School of Economics and Finance

Faculty of Commerce, Law, and Management

AN ANALYSIS OF FRAUD DETECTION USING BENFORD'S LAW AND THE BIAS RATIO

by

BHAVIK GOVAN
1876080

Supervisor: MR JAMES BRITTEN

Date of Submission:
31 March 2024

SCHOOL OF ECONOMICS AND FINANCE

Declaration Regarding Plagiarism

I (Full name & surname):	Bhavik Govan
Student number:	1876080

Declare the following:

1. I understand what plagiarism entails and am aware of the University's policy in this regard.

2. I declare that this assignment is my own, original work. Where someone else's work was used (whether from a printed source, the Internet or any other source) due acknowledgement was given and reference was made according to departmental requirements.

3. I did not copy and paste any information directly from an electronic source (e.g., a web page, electronic journal article or CD ROM) into this document.

4. I did not make use of another student's previous work and submitted it as my own.

5. I did not allow and will not allow anyone to copy my work with the intention of presenting it as his/her own work.

Bhavik Govan

31/03/2024

Abstract

This study explores the detection of fraud within the South African hedge fund industry through the utilisation of the bias ratio and Benford's law. An examination is conducted on a sample consisting of 83 hedge funds, encompassing both Qualifying Investor Hedge Funds (QIFs) and Retail Investor Hedge Funds (RIFs), to identify potential anomalies. Six funds with elevated bias ratios are flagged for further scrutiny, indicating possible fraudulent activities. Benford's law is applied to corroborate these findings, revealing non-conformity in all but one of the flagged funds. The study emphasises the importance of a multifaceted approach to fraud detection, combining various metrics and methodologies to enhance the overall understanding of a hedge fund's returns. While the bias ratio and Benford's law offer valuable insights, their application requires careful consideration of fund type and strategy. Regulatory intervention and investor vigilance are essential for safeguarding against fraudulent activities in the hedge fund industry.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

1 INTRODUCTION

The hedge fund industry is on the rise globally with assets under management (AUM) exceeding US$4 trillion by the end of 2021 (Chatterjee, 2022). The industry is projected to experience growth, rising from US$4.74 trillion in 2024 to US$5.47 trillion by 2029, representing a compound annual growth rate (CAGR) of 3.14% during the forecast period (Modor Intelligence, 2024). In South Africa the hedge fund industry grew its AUM from R86.93 billion at the end of 2021 to R113.01 billion (roughly US$5 to US$6 billion) at the end of 2022 (ASISA, 2023). According to ASISA's statistics, despite further consolidation and closure of funds, the assets saw a 19% growth, resulting in a decrease in the number of hedge funds from 233 to 216 by the end of 2021. These funds can be valuable as part of a well-balanced investment strategy (Sanlam, 2024). The role of hedge funds in financial markets has grown significantly, as the scale of the industry has expanded and its interconnectedness with other major financial market participants has increased (Barth, Joenvaara, Kauppila, & Wermers, 2023). Given the sheer size of hedge funds, they play an important part of the investment industry.

Fraud has cast a shadow over the investment industry, defined by Bollen and Pool (2009) as deceitful practices within hedge funds involving deliberate manipulation, misreporting, or fabrication of financial data to mislead investors and regulators. While large manipulations are clearly illegal, subtler alterations of returns may not always violate regulations (Bollen & Pool, 2012). Moreover, Fraud extends beyond Bollen and Pool’s (2009) definition to encompass a range of deceptive tactics, including inflating portfolio values and returns smoothing (van Vuuren & van Vuuren, 2022), aiming to distort a fund's performance and undermine market integrity. Such activities, including misrepresentation of returns and portfolio value inflation, harm investors and compromise market transparency, thereby highlighting the severity of the issue (van Vuuren & van Vuuren, 2022).

Over the course of history, the investment sector has witnessed numerous corporate scandals and cases of fraud, as managers capitalised on insufficient oversight to partake in unethical behaviours. Notably, the term "Ponzi scheme" originates from the infamous case of Charles Ponzi, who in the early 1900s promised a staggering 50% return in just 45 days, ultimately defrauding thousands of people (Benson, 2009). However, towering above this deception is the most notorious and largest financial fraud, that of Bernard L. Madoff Investment Securities. Bernie Madoff, a hedge fund manager, and prominent figure on Wall Street, orchestrated the largest Ponzi scheme in American history (Henriques, 2011). Madoff manipulated his clients by falsifying his returns and using investments from new clients to pay off earlier ones. Astonishingly, he managed to siphon off approximately $65 billion over several decades before being exposed in 2008 (Quisenberry, 2017). Beyond Madoff’s Ponzi scheme, there were other instances of hedge fund frauds and scandals that have exposed the inherent risks of hedge fund investing. Notable among these were SAC Capital's insider trading scheme (Ahern, 2017), Galleon Group's illicit use of corporate secrets which led to an insider trading scandal (Agapova & Madura, 2013), and Long-Term Capital Management's disastrous market bets (Edward, 1999).[1] All these examples, while they took place in the US, still inflicted substantial losses on investors and tarnished the industry's reputation. Being able to detect such fraud and risky behaviour in advance could mitigate many of the adverse consequences.

Hedge funds, characterised by flexible investment strategies and limited partnerships, aim to generate positive returns by employing speculative practices like leverage and short selling (SEC, 2023; Braun, Riutort, & Roche, 2024). Shadab (2009) posits that the favourable outcomes experienced by hedge fund investors stem from their relatively light regulatory environment, as these funds cater to large institutional investors exempt from extensive reporting requirements (Cumming & Dai, 2010). South Africa has implemented a regulated framework governing hedge funds, encompassing both fund managers and products, positioning it within the ranks of the world's most advanced operational environments (Fairtree, 2016). However, in general, hedge funds are not obligated to publicly disclose information, allowing them to evade substantial disclosure and record-keeping demands (Agarwal, Ruenzi, & Weigert, 2024; Gerritzen, Jackwerth, & Plazzi, 2024). This makes detecting fraud within the hedge fund industry challenging (van Vuuren & van Vuuren, 2022). While concerns about hedge funds' size and market manipulation potential have led to calls for stricter regulations, the effectiveness of such measures in curbing financial misconduct remains uncertain (Cumming & Dai, 2010).

Fraud within hedge funds involves misreporting returns, with evidence suggesting widespread manipulation to downplay losses and attract capital (Bollen & Pool, 2009). There is a concern

[1] According to Neagle (2024) insider trading greatly influences the stock market and can harm everyday investors. When key figures within a company act on privileged information, it creates market imbalances. This leads to stock prices being influenced by undisclosed data instead of reflecting the true value of the company. Consequently, regular investors may end up buying stocks at inflated prices or selling them below their actual worth. Making investment decisions based on incomplete information, influenced by speculative "tips" or market rumours, can lead to substantial financial losses.

regarding the manipulation of performance measures like the Sharpe ratio, as evidenced by fund managers striving to project an illusion of superior performance (Goetzmann, Ingersoll, Spiegel, & Welch, 2007; Auer, 2013).

Researchers have employed various metrics to identify fraud in hedge funds, with notable examples including the bias ratio, which is recognized for its straightforwardness and precision in detecting return smoothing (Douady, Abdulali, & Adlerberg, 2009; Bollen & Pool, 2012; van Dyk, van Vuuren, & Heymans, 2014; King & van Vuuren, 2016). The bias ratio serves as a critical indicator of potentially fraudulent activity within funds, functioning by comparing fund returns to unbiased returns, particularly around the zero percent return mark (Abdulali, 2013; van Vuuren & van Vuuren, 2022). Higher bias ratios are directly correlated with more smoothed returns, while a fund with normally distributed returns typically exhibits a bias ratio near one. However, relying solely on the bias ratio may yield incomplete results; therefore, additional metrics are often necessary to gain a comprehensive understanding (Douady *et al.*, 2009; van Vuuren & van Vuuren, 2022). Previous research has supported the effectiveness of the bias ratio, particularly when combined with other indicators such as kurtosis, skewness, and the Sharpe and Omega ratios (van Dyk *et al.*, 2014; King & van Vuuren, 2016). Integrating findings from the bias ratio with other metrics can enable the early and reliable detection of fraud, potentially mitigating losses from illegal activities such as return manipulation and insider trading (van Vuuren & van Vuuren, 2022).

One metric that has not yet been extensively tested in the context of hedge fund returns is Benford's Law. Benford's Law is a mathematical principle that predicts that in many naturally occurring datasets, the leading digits follow a specific distribution. This characteristic has been effectively utilised in fraud detection within financial data, highlighting deviations from expected patterns. For example, large Sharpe ratios analysed alongside Benford's Law have been used in auditing to detect manipulation (Durtschi, Hillison, & Pacini, 2004; van Vuuren, 2014). The application of Benford's Law extends to stock indices and finance event studies, providing a valuable tool for identifying anomalous patterns (Ley, 1996; Nigrini, 2015; Ausloos, Ficcadenti, Dhesi, & Shakeel, 2021). By comparing the observed distribution of leading digits in fund returns with the distribution predicted by Benford's Law, potential fraud can be detected.

Researchers have proposed applying Benford's Law for fraud detection in hedge funds, particularly when used in conjunction with the bias ratio (Douady *et al.*, 2009; van Dyk *et al.*, 2014). The selection of Benford’s Law for this study is motivated by its proven effectiveness in identifying irregularities that may indicate manipulation or fraudulent activity (Nigrini, 1999). Hedge funds, known for their complex and often opaque financial strategies (Braun *et al.,* 2024), present a unique challenge for traditional fraud detection methods. These funds frequently engage in sophisticated trading activities, making them particularly vulnerable to subtle forms of financial manipulation. Applying Benford’s Law in this context offers a robust statistical approach to uncovering discrepancies in reported returns that might otherwise go unnoticed. Despite its application in various financial contexts, the use of Benford’s Law in the hedge fund industry remains underexplored, highlighting a significant gap in the literature that this study aims to address.

The impetus for detecting and preventing fraud within the financial industry is driven by several critical factors, including protecting investors from financial losses (Hurwitz, 2019), maintaining market integrity (Pearson & Pearson, 2008), and ensuring legal and regulatory compliance (Johnston, 2015). This study is motivated by these concerns and aims to enhance fraud detection mechanisms using Benford’s Law and the bias ratio. A proactive approach to fraud detection not only mitigates the long-term repercussions of fraud but also bolsters market integrity and safeguards investors' interests (Deloitte, 2022).

While fraud detection methods like the bias ratio, when coupled with other measures, have proven effective in hedge funds, several key questions remain: Can these methods be extended? Can other fraud detection methods, such as Benford’s Law, commonly used in accounting, be applied to hedge funds? This study is the first of its kind to utilise the bias ratio in conjunction with Benford’s Law, aiming to enrich the literature by exploring these two metrics within a South African context. The primary research objectives are to investigate the effectiveness of these tools in detecting fraud within hedge funds, assess their applicability in identifying anomalies and potential fraudulent activities, and evaluate their alignment with market indices and hedge fund returns. The study also aims to contribute to the development of more robust fraud detection measures and enhance the trustworthiness and security of the financial landscape. Following the methodology of King and van Vuuren (2016) and van Vuuren and van Vuuren (2022), the bias ratio and Benford’s Law are first applied to a known case of fraud—Fairfield Sentry, a feeder fund associated with Bernie Madoff Investment Securities. The same methodology is then applied to a sample of South African hedge funds. The results demonstrate that the bias ratio, when used in conjunction with Benford’s Law, can indeed serve as an effective early warning tool for detecting fraud.

The remainder of this study is organised as follows: Section 2 presents a review of the existing literature on hedge funds, exploring their strategies, regulatory landscapes, and the prevalence of fraudulent practices within the industry, providing a foundation for the subsequent analysis. Section 3 delves into the data and methodology applied to examine the hedge fund industry in South Africa, outlining the approach taken in this study. Following this, Section 4 presents the study's results and engages in a discussion of their implications. The study concludes in Section 5, summarising key findings and offering insights for future research and practical applications.

2 LITERATURE REVIEW

2.1 Understanding Hedge Funds

The term "hedge fund" can be misleading as it implies that all funds utilise hedging strategies to mitigate risk, which is not always the case. While some funds employ traditional hedging techniques to reduce risk, others may not hedge exposures at all (CFA Institute, 2023). Many hedge funds have given investors double-digit returns, frequently giving the impression that they are unaffected by changes in the market as a whole and showing comparatively little volatility (Getmansky, Lo, & Makarov, 2004). Typically, to achieve this, one keeps both long and short positions in securities. Because of the simultaneous long and short positions, this approach allows investors to profit from both upward and downward market movements while maintaining market neutrality (Getmansky *et al.,* 2004).

Hedge funds gather capital from investors and employ it collectively with the aim of generating profitable returns. They raise funds from a small number of accredited investors and through private placements (El Kalak, Azevedo, & Hudson, 2016). They represent the investor class that closely resembles textbook arbitrageurs, employing advanced trading strategies (Ben-David, Franzoni, & Moussawi, 2012). These funds have the capability to establish significant short positions, leverage extensively, and utilise derivatives (Sgambati, 2024). Additionally, they lock up investor capital to invest in illiquid assets and pursue convergence trades with uncertain payoff horizons; these activities promote price efficiency and provide liquidity to less liquid markets (Barth *et al.,* 2023). Hedge funds typically exhibit more intricate structural and risk attributes compared to mutual funds and other conventional investment instruments (Getmansky, Lee, & Lo, 2015). Furthermore, numerous hedge funds aim to generate profits across various market conditions by employing speculative investment strategies that are less common among other fund managers (SEC, 2023).

In general, hedge funds are set up as limited partnerships, with managers functioning as general partners and investors as limited partners (Fung & Hsieh, 1999). To ensure that the financial interests of all partners are aligned, hedge fund managers often contribute substantial personal capital, or "skin in the game," into their funds. While these allocations can better align incentives, managers may also strategically allocate their private capital in ways that negatively affect outside investors. Gupta and Sachdeva (2024) find that funds with more inside investment outperform other funds within the same family, suggesting that managers may use insider capital to enhance performance in less scalable strategies while restricting outside investment. Additionally, investors are subject to a performance-based fee, which can result in significantly higher payouts for successful managers compared to the fixed management fee.

2.1.1 Hedge Fund Fee Structures

Two components usually make up the fee structure: an incentive fee based on performance (much like options) and an asset-based (or flat-rate) management fee. Additionally, there are "high-watermark" and "hurdle rate" provisions pertaining to the performance-based fee (El Kalak *et al.*, 2016; Braun *et al.*, 2024).[2, 3] According to Goetzmann, Ingersoll, and Ross (2003), hedge funds adopt fee structures resembling options because of the constrained capacity of hedge fund strategies. Hence, recent strong performance cannot be compensated with fees that rise proportionally with AUM. Previous research has examined the influence of "incentive fees" on hedge fund performance. These fees are significant to investors not just because they directly affect returns, but because they may also influence manager behaviour (Getmansky *et al.,* 2015).

Studying the relationship between incentive fees and risk-adjusted performance, Liang (1999), Ackermann, McEnally, and Ravenscraft (1999), and Edwards and Caglayan (2001) all found a statistically significant positive relationship. Kouwenberg and Ziemba (2007) investigate how

[2] A high-watermark is a clause in a contract that requires losses incurred in a particular year to be taken into account for determining incentive fees. This ensures that incentive fees are only calculated on net profits, which are profits after deducting any cumulative losses from earlier periods (Eurekahedge, 2021).

[3] A hurdle rate is the minimal rate of return required in order for the hedge fund manager to be able to charge a performance fee (Shin, Smolarski, & Soydemir, 2017).

managers' personal investments and incentive fees affect the investing strategies of hedge fund managers. According to their findings, when faced with increased incentive fees, managers who are risk averse have a tendency to increase the risk involved in the fund's strategy. On the other hand, when managers put a substantial portion of their own money (at least 30% percent) in the fund, their propensity to take risks declines significantly.

During the lockup period, investors are prohibited from selling or redeeming their shares, constituting a specific timeframe of restricted liquidity for investors.[4] This period of time, along with notice and redemption periods, are characteristics of hedge funds that impose limits on cash withdrawals, aiming to mitigate liquidity challenges (El Kalak *et al.*, 2016). It has been suggested that implementing such limitations on investors can potentially lead to increased returns through the pursuit of long-term arbitrage strategies. Aragon (2007) finds that hedge funds subject to lockup period constraints tend to yield greater excess returns compared to those without such restrictions. In a different study, Schaub and Schmid (2013) examined how share restrictions affected hedge fund performance during times of crisis and non-crisis. They observed that less liquid funds typically produced higher returns and alphas during non-crisis periods. This suggests that there is a share illiquidity premium that exists to offset the liquidity restrictions. However, this premium turns into an illiquidity discount during a crisis, indicating that heightened managerial discretion may be detrimental during severe financial market crises. Moreover, Aragon (2007) shows that share restrictions and portfolio liquidity of the fund have a negative relationship.

Such restrictions on investor redemptions are enforced by many hedge funds, which makes hedge funds an illiquid investment (Schaub & Schmid, 2013). According to Getmansky *et al.*, (2015), illiquidity stands out as a critical feature in certain hedge-fund investments. Not only does illiquidity often play a central role in financial crises, but it also forms a fundamental aspect of how many financial investments yield returns. While adopting the risk of illiquidity stands as a potent strategy for long-term investors striving to build wealth, it can also lead to catastrophe for investors who underestimate their vulnerability to illiquidity and attempt to sell off particular investments at unfavourable times.

[4] Hedge funds restrict redemption opportunities compared to mutual funds, often with limited redemption intervals (e.g., monthly, quarterly, annually) and a lock-up period of one year or more. During this time, the investor cannot cash in their shares, risking potential loss of value. Additionally, hedge funds may impose redemption fees before allowing cashing out (SEC, 2023).

2.1.2 Hedge Fund Classes and Strategies

Hedge funds can be broadly categorised into three main groups: macro funds, global funds, and relative value funds (Eichengreen & Mathieson, 1999). Macro funds typically take large directional positions in national markets, informed by top-down analyses of macroeconomic and financial variables such as the current account, inflation rate, and real exchange rate. In contrast, global funds engage in global investing but focus on selecting equities based on the prospects of specific companies through a bottom-up approach. Relative value funds aim to predict the relative prices of closely related securities, often employing strategies that capitalize on price discrepancies between them.

Within these broad categories, hedge fund strategies are further grouped by "style" and "location" (Fung & Hsieh, 1999). A fund manager's style may involve taking long or short positions, speculating on specific company events, or adopting a market-neutral approach. The location refers to the asset class in which the hedge fund primarily invests. Connor and Woo (2004) argue that categorising hedge funds by their investment strategies is crucial for comparing performance, risk, and other characteristics. These strategies may range from market-neutral approaches, which exhibit minimal correlation with the overall market, to directional approaches that anticipate specific market movements (Connor & Lasarte, 2003). Decision-making in hedge funds can vary from systematic methods, utilising computer models, to discretionary approaches based on human judgment. Often, hedge funds employ multiple strategies simultaneously, allocating assets internally across these strategies in proportion to the opportunities they present. This categorisation is essential for understanding and comparing the performance and attributes of different hedge funds.

2.1.2.1 Specific Hedge Fund Strategies

Capocci (2013) provides further insight into specific hedge fund strategies that are commonly employed:

- Fund of Funds involves investing in a diversified portfolio of other hedge funds, with the aim of reducing risk by spreading investments across multiple underlying funds. By doing so, investors gain indirect access to a variety of hedge funds, which enhances diversification and transparency.

- Fixed Income hedge funds focus on delivering positive returns across varying market conditions by exploiting price discrepancies in related securities. These strategies often involve significant leverage, which can increase both risk and potential reward.
- Long/Short Equity involves taking long positions in undervalued stocks and short positions in overvalued stocks, allowing managers to benefit from both rising and falling markets. Typically, these strategies include a long bias and may use leverage and technical analysis to enhance returns.
- Market Neutral strategies aim to balance long and short positions to neutralise market risk, often employing quantitative models to exploit pricing anomalies. These portfolios are usually highly diversified, reducing their exposure to broader market movements.
- Single Manager Multi-Strategies involve a single manager implementing a variety of hedge fund strategies within one portfolio, allowing for diversified risk management and dynamic allocation based on prevailing market conditions.

These strategies, as detailed by Capocci (2013), underscore the diversity of approaches within the hedge fund industry. Each strategy offers a unique risk-return profile, tailored to different market conditions and investment objectives.

2.1.3 Regulatory Landscape and Oversight

While it is a common belief that hedge funds operate in a regulatory vacuum, a closer look shows that hedge funds are designed to capitalise on regulatory exemptions (Connor & Woo, 2004). Although hedge funds are designed for knowledgeable, wealthy private investors, Fung and Hsieh (1999) argue that these exemptions are justified because regulations are designed for the general public. In contrast to banks, mutual funds, and other financial establishments, hedge funds function as pooled private investment entities, primarily circumventing regulatory rules. They are not required, for instance, to provide financial markets, regulators, or databases with certain information regarding their asset holdings, performance, or risk management practices. They are known for being the least transparent major player in the financial markets, in part because of this lack of disclosure (El Kalak *et al.,* 2016). While they are subject to laws and regulations, they are still freer than other types of funds (Kurdas, 2009). As outlined by Getmansky *et al.,* (2015), because their techniques are proprietary, hedge funds enjoy greater discretion in their investing mandates and provide investors with less transparency.

Nonetheless, the possibility of fraud and operational hazards in hedge funds are of far greater concern to investors.

Hedge funds traverse varying degrees of regulatory landscapes in different countries globally. Notably, the extent of hedge fund regulation diverges internationally, characterised by key factors such as minimum capital prerequisites for hedge fund managers, distribution limitations, and geographical constraints (Cumming & Dai, 2010). The allure of hedge funds rests in their expansive scope of investment possibilities, facilitated by their relatively unburdened regulatory environment, allowing for flexibility and manoeuvrability (Jorion & Schwarz, 2014).

Over the last two decades, there have been multiple regulatory changes pertaining to hedge funds in South Africa. In 2015, a significant turning point in their regulation occurred when they were properly categorised as financial products (Fairtree, 2016). Qualified Investor Hedge Funds (QIFs) and Retail Investor Hedge Funds (RIFs) are the two primary categories into which hedge funds are divided under this regulatory structure[5,6]. Investors must give one calendar month's notice before withdrawing their funds from QIFs, which are valued monthly. RIFs, on the other hand, let investors withdraw their money within a month and are valued daily (Sanlam, 2024). Under the oversight of the Collective Investment Schemes Control Act (CISCA), these hedge funds are classified as regulated collective investment schemes, akin to unit trust portfolios (ASISA, 2023). Oversight over hedge funds is entrusted to the Financial Sector Conduct Authority (FSCA). South Africa has instituted a regulated framework for hedge funds, which encompasses oversight of both fund managers and products. This places it among the world's most advanced operational environments (Fairtree, 2016). This heightened regulatory attention is perceived as less risky compared to globally-operating hedge funds for two key reasons. Firstly, in the South African hedge fund industry, 90% of the total AUM can be liquidated in ten days or less. Secondly, most hedge funds in South Africa do not have an initial lock-up period (King & van Vuuren, 2016). Foreseeing an improvement in transparency and governance standards, the hedge fund industry in South Africa anticipates that its

[5] Qualified Investor Hedge Funds necessitate a minimum investment of R1 million and are accessible to investors possessing a robust comprehension of hedge fund investment strategies and the accompanying risks (ASISA, 2023).

[6] Retail Hedge Funds are subject to strict regulations concerning their investments and allowable risks, and they are available to all investors capable of meeting the average minimum lump sum investment requirement of R50,000 (ASISA, 2023).

regulatory enhancements will boost investor confidence as the regulations become more entrenched and comprehended (Fairtree, 2016).

2.2 Misreporting Returns and Fraudulent Practices

During the late 1990s and early 2000s, a series of corporate scandals cast a shadow on companies such as Enron (Petrick & Scherer, 2003), WorldCom (Scharff, 2005), and Tyco (Enobong, 2017). These scandals originated from dubious accounting practices, such as manipulating revenue recognition, mark-to-market valuation, and utilising special-purpose vehicles to obscure debt. Similar deceptive trends extended to the hedge fund industry, as evidenced by debacles involving entities like the Bayou Hedge Fund Group that overstated their performance (Muhtaseb & Yang, 2008), the Lancer Management Group that violated anti-fraud provisions (Muhtaseb, 2021), and Bernie Madoff Investment Securities that admitted that it was a giant Ponzi scheme (Quisenberry, 2017). These instances were marked by fund managers inflating portfolio values and, in some egregious cases, fabricating returns. All of this unfolded against a backdrop of minimal oversight (Bollen & Pool, 2012).

As an example, Bollen and Pool (2009) highlight the case of Bayou Management, where in 2003, the hedge fund misrepresented an actual loss of $49 million as a gain of $43 million. While explicit instances of illegal conduct are strictly forbidden, less overt manipulations of returns may not inherently breach the antifraud regulations of the SEC (Bollen & Pool, 2012). In their study, Bollen and Pool (2009) use a discontinuity in the monthly hedge fund return distribution to examine patterns of misreporting in hedge funds. They indicate widespread misreporting by exposing a clear abnormality in the monthly hedge fund return distribution. In particular, hedge fund managers frequently falsify Net Asset Values (NAVs) in order to reduce the frequency of their reported losses. Their findings suggest that managers tend to distort returns strategically, particularly when returns are within their discretion and are less stringently monitored. Cassar and Gerakos (2011) associate the zero discontinuity of returns with a manager’s ability to influence fund asset prices. On the regulatory side of the zero discontinuity, Cumming and Dai (2010) provided some insight, noting that returns in jurisdictions that are allowed to be distributed through wrappers are more likely to contain fraudulent information.[7] Contrary to the notion that a discontinuity at zero in the distribution of hedge fund net returns suggests managers manipulate returns to avoid displaying minor losses, Jorion and Schwarz (2014) illustrate that incentive fees can introduce a kink in the net return distribution, clarifying the observed kink in the large, liquid Long-Short Equity styles. Distribution discontinuities can also be caused by yield capping at zero and asset illiquidity. Jorion and Schwarz (2014) therefore conclude that there is no inherent indication of manipulation in the observed discontinuities in hedge fund returns.

The findings of Bollen and Pool (2009) indicate that certain managers may manipulate returns, particularly in circumstances where they have discretion over fund returns and where their reported returns receive minimal scrutiny. Evidence presented by Agarwal, Daniel, and Naik (2011) highlight a strong correlation between fund flow and the frequency of positive returns. Considering that hedge fund fee structures primarily depend on AUM and profits, managers face substantial incentives to potentially inflate NAVs. This framework fosters an environment wherein managers actively aim to perpetually draw in capital through the presentation of absolute returns, all while striving to attain returns that consistently optimise risk-adjusted performance (Bollen & Pool, 2008). Motivated by incentives linked to AUM, managers may manipulate reported returns to allure and retain investors or, in extreme cases, perpetrate outright fraud (Jorion & Schwarz, 2014). In instances as extreme as Ponzi schemes, managers might exploit misreported returns as the primary method to attract capital influx (Bollen & Pool, 2012).

2.2.1 Hedge Fund Flows and Returns

Numerous researchers have observed a positive empirical relationship between fund flows and recent performance, indicating that investors in hedge funds typically favour positive returns and pull away from unfavourable ones (Goetzmann *et al.*, 2003; Baquero & Verbeek 2009). But the relationship between fund flows and investment performance is usually nonlinear and dependent on a number of variables, including investor perceptions, market conditions, fund limitations, manager alpha, and other characteristics of hedge funds (Getmansky *et al.*, 2015).

[7] The legal or financial frameworks that are used to package and distribute investment products are commonly referred to as wrappers. Fund managers seem to be able to evade the same degree of scrutiny that other distribution channels offer due to these connected products.

High-watermarks and increased investor flow sensitivity to historical performance are related, as Aragon and Qian (2010) show. This sensitivity decreases during periods of underperformance. Using a regime-switching model to look at inflows and outflows separately, Baquero and Verbeek (2009) investigate the relationship between fund flows and performance. According to their research, fund inflows had a somewhat positive correlation with historical performance throughout quarterly periods, whereas fund outflows had a significantly higher positive correlation. But over the course of a year, this pattern varies. Teo (2011) examined a group of liquid hedge funds and found that, when risk variables are taken into account, funds with large net inflows often beat funds with minimal net inflows by 4.79% per year. This suggests that new investment inflows gravitate toward the top performers, hinting at a potential motivation for fund managers to manipulate their reported returns.

2.2.2 Gaming the System

Several metrics have been suggested to assess the effectiveness of active fund management (Goetzmann *et al.*, 2007). Performance measures play a pivotal role in evaluating a fund's financial performance. However, the manipulation of performance measures is a concern. In the absence of further information, fund managers have a motivation to potentially manipulate expected performance scores, as their compensation typically depends on these scores. Consequently, investors choose funds based on their performance scores (Brown, Kang, In, & Lee, 2010).

As an example, the Sharpe ratio was introduced as a metric for evaluating a mutual fund's performance (Sharpe, 1998). While this ratio is appropriate for assessing funds with returns following a normal distribution, hedge fund returns often deviate significantly from this distribution. Eling and Schuhmacher (2007) compare the Sharpe ratio to 12 other performance metrics in a large sample of hedge funds. Their results show that the sample of hedge funds has a uniform rank ordering.

Investors typically allocate capital to funds with the highest Sharpe ratio (Botha, 2007 as cited in King & van Vuuren, 2016). However, in the current body of literature, it is apparent that the Sharpe ratio can be influenced or manipulated (Goetzmann *et al*., 2002, 2007; Auer, 2013). The manipulation of the Sharpe ratio often revolves around emphasising the potential for high returns, thus skewing the distribution towards elevated left-tail risk (Zakamouline & Koekebakker, 2009).

Goetzmann *et al.,* (2007) illustrate how managers aiming to manipulate commonly utilised performance metrics are able to produce notably impressive statistics. They present an example of a basic rebalancing strategy that consistently yields market-beating Jensen's alpha figures, even in scenarios with unrealistically high transaction costs accounting for 20% of each dollar traded. This illustrates how skilful manipulation can distort performance metrics to project an illusion of superior fund performance.

Consequently, fund managers are incentivised to manipulate their returns and hence their performance metrics to create the appearance of superior performance. This strategy enables them to attract more capital and earn significant profits from incentive fees. Detecting and preventing this behaviour is crucial as it would protect investors from the harmful effects of manipulation.

2.3 Uncovering Fraud

Diverse metrics have been employed to spotlight potential instances of fraudulent activity. Bollen and Pool (2012) utilised a set of quantitative algorithms or performance flags to pre-identify an increased risk of hedge fund fraud, solely relying on return data. These performance flags were founded on the detection of suspicious patterns in reported fund returns indicative of fraudulent activities. Bollen and Pool (2008, 2009) carried out analyses in their earlier research, including Gaussian kernel density estimations, time series analysis, and conditional serial correlation and discontinuity analysis. Getmansky *et al.,* (2004) argue that the serial correlation detected in hedge fund returns is frequently attributed not to untapped profit opportunities but to the inclusion of those securities which are illiquid in the fund's portfolio.

In order to expand their research, Bollen and Pool (2009) repurposed a manipulation-proof performance measure (MPPM) developed by Goetzmann *et al.,* (2007) in order to identify possibly suspicious funds. This new metric was based on the MPPM, which was developed from a utility function assessing the implied risk aversion of a fund. In addition, a measure based on the MPPM called the Doubt Ratio was created to identify funds that may have falsified returns or altered scores for performance (Brown *et al.,* 2010).

The Sharpe ratio has been used to help regulators and stakeholders detect fraud by assessing the realism of reported performance, as discussed by Bernard and Vanduffel (2014). In contrast, Douady *et al.,* (2009) argue that the bias ratio, invented by Abdulali (2013), is more effective in identifying fraud than the Sharpe ratio.[8] The bias ratio, simple yet precise, can detect return smoothing and manipulation in hedge fund data. The appeal of employing the bias ratio as a measure lies in its simplicity, requiring only returns as inputs. Nonetheless, it is crucial to note that relying solely on the analysis of the bias ratio may yield incomplete results (van Vuuren & van Vuuren, 2022). Complementary metrics are employed to enhance the depth of understanding regarding the observed outcomes (Douady *et al.,* 2009).

Van Dyk *et al.,* (2014) evaluate the importance of using the bias ratio in addition to the Sharpe ratio when evaluating hedge fund risk and making investment decisions. Their study found that the bias ratio offers additional insights beyond the Sharpe ratio. King and van Vuuren (2016) investigated using the bias ratio to detect financial wrongdoing in hedge funds, comparing it with the Sharpe ratio. They found it effective in identifying fraud, illustrated by its early detection of a known case of fraud. They expanded their study to 20 South African hedge funds, uncovering further potential fraud. A similar study by van Vuuren and van Vuuren (2022) examined a broader dataset, finding that the bias ratio was crucial when combined with the first four statistical moments. Their research emphasised the bias ratio's versatility in detecting anomalies in hedge fund returns. Not only does it identify manipulation, but it also encompasses anomalies within returns themselves, including those related to insider trading.

2.3.1 The Law of Anomalous Numbers

Originally published as The Law of Anomalous Numbers by Benford (1938), Benford's Law identifies a distinctive pattern in the distribution of the leading digits in a variety of numerical data sets. It displays a particular logarithmic pattern found in many naturally occurring datasets which defies uniform predictions.

After analysing stock indices, Ley (1996) found that Benford's law was followed by the 1-day returns on the Standard and Poor's Index (S&P) and the Dow-Jones Industrial Average Index (DJIA). This alignment categorises these indices as anomalous numbers, as Ley (1996) aptly describes. Benford’s law also reveals expected frequency distributions for digits within tabulated data, including daily stock returns and volumes. Nigrini (2015) notes that accounting

[8] The bias ratio was initially filed for a patent in 2006. This revised patent asserts the advantages of U.S. Provisional Patent Application No. 60/859,838, filed on 16 November 2006, titled "Bias Ratio Measuring the Shape of Fraud." The entire content of that application is incorporated herein by reference.

and finance event studies, along with expected and abnormal returns, further support this alignment with Benford's principles. Benford's law was also used by Ausloos *et al.*, (2021) to evaluate S&P 500 daily closing values and log-returns over a timeframe longer than 60 years. Their analysis includes frequencies of significant digits and disaggregated distributions at five levels. They find stronger adherence to Benford’s law in log-returns compared to daily closing values.

The significance of Benford’s law transcends its mathematical elegance, finding practical application in the field of accounting. It has proven instrumental in detecting anomalies (Nigrini, 1999; Durtschi *et al.*, 2004; Tam Cho & Gaines, 2007; Asllani & Naco, 2015) and has been utilised alongside large Sharpe ratios in auditing to flag potential manipulation (Durtschi *et al.*, 2004; van Vuuren, 2014). Moreover, its use in evaluating the accuracy of information disclosed in a company's financial statements has gained widespread recognition (Jianu & Jianu, 2021). Previous researchers have proposed using Benford’s law and the bias ratio to detect fraud (van Dyk *et al*., 2014; Douady *et al*., 2009).

This literature review provides an overview of hedge funds, exploring their strategies, fee structures, regulatory landscapes, and the prevalence of misreporting and fraudulent practices within the industry. Despite the term "hedge fund" suggesting risk mitigation through hedging, not all funds employ such strategies. Instead, they use sophisticated trading methods, often with high leverage and illiquid assets, contributing to market efficiency and liquidity provision. Fee structures, including performance-based incentives, drive fund manager behaviour and may incentivise manipulative practices. Regulatory oversight varies globally, with enhancements in South Africa aiming to increase investor confidence. However, instances of misreporting returns and outright fraud persist, highlighting the need for robust detection methods. Metrics like the bias ratio and Benford's law may offer insights into potential anomalies and fraudulent activities, highlighting the importance of vigilance in safeguarding investor interests.

3 DATA AND METHODOLOGY

3.1 Data

Obtaining hedge fund data presents a significant hurdle due to its limited accessibility. The regulatory framework surrounding hedge funds adds to this challenge, as QIFs typically report their returns monthly and RIFs report daily (Sanlam, 2024). However, these data are often proprietary and difficult to obtain. Despite these barriers, the hedge fund data used in this study were acquired from Hedge News Africa, laying the groundwork for analysis within the constraints of the available information.

Monthly returns spanning from April 1998 to July 2023 were gathered for a comprehensive analysis of South African hedge funds. This inclusive dataset encompassed 88 QIFs and 87 RIFs. Benford's law works best for detecting fraud through lack of conformity in larger datasets (Ausloos *et al.,* 2021). To ensure robustness and reliability of the analysis, only funds with a substantial track record of more than 10 years or 120 months of available data during the specified timeframe were incorporated into the final sample. Consequently, the examination focused on a select group of 83 hedge funds that met these criteria.

Following King and van Vuuren (2016), the JSE All Share Index (ALSI) is used as the "non-fraudulent" comparison index for the analysis to establish a reliable standard to assess the performance of hedge funds throughout the period of study. This benchmark not only served as a crucial reference point but also enabled a substantial measure of whether any fraudulent activities may have occurred or could be suspected within the evaluated data.

3.1.1 Hedge Fund Styles

As discussed in Section 2.1.2, hedge funds in South Africa employ a variety of investment strategies, each with distinct approaches to managing risk and generating returns. Fung and Hsieh (1997) found that the methods employed by hedge funds are highly dynamic and behave differently from that of mutual funds. Hedge funds in South Africa are arranged according to their distinctive methods of investing. By the end of 2022, Long Short Equity was the most widely used hedge fund strategy in South Africa, drawing a sizable amount of capital. In particular, 59.2% of retail capital and 45% of funds from qualified investors were captured by Long Short Equity hedge funds (ASISA, 2023). These types of funds generate returns primarily from equity market positions, regardless of the specific strategy used, which can include both "market neutral" and "long bias" approaches. Conversely, investments in fixed income funds are distributed among derivatives and other products that are sensitive to changes in interest rates. Portfolios that do not rely solely on one asset class to generate investment opportunities over time are comprised of multi-strategy funds. Instead, they integrate a diverse array of strategies and asset classes, avoiding the dominance of any single asset class over an extended period. The strategies used in this data sample are as follows:

- Fund of Funds
- Property Focused Hedge [9]
- Fixed Income
- Long Short Equity
- Market Neutral and Quantitative Strategies
- Single Manager Multi-Strategies

3.2 Methodology

3.2.1 The Bias Ratio

The bias ratio, invented by Abdulali (2013), has been widely recognised as a straightforward yet powerful tool in detecting return smoothing, which is often indicative of fraudulent activity within hedge funds. Its simplicity, relying solely on return data, makes it particularly suitable for studies where data availability may be limited, as is often the case with hedge funds. Given the documented instances of return manipulation in the hedge fund industry (van Dyk *et al.*, 2014; King & van Vuuren, 2016), employing the bias ratio allows this study to effectively uncover subtle yet potentially fraudulent practices that other metrics might miss. This study applies the bias ratio as it aligns with the goal of detecting nuanced forms of fraud in a sector known for its opacity. The bias ratio can be represented mathematically in the following manner:

$$Bias\ Ratio = BR = \frac{Count\ (r_i){:}\, r_i \in [0, +1.0\sigma]}{K + Count\ (r_i){:}\, r_i \in [-1.0\sigma, 0)} \tag{1}$$

Where:

- i represents the time index

[9] Property Focused Hedge typically refers to a hedge fund strategy the concentrates on real estate or property-related investments.

- r_i denotes the return for month i
- σ is the standard deviation
- K is a positive, non-zero constant. $K = 1$ to avoid division by zero.
- $\in$ indicates that r_i is within the closed interval $[0, +1.0\sigma]$ in the case of the numerator and within the half open interval $[-1.0\sigma, 0)$ for the denominator.

Formally, the bias ratio can be represented mathematically as a continuous function:

$$BR = \frac{\int_0^{+\sigma} r\,dr}{K + \int_{-\sigma}^{0} r\,dr} \tag{2}$$

The function adheres to the following guidelines:

$$0 \leq BR \leq n$$

If $r_i \leq 0, \forall i$ then $BR = 0$

If $\forall\, r_i$ such that $r_i > 0, r_i > 1.0\sigma$ then $BR = 0$

As $n \rightarrow \infty$, $BR \rightarrow 1.0$ If the return distribution is normal, and $\mu = 0$, where μ represents the mean of the returns.

The region between $(0, +1\sigma)$ is the numerator of Equation (1). The concentration of returns from zero to one standard deviation away from zero is measured by this. The area under the return histogram from minus one standard deviation away from zero to zero is the denominator of Equation (1). As with the numerator, the denominator indicates the proportion of returns that fall into this range.

An entirely unbiased distribution would demonstrate total normality, resulting in a bias ratio of one (Abdulali, 2006, as cited in van Vuuren & van Vuuren, 2022). This ratio can be used as a measure of the distribution's symmetry and the extent to which returns diverge from an unbiased distribution. While the distribution of the returns demonstrates normality, the presence of fat tails suggests extreme negative and positive returns that surpass the expectations of a normal distribution. Additionally, empirical evidence suggests that the central region of the distribution is narrower than that of a typical normal distribution, signalling leptokurtotic characteristics in the returns (van Vuuren & van Vuuren, 2022).

The bias ratio should be close to one for a fund or portfolio that has a high level of liquidity, such as an equity index, but it will be significantly higher than one for a fund that holds highly illiquid securities (Abdulali, 2013). On the other hand, when dealing with illiquid securities, a large (or rising) bias ratio does not by itself demonstrate return manipulation because illiquid stocks typically produce smoother returns by nature (Le Marois, 2008, as cited in van Dyk *et al.,* 2014). However, it is crucial to understand that return smoothing doesn't always indicate unfair manipulation of the NAV; it might just mean that the value is determined subjectively rather than objectively, perhaps through an in-house process (van Dyk *et al.,* 2014).

Abdulali (2013) claims that the bias ratio varies depending on the strategy used: The bias ratios of funds that use illiquid assets might differ markedly from the indices that represent the underlying asset class. Fund managers may have some discretion in determining the NAV of their illiquid, difficult-to-price assets. A manager may include serial correlation as a side effect when returns are smoothed by marking securities aggressively in poor months and conservatively in good months. The manager has less room to distort data the more liquid the fund's securities are (Abdulali, 2013).

Serial correlation of returns is a common symptom of illiquid securities, therefore serial correlation or an abnormally high Sharpe ratio may be cause for concern. Still, these might not provide a definitive verdict on their own. Prior research shows a statistical association between the liquidity of different asset strategies and their bias ratios, hence endorsing the use of the bias ratio as a measure of return smoothing (Le Marois, 2008, as cited in van Dyk *et al.*, 2014). As such, the bias ratio can be useful in identifying instances of inappropriate illiquid securities and in detecting manipulation of NAV, including subjective pricing. However, it is crucial to evaluate the bias ratio in context of the strategy's liquidity (van Dyk *et al.,* 2014).

3.2.2 Benford's Law

Benford's Law, a mathematical principle that predicts the frequency distribution of digits in naturally occurring datasets, has proven to be a potent tool in forensic accounting and fraud detection (Nigrini, 2015). Its application in financial datasets is particularly compelling given the law's sensitivity to anomalies that may indicate manipulation. In the context of this study, Benford's Law serves as an independent verification tool, complementing the bias ratio by offering a different lens through which to detect irregularities in reported returns. The combination of these two metrics is designed to enhance the robustness of the fraud detection methodology, particularly in an industry where traditional oversight mechanisms may be inadequate.

Benford's law is given as

$$P(d_1) = \log\left(\frac{d_1 + 1}{d_1}\right) = \log\left(1 + \frac{1}{d_1}\right) \quad (3)$$

For $d_1 = 1, 2, 3, \dots, 9$, where log is the base-10 logarithm, and $P(d_1)$ is the probability that the integer has the first non-zero digit d_1. The leftmost non-zero digit of an integer is its first significant digit. The largest digit, 9, should appear as the first digit with the least proportion (4.6%), whereas the smallest digit, 1, should appear as the first digit with the highest proportion (30.1%), according to equation (3). The leading digits will therefore have a distribution as shown in Figure 1.

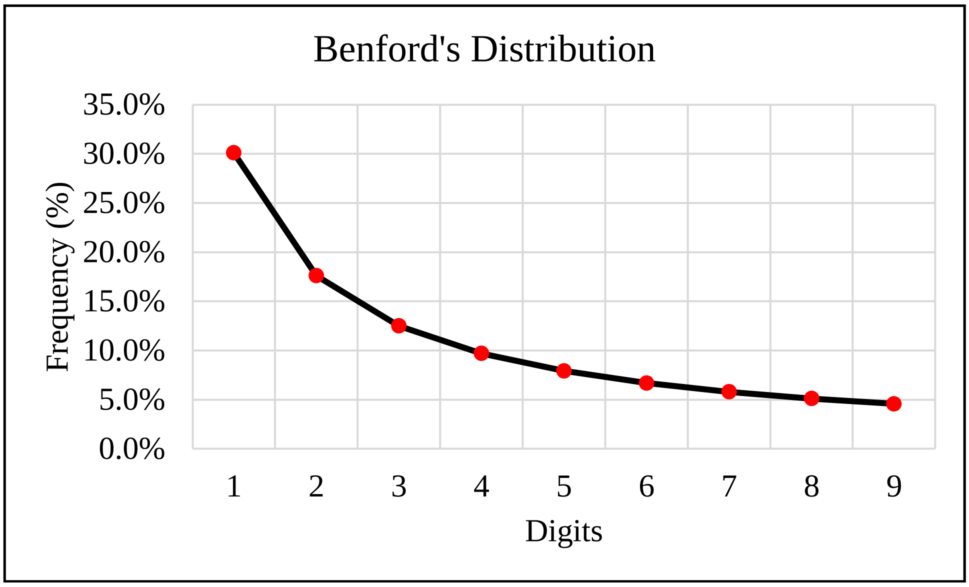

Figure 1: The distribution of digits according to Benford's law

Benford's law for the first digit thus yields N_{d_1}, the number of times the integer $d_1 = 1, 2, 3, \dots, 9$ is seen to occur as the first digit in the dataset:

$$N_{d_1} = N \log\left(1 + \frac{1}{d_1}\right), \qquad d_1 = 1, 2, 3, \dots, 9 \quad (4)$$

Where N is the total number of considered data points.

Benford's law for the second digit can be used to demonstrate the likelihood that $d_2 = 0, 1, 2, 3, \dots, 9$ is encountered as the second digit:

$$P_2(d_2) = \sum_{k=1}^{9} \log\left(1 + \frac{1}{10\,k + d_2}\right) = \log\,[\prod_{k=1}^{9}(\frac{10\,k + d_2 + 1}{10\,k + d_2})] \qquad (5)$$

Ausloos *et al.*, (2021) state that Benford's law is typically accepted in situations where there is no data tampering and no human limitation. Large datasets on deception-prone actions are where it is most useful. While comparing empirical data to these theoretical distributions will not always reveal a "smoking gun," it might provide useful guidance on where to find that gun (Tam Cho & Gaines, 2007). Consequently, if the returns deviate from the predictions made by Benford's law, this could be a sign that fraud may have been committed.

The first step in performing an unbiased assessment of a fund is to calculate its bias ratio. When the bias ratio noticeably exceeds the threshold amount or the benchmark index, the fund is designated for additional examination. The flagged fund is then evaluated using Benford's law. An adherence to Benford's law implies that there is no fraud. On the other hand, departure from Benford's distribution suggests possible deception, necessitating further examination of the fund by the regulatory authority.

3.3 Applying the Methodology to a Known Case of Fraud

To establish the credibility of the bias ratio and Benford's law as effective tools for detecting potential fraud or misrepresentation, this research begins by using these metrics in a retrospective analysis of a known case of fraud, a similar approach was followed by King and van Vuuren (2016). By examining the application of these metrics in a real-world context, the aim is to understand their reliability and sensitivity in uncovering irregularities and deceptive practices within financial data.

The analysis first revisits the case of Bernie Madoff, who garnered widespread attention by consistently generating high returns with remarkably low volatility over an extensive timeframe. In order to obtain these low-risk returns, he allegedly used a split-strike conversion method, which comprises taking on a long position in stocks in addition to a long put and a short call on an equity index to lessen the volatility of the position (Bernard & Boyle, 2009). The astonishing magnitude of his fraud reached approximately $65 billion over several decades, a staggering sum that went undetected until his exposure in 2008 (Quisenberry, 2017).

This research focuses on Fairfield Sentry (FS), a feeder equity hedge fund connected to Bernie Madoff's Investment Securities LLC (BMIS). The S&P 500 Index serves as the benchmark for comparison. FS closely mirrored the returns of BMIS, demonstrating how fraudulent activities can spread throughout the investment sector.

Illustrated in Figure 2, the bias ratio of FS markedly surpasses that of the benchmark index, revealing a significant divergence. From 1998 onward, the bias ratio of FS undergoes a swift ascent, presenting a critical indicator that could have served to detect and avert the ensuing fraud. The bias ratio of the S&P 500 index remained relatively low when compared to that of FS. This revelation finds support in the research conducted by Douady *et al.*, (2009), van Dyk *et al.*, (2014), King and van Vuuren (2016), and van Vuuren and van Vuuren (2022), where the bias ratio, alongside other metrics played a crucial role in uncovering anomalies.

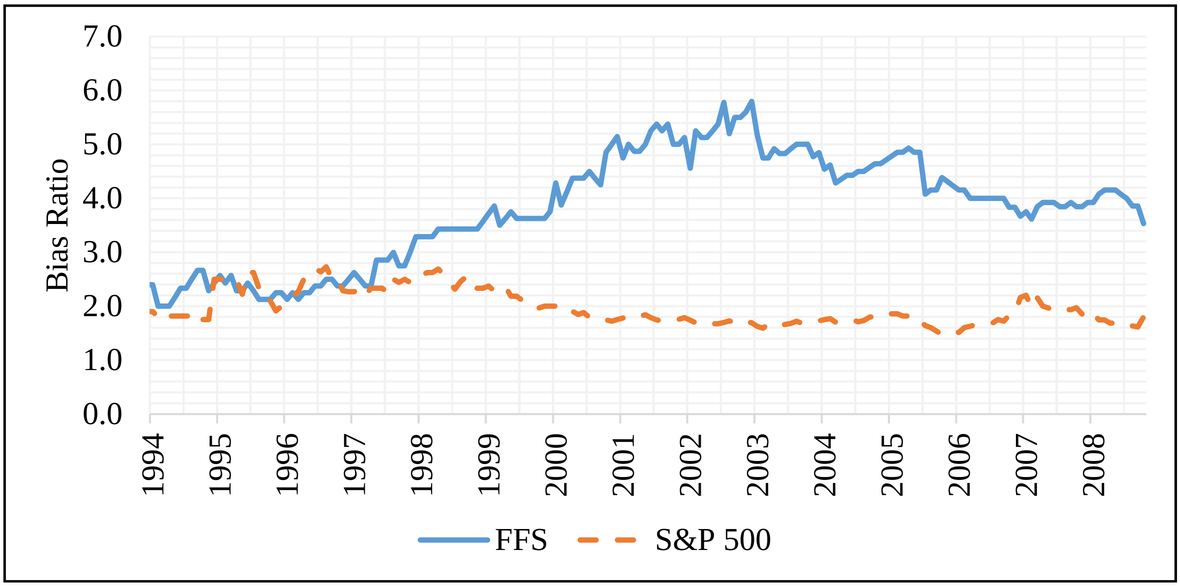

Figure 2: The 36-month rolling bias ratio for FS and the S&P 500 Index using monthly returns.

From 1994 to 2008, spanning the period leading to the collapse of BMIS, this study begins with an examination of the annualised returns. Remarkably, as depicted in Figure 3 (a), the returns of FS predominantly exhibit positivity across the entire timeline and are seldom negative—an accomplishment that may prove challenging for any fund manager. The S&P 500 returns appear to be much more volatile over the period in comparison to FS. There are periods of positive and negative returns. Figure 3 (b) provides a perspective on the annualised standard deviation of FS. Notably, this volatility is subdued when compared to the S&P 500 Index; FS's standard deviation appears tranquil. Hovering around the 2-4% range over the period. This reduced level of volatility for a hedge fund is expected given that BMIS follows a hedged strategy.

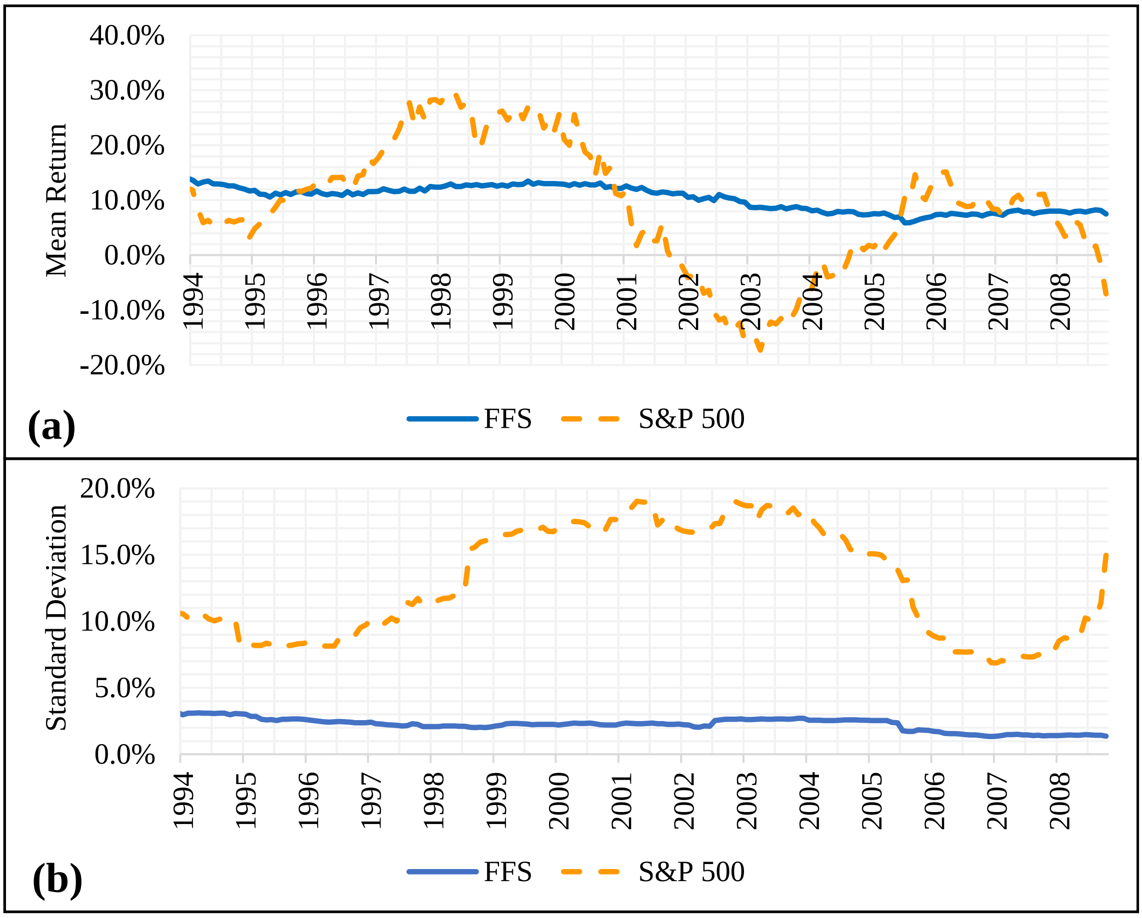

Figure 3: (a) Annualised Mean Returns and (b) Annualised Standard Deviation of the returns of FS and the S&P 500 Index using 36-month rolling data.

2007/8 marked the inception of the global financial crisis, it was a period characterised by a substantial surge in perceived uncertainty regarding market risk and return dynamics. As articulated by Ivashina and Scharfstein (2010), financial market volatility soared to unprecedented levels during this phase. Ben-David *et al.*, (2012) found that it was precisely during this tumultuous era that hedge funds, in response to escalating uncertainty, significantly downsized their equity holdings—a staggering 29% reduction in their aggregate portfolio. This strategic move, aligning with compelled deleveraging, notably involved the liquidation of assets, particularly in volatile and liquid stocks. During such a period one would expect FS or

BMIS to have faced some form of loss, however this was not the case, everything appeared normal during such a chaotic time.

Benford's law was applied to the returns of FS. A visual representation of the distribution of digits of FS can be seen in Figure 4 (a). Prior studies affirm the adherence of the S&P 500 to Benford's law (Ausloos *et al.*, 2021), a confirmation supported by our analysis as seen in Figure 4 (b). The two graphs in Figure 4 indicate that the S&P 500 demonstrates a closer conformance to Benford's law when compared to FSS.

The examination of the first digit of FS's monthly returns, utilising a Chi-squared distribution, yields a noteworthy test statistic of 106.95 with a p-value of 0.00 as shown in Table 1. These statistical findings reveal an anomaly in the FS monthly returns, exemplified in Figure 4 through the distribution of digits. It is evident that the S&P 500 index adheres more closely to Benford's law compared to the FS. The statistical tests for the benchmark, as seen in Table 1, produces a test statistic of 13.74 and a p-value of 0.09 – no anomaly was detected. These findings are consistent with the observations derived from the bias ratio, underscoring a notable difference between the bias ratio of FS and its benchmark. Therefore, the Benford's law tests conducted on the returns provide additional evidence substantiating the presence of an anomaly within FS.

These results suggest that fraudulent activity could have been identified much earlier, particularly during the early 2000s when the bias ratio was on the rise. Had the SEC intervened sooner, it could have mitigated some of the damage caused by BMIS through their deceptive practices. The results also suggest that the application of the bias ratio and Benford's law can indeed serve as potent tools in detecting fraud within hedge funds. In the next section, this methodology is applied to the sample of South African hedge funds.

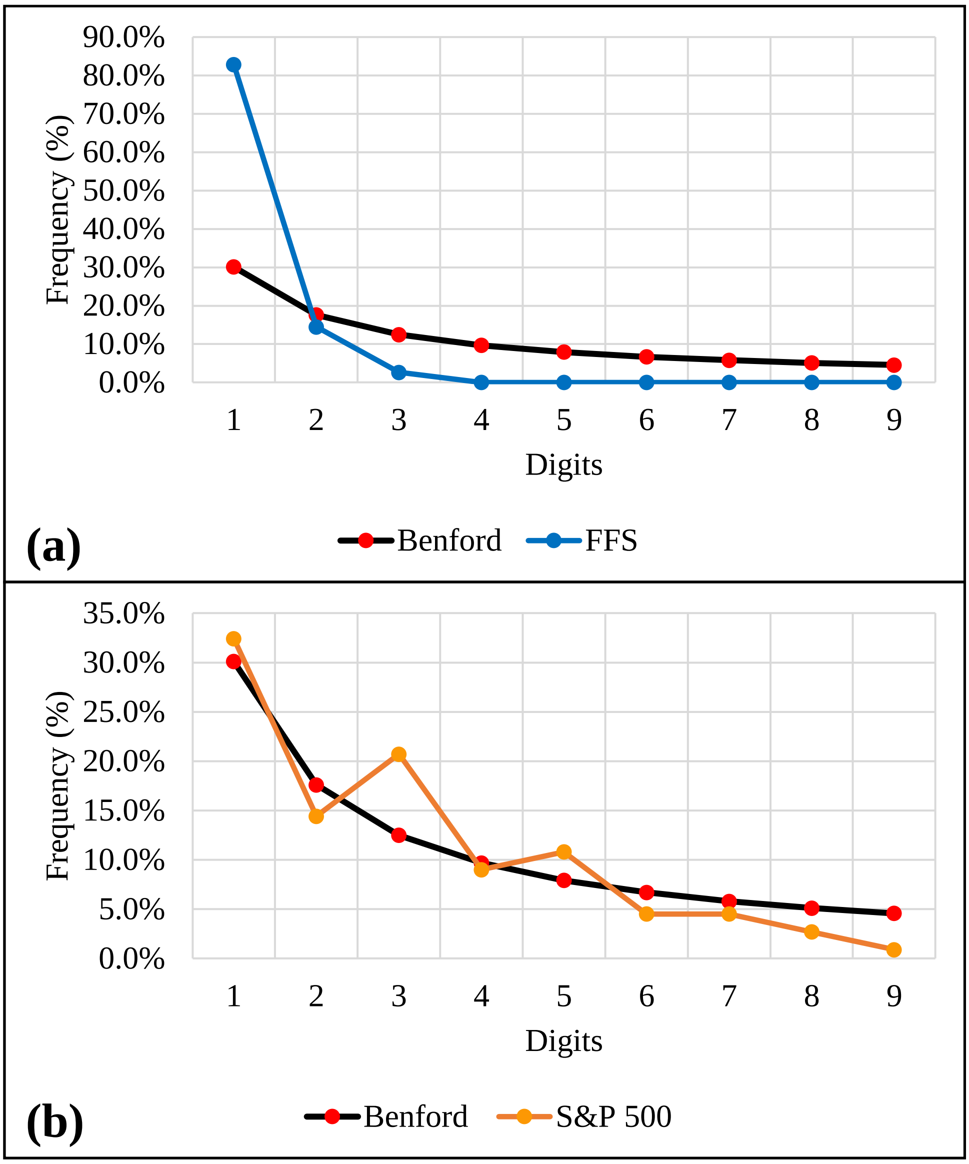

Figure 4: The distribution of the first digit of FS and the S&P 500 monthly returns.

Table 1: Benford's law test statistics for FS and S&P 500 monthly returns

	FS	**S&P 500**
P-Value	0.00	0.09
T-Statistic	106.95	13.74
Conformance with Benford's Law	No	Yes

3.4 South African Hedge Fund Industry Analysis Methodology

The analysis of the hedge fund sample involves two stages. Initially, the bias ratio is computed for each fund in comparison to the benchmark index. Further scrutiny is then directed specifically towards funds with significantly higher bias ratios – these are the flagged funds. This approach is supported by Abdulali (2013), who suggests that elevated bias ratios often signal possible manipulation within the fund. In the second stage, flagged funds undergo a more detailed examination, including tests based on Benford's law. Non-conformance with Benford's law may indicate potentially fraudulent activity, prompting further investigation by regulatory authorities.

Benford's law, in this context, is used to assess whether the empirical distribution derived from observed data significantly deviates from the theoretical distribution based on Benford's expectations. The application of these tests commenced with an examination of the distribution of the first digit of the returns. According to insights from Ausloos *et al.,* (2021), Benford's laws analysis typically focuses on the first digit, and sometimes the second digit, as subsequent digit distributions often exhibit alignment with those preceding them. In this analysis, it is important to highlight two key points: Firstly, according to Benford's law, zero cannot be considered as the first digit because including zero would disrupt the expected logarithmic pattern of leading digits, violating the law's assumptions. Consequently, zero is typically excluded when applying Benford's law to datasets. Secondly, it is crucial to disregard decimal point separators in this context, as emphasised by Ausloos *et al.,* (2021).

4 Results and Discussion

4.1 Empirical Results

Table 2 outlines SA hedge funds by strategy, categorised into six types. Fund of Funds and Long Short Equity are most prevalent, totalling 47 funds. QIFs make up 43 of the total, while RIFs constitute 40. Fixed Income holds 11 funds, and both Market Neutral and Quantitative Strategies and Single-Manager Multi-Strategies each have 11-12 funds. Property-Focused Hedge funds are minimal, with only 2 in the sample. This analysis demonstrates the variety of investment strategies seen in the SA hedge fund market, with a focus on Long Short Equity and Fund of Funds strategies and a discernible difference between QIFs and RIFs.

Table 2: Breakdown of the sample of SA hedge funds by strategy

	QIF	RIF	All Funds
Fund of Funds	17	8	**25**
Property-Focused Hedge	2	0	**2**
Fixed Income	5	6	**11**
Long Short Equity	10	12	**22**
Market Neutral and Quantitative Strategies	5	6	**11**
Single-Manager Multi-Strategies	4	8	**12**
Total	**43**	**40**	**83**

Comparing the summary statistics for hedge funds in Table 3 with those in Table 4 for the JSE ALSI benchmark reveals notable differences in performance and risk measurements. The entire sample of hedge funds contains a total of 15232 data points. The hedge fund sample comprises of 83 funds in total, categorised into QIFs and RIFs. The QIF sample exhibits a higher bias ratio compared to the RIF sample, suggesting potentially greater instances of fraud in the QIF sample. The differences between RIF and QIF are relatively subtle. The average bias ratio across all funds is 2.75 which is over three times greater than the ALSI which had a bias ratio of 0.83.

Additionally, hedge funds demonstrate higher mean annualised returns and lower standard deviations compared to the ALSI, implying potentially more attractive risk-adjusted returns and lower volatility. The standard deviation is quite similar between the RIF and QIF samples. Both categories exhibit relatively low volatility, indicating a certain degree of stability in their return patterns. Conversely, the ALSI shows a lower mean annualised return and higher volatility over the period, indicating a greater market risk. These findings suggest that SA hedge funds, may offer investors the potential for superior returns and reduced volatility compared to the ALSI benchmark.

Table 3: Hedge fund sample summary statistics

	QIF	RIF	All Funds
Number of Funds	43	40	83
Sample Size	8 182	7 050	15 232
Bias Ratio	2.93	2.55	2.75
Mean Annualised Return	0.1169	0.1110	0.1141
Standard Deviation	0.0767	0.0762	0.0765

Table 4: Summary Statistics of the JSE ALSI, the benchmark index.

	ALSI
Sample Size	304
Bias Ratio	0.83
Mean Annualised Return	0.0984
Standard Deviation	0.1903

In the analysis of hedge funds, a subset of six funds were flagged for closer scrutiny due to their notably high bias ratios.[10] The selected funds, denoted as Fund 34, Fund 60, Fund 76, Fund 95, Fund 151, and Fund 375, exhibit distinct characteristics that warrant further attention.

Detailed information and the test statistics of these flagged hedge funds can be seen in Table 5. From this table, it becomes apparent that the QIF sample contained more instances of fraud compared to the RIF sample suggesting that the stricter reporting standards of RIFs make it more difficult for fund managers to misreport or manipulate their returns. From the six funds, four of them follow a Fixed Income strategy, one is a Market Neutral and Quantitative Strategy fund, and the last one is a Single Manager Multi-Strategy fund. All of these funds have a hurdle rate and high-water mark with their performance fee varying from 15% to 20%. One could argue that such high-performance fees coupled with a high-water mark could be motivation for the fund managers to manipulate their returns.

The AUM range sheds light on the size and scope of flagged hedge funds, providing insight into each fund's scale, resources for investment, operational capacity, and market influence. Funds 34 and 95 fall within the R250 million to R500 million range, reflecting substantial AUM in this bracket. Conversely, Funds 76 and 151 operate in the lower R100 million to R250 million range, suggesting a smaller scale. Fund 60 stands out with an AUM exceeding R500 million, indicating significant operations and potentially a large fund size. However, Fund 375's unspecified AUM range may imply varying levels of transparency regarding their asset base.

[10] A fund was flagged if its bias ratio exceeded the threshold of 5.0. King and Van Vuuren (2016) employed a relative threshold of 150% of the category average to identify significant deviations and flag funds. Building on this approach, the threshold of 5.0 in this study was chosen to provide a robust measure for detecting substantial anomalies. This threshold is higher than the category average bias ratios for QIFs and RIFs, ensuring that only significant deviations are flagged. It aligns with statistical practices that use thresholds several standard deviations from the mean to capture outliers.

The flagged hedge funds' test statistics in Table 5 reveal distinctive characteristics across the reported metrics. The bias ratios for Fund 34, Fund 95, and Fund 151 are comparatively high at 11.75, 7.67, and 6.65, respectively, indicating that perhaps the most manipulation has taken place in these funds. Mean annualised returns show variations, with Fund 60 and Fund 151 outperforming others at 0.1102 and 0.1192, respectively. Standard Deviation values suggest varying levels of risk, with Fund 34 having the lowest volatility and Fund 151 having the highest volatility. The high returns and low volatility are somewhat like that of the case study of FS, given that some hedge funds employ hedged strategies one can expect to see lower volatilities and higher returns, it is all dependent on the strategy followed by the fund manager.

Table 5: Flagged funds information and summary test statistics

	Fund 34	**Fund 60**	**Fund 76**	**Fund 95**	**Fund 151**	**Fund 375**
Type	QIF	QIF	QIF	RIF	QIF	RIF
Strategy	Fixed Income	Fixed Income	Market Neutral and Quantitative Strategies	Fixed Income	Single Manager Multi-Strategies	Fixed Income
Management Fee	1%	1%	1%	1%	1%	1%
Hurdle Rate	Yes	Yes	Yes	Yes	Yes	Yes
High Water Mark	Yes	Yes	Yes	Yes	Yes	Yes
Performance Fee	15%	20%	17.5%	20%	15%	20%
AUM Range	R250m - R500m	>R500m	R100m – R250m	R250m - R500m	R100m – R250m	Unspecified
Bias Ratio	11.75	6.35	5.04	7.67	6.65	5.78
Mean Annualised Return	0.0937	0.1102	0.0699	0.1077	0.1192	0.1043
Annualised Standard Deviation	0.0171	0.0399	0.0287	0.0474	0.0756	0.0287

Next each flagged funds metrics are examined closer. The 36-month rolling bias ratio, mean annualised return and annualised standard deviation was calculated for each fund and the benchmark. These metrics are plotted for each fund individually in Figure 5, Figure 6, and Figure 7 respectively. As seen in Figure 5 (a) to (f) the bias ratio for all six funds are significantly higher than the ALSI. The bias ratio of Fund 151 which is a Single Manager Multi-Strategy fund appears to be the highest over the period reaching its peak around 2013 - 2014. The annualised mean returns depicted in Figure 6 (a) to (f) appear much more stable in comparison to the ALSI with the exception of Fund 151 depicted in Figure 6 (e) which appeared to face lower returns at the time of the COVID-19 pandemic. The annualised standard deviations can be seen in Figure 7 (a) to (f), the volatility of all six flagged funds appear to be much lower when compared to the ALSI. Fund 34, Fund 60, Fund 95, and Fund 375 are all Fixed Income. All these funds appear to have very low volatilities and stable returns which makes sense given their respective strategy. Fund 151's standard deviation depicted in Figure 7 (e) appears to resemble that of the ALSI after 2020, however, it is still much lower. From these results, it is clear that the strategy of the fund is important.

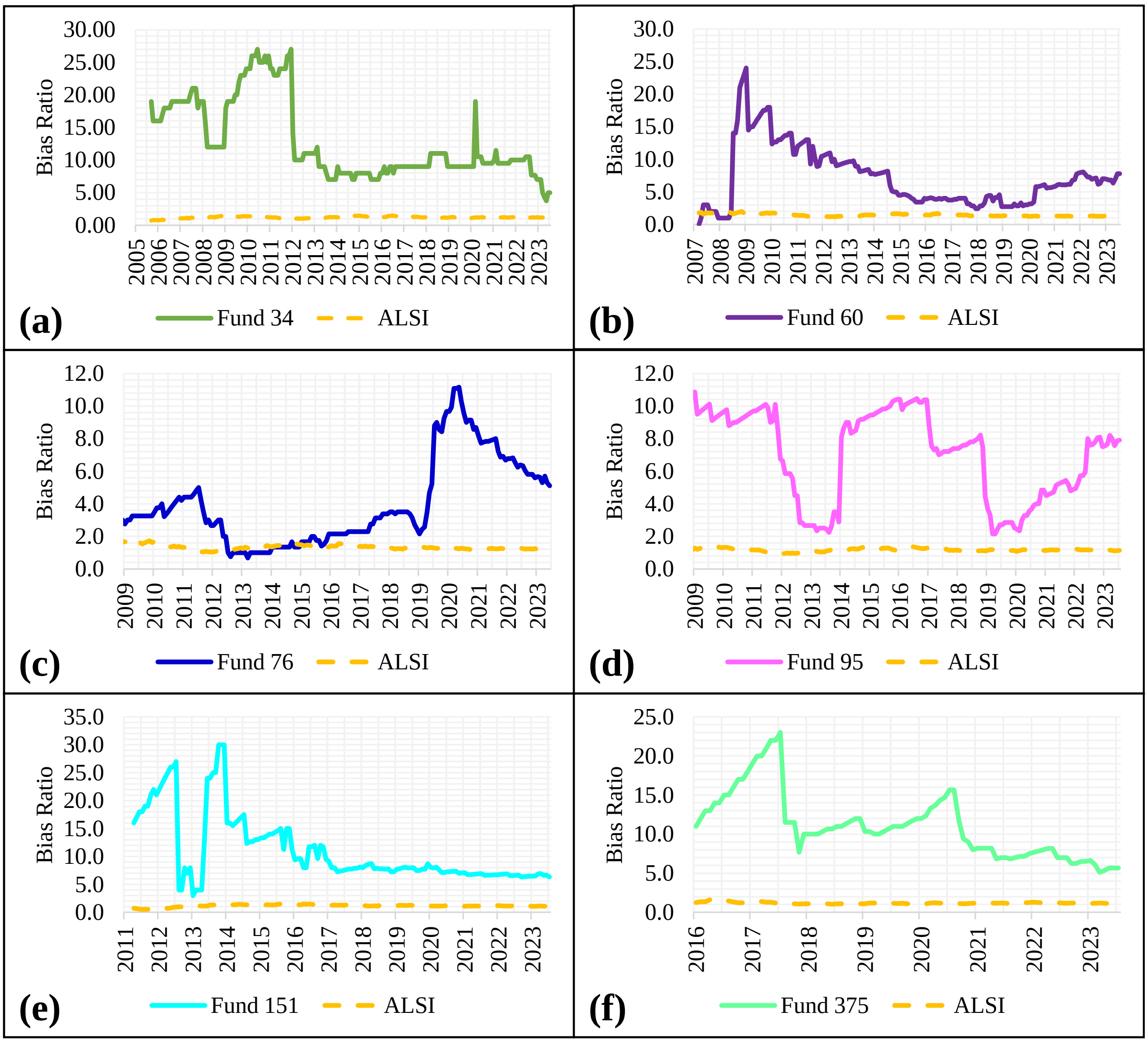

Figure 5: The rolling 36-month bias ratio calculated for each flagged fund. The bias ratio is plotted with the benchmark index.

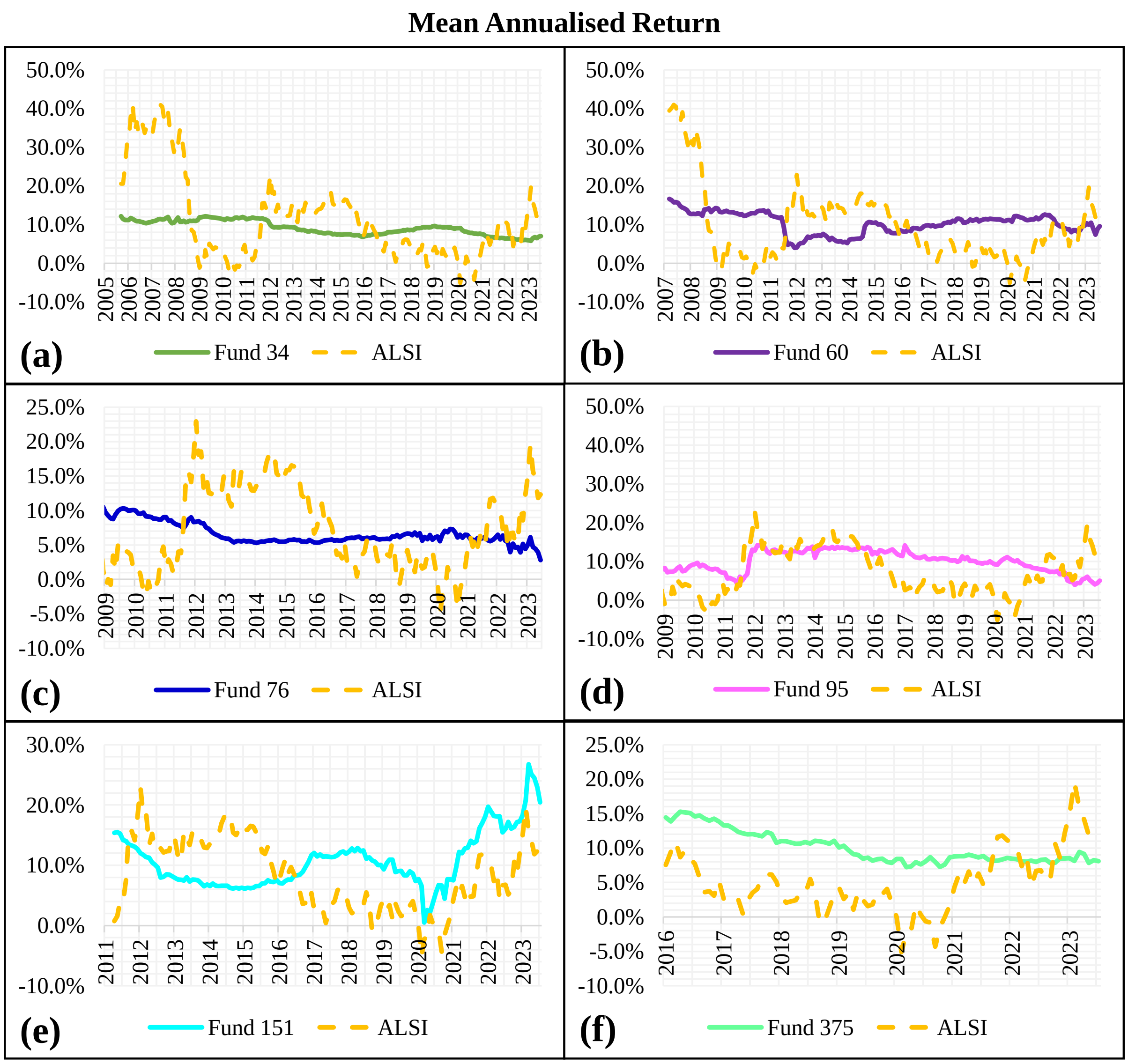

Figure 6: Mean annualised returns for the flagged funds and the benchmark index.

Annualised Standard Deviation

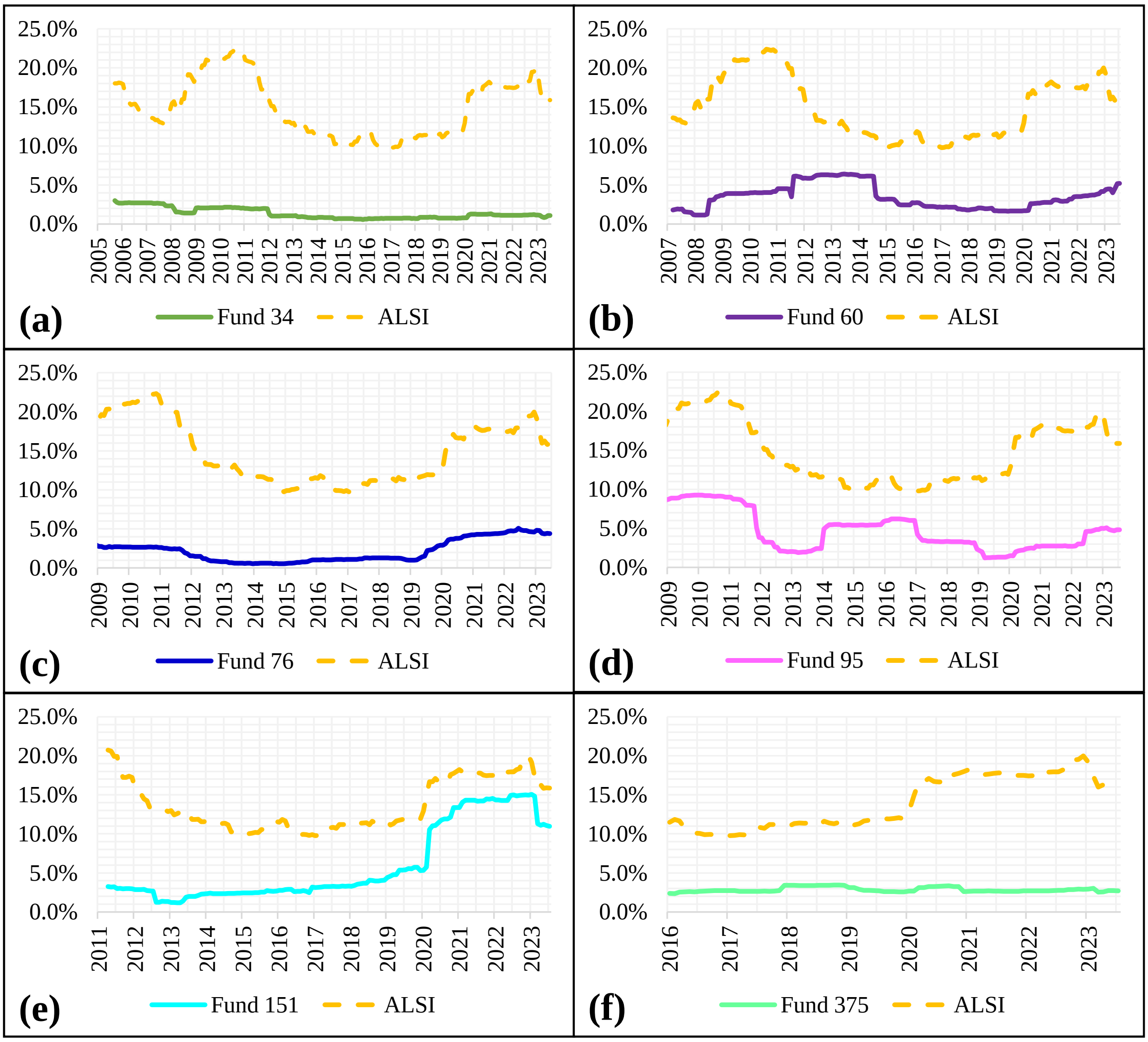

Figure 7: Annualised standard deviation for the flagged funds and benchmark index.

The distribution of the digits of the flagged hedge funds' returns based on Benford's law for the first digit can be seen in Figure 8. From the graphs it is clear that Fund 34 does not follow Benford’s distribution as seen in Figure 8 (a). Visually, it also appears that Fund 151 and Fund 375 conform with Benford’s law as seen in Figure 8 (e) and (f) respectively. The graphs adds another layer of insight to the previously discussed results.

Table 6 displays the statistical tests for the Benford’s law tests. With the exception of Fund 151, the test-statistic values show a substantial departure from Benford's law. Notably, Fund 34, Fund 60, Fund 76, Fund 95, and Fund 375 exhibit Test-Statistic values of 150.07, 47.40, 22.38, 53.12, and 16.95 respectively. The corresponding p-values are below 0.05, indicating a statistically significant deviation from the expected distribution and reinforcing the rejection of conformity with Benford's law for these funds. Interestingly, Fund 151 stands out as conforming to Benford's law, with a higher P-Value of 0.0999, suggesting a closer alignment with Benford's distribution. This finding is perhaps the most interesting, as it could perhaps mean that Fund 151 is not actually engaging in fraudulent activity, and this is a case of a false positive. It has been noted in earlier research that the bias ratio works best when combined with other indicators (Douady *et al.*, 2009; van Dyk *et al.*, 2014; King & van Vuuren, 2016; van Vuuren & van Vuuren, 2022). These combined results highlight the importance of considering additional metrics, such as Benford's law, for an evaluation of hedge funds' performance and potential irregularities.

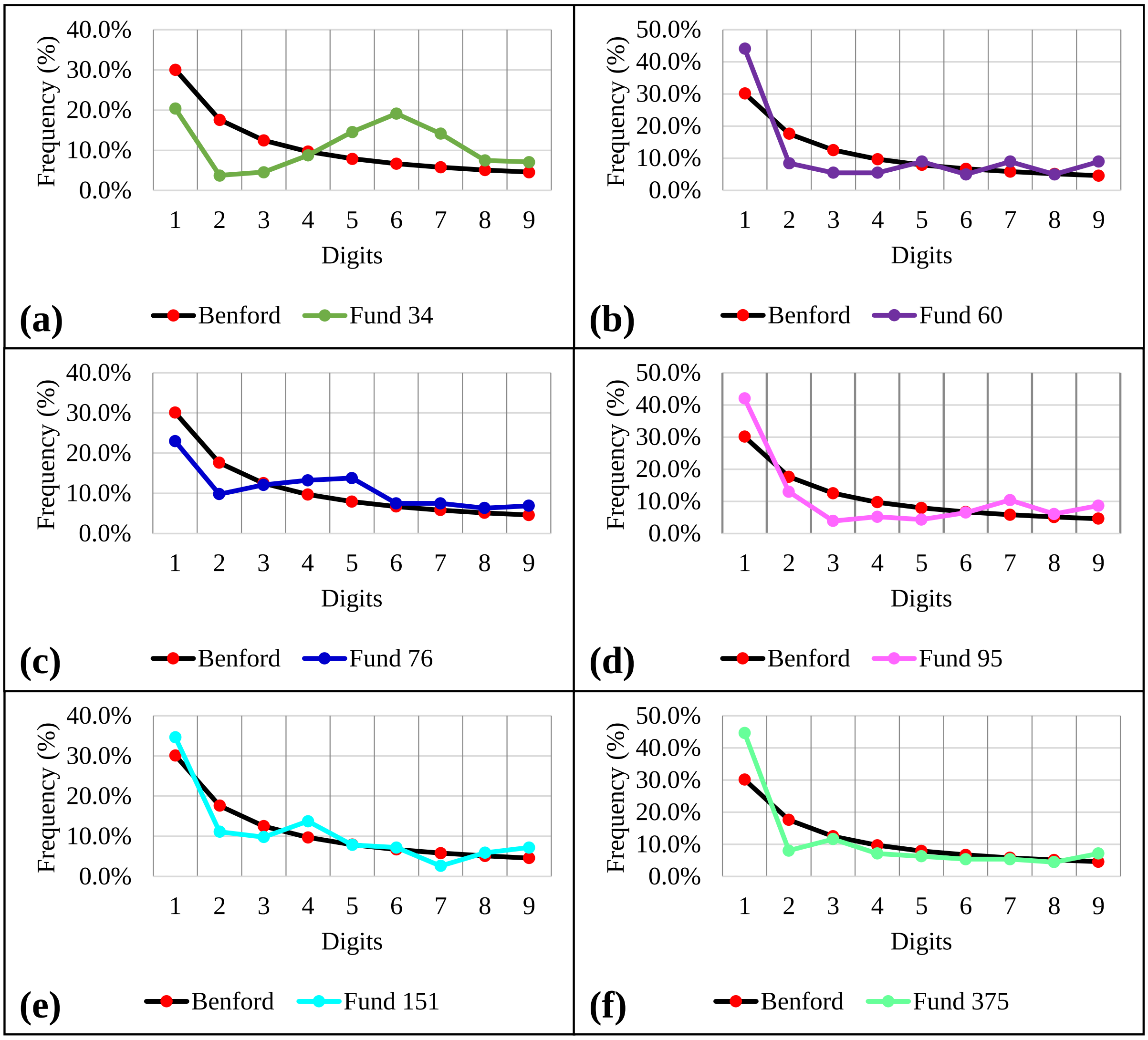

Figure 8: The distribution of the first digit of the monthly returns of the flagged funds.

Table 6: Benford's law analysis of the flagged funds' first digit

	Fund 34	Fund 60	Fund 76	Fund 95	Fund 151	Fund 375
Test-Statistic	150.07	47.40	22.38	53.12	13.36	16.95
P-Value	0.0000	0.0000	0.0043	0.0000	0.0999	0.0306
Conformance with Benford's Law	No	No	No	No	Yes	No

4.2 Discussion

The bias ratio appears to be a useful tool in the analysis of the South African hedge fund data, indicating significant issues within the sample of 83 funds. Notably, six funds were flagged due to their significantly high bias ratio, prompting a deeper investigation into their financial integrity. Of these six funds, four were Fixed Income funds, one was a Market Neutral and Quantitative Strategies Fund, and the last one was a Single Manager Multi-Strategies Fund. Given that the entire sample contained a total of eleven Fixed Income Funds, it is unlikely that almost half of the sample of the Fixed Income funds would be engaged in fraud or misrepresent their returns. Abdulali (2013) claims that alpha-generating methods and returns from the equities and fixed income markets typically have a natural positive skew, producing histograms with a smoothed slope close to zero. Furthermore, total return series with a naturally positive slope approaching zero are also exhibited by fixed income strategies that are characterised by a constant positive return ("carry"). Because negative returns are rare in cash assets like 90-day T-Bills, substantial bias ratios indicate relatively low risk. However, it is important to note that this ratio may be less reliable in hedge funds that keep a sizable cash position and an unleveraged portfolio (Abdulali, 2013). The bias ratio, according to Van Dyk *et al.*, (2014), is a useful tool for spotting NAV manipulation and illiquid securities where they shouldn't be. However, interpretation of the ratio should consider the liquidity of the observed strategy. Since access to the holdings of the individual funds is not available, it becomes challenging to determine whether cash investments and the liquidity of the fund are the actual factors contributing to the substantial bias ratios observed in the fixed income funds.

From the sample of 83 hedge funds, six were flagged - four of which were QIFs and the remaining two were RIFs. As stated previously, hedge funds in SA are regulated by the FCSA and are regulated in terms of the CISCA. According to Fairtree (2016), QIFs resemble traditional industry offerings but with enhanced oversight, targeting sophisticated investors or institutions. These funds have increased leverage limits and more flexible mandates to pursue higher returns. Conversely, RIFs, akin to UCITS hedge funds in Europe, are tailored to a broader investor base. RIFs are priced daily – the regulations around these funds are more prescriptive than that of QIFs. On the other hand, QIFs are priced monthly (Sanlam, 2024). QIF fund managers essentially have greater freedom in managing their funds than is the case of RIFs. Thus, one would expect to find more cases of fraud amongst the QIF funds compared to the RIF sample as is the case.

A more thorough examination of the flagged fund sample reveals that a high bias ratio does not always indicate that a fund is involved in fraudulent activity. After looking at the funds in more detail, the 36-month rolling annualised returns, standard deviation, and bias ratio of each fund were calculated. After that, the distribution of each fund's monthly returns was determined using Benford's law. It was found that only Fund 151, out of the six funds, conformed to Benford's law. This observation enhances the methodology by implying that Benford's law may function as a validated indicator of possible fraudulent activity in hedge funds. This result suggests that Fund 151 did not commit any form of fraud and would not require any investigation by the regulator. However, the other five funds deviated from Benford’s law, indicating the need for additional investigation by the FSCA. Moreover, individual investors should exercise greater caution with these five funds and approach their investment decisions with careful consideration. Of these five funds, four are Fixed Income and one is a Market Neutral and Quantitative Strategies Fund. This breakdown results in 3 QIFs and 2 RIFs, suggesting that regulations may not make much of a difference in terms of preventing fraud; however, the increased regulations in RIFs in the SA industry may have made a difference in the occurrence of fraud.

The elevated bias ratios, combined with the non-conformities to Benford's law observed in the flagged funds, raise suspicions regarding potential fraud. However, drawing definitive conclusions requires thorough investigation by regulatory bodies. The complexity of fraud detection underscores the crucial role of regulatory intervention in validating these findings.

A notable constraint of this research is that the sample dataset predominantly comprises of funds that are currently operational as fraudulent funds often cease to exist (van Vuuren & van Vuuren, 2022). Another limitation of this study, as well as those by van Dyk *et al.*, (2014) and King and van Vuuren (2016), is the inability to ascertain whether a flagged fund is genuinely involved in fraud until after the fact, due to challenges in obtaining relevant hedge fund data and the regulatory landscape in which these funds operate. Consequently, the efficacy of the bias ratio and the results of this study is questioned.

Nevertheless, individual investors may still find value in using the bias ratio and Benford's law as a due diligence tool, albeit with caution. While there remains a possibility that fraud has not occurred in these flagged funds, exercising prudence and avoiding investment in them could safeguard portfolios from potential damage or losses in the event of their involvement in fraudulent activities.

Although it remains challenging to definitively affirm the presence of fraud, the results of this hedge fund analysis align positively with the predictions of earlier researchers, such as van Dyk *et al.*, (2014) and Douady *et al.*, (2009), who advocated for the synergistic use of Benford's law and the bias ratio in hedge fund fraud detection.

As previous researchers have emphasised, the bias ratio can indeed be used as a possible early indicator of fraudulent activity (Douady *et al.*, 2009; van Dyk *et al.*, 2014; King & van Vuuren, 2016; van Vuuren & van Vuuren, 2022). The 36-month rolling bias ratio graphs in Figure 5, demonstrate that once the bias ratio exceeds a certain threshold or experiences a spike, then other complementary tests such as Benford's law should be applied to the fund to bolster fraud detection efforts. Following such an approach could lead to the early detection of fraud.

These findings confirm that the bias ratio needs to be used in conjunction with other metrics, as there is a possibility of false positives. As seen in the FS case, Benford's law can be applied in combination with the bias ratio. By utilising these two tools together, the FS fraud and hence the Bernie Madoff fraud could have been detected in advance. These funds would have been flagged and the regulatory body would need to step in and carry out an investigation into the two funds.

The combination of Benford's law and the bias ratio forms an analytical framework that reveals potential irregularities. This approach appears to be effective when there's enough data for an analysis. In instances where data points are limited, substitute metrics like the first four statistical moments, the Omega ratio, and the Sharpe ratio, as advocated by King and van Vuuren (2016) and van Vuuren and van Vuuren (2022), can serve as complementary tools for fraud detection within hedge funds.

5 CONCLUSION

The hedge fund industry plays an important part of the financial markets, their activities promote price efficiency and provide liquidity to less liquid markets (Barth *et al.*, 2023). While the size of the industry has been on an upward trajectory, a shadow cast by fraud still lingers over the industry and the financial markets. Finding ways to detect fraud are essential in preventing its occurrence, early detection could also lessen the long-term damage caused by fraudulent practices. The bias ratio emerges as a potential tool to detect fraud in hedge funds because of its simplicity and the fact that it requires only returns as inputs (van Vuuren & van Vuuren, 2022). Given the lax regulations on hedge funds, this ratio is suitable given the proprietary nature and scarcity of hedge fund data. Benford’s law is another tool which has been used in the field of accounting to detect fraud. Previous researchers have suggested using it in conjunction with the bias ratio to detect fraud in hedge funds. Owing to the distinct regulatory environment in South Africa, hedge funds are governed by the CISCA and are categorised as regulated collective investment schemes, which are similar to unit trust portfolios (ASISA, 2023).

The potential of combining Benford's law with the bias ratio was demonstrated in this study’s analysis of the hedge funds. Initially used in the notorious FS fraud case, the methodology indicated that both the FS fraud and hence Bernie Madoff's scheme could have been identified as early as the early 2000s, marked by an upward trend in FS’s bias ratio. Extending this methodology to the South African hedge fund industry, a sample of 83 hedge funds spanning from April 1998 to July 2023 were analysed, comprising 40 RIFs and 43 QIFs. Upon application of the bias ratio to this sample, six funds exhibited notably elevated bias ratios, with four among them categorised as Fixed Income funds. Abdulali (2013) provides insightful context, pointing out that cash investments—like 90-day T-Bills—naturally have high bias ratios because of their lower risk profile, which is caused by the rarity of negative returns. While access to the holdings of the individual funds is unavailable, determining whether cash investments and the liquidity of the fund are truly behind the large bias ratios in the Fixed Income Funds becomes challenging. Furthermore, Abdulali (2013) highlights that the bias ratio for hedge funds—which are defined by an unleveraged portfolio with a sizable cash balance—may not always be reliable. Therefore, interpretation of the bias ratio should consider the liquidity of the observed strategy (van Dyk *et al.*, 2014).

Benford’s law was applied to the six flagged funds, and it was found that only one of these funds conformed to Benford's distribution and the remainder did not. This implies that five of the funds have been involved in fraudulent activity and the FSCA would need to step in to carry out a full investigation into these flagged funds. It is important to note that relying solely on the analysis of the bias ratio may yield incomplete results (van Vuuren & van Vuuren, 2022). Complementary metrics are employed to enhance the depth of understanding regarding the observed outcomes (Douady *et al.,* 2009). As a result, evaluating many indicators is essential to avoiding biases and obtaining a thorough grasp of the return distribution of a fund.

Despite the study's limitations in confirming fraud in flagged funds, doubts may linger about the effectiveness of the bias ratio. Nonetheless, individual investors can still benefit from using the bias ratio and Benford's law cautiously for due diligence. These tools can help in selecting hedge funds for investment. Investors should stay vigilant and steer clear of flagged funds—those with high bias ratios and non-conformance to Benford's law. While confirming fraud definitively is still uncertain, this analysis aligns with earlier research by supporting the combined use of Benford's law and the bias ratio in hedge fund fraud detection, as advocated by van Dyk *et al.,* (2014) and Douady *et al.,* (2009).

The integration of the bias ratio and Benford's law not only allows for the identification of potential outliers but also contributes to a more robust evaluation of the integrity and transparency within the hedge fund industry. The examination of the hedge funds using the bias ratio and Benford's law has yielded valuable insights. The study's findings shed light on significant patterns and characteristics within the hedge fund sector, prompting a careful consideration of their implications.

Future research could delve deeper into the application of Benford's law and the bias ratio as tools for due diligence. The bias ratio, serving as an alert system for vigilant investors, can prompt thorough investigations into funds or investments (van Dyk *et al.,* 2014). To gauge the impact of stricter regulations on the hedge fund sector, a comparative analysis akin to this study could be undertaken, incorporating hedge funds from diverse regions and comparing them against a sample from South Africa for a comprehensive evaluation. Furthermore, exploring the applicability of the bias ratio and Benford's law in other investment domains, such as mutual funds, warrants attention. However, it is crucial to note that the bias ratio may not be universally applicable, particularly for funds with substantial cash holdings. Hence, understanding the fund's strategy is paramount when interpreting bias ratio results. Thoughtful consideration of this metric within the broader context of the investment industry is crucial, as it does not offer a one-size-fits-all solution.

Other facets of the investment industry can also be subject to Benford's law, including mutual funds and even individual stocks. The robustness of this law means that it doesn’t need to be applied to returns only, but it can also be applied to the NAV of a fund or the price of a stock. Perhaps it could be used to detect suspicious activity such as insider trading in individual stocks. The distribution of the remaining digits, such as the second, last, and even the first two, can also be examined.

In conclusion, while the bias ratio and Benford's law appear to be valuable in uncovering potential irregularities, the study calls for careful consideration of their application, especially in the context of the type of fund and the strategy followed by the fund. Individual investors may benefit from using these tools to select the funds that they invest in. Furthermore, regulatory bodies must play a role in investigating flagged funds to confirm the presence of fraud. This research, complementing previous studies (Douady *et al.*, 2009; van Dyk *et al.*, 2014; King & van Vuuren, 2016; van Vuuren & van Vuuren, 2022), highlights the necessity of a multifaceted approach to fraud detection.

6 REFERENCES

Abdulali, A. (2006). The bias ratio: Measuring the shape of fraud. Protégé Partners Quarterly Letter, Q3.

Abdulali, A. (2013). *United States Patent No. 8,364,572 B2.*

Ackermann, C., McEnally, R., & Ravenscraft, D. (1999). The performance of hedge funds: Risk, return, and incentives. *Journal of Finance, 54*(3), 833-874.

Agapova, A., & Madura, J. (2013). Impact of the Galleon case on insider trading prior to company issued guidance. *Journal of Applied Finance, 23*(2), 1-16. Retrieved from https://ssrn.com/abstract=2684016

Agarwal, V., Daniel, N., & Naik, N. (2011). Do hedge hund managers manage their reported returns? *Review of Financial Studies, 24*, 3281-3320.

Agarwal, V., Ruenzi, S., & Weigert, F. (2024). Unobserved performance of hedge funds. *The Journal of Finance*.

Ahern, K. R. (2017). Information networks: Evidence from illegal insider trading tips. *Journal of Financial Economics, 125*(1), 26-47. doi:https://doi.org/10.1016/j.jfineco.2017.03.009

Aragon, G. (2007). Share restrictions and asset pricing: Evidence from the hedge fund industry. *Journal of Financial Economics*, *83*(1), 33-58.

Aragon, G. O., & Qian, J. (2009). High-water marks and hedge fund compensation. SSRN. doi:https://dx.doi.org/10.2139/ssrn.1540205

ASISA. (2023, February 27). *Hedge Fund Industry Sees Assets Under Management Jump By An Unprecedented 30% In 2022*. Retrieved from ASISA: https://www.asisa.org.za/media-releases/hedge-fund-industry-sees-assets-under-management-jump-by-an-unprecedented-30-in-2022/

Asllani, A., & Naco, M. (2015). Using Benford's law for fraud detection in accounting practices. *Journal of Social Science Studies, 2*(1), 129-143.

Auer, B. R. (2013). The low return distortion of the Sharpe ratio. *Financial Markets and Portfolio Management, 27*, 299–306. doi:https://doi.org/10.1007/s11408-013-0213-x

Ausloos, M., Ficcadenti, V., Dhesi, G., & Shakeel, M. (2021). Benford's laws tests on S&P500 daily closing values and the corresponding daily log-returns both point to huge non-conformity. *Physica A: Statistical Mechanics and its Applications*, 125969. doi:https://doi.org/10.1016/j.physa.2021.125969

Baquero, G., & Verbeek, M. (2009). A portrait of hedge fund investors: Flows, performance and smart money. Rochester, NY. doi:10.2139/ssrn.773384

Barth, D., Joenvaara, J., Kauppila, M., & Wermers, R. R. (2023). A Bias-Free Assessment of the Hedge Fund Industry: A New Evaluation of Total Assets, Alphas, and the Flow-Performance Relation. Rochester, NY: OFR WP (Office of Financial Research Working Paper) 20-01. doi:10.2139/ssrn.3544181

Ben-David, I., Franzoni, F., & Moussawi, R. (2012). Hedge fund stock trading in the Financial Crisis of 2007-2009. *The Review of Financial Studies, 25*(1), 1–54. Retrieved from http://www.jstor.org/stable/41302012

Benford, F. (1938). The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society, 78*(4), 551–572. Retrieved from http://www.jstor.org/stable/984802

Benson, S. S. (2009). Recognizing the red flags of a Ponzi scheme: Certified Public Accountant. *The CPA Journal, 79*(6), 18-25.

Bernard, C., & Boyle, P. P. (2009). Mr. Madoff's Amazing Returns: An Analysis of the Split-Strike Conversion Strategy. *Journal of Derivatives, 17*, 62-76. doi:10.3905/JOD.2009.17.1.062

Bernard, C., & Vanduffel, S. (2014). Mean–variance optimal portfolios in the presence of a benchmark with applications to fraud detection. *European Journal of Operational Research, 243*(2), 469-480. doi:https://doi.org/10.1016/j.ejor.2013.06.023

Bollen, N. P., & Pool, V. K. (2008). Conditional Return Smoothing in the Hedge Fund Industry. *Journal of Financial and Quantitative Analysis, 43*(2), 267–298.

Bollen, N. P., & Pool, V. K. (2009). Do Hedge Fund Managers Misreport Returns? Evidence from the Pooled Distribution. *Journal of Finance, 64*, 2257-88.

Bollen, N. P., & Pool, V. K. (2012). Suspicious Patterns in Hedge Fund Returns and the Risk of Fraud. *The Review of Financial Studies, 25*(9), The Review of Financial Studies. doi:https://doi.org/10.1093/rfs/hhs085

Botha, M. (2007). A Comparison of South African Hedge Fund Risk Measures. *South African Journal of Economics, 75*(3), 459-477. doi:10.1111/j.1813-6982.2007.00131.x

Braun, M., Riutort, J., & Roche, H. (2024). Hedge fund fee structure and risk exposure. *Economic Modelling, 132*. doi:10.1016/j.econmod.2024.106646

Brown, S. J., Kang, M., In, F. H., & Lee, G. (2010, January 13). Resisting the Manipulation of Performance Metrics: An Empirical Analysis of The Manipulation-Proof Performance Measure. *Finance and Corporate Governance Conference 2010.* SSRN. doi:http://dx.doi.org/10.2139/ssrn.1536323

Capocci, D. (2013). *The complete guide to hedge funds and hedge fund strategies.* London: Palgrave Macmillan.

Cassar, G., & Gerakos, J. (2011). Hedge funds: pricing controls and the smoothing of self-reported returns. *Review of Financial Studies, 24*(5), 1698-1734.

CFA Institute. (2023, August 5). *Hedge Fund Investing & Regulation*. Retrieved from CFA Institute: https://www.cfainstitute.org/en/advocacy/issues/hedge-funds#sort=%40pubbrowsedate%20descending

Chatterjee, S. (2022, January 20). *Global hedge fund industry assets top $4 trillion for the first time*. Retrieved from Reuters: https://www.reuters.com/business/finance/global-hedge-fund-industry-assets-top-4-trillion-first-time-2022-01-20/

Connor, G., & Lasarte, T. (2003). An overview of hedge fund strategies.

Connor, G., & Woo, M. (2004, January). An Introduction to hedge funds. London, UK: Financial Markets Group, The London School of Economics and Political Science. Retrieved from http://fmg.lse.ac.uk

Cumming, D., & Dai, N. (2010). Hedge Fund Regulation and Misreported Returns. *European Financial Management, 16*(5), 829-857.

Douady, R., Abdulali, A., & Adlerberg, I. (2009). The Madoff case: Quantitative beats qualitative. Riskdata.

Durtschi, C., Hillison, W., & Pacini, C. (2004). The effective use of Benford's law to assist in detecting fraud in accounting data. *Journal of Forensic Accounting, 5*, 17-34.

Edward, F. R. (1999). Hedge Funds and the Collapse of Long-Term Capital Management. *Journal of Economic Perspectives, 13*(2), 189-210. doi:10.1257/jep.13.2.189

Edwards, F., & Caglayan, M. (2001). Hedge fund performance and manager skill. *Journal of Futures Markets, 21*(11), 1003–1028.

Eichengreen, B. J., & Mathieson, D. J. (1999). Hedge Funds: What Do We Really Know? International Monetary Fund. Retrieved from https://www.elibrary.imf.org/display/book/9781557758491/9781557758491.xml

El Kalak, I., Azevedo, A., & Hudson, R. (2016). Reviewing the hedge funds literature I: Hedge funds and hedge funds' managerial characteristics. *International Review of Financial Analysis, 48*, 85-97. doi:10.1016/j.irfa.2016.09.008

Eling, M., & Schuhmacher, F. (2007). Does the choice of performance measure influence the evaluation of hedge funds? *Journal of Banking & Finance, 31*(9), 2632-2647. doi:10.1016/j.jbankfin.2006.09.015

Enobong, U. (2017). Implications of Accountants' unethical Behavior and Corporate Failures. *International Journal of Business, 4*(4), 82-94.

Eurekahedge. (2021, May). *High performing hedge funds retain ability to charge performance fees*. Retrieved from Eurekahedge: https://www.eurekahedge.com/Research/News/2108/Hedge-Fund-Fees-Strategy-Profile-May-2021

Fairtree. (2016). *The South African Hedge Fund Industry: An Ever Evolving Landscape.* Retrieved from Fairtree Investments: https://fairtree.com/wp-content/uploads/2016/12/sanne_connect_winter_2016_FINAL.pdf

Fung, W., & Hsieh, A. D. (1997). Empirical characteristics of dynamic trading strategies: The case of hedge funds. *Review of Financial Studies, 10*, 275–302.

Fung, W., & Hsieh, D. A. (1999). A Primer on Hedge Funds. *Journal of Empirical Finance, 6*(3), 309-331. doi:https://doi.org/10.1016/S0927-5398(99)00006-7

Gerritzen, M., Jackwerth, J., & Plazzi, A. (2024). Birds of a feather: do hedge fund managers flock together? *Management Science, 70*(5), 2976-2998.

Getmansky, M., Lee, P. A., & Lo, A. W. (2015). Hedge Funds: A Dynamic Industry in Transition. *Annual Review of Financial Economics, 7*(1), 483-577. doi:10.1146/annurev-financial-110311-101741

Getmansky, M., Lo, A. W., & Makarov, I. (2004). An econometric model of serial correlation and illiquidity in hedge fund returns. *Journal of Financial Economics, 24*(74), 529-609.

Goetzmann, W. N., Ingersoll, J. E., & Ross, S. A. (2003). High-Water Marks and Hedge Fund Management Contracts. *The Journal of Finance, 58*(4), 1685-1718. doi:10.1111/1540-6261.00581

Goetzmann, W. N., Ingersoll, J. E., Spiegel, M. I., & Welch, I. (2002). Sharpening Sharpe Ratios. NBER Working Paper No. w9116. doi:10.3386/w9116

Goetzmann, W. N., Ingersoll, J. E., Spiegel, M. I., & Welch, I. (2007). Portfolio Performance Manipulation and Manipulation-Proof Performance Measures. *The Review of Financial Studies, 20*(5), 1503-1546. doi:https://doi.org/10.1093/rfs/hhm025

Gupta, A., & Sachdeva, K. (2024). Skin or skim? Inside investment and hedge fund performance. *Management Science*.

Henriques, D. B. (2011). *Bernie Madoff, the Wizard of Lies: Inside the Infamous $65 Billion Swindle.* Simon and Schuster.

Hurwitz, M. H. (2019). Focusing on Deterrence to Combat Financial Fraud and Protect Investors. *The Business Lawyer, 75*(1), 1519-1550. Retrieved from https://www.jstor.org/stable/27171056

Ivashina, V., & Scharfstein, D. (2010). Bank Lending During The Financial Crisis of 2008. *Journal of Financial Economics*, 319–338.

Jianu, I., & Jianu, I. (2021). Reliability of financial information from the perspective of Benford’s law. *Entropy, 23*(5), 557. doi:https://doi.org/10.3390/e23050557

Johnston, A. (2015). Regulating Hedge Funds for Systemic Stability: The EU's Approach. *European Law Journal, 21*(6), 758-786. doi:10.1111/eulj.12159

Jorion, P., & Schwarz, C. (2014). Are hedge fund managers systematically misreporting? Or not? *Journal of Financial Economics, 11*(2), 311-327. doi:10.1016/j.jfineco.2013.10.001.

King, J., & van Vuuren, G. (2016). Flagging potential fraudulent investment activity. *Journal of Financial Crime*, 882-901.

Kouwenberg, R., & Ziemba, W. T. (2007). Incentives and risk taking in hedge funds. *Journal of Banking & Finance, 31*(11), 3291-3310. doi:10.1016/j.jbankfin.2007.04.003

Kurdas, C. (2009). Does Regulation Prevent Fraud? "The Case of Manhattan Hedge Fund". *The Independent Review, 13*(3), 325-343.

Le Marois, O. (2013, October 11). *Return smoothing practices: A potential threat for alternative investment growth.* Retrieved from http://www.riskdata.com/resources/hedge_funds_risk_and_replication.html

Ley, E. (1996). On the Peculiar Distribution of the U.S. Stock Indexes' Digits. *The American Statistician, 50*(4), 311-313. doi:https://doi.org/10.1080/00031305.1996.10473558

Liang, B. (1999). On the performance of hedge funds. *Financial Analysts Journal, 55*(4), 72-85.

Modor Intelligence. (2024, 02 01). *Global Hedge Fund Industry Size & Share Analysis - Growth Trends & Forecasts (2024 - 2029).* Retrieved from Modor Intelligence: https://www.mordorintelligence.com/industry-reports/global-hedge-fund-industry

Muhtaseb, M. R. (2021). Does the system reward investors for fraud risk? A clinical analysis. *Journal of Financial Crime, 28*(4), 1106-1123. doi:10.1108/JFC-05-2021-0103

Muhtaseb, M. R., & Yang, C. S. (2008). Portraits of five hedge fund fraud cases. *Journal of Financial Crime, 15*(2), 179-213. doi:10.1108/13590790810866890

Neagle, S. (2024, January 27). *What is Insider Trading and How Can It Affect Your Portfolio?* Retrieved from NASDAQ: https://www.nasdaq.com/articles/what-is-insider-trading-and-how-can-it-affect-your-portfolio

Nigrini, M. J. (1999, May 1). I've got your number. *Journal of Accountancy, 187*(5), 79-83.

Nigrini, M. J. (2015). Persistent Patterns in Stock Returns, Stock Volumes, and Accounting Data in the U.S. Capital Markets. *Journal of Accounting, Auditing & Finance, 30*(4), 541–557. doi:https://doi.org/10.1177/0148558X15584051

Pearson, T. C., & Pearson, J. L. (2008). Protecting Global Financial Market Stability and Integrity: Strengthening SEC Regulation of Hedge Funds. *North Carolina Journal of International Law and Commercial Regulation*, 1.

Petrick, J. A., & Scherer, R. F. (2003). The Enron Scandal and the Neglect of Management Integrity Capacity. *American Journal of Business, 18* (1), 37-50. doi:10.1108/19355181200300003

Quisenberry, W. L. (2017). Ponzi of All Ponzis: Critical Analysis of the Bernie Madoff Scheme. *International Journal of Econometrics and Financial Management, 5*(1), 1-6. doi:10.12691/ijefm-5-1-1

Sanlam. (2024, February 01). *Hedge Funds*. Retrieved from Sanlam Investments : https://www.sanlaminvestments.com/individual/fundsandproducts/hedgefunds#:~:text=Hedge%20funds%20can%20be%20valuable,Investor%20Hedge%20Funds%20(QIHFs).

Scharff, M. M. (2005). WorldCom: A Failure of Moral and Ethical Values. *Journal of Applied Management and Entrepreneurship, 10*(3), 35-47.

Schaub, N., & Schmid, M. (2013). Hedge fund liquidity and performance: Evidence from the financial crisis. *Journal of Banking & Finance, 37*(3), 671-692. doi:10.1016/j.jbankfin.2012.09.019

SEC. (2023, August 5). *Investor Bulletin: Hedge Funds.* Retrieved from SEC: https://www.sec.gov/files/ib_hedgefunds.pdf

Sgambati, S. (2024). *The invisible leverage of the top 1 percent: Absentee debtors and their hedge funds.* CITYPERC Working Paper. Retrieved from https://www.econstor.eu/handle/10419/300389

Sharpe, W. F. (1998). The Sharpe Ratio. *Streetwise–the Best of the Journal of Portfolio Management, 3*, 169-185.

Shin, S., Smolarski, J. M., & Soydemir, G. A. (2017). Hurdle Rates and High-Watermarks: Incentives or Restrictions? *Journal of Accounting and Finance*, *17*(1), 124-143.

Tam Cho, W. K., & Gaines, B. J. (2007). Breaking the (Benford) Law. *The American Statistician, 61*(3), 218-223. doi:10.1198/000313007X223496

Teo, M. (2011). The liquidity risk of liquid hedge funds. *Journal of Financial Economics, 100*(1), 24-44. Retrieved from https://econpapers.repec.org/article/eeejfinec/v_3a100_3ay_3a2011_3ai_3a1_3ap_3a24-44.htm

van Dyk, F., van Vuuren, G., & Heymans, A. (2014). The Bias Ratio As A Hedge Fund Fraud Indicator: An Empirical Performance Study Under Different Economic Conditions. *International Business & Economics Research Journal, 13*(4), 867-896.

van Vuuren, G. (2014). The Bias Ratio – Can Fraud Be Modelled ? *Risk Update*. Retrieved 11 21 , 2023, from https://www.semanticscholar.org/paper/THE-BIAS-RATIO-%E2%80%93-CAN-FRAUD-BE-MODELLED-Vuuren/7564b2a986cfe9a53cc12ebcb12a98d78f1447ae

van Vuuren, J., & van Vuuren, G. (2022). Detecting Investment Fraud Using the Bias Ratio. *Sage Open*, 1-13.

Zakamouline, V., & Koekebakker, S. (2009). Portfolio performance evaluation with generalized Sharpe ratios: Beyond the mean and variance. *Journal of Banking & Finance, 33*(7), 1242-1254. doi:https://doi.org/10.1016/j.jbankfin.2009.01.005