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The modelling and optimisation of aggregate plants, and the use of the Apollo Computer Program

Hayden, John Samuel

2015-01-26
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This expression provides the efficiency curve for the screen, and is shown graphically in Figure 4.3. m is the number of opportunities that a particle has of passing through the screen and is considered proportional to:

an efficiency constant $(k_1)^2$
the length of the screen L
a load factor f

so that $m = (k_1)^2 Lf$.

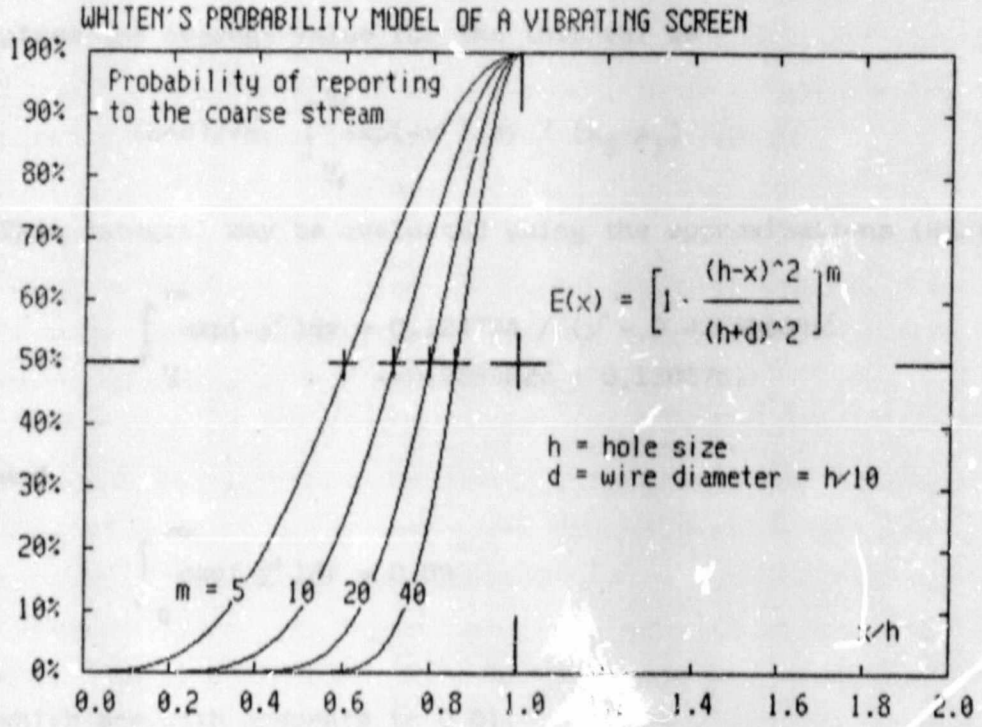


Figure 4.3 Efficiency curves obtained using Whiten's probability model

To use this model for predicting the probability that material in a size interval does not pass through the screen, an average value of the efficiency curve is required for that size fraction. A satisfactory approximation of the average is obtained using, for example, the mean particle-size in that interval. A more accurate value, the unweighted average, is given by

$$\int_{x_1}^{x_2} E(x).dx \;/\; (x_2-x_1) \qquad (4.2)$$

and to facilitate integration, the approximation

$$E(x) = \exp\left[-m(h-x)^2/(h+d)^2\right]$$

may be used. Putting this approximation into equation (4.2) and making the substitution

$$y = \sqrt{m}(x-h)/(h+d)$$

gives the average value for the interval as

$$(h+d)/\sqrt{m} \int_{y_1}^{y_2} \exp(-y^2).dy \;/\; (x_2-x_1)$$

This integral may be evaluated using approximations (Hart,1968)

$$\int_{y}^{\infty} \exp(-y^2)dy = 0.124734 \;/\; (y^3 - 0.43788005y^2 + 0.266982y + 0,138375)$$

and

$$\int_{0}^{\infty} \exp(-y^2)dy = 0.89$$

which are both accurate to 0.01.

According to Whiten, the model provides an adequate description of screen behaviour, except for the sub-mesh residue material of which a percentage was found to report to the coarse stream, possibly due to moisture, which tends to cause fines to cling to larger particles, thereby 'bypassing' the screening mechanism.

CHAPTER 5

LINEAR MODELLING OF AGGREGATE PROCESSING CIRCUITS

Aggregate plants consist primarily of an ordered arrangement of crushers, screens, and stockpiles, linked by means of conveyor belts and chutes.

The mathematical modelling of the two main components of plant-circuits, namely, crushers and screens, has been dealt with in Chapters 3 and 4 respectively. In this chapter, these components are incorporated into a general model of a complete circuit.

The purpose of the model is to provide a means of determining the material tonnages and gradings in all parts of a given plant circuit (i.e. a 'mass-flow analysis'), taking into account the operating characteristics of the various components.

The model is 'general', in that its structure does not prescribe the way in which the crushers and screens must be modelled: Various methods of modelling these components were discussed in the two preceding chapters, and any of these - or other - methods can be used, according to preference. (The Apollo model, described in Chapter 7, is similar in structure to this general model, but is more specific in that it prescribes how the crushers and screens must be modelled.)

In developing the model, the only components that need to be considered directly are the crushers and screens: a crusher acts to transform an incoming material stream into one with an altered size-distribution; a screen serves to split an incoming stream into several new streams, on the basis of size.

Chutes and conveyor belts are modelled only indirectly, to route flows of material "from ... to ...". Also modelled indirectly is the point at which material enters the circuit, and the exit points (the final-product stockpiles).

The other components of the plant do not affect the routing of material or the material grading, and need not be taken into account. Intermediate stockpiles, feed bins and feeders are therefore ignored. (Any other processing facilities, such as hydro-cyclones, gravity-separation systems, etc., can be effectively modelled as either screens or crushers.)

The mathematics of mass-flow analysis is kept simple by assuming that the circuit is linear in character. This implies two important features of the circuit:

- Conservation of mass. What goes into the circuit must come out the other end. The material can be altered in size by crushers, or split along different paths by screens, but the total mass flow-rate in the circuit remains constant.

- Steady-state flow. The periodic build-ups that occur at intermediate stockpiles and in crusher feed-bins, etc., are ignored.

Accordingly, a set of linear equations can be developed to model the circuit. In essence, each material stream in the circuit is represented by a set of n variables making up a vector that describes the size distribution of that stream. Mass-balance equations are then developed at various points in the circuit.

The manner in which the equations are set up is best explained with reference to an example plant-circuit diagram, shown in Figure 5.1. The plant consists of one crusher and two single-deck screens. The various material streams are numbered, to aid identification. The tonnage of raw material entering the circuit, and its size grading, are known.

The material streams are described by vectors S_1, S_2, S_3,..., each with n elements reflecting the tonnages present in each size interval:

$$S_i = (S_{i1}, S_{i2}, \ldots, S_{in}) \qquad \text{for stream } i$$

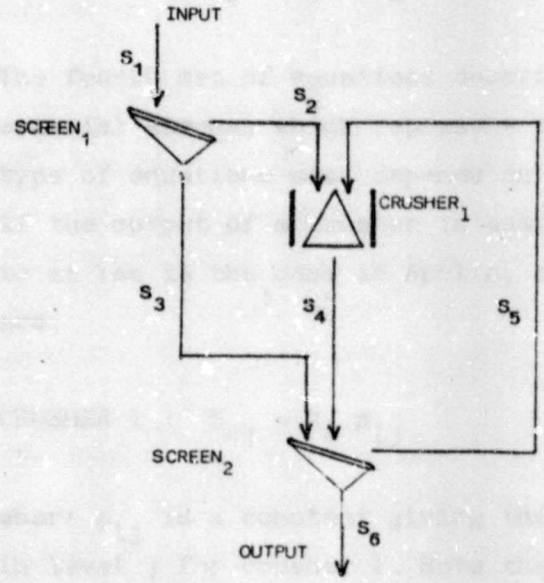


Figure 5.1 Example plant-circuit diagram

The first set of equations are merely to define the raw-material-stream vector S_1 in terms of the tonnages flowing in each size interval:

$$\text{INPUT} \quad : \quad S_{1j} = R.r_j \qquad j = 1,2,\ldots,n$$

R is the known tonnage entering the circuit and r_j is a constant giving the known proportion of raw feed found in size-interval j.

The second set of equations reflect the total tonnages in each material stream in the circuit, obtained by summing the amounts present in each size interval:

$$\text{STREAM}_i \; : \; M_i = S_{i1} + S_{i2} + \ldots + S_{in} \qquad i = 1,2,\ldots,6$$

where M_i is the total tonnage in material-stream i.

The third set of equations define mass balances at each machine in the circuit. The mass balances equate the sum of all tonnages entering a machine to the sum of all tonnages leaving it:

SCREEN 1 : $M_1 = M_2 + M_3$

SCREEN 2 : $M_3 + M_4 = M_5 + M_6$

CRUSHER 1 : $M_2 + M_3 = M_4$

The fourth set of equations describe the size distributions of those material streams which represent the direct output of crushers. The type of equations used depends on the method of modelling the crushers. If the output of a crusher is assumed to be independent of the feed to it (as is the case in Apollo, discussed in Chapter 7), the equations are:

CRUSHER 1 : $S_{4j} = M_4 \cdot p_{1j}$ $\qquad j = 1,2,\ldots,n-1$

where p_{1j} is a constant giving the proportion of total output falling in level j for crusher 1. Note that the amount of residue, in the (n)th level, is determined by subtracting the cumulative tonnage retained on the finest sieve from the total output of the crusher.

[If the crusher has been modelled using a matrix method such as Whiten's, so that the product grading is related to the feed, the equations are:

CRUSHER 1 :

$$\begin{bmatrix} S_{41} \\ S_{42} \\ \cdot \\ \cdot \\ S_{4n-1} \end{bmatrix} = Z. \begin{bmatrix} S_{21} + S_{51} \\ S_{22} + S_{52} \\ \cdot\cdot \quad \cdot\cdot \\ \cdot\cdot \quad \cdot\cdot \\ S_{2n-1} + S_{5n-1} \end{bmatrix}$$

product feed

where Z is an (n-1) x (n-1) transformation matrix, as described in Section 3.3. This method is included here merely for the sake of completeness: to keep things simple, all further discussion in this chapter assumes that crushers are modelled as having output gradings which are independent of their feed gradings.]

The fifth set of equations describe the 'coarse' and 'fine' stream gradings resulting from the screening of material:

OVER SCREEN 1 : $S_{2j} = c_{1j} \cdot S_{1j}$ $\quad j = 1,2,\ldots,n-1$

UNDER SCREEN 1 : $S_{3j} = f_{1j} \cdot S_{1j}$ $\quad j = 1,2,\ldots,n$

OVER SCREEN 2 : $S_{5j} = c_{2j} \cdot (S_{3j} + S_{4j})$ $\quad j = 1,2,\ldots,n-1$

UNDER SCREEN 2 : $S_{6j} = f_{2j} \cdot (S_{3j} + S_{4j})$ $\quad j = 1,2,\ldots,n$

c_{1j} and c_{2j} are screen-separation constants giving the proportions of feed material in level j that pass over Screens 1 and 2 respectively and report to the coarse streams, as described in Chapter 4. Similarly, f_{1j} and f_{2j} describe the proportions of feed material in level j that pass through (under) the screens, reporting to the fine streams. Note that for each screen k and each size-interval j, $c_{kj} + f_{kj} = 100\%$.

The reader will observe that the form of the above equations is not suitable for directly describing a double-deck screen, unless the screen is considered equivalent to two single-deck screens in series. Equations which model double-deck screens more effectively could easily be developed, as was shown in Chapter 4, but, in attempting to keep the overall model simple, are not considered here.

Table 5.1 gives the resulting equations which model the example circuit, assuming that n = 4. The tableau represents m independent equations in m unknowns and is thus easily solved as a set of simultaneous equations.

The size of the tableau, remembering that all screens are single-deck, is determined as:

$$m = (n+1).(\text{streams}) \qquad (5.1)$$
$$= (n+1).(1 + \text{crushers} + 2.\text{screens})$$

Since a typical large plant can easily have as many as 9 crushers and the equivalent of 20 single-deck screens, with n typically equal to about 9, it is evident from equation 5.1 that the required matrix can be very large (500 x 500 = 250 000 elements in this case), too large in fact for straight-forward micro-computer solution. The matrix size can be somewhat reduced, however, by eliminating certain equations and variables due to the appearance of any zero co-efficients: Many

of the screen-separation constants, c_{kj} and f_{kj}, will be zeros if perfect separation is assumed (see Chapter 4). Some zeros may also occur in the raw-material grading, r_j, and in the crusher product-gradings, p_{kj}.

The size of the matrix is also reduced if all double-deck screens are modelled using the equations described in Chapter 4, rather than as pairs of single-deck screens in series. In this case, the number of equations and variables is reduced by (n+1) for every double-deck screen in the circuit.

Significant further reduction of the matrix size is possible, since a substantial number of the equations and variables are, in fact, extraneous. The raw-material grading and all crusher product-gradings are in fact given as input and do not have to be calculated, so that variables and equations describing these streams can be omitted*. In this case, the number of equations and variables is reduced by n.(crushers + 1) + 1 to

crushers + 2(n+1)screens

The resulting matrix is shown in Table 5.2: Needless to say, it looks somewhat different from the original matrix, shown in Table 5.1.

The type of matrix generated by Apollo (Chapter 7) is similar in form to the reduced matrix described above, and it is left to the reader to compare the matrices directly (compare Table 5.2 with Table 7.4, which gives the Apollo matrix for a typical plant).

Note that the set of linear equations can only be solved as long as there are no choices of flow in the circuit. If, for example, material from a machine can be routed to either of two or more destinations or split in some unknown proportion between them, then there exist an infinite number of solutions. The method cannot therefore

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* If the crushers have been modelled using a matrix method such as Whiten's, so that the product gradings are related to the feed gradings, then one cannot reduce the matrix size in this manner.

directly assist in determining the optimum routing of material in the circuit and is accordingly not an optimisation technique. However, should an objective function be developed, it would be possible to use a Linear Programming technique to optimise the flows, as is the case with Apollo.

Note also that, unlike Apollo, the method does not consider the physical capacities of machines and is based merely on the input of some known tonnage into the system. The inclusion of capacities requires inequalities, again pointing to the suitability of applying a Linear Programming technique.

The methods described in this chapter for modelling circuits have been developed by the author, but are by no means unique. For example, Knuttson (1978) describes a computer program used by his company, Svedala Arbra of Sweden, which essentially models circuits in a similar fashion, but does not seem user-friendly in application. And Hodowin et al (1981) have described an algorithm for 'material balancing of mineral processing circuits'. These models have generally been developed by aggregate equipment suppliers to aid in the design and marketing of their products, and, as such, are usually proprietary.

A numerical example showing the equations required to model a simple circuit is now given. Consider the circuit shown in Figure 5.2. Assume that the number of size intervals, n, is 3 (to keep the matrix small), and that the raw material tonnage, crusher grading and screen separation details are as follows:

Size Interval	Raw material (tons/hr)	$Crusher_1$ Grading (% retained)	$Screen_1$ Separation (% to coarse stream)
1	130t/hr	15%	100%
2	65t/hr	30%	65%
3	5t/hr	55%	5%

The equations describing the circuit, and the solution, are given in Table 5.3. (Note that the form of the equations follows the form given in Table 5.1, not the modified form given in Table 5.2.)

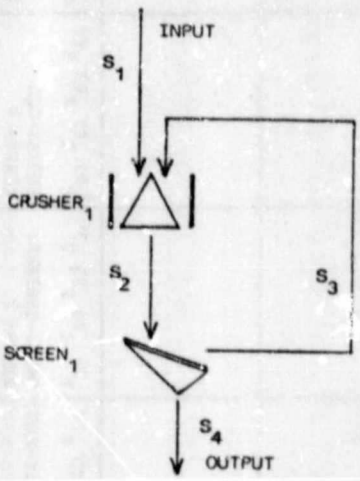


Figure 5.2 Simple circuit used in numeric example

To summarise this chapter: A general model of an aggregate processing circuit has been developed. The model consists of a set of equations which give a mass-flow analysis of a given circuit. The model does have some limitations, however:

- it cannot accommodate choices of flow, and is accordingly not an optimisation technique as such
- it does not take into account the capacities of the machines in the circuit.

Both these limitations are overcome in the Apollo model (Chapter 7) which incorporates, inter alia, (i) an objective function to optimise the routing of material, and (ii) inequalities to model machine capacities.

	STREAM 1 INPUT-SCREEN1 s_{11} s_{12} s_{13} s_{14}	STREAM 2 OVER1-CRUSHER1 s_{21} s_{22} s_{23} s_{24}	STREAM 3 UNDER1-SCREEN2 s_{31} s_{32} s_{33} s_{34}	STREAM 4 CRUSHER1-SCREEN2 s_{41} s_{42} s_{43} s_{44}	STREAM 5 OVER2-CRUSHER1 s_{51} s_{52} s_{53} s_{54}	STREAM 6 UNDER2-OUT s_{61} s_{62} s_{63} s_{64}	TOTALS M_1 M_2 M_3 M_4 M_5 M_6	RHS
INPUT SVE1	1							$R.r_1$
INPUT SVE2	1							$R.r_2$
INPUT SVE3	1							$R.r_3$
INPUT SVE4	1							$R.r_4$
STREAM1	1 1 1 1						-1	0
STREAM2		1 1 1 1					-1	0
STREAM3			1 1 1 1				-1	0
STREAM4				1 1 1 1			-1	0
STREAM5					1 1 1 1		-1	0
STREAM6						1 1 1 1	-1	0
SCREEN1							1 -1 -1	0
SCREEN2							1 1 -1 -1	0
CRUSHER1							1 -1 1	0
CRUSHER1 SVE1				-1			P_{11}	0
CRUSHER1 SVE2				-1			P_{12}	0
CRUSHER1 SVE3				-1			P_{13}	0
OVER1 SVE1	c_{11}	-1						0
OVER1 SVE2	c_{12}	-1						0
OVER1 SVE3	c_{13}	-1						0
UNDER1 SVE1	f_{11}		-1					0
UNDER1 SVE2	f_{12}		-1					0
UNDER1 SVE3	f_{13}		-1					0
UNDER1 SVE4	f_{14}		-1					0
OVER2 SVE1			c_{21}	c_{21}	-1			0
OVER2 SVE2			c_{22}	c_{22}	-1			0
OVER2 SVE3			c_{23}	c_{23}	-1			0
UNDER2 SVE1			f_{21}	f_{21}		-1		0
UNDER2 SVE2			f_{22}	f_{22}		-1		0
UNDER2 SVE3			f_{23}	f_{23}		-1		0
UNDER2 SVE4			f_{23}	f_{24}		-1		0

Table 5.1 Equations to model the circuit shown in Figure 5.1

	STREAM 2 OVER1-CRUSHER1				STREAM 3 UNDER1-SCREEN2				STREAM 5 OVER2-CRUSHER1				STREAM 6 UNDER2-OUT				TOTALS						RHS
	S_{21}	S_{22}	S_{23}	S_{24}	S_{31}	S_{32}	S_{33}	S_{34}	S_{51}	S_{52}	S_{53}	S_{54}	S_{61}	S_{62}	S_{63}	S_{64}	M_2	M_3	M_4	M_5	M_6		
STREAM2	1	1	1	1													-1						0
STREAM3					1	1	1	1										-1					0
STREAM5									1	1	1	1								-1			0
STREAM6													1	1	1	1					-1		0
SCREEN1																	1	1					R
SCREEN2																		1	1		-1	-1	0
CRUSHER1																	1		-1		1		0
OVER1 SVE1	1																						$c_{11}f_1^R$
OVER1 SVE2		1																					$c_{12}f_2^R$
OVER1 SVE3			1																				$c_{13}f_3^R$
UNDER1 SVE1					1																		$f_{11}f_1^R$
UNDER1 SVE2						1																	$f_{12}f_2^R$
UNDER1 SVE3							1																$f_{13}f_3^R$
UNDER1 SVE4								1															$f_{14}f_4^R$
OVER2 SVE1									-1										$c_{21}p_{11}$				0
OVER2 SVE2										-1									$c_{22}p_{12}$				0
OVER2 SVE3											-1								$c_{23}p_{13}$				0
UNDER2 SVE1													-1						$f_{21}p_{11}$				0
UNDER2 SVE2														-1					$f_{22}p_{12}$				0
UNDER2 SVE3															-1				$f_{23}p_{13}$				0
UNDER2 SVE4																-1			$f_{24}p_{14}$				0

Table 5.2 Modified equations to model the circuit shown in Figure 5.1

	STREAM 1 INPUT-CRUSHER1 s_{11}	s_{12}	s_{13}	STREAM 2 CRUSHER1-SCR.1 s_{21}	s_{22}	s_{23}	STREAM 3 OVER1-CRUSHER1 s_{31}	s_{32}	s_{33}	STREAM 4 UNDER1-OUT s_{41}	s_{42}	s_{43}	TOTALS M_1	M_2	M_3	M_4	RHS
INPUT SVE1	1																130
INPUT SVE2		1															65
INPUT SVE3			1														5
STREAM1	1	1	1										-1				0
STREAM2				1	1	1								-1			0
STREAM3							1	1	1						-1		0
STREAM4										1	1	1				-1	0
SCREEN1														1	-1	-1	0
CRUSHER1													1	-1	1		0
CRUSHER1 SVE1				-1										.15			0
CRUSHER1 SVE2					-1									.30			0
OVER1 SVE1				1.00			-1										0
OVER1 SVE2					0.65			-1									0
UNDER1 SVE1				0.00						-1							0
UNDER1 SVE2					0.35						-1						0
UNDER1 SVE3						0.95						-1					0
SOLUTION	130	65	5	48	96	175	48	62	9	0	34	166	200	319	119	200	

Table 5.3 Equations and solution for the circuit shown in Figure 5.2

CHAPTER 6

OPTIMISATION OF AGGREGATE PLANTS USING LINEAR PROGRAMMING

The Apollo model, which is discussed in the next chapter, incorporates two themes:

(i) the analysis of mass flows in a plant circuit

(ii) the economic optimisation of a plant operation, using Linear Programming.

The first theme has been dealt with at length in the preceding chapters. This chapter deals with the second one.

The use of Linear Programming in the economic optimisation of aggregate plants is not new. In 1966, C.F. Fiore of Anglo Alpha Cement Limited did pioneering work in developing a linear optimisation model for the Coedmore Quarry in Durban (Fiore 1966). And in 1968, Fiore and R.T. Rozwadowski demonstrated the implementation of such a process model for optimising production at a lime plant (Fiore et al, 1968). About the same time, C.J. Everett, under the guidance of Fiore, used the technique to critically compare the configuration of an existing plant in the Anglo Alpha Group with an alternative design plant (Everett, 1968).

This chapter describes the Fiore technique in detail, primarily because it forms the basis of the optimisation technique used by Apollo. Also, it serves as a convenient means of introducing to the reader the concept of optimising aggregate plants using Linear Programming.

It must be noted at the outset that the Fiore technique is not concerned with the analysis of mass flows per sé. The method considers that a number of choices exist as to how material can flow in a plant, so that there exist various 'modes' under which a plant can operate.

Each mode yields a characteristic range of product sizes at a certain cost. This information, which can be determined from actual plant-records or by using a mass-flow-analysis method as described in Chapter 5, must be known before the Fiore method can be implemented.

The technique is now described.

Market demand for the various sizes of stone supplied by an aggregate plant invariably does not correspond with the quantities produced by the plant. This problem stems from the characteristic nature in which rock breaks: it simply will not break into only those sizes which the customers desire. The resulting imbalance between demand and production leads to large inventories of unsaleable sizes and a shortage of popular products. Furthermore, it results in increased production costs and reduced income. The objective of the optimisation procedure is to determine which mode or combination of modes will satisfy market demand most economically - in other words, at maximum profit.

The techniques of Linear Programming and the Simplex method are now in widespread use. A linear objective-function is optimised, satisfying a set of constraints. In the particular case of the aggregate plant model developed by Fiore, the objective function represents marginal profit, and the constraints include time available, tonnages produced, and market demand.

The constraints and the objective function are as follows (symbols for variables are represented in emphasised type-face, to distinguish them from symbols for constants):

Time Constraint

Suppose that four different modes of operating the plant have been identified and that the period of operation being considered is 1 week, or 46 hours.

t_q = time operating in mode q, in hours.

If the various modes are assumed to be mutually exclusive, the values of t_q cannot overlap (a plant cannot operate in more than one mode at a time because this would in itself constitute a new mode of operation). Therefore:

$$\sum t_q \leqslant 46$$

Production Constraints

Assume that the plant produces a range of three products, $j = 1,2,3$.

b_{jq} = hourly production-rate of product j, in tons per hour, if operating in mode q (estimated from plant records, or by mass-flow analysis, as discussed in Chapter 5).

B_j = total quantity of product j produced, in tons.

We therefore have for each product j:

$$B_j = \sum_{q=1}^{4} b_{jq} \cdot t_q$$

Market Constraints

For each product, the amount produced, less any surplus (which is dumped), is equal to the amount demanded by the market, less any forfeiture due to shortage.

m_j = market demand for product j, in tons
D_j = amount of product j dumped, in tons
S_j = shortfall of product j, in tons.

We therefore have for each product j:

$$B_j - D_j = m_j - S_j$$

If the various modes are assumed to be mutually exclusive, the values of t_q cannot overlap (a plant cannot operate in more than one mode at a time because this would in itself constitute a new mode of operation). Therefore:

$$\sum t_q \leqslant 46$$

Production Constraints

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For each product, the amount produced, less any surplus (which is dumped), is equal to the amount demanded by the market, less any forfeiture due to shortage.

m_j = market demand for product j, in tons
D_j = amount of product j dumped, in tons
S_j = shortfall of product j, in tons.

We therefore have for each product j:

$$B_j - D_j = m_j - S_j$$

Formulation of the Objective Function

c_q = production cost, operating in mode q, in cents per hour
r_j = selling price of product j, in cents per ton
d_j = cost of dumping product j, in cents per ton
s_j = cost of running short of product j, in cents per ton.

The Marginal Profit is equal to:

total revenue if all product is sold	$\sum r_j B_j$
less: total production costs	$\sum c_q t_q$
less: revenue lost due to dumping	$\sum r_j D_j$
less: costs of dumping	$\sum d_j D_j$
less: costs of running short	$\sum s_j S_j$

The equation to be maximised is thus

$$\sum r_j B_j - \sum c_q t_q - \sum (r_j + d_j) D_j - \sum s_j S_j$$

The resulting linear programming tableau for this example problem is given in Table 6.1.

Variable	t_1	t_2	t_3	t_4	B_1	B_2	B_3	D_1	D_2	D_3	S_1	S_2	S_3	R.H.S.
time	1	1	1	1										≤ 46
production	b_{11}	b_{12}	b_{13}	b_{14}	-1									= 0
	b_{21}	b_{22}	b_{23}	b_{24}		-1								= 0
	b_{31}	b_{32}	b_{33}	b_{34}			-1							= 0
market					1			-1			1			$= M_1$
						1			-1			1		$= M_2$
							1			-1			1	$= M_3$
objective function	c_1	c_2	c_3	c_4	$-r_1$	$-r_2$	$-r_3$	r_1+d_1	r_2+d_2	r_3+d_3	s_1	s_2	s_3	$= -P$
	production costs				sales revenue			costs of dumping & lost revenue			costs of shortage			marginal profit

Table 6.1 Linear programming tableau to determine optimum mode(s) of operation in an aggregate plant

As a numeric example, consider a simplified version of Jankovsky's (1980) application of the model to Ferro (a plant in the Anglo Alpha Group). Here, four independent modes of operation were identified by plant management:

1. standard operating mode
2. adjustment of the setting on a particular crusher to yield increased 37.5mm product
3. crushing surplus small sizes to sand in a proposed sand plant
4. combination of modes 2 and 3.

The amounts of the various sizes produced by the plant, operating in each of the four modes, were derived by plant management from actual records as well as estimates. To keep things simple, variable costs of production were taken as constant, at R1.73 per ton, regardless of mode of operation. This assumption does not quite reflect reality, because, for example, re-crushing small sizes to sand must incur additional production costs per ton. However, the effort required to identify individual costs for each mode was considered to be unwarranted in terms of the effects on final results.

The complete matrix tableau for the problem, and its solution, are shown in Table 6.2. Only the three products that are affected by choice of mode, i.e. sand, small-sizes, and 37.5 mm, are included in the matrix (the plant also produces large amounts of 19 mm and 22 mm stone, but the amounts of production of these sizes are constant, regardless of the mode used). Accordingly, the marginal profit in the solution reflects the contribution from only the three products considered.

The motivation behind the application of the technique to the Ferro plant was to determine the economic effects of the proposed sand plant. The first run of the model considers only modes 1 and 2 (i.e. no sand plant), the solution indicating a maximum monthly profit of **R107 535**, with the plant operating continuously in mode 2. The second run, which includes modes 3 and 4 as well, indicates a maximum monthly profit of **R182 285**, with the plant operating in mode 4 for 98% of the time and in mode 2 for the remainder. The increased operating profits due to the sand plant are thus evident.

As a numeric example, consider a simplified version of Jankovsky's (1980) application of the model to Ferro (a plant in the Anglo Alpha Group). Here, four independent modes of operation were identified by plant management:

1. standard operating mode
2. adjustment of the setting on a particular crusher to yield increased 37.5mm product
3. crushing surplus small sizes to sand in a proposed sand plant
4. combination of modes 2 and 3.

The amounts of the various sizes produced by the plant, operating in each of the four modes, were derived by plant management from actual records as well as estimates. To keep things simple, variable costs of production were taken as constant, at R1.73 per ton, regardless of mode of operation. This assumption does not quite reflect reality, because, for example, re-crushing small sizes to sand must incur additional production costs per ton. However, the effort required to identify individual costs for each mode was considered to be unwarranted in terms of the effects on final results.

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Variable	t_1	t_2	t_3	t_4	B_1	B_2	B_3	D_1	D_2	D_3	S_1	S_2	S_3	RHS
time	1	1	1	1										≤ 1
production														
sand	11 000	11 000	28 642	28 648	-1									= 0
small sizes	24 000	24 000	6 358	6 358		-1								= 0
37.5 mm	20 000	27 110	20 000	27 110			-1							= 0
market														
sand					1			-1			1			= 24 000
small sizes						1			-1			1		= 6 737
37.5 mm							1			-1			1	= 27 110
objective function	-95 150	-107 260	-95 150	-107 260	5.25	4.50	5.00	-5.75	-5.00	-5.50	0	0	0	= P
	production costs (R/month)				sales revenue (R/ton)			dumping costs plus lost revenue (R/ton)			shortage costs (R/ton)			marginal profit (R)

Solutions

Run 1	0	1	N/A	N/A	11 000	24 000	27 110	0	17 263	0	13 000	0	0	107 535
Run 2	0	0.0215	0	0.9785	28 263	6 737	27 110	4 263	0	0	0	0	0	182 285

Table 6.2 Equations and solutions for the Ferro Plant

The relevance of a model of this type obviously depends on the validity of the data used. However, even with acceptable data, the effective-ness of the technique is restricted by practical difficulties associated with two factors: the **identification** and **evaluation** of suitable modes of operation. Some discussion regarding these factors is warranted:

The most logical starting point in **identifying** a set of modes is to speak to plant management, who, more than anyone else, should know their plant and the various ways of operating it. However, as described in Chapter 1, the number of ways in which the various factors which affect plant output can be combined together is very large, and it is unreasonable to expect that plant management will identify more than a few obvious possibilities.

To see how many modes are possible in an average-size plant, suppose that each of six crushers in a circuit has a choice of two settings: these settings control the size gradings of the crusher products as well as the crusher capacities. Suppose also that there exist four points in the circuit where choices exist regarding the routing of material. For example, at a certain point, material may be routed either to a final-product stockpile or to a crusher. There are thus $2^{(6+4)} = 1024$ different modes of operation of the plant. Even if only half of these modes are reasonable and logical from an operational point of view, the number is still large. And, if the model is to provide a reasonably optimal solution, as many modes as possible should be included.

The second practical difficulty is associated with **evaluating** what output and what costs can be expected for each mode. Actual records from a plant can be useful as a starting point and mass-flow-analysis techniques, such as described in Chapter 5, can be applied to determine what the plant will produce in each mode. Needless to say, evaluating each mode individually, especially when a large number of possible modes have been identified, is a frustratingly tedious exercise.

The obvious solution to both these difficulties is to develop a model based on the mass-flow-analysis model discussed in Chapter 5, but extended to include all possible choices of flow in the circuit, the optimal configuration then being determined by an objective function similar to the one described in this chapter. In effect, this is what Apollo achieves.

CHAPTER 7

THE APOLLO MODEL

7.1 INTRODUCTION

In the preceding chapters, two aspects of aggregate-plant modelling, namely, mass-flow analysis and economic optimisation, have been discussed in general terms. In this, the main chapter, the Apollo model, which incorporates both aspects, is described in detail.

The Apollo model is a computer program that essentially does two things: Firstly, it provides a mass-flow analysis of the plant. In other words, given the plant configuration and the operating characteristics of the screens and crushers, it calculates the tonnages in each part of the circuit, and the amount of each product produced. Secondly, making use of flexibility in the circuit (i.e. choices of flow), it determines the optimum routing of material through the circuit, so that the market demand is satisfied at minimum cost. To optimise, the technique of Linear Programming is used.

Essentially, Apollo provides a static simulation of a production period, taking into account virtually all the operating parameters that affect the physical plant. Included in the modelling process are the following factors:

- The market demand for each product
- The available production time
- The performance data of all machines, including crusher product-gradings, screen efficiencies, and all machine capacities
- The plant layout, showing all possible routes of flow in the circuit
- The size grading of the raw material
- Detailed operating costs for the various machines, the raw-material cost, and the costs due to surplus and shortage.

The various aspects of the Apollo model are discussed in this chapter under the following headings:

- Model elements
- Model structure, constraints and objective function
- Information required to set up a model
- Preparing the data and entering it into the computer
- Solution printout and interpretation
- Uses of Apollo
- Conclusions.

Much of what will be discussed applies not only to Apollo but to aggregate modelling in general.

7.2 MODEL ELEMENTS

A good starting point for developing an Apollo model of a crushing plant is the circuit flow diagram. As discussed in Chapter 5, only certain elements of the physical plant need to be considered in the model:

- the point at which material enters the circuit (the 'primary scalper')
- crushers, which transform the material size-gradings
- screens, which separate the material flows
- exit points (the final-product stockpiles).

Chutes and conveyor belts are modelled only indirectly, to route flows of material around the circuit (from ... to ...).

A detailed description of the model elements that make up an Apollo model is given below:

Author Hayden John Samuel
Name of thesis The Modelling And Optimisation Of Aggregate Plants, And The Use Of The Apollo Computer Program. 1986

PUBLISHER:

University of the Witwatersrand, Johannesburg
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