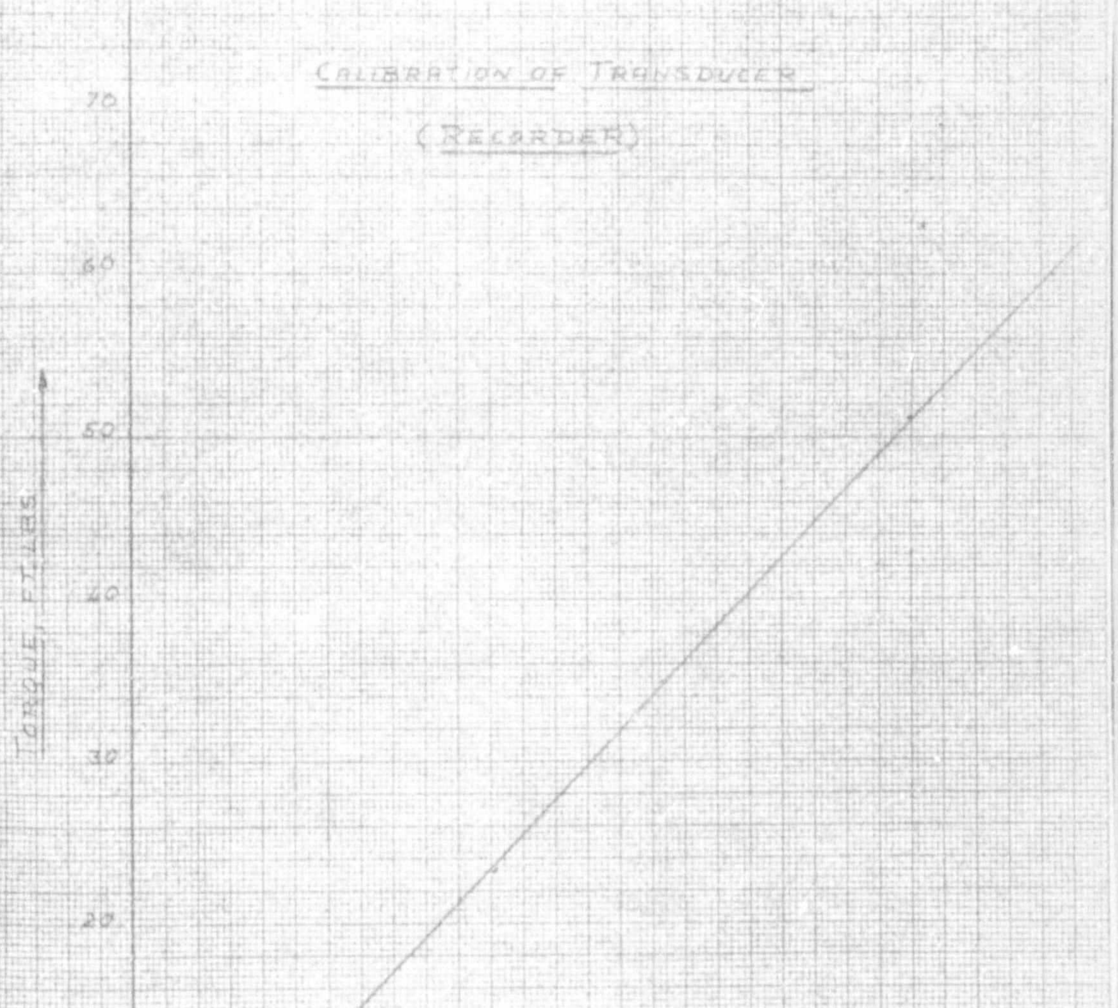

CALIBRATION OF TRANSDUCER
(RECORDER)
TORQUE, FT. LBS
70
60
50
40
30
20
10
0
0.5
1.0
DEFLECTION, CMS

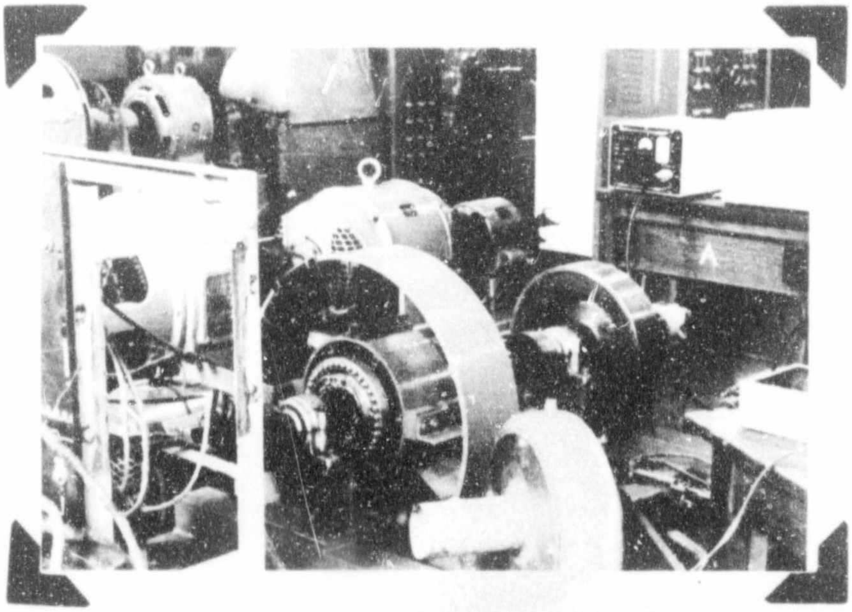


PHOTO A

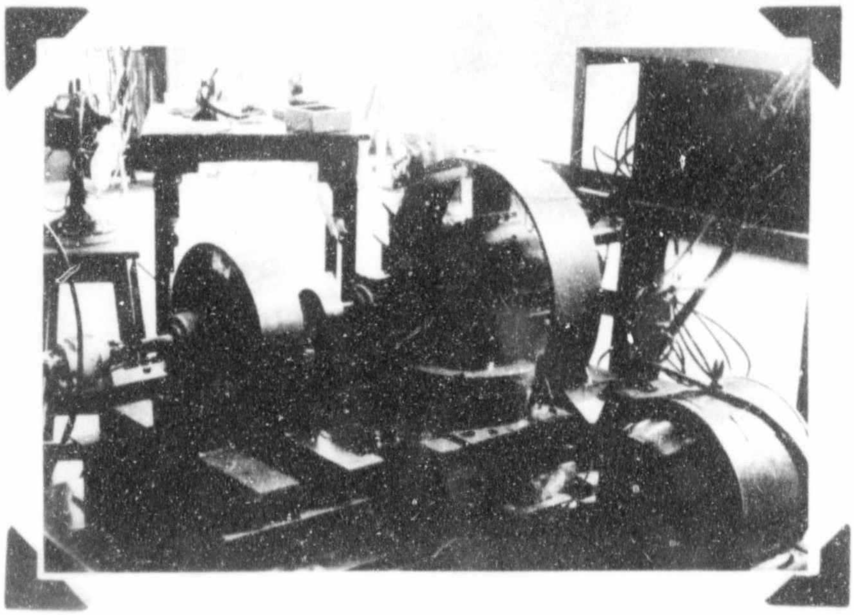
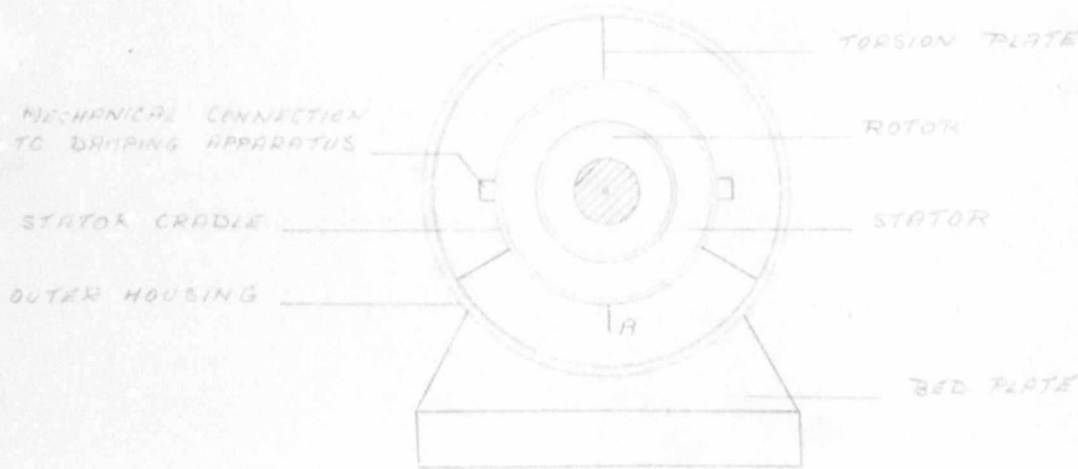


FIG A



TRANSDUCER MEASURES DISPLACEMENT OF
SPINDLE "A"

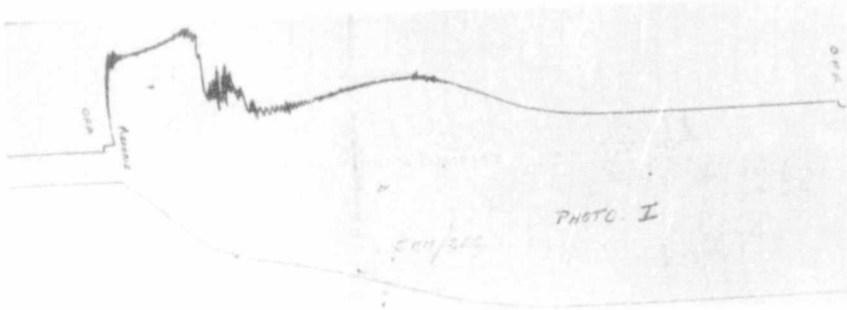


PHOTO I

PHOTO II

PHOTO III

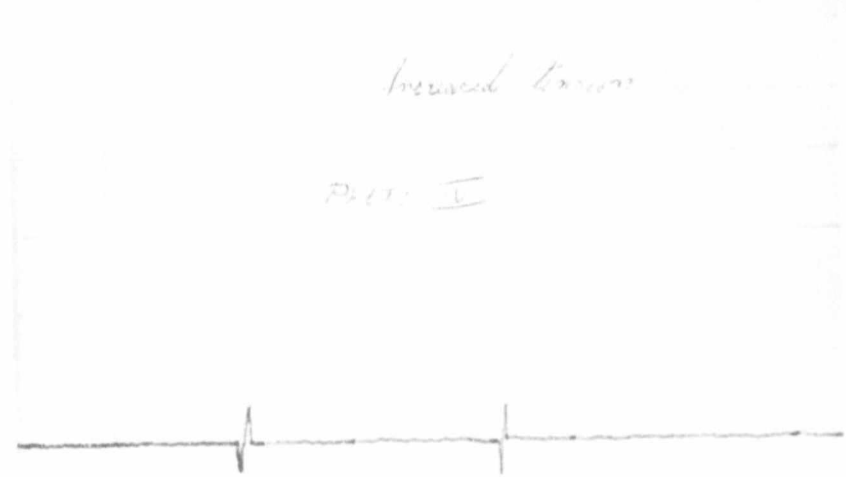


PHOTO IV

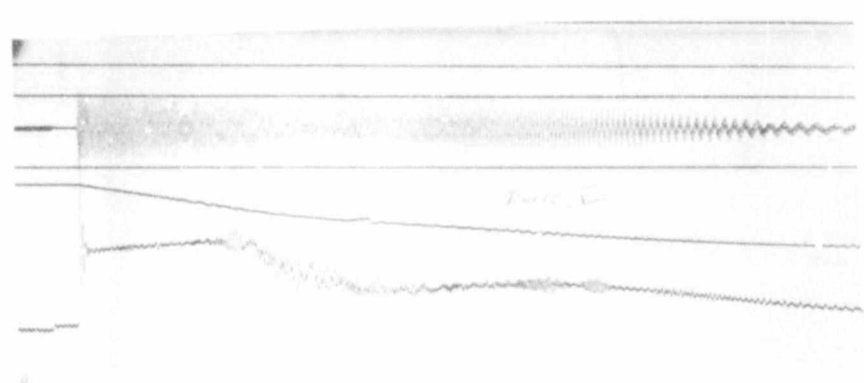


PHOTO V

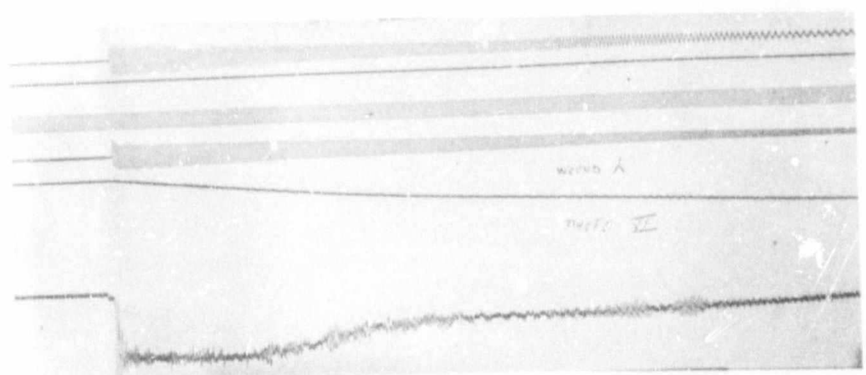


PHOTO VI

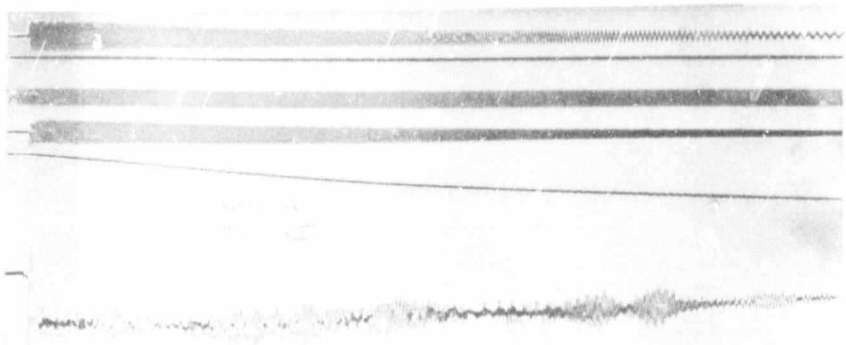


PHOTO VII

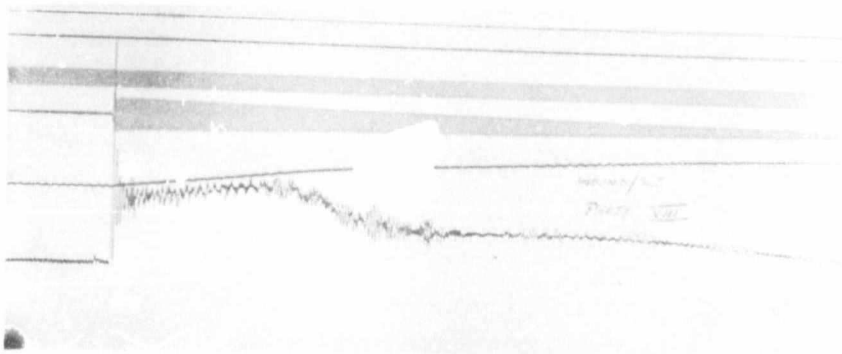


PHOTO VIII

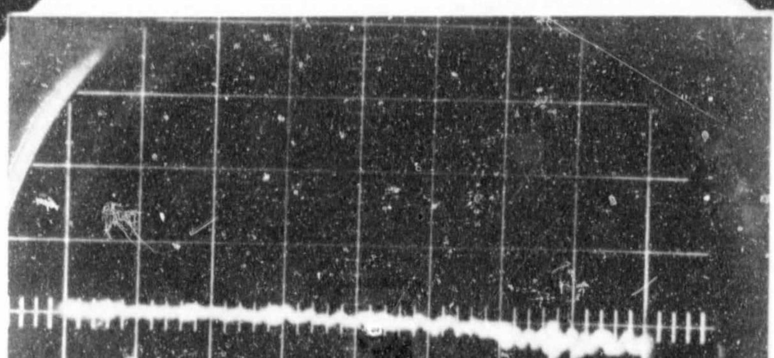
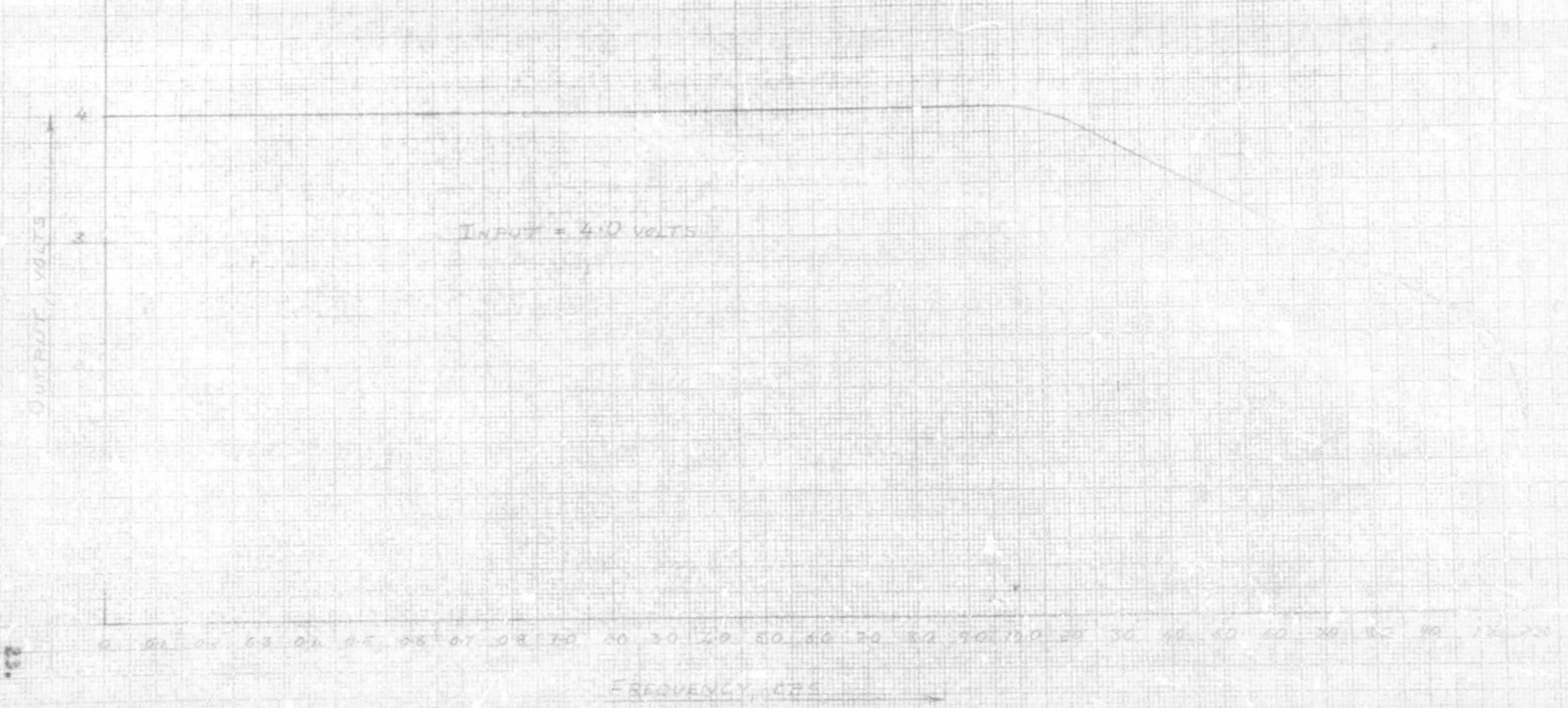


PHOTO IX

FREQUENCY RESPONSE OF LADDER NETWORK

FIG V



TESTING AND CAPABILITIES OF MEASURING APPARATUS.

1. Ultra - Violet Light Recorder.

In order to both test and determine the capabilities of the recording apparatus, it was decided to record speed - time and torque - time graphs for both a squirrel cage and wound secondary machine.

The machine used, being run inverted, had a secondary containing thirty six copper bars, 5.95 inches x 0.65 inches x 0.35 inches, which were held in place by brass end-rings. The copper conductors have 0.25" diameter rods protruding from each end which pass through holes in the end-rings and are thereby bolted into place. It was found that the copper to brass contact was not satisfactory and resulted in unbalance in the secondary. This contact was improved by placing aluminium foil between the two surfaces.

As may be seen by comparing photographs I and II (squirrel cage secondary) there is a large difference between the graphs obtained on the original Brush recording apparatus and that obtained on the ultra-violet light recorder. Because of the larger oscillations occurring in the new recording it was decided to check on the mechanical damping of the apparatus. Damping is obtained by electrically controlled dash-pots placed on either side of the annular structure supporting the secondary. This check was carried out by applying an unit impulse to the supporting structure and observing the output on the ultra-violet light recorder. This was done for various values of damping current.

Damping Current	Conclusion
1.5 amp.	Under damped
2.5 amp.	Critically damped
3.5 amp.	Over Damped

However, even when 2.5 amp. was applied to the damping windings and an unit impulse applied to the structure, there was still found to be a slight oscillation as shown in photo 3. It was thus felt that the damping as designed by M.F.L. Badham (Nov. 1955) was not adequate and these original results were re-checked. The results are shown over page and, as may be seen, are perfectly satisfactory.

It was subsequently discovered, however, that the damping magnets as designed by M.F.L. Badham were for a prototype machine for which they were adequate. It was hoped that the same damping magnets would be suitable for the present machine but the estimation

From page 29 of "The Performance and Design of Direct Current Machines" by Clayton, we have that

$$Kg_1 = \frac{\text{Reluctance with slotted armature}}{\text{Reluctance with smooth armature}}$$

$$= \frac{l_1 + l_3}{l_3} \qquad \text{———————— (1)}$$

and $$Kg_2 = \frac{l_2 + l_3}{l_3} \qquad \text{———————— (2)}$$

Considering (1)

$$l_3 Kg_1 = l_1 + l_3$$

Therefore

$$l_1 = l_3 (Kg_1 - 1)$$

and from (2)

$$l_2 = l_3 (Kg_2 - 1)$$

In using the above formulae, it must be remembered that the "slotted armature reluctance" is an average value as well as being an approximate assumption. Furthermore, the determination of l_1 and l_2 is only a possible approximate method.

Kg_1 and Kg_2 can be determined from the normal methods (see the above mentioned book by Clayton). l_1 and l_2 are equivalent lengths on the assumption that the length varies as a cosine law, despite the relative proportions of the slot opening, slot pitch and gap length.

Now, if the ampere-turns at any point are AT, then the flux density at that point (on the stator) is

$$B = \frac{\mu_0 \ AT}{lg - l_1 \cos Q_1 \theta_1 - l_2 \cos Q_2 (\theta_1 - \omega_s t)}$$

If the ampere turns are produced by an m- phase winding, housed in the stator slots, carrying a current I_1, and having T_1 turns per phase

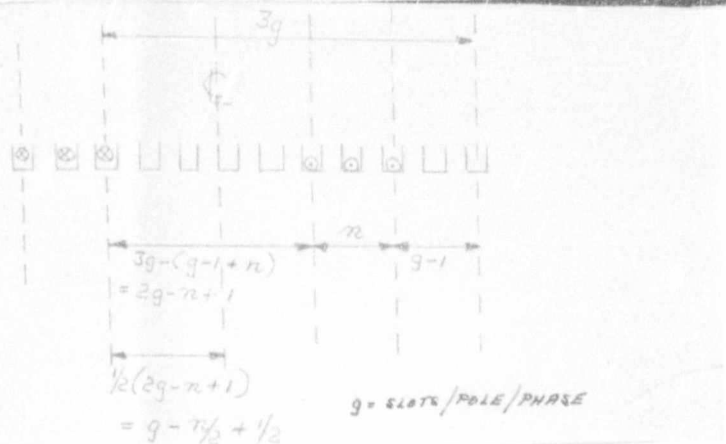
$$AT = \sum \frac{2^{\frac{1}{2}} m I_1 T_1}{q} \text{ Kmq Koq Sin } (wt - q\theta_1)$$

$$\text{where } q = \frac{2 KS m_1 p + p}{b_1}$$

θ_1 is measured from the centre-line of phase one. For any further work this must also be the relative point for reluctance measurements, i.e. the centre of the tooth from which measurement is taken must fall on the centre-line of the phase group of coils. This may not always be possible although it is the case for all integral slot windings.

Footnote:

When considering the qth harmonic, it must be borne in mind that q is the harmonic order based on the fundamental wavelength being the complete circumference of the machine.



The distance to the centre is a whole number of slot pitches provided n is odd. For this condition the centre-line of the phase coincides with the centre-line of a slot. If n is even the distance to the centre is a whole number of slot pitches plus $\frac{1}{2}$ slot pitch, which means that the centre-line of a phase coincides with the centre-line of a tooth. This is the case for a full pitched winding.

Considering the qth harmonic-three phase.

The flux density relative to the stator is

$$Bsq = \frac{\mu o \sqrt{2}.\ 3.\ I_1 T_1\ Kmq\ Keq\ Sin\ (wt - q\theta_1)}{\pi q\ \{\ lg - l_1 \cos Q_1\theta_1 - l_2 \cos Q_2\ (\theta_1 - w_s t)\ \}}$$

$$= \frac{Aq\ Sin\ (wt - q\ _1)}{\pi q\ \{\ lg - l_1 \cos Q_1\theta_1 - l_2 \cos Q_2\ (\theta_1 - w_s t)\ \}}$$

where $Aq = \mu o\ \sqrt{2}\ 3\ I_1\ T_1\ Kmq\ Keq$

$$\text{ie } Bsq = \frac{Aq}{\pi q} Sin\ (wt - q\theta_1) \left\{ \frac{1}{lg - l_1 \cos Q_1\theta_1 - l_2 \cos Q_2(\theta_1 - w_s t)} \right\}$$

$$\doteq \frac{Aq}{\pi q} Sin\ (wt - q\theta_1) \left\{ \frac{1}{lg} - \left[\frac{- l_1 \cos Q_1\theta_1 - l_2 \cos Q_2\ (\theta_1 - w_s t)}{lg^2} \right] \right\}$$

From the Binomial Expression

$$\frac{1}{a + b} = (a + b)^{-1} = a^{-1} - a^{-2}\ b + a^{-3}\ b^2 + \dots\dots\dots$$

$$= \frac{1}{a} - \frac{b}{a^2} + \frac{b}{a^3} + \dots\dots$$

Footnote:

For a full pitched winding n = 0

Hence

$$Bsq = \frac{Aq}{\pi q \; lg} \operatorname{Sin}(wt - q\theta_1) + \frac{Aq \; l_1 \operatorname{Cos} Q_1\theta_1 \operatorname{Sin}(wt - q\theta_1)}{\pi q \; lg^2}$$

$$+ \frac{Aq \; l_2 \operatorname{Cos} Q_2 (\theta_1 - W_s t) \operatorname{Sin}(wt - q\theta_1)}{\pi q \; lg^2}$$

$$\text{Now } \operatorname{Cos} Q_1 \theta_1 \operatorname{Sin}(wt - q\theta_1) = \frac{1}{2}\Big\{\operatorname{Sin}(Q_1\theta_1 + wt - q\,\theta_1) - \operatorname{Sin}(Q_1\theta_1 - wt + q\theta_1)\Big\}$$

$$= \frac{1}{2}\Big[\operatorname{Sin}\{(Q_1 - q)\,\theta_1 + wt\} - \operatorname{Sin}\{(Q_1 + q)\,\theta_1 - wt\}\Big]$$

Similarly

$$\operatorname{Cos} Q_2 \; (\theta_1 - w_s t) \; \operatorname{Sin}(wt - q\theta_1)$$

$$= \frac{1}{2}\Big[\operatorname{Sin}\Big\{(Q_2 - q)\theta_1 + (w - Q_2 W_s)\,t\Big\} - \operatorname{Sin}\Big\{(Q_2 + q)\theta_1 - (w + Q_2 W_s)t\Big\}\Big]$$

Hence,

$$Bsq = \frac{Aq}{\pi q}\Big[\frac{1}{lg}\operatorname{Sin}(wt - q\theta_1) + \frac{l_1}{2lg^2}\Big[\operatorname{Sin}((Q_1 - q)\theta_1 + wt) - \operatorname{Sin}((Q_1 + q)\theta_1 - wt)\Big] + \frac{l_2}{2lg^2}\Big[\operatorname{Sin}((Q_2 - q)\theta_1 + (w - Q_2 w_s)t) - \operatorname{Sin}((Q_2 + q)\theta_1 - (w + Q_2 w_s)t)\Big]\Big] \quad - \quad (1)$$

Hence, each harmonic produces five waves as given below. (All considered relative to the stator).

1 Wave of magnitude $\frac{Aq}{\pi q \; lg}$ having 2q poles and a velocity $+ \frac{w}{q}$ radians per second.

2 ——— $\frac{Aq \; l_1}{2\pi q \; lg^2}$ — $2\,(Q_1 - q)$ ——— $-\frac{w}{Q_1 - q}$ ———

3 ——— $\frac{Aq \; l_1}{2\pi q \; lg^2}$ — $2\,(Q_1 + q)$ ——— $+\frac{w}{Q_1 + q}$ ———

4 ——— $\frac{Aq \; l_2}{2\pi q \; lg^2}$ — $2\,(Q_2 + q)$ ——— $-\frac{w - Q_2 W_s}{Q_2 - q}$ ———

5 ——— $\frac{Aq \; l_2}{2\pi q \; lg^2}$ — $2\,(Q_2 + q)$ ——— $+\frac{w + Q_2 W_s}{Q_2 + q}$ ———

Consider the effect of the following :

1 Synchronous wave $q = p$
2 Stator slot harmonics $q = \pm Q_1 + p$
3 Rotor slot harmonics $q = \pm Q_2 + p$

1. $q = p$ Substitute p for q in equation (1) giving five waves as follows.

	MAGNITUDE	NUMBER OF POLES	VELOCITY (mech rads per sec)
1.	$\frac{A_1}{\pi p l g}$	$2p$	$+ w/_p$
2.	$\frac{A_1 l_1}{2\pi p l g^2}$	$2(Q_1 - p)$	$- \frac{w}{Q_1 - p}$
3.	$\frac{A_1 l_1}{2\pi p l g^2}$	$2(Q_1 + p)$	$+ \frac{w}{Q_1 + p}$
4.	$\frac{A_1 l_2}{2\pi p l g^2}$	$2(Q_2 - p)$	$- \frac{w - Q_2 w_s}{Q_2 - p}$
5.	$\frac{A_1 l_2}{2\pi p l g^2}$	$2(Q_2 + p)$	$+ \frac{W + Q_2 w_s}{Q_2 + p}$

2a) Substitute $q = Q_1 + p$.

	MAGNITUDE	NUMBER OF POLES	VELOCITY (mech rads per sec)
1.	$\frac{A_2}{\pi(Q_1 + p)\, lg}$	$2(Q_1 + p)$	$+ \frac{w}{Q_1 + p}$
2.	$\frac{A_2 l_1}{2\pi(Q_1 + p)\, lg^2}$	$2p$	$+ w/_p$
3.	$\frac{A_2 l_1}{2\pi(Q_1 + p)\, lg^2}$	$2\,(2Q_1 + p)$	$+ \frac{w}{2Q_1 + p}$
4.	$\frac{A_2 l_2}{2\pi(Q_1 + p)\, lg^2}$	$2(Q_2 - Q_1 - p)$	$- \frac{w - Q_2 w_s}{Q_2 - Q_1 - p}$
5.	$\frac{A_2 l_2}{2\pi(Q_1 + p) lg^2}$	$2(Q_2 + Q_1 + p)$	$+ \frac{w + Q_2 w_s}{Q_2 + Q_1 + p}$

2b) Substitute $q = -Q_1 + p$

	MAGNITUDE	NUMBER OF POLES	VELOCITY (mech rads per sec)
1.	$\frac{A_3}{\pi(-Q + p)\, lg}$	$2(-Q_1 + p)$	$+ \frac{w}{-Q_1 + p}$
2.	$\frac{A_3 l_1}{2\pi(-Q_1 + p) lg^2}$	$2(2Q_1 - p)$	$- \frac{w}{2Q_1 - p}$
3.	$\frac{A_3 l_1}{2\pi(-Q_1 + p) lg^2}$	$2p$	$+ \frac{w}{p}$
4.	$\frac{A_3 l_2}{2\pi(-Q_1 + p) lg^2}$	$2(Q_2 + Q_1 - p)$	$- \frac{w - Q_2 w_s}{Q_2 + Q_1 - p}$
5.	$\frac{A_3 l_2}{2\pi(-Q_1 + p) lg^2}$	$2(Q_2 - Q_1 + p)$	$+ \frac{w + Q_2 w_s}{Q_2 - Q_1 + p}$

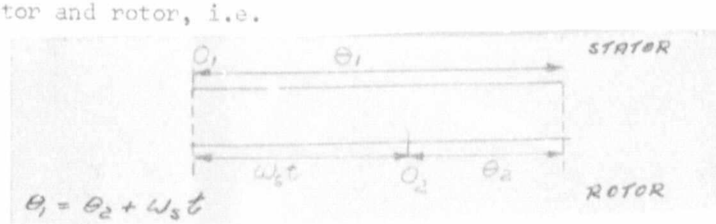
3a) Substitute $q = +Q_2 + p$

1.	$\frac{A_4}{\pi(Q_2 + p)lg}$	$2(Q_2 + p)$	+	$\frac{w}{Q_2 + p}$
2.	$\frac{A_4 l_1}{2\pi(Q_2 + p)lg^2}$	$2(Q_1 - Q_2 - p)$	-	$\frac{w}{Q_1 - Q_2 - p}$
3.	$\frac{A_4 l_1}{2\pi(Q_2 + p)lg^2}$	$2(Q_1 + Q_2 + p)$	+	$\frac{w}{Q_1 + Q_2 + p}$
4.	$\frac{A_4 l_2}{2\pi(Q_2 + p)lg^2}$	$2p$	+	$\frac{(w - Q_2 w_s)}{p}$
5.	$\frac{A_4 l_2}{2\pi(Q_2 + p)lg^2}$	$2(2Q_2 + p)$	+	$\frac{w + Q_2 w_s}{2Q_2 + p}$

3b) Substitute $q = -Q_2 + p$

1.	$\frac{A_5}{\pi(-Q_2 + p)lg}$	$2(-Q_2 + p)$	+	$\frac{w}{-Q_2 + p}$
2.	$\frac{A_5 l_1}{2\pi(-Q_2 + p)lg^2}$	$2(Q_1 + Q_2 - p)$	-	$\frac{w}{Q_1 + Q_2 - p}$
3.	$\frac{A_5 l_1}{2\pi(-Q_2 + p)lg^2}$	$2(Q_1 - Q_2 + p)$	+	$\frac{w}{Q_1 - Q_2 + p}$
4.	$\frac{A_5 l_2}{2\pi(-Q_2 + p)lg^2}$	$2(2Q_2 - p)$	-	$\frac{w - Q_2 w_s}{2Q_2 - p}$
5.	$\frac{A_5 l_2}{2\pi(-Q_2 + p)lg^2}$	$2p$	+	$\frac{w + Q_2 w_s}{p}$

The wave of flux density Bsq given on page 56 equation (1) is that referred to the stator. The wave referred to the rotor can be obtained by realising that after a time t the rotor has moved an amount $w_s t$ and hence this is the displacement between the zeros on both stator and rotor, i.e.



Substituting in equation (1) the equation for Bsq becomes, relative to the rotor

$$Brq = \frac{Aq}{\pi q}\left[\frac{1}{lg}\left\{Sin\ (wt - q\ (\theta_2 + w_s t)\right\} + \frac{l_1}{2lg^2}\left[Sin\left\{(Q_1 - q)X(\theta_2 + w_s t)\right.\right.\right.$$

$$+ wt\} - \text{Sin}\left\{(Q_1 + q)(\theta_2 + w_s t) - wt\right\}\Big] + \frac{1_2}{21g^2}\Big[\text{Sin}\left\{(Q_2 - q)(\theta_2 + w_s t)\right.$$

$$\left. + (w - Q_2 w_s)t\right\} - \text{Sin}\left\{(Q_2 + q)(\theta_2 + w_s t) - (w + Q_2 w_s)\, t\right\}\Big]\Big\}$$

$$\text{Sin}\,(wt - q\,(\theta_2 + w_s t)) = \text{Sin}\,\left\{(w - qw_s)\,t - q\theta_2\right\}$$

$$\text{Sin}\left\{(Q_1 - q)(\theta_2 + w_s t) + wt\right\} = \text{Sin}\left\{(Q_1 - q)\,\theta_2 + \left[(Q_1 - q)w_s + w\right]\,t\right\}$$

$$\text{Sin}\left\{(Q_1 + q)(\theta_2 + w_s t) - wt\right\} = \text{Sin}\left\{(Q_1 + q)\theta_2 + \left[(Q_1 + q)w_s - w\right]t\right\}$$

$$\text{Sin}\left\{(Q_2 - q)(\theta_2 + w_s t) + (w - Q_2 w_s)t\right\} = \text{Sin}\left\{(Q_2 - q)\theta_2 + (w - qw_s)\,t\right\}$$

$$\text{Sin}\left\{(Q_2 + q)(Q_2 + w_s t) - (w + Q_2 w_s)t\right\} = \text{Sin}\left\{(Q_2 + q)\theta_2 - (w - qw_s)t\right\}$$

Thus

$$Brq = \frac{Aq}{\pi q}\left\{\frac{1}{1g}\ \text{Sin}\left[(w - qw_s)\,t - q\theta_2\right]\right.$$

$$+ \frac{1_1}{21g^2}\left[\text{Sin}\left[(Q_1 - q)\theta_2 + ((Q_1 - q)\,w_s + w)t\right] - \text{Sin}\left((Q_1 + q)\theta_2 + ((Q_1 + q)\,w_s - w)\,t\right)\right]$$

$$+ \frac{1_2}{21g^2}\left[\text{Sin}\left((Q_2 - q)\theta_2 + (w - qw_s)t\right) - \text{Sin}\left((Q_2 + q)\theta_2 - (w - qw_s)t\right)\right] \quad (2)$$

It is seen, that observed from the rotor the flux density wave is associated with three frequencies.

$$fr_1\,q = \frac{w - qw_s}{2\pi}\ ;\quad fr2q = \frac{(Q_1 - q)\,w_s + w}{2\pi}\ ;\quad fr3q = \frac{(Q_1 + q)\,w_s - w}{2\pi}$$

From equation (1) however, it is seen that the flux density wave as seen from the stator is associated with 3 frequencies

$$fs_1\,q = \frac{w}{2\pi}\ ;\quad fs2q = \frac{w - Q_2 w_s}{2\pi}\ ;\quad fs3q = \frac{w + Q_2 w_s}{2\pi}$$

Note : Rotor frequencies fr2q and fr3q depend on stator teeth

Stator frequencies fs2q and fs3q depend on rotor teeth

No rotor frequency is produced by rotor teeth.

No stator frequency is produced by stator teeth.

Considering the equation for the flux it is immediately obvious that it can be extended by either considering further terms in the Binomial expansion, or by considering further terms in the Fourier series for the effect of both primary and secondary slots on the airgap, or a combination of both.

In order to determine the extent to which these two "expansions" should be taken it was decided to evaluate numerically the various terms in the expression for the flux. It may be seen from page 55 that all the terms in the expression are known, or can be calculated, apart from I_1. I_1, however, can easily be measured.

The numerical calculation of the flux components follows.

Numerical Calculations:

For all the following calculations it must be remembered that the machine was being run inverted.

Inside diameter of machine		= 17.1 cms.
Rotor diameter		= 17.05 cms.
Outside diameter of stator		= 26.1 cms.
Primary slot pitch	$= \dfrac{\pi \times 17.05}{24}$	= 2.24 cms.
Secondary slot pitch	$= \dfrac{\pi \times 17.1}{36}$	= 1.49 cms.
Primary and secondary slot openings		= 0.318 cms.
Length of airgap	= 0.0456 cms.	
$\dfrac{\text{Slot width}}{\text{Gap width}}$	$= \dfrac{0.318}{0.0456}$	= 6.96

Therefore, from Carter's Air gap contraction Co-efficient Curve

$\delta = 0.63$

Hence

$$Kg_1 = Kg_p = \frac{2.24}{2.24 - 0.63 \times 0.318} = 1.1$$

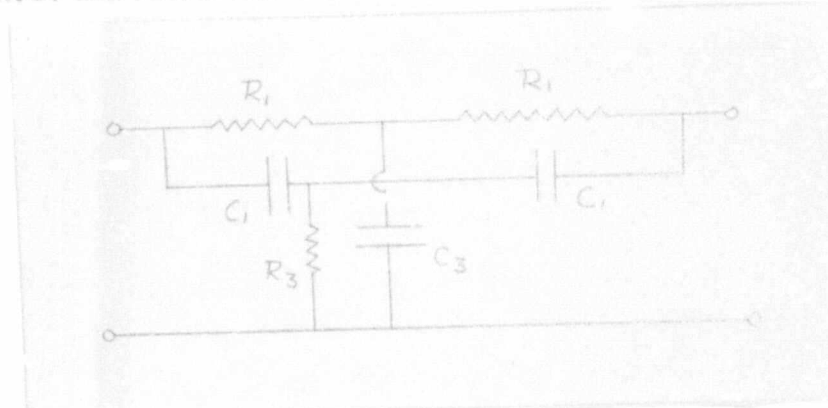
$$Kg_2 = Kg_s = \frac{1.49}{1.49 - 0.63 \times 0.318} = 1.16$$

$$l_1 = l_g (Kg_1 - 1)$$

$$= 0.0456 \times 0.1 = 0.00456 \text{ cms.}$$

Design of Parallel - T Filter for Reducing Hum to C.R.O.

The carrier frequency was found to be objectionable when using the C.R.O. and thus it was decided to filter out the 2320 cps. carrier.



f_o = 2320 (measured oscillator frequency)

$C_1 = C_2 = 0.0102$ micro Farad

$$2\pi \times 2320 = \frac{10^6}{.0102 \times R_1}$$

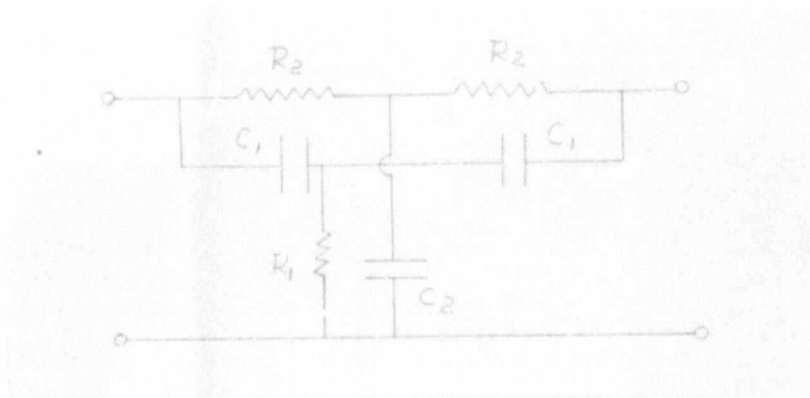
$$\text{i.e. } R_1 = \frac{10^6}{.0102 \times 2\pi \times 2320}$$

$$= 6630 \text{ OHMS.}$$

C_3 must be 0.0204 micro Farad

R_3 must be 3315 OHMS.

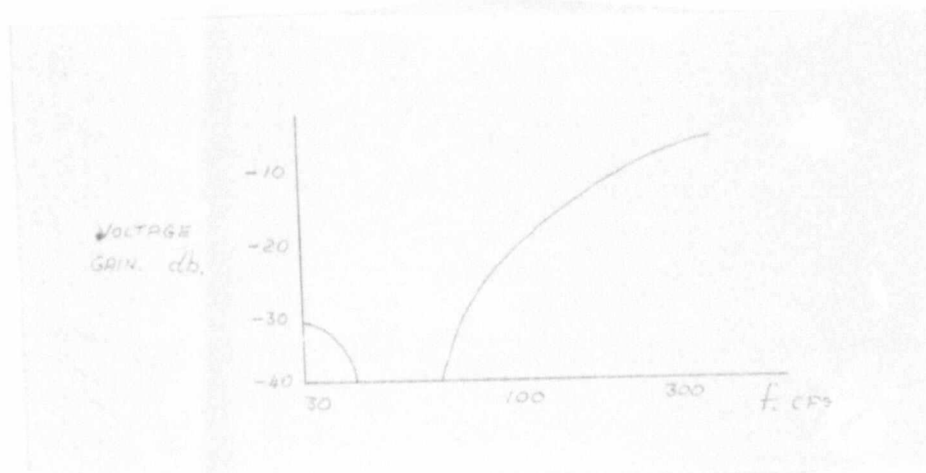
Parallel T Filter to Eliminate 50 cps Hum.



C_1 = 0.001 micro Farad

C_2 = 0.004 micro Farad

R_1 = R_2 = 2 $M\Omega$



$$f = \frac{1}{2 \times 2^{\frac{1}{2}} \pi \; R_2 C_1}$$

$$= \frac{1}{2 \times 2^{\frac{1}{2}} \times 2.0 \times 10^{+6} \times 0.001 \times 10^{-6} \pi}$$

$$\neq \frac{10^3}{18.0} \neq 53 \text{ cps.}$$

Input impedance too high

R_1 must be reduced.

Reducing R_2

We have that:

$$C_1 = C_2 \qquad R_2 = 4R_1$$

$$\text{Let } C_1 = 1 \mu F$$

Therefore:

$$50 = \frac{1}{2^{\frac{1}{2}} \pi \; R_2 \times 1 \times 10^{-6}}$$

$$\text{i.e.} \qquad R_2 = 4.5 \text{ K}$$

Therefore:

$$R_1 = 1.1 \text{ K OHM.}$$

Even with this smaller value of R_1 the filter was found to have too large an attenuation.

CALIBRATION OF APPARATUS

Substituting for the above values of q

q	Keq
-34	-1
-22	+1
-10	-1
2	+1
14	-1
26	+1

Consider the synchronous wave, and the stator and rotor harmonics. Referring to pages 57 and 58, make the following substitutions.

Let $\dfrac{A_1}{\pi \, p l g} = X \qquad \dfrac{A_3}{\pi(-Q_1 + p) \, lg} = Z$

$$\frac{A_2}{\pi(Q_1 + p) \, lg} = Y \qquad \frac{A_5}{\pi(-Q_2 + p) \, lg} = V$$

$$\frac{A_4}{\pi(Q_2 + p) \, lg} = W \qquad \frac{l_1}{2lg} = K \text{ and } \frac{l_2}{2lg} = L$$

Thus, the magnitudes given on pages 57 and 58 can be written as follows.

I) $q = p$

1) X
2) XK
3) XK
4) XL
5) XL

II) $q = Q_1 + p$

1) Y
2) YK
3) YK
4) YL
5) YL

III) $q = -Q_1 + p$

1) Z
2) ZK
3) ZK
4) ZL
5) ZL

IV) $q = -Q_2 + p$

1) W
2) WK
3) WK
4) WL
5) WL

V) $q = -Q_2 + p$

1) V
2) VK
3) VK
4) VL
5) VL

By writing the magnitudes in this manner, the similarity between the various harmonics can immediately be discerned.

Now, for the machine used we have that $Q_1 = 24$, $Q_2 = 36$ and $p = 2$. Also, for the waves to be considered we have the following.

q	Kmq	Keq
2	0.765	+ 1
26	-0.965	+ 1
-22	-0.965	+ 1
35	0.255	- 1
-34	-0.255	- 1

Also, $K = \dfrac{l_1}{2l_g} = \quad 3.90 \times 10^{-2}$

and $\quad L = \dfrac{l_2}{2l_g} = \quad 6.34 \times 10^{-2}$

By substituting the above values in the various values of Aq given on pages 57 and 58 we can determine the various magnitudes of A_1, A_2, A_3, A_4 and A_5, and hence the magnitudes of the various waves.

I. $\quad A_1 = 4\pi \times 10^{-7} \times 2^{\frac{1}{2}} \times 3 \times 96 \times 0.965 \times i \times I_1$

$\quad\quad = 4940 I_1 \times 10^{-7}$

Thus the magnitudes are

1) $\quad 4940 I_1 \times 10^{-7} \times \dfrac{1}{\pi \times 2 \times 57.6 \times 10^{-3} \times 10^{-2}}$

$\quad\quad = 13.65 I_1 \times 10^{-2}$

2) $\quad 0.538 I_1 \times 10^{-2}$

3) $\quad 0.538 I_1 \times 10^{-2}$

4) $\quad 0.865 I_1 \times 10^{-2}$

5) $\quad 0.865 I_1 \times 10^{-2}$

II. $A_2 = 4\pi \times 10^{-7} \times 2^{\frac{1}{2}} \times 3 \times 96 \times 1 \times I_1 \times (-0.965)$

$= $ [illegible] $I_1 \times 10^{-7}$

1) $-$ [illegible] $\times 10^{-2}$

2) [illegible]$.0415I_1 \times 10^{-2}$

3) $-0.0415I_1 \times 10^{-2}$

4) $-0.0665I_1 \times 10^{-2}$

5) $-0.0665I_1 \times 10^{-2}$

III. $A_3 = -4940I_1 \times 10^{-7}$

1) $1.24I_1 \times 10^{-2}$

2) $0.0492I_1 \times 10^{-2}$

3) $0.0492I_1 \times 10^{-2}$

4) $0.0788I_1 \times 10^{-2}$

5) $0.0788I_1 \times 10^{-2}$

IV. $A_4 = -1305I_1 \times 10^{-7}$

1) $-0.19I_1 \times 10^{-2}$

2) $-0.0075I_1 \times 10^{-2}$

3) $-0.0075I_1 \times 10^{-2}$

4) $-0.012I_1 \times 10^{-2}$

5) $-0.012I_1 \times 10^{-2}$

V. $A_5 = 1305I_1 \times 10^{-7}$

1) $-0.245I_1 \times 10^{-2}$

2) $-0.00968I_1 \times 10^{-2}$

3) $-0.00968I_1 \times 10^{-2}$

4) $-0.0155I_1 \times 10^{-2}$

5) $-0.0155I_1 \times 10^{-2}$

Tabulating these results for the five waves:

	I	II	III
	$q = p$	$q = Q_1 + p$	$q = -Q_1 + p$
1.	$13.55 \times 10^{-2} I_1$	$-1.05 \times 10^{-2} I_1$	$1.24 \times 10^{-2} I_1$
2.	$0.538 \times 10^{-2} I_1$	$-0.0415 \times 10^{-2} I_1$	$0.0492 \times 10^{-2} I_1$
3.	$0.538 \times 10^{-2} I_1$	$-0.0415 \times 10^{-2} I_1$	$0.0492 \times 10^{-2} I_1$
4.	$0.865 \times 10^{-2} I_1$	$-0.0665 \times 10^{-2} I_1$	$0.0788 \times 10^{-2} I_1$
5.	$0.865 \times 10^{-2} I_1$	$-0.0665 \times 10^{-2} I_1$	$0.0788 \times 10^{-2} I_1$

	IV	V
	$q = Q_2 + p$	$q = Q_2 + p$
1.	$-0.19 \times 10^{-2} I_1$	$-0.245 \times 10^{-2} I_1$
2.	$-0.0075 \times 10^{-2} I_1$	$-0.00968 \times 10^{-2} I_1$
3.	$-0.0075 \times 10^{-2} I_1$	$-0.00968 \times 10^{-2} I_1$
4.	$-0.012 \times 10^{-2} I_1$	$-0.0155 \times 10^{-2} I_1$
5.	$-0.012 \times 10^{-2} I_1$	$-0.0155 \times 10^{-2} I_1$

All values are in Webers per square metre.

At this stage it was decided to obtain practical measurements of both the magnitude and the frequency of the flux in the rotor (secondary). In order to obtain these practical results a search coil of ten turns was placed around one of the rotor teeth. The end rings of the rotor were removed and the primary was driven by means of a D.C. motor and a twin V-belt drive. The primary was excited with a pure sinusoidal voltage of 124 volts; the exciting current being 2.5 amps. The primary used had straight slots and was star connected.

The output of the search-coil was read on both a Radiometer and a Marconi Wave Analyser, and the speed was kept constant by means of an accurately calibrated strobo-tachometer. A table of the values of magnitude and frequency obtained for various values of speed is shown overleaf.

1. 286 rpm.

Frequency	m V	Frequency	m V
Radiometer		Marconi	
44 cps	1000	44 cps	950
95	110	95	95
125	48	125	40
145	320	145	300
245	60	245	58
260	20	260	18
305	10	305	13
370	16	370	22
475	14	475	15
560	30	560	30

2. 856 rpm.

Frequency	m V	Frequency	m V
Radiometer		Marconi	
25 cps	520	20 cps	500
68	20	68	20
156	110	155	110
195	220	195	200
240	20	240	20
280	40	280	40
325	20	325	20
360	870	362	800
430	60	420	55
490	28	490	28
660	960	660	850
710	180	700	160
1050	72	1050	70

3. 1080 rpm.

Frequency	m V	Frequency	m V
Radiometer		Marconi	
14 cps	320	14 cps	310
45	12	45	12
130	2.3	130	2
170	8.0	170	9
210	120	210	130
230	360	220	225
440	1020	450	980
475	60	475	55
610	20	610	10
645	30	645	25
840	100	840	110
875	200	875	[illegible]
1200	20	1250	80
1300	92		
1700	88	1700	100

4. 1500 rpm.

Frequency	m V	Frequency	m V
Radiometer		Marconi	
200 cps	7	200 cps	7
300	500	300	400
600	1020	600	1010
880	80	880	70
1200	400	1200	350

5. 0 rpm

Frequency	m V	Frequency	m V
Radiometer		Marconi	
50 cps	1310	50 cps	1350
150	30	150	35
250	42	250	40

It must be stated that the above readings were only recorded after a large number of tests had been carried out. In the original readings large variations were encountered for the following reasons.

1. Slight variations in speed caused erroneous readings, and in the final tests the speed was continuously monitored and corrected.

2. The position of the search coil on the tooth affected the magnitude of the results obtained. For the results recorded above the search coil was wound as uniformly as possible, and was positioned as close as possible to the root of the rotor tooth.

3. Although the speed was held as accurately as could be obtained to the desired value, at certain frequencies there was a continuous fluctuation, or beat, of the millivoltmeter, which made accurate reading extremely difficult.

All leads from the search coil to the Wave Analysers were carefully screened and earthed, as it was found that unearthed leads altered the readings radically.

Footnote:

With the instruments used the speed could be held to within 0.5% of the desired value.

In order to check the values obtained for the flux density on page 65 the following calculations, dealing only with the fundamental wave, can be made.

As before

Machine diameter = 17.1 cms.

Machine length = 12.7 cms (5" iron length)

Number of turns 96, Kw = 0.965 x 1 = 0.965.

Length of airgap = 0.576 cms.

The test results obtained on page 66 were obtained with 124 volts applied to the winding. This is line voltage with the winding star connected. The magnetising current was 2.5 amps.

$$E = 4.44 \times \phi\, f\, T\, Kw.$$

$$= 4.44 \times \frac{2}{\pi}\ \hat{B}\ l\ J \times f\ T\ Kw$$

$$= \sqrt{2}\,\pi \times \frac{2}{\pi}\ \hat{B} \times 12.7 \times 10^{-2} \times 13.4 \times 10^{-2} \times 50 \times 96 \times .965$$

$$\text{where } J = \frac{\pi D}{2p} = \frac{\pi \times 17.1}{2 \times 2} = 13.4 \times 10^{-2} \text{metres}$$

$$\text{i.e. } E = \hat{B} \times 223$$

$$\text{Thus } \hat{B} = \frac{124}{\sqrt{3}} \times \frac{1}{223} = 0.321 \text{ wb/m}^2$$

$$\text{Also } \hat{AT} = \frac{6\sqrt{2}}{\pi} \times \frac{IT}{2p}$$

$$\therefore \hat{B} = \frac{6\sqrt{2}IT}{2p\pi} \times Kw \times \frac{1}{lg} \times \mu o$$

$$= \frac{3\sqrt{2} \times 96 \times 0.965 \times 11.4 \times 10^{-7}\ I_1}{\pi \times 2 \times 57.6 \times 10^{-5}}$$

$$\text{i.e. } \hat{B} = 13.65 \times 10^{-2}\ I.$$

This figure agrees with the value given on page 63.

This figure can now be checked with the above value for B by putting I_1 = 2.5 amp.

$$\text{i.e. } \hat{B} = 13.65 \times 10^{-2} \times 2.5$$

$$= 0.341 \text{ wb/m}$$

This agrees with the above value of 0.321 wb/m when it is remembered that the ampere turns for the "iron circuit" have been ignored in the above calculations.

In order to obtain the voltage in the search coil the average value of the flux density over the span of the coil must be found and be multiplied by the area of the coil.

In the preceeding work it may be seen that every wave has the following form

$$b = K \operatorname{Sin} (A\theta_1 + \Omega t) \quad \text{wb/sq. metre.}$$

where K, A and Ω may have different values for each wave.

If the coil has a span β (mech. measure) then the average value of the flux density is :-

$$\frac{1}{\beta} K \int \operatorname{Sin} (A\theta_1 + \Omega t) \, d\theta_1$$ the limits being taken over the span of the coil.

Now, the flux threading the coil will be a time varying quantity and different positions of the coil will mean different phase displacements of the time varying quantity. The limits can thus be arbitrarily chosen, and a simple expression follows if we consider the coil to be placed with its centre line at $\theta_1 = 0$, i.e. the limits of the integration will be from $-\beta/2$ to $+\beta/2$. This is still considering a coil on the primary.

Hence, the average value of the flux density is

$$\frac{1}{\beta} K \int_{-\beta/2}^{+\beta/2} \operatorname{Sin} (A\theta_1 + \Omega t) \, d\theta_1$$

The area of the coil is given by $\frac{\beta}{2\pi} . \pi Dl$ sq. metres.

($\frac{\beta}{2\pi}$ since β is span in mech. measure).

$$= \frac{\beta Dl}{2}$$

Hence, flux linking the coil is

$$\frac{\beta Dl}{2} . \frac{1}{\beta} . K \int_{-\beta/2}^{+\beta/2} \operatorname{Sin} (A\theta_1 + \Omega t) \, d\theta_1$$

$$= \frac{Dlk}{2} . \frac{1}{A} \Big[- \operatorname{Cos} (A\theta_1 + \Omega t) \Big]_{-\beta/2}^{+\beta/2}$$

$$= \frac{Dlk}{2A} \Big[- \operatorname{Cos} (\frac{A\beta}{2} + \Omega t) + \operatorname{Cos} (-\frac{A\beta}{2} + \Omega t) \Big]$$

$$= \frac{Dlk}{2A} \Big[\operatorname{Cos} (\Omega t - \frac{A\beta}{2}) - \operatorname{Cos} (\Omega t + \frac{A\beta}{2}) \Big]$$

$$= \frac{Dlk}{A} \operatorname{Sin} \frac{A\beta}{2} \operatorname{Sin} \Omega t.$$

With reference to the secondary, the same process can be carried out considering that the centre line of the coil was positioned at $\theta_2 = 0$. As may be seen by examining the wave equations, the A's are the same in each case but the Ω's are different for primary and secondary. This is to be expected physically as the number of "poles" of the wave must be the same looked at from either the primary or secondary side, but the frequencies will be different. Hence, the flux linking a coil on the secondary will be

$$\frac{Dlk}{A} \quad \mathrm{Sin} \ \frac{A\beta}{2} \ \mathrm{Sin} \ \Omega_2 t \quad \text{———} \quad \mathrm{I}$$

There remains the value to be given to β for the search coil. In the first instance it may be assumed that all the flux goes down a tooth, i.e. all the flux over a slot pitch of the member concerned goes down the tooth and hence links the search coil. This assumption is not strictly valid and of course conflicts with the idea of the reluctance variation given at the beginning of the work. It also conflicts with the results obtained for search coils in various positions on the tooth, but it is a good working assumption to start with. Thus, for the primary (24 slots) $\beta = \frac{360}{24} = 15°$, and for the secondary (36 slots) $\beta = \frac{360}{36} = 10°$.

If we consider the fundamental only

$K = 0.341$ --- for 2.5 amp current

$A = 2 \ (q = p = 2) \qquad \beta = 10°.$

$\therefore$ Mag. of the flux (peak value) $= \frac{17.1 \times 12.7 \times 10^{-4} \times 0.341}{2} \ \mathrm{Sin} \ 10$

$= \frac{17.1 \times 12.7 \times 0.341 \times 0.174}{2} \times 10^{-4}$

$= 0.645 \times 10^{-3}$ webers.

When the machine is standing still the frequency of the fundamental (as seen from the secondary) is 50 cps.(Ω for the fundamental is $(W - qw_s)t$ which is w for $w_s = 0$)

The emf produced in a ten turn search coil must therefore be

$E = 4.44 f \phi_s T$

$= 4.44 \times 50 \times 0.645 \times 10^{-3} \times 10$

$= 1.43$ volts.

This agrees reasonably well with the experimental results on pp. 67.

Hence, in a similar way to the work done on pp 56 ~~and~~ one can obtain, as far as the secondary is concerned, five main waves with the following

	Mag.	Pole pairs	Frequency
1.	$Aq/_{\pi q l g}$	q	$\frac{W - qW_s}{2\pi}$
2.	$\frac{Aql1}{2\pi qlg^2}$	Q_1-q	$\frac{(Q_1-q)\ W_s + W}{2\pi}$
3.	$\frac{Aql_1}{2\pi qlg^2}$	Q_1+q	$\frac{(Q_1+q)\ W_s - W}{2\pi}$
4.	$\frac{Aql_2}{2\pi qlg^2}$	Q_2-q	$\frac{W - qW_s}{2\pi}$
5.	$\frac{Aql_2}{2\pi qlg^2}$	Q_2+q	$\frac{W - qW_s}{2\pi}$

(See pages 57 and 58)

As before, consider the following

Synch. wave	$q = p$
Stator slot	$q = \pm Q_1+p$
Rotor slot	$q = \pm Q_2+p$

$\underline{q = p}$

1.	$\frac{A_1}{\pi p\ lg}$	p	$\frac{W - pW_s}{2\pi}$
2.	$\frac{A_1 l_1}{2\pi p\ lg^2}$	Q_1-p	$\frac{(Q_1-p)W_s + W}{2\pi}$
3.	$\frac{A_1 l_1}{2\pi p\ lg^2}$	$Q_1 + p$	$\frac{(Q_1 + p)\ W_s - W}{2\pi}$
4.	$\frac{A_1 l_2}{2\pi p\ lg^2}$	$Q_2 -p$	$\frac{W - pW_s}{2\pi}$
5.	$\frac{A_1 l_2}{2\pi p\ lg^2}$	$Q_2 +p$	$\frac{W - pW_s}{2\pi}$

$\underline{q = +Q_1 +p}$

1.	$\frac{A_2}{\pi(Q +p)lg}$	$Q_1 +p$	$\frac{W - (Q_1 +p)W_s}{2\pi}$

2..	$\frac{A_2 l_1}{2\pi(Q_1+p)lg^2}$	$-p$	$\frac{-pw_s + w}{2\pi}$
3.	$\frac{A_2 l_1}{2\pi(Q_1+p)lg^2}$	$2Q_1 + p$	$\frac{(2Q_1 + p)w_s + w}{2\pi}$
4.	$\frac{A_2 l_2}{2\pi(Q_1+p)lg^2}$	$Q_2 - Q_1 - p$	$\frac{w - (Q_1+p)w_s}{2\pi}$
5.	$\frac{A_2 l_2}{2\pi(Q_1+p)lg^2}$	$Q_1 + Q_2 + p$	$\frac{w - (Q_1+p)w_s}{2\pi}$

$q = -Q_1 + p.$

1.	$\frac{A_3}{\pi(-Q_1+p)lg}$	$-Q_1 + p$	$\frac{w - (-Q_1+p)w_s}{2\pi}$
2.	$\frac{A_3 l_1}{2\pi(-Q_1+p)lg^2}$	$2Q_1 - p$	$\frac{(2Q_1-p)w_s + w}{2\pi}$
3.	$\frac{A_3 l_1}{2\pi(-Q_1+p)lg^2}$	p	$\frac{pw_s - w}{2\pi}$
4.	$\frac{A_3 l_2}{2\pi(-Q_1+p)lg^2}$	$Q_2 + Q_1 - p$	$\frac{w - (-Q_1+p)w_s}{2\pi}$
5.	$\frac{A_3 l_2}{2\pi(-Q_1+p)lg^2}$	$Q_2 - Q_1 + p$	$\frac{w - (-Q_1+p)w_s}{2\pi}$

$q = Q_2 + p$

1.	$\frac{A_4}{\pi(Q_2+p)lg}$	$Q_2 + p$	$\frac{w - (Q_2+p)w_s}{2\pi}$
2.	$\frac{A_4 l_1}{2\pi(Q_2+p)lg^2}$	$Q_1 - Q_2 - p$	$\frac{(Q_1-Q_2-p)w_s + w}{2\pi}$
3.	$\frac{A_4 l_1}{2\pi(Q_2+p)lg^2}$	$Q_1 + Q_2 + p$	$\frac{(Q_1+Q_2+p)w_s - w}{2\pi}$
4.	$\frac{A_4 l_2}{2\pi(Q_2+p)lg^2}$	p	$\frac{w - (Q_2+p)w_s}{2\pi}$
5.	$\frac{A_4 l_2}{2\pi(Q_2+p)lg^2}$	$2Q_2 + p$	$\frac{w - (Q_2+p)w_s}{2\pi}$

1080 rpm.

k.	A.	Sin $A\beta/2$	ϕwb x 10^{-3}	f.	E.mV.
.341	2	0.174	.645	14	402
.0135	22	.934	.0124	446	246
.0135	26	.766	.00865	417	160
.216	34	.174	.00239	14	1.49
.0216	38	-.174	-.00214	14	-1.33
-.0275	26	.766	-.0176	417	-326
-.00104	2	.174	-.00197	14	-1.23
-.00104	50	-.9[illegible]	.000563	950	23.7
-.00166	10	.766	-.00237	417	-42.2
-.00166	62	-.766	.000287	417	5.32
.031	22	.934	.0286	346	440
.00123	46	-.766	-.00045	1200	-24
.00123	2	.174	.00234	14	1.46
.00197	58	-.939	-.00694	346	-107
.00197	14	.939	.00287	346	44
-.00475	38	-.174	.000472	634	13.3
-.000187	14	.939	-.000275	302	-3.66
-.000187	62	-.766	.00000502	965	.214
-.0003	2	.174	-.000568	634	-16
-.0003	74	.174	-.000153	634	-4.3
-.00612	34	.174	-.00068	562	-17
-.000242	58	-.939	.00085	1092	41.2
-.000242	10	.766	-.000402	130	-2.32
-.000388	70	-.174	.00002	562	.5
-.000388	2	+.174	-.000735	562	-18.3

Resultants:

f.	E.mV.
14	402.4
446	246
417	(-)203
950	23.7
346	377
1200	(-)24

Resultants:

f.	E.mV.
634	(-)7
302	(-)3.7
965	.214
562	(-)34.8
1092	41.2
130	(-)2.32

In all cases, K, A, Sin A $\beta/2$, and ϕ are independant of the speed. Hence, in each case firstly f. and finally E must be calculated. The method of calculation is as shown in the above two examples and the results given are the final resultants.

$$E = 4.44\ \phi\ f.\ T. = 44.4\ \phi\ f.$$
$$= K. \times f. \text{ in each case.}$$

1500 rpm.

f.	E.mV.
600	34.6
1300	32.5
500	544.5
1200	(-)24.0
900	(-)9.92
400	(-)4.85
1500	57.2
800	(-)49.5
200	(-)3.58

856 rpm.

21.4	594.2
364	200
321	(-)280.63
764	19.10
264	277.2
706	(-)14.10
492	(-)2.44
250	(-)3.02
835	0.186
435	(-)26.91
878	33.2
93	(-)1.66

256 rpm.

f.	E.mV.
41.5	1195.2
143.5	79.0
61	(-)53.4
262.5	6.57
43.5	47.33
245.5	(-)4.9
111.5	(-)1.23
109.5	(-)1.33
213.5	0.0426
94.5	(-)5.85
296.5	11.20
-7.5	0.0134

Comparison of Practical and Theoretical Results.

The following tables show the comparison of the practical and the theoretical results for each particulor value of speed.

1. 256 rpm.

Practical Frequency	Practical mV	Theoretical Frequency	Theoretical mV
44	975	41.5	1193
95	100	43.5	47
125	44	61	53
145	310	94.5	6
245	59	143	79
260	19	245	5
305	11	262	7
370	19	296.5	11
475	15		
560	30		

2. 856 rpm.

Practical Frequency	Practical mV	Theoretical Frequency	Theoretical mV
22	510	21.4	592
68	20	93	2
156	110	250	3
240	20	321	230
280	40	344	200
325	20	435	27
361	835	492	3
425	58	706	14
490	28	764	19
660	905	878	33
705	170		
1050	71		

3. 1080 rpm.

Practical Frequency	Practical mV	Theoretical Frequency	Theoretical mV
14	315	14	402
45	12	130	3
170	9	302	4
210	125	346	377
225	242	417	203
445	1000	446	246
475	58	562	35
610	15	634	7
645	28	950	24
840	105	1092	41

	Practical		Theoretical	
	Frequency	mV	Frequency	mV
	1080 rpm. (continued)			
	875	198	1200	24
	1225	50		
	1300	92		
	1700	94		
4.	1500 rpm.			
	200	7	200	4
	300	450	400	5
	600	1015	500	545
	880	75	600	35
	1200	375	800	50
			900	10
			1200	24
			1300	33
			1500	57
5.	0 rpm.			
	50	1330	50	1486
	150	33		
	250	41		

From these results, it is immediately obvious that apart from the fundamental wave in each case, there is little correlation between the practical and theoretical results.

If one goes back to the original assumption that the effect of the primary slots can be considered by increasing the airgap, and that this increase varies as a cosine law from the centre line of one stator tooth, then an attempt to obtain better correlation of results would be to take a further term in the Fourier expansion of the airgap.

Therefore increased length due to stator is

$$l_1 - l_1 \cos Q_1 \theta_1 + \frac{1}{3} l_1 \cos 3 Q_1 \theta_1 \qquad \text{and}$$

increased length due to secondary slots is

$$l_2 - l_2 \cos Q_2 (\theta_1 - W_s t) + \frac{1}{3} l_2 \cos 3 Q_2 (\theta_1 - W_s t)$$

Therefore total length is

$$l_3 + l_1 - l_1 \cos Q_1 \theta_1 + \frac{1}{3} l_1 \cos 3 Q_1 \theta_1 + l_2 - l_2 \cos Q_2 (\theta_1 - W_s t) + \frac{1}{3} l_2 \cos 3 Q_2 (\theta_1 - W_s t)$$

$$= lg - l_1 \cos Q_1 \theta_1 - l_2 \cos Q_2 (\theta_1 - W_s t) + \frac{1}{3} \left(l_1 \cos 3Q_1 \theta_1 + l_2 \cos 3 Q_2 (\theta_1 - W_s t) \right)$$

Where $lg = l_1 + l_2 + l_3$

Here

$$B^1 sq = \frac{Aq}{\pi q} \sin (wt - q\theta_1) \left\{ \frac{1}{lg - l_1 \cos Q_1 \theta_1 - l_2 \cos Q_2 (\theta_1 - W_s t) + \frac{1}{3} \left(l_1 \cos 3Q_1 \theta_1 + l_2 \cos 3Q_2 (\theta_1 - W_s t) \right)} \right\}$$

$$\doteqdot \frac{Aq}{\pi q} \sin (wt - q\theta_1) \left\{ \frac{1}{lg} - \frac{-l_1 \cos Q\theta_1 - l_2 \cos Q_2 (\theta_1 - W_s t) + \frac{1}{3} l_1 \cos 3Q_1 \theta_1 + \frac{1}{3} l_2 \cos 3Q_2 (\theta_1 - W_s t)}{lg^2} \right\}$$

$$= \frac{Aq}{\pi q\, lg} \sin (wt - q\theta_1) + \frac{Aq\, l_1 \cos Q_1 \theta_1 \sin (wt - q\theta_1)}{\pi q\, lg2} + \frac{Aq\, l_2 \cos Q_2 (\theta_1 - W_s t) \sin (wt - q\theta_1)}{\pi q\, lg^2}$$

$$- \frac{Aq\, l_1 \cos 3Q_1 \theta_1 \sin (wt - q\theta_1)}{3\pi q\, lg^2} - \frac{Aq\, l_2 \cos 3Q_2 (\theta_1 - W_s t) \sin (wt - q\theta_1)}{3\pi q\, lg^2}$$

$$= Bsq - \left[\frac{Aq\, l_1 \cos 3Q_1 \theta_1 \sin (wt - q\theta_1)}{3\pi q\, lg^2} + \frac{Aq\, l_2 \cos 3Q_2 (\theta_1 - W_s t) \sin (wt - q\theta_1)}{3\pi q\, lg^2} \right]$$

$$\cos 3Q_1 \theta_1 \sin (wt - q\theta_1) = \frac{1}{2} \left\{ \sin (3Q_1 \theta_1 + wt - q\theta_1) - \sin (3Q_1 \theta_1 - wt + q\theta_1) \right\}$$

$$= \frac{1}{2} \left[\sin \left\{ (3Q_1 - q)\theta_1 + wt \right\} - \sin \left\{ (3Q_1 + q)\theta_1 - wt \right\} \right]$$

$$\cos 3Q_2 (\theta_1 - W_s t) \sin (wt - q\theta_1)$$

$$= \frac{1}{2} \left[\sin \left\{ (3Q_2 - q)\theta_1 + (w - 3Q_2 W_s)t \right\} - \sin \left\{ (3Q_2 + q)\theta_1 - (w + 3Q_2 W_s)t \right\} \right]$$

$$\therefore B^1 sq = Bsq - \frac{Aql_1}{6\pi q\, lg^2} \left[\sin \left\{ (3Q_1 - q)\theta_1 + wt \right\} - \sin \left\{ (3Q_1 + q)\theta_1 - wt \right\} \right]$$

$$- \frac{Aql_2}{6\ q\ lg^2} \left[\sin \left\{ (3Q_2 - q)\theta_1 + (w - 3Q_2 W_s)t \right\} - \sin \left\{ (3Q_2 + q)\theta_1 - (W + 3Q_2 W_2)t \right\} \right]$$

Rewriting the expression with respect to the secondary,

i.e. putting $\theta_1 = \theta_2 + W_s t$ (see page)

$$B^1rq = Brq - \frac{Aql_1}{6\pi q l g^2}\left\{\operatorname{Sin}\left[(3Q_1 - q)\theta_2 + ((3Q_1 - q)W_s + W)t\right]\right.$$

$$\left.- \operatorname{Sin}\left[(3Q_1 + q)\theta_2 + ((3Q_1 + q)w_s - w)\,t\right]\right\} - \frac{Aql2}{6\pi q l g^2}\left[\operatorname{Sin}\left\{(3Q_2 - q)\theta_2 + (w - qw_s)t\right\} - \operatorname{Sin}\left\{(3Q_2 + q)\theta_2 - (w - qw_s)t\right\}\right]$$

Hence, extending the Fourier series to include three terms we find that each harmonic produces nine waves; the original five as given on page 56, plus an additional four as given below.

	Magnitude	Pole Pairs	Frequency
6.	$\frac{Aql_1}{6\pi q l g^2}$	$3Q_1 - q$	$\frac{(3Q_1 - q)w_s + w}{2\pi}$
7.	$\frac{Aql_1}{6\pi q l g^2}$	$3Q_1 + q$	$\frac{(3Q_1 + q)w_s - w}{2\pi}$
8.	$\frac{Aql_2}{6\pi q l g^2}$	$3Q_2 - q$	$\frac{w - qw_s}{2\pi}$
9.	$\frac{Aql_2}{6\pi q l g^2}$	$3Q_2 + q$	$\frac{w - qw_s}{2\pi}$

As before we must consider the harmonics :

Synchronous	$q = p$
Rotor slot	$q = \pm Q_2 + p$
Stator slot	$q = \pm Q_1 + p$

I. $q = p$

	Magnitude	Pole Pairs	Frequency
6.	$\frac{A_1 l_1}{6\pi p l g^2}$	$3Q_1 - p$	$\frac{(3Q_1 - p)\, w_s + w}{2\pi}$
7.	$\frac{A_1 l_1}{6\pi p l g^2}$	$3Q_1 + p$	$\frac{(3Q_1 + p)\, w_s + w}{2\pi}$
8.	$\frac{A_1 l_2}{6\pi p l g^2}$	$3Q_2 - p$	$\frac{w - p w_s}{2\pi}$
9.	$\frac{A_1 l_2}{6\pi p l g^2}$	$3Q_2 + p$	$\frac{w - p w_s}{2\pi}$

II. $q = Q_1 + p$

	Magnitude	Pole Pairs	Frequency
6.	$\frac{A_2 l_1}{6\pi (Q_1 + p) l g^2}$	$2Q_1 - p$	$\frac{(2Q_1 - p)\, w_s + w}{2\pi}$
7.	$\frac{A_2 l_1}{6\pi (Q_1 + p) l g^2}$	$4Q_1 + P$	$\frac{(4Q_1 + p)\, w_s + w}{2\pi}$
8.	$\frac{A_2 l_2}{6\pi (Q_1 + p) l g^2}$	$3Q_2 - Q_1 - p$	$\frac{w - (Q_1 + p)\, w_s}{2\pi}$
9.	$\frac{A_2 l_2}{6\pi (Q_1 + p) l g^2}$	$3Q_2 + Q_1 + p$	$\frac{w - (Q_1 + p)\, w_s}{2\pi}$

III. $q = -Q_1 + p$

	Magnitude	Pole Pairs	Frequency
6.	$\frac{A_3 l_1}{6\pi (-Q_1 + p) l g^2}$	$4Q_1 - p$	$\frac{(4Q_1 - p)\, w_s + w}{2\pi}$
7.	$\frac{A_3 l_1}{6\pi (-Q_1 + p) l g^2}$	$2Q_1 + p$	$\frac{(2Q_1 + p)\, w_s + w}{2\pi}$
8.	$\frac{A_3 l_2}{6\pi (-Q_1 + p) l g^2}$	$3Q_2 + Q_1 - p$	$\frac{w - (-Q_1 + p)\, w_s}{2\pi}$
9.	$\frac{A_3 l_2}{6\pi (-Q_1 + p) l g^2}$	$3Q_2 - Q_1 + p$	$\frac{w - (-Q_1 + p)\, w_s}{2\pi}$

IV. $q = Q_2 + p$

	Magnitude	Pole Pairs	Frequency
6.	$\frac{A_4 l_1}{6\pi (Q_2 + p) l g^2}$	$3Q_1 - Q_2 - p$	$\frac{(3Q_1 - Q_2 - p)\, w_s + w}{2\pi}$
7.	$\frac{A_4 l_1}{6\pi (Q_2 + p) l g^2}$	$3Q_1 + Q_2 + P$	$\frac{(3Q_1 + Q_2 + p)\, w_s + w}{2\pi}$

8. $\dfrac{A_4 l_2}{6\pi(Q_2+p)lg^2}$ $\quad 2Q_2 - p \quad$ $\dfrac{W-(Q_2+p)\,w_s}{2\pi}$

9. $\dfrac{A_4 l_2}{6\pi(Q_2+p)lg^2}$ $\quad 4Q_2 + p \quad$ $\dfrac{w-(Q_2+p)\,w_s}{2\pi}$

V. $\quad q = -Q_2 + p$

6. $\dfrac{A_5 l_1}{6\pi(-Q_2+p)lg^2}$ $\quad 3Q_1 + Q_2 - p \quad$ $\dfrac{(3Q_1+Q_2-p)\,w_s + w}{2\pi}$

7. $\dfrac{A_5 l_1}{6\pi(-Q_2+p)lg^2}$ $\quad 3Q_1 - Q_2 + p \quad$ $\dfrac{(3Q_1-Q_2+p)\,w_s + w}{2\pi}$

8. $\dfrac{A_5 l_2}{6\pi(-Q_2+p)lg^2}$ $\quad 4Q_2 - p \quad$ $\dfrac{w-(-Q_2+p)\,w_s}{2\pi}$

9. $\dfrac{A_5 l_2}{6\pi(-Q_2+p)lg^2}$ $\quad 2Q_2 + p \quad$ $\dfrac{w-(-Q_2+p)\,w_s}{2\pi}$

As done before on page 75 the values of k, A, Sin $\frac{A\beta}{2}$, $\phi = \frac{Dlk}{A}$ Sin $\frac{A\beta}{2}$, f and E = 4.44 ϕfT must be determined for the various values of speed.

<u>0 rpm</u> :

k.	A.	Sin Aβ/2	φwb.	f.	E mV.
.0045	70	-.174	-24.3×10^{-8}	50	- 0.54
.0045	74	.174	23.2×10^{-8}	50	0.515
.0072	106	.174	25.6×10^{-8}	50	0.568
.0072	110	-.174	-24.7×10^{-8}	50	- 0.548
-.00035	46	-.766	12.6×10^{-8}	50	0.28
-.00035	98	.766	-5.85×10^{-8}	50	-0.13
-.00055	82	.766	-11.1×10^{-8}	50	-0.246
-.00055	134	-.766	6.8×10^{-8}	50	0.151
.00041	94	.939	8.9×10^{-8}	50	0.197
.00041	50	-.939	-16.7×10^{-8}	50	-0.371
.00066	130	-.939	-10.4×10^{-8}	50	-0.231
.00066	86	.939	15.6×10^{-8}	50	0.346

-.000062	34	.174	-2.07×10^{-8}	50	-0.046
-.000062	110	-.174	0.213×10^{-8}	50	0.00473
-.0001	70	-.174	0.54×10^{-8}	50	0.012
-.0001	146	.174	-0.258×10^{-8}	50	-0.00573
-.000081	106	.174	-0.288×10^{-8}	50	-0.0064
-.000081	38	-.174	0.805×10^{-8}	50	0.0179
-.00013	142	-.174	0.345×10^{-8}	50	0.00765
-.00013	74	.174	-0.664×10^{-8}	50	-0.0147

Resultant:	f.	E mV
	50	0.04

Going through the above working for 1030, 1500, 856 and 256 rpm., the following results were obtained.

Resultants	f.	E mV
	1310	-14.1
	1380	14.2
	14	.006
	880	.492
	1810	4.72
	417	-0.82
	1740	6.67
	950	-7.05
	346	0.8
	660	-0.605
	2020	0.191
	634	-.079
	1990	-0.254
	734	0.262
	562	-0.079

From these above results, it is immediately obvious that only the first three waves, i.e. $q = p$, $q = \Omega_1 + p$ and $q = -\Omega_1 + p$ need be considered, and even here the resultants are extremely small; thus hardly affecting the theoretical results obtained already.

256 rpm.

Resultants	f.	E m V.
	347	- 3.74
	364	3.75
	41.5	.015
	245	1.37
	61	- 0.12
	45.5	0.18
	26	- .133
	43.5	0.10
	466	- 1.21

As expected, these above results, for the first three waves are small enough to be neglected. In an attempt to obtain a more accurate result it was decided to consider a further term in the Binomial Expansion, given on page 55.

Considering a further term in the Binomial Expansion:-

$$B^{II}sq = Bsq \; \frac{l_1^{\,2}}{4lg^3} \left[\left[\text{Sin}\left\{(2Q_1 - q)\theta_1 + wt\right\} + 2\,\text{Sin}\left\{wt - q\theta_1\right\} - \right.\right.$$

$$\left. \text{Sin}\left\{(2Q_1 + q)\theta_1 - wt\right\}\right] + \frac{l_1 l_2}{2lg^3}\left[\text{Sin}\left\{(Q_1 + Q_2 - q)\theta_1 + (w - Q_2 w_s)t\right\}\right.$$

$$- \text{Sin}\left\{(Q_1 - Q_2 + q)\theta_1 - (w + Q_2 w_s)t\right\} + \text{Sin}\left\{(Q_1 - Q_2 - q)\theta_1 + (w + Q_2 w_s)t\right\}$$

$$- \text{Sin}\left\{(Q_1 + Q_2 + q)\theta_1 - (w - Q_2 w_s)t\right\} + \frac{l_2^{\,2}}{4lg^3}\left[\text{Sin}\left\{(2Q_2 - q)\theta_1\right.\right.$$

$$\left.\left.\left. + (w - 2Q_2 w_s)t\right\} + 2\text{Sin}\left\{wt - q\theta_1\right\} - \text{Sin}\left\{(2Q_2 + q)\theta_1 - (w + 2Q_2 w_s)t\right\}\right]\right] \frac{Aq}{\pi q}$$

In order to obtain the above expression with respect to the secondary, we must put $\theta_1 = \theta_2 + w\,t$ (As seen on page 53)

$$\text{Hence } B^{II}rq = Brq + \frac{Aq}{\pi q}\left[\left[\frac{l_1^{\,2}}{4lg^3}\left[\text{Sin}\left\{(2Q_1 - q)\theta_2 + \left((2Q_1 - q)\,w_s + w\right)t\right\}\right.\right.\right.$$

$$\left. + 2\text{Sin}\left\{t(w - qw_s) - q\theta_2\right\}\right] + \frac{l_1 l_2}{2lg^3}\left[\text{Sin}\left\{(Q_1 + Q_2 - q)\theta_2 + \left((Q_1 - q)w_s + w\right)t\right\}\right.$$

$$- \text{Sin}\left\{(Q_1 - Q_2 + q)\theta_2 + \left((Q_1 - 2Q_2 + q)\,w_s - w\right)t\right\}$$

$$+ \text{Sin}\left\{(Q_1 - Q_2 - q)\theta_2 + \left((Q_1 - q)w_s + w\right)t\right\}$$

$$- \text{Sin}\left\{(Q_1 - Q_2 + q)\theta_2 + \left((Q_1 + 2Q_2 + q)w_s - w\right)t\right\}$$