



A genetic algorithm for temporal and spatial alignment of long- and medium-term mine production scheduling for open-pit mines

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ABSTRACT

Open-pit mine production scheduling essentially comprises long-term (LT), medium-term (MT) and short-term (ST) schedules, which have typically been optimized in isolation to each other. However, by independently optimizing these schedules, temporal and spatial scheduling misalignment between consecutive schedules occurs and can lead to lower mining project net present values (NPVs). Therefore, it is important to integrate these schedules for improved scheduling alignment. Accordingly, this paper developed a mixed integer programming (MIP) model that integrates LT and MT production schedules to improve scheduling alignment between LT and MT schedules compared to separately optimizing the schedules. The model was solved using a genetic algorithm (GA), which is a stochastic algorithm. The combined MIP model and GA approach was tested on a Geovia Surpac® block model and generated a 2.20 % higher NPV than for the isolated LT schedule. Using the same input parameters on MineLib, the approach was validated by comparing its results to the best-known feasible linear programming (LP) relaxation solutions obtained using a TopoSort heuristic algorithm. For the Newman, Zuck Small and KD block models, the approach generated comparable NPVs, which were 4.90 % lower, 10.90 % higher, and 2.36 % lower, respectively. However, for the four block models, the approach achieved 100 % temporal alignment between LT and MT production schedules, while the isolated schedules had temporal misalignment ranging between 86.22 % and 105.47 %. Therefore, this paper's contribution is on incorporating temporal and spatial alignment between LT and MT production schedules to achieve LT objectives at the MT horizon.

1. Introduction

The open-pit mine planning process broadly consists of two components which are mine design and mine production scheduling. The design of an open-pit mine is based on determining the ultimate pit limit (UPL) as the first optimization step of the mine planning process (Askari-Nasab and Awuah-Offei, 2009). The UPL is determined by maximizing the undiscounted value of a mineral deposit subject to constraints such as block precedence constraints (Askari-Nasab and Awuah-Offei, 2009). The open-pit mine production scheduling (OPMPS) process determines the best extraction sequence subject to production scheduling constraints, while concomitantly minimizing operational costs and risks associated with mining activities (Otto, 2019). The production scheduling constraints include geological, economic and operational constraints. OPMPS seeks to maximize the net present value (NPV) of a mining project by defining the sequence in which each

material parcel within the UPL should be extracted and sent to different destinations such as stockpiles, processing plant or waste dumps (Behrang et al., 2014; Muke et al., 2021; Fathollahzadeh et al., 2021). Open-pit mine planning is conducted at essentially three stepwise planning levels namely, strategic or long-term (LT); tactical or medium-term (MT) and operational or short-term (ST) horizons. The corresponding scheduling horizons are LT, MT and ST production scheduling, which are often optimized in isolation to each other.

LT production scheduling aligns with a company's strategic objectives in defining an optimal LT extraction sequence presented on a yearly basis over the life of mine (LOM) horizon (Osanloo et al., 2008). The LT production schedule is the basis from which the MT production schedule is developed. The MT production schedule is characterized by increased granularity of periods (half-yearly, quarterly or monthly basis) and more detailed information but presented over a one-to five-year horizon (Osanloo et al., 2008). MT production scheduling is the basis

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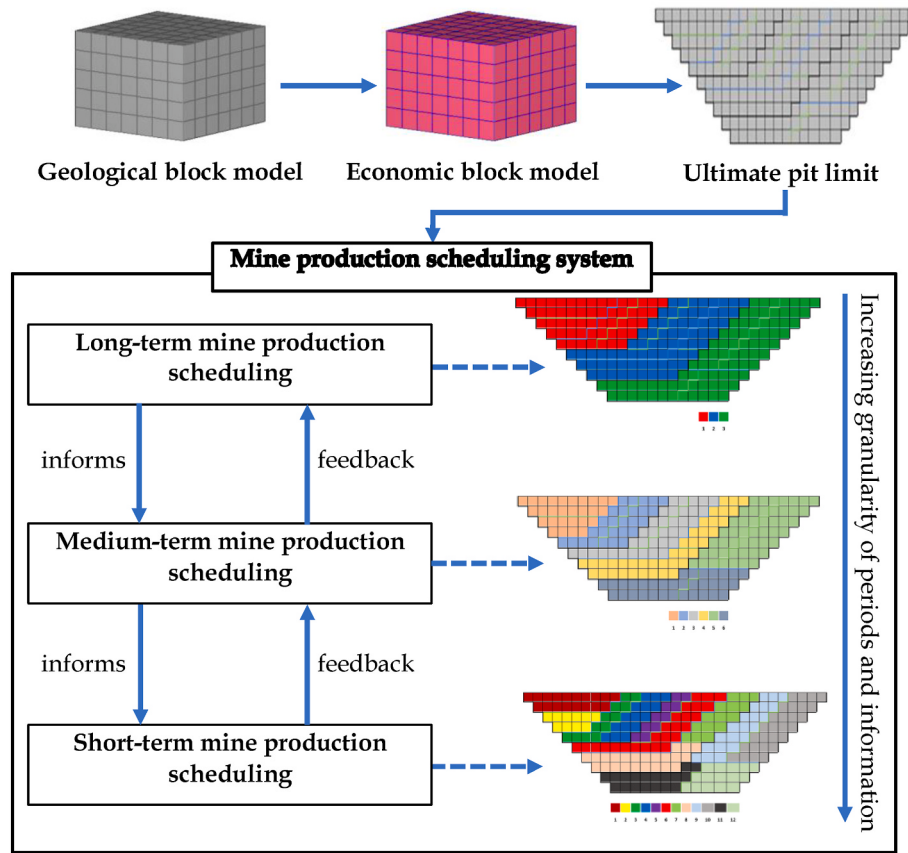


Fig. 1. A high-level generic open-pit mine production scheduling system highlighting the connections between LT, MT and ST production scheduling stages (Muke, 2025).

from which ST production scheduling is subsequently developed. The ST mine production schedule breaks down the MT extraction sequence into either monthly, weekly, daily or shift schedules that are more detailed and contain additional operational requirements (Levinson and Dimitrakopoulos, 2023). Osanloo et al. (2008) suggested that a ST schedule should be presented up to a one-year horizon and must be aligned to the MT production schedule, which in turn must be aligned to the LT production schedule. However, due to complexities and uncertainties encountered in transitioning from one scheduling horizon to the next, changes in schedules must be implemented through feedback loops connecting successive horizons, thus, requiring continual updating of schedules (Otto, 2019). Fig. 1 illustrates a high-level generic open-pit mine production scheduling system linking LT, MT and ST mine production scheduling, including feedback loops.

OPMPS is a complex and dynamic optimization problem because it involves many variables and parameters that have inherent uncertainty (Campeau et al., 2022). The uncertainty is associated with geological, operational and economic factors. Therefore, optimization techniques used to solve the problem should consider this inherent uncertainty. Stochastic optimization techniques are now being increasingly used to solve the OPMPS problem as they incorporate uncertainty in the solution process (Lamghari and Dimitrakopoulos, 2018). Among stochastic techniques, metaheuristic techniques are increasingly being used to solve stochastic problems (Yang, 2011). Metaheuristic techniques aim to find optimal or near-optimal solutions across a large search space and often achieve this aim much quicker than exhaustive deterministic or exact techniques (Gandomi et al., 2013). Solution procedures of metaheuristic techniques involve iteration and employ stochastic operations to modify initial candidate solutions obtained through random sampling from the solution search space (Bandaru and Deb, 2016).

Metaheuristic techniques have proved to be efficient in solving the

OPMPS problem, but have mostly been used to separately optimize LT, MT and ST production schedules (Alipour et al., 2022; Azzamouri et al., 2019; Upadhyay and Askari-Nasab, 2018). When production schedules are optimized in isolation to each other, this can lead to the creation of sub-optimal and misaligned production schedules that can result in mineral resource sterilization, reduced economic value, missed production targets, increased costs, and missed opportunities for optimization (Levinson and Dimitrakopoulos, 2023; Lamghari and Dimitrakopoulos, 2018). To address this shortcoming, it is essential to develop an integrated production scheduling system that seamlessly links the three scheduling stages to avoid scheduling misalignment both temporally and spatially. Otto and Musingwini (2020) presented a framework that utilized manual feedback loops for improving the intertemporal and spatial alignment in the execution of mine plans at the three different LT, MT and ST horizons for an open-pit mine. However, their work did not focus on production scheduling optimization but provided useful practical insights on the feedback loop concept for linking the LT, MT and ST production schedule horizons. Levinson and Dimitrakopoulos (2023) presented an optimization approach that directly integrated LT and ST mine production scheduling with reinforcement learning and stochastic programming. However, by skipping the MT schedule, a smooth transition from LT to ST scheduling cannot occur because mine production scheduling is a sequential and structured process from LT to MT and to ST production scheduling. In addition, the performance metrics for LT and ST production scheduling are not easily aligned because ST production scheduling tends to focus on minimizing deviations from production targets and efficiencies, while the LT production scheduling focuses on NPV maximization. Since LT and MT scheduling both share NPV maximization as a performance metric, it is important that the two scheduling stages must also be integrated. The preceding observations indicate that MT production scheduling, which

Table 1

List of indices used in the MIP scheduling model.

Index	Description
i	Mining or source location, $i \in \mathcal{M}$.
t	Yearly LT scheduling periods, $t \in T^l$.
ℓ	Half-yearly MT scheduling periods, $\ell \in T^{mt}$.
n	Block identification, $n \in \{1, \dots, \mathcal{N}\}$.
c	Material class attribute, $c \in \mathcal{C} = \{\text{ore}, \text{waste}\}$.
r	Block predecessors, $r \in \Gamma$.
lt	LT duration of annual scheduling periods.
mt	MT duration of half-yearly scheduling periods.

Table 2

Sets of entities used in the MIP scheduling model.

Notation	Description
$\mathcal{M}$	Set of mining or source locations.
$\mathcal{C}$	Set of material classes, c .
$\mathcal{N}$	Set of blocks in the block model for each mining location, i .
Γ	Set of direct block predecessors for each block, n .
T^l	Set of long-term yearly scheduling periods.
T^{mt}	Set of medium-term half-yearly scheduling periods.

is tactical, has both quasi-strategic and quasi-operational characteristics since it must link LT and ST production schedules. Therefore, this paper seeks to fill the smooth transition gap left by [Levinson and Dimitrakopoulos \(2023\)](#), while taking cognizance of the work by [Otto and Musingwini \(2020\)](#) in linking LT and MT production scheduling, albeit manually.

Apart from the need for a mine production scheduling system to be integrated, it must also be dynamic, which means it must automatically update the scheduling stages whenever changes occur. Mine production scheduling that does not consider changes may lack optimality, resulting in sub-optimal NPVs ([Tolouei et al., 2020](#)). A dynamic mine production scheduling process must be updated regularly so that subsequent ST scheduling can feedback new information to MT scheduling, which in turn should also feedback changes to LT scheduling. Therefore, this paper presents a mixed integer programming (MIP) model that was solved using a genetic algorithm (GA), to enable incorporation of geological, operational and economic uncertainties and dynamically integrate LT and MT production scheduling. Since GA is a stochastic algorithm, the combined MIP model and GA approach developed in this paper, could be run several times, each time producing a different result. The GA's characteristic strategies such as its ability to reject, repair, modify, and penalize solutions, make it well-suited for problems with complex search spaces like the OPMPs problem. Recently, GA has been used to solve the mine production scheduling problem due to its ability to handle large-scale optimization problems. The following five examples highlight this application of GAs. [Ruiseco \(2016\)](#) developed a GA technique to optimize the production scheduling for an operating mining bench under constraints such as ore grade, equipment availability, mining costs and processing costs. GA was also applied by [Alipour et al. \(2017\)](#) to solve the production scheduling problem of a hypothetical open-pit copper orebody in two-dimensional (2D) space under access, mining rate and processing rate constraints. [Ahmadi and Shahabi \(2018\)](#) compared the performance of GA to that of Lane's method in optimizing processing cut-off grade by targeting high-grade ore at the beginning of the mining operations. [Muke et al. \(2021\)](#) developed a GA-based model to optimize the LT production scheduling process of open-pit mining in a three-dimensional (3D) space subject to mining capacity, processing capacity, ore grade and block precedence constraints. [Alipour et al. \(2022\)](#) developed a GA-based solution to solve the LT production scheduling problem, which was expressed as a conventional integer

programming problem integrating forecasted copper prices based on stochastic differential equations.

The combined MIP model and GA approach developed in this paper to integrate LT and MT production scheduling was coded and executed in Python programming language. Python was selected for programming because among other reasons, it is a freely available open-source programming language which is efficient in handling large datasets and complex computations such as those encountered in OPMPs.

2. Formulation of MIP model for integrating LT and MT production scheduling

This paper presents a new MIP model, which was solved using GA to address production uncertainties associated with meeting targets set for mining rate, processing rate and ore grade. To formulate the MIP model requires the determination of decision variables and parameters. Decision variables are controllable inputs whose values are determined by solving the MIP mathematical model, while parameters are known as uncontrollable inputs whose values must be added to the mathematical model by the user ([Rardin, 2017](#)). [Tables 1–4](#) describe the indices, set of entities, parameters and variables that were used for the MIP model formulation.

Two aspects were considered in developing the MIP model. The first aspect consists of integrating mine production scheduling to connect LT and MT schedules. This is to ensure scheduling alignment by connecting scheduling parameters such as mining and processing rates between LT and MT stages. The second aspect consists of implementing scheduling changes in time and space to holistically capture production uncertainties. For instance, a change of the mining rate and/or processing rate in the MT schedule can also affect the LT production schedule. Therefore, a dynamic integrated mine production scheduling system should simultaneously align scheduling stages and update the whole scheduling system to consider the changes. [Fig. 2](#) illustrates a conceptual framework for a dynamic integrated LT and MT mine production scheduling system.

The framework comprises six steps. Step I consists of entering input scheduling parameters into the LT scheduling stage, which informs MT scheduling parameters in Step III. Step II employs the GA to optimize the LT mine production scheduling process, which is then connected to the MT production scheduling stage in Step IV. Step III consists of collecting LT scheduling results and adjusting MT input parameters to feed the MT scheduling stage. Step IV employs the GA to optimize the MT production scheduling process, which is connected back to the LT scheduling stage. Step V consists of collecting MT scheduling results and LT scheduling parameters. Step VI consists of updating the block model and scheduling parameters to feedback to the LT scheduling stage. Therefore, this framework provides a synchronized development of the LT and MT mine production scheduling model. It enables decision makers to evaluate different production sequences by ensuring seamless communication between the LT and MT scheduling stages. Additionally, the framework improves efficiency of production scheduling by minimizing information gaps encountered when using separately optimized production schedules.

2.1. Objective function

The objective function in Equation (1) aims to maximize NPV and integrate LT and MT production scheduling stages subject to various production scheduling constraints. The model applies scheduling penalties to account for deviations from scheduled production targets and extraction sequences. These penalties provide a mechanism to address potential disruptions or discrepancies that might arise during mining operations. The penalties were created with the view that when the

Table 3
Parameters used in the MIP scheduling model.

Parameter	Description
MT_n	Total mass of a block, $n \in \mathcal{N}$.
OT_n	Quantity of ore material contained in a block, $n \in \mathcal{N}$.
OC_n	Total mass of metal ore content in a block, $n \in \mathcal{N}$.
WT_n	Quantity of waste material contained in a block, $n \in \mathcal{N}$.
$g_{n,c}$	Grade quality of ore material class, $c \in \mathcal{C}$ in a block, $n \in \mathcal{N}$.
$V_{n,t}^t; V_{n,t}^{mt}$	Economic value of a block, n in period, t and ι , respectively.
$v_n^t; v_n^{mt}$	Discounted economic value generated by extracting block, n in period, t and ι , respectively, where, $v_n^t = V_{n,t}^t / (1 + d)^t$; $v_n^{mt} = V_{n,\iota}^{mt} / (1 + d/2)^\iota$.
d	Economic discount rate per annum.
rd	Risk discount rate, for deferring risk to another production period.
$p_{c,i}^t; p_{c,i}^{mt}$	Discounted selling price $P_{c,i,t}^t, P_{c,i,\iota}^{mt}$, per unit of ore material class produced and recovered at mining location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively, where, $p_{c,i}^t = P_{c,i,t}^t / (1 + d)^t$; $p_{c,i}^{mt} = P_{c,i,\iota}^{mt} / (1 + d/2)^\iota$.
$cm_{c,i}^t; cm_{c,i}^{mt}$	Discounted unit mining costs $CM_{c,i,t}^t, CM_{c,i,\iota}^{mt}$ for mining ore and waste material class, $c \in \mathcal{C}$ from location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively, where, $cm_{c,i}^t = CM_{c,i,t}^t / (1 + d)^t$; $cm_{c,i}^{mt} = CM_{c,i,\iota}^{mt} / (1 + d/2)^\iota$.
$cp_{c,i}^t; cp_{c,i}^{mt}$	Discounted unit processing costs $CP_{c,i,t}^t, CP_{c,i,\iota}^{mt}$ for processing ore material class, $c \in \mathcal{C}$ from location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively, where, $cp_{c,i}^t = CP_{c,i,t}^t / (1 + d)^t$; $cp_{c,i}^{mt} = CP_{c,i,\iota}^{mt} / (1 + d/2)^\iota$.
$cs_{c,i}^t; cs_{c,i}^{mt}$	Discounted unit selling costs $CS_{c,i,t}^t, CS_{c,i,\iota}^{mt}$ for selling ore material class, $c \in \mathcal{C}$ from location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively, where, $cs_{c,i}^t = CS_{c,i,t}^t / (1 + d)^t$; $cs_{c,i}^{mt} = CS_{c,i,\iota}^{mt} / (1 + d/2)^\iota$.
$r_{c,i}^t; r_{c,i}^{mt}$	Recovery of ore material class, $c \in \mathcal{C}$ at location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively.
$pc_{i,lt}^{lt,rm}; pc_{i,lt}^{mt,rm}$	Discounted mining rate penalty costs $PC_{i,lt}^{lt,rm}, PC_{i,\iota}^{mt,rm}$ for deviating from the defined LT and MT mining rates at location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively. Where, $pc_{i,lt}^{lt,rm} = PC_{i,lt}^{lt,rm} / (1 + rd)^t$; $pc_{i,\iota}^{mt,rm} = PC_{i,\iota}^{mt,rm} / (1 + rd/2)^\iota$.
$pc_{i,lt}^{lt,ro}; pc_{i,lt}^{mt,ro}$	Discounted processing penalty rate costs $PC_{i,lt}^{lt,ro}, PC_{i,\iota}^{mt,ro}$ for deviating from the defined LT and MT processing rates at location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively. Where, $pc_{i,lt}^{lt,ro} = PC_{i,lt}^{lt,ro} / (1 + rd)^t$; $pc_{i,\iota}^{mt,ro} = PC_{i,\iota}^{mt,ro} / (1 + rd/2)^\iota$.
$pc_{i,lt}^{lt,rg}; pc_{i,lt}^{mt,rg}$	Discounted grade target penalty costs $PC_{i,lt}^{lt,rg}, PC_{i,\iota}^{mt,rg}$ for deviating from the defined LT and MT average grade target at location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively. Where, $pc_{i,lt}^{lt,rg} = PC_{i,lt}^{lt,rg} / (1 + rd)^t$; $pc_{i,\iota}^{mt,rg} = PC_{i,\iota}^{mt,rg} / (1 + rd/2)^\iota$.
$pc_{i,lt}^{mt,rm}$	Discounted integrated MT mining rate penalty costs $PC_{i,lt}^{mt,rm}$ for deviating from LT mining rate scheduled, at location, i in period, $\iota \in T^{mt}$, where, $mt = lt/2$; $pc_{i,lt}^{mt,rm} = PC_{i,lt}^{mt,rm} / (1 + rd/2)^\iota$.
$pc_{i,lt}^{mt,ro}$	Discounted integrated MT processing rate penalty costs $PC_{i,lt}^{mt,ro}$ for deviating from LT processing rate scheduled, at location, i in period, $\iota \in T^{mt}$, where, $mt = lt/2$; $pc_{i,lt}^{mt,ro} = PC_{i,lt}^{mt,ro} / (1 + rd/2)^\iota$.
$pc_{i,lt}^{mt,rg}$	Discounted integrated MT grade target penalty costs $PC_{i,lt}^{mt,rg}$ for deviating from LT grade target scheduled, at location, i in period, $\iota \in T^{mt}$, where, $mt = lt/2$; $pc_{i,lt}^{mt,rg} = PC_{i,lt}^{mt,rg} / (1 + rd/2)^\iota$.
$L_i^{lt,rm}; L_i^{mt,rm}$	Lower bound for lt and mt mining rate at location, $i \in \mathcal{N}$ in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively.
$U_i^{lt,rm}; U_i^{mt,rm}$	Upper bound for lt and mt mining rate at location, $i \in \mathcal{N}$ in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively.
$L_i^{lt,ro}; L_i^{mt,ro}$	Lower bound for lt and mt processing rate at location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively.
$U_i^{lt,ro}; U_i^{mt,ro}$	Upper bound for lt and mt processing rate at location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively.
$L_i^{lt,rg}; L_i^{mt,rg}$	Lower bound for lt and mt average ore grade for processing at location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively.
$U_i^{lt,rg}; U_i^{mt,rg}$	Upper bound for lt and mt average ore grade for processing at location, i in period, $t \in T^t$ and $\iota \in T^{mt}$; respectively.

solution was further away from feasibility from a mining engineering perspective, it would be penalized more. For consistency, the unit of measurement for a penalty function is the same as the unit of measurement for the associated constraint.

The objective function (Equation (1)) has four parts. Part I calculates the discounted cash flows of the blocks scheduled per period of the integrated LT and MT mine production scheduling system. The NPV of

$$\begin{aligned}
 \text{Max } Z_{mt}^t = & \underbrace{\left[\sum_{n=1}^N \sum_{t=1}^{T^t} \left(\sum_{n=1}^{TNB^t} \sum_{t=1}^{T^{mt}} \left(v_{n,t}^{mt} \times X_{n,t}^{mt} \right) \right) \times X_{n,t}^t \right]}_{\text{Part I}} \\
 & - \underbrace{\sum_{t=1}^{T^t} \left(pc_{i,lt}^{lt,rm} \times \phi_{i,lt}^{lt,rm}(x) + pc_{i,lt}^{lt,ro} \times \phi_{i,lt}^{lt,ro}(x) + pc_{i,lt}^{lt,rg} \times \phi_{i,lt}^{lt,rg}(x) \right)}_{\text{Part II}} \\
 & - \underbrace{\sum_{t=1}^{T^{mt}} \left(pc_{i,lt}^{mt,rm} \times \phi_{i,lt}^{mt,rm}(x) + pc_{i,lt}^{mt,ro} \times \phi_{i,lt}^{mt,ro}(x) + pc_{i,lt}^{mt,rg} \times \phi_{i,lt}^{mt,rg}(x) \right)}_{\text{Part III}} \\
 & - \underbrace{\sum_{t=1}^{T^{mt}} \left(pc_{i,lt}^{mt,rm} \times \phi_{i,lt}^{mt,rm}(x) + pc_{i,lt}^{mt,ro} \times \phi_{i,lt}^{mt,ro}(x) + pc_{i,lt}^{mt,rg} \times \phi_{i,lt}^{mt,rg}(x) \right)}_{\text{Part IV}}
 \end{aligned} \tag{1}$$

Table 4
Mixed integer programming of LT and MT mine scheduling decision variables.

Variable	Description
$X_{i,n,t}^{lt}, X_{i,n,t}^{mt}$	Binary variable for each block, $n \in \mathcal{N}$ mined from mining location, i , and takes a value of one if block, n is mined in period, $t \in T^{lt}$ or $t \in T^{mt}$; or zero otherwise.
$RM_{i,t}^{lt}, RM_{i,t}^{mt}$	Reference continuous variable representing the actual mining rate achieved at location, i during period, $t \in T^{lt}$ and $t \in T^{mt}$.
$RO_{i,t}^{lt}, RO_{i,t}^{mt}$	Reference continuous variable representing the actual quantity of ore achieved at mining location, i to be sent to destination, i during period, $t \in T^{lt}$ and $t \in T^{mt}$.
$RW_{i,t}^{lt}, RW_{i,t}^{mt}$	Reference continuous variable representing the actual quantity of waste achieved at mining location, i to be sent to destination, i during period, $t \in T^{lt}$ and $t \in T^{mt}$.
$RG_{i,t}^{lt}, RG_{i,t}^{mt}$	Reference continuous variable representing the actual weighted average ore grade achieved at mining location, i during period, $t \in T^{lt}$ and $t \in T^{mt}$.
$TNB_{i,t}^{lt}, TNB_{i,t}^{mt}$	Reference integer variable representing the total number of blocks achieved at location, $i \in \mathcal{N}$ during each LT and MT scheduling period, $t \in T^{lt}$ and $t \in T^{mt}$.
$TOB_{i,t}^{lt}, TOB_{i,t}^{mt}$	Reference integer variable representing the total number of ore blocks achieved at location, i during each LT and MT scheduling period, $t \in T^{lt}$ and $t \in T^{mt}$.
$TWB_{i,t}^{lt}, TWB_{i,t}^{mt}$	Reference integer variable representing the total number of waste blocks achieved at location, $i \in \mathcal{N}$ during each LT and MT scheduling period, $t \in T^{lt}$ and $t \in T^{mt}$.
$\phi_{i,t}^{lt, RM}(x); \phi_{i,t}^{mt, RM}(x)$	Penalty costs function for deviating from mining rates at location, $i \in \mathcal{N}$ during each LT and MT scheduling period, $t \in T^{lt}$ and $t \in T^{mt}$.
$\phi_{i,t}^{lt, RO}(x); \phi_{i,t}^{mt, RO}(x)$	Penalty costs function for deviating from processing rates at location, $i \in \mathcal{N}$ during each LT and MT scheduling period, $t \in T^{lt}$ and $t \in T^{mt}$.
$\phi_{i,t}^{lt, RG}(x); \phi_{i,t}^{mt, RG}(x)$	Penalty costs function for deviating from ore grade targets at location, $i \in \mathcal{N}$ during each LT and MT scheduling period, $t \in T^{lt}$ and $t \in T^{mt}$.
$\phi_{i,t}^{mt, RM}(x)$	Integrated penalty costs function of MT mining rate for deviating from actual LT mining rate at location, $i \in \mathcal{N}$ during period, $t \in T^{mt}$.
$\phi_{i,t}^{mt, RO}(x)$	Integrated penalty costs function of MT processing rate for deviating from actual LT processing rate at location, $i \in \mathcal{N}$ during period, $t \in T^{mt}$.
$\phi_{i,t}^{mt, RG}(x)$	Integrated penalty costs function of MT grade target for deviating from actual LT grade target at location, $i \in \mathcal{N}$ during period, $t \in T^{mt}$.

annual cash flows for LT production scheduling is calculated using the annual discount rate, while the NPV of half-yearly cash flows for MT production scheduling is calculated using half of the annual discount rate, as illustrated in Table 3. Therefore, Equation (1) sums up half-yearly MT discounted cash flows into yearly LT discounted cash flows. Parts II and III apply penalty costs associated with deviations from minimum and maximum mining rates, processing rate and ore grade targets of LT and MT scheduling, respectively. Part IV applies integrated penalty costs associated with half-yearly MT mining and processing rates and ore grade targets misalignments from the actual yearly LT mining rate, processing rate and ore grade targets.

Penalty costs function for deviating from mining rate, processing rate and ore grade target during each LT and MT scheduling period are calculated in Equations (2)–(4), respectively:

$$\phi_{i,t}^{lt, RM}(x) = \max\{0, -f_t^{lt, RM}(x) + L_{i,t}^{lt, RM}, f_t^{lt, RM}(x) - U_{i,t}^{lt, RM}\}$$

$$\phi_{i,t}^{mt, RM}(x) = \max\{0, -f_t^{mt, RM}(x) + L_{i,t}^{mt, RM}, f_t^{mt, RM}(x) - U_{i,t}^{mt, RM}\} \quad (2)$$

$$\phi_{i,t}^{lt, RO}(x) = \max\{0, -f_t^{lt, RO}(x) + L_{i,t}^{lt, RO}, f_t^{lt, RO}(x) - U_{i,t}^{lt, RO}\}$$

$$\phi_{i,t}^{mt, RO}(x) = \max\{0, -f_t^{mt, RO}(x) + L_{i,t}^{mt, RO}, f_t^{mt, RO}(x) - U_{i,t}^{mt, RO}\} \quad (3)$$

$$\phi_{i,t}^{lt, RG}(x) = \max\{0, -f_t^{lt, RG}(x) + L_{i,t}^{lt, RG}, f_t^{lt, RG}(x) - U_{i,t}^{lt, RG}\}$$

$$\phi_{i,t}^{mt, RG}(x) = \max\{0, -f_t^{mt, RG}(x) + L_{i,t}^{mt, RG}, f_t^{mt, RG}(x) - U_{i,t}^{mt, RG}\} \quad (4)$$

where functions $f_t^{lt, RM}(x) = RM_{i,t}^{lt}$; $f_t^{mt, RM}(x) = RM_{i,t}^{mt}$; $f_t^{lt, RO}(x) = RO_{i,t}^{lt}$; $f_t^{mt, RO}(x) = RO_{i,t}^{mt}$; $f_t^{lt, RG}(x) = RG_{i,t}^{lt}$; $f_t^{mt, RG}(x) = RG_{i,t}^{mt}$. Integrated penalty costs functions of MT mining rate, processing rate and ore grade target for deviating from actual LT mining rate, processing rate and ore grade are calculated in Equations (5)–(7), respectively:

$$\phi_{i,t}^{mt, RM}(x) = \max\{0, -f_t^{mt, RM}(x) + L_{i,t}^{mt, RM}, f_t^{mt, RM}(x) - U_{i,t}^{mt, RM}\} \quad (5)$$

$$\phi_{i,t}^{mt, RO}(x) = \max\{0, -f_t^{mt, RO}(x) + L_{i,t}^{mt, RO}, f_t^{mt, RO}(x) - U_{i,t}^{mt, RO}\} \quad (6)$$

$$\phi_{i,t}^{mt, RG}(x) = \max\{0, -f_t^{mt, RG}(x) + L_{i,t}^{mt, RG}, f_t^{mt, RG}(x) - U_{i,t}^{mt, RG}\} \quad (7)$$

The degree of integration and the level of reconciliation of the scheduling system are the two most important aspects of mine

production scheduling (Otto and Musingwini, 2020). According to Bester et al. (2016), reconciliation is a process of identifying, analyzing, and reporting any discrepancies between scheduled values and actual results. Discrepancies consist not only of measuring what has been scheduled versus what has been actually mined but also of measuring the compliance between scheduling stages. Compliance of an integrated mine scheduling system involves the development of temporal and spatial scheduling alignment between scheduling stages. Temporal scheduling compliance ensures that there is alignment in scheduling stages across periods. Spatial scheduling compliance identifies the location of mining areas at each scheduling stage and implements the sequence in which these mining areas should be mined. Spatial scheduling compliance to align LT and MT production schedules and temporal scheduling compliance are expressed by Equations (8) and (9), respectively:

$$\begin{cases} N = \sum_{t=1}^{T^{lt}} TNB_{i,t}^{lt} \\ TNB_{i,t}^{lt} = \sum_{t=1}^{T^{mt}} TNB_{i,t}^{mt}; \end{cases} \quad (8)$$

$$T^{lt}(t) \subset T^{mt}(\iota). \quad (9)$$

Therefore, the economic alignment between LT and MT production schedules can be expressed by Equation (10):

$$v_{n,t}^{lt} = \sum_{n=1}^{TNB_{i,t}^{lt}} \sum_{t=1}^{T^{mt}} (v_{n,t}^{mt} \times X_{n,t}^{mt}) \quad (10)$$

where:

$$v_{n,t}^{lt} = \left[\underbrace{\sum_{n=1}^N \sum_{t=1}^{T^{lt}} OT_{i,n}^{lt} \times g_{n,c} \times r_{c,i,t} \times (p_{i,c,t}^{lt} - cs_{i,c,t}^{lt})}_{\text{Part I}} \right] - \left[\underbrace{\sum_{n=1}^N \sum_{t=1}^{T^{lt}} (OT_{i,n}^{lt} \times cp_{i,c,t}^{lt}) + (OT_{i,n}^{lt} + WT_{i,n}^{lt}) \times cm_{i,c,t}^{lt}}_{\text{Part II}} \right] \quad (11)$$

Most optimum solutions to real-world problems cannot often be obtained without recognizing constraints impacting the decision variables. Optimization of the integrated objective function (Equation (1)) is subject to several constraints to ensure that LT objectives are achieved at the MT horizon. Section 2.2 presents these constraints.

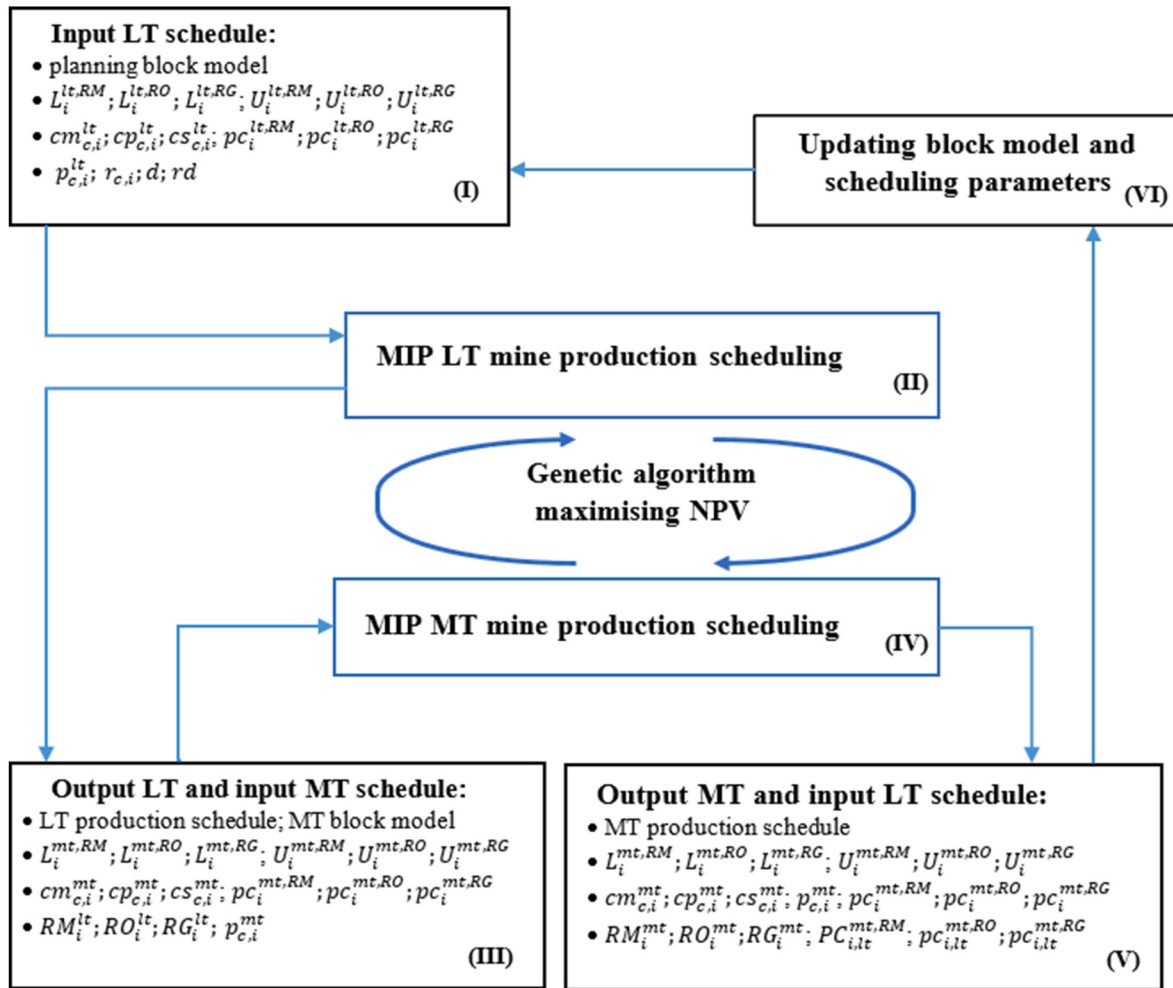


Fig. 2. A conceptual framework for dynamically integrating LT and MT mine production scheduling.

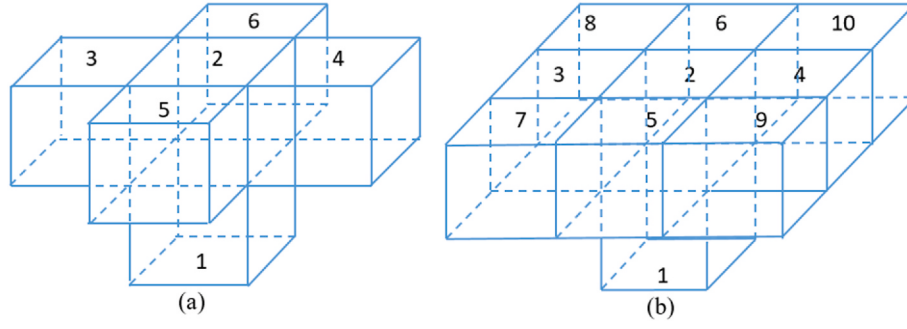


Fig. 3. Graphical views of block predecessors indicating a 1:5 pattern (a) and 1:9 pattern (b).

2.2. Constraints

2.2.1. Integrated LT and MT mining rate constraints

Constraints in Equations 12 and 13 represent the integrated LT and MT mining rate constraint referring to the limitations imposed on the total mining tonnage during each scheduling period. The cumulative half-yearly MT mining rate must be equal to the corresponding yearly LT mining rate to ensure scheduling alignment. However, if the minimum and maximum mining rates are not satisfied, then mining penalty functions in Equations 14 and 15 are used to adjust the MT and LT mining rate constraint. The constraint in Equation (16) links minimum and maximum mining rates, while the constraint in Equation (17) links actual mining rates achieved, between LT and MT schedules.

$$RM_{i,t}^{mt} = f_{t,lt}^{mt,rm}(x) = \sum_{n=1}^{T^{lt}} \sum_{n=1}^{TNB^{lt}} \sum_{\ell=1}^{T^{mt}} (OT_{i,n}^{mt} + WT_{i,n}^{mt}) \times X_{n,\ell}^{mt} \quad (12)$$

$$RM_{i,\ell}^{mt} = f_{\ell}^{mt,rm}(x) = \sum_{n=1}^{TNB^{lt}} \sum_{\ell=1}^{T^{mt}} (OT_{i,n}^{mt} + WT_{i,n}^{mt}) \times X_{n,\ell}^{mt} \quad (13)$$

$$\phi_{i,t,\ell}^{mt,rm}(x) = \max\{0, -f_{t,lt}^{mt,rm}(x) + L_{i,t}^{lt,rm} \cdot f_{\ell,lt}^{mt,rm}(x) - U_{i,t}^{lt,rm}\} \quad (14)$$

$$\begin{cases} -f_{t,lt}^{mt,rm}(x) + L_{i,t}^{lt,rm} \leq 0 \\ f_{t,lt}^{mt,rm}(x) - U_{i,t}^{lt,rm} \leq 0 \end{cases} \quad (15)$$

$$L_i^{mt,rm} = \frac{L_i^{lt,rm}}{T^{mt}}; U_i^{mt,rm} = \frac{U_i^{lt,rm}}{T^{mt}} \quad (16)$$

$$RM_{i,t}^{lt} = \sum_{\ell=1}^{T^{mt}} RM_{i,\ell}^{mt} \quad (17)$$

2.2.2. Integrated LT and MT processing rate constraints

Constraints in Equations 18 and 19 represent the integrated LT and MT processing rate limitations imposed on the ore tonnage during each scheduling period. Ore tonnage realization in each scheduling period influences scheduling decisions since it directly impacts revenue generation and resource utilization strategies. A careful consideration of these rates ensures that extraction activities are carried out within feasible limits without compromising the quality or quantity of extracted ore. The cumulative half-yearly MT processing ore rate must be equal to the corresponding LT processing ore rate to ensure scheduling alignment. However, if the minimum and maximum processing rates are not satisfied, then processing penalty functions in Equations 20 and 21 are used to adjust the MT and LT processing rate. The constraint in Equation (22) links minimum and maximum processing rates, while the constraint in Equation (23) links actual processing rate achieved between LT and MT schedules.

$$RO_{i,t,lt}^{mt} = f_{t,lt}^{mt,ro}(x) = \sum_{n=1}^{T^{lt}} \sum_{n=1}^{TNB^{lt}} \sum_{\ell=1}^{T^{mt}} OT_{i,n}^{mt} \times X_{n,\ell}^{mt} \quad (18)$$

$$RO_{i,\ell}^{mt} = f_{\ell}^{mt,ro}(x) = \sum_{n=1}^{TNB^{lt}} \sum_{\ell=1}^{T^{mt}} OT_{i,n}^{mt} \times X_{n,\ell}^{mt} \quad (19)$$

$$\phi_{i,t,\ell}^{mt,ro}(x) = \max\{0, -f_{t,lt}^{mt,ro}(x) + L_{i,t}^{lt,ro} \cdot f_{\ell,lt}^{mt,ro}(x) - U_{i,t}^{lt,ro}\} \quad (20)$$

$$\begin{cases} -f_{t,lt}^{mt,ro}(x) + L_{i,t}^{lt,ro} \leq 0 \\ f_{t,lt}^{mt,ro}(x) - U_{i,t}^{lt,ro} \leq 0 \end{cases} \quad (21)$$

$$L_i^{mt,ro} = \frac{L_i^{lt,ro}}{T^{mt}}; U_i^{mt,ro} = \frac{U_i^{lt,ro}}{T^{mt}} \quad (22)$$

$$RO_{i,t}^{lt} = \sum_{\ell=1}^{T^{mt}} RO_{i,\ell}^{mt} \quad (23)$$

2.2.3. Integrated LT and MT ore grade target constraints

Constraints in Equations 24 and 25 represent the integrated LT and MT ore grade target limitations imposed on the quality of ore material based on the geological variability during each scheduling period. Different areas within a block model may have different mineral content or quality, thus, requiring careful scheduling to optimize production while minimizing costs. To maximize the objective function through efficient utilization of a mineral resource, the integrated mine production scheduling system must ensure a balance between extracting high-grade ore efficiently and maintaining sustainable plant feed over the LOM. Geological data and economic conditions are analyzed to determine the best extraction sequence within each period to meet plant requirements. This is achieved by setting minimum and maximum ore grade limits. The minimum ore grade limit defines a threshold below which extraction becomes uneconomic; thus, only blocks with a grade equal to or above the threshold meet processing requirements. On the other hand, the maximum ore grade limit defines a threshold that restrains the extraction of blocks with excessively high ore grades to avoid surpassing downstream processing capabilities or result in inefficiencies in ore recovery. The application of ore grade control at each stage of production scheduling enables a balanced production rate while maintaining consistent ore quality requirements for a sustainable production scheduling system.

$$RG_{i,t,lt}^{mt} = f_{t,lt}^{mt,rg}(x) = \sum_{n=1}^{T^{lt}} \sum_{n=1}^{TNB^{lt}} \sum_{\ell=1}^{T^{mt}} \frac{(OT_{i,n}^{mt} \times g_{n,c}) \times X_{n,\ell}^{mt}}{OT_{i,n}^{mt} \times X_{n,\ell}^{mt}} \quad (24)$$

$$RG_{i,\ell}^{mt} = f_{\ell}^{mt,rg}(x) = \sum_{n=1}^{TNB^{lt}} \sum_{\ell=1}^{T^{mt}} \frac{(OT_{i,n}^{mt} \times g_{n,c}) \times X_{n,\ell}^{mt}}{OT_{i,n}^{mt} \times X_{n,\ell}^{mt}} \quad (25)$$

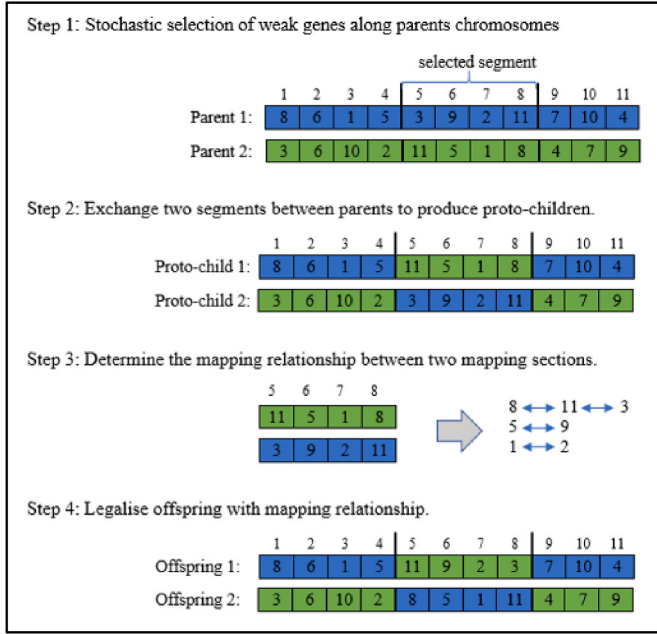


Fig. 4. Illustration of a partial-mapped crossover genetic procedure (Gen et al., 2008).

$$\phi_{i,lt}^{mt,RG}(x) = \max\{0, -f_{i,lt}^{mt,RG}(x) + L_{i,lt}^{lt,RG}, f_{i,lt}^{mt,RG}(x) - U_{i,lt}^{lt,RG}\} \quad (26)$$

$$\begin{cases} -f_{i,lt}^{mt,RG}(x) + L_{i,lt}^{lt,RG} \leq 0 \\ f_{i,lt}^{mt,RG}(x) - U_{i,lt}^{lt,RG} \leq 0 \end{cases} \quad (27)$$

$$L_i^{mt,RG} = \frac{L_i^{lt,RG}}{T_{mt}^{mt}}, U_i^{mt,RG} = \frac{U_i^{lt,RG}}{T_{mt}^{mt}} \quad (28)$$

$$RG_{i,t}^{lt} = \sum_{\tau=1}^{T_{mt}} RG_{i,\tau}^{mt} \quad (29)$$

The cumulative half-yearly MT average ore grade must be equal to the corresponding processing LT average ore grade to ensure scheduling alignment. When the minimum and maximum ore grade targets are violated, then penalty functions in Equations 26 and 27 are used to align the MT and LT ore grades to comply with the grade constraint. The constraint in Equation (28) links minimum and maximum ore grades, while the constraint in Equation (29) links actual ore grades achieved between LT and MT schedules.

2.2.4. Integrated LT and MT block precedence relationship constraints

Block precedence constraints in Equations 30 and 31 must be embedded within both the LT and MT schedules. Two patterns are usually defined to express the block precedence constraints regardless of the scheduling stage. A 1:5 pattern in Equation (30) requires the removal of 5 blocks directly situated above a block and a 1:9 pattern in Equation (31) requires the removal of 9 blocks directly situated above a block, as shown in Figures (3a) and (3b), respectively. These constraints are important in optimizing extraction sequences and generating feasible schedule solutions. They fundamentally govern the extraction sequence of mineral blocks within a given period by ensuring that the extraction sequence adheres to certain predefined conditions such as pit slope angle for guaranteeing slope stability. Moreover, if a scheduling priority is defined, the block precedence constraint ensures that blocks are

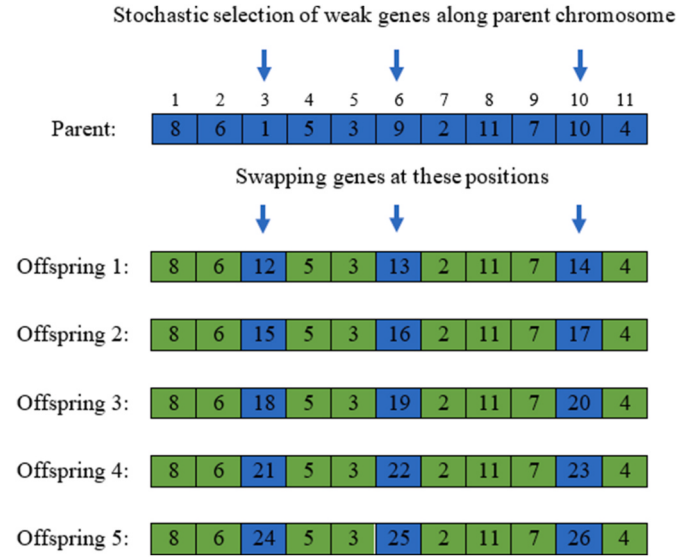


Fig. 5. Illustration of the heuristic mutation genetic procedure (Gen et al., 2008).

mined in a sequence that maximizes economic returns while minimizing waste generation or delaying access to future areas with excessively high grades.

$$5X_{ijk}^t - \sum_{r=1}^{T_{lt}} (X_{i-1,j,k+1}^r + X_{i,j-1,k+1}^r + X_{i,j,k+1}^r + X_{i,j+1,k+1}^r + X_{i+1,j,k+1}^r) \leq 0, \forall t, i, j, k \quad (30)$$

$$9X_{ijk}^{lt} - \sum_{r=1}^{T_{lt}} (X_{i-1,j-1,k+1}^r + X_{i-1,j,k+1}^r + X_{i-1,j+1,k+1}^r + X_{i,j-1,k+1}^r + X_{i,j,k+1}^r + X_{i,j+1,k+1}^r + X_{i+1,j-1,k+1}^r + X_{i+1,j,k+1}^r + X_{i+1,j+1,k+1}^r) \leq 0, \forall t, i, j, k \quad (31)$$

To improve the practicability of the schedule in addressing short-term operational requirements, an additional operational constraint is applied to the MT schedule, defined by the number of benches bh along the k direction to be mined in each MT scheduling period. Let bt be the top bench of MT blocks from the surface, and bb be the bottom bench of MT blocks in period t . The number of benches to mine in each MT schedule is constrained by Equation (32).

$$\sum_{bh=bt-1}^{bb} (RM_{bhk}^{mt}) \leq RM_{i,\tau}^{mt}, bh_k \in [bt, bb], \forall k, \tau \quad (32)$$

where RM_{bhk}^{mt} is the tonnage of material (ore and waste) on bench bh_k for the MT schedule in period τ . The maximum number of benches is attained when the cumulative tonnage satisfies the MT mining rate constraint.

2.2.5. Integrated LT and MT mineral reserves constraints

Constraints in Equations 33 and 34 represent the alignment of the Mineral Reserves constraint in the integrated mine scheduling system. The constraint in Equation (33) links the total number of blocks in the LT and MT schedules, while the constraint in Equation (34) links the total number of blocks from the UPL to LT schedule. Mineral Reserve constraints refer to the estimated amount of economic mineral blocks within the UPL, and are used as the foundation upon which all extraction schedules are built. The challenge lies in developing a production

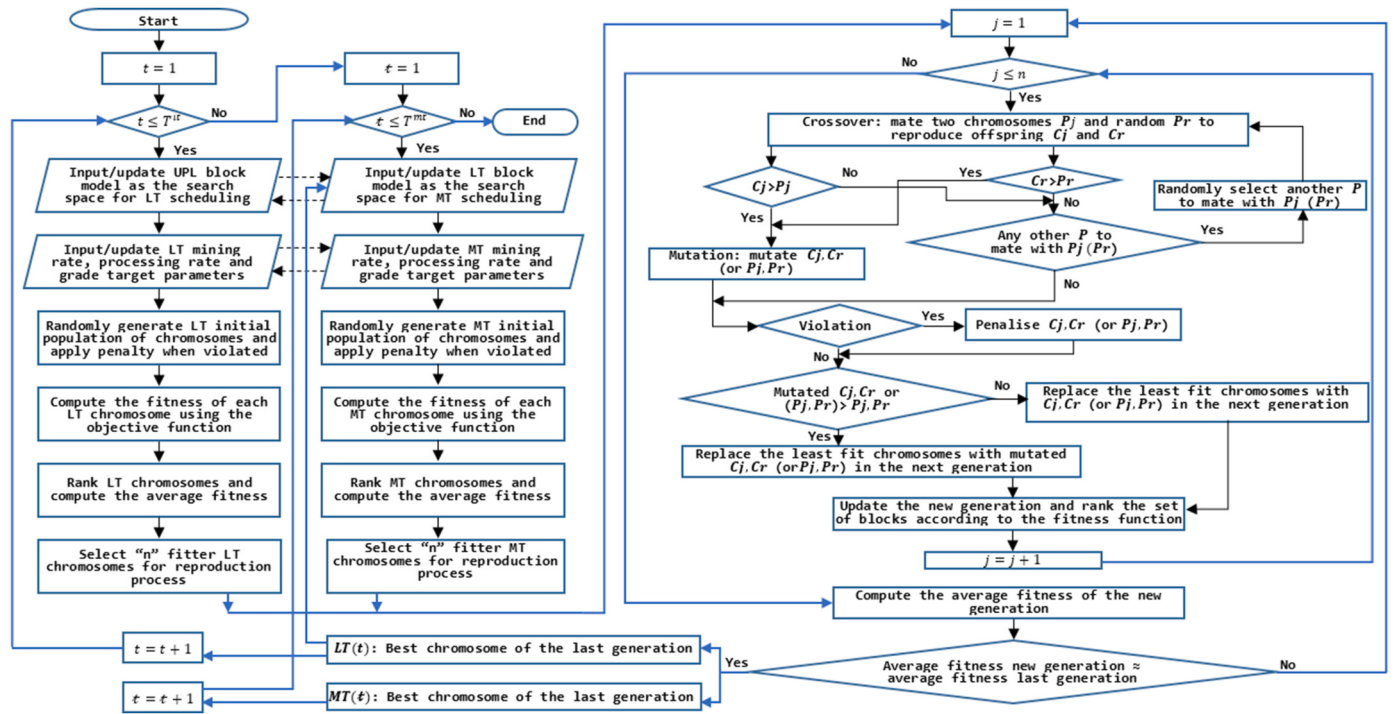


Fig. 6. Framework of the genetic algorithm for solving the MIP production scheduling system.

Table 5

Description of GA input parameters for implementing the combined MIP model and GA approach.

Parameter	Description
Initial population of chromosomes	The algorithm uses a random block selection to create chromosomes that comply with the block precedence constraint.
Population size	The size of the population is equal to 100 chromosomes for better coverage of the solution space.
Chromosome length	The length of chromosomes varies between the minimum and maximum mining rate constraints, measured in number of blocks.
Selection probability for crossover	The probability of a chromosome being selected for crossover is proportional to the fitness value of the chromosome.
Partial-mapped crossover (PMX) rate	This is proportional to the number of weak blocks along two parent chromosomes. Blocks are crossed over together with their block predecessors to comply with the block precedence constraint.
Heuristic mutation rate	This is proportional to the number of weak blocks along a chromosome. A block is mutated together with its block predecessor to comply with the block precedence constraint.
Mining penalty cost	This is equal to the total fitness value of violating blocks (ore and waste) along a chromosome.
Processing penalty cost	This is equal to the total fitness value of violating ore blocks along a chromosome.
Grade penalty cost	This is equal to the total fitness value of violating ore blocks considering grade value along a chromosome.

scheduling system that maximizes block extraction while remaining within the limits set by the total Mineral Reserves. The integrated scheduling system must ensure that in every scheduling stage, a block is mined at most once.

$$TNB_t^{lt} = \sum_{i=1}^{T^{lt}} TNB_{i'}^{mt} \quad (33)$$

$$N = \sum_{t=1}^{T^k} \sum_{i=1}^{T^{mt}} TNB_{i'}^{mt} \quad (34)$$

Table 6

Description of input parameters for the Newman, Zuck Small, KD and Geovia Surpac® block models.

Parameter	Model instances			
	Geovia Surpac®	Newman	Zuck Small	KD
$\mathcal{N}$	12,174	1060	9400	14,153
Γ_n	9	5	32	25
d	0.1	0.08	0.1	0.15
$r_{c,lt}, r_{c,lt'}$	0.95	—	—	—
$L_i^{lt, RM}$	3.4 Mt	1.4 Mt	—	—
$U_i^{lt, RM}$	3.8 Mt	2.0 Mt	≤60.0 Mt	Unlimited
$L_i^{lt, RO}$	0.65 Mt	0.9 Mt	—	—
$U_i^{lt, RO}$	1.0 Mt	1.1 Mt	≤20.0 Mt	10.0 Mt
$L_i^{mt, RM}$	1.7 Mt	0.7 Mt	—	—
$U_i^{mt, RM}$	1.9 Mt	1.0 Mt	≤30.0 Mt	Unlimited
$L_i^{mt, RO}$	0.325 Mt	0.45 Mt	—	—
$U_i^{mt, RO}$	0.5 Mt	0.55 Mt	≤10.0 Mt	5.0 Mt
$g_{n,c,h}$	0.27 g/t	—	—	0.317 Cu%
$p_{c,lt}^{lt}$	1,900USD/t	—	—	—
$p_{c,lt'}^{mt}$	1,900USD/t	—	—	—

3. Applying GA to solve the MIP production scheduling model

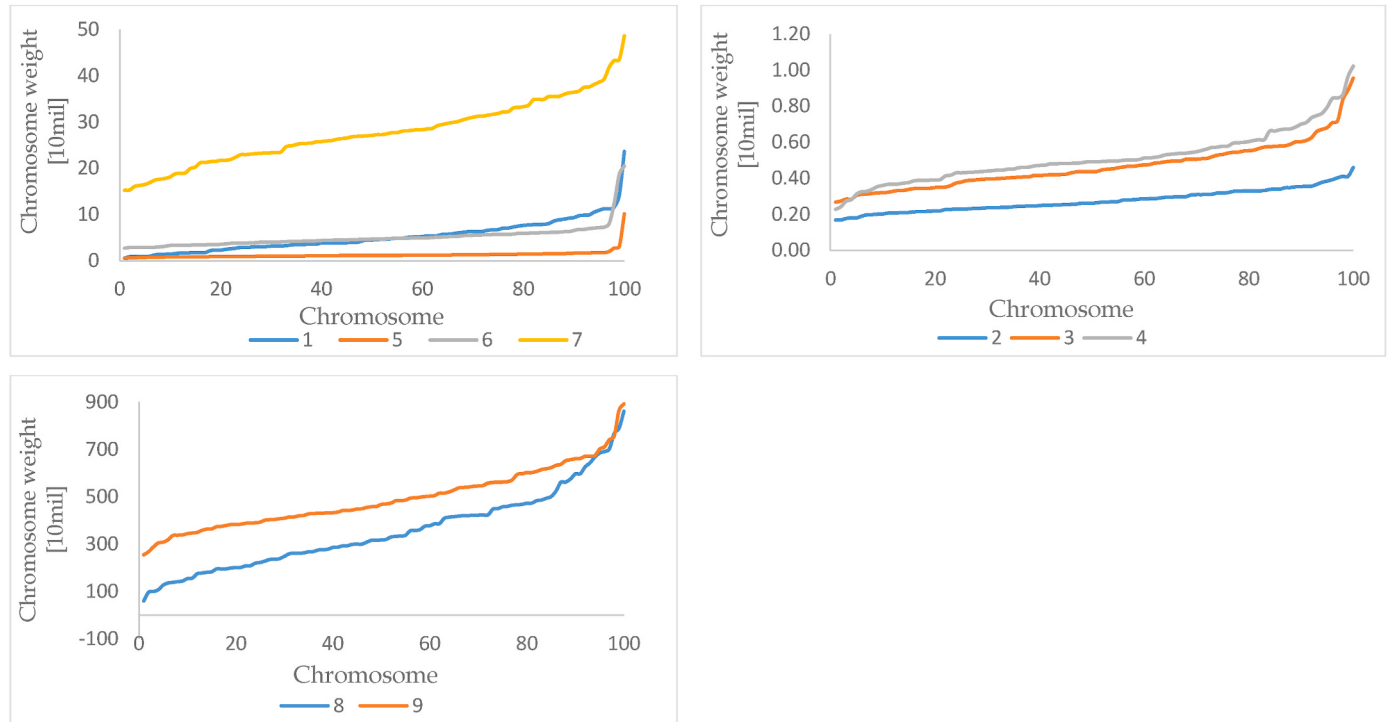
3.1. Concept of the genetic algorithm

GA, which is a stochastic algorithm, is part of a group of evolutionary algorithms which are used to solve complex problems by employing the principles of biological evolution and hereditary processes. Evolutionary algorithms which are part of metaheuristic techniques, imitate the natural behavior of certain species to find optimal solutions (Mitchell, 1999). By utilizing probabilistic or stochastic methods, evolutionary algorithms can quickly reach optimal or near-optimal solutions compared to algorithms based on exact methods. Although evolutionary algorithms do not guarantee optimum solutions due to the randomness of the search method approach, they are still valuable in finding near-optimal solutions for problems that are intractable for exact

Table 7

Isolated LT production scheduling results obtained using the combined MIP model and GA approach on the Geovia Surpac® block model.

Period [Year]	No. Of blocks	Total tonnage [tonne]	Ore tonnage [tonne]	Waste tonnage [tonne]	Stripping ratio	Average grade [g/t]	Cash flow [US\$ mil]
t	$TNB_{i,t}^t$	$RM_{i,t}^t$	$RO_{i,t}^{mt}$	$RW_{i,t}^t$	$\frac{RW_{i,t}^t}{RO_{i,t}^{mt}}$	$RG_{i,t}^t$	$V_{n,t}^t$
1	1350	3,748,997	876,326	2,872,671	3.28	1.69	30.49
2	1350	3,767,982	837,150	2,930,832	3.50	1.77	28.78
3	1350	3,770,640	708,024	3,062,616	4.33	1.91	21.68
4	1321	3,689,466	681,032	3,008,434	4.42	1.92	21.28
5	1297	3,620,887	696,737	2,924,150	4.20	1.75	18.38
6	1266	3,531,064	707,112	2,823,952	3.99	1.93	27.08
7	1255	3,499,745	804,303	2,695,442	3.35	1.96	39.99
8	1250	3,485,049	850,747	2,712,302	3.19	2.32	51.87
9	1250	3,485,807	923,885	2,483,922	2.68	2.25	80.16
10	485	1,352,469	369,553	982,916	2.66	2.27	26.59
Total/average	12,174	33,952,106	7,454,869	26,497,237	3.55	1.98	346.30
NPV [US\$ mil] at a discount rate of 10 %							197.99

**Fig. 7.** Weights of chromosomes generated in the first generation for each scheduling Periods 1 to 9 of the LT production schedule.

algorithms to solve (Yang, 2011). GA operates by evolving the fitness of a population solution from generation to generation. Initially, the algorithm randomly generates a set of potential solutions in the form of chromosomes. A randomly selected block is selected with its block predecessors to comply with the block precedence constraint. If the tonnage of a set of blocks is inadequate for the mining rate to fall between the minimum and maximum mining rates, the algorithm selects another block and its block predecessors and add them to the previous set to form a chromosome. The probability of selecting a block follows an exponential progression probability where the exponential weight of each block in each scheduling period is calculated by Equation (35) and the exponential probability is calculated by Equation (36) (Baron, 2014):

$$w_n^{t,\prime} = e^{V_n^{t,\prime}} \quad (35)$$

$$p^{t,\prime}(n) = \frac{w_n^{t,\prime}}{\sum_{n=1}^N w_n^{t,\prime}}, \forall t \in T^{dt} \text{ and } \prime \in T^{mt} \quad (36)$$

and

$$\sum_{n=1}^N p^{t,\prime}(n) = 1 \quad (37)$$

where $w_n^{t,\prime}$ is the exponential weight for block n in period t ($\prime$), which increases exponentially with the fitness or economic value $V_n^{t,\prime}$ of the block. N is the total number of blocks in the search space in period t ($\prime$). $p^{t,\prime}(n)$ is the exponential probability of block n for its selection in period t ($\prime$). The exponential probability of blocks changes over period t ($\prime$), making the model stochastic. Unlike probabilistic models where future steps are not influenced by previous outputs, stochastic models enable previously selected blocks to impact next selections as the probability of blocks is being adjusted to satisfy Equation (37). The combination of the MIP model and GA approach developed in this paper generates stochastic results.

A chromosome is depicted as a string of mining blocks representing a feasible solution that complies with the block precedence constraint and

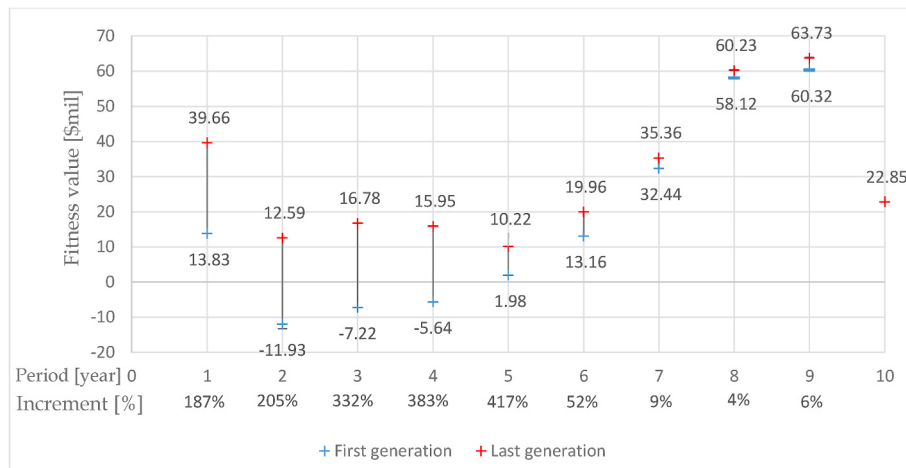


Fig. 8. Fitness value increment from the first to the last generation.

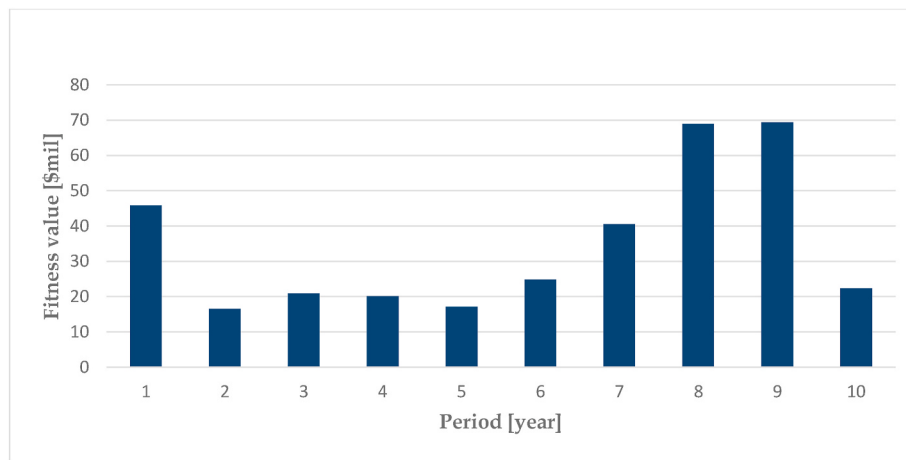


Fig. 9. Best chromosome fitness value per period for the integrated LT and MT production scheduling optimization.

its fitness value is improved by applying genetic operators. Each block along the chromosome represents a gene and is distinguished by its block number or block identification. Block attributes such as block economic value, ore grade and tonnage, mimic the biological characteristics of the chromosome. The length of a chromosome is determined by the number of blocks it contains so that it can comply with the mining rate constraint in each scheduling period of the LT and MT schedules. The fitness value of a chromosome is the cumulative value of block economic values. The algorithm ranks and selects parent chromosomes proportional to their exponential weight $w_{ch}^{t'}$, which is a cumulative weight of blocks comprising the chromosome. The higher the weight a chromosome has, the greater the chance of it being selected to be included in the crossover and mutation processes and produce offspring chromosomes (Mitchell, 1999).

Parent chromosomes pass on their fitness characteristics to their offspring by applying genetic operators such as crossover, mutation and elitism (Mitchell, 1999). The crossover concept is used to pair two parent chromosomes, creating a new generation of chromosomes that is better than the parent's generation. The mechanism of creating a new generation consists of exchanging certain traits from the parents and incorporates mutation and elitism to accelerate evolution. Mutation identifies weaker genes in the chromosome and replaces them with stronger ones with better fitness values, while elitism selects chromosomes with the highest fitness values (Gen et al., 2008). These genetic operations are repeated in the next generation to create another generation until no more improvement in fitness values of chromosomes can

be made.

The combined MIP model and GA approach presented in this paper was solved by employing the partial-mapped crossover (PMX) or double-point crossover proposed by Goldberg and Lingle (1985). In this process, both parent chromosomes are divided into segments of blocks, and these blocks are then exchanged between parent chromosomes to create one or more offspring. Fig. 4 illustrates the genetic procedure of the PMX. The PMX genetic operator improves the exploration of the search space by promoting genetic diversity, thus, potentially improving the performance of the algorithm in finding optimal or near-optimal solutions (Gen et al., 2008).

In Fig. 4, blocks at Positions 5, 6, 7 and 8 are selected in Step 1 to be included in the crossover operation. Step 2 crosses over selected segments between Parent Chromosomes 1 and 2 to produce Proto-children 1 and 2. Step 3 determines the mapping relationship between genes of crossed-over segments, where, for example, Gene 11 is mapped to Gene 3 at Position 5, and Gene 8 is mapped to Gene 11 at Position 8. Step 4 then legalizes chromosome segments to make each chromosome unique by removing duplicate genes and produces Offsprings 1 and 2. Offspring 1 inherits Genes 11 and 3 from Parent 2 and these are placed at Positions 5 and 8, respectively. Offspring 2 inherits Genes 8 and 11 from Parent 1, and these are placed at Positions 5 and 8, respectively.

The weight of a block along a chromosome is calculated by Equation (38):

Table 8
Integrated LT and MT production scheduling results obtained using the combined MIP model and GA approach on Geovia Surpac® block model.

Period [year]	t	No. Blocks		Total tonnage [tonne]		Ore tonnage [tonne]		Waste tonnage [tonne]		Stripping ratio		Average grade [g/t]		Cash flow [US\$ mil]	
		LT	MT	LT	MT	LT	MT	LT	MT	LT	MT	LT	MT	LT	MT
1	1	1350	675	3,890,298	1,876,776	1,015,381	586,762	2,874,917	1,359,541	2.83	2.32	1.98	2.13	45.76	36.25
	2		675		1,874,467		428,618		1,515,376		3.54		1.77		9.52
	3	1350	675	3,765,364	1,885,423	831,348	359,077	2,934,016	1,526,346	3.53	4.25	1.86	1.98	16.52	5.04
	4		675		1,879,941		472,271		1,407,670		2.98		1.76		11.48
	5	1350	675	3,770,472	1,890,531	722,014	282,806	3,048,458	1,607,725	4.22	5.68	1.87	1.26	20.86	4.08
	6		675		1,879,941		439,208		1,440,733		3.28		2.26		16.78
	7	1338	675	3,737,067	1,881,478	700,807	471,537	3,036,260	1,409,941	4.33	2.99	1.89	2.31	20.08	25.48
	8		663		1,855,589		229,270		1,626,319		7.09		1.03		-5.40
	9	1300	675	3,629,185	1,888,026	696,222	226,266	2,932,963	1,661,760	4.21	7.34	1.87	1.82	17.06	-8.68
	10		635		1,741,159		469,956		1,271,203		2.70		1.89		25.74
2	6	1266		3,532,843		685,516		2,847,327		4.15		1.98		24.79	
	7	1252		3,490,502		781,383		2,709,119		3.47		1.83		40.54	
	8	1252		3,419,977		835,638		2,584,339		3.09		2.01		68.93	
	9	1250		3,416,345		847,654		2,568,691		3.03		2.16		69.37	
	10	466		1,300,053		338,906		961,147		2.84		2.26		22.39	
	Total/Ave	12,174	6698	33,952,106	18,653,331	7,454,869	3,965,771	26,497,237	14,826,614	3.55	3.74	1.98	1.82	346.31	120.29
	NPV [US\$ mil] at an annual discount rate of 10 %													200.24	97.33
	NPV [US\$ mil] at an annual discount rate of 10 % (integrated LT and MT schedule)													202.34	

$$w_{n, ch}^{f, \prime} = \frac{1}{e^{\frac{f_{n, ch}^{\prime}}{V_{n, ch}^{\prime}}}} \quad (38)$$

where $w_{n, ch}^{f, \prime}$ is the exponential weight of block n along a chromosome.

The paper also applied heuristic mutation proposed by Gen and Cheng (2000), which is a powerful genetic operator for improving chromosome diversity. Fig. 5 illustrates the genetic procedure of the heuristic mutation. During heuristic mutation, new potential solutions are generated by modifying a chromosome to produce multiple mutated chromosomes. This process involves swapping some weak genes, to explore new regions of the solution space and potentially find better solutions. The purpose of heuristic mutation is to introduce diversity into the population and prevent premature convergence on sub-optimal solutions. By allowing exploration beyond what is dictated by crossover operations, heuristic mutation can help improve the overall performance of the GA in finding optimal or near-optimal solutions to complex problems (Gen et al., 2008), such as the OPMPs problem.

In Fig. 5, Genes 1, 9 and 10 are selected at Positions 3, 6 and 10, respectively. These genes are then swapped with stronger genes to produce five mutated chromosomes which are fitter than the parent chromosome. As stated earlier, crossover and mutation operations are repeated in the next generation to create another generation until no more improvement in fitness values of chromosomes can be made.

3.2. Framework of the genetic algorithm framework applied to the MIP production scheduling model

Fig. 6 illustrates the genetic algorithm framework which was applied in solving the MIP scheduling model. The appendix shows the pseudo-code for the framework. The framework includes links that connect the LT to the MT production scheduling, and feedback loops from MT to LT scheduling to update the production scheduling system. Once the termination condition is met to deliver the solution for a scheduling period, the algorithm moves to the next scheduling period until the last period is executed. The algorithm starts by running the LT scheduling process, which triggers the MT scheduling process through the connecting links. Feedback loops enable updating LT scheduling based on changes to the MT scheduling. After several generations, the solution in each period is found when the average fitness of the new generation is approximately equal to the average fitness of the last generation. The best solution is the highest value in the last generation. The algorithm ends when the MT scheduling process is complete. In case any changes occur on mining, processing or grade target parameters; at either LT or MT scheduling, the algorithm must rerun from LT scheduling to apply the changes and update the whole system.

The combined MIP model and GA approach requires to be set up with GA parameters to enable the implementation of the algorithm. GA parameters affect both the performance and results of the model. Table 5 describes the GA input parameters used in this paper.

4. Experimental results and discussion

4.1. Data description and isolated LT production scheduling results presentation

Four datasets were used in this paper to test and validate the practicability of the combined MIP model and GA approach. The test was conducted on a Geovia Surpac® block model dataset that represents a shallow gold mineral deposit (Geovia, 2024). The size of blocks is 10m × 10m × 10m, and they contain descriptive attributes such as grade, lithology, density, mining cost, processing cost, commodity price, and block economic value (Geovia, 2024). The other three datasets are the Newman, Zuck Small and KD block models which were obtained from the MineLib database (MineLib, 2012). The MineLib database is freely available in the public domain to be used for research experiments. For

Table 9

Results from running the GA five times on each of the four block models.

	Geovia			Newman			Zuck Small			KD		
	LOM [year]	NPV [US\$ mil]	Computation time [min]	LOM [year]	NPV [US\$ mil]	Computation time [min]	LOM [year]	NPV [US\$ mil]	Computation time [min]	LOM [year]	NPV [US\$ mil]	Computation time [min]
Run 1	10	201.69	46.59	4	22.01	0.23	11	935.38	98.03	6	397.27	161.57
Run 2	10	202.04	48.67	4	22.12	0.25	11	967.09	112.35	6	380.96	118.25
Run 3	10	202.29	52.97	4	22.07	0.26	11	941.32	94.35	6	384.99	294.37
Run 4	10	202.34	50.64	4	22.55	0.34	11	956.19	105.37	6	388.30	305.10
Run 5	10	201.61	49.69	4	22.22	0.37	11	951.00	106.19	6	390.58	123.71
Average	10	201.99	49.71	4	22.19	0.29	11	950.20	103.26	6	388.42	200.60

validation, the same input parameters for the three MineLib block models were also utilized in the combined MIP model and GA approach. The data pertaining to input parameters includes mining and processing rates (minimum and maximum rates), ore grade as a geological parameter, economic parameters (discount rate and block economic value) and geotechnical parameters (slope angle defined by the maximum number of direct block predecessors of each block). The description of input data for the four block models is presented in Table 6.

4.2. Results of the combined MIP model and GA approach applied to block models

When running the GA algorithm, the size of the population of chromosomes was set to 100 and each chromosome complies with the block precedence constraint to reduce the extent of the solution space. The application of the combined MIP model and GA approach to the Geovia Surpac® block model generated a 10-year LT production schedule. The crossover rate varied between 0.0148 and 0.0160 for the LT optimization and between 0.0296 and 0.0320 for the MT optimization. The mutation rate varied between 0.007 and 0.008 for the LT optimization and between 0.014 and 0.016 for the MT optimization. Table 7 presents the results of the production schedule obtained when optimizing the LT schedule in isolation for the Geovia Surpac® block model.

The application of the combined MIP model and GA approach on the Geovia Surpac® block model generated an integrated LT and MT production schedule of 10 years. Fig. 7 illustrates weights of chromosomes generated in the first generation of each period in ascending order, from the least fit to the fittest chromosome. The optimization process evolves these chromosomes through the use of genetic operators to reach their highest fitness values. Chromosomes with higher weights have a higher chance of being selected for crossover and mutation processes. The algorithm returned Period 10 without being optimized because the period represents the last period of the scheduling process.

The average fitness value of a generation increases until no more improvement in the fitness value could be made. Fig. 8 shows the average fitness for the first and the last generation for scheduling Periods 1 to 9. The model improved negative average fitness values to positive fitness values by 205 %, 332 % and 383 % in Periods 2, 3 and 4, respectively. This shows the optimization strength of the combined MIP model and GA approach. The highest improvement was recorded in Period 5 with an improvement of about 417 %, while the lowest improvement was 4 % from the first to the last generation in Period 8. The average fitness of the first generation was improved by 187 %, 52 %, 9 % and 6 % in Periods 1, 6, 7 and 9. The algorithm may yield different average fitness values and different numbers of generations despite using the same inputs due to the randomness of the solution procedure. For each scheduling period, the best chromosome of the last generation represents the best scheduling solution for that period.

The fitness values or cash flows of the best solutions for each scheduling period are shown in Fig. 9. It can be noted that the GA algorithm provides positive cash flows during each scheduling period.

This demonstrates the effectiveness of the GA algorithm in producing a sustainable production scheduling. The integrated LT and MT scheduling results obtained using the GA algorithm are presented in Table 8. Considering the LT horizon of 10 years and its yearly scheduling period length, the MT breaks down the yearly LT schedule into scheduling periods of six months long over the first five years of scheduling. Therefore, the half-yearly MT production schedule provides an extraction sequence that optimally splits each annual mining area defined in the LT schedule, into two mining areas.

Comparing the integrated LT and MT scheduling results in Table 8 with results in Table 7 from LT scheduling results when the LT schedule is optimized in isolation, the following observations can be noted.

- The results show that the integrated solutions comply with mineral reserves (total number of blocks equal to 12,174) and mining rates (ranging between 3.4 Mt and 3.8 Mt for LT, and between 1.7 Mt and 3.4 Mt for MT), which are considered hard constraints, from Years 1–9.
- The integrated LT and MT schedule complies with the processing rate (soft constraint) from Years 2–9 (ranging between 0.65 Mt/year and 1.0 Mt/year for LT, and between 0.325Mt/half-year and 0.5Mt/half-year for MT). Year 1 exceeds the annual maximum processing rate limit by 15.38 Kt of ore, which can be stockpiled for future processing. This is also noted for the MT schedule where Half-Year 1 has an excess of 86.76 Kt of ore compared to the 6-month maximum processing rate limit (500Kt/half-year).
- A processing penalty cost was incurred in Year 1 of the LT schedule where the processing rate limit was violated by 15.38 Kt resulting in a processing penalty cost of US\$693,000 (15.38 x 45,058), which was subtracted from the cash flow in Year 1, and added to the cash flow in Year 2. Similarly, 86.76 Kt of excess ore in MT schedule Half-Year 1 resulted in a processing penalty cost of US\$5,360,000 (86.76 x 61,779), which was subtracted from the cash flow in Half-Year 1, of which US\$4,667,000 was added to the cash flow in Half-Year 2 and US\$693,000 was added to the cash flow in Half-Year 3.
- During the first five years, the results show that LT objectives are met on MT horizons in all metrics. For example in Year 1, the LT 1350 blocks are split into 675 blocks and 675 blocks; the LT mining rate 3.89 Mt is split into 1.88 Mt and 1.87 Mt; the LT processing rate 1.015 Mt is split into 0.59 Mt and 0.43 Mt; the LT average grade of 1.98 g/t is split into 2.13 g/t and 1.77 g/t weighted average grades; and the LT cash flow of US\$45.76mil is split into US\$36.25mil and US\$9,52mil.
- During the first five years, the results show that the LT scheduling is aligned to the MT scheduling and that MT scheduling feeds back to the LT scheduling. For example, a combined MT mining rate achieved in the same year always results in the same LT mining rate.
- The NPV for the integrated LT and MT schedule over 10 years of operation was US\$202.34mil which is 2.20 % higher than the LT optimization NPV of US\$197.99mil.
- The results indicate that temporal and spatial scheduling compliances are satisfied as opposed to when the LT is optimized in isolation. For example, the same number blocks (6698 blocks) are

Table 10
Summary of the best run results of the combined MIP model and GA approach applied to four block models.

Block model	No. blocks	LOM [year]	Chromosome size	Crossover rate	Mutation rate	Population size	Isolated LT/Integrated LT-MT	Computation time [min]	NPV [US\$ mil]	Scheduling compliance LT and MT [%]			
										TopoSort	GA	RM	V
Geovia Surpac®	12,174	10	1250–1350	0.0148–0.0160	0.0070–0.0080	100	Isolated LT	34.28	N/A	197.99	100.48	101.85	111.95
Newman	1060	4	250–350	0.0343–0.0480	0.0171–0.0240	40	Integrated LT-MT	50.64	N/A	202.34	100.00	100.00	100.00
							Isolated LT	0.23	23.65	21.66	93.93	98.75	100.28
Zuck Small	9400	11	650–900	0.0733–0.1015	0.0367–0.0508	80	Integrated LT-MT	0.37	N/A	22.50	100.00	100.00	100.00
							Isolated LT	84.02	872.37	624.01	86.22	83.93	64.04
KD	14,153	6	2100–2400	0.0217–0.0247	0.0108–0.0125	100	Integrated LT-MT	112.35	N/A	967.09	100.00	100.00	100.00
							Isolated LT	135.07	406.87	323.98	105.47	108.98	110.25
							Integrated LT-MT	161.57	N/A	397.27	100.00	100.00	100.00

scheduled yearly, and the same blocks are scheduled half-yearly during the first five years of operation.

- The results of the integrated schedule indicate that the alignment between LT and MT scheduling stages had production scheduling compliance ratio and cash flow compliance ratio equal to 1 in each period. The compliance ratios were determined using Equations (39) and (40), respectively (Otto and Musingwini, 2020). The mine production compliance is an intertemporal and spatial metric for measuring the total actual material mined, both in and out of the integrated scheduling system, as a percentage of the scheduled material for the scheduling periods under review.

$$LT \text{ to } MT \text{ production compliance} = \frac{\text{Total actual } MT \text{ material mined}}{\text{Total } LT \text{ material scheduled}} \quad (39)$$

$$LT \text{ to } MT \text{ cash flow compliance} = \frac{\text{Total actual } MT \text{ cash flow}}{\text{Total } LT \text{ cash flow}} \quad (40)$$

If the ratio in Equation (39) is greater than 1, it indicates that more material has been mined than initially scheduled for the LT or MT schedule, suggesting potentially accelerated production. If the ratio is less than 1, it implies that less material has been mined than scheduled, which may indicate delays or slower progress in meeting the LT or MT production targets. A similar logic applies to the interpretation of the cash flow compliance ratio results. The compliance ratios will vary from period to period, hence an overall average compliance ratio is then calculated to get a holistic view. The integrated model can be expected to generate better compliance than the isolated case, because it enforces compliance by only selecting blocks to be mined within each MT period to be from the optimized set of blocks from the LT schedule.

Since the GA algorithm is stochastic, it was run five times on the MIP model, each time generating a different result for each block model. The worst, best and average results are reported in Table 9. From Table 9, the computation time generally increases as the block model size increases.

To validate the combined MIP model and GA approach it was necessary to compare its results to the best-known feasible solutions reported by MineLib obtained when the production scheduling problem was formulated as a linear programming (LP) relaxation model and solved using the TopoSort algorithm. TopoSort is a heuristic method used for ordering nodes of a directed acyclic graph in a linear sequence. It generates a sequence that respects dependencies, making it useful for tasks like scheduling (Pang et al., 2015). The same input parameters used on each respective MineLib block model were also applied in the combined MIP model and GA approach.

Table 10 shows a summary of the best run results of the combined MIP model and GA approach, and the LP relaxation model and TopoSort approach applied to the Geovia Surpac®, Newman, Zuck Small, and KD block models.

These results demonstrate that in the Zuck Small case study, the integrated LT and MT production schedule generated from the combined MIP model and GA approach achieved a higher NPV (US\$967.09mil) than the NPV (US\$872.37mil) for the isolated LT production scheduling generated from the LP relaxation model and TopoSort approach. In the Newman and KD case studies the combined MIP model and GA approach achieved lower NPVs (US\$22.50mil and US\$397.27mil) compared to the NPVs (US\$23.65mil and US\$406.87mil) for the best-known feasible production scheduling solutions obtained from the MineLib website where the production scheduling problem was formulated as a LP relaxation model and solved using the TopoSort algorithm, respectively. These results indicate that if other runs of the combined MIP model and GA approach are conducted, they may generate better solutions in the case studies where it initially underperformed due to the stochastic nature of the GA algorithm.

The advantage of production scheduling solutions from the combined MIP model and GA approach is the achievement of 100 % production scheduling compliance with the alignment of the LT and MT production schedules for the four case studies used in this paper. The

total material mined compliance for the isolated LT and MT production schedules had 93.93 %, 86.22 % compliance on Newman and Zuck Small, respectively; indicating delayed MT production progress in meeting LT production targets. However, on KD and Geovia Surpac® block models the isolated LT and MT production schedules achieved 105.47 % and 100.48 % compliance, respectively; indicating accelerated MT production progress in meeting LT production targets. Therefore, the production scheduling misalignment is characterized by either delayed or accelerated scheduling which could cause Mineral Resource sterilization or reduce NPV.

Table 10 indicates that the performance of GA is affected by GA parameters. For example, GA produces better solutions when higher crossover and mutation rates are used. The table also confirms that generally, computation time increases with larger population size (i.e., larger block model). Although the combined MIP model and GA approach produced acceptable solutions, it has some drawbacks such that multiple runs may be needed before finding satisfactory solutions. This is due to the stochastic nature of GA. An additional limitation is that the combined MIP model and GA approach optimizes the production scheduling process period-by-period, and this may lead to a period's scheduling solution being trapped on a local optimum, unless GA operations are further fine-tuned to improve the diversity of its search capability.

5. Conclusions

This paper presented a new MIP model and GA approach to solve an integrated LT and MT production scheduling problem in open-pit mine planning, as opposed to when these scheduling stages are optimized in isolation. The approach ensures that LT scheduling informs MT scheduling which then feeds back to the LT scheduling for subsequent updating as changes occur. The problem was then solved with an objective of maximizing NPV while considering mining rate, processing rate, ore grade target, block precedence, bench leads and Mineral Reserves constraints. The combined MIP model and GA approach was coded in Python and tested by implementing it on a Geovia Surpac® block model and validated on three other block models downloaded from the MineLib website. The best-known feasible solutions on MineLib that were obtained from solving a production scheduling LP relaxation model using a TopoSort heuristic algorithm were used to validate the combined MIP model and GA approach developed and presented in this paper. Although the combined MIP model and GA approach produced promising results, the approach may require further fine-tuning of GA operations and the running of multiple runs to achieve better solutions. This is due to the random generation of the initial population used in the optimization procedure from which the optimal solution is subject to.

The results from the Geovia Surpac® block model showed that the

combined MIP model and GA approach improves NPV by 2.20 % compared to isolated optimization of LT production scheduling. A comparison of NPV results was also made between the combined MIP model and GA approach and the combined LP relaxation model and TopoSort algorithm. The combined MIP model and GA approach outperformed the LP relaxation model and TopoSort approach by 10.90 % on the Zuck Small block model but underperformed by 4.90 % and 2.36 % on the Newman and KD block models, respectively. However, the combined MIP model and GA approach produced an integrated schedule that ensured LT objectives are achieved at MT horizons resulting in 100 % scheduling alignment between LT and MT production schedules. Future research will focus on extending the LT and MT scheduling approach to fully integrate all three LT, MT and ST scheduling stages, and consider incorporating operational constraints and stockpiling options to manage cases where misalignment occurs between any two consecutive scheduling stages.

CRedit authorship contribution statement

Pathy Muke: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Tinashe Tholana:** Writing – review & editing, Writing – original draft, Methodology, Conceptualization. **Cuthbert Musiwini:** Writing – review & editing, Writing – original draft, Supervision, Methodology, Conceptualization. **Montaz Ali:** Writing – review & editing, Validation, Methodology, Formal analysis, Conceptualization.

Declaration of generative AI and AI-assisted technologies

During the preparation of this paper the author(s) did not use any generative AI and AI-assisted technologies.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

The combined MIP model and GA approach was coded and executed in Python programming language. The pseudo-code algorithm is presented as Algorithm 1.

Algorithm 1. Pseudo-code algorithm of the combined MIP model and GA approach.

1	# Initialise parameters
2	input/update = { $s, g, L_i^{lt, RM}, L_i^{mt, RM}, U_i^{lt, RM}, U_i^{mt, RM}, L_i^{lt, RO}, L_i^{mt, RO}, U_i^{lt, RO}, U_i^{mt, RO}, L_i^{lt, RG}, L_i^{mt, RG}, U_i^{lt, RG}, U_i^{mt, RG}$ }
3	time period $t = 1, t = 1$
4	solution LTt (or MTt) = []
5	
6	# Main loop
7	while LT (MT) search_space s size > $U_i^{lt, RM}(U_i^{mt, RM})$:
8	# Initialise population
9	while size of population $g < 40$:
10	chromosome $ch = []$
11	while $L_i^{lt, RM}, L_i^{mt, RM} > \text{length}(ch)$ or $\text{length}(ch) < U_i^{lt, RM}, U_i^{mt, RM}$:
12	select random ore block [b] from s
13	block_list = $b.append(\text{block}[b] \text{ predecessors})$
14	$ch.append(\text{block_list})$
15	return ch
16	end while
17	
18	$g.append(ch)$
19	return $g = \text{set of } ch$
20	end while
21	
22	for each ch in G :
23	if constraint violation = Yes:
24	$ch.penalty = \text{penalty_cost}(ch)$
25	$ch.fitness = \text{evaluate_fitness}(ch)$
26	evaluate average_fitness of G
27	
28	# genetic operators
29	while termination_condition is not met:
30	select n fitter parents Pj of g
31	for each parent Pj :
32	select randomly Pr from g
33	offspring (Cj, Cr) = crossover(Pj, Pr)
34	offspring (Cj, Cr) = mutate(Cj, Cr)
35	
36	if constraint violation = Yes:
37	$offspring.penalty = \text{penalty_cost}(Cj, Cr)$
38	$offspring.fitness = \text{evaluate_fitness}(Cj, Cr)$
39	
40	
41	if $offspring.fitness(Cj, Cr) > \text{parents.fitness}(Pj, Pr)$:
42	replace two weak ch in new generation g with (Cj, Cr)
43	return best ch in the last generation $g = LTt$ (or MTt)
44	end while
45	
46	solution.append(LTt (or MTt))
47	search space $s = s.remove(LTt \text{ (or } MTt))$
48	time period $t += 1$, (or $t += 1$)
49	
50	return solution
51	end while
52	solution = solution.append(s)
53	end
54	

Data availability

Pseudo code shared.

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