UNIVERSITY OF THE WITWATERSRAND

Thermal Analysis of Continuously Moving Solid and Porous Fins Using Approximate Analytical Methods

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Declaration of Authorship

I, Partner Luyanda Ndlovu, declare that this thesis titled, 'Thermal Analysis of Continuously Moving Solid and Porous Fins Using Approximate Analytical Methods' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.

Signed:

Date: 04 November 2019

"Simple laws can very well describe complex structures. The miracle is not the complexity of our world, but the simplicity of the equations describing that complexity."

Sander Bais.

UNIVERSITY OF THE WITWATERSRAND

Abstract

Faculty of Science
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Doctor of Philosophy

Thermal Analysis of Continuously Moving Solid and Porous Fins Using Approximate Analytical Methods

by Partner Luyanda NDLOVU

In various industrial and engineering applications, fins (extended surfaces) are frequently adopted to enhance the rate of heat dissipation between a system and its surroundings. The heat transfer mechanism of a fin is to conduct heat from a heat source to the fin surface via conduction, and then dissipate heat to the surrounding fluid via convection, radiation, or simultaneous convection-radiation. In order to improve the rate of heat transfer through finned surfaces, it is necessary to understand a fin's dynamic response to change in temperature. The study of heat transfer though fins continues to be of scientific interest and recently, the study of moving fins has attracted a lot of research interests. The study of heat transfer through fins is modeled by differential equations. In search for solutions to differential equations arising in physics and engineering, analytical methods are very useful as it is difficult if not impossible to find the exact solutions. In recent years, the availability of faster processing equipment further means that we are able to compute analytical solutions to highly nonlinear equations that are more accurate in representing the physical phenomena. The modeling of heat transfer through fins reduces the experimental costs and gives insight into various parameters influencing the heat transfer process. In this thesis, the Variational Iteration Method (VIM) and the Differential Transform Method (DTM) are used to solve the nonlinear boundary value problems describing heat transfer in solid and porous fins undergoing convective-radiative heat dissipation. Validation of analytical solutions is also obtained by comparison with numerical solutions. The aim is to derive mathematical models describing heat transfer though fins, analyze the impact of the embedding thermo-physical parameters, compare the accuracy and computational efficiency of these two modern day analytical methods. The study of porous fins is performed using Darcy's model to formulate the governing heat transfer equations. As far as we know, the transient study of heat transfer through moving fins has not been performed anywhere in literature. Related work on finned heat transfer is modeled using steady state models with the assumption that the transient response dies out quickly. Since a broad range of governing parameters are investigated, the results could be useful in a number of industrial and engineering applications.

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Contents

A dedication to my family Shandré and Uminathi.

List of Research Papers

Authors' publications, as submitted for PhD by Publication:

1. P.L. Ndlovu and R.J. Moitsheki, Predicting the temperature distribution in longitudinal fins of various profiles with power law thermal properties using the Variational Iteration Method, *Defect and Diffusion Forum*, 387: 403-416, 2018.
2. P.L. Ndlovu and R.J. Moitsheki, Analysis of temperature distribution in radial moving fins with temperature dependent thermal conductivity and heat transfer coefficient, *International Journal of Thermal Sciences*, 10.1016/j.ijthermalsci.2019.106015, 2019.
3. P.L. Ndlovu and R.J. Moitsheki, Analysis of transient heat transfer in radial moving fins with temperature-dependent thermal properties, *Journal of Thermal Analysis and Calorimetry*, 10.1007/s10973-019-08306-5, 2019.
4. P.L. Ndlovu and R.J. Moitsheki, Thermal analysis of natural convection and radiation heat transfer in moving porous fins, *Frontiers in Heat and Mass Transfer*, 12(7), 2019.
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6. P.L. Ndlovu and R.J. Moitsheki, Effect of variable thermal conductivity calibration functions on fin temperature distribution, *Defect and Diffusion Forum*, 393: 47-58, 2019.
7. P.L. Ndlovu, Numerical analysis of transient heat transfer in radial porous moving fin with temperature dependent thermal properties, *Journal of Applied and Computational Mechanics*, 6(1): 137-144, 2020.
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Authors' previously published work that is related to the current research:

1. P.L. Ndlovu and R.J. Moitsheki, Application of the two-dimensional differential transform method to heat conduction problem for heat transfer in longitudinal rectangular and convex parabolic fins, *Communications in Nonlinear Science and Numerical Simulation*, 18: 2689-2698, 2013.

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4. P.L. Ndlovu and R.J. Moitsheki, Analytical Solutions for Steady Heat Transfer in Longitudinal Fins with Temperature-Dependent Properties, *Mathematical Problems in Engineering*, Vol 2013, Article ID 273052, 14 pages, 2013.

Research conference participation:

1. 12^{th} European Symposium on Thermal Analysis and Calorimetry (ESTAC12), August 2018, Brasov, Romania.

2. Mathematics in Industry Study Group (MISG), January 2012, Cape Town, South Africa.

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Chapter 1

Introduction

In the study of heat transfer, fins are extended surfaces that are used to increase the surface area for heat transfer between a heated body and a cooler ambient fluid. The amount of conduction, convection and radiation of a heat source determines the amount of heat that it transfers. The heat transfer from a heated body can be enhanced by increasing the temperature gradient between its surface and the cooler environment, increasing the convection heat transfer coefficient, or increasing the surface area of the heated body. Research shows that it is not feasible or economically viable to increase the first two [1, 2]. Hence, adding a fin to a heated body increases the surface area for heat transfer and it is found to be an economical solution to heat transfer problems. The use of fins is seen in various engineering applications such as heat exchangers, computer processors, space nuclear reactor power systems, oil carrying pipelines and many more.

The fundamentals of heat transfer analysis from finned surfaces involves solving second-order differential equations in various coordinate systems. The subject of extended surface heat transfer is one where analytical methods have been very successful in providing design information for a variety of geometries, some of which are very complex. The assumptions of constant thermo-physical parameters and uniform heat transfer coefficient reduces the complexities of the governing energy balance equations. These assumptions further allow closed form analytical solutions for a number of cases. This heat transfer coefficient can be modeled, most simply, as a constant, in which the governing second-order differential equation is easy to solve [3]. When the more realistic definition of thermal properties as a function of temperature is employed, the problem becomes nonlinear and is considerably more difficult to solve [4, 5]. This nonlinearity is exaggerated when the solid-fluid interface encounters a phase change in the form of evaporation or condensation. A brief review of published works that is of immediate relevance to this thesis is presented next.

1.1 Literature review

Mathematical models describing heat transfer in fins of different profiles have been studied extensively in literature [6–24]. A detailed study of fin temperature distribution for various embedding parameters is provided for a triangular fin [25], rectangular radial fin [26] and an exponential fin [27]. Fins may be applied in a stationery system [28] or in motion as in the case of an electric motor and bike engines [29]. In recent years, the study of moving fins has drawn the attention of many researchers. Turkyilmazoglu [30] obtained exact formulas for thermal features like the distribution of temperature as well as the efficiency of exponential moving fins. An approximate analytical solution for convection-radiation from a continuously moving fin with temperature-dependent thermal conductivity was developed by Aziz and Khani [31]. They concluded that radiative cooling becomes more effective for stronger radiation and consequently lowering the temperature in the fin. Ming et al. [32] numerically studied conjugate heat transfer from a continuously moving flat plate by employing the cubic spline collocation. The heat transfer of moving fins through a non-Newtonian fluid was first analytically studied by Fox et al. [33]. Recently, Ma et al. [34], simulated a combined conductive, convective and radiative heat transfer in irregular moving porous fins using analytical methods.

The use of porous fins as a passive method for heat transfer enhancement has attracted a lot of research interests following the pioneer work of Kiwan and Al-Nimr [35–37]. The steady state thermal analysis of natural convection and radiation in porous fins was analytically conducted by Moradi et al. [38]. These authors analyzed both infinite and finite fins with rectangular profile using Homotopy Analysis Method (HAM) to solve the governing equations. An analysis of heat transfer in a porous fin with temperature-dependent thermal conductivity and internal heat generation was conducted using spectral collocation by Sobamowo [39]. This author used numerical solutions to investigate the effects of various parameters on the thermal performance of the porous fin. The results showed that increasing the porosity parameter also increases the rate of heat transfer from the base of the fin and consequently improve the efficiency of the fin. Collocation Method (CM) and the Homotopy Perturbation Method (HPM) were presented by Hoshyar et al. [40] to determine the temperature distribution in a porous fin with temperature-dependent internal heat generation. Their results were validated by using the Runge-Kutta fourth order method. They discovered that as the buoyancy effects become stronger, the local temperature in the fin decreases. Following similar work undertaken by Hoshyar et al. [40], Oguntala and Abd-Alhameed [41] applied the Haar Wavelet Collocation Method (HWCM) to analyze the thermal performance of porous fins.

Theoretical studies in extended surface heat transfer are described by nonlinear boundary value problems. In most cases, these problems do not have an exact solution associated with them. As such, much attention has been devoted to the newly developed methods to construct approximate analytic solutions. The applicability of the Collocation Spectral Method (CSM) for solving nonlinear heat transfer problems was demonstrated in a convective-radiative fin with temperature-dependent properties by Sun et al. [42]. Cuce and Cuce [43], studied the the efficiency and effectiveness assessment of longitudinal porous fins using Homotopy Perturbation Method (HPM). A modified Homotopy Analysis Method (HAM) was proposed by Motsa et al. [44]. The newly developed Spectral Homotopy Analysis Method (SHAM) seeks to remove some restrictive assumptions related to the implementation of standard HAM. The Optimal Homotopy Analysis Method (OHAM) was first introduced by Esmaeilpour and Ganji [45] to solve Jeffery-Hamel problem. In this thesis, the VIM and DTM are used to solve the nonlinear boundary value problems describing heat transfer in solid and porous fins undergoing convective-radiative heat dissipation.

1.2 Variational Iteration Method (VIM)

The VIM was introduced by Ji-Huan He [46, 47] in 1999. This method is a modification of the general Lagrange multiplier method and provides analytical solutions to linear, nonlinear, initial and boundary value problems. The method applies a correctional function to the problem which is constructed using a Lagrange multiplier. The initial approximation is determined by the initial condition or one of the boundary conditions. The optimal Lagrange multiplier for the problem is determined by applying the stationary condition to the correction functional and the Lagrange multiplier is chosen based on the order of the problem at hand. The solution procedure is iterative and is improved at each iteration using the previous solution. This generates an infinite series solution which generally converges to the exact solution of the problem. Several problems in fluid mechanics have been solved using the VIM such as, the Euler-Bernoulli beam [48], the evolution equations [49], the gas dynamic equation [50], the KdV equation [51], the Sawada-Kotera equations [52] and the Sturm-Liouville equations [53]. The advantages of the VIM, are that the problem can be solved without any discretization or transformation and is free from round-off errors.

1.3 Differential Transform Method (DTM)

The DTM was first introduced by Zhou [54] in 1986. It is an iterative technique initially designed to solve linear and nonlinear problems in electric circuits. In 1999, Chen and Ho [55], developed a two-dimensional DTM which can be used for solving differential and integral equations. This method generates an analytical solution based on Taylor series expansions. The DTM is based primarily on the Taylor series method, however, at higher orders the DTM differs from the Taylor series method in the way the coefficients are computed. The Taylor series method requires computing coefficients using the initial data and the differential equation, which requires more computational work, while the DTM iteratively obtains the Taylor series equations. The principle behind the method is to apply a differential transform to the original equation. Thereafter, the equation is simplified by applying certain theorems of the differential transform theory [56]. Finally, an inverse differential transform is applied to the simplified equation resulting in an iteration formula for the problem. Ayaz [57], showed that the DTM is better equipped to solve highly nonlinear problems than the Taylor series method. The DTM does not require linearization or discretization and can easily produce closed form solutions [55]. The DTM has been used to solve various problems in applied mathematics and physics, such as systems of differential equations [58, 59], the Schrödinger equation [60] and the Emden-type equations [61]. The drawbacks of the DTM are the small convergence regions of the truncated series solutions and does not exhibit periodic behaviour. Several improvements have been made over the years by Odibat and Momani [62], who generalised the method in order to improve convergence using the Caputo fractional derivative. Momani and Ertürk [63] applied Laplace transforms and Padé approximations to the DTM in order to study the periodic behaviour of the solutions and improve the accuracy of the DTM solution in a larger region. The modifications made above have provided more accurate series solutions when compared to other methods.

1.4 VIM versus DTM

An important advantage of the VIM over DTM is that the method provides successive approximate solutions iteratively, while DTM generates components of the approximate solution and require summation to provide the approximate solution. The ease of computing the correction function in the VIM as compared to applying the differential transform theorems in the DTM shows that the VIM is a simpler and more efficient method. The disadvantage of the VIM is the limited convergence region of the truncated series solution [64]. Abassy et al. [65] showed, for severely nonlinear problems, that VIM may produce unnecessary terms or unneeded computations which may cause a divergence of the solution and increase computation time. These limitations have been addressed by some authors and modified variational iteration methods have been developed over time, such as using the Padé technique and Laplace transforms, to eliminate unnecessary computations [66–68]. These modifications have made the VIM one of the most useful methods in order to obtain analytical solutions to a variety of problems. The application of the two methods described above can be found in a series of publications dedicated to this thesis in the next chapter. The computational efficiency of the two methods is compared on the first paper, and depending on the problem at hand, one of the methods was adopted to analyze heat transfer on the preceding publications.

1.5 Thesis by Publication layout

Following the requirements for a degree of Doctor of Philosophy (PhD) by Publication, this thesis is structured as follows;

- In Chapter 1, a brief overview of extended surface hear transfer is given. The theory behind the need to model heat transfer in moving fins of various profiles is explained. Discussions on applications of analytical methods to solve complex heat transfer problems are given and related previous work is cited.
- In Chapter 2, a series of journal publications relating to the topic at hand are given. The journal articles are presented in a logical manner, where the proceeding article builds from the preceding article.
- In Chapter 3, an overarching conclusion summarizing the key findings and future work is presented.

This thesis incorporates eleven publications in peer-reviewed journals that make a significant contribution to knowledge within the field of extended surface heat transfer.

Chapter 2

Thesis Publications

In this thesis, thermal properties such as heat transfer coefficient and thermal conductivity are considered to be temperature dependent. This assumption results in nonlinear deterministic models that are difficult to solve exactly. Hence, approximate analytical solutions are sought throughout this thesis. The effects of parameters, appearing in dimensionless models, on temperature profile (or temperature distribution) in the fin are studied. The articles in this thesis are linked together and follow in a logical manner to maintain the overall aims of this study.

Heat transfer through solid stationary longitudinal fins of various profiles is studied in Article I. The applicability of VIM and DTM to heat transfer problems is analyzed, followed by the convergence and accuracy analysis of the two analytical methods. Following the results of Article I, steady state analysis of moving radial fins is performed using VIM in Article II. The VIM is applied to generate approximate analytical solutions and the impact of the embedding parameters on heat transfer is illustrated and explained. The transient response to change in temperature on radial moving fins of rectangular and hyperbolic profiles is considered in Article III. The analytical solutions are generated using the two-dimensional DTM and validated using the inbuilt numerical solver in Maple. The effects of porous media on the rate of heat transfer in moving porous fins undergoing natural convection and radiation heat transfer is analyzed in Article IV. The study is performed using Darcy's model to formulate the governing equations. To investigate the impacts of convection, radiation, porosity and advection on heat transfer in a single model, a study of transient heat transfer through a triangular moving porous fin is performed in Article V.

Article I assumes a power law variance of thermal conductivity with temperature. Articles II, III and V assume a linear variance of thermal conductivity with temperature. In Article VI, a logistic thermal conductivity calibration function is considered and compared to other three traditional functions. The proposed function is found to result in lower dimensionless temperatures inside the fin. A highly nonlinear model describing heat transfer in a radial porous moving fin is analyzed in Article VII. The problem is solved numerically and analytically by applying the numerical solver in MATLAB and the two-dimensional DTM respectively. Following the work in Article VII, heat transfer in a rectangular moving fin with temperature dependent surface emissivity is analyzed in Article VIII using the two-dimensional DTM. The efficiency of porous moving fins is demonstrated in Article IX. The analysis is performed using VIM to generate the analytical solutions which are further validated using the Runge-Kutta fourth order method. The significance of a fin profile together with the convective-radiative fin tip boundary condition are investigated in Article X. The impact of fin profile is further investigated for seven different fin profiles in Article XI. The fin surface is assumed to be gray with a constant emissivity to simplify the governing model.

Original copies of articles briefly described above are presented next and a detailed discussion on their contribution is provided in the conclusions chapter.

2.1 Journal Article I

P.L. Ndlovu and R.J. Moitsheki, Predicting the temperature distribution in longitudinal fins of various profiles with power law thermal properties using the Variational Iteration Method, *Defect and Diffusion Forum*, 387: 403-416, 2018.

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Predicting the Temperature Distribution in Longitudinal Fins of Various Profiles with Power Law Thermal Properties Using the Variational Iteration Method

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Abstract. In this article, the variational iteration method (VIM) is used to analyze heat transfer in longitudinal fins of various profiles with temperature dependent thermal conductivity and heat transfer coefficient. In order to show the efficiency of the VIM, the results obtained using the VIM are compared with the previously obtained results using the differential transform method (DTM) and Lie group analysis of a fin problem with the rectangular profile. After establishing confidence in VIM, heat transfer is analyzed to determine the temperature profiles in longitudinal fins of various profiles with power law thermal properties. The temperature distribution is compared between various profiles. The effects of some physical parameters such as the thermo-geometric fin parameter and thermal conductivity gradient on temperature distribution are illustrated.

Introduction

A finned surface has an enhanced mechanism of transferring heat between a primary surface and its surrounding medium. In particular, fins are used in various industrial applications such as the cooling of computer processors, air-cooled automobile engines, oil carrying pipe lines and many more. The study of heat transfer though fins has been documented extensively in literature by Kraus *et al.* [1]. The study of heat transfer in fins continues unabated. Exact solutions for special cases are obtained for example in [2, 3].

The use of analytical methods to solve linear and nonlinear differential equations arising in heat transfer has been accepted by many researchers. In recent years, different methods have been introduced to provide semi-analytical solutions to heat equations. The Homotopy Perturbation Methods have been studied and applied by many researchers [4, 5]. Perturbation Methods depend on small parameter values which are difficult to be find for real life non-linear problems. The Adomian Decomopsition Method (ADM) [6], Homotopy Analysis Method (HAM) [7], Differential Transform Method (DTM) [8, 9] and the Variational Iteration Method (VIM) [10] have also been used to provide analytical solutions for these kinds of differential equations and in the determination of fin efficiency. The VIM was introduced by He in 1999 [11, 12]. This method has been widely used by may researchers literature due to its faster convergence and easy applicability.

A substantial amount of research work in the study of heat transfer through finned surfaces focuses on the case where thermal conductivity depends linearly on temperature and constant heat transfer (see e.g. [13]) and power-law temperature-dependent heat transfer coefficient (see e.g. [14]). Approximate analytical solutions for heat transfer in convective straight fins with temperature dependent thermal conductivity and constant heat transfer were constructed using VIM in [15]. The efficiency of steady state heat transfer in longitudinal moving fins with trapezoidal profile is studied in [16].

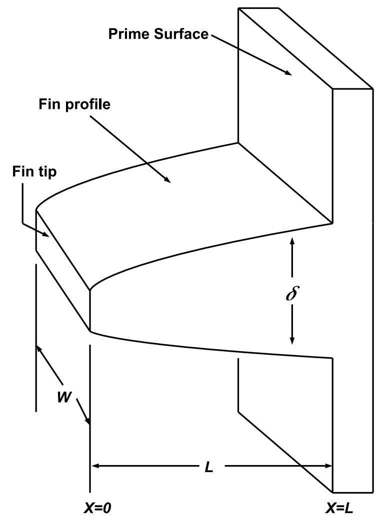

Fig. 1: Schematic representation of longitudinal fin of an unspecified profile.

In this paper, the variational iteration method is used to solve the nonlinear boundary value problem describing heat transfer through an extended surface where the thermal conductivity and heat transfer coefficient are given by power law functions of temperature. The shortcomings of both analytical methods are also noted and the VIM results are compared with the previously obtained results using DTM. The VIM has no transformations, perturbations and decompositions which makes it easy to apply to nonlinear equations arising in heat transfer.

This article is organized as follows; the mathematical background of the problem under consideration is described in Section 2. A brief discussion on the fundamentals of the VIM will be provided in Section 3. Approximate analytical solutions are constructed in Section 4 and Section 5. Lastly, some discussions based on the results obtained are provided in Section 6.

Fin Problem Formulation

We consider a physical system consisting of a longitudinal one dimensional fin of cross-sectional area A_c as shown in Figure 1. The perimeter of the fin is denoted by P and its length by L. The fin is attached to a fixed prime surface of temperature T_b and extends to an ambient fluid of temperature T_a. Based on the one dimensional heat conduction, the energy balance equation is then given by [1],

$$A_c \frac{d}{dX}\left(\frac{\delta_b}{2}F(X)K(T)\frac{dT}{dX}\right) = PH(T)(T-T_a), 0 \leq X \leq L \tag{1}$$

where K and H are the non-uniform temperature dependent thermal conductivity and heat transfer coefficients, respectively (see [1, 8]), T is the temperature distribution and X is the space variable, $F(X)$ is the fin fin profile. The length of the fin is measured from the tip to the prime surface as shown in Figure 1. Since the fin tip is adiabatic, Eqn. (1) admits the following boundary conditions,

$$T(L) = T_b \text{ and } \left.\frac{dT}{dX}\right|_{X=0} = 0. \tag{2}$$

Introducing the following dimensionless variables,

$$x = \frac{X}{L},\ \theta = \frac{T-T_a}{T_b-T_a},\ h = \frac{H}{h_p},\ k = \frac{K}{k_a},\ \text{and } M^2 = \frac{Ph_pL^2}{A_ck_a},\ f(x) = \frac{\delta_b}{2}F(X), \tag{3}$$

where k_a is the thermal conductivity at the ambient temperature and h_p is the heat transfer at the prime surface, reduces Eqn. (1) to

$$\frac{d}{dx}\left[f(x)k(\theta)\frac{d\theta}{dx}\right] - M^2\theta h(\theta) = 0,\ 0 \le x \le 1. \tag{4}$$

In Eqn. (4) θ is defined as the dimensionless temperature, x as the dimensionless space variable, k is the dimensionless thermal conductivity, h is the dimensionless heat transfer coefficient and M is the geometric fin parameter. The dimensionless boundary conditions then become,

$$\theta(1) = 1,\ \text{at the prime surface} \tag{5}$$

and

$$\left.\frac{d\theta}{dx}\right|_{x=0} = 0,\ \text{at the fin tip.} \tag{6}$$

We assume that the thermal conductivity and heat transfer coefficient are given by the power law

$$H(T) = h_p\left(\frac{T - T_a}{T_b - T_a}\right)^n, \tag{7}$$

$$K(T) = k_a\left(\frac{T - T_a}{T_b - T_a}\right)^m, \tag{8}$$

where exponents n and m are real constants. The values of n may vary as between -6.6 and 5; however, in most practical applications, it lies between -3 and 3 [17]. In dimensionless variables, we have $h(\theta) = \theta^n$ and $k(\theta) = \theta^m$. The fin equation under consideration is then given by

$$\frac{d}{dx}\left[f(x)\theta^m\frac{d\theta}{dx}\right] - M^2\theta^{n+1} = 0,\ 0 \le x \le 1. \tag{9}$$

Notice that equation (9) may be transformed into

$$\frac{d}{dx}\left[f(x)\frac{dw}{dx}\right] - \frac{M^2}{n+1}w = 0,\quad n \ne -1, \tag{10}$$

by change of variable $w = \theta^{n+1}$. Notice that the exact solutions exist for (i) rectangular profile in terms of hyperbolic functions [2] and (ii) exponential profile in terms of Bessel functions [3]. Furthermore, the concave parabolic profile results in an Euler equation, and the triangular and cubic profiles have exact solution given in terms of Bessel functions.

He's Variational Iteration Method

Eqn. (9) can be written as [10]

$$L\theta + N\theta = g(x), \tag{11}$$

where L and N are linear and nonlinear operators, respectively, and $g(x)$ is the source inhomogeneous term. In his papers, [11, 12], He proposed the variational iteration method where a correctional functional for Eqn. (11) can be written as

$$\theta_{n+1}(x) = \theta_n(x) + \int_0^x \lambda(t)(L\theta_n(t) + N\tilde{\theta}_\theta(t) - g(t))dt, \tag{12}$$

where λ is the general Lagrange multiplier, which can be be identified optimally via the variation theory, and $\tilde{\theta}_n$ is a restricted variation, which means $\delta\tilde{\theta}_n = 0$ [18]. The Lagrange multiplier can be a constant or a function depending on the order of the deferential equation under consideration. The variational iteration method should be employed by following two essential steps. First we determine the Lagrange multiplier by considering the following second order differential equation,

$$\theta''(x) + a\theta'(x) + b\theta(x) = g(x), \;\; \theta(0) = \alpha, \;\; \theta'(0) = \beta, \tag{13}$$

where a and b are constants. The VIM admits the use of a correctional function for this equation as follows,

$$\theta_{n+1}(x) = \theta_n(x) + \int\limits_0^x \lambda(t)(\theta_n''(t) + a\tilde{\theta}_n'(t) + b\tilde{\theta}_n(t) - g(t))dt. \tag{14}$$

Taking the variation on both sides of Eqn. (14) with respect to the independent variable θ_n gives,

$$\frac{\delta\theta_{n+1}}{\delta\theta_n} = 1 + \frac{\delta}{\delta\theta_n}\left(\int\limits_0^x \lambda(t)(\theta_n''(t) + a\tilde{\theta}_n'(t) + b\tilde{\theta}_n(t) - g(t))dt\right), \tag{15}$$

or equivalently

$$\delta\theta_{n+1}(x) = \delta\theta_n(x) + \delta\left(\int\limits_0^x \lambda(t)(\theta_n''(t) + a\tilde{\theta}_n'(t) + b\tilde{\theta}_n(t) - g(t))dt\right), \tag{16}$$

which gives

$$\delta\theta_{n+1}(x) = \delta\theta_n(x) + \delta\left(\int\limits_0^x \lambda(t)(\theta_n''(t)dt\right), \tag{17}$$

obtained upon using $\delta\tilde{\theta}_n = 0$ and $\delta\tilde{\theta}_n' = 0$. Evaluating the integral of Eqn. (17) by parts gives,

$$\delta\theta_{n+1} = \delta\theta_n + \delta\lambda\theta_n' - \delta\lambda'\theta_n + \delta\int\limits_0^x \lambda''\theta_n dt, \tag{18}$$

or equivalently

$$\delta\theta_{n+1} = \delta(1 - \lambda'|_{t=x})\theta_n + \delta\lambda\theta_n' + \delta\int\limits_0^x \lambda''\theta_n dt. \tag{19}$$

The extreme condition of θ_{n+1} requires that $\delta\theta_{n+1} = 0$. Equating both sides of Eqn. (19) to 0, yields the following stationary conditions

$$1 - \lambda'|_{t=x} = 0, \tag{20}$$

$$\lambda|_{t=x} = 0, \tag{21}$$

$$\lambda''|_{t=x} = 0. \tag{22}$$

This in turn gives

$$\lambda = t - x. \tag{23}$$

In general, for the n^{th} order ordinary differential equation, the Lagrange multiplier is given by,

$$\lambda = \frac{(-1)^n}{(n-1)!}(t-x)^{n-1}. \tag{24}$$

Having determined λ and substituting its value into (12) gives the iteration formula

$$\theta_{n+1}(x) = \theta_n(x) + \int_0^x (t-x)(\theta_n''(t) + a\theta_n'(t) + b\theta_n(t) - g(t))dt, \tag{25}$$

The iteration formula Eqn. (25), without restricted variation, should be used for the determination of the successive approximations $\theta_{n+1}(x),\ n \geqslant 0$, of the solution $\theta(x)$. Consequently, the solution is given by

$$\theta(x) = \lim_{n\to\infty} \theta_n(x). \tag{26}$$

The Fin Temperature Distribution

To show that VIM has faster convergence over DTM, the results from the two series solutions are benchmarked against the exact solution for the special case $n = m$ [2] for a longitudinal rectangular fin profile. For the case where $n \neq m$ and other fin profiles, we construct the series solutions using He's VIM.

Heat transfer in fins of rectangular profile The exact solution for for Eqn. (9) was calculated using Lie group analysis for a rectangular fin profile where $n \neq m$. The fin profile is rectangular when $f(x) = 1$ on Eqn. (9). As a starting approximation for the VIM solution to Eqn. (9), $\theta_0(x)$ is assumed to be a constant [19]. Considering the case where $m = n = 1$, The first three iterations using the VIM Solution are given below,

$$\theta_0(x) = c, \tag{27}$$

$$\theta_1(x) = c + \frac{M^2c^2x^2}{2}, \tag{28}$$

$$\theta_2(x) = c + \frac{c^2M^2x^2}{2} - \frac{c^3M^2x^2}{2} + \frac{c^3M^4x^4}{12} - \frac{c^4M^4x^4}{8} + \frac{c^4M^6x^6}{120}, \tag{29}$$

$$\theta_3(x) = c + \frac{3c^2M^2x^2}{2} - \frac{3c^3M^2x^2}{2} + \frac{c^4M^2x^2}{2} + \frac{c^3M^4x^4}{4} - \frac{19c^4M^4x^4}{24} + \frac{5c^5M^4x^4}{8} - \frac{c^6M^4x^4}{8} + \cdots \tag{30}$$

$$\vdots$$

The third iteration of the VIM gives higher-order powers of the independent variable. A series solution of order $O(14)$ was found to be close to the exact solution with a very small standard error [9]. The DTM produced a polynomial solution agreeable to the exact solution at the 14^{th} iteration. This is computationally expensive. An alternative and less expensive approximate expression for temperature distribution using VIM for the problem under consideration is then given by

$$\theta(x) \cong \theta_5(x). \tag{31}$$

In Eqn. (27), the constant c is the temperature at the fin tip and must lie in the interval $[0, 1]$. Since c is assumed to be a constant as the initial guess, it automatically satisfies the derivative boundary condition. As seen from succeeding equations (27)-(30), additional terms include x with the power of two or more. This also satisfies the derivative boundary condition at $x = 0$. Due to this fact, to obtain the value of c, we substitute the boundary condition (5) into (31) at the point $x = 1$. Similarly, the recurrence relation of VIM Solution to (9) for $m = n = 2$ is given below,

$$\theta(x) = c + \frac{3c^2M^2x^2}{2} - \frac{3c^5M^2x^2}{2} + \frac{c^7M^2x^2}{2} + \frac{c^5M^4x^4}{8} - \frac{3c^7M^4x^4}{2} + \frac{5c^9M^4x^4}{4} - \frac{c^{11}M^4x^4}{4} + \cdots \tag{32}$$

The constant c may be obtained using the boundary condition at the fin base. The comparison of the exact solutions, VIM solutions and DTM solutions are reflected in Figures 2 and 3 for different values of n. The VIM solutions are closely aligned to exact solutions within the first five iterations.

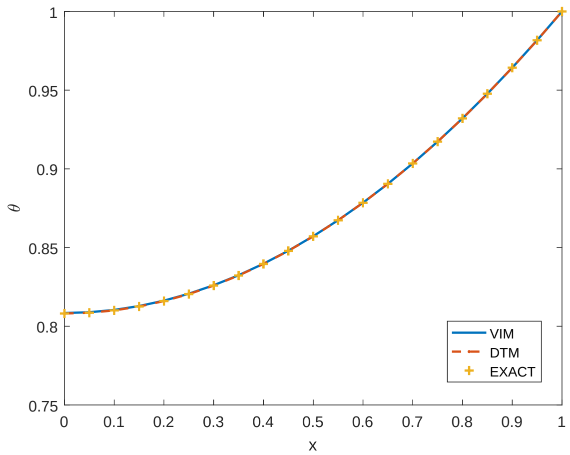

Fig. 2: Comparison of VIM, DTM and Exact solutions for $n = m = 1$, $M = 0.7$.

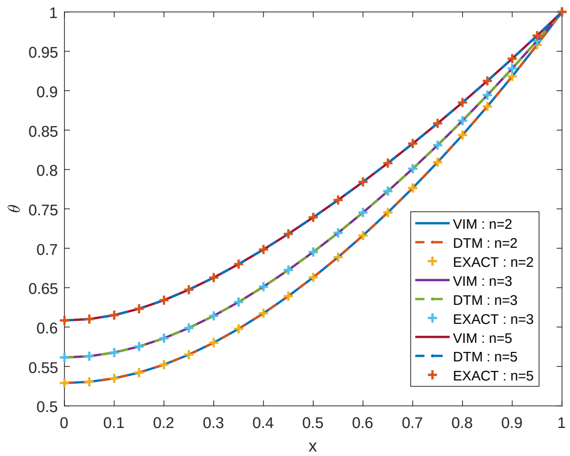

Fig. 3: Temperature distribution in a rectangular fin for varying values of n, $M = 1.7$.

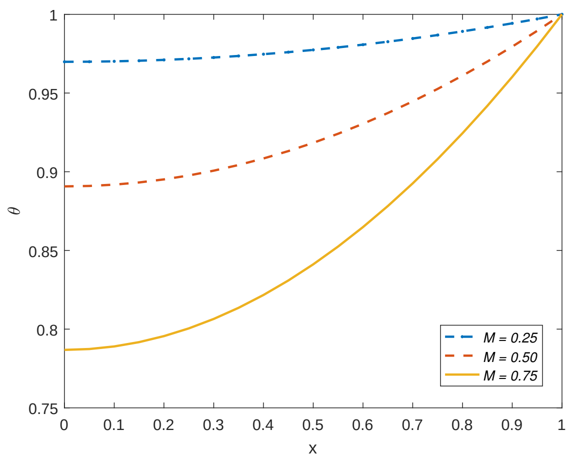

Fig. 4: Temperature distribution in a rectangular fin, $n = 2$, $M = 0.7$.

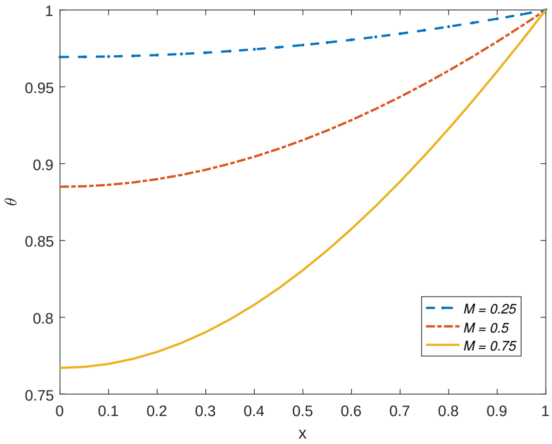

Fig. 5: Temperature distribution in a rectangular fin, $(n, m) = (2, 1)$.

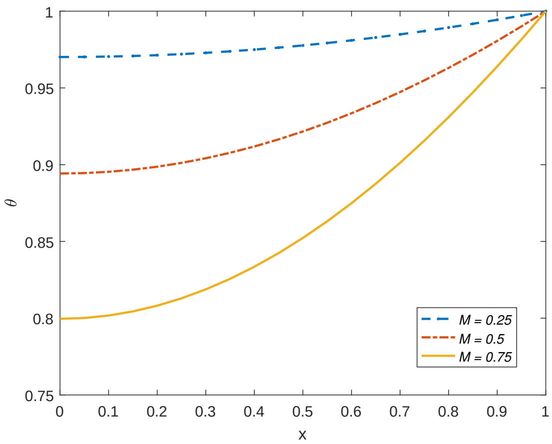

Fig. 6: Temperature distribution in a rectangular fin, $(n, m) = (1, 2)$.

Having established reliance and confidence on VIM solutions, the case where thermal conductivity and heat transfer coefficient are given by distinct exponents is considered. When $n \geq 2$, this indicates that the fin is subject to nucleate boiling, and radiation into free space at zero absolute temperature. Considering the two cases below and applying the correctional function to equation (9), the analytical solutions lend themselves as given below;

Case $(n, m) = (2, 1)$

$$\theta(x) = c + \frac{3c^2M^2x^2}{2} - \frac{3c^4M^2x^2}{2} + \frac{c^6M^2x^2}{2} + \frac{c^3M^4x^4}{4} - \frac{17c^5M^4x^4}{12} + \frac{5c^7M^4x^4}{4} - \frac{c^9M^4x^4}{4} + \cdots \quad (33)$$

Case $(n, m) = (1, 2)$

$$\theta(x) = c + \frac{3c^3M^2x^2}{2} - \frac{3c^4M^2x^2}{2} + \frac{c^5M^2x^2}{2} + \frac{3c^5M^4x^4}{8} - \frac{7c^6M^4x^4}{8} + \frac{5c^7M^4x^4}{8} - \frac{c^8M^4x^4}{8} + \cdots \quad (34)$$

The constant c is obtained by solving the appropriate $\theta(x)$ at the fin base boundary condition. The analytical solutions in (33) and (34) are depicted in Figures 5 and 6, respectively.

Heat transfer in fins of convex parabolic profile The fin profile if convex parabolic when $f(x) = \sqrt{x}$. The DTM could not construct the solution to the convex parabolic fin profile without the use of an alternative substitution, (see [8, 9]). Here, we show the direct construction of the solution

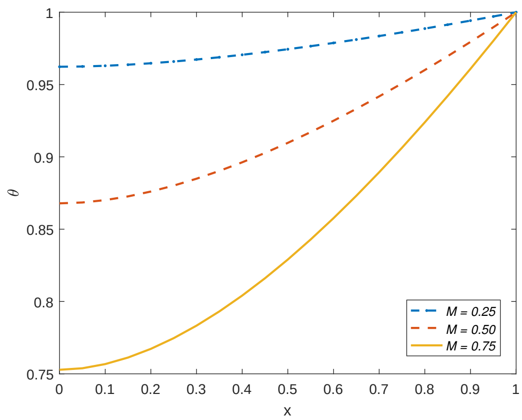

Fig. 7: Temperature distribution in a convex parabolic fin, $M = 0.7$.

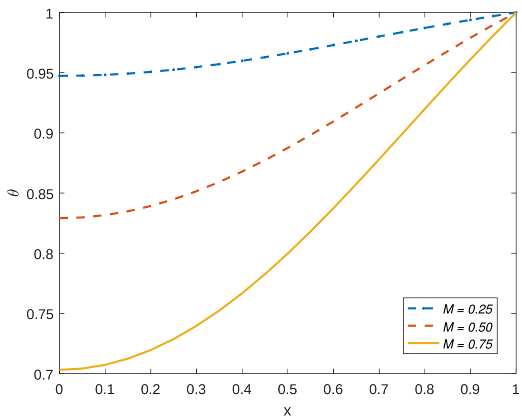

Fig. 8: Temperature distribution in a concave parabolic fin, $M = 0.7$.

using the variational iteration method. Following a similar methodology used in the construction of the analytical solution for the rectangular fin profile, the analytical solution for the convex parabolic profile is given below and the solution is also depicted in Figure 7.

$$\theta(x) \cong c + \frac{3c^2M^2x^2}{2} - \frac{6c^3M^2x^{5/2}}{5} + \frac{c^4M^2x^3}{3} + \frac{c^3M^4x^4}{4} - \frac{643c^4M^4x^{9/2}}{945} + \frac{23c^5M^4x^5}{50} - \frac{4c^6M^4x^{11/2}}{55} + .. \tag{35}$$

Heat transfer in fins of concave parabolic profile The fin profile if concave parabolic when $f(x) = x^2$. The analytical solution for the concave parabolic profile is given below and the solution is also depicted in Figure 8.

$$\theta(x) \cong c + \frac{3c^2M^2x^2}{2} - \frac{3c^3M^2x^4}{4} + \frac{c^3M^4x^4}{4} + \frac{c^4M^2x^6}{6} - \frac{22c^4M^4x^6}{45} + \frac{17c^4M^6x^6}{360} + \frac{c^5M^4x^8}{4} + \cdots \tag{36}$$

Heat transfer in fins of triangular profile The fin profile if triangular when $f(x) = x$. The analytical solution for the concave parabolic profile is given below and the solution is also depicted in Figure 9.

$$\theta(x) \cong c + \frac{3c^2M^2x^2}{2} - c^3M^2x^3 + \frac{c^4M^2x^4}{4} + \frac{c^3M^4x^4}{4} - \frac{3c^4M^4x^5}{5} + \frac{13c^5M^4x^6}{36} + \frac{17c^4M^6x^6}{360} + \cdots \tag{37}$$

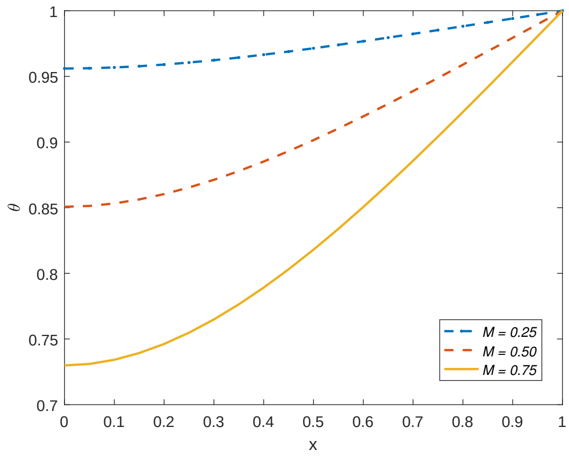

Fig. 9: Temperature distribution in a triangular fin, $M = 0.7$.

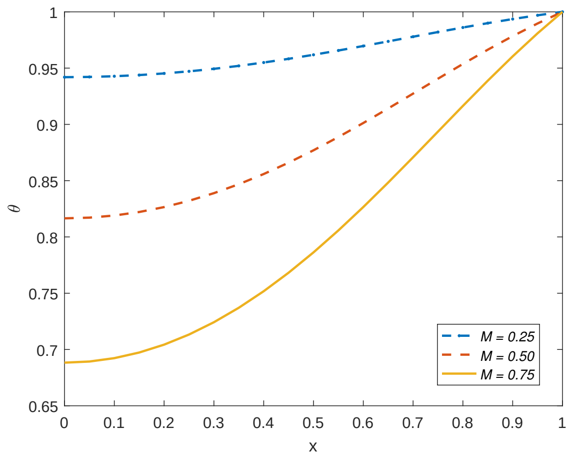

Fig. 10: Temperature distribution in a cubic fin, $M = 0.7$.

Heat transfer in fins of cubic profile The fin profile if cubic when $f(x) = x^3$. The analytical solution for the concave parabolic profile is given below and the solution is also depicted in Figure 10.

$$\theta(x) \cong c + \frac{3c^2M^2x^2}{2} + \frac{c^3M^4x^4}{4} - \frac{3c^3M^2x^5}{5} + \frac{17c^4M^6x^6}{360} - \frac{29c^4M^4x^7}{70} + \frac{c^4M^2x^8}{8} + \frac{11c^5M^8x^8}{3360} + \cdots \tag{38}$$

Aggregated fin profiles In Figure 11, we compare the solutions generated by VIM for various fin profiles. We note that the rectangular fin profile retains more heat. The cubic profile dissipate heat to the ambient fluid better when compared to other profiles. It turns out that the fins of curved profiles perform better than those of a rectangular profile.

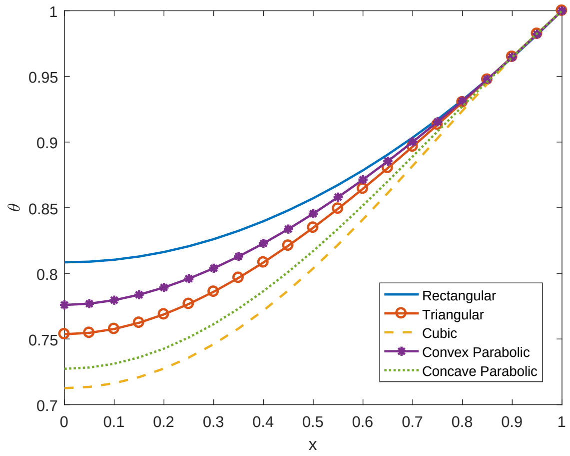

Fig. 11: Temperature distribution in fins of various profiles, $M = 0.7$.

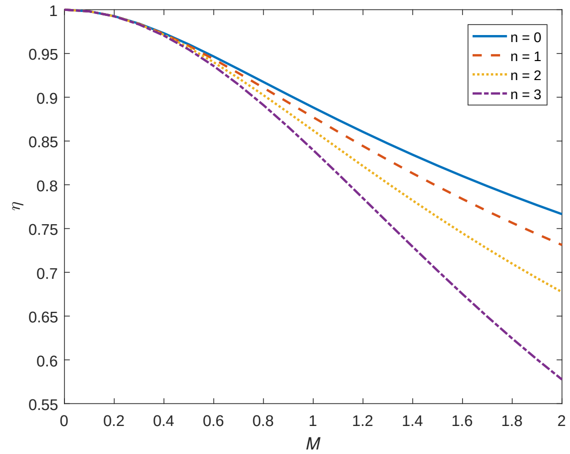

Fig. 12: Fin efficiency of a longitudinal rectangular fin.

Fin Efficiency and Heat Flux

Fin efficiency The heat transfer rate from a fin is given by Newton's second law of cooling,

$$Q = \int_0^L PH(T)(T - T_a)dx. \tag{39}$$

Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin were at the base temperature and is given by (see e.g. [1])

$$\eta = \frac{Q}{Q_{ideal}} = \frac{\int_0^L PH(T)(T - T_a)dX}{Ph_b L(T_b - T_a)}. \tag{40}$$

In dimensionless variables we have

$$\eta = \int_0^1 \theta^{n+1} dx. \tag{41}$$

The fin efficiency for different values of n in depicted in Figure 12.

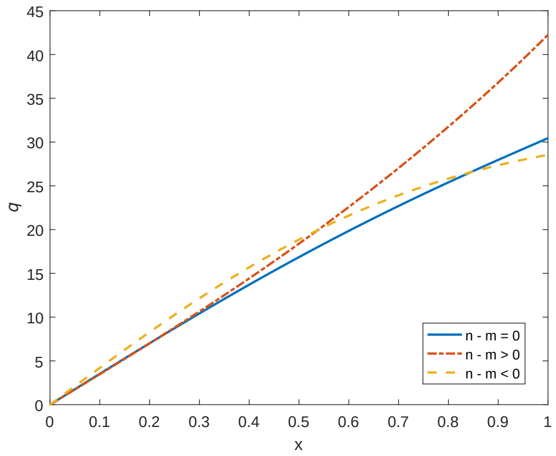

Fig. 13: Base heat flux in a longitudinal rectangular fin, $Bi = 0.01$, $M = 0.65$.

Heat Flux The fin base heat flux is given by the Fourier's law

$$q_b = A_c K(T) \frac{dT}{dx}. \tag{42}$$

The total heat flux of the fin is given by [1]

$$q = \frac{q_b}{A_c H(T)(T_b - T_a)}. \tag{43}$$

Introducing the dimensionless variable as described in § , implies

$$q = \frac{1}{Bi} \frac{k(\theta)}{h(\theta)} \frac{d\theta}{dx}, \tag{44}$$

where the dimensionless parameter $Bi = \dfrac{h_b L}{k_a}$ is the Biot number. Eqn. (44) becomes

$$q = \frac{1}{Bi} \theta^{m-n} \frac{d\theta}{dx}. \tag{45}$$

Not surprisingly, heat flux in one dimensional fins is higher given values $Bi \ll 1$. The heat flux in Eqn. (45) is plotted in Figure 13 and Figure 14 .

Discussion

We have observed an alignment between the analytical solutions generated by the VIM and the exact solutions obtained by Moitsheki *et al.* [2]. The results of the benchmark analysis are depicted in Figures 2 and 3. This confirms that the VIM converges faster and can provide accurate results with minimum computational effort. For all the cases of n considered in the analytics of the rectangular profile, we also see an alignment between the two semi-analytical methods of solutions. In this paper, we have shown that the VIM converges faster when compared to DTM. However, for problems where the number of iterations can be computed as high as as 13 without running short of memory in computer simulations, the DTM outperforms the VIM interms of accuracy.

In the Figure 3, we observe that the fin temperature increases with the increasing values of n. Also, in Figures 5, 7, 8, 9 and 10, we see that the fin temperature increases with the decreasing values of the thermo-geometric fin parameter across fins of different profiles. Furthermore, it appears that the fin with rectangular profile performs least in transferring the heat from the base, since the temperature in

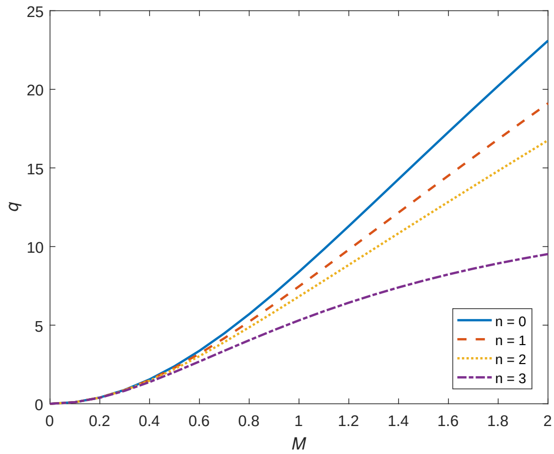

Fig. 14: Base heat flux in a longitudinal rectangular fin, $Bi = 0.01$, $M = 1.70$.

such a fin is much higher than that of other fin profiles. In other words, heat dissipation to the fluid surrounding the extended surface is much faster in longitudinal fins of cubic and concave parabolic profiles. This comparison of fin performance of under different profiles is shown in Figure 11. It also turned out that fins of concave parabolic profile loose heat much faster, followed by triangular and convex parabolic fins. In Figure 12, the fin efficiency decreases with increasing thermo-geometric fin parameter. Also, fin efficiency increases with decreasing values of n. Figures 13 and 14 depict the heat flux at the fin base. The amount of heat energy dissipated from the fin base is of immense interest in engineering [20]. The fin temperature is higher when $n - m > 0$, that is, when heat transfer coefficient is higher than the thermal conductivity. The heat flux also increases with the decreasing values of the the power n. It is easy to show that the thermo-geometric fin parameter is directly proportional to the aspect ratio (extension factor) with square root of the Biot number being the proportionality constant. As such shorter fins are more efficient than longer ones. Else, the increased Biot number results in less efficient fin whenever the space is confined, that is where the length of the fin cannot be increased.

Conclusion

In this study, we have successfully applied He's VIM to highly nonlinear problems arising in heat transfer through longitudinal fins of various profiles. We have studied the effects of the parameters appearing in the model on the temperature distribution in a longitudinal rectangular fin. It was stated in [8, 9] that the DTM stuggles to find solutions for fin profiles given by fractional exponents. The VIM has been able to provide approximate analytical solutions across these fin profiles. A comparison to the models exact solution obtained in [2], shows that the VIM has faster convergence when compared to the DTM. However, as the number of approximations increases beyond 13, we observe that the DTM provides more accurate solutions when compared to the VIM.

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2.2 Journal Article II

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Analysis of temperature distribution in radial moving fins with temperature dependent thermal conductivity and heat transfer coefficient

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ABSTRACT

In this article, a model describing heat transfer through radial moving fins of hyperbolic and rectangular profiles is studied. The thermal conductivity and heat transfer coefficient are assumed to be given by linear and power law functions of temperature respectively. The temperature distribution along the fin is given by a closed-form solution obtained by applying the Variational Iteration Method (VIM). The impact of the embedding parameters on temperature distribution is illustrated and explained. The approximate analytical solutions are validated using numerical results obtained by applying the Runge-Kutta fourth order method. A good agreement is observed between the analytical and numerical solutions. The results further show that the rate of heat transfer to the ambient fluid increases with an increase in the horizontal flow field induced by the moving fin.

1. Introduction

Extended surfaces (also known as fins) are used to enhance heat dissipation from a hot body to its convective, radiative, or convective-radiative fluid. In particular, fins are used in various industrial applications such as oil carrying pipelines, space nuclear reactor power systems and many more. A well-documented review of heat transfer through extended surfaces is given by Kraus et al. [1], Kern and Kraus [2]. Furthermore, a detailed study on fin temperature distribution for various embedding parameters is provided for a triangular fin [3], rectangular radial fin [4], an exponential fin [5] and a trapezoidal fin cross section [6]. Fins may be applied in a stationery system [7] or in motion as in case of electric motor and bike engines [8]. In recent years, the use of fins for heat enhancement and cooling of electronic devices has become popular [9,10]. The search for solutions to models describing heat transfer through fins also continues unabated. The heat transport from a moving fin has attracted the attention of many researchers. This phenomenon is an important problem that occurs in a number of industrial applications such extrusion, hot rolling, casting and many more. The speed of the moving fin is related to its application [11], where velocity of the fin can be very low such as in crystal growth or higher as in optical fiber drawing. Several studies have been performed on heat transfer through moving fins. The effect of temperature-dependent thermal conductivity of the moving fin and radiation effect have been studied by Aziz and Khani [12]. Ming et al. [13] studied conjugate heat transfer from a continuous moving flat plate numerically by employing the cubic spline collocation. Investigation of mixed convection heat transfer along a continuously moving heated vertical plate with suction and blowing was conducted by Al-Sanea [14].

Many scientific problems and physical phenomena are modeled by nonlinear ordinary or partial differential equations. In most cases, these problems do not have an exact solution associated with them. As such, much attention has been devoted to the newly developed methods to construct approximate analytic solutions of nonlinear equations. Radial fins have been investigated numerically [15], analytically [16] and symmetrically [17]. The solutions derived were found to be useful in studying the thermal parameters influencing heat transfer though extended surfaces. In addition, mathematical models describing heat transfer in fins of different profiles have been studied extensively [18,19]. The applicability of the Collocation Spectral Method (CSM) for solving nonlinear heat transfer problems was demonstrated in a convective-radiative fin with temperature-dependent properties by Sun et al. [20].

A Modified Homotopy Analysis Method (HAM) was proposed by Motsa et al. [21]. The newly developed Spectral Homotopy Analysis Method (SHAM) seeks to remove some restrictive assumptions related to the implementation of standard HAM. The Optimal Homotopy Analysis Method (OHAM) was first introduced by Esmaeilpour and

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Nomenclature

c	specific heat (J/kg·K)
h_b	heat transfer coefficient at the fin base (W/m^2·K)
k_a	thermal conductivity at ambient temperature (W/m·K)
U	velocity of the fin (m/s)
r	dimensionless fin radius
A_c	cross-sectional area (m^2)
L	fin length (m)
T_a	ambient temperature (K)
P	fin perimeter (m)
$\mathscr{L}$	linear VIM operator
H	heat transfer coefficient (W/m^2·K)
K	thermal conductivity (W/m·K)
T	temperature (K)
R	fin radius (m)
M	thermo-geometric fin parameter
r_b, r_t	inner and outer radii (m)
$F(R)$	fin profile
T_b	fin base temperature (K)
Pe	Peclet number
$\mathscr{N}$	nonlinear VIM operator

Greek symbols

δ_b	base fin width (m)
ρ	density (kg/m^3)
θ	dimensionless temperature
β	thermal conductivity gradient
α	thermal diffusivity (m^2/s)
λ	Lagrange multiplier

Ganji [22] to solve Jeffery-Hamel problem. Ganji and Dongonchi [23] obtained approximate analytical expressions for temperature distribution in a longitudinal fin with temperature dependent thermal properties and internal heat generation using the Differential Transform Method (DTM). The VIM was applied to study the temperature distribution in fins of various profiles [24]. The solutions were validated using the exact solutions obtained by employing symmetry analysis. The VIM was introduced by He in 1999 [25,26] and has been widely used by many researchers.

In this paper, the VIM is applied to determine approximate analytical solutions to the boundary value problems describing heat transfer in radial moving fins. The mathematical modeling of the problem under consideration is described in § 2. A brief discussion on the fundamentals of the VIM is be provided in § 3. In § 4 we provide the approximate analytical solutions for temperature distribution and the benchmark analysis. The fin efficiency and heat flux are given in § 5 The results are discussed in § 6 and the concluding remarks are provided in § 7.

2. Fin problem formulation

We consider a radial one-dimensional moving fin of cross-sectional area A_c while it moves horizontally with a constant velocity U as depicted in Fig. 1. The fin is attached to a base surface of temperature T_b and extends into a fluid of temperature T_a. The perimeter of the fin is denoted by P and its by $L = r_b - r_t$. We may assume, without loss of generality, that at the tip of the fin, $r_t = 0$. The fin thickness at the prime surface is given by δ_b and its profile is given by $F(R)$. The fluid surrounding the moving fin is heated by the convective fin and induces a horizontal flow field (advection). We assume that the initial transient heat transfer, which occurs when the material is exposed to the ambient temperature, dies out quickly and the heat transfer takes place under steady state conditions. This assumption has been made in several studies [27,28]. The energy balance equation for a radial moving fin is then given by an ordinary differential equation (ODE),

$$\frac{1}{R}\frac{d}{dR}\left[R\frac{\delta_b}{2}F(R)K(T)\frac{dT}{dR}\right] - \frac{P}{A_c}H(T)(T-T_a) - \frac{\delta_b}{2}F(R)\rho c U\frac{dT}{dR} = 0,\quad r_b \le R \le r_t, \tag{2.1}$$

where K and H are the non-uniform thermal conductivity and heat transfer coefficient, respectively, ρ is the density of the material, c is the specific heat capacity, T is the temperature distribution and R is the space variable. Assuming that the fin tip is adiabatic (insulated) and the base temperature is kept constant, then the boundary conditions are given by Ref. [1],

$$T(t, r_b) = T_b \text{ and } \left.\frac{dT}{dR}\right|_{r=r_t} = 0, \tag{2.2}$$

where r_b and r_t are the radii of the fin base and fin tip. Introducing the following dimensionless variables,

$$r = \frac{R}{L},\ \theta = \frac{T-T_a}{T_b-T_a},\ f(r) = \frac{\delta_b}{2}F(R),\ k = \frac{K}{k_a}$$
$$h = \frac{H}{h_b},\ M^2 = \frac{Ph_bL^2}{k_aA_c},\ \alpha = \frac{k_a}{\rho c},\ \text{and } Pe = \frac{UL}{\alpha}, \tag{2.3}$$

reduces equation (2.1) to

$$\frac{1}{r}\frac{d}{dr}\left[rf(r)k(\theta)\frac{d\theta}{dr}\right] - M^2h(\theta)\theta - f(r)Pe\frac{d\theta}{dr} = 0,\quad 0 \le r \le 1, \tag{2.4}$$

and equation (2.4) admits the following boundary conditions,

$$\theta(1) = 1, \tag{2.5}$$

$$\left.\frac{d\theta}{dr}\right|_{r=0} = 0. \tag{2.6}$$

The dimensionless variable M is the thermo-geometric fin parameter, θ is the dimensionless temperature, r is the dimensionless space variable, k is the dimensionless thermal conductivity, k_a is the thermal conductivity of the fin at ambient temperature, h_b is the heat transfer

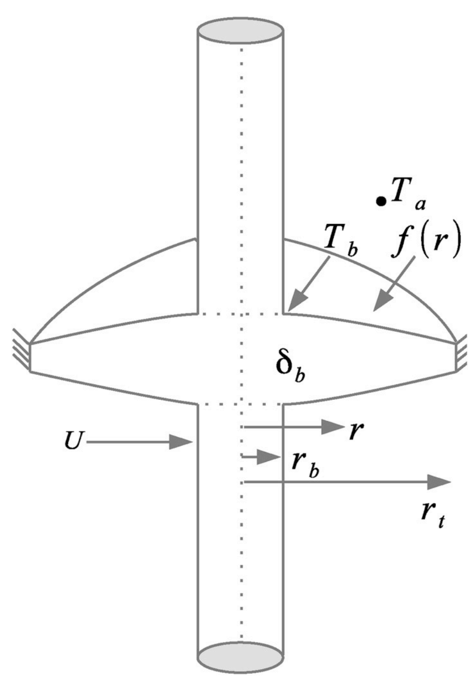

Fig. 1. Schematic representation of a radial fin with arbitrary profile $f(r)$.

coefficient at the fin base, Pe is the Peclet number which represent the dimensionless speed of the moving fin, α is the thermal diffusivity of the fin. For most industrial application, the heat transfer coefficient maybe given as a power law [29],

$$H(T) = h_b\left(\frac{T - T_a}{T_b - T_a}\right)^n, \tag{2.7}$$

where n and h_b are constants. The constant n may vary between -6.6 and 5. However, in most practical applications it lies between -3 and 3 [29]. The exponent n represents laminar film boiling or condensation when $n = -1/4$, laminar natural convection when $n = 1/4$, turbulent natural convection when $n = 1/3$, nucleate boiling when $n = 2$, radiation when $n = 3$ and $n = 0$ implies a constant heat transfer coefficient. In dimensionless variables we have $h(\theta) = \theta^n$. Also, for many engineering applications, the thermal conductivity may depend linearly on temperature, that is

$$K(T) = k_a[1 + \gamma(T - T_a)]. \tag{2.8}$$

The dimensionless thermal conductivity is given by the linear function of temperature is $k(\theta) = 1 + \beta\theta$, where the thermal conductivity gradient is $\beta = \gamma(T - T_a)$. As such the governing equation is given by

$$\frac{1}{r}\frac{d}{dr}\left[rf(r)(1 + \beta\theta)\frac{d\theta}{dr}\right] - M^2\theta^{n+1} - f(r)Pe\frac{d\theta}{dr} = 0,$$
$$0 \leq r \leq 1. \tag{2.9}$$

For a radial moving fin with rectangular profile, $f(r) = 1$ and $f(r) = r_b/r$ for the hyperbolic profile. We apply the VIM to construct the approximate analytical solutions to (2.9), subject to boundary conditions (2.5) and (2.6).

3. He's variational iteration method

To illustrate the basic idea of the method we consider the following nonlinear equation:

$$\mathscr{L}\theta + \mathscr{N}\theta = g(r), \tag{3.1}$$

where $\mathscr{L}$ and $\mathscr{N}$ are linear and nonlinear operators, respectively, and $g(r)$ is the source inhomogeneous term [30]. In his papers [25,26], He proposed the variational iteration method where a correctional functional for equation (3.1) can be written as

$$\theta_{m+1}(r) = \theta_m(r) + \int_0^r \lambda(t)(\mathscr{L}\theta_m(t) + \mathscr{N}\tilde{\theta}_m(t) - g(t)) \tag{3.2}$$

where λ is the general Lagrange multiplier, which can be identified optimally via the variation theory, and $\tilde{\theta}_m$ is a restricted variation, which means $\delta\tilde{\theta}_m = 0$ [31]. The Lagrange multiplier can be a constant or a function depending on the order of the deferential equation under consideration. In general, for the m^{th} order ordinary differential equation, the Lagrange multiplier is given by,

$$\lambda = \frac{(-1)^m}{(m-1)!}(t - r)^{m-1}. \tag{3.3}$$

Equation (3.2) can be re-written as follows:

$$\theta_{m+1}(r) = \theta_m(r) + \int_0^r \frac{(-1)^m}{(m-1)!}(t - r)^{m-1}(\mathscr{L}\theta_m(t) + \mathscr{N}\theta_m(t) - g(t)) \tag{3.4}$$

The iteration formula equation (3.4), without restricted variation, should be used for the determination of the successive approximations $\theta_{m+1}(r), \quad m \geq 0$, of the solution $\theta(r)$. Consequently, the solution is given by

$$\theta(r) = \lim_{m\to\infty} \theta_m(r). \tag{3.5}$$

4. The fin temperature distribution

In this section, we apply the VIM to derive the approximate analytical solution for a hyperbolic profile and the results for a rectangular profile are graphically presented. As a starting approximation for the VIM solution to equation (2.9), $\theta_0(r)$ is assumed to be a constant [32,33]. The fin is of hyperbolic profile when $f(r) = 1/r$. The first two iterations using the VIM solution given $n = 2$ are as follows,

$$\theta_0(r) = a, \tag{4.1}$$

$$\theta_1(r) = a + \frac{1}{6}a^3M^2r^3, \tag{4.2}$$

$$\theta_2(r) = a - \frac{1}{6}a^3(-1 + a\beta)M^2r^3 + \frac{1}{24}a^3M^2Per^4 + \frac{1}{360}a^5(6 - 5a\beta)M^4r^6$$
$$+ \frac{1}{864}a^7M^6r^9 + \frac{1}{28512}a^9M^8r^{12}, \vdots \tag{4.3}$$

The third iteration of the VIM gives higher-order powers of the independent variable. A series solution of order $O(14)$ was found to be close to the exact solution with a very small standard error using the differential transformation and variational iteration [24]. An approximate expression for temperature distribution for the problem under consideration is then given by

$$\theta(r) \cong \theta_5(r). \tag{4.4}$$

The approximate solutions (4.1)–(4.3) all satisfy the derivative boundary condition assigned at the fin tip. To obtain the value of the constant a, we substitute the boundary condition (2.5) into (4.4) at the point $x = 1$. Similarly, the recurrence relation of VIM Solution to (2.9) is given below

$$\theta(r) = a - \frac{1}{6}a^3(-1 + a\beta)M^2r^3 + \frac{1}{24}a^3M^2Per^4 + \frac{1}{360}a^5(6 - 5a\beta)M^4r^6$$
$$+ \frac{1}{864}a^7M^6r^9 + \frac{1}{28512}a^9M^8r^{12} + \cdots \tag{4.5}$$

A benchmark analysis of the VIM solutions and the numerical solution (Runge-Kutta fourth order method) is depicted in Fig. 2. It is observed that the VIM solutions are closely aligned to numerical solutions within the first five iterations.

For all analytical results depicted in the Figures in this article, the following values of parameters are used unless stated otherwise as indicated in the graphs below.

$$n = 2, \; \beta = 1, \; M = 0.75, \; \; Pe = 3. \tag{4.6}$$

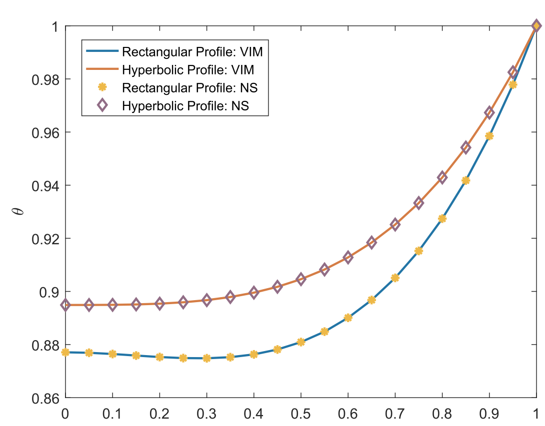

Fig. 2. Temperature distribution along the radial fin for varying fin profiles, $n = 2$, $\beta = 1$, $M = 0.75$. $Pe = 3$.

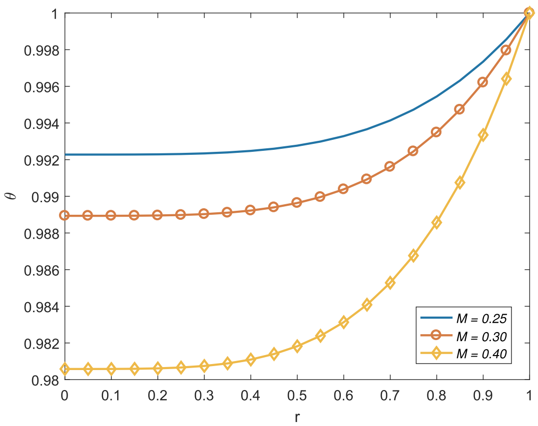

Fig. 3. Temperature distribution along the radial fin for varying M.

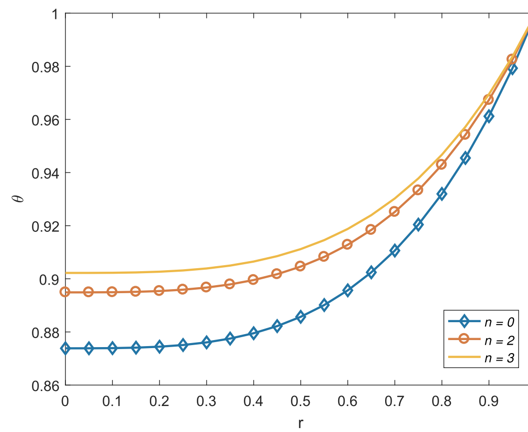

Fig. 5. Temperature distribution along the radial fin for varying n.

5. Fin efficiency and heat flux

5.1. Fin efficiency

The heat transfer rate from a fin is given by Newton's second law of cooling,

$$Q = \int_0^L PH(T)(T - T_a)dR. \tag{5.1}$$

Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin were at the base temperature and is given by (see e.g. Ref. [1])

$$\eta_= \frac{Q}{Q_{ideal}} = \frac{\int_0^L PH(T)(T - T_a)dR}{Ph_b L(T_b - T_a)}. \tag{5.2}$$

In dimensionless variables we have

$$\eta = \int_0^1 \theta^{n+1} dr. \tag{5.3}$$

The fin efficiency is depicted in Fig. 7.

5.2. Heat flux

The fin base heat flux is given by the Fourier's law

$$q_b = A_c K(T)\frac{dT}{dR}. \tag{5.4}$$

The total heat flux of the fin is given by Ref. [1].

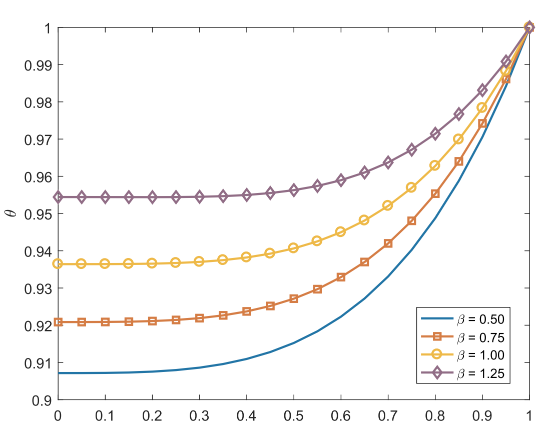

Fig. 4. Temperature distribution along the radial fin for varying β.

$$q = \frac{q_b}{A_c H(T)(T_b - T_a)}. \tag{5.5}$$

Introducing the dimensionless variable as described in § 1, implies

$$q = \frac{1}{Bi}\frac{k(\theta)}{h(\theta)}\frac{d\theta}{dr}, \tag{5.6}$$

where the dimensionless parameter $Bi = \frac{hL}{k_a}$ is the Biot number. Not surprisingly, heat flux in one dimensional fins is higher given values $Bi \ll 1$. The heat flux in equation (5.6) is plotted in Fig. 9.

6. Discussion

This section discuses the impact of the embedding parameters on the temperature distribution in radial moving fins of hyperbolic and rectangular profiles. The governing equations were solved analytically using He's VIM and validated numerically using Runge-Kutta fourth order method. Fig. 2 shows that approximate analytical solutions are in good agreement with the numerical results. We also observe that the rectangular fin profile is more efficient in dissipating heat that the hyperbolic profile.

Fig. 3 shows that the fin temperature decreases with the increasing values of the thermo-geometric fin parameter M. For $M = 0$, this indicates the absence of convection while $M = 1$ indicates that convection and conduction are at equal rates. Note that the thermo-geometric fin parameter $M = (Bi)^{1/2}E$, where $E = L/\delta$ is the aspect ratio or the extension factor. Evidently, small values of M correspond to the relatively short and thick fins of high conductivity and high values of M correspond to longer and thin fins of poor conductivity [34]. A fin is an excellent dissipater at small values of M. As M increases the convective heat loss increases and the temperature profile becomes steeper reflecting high base heat flow rates [35].

In Fig. 4 we observe that the temperature in the fin increases with the increasing values of the thermal conductivity gradient β. Note that $\beta = 0$ implies that the thermal conductivity is a constant given by k_a. The curves with $\beta = 0$, correspond to the fin material whose thermal conductivity increases as temperature increases from T_a to T_b, [35]. In Fig. 5 we observe that the temperature distribution of the fin is an increasing function of exponent n. Fig. 6 illustrates the effect of the Peclet number on the temperature distribution in the fin. As Pe increases, (fin moves faster), the temperature inside the fin decreases rapidly due to increased advective effect on the surface of the fin. We also observe that for $Pe = 0$, which represents a stationery fin, the cooling of the fin takes more time as shown by higher temperatures when compared to a moving fin. The fin efficiency is shown in Figs. 7 and 8. In Fig. 7, we note that the fin efficiency decreases with increasing values of the thermo-geometric fin parameter and with decreasing values of the exponent of the heat transfer coefficient. Fig. 8 further shows that the fin

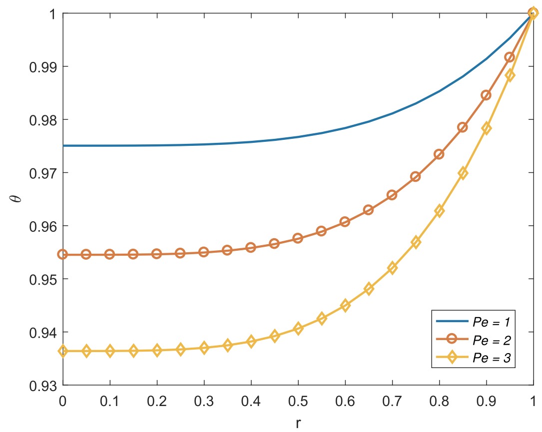

Fig. 6. Temperature distribution along the radial fin for varying *Pe*.

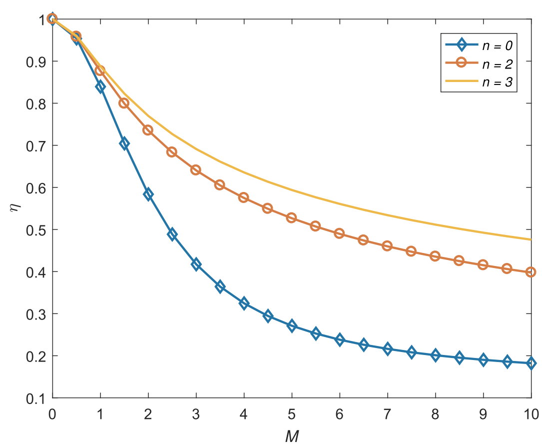

Fig. 7. Fin efficiency of a longitudinal radial moving fin, $Pe = 3$.

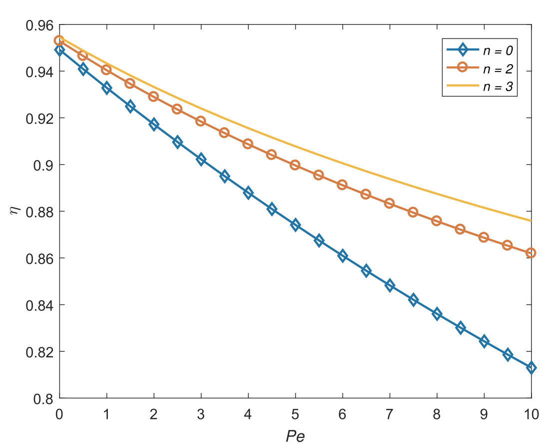

Fig. 8. Fin efficiency of a longitudinal radial moving fin, $M = 0.75$.

temperature gradually diminishes with the increasing values of the Peclet number. The amount of heat energy dissipated from the fin base is of immense interest in engineering [36]. From the heat flux graph, Fig. 9, we note that the parameter q increases with M for all the values of the exponent n considered.

7. Concluding remarks

In this study, we have successfully performed thermal analysis of radial moving fins. The resulting differential equations were solved using He's VIM and the analytical solutions were benchmarked against

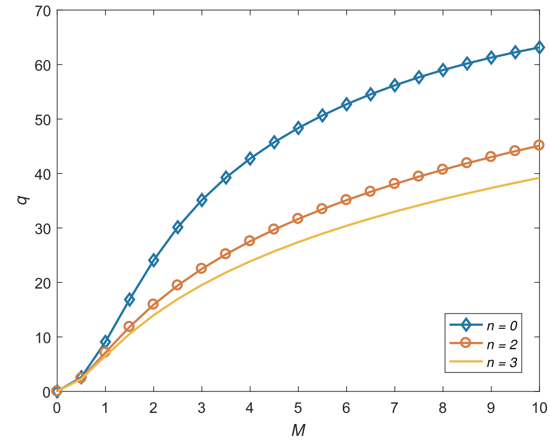

Fig. 9. Heat flux in a longitudinal radial fin.

the numerical solutions. The impact of the thermo-physical parameters was illustrated and explained. The radiation heat transfer was assumed to occur at high temperatures as described by the exponent of heat transfer coefficient. We have also shown that analytical solutions are important in describing the thermo-physical properties of heat transfer equations. The explicit solutions provide an easy way of evaluating fin base thermal conductance and fin efficiency, which would be difficult using numerical schemes. Furthermore, we have incorporated the advection term on the radial fin model, which is a result of the continuous fin movement. An increase in the speed of the moving induces a horizontal flow field which in turn increases heat dissipation from the moving fin. We observe that the temperature inside the decreases with the increasing values of the Peclet number and the thermo-geometric fin parameter. Where cooling is required, higher values of Peclet number and thermo-geometric parameter will be desirable.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ijthermalsci.2019.106015.

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2.3 Journal Article III

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Analysis of transient heat transfer in radial moving fins with temperature-dependent thermal properties

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Abstract

In this article, thermal analysis of radial moving fins of hyperbolic and rectangular profiles is performed. A time-dependent nonlinear differential equation modeling heat transfer in a radial moving fin is considered. The thermal conductivity and heat transfer coefficient are modeled as linear and power-law functions of temperature, respectively. The analytical solutions are generated using the differential transform method which is an analytical solution technique that can be applied to various types of differential equations. The accuracy of the analytical solutions is validated by benchmarking against the numerical solutions obtained by applying the inbuilt numerical solver in Maple (*pdsolve*). A good agreement is observed between the analytical and numerical solutions. The effects of embedding parameters on the dimensionless temperature and heat transfer rate are analyzed. The results show that the fin rapidly dissipates heat to the surrounding fluid with an increase in the speed of the moving fin, that is, an increase in the values of the Peclet number. The results also show that the heat transfer through a rectangular fin is more efficient when compared to a fin of hyperbolic profile.

Keywords Moving fin · Differential transform method · Thermal conductivity · Heat transfer · Fin tip temperature

List of symbols

c	Specific heat ($\mathrm{J\,kg^{-1}\,K}$)
H	Heat transfer coefficient ($\mathrm{W\,m^{-2}\,K}$)
h_b	Heat transfer coefficient at the fin base ($\mathrm{W\,m^{-2}\,K}$)
K	Thermal conductivity ($\mathrm{W\,m^{-1}\,K}$)
k_c	Thermal conductivity at ambient temperature ($\mathrm{W\,m^{-1}\,K}$)
t	Time (s)
T	Temperature (K)
U	Velocity of the fin ($\mathrm{m\ s^{-1}}$)
R	Fin radius (m)
r	Dimensionless fin radius
M	Thermo-geometric fin parameter
A_c	Cross-sectional area ($\mathrm{m^2}$)
r_b, r_t	Inner and outer radii, respectively (m)
L	Fin length (m)
$F(R)$	Fin profile
T_a	Ambient temperature (K)
T_b	Fin base temperature (K)
P	Fin perimeter (m)
Pe	Peclet number
$\Phi(t,x)$	Transformed analytical function
$\phi(t,x)$	Original analytical function

Greek symbols

σ	Boltzmann constant ($\mathrm{W\,m^{-2}\,K^4}$)
δ_b	Base fin width (m)
β	Thermal conductivity gradient
ρ	Density ($\mathrm{kg\,m^{-3}}$)
τ	Dimensionless time
α	Thermal diffusivity ($\mathrm{m^2\,s^{-1}}$)
θ	Dimensionless temperature

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Introduction

An increasing number of engineering applications are concerned with energy transportation by requiring a rapid movement of heat away from a heat source. Fins are extensively used in various industrial applications such as refrigeration, air conditioning, heat exchanger, oil carrying pipelines and chemical processing equipment [1, 2]. A fin is commonly used to enhance heat transfer from a heat source by its thermal conduction mechanism and then dissipate the heat to an ambient fluid by convection or radiation. The rate of heat transfer depends on the thermal conductivity of the fin material and the surface heat transfer coefficient, both of which may be temperature dependent [3, 4]. The models describing heat transfer in fins are given in terms of differential equations. The steady state heat transfer in fins is governed by the ordinary differential equations (ODEs), and a number of studies have been devoted to such models [5]. In many other engineering applications, transient heat transfer is studied and is modeled by partial differential equations (PDEs) [6]. Thermal analysis of a radial porous fin for convection and radiation heat transfer was performed using spectral method by Darvishi and Khani [7]. Their results suggest that radiation transfers more heat than a similar model without radiation. Sobamowo [8] studied heat transfer in a longitudinal rectangular fin with temperature-dependent thermal conductivity and internal heat generation using finite difference method. The author found out that fin temperature distribution, the total heat transfer, and the fin efficiency are significantly affected by the thermo-geometric parameters of the fin.

Most of the work done on extended surface heat transfer is attributed to stationery fin problems. The study of heat transfer in moving fins has aroused in recent years and has drawn the attention of many researchers. Recent research in this field demonstrates that moving fins of different profiles [9] are significant as well as when they are exposed to a heat source [10]. Torabi et al. [11] derived approximate explicit analytical expressions for steady state temperature distribution in a moving fin experiencing simultaneous convective-radiative surface heat loss. Thermal analysis of a moving porous fin in natural convection environment was studied analytically by Moradi et al. [12]. Turkyilmazoglu [13] constructed exact solutions for heat transfer through exponential fins in movement and under the influence of heat generation (absorption).

A large number of accurate approximation techniques have been devoted to solving differential equations modeling heat transfer through fins of various profiles. An annular fin of hyperbolic profile with temperature-dependent thermal conductivity was studied using pseudospectral method [14]. Zhou [15] presented the differential transformation method (DTM) in 1986 which is an analytical method based on Taylor series expansion. The main advantage of this method is its direct applicability to solve the differential equations without any discretization or perturbation. DTM also has high accuracy making it an effective and precise method for solving nonlinear ordinary and partial differential equations. In 1999, the variational iteration method (VIM) was proposed by He [16] and He and Wu [17]. The method introduces a reliable and efficient process for a wide variety of scientific and engineering applications. Babaelahi and Raveshi [18] successfully applied two analytical methods, DTM and optimal homotopy asymptotic method (OHAM), to solve radiating fin heat transfer problems in aerospace applications. The analytical results were validated using numerical data obtained through the fourth order Runge–Kutta method. Aziz and Khani [19] applied the homotopy analysis method (HAM) to generate an analytical solution for heat transfer in a moving fin of variable thermal conductivity. The analytical solution was found to be accurate to at least three decimal places for a wide range of values of the parameters that are commonly encountered in thermal processing. Roy et al. [20] derived local fin temperature fields and efficiencies using Adomian decomposition method (ADM). It was found that the ADM results were in good agreement with the results available in the literature using Galerkin method (GM). Ndlovu and Moitsheki [21] recently performed thermal analysis of natural convection and radiation heat transfer in moving porous fins of rectangular and exponential profiles. The analytical solutions describing temperature distribution were constructed using the DTM and validated by the numerical solutions obtained by applying the *pdsolve* in Maple. Fallo et al. [22] applied the three-dimensional DTM for the first time to study heat transfer in a cylindrical spine fin with variable thermal properties. Aziz and Makinde [23] applied a two-dimensional heat conduction model to obtain the thermal performance and entropy generation in an orthotropic convection pin fin used in advanced light weight heat sinks. The study illustrated the simultaneous realization of the least material and minimum entropy generation pin fin designs. Mhlongo et al. [24] applied the local and nonlocal symmetry techniques to study transient response of rectangular fins to step change in base temperature and base heat flux. The authors showed that the governing equations can be reduced to the tractable Ermakov–Pinney equation.

In this work, transient thermal analysis of radial moving fins of rectangular and hyperbolic profiles with temperature-dependent properties is performed. The thermal conductivity and heat transfer coefficient are regarded as linear and power-law functions of temperature, respectively. This problem is of fundamental importance in the thermal processing of materials [25, 26]. The 2D-DTM is used to provide approximate analytical solutions of the present problem. The results to be presented will highlight the effects of the thermo-geometric fin parameter, M, Peclet number, Pe, thermal conductivity gradient, β, dimensionless time parameter, τ and radial fin profile, $f(r)$, on temperature distribution. This article is organized as follows: the mathematical background of the problem under consideration is described in "The mathematical model for a radial moving fin" section. A brief discussion on the fundamentals of the 2D-DTM will be provided in "Fundamentals of the two-dimensional differential transform method" section. We provide analytical results and discussions in "Results and discussion" section. Lastly, we provide the concluding remarks in "Conclusions" section.

The mathematical model for a radial moving fin

We consider a radial one-dimensional moving fin of cross-sectional area A_c while it moves horizontally with a constant velocity U as depicted in Fig. 1. The fin is attached to a base surface of temperature T_b and extends into a fluid of temperature T_a. The perimeter of the fin is denoted by P and the length of fin by $L = r_b - r_t$. We may assume, without loss of generality, that at the tip of the fin, $r_t = 0$. The fin thickness at the prime surface is given by δ_b and its profile is given by $F(R)$. The energy balance for a radial moving fin is given by a partial differential equation (PDE) [1, 2, 4]:

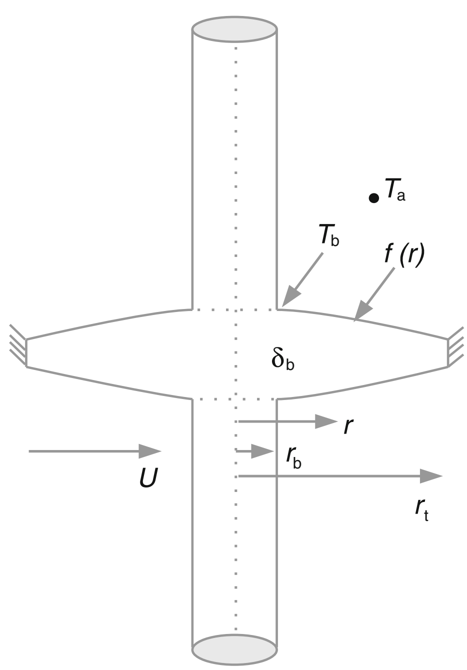

Fig. 1 Schematic representation of a radial fin

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\delta_b}{2}F(R)K(T)\frac{\partial T}{\partial R}\right) - \frac{P}{A_c}H(T)(T - T_a) - \frac{\delta_b}{2}F(R)\rho c U\frac{\partial T}{\partial R}, \quad r_b \le R \le r_t, \tag{1}$$

where K and H are the non-uniform thermal conductivity and heat transfer coefficient, respectively, ρ is the density of the material, c is the specific heat capacity, T is the temperature distribution, t is the time, and R is the space variable. Assuming that the fin tip is adiabatic (insulated) and the base temperature is kept constant, then the boundary conditions are given by [1],

$$T(t, r_b) = T_b \text{ and } \left.\frac{\partial T}{\partial R}\right|_{r=r_t} = 0, \tag{2}$$

where r_b and r_t are the radii of the fin base and fin tip, respectively, and initially the fin is kept at the ambient temperature [3, 4],

$$T(0, R) = T_a. \tag{3}$$

Introducing the following dimensionless variables,

$$r = \frac{R}{L}, \quad \tau = \frac{k_c t}{\rho c L^2}, \quad \theta = \frac{T - T_a}{T_b - T_a}, \quad f(r) = \frac{\delta_b}{2}F(R),$$
$$k = \frac{K}{k_c}, \quad h = \frac{H}{h_b}, \quad M^2 = \frac{P h_b L^2}{k_c A_c} \text{ and } Pe = \frac{UL}{\alpha}, \tag{4}$$

where $\alpha = k_c/\rho c$ reduces Eq. (1) to

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(r f(r) k(\theta)\frac{\partial \theta}{\partial r}\right) - M^2 h(\theta)\theta - f(r) Pe \frac{\partial \theta}{\partial r}, \quad 0 \le r \le 1, \tag{5}$$

and Eq. (5) admits the following boundary conditions,

$$\theta(\tau, 1) = 1, \tag{6}$$

$$\left.\frac{\partial \theta}{\partial r}\right|_{r=0} = 0, \tag{7}$$

and the initial condition becomes,

$$\theta(0, r) = 0. \tag{8}$$

The dimensionless variable M is the thermo-geometric fin parameter, θ is the dimensionless temperature, r is the dimensionless space variable, k is the dimensionless thermal conductivity, k_c is the thermal conductivity of the fin at ambient temperature, h_b is the heat transfer coefficient at the fin base, Pe is the Peclet number which represents the dimensionless speed of the moving fin, and α is the thermal diffusivity of the fin. For most industrial application, the heat transfer coefficient may be given as a power law [27],

$$H(T) = h_b\left(\frac{T - T_a}{T_b - T_a}\right)^n, \tag{9}$$

where n and h_b are constants. The constant n may vary between -6.6 and 5. However, in most practical applications it lies between -3 and 3 [27]. The exponent n represents laminar film boiling or condensation when $n = -1/4$, laminar natural convection when $n = 1/4$, turbulent natural convection when $n = 1/3$, nucleate boiling when $n = 2$, radiation when $n = 3$ and $n = 0$ implies a constant heat transfer coefficient. In dimensionless variables, we have $h(\theta) = \theta^n$. Also, for many engineering applications, the thermal conductivity may depend linearly on temperature [3], that is

$$K(T) = k_c[1 + \gamma(T - T_a)]. \tag{10}$$

The dimensionless thermal conductivity given by the linear function of temperature is $k(\theta) = 1 + \beta\theta$, where the thermal conductivity gradient is $\beta = \gamma(T - T_a)$. As such, the governing equation is given by

$$\frac{\partial\theta}{\partial\tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(rf(r)(1+\beta\theta)\frac{\partial\theta}{\partial r}\right) - M^2\theta^{n+1} - f(r)Pe\frac{\partial\theta}{\partial r}, \quad 0 \le r \le 1. \tag{11}$$

For a radial moving fin with rectangular profile, $f(r) = 1$ and $f(r) = r_b/r$ for the hyperbolic profile. We apply the 2D-DTM to construct the series solutions to (11), subject to the initial and boundary conditions (6)–(8).

Fundamentals of the two-dimensional differential transform method

In this section, the basic idea underlying the two-differential transform method (2D-DTM) is briefly introduced. If function $\theta(t, x)$ is analytic and differentiated continuously with respect to time and the spatial variable x in the domain of interest, then we let

$$\Phi(\kappa, s) = \frac{1}{\kappa!s!}\left[\frac{\partial^{\kappa+s}\phi(t,x)}{\partial t^\kappa \partial x^s}\right]_{(0,0)}, \tag{12}$$

where the spectrum $\Phi(\kappa, s)$ is the transformed function, which is also called the T-function (see [28, 29]). The differential inverse transform of $\Phi(\kappa, s)$ is defined as

$$\phi(t,x) = \sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty}\Phi(\kappa,s)t^\kappa x^s, \tag{13}$$

and from Eqs. (12) and (13) it can be concluded that

$$\phi(t,x) = \sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty}\frac{1}{\kappa!s!}\left[\frac{\partial^{\kappa+s}\phi(t,x)}{\partial t^\kappa \partial x^s}\right]_{(0,0)} t^\kappa x^s. \tag{14}$$

In real applications, the function $\phi(t, x)$ is expressed by a finite series, and Eq. (13) can be written as:

$$\phi(t,x) = \sum_{\kappa=0}^{m}\sum_{s=0}^{n}\Phi(\kappa,s)t^\kappa x^s, \tag{15}$$

Equation (15) implies that

$$\phi(t,x) = \sum_{\kappa=m+1}^{\infty}\sum_{s=n+1}^{\infty}\Phi(\kappa,s)t^\kappa x^s, \tag{16}$$

is negligibly small.

Some of the useful mathematical operations performed by the differential transform method are given in Table 1.

The Kronecker delta function $\delta(\kappa - s)$ is given by

$$\delta(\kappa - s) = \begin{cases} 1 & \text{if } \kappa = s \\ 0 & \text{if } \kappa \ne s. \end{cases}$$

Results and discussion

In this section, we provide analytical solutions for the problem described in "Introduction" section. Using 2D-DTM to solve PDEs consists of three main steps. The steps are: transforming the PDE into algebraic equations, solving the equations, and inverting the solution of algebraic equations to obtain an infinite series solution or an approximate solution. The governing equation for a hyperbolic radial fin profile is given by,

$$\frac{\partial\theta}{\partial\tau} = \frac{1}{r}\frac{\partial}{\partial r}\left((1+\beta\theta)\frac{\partial\theta}{\partial r}\right) - M^2\theta^{n+1} - \frac{1}{r}Pe\frac{\partial\theta}{\partial r}, \quad 0 \le r \le 1. \tag{17}$$

We cross-multiply Eq. (17) by r making it susceptible to DTM analysis. Taking the two-dimensional differential

Table 1 Fundamental operations of the differential transform method

Original function	Transformed function
$\phi(t,x) = f_1(t,x) \pm f_2(t,x)$	$\Phi(\kappa,s) = F_1(\kappa,s) \pm F_2(\kappa,s)$
$\phi(t,x) = \alpha f(t,x)$	$\Phi(k,s) = \alpha F(\kappa,s)$
$\phi(t,x) = \frac{\partial f(t,x)}{\partial t}$	$\Phi(\kappa,s) = (\kappa+1)F(\kappa+1,s)$
$\phi(t,x) = \frac{\partial f(t,x)}{\partial x}$	$\Phi(\kappa,s) = (s+1)F(\kappa,s+1)$
$\phi(t,x) = \frac{\partial^2 f(t,x)}{\partial t^2}$	$\Phi(\kappa,s) = (\kappa+1)(\kappa+2)F(\kappa+2,s)$
$\phi(t,x) = \frac{\partial^2 f(t,x)}{\partial x^2}$	$\Phi(\kappa,s) = (s+1)(s+2)F(\kappa,s+2)$
$\phi(t,x) = t^m x^n$	$\Phi(\kappa,s) = \delta(\kappa-m)\delta(s-n)$
$\phi(t,x) = x^m \exp(at)$	$\Phi(\kappa,s) = \frac{a^\kappa}{\kappa!}\delta(s-m)$

transform of Eq. (17) for $n = 2$, we obtain the following recurrence relation

$$\sum_{i=0}^{\kappa}\sum_{j=0}^{h}(i+1)\Theta(i+1,h-j)\delta(\kappa-i)\delta(j-1)$$
$$= (h+1)(h+2)\Theta(\kappa,h+2)$$
$$+\beta\sum_{i=0}^{\kappa}\sum_{j=0}^{h}\Theta(\kappa-i,j)(h+1-j)(h+2-j)\Theta(i,h+2-j)$$
$$+\beta\sum_{i=0}^{\kappa}\sum_{j=0}^{h}(j+1)\Theta(\kappa-i,j+1)(h+1-j)\Theta(i,h+1-j)$$
$$-M^2\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{z=0}^{\kappa-i-l}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\sum_{t=0}^{h-j-p}$$
$$\Theta(i,h-j-p-t)\Theta(l,j)\Theta(z,p)\delta(\kappa-i-l-z,t)\delta(t-1)$$
$$-Pe(h+1)\Theta(k,h+1), \tag{18}$$

where $\Theta(\kappa,h)$ is the differential transform of $\theta(\tau,r)$. Taking the two-dimensional differential transform of the initial condition (8) and boundary condition (7), we obtain the following transformations, respectively,

$$\Theta(0,h) = 0, \quad h = 0,1,2,\ldots \tag{19}$$

$$\Theta(\kappa,1) = 0, \quad \kappa = 0,1,2,\ldots. \tag{20}$$

We consider the other boundary condition as follows,

$$\Theta(\kappa,0) = a, \quad a \in \mathbb{R}, \quad \kappa = 1,2,3,\ldots \tag{21}$$

where the constant a can be determined from the boundary condition (6) at each time step after obtaining the series solution. Substituting Eqs. (19)–(21) into (18), we obtain the following,

$$\Theta(c,2) = 0, \; c = 1,2,3,\ldots \tag{22}$$

$$\Theta(1,3) = \frac{a}{3} \tag{23}$$

$$\Theta(2,3) = \frac{1}{6}a(3-2a\beta) \tag{24}$$

$$\Theta(3,3) = \frac{1}{6}a(4+a(\beta(-5+2a\beta)+aM)) \tag{25}$$

$$\Theta(1,4) = \frac{aPe}{12} \tag{26}$$
$$\vdots$$

Substituting Eqs. (19)–(26) into (15), we obtain the following closed form of the solution,

$$\theta(\tau,r) = a\tau + a\tau^2 + \frac{a}{3}\tau r^3 + \frac{1}{6}a(3-2a\beta)\tau^2 r^3 + a\tau^3$$
$$+\frac{1}{6}a(4+a(\beta(-5+2a\beta)+aM))\tau^3 r^3$$
$$+a\tau^4 + \frac{aPe}{12}\tau r^4 + \frac{1}{24}aPe(3-4a\beta)\tau^2 r^4$$
$$+\frac{1}{24}aPe(4+a(2\beta(-5+3a\beta)+aM))\tau^3 r^4$$
$$+\ldots \tag{27}$$

The constant a can be determined from the boundary condition (6) at each time step. To obtain the value of a, we substitute the boundary condition (6) into (27) at the point $r = 1$. Thus, we have,

$$\theta(\tau,1) = a\tau + a\tau^2 + \frac{a}{3}\tau + \frac{1}{6}a(3-2a\beta)\tau^2 + a\tau^3$$
$$+\frac{1}{6}a(4+a(\beta(-5+2a\beta)+aM))\tau^3$$
$$+a\tau^4 + \frac{aPe}{12}\tau + \frac{1}{24}aPe(3-4a\beta)\tau^2$$
$$+\frac{1}{24}aPe(4+a(2\beta(-5+3a\beta)+aM))\tau^3$$
$$+\cdots = 1 \tag{28}$$

We then obtain the expression for $\theta(\tau,r)$ upon substituting the obtained value of a into Eq. (27). Using the first 20 terms of the power series solution, we benchmark the series solutions to the numerical solution obtained through employing the Maple package. The *pdsolve* in Maple uses FDM to discretize the differential equation. The technique is a centered implicit scheme that is capable of finding solutions to higher-order partial differential equations. As seen from the benchmark analysis in Fig. 2, the series

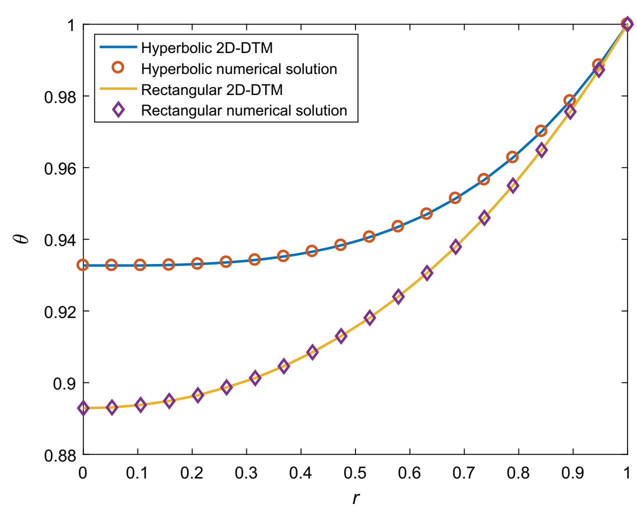

Fig. 2 Temperature distribution in differing fin profiles

solution is in agreement with the numerical solution across all profiles. With the confidence obtained from the benchmark analysis, we plot the solution (27) for various parameters as shown in subsequent figures. For all analytical results reported in this article, the following values of parameters are used unless stated otherwise as indicated in the graphs below.

$$n = 2,\ = 0.75,\ Pe = 3,\ \beta = 1,\ \tau = 0.60 \tag{29}$$

The governing equation for a rectangular radial fin profile is given by,

$$\frac{\partial\theta}{\partial\tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(r(1+\beta\theta)\frac{\partial\theta}{\partial r}\right) - M^2\theta^{n+1} - Pe\frac{\partial\theta}{\partial r}, \quad 0 \le r \le 1. \tag{30}$$

Following a similar approach performed for the hyperbolic profile, we construct series solution for $n = 2$,

$$\begin{aligned}\theta(\tau, r) = {} & a\tau + a\tau^2 + \frac{a}{2}\tau r^2 + \frac{1}{2}a(3-2a\beta)\tau^2 r^2 + a\tau^3 \\ & + \frac{1}{9}aPe\tau r^3 + \frac{1}{18}aPe(3-4a\beta)\tau^2 r^3 \\ & + \frac{1}{4}a(4 + a(\beta(-5+2a\beta) + aM))\tau^3 r^2 \\ & + \frac{1}{18}aPe(4 + a(2\beta(-5+3a\beta) + aM))\tau^3 r^3 \\ & + \ldots\end{aligned} \tag{31}$$

The analysis of the impact of thermo-physical parameters is performed on a fin undergoing nucleate film boiling ($n = 2$). The impact is the same across the different profiles. Figure 3 shows that as time increases, the temperature on the fin increases approaching unity even at small time scales and this corresponds to the imposed boundary conditions. The impact of the change in the thermal conductivity gradient is depicted in Fig. 4. We observe that as the values of β decrease, the temperature in the fin decreases signifying an increased heat loss to the ambient fluid. Figure 5 shows the effect of the nature of heat transfer on transient temperature distribution. The results show that the temperature distribution of the fin is an increasing function of exponent n. Figure 6 illustrates the effect of the Peclet number on the temperature distribution in the fin. As Pe increases (fin moves faster), the temperature inside the fin decreases rapidly due to increased advective effect on the surface of the fin. We also observe that for $Pe = 0$, i.e., stationery fin, the cooling of the fin takes more time as shown by higher temperatures when compared to a moving fin. One of the important parameters influencing temperature distribution is the thermo-geometric fin parameter M. Figure 7 depicts the effect of this parameter on temperature distribution. The figure confirms that for a fin with a high value of M, the temperature distribution to the ambient fluid is more effective. Counter-intuitively, for fin with a low value of M, the temperature near the fin base reaches its maximum with reduced heat dissipation. The temperature variation at the fin tip is shown in Fig. 8. As expected, the fin tip temperature gradually increases as time evolves. We observe that the temperature distribution along the three-dimensional spaces corresponds to the boundary conditions and to all the results described above and the graphs are depicted in Figs. 9 and 10.

Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin was subjected to the base temperature. The fin heat transfer rate, Q, is given by the heat conduction rate at the fin base [30, 31], that is

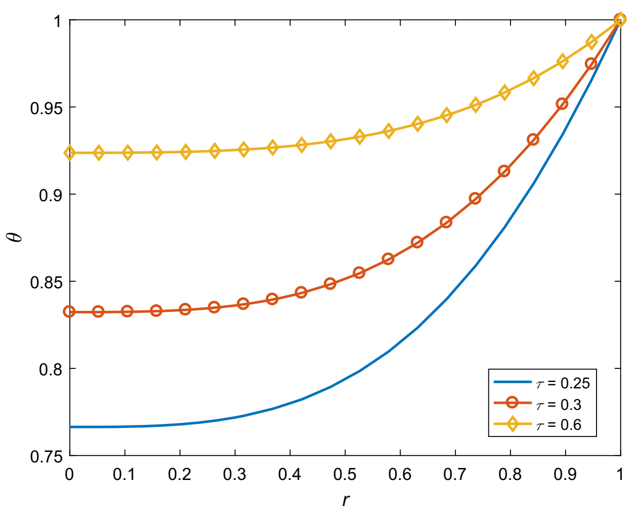

Fig. 3 Temperature distribution in a radial hyperbolic fin for varying time, τ

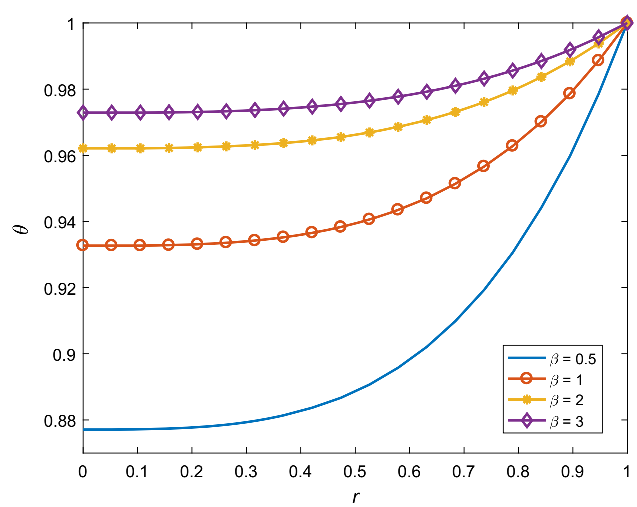

Fig. 4 Effect of the thermal conductivity gradient, β, on temperature distribution

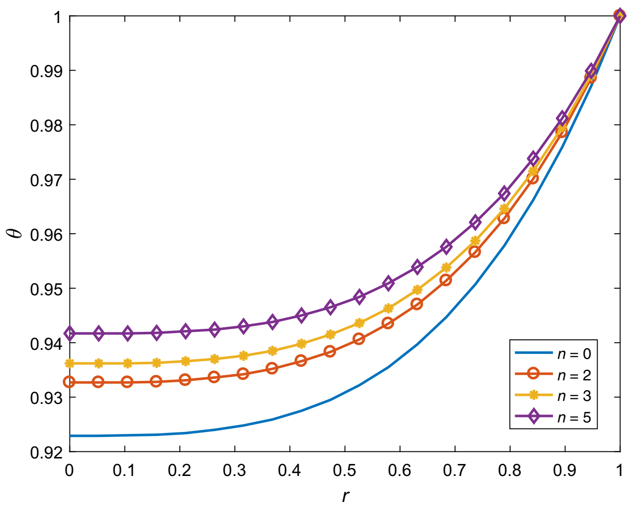

Fig. 5 Effect of varying values of power, n, on temperature distribution

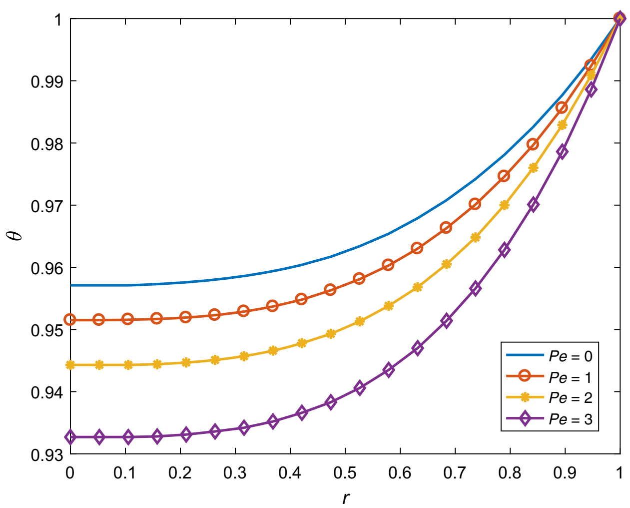

Fig. 6 Effect of varying speeds, Pe, on temperature distribution

$$Q = A_c K(T) \frac{\partial T}{\partial R}. \tag{32}$$

The ideal heat transfer rate obtained if the entire fin is kept at the base temperature is given by,

$$Q_{\text{ideal}} = PLH(T)(T - T_a) + \rho c U L A_c \frac{\partial T}{\partial R}, \tag{33}$$

and the fin efficiency in dimensionless variables is then given by,

$$\eta = \frac{Q}{Q_{\text{ideal}}} = \frac{(1 + \beta\theta)\frac{\partial\theta}{\partial r}\big|_{r=1}}{M^2 + Pe\frac{\partial\theta}{\partial r}\big|_{r=1}}. \tag{34}$$

The rate of heat transfer at the fin base for varying parameters is depicted in Figs. 11 and 12. The fin efficiency is shown in Fig. 13. The heat transfer rate from the

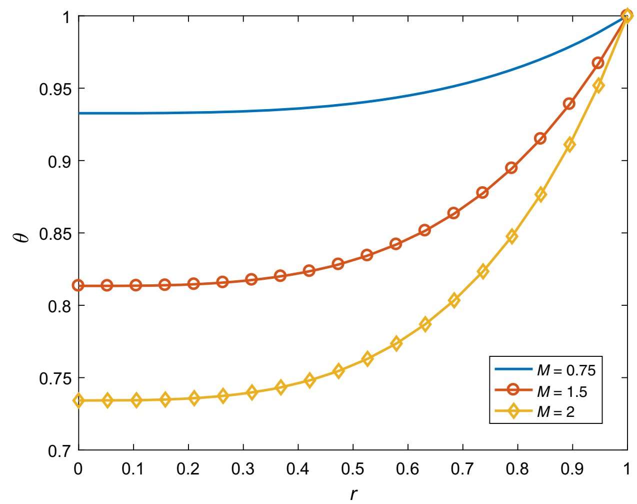

Fig. 7 Effect of the thermo-geometric parameter, M, on temperature distribution

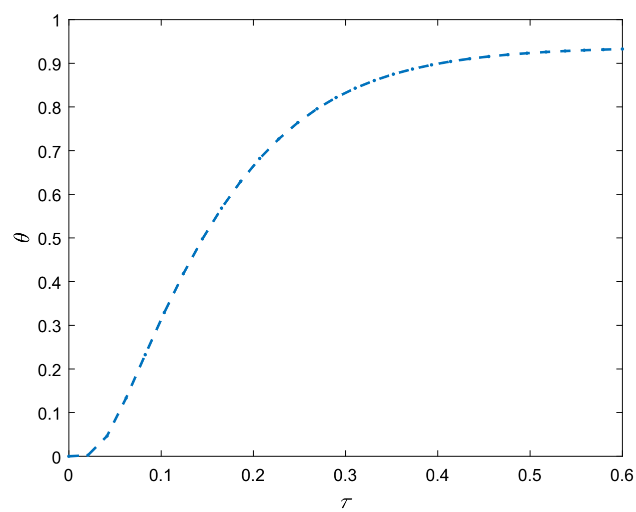

Fig. 8 Temperature distribution at the fin tip

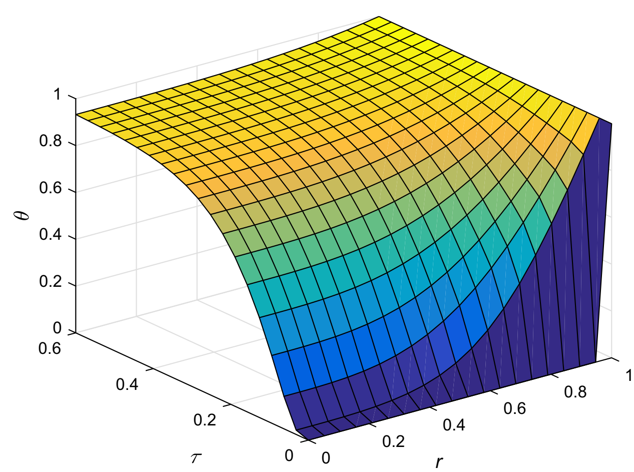

Fig. 9 Transient temperature distribution in a radial hyperbolic fin

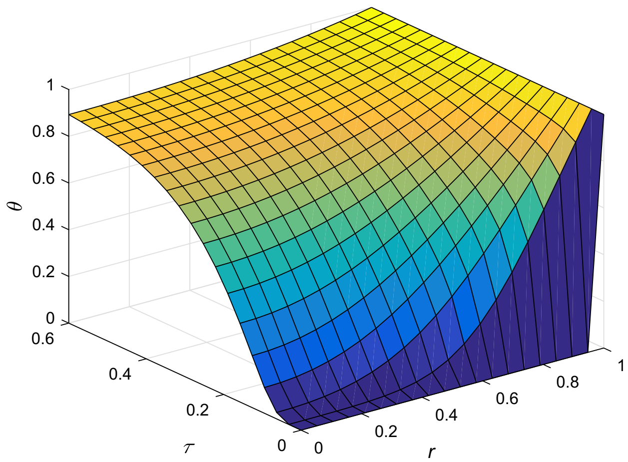

Fig. 10 Transient temperature distribution in a radial rectangular fin

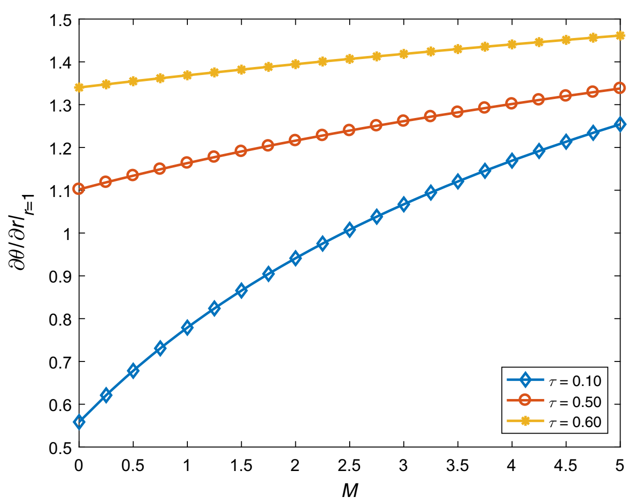

Fig. 11 Heat transfer gradient in a hyperbolic radial fin for varying fin time, τ and M

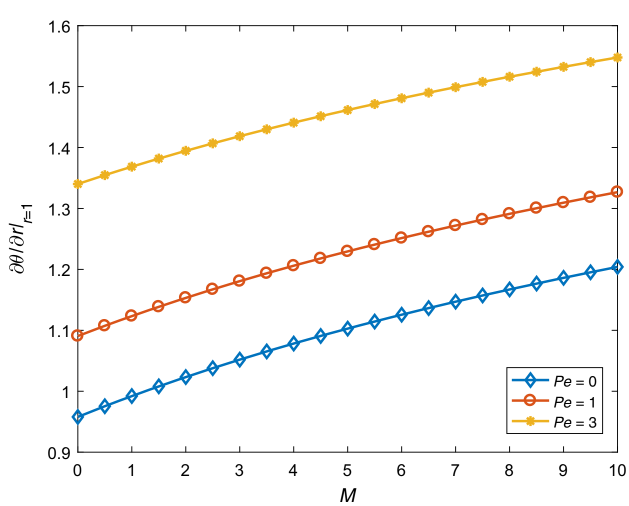

Fig. 12 Heat transfer gradient in a hyperbolic radial fin for varying fin speed, Pe and M

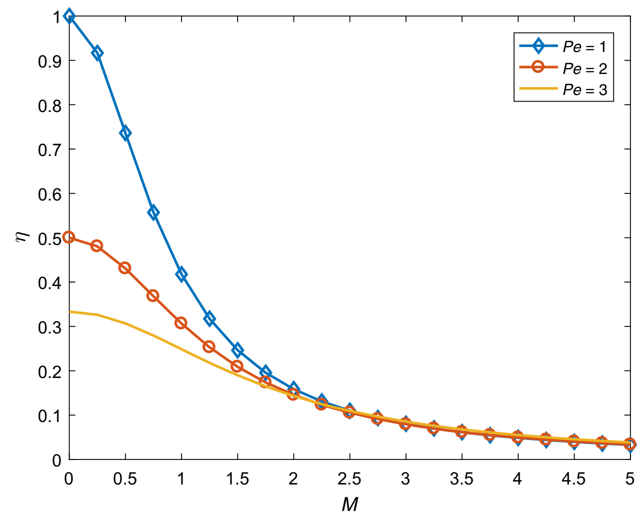

Fig. 13 Efficiency of a hyperbolic radial fin

fin base plays an important role in the fin efficiency. Figure 11 shows that rate of heat transfer from the fin base increases as time evolves. This is attributed to the fact that the fin is initially kept at ambient temperature and with thermal conduction occurring between the heated body and the fin, it is expected to see an increase in heat transfer rate with increasing time. Figure 12 shows that an increase in the speed of the moving fin leads to an increase in the rate of heat transfer. A moving fin induces a horizontal flow field (advection) which in turn leads to an increase in temperature gradient. Across Figs. 11 and 12, we observe that an increase in the convection-radiation parameter, M, leads to an increase in the rate of heat transfer. In Fig. 13, we note that the fin becomes more effective with an increase in the values of the thermo-geometric fin parameter and the non-dimensional speed of the moving fin.

Conclusions

It is difficult and sometimes impossible to construct exact solutions for highly nonlinear equations describing heat transfer through extended surfaces. However, since the DTM technique is more effective and produces more accurate results [3, 5], we proceed by applying the 2D-DTM to a heat conduction problem for heat transfer in radial moving fins. The results show that fin profile has an important role in temperature distribution especially near the fin tip. Also, we introduced the effect of the speed of the moving fin and noted that it affects the rate at which the fin dissipates heat to the ambient fluid. Furthermore, the results show that the fin surface temperature is greatly dependent on the nature of the heat transfer. To our knowledge, the 2D-DTM has not yet been applied to fin problems for transient heat flow in moving fins.

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2.4 Journal Article IV

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THERMAL ANALYSIS OF NATURAL CONVECTION AND RADIATION HEAT TRANSFER IN MOVING POROUS FINS

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ABSTRACT

In this article, the Differential Transform Method (DTM) is used to perform thermal analysis of natural convective and radiative heat transfer in moving porous fins of rectangular and exponential profiles. This study is performed using Darcy's model to formulate the governing heat transfer equations. The effects of porosity parameter, irregular profile and other thermo-physical parameters, such as Peclet number and the radiation parameter are also analyzed. The results show that the fin rapidly dissipates heat to the surrounding temperature with an increase in the values of the porosity parameter and the dimensionless time parameter. The results also show that the heat transfer rate in an exponential profile with negative power factor is much higher than the rectangular profile.

Keywords: *Analytical Solutions, DTM, Darcy's model, heat transfer, fins*

1. INTRODUCTION

Fins are extensively used in many heat transfer applications to enhance heat dissipation from a heated surface. An increased rate of heat transfer with reduced size and cost of fins is the main target for a number of engineering applications such as heat exchangers, conventional furnaces and gas turbines. Some engineering applications such as hydroplane and motorcycle also require a lighter fin with a higher rate of heat transfer. Increasing the heat transfer mainly depends on the heat transfer coefficient, the surface area available and the temperature difference between the surface and surrounding fluid. Extensive studies have been conducted to find an optimum shape and size for the fins to increase the heat transfer rate in different applications. The use of porous fins as a passive method for heat transfer enhancement has attracted a lot of research interests following the pioneer work of Kiwan and Al-Nimr (2001). Also, the research interests have aroused following the further studies by Kiwan (2007); Kiwani and Zeitoun (2008) on the thermal performance analysis of porous fins in natural convection environment. Following the work of Kiwan, most researchers have introduced other thermo-physical parameters to further improve the heat transfer from fins. Recently, a vast amount of research studies have been carried out by various authors in porous medium to improve the heat transfer process through fins.

The heat transfer rate by using porous medium can be further enhanced by increasing the effective area for the heat transfer by convection. The steady state thermal analysis of natural convection and radiation in porous fins was analytically conducted using Homotopy Analysis Method (HAM) by Moradi et al. (2014). These authors analyzed both infinite and finite fins with rectangular profile. Jooma and Harley (2017) studied a porous model for the radial fin. Contrary to their findings, the one dimensional DTM yielded approximate solutions which converge after thirteen iterations without requiring extra memory for construction of steady state solutions. Razelos and Kakatsios (2000) investigated the optimum dimensions of convecting-radiating rectangular fins. They studied the influence of all dependent parameters on optimization and performance of rectangular fins. They also investigated the effect of the temperature-dependent thermal conductivity and emissivity on the temperature profile.

In recent years, the heat transfer from a moving fin has attracted the attention of various researchers. This phenomenon is significant in numerous industrial applications such as extrusion, hot rolling, glass fiber drawing and casting. Turkyilmazoglu (2018) studied heat transfer from moving exponential fins with internal heat generation. Exact formulas for the thermal features like the distribution of temperature as well as the efficiency were derived and analyzed. The heat transfer of moving fins through a non-Newtonian fluid was first analytically studied by Fox et al. (1969). The effect of temperature-dependent thermal conductivity of the moving fin and radiation effect have been studied by Aziz and Khani (2011). They applied the HAM to solve governing equations and compared the analytical and numerical results. An analysis of heat transfer in a porous fin with temperature-dependent thermal conductivity and internal heat generation was conducted using Chebychev Spectral Collocation Method (CSCM) by Sobamowo (2017). This author used numerical solutions to investigate the effects of various parameters on the thermal performance of the porous fin. The results showed that increasing the porosity parameter also increases the rate of heat transfer from the base of the fin and consequently improve the efficiency of the fin. Colloca-

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tion Method (CM) and the Homotopy Perturbation Method (HPM) were presented by Hoshyar et al. (2015) to determine the temperature distribution in a porous fin with temperature-dependent internal heat generation. Their results were validated by using the Runge-Kutta fourth order method. They discovered that as the buoyancy effects become stronger, the local temperature in the fin decreases. Following similar work undertaken by Hoshyar et al. (2015), Oguntala and Abd-Alhameed (2017) applied the Haar Wavelet Collocation Method (HWCM) to analyze the thermal performance of porous fins. A simulation of combined conductive, convective and radiative heat transfer in moving irregular porous fins of various profiles was performed by Ma et al. (2017) using Spectral Element Method (SEM). Besides, the effects of porous materials, profiles and various thermo-physical parameters on non-dimensional, the authors also investigated the temperature and volume adjusted fin efficiency. Gorla and Bakier (2011) solved a second order non-linear ordinary differential equation (ODE) describing heat transfer in a stationary porous fin using the Runge-Kutta fourth order method. It is found that in their analysis that radiates and convects transfers more heat than that dissipates heat by convection only. Kundu and Lee (2016) developed new analytical analysis by using double DTM to determine the temperature distribution in annular step porous fins subject to a mobile condition. They concluded that the porous fins have an ability to transfer more heat compared to the solid fin for an equal mass of fins at an optimum condition.

In this study, the DTM is applied to solve nonlinear heat transfer equation of a moving porous fins of rectangular and exponential profiles to determine temperature distribution along the fin. The effect of radiation on the performance of porous fin is studied in natural convection environment. We introduce a time factor to the governing equations resulting in highly nonlinear second order partial differential equations (PDEs). This improvement has not been developed in the related literature. The effect of porosity and radiation parameter, temperature ratio and Peclet number on dimensionless temperature and heat transfer rate is studied. The DTM is the most dominant method of analyzing temperature distribution through fin equations as it difficult to construct exact solutions. The DTM is an analytical method based on Taylor series expansion and was first introduced by Zhou (1986) in 1986. Zhou used the DTM to solve explicitly the linear and nonlinear initial value problems that arise in electrical circuits. Chen and Ho (1999), developed a two-dimensional DTM which can be used for solving differential and integral equations. Ayaz (2000), showed that the DTM is better equipped to solve highly nonlinear problems than the Taylor series method. The DTM has been used to solve various problems in applied mathematics and physics such as systems of differential equations (Kanth and Aruna, 2008).

This article is organized as follows; the mathematical background of the problem under consideration is described in § 2. A brief discussion on the fundamentals of the 2D DTM will be provided in § 3. We provide analytical solutions in § 4. Lastly, we provide some discussions based on the results obtained in § 5 and concluding remarks in § 6.

2. MATHEMATICAL FORMULATION

We consider a moving porous fin of length L and cross-sectional area A_c while it moves horizontally with a constant velocity U along an axial coordinate X as depicted in Fig. 1. The fin surface is exposed to an environment of temperature T_a and the base temperature of the fin is $T_b > T_a$. As the fin is porous, it allows fluid to penetrate through it, which enhances the convective heat transfer, The fin thickness at the prime surface is given by δ_b and its profile is given by $F(X)$. The following assumptions are considered for the problem under consideration:

- The fin tip is adiabatic.
- The temperature inside the fin is only function of t and X.
- The physical properties of both solid wall and fluid are constant.
- The porous medium is homogeneous, isotropic and saturated with a single-phase fluid.
- The fluid and porous medium are not in local thermodynamic equilibrium.
- For the interaction between the porous medium and fluid, the Darcy's model is applied

The energy balance to the slice segment of the fin thickness Δx, as depicted in Fig. 1, requires:

$$\begin{aligned} I &= q(X) - q(X+\Delta X) - \dot{m}c(T-T_a) \\ &\quad - P\Delta X \varepsilon\sigma(T^4 - T_a^4) - \rho c A_c U \Delta X \frac{dT}{dX} \end{aligned} \tag{1}$$

where ρ is the density of fluid; c is the specific heat of fluid; P is the periphery of the fin; ε and σ are the emissivity and Boltzman constant respectively; T the temperature of the fin. q is the heat flux of conduction. The time rate of change of internal energy in the volume element I is given by,

$$I = \rho c A_c \frac{dT}{dt}\Delta X. \tag{2}$$

By Fourier's law of conduction,

$$q = -k_c A_c \frac{dT}{dX} \tag{3}$$

where k_c is the thermal conductivity of the fin. The mass flow rate of fluid $\dot{m}$ passing through the porous material is expressed by (Kiwan, 2007):

$$\dot{m} = \rho \nu_w W \Delta X \tag{4}$$

where W is the width of the fin. The value of ν_w should be estimated from the consideration of the flow through the porous medium. Darcy's model then yields:

$$\nu_w = \frac{gK\beta}{\upsilon}(T(X) - T_a). \tag{5}$$

Darcy's model stimulates the fluid-solid interaction in the porous medium. Here, g is the gravitational acceleration, K is the permeability, υ and β are the kinematic viscosity and thermal expansion coefficient respectively. Substituting Equations (2) − (5) into Equation (1), the following nonlinear partial differential equation governing the temperature distribution in the fin is obtained:

$$\begin{aligned} \rho c \frac{\partial T}{\partial t} &= \frac{\delta_b}{2}\frac{\partial}{\partial X}\left(k_c F(X)\frac{\partial T}{\partial X}\right) - \frac{\rho c g K \beta W}{\upsilon A_c}(T-T_a)^2 \\ &\quad - \frac{P\sigma\varepsilon}{A_c}(T^4 - T_a^4) - \frac{\delta_b}{2}F(X)\rho c U \frac{\partial T}{\partial X}, \; 0 \le X \le L, \end{aligned} \tag{6}$$

where t is the time. $\delta_b F(X)/2$ is introduced to the energy balance equation to cater for varying fin profiles. Assuming that the fin tip is adiabatic (insulated) and the base temperature is kept constant, then the boundary conditions are given by (Kraus et al., 2001; Kern and Kraus, 1972),

$$T(t, L) = T_b \text{ and } \left.\frac{\partial T}{\partial X}\right|_{X=0} = 0, \tag{7}$$

and initially the fin is kept at the ambient temperature (Ndlovu and Moitsheki, 2013),

$$T(0, X) = T_a. \tag{8}$$

Introducing the following dimensionless variables,

$$\begin{aligned} &\theta = \frac{T-T_a}{T_b-T_a},\; x = \frac{X}{L},\; \tau = \frac{k_c t}{\rho c L^2},\; f(x) = \frac{\delta_b}{2}F(X), \\ &\alpha = \frac{k_c}{\rho c},\; Pe = \frac{UL}{\alpha},\; N_p = \frac{gK\beta W L^2}{\upsilon\alpha A_c}(T_b - T_a), \\ &N_r = \frac{P\varepsilon\sigma L^2}{k_c A_c}(T_b - T_a)^3 \text{ and } N_T = \frac{T_a}{T_b - T_a} \end{aligned} \tag{9}$$

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reduces Eqn. (6) to

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial x}\left(f(x)\frac{\partial \theta}{\partial x}\right) - N_p\theta^2 - N_r((\theta + N_T)^4 - N_T^4)$$
$$- f(x)Pe\frac{\partial \theta}{\partial x},\ 0 \le x \le 1, \tag{10}$$

and Eqn. (10) admits the following boundary conditions,

$$\theta(\tau, 1) = 1,\ \tau > 1, \tag{11}$$

$$\left.\frac{\partial \theta}{\partial x}\right|_{x=0} = 0, \tag{12}$$

and the initial condition becomes,

$$\theta(0, x) = 0. \tag{13}$$

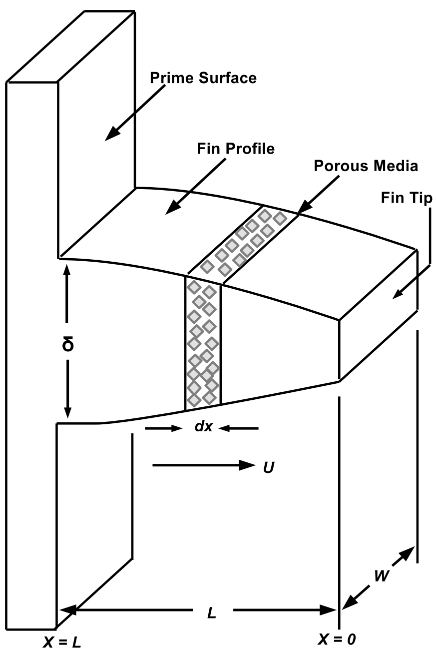

Fig. 1. Schematic representation of a longitudinal porous fin of an unspecified profile.

Here, N_r is the radiation parameter; N_T is the dimensionless ratio of the surrounding temperature T_a to the difference of the base temperature T_b and surrounding temperature; N_p is the porosity parameter that represents the effect of permeability of the porous medium beside the natural convection effects. The dimensionless variable Pe is the Peclet number which represent the dimensionless speed of the moving fin and $Pe = 0$ represents a stationary fin; θ is the dimensionless temperature, x is the dimensionless space variable, τ is the non dimensional time.

3. FUNDAMENTALS OF THE TWO-DIMENSIONAL DIFFERENTIAL TRANSFORM METHOD

In this section, the basic idea underlying the two-dimensional DTM is briefly introduced. If function $\theta(t, x)$ is analytic and differentiated continuously with respect to time and the spatial variable x in the domain of interest, then we let

$$\Phi(\kappa, s) = \frac{1}{\kappa! s!}\left[\frac{\partial^{\kappa+s}\phi(t, x)}{\partial t^\kappa \partial x^s}\right]_{(0,0)}, \tag{14}$$

where the spectrum $\Phi(\kappa, s)$ is the transformed function, which is also called the T-function (see Kangalgil and Ayaz (2009); Ayaz (2000)). The differential inverse transform of $\Phi(\kappa, s)$ is defined as

$$\phi(t, x) = \sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty}\Phi(\kappa, s)t^\kappa x^s, \tag{15}$$

and from Eqns. (14) and (15) it can be concluded that

$$\phi(t, x) = \sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty}\frac{1}{\kappa! s!}\left[\frac{\partial^{\kappa+s}\phi(t, x)}{\partial t^\kappa \partial x^s}\right]_{(0,0)} t^\kappa x^s. \tag{16}$$

In real applications, the function $\phi(t, x)$ is expressed by a finite series, and Eqn. (15) can be written as:

$$\phi(t, x) = \sum_{\kappa=0}^{m}\sum_{s=0}^{n}\Phi(\kappa, s)t^\kappa x^s, \tag{17}$$

Equation (17) implies that

$$\phi(t, x) = \sum_{\kappa=m+1}^{\infty}\sum_{s=n+1}^{\infty}\Phi(\kappa, s)t^\kappa x^s, \tag{18}$$

is negligibly small.

Some of the useful mathematical operations performed by the differential transform method are given in Table 1 .

Table 1: Fundamental operations of the differential transform method

Original function	Transformed function
$\phi(t, x) = f_1(t, x) \pm f_1(t, x)$	$\Phi(\kappa, s) = F_1(\kappa, s) \pm F_2(\kappa, s)$
$\phi(t, x) = \alpha f(t, x)$	$\Phi(k, s) = \alpha F(\kappa, s)$
$\phi(t, x) = \frac{\partial f(t,x)}{\partial t}$	$\Phi(\kappa, s) = (\kappa + 1)F(\kappa + 1, s)$
$\phi(t, x) = \frac{\partial f(t,x)}{\partial x}$	$\Phi(\kappa, s) = (s + 1)F(\kappa, s + 1)$
$\phi(t, x) = \frac{\partial^2 f(t,x)}{\partial t^2}$	$\Phi(\kappa, s) = (\kappa + 1)(\kappa + 2)F(\kappa + 2, s)$
$\phi(t, x) = \frac{\partial^2 f(t,x)}{\partial x^2}$	$\Phi(\kappa, s) = (s + 1)(s + 2)F(\kappa, s + 2)$
$\phi(t, x) = t^m x^n$	$\Phi(\kappa, s) = \delta(\kappa - m)\delta(s - n)$
$\phi(t, x) = x^m \exp(at)$	$\Phi(\kappa, s) = \frac{a^\kappa}{\kappa!}\delta(s - m)$

The Kronecker delta function $\delta(\kappa - s)$ is given by

$$\delta(\kappa - s) = \begin{cases} 1 & \text{if } \kappa = s \\ 0 & \text{if } \kappa \neq s. \end{cases}$$

4. DERIVATION OF ANALYTICAL SOLUTIONS

In this section we provide analytical solutions for the problem described in § 2. Using 2D-DTM to solve PDEs consists of three main steps. The steps are: transforming the PDE into algebraic equations, solving the equations, and inverting the solution of algebraic equations to obtain an infinite series solution or an approximate solution. The fin profile is rectangular when $f(x) = 1$ and exponential when $f(x) = e^{\eta x}$ where η is a constant. Taking the two-dimensional differential transform of Eqn. (10)

given a rectangular profile, we obtain the following recurrence relations,

$$\begin{aligned}
&(\kappa+1)\Theta(\kappa+1,h)=(h+1)(h+2)\Theta(\kappa,h+2)\\
&-N_p\sum_{i=0}^{\kappa}\sum_{j=0}^{h}\Theta(i,h-j)\Theta(\kappa-i,j)\\
&-4N_rN_T^3\Theta(k,h)-6N_rN_T^2\sum_{i=0}^{\kappa}\sum_{j=0}^{h}\Theta(i,h-j)\Theta(\kappa-i,j)\\
&-4N_rN_T^2\beta\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\Theta(i,h-j-p)\Theta(l,j)\Theta(\kappa-i-l,p)\\
&-N_r\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{z=0}^{\kappa-i-l}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\sum_{t=0}^{h-j-p}\\
&\Theta(i,h-j-p-t)\Theta(l,j)\Theta(z,p)\Theta(\kappa-i-l-z,t)\\
&-Pe(h+1)\Theta(k,h+1),
\end{aligned}\tag{19}$$

where $\Theta(\kappa,h)$ is the differential transform of $\theta(\tau,x)$.

Taking the two-dimensional differential transform of the initial condition (13) and boundary condition (12) we obtain the following transformations respectively,

$$\Theta(0,h)=0,\ h=0,1,2,\ldots\tag{20}$$

$$\Theta(\kappa,1)=0,\ \kappa=0,1,2,\ldots.\tag{21}$$

We consider the other boundary condition as follows,

$$\Theta(\kappa,0)=a,\ a\in\mathbb{R},\ \kappa=1,2,3,\ldots\tag{22}$$

where the constant a can be determined from the boundary condition (11) at each time step after obtaining the series solution.

Substituting Eqns. (20)-(22) into (19) we obtain the following,

$$\Theta(1,2)=c+2cN_rN_T^3\tag{23}$$

$$\Theta(2,2)=\frac{1}{2}(3c+c^2N_p+6c^2N_rN_T^2+4cN_rN_T^3)\tag{24}$$

$$\Theta(1,3)=\frac{1}{3}(Pe(c+2cN_rN_T^3))\tag{25}$$

$$\Theta(2,3)=\frac{1}{6}(Pe(3c+c^2N_p+6c^2N_rN_T^2+4cN_rN_T^3))\tag{26}$$

$$\vdots$$

Substituting Eqns. (20)-(26) into (17) we obtain the following closed form of the solution,

$$\begin{aligned}
\theta(\tau,x)&=a\tau+a\tau^2+a+2aN_rN_T^3\tau x^2\\
&+\frac{1}{2}(3a+a^2N_p+6a^2N_rN_T^2+4aN_rN_T^3)\tau^2x^2\\
&+a\tau^3+\frac{1}{3}(Pe(a+2aN_rN_T^3))\tau x^3\\
&+\frac{1}{6}(Pe(3a+a^2N_p+6a^2N_rN_T^2+4aN_rN_T^3))\tau^2x^3\\
&+\ldots
\end{aligned}\tag{27}$$

The constant a can be determined from the boundary condition (11) at each time step. To obtain the value of a, we substitute the boundary condition (11) into (27) at the point $x=1$. Thus, we have,

$$\begin{aligned}
\theta(\tau,1)&=a\tau+a\tau^2+(a+2aN_rN_T^3)\tau\\
&+\frac{1}{2}(3a+a^2N_p+6a^2N_rN_T^2+4aN_rN_T^3)\tau^2\\
&+a\tau^3+\frac{1}{3}(Pe(a+2aN_rN_T^3))\tau\\
&+\frac{1}{6}(Pe(3a+a^2N_p+6a^2N_rN_T^2+4aN_rN_T^3))\tau^2\\
&+\ldots=1
\end{aligned}\tag{28}$$

We then obtain the expression for $\theta(\tau,x)$ upon substituting the obtained value of a into equation (27). Following a similar approach, we obtain the following solution for the exponential fin profile,

$$\begin{aligned}
\theta(\tau,x)&=a\tau+a\tau^2+(a+2aN_rN_T^3)\tau x^2\\
&+\frac{1}{2}(3a+a^2N_p+6a^2N_rN_T^2+4aN_rN_T^3)\tau^2x^2\\
&+a\tau^3+\frac{1}{3}(aPe+2aPeN_rN_T^3-2a\eta-4aN_rN_T^3\eta)\tau x^3\\
&+(2a+a^2N_p+6a^2N_rN_T^2+2a^3N_rN_T^2+2aN_rN_T^3)\tau^3x^2\\
&+\ldots
\end{aligned}\tag{29}$$

Using the first 20 terms of the power series solution, we benchmark the series solutions to the numerical solution obtained through employing the Maple package. The pdsolve in Maple uses Finite Difference Method (FDM) to discretize the differential equation. The technique is a centered implicit scheme that is capable of finding solutions to higher-order partial differential equations. As seen from the benchmark analysis in Fig. 2, the series solution is in agreement with the numerical solution across all profiles.

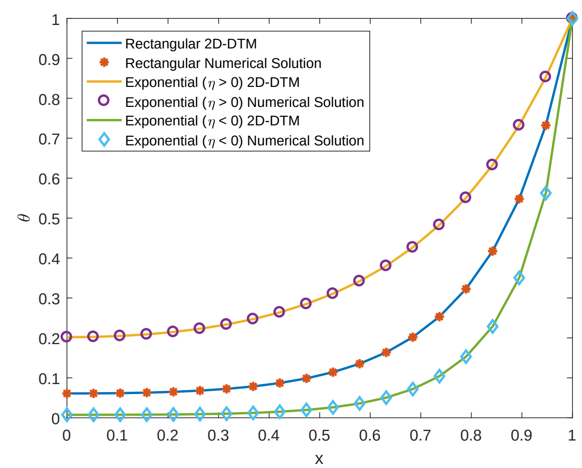

Fig. 2. Temperature distribution in differing fin profiles.

With the confidence obtained from the benchmark analysis, we plot the solutions for (27) and (29) for various parameters as shown in Figs. below. For all analytical results reported in this article, the following values of parameters are used unless stated otherwise as indicated in the graphs below.

$$N_p=10,\ N_r=5,\ N_T=0.10,\ Pe=3,\ \tau=0.6\tag{30}$$

5. SOME DISCUSSIONS

In this article, we have established analytical expressions for determination of thermal performance in moving porous fins with three different profiles. Figs. 3 and 4 shows that as time evolves, the fin temperature increases across different profiles as a result of thermal conduction. Fig. 5 depicts the variation of temperature distribution with varying values of N_T. We note that the temperature on the fin decreases with the increasing values of N_T. This signifies that the heat transfer rate reaches its peak with the increasing values of N_T while other parameters are kept constant. The only way to increase N_T is through changing the value of the ambient temperature for a fixed base temperature. The effect of the porosity parameter N_p is shown in Fig. 6. Expectedly, as the values of the porosity parameter increase, the heat transfer rate to the ambient fluid is increased as evidenced by the rapid decrease of the dimensionless

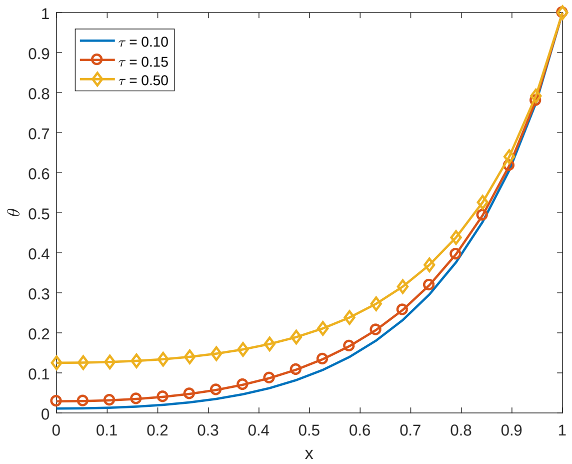

Fig. 3. Temperature distribution in a longitudinal rectangular fin for varying time, τ.

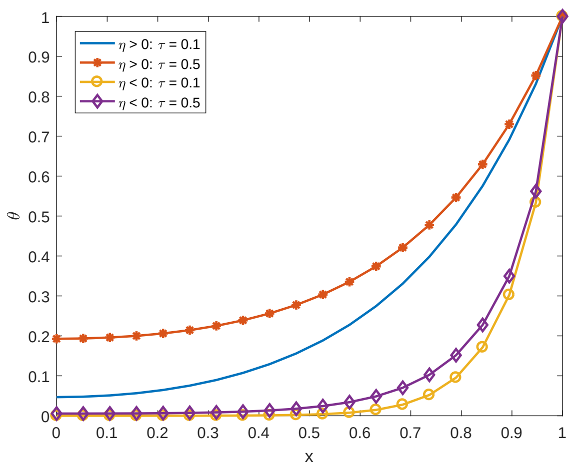

Fig. 4. Temperature distribution in a longitudinal exponential fin for varying time, τ.

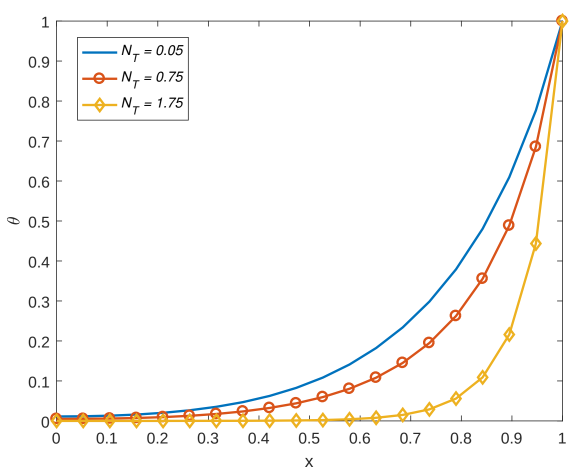

Fig. 5. Effect of the dimensionless ratio of temperature, N_T, on a rectangular fin temperature distribution.

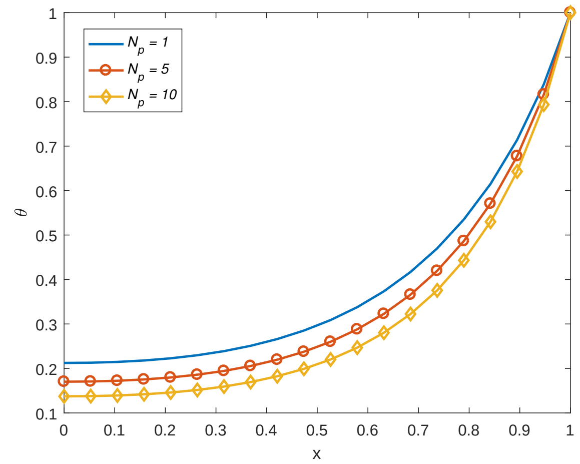

Fig. 6. Effect of the porosity parameter, N_p, on a rectangular fin temperature distribution.

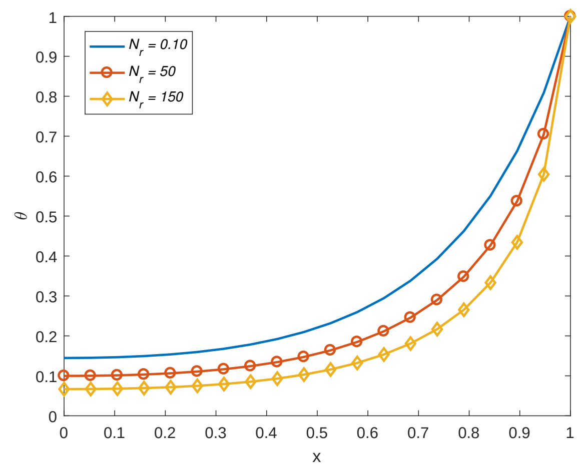

Fig. 7. Effect of the radiation parameter, N_r, on a rectangular fin temperature distribution.

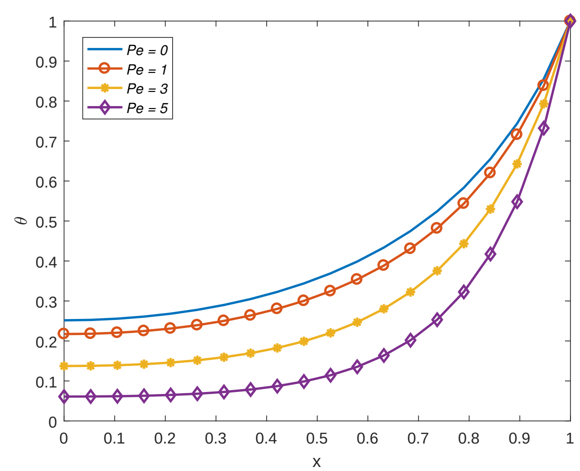

Fig. 8. Effect of varying speeds, Pe, on a rectangular fin temperature distribution.

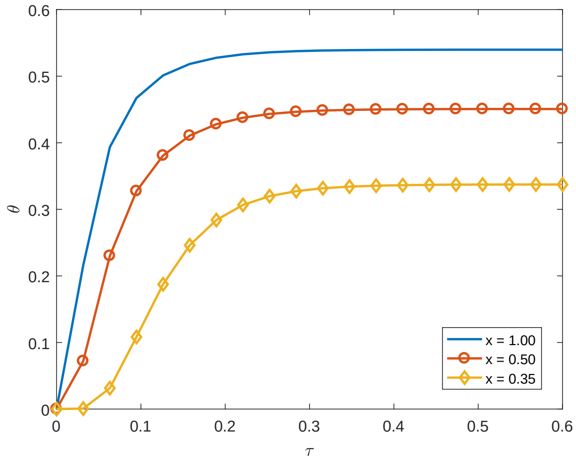

Fig. 9. Temperature distribution for differing axial values in a rectangular fin.

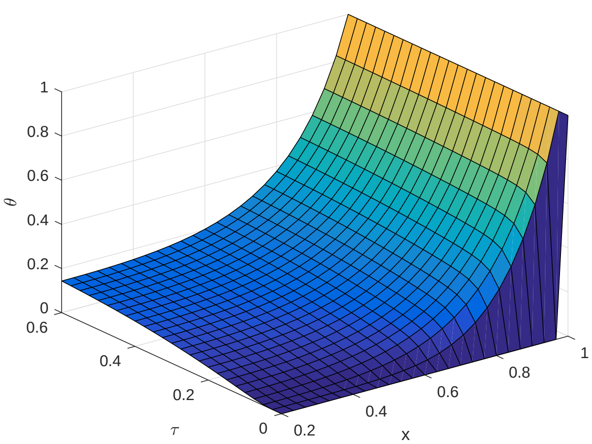

Fig. 10. Transient temperature distribution in a longitudinal rectangular fin.

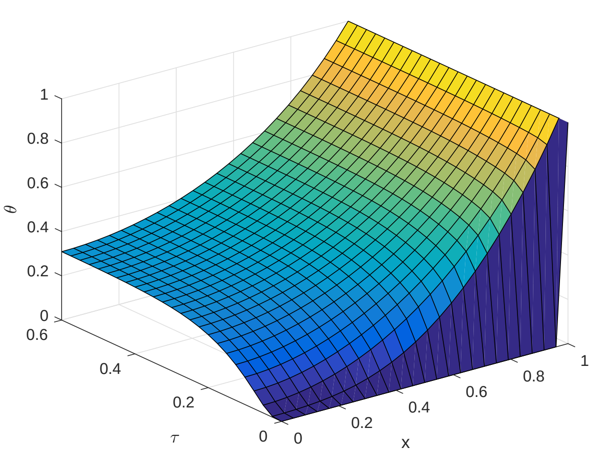

Fig. 11. Transient temperature distribution in a longitudinal exponential fin, $\eta = 2$.

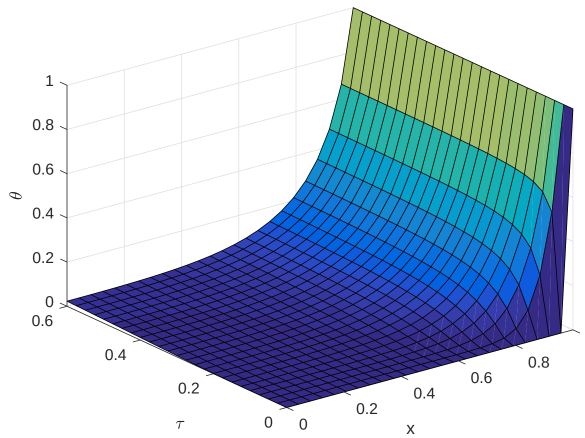

Fig. 12. Transient temperature distribution in a longitudinal exponential fin, $\eta = -2$.

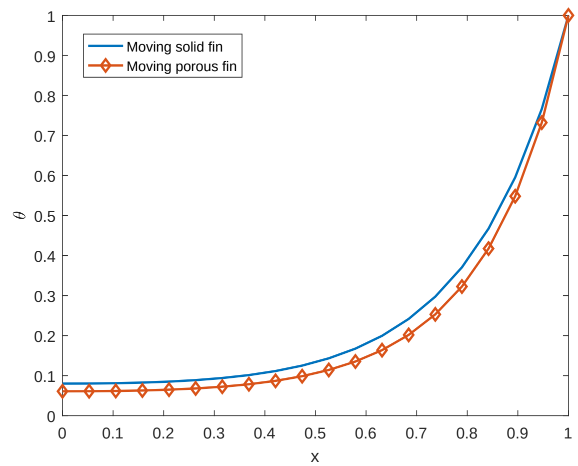

Fig. 13. Effect of different porous media in a longitudinal rectangular fin.

temperature inside the fin. In the case of a porous fin exposed to natural convection, higher porosity due to changing of permeability included in N_p results in higher rate of heat transfer. Physically, this can be caused by the effect of larger area of porous medium. Fig. 7 shows the effect of the radiation parameter N_r on temperature distribution. We note that as radiation gets stronger, the fin loses heat to the ambient fluid effectively as shown by the decreasing temperatures inside the fin. It can be deduced from the results that thermal energy transfers by radiation enhance heat transfer rate from fin. The intensity of such energy flux depends upon the temperature difference between fin and ambient and the nature of fin's surface.

The impact of the speed of the moving fin as defined by the Peclet number Pe is shown in Fig. 8. We also observe that for $Pe = 0$, i.e., stationery fin, the cooling of the fin takes more time as shown by higher temperatures when compared to a moving fin. Physically, this is the ratio of the rate of advection of thermal energy by the flow to the rate of thermal energy diffusion driven by an appropriate gradient. As Pe increases, that is, the fin moves quicker, the advection effect becomes stronger and the fin dissipates heat rapidly as indicated by the lower temperatures on inside the fin. Consequently, where cooling is required, the higher value of Peclet number is desirable. Fig. 9 shows the variation in temperature distribution for different values of the axial parameter x, that is, the temperature variation along the fin profile. We see that as the the axial variable increases, i.e, as we approach the fin base, the temperature inside the fin increases. The comparison between the fin of rectangular profile and that of an exponential profile is show in Fig. 2. We see that the fin of exponential profile with a negative power is more efficient in dissipating heat to the surrounding fluid. The validation of the results and the boundary conditions are shown as three dimensional plots in Figs. 10, 11 and 12. Lastly, Fig. 13 shows that the moving porous fin dissipates heat faster and efficiently to the the surrounding fluid that the moving solid fin.

6. CONCLUDING REMARKS

In this study, we have illustrated that the temperature distribution in the moving porous fin is highly dependent upon the fin profile. This is an important finding for fin design as we noted that an exponential porous fin is more efficient than the solid fin. We also discovered that the temperature distribution in a moving porous fin can be affected by changes in the underlying physical parameters. The DTM was used to perform thermal analysis as it is the most dominant analytical method for these kinds of problems and has been found to converge faster to the exact solutions with a appropriate number of iterations considered.

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NOMENCLATURE

c	specific heat (J/kg · K)
h	heat transfer coefficient (W/m^2 · K)
k	thermal conductivity (W/m · K)
k_c	thermal conductivity (W/m · K)
t	time (s)
T	temperature (K)
U	velocity of the fin (m/s)
X	space variable (m)
x	dimensionless space variable (m)
A_c	cross-sectional area (m^2)
L	fin length (m)
L	fin profile
W	fin width
T_a	ambient temperature (K)
T_b	fin base temperature (K)
P	fin perimeter
K	permeability of the porous medium
Pe	Peclet number
$\Phi(t, x)$	transformed analytical function
$\phi(t, x)$	original analytical function
q	heat flux (W/m^2)
$\dot{m}$	mass flow rate of fluid (Kg/s)
N_r	radiation parameter
N_T	temperature ratio
N_p	porosity parameter
g	gravitational acceleration (m/s^2)
Greek Symbols	
σ	Boltzmann constant (W/m^2 · K^4)
δ_b	base fin width (m)
ε	total emissivity
β	thermal expansion coefficient (K^{-1})
ρ	density (kg/m^3)
τ	dimensionless temperature
ρ	density (kg/m^3)
α	thermal diffusivity (m^2/s)
υ	kinematic viscosity (m^2/s)
θ	dimensionless temperature

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2.5 Journal Article V

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Analytical Study of Transient Heat Transfer in a Triangular Moving Porous Fin with Temperature Dependant Thermal Properties

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Abstract. The present work investigates heat enhancement through a moving porous fin of triangular profile. The interaction between the fluid and the porous media in not locally thermodynamic equilibrium. The transport through the porous medium is derived using Darcy's model. The calculations are carried out using the two-dimensional Differential Transformation Method (2D DTM), which is an analytical solution technique that can be applied to various types of differential equations. The approximate analytical solutions are validated using the numerical solution obtained via the inbuilt numerical solver in MATLAB (*pdepe*). The results reveal that the 2D DTM can achieve accurate results in predicting the solution of such problems. The impact of the thermal parameters on temperature distribution is performed using the closed-form solution generated by DTM.

Introduction

Heat enhancement from fins has become an important factor that has captured the interest of many researchers. Increasing the heat transfer mainly depends on the heat transfer coefficient, the surface area available and the temperature difference between the surface and surrounding fluid. Fins are used as heat dissipators by increasing the surface area of heated surface that is exposed to an ambient fluid. In particular, fins are used in gas turbines, space nuclear reactor power systems, computer processors and many more. The use of porous fins as a passive method for heat transfer enhancement has attracted a lot of research interests following the work of Kiwan and Al-Nimr [1]. Also, the research interests have aroused following further studies by Kiwan [2, 3] on the thermal performance analysis of porous fins in natural convection environment. Following the work of Kiwan, most researchers have introduced other thermo-physical parameters to further improve the heat transfer from fins. The heat transfer rate through a porous medium can be further enhanced by increasing the effective area for the heat transfer by convection. The heat enhancement from porous fins can be studied in a stationery system [4] or in a moving system [5]. Razelos and Kakatsios [6] investigated the optimum dimensions of convective–radiating rectangular fins. They studied the influence of all dependent parameters on optimization and performance of rectangular fins. They also investigated the effect of the temperature-dependent thermal conductivity and emissivity on the temperature profile. Hamdan and Al-Nimr [7] studied a steady, fully developed laminar forced convection heat augmentation via porous fins in isothermal parallel-plate duct. The Darcy-Brinkman-Forchheimer model was applied to model the flow inside the porous fins. Their study reports the effect of several operating parameters on the flow hydrodynamics and thermal characteristics.

A need for a deeper understanding of the transport mechanism of the momentum and the heat transfer in porous media continues unabated. Porous media has a wide range of applications such ground water flow, natural gas production, lubrication [8], laser mirrors and powerful electronics [9]. Most of the analytical studies of the fluid flow and the heat transfer in porous media adopt the Darcy's law [10, 11]. Oguntala and Abd-Alhameed [12] applied the Haar Wavelet Collocation Method (HWCM) to analyze the thermal performance of porous fins. A simulation of combined conductive, convective and radiative heat transfer in moving irregular porous fins of various profiles was performed by

Ma et al. [13] using Spectral Element Method (SEM). Gorla and Bakier [25] solved a second order non-linear ordinary differential equation describing heat transfer in a stationary porous fin using the Runge–Kutta fourth order method. It is found in their analysis that radiates and convects transfers more heat than that dissipates heat by convection only. Kundu and Lee [15] developed new analytical analysis by using double DTM to determine the temperature distribution in annular step porous fins subject to a mobile condition. They concluded that the porous fins have an ability to transfer more heat compared to the solid fin for an equal mass of fin at an optimum condition. Aziz and Makinde [16] applied a two-dimensional heat conduction model to obtain the thermal performance and entropy generation in an orthotropic convection pin fin used in advanced light weight heat sinks. The study illustrated the simultaneous realization of the least material and minimum entropy generation pin fin designs. Ndlovu and Moitsheki [17] recently performed thermal analysis of natural convection and radiation heat transfer in moving porous fins of rectangular and exponential profiles. The analytical solutions describing temperature distribution were constructed using the DTM and validated by the numerical solutions obtained by applying the $pdsolve$ in Maple. Mhlongo [18] et al. applied the local and nonlocal symmetry techniques to study transient response of rectangular fins to step change in base temperature and base heat flux. The authors showed that the governing equations can be reduced to the tractable Ermakov-Pinney equation.

In this article, the 2D DTM is applied to solve the nonlinear heat transfer equation describing the temperature distribution in a moving porous fin with temperature dependent thermal properties. The approximate analytical solutions from the present model are validated using the numerical results obtained by employing the inbuilt numerical solver in MATLAB ($pdepe$). The DTM is an analytical method based on Taylor series expansion and was first introduced by Zhou [19] in 1986. Zhou used the DTM to solve explicitly the linear and nonlinear initial value problems that arise in electrical circuits. Chen and Ho [20], developed a two-dimensional DTM which can be used for solving differential and integral equations. Ayaz [33], showed that the DTM is better equipped to solve highly nonlinear problems than the Taylor series method. The DTM has been used to solve various problems in applied mathematics and physics such as systems of differential equations [22]. Fallo et al. [23] applied the 3D DTM for the first time to study heat transfer in a cylindrical spine fin with variable thermal properties. Chiba et al. [24] solved the one-dimensional phase change problem in a slab of finite thickness using the 2D DTM. The numerical results illustrated that the DTM is useful as a new analytical method for solving the phase change problem in a slab with temperature-dependent parameters.

This article is organized as follows; the mathematical background of the problem under consideration is described in Section 2. A brief discussion on the fundamentals of the 2D DTM will be provided in Section 3. We provide analytical solutions in Section 4. Lastly, we provide some discussions based on the results obtained in Section 5 and concluding remarks in Section 6.

Mathematical Problem Formulation

We consider a one dimensional triangular moving porous fin of length L, with cross-sectional area A_c and perimeter P while it moves horizontally with a constant velocity U as depicted in Figure 1. The fin surface is exposed to a convective and radiative environment of temperature T_a and the base temperature of the fin is $T_b > T_a$. The fin thickness at the prime surface is given by δ_b and its profile is given by $F(X)$. The fluid surrounding the moving fin is heated by the convective-radiative fin and induces a horizontal flow field (advection). The radiative component would be more prominent if fin undergoes natural convection or if the forced convection is rather weak or absent. The surface of the moving fin is assumed to diffuse with a constant emissivity ε. The porous medium is assumed to be isotropic, homogeneous and saturated with a single-phase fluid. The fluid and solid matrix are not at local thermal equilibrium and the fluid-solid interaction in the porous medium is stimulated by Darcy's model. The energy balance equation for the moving porous fin losing heat by simultaneous convection and radiation can be expressed as

$$\rho c\frac{\partial T}{\partial t}=\frac{\delta_b}{2}\frac{\partial}{\partial X}\left[F(X)K(T)\frac{\partial T}{\partial X}\right]-\frac{PH(T)}{A_c}(T-T_a)-\frac{\dot{m}c}{A_c}(T-T_a)$$
$$-\frac{\varepsilon\sigma P}{A_c}(T^4-T_a^4)-\frac{\delta_b}{2}F(X)\rho cU\frac{\partial T}{\partial X},\ 0\le X\le L. \tag{1}$$

The mass flow rate of the fluid passing through the porous media is expressed by [25],

$$\dot{m}=\rho\nu_w W\Delta X \tag{2}$$

The velocity of the fluid passing through the porous media is given by Darcy's model,

$$\nu_w=\frac{gKA}{\upsilon}(T-T_a). \tag{3}$$

For most materials, the thermal conductivity varies linearly with temperature, that is,

$$K(T)=k_a[1+\gamma(T-T_a)], \tag{4}$$

and heat transfer coefficient may be given by a power law function of temperature,

$$H(T)=h_b\left(\frac{T-T_a}{T_b-T_a}\right)^n. \tag{5}$$

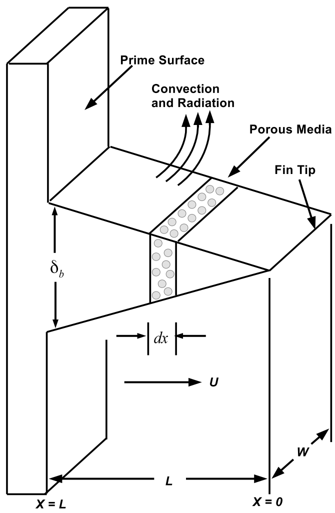

Fig. 1: Schematic representation of a triangular moving porous fin.

Here, k_a is the thermal conductivity of the fin at ambient temperature, γ is a measure of thermal conductivity variation with temperature, h_b is the heat transfer coefficient at the fin base and n is a constant. The constant n may vary between -6.6 and 5. However, in most practical applications it lies between -3 and 3 [26]. The exponent n represents laminar film boiling or condensation when $n=-1/4$, laminar natural convection when $n=1/4$, turbulent natural convection when $n=1/3$, nucleate boiling when $n=2$, radiation when $n=3$ and $n=0$ implies a constant heat transfer coefficient. The third term on the right hand side of equation (1) represents the fluid-solid interaction in the porous medium (see e.g. [27, 28]). Here, g is the gravitational acceleration, K is the permeability, υ and A are the kinematic viscosity and thermal expansion coefficient respectively, ρ is the density of fluid; c is the specific heat of fluid; ε and σ are the emissivity and Boltzman constant respectively.

Assuming that the fin tip is adiabatic (insulated) and the base temperature is kept constant, then the boundary conditions are given by Kraus et al. [29] (see also [30, 31],

$$T(t, L) = T_b \text{ and } \left.\frac{\partial T}{\partial X}\right|_{X=0} = 0. \tag{6}$$

Initially the fin is kept at the ambient temperature,

$$T(0, X) = T_a. \tag{7}$$

Introducing the following dimensionless variables,

$$\begin{gathered} x = \frac{X}{L},\ \tau = \frac{k_c t}{\rho c L^2},\ \theta = \frac{T}{T_b},\ \theta_a = \frac{T_a}{T_b},\ f(x) = \frac{\delta_b}{2}F(X),\ k = \frac{K}{k_c}, \beta = \gamma T_b,\ \alpha = \frac{k_c}{\rho c}, \\ N_c = \frac{P h_b T_a^n L^2}{k_a A_c (T_b - T_a)^n},\ N_r = \frac{\varepsilon \sigma P L^2 T_b^3}{k_a A_c},\ N_p = \frac{\rho c g K A W L^2 T_b}{\upsilon k_a A_c}, \text{ and } Pe = \frac{UL}{\alpha}, \end{gathered} \tag{8}$$

reduces the energy equation (1) to

$$\begin{aligned} \frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial x}\left[x\{1 + \beta(\theta - \theta_a)\}\frac{\partial \theta}{\partial x}\right] - N_c(\theta - \theta_a)^{n+1} - N_p(\theta - \theta_a)^2 - N_r(\theta^4 - \theta_a^4) \\ - xPe\frac{\partial \theta}{\partial x},\ 0 \leq x \leq L. \end{aligned} \tag{9}$$

The prescribed boundary conditions are given by,

$$\theta(\tau, 1) = 1 \text{ and,} \tag{10}$$

$$\left.\frac{\partial \theta}{\partial x}\right|_{x=0} = 0, \tag{11}$$

and the initial condition becomes,

$$T(0, x) = 0. \tag{12}$$

Fundamentals of the Two-Dimensional Differential Transform Method

In this section, the basic idea underlying the two-dimensional DTM is briefly introduced. If function $\theta(t, x)$ is analytic and differentiated continuously with respect to time and the spatial variable x in the domain of interest, then we let

$$\Phi(\kappa, s) = \frac{1}{\kappa! s!}\left[\frac{\partial^{\kappa+s}\phi(t, x)}{\partial t^\kappa \partial x^s}\right]_{(0,0)}, \tag{13}$$

where the spectrum $\Phi(\kappa, s)$ is the transformed function, which is also called the T-function (see [32, 33]). The differential inverse transform of $\Phi(\kappa, s)$ is defined as

$$\phi(t, x) = \sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty}\Phi(\kappa, s)t^\kappa x^s, \tag{14}$$

and from equations (13) and (14) it can be concluded that

$$\phi(t,x)=\sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty}\frac{1}{\kappa!s!}\left[\frac{\partial^{\kappa+s}\phi(t,x)}{\partial t^{\kappa}\partial x^{s}}\right]_{(0,0)}t^{\kappa}x^{s}. \tag{15}$$

In real applications, the function $\phi(t,x)$ is expressed by a finite series, and equation (14) can be written as:

$$\phi(t,x)=\sum_{\kappa=0}^{m}\sum_{s=0}^{n}\Phi(\kappa,s)t^{\kappa}x^{s}, \tag{16}$$

Equation (16) implies that

$$\phi(t,x)=\sum_{\kappa=m+1}^{\infty}\sum_{s=n+1}^{\infty}\Phi(\kappa,s)t^{\kappa}x^{s}, \tag{17}$$

is negligibly small.

Some of the useful mathematical operations performed by the differential transform method are given in Table 1 .

Table 1: Fundamental operations of the differential transform method

Original function	Transformed function
$\phi(t,x)=f_1(t,x)\pm f_2(t,x)$	$\Phi(\kappa,s)=F_1(\kappa,s)\pm F_2(\kappa,s)$
$\phi(t,x)=\alpha f(t,x)$	$\Phi(k,s)=\alpha F(\kappa,s)$
$\phi(t,x)=\frac{\partial f(t,x)}{\partial t}$	$\Phi(\kappa,s)=(\kappa+1)F(\kappa+1,s)$
$\phi(t,x)=\frac{\partial f(t,x)}{\partial x}$	$\Phi(\kappa,s)=(s+1)F(\kappa,s+1)$
$\phi(t,x)=\frac{\partial^2 f(t,x)}{\partial t^2}$	$\Phi(\kappa,s)=(\kappa+1)(\kappa+2)F(\kappa+2,s)$
$\phi(t,x)=\frac{\partial^2 f(t,x)}{\partial x^2}$	$\Phi(\kappa,s)=(s+1)(s+2)F(\kappa,s+2)$
$\phi(t,x)=t^m x^n$	$\Phi(\kappa,s)=\delta(\kappa-m)\delta(s-n)$
$\phi(t,x)=x^m e^{at}$	$\Phi(\kappa,s)=\frac{a^\kappa}{\kappa!}\delta(s-m)$
$\phi(t,x)=x^m\sin(at+b)$	$\Phi(\kappa,s)=\frac{a^\kappa}{\kappa!}\delta(s-m)\sin(\frac{\kappa\pi}{2}+b)$
$\phi(t,x)=x^m\cos(at+b)$	$\Phi(\kappa,s)=\frac{a^\kappa}{\kappa!}\delta(s-m)\cos(\frac{\kappa\pi}{2}+b)$

The Kronecker delta function $\delta(\kappa-s)$ is given by

$$\delta(\kappa-s)=\begin{cases}1 & \text{if } \kappa=s\\ 0 & \text{if } \kappa\neq s.\end{cases}$$

Analytical Solution

In this section we provide an analytical solution to the problem under consideration. Using 2D-DTM to solve PDEs consists of three main steps. The steps are: transforming the PDE into algebraic equations, solving the equations, and inverting the solution of algebraic equations to obtain an infinite series solution or an approximate solution. Taking the two-dimensional differential transform of equation (9), given $n=2$, we obtain the following recurrence relation,

$$(\kappa+1)\Theta(\kappa+1,h) = (1-\beta\theta_a)\sum_{i=0}^{\kappa}\sum_{j=0}^{h}(h+1-j)(h+2-j)\Theta(i,h+2-j)\delta(j-1)\delta(k-i)$$
$$+ (1-\beta\theta_a)(h+1)\Theta(k,h+1)$$
$$+ \beta\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{j=0}^{h}\sum_{p=0}^{h-j}(h+1-j-p)(h+2-j-p)\Theta(i,h+2-j-p)\Theta(l,j)\delta(p-1)\delta(k-i-l)$$
$$+ \beta\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{j=0}^{h}\sum_{p=0}^{h-j}(h+1-j-p)(j+1)\Theta(i,h+1-j-p)\Theta(l,j+1)\delta(p-1)\delta(k-i-l)$$
$$+ \beta\sum_{i=0}^{\kappa}\sum_{j=0}^{h}(h+1-j)\Theta(i,h+1-j)\Theta(k-i,j)$$
$$- N_c\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\Theta(i,h-j-p)\Theta(t,j)\Theta(k-i-t,p) + 3N_c\theta_a\sum_{i=0}^{\kappa}\sum_{j=0}^{h}\Theta(i,h-j)\Theta(k-i,j)$$
$$- 3N_c\theta_a^2\Theta(k,h) - N_p\left(\left(\sum_{i=0}^{\kappa}\sum_{j=0}^{h}\Theta(i,h-j)\Theta(k-i,j)\right) - 2\theta_a\Theta(k,h) + \theta_a^2\right)$$
$$- N_r\sum_{i=0}^{\kappa}\sum_{t=0}^{\kappa-i}\sum_{z=0}^{\kappa-i-t}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\sum_{l=0}^{h-j-p}\Theta(i,h-j-p-l)\Theta(t,j)\Theta(z,p)\Theta(k-i-t-z,l) + N_r(\theta_a^4)$$
$$- Pe\sum_{i=0}^{\kappa}\sum_{j=0}^{h}(h+1-j)\Theta(i,h+1-j)\delta(j-1)\delta(k-i), \tag{18}$$

where $\Theta(\kappa,h)$ is the differential transform of $\theta(\tau,x)$.

Taking the two-dimensional differential transform of the initial condition (12) and boundary condition (11) we obtain the following transformations respectively,

$$\Theta(0,h) = 0,\ h = 0,1,2,\dots \tag{19}$$

$$\Theta(\kappa,1) = 0,\ \kappa = 0,1,2,\dots. \tag{20}$$

We consider the other boundary condition as follows,

$$\Theta(\kappa,0) = a,\ a\in\mathbb{R},\ \kappa = 1,2,3,\dots \tag{21}$$

where the constant a can be determined from the boundary condition (10) at each time step after obtaining the series solution.

Substituting equations (19)-(21) into (18) we obtain the following,

$$\Theta(1,2) = -\theta^2N_p + \theta_a^4N_r, \tag{22}$$

$$\Theta(2,2) = -\frac{1}{2}\theta_a^2(-N_p+\theta_a^2N_r)(2(-5+Pe)+\theta_a(9\beta-2N_p+3\theta_aN_c)), \tag{23}$$

$$\Theta(1,3) = -\theta^2N_p + \theta_a^4N_r, \tag{24}$$

$$\Theta(2,3) = -\frac{1}{2}\theta_a^2(N_p-\theta_a^2N_r)(17+2\theta_a(-8\beta+N_p)-3Pe-3\theta_a^2N_c) \tag{25}$$

$$\vdots$$

Substituting equations (19)-(25) into (16) we obtain the following closed form of the solution,

$$\begin{aligned}\theta(\tau,x) &= a\tau + a\tau^2 - (\theta^2 N_p - \theta_a^4 N_r)\tau x^2 \\ &- \frac{1}{2}\theta_a^2(-N_p + \theta_a^2 N_r)(2(-5+Pe) + \theta_a(9\beta - 2N_p + 3\theta_a N_c))\tau^2 x^2 + a\tau^3 \\ &- \theta^2 N_p + \theta_a^4 N_r \tau x^3 - \frac{1}{2}\theta_a^2(N_p - \theta_a^2 N_r)(17 + 2\theta_a(-8\beta + N_p) - 3Pe - 3\theta_a^2 N_c)\tau^2 x^3 \\ &+ \ldots \end{aligned} \quad (26)$$

The constant a can be determined from the boundary condition (10). To obtain the value of a, we substitute the boundary condition (10) into (26) at the point $x = 1$. Thus, we have,

$$\begin{aligned}\theta(\tau,1) &= a\tau + a\tau^2 - (\theta^2 N_p - \theta_a^4 N_r)\tau \\ &- \frac{1}{2}\theta_a^2(-N_p + \theta_a^2 N_r)(2(-5+Pe) + \theta_a(9\beta - 2N_p + 3\theta_a N_c))\tau^2 + a\tau^3 \\ &- \theta^2 N_p + \theta_a^4 N_r \tau - \frac{1}{2}\theta_a^2(N_p - \theta_a^2 N_r)(17 + 2\theta_a(-8\beta + N_p) - 3Pe - 3\theta_a^2 N_c)\tau^2 \\ &+ \ldots = 1 \end{aligned} \quad (27)$$

We then obtain the expression for $\theta(\tau, x)$ upon substituting the obtained value of a into equation (26). Using the first 25 terms of the power series solution, we benchmark the series solutions to the numerical solution obtained through employing the inbuilt numerical solver in MATLAB ($pdepe$). As seen from the benchmark analysis in Figure 2, the series solution is in agreement with the numerical solution.

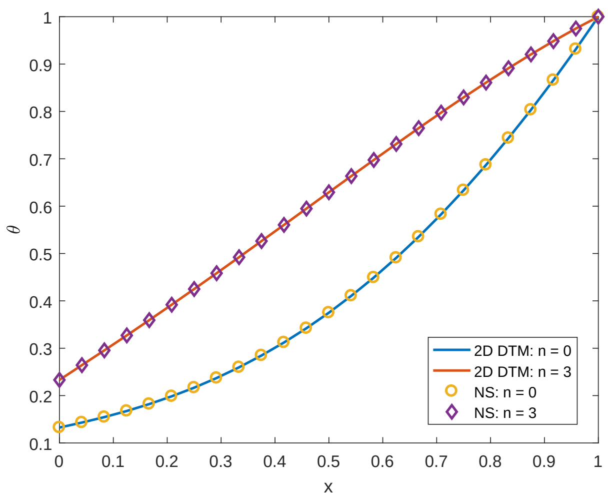

Fig. 2: Temperature distribution in a porous moving fin for varying n.

With the confidence obtained from the benchmark analysis, we plot the solution (26) for various parameters as shown in Figures below. For all analytical results depicted in the Figures in this article, the following values of parameters are used unless stated otherwise as indicated in the graphs below.

$$n = 2,\ \beta = 1, N_c = 4,\ N_p = 0.30,\ N_r = 0.25, \theta_a = 0.10,\ Pe = 3,\ \tau = 0.80 \quad (28)$$

Fin Efficiency

Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin was subjected to the base temperature. The fin heat transfer rate, Q, is given by the heat conduction rate at the fin base [35, 36], that is

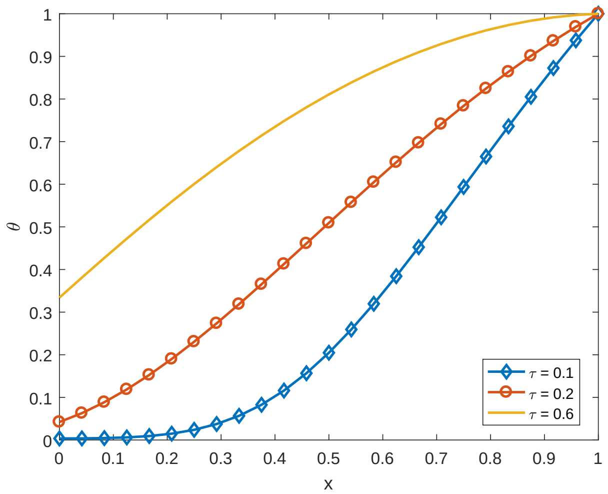

Fig. 3: Temperature distribution in a porous moving fin for varying time, τ, $n = 1$

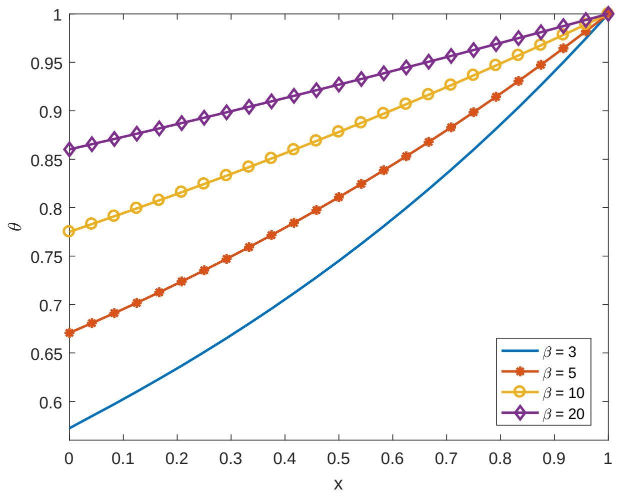

Fig. 4: Effect of the thermal conductivity gradient, β, on temperature distribution.

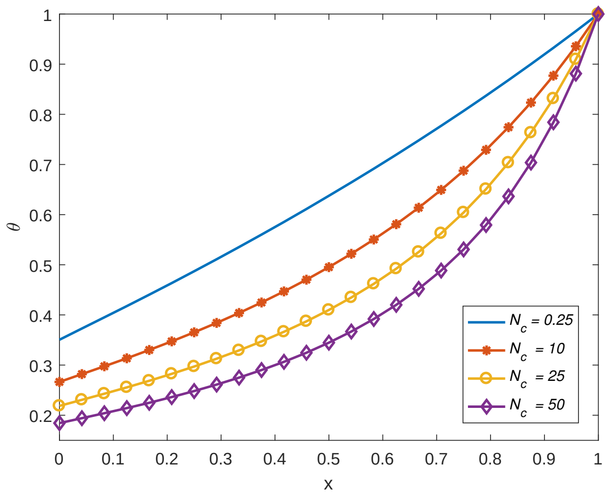

Fig. 5: Effect of the thermo-geometric parameter, N_c, on temperature distribution.

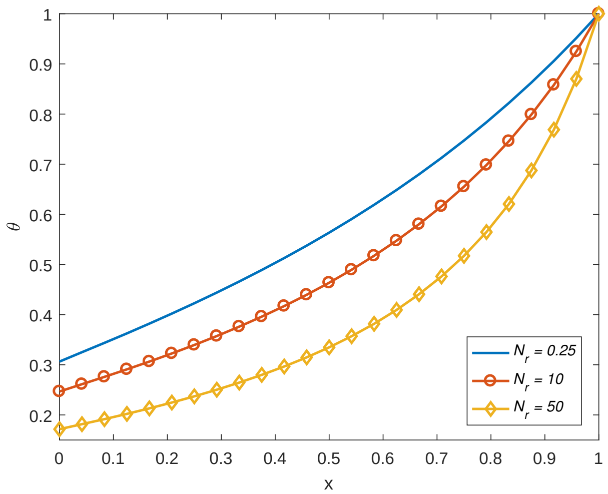

Fig. 6: Effect of the thermo-geometric parameter, N_r, on temperature distribution.

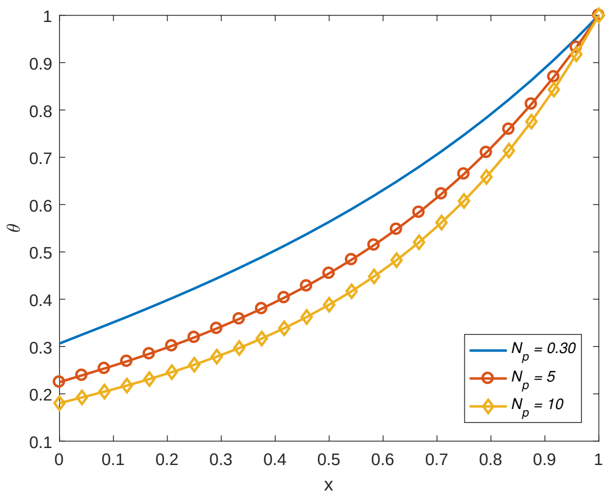

Fig. 7: Effect of the thermo-geometric parameter, N_p, on temperature distribution.

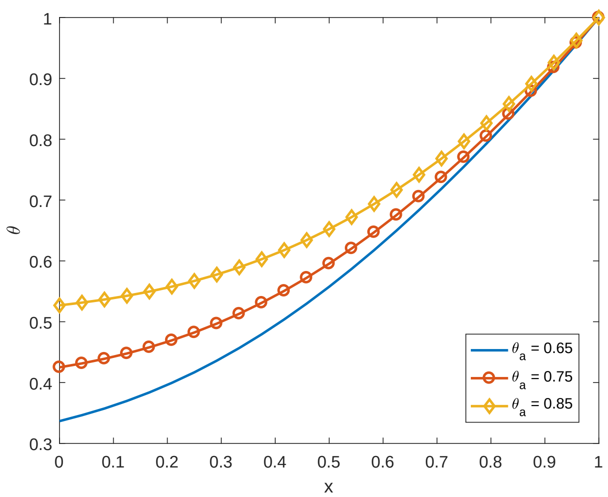

Fig. 8: Effect of the thermo-geometric parameter, θ_a, on temperature distribution.

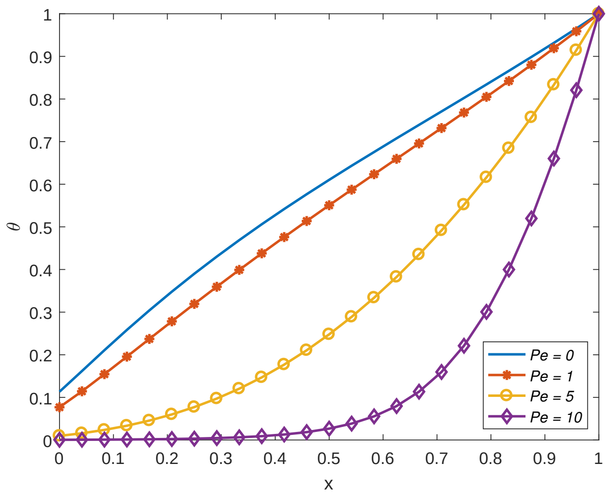

Fig. 9: Effect of varying speeds, Pe, on temperature distribution, $n = 1$.

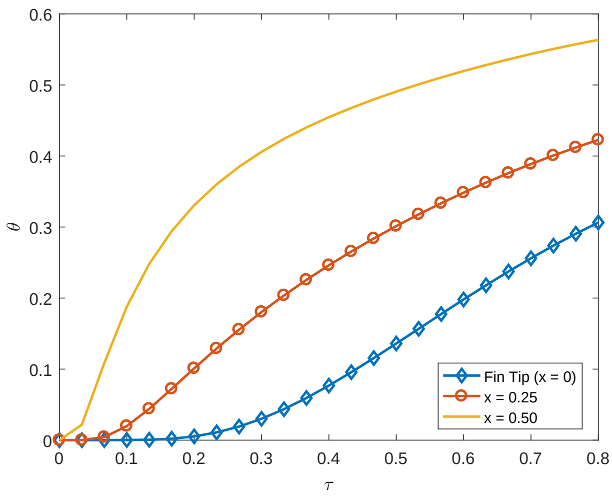

Fig. 10: Effect of the thermo-geometric parameter, x, on temperature distribution.

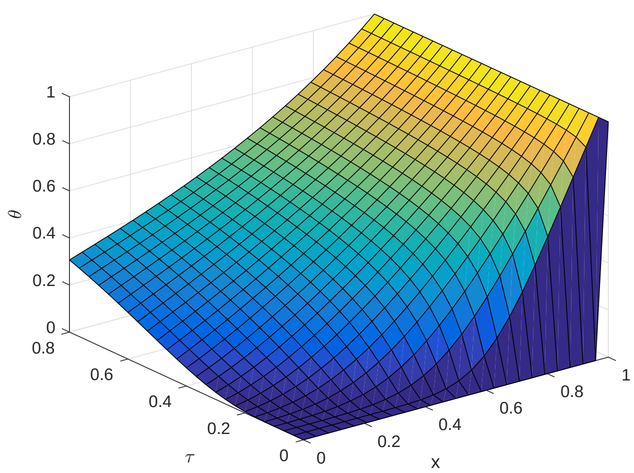

Fig. 11: Transient temperature distribution in a porous moving fin.

$$Q = A_c K(T) \frac{\partial T}{\partial X}. \tag{29}$$

The ideal heat transfer rate obtained if the entire fin is kept at the base temperature is given by,

$$Q_{ideal} = PLH(T)(T - T_a) + \dot{m}cL(T - T_a) + \varepsilon\sigma PL(T^4 - T_a^4) + \rho c U A_c \frac{\partial T}{\partial X}, \tag{30}$$

and the fin efficiency in dimensionless variables is then given by,

$$\eta = \frac{Q}{Q_{ideal}} = \frac{\{1 + \beta(\theta - \theta_a)\} \frac{\partial \theta}{\partial x}\big|_{x=1}}{N_c(1 - \theta_a)^{n+1} + N_p(1 - \theta_a)^2 + N_r(1 - \theta_a^4) + Pe \frac{\partial \theta}{\partial x}\big|_{x=1}}. \tag{31}$$

The rate of heat transfer at the fin base for varying parameters is depicted in Figures 12 , 13 and 14. The fin efficiency is shown in Figure 15.

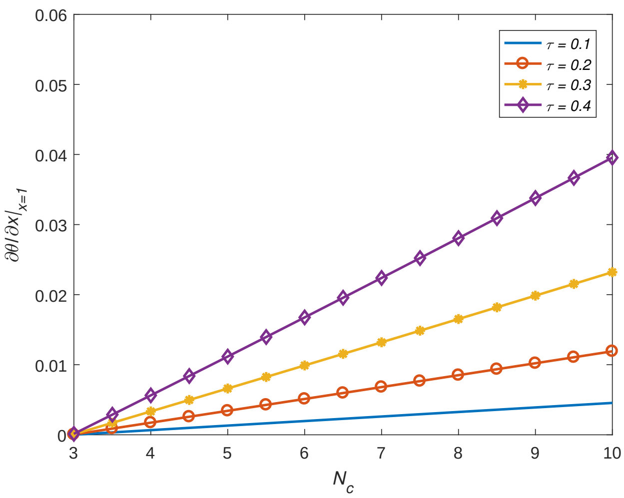

Fig. 12: Heat transfer gradient in a triangular porous fin for varying fin time, τ and N_c.

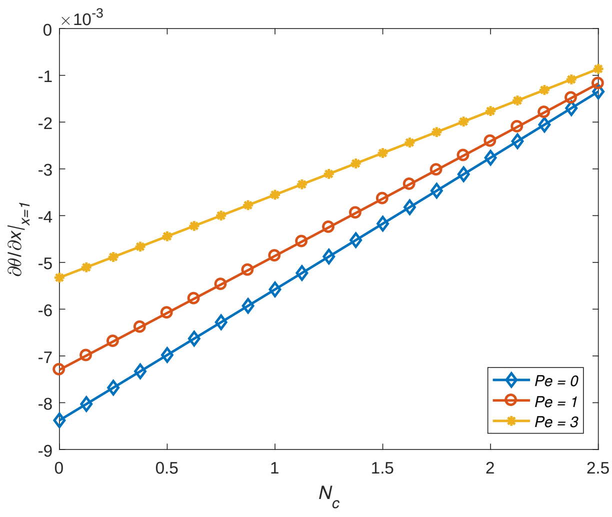

Fig. 13: Heat transfer gradient in a triangular porous fin fin for varying fin speed, Pe and N_c.

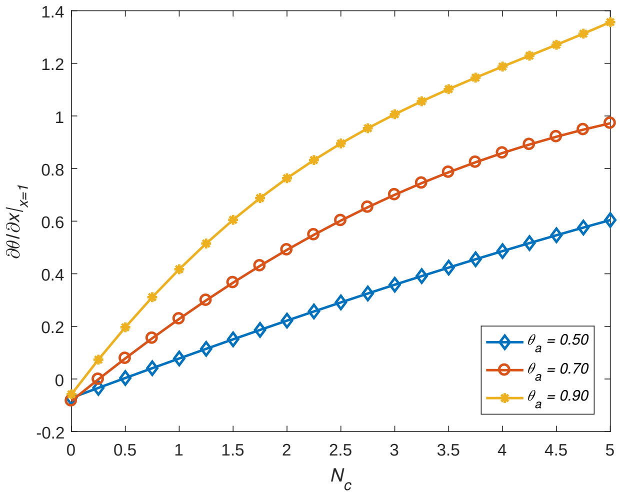

Fig. 14: Heat transfer gradient in a triangular porous fin fin for varying ambient temperature, θ_a and N_c.

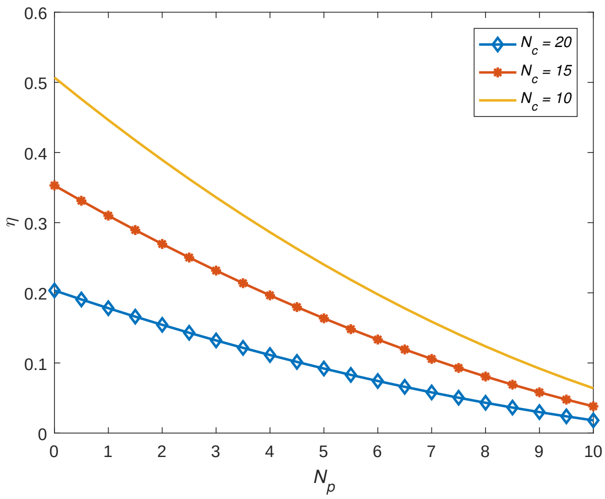

Fig. 15: Fin efficiency of a triangular porous fin.

Results and Discussion

The energy equation (9) has been studied extensively and the impact of all the embedding parameters on temperature distribution has been graphically presented. Figure 2 shows the effect of the nature of heat transfer on transient temperature distribution and also gives a comparison of the 2D DTM and the numerical results. It is observed that the approximate analytical solutions are in good agreement with the numerical results. We also note that the temperature distribution of the fin is an increasing function of exponent n. Figure 3 shows that the fin temperature increases with time. It has been shown that the transient temperature profile approaches the steady state solution as time evolves [30].

Figure 4 shows the impact of the change in thermal conductivity gradient. We observe that as the values of β decreases, the temperature in the fin decreases signifying an increased heat loss to the ambient fluid. Increasing the thermal conductivity gradient enhances heat conduction from the fin base and leads to increased temperatures inside the fin.

Figure 5 illustrates the temperature distribution for varying values of the convective-conductive parameter N_c. For $N_c = 0$, this indicates the absence of convection while $N_c = 1$ indicates that convection and conduction are at equal rates [34]. An increase in the values of the thermo-geometric fin parameter leads to a decrease in temperature in the fin. A similar result is observed in Figure 6 which represents the impact of the radiation parameter N_r on temperature distributions of a moving porous fin. Figure 7 depicts the results of varying the porous media parameter N_p. We observe that the temperature decreases with an increasing values of the porous parameter. The high rate of heat transfer is a result of higher porosity due to changes in permeability included in N_p. Furthermore, this can be caused by the an effect of a larger porous medium area.

Figure 8 shows the variation in temperature distribution for varying values of the convective-radiative sink parameter θ_a. The ambient temperature plays an important role in transient heat dissipation. Higher ambient temperature lead to a reduced heat transfer rate as indicated by high values of fin temperature for higher values of the convective-radiative sink temperature.

Figure 9 illustrates the effect of the Peclet number on the temperature distribution in the fin. As Pe increases, (fin moves faster), the temperature inside the fin decreases rapidly due to increased advective effect on the surface of the fin. We also observe that for $Pe = 0$, i.e., stationery fin, the cooling of the fin takes more time as shown by higher temperatures when compared to a moving fin. Figure 10 shows the temperature variation along the fin profile. The fin temperature gradually increases as the axial parameter x increases and this is in line with the imposed boundary condition at the fin base. Lastly, Figure 11 we observe that the temperature distribution along the three dimensional spaces corresponds to the boundary conditions and to all the results described above.

The heat transfer rate from the fin base plays an important role in the fin efficiency. Figure 12 shows that rate of heat transfer from the fin base increases as time evolves. This is attributed to the fact that the fin is initially kept at ambient temperature and with thermal conduction occurring between the heated body and the fin, it is expected to see an increase in heat transfer rate with increasing time. Figure 13 shows that an increase in the speed of the moving fin leads to an increase in the rate of heat transfer. A moving fin induces a horizontal flow field (advection) which in turn leads to an increase in temperature gradient. Across Figures 12, 13 and and 14, we observe that the an increase in the convection-radiation parameter, N_c, leads to an increase in the rate of heat transfer. In Figure 15, we note that the fin becomes more effective with an increase in the values of the convective-radiative fin parameters.

Conclusions

The 2D DTM was successfully applied to study transient heat transfer in a longitudinal moving fin subject to convective and radiative heat losses. The closed-form solution of the governing highly nonlinear equation was obtained and the embedding parameters were illustrated and explained. The accuracy of the analytical solution was validated using the numerical solution and the results are found to be in agreement.

Furthermore, we also introduced the effect of the speed of the moving fin and noted that it affects the rate at which the fin dissipates heat to the ambient fluid. Also, the results show that the fin surface temperature is greatly dependent on the nature of the heat transfer. To our knowledge, 2D DTM has not yet been applied to fin problems for transient heat flow in moving fins.

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2.6 Journal Article VI

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Effect of Variable Thermal Conductivity Calibration Functions on Fin Temperature Distribution

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Keywords: Heat transfer, variable thermal conductivity, VIM, analytical solutions.

Abstract. In this article, we introduce a new thermal conductivity calibration function in modeling heat transfer through extended surfaces. The variable thermal conductivity functions are studied on a stand alone basis and further compared to one another. The calculations are carried out using the Variational Iteration Method (VIM) which is an analytical solution technique. The series solutions are bench-marked against the numerical results obtained by applying the Runge-Kutta fourth order method coupled with shooting technique. The effects of some physical parameters such as the thermo-geometric fin parameter and thermal conductivity gradient, on temperature distribution are illustrated and explained.

Introduction

Fins are extended surfaces used to dissipate heat from a heated body to its ambient fluid. In particular, fins are used various applications such as in computer processors, space nuclear power systems, oil carrying pipelines and many more. There are many well documented mathematical models which describe the heat transfer in fins of different geometries and profiles with a variety of boundary conditions [1, 2]. The study of heat transfer through extended surfaces is increasing as evidenced by more recent publications in this field. Fins of various profiles were studied by Ndlovu and Moitsheki [3, 4] using various analytical methods to derive the approximate analytical solutions for temperature distribution. Some interesting conclusions were made by the authors regarding the fin profiles that are efficient in rapid heat dissipation. Recently, the study of moving and porous fins has attracted a lot of interest from various researchers. A study of triangular moving fin with multiple variable thermal properties was performed by Kanti Roy et al. [5]. The results were found to be in agreement with other similar studies found in literature. Heat transfer enhancement and efficiency in the moving longitudinal fins of trapezoidal cross section was studied by Turkyilmazoglu [6]. The author further showed that certain type of trapezoidal fins may have advantageous fin design features. The steady state thermal analysis of natural convection and radiation in porous fins was analytically conducted using Homotopy Analysis Method by Moradi et al. [7]. Aziz and Makinde [8] applied a two-dimensional heat conduction model to obtain the thermal performance and entropy generation in an orthotropic convection pin fin used in advanced light weight heat sinks. The study illustrated the simultaneous realization of the least material and minimum entropy generation pin fin designs. Mhlongo et al. [9] applied the local and nonlocal symmetry techniques to study transient response of rectangular fins to step change in base temperature and base heat flux. The authors showed that the governing equations can be reduced to the tractable Ermakov-Pinney equation.

The ordinary differential equations (ODEs) and partial differential equations (PDEs) arising in the study of heat transfer are highly nonlinear and difficult to solve exactly. This makes it difficult to find their exact solutions. Numerous studied have been devoted to finding approximate analytical solutions to these kinds of problems. Ndlovu and Moitsheki [10] applied the Differential Transform Method (DTM) to transient heat conductions problems arising in extended surface heat transfer. The results were found to provide a significant improvement not only by constructing series solutions, also by studying the effects of all the parameters appearing in the model on temperature profile. The applicability of the Collocation Spectral Method (CSM) for solving nonlinear heat transfer problems was demonstrated in a convective-radiative fin with temperature-dependent properties by Sun et al. [11]. Ahmadi at al [12] studied the steady-state natural convection heat transfer from vertically-mounted rectangular fins numerically and experimentally using FLUENT software. Fallo et al. [13] applied the 3D DTM for the first time to study heat transfer in a cylindrical spine fin with variable thermal properties. Makinde and Moitsheki [14] applied the Adomian Decomposition Method (ADM) and VIM coupled with Pade approximation technique to study heat transfer characteristics of natural convection around a vertical vertical permeable flat surface embedded in a saturated porous medium. The VIM was introduced by He in 1999 [15, 16]. This method is widely used in literature dure to its faster convergence and easy applicability.

The pursuit of analytical solutions for the heat conduction equations are of intrinsic scientific interest. In this paper, the VIM is employed to determine the analytical solutions to the boundary value problem describing heat transfer in a straight fin. The mathematical modeling of the problem under consideration is described in Section 2. A brief discussion on the fundamentals of the VIM is be provided in Section 3. In Section 4 we provide analytical solutions for the heat transfer in fin for various thermal conductivity calibration functions. Furthermore, we describe the fin efficiency and the heat flux in Section 5. The results are discussed in Section 6 and the concluding remarks are provided in Section 7.

Mathematical Problem Formulation

We consider a straight fin of temperature-dependent thermal conductivity and a cross-sectional area A_c. The perimeter of the fin is denoted by P and its length by L. The fin is attached to a fixed prime surface of temperature T_b and extends to an ambient fluid of temperature T_a. The fin thickness at the prime surface is given by δ. We assume that the convective flow in the surrounding medium provides a constant heat transfer coefficient h over the entire surface of the fin. As the fin experiences a large change in its temperature, the change in its thermal conductivity cannot be a constant. Based on the one dimensional heat conduction, the energy balance equation is then given by (see e.g. [1])

$$A_c \frac{d}{dX}\left(K(T)\frac{dT}{dX}\right) = Ph(T - T_a),\ 0 \le X \le L \tag{1}$$

where K is the non-uniform temperature dependent thermal conductivity, h is the constant heat transfer coefficient, T is the temperature distribution and X is the space variable. The length of the fin is measured from the tip to the prime surface as shown in Figure 1. Assuming that the fin tip is adiabatic (insulated) and the base temperature is kept constant, then the boundary conditions are then given by

$$T(L) = T_b \text{ and } \left.\frac{dT}{dX}\right|_{X=0} = 0. \tag{2}$$

Introducing the following dimensionless variables,

$$x = \frac{X}{L},\ \theta = \frac{T - T_a}{T_b - T_a},\ k = \frac{K}{k_a},\ M^2 = \frac{PhL^2}{A_c k_a} \tag{3}$$

reduces equation (1) to

$$\frac{d}{dx}\left[k(\theta)\frac{d\theta}{dx}\right] - M^2\theta = 0,\ 0 \le x \le 1. \tag{4}$$

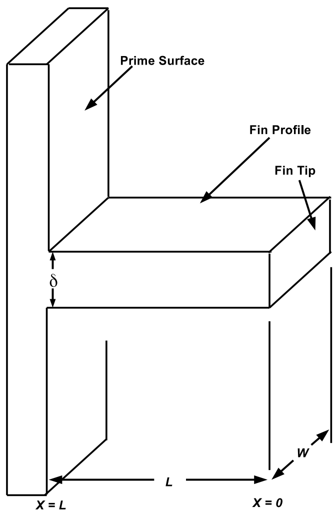

Fig. 1: Schematic representation of a longitudinal fin.

Here θ is the dimensionless temperature, x is the dimensionless space variable, k is the dimensionless thermal conductivity, k_a defined as the thermal conductivity at the ambient temperature and M is the thermo-geometric fin parameter. The dimensionless boundary conditions then become,

$$\theta(1) = 1, \text{ at the prime surface} \tag{5}$$

and

$$\left.\frac{d\theta}{dx}\right|_{x=0} = 0, \text{ at the fin tip.} \tag{6}$$

We consider the four distinct cases of the thermal conductivity as follows in which the last one is new:

(a) the linear function [3]

$$K(T) = k_a[1 + \gamma(T - T_a)], \tag{7}$$

(b) the power power law function [4]

$$K(T) = k_a\left(\frac{T - T_a}{T_b - T_a}\right)^m, \; m \neq 0, \tag{8}$$

(c) the exponential function [17]

$$K(T) = k_a \mathrm{e}^{\beta_1\left(\frac{T-T_a}{T_b-T_a}\right)}, \; \beta_1 \neq 0, \tag{9}$$

(d) the logistic function

$$K(T) = \frac{k_a}{1 + \mathrm{e}^{\beta_2\left(\frac{T-T_a}{T_b-T_a}\right)}}, \; \beta_2 \neq 0, \tag{10}$$

He's Variational Iteration Method

To illustrate the basic idea of the method we consider the following nonlinear equation:

$$\mathcal{L}\theta + \mathcal{N}\theta = g(x), \tag{11}$$

where $\mathcal{L}$ is a linear operator, $\mathcal{N}$ are linear and nonlinear operators, respectively, and $g(x)$ is the source inhomogeneous term [18]. In his papers, [15, 16], He proposed the variational iteration method where a correctional functional for equation. (11) can be written as

$$\theta_{n+1}(x) = \theta_n(x) + \int_0^x \lambda(t)(\mathcal{L}\theta_n(t) + \mathcal{N}\tilde{\theta}_n(t) - g(t))dt, \tag{12}$$

where λ is the general Lagrange multiplier [14, 19], which can be be identified optimally via the variation theory, and $\tilde{\theta}_n$is a restricted variation, which means $\delta\tilde{\theta}_n = 0$ [20]. The Lagrange multiplier can be a constant or a function depending on the order of the deferential equation under consideration. In general, for the j^{th} order ordinary differential equation, the Lagrange multiplier is given by [4, 18],

$$\lambda = \frac{(-1)^j}{(j-1)!}(t-x)^{j-1}. \tag{13}$$

Equation (12) can be re-written as follows:

$$\theta_{n+1}(x) = \theta_n(x) + \int_0^x \frac{(-1)^j}{(j-1)!}(t-x)^{j-1}\big(\mathcal{L}\theta_n(t) + \mathcal{N}\theta_n(t) - g(t)\big)dt, \tag{14}$$

The iteration formula equation (14), without restricted variation, should be used for the determination of the successive approximations $\theta_{n+1}(x),\ \ n \geqslant 0$, of the solution $\theta(x)$. Consequently, the solution is given by

$$\theta(x) = \lim_{n\to\infty} \theta_n(x). \tag{15}$$

The Fin Temperature Distribution

As a starting approximation for the VIM solution to equation (4), $\theta_0(x)$ is assumed to be a constant [21]. The different calibration functions are analyzed on a stand alone basis in the following subsections.

Linear function The dimensionless thermal conductivity given a linear function of temperature is $k(\theta) = 1 + \beta\theta$, where $\beta = \gamma(T_b - T_a)$ and equation (4) reduces to:

$$\frac{d}{dx}\left[(1+\beta\theta)\frac{d\theta}{dx}\right] - M^2\theta = 0,\ 0 \le x \le 1. \tag{16}$$

The first four iterations using the VIM solution technique are given below,

$$\theta_0(x) = a, \tag{17}$$

$$\theta_1(x) = a + \frac{1}{2}aM^2x^2, \tag{18}$$

$$\theta_2(x) = a - \frac{1}{2}a(-1+a\beta)M^2x^2 - \frac{1}{24}a(-1+3a\beta)M^4x^4 \tag{19}$$

$$\begin{aligned}\theta_2(x) = a &+ \frac{1}{2}a(1 - a\beta + a^2\beta^2)M^2x^2 - \frac{1}{24}a(-1+5a\beta - 9a^2\beta^2 + 3a^3\beta^3)M^4x^4 \\ &- \frac{1}{720}a(-1+18a\beta - 60a^2\beta^2 + 45a^3\beta^3)M^6x^6 - \frac{1}{1152}a^2\beta(1-3a\beta)^2M^8x^8\end{aligned} \tag{20}$$

$$\vdots$$

A series solution of order $O(14)$ was found to be close to the exact solution with a very small standard error using the differential transformation and vaiational iteration [4]. An approximate expression for temperature distribution for the problem under consideration is then given by

$$\theta(x) \cong \theta_7(x). \tag{21}$$

In equation (17), the constant a is the temperature at the fin tip and must lie in the interval $[0, 1]$. Since a is assumed to be a constant as the initial guess, it automatically satisfies the derivative boundary condition. As seen from succeeding equations (18)-(20), additional terms include x with the power of two or more. This also satisfies the derivative boundary condition at $x = 0$. Due to this fact, to obtain the value of a, we substitute the boundary condition (5) into (21) at the point $x = 1$. Similarly, the recurrence relation of VIM solution to (16) is given below,

$$\begin{aligned}\theta(x) = a + \frac{1}{2}a(1 - a\beta + a^2\beta^2)M^2x^2 - \frac{1}{24}a(-1 + 5a\beta - 9a^2\beta^2 + 3a^3\beta^3)M^4x^4 \\ - \frac{1}{720}a(-1 + 18a\beta - 60a^2\beta^2 + 45a^3\beta^3)M^6x^6 - \frac{1}{1152}a^2\beta(1 - 3a\beta)^2M^8x^8 + \cdots\end{aligned} \tag{22}$$

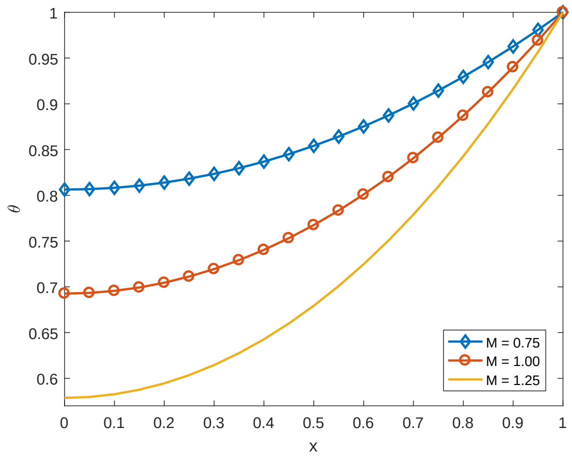

Fig. 2: Temperature distribution along the fin of linear thermal conductivity for varying M, $\beta = 0.25$.

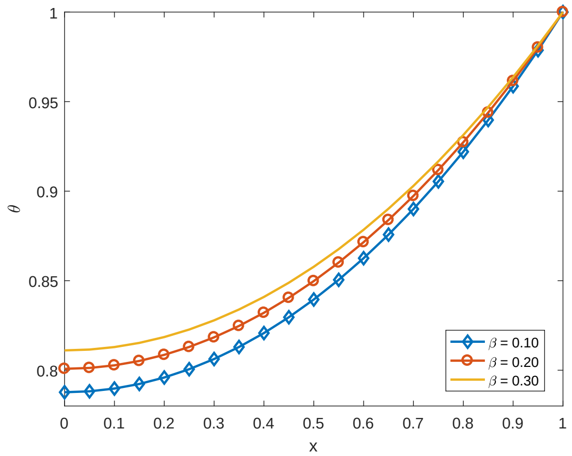

Fig. 3: Temperature distribution along the fin of linear thermal conductivity for varying β, $M = 0.75$

Power Law Function In dimensionless variables we have $k(\theta) = \theta^m$ and equation (4) reduces to:

$$\frac{d}{dx}\left[\theta^m \frac{d\theta}{dx}\right] - M^2\theta = 0,\ 0 \leq x \leq 1. \tag{23}$$

Following a similar approach to construct the approximate analytical solution using VIM, the recurrence relation given a power law calibration function is shown below,

$$\theta(x) = a + \frac{a^{1+m}}{1+m} + aM^2x^2 + \frac{1}{24}aM^4x^4 - \left(\frac{a(2+M^2x^2)(a+\frac{1}{2}aM^2x^2)^m}{2(1+m)}\right) + \cdots \tag{24}$$

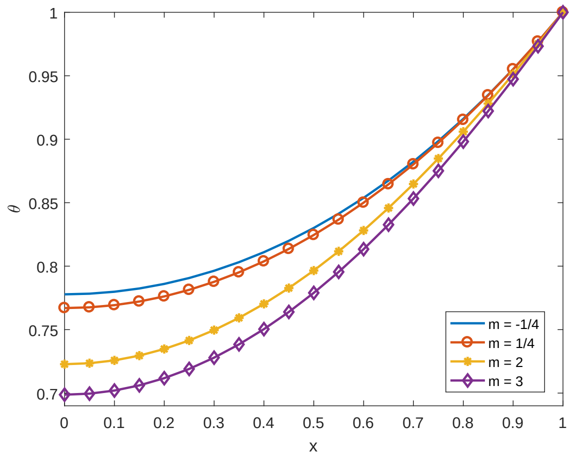

Fig. 4: Temperature distribution along the fin of power law thermal conductivity for varying m, $M = 0.75$

Exponential Function In dimensionless variables we have $k(\theta) = \mathrm{e}^{\beta_1\theta}$ and equation (4) reduces to:

$$\frac{d}{dx}\left[\mathrm{e}^{\beta_1\theta} \frac{d\theta}{dx}\right] - M^2\theta = 0,\ 0 \leq x \leq 1. \tag{25}$$

Similarly, the recurrence relation of VIM solution to equation (25) is given below,

$$\theta(x) = c + \mathrm{e}^{\frac{a\beta_1}{b}} - \frac{\mathrm{e}^{a\beta_1 + \frac{1}{2}a\beta_1 M^2x^2}}{b} + aM^2x^2 + \frac{1}{24}aM^4x^4 + \cdots \tag{26}$$

Logistic Function In dimensionless variables we have $k(\theta) = 1/1+\mathrm{e}^{\beta_2\theta}$ and equation (4) reduces to:

$$\frac{d}{dx}\left[\frac{1}{1+\mathrm{e}^{\beta_2\theta}} \frac{d\theta}{dx}\right] - M^2\theta = 0,\ 0 \leq x \leq 1. \tag{27}$$

Similarly, the VIM solution to equation (27) is given below,

$$\theta(x) = a + \frac{1}{2}aM^2x^2 + \frac{1}{24}aM^4x^4 - \frac{\log[1+\mathrm{e}^{a\beta_2}]}{\beta_2} + \frac{\log[1+\mathrm{e}^{\frac{1}{2}a\beta_2(2+M^2x^2)}]}{\beta_2} \cdots \tag{28}$$

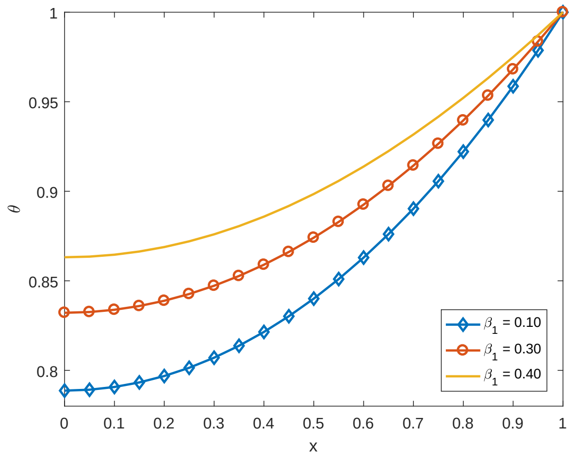

Fig. 5: Temperature distribution along the fin of exponential thermal conductivity for varying β_1, $M = 0.75$

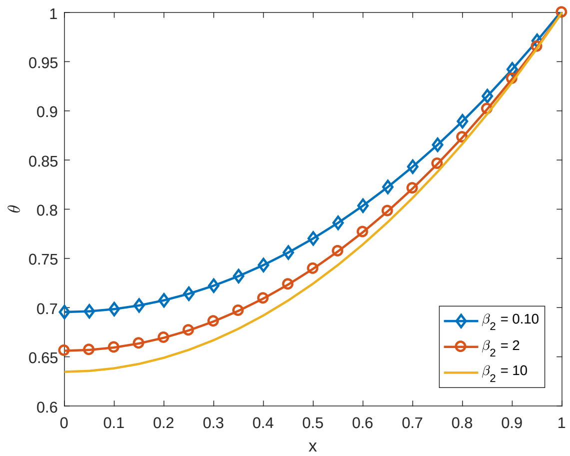

Fig. 6: Temperature distribution along the fin of logistic thermal conductivity for varying β_2, $M = 0.75$

Aggregated Analysis In this section we compare the temperature distribution for four thermal conductivity calibration functions as shown in the Figure 7. Table 1 gives a quantitative view of the benchmark analysis between the analytical solution and the numerical solution for a model with linear thermal conductivity calibration function.

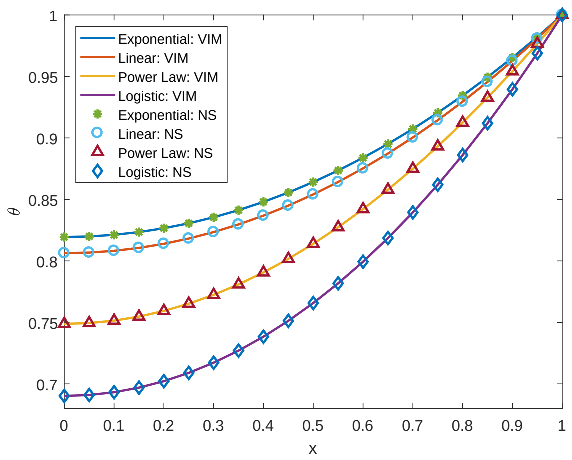

Fig. 7: Temperature distribution along the fin for varying calibration functions, $M = 0.75$, $m = 1$, $\beta = \beta_1 = \beta_2 = 0.25$.

Fin Efficiency and Heat Flux

Fin Efficiency The heat transfer rate from a fin is given by Newton's second law of cooling,

$$Q = \int_0^L Ph(T - T_a)dX. \tag{29}$$

Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin were at the base temperature and is given by (see e.g. [1])

$$\eta = \frac{Q}{Q_{ideal}} = \frac{\int_0^L Ph(T - T_a)dX}{PhL(T_b - T_a)}. \tag{30}$$

In dimensionless variables we have

$$\eta = \int_0^1 \theta dx. \tag{31}$$

The fin efficiency is depicted in Figure 8.

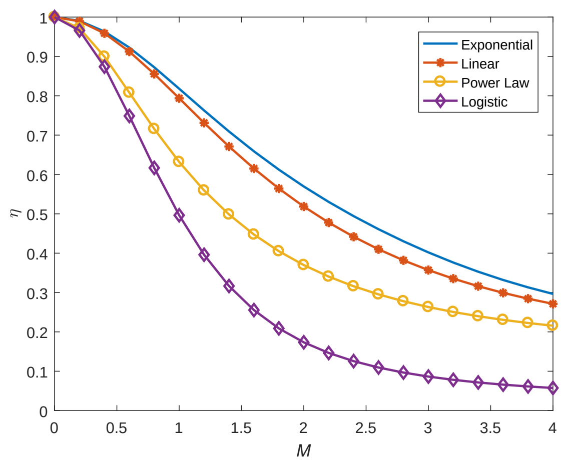

Fig. 8: Fin efficiency of a fin for varying calibration functions, $m = 1$, $\beta = \beta_1 = \beta_2 = 0.25$.

Table 1: Results of the VIM and Numerical Solutions for $\beta = 0.25,\ M = 0.75$

x	VIM	RKM	Error
0	0.80742475	0.80747205	0.00004730
0.1	0.80931523	0.80936199	0.00004676
0.2	0.81499104	0.81503619	0.00004515
0.3	0.82446528	0.82450774	0.00004246
0.4	0.83775967	0.83779843	0.00003876
0.5	0.85490442	0.85493849	0.00003407
0.6	0.87593796	0.87596643	0.00002847
0.7	0.90090670	0.90092876	0.00002206
0.8	0.92986467	0.92987967	0.00001500
0.9	0.96287310	0.96288064	0.00000754
1.0	1.00000000	1.00000000	0.00000000

Heat Flux The fin base heat flux is given by the Fourier's law

$$q_b = A_c K(T)\frac{dT}{dx}. \tag{32}$$

The total heat flux of the fin is given by [1]

$$q = \frac{q_b}{A_c h(T_b - T_a)}. \tag{33}$$

Introducing the dimensionless variable as described in equations 3, implies

$$q = \frac{1}{Bi}(1+\beta\theta)\frac{d\theta}{dx}, \tag{34}$$

where the dimensionless parameter $Bi = \dfrac{hL}{k_a}$ is the Biot number. The heat flux in equation (34) is plotted in Figure 9 and Figure 10 .

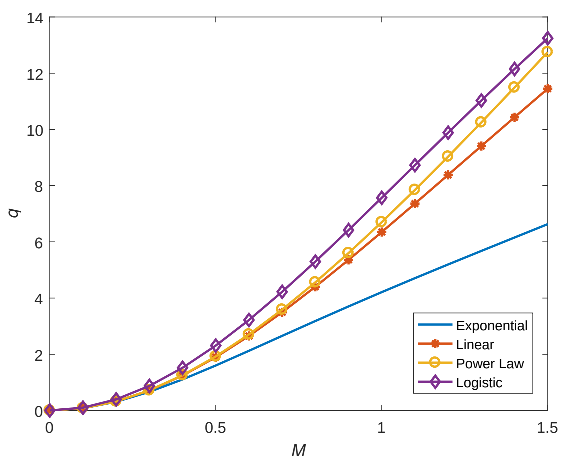

Fig. 9: Heat flux in a fin for varying calibration functions, $m = 1$, $\beta = \beta_1 = \beta_2 = 0.25$.

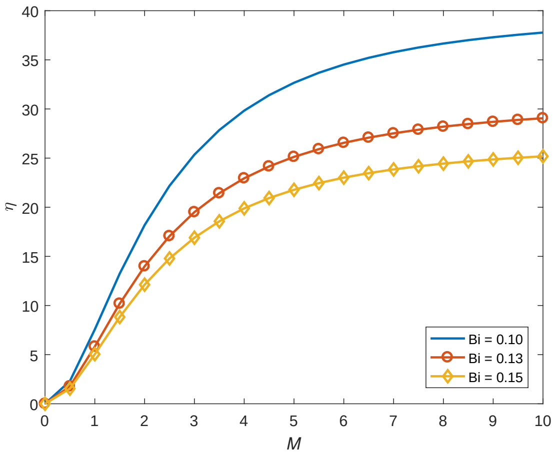

Fig. 10: Heat flux in fin with logistic thermal conductivity, $\beta_2 = 0.25$.

Discussion

This study has resulted in some interesting results. In Figure 2 we observe that the fin temperature decreases with the increasing values of the thermo-geometric fin parameter. Note that the thermo-geometric fin parameter $M = (Bi)^{1/2}E$, where $E = L/\delta$ is the aspect ratio or the extension factor. Evidently, small values of M correspond to the relatively short and thick fins of high conductivity and high values of M correspond to longer and thin fins of poor conductivity [22]. A fin is an excellent dissipator at small values of M. In Figure 3 we observe that the temperature in the fin increases with the increasing values of the thermal conductivity gradient β. Note that $\beta = 0$ implies that the thermal conductivity is a constant given by k_a. The curves with $\beta = 0$, correspond to the fin material whose thermal conductivity increases as temperature increases from T_a to T_b, [23]. Figure 4 shows that as the power constant of the thermal conductivity increases, the temperature inside the fin decreases. An increase in the m, results in an increase in heat loses at the fin base and as a result, the fin dissipates heat to the ambient fluid rapidly. Figure 5 shows the temperature distribution for varying thermal conductivity gradient in an exponential thermal conductivity calibration function. We observe a similar result as that of linear calibration function. However, Figure 6 shows the converse of the other two thermal conductivity gradient. Given a logistic calibration function for thermal conductivity, the temperature inside the fin increases with increasing values of the thermal conductivity gradient. This correspond to the fin material whose thermal conductivity decreases as temperature increases from T_a to T_b.

The aggregated analysis is shown graphically in Figure 7. We note that the approximate analytical solutions are in agreement with the numerical solutions. We also observe that the logistic calibration function leads to lower temperatures in the fin when compared to the other three functions. This is a useful result as it gives alternate choice of modeling heat transfer to suit the prevailing conditions given in the concluding remarks. In Figure 8, fin efficiency decreases with increasing thermogeometric fin parameter across all thermal conductivity calibration functions. Figures 9 and 10, display the heat flux at the fin base. The amount of heat energy dissipated from the fin base is of immense interest in engineering [24]. We note that the heat flux across the fin length increases with increasing values of the thermogeometric fin parameter.

Concluding Remarks

In this study, we have successfully performed thermal analysis of a straight fin with varying thermal conductivity functions. The resulting differential equations were solved using He's VIM and the solutions were bench-marked against the numerical results obtained by applying the Runge-Kutta fourth order method coupled with shooting technique. The results show that different calibration functions of thermal conductivity in turn lead to varying temperature distributions along the fin. The newly introduced calibration function maybe be useful under conditions where the modeled temperature distribution is over predicting the actual temperature distribution. This is shown on the aggregated temperature distribution graph where the choice of the logistic function results in lower temperatures in the fin when compared to the other three dominant functions found in literature.

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2.7 Journal Article VII

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Research Paper

Numerical Analysis of Transient Heat Transfer in Radial Porous Moving Fin with Temperature Dependent Thermal Properties

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Abstract. In this article, a time dependent partial differential equation is used to model the nonlinear boundary value problem describing heat transfer through a radial porous moving fin with rectangular profile. The study is performed by applying a numerical solver in MATLAB (*pdepe*), which is a centered finite difference scheme. The thermal conductivity and fin surface emissivity are linearly dependent on temperature while the heat transfer coefficient is given by power law function of temperature. The effects of thermo-physical parameters, such as the Peclet number, surface emissivity coefficient, power index of heat transfer coefficient, convective-conductive parameter, radiative-conductive parameter and non-dimensional ambient temperature on temperature are studied.

Keywords: Numerical analysis, Heat transfer, Thermal conductivity, Moving fin, Fin tip temperature.

1. Introduction

The study of heat transfer through fins (extended surfaces) is of immense interest in various industrial applications. An increasing number of engineering applications are concerned with energy transportation by requiring a rapid movement of heat away from a heat source. Fins are used to dissipate heat from a heated body and are seen in various applications such as heat exchangers, oil carrying pipelines and chemical processing equipment [1, 2]. Various studies have been devoted to finding efficient and effective ways of heat transportation through finned surfaces. Recent studies on porous stationery fins [3] and moving porous fins [4] have shown great improvements in the rate of heat transfer through fins when compared to solid stationary fins. Furthermore, the fin profile has been shown to have an impact on the rate of heat transfer. Ndlovu and Moitsheki [5, 6] studied the steady-state and transient behavior of fins with various profiles using analytical solutions. Mosayebidorcheh et al. [7] studied the transient heat transfer through radial fins of triangular, hyperbolic and rectangular profiles by applying hybrid analytical-numerical techniques.

In recent years, the study of heat transfer through moving fins has attracted the interests of many scholars. Roy et al. [8] studied the steady heat transfer through a triangular moving fin using a decomposition analytical solution. Turkyilmazoglu [9] further studied the heat transfer through exponential moving fins with internal heat generation. The temperature distribution profile was compared to that of a rectangular profile. Singla and Das [10] studied an inverse problem to estimate the speed of a moving fin using the binary-coded generic algorithm. Dogonchi and Ganji [11] applied differential transformation to study the simultaneous convection-radiation heat transfer through a moving fin with heat generation. Their results showed that the temperature inside the fin decreases with an increase on the speed of the moving fin. To the best knowledge of the author, the transient behavior of moving fins has been only studied by Ndlovu and Moitsheki [12]. Numerous studies in literature focused on the steady-state fin problems.

In this work, we study the heat transfer through a radial porous moving fin with temperature dependent thermal properties. The mathematical model is a highly nonlinear second order partial differential equation which is solved numerically using a built-in numerical solver in MATLAB. Numerical schemes have been used in various researches to benchmark the analytical results employed in the study of heat transfer [13, 14]. Moitsheki and Harley [15] applied classical Lie point symmetries to study the heat transfer through fins of various profiles. However, the resulting boundary value problem was not invariant under any Lie point symmetry and the built-in numerical solver in MATLAB was applied to study the problem. This paper is a great improvement of many studies which consider steady-state fin problems with the assumption that the transient thermal behavior dies out quickly [16, 17]. The current research paper considers a highly nonlinear boundary value problem including the porosity parameter, moving fin parameter, temperature dependent thermal conductivity, temperature dependent heat transfer coefficient, temperature dependent surface emissivity, radiation and convection parameters, all in a single model. The problems considering such nonlinearities are of fundamental importance in the thermal processing of materials [18, 19].

This article is organized as follows; the mathematical background of the problem under consideration is described in Section 2. A brief discussion on the fundamentals of the numerical solver is introduced in Section 3. The author provides numerical results and discussions in Section 4. Lastly, concluding remarks are presented in Section 5.

2. Mathematical Model

The model formulation considers a one-dimensional radial porous moving fin of cross-sectional area A_c while it moves horizontally with a constant velocity U as depicted in Fig. 1. The fin is attached to a base surface of temperature T_b and extends into an ambient fluid of temperature T_a. The perimeter of the fin is denoted by P and the length of fin by $L = r_b - r_t$. We assume that at the tip of the fin, $r_t = 0$ and the fin thickness at the prime surface is given by δ_b. The fluid surrounding the moving fin is heated by the convective-radiative fin and induces a horizontal flow field (advection). The porous medium is assumed to be isotropic, homogeneous and saturated with a single-phase fluid. The fluid and solid matrix are not at local thermal equilibrium and the fluid-solid interaction in the porous medium is stimulated by Darcy's model. The energy balance equation for the moving porous fin losing heat by simultaneous convection and radiation can be expressed as, (see also, [1, 12]):

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left(RK(T)\frac{\partial T}{\partial R}\right) - \frac{P}{A_c}H(T)(T-T_a) - \frac{\dot{m}c}{A_c}(T-T_a) - \frac{\varepsilon(T)\sigma P}{A_c}\left(T^4 - T_a^{\,4}\right) - \rho c U\frac{\partial T}{\partial R}, r_b \le R \le r_t . \tag{1}$$

The mass flow rate of the fluid passing through the porous media is given by [3]:

$$\dot{m} = \rho v_w W \Delta R . \tag{2}$$

The velocity of the fluid passing through the porous media is given by Darcy's model,

$$v_w = \frac{gKA}{\vartheta}(T-T_a). \tag{3}$$

For most materials, the thermal conductivity may be given by a linear function of temperature, that is,

$$K(T) = k_c\left[1+\gamma(T-T_a)\right], \tag{4}$$

the surface emissivity is also assumed to vary linearly with temperature,

$$\varepsilon(T) = \varepsilon_a\left[1+\mu(T-T_a)\right], \tag{5}$$

and the heat transfer coefficient may be given by a power law function of temperature,

$$H(T) = h_b\left(\frac{T-T_a}{T_b-T_a}\right)^n \tag{6}$$

The fin tip is assumed to be adiabatic (insulated) and the base temperature is kept constant. The boundary conditions are then given by [1],

$$T(t,R) = T_b \tag{7}$$

and

$$\frac{\partial T}{\partial R}\Big|_{R=0} = 0. \tag{8}$$

Initially, the fin is kept at the ambient temperature [6, 7],

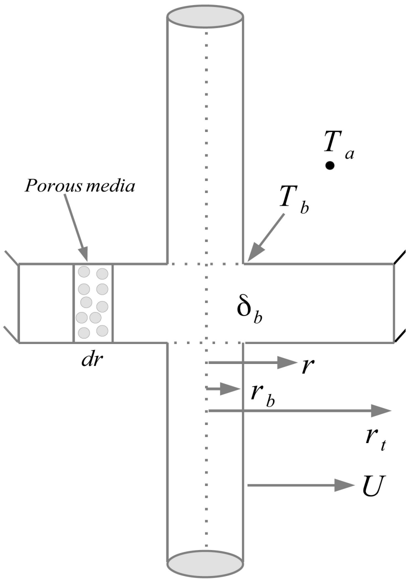

Fig. 1. Schematic representation of a radial fin.

$$T(0,R)=0. \tag{9}$$

Here, r_b and r_t are the radii of the fin base and fin tip, respectively. The thermal conductivity of the fin at ambient temperature is k_c, γ is a measure of thermal conductivity variation with temperature, μ is a measure of surface emissivity variation with temperature, ρ is the density of fluid, c is the specific heat of fluid, ε and σ are the emissivity and Boltzman constant respectively, h_b is the heat transfer coefficient at the fin base and n is a constant. In most practical applications, the convection coefficient n lies between -3 and 3 [20]. The exponent n represents nucleate boiling when $n=2$, radiation when $n=3$ and $n=0$ implies a constant heat transfer coefficient. For the porous media formulation, g is the gravitational acceleration, K is the permeability, ϑ and A are the kinematic viscosity and thermal expansion coefficient respectively. Introducing the following dimensionless variables,

$$\begin{gathered}r=\frac{R}{L},\tau=\frac{k_c t}{\rho c L^2},\theta=\frac{T}{T_b},\beta=\gamma T_b,\zeta=\mu T_b,k=\frac{K}{k_a},h=\frac{H}{h_b},\alpha=\frac{k_c}{\rho c},\\ N_c=\frac{P h_b T_a^{\,n} L^2}{k_c A_c\left(T_b-T_a\right)^n},N_r=\frac{\varepsilon_a \sigma P L^2 T_b^3}{k_c A_c},N_p=\frac{\rho c g K A W L^2 T_b}{\vartheta k_c A_c},Pe=\frac{UL}{\alpha},\end{gathered} \tag{10}$$

reduces Eq. (1) to,

$$\frac{\partial\theta}{\partial\tau}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\left\{1+\beta\left(\theta-\theta_a\right)\right\}\frac{\partial\theta}{\partial r}\right]-N_c\left(\theta-\theta_a\right)^{n+1}-N_p\left(\theta-\theta_a\right)^2-N_r\left[1+\zeta\left(\theta-\theta_a\right)\right]\left(\theta^4-\theta_a^4\right)-Pe\frac{\partial\theta}{\partial r},0\le r\le 1, \tag{11}$$

and Eq. (11) admits the following boundary conditions,

$$\theta(\tau,1)=1, \tag{12}$$

$$\frac{\partial\theta}{\partial r}\Big|_{r=0}=0, \tag{13}$$

and the initial condition becomes,

$$\theta(0,r)=1. \tag{14}$$

The dimensionless variable θ represents the temperature, θ_a denotes the dimensionless ambient temperature, r is the dimensionless space variable, τ is the dimensionless time variable, β is the thermal conductivity gradient, ζ is the surface emissivity gradient, N_c is the convection-conduction parameter, N_r is the radiation-conduction parameter, N_p is the porosity parameter, Pe is the Peclet number which represents the dimensionless speed of the moving fin and α is the thermal diffusivity of the fin.

3. Fundamentals of the Numerical Solver

MATLAB (*pdepe*) specifies the partial differential Eq. (11) in the following form,

$$c(r,t,\theta,\theta_r)\theta_t = r^{-m}\frac{\partial}{\partial r}\left(r^m b(r,t,\theta,\theta_r)\right) + s(r,t,\theta,\theta_r) \tag{15}$$

with boundary conditions,

$$p(r_l,t,\theta) + q(r_l,t)\cdot b(r_l,t,\theta,\theta_r) = 0 \tag{16}$$

$$p(r_r,t,\theta) + q(r_r,t)\cdot b(r_r,t,\theta,\theta_r) = 0 \tag{17}$$

where r_l represents the left endpoint of the boundary and r_r represents the right endpoint of the boundary, and initial condition,

$$\theta(0,r) = f(r). \tag{18}$$

Based on the problem under consideration, (Eq. (11)), we have,

$$c(r,t,\theta,\theta_r) = 1 \tag{19}$$

$$f(r,t,\theta,\theta_r) = \left\{1+\beta(\theta-\theta_a)\right\}\frac{\partial\theta}{\partial r} \tag{20}$$

$$s(r,t,\theta,\theta_r) = -N_c(\theta-\theta_a)^{n+1} - N_p(\theta-\theta_a)^2 - N_r\left[1+\zeta(\theta-\theta_a)\right]\left(\theta^4-\theta_a^4\right) - Pe\frac{\partial\theta}{\partial r}. \tag{21}$$

The boundary conditions are given as follows,

$$p(0,t,\theta) = 0 \tag{22}$$

$$p(1,t,\theta) = \theta - 1 \tag{23}$$

$$q(0,t,\theta) = 1 \tag{24}$$

$$q(1,t,\theta) = 0, \tag{25}$$

and the initial condition becomes,

$$f(r) = 0. \tag{26}$$

The code below executes the numerical solution for the problem under consideration,

```
function pdesolver()
m = 1;
x = linspace(0,1,20);
t = linspace(0,0.6,20);
% Define analytical parameter values here
solution = pdepe(m,@pdex1pde,@pdex1ic,@pdex1bc,x,t,[],b,A,S,n,G,R,Z,P);
% Model specification function
function[c,f,s] = pdex1pde(x,t,u,DuDx,b,A,S,n,G,R,Z,P)
c = 1;
f = (1+b*(u-A))*DuDx ;
s = -S*(u-A)^(n+1)-G*(u-A)^2-R*(1+Z*(u-A))*(u^4-A^4)-P*DuDx;
% Defining the initial condition
function u0 = pdex1ic(x,b,A,S,n,G,R,Z,P)
u0 = 0;
% Defining the boundary conditions
function[pl,ql,pr,qr] = pdex1bc(xl,ul,xr,ur,t,b,A,S,n,G,R,Z,P)
pl = 0;
ql = 1;
pr = ur-1;
qr = 0;
```

The variables on the above code represent the model parameter values. Unless stated otherwise, the following values are used in generating the numerical results; $u=\theta$, $b=\beta=1$, $A=\theta_a=0.50$, $S=N_c=0.75$, $n=2$, $G=N_p=0.50$, $R=N_r=4$, $Z=\zeta=0.75$, $P=Pe=3$.

4. Results and Discussion

In this section, we provide the numerical analysis of the model covering all embedding parameters. Some interesting results are observed, and we explain the physics behind the effects of varying the thermo-physical parameters. Figure 2 shows the temperature distribution in a radial fin for varying time. We observe that the temperature inside the fins increases as the time increases and eventually reaches steady state. Figure 2 also shows a benchmark analysis of validating the numerical scheme against analytical solutions generated by two-dimensional differential transform method (DTM). We observe that the numerical and analytical solutions are in agreement. Figure 3 depicts the effect of the speed of moving fin on the temperature distribution. As Pe increases, (the speed of the moving fin increases), the temperature inside the fin decreases rapidly due to increased advective effect on the surface of the fin. We also observe that for $Pe=0$, i.e. stationary fin, the cooling of the fin takes more time as shown by higher temperatures when compared to a moving fin. In Figure 4, we observe that for $n\geq 3$ the temperature inside the fin does not change significantly when other parameters are kept constant. For $n=3$, the radiation becomes more prominent and any further increase in the convection exponent n has no influence on the fin temperature. This is a result of radiation being modeled by an additional term N_r .

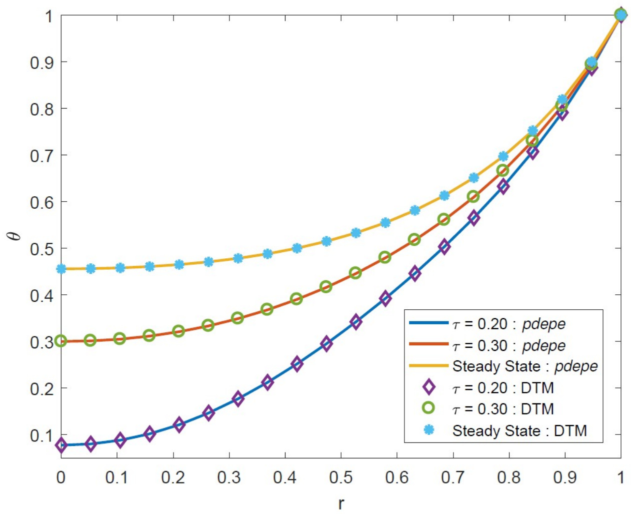

Fig. 2. Temperature distribution along a radial porous moving fin for varying time.

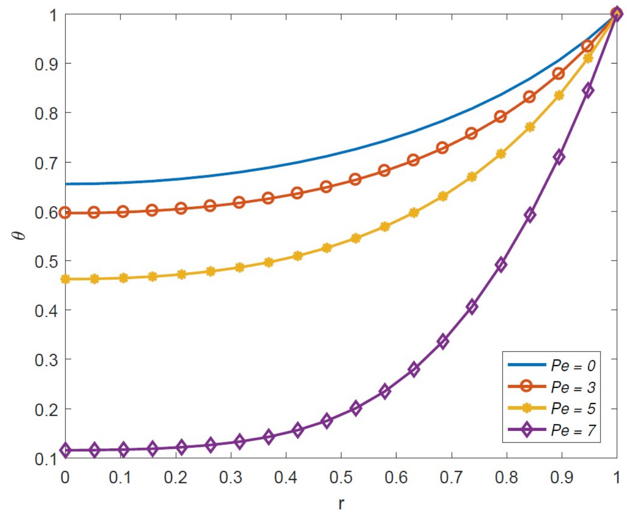

Fig. 3. Temperature distribution along a radial porous moving fin for varying fin velocity.

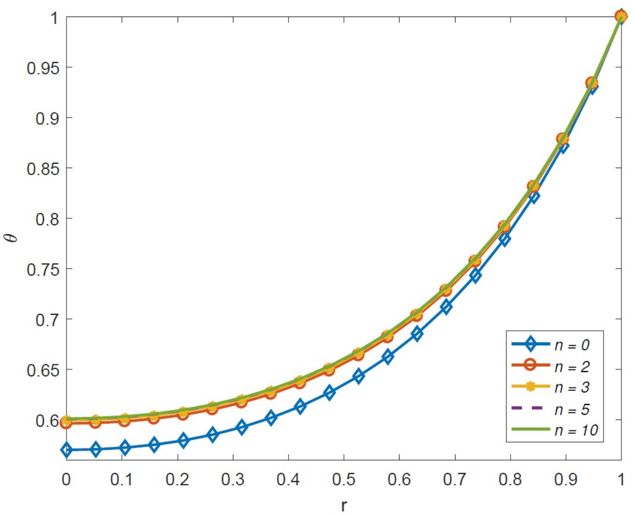

Fig. 4. Effect of varying the power of convection-conduction term on temperature distribution.

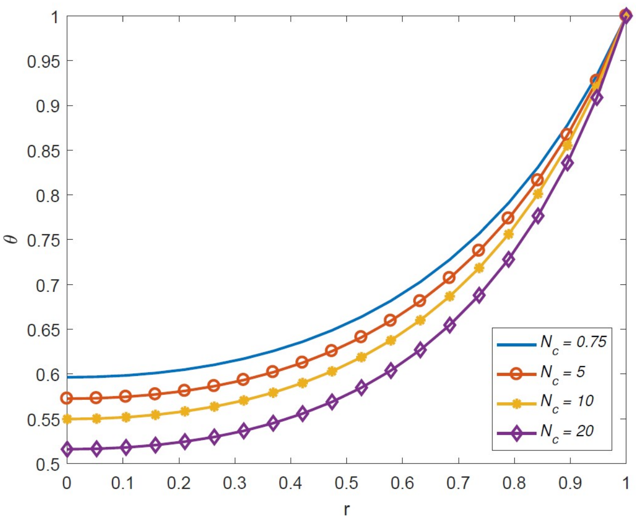

Fig. 5. Effect of varying the convection-conduction parameter on temperature distribution.

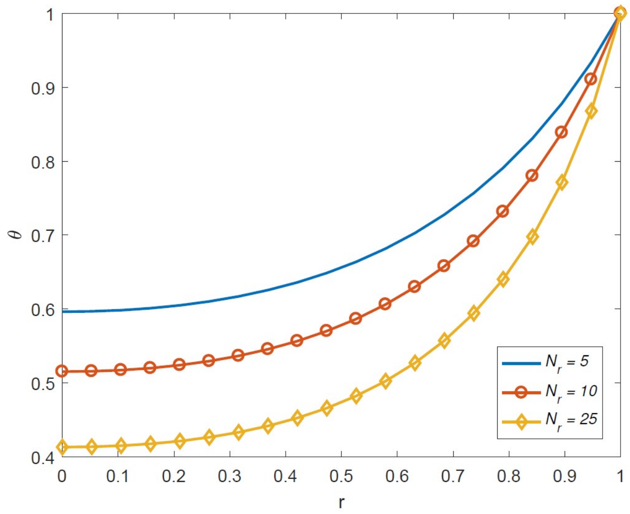

Fig. 6. Effect of varying the radiation-conduction parameter on temperature distribution.

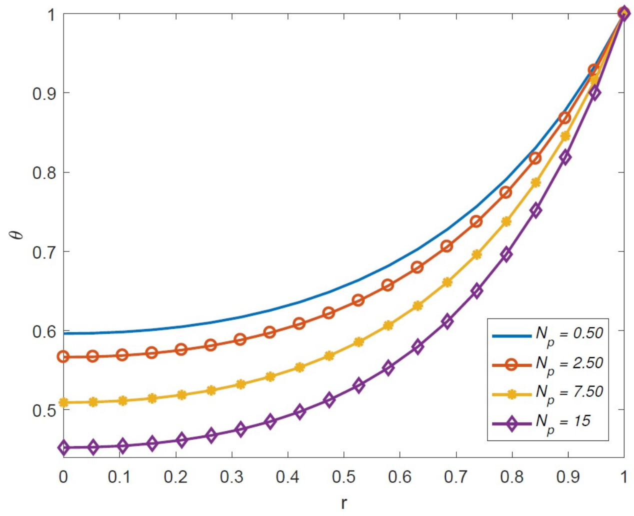

Fig. 7. Effect of varying the porous media parameter on temperature distribution.

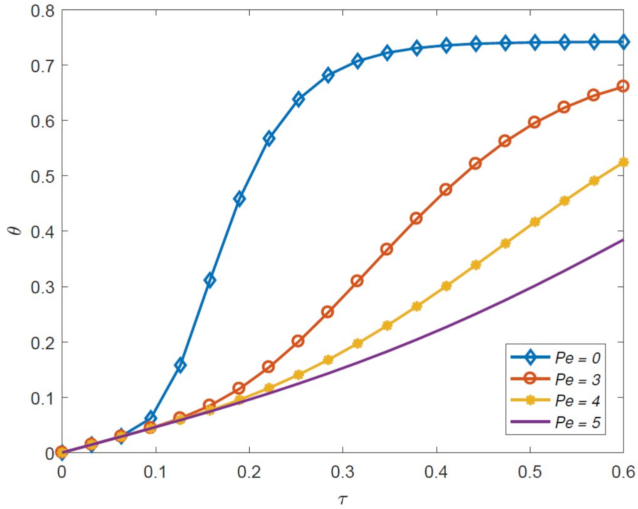

Fig. 8. Temperature distribution on a radial porous moving fin tip for varying fin velocity.

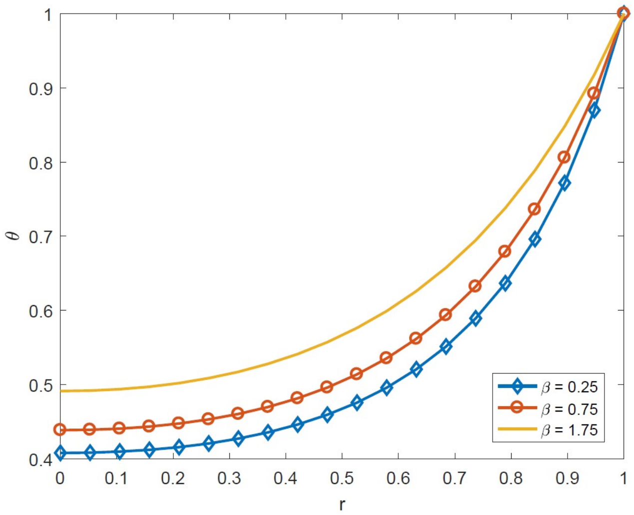

Fig. 9. Effect of varying the thermal conductivity gradient parameter on temperature distribution.

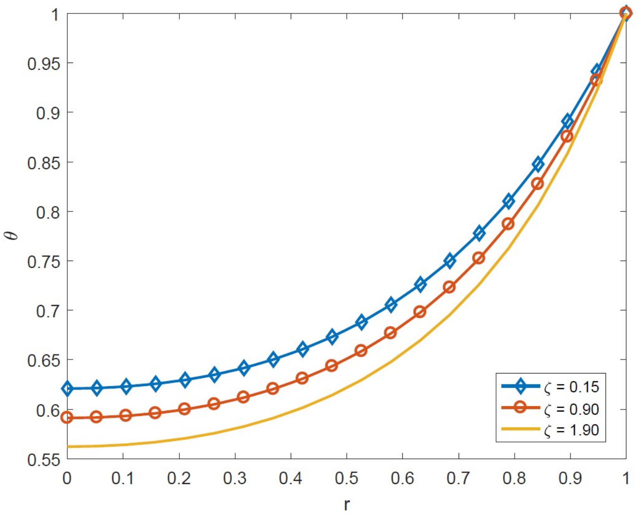

Fig. 10. Effect of varying the surface emissivity gradient parameter on temperature distribution.

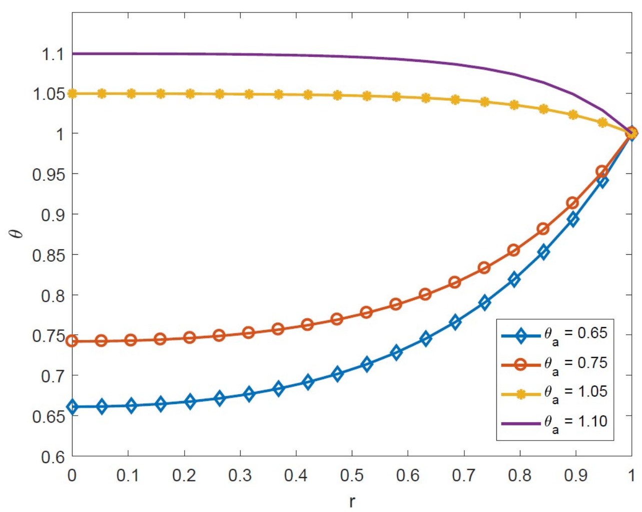

Fig. 11. Effect of varying the ambient temperature parameter on temperature distribution.

Figure 5 illustrates the temperature distribution for different values of the convective-conductive parameter N_c. For $N_c = 0$ this indicates the absence of convection while $N_c = 0$ indicates that convection and conduction are at equal rates [8]. An increase in the values of the thermo-geometric fin parameter leads to a decrease in temperature in the fin. A similar result is observed in Fig. 6 which represents the impact of the radiation parameter $N_r = 0$ on the temperature distribution. Figure 7 shows the effect of varying the porous media parameter $N_p = 0$ on the temperature distribution. We observe that the temperature decreases with an increasing value of the porous parameter. The high rate of heat transfer is a result of higher porosity due to changes in permeability parameter K included in $N_p = 0$.

In Fig. 8, as expected, the fin temperature increases with increasing values of the Peclet number representing the speed of the moving fin and the dimensionless time. We also observe that the transient fin behavior dies out quickly on a stationary fin when compared to a moving fin. Figure 9 shows the effect of thermal conductivity gradient on the temperature distribution. We observe that as the values of β decreases, the temperature in the fin decreases signifying an increased heat loss to the ambient fluid. Increasing the thermal conductivity gradient enhances heat conduction from the fin base and leads to increased temperatures inside the fin. The effect of varying surface emissivity parameter ζ is depicted in Fig. 10. As expected, we observe that as the surface emissivity increases, the radiative heat loss decreases resulting in lower fin temperatures.

Figure 11 shows the variation in the temperature distribution for varying values of the ambient temperature parameter θ_a. The ambient temperature plays an important role in transient heat dissipation. Higher ambient temperature leads to a reduced heat transfer rate as indicated by high values of fin temperature for higher values of the ambient temperature parameter. Lastly, Fig. 12 depicts the temperature distribution along the three-dimensional spaces and this corresponds to the imposed boundary conditions and to all the results described above.

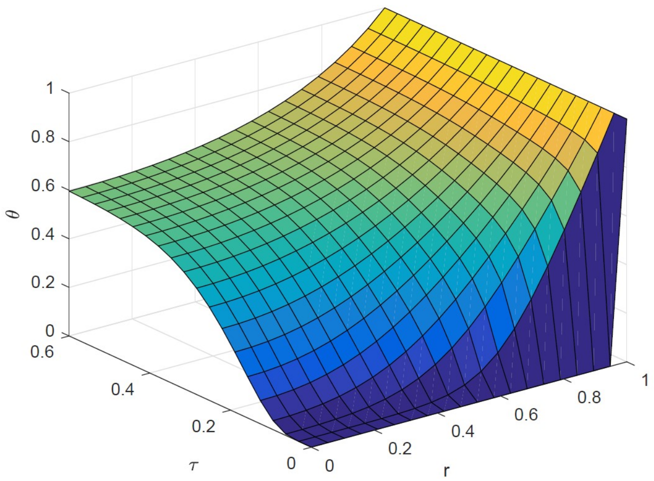

Fig. 12. Transient temperature distribution along a radial porous moving fin.

5. Concluding Remarks

In this study, we derived a mathematical model describing heat transfer through a radial porous fin of a rectangular profile as it moves with a constant velocity. The analysis was performed using built-in numerical scheme in MATLAB software package. MATLAB does not require a compiler to execute and this increases the productivity as well as coding efficiency. Furthermore, the software has inbuilt library of neural networks, fuzzy logic, Simulink and many more, making it easy to handle highly nonlinear problems. In this study, it was observed that the rate of heat transfer through extended surfaces can be influenced by the embedding parameters. An increase in the speed of the moving fin as well as an increase in fin permeability led to increased fin efficiency. The results were in line with some of the studies performed for similar models. The impact of various model parameters on temperatures distribution were illustrated graphically and discussed.

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Conflict of Interest

The author declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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2.8 Journal Article VIII

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A study of transient heat transfer through a moving fin with temperature dependent thermal properties

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Keywords: Heat transfer, analytical solutions, DTM, extended surface

Abstract. In this article, heat transfer through a moving fin with convective and radiative heat dissipation is studied. The analytical solutions are generated using the two-dimensional Differential Transform Method (2D DTM) which is an analytical solution technique that can be applied to various types of differential equations. The accuracy of the analytical solution is validated by benchmarking it against the numerical solution obtained by applying the inbuilt numerical solver in MATLAB ($pdepe$). A good agreement is observed between the analytical and numerical solutions. The effects of thermo-physical parameters, such as the Peclet number, surface emissivity coefficient, power index of heat transfer coefficient, convective-conductive parameter, radiative-conductive parameter and non-dimensional ambient temperature on non-dimensional temperature is studied and explained. Since numerous parameters are studied, the results could be useful in industrial and engineering applications.

Introduction

An increasing number of engineering applications are concerned with energy transport requiring rapid heat dissipation. To increase the heat transfer rate from a heated surface, fin (or extended surface) assembly is commonly used. The use of fins is seen in various industrial applications such as oil carrying pipelines, space nuclear reactor power systems and many more. The heat transfer mechanism of a fin is to conduct heat from a heat source by thermal conduction, and then dissipate heat to an ambient fluid by the effect of thermal convection and radiation. An extensive review of extended surface heat transfer is given by Kern and Kraus [1] and Kraus et al. [2]. A brief review of published work that is of relevance to this article is presented next.

A detailed study of temperature distribution in a moving fin is provided for various embedding parameters in a triangular fin [3], rectangular fin [4], an exponential fin [5] and a trapezoidal cross section [6]. Singla and Das [7] studied an inverse problem to estimate the speed of a moving fin using the binary-coded generic algorithm. Sun et al. [8] applied the Spectral Collocation Method (SCM) to derive the solution for a convective-radiative heat transfer through a moving rod with variable thermal conductivity. Sun and Xu [9] further applied the SCM to predict the temperature distribution in moving fins of complex cross-sections. Recently, Ma et al. [10] simulated a combined conductive, convective and radiative heat transfer in irregular moving porous fins using Spectral Element Method (SEM). Investigation of mixed convection heat transfer along a continuously moving heated vertical plate with suction and blowing was conducted by Al-Sanea [11]. The radiation effects are quite significant in various engineering and industrial processes especially in the design of reliable equipments, nuclear plants and gas turbines. Razelos and Kakatsios [12] investigated the optimum dimensions of convecting-radiating rectangular fins. They studied the influence of all dependent parameters on optimization and performance of rectangular fins. They also investigated the effect of the temperature-dependent thermal conductivity and emissivity on the temperature profile. An approximate analytical solution for convection-radiation heat transfer from a continuously moving fin with temperature-dependent thermal conductivity was developed by Aziz and Khani [13]. Torabi and Zhang [14] investigated thermal performance of a convective-radiative straight fin by considering different profiles containing nonlinearities and further studied the fin efficiency. Sun et al. [15] studied the different temperature-dependent thermal properties for predication of heat transfer in a convective-radiative fin. Kundu and Wongwises [16] presented a decomposition analysis of a convective-radiating fin with one side of the primary surface being heated by a fluid with high temperature. Aziz and Makinde [17] applied a two-dimensional heat conduction model to obtain the thermal performance and entropy generation in an orthotropic convection pin fin used in advanced light weight heat sinks. The study illustrated the simultaneous realization of the least material and minimum entropy generation in pin fin designs. Mhlongo et al. [18] applied the local and nonlocal symmetry techniques to study transient response of rectangular fins to step change in base temperature and base heat flux. The authors showed that the governing equations can be reduced to the tractable Ermakov-Pinney equation.

The DTM is a semi-numerical-analytical method based on the Taylor series expansion and was first proposed by Zhou [19] in 1986 for the solution of linear and nonlinear initial value problems that appear in the analysis of electrical circuits. This method was subsequently used to obtain analytical solutions of various types of higher-order differential equations. Torabi et al. [20] applied the DTM to derive approximate explicit analytical expressions for the temperature distribution in a moving fin with temperature-dependent thermal conductivity and experiencing simultaneous convective-radiative surface heat loss. Moradi and Rafiee [21] extended the work of Torabi et al. to study a similar model for various fin profiles. Ndlovu and Moitsheki [22] successfully applied the 2D DTM to a transient heat conduction problem for heat transfer in longitudinal rectangular and convex parabolic fins. Some interesting results were obtained and the effects of the parameters appearing in the model on the temperature distribution were illustrated and explained. Mosayebidorcheh et al. [23] studied transient thermal behavior of radial fins of rectangular, triangular and hyperbolic profiles. These authors applied the Hybrid Differential Transform Method-Finite Difference Method (DTM-FDM) to generate the numerical solutions to the problem. Fallo et al. [24] applied the 3D DTM for the first time to study heat transfer in a cylindrical spine fin with variable thermal properties. Hatami et al. [25] applied the Differential Quadrature Method (DQM) together with the DTM to study magnetohydrodynamic (MHD) two-phase Couette flow analysis for fluid-particle suspension between moving parallel plates. To the best our knowledge, no research to date has been performed to study transient heat transfer in a moving fins. Numerous studies have been devoted to steady state analysis with the assumption that the transient response dies out quickly [26, 27].

In this article, 2D DTM is applied to study simultaneous conductive, convective and radiative heat transfer in a moving fin of rectangular profile. Thermal conductivity, heat transfer coefficient and surface emissivity are all temperature dependent. The problem formulation is presented in Section 2. A brief discussion on the fundamentals of the 2D DTM is provided in Section 3. The validation of the analytical solutions together with analytical results are given in Section 4. Lastly, we provide some discussions based on the results obtained in Section 5 and the conclusions summarized are Section 6.

Problem formulation

We consider a one dimensional rectangular moving fin of length L, with cross-sectional area A_c, thickness δ and perimeter P while it moves horizontally with a constant velocity U as depicted in Figure 1. The fin surface is exposed to a convective and radiative environment of temperature T_a and the base temperature of the fin is $T_b > T_a$. The fluid surrounding the moving fin is heated by the convective-radiative fin and induces a horizontal flow field (advection). The radiative component would be more prominent if fin undergoes natural convection or if the forced convection is rather weak or absent. The energy balance equation for the moving fin losing heat by simultaneous convection and radiation can be expressed as

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial X}\left[K(T)\frac{\partial T}{\partial X}\right] - \frac{PH(T)}{A_c}(T - T_a) - \frac{P\sigma\varepsilon(T)}{A_c}(T^4 - T_a^4) - \rho cU\frac{\partial T}{\partial X},\ 0 \leq X \leq L. \quad (1)$$

For most materials, the thermal conductivity varies linearly with temperature, that is,

$$K(T) = k_a[1 + \gamma(T - T_a)], \tag{2}$$

the surface emissivity is also assumed to vary linearly with temperature,

$$\varepsilon(T) = \varepsilon_a[1 + \eta(T - T_a)], \tag{3}$$

and heat transfer coefficient may be given by a power law function of temperature,

$$h(T) = h_b\left(\frac{T - T_a}{T_b - T_a}\right)^n. \tag{4}$$

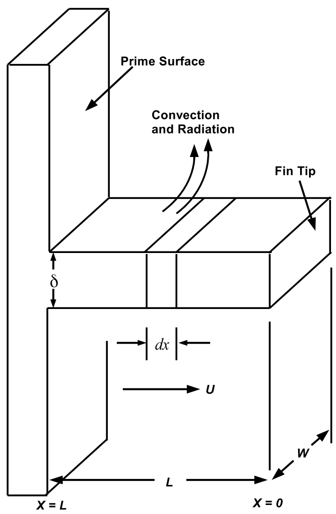

Fig. 1: Schematic representation of a rectangular moving fin.

Here, k_a is the thermal conductivity of the fin at ambient temperature, γ is a measure of thermal conductivity variation with temperature, ε_a is the surface emissivity of the fin at ambient temperature, η is a measure of surface emissivity variation with temperature, ρ is the density of fluid; c is the specific heat of fluid; ε and σ are the emissivity and Boltzman constant respectively, h_b is the heat transfer coefficient at the fin base and n is a constant. The constant n may vary between -6.6 and 5. However, in most practical applications it lies between -3 and 3 [28]. The exponent n represents laminar film boiling or condensation when $n = -1/4$, laminar natural convection when $n = 1/4$, turbulent natural convection when $n = 1/3$, nucleate boiling when $n = 2$, radiation when $n = 3$ and $n = 0$ implies a constant heat transfer coefficient. Assuming that the fin tip is adiabatic (insulated) and the base temperature is kept constant, then the boundary conditions are given by [2] et al.,

$$T(t, L) = T_b \text{ and} \tag{5}$$

$$\left.\frac{\partial T}{\partial X}\right|_{X=0} = 0. \tag{6}$$

Initially the fin is kept at the ambient temperature,

$$T(0, X) = T_a. \tag{7}$$

Introducing the following dimensionless variables,

$$\begin{gathered} x = \frac{X}{L},\ \tau = \frac{k_a t}{\rho c L^2},\ \theta = \frac{T}{T_b},\ \theta_a = \frac{T_a}{T_b},\ \zeta = \eta T_b,\ \beta = \gamma T_b, \\ N_c = \frac{P h_b T_b^n L^2}{k_a A_c (T_b - T_a)^n},\ N_r = \frac{P \varepsilon_a \sigma L^2 T_b^3}{k_a A_c},\ \alpha = \frac{k_a}{\rho c},\ \text{and } Pe = \frac{UL}{\alpha}, \end{gathered} \tag{8}$$

reduces the energy equation (1) to

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial}{\partial x}\left[\{1+\beta(\theta-\theta_a)\}\frac{\partial\theta}{\partial x}\right] - N_c(\theta-\theta_a)^{n+1} - N_r[1+\zeta(\theta-\theta_a)](\theta^4-\theta_a^4)$$
$$- Pe\frac{\partial\theta}{\partial x},\ 0\le x\le 1. \tag{9}$$

The prescribed boundary conditions are given by,

$$\theta(\tau,1) = 1 \text{ and} \tag{10}$$

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=0} = 0. \tag{11}$$

and the initial condition becomes,

$$\theta(0,x) = 0. \tag{12}$$

The dimensionless variable θ represents the temperature, θ_a represents the dimensionless ambient temperature, x is the dimensionless space variable, τ is the dimensionless time variable, β is the thermal conductivity gradient, ζ is the surface emissivity gradient, N_c is the convection-conduction parameter, N_r is the radiation-conduction parameter, Pe is the Peclet number which represent the dimensionless speed of the moving fin, α is the thermal diffusivity of the fin.

Fundamentals of the two-dimensional differential transform method

In this section, the basic idea underlying the two-dimensional DTM is briefly introduced. If function $\theta(t,x)$ is analytic and differentiated continuously with respect to time and the spatial variable x in the domain of interest, then we let

$$\Phi(\kappa,s) = \frac{1}{\kappa!s!}\left[\frac{\partial^{\kappa+s}\phi(t,x)}{\partial t^\kappa\partial x^s}\right]_{(0,0)}, \tag{13}$$

where the spectrum $\Phi(\kappa,s)$ is the transformed function, which is also called the T-function (see [29, 30]). The differential inverse transform of $\Phi(\kappa,s)$ is defined as

$$\phi(t,x) = \sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty}\Phi(\kappa,s)t^\kappa x^s, \tag{14}$$

and from equations (13) and (14) it can be concluded that

$$\phi(t,x) = \sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty}\frac{1}{\kappa!s!}\left[\frac{\partial^{\kappa+s}\phi(t,x)}{\partial t^\kappa\partial x^s}\right]_{(0,0)} t^\kappa x^s. \tag{15}$$

In real applications, the function $\phi(t,x)$ is expressed by a finite series, and equation (14) can be written as:

$$\phi(t,x) = \sum_{\kappa=0}^{m}\sum_{s=0}^{n}\Phi(\kappa,s)t^\kappa x^s, \tag{16}$$

Equation (16) implies that

$$\phi(t,x) = \sum_{\kappa=m+1}^{\infty}\sum_{s=n+1}^{\infty}\Phi(\kappa,s)t^\kappa x^s, \tag{17}$$

is negligibly small.

Table 1: Fundamental operations of the differential transform method

Original function	Transformed function
$\phi(t,x) = f_1(t,x) \pm f_2(t,x)$	$\Phi(\kappa,s) = F_1(\kappa,s) \pm F_2(\kappa,s)$
$\phi(t,x) = \alpha f(t,x)$	$\Phi(k,s) = \alpha F(\kappa,s)$
$\phi(t,x) = \frac{\partial f(t,x)}{\partial t}$	$\Phi(\kappa,s) = (\kappa+1)F(\kappa+1,s)$
$\phi(t,x) = \frac{\partial f(t,x)}{\partial x}$	$\Phi(\kappa,s) = (s+1)F(\kappa,s+1)$
$\phi(t,x) = \frac{\partial^2 f(t,x)}{\partial t^2}$	$\Phi(\kappa,s) = (\kappa+1)(\kappa+2)F(\kappa+2,s)$
$\phi(t,x) = \frac{\partial^2 f(t,x)}{\partial x^2}$	$\Phi(\kappa,s) = (s+1)(s+2)F(\kappa,s+2)$
$\phi(t,x) = t^m x^n$	$\Phi(\kappa,s) = \delta(\kappa-m)\delta(s-n)$
$\phi(t,x) = x^m e^{at}$	$\Phi(\kappa,s) = \frac{a^\kappa}{\kappa!}\delta(s-m)$

Some of the useful mathematical operations performed by the differential transform method are given in Table 1 .

The Kronecker delta function $\delta(\kappa - s)$ is given by

$$\delta(\kappa - s) = \begin{cases} 1 & \text{if } \kappa = s \\ 0 & \text{if } \kappa \neq s. \end{cases}$$

Derivation of analytical Solutions

In this section we provide an analytical solution to the problem under consideration. Using 2D-DTM to solve PDEs consists of three main steps. The steps are: transforming the PDE into algebraic equations, solving the equations, and inverting the solution of algebraic equations to obtain an infinite series solution or an approximate solution. Taking the two-dimensional differential transform of equation

(9), given $n = 2$, we obtain the following recurrence relation,

$$
\begin{aligned}
(\kappa+1)\Theta(\kappa+1,h) = {} & (1-\beta\theta_a)(h+1)(h+2)\Theta(k,h+2) \\
& + \beta\sum_{i=0}^{\kappa}\sum_{j=0}^{h}(h+1-j)(h+2-j)\Theta(k-i,j)\Theta(i,h+2-j) \\
& + \beta\sum_{i=0}^{\kappa}\sum_{j=0}^{h}(j+1)\Theta(k-i,j+1)(h+1-j)\Theta(i,h+1-j) \\
& - N_c\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\Theta(i,h-j-p)\Theta(t,j)\Theta(k-i-t,p) \\
& + 3N_c\theta_a\sum_{i=0}^{\kappa}\sum_{j=0}^{h}\Theta(i,h-j)\Theta(k-i,j) + N_c\theta_a^3 \\
& - N_r\zeta\sum_{i=0}^{\kappa}\sum_{t=0}^{\kappa-i}\sum_{z=0}^{\kappa-i-t}\sum_{q=0}^{\kappa-i-t-z}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\sum_{l=0}^{h-j-p}\sum_{w=0}^{h-j-p-l} \\
& \Theta(i,h-j-p-l-w)\Theta(t,j)\Theta(z,p)\Theta(q,l)\Theta(k-i-t-z-q,w) \\
& - N_r(1-\theta_a\zeta)\sum_{i=0}^{\kappa}\sum_{t=0}^{\kappa-i}\sum_{z=0}^{\kappa-i-t}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\sum_{l=0}^{h-j-p} \\
& \Theta(i,h-j-p-l)\Theta(t,j)\Theta(z,p)\Theta(k-i-t-z,l) \\
& + N_r(\zeta\theta_a^4\Theta(k,h) - \theta_a^4(1+\zeta\theta_a)) - Pe(h+1)\Theta(k,h+1), \qquad (18)
\end{aligned}
$$

where $\Theta(\kappa, h)$ is the differential transform of $\theta(\tau, x)$.

Taking the two-dimensional differential transform of the initial condition (12) and boundary condition (11) we obtain the following transformations respectively,

$$\Theta(0,h) = 0,\ h = 0,1,2,\ldots \qquad (19)$$

$$\Theta(\kappa,1) = 0,\ \kappa = 0,1,2,\ldots. \qquad (20)$$

We consider the other boundary condition as follows,

$$\Theta(\kappa,0) = a,\ a\in\mathbb{R},\ \kappa = 1,2,3,\ldots \qquad (21)$$

where the constant a can be determined from the boundary condition (10) at each time step after obtaining the series solution.

Substituting equations (19)-(21) into (18) we obtain the following,

$$\Theta(1,2) = \theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c), \qquad (22)$$

$$\Theta(2,2) = \frac{1}{2}\theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c)(13 - 12\theta_a\beta - 3Pe + \theta_a^4\zeta N_r - 3\theta_a^2 N_c), \qquad (23)$$

$$\Theta(1,3) = \theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c), \qquad (24)$$

$$\Theta(2,3) = \frac{1}{2}\theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c)(21 - 20\theta_a\beta - 4Pe + \theta_a^4\zeta N_r - 3\theta_a^2 N_c) \qquad (25)$$

$$\vdots$$

Substituting equations (19)-(25) into (16) we obtain the following closed form of the solution,

$$\begin{aligned}\theta(\tau,x) &= a\tau + a\tau^2 + (\theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c))\tau x^2 + (\theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c))\tau x^3 \\ &+ \frac{1}{2}\theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c)(13 - 12\theta_a\beta - 3Pe + \theta_a^4\zeta N_r - 3\theta_a^2 N_c)\tau^2 x^2 + a\tau^3 \\ &+ \frac{1}{2}\theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c)(21 - 20\theta_a\beta - 4Pe + \theta_a^4\zeta N_r - 3\theta_a^2 N_c)\tau^2 x^3 + \ldots \end{aligned} \quad (26)$$

The constant a can be determined from the boundary condition (10). To obtain the value of a, we substitute the boundary condition (10) into (26) at the point $x = 1$. Thus, we have,

$$\begin{aligned}\theta(\tau,1) &= a\tau + a\tau^2 + (\theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c))\tau + (\theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c))\tau \\ &+ \frac{1}{2}\theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c)(13 - 12\theta_a\beta - 3Pe + \theta_a^4\zeta N_r - 3\theta_a^2 N_c)\tau^2 + a\tau^3 \\ &+ \frac{1}{2}\theta_a^3(\theta_a(1+\theta_a\zeta)N_r + N_c)(21 - 20\theta_a\beta - 4Pe + \theta_a^4\zeta N_r - 3\theta_a^2 N_c)\tau^2 + \ldots = 1 \end{aligned} \quad (27)$$

We then obtain the expression for $\theta(\tau,x)$ upon substituting the obtained value of a into equation (26). Using the first 25 terms of the power series solution, we benchmark the series solutions to the numerical solution obtained through employing the inbuilt numerical solver in MATLAB (*pdepe*). As seen from the benchmark analysis in Figure 2, the series solution is in agreement with the numerical solution.

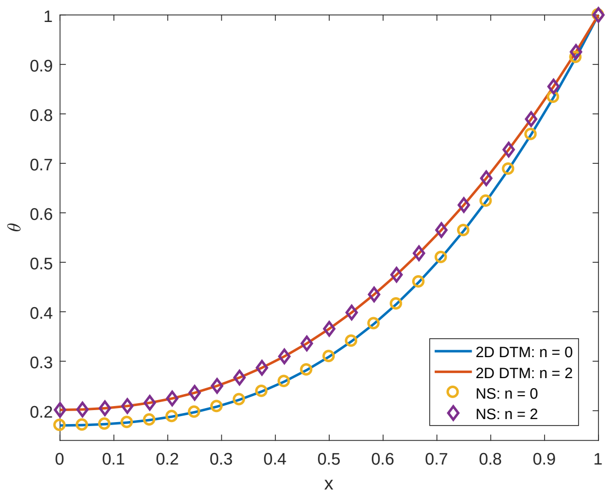

Fig. 2: Temperature distribution in a moving fin for varying n.

With the confidence obtained from the benchmark analysis, we plot the solution (26) for various parameters as shown in Figures below. For all analytical results depicted in the Figures in this article, the following values of parameters are used unless stated otherwise.

$$n = 2,\ \beta = 1, N_c = 4,\ N_r = 0.25, \theta_a = 0.10,\ Pe = 3,\ \tau = 0.30 \quad (28)$$

Results and Discussion

The results have been analyzed graphically and this is useful in terms of discussing the thermo-physical parameters presented by the solutions. This section describes the impact of the embedding parameters on the temperature distribution of a moving fin of rectangular profile. The governing equations were

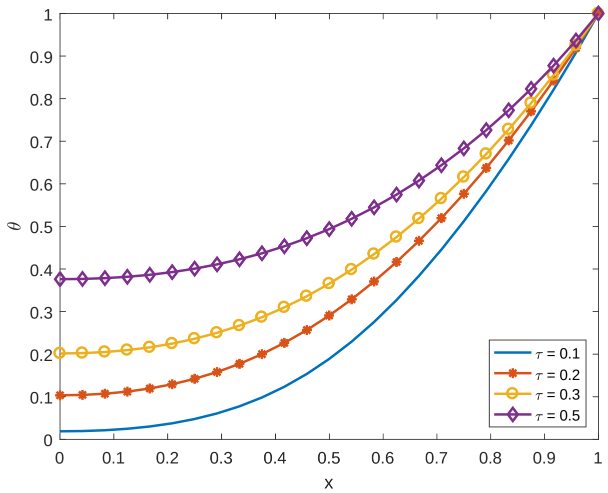

Fig. 3: Temperature distribution in a moving fin for varying time, τ.

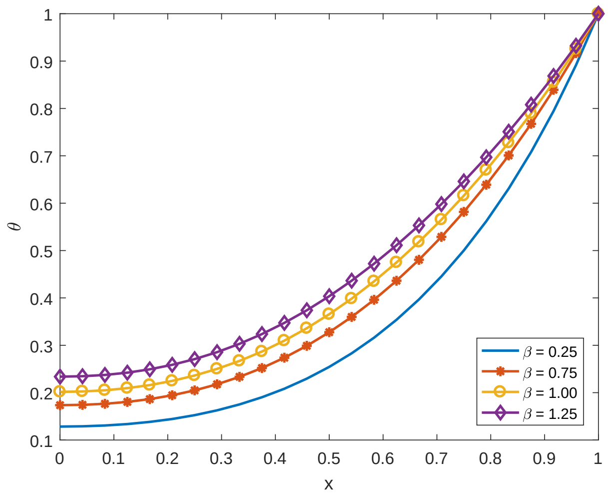

Fig. 4: Effect of the thermal conductivity gradient, β, on temperature distribution.

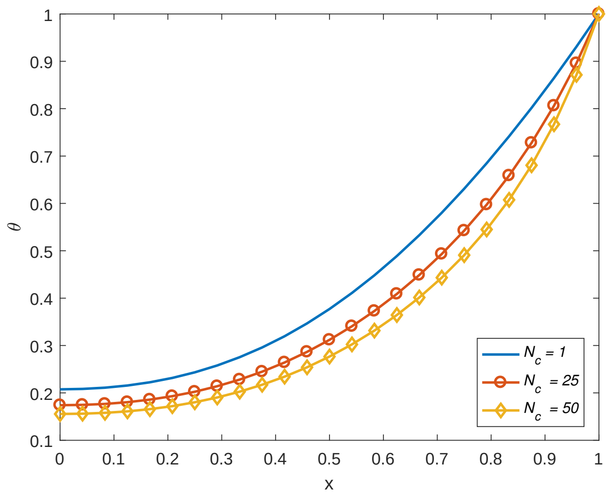

Fig. 5: Effect of the thermo-geometric parameter, N_c, on temperature distribution.

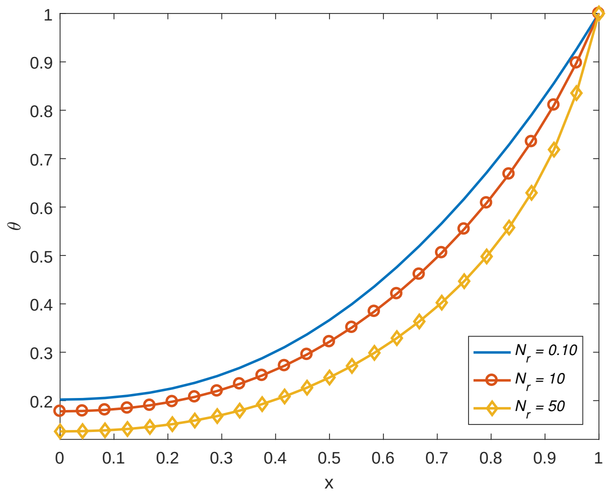

Fig. 6: Effect of the thermo-geometric parameter, N_r, on temperature distribution.

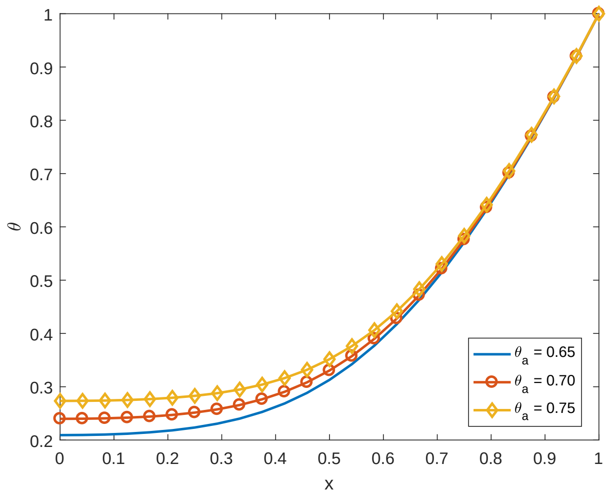

Fig. 7: Effect of the thermo-geometric parameter, θ_a, on temperature distribution.

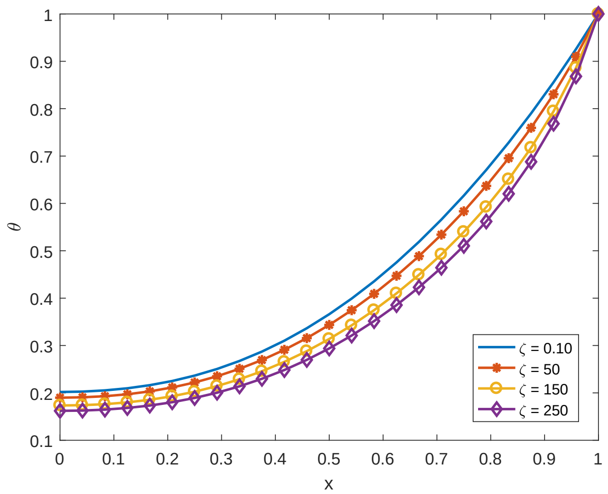

Fig. 8: Effect of the surface emissivity conductivity, ζ, on temperature distribution.

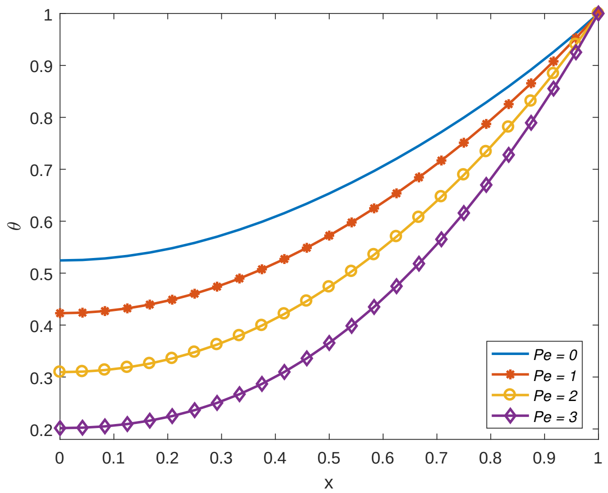

Fig. 9: Effect of varying speeds, Pe, on temperature distribution..

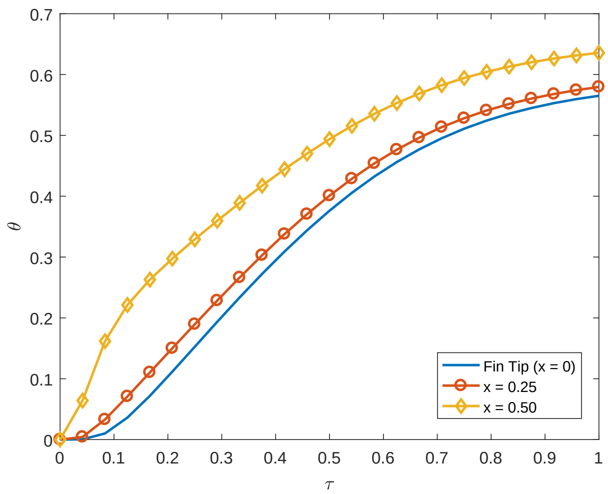

Fig. 10: Temperature distribution along the fin.

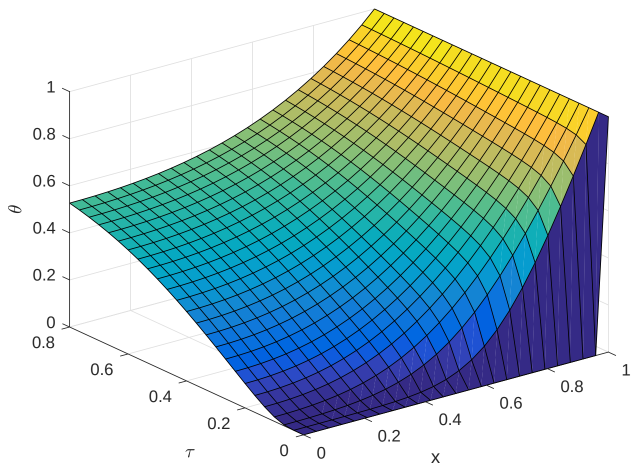

Fig. 11: Transient temperature distribution in a moving fin.

solved analytically using the 2D DTM and validated numerically using the the inbuilt numerical solver in MATLAB ($pdepe$). Figure 2 shows the effect of the nature of heat transfer on transient temperature distribution and also gives a comparison of the 2D DTM and the numerical results. It is observed that the approximate analytical solutions are in good agreement with the numerical results. We also note that the temperature distribution of the fin is an increasing function of the exponent n. Figure 3 shows that the fin temperature increases with time. It has been shown that the transient temperature profile approaches the steady state solution as time evolves [22]. Figure 4 shows the impact of the change in thermal conductivity gradient. We observe that as the values of β decreases, the temperature in the fin decreases signifying an increased heat loss to the ambient fluid. Increasing the thermal conductivity gradient enhances heat conduction from the fin base and leads to increased temperatures inside the fin.

Figure 5 illustrates the temperature distribution for varying values of the convective-conductive parameter N_c. For $N_c = 0$, this indicates the absence of convection while $N_c = 1$ indicates that convection and conduction are at equal rates [3]. An increase in the values of the thermo-geometric fin parameter leads to a decrease in temperature in the fin. A similar result is observed in Figure 6 which represents the impact of the radiation parameter N_r on temperature distributions of a moving fin. Figure 7 shows the variation in temperature distribution for varying values of the convective-radiative sink parameter θ_a. The ambient temperature plays an important role in transient heat dissipation. Higher ambient temperature lead to a reduced heat transfer rate as indicated by high values of fin temperature for higher values of the convective-radiative sink temperature. The effect of varying surface emissivity parameter ζ is depicted in Figure 8. As expected, we observe that as the surface emissivity increases, the radiative heat loss decreases resulting in lower fin temperatures.

Figure 9 illustrates the effect of the Peclet number on the temperature distribution in the fin. As Pe increases, (fin moves faster), the temperature inside the fin decreases rapidly due to increased advective effect on the surface of the fin. We also observe that for $Pe = 0$, i.e., stationery fin, the cooling of the fin takes more time as shown by higher temperatures when compared to a moving fin. Figure 10 shows the temperature variation along the fin profile. The fin temperature gradually increases as the axial parameter, x, increases and this is in line with the imposed boundary condition at the fin base. Lastly, Figure 11 we observe that the temperature distribution along the three dimensional spaces corresponds to the boundary conditions and to all the results described above.

Concluding Remarks

The 2D DTM was successfully applied to study transient heat transfer in a longitudinal moving fin subject to convective and radiative heat losses. The closed-form solution of the governing highly nonlinear equation was obtained and the embedding parameters were illustrated and explained. The accuracy of the analytical solution was validated using the numerical solution.

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2.9 Journal Article IX

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Steady state heat transfer analysis in a rectangular moving porous fin

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Abstract

In this article, the variation of temperature distribution and fin efficiency in a porous moving fin of rectangular profile is studied. This study is performed using Darcy's model to formulate the governing heat transfer differential equation. The approximate analytical solution is generated using the Variational Iteration Method (VIM). The power series solution is validated by benchmarking it against the numerical solution obtained by applying the Runge-Kutta fourth order method. A good agreement between the analytical and numerical results is observed. The effects of porosity parameter, Peclet number and other thermo-physical parameters, such surface power index of heat transfer coefficient, convective-conductive parameter, radiative-conductive parameter, thermal conductivity gradient and non-dimensional ambient temperature on non-dimensional temperature are also studied and explained. The results indicate that the fin rapidly dissipates heat to the ambient temperature with an increase in the Peclet number, convection-radiation parameters and the porosity parameter.

Keywords: Heat transfer, porous fin, Darcy's model, analytical solutions, fin tip temperature

1. Introduction

In many thermal engineering applications, fins are extensively used to improve the rate of heat dissipation from a heated surface. Traditional applications of heat transfer in porous media include heat exchangers, reactor cooling and solar collectors [1]. The concept of heat transfer through porous fins was introduced by Kiwan and Al-Nimr [2] and further improved the heat transfer model formulation through porous media by using Darcy's model [3, 4]. Following the work of Kiwan and Al-Nimr, numerous studies have been performed to understand the concept of heat transfer through prorous fins. Patel and Meher [5], performed thermal analysis of a finite-length porous fin with insulated tip and showed the effect of governing parameters on temperature distribution. It was concluded that porous fins are more efficient in dissipating heat to the ambient fluid when compared to solid fins [6, 7]. An analytical model for determination of the performance and optimization of porous fins with respect to the different models of prediction was presented by Kundu and Bhanja [8]. Magnetohydrodynamic (MHD) effect on a rectangular porous fin was investigated by Taklifi et al. [9]. These authors concluded that by imposing MHD in a system except near the fin tip, heat transfer rate from the porous fin decreases. Collocation Method (CM) and the Homotopy Perturbation Method (HPM) were presented by Hoshyar [10] to determine the temperature distribution in a porous fin with temperature-dependent internal heat generation. Their results were validated using the Runge-Kutta fourth order method. They discovered that as the buoyancy effects become stronger, the local temperature in the fin decreases. A simulation of combined conductive, convective and radiative heat transfer in moving irregular porous fins of various profiles was performed by Ma et al. [11] using Spectral Element Method (SEM). Besides, the effects of porous materials, profiles and various thermo-physical parameters on non-dimensional temperature distribution, the authors also investigated the temperature and volume adjusted fin efficiency.

The radiation effects are quite significant in various engineering and industrial processes especially in the design of reliable equipments, nuclear plants and gas turbines. Razelos and Kakatsios [12], investigated the optimum dimensions of convecting-radiating rectangular fins. They studied the influence of all dependen-

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Nomenclature

c	specific heat (J/kg · K)	H	heat transfer coefficient (W/m^2 · K)
h_b	heat transfer coefficient at the fin base (W/m^2 · K)	K	thermal conductivity (W/m · K)
k_a	thermal conductivity at ambient temperature (W/m · K)	T	temperature (K)
U	velocity of the fin (m/s)	X	space variable (m)
x	dimensionless space variable	N_c	convection-conduction parameter
N_r	radiation-conduction parameter	N_p	porosity parameter
A_c	cross-sectional area (m^2)	W	fin width (m)
L	fin length (m)	R	permeability of the porous medium
$\dot{m}$	mass flow rate of fluid (Kg/s)	g	gravitational acceleration (m/s^2)
Y	thermal expansion coefficient (K^{-1})	T_a	ambient temperature (K)
T_b	fin base temperature (K)	P	fin perimeter (m)
Pe	Peclet number	$\mathcal{L}$	linear VIM operator
$\mathcal{N}$	nonlinear VIM operator		

Greek symbols

δ_b	base fin width (m)	σ	Boltzmann constant (W/m^2 · K^4)
ε	emissivity	υ	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)	α	thermal diffusivity (m^2/s)
θ	dimensionless temperature	λ	Lagrange multiplier

t parameters on optimization and performance of rectangular fins. They also investigated the effect of the temperature-dependent thermal conductivity and emissivity on the temperature profile. Ndlovu and Moitsheki [13] performed thermal analysis of natural convection and radiation heat transfer in moving porous fins of rectangular and exponential profiles. The impact of thermo-physical parameters was investigated and it was concluded that the porous effects of the fin as well as the speed of the moving fin had a greater influence on the rate of heat dissipation from the moving fin. Following similar work, Ndlovu [14] further studied the impact of porous media in a triangular fin with convection and radiation modeled as separate terms. The author has shown that the the combined effects of porosity, convection, radiation and the velocity of the fin play a pivotal role in extended surface heat transfer. An approximate analytical solution for convection-radiation from a continuously moving fin with temperature-dependent thermal conductivity was developed by Aziz and Khani [15]. They concluded that the radiative cooling becomes more effective for stronger radiation and consequently this lowers the temperature in the fin. Moradi et al. [16], examined thermal performance of straight porous fins of triangular profile with temperature-dependent thermal conductivity in the presence of convection and radiation heat transfer. The impact of various embedding parameters was investigated and graphically analyzed. The mathematical models describing heat transfer through extended surfaces are highly nonlinear imposing difficulties in finding their exact solutions. Approximate analytical solutions and numerical solutions are widely used in literature as alternate methods for studying these kind of problems. Cuce and Cuce [17], studied the the efficiency and effectiveness assessment of longitudinal porous fins using Homotopy Perturbation Method (HPM). A Modified Homotopy Analysis Method (HAM) was proposed by Motsa et al. [18]. The newly developed Spectral Homotopy Analysis Method (SHAM) seeks to remove some restrictive assumptions related to the implementation of standard HAM. The Optimal Homotopy Analysis Method (OHAM) was first introduced by Esmaeilpour and Ganji [19] to solve Jeffery-Hamel problem. Mosayebidorcheh et al. [20], studied transient thermal behavior of radial fins of rectangular, triangular and hyperbolic profiles by applying the hybrid method of Differential Transform Method and Finite Difference Method (DTM-FDM). Multiple variable thermal properties were studied for a triangular moving fin using a modified decomposition solution by Kanti Roy et al. [21]. The VIM was applied to study the temperature distribution is fins of various profiles by Ndlovu and

Moitsheki [22]. The solutions were validated using the exact solutions obtained by employing symmetry analysis. The VIM was introduced by He in 1999 [23, 24] and has been widely used by many researchers.

As far as we know, a mathematical model describing steady heat transfer in a rectangular porous moving fin experiencing simultaneous convective and radiative heat losses has not been applied in literature. There are few applications of the VIM to solve these kinds of problems and here, it is shown that the VIM can produce desirable solutions to heat transfer problems. In this paper, the VIM is used to solve the nonlinear boundary value problem describing heat transfer through a porous fin. The mathematical background of the problem under consideration is described in § 2. A brief discussion on the fundamentals of the VIM will be provided in § 3. Analytical solutions are provided in § 4 and § 5. Lastly, some discussions based on the results obtained are provided in § 6 and concluding remarks in § 7.

2. Fin problem formulation

Consider a one dimensional rectangular moving porous fin of length L, with cross-sectional area A_c and perimeter P while it moves horizontally with a constant velocity U as depicted in Figure 1. The fin surface is exposed to a convective and radiative environment of temperature T_a and the base temperature of the fin is T_b, $(T_b > T_a)$. The fluid surrounding the moving fin is heated by the convective-radiative fin and induces a horizontal flow field (advection). The radiative component would be more prominent if fin undergoes natural convection or if the forced convection is rather weak or absent. The porous medium is assumed to be isotropic, homogeneous and saturated with a single-phase fluid. It is assumed that the initial transient heat transfer, which occurs when the material is exposed to the ambient temperature, dies out quickly and the heat transfer takes place under steady state conditions. This assumption has been made in several studies [25, 26]. The fluid and solid matrix are kept at local thermal equilibrium. The surface of the moving fin is assumed to diffuse with a constant emissivity ε and fluid-solid interaction in the porous medium is simulated by Darcy's model. The energy balance equation for the moving porous fin losing heat by simultaneous convection and radiation can be expressed as

$$\frac{d}{dX}\left[K(T)\frac{dT}{dX}\right] - \frac{PH(T)}{A_c}(T - T_a) - \frac{\dot{m}c}{A_c}(T - T_a) - \frac{\varepsilon\sigma P}{A_c}(T^4 - T_a^4) - \rho c U\frac{dT}{dX} = 0,\ 0 \le X \le L. \quad (2.1)$$

The mass flow rate of the fluid passing through the porous media is given by [7]:

$$\dot{m} = \rho v_w W \Delta X \quad (2.2)$$

The velocity of the the fluid passing through the porous media is given by Darcys model [2],

$$v_w = \frac{gRY}{\upsilon}(T - T_a). \quad (2.3)$$

For most materials, the thermal conductivity varies linearly with temperature [27], that is,

$$K(T) = k_a[1 + \gamma(T - T_a)], \quad (2.4)$$

and heat transfer coefficient may be given by a power law function of temperature [28],

$$H(T) = h_b\left(\frac{T - T_a}{T_b - T_a}\right)^n. \quad (2.5)$$

Here, k_a is the thermal conductivity of the fin at ambient temperature, γ is a measure of thermal conductivity variation with temperature, h_b is the heat transfer coefficient at the fin base and n is a constant. The constant n may vary between -6.6 and 5. However, in most practical applications it lies between -3 and 3 [29]. The exponent n represents laminar film boiling or condensation when $n = -1/4$, laminar natural convection when

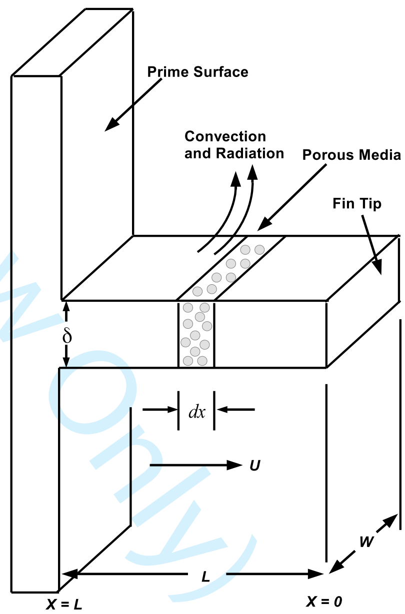

Figure 1: Schematic representation of a rectangular moving porous fin.

$n = 1/4$, turbulent natural convection when $n = 1/3$, nucleate boiling when $n = 2$, radiation when $n = 3$ and $n = 0$ implies a constant heat transfer coefficient. The third term on equation (2.1) represents the fluid-solid interaction in the porous medium (see e.g. [5, 30]). Here, g is the gravitational acceleration, R is the permeability, υ and Y are the kinematic viscosity and thermal expansion coefficient respectively, ρ is the density of fluid; c is the specific heat of fluid; ε and σ are the emissivity and Boltzmann constant respectively. Assuming that the fin tip is adiabatic (insulated) and the base temperature is kept constant, then the boundary conditions are given by [31] (see also [20, 27]

$$T(L) = T_b \text{ and } \left.\frac{dT}{dX}\right|_{X=0} = 0. \tag{2.6}$$

Introducing the following dimensionless variables [13, 14],

$$\begin{gathered} x = \frac{X}{L},\ \theta = \frac{T}{T_b},\ \theta_a = \frac{T_a}{T_b}, k = \frac{K}{k_a},\ \beta = \gamma T_b, \\ h = \frac{H}{h_b},\ N_r = \frac{\varepsilon\sigma P L^2 T_b^3}{k_c A_c},\ N_c = \frac{P h_b T_b^n L^2}{k_a A_c (T_b - T_a)^n}, \\ N_p = \frac{\rho c g K A W L^2 T_b}{\upsilon k_a A_c},\ \alpha = \frac{k_a}{\rho c} \text{ and } Pe = \frac{UL}{\alpha}, \end{gathered} \tag{2.7}$$

reduces the energy equation (2.1) to

$$\begin{aligned} &\frac{d}{dx}\left[\{1 + \beta(\theta - \theta_a)\}\frac{d\theta}{dx}\right] - N_c(\theta - \theta_a)^{n+1} - N_p(\theta - \theta_a)^2 \\ &- N_r(\theta^4 - \theta_a^4) - Pe\frac{d\theta}{dx} = 0,\ 0 \le x \le L, \end{aligned} \tag{2.8}$$

The prescribed boundary conditions are given by,

$$\theta(1) = 1 \text{ and,} \tag{2.9}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = 0. \tag{2.10}$$

The dimensionless variable θ represents the temperature, θ_a represents the dimensionless ambient temperature, x is the dimensionless space variable, β is the thermal conductivity gradient, N_c is the convection-conduction parameter, N_r is the radiation-conduction parameter, N_p is the porosity parameter, Pe is the Peclet number which represent the dimensionless speed of the moving fin, α is the thermal diffusivity of the fin.

3. He's Variational Iteration Method

To illustrate the basic idea of the method, consider the following nonlinear equation:

$$\mathcal{L}\theta + \mathcal{N}\theta = g(x), \tag{3.1}$$

where $\mathcal{L}$ and $\mathcal{N}$ are linear and nonlinear operators, respectively, and $g(x)$ is the source inhomogeneous term [32]. In his papers, [23, 24], He proposed the variational iteration method where a correctional functional for equation (3.1) can be written as

$$\begin{aligned} \theta_{m+1}(x) = \theta_m(x) + \int_0^x &\lambda(t)(\mathcal{L}\theta_m(t) \\ &+ \mathcal{N}\tilde{\theta}_m(t) - g(t))dt, \end{aligned} \tag{3.2}$$

where λ is the general Lagrange multiplier, which can be be identified optimally via the variation theory, and $\tilde{\theta}_m$is a restricted variation, which means $\delta\tilde{\theta}_m = 0$ [33]. The Lagrange multiplier can be a constant or a function depending on the order of the deferential equation under consideration. In general, for the m^{th} order ordinary differential equation, the Lagrange multiplier is given by,

$$\lambda = \frac{(-1)^m}{(m-1)!}(t - x)^{m-1}. \tag{3.3}$$

Equation (3.2) can be re-written as follows:

$$\begin{aligned} \theta_{m+1}(x) = \theta_m(x) + \int_0^x &\frac{(-1)^m}{(m-1)!}(t - x)^{m-1}(\mathcal{L}\theta_m(t) \\ &+ \mathcal{N}\theta_m(t) - g(t))dt, \end{aligned} \tag{3.4}$$

The iteration formula equation (3.4), without restricted variation, should be used for the determination of the successive approximations $\theta_{m+1}(x),\ m \geqslant 0$, of the solution $\theta(x)$. Consequently, the solution is given by

$$\theta(x) = \lim_{m\to\infty} \theta_m(x). \tag{3.5}$$

4. The Fin Temperature Distribution

As a starting approximation for the VIM solution to equation (2.8), $\theta_0(x)$ is assumed to be a constant [34, 35]. The first two iterations using the VIM Solution are given below,

$$\theta_0(x) = a, \tag{4.1}$$

$$\begin{aligned} \theta_1(x) = a - &\frac{1}{2}(-(\theta_a - a)^2 N_p \\ &+ (\theta_a^4 - a^4)N_r + (\theta_a - a)^3 N_c)x^2, \end{aligned} \tag{4.2}$$

$$\begin{aligned} \theta_2(x) = a - &\frac{1}{2}(\theta_a - a)(1 + \beta\theta_a - a\beta)(-\theta_a N_p \\ &+ aN_p + \theta_a^3 N_r + \theta_a^2 a N_r + \theta_a a^2 N_r + a^3 N_r \\ &+ (\theta_a - a)^2 N_c)x^2 + \frac{1}{6}(a - \theta_a)Pe(\theta_a^3 N_r \\ &+ \theta_a^2(aN_r + N_c) + a(N_p + a(aN_r + N_c)) \\ &- \theta_a(N_p + a(-aN_r + 2N_c)))x^3 + \cdots \end{aligned} \tag{4.3}$$

$$\vdots$$

The fifth iteration of the VIM gives higher-order powers of the independent variable. A series solution of order $O(14)$ was found to be close to the exact solution with a very small standard error using the differential transformation and variational iteration [22]. An approximate expression for temperature distribution for the problem under consideration is then given by

$$\theta(x) \cong \theta_5(x). \tag{4.4}$$

The approximate solutions (4.1)-(4.3) all satisfy the derivative boundary condition assigned at the fin tip. To obtain the value of the constant a, substitute the boundary condition (2.9) into (4.4) at the point $x = 1$. Similarly, the recurrence relation of VIM Solution to (2.8) is given below

$$\begin{aligned}\theta(x) = a - \frac{1}{2}(\theta_a - a)(1 + \beta\theta_a - a\beta)(-\theta_a N_p + aN_p \\ + \theta_a^3 N_r + \theta_a^2 a N_r + \theta_a a^2 N_r + a^3 N_r + (\theta_a - a)^2 N_c)x^2 \\ + \frac{1}{6}(a - \theta_a)Pe(\theta_a^3 N_r + \theta_a^2(aN_r + N_c) + a(N_p \\ + a(aN_r + N_c)) - \theta_a(N_p + a(-aN_r + 2N_c)))x^3 \\ + \cdots\end{aligned} \tag{4.5}$$

A benchmark analysis of the VIM solutions and the the numerical solution (Runge-Kutta fourth order method) is depicted in Figure 2. It is observed that the VIM solutions are closely aligned to numerical solutions within the first five iterations.

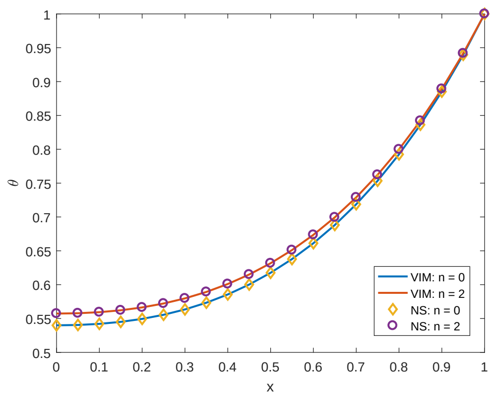

Figure 2: Temperature distribution along the fin for varying n.

For all analytical results depicted in the Figures in this article, the following values of parameters are used unless stated otherwise,

$$\begin{aligned}\beta = 0.25,\ N_c = 0.25,\ N_p = 0.25, \\ n = 2,\ N_r = 4,\ \theta_a = 0.25,\ Pe = 3.\end{aligned} \tag{4.6}$$

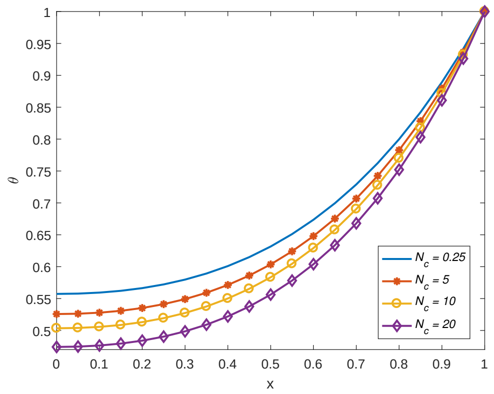

Figure 3: Temperature distribution along the fin for varying N_c.

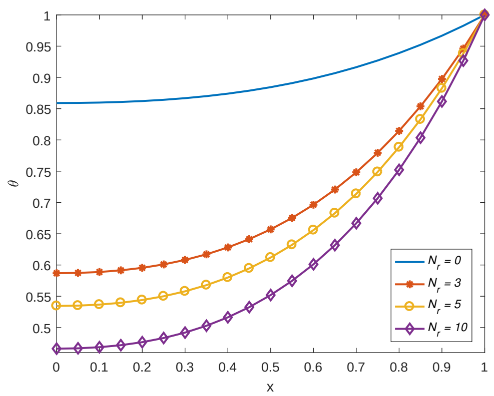

Figure 4: Temperature distribution along the fin for varying N_r.

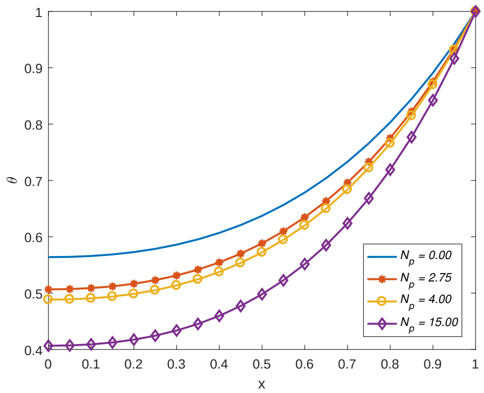

Figure 5: Temperature distribution along the fin for varying N_p.

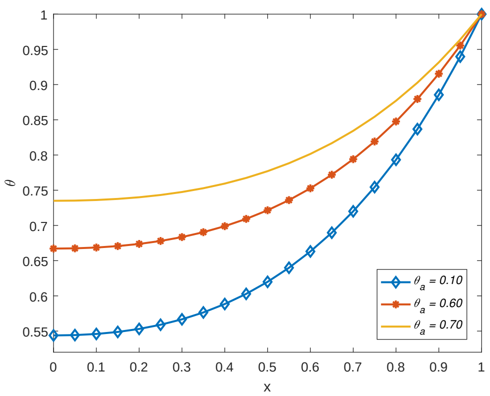

Figure 6: Temperature distribution along the fin for varying θ_a.

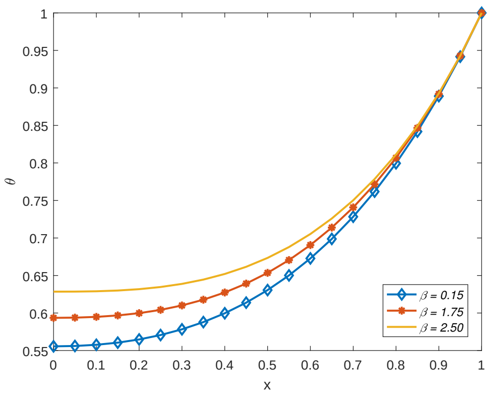

Figure 7: Temperature distribution along the fin for varying β.

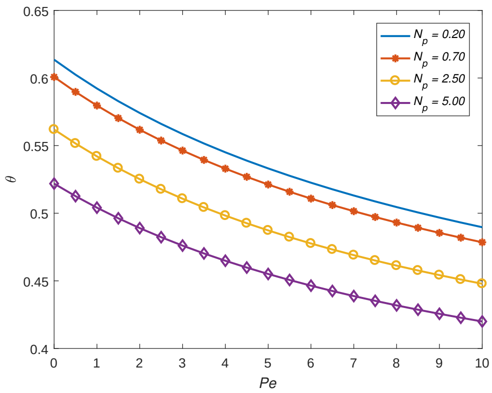

Figure 8: Temperature distribution at the fin tip.

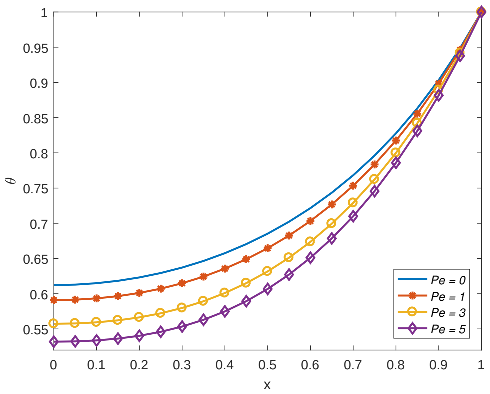

Figure 9: Temperature distribution along the fin for varying Pe.

5. Fin Efficiency

Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin was subjected to the base temperature (see e.g. [36, 37]). The total heat transfer rate is given by,

$$Q = \int_0^L \Big[PH(T)\big(T - T_a\big) + \dot{m}\big(T - T_a\big) + \varepsilon\sigma P\big(T^4 - T_a^4\big) + \rho c U\big(T - T_a\big)\Big]dX, \qquad (5.1)$$

The ideal heat transfer rate obtained if the entire fin is kept at the base temperature is given by,

$$Q_{ideal} = PLH(T)\big(T - T_a\big) + \dot{m}L\big(T - T_a\big) + \varepsilon\sigma PL\big(T^4 - T_a^4\big) + \rho c UL\big(T - T_a\big). \qquad (5.2)$$

In dimensionless variables, Q and Q_{ideal} are given as follows,

$$Q = \int_0^1 N_c\left(\theta - \theta_a\right)^{n+1} dx + \int_0^1 N_p\left(\theta - \theta_a\right)^2 dx + \int_0^1 N_r\left(\theta^4 - \theta_a^4\right)dx + \int_0^1 Pe\left(1 - \theta_a\right)dx, \quad (5.3)$$

$$Q_{ideal} = N_c\left(1 - \theta_a\right)^{n+1} + N_p\left(1 - \theta_a\right)^2 + N_r\left(1 - \theta_a^4\right) + Pe\left(1 - \theta_a\right), \quad (5.4)$$

and the fin efficiency is calculated as follows,

$$\eta = \frac{Q}{Q_{ideal}}. \quad (5.5)$$

The fin efficiency is depicted in Figure 10 and Figure 11.

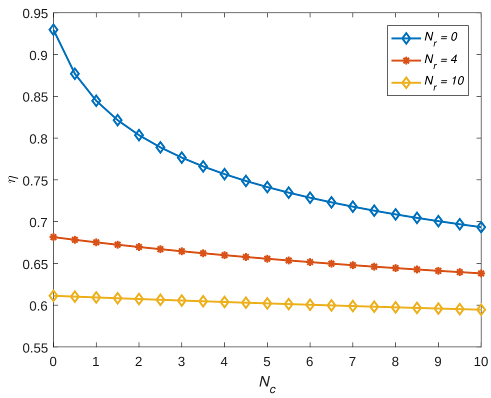

Figure 10: Fin efficiency of a longitudinal rectangular porous fin for varying N_r.

6. Discussion

In this article, a study of heat transfer through a rectangular moving porous fin is performed using an approximate analytical solution generated by VIM. The effect of thermo-physical parameters on temperature distribution are investigated. Figure 2 shows the effect of the nature of heat transfer on temperature distribution and also gives a comparison of the analytical solutions to numerical results. It is observed that the approximate analytical solutions are in good agreement with the numerical results. The average and maximum percentage differences between the numerical and approximate solutions are 0.0034% and 0.0008% respectively. Figure 2 further shows that the temperature distribution of the fin is an increasing function of exponent

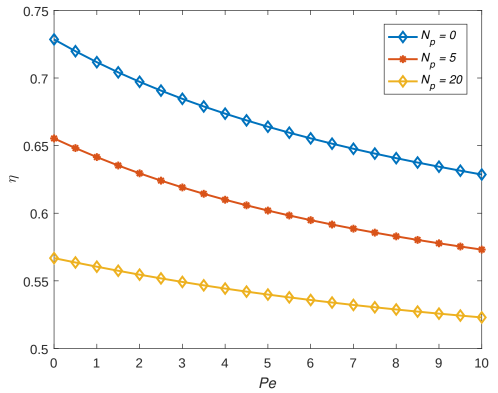

Figure 11: Fin efficiency of a longitudinal rectangular porous fin for varying N_p.

n. Figure 3 shows the temperature distribution for varying values of the convective-conductive parameter N_c. For $N_c = 0$, this indicates the absence of convection while $N_c = 1$ indicates that convection and conduction are at equal rates [21]. An increase in the values of the thermo-geometric fin parameter leads to a decrease in temperature in the fin. A similar result is observed in Figure 4 which represents the impact of the radiation parameter N_r on temperature distributions in a moving porous fin. Figure 5 depicts the results of varying the porous media parameter N_p. It is observed that the temperature decreases with increasing values of the porous parameter. The high rate of heat transfer is a result of higher porosity due to changes in permeability included in N_p. Furthermore, this can be caused by the effect of a larger porous medium area. Figure 5 further shows a comparison on the rate of heat transfer between a porous and non-porous fin, where $N_p = 0.00$ represents a non-porous fin (solid fin). As expected, higher fin temperatures are observed on a non-porous fin when compared to a porous fin. Figure 6 shows the variation in temperature distribution for varying values of the convective-radiative sink parameter θ_a. Higher ambient temperature lead to a reduced heat transfer rate as indicated by high values of fin temperature for higher values of the convective-radiative sink temperature. Figure 7 the impact of the change in thermal conductivity gradient. It is observed that as the values of β decreases, the temperature in the fin decreases signifying an increased heat loss to the ambient fluid. Increasing the thermal conductivity gradient enhances heat conduction from the fin base and leads to increased temperatures inside the fin.

The variation of temperature distribution at the fin tip is depicted in Figure 8. It is observed that the fin tip temperature decreases with an increase in the values of the porosity parameter and the speed of the moving fin. Figure 9 illustrates the effect of the Peclet number on the temperature distribution in the fin. As Pe increases, (fin moves faster), the temperature inside the fin decreases rapidly due to increased advective effect on the surface of the fin. It is also observed that for $Pe = 0$, which represents a stationery fin, the cooling of the fin takes more time as shown by higher temperatures when compared to a moving fin. The fin efficiency is shown in Figure 10 and Figure 11. Figure 10 shows that the fin becomes more effective with an increase in the values of the convective-radiative fin parameters. Figure 11 further shows that the fin temperature gradually diminishes with an increase in the speed of the moving fin.

7. Concluding remarks

In this study, the temperature distribution along a rectangular porous fin have been successfully investigated. The resulting differential equations were solved using He's VIM and the analytical solutions were benchmarked against the numerical solutions. The impact of the thermo-physical parameters was illustrated and explained. It was also shown that analytical solutions are important in describing the physical properties of heat transfer equation as they give an analytical closed form solution.

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2.10 Journal Article X

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The Significance of Fin Profile and Convective-Radiative Fin Tip on Temperature Distribution in a Longitudinal Fin

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Abstract. In this article, the one dimensional nonlinear transient heat transfer through fins of rectangular, convex parabolic and concave parabolic is studied using the two dimensional Differential Transform Method (2D DTM). The thermal conductivity and heat transfer coefficient are modeled as linear and power law functions of temperature respectively. The fin tip dissipate heat to the ambient temperature by convection and radiation. A comparison is made between the proposed convective-radiative fin tip boundary condition and the adiabatic (insulated) fin tip boundary condition which is widely used in literature. It is found that the fin with a convective-radiative tip dissipates heat to the ambient fluid at a faster rate when compared to a fin with an insulated tip. The results further show that the longitudinal fins of parabolic profiles dissipate more heat when compared to the conventional rectangular fin profile. The accuracy of the analytical method is demonstrated by comparing its results with those generated by an inbuilt numerical solver in MATLAB. Furthermore, a wide range of thermo-physical parameters are studied and their impact on the temperature distribution are illustrated and explained.

Introduction

The transient response of fins (or extended surfaces) plays a pivotal role in a wide range of applications including heat exchangers, air cooled turbine blades and the cooling of circuit boards. Besides these traditional applications, fins have also proven to be effective in heat-rejection systems in space vehicles and in the cooling of electronic components [1]. Fins enhance heat dissipation by increasing the surface area for heat transfer between a heated surface and a cooler ambient fluid. A vast amount of work related to heat transfer through extended surfaces has been devoted to steady state analysis with the assumption that the transient response dies out quickly [2, 3]. In many other engineering applications, steady state models fail [4] and as such transient heat transfer is studied. The transient state heat transfer is modeled by partial differential equations (PDEs).

There are numerous publications that have appeared recently in the literature investigating various properties of finned heat transfer. Mueller Jr and Abu-Mulaweh [5] experimentally determined the temperature distribution along a fin cooled by natural convection and radiation. They showed that the heat loss due to radiation is typically 15-20% of the total heat loss. As such, the radiation heat losses needs to be incorporated when modeling heat transfer through fins. Malekzadeh et al. [6, 7] presented the Differential Quadrature Element Method (DQEM) to maximize the rate of heat transfer for any volume of the fin under the effects of convection and radiation. Razelos and Kakatsios [8] investigated the optimum dimensions of convective and radiative rectangular fins. They studied the influence of all dependent parameters on optimization and performance of rectangular fins. They also investigated the effect of the temperature-dependent thermal conductivity and emissivity on the temperature profile. An approximate analytical solution for convection and radiation from a continuously moving fin with temperature-dependent thermal conductivity was developed by Aziz and Khani [9]. Torabi and Zhang [10] investigated thermal performance of a convective-radiative straight fin with the consideration of different profiles containing nonlinearities and further studied the fin efficiency. Sun et al. [11] studied the different temperature-dependent thermal properties for predication of heat transfer in a convective-radiative fin. Kundu and Wongwises [12] presented a decomposition analysis of a convective-radiating fin with one side of the primary surface being heated by a fluid with high temperature which they found necessary for rapid heat dissipation. The effect of variable thermal-conductivity and other thermo-physical parameters on the rate of heat transfer was studied by Animasaun et al. [13-17].

Most of the published work in extended surface heat transfer problems is related to heat transfer through stationery fins. The study of heat transfer in moving fins has aroused in recent years and has drawn the attention of many researchers. Recent research demonstrates that moving fins of different profiles [18] are significant as well as when they are exposed to a heat source [19]. Torabi et al [20] derived approximate explicit analytical expressions for steady state temperature distribution in a moving fin experiencing simultaneous convective-radiative surface heat loss. Thermal analysis of a moving porous fin in natural convection environment was studied analytically by Moradi et al [21]. Turkyilmazoglu [22] constructed exact solutions for heat transfer through exponential fins in movement and under the influence of heat generation (absorption).

The DTM is a semi-numerical-analytical method based on Taylor series expansion and was first introduced by Zhou [25] in 1986. Zhou used the DTM to solve explicitly the linear and nonlinear initial value problems that arise in electrical circuits. Chen and Ho [26] developed the 2D DTM which can be used for solving various kinds of differential equations. At higher orders, the DTM differs from the Taylor series method in the way the coefficients are computed. The Taylor series method requires computing coefficients using the initial data and the differential equation which requires more computational work while the DTM iteratively obtains the Taylor series equations [27]. Ayaz [28] showed that the DTM is better equipped to solve highly nonlinear problems than the Taylor series method. The DTM has been used to solve various problems in applied mathematics and physics such as systems of differential equations [29]. In this article, the mathematical background of the problem under consideration is described in Section 2. A brief discussion on the fundamentals of the 2D DTM is provided in Section 3. The analytical solutions and graphical representations of the impact of thermo-physical parameters is provided in Section 4. Lastly, some discussions based on the results obtained are provided in Section 5 and concluding remarks are provided in Section 6.

Problem Formulation

Consider a longitudinal one dimensional fin of length L, with cross-sectional area A_c and base thickness of δ as depicted in Figure 1. The fin surface is exposed to a convective and radiative environment of temperature T_a, the base temperature is given by T_b where $T_b > T_a$ and its profile is denoted by $F(X)$. The role of radiation component could be more prominent if the forced convection is weak or absent or when only natural convection occurs. The energy balance equation for the fin losing heat by simultaneous convection and radiation can be expressed as (see e.g. [23])

$$\rho c\frac{\partial T}{\partial t} = \frac{\delta}{2}\frac{\partial}{\partial X}\left[F(X)K(T)\frac{\partial T}{\partial X}\right] - \frac{PH(T)}{A_c}(T - T_a) - \frac{P\varepsilon\sigma}{A_c}(T^4 - T_a^4),\ 0 \leq X \leq L, \quad (1)$$

where K is the non-uniform thermal conductivity depending on the temperature (see [23, 24]), P is the periphery of the fin, ε is the emissivity, σ is the Stefan-Boltzmann constant, ρ is the density of the material, c is the specific heat capacity, T is the temperature distribution, t is the time and X is the space variable. For most materials, the thermal conductivity varies linearly with temperature, that is,

$$K(T) = k_a\big[1 + \gamma(T - T_a)\big], \quad (2)$$

and heat transfer coefficient may be given by a power law function of temperature,

$$H(T) = h_b\left(\frac{T - T_a}{T_b - T_a}\right)^n \quad (3)$$

Here, k_a is the thermal conductivity of the fin at ambient temperature, γ is a measure of thermal conductivity variation with temperature, h_b is the heat transfer coefficient at the fin base, n is a constant. The constant n may vary between -6.6 and 5. However, in most practical applications it lies between -3 and 3 [30]. The exponent n represents laminar film boiling or condensation when $n = -1/4$, laminar natural convection when $n = 1/4$, turbulent natural convection when $n = 1/3$, nucleate boiling when $n = 2$, radiation when $n = 3$ and $n = 0$ implies a constant heat transfer coefficient. The adiabatic (insulated) fin tip boundary condition ($\partial T(t,0)/\partial X = 0$) that is widely used in literature is relaxed in this study and a convective-radiative fin tip boundary condition is applied. The boundary conditions are then given by,

$$T(t,L) = T_b \text{ and} \tag{4}$$

$$-K(T)A_c\frac{\partial T}{\partial X}\bigg|_{X=0} = H(T)A_c\big(T(t,0)-T_a\big) + \varepsilon\sigma A_c\big(T^4(t,0)-T_a^4\big). \tag{5}$$

Initially the fin is kept at the ambient temperature [4],

$$T(0,X) = T_a. \tag{6}$$

Introducing the following dimensionless variables,

$$\begin{gathered} x=\frac{X}{L},\ \tau=\frac{k_a t}{\rho c L^2},\ \theta=\frac{T}{T_b},\ \theta_a=\frac{T_a}{T_b},\ f(x)=\frac{\delta_b}{2}F(X),\ \beta=\gamma T_b,\ k=\frac{K}{k_a},\ h=\frac{H}{h_b}, \\ N_c=\frac{P h_b T_b^n L^2}{k_a A_c\big(T_b-T_a\big)^n},\ N_r=\frac{\varepsilon\sigma L^2 T_b^3}{k_c A_c},\ Bi_c=\frac{h_b T_b^n L}{k_a\big(T_b-T_a\big)^n} \text{ and } Bi_r=\frac{\varepsilon\sigma T_b^3 L}{k_c} \end{gathered} \tag{7}$$

reduces equation (1) to

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial}{\partial x}\left[f(x)\big\{1+\beta\big(\theta-\theta_a\big)\big\}\frac{\partial\theta}{\partial x}\right] - N_c\big(\theta-\theta_a\big)^{n+1} - N_r\big(\theta^4-\theta_a^4\big),\ 0\le x\le 1. \tag{8}$$

The prescribed boundary conditions are given by,

$$\theta(\tau,1) = 1 \text{ and} \tag{9}$$

$$\big\{1+\beta\big(\theta(\tau,0)-\theta_a\big)\big\}\frac{\partial\theta}{\partial x}\bigg|_{x=0} - Bi_c\big(\theta(\tau,0)-\theta_a\big)^{n+1} - Bi_r\big(\theta^4(\tau,0)-\theta_a^4\big) = 0. \tag{10}$$

and the initial condition becomes,

$$\theta(0,x) = 0. \tag{11}$$

The dimensionless variable θ represents the dimensionless temperature, x is the dimensionless space variable, τ is the dimensionless time variable, β is the thermal conductivity gradient, $f(x)$ is the dimensionless fin profile, N_c is the convection-conduction parameter, N_r is the radiation-conduction parameter, Bi_c is the convection-conduction parameter at the fin tip, Bi_r is the radiation-conduction parameter at the fin tip.

Fundamentals of the Two-Dimensional Differential Transform Method

In this section, the basic idea underlying the two-dimensional DTM is briefly introduced. If function $\theta(t,x)$ is analytic and differentiated continuously with respect to time and the spatial variable x in the domain of interest, then let

$$\Phi(\kappa,s) = \frac{1}{\kappa!s!}\left[\frac{\partial^{\kappa+s}\phi(t,x)}{\partial t^\kappa\partial x^s}\right]_{(0,0)}, \tag{12}$$

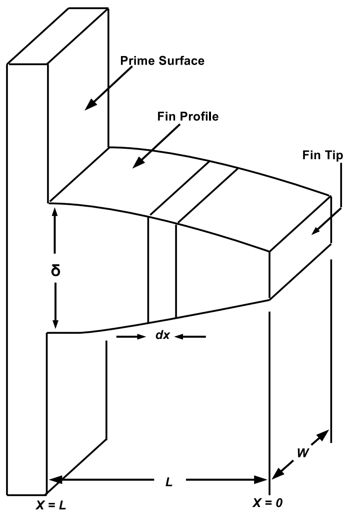

Fig. 1: Schematic representation of a longitudinal fin of an unspecified profile.

where the spectrum $\Phi(\kappa, s)$ is the transformed function, which is also called the T-function (see [27, 28]). The differential inverse transform of $\Phi(\kappa, s)$ is defined as

$$\phi(t,x) = \sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty} \Phi(\kappa, s) t^{\kappa} x^{s}, \tag{13}$$

and from equations (12) and (13) it can be concluded that

$$\phi(t,x) = \sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty} \frac{1}{\kappa! s!}\left[\frac{\partial^{\kappa+s}\phi(t,x)}{\partial t^{\kappa}\partial x^{s}}\right]_{(0,0)} t^{\kappa} x^{s}. \tag{14}$$

In real applications, the function $\phi(t,x)$ is expressed by a finite series, and equation (13) can be written as:

$$\phi(t,x) = \sum_{\kappa=0}^{m}\sum_{s=0}^{n} \Phi(\kappa, s) t^{\kappa} x^{s}, \tag{15}$$

Equation (15) implies that

$$\phi(t,x) = \sum_{\kappa=m+1}^{\infty}\sum_{s=n+1}^{\infty} \Phi(\kappa, s) t^{\kappa} x^{s}, \tag{16}$$

is negligibly small.

Some of the useful mathematical operations performed by the differential transform method are given in Table 1 .

The Kronecker delta function $\delta(\kappa - s)$ is given by

$$\delta(\kappa - s) = \begin{cases} 1 & \text{if } \kappa = s \\ 0 & \text{if } \kappa \neq s. \end{cases}$$

Derivation of Analytical Solutions

In this section, the analytical solutions for the problem described in Section are generated. Using 2D DTM to solve PDEs consists of three main steps. The steps are; transforming the PDE into algebraic equations, solving the equations, and inverting the solution of algebraic equations to obtain an infinite

Table 1: Fundamental operations of the differential transform method.

Original function	Transformed function
$\phi(t,x) = y_1(t,x) \pm y_2(t,x)$	$\Phi(\kappa,s) = Y_1(\kappa,s) \pm Y_2(\kappa,s)$
$\phi(t,x) = \alpha y(t,x)$	$\Phi(k,s) = \alpha Y(\kappa,s)$
$\phi(t,x) = \frac{\partial y(t,x)}{\partial t}$	$\Phi(\kappa,s) = (\kappa+1)Y(\kappa+1,s)$
$\phi(t,x) = \frac{\partial y(t,x)}{\partial x}$	$\Phi(\kappa,s) = (s+1)Y(\kappa,s+1)$
$\phi(t,x) = \frac{\partial^2 y(t,x)}{\partial t^2}$	$\Phi(\kappa,s) = (\kappa+1)(\kappa+2)Y(\kappa+2,s)$
$\phi(t,x) = \frac{\partial^2 y(t,x)}{\partial x^2}$	$\Phi(\kappa,s) = (s+1)(s+2)Y(\kappa,s+2)$
$\phi(t,x) = t^m x^n$	$\Phi(\kappa,s) = \delta(\kappa-m)\delta(s-n)$
$\phi(t,x) = x^m e^{at}$	$\Phi(\kappa,s) = \frac{a^\kappa}{\kappa!}\delta(s-m)$

series solution or an approximate solution. The fin profile is rectangular when $f(x) = 1$, convex-parabolic when $f(x) = \sqrt{x}$ and concave-parabolic when $f(x) = x^2$. Taking the two-dimensional differential transform of equation (8) with $f(x) = 1$, the following recurrence relation is obtained

$$
\begin{aligned}
(\kappa+1)\Theta(\kappa+1,h) = {} & (1-\beta\theta_a)(h+1)(h+2)\Theta(\kappa,h+2) \\
& + \beta\sum_{i=0}^{\kappa}\sum_{j=0}^{h}\Theta(\kappa-i,j)(h+1-j)(h+2-j)\Theta(i,h+2-j) \\
& + \beta\sum_{i=0}^{\kappa}\sum_{j=}^{h}(j+1)\Theta(\kappa-i,j+1)(h+1-j)\Theta(i,h+1-j) \\
& - N_c\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\Theta(i,h-j-p)\Theta(t,j)\Theta(k-i-t,p) \\
& + 3N_c\theta_a\sum_{i=0}^{\kappa}\sum_{j=0}^{h}\Theta(i,h-j)\Theta(k-i,j) - 3N_c\theta_a^2\Theta(k,h) \\
& - N_r\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{z=0}^{\kappa-i-l}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\sum_{t=0}^{h-j-p} \\
& \Theta(i,h-j-p-t)\Theta(l,j)\Theta(z,p)\Theta(\kappa-i-l-z,t) + N_r(\theta_a^4),
\end{aligned}
\tag{17}
$$

where $\Theta(\kappa,h)$ is the differential transform of $\theta(\tau,x)$.

Taking the two-dimensional differential transform of the initial condition (11) and boundary condition (10) results in the following transformations respectively,

$$\Theta(0,h) = 0,\ h = 0,1,2,\ldots \tag{18}$$

$$\{1+\beta(\Theta(\kappa,0)-\theta_a)\}\Theta(\kappa,1) = Bi_c(\Theta(\kappa,0)-\theta_a)^{n+1} + Bi_r(\Theta^4(\kappa,0)-\theta_a^4), \tag{19}$$

A constant is assigned to one of the transformed boundary condition as follows,

$$\Theta(\kappa,0) = a,\ a\in\mathbb{R},\ \kappa = 1,2,3,\ldots \tag{20}$$

where the constant a can be determined from the boundary condition (9) at each time step after obtaining the series solution. Substituting equations (18)-(20) into (17) generates the following recurrence relations,

$$\Theta(1,2) = \theta_a^3(\theta_a N_r + N_c) \tag{21}$$

$$\Theta(2,2) = -\frac{1}{2}\theta_a^3(\theta_a N_r + N_c)(-13 + 12\theta_a\beta + 3\theta_a^2 N_c) \tag{22}$$

$$\Theta(1,3) = \theta_a^3(\theta_a N_r + N_c) \tag{23}$$

$$\Theta(2,3) = -\frac{1}{2}\theta_a^3(\theta_a N_r + N_c)(-21 + 20\theta_a\beta + 3\theta_a^2 N_c) \tag{24}$$

$$\begin{aligned}\Theta(3,2) = \frac{1}{3}\Bigg[&\theta_a^4 N_r - 6\theta_a^3(1-\theta_a\beta)(\theta_a N_r + N_c)(-31 + 30\theta_a\beta + 3\theta_a^2 N_c)\\
&+ \beta\Bigg(12a\theta_a^3(\theta_a N_r + N_c) + 2\theta_a^6(\theta_a N_r + N_c)^2\\
&+ \frac{6\theta_a^3(\theta_a N_r + N_c)((a^4-\theta_a^4)Bi_r + (a-\theta_a)^3 Bi_c))}{1+(a-\theta_a)\beta}\Bigg)\\
&+ \beta\Bigg(4\theta_a^6(\theta_a N_r + N_c)^2 + \frac{6\theta_a^3(\theta_a N_r + N_c)((a^4-\theta_a^4)Bi_r + (a-\theta_a)^3 Bi_c)}{1+(a-\theta_a)\beta}\Bigg)\\
&- N_c\Bigg(-\theta_a^3 - \frac{3}{2}\theta_a^5(\theta_a N_r + N_c)(-13 + 12\theta_a\beta + 3\theta_a^2 N_c)\\
&- 3\theta_a\Bigg(2a\theta_a^3(\theta_a N_r + N_c) + \frac{((a^4-\theta_a^4)Bi_r + (a-\theta_a)^3 Bi_c)^2}{(1+(a-\theta_a)\beta)^2}\Bigg)\Bigg)\Bigg]\end{aligned} \tag{25}$$

$$\vdots$$

Substituting equations (18)-(25) into (15) results in the following closed form of the solution,

$$\begin{aligned}\theta(\tau,x) = {}& a\tau + a\tau^2 + \theta_a^3(\theta_a N_r + N_c)\tau x^2\\
&- \frac{1}{2}\theta_a^3(\theta_a N_r + N_c)(-13 + 12\theta_a\beta + 3\theta_a^2 N_c)\tau^2 x^2 + a\tau^3\\
&+ \theta_a^3(\theta_a N_r + N_c)\tau x^3 - \frac{1}{2}\theta_a^3(\theta_a N_r + N_c)(-21 + 20\theta_a\beta + 3\theta_a^2 N_c)\tau^2 x^3\\
&+ \frac{1}{3}\Bigg[\theta_a^4 N_r - 6\theta_a^3(1-\theta_a\beta)(\theta_a N_r + N_c)(-31 + 30\theta_a\beta + 3\theta_a^2 N_c)\\
&+ \beta\Bigg(12a\theta_a^3(\theta_a N_r + N_c) + 2\theta_a^6(\theta_a N_r + N_c)^2\\
&+ \frac{6\theta_a^3(\theta_a N_r + N_c)((a^4-\theta_a^4)Bi_r + (a-\theta_a)^3 Bi_c))}{1+(a-\theta_a)\beta}\Bigg)\\
&+ \beta\Bigg(4\theta_a^6(\theta_a N_r + N_c)^2 + \frac{6\theta_a^3(\theta_a N_r + N_c)((a^4-\theta_a^4)Bi_r + (a-\theta_a)^3 Bi_c)}{1+(a-\theta_a)\beta}\Bigg)\\
&- N_c\Bigg(-\theta_a^3 - \frac{3}{2}\theta_a^5(\theta_a N_r + N_c)(-13 + 12\theta_a\beta + 3\theta_a^2 N_c)\\
&- 3\theta_a\Bigg(2a\theta_a^3(\theta_a N_r + N_c) + \frac{((a^4-\theta_a^4)Bi_r + (a-\theta_a)^3 Bi_c)^2}{(1+(a-\theta_a)\beta)^2}\Bigg)\Bigg)\Bigg]\tau^3 x^2\\
&+ \dots\end{aligned} \tag{26}$$

The constant a can be determined from the boundary condition (9) at each time step. To obtain the value of a, the boundary condition (9) is substituted into (26) at the point $x = 1$. The expression for $\theta(\tau, x)$ is then obtained upon substituting the derived value of a into equation (26). Using the first 20 terms of the power series solution, the series solutions are benchmarked against the numerical solution obtained through employing the inbuilt numerical solver in MATLAB ($pdepe$). As seen from

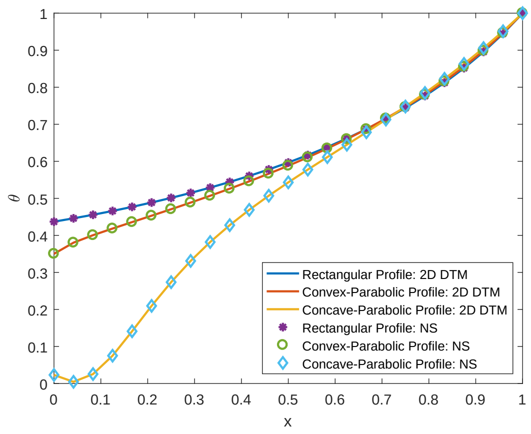

Fig. 2: Temperature distribution in a longitudinal rectangular fin for varying time, τ.

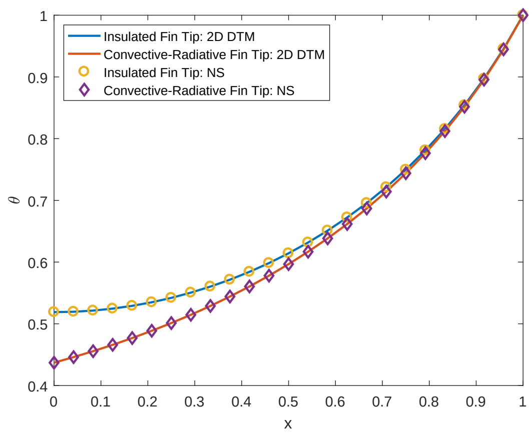

Fig. 3: Variation of temperature distribution at the fin tip.

the benchmark analysis in Figure 2 and Figure 3, the series solution is in agreement with the numerical solution across all profiles.

With the confidence obtained from the benchmark analysis, the solution (26) is plotted for various parameters as shown in Figures 6-13. For all analytical results depicted in the Figures in this article, the following values of parameters are used unless stated otherwise.

$$n = 2,\ \beta = 1,\ N_c = 0.25,\ N_r = 10,\ Bi_c = 0.25,\ Bi_r = 10,\ \theta_a = 0.10,\ \tau = 0.40 \qquad (27)$$

Some Discussions

This section describes the impact of some the embedding parameters on the fin temperature distribution. The governing equations were solved analytically using the 2D DTM and validated numerically using the the inbuilt numerical solver in MATLAB ($pdepe$). Figure 2 shows that the concave-parabolic profile is more efficient in transferring heat to the ambient fluid when compared to the rectangular and convex parabolic profiles. Figure 3 compares the adiabatic (insulted) fin tip boundary condition to the proposed convective-radiative fin tip boundary condition. As expected, the temperature in the fin is lower given the convective-radiative fin tip boundary condition. The analytical solutions and the numerical solutions are in good agreement as depicted in Figure 2 and Figure 3.

Figure 4 shows that the fin temperature increases with time. As time continues to increase, the temperature profile reaches a steady state. Figure 5 illustrates the temperature distribution for varying values of the convective-conductive parameter N_c. For $N_c = 0$, this indicates the absence of convection while $N_c = 1$ indicates that convection and conduction are at equal rates [31]. An increase

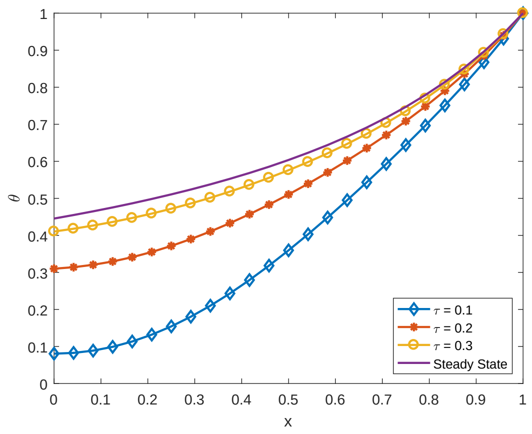

Fig. 4: Temperature distribution in a longitudinal rectangular fin for varying time, τ.

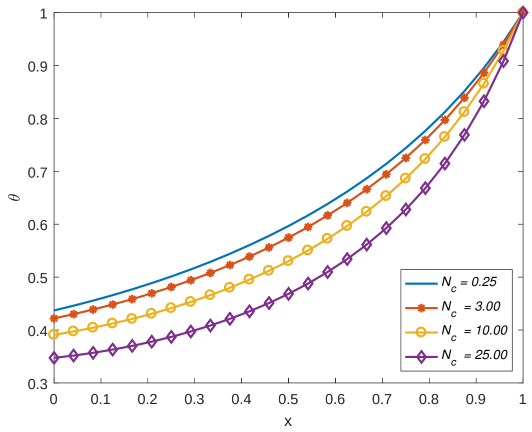

Fig. 5: Effect of the convection-radiation parameter, N_c, on temperature distribution.

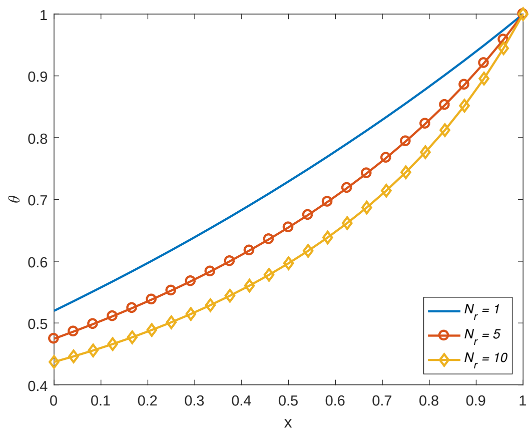

Fig. 6: Effect of the convection-radiation parameter, N_r, on temperature distribution.

in the values of the thermo-geometric fin parameter leads to a decrease in temperature in the fin. A similar result is observed in Figure 6 which represents the impact of the radiation parameter N_r on fin temperature distribution. Figure 7 shows the temperature variation along the fin profile. The fin temperature gradually increases as the axial parameter x increases and this is in line with the imposed boundary condition at the fin base. Figure 8 shows the impact of the change in thermal conductivity gradient. As the values of β decreases, the temperature in the fin decreases signifying an increased

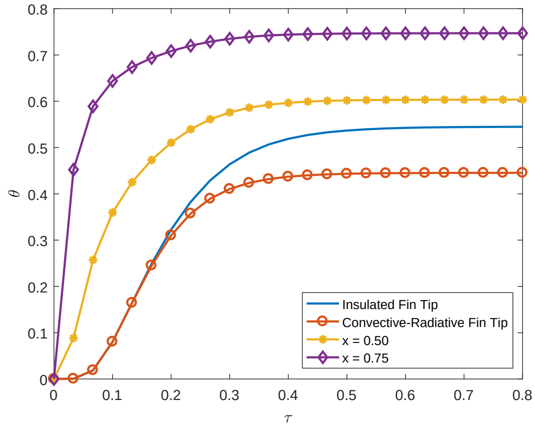

Fig. 7: Temperature distribution for differing axial values.

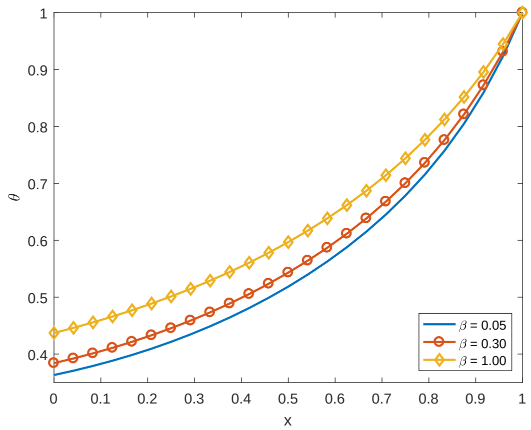

Fig. 8: Effect of the variable thermal conductivity parameter, β, on temperature distribution.

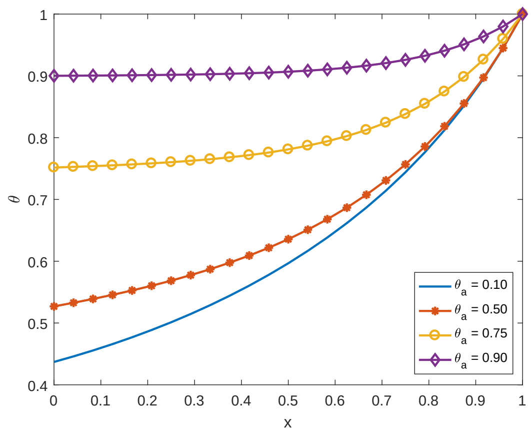

Fig. 9: Effect of convection-radiation sink temperature, θ_a, on temperature distribution.

heat loss to the ambient fluid. Increasing the thermal conductivity gradient enhances heat conduction from the fin base and leads to increased temperatures inside the fin. Figure 9 shows the variation in temperature distribution for varying values of the convective-radiative sink parameter θ_a. The ambient temperature plays an important role in transient heat dissipation. Higher ambient temperature lead to a reduced heat transfer rate as indicated by high values of fin temperature for higher values of the convective-radiative sink temperature. Figure 10 shows the effect of the nature of heat transfer on

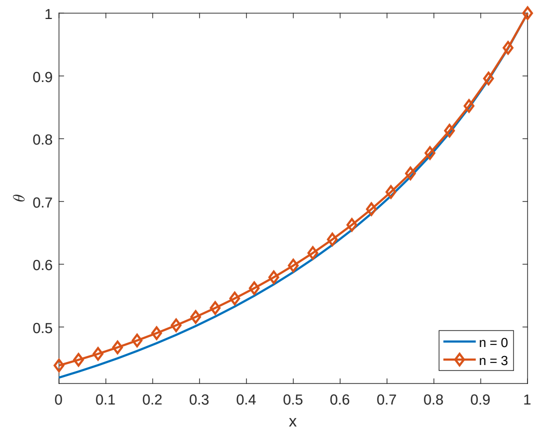

Fig. 10: Effect of the nature of heat transfer parameter, n, on temperature distribution.

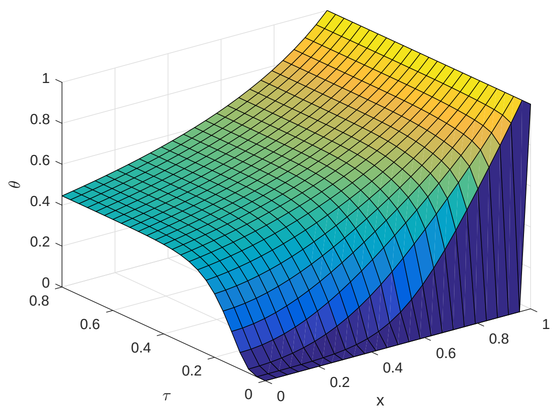

Fig. 11: Transient temperature distribution in a longitudinal rectangular fin.

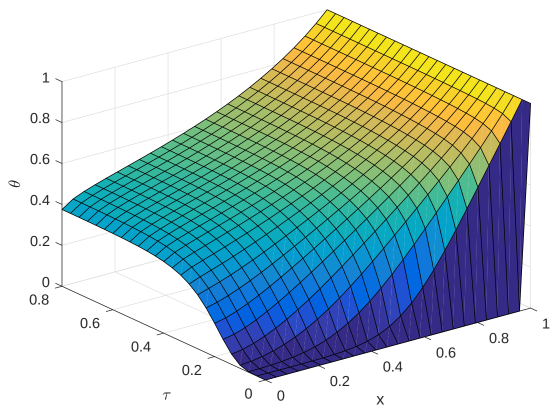

Fig. 12: Transient temperature distribution in a longitudinal convex-parabolic fin.

transient temperature distribution. The temperature distribution of the fin is an increasing function of exponent n. Lastly, the temperature distribution along the three dimensional spaces corresponds to the imposed initial and boundary conditions as depicted in Figures 11, 12 and 13.

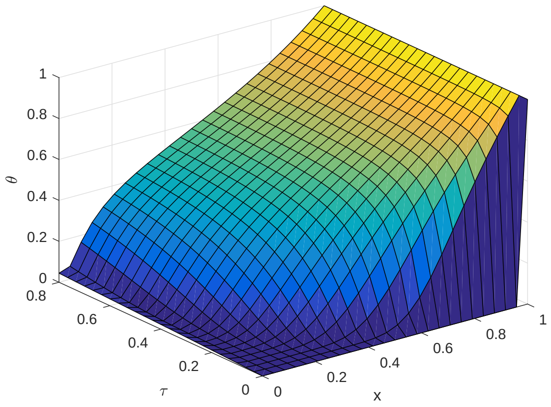

Fig. 13: Transient temperature distribution in a longitudinal concave-parabolic fin.

Concluding Remarks

The 2D DTM was successfully applied to solve the nonlinear boundary value problems describing transient heat transfer in fins undergoing convective-radiative heat dissipation. The results illustrates how the temperature distribution in a fin can be affected by changes in the underlying parameters. The results also provides insights into some physical parameters that can be chosen wisely to induce heat transfer through finned surfaces to desirable temperatures. Furthermore, the proposed convective-radiative boundary condition at the fin tip provide improvements in modeling heat transfer through extended surfaces. The DTM continues to prove that it can provides excellent approximate solutions to the complex nonlinear heat transfer equations.

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2.11 Journal Article XI

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Classifications Go to www.ams.org/msc or www.aip.org/pacs/pacs2010/individuals/pacs2010_regular_edition / to find your classifications:	Partial differential equations, boundary value problems, Partial differential equations, initial value and time-dependent initial-boundary value problems, Methods for differential-algebraic equations
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Analysis of a convective-radiative continuously moving fin with temperature dependent thermal conductivity

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Abstract. In this article, the differential transform method (DTM) is used to solve the nonlinear boundary value problems describing heat transfer in continuously moving fins undergoing convective-radiative heat dissipation. The thermal conductivity is variable and temperature dependent. The surface of the moving fin is assumed to be gray with a constant emissivity ε. The flow in the surrounding medium provides a constant heat transfer coefficient h over the entire surface of the moving fins. The effects of some physical parameters such as the Peclet number, Pe, thermal conductivity parameter, β, convection-conduction parameter, N_c, radiation-conduction parameter, N_r , dimensionless convection-radiation sink temperature, θ_a, on the temperature distribution are illustrated and explained.

Keywords. Moving fin, differential transform method, variable thermal conductivity, convection-radiation heat transfer, fin tip temperature.

1 Introduction

In most industrial processes such as casting, hot rolling and extrusion, the thermal processing of the material being manufactured is performed while the material is in continuous motion relative to its ambient fluid. This allows the material to be cooled to a desirable temperature before it is cooled or spooled [1, 2]. To enhance such cooling, the surface area of the material is increased by the use of fins that extend to an ambient fluid. Fins are solids that extends from an object to increase the rate heat transfer. Fins are used to enhance heat dissipation from a hot surface to its convective, radiative, or convective-radiative surface [3, 4]. Recently, the study of heat transfer in moving fins has been increasing enormously and it has been found that the heat transfer coefficient between the heated surface and its adjacent fluid can be a constant [5]. As the material undergoing the treatment experiences a large change in its temperature during the process, the change in its thermal conductivity is temperature dependent [6]. Problems arising in engineering has this type of dependence and a large number of studies have been devoted to the analysis of fin performance under such circumstances [7–11]. Transient response of fins of various profiles have been studied by Ndlovu and Moitsheki [12, 13]. These authors contributed to the study of heat transfer by constructing conservation laws, Lie point symmetries and providing analytical solutions to these nonlinear equations. Aziz and Khani [14] studied convection-radiation of a continuously moving fin of variable thermal conductivity using homotopy analysis method (HAM) and analyzed the effect of several parameters. In this study, the DTM is used to analyze transient response of fins of various profiles. Some related studies where the DTM was applied are presented next.

The DTM is a semi-numerical-analytical method based on Taylor series expansion and was first introduced by Zhou [15] in 1986. Zhou used the DTM to solve explicitly the linear and nonlinear initial value problems that arise in electrical circuits. Chen and Ho [16], developed a two-dimensional DTM which can be used for solving differential and integral equations. At higher orders, the DTM differs from the Taylor series method in the way the coefficients are computed. The Taylor series method requires computing coefficients using the initial data and the differential equation which requires more computational work while the DTM iteratively obtains the Taylor series equations [17]. Ayaz [18], showed that the DTM is better equipped to solve highly nonlinear problems than the Taylor series method. The DTM has been used to solve various problems in applied mathematics and physics such as systems of differential equations [19]. As far as we know, the the transient study of heat transfer in continuously moving fins having a constant heat transfer coefficient has not been performed anywhere in literature.

This article is organized as follows; the mathematical background of the problem under consideration is described in § 2. A brief discussion on the fundamentals of the 2D DTM will be provided in § 3. We provide analytical solutions in § 4. Lastly, we provide some discussions based on the results obtained in § 5 and concluding remarks in § 6.

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2 Problem formulation

We consider a longitudinal one dimensional moving fin of length L, with cross-sectional area A_c while it moves horizontally with a constant velocity U as depicted in Figure 1. The fin surface is exposed to a convective and radiative environment of temperature T_a and the base temperature of the fin is $T_b > T_a$. The fin thickness at the prime surface is given by δ_b and its profile is given by $F(X)$. The local heat transfer coefficient h along the fin surface is constant and the surface of the moving fin is assumed to be gray and diffuse with constant emissivity ε. The role of radiation component could be more sensible if the force convection is weak or absent or when only natural convection occurs. Since the material experience a large change in its temperature during thermal process, the thermal conductivity varies with the temperature. For most materials, the thermal conductivity varies linearly with temperature [7, 8], that is,

$$K(T) = k_c[1 + \gamma(T - T_a)]. \tag{1}$$

The energy balance equation for the fin moving with a constant speed and losing heat by simultaneous convection and radiation can be expressed as (see e.g. [3])

$$\rho c\frac{\partial T}{\partial t} = \frac{\delta_b}{2}\frac{\partial}{\partial X}\left(F(X)K(T)\frac{\partial T}{\partial X}\right) - \frac{ph}{A_c}(T - T_a) - \frac{\varepsilon\sigma p}{A_c}(T^4 - T_a^4) - \frac{\delta_b}{2}F(X)\rho c U\frac{\partial T}{\partial X},\ 0 \leq X \leq L, \tag{2}$$

where K is the non-uniform thermal conductivity depending on the temperature (see [3, 4]), p is the periphery of the fin, T_a is the ambient temperature, ε is the emissivity, σ is the Stefan-Boltzmann constant, ρ is the density of the material, c is the specific heat capacity, T is the temperature distribution, t is the time and X is the space variable. Assuming that the fin tip is adiabatic (insulated) and the base temperature is kept constant, then the boundary conditions are given by [3],

$$T(t, L) = T_b \ \text{ and } \ \left.\frac{\partial T}{\partial X}\right|_{X=0} = 0, \tag{3}$$

and initially the fin is kept at the ambient temperature,

$$T(0, X) = T_a. \tag{4}$$

Introducing the following dimensionless variables,

$$x = \frac{X}{L},\ \tau = \frac{k_c t}{\rho c L^2},\ \theta = \frac{T}{T_b}, \theta_a = \frac{T_a}{T_b}, \quad f(x) = \frac{\delta_b}{2}F(X),\ k = \frac{K}{k_c}, N_c = \frac{hpL^2}{k_c A_c}, \quad N_r = \frac{\varepsilon\sigma p L^2 T_b^3}{k_c A_c}, \alpha = \frac{k_c}{\rho c},\ \text{and } Pe = \frac{UL}{\alpha}, \tag{5}$$

reduces equation (2)

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial}{\partial x}\left(f(x)k(\theta)\frac{\partial\theta}{\partial x}\right) - N_c(\theta - \theta_a) - N_r(\theta^4 - \theta_a^4) - f(x)Pe\frac{\partial\theta}{\partial x},\ \ 0 \leq x \leq 1, \tag{6}$$

and equation (6) admits the following boundary conditions,

$$\theta(\tau, 1) = 1,\ \tau > 1, \tag{7}$$

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=0} = 0, \tag{8}$$

and the initial condition becomes,

$$\theta(0, x) = 0. \tag{9}$$

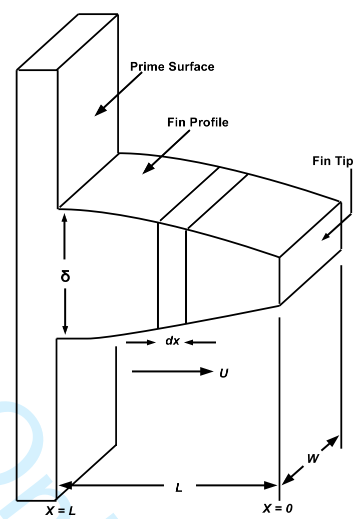

Figure 1. Schematic representation of a longitudinal fin of an unspecified profile.

The dimensionless variable Pe is the Peclet number which represent the dimensionless speed of the moving fin, θ is the dimensionless temperature, x is the dimensionless space variable, $k(\theta) = 1 + \beta(\theta - \theta_a)$ is the dimensionless thermal conductivity, where the thermal conductivity gradient is $\beta = \gamma T_b$, k_c is the thermal conductivity of the fin at ambient temperature, N_c is the convection-conduction parameter, N_r is the radiation-conduction parameter and α is the thermal diffusivity of the fin.

3 Fundamentals of the two-dimensional differential transform method

In this section, the basic idea underlying the two-differential transform method (2D-DTM) is briefly introduced. If function $\theta(t, x)$ is analytic and differentiated continuously with respect to time and the spatial variable x in the domain of interest, then we let

$$\Phi(\kappa,s)=\frac{1}{\kappa!s!}\left[\frac{\partial^{\kappa+s}\phi(t,x)}{\partial t^{\kappa}\partial x^{s}}\right]_{(0,0)}, \qquad (10)$$

where the spectrum $\Phi(\kappa,s)$ is the transformed function, which is also called the T-function (see [17, 18]). The differential inverse transform of $\Phi(\kappa,s)$ is defined as

$$\phi(t,x)=\sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty}\Phi(\kappa,s)t^{\kappa}x^{s}, \qquad (11)$$

and from Eqs (10) and (11) it can be concluded that

$$\phi(t,x)=\sum_{\kappa=0}^{\infty}\sum_{s=0}^{\infty}\frac{1}{\kappa!s!}\left[\frac{\partial^{\kappa+s}\phi(t,x)}{\partial t^{\kappa}\partial x^{s}}\right]_{(0,0)}t^{\kappa}x^{s}. \qquad (12)$$

In real applications, the function $\phi(t,x)$ is expressed by a finite series, and Eqn. (11) can be written as:

$$\phi(t,x)=\sum_{\kappa=0}^{m}\sum_{s=0}^{n}\Phi(\kappa,s)t^{\kappa}x^{s}. \qquad (13)$$

From Eqn. (13), we assume that

$$\phi(t,x)=\sum_{\kappa=m+1}^{\infty}\sum_{s=n+1}^{\infty}\Phi(\kappa,s)t^{\kappa}x^{s}, \qquad (14)$$

is negligibly small.

Some of the useful mathematical operations performed by the differential transform method are given by the following theorems [20–22].

Theorem 1. *If* $\phi(t,x)=x(\tau,x)\pm z(t,x)$, *then* $\Phi(\kappa,s)=X(\kappa,s)\pm Z(\kappa,s)$.

Theorem 2. *If* $\phi(t,x)=\alpha x(t,x)$, *then* $\Phi(k,s)=\alpha X(\kappa,s)$.

Theorem 3. *If* $\phi(t,x)=\frac{\partial\phi(t,x)}{dt}$, *then* $\Phi(\kappa,s)=(\kappa+1)\Phi(\kappa+1,s)$.

Theorem 4. *If* $\phi(t,x)=\frac{\partial\phi(t,x)}{dx}$, *then* $\Phi(\kappa,s)=(s+1)\Phi(\kappa,s+1)$.

Theorem 5. *If* $\phi(t,x)=\frac{\partial^{r+q}\phi(t,x)}{\partial t^{r}\partial x^{q}}$, *then* $\Phi(\kappa,s)=(\kappa+1)(\kappa+2)\ldots(\kappa+r)(s+1)(s+2)\ldots(s+q)\Phi(\kappa+r,s+q)$.

Theorem 6. *If* $\phi(t,x)=x(t,x)z(t,x)$, *then* $\Phi(\kappa,s)=\sum_{i=0}^{\kappa}\sum_{j=0}^{s}X(i,s-j)Z(\kappa-i,j)$.

Theorem 7. *If* $\phi(t,x)=t^{m}x^{n}$, *then* $\Phi(\kappa,s)=\delta(\kappa-m)\delta(s-n)$.

Theorem 8. *If* $\phi(t,x)=x^{m}\exp(at)$, *then* $\Phi(\kappa,s)=\frac{a^{s}}{s!}\delta(\kappa-m)$.

The Kronecker delta function $\delta(\kappa-s)$ is given by

$$\delta(\kappa-s)=\begin{cases}1 & \text{if } \kappa=s\\ 0 & \text{if } \kappa\neq s.\end{cases}$$

4 Derivation of analytical Solutions

In this section we provide analytical solutions for the problem described in § 2. Using 2D-DTM to solve PDEs consists of three main steps. The steps are: transforming the PDE into algebraic equations, solving the equations, and inverting the solution of algebraic equations to obtain an infinite series solution or an approximate solution.

4.1 Heat transfer in fins of rectangular profile

The fin profile is rectangular when $f(x)=1$. Taking the two-dimensional differential transform of Eqn. (6) with $f(x)=1$, we obtain the following recurrence relation

$$\begin{aligned}&(\kappa+1)\Theta(\kappa+1,h)=(h+1)(h+2)\Theta(\kappa,h+2)\\&-\beta\theta_a(h+1)(h+2)\Theta(\kappa,h+2)\\&+\beta\sum_{i=0}^{\kappa}\sum_{j=0}^{h}\Theta(\kappa-i,j)(h+1-j)(h+2-j)\\&\Theta(i,h+2-j)+\beta\sum_{i=0}^{\kappa}\sum_{j=0}^{h}(j+1)\Theta(\kappa-i,j+1)\\&(h+1-j)\Theta(i,h+1-j)\\&-N_r\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{z=0}^{\kappa-i-l}\sum_{j=0}^{h}\sum_{p=0}^{h-j}\sum_{t=0}^{h-j-p}\Theta(i,h-j-p-t)\\&\Theta(l,j)\Theta(z,p)\Theta(\kappa-i-l-z,t)+N_r(\theta_a^4)\\&-N_c(\Theta(\kappa,h)-\theta_a)-Pe(h+1)\Theta(k,h+1),\end{aligned} \qquad (15)$$

where $\Theta(\kappa,h)$ is the differential transform of $\theta(\tau,x)$.

Taking the two-dimensional differential transform of the initial condition (9) and boundary condition (8) we obtain the following transformations respectively,

$$\Theta(0,h)=0,\ h=0,1,2,\ldots \qquad (16)$$

$$\Theta(\kappa,1)=0,\ \kappa=0,1,2,\ldots. \qquad (17)$$

We consider the other boundary condition as follows,

$$\Theta(\kappa,0)=a,\ a\in\mathbb{R},\ \kappa=1,2,3,\ldots \qquad (18)$$

where the constant a can be determined from the boundary condition (7) at each time step after obtaining the series solution.

Substituting Eqns. (16)-(18) into (15) we obtain the following,

$$\Theta(1,2) = \theta_a(N_r\theta_a^3 + N_c) \tag{19}$$

$$\Theta(2,2) = -\frac{1}{2}\theta_a(-13 + 12\beta\theta_a + 3Pe + N_c)(N_r\theta_a^3 + N_c) \tag{20}$$

$$\Theta(1,3) = \theta_a(N_r\theta_a^3 + N_c) \tag{21}$$

$$\Theta(2,3) = -\frac{1}{2}\theta_a(-21 + 20\beta\theta_a + 4Pe + N_c)(N_r\theta_a^3 + N_c) \tag{22}$$

$$\vdots$$

Substituting Eqns. (16)-(22) into (13) we obtain the following closed form of the solution,

$$\begin{aligned}
&\theta(\tau, x) = a\tau + a\tau^2 + \theta_a(N_r\theta_a^3 + N_c)\tau x^2 \\
&- \frac{1}{2}\theta_a(-13 + 12\beta\theta_a + 3Pe + N_c)(N_r\theta_a^3 + N_c)\tau^2 x^2 \\
&+ a\tau^3 + \theta_a(N_r\theta_a^3 + N_c)\tau x^3 \\
&- \frac{1}{2}\theta_a(-21 + 20\beta\theta_a + 4Pe + N_c)(N_r\theta_a^3 + N_c)\tau^2 x^3 \\
&+ \ldots
\end{aligned} \tag{23}$$

To obtain the value of a, we substitute the boundary condition (7) into (23) at the point $x = 1$. Thus, we have,

$$\begin{aligned}
&\theta(\tau, 1) = a\tau + a\tau^2 + \theta_a(N_r\theta_a^3 + N_c)\tau \\
&- \frac{1}{2}\theta_a(-13 + 12\beta\theta_a + 3Pe + N_c)(N_r\theta_a^3 + N_c)\tau^2 \\
&+ a\tau^3 + \theta_a(N_r\theta_a^3 + N_c)\tau \\
&- \frac{1}{2}\theta_a(-21 + 20\beta\theta_a + 4Pe + N_c)(N_r\theta_a^3 + N_c)\tau^2 \\
&+ \ldots = 1
\end{aligned} \tag{24}$$

We then obtain the expression for $\theta(\tau, x)$ upon substituting the obtained value of a into equation (23). Using the first 20 terms of the power series solution, we benchmark the series solutions to the numerical solution obtained through employing the Maple package. The pdsolve in Maple uses FDM to discretize the differential equation. The technique is a centered implicit scheme that is capable of finding solutions to higher-order partial differential equations. As seen from the benchmark analysis in Figure 2, the series solution is in agreement with the numerical solution across various profiles.

With the confidence obtained from the benchmark analysis, we plot the solution (23) for various parameters as shown in Figures 3, 4, 5, 6, 7, 8, 9, 10 and 11.

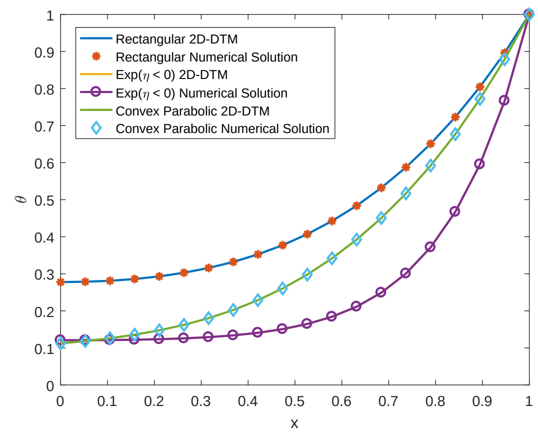

Figure 2. Temperature distribution in differing fin profiles.

For all analytical results reported in this article, the following values of parameters are used unless stated otherwise as indicated in the graphs below.

$$\begin{aligned}
&\theta_a = 0.10,\ N_c = 4,\ N_r = 4, \\
&Pe = 3,\ \beta = 1,\ \tau = 1.20
\end{aligned} \tag{25}$$

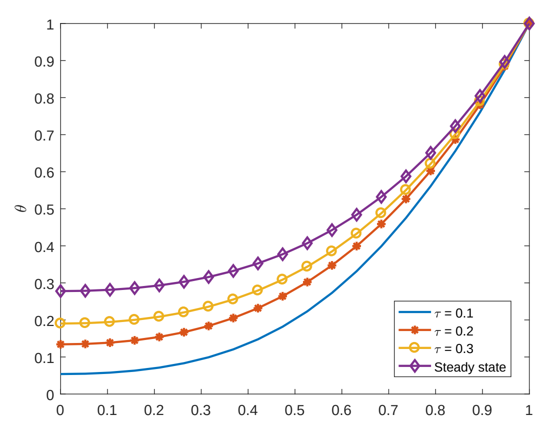

Figure 3. Temperature distribution in a longitudinal rectangular fin for varying time, τ.

4.2 Heat transfer in fins of exponential profile

The fin profile is exponential when $f(x) = e^{\eta x}$. We show the transformation of the exponential profile by the DTM and give only explicit solutions to the remaining fin profiles. Taking the two-dimensional differential transform of Eqn. (6) with $f(x) = e^{\eta x}$, we obtain the following solution

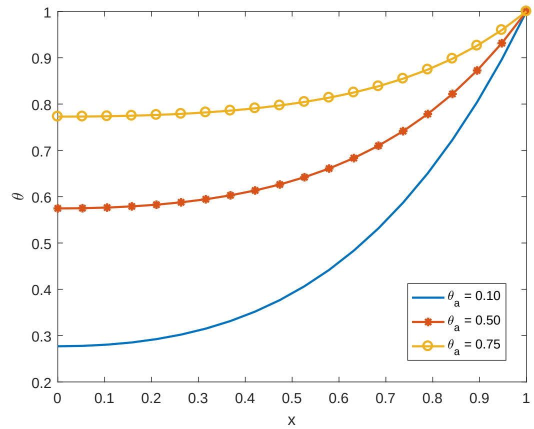

Figure 4. Effect of convection-radiation sink temperature, θ_a, on temperature distribution.

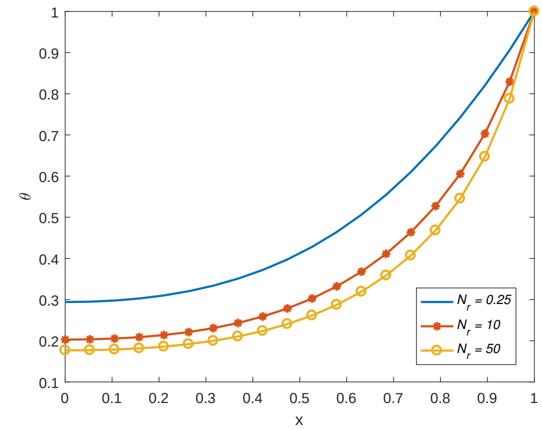

Figure 7. Effect of the convection-radiation parameter, N_r, on temperature distribution.

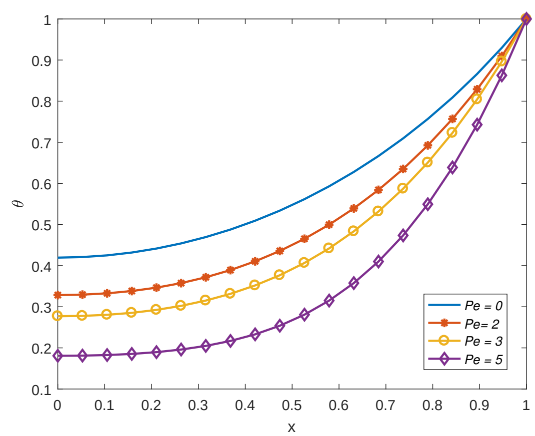

Figure 5. Effect of varying speeds, Pe, on temperature distribution.

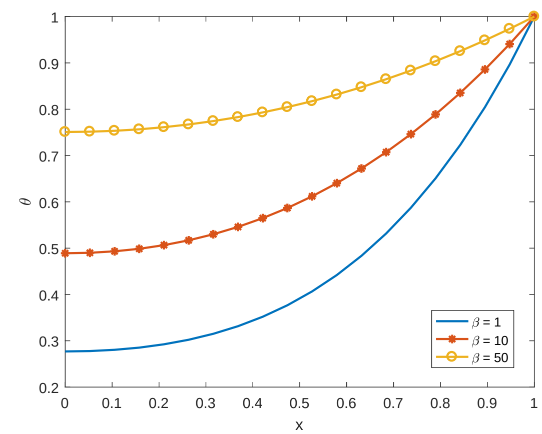

Figure 8. Effect of the variable thermal conductivity parameter, β, on temperature distribution.

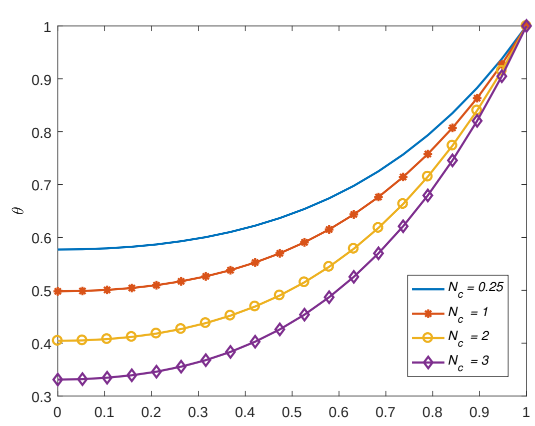

Figure 6. Effect of the convection-radiation parameter, N_c, on temperature distribution.

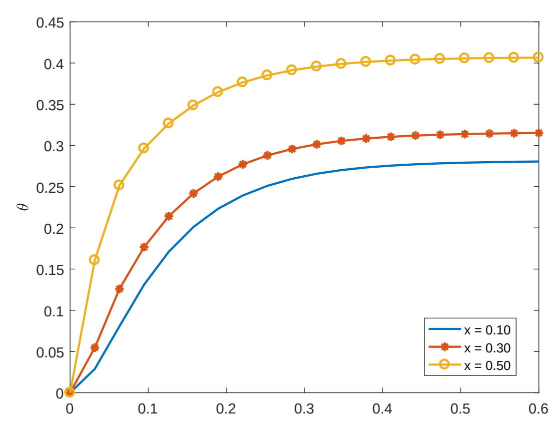

Figure 9. Temperature distribution for differing axial values.

$$\theta(\tau,x) = a\tau + a\tau^2 + \frac{-2a + N_r\theta_a^4 - aN_c + N_c\theta_a}{2(-1-\beta+\beta\theta_a)}\tau x^2 + \frac{-3a + N_r\theta_a^4 - aN_c + N_c\theta_a}{2(-1-\beta+\beta\theta_a)}\tau^2 x^2 + a\tau^3 + \dots \quad (26)$$

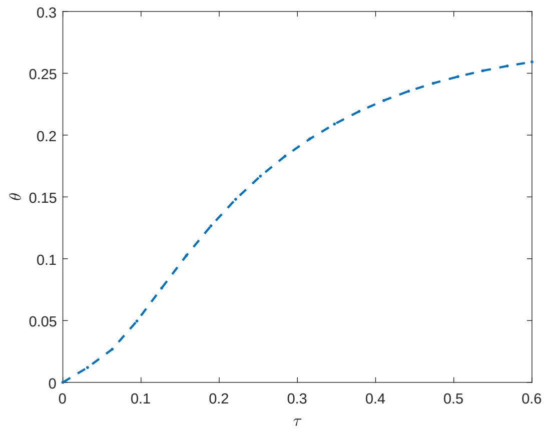

Figure 10. Variation of temperature distribution at the fin tip.

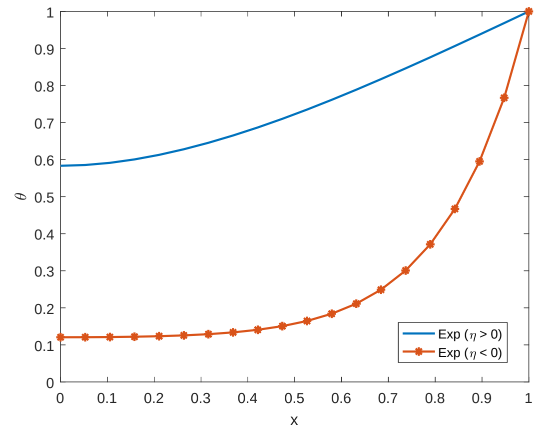

Figure 12. Temperature distribution in a longitudinal exponential fin, $\eta = 2$ and $\eta = -2$.

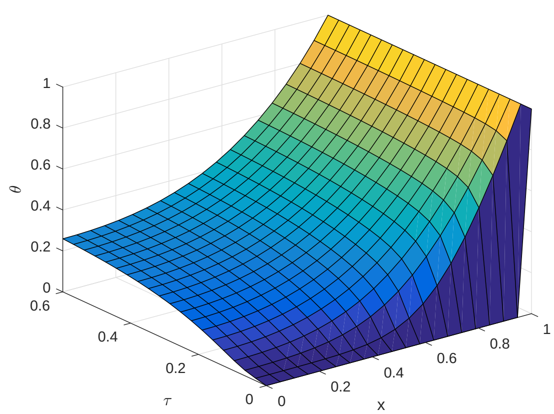

Figure 11. Transient temperature distribution in a longitudinal rectangular fin.

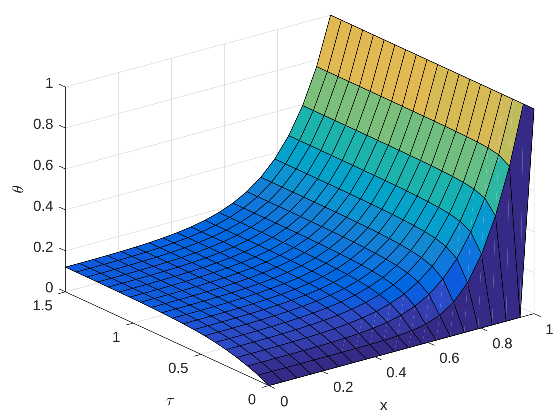

Figure 14. Transient temperature distribution in a longitudinal exponential fin, $\eta = -2$.

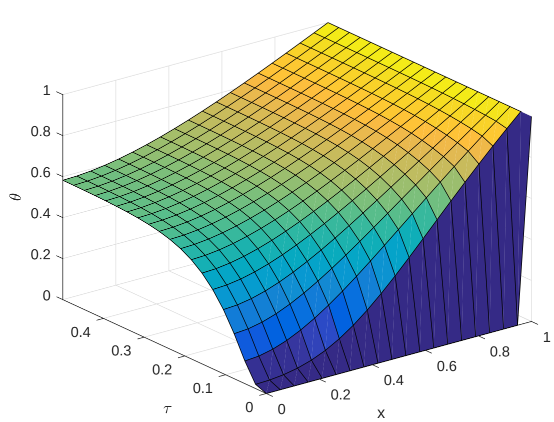

Figure 13. Transient temperature distribution in a longitudinal exponential fin, $\eta = 2$.

4.3 Heat transfer in fins of convex parabolic profile

The fin profile is convex parabolic when $f(x) = \sqrt{x}$. Following a similar approach undertaken by Ndlovu and Moitsheki [12], we substitute $y = \sqrt{x}$ to eliminate fractional powers. We then apply the DTM to the resulting differential equation. The analytical solution of the convex parabolic fin profile is given below,

$$\begin{aligned}\theta(\tau,x) &= a\tau + a\tau^2 - \frac{2\theta_a(N_r\theta_a^3+N_c)}{-1+\beta\theta_a}\tau x \\ &+ \frac{2\theta_a(-1+\beta(a+\theta_a))(N_r\theta_a^3+N_c)}{(-1+\beta\theta_a)^2}\tau^2 x + a\tau^3 \\ &- \frac{2(N_r\theta_a^4+N_c\theta_a-a(2+N_c))}{3(-1+\beta\theta_a)}\tau x^{3/2} + \dots \qquad (27)\end{aligned}$$

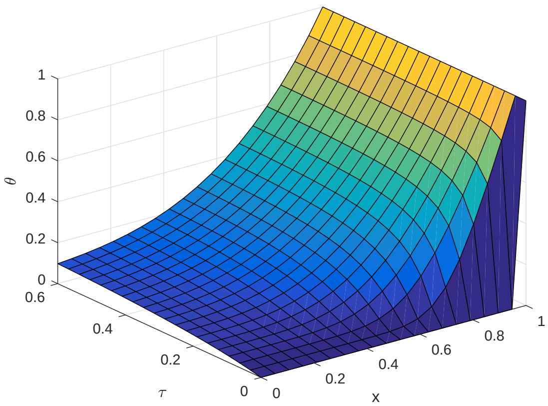

Figure 15. Transient temperature distribution in a longitudinal convex parabolic fin.

4.4 Heat transfer in fins of concave parabolic profile

The fin profile is concave parabolic when $f(x) = x^2$. The analytical solution of the concave parabolic fin profile is given below,

$$\begin{aligned}
&\theta(\tau, x) = a\tau + a\tau^2 + \theta_a(N_r\theta_a^3 + N_c)\tau x^2 \\
&- \frac{1}{2}\theta_a(-7 + 6\beta\theta_a + N_c)(N_r\theta_a^3 + N_c)\tau^2 x^2 + a\tau^3 \\
&+ \theta_a(N_r\theta_a^3 + N_c)\tau x^3 + \ldots
\end{aligned} \tag{28}$$

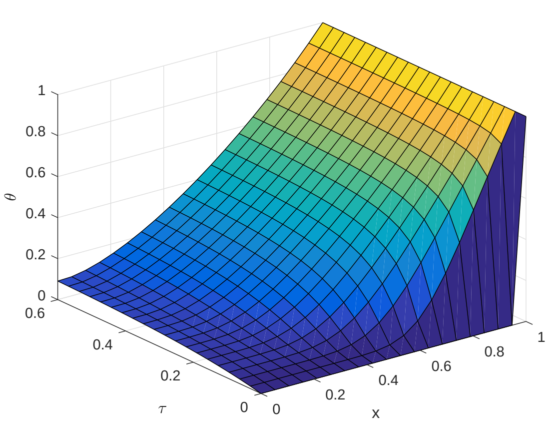

Figure 16. Transient temperature distribution in a longitudinal concave parabolic fin.

4.5 Heat transfer in fins of cubic profile

The fin profile is concave parabolic when $f(x) = x^3$. The analytical solution of the cubic fin profile is given below,

$$\begin{aligned}
&\theta(\tau, x) = a\tau + a\tau^2 + \theta_a(N_r\theta_a^3 + N_c)\tau x^2 \\
&- \frac{1}{2}\theta_a(-1 + N_c)(N_r\theta_a^3 + N_c)\tau^2 x^2 \\
&+ \theta_a(N_r\theta_a^3 + N_c)\tau x^3 + a\tau^3 \\
&- \frac{1}{2}\theta_a(-9 + 8\beta\theta_a + N_c)(N_r\theta_a^3 + N_c)\tau^2 x^3 + \ldots
\end{aligned} \tag{29}$$

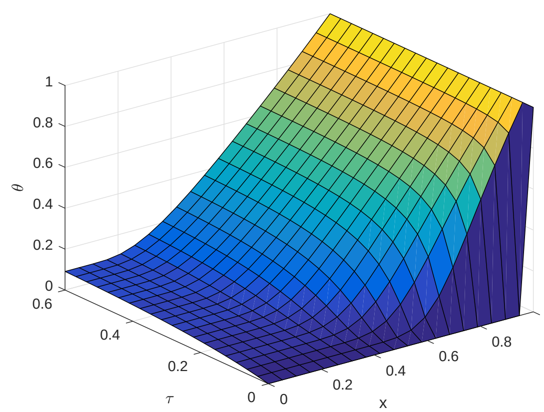

Figure 17. Transient temperature distribution in a longitudinal cubic fin.

4.6 Heat transfer in fins of triangular profile

The fin profile is concave parabolic when $f(x) = x$. The analytical solution of the triangular fin profile is given below,

$$\begin{aligned}
&\theta(\tau, x) = a\tau + a\tau^2 + \theta_a(N_r\theta_a^3 + N_c)\tau x^2 \\
&- \frac{1}{2}\theta_a(-10 + 9\beta\theta_a + 2Pe + N_c)(N_r\theta_a^3 + N_c)\tau^2 x^2 \\
&+ \theta_a(N_r\theta_a^3 + N_c)\tau x^3 + a\tau^3 + \ldots
\end{aligned} \tag{30}$$

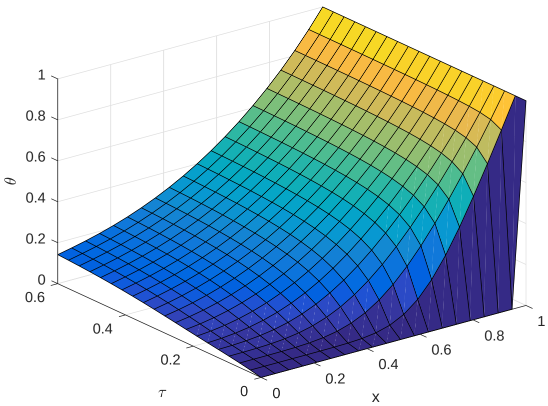

Figure 18. Transient temperature distribution in a longitudinal triangular fin.

4.7 Aggregated fin profiles

In Figure 19, we compare the solutions generated by DTM across various fin profiles.

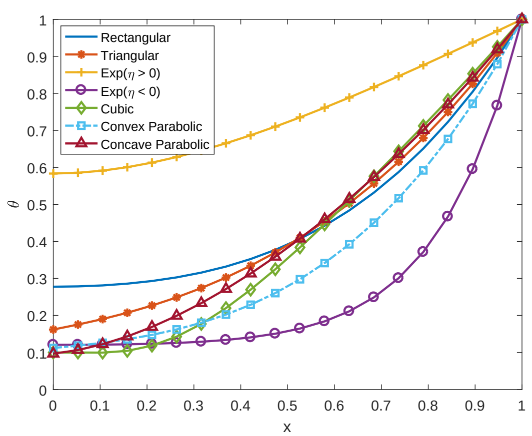

Figure 19. Temperature distribution in fins of various profiles.

5 Some discussions

Figure 3 shows that the fin temperature increases with time. It is to be noted that as time continues to increase, the temperature profile reaches a steady state. Figure 4 shows that as the convection-radiation sink temperature increases, the heat loss from the moving fin decreases as indicated by increasing high temperatures within the fin. Figure 5 illustrates the effect of the Peclet number on the temperature distribution in the fin. As Pe increases, (fin moves faster), the temperature is higher as the time for which the fin is exposed to the ambient fluid is shortened. We also observe that for $Pe = 0$, i.e., stationery fin, the cooling of the fin takes more time as shown by higher temperatures when compared to a moving fin. The effect of the convection-radiation parameter, N_c, and the radiation-conduction parameter, N_r, are depicted in Figures 6 and 7 respectively. As both N_c and N_r increases, i.e., as the convection and radiation become stronger, the cooling of the fin becomes more effective promoting lower temperatures within the fin. We note that higher values of the radiation-conduction parameter are needed to induce cooling of the fin when compared to convection-conduction parameter. Figure 8 shows the effect of the variable thermal conductivity parameter β on temperature distribution. As the parameter β increases, the thermal conductivity of the fin also increases as expected. This is illustrated by a gradual increase on the fin temperature. Figure 9 shows that as time increases, the temperature on the fin approaches unity even at small time scales and this corresponds the imposed boundary conditions. The temperature variation at the fin tip is shown in Figure 10. As expected, the fin tip temperature gradually increases as time evolves. We observe that the temperature distribution along the three dimensional spaces corresponds to the boundary conditions and to all the results described above and the graphs are depicted in Figures 11, 13, 14, 15, 16, 17 and 18.

Figure 12 shows the impact of the power of the the exponent on the exponential fin profile. We observe that for negative exponent, the heat dissipation to the ambient fluid is much greater that when the exponent is positive. Furthermore, Figure 19 shows that the fin of exponential profile ($\eta > 0$) performs the worst in transferring the heat from the base and heat dissipation to the fluid surrounding the extended surface is much faster in longitudinal fins of convex parabolic profiles and exponential fin profile ($\eta < 0$).

6 Concluding remarks

We have successfully applied the differential transform method to solve the nonlinear boundary value problems describing heat transfer in a continuously moving fin undergoing convective-radiative heat dissipation. The results illustrates how the temperature distribution in a moving fin can be affected by changes in the underlying parameters. The results provides some insights into some physical parameters that can be chosen wisely to induce heat transfer through finned surfaces to desirable temperatures. The DTM continues to prove that it can provides excellent approximate solutions to the complex non-linear heat transfer equations.

References

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Chapter 3

Conclusions and future work

This thesis provides various insights into models that describe heat transfer through extended surfaces. The difficulty in solving the resulting mathematical models is evidenced by the use of various analytical and numerical methods found in literature. Prior to deriving the governing heat transfer equations, a study was performed to assess the convergence of the analytical methods that were further adopted as the methods of solutions in the resulting series of publications. Article I provides a benchmarking analysis where a model with known exact solutions was solved using the two analytical methods (VIM and DTM). The two methods provided power series solutions that were close to the exact solution. It was found that the VIM converges faster when compared to the DTM and can provide accurate results with minimum computational effort. However, it was noted that the VIM produces unnecessary term and increases computation time. For problems where the number of iterations can be computed as high as as thirteen without running short of memory in computer simulations, the DTM outperforms the VIM in terms of accuracy. In this regard, the VIM was used the solve the simplistic steady state heat transfer problems and the DTM was applied to generate solutions to transient problems. With the confidence obtained from the benchmark analysis in Article I, the VIM was applied to analyze steady state heat transfer in radial moving fins with temperature dependent thermal conductivity and heat transfer coefficient. The results of this analysis are presented in Article II. The heat transfer in radial moving fins was modeled for the first time in this article and it was found that the rate of heat transfer to the ambient fluid increases with an increase in the horizontal flow field induced by the moving fin. The approximate analytical solutions generated by VIM were validated using numerical results obtained by applying the Runge-Kutta fourth order method. A good agreement was observed between the analytical and numerical solutions signifying that the VIM was indeed a suitable method for the described problem.

The model given in Article II was derived with the assumption that the transient (time dependent) response of fins dies out quickly. In Article III, this assumption was relaxed and transient models describing heat transfer in radial fins of rectangular and hyperbolic profiles were derived. The analytical solutions were generated using the DTM and validated by benchmarking against the numerical solutions obtained by applying the inbuilt numerical solver in Maple (*pdsolve*). The results showed that the heat transfer through a rectangular fin is more efficient when compared to a fin of hyperbolic profile. It is important to note that the rate of heat transfer was found to be an increasing function of time. Similarly, it was found that the fin becomes more effective with an increase non-dimensional speed of the moving fin. Following success in the study of heat transfer in radial fins, it was found viable to analyze the heat transfer in porous fins in motion. The transport through the porous medium was derived using Darcy's model. The DTM was used to perform thermal analysis of natural convective and radiative heat transfer in moving porous fins of rectangular and exponential profiles. The results of this analysis are presented in Article IV. The study illustrated that the rate of heat transfer was faster in porous moving fins when compared to solid moving fins. The temperature distribution in the moving porous fin was found to be highly dependent upon the fin profile. This is an important finding for fin design as it was noted that exponential fins with negative powers (e^{-r}) are more efficient than the traditional rectangular fins.

To illustrate the impact of convection in heat transfer, the assumption of natural convection given in Article IV was eliminated. Article V derives a mathematical model describing heat transfer in a triangular moving porous fin with temperature dependent thermal properties. The approximate analytical solutions generated by DTM were validated using the numerical solution obtained via the inbuilt numerical solver in MATLAB (*pdepe*). The results from the analysis indicate that an increase in the values of the conduction-convective fin parameter leads to a decrease in fin temperature. This variation could not be explicitly analyzed in Article IV, as the heat exchange was occurring in natural convection. In all the articles presented thus far, it was assumed that the thermal conductivity varies linearly with temperature. The majority of the studies in literature use this type of dependence and in Article VI, alternative thermal conductivity calibration functions are given. The results show that different calibration functions of thermal conductivity indeed lead to varying temperature distributions along the fin. The logistic calibration function results in lower temperatures in the fin when compared linear, power-law and exponential functions. The use of a logistic calibration function may be be useful under conditions where the modeled temperature distribution is over predicting the actual temperature distribution obtained from experimental results.

Following the successful modeling of heat transfer on radial fins in Article I and Article II, a highly nonlinear model for transient heat transfer in rectangular radial porous fin was analyzed using an inbuilt numerical solver in MATLAB (*pdepe*). The thermal conductivity and fin surface emissivity were considered to be linearly dependent on temperature, while the heat transfer coefficient was assumed to be given by a power law function of temperature. The results of this analysis are presented in Article VII. The article analyzes the effect of varying surface emissivity parameter on fin temperature distribution. It was observed that as the surface emissivity increases, the radiative heat loss decreases resulting in lower fin temperatures. A similar result is observed in Article VIII which studies heat transfer in a non-porous fin using the DTM. The steady state version of Article VIII is given in Article IX. The results of the article provides some insights into some physical parameters that can be chosen wisely to induce heat transfer through finned surfaces to desirable temperatures.

Various studies in literature assumes that the fin tip is adiabatic (insulated) making it easy to solve the resulting nonlinear boundary value problems. Article X relaxes this assumption and considers a convective-radiative fin tip. It was found that the rate of heat transfer was lower on a fin with an insulated tip when compared to a non-insulated tip. Where the modeled (theoretical) heat transfer is misaligned with the actual experimental heat transfer, adjusting the convective-radiative fin tip parameters can provide profound calibrations. Lastly, an vital contribution in terms of fin design is given in Article XI. The article studied heat transfer in seven different fin profiles. The results indicate that heat dissipation to the fluid surrounding the extended surface is much faster in longitudinal fins of convex parabolic profiles and exponential fin profile with a negative power (e^{-r}).

In conclusion, the VIM and DTM were successfully applied to study theoretical heat transfer in fins of various profiles. Various models were derived and a series of articles with different targets were produced. Across all presented models, it was found that the temperature distribution in finned surfaces can be influenced by various embedding thermo-physical parameters. Since various models were derived and the impact of the physical parameters was extensively analyzed, the results could be useful in fin design and other engineering applications. Future work might involve modeling n-dimensional heat transfer in moving fins. The availability of analytical solutions that solve n-dimensional problems may necessitate the study.

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