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Investigating challenges that Grade 11 mathematics learners face when translating from word problems to linear algebraic representations

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CHAPTER ONE

INTRODUCTION TO THE STUDY

1.1 Problem Statement

The challenges associated with learning Mathematics are multifaceted and one of the major obstacles that learners encounter is that of the language used in articulating mathematical ideas during the learning process. In particular, research in Mathematics education has shown that word problem solving is one critical area where most learners find it difficult to come to terms with the language of Mathematics (Clement, 1982; Essien, 2011; Wollman, 1983; Setati, 2005). These challenges relate to their inadequate knowledge of specialised mathematical language that includes technical, non-technical words and the symbolic notation that characterise the discourse of Mathematics (Dale & Cuevas, 1992; Moschkovich, 2003; Pirie, 1998). A number of research studies have documented and proffered a distinction between Mathematical English (ME) and Ordinary English (OE) (Pilar, 1999; Margolin & Ilany, 2010; Pirie, 1998; Kane, 1967). The above researchers have argued that mathematical language is unambiguous, precise and less flexible as opposed to natural language, which can be ambiguous as it contains words with multiple meanings. Research work has further shown that moving from everyday language to mathematical language can result in errors that generally stem from the misconception that mathematical symbolic language directly represents everyday language and vice versa (Pillar, 1999). Given the above distinction between mathematical and ordinary language, it is therefore, incumbent upon problem solvers to possess appropriate linguistic knowledge and skills for them to achieve high levels of comprehension when dealing with mathematical texts.

More importantly, research in word problem solving has further shown that there exists an interdependent relationship between mathematical understanding and the language of Mathematics as the latter is the medium through which mathematical ideas are communicated (Austin & Howson, 1979; Ernest, 1987; Durkin & Shire, 1991). The challenge in articulating mathematical concepts and ideas according to Driscoll (1999)becomes acute for most learners especially those learning algebra because modelling of problem situations require translating from everyday language to algebraic expressions. Apart from being conversant withthe knowledge of content,specific specialised mathematical vocabulary and notation, Laborde, Conroy, De Corte, Lee, Pimm (1990) and Essien(2011) further argues that learners must also be equipped with sufficient skills in reading, writing, as well as speaking mathematically. The above arguments therefore, point to the need by Mathematics teachers to tailor their teaching

approaches that holistically support learners to develop their linguistic skills in dealing with word problem solving.

In light of the above discussion, the major aim of this research study was to investigate errors and their possible sources in learners' work when they translate mathematical word problems into their equivalent linear algebraic representations. More precisely, the investigations of errors focussed on learners' ability in correctly interpreting linguistic situations depicted in word problems and then present them in their equivalent abstract mathematical structures. The abstract mathematical structures include symbols, equations, expressions, specialised vocabulary, syntax, and spatial positioning of numbers that depict the given situation in a word problem (Harris & VanDevender, 1990; Margolin & Ilany, 2010). In this study, the definition of a *mathematical word problem* is that of a problem expressed in verbal written language.Algebraic representations in this study include equations, inequality equations, and expressions. To that end, this research study attempted to provide answers to the following research questions:

- What type of errors do learners make when translating from word problems to linear algebraic representations?
- What are the possible sources of these identified errors?

1.2Why the Focus is on Word problems?

The desire to investigate learners' challenges in translating word problems to linear algebraic representations has been influenced by some research findings done in South Africa. Firstly, findings from the Trends in International Mathematics and Science Study conducted in 1995 revealed that South African learners learning in a second language had significant language and communication problems as they could not communicate their answers in written form (Howie & Hughes, 1998). In another study, Howie's (2003) secondary analysis (TIMSS-Repeat), it was reported that South African learners had a limited knowledge of technical vocabulary of Mathematics. This limited knowledge impacted significantly on their ability to understand and communicate their answers using English particularly on questions involving word problems. Some factors such as the socio-economic standing of the learners, the language often spoken at learners' homes and the location of the school among others were also investigated in the study and were found to have an impact on learner achievement levels in Mathematics. By carrying out this study, further knowledge might be gained with regard to the extent to which learners' language skills both spoken and written impact on learners' achievement levels in Mathematics.

The second compelling reason to undertake this research study was influenced by my observations on learner performance in Mathematics at my school over a period of five years. The school has been categorised as a poor performing school for the past five years due to poor matriculation results they get each year. Grade 12 examination results in Mathematics have never exceeded fifty percent (50%) over this period. The other worrying observation I made was that the majority of learners (80%) had difficulties in communicating mathematical ideas using English, which is the language of teaching and learning (LoLT). An investigation of learner challenges elicited through errors they make may be an appropriate initial step to take in order to devise teaching strategies that may ease their challenges in all areas of Mathematics learning.

Finally, the reason for undertaking this research study was also an attempt to find out how learners' language difficulties interact with their knowledge of algebra since it is an important topic in the School Mathematics Curriculum. The National Curriculum and Assessment Policy Statement (CAPS) (see Department of Basic Education, 2012 p. 4) outlines some of the salient skills related to the modelling of word problems that Mathematics learners are expected to acquire during the course of their learning. These skills include:

(a) The development of algebraic manipulative skills that recognise the equivalence between different representations of the same concept,
(b) Being able to represent and describe situations in algebraic language, formulae and expressions,
(c) The ability to analyse and interpret equations that describe a given situation and,
(d) The ability to effectively communicate mathematical ideas using visual, symbolic and/or language skills in various modes.

With the curriculum having such a focus and strong emphasis on the algebraic knowledge that learners should possess, it is educationally significant to investigate learner challenges and errors that they make when translating from word to equivalent algebraic representations.

1.3 Significance of the Study

It is important to note that most research studies that have been reported in literature have extensively investigated the student-professor problem (Clement, 1982; Clement, Lochhead, & Monk, 1981) where the majority of learners make reversal errors. However, to widen my understanding of other errors that learners make when translating words to algebraic equation, I included three categories of word problems in this study and these are: de-contextualised word problems, 'additive', and 'multiplicative' compare word problems. The occurrence and frequency of errors under each category of word problems may give Mathematics teachers clues as to

which type of word problems needs more attention and what teaching strategies can be adopted to reduce these errors.

Generally, error analyses in both computational tasks and word problems are important to Mathematics education because they provide useful information for effective teaching and learning. They not only indicate what goes wrong, but also suggest to us that what we do may lead learners to make errors. In addition, they (errors) suggest ways we can help students eradicate their misconceptions and misunderstandings (Olivier, 1989). Error patterns often reveal underlying misunderstandings of mathematical concepts, lack of problem-solving strategies, and/or immature problem-solving strategies (Babbitt, 1990). Corder (1982) provides a theoretical justification for the study of errors as he argues that errors help us to understand how learners learn, process and reproduce information contained in mathematical tasks.

Based on the above discussion, the research findings from this research study therefore, may be helpful to me as a Mathematics educator/researcher and other Mathematics educators in schools with similar learner profiles as my school. They will be informed about possible linguistic challenges that result in learners making errors when translating word problem to linear algebraic representations. Possible learner challenges may include their level of linguistic knowledge and skills in translating word problems to linear algebraic representations, type of errors they make and their possible sources. The research findings will also benefit curriculum designers, Mathematics researchers and the Department of Education policy makers to come up with policies and strategies that are tailored to promote effective learning of Mathematics.

1.4 Definition of Terms

Schema: A schema is an organized structure 'consisting of certain elements and relations' specific to a situation (Mayer, 1999, p. 228). Schemata knowledge is mainly used for identification of problem types during problem solving. Schematic knowledge represents the template-like structure of one problem as it relates to another problem. This schemata knowledge deals with the problem-solvers' required knowledge structure of the components of the problem at hand.

Metacognition: In this study**,** I use the term 'metacognition'to refer to higher order thinking that enables understanding, analysis and control of one's cognitive processes in problem solving.More importantly, metacognitionhelps the problem solver to recognise the presence of a problem that needs to be solved, to discern what exactly the problem is, and to understand how to reach the goal (solution) (Flavell, P. H. Miller & S. A. Miller, 2002).A variety of

metacognitive processes/skillsinclude regulatory activities of planning, monitoring, testing, revising and evaluating throughout problem solving, especially in making the mental representation, selecting and assessing the effectiveness of the strategies employed (Flavell, 1992).

Word Problem: A word problem is defined as an independent unit of text that comprises a question sentence and a speech event (Nesher, 1998; Nesher & Katriel, 1977). In this sense, the textual unit might describe an event from daily life situation and this description is intended to elicit the appropriate mathematical operations specific to solving the word problem. In this study, the definition of a *mathematical word problem* is that of a problem presented in verbal written language describing a mathematical situation that has to be represented by a mathematical equation.

Translation: Translationis defined as the act of recognizing and connecting related quantities, functions and structures in two modes of representation (Brewer and Nakamura, 1984). In other words, the process involves translating from a given situation to a mathematical model (i.e. equation). In this study, learners translated the verbal mathematical language in word problems to algebraic representations (i.e. equations, inequalities, and expressions).

Expository texts: Theseare textsthat utilise clear, focussed language, moves from general facts to specific ones, and has specific structures that are designed to inform or explain concepts especially in content areas for example Mathematics. The ability to recognize text structure is seen to enhance the learners' ability to comprehend and recall the information read (Meyer, Brandt&Bluth, 1980; Anderson &Armbruster, 1984). In the present study, I consider the language structure used in word problems as expository texts.

1.5 Structure of the study

The research study is organised into five chapters, a list of references and appendices. Chapter One is an introduction to the study where I exposed the key research questions that guide the study. Chapter Two deals with the conceptual framework that guides the study and the literature review. The research design and methodology is the focus of Chapter Three. Chapter Four focuses on data analysis, findings, discussion of results andprovides answers to the two research questions. In the last chapter, I outline the main findings of the study, recommendations, future research, and my own reflections about the study.

1.6 Conclusion

In this chapter,I have provided a global picture of what the study intends to investigate. I have done so by articulating the problem statement, research questions, why I focussed on word problems, significance of the studyand finally the structure of the study.The next chapter reviews the conceptual framework and the literature related to the study.

CHAPTER TWO

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 Introduction

In this chapter, I begin by exposing the conceptual framework that underpins my research study. The conceptual framework locates the research study in the domain of word problem solving with a specific focus on the cognitive processes undertaken when transforming algebra word problems into a system of linear algebraic representations. A detailed outlook of the salient areas of literature review that discuss both past research work and contemporary studies that has a focus in Mathematics language research follows. In this discussion of literature, the link among components of mathematical language namely words, symbols and numerals are explored to understand how these interact and contribute to the errors that learners make when solving word problems.

2.2 Conceptual Framework

The conceptual framework that guides my research study is informed by Mayer's (1985; 1999) two stage model of problem solving that comprises two distinctive cognitive processes namely problem representation and problem solution. The problem representation and problem solution stages elaborate the cognitive processes that are critical in understanding the translation and integration processes that take place during the transformation of word problems to their equivalent symbolic form. Mayer (1985; 1987) posits that problem solvers are likely to go through these two distinct stages during word problem solving. Each stage is composed of specific activities that make use of distinctive skills and different types of knowledge (i.e., linguistic, semantic, schematic, strategic and procedural). According to Mayer (1999), these types of knowledge at each specific stage are essential in successfully solving word problems. Mayer's two-stage model of problem solving in this study therefore, provides a conceptual framework for investigating learner errors they made as they translated word problems to algebraic representations. Figure 1 below shows the two stages and the specific types of knowledge utilised in each stage.

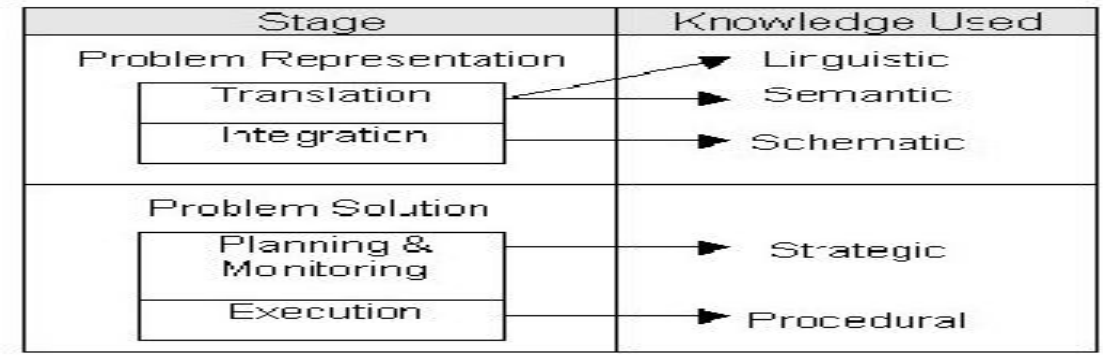


Figure 2.1: Stages in algebra word problem solving (Mayer, 1985; 1999).

2.2.1 The Problem Representation Stage

This stage consists of two processes namely the problem translation and problem integration. These two processes according to Mayer (1985; 1999) encompass the entire transformation of algebra word problems into a system of equations. Mayer (1985; 1999) asserts that the cognitive activities undertaken within these two processes capture the work done by the problem solver during problem solving.

The problem translation process consists of reading, decoding the text of the word problem, and then translating each proposition of the problem into a mental representation in the memory (Mayer, 1985; 1999).The translation process begins with reading the problem for understanding and taps into the linguistic knowledge of the solver (Mayer, 1985; 1999). Linguistic knowledge (i.e. of the English language) at this stage is used to comprehend the words' meanings in the textual description, while semantic knowledge means factual knowledge in the domain of Mathematics knowledge. Factual knowledge comes into play in this step, where the solver must use his/her prior knowledge to connect ideas or concepts that may not be explicit in the context of the story. In some cases, this factual knowledge constitutes specific conceptual knowledge for example knowing that inequalities have more than one solution. Mayer (1985) contends that many learners have difficulties in interpreting linguistic expressions especially those that contain relational statements for example "*twice as old*", "*more than*" *and* "*2 less than*" among others. These relational statements are those that express a quantitative relation between two variables and according to Mayer (1982, 1999), many students end up adopting a word order matching strategy to translate them.

The problem integration process takes the basic understanding of the text and then generates or builds a coherent mental representation of the problem situation taken from the word problem (Mayer, 1985;1999). In other words, the ultimate goal in understanding a problem is to build a situation model that is, a mental representation of the situation being described in the

problem(Kintsch & Greeno, 1985,Mayer & Hegarty, 1996). Further, the process of integration also requires that the problem solver select relevant information from the problem statement (i.e., the givens, goals, unknowns and relations in a problem), organize them into a coherent representation and make necessary inferences (Mayer 1992).

The problem integration process utilises schematic knowledge. A solver must access prior knowledge about similar problem types, discerning how the problem at hand relates to problems solved in previous situations. Moreover, schematic knowledge allows a learner to determine the category of a problem at hand. Mayer (1982) affirms that most students make more errors in recalling rare problem types (i.e., those appearing infrequently in textbooks) than common problem types (i.e., those appearing frequently in textbooks). Efforts to bring the internal representation of the problem situation to the external realm are made here (Mayer, 1985). During the integration process for example, variables (e.g., letters) may be manipulated to show the actions of a problem, or a solver might draw a picture or a diagram of the elements of the problem (Mayer, 1992). At this step, students can vary widely in the level of experience they have with different types of word problems (Mayer, 1985). Eventually, the situation model is to be integrated and quantified to create a problem model, which represents algebraic representations (i.e. equations and expressions in this study).

2.2.2 The Problem Solution Stage

After problem translation, integration and the identification of a problem model, the next stage is planning and monitoring in which the student devises a solution plan and keeps track of how well it works during problem solving. At this stage, metacognitive skills are needed in monitoring understanding of the problem (Schoenfeld 1985; Flavell, 1992). Planning is based on strategic knowledge, that is, general strategies such as finding a related problem, restating the problem in a different way and breaking the problem into subgoals (Mayer 1992, Schoenfeld 1985). The last process is solution executing, that is, carrying out a solution procedure. Solution executing requires procedural knowledge, that is, algorithms such as how to add, subtract, multiply and divide (Mayer, 1992).

However, the major focus of analysis in this study was mainly on the representational aspect of word problems into their equivalent symbolic form and not the computational aspect of word problem solving. Therefore, I considered the algebraic representations written by learners to be the solutions thatI eventually analysed for errors reflected in them.

Using Mayer's (1999) two stage model of problem solving as a conceptual framework for error analysis, the absence or presence of the three types of knowledge (i.e., linguistic, semantic and schematic) provided an analytical lens in detecting errors in learners' written work. Table 2 below gives a summary of these three types of knowledge essential for problem solvers to master the Problem representation stage (Mayer, 1999 p.85). Since the focus of analysis is on learners' written algebraic representations rather than on computational work, strategic and procedural knowledge have been omitted in the Table 2 below.

Table 2.1: Summary of knowledge types essential in each problem solving stage.

stage	process	Type of knowledge used
Problem Representation	translation	*Linguistic knowledge*: essential in understanding the syntactical structure of sentences used to express the problem situation (i.e. in English) for the problem solver to be able to form appropriate equations. Reading comprehension and text decoding skills are required.
		Semantic knowledge: factual knowledge related to the situation or knowledge of specific concepts. Semantic knowledge involves knowing how linguistic symbols behave in relation to the concepts they refer to (i.e. their denotations) or their senses (i.e. their connotations).
	Integration	*Schematic knowledge*: used to integrate the information in the problem in a coherent way in order to build an accurate mathematical representation. The solver needs to understand how the statements in the problem fit together and to have knowledge of the problem types

2.2.3 Connecting Mayer's (1985; 1999) Theoretical Model to the Study

I view Mayer's theoretical model of problem solving as relevant since it provides a concise description of cognitive processes that characterise initial steps undertaken in word problem solving. The theory exposes critical linguistic skills and knowledge types that problem solvers should possess for them to successfully engage in solving word problems. For instance, linguistic and semantic knowledge types are used to decode the meaning of words, phrases, and relational statements in algebraic word problem solving. The presence or absence of such knowledge types or skills can therefore, provide pointers to individual challenges that learners face during word problem solving. More importantly, errors detected in learners' written algebraic representations can also be described, explained and related to these knowledge types. The next sections below discuss the literature review of the study.

2.3 Review of Literature

2.3.1 The Nature of Mathematical Language

> One of the most fundamental aspects of all cultures is language, and it should be of serious concern that so many mathematics education researchers appear to have paid little more than lip service to the centrality of language factors in all aspects of mathematics teaching and learning (Ellerton & Clarkson, 1996:1017).

The centrality of the language of Mathematics in the development of conceptual understanding of various concepts in the field of Mathematics learning cannot be ignored since language is the medium through which Mathematics is communicated meaningfully (Austin & Howson, 1979; Ernest, 1987; Durkin & Shire, 1991). Further to this notion, the above researchers argue that the formation of concepts and development of new forms of thought are directly related to the level of mastery one has in the language of Mathematics.Nevertheless, before delving much into the discussion of literature, an understanding of what mathematical language consists of is important. Halliday (1975: 65) provides concise description of what characterises mathematical language through her definition of a 'mathematical register', whichhe views as:

> a set of meanings that is appropriate to particular function of a language, together with words and structures, which express these meanings.

Other language researchers such as Pimm (1991) concur with Halliday's (1978) views when they observe that although Mathematics is a language, it cannot be treated as a language on its own like French or Arabic as no group of people has ever been known to have Mathematics as its first language (pp 17). On the same note, Gorgorió and Planas (2001) weigh in as they further argue that Mathematics is not a universal language and it is incorrect to believe that it is free from linguistic issues (Barwell, Barton & Setati, 2007; Verzosa & Mulligan, 2013). It is therefore, reasonable to suggest that one function for which Mathematics language can be understood and utilised is in its ability to express mathematical ideas and meanings in mathematical activities in the same process developing a mathematical register. On the other hand, Margolin and Ilany (2010) describe mathematical language as a language of symbols, concepts, definitions and theorems and according to them, these mathematical components need to be learnt and mastered as learners move through grade levels.

Mathematical language (i.e. register) also consists of its own syntax, which deals with configuration rules according to which sentences and words are constructed (Moschkovich, 2007; Martiniello, 2008). The syntax of mathematical language includes lists of symbols, configuration rules for constructing language patterns, axioms, a deductive system and theorems (Margolin & Ilany, 2010;Pimm, 1991; Capraro et al., 2007). Given the nature of mathematical

language as described above, it is imperative that success in Mathematics learning requires learners to familiarise themselves with a variety of words,their order of placement in sentences (syntactical structure)and phrases especially in word problems. Pimm (1991: 20) provides an informative insight into the complexities that characterise mathematical language and he observes that:

> Mathematics, when spoken, emerges in a natural language, when written, it makes varied use of a complex, rule governed writing system mainly separate from that of natural language into which it can be read. For instance, the mathematical writing system is non-alphabetic and complex clusters of symbols are formed by a wider range of principles than our writing system where words are formed solely by putting letters next to each other.

The challenges encountered in mastering the language of Mathematics are manifested in the above quote. The interplay among words, symbols, and numerals in mathematical texts demands greater attention and understanding from doers of Mathematics. Pimm (1991) further expounds the above idea by arguing that one major aim of learning Mathematics is an attempt to gain control of the Mathematics register to be able to speak and behave like what mathematicians do. However, research in Mathematics education has shown that it takes a concerted effort for learners to realize that level of proficiency and skills especially when dealing with word problem solving in which translations from words to algebraic representations are at the core of the mathematical activity.

Pilar (1999), Margolin andIlany (2010) further characterisemathematical language by pointing out how it differs from natural language in terms of preciseness in meanings when it comes to reading mathematical texts with comprehension. The above researchers posit that every assertion in mathematical language is unambiguous, precise and less flexible as opposed to natural language which incorporates words with multiple meanings. Besides the presence of many words with multiple meanings, many different English words and phrases have a single mathematical expression for example the word *subtract* which may be represented by '*take away*', '*less than*' and '*minus*' (Pimm, 1991). In noting the prevalence of such words in both language discourses, Kane (1967) categorically distinguishes between mathematical and ordinary language when he argued that Mathematical English (ME) and Ordinary English (OE) are sufficiently dissimilar that they require different skills and knowledge on the part of readers to achieve appropriate levels of reading comprehension. In other words, learners should be proficient in both languages to achieve substantial understanding when dealing with specialised language in mathematical texts.

Other researchers have provided a distinction between Mathematical English (ME) and Ordinary English (OE) (Shuard & Rothery, 1984; Noonan, 1990; Setati, 2005). In their discussions, the above researchers reveal how the blending of the two forms of language brings with it substantial challenges in learning Mathematics especially for English second language speakers. They contend that Mathematics language, in its spoken form is blended with ordinary language and consequently the distinction between the two languages is often blurred. According to Shuard and Rothery (1984) and Noonan (1990), words used in Mathematical English fall into three main categories namely: (1) words that have the same meaning in ME and in OE, (2) words that have the same meaning only in ME, and (3) words that occur in both OE and ME but have a meaning in ME, which is different from their meaning in OE. Noonan (1990) argues that words in Mathematical English tend to cause reading and comprehension challenges mainly because learners do not often encounter these words in any other contexts other than in Mathematics. The situation is further exacerbated by the inclusion of ordinary words in mathematical texts that are intended to explain mathematical concepts. The paragraphs below elaborate the distinction and the contexts in which these words are applicable in mathematical texts.

2.3.2 Non-technical and Technical Vocabulary

Learning mathematics is characterised by integrated activities such as reading, writing and decoding meanings of words with multiple meanings (Adams, 2003; Ellerton & Clarkson, 1996; Dale & Cuevas, 1987;Schleppegrell, 2007; Rupley & Slough, 2010). Adams (2003) identifies words with multiple meaningsas words that are used both in Mathematics and in everyday contexts and interactions. However, the meanings of these words serve mathematical purposes when being used in the Mathematics discourse. Non-technical terms which fall in the OE category are used in daily conversations and writing (Moschkovich, 2003). Examples of such words which feature in this category are ‘product’, ‘yard’, ‘table’, ‘increase’, ‘of’ among others and these words assume different meanings depending on the context of use. Consequently, these are situations where the problematic nature of mathematical language is visible as learners are unable to integrate new concepts in their learning using these words (Adams, 2003). The reason for this drawback according to Pimm (1981) emanates from the fact that Mathematics attaches specialised meanings to everyday words that already have their own meaning.

On the other hand, technical vocabularyis derived from Mathematical English and have specialised meanings in Mathematics (Halliday, 1975). Such words include multiplication, prime number and perpendicular lineamong many others (Adams, 2003; Zevenbergen, 2000). Dale and Cuevas (1987) argue that apart from teaching learners technical vocabularyas stand-alone words teachers must encourage learners to learn definitions of technical terms within particular

mathematical contexts. Trigger words such as *more, less, got,* and *take away* among others are categorised under non-technical vocabulary and their function is to provide clues to the appropriate operation to take during problem solving (Zevenbergen, 2000). The understanding of the meaning of these words in mathematical texts is critical especially when translating word problems to their equivalent algebraic representations (Jones, 1982). In this study, the learners' mathematical interpretations of the words such as 'more' 'less', older and greater than in terms of the appropriate operation symbols they suggest was under investigation. Having discussed the presence of technical and non-technical vocabulary in mathematical texts, the paragraphs below point to some delicate research findings on learners' approach in using them when answering word problems.

Inone research study, Mangan (1989) identified ‘key words’ that students seemed to respond to when approaching word problems. For example, 'altogether' suggested addition and 'left' suggested subtraction(pp. 120 – 122*)*, suggesting that the structure of the problem appeared to lead to certain actions from the students. De Corte and Verschaffel (1989: 85) coin these types of problems as “semantically different problem types” to which students respond according to cues in the problem wording. Hegarty, Mayer, and Monk (1995) found that unsuccessful problem-solvers are more likely to use the 'direct-translationtechnique', which in essence is a strategy in which ‘unsuccessful’ students base their solutions on numbers and key words in word problems. Other researchers have argued that many learners cannot cope with high school algebra demands mainly because they will be coming from an arithmetic background from lower grades (Bednarz & Janvier, 1996; Capraro & Joffrion, 2006).

In a similar study, Capraro and Joffrion (2006) investigated 668 middle school students on the extent to which they showed facility with translating from English language to mathematical symbols or vice versa using conceptual or procedural indicators as measures of comprehension. Restricting their analysis on three selected questions, they found that only 58 (9%) of all the students had managed to answer the three questions correctly and they concluded that these students were not conceptually or procedurally ready to translate from written word to mathematical symbols. One of the three questions could be translated word for word (Mestre, 1988) into symbolic form to arrive at a correct solution but they could not manage. This further supported the findings of MacGregor and Stacey (1993) who noted that, “in test items designed so that the syntactic translation would produce a correct equation, most students did not translate words to symbols sequentially from left to right, but tried to express the meaning and wrote incorrect equations” (p. 217).

2.3.3 The role of Symbolism in School Mathematics

The language of Mathematics is also characterised by numerous symbolsthat enable communication and logical presentation of concepts that words alone cannot (Marks & Mousley, 1990; Latu, 2004; Pimm, 1981). In this sense, symbolism is seen to be pivotal in the creation and development of Mathematics (Ernest, 1987). Operation symbols include among others $(+, -, \div, \times, =, \geq, \leq, \neq)$ and one of their roles is to indicate mathematical operations during word problem solving. Marks and Mousley (1990) argue persuasively that symbols are purposely established in Mathematics to make sense of and to standardize mathematical meanings and concepts.

For instance, the symbol (+) is associated with addition, sum, plus, increase and combine depending on the context in which it is used. Wheeler (1989) goes further to unravel the problematic transition from arithmetic procedures to algebraic ones by posing the question: "*What happens to one's interpretation of the plus sign* $(+)$ *. . . when it is placed between two symbols which cannot be combined and replaced by another symbol?"* (p. 324). In such instances, the question at stake is what learners think of the algebraic expression $a + b$ after years of being able to compress expressions such as $3 + 5$ into the single number 8. The primacy of understanding the language of symbolism in Mathematics in these circumstances becomes a necessity especially when learners attempt to translate a given mathematical text into mathematical language using appropriate symbols and terminology (Capraro & Joffrion, 2006).

A number of difficulties experienced by students in translating the verbal expressions into algebraic expressions were emphasized in studies that investigated learners' knowledge in algebra. In one such study for instance, Rosnick (1981) noted that the transition from verbal expressions into the algebraic expressions is difficult for students of every age. In support of Rosnick's (1981) study, MacGregor and Stacey (1994) conducted a study researching the difficulties students in the 7-10 age group experienced in learning algebra. In their study, they found that three main obstacles against forming the algebraic notations were namely: the meanings ofletters, the belief that an expression containing an operation sign should be simplified to asingle "answer" without an operation sign and lack of awareness of the need forparentheses. In the study, the students were asked to write the algebraic expression corresponding to the verbal expression "Add 5 to an unknown number x, and then multiply the result by 3". Although the rate of the success in giving the correct answer increases according to age, it was determined to be generally low. The researchers stated that the students do not remember to use parentheses and they do not know how the use of parentheses affects the explanations.

From the above discussion, it is imperative that learners need to understand the contextual meaning of various symbols that dictate and direct mathematical operations. Unless learners are able to know what letters and/or symbols in an algebraic expression represent in natural language, there is a tendency for them to misunderstand the essence of that aspect of mathematics in their real life. With regard to the conceptualisation of mathematical symbolism, Brodie (1989: 49) argues that:

> Effective use of mathematical symbolism requires that the reader, or student, understands the relevant mathematical concept, translates it into a suitable symbolic representation, can manipulate the symbols, and then translate them back into meaningful concepts.

Mathematical activities in most Mathematics classrooms are carried out within the context described above in terms of symbolic manipulation. However, little effort is made to demonstrate to learners where these symbols come from and their correct interpretation in different contexts. To that end, Brodie (1989) observes that unless learners understand the functions and origins of symbols, Mathematics will remain inaccessible to most of them (p. 9).

MacGregor and Price (1999) on the same issue of symbol manipulation, discuss what they refer to as symbol awareness in learning Mathematics. According to them, symbol awareness includes knowing that numerals, letters and other mathematical signs can be treated as symbols detached from real-world referents. In that case, they argue that symbols can be manipulated to rearrange or simplify an algebraic expression, regardless of their original referents. However, experience working with learners has shown that most of them lack conceptual understanding of what symbols really stand for. (*My emphasis*). The problem becomes more acute especially when manipulations involve groups of symbols where the use of parenthesis is needed. This brings to fore another aspect of symbol awareness that of knowing that groups of symbols for example $(x + 2)$ can be treated and used as basic meaning- units for the purposes carrying out manipulations.

One symbol that is complex and difficult for learners to comprehend according to research in Mathematics education is the presence of the equals sign $(=)$ in the Mathematics discourse (Kieran, 1981; Clement, 1982;Herscovics & Kieran, 1980). The equals sign, which is a central mathematical idea, is used to indicate the equality of the values of two expressions. When a variable $\boldsymbol{x}$ is involved, the equals sign may denote the equivalence of two functions (equal values for all values of $\boldsymbol{x}$), or it may indicate an equation to be solved, that is, finding all values of $\boldsymbol{x}$ for which the functions take the same value. Research studies have shown that learners do not comprehend the appropriate contextual use of equality in the formulation of equations especially from word problems (Duru & Koklu, 2011; Clement, 1982). Research has also shown that

learners tend to think of an equals sign not as a statement of equivalence but as a signal to perform an operation, presumably based on experience in the elementary school years with problems such as $8 + 4 = ?$ (Kieran, 1981). Mathematics researchers have attributed this tendency to children's experience in executing arithmetic operations and writing down an answer immediately to the right of an equals sign (Capraro & Joffrion, 2006; Kieran, 1981).

Further, on conceptualising the meaning of the equals sign, expressions like $4 + 3$ can be construed in two ways, that is, arithmetically or algebraically. Arithmetically it is a command to execute an operation, reflecting *procedural* (Kieran, 1992), *operational* (Sfard, 1995, McNeil & Alibali, 2005) or *canonical* (Sherman & Bisanz, 2009) expectations. On the other hand, algebraically it is an object on which other operations may be performed, reflecting *relational* (Kieran, 2006), *structural* (Sfard, 1995, McNeil & Alibali, 2005) or *non-canonical* (Sherman & Bisanz, 2009) expectations. However, as discussed above, the greatest challenge emanates from many learners' early experiences of the arithmetic discourse that reinforce the belief that the equals sign is a command to operate rather than an assertion of equality between two expressions. Moreover, this challenge according to Sherman and Bisanz(2009),the problem is exacerbated by the unfortunate collusion of explanations from both teachers and textbooks.

On the primacy of symbolic knowledge, Pimm (1987) further argues that mathematical symbols act as windows through which learners can access mathematical concepts and fundamental ideas in problem solving. For instance, he contends that symbols such as the equals and the inequality signs act as logical connectives in relational algebraic representations. Similarly, Adams (2003) observes that mathematical symbols are efficient means of showing what words say and how the numerals are to be responded to according to the words.For example, the operation '$8 + 3$' consists of two numbers and one mathematical symbol, which directs what is to be done. In such instances, learners are likely to understand the numbers presented. However, without understanding the meaning of the addition symbol, either the operation of addition cannot be done at all or if done, it will be done inappropriately. Consequently, if learners memorize the result of this operation, there are minimal chances that they would conceptually grasp the use and underlying meaning of the symbol ' + ' without adding or combining two different groups of objects (Duru & Koklu, 2011).

2.3.4 Communication and Mathematical Language

Language is used as a communication tool and it facilitates the transmission of mathematical knowledge, values and beliefs as well as cultural practices (Garegae, 2007; Gorgorio & Planas, 2001). Language is, on the other hand, viewed as a form of communication that enables effective

interaction within a classroom setting between teachers and learners, among learners themselves and learners and recommended textbooks (Smith & Ennis, 1961). Apart from allowing effective communication within the classroom environment, The National Council of Teachers of Mathematics (NCTM) (2000) pinpoints the primacy of communication in terms of its benefits:

> There are dual benefits when mathematics education is rich in communication: students communicate to learn mathematics and learn to communicate mathematically (p. 60).

The above quote resonates with the major goals of effective communication within a Mathematics classroom setting one of which is the gradual mastery of the Mathematics register (Halliday, 1978). Communication in Mathematics according to Pirie (1998), entails the ability to translate the verbal/written form of mathematical language to its equivalent symbolic form. Pirie (1998) observes and summarises the implications of this ability namely that:

- it indicates understanding between symbols and words during problem solving
- it indicates communicational competency in Mathematics as learners gradual replace Ordinary English language with specialised mathematical terms and
- it bring the realisation to learners that the verbaland symbolic representationsdo not always match for example subtracting *two* from *three* is not represented as $-2\ (3)$ *but* $3-2$.

2.4 The Role of Reading Comprehension in Answering Word Problems

Reading skills in Mathematics areregarded as a fundamental facet to Mathematics learning as they help learners to make meaning of various concepts that define school Mathematics. Researchers such as Allington (2001) and Combs (2002) observe that one important indicator in determining the quality of learners' reading is their level of comprehension of mathematical texts. Comprehension as a cognitive skill entails understanding both the context and content of what is read in such a way that the reader puts into action what is understood (Duru & Koklu, 2011). Similarly, Harste et al (1984: 90) point out that comprehension 'is seen as a process of sense-making in light of or through assimilation and accommodation of cognitive structures'. Therefore, reading with comprehension in Mathematics requires learners to possess both linguistic comprehension skills (Mayer, 1999; Bernardo, 2002) and the knowledge of the 'language of Mathematics' which consists of words, symbols and numerals (Duru & Koklu 2011; Adams 2003). In particular, the knowledge of vocabulary is critical in reading mathematical text with comprehension, as readers must recognise the meaning of many previously encountered words, symbols and information organisers to understand the text at hand (Miller & Smith, 1994; Anderson & Pearson, 1984).

In light of the above discussion, it should, however, be noted that there is a marked difference between reading and comprehending a mathematical text (Duru & Koklu 2011). For example, to elaborate more on the difference between two cognitive actions, the mathematical sentence '$2\pi r$' for instance may be interpreted in more than one way. Literally, the statement may be read as 'two-pie-r' in daily language whereas in mathematical language, the underlying meaning would be 'two multiplied by a constant pi and variable r'. According to Duru and Koklu (2011), the statement may also be mathematically interpreted as 'the circumference of a circle with radius r' or as 'Circumference of a circle is proportional to the radius and the ratio of the circumference to that of the diameter is always equal to a constant pi' (p. 449). Consequently, learners who read and understand the mathematical meaning of the mathematical statement '$2\pi r$' are more likely to succeed in interpreting questions that require the application of the concept. On the contrary, those learners whose algebraic language is divorced from their natural language may only apprehend the string of three symbols without making sense out of them.

Similarly, other research studies have gone further to unravel the construct of reading and then investigated its intricate relationship with success in Mathematics problem solving. Stothad and Hulme (1996) define reading as the interaction of two distinct processes namely decoding and comprehension. Decoding is seen as a highly automated activity which, in turn, releases attention directed to comprehension (LaBerge & Samuels, 1974). Other researchers like Yuill and Oakhill (1991) contend that good comprehension is unlikely to be realised in the absence of reading fluency (correct words read per unit of time) although the reverse is not necessarily true. In fact, it can be seen that the purpose of reading shifts from developmental reading towards understanding expository texts (Meyer, Brandt& Bluth, 1980) that are introduced in the middle to high school grades. In higher grades, reading is mainly seen as act of learning where learners make considerable effort to remember key concepts and integrate new learning with the aid of prior knowledge as they engage in problem solving. Research has further shown that the relationship of English reading skills and mathematics performance might not even be linear, that is, there is a minimum reading level associated with improved performance (Beal, Adams & Cohen, 2010). For the purpose of reading mathematical texts with comprehension, Richardson and Morgan (1990) concludes by observing that if readers focus on meanings of words rather than on the recognition of words, they will be positioned to think and learn about content itself rather than about reading the material.

Other research studies have discussed challenges that slower readers encounter when it comes to reading mathematical texts with comprehension (Bye, 1975; Yuill & Oakhill, 1991;Marston, 1989). Results from these studies have shown that slower readers tend to have lower

comprehension of concepts being illuminated in mathematical texts since they make fewer inferences from the written material. This brings to fore the role played by memory in reading mathematical texts with the purpose of understanding. Yuill and Oakhill (1991) maintain that since there is competition for short-term memory space, learners who process words slowly may be limited in their ability to integrate both the decoding and comprehension of written words. Consequently, this slow processing of words and/or phrases incapacitates other cognitive processes necessary for understanding. Barney (1972) concludes by noting that some learners read narratives so slowly and laboriously to an extent that before they come to the words at the end of the sentence, they would have forgotten those at the beginning.

Although the major aim of the present study was not directly concerned with making conclusions on how the results were influenced by the use of participants (i.e. English 2nd language speakers), the following research finding may be relevant. Barton and Neville-Barton (2003) carried out an investigation that focussed specifically on the dynamics of learning Mathematics at the university level for students who did not have English as their first language often referred in literature as English as an additional language students (EAL). The participants of their study were eighty-first year undergraduate Mathematics students. The researchers reported among their results that in comparison with the students for whom English is their first language, "EAL students experience a 10% disadvantage in overall performance through a lack of understanding mathematical text". In addition to this, the authors also observed that **"**technical mathematical discourse is a more important factor than general English and that EAL students unjustifiably rely on symbolic modes to make up for textual disadvantages'. The implication of this finding suggests that many students of Mathematics may never realise the seemingly confusing semantics of mathematical language if they are only exposed to and equipped with essential syntactic procedural skills of manipulating algebraic expressions.

2.4.1 The Role of Prior Knowledge in Word Problem Solving

The role and use of prior knowledge as a comprehension strategy is viewed as pivotal in reading Mathematics with understanding (Allington, 2001; Carter & Dean, 2006). According to these researchers, a majority of learners may have sufficient knowledge about words, symbols and problems in mathematical texts, but may fail to conceptually connect them to one another. They further argue that these learners in most instances have challenges on how to define words contextually resulting in them being unable to comprehend tasks. In such cases, activating prior knowledge on the meaning of words and phrases during reading mathematical texts becomes important as it leads to understanding mathematical concepts at hand, how they build on one another and how they are related (Barton, Heidema & Jordan, 2002).

. In this study, the term "*prior knowledge*" embraces different types of knowledge acquired in various domains for example, general world knowledge, domain-specific factual knowledge, conceptual knowledge, and metacognitive knowledge (Schneider & Bjorklund, 2003). On the other hand, learning Mathematics relies heavily on conceptual understanding of how concepts relate and build on one another and the possession of prior knowledge plays a vital role in this regard (Capraro & Joffrion, 2006). The primacy of activating prior knowledge therefore,can be seen as aiding learners to read mathematical text with comprehension as they relate meanings of words, phrases, and symbols in appropriate contexts.

Similarly, during problem-solving activities, Van de Walle, Karp and Bay-William (2009) observe that students often reflect on the mathematical ideas in the tasks, formulating ideas more likely to be assimilated with their prior knowledge during these activities. These authors further argue that while solving problems, students are constructing and restructuring their own knowledge and will be actively engaged in cognitive processes such as problem-solving, reasoning, communication, connections and representation. In all these cognitive processes, it is imperative that reading comprehension where learners access the language of mathematics presented through multiple semiotic systems is needed (de Oliveira & Cheng, 2011; O'Halloran, 2005; Schleppegrell, 2007; 2010). According to de Oliveira and Cheng (2011), multiple semiotic systems characterising mathematical language involve (a) the use of natural language that introduces, contextualises, and describes a mathematical problem; (b) symbolism that is used for finding the solution of the problem; and (c) visual images that deal with visualising the problem graphically or diagrammatically (Mayer, 1999).In view of the above processes, Aiken (1972) concludes by arguing that there is a high correlation between reading comprehension and mathematical problem solving.

2.4.2 The Role of Conceptual Understanding and Procedural Knowledge in Algebra

Conceptual knowledge is defined as knowledge that is rich in relationships (Kilpatrick et al, 2001). It relates both to the principles that refine understanding of Mathematics and to the interconnections between ideas that explain as well as give meaning to mathematical procedures (Ashlock, 2001; Faulkenberry, 2003). Conceptual understanding is similar to comprehension in reading with reading comprehension defined as making sense of what is read in mathematical texts. Capraro and Joffrion (2006) observe the primacy of conceptual understanding in solving word problems when they argue that children who possess such understanding can easily decode the meaning of words accurately and can translate words into mathematical representations using symbols. In view of the above discussion, conceptual understanding in this study is interpreted as

the sense making of the meaning of words, phrases in the word problem and the subsequent correct translation of word problems into algebraic representations.

Procedural knowledge on the other hand is characterised as that which enables one to quickly and effectively solve problems (Schneider & Stern, 2010: 178; Kilpatrick et al., 2001). Therefore, procedural mathematical understanding can be described as a mastery of computational skills blended with the familiarity of procedures, rules and algorithms in problem solving. However, in many instances, this familiarity with mathematical procedures is without explicit reference to mathematical ideas (Ashlock, 2001: 8). It should, however, be noted that mathematical procedural knowledge is usually specific to particular tasks, while mathematical conceptual knowledge is often more generic. Though the distinction between conceptual understanding and procedural knowledge was elaborated above, Star (2002) strongly argues that the two types of knowledge are not distinct entities when it comes to problem solving. In other words, these two types of knowledge work in unison during word problem solving.

2.4.3 Role of Metacognition in Problem Solving

As highlighted in Chapter 1, the role of metacognitive skills is seen to play a major role in many types of cognitive activity such as oral communication of information, reading comprehension, attention and memory (Flavell et al, 2002). More importantly, research in metacognition for reading comprehension purposes has unravelled the following findings (a) minimal effort is needed to decode words, which frees up a great deal of cognitive capacity for comprehension, for both words and ideas that are represented by phrases, sentences, and paragraphs (b) cognitive capacity is put to effective use to metacognitively focus on knowing that comprehension is built by relating what is read to prior knowledge (c) metacognition thus facilitates prediction by the reader about what might be coming up in the text and summarize what is being read and (d) metacognition used by readers alerts them to when ideas are confusing and how to respond to fix-up strategies, such as rereading, diagramming (Mayer, 1999), searching for patterns, and identifying key concepts from words used in sentences (Pressley, 2002b). In view of the above findings, it is imperative that problem solvers must be able to monitor their comprehension as they read mathematical texts, realise whether they have understood what they are reading and if not, they must be able to regulate their comprehension of the text (Sencibaugh, 2007). Further research has shown that metacognitive skills and strategies have been found to be important for critical thinking skills (Ku & Ho, 2010), a key component for academic success in higher secondary grades. In light of the above, Uwazurike (2010) argues for teaching methods that incorporate and model metacognitive knowledge and strategies to learners of Mathematics.

2.4.4 Errors in Translating Linear Equations

As discussed above, learners encounter a myriad of linguistic challenges in correctly interpreting or representing words into equivalent symbolic language. In fact, like any other language, Mathematics has its own grammatical rules and syntactical structures that can be difficult for students to master (Esty, 1992; Dale & Cuevas, 1992; Pilar, 1999). For this reason, many learners struggle to translate word problems to symbolic form and when they do, many errors observed may be due to a host of reasons some of which are discussed below.

The primary source of difficulty for learners in solving word problems is translating the story into appropriate algebraic expressions (Mayer, 1982). This involves a triple process of assigning variables, noting constants and representing relationships among variables. However, the specifics of algebraic translation errors have not been examined as closely as the translation errors associated with arithmetic word problems (Mayer, 1982; Bishop, Filloy, & Puig, 2008). The above researchers point out that most translation errors result from the semantic structure and memory demands needed to understand the entire message in the word problem. Hinsley et al (1977) showed that schemas guide the translation of algebraic word problems and these schemas are mental representations of the similarities among categories of problems. Failure by learners to coordinate constituent parts of a word problem especially during the processing of relational statements results in them making errors. Another example of a common error in translating word problems to algebraic representations is that documented in literature as the reversal error (Clement, 1982; Mestre, 1988). In executing the translation process, learners simply translate English sentences into mathematical expressions, simply moving from left to right without awareness of syntactic and semantic relationships (Powell et al., 2009). For example, “three less than a number” is wrongly interpreted by many students as “3 – X", which is in fact X – 3 since mathematically the words “less than” indicates subtracting 3 from X.

The study by Newman (1977) further confirmed that the majority of errors learners make occur in the processing (translation) stages of the word problems or the stages before that (i.e. reading, comprehension and the transformation). Mayer (1987) further observes that the above three stages are heavily dependent on reading skills of learners. In particular, the translation stage utilises linguistic knowledge to interpret each mathematical statement for formulation into the learners’ internal representations (Mayer, 1999). Consequently, failure by the learner to accurately read and understand the problem may be responsible for a significant number of errors found in Mathematics tasks (Clements, 1980).As the above discussion has shown, reading, comprehension and recall skills especially in word problem solving are a prerequisite if success

in Mathematics is to be realised. To elaborate further on errors associated with the translation stage, the paragraph below gives global picture of some of them.

In her research study investigating stages where errors were most likely to occur, Newman (1977) found that 47% of her learners categorised as low-achievers in Grade 6 ($N = 124$) made errors prior to the process stage (of which 12% were at the transformation/translation stage). In another study, Clements (1980) found that fewer errors were made at the two lower levels (i.e. reading and comprehension stages); one-quarter of the errors were at the transformation stage. From the above statistics, it can be seen that half of the errors made occurred before the application of process skills. In other research studies carried out with primary and junior secondary school children in Melbourne Australia, by Casey (1978), and Clements (1980), similar results were obtained with 50% of errors first occurring at the reading, comprehension and transformation stages. However, in another study by Clarkson (1983), contrasting results surfaced as he found that only 15% of initial errors made by the 10^{th} and 11^{th} Grade students occurred at any one of the first three stages of word problem solving. These contrasting results raised further questions on whether Newman's analysis procedure when applied at different grade levels and in different cultural contexts would result in different error profiles.

Finally, other specific type of errors found on learners' work when translating from word to algebraic representations are elaborated in a study carried out byMacGregor and Stacey (1993).The two researchers investigated the cognitive models of the students in the 8-10 age group in forming the equations corresponding to the verbal expressions. In their study, they posed a problem, as "s and t are numbers. The number s is eight more than t. Write an equation showing the relation between s and t." They found out that the students madethree major categories of errors while they formed the equations corresponding to the givenverbal expression. The three major categories of errors were: (1) writing expressions, usuallyproducts or totals, instead of equations such as$s(8 - t)$ or $8s$ x t, (2) writinginequalities, such as $s8 > t$and (3) writing reversed equations, such as $t = s + 8$.From the above findings, the two above researchers concluded that the verbal expressions consist of syntactic and semantic processes as well as other processes, so the errors made cannot be explained merely by syntactictranslation.

2.5 Conclusion

In this chapter,I have exposedthe conceptual framework that guides my study as well as the literature related to the language of Mathematics. The description of the nature of mathematical language has been discussed with a bias towards the components that constitute the language of

Mathematics namely Mathematical English (ME) and Ordinary English (OE). In these categories, the presence of technical, non-technical vocabulary and the role of symbolism in representing and understanding mathematical texts have been exposed. In line with the objective of the study, literature focussing on mathematical communication, the primacy of reading comprehension, the role of procedural and conceptual knowledge in mathematical understanding was discussed. The chapter also reported on some research studies that focussed on learners' challenges related tosolving word problems and types of errors they (learners) made when translating from verbal written mathematical language to algebraic representations. The next chapter describes the research design and methodology used in the study.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

In this chapter, I explain the main methodological constructs that were employed in various stages of the study and later unite them together to create an overall summary of the methodology. This discussion presents the research methods used in the study, sampling procedures, data collecting site, participants, research instruments, the pilot study, data analysis methods, rigour and ethical issues observed.

3.2 Mixed-Method Sequential Explanatory Design

A mixed-method sequential explanatory design was used in this research study. By definition, a mixed-method is a procedure for collecting, analysing, and "mixing" or integrating both quantitative and qualitative data at some stage of the research process within a single study for the purpose of gaining a better understanding of the research problem in question (McMillan & Schumacher, 2010; Opie, 2004). The rationale for mixing both kinds of data within a single study is therefore, intended to capture the salient details or emerging trends on the phenomena under investigation. In such instances, quantitative and qualitative methods when used in combination complement each other and allow for a more robust analysis, taking advantage of the strengths of each method (Miles & Huberman 1984; Opie, 2004, McMillan & Schumacher, 2010). According to Opie (2004), this combination of data collecting methods results in a stronger research design that yields more valid and reliable findings.

The quantitative phase of data collection in a mixed-method sequential explanatory research design is characterised by the abstraction of data (i.e. on learner performance) from participants into statistical representations in form of graphs, charts and tables (Opie, 2004). In line with the steps followed in a mixed-method sequential explanatory research design, quantitative data from the written test were initially collected, marked and learners' responses classified as correct or incorrect. In this study, the purpose of quantitative data analysis was to provide an insight into understanding how learners responded within and across test items with the intention of identifying and quantifying types of errors, (i.e. syntactic, semantic, and schematic) observed in their written algebraic representations.

The qualitative phase is where interview data were collected and analysed second in the sequence to help in understanding and elaborating in detail observed errors conspicuous in learners' incorrectly written algebraic representations. The qualitative approach on the other hand

focuses on a holistic view of what is being studied using observation, questionnaires, and written documents (Cohen, Manion& Morrision, 2007). The qualitative approach in this study was intended to allow an in-depth probing on selected test items where language difficulties were perceived to be acute and responsible for errors observed in learners' work. The analysis of data from interviews allowed me to understand the research problem through learner views, which suggested why they had written the algebraic representations the way they did.

Mixed method researchers believe that they can get richer data and strong evidence for knowledge claims by mixing both qualitative and quantitative methods rather than using a single method (Johnson & Christensen, 2008; Creswell, 1998). Johnson and Christensen (2008) listed five major purposes of selecting a mixed method design namely: triangulation, complementarity, development, initiation and expansion. For the purposes of this study, only triangulation and complementarity are relevant and are addressed.

Triangulation is the term used to indicate the use of multiple sources of evidence to claim a result with confidence. According to Johnson and Christensen (2008), the use of this method has the overall effect of increasing the credibility or trustworthiness of the findings obtained from the research study. In this study for example, I used learners' written work and interview data on word problem solving and field notes which I took during the interview sessions to triangulate the data. This helped me to understand learners' thinking processes and to identify different types of errors they made. The term complementarity is used to elaborate and understand the overlapping and different facets of the phenomenon under study (Johnson & Christensen, 2008). In other words, complementarity seeks a clarification of the results from one method with results from the other method. For example, to clarify and further elaborate the results of learners' response to test items, I made use of interviews, which acted as a confirmatory procedure to verify sources of errors made by learners.

The strength and weakness of the mixed methods design have widely been well documented in literature (Miles &Huberman 1984; McMillan & Schumacher, 2010). The advantages in using such a design include its straightforwardness and possible opportunities that it avails in the exploration of quantitative results in more detail. Such designs can also be useful in cases where unexpected results arise from the quantitative phase of data analysis (Morse, 1991). However, limitations in using this design include prolonged periods and feasibility of essential resources to collect and analyse both types of data.

3.3 Data Collection Site

Data collection was done at a secondary school located in Gauteng West District. I chose the school as a site of study because of its accessibility since I am a Mathematics educator at the school.

3.3.1 Participants

The data for the research study were collected in the third term of 2013 from 40 learners in one of the Grade 11 Mathematics classes at my school. The chosen class was an already established class hence there was no random sampling of participants. In the research domain, this is referred to as convenience sampling (McMillan & Schumacher, 2010: 137). Out of the 40 learners, 18 were boys and 22 were girls. The age of learners ranged from 16 to 18 years. Each learner spoke at least one of the three languages namely Isi Xhosa, Isi Zulu and Setswana. About 60% of the learners came from three informal settlements and the rest from the surrounding townships. The majority of these learners came from poor socioeconomic backgrounds as well as from single parent homes. All learners in this study were English second language speakers who completed their primary education in the surrounding primary schools in the same township where my school is located.

3.3.2 Data Collection Methods

Two methods used to collect data in this study were through the test and interviews. The test and interviews done with the selected participants were intended to provide the sources of data that were used to answer the following research questions as shown in Table 1 below.

Table 3.2: Research Questions and Sources of Data

1. What errors do learners make when translating from word problems to linear algebraic representations?	test
2. What are the possible sources of these identified errors?	interviews

3.3.3The Test

A test is a form of assessment that consists of a standard set of questions that are administered to participants to answer either on paper or on computer in order to complete a cognitive task (McMillan & Schumacher, 2010). According to the above authors, the cognitive task in the test may focus on a number of attributes of participants for example what a participant (1) knows (achievement) (2) is able to learn (aptitude) or (3) is able to do (skills). In this study, the major focus of the test was to investigate learners' linguistic knowledge and skills in translating from verbal written language in word problems to linear algebraic representations.

The test was administered to 40 Grade 11 Mathematics learners. The test consisted of 15 test items of which 7 were multiple-choice questions (Section A) and 8 were open-ended questions (Section B) (see appendix A). Section A of the test included questions that required learners to select an appropriate word problem for a given algebraic equation (i.e., question 4) and to select an algebraic equation/expression for a given word problem (i.e. questions 1, 2, 3, 5, 6 and 7). Section B of the test consisted of open-ended questions and their purpose was to enable me to explore learners' abilities and skills in freely expressing their mathematical thoughts in writing algebraic representations. In particular, open-ended questions required learners to write an appropriate inequality equation for a given mathematical statement (question9) and to write an appropriate algebraic equation for a given mathematical statement (questions 8, 11, 12, 13, 14 and 15).

The learners' comments expressed during the interviews were triangulated with the answer choices they had selected in the multiple-choice questions and algebraic representations written on open-ended questions**.** The reason for this was mainly to enable me to identify possible links of error patterns emerging from both sections of the test. The learners' incorrectly written algebraic representations also helped me in developing appropriate questions for the interviews. In other words, the nature of an incorrect response from a particular question provided me with clues as to which types of interview questions were appropriate in order to get as much information as was possible from learners. I also prepared analytical notes on each of the 40 participants' written responses in order to keep track of any interesting emerging patterns in learners' thinking processes. The analytical notes mainly focused on the way learners presented their written algebraic representations and other mathematical ideas they had used when writing their answers.

3.3.4 The Interview

Opie (2004) argues that the purpose of interviews should be to encourage respondents to develop their own ideas, feelings, insights, expectations, or attitudes and to say what they think. It is from this basis that questions used in interviews were tailored to achieve the above goal. To investigate learner errors and their sources while translating from word problems to linear algebraic representations, I interviewed each of the five learners in 30-minute sessions. The major purpose of interviews was then to ascertain observed errors and their possible sources drawing from learners' explanations, arguments and propositions as they defended their selected or written algebraic representations. These explanations from learners provided rich data for analysis as more learner misunderstandings were detected during these exchanges.

As a starting point in the interviews, each learner was asked to read a question before attempting to explain his/her written response. In addition, learners were asked to 'think aloud' (Newman, 1977) or even to sketch diagrams (Mayer, 1985) in order to help them explain their mathematical thinking. During the interview sessions, I prompted learners with "explain further" "what do you mean", "give an example", and "how" or "why" questions whenever I felt the need to fully understand what they were saying. If I felt that the learner had not understood my question, I went ahead and rephrased my question as well as providing examples to make the question clearer. All the five interviews were audio-recorded, transcribed and presented in form of excerpts as shown in appendices $7-10$. In this study, transcribed excerpts from learner interviews are coded as [$\mathbf{L_1}$ - Learner 1 (and given a pseudo name) and = **R** - Researcher].

3.3.5 Criteria Used in Selecting Learners for Interviews

After administering the text to learners, I numbered all learners' scripts from $\mathbf{L_1}$ to $\mathbf{L_{40}}$ for the purposes of identifying the sample of learners who could be considered for interviews. I then selected a sample of five learners (3 boys and 2 girls) whose written work was used for analysis of errors elicited in both their selected multiple-choice answers and their written algebraic representations. The criteria for selection were based on two principles namely: learners who had written incorrect algebraic representations and were willing to participate in the interview and those who had attempted all the test items. Learners selected for interviews according to gender were coded and given pseudo names **Retha**, **Steve**, **Tanatswa**, **Tsidi** and **Thabo**.

3.4 Pilot Study

A pilot study is essential to refine instruments and to identify any other problems in the design of a research project. Emphasizing the importance of carrying out a pilot study, Oppenheim (1992: p.47) asserts that "Piloting can help us not only with the wording of questions but also with procedural matters such as the design of a letter of introduction, the ordering of question sequences and the reduction of non-response rates." Prior to administering the final test instrument to my Grade 11 learners, I conducted a one-phase pilot study at the beginning of July 2013 to gather as much information as possible to improve the validity and reliability of the test instrument. I selected 15 Grade 11 Mathematics learners from another class at my school to answer the pilot questions. In order to identify questions which were ambiguously phrased, learners wrote down what they thought was unclear and gave me suggestions as shall be explained below. Since I wanted all questions to be responded to, all ambiguous terms and phrases were noted and rewritten with the help of my supervisor and a colleague at the university who is a specialist Mathematics educator. All unclear questions were reformulated to suit the

type of English language synonymous with current textbooks used in the South African Mathematics curriculum. Explanations of changes made resulting from piloting the test instrument are elaborated below.

3.4.1 Instructions to Participants

In the pilot question paper, I had included six instructions that learners were supposed to read and adhere to but that was never the case as only four learners managed to follow them correctly. For example, one of the instructions required them to answer all questions on a specially prepared answer sheet and to write down phrases and words they did not understand on each question. As discussed above, only four learners used that specially prepared answer sheet and the rest of the learners ignored it. In the multiple-choice questions, learners circled the letters representing their answer choices. From this observation, I decided that learners answer the questions by circling the correct answer on the question paper. The other instruction removed from the list was that which explained the purpose of test. Five learners suggested that by informing them that the test was not for assessment purposes, most of them would not take it seriously hence I removed the instruction. With these adjustments, only four brief instructions were eventually indicated on the final question paper. On the day I gave the test, I spent the first five minutes explaining the instructions on how they were supposed to answer the questions.

3.4.2 Choice of Questions Selected

One of the three questions I had included in the pilot test required learners to write a verbal statement representing the equation "$4x + 25 = 73$". Only one learner wrote the correct verbal mathematical statement, that is, "25 more than 4 times a certain number is equal to 73". The rest of the learners wrote this statement describing the operations on numbers and letters in a literal direct translation format, that is, $4\ (x)$ plus $25 = 75$.With the advice from my supervisor, I removed two of these types of questions from the test instrument.

My supervisor further advised me to categorise questions that almost tested similar skills to avoid contradictions in my analysis and conclusions. This was after I had discussed the results of the pilot test with him. With this in mind, I included four questions that tested learners' ability to translate word problems that dealt with quantitative comparisons often referred in literature as 'compare word problems'. I further grouped these 'compare word problems' into two categories namely: 'additive compare word problems' and 'multiplicative compare word problems'. The other category of questions consisted of two de-contextualized word problems. The decision to administer 15 questions instead of only nine (selected for analysis) in the test was necessitated by the background knowledge I had of my learners in terms of their commitment to schoolwork.

Experience working with them had shown that, given few questions, they would not treat the task as important as I would have wanted them to. With that in mind, I further explained to them that this was an important test that would be used in assessing their language problems to enable me to prepare them for their Grade 12 examinations in the following year.

I therefore grouped nine selected questions out of the total of 15, which appeared in the test for analysis and placed them into four categories as follows: **Category A** was composed of questions**1**, **3**, and **7**and these were multiple-choice questions. I grouped questions in **Category B** into three subcategories namely: **Categories: B1** - de-contextualised word problems (**9**and**13**); **B2** - 'additive' compare word problem (**11**and**12**) and **B3** - 'multiplicative' compare word problems (**14** and**15**). Two major reasons for selecting these word problems for analysis were that these questions were rich in both their syntax and semantic structures such that they provided a rich source of data for analysing errors associatedwith the translation process. In addition, these were word problems that needed learners' correct interpretation and adequate understanding of the language used for them give correct responses. Secondly, these questions have been widely used in many research studies that focussed on errors committed when translating from word to algebraic representations (Duru & Koklu, 2011; Capraro & Joffrion, 2006; Clement, 1982). More importantly, these questions would help me to compare and contrast the type of errors committed by my learners to those errors documented in past research studies.

I left out the last three questions in the test (see appendix A) that sort to elicit learners' perceptions about their challenges and other comments that related to questions they had answered. A closer analysis of learners' comments relating to these three questions showed that they (comments) did not add value to the overall research findings. For instance, the majority of learners wrote general comments such as *"I had problems with all questions or the test was difficult*". For that reason, I did not consider this section of the test in my final data analysis.

3.4.3 Rationale for Grouping Word Problems into Categories

The major focus of this research study as discussed in Chapter One was to investigate and identify errors that learners made when translating from verbal written mathematical language to linear algebraic representations (i.e. equations, inequality equations, and expressions). The rationale for grouping questions into four categorises was therefore, an attempt to compare, contrast, and quantify specific errors that were conspicuous among the error categories. For instance, word problems in category **B2** often referred to in literature as 'additive' compare

problems[1] (Lewis & Mayer, 1987; Kintsch & Greeno, 1985; Jones, 1982) were considered. These 'additive' compare word problems investigated learners' understanding and mathematical interpretation of the two relational terms 'more' and 'older'. Word problems considered in Category **B3** are those referred to in literature as 'multiplicative' compare word problems[2] (Mayer, 1982). These word problems investigated learners understanding of relational equivalence between two quantities where the key relational terms such as '8 times as many as...' and 6 times greater than...' appeared.

3.5 Data Analysis

Data analysis is described as a systematic search for meaning such that the qualitative and/or quantitative data observed may be communicated to others in understandable ways (Hatch, 2002). Analysis of data involves organizing and interrogating data in ways that allow the researcher to see patterns, identify themes discover relationships, develop explanations, make interpretations, mount critiques, or generate theories (Hatch, 2002: 148). One of the major aims of data analysis, therefore, is an attempt to understand the phenomena under investigation and to be able to provide answers to the research questions. In the data analysis section, both quantitative and qualitative techniques were used to analyse errors in the learners' written work.

In order to understand the type of errors that learners made, I analysed and noted the mathematical content in each question and related this content to the types of knowledge (i.e. syntactic, semantic and schematic) as espoused in Mayer's (1985; 1999) theoretical model of problem solving. Research work done on word problem solving has identified problem representation as the most critical stage where the majority of learners falter (Mayer, 1985, 1999; Clement, 1982; Macgregor & Stacey, 1993). Based on this fact, I therefore considered an error investigation on learners' written responses based on these three types of knowledge.I then used the three types of knowledge as a basis to generate categories of errors from learners work on open-ended questions as discussed in Section 3.6.2 below.

[1] Additive (algebraic) compare problems are characterised by the presence of a relational statement, i.e., sentences that express the value of one variable in terms of another and an operation of addition or subtraction is suggested.

[2] A multiplicative (algebraic) compare word problem is the one with relational propositions that involve the comparison of two or more quantities through multiplication e.g. the student-professor problem (*Clement, Lochhead & Monk, 1981*).

3.5.1 Analysis of Learners' Performance on Multiple-Choice Questions

To capture the general outlook of learners' performance on multiple-choice test items, quantitative techniques of data analysis were used. Quantitative techniques include the use of descriptive statistical methods such as quantifying learners' responses into categories, calculating the frequencies in each category and finally, converting the frequencies into percentages (McMillan &Schumacher, 2010). For example, each multiple-choice question had four possible answer choices of which one was correct. The number of learners who selected a particular answer choice was noted and the respective frequency was converted into a percentage for easy interpretation (see appendix E). Learners' selected answer choices showing the highest response rate for incorrect answers were noted.

3.5.2 Error Analysis – Open-ended Questions

I usedtypological analysis model of data analysis to analyse the test and interview data (Hatch, 2002). According to LeCompte and Preissle (1993) as cited in Hatch (2002 p. 152), typological analysis involves "dividing everything observed into groups or categories based on the basis of some canon for disaggregating the whole phenomenon under". According to Hatch (2002), "Typologies are generated from theory, common sense, and/or research objectives, and initial data processing happens within those typological groupings" In this study, the typologies I identified emanated from the three types of knowledge essential in the problem representation stage of Mayer's (1999) model of problem solving. The types of knowledge are namely: syntactic, semantic, and schematic.

Hatch (2002) further affirms that typological analysis only has utility when initial groupings of data and beginning categories for analysis are easy to identify and justify (p. 152). In selecting these typologies therefore, I hypothesized that lack of these knowledge types on part of learners was more likely to impact negatively on their performance in correctly translating words into algebraic representations. Table 3 below gives a summary of error categories identified from typologies generated from three types of knowledge as proposed by Mayer's (1999) model of problem solving. In this study, **E1**, **E2** and **E3** denote error categories under which syntactic, semantic and schematic errors were categorised respectively. **STE, SE**and **SCH** denote syntactic, semantic and schematic errors respectively coded in learners' incorrectly written algebraic representations.

Table 3.3: Error Categories Identified from Typologies

Stage	Step	Knowledge types	Typologies identified	Error Category
Problem representation	Translation	Syntactic Semantic	Reading comprehension skills Understanding mathematical language	**E1**- Syntactic Errors **(STE)** **E2** - Semantic Errors **(SE)**
	Integration	Schematic	Knowledge of problem types (i.e. equations)	**E3** - Schematic Errors **(SCE)**

3.6 Definition of Error Descriptors

The section below gives a detailed description of error categories that I used as a basis for beginning error analysis from learners' written work in open-ended questions. The error descriptors defined below were initially used as a basis to categorise learners' incorrectly written algebraic representations into appropriate error categories **E1**, **E2** and **E3**.

3.6.1 Category E1 - Syntactic Errors[3]

These errors are observed when the given problem is given a direct translation (i.e. word order matching) as it is structured. In other words, a learner simply assumes that the order of key words in the problem corresponds to the order of the symbols appearing in the equation. The strategy used to translate does not take into account the meaning of the verbal written mathematical language used in the problem (Clement, 1982).

Example: **Question 9**: Write a symbolic representation of the statement: *"Half of* 5 *less than all numbers are less than or equal to 6"*.

In this example, a learner's response such as "$\frac{5}{2} < x \leq 6$" suggests as a direct translation of words in the verbal statement to its symbolic equivalent form (i.e. word order matching). Using this syntactic translation strategy, the response does not take into account the contextual meaning of the words in the question. For instance, $'½'$should multiply '5 less than all numbers' (i.e., ½ $(x - 5)$ not ½ $(5) < x \leq 6$).

3.6.2 Category E2 - Semantic Errors

These errors result when the *meaning* implied in the learners' translated algebraic representation is not the same as that implied by the verbal statements in the word problem. In other words, a

[3]Syntactic errors result when equations are formed by writing the mathematical symbols from left to the right in order in place of the words in the verbal expressions (i.e. word order matching)

semantic error[4] is committed when a written algebraic equation indicates that the learner misunderstood the language (i.e. words, symbols or phrases) used in the problem (Clement, 1982).

Example: (see Question 9 above). A response such as "$\frac{x}{5} - x \geq 6$" disregards the meaning of what is implied in the original verbal written statement that is, "½" should multiply $(x - 5)$ and not "$\frac{1}{5}$" to multiply "x" only. Further, the correct symbol should be "$\leq$" not "$\geq$". Theimplication is that the learner does not understand the meaning of the language used in the problem.

3.6.3 Category E3 - Schematic Errors

This error is coded when the written algebraic representation does not have a logical link to the implied mathematical content in the question. The written algebraic representation shows that the learner is unable to retrieve relevant mathematical knowledge or information implied in the word problem (Fong, 1995). The implication of these errors is that any knowledge or information retrieved from the word problem has no connection or link to the question. In other words, the learner lacks specific knowledge of problem types.

Example: (see Question 9 above): a response such as "$\frac{1}{5} > x \leq 6$" is considered a meaningless mathematical inequality equation since it gives two contradicting solution sets. One set consists of all values of 'x' strictly less than '$\frac{1}{5}$' and the other set give values of 'x' less or equal to 6. The algebraic representation suggests that the learner lacks the appropriate schema for inequality equations.

3.6.4 Procedures Undertaken in Analyzing Open-ended Questions

To analyse learners' errors on selected open-ended questions, I first wrote all different types of incorrectly written algebraic representations for each question from all of the 40 learners who wrote the test (see appendix D). I then carefully coded all the translated written algebraic representations that indicated similar learner misunderstandings for each selected question. Based on the definition of error descriptors as described in **Section 3.8.3** above, I then placed the coded incorrectly written algebraic representations into error categories (i.e., **E1**, **E2**, and **E3**). After categorizing learners' incorrectly written algebraic representations into **E1**, **E2**, and **E3** error categories, further subcategories of errors related to the way these algebraic representations were presented in writing emerged. In other words, this final stage of data analysis focussed on

[4]Semantic errors are coded when the written mathematical representation (i.e. equation) shows learners' misunderstanding of the mathematical language used (i.e. words, phrases, or incorrect use of symbolic notation).

the mathematical structure of these incorrect algebraic representations undererror categories **E1**, **E2**, and **E3**. The subcategories of errors identified were eventually described and named.

3.7 Phases of Data Analysis Observed in the Study

The diagram below (Fig 3) depicts the connection between data collection methods and sources of data in this study. In the diagram below, the one-way arrow connecting the quantitative and the qualitative phases indicates that the research process is sequential. Data from the quantitative phase provided a picture in terms of learner performance on different questions selected for deeper analysis. However, during the error analysis, I had to move back and forth between the quantitative and qualitative data sources (see the two-way arrow) in cases where I needed further clarification and understanding

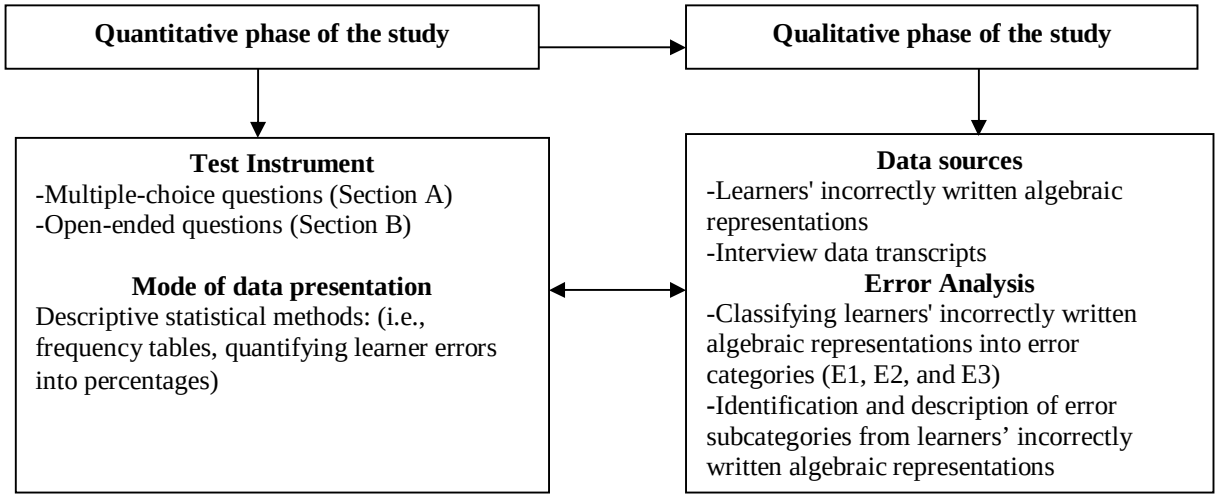


Figure 3.2: Two phases of data analysis

3.8 Rigor

According to McMillan and Schumacher (2010: 330), validity in qualitative research designs is the degree to which the interpretations have mutual meanings between the participants and the researcher. In other words, the questions at stake are: Do qualitative researchers actually observe what they think they are seeing? Do inquirers actually hear the meanings that they think they hear? This brings to fore the idea of credibility or the extent to which the findings can be believed (Silverman, 2001). Therefore, for the research community to believe what has been reported, Maxwell (1992) posits that qualitative researchers' first concern lies in the factual accuracy of their account (p. 285) and refers to this as descriptive validity. In other words, they should aim not to distort things that they see or hear for them to be believed. In this study, descriptive validity was achieved through the use of audio recording and verbatim transcription. As I investigated both errors and their sources from learners' written work, I continuously

referred to taped data (whenever I was in doubt) in an attempt to get more sense of what learners said. In my reporting, I used exact quotes from learners' verbal statements in an attempt to bring to fore their actual mathematical thinking.

Reliability in qualitative research refers to the extent to which research findings can be replicated. To be more precise, the question is, if the study is to be repeated, would it yield the same results? (Merriam, 1998). However, this is impractical in social sciences in which human behaviour is never static. Nevertheless, reliability can be achieved if the researcher documents his or her procedure and demonstrates that categories generated were used consistently (Silverman, 2000 p. 188). In this study, this entailed giving a detailed description of how I coded and classified the data illuminated from learners' written work in a manner that is systematic and justified to the reader. I am convinced that my explanations pertaining to data coding and analysis of errors are clearly stated in this study.Further, in classifying incorrect learners' responses into error categories, I requested my colleague at the university to repeat the same process in order to check whether we agreed on classification of errors observed in learners' work. This is referred to as inter-rater reliability (McMillan and Schumacher, 2010). In fact, our classifications of learners' incorrectly written algebraic representations into error categories were not significantly different from each other.

With reference to the test as a research tool, its validity is equally important as its reliability. In other words, if a test does not serve its intended purpose, it is therefore, considered invalid. According to McMillan and Schumacher (2010: 173),validity of a research tool is a judgement of the appropriateness of a measure for specific inferences or decisions that result from the scores generated. This suggests that validity, as a concept is situation specific since it takes into account the purpose, the sample/population and the context in which the measurement takes place. In this study, the intention of the test used was to measure the learners' abilities to read, comprehend, and translate word problems to algebraic representations. Therefore, the content validity of the test was an important factor. As a measure to preserve content validity ofthe test, questions used were adapted with minor modifications from past research studies that investigated translation of word problems to algebraic representations (i.e., Duru & Koklu, 2011; Capraro & Joffrion, 2006; Essien, 2011). The appropriateness of the test items I selected wasalso verified and approved by my supervisor.

On the other hand, reliability of a test refers to the extent to which a test if administered, gives consistent results across a range of settings when used by other researchers (Wellington, in Opie, 2004: p. 65-66). In other words, reliability is conceived of as the extent to which measures from a test are free from error (McMillan & Schumacher, 2010: 179). In this sense, if the instrument

has little error (i.e. the questions are unambiguous and well constructed), then the instrument may be deemed reliable. In this study, all questions though adapted from past research work mirrored the content of the Mathematics textbooks used by the learners in the Further Education Training (FET) band. Further, these questions had been used successfully in past research studies that investigated learner errors when translating from word problems to algebraic representations. Piloting the test instrument also brought about substantial changes to the construction of some of the question items, which had proved difficult and ambiguous for learners to understand. These changes undertaken improved the reliability of the test as it removed those ambiguous parts of the questions prior to administering the test instrument.

3.8.1 Ethical Issues

In adherence to the requirements of undertaking research using learners, the purpose of the research was clearly explained to all participants and their parents. Consent letters were sent to parents whose children took part in the research study requesting their permission to allow them to participate in the study. In the letters given to both parents and learners, the purpose and reasons for the study and the possible benefits resulting from their participation was explicitly stated. The consent letters sent to all parties clearly stated that participation was voluntary and the anonymity and confidentiality of participants was guaranteed. My application for ethical clearance from the Ethics Clearance Committee at the University of the Witwatersrand categorically described how the rights of participants in the research study were to be respected and protected. Both the school principal and the Gauteng Department of Education granted me permission to conduct the study at my school. In seeking permission from the school authorities to conduct the study, I indicated in my application the willingness to share the research findings with both my colleagues in the Mathematics Department as well as the school principal.

3.8.2 Overview of the Data Collection Plan

Table 5 below summarises the data collection plan for this study and it states the type of research tools used, the date the instrumentswere administered, size of sample selected and what the instruments measured.

Table 3.4: A summary of data collection plan

Instrument	Date	Sample size	Purpose of the Instrument
Pilot test	03/07/2013	15 learners (9 girls and 6 boys) - from a separate Grade 11 class in the same school	Checking the appropriateness of the test items in terms of wording, possible learner difficulties and helping to decide on the duration of the test.
Test	05/08/2013	40 learners (18 boys & 22 girls) - from a Grade 11 Mathematics class selected to participate in the study	To investigate learners' incorrectly written algebraic representations and type of errors they made
Interviews	12/08/2013 to 16/08/2013	selection of participants based on analysis of the test 5 learners (3 boys and 2 girls)	To get in-depth information pertaining to the learners' thinking processes as well to confirm and ascertain observed errors and their possible sources

3.9 Conclusion

This chapter has discussed issues pertaining to the research design, the sample size, sampling procedure, research instruments, data collection methods and method of data analysis for this study. Issues of reliability and validity of the research tools have been discussed. Ethical procedures undertaken have also been discussed. The next chapter focuses on the data analysis and findings.

CHAPTER FOUR

DATA ANALYSIS AND FINDINGS

4.1 Introduction

This chapter presents data analysis and findings that emerged from the analysis of learners' selected answer choices in the multiple-choice questions and their written algebraic representations on open-ended questions. Two sets of data analysed were from the test and subsequent interviews done with five learners. In analysing data from both the multiple choice and open-ended questions, I first used quantitative techniques that included the use of frequency tables depicting learner errors in order to get an overall picture of how learners performed in all selected test items. As explained in Section 3.4.2,word problems in the multiple-choice sectionconsisted of questions **1**, **3**, and **7** and these were selected for deeper analysis in investigating sources of errors that learners committed**.** Error analysis on learners' responses to multiple-choice questions depended on data from the interviews where they expressed their mathematical thinking as they explained their answer choices. Five learners **Retha**, **Steve**, **Tanatswa**, **Tsidi**, and **Thabo** (not their real names) were selected for interviews.

As discussed in Section 3.4.2, all questions selected for analysis in this study were categorised in two main categories **A** and **B**. Category **A** was composed of three multiple-choice questions, Category **B** consisted of algebraic word problems, which were classified into three subcategories **BI**, **B2**, and **B3.** An attempt to provide answers to the research questions is done in the discussion of errors and their sources under each category of word problems. This research study explored the following research questions:

- What type of errors do learners make when translating from word problems to linear algebraic representations?
- What are possible sources of these identified errors?

4.2 Analysis of Errors in Multiple-choice Questions

With reference to Table 5 below, the column under 'category' describes categories of word problems in the multiple-choice section of the test. The column under 'nature of translation' indicates what each respective question asked from learners. The third column under Category **A** indicates the questions selected and the percentage of learners who selected each of the four possible answer choices in each multiple-choice question. Table 5 below shows the sample of word problems selected for analysis in the multiple-choice section of the test.

Table 4.5: Sample of selected multiple-choice questions under Category A

Category	Nature of translation	Question (%) - % of learners who chose the response)
A Multiple-choice questions (*Learners selected a correct response from 4 possible choices*)	From verbal written language to symbolic language (i.e. algebraic equations)	**1**. *A number **n**is the product of half of itself and 6 less than itself.* Therefore the value of ***n***is given by A)**n**= $(n-6)+\frac{1}{2}$ **(5%)**B) $\boldsymbol{n} = (n-6) \cdot \frac{n}{2}$ **(20%)**C) $\boldsymbol{n} = \frac{(n-6)}{2}$**(17.5%)**D) $n = \frac{n}{2} - 6$ **(57.5%)** **3**. *There are 'n' desks in our classroom. Of these desks, 1 is used by only 1 student and each of the remaining is used by* 2 *students.* Which of the following appropriately represents the number of students in our classroom? **A**) $2n-1$ **(20%)B**) $\frac{n}{2}-1$ **(20%) C)** $2n+1$ **(55%) D)** $\frac{n}{2}+1$ **(2.5%)** **7**. When 4 times the first number is added to 3 less than the second number, 21 is obtained. 2 more than two times the first number is equal to 51 more than the 5 times the second number'. **A)**$4x-(y-3)=21$ **(12.5%)B)**$4x+(y-3)=21$ **(15%)** $(2x+2).5 = 51(2x+2)-5y=51$ **C**) $4x+(y-3)=21$**(2.5%)*D*)** $4x+(y-3)=21$ **(67.5%)**$(2x-2)-5y=51(2x+2)+5y=51$

The multiple-choice section is where learners showed their understanding of verbal written mathematical language by way of selecting the correct algebraic representations from the four possible answer choices provided in each question item. In this study, learners' linguistic, semantic and schematic knowledge (Mayer, 1985, 1999) essential in translating from word problems to algebraic representations was under focus in the analysis of errors in their written work. In the multiple-choice section, interviews provided the best opportunity to gain insight into learners' mathematical thinking processes. For this reason, this first analysis phase of learners' work heavily depended on what they said as they explained their selected answer choices. Key concepts, words, or phrases as depicted in the content analysis of each question were used to frame the type of questions asked during interview sessions. In addition, the type of understanding needed to answer each selected question correctly was related to learners' selected responses (see appendix C). The next sections discuss learner views, their misunderstandings and possible sources of errors reflected in their selected answer choices. The discussion focuses on providing answers to research question 2 using data from interviews and learners' selected responses to multiple-choice questions

4.2.1 Source of Errors (1):Learners' Lack of Vocabulary Knowledge

In the first question selected for deeper analysis, learners were required to select a correct expression for the mathematical statement *"A number n is the product of half of itself and* 6 *less than itself. Therefore, the value of n is given by:* Results from Table 5 show that only 8 (**20**%) of the learners marked the correct choice **B:** $\boldsymbol{n = (n-6).\frac{n}{2}}$**.** The most popular incorrect answer choice was ***D:*** $\boldsymbol{n = \frac{n}{2} - 6}$ which the majority of learners 23 (**57.5**%) out of 40 learners chose. Considering the linguistic components of the question and the learners' reflections on the verbal part of the written mathematical statement, it was clear that the meaning of words implied in the mathematical text were not immediately familiar to them. In particular, they ignored to use the meaning of the key words 'product' and some could not decode the contextual meaning of the word to identify the appropriate algebraic equation. The following learners' arguments captured in interviews done on question **1** provide an insight into possible sources of errors that may have attributed to their failure to identify a correct response.

In excerpts**1** and **2**, both **Retha** and **Steve** admitted that they did not make use of the meaning suggested by the word 'product' and they further could not remember its contextual meaning as it is used in Mathematics. In her response to whether she knew the meaning of 'product', **Retha**, replied, *"No, I didn't use the word product...I mean its meaning*" (**excerpt 1: line 8**). In line**10** on the same except she professed ignorance as to the meaning of the word 'product' when she responded, "*Um...I can't remember sir, but we were taught about the word I think in Grade 9"*. Similar responses came from **Steve** (**excerpt 2: line 6, 12 and 16**) and this suggests that the whole meaning of the mathematical text was lost because of misinterpreting the meaning of the word 'product'. On the other hand, **Tanatswa** (**excerpt 3: line 4 and 6**) read the word 'product' but mistook it to mean 'total' and this instantly altered the mathematical meaning of the whole word problem. When asked whether 'product' meant 'total', he argued, "*Um...it's like* $'n'$ *is the total of half of itself, then I changed product to total so I could understand and that's why I chose* $\frac{n}{2}$ *minus* 6". Like **Retha** and **Steve** as described above, **Tanatswa**misunderstood the meaning of the key word 'product' and in **line 14**, he showed faint memories of when he had encountered the word.

On the primacy of vocabulary knowledge, Hofstetter (2003) observes that being unfamiliar with one key term in one language is all it takes a student to fail a question item. The researcher further argues that this happens in cases when that term is specific to the register of a content area in that language. In this question, two terms were supposed to multiply each other (i.e. $\frac{n}{2}$and $n - 6$) and this fact lay in the mathematical meaning of two words that is, the word *'product'*

and the word *'and*'. It therefore points to the fact that learners who chose **D** as their answer could not capture the meaning of the word problem because of misunderstanding the meaning of the word 'product'. In their discussion on the primacy of vocabulary knowledge, Cuevas (1984), Anderson and Pearson (1984) argue that readers of mathematical text must recognise the meaning of many previously encountered words, symbols and information organisers for them to understand the mathematical text at hand. Given the learner challenges discussed above, it is therefore imperative that success in translating from word to symbols requires learners to activate their prior knowledge on the meaning of technical terms used in word problems. These mathematical words according to Adams (2003) dictate the choice of appropriate symbols and operations to use when forming equations from verbal written mathematical statements.

Similarly, research in Mathematics language has acknowledged the difficulty that academic texts pose to the learners. Anderson and Pearson (1984), Allington (2001) and Combs (2002) argue that reading with comprehension depends not only on the readers' general background knowledge regarding the mathematical text at hand, but also on their familiarity with the specialized language used in the text. The above argument serves to illustrate the strong relationship between reading comprehension and vocabulary knowledge (Capraro & Joffrion, 2006; Baumann & Kameenui, 1991; Stanovich, 1986) although other factors such as mathematical abilities (Reed, 1984) play a part in answering word problems.

With reference to Mayer's theoretical model, the translation stage of problem solving demands linguistic knowledge, which is essential in interpreting the verbal written mathematical language in the texts. This involves learners restating the givens and goals as depicted in the verbal text in their own terms and being able to recognise the meaning of various mathematical terms (Mayer, 1985; 1999). However, data from interviews showed substantial evidence that learners were unable to grasp the mathematical message stated in the word problem. In other words, learners failed to undertake important cognitive processes at the problem translation stage where the purpose of reading is mainly to understand the problem structure using linguistic skills that includes decoding meanings of the key words used in the problem (Mayer, 1999; Yuill & Oakhill, 1991). Based on the above argument, it is, therefore, reasonable to suggest that the main source of learners' erroneous selected responses was due to lack vocabulary knowledge.

4.2.2 Source of Errors (2): Learners' Lack of Symbolic Knowledge

Question 3 selected for deeper analysis asked learners to select an appropriate algebraic expression that represented the total number of students in a classroom (see Table 5). The most popular incorrect response was choice **C:** $\boldsymbol{2n + 1}$ with $22\ (55\%)$ out of $\mathbf{39}$ learners who

attempted the question. The question required conceptual understanding of algebra for example knowing that variables (i.e., letters) represent unknown quantities (MacGregor & Price, 1999). The notion of excluding "1 *desk*" from "n" desks (an unknown) and then the remaining desks (i.e., $n-1$) being used by the rest of the students proved difficult for learners to conceptualize when asked to write it in symbols. **Excerpt 5** (**lines 5-12**) below illustrates **Steve's** difficulties in conceptualizing the fact that $'n-1'$ (in symbolic form) represented the number of desks left after one was removed.

5 - R: So, let's say there are *'n desks'* in the classroom, and we are told that only one is used by one student, so how many desks are left in the classroom?
6 - Steve: It's n desks because there is unknown number of desks in the class, so it's difficult to know.
7 - R: Is it 'n' desks left? Let's say we had '10 desks' and we take one away. How many are left?
8 - Steve: There are nine left
9 - R: So let's go back to the idea of 'n desk'. If we take one, how many are left?
10 - Steve: Um...is it '$n-1$' sir? Yeah, it's '$n-1$' desks I think.
11 - R: Can you tell me, what the '$n-1$' represents in terms of desks in the classroom?
12 - Steve: It's total number of desks in the classroom and the one used by one student as well.

Conceptualizing the idea that 'n' desks represented a certain number of desks in the classroom (though unknown) proved elusive at first to all learners interviewed. It was after a given numerical example that they slowly began to realize that one needed to subtract '1' from 'n' to get '$n-1$'. It is interesting to note that even after learners managed to write '$n-1$' as desks remaining, they still could not understand what this number represented as revealed by **Steve**'slast verbal statement in **line 12** above. The challenge for **Steve** here was his failure to engage himself into the algebraic mode of thinking rather than the arithmetic one, which deals with numerical values (Bednarz & Janvier, 1996; Capraro & Joffrion, 2006). The same can be said about **Tanatswa**'s lack of schema (Hinsley et al, 1977) for symbol use in algebraic word problemsin **excerpt 6**. When asked about what was left after taking one desk from 'n' desks, **Tanatswa** responded "*There isn't left, no desk is left or I can say we don't know*" (**line 12**). Here the source of the erroneous response indicates lack of schema for understanding algebraic symbols (Mayer, 1999).

Another source of error observed was that of not realising the fact that $'n-1'$ enclosed in the brackets represents a single number in algebra depending on the context of use. When asked to write the total number of students in the class, **Retha** (**excerpt 4: line 12**) responded,"*I'm getting confused by the* '$n-1$' *desks now. Okay, it must be* '$1+2$ *multiplied by the remaining desk. Sir, should I take* '$n-1$'*as one number? That is what is confusing me*". When asked to put on paper the expression that represented the number students who sat on two desks, **Steve(except 5: line**

10) simply wrote $'2n - 1'$ after indicating that 2 was multiplying $'n - 1'$. It is imperative from this type of response to suggest that the learner lacked symbol awareness. According to MacGregor and Price (1999), symbol awareness includes knowing that groups of symbols can be used as basic meaning-units. For question 3 in particular, $(n - 1)$ was supposed to be conceptualised as a single quantity for the purposes of algebraic manipulation. Owing to this confusion, the source of error in this particular case was largely due to lack of symbol awareness.

Lastly, for both **Retha** and **Tanatswa**, apart from the lack of symbol awareness, the language used in the problem confused them as revealed by their reasoning in the initial stages of the interviews. Asked why she chose response **B**, **Retha** responded, "*I said, one is used by only one student and the rest are used by two students, so I subtracted one from* $\frac{n}{2}$ *students. I subtracted one student from* $\frac{n}{2}$ *because each of the remaining desks is used by two students, there are sharing, so I divided n by* 2*"*. The same sentiments came from **Tanatswa** in **excerpt 6**: **line 2**. This response gives the impression that both **Rethaand Tanatswa** misunderstood the contextual meaning of the phrase '*...and each of the remaining is used by* 2 *students'* and to them, the phrase suggested dividing $'n'$ by$'2'$. By reasoning in this manner, it is imperative that both learnersassociated the word 'sharing' with division though the context required multiplying the number of desks by $'2'$. Wagner and Parker (1993) provide a caveat on the dangers of over-reliance on the use of key words to understand mathematical texts. They stated, "Though looking for key words can be a useful problem-solving heuristic, it may encourage over-reliance on a direct, rather than analytical, mode for translating word problems into equations" (p. 128). In light of the above analysis, the source of error in the incorrect response was due to misunderstanding the contextual meaning of the phrase 'is used by' because to them the phrase suggested 'sharing'.

With reference to Mayer's (1999) theory on problem solving, data from interviews provides ample evidence that the majority of them failed at the problem translation stage, which begins with reading the problem for understanding. Firstly, understanding the words' meanings from the mathematical text is regarded as a fundamental cognitive process that results in successfully translating word problems to algebraic representations (Mayer, 1999). Evidence from interviews suggests that learners struggled with the problem structure of the question that included conceptualising in quantitative terms the number of desks represented by $'n'$ and $'n - 1'$. Misunderstanding the function and purpose of symbol use in algebra negatively affected learners' understanding of the mathematical text. In other words, the majority of learners were not yet conceptually ready to reason algebraically in addition to their perceived language difficulties. Consequently, results from the interviews showed that these learners were unable to build

theappropriate mental representations of the situation stated in the question for them to identify the correct equation.

4.2.3 Source of Errors (3): Learners' lack of Meta-cognitiveSkills

In question 7, learners were required to find a corresponding algebraic expression to the statement: *'When* 4 *times the first number is added to* 3 *less than the second number,* 21 *is obtained.* 2 *more than two times the first number is equal to* 51 *more than* 5 *times the second number'*. In this question, learners were supposed to write two equations with two variables. Only 6 (15%) out of 39 learners marked the correct choice B:

$$4x + (y - 3) = 21$$

$$(2x + 2) - 5y = 51$$

The most popular incorrect answer choice was **D** chosen by 27 (67.5%) out of 39 learners who attempted the question. Learners understood the first part of the question when they realised that two variables were required to construct the system of equations. The problem surfaced as most learners could not use their algebraic knowledge and procedural skills to realise that "*…is equal to* 51 *more than* 5 *times the second number*" could be rewritten with $5y$ as $(-5y)$ on the left hand side of the algebraic equation (i.e., $2x + 2 - 5y = 51$). This is evident in **Tsidi**'s response when asked to reflect on her challengeas she explained, *"I looked for the second part on all equations to see if I could find* $(\ldots = 51 + 5y)$ *to the right side of the equation, but I could not. I ended up choosing answer D because* $5y$ *was being added*" **(excerpt 8: line 12).**

The comment from **Tsidi** shows that it was more of procedural knowledge and skills in dealing with algebraic terms in equations than the application of linguistic knowledge that was required to understand the problem. In other words, she forgot how terms in equations change their signs, as they cross the equal sign. The same challenge was observed from **Retha** who despite writing down the correct set of equations, looked at the answer choices and simply said, "*There is no correct answer given in this question".* As discussed above, **Retha** could not immediately reason that $(5y)$can be moved from the left to become $(-5y)$. Two sources of errors that led to **Retha** and **Tsidi**'s selecting incorrect answer choices can be deduced from both **excerpts7**and **8**.Firstly, these learners were unable to monitor their thinking processes to see whether the equation they had chosen made sense in relation to sentence meaning (Wollman, 1983). This cognitive process is referred to as metacognition, and is defined as the ability to think about thinking, the self-awarenessof problem solving and the ability to monitorand control one's mental processing (Schoenfeld, 1985; Flavell, 1992). Secondly, the other thing noted was their eventually guessing of the answer choice without actually thinking about whether their selected answer made sense.

This is evident in **Retha**'s last verbal response (**excerpt 7: line 10**) "... *When I could not find the answer I ended up choosing D, when I saw* $'+5'$. *...to be honest Sir, I rushed to answer without understanding the way in which terms were arranged in the question".* In view of **Retha**'s argument, the source of error in this case was conceived as 'guessing without reasoning'.

In a general perspective, a review of the learners' responses in interviews provides substantial evidence that they had challenges in understanding the mathematical language in word problems from the multiple-choice section. In particular, data from the interviews elicited three main sources of errors that may have influenced learners to identify inappropriate algebraic representations. Firstly, learners showed insufficient knowledge of vocabulary as used in **ME**[5] contexts (question 1), secondly, they lacked symbol awareness (question 3), and thirdly, they were unable to check the sensibility of their selected response (question 7).

Learners' lack of linguistic knowledge that relates to vocabulary knowledge (i.e. misunderstanding the mathematical meaning of the words 'product' and 'and') falls under error category **E2** (semantic errors) as defined in Section 3.6.3.2. With reference to learners' lack of symbol awareness, selected incorrect algebraic representations showed learners' naivety in carrying out algebraic processes essential in word problem solving. On failure to check the sensibility of selected answer choice, data from interviews suggest that learners ended up guessing answers without proper reasoning. This might have been as result of frustration as they could not immediately see the response with $(+5y)$ on the same side as $'=51'$ in question 7. The next section focuses on Open-ended Questions where learners freely expressed both their linguistic knowledge and skills in translating verbal written mathematical statements to their equivalent symbolic form. The discussions in the sections that follow attempts to provide answers to both research questions 1 and 2 as stated in Section 4.1 above.

4.3 Classification of Incorrectly Written Algebraic Equations into Error Categories

*(De-contextualised Word Problems-*9 & 13*)*

In Table 6 below, learners' incorrect algebraic representations for each question were carefully examined, coded, and categorised based on similar learner thinking processes. These incorrectly written algebraic representations are indicated in Table 6as '**examples**' under each error category **E1**(syntactic errors), **E2**(semantic errors), and **E3**(schematic errors). The categorisation of these incorrect algebraic representations into error categories was guided by the propositions as outlined in error descriptor definitions presented in Section 3.6.3.

[5]**ME** is an acronym for 'Mathematical English'

Based on the above method of categorisation, the frequencies (**F**) of coded learners' incorrectly written algebraic representations under each error category are shown in Tables 6 below[6]. The total frequency of incorrectly written algebraic representations for each question under each error category is indicated in the **sub-total** row in Table 6. The combined frequency of all coded incorrectly written algebraic representations for the two questions under each error category is indicated in the second last row of Table 6. Finally, the last row in Table 6 expresses the total frequencies under each error category as a percentage of the total number of errors ($\boldsymbol{n = 83}$)[7]. Abbreviations **STE**, **SE** and **SCE** in the table below and subsequent tables in this chapter refer to syntactic, semantic and schematic errors respectively recorded and presented as 'examples' as shown in Table 6.

Table 4.6: Classification of incorrectly written algebraic equations into error categories

	Error Categories					
Question	**E1 - (**STE**)**	**F**	**E2 - (**SE**)**	**F**	**E3 -** (SCE)	**F**
	'examples'		'examples'		'examples'	
9) **Write a symbolic representation of the statement**: *"half of 5 less than all numbers are less than or equal to 6".*	1) $5/2 < x \leq 6$	7	1) $x - 5/2 = 6$ 2) $5/2 - x \leq 6$ 3) $x/2 - 5 \leq 6$	6 11 4	1) $5/2 \geq 6$ 2) $1/5 > x \leq 6$	9 3
Sub-total		7		21		12
13) If the product of 2 and 4 is subtracted from twice a certain number and then increased by 4, the result is 22. **Write an algebraic equation that corresponds to the above statement**.	**1)**(2**x**4)-2**x**+4=22*	5	**1**) (4**x**2)-2**x**+4=22 **2)** 2(2**x**-4)+4 = 22 **3)** 2**x**–8+32 = 22 **4)**(2+4)-2**x**+4= 22	7 8 3 14	**1**) 22+4+4+2+2=34	6
Sub-total		5		32		6
Total frequency (Q: 9 &13)		**12**		**53**		**18**
Total frequency as(%) of all errors (**83**) committed for 2 questions in Category **B1**.		**14.4 (%)**		**63.9 (%)**		**21.7 (%)**

**Note: the written response: $'(2x4) - 2x + 4 = 22'$ was classified as both a syntactic and semantic error.*

Results from Table 6 above indicate that for 'de-contextualised' word problems, the highest frequency of errors (i.e. $\mathbf{63.9\%}$) were related to learners' misunderstanding of the semantic structure of verbal written mathematical statements. In other words, the majority of learners had serious challenges in interpreting the mathematical meanings of words and phrases found in the word problems. Based on these results, the next section discusses how different interpretations of verbal mathematical statements depicted in word problems may be a potential source of errors

[6]The process of categorizing learners' incorrectly written algebraic equations was done in a similar way for questions in category **B2** and **B3** as shown in Table 8 and 10. (see **Section 4.4** below)

[7] **(n = 83)** constitutes the total number of errors (i.e. syntactic, semantic, and schematic) coded in the learners' incorrectly written algebraic representations for the two de-contextualised word problems.

illuminated in learners' incorrectly written algebraic representations. The discussion done under each category of word problems ends with a summary of specific subcategories of errors that were observed under error categories **E1**, **E2**, and **E3**. The subcategories of errors identified were related to those illuminated in both contemporary and past research works.

4.3.1 Errors and their Sources [De-contextualised word problems]

In an attempt to provide answers to research questions 1 and 2 based on the analysis done on de-contextualised word problems, error analysis on learners' written work focussed on two aspects. Firstly, learners' incorrectly written algebraic representations were analysed based on the way learners presented them (i.e. their structure). Secondly, the analysis focussed on data from interviews, where learners' mathematical thinking processes were exposed. The views reflected in their arguments during interviews provided clues for possible sources of errors identified in their incorrectly written algebraic representations. In other words, interviews were used as a confirmatory procedure to ascertain learners' misunderstandings reflected in their incorrectly written algebraic representations.

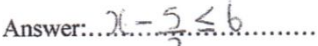
4.3.1.1 Errors and their Sources in Interpreting Mathematical Texts

Results from Table 6 above provide evidence that points to the problematic nature of interpreting verbal mathematical texts with understanding. In other words, what we see written (i.e. algebraic representations) largely depends on how one interprets the linguistic components appearing in the word problem. Out of all **83**errors identified and classified into three error categories **E1**, **E2**, and **E3**, 53 (**63**.**9**%) were related to learners' lack of semantic knowledge (Morris, 1938). This component of knowledge type (i.e. semantic) is central in enabling successful translation processes from verbal written mathematical language to algebraic equations (Mayer, 1999). Accordingly, the grasp of semantic knowledge is viewed as pivotal in enabling the problem solver's ability to discern and identify both meaning of symbols and words. More importantly, semantic knowledge constitutes factual knowledge that relates to specific situations or knowledge of specific concepts (Mayer, 1999). The knowledge of specific concepts implicitly or explicitly embedded in the verbal written form helps learners to understand the link between verbal statements and their equivalent representations (Bernardo, 2002).

4.3.1.2 Learners' understanding of Words*'of', 'less', 'and' and 'product'*

Question**9**shown in Table 6 above posed serious challenges to learners as they could not grasp both the syntax and the semantic structure of the verbal written mathematical statement *'Half of* 5 *less than all numbers are...*' In their written responses, **Retha**, **Steve**, and **Tsidi**wrote the

following algebraic representations respectively representing the statement: "*half of* 5 *less than all numbers are less than or equal to* 6*"*.

(1)  (2)  (3)

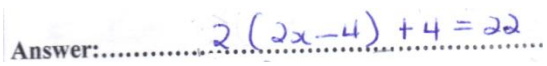
Retha's response was classified under category **E2** (semantic errors) as the response indicates that she did not understand the contextual meaning of the key word 'of' in relation to its meaning as used in the statement. However, she knew that the word 'of' suggested the operation of multiplication but did not consider its context of use (see **line 6**). As evident in **excerpt 9: (line 8), Retha** argued that $\frac{1}{2}$was multiplying $'5'$ only although she had read the whole question twice. The same misunderstanding was evident from **Steve** (**excerpt 10: line 12 & 16**) as he also argued, "$\frac{1}{2}$*is multiplying* 5 *only"*. The explanation for this confusion depends on how one reads and interprets verbal written mathematical statements in the word problem before writing the final algebraic equation. **Retha** and **Steve** thought the word 'of' only implied multiplying $'\frac{1}{2}'$with $'5'$ and not$'\frac{1}{2}'$ with the whole expression $'x - 5'$. In other words, for them it was ' 'half of 5' (i.e. $\frac{5}{2}$) that was less than 'all numbers' (represented by $\boldsymbol{x}$). However, based on this argument, **Steve**'s written algebraic inequality equation can be considered correct depending on the way he read the statement. Here, it is important to see how different interpretations on the same verbal written statement may lead to different algebraic equations. As the above analysis shows, errors committed in such translation processes are only verifiable through interviews as shown in the case of **Retha** and **Steve**. The analysis above shows that the source of error in both incorrectly written algebraic representations was largely due to their failure to read, interpret, and correctly integrate pieces of linguistic components in the problem statements. The subcategory of error was coded an 'interpretation error'.

A similar misunderstanding is revealed in **Tsidi**'s algebraic representations (i.e. 3) shown above. Here the problem was her inability to correctly understandthe syntactical structure in the first part of the sentence where she interpreted the phrase 'less than all numbers' to mean the same as 'less all numbers'. This is evident in **Tsidi**'s line of thinking (**excerpt 11: line 2**) when she argued, "*I multiplied* $\frac{1}{2}$by5 *and I got*$\frac{5}{2}$. *I then subtracted* '*x*' *standing for all numbers, and then I put the symbol less than or equal to* 6*"*. Here experience in reading linguistically packed mathematical texts is fundamental. Such texts require a slower reading rate than everyday language text as well as multiple readings for the purposes of understanding (Bye, 1975; Bernardo, 2002; Yuill & Oakhill, 1991). In mathematical language context, the word 'less' is not

used in the same way as 'less than'. The former primes the operation of subtraction (which **Tsidi** did) whereas in the context of how the sentence is structured, the latter suggests the use of an inequality symbol $(<)$. However, using similar reasoning in subtracting 'all numbers less', **Tsidi** still showed inexperience in reading specialized language used in mathematical texts. Due to this misunderstanding, **Tsidi**'s written algebraic inequality equation was classified under error category **E2** (semantic errors) and the subcategory of error was coded a 'comprehension error'. The source of error was due to misunderstanding the semantic structure of the language used in the text of which the word 'of' was key in coming up with a correct algebraic representation.

MacGregor and Stacy (1999) strongly argue that verbal mathematical statements and expressions may have more than one interpretation depending on how structural relationships or referential terms are interpreted by a reader. As seen in the above arguments, learners understood the statement 'half of 5 less than all numbers...' without considering the mathematical implication and use of the word 'of'. In mathematical terms, terms appearing in front of the word 'of' are to be multiplied and in this case, it was supposed to be $'\frac{1}{2}x(x-5)'$. This scenario provides ample evidence on the potential ambiguities that are associated with the interpretation of mathematical texts resulting in learners making errors in their written algebraic equations. It also raises the issue of syntactic awareness on readers of mathematical texts. Syntactic awareness as used in this study relates to how the syntactic structure in algebraic representations controls both meaning and making inferences from verbal written mathematical texts (MacGregor & Price, 1999).

The same challenge concerning the potential ambiguities that are synonymous with interpreting mathematical statements and expressions was evident in question **13** (see Table 6). In this question, learners misinterpreted the message in the problem because they could not use their syntactic knowledge in carefully analysing the sentence structure in order to write the correct algebraic equation. Particularly in this question, the contextual meaning of the words 'product', and 'and' in relation to how they were supposed to be interpreted proved difficult for them to conceptualize. On this question, 8 $(\mathbf{20\%})$ of the learners wrote the following algebraic equation depicting the verbal written mathematical statement "*If the product of* 2 *and* 4 *is subtracted from twice a certain number and then increased by* 4*, the result is* 22.



For example, when asked why she put $'2'$ outside the brackets **Retha(excerpt 12: line 24)**explained " *Sir, I think I didn't understand or remember what the word product meant but I thought the* 2 *is multiplying* $'4'$ *that was being subtracted from* $2x$".

The challenges in interpreting mathematical text are noticeable here from **Retha**'s written algebraic equation shown above. If the statement is interpreted as the product of 2 and '4 *is subtracted from twice a certain number*' while using the context of the first 'and' as used in Mathematics to prime the operation of multiplication, her written algebraic equation would make sense. Similarly, in this question, much depends on how one interprets the word 'product' and the word 'and' immediately after the number ′2′. In other words, understanding the entire verbal written statement depends on the ability to identify which terms forms the product. In this case, it is supposed to be understood as the product of '2 and 4' that should be subtracted from twice a certain number not the product of 2 and '4 is subtracted from a twice a certain number'. In such word problems, experience in reading linguistically packed expository mathematical texts is a requirement (Meyer, Brandt& Bluth, 1980).

Based on this argument, the issue of reading expository mathematical texts (Meyer, Brandt &Bluth, 1980) designed to inform or explain implicit concepts becomes a challenge. The challenge posed in such type of texts is that less proficient readers tend to retrieve information from verbal mathematical statements in seemingly random ways (Meyer et al, 1980). The above analysis in learners' arguments reveals this fact and this further point to the myriad of challenges learners face when translating words to algebraic representations. As results from Table 6show, these challenges were revealed in learners' incorrectly written algebraic representations, which did not match the meaning implied in the verbal written mathematical statements (Clement 1982). Guided by the definition of error descriptors in Section 3.6.3, **Retha**'s incorrectly written algebraic equation was classified under error category **E2** (semantic errors). The source of error was due toherfailure to integrate the relevant pieces of information in the entire verbal written statement (Mayer, 1985; 1999; Hinsley et al, 1977). Understanding the verbal written statements in the question as discussed above depended largely on correctly interpreting the words ' product' and 'and'.

Misinterpretation of mathematical texts leading to incorrect translations from verbal written mathematical language to algebraic equations is also compounded by having insufficient knowledge pertaining to mathematical vocabulary (i.e. referential terms) as used in **ME** (MacGregor & Price, 1999; Pimm, 1981; Adams, 2003). Analysis of data from interviews provided substantial evidence to that fact. When asked the meaning of what the term 'product' meant in question 13, **Retha**(**excerpt 12**: **line 6**) responded, "*It's the final answer Sir"*. When another term popularly used in word problem solving (i.e. difference) was mentioned (**line 7**), **Retha** quickly remembered what product meant. The failure to recognise the meaning of

vocabulary terms used in **ME** indicated their inability to activate their prior knowledge on the mathematical meaning of these terms (Duru & Koklu, 2011).

Generally, the word 'product' is an example of a word with multiple meanings (Ellerton & Clarkson, 1996; Dale & Cuevas, 1987). In other words, the word has specific meanings in both **OE** and **ME**[8]. The challenge faced in the understanding such words is largely attributed to the fact that Mathematics attaches specialised meanings to such everyday words, which in turn already have their own meaning in **OE** (Adams, 2003; Noonan,1990; Kane, 1967). This may be the reason why learners reading mathematical texts with such words get confused as to the actual meaning of these words. Kane (1970) sums up the primacy of having a sound knowledge of such words when he argues that Mathematical English and Ordinary English are sufficiently dissimilar that they require different skills and knowledge on the part of readers to achieve appropriate levels of reading comprehension. As results from Table 6 show, most of these learners either did not know the meaning or completely ignored to make use of the meaning of these words. Consequently, misunderstanding the meaning of these words resulted in learners writing incorrect algebraic representations. The highest frequency of errors related to semantic knowledge in Table 6 provides evidence to that fact.

In another similar case with **Steve** (**excerpt 13: line 7 - 10**), the following exchanges took place providing evidence that semantic knowledge (Mayer, 1999) plays a role in the translation process from verbal mathematical language to algebraic equations. **Steve**presented his written response to **question 13** as follows:



7 - R: My next question is, where did you get this 32 that you added?

8 - Steve: Okay, I thought about it like..if the product is eight…then I multiplied this product by 4 since we are told in the question that it's increased by 4.

9 - R: Then are you saying that 'increased by 4' means we have to multiply by 4?

10 - Steve: Yes, I think so…. that's why I multiplied by 4 to get 32.

For both **Retha** and **Steve**, the meaning of the verbal mathematical statements were lost largely due to misunderstanding the meaning of the word 'product' as revealed by the incorrect algebraic equations they wrote. For **Retha**, 'product' meant final answer and for **Steve** it meant 'to increase'. It is further interesting to see why relying with 'key' words may prove futile in an attempt to understand the message in the mathematical text (Clement & Bernhard, 2005; Wagner &Parker, 1993; Chinn, 2004). Concentrating on key words according to these researchers

[8]Acronyms 'OE' and 'ME' denotes Ordinary English and Mathematical English respectively.

subverts mathematical understanding leading to incorrect solutions as learners fail to check the sense implied in situations being described in word problems. In an attempt to gain more insight into **Steve**'s argument on why he added $'32'$ instead of $'4'$, he surprisingly changed his earlier explanation of taking the meaning of 'product' to mean 'increased'. In his argument, **(line 16)** he explained " *No Sir...I think I actually wrote* $'32'$ *because the question says …it's increased by* 4 *and I know that the word 'by' means multiplication.* To him and at this later stage, it was the meaning of word 'by' that made him to multiply $'8$ and $4'$to get 32. However, in the verbal written mathematical statement in question**13**, the word 'increased' precedes the word 'by'. In other words, the word 'by' indicates that the number $'4'$ is the one to be added rather than to be multiplied by $'8'$. Similarly, on the same question **(**i.e.13), **Tanatswa** could not come to terms with the meaning of the phrase 'the product of 2 and $4'$ and his written response is shown below:



Tanatswa's understanding of the meaning of the phrase 'the product of 2 and 4' is revealed in **excerpt 14 (line 5 - 8)** below.

5 - R: What do you understand by the phrase 'the product of 2 and 4'?

6 - Tanatswa: Product of 2 and 4 means 2 plus 4 because it's written 2 and 4 so I added them,

7 - R: In Mathematics, does product mean addition?

8 - Tanatswa: No sir, I mean out of 2 and 4, what comes out of it.

As discussed in Section 4.2.1 above, unfamiliarity with one key term used in the text results in learners committing semantic errors that are largely due to misunderstanding the language used in the problem (Clement, 1982). From the above discussion with **Tanatswa**, it is evident that he did not make use of the meaning of the term 'product' but focussed more on the term 'and' which to him meant addition. As seen in his response, he added $'2$ and $4'$ instead of multiplying showing that he misunderstood the language of the verbal written mathematical statement. Hence, his incorrectly written algebraic equation was categorised under error category **E2** (semantic errors). The subcategory of error was coded a 'comprehension error'. The source of this error was due to lack of vocabulary knowledge.

Another misunderstood phrase in question **13** was 'is subtracted from' implying that it was $'2 \times 4'$ (i.e. the product) that was supposed to be subtracted from $'2x'$. In this question, **Tsidi** wrote:

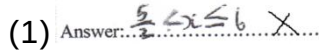
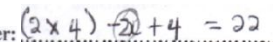


The incorrectly written algebraic equation shows that **Tsidi** understood all key words except the phrase 'is subtracted from'. Asked to reflect on her written response, **Tsidi(excerpt 15: line 8)** was quick to realize her mistake as she explained,"*Oh, it's the product of* '2 *and* 4' *that is being subtracted from* '2*x*' *I now realize. I actually did not read carefully to see that it was the product that was being subtracted from* 2*x"*. Judging from the way she quickly realisedher mistake, one possible explanation for her erroneous algebraic equation is proffered here. The error can be conceived of as a reading error given that she did not take care of the contextual meaning of the word 'from'. The word 'from' in **ME** indicates the order in which terms subtract each other. As data from interviews showed, such errors became prevalent in learners' written algebraic equations mainly because they could not actively monitor their understanding as well as checking the reasonableness of their written equations. Based on this argument, the source of the error was conceived of as 'learners' lack of metacognitive skills'.

As Mayer (1999) maintains, linguistic knowledge, which includes vocabulary knowledge, is required to comprehend the words' meanings during the translation stage of problem solving. It would then be highly unlikely for a problem solver to succeed at the problem integration step, which requires a problem solver to build a coherent mental representation of the problem situation (Mayer, 1999). More importantly, the problem integration process demands the problem solver to understand how problem statements in the mathematical text fit together so that the entire word problem is understandable. However, this process becomes difficult if words' meaning are not familiar to the learner (Mayer, 1999).

4.3.1.3 Learners' Use of Word-order-Matching Strategy

Having discussed the potential ambiguities that are synonymous with interpreting mathematical texts, inappropriate algebraic equations maybe attributed to other factors partly related to the phenomena discussed below. The tendency by learners to take the order of key words in a verbal statement as they appear and form equations is one such strategy that results in what has been referred to as word order matching (Clement, 1982; Mestre, 1988). As research in word problem solving has documented, this strategy does not take into account the meaning of the verbal written mathematical statements. For example, in questions **9** and $\mathbf{13},7(\mathbf{17}.\mathbf{5}\%)$ and $5\ (\mathbf{12}.\mathbf{5}\%)$ respectively wrote the following algebraic equations:

(1)  (2) 

The first algebraic inequality (1) suggests that the learner assumed that the order of key words in the statement corresponded with the order of symbols needed to write the algebraic equation

(Clement, 1982). Reading sequentially from left to right (Mestre, 1988) we have, 'half of $5'(\frac{5}{2})$ 'less than all numbers' $(< x)$ 'are less than or equal to $6'(\leq 6)$. This illustrates learners' inexperience in reading mathematical texts with comprehension whereby they fail to realise that the verbal and the symbolic representations do not always match (Pirie, 1998). In such cases as discussed above, learners wrote strings of symbols matching words as they appeared in the verbal written statements. Consequently, the error subcategory in algebraic representation (1) was coded 'writing solution sets' instead of 'inequality equations'. The source of error was due to learners' use of word order matching strategy, which reflects learners' inability to interact with specialised language used in mathematical texts

Similarly, the incorrectly written algebraic equation (2) reveals evidence of word order matching from the way it is structured. In this example, the learner started with the product of '2 and 4' and went on to subtract $'2x'$ instead of vice versa. It is reasonable to suggest that learners who wrote equation (2) misunderstood the phrase 'is subtracted from $2x'$ to mean simply 'subtract $2x'$. This example serves to illustrate the fact that failure to integrate relevant information across verbal written mathematical statements in the word problem is more likely to result in incorrect responses (Mayer, 1985; 1999). Cognisant of this misunderstanding, the written equation $'(2 \times 4) - 2x + 4 = 22'$ was categorised under both **E1** and **E2** error categories representing syntactic and semantic errors respectively. The subcategory of error in this written algebraic equation (2) was coded a 'comprehension error' and the source of error was largely due to lack of reading comprehension skills.

The last section under de-contextualised word problems provides a brief discussion on learners' written algebraic equations that indicated no logical link to the relevant mathematical knowledge or underlying concepts depicted in the problem. These included equations that contained only numerals or those that elicited lack of knowledge of problem types and these were classified under the error category **E3** (schematic errors). Representations (1), (2) and (3) below were written by 9 (**22.5%**), 6 (**15%**)and 3 (**7.5%**) learners respectively and these written 'examples' reflect learners' lack of schema for algebraic equations.

(1) $5/2 \leq 6$ (2) Answer: $22+4+4+2+2 = 34$ / $34-22 = 12$ the number (3) $\frac{1}{5}?x \leq 6$

Using the definition of the schematic error descriptor in Section 3.6.3, the first and second representations contain numerals only and they do not reflect learners' grasp of concepts related to algebraic equations. Given the mathematical content illuminated in verbal mathematical statements for questions**9**and**13**, the representations provide evidence that they (learners) lacked

knowledge of problem types (Mayer, 1999). In other words, the use of numerical values in forming equations showed that learners were not yet conceptually ready to translate from verbal mathematical language to symbolic representations. Writing numerals instead of algebraic equations according to Capraro and Joffrion (2006) is an indication that learners do not understand the meaning of letters representing variables in an equation.

For instance, representation(3) above is contradictory in both its structure and meaning in that values of $'x'$ are both depicted as less than $' \frac{1}{5} '$ and less than or equal to $'6'$. Further, the implied mathematical message in question 9 required learners to translate the word problem to an inequality rather providing a solution set as suggested in (1) and (3) above. This brings to fore the primacy of utilising schematic knowledge at the problem representation stage where problem solvers discern how the problem at hand may be related to problems solved in the past (Mayer, 1982; 1985, 1999). However, for these learners, little suggests that they had interacted with tasks involving translating from word to algebraic equations. Based on the above analysis, representations 1, 2, and 3 were coded 'incongruent responses' under error category **E3** (schematic errors). Consequently, the subcategory of error observed from these incongruent responses was coded 'assigning numerical values to variables' and the source of error was due to lack of schema for problem types (i.e. equations)

Based on data from interviews, the sources of errors described above largely contributed in making the verbal written mathematical language in word problems difficult to read and interpret. Results from Table 6 in conjunction with learners' arguments, showed that written algebraic equations largely depends on how a reader interprets the linguistic components implicit or explicit in the verbal mathematical text. To that end, identified semantic errors in written algebraic equations were largely due to learners' inexperience in dealing decisively with the semantic features of verbal written mathematical language.

In an attempt to answer the research questions 1 and 2 proposed in this research study, Table 7 below provides a summary of subcategories of errors, and their possible sources for de-contextualised word problems**.**

Table 4.7: Summary of error subcategories and their possible sources

Error category	Subcategory of errors	Possible source of errors
E1 Syntactic errors	**1**. Writing a solution set instead of an inequality equation	**1.1**-The use of word order matching strategy (syntactic translation) which shows learners' inability to interact with highly specialized mathematical language.
E2 Semantic errors	**1**. Interpretation error **2**. Comprehension error **3**. Reading error	**1.1**- Learners' failure to read, interpret, and correctly integrate pieces of linguistic components in the problem statements. **2.1**-Learners' insufficient grasp of vocabulary terminology used in both **OE** and **ME** contexts **2.2**-Failure by learners to activate their prior knowledge on the meaning of mathematical words used in **ME**. **2.3**-Lack of reading comprehension skills **3.1**-Failure by learners to undertake metacognitive processes when reading mathematical texts
E3 Schematic errors	**1.** Assigning numerical values to variables	**1.1**-Lack of schema for problem types (i.e. algebraic equations

The next section below discusses learner challenges, errors and their possible sources pertaining to 'additive' compare word problems. As discussed in Sections 4.2.1, 4.2.3, and 4.3.1.1, the interpretation of what is read depends on a number of factors for example, experience in dealing with specialised language of Mathematics and the ability to reflect on the task at hand. Reflection on what is being read and understood in the task as the interview data revealed is a panacea in enabling one to come up with correct algebraic representations. The section below provides answers to research questions 1 and 2 based on the error analysis done on 'additive' compare word problem.

4.4 Classification of Incorrectly Written Algebraic Equations into Error Categories

('Additive' Compare Word Problems - 11 &12*)*

As discussed in Section 4.1 and 4.11,the word problems discussed in this section are those referred to in literature as additive 'compare' word problems (Lewis & Mayer, 1987; Jones, 1982). The word problems in this category were characterised by verbal comparative phrases such as 'more than' and 'older than' which primes the operation of addition (Jones, 1982). **E1**, **E2** and **E3** denote error categories into which learners' incorrectly written algebraic representations reflecting syntactic, semantic, and schematic errors were categorised. '**Examples'** under each error category are incorrect algebraic equations reflecting similar misunderstandings and these were coded and classified as shown in Table 8 below.

Table 4.8: Classification of incorrectly written algebraic equations into error categories

Question	Error Categories					
	E1- (STE**)** examples	**F**	**E2 -** (SE) examples	**F**	**E3 -** (SCE) examples	**F**
*11) Tachi is exactly one year older than Bill. Let **T** stand for Tachi's age and **B** stand for Bill's age.* **Write an equation to compare Tachi's age to Bill's age?**			**1)** T + 1= B**2**) T > B **3**) T = 1B	6 5 4	**1**) $T_1 = B_2$ **2**) $x + 1 = x$ **3**) T > 1 > B **4**) T + ***x*** - B	6 2 1 1
Sub-total		0		15		10
*12) Agnes's money is R2 more than Thabo's money. Let **A** represents the amount of Agnes's money and **T** represents the amount of Thabo's money.* ***Show the relationship between amounts of Agnes's money and Thabo's money with mathematical symbols.***			**1)** A + 2 = T **2)** 2A = T **3)** A = 2T **4**) 2A > T	6 3 6 10	**1)** A + 2T **2)** $A_{2x} > T_x$ **3)**2A > 2 >T **4)** $2x + x = x$	4 2 1 2
Sub-total		0		25		9
Total frequency (Question 9 &13)		**0**		**40**		**19**
Total frequency as(%) of all errors (**59**) committed for 2 questions in Category **B2**.		**0 (%)**		**67.8 (%)**		**32.2 (%)**

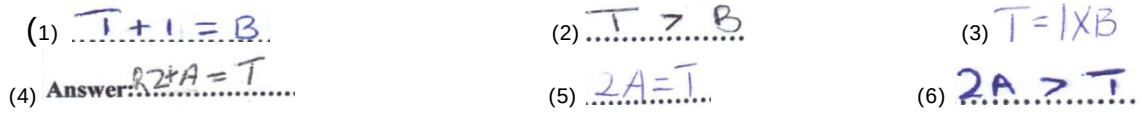
For 'additive' compare word problem, the total number of semantic and schematic errors coded in learners' written incorrect algebraic equations were 59 (**i.e.** $n = 59$). Results in Table 8 show that learners committed more semantic (67.8%) than schematic errors (32.2%) for both questions 11 and 12. Section 4.4.1 below discusses learner challenges in interpreting verbal mathematical statements from compare word problems based on the results in Table 8. Central to the discussion is the identification of errors and their possible sources tapping evidence from what learners wrote and how they interpreted verbal written mathematical language depicted in word problems. The discussion in Section 4.4.1 below attempts to provide answers to research questions 1 and 2 based on analysis done on 'additive' compare word problem.

4.4.1 Errors and their Sources ['Additive' compare word problem]

The column under error category **E1** (syntactic errors) for both questions **11** and **12** (see Table 8) is blank since for these word problems, translating word for word sequentially from left to right yields a correct algebraic equation (Mestre, 1988). For example, following the order of words in question **11** we have, Tachi (T) is (=) exactly one (1) year older than (+) Bill (B). Surprisingly, among the learners who wrote the correct algebraic equations, none of them used the word order matching strategy in both questions. The results in both questions resonate with research findings from MacGregor and Stacy (1993) who reported that in questions in which syntactic translation strategy would yield a correct answer, learners still wrote reversed equations. In light of these findings, they concluded that cognitive models used by learners to write mathematical equations are not linear in form but very complex and for that reason, they advocated for further research to find out how they use these models when translating from verbal written statements to algebraic equations.

4.4.1.1 Learners' Understanding of Words *'more than' and 'older than'*

Results from Table 8 above elicit the challenges that learners face when reading mathematical texts that contain words from everyday language, for example, 'more than' and 'older than' (Jones, 1982). As discussed in Sections 4.2.1, 4.2.2, 4.2.3 and 4.3.1above,the algebraic representations that learners write depends on how they interpret the verbal mathematical statements in the word problem (MacGregor & Price, 1999). The correct interpretation of these verbal mathematical statements largely depends on learners' proficiency in mathematical language use and their metacognitive skills during word problem solving. For both questions **11** and **12**, more errors in learners' incorrectly written algebraic representations **(67.8%)** were largely due to lack of semantic knowledge as the discussion below illustrates. However, learner interviews were done mainly on question**12** as few learners **(15%)** only managed to write the correct algebraic equation as compared to question **11** with **34.2%**. '**Examples**' of incorrect algebraic representations classified under error category **E2** (i.e. semantic errors) are presented below:



About 6 **(15.8%)** out of **38** learners who attempted question **11** wrote equation (1) with reversed variables. In this particular question, learners were supposed to understand the relationship between two variables whose actual quantitative relationship was not given (i.e. Tachi and Bill's ages). However, the key phrase was ' exactly one year older than' which suggested that given Bill's age, Tachi's age will always be one year older than him. The same incorrect reasoning is illustrated in algebraic equation (4) where 6 **(15%)** out of 40 learners made the same error on question**12**. However, in question**12**, the meaning of the phrase 'more than' confused many learners as they could not interpret it in its context of use. For instance, they were unable to integrate the relevant pieces of information stated in the word problem such as knowing who between the two was older than the other.

In both questions **11** and **12**, learners placed the 'number' on the same side with the letter associated with the larger group or quantity resulting in a reversed equation. According to Clement (1982), most learners who write such reversed equations are aware of the larger quantity between the two but they are unable to express it correctly as an equation. Clement (1982) argues that learners who write such reversed equation would have adopted the static comparison strategy where they juxtapose the large number with the letter representing the larger group. In such instances, the given letters act not as variables but names of objects and the

underlying meaning of the equal sign is that of association rather showing a proportional relationship (Wollman, 1983). In light of the above discussion, the subcategory of error in the incorrectly written algebraic equations (1) and (4) was coded a reversal error. The source of the reversal error was the learners' use of the static comparison strategy where they did not understand the underlying meaning of the equal sign, which specifically maintains the quantitative sameness in the resulting algebraic equation.

Algebraic inequalities (**2**) and (**6**) revealed that learners ignored or misunderstood what the questions required them to do as they wrote inequalities rather than equations. In other words, the learners failed at the problem translation step (Mayer, 1999) where they were supposed to read carefully in order to understand the relevant details stated in the question. The learners' confusion in these two questions both illustrate learners' inability to read expository texts with understanding (Meyer, Brandt& Bluth, 1980) as well as their inability to reflect on their written equations to check whether they make sense.

Relational statements with phrases such as 'more than' appear easy to express and understand in natural language but according to MacGregor & Stacy (1993), they must first be paraphrased, reorganized or reinterpreted before writing a mathematical equation. The two researchers further point out that being based on comparison (i.e. more than or older than) rather than equality, the models of the equations do not have a logical form of an equation. This may be the reason why learners wrote these incorrect inequality equations. Consequently, the subcategory of error in algebraic equation (2) and (6) under error category **E3** (semantic errors) was coded 'writing inequalities rather than equations'. The source of error was due to learners' failure to check the sensibility of the equations they wrote.

An interview with **Tanatswa** (**excerpt 17**) illustrated the fact that correct interpretation of verbal mathematical statements is critical to successfully coming up with a correct algebraic equation (MacGregor & Stacy, 1993). Here semantic knowledge that enables learners to understand the problem structure by discerning the meaning of words was required (Morris, 1938). For instance, when **Tanatswa** was asked to state who had more money between Agnes and Thabo (**line 5**), he quickly replied, "*It's Agnes because her money is doubled since it is mentioned that Agnes's money is R2 more than Thabo's".* The challenge with understanding the contextual meaning of the phrase 'more than' is conspicuous here as shown in equation (5). Jones (1982) distinguishes three mathematical contexts in which the phrase 'more than' is used. The first one is the 'comparative context' in which learners must decide which of the two given quantities is greater than the other and this requires understanding the meaning of the word 'more' in isolation. The

second context requires the factual knowledge (Mayer, 1999) that 'more' in Mathematics primes the operation of addition in a direct way. According to Jones (1982), the operation of addition has to be identified from the verbal formulation of the problem statement of which **Tanatswa** did not. The third context, however not applicable to this study, requires the learner to add or subtract in an indirect way.

The above discussion suggests that learners misunderstood the contextual use of the phrase 'more than' in terms of how it was supposed to be interpreted in mathematical terms. Based on **Tanatswa**'s argument that 'more than' meant 'double' (i.e. multiply *A* by 2), the subcategory of error elicited in the algebraic equation (5) was coded a 'comprehension error' and the source of error was lack of understanding of the contextual meaning of the phrase 'more than'. The source of error broadly falls under learners' lack of vocabulary knowledge as used in **ME**.

Under the schematic error category, learners wrote all sorts of algebraic equations that reflected little understanding of algebraic equations and most of these were classified as 'incongruent responses'. As discussed in Section 4.3.2 such responses have no logical link to both content and the linguistic demands as espoused in propositions stated in the word problems. Below are '**examples**' of algebraic equations that fell into error category **E3** (schematic errors):

(1) $x+1=x$ (2) $Tx-B$ (3) $T_1=B_2$

(4) $A_2x<T_4$ (5) $2+x=x$ (6) $A+2T$

Incorrectly written algebraic equations (1), (2), and (3) for question **11** and (4), (5), and (6) for question **12** illustrate learners' naivety with the concept of algebraic equations. Learners who wrote these equations were not conceptually ready to translate from verbal mathematical statements to algebraic equations (Duru & Koklu, 2011). Though their problem of linguistic deficiencies cannot be ignored, it is imperative that these learners failed to retrieve relevant mathematical knowledge essential in forming algebraic equations. A closer look at equations **(1)** and **(5)** shows that learners ignored the givens, goals, unknowns and relations stated in the word problem (Mayer, 1982; 1999). In other words, they went on to use a variable (i.e., letter $'x'$) that was not stated in both problems. In an interview with **Retha** (**excerpt 15)**, it was interesting to note that learners always resort to symbols that they were first introduced to when solving equations in earlier grades. Asked to explain why she used $'x'$ instead of **A** and **T** in question **12**, **Retha**replied, "*Sir it's like I forgot to use the letters **A** and **T** as was required in the question...I did not read it carefully Sir, I thought I could use* 'x' *as we always do in equations"* (**line 10 & 12)**. This response from **Retha** suggests interference from previous learning where variables$'x'$ and $'y'$ are frequently used when introducing equations to learners in earlier grades (Duru

&Koklu, 2011). Without the use of the given variables ***T*** and **B** in question **11** and ***A*** and ***T*** in question **12**, these equations were rendered meaningless since they do not illustrate the correct magnitude of the quantitative relationships espoused in the problem. On the other hand, equations **(3)** and **(4)** are examples where learners used their own symbolic notation, which were meaningless considering the content stated in the questions. Accordingly, the subcategory of error in the incorrectly written equations was coded 'meaningless equations' and the source of errors was largely due to lack of schema for 'problem types' (i.e., equations) and interference from previous learning.

Another interesting response, equation (6) written by 4 (**10**%) out of all 40 participants showed that learners were unfamiliar with the meaning of two phrases 'more than" and 'show the relationship between'. This is conspicuous in **Steve**'s argument **(excerpt 17)** when asked why he had written $'A + 2T'$, and he explained " *Sir, I said Agnes has R*2 *more than Thabo's money … and this means that Agnes will have 'A' money and Thabo will have* 2 *times more money than Agnes, so my answer is* $A + 2T''$ (**line 4 & 16**). As discussed above, the incorrect interpretation 'more than' (Jones, 1982) was conspicuous as **Steve** took the phrase 'more than' to mean 'multiply'. Further to that, his last words "... *so my answer is* $A + 2T''$ is a revelation that the concept of an equation was non-existent in his thinking. Research has shown that writing of equations as totals is evidence of a model of two quantities viewed simultaneously (MacGregor & Price, 1999). According toClement, Narode, and Rosnick (1981), students who write a total may be putting down symbols to represent all the components of the model. In doing so they forget to check whether the resultant expression depicts the requirement of the question at hand, which is to form an equation.

When I further asked **Steve** what the term 'relationship' meant, he professed ignorance at first but eventually replied "*Oh, no sir I think relationships are connected by the equal sign… I mean they are equations".* From his response, it was apparent that the concept of 'equations' as suggested by the key word 'relationship' was lacking. In other words, he lacked the knowledge of problem types (Mayer, 1999). Accordingly, the subcategory of error noted in the incorrectly written algebraic representations was coded 'writing expressions as totals instead of equations' (i.e. 2 and 6). The source of error was largely due to both lack of vocabulary knowledge (i.e. the word 'relationship' and phrase 'more than') and specific knowledge of problem types (i.e., linear algebraic equations).

Table 9 below presents a summary of errors, subcategory of errors and their possible sources under **E2** and **E3** error categories for 'additive' compare word problems.

Table 4.9: Summary of errors subcategories and their possible sources

Error	Subcategory of errors	Possible source of errors
E2 Semantic errors	**1**. Reversal error	**1.1**-Use of static comparison strategy **1.2**-Misunderstanding the underlying meaning of the equal sign (=)
	2. Comprehension error	**2.1**- Misinterpretation of the contextual meaning of the phrase 'more than' as used in **ME**.
	3. Writing inequalities rather than equations	**3.1**-Failure by learners to check whether the equation written makes mathematical sense.
E3 Schematic errors	**1**.Writing meaningless equations	**1.1**-Insufficient schema for algebraic equations. **1.2**-Interference from previous learning (i.e., use of other variables not stated in the word problem)
	2. Writing expressions as totals instead of equations.	**2.1**- Lack of both vocabulary knowledge and reading comprehension skills **2.2**- Inadequate schema for problem types (i.e., linear equations)

In an attempt to provide answers to research questions 1 and 2 for this study, Section 4.5 below discusses learner challenges, errors and their possible sources pertaining to 'multiplicative' compare word problems.

4.5 Classification of Incorrectly Written Algebraic Equations into Error Categories

('Multiplicative' Compare Word Problems - 14 & 15*)*

As discussed in Section 4.1.1, word problems discussed in this section are referred to as 'multiplicative' compare word problems (Mayer, 1982). These word problems investigated learners' understanding of the relational equivalence between two variables whose quantitative relationship was not given. As explained in Section 4.3 and 4.4, **E1**, **E2**, and **E3** are error categories into which incorrectly written algebraic representations (i.e., examples) were categorised. In Table 10 below, examples $'8b = g'$ and $'6S = P'$ were classified under both **E1** and **E2** error categories[9]. The two incorrectly written algebraic equations may be viewed as both a direct translation of key words in the verbal written statements (syntactic translation) and a misunderstanding of the meaning suggested by the relational terms.

[9] Both these equations show reversed variables (reversal error) and misunderstanding of the relational terms used in the word problems.

Table 4.10: Classification of incorrectly written algebraic equations into error categories

Question	Error categories					
	E1 -(STE) 'examples'	**F**	**E2** - (SE) 'examples'	**F**	**E3** - **(**SCE) 'examples'	**F**
14. In a certain school,there are eight times as many boys as there are girls. **Write an algebraic equation that represents the above situation.**	1) 8b = g	3	1) 8b = g 2) x = 8g + b 3)8B ≠ G	3 10 2	1) b + 8g 2) ***x*** = 8***x*** + ***x*** 3) x/8– G	2 4 6
Sub-total		3		15		12
Write an algebraic equation using the variables *S* and *P* that will represent the following situation: "In a certain university, the number of students is 6 times greater than the number of professors" Use **S** for the number of students, and **P** for the number of professors.	1) 6S = P	6	1) 6S = P 2**)** 6 = S + 6P 3) 6S > P 4) 6S < P	6 4 12 3	1) ***x*** + 6S ≥P 2) 6S(***x*** > P) 3) ***x*** = 6 > ***x***	3 2 2
Sub-total		6		25		7
Total frequency (Questions 14 & 15)		**9**		**40**		**19**
Total frequency as(%) of all errors (**68**) committed for 2 questions in Category **B3**.		**13.2 (%)**		**58.8 (%)**		**27.9 (%)**

*Note: $'8b = g'$ and $'6S = P'$ were classified under both **E1** and **E2** error categories.

Table 10 above shows that the total number of errors (i.e. syntactic, semantic, and schematic) recorded in all three error categories (i.e. **E1**, **E2**, and **E3**) for 'multiplicative' compare word problems, were 68 (i.e.,$\boldsymbol{n = 68}$). Out of the 68, 40 $(\mathbf{58.8\%})$ of those errors were however largely related to learners' insufficient grasp of specialised mathematical language particularly the relational phrases 'eight times as many as' and '6 times greater than'. Using data from interviews, the paragraphs below unravel some of the linguistic challenges that these relational terms posed to learners in understanding the proportional relationships between boys and girls (question **14**) and students and professors (question **15**). Central to the discussion below is the identification of subcategories of errors and their source in learners' incorrectly written algebraic representations thereby providing answers to research questions 1 and 2 under 'multiplicative' compare word problems.

4.5.1 Errors and their Sources ['Multiplicative' compare word problems]

Two major sources of error processes were conspicuous in both questions **14** and **15** and these were the learners' application of word order matching strategy and static comparison approach. However, as discussed in Sections 4.2.1, 4.2.3, and 4.3.1.1success in the translation process depends much on how learners interpret verbal written mathematical language in word problems. Under the syntactic error category 3 (7.5%) and 6 (15%) out 40 learners who attempted questions**14**and **15** respectively, wrote the following incorrect algebraic equations shown below:

(1) 8B = G (2) 6S = P

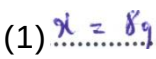
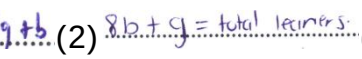
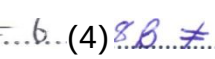
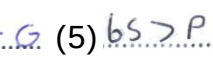
Both (1) and (2) are examples of equations where variables are reversed (Clement, 1982; Mestre, 1988), the reason being that, learners could not sufficiently build a coherent mental representation of the situation described in terms equalising the two quantities. Here as well, both the underlying conceptual meaning of the equal sign (Herscovics & Kieran, 1980) and the knowledge of the function of variables was required. An interview with **Thabo** (**excerpt 19)** on question **14** provides evidence that suggests learners' misunderstanding of the underlying concept of equal sign (=) as used in equations. Firstly, when tasked to elaborate on his written algebraic equation (1), **Thabo**explained, " *I wrote '8b = g' sir. When I read it, I know there must be more boys than girls, so I wrote* 8*b* = 1*g" (***line 2)**. Whenfurther asked what '**B**' and '**G**' stood for he replied, "*It is for boys and girls as stated in the question sir"* (**line 4**).

Both **Thabo**'sverbal response and the incorrectly written algebraic equation provide evidence of the static comparison approach in use. In this approach as discussed in Section 4.4.2 above, **Thabo** understood the fact that there were more boys compared to girls but he could not write his equation correctly. In using such a strategy according to Wollman, (1983) **B** (boys)and **G** (girls) **(**as letters)do not represent varying algebraic symbols but rather graphic representations of individual objects or labels. The fate of the equals sign in such a process is no longer that of an algebraic symbol that represents a mathematical relationship but instead represents the phrase 'for every'. In other words, the ultimate interpretation of an equal sign denotes association or correspondence rather than equality (Wollman, 1983; Clement, 1982). Based on the discussion above, the error subcategory in equations (1) and (2) was coded 'a reversal error'. The source of error was due the learners adopting a static comparison approach, which does not take into account the underlying meaning of the equal sign and variables in this case are treated as labels.

On the other hand, equations (**1**) and (**2**) reflect learners' misinterpretation of relational phrases '8 times as many as' and '6 times greater than' when forming equations. Although the two word problems were contextual in nature suggesting the obvious quantitative relationship between professors and students, this did not help learners in using their common sense. However, it was interesting to note that learners did not reflect on their final written equations to check whether they made sense by methods such as trial and error. This is evident in the exchanges that took place in **excerpt 19** (**line 7 - 14**) where **Thabo**finally realised that the equation ' 8b = g' was giving him opposite results to those he thought were correct. On this note, we are reminded of the fact that mathematical language when spoken emerges in natural language but when written, it makes varied use of a complex and rule governed writing system that is different from that of natural language into which it can be read (Pimm, 1991 p. 20). This fact is summed up in **Thabo**'s response when asked to reflect on his mistake "*I didn't read it correctly sir, plus I didn't*

test it as we did together. I wrote ′8b′ because I thought for every ′8 boys′ there is '1 girl'. That's why I wrote 8b = g" (**excerpt 18**: **line 18**). This response further suggests that **Thabo** syntactically translated the statement into an equation by replacing key words (i.e. 8 times) with mathematical symbols (i.e. b and g) sequentially from left to right to get ′8b = g′ (Clement, 1982; Mestre, 1988). Based on **Thabo**'s reasoning, the source of the reversal error was due to both syntactic translation and failure to check whether the written equations made sense.

Some of the incorrectly written algebraic equations classified under the semantic error category are shown below.

(1)  (2)  (3)  (4)  (5) 

In algebraic equations (**1**), (**2**), and (**3**) for questions**14** and **15** respectively, learners connected the two variables in the equation as 'additive totals'. In doing so, they ignored the proportional relationships suggested by the verbal written relational statements, which required them to connect the variables in form of a meaningful equation relating the two variables. In other words, '**B**′ and ′**G**' or ′**S**′ and ′**P**′ were supposed to be connected by an equal sign depicting the magnitude of their variation. Instead, they ended up building equations that represented incorrect totals. According to Kieran (1981), this way of thinking indicates that learners mistakenly view the equal sign not as a statement of equivalence but a signal to perform an operation. He further points out that this misunderstanding can be traced back from learners' experience from elementary school years where they evaluate problems such as ′8 + 4 = ?. Accordingly, the source of error in equations (**1**), (**2**), and (**3**) was largely due to misunderstanding the language of relational statements. In light of this misunderstanding, the subcategory of error in the three equations was coded 'forming additive totals from proportional relationships'.

In equations (**4**) and (**5**), learners wrote inequalities instead of algebraic equations. As has been discussed above, the underlying meaning of equal sign and the language that included relational phrases proved difficult for them to understand. About 12 (**30%**) of learners who answered **question 15**, used the inequality symbol ′ > ′ to connect the variables '**S**' and '**P**' mainly because of the key word 'greater' that appeared in the question. When asked to reflect on why he had used the 'greater' than symbol on question **15**, **Tanatswa** responded, "*SirI saw the word 'greater than', and I thoughtof an inequality equation was the one needed"* (**excerpt 20: line 24**). However, apart from being distracted by the phrase 'greater than', **Tanatswa** also misinterpreted '**6S**' to mean '6 students' and ′**P**′ to mean '1 professor' instead of taking the two letters as varying quantities. This provides evidence, which suggests that learners were unfamiliar with reading

mathematical texts that contains relational statements. Consequently, the subcategory of error in incorrectly written algebraic representations (**4** and **5**) was coded, 'writing inequalities instead of equations. The source of error as revealed from **Tanatswa**'s argument was due to the inappropriate use of key word identification strategy as a problem solving heuristic. Lastly, under error category **E3** (schematic errors), the following are examples of equations that some of learners wrote.



All the three incorrect constructed responses shown above reveal learners' limited knowledge of forming meaningful algebraic equations from word problems. In other words, learners who wrote these statements were not yet ready to deal with concepts in the discourse of algebra (Duru & Koklu, 2011). These three meaningless algebraic representations provide substantial evidence that learners had insufficient schema for problem types (i.e. algebraic equations). The subcategory of error noted in the way these incorrectly written algebraic representationsare structured was coded 'meaningless equation/expressions' and the source of errors was largely due to insufficient schema for algebraic equations.

Table 10 below summarises the subcategory of errors and their possible source under **E1**, **E2** and **E3** error categories for 'multiplicative' compare word problems. The summary of errors that learners made and their possible sources provide answers to research questions 1 and 2 for the analysed learner' work on 'multiplicative' word problem.

Table 4.11: Summary of error subcategories and their possible sources

Error category	Subcategory of error	Possible source of error
E1 Syntactic errors	**1.** Reversal error	**1.1**-Use of static comparison strategy **1.2**-Misunderstanding the underlying meaning of the equal sign **1.3**-Treating variables as labels or names of objects **1.4**-Failure by learners to undertake metacognitive processes when reading mathematical texts (i.e., check whether the equation written makes mathematical sense)
E2 Semantic errors	**1.**Forming additive totals from proportional relationships	**1.1**-Lack of linguistic skills in interpreting relational statements **1.2**- Misunderstanding the underlying meaning of the equal sign
	2. Writing inequalities rather than equations	**2.1**-Inappropriate use of key word identification strategy as a problem solving heuristic
E3 Schematic errors	**1.** Writing meaningless equations	**1.1** Insufficient schema for problem types (i.e. algebraic equations)

Finally,the next section presents a summaryoffindings drawn from the error analysis done across all categories of word problems selected in this study. The summary includes a comparison of collated results from frequency tables in Sections 4.3, 4.4, and 4.5 combined together. In other words, quantified errors (i.e. syntactic, semantic, and schematic) coded under each category of word problems areanalysed, compared and summarised.

4.6 Overall Summary of Results

Table 11 below provides a summary of frequency counts and percentages of syntactic, semantic, and schematic errors recorded in learners' incorrectly written algebraic representations under each error category for de-contextualised word problems, 'additive' and 'multiplicative' compare word problems. As discussed in Sections 3.6.4 and 4.3, learners' incorrectly written algebraic representations reflecting similar misunderstandings were categorised under the appropriate error category (i.e. **E1**, **E2** and **E3**). This initial process of categorisation was guided by the proposed definitions of error descriptors I presented in Section 3.8.3. In Table 11 below, **B1**, **B2** and **B3** denote categories of word problems selected extensive analysis in this study. **E1**, **E2** and **E3** denote categories of errors into which learners' incorrectly written algebraic representations reflecting syntactic, semantic and schematic errors were categorised. The number in the brackets (∗) denotes the number of errors under each error category expressed as a percentage of the total errors committed for that particular category of word problems.

Table 4.12: Frequency of errors per category of word problems as a percentage

Category of Word problems	**Error category and frequency of errors as a percentage**			
	E1-STE (%)	**E2-SE (%)**	**E3-SCE (%)**	**Total errors per question category (%)**
B1	12 (14.5)	53 (63.8)	18 (21.7)	**83 (39.5)**
B2	0 (0)	40 (67.8)	19 (32.2)	**59 (28.1)**
B3	9 (13.3)	40 (58.8)	19 (27.9)	**68 (32.4)**
Total errors per error category (%)	**21 (10)**	**133 (63.3)**	**56 (26.7)**	$*N = 210$[10]

[10] Note: *N = 210 is the total number of syntactic, semantic and schematic errors coded in learners' incorrectly written algebraic representations for all categories of word problems combined.

An overallanalysis of results in Table 11 above revealed the following findings for this study:

a) Out of 210 ($\boldsymbol{N = 210}$) errors (i.e. syntactic, semantic, and schematic) recorded and combined for all categories word problems, 21 $(\mathbf{10}\%)$, 133 $(\mathbf{63.3}\%)$, and 56 $(\mathbf{26.7}\%)$ were related to syntactic, semantic, and schematic knowledge respectively. According to Mayer's (1999) two-stage model of problem solving, the correct translation process from words to algebraic representations depends largely on the sufficient grasp of these knowledge types. Results from Table 11 suggest that learners had more challenges in understanding the language used in the problem as shown by the 133 $(\mathbf{63.3}\%)$ of semantic errors out of the total $\mathbf{210}$ errors committed in all three categories of word problems. The high percentage of semantic errors reflects challenges associated with lack of proficiency and experience in reading and understanding expository mathematical texts (Meyer, Brandt& Bluth, 1980).

b) 56 $(\mathbf{26.7}\%)$ of all errors made by learners were related to insufficient schema of algebraic concepts reflecting the fact that learners were not yet conceptually ready to deal with a variety of mathematical discourse features that relate to conceptualising the fact that unknown quantities may be represented by variables (i.e. letters).

c) Results from Table 11 further indicate that learner responses on de-contextualised word problems recorded the highest frequency of errors (i.e. syntactic, semantic and schematic) amounting to about $\mathbf{39.5}\%$ of all errors committed. The reasons that may be attributed to such a high number of errors in this category of word problems are twofold. Firstly, these word problems do not suggest any context familiar to learners (Wollman, 1983) such that they (learners) could derive clues or make use of their common sense. Secondly, the sentences in the word problems were structured using specialised language, which is precise and less flexible as opposed to natural language, which is relatively easy to interpret (Pillar, 1999). It also goes on to show that these word problems needed learners to be proficient readers with high interpretive linguistic skills in dealing with specific uses of syntax and semantic features inherent in word problems (Dale & Cuevas, 1987). From the analysis of learners' explanations and arguments during interviews, it was imperative that the majority of them did not realize the fact that verbal and the symbolic representations do not always match (Pirie, 1998).

A pictorial analysis of the frequency of errors (i.e. syntactic, semantic, and schematic) per each category of word problems is shown in **Fig 3** below.

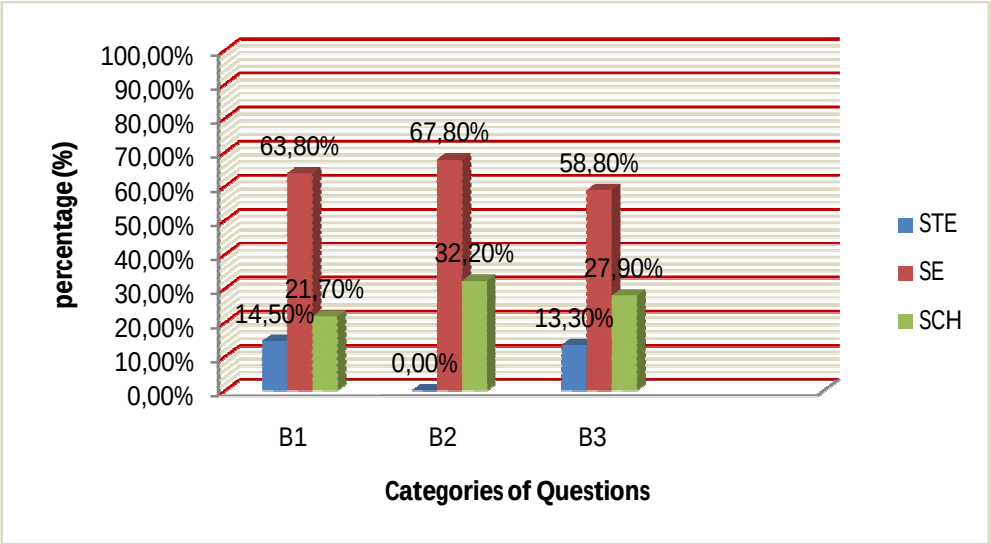


Fig 4.3: Percentage of errors (i.e. syntactic, semanticand schematic) in each category of word problems.

The pictorial presentation of results pertaining to the frequency of errors committed in each category of word problems illustrated in **Figure 3** above suggests that:

a) Learners' challenges in translating verbal mathematical language to algebraic representations for all categories of word problems were compounded by learners' insufficient grasp of specialised mathematical language that characterise the discourse of Mathematics. In particular, learners' lack of semantic knowledge (Mayer, 1999) was evident as shown by their inability to understand the contextual meanings of words such as 'product' and 'more than'. The high frequencies of semantic errors in all three categories of word problems bear testimony to that fact (i.e. $\mathbf{B1} - 63.8\%, \mathbf{B2} - 67.8\% \; and \mathbf{B3} - 58.8\%$). In addition, lack of experience in reading and correctly interpreting the contextual use of words as they appeared in the word problemswas a critical factor contributing to the high percentage of semantic errors in all three categories of word problems as shown in Fig 3.

b) The highest frequency of semantic errors $(\mathbf{67.8\%})$ was recorded in the 'additive' compare word problems. The reason for this may be due to fact that these word problems contained key words 'more than' and 'older than' which according MacGregor and Stacy (1993), simulate semantic features of a situation but not the mathematical form. Hence, being based on comparison rather than equality, most learners failed to form correct algebraic equations. The highest frequency of schematic errors $(\mathbf{32.2\%})$ was recorded in same category of word problems where learners wrote expressions (i.e. totals), inequalities rather than equations and meaningless expressions. This high percentage of schematic errors further supports the above observation that the inclusion of words such as 'more than' and 'older than' in word problems do not suggest forming mathematical equations (see Table 8: Section 4.4).

c) Despite inadequate linguistic skills in decoding meanings of words and phrases from the verbal written mathematical language in word problems, all learners interviewed never attempted to test whether their written equations made sense based on the information stated in the questions. In addition, they never used any problem solving strategy for the purposes of gaining more understanding such as recording the givens, the unknowns, manipulating symbols, and making use of trial and error methods to test the validity of their equations. This substantially contributed to their challenges in translatingfrom words to algebraic representations. However, one important and encouraging observation I made during interview sessions was that, given the appropriate guidance and instruction in word problem solving, the majority of learners can perform highly in tasks that require translating from word to algebraic representations. This was evident from the way they (learners) promptly realised their mistakes and corrected themselves thereafter.

4.7 Conclusion

In this chapter, a detailed analysis and discussion of errors on learners' preferred answer choices in the multiple-choice questions was presented. Sources of errors elicited from the interview data were documented based on what learners said. The error analysis done on learner responses to open-ended questions was discussed and results presented in Tables 6, 8, and 10. A summary of the subcategories of errors and their possible sources from learners' written work was presented at the end of each section in the form of a summary table. Finally, results from error analysis in Sections 4.3, 4.4, and 4.5 were collated (i.e. Table 11), analysed, and a summary of findings presented**.** The next chapter presents a conclusion to the research study.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The overarching objective of this study was to explore and document types of errors and their possible sources when learners translate from the verbal written mathematical language in word problems to algebraic representations. In order to get a global picture of the types of errors in learners' written work, I classified the selected word problems into four categories namely: Category A (multiple-choice questions), Category B1 (de-contextualised word problems), Category B2 ('additive' compare word problems), and Category B3 ('multiplicative' compare word problems). In an attempt to gain access into learners' mathematical thinking given the errors reflected in their incorrectly written algebraic representations, interviews and the test were central in achieving that end. In light of the above discussion, the study explored the following research questions:

- What type of errors do learners make when translating from word problems to linear algebraic representations?
- What are possible sources of these identified errors?

The next section presents a summary of the main findings that emerged from this research study after data from both the written test and interviews were analysed for errors.

5.2 Summary of Main Findings

Learners' performance on both the multiple-choice and the open-ended questions provided me with an insight into errors that learners committed as they translated from verbal mathematical language to algebraic representations. Error analysis in this study mainly focussed on the mathematical structure of the algebraic representations (i.e. equations and inequality equations) that learners wrote and these representations reflected their understanding of the language used in the word problems. In what follows, I engage with the key findings that emerged from the study.

a) **Key Finding 1**: **Errors andtheRole of Semantic Knowledge in Problem Solving**

Out of the three categories of errors (syntactic, semantic, and schematic) investigated, results from the data analysis showed that semantic errors were the most prevalent in learners' written algebraic representations. Semantic errors according to Clement (1982) are a reflection of learner misunderstandings on different language components that constitute mathematical language. Firstly, data analysis from interviews showed that, the major source of semantic errors were due

to learners' lack of vocabulary knowledge (Miller & Smith, 1994; Cuevas, 1984) and their failure to activate their prior knowledge on words and phrases appearing in the word problems (Allington, 2001; Carter & Dean, 2006). Examples of such words in this study included 'product', 'more than', 'increased by' and relational phrases such as '8 times as many' and '6 times greater than'. Misunderstanding the mathematical meaning of these terms as data analysis from interviews showed, resulted in learners writing algebraic representations that did not match the given information in the word problems. Research in word problem solving has also shown that, being unfamiliar with the mathematical meaning of one key term is all it takes a learner to miss the relevant important information in order to come up with a correct algebraic equation (Hofstetter, 2003). For instance, in multiple-choice questions, one learner understood the word 'product' to mean 'total' and in de-contextualised word problems, another learner thought 'increased by' meant to 'multiply'. This brings to fore the importance of semantic knowledge, which is critical in discerning and identifying the meaning of symbols and words appearing in word problems (Morris, 1938; Duru & Koklu, 2011; Clement, 1982).

Secondly, apart from the lack of vocabulary knowledge used in the word problem, data from interviews also showed that learners committed semantic errors due to their inexperience in interacting with expository text structures that are designed to explain or inform (Meyer, Brandt&Bluth, 1980). This was conspicuous especially in de-contextualised word problems (i.e. questions 9 and 13) where they struggled to cope with the grammar and syntax used to phrase these word problems. Results from data analysis indicated that de-contextualised word problems recorded the highest number of all three categories of errors (i.e. syntactic, semantic and schematic). Of concern was learners' lack of syntax awareness (MacGregor & Price, 1999), which enables one to make correct judgements on how syntactic structures of verbal written mathematical language is used to understand how symbols should be arranged in formulating equations particularly for this category of word problems. On this aspect, errors were committed as learners did not approach or read the word problems with any particular plan of action. Of more concern was their disregard in taking cognisant of the contextual meaning of words such as 'and', 'of' and 'more than' as used in **ME**. For example, words such as 'and' and 'of' in Mathematics provide important clues as to which terms are being referred to as well as the appropriate mathematical operation to be undertaken in forming the algebraic representations.

Thirdly, other contributing factor leading learners to commit semantic errors were their inability to monitor their thinking processes (metacognition) as well as reflecting on the appropriateness of their selected answer choices or written algebraic representations for all categories of word problems (Schoenfeld, 1985; Flavell, 1992). Data from interviews showed that none of the

learners interviewed tested any of their written equations to check whether they made sense. In cases where learners could not identify the correct algebraic representation (i.e. multiple-choice section), they got frustrated and ended up guessing answers without proper reasoning.

Lastly, data from interviews further indicated that, learners failed to make inferences to the meaning of all the words appearing in word problem but instead, they picked one word or phrase they thought was key in formulating algebraic equations. For example, in the multiple-choice section one learner misinterpreted the key phrase '...one desk is used by 2 students' to suggest two students were sharing the desk (**see excerpt 4: line 4**). This strategy according to Wagner and Parker (1993), can be a useful problem-solving heuristic, but may end up encouraging over-reliance on a direct, rather than analytical mode for translating word problems into equations. Interaction with learners during interviews provided evidence that learners were using this strategy unsuccessfully. All factors mentioned above largely contributed to the highest frequency of semantic errors coded in learners' incorrectly written algebraic representations.

b) **Key Finding 2**: **Role of Symbol Awareness in Solving Word problems**

Another key finding emerging from the study was that learners lacked symbol awareness (MacGregor & Price, 1999). In other words, they were not conceptually ready to engage with concepts and the language used in algebra particularly for word problems in the multiple-choice section. On this aspect, learners lacked adequate knowledge in conceptualising thatletters can be treated as symbols representing unknown quantities. For example, learners could not understand in quantitative terms the notion of $'n - 1'$ as the number of desks remainingafter one desk used by one student was removed from the total of $'n'$ desks. In addition, they struggled to deal with $'n - 1'$ as a single number when asked to form an expression and then simplify it to get the total number of students in the class.

c**) Key Finding 3**: **Critical Stages in Word Problem Solving**

Key findings (a), and (b) above in conjunction with data analysis on interviews provide substantial evidence that the majority of errors learners committed occurred during the problem translation process. The problem translation process includes activities such as reading the text for understanding where the problem solver constructs an individual mental representation for each proposition stated in the word problem (Mayer, 1999). These findings resonate with those from Newman's (1977); Case (1978); Clements (1980) research work done on errors related to word problem solving. The above researchers found that about 50% of all errors committed by learners occurred in the problem translation stage.

Another key finding that emerged from the study was that learners showed lack of problem solving skills such as using sketch diagrams to help them build appropriate mental representations of the situation described in word problems (Mayer, 1999). They (learners) admitted that they did not make use of trial and error methods to check the functionality of their written equations for both 'additive' and 'multiplicative' compare word problems. As Mayer's model of problem solving suggests, these activities are critical in the problem integration process where the problem solver integrates the information across sentences. Unfortunately, all learners interviewed in these categories of word problems indicated that they were unaware of the importance of undertaking some of these word problem solving strategies during the translation process.

d) **Key Finding 4**: **Syntactic Translation**

For 'additive' compare word problems in which the use of word order matching strategy (syntactic translation) would yield a correct algebraic equation, none of the participants including those who wrote the correct equation used the strategy. This resonates with findings from MacGregor & Stacy (1993) who reported that in questions in which syntactic translation strategy would yield a correct answer, learners still wrote reversed equations. In addition, learners' written algebraic equations for 'additive' compare word problems revealed that the reversal error (often associated with syntactic translation) occurs to a wider class of equations than was previously considered. The present study has shown that it occurs not only in multiplicative items but also in additive items, relating to very familiar situations (i.e. in 'additive' compare word problem).

5.3 Recommendations

One important observation noted from this study was the learners' inability to come to terms with language of Mathematics, which consists of technical terms, unfamiliar uses of syntax and semantics that need careful reading and comprehension skills. Learners' inexperience in dealing decisively with these various mathematical discourse features raises awareness of the need to revisit and engage in classroom pedagogical practices that provide opportunities to learners to master the language of Mathematics. As the findings from the study has revealed, for word problems that clearly have a linguistic component, the application of mathematical knowledge and skills is substantially hampered or even constrained by whether learners can effectively undertake requisite linguistic processes such as monitoring their understanding as they read mathematical texts. Therefore, to harness linguistic challenges that contribute to errors that

learners commit when translating from word to algebraic representations, the following pedagogical practices are suggested.

Firstly, teachers should bring awareness to learners that mathematical language has its own register and that some everyday words used in this register have specific meanings in mathematics. This can be achieved through engaging learners in Mathematics lessons where they should be asked to write contextual definitions of all new words that appear in every topic they do. Mathematics teachers should place more emphasis on the use of mathematical terms such as 'more than', 'less than', 'equal to', 'sum or difference', and ' greater than' in order to improve student's deficiencies on how these words are interpreted in the process of forming equations. The corresponding symbols of the above mentioned terms should be presented to the learners as well. In this way, teachers would be encouraging students to start recognising and mastering such words including understanding their context of use in mathematical texts. Secondly, vocabulary tests may be administered regularly as part of assessments during the course of the learning period as this may help learners to conceptualise mathematical ideas, which are explained using specialised mathematical language. For instance, the present study has shown through the analysis of test and interview data that vocabulary knowledge has a role to play in the successful translation of word problems into their equivalent symbolic representations.

Thirdly, Mathematics educators should engage learners during discussions on word problem solving wherein they model a wide array of strategies important for learners to make sense of the text at hand. Such strategies among others may include teaching learners to be aware of how grammatical structures and word order in mathematical texts provide important clues that lead to correct formulation of equations. For example, when students are instructed to translate word for word from the text to the algebraic equation, it must be emphasized to them that they must 'flip it' (i.e. terms in the equation) when using 'less than'or 'more than'(Capraro & Joffrion, 2006). For instance, when asked to represent '5 less than x' in symbols, they must realise that it should be 'flipped' and written as $'x - 5'$. However, the approach must be practised with a variety of word problems so that learners understand the idea behind the technique.

Learners must be encouraged to actively monitor their understanding during the problem solving and to undertake multiple readings in parts that may appear confusing. This can be achieved when teachers take time to discuss different type of word problems where learners are asked to interpret and explain the meaning of each sentence in word problems. Learners should also be encouraged to test their formulated equations using numerical values immediately after writing them to check whether they make sense. In addition, word problems given out to students should

be attended to by mathematics teachers through immediate scoring, identification of particular errors committed and their possible sources, and giving adequate corrections to students in form of feedback. The present study has also shown that learners eventually noted mistakes in their incorrectly written algebraic representations and went on to correct themselves during interview sessions.

Finally, English and Mathematics teachers should work together to help students understand words that are found in both **OE** and **ME** by way of making references to their specific meanings in both discourses. In addition, Mathematics teachers should set up teaching environments in which conventional mathematical language migrates from the teacher to the students. In such cases, it is likely that meanings that students construct may ultimately descend from those captured through the kind of language the teacher uses.

5.3.1 Future Research

Based on the findings from the present study which investigated learners' ability to translate from words to algebraic representations, it is imperative that learners had more challenges in interpreting the verbal written mathematical language used in the word problems. It can, therefore, be conjectured that inadequate comprehension of the language of the problem has a role to play in the successful transformation of the verbal mathematical statements into their equivalent symbolic representations. In light of the above, further research could be directed towards investigating the influence of English language as both a medium of instruction (LoLT) and that used in word problem solving. Research in word problem solving could also investigate errors that learners make when translating word to algebraic representations, for example, using learners who speak English as their first language and those who speak English as a second language to compare and contrast types of errors they both commit.

Having explored errors that learners committed, further research could be directed towards finding effective teaching strategies that can be adopted to reduce the occurrence of these errors. Any identification of errors is worthless unless we make suggestions to overcome them. For instance, data from interviews showed that learners quickly realised their faulty lines of thinking or understanding immediately after being asked to explain their responses. This observation, therefore, needs attention and further research to understand how learners can be helped to overcome such learning behaviour. The study can also be replicated using appropriate word problems to learners at primary school levels to understand challenges that might give directions in improving teaching methods at secondary school levels. The main purpose of these

investigations will, therefore, be to devise, improve and suggest teaching strategies and materials that can improve the success rate in word problem solving.

5.3.2 Reflections

One observation coming out of this study was learners' immediate realisations of their errors during and after interview sessions. It was interesting to note that after engaging them to defend their selected or written algebraic representations they subsequently applied their new, correct understandings in checking and fixing their previous errors right on the spot. The interviews helped support or refute my initial interpretations of the data as well as provided unanticipated insights into student thinking that were not evident from students' written or selected responses. For instance, learners' incorrect written algebraic representations initially suggested that they were completely overwhelmed by the linguistic demands of the word problem. However, upon interviewing them, I was surprised that they knew most of the mathematical facts but were unable to activate their prior knowledge on them. This was the case in all categories of word problem selected in this study. In addition, the immediate realisation of their mistakes during interviews supports the view that if learners are given an opportunity to "think aloud" (Newman, 1977) as they solve word problems, their understanding may be greatly improved and their chances of making errors minimised.

It was also important to realise that learners bring different views and interpretations on certain concepts that might appear obvious especially when these concepts are being described using verbal written mathematical language. For instance, one would be led to think that word problems that are contextual in nature are easier to understand for example the student-professor problem (Clement, 1982; Clement, Lochhead, & Monk, 1981). However, as the interview data revealed, that was not the case. This observation then brings the realisation to Mathematics teachers that simply using mathematical terminology during instruction without checking whether learners understand them can be detrimental to the goals for which Mathematics learning is intended to achieve. In this study, I found the use of interviews to be effective as they helped me to understand learners' mathematical thinking processes as I investigated errors in both their incorrect selected responses and written algebraic representations.

5.3.3 Limitations of the Study

Several limitations may have influenced the outcomes and interpretations of findings in this study. Of primary concern was the small sample size of five learners that I interviewed in all four categories of word problems. I would have wished to interview more learners especially those whose incorrectly written algebraic representations were categorised as reflecting schematic

errors to gain deeper understanding of their level of thinking. However, due to the tight schedule of school interventions programmes throughout the weeks during data collection, I could not find enough time to interview more learners. Finally, because of my inability to speak the learners' home languages, and the fact that all interviews were done using English only, it was difficult to capture the entire learners' mathematical thinking processes and misunderstandings.

5.4 Conclusion

In this chapter, I have articulated a number of issues including the research questions, main findings, recommendations inferred from observations made in the study, my own reflections, and recommendations for further research. More importantly to note, the present study has attempted to add knowledge related to learners' challenges in translating word to linear algebraic representations to the already growing body of literature in this area of research. In conclusion, the following quote summarises salient characteristics of mathematical language that should draw attention to Mathematics research communities to prioritise research in this critical area of word problem solving:

> *'Mathematics education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language' (Durkin & Shire, 1991, p. 3).*

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APPENDIX A: **Test Instrument**

Test /August 2013 Grade 11 Time: 55 minutes [……]

Participant's Profile

Please complete the information below using a pencil
Gender:…… Grade:……. Age.....
School:.....

Instructions:

1. Carefully read all the instructions before answering the question paper.

2. Section A consists of 7 Multiple Choice Questions. You are required to circle the correct answer.

3. Section B consists of Open ended questions from 8 to 16. Write the required response in the spaces provided.

4. Answer all QUESTIONS in both SECTIONS.

Section A: Multi-choice Questions.

1. A number *n*is the product of half of itself and 6 less than itself. Therefore the value of *n* is given by

A) n= $(n-6)+\frac{n}{2}$ B) n= $(n-6).\frac{n}{2}$ C) n= $\frac{(n-6)}{2}$ D) $n=\frac{n}{2}-6$

2. There are *n* Girl Scouts marching in a parade. There are 6 girls in each row. Which expression could you use to find out how many rows of Girl Scouts are marching in the parade?

A) $n-6$ B) $6n$ C) $6n$ D) $\frac{n}{6}$

3. There are '*n*' desks in our classroom. Of these desks, 1 is used by only 1 student and each of the remaining is used by 2 students. Which of the following appropriately represents the number of students in our classroom?

A) $2n-1$ B) $\frac{n}{2}-1$ C) 2n + 1 D) $\frac{n}{2}+1$

4. Which of the following word problems corresponds to the equation '$\frac{x}{2}+5=24$'?

A) 2 times 5 less than a number is equal to19. B) 5 more than 2 times a certain number is equal to19

C) 5 more than half of a certain number is equal to 24. D) 5 less than half of a certain number equals 24

5. Which of the following algebraic equations appropriately represents the situation?

'Tebogo has 4 times as many marbles as Kaya and they have 40 marbles in total.'?

$A)\ 4x=40 \quad B)\ x+4=40 \quad C)\ x+4x=40 \quad D)\ 4x+40=x$

6. Which of the following representations best describes the statement 'the product of 3 and 4 more than certain numbers are greater than 7'?

$A)\ 3x+4=7 \quad B)\ 3(x+4)>7 \quad C)\ 4x+3<7 \quad D)\ 4(x+3)\geq 7$

7. Which of the followings pair is a correct algebraic representation for this statement below?

'When 4 times the first number is added to 3 less than the second number, 21 is obtained. 2 more than two times the first num**ber is equal to 51 more than** 5 times the second number'.

A) $4x-(y-3)=21$ B) $4x+(y-3)=21$ C) $4x+(y-3)=21$ D) $4x+(y-3)=21\ (2x+2).5=51(2x+2)-5y=51(2x-2)-5y=51(2x+2)+5y=51$

Section B: Open-Ended Questions

Instructions: Read each question carefully and write the appropriate answer in the space provided.

8. Write down the algebraic equation corresponding to the statement '4 more than 3 times of a number is 16'.

Answer:……

9. Write down a symbolic representation of the statement 'Half of 5 less than all numbers are less than or equal to 6'.

Answer:……

10. Write an appropriate verbal statement *(IN WORDS)* for the algebraic statement ' $\frac{x}{2} - 4 \geq 7$'

Answer:……

11. Tachi is exactly one year older than Bill. Let *T* stand for Tachi's age and *B* stand for Bill's age.
Write an equation to compare Tachi's age to Bill's age?

……

12. Agnes's money is R2 more than Thabo's money. Let A represents the amount of Agnes's money and T represents the amount of Thabo's money. Show the relationship between amounts of Agnes's money and Thabo's money with mathematical symbols.

Answer:……

13. If the product of 2 and 4 is subtracted from twice a certain number and then increased by 4, the result is 22. Write an algebraic equation that corresponds to the above statement.

Answer:……

14. In a certain school, there are eight times as many boys as there are girls. Write an algebraic equation that represents the above situation.

Answer:……

15. Write an algebraic equation using the variables *S* and *P* that will represent the following situation: "In a certain university, the number of students is 6 times greater than the number of professors". Use **S** for the number of students, and **P** for the number of professors.

Answer:……

16 a. From question 1 up to 15, write *WORDS, PHRASES OR SYMBOLS* that you feel were difficult to understand. Indicate the question number from which these words, phrases or symbols are found.

……
b. What do you think is important in solving word problems?

……
c. Do you enjoy solving word problems? If yes give a reason why you enjoy solving them. If no, give a reason why you do not enjoy solving them.

.....

END

APPENDIX B: Categories of Questions selected for analysis

Category	Nature of translation	Question
A Multiple-choice questions (*Learners selected a correct response from 4 possible choices*)	From verbal written language to symbolic language (i.e. algebraic equations)	**1**. *A number **n**is the product of half of itself and 6 less than itself.* Therefore the value of ***n***is given by A) **n**= $(n-6) + 1/2$ B) $\boldsymbol{n} = (n-6).n/2$(C) $\boldsymbol{n} = (n-6)/2$ D**)** $n = n/2 - 6$ **3**. *There are 'n' desks in our classroom. Of these desks, 1 is used by only 1 student and each of the remaining is used by 2 students.* Which of the following appropriately represents the number of students in our classroom? $\boldsymbol{A}) 2n-1$ $\boldsymbol{B}) n/2-1$ $\boldsymbol{C}) 2n+1$ $\boldsymbol{D}) n/2+1$ **7**. When 4 times the first number is added to 3 less than the second number, 21 is obtained. 2 more than two times the first number is equal to 51 more than the 5 times the second number'. $\boldsymbol{A}) 4x-(y-3) = 21$ $\boldsymbol{B}) 4x+(y-3) = 21$ $\boldsymbol{C}) 4x+(y-3) = 21$ $\boldsymbol{D}) 4x+(y-3) = 21$ $(2x+2).5 = 51$ $(2x+2)-5y = 51$ $(2x-2)-5y = 51$ $(2x+2)+5y = 51$
B	**Open- ended questions**	
B1 Algebraic word problems *(de-contextualised)*	From verbal written language to symbolic language (i.e. algebraic equations)	**9.** Write down a symbolic representation of the statement '*Half of 5 less than all numbers are less than or equal to 6*' **13**. *If the product of 2 and 4 is subtracted from twice a certain number and then increased by 4, the result is 22*. Write an algebraic equation that corresponds to the above statement. (**12.5%)**
B2 Additive Compare word problems	From verbal written language to symbolic language (i.e. algebraic equations).	**11**. *Tachi is exactly one year older than Bill. Let T stand for Tachi's age and B stand for Bill's age*. Write an equation to compare Tachi's age to Bill's age **(34.2%)** **12.** *Agnes's money is R2 more than Thabo's money. Let **A** represents the amount of Agnes's money and **T** represents the amount of Thabo's money.* Show the relationship between amounts of Agnes's money and Thabo's money with mathematical symbols. **(15%)**
B3 Multiplicative compare word problems	From verbal written language to symbolic language (i.e. algebraic equations).	**14**. *In a certain school, there are eight times as many boys as there are girls.* Write an algebraic equation that represents the above situation. **(32.5%)** **15**. Write an algebraic equation using the variables ***S*** and ***P***that will represent the following situation: "*In a certain university, there are six times as many students as they are professors"* Use S for the number of students, and P for the number of professors. (**20%)**

APPENDIX C: Content analysis of each selected question item

Questions category	Questions	Relevant mathematical knowledge and understanding required
A	**1**	Understanding the words: 'product'; 'half of itself' and '6 less than itself'
	3	The notion of 'n desks', 'I desk is used by only I student', and 'each remaining is used by 2 students'. **Two methods**: 1) its $2(n-1)+1=2n-2+1=(2n-1)$ students OR 2 per desk translate to 2n. But 1 student occupies one desk meaning its $(2n-1)$students in class
	7	Correct interpretation of phrases: '4 times the first number……3 less than the second number, 21 is obtained'. '2 more than two times the 1st number …….51 more than 5 times the 2nd number' into meaningful algebraic equations.
B1	9	'half of'…implying 'multiply by $(\frac{1}{2})$ or divide by 2'…'5 less than all numbers' $\equiv (x-5)$. The phrase '***all numbers***' implies the use of inequality symbol $(\leq)$.
	13	-Correct interpretation of terms/phrases: 'product', 'is subtracted from', 'twice a certain number', 'increased by 4', 'result is 24'.
B2	11	Transposing variables incorrectly or *reversing variables* i.e. T – 1 = B or $B-1=T$. Using constants when attaching age to Tachi and Bill (arithmetic stage of operation) as opposed to operating in algebraic mode. Direct translation abilities by learners under scrutiny i.e. '…is exactly one year older than $\equiv (+1)$. Checking the sensibility of the written algebraic equation.
	12	Transposing variables incorrectly or *reversing variables* i.e. $A+2=T$ or $A=T-2$ and giving responses in form of *additive totals* i.e. $x=A+2+T$. Understanding the concept '***R2 more than'*** to imply add R2 to Thabo's money.
B3	14	*Additive totals* as responses i.e. $x=b+8g$ or total $=b+8g$. Misinterpretation of letters ***b*** and ***g*** as labels (e.g. seen as 1 boy and 8 girls) rather than standing for the *number* of boys and *number* of girls. Reversal error under the spotlight.
	15	*Additive totals* as responses i.e. $x=S+6P$ or total $=S+6P$ or total $=6S+P$. Misinterpretation of letters ***S*** and ***P*** as labels (e.g. seen as 1 student and 6 professors) rather than standing for the *number* of students and *number* of professors. Reversal error under the spotlight.

Key: $(\equiv)$ means 'equivalent to'

APPENDIXD: Student response categories for selected word problems.

Question 9	Question 11	Question 12	Question 13	Question 14	Question 15
1) $x-\frac{5}{2} = 6$	**1)** T = B + 1*	**1)** T = A + 2	**1)** 2(2**x** - 4) = 22	**1)** b = 8g*	**1)** S = 6P*
2) $x- \frac{5}{2} \leq 6$	**2)** B = T -1*	**2)** A + 2 = T	**2)** 2(4 - 2**x**) + 4 = 22	**2)** 8G = B*	**2)** S = 6**x**
3) $\frac{5}{2} - x \leq 6$	**3)** B + 1= T*	**3)** A = T + 2*	**3)** 2(**x** - 4) = 22	**3)** Boys = 8 girls*	**3)** 6S = P
4) $\frac{5}{2} < x \leq 6$	**4)** T – B =1*	**4)** A – 2 = T*	**4)** 2**x**(2 + 4) + 4 = 22	**4)** Let **x** be girls:8**x** = b*	**4)** S = Ps
5) $\frac{5}{2} < x = 6$	**5)** B = T + 1	**5)** T + 2 = A*	**5)** 2(2 - 2**x**) + 4 = 22	**5)** 8B = G	**5)** S = 6**x** + P
6) $\frac{5}{2} < x \geq 6$	**6)** B + **x**= T	**6)** 2**x**+ T= A	**6)** 2(2 – **x** + 2) = 22	**6)** x = 8g + b	**6)** u = 6s + p
7) $\frac{1}{5} > x \leq 6$	**7)** T + 2B = 2	**7)** A = T – 2	**7)** 2**x** – 8 + 32 = 22	**7)** u = 8b + g	**7)** 6 = S + 6P
8) $\frac{5}{2} x \leq 6$	**8)** T + B = T_1	**8)** A + 2 = T	**8)** 2**x** + 8 – 32 = -22	**8)** b = 8b + g	**8)** **x** = 6 > **x**
9) $\frac{5}{2} \geq 6$	**9)** T = 1 –B	**9)** A = 2**x**	**9)** 2**x** – 6 + 4 = 22	**9)** total = 8b +g	**9)** 6S + P
10) $\frac{5}{x} - x \leq 6$	**10)** 1T = B	**10)** 2A = T	**10)** 2**x** - (2 **x** 4) + 4 = 22	**10)** **x** = 8**x** + **x**	**10)** S(6P + P)
11) $\frac{x}{2} - 5 \leq 6$	**11)** T = 1B	**11)** 2T = A	**11)** 2**x** + 4 – 2 + 4 = 22	**11)** b = 8**x** + b	**11)** 6S > P
12) $\frac{x}{2} - 5 = 6$	**12)** $T_1 = B_2$	**12)** A > T	**12)** 2**x** - 2 - 4 + 4 = 22	**12)** **y** = 8**x** – **x**	**12)** S6 < P
13) $\frac{x}{2} 5 < x \leq 6$	**13)** 2T = B	**13)** 2A > T	**13)** 8 - 2**x** + 4 = 22	**13)** **x** = 8**x**	**13)** S6 > P
14) $\frac{x}{5} \leq 6$	**14)** **x**+1= **x**	**14)** A < T	**14)** (4 **x** 2) – 2 **x** 4 = 22	**14)** gb = 8g + b	**14)** 6S ≥ P
15) $\frac{x}{5} - x \geq 6$	**15)** $T_X \neq B$	**15)** $A_2 < T_1$	**15)** (2 **x** 4) – **x**+ 4 = 22	**15)** 8 = 8b + g	**15)** 6S < P
16) $\frac{1}{2}.5 - x \leq 6$	**16)** T > B	**16)** $A_{2x} > T_x$	**16)** (2 **x** 4) - 2(13) + 4 = 22	**16)** 8 = B + 8G	**16)** S > 6P
17) $\frac{1}{2}.5 < x \geq 6$	**17)** T > B = 1	**17)** 2A > 2 >T	**17)** (2 + 4) - 2**x**+ 4 = 22	**17)** 8**x** + **x**	**17)** S ≥ 6P
18) $\frac{x-5}{2} \leq 6$ *	**18)** T > 1 > B	**18)** A + 2T	**18)** 2 - 4**x**+ 4 = 22	**18)** 8g + b	**18)** **x** + 6S ≥P
	19) T + **x** - B	**19)** 2A + T	**19)** 4**x**+ 4 – 2 – 22 = 4**x**	**19)** $\frac{x}{8}$ – G	**19)** 6S(**x** > P)
		20) 2**x**+ **x**	**20)** 4 – 2 – **x**+ 4 = 22	**20)** 8(**x** + g)	**20)** $\frac{x}{5}$ > P
		21) $\frac{2A}{T}$	**21)** 22 = **x** - 2(2 + 4) + 4	**21)** 8B + 8G	
		22) A has 2R and > T	**22)** 2 + 4 – 4 - 2**x**+ 4 = 22	**22)** **x** + 8	
			23) 22 + 4 + 4 + 2 + 2 = 32	**23)** 8B ≠ G	
			24) 2(4 - 4) = 22	**24)** 8B > G	
			25) (16 **x** 2) - (2+4) – 4 = 22	**25)** 8**x** > **x**	
			26) 24 – 6 + 4 = 22		

APPENDIX E: Distribution of responses to multiple-choice questions.

Response	Q 1 (%)	Q 3 (%)	Q 7 (%)
A	5	20*	12,5
B	20*	20	15*
C	17,5	55	2,5
D	57,5	2,5	67,5
No responses	0	2,5	2,5

Note: *Students giving correct responses.

APPENDIX F: Expected responses and (%) of learners with correct responses.

Question	Expected response*	Number of learners with correct responses	%
9	½(x - 5) ≤ 6 or (x-5)/2 ≤ 6	0	0
11	T = 1 + B or T - 1 = B	13 (0ut of 38 learners)	34,2
12	A = R2 + T or A - R2 = T	6	15
13	2x-8+4=22 or 2x-(2 x 4)+4=22	5	12,5
14	b = 8g or 8g = b	13	32.5
15	S = 6P or 6P = S	8	20

APPENDIX G:(Interview excerpts 1 - 8)

Interview transcripts for learners

KEY

$L_{1:}$ Learner 1; R: Researcher; (...) - learner taking time thinking

L_1-Retha; **L_2**- Steve; **L_4**- Tanatswa; **L_5**- Tsidi; **L_{20}**- Thabo*(all pseudo names***)**

Multiple-choice Questions

Question 1: *A number n is the product of half of itself and 6 less than itself. Therefore, the value of n is given by,*

A) $n = (n - 6) + \frac{n}{2}$ ***B)*** $n = (n - 6).\frac{n}{2}$ ***C)*** $n = \frac{(n-6)}{2}$ ***D)*** $n = \frac{n}{2} - 6$

L_1 (Retha) - Excerpt 1

1- R: Okay, from reading the question, what do you think are key words?

2 - L_1: Its 'product of half of itself' and '6 less than itself'.

3- R: Then I see you chose **D**, why did you choose **D**?

4- L_1: I read that a number 'n' is the product of half itself and I multiplied 'n' by half to get$\frac{n.}{2}$

5- R: Which part of the statement is telling to divide n by 2?

6- L_1: It's half of itself Sir and itself must be the $'n'$ being referred to.

7- R: So did you use the word 'product' in thinking what you were supposed to do?

8 - L_1: No, I didn't use the word product...I mean its meaning.

9 - R: Do you understand the mathematical meaning of the word 'product'?

10 - L_1: Um...I can't remember sir, but we were taught about the word I think in Grade 9

11 - R: Can you identify words used in the question that are also associated with multiplying?

12 - L_1: I don't know sir,

13 - R: Okay, the word 'sum' in maths means addition, so can you now remember what product means.

14 - L_1: Oh... I remember now Sir...product must mean 'times'

15 - R: Okay, can you read again and identify the correct answer.

16 - L_1: Um...it must be **B**, I can see it now.

17 - R: So can you reflect on what you should have done.

18 - L_1: I should have interpreted words 'product', 'half of itself' and 'six less than itself ' correctly.

19 - R: Okay, thank you.

L_2 (Steve)- Excerpt 2

1 - R: Can you read the question and tell me the answer you chose and why?

2 - L_2: Yes Sir, I chose **D** because I saw the part, which said 'half of itself', and I said its $\frac{n}{2}$ since its half of this unknown number. I also saw '6 *less than itself*' and I subtracted it from the half, which is multiplying n.

3 - R: Did you understand that part which said 'and 6 less than itself'?

4 - L_2: No Sir…I didn't understand that part well especially, what the second 'itself ' is referring to.

5 - R: Okay, do you understand what the word 'product' meant in the first part of the question?

6 - L_2: No, I do not understand what it means but I read it I think I ignored it

7 - R: Okay, what is this 'half of' referring to?

8 - L_2: To an unknown number $'n'$ over two,

9 - R: And, what is this 'itself' referring to?

10 - L_2: A number 'n', which is unknown

11 - R: In maths, the word sum means addition, so can you remember what the word 'product' means?
12 - L_2: Yeah...it means you are multiplying, is it?
13 - R: Yes, that's true. Now, read again and tell me which quantities are multiplying each other.
14 - L_2: Um...No sir, the answer is **B** because '6 less than itself ' means $'n - 6'$ and it's multiplying $\frac{'n'}{2}$.
15 - R: So what can you say confused you in this statement?
16 - L_2: The word 'itself' appearing twice did confuse me so much. I ignored the word '*product*' and the meaning of this word made me to select **D** where we don't have anything multiplying each other.
17 - R: Do you now understand what the word 'product', 'itself', and '6 less than itself' mean now?
18 - L_2: Yes sir, I now remember what the word product mean and the correct answer must be **B**.
19 - R: Thank you very much for your time.

L_4 (Tanatswa)- Excerpt 3

1 - R: How did you understand this first question? Can you explain?
2 - L_4: I chose D as my answer Sir. Yes sir, n is the product, which is equal to the sum of half of itself.
3 - R: Is it the sum of or the product of the same thing?
4 - L_4: Um... It's like n is the total of half of itself, then I changed product to total so I could understand and that's why I chose $\frac{n}{2}$ minus 6.
5 - R: So you changed product to total. By the way, do you understand what the word 'product' means?
6 - L_4: Yeah...I don't understand this word product and I just thought it should mean total.
7 - R: So what did you understand with this word 'itself'? What is it referring to?
8 - L_4: It's referring to '$n - 6$'.
9 - R: Read again, does the word 'itself refer to '$n - 6$' or to 'n'?
10 - L_4: Let me see…um...no, it is referring to 'n'.
11 - R: Can you say what your actual problem in this question was?
12 - L_4: The word 'itself' sir, and the word 'product'. The word itself confused me because it appeared twice and I thought it wasn't the 'n' being referred to in the last part of the statement that read '6 less than itself'.
13 - R: Have you ever met the word 'product' up to this point?
14 - L_4: No, I think I have forgotten it, if I remember I met this word in grade 8 or 9.
15 - R: So can you go word by word and try to get the answer?
16 - L_4: Since product means multiply, and '6 less than itself ' times 'half n' so the answer becomes **B**.
17 - R: Yes,that's right. Thank you.

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Question 3: *There are 'n' desks in our classroom. Of these desks, 1 is used by only 1 student and each of the remaining is used by 2 students. Which of the following appropriately represents the number of students in our classroom?*

A) $2n - 1$ **B)** $\frac{n}{2} - 1$ **C)** $2n + 1$ **D)** $\frac{n}{2} + 1$

L_1 (Retha)- Excerpt 4

1 - R: I can see you chose **D**. Did you use any strategy or did you write anything down before choosing your answer?
2 - L_1: No, I didn't write anything down but I looked at the answer that made more sense and I chose **B**
3 - R: Why did you choose '$\frac{n}{2} - 1$', can you explain?

4 - L_1: I said, one is used by only one student and the rest are used by two students, so I subtracted one from $\frac{n}{2}$ students. I subtracted one student from $\frac{n}{2}$ because each of the remaining desks is used by two students, there are sharing, so I divided n by 2.
5 - R: Read the question again and tell me what it wants?
6 - L_1: Oh...Sir, it's the students not the desks that we want, I realise now.
7 - R: Okay is it the students or the chairs that gives us the number of students?
8 - L_1: Yes Sir, we should add one student to whatever number of students remaining.
9 - R: Okay, if there are n desks, and I take away one desk, how many are remaining?
10 - L_1: There are '$n-1$' desks left.
11 - R: So, let's get the total number of students in the class now. Remember, there is one student who sits on one desk and two students use the other remaining ones. So can you write the correct statement in terms of n?
12 - L_1: I'm getting confused by the '$n-1$'desks. Okay, it must be '$1+2$ multiplied by the remaining desk. Sir, should I take '$n-1$'as one number? That is what is confusing me.
13 - R: If I knew 'n', could we have one number?
14 - L_1: Yes, so I multiply 2 by '$n-1$', so it's must be '$1+2(n-1)=1+2n-2=2n-1$. So the answer is A.
15 - R: So how can you reflect on what you think was your problem?
16 - L_1: So my problem is I should take the remaining desks '$n-1$'as one number. My problem again was the phrase 'of these desks, one is only used by one student'. I kept on forgetting what was remaining since the number of desks was not given as a number but a letter n.
17 - R: What else?
18 - L_1: Also the phrase, '*each of the remaining is used by two students*'. I then divided the 'n' by 2 thinking that two students were sharing these desks. Lastly, I subtracted one from $\frac{n}{2}$.
19 - R: Lastly, do you think you can get the correct answer without putting a diagram or a sketch down?
20 - L_1: From our discussion, I can't see one getting the correct answer without a sketch diagram and so you can't see a reasonable answer but you have to write things down.
21 - R: Okay, thank you very much.

L_2 (Steve) - Excerpt 5

1 - R: Can you show me the answer you chose and explain how you arrived at your answer?
2 - L_2: I chose **C**, sir.
3 - R: What do you understand by the statement '….*only one is used by one student*'?
4 - L_2: I think it's... one student uses this desk.
5 - R: So, let's say there are '*n desks*' in the classroom, and we are told that only one is used by one student, so how many desks are left in the classroom?
6 - L_2: Its n desks because there are unknown number of desks in the class, so it's difficult to know.
7 - R: Is it 'n' desks left? Let's say we had '10 desks' and we take one away. How many are left?
8 - L_2: There are nine left sir,
9 - R: So let's go back to the idea of 'n desk'. If we take one, how many are left?
10 - L_2: Um...is it '$n-1$' sir? Yeah, it's '$n-1$'desks I think.
11 - R: Can you tell me, what does the '$n-1$' represents in terms of desks in the classroom?
12 - L_2: Its total number of desks in the classroom and the one used by one student as well.
13 - R: Think about what you did in the example of the 10 desk we discussed earlier. What were the nine desks referring to?

14 - L_2: Oh, no sir, I know now. The '$n-1$' desks refers to those minus the one which is used by one learner.
15 - R: Okay, now let us go to the second part '…and 2 students' use each of the remaining desks. So how many students are going to sit on the remaining desks?
16 - L_2: It's '$2n-1$' students Sir.
17 - R: Why 'minus one'
18 - L_2: I'm subtracting the one, which the one is sitting alone.
19 - R: You said there are '$n-1$' desks and each one of them is used by two learners. Therefore, how many learners are going to sit on all the '$n-1$' desks? What do you do to '$n-1$' to get the total number of students sitting these desks?
20 - L_2: I multiply it by 2 to get $2n-1$. *(He writes down the expression $'2n-1'$)*
21 - R: Take note of what you are multiplying by 2. Is it the 'n' only or it is the whole part '$n-1$'?
22 - L_2: Oh, Sir, I must *put brackets* and then I get '$2n-2$'.
23 - R: Now we want to answer the question. Can you now write the total number of students in the class?
24 - L_2: Its '$2n-2-1$' students and we subtract the one student who sits alone.
25 - R: To get the total do we subtract or add one?
26 - L_2: Ah, we must add sir, not subtracting. So it's '$2n-2+1$' and we get... um...$2n-1$ students.
27 - R: So tell me, when you were answering this question, did you write anything down in order to understand the question like what we have just been doing?
28 - L_2: No sir, I just looked at the key words and I thought the total number of students in the class is '$2n+1$'. I didn't think much about the meaning of the parts of the question. Now I know in such questions, I have to look at parts of the questions and draw diagrams at times.
29 - R: Okay, thank you very much.

L_4 (Tanatswa) - Excerpt 6

1 - R: Now in this problem did you use any strategy in the process of selecting your answer?
2 - L_4: Yes, what I did is that, since one student uses one desk and two students use the remaining desks, I then divided n desks by 2 plus '1'to get the number of students in the class.
3 - R: Why are you dividing 'n' desks by 2 then?
4 - L_4: It is because two students remaining are sharing one desk.
5 - R: Okay, how many students sit on one desk?
6 - L_4: Two,
7 - R: Then on two desks?
8 - L_4: It's four,
9 - R: How did you get four? Are you dividing?
10 - L_4: No, I see now, I'm multiplying not dividing.
11 - R: Okay, if we take one desk from 'n' how many are left?
12 - L_4: There isn't left, no desk is left or I can say we don't know.
13 - R: There are 'n' desk remember, take one from n what are you left with?
14 - L_4: Okay, is it '$n-1$' desks.
15 - R: Let's count the students in the class now. We have two types of desks, one learner occupies one desk, and two learners occupy the rest. So what is the total number of students?
15 - L_4: So on the remaining desks, we have 2 multiplied '$n-1$' and we subtract one who sits one desk and we get '$2n-1$' students.
16 - R: How did you multiply, you multiplied n by 2 only and you left the '1'not multiplied. Why?
17 - L_4: Oh, sir I should put brackets by the way and it becomes '$1+2n-2$' = $'2n+1'$.

18 - R: Do your maths properly please, simplify it correctly.

19 - L_4: Um...it must be '$2n-1$', I see now.

20 - R: So what was your problem in this question?

21 - L_4: This statement '*only one desk is used by only one student'*, I could not get it well so I ended up confused because I didn't know how to subtract 'one' desk from an unknown number of desks. In fact I if I remember well, I ignored it.

22 - R: What else confused you?

23- L_4: The other part is the statement, which says 'the remaining is used by 2 students'. I thought I should divide n by 2 since there are sharing the desk so I chose answer **D**.

24 - R: So what have you leant from our discussion?

25 - L_4: When answering these problems, I think I should have used diagrams to represent each part of the problem statement in order to understand the whole problem. Just using key words doesn't help because in choosing **D**, I also saw '$+1$' and I thought it represents the one student who was sitting alone.

26 - R: Okay, good, thank you very much.

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Question 7: Which of the followings pair is a correct algebraic representation for this statement:'*When **4 times** the first number is added to **3 less than the second number**,* 21 *is obtained. **2 more than** two times the first number is equal to* 51***more than 5 times** the second number'.*

$A)\ 4x-(y-3)=21$ $B)\ 4x+(y-3)=21$ $C)\ 4x+(y-3)=21$ $D)\ 4x+(y-3)=21(2x+2).5=51$ $(2x+2)-5y=51$ $(2x-2)-5y=51$ $(2x+2)+5y=51$

L_1 (Retha) - Excerpt 7

1 - R: Let's look at the second statement 'two more than two times the first number is equal to 51 more than 5 times the second number' Can you write it down please?

2 - L_1: (*she writes)*'$2x+2=51+5y$'

3 - R: Now look at the set of answers and tell me which one is the correct answer? Look carefully.

4 - L_1: There is no correct answer given in this question.

5 - R: Is there anything you can do to the equation you have written so that it looks like any one of the given ones from the answer choices?

6 - L_1: Sir, I cannot see it...

7 - R: Look at the format that is the position of all terms used in all the four choices.

8 - L_1: Oh, I see Sir if I transpose the '$5y$' from this side (right) to this side (left) of the equal sign it is going to be $(-5y)$, so the answer is **B** not **D**.

9 - R: Yes, that's right. Therefore, what do you think was the problem in this question?

10 - L_1: Sir, the problem was that I didn't look carefully at the answer choices and the fact that '$5y$' was transposed to the left to become negative. When I could not find the answer I ended up choosing **D** when I saw $'+5'$.I think it was necessary to play around with terms transposing them. To be honest Sir, I rushed to answer without understanding the way in which terms were arranged in the question.

11 - R: Okay, thank you very much.

L_5 (Tsidi) - Excerpt 8

1 - R: Let's go to number 7. Can you do the first part of the question?

2 - L_5: (*He writes it correctly*) $4x+(y-3)=21$.

3 - R: Look at the second part of the statement '2 *more than two times the first number is equal to* 51 *more than* 5 *times the second number*'. Write it symbols.

4 - L_5: He writes '$51 + 5y$'
5 - R: Write the second part of the question in full now and choose which answer is correct from those given.
6 - L_5: He writes '$2 + 2x = 51 + 5y$'
7 - R: Now from the choices we have which one is the correct one.
8 - L_5: It's **B** (*pointing to the correct answer)*
9 - R: Okay, can you explain why **B** is the correct choice?
10 - L_5: Sir, if we transpose this negative $5y$ $(-5y)$ to this side, it becomes plus $5y$ $(+5y)$.
11 - R: Good, now can you reflect on our discussion in relation to your mistake.
12 - L_5: I looked for the second part on all equations to see if I could find $(\ldots. = 51 + 5y)$ to the right side of the equation, but I could not. I ended up choosing answer **D** because $5y$ was being added.
13 - R: Okay thank you very much for your time.
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APPENDIX H:(Interview excerpts 9 - 15)

De-contextualised word problems: Questions [9 & 13]

Question 9: *Write down a symbolic representation of the statement 'Half of 5 less than all numbers are less than or equal to 6'.*

L_1 (Retha)- Excerpt 9

1 - R: Can you read the question and write show me the answer that you wrote?

2 - L_1:*She reads the question* and writes '$x-\frac{5}{2} = 6$'. Um ….*'x'* is representing all numbers that we are talking about.

3 - R: So in mathematics, if I say '*half of*' what do you write or what does it mean?

4 - L_1: Sir, um… it means whatever number you have is being multiplied by *'½'*.

5 - R: So what does 'of' mean?

6 - L_1: It means you multiply

7 - R: Okay, so what is half going to multiply? Look at the statement after '*half of*'

8 - L_1: It's going to multiply 5 that is, ½ of 5 (*she writes* $'\frac{1}{2} \times 5'$

9 - R: Can you read the whole statement and end before it says '*less than or equal to* 6'?

10 - L_1: She reads "*half of*5 *less than all numbers*"

11 - R: Now, what do you understand by that? Can you write that statement in symbols?

12 - L_1: Um… let me think Sir.

13 - R: Okay, for example if I say 5 less than 10, what does that mean?

14 - L_1: She writes '$5 - 10$'

15 - R: Okay, I mean 5 less than 10 not 10 less than 5, get it right here.

16 - L_1: Oh... it means '$10 - 5$', which gives us 5.

17 - R: Okay, that is the correct way of speaking in mathematics we do not usually follow the order of the words as they appear. So, tell me, what does the word '*less*' mean in this statement?

18 - L_1: It means subtract, yeah...it means you are subtracting.

19 - R: Right, let's go back to our question now. So what is '*half*' going to multiply?

20 - L_1: "5 less than all numbers"

21 - R: Right, how do you represent '5 less than all numbers'?

22 - L_1:*(She reads again the statement.)* We are going to say '$5 - x$' (*She writes it*).

23 - R: Is it '$5 - x$' or '$x - 5$'? Remember what we discussed earlier.

24 - L_1: It's '$x - 5$'... yes Sir (*she writes it)*

25 - R: So write the whole statement in symbols and end before 'less than or equal to 6'.

26 - L_1: She writes '$\frac{1}{2}x - 5$'.

27 - R: Now, if you write it like that... check again, what half is multiplying?

28 - L_1: Oh... no, sir, I think we should put '$\frac{1}{2}$' outside the brackets.

29 - R: Now read the whole statement and then see whether it makes sense to you.

30 - L_1: (*She struggles to separate parts of the statement i.e., the part 'half of' and '5 less than all numbers' making it difficult for her to make sense of the symbolic statement). ...um let me think...*

31 - R: Okay, let's continue after '5 less than all numbers'. What symbol is represented by '*less than or equal to* 6'?

32 - L_1: I don't know how to call it, but it's like this sir. (*She writes the symbol* $\leq$*).*

33 - R: Now, in your answer you wrote ' = '. If you can remember, why did you write ' = ' sign?

34 - L_1: Um... sir, I saw the equal to 6 part and then I thought I should put ' = ' and I read the '*half of* 5 *less than all numbers are less than'* as one statement. That's why I ignored the last 'less than part' before the 'equal to 6' part.
35 - R: Now, tell me, say this symbol in words (i.e., ≤).
36 - L_1: It's less than or equal to symbol
37 - R: So, what name do we give to such symbols as ' ≤ ' in mathematics?
38 - L_1: Um...I have forgotten, the name cannot click sir.
39 - R: Have ever met the term '*inequality' symbols.*
40 - L_1: Oh…yes sir, I now remember, these are inequality sign.
41 - R: Now read the whole statement and write the complete symbolic statement.
42 - L_1:*(she writes ½* $(x- 5) \leq 6$*)*. I can reflect now on my mistake: I said, 'half of 5 ($\frac{5}{2}$) and less than all numbers, I wrote$\frac{5}{2}-x$. I had problems with reading the order of words correctly.
43 - R: Thank you!

L_2 (Steve) - Excerpt 10

1- **R:** Can you show me what you wrote on question 9 and explain why you wrote that equation?
2 - L_2: Um...what I did sir is that (*he writes on a paper*) I wrote $x-\frac{5}{2} \leq 6$ because the $'x'$ is the unknown... I don't know it and the 5 is the number that is being subtracted since it is negative and everything is less than or equal to 6.
3 - R: Okay, let's see if that makes sense mathematically. Right, can you read again the first part of the question? Let us concentrate on that part for now.
4 -L_2:(*He reads the first part again)*
5 - R: Okay, what do you understand by the phrase "half of" in mathematics?
6 - L_2: Half of means a number over two or a number divided by two for example half of x means $'\frac{x}{2}'$ (*he writes*$\frac{x}{2}$).
7 - R: Okay, so what does "of" mean in mathematics?
8 - L_2: It means…multiply, so half is multiplying...5 and we subtract it from x. (*he writes* $x-\frac{5}{2} \leq 6$)
9 - R: Is this the answer that you wrote?
10 - L_2: Yes, I wrote that answer.
11 - R: Okay, let's go back again and consider the phrase "half of" separately. What is ½multiplying?
12 - L_2:½ is multiplying 5 only (*he writes ½* × 5*).*
13 - R: Okay, let's leave '½of'. Read the next statement after '½of'.
14 - L_2: Um… It's "5 less than all numbers"
15 - R: Can you represent "5 *less than all numbers"* in mathematical symbols?
16 - L_2:…um…. it is $'x-\frac{5}{2}'$
17 - R: Is the ½ multiplying 5 only or it is multiplying the whole of "5 less than all numbers"?
18 - L_2: Oh… no it's multiplying "5 *less than all numbers*" and I should write $'x- 5'$ to represent 5 less than all numbers (*he writes* $x- 5$*).*
19 - R: Now write the whole mathematical sentence involving ½ and "5 less than all numbers"
20 - L_2:$½x- 5$*(he writes it down*).
21 - R: Remember what ½ is multiplying. You have told me earlier what $\frac{1}{2}$is multiplying. Can think again how you should write it correctly if ½ is multiplying "5 *less than all numbers*"?
22 - L_2: Oh sir…I think I should include brackets since ½ is multiplying everything. So it's ½ $(x- 5)$.
23 - R: That's correct, so what comes next after this first part?

24 - L_2: The inequality symbols represented by the phrase "*less than or equal to*" and its *' ≤ '*

25 - R: That is right, now you can see the sense in the question. So the question was saying "*half of 5 less than all numbers are less than or equal to* 6*"*. Now can you write the final solution that includes all the parts including the symbols?

26 - L_2: Um…½ $(x- 5) \leq 6$

27 - R: Good, now if you look at the whole statement, which word or phrase tells you that the inequality equation does not have one solution, that is, *'x'*does not have a single solution.

28 - L_2: Less than or equal to 6.

29 - R: Is it all? Which other word do you think suggests there are many solutions? Read again and think about what we have discussed.

30 - L_2: Is it '*all numbers*'...I am not sure.

31 - R:That's a good guess. Thank you very much.

L_5 (Tsidi)- Excerpt 11

1 - R: How did you do this one?

2 - L_5: I multiplied $\frac{1}{2}$by 5 and I got$\frac{5}{2}$. I then subtracted 'x' standing for all numbers, and then I put the symbol less than or equal to 6 (i.e. ≤). (*He writes* '$\frac{5}{2}-x \leq 6$*'*)

3 - R: Do you understand what the statement 'half of' mean?

4 - L_5: it means you are multiplying

5 - R: Okay, what is it multiplying? Read the statement after half of and stop before 'are less...'

6 - L_5: Oh...its multiplying '5 less than all numbers' I now realize.

7 - R: What do you realize?

8 - L_5: I read it once not separating the statements. It must be this half ($\frac{1}{2}$) that is supposed to multiply the statement '5 less than all numbers'

9 - R: Now can you write the whole statement correctly

10 - L_5: Yes Sir, its $\frac{1}{2}$**x** $(x - 5)$

11 - R: Think carefully about how we write algebraic statements if we are multiplying. Should we leave like that?

12 - L_5: ...um...no we should include brackets sir, I must write, $\frac{1}{2} \times (x - 5) \leq 6$. (*He laughs*)

13 - R: Now if you look at this inequality symbol $(\leq)$, which words tells you that you are talking about many answers when we solve this equation?

14 - L_5: I don't know which word sir, should I guess? Its numbers…

15 - R: Partly yes, but it's the word ALL numbers (to suggest many solutions). Okay if I write $x \leq 10$, how many solutions do we have in this situation?

16 - L_5: There are 10, ooh...they are 9 sir…No there are many solutions I realize (*he laughs*)

17 - R: So you read the whole statement without noticing individual parts. Thank you very much for your time.

Question 13: *If the product of 2 and 4 is subtracted from twice a certain number and then increased by 4, the result is 22.* Write an algebraic equation that corresponds to the above statement. Find the number.

L_1 (Retha)- Excerpt 12

1 - R: Now let us begin. Can you read the question and show me what you wrote as an answer to this question.

2 - L_1: (*she reads the question and writes*$2(2x- 4) + 4 = 22$.

3 - R: Okay, let's start with the first part of the question. Now, what do you understand by the phrase 'product of 2 and 4'?
4 - L_1: Um... let's see... I'm not sure sir,
5 - R: Okay, what is the meaning of the word 'product' mean in mathematics?
6 - L_1: It's the final answer Sir,
7 - R: Is it the final answer? Okay, if I say sum and difference what I should I do.
8 - L_1: Sum... I should add and difference I should subtract.
9 - R: So, what is the meaning of the word product?
10 - L_1: You multiply (*she laughs as she remembers*).
11 - R: Is it that you had forgotten or what? Okay, now let's start again, what is the product of 2 and 4?
12 - L_1: its 8 Sir.
13 - R: What are we going to do with the product of 2 and 4?
14 - L_1: We are subtracting it from twice a certain number.
15 - R: How do you write '*twice a certain number in symbols'*?
16 - L_1: (she writes $2x$)
17 - R: Okay, what are subtracting from twice a certain number?
18 - L_1: We are subtracting the product of 2 and 4, which is 8.
19 - R: Continue reading from twice a certain number.
20 - L_1:*She reads*...and it is increased by 4…
21 - R: What does the word increase mean or tell you to do in mathematical terms?
22 - L_1: I think it means we add 4 and it is equal to 22.
23 - R: So, why did put 2 outside the brackets as if you were multiplying it with 'twice' a certain number subtract 4
24 - L_1: Sir, I think I didn't understand or remember what the word product meant but I thought the 2 is multiplying '4 that is being subtracted from $2x$.
25 - R: Okay, now reflect on the challenges you faced on this problem.
26 - L_1: I did not understand the words product of 2 and 4 meant the two were multiplication each other, so I took 2 outside the bracket like this *(showing it*) and put '$2x - 4$' inside the bracket and then added 4 to equals 22.
27 - R: Okay, is it this part that says 'the product of 2 and 4' that confused you?
28 - L_1: Yes Sir, this means I should learn how to read the question more than once to understand it and learn meaning of words.
29 - R: That's good, you have now learnt. Thank you for your time

L_2 (Steve) - Excerpt 13

1 - R: Can you read the question and show me the answer that you wrote.
2 - L_2: Writes the answer '$2x - 8 + 32 = 22$'.
3 - R: What do you understand by the phrase '*the product of* 2 *and* 4', in fact what does it tell you to do?
4 - L_2: It's a number that you get when we multiply 2 and 4.
5 - R: And what do you understand by the phrase, '*…the product is subtracted from twice a certain number'?*
6 - L_2: So if this product is subtracted from twice a certain number, this product must be negative.
7 - R: My next question is, where did you get this 32 that you added?
8 - L_2: Okay, I thought about it like...If the product is eight…then I multiplied this product by 4 since we are told in the question that it's increased by 4.
9 - R: Then are you saying that 'increased by 4' means we have to multiply by 4?

10 - L_2: Yes, I think so…. that's why I multiplied by 4 to get 32.
11 - R: I want you to read again the statement immediately after the phrase '*the product is subtracted from twice a certain number'* and tell me what you are going to do next.
12 - L_2: I must increase it by 4.
13 - R: Okay, so if you increase by 4, what are you supposed to do?
14 - L_2: No…um, I must add by 4.
15 - R: So you can see the difference between what you wrote and what you have just said. So what were you supposed to write then instead of 32?
16 - L_2: No Sir...I think I actually wrote 32 because the question says …it's increased by 4 and I know that the word '*by'* means multiplication.
17 - R: So you took '*by*' to mean multiplication. Let's see, for example if I increase my money by *R*5, what does that mean, do you think I should multiply or add there?
18 - L_2: Um...I should add *R*5 to my money.
19 - R: Okay, so in our question, what should you add then?
20 - L_2: I should add 4.
21 - R: So, can you write the correct answer as a whole now?
22 - L_2: He writes '$2x - 8 + 4 = 22$' correctly
23 - R: Thank you very much for your time.

L_4 (Tanatswa) - Excerpt 14

1 - R: Can you read the question and then show me what you wrote.
2 - L_4: (*He reads and then writes)'*$22 = x - 2(2 + 4) + 4$'
3 - R: Now, why did you start with 22 and not the other way round?
4 - L_4: Because the outcome of the equation is 22, so I had to start with it. It's like the conclusion and it's easier for me to start with the results.
5 - R: What do you understand by the phrase '*the product of* 2 *and* 4'?
6 - L_4: Product of 2 and 4 means 2 plus 4 because it's written 2 and 4 so I add them.
7 - R: In mathematics, does product means addition.
8 - L_4: No sir, I mean out of 2 and 4, what comes out of it.
9 - R: Okay, read it again. Try to think, again what does product means in maths?
10 - L_4: What these two numbers 2 and 4 give you.
11 - R: So what does the product of 2 and 4 give you?
12 - L_4: It gives me 6.
13 - R: So are you saying the product of 2 and 4 is adding and getting 6? Think again.
14 - L_4: Um...I think it means adding, I'm not sure now.
15 - R: What does sum mean in mathematics?
16 - L_4: It means you find the total or…um... add.
17 - R: Say one thing, if its total, what are you doing to get the total?
18 - L_4: I'm are adding,
19 - R: So what is the sum of 2 and 3?
20 - L_4: We find the total…add 2 and 3 and we get 5
21 - R: What does the word difference mean in mathematical terms?
22 - L_4: ...um it means to subtract...
23 - R:. From our discussion, sum means adding, difference means subtracting. So what does '*product'* mean then?
24 - L_4: It must mean multiplication
25 - R: So what's the product of 2 and 4?
26 - L_4: It is 8

27 - R: Let's go back to the question. How do you write twice a certain number?
28 - L_4: (*Writes '$2x$'*)
29 - R: So what's being subtracted from $2x$?
30 - L_4: The product of 2 and 4 which is 8 is subtracted from 'x'
31 - R: Is it $x$or $2x$?
32 - L_4: No its $2x$ sorry,
33 - R: What follows?
34 - L_4: Then it is increased by 4 and we add 4.
35 - R: Can you write the whole statement correctly now
36 - L_4: He writes '$(2 \times 4) - 2x + 4$'
37 - R: No, be careful. Read again to determine what is subtracted and from which term?
38 - L_4: Oh… I got it now Sir. (*He writes* $2x - (2 \times 4) + 4 = 22$*).*
39 - R: Yes, now reflect from this discussion what you think was a problem.
40 - L_4: My problem was that I didn't make use of the word product to mean multiplication of 2 and 4. Instead I added 2 and 4.Secondly, I missed reading the word twice from a certain number and that's why I wrote 'x' instead of multiplying it by 2.
41 - R: Thank you for your time.

L_5 (Tsidi)- Excerpt 15

1 - R: How did you do this one? Can you write down your answer?
2 - L_5: I wrote '$(4 \times 2) - 2x + 4 = 22$.
3 - R: Why did you write it like this?
4 - L_5: Sir I can write it correctly now, he writes '$2x - (4 \times 2) + 4 = 22$'.
5 - R: Can you now reflect on why have you changed from what you did earlier?
6 - L_5: Eish… sir...
7 - R: Okay, what is happening to the product of 4 and 2? Read again please carefully. What is being subtracted from the other?
8 - L_5: Oh, it's the product of '2 *and* 4' that is being subtracted from '$2x$' I now realize. I actually did not read carefully to see that it was the product that was being subtracted from '$2x$'.
9 - R: Thank you!
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APPENDIX I:(Interview excerpts 16 - 18)

Additive Compare word problem: Questions [11 & 12]

Question 12: *Agnes's money is R2 more than Thabo's money. Let A represents the amount of Agnes's money and T represents the amount of Thabo's money.* Show the relationship between amounts of Agnes's money and Thabo's money with mathematical symbols.

L_1 (Retha) - Excerpt 16

1 - R: Now can you read the question and show me what you wrote down in this question.
2 - L_1: (She reads fluently and writes $2x + x = x$).
3 - R: Okay, why did you write '$2x + x = x$', can you explain your answer?
4 - L_1: Sir, what I did was that I wrote Agnes has R2 more than Thabo, so it's will be 2 plus whatever Thabo has and this equals x.
5 - R: Okay, let me take you bit by bit. Firstly, did you understand what the question wanted you to do?
6 - L_1: Yes sir, I was supposed to write the mathematical statement using mathematical symbols.
7 - R: Okay, what do you understand by the word 'relationship'?
8 - L_1: Um..., Sir, I think of an equation.
9 - R: Now, let put Agnes here and Thabo there on the other, side. Did you understand what **A** and **T** were supposed to mean in the equation that you were to form?
10 -L_1: Um... Sir it's like I forgot to use the letters **A** and **T** as was required in the question.
11 - R: What might be the reason why you used 'x' instead of A and T?
12 - L_1: I did not read it carefully Sir, I thought I could use 'x' as we always do in equations.
13 - R: Now, between the two learners, who has more money than the other?
14 - L_1: It's Agnes
15 - R: By how much?
16 - L_1: By R2
17 - R: Which word in the question shows that Agnes has more money than Thabo?
18 - L_1: It's written here 'more than'
19 - R: What mathematical symbol is associated with the word more?
20 - L_1: It's an inequality symbol
21 - R: Okay, let's think of the meaning of this word in terms of operation signs. For example, if I have R2 more than you, then what operation sign do you use to get the amount of money that I have
22 - L_1: I' m going to add, so I use an addition sign.
23 - R: So you see the relationship between the amounts of money of Agnes and Thabo. Now I want you to write this relationship using A and T.
24 - L_1: (*She reads as it is in the question but without writing the equation but in words*)
25 - R: Wait, I want you to write it as an equation that is, Agnes's money in relation to Thabo's money.
26 - L_1: Oh...sir I remember now, I must say R2, plus whatever Thabo has equal to Agnes's money. (*She writes* $R2 + T = A$).
27 - R: Good, right, how have you thought about it now? What made change what you had written before?
28 - L_1: Sir, I can't say $2A$ because in between there is a multiplication sign, so it's not 2 multiplied by whatever Agnes plus Thabo has but it must be 2 plus to whatever Thabo has equal to whatever Agnes has.
29 - R: Now this is the correct answer, good! Now reflect on what we have discussed.

30 - L_1: I did not use the letters that I was told to use in the question to represent the relationship and I did not read the question carefully to understand what was meant.
31 - R: That's good, thank you very much for your time.

L_2 (Steve) - Excerpt 17

1 - R: Can you read the question first and then explain how went about getting your answer?
2 - L_2: My answer is $A + 2T$ (*he writes his answer on the sheet of paper provided)*
3 - R: Can you tell me then how you went about it?
4 - L_2: I said...Agnes has $R2$ more than Thabo's money and this means that Agnes will have A money and Thabo will have 2 times more money than Agnes. So my answer is $A + 2T$ (*he writes the answer on the piece of paper provided*)
5 - R: Okay, let us look at it bit by bit. Between the two learners, who has more money than the other?
6 - L_2: Agnes has more money than Thabo.
7 - R: By how much?
8 - L_2: By $R2$... I mean Agnes has $R2$ more than Thabo.
9 - R: To start with, in mathematics, what are you required to do if asked to show the relationship between two quantities?
10 - L_2: Um...I am not sure.
11 - R: Think again about the term relationships in mathematics if we relate two quantities using letters.
12- L_2: Um...I think relationships are connected by equal sign…like in equations.
13 -R: Okay that is right. Now what do you understand by the phrase '*more than'*? If I say I have $R2$ more than you have, what does that mean?
14 - L_2: Um...you have more money than I have by $R2$.
15 - R: Now let me give you a numerical example. Let's say I have $R5$, and you have $R2$ more that what I have, then how much money do you have?
16 - L_2: I will have my money multiplied by your money sir…
17 - R: Okay, from the statement, does the word '*more than'* mean multiplying?
18 - L_2: Yes, I think so.
19 - R: Okay, let's go to the question again. What is the difference between the amounts of money these two learners have?
20 - L_2: It is $R2$
21 - R: Okay, let us have A on the left hand side and T on the right hand side. Now, remember who has more money than the other.
22 - L_2: ...oh, sir I remember now. '*More than'* is adding not multiplying…. I now get the sense.
23 - R: Right, if we are to write the equation connecting the two amounts of money Agnes and Thabo have which side do we put $R2$ for the equation to make sense.
24 - L_2: (*He writes 'A + R2 = T')*
25 - R: Check if the equation must make sense, given that Agnes (A) has $R2$ more than Thabo (T). So, which side should the $R2$ be?
26 - L_2: On the left side...
27 - R: Okay, if $R2$ is on the left hand side, who then has more money between Agnes and Thabo?
28 - L_2: Oh...It should be on the right side I see... (*He writes A = 2 + T).*
29 - R: Read the question once more and explain what A and T stand for in this question?
30- L_2: 'A' stand for Agnes and 'T' stand for Thabo.
31- R: Read again and look at the equation that you have written. What do these letters stand for?
32 - L_2: …um... sir, these letters are referring to... amounts of money that they both have.
33 - R: So, what was confusing you in this question?

34 - L_2; The words '*relationship*' and '*more than*' I didn't understand them. I thought '*more than*' means I have to multiply 2 by Thabo's money to get Agnes's amount of money.
35 - R: Thank you very much.

L_4 (Tanatswa) - Excerpt 18

1 - R: Can you read the Question 12 and show me what you wrote down?
2 - L_4: He writes '$2A = T$'
3 - R: Firstly, do you understand what the question requires you to do?
4 - L_4: Yes Sir, it requires me to write that Agnes has double money than Thabo.
5 - R: Now my first question is, between the two people, who has more money between the two?
6 - L_4: It's Agnes because her money is doubled since it is mentioned that Agnes's money is $R2$ more than Thabo.
7 - R: Does 'more than' means double?
8 - L: Yes Sir,
9 - R: Okay, if I say I have more than you, does it mean that I have doubled the money?
10 - L_4: Um...I mean, if it is $R2$ more than Thabo's money, then Thabo's money has been doubled to get Agnes's money.
11 - R: Okay, let's look at this example. Let us say Agnes has $R5$, how much has Thabo?
12 - L_4: Thabo has $R3$
13 - R: Are we doubling the amount in this case?
14 - L_4: No, we adding $R2$ to Thabo's money to get Agnes's money or subtract $R2$ from Agnes's money to get Thabo's money.
15 - R: So if Agnes's money is $R2$ more than Thabo, are we doubling then?
16 - L_4: Um...Sir no, we are adding not doubling.
17 - R: Is it the reason why you might have written this answer '$2T = A$'?
18 - L_4: Yes Sir.
19 - R: Can you write the correct equation given that Agnes has R2 more than Thabo?
20 - L_4: *(he writes)* '$2 + T = A$'
21- R: Good. Can you make some reflections on this question? What was your problem?
22 - L_4: I didn't understand the phrase 'more than' and how I could use it to form an equation.
23 - R: Okay thanks very much for your time!

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APPENDIX J:(Interview excerpts 19 - 20)

Multiplicative Compare word problems: Questions [14 & 15]

Question 14:*In a certain school, there are eight times as many boys as there are girls.* ***Write an algebraic equation that represents the above situation.***

L_{20} (Thabo)- Excerpt 19

1 - R: Can you show me the equation you wrote and explain why you wrote that equation.
2 -L_{20}: I wrote $'8b = g'$ sir. When I read it, I know there must be more boys than girls, so I wrote $'8b = 1g'$.
3 - R: Okay, if you look at the equation you wrote, what does 'b' and 'g' stand for?
4 - L_{20}: It is for boys and girls as stated in the question sir...
5 - R: Okay, did you ever test whether the equation gives you the correct number of boys if you are given the number of girls?
6 - L_{20}: No I didn't test it sir, in fact I read it as I have said and saw that for every 8 boys there is '1' girl.
7 - R: Okay, let's put $'b'$ on this left hand side and $'g'$ on the other side. Now, if there is one '1' girl, how many boys are there?
8 - L_{20}: ...um there are 8 of them sir.
9 - R: If there are $'2'$ girls, how many boys are there?
10 - L_{20}: ...there are 16 boys.
11 - R: Now, what are you doing to the number of 'boys' if you are given the number of 'girls'?
12 - L_{20}: We multiply the number of 'girls'.
13 - R: The look at your equation to check if you can use it to get the number of boys as we you indicated in the examples I gave you.
14 -L_{20}: ...um, no sir if I say $'2'$ girls the equation does not give me 16 boys, I can see now. But If I say $'b = 8g'$ then it works.
15 - R: Let's go back to what you said earlier. Is the $'b'$ and 'g' standing for boys and girls or number of boys and girls?
16 - L_{20}: Oh, 'b' and 'g' stands for the number of learners. I remember now when we are substituting in equations.
17 - R: Now can you write the correct equation and explain what you think made you to write $'8b = g'$.
18 - L_{20}: I didn't read it correctly sir, plus I didn't test it as we did together. I wrote $'8b'$ because I thought for every '8boys' there is $'1'$girl. That's why I wrote $'8b = g'$.
19 - R: Can you suggest what you have to do when solve such types of problems.
20 - L_{20}: I think I should check if the equation I write gives me the correct values if I substitute one to get another value.
21 - R: Thank you very much for your time.

Question 15: Write an algebraic equation using the variables *S* and *P* that will represent the following situation: "*In a certain university, the number of students is 6 times greater than the number of professors" Use S for the number of students, and P for the number of professors.*

L_4 (Tanatswa) - Excerpt 20

1 - R: Now, can you read the question and show me what you wrote?
2 - L_4: (*he reads and writes* $6S>P$).

3 - R: Okay, let's write S and P on the other side. Now if you look at this university, which group of people are more than the other?
4 - L_4: It should be the students, Sir using common sense...
5 - R: Now let's say, I want to know the number of students at this university, what do I do?
6 - L_4: We multiply 6 by P because there are 6 times more students than professors do,
7 - R: Okay, let's say there are 6 students, how many professors are there?
8 - L_4: Um…there must be one...
9 - R: What about if we have 12 students?
10 - L_4: Two
11 - R: So, if you want to get the number of students what do you do to the number of professors.
12 - L_4: We multiply the number of professors by 6.
13 - R: Now if I want an equation not an inequality that connects the number of students and professors, how do we write it?
14 - L_4: Its $6S = P$.
15 - R: Check again the meaning of your equation to see if makes sense. Why did you multiply S by 6?
16 - L_4: I did this because for every 6 students we get one professor. Isn't it what we said earlier?
17 - R: Okay, let's see if what we did earlier is true for this equation, you have written. Now, let's say we have 6 students, how many professors do you have?
18 - L_4: Um... we have 6 times 6 which equals 36 professors...No sir, it does not make sense now. I am confused now.
19 - R: Your answer now is contradicting your earlier reasoning to say there should always be more professors than students are. Can you see that?
20 - L_4: Oh...if I multiply P by 6...yes! I get more students. So it is $S = 6P$... its tricky sir.
21 - R: Remember an equation gives the value of one quantity in terms of the other. Here we are talking about the number of students and the number of professors.
22 - L_4: ...um...I was thinking wrongly. The way we speak is not the way we write like I did. Yes I can see why it must $S = 6P$.
23 - R: Now reflect back and tell me why did you write with the symbol greater?
24 - L_4: Sir, I saw the word greater than and I thought of an inequality equation was the one needed
25 - R: What else can you say?
26 - L_4: No, I did not think carefully and that's why I made that mistake of putting 6 in front of S. I should have multiplied P by 6. I should have also reasoned in the way we discussed above.
27 - R: Did you see the phrase '*write an equation' in the question*?
28 - L_4: No, I read it well, I just saw the word greater than, and that's why I used the inequality symbol.
29 - R: Thank you very much.

APPENDIX K: Study Consent Documents

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

For administrative use:
Reference no. D2014/031

GDE RESEARCH APPROVAL LETTER

Date:	**27 May 2013**
Validity of Research Approval:	**27 May 2013 to 20 September 2013**
Name of Researcher:	**Madzorera A.**
Address of Researcher:	**No 11; Boston Villas**
	Benson Street
	Randfontein
	1759
Telephone Number:	**011 414 0360 / 073 352 2099**
Fax Number:	**011 414 5702**
Email address:	**madzoreraa@yahoo.com**
Research Topic:	**Investigating challenges that learners face when translating from word problems to algebraic expressions in a Grade 11 multilingual Mathematics classroom**
Number and type of Institutions:	**ONE Secondary School**
District/s/HO	**Gauteng West**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

2013/05/29

1

Making education a societal priority

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

.....

Dr David Makhado
Director: Knowledge Management and Research

DATE: 2013/05/29

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za

The Principal

Thuto-Lehakwe Secondary School

P.O. Box 5231. Mohlakeng 1759

21 April 2013

RE: REQUEST TO CONDUCT A RESEARCH STUDY IN YOUR SCHOOL

My name is Madzorera Andrew and I am a Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on investigating the challenges that learners face when translating from word problems to linear algebraic equations/expressions in a Grade 11 Multilingual Mathematics Classroom.

My research involves two phases where I will administer test consisting of questions involving translating from word problems to algebraic equations/expressions. A sample of five learners will be selected and interviewed to provide more insight into the challenges these learners face as well as the nature of their mathematical thinking.

The reason why I have chosen your school is because of its accessibility and the fact that I am a mathematics educator at the school. I was wondering whether you would mind if I can get permission to carry out my research study at your school.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study. The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed within 3-5 years after completion of the project.

Please let me know if you require any further information.
I look forward to your response as soon as is convenient.

Yours sincerely,
Andrew Madzorera

Name of Principal: D. NGOBENI Signature: .....
Date: 22.04.2013

Wits School of Education
27 St Andrews Road, Parkton, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa
Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

Student Number: 509288
Protocol Number: 2013ECE048M

Date: 24 July 2013
Dear Andrew Madzorera

Application for Ethics Clearance: Master of Science

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

Investigating Challenges that Grade 11 Mathematics Learners Face when Translating from Word Problems to Linear Algebraic Representations.

The committee recently met and I am pleased to inform you that clearance was granted.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page. The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely

MMabeta

Matsie Mabeta
Wits School of Education
011 717 3416
CC Supervisor: Dr. T Essien

University of the Witwatersrand

WitsSchool of Education

Information Sheet for Parents

My name is **Andrew Madzorera**. I am a researcher studying Masters Degree in Science at the University of the Witwatersrand.

I am carrying out a study on **challenges that learners face when solving word problems** at a school in Gauteng West mainly looking at **Investigating Challenges that Grade 11 Learners Face when Translating from Word Problems to Linear Algebraic Representations**. My research should not only benefit the institutions where it is conducted, but also the South African educational system in improving the teaching and learning of Mathematics.

I would like to interview learners who would have interesting response to some of the items in test to elicit more concerning their mathematical thinking.

The time to answer the test is 55 minutes. The time taken to interview each learner will be 15 minutes.

I would like to make it clear that participation in this study is voluntary, no harm is envisaged, and all information will be treated as confidential and names not known. Participants can choose to accept or decline to answer any questions, and can withdraw from the study at any given time. I hope to publish part or all the results of this study in academic journals. In order to maintain anonymity and confidentiality, all names I use will be pseudonyms.

I will provide participants with a summary of my research results on completion if they would like me to.

Name: Andrew Madzorera

Signature: _____

Date: 10-04-2013

University of the Witwatersrand

WitsSchool of Education

Informed Consent Form for Conducting Research in mathematics classrooms

Research Topic: Investigating Challenges that Grade 11 Mathematics Learners Face when Translating from Word Problems to Linear Algebraic Representations

Parents' Informed Consent

1. I hereby confirm that I have been informed by the researcher, Mr. A Madzorera about the nature of the study.
2. I have also received, read and understood the Information and Consent sheets regarding the educational study.
3. I am aware that the information my child gives will be processed without mentioning his/her real name.
4. In view of the requirements of the research, I agree that the data collected during this study can be processed in a computerized system by the researcher.
5. My child can at any stage, without prejudice, withdraw his/her participation in the study.
6. I have had sufficient time to ask questions and (of my free will) allow my child to join the study.

Name: _____

Signature: _____

Date: _____

Consent Form Learners' Interview

Please fill in the reply slip below if you agree to be interviewed. I will use your answers to my questions for my study called:

_____

Investigating Challenges that Grade 11 Mathematics Learners Face when Translating from Word Problems to Linear Algebraic Representations.

_____

Permission for interview

My name is: _____

I would like to be interviewed for this study YES/NO

I know that Mr A Madzorera will keep my information confidential YES/NO

I know that I can stop the interview at any time and don't have

to answer all the questions asked YES/NO

Sign_____ Date_____

Contact person:

NAME: Mr A Madzorera

ADDRESS: 6579 Kent Masire Street. P.O. Box 5231 Mohlakeng

TEL NUMBER: 011 414 0360

University of the Witwatersrand
WitsSchool of Education

Informed Consent Form for audio recording in mathematics classrooms

Research Topic: Investigating Challenges that Grade 11 Mathematics Learners Face when Translating from Word Problems to Linear Algebraic Representations

Parents' Informed Consent for audio recording

1. I accept that my child can be audio recorded during classroom interactions. yes/no (Tick one)

2. I have also received, read and understood the Information and Consent sheets regarding the educational study.

3. I understand that the researcher will keep all raw data under lock and key for a period of up to 5 years. After this, the raw data will be destroyed. Yes/no (Tick one)

4. In view of the requirements of the research, I agree that the data collected during this audio recording can be processed in a computerized system by the researcher. Yes/no (Tick one)

Name: _____
Signature: _____
Date: _____

Consent Form for Learners Documents

Please fill in the reply slip below if you agree to have the following documents: written responses to the test questionnaire used for my study called:

_____

Investigating Challenges that Grade 11 Mathematics Learners Face when Translating from Word Problems to Linear Algebraic Representations.

_____

Permission for documents (written responses to the test)

My name is: _____

I agree that my written responses to the test can be used for this study only YES/NO

I know that Mr Madzorera will keep my information confidential YES/NO

Sign_____ Date_____

Contact person:

NAME: Mr A Madzorera

ADDRESS: 6579 Kent Masire Street. P.O. Box 5231 Mohlakeng.

TEL NUMBER: 011-414-0360