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Optimizing Structures with Semi-Rigid Connections Using the Principle of Virtual Work

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Abstract

In this paper, the virtual work optimization method (VWOM) has been generalised to consider structures with semi-rigid connections. The VWOM is an automated method that minimizes the mass of a structure with a given geometry, multiple deflection criteria and load cases, while adhering to Design code requirements. In the optimization process, members are selected from a discrete database to meet all strength and stiffness criteria. Connections are modelled using rotational springs, allowing some moment transfer. The rotational stiffness of each connection can be varied form rigid to pinned. The example of a pitched roof frame is used to explain the method. Two case studies are considered: (i) a three-storey two-bay and (ii) a four-storey three-bay office building. The VWOM produced results up to 26.7% lighter than results in the literature. Furthermore, the structures were optimized for a range of rotational stiffness, where all connections in the structure were assumed to have the same rotational stiffness. Characteristic jumps in the optimized mass versus rotational stiffness were observed.

Keywords Virtual work optimization method · Semi-rigid joints · Automated member selection · Discrete sections · Design

1 Introduction

Design codes require design of structures to meet both deflection and strength criteria. The objective of lighter, cost-effective structures leads engineers to optimize designs. It is not always clear where the structure should be stiffened and strengthened. The question whether the use of semi-rigid or rigid joints will result in a lighter structure remains open. In practice, the engineer uses past experience and intuition to arrive at a solution; generally a solution is assumed after a few iterations. A tedious iterative trial and error approach is followed. Many optimization methods have been presented in the literature to deal with this problem.

Methods based on the principle of virtual work have been used by Chan (1992) and Park and Park (1997) to optimize tall buildings. Barrar et al. (2009) optimized steel buldings and focused on the effect of wind loading. However, these authors neglected strength constraints and considered only deflection criteria. Methods published by Makris and Provatidis (2002) and Makris et al. (2006) use strain energy criteria. Member selection is performed based on determining either cross-sectional areas for trusses or second moment of areas for frames, but not both simultaneously. Most structures cannot be analyzed by such methods because bending, torsion and axial forces occur in members simultaneously. Patnaik et al. (1997) proposed a methodology of satisfying stress constraints and then reducing deflections, but considered only trusses. Elvin et al. (2009) developed an iterative virtual work optimization method for structures with fixed topologies subject to a single deflection constraint and one load case. These restrictions were lifted in Elvin and Walls (2010) who assumed that all connections are rigid. In practice rigid connections are difficult and expensive to implement.

In design, either rigid or frictionless pin joints are assumed in global frame analysis. The implementation of joint behavior in the design must satisfy these assumptions. In reality, joints have some stiffness. The contribution of joint stiffness to the economy of the structure is unknown. Modern steel design standards such as Eurocode 3 (2005), allows for the use of semi-rigid joints. Weynand et al. (1998) and Steenhuis et al. (1994) have found that large cost savings are possible in terms of detailing and manufacturing of semi-rigid connections.

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A number of methods exist to analyze the rotational stiffness of a beam or column end connection. The most common used includes the penalty and the Lagrange multiplier methods, both described for example by Bathe (1996). Chen and Lui (1991) presented a method to analyze semi-rigid connections where the moment transferred is dependant on the relative rotation between connected members.

This paper generalizes the Virtual Work Optimization Method (VWOM) presented by Elvin and Walls (2010), for optimizing steel structures, by considering rotational stiffness of joints. The VWOM is an automated, iterative method that considers multiple load cases and deflection criteria. The principle of virtual work is utilized in the optimization process. Structural sections are selected from a predefined database to satisfy both strength and deflection criteria. For every iteration, members are chosen to provide the highest overall deflection reduction per unit mass increase. Once all constraints are satisfied using the minimum amount of material, the result is assumed to be the optimal structure.

The layout of this paper is as follows: Semi-rigid and rigid connections are discussed first. Secondly, a method to analyze semi rigid connections is presented. Third, building code requirements are presented. Next, the VWOM is explained by optimizing a simple pitched roof frame. Finally, to demonstrate the VWOM, two case studies are given: (i) a three-storey two- bay and (ii) a four-storey three-bay office building. Where possible the results are compared to those found in the literature.

2 Semi-Rigid Connections

The traditional approach to steel frame design neglects the actual behavior of connections and considers two extreme cases: pinned and fixed connections (The Green Book 2012). These approaches simplify analysis and design procedures. However, the predicted frame response may not be realistic (Simoes 1995). The term semi-rigid is commonly used to describe the connection behaviour between the two extremes. The interested reader is referred to the references in this section for more details.

2.1 Rigid Versus Semi-Rigid Connections

Semi-rigid connections are designed to resist vertical shear and axial forces. Thus from a theoretical point of view, a semi-rigid connection is a pin, but in reality these joints have some rotational stiffness and will therefore transfer some moment (The Green Book 2012). An example of a semi-rigid connection is provided in Fig. 1. These connections take a relatively short time to design.

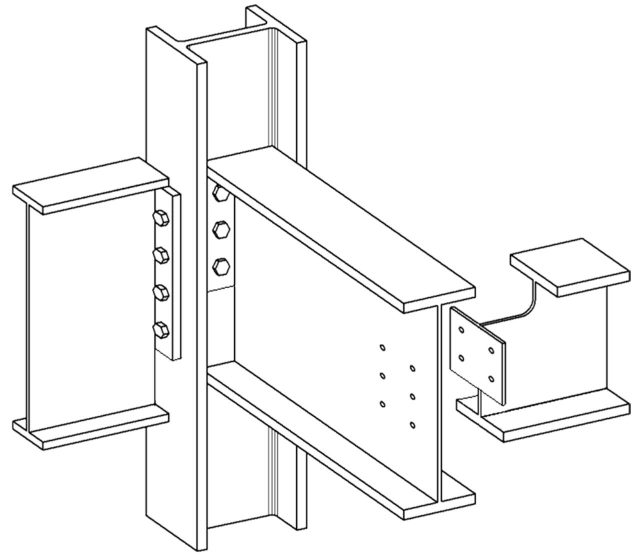

Fig. 1 Simple beam end connections [adapted from the Simple joints to Eurocode (2011)]

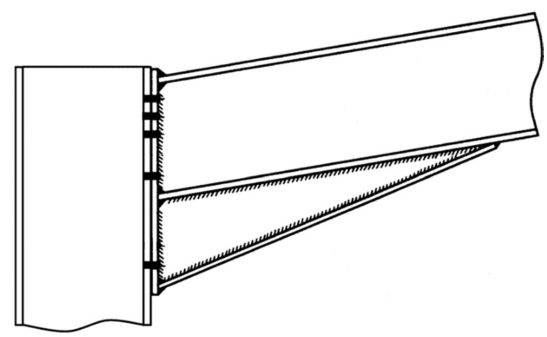

Fig. 2 Rigid joint [adapted from the Moment-Resisting Joints to Eurocode (Eurocode 3 2013)]

Rigid connections transmit bending moment, shear and axial forces from one member to another. An example of a rigid connection is shown in Fig. 2.

It must be noted that rigid joints are not rigid or infinitely stiff (Weynand et al. 1998).

2.2 Economy

Semi-rigid connections are more economical than rigid connections as less material and much less labour and time is required in fabrication (The Green Book 2012). Weynand et al. (1998) have shown that using semi-rigid connections results in a fabrication cost saving of up to 28% due to less man-hours required, thinner plates, fewer bolts and smaller welds. In addition, structures with semi-rigid connections have additional load carrying capacity, as the moments that they can resist is not accounted for in the analysis.

2.3 Analysis of Semi-Rigid Connections

The analysis of semi-rigid connections needs to incorporate the rotational stiffness of the joint. For simplicity, and in general, design only considers two extremes: rigid and pinned joints. For the rigid and the pinned cases, the matrix methods for frames and trusses can be used as described for example by Kassimali (2012) and Bathe (1996). The frame method assumes that all joints are infinitely stiff and the truss method assumes that no moment is transferred by the joints (frictionless pins). It is however important to consider the connection rotational stiffness to ensure a more realistic model (Weynand et al. 1998).

The rotational stiffness of the connection at the beam ends can be represented by a rotational spring as shown in Fig. 3. The spring has zero volume and connects two members in the joint. If the spring has zero stiffness, the connection is a pin; if the spring is infinitely stiff, the connection is rigid.

M_A and M_B are the beam A-B end moments. θ_A and θ_B are rotations at the ends of beam A-B measured with respect to the connecting beam. θ_{rA} and θ_{rB} are the relative rotations at the ends of beam A-B. k_A and k_B are the springs stiffness given by:

$$k_A = \frac{M_A}{\theta_{rA}} \tag{1a}$$

$$k_B = \frac{M_B}{\theta_{rB}} \tag{1b}$$

By substituting the end-rotation θ_A and θ_B with $(\theta_A - \theta_{rA})$ and $(\theta_B - \theta_{rB})$, the relationship between the end-rotation and the end-moments of a beam can be written (Hayalioglu et al. 2004):

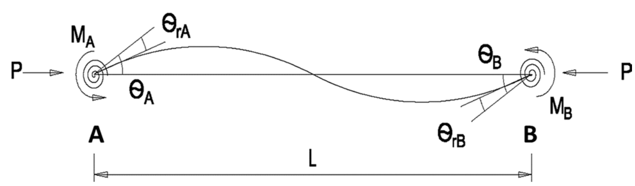

Fig. 3 Member with rotational springs (Hayalioglu et al. 2004)

$$M_A = \frac{EI}{L}\left[4\left(\theta_A - \frac{M_A}{k_A}\right) + 2\left(\theta_B - \frac{M_B}{k_B}\right)\right] \tag{2a}$$

$$M_B = \frac{EI}{L}\left[4\left(\theta_B - \frac{M_B}{k_B}\right) + 2\left(\theta_A - \frac{M_A}{k_A}\right)\right] \tag{2b}$$

where E is the modulus of elasticity, I the second moment of area and L is the length of the beam A-B.

Chen and Lui (1991) showed that the stiffness matrix of the beam A-B $[q]$ incorperating connection flexibility in local coordinates can be written as:

$$[q] = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & (r_{ii}+2r_{ij}+r_{jj})\frac{EI}{L^3} & (r_{ii}+2r_{ij})\frac{EI}{L^2} & 0 & -(r_{ii}+2r_{ij}+r_{jj})\frac{EI}{L^3} & (r_{ii}+2r_{ij})\frac{EI}{L^2} \\ 0 & (r_{ii}+2r_{ij})\frac{EI}{L^2} & r_{ii}\frac{EI}{L} & 0 & -(r_{ii}+2r_{ij})\frac{EI}{L^2} & r_{ij}\frac{EI}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -(r_{ii}+2r_{ij}+r_{jj})\frac{EI}{L^3} & -(r_{ii}+2r_{ij})\frac{EI}{L^2} & 0 & (r_{ii}+2r_{ij}+r_{jj})\frac{EI}{L^3} & -(r_{ii}+2r_{ij})\frac{EI}{L^2} \\ 0 & (r_{ii}+2r_{ij})\frac{EI}{L^2} & r_{ij}\frac{EI}{L} & 0 & -(r_{ii}+2r_{ij})\frac{EI}{L^2} & r_{jj}\frac{EI}{L} \end{bmatrix} \tag{3}$$

where r_{ii}, r_{jj} and r_{ij} are given by:

$$r_{ii} = \frac{1}{k_R}\left(4 + \frac{12EI}{Lk_B}\right) \tag{4a}$$

$$r_{jj} = \frac{1}{k_R}\left(4 + \frac{12EI}{Lk_A}\right) \tag{4b}$$

$$r_{ij} = \frac{2}{k_R} \tag{4c}$$

where k_R is the rotational spring constant:

$$k_R = \left(1 + \frac{4EI}{Lk_A}\right)\left(1 + \frac{4EI}{Lk_B}\right) - \left(\frac{EI}{L}\right)^2\left(\frac{4}{k_A k_B}\right) \tag{5}$$

It is important to note that the rotational spring stiffness is a function of the member stiffness parameters EI/L.

The structural system stiffness matrix $[Q]$ in global coordinates is obtained in a standard way by superimposing member stiffness matrices which contain geometric and connection stiff nesses (Hayalioglu et al. 2004). Deflection of the nodes, D, is calculated from:

$$[Q][D] = [F] \tag{6}$$

where $[F]$ is the nodal force vector.

3 Building Code Requirements

According to building codes such as Eurocode 3 (2005) the rotational stiffness of a joint can be determined from the flexibilities of its basic components, represented by an

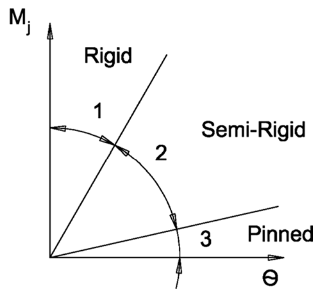

Fig. 4 Classification of joints by Stiffness (Eurocode 3 2005)

elastic stiffness coefficient k_j where j is the join node number. The rotational stiffness k_j of a beam to column connection or a splice, carrying a moment in the joint, may be obtained by the nonlinear equation[1] (Eurocode 3 2005):

$$k_j = \frac{Ez^2}{\mu \sum_i \frac{1}{c_i}} \tag{7}$$

where z is the lever arm, μ is the stiffness ratio of the initial rotational stiffness $k_{j,ini}$ and k_j, and c_i is the stiffness coefficient. For a more detailed explanation, the reader is referred to Eurocode 3 (2005). The joint stiffness is a function of the connection geometry.

Equation (7) holds if the axial force in the connected member does not exceed 5% of the design resistance of its cross-section.

Three zones of connection stiffness are specified in Eurocode 3 (2005) as shown in Fig. 4. In this figure, the moment applied to the joint is plotted against rotation.

The different zones are classified as:

- Zone 1: "Rigid" (i.e. very high rotational stiffness), defined when $k_j \geq \frac{8EI}{L}$
- Zone 2: "Semi-Rigid", defined when $\frac{8EI}{L} \geq k_j \geq \frac{0.5EI}{L}$
- Zone 3: "Pinned" (i.e. very low rotational stiffness), defined when $k_j \leq \frac{0.5EI}{L}$

Eurocode 3 (2005), stipulates that the behaviour of semi-rigid joints needs to be taken into account when the rotational stiffness of the connection lies within Zone 2. It further requires that a semi-rigid joint shall be designed to be capable of transmitting internal forces and moments as obtained from the analysis.

4 Virtual Work Optimization (VWO) Method with Semi-Rigid Connections

Elvin et al. (2009) and Elvin and Walls (2010) developed a method to optimize structures subjected to multiple deflection constraints and load cases. Variable numbers of section changes were allowed per iteration. Frame analyses were conducted both before reducing deflections and before selecting sections to satisfy strength requirements. However, Elvin et al. (2009) assumed that all joints were rigid. The current work incorporates the effect of joint stiffness when optimizing the overall mass of the structure.

The optimization process in this paper can be summarized as follows: first, connection stiffness is assigned. Second, members are chosen to satisfy strength requirements. Third, members most effective in reducing deflections are identified and changed in an iterative manner until all deflection and strength criteria are satisfied. This process is repeated for different connection rotational stiff nesses. Although the method concentrates on 2D structures, its application to 3D structures is identical.

To explain how the method works, a pitched roof frame with only four members is optimized (Fig. 5). This structure is subject to deflection constraints and strength requirements. The maximum vertical deflection of the roof apex is limited to span/400 (25 mm) under serviceability limit state. The maximum horizontal sway of the columns is limited to height/200 (20 mm). Members are chosen to comply with the South African Structural Steel Code (SANS1062-1 2005) using grade 350 W steel.

In general, any design code and grade of steel can be used. To demonstrate versatility, the American Institute of Steel Construction (AISC) W sections database is utilized. In some cases, the South African Institute of Steel Construction (SAISC) I sections are also used for comparison. The rafters and columns are grouped into two distinct groups. All members in a group will be adjusted simultaneously. Grouping is important from a constructability point of view.

4.1 Input: Setting Optimization Parameters

The structure's geometry, loading, deflection requirements, the design code that needs to be used, the effective length of each member and the stiffness of each connection k are required as the input to the optimization process. Critical points at which deflection have to be limited are defined by the user, for example see the circled nodes in Fig. 5. The automated optimization process can now start.

[1] In Eurocode 3 (2005) the rotational stiffness of a connection k_j, is denoted as S_j, and the stiffness coefficient c_i is denoted as k_i.

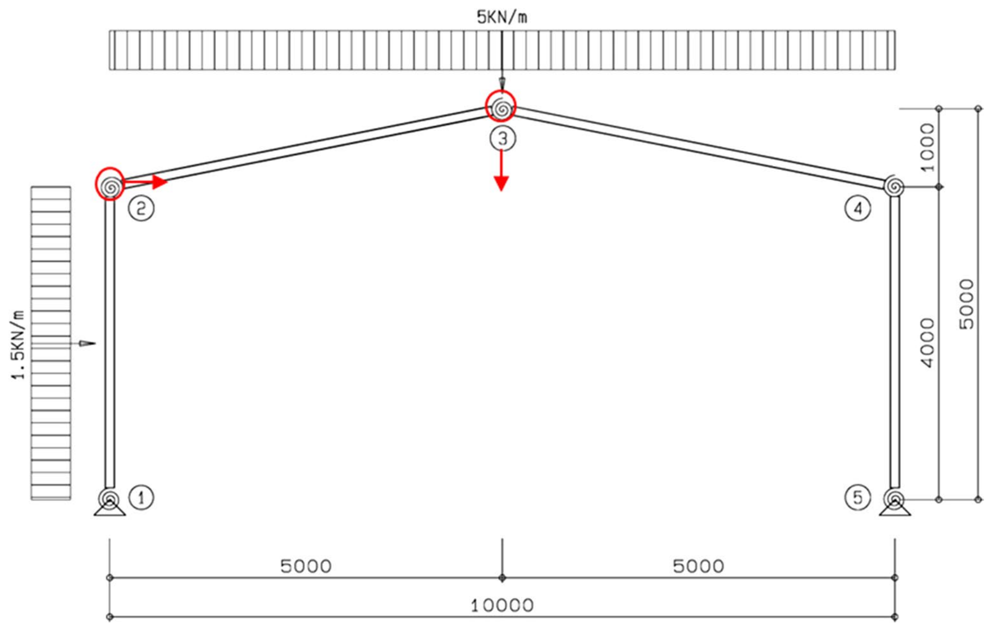

Fig. 5 Pitched roof frame with semi-rigid connections [geometry from Elvin and Walls (2010)]. Numbers in circles refer to node numbers

4.2 Step 1: Satisfying Strength Requirements

All load cases and combinations are considered. Members in a group are re-sized after each iteration, accounting for the redistribution of forces that occur as the structure is changed. The lightest possible member for the group satisfying building code strength requirements is chosen from the pre-specified database. Members are selected such that the mass converges after the first member satisfies strength requirements. This is different to Elvin and Walls (2010) who iterated until the structure's mass converged, which in some instances found only a local minimum.

4.2.1 Pitched Roof Frame: Choosing Initial Sections

For the pitched roof frame, only one strength iteration was needed. The sections selected were W6×12 for the columns and W5×16 for the rafters.

4.3 Step 2: Reducing Deflections

Deflection constraints are tested only at critical nodes. The deflection reduction process starts if node deflections exceed the pre-specified limits. To determine which members should be changed, the strain energy for each member/group is considered.

4.3.1 The Principle of Virtual Work

A unit virtual load is applied to the structure at the node where the deflection had to be limited, i.e. the critical point. Internal forces are calculated. δ_i represents the deflection contribution at node i. The extent of the contribution to the deflection is governed by the member's stiffness and internal forces. The total deflection at the critical point, Δ, is calculated by the summation of all deflection contributions δ_i (Elvin and Walls 2010). Note that the analysis is based on elastic analysis and plastic hinges are not predicted to form in the strucutre.

The deflection contribution of each member i, in a three dimensional structure with semi-rigid joints is calculated using the principle of virtual work:

$$\delta_i = \frac{Ff}{EA}L + \int_L \frac{Mm}{EI}dx + \int_L \frac{Qq}{GA}dx + \int_L \frac{Tt}{GJ}dx + \frac{M^2m^2}{4k^2} \tag{8}$$

A load system is required for each critical deflection point. The virtual load system's internal axial, shear forces, bending and torsional moments are represented by f, q, m and t. The real load system's internal axial, shear forces, bending and torsional moments are represented by F, Q, M, and T. Integration is performed over the total length, L, of each member. Material and geometric section properties are: Young's modulus, E, shear modulus, G, cross-sectional area, A, second moment of area, I, polar second moment of area, J and the spring rational stiffness k.

For two dimensional structures, with semi rigid joints, Eq. (8) reduces to:

$$\delta_i = \frac{Ff}{EA}L + \int_L \frac{Mm}{EI}dx + \frac{M^2m^2}{2k} = \delta_i^{Axial} + \delta_i^{Moment} + \delta_i^{Springs} \tag{9}$$

The deflection contribution consists of the δ_i^{Axial} axial, δ_i^{Moment} moment and $\delta_i^{Springs}$ spring components. In regular steel members, the shear component is small compared to the axial and bending energy terms and is thus neglected.

It is interesting to note that members with a high deflection contribution may already have a large mass, possibly requiring a substantial mass increase to be stiffened, and would thus not necessarily be the best member to change. The efficiency of making any member change is discussed next.

4.3.2 Predicting the Effects of Section Changes

The deflection contribution of a member is inversely proportional to its area and second moment of area. The new deflection contribution of the member's section about to be altered would be changed in proportion to the ratio of sectional properties. The new deflection contribution can be predicted as:

$$\delta_i^{new} = \frac{A_i}{A_i^{new}}\delta_i^{Axial} + \frac{I_i}{I_i^{new}}\delta_i^{Moment} + \frac{k_i}{k_i^{new}}\delta_i^{Springs} \tag{10}$$

An efficient section change is one that results in a large deflection decrease at all critical points for each unit mass increase. It is necessary to ascertain the effect of any change relative to all critical points. The best overall change is determined by quantifying the effect of the change without having to solve the stiffness matrix again.

The efficiency of a section change for N critical points is defined by Elvin and Walls (2010):

$$Efficiency = \frac{\sum_{j=1}^{N}\left(\Delta_j^{Decrease} \Big/ \Delta_j^{Target}\right)}{\Delta M} \tag{11}$$

where $\Delta_j^{Decrease}$ is the deflection decrease at a critical point j as a result of a section change and Δ_j^{Target} is the target deflection of a node for a specific load case. ΔM is the change in mass of the group (Elvin and Walls 2010). The efficiency of a section change can be described as the fraction of deflection reduction that will occur per unit mass increase.

4.4 Step 3: Adjusting Member Sections

Every group of members is replaced by suitable sections from a pre-selected database. A suitable section to be chosen from is one which has both a larger cross-sectional area and second moment of area than the current section. The structure is not analyzed for each change. Equations (9) to (11) are used instead to predict the effect of adjusting section properties by calculating efficiencies. There is a linear relationship between the number of changes investigated and the number of suitable sections for each group (Elvin and Walls 2010).

For this method, increasing the section database size or the number of groups, does not result in the typical exponential increase in computational costs. Large section databases and number of groups can therefore be used with very little effect on computation time.

After suitable changes have been tested, the one with the highest efficiency is selected and changed. Member groups adjusted to reduce deflections are evaluated for strength requirements. These members are labeled deflection dependent members and will not be substituted by lighter members during Step 1 in subsequent iterations. Note that strength dependent members can be replaced by lighter sections during Step 1 when forces redistribute. Equation (7) ensures that all critical point deflections are reduced simultaneously. It has been observed that their target deflections are reached at approximately the same time. This prevents parts of the structure being over-stiffened.

4.4.1 Reducing Deflections in the Pitched Roof Frame

In the first deflection iteration, for the pitched roof frame example, the effect of changing columns and rafters to any W section is investigated. The most efficient change found is to replace the W5×16 rafters with a W6×16 section, leading to the structure's mass increasing by 2.1 kg. Although axial strain energy has been taken into account, it is small and is not shown in Table 1.

Table 1 Material propertied for the pitched roof frame's rafter after iteration 1

Member property	Initial member: W5×16	New member: W6×16
I_x ($\times 10^6$ mm^4)	8.98	13.5
A ($\times 10^3$ mm^2)	3.06	3.09

4.5 Step 4: Satisfying all Deformation and Strength Constraints

Steps 1–3 are repeated until the deflection and strength criteria are satisfied. The number of iterations required to produce an optimized structure is dependent on the size of the structure, the number of groups, the number of critical nodes and the amount that critical node deflections have to be reduced. For example, primarily strength dependent structure will require few iterations.

It is evident that making only one group section change per iteration usually produces the optimal structure. The over stiffening of members is prevented as the effect of any

Table 2 Summary of the optimization results for the pitched roof frame

Iteration	Changed			Efficiency (%)	New mass (Ton)
	Group	From	To		
1	Rafters	W5×16	W6×16	92.81	0.393
2	Rafters	W6×16	W12×16	93.90	0.395
3	Columns	W6×12	W8×13	10.54	0.408
4	Columns	W8×13	W12×14	10.90	0.423

Table 3 Results comparison for the pitched roof frame

Mass (tons)		
Connection type	Semi-Rigid (0.5*EI*/*L*)	Rigid (8*EI*/*L*)
Elvin et al. (2009) (AISC sections)	–	0.521
VWOM (AISC W sections)	0.511	0.487
VWOM (SAISC I sections)	0.467	0.467

section change on the rest of the structure is determined before more adjustments are made. Multiple section changes can however be made per iteration to accelerate the process. This may be necessary for large structures with numerous member groups.

4.5.1 Results: Pitched Roof Frame

The pitched roof frame requires one strength and three deflection iterations to produce the optimal solution. The final horizontal and vertical deflections in the structure are 3.9 and 20.1 mm. This satisfies the target deflection constraints of 20 and 25 mm. Note that target deflections are seldom met exactly. Table 2 summarizes the optimization results of the pitched roof frame.

Figure 6 illustrates the increase in mass as the deflection of node 3 decreases with each iteration.

Results obtained by the VWOM and those obtained by Elvin and Walls (2010) are compared in Table 3. The results obtained here are 10.7% lighter. This is due to a new strength algorithm. The current strength algorithm ensures the lightest new member is chosen from the database. Elvin and Walls (2010) chose the next suitable member, which was not always the lightest.

4.5.2 Varying the Rotational Stiffness of the Joints

The connection rotational stiffness of all joints is varied from $k = 15 \times 10^3$ kNm/rad to infinity (rigid), as indicated in Fig. 7. The three connection stiffness zones from Eurocode 3 (2005), "Pinned", "Semi-Rigid" and "Rigid" are indicated in the figure.

It is evident that the mass of the optimized structure decreases in steps as the rotational stiffness of the joints are increased. The stiffer joints assist in resisting deflection.

Although a lot more work is required, Fig. 7 suggests that the boundary definitions, and perhaps the method of definition, of "semi-rigid" and "rigid" are not correct.

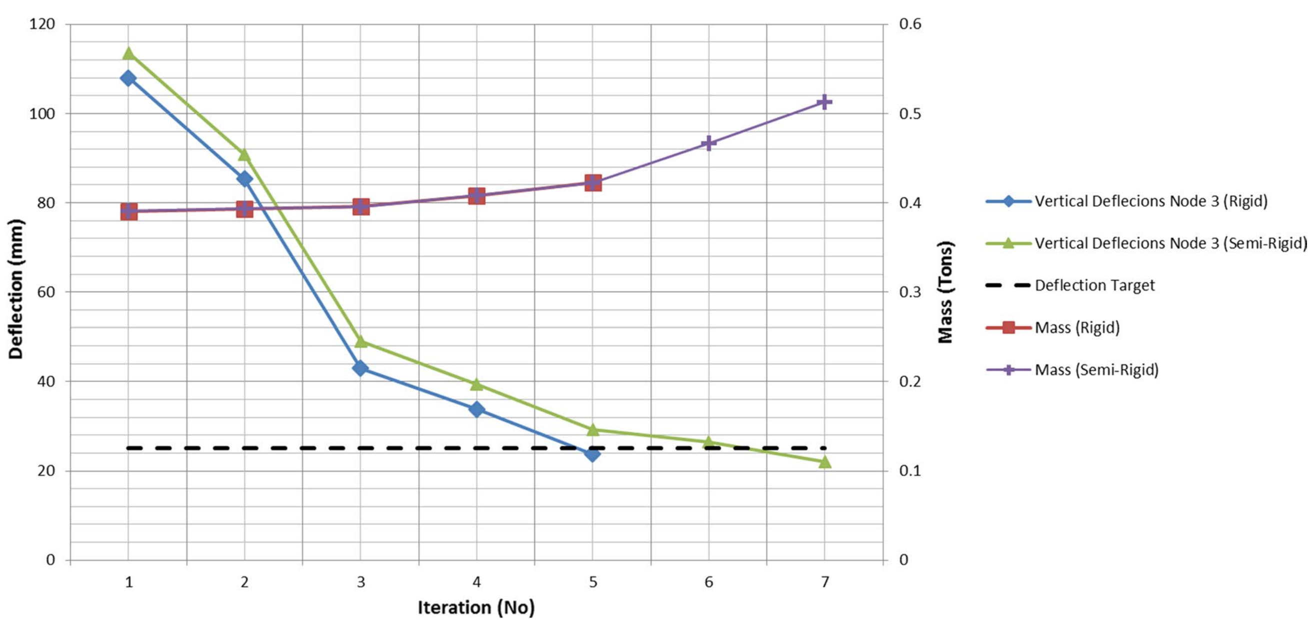

Fig. 6 Portal frame: Deflection vs iteration for Node 3

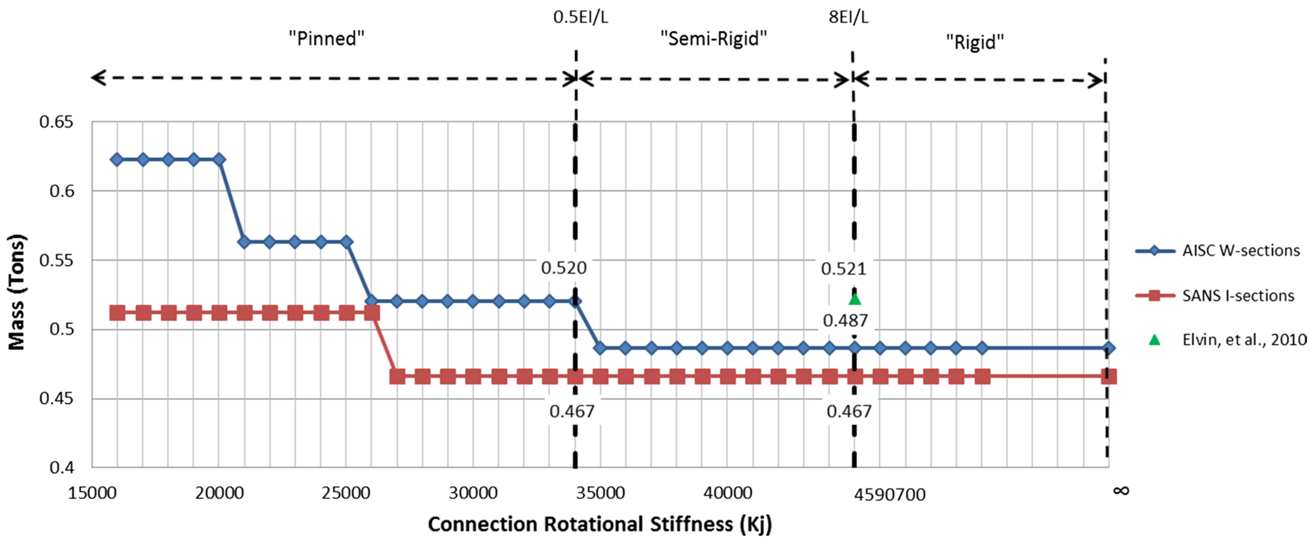

Fig. 7 Pitched roof frame, with semi-rigid connections—Mass versus connection stiffness

Fig. 8 Three-storey, two-bay building [after Simoes (1995)]. Member groups are identified by the numbers in rectangles. Numbers in circles refer to node numbers

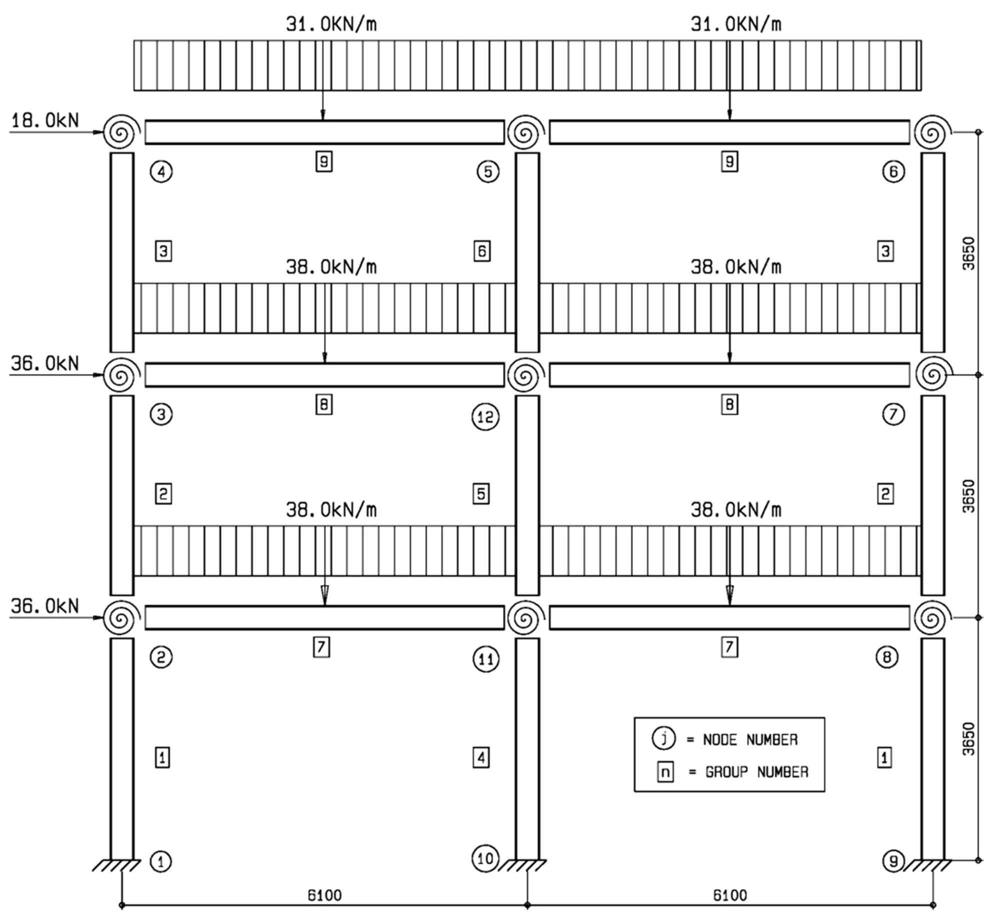

Table 4 Three-storey, two-bay building- Summary of results

Mass (tons)		
Connection type	Semi-Rigid (0.5*EI/L*)	Rigid (8*EI/L*)
Simoes (1995) (IPE sections)	4.210	3.752
VWOM (IPE sections)	3.127	3.10
VWOM (IPE sections, full database)	3.010	3.00

Table 5 Three-storey, two-bay building- Final design sections

European wide flange beams				
Group no	Semi-Rigid connections (0.5*EI/L*)		Rigid connections (8*EI/L*)	
	Simoes (1995)	VWOM	Simoes (1995)	VWOM
1	IPE 360	IPE 300	IPE 360	IPE 300
2	IPE 330	IPE 300	IPE 330	IPE 300
3	IPE 300	IPE 300	IPE 330	IPE 300
4	IPE 550	IPE 450	IPE 500	IPE 360
5	IPE 330	IPE 330	IPE 330	IPE 300
6	IPE 240	IPE 270	IPE 240	IPE 240
7	IPE 360	IPE 330	IPE 450	IPE 330
8	IPE 360	IPE 330	IPE 400	IPE 330
9	IPE 330	IPE 300	IPE 360	IPE 300
Mass (Tons)	4.210	3.127	3.752	3.086
Difference	26.7%		17.1%	

5 Case Studies

The optimization of two different structures are considered to demonstrate the VWOM: (1) a three-storey, two-bay building and (2) a four-storey, three-bay office building. These structures were chosen as they have been optimized in the literature. Here the examples are optimized using the VWOM assuming grouping and material properties identical to those found in the literature. Thus the grouping itself might not be optimal. Structures are designed in accordance with the South African Steel Code (SANS 1062-1 2005).

5.1 Case Study 1: Three-Storey, Two-Bay Building

Simoes (1995) optimized a three-storey, two-bay office building with semi-rigid jonts. Figure 8 illustrates the loading, geometry and grouping of the structure as reported by Simoes (1995). Horisontal deflection is limited to height/400. Simoes (1995) utilised the Segmental Optimum Design method, which uses linear programing, to optimize the structure.

Results obtained by Simoes (1995) are compared to the VWOM. This reference used IPE sections with a yield stress of 355 MPa and a Young's modulus of 206 GPA.

Results obtained by the VWOM and by Simoes (1995) are summarized in Table 4. The structure was optimized using both the standard IPE Section and the extended IPE Section databases.

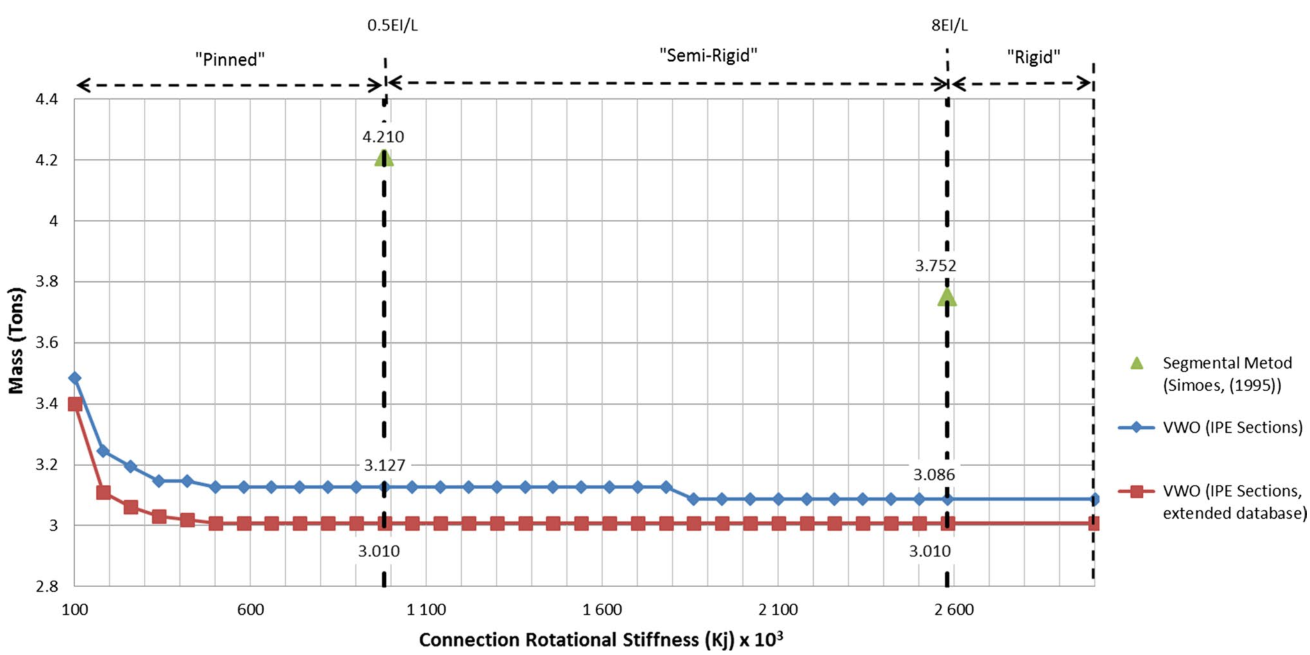

Fig. 9 Three-storey, two-bay building—Mass versus connection stiffness

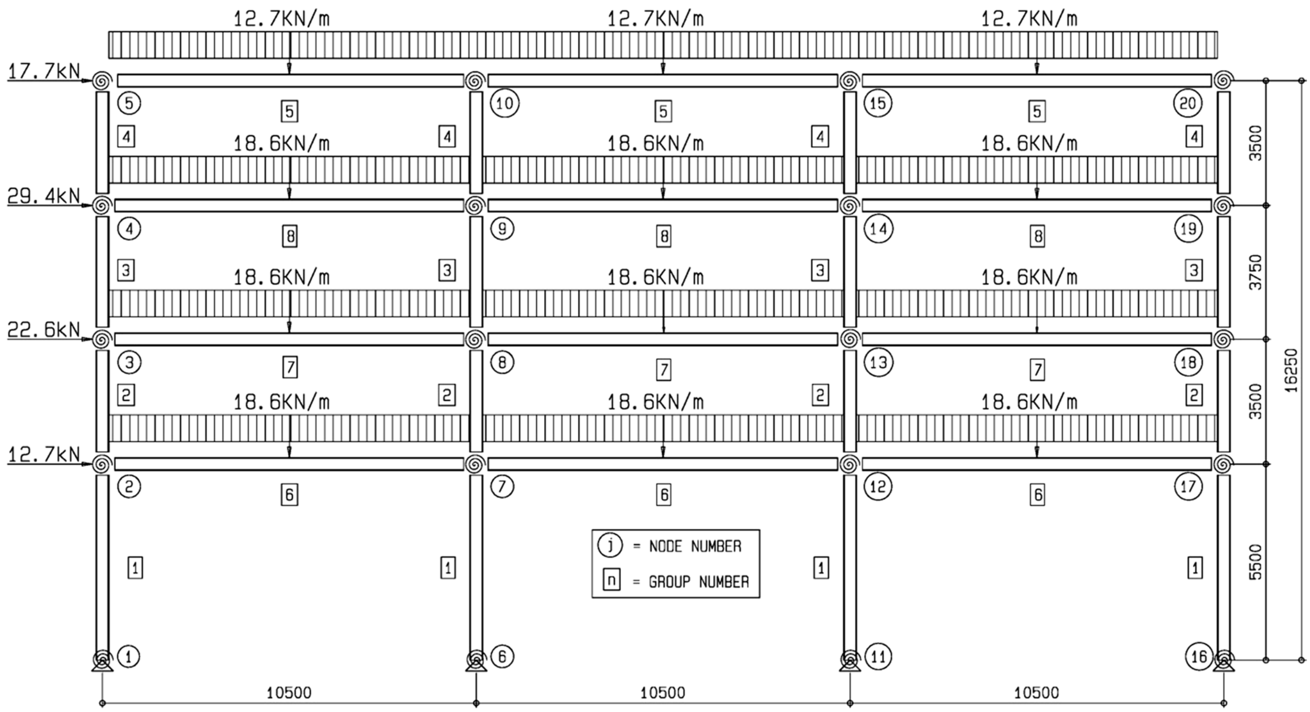

Fig. 10 Four-storey, three-bay office building with semi-rigid joints (Hayalioglu et al. 2004). Member groups are identified by the numbers in rectangles. Numbers in circles refer to node numbers

Note that Simoes (1995) only produced results for two connection stiffness parameters, i.e. rigid and semi-rigid. Results obtained by the VWOM are 26.7 and 17.1% lighter than obtained by Simoes (1995) for the semi-rigid and rigid cases.

Final design sections obtained by Simoes (1995) and the VWOM are given in Table 5.

If the connection rigidity is varied, and the structure is optimized for each connection stiffness, Fig. 9 is produced. Results obtained by Simoes (1995) are included for comparison.

It is interesting to note that Fig. 9 exhibits characteristic steps as the connection stiffness of connections in the structure is increased. As expected, the higher the rotational stiffness of the joints, the lower the optimized mass of the structure. However, the cost of the connections increase as the connection rotational stiffness increases, since rigid connections are more labor intensive, or larger in size and require thicker plates and more welds. Also note that the VWOM optimized results for the given structure with semi-rigid joints is only 1.32% heavier than for the rigid case.

5.2 Case Study 2: Four-Storey, Three-Bay Office Building

Hayalioglu et al. (2004) optimized the three-storey, single-bay frame shown in Fig. 10. The geometry, loading and member grouping used are as indicated in Fig. 10. The Young's modulus of steel was taken as 206 GPa. The inter-storey drift was limited to height/320.

Hayalioglu et al. (2004) used the Turkish Building Code for Steel Structures (TS648 1980) to design the members in an interacive optimization process. This process gives the designer the chance to change parameters such as connection rotational stiffness's and member cross-sectional areas. The results obtained using VWOM are compared to the results obtained by Hayalioglu et al. (2004). In this case study, strength design is in accordance with The South African

Table 6 Results comparison for the four-storey, three-bay office building

Mass (tons)		
Sections	European wide flange beams	
Connection type	Semi-Rigid (0.5*EI/L*)	Rigid (8*EI/L*)
Hayalioglu et al. (2004)	10.482	7.870
VWOM	7.835	7.600

Table 7 Final design sections for the four-storey, three-bay office building

European wide flange beams				
Group no	Semi-Rigid connections (0.5*EI/L*)		Rigid connections (8*EI/L*)	
	Hayalioglu et al. (2004)	VWOM	Hayalioglu et al. (2004)	VWOM
1	HE 500AA	HE 260AA	HE 400AA	HE 260AA
2	HE 340AA	HE 320AA	HE 300AA	HE 320AA
3	HE 340AA	HE 280AA	HE 300AA	HE 240A
4	HE 200AA	HE 240A	HE 220AA	HE 240A
5	HE 240AA	HE 260AA	HE 500AA	HE 220AA
6	HE 280AA	HE 260AA	HE 340AA	HE 260AA
7	HE 280AA	HE 260AA	HE 340AA	HE 220AA
8	HE 280AA	HE 180A	HE 340AA	HE 180AA
Mass (Tons)	10.482	7.835	7.870	7.600
Difference	25.3%		3.4%	

code (SANS 1062-1 2005). The European wide flange beam (i.e., HE sections) database is used. The steel is assumed to have a yield stress of 235 MPa.

The VWO process required 19 deflection iterations to optimize the structure. The number of section changes per iteration was set to one. The results are compared in Table 6. The VWOM solutions for semi-rigid and rigid connections are 25.3 and 3.4% lighter than those obtained by Hayalioglu et al. (2004).

Design sections as optimized by the VWOM and Hayalioglu et al. (2004) are listed in Table 7.

Figure 11 plots the mass of the structure versus the connections' stiffness. The connection rotational stiffness is varied from $k = 60 \times 10^3$ kNm/rad to infinity, i.e. rigid.

As before, it is apparent that the higher the rotational stiffness of the joints the lower the optimized mass of the structure. Note that the optimized structure with semi-rigid joints is only 3.1% heavier than the structure with rigid joints.

6 Conclusion

This paper presented an automated, iterative method for optimizing structures subject to multiple deflection criteria and load cases with semi-rigid connections. The method minimizes overall structural mass while satisfying strength and deflection criteria. The principle of virtual work guides the process of making the most efficient changes when selecting members to resist deflections.

The two case studies, (1) a three-storey, two-bay and (2) a four-storey, three-bay buildings demonstrate the efficacy of the VWOM in optimizing structures, while also satisfying strength criteria and taking semi-rigid joints into account. The VWOM produced results up to 26.7% lighter than those

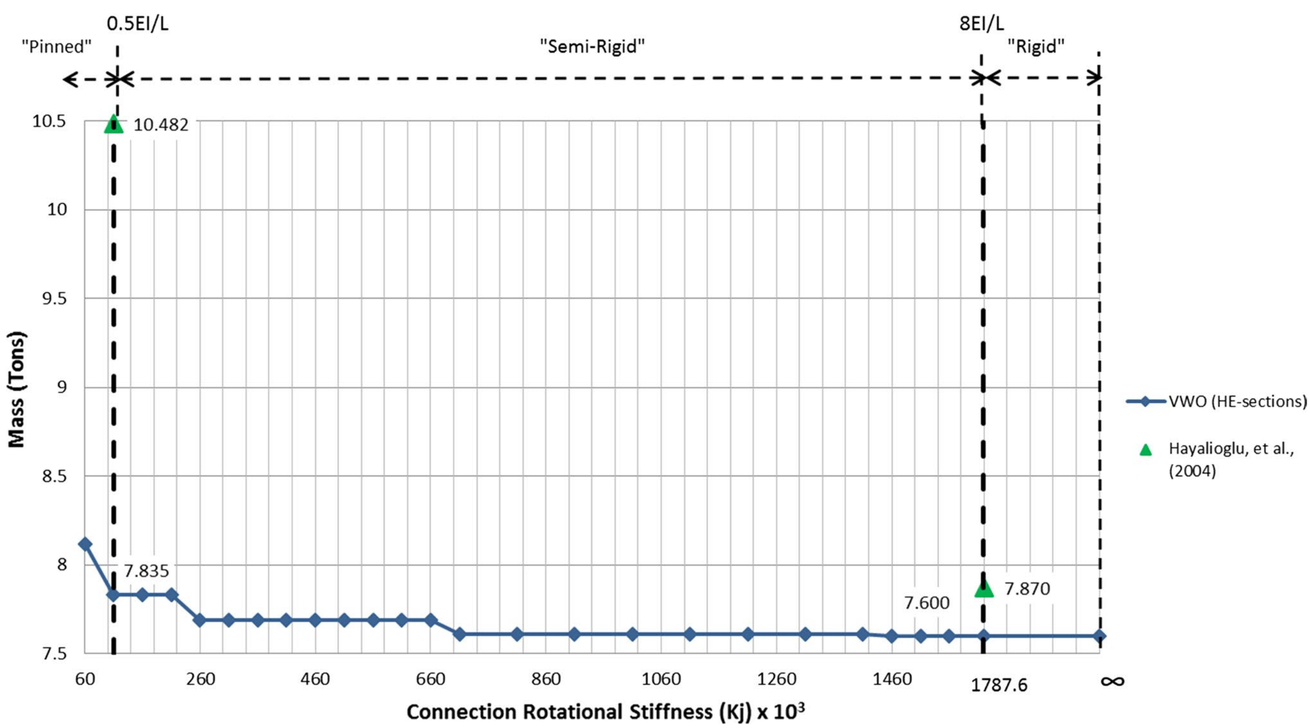

Fig. 11 Four-storey, three-bay office building –Mass versus connections stiffness

found in the literature. The method does not require calibration and is applicable to all structures.

As the rotational stiffness of the joints is increased (from Zone 2 to Zone 1 as defined in Eurocode 3 (2005) classification), the cost of connections increases. At the same time, the optimized mass of the structure decreases. The difference in optimized mass between structures with rigid and semi-rigid joints, defined in Eurocode 3 (2005), can be as little as 3.1%.

Future research will concentrate on studying grouping the connection rotational stiff nesses. Minimizing structures' cost as opposed to mass will be investigated. The section database entries and their distribution have an effect on the solution obtained. This effect should be characterized. Finally, research will be conducted to determine the optimization path followed and how this can be improved.

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References

Barrar, C., Charney, F., Easterling, W., & Moen, C. (2009). *Structural optimization using the principle of virtual work and an analytical study on metal buildings*. Virginia: Virginia Polytechnic Institute and State University.

Bathe, K. (1996). *Finite element procedures*. New Delhi: Askotek. Ghosh, PHI Learining Private Limited.

Chan, C. (1992). An optimality criteria algorithm for tall steel building design using commercial standard sections. *Structural Optimization, 5,* 26–29.

Chen, W., & Lui, E. (1991). *Sustainability design of steel frames*. Bocaration: CRC Press Inc.

Elvin, A., & Walls, R. (2010). Optimizing structures subjected to multiple deflection constraints and load cases using the principle of virtual work. *Journal of Structural Engineering, 136*(11), 1444–1452.

Elvin, E., Walls, R., & Cromberge, D. (2009). Optimising structures using the principle of virtual work. *Journal of the South African Instruction of Civil Engineers, 51*(3), 11–19.

Eurocode 3, M.-R. J. T. (2013). s.l.: British Standards Institute.

Eurocode 3. (2005). *Design of steel structures—part 1-8: Design of Joints*. Brussels: European Committee for Standardisation.

Hayalioglu, M. S., Degertekin, S., & Gorgun, H. (2004). Design of simi-rigid planar steel frames according to Trukish steel design code. *Journal of Engineering and Natural Sciences*, *25*(2), 101–116.

Kassimali, A. (2012). *Matrix analysis of structures*. Stamford: Cengage Learning.

Makris, P., & Provatidis, C. (2002). Weight minimisation of displacement-constrained truss structures using a strain energy criterion. *Computer Methods in Applied Mechanics and Engineering, 191,* 2159–2177.

Makris, P., Provatidis, C., & Rellakis, D. (2006). Discrete variable optimization of frames using a strain energy criterion. *Struct Multidisc Optim, 31,* 410–417.

Park, H., & Park, C. (1997). Drift control of high-rise buildings with unit load metho. *The Structural Design of Tall Buildings, 6,* 23–25.

Patnaik, S., Gendy, A., Berke, L., & Hopkins, D. (1997). Modified fully utilized design (MFUD) method for stress and displacement constraints. *NASA Technical Memorandum*, *4743*, 1–24.

SANS1062-1 (2005). *The structural use of steel. Part 1: Limit-state design of hot-tolled steelwork. South African National Standard.* s.l.: South African National Standard.

Simoes, L. (1995). Optimiztion of frames with semi-rigid connections. *Computers & Structures, 60*(2), 531–539.

Simple joints to Eurocode, 3. (2011). s.l.:British Standards Institute.

Steenhuis, M., Gresnigt, N., & Weynand, K. (1994). *Pre-design of semi-rigid joint in steel frames* (pp. 131–140). s.n.: Chxech Republic.

The Green Book. (2012). *Structural steel connections—fifth edition.* s.l.: South African Institute of Steel Construction.

The Red Book. (2005). *South African steel instruction handbook—fifth edition.* s.l.: South African Institute of Steel Construction.

TS648. (1980). *Building code for Steel Structures.* Ankara: Turkish Institute of Standards.

Weynand, K., Jaspart, J., & Steenhuis, M. (1998). Economy studies of steel building frames with semi-rigid joints. *Journal of Constriction Steel Research, 46*, 1–3.