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UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Master of Education by Coursework

Investigating Grade 8 Learners' Algebraic Problem-Solving Skills when Engaging with Real-life problems.

A research report prepared by

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A research study submitted to the Wits School of Education, Faculty of Humanities, University of Witwatersrand in March 2025 in fulfilment of the requirements for the degree of

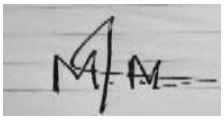
MASTER OF EDUCATION (M Ed)

Declaration.

I, **Mechaela Meyer (1843496),** hereby declare that this research project titled, ***Investigating Grade 8 Learners' Algebraic Problem-Solving Skills when Engaging with Real-life Problems*****],** is entirely my own work. It is being submitted as part of the degree in Senior / FET Phase Mathematics at the University of the Witwatersrand, Johannesburg. This research report has not been submitted for any other degree or examination at any other university. According to the APA style of citation, I have provided a comprehensive list of references that are firstly in alphabetical order and secondly in a chronological order according to year of publication by an author. I understand the consequences should the university discover that the work submitted is not my own or fail to acknowledge any of the sources or ideas which are not of my own.

Sincerely

M.C. Meyer



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List of Acronyms

ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
NCSM	National Council of Supervisors of Mathematics
NCTM	National Council of Teachers of Mathematics
RME	Realistic Mathematics Education
SACMEQ	Southern African Consortium for Monitoring Educational Quality
TIMSS	Trends in International Mathematics and Science Study
VHPOL	Van Hiele Phases of Learning

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Chapter 1: Introduction, Context and Background.

"Students who learn algebra as a relational system are better equipped to understand and apply mathematical concepts, leading to deeper comprehension and improved problem-solving skills."

James Kaput (2008)

1. Introduction:

The development of the information communications technology industry increasingly makes us aware of the importance of having and incorporating a grounded understanding of mathematics in our everyday lives. Slow progress has been made in South African secondary schools to alleviate the narrative that learning mathematics is an individual and unnecessary experience. The 2011 Trends in International Mathematics and Science Study (TIMSS) provided more evidence showing that South African learners perform disappointingly in mathematics. The TIMMS (2011) showed that South African learners have the "lowest performance among all 21 middle-income countries that participated" (McCarthy & Oliphant, 2013, p. 3). The growing number of parents enrolling their children in private mathematics tutoring suggests that many parents have greater confidence in private tuition than in public-school mathematics teaching.

The National Curriculum Statement emphasizes the need to foster an active and critical engagement with learning, as opposed to the passive memorisation and uncritical acceptance of predetermined knowledge (DBE, 2011). The role of the teacher is thus to cultivate learners' creative and critical thinking abilities, thereby preparing them to navigate problem-solving and decision-making effectively in their developmental journey.

In the senior phase (Grades 7, 8 and 9), learners in South African schools are introduced to formal symbolic algebra. With predominant research done in these grades (Molina et al., 2016; Warren et al., 2016; Pincheira &Alsina, 2021) much detail has been provided to inform the way forward for mathematical learning.

A recurrent theme is that the goal of algebra to promote thinking habits in mathematics classrooms must aim to reflect a way of working with mathematical objects, relationships between these

objects, and situations to promote algebraic reasoning associated with mathematics problem-solving real-life activities. In the context of the South African schooling system and in accordance with the Curriculum and Assessment Policy Statement (2011) for senior phase mathematics, acquiring mathematical knowledge and specifically in algebra, may be described as a system of interrelated concepts which allows learners to make connections across the constituents (i.e. variables, like terms or unlike terms) of algebra (DBE, 2011).

1.1 **Algebra and Algebraic Reasoning: An Introduction and Background.**

According to the (American) National Council of Teachers of Mathematics:

> Algebra is the key ingredient in the recipe for understanding mathematics, as it provides the language and tools necessary to express and solve problems across various mathematical domains. (Smith, Bill & Raith, 2018).

Smith, Bill, and Raith, (2018) emphasizes that a strong grasp of algebraic concepts is essential for learners to navigate through other areas of mathematics effectively. It is therefore not surprising that research on algebra and algebraic reasoning is an on-going topic of interest within the mathematics community, from foundation level to tertiary level. Warren, Trigueros, and Ursini (2016) observe that the challenges encountered by young learners in developing algebraic reasoning are similar to those faced by students when they commence formal algebra. Within the mathematics domain, algebraic reasoning plays an important role in fostering habits of mind that enhances problem solving and generalisation skills in young learners. Du Plessis (2018), in a study focused on Early Algebra at foundation level, argues that a focus on structure in mathematical engagement enhances the relational understanding of young learners, and serves to inform the conceptualisation of further algebraic learning. Warren et al (2016) argued that strengthening learners' understanding of variables enhances their ability to generalise and use advanced algebraic notation, while fostering awareness of the dynamic relationships between unknowns and variables.

Molina et al (2017) attests that common errors in secondary school mathematics are foregrounded when learners work with algebraic symbolic and real-life problem-solving situations. These problem types are commonly linked to learners' prior knowledge and relational understanding when working with unknown quantities in arithmetic (Kaput, 2008). This clearly signals that the

common errors made by secondary school mathematics learners are reflective of the lack in sufficient foundational mathematical thinking established during the early years of mathematics learning.

Algebra finds its foundation on two main pillars: "variables as *general aspect entities* that can be represented in different ways, and *structure sense* of algebra" (Warren et al, 2016, p.74). Both aspects are deeply embedded in any meaningful algebraic activity (Du Plessis, 2018, Kaput, 2008). A growing consensus among researchers, therefore, is on the importance of offering learning experiences that foster algebraic thinking from the earliest stages. The promotion of algebraic reasoning skills in mathematics classrooms must involve an explicit attempt to enhance a structural view which underlies mathematical exploration (Pincheira et al, 2021, Du Plessis, 2018, Kaput 2008, Warren, 2011).

Prior to learners engaging in sophisticated algebraic problem solving, they must develop a habit of mind that emphasizes the relational and structural aspects of algebra (Carpenter & Franke, 2001, Du Plessis, 2018). When learners understand how to manipulate expressions based on their structural properties rather than memorising procedures, they become more skilled at applying their knowledge to more advanced problems. This foundational skill set is crucial for developing mathematical proficiency and confidence as they progress to more sophisticated problem-solving.

1.2. Problem Statement.

South African learners underperform in mathematics, and this is due to a lack of foundational knowledge in mathematics (Mabena et al, 2021). Most learners struggle with algebra because of residing gaps in their foundational mathematical knowledge, making it difficult for them to grasp more complex concepts such as algebraic problem-solving activities embedded in a contextual setting. Furthermore, in South African schools, English is the primary language of instruction, yet for many learners it is not their first language, which poses challenges in interpreting real-life algebraic problem-solving questions (Mabena et al, 2021). Research attests that primary school mathematics classes focus on activities leading toward the development of a function or variable concept in algebra and very little emphasis is placed on developing learners' generalisation and inferring skills which is assessed in algebraic problem-solving activities (Van

Larren & Moore-Russo, 2012, Du Plessis, 2018). This poses a challenge for the introduction to formal algebra taught in the senior phase as a gap is created when conceptual understanding of algebraic word problems is seldom or hardly ever assessed during the earlier grades.

In 2021, the diagnostic report of the annual national assessment (ANA) reflected that the group performance of senior phase learners was below 49,3% and at an individual level, below 40%. This constitutes to 'Not Achieved' on the achievement scale provided by the DBE. The result from this study suggests that learners at the senior phase level (and consequently, the further education and training level) have significantly underperformed in mathematics over the last decade. TIMSS (2011) as well as the Southern African Consortium for Monitoring Educational Quality (SACMEQ) provides insight into why our learners underperform. Their claim is that South African learners reflect poor results in mathematics due to an inability to reason mathematically, and this being a prerequisite for mathematical learning.

1.3. Purpose Statement.

This research has at its core, the investigation of the impact that *model strategies*[1] have on learner knowledge to work relationally with mathematical objects and recognize patterns and structures that may enhance reasoning skills in mathematics. The aim of this study is to examine the transition that grade 8 learners make after using model strategies in algebraic problem-solving. This research engaged with and analysed the mathematical methods employed by Grade 8 learners when solving real-life mathematics problems.

Learners who engage with algebra through relational learning tend to demonstrate greater confidence in their mathematical abilities They are also more likely to transfer their knowledge to new problems and real-world situations, which is essential for developing critical thinking skills in mathematics. The researcher, based on the results of a pre-test, designed an intervention to enhance algebraic reasoning skills and activate efficient problem-solving when dealing with contextual problems.

1.4. Using Models in Mathematics Problem-Solving.

The concept of modelling in mathematics has been continuously researched with the aim of enhancing learners' ability to solve problems in real-world situations through using mathematical structures embedded in such models (Erbas & others, 2014). Modelling in mathematics is described as a "focus on structural features and functional principles of objects or situations in real life" (Erbas & Kertil, 2014, p.1622). By using models as visual mediators, learners may break down complex problems into smaller, more manageable components, making abstract algebraic concepts more concrete and accessible.

A mathematical model as a tool for learning is a convergence of structural order, representations, mathematical relations, and operations which come together to help solve real-world problems. Van den Heuvel-Panhuizen (2003) argues that the use of models promotes progression and, when used effectively, enables learners to comprehend mathematics through analysing alternative options. Van den Heuvel-Panhuizen (2003) asserts that learners' capacity to solve complex problems is closely linked to the context in which these problems are introduced.

Incorporating model strategies in algebra instruction is believed to promote relational understanding, which extends beyond memorisation and procedural knowledge. Relational understanding enables learners to grasp why mathematical rules apply to specific solution procedures helping them recognize connections between concepts and mathematical objects (Skemp, 1976). Consequently, relational understanding is regarded as a form of higher-order thinking, as it enables learners to construct meaningful connections among mathematical concepts rather than merely executing mechanical procedures.

It was thus useful to introduce the leaners in this research to the development of various solution models and strategies for solving algebraic word problems. Valuable insights were gained into how the participants used and produced models to solve algebraic real-life problems

In this research, a model was used to represent problem situations, while strategies were integrated to guide learners' engagement with these models. This approach supported the development of algebraic reasoning and enhanced relational understanding in mathematical problem-solving.

1.5. Research questions.

The following research questions guided the study:

a) What are the algebraic problem-solving strategies that Grade 8 learners engage in when solving real-world problems?
b) How do the use of model strategies, if any, vary among Grade 8 learners?
c) How do the problem-solving approaches introduced during the three-week intervention enable learners to develop more relational approaches to problem-solving?

1.6. Rationale for the Study.

This study focused on investigating potential transitions in learners' approaches, particularly the use of models as visual mediators in problem solving. It aimed to support effective strategies that enable learners to recognize the relational aspects of algebraic processes. Lessons were delivered using various approaches that incorporated models and appropriate strategies for solving algebraic real-life problems effectively.

Kaput (2008) maintains that when learners grasp algebraic concepts relationally, they develop stronger mathematical reasoning abilities. This approach contrasts with traditional methods that often focus on rote memorisation and procedural tasks without emphasizing the connections between different mathematical objects and ideas (Du Plessis, 2018). The foundations of mathematical thinking, and in particular, the ability to solve algebraic real-life problems, are activated by the relational understanding which is key in enhancing the structural approach to learning algebra (Du Plessis, 2018, Warren, 2011).

It is therefore essential to expose learners to various problem-solving approaches that cultivate the necessary habits of mind to guide their processes when encountering more advanced algebraic problems. Experience as an educator in a South African township school demonstrated that learners, for whom English was typically a third or fourth language, often struggled to translate linguistic expressions into correct algebraic statements. Consequently, the research aimed to support these learners by providing strategies that facilitated this transformation.

A pre- and post-test model was used for data collection, as it provided significant insight into the methods learners employed and whether the intervention lessons shifted their approaches toward more efficient problem-solving skills.

The results of this research informed future pedagogical approaches aimed at assisting learners with problem-solving, particularly in real-life contexts.

The next chapter elaborates on the theoretical underpinnings of this research.

Chapter 2: Learning Theories

"Effective teaching of algebra requires not only a deep understanding of the content but also an awareness of how students learn and think about mathematics. A well-designed pedagogical approach can bridge the gap between students' prior knowledge and new concepts, fostering a more profound understanding and retention."

(Stein & Smith, 2018).

2. Introduction.

This chapter outlines the theories, models, and conceptual perspectives that underpin this research, offering a lens through which the problem under investigation is examined.

The two learning theories that informed the pedagogical approach followed in this study are discussed. The first is the notion Realistic Mathematics Education (RME) and the second is taken from the Van Hieles' Phases of Learning (VHPOL). Both these learning theories provide insights into effective teaching strategies in mathematics education, focusing on developmental approaches that enhance learner understanding of key mathematical concepts.

Thereafter, the compatibility of the two learning theories was discussed, followed by a brief summary.

2. Pedagogical Approaches drawn on for the Intervention Lessons

2.1 Realistic Mathematics Education (RME)

RME encourages learners to construct mathematical models grounded in real-life contexts, fostering a process of *mathematization*—the translation of real-world situations into mathematical structures. This allows learners to build upon prior knowledge and develop deeper reasoning (Gravemeijer & Doorman, 1999). Freudenthal (1991) emphasises that mathematization involves more than application; it requires meaningful transformation of situations into analysable mathematical constructs. RME distinguishes between two types of mathematization (Van den Heuvel-Panhuizen, 2001): horizontal and vertical.

2.1.1 Horizontal Mathematization

Horizontal mathematization refers to informal strategies learners use to solve contextual problems. In this research, these strategies served as a foundation for identifying relational elements and nurturing reasoning skills essential to algebraic thinking.

2.1.2 Vertical Mathematization

Vertical mathematization involves recognising abstract mathematical structures, forming connections between concepts, and applying symbolic reasoning (Van den Heuvel-Panhuizen, 2001; Barnes, 2005). As learners transitioned from informal to formal strategies, they refined their reasoning by articulating and applying efficient algebraic methods (Polotskaia et al., 2022).

2.1.3 Integrating Both Forms of Mathematization

Both forms of mathematization are critical and exist on a continuum of understanding (Van den Heuvel-Panhuizen, 2001). Unlike traditional approaches that use context to consolidate concepts, RME integrates context throughout the learning process. The intervention was designed to guide learners in modelling algebraic real-life problems through structured engagement and teacher support. Learners developed, discussed, and refined their own model strategies—an approach aimed at fostering both horizontal and vertical mathematization.

2.2 The Van Hiele Phases of Learning (VHPOL)

Originally developed for geometry, Van Hiele's model is equally applicable to algebra. It recognises that intuitive, often non-verbal, understanding precedes formal reasoning (Van Hiele, 1986). Learners in multilingual contexts, such as South Africa, may struggle to verbalise their mathematical thinking, which reflects the challenge of "thinking without words" (Papademetri-Kachrimani, 2012). It was therefore essential for the teacher-researcher to listen attentively and support the development of learners' mathematical language.

The VHPOL model highlights that learning is influenced by learners' environments, exploratory experiences, and guided instruction. This research applied the first three phases to support the development of more sophisticated model strategies.

2.2.1 Phase 1: Information Gathering

The teacher-researcher facilitated inquiry-based tasks to stimulate critical thinking and pattern recognition. Learners engaged in discussion and exploration, allowing the teacher-researcher to

observe their reasoning, identify prior knowledge, and correct their mathematical register (Dongwi, 2014).

2.2.2 Phase 2: Guided Orientation

Through structured guidance, learners explored key concepts and relational structures. The teacher-researcher posed strategic questions and corrected language use while introducing new algebraic terminology.

2.2.3 Phase 3: Explication

Learners articulated their evolving understanding using formal algebraic language. The teacher-researcher consolidated learning by leading discussions, refining terminology, and highlighting conceptual connections (Dongwi, 2014).

2.3 Integrating the Realistic Mathematics Education Framework and the Van Hiele Phases of Learning as the pedagogical approach for the intervention workshop.

The integration of Realistic Mathematics Education (RME) and the Van Hiele Phases of Learning (VHPOL) was central to this research, as it fostered relational understanding and critical thinking in the development of learners' algebraic and problem-solving skills. As learners engaged with contextualised problems through the RME approach (Van den Heuvel-Panhuizen & Drijvers, 2014)., they simultaneously progressed through the cognitive phases outlined in the VHPOL model.

RME offered meaningful contexts for exploring algebraic ideas, supporting learners' gradual movement from intuitive understanding to abstract reasoning. Both RME and VHPOL emphasise the transition from concrete experiences to formal mathematical thinking, making them complementary in guiding learners toward a deeper relational understanding of- and structural approach to algebra.

Effective mathematization in RME requires learners to reflect on real-life problems, extract relevant mathematical relational information and structures, and apply them purposefully. This process aligns with the VHPOL's focus on guided progression through phases of learning , underscoring the importance of teacher support in helping learners verbalise and formalise their thinking. Together, these learning theories grounded the pedagogical approach taken in the intervention lessons. It situated mathematics instruction in relevant contexts while systematically

building learners' capacity for abstract reasoning. Table 2 summarises the alignment of the two learning theories.

Table 2.1: Alignment between Realistic Mathematics Education and the Van Hiele Phases of Learning

Realistic Mathematics Education	Van Hiele Phases of Learning
Encourages learners to build and apply mathematical models in real-life contexts	Learners often understand problem components but struggle to articulates their thinking, especially in multilingual contexts
Connects mathematical concepts to everyday situations, supporting deeper understanding	Mathematical development depends on cultural context, individual exploration, and guided learning
Defines *mathematization* as the process of transforming contextual problems into mathematical form—both horizontal and	Phases 1 and 2 focus on guided exploration and hands-on engagement with contextual problems under teacher facilitation
Provides structured guidance to help learners Identify relationships and generalize strategies	In Phase 3, learners apply formal reasoning, symbolic representation, and deductive thinking to algebraic problems

The two learning theories (RME & VHPOL) together supported the activation of learners' relational understanding and concomitant move towards a grounded algebraic approach. RME's focus on contextualised problem-solving enabled learners to engage meaningfully with real-life algebraic tasks, allowing them to reinvent mathematical concepts and processes through practical exploration.

Integrating the learning theory penned by Van Hiele (1986), the intervention employed Phases 1 to 3 of the VHPOL model viz information gathering, guided orientation, and explication. This integrated approach was very useful to follow and support learners' progression from intuitive understanding to formal expression. Through structured discussions and teacher-guided instruction, learners refined their reasoning, clarified misconceptions, and articulated mathematical ideas using appropriate algebraic language.

This integration of RME and VHPOL activated a clear relational understanding and promoted deeper engagement with algebraic structures, resulting in more effective and reflective problem-solving skills.

2.4. In Summary:

This chapter explored the theoretical foundations of this research. Focusing on the informal strategies learners used in mathematical reasoning, two key learning theories guided this research. RME and the VHPOL shaped the lenses through which the instruction in the intervention was conducted. These learning theories provided a strong basis for identifying and developing the reasoning skills necessary for understanding algebraic relationships.

In this research, learners were required to use mathematical symbols and language to find efficient methods for solving real-life problems. A core feature of RME is its emphasis on contextual problems, which are seamlessly integrated into instruction rather than being used solely for practice, as seen in traditional approaches. When learners engage in mathematizing algebraic real-life problems, they gain valuable opportunities to reinvent mathematical concepts and processes through practical exploration.

This research used the VHPOL learning theory to assess whether learners not only recognise the individual components of a real-life problem but also how well they can articulate their understanding. Furthermore, this research specifically applied Phase 1 (Information Gathering), Phase 2 (Guided Orientation), and Phase 3 (Explication) of the Van Hiele model, as these phases provided insight into how learners, with the guidance of the teacher-researcher, became more efficient problem solvers.

The theoretical perspectives underpinning this research emphasized discussion-based learning between the learners and the teacher-researcher. This interactive approach allowed learners to explore algebraic concepts while engaging in peer discussions to develop their reasoning skills. As they are encouraged to exchange ideas, misconceptions can be addressed in real time, ensuring that learners first engaged with the problem before receiving corrective guidance. This process helped them become more familiar with new concepts and express their understanding using appropriate algebraic language.

Integrating both learning theories, RME and the VHPOL, is essential for this research, as it fostered relational understanding in the development of algebraic thinking based on understanding underlying structural approaches in the real-world problems they were presented with. As learners engaged in realistic problem- solving tasks within the RME theory of learning, they strengthened their algebraic reasoning skills and gradually progressed through the VHPOL.

The subsequent chapter expands on the literature review that underpinned this research.

Chapter 3: Literature Review

"*Algebra is not merely a set of rules for manipulating symbols; it is a way of thinking that involves understanding relationships and structures. Developing algebraic reasoning enables students to engage with mathematical problems in a meaningful way, fostering deeper comprehension and problem-solving abilities."*

David Tall (2009).

3.1. Introduction

In this chapter, a discussion is provided on the contextual problems South African schools faced regarding the algebraic mathematical performance of Senior Phase learners. The notions of relational understanding and relational reasoning were then discussed, along with how these concepts assisted in enhancing problem-solving skills. The focus subsequently shifted to algebraic reasoning and its importance for problem solving, which in turn became a tool for activating relational reasoning skills. An algebraic problem-solving approach to solving word problems was also discussed, and model strategies to deepen mathematical understanding were examined. The chapter concluded with a short summary.

3.2. The context in South African secondary mathematics classroom

Promoting an active and critical learning environment is important for teachers when considering the ideal scaffolding of new concepts and skills in the algebra class. However, apart from acknowledging the learner's needs in the mathematics classroom and incorporating responsive thinking and teaching, learners may find it challenging to develop the desired skills stipulated in CAPS (DBE, 2011). The CAPS document encourages teachers to consider that "the mathematics period should also provide for supporting learners experiencing barriers to learning, providing enrichment activities for highflyers, setting rich assessment activities, etc." (DBE, 2011, p. 12). This means that within the 45 minutes to 1 hour lesson, the teacher is expected to check for homework, revise prior learning, teach a new lesson, and provide classwork activities to ensure that learners understood the new mathematical content.

In the context of educational reform, particularly in secondary mathematics education, it is crucial to recognize the factors contributing to teachers' resistance. As McCarthy and Oliphant (2013) highlight, the perception of one's effectiveness can significantly influence a teacher's willingness to engage in professional development and embrace changes in pedagogical practices. This notion raises important questions about motivation and self-efficacy among educators.

To inform the pedagogical approaches employed in this research, the teacher-researcher sought to develop learners' algebraic relational reasoning and relational understanding skills. The overarching goal was to cultivate robust habits of mind in approaching algebraic word problems.

3.3. Relational Understanding and Relational Reasoning.

Relational understanding refers to a substantial comprehension of mathematical concepts, where learners grasp not only *how* to perform mathematical operations but also *why* these operations are appropriate to use. Skemp (1976) points out that relational understanding is knowing both what to do and why it works . This type of understanding allows learners to connect different mathematical ideas and apply them flexibly in various contexts. Downes (2008) claims that the discovery of variance in numbers and the understanding of how to operate with and manipulate mathematical operations is influenced by the relational understanding learners are equipped with while engaging in mathematical processes needed in algebra. A solid understanding of these variances assists learners to use model strategies efficiently and accurately when engaging in problem-solving activities. According to Skemp (1976), relational understanding occurs when learners not only attempt to comprehend new material in a relational manner but also proactively engage in seeking out additional content and exploring unfamiliar domains

Relational reasoning involves the ability to recognize relationships among mathematical entities and use these relationships to solve problems. It emphasises the cognitive processes involved in *making connections* between different pieces of information or concepts. Hattie and Donoghue (2016) describe relational reasoning as the ability to identify patterns and establish connections, thereby facilitating flexible and adaptive thinking in mathematical contexts. When developing learners relational reasoning ability, a specific approach to learning mathematics takes shape. Relational reasoning goes beyond understanding the variance of numbers but also, it is assessed by the ability of learners to use these variances in a problem-solving context.

In the process of developing relational reasoning, numbers and number relations should be studied

from a structural point of view. Furthermore, this is achieved when the structure within the mathematics domain is viewed as the process of exploring numbers and space relationally, which subsequently promotes reasoning that emphasizes the underlying associations of objects rather than the objects themselves (Warren, 2011).

Relational reasoning develops through first acquiring sound relational understanding of the object you are working with. According to Hattie and Donoghue (2016), the essential difference lies in their focus: relational understanding is about *comprehending* the underlying principles of mathematics, while relational reasoning is about *applying* those principles through recognising relationships (emphasis added by author). One can have a strong relational understanding without necessarily being adept at relational reasoning if they struggle to apply their knowledge flexibly.

Evidence of relational learning is categorised by how the learner formulates a schema from which the learner can produce a variety of processes through moving from an initial point within their own schema to any finishing point (Skemp, 1976). Therefore, reliance on rote application of rules or procedures would limit learners should they be presented with a higher order algebraic equation without having a solid relational understanding and the ability to activate their relational reasoning. It is through relational understanding and reasoning that underlying algebraic principles, connections to algebraic operations, and its flexible application in problem-solving will be better comprehended (Kilpatrick, Swafford & Findell, 2001).

3.4 Problem solving through enhancing relational understanding and reasoning.

Abdulhamid and Venkat, (2018, p. 6) note that "gaps in the mathematical practise base are associated with reluctance to explore students' alternative mathematical solution strategies", thus effectively ignoring the importance of relational reasoning and critical thinking as a mathematical activity. The low performance rate stems from the lack of specific interest in the use of efficient problem-solving strategies that move beyond mere application of simple procedures but engaging with the relational attributes when unpacking algebraic structures that provides alternative pathways to problem-solving. Attaining sufficient skills in contextual algebraic problems-solving and calculations in the senior phase, is recognised as a gatekeeper to more advanced mathematics and opportunities (Appah et al, 2020).

Algebra is formally established as a senior phase course in high school. The proposal that algebraic thinking should be introduced in the primary grades, referred to as 'early algebra' (EA) in literature, has been widely accepted (Du Plessis, 2018). The South African CAPS (DBE, 2011)

and its accompanying documents emphasise that learners are expected to comprehend the underlying logic of patterns. Moreover, they assert that concentrating on this logical structure serves as a foundation for the development of algebraic thinking skills.

This can be achieved through focusing on the relational understanding and reasoning of leaners when they engage in a structural approach in mathematical problem-solving (Du Plessis, 2018). Little time is spent on enhancing mathematical reasoning assessed in the form of relations between variables or sets of values. According to the NCTM (2014), effective mathematics instruction necessitates a deep understanding of the interconnections among mathematical concepts and their underlying relationships. Additionally, research highlights that learners frequently encounter difficulties in comprehending mathematical relationships, due to an inadequate focus on reasoning and justification within their learning experiences (Hattie & Timperley, 2007).

A thorough understanding of unknown quantities and variables, alongside the ability to manipulate and apply operations on them, is fundamental for engaging with the structural and relational properties of numbers (Van den Heuvel-Panhuizen, 1999; Du Plessis, 2018; Warren, 2011). Thus, one can argue that algebraic reasoning introduces learners to the concept of variables, which are symbols used to represent unknown values and the relationships between them.

While both concepts of understanding and reasoning were crucial for effective mathematical learning in this research, they served different purposes: relational understanding provided a foundation for knowledge, whereas relational reasoning enabled the application of that knowledge in varied contexts. In this research, this abstraction allowed learners to organise model representations and formulate equations that described real-world situations or complex mathematical relationships.

3.5. Algebraic Reasoning and its importance for problem solving

Algebraic reasoning comprises two primary components: the cultivation of mathematical problem-solving techniques and the exploration of foundational algebraic concepts (Kriegler, 2008). Mathematical problem-solving techniques are categorised as analytical habits of the mind and, are organised around three central sub-topics: problem-solving skills, representation skills and quantitative reasoning skills (Kriegler, 2008).

First, problem solving skills are vital for learners to effectively analyse and approach mathematical challenges. In algebraic reasoning, this means being able to identify relevant algebraic concepts

and formulate expressions or equations that accurately represent a given problem (Kriegler, 2008). This analytical approach fosters a deeper understanding of mathematical relationships and enhances problem-solving capabilities (Kriegler, 2008). Then, representation skills are described as the ability to represent mathematical ideas in various forms such as models, graphs, tables, and equations. This ability to organise information is a crucial skill necessary for critical reflection when dealing with problem solving in algebra. This can be seen when learners are assessed to translate real-world situations into algebraic expressions, which requires robust organising and representation skills (Kriegler, 2008). Lastly, quantitative skills are skills involving the understanding and application of numerical concepts within different contexts. In the realm of algebraic reasoning, quantitative reasoning enables learners to interpret and manipulate numbers and variables effectively. This understanding is important for solving algebraic equations and interpreting the significance of mathematical results, thereby strengthening learners' ability to engage with foundational algebraic concepts, and thereby complimenting their algebraic reasoning skillset (Kilpatrick et al, 2001).

Foundational algebraic reasoning habits of mind provide the framework within which mathematical problem-solving techniques are developed. This positions algebra as generalised arithmetic, a mathematical language, and a tool for exploring functions and engaging in mathematical modelling (Kriegler, 2008). Algebraic reasoning therefore encompasses more than the manipulation of symbols; it also entails an understanding of the relationships between quantities and the structural characteristics of mathematical entities (Kieran, 2004). This emphasizes two critical aspects of algebraic reasoning: the relational understanding necessary for grasping how different mathematical entities interact and the structural reasoning required to manipulate these entities effectively. In this sense, the relational and structural components converge into the idea of sound algebraic reasoning.

This research used the components discussed above to construct a schema characterized by the habits of mind that assisted learners in developing their own model strategies when solving real-life problems.

In an article written by the National Research Council et al (2005), structuring in mathematics entails learners integrating their knowledge into a unified framework. This process involves identifying connections between concepts and procedures, organizing information into categories, and uncovering the fundamental principles that govern mathematical phenomena. In short, having

a well organised schema that enables efficient algebraic reasoning ability.

3.6. Problem-solving: a tool in activating relational reasoning in algebra.

Relational reasoning in algebra develops abstract skills like fluency which speaks to the underlying ability to use reasoning as a cognitive tool with which the learner extracts relevant structural aspects (Rittle-Johnson & Schneider, 2015). Relational understanding is essential for developing fluency in algebraic problem solving; it allows learners to see the connections between different mathematical concepts and to apply their relational reasoning skills effectively (Wu, 2018). This aligns with the notion that reasoning serves as a cognitive tool, facilitating deeper comprehension and application of mathematical principles.

Being competent in algebraic relational reasoning implies being competent in algebraic approaches when engaging in problem-solving activities. This means the learner can extract information from a given word problem and then identify that "an invariant difference between two values is considered, and thus to find an unknown member of another pair" (van Dooren, et al, 2010, p. 3). Rittle-Johnson and Schneider (2015) maintains that learners who engage in relational reasoning are better able to identify and utilize underlying structures in mathematics, leading to improved problem-solving abilities. Relational reasoning is thus crucial for developing abstract skills such as fluency in algebra, enabling learners to leverage their cognitive tools effectively when navigating complex mathematical concepts.

Problem-solving is an example of an abstract skill linked to the contextualisation of numbers and, in algebra, the contextualisation of variables. However, with mathematics usually taught as a monotonous and procedural activity in South African classrooms, problem-solving in mathematics is often overlooked or taught as a mechanistic learning process – In this approach, the teacher delivers explanations of the material, provides illustrative examples, assigns exercises for learners to complete, reviews their responses, and briefly discusses problem-solving strategies that learners subsequently replicate (Widodo et al., 2020).

Fundamentally, the thinking processes of learners when engaging with mathematics were crucial in assessing their "ability to connect mathematical information" (Widodo et al., 2020, p. 2). Consequently, the unsatisfactory performance of learners in attempting problem-solving questions was linked to their lack of understanding of mathematical concepts and their inability to apply relational reasoning, let alone relational understanding.

This research endeavoured to enhance learners' relational understanding of real-life problems to support their relational reasoning abilities and to activate the successful application of algebraic reasoning skills in solving these problems.

3.7. Solving word problems: An algebraic problem-solving approach.

Algebraic problem-solving is the process of using algebraic methods to find solutions to mathematical problems, typically involving unknown quantities represented by variables (Schoenfeld, 1985). Since problem-solving comprises of a variety of cognitive processes and strategies by applying mathematical knowledge, implementing algebraic problem-solving activities in a lesson is an essential tool in helping the teacher identify if the learner uses algebraic concepts.

Stein and Smith (2019) argues that engaging learners in problem-solving tasks not only enhances their mathematical skills but also provides insights into their conceptual understanding and algebraic reasoning processes. They posit that such tasks act as a window into learners' thinking, allowing educators to assess their grasp of mathematical concepts effectively (Stein & Smith, 2019). This underscores the notion that algebraic problem-solving tasks not only develop mathematical skills but also serve as diagnostic tools for educators to evaluate learners' conceptual understanding of the algebraic process.

Efficiently answering an algebraic contextualised is evidenced when the learner "translates and interprets mathematical information embedded in text" (Boaler & Staples, 2008, p. 73). For teaching algebraic word problems, it is important to acknowledge that when introducing the concept, the use of model strategies provides the learner with tangible representations of abstract mathematical concepts, which simplifies learners' understanding and enables more flexible application of these concepts. (Ferrini-Mundy, 2000).

For this reason, introducing model strategies to learners may serve as essential cognitive tools that support learners in navigating the complexities of algebraic word problems effectively. Solution models provide a systematic way to break down complex problems into manageable parts, making it easier for learners to analyse and solve them. Healy and Hoyles (1999) argues that visual methods, like model strategies, used in tasks focused on generalising numeric sequences can significantly aid in the development of algebraic expressions, effective use of variables, and the conceptual understanding of algebraic problem-solving activities.

Schoenfeld (1985) outlines five key strategies in algebraic problem-solving: identifying unknowns and representing them with variables; translating verbal information into algebraic expressions; applying algebraic techniques to solve equations; interpreting solutions within context; and, finally, generalising by formulating patterns and rules from solved problems. Table 3 below illustrates these problem-solving strategies, and the theoretical aspects enacted in each of the strategies proposed by Schoenfeld.

Table 3: Linking the problem-solving strategies proposed by Schoenfeld (1985) and the theoretical underpinnings.

Strategy proposed by Schoenfeld	**Theoretical underpinnings**
Recognising the unknown quantities in a problem and represent them with variables.	Relational understanding. Horizontal mathematization.
Setting up equations by translating verbal or written information into algebraic equations or expressions.	Relational reasoning. Structural composition of the context. Vertical mathematization.
Applying algebraic techniques to solve equations.	Algebraic reasoning and algebraic problem-solving skills are activated.
Interpreting solutions in the context of the given problem.	Relational reasoning. Critical mathematical thinking.
Transitioning to a process of formulating generalisations.	Algebraic habits of mind are supported for future problem-solving activity.

Applying appropriate strategies to solving algebraic problems and comprehending the connections that are made are central to formal algebra taught in Grade 8. This research puts forth a hypothesis that through the use of effective model strategies in algebraic problems, relational attributes and algebraic problem-solving skills will develop in Grade 8 learners who experience persistent difficulties in algebra.

3.8. Making deeper connections through model strategies in problem solving.

Classroom practices in South African schools do not always "privilege concrete models of representation, which inhibit more abstract ways of working with number" (Ensor et al, 2009, p.8). Adler and Ronda (2016) have studied educational methodologies and their effectiveness in the context of South Africa. They argue that in many South African classrooms, there is a reliance on concrete representations that often limits learners' ability to engage with more abstract mathematical concepts, thereby inhibiting their overall numerical fluency, and emphasize the need for a balanced approach that incorporates both concrete and abstract representations to enhance mathematical learning.

Lesh and Doerr (2003) emphasizes the role of modelling strategies in mathematics education and its impact on learner understanding and skill development. They maintain that modelling serves as an effective pedagogical strategy. When learners create models based on real-life scenarios, they are not merely learning to solve specific problems, they are also cultivating a deeper understanding of relationships between variables and mathematical structures (Lesh & Doerr, 2003). The process of abstraction involved in activating model strategies assists leaners in recognising patterns and relationships, which is fundamental to both relational and algebraic reasoning (Lesh & Doerr, 2003).

The teacher-researcher considered that implementing mathematics model strategies in lessons could stimulate learners' thinking about mathematical processes, as these strategies provide an organizing framework for learners who experience difficulties with algebraic problem solving. Lesh and Doerr (2003) concurs with the view that mathematical model strategies provide a framework that helps learners organize

their thinking about complex problems, allowing them to make connections between different mathematical concepts and processes, which is especially beneficial for those who find algebraic problem solving challenging.

Employing model-based strategies in the solution of algebraic problems enhances the comprehension of new operations and mathematical objects, offering multiple benefits. Notably, this approach facilitates the assessment of learners' relational understanding of algebra while fostering the development of relational reasoning, which they often apply when engaging with algebraic word problems (Chazan, 1993).

3.9 Algebraic formalisation and symbolic representation

In this research, the work of Boero (2002) emphasizes the significance of both cognitive development and contextualized learning, while the VHPOL describe the stages through which learners advance as they enhance their understanding of algebra, progressing from concrete representations to abstract symbols. RME integrates smoothly with the VHPOL approach by using contextualized problem-solving activities that help learners build algebraic understanding through real-world tasks, while gradually guiding them toward more formal

Boero (2002) defines algebraic formalisation as the process by which learners move from informal reasoning and problem-solving methods to the use of formal algebraic expressions and operations. This involves learning to represent mathematical situations symbolically, apply algebraic rules to manipulate these symbols, and identify or generalise patterns and relationships. Boero (2002) emphasizes that this process involves both cognitive activities, which relate to thinking, reasoning, and problem-solving, and semiotic activities, which involve the use of symbols, representations, and signs to express and communicate mathematical ideas, enabling learners to build meaning while engaging with the abstract structures of algebra. In this research, algebraic formalisation was examined as a vital component of algebraic problem-solving, as it provided a structured and universal language for representing, analysing, and solving mathematical problems (Boero, 2002).

3.10. In Summary

The teacher-researcher examined the contextual challenges faced by South African schools regarding the algebraic performance of Senior Phase learners. Time constraints, teacher attitudes, and learners' needs were identified as key issues within the context of mathematics teaching in South African classrooms.

Relational understanding and relational reasoning is foregrounded as essential skills that will assist learners in problem-solving. The development of sound algebraic reasoning skills relies on the ability of the learners' critical thinking skills and ability to use the relational and structural attributes of real-life problems to find solutions to algebraic problems.

Algebraic reasoning skills and problem-solving abilities serve as essential foundations for enabling learners to apply model strategies effectively in solving algebraic word problems. This approach fosters learners' ability to organise relational attributes and facilitates access to problem-solving for those who experience difficulties in addressing contextual problems.

In the chapter that follows, the teacher-researcher elaborates on the methodological aspects of the research.

.Chapter 4: Research Methodology

"The methodology section of a research proposal or report provides a detailed description of how the research was conducted, including the design, participants, instruments, and procedures used to collect and analyse data"

(John W. Creswell, 2014).

4.1. Introduction

The purpose of this research is to investigate Grade 8 learners' algebraic problem-solving skills when engaging with real-life problem-solving activities. Chapter 3 elaborated the elements of algebraic reasoning, comprising of two primary components: the cultivation of mathematical problem- solving techniques and the exploration of foundational algebraic concepts (Kriegler, 2008). For this research, the use of solution models, such as tables, patterns, and arrays, served as essential tools that supported learners in navigating the complexities of algebraic word problems effectively (Gerald, 2014). Model strategies provide a systematic way to break down complex problems into manageable parts, making it easier for learners to analyse and solve them. This approach enhances the flexibility of the learners' mental strategies and supports the formation of good algebraic reasoning skills. Applying appropriate strategies to algebraic problems based on sound relational understanding (Skemp, 1976) and relational reasoning (Hattie & Donoghue, 2016) for the purposes of making connections, is central to formal algebra in Grade 8. Therefore, this was the focus of this research.

This chapter outlined the proposed methodology and explained why it was selected as the central focus. The chapter further compared different research designs to illustrate why the teacher-researcher employed an intervention study as the chosen research method. First, the proposed research design—a mixed-methods approach—will be outlined, followed by a detailed background on the effectiveness

and relevance of using an intervention study as the core method of this project.

4.2 An intervention study: the research focus for this project.

Intervention studies, often referred to as an experimental study, has a rich history rooted in the fields of medicine, psychology, and education (Scott & Morrison, 2005).

The main objective of an intervention study is to assess whether a particular intervention results in a measurable change in an outcome variable. Researchers manipulate one or more independent variables to examine their effects on dependent variables (Shadish, Cook, & Campbell, 2002). The important aspect of an intervention study is that it is concerned with the details of information collected and the nature of the data collected.

Intervention studies involve the deliberate introduction of a new, improved, or alternative methods by the investigator. These methods are compared to existing or standard approaches, and their effects are assessed based on relevant outcome measures (Hsieh et al., 2005).

For this research, the key outcome led to tentative generalisations, whereby the validity of the hypothesis was tested. As mentioned in chapter 3, this research puts forth a hypothesis that through the use of effective model strategies in algebraic problems, relational attributes and algebraic problem-solving skills will develop in Grade 8 learners who experience persistent difficulties in algebra.

The teacher-researcher used an intervention study since the research was concerned with the detailed information collected about leaners problem solving skills. The research gathered data from a group of learners in the form of a pre-intervention test, where real-life problems were given to the group to solve. The data obtained from the pre-test served as a baseline. The initial data analysis informed the design of intervention lessons aimed at enhancing problem-solving skills in real-life contexts. Following the intervention, a post-test assessed changes in learners' approaches to algebraic word problems.

4.3. The strengths and weaknesses of intervention studies.

After establishing the reason for intervention studies in the education realm. It is important to understand both the benefits and drawbacks of intervention studies in order to recognize their implications and determine the most appropriate contexts for their implementation. Fundamentally, intervention studies are social enquiries, particularly when they focus on social, educational, or behavioural outcomes and therefore are best utilized in small-scale studies like a classroom-based study. Intervention studies also show strength in addressing social problems or improve outcomes in specific populations, such as enhancing educational achievement (Shadish, Cook, & Campbell, 2002). In the realm of mathematics education, an intervention study provides empirical evidence which can inform instructional strategies and curriculum development. By assessing the impact of specific interventions, educators can make data-driven decisions that enhance teaching and learning in mathematics (Creswell, 2014).

Intervention studies, however, may exhibit limitations in capturing long-term effects, as their outcomes are often assessed over short periods. This limitation can lead to an incomplete understanding of the intervention's overall impact (Hernán & Robins, 2016). However, intervention studies draw inferences by transferring findings from one context to another, with the validity of this transfer depending on how well the study aligns with its intended purpose (Creswell, 2014). Intervention studies may reflect constraints in this sense because the findings from intervention studies may not be easily generalizable to broader populations due to specific sample characteristics or settings. If the study sample is not representative, the results may not apply to other groups (Shadish, Cook, & Campbell, 2002).

Intervention studies as a research tool may present certain limitations, as their utility depends on how readers interpret and apply the findings for research outcomes or other purposes. More specifically, such studies are constrained by their focus on exploring the effects of interventions on performance rates, which aligns with the broader aims of social inquiry that seek to investigate the complexities of human

behaviour and social dynamics (Bryman, 2016)

In light of the above, careful consideration was given to the aim of the research and to how the selected method would effectively address the research questions.

4.4. Merged Methods: The research design as an analytical strategy throughout this research.

The teacher-researcher analysed the intervention lessons using a merged-methods model approach, in which qualitative and quantitative data analyses were integrated throughout the study. Quantitative methods were employed to support the qualitative discussion of the results. In the merged approach, qualitative and quantitative data are combined in a single analysis, often by transforming one data type into the other, allowing for direct comparison and integrated interpretation (Creswell & Plano Clark, 2018, p. 68). Blending their strengths provides a more comprehensive understanding of complex educational phenomena (Tashakkori et al., 2021). Thus, merging qualitative and quantitative data provides deeper insights than analysing each type separately. This integration enhances and broadens the scope of the research by ensuring a more comprehensive dataset and strengthening the justification for employing multiple forms of data collection (Creswell, 2009). Merged methods thus emphasize the seamless integration of data collection, analysis, and interpretation to produce richer, more nuanced insights (Johnson & Christensen, 2020). Additionally, it allows researchers to explore unexpected patterns by iteratively analysing both numerical and narrative data, leading to more robust conclusions (Bryman, 2016).

Prior to implementing a merged-methods approach, it is essential to consider both the practicality and technical feasibility of data acquisition. Given the hybrid nature of merged-methods, careful attention was paid to how effectively the chosen approaches address the research problem and whether they were appropriate and valid for the context. This required aligning research questions with the methods most suited to answering them, either individually or in combination (Scott & Morrison, 2005).

4.5 Research instruments.

The teacher-researcher implemented three intervention lessons designed to provide learners with opportunities to engage in algebraic reasoning using specific strategies. Mental warm-up exercises, such as True or False statements, were incorporated to present learners with algebraic expressions or equations (e.g., $2(x + 6) = 2x + 12$) and prompt them to determine the validity of said expressions. This approach fostered critical engagement with key algebraic properties and operations, thereby not only preparing learners for deeper algebraic reasoning but also establishing a supportive and collaborative classroom environment that encouraged active participation and meaningful discussion. Furthermore, it enabled the teacher-researcher to assess the extent of learners' conceptual and relational understanding in order to discern whether responses reflected genuine reasoning or mere guessing. Throughout this intervention learners were expected to provide a reason for choosing a specific solution strategy and present a short argument for why their choice was appropriate.

The intervention program was designed to target specific proficiencies essential for algebraic problem solving, namely contextualisation, strategic calculation, and structuring through algebraic reasoning. Contextualisation involved learners applying mathematical reasoning to real-world situations, drawing on both cognitive processes and imaginative capacities to make meaningful connections between abstract concepts and lived experiences (Radford, 2003). Strategic calculation referred to the use of efficient, flexible problem-solving strategies that reflect relational understanding - knowing not only how to perform a procedure but also why it works (Skemp, 1976) - thereby enabling learners to adapt methods to novel situations. Structuring through algebraic reasoning focused on identifying, representing, and leveraging underlying mathematical relationships (relational reasoning) to solve problems without over-reliance on calculators (Carpenter, Franke, & Levi, 2003). This process often required structural extension (Du Plessis, 2018), where learners generalised familiar patterns or relationships to new and more complex contexts, thereby deepening their conceptual grasp and supporting

transfer of knowledge (Radford, 2014; Venkat & Adler, 2012).

4.5.1 The Concept of a Solution Model

In mathematics education, a solution model refers to the representation—either mental or external—of the steps, relationships, and reasoning a learner uses to arrive at a solution for a given problem (Hegarty, Mayer, & Monk, 1995). It is more than simply arriving at an answer; it is the structured pathway through which a problem is conceptualised, broken down, and resolved. Solution models may be expressed in multiple forms, such as diagrams, algebraic expressions, verbal explanations, tables, or physical manipulatives (Goldin, 2003).

A solution model is closely tied to relational understanding (Skemp, 1976) because it embodies not just the procedure but the reasoning behind it. Learners with strong relational understanding are able to construct generalised solution models that can be adapted to novel problems, which aligns with relational reasoning - the ability to identify structural similarities across tasks (Carpenter, Franke, & Levi, 2003). Furthermore, solution models can act as vehicles for structural extension (Du Plessis,2018), as learners extend known solution pathways to more abstract or complex contexts (Radford, 2014).

From a semiotic perspective, solution models are external manifestations of internal cognitive activity; they provide visible evidence of the learner's thinking processes and allow for reflection, refinement, and teacher scaffolding (Radford, 2003; Duval, 2006). In a RME informed classroom, multiple means of representation and expression encourage learners to develop and share diverse solution models, which can improve accessibility and promote deeper conceptual understanding across varied learner profiles.

4.5.2 Applying Solution Models to Real-life Problems:

To assess learners' problem-solving approaches to real-life problems, solution

models were employed through word problems that incorporated familiar contexts. This approach aimed to enhance learners' appreciation of mathematics as relevant and meaningful, while supporting their relational understanding of problem statements. The intervention consisted of four lessons, each emphasizing the recognition and interpretation of rules or relationships expressed symbolically, as well as the careful analysis of problems to determine and work with the relational attributes embedded between numerical elements. The lessons were implemented sequentially over a three-week period during Term 3. The intervention lessons employed a balanced instructional approach, alternating between teacher-led sessions and independent learner activities with oversight by the teacher. Each lesson concluded with a worksheet for independent completion, administered after the teacher-led discussion, during which the teacher-researcher addressed misconceptions and common errors. Each lesson was allocated approximately 45 minutes and consisted of a brief mental warm-up (up to 5 minutes), followed by a teacher-led explanation (15 to 20 minutes) during which solution strategies were discussed.

To create a multi-cultural environment, the teacher-researcher intended to implement games that will illicit the strategy taught for the lesson of the day. For instance, True or False, which assists to illicit the critical analyses and evaluation of algebraic properties and operations. This approach was used during the mental warm-up for the intervention lesson that followed. This game assisted in developing the knowing why and knowing when, a critical aspect in promoting algebraic relational reasoning. Skemp (1976) attests that learning consists of formulating a schema from which the learner can produce a variety of processes through moving from an initial point within his or her schema to any finishing point .

4.6. Pre- and post-tests as a research instrument.

Tests were utilized to measure learners' understanding and development at specific stages of the problem-solving process. For this research, the teacher-researcher employed standardized tests, as this type of assessment is tailored to small groups.

The standardized tests in this study were criterion-referenced, adapted from the CAPS prescribed Mind in Action series textbook (Phillips, p. 97-106, 2014) which, according to Scott & Morrison (2005), involved assessing an individual's performance against a predetermined set of indicator descriptors.

For the pre-test (see page 94), the teacher-researcher drew on learners' ability to recall fundamental skills necessary for performing and interpreting relational reasoning through word problems and strategic calculations in algebra. According to the Department of Basic Education (DBE) (2011), learners in Grade 8 mathematics classrooms are expected to engage with complex relationships between numbers within given patterns for generalisation, thereby developing more advanced skills to transition from an understanding of numbers to variables, particularly for expanding and simplifying algebraic expressions when solving equations. This pre-test served as a tool to provide an initial measure of learners' existing knowledge, skills, and attitudes prior to the intervention (Majka, 2024).

The post-test was administered following the intervention lessons (see page 95). The primary focus of this assessment was to evaluate whether the Grade 8 learners had developed a deeper understanding of the relational reasoning required to comprehend algebraic concepts and whether improvements were evident in their algebraic problem-solving skills. According to Stein et al. (2000), varying the difficulty of test items in post-tests is essential for accurately capturing the full scope of learners' mathematical understanding. Their cognitive demand framework differentiated between tasks requiring basic recall, procedural knowledge, and higher-order mathematical reasoning. Consequently, the test was designed with questions arranged progressively from less complex to more complex to accommodate the diverse needs of both high-attaining and low-attaining learners and also give an accurate reflection whether the intervention lessons deepened relational understanding of the tasks and not from mere recall of the methods used in the pre-test.

Tests are a tool used in the education environment to identify the level of progression of learners at a specific period. Tests convey characteristics of reliability

and validity which has pros and cons. First, reliability refers to the measure of accuracy in relation to the purpose it intends to measure. Validity refers to the extent to which the items in a test accurately measure the intended learning goals (Scott & Morrison, 2005). Furthermore, tests may lack sufficiency when the constructs being assessed are not robustly defined or measured, potentially resulting in an incomplete or incoherent representation of the concept under investigation (Scott & Morrison, 2005). With this in mind, the tests in this research went through cycles of design until it became clear that they will address the research questions sufficiently.

4.7. Design and Use of an evaluative tool to assess problem-solving skills.

The tests in this study were analysed using an evaluative problem-solving skills framework developed by Tong and Han (2020). This tool was designed to assess students' problem-solving abilities in mathematics and was applied in a study involving Grade 8 learners working with first-degree equations in one unknown. Their findings indicated that learners taught using this evaluative framework showed significant improvements in problem-solving performance compared to those taught through traditional instructional methods (Tong & Han, 2020).

In this research, the Tong and Han (2020) tool focused on four essential components. The first, problem identification, evaluated learners' competence in accurately recognizing and defining the problem being addressed. The second component, strategy selection, assessed the suitability and effectiveness of the strategies chosen to approach the problem. During the implementation phase, the execution of the selected strategies was observed, along with the correctness of the procedures followed. The final phase, solution review, considered learners' capacity to reflect on the obtained solution, including verifying results in the context of the problem and exploring alternative methods to solve the problem.

Table 4: Evaluation criteria of problem-solving skills.

Component Capacity	Evaluation criteria	Scoring points

Ability to detect the given problem	Identify the algebraic model that needs to be solved	+ 1
	Incorrect identification of algebraic problem	0
	Not fully understanding the problem, and small flaws	+ 1
	Correct understanding	+2
	Collect information related to the problem (list the mathematics facts and figures related to the problem	+1
	Incomplete collection of relevant information related to the problem	0
	Information was collected but incomplete or inaccurate	+½
	Collect information fully and accurately and convert given problem information to the mathematical model.	+1
	Contact relevant knowledge and information.	+1
Ability to propose solutions	Express the problem in the mathematical language. Find a solution to solve math problem	+1
	Wrong, ineffective solution.	0
	Correct solution but with small flaws.	+ ½
	Correct and detailed solution.	+ 1
Ability to solve problems.	Correct calculation but the solution model and tool are not optimal.	+ 1
	Correct calculation but the solution model and tool are not optimal.	+ 2
Ability to evaluate performance results	Answer the request of the problem.	+ 1

4.8. Target population

The teacher-researcher selected a school located in the south-western townships of Johannesburg, specifically Soweto. This choice of purposive sampling was informed by the teacher-researcher's employment at the school and ensured for easy access. The school is classified as a Quantile 2 institution, characterized by a multicultural student body and generally functioning as a well-resourced public school. The sample consisted of nine Grade 8 learners in the mathematics class, consisting of five boys and four girls. These targeted learners were actively involved in the intervention lessons and constituted the learner intervention group.

Data was collected after school hours to provide a focused environment free from regular class interruptions, allowing for more in-depth and focused engagement. The selected learners represented a diverse range of mathematical abilities, comprising three average performers, four above-average performers, and two below-average performers. This varied sample was chosen to capture a broad spectrum of learner responses and to better understand how the intervention impacted students with differing proficiency levels.

The exploration of the results between pre-test and post-test results was deemed effective if a positive shift was observed from the pre-test to the post-test outcomes. The percentage shift in improvement following the intervention lessons was analysed quantitatively using an evaluative problem-solving skills tool developed by Tong and Han (2020). Given that the study focused on the relational reasoning and problem-solving techniques employed by Grade 8 learners when solving real-life algebraic problems, the evaluative tool considered four key aspects: (a) the ability to identify the given problem, (b) the ability to propose solutions, (c) the ability to solve problems, and (d) the ability to evaluate performance outcomes (See Table 4).

During the intervention lessons, the teacher-researcher emphasized contextual word problems, a central feature of Realistic Mathematics Education (RME) and an indicator of algebraic relational understanding. In RME, context problems serve as the foundation of mathematical activity (van den Heuvel-Panhuizen, 2001), with guided reinvention enabling learners to reconstruct knowledge through exploration and problem-solving. This approach complemented the VHPOL phase 2: guided orientation, as both stress scaffolding learners' reasoning and actively involving them in constructing algebraic understanding. While VHPOL phases one to three supported learners' thought processes, RME focused on developing and applying mathematical models to meaningful real-life situations. To evaluate the effectiveness of these approaches in fostering algebraic reasoning, the teacher-researcher qualitatively analysed intervention worksheets and transcriptions and applied Tong and Han's (2020) problem-solving skills tool. Guided facilitation

further addressed misconceptions, cultivated habits of mind, and encouraged solution models that strengthened learners' relational understanding in algebraic word problems.

The worksheets collected was marked to inform the teacher-researcher on the progress learners have attained at different stages of the learner intervention lessons program and measure the extent to which the intervention was successful. Through a process of quantitative analysis, comparing individual learners' pre-test and post-test results, an assessment of their combined abilities in solving algebraic word problems was realised. Quantitative methods were deemed most appropriate for this comparison as they allow for objective measurement of performance changes and facilitate clear visualization of trends and patterns (Scott & Morisson, 2005). The data was presented using bar and line graphs, which effectively illustrated individual progress and variations across the intervention period, thereby enabling a comprehensive interpretation of learners' development.

4.9. Rigour.

Reliability and validity are key indicators of the rigour of research processes and the credibility of findings (Roberts & Priest, 2006). Reliability, in particular, is anchored in the trustworthiness of a study and is influenced by several factors, including the formulation of the research question, the methods and timing of data collection, the selection of learners, the analytical procedures employed, and the conclusions drawn (Roberts & Priest, 2006)

To maintain reliability over the three-week research period, the teacher-researcher implemented measures to support consistent learning outcomes. During the intervention lessons, learners were guided to use relational thinking strategies, which were reinforced through mental warm-ups and practice tasks. These strategies enabled learners to apply the acquired skills effectively when attempting the post-test questions three weeks later.

Scott and Morrison (2005) describe validity in five categories: (a) content validity, (b) predictive validity, (c) concurrent validity, (d) construct validity, and (e) face validity. For this study, content validity was particularly important as it assessed

whether the test items accurately measured whether the intended goals were attained. In the context of this research, this referred to measuring learners' improvement in problem-solving and their recognition of relational reasoning in algebra from the pre-test to the post-test and during intervention lessons.

Muhammad (2008) emphasizes that validity reflects the extent to which research measures what it was intended to measure and how accurately the results reflect the targeted constructs. To uphold validity, the teacher-researcher carefully selected pre-test and post-test questions, as well as structured the intervention lessons, to ensure fairness and alignment with the study objectives. Moreover, Richland, Kornell, and Kao (2009) demonstrated that even unsuccessful pretests can enhance subsequent performance, particularly when post-tests include complex or transfer-based questions. Their findings support designing tests with questions ranging from lower to higher levels of complexity to foster deeper learning, relational discovery, and a stronger understanding of mathematical processes, concepts, and techniques. For this reason, the post-test in this research varied in difficulty.

4.10. Ethical considerations.

As outlined in Chapter 1, ethics in research provide a set of principles that guide the design and implementation of research projects. Researchers are expected to adhere to established codes of conduct, particularly when collecting data or engaging with learners and research instruments. Ethical considerations are essential for protecting the rights of learners, ensuring the validity of findings, and upholding scientific integrity (Bhandari, 2021). One key aspect is obtaining informed consent, which requires learners to be made aware of the nature and purpose of the research. However, Scott and Morrison (2005) note that this process is not always absolute, as researchers may be unable to provide learners with a complete account of the study due to the evolving nature of many research designs.

Ethical considerations in this study were designed to respect the rights of learners, enhance the validity of the research, and uphold scientific integrity (Bhandari, 2021). Parental consent was obtained through formal consent forms, while learners were provided with assent forms that assured them of their voluntary participation

and the confidentiality of their data. To protect anonymity, all identifying information related to the learners and the school was removed. The research site was described in general terms as a public secondary school in Soweto, and learners were assigned unique codes (e.g., Learner1 to Learner9) for use during data collection, analysis, and reporting. Pseudonyms were applied where learner responses were included to further safeguard identities. The teacher-researcher maintained clear boundaries between professional and research roles, ensuring that participation was voluntary and learners' privacy was respected throughout the study. The teacher-researcher accessed all transcriptions exclusively, and protective measures were implemented by coding the data according to the problem-solving evaluation criteria (see Table 4), thereby avoiding the collection of specific details that could potentially reveal learner identities. Furthermore, learners retained the right to withdraw from the study at any stage without any negative consequences to their academic progress.

4.11. Limitations to selecting a larger sample.

The limitation of this research was the small sample size. Since only nine learners were selected to participate in this small-scale research, the findings were not generalisable to larger populations and may have limited applicability for policy development. Additionally, the school's curriculum and schedule had the potential to interfere with the implementation of the intervention lessons. To address this challenge and given that the research took place at the school where the teacher-researcher was employed, the intervention lessons were scheduled as extra-curricular activities conducted after school hours.

4.12. Summary.

Chapter 4 primarily focused on the methodology employed in this study, offering a

detailed rationale for selecting an intervention study as the research design. This approach was chosen due to its suitability for collecting in-depth data relevant to the research objectives. Given that the data analysis incorporated the merged methods approach, an evaluative tool was utilized to assess the problem-solving strategies learners applied during the intervention lessons. Additionally, bar graphs were employed to visually represent learners' progress from the pre-test to the post-test, as well as to illustrate their combined abilities.

Chapter 5 offered an in-depth analysis of the collected data, examining learner performance by comparing results from the pre-test and post-test. Additionally, the chapter identified emerging patterns observed during the intervention lessons, supported by a detailed transcription and analysis of the digital recordings. This comprehensive approach allowed for a nuanced understanding of learners' use of solution models, their problem-solving development and management throughout the research.

Chapter 5: Findings and Discussion.

"The greatest obstacle to learning is not ignorance but the illusion of knowledge."

Daniel J. Boorstin

This chapter presents the results of the research and provides a critical analysis of the findings in relation to the research objectives and existing literature. It serves as the core of the research, where data is systematically analysed, interpreted, and contextualised to derive meaningful insights.

5.1. Introduction.

This chapter introduces the reader to the findings of the research, during an intervention study programme in September/October 2024. In the previous chapter, the teacher-researcher had outlined the methodological processes which informs the findings of this research and, staying within the merged methods research approach, this chapter provides the results of the pre-test which was conducted before the intervention lessons. Then, a post-test was conducted to assess the knowledge gained, if any, after the three-week intervention program. Initially, a pre-test was given to the learners to draw on their ability to recall fundamental skills needed to apply and interpret algebraic word problems. At the end of the intervention lessons, a post-test was handed to the learners which evaluated whether the grade 8s have achieved fluency in effective problem solving through activating relational understanding and relational reasoning when solving algebraic real-life problems. The teacher-researcher provides an overview of the research based on the analysis of the intervention lessons and the pre- and post-test results.

Throughout this chapter the following three research questions will be answered:

a) What are the algebraic problem-solving strategies that Grade 8 learners engage in when solving real-world problems?
b) How does the use of model strategies, if any, vary among Grade 8 learners?
c) How do the problem-solving approaches introduced during the three-week intervention enable learners to develop more relational approaches to problem-solving.

This research is grounded in the learning theories of the Van Hieles' (1986) phases of learning and the principles of realistic mathematics education, as articulated by Van den Heuvel-Panhuizen (1999). Specifically, the research focused on enhancing relational understanding and reasoning, structuring, and the use of model strategies as core approaches for effective problem-solving. These proficiencies serve as the essential criteria that shape and guide the research questions, whilst highlighting their pivotal role in understanding and improving problem-solving approaches with grade 8 learners.

5.2. Individual Learner Pre-Test scores.

The graph below represents the individual learner results from the pre-test as assessed using Tong and Han (2020) evaluative tool.

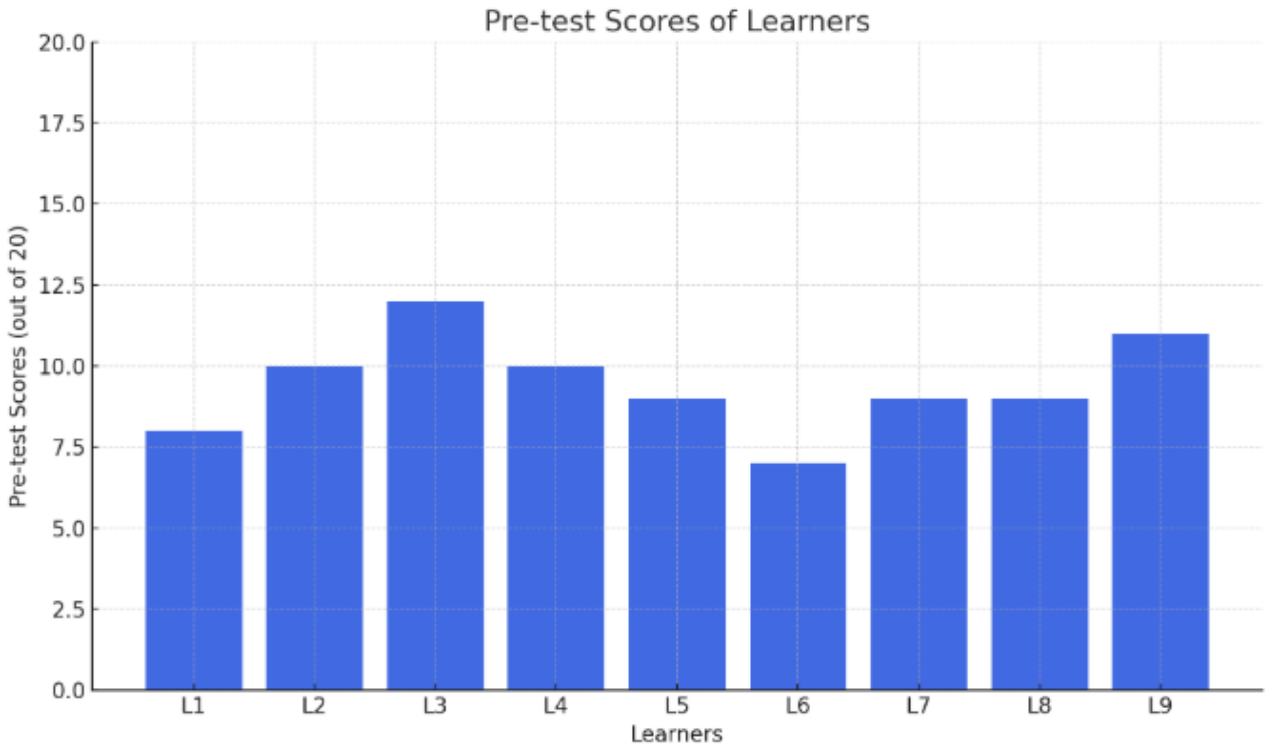


Figure 5.1. The Individual Learner Pre-Test Results.

The pre-test results showed that Grade 8 learners had limited proficiency in contextualised algebraic problem-solving. Seven out of nine learners experienced difficulty connecting abstract mathematical concepts to real-life contexts. This suggested that learners had not been given sufficient opportunities to mathematise real-world situations, weakening their ability to interpret and solve contextual problems.

In response, teaching approaches were adjusted to prioritise conceptual understanding and meaningful engagement with problem contexts, rather than relying solely on memorised procedures.

5.2.1. Individual Learner Pre-test analysis.

An analysis of learners' problem-solving abilities revealed that six out of nine learners were able to detect the central problem, with average scores ranging between four and five out of ten possible points. Learner five performed well by achieving the highest score of eight out of ten in this category. Despite this, eight of the nine learners were unable to accurately translate the contextual information into a mathematically appropriate model. Only learner five demonstrated proficiency in this regard, achieving a modelling score of seven out of ten. Consequently, the overall quality of proposed solutions were weak, reflecting underdeveloped problem-solving skills. Although six learners were able to extract most of the relevant information and partially represent it mathematically, only learner five was able to correctly identify the quantitative relationships, indicating a more advanced capacity to evaluate and respond to algebraic word problems effectively (see Figure 5.2).

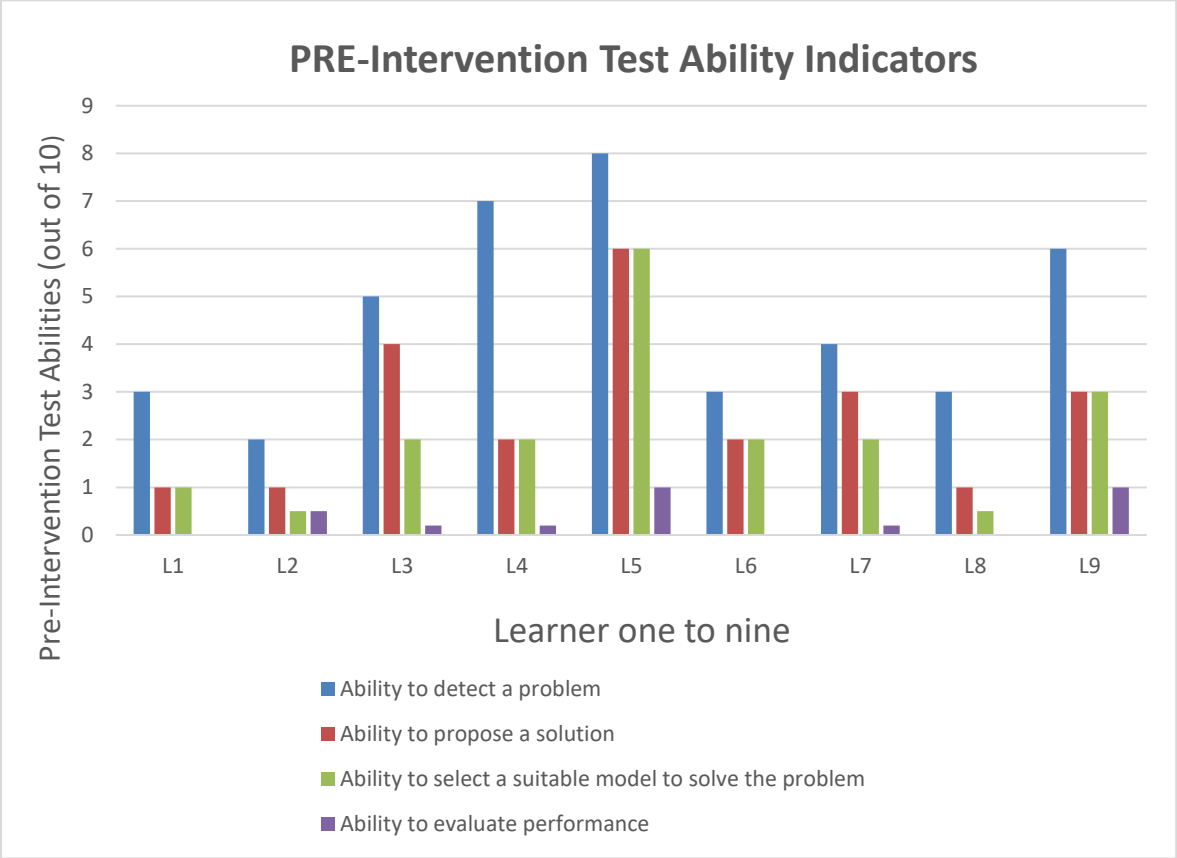


Figure 5.2. Individual Learner abilities in real-life problems Pre-Intervention test using Tong and Han (2020) evaluative tool for problem-solving.

5.3. Comparing the pre-test and post-test scores.

Figure 5.3. below shows that the highest score achieved in both the pre-test and post-test was 14 out of 20. Additionally, most learners achieved similar results on both tests. Notably, Learner four demonstrated the most significant improvement, increasing their score from the pre-test (10 out of 20) to the post-test (14 out of 20).



Figure 5.3. Comparison of individual learner pre and post-test scores.

The pre-test and post-test results were evaluated using the Tong and Han (2020) problem-solving assessment tool, with the outcomes presented in a statistical bar graph (see Figure 5.3). This graph clearly illustrates the learners' results from the pre-test to the post-test on the same graph.

From this graph it is clear that L1, L2 , L6, L7 and L8 had no significant improvement from the pre- to post-test, while L4 improved satisfactory in their algebraic problem-solving skills. The lower post-test performance of five learners compared to their pre-test results can be attributed to the rapid introduction of a relational approach to understanding algebraic concepts, which may have caused cognitive overload and confusion. The transition from reliance on procedural methods, such as the application of memorised rules, to relational reasoning involving the interpretation of variables, expressions, and relationships in unfamiliar contexts likely required more time for consolidation. Insufficient practice and reinforcement may have hindered the effective application of newly acquired knowledge, resulting in decreased performance. Learner L3, L5, and L11 achieved the same score in their pre- and post- intervention tests.

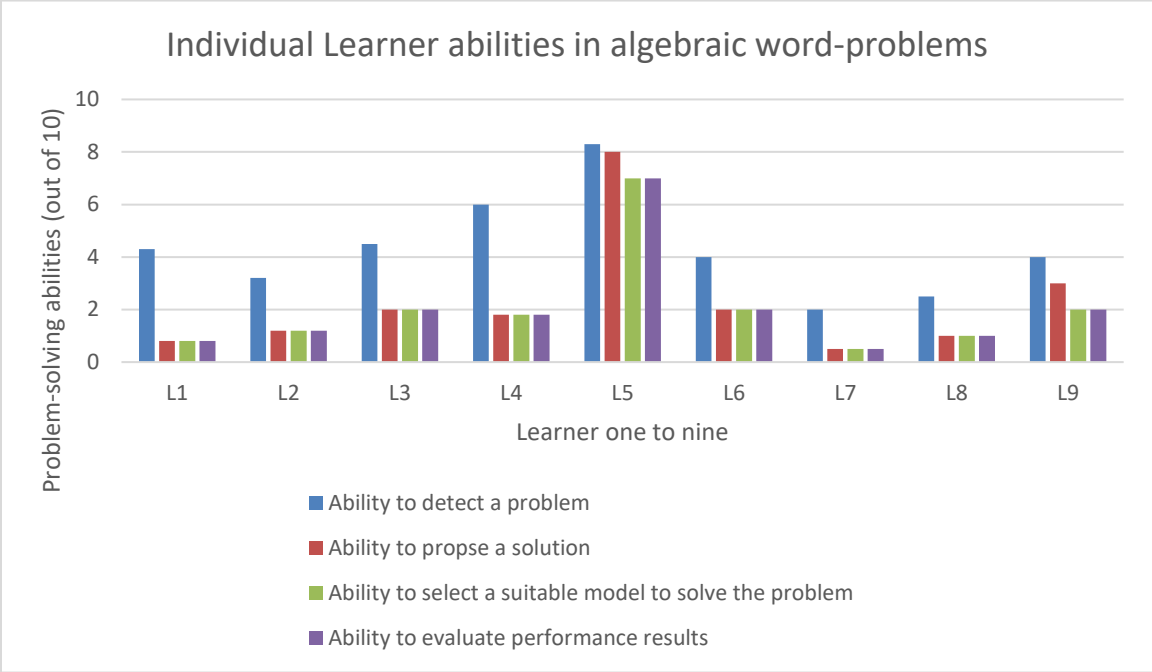


Figure 5.4. Comparison of individual problem-solving abilities in the pre- and post-test.

Figure 5.4. illustrates the abilities in bar graph format of individual learners as assessed using the Tong and Han (2020) evaluative tool, based on four criteria: (a) the ability to identify the problem, (b) the ability to select an appropriate model for solving the problem, (c) the ability to propose a solution, and (d) the ability to evaluate their performance by accurately providing the final answer.

Figure 5.5 below illustrates the ability of the nine learners to propose a solution using a suitable model. Five out of nine learners demonstrated this proficiency below 2 out of 10. This statistic is troubling, given that the study was fundamentally designed to strengthen learners' skills in solving contextualised algebraic problems. Observantly, the five learners may be able to select a suitable model because this step often relies on recognising familiar structures, keywords, or relationships between quantities, which can be matched to a known equation or expression. However, proposing a solution requires deeper conceptual understanding, including manipulating algebraic expressions accurately, applying operations in the correct sequence, and interpreting the meaning of the solution within the problem context. This brings to mind that learners may believe they understand a concept when, in reality, they have not yet developed the deeper comprehension necessary for applying that

knowledge effectively in real-world problem-solving. This disconnect makes it challenging for them to translate theoretical knowledge into practical problem-solving skills, as argued by Stein et al(2018). These visual representations (see Figure 5.5) effectively highlight the comparisons between the pre-test and post-test results, specifically in the context of solving algebraic word problems.

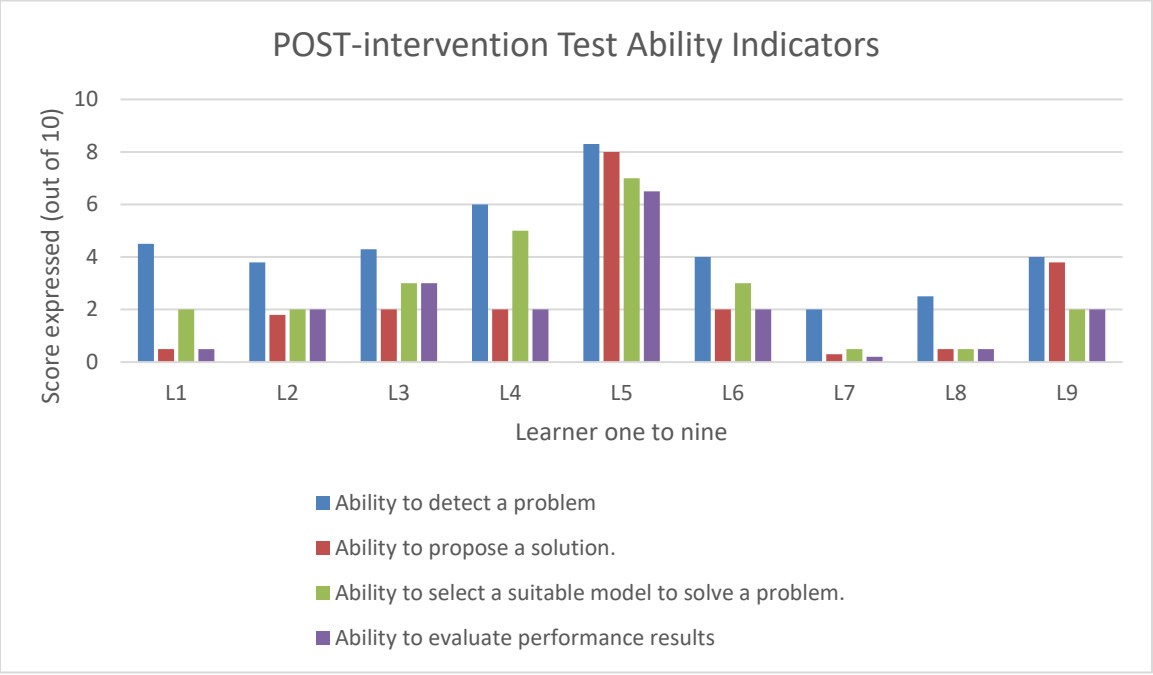
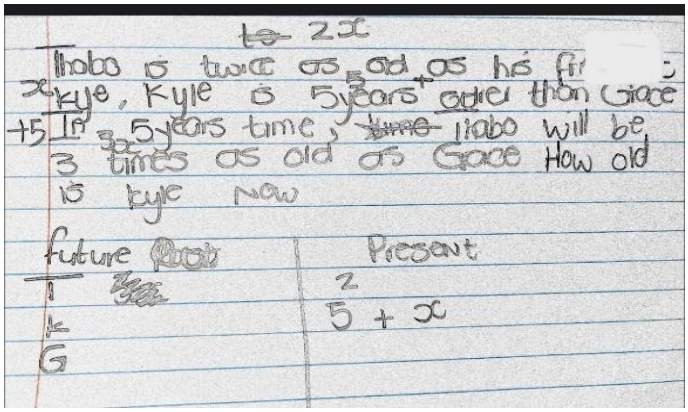


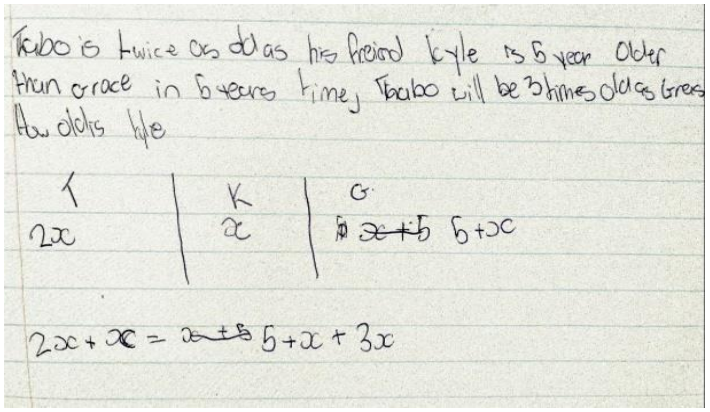
Figure 5.5. Individual learner abilities in real-life problems POST-Intervention test using Tong and Han (2020) evaluative tool for problem-solving.

Furthermore, to identify an appropriate problem-solving strategy, such as converting information about the mathematical model and relating the quantities, many learners displayed operational errors. These errors hindered their ability to develop a complete solution to the problem and limited their flexibility and critical thinking in problem-solving (Van den Heuvel-Panhuizen, 2003).

This phenomenon is illustrated in Extract A and B in Figure 5.6. which highlight the post-test responses of L4 and L5.

Problem solving Question: *Thabo is twice as old as his friend Kyle. Kyle is five years older than Grace. In five years, Thabo will be three times as old as Grace. How old is Kyle now ?*



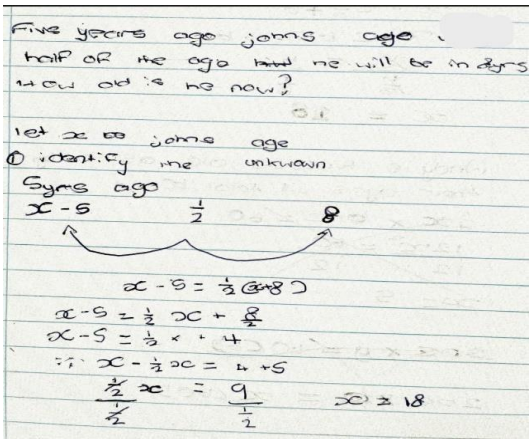


Extract A: Learner 4's response Extract B: Learner 5's response.

Figure 5.6. The post-intervention responses from Learner 4 and 5.

Even though many learners did not know how propose a solution and thereafter solve it, they still displayed sufficient knowledge in detecting the problem which resulted in the learners' failing to create a relationship between the real-world scenario and the mathematical representation. One aspect of RME is the ability to apply real-world scenarios in diverse context and, Extract C below, reflected L5's post-test with the structured answer.

Problem solving question: *Five years ago, John's age was half of the age he will be in eight years. How old is he now ?*



Extract C: Learner 5's post-test with a structured answer

Moreover, L5's accurate response demonstrated competence by successfully identifying the problem accurately, recording relevant relational information using algebraic symbolisation, and constructing an effective model to solve the variable to determine the unknown. This process enhanced her ability to recognize relationships and devise a strategy to solve the word problem. By modelling the connection between known and unknown quantities, L5 was able to develop a mathematical equation, bridging the real-world scenario with its algebraic representation. This approach further deepened L5's understanding of the relationship between variables and improved comprehension of the problem's structure (Blum and Ferri, 2009).

It was demonstrated through the post-test analysis that the models introduced during the three-week intervention lessons assisted the learners in developing strategies when detecting the problem and gathering the relevant information accurately. However, the gap between the learners' ability to propose a suitable solution and thereafter present it using a model which accurately depicts the problem, is still large. A frequent challenge throughout the learners' post- test involves converting verbal statements into algebraically representative expressions. According to Clement (1982), errors such as misinterpreting key terms, misunderstanding operations, or overlooking relationships between variables often result in incorrect equations.

Additionally, it would be useful to tabulate specific competencies of learners from the pre – to the post-test. The evaluative table below (see Table 5.1) reflects the results of the learners relational reasoning in their approach to algebraic word-problems.

Table 5.1. Evaluative criteria of Problem-solving skills in the post-test.

		Problem-solving skills in the Post-Test .	
Ability to detect given problem		7/9 learner could find out, determine the problem to be solved.	2/9 learners could not find out, determine the problem to be solved.
Possible: $\frac{20}{20}$	Actual: $\frac{10}{20}$		
Ability to propose solutions		5/9 learners could convert the problem information to the mathematical model	4/9 learners could not convert the problem information to the mathematical model
Possible: $\frac{20}{20}$	Actual: $\frac{11}{20}$		
Ability to solve the problem		8/9 learners could calculate correctly and use an optimal solution model and tool	1/9 learners could not calculate correctly and use an optimal solution model and tool
Possible: $\frac{20}{20}$	Actual: $\frac{16}{20}$		
Ability to evaluate results		5/9 learners could review and answer the problem.	4/9 learners could not review and answer the problem.
Possible: $\frac{20}{20}$	Actual: $\frac{11}{20}$		

The implications of these findings are useful in providing insight into the misconceptions learners have when answering problem solving questions. Lesh and Doerr (2003) acknowledge that under guided-orientation and a collaborative environment, learning mathematics through meaningful tasks develops the necessary mathematical reasoning to answer the problem-solving question. However, this would be fruitless if there is a disconnect between new knowledge and already established knowledge. Bransford, Brown and Cocking (2000) posits that learning specific concepts and comprehending them (i.e. algebraic terms, symbols etc) at a foundational level imposes new patterns of organising knowledge in a relational manner when assessed in a different context for example in algebraic word problems (see Annexture 1 and 2 on p93 and p94). Challenges such as not completely relating information into a mathematical model and therefore proposing solutions reflect potential misconceptions in relationally organising information.

5.4. Initial algebraic problem-solving strategies.

The focus on the algebraic problem-solving strategies that Grade 8 learners typically use before participating in the intervention program aims to assess their existing skillset. This assessment provides valuable insight into their current understanding and highlights areas where the intervention program can address and potentially correct any misconceptions.

To analyse the types of algebraic problem-solving strategies from the pre-test, the teacher-researcher considered the various stages relative to algebraic formalization (Boero, 2002). The reason for this is that the Van Hiele (1986) phases of learning and Van den Heuvel-Panhuizen's (2003) realistic mathematics education, which form the learning theories for this research, enhance Boero's (2002) work on algebraic formalization by providing insights into how learners transition from informal to formal mathematical reasoning, particularly in algebra.

5.4.1. The misapplications of arithmetic operations.

Pre-Test Focus Question 2.1.

Erin is 5 years older than Sina. Sina is 7 years older than May. Their ages add to 31 years.
How old are each of them?
Solution:

Let May be x years old:

E S M

$x + 7 + 5$ $x + 7$ x

$\therefore x + 7 + 5 + x + 7 + x = 31$

$\therefore 3x + 19 = 31$

$\therefore 3x = 31 - 19$

$\therefore 3x = 12$

$\therefore x = 4.$

Therefore, Erin is 16 years old, Sina is 11 years old, and May is 4 years old.

5.4.2. Relational identification as a problem-solving strategy.

Pre-test focus Question 2.3.

Carla, Dee, and Mpho are three girls. Carla scores 5 goals. Dee scored twice as many goals as Mpho. Altogether, the three scored 17 goals. How many goals did each girl score?

Solution:

Let Mpho score x goals:

C D M

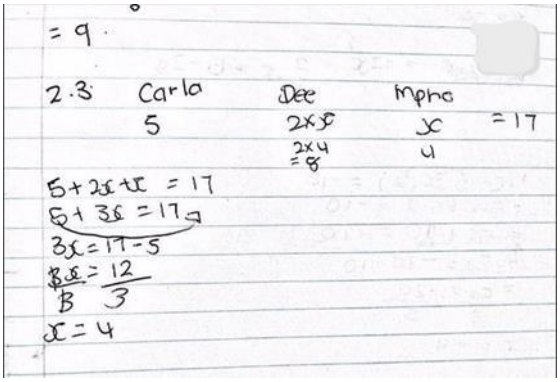
5 $2x$ x

$\therefore 5 + 2x + x = 17$

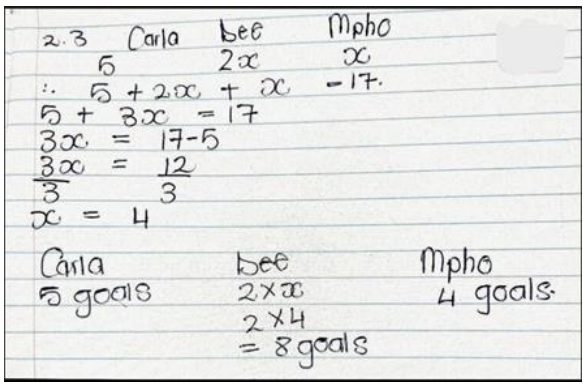
$\therefore 3x = 12$

$\therefore x = 4$.

Carla scored 5 goals; Dee scored 8 goals and Mpho scored 4 goals.



Extract D: Learner 3's response



Extract E: Learner 4's response

Figure 5.7: Learners' 3 and 4 response in Pre-Test focus Question 2.3.

Figure 5.7. above illustrates that the approach used by the L3 and L4 in answering this question displayed sufficient understanding of a relational foundation for solving the given problem. For example, recognizing the relationships between quantities and correctly assigning the "unknown" quantity laid the groundwork for activating an algebraic symbolisation in order to solve the problem. This, in turn, enabled L3 and L4 to develop a consistent habit of mind, which could be applied to a broader range of problems. Such an approach is especially significant in algebra, where abstraction plays a central role when solving word-problems.

Building on this observation, the use of model strategies in algebraic word problems was analysed during the three-week intervention lessons. The analysis focused on two central themes: (1) the misuse of operations and (2) the role of relational understanding in identifying relationships between unknowns.

5.5. How do the utilisation patterns of models vary among Grade 8 learners ?

Analysis of learner performance revealed distinct patterns in the utilisation of models for problem solving. Of the nine Grade 8 learners, five demonstrated the ability to select and apply a suitable model to solve algebraic word problems. These learners showed proficiency in recognising

underlying relationships, translating problem contexts into algebraic expressions, and manipulating those expressions to reach a valid solution. In contrast, four learners were unable to effectively utilise a model, often struggling with identifying key relationships or misapplying operations once a model was selected.

The variation in model utilisation patterns suggests differing levels of algebraic relational understanding across the group. Learners who successfully employed models exhibited stronger conceptual links between problem contexts and algebraic representations, whereas those who did not displayed fragmented or procedural thinking. Understanding these patterns is crucial for identifying targeted intervention strategies that support all learners in developing flexible and meaningful modelling skills. Each intervention lesson was therefore analysed using the evaluative problem-solving skills tool developed by Tong and Han (2020) to capture these variations in depth.

5.6. The Hypothesis.

Modelling algebraic word-problems is used appropriately when problem-solving is regarded as a process of leveraging knowledge gained from previous problems to address new and unfamiliar situations, involving questioning, evaluating circumstances, interpreting results, and employing trial and error (NCSM, 1989; cited in Farida et al, 2018). In this research, modelling word problems using algebraic symbolisation is essential. Ersoy (2016) emphasizes that meaningful learning emerges from engaging in problem-solving activities.

The activities used in the intervention lessons were real-world problems associated with the first-degree linear equations in one unknown. During the intervention lessons, the data collected were in the form of transcribed digital recordings and study sheets with class activities that needed to be completed. The selection of activities focused on problem-solving aspects differentiated into three characteristics of algebraic relations, namely: (a) word problems involving age, (b) word problems involving consecutive integers and, (c) word problems involving fractions. The proposed hypothesis suggested that these problem-solving lessons would improve learners ' higher-order thinking skills and promote their capacity for relational reasoning by utilizing "the power of mathematical ideas, knowledge, and skills" (Tong & Han, 2020, p. 568).

To introduce the series of intervention lessons, the teacher-researcher revised the mathematical

register necessary to answer these questions e.g. “the sum of” or “consecutive integers” or “the difference of” through discussion with the learners. Then, using Schoenfeld (1985) algebraic problem- solving strategies as the structure of problem-solving model, the teacher-researcher, using the principles of guided-orientation, supported the learner’s acquisition of comprehension skills that will later enable them to solve the word-problem and reason effectively.

5.6.1. The methodology and contents of each intervention lesson.

Lesson 1: algebraic word-problems involving age (see Appendix A, lesson 1)

Problem: Five years ago, John’s age was half of the age he will be in 8 years from now. How old is he now?

Solution:

Assume that John’s age **now** is x years old. Five years ago: $x - 5$

In eight years from now: $x+8$

$x - 5 = \frac{1}{2}(x + 8)$

$\therefore 2x - 10 = x + 8$

$\therefore 2x - x = 8 + 10$

$\therefore x = 18$

So, John is 18 years old now.

1.1. What does the problem require you to find? List the different quantities and a formula for the relations between the quantities?

1.2. What information does the problem already provide? Create a table to list the information and identify what quantity will be used as the “unknown.”

1.3. Present the solution

1.4. In your opinion, do you think the solution satisfies the condition(s) of the problem? Is there any other way to solve this problem?

Lesson 2: algebraic word-problems involving consecutive integers (see Appendix A, lesson 2)

Problem: Find four consecutive odd integers whose sum is 152.

Let x be the first odd integer in this sequence:

$\therefore x + (x + 2) + (x + 4) + (x + 6) = 152$

$\therefore 4x + 12 = 152$

$\therefore 4x = 140$

$\therefore x = 35$

Check: $35 + 37 + 39 + 41 = 152$

The odd integers are: 35, 37, 39, 41.

2.1. What does the problem require? Which quantities were included? What would be the proposed formula for the relation between them?

2.2. Create a model representation of the information. In this way, it organizes the data in a clear, structured manner, facilitating the formulation of equations and solutions.

2.3. Solve the problem.

2.4. Consider and select the results to present an answer to the initial problem.

Lesson 3: algebraic word-problems involving fractions (see Appendix A, lesson 3)

Problem: Two thirds of a number is 14. What is the number?

Solution:

Let the number be x:

$$\therefore \frac{2}{3}x = 14$$

$$\therefore \frac{2}{3}x \times 3 = 14 \times 3$$

$$\therefore 2x = 42$$

$$\therefore x = 21$$

The number is 21.

3.1. What does the problem require? Which quantities were included? What would be the proposed formula for the relation between them?

3.2. Create a summary sheet or tabulate the information. In this way, it organizes the data in a clear, structured manner, facilitating the formulation of equations and solutions.

3.3. Solve the problem.

3.4. Consider and select the results to answer to the initial problem.

5.6.2. Analysis of Post-intervention Lesson 1.

The following were the results obtained from analysing learners work in class activity 1.

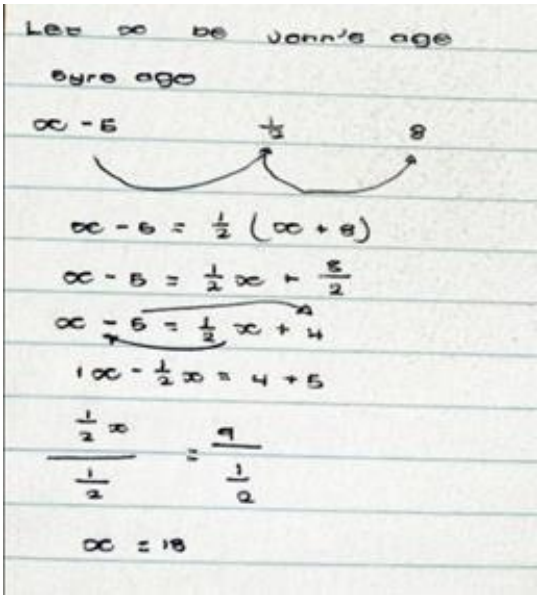
Type 1 involving age of one person: *Five years ago, John's age was half of the age he will be in 8 years. How old is he now?*

Type 2 involving age of more than one person: *Tanya is 4 years younger than Marcus. Twenty years ago, Marcus's age was 13 years more than half the age of Tanya. How old are both Tanya and Marcus now?*

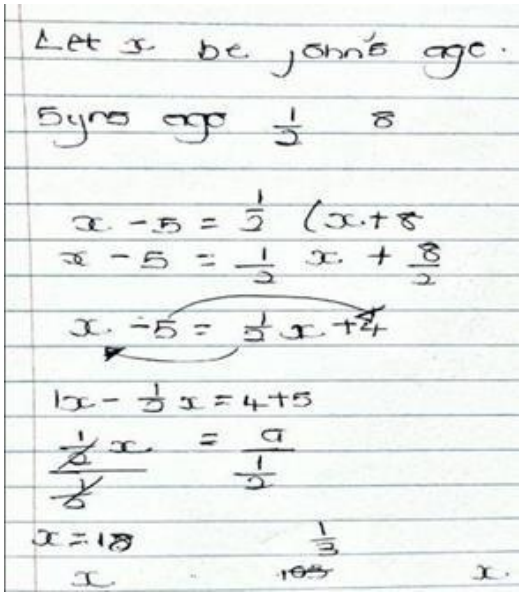
During this lesson, it was observed that the learners developed the skill of symbolic representation.

For instance, as demonstrated in both Extracts F and G, the learners successfully translated the language of the word problem using relational reasoning to write down algebraic expressions and equations. This indicates that their algebraic relational understanding (Skemp,1976) has advanced to include the assignment of variables to represent unknown quantities (Schoenfeld, 1985, Lesh & Doerr, 2003). The statement of *"Let x represent John's age"* became a habit of mind to systematically approach and solve the given problem by moving from unknown quantity to defining a variable to represent it (Boero, 2002).

Problem solving question: *Five years ago, John's age was half of the age he will be in 8 years. How old is he now ?*



Extract F: Learner 3's response



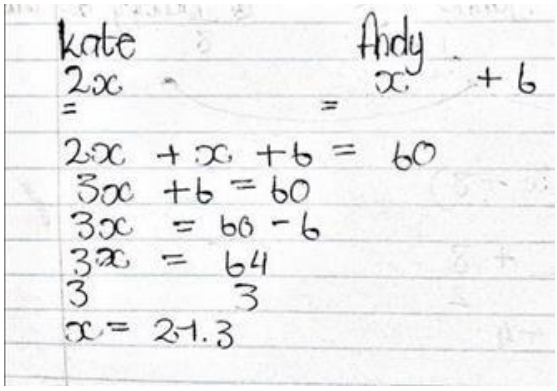
Extract G: Learner 4's

Figure 5.8: Learners' 3 and 4 responses to the activities involving age of one person.

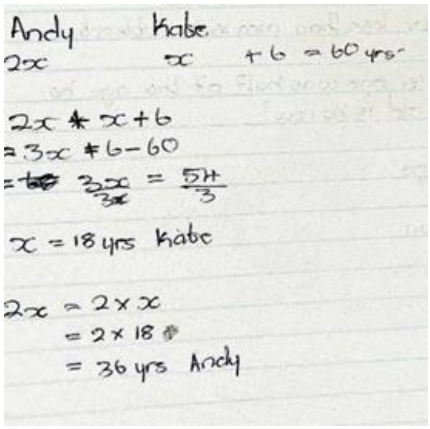
Furthermore, the teacher-researcher became aware that the questions involving the ages of more than one person were more likely to be challenging and as a result answered unsuccessfully (See Figure 5.8). This may be due to a significant gap between the learners' current relational

The problem: *Andy is twice as old as Kate. In six years, time, their ages total sixty. How old is*

each of them now?



Extract H: Learner 1's response



Extract I: Learner 2's response

Figure 5.9: Learners' 1 and 2 responses to the activities involving ages of two persons.

understanding (Skemp, 1976) and the level of relational reasoning (Downes, 2008) required to solve such problems. Consequently, none of the learners provided a complete and correct response to this question. Hattie and Donoghue (2016) warned that some learners could have a well- developed relational understanding but struggle to activate the necessary relational reasoning that assist them in algebraic problem-solving. Table 5.2. below shows a summary of the quantitative analysis of learner responses, based on the scores in activity 1 from lesson 1 (Tong & Han, 2020, p. 568).

Table 5.2. Scores attained in Lesson 1 related to the problems on age in Lesson 1

		Involving one person's age	**Involving more than one person's age.**
Ability to detect given problem		7/9 learners could find out, determine the problem to be solved.	2/9 learners could not find out, determine the problem to be solved.
Possible: $\frac{20}{20}$	Actual: $\frac{14}{20}$		
Ability to propose solutions		7/9 learners could convert the problem information to the mathematical model	2/9 learners could not convert the problem information to the mathematical model
Possible: $\frac{10}{10}$	Actual: $\frac{7}{10}$		
Ability to solve the problem		7/9 learners could calculate correctly and use an optimal solution model and tool	2/9 learners could not calculate correctly and use an optimal solution model and tool
Possible: $\frac{20}{20}$	Actual: $\frac{14}{20}$		
Ability to evaluate results		7/9 learners could review and answer the problem.	2/9 learners could not review and answer the problem.
Possible: $\frac{10}{10}$	Actual: $\frac{7}{10}$		

Overall, the relational understanding and relational reasoning required to identify critical information and construct an equation, resulted in learners showing challenges while solving the mathematical problem. In the field of algebraic reasoning, this study concluded that exploring the extent to which the VHPOL (Van Hiele, 1986) phases 1 to 3 of learning serve as a clearly defined model for assessing the learner levels of thinking. It is the most effective approach for the educator to guide learners towards critical engagement with the information given in the problem.

During phase 1, the teacher-researcher provided inquiry-based activities with the intention for learners to engage in a problem-solving activity and thereafter formulate conjectures of what they have learned. By doing so, engaging directly with activities through discussions during phase 1 informed the teacher-researcher about the prior knowledge the learners possesses and how it may be accessed and enriched. During phase 2, guided-orientation, the learners uncovers connections between concepts using relational understanding to make sense of the subject matter under the carefully planned guidance of the teacher-researcher. At this stage, learners begin to engage with the concepts as they are "given the liberty to exchange ideas during classroom discussions" (Dongwi, 2014 p.45) and this helps learners to enhance their relational reasoning about the problem. During phase 3, explication, the learners are provided with the opportunity to express their understandings of mathematical concepts (Dongwi, 2014) and solve the problems. This phase differs from the guided orientation phase because learners familiarize themselves better with new ideas learned and express them using algebraic appropriate processes and symbolisation (Dongwi, 2014). Every intervention lesson was designed with the above principles in mind.

5.6.3. The effectiveness of the phases of learning (1 to 3) in improving the learners achievement in solving algebraic word-problems.

It was observed after a transcribing the digital recording of lesson 1 that learners in this group achieved lower interpretive scores when evaluating the problem. This highlights the need to adopt diverse learning strategies that move beyond instructor-centred approaches or reliance on rote memorization.

5.6.3.1. Transcription from digital recording Lesson 1: Part 1

The Problem: *Tanya is 4 years younger than Marcus. Twenty years ago, Marcus's age was 13 years more than half the age of Tanya. How old are both Tanya and Marcus now?*

This question relies on a sound relational understanding (Skemp, 1976) of the information, followed by a well-developed relational reasoning ability which will enable the learners to activate a model strategy that will enable them to reach a solution, and effectively interpret its validity by revisiting the context. This is what the process of algebraic reasoning requires (Kriegler, 2008, Kieran 2004, Kaput, 2008).

I will now discuss a short transcript of the discussion related to this problem.

Learner 1: Ma'am, instead of writing it can I explain it?

Researcher: Yes you may.

Learner 1: Ma'am you must, (*scratching his head*) eish…. If Marcus was 13 years old, you plus the 13 with the 20 and it will equal to 33. Then 33 minus the 4 years

of Tanya will be 29. Which means Tanya is 29 years old and Marcus is 33 years old.

Learner 2: But Ma'am what about the half? Learner 1 did not involve the half.

Researcher: Where would you suggest we start off ?
(*Class does not respond*).
We have to identify a starting point from our information.

Learner 2: (*Hesitantly*) …who…what is the unknown….?

Class: (*Choir excitedly*) Marcus is unknown!

(*The researcher smiled, waited a while and then signalled for the learners to continue*)

Learner 3: Ma'am we can…can we use a table…a model table with the past, the…that what is in the future?

Researcher: Okay, that might be a good plan…do we all agree?
(*The class confirms*).

So what now…

Learner 2: Marcus will be *x* ma'am.

Learner 3: I think we minus *x* with 13... That will give us Tanya's age.

An analysis of the digital recording from Lesson 1 highlighted intriguing insights into algebraic reasoning through the interactions among the learners. L1's progression in algebra and relational reasoning revealed a lack in relational understanding and consequently algebraic thinking. Increased engagement with realistic problem-solving tasks led to an increase in conceptual errors. As Hattie and Temperly (2007) points out, "engagement in complex problem- solving tasks often exposes students to their misconceptions, which may initially increase their conceptual errors before they achieve deeper understanding (p92)". L2 appeared to have been ready to use a model strategy based on their understanding, but was struggling to place the half, and therefore immediately noticed when Learner 1 did not use the half.

In contrast, L3 experienced self-discovery by exploring L1's responses, a process supported by guided learning as outlined by Van Hiele (1986). Research has repeatedly shown that collaborative learning environments enhance problem-solving skills and understanding of mathematical processes (Johnson & Johnson, 1999). When leaners engage in discussions, they are exposed to a range of perspectives, approaches and strategies. In this transcription it clearly led to retention of mathematical principles. Although there was not return to the query of the absent ½, the problem-solving moved forward with various contributions.

The conceptual errors in algebraic word problems stem from misunderstandings of problem contexts, challenges in interpreting relationships between quantities, and procedural missteps, often leading to the construction of incorrect equations or misrepresentation of variable relationships (Jupri & Drijvers, 2016). During Lesson 1, learners frequently assigned the same variable to multiple individuals or failed to correctly express unknown relational aspects by applying the variable appropriately to the given problem's condition. For example, they might represent the ages of two individuals without accounting for the specified difference within the context of the problem. Additionally, the VHPOL phase 3 which focuses on explication of

important milestones through emphasising verbalising and reflecting on reasoning during problem-solving, assist with building a schema which learners can access during problem-solving.

In this research, the focus on explication, as seen in the transcript, enabled learners to better articulate and justify their thought processes. However, when attempting age-related problems involving multiple individuals, learners faced difficulties due to a limited understanding of the problem's structure and the algebraic relationships involved. This aligns with Van Hiele's assertion that reflection and formalization are critical components of the learning process (Van Hiele, 1986).

5.6.4. Analysis of worksheet and Lesson 2.

The following were the results obtained from analysing learners work in class activity 2. Type 1: *Find four consecutive odd integers whose sum is 152*.

Type 2: *Find three consecutive even integers such that 6 times the sum of the first and the third integer is 24 greater than 11 times the second.*

Solution:

Let the first even integer be x. Then the next two will be $x + 2$; $x + 4$.

$\therefore 6(x + x + 4) = 11(x + 2) + 24$

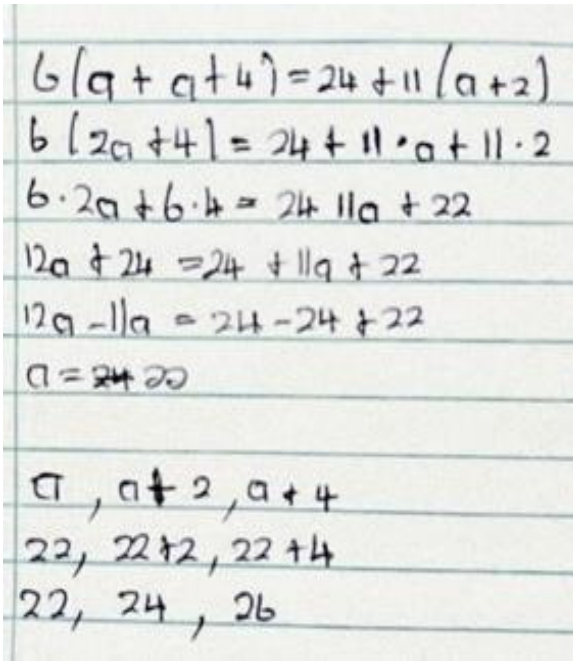
$\therefore 6(2x + 4) = 11x + 22 + 24$

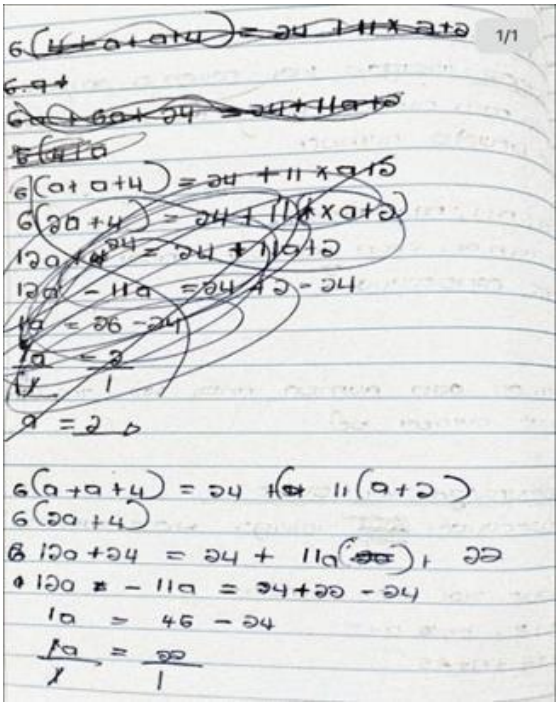
$\therefore 12x + 24 = 11x + 46$

$\therefore x = 22$ ... Therefore, the consecutive integers are: 22, 24,26

Lesson 2 was conducted a week after lesson 1. The results demonstrated a better understanding of the use of modelling and its effectiveness to breakdown a complex word-problem into manageable components. The outcomes of this lesson demonstrated that six of the learners were able to set up and equation and solve the equation correctly. In Figure 5.10. below, we see the solution of some of the learners that provided full resolution. This could be due to the fact that the school had extensively taught algebraic concepts and vocabulary during the contact time of this period

(September 2024) and as stipulated in the Grade 8 mathematics annual teaching plan (DBE, 2011).





Extract J: Learner 5's response

Extract K: Learner 9's response

Figure 5.10: More responses to the activities involving ages of two persons.

During the digital recording, a group discussion took place where one learner (Extract K) incorrectly handled the problem due to not interpreting the word problem correctly, another learner addressed the correct equation (Extract J). Incorrectly interpreting an algebraic word problem could arise from a learner quickly jumping into the calculations without first translating the word problem into an equation, then, only later realise that it would have been answered correctly if they had started by defining the variable(s) first. It appeared that the more the learners detected the problem to be solved i.e. "*Find three consecutive even integers such that 6 times the sum of the first and the third is 24 greater than 11 times the second*", the more connections to the problem were made. For instance, stating that the problems known values were that there are "three consecutive integers" meant that a certain number will follow the next in a coherent pattern for as long as it is added by an "even integer." Therefore, when representing the known information, the learners saw it fit to allocate a variable to any unknown in the problem.

Observing their work progress and the results that followed revealed an improvement in the ability to discern when the learners had developed valuable strategies from the introductory lesson. It appeared that they had solved the word problem by using algebraic representations more efficiently. Solving the questions posed in lesson two (Part 2) seemed straightforward to most learners and resulted in more of them showing confidence to share their responses. Including additional operations such as "*6 times the sum of the first and the third integer*" reflected some doubt in their reasoning and abandoning of their first attempt (see Extract K). Salemeh and Etchells (2016) notes that the same operational difficulties are faced by students when answering problem-solving questions.

Table 5.3. on the next page reveals the results of class activity 2 and therefore proves that group activities during the practice lesson created better learning opportunities for the learners by allowing them to observe and recognise any shortcomings they may have experienced by building onto the contributions of their peers (Johnson & Johnson, 1999).

The Activity: *Find three consecutive even integers such that 6 times the sum of the first and the third integer is 24 greater than 11 times the second*

Table 5.3: Evaluative tool for algebraic problem-solving during Intervention Lesson 2

		Integer problems involving multiple operations	
Ability to detect given problem		6/9 learners could find out, determine the problem to be solved.	3/9 learners could not find out, determine the problem to be solved.
Possible: $\frac{20}{20}$	Actual: $\frac{10}{20}$		
Ability to propose solutions		6/9 learners could convert the problem information to the mathematical model	3/9 learners could not convert the problem information to the mathematical model
Possible: $\frac{20}{20}$	Actual: $\frac{10}{20}$		
Ability to solve the problem		5/9 learners could calculate correctly and use an optimal solution model and tool	4/9 learners could not calculate correctly and use an optimal solution model and tool
Possible: $\frac{20}{20}$	Actual: $\frac{10}{20}$		
Ability to evaluate results		6/9 learners could review and answer the problem.	3/9 learners could not review and answer the problem.
Possible: $\frac{20}{20}$	Actual: $\frac{10}{20}$		

Transcription of Lesson 2: Part 2

Learner 6: Ma'am, when they say, "*6 times the sum of the first and the third integer*", doesn't it mean we must multiply 6 with the first then again multiply 6 with the third.

Learner 2: Yes ma'am, that's how I understand it as well. Then I would have added the answers I got at the end.

Here L6 is clearly referring to the distributive law, clearly exhibiting the ability to verbalise a mathematical process. This is evidence of relation reasoning shaping algebraic reasoning skills that L2 could verbalise and, continue to describe the method she continued with to solve the equations.

Referring to VHPOL Phase 3 (explication), learners in this research discovered that verbalising and reflecting on their reasoning during class discussions could enhance their understanding. However, since English was not the first language for most learners, some struggled with the precise language needed for explication. This often led to the misconception that verbal clarity required overly formal or rigid statements, which sometimes resulted in misunderstanding rather than improved comprehension. Vinner (1997) describes this dilemma as the process of translating English statements into mathematical expressions being loaded with complications, as learners often misinterpret terms that have specific meanings within mathematics. The transcript below serves as an example of such challenges which shift in meaning from English to mathematics.

A transcribed recording of lesson two is found below: (Part 1).

Researcher: Let's talk about the word "consecutive." What does this word mean?

Learner 5: Ma'am, it means something that is next to something. Like the number three is consecutive to two.

Learner 9: Ma'am, like in number patterns…

Researcher: (*Connecting to the idea of a number pattern*)

So, if consecutive numbers form a pattern, what can you tell me about the sequence $n + 1, n + 2, n + 3$.

Learner 3: We must make the variable n…and solve the equation.

(*Almost immediately correcting herself*) No…there is no equation to solve.

Researcher: Anyone else want to tell us what they see?

Learner 5: Ma'am, each time the n is increased by one more.

Researcher: So, if I start at three…let us make $n = 3$, then what can you tell me about $n + 1, n + 2, n + 3$.

Learner 5: You will have $3 + 1, 3 + 2, 3 + 3$. So, the numbers will be 4,5,6.

Learner 9: Ma'am, can we make *n* zero? (*Teacher affirms with a nod*). The … the numbers are 1,2,3….so *n* is an unknown. We can make *n* any value, the numbers stay consecutive right?

Researcher: Well done. So, we agree all consecutive numbers differ by one, no matter what the value is for *n.* Happiness?

(*Class confirms*).

So, what do you think even numbers will look like? What will odd numbers look like?

Learner 9: Even numbers are 2, 4, 6, 8, …. So, the differ by two. (*Class agrees*)

Researcher: So, will it be fine if I write even numbers as $n + 2, n + 4, n + 6$? I will give you a few minutes to think about this.

Learner 5: Ma'am, I think it is the unknown value used that will represent the even number. If *n* is zero, then $n + 2, n + 4, n + 6$ becomes 2, 4, 6. This is even. But if $n = 1$, **….(***Teacher interrupts Learner 5***)**

Researcher: Wait Learner 5…(*To class*) Did everyone hear what Learner 5 is saying? Please repeat what you said, and I want everyone to listen carefully.

(*Learner 5 repeats the statement*)

(*Learner 9 excitedly puts up their hand*)

Learner 3: Ma'am, Learner 5 is correct. I did the same. And I looked at $n + 2, n + 4, n + 6$ as well. If *n* is one, you get 3, 5, 7 which is odd. So, they are both even and odd. You must just choose the unknown *n* right.

Researcher: Okay wait, I don't understand what you mean by they…Is same sequence being both even and odd? Explain to us.

Learner 3: Ma'am…okay. Even numbers and odd numbers all differ by two. I mean consecutive even and odd numbers. So, for $n + 2, n + 4, n + 6$ to be even you start at 0 and get 2, 4, 6 but you can also start at any even value for n…If n is two, you get 4, 6, 8 which are even. And if $n + 1, n + 3, n + 5$ must be even… to be odd...no

even… you must replace the n with an odd value.

This discussion is filled with very interesting and unexpected responses. The learners are in Grade 8 and showed relatively good insight into what was expected of them. When the teacher-researcher asked (not shown here) the class what a variable is, the overwhelming reply was that it is a letter used to represent the unknown quantity.

The teacher-researcher decided to do a recap of the mathematical terms that have been used up to now, which gave rise to learners applying their knowledge to provide answers to the complex question posed by the teacher-researcher. Johnson and Johnson (1999) calls this collaborative learning where they learn from the contributions of their peers and improve on their own thinking. This is clearly shown at the point where L3 immediately self-corrected. Moving the cognitive load of the initial question, the teacher-researcher added a second condition to the notion of consecutiveness, that of even – and odd consecutive numbers.

Here L5 became the thought leader when referring back to the sequence

$n + 2, n + 4, n + 6$ which the teacher-researcher put on the board. They shift the focus immediately to the n and how this behaviour of n changes to switch between odd and even numbers. This is a very good example of well organised relational understanding of the use of variables to represent unknown quantities by L5 and expanded on by L3 where relational reasoning providing a sophisticated answer that includes the second condition of odd and even based on the manipulation or control of n.

5.6.5. Analysis of worksheet 3 :Lesson 3

The following were the results obtained from analysing learners work in class activity 3:

Problem: *Two thirds of a number is* 14. *What is the number*?

Problem Question:

Let x be the "number"

$\therefore \frac{3}{2}\, x = 20 - 8$

Let x be the "number"

$\therefore \frac{4}{2}\, x = 36$

$\therefore \frac{3}{2}x = 12$

$\therefore 3x = 24$

$\therefore x = 8$

Therefore, the number is 8

$\therefore 2x = 36$

$\therefore x = 18$

alternative:

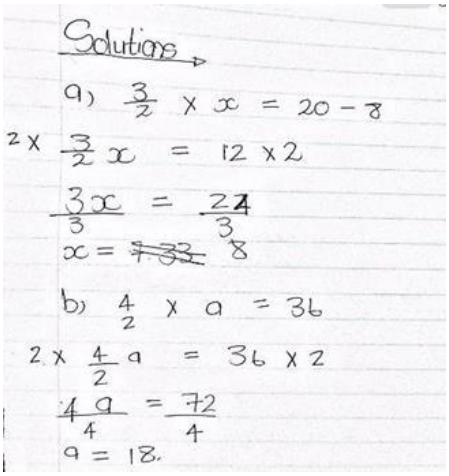
$\therefore \frac{4}{2}x = 36$

$\therefore 4x = 72$

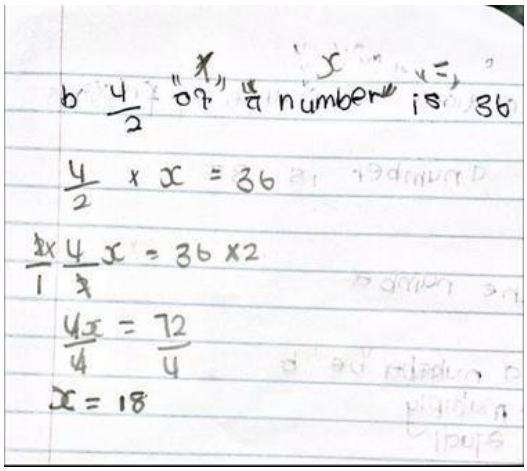
$\therefore x = 18$

Therefore, the number is 18.

Extracts from learners work in Lesson 3:



Extract L: Learner 3's response



Extract M: Learner 1's response

Figure 5.12: Learners 1 and 3 responses to the activities involving fractions.

Intervention lesson three was conducted a week after intervention lesson 2. The lesson was introduced by the teacher-researcher asking the learners to recall different types of fractions that they have learnt about thus far. The teacher-researcher was attempting to access prior knowledge to activate solution processes involving fractions. Bransford (et al. 2000) maintains that in order to understand new concepts in mathematics, it is key that learners must connect them to already

established cognition. This process is of particular importance in mathematics, where prior knowledge can significantly influence problem-solving strategies.

The aim of the lesson was to recognise and solve word problems involving fractions. These can be differentiated into either comparing fractions or multiplying and dividing fractions. The introductory lesson took place in three phases by assigning a variable to suit the equation and represent the unknows (thus using their relational understanding to activate the relational reasoning during the process of horizontal mathematisation), then to translate the English word sum into an equation (vertical mathematisation) and lastly to solve the equation by isolating the variable trough the mathematical approaches that is relevant for this (using algebraic reasoning skills effectively).

The constraint lesson 3 presented was that most of the learners did not understand fractions and how to manipulate them. Therefore, the introductory lesson took place over two days before the activity was assigned to the learners. Below is a transcription of lesson 3 introductory lesson.

Transcription of an extract of Lesson 3:

Learner 1: Ma'am, the first thing I see is that "*a number*" is the unknown so I must put a variable. Then, whenever we see "*of*" it means to multiply and "*is*" means an equal sign. (*Emphasis included by transcriber)*

Researcher: That is interesting Learner 1. Try to turn it into a workable equation that you can use to answer the question with.

Learner 3: I choose a as my variable. But ma'am, since $\frac{2}{3}$ is a fraction, how do we multiply a fraction with the variable. I'm lost. I took the fraction over to the other side and it became a minus, but how do you minus a fraction with a whole number?

Researcher: (*To Learner 3*). Think about… Learner 1, I want you to also think of what I am going to say. Think about the inverse operations you have learnt in algebra…to get rid of multiplication, you do what?

(*Class responds divide*). And to undo division…?

Class: You multiply.

Researcher: And what you do on one side of an equation, you must…

Learner 1: …do to the other side.

According to Van den Heuvel-Panhuizen (2001), misunderstanding the context of a problem frequently occurs when learners concentrate exclusively on symbolic manipulation, neglecting to connect the variable relationally to the real-world scenario presented in the problem. The learners struggled with visualising the fraction in the problem, and what to do with it. It is noted in this case that, while many learners could detect the problem, their focus on relational connections and symbolic manipulation was flawed. Learners often exhibit a higher frequency of errors when solving problems involving fractions due to difficulties in understanding the underlying concepts and applying appropriate procedures (Kerslake, 1986). The learners predominantly relied on rote memorization of fraction operations based on cue words, rather than fostering a conceptual understanding of fractions as parts of a whole. Rittle-Johnson and Schneider (2019) warns that learners often learn to associate specific cue words with particular mathematical procedures, which can lead to a superficial understanding of mathematics that lacks depth and flexibility**.** This reliance created uncertainty when progressing through the problem, highlighting a lack of relational understanding of the factions and the integer. Consequently, this limitation hindered their ability to apply knowledge flexibly across various problems (Van den Heuvel-Panhuizen, 2001).

Furthermore, their inability to model the problem created a barrier, as learners often made errors in fundamental fraction operations, such as not finding a common denominator or misapplying cross multiplication and fraction division. Lesson 3 (as highlighted in Transcription 3 above) revealed that some learners faced challenges in successfully solving the problem due to an insufficient relational understanding of the task.

The following day, after the teacher-researcher reviewed the fundamentals of fractions, the learners demonstrated improved ability to identify and successfully solve the problem during the class activity. However, some learners still expressed uncertainty about their final answers (see Figure 5.7).

Overall, the results of Lesson 3 displayed that three out of the nine learners were still underdeveloped insofar the use of algebra when dealing with fractions are concerned. The tendency for learners to connect cue words with procedures instead of fostering a robust conceptual framework for understanding mathematics, hinders the development of deep relational understanding of mathematics. Learners failed to gather accurate and comprehensive information and did not fully comprehend the significance of manipulating fractions and could not convert information from a given problem to an appropriate mathematical model. Although, five of the learners could partially accomplish the goal of gathering information and discovering the relationships between the known (and unknown) quantities, one learner could successfully accomplish this. The table below reflects these results.

Table 5.4: Evaluative tool for more algebraic problem-solving during Intervention Lesson 3.

		Integer problems involving multiple operations	
Ability to detect given problem		5/9 learners could find out, determine the problem to be solved.	4/9 learners could not find out, determine the problem to be solved.
Possible: $\frac{20}{20}$	Actual: $\frac{10}{20}$		
Ability to propose solutions		5/9 learners could convert the problem information to the mathematical model	4/9 learners could not convert the problem information to the mathematical model
Possible: $\frac{20}{20}$	Actual: $\frac{10}{20}$		
Ability to solve the problem		5/9 learners could calculate correctly and use an optimal solution model and tool	4/9 learners could not calculate correctly and use an optimal solution model and tool
Possible: $\frac{20}{20}$	Actual: $\frac{10}{20}$		
Ability to evaluate results		4/9 learners could review and answer the problem.	5/9 learners could not review and answer the problem.
Possible: $\frac{20}{20}$	Actual: $\frac{10}{20}$		

The implications of these findings are significant for teaching of mathematics. It suggests a need for targeted interventions that focus on enhancing learners' conceptual understanding of fractions before they engage with algebraic problems where fractions are present. The persistent challenge with fractions can originate from various perspectives including weak foundational knowledge and inappropriate instructional methods. Effective teaching strategies must include using real-world applications to build learners' cognition the concept of fractions more thoroughly.

5.7. How do the approaches observed in the post-intervention lesson relate to the solution

models introduced during the three-week intervention program?

From this research, several conclusions can be drawn from the three-week intervention lessons focused on solving problems by building equations using strategies that had as its intention the ability to formulate effective model strategies for algebraic problem-solving. Four important approaches that was underscored during the intervention agenda:

5.7.1. Learning and Practice:

Learners needed to first learn the process of solving word problems and then practice extensively to develop the necessary problem-solving skills grounded in sound relational understanding- and reasoning. This was facilitated by the teacher-researcher using guided orientation, which enhanced structured stages of approaching real-life problems. This approach is supported by Lesh and Doerr (2003) who posits that "learners learn mathematics through engaging in meaningful tasks that require them to make sense of mathematical ideas and relationships, and they develop their understanding through guided exploration and extensive practice".

5.7.2. Active Engagement and Support:

When learners actively verbalized their thought processes, worked with guiding examples, and clearly understood the relevant information, they were more successful in using their relational reasoning in creating and solving algebraic equations. In many cases, their conclusions were clear, as some learners would verify their answers for accuracy. Tools like summary lists or tables helped learners identify the hypotheses and requirements of problems effectively.

5.7.3. Challenges in Algebraic Translation:

Despite progress, some students struggled to translate given information into algebraic expressions, reflecting difficulties in connecting the information to establish equations. This points to a lack of using relational understanding to activate the necessary reasoning to form appropriate algebraic equations and then solving them. This problem will lead to later under-developed algebraic reasoning skills.

5.7.4. Collaborative Learning Benefits:

Using the Tong and Han (2020) evaluative tool for problem-solving, it was evident that group activities during the practice lessons provided valuable learning opportunities. These activities

allowed learners to question each other's reasoning and also building on personal conceptual understanding of personal solution approaches. This collaborative approach fostered both individual and group learning, addressing key challenges in problem-solving, as these were identified by the teacher-researcher.

5.8. In summary:

After discussing teaching problem-solving by setting up equations from real-world scenarios, it can be said that learners apply the process of solving problems through class discussions and activities with the hope of memorising each step instead of focusing on conceptually understanding why and how to solve the problem through focusing on the relational aspects. This resulted in only one out of nine learners moving beyond detecting the problem and gathering information to successfully discovering the relationship between the known and unknown quantities in each problem.

Class discussions promoted opportunities for the learners to learn from each other, possibly correcting their misconceptions and assisted them in strengthening their understanding. This was done through various learners questioning each other's reasoning.

The results from the pre- to post-test (see Figure 5.3) displays that collaborative learning increased the learners confidence in engaging in class discussions and, while Johnson and Johnson (1999) persists that through collaboration in the mathematics classroom the building of "more complete and complex conceptual structures" (Johnson & Johnson, 1999, p.1) are developed, this was not evident in the results of individual performance in the post-test. During the class discussions, the learners' were assigned two responsibilities: first, to maximise their own learning and understanding then, to maximise the learning and understanding of the others. In this study, the latter was more effective than the former. This is clearly noted during class discussion from lesson 1, part 1 (see p.55 and 56) where L2 perceived that she can only reach their learning goal when she assisted in illuminating learner 1's misinformation. Johnson and Johnson (1999) attests that learners seek outcomes that are beneficial those whom they are collaboratively lined with. While collaborative learning was a priority in this research to promotion relational understanding in algebra, it was in vain if individual understanding is not a priority as well.

Resultantly, many learners continued to struggle with setting up equations when assessed

individually in the post-test. While they demonstrated a good understanding of the given information, they faced challenges in connecting the details to formulate an appropriate equation needed to solve for the unknown. The largest constraint took place during lesson 3, where the teacher-researcher had to teach the concepts of fractions over two days as many learners were unaware of the use of and manipulation of fractions.

The learners still demonstrated a consorted effort to solve algebraic word problems using an appropriate model. The results in the post-test revealed that using real-world scenarios (i.e. word problems involving age) has the capacity to stimulate interest in the learners. However, the learners still exhibited operational errors (see p.53), which hindered their ability to develop a comprehensive solution to the problem and restricted their "flexibility and critical thinking in problem-solving" (Van den Heuvel-Panhuizen, 2003).

The subsequent chapter presented a comprehensive summary of the research, outlining key findings and discussing their implications in relation to the three guiding research questions: (1) What algebraic problem-solving strategies did Grade 8 learners engage in when solving real-world problems? (2) In what ways did the use of model strategies, if any, vary among Grade 8 learners? (3) How did the problem-solving approaches introduced during the three-week intervention enable learners to develop more relational approaches to problem-solving? The chapter also offered recommendations based on the study's results, as interpreted by the teacher-researcher.

Chapter 6: Summary, Recommendations and Conclusions.

"Research is the foundation upon which effective teaching and learning are built; it provides educators with the insights needed to adapt them methods and improve student outcomes."

(Smith, J. 2020)

6.1. Introduction.

This final chapter provided a comprehensive summary of the research, highlighted the key findings, discussed their implications, and offered recommendations based on the study's results. It served as a reflective conclusion that synthesised the research journey, evaluated its contributions, and suggested directions for future research.

In Chapter 5, the teacher-researcher focused on developing insights into the use of effective models in answering algebraic problem-solving questions. In this chapter, the teacher-researcher presented an overall conclusion and recommendations for further research.

6.2. Focus of the study.

The focus of this research was to provide an intervention strategy to explore the use of models and algebraic reasoning while working on algebraic problem-solving questions. This project followed a series of 3 intervention lessons accompanied by two tests: the pre-test (given before the intervention lessons) and the post-test (given after the intervention lessons) over a three-week period. These intervention lessons focused on integrating effective models to use when approaching algebraic word-problems. Using techniques like real-life problems in mathematics which aids at integrating the habits of mind in problem-solving assists the learners in recognising the importance of models in algebraic problem-solving. The reason behind this was to provide learners with efficient habits and effective problem-solving strategies that move them beyond depending on recalling basic algebraic "recipes." Thus, allowing them to use the associated properties involved in identifying a solution to a problem using variables to work with unknow

quantities.

Guiding this research were three essential questions:

a) What are the algebraic problem-solving strategies that Grade 8 learners engage in when solving real-world problems?

b) How does the use of model strategies, if any, vary among Grade 8 learners?

c) How do the problem-solving approaches, introduced during the three-week intervention, enable learners to develop more relational approaches to problem-solving?

The concluding chapter was initiated with a discussion of each research question by the teacher-researcher.

6.3. Algebraic problem-solving strategies Grade 8 learners engage in when solving real-world problems?

An analysis of the pre-test scores highlights two central themes, (1) the misapplication of arithmetic operations and (2) the under-use relational understanding and relational reasoning as a problem-solving strategy. Correctly applying operations is an essential part of effective problem-solving, whereas misapplications reveal conceptual errors or misunderstandings that can impede successful problem resolution.

In the case of the learners in this study, the misapplication of arithmetic operations typically arises from a misunderstanding or incorrect application of mathematical principles, leading to errors in problem-solving. This is in line with what Boero (2002) argued about the misalignment of procedural and conceptual knowledge. While six out of nine of the learners were able to complete basic calculations, their ability to apply a structured approach to solving word problems was limited, predominantly due to the failure in translating the language used in the word problem to form accurate algebraic equations which was written in mathematical notation. There was a clear lack in the skill of using model strategies to order the information into a structural algebraic sentence. Therefore, solving the algebraic word problems in the pre-test revealed that only two out of nine of the learners could successfully apply a structured approach to solving word problems. The approach to solving word problems was also based on pre-algebraic methods. For example, the learners understood that a variable representing the unknown needed to be assigned within the

context of the word problem but failed to place it in an appropriate position. This highlighted that some learners lacked the conceptual understanding of the *unknown* quantity in an algebraic word problem, and where to introduce it, solely attempting recipes that worked for them before.

In algebra, relational understanding in word-problems is a fundamental skill in solving algebraic word problems as it allows learners to identify underlying structures and relationships, enabling them to develop strategies for effective problem-solving (Boero, 2002). In the pre-test, two out of the ten learners employed an efficient problem-solving approach which displayed an understanding of a relational foundations for solving algebraic word problems. The remaining learners found it difficult to translate the English into mathematical sentences that will allow them to solve the problems. The pre-test also depicts that the learners shy away from using efficient problem-solving strategies, providing insight on a lack of their conceptual representation of complex relationships within a problem. This made it complicated for learners to understand and work with abstract concepts and break down complex scenarios into smaller and manageable components with which they could work.

6.4 The variation in the use of model strategies

During the intervention lessons, the researcher aimed to emphasize contextual word problems, which constitute a pivotal element of RME and serve as a crucial indicator of algebraic relational understanding. In contrast to conventional pedagogical methodologies that utilize contextual problems primarily to reinforce learning approaches, RME comprehensively integrates algebraic relational understanding and relational reasoning. This integration is designed to direct learners towards model strategies that cultivate beneficial cognitive habits when engaging in problem-solving.

On one hand, seven out of the nine learners could answer the question involving the age of one person with a full and correct response, while three out of the nine learners stated that they lacked the ability to work the formula for the relationships embedded in the information. During this instructional session, it was noted that the learners exhibited a marked enhancement in their capacity for symbolic representation. They successfully translated the linguistic components of word problems into algebraic expressions and equations. This signifies an advancement in their

algebraic relational thinking, particularly in recognizing the critical importance of accurately assigning variables within the context of algebraic word problems.

Nevertheless, when confronted with word problems pertaining to the relationships between the ages of multiple individuals, significant discrepancies emerged between the learners' existing comprehension and the level of relational reasoning necessary to effectively resolve such issues. As a result, none of the learners were able to devise a comprehensive and efficient model strategies for addressing these word problems.

This observation highlighted the need to assist learners in constructing habits of mind that would enable them in developing relational understanding in algebraic problem-solving. One approach used by the researcher during the intervention lessons was group discussions. It was observed after a transcribing lesson 1 digital recording that learners in this group achieved lower interpretive scores when evaluating the problem. This highlights the need to adopt diverse learning strategies that move beyond instructor-centred approaches or reliance on rote memorisation. It became clear that while there has been progress in symbolic representation and variable assignment, challenges remained in relational reasoning, particularly in complex scenarios involving multiple subjects.

During the digital recording of lesson 2, a group discussion took place where one learner incorrectly handled the problem due to not interpreting the word problem correctly, another learner addressed the correct equation. Incorrectly interpreting an algebraic word problem arises from the learner quickly jumping into the calculations without first translating the word problem into an equation, then, only later realise that it would have been answered correctly if they had started by defining the variable(s) first. It appeared that the more the learners detected the problem to be solved i.e. "Find three consecutive even integers such that 6 times the sum of the first and the third is 24 greater than 11 times the second", the more connections to the problem were made. For instance, stating that the problems' known values were that there are "three consecutive integers" meant that a certain number will follow the next in a coherent pattern for as long as it is added by an "even integer." Therefore, when representing the known information, the learners saw it fit to allocate a variable to any unknown in the problem.

During lesson 3, the aim was to recognise and solve word problems involving fractions. These can be differentiated into either comparing fractions or multiplying and dividing fractions. The introductory lesson took place in three phases namely, (1) assign a variable that suits the equation,

(2) translate the word sum into an equation and (3) solve the equation by isolating the variable. It was noted that a significant proportion of the learners exhibited a lack of understanding regarding fractions and the requisite skills for their manipulation. Moreover, Lesson 3 elucidated that when learners focus solely on symbolic manipulation, they fail to establish a relational connection between the variables and the real-world context delineated in the problem.

In this particular case, it is observed that although numerous learners were capable of identifying the underlying issue, their emphasis on relational connections and symbolic manipulation exhibited significant deficiencies. Among the cohort of ten learners, the majority demonstrated a reliance on rote memorisation concerning operations involving fractions, rather than cultivating a robust conceptual understanding of fractions as integral components of a whole. This dependence on memorization engendered uncertainty as they navigated through the problem, thereby underscoring a deficiency in relational comprehension. As a result, this limitation obstructed their capacity to apply their knowledge with flexibility across diverse mathematical problems. Thus, while learners could identify problems, their flawed focus on relational connections and reliance on rote memorization hindered their conceptual understanding and flexible application of fraction knowledge.

During the course of this investigation, the learners exhibited a notable deficiency in their capacity to effectively model algebraic relational problems. This inadequacy served as a significant impediment, as numerous learners frequently committed errors in fundamental operations involving fractions. Such errors included the failure to identify a common denominator, as well as the improper application of cross multiplication and fraction division.

The overall findings from the instructional sessions indicated that three out of the nine learners remained underdeveloped in their mathematical competencies. The primary contributing factor to this underdevelopment was their inability to gather accurate and comprehensive information. Furthermore, these learners did not fully grasp the importance of manipulating fractions and struggled to translate information from a given problem into an appropriate mathematical model.

Despite these challenges, it is noteworthy that five out of the nine
learners were able to partially achieve the objective of collecting relevant information and discerning relationships between known and unknown quantities. Conversely, only one out of the nine learners successfully accomplished this task in its entirety.

It can thus be concluded that the study revealed significant barriers in learners' abilities to model algebraic relational problems due to fundamental misunderstandings and operational errors with fractions.

6.5 How did the problem-solving approaches introduced in the intervention enable learners to develop more relational approaches to problem-solving?

From this research, several conclusions can be drawn regarding the three-week intervention lessons that concentrated on problem-solving through the construction of equations utilizing an effective pedagogical model strategy for these problem-solving activities. It is evident that learners were required to first develop a comprehensive understanding of the methodology for resolving word problems before engaging in extensive practice to cultivate the requisite problem- solving competencies. This learning process was significantly enhanced by the researcher's structured guidance, which encompassed clearly defined stages with the aim of enhancing these strategies.

The findings indicate that learners who actively articulated their cognitive processes, engaged with illustrative examples, and possessed a clear comprehension of pertinent information exhibited greater success in formulating and solving algebraic real-life problems. In numerous instances, their conclusions were unequivocal, as some learners took the initiative to verify their solutions for accuracy. Moreover, tools such as summary lists or tables proved instrumental in aiding learners to effectively identify the hypotheses and requirements inherent in the problems they encountered. Overall, these insights underscore the importance of *structured learning* processes and *active engagement* in enhancing problem-solving skills among learners.

Despite advancements in educational methodologies, certain learners exhibited difficulties in the translation of provided information into algebraic expressions. This struggle underscores a significant challenge in establishing the necessary connections to formulate equations effectively. Such difficulties are indicative of an insufficient level of proficiency in algebraic relational reasoning, which is essential for successful problem-solving within this domain.

Utilizing the evaluative framework established by Tong and Han (2020) for assessing problem-solving capabilities, it became apparent that group activities integrated into practice sessions yielded substantial educational benefits. These collaborative endeavours facilitated an environment

where learners could engage in critical discourse regarding each other's reasoning processes, thereby illuminating gaps in their comprehension. This structured approach to collaborative learning not only enhanced individual understanding but also promoted collective knowledge acquisition, effectively addressing pivotal challenges encountered during problem- solving tasks.

Utilising the Van Hiele phases of learning model alongside Realistic Mathematics Education fundamentally aimed to challenge conventional pedagogical practices in favour of a Vygotskian framework of knowledge construction. This approach sought to actively involve learners in the process of knowledge construction, and it did work well to engage in fruitful classroom discussions.

6.6. Summary of the findings of this Intervention Study.

6.6.1 Improvement needed in mathematical understanding and reasoning skills.

The findings of this research indicate that a majority of learners in the sample were able to identify the 'unknown' variable in each problem presented during both the pre-test and post-test assessments. However, it was observed that over six out of these nine learners struggled to progress beyond this initial identification stage. This limitation suggests that their understanding is primarily superficial, focusing on recognising surface features of the problems rather than fully grasping the underlying procedures or strategies required for effective problem-solving.

This phenomenon reflects a state of partial proficiency among the learners, highlighting specific deficiencies in both their conceptual and procedural knowledge. Notably, there appears to be a significant gap in their ability to utilise relational understanding effectively. This inability hinders their capacity to engage in relational reasoning, which is essential for constructing models that facilitate problem resolution.

This indicates that while learners demonstrated an ability to recognize key elements within problems, their challenges in advancing beyond this point underscore critical areas for improvement in their mathematical understanding and reasoning skills.

6.6.2 Manipulating and reasoning skills related to the concepts of unknows

The findings of the research highlight a significant aspect of the mathematics curriculum and the formulation of mathematical tasks within South African school texts. Notably, approximately three

out of the nine learners demonstrated the ability to integrate efficient and effective problem-solving strategies. This proficiency enabled them to accurately comprehend the relational aspects necessary for addressing mathematical problems and to apply their algebraic reasoning skills effectively, particularly evident in the post-test results.

A critical component of algebraic problem-solving is the identification of unknown values and their representation through variables. This process signifies a crucial step in mathematical reasoning, as it entails introducing variables as placeholders for unknown quantities. Such an approach reflects a foundational understanding of the structure of mathematical problems and indicates an individual's capacity to engage with abstract algebraic processes.

Thus, while a minority of learners exhibited strong problem-solving capabilities linked to sophisticated algebraic reasoning skills, this underscores both the strengths and areas for improvement within the mathematics curriculum in South African schools.

Proficiency in algebra necessitates that learners engage in the manipulation and reasoning of relationships among variables, applying this comprehension to derive solutions. This level of skill was effectively exhibited by three of the learners in the research. Furthermore, during group discussions within the intervention study, many learners concluded that simply recognising a problem diminishes the perceived need to transition from an informal understanding to a more formalised comprehension of algebraic concepts.

True proficiency in algebra involves not only identifying unknowns but also manipulating and reasoning about variable relationships, as demonstrated by three of learners. Group discussions revealed that merely identifying problems may lessen the perceived necessity for transitioning from informal to formal understanding.

6.6.3 Enhancing the ability to engage in relational reasoning practices.

The findings indicate that the learners were not able to clearly articulate their reasoning regarding the algebraic problem, suggesting that they had not yet reached Van Hiele's (1986) Phase of Learning 3: the Explication Phase. Nonetheless, a noteworthy aspect of this case study was the capacity of high-performing learners to identify effective and efficient models for solving word problems with an algebraic foundation. This ability signifies that these learners are beginning to solve problems independently and apply their knowledge across various contexts. Consequently, it suggests that these learners have developed the capability to generalize algebraic principles and transfer

strategies to new problems. This the findings indicate that while learners struggled with articulating their reasoning, high-performing individuals demonstrated emerging independence in problem-solving and the application of algebraic concepts across different scenarios.

6.7. Contributions of this Intervention Study to the body of knowledge.

This intervention study contributes significantly to the existing body of research regarding the effectiveness of model strategies in algebraic problem-solving. It underscores the critical role of nurturing algebraic and relational reasoning habits among learners, which facilitates the application of efficient problem-solving techniques when addressing algebraic word problems. Consequently, this enhancement in reasoning skills leads to an overall improvement in students' mathematical competencies.

The data collected from learners who successfully navigated the word problems, along with the model strategies they utilized, provide educators with valuable insights into preferred and effective problem-solving methods among learners. Furthermore, the study reveals challenges encountered by learners possessing only a partial understanding of model strategies, indicating a need for targeted instructional support.

Emphasizing pedagogical approaches that prioritize relational understanding and reasoning is crucial for fostering a structural approach to algebra. Such an emphasis is instrumental in developing robust algebraic reasoning skills among students.

6.8. Implications for Teaching.

The findings contribute significantly to the body of knowledge by highlighting effective pedagogical strategies that enhance relational understanding and problem-solving capabilities in algebra among learners.

This intervention study (and this project in general) enhances our understanding of effective teaching strategies in algebra by highlighting the importance of a pedagogical approach that activates relational reasoning and providing insights into learners' preferences and challenges in problem-solving.

The research underscores the necessity for teacher training programs to enhance teaching effectiveness specifically in the domains of relational teaching and reasoning in knowledge building. It has been established that educators who incorporate problem-solving skills and models into their instructional practices significantly aid learners in developing a more profound and nuanced relational understanding of algebraic concepts, including unknowns and variables, as well as their applications. By consistently employing models to represent mathematical problems, learners are better equipped to connect abstract algebraic concepts with real-world contexts. This practice not only enriches their conceptual understanding but also improves their proficiency in navigating structured problem-solving within algebra.

In particular, during the senior phase of education, these methodologies enable teachers in implementing appropriate scaffolding strategies within mathematics classrooms. Teaching that enhances relational approaches are instrumental in cultivating flexible problem-solvers who can adapt their knowledge and skills to approach various types of algebraic questions. Furthermore, they enable learners to draw connections between disparate topics or utilise model strategies to decompose complex problems into more manageable components.

The findings from this intervention study suggest that learners with higher proficiency levels are capable of selecting the most effective strategies tailored to the specific characteristics of algebraic problems. This insight can serve as a valuable resource for educators, enabling them to assist students in making informed strategic choices when approaching algebraic tasks.

Furthermore, it is imperative for educators to expose students to a diverse array of models for solving algebraic problems. Instruction should not only focus on the introduction of these models but also emphasize the effective application of such strategies. Establishing a robust foundational understanding of effective models for addressing algebraic word problems is essential for fostering learners' problem-solving skills.

Thus, proficient learners can effectively choose strategies suited to specific algebraic challenges, highlighting the need for teachers to provide varied models and ensure a strong foundational understanding among students.

6.9. Limitations of the Intervention Study.

This intervention study was conducted with a small sample of learners, which restricts the

possibility to generalise the findings to a broader population. However, it is important to acknowledge that the results lay the groundwork for larger-scale research, where the findings could be either confirmed or refuted or even extended to a more diverse sample from a variety of different schools.

6.10. Recommendations

A further study with a larger sample space can be conducted. Moreover, the post-test results can be more robust if learners were exposed to more types of models when answering algebraic word-problems and had the time to familiarise themselves with more advanced questions (part 2 of the pre- and post-tests) and apply the habits of mind that the intervention proposed.

A further recommendation is anchored in supporting the improvement in learner algebraic reasoning through a variety of instructional strategies that encourage deep conceptual and relational understanding and building toward the development of problem-solving skills in algebra. This can be done through integrating different models, such as visual aids, diagrams, or manipulatives, and can assist learners in building a more intuitive grasp of abstract algebraic concepts.

By using model strategies, learners can deconstruct complex problems and gain a clearer understanding of how the various elements of an equation are relationally interconnected. Secondly, by providing learners with opportunities to explore algebraic problems in a collaborative environment can enhance their reasoning skills. It is through group discussions facilitated by the more knowledgeable other where learners can exchange and refine their ideas, recognise patterns, and create more effective strategies for solving algebraic problems.

6.11. Self-Reflection.

This research offered valuable insights into the algebraic reasoning of South African senior phase learners and the strategies they employ to solve word problems. It revealed gaps within the senior phase classroom and provided explanations for the poor performance in algebra at this level. As a teacher-researcher, participating in this research has increased my awareness of the teaching strategies I need to emphasize in the classroom to enhance relational understanding and foster the development of algebraic reasoning skills. Additionally, this project has expanded my knowledge of the critical role of modelling in algebra, particularly in developing learners' problem-solving

skills, and highlighted my role in facilitating this process as a teacher

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Ascent Form

Algebraic Problem-solving through a relational approach intervention

I am happy for my tests answers to be used in the Masters research project. Please tick one of the boxes.

YES ☐ NO ☐

Learner name: _____
Grade: _____
School: _____
Province: _____

Declaration by participant

By signing below, I .....agree to for my test data to be used in the project entitled (*Algebraic Problem-solving through a relational approach intervention).*

I declare that:

- The teacher (researcher) has explained the work I will be doing in class on this project. She has explained this in a language and in a way that I understand.
- I know that I can say no to my test answers being used in the project as I understand that taking part in this project is **voluntary,** and I have not been pressurised to take part.
- My role in this project will be, if I agree to participate, to interact in all activities taking place during the teaching lessons.

Signed at (*place*) ..... on (*date*).....2024.

.....

Signature of participant (Learner)

Tick the Option you choose for tests without my name sharing with journals:

I agree to have my test answers without my name shared with journals during publication of results of this project

☐ Signature_____

I agree that direct quotations from digital recordings/interviews/focus activities/group activities may be used by the researcher in their research project.

☐ Signature_____

OR

I do not agree to have my test answer without my name shared with journals during publication of results of this project

☐ Signature_____

Tick the Option you choose for sharing tests results with researchers beyond the Masters Student:

I do not want my tests answers to be shared with researchers

☐ Signature_____

OR

My tests answers may be shared with researchers beyond the Masters Student for further analysis and future research in a field related to *Mathematics Education Research*.

☐ Signature_____

Consent Form

Algebraic Problem-solving through a relational approach intervention

I am happy for my child's test answers in the *Algebraic Problem-solving through a relational approach intervention* project to be collated and used for research alongside the development purposes detailed above.

Parent/Guardian name:	
Parent signature:	
Learner name:	
Learner grade:	
School:	
Province:	
Date:	

Declaration by parent

By signing below, I..... agree for my child's test results to be included as part of the Masters project entitled (*Algebraic Problem-solving through a relational approach intervention)*.

I declare that:

- I have read this information and consent form, or it was read to me, and it is written in a language in which I am fluent and with which I am comfortable.
- I have had a chance to ask questions, and I am satisfied that all my questions have been answered.
- I understand that the inclusion of my child's test results in this project is **voluntary,** and he/she have not been pressured to take part.
- I may ask that my child not provide access to her test answers at any time and nothing wrong will come of it – He/she will not be penalised or prejudiced in any way.

Signed at (*place*) ..... on (*date*) .....2024.

.....

Signature of Parent/ Guardian

Tick the option you choose for anonymous data sharing with journals:

I agree to have my child's anonymous test answers shared with journals during the publication of the results of this project

☐ Signature_____

OR

I do not agree to have my child's anonymous test answers shared with journals during the publication of the results of this project.

☐ Signature_____

I agree that direct quotations from interviews/focus activities/group activities with my child may be used by the researcher in their research project.

☐ ☐ Signature_____

OR

I do not agree that direct quotations from interviews/focus activities/group activities with my child may be used by the researcher in their research project.

☐ ☐ Signature_____

Tick the option you choose for sharing information with researchers beyond the Maters student:

I do not want my child's test answers to be shared with researchers

☐ Signature_____

OR

My child's test answers may be shared with researchers beyond the Masters Student for further analysis and future research in a field related to *Mathematics Education Research*.

☐ Signature_____

ANNEXTURE 1
PRE-TEST

TIME : 45 Minutes

1) Solve for x in each case below.

1.1. $x + 7 = 2$ (1)

1.2. $5x = 8$ (1)

1.3. $3x + 14 = 5$ (2)

1.4. $\frac{2}{3}x = 6 - 2x$ (3)

1.5. $5(x + 2) = -10$ (3)

[10 marks]

2) Read the statements below and show all the working when calculating your answers.

2.1. Erin is 5 years older than Sina. Sina is 7 years older than May. Their ages add to 31 years. How old are each of them?

(3)

2.2. Sbu runs the same distance each day, Monday to Saturday. On Sunday he goes for a longer run: $12km$. If he runs a total of $54km$ each week, how far does he run on Mondays ? (Show all working)

(3)

2.3. Carla, Dee and Mpho are three girls. Carla scores 5 goals. Dee scored twice as many goals as Mpho. Altogether, the three scored 17 goals. How many goals did each girl score ?

4)

[10 marks]

ANNEXTURE 2
POST-TEST SAMPLE

TIME: 45 Minutes

1) Solve for x in each case below.

1.1. $9-x=4$ (1)

1.2. $45x=15$ (1)

1.3. $5=\frac{x}{2}$ (2)

1.4. $3-4x=-17$ (3)

1.5. $\frac{1}{2}(5-x)=-2$ (3)

[10 Marks]

2) Read the statements below and show how you have calculated your answers.

2.1. Seven times a number is the same as 12 more than 3 times the number. Find the number.

(3)

2.2. Keith weighs $20kg$ more than Beth, while Henry weighs $30kg$ less than and twice as much as Beth. If Keith and Henry weigh the same, how much does Beth weigh (in kg) ?

(3)

2.3. Aunty K gave India some sweets. She gave Kyle twice as many sweets. Jade got 7 more s sweets than India. After the children gave their teacher 4 sweets, together they had 55 sweets left.

How many sweets did each child get?

(4)

TOTAL: 20 Marks

INTERVENTION LESSON 1

Lesson Content/Concept development: TIME: 45 MINUTES

TOPIC: PROBLEM SOLVING USING ALGEBRAIC RELATIONS	SUB-TOPIC: WORD PROBLEMS INVOLVING AGE

Concept and Skills:

- Recognise and interpret rules or relationships represented in symbolic form.
- Read the problem to carefully determine relationships between numbers.

Resources required:

• algebraic problem-solving handout activity from ***Algebra with Pizzazz***
Marcy, S., & Marcy, J. (1989). *Algebra with Pizzazz!: Practice Exercises for First Year Algebra*. Creative Publications, page 45 SEE APPENDIX A

Vocabulary:

- Variable ; 'the sum of' OR 'more than' ; 'the difference of' OR 'less than' Prior-Knowledge:
- Learners can add or subtract like terms.

Introduction: **5min:** Teacher lead a discussion about recognising word problems in algebra which involved age using a 'True or False' game

MENTAL WARM-UP INTSTUCTION: Read each algebraic equation related to age problems carefully. Determine whether each equation is True or False based on standard reasoning about ages. If you state FALSE, justify your answer:

1. **If x represents a person's current age, then "In 5 years, the person's age" can be expressed as:**

   $x + 5$ **[TRUE]**

2. **The sum of ages of two siblings, where the older sibling is n years old and the younger sibling is 3 years younger, can be expressed as:**

   $n + (n - 3) = 2n - 3$ **[TRUE]**

3. **If x is the current age of a person, then "5 years ago, the person was $x + 5x$ years old" can be written as**

   $x - y = 5$ **[FALSE, five years ago should be $x - 5$]**

These problems involved (a) **age of a single person** or (b) **relational aspect of age involved two people** and (c) **relational aspect of age of three or more people.**

How to solve problems involving age of a single person? This is similar to an integer problem therefore, the relationship between numbers (**for example, 'the sum of' ; 'the difference of' ; 'half of' etc)** must be carefully understood and identified by learners.

How to solve problems involving the age of more than one person ?If the word problem consists of solving the age of more than one person, it will be useful to **use a table to help organise the information and write out an equation. The use of a table will be a type of solution model learners engage with to help make sense of word problem.**

Concept development: **20min**

Example 1. **How to solve problems involving age of a single person?**

Five years ago, John's age was half of the age he will be in 8 years. How old is he now ?

Solution:

Phase 1: IDENTIFY AND INTRODUCE THE VARIABLE

Let x be John's age now. Read the question again and put in the relevant expressions above it:

$x - 5$ $\frac{1}{2}$ $x + 8$

Five years ago, John's age was half of the age he will be in 8 years.

Phase 2: FORM AN ALGEBRAIC STATEMENT THAT EXPRESSES THE MATHEMATICAL RELATIONSHIP
Write out an equation.

$$x - 5 = \frac{1}{2}(x + 8)$$

Isolate variable x:

$$x - \frac{1}{2}x = 4 + 5$$
$$\frac{1}{2}x = 9$$
$$x = 18$$

Phase 4: SOLVE THE PROBLEM

ANSWER: John is now 18 years old.

EXAMPLE 2: **How to solve problems involving the age of two people?**

Tanya is 4 years younger than Marcus. Twenty years ago, Marcus's age was 13 years more than half the age of Tanya. How old are both Tanya and Marcus now?

Phase 1: IDENTIFY AND INTRODUCE THE VARIABLE

Tanya or Marcus's present age is currently unknown. BUT Tanya's age is expressed in relation to Marcus's age, so let us express Marcus's age using the variable: h

- Marcus's age: h
- Tanya's age since she is four years younger than Marcus: $h - 4$

Phase 2: SET UP THE SOLUTION MODEL THAT WILL BE FOLLOWED

Set up a table
Fill the table with the information given.
Let x be Marcus's age as his age is unknown. Then, Tanya's present age must be $x + 28$.

NAME	PAST (20 years ago)	PRESENT	FUTURE
Tanya		$h-4$	
Marcus		h	

Let h be Marcus's age as his age is unknown. Then, Tanya's present age must be $h-4$.

The column labelled **Future** will be Tanya and Marcus's age in 6 years time. Therefore, we must '**add 6'** to both Tanya and Marcus's age.

- Tanya: $(h-4)$ – 20 = $h-24$
- Marcus: $h-20$

NAME	PAST (20 years ago)	PRESENT	FUTURE
Tanya	$h-24$	$h-4$	
Marcus	$h-20$	h	

Phase 3: FROM THE MODEL, FORM ALGBERAIC STATEMENTS THAT WILL EXPRESS THE MATHEMATICAL RELATIONSHIPS.

Write the new relationship in an equation. **20 years ago**, Marcus's age was **thirteen years more than half** the age as Tanya

$$\therefore Marcus's\ age\ in\ 20\ years\ ago = \frac{1}{2}\,(Tanya's\ age\ 20 years\ ago) + 13$$

$$h-20 = \frac{1}{2}\,(h-24) + 13$$

$$h-20 = \frac{h}{2} - 12 + 13$$

$$h-20 = \frac{h}{2} + 1$$

$$2(h-20) = 2\,(\frac{h}{2} + 1)$$

$$2h-40 = h+2$$

$$= 42$$

Phase 4: SOLVE THE PROBLEM

- Marcus's present age : 42 years old
- Tanya's present age: $h - 4 = 42 - 4 = 38$ years old

EXAMPLE 3: **How to solve problems involving the ages of three or more people ?**

Thabo is twice as old as his friend Sonwabo. Sonwabo is 5 years older than Grace. In 5 years, Thabo will be three times as old as Grace. How old is Sonwabo now ?

Phase 1: IDENTIFY AND INTRODUCE THE VARIABLE

Sonwabo's present age is currently unknown. Therefore, let us express Sonwabo's age using the expression: x

Fill the table with the information given.

Thabo is ***twice as old*** as his friend Sonwabo. Sonwabo is ***5 years older*** than Grace. In 5 years, Thabo will be three times as old as Grace. How old is Sonwabo now ?

Phase 2: SET UP THE SOLUTION MODEL THAT WILL BE FOLLOWED

Set up a table as a solution model

	Age now	Age in 5 years
Thabo	$2x$	$2x+5$
Sonwabo	x	$x+5$
Grace	$x-5$	$x-5+5$

Fill the table with the information given.

Thabo is ***twice as old*** as his friend Sonwabo. Sonwabo is ***5 years older*** than Grace. In 5 years, Thabo will be three times as old as Grace. How old is Sonwabo now ?

Let x be Sonwabo's age now. Add 5 to get the age in 5 years.

Phase 3: FROM THE MODEL, FORM ALGBERAIC STATEMENTS THAT WILL EXPRESS THE MATHEMATICAL RELATIONSHIPS.

Write the new relationship in an equation using the ages in 5 years. In 5 years,

Thabo will be ***three times*** as old as Grace

$$2x + 5 = 3(x - 5 + 5)$$
$$2x + 5 = 3x$$

Isolate the variable x $2x - 3x$

$-x = -5$

$x = 5$

Phase 4: SOLVE THE PROBLEM

Thabo is now 5 years old

Class activity: **20min**

Participants complete algebraic problem-solving handout activity from ***Algebra with Pizzazz***

Marcy, S., & Marcy, J. (1989). *Algebra with Pizzazz!: Practice Exercises for First Year Algebra*. Creative Publications, page 45 SEE APPENDIX A

INTERVENTION LESSON 2

Lesson Content/Concept development: TIME: 45 MINUTES

TOPIC: PROBLEM SOLVING USING ALGEBRAIC RELATIONS	SUB-TOPIC: WORD PROBLEMS INVOLVING CONSECUTIVE INTEGERS

Concept and Skills:

- Recognise and interpret rules or relationships represented in symbolic form.
- Read the problem to carefully determine relationships between numbers.

Resources required:

- Participants complete algebraic problem-solving handout activity ***Algebra with Pizzazz*** Marcy, S., & Marcy, J. (1989). *Algebra with Pizzazz!: Practice Exercises for First Year Algebra*. Creative Publications**.**, page 46, SEE APPENDIX B

Vocabulary:

- consecutive ; 'consecutive even integers' ; 'consecutive odd integers' Prior-Knowledge:
- Learners know what integers are and understand the conceptual meaning of consecutive.

Introduction: **5min**

Teacher led a discussion in the form of a True and False game about recognising word problems in algebra which involved consecutive integers. These were problems involving (a) **consecutive odd integers** or (b) **consecutive even integers. How to solve problems involving consecutive integers?**

MENTAL WARM-UP INSTRUCTION: Read each algebraic equation carefully. Decide whether it correctly represents the situation about consecutive integers. Write True or False and justify your answer

1. **If x represents the first of three consecutive integers, then the other two can be written as $x + 1$ and $x + 2$.**

SOLUTION: True; the first would be $x + 1$, $x + 2$

2. **If the difference between two consecutive integers is 1, then: $x - (x + 1) = 1$**

SOLUTION: False; difference is $(x + 1) - 1 = 1$

3. **The sum of two consecutive odd integers can be expressed as: $x + (x + 1) = 2x + 1$**

SOLUTION: False; Sum of two consecutive odd integers is $2x + 2$

Consecutive integers are integers that follow in sequence, each number being 1 more than the previous number, represented by n, n + 1, n + 2, n + 3, …, OR n - 1, n – 2 , n – 3, …, where *n* is any integer.

For example: 23, 24 , 25

If we start with an even number and each number in the sequence is 2 more than the previous number, then we will get **consecutive even integers**. **For example**: 16,18, 20, … or 2n , 2n+1, 2n+2,
If we start with an odd number and each number in the sequence is 2 more than the previous number then we will get **consecutive odd integers. For example:** 33, 35, 37 … or 33n, 33n-1, 33n2…
Concept development: **20min**

EXAMPLE 1: Find four consecutive odd integers whose sum is 152.

Phase 1: Identify and introduce the variable.

Let n = 1st Integer

Let $n + 2$ = 2nd Integer

Let $n + 4$ = 3rd Integer

Let $n + 6$ = 4th Integer

Phase 2: Translate the word sum into an equation

Find **four consecutive odd integers** whose **sum is 152**

$\therefore \boldsymbol{n + (n + 2) + (n + 4) + (n + 6) = 152}$

Phase 3: Form algebraic statements that will express the mathematical relationships

$n + (n + 2) + (n + 4) + (n + 6) = 152$

$4n + 12 = 152$

$4n = 152 - 12$

$4n = 140$

$n = 35$

Phase 4: Solve the problem

$n = 35$

$n + 2 = 35 + 2 = 37$

$n + 4 = 35 + 4 = 39$

$n + 6 = 35 + 6 = 41$

ANSWER: Four consecutive odd integers are 35, 37, 39 and 41

$\therefore 35 + 37 + 39 + 41 = 152$

EXAMPLE 2: Find three consecutive even integers such that 6 times the sum of the first and the third is 24 greater than 11 times the second.

Phase 1: Identify and introduce the variable

Let x = 1st Integer

$x + 2$ = 2nd integer x

$+ 4$ = 3rd integer

Phase 2: Translate the word sum into an equation

Find three consecutive even integers such that **6 times the sum of the first and the third** is **24 greater than 11 times the second.**

Rewrite sentence as an equation:

$$\mathbf{6}(\boldsymbol{x} + \boldsymbol{x} + \mathbf{4}) = \mathbf{11}(\boldsymbol{x} + \mathbf{2}) + \mathbf{24}$$

Phase 3: Form algebraic statements that will express the mathematical relationships.

$$6(2x + 4) = 11x + 22 + 24$$

$$12x + 24 = 11x + 46$$

$$x = 22$$

Phase 4: Solve the problem

1st Integer = 22

2nd integer= $x + 2 = 22 + 2 = 24$

3rd integer = $x + 4 = 22 + 4 = 26$

ANSWER: Three consecutive even integers are **22, 24, and 26**

Class activity: **20min**
Participants complete algebraic problem-solving handout activity ***Algebra with Pizzazz***
Marcy, S., & Marcy, J. (1989). *Algebra with Pizzazz!: Practice Exercises for First Year Algebra*. Creative Publications., page 46, SEE APPENDIX B

INTERVENTION LESSON 3

Lesson Content/Concept development: TIME: 45 MINUTES

TOPIC: PROBLEM SOLVING USING ALGBERAIC RELATIONS.	SUB-TOPIC: WORD PROBLEMS INVOLVING FRACTIONS

Concept and Skills:

- Recognise and interpret rules or relationships represented in symbolic form.
- Read the problem to carefully determine the numerator and denominator of a given fraction between numbers.

Resources required:

• Participants complete algebraic problem-solving handout activity from ***Algebra with Pizzazz*** Marcy, S., & Marcy, J. (1989). *Algebra with Pizzazz!: Practice Exercises for First Year Algebra*. Creative Publications., page 50, see APPENDIX C

Vocabulary:

- Increase ; decrease ; denominator and numerator. Prior-Knowledge:
- Learners know diverse types of fractions are.

Mental Warm Up instruction:
Read each word problem and its corresponding algebraic equation carefully. Decide whether the equation correctly represents the situation. Write **True** or **False**. If you stated FALSE, justify your answer.

1. A car travels for 3 hours at a speed of xxx km/h. The distance travelled is represented by $\frac{3}{x}$

SOLUTION: False; Distance = speed x time= $x \times 3 = 3x$

2. A recipe requires $\frac{1}{4}$ cup of sugar for each serving. For x servings, the amount of sugar needed is given by: $\frac{1}{4x}$

SOLUTION: False; should be $\frac{x}{4}$; since $\frac{1}{4}$ per serving $\times$ x servings.

3. A tank drains at a rate of $\frac{1}{6}$ per hour. After x hours, the fraction remaining is: $1 - \frac{x}{6}$

SOLUTION: True; the remaining fraction is full tank $-$ drained amount = $1 - \frac{x}{6}$

Introduction: **5min**

Teacher leads a discussion about recognising word problems in algebra which involves fractions.

This can be problems involving (a) **comparing fractions** or (b) **multiplying and dividing fractions**

TYPES OF FRACTIONS:

Unit fraction = $\frac{1}{1}$ **; Mixed Fraction =** $2\frac{44}{48}$ **;** $\frac{1}{2}$ **; Proper fraction =** $\frac{7}{4}$ **; Improper fraction =**

Concept development: 15**min**

EXAMPLE 1: $\frac{2}{3}$ of a number is 14. What is the number?

Phase 1: Assign variables that suit the equation.

Let x = a number

Phase 2: Translate the word sum into an equation

$\frac{2}{3}$ ***of a number is 14***. What is the number

$14 = x$

Phase 3: Solve the equation by isolating the variable

$14 = \frac{2}{3}x$

$x = 14 \times \frac{3}{2}$

$x = 21$

ANSWER: The number is 21.

Class activity: **25min**

Participants complete algebraic problem-solving handout activity from ***Algebra with Pizzazz***

Marcy, S., & Marcy, J. (1989). *Algebra with Pizzazz!: Practice Exercises for First Year Algebra*. Creative Publications., page 50, see APPENDIX C

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NONMEDICAL)

CLEARANCE CERTIFICATE	PROTOCOL NUMBER: 2024ECE016M
PROJECT TITLE	Investigating Changes in Grade 8 Learners' Algebraic Problem-Solving Through a Relational Approach
INVESTIGATOR	**MECHAELA CHARLSEY MEYER**
SCHOOL/DEPARTMENT OF INVESTIGATOR	WSoE
DATE CONSIDERED	06 May 2024
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	Minimal Risk
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	24 May 2024

CHAIRPERSON

Dr. Lawan Abdulhamid

cc: Dr Jacques Du Plessis

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

_____ _____/_____/_____

Signature **Date**

GAUTENG PROVINCE

Department: Education
REPUBLIC OF SOUTH AFRICA

8141411/2

GDE RESEARCH APPROVAL LETTER

Date:	08 October 2024
Validity of Research Approval:	08 February 2025-30 September 2025 2024/393
Name of Researcher:	Meyer MC
Address of Researcher:	10, 3rd Avenue
	Bovancia Villas, Florida/ Roodepoort, JHB
Telephone Number:	063 152 4051
Email address:	mechaela.meyer@gmail.com
Research Topic:	Investigating Changes in Grade 8 Learners' Algebraic Problem-Solving Through a Relational Approach.
Name of University:	Wits
Type of qualification	Masters
Number and type of schools:	1 Secondary
DistricUs/HO	Johannesburg North

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to

proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted,

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met, Approval may be withdrawn should any of the conditions listed below be flouted:



Making education a societal priority

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