

TALK AND WRITING: TWO DISTINCT MODES OF DOING MATHEMATICS

NICK TAYLOR

Funda Centre, Soweto

1. TALK AND LEARNING

The new G.C.E. mathematics curricula due to be implemented in Britain in 1991 will make explicit provision for the introduction of a number of practices never before formally addressed in the curriculum. The National Criteria, to which all new syllabuses must confirm, specify that teachers will be required to assess the abilities of their students to "respond orally to questions about mathematics, discuss mathematical ideas and carry out mental calculations"(1). Not only is an important role thus accorded to talk in mathematics learning but this quote establishes an implicit link between speech and mental processes.

The importance of verbal processes in learning was first systematically investigated by Vygotsky(2) in the early 1930s. He and his collaborators carried out a series of experiments on the perception, learning and problem solving behaviour of children. Vygotsky observed that in the preverbal stage children's use of tools in solving a problem resemble that of apes, but as soon as the child is able to speak, speech not only becomes an integral part of the solution, but the action becomes transformed and organised along entirely new lines. The more complex the task, the more important the role of speech becomes, and attempts to block this speech are either futile or cause the child to "freeze up".

Vygotsky distinguished between verbal appeals to the object of attention, speech directed toward the adult conducting the experiment (communicative speech) and what Piaget described as egocentric speech (a stream-of-consciousness running commentary). The

interaction between the child and the material world is seen as proceeding through another person; learning is inextricably embedded in a social context and is mediated, directed and structured through speech:

"An intrapersonal process is transformed into an interpersonal one. Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level ... This applies equally to voluntary attention, to logical memory and to the form of concepts. All the higher functions originate in actual relations between human individuals."(3)

In contrast to the views of developmental and cognitive psychology, on the one hand, which view speech as a later acquisition by a subject already capable of directed cognitive activity, and behaviourism, on the other hand, which assumes the tabula rasa view of mind, Vygotsky posits an inextricable link between hand, speech and mind. Speech and other signs are seen as the mediators between thought and action, extending both the hand through the purposive use of tools, and the mind through the structuring of concept formation and memory.

But what is the trigger for, or motivating force behind the development of these psychological functions? Vygotsky distinguishes between what children are capable of doing on their own, and what they can achieve with the assistance of others. He defines the difference between these two levels as the Zone of Proximal Development: this can be seen as a tension between the actual state of development and the immediate potential of the learner. The Zone of Proximal Development is created by learning, through the awakening of a variety of internal developmental processes, which are brought to maturity through interaction between the child and others in her environment.

These ideas find a perfect echo in Barnes' description of a classroom sequence in which a group of 13-year-olds discuss a question set by their science teacher. He concludes:

"In this paragraph we see four of them applying their existing understanding

to the set problem and, through a serious engagement with each other's accounts, clarifying for one another and for themselves some concepts their physics teacher had presented to them in an earlier lesson." (4)

For Barnes, such classification is not an intellectual luxury, but essential to a proper understanding of the concepts by pupils. He also describes how open-ended teacher-initiated questions invite pupils to think aloud, and through the give and take of reciprocal discussions, try out new concepts and modify them in response to the teacher's replies.

In his experimental investigation of the transition from 'interpsychological' to 'intrapsychological' processes, Wertsch points out that Vygotsky and his followers "... do not think it profitable to separate verbal output and study it in isolation. Rather, they insist that speech can be understood only if it is viewed as being part of an ongoing human activity." (5) Thus a whole range of communicative strategies are used by mothers in guiding their 2.1/2 - 4.1/2 year old children through the task of assembling a picture puzzle. The role and nature of speech undergoes a transformation and development in the unfolding of this activity, which Wertsch characterises as a Wittgensteinian language-game.

Wertsch identifies four planes or points along the developmental path along which the child must pass from other- to self-regulation. At first she must learn to interpret the utterances and other signs of her mother: the child must define the task situation in terms of the communicative gestures, words, intonations, etc. of the adult. Later, the child will respond to instructions and suggestions, but will not have developed a sufficient sensitivity to all the inferences and implications of the adult's communicative strategies. In the third stage, the child's interpretation of the adult's utterances has reached a sophisticated level: she is able to follow quite non-explicit directives and is aware of many of the finer rules of the language game. In the fourth stage, the child is operating on the intrapsychological plane, having taken over the regulatory function formerly performed by the adult. The Zone of Proximal

Development for that particular task has been eliminated, and the child's speech is almost entirely egocentric, much of it assuming the same form as utterances in the adult-child interactions of the three earlier planes.

For Bernstein (6) the social aspects of learning do not occur as a set of operations which are constant for all individuals: the family background and other social relations determine to a large extent the perceptions and linguistic codes adopted by any individual. Class plays a strong regulating function in determining whether a subject is oriented towards expressing individual intent, opinions and feelings in a verbally explicit form, or whether this facility is inhibited. The first orientation Bernstein defines as the elaborated code characteristic of the middle class child, while the latter is more commonly adopted by working class children and is labeled a restrictive code. The respective code then determines how and what children learn:

"The net effect of the constraint of a restricted code will be to depress a potential linguistic ability, raise the relevance of the concrete and descriptive levels of response and inhibit generalizing ability in the higher ranges." (7)

The application of Bernstein's class differential to societies in which social stratification is not as rigid as in Britain, has been questioned.(8) But in South Africa there exists a diversity of interpenetrating cultures and sub-cultures, each characterised by a language, dialect or sub-dialect reflecting a particularistic code of communication and behaviour. In this situation, the discourses of science, commerce, academics, computer and other technology, etc., sustained by elaborated linguistic codes play a prominent role in regulating admission to or exclusion from the privileged elite. As Bernstein has pointed out, class is only one of many principles of social stratification and differentiation.

Mathematical knowledge plays a central role in many of these elaborated discourses. In recent years the perception of mathematics as the epitome of neutral, universalistic,

'objective' knowledge has been seriously undermined; this challenge is given a specifically South African context by writers such as Breen(9), Taylor, et al.(10), and Adler(11). Reformist reaction to these initiatives, while differing on the choice of strategies for correcting past imbalances, do not question the need for a thorough reassessment of mathematics education in South Africa (12).

It is not my intention to further the debate here; the point I wish to demonstrate in this paper is that discussion assists in 'doing' mathematics and that the effects of this process are inversely correlated with the ability to communicate verbal and other means. Even if mathematics education is radically restructured so as to eliminate its role as gatekeeper to the professions, and made accessible and relevant to the majority, any future South African society will need citizens artful in the deployment and application of both formal and applied mathematical knowledge.

I want to argue that mathematics learning is facilitated by talk. Through the development of elaborated linguistic codes by means of discussion, school knowledge will become more accessible to a greater number of people.

2. SETTING OF THE STUDY

This paper is based on an extract from a larger study aimed at investigating the effects of visual elements in an educational TV programme on the mathematical performance of a class of Std. 6 students in Soweto.

The TV intervention was preceded by a first set of observations, and followed by a second set. Each set of observations consists of a written test, followed by a personal interview with four of the students. Corresponding questions in the two written tests were matched with respect to task and degree of difficulty.

During each interview, the student was presented with his or her written test and asked to explain how the respective answers had been obtained. The same students were interviewed during each set of observations, the interviews recorded on audio tape and the recordings transcribed.

The parent study is concerned with differences between the two sets of observations; in studying these I became aware of a number of intriguing differences between the written and interview responses within each set of observations. This paper focuses on these intraobservational differences.

3. EXAMPLES OF THE INFLUENCE OF TALK ON MATHEMATICAL PERFORMANCE

A classification of the number and types of differences between the written and verbal responses for the four students interviewed is given in Table 1.

TABLE 1. Changes between written and interview responses during 1st and 2nd set of observations.

3.1 CAMEL

WRITTEN TESTS

1st Put a ring around the bigger fraction in each of the pairs below. The first one is done for you.

a) $\frac{1}{3}$ $\left(\frac{2}{3}\right)$	b) $\frac{4}{9}$ $\left(\frac{7}{9}\right)$	c) $\frac{2}{3}$ $\left(\frac{2}{4}\right)$	d) $\left(\frac{1}{4}\right)$ $\frac{3}{8}$	e) $\frac{58}{59}$ $\left(\frac{59}{60}\right)$
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2nd Put a ring around the bigger fraction in each of the pairs below.

a) $\frac{1}{4}$ $\left(\frac{3}{4}\right)$ b) $\frac{3}{7}$ $\left(\frac{5}{7}\right)$ c) $\frac{3}{5}$ $\left(\frac{3}{4}\right)$ d) $\left(\frac{1}{5}\right)$ $\frac{3}{10}$ e) $\frac{89}{90}$ $\left(\frac{90}{91}\right)$

	Camel		Paul		Valeria		Brenda	
	1st	2nd	1st	2nd	1st	2nd	1st	2nd
Incorrect written answer changed to correct answer	2	2	0	0	1	0	0	0
Incorrect written answer improved upon in interview	2	1	2	2	1	0	0	0
Total number of positive changes during interviews	4	3	2	2	2	0	0	0
Correct written answer altered to incorrect answer	0	0	1	0	1	0	0	1
Number of questions compared in written test and interview modes	19	16	23	22	21	21	21	20
Score on written test (%)	44	44	59	56	41	38	41	47
Rank on written test (N = 35)	2	8	1	1	3	9	3	5

Table 1. Changes between written and interview responses
during 1st and 2nd set of observations

INTERVIEWS

1st interview

3 N: I see. Right let's try question 5. And let's do 5(c) first.

C: ----- Er, two quarters is the biggest here.

N: How do you know that?

C: Because two-quarters is bigger than two-thirds.

N: How do you work that out, Camel? (Long silence while he performs the following calculation; photostat copy reproduced below)

$$\begin{array}{r} 2 \times 4 = 8 \\ 3 \times 4 \quad 12 \end{array} \quad \begin{array}{r} 2 \times 3 = 6 \\ 4 \times 3 \quad 12 \end{array} \quad \begin{array}{r} 8 \\ 12 \end{array} - \begin{array}{r} 6 \\ 12 \end{array}$$

C: Two-quarters is wrong, and two-thirds is right.

N: Okay. Alright. I'm just going to put here question 5 next to this working. That's very nice Camel.

2nd Interview

N: Have a look at 5(d) Camel. (Silence while he writes the following calculation).

$$\begin{array}{r} 1 \times 4 = 4 \\ 5 \times 4 \quad 20 \end{array} \quad \begin{array}{r} 3 \times 2 \\ 10 \times 2 \end{array} \quad \begin{array}{r} 1 \times 2 \\ 5 \times 2 \end{array} \quad \begin{array}{r} 2 \\ 10 \end{array} \quad \begin{array}{r} 3 \\ 10 \end{array}$$

C: I don't know, it's wrong (Obviously referring to his written answer).

ANALYSIS

In each of the two sets of observations, an incorrect answer in the written test is corrected during the interview. Looking at each set of observations in isolation, one may be tempted to conclude that, since Camel is seeing the question for the second time during the interview, it is only natural for him to improve upon his first attempt. But, taking the two sets of observations together, the similarities between the two test answers, on the one hand, and the two interview responses, on the other, are quite striking. It would appear that the interview situation causes his attention to focus on the problem in a qualitatively different way to that which operates when he is working on his own.

The mechanism for this improvement seems to be the demand that he communicate the

process which led to his answer. Although the students were urged to show their calculations on the test paper, Camel preferred to perform the calculations mentally. During each interview, however, when asked to explain his test answers, Camel presented a beautiful written arithmetic refutation of the test response. He clearly has an excellent command of this technique, and one wonders whether he used a mental analogue of the method during the written test, or whether he applied a different method. Unfortunately this question cannot be resolved, but a significant conclusion can be drawn from these observations: when asked to describe his mental processes he produces a better answer than he did when operating on his own.

3.2 PAUL

FIRST INTERVIEW

$$12. \frac{4}{9} = \frac{\triangle}{18} = \frac{\boxed{20}}{\boxed{10}}$$

What number goes in $\triangle$? 6

What number goes in $\square$? 10

N: Okay. Alright, question 12.

P: Question 12? Question 12 I said ----- Question 12 I said six goes into 18 three times. The I ---- in the first block I put 6. Er ----- in the second block I put 10.

N: You put 6 there? Because 6 goes into 18 three times. Why do you say 6 goes into 18 three times?

P:

N: Where does the 3 come from?

P: I make a multiples of 3 and 6.

N: 3 and 6? Why do you do that, Paul?

P: You say ... er ... here meneer. Er ... must suppose to say... er ... er ... 9 into 18 goes twice. Then I must put 9, and I put 6 ...

N: You say 9 goes into 18 twice?

P: Yes, meneer.
 N: And then what do you do?
 P: I put 9.
 N: You put 9 there?
 P: Yes meneer.
 N: In the triangle. And now, in the square?
 P: In the square I put 10.
 N: Where did you get that 10 from, Paul?
 P: I ... I said 10 ... 10 goes into 20 twice and in the square I put 10.
 N: And why do you want it to go twice?
 P: In the first sum I'm sure that I'm supposed to put 20 over 40. Now I put 10. I said I first listed the number that is in is ... below ...
 N: Oh.
 P: Yes, meneer.
 N: So you think it should be 40 there?
 P: Yes, meneer.

ANALYSIS

Paul cannot justify the manner in which he arrived at the answers in the written test. When urged to consider the question more closely, however, he resorts to a highly systematic type of answer, viz:

$$\frac{4}{9} = \frac{\triangle 9}{18} = \frac{20}{\square 40}$$

Furthermore, his 'algorithm' for deducing this answer is not only internally consistent (if the 9 in the triangle is assumed to be correct, then the 40 in the square is correct), but seems to differ from the correct method in one mis-applied step.

Correct argument : 9 times 2 is 18 :

$$\frac{4}{9} = \frac{\triangle}{18}$$

↙ x 2 ↘

therefore 4 times 2 is 8 :

$$\frac{4}{9} = \frac{\triangle 8}{18}$$

↖ x 2 ↗

Paul's Argument:

9 times 2 is 18 :

$$\frac{4}{9} = \frac{\triangle}{18}$$

↙ x 2 ↘

therefore 9 times 2 is 18 :

$$\frac{4}{9} = \frac{\triangle 9}{18}$$

↖ x 2 ↗

Although still not correct, Paul's interview response - based on a clearly articulated algorithm which is very close to the correct method - is considered an improvement over his test answer, for which no systematic derivation is apparent.

3.3 VALERIA

FIRST WRITTEN TEST

9. Thandi and Jill each have slice of bread of the same size. Thandi cuts hers into 6 equal pieces and eats 3 of them. Jill cuts hers into 12 equal pieces and eats 6 of them.

Who eats the most bread? Jill eats the most bread (written)

Why do you think this? Because he cuts into many equal pieces(written)

FIRST INTERVIEW

N: Okay, question 9?

V: Thandi eats the mo the most bread than Jill.

N: How do you know that, Valeria?

V: 'cause sir, Thandi(sic) have the twelve pieces are smaller than the six pieces.

ANALYSIS

Valeria's written answer is based on the assumption that the greater number of pieces represents the larger quantity of bread; this in turn must presume that all the pieces are of equal size. In the interview, she looks at the problem from a totally different perspective: she now realises that when two slices of the same size are cut into six and twelve pieces, each of the six pieces will be larger than each of the twelve pieces. The next step would be to take into account the number of sixths and twelfths which are being compared. Although she has not yet deduced the correct answer, Valeria has achieved the fundamental breakthrough necessary to understanding the problem.

3.4 BRENDA

SECOND WRITTEN TEST

9. Peter and Abdul each have a bar of chocolate of the same size. Peter breaks his into 8 equal pieces and eats 4 of them. Abdul breaks his into 4 equal pieces and eats 2 of them.

Who eats the most chocolate? No one eats the most(written).

Why do you think this? Because the chocolate is the same size difference is that Peter breaks the chocolate into (illegible) pieces

SECOND INTERVIEW

N: Question 9?

B: No one eats more-most.

N: So they eat the same?

B: Yes, because the bar chocolate - they give them the same size

N: Uh-huh, and then?
 B: Please sir ... I'm not sure.
 N: You're not sure?
 B: Yes.
 N: See: Peter breaks his into 8 equal pieces.
 B: Yes.
 N: ... and eats 4 of those.
 B: Yes.
 N: Abdul breaks his into 4 equal pieces and eats 2 of them. Now, you say they'll eat the same.
 B: No.
 N: How ...
 B: They did not eat the same.
 N: They didn't eat the same?
 B: Yes, 'cause Peter eats 4 and Abdul eats 2.
 N: So who eats the most chocolate?
 B: Peter.
 N: Peter eats the most?
 B: Yes.
 N: Okay, he eats 4 pieces and Abdul eats 2 pieces.
 B: Yes.
 N: Are those pieces the same size?
 B: Yes.
 N: They are the same size?
 B: Yes.
 N: Okay. Then ... so you still think Peter eats more?
 B: Yes.
 N: He eats more? Okay.

ANALYSIS

In the written test Brenda gives the correct answer, but changes her mind during the interview, regressing to an incorrect response.

4. FACILITY OF COMMUNICATION

In this section I am interested in describing the relative abilities of the students to communicate. In this context, external speech and the ability to communicate must be distinguished: the latter represents a wider set which includes speech, Camel's written

arithmetic argument quoted under 3.1 above, and the diagrams used by Valeria and illustrated below.

By freezing a conversation, thereby facilitating both a global view and a minute word-by-word analysis, transcripts of personal interviews open the way to the realization of subtle and profound insights not nearly as easily obtainable during the interview itself or from a tape recording of the event. Far more difficult to record on paper are characteristics such as intonation, voice volume, length of pause, gesture, assertiveness, self assurance and language facility. Thus interview must be analysed in conjunction with repeated hearings of the tape recordings.

While wholly in sympathy with Stubbs' ideal that analyses and the data on which they are based should be published side by side in order to motivate the interpretation(13) and allow rival hypotheses to be generated(14), I would contend that it is just not possible to transcribe qualities such as self assertiveness and confidence, upon which most of the character of speech depends. There seems to be no alternative to a novelistic description of these personality factors.

Bernstein based much of his characterisation of the differences between restricted and elaborated codes on a statistical analysis of the pauses - or hesitation phenomena - which punctuated the speech of his subject. For my present purposes such an analysis of the length and frequency of pauses would be inadequate, since much of the significance of these hesitation phenomena resides in their quality and origin. The four children in this study exhibit two types of pause, terminal or linking. Terminal pauses occur in mid-sentence, where the speaker tails off or freezes, unable to continue, and the utterance ends in silence. Linking pauses, which vary in length from between one and ten or fifteen seconds, separate syllables, words, phrases or sentences of the same utterance.

Brenda is the least able to communicate. Her speech is riddled with linking pauses of the order of 5 or 10 seconds, often coalescing into terminal pauses. During the linking pauses she struggles to find words, and the interview is clearly an ordeal for her.

I rank my own performances during the two conversations with Brenda as the worst of the eight interviews. I was often guilty of prompting her to the point of putting words into her mouth, as for example, in my second utterance of the extract quoted under 3.4. Many of my utterances appear as redundant repetitions of her previous statements. In large measure this poor interview technique is traceable to an attempt to penetrate the stone wall of her inability to express herself.

Besides terminating many of her utterances in silence, Brenda often frustrates attempts to probe her mental processes by means of the statements "I don't know ..." (used a total of nine times in the two interviews), and "I was just guessing .." (four times). What is particularly interesting about this type of termination is that it often occurs at the very site where a more articulate interviewee would produce a more constructive answer to that given in the written test. The following comparison illustrates the point.

CAMEL

SECOND WRITTEN TEST

11. Three bars of chocolate must be shared equally between five children. How much will each child get? $\frac{3}{4}$ (written)

SECOND INTERVIEW

N: Tell me how you did question 11, Camel.

C:

N: You said the answer is three-quarters. Where did you get that from? How did you work it out?

C: I said ... er ... when you cut ... er ... three-quarters is wrong. Each children must get a one-fifth.

N: One-fifth?

C: Yes.

N: I see.

BRENDA

SECOND WRITTEN TEST

11. Three bars of chocolate must be shared equally between five children. How much will each child get? $1\frac{1}{2}$ (written)

SECOND INTERVIEW

N: Okay, question 11 Brenda. How do you do that one?

B:Ha ... I don't know this one.

N: You don't know?

B: Yes.

N: Where did you get that one-and-a-half?

B: I was just guessing.

N: You were just guessing?

B: Yes.

N: But what did you think when you guessed?

B: I didn't.

N: You didn't think at all? Where did you get that one-and-a-half from? Is it just any number that you thought of?

B: Yes.

Question 11 is ambiguous in that it is not clear whether the answer required is three-fifths of one bar of chocolate, one-fifth of all the chocolate or one-fifth of each bar. Even if Camel was considering the first of these possibilities, his correct identification of 5 as the denominator marks his interview response as an improvement over his test answer. Brenda's refusal to talk about the problem, on the other hand, ensures that further learning is blocked.

Camel also speaks haltingly, stumbling over linking pauses of the order of to 10 seconds. He communicates decidedly more effectively than Brenda, however: his linking and terminating pauses are less frequent and on only two occasions does he terminate a discussion on a particular problem with "I don't understand this question". In addition, he communicates extremely effectively by means of written arithmetic explanations, as illustrated in 3.1: he uses this technique a total of six times during the two interviews.

Paul has a greater verbal facility and confidence than Camel. While Camel produces his arithmetic algorithms in silence, the extract quoted under 3.3 shows Paul conveying a complex arithmetic argument entirely verbally. Paul's utterance in the following passage aptly illustrates his increasingly sophisticated control of syntax:

PAUL

SECOND WRITTEN TEST

1. This is a loaf of bread. It must be shared equally between 5 children. What fraction of the loaf does each child get? $\frac{2}{10}$ (written)

SECOND INTERVIEW

N: Paul, what I'd like you to do is to try and answer these questions again, but now I want you to talk about it, while you're doing it. So you tell me how you calculated your answers. Will you do that for me, please? And then if you want to write, you can write on this paper, you see. Please. Okay, question 1 Paul.

P: In question 1 I said a loaf of bread ... oh, by the way ... you cut into the slices. You ... you got a ten ... ten slices and ten slices makes a loaf of bread, and then I said five .. five people can share a loaf of bread by getting .. a two-tenth of it.

The passage contains numerous clauses, complexly linked through relations of consequence, qualification and inference. The construction is clearly superior to the kind of staccato phrases, two-clause sentences and single sentence utterances beyond which the

speech of Brenda and Camel. (sic)

In other ways too, Paul is more communicative than Camel and Brenda. Not only are his linking pauses shorter and less frequent, but he is more composed, giving the impression that these pauses are at least as often occasioned by grappling with the mathematical problem as through a painful searching for words. He never terminates a discussion by means of silence or the kind of refusal resorted to by Brenda.

Valeria is quietly spoken, but clearly the most self assured and articulate of the four students. She never terminates a discussion prematurely, while her linking pauses are controlled and deliberate as she carefully considers her words. Nor is Valeria's ability to communicate restricted to verbal facility: on a total of 5 occasions during the two interviews she uses a diagram to illustrate her argument. These elements combine to form the beautifully lucid explanation. (The drawing is progressively constructed during the linking pauses and the unfolding verbal description while the $\frac{1}{5}$ is written during her last sentence).

VALERIA

SECOND WRITTEN TEST

1. This is a loaf of bread. It must be shared equally PICTURE between 5 children. What fraction of the loaf does each child get? $\frac{1}{5}$ (written)

SECOND INTERVIEW

N: Let's start with question 1. If you want to write you can write on there. Don't write on the question paper again, please.

V:I draw this loaf of bread down ...

N: Yes, alright.

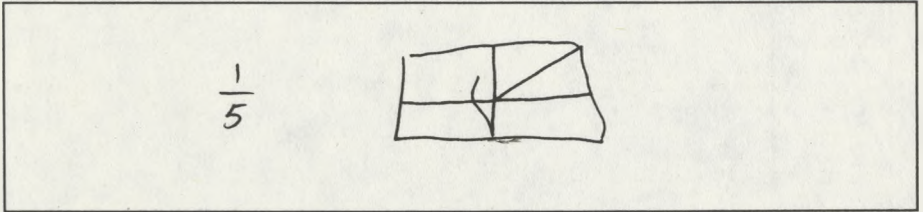
V: Cut it into two equal parts ...

N: Yes?

V: I divide it into five equal parts ...

N: I see.

V: ... and then my answer will be one-over-five.

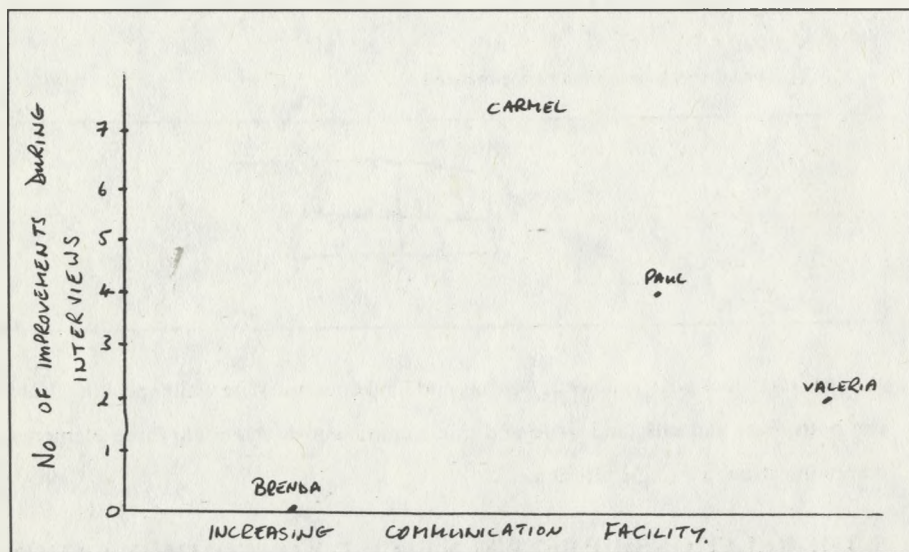


While Camel does not speak while writing, and Paul does not write while speaking, Valeria can both draw and talk, and write and talk simultaneously, blending three elements of communication in a single utterance.

5. THE RELATIONSHIP BETWEEN FACILITY OF COMMUNICATION AND THE DIFFERENCE BETWEEN SOCIAL AND INDIVIDUAL MATHEMATICAL REASONING

The main finding of this study is that, for three of the four students, communicating how he or she solves a set of mathematical problems results in a more successful performance when compared with attempts to solve the same problems when communication with another person is denied. It would appear that articulation of one's processes of solution, primarily through speech but sometimes assisted by written calculations and drawings, produces a clearer focus and more efficient unfolding of these processes, and perhaps even a different choice of method.

This regulatory function of communication does not operate uniformly, but is linked to the facility of communication exhibited by any particular student. If the number of improvements in mathematical performance during the interview is correlated with communication ability, an inversely proportional relationship for Valeria, Paul and Camel



Graph 1. Number of improvements on written test during interview, as a function of facility of communication

emerges. In contrast, the number of improvements shown by Brenda, the least able to communicate of the four, is closest to that of Valeria, the most able. These relationships are illustrated in Graph 1.

The interviews used in the present study differ in certain essential features from the kind of teacher-pupil and pupil-pupil talk which Barnes considers so essential to concept development, and which Vygotsky would presumably have classified as the type of social learning by which the internal development processes are brought to maturation and the zone of proximal development eliminated.

Since they involve the pupil answering questions initiated by an adult in an authority role, the interviews are closer in structure to teacher-pupil interaction than to the mutual give-and-take talk between pupils. But one of the main characteristics of the typical teacher-pupil exchange is missing. Berry (15) described such discourse as consisting of

three main elements: teacher elicitation (dK1), pupil response (K2) and authorisation by the teacher of the pupil response (K1). It is this last element which I studiously avoided during the interviews, thus removing a major strategy for directing the efforts of the pupils in any particular direction. I almost invariably bring the discussion on any particular question to a non-committal closure. ("Okay. Alright"). Perhaps the closest I come to a recognizable K1 is the admiration I express for Camel's effort, at the end of the extract in 3.1 ("That's very nice, Camel"). Even here, however, since I do not tell Camel whether his answer is correct or not, my statement can at best be described as what Muller (16) has described as an underauthorisation.

However, no interview can be neutral. In a study of the dialectic relationship between concept structure and answering questions about the concept, Lehnert and Robertson say: "...modification of prior memories is a natural part of question answering".(17) Since the kind of questions asked in the interview ("Tell me how you calculated ..") differ fundamentally from those asked in the written test ("Calculate ..."), one would expect the two kinds of task to affect the psychological processes of the students in different ways. Thus, because my questions direct the attention of the students to the problems in novel ways, and encourage them to persist with their efforts when they waver, the differences between the interview and test responses must give some indication as to the magnitude of the zone of proximal development.

Camel derives the most benefit from the regulative function of our interviews; he is most dependent on other-regulation. Yet, beside Brenda, he is the least able to communicate, at least in the elaborated code of mathematical discourses.

According to Vygotsky, inner speech is an internalisation of external and egocentric speech. It follows that Camel's lack of facility in external communication reflects a corresponding lack in his self-regulatory processes. Engaging him in external conversation stimulates the

inner speech to function more efficiently than when he is working in isolation.

Valeria, on the other hand, benefits least (excluding Brenda) from other-regulation. She has reached a higher degree of independence in regulating her own solution processes, and this is reflected in her superior communication abilities. Her internal speech operates effectively in reducing the tensions induced by school learning; her Zone of Proximal Development on the tasks set in this study is small.

Paul lies somewhere between Camel and Valeria on Wertsch's continuum between other- and self-regulation, and this could also be predicted from the relative ease with which he maintains external communication.

This brings us to the anomaly of Brenda. Although she is the least able to communicate, she made only one change to her written test performance during the interview, and that in the negative direction. The most likely explanation for this is that her verbal processes or self confidence or both, lie below the minimum threshold necessary for the kind of conversational engagement which will activate her inner processes in working towards a more constructive answer.

6. CONCLUDING REMARKS

The results of this study give support to the view that discussions between teacher and pupils, and amongst pupils stimulate more efficient learning.

Less articulate students benefit most from such practices. Certain particularly uncommunicative students present a problem and need special attention in order to draw them into conversation. The assistance of her peers may play an important role in inducting such a child into the discourse, particularly if the problem is caused by excessive self-consciousness or unfamiliarity with the language of instruction. (18)

In time, as both the inter- and intrapsychological processes are developed through discussion, inner speech becomes increasingly adept at directing the processes of learning and problem-solving.

The U.K. is providing the lead in the incorporation of these ideas into the mathematics curriculum. It would seem that some of South Africa's most pressing problems may derive particular benefit from a move in the same direction.

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