

that measured immediately before the test. (Table IX)

- (ii) Comparing the two leakage measurements made after the triaxial test (Table IX), it appears that they are different to one another, but of more or less the same order of magnitude.
- (iii) As regards the comparison of leakage measurements made under steady conditions and those derived from the test simulating the disturbed conditions of an actual triaxial test, the values in Table X show clearly that the leakage occurring under the disturbed conditions caused by the loading spindle advancing into the test cell, is greater than even the maximum value of leakage measured at any time under steady cell conditions.

In trying to establish the causes of these experimental observations, it was necessary to examine more closely the factors affecting leakage and variations in the rate of leakage from the test cell:

Before commencement of any triaxial test, steady conditions exist in the test cell. These conditions are maintained only by the action of the particular chamber pressure selected for the test. Any leakage occurring usually takes place from the point where the loading spindle enters the test cell. This point was greased prior to positioning the test sample in order to reduce leakage during the test as much as possible. The rate of leakage under these steady cell conditions is indicated by the leakage measurement immediately before commencement of the test.

When the test is commenced, the advance of the loading spindle into the test cell results in an increased chamber pressure which causes a tendency for fluid flow out of the test cell, as has been previously fully discussed (Section 2.4). Simultaneously however, the volume of the test sample is reduced (initially) due to the compressive loading, which results in a reduction in the chamber pressure and hence a tendency for a resulting inflow of fluid into the test cell. The nett effect may be either an increase or decrease in the chamber pressure depending on which effect be the greater, and hence, there is a consequent increase or decrease in the rate of leakage from the test cell (usually however, a decrease). As failure of the test sample is approached, the compressive loading causes an expansion of the sample, resulting in an increased chamber pressure and a tendency for fluid flow out of the test cell. The nett effect then becomes a definite increase in chamber pressure giving rise to a sharp increase in the rate of leakage from the test cell. Grease conditions where the loading spindle enters the test cell are hereby so seriously disturbed that even when steady conditions again exist at the end of the test, the rate of leakage is usually higher than that which existed before the test was commenced. This means that the effect of the increased cell pressure at sample failure is usually greater than any initial reduction in cell pressure which could have occurred while the sample volume was decreasing. Hence, the phenomenon observed as mentioned in (i) above.

Following on the first leakage measurement immediately after the triaxial test, the test for determining the leakage under disturbed cell conditions simulating those existing during an actual test was carried out. Raising the loading spindle in preparation for this test and then subsequently the advance of the loading spindle into the test cell, again disturbs the grease conditions at the point where the loading spindle enters

the/

the test cell, causing the leakage measured after this test to be different to the previous two measurements, but apparently of the same order of magnitude as the leakage measured immediately after the triaxial test. This is the phenomenon observed as mentioned in (ii) above.

Further, Table X shows clearly that the value of leakage determined from this test, is in every case greater than even the maximum value of leakage measured under steady conditions before and after this test. The reason is clearly that during the test an increased rate of leakage results due to the increased chamber pressure caused by the advance of the loading spindle. The fact that the two leakage measurements after the triaxial test generally show little agreement with the measurement carried out before the triaxial test, is due to the former two measurements not being relatively affected by any sample volume change effects.

Hence, it is also clear that the test which it was hoped would simulate the disturbed cell conditions of an actual triaxial test, was not a good representation, as it did not cater for the phenomenon of an initially reduced and then increased chamber pressure due to decrease and increase in sample volume respectively.

However, although the main problem of determining the rate of leakage under the disturbed cell conditions of an actual test has thus not been solved by the information obtained from the leakage measurements that were carried out, it is nevertheless possible, as a result of the foregoing discussions, to obtain a better understanding of the problem which is likely to enable a more reliable estimation of the rate of leakage during an actual test based on the data that was obtained.

The main conclusions from the above discussions are that:

- (i) as long as there is a flow of fluid into the test cell (which is indicated by a slug movement in the capillary tube from right to left, i.e., decreasing slug scale readings), there is a pressure deficiency in the test cell giving rise to this flow, and hence, the rate of leakage from the test cell is less than under the steady cell conditions existing immediately before the start of the test.
- (ii) Immediately the fluid flow starts to be out of the test cell (as indicated by a reversal in the direction of the slug movement in the capillary tube, i.e., increasing slug scale readings), there is a pressure excess in the test cell giving rise to this flow and hence, there is an increase in the rate of leakage from the test cell.

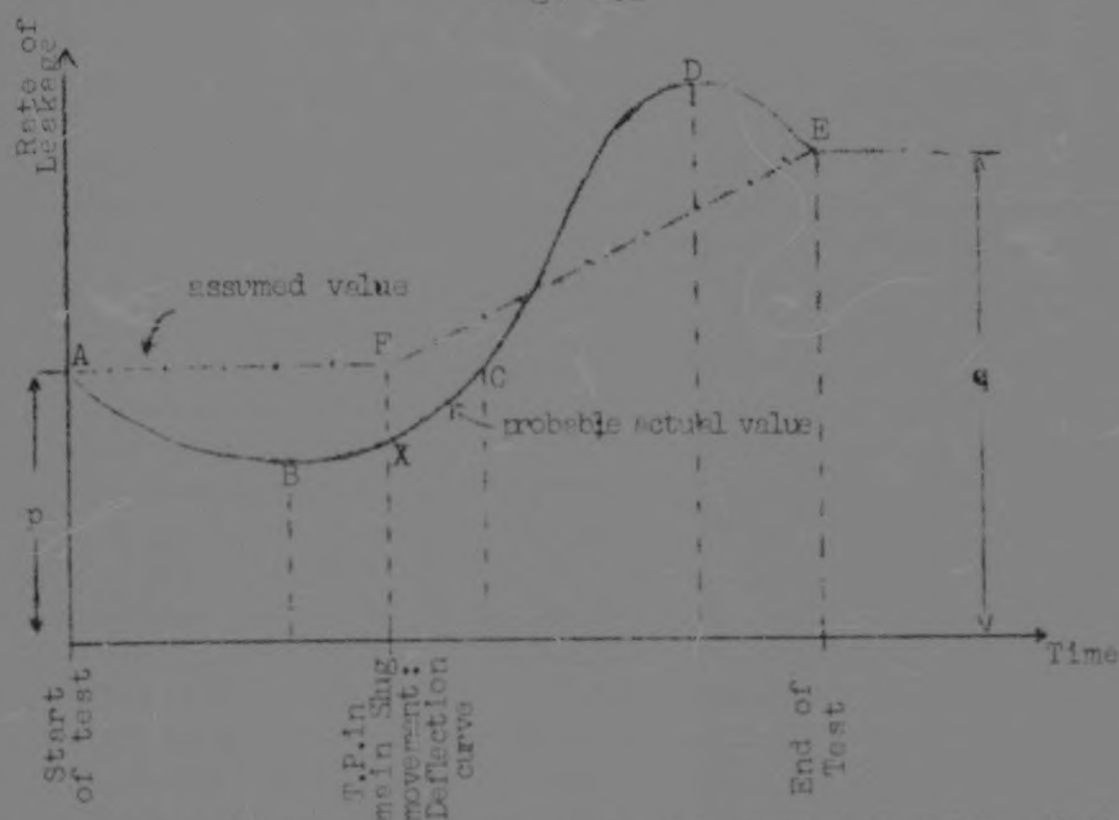
With regard to the Main Slug Movement: Deflection curves, this means that in case over the portion of the curve to the left of the trough-shaped turning point, (i.e. the portion resulting from decreasing slug scale readings), there is a pressure deficiency in the test cell, while over the portion to the right of this turning point, (i.e. the portion resulting from increasing slug scale readings), there is a pressure excess in the test cell. This statement ignores the possible small pressure increase close to the start of the curve resulting from the small peak due to bedding-in effects, which is observed in some cases immediately before the main downward portion of the curve commences — an effect which was fully discussed in Section 7.7.

Hence, it is considered that subsequent to the rate of

leakage/

leakage existing under the steady cell conditions immediately before the start of a test, the rate of leakage varies approximately according to the relationship shown in the diagram below:

Fig. 8.1



In this diagram, p represents the measured value of the rate of leakage under steady cell conditions immediately before test, while q represents the measured value of the rate of leakage under steady cell conditions immediately after test. As soon as the test commences there is a pressure deficiency in the test cell and therefore a resulting decrease in the rate of leakage. This pressure deficiency continues, but gradually decreases as the turning point of the Main Slug Movement: Deflection curve is approached, thereby tending to restore steady pressure conditions in the test cell. At the turning point when steady conditions exist and there is neither inflow nor outflow from the test cell, the rate of leakage would probably be a value which is greater than the minimum value attained (ordinate at B) while the pressure deficiency existed, but probably less than the value before commencement of the test (ordinates at A and C), i.e., some value between points B and C as shown (ordinate at X).

As soon as the turning point in the Main Slug Movement: Deflection curve is passed, there is a pressure excess in the test cell and hence a rapid increase in the rate of leakage to some maximum value as indicated by the ordinate at D. When the test is stopped, steady conditions again set in (pressure excess then reduced to zero), and the corresponding rate of leakage which is the value measured immediately after the test, is somewhat less than the maximum but, according to actual measurements, somewhat greater than the value for the steady conditions existing immediately before test (ordinates at A and C) i.e., possibly as represented by the ordinate at E.

Hence, as an approximation to the curve ABCDE shown, it was decided to assume that the initial measured value of the rate of leakage under the steady cell conditions before test (ordinate at A), remains constant until the turning point in the Main Slug Movement: Deflection curve is reached (which is a known point with regard to time), and then to assume that the rate of leakage increases uniformly to the value measured under the steady cell conditions immediately after test. In

cases/

cases where more than one value was obtained for the rate of leakage after test (See Table IX), the maximum value was used, as it is considered that this value would be closer to the actual maximum value attained at the end of the test, which is represented by the ordinate at D.

The above procedure was followed in all calculations relating to the determination of instantaneous values of Sample Dilation ($\delta V/V$) and Poisson's Ratio (μ). Although the results obtained in this way may not have been strictly accurate, it is considered that they were sufficiently accurate for the purposes required, as in actual fact, the proportion of slug movement due to leakage only in the overall measured value of slug movement due to the three combined effects is small (see calculation sheets, Section 8.5). Hence, an error in the value of slug movement due to leakage only, does not cause a serious error in the value of Sample Dilation or Poisson's Ratio.

However, in order to demonstrate this fact, the calculation of Poisson's Ratio for the Selected Range of each stress:strain curve was carried out, using both the measured value of leakage under the steady cell conditions existing before test, as well as the (maximum) measured value of leakage under the steady cell conditions existing immediately after test. The results show an insignificant difference in the value of μ obtained (see Section 9.5)

The values of μ based on the value of the rate of leakage under steady cell conditions immediately before test, are considered to be more reliable, as the actual rate of leakage during test over the Selected Range corresponds closer to this value than the one relating to conditions at the end of the test. The reason for this is that the Selected Range of the stress:strain curve always falls within that range of the Main Slug Movement: Deflection curve which is to the left of its turning point, i.e., the range over which there is a pressure deficiency in the test cell. Therefore, the actual rate of leakage over the Selected Range is in each case a value which is lower than the value measured under the steady cell conditions immediately before the start of the test.

8.3 Time Effects during Test

The reduction in rate of strain during test, (the causes of which were discussed at the end of Section 7.7), has a very important bearing on the subsequent reduction of test results.

This statement is made, mainly as a result of the effect of a reduction in rate of strain on the calculated value of actual slug movement due to change in volume of the test sample only. The original intention with regard to the latter determination, was merely to combine:

- (i) whatever value of rate of leakage during test (as a slug movement in millimetres/minute) be decided on, with
- (ii) either the nominal or calibrated rate of strain (in units of divisions deflection/minute, i.e., units of 10^{-3} inches/minute), both of which were considered to be constant throughout the duration of a test.

Item (i) divided by item (ii) would give the rate of leakage from the test cell as a slug movement in millimetres/division deflection, which would then be in the same units as obtained for the calculated value of slug movement due to entry of the loading spindle into the test cell (see Appendix V, Item 2),

at/

as well as in the same units as the measured value of total slug movement due to the three combined effects during test (Section 7.3). The subsequent combination of these three rate of slug movement values in order to determine the actual slug movement in millimetres due to volume change of the test sample only over any particular range of strain, would then present no difficulty.

If the rate of strain during test remains constant, this procedure is in order. However, leakage from the test cell is in the first instance dependent on time. Hence, with the method discussed above, this determination may be seriously in error if, while the determination of rate of slug movement due to leakage only (in units of mm./div. deflection) be based on some rate of strain which is considered to be constant throughout the test, the actual rate of strain during part of the test be considerably lower than the value used. The reason for this is that, the lower the rate of strain, the greater is the proportion of actual slug movement due to leakage only in the measured value of total slug movement due to the three combined effects, the values of slug movement due to spindle entry and sample volume change only being independent of time.

Hence, it was necessary to express all measured rates of slug movement (whether they be in units of millimetres/minute or millimetres/division deflection), as actual slug movements in millimetres over the particular range of strain considered. For slug movements measured in millimetres/minute it was thus necessary to determine the actual time interval corresponding to the particular range of strain considered, in order to obtain an actual slug movement in millimetres. Hence also, the need for continuous time recordings during test mentioned in Section 7.5.

In cases where continuous time recordings during test were taken, the actual time interval corresponding to any particular range of strain could be directly obtained. In these cases it was therefore possible to evaluate Sample Dilation ($\delta V/V$) and Poisson's Ratio (μ) for each increase in strain (working from the same starting point each time - usually the corrected start of the stress:strain curve as previously discussed in Section 7.7), and so to obtain curves of instantaneous Sample Dilation:Strain ($\delta V/V : \epsilon$) and instantaneous Poisson's Ratio : Strain ($\mu : \epsilon$).

In cases where continuous time recordings during test had not been taken, (see remarks in this respect in Section 7.5), this was not possible, and it was only possible to determine the value of Sample Dilation and Poisson's Ratio corresponding to the Selected Range of the Stress:strain curve in each case. To enable even these determinations, it was necessary to estimate the value of the actual rate of strain over the Selected Range, in order to obtain the time interval corresponding to the duration of the Selected Range. The basis on which this estimate could be made, is discussed in the following paragraphs.

Careful investigation of the deflection:time curves for the tests in which detailed time recordings were taken (Nos. 1, 3, 4 and 6), revealed the facts shown in Table XI on the following page:

TABLE XI/

as well as in the same units as the measured value of total slug movement due to the three combined effects during test (Section 7.3). The subsequent combination of these three rate of slug movement values in order to determine the actual slug movement in millimetres due to volume change of the test sample only over any particular range of strain, would then present no difficulty.

If the rate of strain during test remains constant, this procedure is in order. However, leakage from the test cell is in the first instance dependent on time. Hence, with the method discussed above, this determination may be seriously in error if, while the determination of rate of slug movement due to leakage only (in units of mm./div. deflection) be based on some rate of strain which is considered to be constant throughout the test, the actual rate of strain during part of the test be considerably lower than the value used. The reason for this is that, the lower the rate of strain, the greater is the proportion of actual slug movement due to leakage only in the measured value of total slug movement due to the three combined effects, the values of slug movement due to spindle entry and sample volume change only being independent of time.

Hence, it was necessary to express all measured rates of slug movement (whether they be in units of millimetres/minute or millimetres/division deflection), as actual slug movements in millimetres over the particular range of strain considered. For slug movements measured in millimetres/minute it was thus necessary to determine the actual time interval corresponding to the particular range of strain considered, in order to obtain an actual slug movement in millimetres. Hence also, the need for continuous time recordings during test mentioned in Section 7.5.

In cases where continuous time recordings during test were taken, the actual time interval corresponding to any particular range of strain could be directly obtained. In these cases it was therefore possible to evaluate Sample Dilation ($\delta V/V$) and Poisson's Ratio (μ) for each increase in strain (working from the same starting point each time - usually the corrected start of the stress:strain curve as previously discussed in Section 7.7), and so to obtain curves of instantaneous Sample Dilation:Strain ($\delta V/V : \epsilon$) and instantaneous Poisson's Ratio : Strain ($\mu : \epsilon$).

In cases where continuous time recordings during test had not been taken, (see remarks in this respect in Section 7.5), this was not possible, and it was only possible to determine the value of Sample Dilation and Poisson's Ratio corresponding to the Selected Range of the Stress:strain curve in each case. To enable even these determinations, it was necessary to estimate the value of the actual rate of strain over the Selected Range, in order to obtain the time interval corresponding to the duration of the Selected Range. The basis on which this estimate could be made, is discussed in the following paragraphs.

Careful investigation of the duration:time curves for the tests in which detailed time recordings were taken (Nos. 1,3,4 and 6), revealed the facts shown in Table XI on the following page:

TABLE XI/

TABLE XI

Test No.	RATE OF STRAIN		AVERAGE RATE OF STRAIN DURING TEST OVER SELECTED RANGE (=b) inch/min.	REDUCTION IN RATE OF STRAIN OVER SELECTED RANGE (=b/a) percent	REMARKS
	NOMINAL inch/min.	CALIBRATED(=a) inch/min.			
1	0.00256	0.002555	0.000933	36.5%	Selected Range for each test includes the pronounced time-lag effects soon after test commencement.
3	0.00256	0.002544	0.000909	31.3%	
6	0.00256	0.002500	0.000903	36.2%	
4	0.01250	0.01282	0.004000	31.2%	
				Av=33.9%	

It was also observed that the Selected Range for each of the four tests listed above, included the range over which the pronounced time-lag effects occurred (discussed in Section 7.7). In the case of the two tests during which continuous time recordings had not been taken (Tests No. 2 and 5), the strain at which the pronounced time-lag effects commenced had been noted down during the original test observations, and hence it was known that their Selected Ranges (as chosen), also include these effects.

It therefore seemed reasonable to conclude that approximately the same reduction in rate of strain had occurred over their respective Selected Ranges, as was found to be the case with Tests No. 1, 3, 4 and 6. As the latter tests had shown an average reduction in rate of strain over their Selected Ranges of 33.9% (calculated with respect to the calibrated rate of strain in each case) with very little variation among individual values, it was decided to adopt a value of 33.9% of the calibrated rate of strain, or, where this was not available, simply 33.9% of the nominal rate of strain, as the actual rate of strain during test over the Selected Range for the two tests concerned.

8.4 Initial Volume of test samples

Under this heading, a clear definition is in the first instance considered to be essential. By initial volume of test samples is meant the volume of test samples as at the commencement of the actual triaxial test, i.e., the sample volume corresponding to zero strain. It is this volume, relative to which all subsequent sample volume changes during test are considered to occur, and hence, it is this volume which is required for substitution in equation 7.5 for the evaluation of Poisson's Ratio.

In approaching the problem of determining this volume, the two factors which have a direct bearing on the initial volume of test samples, need to be considered in detail. These are:

- (i) The volume of a test sample after being extruded from the cutting cylinder. (Let this volume be denoted V_1).
- (ii) the subsequent change in this volume V_1 due to consolidation of the sample in the triaxial cell during the pre-shear condition. Let the volume at the completion of the consolidation process be denoted V_2 .

With regard to item (i) above: It was previously mentioned that the test soil was a rather dry sandy soil, so that in

sliding/

TABLE XI

Test No.	RATE OF STRAIN		AVERAGE RATE OF STRAIN DURING TEST OVER SELECTED RANGE (=b) inch/min.	REDUCTION IN RATE OF STRAIN OVER SELECTED RANGE (=b/a) percent	REMARKS
	NOMINAL inch/min.	CALIBRATED(=a) inch/min.			
1	0.00256	0.002555	0.000933	36.5%	Selected Range for each test includes the pronounced time-lag effects soon after test commencement.
3	0.00256	0.002544	0.000809	31.8%	
6	0.00256	0.002500	0.000903	36.2%	
4	0.01250	0.01282	0.004000	31.2%	
				Av=33.9%	

It was also observed that the Selected Range for each of the four tests listed above, included the range over which the pronounced time-lag effects occurred (discussed in Section 7.7). In the case of the two tests during which continuous time recordings had not been taken (Tests No.2 and 5), the strain at which the pronounced time-lag effects commenced had been noted down during the original test observations, and hence it was known that their Selected Ranges (as chosen), also included these effects.

It therefore seemed reasonable to conclude that approximately the same reduction in rate of strain had occurred over their respective Selected Ranges, as was found to be the case with Tests No. 1,3,4 and 6. As the latter tests had shown an average reduction in rate of strain over their Selected Ranges of 33.9% (calculated with respect to the calibrated rate of strain in each case) with very little variation among individual values, it was decided to adopt a value of 33 $\frac{1}{3}$ % of the calibrated rate of strain, or, where this was not available, simply 33 $\frac{1}{3}$ % of the nominal rate of strain, as the actual rate of strain during test over the Selected Range for the two tests concerned.

8.4 Initial Volume of test samples

Under this heading, a clear definition is in the first instance considered to be essential. By initial volume of test samples is meant the volume of test samples as at the commencement of the actual triaxial test, i.e., the sample volume corresponding to zero strain. It is this volume, relative to which all subsequent sample volume changes during test are considered to occur, and hence, it is this volume which is required for substitution in equation 7.5 for the evaluation of Poisson's Ratio.

In approaching the problem of determining this volume, the two factors which have a direct bearing on the initial volume of test samples, need to be considered in detail. These are:

- (i) The volume of a test sample after being extruded from the cutting cylinder. (Let this volume be denoted V_1).
- (ii) the subsequent change in this volume V_1 due to consolidation of the sample in the triaxial cell during the pre-shear condition. Let the volume at the completion of the consolidation process be denoted V_2 .

With regard to item (i) above: It was previously mentioned that the test soil was a rather dry sandy soil, so that in

sliding/

sliding the cutting cylinder downwards while trimming a test sample and, subsequently, when extruding the test sample from the cutting cylinder, a thin layer of soil viz. that which was previously in contact with the inside face of the cutting cylinder, is loosened from the test sample and is lost. Therefore, the final sample diameter is slightly less than the actual inside diameter of the cutting cylinder and hence, the final sample volume is slightly less than the internal volume of the cutting cylinder.

Regarding item (ii): The consolidation process in the triaxial test cell which test samples are subjected to during the pre-shear condition, is a 3-dimensional effect. As yet, the change in volume of a test sample during this type of consolidation process can only be evaluated with difficulty for the case of saturated samples, but cannot be evaluated in the case of partially saturated samples such as is the case with the test samples being considered.

Thus, as both the quantities concerned cannot be readily evaluated (if at all), it is necessary to consider what arbitrary reduction in the internal volume of the cutting cylinder should be allowed in order to obtain the initial volume of a test sample. Further, as the same cutting cylinder was used throughout for trimming the test samples, it will also be assumed that the initial volume of all test samples was the same throughout.

The internal dimensions of the cutting cylinder were $1\frac{1}{2}$ " dia. x 3" length, thus giving a volume of 86.89 cub.cm. Assuming that on removal from the cutting cylinder a sample is $\frac{1}{64}$ " short in diameter, i.e. only $1\frac{31}{64}$ " in diameter, its volume would be 85.09 cub.cm., thus a reduction of 1.80 cc. As regards the additional decrease in volume due to the consolidation process during the pre-shear condition, it is considered that this reduction, as effected by the chamber pressures used viz. 10lb/sq.in., 20lb/sq.in. and 30lb/sq.in., could not have been appreciable, since, from the normal 1-dimensional consolidation test which was conducted (Section 7.5), the test soil was found to be fairly heavily preconsolidated (p_c approx. 8Ton/sq.ft. - see Fig.18), with an undisturbed compression index value of 0.26.

On the basis of this information, it was decided to adopt a value of 85.0 cub.cm. as the initial volume of all test samples used for the test series. It is considered that a slight error in this value is not likely to cause a serious error in the value of Sample Dilatation ($\Delta V/V$) or Poisson's Ratio subsequently determined.

8.5 Calculation of Poisson's Ratio

On the following pages in this Section, the calculation of Poisson's Ratio for each test has been set out in tabular form; in the first instance, the calculation of Poisson's Ratio as obtained from the Selected Range of each stress:strain curve (giving a value denoted by μ_{sel}), and secondly, the calculation of instantaneous values of Poisson's Ratio (denoted μ_i) for each increase in strain along the stress:strain curve, but always working from the corrected start of the curve (i.e. bedding-in effects ignored).

In both cases the calculations follow along the same lines, which are broadly as follows for each particular range of strain considered:

(i) Determination of the time elapsed over the range considered.

(ii) Determination /

- (ii) Determination of the slug movement (in mm.) due to leakage only.
- (iii) Determination of the slug movement (in mm.) due to entry of the loading spindle into the test cell - using the rate of slug movement due to spindle entry determined as shown in Appendix V, Item 2.
- (iv) Determination of the slug movement (in mm.) due to change in volume of the test sample only, by combining (ii) and (iii) above with the measured total slug movement due to the three combined effects during test.
- (v) Determination of the change in volume of the test sample (δV) in units of 10^{-3} cub.cm. - using the value of the cross-sectional area of the capillary tube employed during the test series which was determined as shown in Appendix V, Item 1.
- (vi) Calculation of the Sample Dilation ($\delta V/V$) - using the initial volume of test samples as 85.0cc. According to the discussion in Section 8.4.
- (vii) Calculation of Poisson's Ratio based on equation 7.5 and using the relevant value of Young's Modulus (i.e. either the particular instantaneous or Esel value) as well as the relevant change in deviator stress value.

More specifically, however, the first calculations appearing are those for μ_{esl} . The calculations are accommodated in 3 pages and, for each test, deal with the various steps in the following order:

First page: Determination of data relevant to the Selected Range of each test only, viz.: the value of Young's Modulus (E_{esl}); the change in deflection (strain; the nett slug movement due to the three combined effects; and the time elapsed over the Selected Range.

Second page: Items (i) to (vi) inclusive, as detailed above.

Third page: Item (vii) above.

Then follow the calculations for μ_i . As previously discussed (Section 8.3), it was only possible to determine μ_i for tests in which continuous time recordings during test had been carried out i.e. Tests No. 1, 3, 4 and 6. The calculations in this case are more lengthy and occupy several pages. On the first calculation pages for each test, the items (i) to (vi) inclusive appear in columns all on one page for each increase in strain being considered. On the subsequent pages, item (vii) is set out in a similar manner, using the instantaneous values of Young's Modulus shown calculated in Section 7.6. The calculation of values of μ_i was in each case not continued very far past the end of the linear range of the stress:strain curve, and particularly not past the peak, as actually Poisson's Ratio becomes a doubtful concept once the limit of the linear range has been exceeded (See Chapter 2). However, the calculation of values of Sample Dilation ($\delta V/V$) were continued over the full range of each test.

For the results of all these calculations, it was possible to prepare curves of instantaneous Sample Dilation : Strain and instantaneous Poisson's Ratio : Strain as mentioned in Section 8.3. These curves are presented in each case subsequent to the μ_i calculation sheets and are shown plotted below the relevant stress:strain curve (bedding-in effects ignored) for each test.

The/

The following more detailed points about the calculations should, however, be noted:

An arrow above figures relating to slug movement should be interpreted in accordance with the meaning given at the head of Table IX, Section 8.2. In the case of the calculations pertaining to μ_{sl} , such arrows have been omitted, and instead, algebraic signs have been inserted in front of these figures. These algebraic signs merely serve to indicate how the figures have been combined in determining the slug movement due to change in volume of the test sample only (item (iv) on the previous page), and were arrived at as follows:

- (a) Slug movement due to leakage from the test cell is from right to left in the capillary tube - denoted $\leftarrow$
- (b) Slug movement due to entry of the loading spindle into the test cell is from left to right in the capillary tube - denoted $\rightarrow$
- (c) Slug movement due to the combined effects of leakage, spindle entry and sample volume change, is from right to left generally (i.e. over the main portion of the Main Slug Movement : Deflection curve), and is denoted $\leftarrow$ in these cases.

Thus, numerically: $(b) + (c) - (a) = M$, say,

gives the slug movement due to change in volume of the test sample only.

In cases where the slug movement (c) above is from left to right (such as sometimes happens at the start of tests), the relevant figure for item (c) has been given a negative sign.

A positive value for M denotes that the slug movement due to change in volume of the test sample only is from right to left in the capillary tube, i.e., the sample has experienced a volume decrease. Hence, also, a sample volume decrease yields a positive value for the Sample Dilatation $\delta V/V$, which is thus in accordance with the sign convention adopted as discussed in Section 7.1.

CALCULATION OF SAMPLE DILATION ($\frac{\Delta V}{V}$) BASED ON THE SELECTED RANGE OF EACH STRESS-STRAIN CURVE*

TEST NUMBER	TIME ELAPSED OVER SELECTED RANGE		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY $\frac{mm}{min}$	RATE OF SLUG MOVEMENT OVER SELECTED RANGE DUE TO SPINDLE ENTRY ONLY $\frac{mm}{div defl}$	NUMBER OF DEFLECTIONS OVER RANGE CONSIDERED	HENCE, SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY mm	% ALL MEASURED SLUG MOVEMENT OVER RANGE DUE TO THREE COMBINED EFFECTS mm	HENCE, SLUG MOVEMENT DUE TO PRESSURE ONLY OVER SELECTED RANGE mm	CROSS SECTIONAL AREA OF CAPILLARY TUBE $sq. mm$	HENCE, CHANGE IN VOLUME ΔV OF SAMPLE OVER RANGE OF STRAIN CONSIDERED cc	INITIAL SAMPLE VOLUME V cc	SAMPLE DILATION $\frac{\Delta V}{V}$ units of 10^{-3}
	MIN	SEC										
1.	13	56	0.92	1.614	13	12.818 39.012	125.25	133.414 107.220	1.9935	265.96 213.74	85.0	3.13 2.51
2.	3	00	1.27	1.614	12.5	3.810 12.000	111.50	127.865 119.675	1.9935	254.90 58.57	85.0	3.00 2.31
3.	9	53	1.54	1.614	8	15.220 26.355	84.50	82.192 71.057	1.9935	163.85 141.65	85.0	1.93 1.67
4.	2	30	0.875	1.614	10	2.187 5.000	68.50	82.433 79.640	1.9935	164.37 158.76	85.0	1.93 1.87
5.	1	46	0.500	1.614	7.5	0.885 1.770	65.00	76.220 75.335	1.9935	151.94 150.18	85.0	1.79 1.77
6.	13	16	0.875	1.614	12	11.60 12.564	114.00	121.750 120.804	1.9935	242.73 240.82	85.0	2.86 2.83

*NOTE WHERE TWO SETS OF FIGURES ARE SHOWN (ONE BELOW THE OTHER), THESE ARE INTENDED FOR COMPARISON PURPOSES:

THE UPPER FIGURES CORRESPOND TO CALCS. BASED ON THE RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY MEASURED IMMEDIATELY BEFORE TEST.

THE LOWER FIGURES CORRESPOND TO CALCS. BASED ON THE RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY MEASURED IMMEDIATELY AFTER TEST.

THE UPPER FIGURES ARE CONSIDERED TO LEAD TO MORE RELIABLE VALUES OF SAMPLE DILATION ($\frac{\Delta V}{V}$) AND POISSONS RATIO (μ)

CALCULATION OF POISSONS' RATIO (μ) BASED ON THE SELECTED RANGE OF EACH STRESS-STRAIN CURVE*

TEST NUMBER	SAMPLE DILATION $\frac{\Delta V}{V}$ units of 10^{-3}	YOUNG'S MODULUS E_{Sel} lb./sq.in	$\frac{\Delta V}{V} \times E_{Sel}$ lb./sq.in	CHANGE IN DEVIATOR STRESS ($\Delta \sigma_d$) $[E \Delta (\sigma_1 - \sigma_2)]$ OVER SELECTED RANGE. lb./sq.in	$\frac{\frac{\Delta V}{V} \times E_{Sel}}{\Delta \sigma_d} = N$	$1 - N$	POISSONS' RATIO (μ_{sel}) $= \frac{1}{2} [1 - N]$
1	3.13	8015	25.079	34.71	0.7225	0.2775	0.1387
	2.51		20.155		0.5807	0.4193	0.2097
2	3.00	7750	23.241	32.29	0.7198	0.2802	0.1401
	2.81		21.752		0.6736	0.3264	0.1632
3	1.93	7697	14.837	20.55	0.7220	0.2780	0.1390
	1.67		12.827		0.6242	0.3758	0.1879
4	1.93	9567	18.500	31.89	0.5801	0.4199	0.2099
	1.87		17.869		0.5603	0.439	0.2198
5	1.79	7212	12.892	18.03	0.7150	0.2850	0.1425
	1.77		12.742		0.7067	0.2933	0.1466
6	2.86	8125	23.202	32.49	0.7141	0.2859	0.1429
	2.83		23.020		0.7085	0.2915	0.1457

* SEE FOOTNOTE ON THE PREVIOUS PAGE.

TEST No 1 CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION(ΔV) (Cont.)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT OVER RANGE TO LEAKAGE ONLY	RATE OF HENCE SLUG MOVEMENT OVER RANGE TO SPINDLE ENTRY ONLY	RATE OF HENCE SLUG MOVEMENT OVER RANGE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS SLUG MOVEMENT DEFLECT OVER RANGE OVER CONSIDERED RANGE TO SPINDLE ENTRY ONLY	CONSIDERED	OBSERVED SLUG MOVEMENT OVER RANGE DUE TO THREE COMBINED EFFECTS	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSS SECTIONAL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOL (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME	SAMPLE DILATION ΔV
	min	sec	mm/sec	mm/sec	mm/sec	mm/sec	mm/sec	mm/sec	mm/sec	mm ²	mm ³ ($\times 10^{-3}$ cc)	cc/cm	cc/cm ³ of ΔV
0.000 - 0.553	15	50	0.92	- 14.260	1.614	16	+ 25.924	+ 144.5	+156.054	1.9935	311.11	85.0	3.66
0.000 - 0.687	16	03	0.92	- 14.766	1.614	17	+ 27.438	+ 154.25	+161.922	1.9935	332.76	85.0	3.91
0.000 - 0.800	16	37	0.92	- 15.288	1.614	19	+ 29.052	+ 164.40	+174.164	1.9935	355.17	85.0	4.18
0.000 - 0.853	17	12	0.92	- 15.824	1.614	19	+ 30.566	+ 174.0	+185.942	1.9935	376.46	85.0	4.43
0.000 - 0.867	17	40	0.92	- 16.254	1.614	20	+ 32.220	+ 180.25	+196.276	1.9935	391.26	85.0	4.60
0.000 - 0.900	18	17	0.92	- 16.920	1.614	21	+ 33.934	+ 192.0	+209.074	1.9935	416.79	85.0	4.90
0.000 - 0.950	19	58	0.92	- 18.370	1.614	24	+ 38.736	+ 219.0	+239.366	1.9935	477.18	85.0	5.61
0.000 - 0.900	21	47	0.92	- 19.810	1.614	27	+ 43.578	+ 243.5	+267.2	1.9935	532.80	86.0	6.27
0.000 - 1.000	22	57	0.92	- 21.114	1.614	30	+ 48.420	+ 266.0	+296.305	1.9935	582.71	86.0	6.86
0.000 - 1.200	25	56	0.92	- 23.858	1.614	36	+ 59.104	+ 300.25	+334.426	1.9935	666.82	86.0	7.84
0.000 - 1.400	28	31	0.92	- 26.236	1.614	42	+ 67.786	+ 331.75	+376.302	1.9935	744.18	86.0	8.76
0.000 - 1.800	31	13	0.92	- 28.720	1.614	48	+ 77.472	+ 356.5	+405.252	1.9935	807.87	86.0	9.50
0.000 - 1.800	33	50	0.92	- 31.126	1.614	54	+ 87.156	+ 378.0	+431.030	1.9935	859.26	86.0	10.11
0.000 - 2.000	36	21	0.92	- 33.442	1.614	60	+ 96.840	+ 399.5	+452.893	1.9935	902.86	86.0	10.62
0.200 - 2.200	37	52	0.92	- 34.838	1.614	66	+106.524	+ 401.0	+472.667	1.9935	942.30	86.0	11.08

TEST No 1 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION $(\frac{\Delta V}{V})$ (Cont.)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	RATE OF SLUG MOVEMENT DUE TO THREE COMBINED EFFECTS	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSS SECTIONAL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOL. (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
	min	sec	mm/min	mm/min	mm/min	mm/min	sq. mm	cub. mm $(\times 10^{-3} \text{ cc})$	cub. cm	units of 10^{-3}
0.000 - 2.400	41	23	0.92	- 36.072	1.614	+ 116.208	+ 409.0	+ 487.136	85.0	11.42
0.000 - 2.800	43	49	0.92	- 40.312	1.614	+ 125.692	+ 413.0	+ 496.580	85.0	11.69
0.000 - 2.900	45	10	0.92	- 42.474	1.614	+ 135.576	+ 414.5	+ 507.602	85.0	11.90
0.000 - 3.000	48	31	0.92	- 44.636	1.614	+ 145.260	+ 414.5	+ 515.124	85.0	12.00
0.000 - 3.200	50	50	0.93	- 47.275	1.614	+ 154.944	+ 414.25	+ 521.919	85.0	12.24
0.000 - 3.400	53	12	1.10	- 58.520	1.614	+ 164.628	+ 412.5	+ 516.609	85.0	12.16
0.000 - 3.600	55	32	1.27	- 70.827	1.614	+ 174.312	+ 409.75	+ 513.835	85.0	12.04
0.000 - 3.800	57	54	1.44	- 83.375	1.614	+ 183.996	+ 406.75	+ 507.370	85.0	11.90
0.000 - 4.000	60	14	1.61	- 95.975	1.614	+ 193.680	+ 404.5	+ 501.205	85.0	11.75
0.000 - 4.200	62	35	1.78	- 111.398	1.614	+ 203.364	+ 400.75	+ 492.716	85.0	11.56
0.000 - 4.400	64	53	1.95	- 125.922	1.614	+ 213.048	+ 396.25	+ 482.775	85.0	11.32
0.000 - 4.600	67	14	2.12	- 142.534	1.614	+ 222.732	+ 392.5	+ 472.698	85.0	11.08
0.000 - 4.800	69	31	2.29	- 159.194	1.614	+ 232.416	+ 388.0	+ 461.222	85.0	10.62
0.000 - 5.000	71	50	2.46	- 176.709	1.614	+ 242.100	+ 384.0	+ 449.391	85.0	10.32
0.000 - 5.200	74	05	2.63	- 194.832	1.614	+ 251.784	+ 380.75	+ 437.692	85.0	10.26

TEST NO 1 : CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO (μ_i)

RANGE OF STRAIN CONSIDERED	SAMPLE DILATION $\frac{\Delta V}{V}$ units of 10^{-3}	YOUNG'S MODULUS E lb/sq in	$\frac{\Delta V}{V} \times E$ lb/sq in	CHANGE IN DEVIATOR STRESS ($\Delta \sigma_d$) $= \Delta(\sigma_1 - \sigma_3)$ OVER RANGE CONSIDERED lb/sq in	$\frac{\Delta V \times E}{V \Delta \sigma_d} = N$	1 - N	POISSONS' RATIO (μ_i) $\frac{1}{2} [1 - N]$
0.000 - 0.033	0.03	7697	0.2309	2.54	0.0909	0.9191	0.4585
0.000 - 0.067	0.06	6313	0.3786	4.23	0.0896	0.9104	0.4552
0.000 - 0.100	0.17	6770	1.1509	6.77	0.1700	0.8300	0.4150
0.000 - 0.133	0.48	7549	3.6235	10.04	0.3609	0.6391	0.3195
0.000 - 0.167	0.73	8383	6.1196	14.00	0.4371	0.5629	0.2815
0.000 - 0.200	1.06	8400	6.9040	16.20	0.5300	0.4700	0.2350
0.000 - 0.233	1.30	8439	11.0357	19.78	0.5579	0.4421	0.2210
0.000 - 0.267	1.56	8401	13.1056	22.43	0.5843	0.4157	0.2078
0.000 - 0.300	1.83	8410	15.3902	25.23	0.6100	0.3900	0.1950
0.000 - 0.333	2.09	8228	17.3219	27.60	0.6276	0.3724	0.1862
0.000 - 0.367	2.35	8161	19.1784	29.95	0.6403	0.3597	0.1798
0.000 - 0.400	2.53	8160	21.4608	32.64	0.6575	0.3425	0.1712
0.000 - 0.433	2.88	8042	23.1610	34.82	0.6652	0.3348	0.1674
0.000 - 0.467	3.16	7972	25.1915	37.23	0.6766	0.3234	0.1617
0.000 - 0.500	3.42	7558	26.9744	38.29	0.6840	0.3160	0.1580

TEST NO 1 : CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO (μ_i) (Cont.)

RANGE OF STRAIN CONSIDERED percent	SAMPLE DILATION $\frac{\Delta V}{V}$ units of 10^{-3}	YOUNG'S MODULUS E lb/sq in	$\frac{\Delta V}{V} \times E$ lb/sq in	CHANGE IN DEVIATOR STRESS ($\Delta \sigma_d$) $= \Delta (\sigma_1 - \sigma_3)$ OVER RANGE CONSIDERED lb/sq in	$\frac{\Delta V \times E}{V \Delta \sigma_d} = N$	POISSONS' RATIO (μ_i) $\frac{1}{2} [1 - N]$
0.000 - 0.533	3.66	7792	28.5187	41.53	0.6867	0.3133
0.000 - 0.567	3.91	7619	29.7903	43.20	0.6826	0.3104
0.000 - 0.600	4.18	7563	31.6133	45.38	0.6956	0.3034
0.000 - 0.633	4.43	7450	33.0035	47.16	0.6998	0.3002
0.000 - 0.667	4.60	7319	33.6674	48.80	0.6896	0.3104
0.000 - 0.700	4.90	7261	35.5789	50.53	0.7000	0.3000
0.000 - 0.800	5.61	6999	32.2644	55.99	0.7013	0.2997
0.000 - 0.900	6.27	6659	41.9145	60.02	0.6967	0.3033
0.000 - 1.000	6.86	6382	43.7805	63.82	0.6360	0.3140
0.000 - 1.200	7.84	5861	45.9502	70.53	0.6834	0.3466
0.000 - 1.400	8.76	5364	45.9986	75.09	0.6234	0.3742
0.000 - 1.600	9.50	4944	45.9680	79.11	0.5937	0.4063
0.000 - 1.800	10.11	4589	45.7049	82.61	0.6616	0.4384
0.000 - 2.000	10.62	4250	45.1862	88.04	0.5310	0.4690

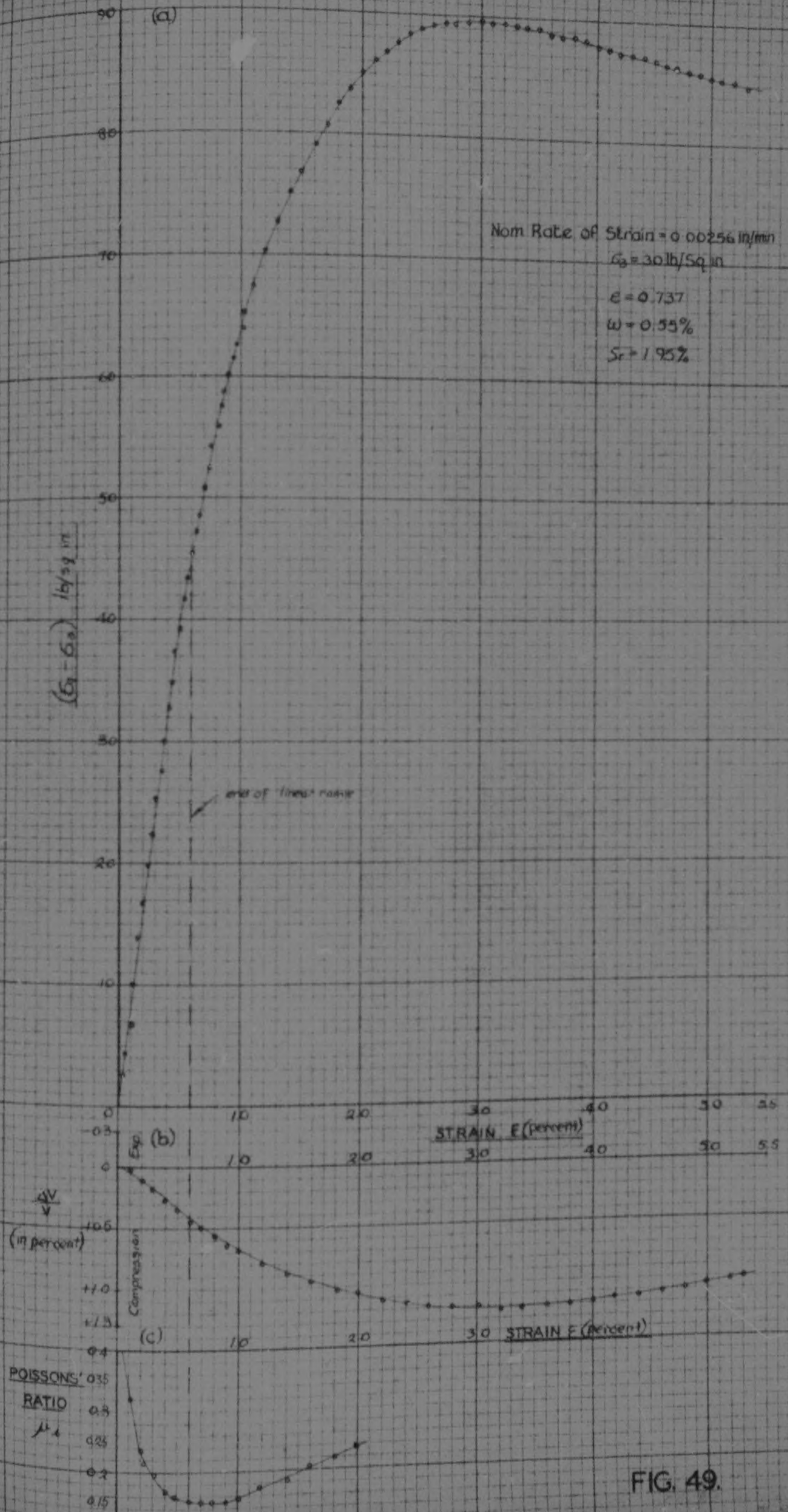


FIG. 49.

TEST NO 3 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION $(\frac{\Delta V}{V})$ (Cont.)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS OF DEFLECT OVER RANGE CONSIDERED DUE TO SPINDLE ENTRY ONLY	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THREE COMBINED EFFECTS	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSS SECTIONAL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOL. (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
	min	sec	mm/min	millimetres	mm/deflect	mm/deflect	millimetres	millimetres	sq mm	cub mm $(\times 10^{-3} \text{cc})$	cub cm	units of 10^{-3}
0.000 - 1.833	31	53	1.54	- 49.100	1.614	55	+ 88.770	+ 303.920	1.9935	605.80	85.0	7.13
0.000 - 2.000	33	53	1.54	- 52.180	1.614	60	+ 95.940	+ 310.160	1.9935	619.30	85.0	7.27
0.000 - 2.167	35	53	1.54	- 55.260	1.614	65	+ 104.910	+ 315.150	1.9935	629.05	85.0	7.39
0.000 - 2.333	37	52	1.58	- 59.830	1.614	70	+ 112.980	+ 317.650	1.9935	635.04	85.0	7.45
0.000 - 2.500	41	51	1.56	- 69.471	1.614	80	+ 129.120	+ 320.399	1.9935	638.72	85.0	7.51
0.000 - 2.833	43	49	1.70	- 74.489	1.614	85	+ 137.190	+ 320.951	1.9935	639.82	85.0	7.53
0.000 - 3.000	45	49	1.74	- 79.722	1.614	90	+ 145.260	+ 322.282	1.9935	642.48	85.0	7.56
0.000 - 3.167	47	45	1.78	- 84.995	1.614	95	+ 153.330	+ 322.335	1.9935	642.87	85.0	7.55
0.000 - 3.333	49	45	1.82	- 90.545	1.614	100	+ 161.400	+ 322.105	1.9935	642.12	85.0	7.55
0.000 - 3.500	51	41	1.86	- 95.130	1.614	105	+ 169.470	+ 320.340	1.9935	639.60	85.0	7.51
0.000 - 3.667	53	41	1.90	- 101.998	1.614	110	+ 177.540	+ 317.042	1.9935	632.02	85.0	7.44
0.000 - 4.000	57	39	1.97	- 113.668	1.614	120	+ 193.680	+ 309.862	1.9935	617.71	85.0	7.27
0.000 - 4.167	59	39	2.01	- 119.996	1.614	125	+ 201.750	+ 305.604	1.9935	609.22	85.0	7.17
0.000 - 4.333	61	38	2.05	- 126.346	1.614	130	+ 209.820	+ 301.222	1.9935	600.49	85.0	7.06
0.000 - 4.667	65	34	2.13	- 139.658	1.614	140	+ 225.960	+ 294.052	1.9935	586.19	85.0	6.90

TEST NO 3 CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO $(\mu)_L$

RANGE OF STRAIN CONSIDERED percent	SAMPLE DILATION $\frac{\Delta V}{V}$ units of 10^{-3}	YOUNG'S MODULUS E lb/sq in	$\frac{\Delta V}{V} \times E$ lb/sq in	CHANGE IN DEVIATOR STRESS $(\Delta \sigma_d)$ $= \Delta(\sigma_1 - \sigma_3)$ OVER RANGE CONSIDERED lb/sq in	$\frac{\Delta V \times E}{V \Delta \sigma_d} = N$	1 - N	POISSONS' RATIO $(\mu)_L$ $\frac{1}{2} [1 - N]$
0.100 - 0.200	0.39	7340	2.6712	7.34	0.3612	0.6088	0.3044
0.100 - 0.233	0.57	7737	4.4101	10.29	0.4286	0.5714	0.2957
0.100 - 0.367	1.51	7865	11.8762	21.00	0.5655	0.4345	0.2172
0.100 - 0.400	1.80	7860	14.1480	23.59	0.6000	0.4000	0.2000
0.100 - 0.467	2.32	7599	17.6207	27.89	0.6318	0.3682	0.1841
0.100 - 0.500	2.55	7478	19.1437	29.91	0.6400	0.3600	0.1800
0.100 - 0.567	3.04	7242	22.0187	33.82	0.6510	0.3490	0.1745
0.100 - 0.700	3.89	6770	26.3353	40.62	0.6483	0.3517	0.1758
0.100 - 0.833	4.82	6344	29.5630	46.50	0.6356	0.3644	0.1821
0.100 - 1.000	5.41	5787	31.1995	51.90	0.6011	0.4006	0.2003
0.100 - 1.167	5.96	5217	31.0933	55.67	0.5565	0.4415	0.2208
0.100 - 1.333	6.39	4702	30.0458	57.97	0.5183	0.4817	0.2408
0.100 - 1.500	6.71	4245	28.4940	59.43	0.4793	0.5207	0.2504
0.100 - 1.667	6.95	3871	26.9035	60.66	0.4435	0.5565	0.2783

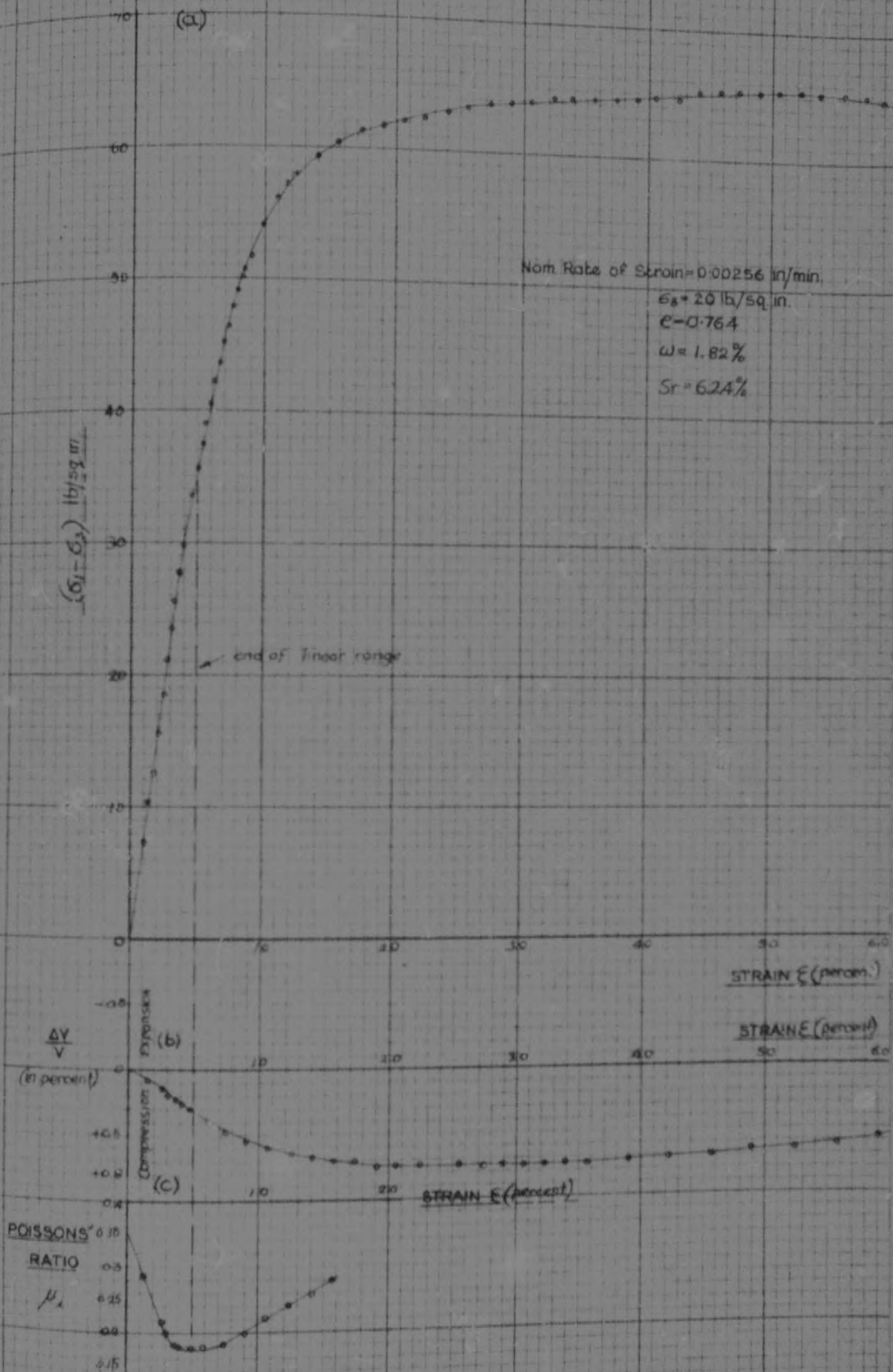


FIG. 50.

TEST NO 4 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION ($\Delta V/V$)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO ENTRY ONLY	RATE OF SLUG MOVEMENT DUE TO DEFLECT OVER RANGE CONSIDERED	OBSERVED OVER RANGE DUE TO THREE COMBINED EFFECTS	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSS SECTIONAL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOL (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
	min	sec	mm/min	mm/min	mm/min	mm/min	mm/min	sq. mm	Cub. mm ($\times 10^{-3}$ cc)	ccm	units of $\frac{\Delta V}{V}$
0.000 - 0.083	00	24	0.400	0.875 - 0.350	1.614	2.8	- 2.75	1.9935	1.86	85.0	0.02
0.000 - 0.167	00	37	0.617	0.875 - 0.540	1.614	5.0	- 7.00	1.9935	1.06	85.0	0.01
0.000 - 0.250	00	45	0.750	0.875 - 0.656	1.614	7.5	- 9.00	1.9935	4.88	85.0	0.06
0.000 - 0.333	00	56	0.933	0.875 - 0.816	1.614	10.0	- 12.00	1.9935	6.63	85.0	0.08
0.000 - 0.417	01	07	1.117	0.875 - 0.977	1.614	12.5	- 14.25	1.9935	9.86	85.0	0.12
0.000 - 0.500	01	24	1.400	0.875 - 1.225	1.614	16.0	- 16.50	1.9935	12.93	85.0	0.15
0.000 - 0.583	01	40	1.667	0.875 - 1.459	1.614	17.5	- 17.00	1.9935	19.61	85.0	0.23
0.000 - 0.567	03	07	3.117	0.875 - 2.727	1.614	20.0	- 7.50	1.9935	43.96	85.0	0.52
0.000 - 0.750	03	30	3.500	0.875 - 3.062	1.614	22.5	+ 13.00	1.9935	92.20	85.0	1.09
0.000 - 0.933	03	48	3.900	0.875 - 3.325	1.614	25.0	+ 30.50	1.9935	134.61	85.0	1.58
0.000 - 0.917	04	10	4.167	0.875 - 3.646	1.614	27.5	+ 51.50	1.9935	193.88	85.0	2.16
0.000 - 1.000	04	26	4.433	0.875 - 3.879	1.614	30.0	+ 68.00	1.9935	224.36	85.0	2.64
0.000 - 1.083	04	46	4.767	0.875 - 4.171	1.614	32.5	+ 84.00	1.9935	263.71	85.0	3.10
0.000 - 1.167	05	03	5.050	0.875 - 4.419	1.614	35.0	+ 99.00	1.9935	301.16	85.0	3.54
0.000 - 1.250	05	21	5.350	0.875 - 4.681	1.614	37.5	+ 114.00	1.9935	338.58	85.0	3.98

TEST No 6 : CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO (14)

RANGE OF STRAIN CONSIDERED percent	SAMPLE DILATION $\frac{\Delta V}{V}$ units of 10^{-3}	YOUNG'S MODULUS E lb./sq. in.	$\frac{\Delta V}{V} \times E$ lb./sq. in.	CHANGE IN DEVIATOR STRESS $\Delta \sigma_d$ $= \Delta(\sigma_1 - \sigma_3)$ OVER RANGE CONSIDERED lb./sq. in.	$\frac{\Delta V \times E}{V \Delta \sigma_d} = N$	1 - N	POISSONS' RATIO (μ_e) $\frac{1}{2} [1 - N]$
0.150 - 0.200	0.25	9020	2,2550	4.51	0.5000	0.5000	0.2500
0.150 - 0.233	0.46	9144	3,7462	6.76	0.5542	0.4458	0.2229
0.150 - 0.267	0.72	7658	5,5138	8.96	0.6154	0.3846	0.1923
0.150 - 0.300	0.94	7953	7,4758	11.93	0.6266	0.3734	0.1867
0.150 - 0.333	1.18	7934	9,4329	14.63	0.6448	0.3552	0.1776
0.150 - 0.367	1.45	8041	11,5525	17.48	0.6682	0.3318	0.1689
0.150 - 0.400	1.76	8152	14,3475	20.38	0.7040	0.2960	0.1480
0.150 - 0.433	2.02	8141	16,4448	23.04	0.7138	0.2862	0.1431
0.150 - 0.467	2.29	8142	18,6452	25.31	0.7224	0.2776	0.1388
0.150 - 0.500	2.44	8186	19,9738	28.65	0.6972	0.3028	0.1514
0.150 - 0.533	2.71	8214	22,2599	31.46	0.7076	0.2924	0.1462
0.150 - 0.567	2.96	8213	24,3105	34.25	0.7098	0.2902	0.1451
0.150 - 0.600	3.13	8200	25,5660	36.90	0.6956	0.3044	0.1522
0.150 - 0.633	3.32	8126	26,9783	39.25	0.6873	0.3127	0.1564
0.150 - 0.667	3.46	8033	27,7942	41.53	0.6692	0.3308	0.1654

CHAPTER 10

CONCLUSIONS

In conclusion, it is necessary to return to the question of the significance of Poisson's Ratio in soils work. For this purpose it is appropriate to consider in the first instance again the applicability of the elastic theory to soils, in the light now also of the information which has come to hand as a result of the practical work conducted, as was described and discussed in Chapters 7, 8 and 9.

A number of additional practical facts, besides those indicated at the beginning of Section 6.1, can now be mentioned which tend to reassure the contention that the elastic theory can be applied to soils.

Firstly, the values of Poisson's Ratio obtained from the tests conducted, were found by using a formula based on elastic theory. Without exception, all the values obtained (i.e. both μ_v and μ_{sel}) fell within the range of values applying to homogeneous, isotropic, elastic materials, viz. zero-0.5 as was discussed in Chapter 3. The values obtained also fell into line with those applying to materials other than metals, such as concrete (μ between 0.083 and 0.125 — see Section 3.3), to which the elastic theory has been applied with reasonable success. That values of Poisson's Ratio in excess of 0.5 were not obtained is due to the fact that in all cases, test samples showed a decrease in volume within the extent of the linear range of the Stress:strain curve as a result of the application of compressive loading, which is a phenomenon in accordance with the requirements of the elastic state (as discussed in Section 3.2). Had values of Poisson's Ratio outside the range zero-0.5 been obtained, it would have served as a strong indication that a soil behaves more in a non-elastic manner and the application of the elastic theory to soils should be treated with reserve.

With regard to the fact that the curves of instantaneous Poisson's Ratio:Strain and instantaneous Young's Modulus:Strain showed (respectively) that both Poisson's Ratio and Young's Modulus are not constant within the linear range of the stress:strain curve as in accordance with the requirements of the elastic theory (see Chapter 2): The causes of these variations have already been fully discussed: in the case of Young's Modulus at the end of Section 7.7 and in the case of Poisson's Ratio in Chapter 9. With regard to Young's Modulus it should be remembered that in each case E was determined as a secant modulus based on actual points plotted, and not on any smooth curve drawn through the plotted points so as to obtain the optimum conditions for enabling a constant value to be determined. It is therefore considered that the variation of both Poisson's Ratio and Young's Modulus within the extent of the linear range as obtained in the case of the tests conducted, does not necessarily tend to invalidate the application of the elastic theory to soils.

However, besides considerations of laboratory and field tests, recent results of actual practical measurements and their comparison with the results of calculations by the theory of elasticity, lend increasing evidence to the contention that the elastic theory can be applied to soils. Firstly a comparison⁽²⁵⁾ of measured and calculated vertical stresses (σ_z) developed in a uniform layer of sandy clay as a result of applied surface pressures, showed very good agreement. The applied pressures were of a transient type similar to those produced by road

traffic/

traffic. Jones (24), in this connection, expresses the view that the basic assumption of elasticity is not correct for soil when it is subjected to slowly applied loads. This is possibly because it is felt that too much consolidation effects can result under these loading conditions. It is not possible to express a view on this opinion on the basis of the tests conducted. However, the results of practical measurements under static loading conditions made by the U.S. Army Corps of Engineers, Waterways Experiment Station (26), (27), (28), which were specifically directed towards the purpose of comparing calculations by the theory of elasticity with the equivalent soil measurements, showed that much better agreement was obtained than seemed generally to have been expected.

It can therefore be concluded that although soils do not comply rigidly in all respects with the requirements of the elastic theory, there is ever-increasing evidence to strengthen the belief that the theory can be applied to soils with reasonable confidence and possibly in all cases of loading, provided (according to the discussion in Section 6.1) the induced stresses are low compared to those required to produce failure i.e. provided the induced stresses are sensibly within the elastic range of loading.

With regard to Poisson's Ratio therefore, it can be concluded that since it is primarily an elastic concept and its meaning is seldom considered other than in relation to the elastic state, it rarely enters in soils work other than in connection with applying the elastic theory to soils. Furthermore, when this is the case, Poisson's Ratio can only enter in 3-dimensional problems of stress and strain, since it was shown in Chapter 4 that the elastic constants do not affect the stress distribution in 2-dimensional cases. The 3-dimensional problem can result from any type of concentrated point loading or finite area loading, such as a rolling wheel load due to rail or road traffic or a structural foundation in the form of a rectangular, square or circular footing. However, as was pointed out in Section 4.4, in the case of stress distribution in 3-dimensional problems, Poisson's Ratio only appears in the expressions for certain of the stress components, the vertical normal stress σ_z which is most frequently used in soils work, being unaffected by it. The Maximum Shear Stress (S_{max}) in 3-dimensional cases is also affected by Poisson's Ratio as was shown in Chapter 6. Yet, for the basic case of a vertical point load, Poisson's Ratio was in general found to be of insignificant effect below a certain comparatively shallow depth in the soil, as the stress distribution (S_{max}) below such depth was not seriously affected by a complete range of values between zero and 0.5 (Fig. 14). Furthermore, on the axis of loading, which is actually of prime importance since the greatest S_{max} values occur on the axis of loading, μ has no very great effect on the values of S_{max} obtained at all depths, as the limits of the complete range of μ values applicable to elastic materials only caused a 14% variation in the corresponding values of S_{max} .

As far as stress distribution in soils is therefore concerned, it appears that Poisson's Ratio is generally not of very great significance. The immediate (elastic) settlement of foundations on soils is, however, more affected by variation in the value of Poisson's Ratio, since the factor $(1 - \mu^2)$ in the equation 4.11 for the settlement of a uniformly loaded rectangular area, (a factor which also appears in the case of the expression for the settlement of the surface of a semi-infinite solid due to a vertical point load - see Ref. 9, p. 376), varies between 0.75 and unity for μ varying between zero and 0.5, and is therefore liable to cause a maximum variation of 25% in the

Immediate

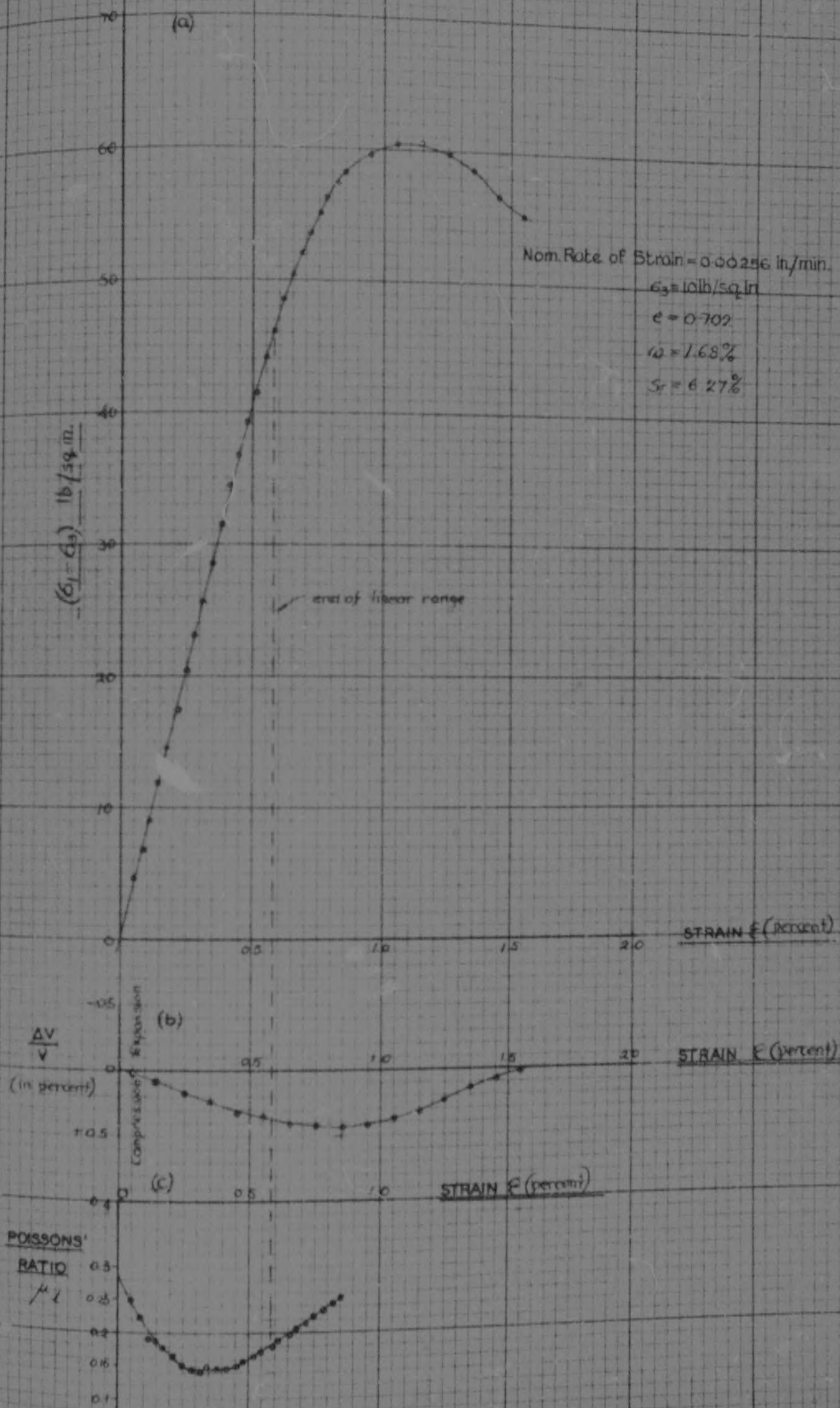


FIG. 52

immediate settlement value obtained. Yet, this effect is ultimately also not very great, since the immediate settlement only forms a part of the total settlement of any foundation, the other part being that due to consolidation effects. Therefore as far as the significance of Poisson's Ratio in the determination of both stress and settlement in soils is concerned, Poisson's Ratio is only of limited significance.

As a result of the discussions in Chapters 6 and 9, the following conclusions can be drawn with regard to the value of Poisson's Ratio applicable in various cases. In the first instance, it was shown in Sections 6.2 and 6.3 that for all types of saturated soils (i.e. both cohesive and non-cohesive), Poisson's Ratio = 0.50 should be used for all calculations by the theory of elasticity. Secondly, in the case of partially saturated soils, the following information is available from Chapter 9:

(a) D.S.I.R. findings:

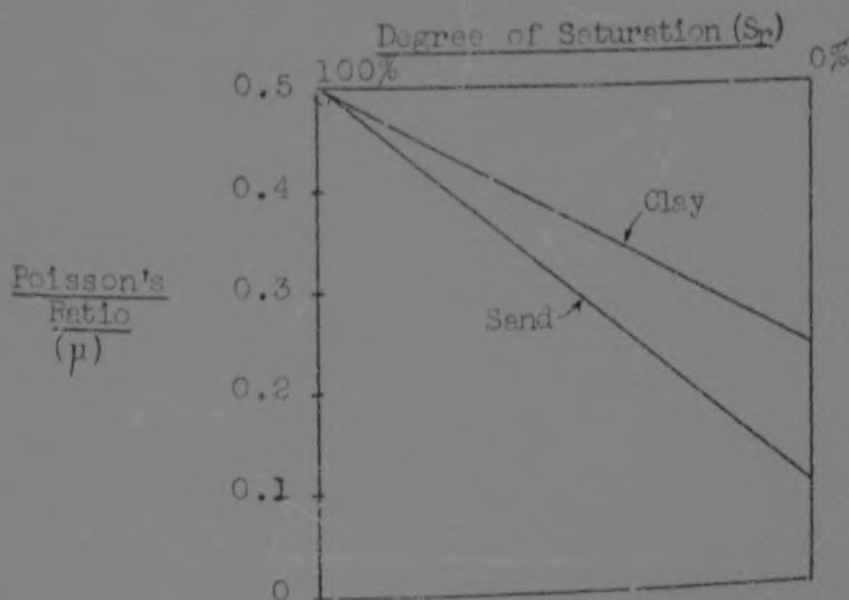
- (i) For a silty clay at a moisture content of approximately 20% (plastic limit), $\mu = 0.45$
- (ii) For the same soil at a moisture content of 5%, $\mu = 0.25$.
- (iii) No values for sands were given, but the view was expressed that the values of μ for sands were below those for clays.

(b) Findings by the U.S. Army Corps of Engineers:

- (i) For a lean clay at a moisture content near the optimum $\mu = 0.50$.
- (ii) For a dry, fairly fine, uniform sand, $\mu = 0.30$.
- (iii) Results (i) and (ii) in this case also indicate that μ values for sands are below those for clays.

(c) Findings from the laboratory work discussed in this report: For a silty sand at a moisture content of 1-2%, μ falls within the range 0.14-0.24.

From the above data it is thus clear that Poisson's Ratio is dependent on moisture content in all types of soils, i.e., according to indications so far, μ decreases with moisture content. Furthermore, it is clear that in the case of partially saturated soils, μ values for sands are below those for clays. Hence, it is suggested that the Poisson's Ratio:Degree of Saturation relationship may be of the nature shown in the diagram below:



In

CHAPTER 9

FINAL DISCUSSION OF TEST RESULTS

The curves of instantaneous Poisson's Ratio (μ_i): Strain (Figs. 49, 50, 51 and 52) for each of the four tests in which it was possible to prepare these curves, show the same basic pattern in each case, viz. that of a trough-shaped curve, but with a short, approximately horizontal portion forming the turning point.

This approximately horizontal portion of the curve indicates that μ_i attains a value which is more or less constant and can be considered as such with little detrimental effect. For convenience, this constant value will be denoted $\mu_i(\text{const.})$. Its actual numerical value was for each test obtained from the relevant calculation sheets as the average of the values for the range over which μ_i can be considered to be constant. This range of values has in each case been marked on the calculation sheets by a vertical line alongside the relevant figures, which extends between arrow heads demarcating the limits of the range.

The curves show clearly that the range of strain over which μ_i is constant, does not always fall fully within that range of strain corresponding to the linear range of the stress:strain curve. In three out of four instances (Figs. 49, 50 and 51), the bulk of the range of strain over which μ_i is constant falls outside the limit of the linear range of the stress:strain curve. Furthermore, before commencement of this range of strain, μ_i decreases in each case from a value generally between 0.3 and 0.5 to the respective $\mu_i(\text{const.})$ value in the region of 0.14 - 0.24, this decrease taking place within the linear range of the stress:strain curve. Subsequent to the $\mu_i(\text{const.})$ range, μ_i again steadily increases in value.

Regarding the latter points mentioned, their causes can be explained by considering again equation 7.5 from which Poisson's Ratio was evaluated:

$$\mu = \frac{1}{2} \left[1 - \frac{\delta V}{V} \cdot \frac{E}{\delta \sigma_d} \right] \dots\dots\dots 7.5$$

or, more accurately,

$$\mu = \frac{1}{2} \left[1 - \frac{\delta V}{V} \cdot \frac{E}{\delta \sigma_d} \right] \dots\dots\dots 9.1$$

$$\text{But Young's Modulus (E)} = \frac{\delta \sigma_d}{\delta \epsilon} \dots\dots\dots 9.2$$

$$\therefore \mu = \frac{1}{2} \left[1 - \frac{\delta V}{V} \cdot \frac{\delta \sigma_d}{\delta \epsilon} \cdot \frac{1}{\delta \sigma_d} \right] \dots\dots\dots 9.3$$

$$= \frac{1}{2} \left[1 - \frac{\delta V}{V} \cdot \frac{1}{\delta \epsilon} \right] \dots\dots\dots 9.3$$

which means that, as V is a constant and $\delta \epsilon$ is the independent variable, the value of Poisson's Ratio obtained is dependent only on the measured value of δV (i.e. the change in volume of the test sample during test) and it cannot be effected by any peak-shaped variation in Young's Modulus (E) as would seem to be possible from equation 7.5.

Further, with regard to the initial decreasing portion of the $\mu_i:\epsilon$ curve, the same equation (9.3) shows that values of μ_i which are too large can be obtained if the value of δV used is too small, i.e., if the value of slug movement due to leakage only which was used for the determination of δV was larger than the actual value during test, since, the slug movement due to spindle entry only is a function only of the independent variable deflection (i.e. strain), and hence, cannot affect the value

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In the case of partially saturated soils therefore, it may be necessary, depending on the nature of the purpose for which a value of Poisson's Ratio is required, to conduct a special test in order to determine the value applicable to the particular soil concerned, or it may suffice to use an approximate value based on the information given above. However, on occasion there may also be overriding considerations, such as when the condition of a soil mass as a whole is considered and it becomes necessary to adopt a particular value of Poisson's Ratio (such as in the case of the Westergaard equations), in order to obtain a result in close agreement with actual conditions.

Finally, it should again be stressed that in all cases the concept of Poisson's Ratio in soils only has a rational meaning when viewed in relation to soil conditions which can be considered to approximate the elastic state to which the theory of elasticity is applicable. Such soil conditions therefore constitute the only justifiable basis on which the significance of Poisson's Ratio in the determination of stress and settlement in soils can on any occasion be considered.

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of δV . In effect, this means that if the assumed value of rate of leakage during test is too large, the resultant value of μ_i is too large and vice versa.

This was actually the case with the reduction of the test results, since as can be seen in Fig.(a), Section 8.1, the rate of leakage during the initial portion of a test was assumed to be a constant value equal to the rate of leakage measured immediately before the start of test, and, as was discussed in Section 8.2, this is a value which is most probably in excess of the actual value of rate of leakage occurring immediately after test commencement. Hence, the fact that values of μ_i which are too large were initially obtained. It also seems likely that while, soon after test commencement, the slug movement due to sample volume change only is still comparatively small in relation to the slug movement due to leakage only (see for example Test No.1), an error in the assumed value of rate of leakage could cause a significant error in the value of μ_i obtained. As the test progresses, and the slug movement due to sample volume change increases in relation to the slug movement due to leakage, an error in the rate of leakage has continually less significant effect on the values of δV and μ_i obtained. On the curve of μ_i vs δV this is manifested by steadily decreasing μ_i values until the $\mu_i(\text{const.})$ range is obtained. It is therefore considered that the values of $\mu_i(\text{const.})$ are the more reliable and will therefore be used.

With regard to the fact that μ_i again increases in value subsequent to the $\mu_i(\text{const.})$ range, it will be noted that this subsequent increase in value occurs simultaneously with the turning point in the Main Slug Movement:Deflection curve being approached, i.e. it occurs simultaneously with the commencement of the phenomenon of sample expansion on approaching failure conditions (see Section 7.7). This phenomenon results in decreasing values of δV (relative to the original sample volume V) and, as can be seen from equation 9.3, is the cause of the subsequent increasing values of μ_i .

However, the fact that the $\mu_i(\text{const.})$ range which precedes the range of increasing μ_i values, extends outside the limits of the linear range of the stress:strain curve, is considered to be due to inclusion of sample volume change due to consolidation effects in the measured value of δV . These consolidation effects occur as a result of the application of the deviator stress σ_d and possibly only become of significant effect near and beyond the end of the linear range and prior to the commencement of the phenomenon of sample expansion on approaching failure conditions. The curving over to the right of the stress:strain curve which commences from the end of the linear range, is considered (61) to be indicative of the inclusion of consolidation effects resulting from the application of the deviator stress. This can possibly be appreciated if it be considered that in some tests the linear range of the stress:strain curve lasted for about 15 minutes. As the test soil was partially saturated, in fact, nearly completely dry (generally less than 2%), and was also of a cohesionless nature, it is difficult to conceive especially as a result of the discussions in Sections 6.2 and 6.3, how some consolidation effects (possibly not in the form of extrusion of pore water from the test sample, but air), could have been avoided within the linear range of the stress:strain curve.

As these consolidation effects represent loss of mass during strain (see Chapter 2), they cannot be included in sample volume changes due to elasticity effects for the purposes of calculating Poisson's Ratio. The effect of their inclusion is to cause a larger value of δV than would otherwise have been the case, and which, according to equation 9.3, is therefore

the/

APPENDIX I

The possible Algebraic Signs of the Moduli of Elasticity E, G and K

(a) Young's Modulus (E):

By Hooke's Law (Chapter 2), $E = \frac{\sigma}{\epsilon}$ (a)

Since, if σ is +ve, ϵ is essentially +ve,
and if σ is -ve, ϵ is essentially -ve,

Therefore, E is always positive.

(b) Modulus of Rigidity (G):

From basic strength of Materials (Ref.2,p.55),

$$G = \frac{\tau}{\gamma} \text{ (b)}$$

Since, if τ is +ve, γ is essentially +ve,
and if τ is -ve, γ is essentially -ve,

Therefore G is always positive.

(c) Bulk Modulus (K):

From equation 3.2 : For fluid pressure conditions
when $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon$ say,

$$\Delta = 3\epsilon \text{ (c)}$$

and as discussed in (a) above:

if σ is +ve, ϵ is +ve, therefore in (c): Δ is +ve when
 σ is +ve,

and if σ is -ve, ϵ is -ve, therefore in (c): Δ is -ve when
 σ is -ve.

Hence, in equation 3.4: $K = \frac{\sigma}{\Delta}$

if Δ is +ve when σ is +ve, then $\frac{\sigma}{\Delta}$ is +ve,
and if Δ is -ve when σ is -ve, then $\frac{\sigma}{\Delta}$ is +ve.

Therefore K is always positive.

the cause of a smaller value of μ_i being obtained, and a delay in the start of the range of increasing μ_i values. Hence, the phenomenon that the $\mu_i(\text{const.})$ range extends outside the limits of the linear range of the stress:strain curve.

Table XII on the next page shows the determination of the proportion of the constant μ_i range falling within the limit of the linear range of the stress:strain curve for each test. The table also compares in adjacent columns the values of μ_{s1} and $\mu_i(\text{const.})$ obtained in each case. It appears that the values of both μ_{s1} and $\mu_i(\text{const.})$ fall within the range 0.138-0.238, the values of $\mu_i(\text{const.})$ being slightly larger than the corresponding values of μ_{s1} in each case.

These values show reasonable agreement with those obtained by Research workers of the D.S.I.R.'s Road Research Laboratory, Hemondsworth, Middlesex, England, (23), (24), (25), as a result of in-situ measurements made of the mechanical properties of soil by vibration methods. This research work is as yet unpublished. It was found that Poisson's Ratio for a silty clay with a liquid limit of 50%, was approximately 0.45 at a moisture content quite near to its plastic limit of 20%. For the same silty clay, Poisson's Ratio was 0.25 at 5% moisture content, thus indicating a decrease in value with decreasing moisture content. For a heavy clay a value a little above 0.45 seemed to be indicated. No data are as yet available for sands, but the opinion was expressed (23) that the values will be below those for clays.

Information obtained from the Director, Waterways Experiment Station, U.S. Army Corps of Engineers, Vicksburg, Mississippi, (26), (27), (28), substantiates these findings of the D.S.I.R., England. Their work has been directed at comparing calculations by the theory of elasticity with the equivalent soil measurements. For a lean clay at a moisture content near the optimum, a Poisson's Ratio of 0.5 seemed to serve very well, while for a dry, fairly fine, uniform sand, a value of 0.3 was indicated, which thus verifies the opinion of the D.S.I.R., workers that the Poisson's Ratio values for sands are below those for clays.

Hence, if, according to the D.S.I.R. findings previously mentioned, the value of Poisson's Ratio for a silty clay drops from 0.45 to 0.25 for a drop in moisture content from near to 20% to 5%, and if, in addition, the values of Poisson's Ratio for sands are below those for clays, then values between 0.138 and 0.238 for a silty sand at 1% - 2% moisture content would seem to be in reasonable agreement. It should especially be noted that all the values of Poisson's Ratio given above apply to partially saturated soil conditions.

In order to ascertain the possible affect of moisture content on the values of Poisson's Ratio determined from the test series, Table XIII was arranged in order of decreasing moisture content values, with the relevant values of Degree of Saturation

(Sr)/

APPENDIX II

Steps in the Final Reduction of the Mathematical Expressions in Flamant's paper (Ref.12)

It is useful to commence by considering in the first instance some basic relationships of the theory of elasticity which are involved in the work to follow in this Appendix. These relationships will only be quoted without their actual derivations being given. However, reference will be made to pages in Timoshenko's "Theory of Elasticity" (Ref.3) wherein the derivations are outlined in full. It is important to point out that Timoshenko uses the same notation as used by Love and other masters of the elastic theory, in that Poisson's Ratio is denoted by the Greek letter ν (Nu), whereas in this report Poisson's Ratio is in all cases denoted by μ . However, since Lamé's elastic constants which appear in Flamant's paper are denoted by λ and μ , Poisson's Ratio will in this Appendix only be denoted by ν , to avoid confusion in the work to follow. Furthermore, the unit volumetric strain or Dilation which throughout this report is denoted by Δ , is denoted by ϵ in Timoshenko's book as well as in Flamant's paper. Therefore the Dilation also will in this Appendix only be denoted by the symbol ϵ .

The first set of basic relationships which are involved in Flamant's work are those given by Timoshenko(3) in his equations (2), p.7, viz., that if with respect to a set of three mutually perpendicular axes OX, OY and OZ the small displacements of particles of a deformed body be resolved into components u , v , and w parallel to these respective coordinate axes, then:

(i) the linear strain components ϵ are expressible by:

$$\epsilon_x = \frac{\partial u}{\partial x}; \epsilon_y = \frac{\partial v}{\partial y}; \epsilon_z = \frac{\partial w}{\partial z} \dots\dots\dots (a)$$

and (ii) the shear strain components γ are expressible by:

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}; \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}; \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \dots\dots\dots (b)$$

The second set of basic relationships which are involved are those given by Timoshenko(3) in his equations (11), p.11, relating the stresses and strains in a 3-dimensional system:

$$\begin{aligned} \sigma_x &= \lambda \epsilon + 2\mu \epsilon_x \\ \sigma_y &= \lambda \epsilon + 2\mu \epsilon_y \\ \sigma_z &= \lambda \epsilon + 2\mu \epsilon_z \end{aligned} \dots\dots\dots (c)$$

wherein:

(1) λ and μ are Lamé's elastic constants as given by:

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \dots\dots\dots (d)$$

$$\text{and } \mu = G = \frac{E}{2(1+\nu)} = \frac{\tau}{\gamma} \dots\dots\dots (e)$$

where E = Young's Modulus
 ν = Poisson's Ratio
 G = Modulus of Rigidity

τ = shear stress
 γ = shear strain

(ii) ϵ is the unit volumetric strain as given by:

$$\epsilon = \epsilon_x + \epsilon_y + \epsilon_z \dots\dots\dots (f)$$

Equations (e) and (f) above are identical to equations 3.5 and 3.2 which appeared earlier in Chapter 3 of this report.

From equation (e) above, it should be noted that:

$$\tau = \mu \gamma$$

whence

TEST No 4 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION $(\frac{\Delta V}{V})$ (Cont.)

OF IN ERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ONLY	NUMBER OF DIVS OVER RANGE CONSIDERED ONLY	HENCE SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE DUE TO THREE COMBINED EFFECTS	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CON- SIDERED	CROSS SECTION - AL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOL. (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
	mm sec	minutes	mm/min	mm/min	mm/deflect	divs	mm/min	mm/min	mm/min	sq mm	cub mm ($\times 10^{-3}$)	cub cm	unit of $\frac{\Delta V}{V}$
1.333	05 35	5.683	0.875	- 4.885	1.614	40.0	+ 64.660	+123.25	+182.925	1.9935	564.66	85.0	4.29
1.417	05 50	5.833	0.875	- 5.104	1.614	42.5	+ 69.595	+132.00	+198.491	1.9935	389.71	85.0	4.58
1.500	05 04	5.067	0.875	- 5.309	1.614	45.0	+ 72.630	+138.00	+205.321	1.9935	409.31	85.0	4.82
1.583	06 19	6.317	0.875	- 5.527	1.614	47.5	+ 76.665	+142.75	+213.888	1.9935	426.39	85.0	5.02
1.667	06 33	6.550	0.875	- 5.731	1.614	50.0	+ 80.700	+144.75	+219.719	1.9935	438.01	85.0	5.15
1.750	06 56	6.933	0.988	- 6.850	1.614	55.0	+ 88.770	+144.00	+225.320	1.9935	450.37	85.0	5.30
2.000	07 21	7.350	1.000	- 6.085	1.614	60.0	+ 96.840	+134.75	+223.505	1.9935	445.56	85.0	5.24
2.167	07 45	7.750	1.212	- 9.393	1.614	65.0	+104.910	+111.00	+206.517	1.9935	411.69	85.0	4.84
2.333	08 06	8.100	1.325	-10.732	1.614	70.0	+112.980	+ 90.75	+192.998	1.9935	384.74	85.0	4.53
2.500	08 27	8.450	1.438	-12.151	1.614	75.0	+121.050	+ 69.25	+177.149	1.9935	353.15	85.0	4.15
2.667	08 51	8.850	1.550	-13.718	1.614	80.0	+129.120	+ 47.00	+162.402	1.9935	323.75	85.0	3.81
2.833	09 13	9.217	1.662	-15.319	1.614	85.0	+137.190	+ 23.50	+145.371	1.9935	289.20	85.0	3.41
3.000	09 36	9.600	1.775	-17.040	1.614	90.0	+145.260	+ 4.50	+132.720	1.9935	264.58	85.0	3.11
3.167	09 59	9.983	1.888	-18.948	1.614	95.0	+153.330	- 25.00	+109.482	1.9935	218.25	85.0	2.57
3.333	10 24	10.400	2.000	-20.200	1.614	100.0	+161.400	- 45.00	+ 95.600	1.9935	190.88	85.0	2.24

TABLE XII: COMPARISON OF VALUES OF POISSONS' RATIO μ_{sec} AND μ_{nst}

TEST NUMBER	STRAIN AT END OF LINEAR RANGE OF STRESS STRAIN CURVE = a percent	STRAIN DUE TO BEDDING-IN EFFECTS = b percent	DIFFERENCE a-b CORRECTED STRAIN AT END OF LINEAR RANGE OF STRESS STRAIN CURVE = c percent	CURVE OF μ_L VS E CORRECTED STRAIN* VALUES (percent)		PROPORTION OF CONSTANT RANGE FALLING WITHIN THE LIMIT OF THE LINEAR RANGE OF THE STRESS STRAIN CURVE		POISSONS' RATIO μ_{sec}	POISSONS' RATIO μ_L (constant)	DIFFERENCE $\mu_L(\text{const}) - \mu_{sec}$
				AT START OF CONSTANT μ_L RANGE = d	AT END OF CONSTANT μ_L RANGE = e	$\frac{c-d}{e-d}$	PERCENTAGE			
1.	0.600	ZERO	0.600	0.500	1.000	$\frac{0.100}{0.500}$	20%	0.1387	0.1535	0.0148
2.	0.833	0.150	0.683	—	—	—	—	0.1401	—	—
3.	0.600	0.100	0.500	0.367	0.733	$\frac{0.133}{0.366}$	36%	0.1390	0.1793	0.0403
4.	1.083	0.500	0.583	0.417	0.833	$\frac{0.167}{0.416}$	40%	0.2099	0.2372	0.0273
5.	0.750	0.260	0.490	—	—	—	—	0.1425	—	—
6.	0.750	0.150	0.600	0.250	0.483	$\frac{0.350}{0.233}$	100%	0.1429	0.1464	0.0035

* STRAIN DUE TO BEDDING-IN EFFECTS SUBTRACTED

TEST NO 4 : CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO (μ)

RANGE OF STRAIN CONSIDERED	SAMPLE DILATION $\frac{\Delta V}{V}$ units of 10^{-3}	YOUNG'S MODULUS E lb/sq in	$\frac{\Delta V}{V} \times E$ lb/sq in	CHANGE IN DEVIATOR STRESS ($\Delta \sigma_d$) $= \Delta(\sigma_1 - \sigma_3)$ OVER RANGE CONSIDERED lb/sq in	$\frac{\Delta V \times E}{\Delta \sigma_d} = N$	$1 - N$	POISSONS' RATIO (μ) $\frac{1}{2} [1 - N]$
0.500 - 0.583	0.23	9470	2.1781	7.26	0.2771	0.7229	0.2619
0.500 - 0.667	0.52	8916	5.1563	16.56	0.3114	0.6886	0.3443
0.500 - 0.750	1.08	9649	10.4198	24.12	0.4320	0.5680	0.2840
0.500 - 0.833	1.58	9339	14.7556	31.10	0.4744	0.5256	0.2698
0.500 - 0.917	2.16	9532	20.5891	39.75	0.5180	0.4820	0.2410
0.500 - 1.000	2.64	9174	24.2194	45.87	0.5280	0.4720	0.2360
0.500 - 1.083	3.10	9154	28.3774	53.87	0.5317	0.4683	0.2342
0.500 - 1.167	3.54	8823	31.2334	58.85	0.5307	0.4693	0.2346
0.500 - 1.250	3.98	8629	34.3434	64.72	0.5306	0.4694	0.2347
0.500 - 1.333	4.29	8298	35.5984	69.12	0.5150	0.4850	0.2425
0.500 - 1.417	4.58	7956	36.4385	72.96	0.4994	0.5006	0.2503
0.500 - 1.500	4.82	7596	36.6127	75.96	0.4820	0.5180	0.2590
0.500 - 1.583	5.02	7291	36.6008	78.96	0.4635	0.5365	0.2662
0.500 - 1.667	5.15	6926	35.6599	80.83	0.4413	0.5587	0.2794
0.500 - 1.833	5.30	6241	33.0773	83.19	0.3976	0.6024	0.3012

(S_r) alongside. From the data presented it is clear, however, that no conclusion can be drawn as to the effect of moisture content or Degree of Saturation on the value of Poisson's Ratio. Insufficient values are available and the range of moisture content is too small to show up any significant trend such as that found by the Road Research Laboratory of the D.S.I.R. For the same reasons no conclusion can be drawn about the effect of void ratio on the value of Poisson's Ratio (Table XIV).

However, Table XIV clearly indicates the important relationship between chamber pressure (σ_3), void ratio (e) and sample dilation ($\delta V/V$) which has also been discussed by Taylor (1.7), viz., that for sandy soils, the nature and magnitude of the volumetric strains ($\delta V/V$) during test, depend on both the chamber pressure used and the void ratio of the sample. In the first instance, the volumetric strain ($\delta V/V$) for a reduction in volume, increases as the chamber pressure increases, and secondly, the lower the void ratio the greater the tendency for a volume increase after the initial volume decrease at the commencement of the test. Hence, for the two effects combined, the lower the void ratio and chamber pressure the less the initial volume decrease of the sample and, subsequently also, the more rapid the tendency to a volume increase. In the case of Tests No. 1 & 3: the effect of the higher chamber pressure in Test No. 1 apparently outweighed the effect of the lower void ratio and resulted in a larger volume decrease and a lesser tendency for a volume increase than in the case of Test No. 3. The effect of a lower void ratio at the same chamber pressure is clearly indicated in the case of Tests No. 3 & 4. In the case of Test No. 6 with both low void ratio and chamber pressure, the phenomenon of sample expansion subsequent to an initial compression to the extent of yielding a sample volume in excess of its original volume at the commencement of the test, was nearly realised, i.e., a change in sign of $\delta V/V$ from that corresponding to a decrease in volume to that corresponding to an increase in volume, was nearly effected.

The effect of variation in chamber pressure at a constant rate of strain, on the Shear Strength obtained, is firstly presented in Table XV. The Mohr circles for each of the three chamber pressures for the two respective rates of strain used, are shown in Fig. 48, and give the values of cohesion (c) and apparent angle of friction (ϕ_a) shown in Table (XV) in the column adjacent to the Shear Strength values. The values of c and ϕ_a are slightly different for the two rates of strain, the higher rate of strain yielding the higher value of apparent angle of friction and the lower value of cohesion. Individual Mohr circles for the case of the faster rate of strain, fit in better with the relevant Mohr rupture line, than in the case of the lower rate of strain. From the following discussion of the data presented in Table XVI, it will be seen that chamber pressure and rate of strain probably have no influence in causing this phenomenon.

The effect of both variation in chamber pressure at a constant rate of strain and variation in rate of strain at a constant chamber pressure (Tables XV and XVI), on the values of

Young's/

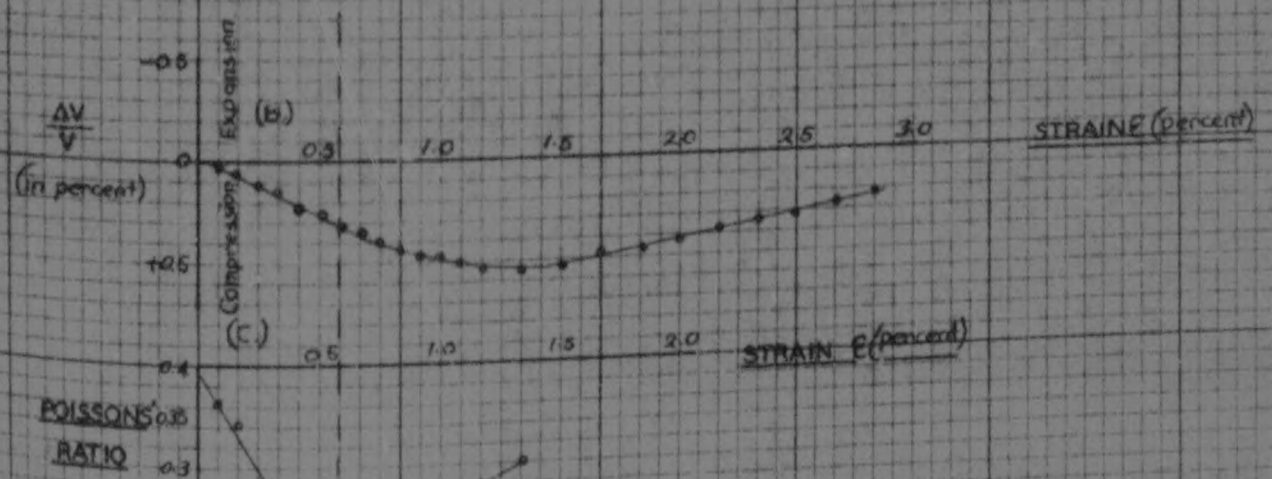
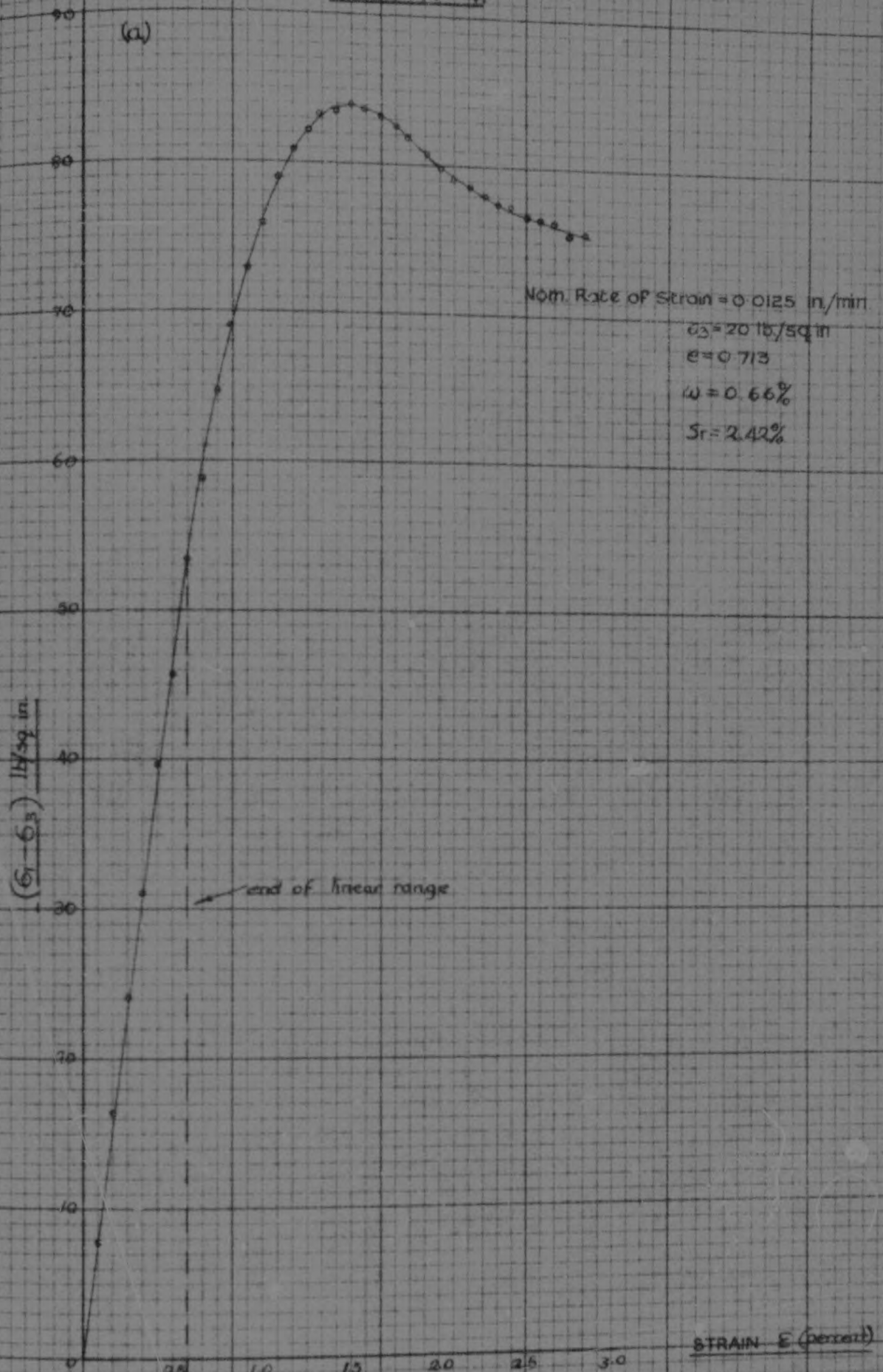
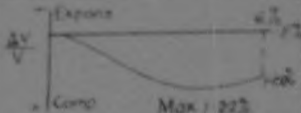
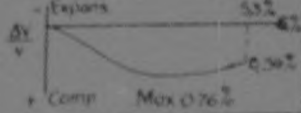
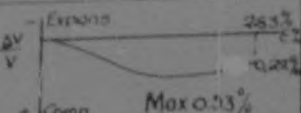
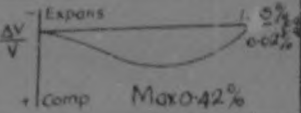


TABLE XIII

TEST NUMBER	INITIAL MOISTURE CONTENT w percent	INITIAL DEGREE OF SATURATION S_r percent	POISSONS' RATIO μ_{rel}	POISSONS' RATIO $\mu_{rel}(\text{constant})$
2	2.290	8.64	0.1401	—
3	1.820	6.24	0.1390	0.1793
5	1.750	6.42	0.1425	—
6	1.680	6.27	0.1429	0.1464
4	0.658	2.42	0.2099	0.2372
1	0.548	1.95	0.1387	0.1535

TABLE XIV

TEST NUMBER	NOMINAL CHAMBER PRESSURE P_c lb/sq in	INITIAL VOID RATIO e	POISSONS' RATIO μ_{rel}	POISSONS' RATIO $\mu_{rel}(\text{constant})$	DETAILS OF $\frac{\Delta V}{V} = E$ CURVE
1	30	0.737	0.1387	0.1535	
3	20	0.764	0.1390	0.1793	
5	10	0.714	0.1425	—	—
4	20	0.713	0.2099	0.2372	
6	10	0.702	0.1429	0.1464	
2	30	0.695	0.1401	—	—

TEST N° 6 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION $(\frac{\Delta V}{V})$

TIME OF RANGE CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS. DEFLECT OVER RANGE CONSIDERED	HENCE: SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THREE COMBINED EFFECTS.	HENCE: SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CON- SIDERED.	CROSS-SECTIONAL AREA OF CAPILLARY TUBE	CHANGE IN VOL. (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
	min.	sec.	mm/min	mm/divs/deflect		mm	mm	mm	sq. mm	cc/mm ³	cc/cm ³	units of 10^{-3}
- 0.033	01	09	0.875	1.614	1	+ 1.614	+ 0.75	+ 1.358	1.9935	2.71	85.0	0.03
- 0.067	01	34	0.875	1.614	2	+ 3.228	+ 0.75	+ 2.607	1.9935	5.20	85.0	0.06
- 0.100	01	58	0.875	1.614	3	+ 4.842	+ 0.25	+ 3.371	1.9935	6.72	85.0	0.08
- 0.133	02	22	0.875	1.614	4	+ 6.456	+ 0.25	+ 4.679	1.9935	9.33	85.0	0.11
- 0.167	02	49	0.875	1.614	5	+ 8.070	+ 0.50	+ 6.150	1.9935	12.17	85.0	0.14
- 0.200	03	20	0.875	1.614	6	+ 9.684	+ 3.75	+ 10.518	1.9935	20.97	85.0	0.25
- 0.233	04	06	0.875	1.614	7	+ 11.298	+ 12.00	+ 19.710	1.9935	39.29	85.0	0.46
- 0.267	03	18	0.875	1.614	8	+ 12.912	+ 26.00	+ 30.774	1.9935	61.35	85.0	0.72
- 0.300	10	40	0.875	1.614	9	+ 14.526	+ 35.00	+ 40.912	1.9935	80.12	85.0	0.94
- 0.333	11	29	0.875	1.614	10	+ 16.140	+ 44.25	+ 50.342	1.9935	100.36	85.0	1.18
- 0.367	12	26	0.875	1.614	11	+ 17.754	+ 55.00	+ 61.845	1.9935	123.29	85.0	1.45
- 0.400	13	05	0.875	1.614	12	+ 19.368	+ 67.00	+ 74.920	1.9935	149.35	85.0	1.76
- 0.433	13	42	0.875	1.614	13	+ 20.982	+ 77.00	+ 85.994	1.9935	171.43	85.0	2.08
- 0.467	14	13	0.875	1.614	14	+ 22.596	+ 87.50	+ 97.569	1.9935	194.50	85.0	2.29
- 0.500	14	55	0.875	1.614	15	+ 24.210	+ 93.00	+ 104.158	1.9935	207.64	85.0	2.44

TABLE XV

TEST NUMBER	NOMINAL RATE OF STRAIN inches/min	NOMINAL CHAMBER PRESSURE σ_3 lb/sq. in.	SHEAR STRENGTH S lb/sq. in.	COHESION (C) AND FRICTION (φ) COMPONENTS OF SHEAR STRENGTH	YOUNGS' MODULUS E_{sel} lb/sq. in.	POISSONS' RATIO μ_{sel}	POISSONS' RATIO μ_{sc} (constant)
1.	0.00256	30	89.6	C = 9.5 lb/sq. in. φ = 30° 08'	8163	0.1387	0.1535
3.	0.00256	20	65.3		7697	0.1390	0.1793
5.	0.00256	10	60.6		8125	0.1429	0.1464
2.	0.0125	30	102.0	C = 9 lb/sq. in. φ = 33° 42'	7750	0.1401	—
4.	0.0125	20	83.9		9567	0.2099	0.2372
5.	0.0125	10	53.0		7212	0.1425	—

TABLE XVI

TEST NUMBER	NOMINAL CHAMBER PRESSURE σ_3 lb/sq. in.	NOMINAL RATE OF STRAIN inches/min	SHEAR STRENGTH S lb/sq. in.	YOUNGS' MODULUS E_{sel} lb/sq. in.	INITIAL MOISTURE CONTENT w percent	INITIAL VOID RATIO e
1.	30	0.00256	89.6	8163	0.548	0.737
2.	30	0.0125	102.0	7750	2.290	0.695
3.	20	0.00256	65.3	7697	1.820	0.764
4.	20	0.0125	83.9	9567	0.655	0.713
6.	10	0.00256	60.6	8125	1.680	0.702
5.	10	0.0125	53.0	7212	1.750	0.714

TEST No 6 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION $(\frac{\Delta V}{V})$ (Cont.)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE DUE TO THREE CC. BINED EFFECTS.	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED.	CROSS SECTION - AL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOL. (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
	min sec.	minutes	mm./min.	mm./min.	mm./min.	mm./min.	mm./min.	mm./min.	sq. mm.	cc. mm. $(\times 10^{-3})$	cc. cm.	units of 10^{-3}
0.000 - 0.533	15 34	15.567	0.875 - 13.621	1.614	1.614	16	+ 25.824	+ 103.50	1.9935	230.65	85.0	2.71
0.000 - 0.567	16 12	16.200	0.875 - 14.175	1.614	1.614	17	+ 27.438	+ 112.75	1.9935	251.21	85.0	2.96
0.000 - 0.600	16 48	16.800	0.875 - 14.700	1.614	1.614	18	+ 29.052	+ 119.00	1.9935	265.84	85.0	3.13
0.000 - 0.633	17 28	17.367	0.875 - 15.196	1.614	1.614	19	+ 30.666	+ 126.00	1.9935	282.02	85.0	3.32
0.000 - 0.667	17 58	17.967	0.875 - 15.721	1.614	1.614	20	+ 32.280	+ 131.00	1.9935	294.16	85.0	3.46
0.000 - 0.700	18 34	18.567	0.875 - 16.246	1.614	1.614	21	+ 33.894	+ 135.50	1.9935	305.30	85.0	3.59
0.000 - 0.733	19 09	19.150	0.875 - 16.756	1.614	1.614	22	+ 35.508	+ 140.50	1.9935	317.47	85.0	3.73
0.000 - 0.767	19 47	19.793	0.875 - 17.310	1.614	1.614	23	+ 37.122	+ 144.75	1.9935	329.05	85.0	3.86
0.000 - 0.800	20 21	20.350	0.875 - 17.806	1.614	1.614	24	+ 38.736	+ 148.00	1.9935	336.76	85.0	3.96
0.000 - 0.833	20 51	20.850	0.875 - 18.244	1.614	1.614	25	+ 40.350	+ 150.00	1.9935	343.09	85.0	4.04
0.000 - 0.867	21 25	21.417	0.875 - 18.740	1.614	1.614	26	+ 41.964	+ 151.50	1.9935	348.31	85.0	4.10
0.000 - 0.900	21 56	21.933	0.875 - 19.191	1.614	1.614	27	+ 43.578	+ 152.00	1.9935	351.63	85.0	4.14
0.000 - 0.933	22 23	22.383	0.875 - 19.585	1.614	1.614	28	+ 45.192	+ 152.12	1.9935	354.30	85.0	4.17
0.000 - 0.967	22 52	22.867	0.875 - 20.009	1.614	1.614	29	+ 46.806	+ 152.12	1.9935	356.67	85.0	4.20
0.000 - 1.000	23 21	23.350	0.884 - 20.641	1.614	1.614	30	+ 48.420	+ 152.00	1.9935	358.39	85.0	4.22

Young's Modulus and Poisson's Ratio found, cannot readily be assessed from the data shown. In the case of Young's Modulus (E_{so}), it seems probable, (also as a result of the discussion in the next paragraph on the data presented in Table XVI), that both the effects mentioned have little influence on the variation in value of Young's Modulus. As regards Poisson's Ratio, insufficient data is available to enable any definite conclusion to be drawn.

Table XVI shows values of Shear Strength (S) and Young's Modulus (E_{so}) for tests conducted at the same chamber pressure (although not at the same rate of strain), with the corresponding values of initial moisture content (w) and initial void ratio (e) for each case. From the data shown, it can clearly be seen that the differences in Shear Strength and Young's Modulus values, can be accounted for very satisfactorily by the respective values of initial void ratio and initial moisture content. In the case of each of the test pairs: Nos. 1 & 2 ($\sigma_3 = 30\text{lb/sq.in.}$); Nos. 3 & 4 ($\sigma_3 = 20\text{lb/sq.in.}$); and Nos. 6 & 5 ($\sigma_3 = 10\text{lb./sq.in.}$), the higher Young's Modulus value occurs together with the lower moisture content value, while in the case of the Shear Strength values for the same test pairs, the higher Shear Strength value occurs together with the lower void ratio value. Where the higher value of moisture content and void ratio occur simultaneously in any one of a pair of tests, the reduction in both Shear Strength and Young's Modulus values is very marked (Tests No. 3 & 5), and conversely Tests No. 4 & 6). It is thus clear that both the values of Shear Strength and Young's Modulus are affected by the initial moisture content and initial void ratio of the sample being tested, with void ratio having a particular effect on Shear Strength and moisture content a particular effect on Young's Modulus. Hence, also, in the case of the test pair Nos. 1 & 2, wherein the higher moisture content is accompanied by the lower void ratio in the case of Test No. 2 and conversely in the case of Test No. 1, the test with the lower void ratio has the higher Shear Strength, and the test with the lower moisture content has the higher value of Young's Modulus. It is thus not necessarily the higher rate of strain which leads to the higher Shear Strength, as would appear to be the case by comparing the Shear Strength values for the test pairs Nos. 1 & 2; and Nos. 3 & 4, especially as the test pair Nos. 6 & 5 shows the higher value of Shear Strength at the lower rate of strain.

As a result of this discussion it is also possible to explain the slight misfit on the Mohr envelope (Fig. 48) in the case of the test group Nos. 1, 3 & 6 which was conducted at a constant rate of strain. The Mohr envelope is tangential to the Mohr circle for Test No. 1, with Tests No. 3 & 6 showing the misfit. Test No. 3 has a higher void ratio (0.764), and Test No. 6 a lower void ratio (0.702) than Test No. 1 (0.737), and hence the relatively lower and higher strengths respectively obtained for these tests, leading to the Mohr circles passing below and above the Mohr envelope respectively. In the case of the test group Nos. 2, 4 & 5, the void ratios were in much closer agreement and hence gave a better fit on the Mohr envelope.

The average void ratio for the test group Nos. 1, 3 & 6 was 0.731 compared with an average of 0.707 for the test group Nos. 2, 4 & 5. Hence, the higher average strength in the case of the latter test group and thus the steeper angle of the Mohr envelope in this case, leading to the difference in apparent angle of friction (ϕ_a) between the two test groups.

An/

An effect which has not been discussed so far, is the restraint at the ends of a sample during test, as a result of the end plates used. These end plates tend to prevent the ends of the sample from straining laterally while the compressive loading is being applied, so that the sample deforms as a cylinder bulging over its midlength only. This causes a different change in volume to be measured than if the ends had been unrestrained. The phenomenon has also been discussed by Bishop and Henkel (29), who feel that at small strains, end restraint is of no importance. As the limit of the linear range in the case of every test in the test series occurred at less than one percent strain, it is considered that the effect of end restraint on the values of Poisson's Ratio determined could not have been serious.

Finally, with regard to the possible effect of test temperature (see Section 7.7) on the values of Poisson's Ratio determined, Table XVII below shows the tests arranged in order of decreasing average test temperature:

TABLE XVII

TEST NUMBER	AVERAGE TEST TEMPERATURE °C	POISSONS' RATIO μ_{sel}	POISSONS' RATIO $\mu_i(\text{const.})$
1	27.25	0.1387	0.1535
2	25.25	0.1401	-
4	24.25	0.2099	0.2372
5	24.0	0.1425	-
3	23.75	0.1390	0.1793
6	20.75	0.1429	0.1464

It is clear that no conclusion can be drawn as to the possible effect of the particular test temperature on the values of Poisson's Ratio determined.

whence equations (b) above can also be written:

$$\tau_{xy} = \mu \delta_{xy}; \quad \tau_{xz} = \mu \delta_{xz}; \quad \tau_{yz} = \mu \delta_{yz} \dots\dots\dots (g)$$

Turning now to Flamant's paper: The following is a closer investigation of the steps omitted in the paper in obtaining the expressions for the stresses N_x , N_z and T_y which in the notation used in Chapter 4 of this report, are the stresses ϵ_x , ϵ_y and ϵ_z . On page 1467 Ref. 12, Flamant firstly obtains the expressions for the displacements u , v , w of any point (x, y, z) below the horizontal surface of the semi-infinite solid subjected to an infinitely extended line loading of P per unit length. These are given as:

$$u = \frac{P}{2\pi\mu} \left(\frac{xz}{x^2+z^2} - \frac{\mu}{\lambda+\mu} \arctan \frac{x}{z} \right)$$

$$v = 0$$

$$w = \frac{P}{2\pi\mu} \left(\frac{z^2}{x^2+z^2} + \frac{\lambda+2\mu}{\lambda+\mu} \log \frac{2\ell}{\sqrt{x^2+z^2}} \right)$$

Hence, based on equations (a):

$$(i) \quad \epsilon_x = \frac{\partial u}{\partial x} = \frac{P}{2\pi\mu} \left[(x^2+z^2)^{-1} \cdot z + xz \left\{ -1(x^2+z^2)^{-2} \right\} 2x - \frac{\mu}{\lambda+\mu} \left(\frac{1}{z} \cdot \frac{1}{1+\frac{x^2}{z^2}} \right) \right]$$

$$= \frac{P}{2\pi\mu} \left[\frac{z}{x^2+z^2} - \frac{2x^2z}{(x^2+z^2)^2} - \frac{\mu}{\lambda+\mu} \left(\frac{1}{z} \cdot \frac{z^2}{x^2+z^2} \right) \right]$$

$$= \frac{P}{2\pi\mu} \left[\frac{(x^2+z^2)z - 2x^2z - \frac{\mu}{\lambda+\mu} \cdot z(x^2+z^2)}{(x^2+z^2)^2} \right]$$

$$= \frac{P}{2\pi\mu} \left[\frac{z(x^2+z^2)(1-\frac{\mu}{\lambda+\mu}) - 2x^2z}{(x^2+z^2)^2} \right]$$

$$= \frac{P}{2\pi\mu} \left[\frac{\lambda}{\lambda+\mu} \cdot \frac{z}{x^2+z^2} - \frac{2x^2z}{(x^2+z^2)^2} \right]$$

$$(ii) \quad \epsilon_y = \frac{\partial v}{\partial y} = 0$$

$$(iii) \quad \epsilon_z = \frac{\partial w}{\partial z} = \frac{P}{2\pi\mu} \left[(x^2+z^2)^{-1} \cdot 2z + z^2 \left\{ -1(x^2+z^2)^{-2} \right\} 2z + \frac{\lambda+2\mu}{\lambda+\mu} \left\{ \frac{(x^2+z^2)^{\frac{1}{2}}}{2\ell} \cdot 2\ell \left(-\frac{1}{2} \right) (x^2+z^2)^{-\frac{3}{2}} \cdot 2z \right\} \right]$$

$$= \frac{P}{2\pi\mu} \left[\frac{2z}{x^2+z^2} - \frac{2z^3}{(x^2+z^2)^2} - \frac{\lambda+2\mu}{\lambda+\mu} \cdot \frac{z}{x^2+z^2} \right]$$

$$= \frac{P}{2\pi\mu} \left[\frac{(x^2+z^2) \cdot 2z - 2z^3 - \frac{\lambda+2\mu}{\lambda+\mu} \cdot z(x^2+z^2)}{(x^2+z^2)^2} \right]$$

$$= \frac{P}{2\pi\mu} \left[\frac{z(x^2+z^2)(2-\frac{\lambda+2\mu}{\lambda+\mu}) - 2z^3}{(x^2+z^2)^2} \right]$$

$$\text{i.e. } \epsilon_z = \frac{\partial w}{\partial z} = \frac{P}{2\pi\mu} \left[\frac{\lambda}{\lambda+\mu} \cdot \frac{z}{x^2+z^2} - \frac{2z^3}{(x^2+z^2)^2} \right]$$

Now by equation (f): $e = \frac{\delta V}{V} = \epsilon_x + \epsilon_y + \epsilon_z$

$$\therefore e = \frac{P}{2\pi\mu} \left[\frac{\lambda}{\lambda+\mu} \cdot \frac{2z}{x^2+z^2} - \frac{2x^2z}{(x^2+z^2)^2} - \frac{2z^3}{(x^2+z^2)^2} \right]$$

$$= \frac{P}{\pi\mu} \dots\dots\dots/$$

$$\begin{aligned}
 &= \frac{P}{\pi \mu} \left[\frac{\lambda}{\lambda + \mu} \cdot \frac{z}{x^2 + z^2} - \frac{z(x^2 + z^2)}{(x^2 + z^2)^2} \right] \\
 &= \frac{P}{\pi \mu} \left(\frac{z}{x^2 + z^2} \right) \left(\frac{\lambda}{\lambda + \mu} - 1 \right) \\
 &= \frac{P}{\pi \mu} \left(\frac{z}{x^2 + z^2} \right) \left(-\frac{\mu}{\lambda + \mu} \right) \\
 &= -\frac{P}{\pi(\lambda + \mu)} \left(\frac{z}{x^2 + z^2} \right)
 \end{aligned}$$

Hence, by equations (c):

$$\begin{aligned}
 (i) \quad \sigma_x &= \lambda e + 2\mu \epsilon_x \\
 &= \lambda \left[-\frac{P}{\pi(\lambda + \mu)} \frac{z}{x^2 + z^2} \right] + 2\mu \left[\frac{P}{2\pi\mu} \left\{ \frac{\lambda}{\lambda + \mu} \frac{z}{x^2 + z^2} - \frac{2x^2 z}{(x^2 + z^2)^2} \right\} \right] \\
 &= -\frac{P}{\pi} \cdot \frac{\lambda}{\lambda + \mu} \cdot \frac{z}{x^2 + z^2} + \frac{P}{\pi} \cdot \frac{\lambda}{\lambda + \mu} \cdot \frac{z}{x^2 + z^2} - \frac{P}{\pi} \cdot \frac{2x^2 z}{(x^2 + z^2)^2}
 \end{aligned}$$

$$\text{i.e.} \quad \sigma_x = -\frac{2P}{\pi} \frac{x^2 z}{(x^2 + z^2)^2} \dots\dots\dots (1)$$

$$\begin{aligned}
 (ii) \quad \sigma_z &= \lambda e + 2\mu \epsilon_z \\
 &= \lambda \left[-\frac{P}{\pi(\lambda + \mu)} \frac{z}{x^2 + z^2} \right] + 2\mu \left[\frac{P}{2\pi\mu} \left\{ \frac{\lambda}{\lambda + \mu} \frac{z}{x^2 + z^2} - \frac{2z^3}{(x^2 + z^2)^2} \right\} \right] \\
 &= -\frac{P}{\pi} \cdot \frac{\lambda}{\lambda + \mu} \cdot \frac{z}{x^2 + z^2} + \frac{P}{\pi} \cdot \frac{\lambda}{\lambda + \mu} \cdot \frac{z}{x^2 + z^2} - \frac{P}{\pi} \cdot \frac{2z^3}{(x^2 + z^2)^2}
 \end{aligned}$$

$$\text{i.e.} \quad \sigma_z = -\frac{2P}{\pi} \frac{z^3}{(x^2 + z^2)^2} \dots\dots\dots (2)$$

For the calculation of the Shear Stress τ_{xz} (i.e. τ_{zx}), the equation $\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$ is firstly used (equation (b))

$$\begin{aligned}
 u &= \frac{P}{2\pi\mu} \left\{ xz(x^2 + z^2)^{-1} - \frac{\mu}{\lambda + \mu} \arctan \frac{x}{z} \right\} \\
 \frac{\partial u}{\partial z} &= \frac{P}{2\pi\mu} \left[(x^2 + z^2)^{-1} \cdot x + xy(-1)(x^2 + z^2)^{-2} \cdot 2z - \frac{\mu}{\lambda + \mu} \cdot \left(-\frac{x}{z^2} \right) \left(\frac{1}{1 + \frac{x^2}{z^2}} \right) \right] \\
 &= \frac{P}{2\pi\mu} \left[\frac{x}{x^2 + z^2} - \frac{2xz^2}{(x^2 + z^2)^2} + \frac{\mu}{\lambda + \mu} \cdot \frac{x}{x^2 + z^2} \right] \\
 &= \frac{P}{2\pi\mu} \left[\frac{x}{x^2 + z^2} \left\{ 1 + \frac{\mu}{\lambda + \mu} \right\} - \frac{2xz^2}{(x^2 + z^2)^2} \right] \\
 &= \frac{P}{2\pi\mu} \left[\frac{\lambda + 2\mu}{\lambda + \mu} \cdot \frac{x}{x^2 + z^2} - \frac{2xz^2}{(x^2 + z^2)^2} \right]
 \end{aligned}$$

$$w = \frac{P}{2\pi\mu} \dots\dots\dots/$$

$$w = \frac{P}{2\pi\mu} \left[z^2 \cdot (x^2+z^2)^{-1} + \frac{\lambda+2\mu}{\lambda+\mu} \log \left\{ 2\ell (x^2+z^2)^{-\frac{1}{2}} \right\} \right]$$

$$\text{So } \frac{\partial w}{\partial x} = \frac{P}{2\pi\mu} \left[z^2 (-1) (x^2+z^2)^{-2} \cdot 2x + \frac{\lambda+2\mu}{\lambda+\mu} \left\{ \frac{(x^2+z^2)^{\frac{1}{2}}}{2\ell} \cdot 2\ell \cdot \left(-\frac{1}{2}\right) (x^2+z^2)^{-\frac{3}{2}} \cdot 2x \right\} \right]$$

$$= \frac{P}{2\pi\mu} \left[-\frac{2xz^2}{(x^2+z^2)^2} - \frac{\lambda+2\mu}{\lambda+\mu} \left(\frac{x}{x^2+z^2} \right) \right]$$

Further, by equations (g):

$$\tau_{xz} = \mu \cdot \gamma_{xz} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$

$$\text{So } \tau_{xz} = \mu \cdot \frac{P}{2\pi\mu} \left[\frac{\lambda+2\mu}{\lambda+\mu} \cdot \frac{x}{x^2+z^2} - \frac{2xz^2}{(x^2+z^2)^2} - \frac{\lambda+2\mu}{\lambda+\mu} \cdot \frac{x}{x^2+z^2} - \frac{2xz^2}{(x^2+z^2)^2} \right]$$

$$\text{ie. } \tau_{xz} = -\frac{2P}{\pi} \frac{xz^2}{(x^2+z^2)^2} \dots\dots\dots (3)$$

It should be noted that in each case the stresses σ_x , σ_z and τ_{xz} finally do not contain terms of Lamé's constants, which in turn contain the moduli of elasticity including Poisson's Ratio.

The equations (1), (2) and (3) can be transformed into Polar coordinates, to bring them into the form given by Terzaghi Ref.8 p.376, (also equations 4.1, 4.2 and 4.3 of this report) as follows:

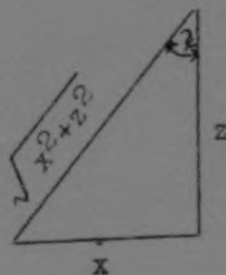
$$\cos \psi = \frac{z}{\sqrt{x^2+z^2}}; \quad \sin \psi = \frac{x}{\sqrt{x^2+z^2}}$$

$$\therefore \cos^4 \psi = \frac{z^4}{(x^2+z^2)^2}; \quad \cos^2 \psi \sin^2 \psi = \frac{x^2 z^2}{(x^2+z^2)^2}; \quad \cos^3 \psi \sin \psi = \frac{z^3 x}{(x^2+z^2)^2}$$

$$\text{Hence in equation (1) : } \sigma_x = -\frac{2P}{\pi} \frac{x^2 z}{(x^2+z^2)^2}$$

$$= -\frac{2P}{\pi z} \frac{x^2 z^2}{(x^2+z^2)^2}$$

$$\text{ie. } \sigma_x = -\frac{2P}{\pi z} \cos^2 \psi \sin^2 \psi$$



$$\text{in equation (b) : } \sigma_z = -\frac{2P}{\pi} \frac{z^3}{(x^2+z^2)^2}$$

$$= -\frac{2P}{\pi z} \frac{z^4}{(x^2+z^2)^2}$$

$$\text{ie. } \sigma_z = -\frac{2P}{\pi z} \cos^4 \psi$$

$$\text{in equation (c) : } \tau_{xz} = -\frac{2P}{\pi} \frac{xz^2}{(x^2+z^2)^2} = -\frac{2P}{\pi z} \frac{xz^3}{(x^2+z^2)^2}$$

$$\text{ie. } \tau_{xz} = -\frac{2P}{\pi z} \cos^3 \psi \sin \psi$$

It should be noted, however, that Terzaghi's sign convention is such that the expressions for the stresses are in each case taken to be positive.

APPENDIX III

(1) Comparison of values of S_{max} as determined from:

(1) $S_{max} = \frac{1}{2} \sqrt{(\sigma_z - \sigma_r)^2 + 4\tau_{rz}^2}$ Eqn 5.3

(2) $S_{max} = \frac{1}{2} \sqrt{(\sigma_z - \sigma_t)^2 + 4\tau_{tz}^2} = \frac{1}{2}(\sigma_z - \sigma_t)$ Eqn 5.5 ($\tau_{tz} = 0$)

TABLE XVII

z	r	$\mu = 0$		$\mu = 0.125$	
		Eqn 5.3	Eqn 5.5	Eqn 5.3	Eqn 5.5
3	0	30.9468	30.9468	29.8415	29.8415
	5	3.1942	0.6027	3.2351	1.6915
	10	0.3495	0.3061	0.3610	0.2165
	20	0.0910	0.1390	0.0520	0.1038
	30	0.0536	0.0706	0.0336	0.0529
	45	0.0288	0.0340	0.0192	0.0255
6	60	0.0175	0.0199	0.0128	0.0150
	0	7.7365	7.7365	7.4604	7.4602
	5	3.2250	2.0427	3.1563	1.9761
	10	0.7983	0.1536	0.8089	0.1749
	20	0.0866	0.0763	0.0897	0.0540
	30	0.0316	0.0525	0.0212	0.0390
10	45	0.0193	0.0294	0.0100	0.0221
	60	0.0132	0.0175	0.0087	0.0131
	0	2.7852	2.7852	2.6857	2.6817
	5	1.9295	1.6011	1.8709	1.5426
	10	0.8751	0.4703	0.8614	0.4581
	20	0.1724	0.0040	0.1776	0.0137
15	30	0.0447	0.0278	0.0471	0.0191
	45	0.0132	0.0222	0.0100	0.0165
	60	0.0080	0.0151	0	0.0113
	0	1.2374	1.2374	1.1929	1.1932
	5	1.0416	0.9549	1.0072	0.9201
	10	0.6736	0.4933	0.6560	0.4758
20	20	0.2201	0.0735	0.2196	0.0803
	30	0.0767	0.0024	0.0787	0.0066
	45	0.0200	0.0127	0.0212	0.0087
	60	0.0032	0.0111	0	0.0081
	0	0.6963	0.6963	0.6717	0.6715
	5	0.6305	0.5984	0.6088	0.5770
30	10	0.4828	0.3995	0.4684	0.3850
	20	0.2284	0.1183	0.2262	0.1148
	30	0.0938	0.0263	0.0943	0.0275
	45	0.0304	0.0032	0.0316	0.0008
	60	0.0100	0.0072	0.0112	0.0050
	0	0.3095	0.3088	0.2984	0.2978
45	5	0.2952	0.2897	0.2845	0.2791
	10	0.2600	0.2387	0.2513	0.2300
	20	0.1692	0.1234	0.1650	0.1190
	30	0.0975	0.0517	0.0960	0.0505
	45	0.0413	0.0119	0.0415	0.0125
	60	0.0200	0.0008	0.0206	0.0017
60	0	0.1377	0.1369	0.1328	0.1320
	5	0.1347	0.1337	0.1301	0.1289
	10	0.1271	0.1218	0.1226	0.1173
	20	0.1022	0.0876	0.0987	0.0843
	30	0.0750	0.0541	0.0728	0.0523
	45	0.0427	0.0231	0.0421	0.0225
60	60	0.0245	0.0096	0.0244	0.0095
	0	0.0772	0.0772	0.0740	0.0744
	5	0.0763	0.0756	0.0731	0.0730
	10	0.0744	0.0724	0.0712	0.0698
	20	0.0648	0.0597	0.0628	0.0575
	30	0.0543	0.0446	0.0526	0.0430
60	45	0.0361	0.0247	0.0353	0.0239
	60	0.0255	0.0136	0.0250	0.0131

z	r	$\mu = 0.25$		$\mu = 0.375$	
		Eqn 5.3	Eqn 5.5	Eqn 5.3	Eqn 5.5
3	0	28.7363	28.7363	27.6311	27.6311
	5	3.3214	0.7801	3.4502	0.8689
	10	0.4229	0.1270	0.5176	0.0375
	20	0.0264	0.0685	0.0483	0.0333
	30	0.0138	0.0351	0	0.0175
	45	0.0105	0.0170	0	0.0084
	60	0.0071	0.0100	0	0.0050
6	0	7.1835	7.1839	6.9073	6.9075
	5	3.0974	1.9094	3.0481	1.8428
	10	0.8308	0.1961	0.8627	0.2174
	20	0.1051	0.0318	0.1291	0.0095
	30	0.0229	0.0254	0.0350	0.0120
	45	0	0.0147	0.0063	0.0023
	60	0	0.0087	0	0.0044
10	0	2.5863	2.5863	2.4868	2.4868
	5	1.8149	1.4841	1.7612	1.4256
	10	0.8514	0.4460	0.8454	0.4339
	20	0.1865	0.0234	0.1990	0.0332
	30	0.0541	0.0103	0.0636	0.0016
	45	0.0122	0.0107	0.0173	0.0050
	60	0	0.0075	0.0055	0.0039
15	0	1.1491	1.1491	1.1053	1.1049
	5	0.9726	0.8852	0.9393	0.8504
	10	0.6395	0.4583	0.6243	0.4408
	20	0.2208	0.0811	0.2239	0.0820
	30	0.0829	0.0107	0.0882	0.0150
	45	0.0240	0.0048	0.0287	0.0008
	60	0.0055	0.0052	0.0095	0.0022
20	0	0.6462	0.6466	0.6215	0.6217
	5	0.5875	0.5555	0.5758	0.5340
	10	0.4551	0.3704	0.4402	0.3559
	20	0.2228	0.1118	0.2214	0.1088
	30	0.0960	0.0286	0.0985	0.0298
	45	0.0339	0.0015	0.0367	0.0040
	60	0.0122	0.0028	0.0150	0.0006
30	0	0.2873	0.2869	0.2761	0.2760
	5	0.2747	0.2685	0.2641	0.2580
	10	0.2432	0.2212	0.2346	0.2125
	20	0.1607	0.1146	0.1567	0.1102
	30	0.0947	0.0493	0.0941	0.0481
	45	0.0421	0.0131	0.0433	0.0137
	60	0.0218	0.0027	0.0230	0.0037
45	0	0.1272	0.1273	0.1225	0.1225
	5	0.1254	0.1241	0.1207	0.1194
	10	0.1182	0.1130	0.1139	0.1086
	20	0.0958	0.0812	0.0925	0.0780
	30	0.0712	0.0505	0.0694	0.0487
	45	0.0415	0.0219	0.0412	0.0213
	60	0.0214	0.0095	0.0250	0.0095
60	0	0.0716	0.0716	0.0692	0.0688
	5	0.0707	0.0704	0.0683	0.0678
	10	0.0691	0.0672	0.0667	0.0646
	20	0.0602	0.0553	0.0581	0.0531
	30	0.0512	0.0413	0.0497	0.0398
	45	0.0343	0.0231	0.0335	0.0223
	60	0.0245	0.0127	0.0245	0.0124

z	r	$\mu = 0.50$	
		Eqn 5.3	Eqn 5.5
3	0	26.5258	26.5258
	5	3.6186	0.9576
	10	0.6299	0.0520
	20	0.0865	0.0018
	30	0.0260	0.0002
	45	0.0078	0.0001
	60	0	0
6	0	6.6312	6.6312
	5	3.0096	1.7761
	10	0.9039	0.2387
	20	0.1570	0.0127
	30	0.0502	0.0016
	45	0.0151	0
	60	0.0063	0
10	0	2.3873	2.3873
	5	1.7091	1.3671
	10	0.8434	0.4217
	20	0.2140	0.0430
	30	0.0751	0.0071
	45	0.0245	0.0008
	60	0.0103	0
15	0	1.0607	1.0607
	5	0.9065	0.8156
	10	0.6107	0.4233
	20	0.2286	0.0827
	30	0.0948	0.0191
	45	0.0335	0.0032
	60	0.0135	0.0006
20	0	0.5968	0.5968
	5	0.5446	0.5125
	10	0.4271	0.3414
	20	0.2209	0.1058
	30	0.1018	0.0310
	45	0.0403	0.0063
	60	0.0180	0.0016
30	0	0.2650	0.2650
	5	0.2543	0.2475
	10	0.2262	0.2037
	20	0.1535	0.1058
	30	0.0938	0.0470
	45	0.0445	0.0143
	60	0.0245	0.0047
45	0	0.1177	0.1177
	5	0.1161	0.1146
	10	0.1095	0.1042
	20	0.0898	0.0748
	30	0.0678	0.0470
	45	0.0412	0.0207
	60	0.0255	0.0095
60	0	0.0660	0.0660
	5	0.0652	0.0652
	10	0.0636	0.0620
	20	0.0561	0.0510
	30	0.0482	0.0382
	45	0.0331	0.0215
	60	0.0245	0.0120

2) Determination of the Points of Intersection of Stress Contour Lines with the Surface $z = 0$ of a Semi-infinite Solid in the case of Stress Contour Diagrams for σ_r , σ_t and S_{max}

(i) The Horizontal Radial Stress σ_r is given by the expression:

$$\sigma_r = \frac{Q}{2\pi} \left[\frac{3r^2 z}{(r^2 + z^2)^{3/2}} - \frac{1 - 2\mu}{r^2 + z^2 + z \sqrt{r^2 + z^2}} \right] \dots\dots\dots 4.8(a)$$

On the plane $z = 0$, this becomes:

$$\begin{aligned} \sigma_r &= \frac{Q}{2\pi} \left[- \frac{(1-2\mu)}{r^2} \right] \\ &= - \frac{Q}{2\pi r^2} (1-2\mu) \end{aligned}$$

$$\text{i.e. } r^2 = - \frac{Q}{2\pi \sigma_r} (1-2\mu)$$

on the plane $z = 0$, σ_r is found by calculation to have negative values for all values of μ .

Hence:

$$r = \pm \sqrt{- \frac{Q}{2\pi \sigma_r} (1-2\mu)} \dots\dots\dots (a)$$

where $\sigma_r = -0.1, -0.2, -0.5$ etc. which therefore renders the term inside the square root a positive quantity.

(ii) The Horizontal Tangential Stress σ_t is given by the expression:

$$\sigma_t = - \frac{Q}{2\pi} (1-2\mu) \left[\frac{z}{(r^2 + z^2)^{3/2}} - \frac{1}{r^2 + z^2 + z \sqrt{r^2 + z^2}} \right] \dots\dots\dots 4.9(a)$$

On the plane $z = 0$, this becomes:

$$\begin{aligned} \sigma_t &= - \frac{Q}{2\pi} (1-2\mu) \left(- \frac{1}{r^2} \right) \\ &= + \frac{Q}{2\pi r^2} (1-2\mu) \end{aligned}$$

$$\text{i.e. } r^2 = + \frac{Q}{2\pi \sigma_t} (1-2\mu)$$

On the plane $z = 0$, σ_t is found by calculation to have positive values for all values of μ .

Hence:

$$r = \pm \sqrt{\frac{Q}{2\pi \sigma_t} (1-2\mu)} \dots\dots\dots (b)$$

(iii) The Maximum Shear Stress S_{max} is given by the expression:

$$S_{max} = \frac{1}{2} \sqrt{(\sigma_z - \sigma_r)^2 + 4\tau_{rz}^2} \dots\dots\dots 5.3$$

On

On substituting the values of σ_z , σ_r and τ_{rz} as given by equations 4.7(e), 4.8(e) and 4.10(e) in the above equation 5.3, and in addition substituting $z = 0$, this gives the Maximum Shear Stress S_{max} on the plane $z = 0$ as:

$$\begin{aligned} S_{max} &= \left[\left\{ -\frac{Q}{2\pi} \left(-\frac{1-2\mu}{r^2} \right) \right\}^2 \right]^{\frac{1}{2}} \\ &= -\frac{Q}{2\pi} \left(-\frac{1-2\mu}{r^2} \right) \\ &= +\frac{Q}{2\pi r^2} (1-2\mu) \end{aligned}$$

$$\text{i.e. } r^2 = +\frac{Q}{2\pi S_{max}} (1-2\mu)$$

On the plane $z = 0$, S_{max} is found by calculation to be positive for all values of μ

Hence:

$$r = +\sqrt{\frac{Q}{2\pi S_{max}} (1-2\mu)} \dots\dots\dots (c)$$

It will be seen that the equations (a), (b) and (c) reduce to the same equation if:

firstly : $-\sigma_r = \sigma_t = S_{max} = K$ is substituted;

secondly: the same value of μ is used in each case.

Hence, for a particular numerical value of any contour, say the contour 1.0 units of force/(units of length)², and the same value of μ , only one calculation is needed to find the value of r for the contour lines $-\sigma_r = \sigma_t = S_{max} = 1.0$ units of force/(units of length)².

The various calculations are shown in tabular form on the next page.

Equation/

Equation: $r = \sqrt{\frac{Q}{2\pi K} (1 - 2\mu)}$ where $-\sigma_r = \sigma_t = S_{max} = K$

TABLE XVIII

K	1-2μ	$\frac{Q}{2\pi}$	$\frac{Q}{2\pi K}$	$\frac{Q}{2\pi K} (1-2\mu)$	$r = \sqrt{\frac{Q}{2\pi K} (1-2\mu)}$
Condition: μ = 0					
0.05	1	159.1549	3183.0989	3183.0989	56.4
0.1	1	"	1591.5494	1591.5494	39.9
0.2	1	"	795.7747	795.7747	28.2
0.5	1	"	318.3099	318.3099	17.8
1.0	1	"	159.1549	159.1549	12.6
2.0	1	"	79.5775	79.5775	8.9
5.0	1	"	31.8310	31.8310	5.6
10.0	1	"	15.9155	15.9155	4.0
Condition: μ = 0.25					
0.025	0.5	159.1549	6366.1977	3183.0989	56.4
0.050	"	"	3183.0989	1591.5494	39.9
0.1	"	"	1591.5494	795.7747	28.2
0.2	"	"	795.7747	397.8873	19.9
0.5	"	"	318.3099	159.1549	12.6
1.0	"	"	159.1549	79.5775	8.9
2.0	"	"	79.5775	39.7887	6.3
5.0	"	"	31.8310	15.9155	4.0
10.0	"	"	15.9155	7.9577	2.8
Condition: μ = 0.375					
0.0125	0.25	159.1549	12732.3954	3183.0989	56.4
0.025	"	"	6366.1977	1591.5494	39.9
0.050	"	"	3183.0989	795.7747	28.2
0.1	"	"	1591.5494	397.8873	19.9
0.2	"	"	795.7747	198.9437	14.1
0.5	"	"	318.3099	79.5775	8.9
1.0	"	"	159.1549	39.7887	6.3
2.0	"	"	79.5775	19.8944	4.5
5.0	"	"	31.8310	7.9577	2.8
10.0	"	"	15.9155	3.9789	2.0

1)

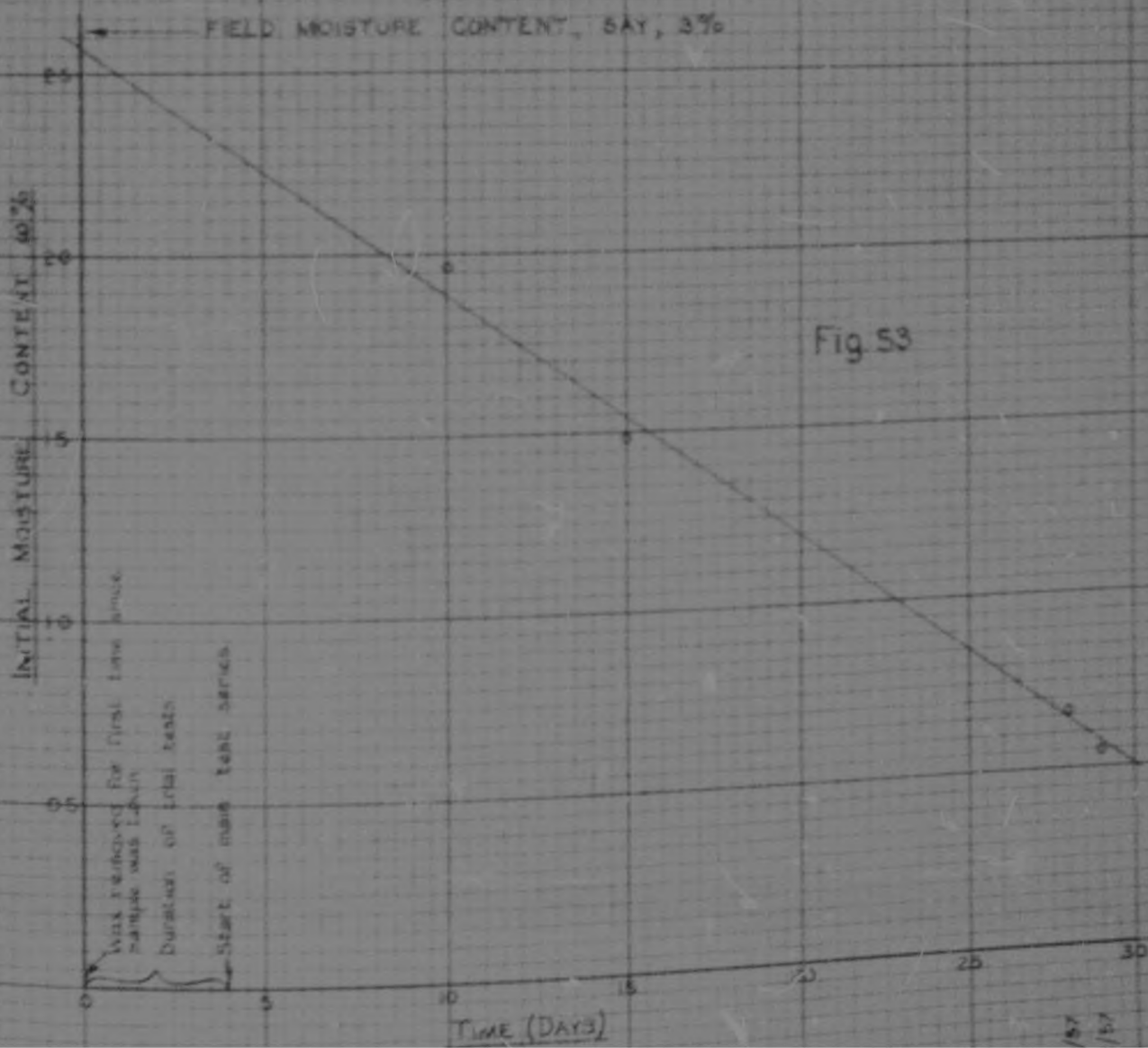
APPENDIX IV
MOISTURE CONTENT DETERMINATIONS

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TABLE XX

DATE	TEST NO.	INITIAL MOISTURE CONTENT			MOISTURE CONTENT AFTER TEST		
		WT. OF OVEN DRIED SOIL = W_s (gm)	WT. OF WATER = W_w (gm)	MOISTURE CONTENT ω %	WT. OF OVEN DRIED SOIL = W_s (gm)	WT. OF WATER = W_w (gm)	MOISTURE CONTENT ω %
22-1-57	2	—	—	—	131.4	3.5	2.740
28-1-57	CONSOLIDOMETER	178.5	3.5	1.951	—	—	—
29-1-57	3	—	—	—	126.2	3.0	2.377
30-1-57	5	—	—	—	129.9	2.7	2.079
31-1-57	6	—	—	—	130.9	2.5	1.910
22-2-57	* 6B	33.77	0.50	1.481	128.1	2.1	1.665
15-2-57	4	48.61	0.32	0.658	130.0	4.8	3.538
18-2-57	1	23.74	0.13	0.548	128.2	4.2	3.276

* This test was not included in the test series reported on



2) DETERMINATION OF INITIAL VOID RATIO (e) AND DEGREE OF SATURATION (S_r)
OF EACH TEST SAMPLE.

TABLE XXI.

TEST NUMBER	WT. OF OVEN DRIED SOIL = W _s gms.	SPECIFIC GRAVITY OF SOIL GRAINS = G _s	HENCE: VOLUME OF SOIL SOLIDS = V _s = $\frac{W_s}{G_s}$ cc	OVERALL VOL OF SOIL SAMPLE = V cc	HENCE: VOLUME OF VOIDS = V _v = V - V _s cc	HENCE: INITIAL VOID RATIO = e = $\frac{V_v}{V_s}$	INITIAL MOISTURE CONTENT = w (From Fig. 53) percent	HENCE: INITIAL DEGREE OF SATURATION = S _r = $\frac{w G_s}{e}$ percent
1.	128.2	2.62	48.74	85.00	35.06	0.737	0.548	1.948
2	131.4	2.62	50.16	85.00	34.84	0.695	2.290	8.636
3.	126.2	2.62	48.17	85.00	36.23	0.764	1.820	6.237
4.	130.0	2.62	49.62	85.00	35.38	0.713	0.658	2.417
5.	129.9	2.62	49.60	85.00	35.40	0.714	1.750	6.422
6.	130.9	2.62	49.95	85.00	35.05	0.702	1.680	8.272
CONSOLIDOMETER	179.5	2.62	68.13	115.85	47.72	0.700	1.951	7.331

APPENDIX V

1) Calibration of Capillary Tube used.

Measured length of capillary tube = $15\frac{9}{32}$ inches.
Weight of porcelain dish used = 38.19 gm.

Expt. No.	Wt. of Mercury to fill Capillary Tube + Wt. of basin.	Temperature
1	48.67 gm.	26.5°C
2	48.64 gm.	
3	48.67 gm.	
4	48.67 gm.	26.7°C
5	48.65 gm.	
Averages:	48.66 gm.	26.6°C

Wt. of Mercury to fill capillary tube = (48.66 - 38.19) gm.
= 10.47 gm.

From Ref. 31, page 1991, by interpolation:
Density of Mercury at 26.6°C = 13.5301 gm/cc.

Hence, Volume of Mercury to fill capillary = $V = \frac{10.47}{13.5301}$ cub.cm.

From above, length of capillary = $15\frac{9}{32}$ " = $\frac{126}{32}$ " x 2.54 cm.

Cross-section Area of capillary = $A = \frac{10.47}{13.5301} \times \frac{32}{489 \times 2.54}$ sq.cm.
= 1.9935×10^{-2} sq.cm.

Hence, Diameter of Capillary = $D = \sqrt{\frac{4A}{\pi}}$
= $2 \sqrt{\frac{1.9935 \times 10^{-2}}{\pi}}$
= 0.1593068 cm.
say, = 1.593 mm.

2) Calculation of Slug Movement due to Spindle Entry only.

Diameter of Loading Spindle = 0.5 inches.

Hence, cross-sectional Area of Spindle = $\frac{\pi \times (0.5)^2}{4}$ sq.in.

Fluid displaced per 1 division deflection (0.001")

$$= \frac{\pi \times (0.5)^2}{4} \times 10^{-3} \text{ cub.in.}$$

$$= \frac{\pi \times (0.5)^2}{4} \times 10^{-3} \times 16.39 \text{ cub.cm.}$$

$$= \frac{\pi \times 16.39}{16} \times 10^{-3} \text{ cub.cm.}$$

$$= 3.2192 \times 10^{-3} \text{ cc.}$$

From Item (1) of this Appendix
Cross-sectional area of capillary tube = 1.9935×10^{-2} sq.cm.
i.e. 1.9935×10^{-2} cub.cm. of fluid displaced from the test cell,
cause a slug movement of 1cm. in the capillary tube.

Hence

Hence, 3.2182×10^{-3} cub.cm. of fluid displaced from the test cell, cause a slug movement of:

$$\begin{aligned} & \frac{3.2182 \times 10^{-3}}{1.9935 \times 10^{-2}} \times 1.0 \text{ cm.} \\ & = 1.6143 \times 10^{-1} \text{ cm.} \\ & = \underline{1.614 \text{ mm.}} \rightarrow \end{aligned}$$

Hence, per unit deflection (10^{-3} in.), the loading spindle advances a distance of 1×10^{-3} inches into the test cell, there being a flow of fluid out of the test cell resulting in a slug movement of 1.614 mm. in the capillary tube, from left to right.

i.e. slug movement due to spindle entry only

$$= \underline{1.614 \text{ mm. per division deflection.}} \rightarrow$$

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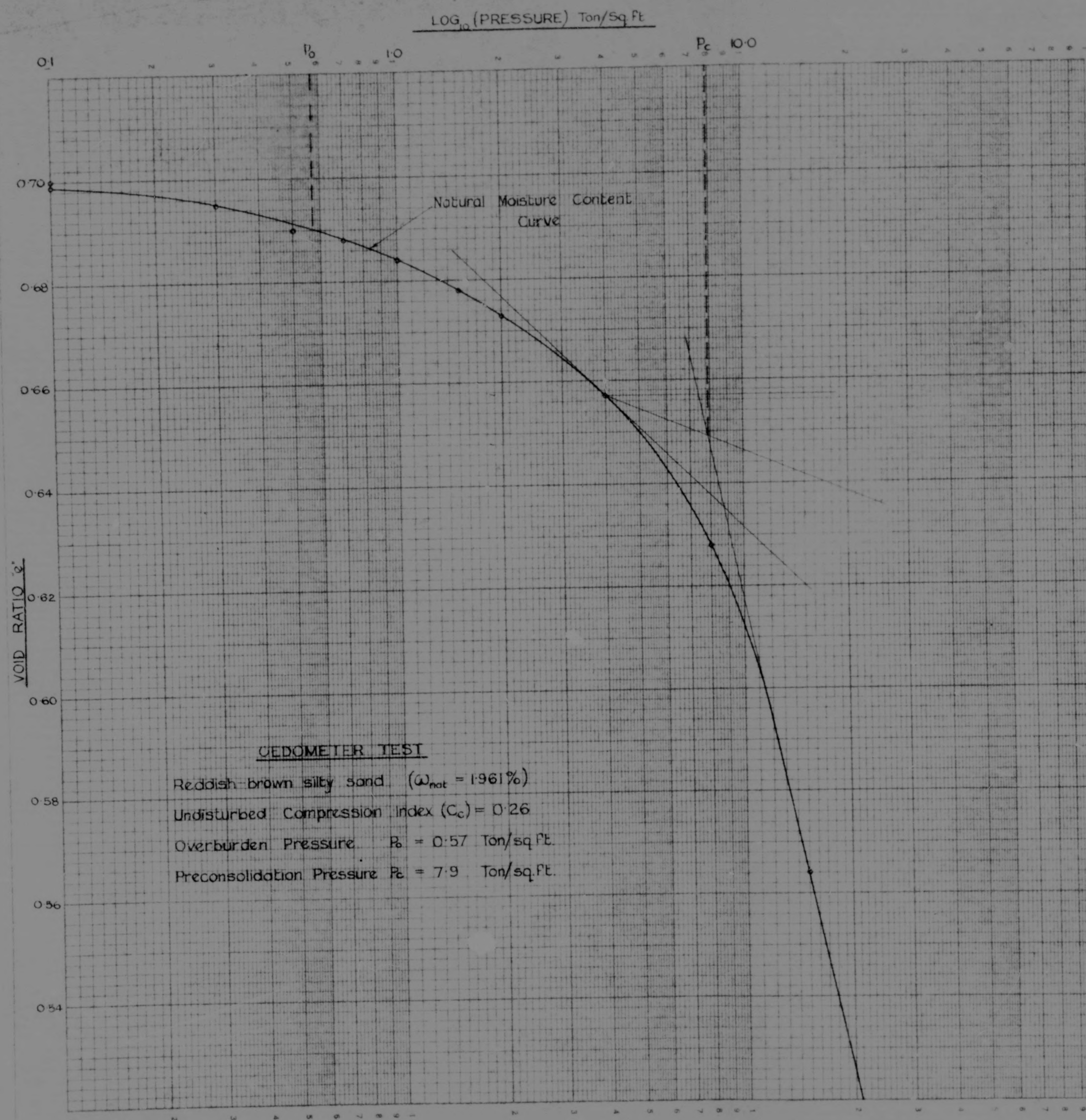


Fig. 18 — Pressure : Void Ratio Curve for the Test Soil
(Normal 1-Dimensional Consolidation Test)

Author Rauch H P

Name of thesis The significance of Poisson's Ratio in the determination of stress and settlement in soils 1958

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