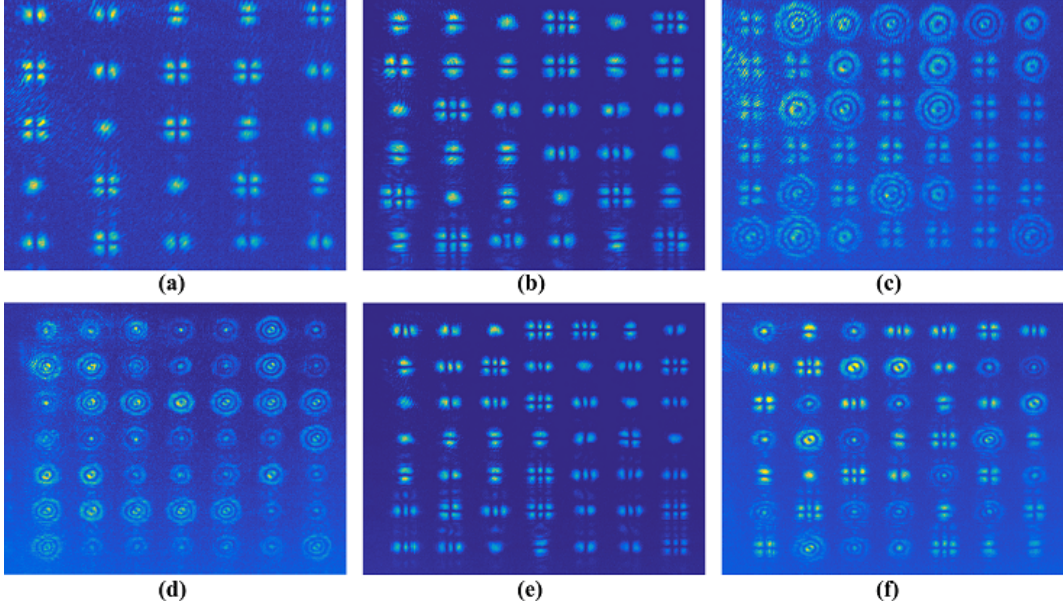


Figure 5.5: Experimental intensity profile of (a) 25, (b) 36, (c) 42 and (d)-(f) 49 multiplexed modes. (a), (b), (d) and (e) shows multiplexed modes from the same basis set,  $LG_p^\ell$  or  $HG_{nm}$  whereas (c) and (f) shows multiplexed modes from both basis. The images only show a maximum of 49 multiplexed modes due to the limitations of the detector.

A simulation of multiplexed beams was carried out using the Fraunhofer approximation of the angular spectrum method (far field), to extend our results to other SLM's screen resolutions. Again, the intensity profile of the simulated mode was correlated against the theoretical. The modes generated through these simulations are, non surprisingly, of higher quality but clearly illustrate a decay in the correlation, as the pixel resolution decreases from  $1700 \times 1700$  to  $500 \times 500$  [Fig. 5.6(a)]. For this plot, a single mode ( $LG_2^1$ ) was generated and its correlation was tracked while decreasing the pixel resolution. The simulated and theoretical images use the same equations however to simulate the decrease in beam quality observed in the experiment, a series of images with a gradual decrease in resolution (i.e. pixelated) were generated. This way, a pixelated image is obtained with each decrease in pixel resolution as illustrated in Fig. 5.6(a) and compared with the initial high resolution

theoretical image to obtain the corresponding correlation value. The insets shows the intensity profile of this modes at various resolutions. We also simulated the correlation as a function of the number of multiplexed modes for different resolutions, as shown in Fig. 5.6(b). As in the experiment, the correlation coefficient decreases as the number of modes increases. The insets of this figure, shows the intensity profile of the simulated modes for a resolution of  $500 \times 500$  pixels as we increased the number of multiplexed modes.

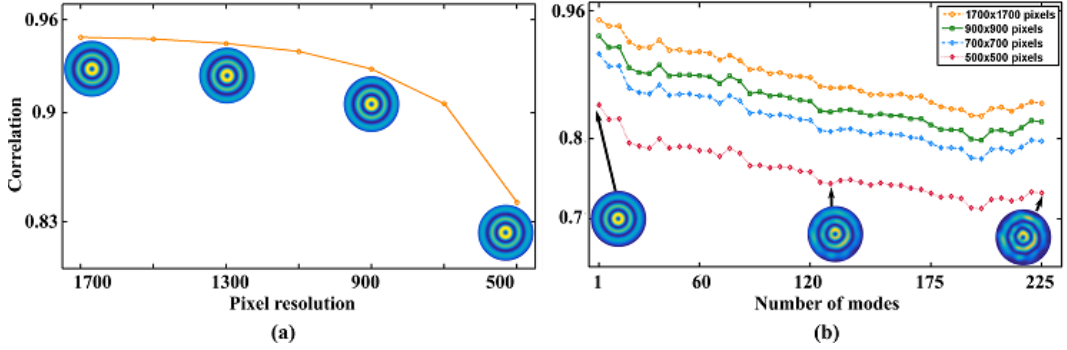


Figure 5.6: (a) Simulated correlation curve for various SLM resolutions. (b) Simulated correlation curves for 225 multiplexed modes using different resolutions. For these simulations we tracked the mode ( $LG_2^1$ , but similar result can be obtained with other modes).

## 5.5 Conclusion

In this Chapter, holographic laser beam shaping using a phase only spatial light modulators was presented. We demonstrated the capability of these devices to generate multiple structured light fields simultaneously. We use two basis sets of spatial modes, Laguerre-Gaussian and Hermite Gaussian, as examples to help demonstrate the multiplexing concept. We firstly considered the experimental generation of our modes using phase-only holograms where the amplitude is left as a free variable. Thereafter we took into account the amplitude, through the generation of our modes using complex amplitude modulation. We showed that of the two beam shaping techniques, complex amplitude modulation reproduces tailored spatial modes of higher quality compared to phase only modulation. We then presented an analysis of the quality of the created spatial modes while increasing the number of multiplexed beams from 1 up to 225. The analysis provided useful information on how the mode quality decreases as the number of multiplexed modes increases with respect to the

SLMs resolution. Furthermore, a theoretical analysis of the mode quality as function of the resolution of the screen was presented. The results revealed, as expected, that the quality of the modes decays as the screen's resolution decreases and therefore fewer number of multiplexed modes can be generated.

## Chapter 6

# Generation of multiple complex vector light fields

### 6.1 Introduction

In Chapter 5, we demonstrated that SLMs can simultaneously generate multiple structured scalar light fields by modulating only the phase or through complex amplitude modulation where we shape both phase and amplitude. However, in our laser beam shaping guide, we did not pay any particular attention to the polarization of the resultant customized spatial modes. As such in this Chapter following the same multiplexing concept, we put forward a novel technique that exploits the superposition principle in optics to enable the simultaneous generation of many vector beams using a single digital hologram. Previously, these complex light fields have been generated through a combination of physical optical elements such as spiral phase plates, q-plates and quarter wave plates. However, such "hard-coded" optical devices present some restrictions such as their inability to generate multiple structured light fields simultaneously. This has stimulated a wide range of generation techniques based on computer controlled devices such as SLMs.

Here we present the experimental realisation of structured vector light fields by shaping the phase, amplitude and polarization of some initial input field using an SLM. Our method is purely digital based on the concept discussed in Chapter 3 section (3.6.1) and allows the generation of a wide variety of vector beams by simply changing the encoded digital hologram. As proof-of-concept, we demonstrate simultaneous generation of vector beams with different polarization states on the High Order Poincaré Sphere (HOPS). To further show the flexibility of our approach,

we on demand create multiple vector Bessel modes using the same experimental set up by simply altering the programmed digital hologram. This generation technique will be of impact in the context of quantum and classical communication, optical micro-manipulation and super resolution microscopy.

## 6.2 Experiment realisation and implementation

To experimentally demonstrate the simultaneous generation of multiple cylindrical vector beams (CVB), we used a horizontally polarized laser and a reflective SLM (Holoeye Pluto) to generate multiplexed scalar beams. A schematic representation of the implemented setup is shown in Fig. 6.1. On the SLM, a set of holograms were multiplexed to generate two sets of scalar fields, each with a unique carrier frequency (grating). The frequency of each grating is carefully selected to separate the multiplexed holograms into two groups, one traveling along path **A** and another along path **B**. To accomplish this, the holograms are multiplexed such that the grating frequencies within each group are relatively small compared to the grating of the two different sets. The beams propagating along path **A** are kept with the same horizontal polarization while the ones traveling along path **B** are rotated to vertical polarization by means of a Half-wave plate (HWP) orientated at  $45^\circ$ . Both sets of beams are recombined into a single set, using a Polarizing Beam Splitter (PBS). To simplify the overlapping of all beams, a judicious choice of the gratings is encoded so that the alignment of one ensures the alignment of the rest. A Quarter-Wave Plate (QWP) set at  $45^\circ$  transforms the states of polarization to left- and right-handed, respectively, to generate in this way multiple CVBs, each with its own polarization state or spatial shape. The resulting CVBs were observed with a CCD camera and their state of polarization analyzed with a linear polarizer placed before the CCD. A further analysis of their quality was performed using the vector quality factor, a measure that assigns the value 1 to pure vector beams and 0 to scalar beams. In this proof-of-principle experiment we restricted ourselves to the generation of vector beams with cylindrical symmetry (CVB) but any other vector beam can be generated by simply modifying the displayed hologram.

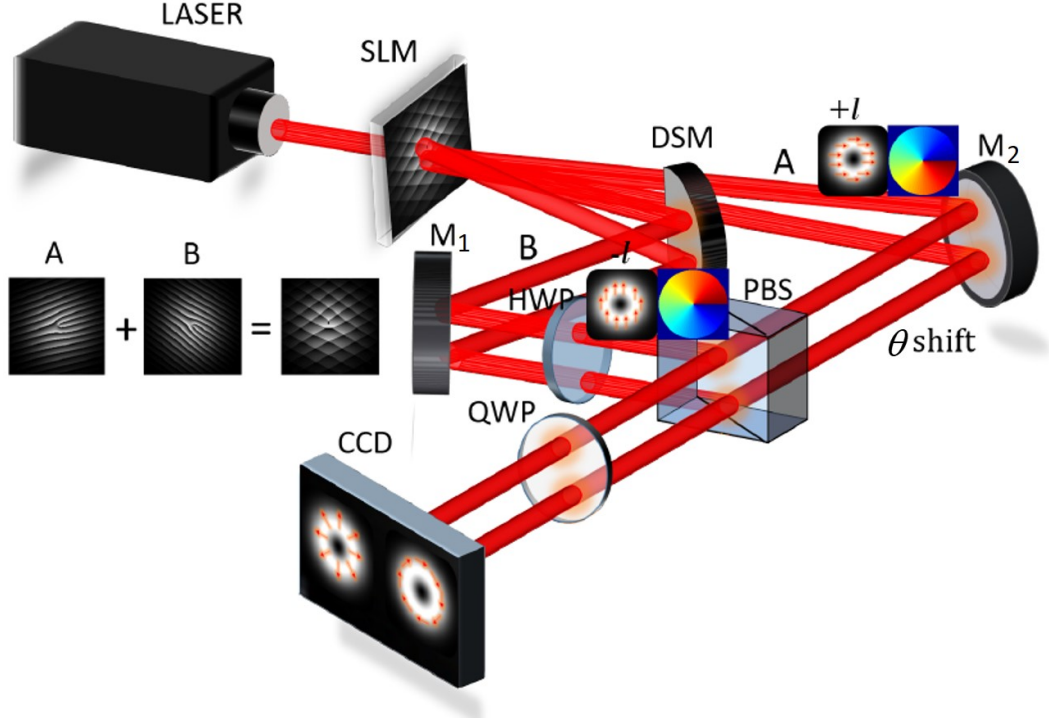


Figure 6.1: Schematic representation of the implemented setup to generate multiple cylindrical vector beams. SLM: Spatial light modulator; DSM: D-shaped mirror;  $M_{1,2}$ : mirrors; PBS: Polarizing beam splitter; QWP: Quarter-wave plate; HWP: Half-wave plate; L: Lens.

### 6.3 Multiplexing of high order Poincaré sphere beams

The Higher Order Poincaré Sphere (HOPS) is a very useful geometrical interpretation of CVB, according to which any vector state can be represented as a point  $(\alpha, \phi)$  on a sphere [133, 145, 51]. In this representation, left and right circularly polarized states are positioned on the poles, CVB are located along the equator and states with elliptical polarizations occupies the rest of the sphere. A mathematical representation can be derived from Eq. 3.33 by choosing,  $\Psi_R^\ell = \cos(\varphi/2)$ ,  $\Psi_L^\ell = \sin(\varphi/2)$  and  $-\alpha_1 = \alpha_2 = \alpha/2$  as,

$$\Psi_{\varphi, \alpha} = \cos\left(\frac{\varphi}{2}\right) e^{i\ell\phi} e^{-i\alpha/2} \hat{e}_R + \sin\left(\frac{\varphi}{2}\right) e^{-i\ell\phi} e^{i\alpha/2} \hat{e}_L, \quad (6.1)$$

where  $\alpha \in [0, 2\pi]$  and  $\varphi \in [0, \pi]$ . In this representation, each  $\ell$  value gives rise to a unique HOPS. The digital method presented here enables the generation of any CVB on the full Poincaré sphere. Moreover, it allows for the generation of CVB corresponding to different HOPS. For example, by simply varying the phase offset  $e^{i\alpha}$ , we can generate any vector mode along the equator. In addition, we can vary

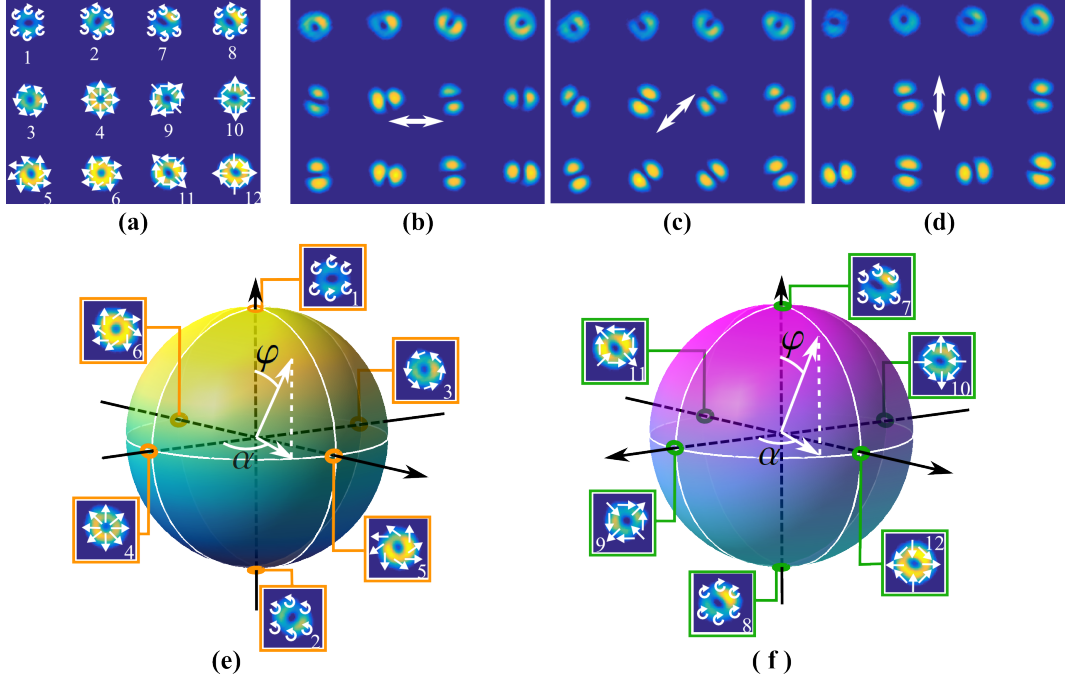


Figure 6.2: (a) Intensity profile of twelve multiplexed CVB from the High Order Poincaré sphere, arrows depict the polarization state for each of them. Polarization state after an analyzer at (b)  $0^\circ$ , (c)  $45^\circ$ , (d)  $90^\circ$ . Modes one to six correspond to a Poincaré sphere defined by a topological charge  $\ell = +1$  (e), whereas modes six to twelve corresponds to a topological charge  $\ell = -1$  (f).

the weighting terms, related to  $\varphi$ , on either path of the interferometer by changing the phase modulation depth of the encoded hologram to move from a full vector mode to a full scalar mode on either pole of the HOPS, as previously demonstrated in [156, 157]. Figure 6.2 illustrates the simultaneous generation of twelve modes, defining two different HOPS, one associated to  $\ell = +1$  [Fig. 6.2(e)] and the other to  $\ell = -1$  [Fig. 6.2(f)]. Multiplexing of these twelve higher order Poincaré beams (HOPB), required the multiplexing of twenty four holograms, to generate twelve beams traveling along path **A** and twelve along path **B**. Table 1 contains specific values  $\alpha$ ,  $\phi$  of each generated mode. This table also shows the phase difference  $\Delta\varphi$  between  $\hat{e}_R$  and  $\hat{e}_L$ , a value that is encoded in the SLM to generate the desired CVBs. Here we also show the frequency values  $(u, v)$  used for each encoded hologram. The values  $(u_A, v_A)$  correspond to the beams encoded along path **A**, where as the values  $(u_B, v_B)$  to those traveling along path **B**.

To test the robustness of our proposed method, we measured the quality of the multiplexed modes using the Vector Quality Factor (VQF) [158, 35]. The VQF

Table 6.1: Experimental values of generated HOPS modes.

| $n$            | 1    | 2     | 3       | 4       | 5       | 6        | 7    | 8     | 9       | 10      | 11      | 12       |
|----------------|------|-------|---------|---------|---------|----------|------|-------|---------|---------|---------|----------|
| $\alpha$       | 0    | 0     | $\pi$   | 0       | $\pi/2$ | $3\pi/2$ | 0    | 0     | $\pi$   | 0       | $\pi/2$ | $3\pi/2$ |
| $\varphi$      | 0    | $\pi$ | $\pi/2$ | $\pi/2$ | $\pi/2$ | $\pi/2$  | 0    | $\pi$ | $\pi/2$ | $\pi/2$ | $\pi/2$ | $\pi/2$  |
| $\Delta\alpha$ | —    | —     | $\pi/2$ | $\pi/2$ | $\pi/2$ | $\pi/2$  | —    | —     | $\pi/2$ | $\pi/2$ | $\pi/2$ | $\pi/2$  |
| $u_A$          | -.28 | —     | -.28    | -.18    | -.28    | -.18     | -.15 | —     | -.15    | -.12    | -.15    | -.12     |
| $v_A$          | .13  | —     | .07     | .07     | .05     | .05      | .13  | —     | .07     | .07     | .05     | .05      |
| $u_B$          | —    | .15   | .12     | .15     | .12     | .15      | —    | .27   | .19     | .27     | .19     | .27      |
| $v_B$          | —    | .15   | .12     | .15     | .12     | .15      | —    | .27   | .19     | .27     | .19     | .27      |

measures the purity of a vector beam, assigning a value 0 to scalar modes and the value 1 to vector modes. The methodology to determine the VQF is further explained in the appendix. From our generated modes, we randomly selected two of them and measured their VQF before and after multiplexing. We deliberately selected a scalar and a vector mode (mode labeled as 8 and mode labeled as 3). For the scalar case, we obtained a value  $VQF_b = 0.01$  before multiplexing and a value  $VQF_a = 0.01$  after multiplexing. For the vector beam we obtained  $VQF_b = 0.98$  and  $VQF_a = 0.96$ , respectively. This strongly indicates that multiplexing of twelve CVB does not affect their purity.

## 6.4 Bell states

As another example to highlight the capabilities of our technique, we multiplexed sixteen Bell states, eight of which were generated with  $\ell = \pm 1$  and eight with  $\ell = \pm 2$ . Bell states are of great relevance in optical communication and quantum computing [29, 48, 159], mathematically they can be represented as,

$$TM = \frac{1}{\sqrt{2}} \left( e^{i\ell\phi} \mathbf{e}_R + e^{-i\ell\phi} \mathbf{e}_L \right), \quad (6.2)$$

$$TE = \frac{1}{\sqrt{2}} \left( e^{i\ell\phi} \mathbf{e}_R - e^{-i\ell\phi} \mathbf{e}_L \right), \quad (6.3)$$

$$HE^e = \frac{1}{\sqrt{2}} \left( e^{i\ell\phi} \mathbf{e}_L + e^{-i\ell\phi} \mathbf{e}_R \right), \quad (6.4)$$

$$HE^o = \frac{1}{\sqrt{2}} \left( e^{i\ell\phi} \mathbf{e}_L - e^{-i\ell\phi} \mathbf{e}_R \right) \quad (6.5)$$

Figure 6.3 shows the intensity profile of the 16 Bell states generated by properly adjusting the hologram's topological charge and phase. To generate the  $TM$  modes,

two holograms with opposite topological charges  $-\ell$  and  $+\ell$  were multiplexed on the hologram, with frequency grating values similar to the previous case. A similar

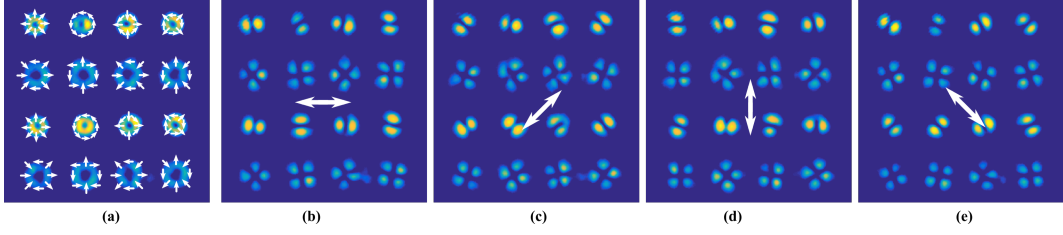


Figure 6.3: (a) Experimental intensity profile of sixteen multiplexed CVB with eight orthogonal polarization states, indicated by the arrows, for  $\ell = 1$  and  $\ell = 2$ . Intensity profile of the CVB when a linear polarized is placed at (b)  $0^\circ$ , (c)  $45^\circ$ , (d)  $90^\circ$  and  $135^\circ$ .

procedure is applied to generate the mode  $TE$ , but in this case, a  $\pi$  phase offset between both beams is digitally encoded on the hologram. To generate the  $HE^o$  and  $HE^e$  modes, we simply interchange the  $\ell$  values in the hologram. The intensity profile after an analyzer orientated at  $\varphi = 0^\circ$ ,  $45^\circ$ ,  $\varphi = 90^\circ$  and  $135^\circ$ , inserted before the CCD, are shown in Figs. 6.3(b), 6.3(c), 6.3(d) and 6.3(e) respectively. In this case, the VQF values before and after multiplexing for a randomly selected mode are  $VQF_b = 0.99$  and  $VQF_a = 0.98$ , respectively. Showing that the purity of the generated CVB remains practically constant even after multiplexing.

## 6.5 Generating multiplexed vector Bessel-Gauss beams

To show that our technique can be applied to CVB with other spatial shapes, we multiplexed sixteen Vector Bessel Beams (VBB). While previous methods have used SLMs to generate single VBB [160, 161], here we show the multiplexing of sixteen VBB simultaneously. To generate these modes, we can substitute the amplitude terms  $\Psi_R^{-\ell}$  and  $\Psi_L^\ell$  of Eqn. (3.33) (Chapter 3) by  $J_{-\ell}(k_t\rho)$  and  $J_\ell(k_t\rho)$ , respectively. Here,  $J_\ell(x)$  is the Bessel functions of the first kind and  $k_t$  is the transverse component of the wave vector. Bessel beams of any order can be easily generated by encoding a digital axicon on an SLM[106]. To generate multiple VBB simultaneously, we proceed in an analogous way as before by first generating two sets of scalar Bessel beams,  $J_{\pm\ell}(k_t\rho)$ . Experimentally, we can not generate a pure Bessel beam, as it will carry infinite energy. Instead, we generate a Bessel-Gaussian beam, by multiplying the term  $J_{-\ell}(k_t\rho)$  by a Gaussian envelope  $\exp(-\rho^2/\omega_0^2)$ , where  $\omega_0$  is the beam width.

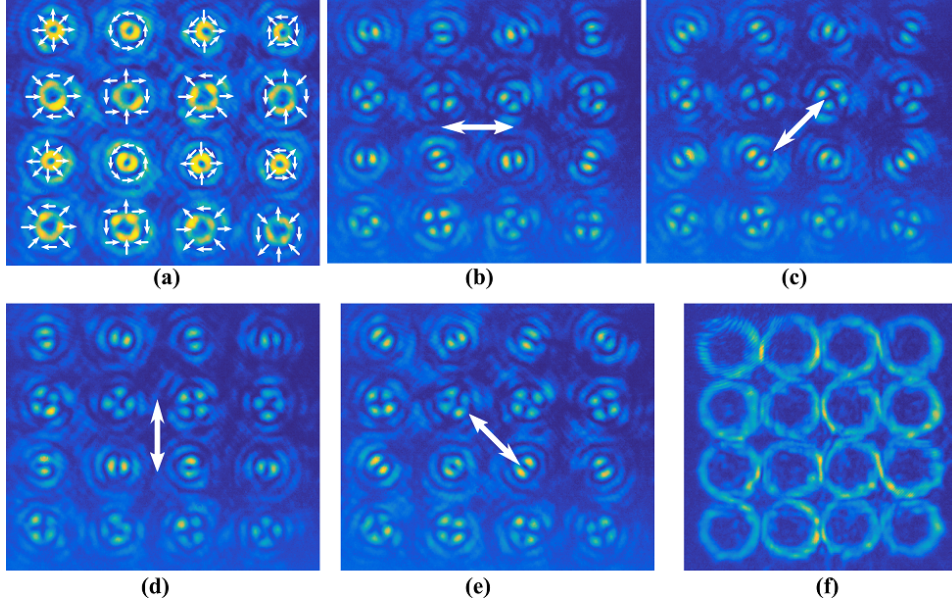


Figure 6.4: (a) Experimental intensity profiles of sixteen multiplexed vector Bessel beams with eight orthogonal polarization given by  $\ell = \pm 1$  and  $\pm 2$ . Corresponding intensity profile when an analyzer at (b)  $0^\circ$ , (c)  $45^\circ$ , (d)  $90^\circ$  and (e)  $145^\circ$  is placed before the CCD camera. (f) Far field intensity profile of Bessel beams.

Figure 6.4(a) shows a set of sixteen multiplexed vector Bessel modes with  $\ell = \pm 1$ . The corresponding intensity profile for the analyzer placed at  $\varphi = 0^\circ$ ,  $\varphi = 45^\circ$ ,  $\varphi = 90^\circ$  and  $\varphi = 135^\circ$  are shown in 6.4(b), 6.4(c), 6.4(d) and 6.4(e), respectively. Moreover, as it is well-known, the intensity profile of a Bessel beam forms a delta ring in the far field. This provides with a straight forward way to confirm that in fact, we are generating Bessel beams, as illustrated in Fig. 6.4(f).

## 6.6 Conclusion

In this Chapter we presented a novel method to generate and multiplex vector modes using a single SLM. This method is purely digital and is based on the multiplexing concept (discussed in Chapter 3) implemented with a single hologram displayed on an SLM. Multiple holograms with unique grating frequencies are superimposed to form a single hologram such that the multiplexed beams are propagated along two separate paths before recombination by interferometric means generating the desired VVB. The digital implementation of our technique provided on demand full control of each beam's amplitude weighting, spatial shape and phase, enabling the generation of arbitrary vector beams on the higher order Poincaré sphere. We

showed the on demand simultaneous creation of 16 VVB on the HOPS, however our method allows the generation of many more and is not limited to vector vortex beams as was demonstrated with vector Bessel beams. Finally, it should also be considered that any beam shaping method based on a multiplexing approach suffers from an intensity decrease after the addition of each hologram, which is proportional to the number of multiplexed beams. Hence, the maximum number of beams that can be multiplexed on a single hologram depend among other things on the specific device as was discussed in Chapter 5.

## Chapter 7

# Holographic optical trapping with structured light fields

### 7.1 Introduction

Optical manipulation studies continue to be topical and have gained momentum over the years particularly the application of structured light fields to manipulate micron sized objects [6]. These structured light fields can be created by several different methods as was discussed previously. In this Chapter we demonstrate optical trapping and tweezing using some of the customized light fields presented in the previous Chapters. We begin with a simply experiment (section 7.1.1) where we test our holographic optical trap by creating multiple traps using laser beam multiplexing.

Secondly, a holistic classical and quantum toolkit to analyse the dynamics of vector flat-top beams during propagation is presented. For the classical case, spatially resolved Stokes measurements (presented in Chapter 2) were performed to determine the polarization structure of the vector flat-top beam. With respect to the quantum case, we employ the vector quality factor (VQF) (detailed explanation in the appendix) which gives a quantitative measure of the vector nature of our beam. We then describe the experimental realisation of these shape invariant vector flat-top beams and employ our toolkit to characterise and analyse our beam. Thereafter we demonstrate the versatility of these beams in an optical trapping and tweezing application.

Finally, based on the novel experimental technique for realizing multiple arbitrary vector light fields presented in Chapter 6, we present a new tool in which we

employ vector beams in optical tweezers to form a vector holographic optical trap (HOT). As a proof of principle, we present a vector HOT with arrays of digitally controlled Higher-Order Poincaré Sphere (HOPS) beams. Our digital holographic approach allows independent manipulation of each vector beam such that arbitrary HOPS states can be attained. Following the generation technique presented in Chapter 6, we direct multiple vector beams into an optical tweezers system. A classical analysis of the created array of vector beams to determine each beam's polarization state is presented. We demonstrate trapping and tweezing with customized arrays of HOPS beams comprising scalar and cylindrical vector beams, including radially and azimuthally polarized beams simultaneously in the same trap. As an immediate application we demonstrate simultaneous real-time calibration with linear, radial and azimuthal states allowing real-time comparison of the individual beam's transverse trapping strengths. Similar to the generation method presented in Chapter 6, this approach is general enough to allow the realisation of arbitrary vector beams in the trapping plane and can further be implemented with DMDs which have fast refresh rates [162].

### 7.1.1 Holographic multiplexing in optical tweezers

Structured light fields have been employed in optical trapping and tweezing to demonstrate multiple beam trapping and manipulation [28, 43]. To test our holographic optical tweezers system we generated multiple optical traps using an SLM, as shown in Fig. (7.1). By multiplexing a set of 9 holograms we created 9 beams from a single input beam incident onto the SLM forming 9 optical traps in the trapping well. The resultant 9 traps can be independently manipulated by altering the encoded hologram thereby moving each trapped particle ( $2\text{ }\mu\text{m}$  in size). We further show the dynamics of our holographic optical tweezers by rearranging the particles in such a way that the WITS logo is generated as shown in Fig. (7.1). During the process of re-arranging the beads, each beam was moved with a spatial step size of approximately half the bead diameter ( $1\mu\text{m}$ ) to ensure that beads do not escape from each trap. We observed that for a step size greater than  $4\mu\text{m}$  the beads will escape from the trap since at this distance the beads will be beyond the attractive force of the laser beam intensity. The process of re-arrangement process could be achieved at an optimum time of 135s and this was limited by the SLM's refresh rate.

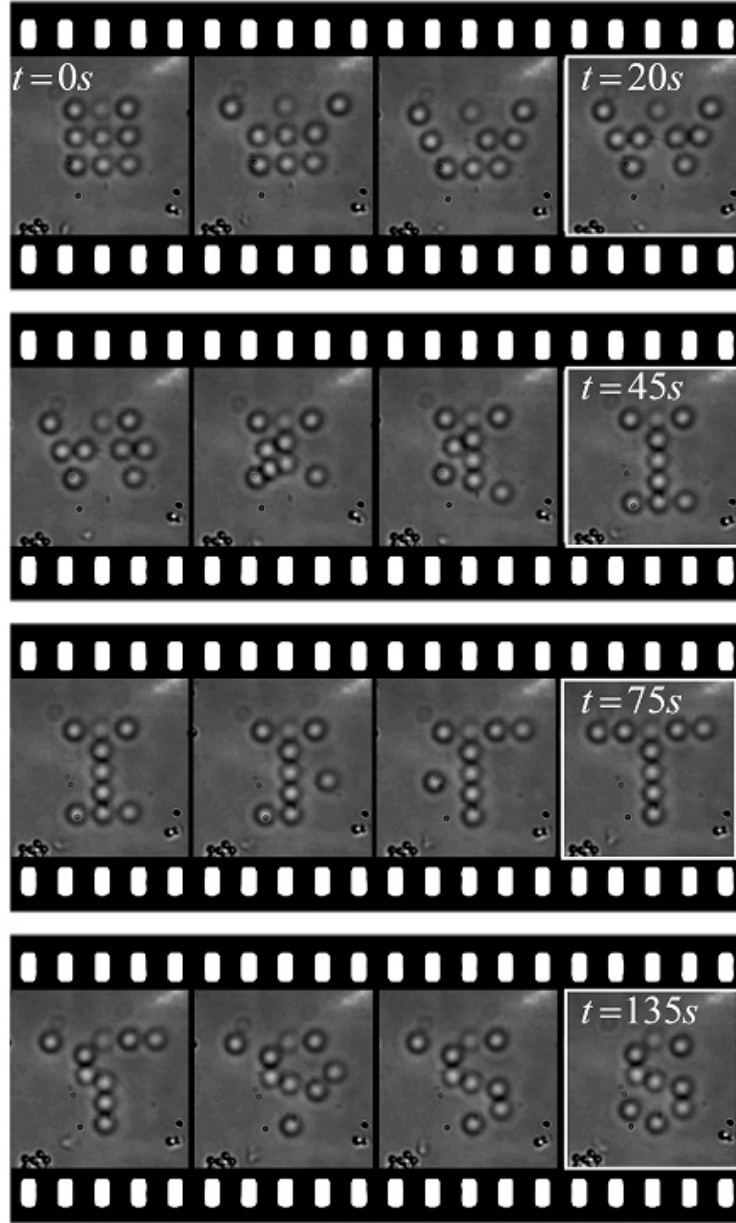


Figure 7.1: Experimental frames showing  $2\ \mu\text{m}$  silica beads initially trapped in a  $3 \times 3$  array at  $t = 0\text{s}$  and then holographically manipulated to spell each letter of the WITS logo. The complete process of arranging the particle to spell each letter took  $t=135\text{s}$ . In the first row, second column the trap holds two beads such that the bead is of lower contrast compared to the other traps with a single bead each.

## 7.2 Creation and analysis of propagation invariant flat-top beams

In this section we present the toolbox to analyse vector flat-top beams and as our primary novel step, we apply quantum measurements to understand the vector prop-

erties of these beams. The following section then describes the experimental generation of these propagation invariant vector flat-top beams

The usual analysis of vector beams makes use of the Stokes measurements to determine the degree of polarisation at each point on the beam [47]. We will later show examples of this measurement which we deem well known and therefore not required to be explained here in detail. Suffice to say that the usual polarisation projections allows one to calculate the Stokes parameters (discussed in Chapter 2) following

$$\begin{aligned} S_0 &= I_H + I_V, \\ S_1 &= I_H - I_V, \\ S_2 &= I_D - I_A, \\ S_3 &= I_L - I_R. \end{aligned} \tag{7.1}$$

Instead we introduce a quantitative measure of the vector nature of the beam, the vector quality factor [35], by employing quantum tools of non-separability to our vector beam [158]. This approach exploits the similarities between quantum entanglement and non-separable states of classical light [163, 164, 17, 165]. The VQF may be found by performing a state tomography of the vector beam at any location, and thus provides a quantitative measure of how the vector nature of the beam changes during propagation. The approach requires inner product measurements on each polarisation component of the field in order to reconstruct the density matrix, from which the non-separability of the beam, i.e., its vector quality factor (VQF), can be found from

$$\text{VQF} = \text{Re} \left( \sqrt{1 - \sum_{i=1}^3 X_i^2} \right), \tag{7.2}$$

where the  $X_i$  parameters can be calculated from twelve projective measurements comprising six spatial intensities for each polarisation state, given by [35]

$$\begin{aligned} X_1 &= (I_{13} + I_{23}) - (I_{15} + I_{25}), \\ X_2 &= (I_{14} + I_{24}) - (I_{16} + I_{26}), \\ X_3 &= (I_{11} + I_{21}) - (I_{12} + I_{22}). \end{aligned} \tag{7.3}$$

with the  $I_{ij}$  illustrated graphically in Fig. (7.2). The VQF varies from 0 for scalar beams to 1 for vector beams. For the case of Eqn. (3.32) presented in Chapter 3,

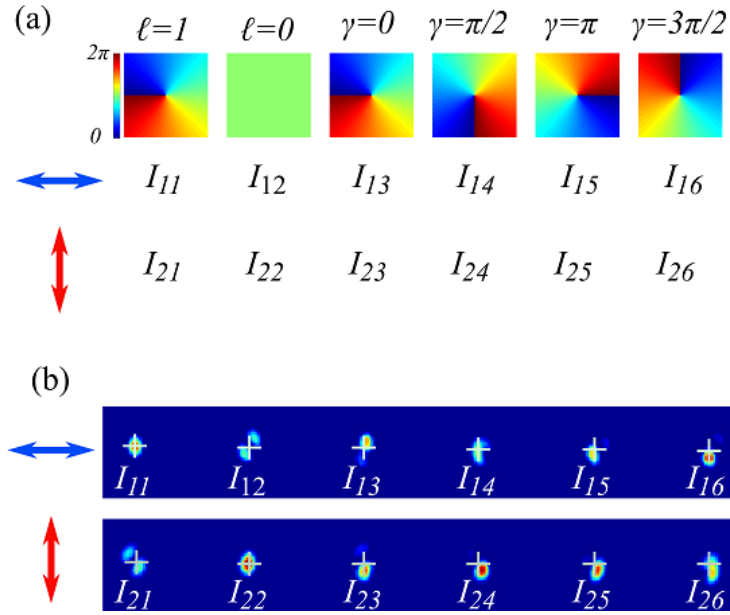


Figure 7.2: (a) Illustration of the twelve projective measurements needed to calculate the VQF: six spatial projections (shown as holograms) for each of two polarization states. An example of an actual measurement is shown in (b). Experimental data where only the on-axis intensity (measured at the white crosshairs) is used to determine the desired parameters  $I_{ij}$  from the inner product of the projections on SLM2 and the incoming beam using lens L4, as shown in Fig. 7.3

the calculated VQF is  $\text{VQF} = 2|\sqrt{\alpha(1-\alpha)}|$ ; for the desired flat-top beam when  $\alpha = 0.5$  we expect a perfect vector state with  $\text{VQF} = 1$  throughout its propagation in free-space.

### 7.2.1 Generation of vector flat-top beams

The experimental setup used to create our vector flat-top beams is shown in Fig. 7.3. Our vector flat-top beam was generated by illuminating a vortex hologram (an example is given in the inset of Fig. 7.3) with an expanded Gaussian laser beam ( $\lambda = 532$  nm). The Gaussian beam was expanded passed through a combination of wave plates and a polariser to control its linear state of polarisation, and then directed onto the liquid crystal display (LCD) of a SLM (HoloEye, PLUTO-VIS, with  $1920 \times 1080$  pixels of pitch  $8\mu\text{m}$  and calibrated for a  $2\pi$  phase shift at 532 nm) labeled SLM1. The LCD was addressed with a hologram of an azimuthal phase of topological charge  $\ell = 1$  without any grating.

The energy into the first diffracted order is a function of the incoming beam's

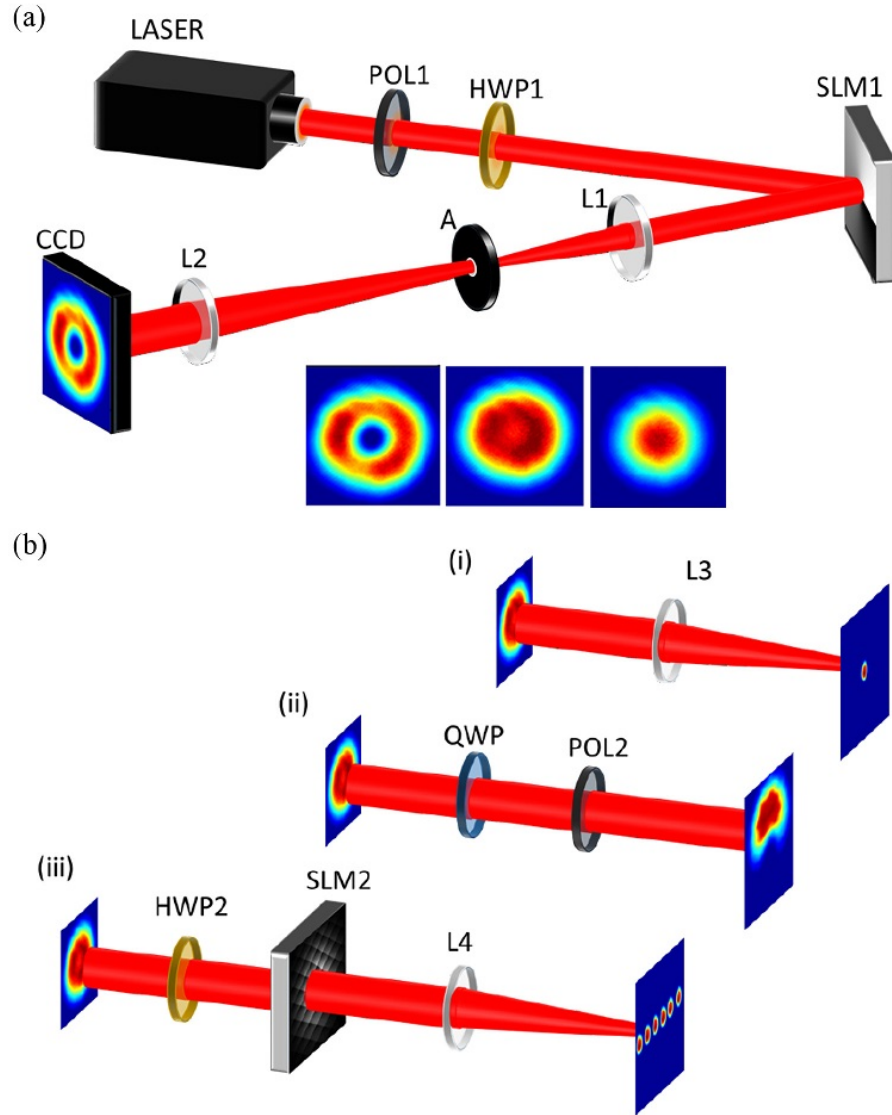


Figure 7.3: (a) Experimental set-up to create, propagate and analyse vector flat-top beams. A spatial light modulator (SLM1) is used to modulate the incident beam with the desired mode being created after some spatial filtering using a pin-hole (A) between two lenses which relay the plane of the SLM to the detector. Insets show evolution from vortex to Gaussian with the flat-top as the intermediate state when the incident polarization is rotated from horizontal to vertical. (b) The characterisation comprised three approaches: (i) a propagation test with the aid of lens L3, (ii) a Stokes analysis with the QWP and POL2, and (iii) a quantum inspired measurement to obtain the vector quality factor, using polarisation and spatial mode projections with HWP2 and SLM2.

polarisation and so usually the incoming polarisation is set to one orientation (horizontal in our experiment). Ideally, there is no diffraction for the vertical polarisation component and 100% for the horizontal due to the birefringence of the liquid crystals themselves [16]. This fact was exploited to control  $\alpha$  in our superposition state (the weighting of the two modes) by controlling the incoming beam's polarisation. In fact SLMs are not 100% efficient but even this can be compensated for by simple adjustment of the polariser's angle. The output beam was then a superposition of the horizontally polarised (diffracted) vortex beam and the vertically polarised (undiffracted) Gaussian beam. The result of this polarisation control for vector mode

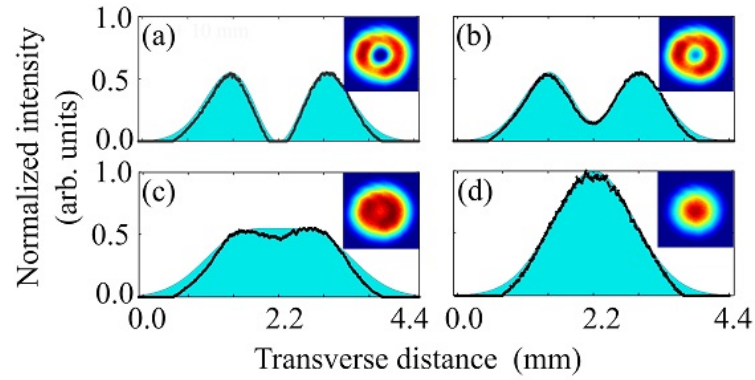


Figure 7.4: Measured intensity profiles as the polarisation state of the initial beam was altered, showing the expected evolution from (a) vortex to (d) Gaussian with a flat-top beam as an intermediate superposition state shown in (c). The experimentally measured cross-sections (black data points) are shown with the theoretical profiles in purple, with the insets showing measured 2D intensities.

control is shown in Fig.( 7.4), where it is evident that indeed we are able to create a good approximation to a flat-top beam. On close inspection, one sees that while the beams are rotationally symmetric there is a slight intensity variation, particularly in the vortex beams. This is because even small offsets in the vortex center, or any minor misalignment, will result in intensity variations about the ring of light. In our case, this is negligible as seen by the quality of the flat-top beam. Note that in the experimental set-up we employed an aperture. The reason for this, which is not highlighted in prior flat-top studies, is that contrary to theory the programming of a vortex pattern on the SLM does not give rise to the desired  $LG_{01}$  mode but instead a mode of the form  $LG_{00} \exp(i\ell\phi)$ . The latter is missing the required amplitude term of the  $LG_{01}$  mode and is referred to as a hypergeometric beam [166] which may be expressed as a superposition of many Laguerre-Gaussian modes. We have recently

shown that the impact of this is the emergence of many radial modes in addition to the desired  $p = 0$  mode and hence a reduced energy into the  $LG_{01}$  mode structure [167]. We illustrate the effect this has in Figs. (7.5 (a)) and (7.5 (b)). We see that the measured modes have an unwanted radial structure, which can be removed to produce the desired mode structure shown in the insets.

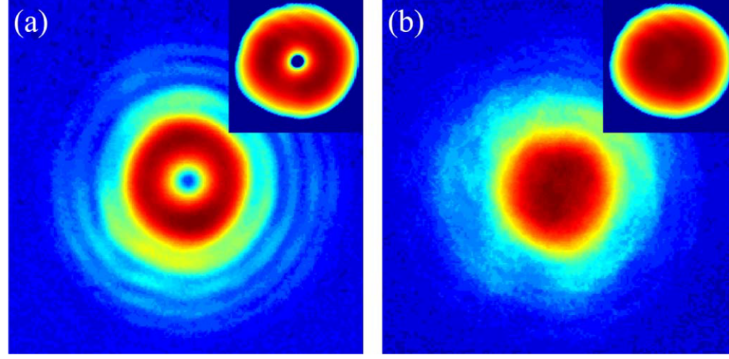


Figure 7.5: (a) Vortex beam with many rings, indicative of several radial modes. (b) The superposition of the many radial modes results in a flat-top beam with an evident halo. By employing a spatial filter, the radial modes can be removed, producing the desired beam shapes shown in the insets.

We therefore employ a spatial filter to remove the unwanted  $p$  modes producing the desired mode after the filter, albeit with less energy. This is possible because the mode sizes follow the relation  $\omega_{p\ell} = \sqrt{2p + \ell + 1}\omega_0$ , where  $\omega_0$  is the Gaussian beam width. This implies that modes of  $p > 0$  for a given  $\ell$  can easily be filtered by judicious selection of the aperture radius, since they are all bigger than the  $p = 0$  structure. In our scheme this energy reduction can be compensated for by adjusting the angle of the polariser to bring  $\alpha$  back to the desired weighting.

After the required filtering to produce the desired beam, the vector state of this beam was analysed by a sequence of Stokes measurements and VQF measurements, with the results for the Stokes measurements shown in Fig.( 7.6). The experimental results agree excellently with theoretical predictions. The VQF measurements likewise agree, with  $VQF = 0.97$  (experiment) compared to  $VQF = 1$  (theory), both are unchanged during propagation [an example measurement is shown in Fig. (7.2 (b))]. Next we tested the propagation of the vector flat-top beam by passing the beam through a focusing lens. The result is shown in Fig. (7.9), confirming the propagation dynamics of the beam. We confirm that the beam remains shape invariant during propagation from the near to the far field (see Fig. (7.7)). From a

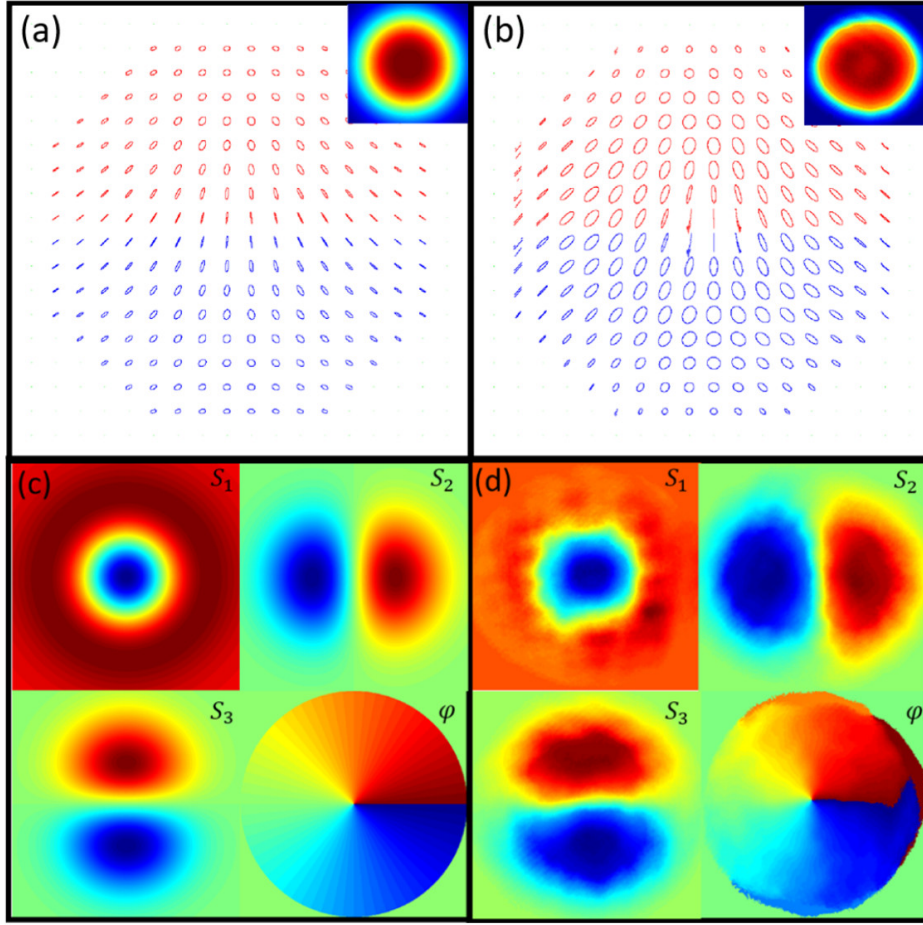


Figure 7.6: (a) Calculated and (b) measured polarisation state of the beam via a Stokes projection with (c) and (d) showing the theoretical and measured Stokes parameters and extracted inter-modal phase (our azimuthal phase). The slight differences in the experimental results from the theory can be attributed to imperfections in the experimental setup as the beam propagates through the optics.

beam propagation analysis (as shown in Fig. 7.8), we find an experimental  $M^2 = 1.42$ , which is in very good agreement (within 5%) with the theoretical value of  $M^2 = 1.5$ . The cross-sections of the beam at each plane are shown in Fig.(7.9).

### 7.2.2 Vector flat-top optical trap

An optical trap was constructed to compare the trapping ability of the Gaussian and flat-top beams. Both beams were passed through an objective lens of magnification  $100\times$  and focused into a trapping well of a depth of  $400\text{ }\mu\text{m}$  that was filled with glass beads of a diameter of  $2\text{ }\mu\text{m}$  that were diluted in water. The resulting images of the trapped beads were captured by an inverted vertical white light illumination

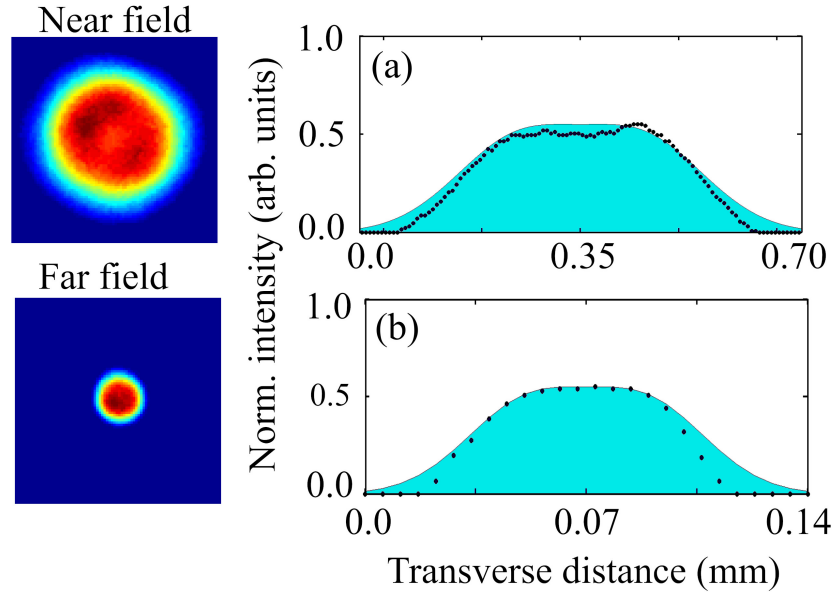


Figure 7.7: Experimental confirmation that the vector flat-top beam remains shape invariant both in the (a) near field and (b) the far field. The experimentally measured cross-sections (black data points) are shown with the theoretical profiles in purple, with the left panels showing measured 2D intensities.

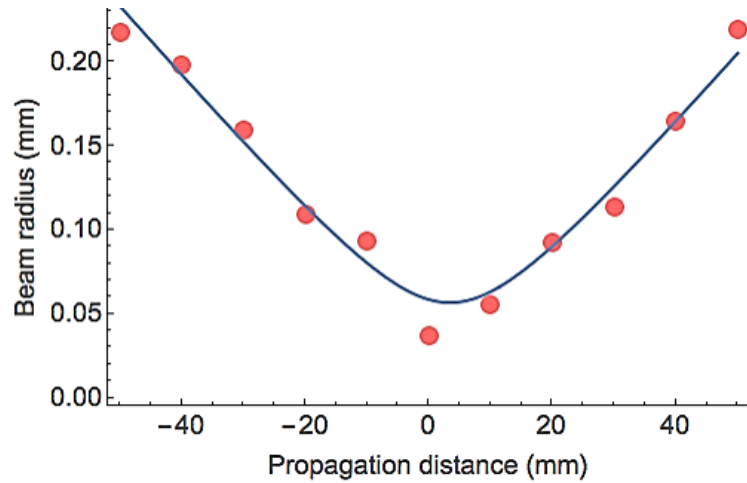


Figure 7.8: Propagation analysis of the vector flat-top beam. The beam radius was calculated using a second moment approach, shown as the data points, and plotted with the probable propagation trajectory (solid curve).

system with the CCD detector below the trap. With this arrangement, we were able to demonstrate successful trapping with both beams, an example of which is shown in Fig. 7.10 for the case of the flat-top beam. To quantitatively compare the two beams, we performed a drag force calibration test. This involved establishing a controlled velocity of the medium with a piezo driven translation stage. The

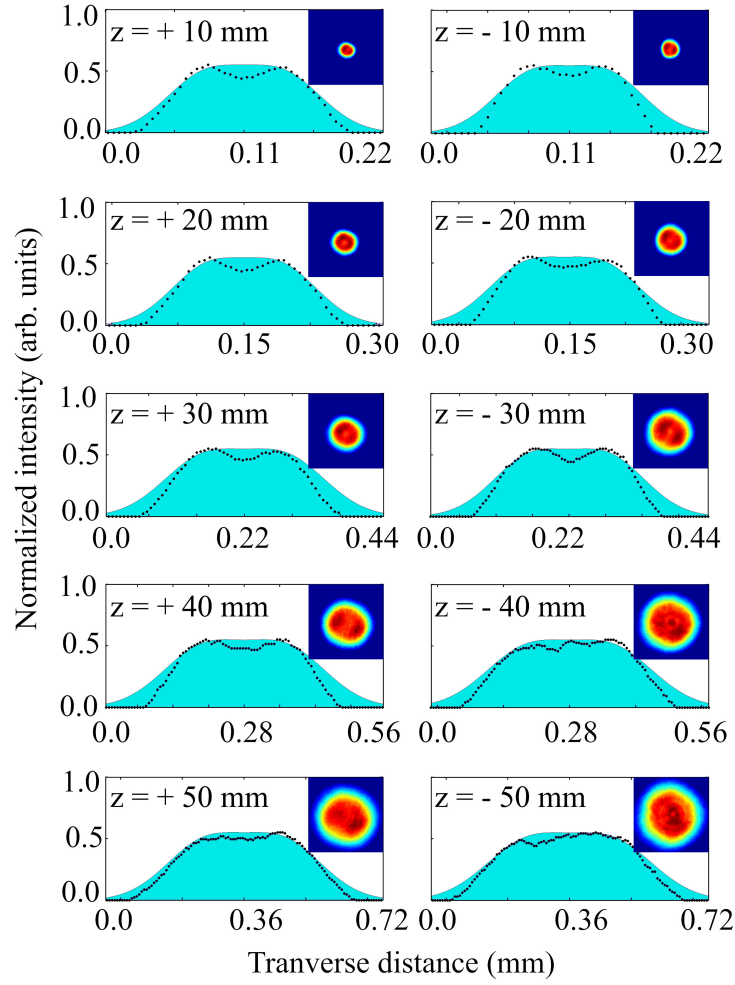


Figure 7.9: Experimental confirmation that the vector flat-top beam remains shape invariant during propagation. The experimentally measured cross-sections (black data points) are shown with the theoretical profiles in purple, with the insets showing measured 2D intensities.

resulting escape velocity of the bead could be recorded for a given delivered power level and used to determine the drag force from which the trapping strength could easily be inferred. The results of this are shown in Fig. 7.11. Here, we see that the trapping strength of the two cases are similar. This is understandable: while the flat-top beam has a larger gradient in intensity, it has a lower peak value, resulting in a comparable force to that of the Gaussian case. This may be advantageous in trapping delicate structures, e.g., living cells, since the required trapping strength is achieved at lower peak intensity, thus reducing the likelihood of thermal damage.

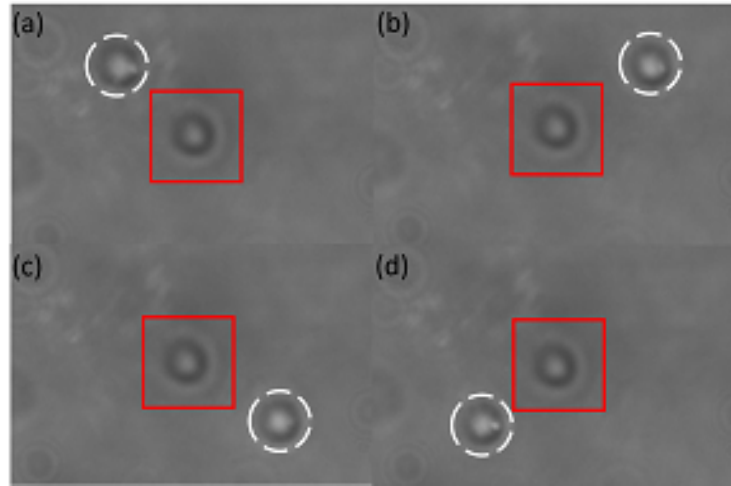


Figure 7.10: We illustrate with example frames the use of a vector flat-top beam to trap micrometer sized glass beads. The frames show the movement of a trapped bead (white dashed circle) in a squared trajectory around an untrapped bead (red square) stuck onto the glass slide.

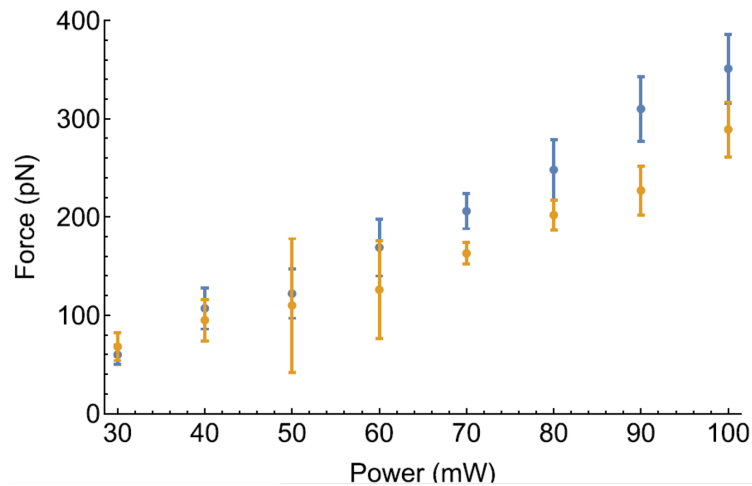


Figure 7.11: Here, we show the results of a drag force calibration test, showing that the Gaussian beam (blue) performs only marginally better than the flat-top beam (orange). Each data point is an average of six repeats. The similarity in the plots is expected given that the gradient of the edge in the flat-top beam is higher, but the peak intensity is lower, hence, negating the benefit.

### 7.2.3 Discussion

The polarisation structure of the beams investigated here can be altered to the left/right basis from the horizontal/vertical; one then immediately recognises that they have been identified and characterised as exotic morphologies [168–170], having

found many applications in the field of singular optics [171]. Recently it has even been suggested that such beams may offer some resilience to turbulence [172]. Should this be true, the propagation invariance and turbulence invariance would open new avenues to explore. An as yet unexplored field would be that of the preservation of entangled states with this superposition: if there is no modal coupling in the noisy channel (e.g., turbulence) as is the case in free-space, then the non-separability would likewise remain invariant through the quantum channel.

Here we have considered the use of these beams in free-space and under paraxial conditions with focusing. For completeness we point out that under very tight focusing one would expect the vector state to change when the conditions become non-paraxial [173]. This effect could likewise do with further study.

To summarize, we have presented a simple method to generate vector flat-top beams. Our experimentally generated beam presented a good approximation to the analytical solution as confirmed by the  $M^2$  values of 1.42 (experimental) and 1.5 (theory). A modern approach to investigate the properties of our created vector flat-top beam by characterizing the propagation dynamics, polarization and vector state was presented. From our results we showed that one can tailor the flat-top beam to be shape invariant during propagation. Furthermore, we introduced a holistic and new approach that utilises classical and quantum concepts to analyse such beams. Using this tool kit we confirmed invariance in the polarization and vector nature of the vector flat-top beams. Finally, we demonstrate the versatility of our created vector flat-top in an optical trapping and tweezing application however can also be useful in the field of material processing. The presented analysis tool kit will be invaluable for characterising and investigating the classical and quantum like properties of such structured light fields which have been found useful in numerous laser based applications.

### 7.3 A vector HOT with multiple HOPS beams

A schematic representation of the experimental setup used to generate arrays of optical traps with multiple vector beams is illustrated in Fig. 7.12. In our implementation, we used a horizontally polarized laser beam ( $\lambda = 532$  nm, Coherent Verdi G) incident on a reflective SLM (Holoeye Pluto) aligned to modulate horizontal polarization for multiple vector beams generation. The inclusion of a telescope (lenses L1 and L2 in the figure;  $f1 = 500$  mm,  $f2 = 1000$  mm) is a crucial step

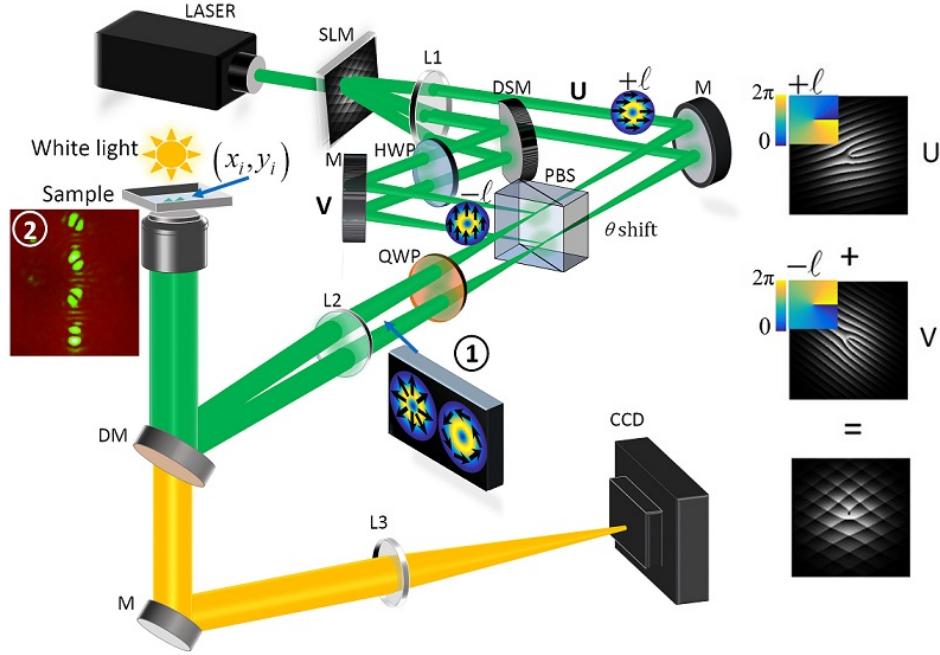


Figure 7.12: Schematic illustration of the experimental set-up used to simultaneously create multiple HOPS beam optical traps. An example multiplexed hologram pair used to generate two scalar beams traversing along two separate paths is shown in the insert on the right. Insert 1 shows example HOPS beams with radial and azimuthal polarization distributions. The generated vector beams were simultaneously directed into the microscope objective to create multiple optical traps. An inverted microscope comprising white light source, dichroic mirror, lens and CCD camera was used to image the trapped beads. Insert 2 shows an experimental image of a vertical array of vector beams in the trapping plane as observed on the CCD through back reflection off the sample container.

required to send the HOPS beam into the optical trap. On one hand, by placing the lenses following a 4f configuration, it images the SLM plane at the front focal plane of the microscope objective (100 $\times$  Nikon oil immersion, N100X-PFO, NA 1.3). By doing this, unique propagation angles (as programmed on the SLM) will translate into unique  $(x_i, y_i)$  positions in the trapping plane: beams from the SLM propagate at differing angles which are mapped to positions with L1. The set-up converges the light at the PBS for interferometric recombination to produce the desired vector beams. To recreate the position control in the trap, L2 maps position to angle such that each vector beam incident at the objective will be focused to a unique posi-

tion. On the other hand, focal length of the lenses of the telescope were chosen to ensure a  $2x$  magnification to expand the beams and fill the entrance pupil of the microscope objective, ensuring tight focusing, then providing the necessary intensity gradient for trapping. A half wave plate (HWP) was introduced along path **V** to rotate the polarization from horizontal to vertical such that coaxial superposition of orthogonal polarization states was attained. The introduction of a HWP results in an optical path difference and potentially undesired phase delays which may cause a rotation in the polarization states of the generated vector beams. Since our created vector beams were cylindrical symmetric, the rotational effect cannot be observed. Nevertheless, to keep the system general, the optical path difference was corrected for by performing translation adjustments of the mirror in path **U**, countering the unwanted phase shift. The objective focused the beams into a sample container of depth  $400\text{ }\mu\text{m}$  holding  $2\text{ }\mu\text{m}$  diameter silica beads diluted in deionized water. An inverted microscope setup with an illuminating white light-emitting diode and a CCD camera was used to image the trapped beads. A dichroic mirror (DM) was used to reflect the vector beams into the objective while allowing white light from the light emitting diode to pass through to the CCD camera.

### 7.3.1 Arrays of HOPS beams

Using our approach, we created a square array of  $3 \times 3$  HOPS beams with topological charges  $\ell = 1$  and  $\ell = 2$  for 9 arbitrarily chosen polarization structures, and delivered these through the system. The intensity of the experimentally generated array of HOPS beams is shown in Fig. 7.13 (a) while a single shot experimental image of the polarization map determined using Stokes measurement is presented in Fig. 7.14. Intensity differences between vector beams in the bottom row in comparison to those at above rows arise from the distinct deflection angles out of the SLM, such that the more intense are less deflected from the undiffracted zero order. Importantly, by adjusting the phase modulation depth on the encoded hologram, equal intensity distributions can be easily attained. All 9 beams were delivered to the desired locations  $(x_i, y_i)$  in the trapping plane, here designed to be in a square array. Table 7.1 contains the values of the beam positions at the trapping plane as observed on the image, their corresponding grating frequencies  $(x_u, y_u)$  and  $(x_v, y_v)$  for paths **U** and **V**, respectively, and the calculated angles relative to the undiffracted zero order beam as determined according to Eqn. 3.35 in Chapter 4. These values

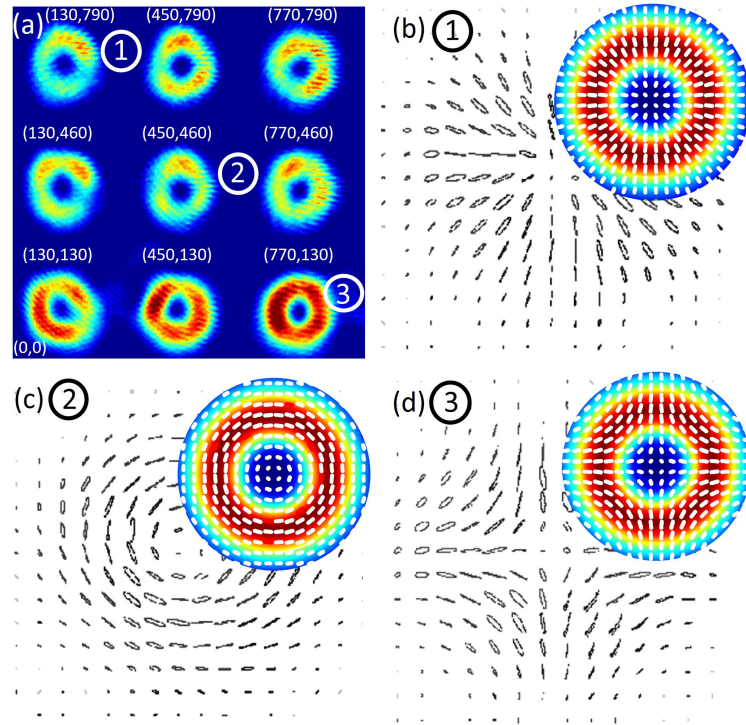


Figure 7.13: (a) Experimental image of a  $2\text{D}$  array of  $3 \times 3$  HOPS beams (topological charges of  $\ell = 1$  and  $\ell = 2$ ) at the desired locations in the  $(x, y)$  plane. (b)-(d) Shows experimentally determined polarization maps of beams **1**, **2** and **3**, confirming the vector nature of the created beams. The inserts are theoretically simulated intensity and polarization distributions of the beams.

clearly show that by modifying grating frequencies in the hologram, we can chose the arrangement of the vector beams at will. To determine the quality of each HOPS beam, we performed a spatially resolved Stokes measurement [174, 47]. Figure 7.13 (b)-(d) shows the measured polarization distributions of beams **1**, **2** and **3** of the 9 multiplexed vector beams. The inserts show the corresponding theoretically generated intensity and polarization distributions, which are in good agreement with the experimental results confirming the creation of vector beams. Because the beams are created in a  $2\text{D}$  array, the measurements could be performed simultaneously on all 9 beams.

### 7.3.2 2D Optical traps with arrays of HOPS beams

In previous sections we described how to create arrays of reconfigurable HOPS beams; now we wish to employ this in a vector HOT. To this end, we projected

| Position $(x_i, y_i)$ (pixels) | $\theta$ (rad) | $x_u$ | $y_u$ | $x_v$ | $y_v$ |
|--------------------------------|----------------|-------|-------|-------|-------|
| (130,790)                      | 0.008          | -0.05 | 0.15  | 0.21  | 0.15  |
| (450,790)                      | 0.008          | -0.08 | 0.15  | 0.18  | 0.15  |
| (770,790)                      | 0.008          | -0.11 | 0.15  | 0.15  | 0.15  |
| (130,460)                      | 0.005          | -0.05 | 0.10  | 0.21  | 0.10  |
| (450,460)                      | 0.005          | -0.08 | 0.10  | 0.18  | 0.10  |
| (770,460)                      | 0.005          | -0.11 | 0.10  | 0.15  | 0.10  |
| (130,130)                      | 0.001          | -0.05 | 0.05  | 0.21  | 0.05  |
| (450,130)                      | 0.001          | -0.08 | 0.05  | 0.18  | 0.05  |
| (770,130)                      | 0.001          | -0.11 | 0.05  | 0.15  | 0.05  |

Table 7.1: Positions, angles and grating frequencies for the created  $3 \times 3$  square array of HOPS beams .

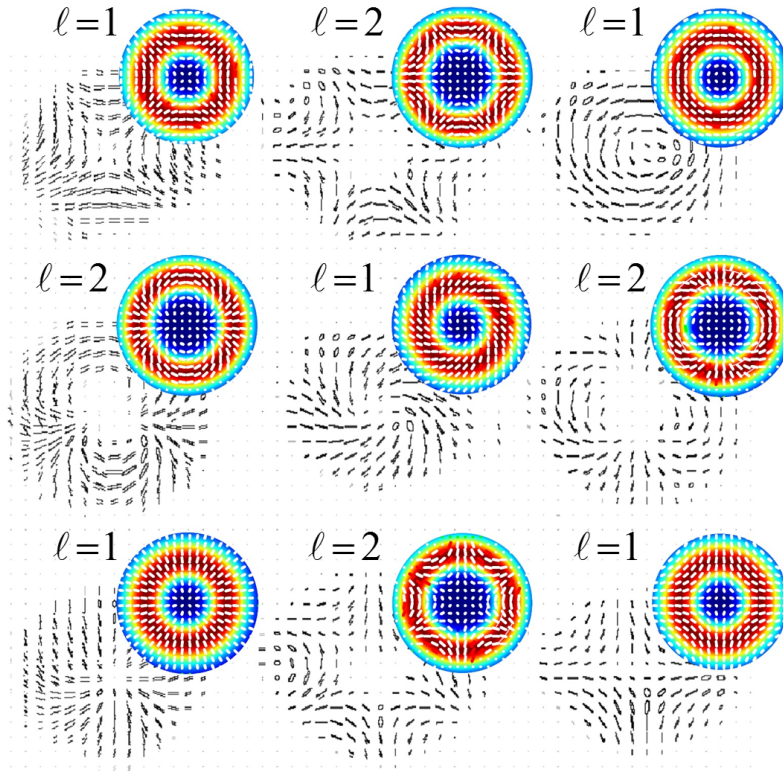


Figure 7.14: A single shot experimental image of the polarization maps of a multiplexed 2D array of  $3 \times 3$  HOPS beams (topological charges of  $\ell = 1$  and  $\ell = 2$ ). The insets are theoretical simulations of the expected polarization distribution.

an array of created vector beams into the entrance pupil of the objective with each HOPS beam at an appropriate incident angle so as to produce the desired 2D array

of traps. By way of example, we trapped beads in a triangular array, illustrated in Fig. 7.15 (a) and shown experimentally in (b). Figure 7.15(b) shows six particles of diameter  $\approx 2 \mu\text{m}$ , trapped with  $\approx 10 \text{ mW}$  of average power arranged in a triangular configuration. Since our approach allows the independent creation of any arbitrary polarization state on the HOPS, we demonstrated multiple optical traps with six arbitrary positions on the HOPS, as shown in Fig. 7.15 (c). Here we also make clear that the vector HOT can also reproduce traditional scalar HOTs: two of the six beams in the triangular array are in fact scalar OAM beams from the poles of the HOPS. All beam insets in Fig. 7.15 (b) are experimentally measured intensity profiles, and insets in (c) show the intensity after a linear polarizer, as explained in the caption.

### 7.3.3 Optical forces in our vector HOT

Finally, as a proof-of-concept application of our technique, we trap three beads simultaneously in a linear array and performed a drag force test. We create HOPS beams that are radially, azimuthally and linearly polarized at the plane of the trap, as shown in Fig. 7.16. Such a configuration allows for the direct comparison of the trap stiffness for each beam. First, we corroborated that the trapping strength of each beam is position independent. For this, we interchange beam **1** (azimuthal) with beams **2** (linearly) and **3** (azimuthal). That is, from position  $x$  to position  $y$  and  $z$ . We find that the outcome was independent of position. Radially polarized beams exert the weakest trapping force, as compared to azimuthally and linearly polarized beams which remain trapped for higher speeds of  $\sim 55 \mu\text{m/s}$  and  $64 \mu\text{m/s}$ , respectively. Some frames of this test are shown in Fig. 7.16 and the associated media file.

We measured the lateral trapping strength of each beam, with the results shown in Table 7.2. From the results we observe that the linearly polarized scalar beam has the strongest transverse trapping force, with the radially polarized beam the least. These results are in agreement with previous reports [125, 126, 123].

### 7.3.4 Discussion

The approach we have outlined allows arbitrary arrays of vector beams to be created and delivered to a trap, resulting in the demonstration of a vector HOT with multiple beams. Importantly, as we have shown, the vector HOT can be reduced

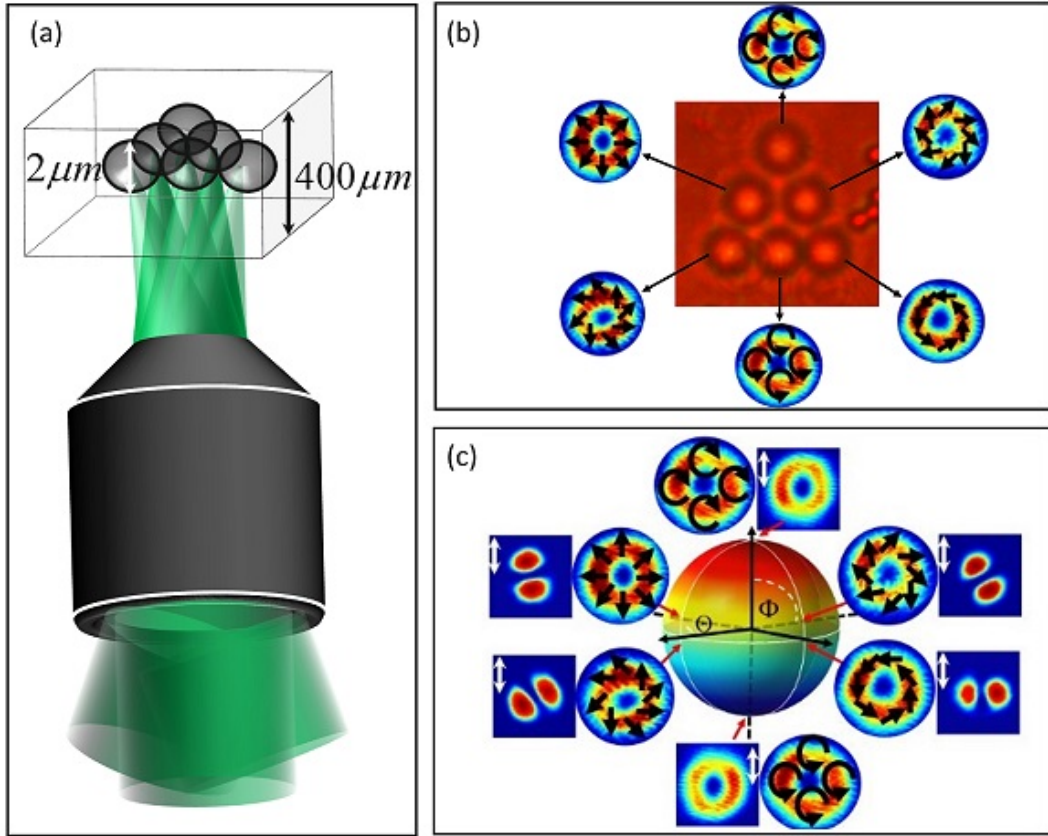


Figure 7.15: (a) Schematic of our vector HOT, with  $2\ \mu\text{m}$  silica beads to be trapped in a triangular pattern inside a trapping well of height  $400\ \mu\text{m}$ . (b) Experimental image of the six trapped beads with the corresponding measured intensity of each trapping HOPS beam, overlaid with the respective polarization structure. The HOPS beams can be geometrically represented on the HOPS as shown in (c). Insets in (c) show the measured intensity profiles, as well as those after projection through a linear polarizer set to vertical transmission, confirming the state of each mode. Note that here we have demonstrated simultaneous trapping with vector and scalar OAM beams with the same HOT.

to a traditional scalar HOT by simply switching one hologram off, and is versatile enough to allow both scalar and vector beams to be used in the same trap. As with traditional HOT, the hologram efficiencies must be adjusted to ensure the power delivered to each trap is as desired. In our experiments, we paid particular attention to this fact, particularly in the trapping stiffness tests, to ensure that only the polarization structures differed and that each beam has the same power at the trap. Related to this is the importance of due care with choice of optics in the delivery system in order to maintain the desired polarization structure of the beams in the

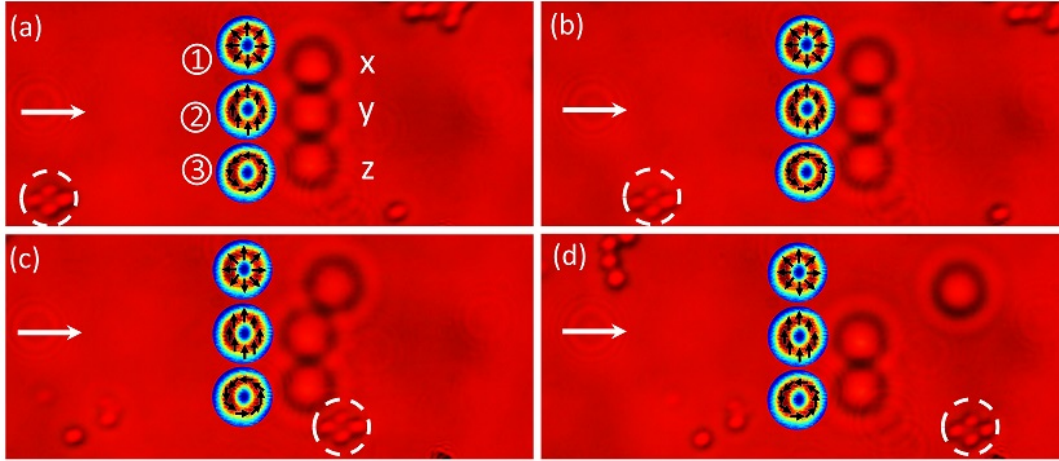


Figure 7.16: Real-time calibration of multiple vector beam traps consisting of radial, linear and azimuthal polarization states. At some set stage velocity the weakest radially polarized trap (bead 1) results in the bead escaping while the beads in the stronger traps (beads 2 and 3) remain trapped. Arrows show the direction of motion of the stage while the particle in the white dotted circle serves as a stationary reference.

vector HOT.

In this work, we have demonstrated the vector HOT using SLMs, HOPS beams and 2D arrays of traps. The SLM may be replaced with equivalent technology for faster refresh rates, e.g., digital micro-mirror devices (DMDs). Similarly, it should be noted that the HOPS beams are an example only and that any arbitrary vector beam can be created at each location in the HOT array. Moreover, by switching off one component of the vector field at the SLM, the well-known scalar holographic trap can be reproduced, and as we have shown, vector and scalar beams may be produced at the same time in the same trap array. Given the many applications of scalar holographic traps, it is reasonable to assume that this new vector holographic trap too will find many applications. For example, in the trapping of gold nanoparticles for plasmonic studies where vector beams have known benefits. Other applications are inducing optical chirality in form birefringent structures, exploiting known vector trapping efficiency to adjust trapping strengths, e.g., in biological matter control with the same modal conditions (power and intensity profile) but differing polarizations for differing trap strengths. This device can also be employed to execute fibre traps with vector modes, the natural modes of most fibres, and even

| Power ( $mW$ ) | Linear         | Azimuthal     | Radial         | Hybrid odd     | Hybrid even    |
|----------------|----------------|---------------|----------------|----------------|----------------|
| 40             | $2.7 \pm 0.4$  | $2.4 \pm 0.3$ | $1.8 \pm 0.3$  | $2.5 \pm 0.2$  | $2.2 \pm 0.3$  |
| 35             | $2.1 \pm 0.3$  | $1.6 \pm 0.5$ | $1.4 \pm 0.4$  | $1.7 \pm 0.5$  | $1.5 \pm 0.4$  |
| 30             | $1.6 \pm 0.2$  | $1.2 \pm 0.2$ | $1.1 \pm 0.1$  | $1.3 \pm 0.2$  | $1.2 \pm 0.1$  |
| 25             | $1.2 \pm 0.2$  | $1.0 \pm 0.2$ | $0.9 \pm 0.2$  | $1.1 \pm 0.1$  | $1.0 \pm 0.2$  |
| 20             | $0.8 \pm 0.2$  | $0.6 \pm 0.1$ | $0.5 \pm 0.1$  | $0.7 \pm 0.1$  | $0.5 \pm 0.1$  |
| 15             | $0.5 \pm 0.03$ | $0.4 \pm 0.1$ | $0.4 \pm 0.04$ | $0.5 \pm 0.05$ | $0.4 \pm 0.04$ |

Table 7.2: Transverse trapping stiffness ( $pN/\mu m$ ) as a function of laser power ( $mW$ ).

in fundamental studies of light-matter interaction to compare phase singularities with polarization singularities (scalar OAM being an example of a phase singularity with the vector HOPS having a polarization singularity). Finally, by following the advances of scalar HOT, it will be possible to adjust the trapping plane of each vector beam by adding individual digital lenses to each beam [16] for the realization of 3D traps [175]. We believe that since our system can produce both vector and scalar beams holographically at the same time, the addition of the polarization degree of freedom through vector beams will open new applications to follow.

To summarize, we introduced and demonstrated a new tool to manipulate micron sized particles in any 2D arrays using HOPS beams. As examples we showed vector beams arranged in square, triangular and linear arrays. External characterization of the created arrays of vector beams to determine their polarization state was presented. Optical trapping using multiple vector beams was demonstrated in a which the trapped particles were arranged in a triangular and linear array. We showed independent digital control of each vector beam by manipulating the encoded hologram on the SLM. importantly, we demonstrated that several states on the HOPS can be obtained digitally, including those associated with the traditional scalar HOTs. Furthermore, we showed that one can on-demand adjust the transverse optical forces inside the optical trap by changing the polarization state of the trapping beam. This enabled the simultaneous realization of many traps with different trapping strength for the same laser power. However our vector HOT tool may be further advanced and will prompt interest from the structured light and optical manipulation communities alike.

## 7.4 Conclusion

In this Chapter we demonstrated the dynamics of a holographic optical trap by generating 9 traps which were then digitally manipulated into different configurations. An all inclusive toolkit to analyse the polarization and vector properties of structured light fields was presented with the shape invariant vector flat-top beam as our chosen customized light beam. From our analysis, the vector state as well as the spatial profile of our vector flat-top are preserved during propagation. We then employed this beam in an optical trap and compared its trapping strength to that of a Gaussian beam. Finally, we presented a new holographic optical trap which allows the use of both scalar and vector light fields in the same trap simultaneously. The HOT was shown to be purely digital and enabled the realisation of arbitrary polarization states in the trap as described on the HOPS. We further showed that one can digitally tailor the trapping strength by simply changing the polarization state and this was achieved digitally without any mechanical adjustment of the physical optics.

## Chapter 8

# General conclusions and outlook

### 8.1 Conclusions

We have presented in detailed the creation and analysis of structured light fields in scalar and vector form. In our discussions we considered the theoretical concepts as well as the experimental aspects to obtain these customized light beams. The versatility of these light beams was then demonstrated in an optical trapping and tweezers application.

We began in the first Chapter with a motivation and overview of our study which was followed by a detailed literature review of the different methods of creating structured scalar and vector light fields. A brief discussion on the previous work in which these custom light fields were employed in optical tweezers was also presented.

We then presented the Helmholtz wave equation within the paraxial approximation as well as the corresponding solutions such as Laguerre-Gaussian, Hermite-Gaussian, Bessel-Gauss and flat-top beams which were our structured light fields of choice. In addition we consider the vector nature of light and its relation to vector light fields. A geometric representation of vector light waves on the higher order Poincaré sphere was presented together with how any arbitrary polarization state can be accessed. As an example, we chose cylindrical vector vortex beams as these have recently stimulated a lot of research interest [6, 17].

Following the theory associated with our structured light fields of choice presented in Chapter 2, we outlined in Chapter 3 the process of digital hologram generation for the creation of our desired modes. To obtain these holograms, we considered phase-only and complex amplitude modulation as our beam shaping techniques. A discussion on obtaining multiple structured light beams (multiplexing) simulta-

neously using these two beam shaping methods was presented. In addition, we described a theoretical concept of creating shape invariant flat-top beams. To this end, analytical simulations demonstrating shape invariance of vector flat-top beams were presented. Following the multiplexing approach of generating multiple beams, we presented a novel concept of realising multiple vector beams using a single hologram. Since the resultant light fields were to be employed in an optical tweezers system, the fundamentals of optical trapping were then briefly described in Chapter 4.

Using digital holography as well as the Laguerre-Gaussian and Hermite-Gaussian beams as our example basis set, we then experimentally demonstrated the multiplexing concept in Chapter 5. A comparison of the phase-only and complex amplitude modulation beam shaping methods was conducted and we showed that of these two techniques, complex amplitude modulation reproduces optical modes with higher qualities as it modulates both the phase and amplitude of the incident beam. However this method suffers from being less efficient as most of the input beams energy is distributed to higher orders or the zero order [15]. From our quantitative analysis of the mode quality as function of the resolution of the screen's digital device we demonstrated multiplexing of approximately 200 modes. From the analysis we revealed, as expected, that the quality of the modes decays as limited by the SLM resolution as the number of multiplexed modes increased.

In Chapter 6, we then presented a novel method to generate and multiplex vector modes using a single SLM. This technique was based on the multiplexing concept implemented in Chapter 5 and employed an interferometric approach. By multiplexing several holograms, it was possible to split an input light beam into many more, each with their own spatial shape and phase. The multiplexed holograms were split into two groups to generate two sets of beams travelling along two spatially separated paths where each set is endowed with left and right polarizations, respectively, and then interferometrically recombined to generate the desired set of cylindrical vector beams (CVB). We further demonstrated that the digital nature of this method allows full control of each beam's amplitude weighting, spatial shape and phase, which enables the generation of arbitrary vector beams with various spatial shapes and polarization distribution. For this work, we only generated cylindrical vector beams but our technique can be applied to any other geometry. Although we only showed the generation of sixteen CVB, this technique allows for the generation of

many more.

Finally in Chapter 7, we demonstrated the application of our customized structured light fields in an optical tweezers system. As a first test, we used laser beam multiplexing to generate multiple optical traps. We presented a modern holistic tool-set which we used to confirm that vector flat-top beams are shape invariant during propagation and their vector nature is also preserved. Thereafter, our experimentally generated shape invariant vector flat-top beam was then characterised, analysed and then employed in an optical trap. This demonstrated that our vector flat-top beam can be easily delivered through standard optical set-ups involving focussing lenses with out alteration of its shape. In addition, using our quantum toolkit we revealed the non-separability of our beam which remained unchanged during propagation.

We then presented a novel vector holographic optical trap for the delivery of 2D arrays of traps using HOPS beams. We created example arrays (square, triangular and linear) with combinations of scalar and vector HOPS beams, characterized the beams externally to the trap, and then demonstrated successful trapping with them. Each HOPS beam in the array was independently controlled by a SLM and spanned a wide range of positions on HOPS, including those associated with conventional scalar HOTs. By this approach, we were able to tailor on-demand the 3D shape of the optical forces at specific locations inside the optical trap, enabling the simultaneous realization of many traps with different trapping strength for the same laser power.

## 8.2 Future work

This work demonstrated novel techniques and implementation methods of creating structured scalar and vector light fields and then applied such custom light beams in optical trapping and tweezers. However, it is worth while to investigate the nature of these custom modes under tight focusing where the paraxial approximation ceases to apply. In particular the polarization structure of the vector light fields under tight focusing, since changes in the polarization state of the trapping beam alters the trapping strength of the optical tweezers. Some theoretical studies along this line have recently began to emerge [130] with not much experimental work being reported.

We only demonstrated the use of our created structured light fields on a micro-scale with respect to optical trapping, the implementation of these beams can be

extended to the nano-scale. This will be of particular interest in the field of nanotechnology in particular carbon nano-tube which are currently being investigated with the aim of making nano-devices [176]. A combination of these structured light fields and optical tweezers can provide a non-invasive method for nano-device assembly together with the use of a laser beam to weld these nano-devices together.

## Appendix A

# Vector quality Factor

The vector quality factor is a quantitative analysis of the vectorness of a vector beam whose scale lies between 0 for completely scalar and 1 for a purely vector state. The VQF takes advantage of the similitude between quantum entanglement and non-separability of phase and polarization in vector beams. It is defined using quantum mechanics tools as,

$$\text{VQF} = \text{Re}(C) = \text{Re}\left(\sqrt{1 - s^2}\right), \quad (\text{A.1})$$

where  $C$  is the concurrence [177] which is a measure of entanglement or the degree of non-separability between two states.

The VQF ranges from 0 for scalar beams to 1 for vector beams. The parameter,  $s$  defined by,

$$s = \left(\sum_i \langle \sigma_i \rangle^2\right)^{1/2} \quad (\text{A.2})$$

is the length of the Bloch vector and  $\langle \sigma_i \rangle$  represents the expectation values of the Pauli operators for  $i = \{1, 2, 3\}$ . These are obtained by a set of 12 normalized, on-axis intensity measurements, six identical measurements for two different basis states [35, 158]. For our experiment we chose the circular polarisation basis and project these into the orbital angular momentum (OAM) basis. For this, the two circular polarization (left and right) of the vector beam, are projected into a set of six holograms with topological charge  $\ell$ ,  $-\ell$  and four superposition states given by  $\exp(i\ell\phi) + \exp(i\gamma)\exp(-i\ell\phi)$  with  $\gamma = \{0, \pi/2\ell, \pi\ell, 3\pi/2\ell\}$ , as illustrated in Fig. A.1 The expectation values  $\langle \sigma_i \rangle$  are calculated from the twelve intensity measurements

Table A.1: Normalized intensity measurements  $I_{mn}$  to determine the expectation values  $\langle\sigma_i\rangle$ .

| Basis states | $\ell = 1$                                                                        | $\ell = -1$                                                                       | $\gamma = 0$                                                                      | $\pi/2$                                                                           | $\pi$                                                                              | $3\pi/2$                                                                            |                                                                                     |
|--------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|              |  |  |  |  |  |  |  |
| $L$          | $I_{11}$                                                                          | $I_{12}$                                                                          | $I_{13}$                                                                          | $I_{14}$                                                                          | $I_{15}$                                                                           | $I_{16}$                                                                            |                                                                                     |
| $R$          | $I_{21}$                                                                          | $I_{22}$                                                                          | $I_{23}$                                                                          | $I_{24}$                                                                          | $I_{25}$                                                                           | $I_{26}$                                                                            |                                                                                     |

as,

$$\langle\sigma_1\rangle = (I_{13} + I_{23}) - (I_{15} + I_{25}), \quad (\text{A.3})$$

$$\langle\sigma_2\rangle = (I_{14} + I_{24}) - (I_{16} + I_{26}), \quad (\text{A.4})$$

$$\langle\sigma_3\rangle = (I_{11} + I_{21}) - (I_{12} + I_{22}). \quad (\text{A.5})$$

and represented in Table A.1.

To determine the VQF experimentally, we measure the on-axis intensity values  $I_{mn}$  with  $m, n \in \{1, 2, 3\}$ . A multiplexing approach allowed us to measure six intensity values simultaneously corresponding to one of the two polarizations. To change from one polarization basis to another, we used a quarter wave plate ( $\lambda/4$ ) set to  $\pm 45^\circ$  in combination with a polarisation sensitive spatial light modulator SLM<sub>2</sub>, and OAM projections by a phase pattern on SLM<sub>2</sub>. Figures A.1 show typical

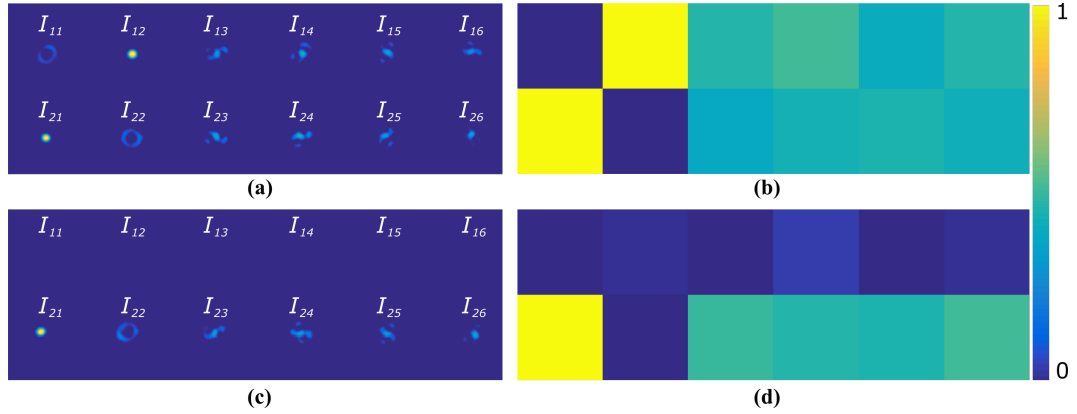


Figure A.1: Common intensity patterns acquired with a CCD camera for a vector (a) and a scalar (c) beam. Normalized intensity values from which the VQF is determined for a vector (b) and a scalar (d) beam.

intensity distribution obtained experimentally for pure vector beams A.1(a) and a scalar beams A.1(c). For the scalar case, the beam has a homogeneous polarization state (i.e. LHC or RHC) as such only a single row of intensities is obtained (Fig

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A.1(c)). While for the vector case where the polarization state of the beam is inhomogeneous (i.e. a superposition of LHC or RHC) two rows of intensities are obtained as shown in Fig. A.1(a). Figure A.1(b) and A.1(d) shows the intensity values, properly normalized and arranged in the form of Table A.1., from which the VQF can be easily computed.

In principle, for these VQF measurements one should take only the intensity value of the central pixel, however the background noise causes fluctuations of this value. As such, an average of many measurements of that single pixel intensity has to be obtained. Another solution would be to take the average over a given area around the central pixel. While in the former case, the larger the sampling the more accurate the value will be, this is not the case for the last one. Along this line, there is no explicit rule about the area over which an average can be obtained as it depends on other factors such as the focal length of the Fourier lens. The smaller the focal length, the smaller the focused spot will be, which means that the number of pixels we can use cannot be large in order to avoid interference from the surrounding area of the intensity distribution. Hence, the larger the focal length of the lens, the more pixels we can use in the average.

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