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FROM COHERENCE IN THEORY TO COHERENCE IN PRACTICE: A STOCK-TAKE OF THE WRITTEN, TESTED AND TAUGHT NATIONAL CURRICULUM STATEMENT FOR MATHEMATICS (NCSM) AT FURTHER EDUCATION AND TRAINING (FET) LEVEL IN SOUTH AFRICA:

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A thesis submitted to the Wits School of Education, Faculty of Humanities, University of the Witwatersrand in fulfillment of the requirements for the degree of Doctor of Philosophy

Johannesburg

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Abstract and Keywords

Initiatives in many countries to improve learner performances in mathematics in poor communities have been described as largely unsuccessful mainly due to their cursory treatment of curriculum alignment. Empirical evidence has shown that in high achieving countries the notion of coherence was strongly anchored in cognitively demanding mathematics programs. The view that underpins this study is that a cognitively demanding and coherent mathematics curriculum has potential to level the playing field for the poor and less privileged learners. In South Africa beyond 1994, little has been done to understand the potential of such coherent curriculum in the context of the NCSM. This study examined the levels of cognitive demand and alignment between the written, tested and taught NCSM. The study adopted Critical Theory as its underlying paradigm and used a multiple case study approach. Wilson and Bertenthal's (2005) dimensions of curriculum coherence provided the theoretical framework while Webb's (2002) categorical coherence criterion together with Porter's (2004) Cognitive Demand tools were used to analyse curriculum and assessment documents. Classroom observations of lesson sequences were analysed following Businskas' (2008) model of forms of mathematical connections since connections of different types form the bases for high cognitive demand (Porter, 2002). The results indicated that higher order cognitive skills and processes are emphasized consistently in the new curriculum documents. However, in the 2008 examination papers the first examinations of the new FET curriculum, lower order cognitive skills and processes appeared to be emphasized, a finding supported by Umalusi (2009) and Edwards (2010). Classroom observations pointed to teachers focusing more on rote learning of both concepts and procedures and less on procedural and conceptual understanding. Given the widespread evidence of the tested curriculum impacting on the taught curriculum, this study suggests that this lack of alignment between the advocated curriculum on one hand, the tested and the taught curricula on the other, needs to be investigated further for it endangers the teaching and learning of higher order cognitive skills and processes in the FET mathematics classrooms for the poor and less privileged. Broader evidence suggests that this would work against efforts towards supporting the upward mobility of poor children in the labour market.

Keywords:

curriculum coherence, alignment index, mathematical connections, critical theory, cognitive demand



DECLARATION

I declare that this thesis is my own unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

Signature of candidate .....

Michael Kainose Mhlolo

…….. day of September in the year **2011.**

DEDICATION

This work is dedicated to our son Brilliant Mhlolo who turns six as I submit this thesis for examination. You are special to us!!!





PUBLICATIONS AND PRESENTATIONS EMANATING FROM THIS RESEARCH

Lelliot, A., Mwakapenda, W., Doidge, M., du Plessis, J., Mhlolo, M. K., Msimanga, A., et al. (2009). Issues of teaching and learning in South Africa: A disjunction between curriculum policy and implementation. *African Journal of Research in MST Education* (Special Issue), 47 - 64.

Mhlolo, M. K. (2008a). *Studying the implementation of the Further Education and Training (FET) curriculum in South African mathematics classrooms: Shifting teacher identities, promising avenues and blind pathways for mathematics teaching.* Paper presented at the SAARMSTE Southern African Association for Research in Mathematics, Science & Technology Education Maseru - Lesotho.

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Abbreviations

AI	Alignment Index
AMESA	Association of Mathematics Educators of South Africa
CAPS	Curriculum and Assessment Policy Statement
CASS	Continuous Assessment
CAT	Common Tasks for Assessment
DfID	Department for International Development
DOE	Department of Education
DR	Different Representations
FET	Further Education and Training
GET	General Education and Training
GNP	Gross National Product
HLM	Hierarchical Linear Model
HOCS	Higher Order Cognitive Skills
IAM	Instructional Activities and Materials
ICC	Implementation of Curriculum Change
ICME	International Commission of Mathematics Education
ICMI	International Commission of Mathematics Instruction
IES	Institute for Educational Sciences
IM	Implication
IOC	Instruction Oriented Connection
LO	Learning Outcome
METE	Mathematics Education Traditions of Europe
MR	Mathematical Reasoning
NCS	National Curriculum Statement
NCSM	National Curriculum Statement for Mathematics
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
NRST	Norm Referenced Standardised Test
NSC	National Senior Certificate
NSF	National Science Foundation
OBE	Outcomes Based Education
P	Procedure
PhD	Doctor of Philosophy
PWR	Part whole relationship
Q1	Quintile 1
SAARMSTE	Southern African Association of Researchers in Mathematics Science and Technology Education
SK	Structural Knowledge
TIMSS	Trends in Mathematics and Science Studies

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CHAPTER ONE - INTRODUCTION

1.1 NATURE OF THE PROBLEM

Recent curriculum reforms in mathematics and science in many developing countries including South Africa, come as an attempt to address issues of inequity and exclusion both educational and social. In South Africa the introduction of outcomes based education (OBE) in 1997 was intended to redress the legacy of apartheid by promoting the development of high skills throughout the school-leaving population in order to prepare South Africa's workforce for participation in an increasingly competitive global economy (Le Grange, 2007). Given that in South Africa mathematics learners from communities which were previously marginalized during the apartheid era continue to be outperformed by those from the previously advantaged communities, researchers argue that despite many of the new education policies being acclaimed by international experts as some of the best in the world, there is little evidence that the goals of transformation, including redress, equity and democracy, have been achieved in practice (Fleisch, 2008; Sayed & Jansen, 2001).

Lamenting the credibility of Senior Certificate pass rates in South Africa against public opinion that standards had dropped Muller (2005, p. 43) wrote;

> The largely invisible outcome, invisible to school educators, that is though not invisible to employers or university admission officers, was that the schooling system was emitting a cohort or two which had reduced opportunities to demonstrate higher-level cognitive skills, had possibly not even been taught them and, in far too many cases, therefore did not have them.

The contention was that standards had actually dropped despite the upward trend in pass rates post democracy. Muller supported this claim with empirical evidence from Umalusi research forums which analysed the cognitive demand levels of the papers set in 1992, 1993, 1999 and 2003. Muller argued that the results were 'unambiguous': school expectations as expressed by the complexity of the examination questions had declined

because of the level of cognitive demand which was low. Muller (2005, p. 44), posited that, whether advertently or inadvertently, low cognitive demand and challenge was a threat to the learning health of the nation and suggested that it should be addressed;

(i) in the curricular statements

(ii) in the manuals of exemplars

(iii) in textbooks and learning materials

(iv) in examination papers; and

(v) in marking standards

To Lolwana (2005), the issue of the standards of the school leaving or matriculation certificate could not be separated from the curriculum. Her argument was that while the issue of standards was not a simple one, as standards meant different things to different people, it could still be assumed that there would be consensus in the view that the notion of standards must include some agreement on (a) the nature of the content or knowledge to be acquired (b) the amount or volume and depth of that content and (c) the cognitive skills to be acquired in the process of learning (Lolwana, , op cit). Literature has shown that while the tested curriculum exerts more pressure on the taught curriculum, the key to the standards debate lay in the curriculum as a whole (intended, tested and taught) more than in the examinations (tested) as a component of the curriculum (Trumpelmann, 1991). According to Trumpelmann (op cit), not only was there a need for an overall strategy for checking the alignment of the stages but there was also a need for regular and systematic checks on the cognitive demand health at the different stages of the curriculum process. Given that the National Curriculum Statement for Mathematics (NCSM) came into operation after these observations had long been made, this research examined the cognitive demand levels of and the alignment between the written, the tested and the taught curriculum for mathematics at FET level. This also followed empirical evidence from large scale studies internationally which bring the taught curriculum within the alignment framework and suggest that an aligned, as well as cognitively demanding curriculum across written, tested and taught could be one possible way of leveling the playing field for the poor and the minority students as it has potential to reduce the achievement gap (Squires, 2009).

1.2 SCOPE AND BOUNDARIES

Using a battery of cognitive demand tools this research analysed the content of the written, the tested and the taught national curriculum statement for mathematics. In this study the term 'cognitive demand' is used in two ways to describe learning opportunities in line with Zurawsky's (2006), recommendations.

> The first way is linked with the curriculum policy – how much math? The second way relates to how much thinking is called for in the classroom. Routine memorisation involves low cognitive demand, no matter how much advanced the content is. Understanding mathematical concepts involves high cognitive demand, even for the basic content (Zurawsky, 2006, p. 1).

Guided by this definition (discussed further in chapter 2), the overview document, the mathematics subject statement, its learning programme guidelines and its assessment standards were analysed in order to establish the levels of cognitive demand and the alignment between those components of the written curriculum. The 2008 exemplar papers (the first papers published to give teachers an indication of the reformed FET curriculum introduced in 2006) were also analysed using the same cognitive demand tools in order to establish whether they were testing at the same level of cognitive demand as was intended in the written curriculum. Four Grade 11 mathematics teachers were then observed each teaching a series of five lessons related to algebraic topics. Three of the teachers were teaching LO2 (Functions and Algebra) and one teacher was teaching LO1 (Number Patterns). A more detailed justification for this choice is made in chapter three – Research Design and Methodology. A total of 300 pages of transcribed data were generated from the video recordings of classroom observations.

The focus of the classroom observations was on the teachers due to my research interest in how they translated the written curriculum into practice. The aim was to establish to what extent these mathematics teachers' approaches created opportunities for pupils to learn higher order processes and cognitive skills in mathematics. In terms of teacher strategies that enhance learner development of higher order cognitive and process skills, lessons learnt from the TIMSS studies are that high cognitive demand mathematics programs generally deviate in important ways from the "normal" approaches to

mathematics instruction and classroom practice (Zurawsky, 2006). The countries with high scores in mathematics focused on building connections among mathematics ideas, facts, and procedures. Higher performing countries avoided reducing mathematics tasks to mere procedural exercises involving basic computational skills, and they placed greater cognitive demands on students by encouraging them to focus on concepts and connections among those concepts in their problem solving. Task rigour was maintained when teachers pressed for justifications, explanations and meaning through questioning or other feedback (Hiebert et al., 2003; Stigler & Hiebert, 2004). Borrowing from these notions, the questions that guided classroom observations were; 'In what ways does the teacher work to promote either more procedural exercises involving basic computational skills or towards encouraging learners to make connections between concepts and/or procedures? In what ways does the teacher press for, or provide justification and explanations for mathematical decisions made?' Analysis of the teachers' lesson transcripts was framed around the conceptual/procedural framework, guided by Businskas' (2008), model of types of mathematical connections. The justifications and explanations that teachers gave, analysed within this framework, were then used as indicators of the quality in terms of the levels of cognitive demand of the connections made.

The rationale for moving from document analysis, to test analysis and then to classroom observations was premised on the view that knowing the cognitive demand of the written curriculum is important because the written curriculum is the starting point and target for both the tested and the taught. Knowing the cognitive demand of the tested curriculum is important because student achievement is measured only for the content tested. Knowing the cognitive demand of the taught is important because, other things being equal, students in situations of poverty in particular, tend to have opportunities to learn only what they are taught (Squires, 2009). It has also been argued that the cognitive demand of the taught curriculum is a powerful predictor of variance in student achievement gains, and helps explain a portion of the achievement gap between students from different backgrounds (Mullis, Martin, & Foy, 2008). Measures of cognitive demand levels of the

written, the tested and the taught curriculum therefore provide a window into investigating the quality of implementation of a new curriculum.

1.3 THE CONTEXT OR SETTING OF THE STUDY

While many examples have been used to highlight the nature of discrimination in the colonial system of education common in the African countries prior to democracy, perhaps Verwoerd's (1954), speech at the Second Reading of the Bantu Education Bill in South Africa stands out as the most infamous illustrations of what necessitated change, especially in mathematics education, for many African countries including South Africa;

> When I have control over native education I will reform it so that the natives will be taught from childhood to realize that equality with Europeans is not for them … What is the use of teaching the Bantu mathematics when it cannot use it in practice? (House of Assembly Debates, vol. 78 Aug. – Sept. 1953, p3585)

Consistent with this view towards the education of the natives, the apartheid system in South African education, sought to discriminate and differentiate people by preparing learners differently for the positions they were expected to occupy in social, economic and political life (Chisholm, 2005). Post-apartheid curriculum reforms in mathematics and science post democracy in South Africa make claims that they come as an attempt to address such issues of inequity and exclusion both educationally and socially. Common within these curricula is their emphasis on school systems targeting the development of students' higher-order cognitive and process skills, especially in mathematics and science (Edwards, 2010). The justification for targeting such higher order cognitive and process skills is that they are viewed as being inextricably intertwined with the ability to solve real life problems. In the case of South Africa this was also in line with the two principles of social transformation and high knowledge and high skills drawn from the new constitution.

In pursuit of those objectives, South Africa has experienced enormous changes in the field of education in the last decade evidenced by the introduction of Continuous Assessment (CASS) in 1996 with curriculum 2005 (C2005) being launched in 1997, followed by the Revised National Curriculum Statement at GET phase in 2000, followed

by the National Curriculum Statement (NCS) at FET phase in 2006. The most recent proposed changes are contained in the Curriculum and Assessment Policy Statement (CAPS) (2010). However, despite many of these new education policies being acclaimed by international experts as some of the best in the world, there is little evidence that the goals of transformation, including redress, equity and democracy, have been achieved in practice (Sayed & Jansen, 2001). The South African education system has continued to be criticized for failing close to 60% of the candidates in general and 90% of Black students in particular (Motala & Pampallis, 2007). In a recent report to the Council of Education Ministers the South African Department of Basic Education highlighted the persistently strong correlation between poverty and performance (Department of Basic Education, 2010). On the international scene South Africa has continued to perform poorly in all the TIMSS studies that it has participated in (3 times) since 1995. Following this negative trend further internal disaggregating of the learners' scores by type of school has been undertaken by local researchers (Long, 2007a). The results of this disaggregation showed that the distribution of students' marks on TIMSS correlated strongly with racial groupings, and ultimately, those groups of learners who were previously marginalized during the apartheid era continued to be outperformed by the previously advantaged learners (Fleisch, 2008). All this has led researchers to lament that whilst the post apartheid curricula for Black and White schools in South Africa at intended curriculum level are technically similar, apartheid continues to show its ugly face. This is untenable in a country where efforts are being made to reverse such disparities. Without a deep understanding of how mathematics and science education in the post apartheid era addresses issues of exclusion/inclusion, it is likely that policy approaches will continue to misinterpret the challenges of inclusion.

Schmidt, Wang and McKnight (2005), posited that there were lessons that could be learnt from such studies as TIMSS that might help countries like South Africa to respond to challenges they faced in their Mathematics education. They argued that participating on the international studies such as TIMSS did not only provide opportunities for participating countries to evaluate their Mathematics and Science programs in an international context, but pointed out that empirical evidence also suggested that the top

achieving countries on TIMSS have cognitively demanding and coherent or aligned mathematics curricula. They also cited several studies showing that such curricula had potential to level the playing field for the poor and minority students and reduce the achievement gap between the poor and the rich (Gentile & Lalley, 2003; Kulik, Kulik, & Bangert-Drowns, 1990; Squires, 2005a, 2005b, 2009; Wishnick, 1989). The major policy implication was that if countries were serious about reducing inequality, it would be important to provide all students with a challenging Mathematics curriculum that was coherent, focused and cognitively demanding not only by an individual country's own sense of what this might mean, but by international standards.

But despite the observation that alignment can "cancel out" more traditional predictors of student achievement such as socioeconomic status, gender and race, some researchers suggest that little attention is given to alignment issues by many curriculum models (Squires, 2005a, 2005b, 2009). In South Africa post 1994 this concern was first raised by the Curriculum Review Committee as they commented that implementation of Curriculum 2005 had been confounded by lack of curriculum alignment between the standards and assessment (DoE, 2000). Zeroing in on the exact nature of this misalignment Muller (2004), lamented the decline of cognitive demand in the Grade 9 examination papers as the major contributing factor to the drop in standards. In the context of the NCS Edwards (2010), did an analysis of alignment and the cognitive demand levels of the revised Physical Science Curriculum at FET level and noted that the examination papers from 2008 – 2009 were mostly testing lower order skills and processes. His recommendation thereof was that more needed to be done to understand specifically the cognitive demand levels and alignment thereof between the written, the tested and the taught components of the National Curriculum Statement (NCS). Despite the rapid transformation of the school curriculum after 1994, researchers consistently point to this gap as a possible explanation for poor learner performances. Within this doctoral study I hoped to contribute to this identified gap in studies focused on alignment as a means of identifying and understanding some of the root causes of poor performance by those learners from previously disadvantaged communities.

1.4 BACKGROUND AND MOTIVATION TO THE STUDY

As a mathematics teacher educator in one of the Teacher Training Colleges in Zimbabwe, I found myself faced with the challenge of implementing a new curriculum in 2005. This new policy shift came about as a result of a recommendation from a study which was carried out by the Nziramasanga Commission of 1998. From its findings, and other findings prior to this commissioned research, there was overwhelming evidence that a great number of pupils did not like mathematics and were not doing well in the subject (Kilborn, Dhliwayo, Gudza, & Ngaru, 1996). This, in the commission's view, was mainly because of a teaching force, especially at primary level, with inadequate skills to cope with the demands of the subject. The commission then recommended, among other strategies, some changes in the way primary school teachers were to be recruited and trained. But while the new policy was explicit on the image of the 'new teacher', it did not specify "how" the new mathematics teacher would be trained thereby creating again, a 'dislocation' between policy and practice.

In my case, I had to find a way of coping with the new policy expectation, a challenge that consequently gave birth to my interest in reform research generally and mathematics education reform in particular. This interest however, was only catapulted into becoming a reality when I was offered an opportunity to study as a PhD Fellow working on a DfID funded 'Implementing Curriculum Change' (ICC) project in South Africa. Curriculum change in mathematics in South Africa, which the ICC project investigated, is similar to the Zimbabwean experience in a number of ways. Firstly, the reforms in both cases were triggered by a trend of poor performances in mathematics, especially by learners from previously disadvantaged communities, and so both have a specific intention to contribute towards the improvement of teaching/learning of mathematics in such environments. Secondly, in both reforms researchers have pointed to a possible policy 'dislocation' between the written and the taught curriculum, pointing to the need to investigate the challenges of implementing reforms in mathematics education.

1.5 IMPORTANCE OF THE STUDY

In discussing the importance of curriculum alignment studies in general and this one in particular, this study acknowledges that the claims made about the importance of alignment studies are contestable and as such alignment represents a necessary but not sufficient ingredient in the recipe for greater student achievement (Roach, Niebling, & Kurz, 2008). On one side there is a preponderance of research that has established strong links between a student's socio economic status, teacher effect, and gender as predictors of success on norm referenced standardized tests (Brophy, 1986; Roach, et al., 2008). On the other side, there is also some semblance of logic that seems to hold in the argument: "If learner performances were greatly influenced by socioeconomic factors then one would have expected a positive correlation between learner performances and the gross national products (GNP) of their respective countries on the international scene. Why for example, would countries such as USA and South Africa continue to be outperformed by their counterparts with a much lower (GNP)?" This kind of logic became one of the driving forces behind research that tried to unravel the socioeconomic, cultural and instructional factors that explain why children in certain countries dramatically outperform their counterparts in economically comparable countries. More recently research has shown that, contrary to the long standing view about the power of demographics on learner performance, alignment was more powerful in predicting student achievement (Wishnick, 1989) and that alignment effect was more powerful for low achievers than for high achievers (Porter, Smithson, Blank, & Zeidner, 2007; Schmidt & Prawat, 2006; Squires, 2009). In one of the most complex alignment studies undertaken, the relative effects of seventeen critical factors that contribute to learner performances were examined and the results in summary were as follows:

(a) socioeconomic status accounted for only 1% of the norm referenced standardised test (NRST) performance variance while the alignment effect accounted for 36.72% on the same performance scale

(b) Taken as a whole, other variables (gender, teacher effect, and socioeconomic status) accounted for only 3% of the NRST performance variance
(c) The alignment effect was more powerful for low achievers than for high achievers
(d) The lower the degree of instructional alignment the higher the influence of demographic variables (Wishnick, 1989, p. 154).

According to Squires (2009), these were stunning findings given the vast majority of research that shows a strong connection between demographic factors and performance. These results provided compelling evidence to support the view that curriculum alignment was a potential criterion that could provide results negating effects of race, socioeconomic status and gender. In South Africa researchers have also shown a correlation between poverty and performance (Department of Basic Education, 2010) and according to van der Berg (2007), in such circumstances government policy typically moves to what is thought to be the next best thing – providing added resources to those schools. Yet broad evidence from the experience in the United States and the rest of the world suggests that this is an ineffective way to improve quality (van der Berg, op cit). Resource inputs to improve educational quality may first require some other conditions for quality education to be met such as curriculum alignment. All these observations suggest that alignment could be a possible lens through which to study poor performance of learners in less privileged schools in South Africa. However, TIMSS results suggest it is only when the aligned curriculum is also cognitively demanding that positive effects can be noticed and it is in this context that this study is important.

This study drew from the large scale (ICC) project whose objective was to understand the potential impact of mathematics and science curriculum reforms on the quality of education in previously disadvantaged schools. Specifically the PhD study was interested in understanding the level of cognitive demand and curriculum alignment in the NCSM at FET level. The broader significance of my study is also in its concern with the opportunity to learn mathematics i.e. the way mathematics as a subject is actually taught in the classroom traced back and compared with how content is prescribed in the official

curriculum statements. Research has shown that teachers' ability to create opportunities for pupils to learn cognitively demanding mathematics is a good predictor of mathematical achievement or empowerment especially of the learners from previously disadvantaged communities (Squires, 2009).

Alignment studies are also important in the context of a changed curriculum as such studies guide teaching and learning. For example, if there is no alignment between what is taught as specified in the content standards and what is tested, then schools may well teach to the test and ignore the desired content standards. The implication thereof would be that if cognitively demanding content was not being assessed it was probably not being taught or learned even though it was espoused. On the other hand if schools teach according to the desired cognitively demanding assessment standards and the tests are not aligned, the learners' results may give a false impression of the students' performance relative to the desired content standards. The negative impact on remedial action may be great because the real cause of the problem may not be addressed (Edwards, 2010).

Borrowing from alignment research elsewhere, this study was premised on the view that there are benefits that accrue both at the systemic level and at the specific subject level when alignment is present. At the systemic level, standards based school reforms, which are presented as national curriculum in South Africa and many other countries, define a vision of what is important for a country's children to learn. According to Schmidt et al., (2005) standards based school reforms are based on an assumption that the education system should be guided by content standards defining what it is that students should be expected to know or do. If content standards are seen as policy instruments used to articulate the vision, or framework, of a subject-matter discipline to its educational system, then one of the most important questions that should be asked is whether those standards reflect a coherent framework. If they do not, the result will be a 'splintered vision', something more analogous to the concept of 'diffusion' in science indicating lack of order or direction.

The importance of curriculum alignment is also seen in that a poorly aligned curriculum is likely to result in underestimation of the effect of instruction on learning. Teachers may be 'teaching up a storm', but if what they are teaching is neither aligned with the state standards nor the state assessments; then their teaching is in vain. Anderson (2002, p. 259), puts this in the form of a metaphor; "This is the educational equivalent of a tree falling in the forest with no one around… no demonstrated learning, no recognised teaching". Literally this means the falling tree (high pass rates claimed from the assessments) is making a noise which no one really pays attention to. Proper curriculum alignment enables us to understand the differences in the effects of schooling on the students' achievement. Even in an accountability focused curriculum environment, curriculum alignment is still central to such accountability programs. If students are going to be held accountable for their learning, then schools must also be held accountable by demonstrating that they provide students with opportunities to learn to meet the standards that have been set. Anderson (2002), for example, cited an example of a lawsuit that was successfully filed against the state of Florida in 1979 where the argument was that it was unconstitutional to deny high school diplomas to students who had not been given the opportunity to learn the material covered on an assessment that was a requirement for graduation.

Teachers are also likely to give more importance to curriculum documents if alignment exists because the documents, which are central in this scenario, will be more useful in teaching students. Alignment can also improve the effectiveness and efficiency of the school system by providing feedback on standards that need more work, so money and time can be allocated based on need. In an aligned system student progress is more easily mapped through different assessments tools at different levels of the system.

1.6 PROBLEM STATEMENT AND OBJECTIVES

In South Africa post democracy, Senior Certificate pass rates rose steadily from a 49% in 1998 to 73% in 2003 then slightly dropped to 71% in 2004 (Ndaba, 2005). While the department of education patted itself on the back for this great achievement serious

doubts were expressed about the extent to which these results reflected an actual improvement in the performance and quality of candidates (Ndhlovu, 2004). Authoritative sceptics argue that while the pass rates are rising the system has not done enough to make a rise in passes credible, hence current assessment is not telling the public what it ought to know (Foxcroft, 2004; Lolwana, 2005; Loock & Grobler, 2005; Muller, 2004; Ndaba, 2005; Ndhlovu, 2004; Taylor, Muller, & Vinjevold, 2003). Public opinion often suggests that standards must have dropped such that it has become easier to pass or do well but the consequences are detrimental especially for the learners from previously disadvantaged societies which current reforms purport to serve (Lolwana, 2005; Muller, 2004). It is in light of such conflicting observations that some researchers have questioned the validity of the claims in many of the new education policies in South Africa about redress, equity and democracy for there is little evidence to show that such goals of transformation have been achieved in practice (Sayed & Jansen, 2001). Without a deep understanding of how mathematics education in the post apartheid era addresses issues of exclusion/inclusion, it is likely that policy approaches will continue to misinterpret the challenges of inclusion. According to Muller (2004), all these concerns point to the question of standards – have the South African expectations as expressed by their level of cognitive demand declined?

Given these conflicting observations this study aimed at:

1. Examining the level of cognitive demand in the mathematical knowledge and skills as articulated in the written and tested NCSM within the reformed FET curriculum introduced in 2006.
2. Examining the level of alignment within the written and between the written and tested curriculum.
3. Observing 4 Grade 11 mathematics teachers teach a series of algebra focused lessons in order to determine (a) the extent to which their practices created opportunities for pupils to learn higher order cognitive processes and skills and (b) the nature of alignment of the taught curriculum with the written and the tested curricula.

1.7 RESEARCH PARADIGM

The main research paradigm for several centuries has always been that of Logical Positivism. This paradigm is based on a belief in an objective reality. However, reform researchers have argued that positivist science has proved to have some deficiencies when it has been removed from the closely defined laboratory setting and asked to cope with the kind of organized complexity facing humanity in the real world (McKenna, 2010). Although there are many other paradigms which researchers have adopted in trying to understand complexities of the real world, this research adopted critical theory as its broad paradigm. The critical paradigm aims not only to understand the structural shaping of experience but to do so with a view to effect change. The study aimed at understanding curriculum coherence within the South African context with the belief that the results thereof can contribute to making desirable change (McKenna, 2010). In terms of curriculum theory, the critical approach has a concern with the emancipatory function of teaching and learning. The curriculum statements make claims that new reforms target the poor and minority groups and by studying the way the new reforms are being implemented in the target communities, I hoped to understand in depth the extent to which the new reforms opened up opportunities to grapple with higher order cognitive processes and skills.

Realist/critical theory approaches tend to rely on a combination of qualitative and quantitative methods and usually incorporate methods such as interviews, observations and analyzing texts to elicit participants' ways of knowing and seeing. This study used both quantitative and qualitative tools to measure the levels of cognitive demand as well as levels of alignment. Document analysis and video recording were also used as the data collection methods. Research which falls into the realist paradigm category is usually conducted in more natural settings and so more situational or contextual data is collected. In this study Grade 11 mathematics teachers were observed teaching in their normal set ups without manipulating their environment in any way. Research designs associated with this paradigm provide opportunities for discovery (emergent knowledge) as opposed to manipulating the environment and proceeding by testing an a priori hypothesis. The

study did not go into the classrooms with an a priori hypothesis to test but the analysis of the written curriculum and the tested curriculum did provide comparative lenses for the empirical data.

1.8 OUTLINE OF CHAPTERS

The following is a brief outline of the chapters of this research.

1.8.1 CHAPTER ONE - INTRODUCTION

Chapter one introduces the reader to the problem of poor performance in mathematics of learners from previously disadvantaged communities in South Africa. It then provides empirical evidence to show that the poor continue to be outperformed to date. This is despite claims made by policy makers that new reforms come as an attempt to address such issues of inequity. There is empirical evidence that suggests that alignment has potential to improve performance by previously disadvantaged learners and that when curricula are cognitively demanding and aligned learners are likely to learn effectively and improve their performances as a result. This justifies why this study was interested in having alignment with higher cognitive demand as a focus. This was all premised on the view that curriculum alignment was a potentially cost effective criterion that could provide results limiting effects of race, socioeconomic status and gender.

1.8.2 CHAPTER TWO – LITERATURE REVIEW

This chapter begins with an overview of the history of curriculum development. It traces this to the time of and cites the contributions made by Tyler (1949), and Bruner (1960), which have endured to this day. Tyler for example identified four critical questions that must be asked or answered when a curriculum is developed and the literature review shows how these design features are evident in the 2006 South African National FET Curriculum Statement for Mathematics. His model was organised around the following four corresponding principles: (a) defining goals, (b) establishing corresponding learning experiences, (c) organising learning experiences to have a cumulative effect and (d) evaluating outcomes. The fact that they must be corresponding suggests that coherence

must be considered at design stages. Bruner (1960), has advocated that fundamental ideas must be identified, and once identified, they must allow a student to move from a primitive and weak grasp of the subject matter to a stage in which s/he has a more refined and powerful grasp. This process of allowing a student to move from a primitive and weak grasp of the subject to a more powerful grasp is a key characteristic feature of developmental coherence. Following on from this contribution by Bruner (op cit), the literature review argues that developmental coherence, defined in terms of the deepening and extension of understanding of content, should also be examined. That the fundamental ideas should be constantly revisited and reexamined so that understanding deepens and extends over time, also points to the need to examine the cognitive demand of the mathematical knowledge and skills in the curriculum. Within the alignment studies literature, coherence and cognitively deeper understanding are described in relation to the following aspects: making conceptual links (Schmidt & Prawat, 2006), evolving from particulars to deeper structures (Schmidt, Wang & McKnight, 2005) and descriptions of the successively more sophisticated ways of thinking (Wilson & Draney, 2009). While it is acknowledged that today curriculum alignment has become more complex, Howard (2007), argued that regardless of the theoretical orientation or practical perspective, hearkening back to Bruner, Tyler and others before them, curriculum writers still emphasize the importance of curriculum coherence which should not only be sequential, but should enable students to make ever-deepening inquiries into central concepts and principles.

By way of contextualizing the study, and explaining the gap in knowledge that this study aims to address, the literature review cites the South African researchers who have argued that pass rates are rising but standards are dropping (Muller, 2004; Taylor, et al., 2003), that this was due to misalignment between the curriculum and assessment (Chisholm, 2000) as well as the level of cognitive demand which was declining in the examination papers, (Muller, 2004), that this was more detrimental to learners from poor communities (Department of Basic Education, 2010; Fleisch, 2008; Long, 2007a) and that this level of cognitive demand needed to be investigated further (Edwards, 2010).

This background flows into the central concern in this study - the extent to which high levels of cognitive demand were evident in the National Curriculum Statement and the degree of alignment between this written curriculum, the tested and the taught curricula in terms of cognitive demand. In the literature review, I then examine how others have researched curriculum coherence, and this leads to the adoption of Wilson and Bertenthal's (2005), coherence framework as the key conceptual framework for this study. The justification for its choice is given in more detail in the literature review chapter. The framework posits that a successful system of standards based education is horizontally coherent if the curriculum, instruction and assessment are all aligned with the standards and target the same goals for learning. The system is developmentally coherent if it takes into account what is known about how students' mathematical understanding develops over time. It is vertically coherent if the curriculum instruments accorded with school practice. While Wilson and Bertenthal's (2005), framework provides a description of these types of alignments and uses the term curriculum in a more restricted sense with specific reference to the content as specified in the written curriculum, Squires' (2009) provided a model showing the relationship between the different components of a curriculum. The model views the term curriculum in a more embracive sense with curriculum/content, textbooks and the standards as constituting the written curriculum, the assessment (standardised tests, curriculum embedded tests) as constituting the tested curriculum and instruction (lesson plans, teachers' instructional strategies and mastery learning students' assignments) as constituting the taught curriculum. It is this Squires' (op cit) broader model of the written, the tested and the taught curriculum that this study adopted but restricted its focus only on the level of coherence between the content and standards (written curriculum), the standardised tests (tested curriculum) and the teacher's instructional strategies (taught curriculum) in the context of the South African NCSM at FET level. The literature review finally sets the study in the critical paradigm and adopts a multiple case study design.

1.8.3 CHAPTER THREE – RESEARCH DESIGN AND METHODOLOGY

This chapter starts by justifying why the multiple-case design was adopted for this study. Document analysis is employed as a method of collecting data. Porter's (2007), alignment indices (quantitative techniques), are introduced and explained. These are used in judging the overall level of cognitive demand and alignment within the documents themselves (internal consistency) and in relation to the examination papers (external consistency). Webb's (2005), categorical coherence criterion (qualitative technique) is used to support the cognitive demand analyses as it compares the content within specific categories of the written curriculum and the examination papers with the aim of identifying and explaining in which categories content is of a low/high cognitive demand. Video recording is adopted and justified as the data source for the classroom observations of the four grade 11 teachers. Businskas' (2008), model of mathematical connections is used to analyse teachers' utterances in relation to their potential to enable learners' development of higher order cognitive skills and processes. To ensure validity and reliability in this study cognitive demand tools were discussed, contextualized, tried and tested by three experts in mathematics and then pilot tested by the researcher, details of which are also provided in chapter 3. That the study employed a 'multiple case design' approach also ensured validity and reliability in that each case is viewed as an experiment and the presumption was that the greater the number of case studies that show replication the greater the rigour with which theory would have been established. The ethical considerations that were envisaged were also discussed in detail together with the steps taken to ensure that the research would be conducted within an "ethic of respect" to those who participate.

1.8.4 CHAPTER FOUR – RESULTS OF DOCUMENT ANALYSIS

This chapter starts by addressing the first research question of this study "What is the level of cognitive demand of the mathematics knowledge and skills in the NCSM at FET level?" Judging by the data counts in the different categories of the cognitive demand matrices used to analyse the curriculum documents, the notion of the NCSM placing emphasis on higher order skills and processes such as investigating, generalizing, problem-solving and proving is explored and discussed in detail. In addressing the second question of whether or not the curriculum documents were coherent in articulating this message, the results were analysed in terms of internal consistency as well external coherence. In terms of internal consistency, sequential as well as hierarchical development of content in the content standards from one grade to the other, were also analysed. By way of measuring the levels of external consistency, alignment indices were calculated between the examination papers and the content standards and detailed discussions followed thereof. Webb's categorical criterion is also used to explain the levels of alignment and the results were also compared with observations made, in Physical Science, by Edwards (2010), and in Mathematics by Umalusi (2009), after extensive analyses by groups of experts in mathematics education. Using a categorical-concurrence criterion, areas where differences are more pronounced were identified.

1.8.5 CHAPTER FIVE – EMPIRICAL RESULTS

The aim of classroom observations was to examine both qualitatively and quantitatively how teachers structured students' opportunities for learning higher order skills and processes. In this chapter teacher utterances were categorized according to Businskas (2008), types of connections with three levels of cognitive demand measured and scored as follows: level 0 if the connection was mathematically faulty, level 1 if the connection was mathematically acceptable but lacked further articulation and/or justification, and

level 2 if the connection was mathematically acceptable and was accompanied with further articulation and/or justification.

Given that the instructional representations that students encounter define the formal opportunities for learning about the subject content, one of the key focuses of the analyses of teacher utterances was on the connections that the teachers enabled/constrained through different representations. Ball (2003), has suggested that effective teachers of mathematics have to use mathematically appropriate and comprehensible definitions, represent ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process. Research into classroom interaction has also shown that teachers are constantly engaged in this process of defining and constructing a mental image of some mathematical object and using instructional representations in the process (Businskas, 2008; McDiarmid, Ball, & Anderson, 1989b). McDiarmid et al. (1989b) have argued that good instructional representations correctly and appropriately represent the substance and the nature of the subject being taught. They further posited that precision of definitions and lack of ambiguity in statements was a fundamental principle of mathematics learning. All these factors were considered in the analyses of teacher utterances and the detailed discussions that followed thereof.

1.8.6 CHAPTER SIX DISCUSSION AND CONCLUSION

In trying to establish whether the tested curriculum is aligned with the written curriculum, the question this study was trying to address concerned the validity of the testing instruments in relation to the higher order skills and processes that are targeted in the curriculum. Classroom observations were concerned about how teacher practices were responding to the targeted higher order cognitive skills and processes. In this chapter the discussion focuses on the findings on alignment and the degree and nature of higher order cognitive skills and processes that were present within the written documents, the examination papers and the teacher utterances. The discussion provides some implications of the findings in terms of mathematical teaching and learning in less privileged classes that were observed.

CHAPTER TWO - LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

2.1 INTRODUCTION

Given that this study focuses on the level of alignment in the NCSM, tracing the historical origins and the development of curriculum coherence over the years was considered valuable in a number of ways. The understanding of what has consistently formed the logic underlying alignment was critical in shaping the focus of this study and in supporting some of the arguments that were raised in the process. Foundational works on curriculum coherence are often traced back to the work of Tyler (1949), and Bruner (1960). These two authors' work, among the first books on curriculum to be published, continue to be discussed and used for their sound foundations and because the ideas therein have been amongst the most enduring (Howard, 2007).

2.1.1 TYLER'S CURRICULUM MODEL

In 1949 Tyler published his classic text on curriculum development with a model which has come to be known as the *product process* in the history of curriculum development (Howard, 2007). The model was organised around the following four corresponding principles: (a) defining goals, (b) establishing corresponding learning experiences, (c) organising learning experiences to have a cumulative effect and (d) evaluating outcomes. For almost 30 years after Tyler's publication, his principles remained the accepted approaches to curriculum development. Although the same principles are now applied to newer ideas and considerations that either extend or reinterpret them, they still guide the essential questions of curriculum development today. While the term alignment was not used explicitly in Tyler's product model there are 'catch' words which point in that direction of alignment. For example, when one considers that 'corresponding learning experiences' had to be established with the 'defined goals' as the reference point, the

need for alignment between those two components of the curriculum is implied. Not only were the learning experiences to be corresponding with the defined goals but these learning experiences were also supposed to be organised in such a manner that they would 'have a cumulative effect'. Tyler, (1977) followed his earlier proposition arguing that;

> The primary educational function of organisation is to relate the various learning experiences which together comprise the curriculum so as to produce the maximum cumulative effect in attaining the objectives of the school. The significant question to ask about any scheme of organisation is: How adequately does it provide reinforcement of the several learning experiences so that they produce a maximum cumulative effect? (p. 48).

In the catch words "corresponding" or "relating the various experiences" as well as "maximum cumulative effect", the notion of both *horizontal* as well as *developmental coherence* respectively began to emerge, and these are concepts around which the research questions for this study were formulated. Both concepts are discussed in more detail later in this chapter.

2.1.2 BRUNER'S CURRICULUM MODEL

A decade after Tyler published his classic text, Bruner met with a group of scientists, scholars and educators in 1959 to "examine the fundamental processes involved in imparting to students a sense of the substance and method of science" (Bruner, 1960, p. xvii). Although their concern was in the improvement of science education, important themes emerged from these meetings that were also to have major implications not only for science education, but for education in general. For example Bruner advocated that fundamental ideas must be identified, and once identified, they must allow a student to move from a primitive and weak grasp of the subject matter to a stage in which s/he has a more refined and powerful grasp. Bruner also advocated that as times goes by, students return again and again to the fundamental ideas, building on them, making them more complex, and understanding them more fully. These ideas had earlier been cited in the work of Whitehead (1929), who wrote;

> Let the main ideas which are introduced into a child's education be few and important, and let them be thrown into every combination possible. The child should make them his own, and should understand their application here and now (p. 2)

In these propositions the notion of *developmental coherence* mentioned above, re-emerged and this again is a concept around which my research questions 2 and 3 were formulated. Generally this type of coherence looks at whether both the written curriculum and the taught curriculum take into account the abilities and understanding that are needed for learning to progress at each stage of the process of mathematical development. Since then researchers have continued to contextualize and redefine what is desirable, essential or important for learners to know but the idea of coherence remains evident in all current efforts towards the 'fundamental processes' of learning. Currently, an influential book on curriculum development is by Wiggins and McTighe (2005). They call their approach to understanding curriculum design, 'backward design' and they cite the Tyler model as providing the logic behind their new idea. However, their design avoids the mechanistic predisposition of behaviorism in Tyler's work and offer ideas incorporating formative assessment with latest thinking in assessment. They name their design 'backward' because it starts with the end, the desired results first, then works backwards through the determination of acceptable evidence to a curriculum based on acceptable or desirable goals. In determination of desired results, Wiggins and McTighe's (2005), also reference Bruner (1960), reiterating his idea that these essential concepts and principles are what should 'anchor' the curriculum, whether it be a unit of study, a course, or a major field comprised of a number of courses. This idea of anchoring the curriculum again points in the direction of coherence in that the essential concepts are used as pillars that provide stability, or security or connection between the different curriculum components.

2.2 COGNITIVELY DEMANDING KNOWLEDGE AND SKILLS

A difficulty arises in trying to use the essential concepts to anchor the curriculum, because no consensus has so far been reached on value laden issues related to what constitutes 'essential concepts', 'important mathematics', 'fundamental ideas', and 'desired results'. Indeed research reveals important differences in how school systems define such learning goals. For example, Muller and Subotzky (2001), say the question of what knowledge is needed by millennial citizens and their societies in these rapidly changing times is asked frequently. What is most striking is that the answers tend to fall

into two mutually exclusive categories. The first category provides answers in terms of cultural, political or moral knowledge and skills. The second category, growing increasingly influential, provides an answer in terms of skills and knowledge for economic productivity – the cognitively demanding knowledge and skills purportedly required for a rapidly changing world. Some writers encapsulate this increasing salience and reach of knowledge in modern life with the term knowledge society in which 'science and technology have extensively heightened society's capacities to act upon itself, its institutions and its relations to the natural environment' (Stehr, 1994). Within this context, mathematics gains its high status in industrialized nations because of its socio-economic utility as a form of what has also been referred to as 'technical/administrative knowledge' (Apple, 1992). The accumulation and control of this knowledge was essential in science-based industries. So it is not merely a question of access to knowledge becoming important to all citizens in modern society, but of access to and command of the marginal additions to knowledge becoming key (Muller & Subotzky, 2001). This suggests that questions must be asked about the quality of knowledge being accessed by citizens for according to Lewis and Smith (1993),

> Learning to be effective in higher order thinking is important for everyone; it is not a frill, nor is it a skill that only "gifted children" can or need to develop. Any time an individual is faced with a perplexing situation or a situation where it is necessary to decide what to believe or do, higher order thinking is necessary (p. 136)

Despite this realisation of the need for such quality in knowledge and skills for life in general or for economic productivity in particular, many African governments, with a firm foundation in an egalitarian philosophy, pursue a democratic agenda in which they strive in principle for equality in social, political, economic and educational rights and opportunities. Consistent with this view, social programs (including education) planned for their citizens are based on a notion of parity, fairness, impartiality or equal opportunity for all – comprising what Muller and Subotzky (2001), referred to as cultural, political and moral knowledge and skills. In South Africa post democracy, Lolwana (2005), points to a similar political massification of education. She posits that the new democratic government, acutely aware of the inequalities and divisions in the education system, prioritized its effort to equalize inputs into the system, to remove racial divisions,

and to be inclusive in all decision-making structures, including the standards-setting and evaluation processes of the Senior Certificate examinations.

However, Apple (1992), warns that too much democracy culturally and politically could be one of the major causes of "our" declining economy and culture. An over-emphasis on distribution tends to detract from the equally powerful demands placed on the education system to produce technically oriented knowledge that can be controlled and utilized by the economy. With specific reference to mathematics, Diezmann and Watters (2002), argue that the concept of equal opportunity for all seems to be ill defined to mean basic numeracy. Similar sentiments are echoed by Muller and Subotzky (2001), who posit that the notion of access to education for all resembles political knowledge (education as a human right) which was and largely is a 'low-skill' one, by which is meant that a small minority attains high skills (those who attend special/private schools), while a large majority who attend mostly public schools attains fairly mediocre skills.

Apple (1992), argues that it was not only in the treatment of race, gender, and class differences in schools that society should be cognisant of inequalities but this realisation needed to be extended to some of the predominant uses of mathematical knowledge. In a knowledge economy, massification of education tends to reproduce inequalities all over again albeit in a different form. A 'unified high-skill' educational transformation could change that and lead the economy and its society toward winning nationhood.

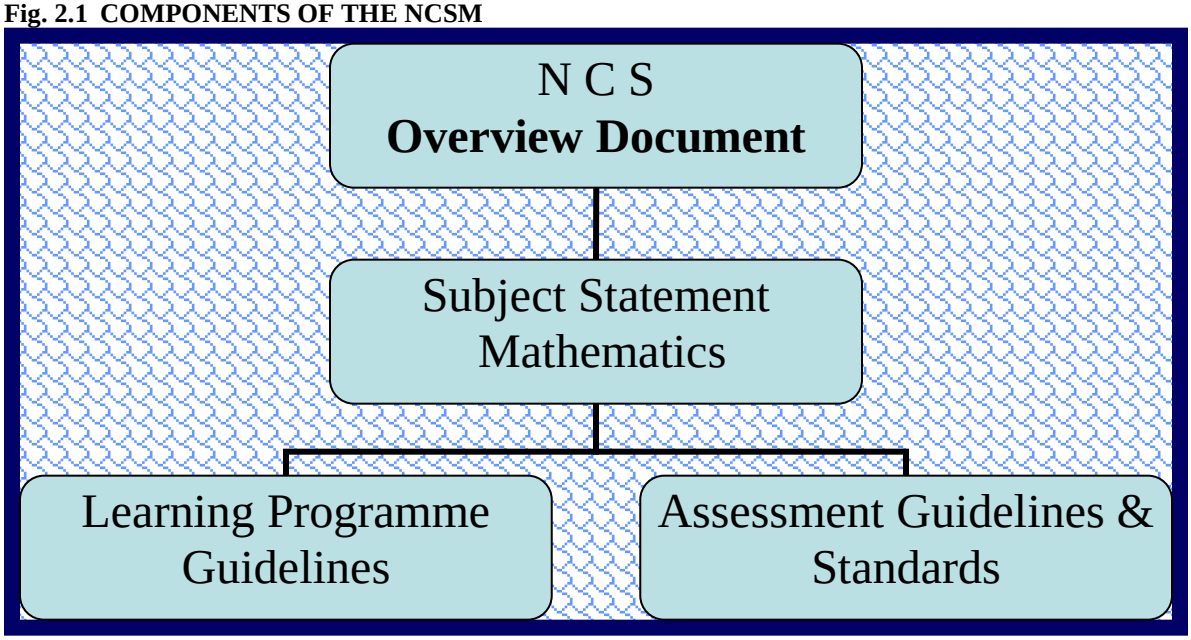
Across the world developed and developing countries have in recent years revised their school and higher education curricula to take account of the knowledge and skills required to participate in a globalizing twenty-first century world (Vinjevold, 2005). In these countries, there appears to be some convergence on the view that when describing any content of instruction with the goal of building an indicator with a strong predictive value for gains in student achievement, that content must be described not only by the particular topics covered but also by the cognitive demand levels of the activities which students are to be engaged in with those topics (Porter, 2002). So while there might be no consensus on what constitutes 'essential concepts', 'important mathematics',

'fundamental ideas', and 'desired results' this study takes the view that higher order cognitive skills and processes are desirable for more equitable educational outcomes and for economic productivity. There is empirical evidence to suggest that of the two categories of knowledge comprising on one hand political, cultural and moral knowledge and skills, and on the other higher order skills and knowledge for economic productivity, the latter is more prominently espoused in many educational reforms.

2.3 FOUNDATION OF OUTCOMES BASED CURRICULUM

The first concern for this study emanating from this historical development of curriculum coherence had to do with how the NCSM was designed. Reform of the curriculum in many parts of the world has centred on standards-based systems of education. Outcome-based education, which is a typical example of such standards-based systems, has formed the foundation for curriculum reform in South Africa post 1994. Borin et al., (2008) posit that (OBE) has its roots in strategic planning, where descriptions of the future conditions that students are likely to encounter, serve as the starting point for its design which helps to guide the establishment of significant outcomes. Four key design elements characterize (OBE) and these are (a) focus on significant outcomes, (b) design curriculum to achieve outcomes, (c) set high expectations for achievement and (d) provide multiple opportunities to receive instruction and demonstrate learning. The four elements which characterize (OBE) are reminiscent of the simple, logical and rational model originating from Tyler's model (Borin, et al., 2008). In the case of South Africa, these components are evident in the structure of the National Curriculum Statement for Mathematics (NCSM) which in this study was conceptualized as follows:

Fig. 2.1 COMPONENTS OF THE NCSM



According to the South African Government Gazette of 6 October 2003, the FET NCS Grades 10 – 12 (General) consists of the following documents: (a) the overview document (b) the subject statement (c) the learning programme guidelines and (d) the assessment guidelines. Redolent of the Tyler (1949) model of curriculum design, the foundation or philosophy is articulated in the overview document**.** The content/competencies or learning goals which identify what we want students to learn are articulated in the various subject statements**.** The approach or learning activities which identify how students will learn what we want them to learn and which specifies how this learning has to happen, are all contained in the learning programme guidelines and the assessment, which shows how this learning has to be evaluated or how we will know how students have achieved the intended goals is articulated in the assessment guidelines.

Thus there is a prima facie rational logic in the design features of the curriculum with an assumption of a well aligned backward design from goals into curriculum, into assessment and into classroom practices. Yet learners continue to perform badly despite what appears to be well aligned design features at the document level. This disparity is cause for concern in the case of South African reforms post 1994 as can be evidenced by this quote from Sayed and Jansen (2001);

> Despite many of our new education policies being acclaimed by international experts as some of the best in the world, there is little evidence that the goals of transformation, including redress, equity and democracy, have been achieved in practice (p. 2).

This suggested that translating policy coherence into improved instructional coherence and student learning was not only elusive but was more complex than just the designing of the curriculum components. Falling back on the historical development the question that kept ringing was; "If curriculum coherence has been valued for such a long time, and its design features continue to feature in current curriculum reforms in South Africa, then what other possible explanations are there to justify why learners continue to perform way below expectations in mathematics?" This suggested that an in-depth exploration of alignment, incorporating higher order cognitive demand within and across the written, into the tested and into the taught curricula, would be useful.

2.4 THE CONCERN AND THE GAP

This study acknowledges that many approaches have been developed for assessing alignment between the different components of a curriculum. Webb (1997), for example developed four different criteria for judging alignment i.e. categorical concurrence, range of knowledge, balance of representation and depth of knowledge. The Webb alignment method refers to the broadest level of content expectations as "standards," the intermediate level of content as a "goal" and the most specific level as "content objectives." Within this broadest level of content expectations or standards, Webb identified four categories as follows:

(a) Number Sense

(b) Algebraic Operations

(c) Geometry – Solid

(d) Data Analysis and Statistics (Webb, 2002).

Categorical concurrence indicates whether the same or consistent categories of content appear in both the content standards and the assessment items. This measurement is made after the broad levels of content expectations have been refined to the most specific level of content objectives. Range-of-knowledge correspondence indicates whether the span of knowledge expected of students by a standard is the same as, or corresponds to, the span

of knowledge that students need to answer assessment items or activities correctly. Balance of representation provides an index of the degree to which one curriculum objective is given more emphasis on the assessment than another. Lastly, depth-of-knowledge consistency is intended to represent the level of complexity required by the objectives and assessment items. The depth-of-knowledge criterion indicates whether what is elicited from students on an assessment is as complex for the content area as what students are expected to know and do as stated in the model academic standards.

While all these attributes were taken into consideration as aspects of horizontal alignment in chapter 4 of this study, the structure of the whole study was particularly concerned with understanding the levels of alignment between the different components of the NCSM with respect to the last attribute i.e. the depth of knowledge consistency criterion. This was consistent with the earlier position taken in this study that cognitively demanding knowledge and skills are particularly critical for economic productivity. This focus was also in line with the problem statement which raised concerns about falling standards - that school expectations had declined in terms of cognitive demand and that this posed a threat to the learning health of the nation especially those from previously disadvantaged communities.

In trying to locate a possible gap in knowledge, the literature search for this study, cognisant of the numerous alignment studies that have been documented, was guided by and focused specifically on alignment studies both internationally and locally that:

1. have been guided by this criterion of quality defined in terms of depth of knowledge or cognitive demand levels.
2. have investigated the power of instructional alignment in relation to the power of demographics that have usually explained differences in learner achievement in mathematics.
3. have shown how aligned curriculum can level the playing field for the poor and minority students and reduce the achievement gap in mathematics.

2.4.1 INTERNATIONAL LITERATURE SEARCH

Internationally, the work of Bloom (1976), is cited among the earliest which focused on the power of alignment. According to Squires (2009), this is the first look at a system of curriculum that has been shown to improve results. Bloom (1976) believed that a scope and sequence of learning tasks could be designed so that higher learning outcomes were ensured for all and not just some students. His mastery learning theory centred around the alignment of curriculum embedded tests to the written and the taught curriculum and emphasized that the scope and sequence of instructional tasks – the curriculum – would make a difference in students' performances and that all students could reach high standards. Three common elements of the mastery learning system were:

1. explicit instructional objectives, hierarchically sequenced, which students are expected to obtain
2. criterion-referenced assessment to evaluate and provide feedback on the achievement of those objectives
3. remedial instruction for students who did not achieve the desired standard of performance (Gentile & Lalley, 2003, p. 156)

Findings from and further reviews of mastery learning research indicated that mastery learning is successful at raising achievement levels of approximately 80% of students to the high levels now enjoyed by only 20% of students (Squires, 2009). The effect size was such that an average 50^{th} percentile student would move up to about the 77^{th} percentile in achievement (Block & Burns 1989 p.28). Bloom's mastery learning theory of instructional quality implies that the school controls factors that affect student outcomes i.e. outcomes are not predetermined by race, culture or socioeconomic status, but are under the control of the school.

Cohen (1987) also focused his studies specifically on the alignment aspects and coined his work 'instructional alignment'. Central to his theory was that lack of excellence in American schools was not caused by ineffective teaching but mostly by misaligning what teachers teach, what they intend to teach, and what they assess as having been taught. His findings were that misalignment of instruction to testing caused low-aptitude students to

fail, while high-aptitude students succeeded. When instruction and assessment were aligned during sample lessons both low and high aptitude students scored well on curriculum embedded tests. The quantitative effect of this alignment on achievement was measured and showed that a student scoring at a 50^{th} percentile would increase to between 84^{th} and 98^{th} percentile. This measure also clearly showed that alignment was generally more important for low-aptitude students than for high-aptitude students, with low aptitude students making greater gains when alignment was present.

Wishnick's (1989), study proposed to investigate the power of instructional alignment compared to the power of demographics that have usually explained significant amounts of norm referenced standardised achievement test (NRST) scores variance. This followed a preponderance of research that established strong links between students' socioeconomic status, teacher effect, and gender as predictors of success on norm referenced standardised tests. The question that guided Wishnick's study was 'If the curriculum and assessments are aligned, will this correlation still hold true?' He then identified seventeen critical features that contribute to alignment and analysed their effects on achievement. The findings of the study in a nutshell were that:

- Alignment is more powerful in predicting student achievement than socioeconomic status, gender, or teacher.
- Socioeconomic status accounted for only 1% of the NRST performance variance while the alignment effect accounted for 36.72% on the same performance scale. This means that whether a student received free or reduced lunch had almost nothing to do with how well s/he scored on the NRST.
- The alignment effect is more powerful for low achievers than for high achievers. Low achievers do better when instructional outcomes are clear and instruction is congruent with post-instructional assessment.
- Taken as a whole, other variables (gender, teacher effect and socioeconomic status) accounted for only 3% of the NRST performance variance

- The lower the degree of instructional alignment the higher the influence of demographic variables (Wishnick, 1989, p. 154).

Guided by similar interests, Porter and colleagues in the 'Reform Up Close' project (1993, 1994) studied high schools which were implementing change in curriculum. To describe the enacted (taught) curriculum across the schools they employed a detailed and conceptually rich set of descriptors that were organised into three dimensions: topic coverage, cognitive demand and mode of presentation (Porter & Smithson, 2001). Topic coverage included ninety-four categories (for example, ratio, volume, expressions, and relations between operations) and there were seven descriptors for modes of representation i.e. exposition, pictorial models, concrete models, equations/formulas, graphical, laboratory work, and fieldwork. Cognitive demand included nine descriptors: memorize, understand concepts, collect data, order/compare/estimate, perform procedures, solve routine problems, interpret data, solve novel problems, and build/revise proofs. Of particular importance to this doctoral study are the cognitive demand descriptors which were later compressed into five levels of cognitive demand as discussed in more detail in the methodology chapter.

Porter found that content of mathematics and science courses appeared not to have been compromised by increased enrollments (in more difficult courses) but that the enacted curriculum in high school mathematics and science was not at all in alignment with the curriculum reform toward higher-order thinking and problem solving for all students (Porter, et al., 2007). When they controlled for prior achievement and students' poverty levels using an HLM (Hierarchical Linear Model) they were able to demonstrate a strong positive and significant correlation (.49) between the content of instruction and student achievement gains confirming earlier findings that aligned instruction is linked to increased student outcomes (Porter, et al., 2007). These studies suggest an emancipatory approach to curriculum alignment issues as they show with statistical confidence that disadvantaged children suffer disproportionately from incoherent curricula (Schmidt et al., 2002; Squires, 2009), suggesting almost a cause and effect relationship between incoherent curricula and poor performance of learners in previously disadvantaged

communities. The recommendation made by Porter (2002) for example, was that if poor and minority children are to have epistemic access to education or receive a high quality, standards-based education – and ultimately reduce the achievement gap – then the instruction they receive must be cognitively demanding and aligned with the state content standards.

Of the international alignment studies, the TIMSS project is so far the largest and most rigorous cross-national set of studies of curriculum and alignment yet undertaken. It has probably been the most influential in terms of curriculum change globally hence it gets relatively more attention in this literature review. The TIMSS project has uncovered some key conditions that make up what appears to be necessary, though not sufficient, conditions for the realization of higher achievement in mathematics and science for large numbers of school-children. Notable differences between the intended/written, the tested/assessed and the taught/enacted curricula of countries exhibiting high levels of mean student achievement in mathematics and those of countries with lower mean achievement levels point to key elements common among most high achieving countries that are not shared by most low-achieving countries (W. H. Schmidt, et al., 2005). In high achieving nations, when goals first enter the curriculum they receive concentrated attention with the expectation that they can be mastered and that students can be prepared to attain a new set of progressive goals in ensuing grades. The view is that demanding standards require more sophisticated content taught in depth, as students make progress through the grades. In high achieving countries, rigorous standards go through a dynamic process of focused and coherent transitions from simple to increasingly more complex content and skills. So in most of these countries, each new grade sees a new set of curricular goals receiving concentrated attention to prepare for and build toward mastering more challenging goals yet to come.

Analyses of data from this mammoth study of students in over forty countries teased out some of the variables that affected performance in mathematics and science. According to Squires (2009), one of the guiding questions of the analyses was “If students in Japan and Thailand perform better in mathematics than those in the United States, what helps to

produce those differences?" The results showed that gross national product (GNP) as a measure of a country's wealth was not strongly related to achievement gains in either mathematics or science (Schmidt et al., 2001). These findings also noted little relationship between socioeconomic status and student outcomes when alignment was controlled confirming earlier findings that the curriculum alignment was more important than socioeconomic status in determining learning gains.

In terms of design, the TIMSS project has developed a reputation of a well designed study, "valued for its rich comparative information about educational systems, curriculum, school characteristics, and instructional practices" (Wang2001, p. 15). The design allows researchers to link variation in learners' achievement scores with the characteristics of an educational system for the purpose of improving specific components of the system (Schmidt et al., 2001). For example an analysis of TIMSS data indicated that the scores of white students in the United States were exceeded by only three other nations compared to black American school children who were beaten by every single nation (Berliner, 2001). In South Africa a similar disaggregation of the learners' scores by type of school also showed that distribution of marks on TIMSS correlated strongly with racial groupings with Black Schools being out performed by White Schools (Long, 2007b). These results highlight the need for within country analysis to identify community groups who are disadvantaged in school systems intended to provide opportunities for all learners, irrespective of their background characteristics. Such an analysis when linked with further qualitative contextual investigations, can inform policies intended to address inequalities and provide quality education for all (Frempong, 2010).

Another lesson that can be learnt from TIMSS is that despite the myriad of content standards in mathematics and lack of an agreed upon or universal list of such standards, these can be described in terms of performance standards defined in terms of cognitive demand levels where there is relatively less controversy/disagreement. TIMSS developed a list of mathematics and science performance standards so that curriculum from various nations could be described, compared and analysed. Empirical evidence suggests that

there is relative convergence on the view that excellence must be demanded of all learners and that content must reflect quality measured by the cognitive demand levels of the activities which students are to be engaged with those topics (Porter, 2002). In developing the performance standards, the TIMSS framework used three dimensions i.e. content, performance expectations and general perspectives to analyse mathematics curricula in the participating countries. Under performance expectations, the student behaviours used to define the mathematics framework were classified into the following four cognitive domains: knowing facts and procedures, using concepts, solving routine problems, and reasoning (Mullis, et al., 2003) with their corresponding descriptors as follows:

Table 2.1 TIMSS FRAMEWORK OF PERFORMANCE EXPECTATIONS (Mullis et al., 2003)

COGNITIVE DOMAINS	***Knowing Facts and Procedures***	***Using concepts***	***Solving routine problems***	***Reasoning***
DESCRIPTORS	**Recall** **Recognise/identify** **Compute** **Use tools**	**Know** **Classify** **Represent** **Formulate** **Distinguish**	**Select** **Model** **Interpret** **Apply** **Verify/check**	**Hypothesize/conjecture/predict** **Analyse** **Evaluate** **Generalise** **Connect** **Synthesize/integrate** **Solve non-routine problems** **Justify/prove**

In general the cognitive complexity of tasks in these cognitive domains, increased from one broad cognitive domain to the next with the first column being the least demanding end of the cognitive spectrum and the last column being the most demanding end (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005). While the TIMSS study placed solving routine problems as a domain label at a higher level 3 of this cognitive demand continuum, other rubrics consider it as a descriptor of a lower level skill. However the descriptors used in that domain (apply, verify etc) would still fit into level 3 (Strategic Thinking) according to Webb's (2002), depth-of-knowledge levels. This just confirms why Cuban (1984), referred to the defining of thinking skills, as a 'conceptual swamp' because of the considerable variations that exist about these mathematical tools. Suffice it to say that this also provided a resource to tap from in terms of construction of a

cognitive demand tool for document analysis in my study. The intent in the TIMSS' cognitive demand continuum was to allow for a progression from knowing a fact, procedure, or concept to using that knowledge to solve a problem, and from use of that knowledge in uncomplicated or familiar situations to the ability to engage in systematic reasoning. Mullis et al., (2003) cautioned that cognitive complexity should not be confused with item difficulty for it was possible to create easy or challenging items for nearly all the cognitive skills listed above. However, it was expected that item difficulty should not affect the designation of the cognitive skill.

In the countries participating in the TIMSS 1999 video study for example, the mathematical tasks presented during lessons were analysed following similar cognitive demand levels. The findings were that in top performing countries such as Japan for example, 17% of the content was at low complexity level and 39% at high complexity level; whereas in Australia for example, 77% of the content was at low complexity level and only 8% at high complexity level (Hiebert et al., 2003, p. 71). In the TIMSS 1999 video study also, the ability to maintain the high-level demands of cognitively challenging tasks during instruction was the central feature that distinguished classroom teaching in countries where students exhibited high levels of mathematics performance (Silver, 2009).

In the TIMSS studies, while there were differences in terms of content standards among the top performing countries, indications were also that in each case choices had been made so that each country's curriculum was coherently organised (Shepard, Hannaway, & Baker, 2009). For example, the mathematics and science curricula in the Czech Republic, Japan, Korea and Singapore (top performing countries) reflected a hierarchical sequencing of topics designed to move progressively toward more advanced topics and deeper understanding of the structure of the discipline (Shepard, et al., 2009).

In terms of assessment and consistent with the view that assessment should provide evidence that cognitively demanding goals have been achieved; high achieving countries also designed their tests framed around different levels of cognitive demand. Certain

assumptions are made when tests are designed according to different levels of cognitive demand one of which is that any assessment of mathematics learning should first and foremost be anchored in that which has been identified as desirable or important mathematics (National Research Council, 1993). The second assumption points again to the quality of the assessment. According to Zurawsky (2006), there can be no equity in assessment as long as excellence is not demanded of all; if we want excellence, the level of expectation must be set high enough so that, with effort and good instruction, every student will learn the important mathematics. Because assessment is key to determining what students learn and how teachers teach, it must be reshaped in a manner consistent with the new vision of teaching and learning. Improved assessment is required to complement and support the changes under way in mathematics education, both in the kinds of mathematics that are taught and in the ways in which they are taught. Assessment can contribute to students' opportunities to learn important mathematics only if they are based on standards that reflect high-expectations for all students.

With regard to the plan of instruction, high performing countries avoided reducing mathematics tasks to mere procedural exercises involving basic computational skills (Schmidt & Prawat, 2006). The fundamental premise of such educational reforms that focused on cognitively demanding curriculum was that the intended curriculum served to support the creation of opportunities for students to learn. To be effective in promoting learning the view was that the enactment of the intended curriculum would neither simply be a matter of covering the content specified in the curriculum, nor just a matter of the amount of time devoted to teaching them. High performing countries therefore placed greater cognitive demands on students by encouraging them to focus on concepts and connections among those concepts in their problem solving (Squires, 2009). The findings showed that in classrooms in which instructional tasks were set up and enacted at high levels of cognitive demand, students did better on measures of reasoning and problem-solving than did students in classrooms in which such tasks declined into merely following the rules, usually with little understanding. In successful classrooms, task rigor was maintained when teachers pressed for justifications, explanations and meaning through questioning or other feedback (Zurawsky, 2006).

In summary a number of important messages come through from all these alignment studies most important of which are:

(a) Alignment is more powerful in predicting student achievement than race, gender or socioeconomic status.

(b) Curriculum alignment was important for low-aptitude students than for high-aptitude students with low aptitude students making greater gains when alignment was present.

(c) This alignment should be anchored in cognitively demanding performance standards.

(d) When anchored in cognitively demanding standards curriculum had potential to reduce the achievement gap between the previously advantaged and previously disadvantaged learners.

(e) If we wanted students to develop the capacity to think, reason and problem solve then classroom practices need to be designed to give students opportunities to learn cognitively complex content.

(f) It was possible to use curriculum alignment as an emancipatory approach to address inequalities in the educational needs of previously disadvantaged learners.

In conclusion, empirical evidence internationally has continued to make similar recommendations about demanding standards, about levels of expectations being set high, about more sophisticated content being taught in depth as learners progressed from one grade to the other and about placing greater cognitive demand on students. These features of the written, the tested and the taught curriculum have resulted in the specification of many of the key features of curricula that would promote high achievement. This international literature search helped focus this study on the examination of quality defined in terms of cognitive demand within the written, tested and taught NCSM and to analyse whether these curriculum components were organised in a manner that would ‘anchor the curriculum’ around the cognitively demanding knowledge and skills and enable learners especially from previously disadvantaged communities to deepen their understanding of such concepts.

2.4.2 SOUTH AFRICAN LITERATURE REVIEW

In South Africa post democracy outcomes based education (OBE) made similar quality claims in that 'high expectations for achievement' had to be set (Borin, et al., 2008). The question that then guided the local literature search was "What empirical evidence is there to show that high expectations of achievement exist in the mathematics policy rhetoric and in specifications?"

The literature search revealed that generally when a new reform is introduced there are certain buzz words which are identified with it and these tend to sway researchers in a particular direction. For example Sayed et al., (2007) posited that the prolific writer Jonathan Jansen and his proposition that '(OBE) would fail', provoked so much debate which swayed educational researchers into two camps with some supporting this view and others trying to allay the fears thereof. According to Breen (2005), the research terrain in South African education post democracy has tended to be motivated and informed by concerns that cut across subject areas and so the immediate questions considered how the organization of learning in schools reflected these broader educational aims.

However, attention to issues of coherence in mathematics education curricula began to rise amidst concerns that pass rates were rising but standards were dropping (Muller, 2004; Taylor, et al., 2003). Misalignment of the curriculum (Chisholm, 2000) as well as the level of cognitive demand which was declining in the examination papers, (Muller, 2004, 2005; Umalusi, 2009) were raised as key issues within this problem. Analyses of the nature and pattern of these poor performances show that this was more detrimental to learners from poor communities (Department of Basic Education, 2010; Fleisch, 2008; Long, 2007a; Reddy, 2006). This suggested that levels of cognitive demand needed to be investigated further in the intended/written, the tested/assessed and the taught/enacted curriculum (Edwards, 2010). Edwards (op cit) then followed this up with a study on the levels of cognitive demand and coherence between the Physical Science Curriculum and

the exemplar papers for 2008, the examination papers for 2008 and 2009 respectively. His document analysis revealed that the curriculum was aligned with all the exemplar and the examination papers but the concern was that the focus in each of those documents was on lower order cognitive and process skills. His recommendation was that this should be investigated in the other curriculum statements, in their manuals of exemplar papers, in their textbooks and learning materials, in their examination papers and in their marking standards. Reaffirming these concerns, Lolwana (2005), proposed not only for regular and systematic checks on the cognitive demand health at the different stages of the curriculum processes, but also pointed to the need for an overall strategy for checking the alignment of all the stages.

What emerges from the South African literature search is that although attempts have been made to attend to issues of alignment among standards, assessment, textbooks and classroom practice, international literature would argue that these are 'skeletal match-ups', outlining similar topics without addressing deeper issues of conceptual congruence between challenging curricular goals and the underlying structure of pre-requisite topics and skills needed to achieve them (Shepard, et al., 2009). They attribute these skeletal match-ups to a highly political process of developing encyclopedic content standards that often left out more complex, discipline-based expertise about how knowledge, skills, and conceptual understanding can be developed together in a mutually reinforcing way. They further posit that in the standards negotiation process, these more complex understandings are usually replaced by inclusive but disorganized lists of topics. Fuhrman (1993), had earlier pointed in this direction as he proposed that the idea of coherent policy should not be about consistency for its own sake but consistency in service of 'high quality goals' for student learning.

But judging by the focus of the studies done in South Africa post 1994, there appeared to be a paucity of studies focusing on curriculum coherence in mathematics, driven by cognitively demanding standards, as a possible approach to addressing issues of poor performance by learners in the previously disadvantaged communities. Considering that the idea of coherent policy was not about consistency for its own sake but that there was

empirical evidence to suggest that there were gains in student performance when curriculum was consistent in service of 'high quality goals', this study saw a potential gap in knowledge that could be filled. Alignment studies are therefore important as South Africa is a developing country with disparities of educational access that learners continue to experience as a result of inequitable policies of the past. Not only are alignment studies important in addressing the concerns of the learners from poor and minority groups, but in the context of changed curriculum they may also give an indication of the reform efforts when the assessment results are published (Edwards, 2010).

2.5 THE PURPOSE OF THE STUDY

It is in this context that the research questions for this study were not phrased to answer more general 'laundry list' matches or 'skeletal-match ups' between the different curriculum components. Instead, the purpose of this study was to critically examine to what extent the different components of the written, the tested and the taught mathematics curriculum were cognitively demanding and then understanding to what extent the components were organised around those cognitively demanding knowledge and skills in order to enhance teaching and learning of mathematics with a focus on schools serving communities which were previously disadvantaged.

2.6 THEORETICAL FRAMEWORK- (Bernstein's pedagogic device)

In problematising this research, I highlighted how the apartheid system in South African education, used mathematics to discriminate and differentiate people. I then made reference to post-apartheid claims that curriculum reforms come as an attempt to address such issues of inequity and exclusion both educationally and socially. Common within these curricula is the emphasis on school systems targeting the development of students' higher-order cognitive and process skills, especially in mathematics and science. Though couched in different phraseology, the consistent message that comes out from literature could be summarised in Apple's (2003) words that the national curriculum is 'not so much being implemented' in schools as being 'recreated', not so much 'reproduced' as

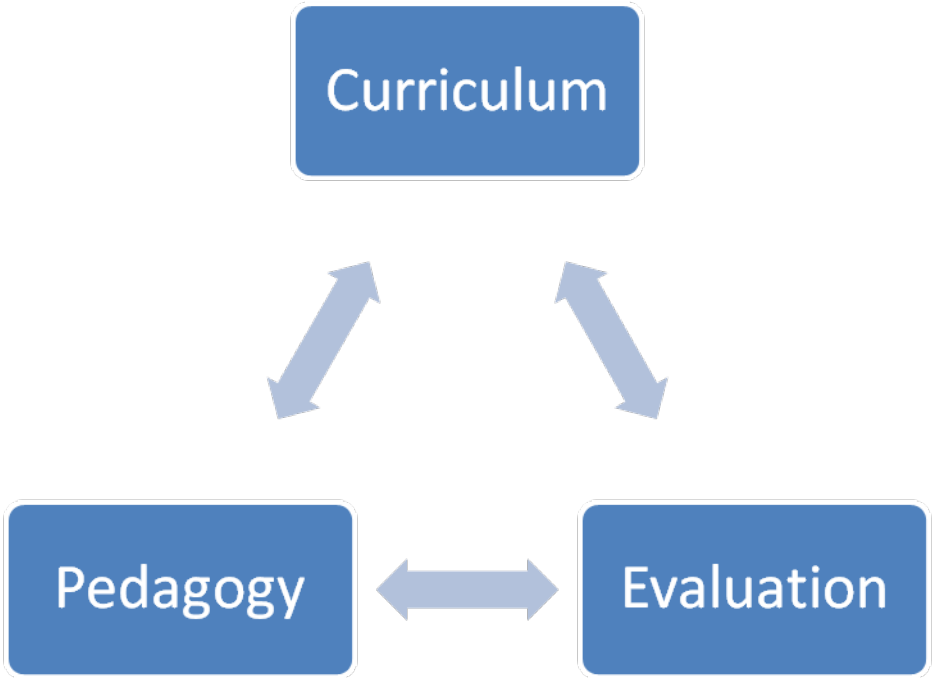
'produced'. This is so because curriculum policy is not a coherent policy as it represents conflicting arguments which become visible when analysing the discourses represented in the policy process i.e. from policy production through to its implementation. Westbury (2003), posits that the term "curriculum" must always be seen as symbolizing a loosely-coupled system of ideologies, symbols, discourse, organisational forms, mandates, and subject and classroom practices. According to Bowe, Ball and Gold (1996) as cited in (Ketlhoiwe, 2005), practitioners do not confront policy texts as naïve readers, they come with history, with experience, with values and purposes of their own and they have vested interests in the meaning of policy. This suggests that policy writers therefore cannot control the meanings of their text. Curriculum questions are situated on a macro, meso and micro levels and they represent contesting and conflicting perspectives which are important in order to understand the implementation process in relation to the intended curriculum.

By raising the question from coherence in theory to coherence in practice, this study attempts to address these complexities of policy formation, its distribution and its implementation. The presumption is that policy is not simply received and implemented but rather that it is subjected to a complex process of interpretation and then recreation. This whole idea of tracing the curriculum process from formulation to implementation was meant to address three important issues (1) to what extend are the intended, assessed and enacted curricula speaking with the same voice in relation to high-order cognitive skills (2) – to what extent are the higher order cognitive skills likely to be achieved in practice and (3) are the goals of transformation, including redress, equity and democracy therefore being achieved in the previously disadvantaged schools? These questions resonate with Bernstein's(2000a) notion of the pedagogic device, which helps to illustrate the multiple and complex relations which intervene in the production and reproduction of educational goals in the various fields. In his pedagogic device Bernstein was concerned with the production, the distribution and evaluation of official knowledge and how this knowledge is related to structurally determined power relations.

One of his conceptualisations of this pedagogic device was through what he identified as the three pillars of public education. Bernstein (1977) presented a structuralist view of

education when he posited that there are three pillars of public education, these being: curriculum, pedagogy and evaluation.

Fig. 2.2 Bernstein's Three Pillars of Education



In explaining these three pillars of education, Bernstein (ibid) wrote;

> *Formal educational knowledge can be considered through three message systems: curriculum, pedagogy and evaluation. Curriculum defines what counts as valid knowledge, pedagogy is what counts as valid transmission of knowledge, and evaluation defines what counts as a valid realization of this knowledge on the part of the taught (p. 85)*

Bernstein's concept of the pedagogic device provides a way of describing the internal construction of any pedagogic communication of knowledge through there hierarchical and interrelated sets of rules. These rules specify the transmission of suitable contents under time and context, and perform the significant function of monitoring the adequate realisation of the pedagogic discourse. The rules operate over three fields each of which have their own 'rules of access, privilege and special interest':

- The field of production where the new knowledge discourses are generated

- The field of recontextualisation where discourses appropriated from the field of production are recontextualised, simplified and transformed into a new pedagogic discourse; and

- The field of reproduction where recontextualised discourses are transformed a second time for general consumption, where pedagogy and curriculum are actually enacted in schools (Apple, 2003; Parker, 2004).

In relation to my study, I am raising research question (1) in relation to the cognitive demand of the mathematical content as articulated in the written curriculum (field of production), research question (2) in relation to the cognitive demand of the mathematical content as articulated in the examination papers (field of recontextualisation) and research question (3) in relation to the cognitive demand of mathematical content as enabled/constrained during classroom interactions (field of reproduction). Bernstein (2000b) notes that when discourse moves from the original site of production to a new position a transformation happens. In that process there may well be contradictions, cleavages and dilemmas created between these fields and his pedagogic device theoretically models this distinction and the potential discursive gaps between these fields. In other words, Bernstein argues that disciplinary knowledge does not equal the educational knowledge of that discipline

Explaining this recontextualising process his view was that it entails a principle of de-location, which involves selective appropriation of discourse from the field of production, and a principle of relocation of that discourse as a legitimate discourse within the recontextualising field. He further suggests that in the processes of de-location and relocation, the original discourse can undergo ideological transformation according to the play of the specialised interests in the recontextualising field. Further explaining these ideas about the curriculum process, Neves (2004) says that the text of any curriculum represents the official pedagogic discourse (OPD) produced in the official recontextualising field (Department of Education) and is the result of multiple influences from state, symbolic control and economy together with international influences. The text is subjected to recontextualising processes in the pedagogic recontextualising field when

it is used, for example in the construction of tests, textbooks or in professional development programmes. It is then further transformed through the pedagogic discourse of reproduction (PDR). Both the curriculum text (OPD), the tests, textbooks or professional development text (PDR) are recontextualised in the reproduction context, at the level of the teacher's pedagogic practice in the classroom. The model illustrates that when pedagogic discourses produced at the level of the official and pedagogic recontextualising fields are incorporated and developed into pedagogy at the transmission level, they still undergo a recontextualising process, which is influenced by the specific context of each school, community context, and the pedagogic practices of the teacher. In this way the discourse reproduced in the schools and classrooms is influenced by the relationships (school community; teacher-learner), which characterise its specific transmission contexts (Neves, 2004).

What is critical is that Bernstein was concerned with more than the description of the production and transmission of knowledge; he was concerned with the question of education and inequality that form the original basis of current curriculum reforms in many countries and applied his theory to understand the education process and its consequences for different groups. He was concerned not with the way in which such functioning leads to consensus but with how it forms the basis of privilege and domination. Thus Bernstein's theory of education should be understood in terms of the concepts of classification, framing and evaluation, and their relationship to the structural aspects of his sociological project (Sadovnik, 2001). Whereas classification is concerned with the organization of knowledge into the curriculum, framing is related to the transmission of knowledge through pedagogic practices. Framing refers to the location of control over the rules of communication and, according to Bernstein if classification regulates the voice of a category then framing regulates the form of its legitimate message. Furthermore frame refers to the degree of control teacher and pupil posses over the selection, organization, pacing and timing of the knowledge transmitted and received in the pedagogical relationship. Therefore, strong framing refers to a limited degree of options between teacher and students, weak framing implies more freedom.

Reflecting on Bernstein's ideas within the South African context, Parker (2004) highlighted that in each arena ideological struggle takes place as different agents and agencies attempt to dominate the distribution, recontextualisation and evaluation of pedagogic discourse at different levels of the system. The pedagogic device as outlined by Bernstein thus generates 'a symbolic ruler of consciousness', the question becomes thus; whose ruler, what consciousness? Drawing on this Parker (2004) asks, does the state control the pedagogic device through the policy they generate; or do the various recontextualisers do that?

2.7 RESEARCH PARADIGM

2.7.1 INTRODUCTION

Paradigms act as perspectives providing a rationale for research and commit the researcher to particular methods of data collection, observation and interpretations. Research paradigms can be seen as descriptions of views of how knowledge is constructed, that is, of what counts as truth (McKenna, 2010). Researchers generally posit that it is on the issues of "the nature of reality, the nature of knowledge, and the concept of truth" that paradigm designations differ. Because of these different views to the nature of knowledge many paradigms have been identified with other scholars arguing that research can either be qualitative or quantitative and nothing else (Guba & Lincoln, 1994). However, with quantitative and qualitative being positioned at the opposite ends of the continuum many other paradigms have emerged in between. The names of research paradigms associated with each of these views to the nature of knowledge vary from textbook to textbook because of the "untidy reality" of research. However, Habermas (1972), posited that knowledge was constructed according to three fundamental human interests, namely the "technical" the "practical" and the "emancipatory" interests. The paradigms associated with each of those three human interests are:

(a) positivism (technical)

(b) interpretivism (practical)

(c) critical theory (emancipatory)

Because each discipline's academic literacies have evolved out of particular views of knowledge, it is useful to consider paradigms in terms of the possible approaches to views of knowledge construction that become evident in the various approaches to curriculum studies. Below are brief descriptions of the philosophical assumptions associated with each view to the nature of knowledge.

2.7.2 THE POSITIVIST PARADIGM

The technical paradigm, also known as rational curriculum planning (Knight, 2001), or positivist paradigm (McKenna, 2010), assumes there is a logical way of proceeding, redolent of scientific method. This systematic approach begins with specifying goals, proceeding to objectives, thence to curriculum, instruction, assessment of learning and then evaluation. The positivist paradigm is often termed the "default paradigm" and its assumptions are frequently used as the criteria against which all research is assessed (McKenna, 2010). This paradigm identifies a reality that can be discovered, measured and manipulated. Knowledge is seen to be value-free and neutral and is attained by the objective observation of reality, which is out there. In this approach, the curriculum could be simplified to the following equation: objectives + inputs = outputs. Positivist studies in curriculum development are usually concerned with being able to predict and control the environment redolent of the Tyler model that has been discussed earlier. The immediate, measurable and methodological aspects of the curriculum are valued highly. Although the critiques of the positivist paradigm have been influential, victory has often gone to it because it has a common sense quality that fits well with the managerialisms that have dominated the public sector (Knight, 2001), and because it plays well as a populist political position.

A key objection to Tyler's positivist approach, could be Knight's (2001) critic of positivist approaches to curriculum processes as he argued;

> …there is nothing wrong with having goals and expressing them as open outcomes. Trouble comes when precise outcomes are linked with indeterminate processes, when they are expressed in generic terms as if achievements were independent of context; when attempts are made to deck them with false precision; and when it is assumed that anything important can be described by precise, generic outcomes (p. 379).

Critics of the positivist paradigm then suggested other alternatives to the view of knowledge, thereby giving birth to the emergence of many other paradigms.

2.7.3 THE INTERPRETIVIST PARADIGM

Interpretivism and constructivism are related approaches to research which emerged from critiques of positivism in the social sciences and in this study they are used interchangeably. They are paradigms which are usually used to discuss research associated with practical interests and the purpose of research in the interpretive paradigm is to understand a specific context as it is without manipulating the environment as would be the case in the positivist paradigm. The practical interest relates to the desire to take the "right action (practical action) within a particular environment" (Grundy, 1987, p. 13). The practical interest "generates knowledge in the form of interpretive understanding which can inform and guide practical judgement"(Carr & Kemmis1986, p. 135). In this paradigm reality is seen as a construction, which is relative to its context – as such this paradigm does not attempt to generalise or replicate. It is context driven and curriculum design within this paradigm thus tries to understand teaching and learning in terms of the environment in which they take place. Knowledge, here, is seen to be a process of making meaning through interaction. The curriculum is not viewed as a linear equation but is rather seen as an ongoing activity shaped by interaction between the educator, the learner, classroom and the broader context. While positivism, as a research design seeks to control the environment, research in the interpretive paradigm seeks to extend human understanding thereof so that we can exist harmoniously within it.

2.7.4 THE CRITICAL THEORY

According to Clark (1999) critical theory is best described using Ortner's (1993) phrase as an "issues-oriented ethnography" as it seeks to explore the ways in which societal issues and their contradictions are worked out in the context of complex "lived" lives that are situated with reference to class, race, place, gender, and other identifications. Critical or Realist Paradigms have emerged more recently and in the context of the debate about

validity of interpretive research methods and the need for appropriate criteria for evaluative qualitative research (Silverman2001). Critical realists assume that there are real world objects apart from the human knower i.e. there is an objective reality. Because critical theory brings a specific standpoint and theoretical orientation to its research questions, it cannot be said to be humanistic in the sense that usually defines qualitative research. While qualitative, interpretive research foregrounds the meanings research participants ascribe to their own actions, critical researchers seek analytically to place such actions in a wider context that is limited by economic, political, ideological forces, forces that might otherwise remain unacknowledged. Critical theorists thus require a greater measure of autonomy from the persons studied, or to use anthropological terms, a more 'etic' (outsider) than 'emic' (insider) position from which to analyze and construct arguments (Clark, 1999).

Research that aspires to be critical seeks, as its purpose of inquiry, to confront injustices in society. Critical researchers assume that the knowledge developed in their research may serve as a first step towards addressing such injustices. As an approach with a definite normative dimension, the research aims for a transformative outcome, and thus is not interested in 'knowledge for knowledge's sake' but to do so in order to effect change. Some critical researchers, in fact argue that a 'neutral' stance toward research can too easily play into the conservative agendas of those who would rather preserve than challenge the status quo (Ferguson & Golding, 1997). With specific reference to education, critical theorists such as Carr and Kemmis (1986), suggest that the critical approach has a concern with the emancipatory function of teaching and learning. The curriculum would be scrutinized for ingrained power relations. The questions asked of the curriculum would be "whose interests are served by the curriculum, what curriculum would promote greater equity, emancipation and social justice, how is power distributed in the teaching learning process and how can it be more equitably distributed" (Grundy, 1987, p. 122). According to McKenna (2010), in the case of a critical approach to an outcomes-based curriculum, great emphasis would be placed on determining who is being served by the outcomes selected and in whose interests the assessment criteria are designed.

In terms of their ontological and epistemological standpoints, realist perspectives are grounded in a theoretical belief that our knowledge of reality is imperfect and that we can only know reality from our perspectives of it. Attaining truth or objectivity is impossible, but is a goal that all research should strive for as this is believed to lead to more rigorous research (Silverman, 2001). Because our ability to know this reality is imperfect, claims about this reality must be subject to wide and critical examination to achieve the best understanding of reality possible. Objectivity remains as an ideal that researchers attempt to attain through careful sampling and specific research techniques. It is also possible to evaluate the extent to which objectivity is attained and this is achievable through evaluation of arguments in light of a community of scholars and researchers of which the researcher is part. By posting a reality that can separate the subject and object, the realist paradigm provides an objective reality against which researchers can compare their claims and the extent to which they ascertain truth. This is sometimes called credibility or trustworthiness of an account. In contrast to some humanistic qualitative researchers who rely upon the claims of science to affirm their study's validity, critical researchers distance themselves from methodologies that are imported from the natural sciences. Qualitative research that emerges from a critical perspective is often viewed as being at the meta-theoretical level, which may encompass and draw from other paradigms, offering an explanation of the workings of power that are often unexamined in logical positivist approaches (with their focus on causal relations between variables) and in humanistic approaches (with their focus on human explanations of actions or meanings) (Clark, 1999). Critical research often encounters from its audience less perceived need to argue for a study's validity using terms imposed from logical positivism. The test of validity in critical research is directly related to its stated purpose of inquiry. The research is therefore valid to the extent that the analysis provides insights into the systems of oppression and domination that limit human freedoms, and on a secondary level, in its usefulness in countering such systems.

Realist approaches tend to rely on a combination of qualitative and quantitative methods and usually incorporate methods such as interviews, observations and analyzing texts to

elicit participants' ways of knowing and seeing. Research which falls into the realist paradigm category is usually conducted in more natural settings and so more situational or contextual data is collected. Research designs associated with this paradigm provide opportunities for discovery (emergent knowledge) as opposed to manipulating the environment and proceeding by testing an a priori hypothesis. In terms of data analysis, critical researchers assume that their task is to expose the hidden assumptions that guide both research respondent statements and often, initial analyses of data. Researchers therefore bring a level of scrutiny to their task that includes rooting out the meanings of what is left unsaid as well as that which is stated. The research is verified as other members of the research community offer corroboration that has come from their own research experiences.

2.7.5 THE RATIONALE FOR ADOPTING THE CRITICAL PARADIGM

The broad use of paradigms in discussions of curriculum design is not in order to judge which paradigm is best but rather the question should be understood as a matter of values and ethical choice. One has to choose which paradigms to work within and does so on the basis of his/her values (Luckett, 1995). When one adheres to a new paradigm one adopts a new way of observing, reflecting on and describing the world. For example Connole (1998), reminds researchers that beneath this jargon of (positivist, interpretivist and critical) paradigms, there is a familiarity to each of them:

> In the everyday world of less than strictly scientific enquiry it is possible to see all of these approaches at work. Most of us are inclined to empiricism when deciding which bank will lend us money most cheaply or where to insure our car. When we are trying to understand a friend who is recounting an upsetting incident we are much more likely to operate in an interpretive mode. The appearance of a politician on our television screen tends to trigger a shift into the critical approach as we pr(OBE) for distortions and hidden agendas. When questioning the tenacity of gender roles in the division of housework we may want to adopt a deconstructionist approach towards our own ambivalences. Thus none of these approaches is wholly unfamiliar (p. 21).

Consistent with this view, a number of researchers find the quantitative-qualitative continuum idea attractive because rather than dividing paradigms into two separate groups (e.g. positivism is quantitative; interpretivism is qualitative) it asserts that there is no 'right' paradigm (Niglas, 2007; Onwuegbuzie, 2000). It was after considering the ontological and epistemological assumptions behind each paradigm together with these

guiding principles that a critical paradigm was adopted in this study. The overall aim of this study was to understand how curriculum alignment, as a change strategy, might contribute to improved learner performances in mathematics in the previously disadvantaged schools of South Africa. Through his pedagogic discourse Bernstein presented a complex analysis of the recontextualisation of knowledge through the pedagogic device, arguing that schools reproduce what they are ideologically committed to eractidicating i.e social class advantages in schooling and society. This study is concerned with the production, distribution and reproduction of official mathematical knowledge and how this knowledge is related to structurally determined power relations. According to Stenhouse (1975), a curriculum is an attempt to communicate the essential principles and features of an educational proposal in such a form that it is open to critical scrutiny and capable of effective translation into practice. He suggests that a curriculum is rather like a recipe in cookery. It can be criticised on nutritional or gatronomic grounds – does it nourish the students and does it taste good- and it can be criticised on the grounds of practicality.

Curriculum documents and programs are so constructed not just because of what is considered to be the best for the learner but because curriculum is social and political process. Critical theory inquires into the taken-for-grantedness of situations, interactions and experiences, and exposes both enabling and constraining issues (Thompson, 2003). In education critical theory also known as critical pedagogy can provide a theoretical and practical lens through which to understand and anlyse educational change processes and to see important aspects of them that are overlooked in more technical-rational explanations. Critical pedagogy studies the role which schools play in maintaining the social stratification of society, and the possibilities for social change through the schools. "Critical pedagogy is both a way of thinking about and negotiating through praxis the relationship among classroom teaching, the production of knowledge, the larger institutional structures of the school, and the social and material relations of the wider community, society, and nation state."

Common questions for the critical educator include:

(1) What knowledge is most worth?

(2) Whose knowledge is most important?

(3) What knowledge should/should not be taught?

(4) How does the structure of the school contribute to the social stratification of our society?

(5) What is the relationship between knowledge and power?

(6) What does this imply for our children?

(7) What is the purpose of schooling – is it to ensure democracy or to maintain the status quo and support big business?

(8) How can teachers enable students to become critical thinkers who will promote true democracy and freedom?

Justifying crtical pedagogy Shor, argues that once we accept education's role as challenging inequality and dominant myths rather than as socialising students into the status quo, we have a foundation needed to invent practical methods. Critical pedagogy, then is defined by what it does—as a pedagogy which embraces a raising of the consciousness, a critique of society, as valuing students' voices, as honouring students' needs, values and individuality, as a hopeful, active pedagogy which enables students to become truly participatory members of society who not only belong to the society but who can and do create and re-create that society, continually increasing freedom. Marcuse states that liberation "presupposes a knowledge and sensibility which the established order, through its class system of education, blocks for the majority of the people. Freire states that there is no such thing as a neutral educational process." Education either functions as an instrument that is used to facilitate the integration of the younger generation in to the logic of the present system and bring about conformity to it, or it becomes 'the practice of freedom' the change or means by which men and women deal critically and creatively with reality and discover how to participate in the transformation of the world."

Consistent with the critical paradigm this research's aim was not only to understand the structural shaping of curriculum alignment but to do so in order to effect change. Although the research started with a document analysis of the NCSM, the major part of the study was carried out in schools from previously disadvantaged communities. It was motivated by a claim in the NCSM that suggests that curriculum reforms in mathematics and science in South Africa, come as an attempt to address issues of inequity and exclusion both educational and social. According to Le Grange (2007), the introduction of (OBE) in South Africa was intended to redress the legacy of apartheid by promoting the development of skills throughout the school-leaving population in order to prepare South Africa's workforce for participation in an increasingly competitive global economy.

> The new curriculum is designed to embody the values, knowledge and skills envisaged in the constitution of the new democratic South Africa. It provides learners with the opportunity to perform at the maximum level of their potential and focuses on high levels of knowledge and skills, while promoting positive values and attitudes DoE (2008a, p. 2).

Contrary to this claim, Edwards (2010), shows how the 2008 pass rates of 62,5% declined in 2009 to a 60.6%. The Department of Basic Education (2010), did a further disaggregation of the 2009 results and showed how they correlated very strongly with the poverty levels with the learners from the previously disadvantaged communities hardly making it through the system. International participation on TIMSS and a further disaggregation of the scores done by the Human Sciences Research Council had earlier revealed that performance of learners correlated strongly with the racial groupings and ultimately those groups of learners who were previously marginalized during the apartheid era continue to be outperformed by the previously advantaged learners (Long, 2007a).

Consistent with the critical paradigms, the concern of this research related to understanding the nature and degree of access to higher order cognitive knowledge and skills across a sample of historically disadvantaged schools. Empirical evidence has shown that in our global economy and democratic society, limiting mathematics education to selected students, whether deliberately or unintentionally, was unacceptable (Zurawsky, 2006), because college and workforce require the same levels of readiness in

mathematics in all students irregardless of their race, gender or any other social classification. The view was that all students require a greater level of "cognitive demand" in mathematics. It is in this context that, after noticing the "rise" in pass rates against public opinion that standards were falling in South Africa, Umalusi (2005), raised the question: 'Have our expectations as expressed by the level of cognitive demand declined?' According to Muller (2004), if this did not become the fundamental question that we attempted to answer, then it should not be surprising that 'we' appear to be making little headway with increasing the attainment levels of candidates in subjects where specialised skills are required like Mathematics.

This research hoped to contribute to the change process by attempting to answer this question. The first research question aimed at examining the level of cognitive demand in the NCSM using data from the lineage of curriculum texts. Because the critical paradigm aims at objective reality, the research employed the use of cognitive demand tools (quantitative) to judge the levels of cognitive demand and alignment in the mathematics curriculum documents borrowing from Porter's (2002), alignment studies cited earlier. To ensure credibility or trustworthiness of this categorization, the coding was done following some rubrics in the form of cognitive demand tables. The coding followed debriefing with experts on the use of these cognitive demand tools - proponents of which define, as well as exemplify, the content that would fit into each of the categories of the cognitive demand tables. In objective reality there is a presumption that any other interested person using the same tools should obtain the same data.

The second research question followed up from the analysis of the mathematics content in the written documents and examined the levels of cognitive demand in and alignment with the 2008 exemplar papers. Alignment in literature has been defined as the degree to which curriculum components are in agreement and serve in conjunction with one another to guide the system towards students learning what they are expected to know and do (Webb, 2002). With reference to the alignment checks in the South African context, many reports point to a tick list approach to ascertaining whether all assessment standards had been covered, rather than considering the quality of the assessment

procedures and whether the appropriate content had been covered. Yet the emphasis should be on the quality of the relationship between the two (Edwards, 2010), and in this study this quality criterion was the motivating factor for examining the cognitive demand levels of the exemplar paper.

Because the study aimed at objective reality it employed the services of an examiner and a moderator of mathematics papers to interpret and contextualize the cognitive demand tools. The team of experts worked with the content in both the examination papers and the assessment standards to exemplify what would count as a task requiring a learner to memorize or apply routine procedures or such other knowledge level. This again was consistent with the assumption in the critical paradigm that objective reality could be achievable through evaluation by a community of scholars (Silverman, 2001). Mathematical tools were also used to measure the levels of agreement amongst the researcher and the experts i.e. an inter-rater reliability was calculated. After the coding was complete, mathematical tools were employed again to calculate the level of alignment (alignment index) between the content in the assessment standards with the content of the examination papers. Details of these strategies can be found in the methodology chapter but suffice it to say all these were measures aimed at getting as close as possible to objective reality thereby improving the credibility and trustworthiness of the study.

Consistent with the view that all students require a greater level of "cognitive demand" in mathematics, the third research question was concerned with how teachers created opportunities for learners to learn the higher order cognitive skills and processes in mathematics. Similar quantitative tools were also employed in the coding of video data. To ensure credibility and trustworthiness of the coding tools they were presented to other scholars during PhD seminars and professional conferences such as ICME, SAARMSTE and AMESA so that they were critiqued and sharpened. These are examples of external audits suggested as strategies associated with the critical paradigms. After the mathematical calculation of the different numerical indicators of alignment, more qualitative techniques were then employed in describing other non numerical indicators

e.g. sequencing of content in the curriculum documents as well as in the classroom interactions. The research employed a multiple case study as an approach and document analysis and observation as methodologies for collecting data from the documents and the classrooms respectively. All these are methodological processes that are associated with the critical paradigm.

2.8 THE CURRICULUM AS A SYSTEM THAT MUST COHERE

There are four main views about curriculum alignment, which are common in literature i.e. (a) incoherence is inevitable (b) the subsystems that comprise the whole must work well both independently and together for the system to function as intended (c) curriculum coherence should serve as an accountability tool, and (d) curriculum coherence should guide effective teaching and learning. Each of these views reflects a different philosophy and has a different implication on implementation as discussed below.

The first view to curriculum coherence is that incoherence is inevitable after all. Scholars who hold this view posit that the rather confused and contradictory nature of messages in policy documents is deliberate and inevitable because standards act as a 'slogan system' under which educators, the public, and funding agencies of varying political and ideological persuasions can fit under the umbrella (Apple, 1992a). Apple posits that if they are to be effective, 'slogan systems' must have a 'penumbra of vagueness' so that each of these groups, groups often at odds with each other, can believe that "there is something in it for us". The implication thereof for implementation was that curriculum coherence was simply an inevitable consequence of multiple and often competing interests.

However Apple, (1992a), was quick to point out that successful 'slogan systems' cannot be too vague, they need to be specific enough to offer something to practitioners. If they do not, the result will be a 'splintered vision' analogous to the concept of "diffusion" in science. In such an environment teachers face a configuration of demands that often contradict one another. This second view follows from a systems perspective that posit

that education is a system that is composed of subsystems, or parts, that each serve their own purposes but also interact with other parts in ways that help the larger system to function (Wilson & Bertenthal, 2005). Because the system and its subsystems are organised around a specific goal, the subsystems that comprise the whole must work well both independently and together for the system to function as intended. “The very nature of organisations argues that we succeed when all parties are rowing in the same direction” (Schmoker & Marzano, 2000, p. 21). Viewed in this context of a system and its subsystems, and consistent with the three primary components of a written curriculum, tested curriculum and taught curriculum some scholars posit that a system naturally functions effectively if those components are all aligned with each other. Consistent with this view, performance-based standards are established in such a way that they are attached to powerful stakes such as progress through and graduation from school, admission to higher education and access to employment opportunities and training. Consequently performance-based standards become self-regulatory systems in that the powerful stakes promote the achievement of desired outcomes without having to resort to coercion, which was the norm in previous reform periods (Cohen & Hill, 1998). The result is that curriculum coherence is taken as a function of management (Finley, 2000) with no specific strategy being employed to check that the subsystems are indeed working together for the common good.

The third and perhaps the most common view to curriculum coherence is that it should serve as an accountability tool (Finley, 2000). Because education is heavily funded by the state from tax-payers’ money, policy makers have a responsibility to account for such funds and standardized test results have always been used as evidence that the tax-payers’ money was used effectively and efficiently. Where coherence has been used in this sense, educational assessment has been driven largely by accountability concerns rather than for educational priorities. This view is usually popular with politicians since it gives them the political mileage that they badly need (Finley, 2000). It is common in such cases that tests are made easy, tests results only reflect norm referenced grades and not criterion referenced grades, test results are usually scaled upwards to conceal actual performance and examination results are announced focusing on quantity or ‘bean counting’ rather

than quality. In the absence of expressly articulated educational principles to guide assessment, political, technical and practical criteria then become the de facto ruling principles (National Research Council, 1993).

The last view to curriculum coherence is that it should guide effective teaching and learning. One strategy of the last decade was the push for coherence in educational policy with the expectation that aligned policy would result in better teaching and learning (Herman & Webb, 2007). Rather than viewing coherence as a management or an accountability tool, the belief in this view was that curriculum alignment should be a normal part of the process of planning teaching/learning activities. The argument was that if standards are seen as policy instruments used to articulate the vision, or framework, of a subject-matter discipline to its educational system, then it was also important that those standards reflect a coherent framework. In this view coherent policy means giving a sense of direction to the educational system by specifying policy purposes, it means establishing high-quality goals about what students should know and be able to do and then coordinating policies that link the goals. The concern becomes quality rather than quantity. Assessment tools are then designed in such a way that they measure deep understanding of concepts and processes. The premise is that it is only when assessments are aligned with both standards and classroom instruction that assessment results can provide sound information about how well teachers are doing in helping students to attain the standards. This view of alignment represents a promising framework for analyzing the extent to which components of the educational system are coordinated, and its measurement has the potential to provide empirical evidence of the potential of classroom instruction to influence student achievement (Roach, et al., 2008). Because this study aimed at finding ways of improving learner performances in the context of new reform, it took this view of curriculum coherence as a tool for improving instruction and student learning. Drawing from the literature, the next sections discuss some of the different forms of coherence that a curriculum can take.

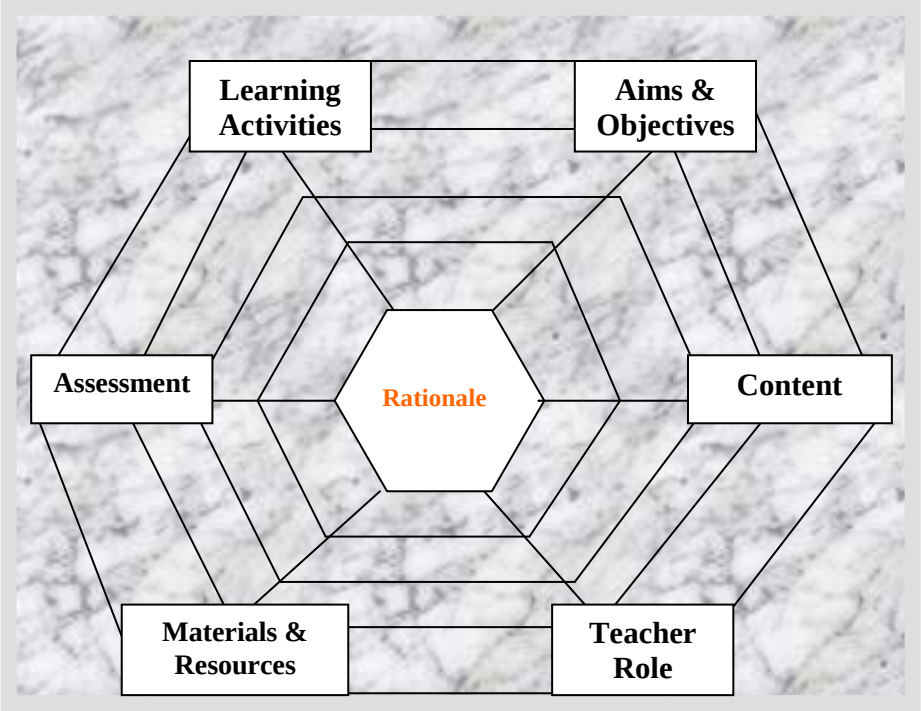
2.8.1 ALIGNMENT/COHERENCE DEFINED

According to Stenlund (2007), the notion of curriculum alignment has recently become one of the most important principles of education reform. Curriculum alignment is typically understood as a systems approach to the development and evaluation of a curriculum. A systems approach has three basic components: inputs, process and output. Similarly Broski (1976), lists three steps in the development of a curriculum: define, develop and evaluate leading to the three major elements of the triad i.e. content, instruction and assessment. Alignment in this sense means that the three functions are directed toward the same ends and reinforce each other rather than working at cross-purposes (Pellegrino, 2006) and if any of the functions is not well synchronized with the others, it will disrupt the balance and skew the educational process. Alignment was being used to characterize the agreement or match among a set of documents or multiple components of an educational system. This view of curriculum alignment is also echoed in this definition:

> A good teaching system aligns teaching method and assessment to the learning activities stated in the objectives, so that all aspects of this system are in accord in supporting appropriate student learning. This system of constructive alignment is based on the twin principle of constructivism in teaching and alignment in learning (Biggs1999, p. 11).

Today, alignment of the components of a curriculum has become more complex and one of the models used to conceptualize this complexity is a spider web configuration as shown in figure 2.2, illustrating not only their interconnectedness, but also their vulnerability (Webb, 2005).

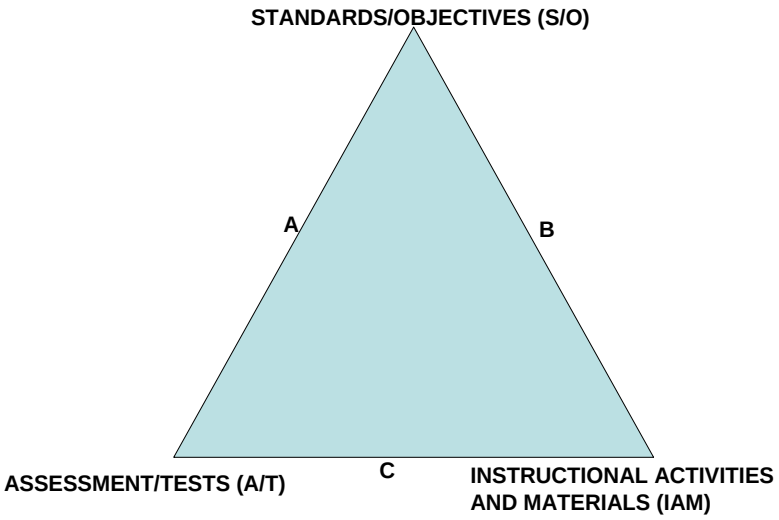
Fig. 2.3 SPIDER WEB RELATIONSHIPS (Ottenvanger, van de Akker, & de Feiter, 2007)



Although emphasis on specific components may vary from time to time, at some point, alignment of all the components has to occur to create and maintain coherence. The spider web illustrates a familiar observation and seems a very appropriate metaphor for understanding curriculum development and analysis. It points to the complexity of efforts to improve the curriculum in a balanced, consistent and coherent manner in that pulling at one or more of the strings of the web will cause the rest of the web to shift (Ottenvanger, et al., 2007). However, if the other strings do not move along adequately, the tension in the web may cause it to break. This is similar to what would happen with the components of the curriculum if they were not aligned hence point to the need to ensure coherence within the components.

While curriculum alignment has been modelled in many different ways, the common conceptualisation has been the three components model comprising the written, the taught and the tested curriculum (English, 1992).

Fig. 2.4 CURRICULUM ALIGNMENT (Anderson, 2002)



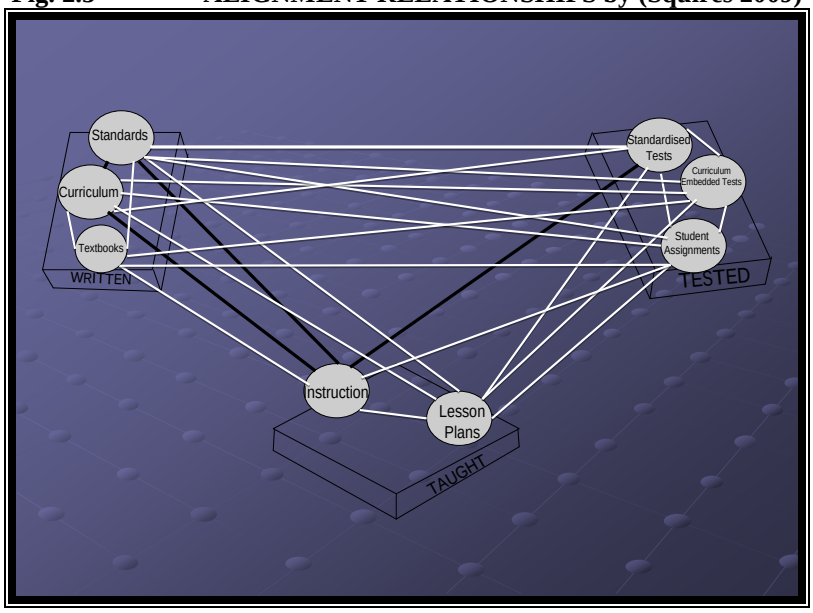
Anderson (2002), provided this alignment view in the form of a triangle which shows the relationships between the three primary components of a curriculum i.e. (Side A) objectives or standards, (Side B) instructional activities and supporting materials, and (Side C) assessments. The sides of the triangle represent the relationships between pairs of components: (Side A) objectives with assessment, (Side B) objectives with instructional activities and materials, and (Side C) assessments with instructional activities and materials. According to Anderson (2002), curriculum alignment is represented by the entire triangle and so requires a strong link between objectives and assessment, between objectives and instructional activities, and between assessments and instructional activities.

Current views on curriculum alignment however find this definition of coherence insufficient because of its reference to a general curriculum framework which could imply mere alignment of a 'laundry list' of content without consideration of other important aspects of the specific subject area (Schmidt, Wang & McKnight, 2005; Wilson & Bertenthal, 2005). In fact this 'laundry list' view of alignment is cited by Fullan (1996), as one of the possible reasons why the systemic reform approach had limited impact on school practice. Similarly, Squires (2009), suggests that alignment is

becoming more precise such that the meaning has been further differentiated so that a match is not just a match of one set of content to another but also to other characteristics, such as balance, range and level of difficulty. In a subject like mathematics for example, lack of coherence might manifest itself in the introduction of a topic before the prerequisite knowledge that makes a reasonable understanding of the topic possible (sequencing of topics) – suggesting that besides the cursory correspondence between content in the different components of the curriculum, coherence or alignment should also take into consideration logical sequencing of that content. Lack of coherence might also manifest itself in the introduction of content (sequencing of content) within the same topic before the prior content knowledge that makes a reasonable understanding of the subject matter possible. This seems to suggest that curriculum coherence was more complex than just the two dimensional linear relationships evident in both the spider web and the triangular configurations above.

Squires' (2009), model provided a three dimensional alignment matrix that he used to organise literature review of the various ways that researchers had studied curriculum alignment. In this model the written curriculum comprises the textbooks, the curriculum (subject statement) and the assessment standards. The taught curriculum comprises the actual instructions and the lesson plans. The tested curriculum comprises the standardized tests, the curriculum embedded tests and students' assignments.

Fig. 2.5 ALIGNMENT RELATIONSHIPS by (Squires 2009)



This alignment matrix does not only reaffirm the complexity of curriculum alignment that Webb's spider web depicted but it presents that complexity in a different form. It can be argued that a three dimensional model depicted in Squires' alignment matrix does not only have the potential to capture the horizontal alignment between different components of the curriculum but it also has the potential to allow analysis of the other forms of alignment like the vertical alignment to happen. Squires did not however use this model to study curriculum alignment but the matrix was a product of superimposing different research studies that have focused on different aspects of curriculum alignment. According to Squires (2009), any of these categories can be aligned to each other depending on the objectives of the study, highlighting not only the complexity of the concept of curriculum alignment but also the possibilities of handling such complexity. Consistent with the three research questions of this study, a path was also carved within this maze and that path is highlighted by the bold lines connecting curriculum/subject statement and standards, (written curriculum) standardized assessment (tested curriculum) and instruction (taught curriculum). This also followed Anderson's (2002), postulation that these were the three primary components of a curriculum. Note that there is also a bold line underneath the white line joining the written/content standards and the standardized tests which does not look evident in the above diagram but which comes out clearly in the model for this study in chapter 3.

2.8.2 WORKING DEFINITION OF CURRICULUM ALIGNMENT

While the concept of coherence has been used in different ways as discussed above, Wilson and Bertenthal (2005), provide a definition that appears to capture the current view that alignment is not just a match of one set of content to another but also to other characteristics such as logical and hierarchical sequencing. They start off with the general view that is shared by many researchers, that a system, such as a curriculum, is considered coherent if the subject statement or objectives, instruction and assessment are all aligned with each other. Wilson and Bertenthal (op cit) then go deeper into this relationship by using such terms as 'horizontal coherence', 'vertical coherence' and 'developmental coherence' to distinguish aspects of alignment within a curriculum. They

contend that a successful system of standards based education is horizontally coherent if the curriculum, instruction and assessment are all aligned with the standards, target the same goals for learning, and work together to support student developing mathematical proficiency. The system is developmentally coherent if it takes into account what is known about how students' mathematical understanding develops over time and the mathematical content knowledge, abilities and understanding that are needed for learning to progress at each stage of the process. There are two common views to vertical alignment both of which contributed to the framing of this study. The first view shared by Wilson and Bertenthal (op cit), is that an educational system is vertically coherent if the curriculum instruments accorded with school practice i.e. there is shared understanding (policy-makers, parents, teachers, students etc.) of the goals for mathematics education that underlie the standards, as well as consensus about the purposes and uses of the standards. The second view is that vertical alignment examines whether standards at one grade level are built upon standards at the previous grade levels (Squires, 2009).

2.8.3 WORKING DEFINITION OF HORIZONTAL COHERENCE

An education system is generally composed of many interconnected, mutually reinforcing components, including curriculum, assessment, teacher professional development and research and evaluation (Howard, 2007), each of which influences and is influenced by the other components. Horizontal coherence has generally been viewed as the degree to which the pieces of a curriculum work together, provide support for teaching and learning, and convey consistent messages to learners (Case, 2005). Horizontal coherence refers to the situation in which components such as curriculum, instruction, standards, and assessments are all grounded in a common model of cognition, learning and representation.

This type of coherence takes various forms some of which point to internal consistency (links between the different documents that constitute the NCSM) and some which are indicators of external consistency (e.g. examination papers). Starting with those that point to internal consistency, one such type is what Webb (2005), referred to as sequential

development. According to Squires (2009), sequential development means developing documents in sequence so that the first document (e.g. state standards) is aligned and used as reference for the second document (e.g. curriculum frameworks or assessments). A similar approach of ensuring this horizontal alignment is one which creates common descriptions of a curriculum, then analyses the alignment between the common descriptions and other parts of the educational system, such as standards, assessments, and instructional plans (La Marca, 2001; Porter, et al., 2007). According to Squires (op cit) none of these approaches had specific criteria for judging alignment; in many cases it was just alignment based strictly on the content of the standards and that of the assessment.

Some of the attributes that could also be analysed in this form of coherence include categorical coherence i.e. the same category of content appears both in the standards and in the other documents. For example, if learner-centred philosophy is at the heart of the curriculum , it should be clear how this approach is reflected in other components of the curriculum such as instructional materials and assessments; similarly if "problem solving" appeared as a major heading in the standards, one would expect "problem solving" to be a major heading and infused in the other specifications (Anderson, 2002). Balance of representation is yet another attribute that could be analysed under this type of horizontal coherence. Balance of representation provides an index of the degree to which one curriculum objective is given more emphasis on the assessment than another. This is premised on the view that all the standards need to be consistently represented in the different documents. Another view of internal consistency that has been identified by researchers has to do with the order in which content is arranged in a learning programme. Following this view of curriculum coherence, Schmidt et al. (2005, p. 529) say,

> We define content standards, in the aggregate to be coherent if they are articulated over time as a sequence of topics and performances consistent with the logical and, if appropriate, hierarchical nature of the disciplinary content from which the subject-matter derives. That is, what students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organised and generated within that discipline. This implies that "to be coherent" a set of content standards must evolve from particulars to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (p. 528)

The general view that researchers hold is that if content standards are not based on the progressive structure that is reflective of the discipline then they are likely to appear arbitrary and will look more like a 'laundry list' of topics. A subtler aspect in which this lack of coherence might manifest itself is in the introduction of a topic in the written curriculum before the pre-requisite knowledge that makes a reasonable understanding of that topic possible. Elsewhere this view is treated as developmental coherence (Wilson and Bertenthal, 2005) but in this study it is being taken in this context as an aspect of internal consistency.

With specific reference to the second view of horizontal coherence the concern is with taking an external audit of the curriculum components against the many other interconnected, mutually reinforcing components, including textbooks, assessment, teacher professional development and research and evaluation. Because of the impact assessment has on teaching and learning the concern with this relationship has mainly been with the extent to which assessment measures the important curricular objectives. From this perspective, horizontal curriculum alignment has been defined as the extent of agreement between academic standards documents and the assessment(s) used to measure student achievement of those standards (Bhola, Impara, & Buckendahl, 2003). Roach, Niebling, & Kurz (2008), view this particular type of alignment as the extent to which curricular expectations and assessments are in agreement and work together to provide guidance for educators' efforts to facilitate students' progress toward desired academic outcomes. Webb (2002), viewed this alignment as the degree to which expectations and assessment are in agreement and serve in conjunction with one another.

Consistent with the research questions and cognisant of the fact that within the time frame of a PhD study it might not be possible to analyse in more detail the many interconnected, mutually reinforcing components of an education system, this study also broadly adopted this rather limited but more powerful view of horizontal coherence i.e. the extent to which assessments are aligned with standards/objectives in the NCSM. This followed empirical evidence that also suggests that most approaches have focused only

on measuring the extent to which assessments are aligned with standards/objectives (Anderson, 2005; Bellis, 1999; Bhola, et al., 2003). While the major focus was on the external links between the taught curriculum and the tested curriculum the discussions however touched though slightly, on internal consistency as discussed above especially within the curriculum documents.

2.8.4 RATIONALE FOR HORIZONTAL COHERENCE

From a systems perspective education is viewed as a system that is composed of subsystems, or parts that help the larger system to function. The subsystems that comprise the whole must work well both independently and together for the whole to function as intended. The very nature of organisations argues that we succeed when all parties are rowing in the same direction.

Although horizontal coherence has been defined in different ways as discussed earlier, in this study the focus is on the links between the written curriculum and the tested curriculum. There are two basic reasons why this type of coherence has attracted so much attention from researchers, including this particular study. Firstly from a rational curriculum planning perspective, in stating the objectives of a curriculum a claim is made about what the learners will do and this must be validated through assessment lest those claims remain rhetoric. According to Knight (1995, p. 13):

> In writing a mission statement, a programme `plan or a validation document, skilled drafting allows us to lay claims to a wonderland of concepts, skills, competences and the like, of which our students are to be made citizens. However, for those who want to know about the quality of a course, programme or institution, the test is whether these goals are assessed and how well they are assessed. In a sense, the way students are assessed is the 'DNA evidence' of their learning experiences. …if there is no evidence of appropriate assessment, then the DNA evidence belies the claim. At best, the absence of assessment suggests that our intentions have not been completely realised. At worst, it says that our intentions were rhetoric, for the benefit of auditors, not students

Another justification for this focus is that there is not only the perception that student learning is closely linked to assessment, but also strong evidence to show that students indeed learn strategically in order to maximize their chances of obtaining good grades. Matching the objectives of assessment to the objectives of the subject or course, is part of the alignment suggested by Biggs (1996), to enhance student learning. Research has

demonstrated how assessments exert direct and indirect control over curriculum and teaching practice at different levels of the school system. According to Biggs (2003), "backwash" happens when the assessment determines what and how students learn more than the curriculum does. Brown & Knight (1994) share a similar view as they also claim that assessment shapes the curriculum as it defines what students regard as important and how they spend their time. In other words students will learn for the assessment and according to Biggs (2003), "They would be foolish if they didn't" (p. 141). Viewed in this way tests therefore act as 'traps' into which both teachers and students find it difficult to escape. So what is assessed determines what is taught and what is learnt. Brown and Knight (1994), capture all this in their claim that:

> … Assessment defines what students regard as important, how they spend their time, and how they come to see themselves as students and then as graduates. It follows, then that it is not the curriculum which shapes assessment, but assessment which shapes the curriculum …..(p. 12).

While the logic on the need to link standards with assessments looks simple, its implications are quite profound (National Research Council, 1993). The metaphor 'You can't fatten a hog by weighing it' has been used several times to point to some of the dilemmas of educational assessment. Experience reveals enormous gaps between current assessment practices and the new goals for mathematics education. There is general consensus that for education to be effective, curriculum, instruction and assessment must harmonize for their mutual support. However, the path from general consensus to specific assessment is far from clear.

A number of weaknesses, which I will in more detail explain shortly, have been identified in the current assessment practices. One of the criticisms has been that educational assessment has been driven largely by practical and technical concerns rather than educational priorities (National Research Council, 1993). For too long a narrow focus on efficiency and reliability has meant that examinations required students to perform a large number of small tasks rather than engage in complex problem solving or such other important mathematical skill. The concern for test developers have been more about coverage where tests are designed by following a check-off lists of mathematical topics. In the absence of expressly articulated educational principles to guide assessment,

technical and practical criteria have become de facto ruling principles (National Research Council, 1993). Current reforms recognise that students must learn to reason, create models, prove theorems, and argue points of view but current assessment practices do not support this vision and often work against it.

Assessment can be the engine that propels reform forward, but only if education rather than measurement is the driving force. Pointing again to the need for identifying the important concepts, researchers argue that important mathematics must shape and define the content of assessment. Rather than forcing mathematics to fit assessment, assessment must be tailored to the mathematics that is important to learn. The goal ought to be assessment tasks that elicit student work on the meaning, process, and uses of mathematics. To be effective as part of the educational process, assessment should be seen as an integral part of learning and teaching rather than as the culmination of the process. However, there is an observation that current assessment practices provide little information about whether students have developed the skills and concepts they need to live and work in the 21st century

Given this situation, ensuring that assessment measures the intended learning processes and outcomes seems appropriate in order to encourage students to learn what the curriculum would like them to learn (Santhanam, 2002). So, in a way assessment should address precisely those performances that are valued and conversely the performance valued in an assessment system should provide a model of the goals of the curriculum. Due to this relationship between standards/objectives and assessment and the power or control that assessment exerts on the curriculum system, reform researchers argue that: "If we do not model in our assessment all the learning outcomes that we value, then our curriculum will degenerate to reflect our impoverished assessment"(Clarke, 1996, p. 329). Yu, Kennedy, Fok, & Chan (2006), confirm the need for this link as they warn that curriculum change will not be effective without making corresponding changes in assessment and to them the quickest way to change student learning is to change the assessment system.

2.8.5 WORKING DEFINITION OF VERTICAL COHERENCE

What emerges from the literature is that vertical alignment can occur at a macro, meso or micro level of an education system. At a macro level vertical alignment implies a connection between policies and initiatives at various levels of governance (Case, 2005). The view is that national policies should guide and be coordinated with those at the state, provincial, and district levels so that resources allocated at the national level are appropriately applied at the state and local levels to have maximum impact on schools and classrooms. From this macro perspective, vertical alignment refers to the coordination of policies up and down structural layers; it connects and aligns policies and programs through the hierarchical levels of the system (Howard, 2007). According to Case (2005), standards and assessment represent only one part of an education system the other parts include curricula, textbook content, the opinions of stakeholders, classroom instruction and student achievement outcomes. An education system is vertically coherent if there is shared understanding (policy-makers, parents, teachers, students etc.) of the goals for mathematics education that underlie the standards, as well as consensus about the purposes and uses of the standards (Wilson & Bertenthal, 2005). In terms of how the three research questions for this study were formulated, it could be argued that the whole study was focusing on vertical coherence at a macro level i.e. the manner in which the written or intended curriculum is translated into classroom practice.

At a meso level, if one views the progression of a learner from one grade up the ladder to the next grade as a process requiring coordination, then vertical alignment articulates the logical and consistent order for teaching the standards-based content in a subject area from one grade level or course to the next (Case & Zucker, 2005). According to Squires (2009), four questions guide studies into vertical alignment at this level i.e. (a) What level of concurrence is there between objectives for the two successive grades? (b) To what extent do comparable objectives increase in depth from one grade to the next? (c) To what extent does the range of content increase from one grade to the next? (d) How does the balance of representation change from one grade to the next?

At a micro level one can also argue that in a subject such as mathematics where content is mostly hierarchical, the same questions that were raised by Squires (2009), can be raised about progression from one topic or content to the other. Consistent with that view, I argue that vertical alignment can also articulate the logical and consistent order for teaching the standards-based content in a subject area from one topic/content to another. In the document analysis it is this vertical alignment at grade and topic level that was found more useful in this study.

2.8.6 WORKING DEFINITION OF DEVELOPMENTAL COHERENCE

Developmental coherence emerged recently as deficiencies were noticed in the traditional view which according to Wilson (2009), starts with a limited idea of the way in which curriculum might inform instruction. The concern was that standards should take into consideration the patterns of development of students as they progress from novices to experts in a particular discipline. With specific reference to mathematics, Wilson and Bertenthal (2005), posit that a system of education is developmentally coherent if it takes into account what is known about how students' mathematical understanding develops over time and the mathematical content knowledge, abilities and understanding that are needed for learning to progress at each stage of the process. This view suggests that developmental coherence could be viewed as (a) 'a picture of the path students typically follow as they learn, (b) a description of the skills, understandings and knowledge in the sequence in which they typically develop (Masters & Forster, 1996)' or (c) 'descriptions of the successively more sophisticated ways of thinking about an idea that follow one another as students learn'. These views suggest that developmental coherence should exist both in the curriculum documents in terms of the way mathematical content is sequenced as well as in classroom practices in terms of how that mathematics content could best be taught. This again confirms the complexity of curriculum alignment that Squires (2009), alluded to earlier as one can notice aspects of both vertical and horizontal coherence that have been discussed in the preceding sections.

In the curriculum documents developmental coherence suggests an orderly development and sequencing of content and mathematical experiences for students (NCTM, 2009). Such progressions include careful sequencing of content, developing skills, identifying connections across mathematical strands, using multiple representations, and relating the mathematics to its applications (NCTM, 2009). This study had no intention to pursue this view of developmental coherence in detail, as it was assumed that issues of content sequencing would be dealt with sufficiently as an aspect of horizontal coherence as discussed earlier.

In this study it is the second view of developmental coherence that is taken i.e. developmental coherence in relation to classroom practices. With specific reference to what goes on in the classroom, Wilson and Draney (2009) advance a position regarding developmental coherence by focusing on the idea of learning progressions which they define as :

> … descriptions of the successively more sophisticated ways of thinking about an important domain of knowledge and practice that can follow one another as children learn about and investigate a topic over a broad span of time. They (learning progressions) are crucially dependent on instructional practices if they are to occur. (p. 7)

This idea of learning progressions suggests an orderly development and sequencing, of content and mathematical experiences for students. Wang and Murphy (2004), share similar views in what they have coined 'instructional coherence' which they defined as causally linked activities/events in terms of the structure of the instructional content and the meaningful discourse reflecting the connectedness of topics, which benefit students' learning of mathematics. This also seems to resonate with 'didactic coherence' a term coined by Andrews (2009), and defined as the logic implicit in the sequencing of concepts, and the extent to which learners are offered connected and integrated experiences of mathematics. According to Silverman and Clay (2009), when the focus is on developmental coherence, the emphasis is not just on doing and learning the mathematics, but rather on developing a scheme of understanding within which a variety of mathematical ideas are connected and that can serve as a conceptual anchor for mathematics instruction. This view takes into consideration the teacher's ability to create opportunities for learners to acquire a profound/deeper understanding of fundamental

mathematics which Ma (1999), defined as a well-organised mental package of highly connected concepts and procedures evidenced by knowing how and also why the sequence of steps in any computation makes sense. So to be able to design and deliver a developmentally coherent lesson a teacher must therefore carefully delineate key mathematical concepts and their associated procedures, identify what children at various stages understand and what they struggle to learn, and then create opportunities for children's deep acquisition of both concepts and procedures (Rittle-Johnson & Alibali, 1999).

2.8.7 DEVELOPMENTAL COHERENCE IN CLASSROOM CONTEXTS

The justification for developmental coherence is premised on the view that education cannot be planned without some reference to development (Kelly2009), and that formal education cannot take place without the adoption of some stance towards development (Blyth, 1984:7). One of the strengths claimed for the developmental view to curriculum approaches is its central concern with individual empowerment; what Bernstein (1996) called 'competence' as opposed to a 'performance' mode of pedagogic practice. It sees the individual as an active being who is entitled to have control over his/her destiny or act autonomously and consequently sees education as a process by which the development of the child's ability to act autonomously becomes a central feature. If formal education is conceived as some kind of guided development to bring about certain changes in pupils' behaviours then the key issue becomes the nature of that guidance (Kelly, 2009).

There is literature that supports developmental coherence in any learning environment in general and in mathematics classrooms in particular. Lambert & McCombs (1998), for example, identified the goal of learning in general as the development of meaningful, coherent representations of knowledge, constructed through linking new information with existing knowledge in meaningful ways. With specific reference to mathematics, Romberg and Kaput (1999), posit that learning mathematics in a meaningful way requires focusing on important mathematical ideas and assisting students to organise these ideas into a coherent whole. This coherence, they argue, provides students with a unified conceptual understanding of the domain of mathematics. This seems consistent with a

'competence' as opposed to a 'performance' mode of pedagogic practice as propounded by Bernstein (1996).

Schmidt et al. (2005), also show the importance of developmental coherence. They argue that if one of the major purposes of schooling is to help students develop an understanding of the various subject matters deemed important by society, such as mathematics and science, then the definition of 'understanding' is important to examine, as a way of viewing the delivery of each discipline intended for schooling. Schmidt et al. (ibid) posit that the goal of helping students understand the subject is facilitated by making visible to them an emerging and progressive sense of its inherent structure. Bruner described this as:

> … opting for depth and continuity in our teaching … to give ….[the student] the experience of going from a primitive and weak grasp of some subject to a stage in which he has a more refined and powerful grasp of it (p. 334)

Viewing it as a curriculum delivery issue and pointing to the need to investigate developmental coherence in the mathematics classrooms in South Africa, the 2009 curriculum review report argued that the new curriculum was never researched or properly trialled and as a result there was a fair amount of criticism of curriculum delivery and implementation (DoE, 2009). Umalusi (2009) also reported that at the FET level, while the desired sequencing of content and skills was clear, this was not always the case for the means of achieving this progression in practice.

2.8.8 TYPES OF COHERENCE AND MY RESEARCH QUESTIONS

Having provided some working definitions of the different types of alignment with the intention of falling back on them to answer the three research questions of this study, this section provides the mapping of the questions and the types of coherence as was envisaged:

Table 2.2 RESEARCH QUESTIONS AND TYPES OF ALIGNMENT

Research Question		Horizontal Alignment	Vertical Alignment	Developmental Alignment
1	What levels of cognitive demand are evident in the mathematical knowledge and skills as articulated in written components of the NCSM?		X	X
2	To what extent are the written and the tested components of the NCSM aligned in terms of the cognitive demand levels?	X		
3	To what extent do Grade 11 teacher practices create opportunities for pupils to learn higher order cognitive processes and skills?			X

The concern in the first research question was with vertical and developmental coherence as the documents were analysed in terms of the cognitive demand levels as well as internal consistency (links between the documents that constitute the NCSM). The second research question was concerned with horizontal coherence as it analysed the external consistency between the standards and the examination papers. In the classroom observations developmental coherence was considered primarily in terms of how teachers sequenced the content for learners during their teaching/learning. This was the concern in the third research question of this study.

This study did not intend to develop a specific research question addressing vertical coherence at a macro level for some important reasons. In the interest of maintaining the scope of this research within manageable limits, the view taken was that the issue to do with there being a shared understanding between policy makers, teachers, parents and learners could constitute a large scale research by itself. However, through the way the teachers implemented the curriculum it was presumed possible to make some inferences about this shared understanding especially between policy intentions and the teachers'. Another reason was that, while there is general consensus on the benefits that accrue from the alignment of expectations and assessment, the curriculum resides in the

realization of these expectations in practice (Barnes, Clarke, & Stephens, 2000). The argument put forth was that an analysis of the system's existing documentation of curriculum expectations and assessment arrangements only determined the degree of alignment in a 'narrow' sense. The recommendation was that researchers should move beyond document analysis and examine what teachers actually do in the classroom. These observations suggest that it is more important to understand how the written curriculum translates into practice than to understand what sense teachers make of the curriculum.

Consistent with those views, Wilson and Bertenthal's (2005) framework of horizontal, vertical and developmental coherence was considered broadly sufficient to analyse both the theoretical aspects (internal consistency) of the curriculum as articulated in the policy documents and how the rhetoric in the policy documents is being assessed (external consistency) and also how it finally translates into practice.

2.8.9 WORKING DEFINITION OF COGNITIVE DEMAND

Because this study intended to make judgments about levels of cognitive demand in the components of the NCSM it was also important to have an understanding of this cognitive demand concept. According to Edwards and Dall'Alba (1981), no single theory of learning was found to be adequate to allow the articulation of a definition of cognitive demand. Each theory reflected some distinctions but failed to account satisfactorily for others perceived to be of importance. This according to Edwards and Dall'Alba (op cit), was in agreement with the conclusion reached by Bloom (1956), when attempting to construct the Taxonomy of Educational Objectives – Cognitive Demand;

> We were reluctantly forced to agree with Hilgard (1948) that each theory of learning accounts for some phenomena very well but is less adequate in accounting for others. What is needed is a larger synthetic theory of learning than at present seems to be available (Bloom1956, p. 17).

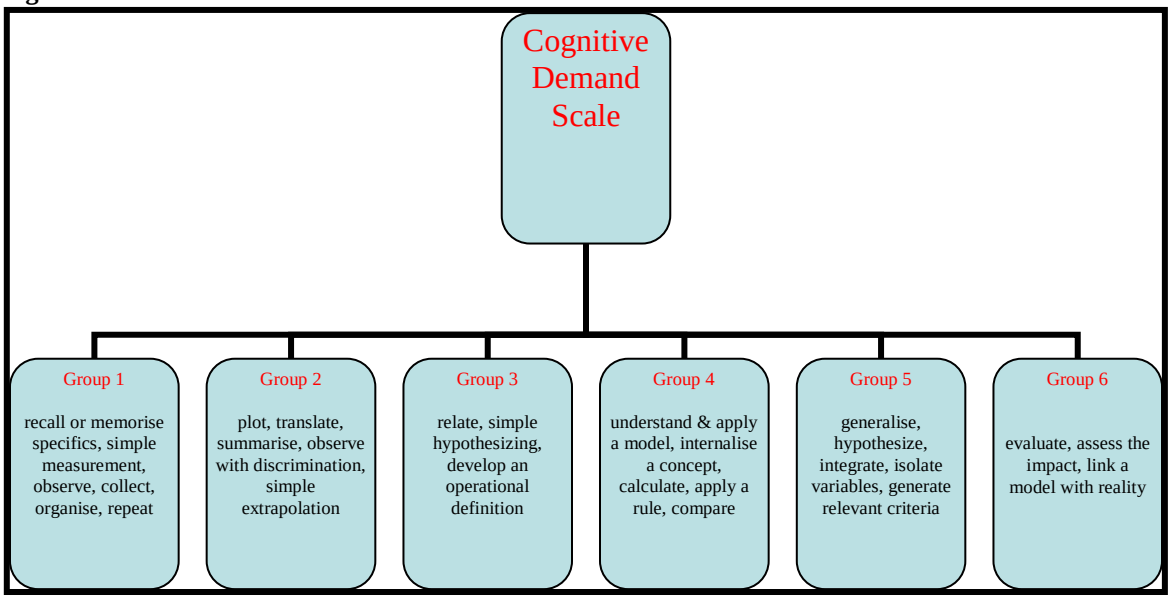
In the absence of an adequate theory, Edwards and Dall'Alba's (op cit) concept of cognitive demand drew from a range of theorists because to them education was a practical discipline, much richer than any one theory could encompass (Tamir 1980). Because this study examined levels of cognitive demand in the documents as well as in

the classroom activities, the entry point into the term cognitive demand was used in two ways to describe learning opportunities in line with Zurawsky's (2006) recommendations.

> The first way is linked with the curriculum policy – how much math? The second way relates to how much thinking is called for in the classroom. Routine memorisation involves low cognitive demand, no matter how much advanced the content is. Understanding mathematical concepts involves high cognitive demand, even for the basic content (Zurawsky, 2006, p. 1).

Moving deeper into this concept, Edwards and Dall'Alba (1981), defined cognitive demand as the demand which is placed on cognitive abilities, through the dimensions of complexity, openness, implicitness and level of abstraction. They then provided a scale of cognitive descriptors that formed the foundation for developing a tool for this study. Their scale of cognitive demand was derived from an analysis of and ranking of tasks in order of increasing cognitive demand according to the cognitive demand construct as defined. Although this model is not exhaustive in terms of what their analysis revealed, their observations were that tasks appeared to fall into six distinct groups, corresponding to six levels of cognitive demand with the following descriptors;

Fig. 2.6 COGNITIVE DEMAND LEVELS WITH DESCRIPTORS



While the definition together with the descriptors gives a general guide on what constitutes a cognitive demand skill, the next challenge was to distinguish which of these descriptors constituted lower order and higher order respectively.

2.8.10 DEFINING HIGHER ORDER AND LOWER ORDER THINKING

Cognitive scientists have hypothesized different levels of knowledge since Bloom's work 50 years ago but there is overall lack of common terminology. However, according to Lewis and Smith (1993), there is general agreement that lower order and higher order thinking skills can be distinguished but for a given individual the need to use higher order thinking will depend upon the nature of the task and the person's intellectual history. Higher order and lower order skills have been described in different ways a few of which will be discussed here.

Maier (1933), for example used the terms reasoning or productive behaviour in contrast with learned behaviour or reproductive thinking to distinguish between higher order and lower order thinking respectively. His definition of productive and reproductive thinking provided a useful distinction between lower order and higher order thinking. Through experiments, Maier (op cit) found that learned behaviour came from contiguous experiences with previous repetitions of the relationships involved in the learned behaviour patterns. However, behaviour integrations that are made up of two or more isolated experiences are qualitatively different, arise without previous repetitions and consequently are new. This constitutes "reasoning" which is used to solve problems. For Maier (1933), a problem arises when behaviour is blocked because a desired end is not at once attainable. Another view of higher order thinking which is linked to problem solving is offered by the Commission on Science Education of the American Association for the Advancement of Science, who argue that a problem solving activity consists of basic and integrated processes (Lewis & Smith, 1993). The basic processes include observing, measuring, inferring, predicting, classifying, and collecting and recording data. The integrated processes include interpreting data, controlling variables, defining operationally, formulating hypotheses and experimenting. The hierarchy represented by

the basic processes and the integrated processes suggest a difference between lower order and higher order thinking skills.

In further defining higher order thinking Bartlett (1958), extended the idea of integrating past experience by using the term gap filling (Lewis & Smith, 1993). Bartlett (op cit) believed that thinking involved one of three gap filling processes: interpolation (the filling of information that is missing from a logical sequence), extrapolation (extending an incomplete argument or statement), reinterpretation (rearrangement of information to effect a new interpretation). Bartlett (1958), defines thinking as:

> ...the extension of evidence in accord with that evidence so as to fill up gaps in the evidence: and this is done by moving through a succession of interconnected steps (p. 75).

From observations in classrooms and interviews, Newman (1990), concluded that lower order thinking demands only routine or mechanical application of previously acquired information such as listing information previously memorized and inserting numbers into previously learned formulas. In contrast, higher order thinking "challenges the student to interpret, analyze, or manipulate information" (Newman, 1990, p. 44).

Newman (1990), also made an important point that since individuals differ in the kinds of problems they find challenging, higher order thinking is relative. A task requiring higher order thinking by one individual may require only lower order thinking by someone else. Accordingly, "to determine the extent to which an individual is involved in higher order thinking, one would presumably need to know something about the person's intellectual history" (p. 45). For my study, Newman's point adds another important dimension, to the understanding of higher order thinking, which allows analysis of developmental coherence as seen in descriptions of successively more sophisticated ways of thinking from one grade to the other. Because cognitive demand descriptors are not grade specific, what Newman (op cit) points to is that a learner at Grade 11 for example may be asked to 'define' and yet a learner at Grade 12 may also be asked to 'define' but in the absence of the mathematical idea to be defined one could not possible judge that defining is a lower order cognitive skill because the idea could be of a lower order cognitive demand at Grade 12 level but defining that same concept for a Grade 4 learner might be a higher

order cognitive demand. So instead of looking at descriptors such as ‘define’ or ‘compare’ or ‘identify’ in isolation, (which are all lower order cognitive demand descriptors) Newman’s point suggests deeper analysis be made of the (what) mathematical idea that is being defined, identified or compared and at what grade level and by who. In my study such analysis was critical in order to make qualitative judgements about developmental coherence from one grade to the other which when taken in Newman’s context seems to be based on the presumption that ‘defining’ or identifying’ gets more cognitively demanding as one progresses from one grade to the next.

Newman’s point also suggests that one would presumably need to know something about the person’s intellectual history in order to determine the extent to which an individual is involved in higher order thinking. This study however did not intend to have a major focus on whether or not learners were actually involved in higher order thinking. Instead the major focus of classroom observations was on analysing the strategies used or the extent to which the teachers created opportunities that enabled/constrained learners developing these higher order skills and processes. However learner responses observed within the teachers’ video were used in some instances to support arguments raised within those analyses of teacher strategies. In the document analysis the major focus was on cognitive demand descriptors in relation to the cognitive demand levels of content in the curriculum documents and in the examination papers details of which are given in the research design chapter.

For the purposes of document analysis the challenge was to develop a tool that could be used to make such judgements. In doing so, this study examined a number of frameworks that have been used to distinguish different levels of cognitive demand in mathematical tasks, some of which have been discussed earlier in this chapter. Although these frameworks differed in many ways, they are quite consistent in distinguishing the extremes (lower order and higher order) of cognitive demand (Ginsburg, et al., 2005; Mullis, et al., 2003; Silver, Mesa, Morris, Star, & Benken, 2009). Low-demand tasks exclusively involve recalling, remembering, implementing, or applying facts and

procedures which is in contrast to high-demand tasks, which require students to analyze, create, or evaluate facts, procedures, and concepts or to engage in metacognitive activity. A great deal of discussion has gone into how many levels of cognitive demand should be made, what the distinctions should be and how they should be defined. However in Porter's view (2002), perfect clarity is not achievable. What was more important was to have distinctions of cognitive demand descriptors that were understood in the same way by each of the raters. Porter et al., (2007) took over 20 years developing such a cognitive demand tool which this study found useful and adapted as follows:

Table 2.3 LANGUAGE FREQUENTLY ASSOCIATED WITH PERFORMANCE GOALS (Porter, 2002, p. 13)

Lower order skills/procedures		**Higher order skills/procedures**		
A	**B**	**C**	**D**	**E**
Memorize facts, definitions, formulas	Perform procedures/solve routine problems	Communicate understanding of concepts	Solve non-routine problems/make connections	Conjecture, generalize, prove
Recognise **Identify** **Recall** **Recite** **Name** **Tell**	**Do computations** **Make observations** **Take measures** **Compare** **Develop fluency**	**Communicate mathematical ideas** **Use representations to model mathematical ideas** **Explain findings and results from statistical analyses** **Explain reasoning** **Describe** **Select**	**Apply and adapt a variety of appropriate strategies to solve non-routine problems** **Apply mathematics in context outside mathematics** **Analyze data, recognize patterns** **Explore** **Judge**	**Complete proofs** **Make and investigate mathematical conjectures** **Infer from data and predict** **Determine the truth of a mathematical pattern or proposition**

Notice that Porter's tool only had five categories or levels of cognitive demand but did not explicitly distinguish them between lower order and higher order. But borrowing

from the literature on the definition of higher order and lower order thinking it was possible to link the descriptors given in Porter's tool with those proposed for lower order and higher order thinking respectively. This distinction was critical in this study in order to analyse and confirm/refute claims made in the policy rhetoric about targeting higher order knowledge and skills.

2.9 ANALYTICAL FRAMEWORK

Research on instructional practice in mathematics classrooms has identified a number of activities that facilitate learners' development of higher order concepts, skills and processes. Cognitively undemanding activities included recalling facts and applying well-rehearsed procedures to answer simple questions quickly and efficiently without much attention to explanation, justification, or the development of meaning (Stigler & Hiebert, 1999). Such pedagogy is at odds with current conceptualisations of how people learn best when the goal is developing understanding which offer learners an opportunity to develop proficiency with complex high-level cognitive processes (Stigler & Hiebert, 1999, Silver, et al., 2009). On the other hand Zurawsky (2006), posits that cognitively demanding activities include using procedures and algorithms with attention to practices such as conjecturing, justifying, explaining and interpreting (procedural/conceptual dichotomy). Empirical evidence shows that high performing countries on TIMSS avoided reducing mathematics tasks to mere procedural exercises involving basic computational skills as they placed greater cognitive demands on students by encouraging them to focus on concepts and connections among those concepts in their problem solving (Zurawsky, 2006). High achieving countries created opportunities for learners to make connections by following through on the rich potential implied within the problem statements. They also gave the students an opportunity to work on problems that required them to construct relationships among ideas, facts and procedures and to engage in mathematical reasoning such as conjecturing generalizing and verifying.

Consistent with how developmental coherence was linked with deeper understanding of mathematical concepts and procedures, and how high and low cognitively demanding activities also reflect the procedural/conceptual frame, Skemp's (1978) distinction

between instrumental understanding and relational understanding seemed plausible as an *a priori* analytical framework. Skemp's views are mirrored in Hiebert and Lefebvre's (1986) notion of conceptual orientations to knowledge versus procedural orientations which is a key framework that has been related to quality within the mathematics education terrain and that also speaks in favour of competence as opposed to mere performance. These orientations have origins in several theories of learning and cognition which posit that our behaviour is shaped by at least two different kinds of knowledge: one providing an abstract understanding of the principles and relations between pieces of knowledge in a certain domain, and another one enabling us to quickly and efficiently solve problems. In recent empirical research in mathematics learning the former is frequently named conceptual knowledge, while the later is labelled procedural knowledge (Baroody, 2003; Schneider & Stern, 2010). Hiebert and Lefebvre (1986) defined procedural knowledge thus:

> Procedural knowledge […] is made up of two distinct parts. One part is composed of the formal language, or symbol representation system of mathematics. The other part consists of the algorithms, or rules, for completing mathematical tasks. The second part of procedural knowledge consists of rules, algorithms or procedures used to solve mathematical tasks. They are step-by-step instructions that prescribe how to complete tasks. A key feature of procedures is that they are executed in a predetermined linear sequence. It is the clearly sequential nature of procedures that probably sets them apart from other forms of knowledge (p6).

On the other hand:

> Conceptual knowledge is characterised most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. In fact, a unit of conceptual knowledge cannot be an isolated piece of information; by definition it is a part of conceptual knowledge only if the holder recognizes its relationship to other pieces of information. (p3-4)

For decades, researchers in the field of mathematics education and cognitive psychology have been interested in the relationship between conceptual and procedural knowledge. A number of questions have been asked about how these two types of knowledge are conceived and how they are linked, including which one develops first and whether one is necessary for the development of the other. Despite a long history of research on the relations between conceptual and procedural knowledge, the conflicting theoretical viewpoints have not converged on a universally agreed upon position but rather have been subject of ongoing debates (Schneider & Stern, 2010).

It is not the intention to delve much into the conflicting viewpoints that emerge from such debates, but rather to take a position which forms the springboard for this study. Generally procedural and conceptual knowledge have been discussed frequently as if they existed on a continuum. The predominant assumption in the field was that at one endpoint of this continuum lie procedural knowledge – identified when skills became routine and could be executed with fluency, in other words, when such knowledge had become "automatized" (Newton & Star, 2009; Star, 2005). At the other endpoint of the continuum lie acquisition of concepts identified when factual or principled knowledge could be used to recognize, identify, explain, evaluate, judge, create, invent, compare and choose i.e. when such knowledge was understood. Star (2005), and his colleagues saw a number of deficits in this view where procedural knowledge was oversimplified to mean rote learning and how conceptual knowledge was also oversimplified to mean deep understanding. Many other terminological distinctions that have been introduced to refer to different aspects of knowledge (e.g. generic/domain specific, formal/informal, elaborated/compiled, implicit/explicit, and tacit/inert) have not been spared this criticism as they also reflect this tendency to collapse many dimensions into one.

Star (2000), Star and Seifert (2002), proposed that there are different ways in which one can know mathematical concepts and procedures and to them skillful execution in mathematics could mean two very different things. On one hand, skillful execution involves being able to use procedures rapidly, efficiently, with minimal error, and with minimal conscious attention; in other words to execute a procedure automatically or by rote. On the other hand, being "skilled" means being able to select appropriate procedures for particular problems, modifying procedures when conditions warrant, and explaining or justifying one's steps to others; i.e. is to execute a procedure thoughtfully or deliberately, rationally, mindfully or intelligently. Star and Seifert (2002), then suggested that there could be other ways in which a procedure could be executed other than rote, some of which could be characterized as "intelligent" or even indicative of procedural understanding. They further argued that there was some evidence to suggest that such knowledge exists. Similarly they argued that there could also be qualitatively different

ways in which one can know mathematical concepts as these concepts can be "memorized" at one extreme or "understood" on the other.

To challenge what they viewed as a 'simplistic' view to procedural and conceptual understanding and to suggest alternative ways in which this topic could be conceptualized and studied Star (2000), analysed studies done from 1983 – 1996 on the relationship between concepts and procedures and noted that almost all the studies were from the topic areas of counting, single-digit addition, multi-digit addition and fractions – all areas of study in elementary school. Tapping from this analysis of these empirical studies on the topic, the researcher argued that there was a notable reliance on elementary school mathematics as the predominant domain of inquiry in the study of procedural and conceptual knowledge and that there was an equally notable absence of studies of the development of procedural and conceptual knowledge in algebra, geometry, and calculus. A common perception from the analysed studies was that in the absence of conceptual knowledge of place value for example, a student could only know how to add multi-digit numbers by rote. Similarly in the absence of conceptual knowledge of fractions, a student could only know the procedure of adding fractions by rote. The researcher posited that research emphasis on elementary school procedures tended to obscure the existence of what he termed 'procedural understanding' through the exclusive focus on knowledge of procedures as necessarily superficial or rote. He further posited that it was also difficult to conceive of having understanding of a procedure when one only considered the relatively simple and short procedures learned in elementary school.

Star and Rittle-Johnson (2009), then followed their theory that there are different ways in which one can know mathematical concepts and procedures with a study on high school mathematics focusing on more complex and abstract procedures such as those learned in algebra and calculus and hypothesized that it was possible to conceive of a non-conceptual yet deep way in which a procedure could be known – something they coined 'procedural understanding'. Results from their research suggested that such knowledge existed and their proposition was that distinguishing between knowledge types and depth of knowledge had the potential to illuminate alternative ways in which conceptual and

procedural knowledge could be known and understood (Star, 2000, 2005; Star & Rittle-Johnson, 2009; Star & Seifert, 2002). Many developmental psychologists have found it useful to treat knowledge not as a unitary construct but as differentiated into at least two kinds of knowledge: (a) conceptual knowledge, facilitating understanding of abstract principles, and (b) procedural knowledge, assisting in solving concrete problems (Schneider & Stern, 2010) both of which can cognitively be represented at a superficial as well as at a deep level. It is this conceptualisation of procedural and conceptual understanding that this study takes.

Star (2005), maintained his earlier proposition that deep procedural knowledge would be knowledge of procedures that is associated with comprehension, flexibility and critical judgment and that it is distinct from but possibly related to knowledge of concepts. He then defined procedural understanding as he had suggested earlier thus:

> … to understand a procedure is to have a planning knowledge – knowledge of such things as the order of steps, the goals and sub-goals of steps, the environment or type of situation in which the procedure is used, constraints imposed upon the procedure by the environment or situation, and any heuristics or common sense knowledge which are inherent in the environment or situation. This knowledge is abstract and deep, but not necessarily conceptual …..(Star, 2000, p. 6)

This definition seems to differentiate between 'procedural understanding' and 'surface/superficial procedural knowledge' terms which have always been viewed as being synonymous with low level understanding. Procedural understanding as seen by Star (op cit) seems to resonate with VanLehn and Brown's (1980); teleological semantics of a procedure which they defined as;

> …knowledge about [the] purposes of each of its parts and how they fit together….Teleological semantics is the meaning possessed by one who knows not only the surface structure of a procedure but also the details of its design (p. 95).

According to this view a procedure can be cognitively represented on multiple levels. On a very superficial level, a procedure may be represented simply as a chronological list of actions or steps; on a more abstract level, a procedure can include planning knowledge in its representation. Planning knowledge includes not only the surface structure but also "the reasoning that was used to transform the goals and constraints that define the intent of the procedure into its actual surface structure" (VanLehn & Brown, p. 107).

Along the memorized and understood continuum the predominant view has also been that superficial/rote/memorization of concepts/procedures is "bad" while deep understanding of both is "good." While there might be fewer disputes on the view that deep understanding of either procedures or concepts is good, the 'rote is bad' view has also been a subject of contestation. Empirical evidence suggests that under some circumstances children learn important mathematical ideas through rote e.g. counting, and later develop an understanding of the concepts underlying it. According to Rittle-Johnson and Alibali (1999), several studies have shown that children count or even add starting by rote but often understand the principle of commutativity for addition even though they never received instruction on the principle. Because children do not receive instruction on counting principles and commutativity principles, understanding of these principles is probably abstracted from their experiences. These studies suggest that rote learning is only bad if it does not eventually result in deeper understanding. However while some studies have shown that procedural knowledge could lead to gains in deep conceptual understanding and vice versa (Rittle-Johnson & Alibali, 1999), the strengths of their influences might not be symMatrical. There is more empirical evidence to suggest that gains in deep conceptual understanding led to fairly consistent improvements in procedural understanding. It is against such empirical evidence that Star (2000), proposed the broadening of the current conception of procedural/conceptual knowledge to include the depth of knowledge at endpoint of acquisition of both concepts and procedures.

Consistent with this view and in an attempt to describe knowledge more parsimoniously some researchers' proposition was for separating the characteristics of knowledge into *type* of knowledge (procedural/conceptual) and *quality* of knowledge (superficial/deep) (Star, 2000). The notion of quality of knowledge seems to resonate with the view that understanding exists on a continuum;

> ... Everyone understands to some degree anything that they know about. It also follows that understanding is never complete; for we can always add more knowledge, another episode, say, or refine an image, or see new links between the things we know already (White & Gunston, 1992, p. 6)

Depth of knowledge refers to the extent that knowledge is firmly anchored in a person's knowledge base and the dimensions of depth of knowledge are surface (superficial) versus deep, with the implication that surface is poor and deep is good (Star, 2000). Deep level knowledge of both concepts and procedures is preferred because it is associated with comprehension and abstraction and with critical judgment and evaluation. Deep-level knowledge has been structured and stored in memory in a way that is maximally useful for the performance of tasks, while surface-level knowledge is associated with rote learning, reproduction, and trial and error. Deep understanding of both procedural and conceptual knowledge should be the ultimate goal and priority of all mathematics learning as it refers to an integrated and functional grasp of mathematical ideas. Students with deep understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful. They have organised their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know (Kilpatrick, Swafford, & Findell, 2001).

With specific reference to LO2 (Functions and Algebra) which is one of the focus areas of this study, the NCSM suggest that both procedural and conceptual knowledge is valued, but at a deeper level as evidenced by this statement:

> It is important that the Learning Programme provides for appropriate experiences of these problem types, and that it develops the underlying concepts and techniques to enable learners to experience the power of algebra as a tool to solve problems. The emphasis is on the objective of solving problems and not on the mastery of isolated skills (such as factorization) for their own sake (Department of Education, 2003, p. 13).

Star (2000), provided a categorization of the types of knowledge and the quality of knowledge respectively as discussed above:

Table 2.4 QUALITIES OF PROCEDURAL AND CONCEPTUAL KNOWLEDGE **(Star2000).**

KNOWLEDGE TYPE	**KNOWLEDGE QUALITY**	
	Superficial	**Deep**
Procedural	Automatized procedures	Procedural understanding
Conceptual	Memorized concepts	Conceptual understanding

This study took the view that competence in a mathematical domain requires knowledge of both concepts and procedures (Rittle-Johnson, Siegler, & Alibali, 2001). Both procedural and conceptual knowledge are therefore important and must be inculcated in the classroom. Both can also be associated with degrees of quality with both categories possibly having 'superficial' as well as 'deep' knowledge. In this study another view taken was that the relations between conceptual and procedural knowledge are bidirectional suggesting that improved procedural knowledge can lead to improved conceptual and vice versa. This follows empirical evidence that has shown that knowledge of concepts and knowledge of procedures are positively correlated although their influence on one another may not be equivalent (Rittle-Johnson & Alibali, 1999). This suggests that the two might be learned in tandem rather than independently with no fixed order in the acquisition of the two but with deep understanding of both concepts and procedures being the more preferred endpoint of acquisition (Rittle-Johnson, Star, & Durkin, 2010). Premised on the view that procedural and conceptual understanding should complement each other for meaningful understanding with deep understanding of both concepts and procedures being the more preferred endpoint of acquisition, this study became more concerned about quality of teacher utterances rather than focusing on the distinction as to whether the utterances or actions were rooted in conceptual or in procedural knowledge. What is then fore-grounded are the connections that teachers enable/constrain and the extent to which such connections in turn enable/constrain deep conceptual and procedural understanding.

2.10 INDICATORS OF QUALITY OF MATH KNOWLEDGE

In developing the analytical tool, this study borrowed from a number of researchers including Businskas(2008), Kilpatrick et al., (2001), National Council of Teachers of Mathematics (1989), Sierpinska (1996) before moving to connections. Businskas (2008) was one of the researchers who seemed to use the term 'conceptual understanding' to mean deep understanding of both procedural as well as conceptual knowledge as discussed earlier. There is evidence in her model of mathematical connections (as will be seen later) that suggests both conceptual and procedural connections were identified in

her study yet she continued to describe the deep understanding of both as 'conceptual understanding'. Consistent with the view that both procedural and conceptual knowledge can be associated with degrees of quality with both categories possibly having 'superficial' as well as 'deep' knowledge, Businskas model was analysed in more detail as a potential tool but the term 'deep understanding' will be used in place of her 'conceptual understanding'. Businskas (2008) posited that while deep understanding could be visualized from different perspectives, describing it in terms of making mathematical connections was evident in the work of several well respected mathematics educators. Empirical evidence shows that high performing countries on TIMSS for example placed greater cognitive demands on students by encouraging them to focus on concepts and connections (Andrews, 2009). According to Zurawsky (2006), elevated thinking processes come into play when students focus on mathematical concepts and connections among those concepts. High-level cognitive processes require emphasis on reasoning about and connecting ideas and solving complex problems (Silver, et al., 2009). According to the NCTM, (2000) making connections is usually treated as synonymous with (or perhaps an indicator of) "deeper and more lasting understanding." Barmby et al., (2009), echo similar sentiments as they contend that;

> In order to examine someone's understanding of a mathematical concept, it is important that we examine the connections that a person makes to that concept. Of course, we cannot see these internal connections directly; rather, we must observe the connections that a person can demonstrate and then infer understanding from these. ….However it is not just a case of looking at the number of connections but the quality or strength of the connections as well. (pp. 5 - 6)

Both the research literature and the pedagogical literature stress the value and importance of making mathematical connections, the rationale being that making connections will allow students to better understand, remember, appreciate and use mathematics. These views from international literature point to mathematical connections as a potential indicator of quality featuring within 'deep conceptual' and 'deep procedural' understandings. This led to the question of whether there was similar evidence in the NCSM.

2.11 CONNECTIONS AND THE NCSM

With specific reference to the NCSM at FET level in South Africa, Mwakapenda (2008), argued that relationships, hence connections were at the heart of the definition of mathematics and that the curriculum makes connections among the key elements of the learning outcomes and experiences to be gained by learners. Below is a table that suggests this focus from Grade 10 right through to Grade 12 (Mwakapenda, 2008, p. 191)

Table 2.5 COMPETENCE DESCRIPTIONS FOR LEARNER ACHIEVEMENT

BY THE END OF GRADE 10 THE LEARNER WITH MERITORIOUS ACHIEVEMENT CAN	MAKE CONNECTIONS AMONG BASIC MATHEMATICAL CONCEPTS (DOE, 2003 P. 74)
By the end of grade 11 the learner with meritorious achievement can	make connections between important mathematical ideas from this and lower grades (DoE, 2003 p. 75)
By the end of grade 12 the learner with satisfactory achievement can	make connections across important mathematical ideas and provide arguments for inferences (DoE, 2003 p. 77)
By the end of grade 12 the learner with outstanding achievement can	synthesize across different outcomes and make connections with other subjects (DoE, 2003 p. 78)

Mwakapenda (2008), also analysed the Learning Programme Guidelines, and showed that from Learning Outcomes 1 right through to Learning Outcome 4 there were statements that suggested the prevalence of connections across learning outcomes. Phrases and words such as generalizing, identifying patterns, modelling, integration within and across subjects, identifying rules and relationships, which are prevalent in the document all, suggest making connections in one way or the other. Parker (2004) made similar observations when she commented:

> Whereas the earlier curriculum was very much product oriented working on the basis of 'received' knowledge, this curriculum is not – it is more practice oriented and focused on producing "connected knowledge". It focuses on the practices of mathematics (e.g. investigating, making conjectures, justifying, generalising) rather than simply the skills (e.g. factorising) and products (e.g. laws of exponents); and on making meaning not only through problem solving contexts, but also within the structure of mathematics itself (p. 6)

Mwakapenda (2008) then concluded that the NCSM in South Africa, was:

> ... replete with demands upon educators and learners for making connections, producing representations and working in integrated modes within mathematics and across curriculum disciplines(p. 201).

In summary, this seemed to suggest that one possible indicator of the development of deep understanding or lack thereof within the learners would be the manner in which the teacher created opportunities for learners to make such connections. With specific reference to the creation of opportunities for learners to make mathematical connections, Weinberg (2001) acknowledges that learners might make connections spontaneously but "we cannot assume that the connections will be made without some intervention"(p. 26). The implied role for the teacher was to act in ways that would promote learners' making of these mathematical connections.

While Mwakapenda (2008), acknowledged that the emphasis on connections therein was consistent with developments in mathematics education globally, he however pointed to the same important gap that appears not to have been recognised both in theory and practice which has to do with the preparedness of teachers to work in a reformed curriculum that demands making mathematical connections. He commented that the practices that are being asked of teachers are often difficult to define, and that they require a substantive re-orientation not only of teachers' practices but also of their beliefs about mathematical ideas, teaching and learning. Similarly Adler and Reed (2002) argued that teachers needed to develop a specialised pedagogy "for the complex task of transforming this knowledge into appropriate opportunities for learning in school"(p. 151). It is in this context that this study tried to understand the nature of the strategies that FET mathematics teachers employed and to what extent such strategies enabled/constrained learners' abilities to make connections.

2.12 SUMMARY

This chapter started off from the history of curriculum alignment. It traced this to the time of and cited the contributions made by Tyler (1949) and Bruner (1960) which have endured to this day. Tyler for example identified four critical questions that must be

asked or answered when a curriculum is developed and the literature review shows how these design features are evident in the National Curriculum Statement for Mathematics. His model was organised around four corresponding principles which suggest coherence must be considered at design stages. Bruner then advocated that fundamental ideas must be identified, and once identified, they must allow a student to move from a primitive and weak grasp of the subject matter to a stage in which he has more refined and powerful grasp. But the literature shows lack of consensus on what constitutes fundamental or important mathematical ideas.

The recommendations thereof were that important goals should instead be described in terms of quality defined in terms of cognitive demand. These should then anchor the curriculum in that all its components should target such higher order thinking knowledge and skills. A preponderance of the literature articulates the need for learners to have higher order knowledge and skills in mathematics. The justification was that these were skills which prepared the learner for the field of work and so empowered the learners both economically and socially. These were also skills needed by any nation for economic development.

Having established what should anchor the curriculum, the design features of the NCSM were then analysed. There was prima facie evidence of rational logic in the design features and these were acclaimed internationally. The concern then became alignment of the curriculum components with a depth of knowledge criterion. This then guided the literature search on such alignment studies. International literature showed that lack of excellence in American schools was caused by misalignment between the written, the tested and the taught curriculum justifying why curriculum alignment was important. There is also empirical evidence which shows that alignment was a more powerful predictor of student achievement than demographic factors and that this alignment effect on student performance was more powerful for low achievers. These findings suggested that alignment could be used as a lens through which researchers could look into issues of inequalities which manifest in poor performances of learners from previously disadvantaged communities. This international literature review also provided a guide on

the descriptors of cognitive demand that formed the bases of search for cognitive demand tools used by other researchers.

The South African literature search showed that post democracy researchers were more concerned about educational issues that cut across disciplines pointing to the need for regular checks especially on alignment and the cognitive demand health of the education system and its subsystems. The literature search for such alignment studies showed that there appeared to be a paucity of studies on alignment driven by the quality of knowledge measured in terms of cognitive demand. This then provided a rationale for the study examining the extent to which levels of cognitive demand were evident in the National Curriculum Statement and the justification for curriculum coherence.

The literature review set the study in the critical paradigm which is driven by an emancipatory objective and attempts to challenge the status quo. Curriculum rhetoric makes claims about empowering the previously disadvantaged learner through an education system that targets higher order knowledge and skills yet such claims do not appear to translate into reality. A variety of views to curriculum alignment are discussed with some researchers arguing from a political perspective that misalignment was inevitable after all because stakeholders have incompatible expectations about what an educational system should deliver. Those arguing from a systems perspective put it that subsystems of a whole should work together for the common good of the whole. Researchers arguing from an emancipatory perspective put it that in order to redress inequalities, the educational goals must be guided by a quality criterion measured in terms of cognitive demand. This then justified why this study was located in the critical theory paradigm.

Some working definitions of curriculum coherence, horizontal coherence, vertical coherence and developmental coherence are then offered. Cognitive demand descriptors are identified and the definition of lower order and higher order thinking is given. The next chapter details the methodology and the data collection tools for this study.

CHAPTER THREE - RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

Literature suggests that the way researchers develop their research designs is fundamentally affected by a number of factors which included the specific interest of the researcher (Dooley, 2002; Noor, 2008; Rowley, 2002; Yin, 1994; Zainal, 2007). Rowley (2002) for example sees a research design as the logic that links the data to be collected and the conclusions to be drawn to the initial questions of a study. It ensures coherence. The design of a study can be thought of as a blueprint detailing what will be done and how this will be accomplished. Research design involves determining how a chosen method will be applied to answer the chosen research question(s). Key aspects of research design include: research methodology, participant/sample collection and assignment, (if different conditions are being explored); and data collection procedures and instruments.

The specific interest of this study was in understanding the level of cognitive demand and coherence in and between the different components of the NCSM. It was triggered by a combination of observations both locally and internationally. Locally (in South Africa) it was observed that while pass rates were rising, standards were falling in mathematics. Internationally, statistical analysis showed that poor performances in over 40 countries that took part in the TIMSS studies including South Africa could be attributed to the level of coherence in the curricula for those respective countries. This study then became interested in curriculum coherence as a specific phenomenon and aimed at using it as a lens to see how theory translates into practice in the South African classroom context. The study hoped to achieve this not by controlling variables (as was the case in the large scale TIMSS studies) but rather by observing all of the variables and their interacting

relationships across a small sample of schools. This was premised on the view that while it was worthwhile to have a general picture as was established by the large scale studies, it is sometimes even more important to understand specific cases and ensure a more holistic and in-depth approach to research in a specific context.

Consistent with this view, the case study design was considered more appropriate for this study and a number of reasons were identified in order to justify why. Unlike quantitative analysis which observes patterns in data at the macro level on the basis of the frequency of occurrence of the phenomenon being observed, case studies observe the data at the micro level (Rowley, 2002). Basically a case study is an in-depth study of a particular situation rather than a sweeping statistical survey. It is a method used to narrow down a very broad field of research into easily researchable topic. Whilst it might not answer a question completely, it has the potential however to give some indications and allow further elaboration and hypothesis creation on the subject. The case study research design is useful for testing whether scientific theories and models actually work in the real world. For psychologists, anthropologists and social scientists case study designs have been regarded as a valid method of research for many years (Yin, 1994).

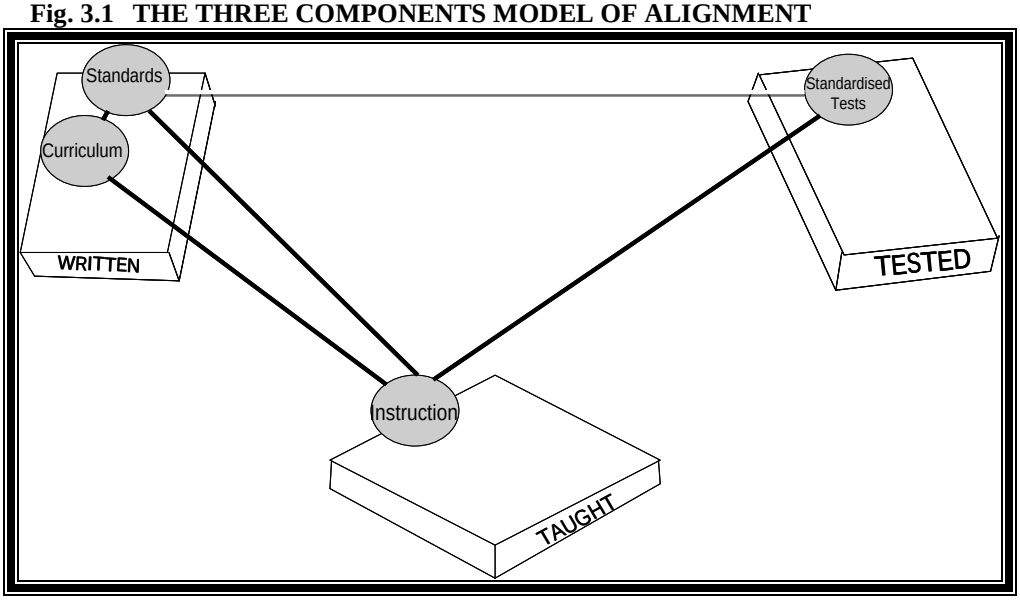
Case studies can involve single or multiple cases. If two or more cases are shown to support the same theory, replication can be claimed. In analytical generalisation, each case is viewed as an experiment, and not a case within an experiment. The greater the number of case studies that show replication the greater the rigour with which a theory has been established. Multiple case designs are preferred on the basis of this replication logic because multiple cases can be regarded as equivalent to multiple experiments. The more cases that can be marshalled to establish or refute a theory, the more robust are the research outcomes. A frequent question is how many cases should be included in a multiple case study? There is no simple answer to this question but the caution from literature was that cases needed to be carefully selected so that they either produce similar results (literal replication), or produce contrasting results but for predictable reasons (theoretical replication) (Rowley, 2002).

The unit of analysis is the basis for the case. It may be an individual person, or an event, (such as a decision, a programme, an implementation process or organisational change), or an organisation or team or department within the organisation. It may sometimes be difficult to identify the boundaries of the unit of analysis. A key issue is that the case study should only ask questions about the unit of analysis, and any sub-units; sources of evidence and the evidence gathered are determined by the boundaries that define the unit of analysis. In the next section the multiple-case research design is described in more detail and justification is also provided as to why it was considered appropriate for this study.

3.2 A MODEL OF THE STUDY

Curriculum has many different components and meanings but within those versions the most common elements are those of the three components-model comprising the written, the taught and the tested curriculum (English1992). According to Squires (2009), this three-components-model provides a first cut in examining the problem of alignment and consistent with how the research questions were conceptualized in this study, it is this three-model-configuration that was adapted. Squires' model had to be adapted because it captures the many components of the curriculum and their possible linkages at the same time. However while this study acknowledged the complexity, there was need to strike a balance between complexity and manageability considering the time constraints of a PhD study. Squires (2009), suggested that the various alignments as depicted by his model (on page 58) present different levels of complexity and the definitions of such alignments also take into account the different time perspectives (daily for lesson planning versus yearly or longer for state standards). It was after taking all these factors into consideration that the model below was adopted for this study.

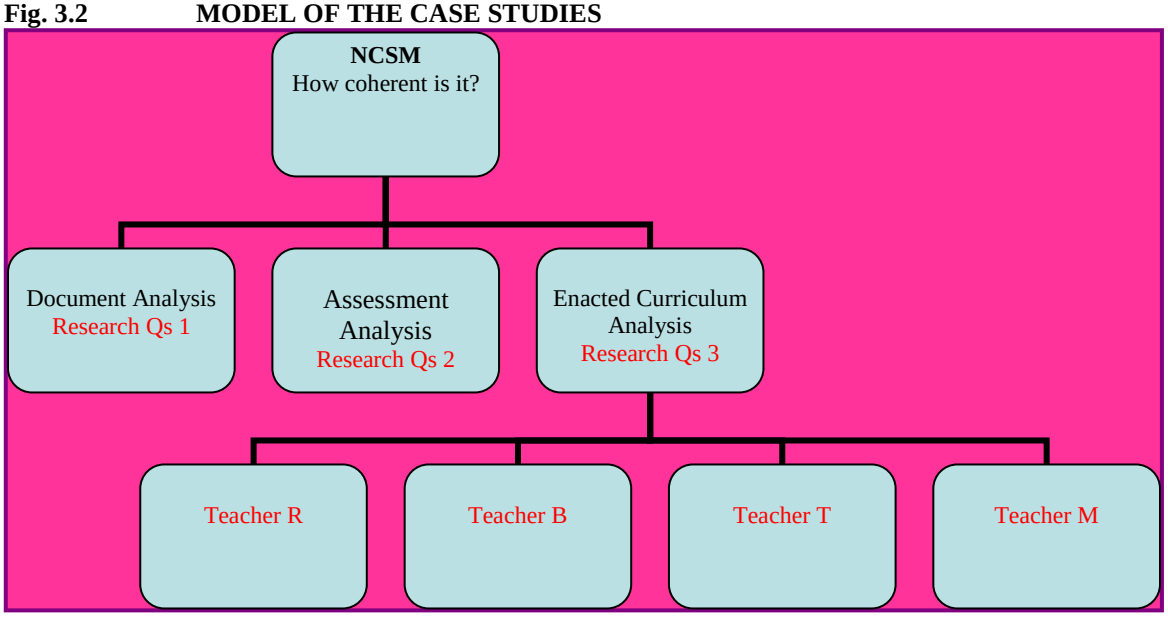
Fig. 3.1 THE THREE COMPONENTS MODEL OF ALIGNMENT



The three dimensional cuboids in the model allowed the examination of the cognitive demand levels within each of the three components, the written, the tested and the taught curriculum. This was consistent with the three research questions which were raised in this study as well as Bernstein's three pillars of education. The written curriculum in the context of the NCSM comprises the subject statement (which Squires loosely refers to as curriculum) and the standards (which the NCSM refers to as assessment standards). In the context of the NCSM these two complement each other in articulating the content or what the learners are expected to know hence they represent the written curriculum in the model above. Broadly the other components that would normally constitute the written curriculum such as the learning programme guidelines and the assessment guidelines were not considered in this model as some of them (e.g. learning programme guidelines) focus more on strategies for delivering content whereas the focus of my document analysis was on cognitive demand levels of the content. However the analyses will cite examples from any of these other documents where appropriate to support certain arguments raised. In as far as the taught curriculum is concerned, the specific focus was on actual instruction, and hence the lesson plans and learner assignments were also left out. Similarly with the tested curriculum the interest was with the exemplar papers for 2008 as they were the first standardised tests available for the revised FET mathematics curriculum – hence curriculum embedded tests and learner exercises were also left out.

Mapping the model with the multiple cases the study was conceptualized as follows:

Fig. 3.2 MODEL OF THE CASE STUDIES



Because the unit of analysis could be an individual person or a document depending on what the researcher desired to focus on (Zainal, 2007), this study considered the written curriculum as contained in the different documents of the NCSM as a unit of analysis that could be studied in its own right. Similarly the tested curriculum as seen through the examination papers was also considered as a unit of analysis in its own right and finally the enacted curriculum as seen through the way in which the 4 selected teachers taught algebra related topics were considered as four multiple cases at the enacted curriculum level of the study. Across all these units, there was a focus on higher order cognitive demand and the level of coherence.

3.3 DATA COLLECTION METHODS USED

3.3.1 PREAMBLE

According to Rowley (2002), case study research can be based on any mix of quantitative and qualitative approaches. Tools used in this type of data collection are usually surveys,

interviews, document analysis and observation, although standard quantitative measures such as questionnaires are also used. Consistent with the title of my study "from coherence in theory to coherence in practice…" the research questions were also conceptualized at both the theoretical level and at the practical level. Similarly my data collection methods and instruments were also conceptualized at the theoretical level i.e. those that could be used to establish alignment among the documents, as well as at the implementation level i.e. those that could be used to measure level of coherence between the documented vision and the classroom practices.

3.3.2 DOCUMENT ANALYSIS AS MY METHOD FOR QUESTIONS 1 AND 2

At the policy or theoretical level, Roach et al., (2008) outlined three methods for establishing the alignment among the policy elements of curriculum i.e. content, instruction and assessment systems. These are (a) sequential development (b) expert review, and (c) document analysis. Sequential development involves creation and acceptance of one policy element, which subsequently serves as a "blueprint" for the creation of additional policy elements. The process of expert review involves the convening of a panel of content experts to review the policy elements and determine the extent of their "match" or alignment. Document analysis involves the coding and analysis of documents that represent the different policy elements. Of these three approaches, document analysis was adopted because the other two methods are useful at curriculum design and developmental level. In other words, they help in creating the documents while on the other hand document analysis works with already existing documents.

The study employed document analysis as a way of collecting data required to answer the first two research questions that were conceptualized at the theoretical level. As implied by its name, document analysis, also commonly referred to as content analysis or extant data analysis, refers to the processes of locating and analyzing facts or trends in already existing documents (Witkin & Altschuld, 1995). It is the gathering of information used in a formal description of the text, studying and analyzing the content and then processing and understanding of the contents in the documents so that conclusions may be drawn. According to Pershing (2002a), document analysis therefore refers to the analysis of any

type of document for the purpose of gathering facts. With specific reference to rational curriculum planning and design, Knight (1995), refers to this process as curriculum auditing (of the intended curriculum) and argued that it is a good way to stimulate discussion about curriculum coherence. This kind of analysis can be used to draw inferences about the degree to which pedagogic practices are likely to be in sync with the curriculum goals which have been framed in response to the expectations of the citizens for whom policy is designed. The choice of this method was premised on the view that documents can contribute a different level of analysis on the gap between official policy and practice (Bryman, 1989).

3.3.3 ANALYTICAL TOOL FOR RESEARCH QUESTIONS 1 AND 2

In looking for coherence through the curriculum documents, the guiding principle was to understand the key goals underlying the reformed FET mathematics curriculum and inevitably the first question to be addressed was; "What are the desirable goals in this change and of what quality are they?" The term 'desirable' was being used in this context of something that was being championed or advocated for. The cognitive demand tools would in this case determine what was advocated for or the desirable goals in the new curriculum. This would be achieved through the analysis of content standards and observing where the emphasis was with specific attention to lower/higher order skills and procedures. If the content standards were clustered say around the lower order or the higher order skills and processes; then that which was emphasized would constitute the 'advocated for' or 'desirable goals' of the curriculum in this study. The analysis would then move on to find out how aligned the documents (theory) and classroom interactions (practice) were with reference to the desirable goals. The decision for this single focus approach to what is 'desirable' was guided by empirical evidence which suggests that most approaches to study alignment begin with one set of standards/goals and then measure the extent to which assessments are aligned to that specific set of standards/goals (Liang & Yuan, 2008). Their findings from such approaches provide important lessons for those pursuing alignment analyses in standards-based reform in that the analysis remains focused.

In the South African context, the claim made in the curriculum is that the critical principles of social transformation and high knowledge and high skills were drawn from the new constitution of the Republic. It is further claimed that the mastery of these higher level mathematical skills and knowledge depended to a large extent on mathematical processes such as investigating patterns, formulating conjectures, arguing for the generality of such conjectures and formulating links across the domains of mathematics to enable critical thinking (DoE, 2008a, p. 69). The Learning Programme Guidelines (Department of Education, 2008, p. 11) specify that;

> As a way to achieve the mathematics learning outcomes, teaching and learning in mathematics focuses on the development of learners towards the four Learning Outcomes. Central to the attainment of the learning outcomes is the development of mathematical process skills e.g. investigating, conjecturing, organizing, analyzing, proving, problem solving, modelling.

The cognitive demand tools designed by Edwards and Dall'Alba (1981) as well as those by Porter's (2002, 2004, 2007) categorise these skills as belonging to the higher order level. The interest in this study was to find out to what extent the curriculum documents were consistent in articulating this message and how far teacher practices were creating opportunities for learners to develop the high knowledge and skills.

While Wilson and Bertenthal (2005), Ottenvanger et al., (2007), and Squires (2009), have all contributed to the framework for understanding curriculum alignment in this study, their frameworks offered less precise tools for measuring the extent to which alignment of cognitively demanding mathematics is evident in the policy documents. Because of the centrality of alignment to current policy logic, for the past 25 years researchers have been trying to develop such tools for measuring curriculum content and alignment. Porter et al., (2002, 20042007) have developed such a tool, with descriptors as discussed before, which differs from other efforts to measure the content of instruction and alignment in two important ways. It is premised on the increasing evidence that estimating curriculum alignment based on both knowledge and cognitive processes is superior to other methods of estimating alignment (Gamoran, Porter, Smithson & White1997).

These researchers have argued that;

> ...to predict student achievement gains from knowledge of the content of instruction, a micro-level description of content that looks at cognitive demands by type of knowledge is the most useful approach considered to date (p. 331).

They further posit that a micro-level description of content was needed in order to be able to make more precise alignment decisions (Porter, et al., 2007). Porter's tools work on the presumption that independent and replicable descriptions of the content of instructional practice and instructional material can be made. A single language for measuring content ensures description at a consistent level of depth and specificity. The single language then generates data counts that allow alignment to be measured across a large number of instructional materials and instructional practices. The procedure developed by Porter has demonstrated a strong relationship between alignment and student achievement gains and is one of the few approaches to alignment analyses approved by both the Institute for Education Sciences (IES) and the National Science Foundation (NSF) (Webb2005).

Using that frame this study then analysed the content (assessment) standards of the NCSM at Grade 11 level. Using an extract from the assessment standards for Grade 11 the table below shows how each of the assessment standards were categorized following Porter's tool and how the total of 10 data counts for LO1 (Number and Number Relationships) were arrived at. The interpretation given to 11.1.1 for example was that 'understanding that not all numbers are real' required the learners to recall, recognise, identify - hence memorise (coded A) the definition and characteristic features of a real number and perhaps recall the existence of the set of numbers referred to as the imaginary numbers. In 11.1.2, 11.1.3 and 11.1.4 the interpretation given was that these were examples were learners are required to compute or solve routine problems – hence perform procedures (coded B). Both 11.1.2(c) and 11.1.5 required the learners to use representations to model mathematical ideas or to describe – hence communicating understanding of concepts (coded C). In 11.1.6 this was a case of solving non-routine problems (coded D). Lastly in 11.1.3 (a),(b) and (c) the assessment standards required the learners to determine the truth of a mathematical pattern, make and investigate

mathematical conjectures, provide proof of a conjecture – hence (coded E). All the other assessment standards for the remaining three learning outcomes (LO2, LO3, and LO4) were coded in a similar way.

Table 3.1 How coding of assessment standards for Grade 11 was done

WE KNOW THIS WHEN THE LEARNER IS ABLE TO:	
Assessment standard	**code**
11.1.1 Understand that not all numbers are real	**A**
11.1.2 (a) simplify expressions using the laws of exponents for rational exponents	**B**
(b) Add, subtract, multiply and divide simple surds	**B**
(c) demonstrate an understanding of error margins	**C**
11.1.3 (a) Investigate number patterns hence	**E**
(b) make conjectures and generalizations	**E**
(c) provide explanations and justifications and attempt to prove conjectures	**E**
11.1.4 Use simple compound growth formulae to solve problems	**B**
11.1.5 demonstrate an understanding of different periods of compound growth and decay	**C**
11.1.6 solve non-routine unseen problems	**D**

This resulted in a mathematical process oriented categorization, based on Porter's tools. The table then ended with 1 data count in column A, 3 data counts in column B, 2 data counts in column C, 1 data count in column D and 3 data counts in column E for LO1 (Number and Number Relationships). The complete table of all data counts for all the learning outcomes is shown below.

Table 3.2 Matrix for Mathematics Assessment Standards Grade 11(adapted from Porter, 2002)

LEARNING OUTCOME	CATEGORY OF COGNITIVE DEMAND						
	Lower Order			Higher Order			
	Memorize A	Perform procedures B	Total	Communicate understanding C	Solve non-routine problems D	Conjecture/ generalize/ prove E	Total
(Number and Number Relationships)hips	1	3	4	2	1	3	6
Functions and Algebra	3	4	7	5	5	5	15
Space, Shape and Measurement	0	1	1	0	10	9	19
(Data Handling and Probability)	2	2	4	4	3	3	10
TOTAL	**6**	**10**	**16**	**11**	**19**	**20**	**50**

So, the way the cognitive demand table above is interpreted is that columns A and B are the constituencies of lower order skills and processes while columns C, D and E constitute higher order skills and processes based on the definitions of higher/lower order thinking as discussed in the previous chapter. There were 6 scores for example, in the assessment standards for (Number and Number Relationships) that were identified as higher order requiring learners either to communicate understanding, problem solve or conjecture, generalize and prove. If one were to take totals of columns C (11), D (19) and E (20) for example, it might be clear how this tool was useful in terms of answering the first research question for this study i.e. “What is the level of cognitive demand in the mathematical concepts, processes and skills as articulated in the written and tested NCSM at FET level?” Not only did this tool enable this study to answer this first research question, but the same tool also became useful in answering the second research question, “How coherent are the curriculum documents as they articulate these skills and

knowledge?" This would be achieved through comparison of the cognitive demand scores from another table similar to this one but with scores for the exemplar papers.

The central idea behind Porter's tool is the development of a uniform language which makes it possible to build what he has called an 'alignment index' (Porter, 2002). His alignment index P was calculated using the following formula:

Alignment Index $P = 1 - \frac{\sum |x - y|}{2}$, where ***x*** denotes cell proportion in one matrix and ***y*** denotes cell proportion in another matrix. So in this case once the content matrix for the Assessment Standards (written document) was done another matrix for the examination papers (Standardized tests) was developed using the same cognitive demand table. The corresponding cells in each of the two matrices were then compared to arrive at the alignment index. Measuring alignment became a question of the extent to which the proportions in one content matrix matched the proportions in another content matrix. The possible values of this alignment index range from 0 (no alignment at all) to 1.0 (perfect alignment). The argument in favour of this type of alignment analysis was that it provided a relatively precise mathematical procedure for calculating the degree of alignment or similarity between any two descriptions employing the same descriptive language (Edwards2010; Squires, 2009).

3.3.4 VIDEO RECORDING TO ANSWER QUESTION 3

Traditionally, attempts to measure classroom teaching have used teacher questionnaires because they are economical, simple to administer and usually can be transformed easily into data files ready for statistical analysis. However using questionnaires has its own limitations which researchers believe could be overcome by direct observations of classrooms. According to Rich and Hannafin (2009), video technologies afford largely untapped potential to support and document the processes of teaching practices. These tools provide potentially important methods for scrutinizing instructional decisions within specific teaching contexts. Video is generally thought to be a valuable medium for exploring teaching and learning because it captures much of the richness of the class setting (Seago, 2004). There is widespread agreement that researchers and teachers will

gain more from watching authentic, realistic classrooms than from watching staged interactions (Sherin, Linsenmeier, & van Es, 2009). According to Hiebert at al.,(2003), video offers a promising alternative for studying teaching in that the method has significant advantages over other means of recording data for investigating teaching. Some of the advantages are that video;

(1) enables the study of complex processes like teaching/learning
(2) enables coding from multiple perspectives
(3) can provide the time and space needed to reflect on classroom-interactions
(4) stores data in a form that allows new analyses at a later time
(5) facilitates integration of qualitative and quantitative data
(6) facilitates communication of the results

Because of these anticipated benefits, video recording was chosen as the method for collecting data to answer the research question 3 that was conceptualized at the practical level.

3.3.5 ANALYTICAL TOOL FOR QUESTION 3

The aim of classroom observations was to examine both quantitatively and qualitatively how teachers structured students' opportunities for learning the higher order skills and processes. In this regard opportunity for learning was less a measure of curriculum coverage, than an analysis of the didactic strategies exploited and mathematical knowledge and skills encouraged by teachers (Andrews, 2009). Guided by an important finding from the TIMSS Video Study and the literature detailed in the previous chapter, that explicitly making mathematical connections during mathematics class positively impacted students opportunities to learn (Hiebert et al.2003) the analytical tool focused on this aspect as a key feature within higher order cognitive demand. In developing an analytical tool for identifying mathematical connections in the teacher utterances and activities, this study first analysed a number of conceptualizations of mathematical connections from different perspectives. What was evident from literature were the following conceptualisations; equivalent representations in mathematics as a form of connection (Hodgson, 1995; Weinberg, 2001), abstraction as a form of connection (Noss & Hoyle, 1996), concept to concept links (Zazkis, 2000), unifying themes as a form of

connection (Coxford, 1995), modelling as a form of connection (National Council of Teachers of Mathematics, 1989), problem solving as a form of connection (Evitts, 2004) and different functional representations as another form of mathematical connections (Boaler, 2002).

Businskas (2008), suggested a model with 5 forms of mathematical connections which encompassed these orientations. Her proposition was that basically connections could be viewed from three perspectives (a) as a relationship between mathematical ideas (b) as a relationship that is constructed by the learner and (c) as a process that is part of the activity of doing mathematics. She further argued that all three ways of considering connections were viable but her specific interest was in connections as an idea, as a product of mental activity from the point of view of the teacher. Businskas' (2008) study was underpinned by the view that teachers have to understand mathematics as an interrelated web of ideas (*connections as a feature of mathematics*) themselves before they can devise strategies and examples (*connections as a process of making mathematical relationships*) that would make it easier for students in turn, to build such relationships. The questions guiding her study were: "How do secondary mathematics teachers conceptualize mathematical connections? What are the characteristics of the explicit mathematical connections that teachers are able to articulate?" From her study she was then able to summarize the types of mathematical connections that teachers articulated with illustrative examples, hence developed this model inductively as follows:

Table 3.3 FORMS OF MATHEMATICAL CONNECTIONS

	FORM OF CONNECTION	***CODE***	***DESCRIPTION***	***EXAMPLE***
1	Different Representation	**DR**	Alternate representation: i.e. **A** is an alternate representation of **B**	The two representations are from different modes like symbolic, graphic, pictorial, manipulative, verbal, written etc.
			Equivalent Representation: i.e. **A** is equivalent to **B** - equivalent here is used to distinguish from representations in different forms and so refers to concepts that are represented in different ways within the same form of representation	3 + 2 is equivalent to 2 +3 is equivalent to 5; $f(x) = ax^2 + bx + c$ is equivalent to $f(x) = a(x – p)^2 + q$. In these examples both **A** and **B** are symbolic
2	Part-whole Relationship	**PWR**	**A** is included in (is a component of) **B; B** includes (contains) **A** i.e. this is a hierarchical relationship between two concepts	A vertex is a component of a parabola (and a parabola contains a vertex)
			A is a generalisation of **B; B** is a specific instance (example) of **A**. This is another kind of a hierarchical relationship	$ax^2 + bx + c = 0$ is a generalisation of $2x^2 + 7x + 3 = 0$ *(quadratic equations)* Part-whole relationships include examples, inclusions and generalisations.
3	Implication	**IM**	**A** implies **B** (*and other logical relationships*) i.e. this connection indicates a dependence of one concept on another in some logical way. *If ....Then....*	The degree of an equation determines the maximum number of possible roots
4	Procedure	**P**	**A** is a procedure used when working with object **B**	Making a tree diagram is a procedure used to describe a sample space (probabilities).
5	Instruction-oriented Connection	**IOC**	**A** and **B** are both prerequisite concepts/skills that must be known in order to understand/learn **C.** This form of connection also includes extension of what students already know - linking new concept to prior knowledge	Factors and multiples are concepts that must be known in order to understand working with fractions

It is important to note that Businskas (op cit) did not provide a table with codes like the one above – both the table and the codes were added by me as I will elaborate shortly. The codes were developed from the phrases that she used to describe these connections e.g. code IOC from Instruction Oriented Connections. She however provided both a description and an example for each form of connection that appeared identifiable in a

typical mathematics classroom situation following the analyses she made of teachers' responses.

Businskas' goal was to identify emerging themes in teachers' understanding of mathematical connections in the context of thinking about their practice. The assumption was that it was possible to identify connections and then explicitly describe all their relationships to diagram a web similar to a concept map which could then become the reference point for instruction and assessment. My study could be viewed as extending Businskas work in that I was interested in looking at how teachers acted in ways that promoted learners' building of mathematical connections as conceptualized by teachers in Businskas' (2008) study. In that view these types of connections became the reference point for trying to understand instruction in the context of new reform in some selected South African FET schools.

The decision to make Businskas connections as the reference point was reached after considering that these were descriptions of the mathematical connections that were prevalent in teachers' descriptions of their conceptualisations of mathematical connections based primarily on their classroom experiences with learners i.e. classroom based. So I saw potential in that these connections were likely to feature prominently in my participants' lessons thereby allowing richer analyses of how the mathematical connections were enabled or constrained.

Another reason was that the model appeared robust within Businskas' study. The connections identified were not constrained by the topic as the study was able to capture a wide range of mathematical connections that teachers articulated across a span of mathematical topics from number theory, geometry, trigonometry, algebra calculus and probability (Businskas, 2008). The model was field tested and was found to be applicable. It also attempted to bring mathematical connections to a fine grain level (which teachers need in the South African classrooms) when compared with prior research with similar foci e.g. the mathematics education traditions of Europe (METE) project (Andrews, 2009). This argument can be supported using one example from the

METE project where a code **(SK)** for structural knowledge was developed. (SK) was identifiable through teacher emphasizing connections between different mathematical entities but the term 'different mathematical entities' seems rather too broad yet in Businskas model the same notion of structural knowledge appears to have been captured more precisely by breaking it down into specific forms of connections, supporting the argument that the model has finer grains of mathematical connections.

Another reason for opting to start with Businskas' model was its potential to capture teachers' pedagogical content knowledge of connections through the different definitions or representations that they made. Prior research e.g. METE project (Andrews, 2009), Learner Perspective Study (Clarke & Hoon2005) and TIMSS 1995 & 1999 Video studies (Hiebert et al.2003) which also used this higher order cognitive demand framework seemed to work on the presumption that teacher's subject matter knowledge was generally good. In fact in the TIMSS Video Studies the teachers who took part were exemplary mathematics teachers from high achieving countries and in the METE project the researchers were explicit in that the sequences of lessons were taught by teachers regarded locally as effective in the manner of the learner's perspective study (Andrews, 2009). Within the NCSM itself, Parker (2004), posited that there was this same presumption:

> .... The national curriculum statement Grades 10 – 12 ...visualizes teachers who are qualified, competent, dedicated and caring. They will be able to fulfil the various roles outlined in the Norms and Standards (DoE, 2003b, p. 5)

However, in the South African educational terrain literature abounds that suggest teacher's subject matter knowledge is weak (Adler, 2009; Brodie & Pournara, 2005; Graven, 2005; Harley & Wedekind, 2004; Howe, 1999; Long, 2007a; Taylor & Vinjevold, 1999). Considering that literature also suggests that studying teachers' pedagogical efforts to promote the making of mathematical connections necessitates considering the intersection of three frameworks – their subject matter knowledge, their general pedagogical knowledge and their specific pedagogical content knowledge (Shulman, 1986), it was considered important for this study to look for a model that would capture teacher's subject matter knowledge of the mathematical connections.

A number of indicators were available for looking into the teachers' subject matter knowledge of the connections. Ferrini – Mundy et al. (2007) for example, posited that in mathematics education research it was common and plausible to assume that secondary school mathematics teachers have had some exposure to more advanced versions of mathematical ideas than those that appear in the secondary curriculum. In that sense teachers had to 'trim' mathematical or contextual content in a way that was mathematically acceptable but which also left intact the content to be learned. Ferrini – Mundy et al (op cit) see this trimming as;

> … a transformation of mathematical ideas from a more advanced or rigorous form to a form that preserves the essence but that will be accessible to students, considering their backgrounds, understanding, and knowledge. Trimming involves scaling down, and intentionally and judiciously omitting detail and modifying levels of rigour, and also being able to judge when a student, or textbook presentation, is trimming, and if so, whether the trimming is appropriate. It includes reducing one's own mathematical understanding to a form that is accessible for a student, or that connects well to the mathematics students bring to the context. Trimming includes the interpretation and judgment a teacher must use in considering a textbook's treatment of a mathematical concept or process. (p. 41 - 42)

Closely related to this notion of trimming, McDiarmid, Ball and Anderson, (1989b) argued that teachers were constantly engaged in a process of constructing and using instructional representations of subject matter knowledge which define students' formal opportunities for learning about the subject matter. These instructional representations form a wide range of models including verbal, symbolic, graphical, concrete representations as well as definitions, activities, questions, examples and analogies. It is through the representations they select and the ways they use them that teachers convey messages to their pupils about both the substance and nature of the subject matter they teach, hence trimming. This notion of trimming could also be related to Ma's (1999) view of having a "clear idea of what is the simplest form of a certain mathematical idea" (p. 47). Key in this idea of trimming is that whatever mathematical decisions are made important mathematical features must be retained while reducing complexity in ways that made the content accessible to students. In trimming there is also need to anticipate later mathematical ideas that students will encounter. In elementary mathematics for example, a frequent example is how the adage "multiplying makes bigger" can cause problems for

students when they later encounter multiplication of whole numbers by fractions between zero and one.

From an initial analysis of data in my study it would appear this notion of trimming mathematical ideas for student access would be accommodated very well in Businskas (op cit), category of connections through different presentations **(DR)** – a category comprising 2 subcategories i.e. alternate representation and equivalent representation as discussed within the table of connections earlier. Within the alternate representation subcategory, the two representations of a concept used are from different modes like from verbal to symbolic or graphic to pictorial or manipulative to written etc. On the other hand, within the equivalent representation subcategory, Businskas says equivalent here is used to distinguish from representations in different modes/forms and so refers to concepts that are represented in different ways within the same form of representation. She cites definitions of concepts when given from verbal to verbal or from written to written using different descriptors or words as an example of equivalent representation. Teachers give such equivalent and alternate representations of concepts more often in their interaction with the learners as McDiarmid, Ball and Anderson, (1989b) explained in detail.

3.3.6 REFINING THE CODES FOR THE ANALYTICAL TOOL 3

The point of departure though was that Businskas was interested in connections from a static perspective as mental objects that teachers construct hence one could not possibly make judgement about the quality or depth-of-knowledge of that connection. However, in my study I was interested in connections from a dynamic view as a process of making such connections hence linking connections to conceptual/procedural depth. This would enable me to explore these mathematical connections and the quality of such connections (higher order/lower order levels) in the context of teacher's actual practice. This followed from the view that much of the mathematics education literature that describes useful activities for promoting the making of connections seems based, at least implicitly, on a view of making connections as a process (Businskas, 2008). Taking this view meant

adjusting certain components of the model suggested by Businskas to allow judgment of the quality in terms of higher order or lower order levels of the connections.

In developing these adjustments this study also borrowed some ideas from the METE project which had a similar focus. In the METE project Andrews (2009) identified mathematical reasoning **(MR)** where the teacher encouraged learners' development and articulation of justification and argumentation. This notion of justification and argumentation appeared to be critical in my cognitive demand framework in that it is explicitly espoused in the NCSM and it also appeared key to the distinction between lower order cognitive demand and higher order cognitive demand knowledge. Deep understanding or higher order cognitive demand is associated with showing how and why something works in mathematics and if the 'why' or justification was not explicit or implicit then the utterance/activity was considered superficial or of lower order cognitive demand e.g. just telling learners what to do without worrying about why it works i.e. just automating the routine**.** Without necessarily including an extra **(MR)** code into my model, but building on the METE project code design, the notion of justification was infused into the coding following the higher order thinking descriptors discussed within the previous chapter. So this notion of articulation, justification and argumentation was then considered an indication of higher order cognitive demand or deeper understanding of knowledge and skills and coded at level 2. In the TIMSS Mathematics Quality Analysis Group, Hiebert et al., (2003) used a similar coding system although their focus was on the extent to which a lesson included some development of the mathematical concepts and procedures. Their presentation ratings also took into account the quality of mathematical arguments. Higher ratings meant that sound mathematical reasons were provided by the teacher (or students) for concepts and procedures. Mathematical errors made by the teacher reduced the ratings. A rating of 1 indicated a lesson that was descriptive or routinely algorithmic with little mathematical justification provided by the teacher or students for why things work the way they do. A rating of 5 indicated a lesson in which the concepts and procedures were mathematically motivated, supported, and justified by the teacher or students (Hiebert et al.2003).

However, this coding or rating of 1 - 5 lacked clarity in contextualizing it, especially in the absence of examples of what would constitute a connection that is partially developed 2, moderately developed 3, or substantially developed 4. On the other hand, Andrews' (2009) coding of absent 0 and present 1 appeared problematic in that coding an absent 0 connection had the potential of bringing in subjective analyses. Coding 1 present would also be insufficient in terms of measuring the degree or quality of the connections. This study then struck some balance between these many models and developed its own 3 level coding scheme borrowing from these other models. Because this study considered that it was going to be subjective and problematic to record something that was absent as 0, in this study coding at level 0 would imply a mathematical connection that was present but mathematically problematic, level 1 would imply present and mathematically correct but superficial or routinely algorithmic with no further explanation or justification and level 2 present, correct and articulated (deep).

Because of the complexity inherent in mathematical concepts, this study acknowledges that the model developed here could not possibly capture all the mathematical connections that could be made by teachers as they interact with their learners. Empirical evidence has shown that because of the one-to-many relationships between mathematical concepts (Ma, 1999), it was not possible to account for every conceivable form of teacher action and objective in relation to these connections (Andrews, 2009). However judged by their prevalence in teachers' descriptions, these mathematical connections were considered a sufficient reference point for this study for getting at the range of cognitive demand levels opened up across a sequence of lessons by each teacher.

3.3.7 EXAMPLES OF CODING USING LIVE DATA

Here are some examples of how live data for this study were coded in accordance with these three levels and in relation to each of Businskas' five types of connections. In one of the episodes from the live transcribed data, the teacher was trying to define the word "calculus" to the learners and she said;

> *Say for instance I mean it's calculus it has the word calculate within it. Ok. So we will be calculating something but there are rules that we need to follow.*

I found such an utterance easy to place within the connections through different representation **(DR)** category suggested by Businskas above as the teacher attempted to trim the concept 'calculus' for the learners. However, as discussed earlier trimming includes reducing one's own mathematical understanding to a form that is accessible for students giving them a clear idea of what is the simplest form of a certain mathematical idea. Considering that mathematically sound definitions are critical for a deep understanding, this trimmed definition of calculus appears to be problematic in that it does not seem to offer learners a clear idea of what is the simplest form of calculus as a mathematical concept. Considering that the whole set of lessons for this teacher for the whole week were on calculus and that no other definition was given later, it is doubtful whether the definition given by the teacher transformed the mathematical idea of calculus from a more advanced or rigorous form, which it is, to a form that was accessible to the students. The teacher's utterance was then coded **(DR0).**

In another example the teacher was dealing with factorisation of polynomials and in this specific instance it was a quadratic expression which had to be factorised. So the teacher says;

> *What we are going to do here is, we have got the following expression on the board.* (*The teacher had written* $a^2 + 14a + 48$ *on the board). We want to choose factors which when you add them they give us +14a and when we multiply them they give us +48. Which are the two factors?*

This utterance was considered to be in the procedure (**P)** category since the teacher was focusing on the process or method of identifying factors of 48 that would lead to the correct factorization of the quadratic expression. The utterance was coded as **P1** as it was considered mathematically acceptable but lacking precision in language because no factors of *48* can add up to *14a* but there are factors that can add up to its coefficient *14* that is *8* and *6*. There was also no justification for the procedure, instead the focus appeared to be on 'what to do', but not 'why' we do it.

Prior research which has employed coding of mathematical interactions in classroom situations acknowledge that such methods of data collection inevitably result in multiple coding but importantly, the affordance of multiple coding allowed for important distinctions in teacher behaviours to be highlighted (Andrews, 2009). In this study multiple coding was also inevitable and this last example shows how double coding was accommodated within the same utterance.

This teacher's lessons were focusing on expansion of brackets and then collecting and dealing with the like terms in the process of simplifying the expanded algebraic expression. At this stage the two brackets had been expanded and the teacher was now dealing with the collection of like times in $2a^3 + a^2 + a - a - 2a^2 - 1$. The final answer was presented as $2a^3 - a^2 - 1$ but while most learners in that class saw how the $+ a - a$ eliminated each other some learners seemed unclear as to where the $- a^2$ (middle term) was coming from. Here is what one of the learners said;

> *Learner:* *U a^2 kee? So where is that a^2coming from?*

So the explanation that was given is as follows;

> $2a^3 + a^2 - 2a^2 - 1$. *You see (learner's name). You got the $2a^3$ then you have the $+a^2$ then the $-2a^2$. It's just like saying 1 – 2 its -1. So there it has got to be $- a^2$.*

This analogy shows at least two equivalences as follows; a^2 is equivalent to $1a^2$ and $-1a^2$ could also be written as $-a^2$. Judged by the learners' nodding of heads in agreement after this analogy was given, it can be argued that the analogy (equivalent representation) reduced the complexity of the mathematical idea in ways that made the content accessible to students hence it was coded **DR2**. The same analogy was also as an example of part-whole relationship PWR according to Businskas model. Part-whole relationships include examples, inclusions and generalisations of the form **A** is a generalisation of **B; B** is a specific instance (example) of **A**. In this analogy $a^2 - 2a^2$ was viewed as a generalisation of which $1 - 2$ is an example. Both of them are generally showing the same reasoned idea of 1 a single item/thing minus 2 of the same items/things. Consistent with the view that high-level cognitive processes require emphasis on reasoning about and connecting ideas

this analogy was also considered an instance of such reasoning and connection of 1 -2 with $a^2 - 2a^2$ hence it was coded as **PWR2.**

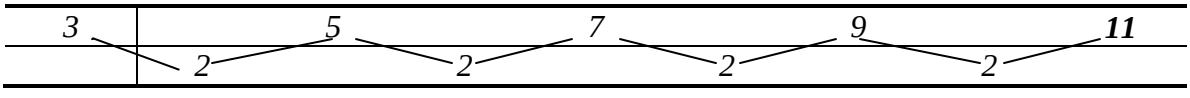
Here is another episode from a lesson which was focusing on number patterns:

> *Teacher: Right suppose you are given a list of numbers starting with;* (Teacher writes on the board 3; 6; 9 ;). *Somebody tell me the next number. (One learner says 12) Teacher: 12 he says the next number will be 12. Anybody who does not agree?* (Teacher puts his hand up and pauses, no disagreement from the learners; teacher folds his arms and asks). *But how do we know its 12? Suppose somebody comes in from a distance and says its 13. Why 12 why not 13? (Teacher names and points to a student to give a reason why)*

This episode was considered as an example of an instruction oriented connection **(IOC)** defined in Businskas' model as including extension of what students already know. In this case one can notice the teacher following up on learner responses and building their deeper understanding from what they already know. The teacher did not just accept 12 as a correct answer but wanted learners to support their choice of 12 with reasons. This teacher utterance/episode was considered as a form of representation where the development and articulation of justification and argumentation appeared evident. Consistent with how probing for reasoning and justification has been linked with higher order thinking in the literature review, asking learners for justification why 12 and not any other value was then coded as **IOC2.**

The last example also comes from this same set of lessons on number patterns where the teacher was discussing the following pattern: 3; 5; 7; 9; 11; ….. At this stage of the discussion the general term $T_n = 2n + 1$ had been generated

> *Teacher: If our general term is linear. Alright. OK. (Points to the linear equation $T_n = 2n + 1$). A linear general term will generate a sequence in which the first difference is constant. OK? This first difference is constant.*



You've got 2, 2, 2, and 2 (referring to the constant difference between the consecutive terms)

This episode was considered to be an example of a connection where **A** implies **B.** According to Businskas model this connection indicates a dependence of one concept on another in some *If ....Then....* logical way. In this case **if** it is a linear sequence **then** 1st difference is constant or alternatively **if** 1st difference is constant **then** it is a linear sequence. Because the teacher was not just telling the learners (rote) but did it with articulation and justification the episode was coded as **IM2**.

3.3.8 THE FINAL ANALYTICAL TOOL FOR LIVE DATA

Juxtaposed onto my lower/higher order cognitive demand framework, the first three forms of connections in the Businskas' model **DR, PWR,** and **IM** captured utterances/activities at different levels of conceptual knowledge. The last two forms of connections **P** and **IOC** captured utterances/activities at different levels of procedural knowledge. This follows Forrester and Chinnappan's (2010) view that procedural knowledge includes knowing the algorithm/method **(code P)** applicable to a particular type of a mathematical task, which if followed correctly, is guaranteed to give a correct answer to the task. A second view is that procedural knowledge may also mean the teacher's technique, method of performance or way of accomplishing **(code IOC)** i.e. the manner and ability with which the mathematics teacher employs the technical skills to teach a mathematical concept (Forrester & Chinnappan, 2010).

Consistent with how lower order and higher utterances would be identified as articulated earlier, in the final modified model for this study, if a teacher utterance/activity/episode appeared to recognise an opportunity for a particular mathematical connection e.g. different representation **(DR)** then it would be coded **DR0** if that connection was mathematically problematic. If that particular form of mathematical connection was recognised at a rote/superficial level it would be coded **DR1**, and when recognised with further justification and/or articulation **DR2**. Similarly all the other types of connections would be coded at these three different levels. Consistent with the view that deep understanding of both procedural and conceptual knowledge should be the ultimate goal and priority of all mathematics learning as it refers to an integrated and functional grasp of mathematical ideas, the conceptual/procedural distinction was not fore grounded in the

analytical tool. Its inclusion in the literature review was meant to enable important distinctions in teacher behaviours to be highlighted and richer discussions of the findings to be made. The final version of the analytical model for this study fore-grounded levels of knowledge quality and was then conceptualized thus:

Table 3.4 ANALYTICAL MODEL FOR CASE STUDY 3

FORM OF CONNECTION	LEVELS OF KNOWLEDGE QUALITY		
Code	**0**	**1**	**2**
DR			
PWR			
IM			
P			
IOC			

3.3.9 THE FOCUS OF THE VIDEO RECORDINGS

A series of lessons on expansion of brackets of binomials and trinomials, factorization of binomials and trinomials, calculus and number patterns for the whole week were video recorded and transcribed for each of the four teachers (2 female, 2 male) from three different high schools around Johannesburg. In the transcriptions an utterance/activity was considered as a unit of analysis and both teacher and learner utterances/activities were numbered for easier reference. This utterance/activity was defined in the same manner that Andrews (2009), defined an episode which was that part of a lesson where the teacher's didactic intent remained constant. Thus an utterance/activity could be as short as a teacher's request for a learner's explanation or justification, or it could be as long as a teacher's explanation of a mathematical procedure together with the diagrams/models thereof. But unlike Andrews' (op cit) use of the term episode to mean both didactic and managerial activities, in this study an utterance/activity was considered and coded specifically as a didactic activity.

Of the four sets of lessons which were video recorded, 3 were on LO2 (Functions and Algebra) and one set of lessons was specifically Number Patterns which is a section of LO1 (Number and Number Relationships). There are a number of reasons leading to the choice of and the deliberate bias towards LO2 (Functions and Algebra). In the NCSM, this learning outcome (LO2 Functions & Algebra) is given greater weighting in that the distribution of marks in the Assessment Guidelines stipulates that 105 marks (70%) of the total of 150 marks in Paper 1 should come from LO2 (DoE, 2008b, p. 12). From the document analysis it was also argued that judged by the data counts in table 4.2 which categorized content from the written curriculum that the NCSM placed more emphasis on LO2 (Functions and Algebra).

Besides these quantitative measures of relative emphasis, there are other forms of qualitative evidence in the NCSM that point to the curriculum placing more emphasis on LO2 (Functions and Algebra). The power of the concepts of algebra and functions is evident in the constitution of these concepts as being central to the acquisition of mathematical knowledge and its structures needed for learners to understand their world. For example according to the DoE (2003a, p. 12),

> A fundamental aspect of this outcome is that it provides learners with versatile and powerful tools for understanding their world while giving them access to the strength and beauty of mathematical structure. The language of algebra will be used as a tool to study the nature of the relationship between specific variables in a situation. The power of algebra is that it provides learners with models to describe and analyse such situations.

Within the NCSM there are quite a number of statements which also show how this learning outcome (LO2) cuts across all the four learning outcomes in which FET learners are expected to demonstrate their achievement. Firstly within LO2 itself the relationship between functions and algebra is seen in that algebra serves as a tool for working proficiently in functions, and proficiency in both algebra and functions enables learners to work efficiently in four representations of mathematical activity, namely, *numerical*, *graphical*, *verbal* and *symbolic*. The connections among these four representations are made possible through proficiency in both algebra and functions. Within LO1 (Number and Number Relationships) the role of functions and algebra is seen in that learners are expected to "expand their capacity to represent numbers in a variety of ways and move

flexibly between representations" (DoE, 2003a, p. 12). Within LO3 (Space, Shape and Measurement) the expectation is that for learners to work proficiently in their study of space, shape and measurement they need to use both algebraic and geometric knowledge which enables them to "link algebraic and geometric concepts through analytical geometry" and to "analyse natural forms, cultural products and processes as representations of shape and space" (DoE, 2003a, p. 14). Within LO4 learners engage in collecting, organising, analysing and interpreting data to solve related problems. It is in the area of data analysis and interpretation that learners begin to appreciate concepts learnt in LO2 (Functions and Algebra) as these concepts enable learners to "become critically aware of the deliberate abuse in the way data can be represented to support a particular viewpoint" (DoE, 2003a, p. 14). According to Mwakapenda (2008), there is therefore a requirement that teachers structure learning experiences and situations that develop these key concepts and enable learners to "experience the power of algebra as a tool to solve problems"(DoE, 2003a, p. 13)

International literature also demonstrates the central role played by functions and algebra in the acquisition of mathematical knowledge. An observation made by Star and Rittle-Johnson (2009), was that competence in algebra is increasingly recognized as a critical milestone in students' middle and high school years. Algebra has always represented students' first sustained exposure to the abstraction and symbolism that makes mathematics powerful (Kieran, 1992). In addition to its central role in mathematics, algebra also serves as a critical "gatekeeper" course, in that earning a passing grade has become a de facto requirement for many educational and workplace opportunities (Star & Rittle-Johnson2009). Yet students' difficulties in algebra have been documented in international assessments and there is empirical evidence to suggest that the transition from arithmetic to algebra is a notoriously difficult one with teachers facing numerous challenges (Blume & Heckman, 1997). It was after considering all these factors that the study wanted to find out how teachers were coping in an area which seemed to be emphasized both in the NCSM and in international literature yet notoriously difficult for both learners and teachers.

The choice of a lesson sequence on Number Patterns (in LO1 in the FET curriculum) was guided firstly by the widely accepted belief in mathematics literature of the links that exist between patterns and functions (Radford, 2010; Sfard, 1991; Warren, 2006; Warren & Cooper, 2008). A number of researchers are of the view that abstracting patterns is the basis of structural knowledge (Sfard, 1991; Warren, 2006) with some claiming that when one recognizes the structure of the system one engages in, explains this structure to others by such means as encoding it in a diagram or applying some overarching framework then mathematics exists (Presmeg, 1997). Students begin their study of functions in the primary grades, as they observe and study patterns in nature and create patterns using concrete models. Warren and Cooper (2008), argue that a common approach used for introducing algebra to young adolescents is a 'pattern approach to algebra' where learners explore patterns and express these patterns as functions and algebraic expressions. Radford (2010), shares a similar view that pattern generalisation is considered one of the prominent routes for introducing students to algebra. Students in high school then move to expand their knowledge of algebra as they analyze a variety of different types of number patterns/sequences, including arithmetic and geometric sequences, whose behaviour is then expressed using functional notation. In that sense it can be argued that through functions algebra provides the language in which to communicate the patterns in mathematics. The various number patterns in mathematics are formed by the functions that define the relation between the consecutive numbers in the series hence patterns, relationships, and functions continue to provide a unifying theme for studying mathematics in high school.

There is also a long standing observation that patterns pervade every part of our lives - more so in mathematics - and provide a sense of order in what might otherwise appear a chaotic world. This *pattern-based thinking*, using patterns to analyze and solve problems, is an extremely powerful tool for doing mathematics. Students who are comfortable looking for patterns and then analyzing those patterns to solve problems can also develop deep understanding of new concepts in the same way. Because patterns provide clear insights into mathematical understanding it is not surprising that mathematics as a discipline has often been regarded as the 'science of patterns and relationships' (NCTM,

1989). In the NCSM a similar view of the nature of the discipline is that mathematics is based on observing patterns, with rigorous logical thinking which leads to theories of abstract relations (DoE, 2008a, p. 9).

3.4 SAMPLING

This study employed purposive sampling to arrive at the participants. Kerlinger (1986) described purposive sampling as another type of non-probability sampling where the characteristics of the participants are used as the basis for selection and the number of participants is less important than the criteria used to select them. Purposive sampling is characterised by the use of judgement and a deliberate effort to obtain representative samples by including typical areas or groups in the sample. Purposive sampling groups participants according to preselected criteria relevant to a particular research question.

With respect to the selection of the schools to participate on this study, a number of factors were taken into consideration. The study aimed at understanding how the new curriculum was implemented in the less privileged communities, so such communities formed the population from which to sample the participating schools. The first port of call was the official policy documents on poverty. South African Education Department introduced a new policy in 2007 in which it classified schools and assigned each one a poverty score using data from the community in which the school is located. This "poverty score", was created by assessing "income, unemployment rates and the level of education of the community, which are weighted to assign a poverty score for the community and the school (Sayed & Motala, 2009, p. 3) This score known as a poverty quintile ranges from Quintile 1 (Q1) to Quintile 5 (Q5) with (Q1) being the poorest and (Q5) the least poor and Q1 – Q3 being no fee paying schools in Gauteng Province.

So initially the study intended to identify Q1 – Q3 schools around Johannesburg. However it became apparent that such a criterion would be problematic. While the creation of a national quintile system had been welcomed, it was realized later it had its own problems. In some provinces such as Gauteng, some schools formerly deemed poor now found themselves located in the less poor quintiles (Department of Basic Education,

2010). This meant that the poverty score which considers both poverty of the community and the poverty of the school may not always accurately capture the learner population of the school. Searching from the archives, for other indicators of poverty, two inner city schools and one township school around Johannesburg were identified as possible sites for the study.

These neighbourhoods are known for their high levels of population density, unemployment, poverty and crime. During the apartheid era these townships were created away from the city centre to house mainly black labourers, who worked in mines and other industries in the city. The inner city was later to be reserved for white occupation as the policy of segregation took root. The perennial problems of townships since their inception included poor housing, overcrowding, high unemployment and poor infrastructure. This has seen settlements of shacks made of corrugated iron sheets becoming part of the landscape.

So taking these factors into consideration, schools dealing with these predominantly black communities were identified through shared information from other research projects that were taking place in the Gauteng Schools. Letters requesting for their participation were sent out, preliminary school visits were made to the schools that showed interest and eventually three schools were selected to participate on this study.

3.5 VALIDITY AND RELIABILITY

According to Golafshani (2003), although reliability and validity are tools of an essentially positivist epistemology, these concepts are viewed differently by qualitative researchers who strongly consider the way these concepts are defined in quantitative terms as inadequate. If we see the idea of testing as a way of information elicitation then the most important test of any qualitative study is its quality; hence the difference in purposes of evaluating the quality of studies in quantitative and qualitative is one of the reasons that the concept of reliability is irrelevant in qualitative research. Although reliability and validity are treated separately in quantitative studies, these terms are not viewed separately in qualitative research. Instead, terminology that encompasses both

such as credibility, transferability and trustworthiness is used. In qualitative research, the idea of discovering truth through measures of reliability and validity is replaced by trustworthiness, which is defensible and establishing confidence in the findings (Lincoln & Guba, 1985). Seale (1999), asserts that in qualitative research the term trustworthiness of a research report lies at the heart of issues conventionally discussed as validity and reliability while Lincoln and Guba (1985), argue that sustaining the trustworthiness of a research report depends on the issues, quantitatively, discussed as validity and reliability. So while the terms validity and reliability are an essential criterion for quality in quantitative paradigms, in qualitative paradigms the terms credibility, neutrality, consistency and applicability are essential criteria for quality (Healy & Perry, 2000; Lincoln & Guba, 1985).

If the validity or trustworthiness can be maximized or tested then more credible and defensible results may lead to generalisability which is one of the concepts suggested by Stenbacka (2001) as the structure for both doing and documenting high quality qualitative research. Triangulation is typically a strategy (test) for improving the validity and reliability of research or evaluation of the findings. Mathison (1988), elaborates this by saying;

> Triangulation has risen as an important methodological issue in naturalistic and qualitative approaches to evaluation [in order to] control bias and establishing valid propositions because traditional scientific techniques are incompatible with the alternate epistemology (p. 13).

Triangulation is defined to be a validity procedure where researchers search for convergence among multiple and different sources of information to form themes or categories in a study (Golafshani, 2003). To improve the analysis and understanding of others, triangulation is a step taken by researchers to involve several investigators or peer researchers' interpretation of the data at different time or location. In a related way, a qualitative researcher can use investigator triangulation and consider the ideas and explanations generated by additional researchers studying the research participants.

In terms of ensuring validity and reliability of this study the research employed various quality assurance methods. Firstly the proponents of the cognitive demand tools that were used provided examples of content that would go into each cognitive level which made it

easier for raters to reach consensus thereby increasing the inter-rater reliability. These tools were also presented and sharpened at PhD seminars as well as local and international conferences. The same tools were tried and tested by experts in mathematics at Senior Certificate Level (inter-rater reliability of 0,84) and were finally pilot tested by the researcher. Evidence of credibility is also based on similar findings from other research analysing the new Matric papers e.g. (Umalusi, 2009). Although Edward's (2010) research focused on the Physical Science curriculum, he used exactly the same tool by Porter (2002) in his analysis of the FET curriculum and the results of his alignment research were also used to buttress the findings of this study.

The design of this study was also conceptualized with validity and reliability in mind. As alluded to in the methodology chapter this study employed a multiple case design with four teachers teaching a series of five lessons each for the whole week. It was hoped that if more cases were shown to support the same theory, replication could be claimed. In analytical generalisation, each case is viewed as an experiment, and not a case within an experiment. The greater the number of case studies that show replication the greater the rigour with which a theory has been established. Multiple case designs are preferred on the basis of this replication logic because multiple cases can be regarded as equivalent to multiple experiments. The more cases that can be marshaled to establish or refute a theory, the more robust are the research outcomes.

3.6 ETHICAL CONSIDERATIONS:

Educational researchers have a responsibility to ensure that in whatever research paradigm they work, their research should be conducted within an "ethic of respect" to those who participate (Clarke2004). Good research practice should therefore involve a partnership and whenever possible should be guided by the needs of the participants who should be an important concern to the researcher. So an ethical relationship in research is one that regards the other (participant) as an end and not as a means to an end.

The following were the more general ethical considerations that were envisaged together with the possible strategies the researcher employed in addressing them. I first applied for

ethics clearance from the University Ethics Committee which was granted. Through letters of request to the department of education, the school principals and the participant teachers, the parties to this research went into a contractual agreement that the data collected in this research would remain confidential. I then informed the prospective participants through personal visits to the schools of the aims and objectives of the project then sought their informed consent to participate either as interviewees, respondents to questionnaires, or video recordings' participants of the research. Participants were also informed as to where the data collected and reports thereof would be presented or published and that, unless it was considered of benefit to the participants and only after getting their approval, this research would not identify both people and schools by their names and communication through conferences presentations, journal papers and other forums would observe this anonymity throughout. Research is built on mutual trust, i.e. participants will provide correct information and the researcher also keeps no hidden agendas. This mutual trust was emphasized during communication with the prospective participating teachers and was maintained throughout the research period. The participants were also informed of their right to discontinue participation if they so wished. However the researcher marketed the research and presumably all the four teachers identified saw value in the research evidenced by their continued participation throughout the period of research.

The object of study in this research was teacher classroom practices, which comprise their actions, associated attitudes, beliefs and knowledge with reference to the NCSM at FET level. Because these were issues to do with the 'image of a person' they also raised a number of ethical questions like: "Whose perspective is being documented? Whose practice do we seek to understand and for whose benefit?" Researchers who have done relational educational research advise that an ethical way of doing such research would call for a commitment on the part of the researcher key to which was a commitment to reciprocity (Piquemal, 2000). Reciprocity implies that both the researcher and participants are involved in a relational dialogue in which they share, give and receive. In this study, the researcher informed the participants upfront that this study was a practice oriented analysis of learning where the participant teachers would, in a collegial

manner, be observed teaching. Discussions that would follow thereof were meant to benefit all i.e. the researcher, the participant teacher and the participant learners. Such discussions offered opportunities for both reflective feedback and a launching pad for different and possibly better strategies leading to improved learner performance. Classroom events and not individual subjectivities formed the constituent elements through which the research intended to identify patterns of classroom practices. This in turn would allow both the researcher and the participant teachers to interrogate the curriculum expectations and the classroom settings with respect to practices they afforded and constrained. The question that the research was concerned with was the extent to which the characteristics of higher order cognitive skills and processes as claimed in the national curriculum statement for mathematics were evident in classroom practices and what might be learned from the correspondence or inconsistency in the occurrence of some of the observed classroom patterns. Without an understanding of these processes, attempts to improve teaching practices and learning outcomes in mathematics classrooms in South Africa would have little chances of success.

A number of ethical issues were also envisaged to emerge from both the collection and use of video-based data in this research. For example the presence of a camera intrudes on the natural environment being studied i.e. their privacy and in a way then research influences the researched. So participants in this research were informed of the nature and purpose of the filming to help them allow their privacy to be shared by the researcher and therefore try to 'act naturally' and minimize distraction. Another ethical issue that could emerge from this research is that of ownership of digital video based documents. The issue of the researcher's right to use material generated from such video recordings during conference presentations or other forms of publications in a way impinges on both the participants' intellectual property rights and the need for confidentiality (Schuck & Kearney, 2004). This constraint meant that if I wished to use a clip in which there was potential that learners or teachers could be clearly identified, I would have to find a way to disguise the identity of their faces. This would be achieved through the use of video editing like 'masking'. Another way of that was used to get around the problem was that during filming, I would concentrate on getting video footage of the activity rather than

the learners or the teachers. This was also be achieved by recording them from behind so that they could not be identified. Alternatively, the video recordings could simply be used for analytic purposes only and never be shown to the public.

3.7 SUMMARY

This chapter started by justifying why the multiple-case design was appropriate for this study. Document analysis was employed as a method of collecting data. Porter's alignment indices (quantitative techniques) were proposed for judging the level of cognitive demand and the level of alignment within the documents themselves (internal consistency) and in relation to the examination papers (external consistency). Webb's (2005) categorical coherence criterion (qualitative technique) was proposed to compare the content within the documents themselves and against the examination papers. Video recording was adopted as the method for collecting data in the remaining four cases of teachers. The video recordings were going to focus mainly on LO2 (Functions and Algebra) and LO1 (Number and Number Relationships) with a specific interest on Number Patterns. The justification for this choice was articulated. Businskas' (2008) model of mathematical connections was proposed to analyse teachers' utterances/activities in relation to their potential to enable learners' development of higher order cognitive skills and processes. The next chapter now presents the findings from the document analysis.

CHAPTER FOUR – DOCUMENT ANALYSIS

4.1 INTRODUCTION

In looking for coherence through the curriculum documents, the study borrowed from Pershing (2002b), who advised that a document analysis should start by answering the question: ‘ What performance problem, quality improvement initiative, or evaluation criterion is to be analyzed from the documents or artefacts?’ So the analysis started by identifying what was espoused in the NCSM. The term ‘espoused’ is being used in this context of something that is being championed or advocated for, which was not being advocated for before - hence ‘new’. The justification for this single focus approach to what is ‘new’ is that most approaches to study alignment begin with one set of standards and then measure the extent to which assessments are aligned to that specific set of standards/objectives (Liang & Yuan, 2008). Their findings from such approaches provide important lessons for those pursuing alignment analyses in standards-based reform in that when there is a single focus the analysis also remains focused. This was also in line with Bruner’s (1960) and Whitehead’s (1929), long standing recommendations that the important goals should be identified and that these should be high quality goals which should then anchor the curriculum.

Following from this question and consistent with the overall aim of the study, the second question aimed at examining the extent to which the documents were consistent both internally (*amongst the documents*) and externally (*against the examination papers*) in articulating this vision. Because the same set of cognitive demand tools were used to address both the identification of the espoused mathematics and to ascertain the levels of coherence in the documents, the results for those two questions are presented and discussed concurrently.

4.2 RESULTS 1 AND 2

This section starts with observations made about developmental coherence. The concept of developmental coherence as discussed in the previous sections of this study was viewed from two perspectives. Within the documents it can be viewed in terms of whether the content is based on a progressive or hierarchical structure consistent with the logical nature of the discipline. The decision to look across the three grades (10 – 12) at FET level was taken on the premise that richer analysis of development coherence in terms of logical progression and hierarchical development of content would best be achieved by looking across the grades. An extract of the assessment standards is provided in a table below to highlight some of the observations made from this document analysis. Notice again that in the South African context the term Assessment Standards is not used in the context of the tested curriculum but it actually refers to what would be called content standards in other contexts. In that sense Assessment Standards are actually part of what is being considered as the written curriculum in this study.

The table below is an extract of Assessment Standards compiled from the NCSM (Department of Education, 2003, pp. 16 - 21) across (Grades 10 – 12).

Table 4.1 AN EXTRACT FROM THE NCSM SHOWING SEQUENCING OF CONCEPTS

LO 1	**ASSESSMENT STANDARDS**		
	Grade 10	Grade 11	Grade 12
	We know this when the learner is able to:		
Number and Number Relationships	10.1.1 Identify rational numbers and convert between terminating and recurring decimals. 10.1.2 (a) Simplify expressions using the laws of exponents (b) establish between which 2 integers any simple surd lies (c) round rational and irrational numbers to an appropriate degree of accuracy	11.1.1 Understand that not all numbers are real 11.1.2 (a) simplify expressions using the laws of exponents for rational exponents. (b) Add, subtract, multiply and divide simple surds (c) demonstrate an understanding of error margins	12.1.1 Understand that not all numbers are real 12.1.2 Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real-life problems e.g. growth and decay
	10.1.3 Investigate number patterns and hence (a) make conjectures and generalizations (b) provide explanations and justifications and attempt to prove conjectures 10.1.4 Use simple compound growth formulae to solve problems 10.1.5 demonstrate an understanding of the fluctuating foreign exchange rates 10.1.6 solve non-routine , unseen problems	11.1.3 Investigate number patterns hence (a) make conjectures and generalizations (b) provide explanations and justifications and attempt to prove conjectures 11.1.4 Use simple compound growth formulae to solve problems 11.1.5 demonstrate an understanding of different periods of compound growth and decay 11.1.6 solve non-routine unseen problems	12.1.3 (a) Identify and solve problems involving number patterns, including but not limited to geometric sequences and series (b) Correctly interpret sigma notation (c) Prove and correctly select formula for … 12.1.4 (a) Calculate the value of *n* in the formula (b) Apply knowledge of geometric series to solve….. 12.1.5 Critically analyse investment and loan options… 12.1.6 Solve non-routine , unseen problems

If one looks at the structure of the standards one could immediately notice a prima facie evidence of the concept of sequential development that Webb (2005), referred to. Sequential development meant developing documents in sequence so that the first document is aligned and used as reference for the second document. Looking at this table one could notice that the Grade 10 standards build on to the Grade 11 standards which in turn also build on to the Grade 12 standards. This can be evidenced by the fact that the assessment standards for each grade are placed against those of the next grade allowing a one to one match of the items to be made as one progresses from Grade 10 right through to Grade 12.

Judged by the descriptors of lower and higher order cognitive demand given in the literature earlier, one could notice that there is some evidence of developmental coherence within the grades. For example in Grade 10 the standards get more cognitively demanding from 10.1.1 identify rational umbers, 10.1.2 simplify expressions 10.1.3 investigate number patterns, 10.1.4 use simple compound growth formula to solve problems 10.1.5 demonstrate an understanding of the fluctuating foreign exchange rates 10.1.6 solve non routine problems. Identifying rational numbers would denote a recall skill which is at the lowest end of the cognitive demand scale of the grade 10 content, while solving non routine problems would denote a higher order cognitive skill. Developmental coherence can also be noticed in that as the assessment standards progress from one grade to the next they appear to become more cognitively demanding going up. For example the item Grade 10.1.5 says demonstrate an understanding of the fluctuating foreign exchange rates, Grade 11.1.5 says demonstrate an understanding of different periods of compound growth and decay, Grade 12.1.5 says critically analyse investment and loan options. One can notice that the assessment standards, at least within a learning outcome, articulate concepts and skills that are progressing from lower to higher order both within the grades and from one grade to the other.

This seems consistent with policy rhetoric which suggests conceptual progression in three stages i.e. stage 1 subject framework, stage 2 work schedule and stage 3 lesson plan. According to the Learning Programme Guidelines,

> The subject framework (stage 1) should indicate the increasing depth of difficulty across Grades 10 – 12. Progression in a grade (stage 2) should be evident in the increasing depth of difficulty in that particular grade. Grade specific progression is achieved by appropriately sequencing the groupings of the integrated learning outcomes and assessment standards in the work schedule. In the individual Mathematics classroom (stage 3) increasing depth of difficulty should be shown in the activities and lesson plans (Department of Education, 2008, p. 16).

Pages 22 – 48, of the learning programme guidelines then provide some examples of work schedules for the different grades. Looking through the schedules one could notice that from week 1 through to week 4 there is evidence of promotion of multiple representation (numerical, verbal, graphical and symbolic) and there is a deliberate attempt to promote the development of higher order thinking/reasoning skills (make and test conjectures then generalize the effects of the parameters…). What stands out clear in all the examples is the continuous intention to have progression from lower order to higher order and the multiple representations of mathematical concepts as the espoused approaches to learning and teaching of mathematics.

However within the same structure of the standards one could also notice that there is a clear focus on performance descriptors yet an equally conspicuous lack of specification of content is evident. For example looking at 10.1.3 Investigate number patterns and hence (a) make conjectures and generalizations (b) provide explanations and justifications and attempt to prove conjectures, one can notice that the same descriptors are used at the next grade level 11.1.3 Investigate number patterns and hence (a) make conjectures and generalizations (b) provide explanations and justifications and attempt to prove conjectures. Similarly the last set of standards across the three grades in the table above, 10.1.6 says solve non-routine, unseen problems, 11.1.6 says the same thing solve non-routine, unseen problems and 12.1.6 says exactly the same thing solve non-routine, unseen problems. Falling back on Newman's point as discussed in chapter 2, in the absence of specification of 'what' is being investigated, solved or explained at the different grades policy rhetoric might be constrained in claiming developmental

coherence or progression between grades in such circumstances. So while the learning outcomes are designed to apply across all grades from R to 12, they have been heavily criticised for creating artificial similarities around what is learnt at different levels (Dada, et al., 2009). Critics have pointed to lack of specificity and detail in content because outcomes are specified in a general and often generic way.

This seems to confirm why for the last ten years, (OBE) has been under persistent attack in South Africa. A wide range of both local and international research argues that outcomes inhibit the clear specification of what content, concepts and skills needed to be taught and learnt (Donnelly, 2005; Jansen, 1998; Muller, 2004, 2005; Muller & Subotzky, 2001). According to Donnelly (2005), what results from an (OBE) curriculum are curriculum and assessment descriptors that are often vague, ambiguous, difficult to measure and low in academic content (p. 38). There is evidence that teachers at these different grade levels struggle to make sense of some of these requirements yet one key criteria for considering curricula is the extent to which they make available to teachers statement which are "clear, succinct, unambiguous, measurable, and based on essential learning as represented by subject disciplines" (Donnelly, 2005, p. 8). So while there are some indications of a system that takes into account what is known about how students' mathematical understanding develops over time, the NCSM also appears to lack specificity on the mathematical content knowledge, abilities and understanding that are needed for learning to progress at each stage of the process.

Having said that about the structural links that appear to be evident in the content standards the discussion now moves into more quantitative ways that were used to judge levels of alignment. In doing so the study acknowledges that the Assessment Standards represent only a summarised and therefore small sample of the intended curriculum because of lack of detail alluded to earlier. Similarly, examination papers are also by nature a small sample taken from the intended curriculum. Because this study analysed the level of alignment using mainly the assessment standards and 2008 exemplar papers as the major source documents, inevitably the results are based on a small sample of the intended curriculum. The decision to sample only the 2008 exemplar papers was made

due to the fact that at the time of doing the document analysis, no national examination based on the new FET curriculum had been written yet. Due to its relatively small size, this subsample of the 2008 exemplar papers might not be representative of the entire sample of the FET mathematics curriculum. The results are therefore raw percentages rather than weighted percentages and ratings shown in the figures are descriptive in terms of the relationship that existed between the 2008 exemplar papers and the assessment standards. With no statistical comparisons having been made the figures might therefore have low inferential power. However in order to ensure validity and reliability in the findings they were also corroborated with findings from other research with a similar focus.

When coding data from the content standards and the exemplar papers, two experts were consulted - one who sets papers for FET National Examinations and one who is a moderator of these papers making a total of three raters with the researcher. This followed Lombardi et al., (2010), who recommended that most alignment studies use between three and ten expert raters and the error attribute to these raters should be as small as possible. There was a debriefing with the researcher and the two experts in order to come to a consensus as to what content would be coded under each of the cognitive levels of the tool. Coding of the content standards was relatively straight forward in view of the fact that, in the absence of content, these are stated mainly in the form of descriptors such as identify, simplify, investigate, and provide an explanation and justification for. These descriptors had an almost perfect match with the descriptors in Porter's cognitive demand tool as discussed in chapter 2, (p77) such that there was very little reinterpretation if any to be done by coders.

The Matrix X in table 4.2 below first gives the data counts showing how the items in the Grade 11 curriculum standards were coded with respect to the different categories.

Table 4.2 MATRIX X FOR DATA COUNTS OF ASSESSMENT STANDARDS GRADE 11

LEARNING OUTCOME	CATEGORY OF COGNITIVE DEMAND							
	Lower order level			Higher order level				
	A	B	Sub-totals	C	D	E	F	Totals
(Number and Number Relationships)hips	1	3	**4**	2	1	3	**6**	**10**
Functions and Algebra	3	4	**7**	5	5	5	**15**	**22**
Space, Shape and Measurement	0	1	**1**	0	10	9	**19**	**20**
(Data Handling and Probability)	2	2	**4**	4	3	3	**10**	**14**
TOTAL	**6**	**10**	**16**	**11**	**19**	**20**	**50**	**66**

The decision to use Grade 11 standards was made after taking a number of validity and reliability issues into consideration. Firstly from the document analysis there is evidence that the design features of the assessment standards are such that each grade standards are placed against those of the next grade allowing a one to one match of the items to be made as they progressed from Grade 10 right through to Grade 12. Such a design feature results in similar numbers of items falling into each category on the cognitive demand scale such that any of the grade curriculum standards (grades 10 – 12) which were so sequentially developed would have produced a similar table above - hence validity and reliability of data would not be compromised.

Another consideration was that Porter's alignment index can best be described as a measure of relative emphasis (Porter, et al., 2007) in that their cognitive demand tool does not directly compare content in two different documents. What it does is that it places content for each document into cognitive demand categories based on the

descriptors discussed in chapter 2, then compares the relative emphasis of the two documents. That way it was possible to compare a Grade 1 curriculum with say a Grade 6 curriculum. Similarly my study was not comparing Grade 11 content standards directly with the content in the Grade 12 Exemplar papers but in accordance with this tool, content in Grade 11 curriculum was first placed into cognitive demand categories independent of the exemplar papers then the same was done for the exemplar papers. It was only after doing this that an alignment index (measure of relative emphasis) was calculated. But I have also argued that the structural designs for the Grade 10, 11 and 12 content standards are the same - hence a similar alignment index (measure of relative emphasis) would still have been obtained if any of those grades' 10 – 12 content standards had been compared with the exemplar papers. This should allay fears about credibility of the data and the results thereof.

The other reason for using Grade 11 curriculum standards was that classroom observations were to be done in Grade 11 classes following standard practice in South Africa that recommends that research be done in non examination grades. A Grade 12 is considered as a writing class in the sense that it is where standardised testing at FET (end of phase) level is done. The concern generally is that allowing researchers to use learners/teachers as research participants in such classes might interfere with their preparations for the final examinations. So in terms of tracing the translation of the written/intended curriculum into the taught curriculum the Grade 11 curriculum standards were used so that richer and more informed comparisons would be made between data obtained from classroom observations and what was in the written curriculum.

So going back to the table of coded data above, the way Matrix X in table 4.2 is interpreted is that there are 5 items for example, in the assessment standards for Functions and Algebra, 9 items for Space, Shape and Measurement that require learners to conjecture, generalize and prove. There were two major areas of interest in this table 4.2 i.e. data counts in relation to learning outcomes and data counts in relation to lower order level and higher order levels. The data counts in relation to learning outcomes would enable me to identify in which learning outcome the NCSM placed more

emphasis. The intention was to focus classroom observations on a learning outcome that was considered relatively more important in the NCSM. Analysis of data counts in relation to cognitive demand levels would enable judgement to be made on whether or not high quality goals were espoused.

Looking across the rows of learning outcomes, it would appear the NCSM places more emphasis on LO2 (Functions and Algebra). It has the highest total of 22 counts. Having done this comparison of data counts by learning outcomes the next thing was to attempt to answer the first research question of this study; "What levels of cognitive demand are evident in the mathematical knowledge and skills as articulated in the written curriculum?" Cognitive demand levels are represented by the columns from A lowest to E highest. An analysis of those totals of columns reveals the following A = 6, B =10, C = 11, D = 19, E = 20. The columns A and B comprising data counts at lower order levels have a total of 16 counts while columns C, D and E comprising data counts at higher order levels have a total of 50 counts. The skewness towards these higher order level categories appeared more pronounced in LO3 Space, Shape and Measurement where there were 19 scores in the three categories C, D and E against 1 in column B (lower order level) representing a 95% bias towards these higher order level categories. This data count alone seemed to affirm the pronouncement in the Learning Programme Guidelines (LPG) that:

> As a way to achieve the mathematics learning outcomes, teaching and learning in mathematics focuses on the development of learners towards the four learning outcomes. Central to the attainment of the learning outcomes (LO's) is the development of mathematical process skills e.g. investigating, conjecturing, organizing, analyzing, proving, problem solving, modelling (Department of Education, 2008, p. 11).

However caution should also be taken in the interpretation of these data counts in view of the lack of content specification that was alluded to earlier. Suffice it to say judged by these data counts one can argue that the NCSM's espousal of high order level skills and processes is evident within the assessment standards.

After having completed the coding of data in Matrix X table 4.2, these data were processed for proportional quantification. This quantification process transforms the data

counts into proportional values e.g. cell A1 or (1,1) transforms thus: $\frac{1}{66} = 0$ where 1 is the value in the cell and 66 is the total of such cell values (16 lower order level + 50 higher order level) in the matrix. Notice that the proportional values are given correct to 1 decimal place. This procedure is repeated for all other cells to give another Matrix X in table 4.3 now with proportional values as opposed to data counts. Once completed, the proportional values across all content descriptions for any given document should add up to 1. It is on these proportional values that alignment analyses are conducted (Porter, 2004).

Table 4.3 MATRIX X WITH PROPORTIONAL VALUES

LEARNING OUTCOME	**CATEGORY OF COGNITIVE DEMAND**				
	Lower order level		**Higher order level**		
	Memorize **A**	**Perform procedures** **B**	**Communicate understanding** **C**	**Solve non-routine problems** **D**	**Conjecture/ generalize/ prove** **E**
(Number and Number Relationships)hips	0	0	0	0	0
Functions and Algebra	0	.1	.1	.1	.1
Space, Shape and Measurement	0	0	0	.2	.2
(Data Handling and Probability)	0	0	.1	.1	0

After completing this initial coding, the next task was to code the content in the exemplar papers. Key to this coding process was that comparison of tasks had to be at the level of processes rather than content – which made comparison across Grade 11 curriculum and Grade 12 exemplar papers more possible since the process descriptors are the same right across the three grades 10 – 12 of the FET level. However, coding of content in the

exemplar tasks was not as straight forward as coding of assessment standards since exemplar tasks use descriptors in context e.g. solve the equation $2x + 2 = 12$. In such cases a decision had to be made as to whether such a solution would require a routine procedure (lower order) or it would be non-routine (higher order). In order to achieve this, I also borrowed from Newman's (1990) suggestion that one would need to know something about the person's intellectual history in determining the cognitive demand levels of a task in relation to that particular person. In this case the intellectual history was not measured by the levels of understanding of the Grade 12 learners but the reference point was the Grade 12 curriculum. It was possible to tell what was expected of Grade 12 learners intellectually by looking at the demands of the curriculum at that level hence determine whether or not a task was a lower order or higher order one for a Grade 12 learner. So when coding content in the exemplar papers the cognitive demand descriptors as discussed in the literature review were also used in the context of each task to decide the category into which the task would be placed. This was also consistent with Newman's (1990) point that we needed to analyse the descriptors such as define, identify, name, and compare in conjunction with the mathematical idea 'the what' and 'the who' in order to make appropriate judgement about lower order and higher order skills and processes. This was important because the exemplar papers specify content unlike the curriculum standards which appear to focus more on descriptors of processes than content. Just to have a feel of how the categorization was done across Porter's 5 categories, here are some examples of tasks taken from the 2008 exemplar papers and how they were coded in this study across porter 5 categories.

Table 4.4 EXAMPLES OF CODING OF CONTENT IN EXEMPLAR PAPERS

EXAMPLE	TASK	CATEGORY	
1	Consider the sequence: 2;5;2;9;2;13;2;17;…Write down the next two terms given that the pattern continues	A	Lower order cognitive demand
2	solve for x: $x^2 - 10x = 24$	B	
3	Using a search line and your graph, determine the number of Acuna and Matata minibuses that will yield a maximum profit (*data had been provided in the question*)	C	Higher order cognitive demand
4	Explain why the equation $\frac{x^4+1}{x^4} = \frac{1}{2}$ has no real roots	D	
5	Calculate the amount that must be invested monthly into a sinking fund to cover the replacement cost of the bus in 4 years' time if the interest paid by the financial institution is 9% per annum compounded monthly. Payments are made at the end of each month. (*certain information had been provided*).	E	

Task 1 was interpreted in accordance with the distinction made by Lewis and Smith (1993) between basic (lower order) and integrated (higher order) processes. Their view is that basic processes include observing and inferring and in this case our view was that the task required the student to observe the pattern and then to make an inference about the next two terms – therefore a lower order task. Task 2 was interpreted in accordance with Newman's (1990) definition of a lower order task which he views as demanding only routine or mechanical application of previously acquired information and or formulae. Our view was that this task required the learners to factorise a quadratic expression which could be achieved by inserting numbers into a previously learned formula. This in Maier's (1933) view could be a typical example of a learned behaviour which came from contagious experiences with previous repetitions. Learners are usually drilled on factorization of such types of trinomials hence we considered this task a lower order one as it required them to know/remember the procedure. Task 3 was interpreted in accordance with Lewis and Smith's (1993) view of integrated process (higher order) which required the learner to manipulate, analyze, and interpret data. The view taken in this analysis was that this task required systematic reasoning or communicating understanding on the part of the learners hence a higher order task. Task 4 was in our view requiring the learner to prove that the given equation had no roots and then to justify why this was so. This would be a typical example of a process of defining operationally, solving a non-routine problem, an integrated process according to Lewis and Smith's (1993) which is therefore higher order. Task 5 in our view required the learners to solve a

non-routine problem. I noted from the discussions with the other two raters that with all these varying descriptors, there was no perfect agreement when placing content into the five categories especially between columns A and B (lower order) then columns C, D and E (higher order). However there was a general consensus on the distinction between higher order and lower order tasks. After having agreed generally on how coding would proceed, some items from the exemplar papers were independently pilot coded during this debriefing exercise and then cross checked to see how much the team was in agreement/disagreement. The inter-rater reliability for the three raters was 0.84. This index of dependability is considered quite high (Porter, 2002). The coding then proceeded after that and below is Matrix Yin table 4.5 showing the data counts of the different cognitive demand categories.

Table 4.5 MATRIX Y FOR 2008 GRADE 12 EXEMPLAR PAPERS 1 & 2 - DATA COUNTS

LEARNING OUTCOME	CATEGORY OF COGNITIVE DEMAND					
	Lower order level		Higher order level			Totals
	Memorize	Perform procedures	Communicate understanding	Solve non-routine problems	Conjecture/ generalize/ prove	A+B+C+D+E
	A	B	C	D	E	F
(Number and Number Relationships)hips	2	9	2	0	0	**13**
Functions and Algebra	7	20	2	2	2	**33**
Space, Shape and Measurement	5	22	6	2	1	**36**
(Data Handling and Probability)	1	6	2	1	1	**11**
TOTALS	**15**	**57**	**12**	**5**	**4**	**93**

Having done the categorization of tasks according to the cognitive demand levels the next step was to convert them to their proportional values as was the case with the content standards. Table Y in table 4.6 below shows the result of that conversion into proportional values.

Table 4.6 MATRIX Y FOR 2008 GRADE 12 EXEMPLAR PAPERS 1 & 2 –PROPORTIONAL VALUES

LEARNING OUTCOME	CATEGORY OF COGNITIVE DEMAND				
	Lower order level		Higher order level		
	Memorize A	Perform procedures B	Communicate understanding C	Solve non-routine problems D	Conjecture/ generalize/ prove E
(Number and Number Relationships)hips	0	.1	0	0	0
Functions and Algebra	.1	.2	0	0	0
Space, Shape and Measurement	.1	.2	.1	0	0
(Data Handling and Probability)	0	.1	0	0	0

At this stage it was now possible to attempt to answer the second research question of this study; “To what extent are the written and tested components of the NCSM aligned in terms of cognitive demand levels?” To determine the level of alignment between the two sets of data, a cell-by-cell comparison was then made for each corresponding proportion from the cells of the two proportion matrices X in table 4.3 and Y in table 4.6. The alignment measure between those two cells reports the relative emphasis of instruction content in common as defined by the cognitive demand descriptors. This value can be determined in two ways, which complement each other but yield the same mathematical result. The first procedure that is going to be discussed here is the one that is readily understood and easier to calculate (Roach, et al., 2008). It makes use of the smaller of the two values taken from the comparison of the two corresponding cells. The process is repeated for each pair of cells in the matrices, to end up with a matrix like table 7 below.

Table 4.7 MATRIX FOR ALIGNMENT INDICATOR

LEARNING OUTCOME	CATEGORY OF COGNITIVE DEMAND				
	Memorize **A**	**Perform procedures** **B**	**Communicate understanding** **C**	**Solve non-routine problems** **D**	**Conjecture/ generalize/ prove** **E**
(Number and Number Relationships)hips	0	0	0	0	0
Functions and Algebra	0	.1	0	0	0
Space, Shape and Measurement	0	0	0	0	0
(Data Handling and Probability)	0	0	0	0	0

To obtain the final alignment index the values held in common for each pair of cells (the smaller of the two numbers in the comparison) are then summed across all cells. For my comparisons between the assessment standards and the exemplar papers the resulting alignment value was:

Alignment Index P = 0.1

The second procedure aggregates the absolute value of the difference between each pair of corresponding cells across all the cells of the matrix X in tables 4.3 and matrix Y in table 4.6. This total is then divided by 2 and the result is subtracted from 1 to end up with the alignment index. For my comparisons between the assessment standards and the exemplar papers the resulting alignment value was:

$$AI = 1 - \frac{\sum |x - y|}{2}$$

$$= 1 - \frac{1.6}{2} = 0.2$$

There is a small difference in the two alignment indices resulting from the two routes. Suffice to say that both indices 0.1 and 0.2 reflect a weak alignment. Although two Curriculum, Instructional and Assessment (CIA) components could be in perfect alignment (i.e. a P score of 1), a more typical result is a score between 0 (no match) and 1 (perfect alignment). The researchers who have developed this alignment index have indicated that the larger the value of the index, the better the alignment (Porter, et al., 2007). In view of the fact that this alignment index can best be described as a measure of relative emphasis, it can be argued that there is some disparity between what the NCSM assessment standards emphasize and what is being emphasized in the exemplar papers. However, a low alignment index such as the 0.2 obtained here is not necessarily a bad thing if this is due to the examination including more cognitively demanding items than the standards. There is empirical evidence showing a positive impact of testing on teaching due to the relative emphasis of the topics and cognitive demand (Liu, et al., 2009). This is expected given the 'back wash' effect that has been cited in the literature. Teachers may use the items to adopt more student-centred pedagogies or they will use it to drive their lessons in a teacher-centred way (Edwards2010).

So the determination of an alignment index only marks the beginning of more comprehensive alignment analyses because usually an attempt has to be made to account for the low or high alignment index and to see where the differences in emphasis could be. According to Roach et al., (2008), usually reviewers have to make qualitative judgments considering what objectives in standards seem to be over-assessed and what objectives seem to be under-assessed or not assessed at all.

Having confirmed this position, the next task then was to analyse whether there were some other indicators of horizontal coherence. Webb's (2005), categorical-concurrence criterion was used at the next level of analysis. According to Webb (op cit), an important aspect of alignment between standards and assessment is whether both address the same content categories. The categorical-concurrence criterion provides an indication of alignment if both documents incorporate the same content. This criterion is met if the same or consistent categories of content appear in both documents.

Within this broader context of categorical-concurrence, Edwards (2010), used the concept of discrepancies to analyse ratios of corresponding cells from two alignment matrices developed within the lower order /higher order framework. These discrepancies represent the differences between the **ratios** in the assessment standards table 4.3 (Matrix X) and in the examination papers table 4.6 (Matrix Y). Because the cells for the assessment standards are coming first in each case, negative discrepancies indicate that the assessment standards place less emphasis on that particular content at that particular cognitive level while the exemplar papers place more emphasis on the same content at the same cognitive level. Similarly positive discrepancies indicate that the assessment standards place more emphasis on that particular content at that particular cognitive level while the exemplar papers place less emphasis on the same content at the same cognitive level. A discrepancy of 0 indicates equal emphasis by both the assessment standards and the exemplar papers. The following four tables present the discrepancies by cognitive level in each of the four learning outcomes from (LO1) – (LO4).

Table 4.8 DISCREPANCIES BY LEARNING OUTCOME (LO1), WITH DIRECTION

		Memorise	Perform procedures	Communicate Understanding	Solve non-routine problems	Conjecture/ generalise/
		A	B	C	D	E
(Number and Number Relationships)	AS	0	0	0	0	0
	EX	0	.1	0	0	0
	Discrepancy	**0**	**-0.1**	**0**	**0**	**0**

Starting with table 4.8 above for example, the discrepancies shown in the last row indicate that both the assessment standards and the exemplar papers placed equal emphasis on LO1 (Number and Number Relationships) at all the cognitive levels except performing procedures (column B) where the assessment/content standards placed less emphasis than the exemplar papers.

I need to explain in a little more detail how to interpret the numbers in these tables in order to bring out the meanings and interpretations of the discrepancies thereof. For

example the zeros along the row for assessment (content) standards do not suggest that there were no assessment standards for LO1 Number at all but because the concept of discrepancies is based on comparing ratios these zeros are actually measures of relative emphasis hence ratios. Going back to the Matrix X for content standards with data counts, we would notice that there are 3 items in the row for LO 1 Number & Number Relationships under the cognitive demand column B (Perform procedures). There is a total of 66 items coded in this whole matrix. This number 3 when expressed as a ratio of 66 gives (0,045) which when rounded off to one decimal gives (0) a measure of relative emphasis of the LO1 Number & Number Relationships at this cognitive demand level as compared to total data counts for all the four learning outcomes coded on this matrix. Similarly all the other 0's along this row and any other value in these discrepancy tables were determined in the same way as ratios or measures of relative emphasis. These figures suggest that in terms of higher order skills there was equal emphasis in the two documents being compared but the exemplar papers had more emphasis at lower order skills than the assessment standards with respect to LO1.

Table 4.9 DISCREPANCIES BY LEARNING OUTCOME (LO2), WITH DIRECTION

		Memorise	Perform procedures	Communicate Understanding	Solve non-routine problems	Conjecture/ generalise/
		A	B	C	D	E
Functions and Algebra	AS	0	.1	.1	.1	.1
	EX	.1	.2	0	0	0
	Discrepancy	**-0.1**	**-0.1**	**0.1**	**0.1**	**0.1**

This table shows that in LO2 (Functions and Algebra), the curriculum placed more emphasis at the higher order levels C, D, and E while exemplar papers placed less emphasis at that same level. Exemplar papers placed more emphasis at the lower order levels of memorizing and performing procedures while the content standards placed less emphasis at the same level.

Table 4.10 DISCREPANCIES BY LEARNING OUTCOME (LO3), WITH DIRECTION

		Memorise	Perform procedures	Communicate Understanding	Solve non-routine problems	Conjecture/ generalise/
		A	B	C	D	E
Space, Shape and Measurement	AS	0	0	0	.2	.2
	EX	.1	.2	.1	0	0
	Discrepancy	**-0.1**	**-0.2**	**-0.1**	**0.2**	**0.2**

A striking observation from an analysis of the policy documents, together with these discrepancy matrices, is the curriculum expectation with regards LO3 Space, Shape and Measurement. A look through the assessment standards starting with 11.3.1 on page 33, right through to 11.3.5 on page 35 of the assessment (content) standards reflects a skewed expectation towards categories D and E (highest) of the cognitive demand scale. This is evidenced by the prevalence of higher order cognitive demand descriptors such as, investigate, prove, derive and apply, investigate, generalize and apply, establish and apply. This has the effect of placing these expectations in the columns with the highest levels of the cognitive demand matrix (D + E = 0.4) yet when one looks through the exemplar papers, one notes that out of 36 tasks that were testing this learning outcome, 22 tasks were in the routine procedures (lower level) of the cognitive demand matrix and only three were testing problem solving, hence the cumulative discrepancy of (0.4)

Table 4.11 DISCREPANCIES BY LEARNING OUTCOME (LO4), WITH DIRECTION

		Memorise	Perform procedures	Communicate Understanding	Solve non-routine problems	Conjecture/ generalise/
		A	B	C	D	E
(Data Handling and Probability)	AS	0	0	.1	.1	0
	EX	0	.1	0	0	0
	Discrepancy	**0**	**-0.1**	**0.1**	**0.1**	**0**

In (Data Handling and Probability), there is evidence also that the assessment standards placed more emphasis in communicating understanding and solving non-routine problems than the exemplar papers. There was equal emphasis at both extremes of the cognitive demand levels in this learning outcome. While the discrepancies may look

insignificant when broken down by learning outcomes, the cumulative effect is significant in terms of a skewed combined comparison.

Table 4.12 DISCREPANCIES FOR ALL LO'S BY COGNITIVE LEVEL, WITH DIRECTION

	Memorise	Perform procedures	Communicate Understanding	Solve non-routine problems	Conjecture/ generalise/prove
	A	B	C	D	E
Assessment Standards	0	.1	.2	.4	.3
Exemplar Papers	.2	.6	.1	0	0
Discrepancy	**-0.2**	**-0.5**	**0.1**	**0.4**	**0.3**

An analysis of the cumulative discrepancies in columns C, D and E which focus on the higher order levels of communicating understanding, solving non-routine problems, conjecturing, generalizing and proving seem to reveal some glaring disparities. For example, the sum of proportional values in those 2 cells for assessment standards is 0.7 as compared to a sum of 0 from the corresponding cells from the exemplar papers. This suggests that while the assessment standards place significant emphasis on those higher order skills and processes, the exemplar papers provided very little emphasis at all on these higher order skills and processes in their requirements.

While these results had been obtained in November 2008 before the actual examination paper had been written, six months later Umalusi (2009), made an analysis of both sets of the exemplar papers and the examination papers for 2008 and summarised their findings with this comment:

> …However, the team was interested to note that the shift in the NSC curriculum towards more modelling, problem-solving and a focus on mathematical processes (like conjecturing, justifying, generalising etc.) was not reflected in either the exemplar papers or final papers to the extent that the team had imagined it would be, judging by the contents of the NSC curriculum (p. 70)

This seems to confirm that indeed the NCSM espouses the higher order skills but the tasks in the examination papers were not pitched to the same level of cognitive demand.

With specific reference to the learning outcome 3 Space, Shape and Measurement, Umalusi also made similar observations in that:

> ….the team found that almost all of the items dealing with transformation geometry as well as most of the statistics items in the 2008 NSC exemplar papers and the final papers were relatively easy in terms of cognitive demand. These levels comprise a possible reason as to why the NSC exemplar papers and the final Paper 2 papers were significantly easier than previous High Grade papers (p. 70)

The findings from these analyses help to confirm and therefore support the credibility of my findings in a number of ways. Firstly I argued that an analysis of the structural design of the NCSM shows that the content standards were sequentially developed from one grade to another such that the distribution of content into cognitive demand levels for each grade from Grade 10 – 12 would be similar. Measuring the alignment index of the exemplar papers against the content standards for any of those three grades whose curricula were sequentially developed would not yield any difference. While Umalusi (2009) did not specify which content standards were compared with the 2008 exemplar and examination papers, but assuming they used the Grade 12 content standards or any other, their findings which are similar to mine would still work in support of my argument that any of those three content standards would yield the same alignment results as similar process skills are mentioned across all 3 grades.

The Umalusi findings also confirm results from other quantitative measures from my study, such as discrepancies between cells, which showed that there were significant differences in LO3 Shape, Space and Measurement with exemplar papers focusing on the lower order cognitive levels while the content standards focused on the higher order cognitive levels. This observation was taken into consideration when deciding what the focus of the lesson observations would be, details of which are given in chapter 3.

Although Umalusi did not use a quantitative measure of the overall alignment between examinations and the curriculum, as was the case in this study, they still confirmed that emphasis in the NCSM is on higher order cognitive skills and processes while the 2008 exemplar and examinations papers measured lower order skills and processes.

Umalusi had this to say about the papers being used as models for future examinations:

> While recognizing that the examiners were faced with a difficult task in setting the first round of 'new' mathematics examinations, the team did express concern about some aspects of the 2008 NSC exemplar and final papers as models for future mathematics examinations. The most apparent issue was the lack of sufficient challenging questions to distinguish between achievement levels of the top candidates. In addition, the strong weighting towards skills at the lower end of the cognitive demand type, was of concern (p. 70).

About the team's general concerns with regards the quality of the papers:

> In addition, the team felt that, given the emphasis in the NCSM curriculum documents, the 2008 exemplar and final papers did not give sufficient attention to the following aspects
> - Application and modelling
> - Mathematical processes (e.g. investigating, generalising, conjecturing, justifying
> - Problem solving
> - Communication (expressing arguments, demonstrating reasoning ability (p.71)
>
> The team was also concerned that, given the similarity between the exemplar papers, preliminary examinations, additional exemplars and the final papers, teachers might already have assumed that future examinations will continue to be of this style and standard. The team thus felt that particular care needs to be taken when setting the 2009 exemplar and examinations, to not entrench these qualities (p. 71)

Umalusi (op cit) then put the implications of their findings in the African context and argued that examination results in Africa are high stakes and are the most popular determinant of access to higher education and the world of work. In that sense they hold great significance as a rite of passage, thereby providing incentives and motivation for students to learn. 'Teachers teach for examination success', is a commonly repeated phrase all over Sub-Saharan Africa. Assessment and evaluations that only require students to reproduce facts and definitions will inevitably train students for rote learning and memorization of facts, no matter what the curriculum wishes to aim at. Similarly;

> Assessment and qualifications that only test methodological and social competences lack the achievement of clear exit skills, and have proven to lead to an 'anything goes' attitude (Umalusi, 2009, p. 58)

Putting these findings in the global context, the World Bank Report 2007 also noted that for mathematics in Ghana and South Africa, 90 per cent or more students did not reach even the low international benchmark scale of 400 in the TIMSS 2003 (Mullis, et al., 2008, p. 59). TIMSS intended to mainly measure higher cognitive skills. This same report notes that pass rates in South Africa since 1994 have raised concerns in the country (p.

62). Commenting on the composition of the South African basic education examinations consisting of 75% CASS and 25% CTA, their observation was that students passing such examinations are widely reported to lack basic reading, writing and mathematical skills. They concluded by lamenting; “The validity of the assessment and reliability of the subsequent certificate awarded are questioned and yet to be proven” (Mullis, et al., 2008, p. 65)

4.3 SUMMARY

The results from this document analysis show that in terms of structure the NCSM appears to be internally consistent and developmentally coherent. There is evidence to show that the assessment standards from one grade to the other intend to build on one another hence there is a prima facie evidence of sequential developed. Within each grade, from grade 10 – 12, there is also evidence to show that the assessment standards attempt to articulate concepts and skills that are progressing from lower to higher order.

Comparing the assessment standards with the exemplar papers, an alignment index of 0,1 was obtained from one method and by way of triangulation of the methods 0,2 was obtained using a different method. In each case the alignment index is very low, pointing to some disparity between what the assessment standards appear to emphasize and what the exemplar papers seemed to test. A categorical –concurrence criterion was then used to analyse which learning outcomes could have possibly contributed to this low alignment index. The results showed that there was an almost equal emphasis from both the assessment standards and the exemplar papers in LO1 (Number and Number Relationships). However the discrepancies got more pronounced in LO2 (Functions Algebra), in LO4 (Data Handling and Probability) with the highest discrepancies being observed in LO3 Space, Shape and Measurement. Such discrepancies help to point to areas that need to be revisited when tests are being developed in future. Umalusi (2009) made similar observations about the pronounced difference in emphasis between the assessment standards and the examination papers in relation to LO3 Space, Shape and Measurement.

While the study observed these pronounced differences in emphasis on LO3, this observation however did not influence the decision on what was going to be observed in the classrooms. The interpretation of these discrepancies is that these are just temporary differences in relative emphasis between the written curriculum and the 2008 exemplar papers. They are temporary in the sense that another set of examination papers coming after the 2008 exemplar papers might have shown a pronounced difference in emphasis say in LO4 (Data Handling and Probability). So these discrepancies or differences in relative emphasis are not in anyway an indication of the overall emphasis of the curriculum. They have an implication only for future revision of tests or examination papers. Classroom observations were guided by what was emphasized in the curriculum statement as this was considered to be more sustainable and judged by the totals of data counts in table 4.2, this happened to be LO2 (Functions and Algebra).

4.4 CONCLUSION

This document analysis showed that the NCSM places more emphasis on LO2 (Functions and Algebra). The document analysis also helped to answer the first research question: "What levels of cognitive demand are evident in the mathematical knowledge and skills as articulated in the written curriculum?" Judged by the data counts in table 4.2 one can argue that in the NCSM what stands out are the high order level skills and processes. The second research question was: "To what extent are the written and tested components of the NCSM aligned in terms of cognitive demand levels?" Judged by the two alignment indices of 0.1 and 0.2 respectively, one can argue that the tested component of the NCSM as represented by the 2008-exemplar papers had a weak alignment with the written component of the NCSM. This was due to the exemplar papers testing mainly lower order knowledge and skills as opposed to the espoused higher order knowledge and skills in the written/intended curriculum. Using the categorical-concurrence criterion measured in terms of discrepancies between the assessment standards and the exemplar papers the results also showed that the most pronounced differences were in LO3 Shape, Space and Measurement where the exemplar papers tested more lower order items while the curriculum emphasized higher order skills and processes in that same learning outcome. A similar pattern can be seen in LO2 and LO4.

CHAPTER FIVE – EMPIRICAL RESULTS

5.1 INTRODUCTION

In this section I describe and explore all the four teachers' instructional practice as a response to the research question: "How do FET mathematics teacher practices foster the development of mathematical knowledge and process skills that are espoused in the NCSM?" I compare the utterances and activities with the aim of exploring the relationship between the FET mathematics curriculum expectations and the teachers' instructional practice. I will start with individual teachers' analysis then summary for each teacher followed by an overview analysis of all the four teachers looking for emerging patterns across all four teachers.

5.2 TEACHER 'R'- EXPANSION OF TRINOMIALS

Teacher R's lessons for the whole week were all on LO 2 Functions and Algebra and focused on the expansion of brackets or finding products of binomials and trinomials. All the tasks that learners worked with for the whole week were of the following structures:

1. (a + b)(x + y + z) binomial x trinomial where all three signs are positive
2. (a + b)(x - y + z) binomial x trinomial where two signs are positive
3. (a + b)(x + y - z) binomial x trinomial where two signs are positive
4. (a + b)(x - y - z) binomial x trinomial where one sign is positive
5. (a - b)(x - y - z) binomial x trinomial where all three signs are negative
6. (a – b)(x – y + z) binomial x trinomial where two signs are negative
7. (a – b)(x + y – z) binomial x trinomial where two signs are negative
8. (a – b)(x + y + z) binomial x trinomial where two signs are positive

Below is a table with data excerpts from teacher R's lessons exemplifying how live data was placed into each of the categories and levels. In the same table I also exemplify an

utterance which was numbered but not coded because it was considered to be of no mathematical significance in accordance with how an episode was defined i.e. in this study an utterance/activity/episode was considered and coded specifically as a didactic activity.

Table 5.1 Excerpts from Teacher R's coded utterances

Episode/Utterance/Activity	***Code***	***Comment***
Teacher: *You say yes Mam (referring to a female learner). You got it? All right. Now we agreed here (underlining the $a^3 + b^3$) that this is the difference between two cubes.*	***DR0***	*Defining $a^3 + b^3$ as difference of 2 cubes was considered mathematically problematic because what conceptualisation would learners get of difference*
Learner: *$(3x^2 + xy - 2y^2)(x + 2y)$* *$3x^3 + 6x^2y + x^2y + 2xy^2 - 2xy^2 - 4y^3$* *$3x^3 + 7x^2y - 4y^3$* *Teacher:* *What are you saying about her approach? How did she approach this? She was finding the product of binomials and trinomials using the distributive law. Did she apply the distributive law?*	***DR1***	*Tr was concerned that the learner 'did not' apply the distributive law because the learner did not re-arrange the two polynomials to start with the binomial on the left. This gives a limited understanding of the distributive law i.e. it is associated with a specific arrangement of the polynomials*
Teacher: *Explains that instead of $\frac{1}{3}x^5$ this can be written as $\frac{x^5}{3}$ and instead of $\frac{1}{3}x^3y^2$ we could also write $\frac{x^3y^2}{3}$*	***DR2***	*This was after a lengthy discussion with examples showing a clear understanding of these equivalent representations*
Teacher: *Eeeh, the question says find the product of these binomials, it's a binomial and a trinomial (and she writes on the board) $(a + b)(a^2 - ab + b^2)$ and we must find the product, alright. From what we have been doing all along. By just looking at that expression, by just looking at, by inspection, what is the answer for it?*	***PWR0***	*Before this episode, the teacher had asked the learners to listen attentively so that she could generalise from the specific examples that had been worked with. But despite having asked the learners to listen attentively, Tr still did not say what could be generalized. Now this episode suggests that she wants them to be*

		able to abstract the general from the specific examples that have been worked with. ***Is it possible to find this product by mere inspection?***
Teacher: *I'm going to highlight all the answers. Ok which ones are missing? Let's look at the answer for A. I'm going to write the answers only.* *A:* $2a^3 - a^2 - 1$ *B:* $a^3 - 1$ *C:* $3x^3 + 7x^2y - 4y^3$ *D:* $8a^3 + 27$ *E:* $b^3 - 64$ *F:* $a^3 - 2a^2 + 2a - 1$ *G:* $b^3 - 8b^2 + 32b - 64$ *What is B, (she checks to see that all the answers are there) so all the answers are there? So when you look at those answers what can you say? The power is always a cube, by the first term.*	***PWR1***	*There is some connection though superficial in that the observation made does not appear to be generalisable to all cases e.g. can we say* $2a^3$ *or* $3x^3$ *are cubes?*
Teacher: *(Coming to the rescue) This is addition and this is subtraction and we are starting from the left side to the right side. Which means you have to say* $+a^2 - 2a^2$*, which gives you* $-a^2$*? It's almost like saying 1 – 2 then you get -1.*	***PWR2***	*The teacher was showing how* $a^2 - 2a^2$ *is a generalisation of 1 -2*
Teacher: *Let's do the next one and we want to close this chapter. Here we have got* $(x - y)(x^2 + xy + y^2)$ *then our approach is distributive law then my brackets are* $x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$ *then what is going to be my final answer?*	***IM0***	*In this case this episode came after the teacher had given the impression that learners could be able to get the final answer from observing a certain arrangement of these polynomials. This implied a pre-determinable result just by looking at the structure but Tr had not shown how this could be achieved so the class could not forge ahead from here*
No episodes for this teacher were coded	***IM1***	
No episodes for this teacher were coded	***IM2***	

Tr: *In this case the teacher was showing how to go about the expansion of this bracket* $\left(\frac{2x^3}{3}\right)\left(\frac{1}{2}x^2-\frac{4}{3}xy-\frac{1}{2}y^2\right)$ *(Shows how to arrive at the answer* $\frac{2x^5}{6}$ *)* *Got it? I heard somebody saying Uhuuuu. Then you go to the next one, you are going to do the same* $\frac{2x^3}{3}\ x\ \frac{4xy}{3}$	**P0**	*Procedure is not correct, at the second stage where the teacher ignores the sings and instead of multiplying* $\frac{2x^3}{3}\ x\ -\ \frac{4xy}{3}$ *she multiplies* $\frac{2x^3}{3}\ x\ \frac{4xy}{3}$*. She did it several times and in this case it lead to an incorrect answer.*
Teacher: *Let us look at this question. Question number 3 we are given* $(a + b)(a^2 – ab + b^2)$ *we are still applying the distributive law. Our second step is* $a(a^2 – ab + b^2)\ \ b(a^2 – ab + b^2)$ *Then what is going to be our final answer?*	***P1***	*Again ignoring the signs but in this case it led to a correct answer because the signs which were ignored were positive*
Teacher: *The task was* $(b – 4)(b^2 – 4b + 16)$ *after which she explained how to move to the next step* $b(b^2 – 4b + 16) – 4(b^2 – 4b + 16)$ *explained how to remove the brackets* $b^3 – 4b^2 + 16b – 4b^2 + 16b – 64$ *After that we try to collect the like terms which are* $b^3 – 4b^2 + 16b – 4b^2 + 16b – 64$ *and this is our final answer* $b^3 - 8b^2 + 32b – 64$	***P2***	*This was considered a clear and justified explanation of how to deal with the signs and the like terms – reflects deep understanding of multiplying trinomials*
Teacher: *See where the sign comes in. Right. So when we do calculations here, we don't put those negatives but we are going to multiply when we finish up this. So we say* $\frac{1}{y^2}\ x\ \frac{4xy}{3}$ *and then (pointing to the negative signs of these two fractions*	***IOC0***	*This was coded in the IOC category because the teacher was following up on a learner's work. The two terms that were supposed to be multiplied here were* $-\frac{1}{y^2}\ x\ -\ \frac{4xy}{3}$ *But to instruct learners to leave out the signs until they finish*

in the initial brackets) it is going to be negative multiplied by negative and it is going to give us a positive in the answer. Now what is the answer for this one?		*has already proved futile elsewhere where even the Tr ended up with incorrect answers because she ignored the signs until the end*
Learner: *(a – 1) (a^2 – a + 1)* *a (a^2 – a + 1) – 1 (a^2 – a + 1)* *a^3 – a^2 + a – a^2 + a - 1* *a^3 – 2a + 2a – 1)* *Teacher:* *He could not solve that problem further. Something is missing there. Where did he go wrong? Who can chip in? Who can help (learner's name)? Wait; wait who hasn't been on the chalk board before? The final step needs some attention. Where did he go wrong?*	***IOC1***	*Teacher is following up on student's work points to a specific step (final) but still delegates the responsibility to the class. The class however took quite long to see what was wrong with the final step. The follow up was therefore considered not to be so productive hence coded as IOC1*
No episodes for this teacher were coded	**IOC2**	
Who was in charge for question A that's you (pointing to a learner). Who was in charge for question B stand up? (Pointing to different learners) B stand up, C stand up, D stand up and E stand up. That's we have A, B, C, D. Finding the product of binomials and trinomials applying the distributive law, let us start with A, let us check A. Let us look at their work starting with A. Did he apply the distributive law? (Speaking to LA) They say (class) explaining what you did.	***Not coded at all***	*This was considered as an example of an administrative or managerial activity.*

I will now comment on teacher R's utterances/activities within the different types of connections as discussed in the research design chapter.

Table 5.2 Totals of Teacher R's coded utterances (n = 98)

FORMS OF MATHEMATICAL CONNECTION	LEVELS OF KNOWLEDGE QUALITY			
	Code	**0**	**1**	**2**
Different Representation	**DR**	**28**	**5**	**3**
Part-whole Relationship	**PWR**	**3**	**6**	**2**
Implication	**IM**	**10**	**0**	**0**
Procedure	**P**	**7**	**19**	**10**
Instruction-oriented Connection	**IOC**	**14**	**5**	**0**
Totals		**62**	**35**	**15**

5.2.1 Episodes coded as indicating Different Representation (DR)

A total of 36 utterances by the teacher were coded as indicating connection through different representations. This category captured utterances/activities where the same concept had the potential to or was represented in two or more ways. There are two subcategories which in this model constitute connections through different representations and these are alternate representations and equivalent representations. Alternate representations are those in different modes i.e. it could be from verbal to symbolic and equivalent representations are those in the same mode. The codes used in this category were at three hierarchical levels DR0, DR1, and DR2 as explained earlier, with DR0 as the lowest and DR2 as the highest respectively.

Three utterances/activities were coded at DR2 level and these are #19, #28 and #274. These are utterances which could be associated with forms of representation where learners' development and articulation of justification and argumentation appeared evident. In #19 for example the learner seemed to show a conceptual understanding of how ($a^2 - 2a^2$) gives ($-a^2$) by using an analogy "*It's just like saying 1 – 2 = -1.*"

Considering that some learners in this class had difficulties in seeing what had happened to $2a^2$ and where the $-a^2$ was coming from, it can be argued that this analogy reflects a learner with a connected understanding of concepts. In activity #28, the learner went straight into multiplying within the brackets without rearranging the polynomials to start with the binomial on the left as had been suggested by the teacher i.e.

$(3x^2 + xy - 2y^2)(x + 2y)$
$3x^3 + 6x^2y + x^2y + 2xy^2 - 2xy^2 - 4y^3$
$3x^3 + 7x^2y - 4y^3$

This learner appeared to recognise equivalent representation through the commutative law where a x b is equivalent to b x a. Within those two steps she was also able to efficiently deal with the like terms that eliminate each other and those that do not eliminate each other. When asked to explain her working, she was able to articulate and justify how she got each of the six terms after removing the brackets. Judged by the way she explained herself, it can be argued she appears to show a deep understanding of multiplying polynomials. In #274 the learner explains that instead of $\frac{1}{3}x^5$ this can be written as $\frac{x^5}{3}$ and instead of $\frac{1}{3}x^3y^2$ we could also write $\frac{x^3y^2}{3}$. This again seems to reflect a clear understanding of equivalence.

The main reason why these learners' activities were coded despite the fact that the study focused on teacher utterances was because of the teacher's reaction that followed such representations. For example after activity #28, because the learner did not rearrange the polynomials to start with the binomial on the left, the teacher raised the question for the class: "*Did she apply the distributive law (# 35*)" and the class response was "*No* ". Even after another learner had rearranged the above task to start with the binomial on the left (# 40) as $(x + 2y)(3x^2 + xy - 2y^2)$ the teacher still raised the same question "*Did he apply the distributive law (# 43*)" and the class response was again a "*No (#44)*".

Although initially this appeared more like a procedural issue, with the teacher preferring a certain arrangement of the polynomials, however other comments later revealed that there could be some problems in the way the distributive law was being represented or

conceptualized. For example, here are some of the teacher's comments that followed after one learner had indicated that she understood learner C's method better;

> "*So next time you should read the question because the question says apply the distributive law (# 48). We are following instructions. Ok, Ok if it was just ordinarily finding the product of binomials and trinomials she was, really she was correct but now in brackets there are those finer lines in a question that say we can get the same answer but if it was in an exam I was not going to credit her because she did not follow instructions from the question which is very importan*t (# 50)."

The implications seemed to be that there was another 'ordinary' way of multiplying binomials and trinomials which was not distributive and the way learner C had done the multiplication was not recognised as the distributive law and would not earn marks in an examination. It can be argued that a mathematically sound conceptualisation of the distributive law was critical considering that the whole set of lessons for the whole week was focused on this image. If this was the main objective for the lessons; the question that then comes to mind is "What image did the teacher and the rest of the class have of the distributive law?"

Without going into the complex interpretations of the distributive law, the basic idea is that in mathematics pairs of parentheses or brackets such as ( ), [ ], { }, are one way that is used to group parts of an expression together to show exactly in what order the operation is to be done. Thus, for example, when we write 2(3 + 5), we mean "add 3 to 5 first, then multiply the result by 2." Thus 2(3 + 5) = 2 x 8 = 16. This expression can also be interpreted to mean that the multiplication by 2 is to be done (distributed) to every term inside the brackets. Thus 2(3 + 5) = 2 x 3 + 2 x 5 = 6 + 10 = 16 which is the same final result as before. In the sequence of steps in this example we say that we are expanding the brackets. If we use the symbols a, b, and c for example to represent any three numbers, then the overall process can be symbolized as:

$$a(b + c) = a \times b + a \times c.$$

Since we could have put any real numbers in place of 2, 3, and 5 above, and still have obtained a true equation, we say that multiplication of real numbers distributes over addition of real numbers. It can also be stated in words: The result of first adding several

numbers and then multiplying the sum by some number is the same as first multiplying each separately by the number and then adding the products. In other words, one need not add what is in parenthesis before multiplying as long as one multiplies each of the addends first by the desired multiplier. The sum of these results will be equivalent to the desired expression. The property is true for any number of addends. This rule is called the **distributive law** for multiplication. It shows how multiplication of the bracketed expression by 'a' is "distributed" to all of the terms in the brackets.

Given then an example such as $(3x^2 + xy – 2y^2)$ $(x + 2y)$ that learner C was tasked with, it basically meant that she had to multiply all three terms in the trinomial by both terms in the binomial. The number of products she would get had to be the number of terms in the first factor times the number of terms in the second factor. In this case she would get in all $3 \times 2 = 6$ terms, which may reduce to 5 or less after adding or subtracting the like terms. But from the knowledge of some of the laws of operations we know that no matter in what order multiplication is carried out, the product will always be the same: ab = ba. This is called the commutative law of multiplication and in the example above it can be shown that the trinomial and the binomial can be multiplied in any order (this multiplication is both left and right distributive) and the results are logically equivalent. The learners' utterances/activities seemed to incorporate all these big ideas of dealing with the expansion of brackets of polynomials. However the teacher appeared to be discouraging the development of such connections.

Activities #29, #35, #48, #206 and #294 were all coded as DR1. As discussed earlier, these are activities where connections were being recognised but in a manner that would limit (superficial) the learners' conceptual understanding. In #29 for example, the teacher says:

> *I was even thinking that as you <u>cancel out the like terms</u> you use coloured chalk so that even us the blind people can be able to see that. Right.*

Dealing with the like terms seems to be defined or represented in terms of cancellations of like terms despite the fact that the problem that the learner was dealing with #28 had some like terms which did not 'cancel out'. In #35 the teacher commented on the

approach used in #28 and asked the class "*Did she apply the distributive law*?" While the approach favoured by the teacher might not be mathematically incorrect, it however appears she only accepted her arrangement (binomial x trinomial) as the only correct representation of the distributive law. In #48 the same view is emphasized by the teacher. In #294 and in response to a question raised earlier by a learner as to whether in the fraction $\frac{4x}{3y}$ it was possible to write the 4/3 as $1\frac{1}{3}$ the teacher says: "*So far leave it like that.*" It can be argued that while all these utterances were not mathematically incorrect, they however limited learners' abilities to make mathematical connections hence coded as DR1 since no justification was provided as to why.

The highest number 28 of utterances in this category were coded as DR0 which implies that the connections were recognised in ways that were mathematically faulty. In activities #7, #8, #37, #39, #41, #43, #45, and #50, the main issue was about the definition or representation of the distributive law. The teacher was reluctant to accept both the alternative and/or equivalent representations and preferring to define distributive law as being associated with a specific arrangement. Notice that most of these utterances have also been double coded P1, the argument being that as a procedure putting the binomial on the left will yield a mathematically correct answer but to deny the learners to work with either alternative or equivalent versions would be problematic in terms of their conceptual understanding. In #50 for example, the teacher categorically refused to accept an equivalent representation claiming she would not award marks for it. Given that throughout the week the definition of the distributive law was not explicitly given by the teacher and that some learners' correct versions were discouraged, it can be argued that the teacher might not have created a comprehensive mental image (abstraction) of the distributive law.

In the next set of utterances #93, #98, #100, these were also coded as DR0 because the teacher appeared to represent dealing with like terms as cancellation of like terms. This representation appeared to be problematic in that learners tended to cancel out like terms even where such like terms were not eliminating each other resulting in incorrect answers in a number of tasks like in #93. In utterances #124, #127, #130, and #132, the teacher

seemed to be saying when multiplying binomials and trinomials we are either squaring or cubing and the result was either a difference of two squares or a difference of two cubes. This again appeared to be problematic in that it did not seem to give learners a mathematically acceptable representation of products of polynomials. In #136, and #138 the issue was about a correct equivalent i.e. ($a^3 – a^2b + ab^2 + a^2b – ab^2 – b^3$) would not be equivalent to ($a^3 + b^3$) but the teacher accepted this as the correct answer. This was followed by the teacher defining ($a^3 + b^3$) as a difference of two cubes in #160, #162, and #168. In #170 and #203 the teacher asked the learners to find an answer for or work out ($a^2 – b^2$), a task which appeared to require factorization as opposed to finding products of polynomials, which was the focus of the series of lessons for the whole week. There appears to be a disconnection here because finding an answer for ($a^2 – b^2$) in a context where learners were dealing with expansion of brackets would imply they had to expand the brackets and yet in this case it would appear this might not have been possible. In view of the fact that the teacher did not show an example of how to work with this problem right through the week, despite learners experiencing problems with this specific task, it can be argued that the teacher did not define "finding an answer for" in a manner that would enable learners to conceptualize what was required of that task.

In #253, the issue was about the teacher accepting $(2p^2 + 3q^2)(5p – 6pq + q^2)$ as being equivalent to $10p^3 – 12p^3q + 4p^2q^2 + 15pq^2 – 18pq^3 + 6q^4$ as provided by a learner in #235. In #301 the disconnection appeared to be in the teacher exemplifying x^2 in the middle of a fraction $\frac{1}{2}x^2$ as implying that it is for both the numerator and the denominator and so could be written as $\frac{1x^2}{2x^2}$. In #314, #318 and #337 the issue was about leaving the negative signs out when multiplying resulting in the equivalence being violated.

5.2.2 Episodes coded as Part-Whole Relationship (PWR):

These connections are of the forms: **A** is included in **B** and **A** is a generalisation of **B.** In teacher R's series of lessons, a total of 11 utterances/activities were coded as belonging to this part-whole connection category. The utterances #19 and #24 were coded at the

highest level as PWR2. In both cases the analogy of 1-2 given by the learner to explain $a^2 - 2a^2$ demonstrates how $a^2 - 2a^2$ is a generalisation of $1 - 2$. It can be argued that the learner is demonstrating a conceptual understanding of subtracting algebraic expressions which is connected to the arithmetic from which such subtraction is abstracted.

A total of 6 utterances were coded at the second level of PWR1. These are utterances/activities where generalisations are recognised but they appear to be superficial in that the observations made do not appear to be generalisable to all cases. For example in #112, the power is always a cube, in #114 that is numbers, indeed these generalisations are true in the specific cases in which they were being observed but cannot be generalized to all cases of multiplying binomials and trinomials. In #120 while the teacher emphasized that these generalisations could be observed in 'some' cases, she did not however provide the learners with the characteristics of those 'some' cases which would enable the learners to generalise in such cases hence it can be argued that the connections are again superficial. In #132 and #138 the generalisation appeared to be that when multiplying a binomial and a trinomial 'we' are cubing and the result is a difference of two cubes - again a superficial connection in that this might not apply to all cases. Admittedly this would have been coded differently had the teacher exemplified how the learners could distinguish those cases where the result was always a difference of two cubes and where learners could get numbers at the end.

A total of three utterances #145, #147 and #308, have been coded as PWR0. In #145 for example the teacher says:

> *Listen attentively I want to give you the in conclusion from the past information in the past exercises that we did. I want us to look at this right. Eeeh, the question says find the product of these binomials, it's a binomial and a trinomial (and she writes on the board) $(a + b)(a^2 - ab + b^2)$ and we must find the product, alright. By just looking at that expression, by just looking at, by inspection, what is the answer for it?*

This statement seemed to suggest that she wanted them to be able to abstract the general from the specific examples that have been worked with. But despite having asked the learners to listen attentively, the teacher still did not say what could be generalized which

would enable the learners to find the answer by mere inspection. In #147, the same message was repeated that products of polynomials could be found by mere inspection but she still did not say what it was that learners should look for in order to get that answer. Instead she asked them to work out again in their scribblers #153. In #308 the teacher instructed the learners to put the variable in $\frac{1}{2}x^2$ next to the numerator. While this might be mathematically acceptable there however appears to be some inconsistency in the generalisation. For example in #301 and with reference to the same fraction, the teacher said in the middle would mean it is for both the numerator and the denominator and even represented it thus $\frac{2x^3}{3}$ x $\frac{1x^2}{2x^2}$ but now it means it is for the numerator. In that case the generalisation would contradict itself resulting in learners having an unclear understanding of what happens when the variable is in the middle of a fraction.

Research has shown that generalisation is one of the most fundamental and important mathematical thinking process in that it demonstrates that students are able to analyse problem situations in a variety of different ways (Driscoll, 1999). Kaput (1999), defined generalisation as

> …deliberately extending the range of reasoning or communication beyond the case or cases considered, explicitly identifying and exposing commonality across cases or lifting the reasoning or communication to a level where the focus is no longer on the cases or situations themselves but rather on the patterns, procedures, structures, and relationships across and among them (p. 136)

It is a process that allows us to look beyond the particularities of a mathematical situation and make conclusions such as ‘that’s a case of ...’ or ‘this will always hold true as long as ....’ In all these cases it could be argued that the teacher appeared not to identify explicitly and expose the commonalities across the cases that were being dealt with hence the learners appeared not clear about the generalisations they were expected to make.

5.2.3 Episodes coded as indicating connection through Implication (IM):

In this category no utterances were coded at IM1 and IM2 levels. A total of 10 utterances were all coded at the IM0 level implying that the '*if –then'* connections were implied without making mathematical sense to the learners. For example in #124, #130, #160 and #162, these are utterances where the teacher gave an impression that ***if*** 'we' are finding the product of two binomials or a binomial and a trinomial ***then*** we are squaring or cubing respectively which would not be mathematically true. In #136, the teacher utterance implied some pre-determinable result could be obtained just by looking at the arrangement of the polynomials but still does not say what learners should specifically look for in order to give an answer without working. In #203 and with reference to the expression $a^2 - b^2$, the teacher said "*Remember we said let's work it out. We said let's work it out. Is she correct*?" this was after the learner had given $a^2 - b^2$ as the answer after having 'worked it out'. In this context 'working it out' would imply finding products of polynomials because that was what the lessons were all about but for this task that implication appeared not logically holding because learners could not possibly expand brackets which were not there. In fact the teacher later abandoned this task through to the end of the week and never showed the learners how it could be 'worked out'.

In #301 the activity implied that ***if*** x^2 is in the middle of a fraction ***then*** it is both for the numerator and for the denominator and again this is not mathematically true hence the implication would not hold logically. In #314, #318 and #319 the main issue is about leaving the negative signs out implying e.g. that $\frac{2x^3}{3}$ x - $\frac{4xy}{3}$ was the same as $\frac{2x^3}{3}$ x $\frac{4xy}{3}$. Mathematically such connections do not hold logically because once the negative sign has been left out then the equivalence does not also hold.

5.2.4 Episodes coded as indicating connection through Procedure (P):

These are utterances which reflected a procedural connection i.e. **A** is a procedure when working with object **B**. A highest total of 36 utterances were recorded in this category. According to Businskas (2008), this prevalence of utterances showing presence or lack thereof of procedural connections was to be expected given teachers' embedded view of mathematics as 'doing questions'. Research has shown that whether or not they are aware of it, teachers are constantly engaged in a process of constructing and using instructional representations and that this concept of instructional representations tightened the connection between subject matter and method (procedure). McDiarmid, Ball and Anderson (1989b), then argued that the instructional representations that students encounter define their formal opportunities for learning about the subject – the possible, not the inevitable. It is the degree to which the methods/procedures used by the teacher in representing or formulating the subject matter to make it comprehensible to the learners that helps us to judge an utterance as P0, P1 or P2.

Within my data set this is the category also with the highest number of utterances (10) coded at the highest level of P2. Some of the defining features of this level of procedural connection include competence in using a range (flexibility) of mathematical procedures, ability to select a procedure that would be efficient in solving a specific problem and the ability to explain and justify methods selected for working out a problem.

Evidence of flexibility in the procedure is shown especially in # 28, #30 and #34 where the learner is showing a different arrangement of the trinomials and multiplying without having to break the binomial into two monomials before multiplying as shown in this excerpt.

$(3x^2 + xy – 2y^2)\ (x + 2y)$
$3x^3 + 6x^2y + x^2y + 2xy^2 – 2xy^2 - 4y^3$
$3x^3 + 7x^2y – 4y^3$

In this excerpt one can notice that the binomial was not brought to the left hand side with the trinomial going to the right hand side. The binomial was not broken into two monomials before multiplying. The apparent advantage is that the like terms

automatically come adjacent to each other in the next step so that they become relatively easy to identify and work with. This would be evidence of an ability to select a procedure that would be efficient in solving a specific problem.

In #26, #28, #30 and #34 the procedure is well articulated and justification. Let me use an excerpt which followed the above rearrangement of the polynomials. In this excerpt one can notice evidence of the ability to explain and justify methods selected for working out a problem.

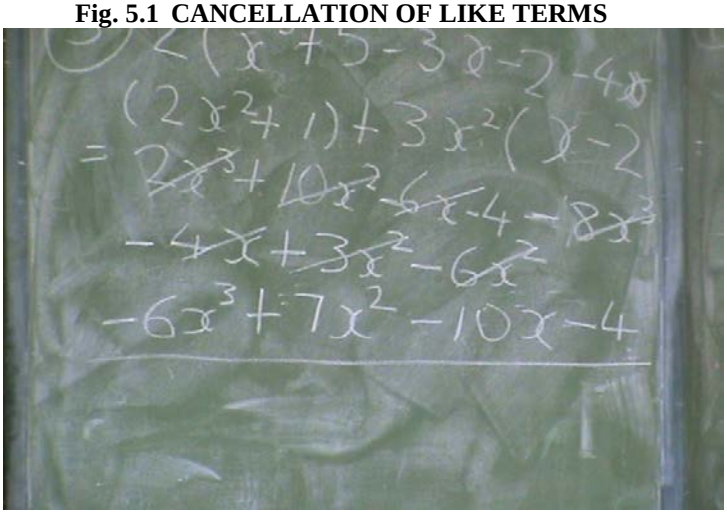
> *I notice that that there is no other groups of x^3 and so this comes down (pointing to the $3x^3$ coming down into the answer section). These two* + $6x^2y + x^2y$ *add up to* $+7x^2y$. *The* $+ 2xy^2$ *and* $- 2xy^2$ *they cancel out because of the signs. The* $-4y^3$ *remains as it is and so this is the final answer.*
> $3x^3 + 7x^2y - 4y^3$.

All these are examples showing a deep procedural understanding of dealing with the expansion of polynomials. In #63, #67 and #73 there appears to be similar precision in the procedure as well as in the answers, especially when working with negative signs which appeared to be problematic elsewhere. In #100, #106 and #237 there is also evidence of an ability to deal with the signs and the like terms which appears also to reflect deep understanding of multiplying polynomials.

A total of 19 utterances were coded at P1 level and these are utterances of a procedural nature which were considered to be mathematically sensible but where the teacher appeared to simply tell the learners what they ought to know with no flexibility in creating access to knowledge. Research has shown that mathematicians use a number of established and accepted procedures to respond to problem situations. The recommendation thereof was that students should learn a repertoire of mathematical procedures so that when confronted with a problem situation, they would have a number of ways of working 'at their fingertips' from which to choose (Sawyer, 2008). P1 coding started at utterances #7 and #8 where the procedure of canceling the like terms out, which the teacher suggested, would yield a correct result when the like terms were such that they would eliminate each other. However, it is not always the case that like terms will eliminate each other hence suggesting that learners have to cancel out like terms had the

potential to inhibit their procedural understanding of how to deal with the like terms when multiplying polynomials in general. There is evidence later in the lesson of where learners were just cancelling the like terms even where such like terms were not eliminating at all as evidenced in the snapshot below.

Fig. 5.1 CANCELLATION OF LIKE TERMS



In #11, #13, #77 and #136 there was lack of precision in dealing with the like terms hence the procedures were partially correct. In #37, #41, #43, #45, #48, #50, #243 and #294 the teacher suggested a specific arrangement or procedure for multiplying polynomials i.e. $(x + 2y)(3x^2 + xy - 2y^2)$ binomial to the left and trinomial to the right. However while the teacher's procedures would yield mathematically correct results there appears to be lack of procedural flexibility in that alternatives or equivalent representations were not accepted. This was also likely to inhibit learners' deep procedural understanding. In #46, #55, #132 and #355 the signs were being ignored hence the procedures and the final results would only be correct if the signs so ignored were positive signs.

A total of 7 utterances were coded at the lowest level (P0) of this category. These utterances are different from those that were previously coded P1 in that these are cases where the procedures were more of rote learning which resulted in getting incorrect answers. In # 93 and #98 the issue was about cancelling of like terms even where they did not eliminate each other. For example in #93 a learner expanded her brackets to $a^3 - a^2$

$+ a - a^2 + a - 1$ after which she then cancelled out the like terms and remained with $a^3 - 1$ as the final answer. It took long for the learners to realize that the meaning of cancelling like terms did not always mean making them disappear as had been suggested earlier and this is evidenced in # 230 where the learner explained her steps in expanding the brackets as follows:

$$(a - b)^2 = (a - b)(a - b)$$
$$= a^2 - ab - ab + b^2$$
$$= a^2 - 2ab + b^2$$

In this case the like terms were identified, collected and subtracted/added but not cancelled and the class commented "*Ohhhoooo we don't cancel the like terms*!" . Although the teacher later warned the learners thus: "*Remember we used coloured chalk to highlight the like terms. You can do the same even in your scribblers; you can tick the like terms so that you don't make a mistake #265*". It would appear there were some learners who still remained with this notion of cancelling out even where the like terms were not eliminating each other. This can be evidenced again by the snap shot (Fig 5.1). It can be seen in the snapshot above that this learner had now mixed the two ideas in that while he continued to cancel out the like terms, he still managed to add and subtract them as opposed to just cancelling and eliminating them like in the previous example.

In #314, #318, #326 and #337 the issue was about leaving negative signs when multiplying polynomials. These utterances were also different from those that were previously coded P1 in that these were cases where such leaving out of the negative signs would also result in getting incorrect answers.

5.2.5 Episodes coded as indicating Instruction-oriented connections (IOC):

These are utterances where the connections were of the form: ***A*** *and* ***B*** *are both prerequisites concepts or skills that must be known in order to understand/learn* ***C.*** *This form of connection also includes extension of what students already know thereby linking new concept to prior knowledge.* According to Kahan, Cooper and Bethea (2003), the mathematical development of a lesson or unit is important for effective teaching. The content should not appear to be a collection of disjointed, isolated topics, and it should be

sequenced so that topics are studied in a sensible order with prerequisite content being taught first or reinforced as needed. In this category no utterance was coded at the highest level (IOC2).

A total of 5 utterances were coded at the second level (IOC1) of this category. This is where the teacher appears to be building on the learners' prior knowledge but somehow does not do so comprehensively. In #21 for example, there is evidence of the teacher building on the learner's prior knowledge although she delegates this responsibility to the class. In #68 when the teacher says "*So far so good …*" this also appears to be building on the learner's confidence by acknowledging what the learner has done correctly so far. In #78, #80, and # 84 the teacher appears to be closely following the learner's work but still delegates the responsibility to the class to confirm whether the work was correct or not.

A total of 14 utterances were coded at the lowest level (IOC0) of the category. In #11 for example the teacher says "*I don't know we are waiting for your answer*" after a learner appeared stuck on how to deal with like terms which did not cancel out. While it might be argued that the teacher was trying to get the learner to think for herself, there is evidence immediately after this comment where the learner just wrote down an incorrect answer. This appears to justify the argument that the learner genuinely needed the teacher's help which did not come in both #11 and #23. In #92 after some debate whether a learner's work was correct or not the teacher says: "*I don't know. I'm looking at my answer here (pointing to her answer book). I don't know what's wrong*." It could also be argued that the teacher appears to check for an answer in her work-book instead of following the learner's work to identify exactly where there is a mistake. In this case the learner's working was quite correct but what appeared interesting was that after checking the answer in her book she still did not confirm that the learner's answer was in fact correct. Although in #103 the teacher confirmed that the answer had been correct long back, in terms of making an instruction-oriented connection it is now not clear why the teacher took so long to confirm a correct answer.

In #153 the teacher also appeared not to build on what the learners had been doing up to that stage. Instead of helping learners after they had expressed in #150 and #152 that they were not sure of how they could find a solution by mere inspection, the teacher gave them another task requiring more working and this did not seem to support her earlier suggestion that learners could find a product of polynomials by mere inspection. In #189 the teacher appeared not to have provided an instruction-oriented connection between expansion of brackets and factorization in that all the examples that had been worked with so far required the learners to expand brackets and none of the tasks required the learners to factorize like in this case of $a^2 - b^2$. Yet the teacher was asking the learners to work it out.

In #233 the teacher whether deliberate or in error appears again not to work with learners' prior knowledge by avoiding working out $a^2 - b^2$ which the learners appear to be struggling with. In #251 the teacher also seems not to be building on the learner's work as it appears she could not specifically locate where the learner went wrong. She says, "*Somewhere she is wrong*" #249 and when the class says "*Where*" #250 she points to the term ($-18pq^3$) which in fact was correct. Later in #253 and #257 after the learner had explained how she got the $-18pq^3$ the teacher says, "*Ok no problem so the answer is very right*." This was despite the fact that the learner's answer in #235 had some errors in it. In #282 the teacher again seems not to be following the learner's working on the board because she asks the class "*Is she on the right track so far*?" but despite that she still goes on to instruct the class to check their work against this same learner's working implying that the work is correct. In #304 the teacher again does not seem to build on the conceptualisation she has given the learners in #301 where she said a variable in the middle of a fraction would mean it is for the numerator and the denominator and even represented it thus $\frac{2x^3}{3} \times \frac{1x^2}{2x^2}$. However, when the learner in #303 wants this confirmed the teacher says "*You want me to give you an answer for this*?" She still does not give an answer despite the class saying "*Yes Mam*" in #305. Instead she goes on in #308 to offer a different representation i.e. the variable is for the numerator. In #330 the teacher says, "*So when we do calculations here, we don't put those negatives but we are going to*

multiply when we finish up this." To instruct learners to leave out the signs until they finish has already proved futile elsewhere where even the teacher herself ended up with incorrect answers because she ignored the signs until the end. In #347 the teacher seems done with the week's lessons on expansion of brackets but what seems interesting is that she points learners' attention to the next activity 5.10 whose focus is on the identification of like terms. In terms of instructional oriented connection this appears to be a typical example where the connections were of the form '***A*** *and* ***B*** *are both prerequisite concepts and skills that must be known in order to understand/learn* ***C*****'**. Identification of like terms is a pre-requisite skill that should have been known in order to understand how to deal with the products of polynomials. Even if this was the next activity in the text-book, learners could not possibly go to it after they had dealt with all these different types of products and be able to make sense of the connections. Ball (2003), suggested that effective teachers of mathematics have to make judgments about the mathematical quality of instructional materials and modify as necessary and this could be one good example where the teacher had to modify the order of the teaching material in the text-book.

5.2.6 Key messages emerging from Teacher R's teaching of the distributive law:

The discussion of what emerges from Tr R's teaching is guided by the following:

(a) what aspects of connections does the teacher appear to handle well
(b) what aspects appear to be problematic
(c) what are the underlying features in each case
(d) how do the strengths and weaknesses relate to the orientations in the curriculum.

The discussion starts by summarizing all the teacher's utterances/activities for the whole week into the different codes under which they were captured. Below is a graph fig. 5.2 that summarizes the coding of teacher R's utterances and activities for the whole week.

Fig. 5.2 SUMMARY OF TEACHER R'S UTTERANCES BY QUALITY OF KNOWLEDGE LEVELS

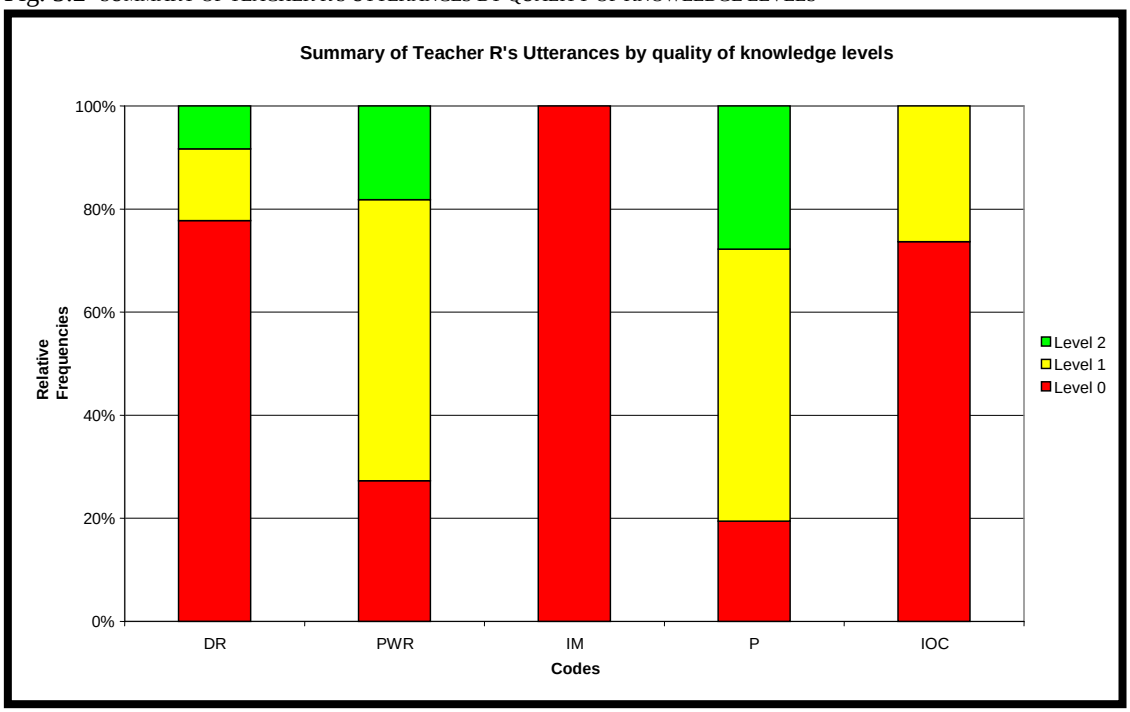
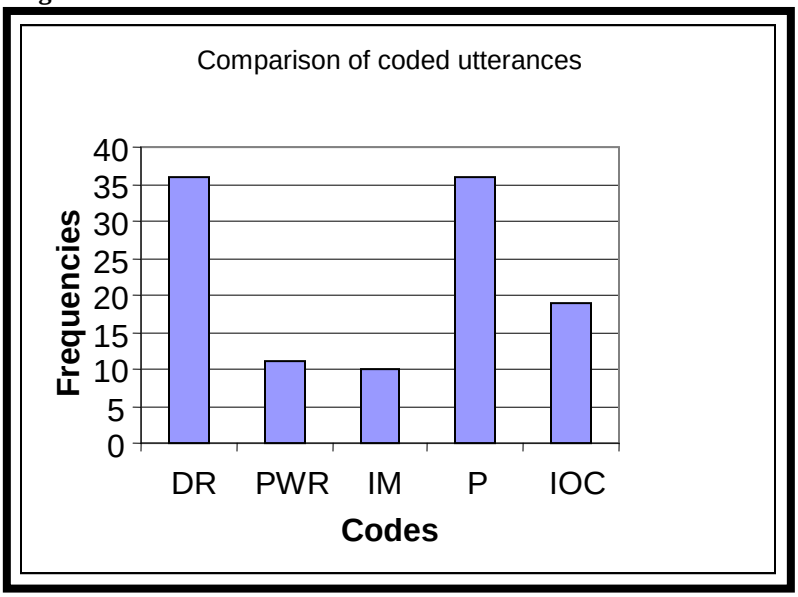


Fig. 5.3 COMPARISON OF Tr R's CODED UTTERANCES



The decision to represent knowledge types in bar charts was not arrived at without considering that knowledge by its nature is an abstract concept and perhaps a continuous variable. However according to McBurney and White (2010) although a variable may be

continuous, its measurement is often discontinuous. They gave an example of height which is a continuous variable but which is generally measured to the nearest metre or centimeter both of which are discontinuous. Similarly they contended that knowledge of psychological research methods for example may be a continuous variable, but it is often measured by the number of items correct on a test – a discontinuous measure. However, they argued that this does not make knowledge a discrete variable because it would be impossible in principle to measure knowledge as finely as one wished (McBurney & White, 2010, p. 123). Similarly procedural or conceptual knowledge may also be a continuous variable but in this study it is being measured by the number of teacher utterances – a discontinuous measure which can well be represented in the form of bar charts. There is also empirical evidence to further support this view as occurrences of conceptual and procedural knowledge and other forms of knowledge have been represented on bar charts by researchers including (Forrester & Chinnappan, 2010; J. Hiebert, et al., 2003; Rittle-Johnson, et al., 2001).

Having justified the use of bar charts in this section of the study I now try to explicate meaning from the figures. An overview of the graph above shows that the highest numbers of utterances, 36 in each, were recorded in the category of connections through different representations and procedural connections respectively. This was to be expected given that research into classroom interaction has shown that teachers are constantly engaged in a process of defining and constructing a mental image of some mathematical object and using instructional representations in the process (Businskas, 2008; McDiarmid, et al., 1989b). However, McDiarmid et al. (1989b) suggested that good instructional representations correctly and appropriately represent the substance and the nature of the subject being taught. They further posited that precision of definitions and lack of ambiguity in statements was a fundamental principle of mathematics learning. Similarly Ball (2003), provided further elaboration and suggested that effective teachers of mathematics have to use mathematically appropriate and comprehensible definitions, represent ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process. Given that the instructional representations that students encounter define the formal opportunities for learning about the subject content

the question that might quickly come to the fore is what stands out as key in this teacher's representation of the distributive law in view of the fact that this was the focus of all the lessons for the whole week. To enable a deeper engagement with the question Fig. 5.1 above shows a further disaggregating of the connections which reflect the relative frequencies within each of the categories under which they were coded. What this relative frequency graph seems to highlight is that connections through different representations both alternate and equivalent (DR) appeared faulty in close to 80% of the cases and restricted in another 10%. However in the procedural connection most of the utterances and activities (57%) fell into the level 1 category and (24%) into the level 2 category indicating that a total of (81%) of them were not faulty.

Besides those two categories, this graph also shows that opportunities for learner procedural and conceptual understanding (level 2) were created in relative fewer utterances i.e. 8% in the different representations category, 19% in the part-whole relationships and 23% in the procedural connections category. Going back to the literature review chapter; one might recall that in the NCSM the espoused view is that mastery of mathematics was key to democratic citizenship and this mastery depended to a large extent on mathematical processes such as investigating patterns, formulating conjectures, arguing for the generality of such conjectures and formulating links across the domains of mathematics to enable critical thinking (Department of Education, 2008a). All these are skills identifiable with deep understanding that are also inextricably intertwined with the ability to solve real life problems. What this graph seems to portray is that on average only 10% of the teacher's utterances had the potential to develop such deep understanding.

A significant number of opportunities for learner rote or superficial understanding (level 1) were created i.e. 12% in the different representations category, 56% in the part-whole relationships, 55% in the procedural connections category and 27% in the instruction-oriented connections category respectively. Let us recall that in level 1 of all the categories, connections were being recognised in a manner that was mathematically acceptable but limited in their potential to develop learners' deeper understanding. For

example the emphasis by the teacher that when multiplying a binomial and a trinomial, the binomial must always come first # 38 and #40, is not mathematically faulty, but maybe in the teacher's view it might have been procedurally efficient yet it could be argued that this could be conceptually limiting in terms of deep understanding of dealing with expansion of polynomials. However this emphasis on procedural fluency or automation without showing the learners why the process worked the way it did could be explained in a number of different ways in the South African context.

One of the observations in mathematical reform literature was that while current reform emphasizes the importance of the interconnectedness among mathematical topics and the connections of mathematics to other domains and disciplines, a number of factors tend to militate against such practices. For example in an assessment driven education system, there is increased demand for accountability on the part of all players including teachers. In some cases, teacher and administrator compensation is based on how well students perform on mandated tests. Standards have therefore become a very high stakes issue in public education with communities insisting on strong performance. With the current increased demand for accountability on the part of the teachers measured by student performance on mandated tests, the task of teaching for conceptual understanding appears overwhelming to teachers who struggle to find enough time to complete even routine duties. This is also compounded by the fact that current assessment practices do not seem to support this vision of teaching for deeper understanding and often work against it. Educational assessment has now been driven largely by practical and technical expedience rather than educational priorities. These constraints of efficiency have led to mathematics assessment following an atomistic approach where low-level skills were emphasized. Yet in a standards based environment common in many nations including South Africa, there is empirical evidence to suggest that there is a lot of teaching to the test with teachers focusing on topics and skills that are included in the examinations and devoting a lot of time to acclimatizing students to examination-type questions (Ottenvanger, et al., 2007). Teaching for the tests in which low-level skills were emphasized appears to have led to a new push for the basics, but unfortunately these new

basics are not the basics needed for future success in the world beyond school (English2008).

In teacher R's case there is evidence that seems to point to this teaching for procedural efficiency in the test. For example, when learners pressurized the teacher to accept alternative representations of the distributive law the teacher in her defence made reference to examinations pointing out that ordinarily those alternatives would be acceptable but she would not award marks if this was in an examination. It would appear that the teacher's concern here was about performance in the examinations rather than competence in the world beyond school hence the use of mnemonic techniques such as 'binomial to the left trinomial to the right' when multiplying polynomials and 'variable in the middle of a fraction must be next to the numerator' when working with fractions. Admittedly mnemonic techniques learned by rote may provide connections among ideas that make it easier to perform mathematical operations in the examination, but they may not lead to deep understanding of both procedures and concepts.

A significant number of opportunities for learners to develop both procedural and conceptual understanding appear to have been lost (level 0) in 78% of the (DR) different representations category, 24% of the (PWR) part-whole relationships, 100% of the (IM) implication category, 20% of the (P) procedural connections category and 72% of the (IOC) instruction-oriented connections category respectively. Utterances coded at this level were characterized mainly by the teacher creating opportunities for a flawed understanding of key mathematical ideas or facts during instruction. According to Kahan, Cooper and Bethea (2003), teacher weaknesses may become manifest in inaccurate mathematical statements, careless and otherwise. In this case such weaknesses were particularly prevalent when the teacher was summarizing the week's lessons by trying to identify patterns in the results obtained from multiplying binomials and trinomials. Some of the teacher's "in conclusions" included; 'When we are multiplying a binomial by another binomial we are squaring (#162), and when we multiply a binomial by a trinomial we are cubing (# 130), and the result is a difference of 2 cubes (#132) and that $a^3 + b^3$ was a product of $(a + b)(a^2 – ab + b^2)$ and an example of a difference of 2 cubes

(#136, #160, #168)'. Hiebert et al., (2003) for example highlighted some characteristic features of Japanese math classrooms which were regarded as indicating some indispensable elements of mathematics classroom instruction that are valued and emphasized. These included highlighting and summarizing the main points. In this teacher's summary of the main points it is not quite clear what she wanted the learners to see as the main ideas about the multiplication of binomials and trinomials. This can be evidenced by the learners themselves admitting they were confused (#150, #152, #184).

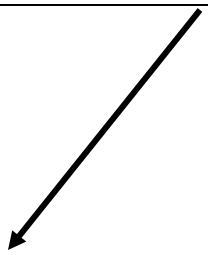
In terms of highlighting and summarizing the main points it would appear there was also faulty representations in particular suggesting gaps in their subject matter knowledge. This could be explained again by the observations made earlier that in South Africa literature abounds that suggest teacher's subject matter knowledge is weak (Adler, 2009; Brodie & Pournara, 2005; Graven, 2005; Harley & Wedekind, 2004; Howe, 1999; Long, 2007a; Taylor & Vinjevold, 1999).

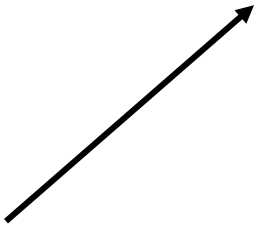
In terms of Teacher R responding to the expectations of the NCSM, one would argue on both the affirmative and the negative. If one considers that assessment comprises an important component of the curriculum and if those assessments are also testing for procedural fluency as was argued before and the teacher is also teaching for procedural fluency then in that sense it can be argued that she is meeting the expectations of the curriculum. However, if one takes deep understanding as a long term goal of the NCSM which cannot possibly be tested in the normal standardized tests then it can be argued that the teacher was unable to get learners to formulate links, identify patterns, and draw valid generalisations about the multiplication of binomials and trinomials.

5.3 TEACHER 'B'- TEACHING CALCULUS

Teacher B's lesson for the whole week were all on LO2 (Functions and Algebra) and focused on Calculus with specific interest on gradient. Below is a table with data excerpts from teacher B's lessons exemplifying how live data was placed into each of the categories and levels.

Table 5.3 Excerpts from Teacher B's coded utterances

Episode	*Code*	*Comment*
And remember here we are talking of gradient of a line. Okay. So be it, this is what It's horizontal, (and teacher draws a horizontal line) *So if you have a horizontal line what does it tell you about the gradient? We have a horizontal gradient.*	***DR0***	*A horizontal line is being defined as having a horizontal gradient*
Look at the gradient of aaaa… let's say y = 2x + 1 the gradient is what its 2 okay a positive 2. It's a value that is greater than what than 0 okay. So the gradient will be increasing okay and if it was y = -2x + 1. Haa. It's negative so it's what its decreasing. Okay. So the gradient of a line when, you talk of a gradient of a line given an equation of a straight line, okay a linear equation the coefficient of x is what is the gradient. Okay. So the sign before the coefficient is the one that tells you the gradient is what positive or it's what negative.	***DR1***	*Here the teacher is able to identify the gradient correctly as the value of m in a standard equation for a straight line i.e. y = mx + c but what does it mean to say when this m-value is positive it is increasing and when it is negative it is decreasing?*
You can either do it in a table form whereby you have your input and your output. So you have what x as the input and then y as the as the output. We are going to substitute the x values into the function and then we will get what the y values. So we have $f(x) = x^2 + 1$. *x -2 -1 0 1 2* *y 5 2 1 2 5* *(The teacher then completes the table of values after which she raises another question)*	***DR2***	*Another form of representing a function which appeared to be well explained*
 *Will all these lines have the same gradient? Haa. It won't. So this one (referring to one whose arrow is pointing up) is it increasing or is it decreasing? It's what its increasing. So if it's increasing we are talking of what greater than what 0 okay. So if it's decreasing (referring to one whose arrow is pointing down) we are talking of less than 0 and if it's horizontal it's what? Haa. If it's horizontal it's close to…If it's increasing it's greater than 0 okay if it's decreasing its less than 0 and if it's horizontal it's what? It's equal …to*	***PWR0***	*This forward slanting line is being given as a case/ example of lines having a decreasing gradient and whether this 'decreasing' was a slip of the tongue to mean negative, in both cases of either decreasing or negative gradient it was mathematically problematic hence it was coded as PWR0*

what to 0.		
	**PWR1**	*This forward slanting line is being given as an example of lines having an 'increasing' gradient. Here the teacher was being given the benefit of doubt in her interpretation of 'increasing' to mean positive gradient hence coded as PWR1*
No episodes for this teacher were coded	***PWR2***	
So here we talk of average gradient but we have an actual gradient between two points and you can only calculate a gradient (of a curve) when you have two points.	***IM0***	*This seems to imply that you cannot calculate the gradient of a curve at a point but only between two points on the curve which mathematically would be problematic for learners*
None of this teacher's episodes was coded	***IM1***	
None of this teacher's episodes was coded	***IM2***	
Motion, chemical reaction, ok, like if you have a tumor and they want see if using radiation will make that tumor to shrink they will use, they will use calculus. So it is a very important mathematic tool. Okay. It was invented in I think in the 17th century. One of the Mathematicians was Newton.	**P0**	*Calculus as a procedure to see if using radiation will make a tumour to shrink*
Ok you used decimals all of you…. (After which the teacher takes the learners' decimal values into consideration and starts working with decimals instead. The challenge came when she tried now to locate the position of these values on the y axis) *0.3 ok now, where is 0.3, ok, -0.3, I usually discourage learners to use decimals, ok because if you have used* $\frac{1}{3}, \frac{1}{2}, -1$ *it won't be as accurate as you want it to be but it will be better than using 0.3 and 0.5 ……..*	***P1***	*Teacher's values are correct but equivalent values discouraged because teacher claims proper fractions are better than decimals in this case. This tends again to limit learners' deep procedural understanding when plotting points on a graph*
You can either do it in a table form whereby you have your input and your output. So you have what x as the input and then y as the as the output. We are going to substitute the x values into the function and then we will get what the y values. So we have $f(x) = x^2 + 1$.	**P2**	***Correct procedure for drawing graphs***
(Prior to this the teacher had stated that a function can be linear or quadratic) ….One x value is related to one y value. It can... A function can be a one to one but it can also be a many to one. So define the word function.	**IOC0**	*Teacher does not seem to bring out the differences between a one-to-one and a many-to-one function building on the examples of linear and quadratic functions that the class had been discussing*

Teacher: How do I identify a linear function? *Learner: Straight line* *Learner: The graphs* *Teacher:* *Is determined by x. If x is to the exponent 1 you are talking of a linear function if it's a square it's what? Quadratic. Okay. That is how you differentiate between the two functions. Okay. So now I have written a function: f(x) = x^2 + 1. This is what linear or quadratic?*	**IOC1**	*Correct but again Tr does not seem to be building on learner's responses because both straight line and graphs would have been acceptable but it looks like she is interested in the exponent of x to distinguish the types of functions*
None of this teacher's episodes was coded	**IOC2**	

5.3.1 Episodes coded as indicating Different Representations (DR):

Two utterances were coded at the highest level DR2 and these are #41 and #44. These are utterances which could be associated with forms of representation where learners' development and articulation of justification and argumentation appeared evident. In #41 for example, the teacher shows a complete table of values as another form of representing a function which had been given on the board in algebraic form. In #44 this table of values is again represented in graphical form which is yet another correct form of representing a function.

Three utterances were coded at the second level DR1 and these are #5, #32 and #82. These are utterances where connections were recognised but in a manner that would limit (superficial) the learners' conceptual understanding. In #5 for example, the teacher defines calculus as having to do with limits, derivatives and differentiation. While these terms are indeed linked to calculus but they are terms that need further defining themselves hence it can be argued that the teacher seemed not to have 'a clear idea of what is the simplest form of this mathematical idea' (Ma, 1999). In other words it would appear she was unable to scale down or trim the concept so that it could easily be accessible to the learners. In fact throughout the series of lessons, the teacher never attempted to define the terms derivative and differentiation. She kept scratching through the definition of limits, functions and domain without exhausting any of them such that the learners would be able to link them with the concept of calculus.

In #32 the teacher says:

> *When you substitute the x value you get a y value so the x value is the input and the y value is what the output.*

This is being given as a definition of a function but it appears to lack mathematical precision especially considering that the teacher had earlier on said a function can be a one-to-one or a many-to-one in (#30). These are critical features of a function which do not seem to come out clearly in the above definition in the absence of further articulation. In #82 the teacher represents a positive gradient in the form of a forward slanting line i.e. / but there seems to be lack of precision again when the teacher claims that the gradient would be increasing in this case. So one would imagine what conceptualisation learners would make of a positive and increasing gradient in a straight line. Considering that earlier on the teacher had said that the gradient of curve changes but that of a straight line is constant, this was likely to confuse the learners leaving them with no clear understanding of the mathematical concept that was being focused on.

Thirteen utterances were coded at the lowest level DR0 and these are #9, #19, #24, #26, #45, #51, #80, #82, #82, #85, #103, #107 and #113. These are utterances where connections were recognised in ways that were mathematically faulty. In the utterances #9 – 45 the issue is about the definition of calculus which is first given (#9) as "*there is the word calculate in it, so we will be calculating*." The question could be; 'What exactly would learners be calculating in calculus and how different would this be from any other calculations that are done in other areas of mathematics which are not calculus?' Later on the teacher defines calculus as having something to do with limits, functions, derivatives and differentiation. At that point the utterances are coded DR1 because indeed these are terms which can be linked to calculus, but one was hoping that each of these terms would be defined in a more comprehensive manner that would enable the learners to see the links. However the teacher kept on introducing another new term to define the previous one. In #19 for example, the teacher asks "*Do you understand the word limit*?" but before this term had been defined the teacher introduces another term in #24 when she says "*By*

the way what is a function?" yet another one in #26 "*What is the domain?"* and back in #45 "*Now coming back to the word limit ….. Now I want to explain the term limit what it means*." Immediately after this the teacher substitutes different values into $f(x) = x^2 + 1$ in what appears to be a haphazard manner which does not seem to bring out the concept of a limit. She then ends the lesson by saying; "*Now we have talked of the word limit and we will get back into the word limit as we go on"* (#46). In short it is doubtful whether learners would have made sense of any of these terms because each one was defined in terms of the next and yet another with the result that no comprehensive definition was given at all.

In #51 the teacher seems to suggest that the gradient is negative (-3) because the points are on the negative side – but negative side of what is not clear. On a Cartesian plane for example, gradient whether positive or negative is determined by the slant of the lines and not by where such lines are positioned. In #80 the teacher seems to suggest that a horizontal line has a horizontal gradient and #82 the utterances suggest that a forward slanting line / drawn from bottom going up has a positive gradient which is increasing while a similar forward slanting line / but drawn from top going down has a negative gradient which is decreasing. It is clear within those same utterances 80 – 82 that the teacher could identify the value of *m* in the standard equation for a straight line $y = mx + c$ as standing for the gradient but it is also clear that the teacher could not exemplify positive and negative gradient using line segments. In #85 the teacher suggests that learners can only calculate the gradient of a curve when they have two points. To define the gradient of a curve in this way would certainly be problematic for the learners as they might never be able to deal with this concept. The gradient of a curve is generally measured/defined at a specific point on the curve.

In #103 the teacher seems to suggest that when learners divide 1 by 0 and get error on their calculators, the mathematical language for that is 'undefined'. In #107 the teacher seems to confirm that -0.33 is less than -0.5. In #113 when comparing the equation $y = \frac{1}{x}$

with the standard equation $g(x) = a.\frac{k}{x} + q$ for a hyperbola the teacher seems to suggest that *a = 1* but "W*e do not have a value for q.*" Yet *q = 0* in this case. All these appear to be faulty mathematical statements which had the potential to limit the learners' deep understanding of the mathematical ideas that were being dealt with in this series of lessons.

5.3.2 Episodes coded as indicating Part-Whole Representations (PWR):

A few utterances (4) were coded in this category for this teacher's series of lessons indicating that she made a few generalisations. This is understandable considering that generalisations are usually made after making a number of observations and identifying patterns thereof. No utterance was coded at the highest level PWR2. Only one utterance #82 was coded at the second highest level PWR1. In this utterance the teacher makes a generalisation that a forward slanting line / but drawn from down going up has a positive gradient and the gradient is increasing. This statement is partially true as indeed such a slanting line has a positive gradient but that the gradient is increasing contradicts a mathematical fact that the gradient on a straight line is constant.

Three utterances were coded at the lowest level PWR0 indicating that there were some mathematical faults in them. In #10 for example, the teacher says; "*So whatever that we do in Math is related to our daily lives.*" This kind of generalisation suggests that Math only has utility value yet mathematics can be done for its own sake (aesthetic value) and not necessarily that it has particular relevance to the everyday. In #82 the teacher makes another generalisation that a forward slanting line / but drawn from top going down has a negative gradient and the gradient is decreasing. In #85 the teacher makes another generalisation about the gradient of a curve as she says;

> *So here we talk of average gradient but we have actual gradient between two points and you can only calculate a gradient when you have two points.*"

5.3.3 Episodes coded as indicating connections through Implications (IM):

A total of only three utterances were all coded at the lowest level (IM0) in this category. In #54 for example the teacher asks a question with reference to the gradient of a curve;

> *If it changes …will we still talk of an actual gradient like when we talk of a straight line graph*? *We will talk of what, average gradient because the gradient is not constant.*

This statement seems to imply that a straight line graph has 'actual gradient' because it has a constant gradient, but it is doubtful whether mathematically there is something called actual gradient. Apparently this term continued to be used right through the series of lessons. In that same utterance the teacher also says;

> *So the gradient at these two points …will be called the actual gradient but knowing that the gradient of a curve changes.*

To suggest that the gradient between any two points on a curve is called actual gradient, implies that there is a constant gradient between these two points and that implication is mathematically faulty as it contradicts the fact that the gradient on a curve changes. In #85 the teacher makes a statement which implies that one cannot calculate the gradient at any given point on a curve since one has to know or be given two points in order to calculate the gradient.

5.3.4 Episodes coded as indicating connections through Procedure (P):

A total of nine utterances were coded in this category. Two of them #40 and #41 were coded at the highest level of P2 basically the teacher was showing correct but different procedures of representing a function.

Two utterances #97 and #110 were coded at the second highest P1 of this category. In #97 for example the teacher shows a correct procedure for calculating an output given an

input of a function. However, an equivalent procedure of working with decimals is discouraged as the teacher says;

> *I usually discourage learners to use decimals, ok because if you have used (proper fractions) it won't be as accurate as you want it to be but it will be better than using (decimals).*

In #110 the teacher suggests that learners can use any of the Cartesian Planes that they have drawn before to plot the graph of any other function.

> *Use any of your Cartesian planes, any that you have in your book, so you are wasting time…*

This was after the teacher had noticed that the learners were trying to draw a Cartesian plane for each of the graphs they had been tasked to draw. However, when using graphic calculators for plotting graphs of functions, learners usually don't see the need for choosing an appropriate scale for the axes since the calculator does that automatically but when plotting the points for a graph manually on a Cartesian plane the skill to choose an appropriate scale on both axes is extremely important as a Cartesian Plane scaled for one graph might not be appropriate for a different function.

Five utterances were coded at the lowest level under this category. In #18 for example, the teacher says,

> *If you have a tumour and they want to see if using radiation will make that tumour to shrink, they use calculus. So it is a very important mathematical tool…*

It is not clear how calculus could be used as a mathematical tool in this process of getting the tumour to shrink. In #45 the procedure used to demonstrate the concept of limits does not seem to be making any mathematical sense. In #49 the teacher suggests that $\frac{y_2 - y_1}{x_2 - x_1}$ is the formula for finding the gradient of a curve. This procedure might not also lead the learners to a correct understanding of the gradient of a curve. This is followed up in #51 with the teacher using coordinates of two specific points to substitute into that formula in order to calculate the gradient but by convention the gradient is calculated at a specific point on the curve hence it can be argued the procedure is problematic. In #54 the teacher comments after calculating the gradient using the two specific points on the curve;

So the gradient at these two points will be what will be…. It will be called the actual gradient but knowing that the gradient of a curve changes. That is why we call it what? We talk of average gradient.

However the teacher still does not show how that 'average gradient' is calculated and how different the gradient of a straight line is from the gradient of a curve.

5.3.5 Episodes coded as indicating Instruction-oriented connections (IOC):

A total of 16 utterances were coded under this category. Of those utterances, none was coded at the highest level IOC2. Two utterances were coded at the second highest level IOC1. In #32 and 36 the teacher suggests correctly that;

We talked about functions. We talked about linear functions. We talked about quadratic functions. Okay. How do I identify a linear function? Is determined by x. If x is to the exponent 1 you are talking of a linear function if it's a square it's what? Quadratic. Okay. That is how you differentiate between the two functions. Okay. So now I have written a function: $f(x) = x^2 + 1$. This is what linear or quadratic?

Prior to these utterances the teacher had just explained that a function can be a one-to-one or a many-to-one but in terms of making an instructional oriented connection it appears she does not build on this fact as she seems not to mention which of those two linear and quadratic would be a one-to-one and many-to-one function.

A total of thirteen utterances were coded at the lowest level IOC0 of this category. In #14 the teacher seems not to recognize that the term calculus has a different meaning in Biology for example from the mathematical one she wanted the learners to focus on. After she had asked how in engineering people could make use of calculus one learner's response was

Mineral mass in the body" in #13.

However she does not appear to build or extend on this knowledge to enable the learners to make a distinction between calculus as in mineral mass in the body and calculus as used in mathematics. In #9, #26, #30, #57, #59 the learners are using statements like *"To explain it more further"* to define calculus and *"It shows us the height above the earth's*

surface" to define gradient and the teacher seems to be regurgitating the same statements as given by the learners without asking the learners to show the connections and neither does she build on that information herself to bring out clearly the meanings of those concepts. In #54 the teacher again does not seem to bring out the differences between a one-to-one and a many-to-one function despite having mentioned earlier on that functions could be distinguished that way. In #57 despite having mentioned that the gradient of a curve changes but is sometimes constant, she does not show where it begins to change and where it becomes constant – all this does not seem to come out yet this was critical information in this series of lessons. In #67 the teacher does not suggest the link between calculus and the ability to go to the moon. In #79 the teacher quickly abandons gradient of curves and goes back to gradient of straight lines again suggesting that the way we calculate those gradients was the same. In #89 the teacher's summary seems not to highlight the main points dealt with in this lesson. In #109 the teacher does not make a follow up despite some learners having noticed that it was false to say that -0.33 is less than - 0.5.

5.3.6 Key messages emerging from Teacher B's teaching of Calculus:

The discussion of what emerges from Tr B's teaching is again guided by the following:

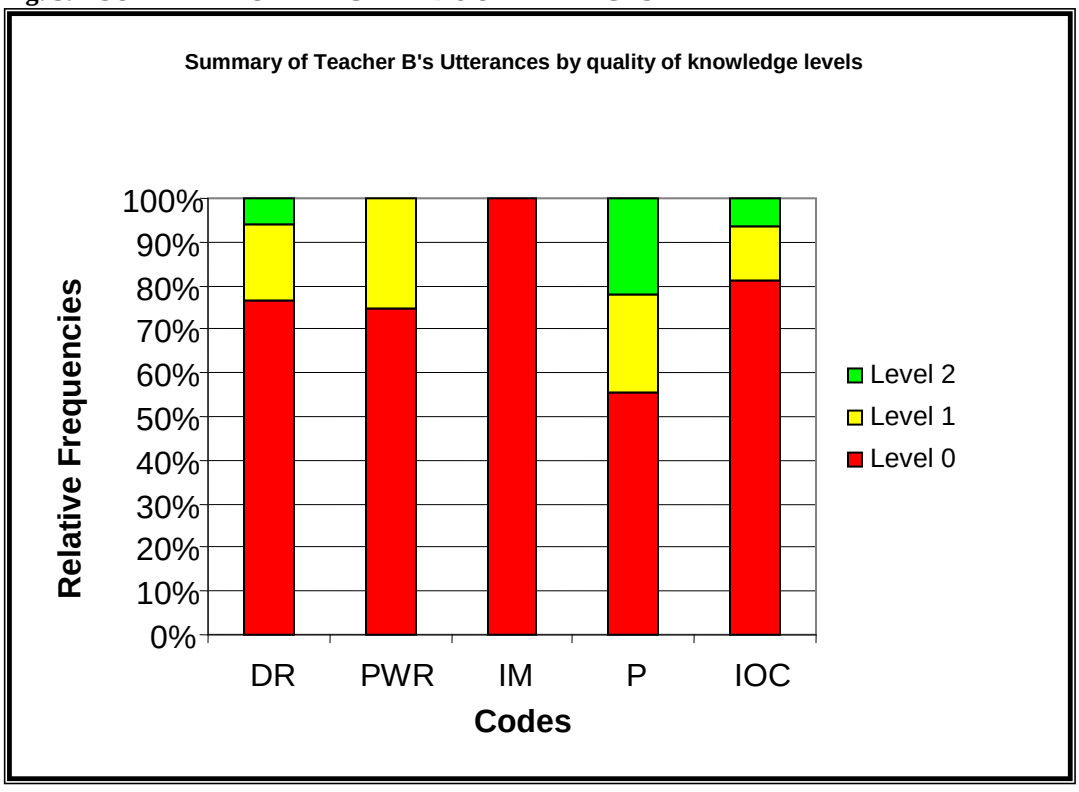
(a) What aspects of connections does the teacher appear to handle well?
(b) What aspects appear to be problematic?
(c) What are the underlying features in each case?
(d) How do the strengths and weaknesses relate to the orientations in the curriculum?

Table 5.4 and Fig. 5.4 below summarize all the teacher's utterances/activities for the whole week into the different codes under which they were captured.

Table 5.4 **Totals of Teacher B's coded utterances** (n = 42)

FORMS OF MATHEMATICAL CONNECTION	LEVELS OF KNOWLEDGE QUALITY			
	Code	0	1	2
Different Representation	**DR**	**13**	**3**	**2**
Part-whole Relationship	**PWR**	**3**	**1**	**0**
Implication	**IM**	**3**	**0**	**0**
Procedure	**P**	**5**	**2**	**2**
Instruction-oriented Connection	**IOC**	**13**	**2**	**1**
Totals		**37**	**8**	**5**

Fig. 5.4 SUMMARY OF TEACHER B's UTTERANCES



The teacher starts the series of lessons by asking the learners for the definition of the word calculus. Learner responses reflect some prior knowledge as they mention that it has to do with limits, derivatives, differentiation and that calculus is used to describe a system of rules. The teacher then summarizes learners' contributions by saying;

> *I mean its calculus it has the word calculate within it. Ok, so we will be calculating something but there are rules that we need to follow.*

One of the fundamental principles of mathematics revolves around explicit definitions of terms and symbols. O'Connor (1999) discusses different types of mathematical definitions and these include stipulative, working, dictionary and formal. Stipulative and working definitions are developed as part of an interaction or an exploratory activity; while dictionary and formal are given by a text. The definitions students used in this conversation can be described as working and stipulative definitions. It could be argued that this is productive for the students are actually participating in an activity that may be closer to the practice of scientists and mathematicians than to school practices of using only dictionary definitions (Ball, 2003).

However, knowing definitions for teaching requires being able to understand and work with them sensibly, treating them in a way that is consistent with the centrality of definitions in doing and knowing mathematics. For example the centrality of definitions of Calculus is that when learners are studying the rates of change in mathematics, they are in the branch of mathematics called Calculus. From the way the teacher summarizes this discussion on the definition of Calculus this seems not to come out as the impression learners might get is that any calculation is in the branch of Calculus because there is the word 'calculate' in it. Knowing how definitions function, and what they are supposed to do, together with also knowing a well accepted definition in the discipline, would equip a teacher for the task of developing a definition that has mathematical integrity and is also comprehensible to students (Ball, Hill & Bass, 2005).

In this discussion it can also be argued that learners were using terms like, limits, derivatives and differentiation that also required further definitions before learners could

really grasp the mathematical object that was targeted. If these terms were compared with a much simpler term of "rate of change" as defining Calculus, it can be argued that they were less likely to enable learners to have a clear conceptual understanding of the term. Empirical evidence has shown that in order to make sense to the learners, definitions must be based on elements that are themselves already defined. A definition of a mathematical object is useless, no matter how mathematically refined or elegant, if it includes terms that are beyond the prospective user's knowledge. The implication is that teachers must be able to choose and develop a definition that is mathematically appropriate and also usable by students at a particular level (Ball, Hill & Bass2005).

The teacher opens the next discussion with a statement;

So now what ever we do in Maths is related to our daily lives.

It might appear this statement was over generalised because not everything we do in mathematics could be linked to the everyday. One of the most important attributes of mathematics, and one that gives it much of its power, is its abstract nature. Using mathematical language with care, and understanding how definitions and precision shape mathematical problem solving and thinking are crucial elements to understanding mathematics.

In the next discussion, it would appear the focus was on the application of Calculus in everyday lives. Teacher utterances and activities here appear to be productive as learners were able to mention population growth, motion, chemical reaction, and shrinking of a tumour which are all examples of rates of change where the concept is applicable. However there is a learner who mentioned that in Engineering, Calculus can be used to find mineral mass in the body. According to Moschkovich (2004), learning mathematics involves in part a shift from everyday to a more mathematical and precise use of language. Students use resources from both everyday and mathematical discourses to communicate mathematically. Learning the mathematical meanings of words describes one important aspect of learning mathematics. This learner for example understands the term Calculus as used in medical terms with reference to '*a hard lump produced by the*

concretion of mineral salts; found in hollow organs or ducts of the body." While the teacher acknowledges the learner's contribution by saying; "*Mineral mass in the body of what, human beings, animals*," she quickly diverts the talk to focus on population growth. Contrasting everyday meanings with the more restricted meaning of the mathematics register points to these multiple meanings as possible sources of misunderstanding in classroom discussions. But everyday meanings are not only obstacles; they are also resources for developing mathematical competence. Mounting evidence from cognitive science research shows that pupils' prior knowledge and beliefs powerfully influence the way they make sense of new ideas. Children's understanding of subject matter is the product of an interaction between the ideas, information and understandings they bring and the new ideas and information that they are presented (Moschkovich, op cit). So their ability to understand new information depends in part on the fit between the new information and the ideas they encounter and the schemata they have developed to assess and organise new information or experience.

The next discussion appeared to be focused on the understanding of the term limit, a word which had earlier on been used to define Calculus. In mathematics, the concept of a "limit" is used to describe the value that a function or sequence "approaches" as the input or index approaches some value. The teacher states this concept quite explicitly as she says;

> *So when x approaches 2 okay the limit of f(x) will be equals to 5*.

However, it is the representation of the concept of 'approaching' that might not have been productive for the learners. The phrase 'approaching a certain value' suggests some systematic trajectory, one that would enable learners to see the link between the verbal representation of a limit and the numerical representation of it. However, asking learners to substitute values for x in the function $f(x) = x^2 + 1$ appeared not to be consistent with this verbal representation as the teacher asked the learners to use 1,9; 1,9999; 2,9; 2,9999; 2,05; 2,025 in that order. Representation is a central feature of the work of mathematics teaching (McDiarmid, et al., 1989b). Skill and sensibilities with representing particular ideas or procedures is as fundamental as knowing their definitions. Teachers need to be

able to use representations skilfully, choose them appropriately and map carefully between given representations.

Going into the next discussion the teacher starts by asking "*What is the purpose of a gradient*?" It is not quite clear here what kind of response the teacher was expecting from the learners but one learner said; "*It shows us the height above the earth's surface*." Although there was more probing from the teacher, the learners appeared not to get what exactly the teacher wanted. In an effort to get learners to respond, the teacher then gave examples like,

> *Let's take a rocket okay. Is the rocket only going to influence the height above sea level? You talk of what in Science gravity – so what is happening there? You know rockets people okay what is happening there? Do you remember McShuttleworth – so what happened there? What happens to the air?"*

The way the teacher is trying to ask questions, explain and get responses from the learners seems to confirm research findings that suggest that teaching requires an awareness and understanding of fundamental mathematical connections (Moschkovich, 2004). It would appear that understanding of gradient through real life situations might be problematic. Judging by the examples she is focusing on, this teacher seems to be working with the notion of gradient as the degree to which something inclines; a slope. However, it is doubtful whether the notion of a negative gradient has a real life equivalent. Empirical evidence has shown that mathematics teachers need to know a great deal more about a slope than the phrase 'rise over run' (McDiarmid, Ball, & Anderson, 1989a). They may need to think about the relationship between slope as a mathematical device and slope as a phenomenon of everyday life if they are to represent the concept in a way that makes sense to pupils. In addition, they need to think about slope as a way of understanding relationships within mathematics – for instance, as a way of representing the covariance of two variables. They may need to see that this concept and related concepts have application in many other fields, from engineering to sociology to economics to business. All these are issues which seem not to be addressed from the manner in which the explanations and questions are coming from the teacher. As a result of similar observations on the problematics of content area, one of the recommendations

was that the standard way to teach calculus for such diverse application is to abstract the material and teach the core principles that apply in all situations (Wilson, Fernandez & Hadaway1993).

In this discussion it can also be observed that there was some misconception in the way negative gradient was presented graphically. Both slanting lines drawn on the board in fact represent a positive gradient yet the teacher seemed to say one of them was showing an increase and another one showing a decrease in the gradient. From the way the teacher is also explaining negative gradient as decreasing and positive gradient as increasing learners might end up with a misconception. Often slope is calculated as a ratio of "rise over run" in which run is the horizontal distance and rise is the vertical distance. This ratio becomes positive (positive gradient) if the change in both the rise and run are positive or when both of them are negative. The ratio becomes negative (negative gradient) when the change in either the rise or run is positive and the other one negative. This does not seem to come out clearly from the way the teacher is talking about increasing gradient and decreasing gradient. However given an equation of a linear function she explains this gradient productively as she says;

> *So the sign before the coefficient is the one that tells you the gradient is what positive or it's what negative.*

Teaching requires the ability to represent ideas and connect carefully across different representations - symbolic, graphical, and geometric (Moschkovich, 2004). It requires knowing ideas and procedures in detail, and knowing them well enough to represent and explain them skilfully in more than one way.

In terms of making a connection between the gradient of a straight line and that of a curve learners might not have been able to make this connection from the way the teacher explained.

> *So here we talk of average gradient (referring to a curve) but we can have actual gradient between two points and you can only calculate a gradient (of a curve) when you have two points."*

It might not be very clear here what the teacher meant by the terms *average gradient* and *actual gradient*. But mathematics is precise and precision is necessary in the doing of mathematics and in the communication of mathematics. The meaning of terms, operations, and symbols of mathematics must be completely unambiguous or else communication gets lost and mathematics slips away. Students who are unsure of what they are talking about cannot hope to solve problems with such ambiguous underpinnings (Wilson, Cooney & Stinson2005). For students and teachers to communicate effectively about mathematics they must all have precise meanings for symbols and terms in common. This is easy to overlook, and often overlooked, in situations where content seems elementary, but this is exactly when that precision should start.

In view of the fact that this lesson was a follow up on the lesson focusing on Calculus it might also be interesting to discuss the connection between these two lessons in terms of logical sequencing of content. It is essential that learners appreciate and make use of the connections of one part of mathematics with another as this ability forms the basis of application of mathematical concepts into other areas of mathematics, into other subjects and into their daily lives. Mathematical reasoning makes use of the structural organisation by which the parts of mathematics are connected to each other and if this reasoning ability is not developed in the students, then mathematics simply becomes a matter of memorizing large numbers of disconnected facts and following a set of procedures without thought as to why they make sense.

Through teacher question in the previous Calculus lesson, learners came up with such terms as limits, derivatives, differentiation which link very well with what was being focused on in this particular lesson. In Calculus the derivative is a measure of how a function changes as its input changes. The derivative of a function at a chosen input value describes the best linear approximation of the function near that input value. The process of finding a derivative is called differentiation. So differentiation could be defined as a method to compute the rate at which a dependent output y changes with respect to the change in the independent input x. This rate of change is called the derivative of y with

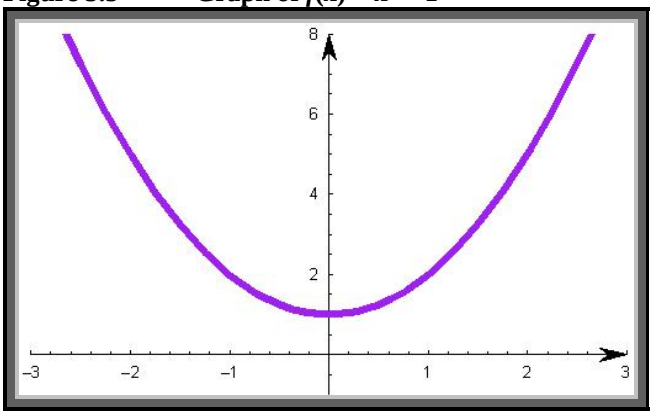
respect to x. If x and y are real numbers, and if the graph of y is plotted against x, the derivative measures the slope of this graph at each point.

The simplest case is when y is a linear function of x, meaning that the graph of y against x is a straight line. In this case, $y = f(x) = m\,x + c$, for real numbers m and c, and the slope m is given by

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

This gives an exact value for the slope of a straight line. In this lesson the teacher's explanation brought this concept out clearly and at that stage the teaching tasks were quite productive. However building on it into getting learners to understand its relationship with the gradient of a curve seemed to be problematic. Firstly for non-linear functions, the rate of change varies along the curve. While this came out clearly in this lesson it would appear learners were then deprived of the connections between positive gradient, negative gradient and zero gradient, which the teacher had talked about, and which could all be exemplified in the quadratic curve that the teacher focused on with the class and which she drew on the board.

Figure 5.5 Graph of $f(x) = x^2 + 1$



On the left hand side of the y axis the curve has a negative gradient, at the turning point the gradient is zero and on the right hand side of the y axis, the curve has a positive gradient. However the teacher did not talk about these concepts in this context and it can be argued that this compromises the learners' abilities to form and to reason with these mathematical structures.

So, while change in y over change in x gives an exact value for the slope of a straight line, if the function f is not linear (i.e. its graph is not a straight line), then the change in y divided by the change in x (the gradient) varies. Differentiation is a method to find the gradient function for that curve which in turn leads to finding an exact value for this rate of change at any given value of x. Graphically the concept could be explained as follows:

Rate of change as a limiting value

Fig. 5.6 The tangent line at [*x, f(x)*]

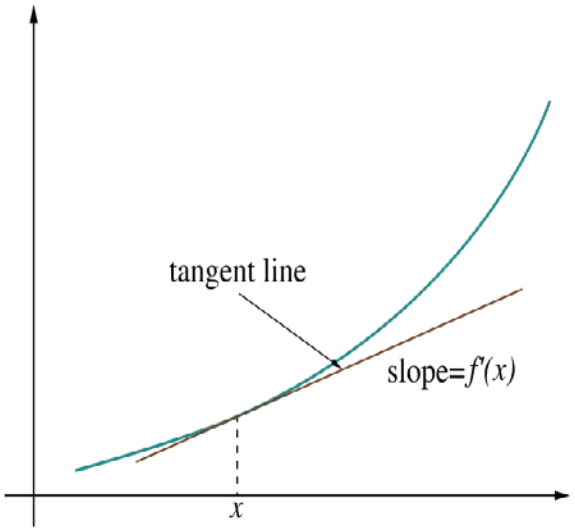


Fig. 5.7 The secant to a curve $y = f(x)$ determined by points $[x, f(x)]$ and $[x+h, f(x+h)]$

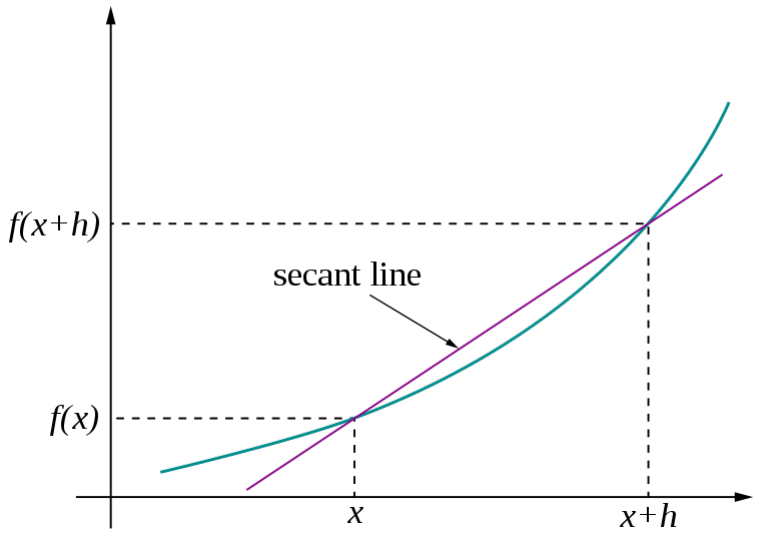
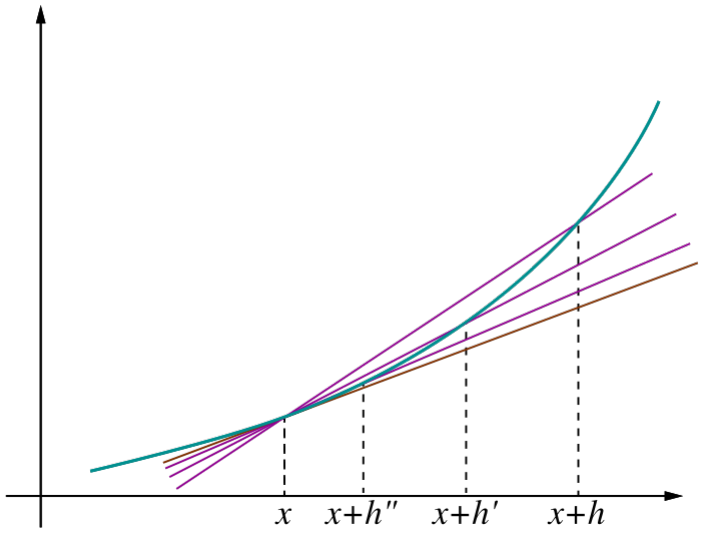


Fig. 5.8 The tangent line as limit of secants



The idea, illustrated by the three figures above, is to compute the rate of change as the limiting value of the ratio of the differences $\frac{\Delta y}{\Delta x}$ as Δx becomes infinitely small. By

moving the two points closer together so that Δy and Δx decrease, the secant line more closely approximates a tangent line to the curve, and as such the slope of the secant approaches that of the tangent. Using differential calculus, one can then determine the limit, or the value that $\frac{\Delta y}{\Delta x}$ approaches as Δy and Δx get closer to zero; it follows that this limit is the exact slope of the tangent.

In this lesson the teacher attempted this procedure and from the teaching tasks it would appear that the concept of limits was meant to lead into learners' understanding of this view of gradient of a curve at a particular point. The procedure above shows that there is a link between a tangent to a curve at a point and a secant of the curve that passes through that same point. The gradient of a secant to the curve $y = f(x)$ through $[c,f(c)]$ and $[x;f(x)]$ is given by the formula: $m = \frac{f(x) - f(c)}{x - c}$. This formula comes directly from the formula for the gradient of a straight line: $m = \frac{y_2 - y_1}{x_2 - x_1}$, and learners were familiar with this formula for gradient of a straight line. However, the process seemed to be unproductive for the learners as the teacher could not show the learners;

(a) how the secant revolved about the fixed point
(b) that the limit of its revolving was the tangent line
(c) that as x gets closer and closer to c (from either side) and the gradient of the secant becomes closer and closer to the gradient of the tangent at $[c;f(c)]$
(d) that the tangent line is the limiting position of the secant
(e) that the limiting value of the gradient of the secant at a point is defined to be the gradient of the tangent at that point or simply the gradient of the curve at that point.

Unfortunately the teacher abandoned the concept of limits before these links were shown and although she promised the learners that "*we will get back to the word limit as we go on*" she never came back to it until the end of the lesson. So it can be argued that learners

were not presented with an opportunity to create these critical connections hence the purpose of talking about limits and where that talk fits in the whole structure might have been blurred to the learners.

In the discussion about the table of values, two critical issues seem to stand out conspicuously in terms of mathematical procedures and reasoning with mathematical concepts. In terms of using appropriate tools and conventions one could start by analysing the comment by the teacher when learners used calculators to complete the table of values;

> *I usually discourage learners to use decimals, ok because if you have used* $\frac{1}{3}, \frac{1}{2}, -1$ *it won't be as accurate as you want it to be but it will be better than using 0.3 and 0.5 ...*

The teachers' comments here seem to suggest that there is one correct way of completing the table of values. This could be evidence that the teacher was trying to control the open ended process (using decimal fractions to plot the coordinates of points on the Cartesian plane) in order to ensure the desired learning outcome for all the students i.e. 'the conventional way of completing tables of values using either whole number values or proper fractions.

In terms of providing reasons why, it is also interesting to note that the teacher's comments seem to suggest that proper fractions are not as accurate as decimals but they are better to use when plotting graphs. However she does not provide the rationale why this is so. This seems to confirm Kelly's (2006) observation that the negotiation of meaning is rare in mathematics classrooms, replaced with statements of "fact" offered by the teacher for acceptance by the students and replicated under examination conditions. Negotiation requires dialogue in the classroom, something that is often difficult to manage, and viewed by many teachers as an inefficient use of limited classroom time. However, classroom environments that foster the production of multiple voices provide a rich learning environment for students and teachers. According to Bruner (1986), mathematical discussions within the classroom that engage multiple voices including

those of teachers, students and other texts support the constant negotiation and recreation of cultural meaning.

In terms of making mathematical explanations which are comprehensible for students using a language which is shared by the community of mathematicians one could analyse the term "error" as used by the teacher to define an undefined number. While the teacher was trying to get the learners to conceptualise a number in the form $\frac{a}{0}$ as an undefined number, learners might have missed an opportunity to conceptualise it correctly as the teacher seemed to associate it with a calculator display of error. According to Harden (2000), the teacher as a representative of the historical practice of mathematics is of utmost importance in structuring the discourse of the classroom to reflect the discourse of communities of mathematicians. In this utterance, it might have been unproductive for the learners to associate an undefined number with "error" as displayed on their calculators especially considering that almost all the students in this class had calculators for daily use. The teacher 'cues' 'undefined' as following from error on calculation rather than from the idea of dividing a number by 0. This is an important concept in mathematics which should have been followed up using different examples so that learners would end up with a generalisable understanding of what this concept was all about.

While the discussion on the plotting of negative decimal fractions progressed, the teacher raised a question; "*Ok between -0.5 and -0.33 which one is bigger?"* The learners' response seems to suggest that -0,5 was bigger and the teacher seems to confirm this response; *"-0.5 so -0.33 is less than - 0.5"* The fundamental principles of mathematics revolve around precision both in the process and in the communication of mathematical ideas. An important point to note is that negative numbers present their own unique challenges which students need to be aware of. The decimal -0.33 is in fact greater than -0,5 and such misconceptions could easily be carried forward into other related areas in mathematics or in other studies.

In the next discussion, the teacher emphasised once more that learners should not use calculators. The task here was for the learners to make a table of values for the hyperbola $y = \frac{8}{x}$. Going through the learners' work there was evidence that some learners had not conceptualised an undefined number. The corresponding value for y when x was 0 was given as 8 by some learners since division without a calculator did not result in the 'error' as denoting an undefined number. This seems to support the argument that the teacher's definition of an undefined number might not have been productive for the learners.

In the next set of utterances the teacher gave instruction that learners could use any of their previous Cartesian planes in the interest of saving time. This again might have been problematic for the students. Some learners then went on and copied the Cartesian plane they had used for the equation $y = \frac{1}{x}$ in the interest of saving time. In terms of modelling for concept formation, it was critical for the learners to understand that when drawing graphs manually, the skill of deciding on an appropriate scale to use has inescapable consequences as each graph demands some critical thinking on what scale to use. Throughout this utterance the teacher seemed to ignore this as she urged learners to use any of the Cartesian planes they had drawn earlier. This seemed to imply that any scale would work with any graph but this might not always be the case as can be evidenced by some of the graphs that were drawn by the learners which failed to bring out the features of a hyperbola.

In winding up the series of lessons, the teacher tried to link the equation $y = \frac{1}{x}$ that the learners had worked on with the standard equation for the hyperbola $g(x) = a.\frac{k}{x} + q$ that had been on the board at the beginning of the lesson. The teacher raised the question;

> "*What is the value for **q**? This graph* $y = \frac{1}{x}$ *ok what is the value for **a**. (Name of student) said **1** and then what is the value for **q**? Haah! The graph is given by*

$g(x) = a.\frac{k}{x} + q$ *(After some few seconds of silence the teacher then went on to say). We do not have a value for* ***q****. Ok. This is the standard graph……"*

For the teacher to suggest there is no value for *q* might not have been productive for the learners in terms of understanding the standard equation for a hyperbola. In fact the values for *a* and *q* were 1 and 0 respectively. In view of the fact that the teacher suggested the next lesson was going to focus on the effects of changes in *a* and *q*, it was critical for learners to have a clear understanding of those values in the standard equation for a hyperbola. This understanding appeared uncertain given the somewhat ambiguous explanation given by the teacher in this discussion.

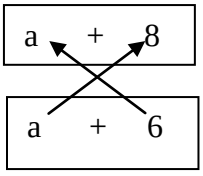
What the relative frequency graph seems to highlight is that connections through different representations both alternate and equivalent and part-whole-representation appeared faulty (level 0) with this teacher in close to 80% of the cases and 100% in the if-then category. A significant number of opportunities for learner rote or superficial understanding (level 1) were created i.e. 20% in the different representations category, 22% in the part-whole relationships, 20% in the procedural connections category and 10% in the instruction-oriented connections category respectively. Let us recall that in level 1 of all the categories, connections were being recognised in a manner that was mathematically acceptable but limited in their potential to develop learners' deeper understanding.

The graph (Fig. 5.4) also shows that opportunities for learner's deep procedural and conceptual understanding (level 2) were created in relative fewer utterances i.e. 5% in the different representations category, 22% the procedural connections category and 6% in the instructional oriented connections. What the graph seems to portray is that on the whole the teacher's utterances had little potential to develop deep procedural and conceptual understanding.

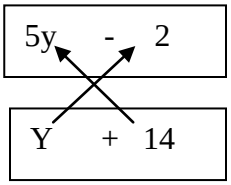
5.4 TEACHER 'T'- FACTORISATION OF TRINOMIALS

Teacher T's lessons for the whole week were all on LO2 (Functions and Algebra) and focused on factorization of trinomials. Below is a table with data excerpts from teacher T's lessons exemplifying how live data was placed into each of the categories and levels.

Table 5.5 EXCERPTS FROM TEACHER T'S CODED UTTERANCES

This was the polynomial to be factorised $7ab - 28a^2b + a^2b + 4a^2$ *and learners had already listed the factors of 28 after multiplying 7 x 4 (the first and last coefficients respectively)* *Teacher:* *Then the next thing was, we have got this number here (putting a circle around -28). That's where the problem is. How do we deal with that number in the middle? We have all the factors of 28 but if we add any of these factors of 28 do we get this number here?*	**DR0**	*The number -28 cannot be defined as a middle term in this case and in any case this would have been -27 if the like terms had been dealt with first.*
 *The next thing we are going to do is we are going to say a x 8 (showing the cross multiplication) and we put 8a here (on the right hand side of the first rectangle). Then we are going to say 6 x a (again showing the cross multiplication) then we put 6a here. Because we cross multiplied these numbers here? So it means the correct factors the first one is this one. Are we together (putting a circle around the factors in the first rectangle and writing) (a + 8) and the second one is what? (a + 6)*	**DR1**	*Lack of precision in presentation because we are not only cross multiplying here but there is also vertical multiplication of terms i.e.* *a x a and 6 x 8*
(Pointing to $a^2 + 14a + 48$*).* *Teacher:*	**DR2**	*This was considered to be a sound definition of a trinomial*

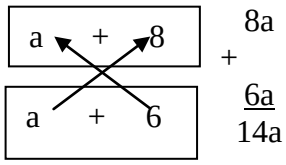
So we are saying this a^2 is a term on its own. Are we together? Then the second 14a it's a term on its own. The next one is 48 it's also a term on its own. So we are saying this is a trinomial because we have got 1, 2, 3 what, 3 terms.		
None of the episodes for the teacher were coded	**PWR0**	
This is how you factorise the trinomials but now we are going to check. Let's check by multiplying. We are going to say a x a = a^2, a x 6 = 6a, 8 x a= 8a and 8 x 6 = 48. What is the next step now?	**PWR1**	*True but some trinomials may not be factorised this way as it turned out later in this series of lessons hence to generalise this way may mislead the learners*
None of the episodes for the teacher were coded	**PWR2**	
None of the episodes for the teacher were coded	**IM0**	
Teacher: *Let's start from here (Now writes a new task on the board n^2 -16mn + $15m^2$. learners then shout the factors of)* 1m 15m 3m 5m -m -15m -3m -5m *These are the possible factors of $15m^2$. So from this list of possible factors we are saying if we add the factors they must give us a what, a -16mn. (So he puts them in the rectangle of factors and does the cross multiplication which does not yield the middle term). So it means these two factors are what? Are wrong ….*	***IM1***	***If** the cross multiplied terms do not yield a sum equal to the middle term **then** the factors are not correct - yes but sometimes the teacher contradicts this by suggesting it is the sum of the coefficients which should yield the middle term. It is for that reason that even in listing the factors here, only the coefficients(1 x 15) were considered yet it should have been $1n^2$ x $15m^2$ - no factors of **m's** here can give a middle term with **mn.** Notice that this episode was also coded P0 because procedurally the products of each pair of listed factors do no give $15m^2n^2$*
None of the episodes for the teacher were coded	**IM2**	
Teacher: *This is what is going to happen. Let me show you how. Let's start with those two factors (pointing to the first 2)*	**P0**	*Ignoring the signs resulting in incorrect answer*

 2y + 70y 72y *Let's check now. They don't give us -13y so it means we cannot use these two factors are we together.*		
Teacher: *Let's try to list down the common factors of 48. As pairs. They must be in pairs.*	***P1***	*Learners at this level should be able to identify factors of 48 that add up to the coefficient of the middle term*
Earlier on from $3t^2 - 9t - 5t + 15$ the learners had been able to factorise thus $3t(t - 3) - 5(t - 3)$ leading to the final answer of $(3t - 5)(t - 3)$ *Teacher:* *I have got a question what about if he wrote something like this (he reverses the middle terms from $3t^2 - 9t - 5t + 15$ and writes)* *$3t^2 - 5t - 9t + 15$* *Is it going to affect our answer? Is it going to affect our answer?*	***P2***	*Probing the learners for procedural flexibility.*
Teacher: *Listen; is it possible that you find an expression like that?*	***IOC0***	*This kind of comment might not be helpful to the learners, in fact there are trinomials which cannot be factorised and this could have been explained in a similar way by asking learners for factors of the last term that add to the middle term*
Teacher: *If we multiply 8 and 6 we get what 48. But before we do anything we have to take note of the sign before this 48. The sign is a what?*	***IOC1***	*Agreed but this could have come much earlier so that the learners could keep that in mind from the word go as they looked for the pairs of factors.*
None of the episodes for the teacher were coded	**IOC2**	

5.4.1 Episodes coded as different representations (DR):

Only one utterance was coded at the highest level DR2. However, this utterance was coded twice because in that same utterance #5 the teacher gives a sound definition of a coefficient and a trinomial.

Eight utterances were coded at the second level DR1 and these are utterances where connections appeared to be made but at a superficial/rote level. In #26 and with reference to the binomials in these rectangles the teacher says; "… *because we cross multiplied these numbers here…"*



As a way of just checking whether the factors are correct, this representation might be acceptable but when checking to see whether the two binomials when expanded can give us the original trinomial, this representation does not seem to connect with the expansion of brackets of (a + 8)(a + 6). The arrows suggest only a cross multiplication of a x 6 and a x 8 yet the a's should also be multiplied and the 8 and 6 should also be multiplied. Hence it might be more meaningful if there could also be arrows showing cross multiplication as well as vertical multiplication for learners to have a deep understanding of what is going on. In #44 the middle term that learners have to check with seems not to be defined precisely/accurately. The teacher lists all the possible factors of $15m^2$ and says;

> *These are the possible factors of $15m^2$. So from this list of possible factors we are saying if we add the factors they must give us a what, a (-16mn)*

But the factors that were identified -m and -15m can only add to -16m and not the -16mn which is the middle term. In #102 the teacher suggests that to reduce $9t^2 – 42t + 45$ the learners have to factorise it first. Here the step of taking out a common factor 3 does not seem to be distinguished from the process of factorization of which it is only a part. To

suggest that we factorise first then we factorise again may confuse the learners. In #140 the way common factors are being defined might be problematic for learners. After taking out the common factor 3 and factorising the inner brackets as follows: 3{t (3t – 5) -3(3t - 5)} the teachers says;

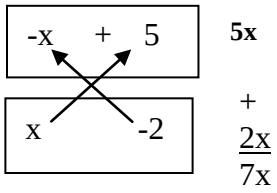
> *So we are going to say the numbers in the brackets are the same, (pointing to the highlighted factors). Can you see them? So we are going to take one of them. Even if you take this one, (pointing to the second bracket) if you choose this one you have also chosen this one.*

The explanation given by the teacher suggests that taking out a common factor is about choice yet learners needed to be able to identify common factors of this nature and connect such a process to say 13x – 8x where the common factor x is taken out and is equivalent to x(13 - 8) = 5x. In # 160 the teacher suggests with reference to swapping the signs of the following factors of -10; *"Let's say they are the same because we just swap the signs."*

5	-2
-5	2

While the swapping of signs might not have an effect on their product, it however has an effect on their sum, which in this context of factorization is related to the middle term. So to suggest that they are the same only in relation to their product of -10 might inhibit the learners' ability to see the connection they also have with the middle term which is critical in this context. After having identified the correct factors of 10 that would add up to 7, in #179 the teacher says; "So *we are going to say* $-x^2 + 5x + 2x - 10$" but does not say why this has to be done. Equivalence needs articulation here i.e. why are we writing the expression this way? In this case it would have been important for learners to see how $-x^2 + 5x + 2x - 10$ *and* $-x^2 + 7x - 10$ were equivalent but that breaking the $+7x$ into those two parts would allow for factorization in parts. In #181 the issue is about how common factors seem to be defined. It would appear "common" is being taken as "what is apparent" hence in dealing with the factorization of $-x^2 + 5x + 2x - 10$ the teacher ends up with $x(-x + 5) + 2(x - 5)$. In this case there was need to look beyond what the

common eye can see so that identifying the not so apparent factors e.g. *x (-x + 5) - 2(-x + 5)*would yield other sets of 'apparent' common factors. In fact this ability to see beyond the eye became such a stumbling block that there was confusion on this task till the end of the lesson which also happened to be the end of the series of lessons. In #200 and with reference to the binomials in the rectangles:



The cross multiplication yields a correct middle term of the trinomial – $x^2 + 7x – 10$ but the rule (*look for factors of -10 which add to 7*) seems not to apply yet this is the rule that the learners have been given right through the series of lessons.

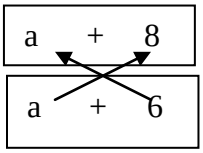
Nine utterances were coded at the lowest level DR0 of this category. In #70 and #72 the issue is about ignoring signs when multiplying resulting in incorrect products. In #70 for example the teacher cross multiplies -14 and y in the rectangles and gets 14y. Clearly one can notice that equivalence is violated through ignoring the sign resulting in an incorrect answer. After factoring 3 out of $9t^2 – 42t + 45$ a learner got $3t^2 – 14t + 15$ and in #104 the teacher suggests this is okay and advises the learner to proceed with the factorization but the two expressions are not equivalent without the 3 being shown outside the bracket i.e. $3(3t^2 – 14t + 15)$. In # 144 and with reference to the polynomial $7ab – 28a^2b + a^2b + 4a^2$, the term $– 28a^2b$ is being defined as the middle term. However, in terms of both its position in the polynomial and its value the term $– 28a^2b$ cannot possibly be defined as a middle term because it does not fall in the middle and in any case this middle term would have been $-27a^2b$ if the like terms had been dealt with first. In #156 the issue is about how the coefficient is being defined in the trinomial – $x^2 + 7x – 10$. With reference to the first term in this trinomial, for the teacher to say;

Let's take this one as a 1 are we together? Let's take the coefficient as what 1

would be misleading to the learners. In #169 the teacher goes back to the same coefficient and says lets go to the coefficient negative. Yet in #187 the coefficient is again taken as positive. The question then would be "What image of a coefficient do the learners get in such circumstances?" In #163 whether by error or what the teacher keeps referring to a coefficient as an exponent.

5.4.2 Episodes coded as indicating part-whole relationships (PWR):

As defined in the model earlier, part whole relationships include examples, inclusions and generalisations. In this set of lessons there was only one utterance coded at the second level PWR1 in this category. In #26 the teacher uses this example;



He then gives a statement which suggests a generalisation that this is the way to factorise trinomials implying that this rectangle method works in all cases. However as it turned out during the series of these lessons, this approach worked well with some and not all trinomials as some required a different approach altogether. Let us recall again that research has shown that generalisation is one of the most fundamental and important mathematical thinking process in that it demonstrates learners' abilities to identify patterns and commonalities across cases that were being dealt with.

5.4.3 Episodes coded as indicating connections through implication (IM):

Only two utterances were coded at level IM1 of this category and in both case utterances #30 and #46 the implication was that if the cross multiplied terms do not yield a sum equal to the middle term then the factors are not correct. While this observation appeared to make mathematical sense for the learners it would also appear that the teacher sometimes contradicted this by suggesting that it is the sum of the factors which should yield the middle term.

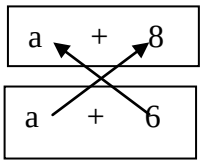
5.4.4 Episodes coded as indicating connections through procedure (P):

Let us recall that these are utterances, which would be showing that **A** is procedure used when working with object **B**. Two utterances were coded at the highest level P2 in this category. In #28 for example, the teacher shows a correct procedure of dealing with the expansion of polynomials and in #118 there is evidence of the teacher probing the learners for procedural flexibility.

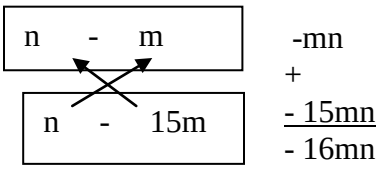
There are ten utterances which were coded at the second level P1 of this category. In # 7 and with reference to the trinomial $a^2 + 14a + 48$ the teacher says to the learners;

> *The next thing is you are going to leave this middle term alone (putting a circle around 14a in the trinomial")* but does not explain why. In #9 he says; "*Let's try to list down the common factors of 48. As pairs. They must be in pairs.*

Procedurally this makes mathematical sense but when one considers the number of pairs of factors involved this might not be economic in terms of time. Learners at this level should be able to identify factors of 48 that add up to the coefficient of the middle term. In#20 the teacher suggests to the learners that they have to look for factors of 48 that will add up to the middle term 14a but such factors do not exist so the issue is about precision in mathematical language. In #26 the teacher suggests that this is the way to factorise trinomials implying that this rectangle method works in all cases.



However as it turned out during the series of these lessons, this approach works well with some and not all trinomials as some require a different approach altogether. In #59 and with reference to the trinomial $n^2 - 16mn + 15m^2$ the teacher has all along given the impression that learners have to look for factors of $+15m^2$ which add up to the middle term i.e. -16mn. The class then agreed that the following were the correct factors.



However when one looks carefully at the terms being added one notices that they are not factors of $15m^2$, so one gets the impression that there was need for a more precise description of the procedure or the rule. In #124 and with reference to the factorization of $9t^2 – 42t + 45$ as t (3t – 5) -3(3t - 5), the 3which has been factored out is left out of this process. In#126 the reason why one of the common factors (3t - 5), has to be outside the brackets of *(3t – 5) (t – 3)* does not seem to be clear. What is in brackets is also another example of common factors but this does not seem to be the message that the teacher is putting across to the learners. Instead he says they have to choose one of the brackets because they are the same. In #181 and with reference to the factorization of $– x^2 + 7x – 10$ from $– x^2 + 5x + 2x – 10$ through to *x (-x + 5) + 2(x – 5)* it would appear as a procedure the focus was on the 'apparent' common factors. However it would appear this task required the learners to go beyond the 'apparent' so that the factorization would result with more apparent common factors.

Eleven utterances were coded at the lowest level P0. In #70, #72, #158, #165, #187 and #200 the issue is about dealing with the negative sign in the multiplication resulting in an incorrect product. In #140 and with reference to the factorization of 3{t(3t – 5) -3(3t - 5)} the teacher suggests taking out the highlighted common factor is about choosing any one of them yet learners needed to see the connection between this not so familiar form of common factors and any others that might be a bit more obvious. In #142, #144 and with reference to the polynomial $7ab – 28a^2b + a^2b + 4a^2$ there was need to rearrange the polynomial starting with $4a^2$ consistent with mathematical convention, but more importantly there was need to deal with the like terms $– 28a^2b + a^2b$ first as these had an effect on the middle term ($-27a^2b$ instead of $-28a^2b$). In #198 the teacher suggests that;

So we are going to play around with these factors

Mathematically, learners cannot be expected to play around with factors; they should have an explicit route that they have to follow in order to be able to factorise these trinomials.

5.4.5 Episodes coded as indicating instructional oriented connections (IOC):

No utterances from this class were coded at the highest level IOC2 of this category. Sixteen utterances were coded at the second level (IOC1). In #12 for example the teacher advises the learners when they are looking for the factors of 48 that;

But before we do anything we have to take note of the sign before this 48.

This is an important hint for the learners but this only came after the learners had gone way into the listing of pairs of factors, all of which were positive while the negative factors were totally ignored. This hint could have come much earlier so that the learners could have kept that in mind from the word go as they looked for the pairs of factors. In #51 and #59 with reference to the trinomial $n^2 - 16mn + 15m^2$ the learners have already identified that 1m and 15m are factors of $15m^2$ but they do not give the middle term when added. Despite this important observation by the learners the teacher still takes those factors and puts them in the rectangles and goes through the whole process of cross multiplication and adding the products only to get +16mn. Because learners had already understood what the rule was, it was important for the teacher to build on that and focus on more critical issues rather than continue with the cumbersome trial and error method. In #55 and #70 the issue is about the instruction on changing position and/or signs of the pairs of factors e.g. 7 and -4 becoming -7 and 4. While changing signs and/or position might not have an effect on the product of the factors learners should however be made aware of the effect this has on the sum of the factors which in this context of factorization has an important relationship with the middle term of the trinomial. Change of position of factors sometimes affects correct factorization in that a specific factor sometimes has to go into a specific bracket. For example in the factorization of $5y^2 – 13y – 28 = (5y + 7)(y – 4)$, changing either the signs or the position of 7 and -4 would result in incorrect factorization. In #82 and related to this same trinomial the teacher does not appear to be consistently building on the rules he has given the learners i.e. look for factors of the last

term that add up to the middle term. These factors of 7 and -4 do not add to the middle term of -13y, but the factors are 'accidentally correct' because incidentally they are also correct factors of -140 (5 x -28) which would have been the more systematic route for this factorization. So in the absence of a systematic procedure the learners are likely to get the correct solution by superficial/rote procedures. In #91 the teacher says to the learners;

> *Today we want to shift from our method to (another) method. So we have got number 19 (on the board)*

The teacher had written $9t^2 - 42t + 45$. The concern in this utterance as well as in #100 is that the teacher seems not to be explicit about the strengths and weaknesses of each method and which one would be best under what circumstances – so learners just shift from one method to another and hopefully find out for themselves what works and what does not. In #104 after the learner had factored out the 3 in $9t^2 - 42t + 45$ the teacher needed to explain what happens to it because the learner ended up with $3t^2 - 14t + 15$ and continued to factorise it without considering the 3 factored out. In #175 and with reference to the trinomial $-x^2 + 7x - 10$ the teacher still wants learners to list all the possible factors of 10 despite the fact that one learner has already identified the factors that add up to 7 the middle term.

A total of eleven utterances were coded at the lowest level (IOC0) of this category. In #66 for example a learner had asked the teacher what would happen if the trinomial had been like:

> *She comes and writes on the board* $n^2 - \frac{16}{mn} + 15m^2$

The teacher's response was;

> *Listen; is it possible that you find an expression like that?"*

For the teacher to give this kind of comment might suggest that all trinomials can be factorised and this might not have been helpful to the learners because in fact there are such trinomials which cannot be factorised. Here was an opportunity to explain in a

similar way by asking learners for factors of the last term that add to the middle term – the rule that they had learnt. Learners would then have concluded by themselves that such a trinomial cannot be factorised. In #84 the teacher gives learners a totally different task $4a^4 – 20a^2b^2 + 9b^4$ from the ones that he has used as examples on the board. However the teacher does not show the links with the worked examples and has not given an explanation on how to deal with this type of a trinomial. This is followed up in #88 by a comment;

> *This side no one has got it right; do something about it*".

So while there is evidence to show that learners are struggling with the task the teacher still says do something about it. Again this might not be helpful to the learners and is further compounded by the fact that the lesson ended without having got feedback from the teacher and on the following day the teacher started on another different task without having shown the learners how to deal with this kind of a problem. In #106 the teacher discourages a learner from continuing with a certain method of factorization by saying;

> *I want the second method. I know you know this one. Let's try the second method. There are so many ways of killing a cat*

This was despite the fact that the learner had not even finished the factorization process raising questions as to whether he actually knew how to factorise using this method. Perhaps the teacher could have waited for the learner to finish thereby confirming whether the learner indeed knew this method after which he would then challenge the same learner or others to use another different method. In #149 after having struggled with the factorization of the polynomial $7ab – 28a^2b + a^2b + 4a^2$ without success, a learner suggests;

> *We arrange starting with* $4a^2 + a^2b – 28a^2b + 7ab$

However the teacher seems to buy this suggestion without realizing that even this re-arrangement would not yield a correct result without the learners identifying that there are two like terms in the middle which had to be dealt with first. In#152 the teacher ends the lesson without resolving this factorization but surprising again the next day's lesson starts with different tasks without this task having been solved. In #192 after having

struggled again with the factorization of the trinomial $-x^2 + 7x - 10$ the teacher says to the learners;

> *In other words this method, some of you they cannot do this method. Are we together? If you are not well prepared to use this method you can use the other what, method.*

This was despite the fact that the teacher had not shown any other method that would work for this problem and that the teacher been working this problem on the board with the whole class. So for him to accuse learners of not being able to use a method was just trying to apportion blame – it seemed like the teacher could not handle trinomials of this nature. In #198 and with reference to the same trinomial the teacher says;

> *So we are going to play around with these factors. So we are saying of these factors which ones will you add to give us what; +7x?*

To instruct learners to play around with factors might not be helpful as an instructional strategy. This can be evidenced by the fact that the lesson ended without having solved this problem #204. Learners cannot play around with factors; instead they should have an explicit and systematic route that they have to follow in the process of factorization of trinomials.

5.4.6 Key messages emerging from Teacher T's teaching

The discussion of what emerges from Tr T's teaching is again guided by the following:

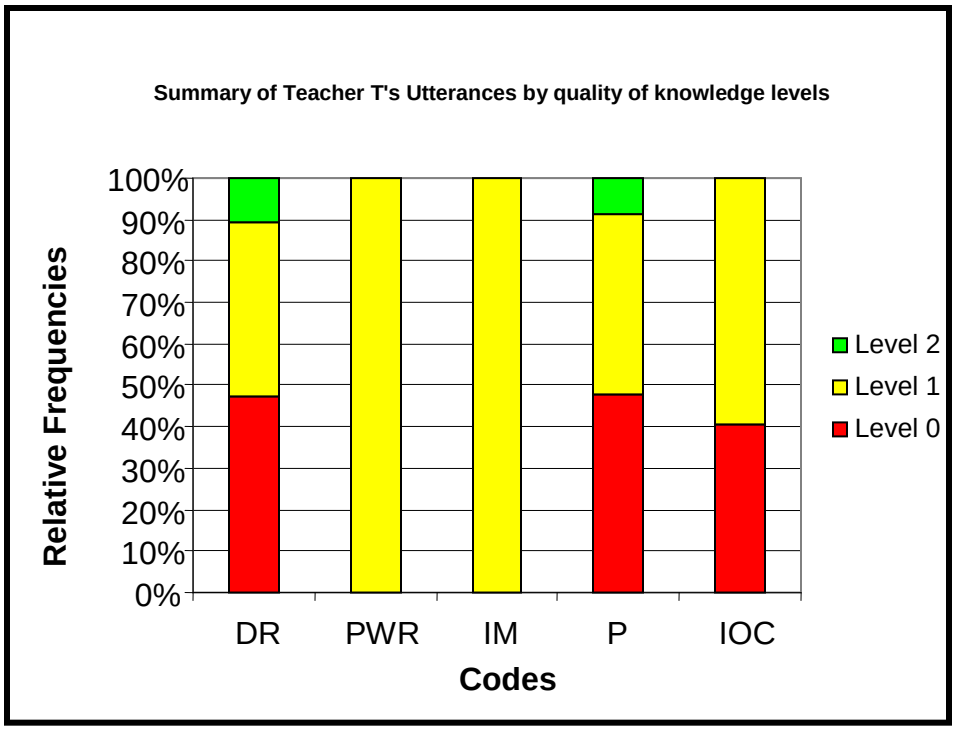
(a) What aspects of connections does the teacher appear to handle well?
(b) What aspects appear to be problematic?
(c) What are the underlying features in each case?
(d) How do the strengths and weaknesses relate to the orientations in the curriculum?

The discussion starts by summarizing all the teacher's utterances/activities for the whole week into the different codes under which they were captured.

Table 5.6 **TOTALS OF TEACHER T'S CODED UTTERANCES**

FORMS OF MATHEMATICAL CONNECTION	LEVELS OF KNOWLEDGE QUALITY			
	Code	0	1	2
Different Representation	**DR**	**9**	**8**	**2**
Part-whole Relationship	**PWR**	**0**	**1**	**0**
Implication	**IM**	**0**	**2**	**0**
Procedure	**P**	**11**	**10**	**2**
Instruction-oriented Connection	**IOC**	**11**	**16**	**0**
Totals		**31**	**37**	**4**

Fig. 5.9 Summary of teacher T's utterances by quality of knowledge levels



This relative frequency graph seems to suggest that learners' opportunities for deep understanding (level 2) were created in only 10% in the DR category and 9% in the P category. A significant number of opportunities for learner rote or superficial understanding (level 1) were created as follows: 42% in the DR category, 100% in the PWR category and 100% in the IM category, 42% in the P category and 60% in the IOC category respectively. This also suggests that the teacher's utterances were mathematically faulty in 48% in the DR category, 49% in the P category and 40% in the IOC category.

One of the major challenges observed in this series of lessons was the teacher's inability to analyse a method so that the learners could appreciate where it was more suitable to be used, what its strengths and weaknesses are and how efficient it was to use. Choice of a method to use seemed to be guided by the teacher opting for that method and not by its appropriateness for that particular task. During this week's observations, the teacher also appeared not to be very comfortable working with negative numbers. The teacher often suggested that leaving out the negative sign would have no effect on the result yet this proved problematic in quite a number of cases. Another observation made was the teacher's inability to deal with unexpected questions from the learners. The question posed by one learner required the teacher to realize that just like in factorising numbers, there are some which are prime whose factors are 1 and the number itself. Similarly we can also have polynomials that cannot be factorised further and the example given by the learner in one of the lessons was a typical example of such a polynomial. However the teacher discarded it as a task that would never be expected. Lastly, this teacher appeared not to build on what the learners already knew. After having explained the rule that when factorising the type of trinomials that they were dealing with, learners had to look for factors of the last term in the trinomial that would add to the middle term, the teacher went on to insist that learners had to list all the factors of that last term. This proved to be cumbersome especially when one considers some numbers like 72 can have a very long list of factors. It was evident that at this level that the learners were able to identify factors of the last term that would add up to the middle term which the teacher could have exploited and moved on thereby being efficient with time.

5.5 TEACHER 'M'- TEACHING NUMBER PATTERNS

Teacher M's lessons for the whole week were on LO1 (Number and Number Relationships)hips and specifically focused on Number Patterns. Below is a table with data excerpts from teacher M's lessons exemplifying how live data was placed into each of the categories and levels.

Table 5.7 EXCERPTS FROM TEACHER M'S CODED LIVE DATA

Episode/utterance/activity	***Code***	***Comment***
Tr: *Right. Then the second one, but did you use any formula? How did you get that general term*? *Learner: I just looked at it* *Tr:* *You just looked at it and then you got it right. Ok good. Yaa it's a method, it's a method as well, you just look at things and then get them right it's a method. Just look at things and then get them right.*	***DR0***	*Just looking at and getting things right is being defined as a method- surely a learner could not possibly look at a sequence and get the general term - at least there should be reasoning that happens*
T: [writes (2)] So our T3 will be equal to 4 [writes = 4] It appears here in the sequence [points to the sequence 1; 2; 4; 8; 16…] Like I say keep it as a recursive formula. We must be able to generate a sequence to find the general formula.	***DR1***	*While the critical components of the recursive formula have been discussed there is nowhere the formula it has been presented in its complete form*
Tr: *Wonderful. That's good. So here it will be Tk-1 Any problem with that? [writes T_{k-1} + 3] So before Tk it becomes equal to Tk-1 + 3 [points to that equation] In other words we are saying a term that comes just before Tk is Tk-1. We agree?*	***DR2***	*A term that comes before k is defined as k – 1- This definition again after some well articulated discussion*
None of the episodes for the teacher were coded	***PWR0***	
Tr: *Ok wonderful, she has made a very good observation here. You see. What she is saying is; do you see kuti there is always a difference of 2 between any two successive terms. There is a difference of 2. Now there is a difference of 2 here and she has*	***PWR1***	*There is a generalisation that can be made about all linear sequences with a constant difference of 2 in that they all become 2n + b and all linear sequences with a constant difference of 3 they all become*

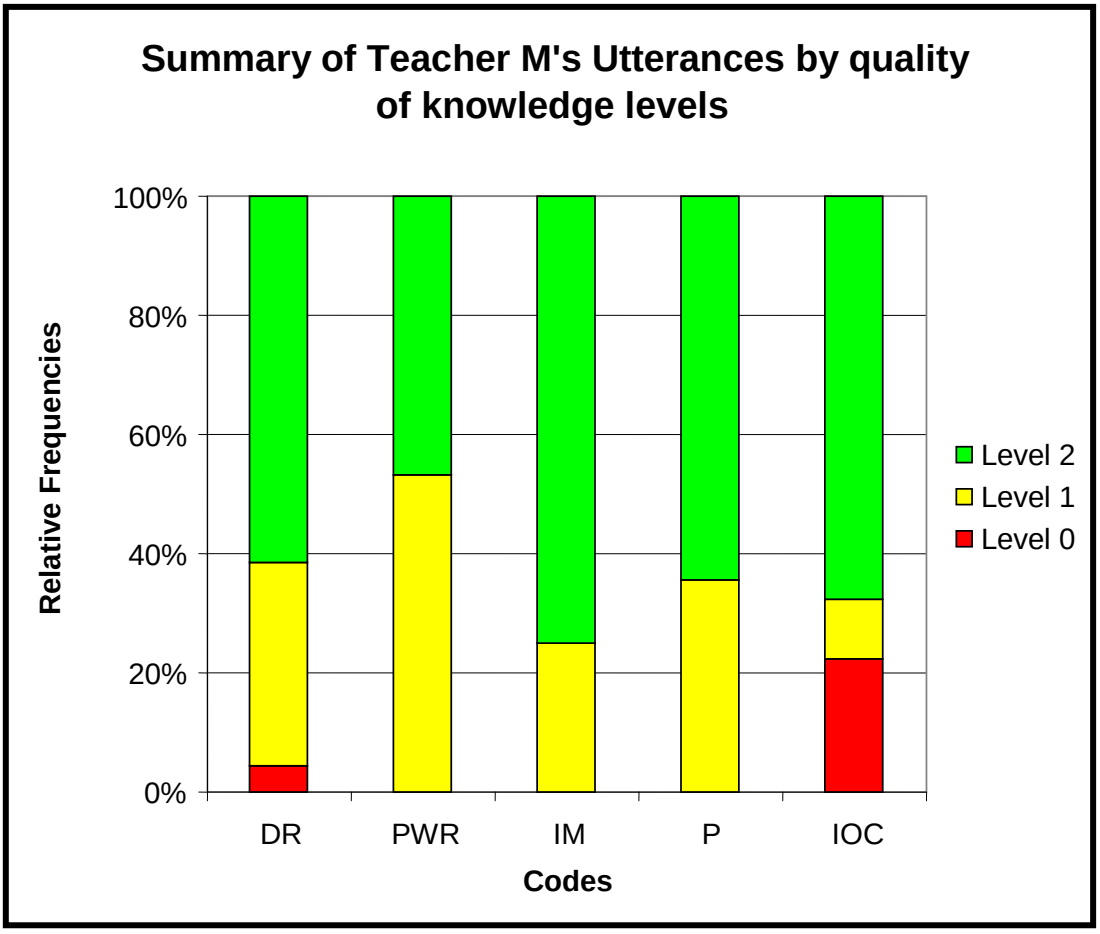
observed ok there is always a 2 here (referring to the 2 in $T_n = 2n+ 1$ of the general term) She also made an observation to say there is a difference of 2 between any two successive terms here (referring to the sequence -4; -2; 0; 2; ...) and now there is a common difference of 2 there is also a 2 here (referring to the 2 in $T_n = 2n – 6$; the general term). Then she came this side and she made an observation to say there is a difference of what of 4 between any two successive terms here (referring to the sequence 3; 7; 11; 15...) and then there is a 4 here (referring to the 4 of $T_n = 4n – 1$; the general term for the sequence.) It's ok, its alright		*3n + b and so on. However this does not appear to be made from the teacher's comments yet it is such a critical observation that the learner has made which would guide them in generating the general terms of the linear sequences. .*
T: It might be quadratic, it might be cubic or it could be something else.	***PWR2***	*Here the teacher is very cautious about the generalisation that can be made after one has observed that the 1st difference is not constant (It might….)*
None of the episodes for the teacher were coded	***IM0***	
T: Right. Sh….., if the differences are not constant, what does that, what does that tell you? *L: Its quadratic* *L: Its cubic* *T: That's correct. This difference here is not constant, what does it tell you?*	***IM1***	*If the 1st difference is not constant we cannot tell whether its quadratic or cubic all we can tell is that it is not linear (precision in mathematical statements)*
Tr: *It suggests linear. If the first difference is constant that suggests a linear general term. How then shall we find… We know definitely that one [points to quadratic equation on the board]? Now how shall we find a general term now? How shall we find a general term? All we know is it takes the form Tn*	***IM2***	*If there is 1st constant difference this suggests a linear general term – this kind of conclusion was arrived at after some well articulated examples had been discussed in detail*
None of the episodes for the teacher were coded	***P0***	
Teacher: *Term number 2 is a 6 term number 3? (teacher says all this while underlining the subscript of T and the value of the term).* T_1 T_2 T_3 T_4 T_5 …T_{10} …	***P1***	*Showing the connection between term value and term number in a sequence as a way of predicting unknown terms in the sequence*

3 6 9 12 15 30 *There appears a pattern, a relationship between the term number and* (pointing to the subscript and the term value) *good what about term number n? What shall term number n be?*		
T: Its okay? (Teacher underlines the general term. It can also be in terms of k. If you got T_k *what will it be in this case (teacher writes on the board* T_k = *)*	***P2***	*Showing flexibility in the way we could write the general term*
*T: suppose we consider the sequence now 1; 8; 27; … (*teacher folds arms and walks around smiling while learners look at the board and discuss but the all agree there is no pattern *)Alright let us put that one on stand by for a while. What about here (teacher writes) 1; 4; 9; 16; …? Do you observe any pattern there?*	***IOC0***	*Sequencing of tasks might not benefit the learners in terms of observing patterns inherent in them- learners had just done sequences whose general term is linear yet this one had a cubic general term.*
T: It's ok; a common difference, a difference of 2 between any two successive terms and we have a 2 here and here the difference between any two successive terms is 4 and we have a 4. Right then how would you come up with this 1 and this -6 (pointing to the second parts of the general term)? Ok, all right ... Yes	IOC1	*Indeed there is a connection between the constant difference in a linear sequence and the coefficient (first constant) 2 in the general term but teacher seems not to articulate why it is so*
T: suppose we have got a sequence of numbers (teacher writes: 4; 7; 10;) T: Do you observe a pattern to find term number ten there that will enable us to predict the tenth term. We want to predict the tenth term. Yes what will the tenth term be? (class shouts 31, 32 - teacher chooses a response from one learner) She says term number ten will be 32. Alright suppose somebody comes in from a distance and says why 32? Why not 31?	***IOC2***	*Following learner's response and probing for justifying*

Table 5.8 **TOTALS OF TEACHER M'S CODED UTTERANCES**

FORMS OF MATHEMATICAL CONNECTION		LEVELS OF KNOWLEDGE QUALITY		
	Code	0	1	2
Different Representation	**DR**	**2**	**15**	**27**
Part-whole Relationship	**PWR**	**0**	**8**	**7**
Implication	**IM**	**0**	**4**	**12**
Procedure	**P**	**0**	**10**	**18**
Instruction-oriented Connection	**IOC**	**9**	**4**	**27**
Totals		**11**	**41**	**91**

Fig. 5.10 **Summary of Teacher M's Utterances**



5.5.1 Episodes coded as indicating Different Representations (DR):

The highest number of this teacher's utterances (44) was coded in this category. Of these 27 were coded at the highest level 2. In # 72 and #74 the teacher comprehensively defines a subscript as symbolised in T_n denoting a term number and distinguishes it clearly from Tn where T is being multiplied by n. In #108 the teacher offers a comprehensive definition of a general term which is given in terms of an unknown where 'we' substitute to generate the sequence. In #746, #752, #762, and #771 the teacher gives valid equivalences of T_1, T_2, T_3, and T_4 respectively in terms of *a* and *b* which appear to have been generated from a comprehensive process of analysing how the quadratic sequence was growing both numerically and algebraically. In #1053, #1060, #1070, #1088, #1094, #1118, #1138, #1168, #1184, #1199, #1203 and #1214 the teacher gives valid equivalences of the differences between successive terms as well as their constant difference in terms of *a* and *b* which also appear to have been generated from a comprehensive process of analysing a quadratic sequence both numerically and algebraically. These equivalences proved critical in helping the learners to generate a system of equations which appeared to form a strong foundation for coming up with the general term or explicit formula for any given quadratic sequence. In #1773, #1861 and #1866 the teacher gives comprehensive definitions and equivalent representations of the recursive formula in relation to linear sequences.

A total of 15 utterances were coded at the second level 1. In #11, #17, #19, the definition of successive numbers offered by the teacher might inhibit learners' deep understanding in that it would appear the teacher does not emphasize the fact that these successive numbers should be following the same order that has been observed in the sequence. In #102 the teacher seems to be defining a general term as in a manner that might give learners the impression that there is only one general term, yet there is a general term for all linear sequences i.e. $T_n = an + b$, then there is a general term for all linear sequences with a common difference of say 3 i.e. $T_n = 3n + b$ then there is a general term for a specific linear sequence e.g. $T_n = 3n + 1$. In #381, #388, and #395 the issue is about the

two constants *a* and *b* in the general term for a linear sequence whose relationship is such that $a + b = T_1$, but from the way the teacher defines them continues to look like this relationship is accidental. The logical connection between them does not appear to come out explicitly yet it is a critical step for learners to be able to generate the general rule. In #403, #405 and #408 the teacher defines linear and quadratic functions in a manner that does not seem to show why they considered as such. In analytical geometry, the term linear function is sometimes used to mean a first degree polynomial function of one variable. These functions are "linear" because they are precisely the functions whose graph in the Cartesian coordinate plane is a straight line. If for example the terms 3; 6; 9; 12; …of a sequence were to be plotted in a Cartesian plane against their term numbers, the resultant graph would be a straight line graph hence such sequences are defined as liner sequences. It is likely that learners might have a superficial understanding of how such sequences get defined as linear or quadratic. In #1864 and #2127, the issue is about the definition of the recursive formula where the teacher emphasises that 'we' need to know at least T_1 for us to have a complete recursive formula but in all the examples that the teacher used none of them shows the formula in its complete form e.g. $T_n = 2T_{n-1}$ where $T_1 = 3$.

There are only two utterances coded at the lowest level 0 suggesting that this teacher was relatively strong in his own understanding of the concepts that he was dealing with in this series of lessons. In #247 the teacher says; *"So the pattern is in the difference"* and it can be argued that such a definition of a pattern might not give learners a clear sense of what a number pattern is. In #295, the teacher seems to suggest that it is a method to just look at a sequence and get things right yet a method is something more strategic than just mere looking and getting answers. In fact it is doubtful that learners could just look at a number sequence without some thought process and get things right.

5.5.2 Episodes coded as indicating Part-whole Relationships (PWR):

A total of 7 utterances were coded at the highest level 2 in this category. In #573 the teacher arrives at an important generalisation with the learners about the connection between the degree of a polynomial for the general term of a sequence and the level at

which the constant difference can be found. For example whenever the polynomial is a 2^{nd} degree as in $T_n = an^2 + bn + c$ the constant difference is always at the second level. In #875 the teacher also makes an important generalisation that whenever the first difference is not constant in a sequence one can always conclude that the sequence is not linear.

It might be quadratic, it might be cubic or it could be something else

In #1097 and #1629 the teacher draws another important generalisation that 2a is always equal to the second difference between terms in a quadratic sequence. Subsequent to that the other generalisations are that in #1637 T_1 is always equal to a + b + c and in #1641 the first difference in any quadratic sequence is always equal to 3a + b. With reference to the recursive rule the teacher draws a generalisation in #1928 that whenever the previous term is being multiplied by say 2 the recursive rule will always be $T_k = 2T_{k-1}$ and T_1 has to be known.

A total of 8 utterances were coded at the second level 1 of this category. In #344 there is a generalisation that is implied from the learner's observation about all linear sequences with a constant difference of 2 that their general term will always be 2n + b and all linear sequences with a constant difference of 3 their general term will always be 3n + b and so on. However the teacher appears not to explicitly put that in algebraic terms yet it is such a critical observation that the learner has made and that would also help the learners when challenged to generate the general term of any linear sequence. In #421 and #504the teacher makes a generalisation that when we have a linear general term the first difference is constant but while this is true the concern was that we could not possibly conclude that there is a constant first difference by just observing one case of a linear sequence hence it could be argued that such a generalisation might have been prematurely or superficially made. In #539 and #546 the learners indeed make valid generalisations, however these might have limited application since the kind of sequences from which such generalisations were drawn do not appear to be commensurate with the curriculum requirements for the learners at this level. In grade 11, learners are expected to investigate number patterns where there is a constant second difference between the consecutive terms i.e. the general term is quadratic. In #618 the teacher draws a valid

generalisation that if the first difference is constant this suggests a linear general term but when the teacher asks for justification why it is linear the class says;

Because its first difference is constant

While this generalisation is mathematically valid, there appears to be some redundancy in that it is using the same semiotic system to generate as well as justify the generalisation i.e. constant difference is being used as an indicator for a linear sequence which in turn is defined in terms constant difference. Radford (2010), posits that the sense of generality achieved through words and gestures is not the same as the one achieved through a formula or a graph. While one semiotic system may provide us with specific ways to signify or to say certain things, another semiotic system may provide us with other or even better ways of signification. Citing the principle of *nonredundancy* and with specific reference to pattern activities, he further argues that the objectification of a mathematical structure behind a pattern that was mediated by words and gestures may be deepened by an activity mediated through other types of signs. This seems to suggest that a different e.g. visual example like graphing the inputs and outputs of the sequence might have enhanced the learners' understanding of a linear sequence. In #863 the teacher makes a valid generalisation that any two equations generated from the relationships of variables in a linear sequence can be used to solve for the unknown variables. However it might not be as simple as any two because there has to be a strategic selection of the two equations that have to be used.

There are no utterances coded at the lowest level 0 again suggesting that this teacher was relatively strong in his own understanding of the concepts that he was dealing with in this series of lessons.

5.5.3 Episodes coded as indicating some connection through implication (IM):

A total of 12 utterances were coded at the highest level 2 of this category. In #533, #607, #612 and #1353 the argument being raised there is that if there is a first constant difference this suggests a linear general term. On the other hand in #870 if the first difference is not constant then all we can conclude is that the general term is not linear. In #945, the teacher guided the learners to argue that if the second difference is constant

then the sequence is quadratic and if the general term is quadratic #964, then $T_1 = a + b + c$. If the first difference in a quadratic sequence is 10 #1118, then $3a + b = 10$ and if the first term of this sequence is 3 in #1138 then $a + b + c = 3$. Lastly in #1373, the teacher guides the learners to argue that if the general term is consistent in generating the correct terms in a sequence, then it is correct. All these are sound mathematical arguments which were critical in laying a strong foundation for generating the general term for these sequences.

Only 4 utterances were coded at level 1 of this category. In all the first three cases #788, #816 and #820 these are follow through statements of the nature $6 – 2b + b = 5 \Rightarrow -2b + b = -1$. In #868 the teacher responds "*That's correct*" after the learners had suggested that if the first difference is not constant then the sequence is quadratic #866 and it is cubic #867. These statements lack mathematical precision in that if the first difference is not constant, all we can be able to tell is that the sequence is not linear but we can conclude that it is either quadratic or cubic.

There are no utterances coded at the lowest level 0 again suggesting that this teacher was relatively strong in his own understanding of the concepts that he was dealing with in this series of lessons.

5.5.4 Episodes coded as indicating some Procedural connection (P):

A total of 18 utterances were coded at the highest level 2 of this category. In # 104 for example the teacher is showing flexibility in the way learners could write the general term. In #168 the teacher is showing the link between the term number (input) and the value of the term (output). In #196, #201, #775, #855, #916, #1184, #1278, #1282, #1366, #1371, #1430, #1440 and #1643 the teacher is showing comprehensive procedures of how the learners could generate the general term of the sequences and how they could test or prove their correctness. In #2024 the teacher gives a comprehensive explanation why T_1 is a critical component of the recursive rule for a number sequence.

A total of 10 utterances were coded at the second level 1 of this category. In #37, #45, and #90 the main reason why these have been coded as P1 is that these are basic procedures which learners at this stage would have been expected to handle without the need for further articulation. In #386, #389 and #696 the way the teacher deals with the relationship of $a + b = T_1$ for a linear sequence and the proof of its general term thereof continues to look as though this is accidental. The mathematical logic of that relationship does not appear to come out explicitly from the teacher's talk. In #651 the relationship of *a = constant difference* with reference to a general term for a linear sequence perhaps needed further articulation in view of the potential it had to confuse the learners. In analytical geometry a linear function can be written in the slope-intercept form as $y = mx + b$ where *m* and *b* are constants. This also happens to be the form in which the general term for a linear sequence was written in this series of lessons. Written in this form the constant *m* is often called the slope or gradient, while *b* is the y – intercept. From the way a linear sequence grows e.g. 3; 7; 11; 15... Learners can notice that we are constantly adding a 4 (constant difference) to the previous term to get to the next term. This seems to suggest that when this same sequence is written in its algebraic form (general term) the constant *b* in the general term should be equal to this 4 (constant difference). However when one looks at the general term for this sequence, this in fact is not the case as can be evidenced that its general term is T*n* = 4n -1. There is evidence of a learner having this confusion in #314 when she said;

> *Yaah. Then why is it that you can't write like T_n = ... + 4? Why do you have to write – 1 that's my question?*

The teacher does not appear to deal with this genuine confusion adequately opting to say in #330 and #332

> *Okay, alright I thought you had made an observation. Okay order, alright okay, let's give somebody else a chance*

Despite this the learner insisted in #331,

> *Sir I do have an observation.....,*

However it would appear the teacher was not getting the learner's argument and therefore continued to ignore this critical disconnection between the two ways of representing a linear sequence right through the series of lesson. In # 1362 the teacher suggests that

learners have to divide both sides of –b = -7 by -1 in order to get to the final stage of working this equation out. While this is mathematically valid it might not be the only way of dealing with such a task because learners could also multiply both sides by -1. Dividing both sides of an equation by -1 proved challenging later on in # 1556 when the learner was presented with this task –c = 0 and wanted to divide both sides by -1 to make the c positive. It would appear it was easier to multiply -1 by 0 than to divide -1 into 0. In #1889 the issue is about the purpose that would be served by a recursive rule in a situation where the general (explicit) term for that sequence had already been found.

There are no utterances coded at the lowest level 0 again suggesting that this teacher was relatively strong in his own understanding of the concepts that he was dealing with in this series of lessons.

5.5.5 Episodes coded as indicating Instructional Oriented Connections (IOC):

A total of 27 utterances were coded at the highest level 2 in this category. In all the following utterances #5, #56, #74, #108, #130, #138 #148, #151, #159, #166, #205, #305, #746, #977, #1005, #1013, #1021, #1220, #1244, #1266, #1351, #1364, #1436, #1767, #1786, #2000 and #2018 the teacher is either offering justification or logical explanation as to why the relationships are the way they are or he is following up on learners' observations and guiding them through questioning so that they move from what they know to what they don't know.

A total of 4 utterances were coded at this second level 1 of this category. In #66 and #359 the teacher is agreeing with an observation made by a learner an indication that he is following what the learner is doing. However he does not offer at this stage some justification or further explanation as why there is this relationship between the constant difference in a linear sequence and the coefficient in the general term i.e. a linear sequence with a constant difference of 3 has a general term of the form 3n + b. In #386 the relationship of a +b = T_1 continues to look as though it is just accidental because the teacher does not at this stage offer some logical explanation as to why there is such a relationship in a linear sequence. In #1704 it appears this is just a common slip of the

tongue where both the teacher and the learners keep referring to a common difference of 3 between the terms as 3 times bigger. It is clear the teacher is following up on what the learners are observing but he does not seem to focus their attention on precision of mathematical statements. However it can be observed from the way the learners are proceeding with their arguments that they mean the next term is 3 bigger than the previous one although they say 3 times bigger.

A total of 9 utterances were coded at the lowest level 0 of this category. In #224, #226 and #233 there appears to be a haphazard order in which the tasks are presented by the teacher i.e. linear sequence, cubic sequence, and quadratic sequence then back to linear sequence - some of which are not in the curriculum at this level. This tended to inhibit learners abilities to identify patterns in these sequence as can be evidence in #227 and #230 where the learners say there is no pattern in the sequences. In #301 and #330 a learner makes an observation that there appears to be no connection between the constant difference in a linear sequence and the constant in the general rule that would generally be referred to as the y-intercept i.e. in the sequence 4; 7; 10; 13;… there is a constant difference of 3 but in its general term of $T_n = 3n + 1$ there is +1 and not +3. The teacher seems to ignore this genuine concern throughout the series of lessons. In #359 a learner suggests a recursive rule but the teacher deliberately or otherwise decides to ignore the learner. In #660 a learner makes an observation that suggests that in a linear sequence the constant difference is equivalent to the a of the $T_n = an + b$ (the general term) but in a quadratic sequence the constant difference does not appear anywhere. The teacher again does not appear to deal with this observation at all. In #1708 the teacher introduces the recursive rule long after the general term or explicit rule has already been dealt with and the question would be what purpose the recursive rule would serve other than just the learners knowing there is something called a recursive formula.

5.5.6 What key messages emerge from Teacher M's teaching of Number Patterns

The discussion of what emerges from Tr M's teaching is again guided by the following:

(a) What aspects of connections does the teacher appear to handle well?

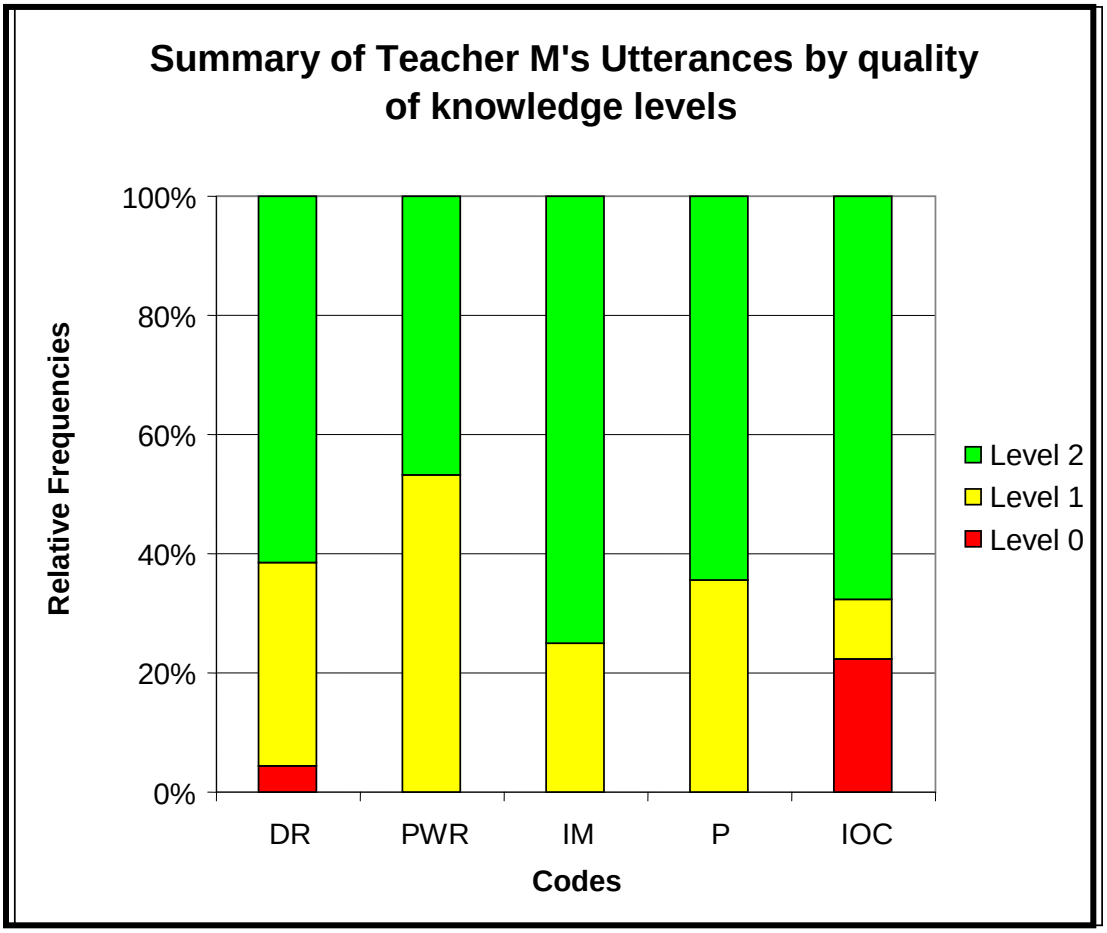
(b) What aspects appear to be problematic?

(c) What are the underlying features in each case?

(d) How do the strengths and weaknesses relate to the orientations in the curriculum?

Let me start by summarizing all the teacher's utterances/activities for the whole week into the different codes under which they were captured.

Fig. 5.11 Summary of Teacher M's Utterances by quality of knowledge



What this relative frequency graph seems to suggest is that teacher M is quite a strong mathematics teacher. This can be evidenced by the fact that most of his utterances (61% in DR category, 46% in the PWR category, 76% in the IM category, 64% in the P category and 68% in the IOC category) were coded in the highest level 2 of these respective categories. This suggests that the teacher created opportunities for learners' deep understanding in most of the utterances in this series of lessons. In fact in this series of teacher M's lessons there were no utterances which were mathematically faulty in the P category, the IM category and the PWR category. Only two utterances were

mathematically faulty in the DR category. Generally most of his definitions of mathematical terms were characterised by well thought out examples which clearly brought out mathematically accurate meanings. The strategies were justified and well explained and learners' observations were pr(OBE)d with questions that solicited clear articulation and reasons why. In terms of responding to curriculum expectations it could be argued that in the majority of cases the teacher utterances and activities had the potential to develop deep understanding of concepts and procedures related to the topic Number and Patterns.

While the teacher appears to be very strong one of the challenges that emerged in his series of lessons was the order in which his tasks were presented especially his decision to deal with the recursive rule towards the end of the week after he had dealt thoroughly with the general term or explicit rule earlier. There is empirical evidence to suggest that as an instruction oriented connection, the recursive rule should have been dealt with before the general term. For example, Warren (2006), posits that approaches used to find the general rule i.e. defining the growing pattern in relation to its position in the pattern, appear to fall into three broad categories. These are;

(a) Using one example to predict the relationship between uncountable examples (**induction or trial and error**).
(b) The additive strategy where connections among consecutive elements are exploited (**recursive** e.g. for each next step you add 3).
(c) Functional strategy where a relationship is formed between the term number and the term value (**general term** e.g. step number multiplied by 2 add 1).

These strategies tend to be hierarchical (Redden, 1996; Stacey, 1989; Warren, 2006) and once students perceive a pattern in a certain way, it is difficult for them to abandon their initial perception. Past research has shown that there is a propensity for students to use (a) one example to predict the relationship between uncountable examples and (b) the additive strategy by connecting consecutive elements (Warren, 2006). Research has shown for example that children's very first experience with sequences come along with the following type of numerical reasoning in which one is asked to fill in a missing number in a chain of numbers such as:

1, 4, 7, 10, _

The fact that each term exceeds the previous one by 3 is an example of a recurrence relation in which each term has a certain relation with the previous term(s). Each term of the sequence is a function of the preceding term. There is evidence from this series of lessons of learners using the differences as their entry point into understanding sequences. For example when learners were given this sequence;

1, 8, 27… and asked if they observed any pattern, the responses were:

#227 that's not a sequence,

#230 no pattern.

In fact the teacher had to abandon this sequence altogether for one reason or the other but immediately when he wrote; 1, 4. 9, 16… on the board the learners were able to visualize the sequence through the differences (3, 5, 7,) even though this sequence could also be seen in terms of squaring the term number. In #314 when the teacher was dealing with the general term of the linear sequence; 3, 7, 11, 15… there was clear evidence that learners continue to see sequences in the recursive as one learner commented;

> *Yaah. Then why is it that you can't write like T_n = ... + 4? Why do you have to write – 1 that's my question?*

This suggests that the teacher should have dealt with the recursive rule before he attempted to find the general terms of the sequences they were dealing with as recursive rules are easy to understand and it has also been shown that this is the general entry point into sequences. There is also evidence to show that learners were experiencing problems with the recursive rule especially after they had already learnt the explicit or general term. This seems to confirm the observation that once students perceive a pattern in a certain way, it is difficult for them to abandon their initial perception.

While recursive sequences are easy to understand, their shortcoming is that they are difficult to deal with. For example for one to get to say the 100^{th} term in a sequence; one would have to first find the first through to ninety-nine terms. The teacher alluded to this difficulty in #305 when he said;

> *… But there are times when it becomes very very difficult to obtain the general term by mere inspection…*

This then would have provided a justification for and an entry point into the general term of the sequence, which expresses the n^{th} term in terms of n and independent of the previous terms of the sequence.

5.6 Key messages emerging from all the four teachers

The discussion of the key messages emerging from the four teachers' teaching of algebra related topics is also guided by the questions:

(a) What aspects of connections do these four teachers appear to handle well?

(b) What aspects appear to be problematics?

(c) What are the underlying features in each case?

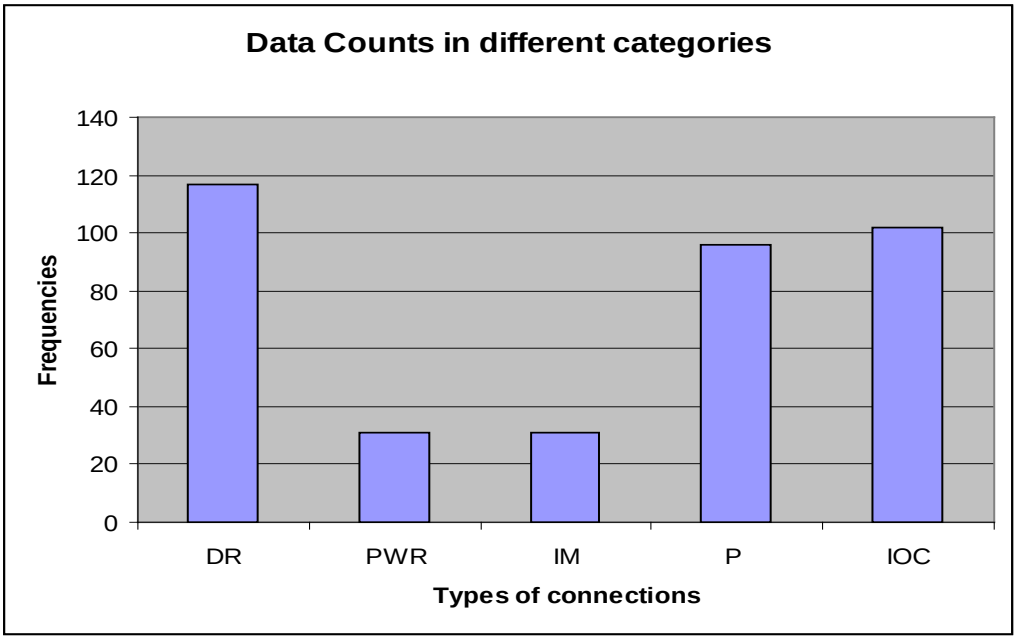
(d) How do the strengths and weaknesses relate to the orientations in the curriculum?

Table 5.9 shows a breakdown of all the utterances into the different categories into which they were coded.

Table 5.9 SUMMARIES OF ALL TEACHERS' UTTERANCES (n = 377)

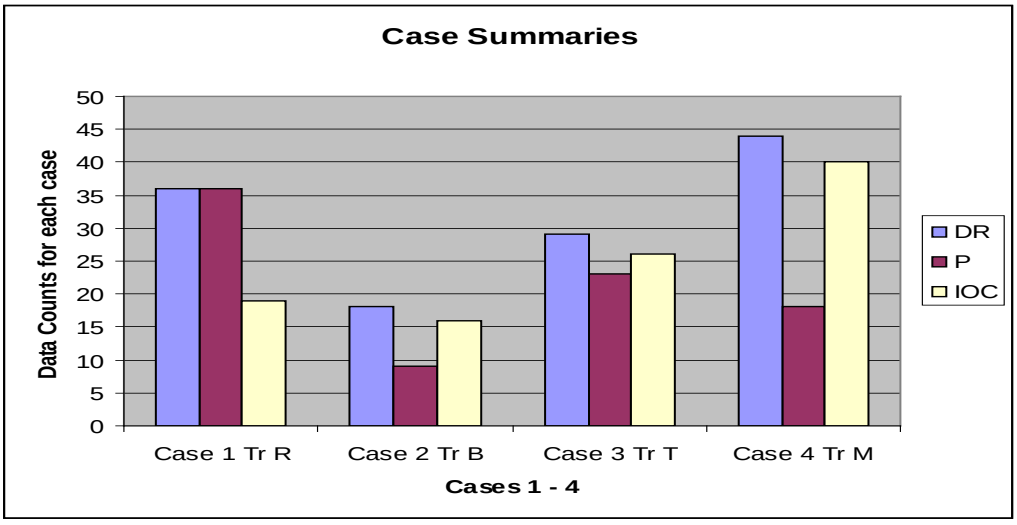
FORMS OF MATHEMATICAL CONNECTION	**LEVELS OF KNOWLEDGE QUALITY**				
	Code	**0**	**1**	**2**	**Total**
Different Representation	**DR**	**52**	**31**	**34**	**117**
Part-whole Relationship	**PWR**	**6**	**16**	**9**	**31**
Implication	**IM**	**13**	**6**	**12**	**31**
Procedure	**P**	**23**	**41**	**32**	**96**
Instruction-oriented Connection	**IOC**	**47**	**27**	**28**	**102**
Totals		**141**	**121**	**115**	**377**

Figure 5.12 Summary of data counts for all four teachers in different categories



What this summary graph reveals is that the highest number of the teachers' connections was created in the Different Representations category, followed by Instructional Oriented connections and Procedural connections respectively. This appears to be a consistent pattern when the teacher utterances were analysed individually as the figure 5.13 confirms.

Figure 5.13 Comparison of individual case summaries for 3 popular categories



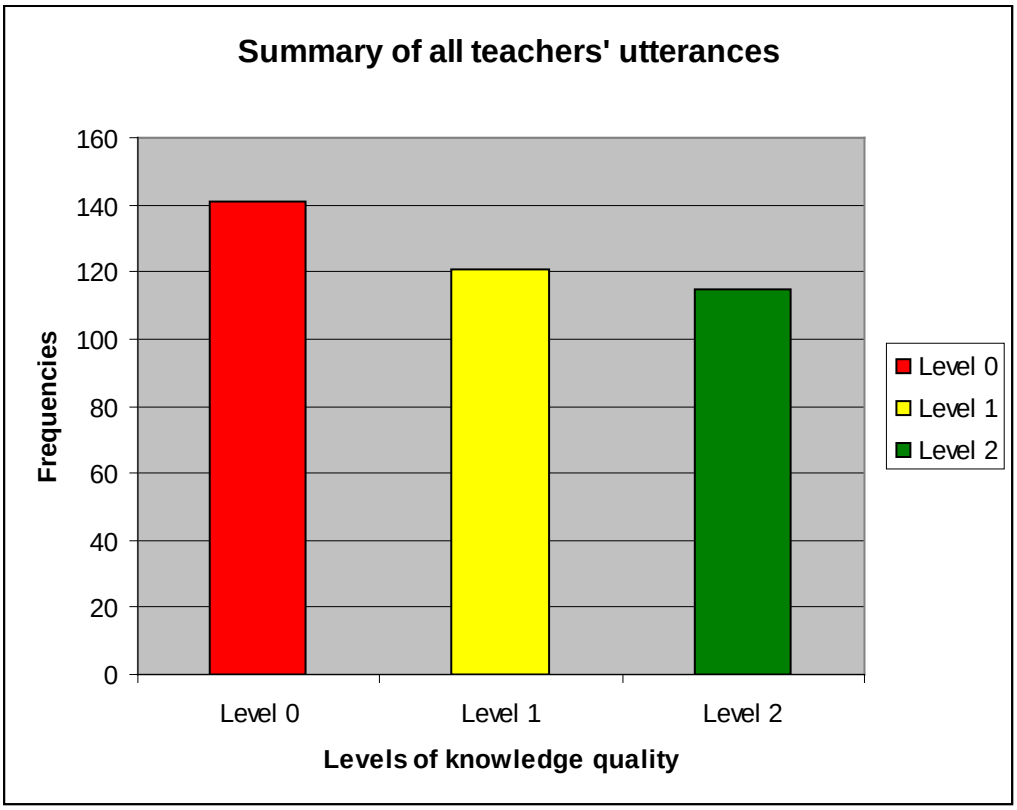
In each of the four cases one can notice that DR has the highest data counts or frequencies. With the exception of case 1 the second highest frequencies are, in all the other three cases, found in the IOC category. Although the study did not have a specific intention of investigating the relative occurrences of these different types of connections this still confirms the strength of case-study design i.e. its ability to capture the emergent and immanent properties of life (Noor, 2008). It also confirms the observation made by Zainal (2007) that case studies allow the investigation of contextual realities and the differences between what was planned and what actually occurred. In this study, these emergent results could be useful especially in planning staff development activities as they point to what teachers do more often in their classrooms.

The emerging findings that the DR, P and IOC categories have the highest number of data counts, are also consistent with earlier classroom observations which have shown that teachers are constantly engaged in a process of defining and constructing a mental image of some mathematical object and using instructional representations in the process (Businskas, 2008; McDiarmid, et al., 1989b). This development of consistent findings, over multiple cases can be considered a very robust finding for according to Zainal (2007), multiple-case design shows evidence through replication rather than sampling logic. By linking several pieces of information from the cases to some theoretical proposition, multiple-case design enhances and supports the results especially where a 'chain of evidence', either quantitatively or qualitatively, are systematically recorded. This also confirms the research assumptions that two or more cases should be included within the same study precisely because the investigator predicted that similar results (replication) would be found. According to Noor (2008), if such replication is indeed found for several cases, this enhances the accuracy, validity and reliability of the results hence we can have confidence in the overall findings.

Having said that about the emerging results, the key question this study raised was whether the NCSM as a standards-based reform was making a difference in the type and/or quality of instruction experienced by students. The major concern was with the quality of connections that teachers made during their interaction with the learners as

they taught algebra related topics. This was judged by levels of knowledge and the summary graph (Fig. 5.14) using figures extracted from Table 5.9 above shows the results of that comparison.

Figure 5.14 Comparison by quality of knowledge

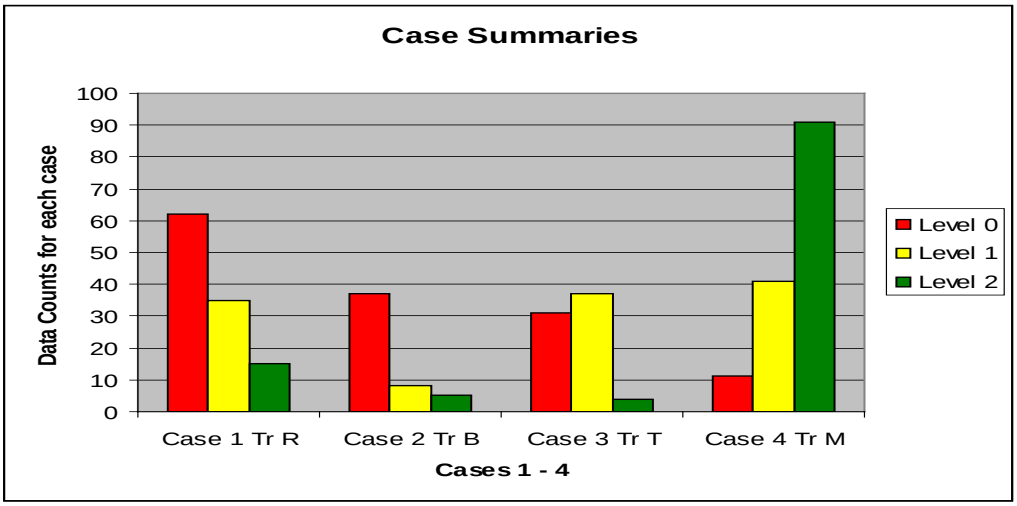


What this summary graph reveals is that the highest number of the four teachers' connections was either faulty (level 0) or superficial (level 1). Apparently the differences in heights amongst the three bars in the graph above might not reveal the magnitude of the problem, but when one considers that level 2 of cognitive demand should be the target of classroom practices then putting the level 0 and 1 bars together one can notice a cumulative 70% off target in the teachers' utterances and activities.

In chapter one this study argued that cognitively demanding knowledge and skills were essential for economic productivity and important for everyone irrespective of their backgrounds. Knowing the cognitive demand of the taught curriculum was also important because other things being equal, students in situations of poverty in particular, tend to have opportunities to learn only what they have been taught. So the cognitive demand of

the taught curriculum is a powerful predictor which helps explain a portion of the achievement gap between students from different backgrounds. Again I address the question of replication of these results by comparing the four individual cases in terms of this quality of knowledge criterion.

Figure 5.15 Comparison of level of knowledge quality by individual case



Looking at Fig. 5.15 one might notice that with the exception of case 4, the lowest frequencies in all the other three cases were again recorded in the targeted level 2 category indicating that minimal opportunities were created for learners to develop higher order thinking skills. The first two bars in each of the first three cases again confirm that those teachers' connections during classroom interactions were either faulty (level 0) or focused at a superficial level (level 1). A further disaggregation of the different categories of connections from Table 5.9 shows that the highest number of faulty (level 0) or superficial (level 1) connections was made again in the DR category followed by the IOC category then the P category. Capturing the importance of all these three forms of connections in conceptual development, one summary of research concluded that the five components used by successful teachers to help students develop mathematical ideas are: attending to perquisites, developing relationships, employing representations, attending to student perceptions, and emphasising the generality of mathematical concepts (Good, Grouws, & Ebmeier, 1983).

5.7 Summary of emerging messages

I will now consider the role played by each of these categories of teacher knowledge in effective classroom practice. Starting with different representations, literature suggests that instructional representations play such an important role in the development of student understanding (Ball, 2003; NCTM, 2009; Shulman, 1986). While there are many examples of representations in literature, with some researchers viewing representations synonymously with the whole process of teaching, in this study representations were viewed specifically in line with Lesh et al., (1987) as an aspect of the process of teaching which provides multiple perspectives for the same concepts or mathematical ideas and these included verbal/written representations of a mathematical idea, symbolic representations such as equations and formulas, graphical representations, manipulatives, and visual representations such as pictures and diagrams. From this perspective, instructional representations can be described as the words, pictures, graphs, objects, numbers, symbols, and contexts (including examples, metaphors, and analogies) that teachers use during instruction to communicate abstract mathematical ideas to students (Berenson & Nason, 2003). Goldin (2002) referred to this view of representations as the signs, objects or actions that symbolize, depict or encode something other than itself.

Empirical evidence suggests that this activity of representing is considered a fundamental and core activity of teaching mathematics (Ball2001) because the ways in which mathematical ideas are represented is fundamental to how people understand and use those ideas (NCTM, 2009). Shulman (1986) also describes the uses of representations within the category pedagogical content knowledge (PCK):

> Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others (p. 9).

Literature also suggests that teachers need to know a variety of instructional representations to use during instruction. Wilson et al., (1987), term this variety of representations as a *representational repertoire* - that consists of the metaphors,

analogies, illustrations, activities, assignments and examples that teachers use to transform the content for instruction. The appropriate use of multiple instructional are considered crucial in students' conceptual development of fundamental understanding (Post & Cramer, 1989). Empirical evidence has shown that effective teachers of mathematics use mathematically appropriate and comprehensible definitions, representing ideas carefully, mapping between a physical or graphical model (Ball, 2003). Because representations of mathematical ideas are so important to conceptual development (Ball, 1991), these representations should be carefully developed but the findings in this study generally point to problems in the manner in which these three of the four teachers represented the mathematical ideas and concepts within the algebra related topics that were observed. Within the South African research terrain Davis and Johnson (2007) also made similar observations that most of classroom time was spent on the exposition of mathematical ideas, principles and definitions within which "mostly teachers briefly referred to definitions but without discussing or explicating the mathematical reasons for the productions of the definitions" (p. 123). Their conclusion was that there was a 'weak curriculum coherence' which could be predicated in their language as the extent of teachers and students operation on grounding ideas, principles and definitions of the contents of a field of knowledge when teaching and learning (Z. Davis & Johnson, 2007). Their findings also concurred with the TIMSS findings that this weak coherence, which limited students' opportunities to learn, persistently characterised many of the system's high-poverty schools (Smith, Smith, & Bryk, 1998). Given that the instructional representations that students encounter define the formal opportunities for learning about the subject content, findings from my study show that opportunities for learners to develop deep understanding of both mathematical procedures and skills were likely to have been lost in the majority of cases.

With reference to procedural connections, in this study connections of this form were defined as A is a procedure used when working with B. Linking representations and procedures, Davis (2004) argued that the resource that enabled one to monitor correct rules or procedures was the knowledge of the mathematical ideas, principles and definitions that function as grounds for those rules and if teachers appear to be providing

students with a great deal of information on what (mathematical ideas and concepts), but students still fail, then it is likely that they are not providing sufficient information on how to do so (the procedure). This suggests that conceptual knowledge (understanding the "what and why") is important for the development of procedural fluency, while fluent procedural knowledge supports the development of further understanding and learning. In that sense, production of a solution to a standard problem requires that one knows both what to do (the mathematical idea or concept) and how to do what needs to be done (the procedure). Unfortunately knowing what to do does not necessarily imply knowing how to do what needs to be done and judged by Table 5.9 which shows that the third highest number of faulty (level 0) or superficial (level 1) connections was made in the P category one could possible infer that it was likely that the teachers observed were not providing sufficient information on how to do the mathematical solutions (the procedure).

With reference to instruction oriented connection, in this study this form of connection was defined in terms of how A and B are both prerequisites concepts/skills that must be known in order to understand C. This form also includes linking new concepts to prior knowledge or extension of what students already know. The importance of taking account of students' ideas is captured in Ausubel's (1968) statement that the most important signle factor influencing learning is what the learner already knows. Fostering better understanding in students requires taking time to attend to the ideas they already have, both ideas that are incorrect and ideas that can serve as a foundation for subsequent learning. Resnick (1988), concluded that without explicit assistance in connecting ideas or procedures people do not usually learn concepts simply by building up pieces of knowledge. His view was that unless materials attend to students' prior knowledge and teachers are alerted to it, the sequence of activities might be inappropriate and further misconceptions may develop or achievement will be diminished partially due to persistent errors. Empirical evidence suggests that students learn efficiently when their teachers first structure new information for them and help them relate it to what they already know, then monitor their performance and provide corrective feedback recitation, drill, practice, or applications activity (Peterson & Leatham, 2009).

CHAPTER SIX – DISCUSSION AND CONCLUSION

6.1 INTRODUCTION

This study was particularly concerned with understanding the levels of alignment between the written, the tested and the taught components of the NCSM with respect to the depth-of-knowledge consistency criterion. This was premised on the view that cognitively demanding knowledge and skills were critical for economic productivity and development. The focus on cognitively demanding knowledge and skills was triggered by concerns raised that school expectations had declined in terms of cognitive demand and that this threatened the learning health of the nation especially those from previously disadvantaged communities. Within this broader concern for alignment three types of alignment were investigated i.e. horizontal alignment, developmental alignment and vertical alignment. Literature on alignment suggests that before any alignment analysis can be done, the starting point should be answering the question: 'What performance problem, quality improvement, or evaluation criterion is to be analysed?' This then led into the first research question of this study.

6.2 RESEARCH QUESTION 1

The question was: "What levels of cognitive demand are evident in the mathematical knowledge and skills as articulated in the written curriculum?" In attempting to answer this research question, content from the Grade 11 Subject Statement and the Assessment Standards for mathematics was placed into categories of a cognitive demand tool. Using a range-of-knowledge criterion to guide the data counts first with respect to learning outcomes, the results showed that LO2 (Functions and Algebra) had the highest number of entries. Judged by that range-of-knowledge criterion it was possible to conclude that the NCSM places more emphasis on LO2 (Functions and Algebra). Within the written curriculum, the Subject Assessment Guidelines also confirm this position evidenced by the rubric on distribution of final examinations marks where 70% of the Paper 1 marks

are allocated to LO2 (Functions and Algebra). International literature also evidence to justify the bias towards functions and algebra. These findings then guided the research further into classroom observation where the focus would be on this LO2 as it was considered to be relatively more important in the NCSM.

Data counts within the cognitive demand tables and in relation to higher order – lower order levels revealed these were distributed in the ratio 4:1 respectively. Judged by this finding it can be argued that the espoused quality of knowledge of mathematics in the NCSM as reflected through the cognitive demand tools is the higher-order cognitive knowledge and skills. This skewness towards higher-order cognitive knowledge and skills was more pronounced in LO3 Space, Shape and Measurement. Both positions of the NCSM espousing higher order knowledge and the skewness in LO3 were also confirmed by other researchers who argued that the curriculum for mathematics targeted the development of higher-order cognitive knowledge and skills in the learners (Umalusi, 2009).

6.3 RESEARCH QUESTION 2

The second question focused on horizontal coherence and concerned the issue of how this official position was articulated through the different curriculum documents. It was phrased thus; “To what extent are the written and tested components of the NCSM aligned in terms of cognitive demand levels?” Within this broader view of horizontal coherence, the analysis started by looking at the internal consistency of content within the written components of the curriculum. The concern was whether or not the content was developed based on a progressive or hierarchical structure consistent with the logical nature of mathematics as a discipline. Judged by the sequential development of the assessment standards from Grade 10 through Grade 11 to Grade 12, the results showed that there was logical progression consistent with the logical nature of mathematics. Judged by the cognitive demand of content within each grade and from one grade to the next, the results also show that there was developmental coherence of a hierarchical nature from lower order to higher order as the assessment standards progressed within grades and from one grade to the next.

Having looked at the different aspects of internal consistency, the second view of horizontal coherence as defined in this study was specifically concerned about external consistency i.e. the level of alignment between the components of the written curriculum with the exemplar papers in terms of cognitive demand. Using a combination of cognitive demand tools overall low alignment indices of 0.1 and 0.2 were established from two different formulae. The results from more detailed analyses show this low alignment index was a result of exemplar papers placing more emphasis at the lower order levels of memorizing and performing procedures while the content standards placed less emphasis at these lower levels but more on higher order skills of generalising, conjecturing and solving non-routine problems. In view of the fact that this alignment index can best be described as a measure of relative emphasis, it can be argued that there is some disparity between what the NCSM assessment standards emphasize and what is being emphasized in the exemplar papers.

With specific reference to horizontal coherence it could therefore be argued that the documents that constitute the NCSM seem to be presenting a coherent message internally but a splintered vision externally. In other words the espoused HOCS seem to be articulated consistently through the policy documents but differently especially between the assessment standards/objectives and exemplar papers. While HOCS are treated as a critical outcome in the subject statement, and espoused through the assessment standards, the national exemplar papers seem to downplay that orientation. The final analysis could be that so far 'the DNA evidence' of the assessments seems to 'belie the claim' that learners will learn HOCS in mathematics.

6.4 RESEARCH QUESTION 3

The third and final research question for this study was concerned with how policy rhetoric translates into practice. In view of the findings from document analysis that higher order knowledge and skills were espoused in the written/intended curriculum the question which had to be answered was; "To what extent do Grade 11 mathematics teacher practices create opportunities for pupils to learn higher order cognitive processes

and skills?" Using an analytical tool developed around five types of connections that earlier research had identified as being prevalent in classroom interactions; the summarised results for all the four teachers show the following distribution of teacher utterances/activities/episodes in their order of prevalence: Different Representations 31%, Instruction Oriented Connections 27%, Procedural Connections 25%, Part-Whole Relationships 8% and Implications 8% respectively. This seems to be consistent with similar research findings that have shown that teachers are constantly engaged in a process of defining and constructing a mental image of some mathematical object and using instructional representations in the process. Given that the instructional representations that students encounter define the formal opportunities for learning about mathematics, and that effective teachers of mathematics have to use mathematically precise and comprehensible definitions, represent ideas carefully mapping between the different modes the question then was of what quality these representations and instruction oriented connections were. A further disaggregation of the relative frequencies within each category showed that 44% of the Different Representations were at level 0 (faulty) and 27% of them were at level 1 (rote/superficial) leaving only 29% at level 2 (higher order or deep understanding). Within the Instruction Oriented connections 46% of them were at level 0 (faulty) and 26% of them were at level 1 (rote/superficial) leaving only 28% at level 2 (higher order or deep understanding). Within the Procedural category 24% were at level 0 (faulty) and 43% of them were at level 1 (rote/superficial) leaving only 33% at level 2 (higher order or deep understanding). These disparities were more pronounced when the disaggregation was done for each individual teacher showing that in some cases different representations appeared faulty in as high as 80% of the teacher's utterances and 10% restricted or rote, leaving only 10% of the utterances with a potential to develop learner's deep understanding of concepts and procedures. This trend was observed in 3 of the 4 teachers and what appears to have pushed the averages up was that in one teacher different representations fell into level 2 (higher order) in 61% of the cases and level 0 (rote/superficial) in 0,5% indicating generally that this was a very strong teacher when compared with the other three. Overall what these results show is that within the series of lessons by the four teachers that were observed in this study

opportunity for pupils to learn higher order cognitive processes and skills was not created in the majority of the teacher utterances and activities.

6.5 IMPLICATIONS FOR IMPLEMENTATION

In trying to establish whether assessments are aligned with the curriculum objectives, the question this study was trying to address was to do with the validity of the testing instruments in relation to the higher order knowledge and skills that are espoused in the curriculum. While there are divergent views on issues of test validity in public examinations, there seems to be some consensus on the backwash effects of such tests. Several studies have shown that high-stakes testing encourages teachers to use methods that conform to the content of the test, which in itself is not a problem according to Biggs (2003). Viewed in this sense, tests that measure complex concepts and extended reasoning are likely to encourage stimulating instruction which result in higher-level thinking and problem solving skills being developed in the learners. The danger however comes when the test measures only simple knowledge and skills. In such cases teaching to the test, as certain studies have shown, results in superficial rote learning and an inevitable deskilling of learners.

In view of this observation that teaching to the test is inevitable, it would mean that alignment or nonalignment between the curriculum objectives and assessment has a number of implications for all the stakeholders. As long as test validity is conceived of in terms of the constructs embodied in the broad curriculum objectives there is little problem because then the tests are measuring what the learners are expected to learn and the public can have confidence that the grades so obtained are likely to be a true reflection of knowledge and skills gained. Nevertheless, when test items are prepared which reflect noncontent objectives, or are nonaligned with the objectives then the validity of these items is questioned. To the extent that flaws in validity are built into the system in order to serve the interests of particular groups, it becomes very difficult to validate that learners learnt what they were supposed to learn.

Students' grades in such high stakes examinations then tend to be misleading the public because of this lack of alignment. More than any other groups, the various institutions of higher education make direct use of public examination results. Students are admitted to these institutions almost entirely on the basis of their performance in public examinations. With specific reference to courses that require mathematics as an entry requirement, if higher order cognitive skills are not tested in the examination, both students and institutions of higher education could be misled by public examination results leading into wrong career choices. This can have devastating effects especially on the part of the students in that both time and other resources are lost when they find out that they cannot cope with the demands of a course which they sincerely believed they were capable of taking. Such students have been known to loose self-confidence and the whole purpose for life. Consequently, higher education institutions have also been known to have insurmountable challenges of having to counsel such students or having to design bridging courses and such other strategies to enable the students to cope. In most cases through-put rates for graduates in such courses tend to drop significantly when compared with the enrolment for first year study. This is not then solving the initial problem of lack of human resources in areas such as Actuarial Sciences, Engineering, Accountancy and Teaching just to name a few.

In terms of impact on curriculum implementation, the view is that there is an organized body of knowledge that students need to know so that society might cohere around a common identity (Crocker, 1991). Assessment is then seen as an important method to determine whether these skills were being learned and if not, it proved the need to return to a 'back to basics' approach to ensure they were being assed and in turn being taught. The impact of assessment on the whole system of education can therefore be seen in that the curriculum is usually reduced to that which can be or has been tested, and this becomes a self-perpetuating loop when what is assessed becomes what is valued, which then becomes what is taught. Hence researchers have argued that no effective curriculum change can take place without changing the assessments regime.

6.6 RECOMMENDATIONS

In terms of making long term recommendations from such alignment studies, Liang & Yuan (2008), cautioned that any examination paper is a sample or snapshot of all that could possibly be tested. The suggestion is that alignment analyses should be an ongoing activity as this constitutes part of quality assurance in the curriculum development process. However, in the short term it is important to note that every public examination has an immediate labeling effect on that particular cohort of learners being tested. Seen in this sense then, the issue of quality assurance in every single public examination paper becomes paramount. Those charged with this responsibility of testing should therefore be cognizant of the fact that if learners are expected to learn higher order cognitive skills then it is imperative that public examinations should test such skills. If not, then that validation process becomes flawed and the public could be misled by the results thereof.

With specific reference to results obtained of alignment using categorical-concurrence criterion, LO3 Space, Shape and Measurement had the highest discrepancies indicating the skewness of the exemplar papers, which was also confirmed by the Umalusi analyses. Such discrepancies are however temporary in that they point to differences in relative emphasis specifically between the 2008 exemplar papers in relation to the curriculum. Making long term recommendations from such discrepancies is therefore constrained unless the trend is observed over a longer period of time. In such cases what continues to be tested becomes the taught curriculum in the classrooms because of the wash back effects. Generally such discrepancies have a short term implication as they help point to areas that need to be revisited when the next tests are being developed. The recommendation would be that such categorical-concurrence analyses should form part of quality assurance and be carried out each time an examination gets set or gets written.

This alignment study has also shown that the NCSM appears to lack specificity on the mathematical content knowledge, abilities and understanding that are needed for learning to progress at each stage of the process. It could also be one of the reasons why there is a low alignment index between the assessment standards and the exemplar papers. In the

absence of the specification of content policy rhetoric might also be constrained in claiming developmental coherence or progression between grades. Lack of content specificity also poses problems for teachers in terms of implementation and it also makes it difficult for one to measure the extent to which learners are achieving curriculum objectives. All these shortcomings point to the need for the curriculum to be specific in terms of content for the benefit of all stakeholders.

In terms of pedagogy, this study does not make any immediate recommendations for practice as there is still a need to do further investigations in order to establish the extent of the prevalence of teacher practices that were observed in this study.

6.7 CONTRIBUTIONS

One contribution that I perceive as coming from my study is the 3 level-model (table 3.4) for analysing mathematical connections that teachers make during their interactions with learners in their everyday practices. In developing this tool for identifying mathematical connections in the teacher utterances and activities, this study first analysed a number of views about mathematical connections from different perspectives. What was evident from literature was that relationships, hence connections were at the heart of the definition of mathematics and that many countries make mathematical connections among the key elements of the learning outcomes and experiences to be gained by learners. However, I found two critical gaps in this literature that my model could possibly fill. Firstly the teachers who took part in studies that suggest that making connections positively impacted on students' opportunities to learn higher order skills, were exemplary mathematics teachers suggesting that both their subject matter knowledge and the connections they enabled were of high quality. In such cases the tools used to capture such connections work on this presumption that the teachers' subject matter knowledge is of high quality hence quality of the connections they enabled is assumed likewise. Businskas (2008), model which formed the basis of my model would be a good example as it talks about different types of connections without any mention or focus on the quality of such connections. Yet in the South African classrooms teachers' subject matter knowledge is reported to be generally weak. I argue that making

mathematical connections during mathematics class positively impacts students' opportunities to learn higher order skills only if the connections are not faulty and are not superficial but mathematically precise and conceptually deep. Considering that literature also suggests that studying teachers' pedagogical efforts to promote the making of mathematical connections necessitates considering the intersection of three frameworks – their subject matter knowledge, their general pedagogical knowledge and their specific pedagogical content knowledge (Shulman, 1986), it was important for this study to develop a model that would capture teachers' subject matter knowledge in relation to the different types of mathematical connections hence I saw the need to build a depth-of-knowledge criterion onto each mathematical connection that was identified by Businskas (2008) before.

The second gap I perceived was that much of the mathematics education literature that describes such mathematical ideas like defining, generalising and making connections does so mostly from a process perspective which does not allow judgment of quality of such definitions, generalisations and making connections in practice. In other words 'defining' is regarded as a lower order process in literature while generalising is regarded as a higher order process. But in practice a definition or a generalisation can be faulty, can be correct but without further articulation, or it can be accurate and well articulated with examples. These levels of quality have an impact on the learners' ability to develop higher order or lower order levels of the connections. Hence I argue that especially in the South African context, there is need for a tool that can capture classroom interactions taking cognizance of all these quality factors. It was from combining contributions from literature on mathematical connections with that on depth-of-knowledge levels, that this study then struck some balance between these many models and developed its own 3 level coding scheme as shown in table 3.4. I see potential in my model to capture such quality in these representations and instruction oriented connections which researchers need especially in the South African classrooms where teacher's subject matter knowledge is reported to be generally weak.

6.8 LIMITATIONS

One of the limitations that I perceive in my study is that document analysis was based on a small sample of the curriculum due to the fact that both the subject statement and the assessment standards lack content specificity. The comparison between the written curriculum and tested curriculum was based primarily on process descriptors of cognitive demand, especially in the written curriculum, such as 'learners will solve non-routine problems'. In such cases the precision in the alignment index might be compromised. However the low alignment between the written curriculum content and the tested curriculum as reflected in the examination papers has been confirmed in other studies - hence the generalisability of the findings with respect to the low alignment between the written curriculum and the tested curriculum might be considered credible.

A second limitation this study was its focus on only one set of the 2008 exemplars, which in turn would limit the generalisability of the alignment indices. Another limitation was the comparison between the Grade 11 curriculum content with Grade 12 exemplar papers posed by the need to trace the written Grade 11curriculum into practice yet Grade 11 is not an examination grade hence no external examination for it. However it has been argued that the three FET grade 10 – 12 curricula are designed from the same process descriptors suggesting that similar results would still have been obtained by comparing any of those three written curricula with the 2008 Grade 12 exemplar papers. This argument can be supported by the fact that similar follow up studies comparing the Grade 12 written curriculum with the same 2008 exemplar papers together with the actual 2008 and 2009 examination papers found similar (low alignment) results thereby adding credibility to the findings of my study.

With specific reference to empirical data from lesson observations, only four teachers were observed teaching and so the generalisability of the findings is only limited to the cases that were studied. This is the nature of case studies. However, the findings lay a foundation for what could be considered in future research especially when one considers

that studies done elsewhere also pointed to teachers focusing more on rote learning of both concepts and procedures and less on procedural and conceptual understanding.

6.9 FUTURE RESEARCH

Moving forward from such an alignment study I see two complimentary possibilities
(a) Extending this study to other areas in order to determine the extent of the prevalence of teacher practices that were observed in this study. This would help to focus the design and development of teacher development programmes.
(b) Running concurrently with the extension of the study into other areas I also see potential in carrying out alignment analyses of the pre-service teacher training programmes and teacher development strategies in order to determine the extent to which these programmes are in line with the expectations of the new reforms.

6.10 REFLECTIONS

In the process of collecting classroom data one thing that struck me throughout the series of lessons was the way teachers followed textbook tasks in the specific order they were presented even where such order might have disrupted the logic inherent in the mathematical idea being focused on. For example in one of the lessons on number patterns the first task was focusing on this linear sequence 3, 5, 7, 9…. and the following task immediately after that was focusing on a cubic sequence 1, 8, 27, 64…Although the terms linear and cubic had not been introduced at this stage, in the context of trying to find a general term for these sequences such an order might not have been logical. To me this raised questions about trimming as discussed in chapter 2 and 3. Trimming includes the interpretation and judgment a teacher must use in considering a textbook's treatment of a mathematical concept or process. The question I remain asking myself after this study is "Do mathematics teachers ever question the order or content of textbook tasks?"

6.11 CONCLUDING REMARKS

In the concluding remarks I go back to Bernstein (2000b) who notes that when discourse moves from the original site of production to a new position a transformation happens. In

this study this transformation appears to be highlighted in the way the written curriculum documents consistently send the message that higher order cognitive skills are espoused yet on the other hand both the pedagogy (taught curriculum) and the examination papers (evaluation of the curriculum) would appear to send a different message i.e. lower order cognitive skills appear to be tested in the examination papers and taught in the mathematics classrooms. This seems to resonate with Bernstein's argument that disciplinary knowledge does not equal the educational knowledge of that discipline because the process of production and transmission of knowledge may well have contradictions, cleavages and dilemmas created between these fields.

In Bernstein's analysis of this process he was concerned with more than the description of the production and transmission of knowledge; he was concerned with the question of education and inequality that form the original basis of current curriculum reforms in many countries. In conceptualising the research questions for this study, I showed how the written curriculum's vision is to address issues of inequity and exclusion by promoting the development of high skills throughout the school-leaving population but as this study showed, both teachers and tests (examinations) constrain the trasmission of such skills hence I argue that the interests of these school-leaving citeznes are not likely to be served due to the contradictions caused during the recontextualisation process.

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