

**EVALUATING THE EFFECTIVENESS OF SELF-DIRECTED
METACOGNITIVE (SDM) QUESTIONING DURING SOLVING OF
EUCLIDEAN GEOMETRY PROBLEMS BY GRADE 11 LEARNERS**

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**A research report submitted to the Faculty of Science, University of the
Witwatersrand, Johannesburg in partial fulfilment of the requirements for
the degree of Master of Science.**

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Signed on 04 June 2017

Plagiarism declaration

I declare that this work is my own unaided work and has not been submitted before for any qualification in any University. It is here submitted for the Degree of Master of Science Education (Mathematics) in the University of the Witwatersrand, Johannesburg.

(Signature of candidate)

_____ day of _____, 2017

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Dedication

I dedicate this work to my wife Chemedzai and my children Evernice, Evidence, Eunice, Ethel, and Elias whose unconditional encouragement supported my efforts in acquisition of knowledge.

In fond memory of my mother Rosemary Madiridze, born in 1952 and passed away in 2009.
May her soul rest in peace.

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Abstract

This research explores the importance of Self- Directed Metacognitive questioning in the solving of Euclidean Geometry problems by grade 11 learners. A quasi-experiment was carried out at an urban school with fifty eleventh grade learners. Most researches in Mathematics Education aimed at unveiling socio-economic factors that hamper mathematics learning. In this research, I suggested that strategies that target the learners' metacognitive development can assist in addressing poor mathematics achievement. Metacognition (thinking about thinking) makes learners drivers of their own cognitive processes so that they can become better doers of the subject

The research answered the question: Does teaching of metacognition to Further Education and Training (FET) learners help them to become better learners of Euclidian Geometry? This question was broken down into the following sub-questions: What is the effect of the use of Self -Directed Metacognitive (SDM) questions on the confidence level and preparedness of learners in the learning of Euclidean Geometry? To what extent does purposeful teaching and learning of metacognitive skills yield positive results in answering Euclidean Geometry questions?

Metacognitive skills helped learners to perform better in problem-solving. This result agrees with previous researchers and is also consistent with the results of earlier investigations showing that achievement in mathematics can be raised through instruction enriched with metacognitive activity. Where previous research dealt with metacognitive training in an implicit manner this research on metacognitive training was done explicitly and it resulted in improved mathematics performance by learners.

CHAPTER 1 : INTRODUCTION

This chapter introduces research that I carried out with the objective of exploring the importance of Self-Directed Metacognitive (SDM) questioning in the solving of Euclidean Geometry problems by grade 11 learners. As the first chapter, it provides context, spells out the statement of the problem, research questions, rationale of the study and the research objective.

1.1 Context

Mathematics education research in South Africa shows that socio-economic factors have the most impact on mathematics and science performance (Crouch & Mabogoane, 1998; Spaul, 2013). These factors are complex and numerous, and “they include a lack of facilities and resources at many schools, large class sizes, inadequate teacher education, poor learner commitment and discipline, inadequate parental involvement, to name but a few”(p.43) according to the Centre of Development and Enterprise (CDE) (Leibbrandt & Finn, 2012). For the past five years, achievement in mathematics (measured using grade 12 NSC results) has been unacceptably low, however there are notable improvements.

Between 2010 and 2013, the percentage of candidates who passed mathematics at 40 per cent has increased from 29.4 per cent to 40.5 per cent (DBE, 2014). This development is positive, but the decreasing number of learners passing mathematics remains a great concern: a decrease from above 260 000 in 2010 to just about 100 000 in 2013 according to the Department of Basic Education (DBE) (DBE, 2013). However, the numbers in 2014 saw a drop in the percentage of candidates who achieved 40% and above dropping from 40.5% in 2013 to 35.1% in 2014 when the first cohort of Curriculum and Policy Statement (CAPS) students confronted Euclidean Geometry.

Additional area of concern is the tendency of more learners selecting to take the less challenging route of mathematical literacy instead of mathematics. Enrolment for mathematics decreased from 263 034 in 2010 to 104 033 in 2013. Over the same period, enrolment in mathematical literacy increased from 280 241 to 323 097 ((Leibbrandt & Finn,

2012). This trend is not good for a country that is so much in need of professionals who are mathematically competent.

In the Gauteng province, the province in which I teach, the picture is equally bleak. In 2015, 37 053 candidates wrote mathematics compared to 72 765 who wrote mathematical literacy (DBE, 2015). Of those who wrote mathematics only 34% attained 50% and more. This shows that there is need for some intervention programmes to correct the current trend. In Gauteng, there are a number of such interventions that include teacher subject content workshops and Secondary Schools Intervention Programmes (SSIPS) where learners attend extra classes.

Mathematics generally challenges learners, but a section that poses serious challenges is geometry. It is studied under two sections: analytical geometry and Euclidean geometry. The first grade 12 CAPS examination in 2014 had three Euclidean geometry questions and in 2015, there were four questions on this topic. According to diagnostic reports published by the DBE (2015), the performance of candidates in questions, relating to Euclidean geometry was not satisfactory. For the 2014 and 2015 examinations, the report identified a number of common errors and misconceptions that were committed by candidates and outlined some recommendations and suggestions to teachers.

Among the suggestions given in the report was that more time must be spent on teaching Euclidean geometry in all grades. In addition, the report recommends that “teachers should encourage learners to scrutinise given information and the diagram for clues about what theorems to use in answering questions” (DBE, 2015). There is no short cut to master skills to answering Euclidean Geometry questions (DBE, 2015). That being the case, learners need continuous and deliberate practice. In light of this, it is evident that learners need to have planning, monitoring and evaluation skills when doing Euclidean geometry. The above stated skills are metacognitive¹ in nature so, learners who are taught metacognition may therefore benefit from developing these skills through being taught them.

Research has been done in the field of psychology to examine how one’s knowledge about oneself and how one’s self-regulation can influence one’s cognitive capabilities like metacognition. Metacognition has become an important phenomenon in the field of education that ignoring its influence in modern day classrooms is injudicious (Yimer & Ellerton, 2009). Flavell (1979) mentioned that educators would rather lay emphasis on content than describe,

¹ Metacognition involves planning, monitoring and evaluation skills. These are discussed at length later.

explain and teach metacognition. In light of this, my research is in the area of metacognition to investigate whether this is a way of assisting learners' heuristic development but more their mathematics problem solving skills development in the area of Euclidean geometry.

Current teaching trends de-emphasise the teaching and practising of algorithms while stressing metacognitive and heuristic skills (NCTM, 2000), cited in Depaepe, Corte and Verschaffel (2010). It is important to stimulate students' metacognitive behaviours as a way of strengthening their ability to reflect as they learn mathematics. Though metacognitive awareness is evident at the age of 4-6 years (Demetriou & Efklides, 1990) its development is a function of age and experience. Flavell (1988) however noted that instruction has more impact on the acquisition of metacognitive skills than growth. This provides the rationale for investigating the value of the instruction of metacognition in mathematics classrooms.

Traditional classroom discourses i.e. procedural discourses, introduce learners to procedures and following procedures leads to rote learning and memorisation (Post, Behr & Lesh, 1982). Although doing mathematics involves some knowledge of algorithms, it also involves a good deal of conceptual understanding in order to know why and how the steps should be taken (Chitera, Kufane & Jumbe, 2012, p.7). For learners to be able to know the 'why' and the 'how' it means they should have control of their thinking and they should be in a position to self-regulate. In this work, I will impart metacognitive knowledge to learners with the expectation that they will make use of this knowledge to develop their mathematics learning skills and enhance their conceptual understanding. When an individual perceives how cognitive processes operate, he/she will be able to "control these processes and use them in a more efficient way by arranging them for more qualified learning" (Ulgen, 2004,).

1.2 Statement of the problem

As reported by the DBE (2015) the teaching and learning of Euclidean geometry requires attention since school learners perform badly at it and their mathematics marks and overall performance is affected by it.

"Metacognitive actions are common in daily life, such as one's decision to use a calendar as a reminder of one's obligations or the choice of a particular strategy when studying for an examination" (Özcan et al., 2015, p. 1416). As noted, metacognition is a daily activity but the concern is that it is practiced without people noticing that they are practising it. The danger is that if people use something without realising it, they might not put in the extra effort that is

needed to make full use of it (Ulgen, 2004). Useful as they are, metacognitive skills are not being emphasised as vital tools for doing mathematics (Ulgen, 2004). The underlying assumption of this work and hypothesis is that explicit teaching of metacognition will result in improved mathematics performance by learners (Lovett, 2008). By teaching the students metacognition, they become more aware of existence and usefulness of metacognitive activities for their learning.

1.3 Research questions

In order to achieve this, the research aims at answering the question:

Does teaching of metacognition to a group of Grade 11 students help them become better learners of Euclidian geometry? Answering the above question will be done by answering the following sub-questions:

1. What is the effect of the use of Self -Directed Metacognitive (SDM) questions on the confidence level and preparedness of learners in the learning of Euclidean geometry?
2. To what extent does purposeful teaching and learning of metacognitive skills yield positive results in answering Euclidean Geometry questions?

1.4 Rationale of the study

Previous studies (Crouch & Mabogoane, 2001; Spaul, 2013) focused on various factors that affect learner academic performance, e.g. class schedules, class sizes, school environment, examination and assessment systems, social background, teacher qualification and experience, etc. Most of these studies had their attention on aspects that were outside of the learner's control. This study may help education officials and teachers to design programmes that aim at improving students' performance through a change of learning approach. Academic intervention can be more than the traditional extra classes where the main task is repetition of learned concepts to enhance memorisation. An intervention will encourage learners to view themselves as drivers of their own cognitive processes rather than vessels into which information must be deposited. The aim of the study was to assess the extent to which Self-Directed Metacognition skills improve grade 11 learners' performance in Euclidean geometry.

1.5 Research objective

The objective of this research is to explore the value of Self-Directed Metacognitive questioning by learners for the solving of Euclidean geometry problems.

1.6 Organisation of this report

This report will comprise a literature review, methodology, results, discussions, conclusions, and recommendations. The literature review has two sections. The first section deals with theories and concepts that guide this research. This is followed by a review of research in role of metacognitive skills in mathematics achievement. The third part of the work elaborates on the methodology that was followed. A discussion of the data precedes the conclusions arrived at. This is followed by recommendations arising from the research.

CHAPTER 2: LITERATURE REVIEW

This chapter consists of two sections. The first section explores the fundamental theoretical constructs upon which my research is based. This includes identifying and defining key concepts of the conceptual framework. This is followed by reviewing research in the field of mathematics education that relates to metacognition.

2.1 Theoretical Framework

The principal goal of education is acquisition of durable knowledge, not just a transient increase in the familiarity of information (Azevedo & Aleven, 2013). Teachers and researchers must discover how to support students' learning of concepts in a manner that ensures long term retention. In this work, attainment of skills that allow long term retention will be referred to as qualified learning.

Qualified learning will be viewed within the constructivist-learning framework. Within this learning framework, "learning is a process of construction in which the students themselves have to be the primary actors" (von Glasersfeld, 1995). Active learning processes (Hiebert, 1992) substitute passive intellectual engagement and memorisation. According to Anthony (2002), active learning processes denote "learning activities in which students are given considerable autonomy and control of the direction of the learning activities". On the other hand Kyriacou and Marshall (1989) argued that, active learning denotes "a quality of the pupils' mental experience in which there is active intellectual involvement in the learning experience characterised by increased insight". This concept of active learning incorporates the notions of mental effort or intentional learning, meaningful learning, and metacognition (Bereiter & Scardamalia, 1989).

Based on this understanding, my research focuses on an intervention strategy that aims at developing metacognition by encouraging students to have intellectual involvement in their tasks by asking questions (Özsoy & Ataman, 2009). Asking questions is a skill that can develop naturally in a person but asking effective questions for learning purposes might need coaching and training. Effective questions are vital because they trigger the thinking process and stimulate imagination (Özsoy & Ataman, 2009).

My theoretical position in this research is situated in the theory of “active learning processes in mathematics” (Hiebert, 1992; Wang, Haertel & Walberg, 1993) taking into account the fact that current learning perspectives incorporate three important assumptions postulated by Anthony (1996). These assumptions are (a) learning is a process of knowledge construction, not of knowledge recording or absorption (b) learning is knowledge-dependent; people use current knowledge to construct new knowledge (c) the learner is aware of the processes of cognition and can control and regulate them. As my research participants involved themselves in Self-Directed Metacognitive (SDM) questioning, they engaged with active learning processes.

The third assumption above relates to learners’ awareness of processes of cognition and their ability to control and regulate them. This brings about notions of metacognition, self-regulation, and self-regulated learning. Since the focus of this research is metacognition, clarity on the meaning of these three concepts is given below.

Over the years, metacognition and self-regulation have been blurred by varying use and theory haziness (Bandura, 1977; Flavell, 1979). The former is rooted in the theoretical foundation of Jean Piaget “and is centred on cognition and matters of the mind” (Dismore, 2008) and is conceptualised as being comprised of two factors: “knowledge and monitoring/regulation”. Knowledge refers to “what individuals know about their own cognition and cognition in general”. On the other hand monitoring/regulation are a set of activities that help students control their learning (Flavell 1979, Schraw & Moshman, 1995). Moshman (1982) regards metacognition as being within the jurisdiction of the mind and is less concerned with the human-environment interaction.

Self-regulation emphasises person-environment interaction, the importance of emotional and behavioural regulation and regulation of motivation (Bandura, 1977) and Dismore (2008) et al. describe it as “the reciprocal determinism of the environment on the person mediated through behaviour”. It consists of three components: “cognition, metacognition and motivation” (Schraw, Crippen & Hartley, 2006) and within it there is intersecting conceptual space between self-regulation and metacognition. Kaplan (2008) explained this overlap in a multidimensional conceptual space where self-regulated action is the umbrella with self-

regulation, metacognition, and self-regulated learning as the main concepts under the umbrella.

The concepts metacognition, self-regulation, and self-regulated learning share an underlying notion of “a marriage between self-awareness and intention to act” (Dismore et al., 2008, p. 404). They are parallel and intertwining constructs that are different but mutually entailed in their functions in human thought and behaviour (Fox & Riconscente, 2008). Despite this interconnectedness, Fox and Riconscente contend that this interconnectedness is not a justification for their treatment as synonymous terms. Instead, they should be treated as subtypes of the same general phenomenon of self-regulated action (Kaplan, 2008). An attempt to establish boundaries between them results in boundaries that are fuzzy and permeable. Instead of using boundaries, Kaplan rather uses dimensions which allow the concepts to gradually transform into each other (Kaplan, 2008). This is evident in the definitions given below. For instance, metacognition is now thought to be a component of self-regulation (Winne & Hadwin, 1998; Zimmerman & Moylan, 2009)

2.2 Definition of terms

2.2.1 Metacognition

Metacognition refers to the knowledge one possesses about one's own cognitive processes (knowledge of cognition) and the monitoring and regulation of these cognitive processes in order to serve a concrete goal (Veenman et al., 2006).

2.2.2 Self-regulation

This is the control that students have over their cognition, behaviour, emotions, and motivation through the use of personal strategies to achieve the goals they have established (Panadero & Alonso-Tapia, 2014)

2.2.3 Self-regulated learning

Self-regulated learning is “an active, constructive process whereby learners set goals for their learning and then monitor, regulate, and control their cognition, motivation, and behaviour, guided and constrained by their goals and contextual features in the environment” (Pintrich 2000a, p. 453).

One of the most comprehensive models that explains and presents how the various associated processes work is the Zimmerman and Moylan model (Zimmerman & Moylan, 2009) as presented in Figure 2.1.

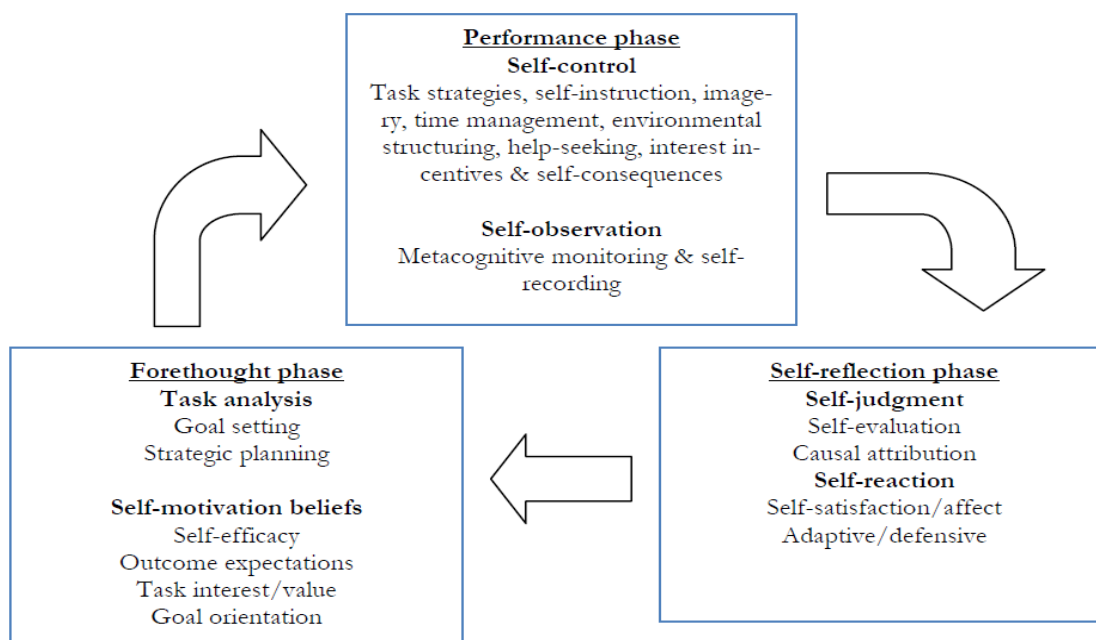


Figure 2. 1. Phases and processes of self-regulation, Zimmerman & Moylan ,2009

In this research, attention was paid to the processes of the performance phase of the model where actual task performance takes place. During performance, it is important that students keep their concentration for two reasons. First, so that their impetus does not decline and second that they have trail of their progress in the direction of their goals (Panadero & Alonso-Tapia, 2014). The two main processes during this phase are self-observation and self-control.

Self-observation involves having a clear understanding of what the task entails; an understanding of what one is doing so that if it is correct one continues and if it is wrong then one changes track. To self-observe, students perform two types of actions: self- monitoring also known as metacognitive monitoring or self-supervision and self-recording (Panadero & Alonso-Tapia, 2014). Metacognitive monitoring helps to compare what is being done against the standards that measure the quality of the process being followed (Winne & Hadwin, 1998). Self-recording is coding of the actions that are being done during task performance to be able to capture things that could go undetected e.g. recording time spent on a task (Panadero & Alonso-Tapia, 2014). An explanation of how learners were taught how to record time spent on a task will be detailed later in the research.

The second process of the performance phase is self-control. Strategies of self-control as shown in Figure 2.2 below are classified as metacognitive and motivational strategies. As revealed in the diagram, the first six tactics fall under metacognitive strategies and these are: task strategies, self-instruction, imagery, time management, environmental structuring and help seeking. The motivational strategies include interest incentives and self-consequences (Panadero & Alonso-Tapia, 2014).

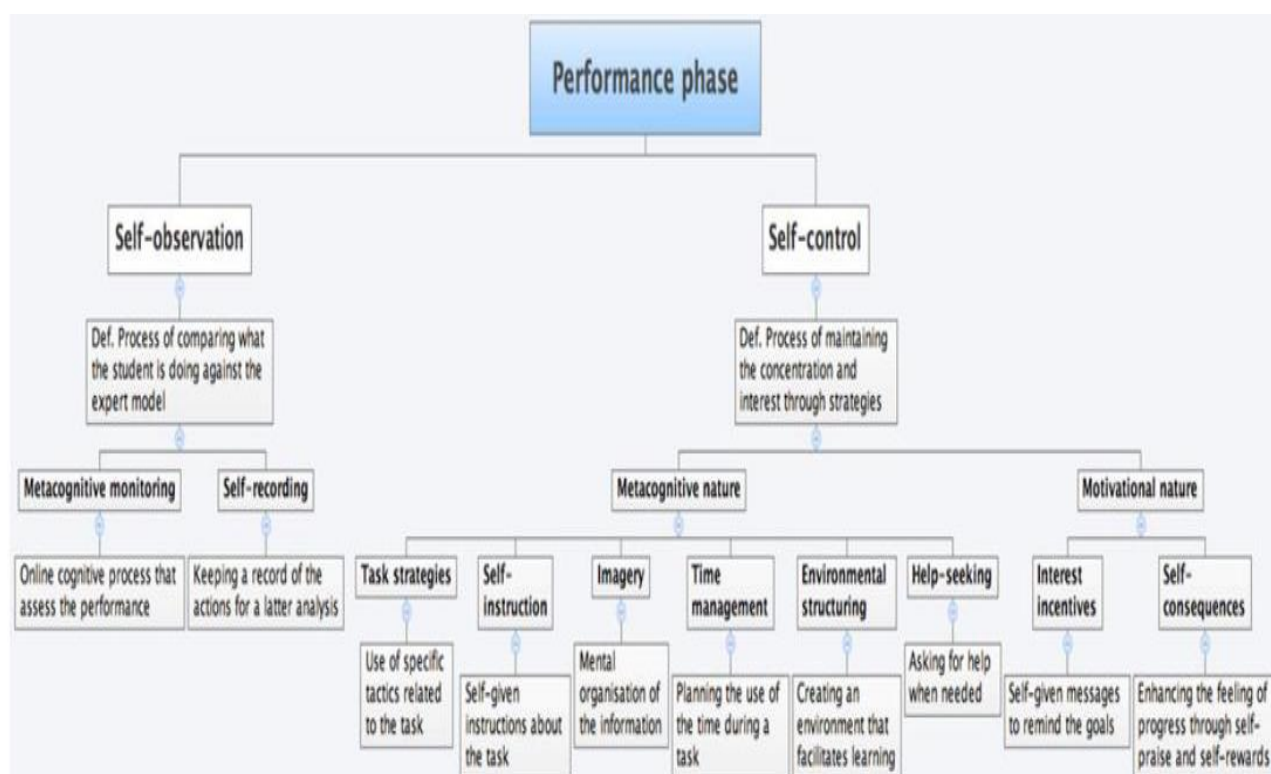


Figure 2.2. The performance phase of the Zimmerman & Moylan model, 2009.

Metacognitive strategies assist students to preserve attentiveness and motivational strategies assist them keep interest in following the task. One important strategy of self-control is self-instruction. This involves self-directed orders or descriptors about the task that is performed. This can be in the form of students asking themselves questions about the steps to take and if they are correct. These types of verbalisations improve learning and are vital for self-regulation (Schunk, 1982).

The above discussion explains the relation between the concepts of self-regulation, metacognition, and self-regulated learning. Since the research focuses on a branch of geometry, below is a brief description of some important aspects about the topic. The description starts with what geometry is, aims of teaching it, why it was chosen in this research, and finally why Euclidean geometry was preferred for this research.

2.3 Geometry in the society

According to Jones (2002), geometry is a fantastic area of mathematics to teach and learn, full of interesting problems and surprising theorems and is open to various approaches. It has a long history, closely linked with the development of mathematics. Jones also stated that it is an “essential part of our cultural experience and a vital component of numerous aspects of life from architecture to design.” More importantly, geometry appeals to our visual, aesthetic, and intuitive senses. Consequently, “it is a topic that can capture the interest of learners, often those learners who may find other areas of mathematics, such as number and algebra, a source of confusion and failure rather than excitement and creativity” (Jones, 2002, p.122).

2.3.1 What is Geometry?

A useful modern definition of geometry is that credited to Sir Christopher Zeeman: “geometry comprises those branches of mathematics that exploit visual intuition (the most dominant of our senses) to remember theorems, understand proof, inspire conjecture, perceive reality, and give global insight” (Jones, 2002, p.4). These skills are transferable and needed for all other branches of mathematics and science. However, these branches do not teach the skills.

Jones (2002) recorded that the term ‘geometry’ originates from two ancient Greek words, one meaning earth, and the other meaning to measure. These words may themselves be derived from the Sanskrit word ‘Jyamiti’. In Sanskrit, ‘Jy a’ means an arc or curve and ‘Miti’ means right perception or measurement (Jones, 2002, p.122). The origins of geometry are “very ancient (it is probably the oldest branch of mathematics) with several ancient cultures (including Indian, Babylonian, Egyptian, and Chinese, as well as Greek) developing a form of geometry suited to the relationships between lengths, areas, and volumes of physical objects” (Jones, 2002, p.122). In ancient times, geometry was used in the measure of land (modern day surveying) and in the building of religious and cultural artefacts.

Around 300 BC, much of the amassed knowledge of geometry was documented in a text that became known as Euclid's *Elements* (Jones, 2002, p.122). Euclid's *Elements* comprise 13 books, and based on its 10 axioms and postulates, several hundred theorems were proved by deductive logic.

2.3.2 Aims of teaching geometry

The Royal Society/JMC (2001) report proposes that the aims of teaching geometry can be summarised as follows:

1. “ To develop spatial awareness, geometrical insight and the skill to envision;
2. To offer a range of geometrical familiarities in 2 and 3 dimensions;
3. To advance familiarity and knowledge of and the capability to use geometrical properties and theorems;
4. To encourage the development and use of conjecture, deductive reasoning and proof;
5. To advance skills of applying geometry through modelling and problem solving in real world settings;
6. To advance useful ICT skills in specifically geometrical circumstances;
7. To generate a positive attitude to mathematics; and
8. To develop an awareness of the historical and cultural heritage of geometry in society, and of the modern applications of geometry”.

Having given the description of geometry, and a list of the aims of teaching the topic, it is now possible to say why I chose it for this research.

2.3.3 Why I chose geometry in the mathematics curriculum?

The CAPS document prescribes seven content areas for the first paper and four content areas for the second paper in the National School Certificate (NSC) grade twelve final examinations. Geometry is examined in the second paper under Analytical and Euclidean geometry. Of the eleven content areas prescribed, this research focused on Geometry for the perceived role that it has in learner mental skills development.

“The study of geometry contributes to development of vital learning skills of visualisation, critical thinking, insight, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof” (Jones, 2002, p.125).

Jones (2002) stated that geometric representations could assist learners to make sense of other areas of mathematics. These areas comprise of “fractions and multiplication in arithmetic, the relationships between the graphs of functions, and graphical representations of data in statistics” (Jones, 2002). In addition to learning of mathematics, spatial reasoning done in

geometry is important in other areas of the curriculum such as science, geography, art, design, and technology. This shows that geometry is not only learnt for itself but for a bigger purpose in learner development.

Geometry also provides a framework within which to teach and learn mathematics. If geometry concepts are well taught and learnt, they awaken inquisitiveness and promote exploration. Such inquisitiveness can develop students' learning and their attitudes towards mathematics.

As students deliberate on problems in geometry, communicate their ideas and develop well-thought-out points of view to support their opinions they develop better communication skills and appreciation of the importance of proof (Jones, 2002). Though proofs are not limited to geometry, it (geometry) is a rich source of opportunities for developing notions of proof. Additionally, Jones (2002) stated that geometry also contributes to learners' spiritual, moral, social, and cultural development.

Teaching geometry well could mean empowering more students to become successful in mathematics. Bursill-Hall (2002) stated that geometry has been studied because it has been held to be "the most exquisite, perfect, paradigmatic truth available to us outside divine revelation." It is the surest, clearest way of thinking available to us. "Studying geometry reveals – in some way – the deepest true essence of the physical world and its teaching trains the mind in clear and meticulous thinking" (Bursill-Hall, 2002, p.1)

2.3.4 Choice of Euclidean geometry

In South African secondary schools, Euclidian geometry has been included and excluded from the Curriculum over the past years with various renovations. The result is that the teaching of Euclidean geometry in schools poses challenges to some mathematics teachers. The National Curriculum Statement's (NCS) framework prior to the current Curriculum and Assessment Policy Statement (CAPS) did not address certain aspects of Euclidian geometry e.g. circle theorems, which are now taught and examinable under CAPS. These concepts were however included in the curriculum before the NCS. Reasons for such developments are

not the subject of this research but it is an interesting observation and one that among others motivated this research to examine the teaching and learning of the topic.

“Mathematics learners, like graduates of higher education, should have critical thinking ability” (Perkins & Murphy, 2006). The ability, however, cannot be obtained quickly. Therefore, the ability in problem solving should always be improved through mathematical learning, and this is likely to improve a critical thinking aptitude (Kosiak, 2004; Perkins & Murphy, 2006). It is also stated that critical thinking may be improved through the learning of geometry (Ruseffendi, 2006). This is in addition to logical thinking which is promoted by Euclidean Geometry (Perkins & Murphy, 2006).

Since Euclidean geometry is a tool for providing logical thinking, this research examined the mathematics policy document to check how the topic is treated and prioritised. In the CAPS document, Euclidean geometry has more weight than other mathematics topics at grade 12 as illustrated in Table 2.1.

Table 2.1. Weighting of content areas in the FET mathematics curriculum (DBE, 2011, p.10)

Weighting of Content Areas			
Description	Grade 10	Grade 11	Grade 12
PAPER 1 (Grades 12:bookwork: maximum 6 marks)			
Algebra and Equations (and inequalities)	30 ± 3	45 ± 3	25 ± 3
Patterns and Sequences	15 ± 3	25 ± 3	25 ± 3
Finance and Growth	10 ± 3		
Finance, growth and decay		15 ± 3	15 ± 3
Functions and Graphs	30 ± 3	45 ± 3	35 ± 3
Differential Calculus			35 ± 3
Probability	15 ± 3	20 ± 3	15 ± 3
TOTAL	100	150	150
PAPER 2: Grades 11 and 12: theorems and/or trigonometric proofs: maximum 12 marks			
Description	Grade 10	Grade 11	Grade 12
Statistics	15 ± 3	20 ± 3	20 ± 3
Analytical Geometry	15 ± 3	30 ± 3	40 ± 3
Trigonometry	40 ± 3	50 ± 3	40 ± 3
Euclidean Geometry	30 ± 3	50 ± 3	50 ± 3
TOTAL	100	150	150

Table 2.1 above shows that Euclidean geometry has more weight than any other content area in grade twelve. About 33% of the marks for the second paper come from Euclidean geometry. This means that good mastery of the topic places a candidate in a good position to pass the paper. For the purposes of accumulating marks in grade twelve mathematics paper two, candidates need to do well in Euclidean geometry.

The same CAPS document shows the importance of the topic by allocating considerable teaching time to it. This is shown in table 2.2 below.

Table 2.2. Prescribed allocation of teaching time per topic (DBE, 2011, p.17)

Terms	Grade 10		Grade 11		Grade 12	
		No. of weeks		No. of weeks		No. of weeks
Term 1	Algebraic expressions	3	Exponents and surds	3	Patterns, sequences and series	3
	Exponents	2	Equations and inequalities	3	Functions and inverse functions	3
	Number patterns	1	Number patterns	2	Exponential and logarithmic functions	1
	Equations and inequalities	2	Analytical Geometry	3	Finance, growth and decay	2
	Trigonometry	3			Trigonometry - compound angles	2
Term 2	Functions	4	Functions	4	Trigonometry 2D and 3D	2
	Trigonometric functions	1	Trigonometry (reduction formulae, graphs, equations)	4	Polynomial functions	1
	Euclidean Geometry	3	MID-YEAR EXAMS	3	Differential calculus	3
	MID-YEAR EXAMS	3			Analytical Geometry	2
Term 3	Analytical Geometry	2	Measurement	1	MID-YEAR EXAMS	3
	Finance and growth	2	Euclidean Geometry	3	Geometry	2
	Statistics	2	Trigonometry (sine, area, cosine rules)	2	Statistics (regression and correlation)	2
	Trigonometry	2	Probability	2	Counting and Probability	2
	Euclidean Geometry	1	Finance, growth and decay	2	Revision	1
	Measurement	1			TRIAL EXAMS	3
Term 4	Probability	2	Statistics	3	Revision	3
	Revision	4	Revision	3	EXAMS	6
	EXAMS	3	EXAMS	3		

The table above shows that a considerable amount of teaching time is allocated to Euclidean geometry from grade ten through to grade twelve. Geometry is allocated sixteen weeks in the Further Education and Training (FET) phase of which nine (56.25%) of those are for Euclidean geometry.

In the next section of this report, I examine some research on metacognition over the past years under the heading empirical findings.

2.4 Literature Review

In the 21st century a lot of work was done to try to explain the learning of mathematics by school children (Munda, Tamui & Kaberia, 2000; Desarrollo, 2007). One of the areas of focus has been the role that metacognition plays in children's learning in general and learning in mathematics in particular (Pintrich, 2002; Özsoy & Ataman, 2009; Breed, Ment, Havenga, Govender, Govender, Dignum & Dignum, 2013; Özcan & Erktin, 2015). "Metacognition refers to the knowledge one possesses about one's own cognitive processes (knowledge of cognition) and the monitoring and regulation of these cognitive processes in order to serve a concrete goal" (Veenman et al., 2006). As highlighted by Pintrich (2002), one of the trademarks of 'psychological and educational theory and research on learning' is that as students learn mathematics they should become more conversant of and in charge of their own reasoning and thinking. By being aware of and being able to control what one thinks and knows, one can be in a better position to master subject content.

Bransford, Brown, and Cocking (1999) postulated, that with development, students become more knowledgeable about cognition in general and as they act on this awareness, they tend to learn better. The above works point to the fact that for one to become an efficient learner, one has to be able to control and be a master of one's own thinking. This control and regulation of thinking and knowing can come on its own with maturity but an academic intervention can be of vital importance in bringing about this phenomenon (Pintrich, 2002). That being the case, this work is based on the assumption that explicit teaching of metacognition to students can improve their mathematics performance (Lovett, 2008).

"Teachers are absolutely willing to invest effort in the instruction of metacognition within their lessons, but they need the 'tools' for implementing metacognition as an integral part of

their lessons, and for making students aware of their metacognitive activities and the utility of those activities” (Veenman, 2006, p. 10). If teachers are willing to do so, then it becomes a potential pedagogical resource. Teaching of metacognition to students makes a lot of sense taking into account that a “satisfactory level of metacognition may compensate for students’ cognitive limitations” (Veenman, 2006). One of the challenges that teachers might face is knowing when, what and how to teach metacognition (Pintrich, 2002). If they do teach it, their next challenge will be how to measure it or to evaluate its impact on students’ performance.

According to Pintrich et al. (2002) teaching of metacognition can be done explicitly or implicitly. The more common of these two approaches is implicit teaching where learners are taught metacognition embedded in subject content – problem solving in action. This way, learners are not made aware that they are receiving metacognitive teaching. This approach works on the assumption that learners will be able to acquire metacognition on their own, however some lack the ability to do so as was discovered in Pintrich’s study with college students. In their work, Pintrich et al. stated that although it is not their expectation that teachers teach metacognitive knowledge in separate courses or separate units, it can be done. They concluded that, because metacognitive knowledge in general is positively linked to student learning, explicitly teaching metacognitive knowledge to facilitate its development is needed (Pintrich et al., 2002, p. 225).

On the other hand, Kuhn (2000) claimed that metacognition does not appear gruffly but that it rather arose and developed under an individual’s cognisant control. Any research that acknowledges this will view the explicit teaching of metacognition as a prerequisite for its development in school children. By teaching it, its appearance ceases to be abrupt and disjoint. Focusing on teaching content alone and ignoring metacognition can be a vice in children’s mathematics development. Content knowledge is an essential but not an adequate trait for solving mathematical problems (Schoenfeld, 1987). This supports the thinking that children should be taught metacognition as a way of assisting them to develop their mathematics solving skills. As stated in Artzt and Armour-Thomas (1997) the main source of children’s difficulty in problem solving is their failure to initiate active monitoring and regulation of their own cognitive processes. These research findings show that teaching of metacognition has a positive impact on children’s performance in mathematics as metacognitive processes shape cognitive activities. Metacognition can also improve students’

confidence as they perform tasks, and provide them with skills to overcome impediments encountered during problem solving (Özcan & Erkin, 2015).

Metacognition can be viewed in the light of two main distinctions: metacognitive knowledge and metacognitive control or self-regulatory processes (Pintrich, 2002). Metacognitive knowledge encompasses knowledge of strategy, task and person variables (Flavell, 1979). Strategic knowledge is knowledge of general strategies for learning, thinking, and problem solving (Pintrich, 2002). Within the constructivist learning framework, one of the strategies that can be used to develop metacognition is by encouraging students to ask questions themselves (Özsoy, 2009). Asking questions is a skill that can develop naturally in a person but asking effective questions for learning purposes might need coaching and training. Effective questions are vital because they trigger the thinking process and stimulate imagination (Özsoy, 2009). Knowledge about cognitive tasks involves such knowledge as knowledge that some tasks are more difficult than others. Lastly, self-knowledge includes one's knowledge of one's strengths and weaknesses. Metacognitive control and self-regulation processes are cognitive processes that learners use to monitor, control, and regulate their cognition and learning (Pintrich, 2002, p. 220).

Having said that metacognition can be looked at from these two perspectives (metacognitive knowledge and metacognitive control), the research in this area is now briefly reviewed. Özsoy and Ataman (2009) studied the influence of metacognitive strategy training on mathematical problem solving success with fifth graders in Turkey. In their case, metacognitive strategy training was done during problem solving. This means, the training was not separated from the actual mathematics content and lessons. Their work focused on strategy training and was carried out using a quasi-experimental design using an experiment and a control group. The study was intended to examine the effect of metacognitive strategy teaching in mathematical problem solving achievement (Özsoy & Ataman, 2009, p. 72). They found out that there was an increase in problem solving skills of the learners who were exposed to metacognitive strategy as opposed to those who were in the control group.

In their study, Özsoy & Ataman established that metacognitive skills can be developed through instruction and that metacognition can be used as a useful tool in order to develop the problem solving skills of students (Özsoy & Ataman, 2009, p. 80). The success of the instructional method used in their study is the motivation for trying to replicate their method in this research. Veenman (2006) claimed that there are three fundamental principles for

successful metacognition instruction: (a) ‘embedding metacognition teaching in the content matter to guarantee connectivity (b) notifying learners about the value of metacognitive activities to make them apply the initial additional effort, and (c) prolonged training to guarantee the smooth and maintained application of metacognitive activity’ (Veenman, 2006, p. 9). However in this research instead of embedding the teaching of metacognition in the mathematics content it was taught separately and overtly in order to make the learners aware that it is an aspect that is separate from their mathematics but aids their learning. On completion of the intervention, students were left to use what they learnt during the training in their mathematics lessons. The model that was used for metacognitive training in this research is the IMPROVE technique of Mevarech and Kramarski (1997).

In Breed et al. (2013) the views of teachers on the use of self-directed metacognitive (SDM) questions was investigated, as well as learners’ experiences in using the SDM questions to direct their thinking during the execution of pair programming tasks (Breed et al., 2013, p. 207). The study was carried out in rural schools of KwaZulu-Natal and the North West provinces of South Africa. It aimed at finding out teachers’ views on the use of SDM questions as a teaching strategy and wanted to see if the use of SDM questions could improve learners’ achievement in computer programming. The main findings of their work are that training in metacognitive skills could contribute to empowering both teachers and learners in self-regulation in the teaching and learning process. It was also realised that the practise of metacognitive strategies was necessary to make it an integral part of the learners’ learning activities (Breed et al., 2013, p. 216).

However, it was discovered that implementation of SDM questions needed a longer time span, more encouragement and focused support on implementation. One very decisive observation made by the study is that besides the need for a longer time span, “use of SDM questions needs to be supported by introducing the method gradually and letting learners experiment with it” (Breed et al., 2013, p. 216). This suggests that learners must be trained to use the method, practise it and understand it before it is brought into the actual problem-solving scenario. These lessons were applied in the intervention phase of this research. Participants were coached to use SDM questions and the IMPROVE method in their mathematics lessons prior to the start of Euclidean geometry classes (focus area of the research).

Özcan & Erktin (2015) investigated students' homework behaviours and the effect of homework assignments enriched with metacognitive questions. This research was done by means of an educational research experiment with a quasi-experimental design with forty-four learners from two grade seven classrooms at a primary school in Turkey (one an experimental class and the other a control class). Students in the experiment group were given homework that was enriched with metacognitive questions and those from the control group were given normal homework (subject content only). The results of the work showed that there was a significant difference between mathematics results of students who were given homework enriched with metacognitive questions and those who were not given such homework (Özcan & Erktin, 2015, p. 1423) Metacognitive enriched homework improved students' achievement. The inclusion of metacognitive concepts in students' work therefore appears to improve their learning as well as their results.

Many methods and tools are used to measure metacognition, e.g. questionnaires, interviews, analysis of think aloud protocols, on-line computer-log file registration and eye-movement registration among others (Veenman, 2006). These methods are classified into on-line and off-line methods where online methods are those where measurements are taken concurrently with task performance. These include think aloud protocols, observation, eye movement registration and log file registrations (Veenman, 2011, p. 206). On the other hand, off-line methods are administered either before or after a task performance. These methods include questionnaires and interviews. In this research, I used both online and offline methods. During lessons, I encouraged learners to use think aloud protocols (online) but after the intervention questionnaires and interviews (off-line), I used to collect data.

The research process will be detailed in the next chapter.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

In this chapter, a description of the research paradigms, the design of the research, the sample and the research tools is provided. Also provided are details of the intervention here referred to as the training programme.

3.1 Research paradigms

“A paradigm is a broad view or perspective of something” (Taylor, Kermode, and Roberts, 2007, p.5). It defines how research could be affected and guided by the researcher’s way of viewing reality and truth. To this effect, Weaver and Olson (2006) stated that “paradigms are patterns of beliefs and practices that regulate inquiry within a discipline by providing lenses, frames and processes through which investigation is accomplished”.

There are five common paradigms that guide research: Interpretivist, Positivist, Critical Realist, Critical Theory and Feminist paradigms. Choice of a paradigm to use is influenced by a number of factors but the most influence comes from the type of research conducted. In this research, I opted for an interpretive view, a paradigm that supports the view that there are several truths and various realities. I acknowledged that different people have diverse insights, needs and experiences.

The interpretivist paradigm uses interpretive approaches which rely heavily on naturalistic methods such as interviewing and observation. These methods guarantee a satisfactory discourse between the researcher and those with whom he/she interacts in order to collaboratively construct a meaningful reality.

I used the paradigm (interpretivist) in a qualitative approach with some numerals as data and most of the analysis being thick descriptions of the data and what it meant. The table below summarises the paradigm used in this research.

Table 3.1. Summary of research paradigm

Characteristic	Interpretive view
Purpose	Researcher interviews participants and acknowledges importance of individual insight.
Beliefs	Numerous truths and realities People have diverse insights, needs and experiences.
Research method	Qualitative
What data is based on	Descriptive, explanatory and contextual words from interviews.

3.2 Research design

In this research, qualitative and quantitative data collection methods were used. These include a generic test, pre and post mathematical tests, unstructured interviews, questionnaires, field notes and observations.

Qualitative research is predominantly fact-finding. It helps gain an understanding of underlying reasons, opinions, and impetuses. In addition, qualitative research offers insights into the problem or helps to develop ideas or hypotheses for potential quantitative research. It assists in uncovering trends in thought and opinions that move deeper into the problem. Qualitative data collection methods include unstructured or semi-structured techniques like focus groups (group discussions), individual interviews, and participation/observations. Another defining feature of qualitative research is sample size which is characteristically small, (fifty learners in this research).

On the other hand, quantitative research is used to quantify a problem by way of generating numerical data or data that can be converted into useable statistics. It can be used to quantify attitudes, opinions, behaviors, etc. – and generalize results from a larger sample population. Research of this nature uses measurable data to formulate facts and uncover patterns in research. Data collection methods are much more structured than qualitative data collection methods and they include various forms of surveys – online surveys, paper surveys, mobile surveys and kiosk surveys, face-to-face interviews, telephone interviews, longitudinal studies, website interceptors, online polls, and systematic observations.

As a way of increasing confidence in the research results, I used triangulation to provide in – depth data and also to address the “consideration of different dimensions of the problem” (Barbour, 2001; Jones & Bugge, 2006). Triangulation is the “application and combination of various research methodologies in one study” (Taylor, Kermode & Roberts, 2007). It is “a means by which the researcher is able to capture a more complete and holistic portrait of the phenomenon under study” (Jones and Bugge, 2006). I used methodological, data and unit of analysis triangulations in this study.

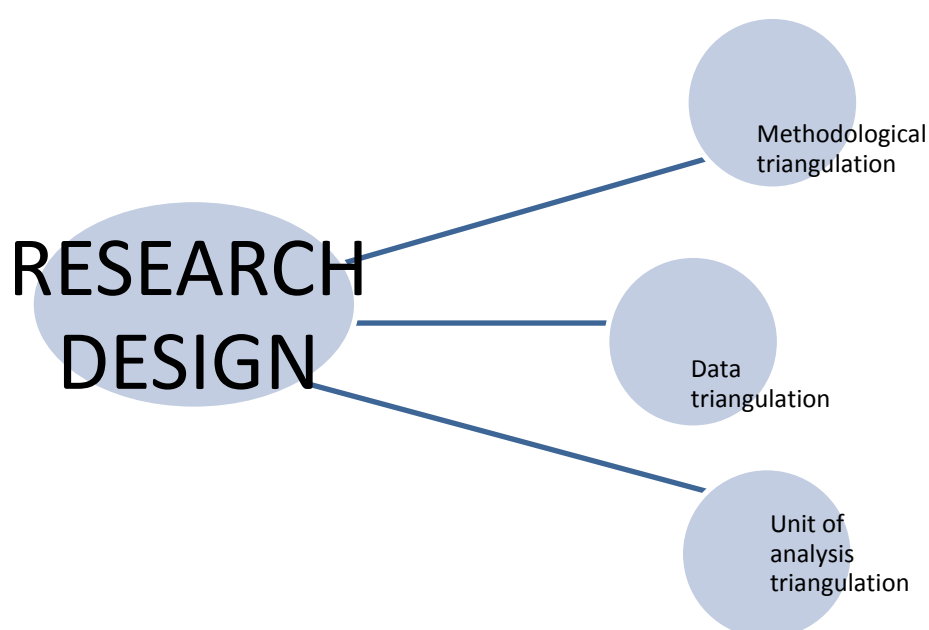


Figure 3.1. Triangulations in the Research Design

With reference to Fig. 3.1, methodological triangulation refers to use of both qualitative and quantitative data collection methods. These involved tests, questionnaires, observations and interviews. Data triangulation refers to use of qualitative and quantitative data in the same research. Unit of analysis triangulation refers to the use of more than one approach to analysis to validate the same set of data (Begley, 1996). Test results were analysed using graphs and SPSS computations. Questionnaire data were looked at using graphs and narrative interpretations.

3.3 Sampling and Design

A quasi-experimental design was used for evaluating the effectiveness and impact of teaching metacognition on students’ performance in Euclidean geometry (Gribbons, et al., 1997). A

mixed methods approach generated both numerical and non-numerical data. A non-equivalent group's design (NEGD) was used. In a NEGD, intact groups that are thought to be similar are used as the experiment and control groups (Gribbons, et al., 1997). The study was carried out at one research site, an urban boys' school in Johannesburg. The school has ninety-seven eleventh grade learners of which fifty-do mathematics and the remainder do mathematical literacy. The research participants were fifty boys from two grade eleven groups. The convenience sample consisted of an experimental group (n=29) and a control group (n=21).

3.4 Validity and Reliability

Joppe (2000) gives the subsequent account of what validity is in research. "Validity determines whether the research truly measures that which it was intended to measure or how truthful the research results are" (Joppe, 2000, p.58).

On the other hand, reliability refers to whether results obtained or conclusions reached could still be reached at if the research was to be repeated several times (Opie, 2004). Joppe (2000) defines reliability as the extent to which results are consistent over time and an accurate representation of the total population under study. If the results of a study can be reproduced under a similar methodology, then the research instrument is considered reliable (p. 1).

3.4.1 Validity

According to Henning, Rensburg and Smit (2004) data itself is neither valid nor invalid. It is the conclusion drawn from it that determines its validity. In order for me to stay on track, I kept on consulting my supervisor and peers with regard to emerging findings and sought clarifications on certain assumptions. During the study, the examination of participants' documents, observing them during the intervention, and recorded notes contributed to validity. I also captured participants' explanations and interpretations of meaning by use of field notes.

Henning (2004) suggested that in order to achieve validity, a researcher should guard against bias and scrutinise both procedures and decisions in a manner that is critical. Critical analysis could be achieved through use of theory from credible theories and research.

In this research, I triangulated the data using different data collection techniques. These include tests, notes, observations, questionnaires and interviews. Convergent data from cross

validation (McMillan & Schumacher, 2010, p.331) amounts to triangulation which increases the credibility of findings by revealing different insights. My research findings concurred with the findings of other researchers in the field of cognition and metacognition

3.4.2 Reliability

According to Opie (2004), reliability in qualitative research encompasses the whole process of data collection together with the findings. It involves consistency in terms of results across a range of similar settings and repetition (Wellington, 2000; Bell, 1999). Research results of studies conducted in classrooms are not easy to replicate. Despite this difficulty, Opie (2004) argues that the process of data collection could itself be subjected to reliability judgement. Opie (2004) suggested that reliability could be achieved through; (a) “test-retest,” using an instrument on the same subjects and then compare results. (b) “equivalent forms” where two equivalent versions of data gathering instrument is applied on the same subjects and (c) “split-half ” which makes use of a single instrument in which the results are then split into two halves which are then compared.

In this study I did not use any of the suggested techniques. Taking into account that it was classroom based research I cannot claim that the study was that reliable. Furthermore, I did not have a chance to replicate the results. However, I used self-examination throughout the research to add credibility to the research. According to McMillan & Schumacher (2010), by posing difficult questions to him or herself, a researcher assumes that he or she cannot be neutral, objective or detached. That being the case there was an inevitable element of subjectivity in the research.

3.5 Ethical Issues

Before I began this research, I applied to the Human Research Ethical Committee (the university’s ethical clearance committee - protocol number included in title page) and the Gauteng Department of Education. From them I sought permission to carry out research in areas of their jurisdiction. After obtaining clearances, I informed parents, learners and the school principal in writing about the research study. I made them aware of the fact that their

privacy would be protected and how the research was going to be carried out (Henning et al, 2004). The said parties gave their consent by completing and signing consent forms.

Anonymity was guaranteed through use of codes in place of participants' real names. All participants were informed of their right to withdraw from the study at any time and stage if they felt to do so.

3.6 Method

The control and the experiment groups were exposed to normal mathematics lessons so that no group was disadvantaged. However, there were certain differences in the activities that the two groups did. Figure 3.2 shows the activities of the two groups during the course of the research.

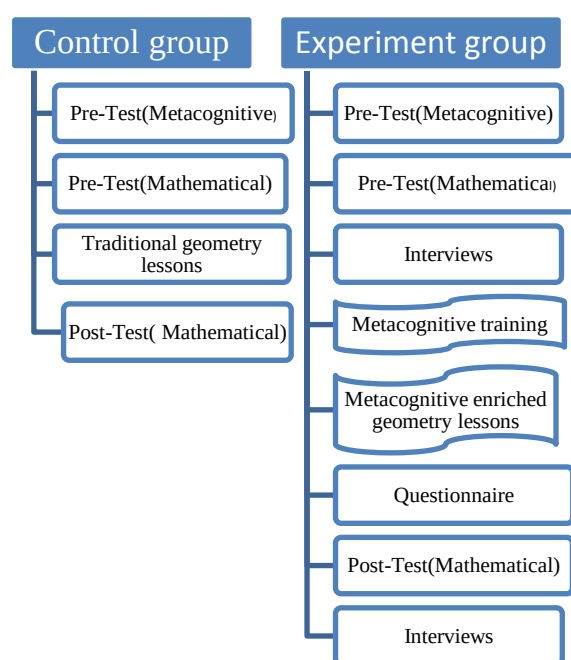


Figure 3.2 : Activities of the control and experiment groups during the research

The participants (both groups) took part in two pre-tests (one generic and one mathematical) and one post-test. The experiment group also completed a questionnaire and did two interviews. The results of the post-test (mathematical), interview and the responses to the questionnaire were used to assess the impact of the program. The nature of the tests is elaborated on below.

One of the instruments that were used in this research is diagnostic tests due to their appropriateness in experimental research (Bertram & Christiansen, 2014). The tests were intended to provide answers to the second research question: To what extent does purposeful teaching and learning of metacognitive skills yield positive results in answering Euclidean Geometry questions? The question sought to check the effectiveness of the intervention in the teaching and learning of Euclidean Geometry by grade 11 learners. The test results of the two groups were compared using an independent t-test using the statistical package for the social sciences (SPSS) software.

To ensure validity of the pre-tests tests, I used a two-way table to construct them. The two-way table matched test objectives with test content (Froese-Germain, 2001). Before administration of all the research instruments, a peer and my supervisor checked them for content and language appropriateness.

The questionnaire collected both quantitative and qualitative data. In addition, it also encouraged anonymity on the part of the participants. This was aimed at them giving honest responses. It was also a better way of reducing interviewer bias because there were “no verbal or visual clues” that could influence a participant (respondent) to answer in a certain way (Robson, 2002). In addition to the tests, ten students were interviewed using unstructured interviews soon after the pre-tests were completed. Other video-recorded unstructured interviews were conducted with seven learners after the post-test was marked and the results known. The reason for use of in-depth unstructured interviews was that the type of information sought was complex information with a higher proportion of opinion-based information (Abawi, 2013).

During the pre-test stage, two tests were given to both groups. The first test was a generic test and the second was mathematical. The purpose of the pre-tests was to determine the equivalence or non-equivalence of the groups before the start of the metacognition teaching intervention. This was an important consideration since the assignment of participants to groups through the mechanism of random assignment was not researcher controlled. As a result, the groups might have been different prior to the study. The NEGD is especially susceptible to the internal validity threat of selection (Gribbons et al., 1997).

The generic test (van Jaarsveld, 2012) consisted of eight PowerPoint slides. Each slide was shown to participants on a white screen for 90 seconds after which it was removed from the screen. Without the slide in their view, students were given 90 seconds to recall the items from the screen from memory. The content covered a variety of skills; language, numerical, and picture content. Each student's response was analysed, followed by ten students being interviewed to find out what they felt about the test and what they thought about such an exercise and the strategies that they incorporated. The memory strategies would help them remember more of what they see and/or hear. The interviews were of an unstructured nature. Also included in the interview were questions to find out what students thought of Euclidean geometry, how much they liked or disliked the topic and their reasons for that. The second interviews were video recorded and then transcribed.

The experiment group was taught both mathematics and metacognition. Mathematics content was taught during normal school times and metacognition was taught outside lesson times. Think-aloud protocols were encouraged and as the learners answered questions they were encouraged to write brief notes to justify what they were doing. They were also encouraged to self-record using some recording sheets designed for the research. On the recording sheets, learners entered enough information to help them do self-recording to facilitate self-evaluation. In addition, think aloud protocols were used during lessons. To encourage these think-aloud habits they were used more frequently in lessons to the extent that it became a habit in the group.

Aspects of metacognition, namely planning, monitoring, and evaluation were taught to learners of the experiment group outside mathematics lessons as a way of stimulating metacognitive behaviours. The intention was to see if learners' perception and achievement in mathematics would improve through learning and stimulation of metacognitive behaviours. The manner in which students perceive mathematics and perceive themselves as learners of mathematics can affect their ability to solve mathematical problems (Artzt & Armour-Thomas, 1997). It can be difficult to change perception about a subject but it might be easy or at least manageable to change self-perception through instruction of metacognition. Students with "high metacognitive skills perform better in problem solving" (Özsoy & Ataman, 2009) and so metacognition is a skill that promotes success in problem solving.

There was an awareness of the challenge that teaching metacognition outside mathematics lessons came with the expectation for learners to transfer and use this knowledge in mathematics lessons. As noted by Stillman and Galbraith (1998), providing an opportunity for metacognitive decisions does not insure that they will be made, but a rich store of knowledge of metacognitive strategies is a prerequisite for productive decision making. It is this rich store that I intended to be created within students and enhance their decision making during problem solving.

Despite the fact that learning metacognition is not part of the learners' syllabus, teaching, and stimulation of metacognitive behaviours was intended to make the learners aware that this knowledge, and skills gained, was meant for use in their mathematics classes and exercises. Flavell (1971) mentioned that metacognition is intentional, conscious, foresighted, and purposeful and directed at accomplishing a goal or outcome. That being the case, if learners are made aware of the importance of metacognitive skills they will consciously and unconsciously make use of it. Now that metacognition is accepted as a quality that is essential in mathematical problem solving (Yimer & Ellerton, 2009) teaching it per se was intended to develop learners' cognitive capabilities in mathematics learning.

Students in the experiment group worked on their metacognitive skills over a period of two months and at the end of that period, a post-test (mathematical) was administered. During the two months, students were taught and exposed to metacognitive stimulating activities some of which were aimed at arousing self-awareness, time management and building self-confidence. As far as time management is concerned, during the metacognitive training, I made learners aware of its importance. The group established the relationship between mark allocation and duration of mathematics examination. We calculated the relationship and named it minutes per mark relationship. Standard mathematics examinations at grade eleven and twelve have durations of three hours with a total mark allocation of one hundred and fifty. Dividing one hundred and eighty minutes by one hundred and fifty marks, (180 minutes /150 marks) results in a time allocation of one minute and twelve seconds for every mark in the examination. Learners were encouraged to time their work according to marks allocated for every question. As they worked, they self-recorded as a way of regulating their work so that they could self-evaluate.

Metacognition spans three different phases which successful thinkers and students should follow (Fogarty, 1994) and these are:

Planning: This involves carefully thinking of and choosing an approach to take when solving a problem.

Monitoring: This involves fix up strategies when meaning breaks down.

Evaluating: Students should evaluate their thinking after completing a task.

As part of the metacognitive training, students were asked to answer questions that related to planning, monitoring and evaluation in line with the Metacognitive Awareness Inventory (MAI). The questions that were given to students to answer as they made some self-assessments are shown in Table 3.3 in the following section on the training programme.

3.7 The training programme

Metacognitive training seems to be an effective method for teaching problem solving (Özcan & Erktin, 2015, p. 1416). Metacognitive training was introduced to the students of the experiment group. The training was done in sessions of thirty minutes each, every day over a period of two weeks. These sessions were deemed crucial in the sense that it was during these sessions that explicit teaching of metacognition was done and thereafter participants practised what they were taught. The sessions were conducted in a normal classroom and they were conducted with the whole experiment group as there was no intention to split the group into smaller groups for individual training. The metacognitive training was aimed at improving metacognitive knowledge and metacognitive skills of the students (Moga Maier, 2011) and this was designed using the IMPROVE model (Mevarech & Kramarski, 1997). This model is based on the principle of Self-Directed Metacognitive questions (SDM) to enhance students' mathematical reasoning (Özcan & Erktin, 2015). Table 3.2 below shows examples of SDM questions.

Table 3.2. Examples of SDM questions

Plan	Monitor	Evaluate
What am I supposed to do?	How am I doing?	How well did I do?
What prior knowledge will help me to do this task?	Am I on the right track	What did I learn?
What should I do first?	How should I proceed?	Did I get the results I expected?

During the first session, students were introduced to the concept of metacognition and the IMPROVE model. This model was explained to the participants by explaining the meaning of each of the letters in the acronym as proposed by Mevarech and Kramarski.

I: Introduction of new concepts

M: Metacognitive questioning

P: Practicing

R: Reviewing

O: Obtaining mastery on the proposed objective

V: Verification

E: Enrichment

The second session focused on a review of IMPROVE where learners were asked to recall and say the meaning of the letters of the acronym, IMPROVE in their own languages, one to another. Though language understanding was not the focus of the session, doing this was intended to bring ownership, confidence, and recognition to all participants.

In the next session, learners were guided in asking metacognitive questions that were classified in the following four categories (Weaver & Kintsch, 1992).

- (a) Comprehension Questions: What is the problem all about?
- (b) Connecting Questions: In what way is the problem similar to or different from previous ones?
- (c) Strategy Questions: What strategies are essential for solving the problem?
- (d) Reflection Questions: How can the strategy and the solution be evaluated?

Asking effective questions is an important aspect of metacognitive training and development. Effective questions are vital because they trigger the thinking process and stimulate imagination (Özsoy, 2009).

When the participants demonstrated that they were effective self-questioners, the training moved on to the level of practising IMPROVE by working out some selected questions on geometry. After two sessions of doing this, the participants were asked to evaluate the

IMPROVE model. On completion of this exercise the participants were urged to make use of the model in all their mathematics lessons.

To reinforce the use of the model participants were each given a copy of Table 3.3 to refer to as they solved Euclidean geometry problems.

Table 3.3. MAI questions adapted from Schraw & Dennison (1994).

Plan	Monitor	Evaluate
What am I supposed to do?	How am I doing?	How well did I do?
What prior knowledge will help me with this task?	Am I on the right track?	What did I learn?
What should I do first?	How should I proceed?	Did I get the results I expected?
What relationships and theorems should I look for in this diagram?	What information is important to remember?	Can I apply this way of thinking to other problems or situations?
How much time do I have to complete this?	Should I move in a different direction?	Is there anything I do not understand- any gaps in my knowledge?
In what direction do I want my thinking to take me?	Should I adjust the pace because of the difficulty?	Do I need to go back through the task to fill in any gaps in understanding?
	What can I do if I do not understand?	How might I apply this line of thinking to other problems?

3.8 Measurement

As already explained, metacognition can be measured using either on-line or off-line techniques. In this study learners were encouraged to use think-aloud, a protocol which is an on-line technique. In addition to thinking aloud as they worked and solved problems they were encouraged to write down explanatory notes which related to what they were doing and why they did it. At the end of the topic a post-test (Euclidean geometry) was administered to the two groups and the results were analysed to look for similarities and differences. In addition to the post-test, the participants (experiment group) were asked to complete a questionnaire (post-training) to gauge their feelings on the effect of metacognitive training, particularly the use of SDM questioning in their learning of Euclidean geometry. Seven learners from the experiment group were also interviewed and video recorded after the post-test was written and marked.

3.9 Data analysis

Data collected through tests were analysed using the independent t-test in SPSS to test for equivalence of means. By comparing the means, the effect of the intervention with the experiment group was examined. On the other hand qualitative data was analysed to generate themes and categories. Learners' confidence was examined using Indicators of confidence by Burton and Platts (2011). These two sets of data were assessed and analysed to answer the research questions.

3.10 Timeline and modus operandi

The research outlined above was carried out during the 2016 academic year and was aimed at evaluating the effectiveness of decontextualized metacognitive training of learners. This was done by allowing learners to use metacognitive questioning skills.

CHAPTER 4: DISCUSSIONS AND ANALYSIS

This chapter comprises the presentation of the data, analysis and discussion of the findings. The data collected is presented in the form of tables, graphs and narrations. This data is of both quantitative and qualitative nature. It is analysed using statistical methods to compare performance of the control and the experiment groups. Qualitative data will be organised and analysed using appropriate techniques. The techniques are not mutually exclusive and were used in combination (Cohen, Manion, & Morrison, 2011) that is organising and presenting in response to particular issues, organising by individuals, by research questions, by instrument, and using a narrative.

4.1 Pre-intervention phase

4.1.1 Pre-test (Generic)

The generic test was done by means of showing learners items on power point slides. Learners viewed and studied each slide for ninety seconds and then the slide was removed. In ninety seconds, they completed a score sheet by filling in the items they saw on the slide.

The score sheets were marked and results for the experiment and control groups recorded in tables as follows.

Table 4.1. Generic test results-Experiment group

Mark interval (experiment)	0-3	4-6	7-9	10-12
Slide 1	0	1	21	6
Slide 2	0	3	18	7
Slide 3	0	5	18	5
Slide 4	0	2	22	4
Slide 5	2	14	11	1
Slide 6	2	17	5	4

Table 4.2. Generic test results-control group

Mark interval (control)	0-3	4-6	7-9	10-12
Slide 1	0	1	16	4
Slide 2	0	3	13	5
Slide 3	0	4	12	5
Slide 4	0	2	16	3
Slide 5	2	11	7	1
Slide 6	1	13	5	2

The two tables show numbers of learners who scored marks in ranges created by the researcher. Below are the slides and pie charts to show a graphical comparison of the results of the two groups.



Figure 4.1: Slide 1 Generic test (van Jaarsveld, 2012)

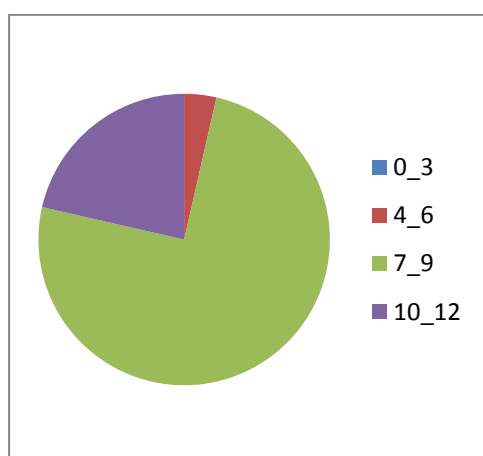


Figure 4.2: Slide 1 results-Experiment

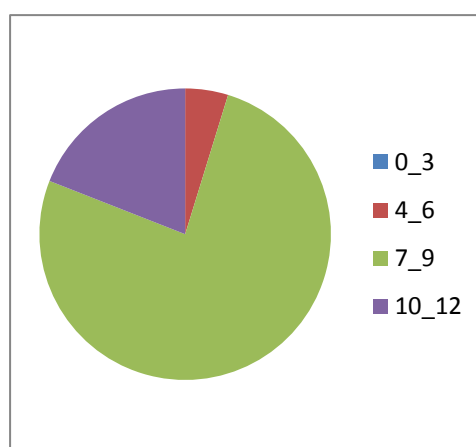


Figure 4.3: Slide 1 results-Control

Seventy five percent and seventy six percent of learners scored between 6 and 9 in the experiment and control groups respectively. No learner scored below four in both groups.



Figure 4.4. Slide 2 Generic tests (van Jaarsveld, 2012)

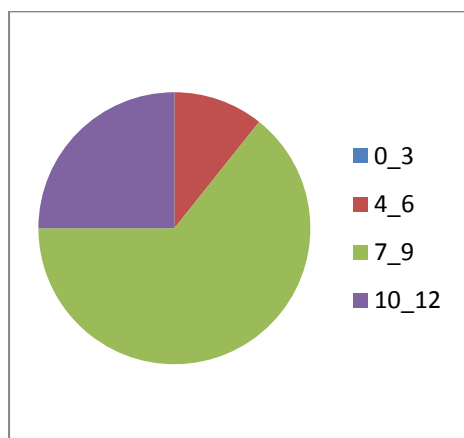


Figure 4.5: Slide 2 results-experiment group

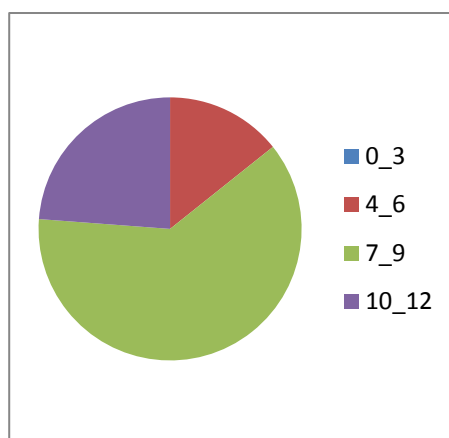


Figure 4.6: Slide 2 results –control group

In this slide, more marks ranged between seven and nine. Like in the first slide, no learner scored zero in this slide. The results of the two groups looked almost the same.



Figure 4.7: Slide 3 generic test (van Jaarsveld, 2012)

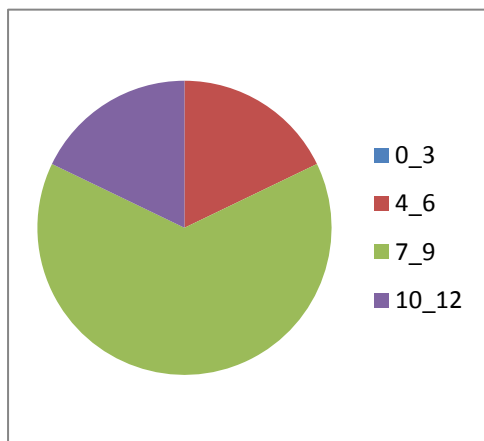


Figure 4.8: Slide 3 results-experiment group

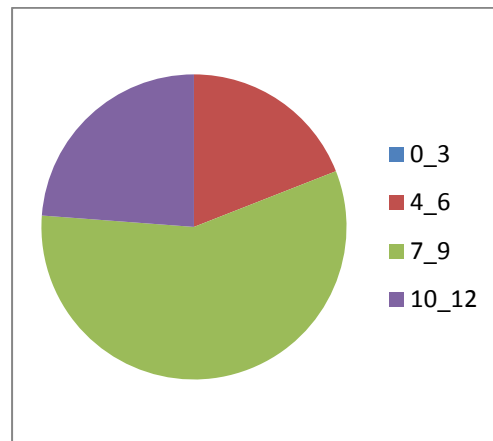


Figure 4.9: Slide 3 results-control group

The modal interval was seven to nine with no learner-scoring zero to three. This interval was also modal in the first two slides.

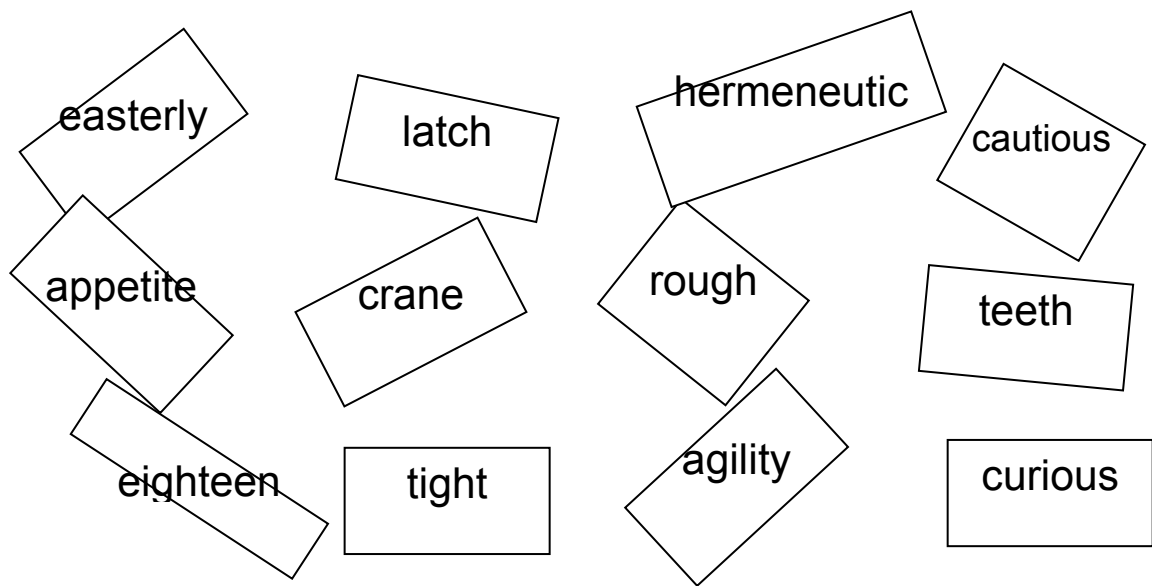


Figure 4.10: Slide 4 generic test (van Jaarsveld, 2012)

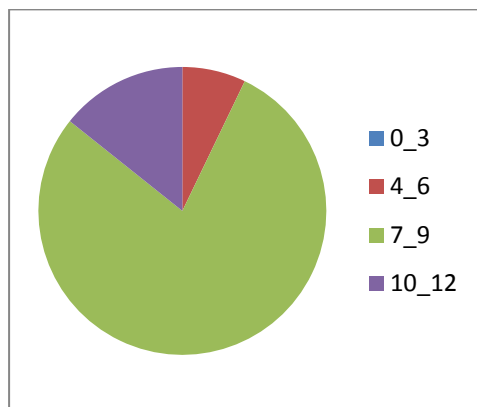


Figure 4.11: Slide 4 results-experiment group

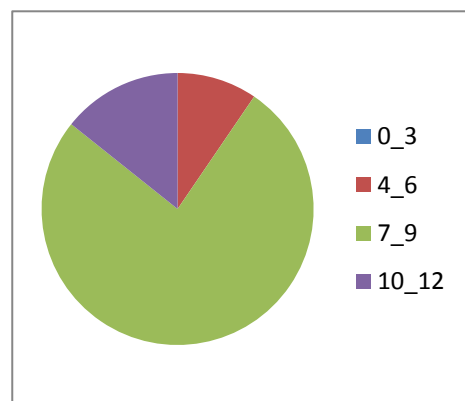


Figure 4.12: Slide 4 results –control group

Like the previous slides, the modal score interval is seven to nine with no learner scoring zero.

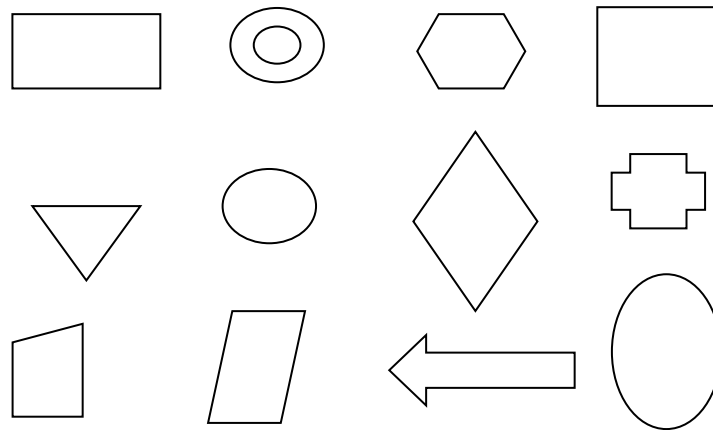


Figure 4.13: Slide 5 generic test (van Jaarsveld, 2012)

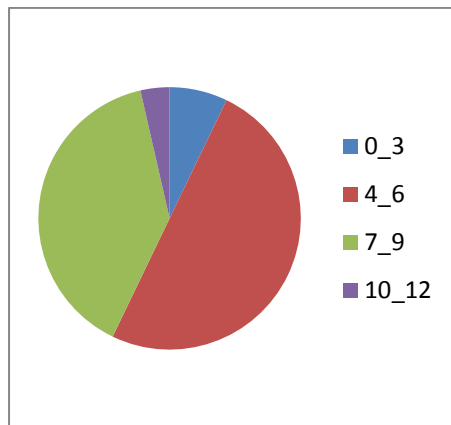


Figure 4.14: Slide 5 results-experiment group

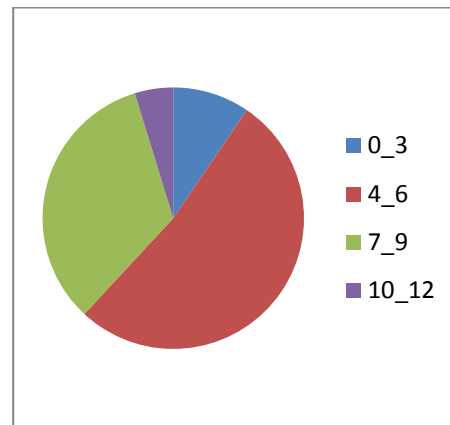


Figure 4.15: Slide 5 results –control group

Unlike the first four slides, in this slide scores of between zero and three were recorded in both the experiment and the control groups. The modal interval changed from seven to nine to four to six.

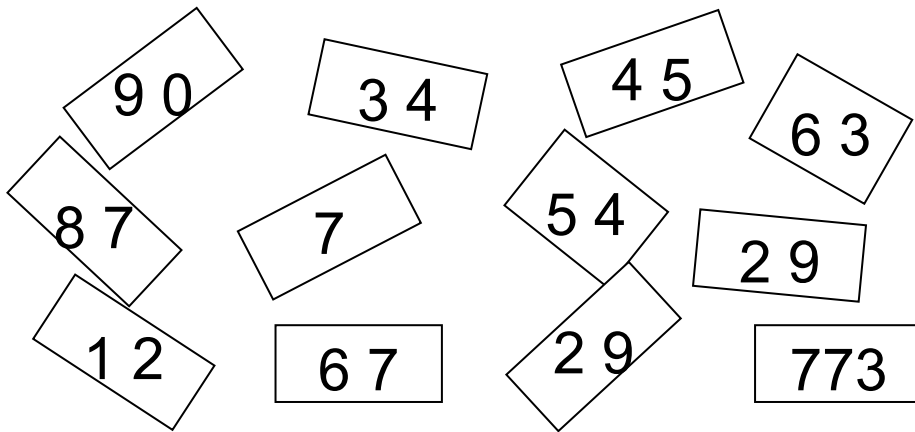


Figure 4.16: Slide 6 generic test (van Jaarsveld, 2012)

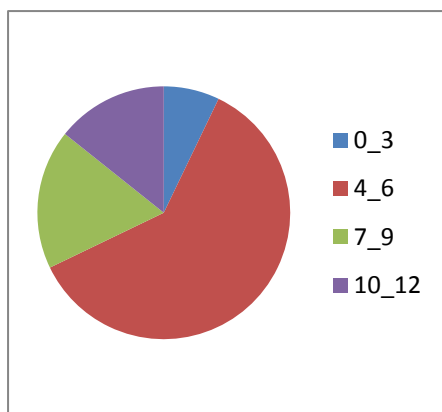


Figure 4.17: Slide 6 results –experiment group

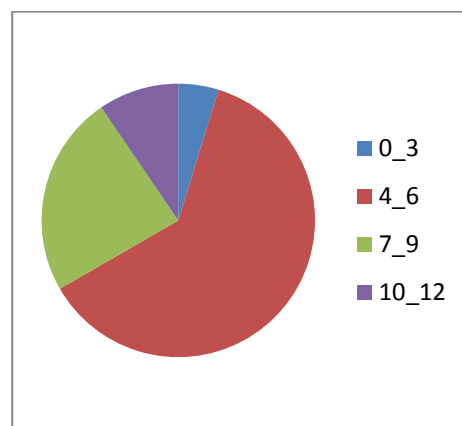


Figure 4.18: Slide 6 results-control group

More learners scored between four and six and very few between ten and twelve. In addition, scores between zero and three were recorded in both groups.

Answers to the generic test showed that most learners did well in remembering coloured objects and words but struggled with geometric shapes and numbers. Of the forty-nine learners, only twenty remembered more than half of the shapes and sixteen remembered half the numbers.

During a follow up interview with ten of the learners, when the participants were asked to say what they perceived to be the purpose of the task, two reasons emerged. Nine learners stated that the exercise tested their ability to remember and one viewed it as a task to check and test speed. When asked to give an opinion about Euclidean geometry, nine except one suggested that it should be removed from the high school mathematics curriculum.

In the view of one of the respondents K, this was a good exercise and a revelation because he learnt that he had to find a way of remembering things. He claimed that he was not aware of how weak his ability to remember was up until he took this exercise. When asked if he thought there could be a strategy that he could use to improve his ability to remember, he said he thought that it could be through repetition and memorising. His response was different from the rest of the interviewees in that he suggested a strategy yet the rest thought that remembering was something natural in a person and that it could not be improved. They also cited their poor ability to remember as the reason why they do not do well in mathematics. The participants referred to Euclidean geometry as a difficult topic in the curriculum

The researcher suggested to learners that remembering could be improved by controlling one's thoughts. This created some anxiety amongst the learners and thus provided a reason for motivating the learners about metacognitive training that followed.

4.1.2 Pre-test (Euclidean geometry)

Both sets of participants were given a mathematics test in Euclidean geometry (Annexure B). The content was derived from work covered in grade 10. The results of the experiment group were compared against those of the control group. Before the application of the t-test the following data characteristics were checked for compliance.

1. The two groups had the same variance i.e. they conformed to the assumption of *homogeneity of variance*.
2. The performance of the groups is *normally distributed*.
3. Each value is sampled *independently* from each other value. No participant had two marks for the same test.

Small-to-moderate violations of assumptions 1 and 2 do not make much difference but it is important not to violate the third assumption (Lane, 2010). There were slight violations to the first two assumptions (see figures 4.21, 4.22, 4.26 & 4.27) but no violations of the third assumption. From a statistical point of view, the data may therefore be used for comparison of the groups 'means. Any differences arising in the means thereafter could therefore be attributed to the intervention.

Below are the pre-test results for the two groups (marked out of a possible mark of 25)

Table 4.3. Pre-test results for the control and experiment groups

Control	11	5	19	4	10	6	11	2	6	3	7	5	8	5	2	5	8	1	5	10	12							
Experiment	12	6	15	6	10	8	13	2	7	7	8	7	2	5	1	5	8	3	8	18	12	11	6	7	5	6	5	1

The above data is presented in box plots to compare distribution. The data sets show very small differences. The graph on the left comes from data from the control group and the counterpart from the experiment group.

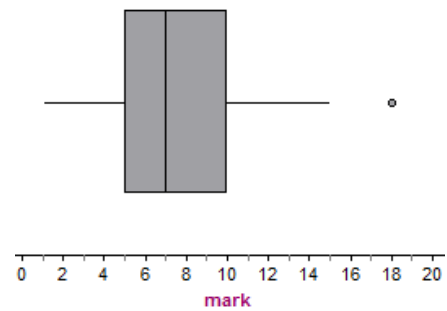
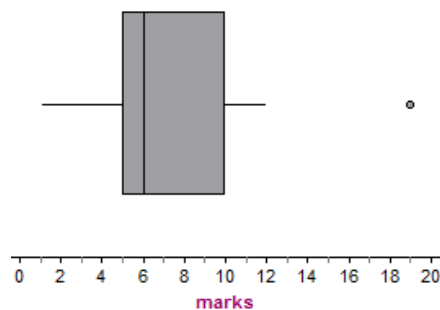


Figure 4.19: Distribution of pre-test marks for control group. Figure 4.20: Distribution of pre-test marks for experiment group

As shown in the box plots, the mean of the experiment group was slightly above that of the control group. The two groups had outliers and their minimum values were equivalent. Since the means were different, I had to check if this difference was significant. Before doing that, the data sets were first tested for normal distribution and the resultant graphs are shown below.

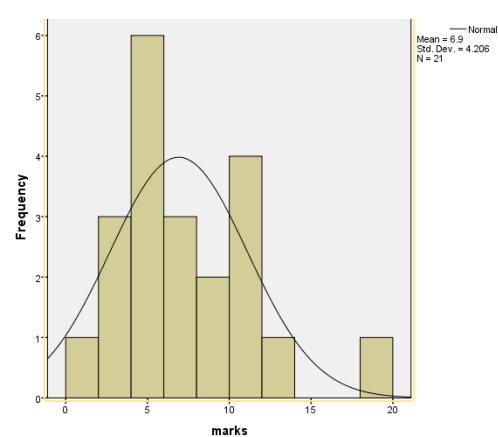
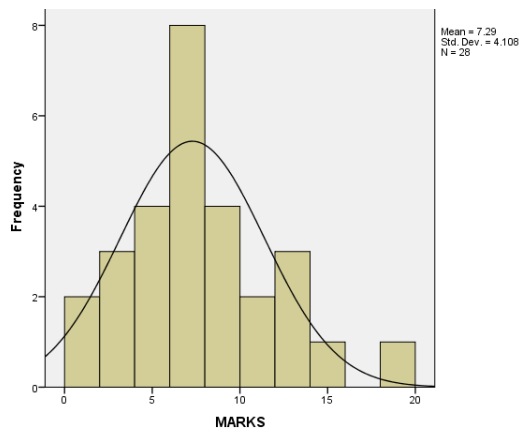


Figure 4.21: Normal distribution curve: pre-test marks (Experiment) Figure 4.22: Normal distribution curve: pre-test (Control)

SPSS computations for independent t-test were made in order to compare the means. The results and the statistical comparisons are shown in Tables 4.4 and 4.5 respectively.

Table 4.4. Group statistics: Pre-test

Group Statistics					
	Group	N	Mean	Std. Deviation	Std. Error Mean
Pre-Mark	0	28	7.29	4.108	.776
	1	21	6.90	4.206	.918

Table 4.5. Independent t-test (Pre-test)

Independent Samples Test									
		Levene's Test for Equality of Variances		t-test for Equality of Means					
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference
PreMark	Equal variances assumed	.046	.832	.318	47	.752	.381	1.198	-2.029 2.791
	Equal variances not assumed			.317	42.677	.753	.381	1.202	-2.044 2.806

Deducing from the above computations, the two groups are equivalent. There is no significant difference between them except that the standard deviation of the control group is slightly above that of the experiment group.

On average, the experiment group is slightly better ($M=7.29$, $SE=0.776$) than the control group ($M=6.9$, $SE=0.918$) during the Euclidean Geometry Pre-Test. However, there was no significant difference between the two groups ($t(47) = 0.318$, $p>0.05$).

4.2 The Intervention Phase

As described in chapter three, there was an intervention with the experiment group. The training was done in sessions of thirty minutes each, on Mondays, Tuesdays and Wednesdays and one hour on Thursdays. This gave a total of five hours of training. After the training, learners were asked to use what they had learnt in their mathematics lessons.

In the first session the purpose of the research (spelt out in the consent forms) was explained to the learners. The researcher took the participants through the IMPROVE model by explaining meanings of letters in the acronym. Emphasis was made on the **M**, metacognitive questioning. A brief explanation was given about the role of questioning in learning. However, this time around the questioning was expected to be directed to oneself more than to someone else.

The second session was more of a repetition of the first session though in this session, learners were more active than before. Learners were asked to sit in pairs and each member used his home language to give a meaning of M, in IMPROVE. There was a lot of fun during the session as there was a lot of dialogue in the multilingual group. The group comprised of eight different languages of which six were South African and two were other African languages. Learner meanings given to metacognitive questioning were more towards self-directed questioning.

In the third session, participants were taken through the process of use of metacognitive strategies of planning and organising, monitoring own work, self-reflecting and directing own learning. This is illustrated as follows:

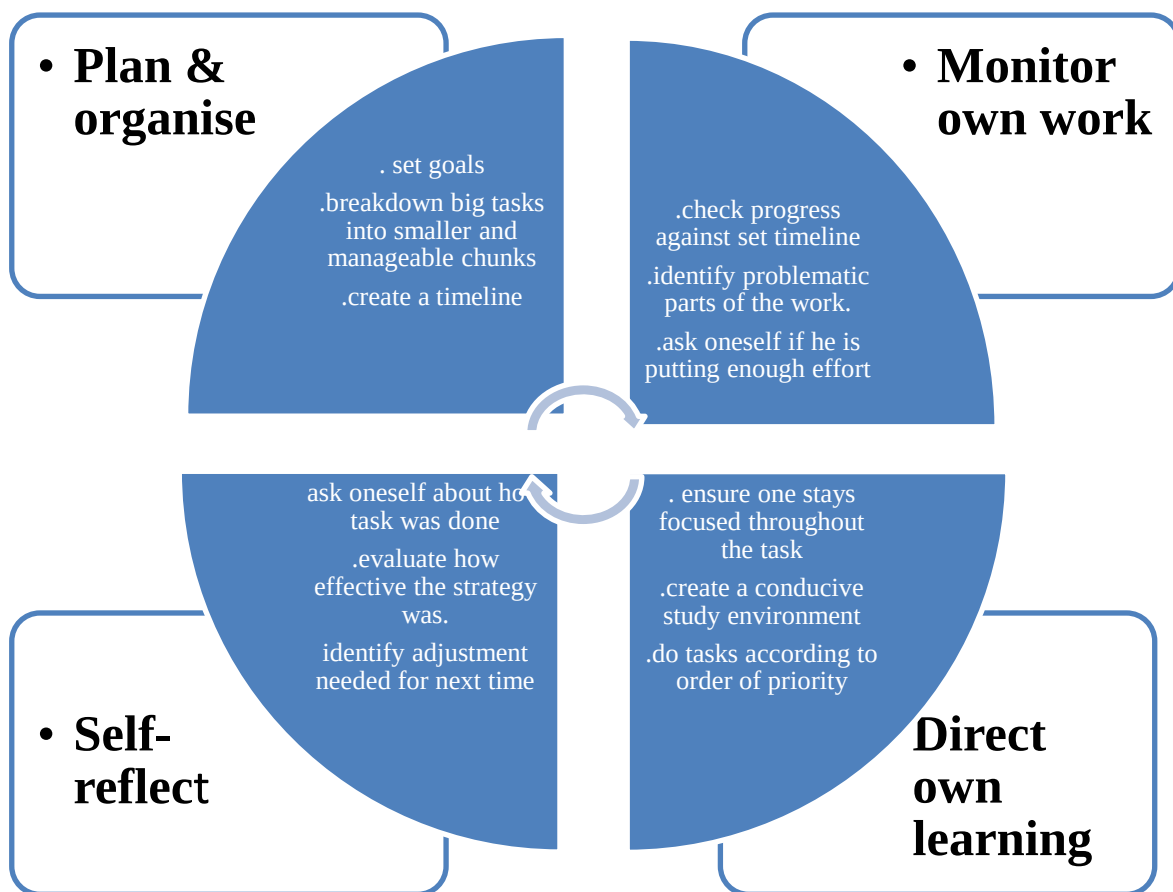


Figure 4.23: Metacognitive processes.

The following session was revision of what was done in the previous session. In addition, learners were asked to practise setting goals and devising time lines. They also each tried to think and say what they considered to be a conducive study environment. In their discussions, they discouraged one another from the use of headphones and or loud music during study.

The fifth session was held on a Monday of the second week. During this session, the group learnt about the role of metacognitive questions. Questions were classified into four categories (Weaver & Kintsch, 1992). These include Comprehension Questions, Connecting Questions, Strategy Questions and Reflection Questions. Following this, learners practised asking effective metacognitive questions for one more session. Thereafter, they worked with IMPROVE to answer geometry questions as practice.

Towards the end of the last session of the training the researcher asked learners to sit in pairs and converse once again in their home languages about their general understanding of what

they learnt during the training. The learners also took this opportunity to pledge to one another their preparedness to use self-directed metacognitive questioning as a learning strategy.

After a reasonable number of practice sessions, the experiment group was encouraged to use SDM questioning during Euclidean geometry lessons. During these lessons, the teacher was at the forefront in using think aloud protocols and as expected all learners in the experiment group adopted the technique with ease. In addition, each learner was encouraged to self-record (write notes) as they worked through Euclidean geometry questions. Self-recording included, self-timing to ensure that time was managed well. Included here is what transpired in one of the Euclidean geometry lessons with the experiment group.

Each of the lessons with the experiment group where they did Euclidean geometry was called an episode. During all episodes, the researcher used think aloud protocols to demonstrate concepts and solve problems with the class. In the episodes, learners were also encouraged to use the same techniques when solving problems.

Episode 9

In episode 9, learners were asked to answer questions in exercise 4 on page 200 in their core textbook (Platinum Mathematics, Learners' book, grade 11) see Annexure F. The questions required use of learnt concepts and Theorems 1 to 5 (stated in the textbook). This episode was a double period (eighty minutes).

As the learners worked out the problems, the teacher moved around the class listening to learner conversations and taking down some notes in a note book. In some instances, the learners would call the teacher to seek clarifications or approval of their methods and answers. From previous episodes learners had learnt that they should take down some notes and write information that could be of help in solving a problem at hand. It was also to their benefit to remember and consult useful sources for clues.

The researcher encountered K, during episode 9, who was attempting question 3, item 3.3.

I looked at his work and read through some of the notes he had for item 3.2. As I went through the notes, K was busy with 3.3 and he was thinking aloud. I stopped reading the

notes and listened to his self- conversation. I still had in my hand the paper with his notes for items 3.1 and 3.2.

As he conversed, he was writing notes on a paper taken from the same examination pad as before. The conversation was a series of questions, notes and answers. In the process, there were physical gestures of approvals and refutals. These included nodding and shaking of his head. I started to record K's self conversation in my notebook. This was possible because he was using think aloud techniques to allow the teacher to follow his thinking.

Conversation with student K

K: This (putting a tick in pencil at an angle on the diagram: ABC) is cut into half. Yes, that is to bisect. Then $B_1 = B_2$. Yaa, $B_1 = x$ so $B_2 = x$. What do I need to do? To see if $B_2 = x$. But there is nowhere where it shows that it is x . So I must find angles related to B_2 using theorems. Let me see (flipping the book going backwards).

He opened to page 195 and stared at the two diagrams under theorem 4. Turned back to page 201 and again to page 195. (refer to Annexure A for diagrams)

K: Angles subtended by the same arc are equal and angle at centre...(pause)..

He turned back to the question and drew a line connecting C to D and sighed.

K: This arc, what is it, I mean chord and yes, it subtends B_2 and E_2 . Yes $B_2 = E_2$.

He marked the two angles with a similar notation and then a tick.

K: But this and this (putting other ticks on B_2 and E_2) are equal but not B_1 . Sir, do I need to find E_2 first?

Since he asked me a question, I stopped writing and attended to him by asking him a question.

T: Why do you want to find it?

K: I see it must be equal to B_2

T: So?

K: Is B_2 not equal to B_1 sir?

T: Why, K?

K: Here they say bisect meaning equal.

T: You are right, so continue.

K: So is it right if I calculate this first?

T: Yes you are right, look at this statement (teacher pointing at the second statement) and see what comes out from it.

K: Ok.

He underlined the term bisects and wrote the word equal underneath.

K: Yaaa sir, I think this is equal to this (pointing at angles on the diagram).

T: Which ones?

K: E_1 and E_2 ?

T: Why?

K: Because, here, it bisects.

T: Write it down then call me when you have it well written.

K: Ok sir.

I left K and moved on to another learner who had his hand up. This learner was KB and he sat two rows behind K. Before I assisted KB, I asked the boys to lower their voices because some were speaking louder than they should.

KB: Sir check if I did well on this one, number 3.2.

I looked at his working and there were many words written on the paper. At the top of the paper he had the word, Data, and it was underlined. Below it there were phrases:

Triangle DEF

Isosceles(circled with a pencil)

Two sides equal

Two angles the same

$$E_1 = F$$

$$E_1 = x.$$

As I went through his work, I came across some notes and I tried to find meanings from those notes. KB had managed to prove that $DE = DF$ by showing that $E_1 = F$.

I took down some notes about KB's workings and moved on to other learners. By this time we were already in the second lesson of the double period.

Most of the learners in the group did well in answering questions 1 and 2. A general observation that I made was that most learners referred to notes in their textbooks, underlined key words, wrote meanings to words, and marked angles with notations. This was a recommendable technique as it helped them to reflect and focus on main ideas and issues. Pages mostly referred to were pages 195, 197 and 198 of the textbook. Page 195 has descriptions and illustrations of theorem 4, page 197 explains theorem 5 and page 198 has notes on how to prove that a quadrilateral is a cyclic quadrilateral(Annexure F).

A class discussion was then held with the teacher clarifying on certain issues of concern and learners asked questions which were answered by means of a general class discussion. During the discussion, learner to learner interactions were common as some learners asked questions and others answered those questions . All this was done by means of teacher coordination. I asked K, to go to the board and answer question 3.3. He used think aloud techniques and managed to prove that $\angle ABC = \angle C$.

Checking on his work, K, used self questioning as a metacognitive strategy of answering the question. His questions could easily be classified into planning, monitoring and evaluation questions. He also used available resources to solve the problem. In the same way KB also answered questions by asking himself questions about what was expected of him by the question, what was given by the question and how he needed to solve the problem.

I concluded the lesson and when the bell rang, I asked learners to leave their books behind for marking and they left for their next lessons. All Euclidean geometry lessons with the experiment group were conducted in this fashion up to the end of the chapter. Notes that I took down were then later used as data.

4.3 Post intervention

4.3.1 Post-test (Euclidean geometry)

After the intervention, another test in Euclidean geometry, was given to the two groups and the results were compared – experiment against control. This was again done using an independent t-test in SPSS. The results conformed to a normal distribution and the scores are independent as they come from different groups, this provides for the reliability of the data.

Below are the post-test results of the two groups. The results were processed and resultant information is presented in the next page

Table4.6. Post-test results for control and experiment groups

Control	11	16	11	16	13	22	17	9	17	8	22	14	7	27	14	17	3	19	12	11	17										
Experiment	22	20	29	28	27	12	36	7	24	8	13	10	19	23	19	20	2	7	12	10	34	29	5	23	15	16	13	23			

Like data from pre-test, this data was also presented in box plots to compare distribution of marks from the two groups as shown below.

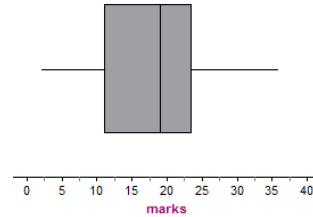
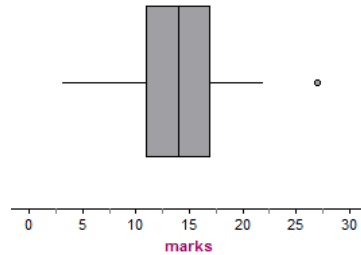


Figure 4.24: Distribution of post- test marks for control group Figure 4.25: Distribution of post- test marks for experiment group

The mean of the experiment group was higher than that of the control group. As was the case during pre-test, the control group had an outlier but this time around, there was no such in the experiment group. A statistical test was done to check if the difference in the two means was significant. Before the statistical test was done, the data sets were tested for normal distribution and the resultant graphs are shown below.

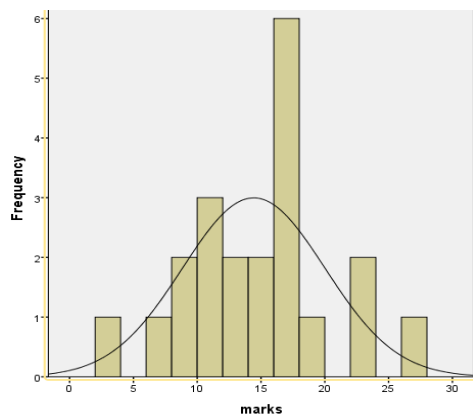


Figure 4.26: Normal distribution curve, post-test (control)

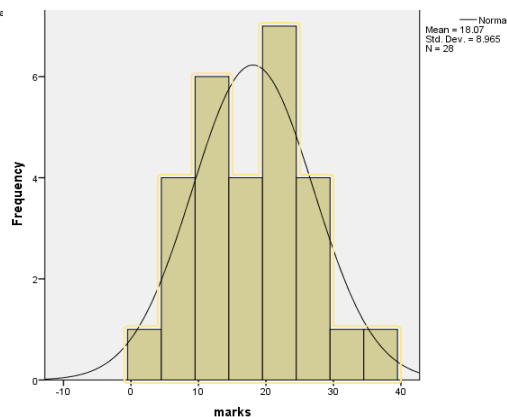


Figure 4.27: Normal distribution curve, post-test (experiment)

An independent t-test was calculated in SPSS to compare the means of the experiment and the control groups. The results are shown in the two tables below.

Table4.7. Group statistics of Post Test

Group Statistics					
	Group	N	Mean	Std. Deviation	Std. Error Mean
Postmark	0	28	18.07	8.965	1.694
	1	21	14.43	5.591	1.220

Table 4.8. Independent t-test, Post test

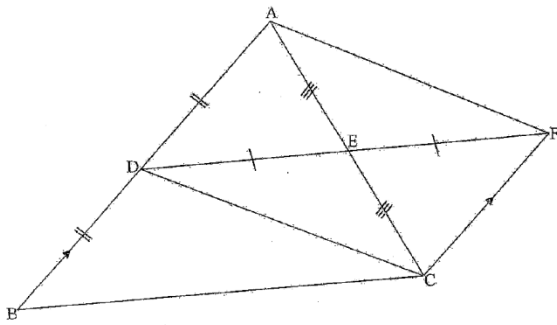
Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
PostMark	Equal variances assumed	6.753	.012	1.636	47	.108	3.643	2.226	-.836	8.121
	Equal variances not assumed			1.745	45.680	.088	3.643	2.088	-.560	7.846

On average, the experiment group performed better ($M=18.07$, $SE=1.694$) than the control group ($M=14.43$, $SE=1.220$) during the Euclidean geometry Post-test. This difference was significant ($t(47)=1.636$; $p<0.05$). It did represent a fairly substantial effect $r=0.232$.

The standard deviation was higher in the experiment group (Std. Deviation= 8.965) than the control group (Std. Deviation= 5.591). This was due to the presence of some very high marks of certain learners in the experiment group. For this reason, the scripts of those top learners were analysed. The reason for doing this was to see the kind of skills that they had gained that made their work a cut above the rest.

4.3.2 Analysis of scripts

An examination of the scripts of those learners showed that during the pre-test they lacked control of their own work. The work was not organised and not enough justifications were given for claims. I analysed the scripts for questions that asked the learners to give proofs since the understanding and reasoning skills developed in the process of generating a proof are highly beneficial (Hanna, 2007, p.15). Furthermore, proofs serve the functions of verification, explanation, systematisation, discovery, communication, and intellectual challenge (De Villers, 2007). Metacognitive skills would be put to test in situations of intellectual challenges. In a question that required proving that $DE = \frac{1}{2} BC$ (pre-test), one of these learners, B, expressed it as follows.



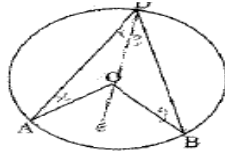
212. DBCF is a parallelogram (AD is parallel to CF)
 213. if DB and AB are equal and AE and EC are equal therefore
 214. $DE = \frac{1}{2} BC$

Showing that DBCF is a parallelogram was done in the previous question, and this is how B continued. However (see alongside), referring to “DB=AB (instead of DB=AD) and AE = EC” may be an innocent mistake, lack of concentration or incorrect reasoning since the proof fails because of the incorrect premise that DB=AB as these line segments do not help prove what needed to be proved.

During the post-test, the same learner B answered a question on proving a theorem in a way that was quite different from his first attempt. The question required to prove that:

$$\hat{AOB} = 2\hat{ADB}.$$

His proof follows.



$\angle AOB = 2(\angle ADB)$
 Construct line DE passing through O .
 Label $\angle ADO = x$ and $\angle BDO = y$.
 $DO = AO = BO$ (radii).
 $\triangle ADO$ and $\triangle BDO$ are isosceles $\triangle$ s.
 $\therefore \hat{A} = x$ and $\hat{B} = y$.
 $180^\circ - x + x = \angle DOA$ and $180^\circ - y + y = \angle DOB$.
 $180^\circ - 2x$ and $180^\circ - 2y$.
 $\angle AOB = x + y$ and $\angle AOB = 360^\circ - (180^\circ - 2x) - (180^\circ - 2y)$.
 $\angle AOB = 360^\circ - 180^\circ - 180^\circ + 2x + 2y$ (collect like terms).
 $\angle AOB = 2x + 2y$.
 $\angle AOB = 2(x + y)$.
 $\therefore \angle AOB = 2(\angle ADB)$.



B showed more focus here indicated by his comprehensive follow-through and giving reasons for his claims. He used some fundamental mathematical habits of mind (Driscoll et al., 2007) evident in his work that include, visualising, justifying claims using words and reasoning in describing relationships. His post-test attempt shows that he is confidently in control of what he is doing.

Another learner, AD, did not attempt a proof in the pre-test. An excerpt of his script is shown alongside.

Question 2
 2.1.1 (500)
 2.1.2 OB is CF
 2.1.3 DE is the midpoint of DF

In the post-test, the same learner, AD, made a lot of effort to answer a question on giving a proof as provided alongside.

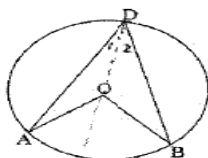
$$\begin{aligned} \angle AOB &= 2\angle ADB \\ \text{Construct a line } OD \text{ to } O \text{ and point } T \\ \triangle OAD &= \triangle ODA \mid \text{isosceles } \triangle \text{ are equal} \\ \triangle ODB &= \triangle OBD \mid \text{isosceles } \triangle \text{ are equal} \\ \text{make } \angle ADO = x \text{ and make } \angle ODB = y \\ \angle ODB \text{ or } \angle DOB \text{ are equal} \mid \text{Sum of } \triangle \text{ is } 180^\circ \\ \angle DOT \text{ is a straight line which means that } 180^\circ - 180^\circ - 2y \text{ or} \\ \angle DOT &= 180^\circ - 180^\circ - 2y \\ \angle AOB &= 2y + 2x \\ \angle ADB &= x + x \\ \angle AOB &= 2(y + x) \\ \text{Divide both side by 2} \\ \angle AOB &= 2\angle ADB \end{aligned}$$

AD confessed during an interview that before metacognitive training, whenever he came across something that he did not know, he would leave it without attempting it. In the pre-test, he left the question unanswered but during the post-test, he proved what needed to be proved. He claimed that his focus and concentration was enhanced by self-questioning himself.

Learner IK's answers were also analysed. In the pre-test, IK showed that he knew how to give a proof. His answer was short but precise although he did not give a reason for his conclusion.

$$\begin{aligned} 2.1.2 \quad DE &= BC \text{ (opp sides of a parallelogram)} \\ DE &= EF \text{ (given)} \\ \therefore DE &\text{ is } \frac{1}{2} \text{ of } BC \end{aligned}$$

In the post-test, he still demonstrated his ability to prove. However, his work was clearer and full of justifications (reasons) for each claim he made.



I will make a construction line from D to O

Let $\hat{D}_1 = x$ and $\hat{D}_2 = y$

$\hat{A} = x$ (AO and DO are radii)

$\hat{B} = y$ (OB and DO are radii)

$\hat{AOD} = 180 - 2x$ (sum of $\angle$ s of Δ)

$\hat{BOD} = 180 - 2y$ (sum of $\angle$ s of Δ)

$\hat{AOB} = 360 - \hat{AOD} - \hat{BOD}$ ($\angle$ s in a rev)

$\hat{AOB} = 360 - (180 - 2x) - (180 - 2y)$

$\hat{AOB} = 360 - 180 + 2x - 180 + 2y$

$\hat{AOB} = 2x + 2y$

$\hat{AOB} = 2(x + y) = 2\hat{ADB}$

Good!

A look at the answers given by AD, B and IK reveals that there was similarity between their answers. It seems that the students had learned and produced the theorem proofs. For this reason, these students were selected for interview to find out if there were particular techniques that they used to produce the proofs. The interviews will be presented latter in this report.

The above-analysed scripts are testimony of the fact that the skill of proving was better during post-test than during pre-test. This was generally true for nineteen of the twenty-eight learners whose pre-test and post-test scripts were analysed.

Aspects that were looked for during the script analysis were: making necessary constructions, establishing correct relationships, stating theorems correctly, giving reasons derived from Euclidean geometry theorems and drawing true conclusions.

4.3.3 Questionnaire Results

The questionnaire was completed by participants soon after the training but before they sat for the post-test. Twenty-eight learners were each given a copy of the questionnaire, which they were asked to take home and complete. I asked the learners to bring back the completed questionnaires the following day of which all the twenty-eight learners did that.

The purpose of the questionnaire was to evaluate the training and check on learner preparedness for the post-test. The skills gained during the training were spotlighted. It was

still unclear what the impact of these skills were on learner preparedness for a test and how they felt in terms of their confidence to do well in the test. None of the participants had an idea of how difficult or how easy the test would be so their responses were considered to be their genuine thoughts and feeling about their preparedness.

The questionnaire comprised of four sets of questions as appear in the following section: Motivated Strategies for Learning (Part A), Metacognition Questions (Part B), and Awareness of Independent Learning questions (Part C) and Questions to evaluate the training programme (Part D).

4.3.3.1 Analytical Framework

The extent to which SDM questioning affects learners' confidence level and preparedness in Euclidean geometry was measured using the confidence indicators of Burton and Platts (2010). Confidence is defined as “the ability to take appropriate and effective action in any situation, however challenging it appears to you or others” (Burton & Platts, 2011, p.10). These indicators are generally applicable to people and the indicators need not be present simultaneously for a person to be regarded as confident. If at any given moment, one of the indicators is present in a person, then the person is deemed confident. Burton and Platts' indicators were used to interpret learners' ratings. This helped to answer the question: What is the effect of the use of Self -Directed Metacognitive (SDM) questions on the confidence level and preparedness of learners in the learning of Euclidean geometry?

The first and third sets of questions (parts A and D) consisted of questions to which respondents were to answer using a rating of one to seven. A rating of four showed a respondent's indifference. Ratings 1, 2, and 3 meant that the respondent was negative and ratings 5, 6, and 7 were considered as positive. Positive ratings were an indication of high confidence and negative ratings showed lack of it.

Indicators of confidence by Burton and Platts (2010)

- A positive mind-set: You have the capacity to stay positive and see the bright side even when you come across hindrances. You have a positive esteem for yourself as well as other people.
- Self-awareness: You recognize what you are good at, how gifted you feel, and how you look and sound to others. You also accept that you are a human being, and you do not expect to be flawless.
- Flexibility in behaviour: You adjust your conduct according to circumstances. You can see the bigger picture as well as giving consideration to details. You consider other people's views in making decisions.
- A willingness to take risks: You have the capability to act in the face of uncertainty – and put yourself on the line even when you don't have the answers or all the skills to get things right.

4.3.3.2 Discussion of Questionnaire Results

The first part of the questionnaire here referred to as part A (Annexure A) comprised of sixteen questions. Learners were required to answer each question by means of a rating. The ratings ranged from one to seven as follows:

1. Not true at all
2. Rarely true
3. A little true
4. Sometimes true
5. Often true
6. Very often true
7. Always true

Table 14 below summarises the participants' responses to the questions in Part A.

Table 4.9. Summary of response to questions in Part A

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
			I prefer class work that is challenging so I can learn new things.	Compared with other students in this class I expect to do well.	I am certain I can understand the theorems and concepts taught in Euclidean Geometry.	I think I will be able to use what I learnt during the training in other topics/subjects.	I am sure I can do an excellent job on the problems and tasks assigned for Euclidean geometry	I think I will receive a good mark in the coming test in Euclidean Geometry.	Even when I do poorly on the test I will try to learn from my mistakes in preparation for the examination.	Compared with other students in this class I think I know a great deal about Euclidean geometry	When I take a test I think about how poorly I am doing.	When I study I put important ideas into my own words.	When I study for the test I try to remember as many Theorems as I can.	When I study for the test I practice saying the important theorems over and over to myself.	Before I begin studying I think about the tasks I will need to do to learn.	I use what I have learned from old homework assignments and the textbook to do new assignments.	When I am studying Euclidean Geometry I try to make connections between all Geometry concepts and theorems that I know.	When studying, I try to connect the things I am studying about with what I already know.
Negative	1	0	0	0	0	1	0	1	0	1	5	0	0	2	0	0	0	0
	2	0	0	1	1	0	2	1	0	1	4	0	2	1	0	0	2	0
Neutral	3	2	3	3	5	0	3	2	0	4	2	4	1	3	3	0	1	0
	4	4	4	4	2	3	7	3	1	7	1	5	4	4	2	2	2	0
Positive	5	4	3	3	5	4	3	5	3	3	2	4	1	2	5	1	4	4
	6	4	1	1	6	7	4	5	4	2	0	3	7	3	3	7	6	6
	7	3	7	7	1	5	1	3	13	1	4	4	3	4	6	6	4	6

The results showed a shift away from students' prior position in terms of liking or disliking geometry as was revealed in the interview before the training. Of the ten students who were interviewed (before training), nine preferred that Euclidean geometry should be removed from the curriculum as they considered it to be difficult for them. In the table above, ratings for questions 3, 5, 6, 8 and 15 which concerned Euclidean geometry showed some positive ratings. This showed that there had been a change in attitude towards the topic in the majority of those who completed the questionnaire.

Another point to observe is in the response to questions relating to expectations on the outcome of the test. This is found in how learners responded to questions 2, 5 and 6. The responses showed that most learners believed that they would do well in the test. They were willing to take risk, an indication that they were optimistic and confident of their preparedness. They were not scared of the result. This was shown in responses to question 7. Thirteen of them responded with a rating of seven to the fact that even if they were to do poorly in the test they would try to learn from their mistakes in preparation for the examination. They seemed to have grasped that mistakes can be used for learning purposes. Positive mind-set, self-awareness, eagerness to develop and a willingness to take risk (Burton & Platts, 2010) were the indicators of confidence displayed by learners who evaluated themselves positively on questions of this questionnaire. This way, I concluded that in fifteen of the sixteen questions presented, 64% of the learners displayed confidence in themselves.

The above data are presented graphically in figure 4.28 below.

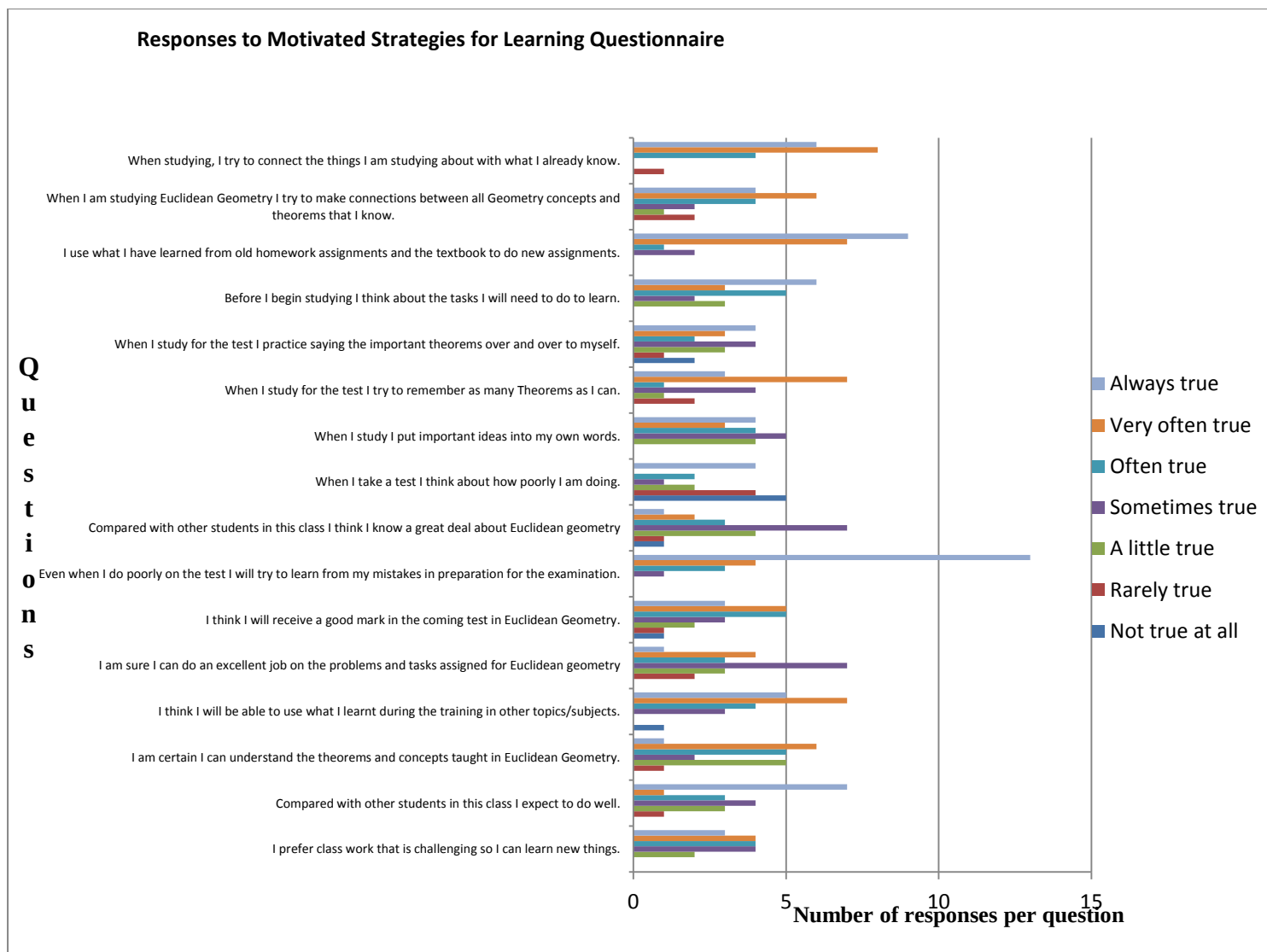


Figure 4.28: Responses to Motivated Strategies for Learning Questionnaire

The second part of the questionnaire (B) was answered by use of YES or NO responses. The purpose was to check how learners evaluated themselves in terms of monitoring their thoughts as they worked through problems. Metacognition involves knowledge about thought processes. If learners could think about what they think, they would be regarded as being self-aware and this is an indicator of confidence (Burton & Platts, 2010). Learner results to this questionnaire were recorded as follows:

Table 4.10. Responses to Metacognition questionnaire

	Yes	No
Whenever I work through a geometry question, I constantly examine my thoughts.	16	3
I pay close attention to the way my mind works.	11	9
As I work, I monitor my thoughts.	15	5
I am constantly aware of my thinking.	13	6
I am aware of the way my mind works when I am thinking through a problem.	10	9
I think a lot about my thoughts.	12	6

A graphical representation of the above results is shown in the Figure 4.29 below.

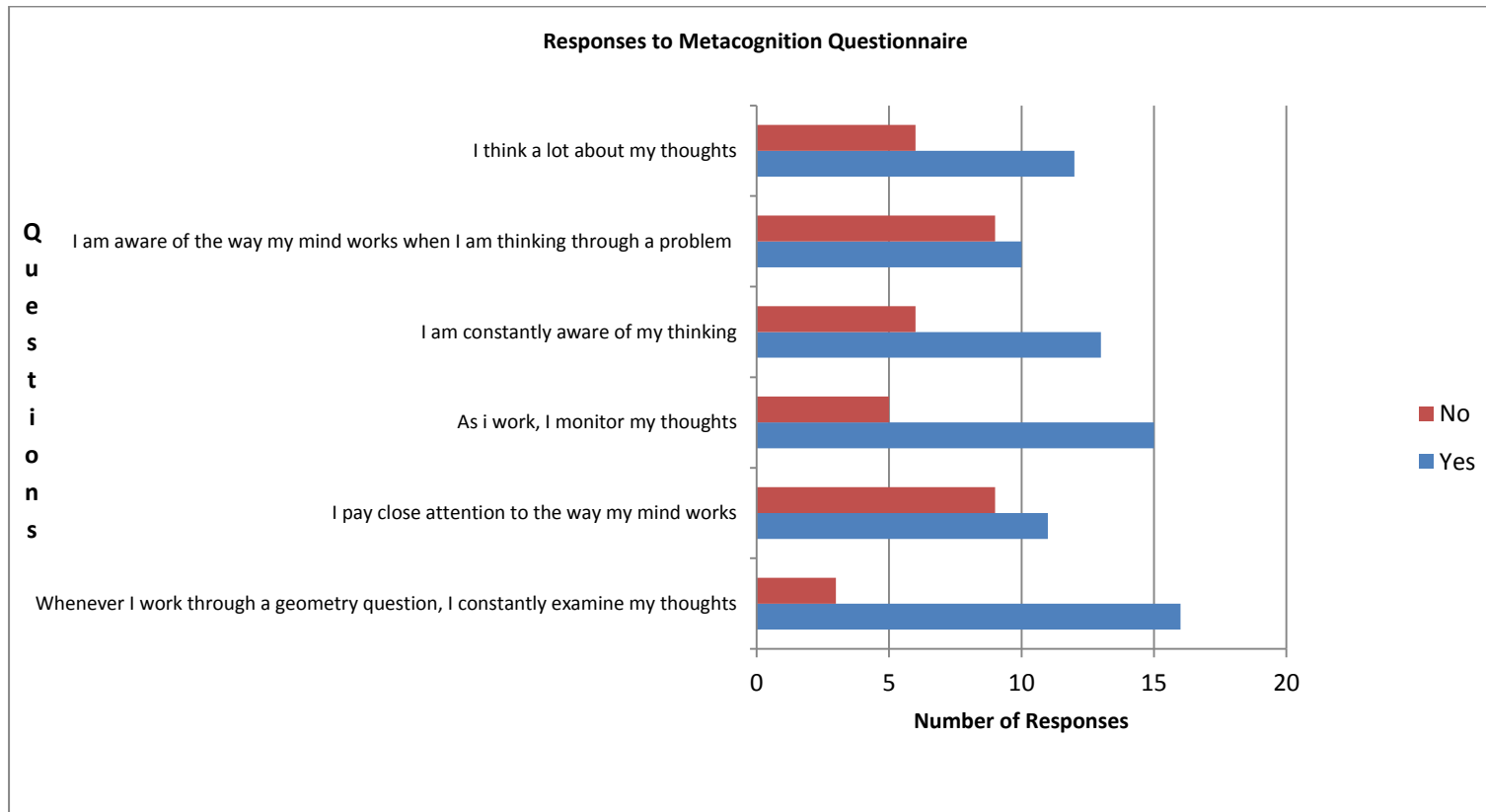


Figure 4.29: Responses to Metacognition questionnaire

The results show that for all the six questions, the most popular response was YES. For every question, the majority of learners who completed this questionnaire suggested that they monitored their thoughts. In this regard, most learners 16, 11, 15, 13, and 10 for questions one to six consecutively evaluated themselves as drivers of their thought processes.

The third part of the questionnaire (C) comprised of ten questions. To these, learners were asked to give ratings from one to seven as they did in part A. Unlike A which asked for ratings on motivated strategies for learning, C asked for ratings on awareness of independent learning strategies. This directly focused on self-awareness as an indicator of confidence. The ten questions are given in the following table and learner responses are presented in the figure after the table.

Table 4.11. Awareness of Independent Learning Inventory Questionnaire

I do not think it is important to feel personally involved in what I am studying.
I ignore feedback from tutors on my method of work.
While working on an assignment I pay attention to whether I am carrying out all parts of it.
While working on an assignment I keep a record of my learning aims.
When I have finished an assignment/test/examination, I do not revise my work to check for mistakes and correctness of my answers.
Sometimes while working together with others on an assignment I get a sudden feeling that I am learning a great deal from them.
When I have worked together with others on an assignment, I do not think about whether the co-operation was useful for me.
When I work together with others, I regularly think about what I learn from them.
I see no reason to talk with others about the usefulness of working together on our studies.
If I find information difficult to understand, I do not try to find a deeper reason for not understanding it.

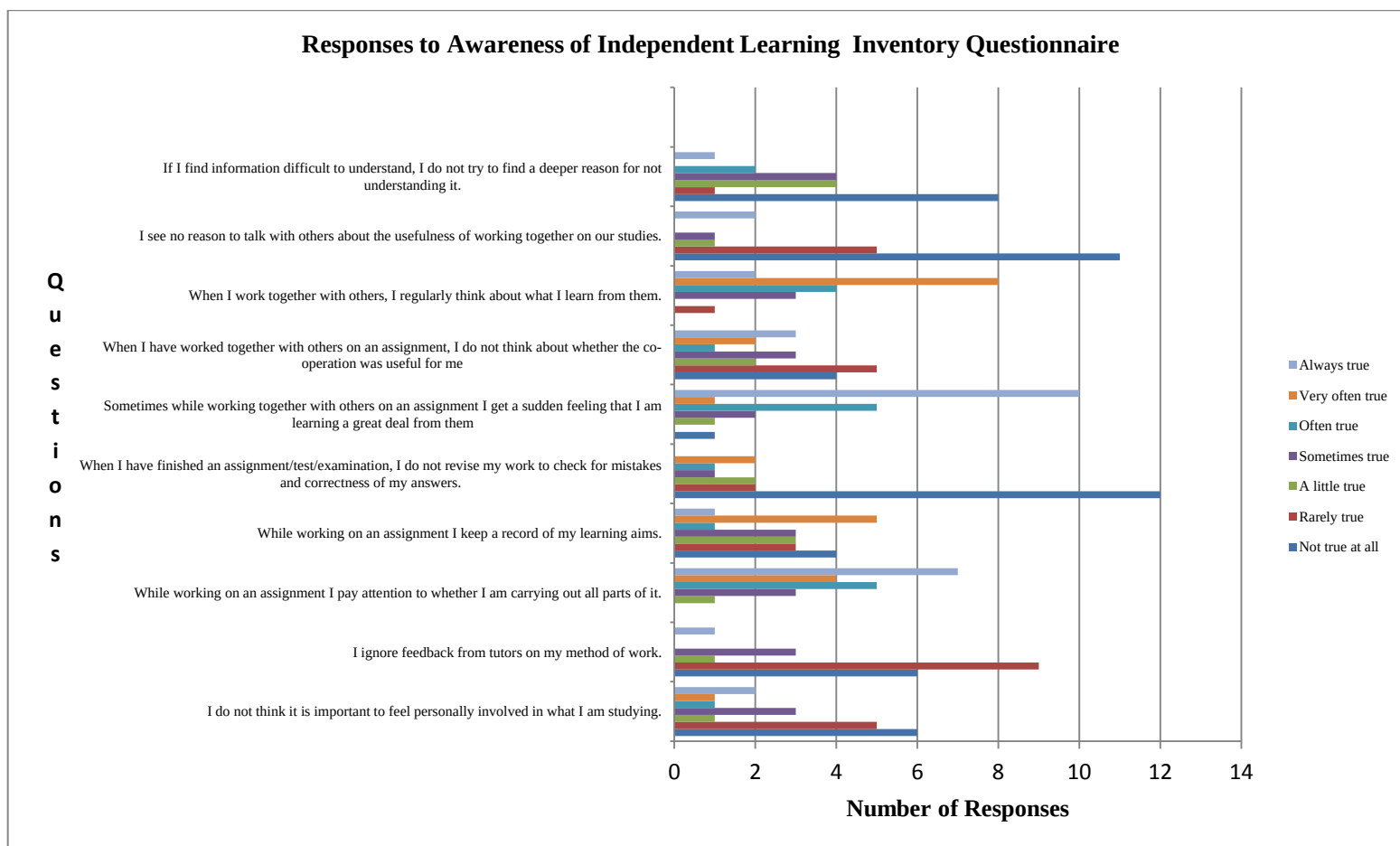


Figure 4.30: Responses to Awareness of Independent learning inventory questionnaire

For all the ten questions, positive evaluations were more dominant than the negative responses. This showed that the respondents displayed self-awareness about their learning strategies. By this, I concluded presence of confidence in the learners.

The last part of the questionnaire (D) consisted of two open ended questions which sought to evaluate the strategy of SDM questioning. Of the nineteen respondents who answered the question on what the main strategy was during the metacognitive training, none was unaware of what it entailed. The two main categories from the answers were:

1. Self-questioning (mentioned nine times)
2. Monitoring thought (mentioned six times)

The other four responses were:

- One respondent gave meaning of acrimony IMPROVE and underlined M.
- One stated that he should value himself.
- One stated that he learnt that he needs to think of all laws and connect them when doing geometry.
- One stated that he learned that he needs to participate more in practising mathematics and do all his homework and assignments.

The above shows that the strategy of SDM questioning was well remembered and/or used. To ascertain learner perception of SDM question as a strategy for learning Euclidean geometry, learners were asked to complete the second part of part D and results presented in the preceding section of the report.

Table 4.12. Classification of responses

Response	Classification
Improve results	Positive
Does not improve results	Negative

All eleven respondents to the question evaluated it positively by stating that if all learners in school were exposed to it, mathematics results would improve significantly

From the participants' responses, the reasons given for the positive evaluation of SDM questioning were as follows:

- It makes one enjoy maths
- Helps to explore new methods
- Encourages one not to rush things
- Improves understanding –reveals ones’ strengths and weaknesses
- Helps recognise mistakes and correct them before it is too late
- Know what is required before answering questions
- Changes the negative attitude towards mathematics – gives encouragement
- Brings analysis and critical thinking
- Enables reasoning and makes understanding easy and fast
- Reveals where one lacks confidence
- Instils confidence and is a strategy for pro-action

In light of the responses given by the respondents, the technique of Self-Directed Metacognitive questioning was perceived as an aid to learning. Though it is not mathematics per se, it was received as a way of learning the subject matter and would improve results in the subject. All the above reasons fit into the indicators of confidence by Burton and Platts showing that SDM questioning had resulted in improved confidence and preparedness for Euclidean geometry tasks.

4.4 Interviews

After the post-test, some follow up interviews were done with seven learners. The interviews were video recorded and then transcribed. During the interviews, the researcher tried to ask all the interviewees similar questions. The questions were aimed at evaluating the effectiveness of SDM questioning as a learning strategy, its applicability in other topics and subjects and general perception on its role on mathematics results in school if it were to be practised by all learners. However IK was not available for interview.

All the interviewees were of the belief that SDM questioning can help improve mathematics results in school. This coincided with the conclusion reached at using responses to the questionnaire. As the interviewees gave their responses, they showed confidence in what they said. Most likely this was due to the fact that they were confident of the topic under discussion. In this regard, confidence was expressed through body language and strong eye contact with the interviewer. A strong eye contact is the single greatest indicator of confidence (Glass, 2012). Eye contact establishes a connection, shows sincerity, and helps to

create a sense of trust between people (Glass, 2012). As the researcher, based on eye contact during the interviews, respondents were sincere in their responses and this gives one reason to trust what they said.

4.4.1 Metacognitive knowledge

As already highlighted, metacognition involves metacognitive knowledge and metacognitive control or self-regulatory processes (Pintrich, 2002). The first aspect to look for in learner interview responses was the presence of metacognitive knowledge. Metacognitive knowledge involves knowledge of strategy, knowledge of task and knowledge of person variables (Flavell, 1979).

- Knowledge of person variables: includes one's knowledge of one's strengths and weaknesses.
- Knowledge of task: Involves such knowledge as knowledge that some tasks are more difficult than others
- Knowledge of strategy: knowledge of general strategies for learning, thinking, and problem solving

The above are the three components of metacognitive knowledge suggested by Flavell (1979). Learners' interview responses were analysed to establish how much they portray these components of metacognitive knowledge. The purpose of doing this analysing was to attempt to answer the first research question: What is the effect of the use of Self -Directed Metacognitive (SDM) questions on the confidence level and preparedness of learners in the learning of Euclidean geometry?

By so doing, the research sought to establish the presence of metacognitive knowledge in learners as this would influence development of academic confidence. The more learners become knowledgeable about cognition, the more they learn better (Bransford, Brown, & Cocking (1999). As explained above, the metacognitive components that were examined are person variables (self-awareness), knowledge of task, and strategic knowledge as explained above.

This research considers self-awareness as one's ability to identify one's weaknesses and strengths, likes, and dislikes as well as being able to notice changes in one's abilities. In the interview, K highlighted that he was weak in mathematics but he also mentioned that he is now better at the subject. He considered himself as a weak mathematics student. He reached this conclusion before he received metacognitive training. In his case, metacognitive training might only have strengthened or reversed the conclusion and or developed the other components of metacognitive knowledge. In addition, he confessed that after undergoing metacognitive training, his approach to content is more organised, structured, and efficient. Below is what he had to say:

K: The **permanent** thing that I remember is that when I **utilised it**, *my thinking with regards to the content is more structural* so I could develop an understanding of everything in *sequence and in context*, as opposed to, my time **before** I adapted to metacognitive training, I utilised an instinctive learning attitude. With that, I could grasp the content much *easier*. The *quality* of my comprehensive attitude towards the content **had changed**.

K used the term, permanent, and there was no direct follow up to find out if he really knew the weight of the term in his response. However, further into the interview, some of his remarks were consistent with permanency. For instance, he mentioned that he embraced metacognition as a general phenomenon with use that goes beyond academia and which he consciously uses in many spheres of life.

In his response, K showed that he was mindful of his inabilities before and his abilities after receiving metacognitive training. He went to the extent of claiming that the quality of his comprehensive attitude had changed. This way he demonstrated mindfulness of knowledge of himself. He also made a declarative statement to claim that metacognitive training helped him to become more prepared for mathematics tasks. In his claim, the preparedness was due to availability of a strategy (SDM) to deal with problem solving.

I used the above technique with all interviewees' responses and discovered that after the training, all seven demonstrated possession of metacognitive knowledge. It is the possession of this knowledge that would be expected to assist the learners in solving Euclidean geometry problems.

4.4.2 Interview results

The interview responses are first presented and analysed per question as tabulated below.

Table 4.13. Responses to what the training programme entailed

Q1 : What the training entailed	
K	• Thinking concerning content.
B	• Critical thinking • When given a question, try to criticise it before doing anything about it
D	• The acronym IMPROVE
M	• It shows me different kinds of shapes that can look similar and it brings a lot of formulae in the head
A	• We were given time to look at objects and then latter tried to remember them. • Oh, the acronym IMPROVE
KB	• The training helps when you get a question. • Ask about the question, list the data, know what to use, what theories and methods to use
AD	• You have to practise to get it perfect • Ask yourself question in order to understand the question

Learner responses differed with KB and AD concurring on asking questions whilst K and B remembered the training for its emphasis on thinking. Responses from these four learners have their similarity in that the thinking referred to by K and B resulted in questioning. As for D and A, they remembered IMPROVE of which during the training, in IMPROVE, emphasis was given to metacognitive questioning (M). That being the case, I concluded that though the learners presented their responses in different styles, they all pointed to questioning.

Table 4.14. Strategy learnt during the training

Q2: Main learning strategy learnt during the training	
K	• Questioning • I would construct questions relative to whatever content I have and then those questions would serve as a conduit to my capacity to understand it in a manner that is individualised
B	• find the main idea of what they want
D	• M. • Metacognitive questioning
M	• To know the formulas
A	• I think the major strategy is actually questioning
KB	• To ask questions about the question that you have
AD	• Self-questioning

All the seven learners agreed on questioning as the learning strategy that they mastered during their metacognitive training.

Table 4.15. Usefulness of SDM

Q3: Does SDM questioning assist learning of Euclidean geometry?	
K	• Yes. • it's very reflective • it should be something that is a part of curricula education
B	• Yes it does
D	• Yes sir, I did, because whenever I question myself. • Maybe to tell whether or not it makes sense.
M	• Yes it does and it helped me get an improved mark in the test
A	• For me it did help me because it managed to help me improve my marks for the term when I used the strategy
KB	• Yes. Its productive
AD	• Very much.

Results in the table clearly show that the interviewed learners agreed that self-directed metacognitive questioning assists in the learning of Euclidean geometry.

Table 4.16. Use of SDM in other topics and subjects

Q4: Use of SDM questioning in other topics and subjects	
K	• I used it in many arenas. It is something that I adapted to quickly. • I did not really assimilate it with reference to mathematics only. • It was something that was very general.
B	• Yes, it can have an influence with the results in maths • So it's not really for maths only
D	• Yes sir, I used it in Trigonometry and analytical geometry • In other subjects, yes sir
M	• I used it in all subjects sir
A	• Yah, it's possible to teach it
KB	• Yes sir, it would be a good idea. It would improve the marks
AD	• It works in all topics and all subjects

There is a general agreement in the responses. To start with, the learners felt that use of SDM questioning can help improve results in mathematics in their school. Not only did they think of mathematics results but results in other subjects as well. All seven conceded that they had used SDM questioning in other subjects and it yielded results.

More than just remembering that *questioning* was the main strategy of the metacognitive training; the interviewees rated the strategy highly. Below is an extract of the researcher's conversation with one of the learners, K.

4.4.2.1 Conversation with K

When asked what the training regarded as the main strategy for learning, this is what K had to say.

K: Questioning. Every time I had received content, I would have, on the premises of metacognitive training I would construct questions relative to whatever content I have and then those questions would also to some degree serve as a conduit to my capacity to understand it in a manner that is individualised. It's much more personal for me to understand it that way so it's easier to apply.

Teacher (T): It's easier to apply

K: Yes sir

T: Ok, so generally what do you say about that strategy? Does it work or it doesn't work, what's your take on it?

During the interview with learner K, his opinion about whether self-questioning works or not, this is what he had to say:

K: Well, I think it's very reflective. I think it should be something that is a part of curricula education because if you are going to have a subject like mathematics that is based on reasoning, it is sensible to have a component of the curriculum that perpetuates the degree of reasoning.

From K's point of view, teaching of SDM questioning should be incorporated in curricula education. He viewed it so positively that he wished it could be taught to all school children. In his view, metacognitive training helped him beyond, Euclidean geometry, beyond mathematics and even beyond school.

K: I used it in many arenas. It's something that I adapted to quickly. I didn't really assimilate it with reference to mathematics only. It was something that was very general. It was very psychological even with my understanding of things beyond academia. It played some role. It really has to do with cognitive capacity. I think that was the pivotal element it affected.

K was of the opinion that, metacognitive training helped him develop a sense of individual learning, a sense to questioning things and an inquisitive attitude in learning. He believed that these attributes help improve understanding. In his words...*the primary element to the issues in terms of education are based on cognition*. He picked out cognition as an element that has to be addressed by education.

To address the said element, one must have some control over cognition. That control is derived from metacognition which the research views as a key factor in teaching and learning mathematics. Based on this, metacognitive training should be considered as part of what learners should be taught in school.

4.4.2.2 Conversation with D

One of the respondents, D, viewed SDM questioning as a self-regulatory technique. He believed that when one arrives at an answer in mathematics, that answer needs to be critiqued for correctness. This critique must be done by the author of the working and the answer should make sense or else it is likely to be wrong:

D: It will be very useful. It will be helpful because, if the question doesn't, the answer, doesn't make sense, the answer you get doesn't make sense to you, then it can probably be wrong. It has to make sense. You have to question yourself. Go over it again and then see, try to reason with it.

From what D said, SDM questioning instilled a sense of self-instruction. By its own nature, self-instruction (self-directed verbalisations) is a prerequisite for qualified learning. “Verbalisations improve learning and are crucial for self-regulation” (Schunk, 1982). In this regard, D claimed that through SDM, he had acquired qualified learning.

All the interviewed learners showed great appreciation of the metacognitive training and the SDM questioning techniques. A (learner) pointed out that self-questioning has become part of his learning aid to the point that he now uses it consciously.

T: In your case, did you ever use it in other subjects?

A: I realise that I did. Actually I do.

T: Which subjects?

A: My Geography, a bit of Life Sciences.

T: And was there an improvement in marks?

A: Big time

By pointing out that he *realised* that he is using SDM questioning in other subjects was an indication that he did it consciously.

All the seven interviewees showed confidence in the use of SDM questioning and its applicability and usefulness for learning.

4.5 Summary

Research data collected by means of the suggested research instruments were analysed and results used to answer the research questions. The first question related to learner confidence and the second to learner performance in Euclidean geometry. The research established that use of SDM questioning enhanced learner confidence and preparedness. This came out from results of the questionnaire and the second interview. In the questionnaire, positive evaluations surpassed negative results. According to the criterion used in this research, positive evaluations pointed to presence of confidence and preparedness whilst negative evaluations revealed otherwise.

An SPSS application was used for statistical computations. These served the purpose of comparing the means of the experiment and the control group. Computation results revealed that, at the start of the research, the two groups had equivalent performance. After the intervention, the experiment group showed more competency than its counterpart. From this I concluded that learners who were exposed to the metacognitive strategy of self-directed metacognitive questioning outperformed their counterparts in the post test. SDM questioning worked as an aid to learning and solving of Euclidean geometry problems by grade 11 learners. In addition, it helped boost their confidence in solving problems.

Comparisons and conclusions were made in this research by means of tables, calculations, graphs, and narrations. A combination of these presentation techniques helped gain a deeper understanding of the phenomenon of SDM questioning as an aid for solving Euclidean geometry problems by grade 11 learners. Data presentation methods used here were useful due to triangulation of data collection tools in this research.

CHAPTER 5: CONCLUSIONS, LIMITATIONS AND RECOMMENDATIONS.

In this chapter conclusions derived from the research, limitations of the research and recommendations are presented.

5.1 Conclusions

This research aimed at exploring the effectiveness of self-directed metacognitive questioning on learner performance during solving of Euclidean geometry problems. This was done by means of an intervention which aimed at improving learner confidence and preparedness for Euclidean geometry tasks. This preparedness would culminate into improved performance in Euclidean geometry. To check the effect of the intervention on performance, a pre-test and a post-test were given to the experiment and the control group. As was revealed by the results of the pre-test, the two groups were equivalent in performance before the study.

After the intervention, a post-test was administered and results thereof compared using independent t- test in SPSS. The means of the two groups showed that the performance of the groups had improved but that of the experiment group had experienced a higher improvement than that of the control group. This signified that the intervention had resulted in improved performance in problem solving in the experiment group. The difference in the means was significant enough to warrant inferring that the experiment group had performed better than the control group. In addition to there being a significant difference between the means, the magnitude of the effect of the intervention had a substantial effect on the results, $r = 0.232$. Using this comparison it can be concluded that the intervention had played a role in the improved performance of the participants to whom it was introduced. Students who experienced metacognitive training (emphasising Self-Directed Metacognitive questioning) performed better than their counterparts did. Metacognitive skills helped learners to perform better in problem solving. Metacognition is a skill that can promote being successful in problem solving (Özsoy & Ataman, 2009).

Metacognitive skills were imparted to the learners in a way that was visible and practical for the learners. This was one of the ways proposed by Pintrich (2002). Metacognitive training was conducted with them after school hours and they were all aware that they were not doing

mathematics, instead, a training that was intended to aid their learning. Results obtained in this research demonstrated that metacognitive skills could successfully be given to learners in an explicit way. More than just the explicit training, the skills helped to improve learners' problem solving in Euclidean geometry. This result is consistent with the results of earlier investigations showing that achievement in mathematics can be raised through instruction enriched with metacognitive activity (Jacobse & Harskamp, 2009; Kapa, 2002; Özsoy & Ataman, 2009; Panaoura, Gagatsis, & Demetriou, 2009).

A review of the literature suggests that attempts to improve mathematics results through metacognitive skills usually take one of two forms: explicit or implicit training. However, because of the time constraints in schools and the need to move through the mathematics curriculum, most mathematics educators have preferred to embed metacognitive training in course content (Kincannon, Gleber, & Kim, 1999), through cooperative learning (Kramarski & Mevarech, 2003), computer assisted instruction (Jacobse & Harskamp, 2009), web pages (Panaoura, Gagatsis, & Demetriou, 2009), and homework (Bembenutty, 2009; Jacobse & Harskamp, 2009; Zimmerman & Kitsantas, 2005). All of these studies succeeded in raising students' mathematics achievement.

What differentiates this research from the rest is its explicit approach to metacognitive training. The training was done after school hours just for a specified time and after that, it was discontinued and learners were allowed to implement the dictates of the training in their lessons. Though conducting the training was, time consuming, as already indicated in other research, doing it this way, made learners accept the skills so gained as general and useful in the whole curriculum. It also helped learners realise that learning required mastery of concepts as well as mastery of strategies for the learning itself.

In addition to problem solving skills, learner confidence and preparedness for tasks in Euclidean geometry also experienced a remarkable improvement due to the intervention. After the intervention (metacognitive training), most participants showed confidence in themselves and were prepared for tasks in Euclidean geometry. This was revealed in their responses to a questionnaire and their responses to follow up interview questions. The acceptance of the strategy of Self-Directed Metacognitive questioning was encouraging. Learners claimed that they used it in other topics in mathematics and in other school subjects. It was interesting to note that other participants evaluated it as a general learning tool that must be taught to all learners by incorporating it in the curriculum. Equipping learners with

metacognitive skills is equipping them with learning tools that make them better doers of mathematics. SDM questioning is a quality that is important in mathematical problem solving.

5.2 Limitations

The fact that this research was conducted in an all-boys school poses some challenges to the results. It is not known what the results could have been had the training been conducted with girls only or a group that consisted of both boys and girls.

5.3 Recommendations

I recommend that subsequent research be carried out with both boys and girls and also that the training programme be longer. Teachers need to be invited to such programmes.

I further recommend that department of education officials responsible for curriculum must be made aware of the role that metacognitive skills play in learner academic achievement especially in mathematics. This can be done through inviting them to participate in research that targets metacognitive skills development as a teaching and learning aid.

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Annexure A: Questionnaire

PART A: [Motivated Strategies for Learning Questionnaire]

Please rate yourself on your *attitude* to each of the situations below. Your rating in each item can be any number from 1 to 7 where 1 means “not at all true of me” and 7 means “always true of me.”

1	2	3	4	5	6	7
Not true at all	Rarely true	A little true	Sometimes true	Often true	Very often true	Always true

1.	I prefer class work that is challenging so I can learn new things.	
2.	Compared with other students in this class I expect to do well.	
3.	I am certain I can understand the theorems and concepts taught in Euclidean Geometry.	
4.	I think I will be able to use what I learnt during the training in other topics/subjects.	
5.	I am sure I can do an excellent job on the problems and tasks assigned for Euclidean Geometry.	
6.	I think I will receive a good mark in the coming test in Euclidean Geometry.	
7.	Even when I do poorly on the test I will try to learn from my mistakes in preparation for the examination.	
8.	Compared with other students in this class I think I know a great deal about Euclidean Geometry.	
9.	When I take a test I think about how poorly I am doing.	

10.	When I study I put important ideas into my own words.	
11.	When I study for the test I try to remember as many Theorems as I can.	
12.	When I study for the test I practice saying the important theorems over and over to myself.	
13.	Before I begin studying I think about the tasks I will need to do to learn.	
14.	I use what I have learned from old homework assignments and the textbook to do new assignments.	
15.	When I am studying Euclidean Geometry I try to make connections between all Geometry concepts and theorems that I know.	
16.	When studying, I try to connect the things I am studying about with what I already know.	

PART B: [Metacognition Questionnaire- MCQ]

Circle Y or N for Yes/No to the following cognitive self-consciousness questions.

1.	Whenever I work through a geometry question, I constantly examine my thoughts.	Y	N
2.	I pay close attention to the way my mind works.	Y	N
3.	As I work, I monitor my thoughts.	Y	N
4.	I am constantly aware of my thinking.	Y	N
5.	I am aware of the way my mind works when I am thinking through a problem.	Y	N
6.	I think a lot about my thoughts.	Y	N

PART C: [Awareness of Independent Learning Inventory]**Rate how true each statement is for you on a scale of 1 to 7.**

1	2	3	4	5	6	7
Not true at all	Rarely true	A little true	Sometimes true	Often true	Very often true	Always true

1.	I don't think it is important to feel personally involved in what I am studying.	
2.	I ignore feedback from tutors on my method of work.	
3.	While working on an assignment I pay attention to whether I am carrying out all parts of it.	
4.	While working on an assignment I keep a record of my learning aims.	
5.	When I've finished an assignment/test/examination, I don't revise my work to check for mistakes and correctness of my answers.	
6.	Sometimes while working together with others on an assignment I get a sudden feeling that I'm learning a great deal from them.	
7.	When I've worked together with others on an assignment I don't think about whether the co-operation was useful for me.	
8.	When I work together with others I regularly think about what I learn from them.	
9.	I see no reason to talk with others about the usefulness of working together on our studies.	
10.	If I find information difficult to understand I don't try to find a deeper reason for not understanding it.	

PART D: [Evaluating use of SDM questioning]

1.	What learning strategy or strategies did you learn from the metacognitive training?
2.	If all learners in your school can use Self Directed Metacognitive (SDM) questioning as a learning strategy, how do you think it can improve mathematics results in the school?

Thank you for your co-operation and participation in answering this questionnaire.

Annexure B: Mathematics Pre- Test

QUESTION PAPER

GRADE 11

MATHEMATICS- GEOMETRY

MARKS : 25

TIME : 30 minutes

This question paper consists of 3 questions.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

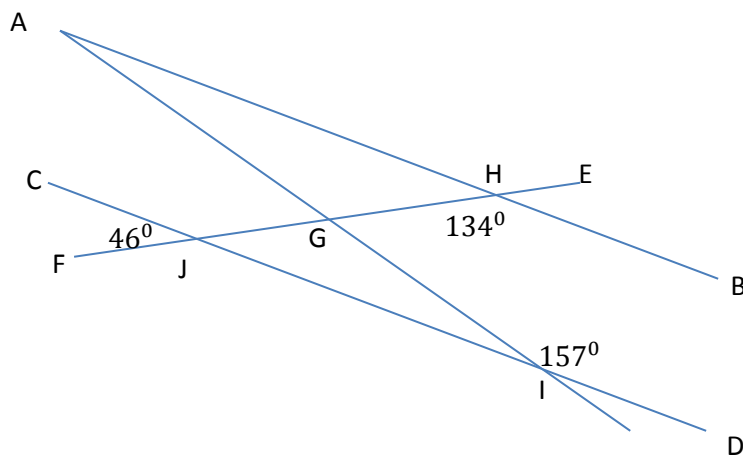
1. This question paper consists of 3 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessary drawn to scale.
9. Write neatly and legibly.

Question 1[9 marks]

Complete the following statements

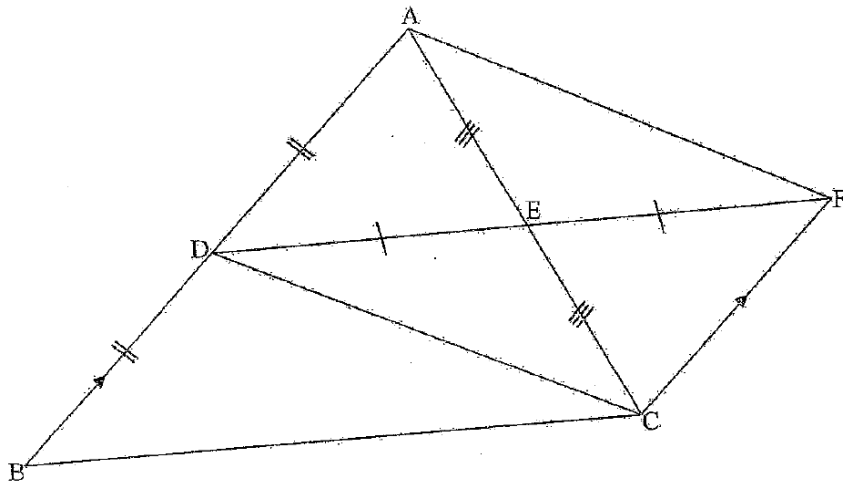
- 1.1.1 Alternate angles are..... (1)
- 1.1.2 A kite is a quadrilateral with two pairs of adjacent sides (1)
- 1.1.3 A square is a rhombus with aangle (1)
- 1.1.4 RHS is a case to prove of triangles (1)
- 1.1.5 Supplementary angles add up to..... (1)

1.2 Prove that line segments AB and CD are parallel to each other. Give reasons for your calculations. (4)



Question 2[10 marks]

In the diagram below, D is the midpoint of side AB of $\triangle ABC$. E is the midpoint of AC. DE is produced to F such that $DE=EF$. $CF \parallel BA$.

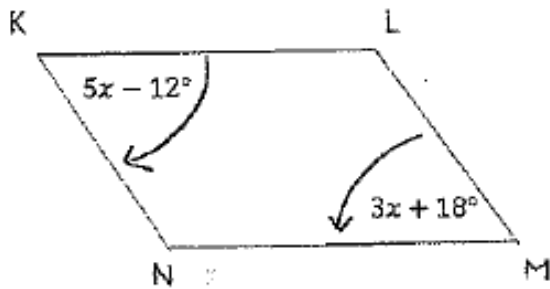


2.1.1 Write down a reason why $\triangle ADE \cong \triangle CFE$. (1)

2.1.2 Write down a reason why DBCF is a parallelogram. (1)

2.1.3 Hence, prove the theorem which states that $DE = \frac{1}{2} BC$. (2)

2.2 In the diagram below, KLMN is a parallelogram. Angles at K and M are given.



2.2.1 Calculate the value of x (3)

2.2.2 Hence calculate the size of angle at L (3)

[10]

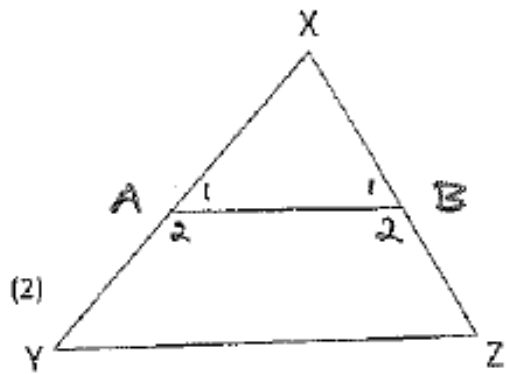
Question 3[6 marks]

In the diagram below, $\hat{A}=50^\circ$ and $\hat{Y} = 50^\circ$

3.1 prove that $AB \parallel YZ$ (1)

3.2 Show that $\triangle XAB \sim \triangle XYZ$ (3)

3.3 Hence, if $XB=2.5$; $XZ =7.5$ and $YZ =5.4$, calculate AB (2)



Total 25 marks

Annexure C: Mathematics Post Test

QUESTION PAPER

GRADE 11

MATHEMATICS- EUCLIDEAN GEOMETRY

MARKS : 40

TIME : 1 hour

INSTRUCTIONS AND INFORMATION TO CANDIDATES

Read the following instructions carefully before answering the questions.

10. This question paper consists of 2 questions.
11. Answer BOTH the questions.
12. Number the answers correctly according to the numbering system used in this question paper.
13. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
14. Answers only will not necessarily be awarded full marks.
15. FIVE diagram sheets are attached at the end of this question paper. Write your name on these sheets in the spaces provided and insert them in your ANSWER SHEET.
16. Write as much Information as possible on the diagram sheets to explain and justify your steps towards your answers.
17. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
18. If necessary, round off answers to TWO decimal places, unless stated otherwise.
19. Diagrams are NOT necessary drawn to scale.
20. Write neatly and legibly.

QUESTION 1[22 marks]

1. 1 Complete the following statements:

1.1.1 The opposite angles of a cyclic quadrilateral are (1)

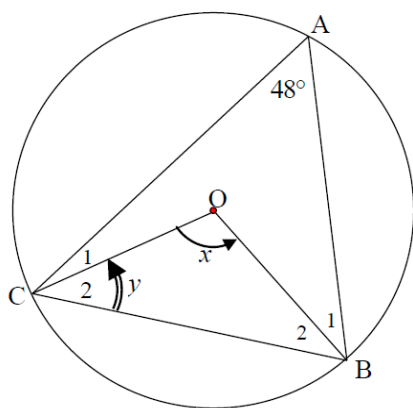
1.1.2 Equal chords subtend..... (1)

1.1.3 The angle between a tangent and a chord are..... (1)

1.1.4 The angle subtended by a chord at the centre of a circle is.....the angle subtended by the same chord at the circumference of the circle. (1)

1.1.5 Tangents to a circle from the same point are..... (1)

1.2 In the diagram, O is the centre of the circle passing through A, B and C. $\hat{CAB} = 48^\circ$, $\hat{COB} = x$ and $\hat{C}_2 = y$

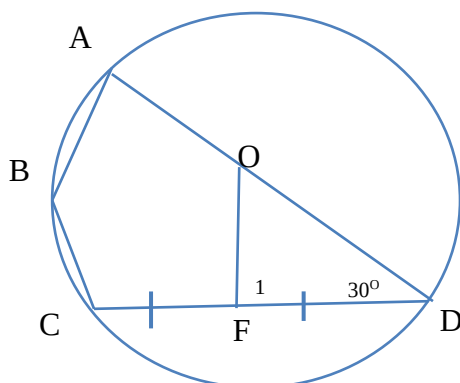


Determine, with reasons, the size of:

1.2.1 x (2)

1.2.2 y (3)

1.3 In the diagram, O is the centre of the circle passing through A, B, C and D. AOD is a straight line and F is the midpoint of chord CD. Angle ODF = 30° and OF are joined.

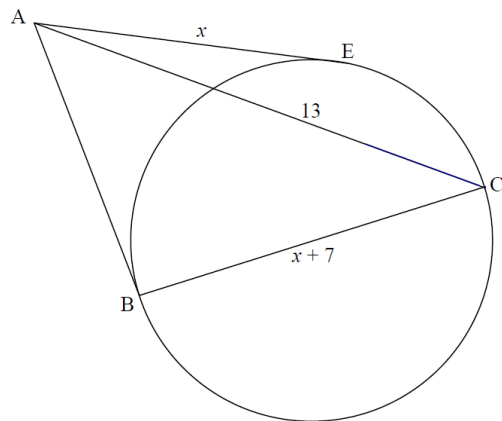


Determine, with reasons, the size of:

1.3.1 F_1 (2)

1.3.2 $\hat{A}BC$ (3)

1.4 In the diagram, AB and AE are tangents to the circle at B and E respectively. BC is a diameter of the circle. $AC = 13$, $AE = x$ and $BC = x + 7$.



Give reasons for the statements below.

1.4.1 Complete the table on DIAGRAM SHEET 3

	Statement	Reason
(a)	$\angle ABC = 90^\circ$	
(b)	$AB = x$	

(3)

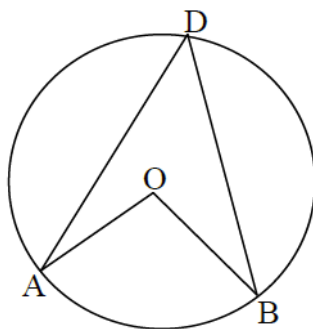
1.4.2 Calculate the length of AB. (4)

[22]

QUESTION 2[18 marks]

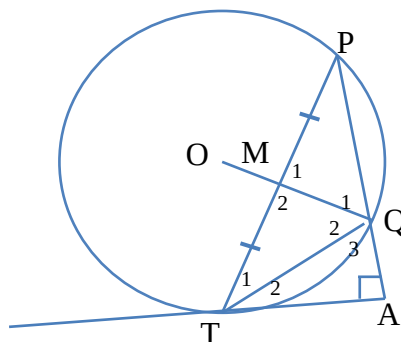
2.1 In the diagram, O is the centre of the circle and A, B and D are points on the circle. Use Euclidean geometry methods to prove the theorem which states that

$$\hat{AOB} = 2\hat{ADB}.$$



(6)

2.2 M is the midpoint of chord PT of a circle with centre O. OQ is a radius passing through M. PQ is produced to intersect tangent TA at A, such that $TA \perp PA$.



Prove that

2.2.1 MTAQ is a cyclic quadrilateral

(4)

2.2.2 $PQ = TQ$

(4)

2.2.3 $\hat{T}_1 = \hat{T}_2$

(4)

[18]

TOTAL 40 MARKS

DIAGRAM SHEET 1

LEARNER'S NAME & SURNAME:

QUESTION 1.2 When answering this question, justify all your steps by writing as much information as possible to explain how you obtained the answers.

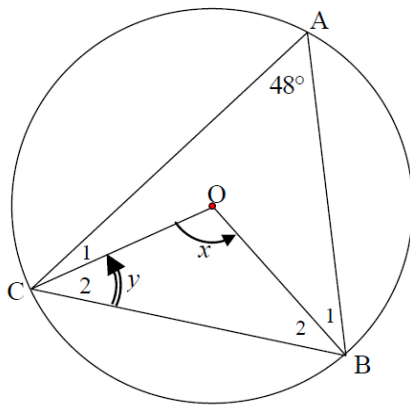


DIAGRAM SHEET 2

LEARNER'S NAME & SURNAME:

QUESTION 1.3 When answering this question, justify all your steps by writing as much information as possible to explain how you obtained the answers.

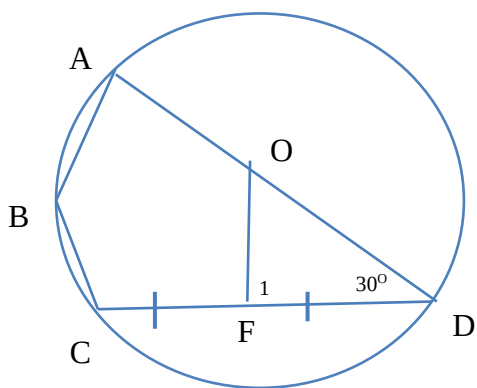
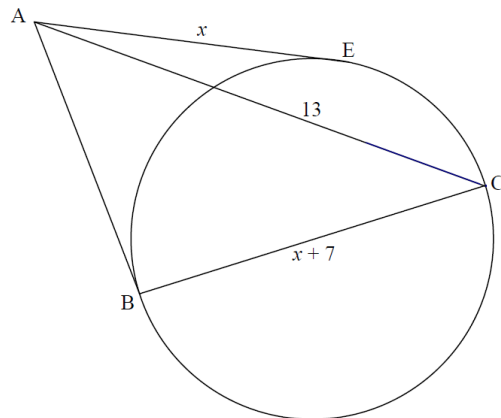


DIAGRAM SHEET 3

LEARNER'S NAME & SURNAME:

QUESTION 1.4 When answering this question, justify all your steps by writing as much information as possible to explain how you obtained the answers.



	Statement	Reason
(a)	$\angle ABC = 90^\circ$	
(b)	$AB = x$	

DIAGRAM SHEET 4

LEARNER'S NAME & SURNAME:

QUESTION 2.1 When answering this question, justify all your steps by writing as much information as possible to explain how you obtained the answers.

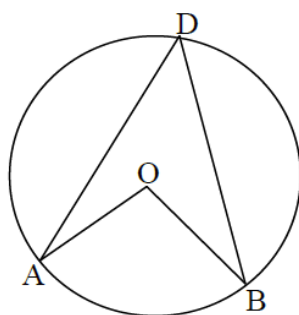
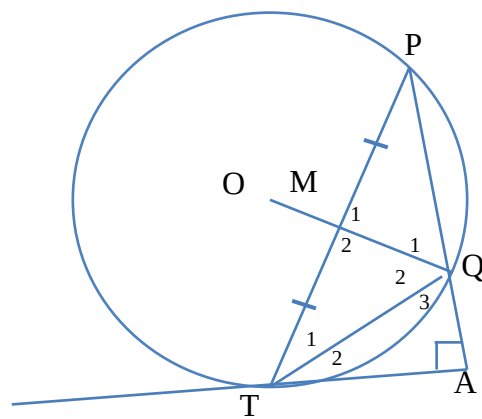


DIAGRAM SHEET 5

LEARNER'S NAME & SURNAME

--

QUESTION 2.2 When answering this question, justify all your steps by writing as much information as possible to explain how you obtained the answers.



Annexure D: Metacognition and Soliloquy Score sheet

	Task 1	Task 2	Task 3 Audio 1	Task 4 Audio 2	Task 5	Task 6	Task 7	Task 8
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								

Score								

Annexure E: Interview Responses (Post -Test)

Interviewee 1(K)

T: Welcome K to this short interview. There are few things that I would like to learn from you about metacognitive training that we did in class. One of the main reasons why I chose you for this interview, I am just looking at your two marks in Euclidean geometry. The first test was 28% and the second test after metacognitive training was 60%. If I look at these two marks they are quite, there is very big difference between them, so that's why I called you so that I can find out from you.

K: The permanent thing that I remember is that when I utilised it, my thinking with regards to the content is more structural so I could develop an understanding of everything in sequence and in context, as opposed to, my time before I adapted to metacognitive training, I utilised an instinctive learning attitude. With that, I could grasp the content much more easier. The quality of my comprehensive attitude towards the content had changed

T: It had changed

K: It had changed

T: Significantly

K: Yes sir

T: What was the main strategy that was there in that training?

K: Questioning

T: Questioning

K: Questioning. Every time I had received content, I would have, on the premises of metacognitive training I would construct questions relative to whatever content I have and then those questions would also to some degree serve as a conduit to my capacity to understand it in a manner that is individualised. It's much more personal for me to understand it that way so it's easier to apply.

T: It's easier to apply

K: Yes sir

T: Ok so generally what do you say about that strategy. Does it work, it doesn't work, what's your take on it?

K: Well, I think it's very reflective. I think it should be something that is a part of curricula education because if you are going to have a subject like mathematics that is based on reasoning, it is sensible to have a component of the curriculum that perpetuates the degree of reasoning.

T: Umm, ok, so you think it must be incorporated in the curriculum

K: Incorporated into all LOs, some, it should be curricula.

T: Ok, so in your case, I remember we used it in Euclidean geometry. Did you try to use it somewhere else outside Euclidean geometry?

K: I used it in many arenas. It's something that I adapted to quickly. I didn't really assimilate it with reference to mathematics only. It was something that was very general. It was very psychological even with my understanding of things beyond academia. It played some role. It really has to do with cognitive capacity. I think that was the pivotal element it affected.

T: So do you think it can be helpful if it is introduced to everyone in school.

K: Yahh definitely, if it can help me especially having considered that I was very flaccid with mathematics, having that kind of, ehh that kind of education in curricula would definitely improve chances of everybody having it because I do think that the primary element to the issues in terms of education are based on cognition. It has to do with the comprehensibility per learner. So if everybody had the sense of an individual learning process, the sense to question things, an inquisitive attitude they would be more likely to assimilate whatever content they are given and be able to utilise that data and morph the data in a way that is corresponsive to their dimina or their manner of thinking so I think it really allows for versatility among the classes. It should be for everybody.

T: Thank you very much K. Thank you for your time.

Interviewee 2(B)

T: Ok B

B: Yes

T: Mmm, I am just. Ok or first thing I want to welcome you to this brief interview. I want to find out your opinion about uh, metacognitive training that we did in class.

B: Yes

T: And one of the reasons why I chose you is, looking at your marks. The mark for the first test on Euclidean geometry was 48% and the mark for the second test was 85% see, which shows there was a very big improvement here. 48% was before the training and 85% was after the training.

B: After the training

T: So can I find out from you. Do you still remember what the training was all about? The metacognitive training. What was it all about?

B: Well sir, it was about ummh, uhh, critical thinking, if I may say

T: Critical thinking

B: Yes uhh, when given a question, try to criticise it first before you further do anything else. Like thinking about what is being asked and uhh thinking about what they want.

T: Mmm

B: And then finally then you can answer a question with given items, like if they want this, then this is related to that you also have.

T: So in that case, what was the main strategy? What is the main strategy that we use here for the critical analysis?

B: The main strategy was to umm, find the main idea of what they want. For what is being asked.

T: Finding the main idea of what is being asked?

B: Yes

T: Ok, so what do you say about the strategy in general?

B: well, the strategy in general, it's uh, it's comprehensive. It's easy to understand and seemingly its practical, when you are asked the question you can't just blip out the answer. You need to think about it first. Then you can answer uh, critically or if I may say or answer uh in such a way that is you understand what is being asked first. Before you can just answer first.

T: Okay, so you remember we were also talking about metacognitive questioning

B: Yes

T: Self- directed questioning, asking yourself questions. That is what you are referring to?

B: Yes

T: Okay, do you think it can work for everyone in school. Suppose we think of introducing it to everyone in school, do you think it can improve the results in maths?

B: Yes, it can have an influence with the results in maths. Uhm because if we teach or try to implement in such an idea to most, because most students will just look like, if it's a maths, if they say $2 + 2$ is four, but you didn't think what if they say $2 + 4$ and then the 2 is in brackets or what so ever. You just looking at the general idea, you don't know the other things behind it, so I think if maybe we teach people and implement such an idea, maybe it will give them the sense of wanting to know more, like if I am being asked something but what else is there within that something. So I think if maybe we introduce especially with maths. With maths you can say I know factorising but then you leave behind some things just like me sometimes I forget how to factorise, but when at the end of the day if I give myself time and try to look at that thing I factorise it better.

T: Umm

B: Then I think everyone else can at least improve on how they deal with mathematics, uh with the mathematical questions.

T: Ok, so besides in maths, do you think it can be applied to other subjects.

B: Yes, somehow, although the other subjects are theoretical. With maths, it's practical; you do write numbers and stuff. With other subjects its theoretical, like if I am being asked uh, what is climatology

T: Ummm

B: I will just go, it's the study of weather, uh weather patterns, but then climatology it's the study and it's still a practise so if I break it further, it talks about climate but then the methodology behind it

T: Umm

B: So there are two things involved, so now that I have applied critical thinking and asking myself about the question which is being asked. I think in it, other subjects it tends, it works also.

T: It can work also.

B: So it's not really partially for maths only,

T: for maths only?

B: Yeah

T: Thank you very much B.

B: Thank you sir

Interviewee 3(D)

T: Ok, thank you D for coming. Uhm, I would like to ask you some very few questions

Nods his head

Maybe it would take 2 or 3 minutes

D: Yes sir

T: It is in connection with metacognitive training. Do you still remember the training that we did in class?

D: Yes sir

T: and the strategies that we used in our Euclidean geometry. What do you still remember about the training?

D: Uhm, the acronym IMPROVE. We spoke about IMPROVE which uhm and you told us what it stands for.

T: No, if you don't mind, you can say if you still remember or whatever you still remember about it.

D: Ok, uhm, you told us that, uhm, Ok a few, a few of them.

T: Mmm

D: M was for metacognitive questioning whereby you have to question yourself when studying, practising Euclidean geometry. You have to question yourself so that you can know. You must be sure of what you are writing.

T: Mmm

D: P, you practise geometry

T: Mmm

D: Uhm, the R, you revise

T: Mmm

D: O, I am not sure what it was

T: Ummm

D: And then the rest yeah, that's about it.

T: Ok, but the otherwise the main one was the M, Metacognitive questioning?

D: Yes Sir (nodding his head)

T: That was the main strategy of the training, neah?

D: Yes sir

T: Did you find it helpful?

D: Yes sir, I did, because whenever I question myself. Maybe to tell whether or not it makes sense.

T: Mmm, oh, ok, so generally what do you say about the strategy of metacognitive questioning?

D: It's a good strategy sir, you see if people can implement it when studying

T: Ummm

D: It will be very useful. It will be helpful because, if the question doesn't, the answer, doesn't make sense, the answer you get doesn't make sense to you, then it can probably be wrong. It has to make sense. You have to question yourself. Go over it again and then see, try to reason with it.

T: Ok, we used the strategy of self-questioning in Euclidean geometry, so in your case did you ever try to use it in other chapters in maths, besides Euclidean geometry?

D: Yes sir, I used it in Trigonometry and analytical geometry

T: Ohh, so you used it in other chapters?

D: Ummm

T: Ok, what do you think if someone comes and then say that maybe it is a strategy that should be taught to everyone in school. Do you think that it can improve the results in maths?

D: Yes sir, it can, because just like I said, if everyone can be sure of what they are writing. Then they are probably correct and the only way to be sure of what you are writing is by questioning yourself.

T: Ok, do you think it can work in other subjects besides maths?

D: In other subjects, yes sir

T: Did you ever try to use it:

D: I do, I used it for physics

T: Ummm

D: And it worked

T: It worked for you in physics?

D: Yes for physics

T: Ok, thank you Daniel, thank you very much. Thank you for your time.

Interviewee 4(M)

T: Welcome M and thank you for coming for this brief interview.

M: Thank you sir

T: I would like to ask you some few questions about the metacognitive training that we did in class and the strategies that we were using in our mathematics lessons of Euclidean geometry. One of the reasons that I chose you is that looking at your two marks for Euclidean Geometry

M: Yes sir

T: The first one before the training, it was 40%, and then the second one after the training was 68.5. I am sure there was an improvement here. So can you tell me, what do you still remember about that metacognitive training?

M: Sir, it shows me different kinds of shapes that can look similar and a lot of, it brings a lot of formulas into head due to the exams.

T: Ok, what was the main strategy that you used during the.....

M: I used.

M: I used, usually I used more shapes because they are easy to capture sir

T: Ummm

M: and then formulas, I just had to know the formulas like form the shape. I used, I have to know all formulas

T: Ummm

M: Like square only, I always used for a triangle I used the opposite sides

T: Ummm

M: To know the formulas

T: Ummm

M: Yes sir

T: Ok, do you still remember the strategy of self-questioning. Asking yourself questions

Mlungisi nods his head

T: How often did you use it during the second test?

M: Uhm sir, I used it when I saw a lot of complex things in the paper.

T: Uhm

M: That's when I used to ask myself questions

T; Uhm

M: Yes sir

T: What do you think about that strategy, do you think it helped you or it doesn't help you?

M: It does sir

T: It does help you

M: Yes sir

T: Do you think it is part of the reason why your mark improved?

M: Yes sir

T: So in the event that someone comes with a suggestion that it must be taught to everyone in school, do you think it can help improve results in maths.

M: Yes sir, the more they ask themselves questions, the more they get more, they get more knowledge towards themselves

T: Ok

M: Yes sir

T: So you think we can introduce it to everyone in school

M: Yes sir

T: Then are you using it in other subjects or you only use it in maths?

M: I use it due to all subjects sir

T: Oh, in all subjects

M: Yes sir

T: And is it working

M: Yes Sir

T: Have you seen some improvement?

M: Yes sir, physics, I improved, I improved my mark

T: Because of self-questioning?

M: Yes sir

T: Ok thank you very much

M: Yes sir

T: Thanks a lot for your time

M: Yes sir

T: Thank you

Interviewee 5(A)

T: Ok, thank you for coming

A nods his head

T: I would like to ask you some very few questions. And the questions relate to the metacognitive training. Do you still remember the metacognitive training that we did in class?

A: yes sir I do

T: And the related strategies that we were using in our Euclidean geometry. What do you still remember about that metacognitive training? What do you still remember from it?

A: Uhm, the time, uhm we were working through the projector

T: Uhm

A: I remember we were to, we were given some certain amount of time to check what's on the projector and then afterwards we tried to remember what was on it.

T: Uhm, no I am not referring to that one. I am referring to the training after that, the training and the strategies that we were using in Euclidean geometry after that

A: Oh, the IMPROVE

T: Exactly, now you remember it?

A: Yaah I know it. Actually I know it

T: Okay, ohh actually you know it.

A: I thought you were referring to....

T: What was the major strategy in that IMPROVE

A: I think the major strategy is actually questioning, why

T: It's questioning?

A: Yah

T: Ok, did it help you.

A: For me it did help me because it managed to help me improve my marks for the term when I used the strategy

T: Oh, generally your term mark improved.

A: It managed to improve though it was not convincing but it managed to raise the mark through the strategy

T: ok, what about your general understanding in Euclidean geometry, how is it now?

A: Yah, my understanding is good, it's only just some problems there and there, but every time I struggle, I use that method. It helps me.

T: It helps you

A: Yah

T: have you ever tried to use it in other topics not necessarily Euclidean geometry?

A: yah, I did try, all the topics, financial, financial maths and everything. It helps

T: Do you think it is a strategy that we can use in school in general for everyone who is doing maths? Can we teach it to everyone?

A: Yaah, it's possible to teach it.

T: You think it can help us

A; because it's not hard to learn for all the strategies.

T: In your case, did you ever use it in other subjects?

A: I realise that I did. Actually I do.

T: Which subjects?

A: My Geography, a bit of Life Sciences.

T: And was there an improvement in marks?

A: Big time

T: big time.

A: Yaah

T: Thank you A. Thank you very much for your time.

Interviewee 6(KB)

T: Thank you K for coming

KB: Yes sir

T: Uhm, I am saying thank you for coming to this short interview.

KB: Ok, thank you sir

T: I don't think it will take long, maybe 2 or 3 minutes we will be done.

KB: Alright sir

T: Ok, I would like to find out from you about the metacognitive training that did in class, and the reason why I chose you is that Uh, I am looking at your two marks for Euclidean geometry.

KB: Yes sir

T: The first mark before the training was 48% and the second mark for the test after the training was 58%

KB: Ummm sir

T: There was a positive movement here. There was an improvement. That's one of the reasons why I chose you.

KB: Ok sir

T: Ok, right, can I find out from you. What do you still remember about the training?

KB: The training helps you in thing, when you get a question.

T: Uhm

KB: To ask questions about the question that you have. List the data that you have

T: Uhm.

KB: And after listing the data, you can then uh, know what to use after then, what theories to use and what mathematical methods to apply.

T: Ok, so you are saying the main strategy there was about asking yourself questions.

KB: Yes

T: Ok, so do you think it helped you during the second test?

KB: Yes it did because I then asked myself questions about the question that I had. Then listed the data and then applied my mathematical skills.

T: Ok so generally what do you think about the strategy. Does it work or it doesn't work?

KB: Yes it does work. It's productive in fact.

T: It helped you

KB: Yes sir

T: So do you think it can be a good idea if we can introduce it to everyone in school.

KB: Yes sir, it would be a good idea. It would improve the marks.

T: It would improve the marks in maths

KB: Yes sir

T: And in your case, we used it in Euclidean geometry. Have you ever tried to use it outside Euclidean geometry like maybe in other chapters in maths?

KB: Yes I used it in physics, during my physics test.

T: Oh, in other subjects as well

KB: Yes sir

T: Did it help you

Interviewee 7(AD)

T: Ehh, welcome AD

AD: Thank you sir

T: Ummm, I just want to ask you some few questions for about 2 or 3 minutes or even less.

AD: No problem sir

T: These are related to metacognitive training. You still remember the training, metacognitive training that we did in class.

AD: The one for practising, asking questions.

T: Yes

AD: Yea I know it

T: Right, what do you still remember about that?

AD: That, uh, you have to practise to make it more perfect. You have to ask your questions, yourself questions in order to understand the question.

T: Ok, did it work for you?

AD: Yea it worked sir

T: alright, and uhm, actually the main reason why I chose you here is that, I a looking at your two marks for Euclidean geometry

AD: Yes sir

T: The first test, it was 24% and then the second test, it was 705, so the first test was before the training and then the second test was after the training.

AD: Yes

T: So can we say this improvement was caused by the training?

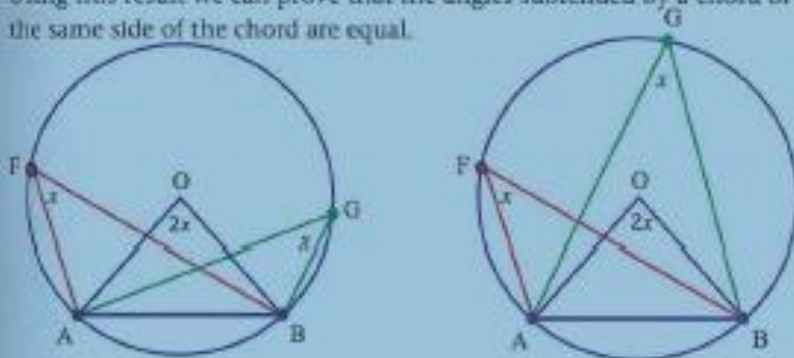
AD: Yes sir because uhm before the training, I just uhm, when I was writing the exam, just read the question, if I don't understand, just move to the.....

Annexure F: Task for episode 9

Theorem 4

In Theorem 3 we proved that the angle subtended by an arc at the centre of the circle is double the size of the angle subtended by the same arc at any point on the circumference of the circle.

Using this result we can prove that the angles subtended by a chord of the circle on the same side of the chord are equal.



Given: A, F, G and B are points on the circle with centre O.

Construction: Join AO and OB.

Required to prove: $\hat{F} = \hat{G}$

Proof:

$$\angle AOB = 2\hat{F} \quad |\angle \text{ at centre}$$

$$\angle AOB = 2\hat{G} \quad |\angle \text{ at centre}$$

$$\hat{F} = \hat{G}$$

The corollary of Theorem 4 is that equal chords subtend equal angles.

In $\triangle OPQ$ and $\triangle OSR$:

$$1) OP = OS \quad |\text{radii}$$

$$2) OQ = OR \quad |\text{radii}$$

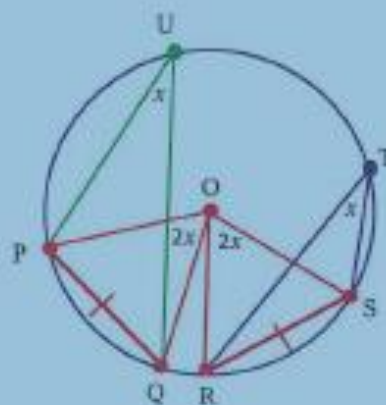
$$3) PQ = RS \quad |\text{given}$$

$$\Rightarrow \triangle OPQ = \triangle OSR \quad |\text{SSS}$$

$$\angle POQ = \angle ROS \quad |\triangle OPQ = \triangle OSR$$

$$\angle POQ = 2\hat{U} \text{ and } \angle ROS = 2\hat{T} \quad |\angle \text{ at centre}$$

$$\hat{U} = \hat{T}$$



$\cong$ is the symbol for congruent.

Congruent means identical in all respects.

KEY WORDS

corollary – deduction based on the result of a theorem

Theorem 5

The opposite angles of a cyclic quadrilateral are supplementary.

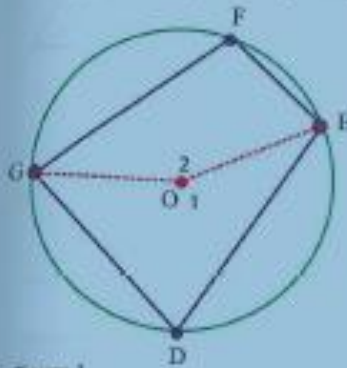


Figure 1

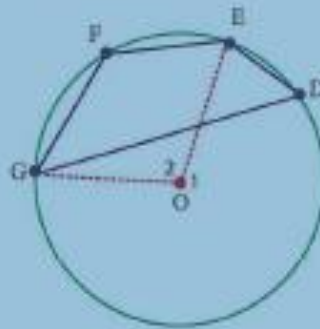


Figure 2

Given: D, E, F and G are 4 points on the circle with centre O.

Construction: EO and GO.

Required to prove: $\hat{D} + \hat{F} = 180^\circ$ and $\hat{D} + \hat{G} = 180^\circ$

Proof: (The proof is the same for both figures.)

Let $\hat{D} = x$

$\hat{O}_2 = 2x$ | $\angle$ at centre

$\hat{O}_1 = 360^\circ - 2x$ | revolution

$\hat{F} = 180^\circ - x$ | $\angle$ at centre

$\hat{D} + \hat{F} = 180^\circ$

$\hat{D} + \hat{G} = 180^\circ$ | $\angle$ sum quad

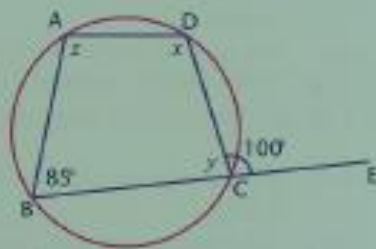
Converse (Theorem 5)

A quadrilateral is cyclic if its opposite angles are supplementary.

Corollary: The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

WORKED EXAMPLE

A, B, C and D lie on the circle and BC is produced to E. $\hat{B} = 85^\circ$ and $\hat{DCE} = 100^\circ$. Determine, with reasons, x , y and z .



SOLUTIONS

$x + 85^\circ = 180^\circ$ | opposite $\angle$ s cyclic quadrilateral

$\Rightarrow x = 95^\circ$

$y = 80^\circ$

straight $\angle$

$z + 80^\circ = 180^\circ$ | opposite $\angle$ s cyclic quadrilateral

$\Rightarrow z = 100^\circ$

or $z = 100^\circ$

exterior $\angle$ cyclic quadrilateral

KEY WORDS

cyclic quadrilateral – a four-sided figure with all four vertices lying on the circumference of a circle
supplementary – add up to 180°

revolution – the sum of the angles around a point

produce – lengthen a straight line, for example, AB produced to C means that ABC is a straight line

exterior angle – outside angle of a polygon formed by a line which has been extended

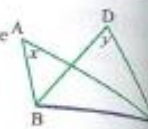
interior angle – inside angle

REMEMBER

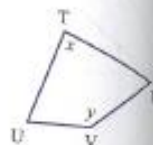
- Angles in any triangle add up to 180° .
- Angles in any quadrilateral add up to 360° .
- Angles around any point add up to 360° .
- Exterior angle of triangle = sum of two interior opposite angles.
- Exterior angle of cyclic quadrilateral = one interior opposite angle.

There are three ways to prove that a quadrilateral is cyclic.

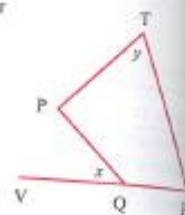
- 1 If a line subtends equal angles at two different points on the same side of the line, then the four points are cyclic.
If $x = y$, then ABCD is a cyclic quadrilateral.



- 2 If one pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.
If $x + y = 180^\circ$, then TUVW is a cyclic quadrilateral.



- 3 If the exterior angle of a quadrilateral equals the interior opposite angle, then the quadrilateral is cyclic.
If $x = y$, then PQRT is a cyclic quadrilateral.



EXERCISE 3

- 1 Complete the statements by filling in the missing words:
 - 1.1 The line drawn from the centre of a circle perpendicular to a chord ...
 - 1.2 The line drawn from the centre of a circle to the midpoint of a chord ...
 - 1.3 If PQ is the perpendicular bisector of chord AB, then PQ passes through ...
 - 1.4 If PQ and JK are the perpendicular bisectors of any two non-parallel chords on the same circle, then PQ and JK will intersect each other and the centre of that circle will lie on their ...
 - 1.5 The angle subtended by a chord at the centre of the circle is ...
 - 1.6 The angles subtended by a chord in the same segment of the circle ...
 - 1.7 The angle subtended by a diameter on the circumference of the circle is always equal to ...
 - 1.8 If a chord subtends a right angle on the circumference of a circle, then the chord is ...
 - 1.9 The opposite angles of a cyclic quadrilateral ...
 - 1.10 If the opposite angles of a quadrilateral are supplementary, then ...
 - 1.11 If a line subtends equal angles at two points on the same side of itself, then ...
 - 1.12 If a line subtends unequal angles at two points on the same side of itself, then ...
 - 1.13 If the exterior angle of a quadrilateral is equal to the interior opposite angle, then ...
 - 1.14 If the exterior angle of a quadrilateral is not equal to the interior opposite angle, then ...
 - 1.15 Equal chords subtend ...

REMEMBER

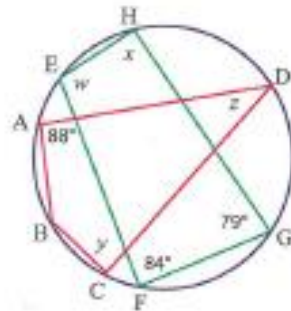
To prove that two lines parallel, show one of the following:

- alternate angles are equal
- corresponding angles are equal
- co-interior angles are supplementary

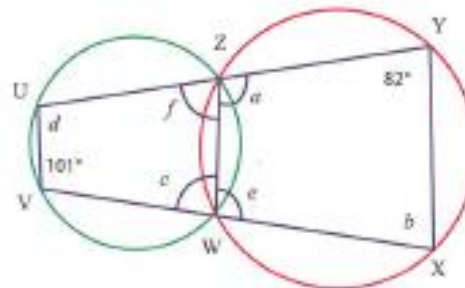
Four points are concyclic if they lie on the same line.

EXERCISE 4

- 1 A, B, C, D, E, F, G and H all lie on the circumference of the circle. Determine, with reasons, w , x , y and z .

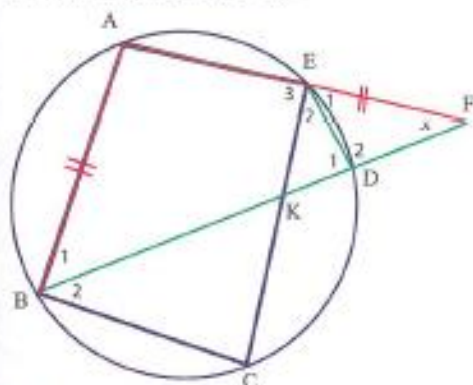


- 2 Two circles intersect at W and Z. U and V lie on the smaller circle, Y and X lie on the larger circle. $\hat{U} = 101^\circ$ and $\hat{Y} = 82^\circ$.

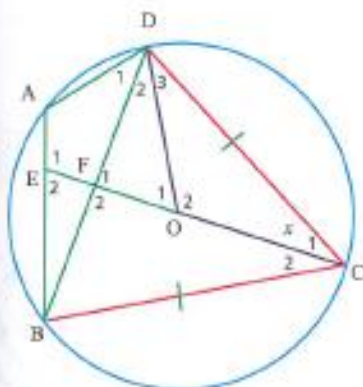


- 2.1 Determine, with reasons, a , b , c , d , e and f .
 2.2 Is $UY \parallel VX$? Justify your answer.
 2.3 Is $UV \parallel YX$? Justify your answer.
 2.4 Is $UVXY$ a cyclic quadrilateral? Justify your answer.

- 3 A, B, C, D and E lie on the circle. $AB = AF$ and DE bisects $\widehat{CE}$. BD and AE produced meet at F and $\hat{F} = x$.



- 3.1 Name two cyclic quadrilaterals.
 3.2 Prove that $DE = DF$.
 3.3 Prove that BF bisects $\widehat{AC}$.
 3.4 Prove that $\widehat{ABC} = \hat{C}$.
 3.5 Is ABCF a cyclic quadrilateral? Justify your answer.
 3.6 Is ABKE a cyclic quadrilateral? Justify your answer.
 3.7 Is $AF \parallel BC$? Justify your answer.
- 4 A, B, C and D are points on the circumference of the circle. COE is a straight line such that E lies on AB. $DC = BC$ and $\hat{C}_1 = x$.



- 4.1 Determine the size of $\hat{B}_2$ and $\hat{D}_2$ in terms of x .
 4.2 Show that EC bisects $\widehat{BCD}$.
 4.3 Prove that $DF = FB$.
 4.4 Show that AEOD is a cyclic quadrilateral.
 4.5 Draw ED and prove that EBOD is a kite.

Dear Principal

My name is **Edwin Madzore** I am a student in the School of Education at the University of the Witwatersrand. I am doing research on

Cognition and Metacognition in Mathematics Teaching and Learning

My research involves teaching and training of metacognition to students as a way of equipping them with skills for learning Euclidean Geometry. Metacognition refers to the knowledge one possesses about one's own cognitive processes (knowledge of cognition) and the monitoring and regulation of these cognitive processes in order to serve a concrete goal. This entails that students should be aware of what they know. They should be able to control and drive their own learning process. I will train the students after school for two weeks before I allow them to use the skills gained during training for doing their geometry.

The training involves defining and explaining to the students the concept of metacognition. During the training, I will coach students on asking effective questions guided by a training model called IMPROVE. In the acronym IMPROVE; my research will put emphasis on M, which stands for Metacognitive questioning. These questions will be classified into questions of planning, questions of monitoring and questions of evaluation. This way, students will gain planning, monitoring and evaluation skills. Planning questions include questions like: What am I supposed to do? What should I do first? The assumption is that if learners do proper planning, they are likely to take a right course of action in solving problems in their mathematics.

I will conduct the training in my classroom and the training does not attract any charge on the part of the school, the students or me. After the two weeks of training, I will allow the students to implement the questioning technique as they do their Euclidean geometry. During the course of the research, participating students will do some pre-tests at the beginning of the research and post- tests at the end as a way of checking the effectiveness of the intervention program. I will also interview them (experiment group) once at the beginning of the research. The interview session will take at most 5 minutes. The interviews will take place in my classroom after school so that they do not interfere with lessons.

The reason why I have chosen your school is because I am a mathematics teacher in the school already. It will be beneficial for the learners to undergo the training as in the process they will become equipped with skills that can help them become better doers of mathematics.

I am inviting your school to participate in this research. It is not compulsory that your school takes part but I am kindly requesting your assistance in this regard.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

Edwin Madzore

ADDRESS: 22 Essex Street, Kensington, Johannesburg, 2094

EMAIL: madzorebest@yahoo.com TELEPHONE NUMBER: 0790579112

DATE: 25/05/2016

Dear Learner

My name is Edwin Madzore and I am a student in the School of Education at the University of the Witwatersrand. I am doing research on

Cognition and Metacognition in Mathematics Teaching and Learning

My investigation involves training students on issues of being managers of their own mind and their learning processes. I will train you after school for two weeks in my classroom. During the training sessions I will help you to learn how to plan, monitor and evaluate your own learning. As part of the research, you will write some tests but these tests will not be for marks and they do not have any effect on your term or year mark. The marks obtained in these tests are only for the purpose of the research and not for classifying you in anyway.

Would you mind if I ask you to take part in my research? This is not a compulsory exercise but I am kindly inviting you to participate in the research.

I need your help with taking part in the training and then later in the use of the methods learned during training in learning your Euclidean Geometry topic. I will also ask you to complete a questionnaire and answer a few interview questions as a way of helping me to evaluate the effectiveness of the training program.

Remember, this is not a test, it is not for marks and it is voluntary which means that you don't have to do it. Also, if you decide halfway through that you prefer to stop, that will be completely your choice and will not affect you negatively in any way.

I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join us in the study should your parents' consent.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

Edwin Madzore

ADDRESS: 22 Essex Street, Kensington, Johannesburg, 2094

EMAIL: madzorebest@yahoo.com TELEPHONE NUMBER: 0790579112

Dear Learner

Please fill in the reply slip below if you are willing to participate in my study called:
Evaluating the effectiveness of Self-Directed Metacognitive (SDM) questioning during solving of Euclidean Geometry problems by grade 11 learners

My name is: _____

Please circle only YES or NO to each of the following questions/statements.

Permission to review/collect documents/artifacts

I agree that my work book can be used for this study only. YES/NO

Permission to observe you in class

I agree to be observed in class. YES/NO

Permission to be interviewed

I would like to be interviewed for this study. YES/NO

I know that I can stop the interview at any time and don't have to answer all the questions asked.

YES/NO

Permission to be video recorded

I would like to be video recorded for this study YES/NO

I know that I can stop the video recording at any time and don't have to answer all the questions asked. YES/NO

Permission for questionnaire/test

I agree to fill in a question and answer sheet or write a test for this study. YES/NO

Informed Consent

I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Signature_____ Date_____

Dear Parent

My name is Edwin Madzore and I am a student in the School of Education at the University of the Witwatersrand. I am doing research on

Cognition and Metacognition in Mathematics Teaching and Learning

My research involves teaching and training students on methods of managing their own learning process. The training that I will give them helps them to be able to plan, monitor and evaluate how they learn. I will do the training during school days and there are no costs involved in doing the training. Training sessions will take place after school so they do not affect your child's lesson times.

The reason why I have chosen your child's class is because the research is based on Senior Secondary School Euclidean Geometry. This targeted topic of Euclidean Geometry is done in grade 11 so the fact that your child is in this grade at the school where the research will be conducted makes your child's class a potential participating group.

Would you mind if I ask you to allow your child to take part in the research? In addition to being trained and observed as he works with the research tools, your child will be asked to answer some interview questions. I will interview him once at the beginning and once at the end of the research. The interview sessions will take at most 5 minutes each. The interviews will take place in my classroom after school so that it does not interfere with lessons. He will also write two tests that are not for marks, and complete a questionnaire related to the research.

Your child will not be disadvantaged in any way. He will be reassured that he can withdraw his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for taking part in this study.

Your child's name and identity will be kept confidential at all times and in all academic writing about the study. His individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely

Edwin Madzore

ADDRESS: 22 Essex Street, Kensington, Johannesburg

EMAIL: madzorebest@yahoo.com TELEPHONE NUMBER: 0790579112

Dear Parent(s)

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called: **Evaluating the effectiveness of Self -Directed Metacognitive (SDM) questioning during solving of Euclidean Geometry problems by grade 11 learners**

I/We, _____ the parent (s) of _____

Please circle only YES or NO to each of the following questions/statements.

Permission to review/collect documents/artifacts

I agree that my child's work book can be used for this study only. YES/NO

Permission to observe my child in class

I agree that my child may be observed in class. YES/NO

Permission to be interviewed

I agree that my child may be interviewed for this study. YES/NO

I know that he/she can stop the interview at any time and doesn't have to answer all the questions asked.

YES/NO

Permission to be video recorded

YES/NO

I agree that my child may be video recorded for this study

YES/NO

I know that he/she can stop the video recording at any time and doesn't have to answer all the questions asked. YES/NO

Permission for questionnaire/test

I agree that my child may fill in a question and answer sheet or write a test for this study.

YES/NO

Informed Consent

I/We understand that:

- My child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- My child does not have to answer every question and can withdraw from the study at any time.
- My child can ask not to be audiotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Signature_____ Date_____