THE OPTIMAL DEPLETION OF A MINERAL DEPOSIT

Samuel Yawson Eshun

A project report submitted to the Faculty of Engineering, University of the Witwatersrand, in partial fulfilment of the requirement of Master of Science in Engineering.

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DECLA.ATION

I declare that this project is my own unaided work. It is being submitted for the Degree of Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

27th day of August 1992.

ABSTRACT

The optimal depletion of a mineral deposit involves the optimisation of all the processes involved in the mining operation. Major parameters in the processes are the scale of the production and cut-off grades. But whereas optimisation of the production level is very well developed and receives much attention in the design of a mine, there seems to be no defined cut-off grades policy during the design. This project studies three cut-off policies: constant (conventional), increasing and declining (decreasing). By subjecting these policies to the same economic conditions, the declining cut-off grades policy has been found to be a better strategy when compared to the widely used constant or conventional policy and the suggested increasing policy. The declining policy was also found to provide more optimal return on investment under reasonably varying price trends than the other policies.

To Him who has promised me: "For I alone know the plans
I have for you, plans for good and not evil,
plans to give you a hope and a future",
to my blessed Lord and Saviour
Jesus Christ.
(Jeremiah 29:11)

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I am thankful to all at the Mining Engineering Department for making me feel very much at home. Professor H.R. Phillips, the head of department, Professor D.G. Krige, Dr. Isobel Clark, Mrs. Dee McKee, Mrs. Alice Kaplan, Mrs. Marilyn Bedborough and Miss Dorothy Nawa, to shorten the long list, have been so helpful.

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CHAPTER 1

INTRODUCTION

1.1 Statement of the Problem

A mining project is, basically, an investment venture with an objective to maximise returns. This objective can be realised when the exploitation process and all parameters leading to it are optimised to maximise the financial results. The cut-off grade is the most basic of such parameters, which is determined in part by the level of production cost and what material can be mined at a profit. The cut-off grade is therefore an important factor in defining the ore reserve. Three main cut-off grade policies have been suggested:

i.) increasing cut-off (Berry[1]),

ii.) constant or conventional cut-off (very widely practised) and

iii.) declining cut-off (Henning[2], Lane[3]).

The purpose of this project is to investigate these three policies and determine which gives the optimal depletion path(s) to maximise the present value of profits.

Factors other than the cut-off grade play an important part in making a mining project profitable. The size of the ore reserve is an important variable in determining the optimum mine size. The mine size, in turn, affects fixed and capital costs, as economies of scale are possible with larger production rates. Also the level of production cost and the price of the metal determine the cut-off (the material that can be mined at a profit) and, therefore, determine the size of the ore reserve. It is realised that each time any of these variables changes, it impacts on the other variables and thus on the financial results.

However for the purpose of this study only the impact of various cut-off grade policies will be investigated, and for the various scenarios of cut-off grades chosen for particular analyses, all of the other variables are assumed to be constant, which may not be necessarily so. Also the deposit used for the analyses is assumed to be well sampled so there is no possibility for new reserves being located in the area, to necessitate further expenditure and possibly the revision of mining plans.

1.2 Organisation of the Report

Chapter One begins with Statement of the Problem, where the justification and the importance of the problem is established together with some limitations of the undertaking. This is followed by a statement on the sources of data, procedures applied and the treatment of all findings. Then a discussion on the theory of optimal depletion, the main subject of the project, is done. The introductory chapter ends with a literature survey carried out on the history and present status of the problem from pioneer work and present authorities in the field of practice.

The central theme is addressed in Chapter Two with an estimation of the reserve to be used in the analysis by geostatistical methods. Structural analyses are done to produce a model semi-variogram for the deposit, followed by kriging and block estimation to determine the mineable reserves.

Chapter Three discusses the strategic mining paths for the deposit, dealing with important considerations like the mining method, mining costs and revenues, production rates and other factors that relate to mining costs and revenues.

The core of the report is in Chapter Four where the various types of cut-off grades are discussed and the pay limit (the economic break-even grade) of the deposit under study is derived. After obtaining the tonnage and average grade above the pay limit, the three cut-off policies are applied to the reserve under study.

After the discussion of the outcome of the analysis of the various policies, Chapter Five continues with the effect of other factors that affect the policies such as costs, grade distribution and price of gold.

Chapter Six is a discussion of the procedure on how the optimum cut-off policy may be implemented in practice and application of the procedure to the model under study.

Conclusions of the study are outlined in Chapter Seven and some recommendations are discussed to end the report.

1.3 Data sources and procedures

The data used to evaluate the ore reserve is simulated chip samples at 2 metres intervals over the entire area, nearly 900 000 samples in total. Due to limitations of the available computer facility to handle this amount of data intervals, intervals of 20 metres were chosen, thereby reducing the data to a manageable 7 920 samples.

Cost data were obtained from publications, company reports and discussions with technical staff in industry.

The cut-off policies were each analysed under the same economic conditions. Various factors which impact on the policies are determined and analysed for their effects.

All computer facilities were provided by the Mining Engineering Department, University of the Witwatersrand, and all software used have been licensed to the same.

1.4 The theory of optimal depletion of exhaustible resources

1.4.1 Introduction

Concern for the perceived dwindling of the world's natural resources has called for optimal depletion of exhaustible resources. In his 1931 paper, Hotelling[4] had this to say about the optimal rate of exploitati.. :

> " The world's disappearing supplies of minerals, forests and other exhaustible assets have led to the demands for regulation of their exploitation. The feeling that these products are now too cheap for the good of future generations, that they are being exploited at too rapid a rate, and that in consequence of their exhaustion they are being produced and consumed wastefully, has given rise to the conservation movement."

It is a strongly held view, otherwise, that as a consequence of efforts of producing nations and large corporations to restrict output and thus raise prices, some resources are being depleted too slowly.[5]

The theory of optimal depletion of an exhaustible resource is an enquiry into the question of whether our exhaustible resources are being depleted too rapidly or too slowly. The origin of this field, natural resource economics, is widely recognised as Hotelling's paper in April, 1931.[6] He gives a list of "intriguing problems" associated with exhaustible resources: How much of the revenue of a mine should be recognised as return on capital, that is how high should the discount rate be, since it determines a large part of the operator's depletion strategy? What is the value of a mine when its content is supposed to be fully known, and what is the effect of uncertainty on estimates? If a mine operator produces too rapidly, he may depress the price, if he produces too slowly his profits, though larger, may be postponed further into the future than the rate of interest warrants. Where then is his optimum? And how does this profitable rate of production vary as exhaustion approaches? Is it more profitable to complete the extraction within a finite time, to extend it indefinitely in such a way that the amount remaining in the mine approaches zero as a limit, or to exploit so slowly that mining operations will not only continue at a diminishing rate forever but leave an amount in the ground which does not approach zero. How should exploitation take place to maximise benefits to the community, and will this compare with the action of the profit-seeking entrepreneur? How can the state, by regulation or taxation, induce the mine-owner to adopt a schedule of production more in harmony with the public good? For all these dynamic systems what becomes of the classical economic theories of monopoly, duopoly and free competition? Further questions can be raised about market failures peculiar to these resources, involving things such as the uncertainty that surround their discovery, the varied characteristics of different deposits, the risks that surround their exploitation, the high capital investment involved - especially in mining, and the lasting effect of current decisions about their use and about the environmental disruption that often accompanies the extraction and use of exhaustible resources.

Conflict about how the community (or a government of a nation) and a private exhaustible-resource exploiting firm (trans-national mining corporation) will define "optimal depletion" is inevitable. Government will prefer to maintain employment for as long as possible for the community, even if it will mean subsidising firms, whereas private firms are motivated by profits and will want to deplete a resource as quickly as the technology and market permit.

In the framework of this project the optimality will be seen from the viewpoint of a privately owned mine owner. The question he seeks to answer is: "How should the cut-off grades of the mine be determined so that the highest possible net present value is obtained?"

1.4.2 Key results in the theory of optimal depletion

There are two conditions that must be satisfied in any optimal depletion regime. The difference between an exhaustible resource and an ordinary good or any other resource is simply that it is limited in quantity and is not reproducible. This means that the extraction and consumption of a unit involves an opportunity cost, defined in terms of economic rent as "the value of a factor to its user, measured by the value of the best alternative use. The opportunity cost is the factor's marginal value product in the alternative use".[7] The opportunity cost, then, must be taken into account in determining how to allocate the resource over time. In particular, instead of the usual efficiency condition,

price = marginal (production) cost

we have

price = marginal (production) cost + opportunity cost.

This is the first condition of optimal depletion.[8] Given a stable demand pattern for a resource the planner, seeking to allocate extraction efficiently over time, will extract more of the resource now if the resource was reproducible, because he does not worry about exhaustion and thus the opportunity cost of depleting the resource now. He will extract a certain quantity now in order to have a positive difference between the price of the resource and the marginal cost of production or extraction. How much will be extracted or how much will be the profit will be determined by the owner's perception of the value of the resource or the opportunity cost forgone in extracting the resource.

The second condition of optimal depletion describes the behaviour of the opportunity cost over time. Opportunity cost, the difference between price and marginal extraction cost, is known by different names in resource economics literature; user cost, royalty, rent, net price and marginal profit. The term royalty will be used in this discussion. A fairly general and simplistic result for the second condition is the case where the costs do not rise with cumulative extraction. It shows that the present value of the royalty must be the same in all periods, or equivalently the undiscounted royalty must rise at the rate of interest. This conclusion was arrived at also by Hotelling in 1931, in what has become the famous "Hotelling Rule", which states that the price of an exhaustible resource must grow at a rate equal to the rate of interest, both along an efficient extraction path and in a competitive resource industry equilibrium. He showed that the competitive resource owner would deplete at a socially optimal rate assuming that he discounts at the same rate as the community would, that is, there is no difference between the private and social discount rates. The Hotelling Rule is just the simplest case because, apart from neglecting extraction costs, it is the royalty not the price that should grow at the rate of interest.

Herfindahl[9] considered the optimal extraction profile of a competitive firm with several deposits of an exhaustible resource, for each of which the average and marginal cost of extraction is constant but costs of extraction differed. This is likened to a group of mines mining deposits of varied characteristics and thus costs structures. He showed that if the firm seeks to maximise its present value then it must begin working on the lowest cost portion in the mining sequence. This general finding was re-examined by Kemp and Long[10] on his assumptions about costs and revenue, and came to the conclusion that if the rate of price increase is less than the rate of interest then the deposit must be extracted in strict sequence, beginning with the lower cost deposit.

In terms of cut-off grade policy the implication, established simply by interchanging the roles of cost and revenue, is that if the rate of rise of price is less than the interest rate, then optimal depletion must mean beginning with the higher revenue or the lower cost deposit. The converse will also hold, for the case where the rate of rise in the price is higher than the rate of interest, resulting in policy that extracts the lower revenue or higher cost at the beginning. These issues are some of the subjects the second condition will address as it follows the path of the royalty over time.

A very important aspect of optimal depletion is the issue of efficiency. Inefficient depletion or wastage in any way cannot be accepted in an optimum depletion model. Efficiency, in this context, requires that there should be no gain in shifting extraction from one period to another, which in turn implies that the returns must be the same for all periods.

In identifying an efficient pattern of exhaustible resource allocation within a market-determined or competitive equilibrium (when there is no gain in shifting extraction in time), some difficulties are identified. Firstly there exist some externalities or market failures that can make an equilibrium inefficient. Two particular kinds of externalities are spillovers, associated with exploitation of a common pool by several different producers, and environmental disruption. The common pool problem is brought about when, say, information is obtained cheaply by observing other operators' actions, for example results of drilling, and thereby depleting adjacent resource without incurring all the accompanying exploration costs. The environment has now become a vital issue, but before it has been cheaply used to the extent that it has been depleted inefficiently. These two externalities lead to inefficient resource depletion.

The second source of divergence between competitive equilibrium and efficient rates of depletion is the disparity between private and social rates of discount. The effect of discounting on depletion, the issue of private versus social or community discount rates and the questions related to the welfare of future generations are discussed later.

Before going further into the discussion, one more point must be made. By optimal depletion is meant efficiency, as discussed already, and is also meant the pattern of depletion that maximises the present value of net benefits from the resource. However, the concept of optimality, i.e efficiency and maximisation of net benefits, will not address the distributional concern of specifying to whom the benefits accrue.

1.4.3 A model of Optimal Depletion

The difference between the initial discussion and the model to be developed here lies in the emphasis on the resource stock. Whereas it has previously been assumed that extraction costs are not affected by the size of the remaining stock, it will now be assumed they are affected. Also production was assumed to have come from a single deposit and no separate producers were distinguished. In what follows, the analysis is first done to determine how much resource may be produced by each of many identical firms under the control of a planner. The planner may be some form of regional or global resource overseer. After this, it is assumed that the firms are local types or specific mines, independently managed, and competitive, seeking to maximise the present value of profits. If the resulting depletion paths turn out to be the same as the planner's, then it shall have been shown that a competitive equilibrium is efficient, subject, of course, to the qualifications concerning potential market failures or externalities noted earlier.

Let the extraction costs for each of n identical resource firms be a function of the firms's output and the stock it holds. In symbols

$$c = c(y_t, X_t)$$

where c is total cost of extraction

y_t is output in period t

X_t is the resource stock in period t.

Then cost can be positively ($\partial c/\partial X > 0$), negatively ($\partial c/\partial X < 0$) or not related to the stock, ($\partial c/\partial X = 0$).

The normal case is the negative relation, that is the smaller the remaining stock the larger the cost of extraction. Thus, by letting cost be a function of stock size, a simple way of capturing the effect on stock and cost for the model has been derived.

Competitive Depletion - Resource Planner

The measure of benefit from a resource or revenue is measured by the willingness to pay for the resource, and given by the integral of the demand for the resource. In symbols, this is

$$\int_0^{ny_t} p(\xi)\, d\xi$$

where ny_t is the total consumption in period t

$p(ny_t)$ is the demand in period t

and ξ is variable of integration.

The net benefit is the difference between the benefit and cost:

$$\int_0^{ny_t} p(\xi)\,d\xi - nc(y_t, X_t)$$

The essence of the depletion problem is that which is extracted and consumed in one period and is not available in another. In making a decision about what to extract in any one period, the planner must look to the effects on benefits and costs in other periods as well. Thus his problem is to maximise the present value of the sum of benefits minus costs over the relevant period. In symbols, this is

$$maximise_{y_0 \cdot\cdot y_t \cdot\cdot y_T} \sum_{t=0}^{T} \frac{\int_0^{ny_t} p(\xi)\,d\xi - nc(y_t, X_t)}{(1+r)^t}$$

subject to the constraint on the cumulative extraction;

$$\sum_{t=0}^{T-1} y_t = \overline{X_0} - \overline{X_t}$$

where X_0 and X_t are known numbers, and the amount extracted from the stock in any period is the difference between the amount of stock at the beginning and end of the period. The constraint can be re-written as

$y_t = X_t - X_{t-1}$ (from t=0 to t=T-1 for each firm).

Applying the Lagrange maximisation technique, the Lagrangean is

$$L = \sum_{t=0}^{T} \frac{\int_0^{ny_t} p(\xi)\,d\xi - nc(y_t, X_t)}{(1+r)^t} + n\sum_{t=0}^{T-1} \mu_t (X_t - X_{t+1} - y_t) + n\alpha(\overline{X_0} - X_0) + n\beta(\overline{X_T} - X_T)$$

where μ_t, α and β are Lagrangean multipliers. Solving, the necessary conditions for the output in period t y_t and the stock

X_t are respectively,

$$\frac{p(ny_t)-\frac{\partial c}{\partial y_t}}{(1+r)^t}-\mu_t=0 \qquad (a)$$

(t = 0 to t = T-1)

and

$$-\frac{\frac{\partial c}{\partial X_t}}{(1+r)^t}+\mu_t-\mu_{t-1}=0 \qquad (b)$$

(t = 0 to t = T-1).

Equation (a) is the first condition of optimal depletion. It says that along the optimal path, the price $p(ny_t)$ is just equal to the marginal cost of extraction $\partial c/\partial y_t$ plus the undiscounted royalty $\mu_t(1+r)^t$. The Lagrangean multiplier, μ_t, is interpreted as the royalty; the value of a unit of the resource "in the ground", in terms of the discounted objective function. Equation (b) is the second condition of optimal depletion that describes the behaviour of the royalty over time. Re-writing it in terms of continuous time instead of discrete, equation (b) becomes

$$\frac{d\lambda_t}{dt}=r\lambda_t+\frac{\partial c}{\partial X_t} \qquad (c)$$

There is a variety of results from the second condition depending on the effect of extraction cost on the size of the remaining stock of the resource.

Supposing that the extraction cost is not affected by the size of the remaining stock, that is $\partial c/\partial y_t = 0$. Then equation (c) confirms that the rate of change in the royalty, $(d\lambda_t/dt)/\lambda_t$, is equal to the rate of interest, r. If, in addition, the marginal cost of extraction is negligible, then from equation (a) the royalty is equal to the price and from equation (c) the rate of price change $(dp/dt)/p$ is equal to the rate of interest. Whereas this result has been found already to be fundamental in economics of exhaustible resources, this analysis shows it to be rather a special case. Production of oil in the Gulf states provides a remarkable approximation, being extracted at a cost of 10 to 20 cents per barrel, certainly a negligible fraction of the selling price ($30 to $40 in 1981). In the larger oilfields cumulative extraction will not affect the cost of extraction. It is noted that the oil price does not rise smoothly at the rate of interest because OPEC decisions do not necessarily coincide with those of a planner seeking to maximise consumer surpluses, or those that would result from a competitive market.

If the extraction cost is affected by the size of the remaining stock, in particular as depletion proceeds from better and more accessible deposits or portions to the poorer and less accessible ones, that is $\partial c/\partial X_t < 0$, then equation (c) shows that the rate of change in royalty is less than the rate of interest. The return on investment generally includes both capital gain and a dividend, but there is no dividend on investment in resources in the ground. Therefore the entire return must come from the capital gain, i.e the rise in the royalty. But when a reduction in the stock has the effect of increasing subsequent extraction costs, then, without considering other factors, holding the resource in the ground and not mining provides a dividend in the form of cost saving.

The intermediate case is that in which the extraction cost is not affected by stock size over a period corresponding to, say, production from a deposit of constant quality, but rises when the deposit is exhausted and production shifts to a poorer deposit. In this case the royalty will rise at the rate of interest along the intensive margin, that is, as the richer deposit is being depleted ($\partial c/\partial X_t = 0$ approaches $d\lambda_t/dt = r\lambda_t > 0$) and then fall at the extensive margin, that is as production shifts to poorer deposits.

Thus far conditions have been derived for optimal depletion of an exhaustible resource. By using the Hamiltonian technique, more general optimal control conditions are derived, which are more general and usable in the rest of the analysis and are summarised in Table 1.4.1.

Competitive Depletion - A Firm

Now we apply the general control conditions (Table 1.4.1) to the problem of an individual competitive firm. The difference between this and the problem of the planner, is that instead of the hypothetical regional or global resource planner seeking to maximise the net social benefits, the decision maker here is a firm seeking to maximise profits. In the framework of the of preceding problem the focus is on one of the n identical firms, now acting independently and in a competitive, price-taking

Table 1.4.1 A comparison of optimal depletion and optimal control conditions.

The depletion problem:

$$Maximise_{|y_t|}\int_0^T [\int_0^{ny_t} p(\xi)\, d\xi - nc(y_t, X_t)]\, e^{-rt} dt$$

subject to

$$\frac{dX_t}{dt} = -y_t, \qquad X_o = \overline{X}_o, \qquad X_t = \overline{X}_t$$

Hamiltonian

$$H = \int_0^{ny_t} p(\xi)\, d\xi - nc(y_t, X_t) - \omega_t y_t$$

Optimal Depletion Conditions	Optimal Control Conditions
$p(ny_t) - \frac{\partial c}{\partial y_t} = 0$	$\frac{\partial H}{\partial y_t} = 0$
$\frac{d\lambda_t}{dt} = r\lambda_t + \frac{\partial c}{\partial X_t}$	$\frac{d\omega_t}{dt} = r\omega_t - \frac{\partial H}{\partial X_T}$
$\frac{dX_t}{dt} = -y_t$	$\frac{dX_t}{dt} = \frac{\partial H}{\partial \omega_t}$

(where $\omega_t = r\lambda_t$)

(p=p(ny)) position.

The firm's problem is

$$maximise_{\{y_t\}}\int_0^T [py_t - c(y_t, X_t)]e^{-rt}dt$$

subject to

$$\frac{dX_t}{dt}=-y_t, \qquad X_0=\overline{X_0}, \qquad X_t=\overline{X_t}$$

the H iltonian is

$$H=py_t-c(y_t,X_t)-\rho_t y_t$$

These necessary conditions are exactly the same as the results in Table 1.4.1. Thus it has been shown that the conditions that describe the profit-maximising depletion for a competitive producer are identical to the conditions that describe the socially optimal path, given a number of assumptions of varying plausibility.

The next part of the discussion will focus on what happens when the assumptions do not hold. Monopoly and other externalities, such as uncertainty about future demand and supply of the resource and the importance of the discount rate in determining the depletion profile, are discussed.

Monopoly

The monopolist's problem, analytically, is exactly the same as the competitive firm. The only difference is in the first condition, because the monopolist will take into account the influence of his own output decision on price. Analyses[a] show that generally the monopolist depletes the resource more slowly. More so if the quantity of the commodity produced (supply) is inelastic, causing big price changes to produce little changes in quantity of the commodity supplied. However depletion may be accelerated if price rises at a rate greater than the interest rate. It can be added that the tendency to deplete slowly would be strengthened where costs rise with cumulative depletion.

As far as a cut-off policy is concerned the monopolist will choose one that will offer him the chance to deplete slowly, except when the mentioned exceptional situation about increasing price does occur.

Uncertainty

Uncertainty can arise from different sources, including resource demand and supply or from both. The fear about uncertainty is in its ability to disrupt revenues expected from depletion. It is typically reflected in economic models as a higher discount rate. Some key uncertainties in the resource market are now discussed.

When the uncertainty is about demand for the resource, then it may be reasonably assumed that the further into the future any extraction decision is, the more uncertain will the demand be. The resource owner is will likely be more certain about present

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[a] This was Hotelling's view and it is supported by the results of more recent studies by Weinstein and Zeckhauser, Kay and Mirrlees[22].

demand than the future. If he wants to avoid future demand risks, he will shift depletion to the present just as it would be if the discount rate were increased. If the demand for the commodity, and thus the revenue from its exploitation, is determined largely by how much is produced, then the resource owner will tend to reduce the risk of low demand by cutting back now to produce more later. That way he can stifle the present demand for his future advantage.

Another kind of uncertainty related to the demand for the resource can tilt depletion to the present. Where the resource is subject to the threat of expropriation or there is the risk that the market may be lost because of the appearance of a lower-cost substitute at some future date.

On the supply side, uncertainty may be largely due to the outcome of exploratory activity (treated later) or knowledge of the exact size of the resource stock.

The impact of uncertainty about future demand on cut-off grade policies will be determined largely by the nature of uncertainty and also by the risk aversion of the owner. But definitely any uncertainty may adversely affect future revenues and will be avoided by taking to a cut-off grade policy that will promote present exhaustion. Uncertainty future supply may be incorporated in the cut-off grade policy by depleting slowly or postponing exhaustion - the "eating a cake of unknown size" syndrome.
The different kinds of uncertainties and their effects are

summarised in Table 1.4.2.

Table 1.4.2 Effect on depletion of different kinds of uncertainty.

Kind of uncertainty	Effect on depletion
Uncertain demand for the resource, with degree of uncertainty related to distance in time from the depletion decision	Shifts depletion to the present
Uncertain demand for the resource, with variation in expected returns related to the quantity of output	Shifts depletion to the future
Uncertainty regarding date of event that will destroy the value of the resource to the owner (expropriation, discovery of a cheaper substitute)	Shifts depletion to the present
Uncertain size of resource stock	Shifts depletion to the future

(Source: Fischer[5])

Discounting

A high discount rate for exhaustible resource projects generally indicates high risk and a low return on investment. It will reduce investment generally and the exploration and development of new deposits specifically. Increase in discount rate will also mean that more will have to be extracted in a short time in order to meet the high discount rate, and will lead to more rapid depletion initially, followed by a reduced rate as a consequence of restricted investment in exploration and development in the initial period. Depletion from known, producing mines will be accelerated but little effort will go into finding and developing new ones. The implication for a cut-off grade policy is that higher discount rate will necessitate a form of "high-grading", by exhausting the profitable portion of the deposit in order to meet the expected high return on the investment.

Instability in the resource markets

Uncertainty of any nature may lead to instability in the resource market, with the consequence that depletion may proceed either rapidly or slowly, being inefficient.

Because the present futures markets are of limited time periods, and do not span the infinite future to enable resource owners sell of their commodity at a determined price over the period of production, resource owners must form expectations about future prices and then act on them to make decisions about how much of the resource to extract in any given time. A plausible way is studying the behaviour of current prices, perhaps from a trend of current and past prices. If a rise in the current price leads to a rise in the expected future price, this will lead to further rise in the current price as resource owners decide to cut back on production to take advantage of the higher price expected in the future. If, in turn, the second-round rise in current price implies a further rise in the future price there may be the possibility of an explosive cycle. The possibility depends on the elasticity of expectations, defined as the percentage change in the expected future price divided by the percentage change in current price. The existence of an equilibrium will depend on the behaviour of the elasticity of expectation, where the expected future price is either too high or too low. Elasticity of expectations greater than one will lead to the explosive cycle, whereas a value of one or less will be consistent with equilibrium of the price path.

Knowledge of cost developments and the trend in the technology for producing the resource and the likely demand for it will set bounds on expected future prices. A cut-off grade policy will be effected to a great extent by the mine owner's perception about future price and his expectation about the present trend on the future. If he anticipates a rise in future price he will choose a policy that will allow him to postpone extraction to the future. If he perceives a lower future price he will deplete now.

Exploration

It has been already suggested that changes in expected future price have a feedback effect on the current price of a resource. A rise in expected future price, for example, will lead suppliers to cut back on current production, implying a rise in current price. The effect of discovery of new deposits of the resource can be understood as an example of the working of the same feedback mechanism. If new deposits are discovered the price expected in the future will fall. This will, in turn, lead suppliers to expand current production, resulting in a fall in current price.

Discovery or exploration can be defined generally as an activity that increases the stock of and thereby lowers extraction costs. Obviously, the discovery of a new deposit will do this, but so can research and development of a new process that makes profitable production of the resource material formerly considered too poor in quality. Purposeful exploration can be built into the model by assuming a function that relates cost to discoveries, just as another function relates cost to extractive output. The outcome of including this in the model is that royalty is equal to the cost of finding another to replace it, in a competitive equilibrium. Previously royalty was interpreted as the benefit of having an additional unit in stock. Where a unit can be added, the benefits just balanced by the cost of adding to it.

So because of exploration efforts, the effect of depletion on the stock is countered, as least in part. The change in stock is equal to the number of units found less the number extracted. If change in stock is positive, the effect is that price rises more slowly or may even fall.

The outcome of an exploratory programme, whether positive or otherwise, may be seen as a reduction of uncertainty about the future supply of the resource. While new discoveries may not necessarily mean quick depletion of existing stocks, it may indicate to resource owners that prices are not going to rise (quickly) in the foreseeable future. A negative outcome will be a clear signal of the possibility of higher price brought about by scarcity. Any cut-off grade policy to be chosen should see the outcome of exploration in this light.

Equity and exhaustible resources

So far the criterion for judging rates of depletion has been that of social efficiency, by means of instruments including taxation. This criterion, however does not address the question of equity, both in the present and later generations. This subject is the main body of welfare economics and will only be briefly discussed.

In the present generation benefits from depletion of a resource may be re-distributed by government instrument and a deliberate policy. But given a positive discount rate, it is possible that a depletion programme that is efficient from the viewpoint of the present generation may leave nothing for the future generations. This raises ethical and welfare considerations which economists are still addressing. The issue is conveniently broken into two parts: social discounting and welfare criteria. With social discounting the debate concerns how to discount (and whether or not to discount) utilities, benefits or otherwise that are projected to accrue to the future generations, and the difference between the social and private discounts. On welfare criteria various schools of thought have emerged, a prominent one being the "maximin" criterion by Rawls[b] intended to apply only within a generation, but which has been extended to evaluate intergenerational programmes.

Much as it is noted that these distributional issues will underscore the optimal depletion mode or the choice of a cut-off grade policy to a very large degree, the complex issues they raise in welfare economics cannot be easily addressed in this project.

1.4.4 Summary

The issue of optimal depletion is becoming an increasingly important concern. It is complex and involves many, hitherto unaddressed areas like intratemporal equity. Nonetheless the issues must be faced. All optimal depletion models must take into account:

a) the opportunity cost or royalty determined by the value placed on the resource or asset, and

b) the behaviour of the opportunity cost along the depletion path with respect to factors such as changes in cost of extraction and prices.

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[b] Rawls, in 1971, suggested that social welfare is determined by the welfare of the least well-off member of society and so social welfare is maximised by maximising the welfare of the least well-off individual. Analytically this is symbolised into

$$\max W = \max [\min (U_1, U_2, \ldots\ldots U_n)]$$

where W is social welfare

U_i is individual i's utility.

1.5 Literature Survey

The issue of cut-off grades has long standing controversies and thus demands precise terminology. In particular, a clear distinction is needed between "cut-off grade" - an operating control - and the terms "pay limit" and "break-even grade" which are grade equivalents of particular specifications. (Taylor[11]). He defines cut-off grade as any grade that for any specified reason is used to separate two courses of action e.g. to mine or to leave, to mill or to dump; also any of a series of grades used to truncate a frequency distribution or to separate mineralised materials into graded fractions. In the same paper[12], he describes optimum cut-off grade for each pair of three main exploitation stages (ore availability, ore handling or treatment and product treatment) as the balancing grade subject to upper and lower limits defined by the two break-even grades. From the three grades so selected, the middle value gives the overall optimum cut-off that will maximise the total undiscounted profit of the mineral inventory.

Dowd[13] applied dynamic and stochastic programming to optimise cut-off grades and production rates, allowing for the forecasting of future market conditions in terms of probability, and used the maximum present value as the optimising criteria. His work serves as the foundation on which a general theory of cut-off grades under real market conditions can be established.

Pasieka and Sotirow[14] outlined a computerised net cash flow approach for practical cut-off grade determinations and observed that in a constantly changing technical and economic environment, the decision to maintain a mining operation at its best economic potential requires an appropriate cut-off grade strategy consistent with short and long term operational and corporate objectives, and based on realistic marketing and metal price forecasts.

Much earlier work related to cut-off grades was done by gentlemen like Berry, Carlisle and Mortimer. Berry[15] applied the present value theory to justify the permanent raising of cut-off grades well above the break-even point. Carlisle[16] in three papers of 1953/54, makes a brave effort to move away from strict orthodoxy but a cost classification introduced serious errors by divorcing mine expenditures from activities requiring them. In a paper on Witwatersrand grade control, Mortimer[17] gave a definition of cut-off grade which cannot be bettered: "the average grade of rock must provide a certain minimum profit per ton milled .... the lowest grade of rock must pay for itself".

Krige[18], in two papers published in 1962, proposed certain downward adjustments on the official South African pay limit policy based on statistical regression theory; in later papers he investigated the effect of taxation systems on break-even grades. He is credited with the statement of two important issues, the first being the highly necessary concept of a "minimum selectable unit" which he determined for particular gold mining conditions. Secondly, he demonstrated that any tax code which permitted deduction of part of revenue from taxable income lowered the break-even grade provided that the average profit of the mine is above some critical value.

Henning[19], in a short concentrated paper, displayed a lot of logic. He assumed that ore parcels conformed to an exponential law (imposing thereby a constant difference between cut-off and average grades), that unmined ore becomes unrecoverable and that costs could be wholly defined as variable, fixed or capital and other sunk categories. He derived a formula for cut-off grade that among other things, i) showed sunk costs to be without direct relevance and, ii) for the first time showed that true maximisation of present value demanded a declining cut-off grade.

Lane[20] must be the pioneer of serious cut-off grade theory. His significant contributions were to distinguish between the three main stages of mining activity with their distinctive costs - ore creation, ore treatment and product treatment - to note that cut-off grades affected the quantitative balance between these stages; and that capacity constraints in each case imposed different break-even requirements. These break-even points served as markers between which the best cut-off grade might float according to particular conditions. In addition he confirmed the need for a declining cut-off if present value were to be maximised.

Wells[31] must also be noted for his paper on optimising mine design in mineral evaluation, in which he observed that in theory it is assumed that cut-off grade of a mine remains constant throughout the life of a mine, but in practice early revenue may be increased (at the expense of reducing cash flow in the later years of the mine's life), by selectively mining to produce milling grade higher than the average tenor of the reserves. He remarked that this same philosophy will continue to be attractive throughout the life of the mine, leading to a progressive deterioration in the mill head grade. In his analysis, he proved that the progressive deterioration in the reserve grade and in the average mining grade over the life of the mine increased the present value ratio, his chosen optimisation criterion. He observed that in practice it is likely that addition of blocks above the economic cut-off grade in the later stages of the mine's life would further increase the present value ratio. He also showed in his analysis how controlled build up to full production capacity, more practicable than assuming full capacity at the start, can favourably affect the return on the investment under certain circumstances because it delays early capital outlays.

Still on the theory of declining optimum cut-off grades, Border[32] has recorded that the use of declining cut-off grades, which has been thought to cause high-grading, is in fact true if an artificially high discount rate is used in the calculation. He observed that the theory of declining cut-off grade is more than 25 years old, but for a number of reasons it is still not widely used in industry. He also notes that the lack of faith in the veracity of estimated grades is not a good reason to abandon any attempt to optimise cut-off grades. In his paper he suggested some optimisation equations and showed how they can be practically integrated into the various stages of converting a prospect into an underground mine. He concludes that perfect application of an optimum cut-off grade policy must ensure that only ore generating more than a minimum net revenue is mined.

CHAPTER 2

ORE RESERVE ESTIMATION

The ore reserve investigated is a typical Witwatersrand low grade, flat, thin reef. The mine has been in operation for the past 20 years. Most of the claim area has been developed and there are few remaining places where the grades are not tested. Fully developed reserves for which detailed grade-tonnage compilation is to be done for the analysis amount to some 8 million tonnes.

From the statistics accumulated over the years, a "regression" effect has been established between the predicted grades and the actual grades, based on comparisons between the sample results taken during development and panel sample results taken during subsequent mining. The relationship indicates that the lower grades are under-estimated and the higher grades over-estimated. It is generally linear on the logarithm. This regression effect is integrated into the grade-tonnage data by adjusting the grade estimate for each selective mining unit.

The geostatistical method was used for the estimation of the reserve. The software employed was the **Geostokos PC Toolkit**[f]. The analysis was simplified because estimation of the reserve is not the main objective of the project.

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[f] This software was developed by Dr. Isobel Clark of the Mining Engineering Department, University of the Witwatersrand.

2.1 Spatial Statistics

The data points used were simulated using "conditional simulation"[23] on the Kimberley reef (Winkelhaak Mine) for chip samples 20 m x 20 m over the study area. The statistics for raw data of the mineralisation are:

Number of sample points : 7920

Mean : 2.92 g/t

Median : 1.5 g/t

Standard deviation : 3.8893 g/t

Variance : 15.1267 $(g/t)^2$

Maximum 61.3 g/t 2

Minimum 0.0 g/t

Range 61.3 g/t

Interquartile range 3.00 g/t

Skewness 2.66

Kurtosis 9.56

2.2 Modelling

Semivariograms of the raw data gave better structures than log transformed data. This could be due to the fact that the underlying ore value distribution is not log normal. The log transformation gave very different statistics, such as:
mean = 0.3568 log g/t, variance = 1.7966 $(\log g/t)^2$,
skewness = -0.2171, kurtosis = -0.7884. The coefficient of

correlation being a high of 3.7569, as compared to only 1.3320 for the raw data.

The semivariogram model chosen for the mineralisation is shown in Figure 2.2.1 and had the following description:

Model : Spherical

Nugget effect : 6 $(g/t)^2$

No. of components : 2

Range of influence of 1 : 80 m

Sill of 1 : 8 $(g/t)^2$ Sill of 2 : 3 $(g/t)^2$

Range of influence of 2 : [illegible]0 m

Anisotropy factor major axis : 1.0

Anisotropy factor minor axis : 0.5455

Angle of rotation : 45°

Cross validation statistics

mean error : - 0.007

standard error of mean : 1.0174

2.3 Kriging - Block Estimation

Kriging over the area on 60 m x 45 m panels generated 1102 blocks. The kriged panels were found to have a distribution with mean = 3.03 and log variance 0.2721. Histogram and probability plot of the distribution are shown in Figure 2.3.1 and Figure 2.3.2.

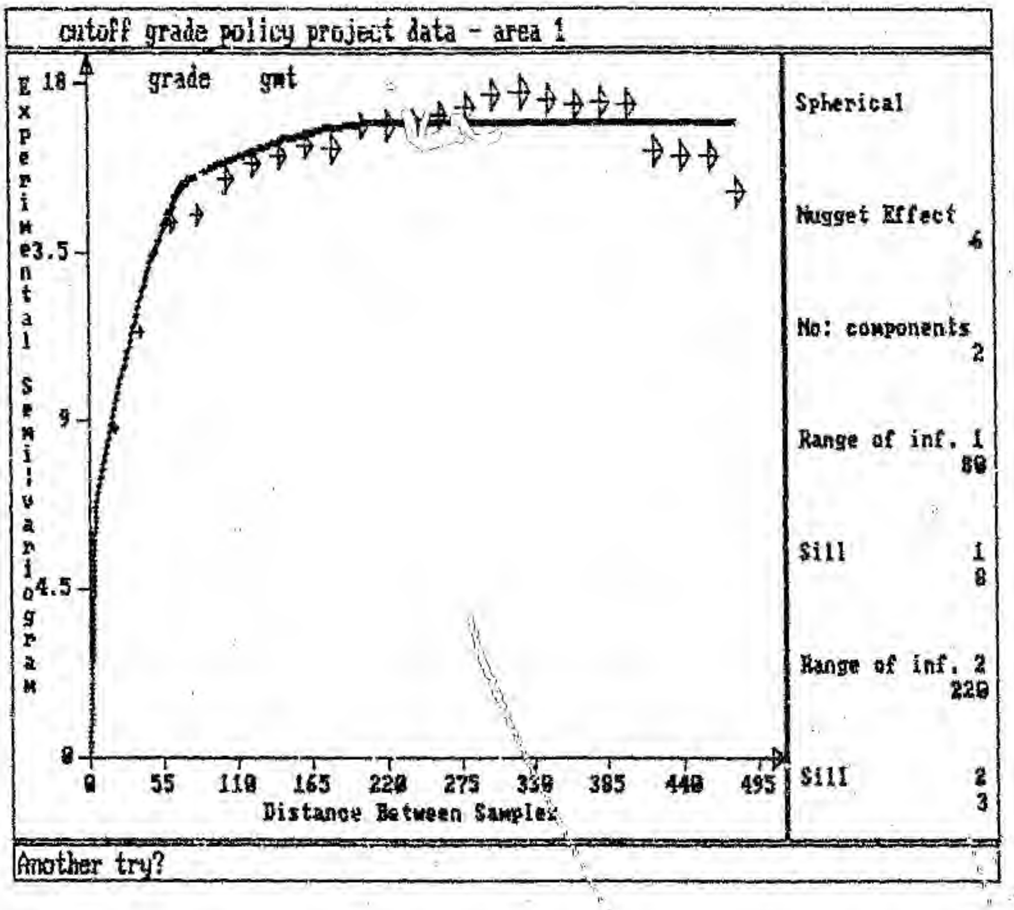

Figure 2.2.1 The model semivariogram of the experimental semivariogram points.

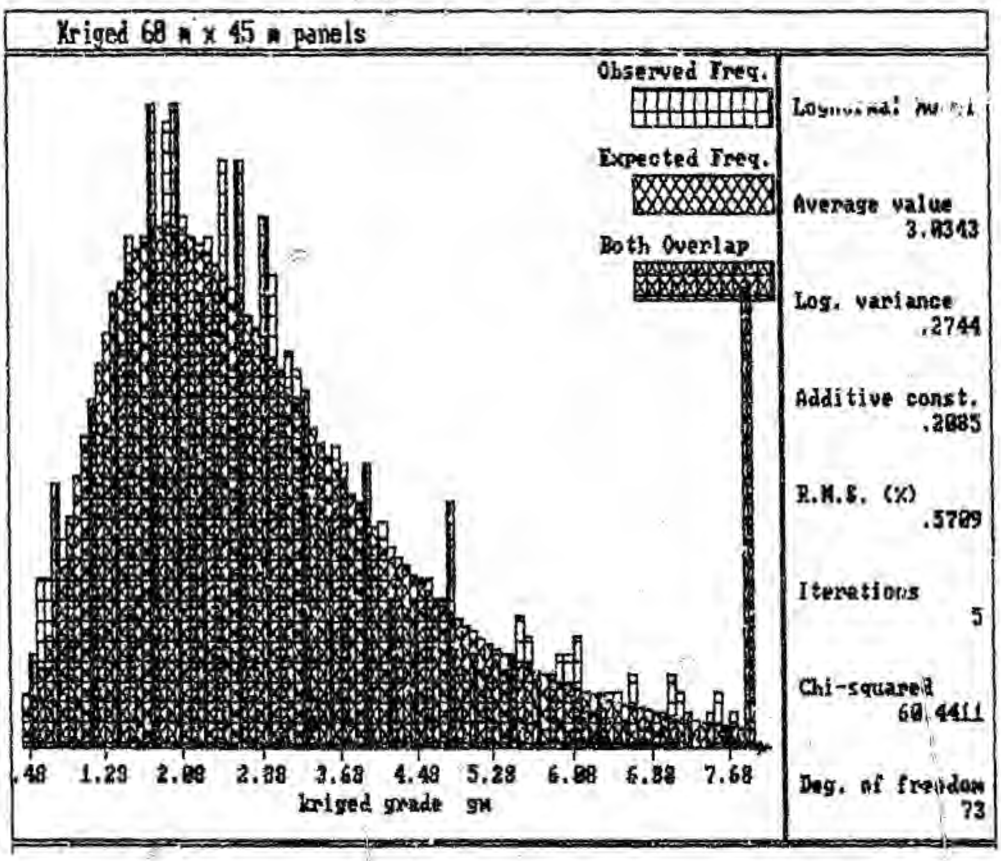

Figure 2.3.1 Histogram of kriged block grades.

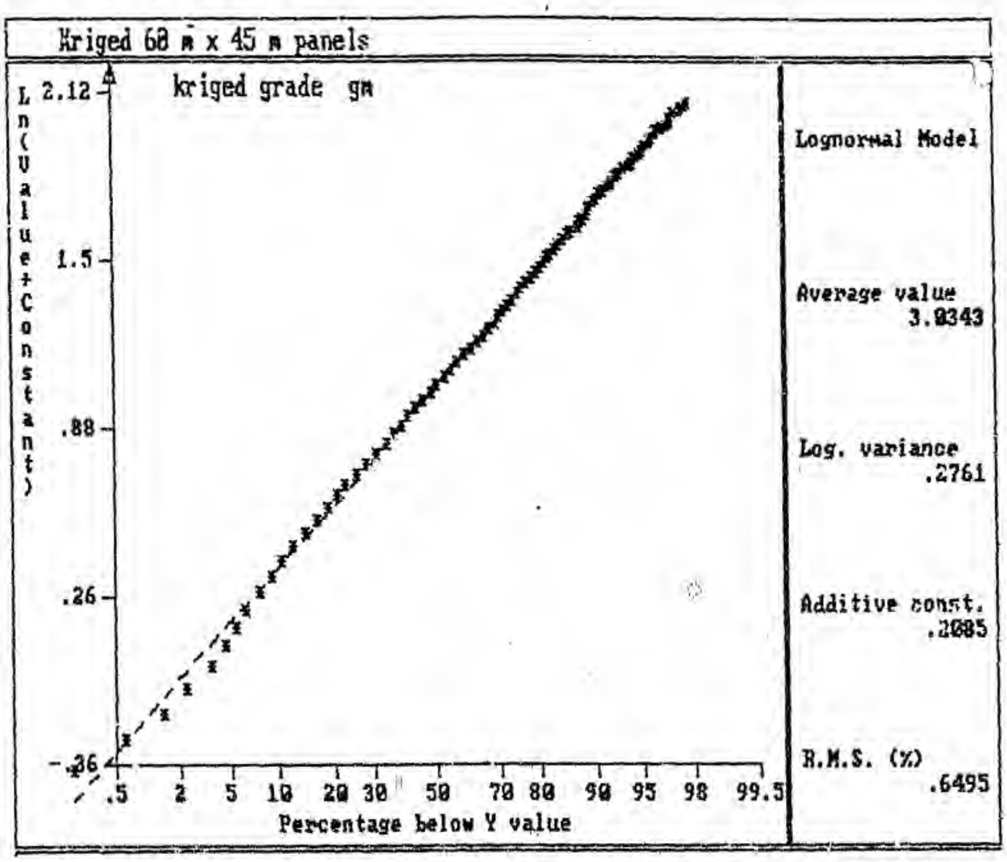

Figure 2.3.2 Probability plot of kriged block grades.

2.4 Grade tonnage curves

The total reserve tonnage is 8,123,000 tonnes with mean grade of 3.02 g/t and a 10 percent chance of falling to 2.93 g/t. Grade tonnage curves are constructed of plots of cut-off grade against recoverable tonnage, and average grade above cut-off pay value is shown in Figure 2.4.1.

The "regression" effect has been observed over the past in mining in the area, i.e. high value panels tend to be overestimated whilst low value panels tend to be underestimated. The result is that planned high values tend to be lower grade, whilst supposedly unpay panels are not mined. This leads to over-mining, producing high tonnages at lower grades. To correct this, a Least Squares correction is combined with the kriging system to produce a simple robust estimator to improve the prediction given by kriging. This "georegression" approach[24] is accounted for in the **Geostokos Toolkit** by flagging a sub-routine which demands a global mean of the area (2.9 g/t) and its associated standard error (0.05024) to adjust the kriged estimate for each block. The other, general, approach is that, because the kriging system is the best linear unbiased etimator, the regression effect is already assumed to be well accounted for by kriging.

The resulting distribution of the "corrected" blocks is shown on a probability plot in Figure 2.4.3 and a histogram in Figure 2.4.2 . The grade-tonnage curve of the "regression-effect" corrected blocks is shown in Figure 2.4.4.

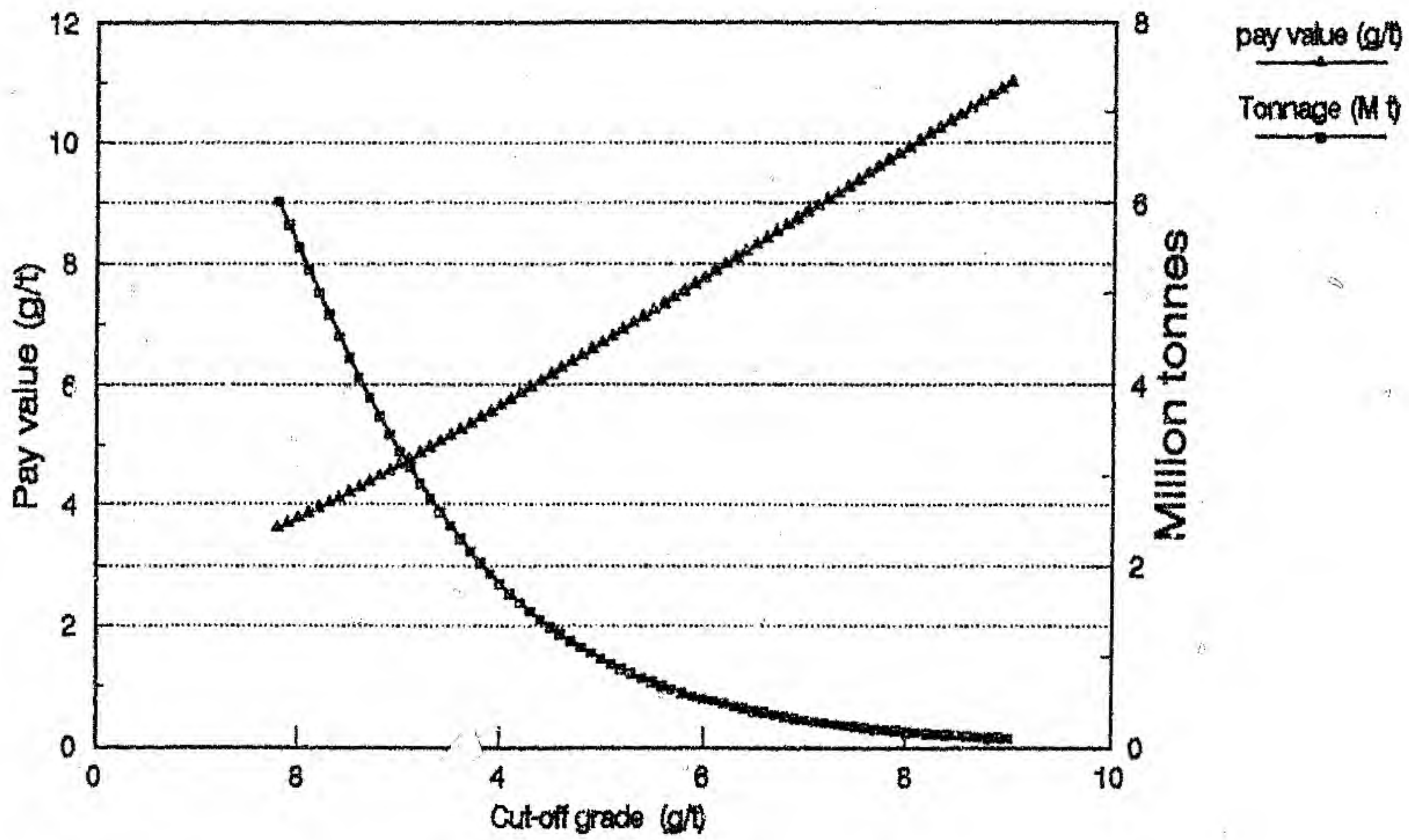

Figure 2.4.1 Grade-tonnage curve of kriged blocks

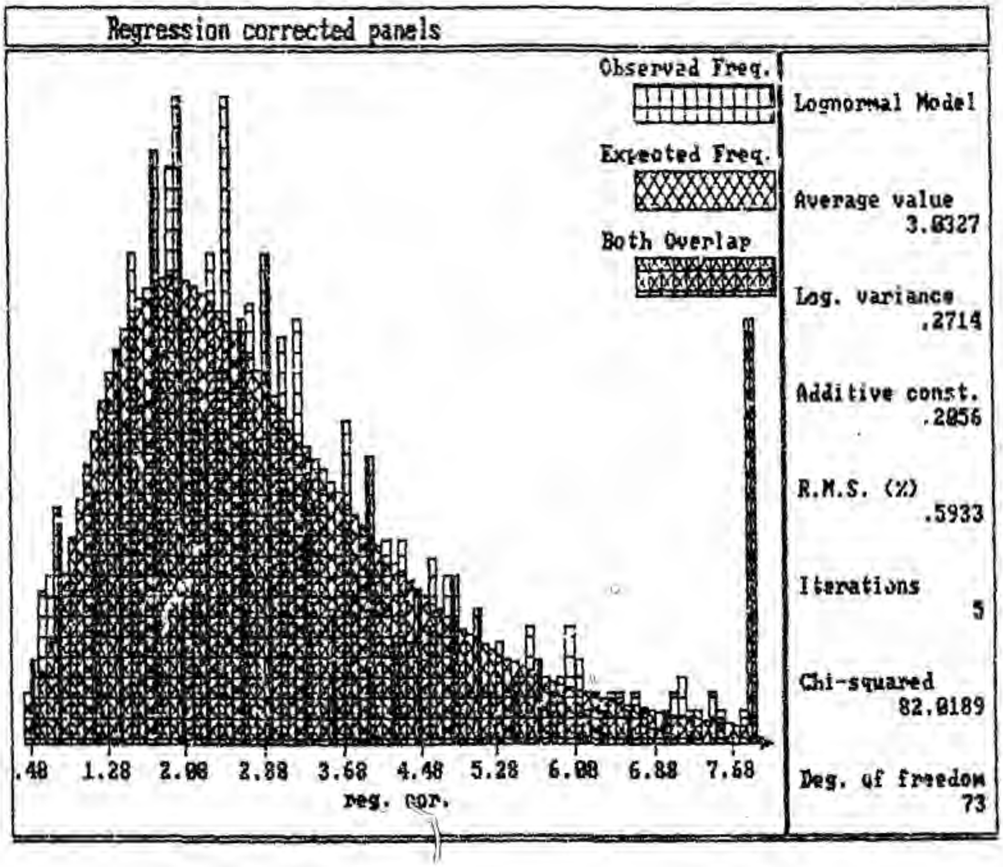

Figure 2.4.2 Histogram of regression corrected block grades.

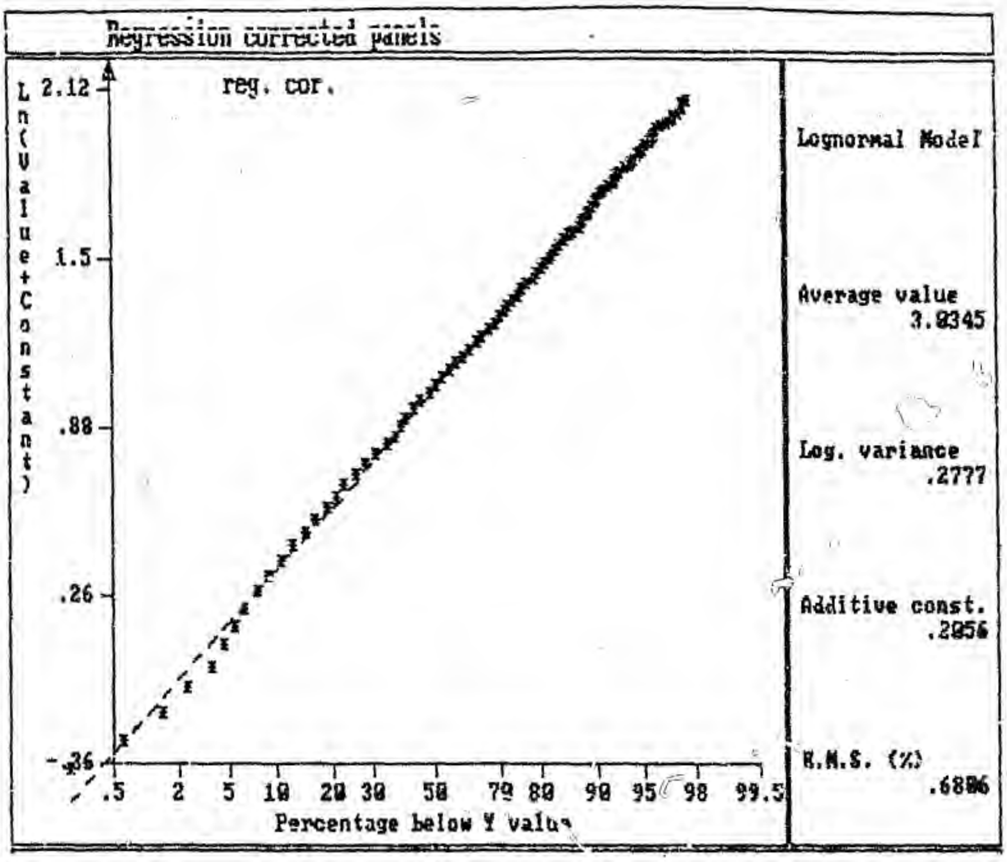

Figure 2.4.3 Probability plot of regression corrected block grades.

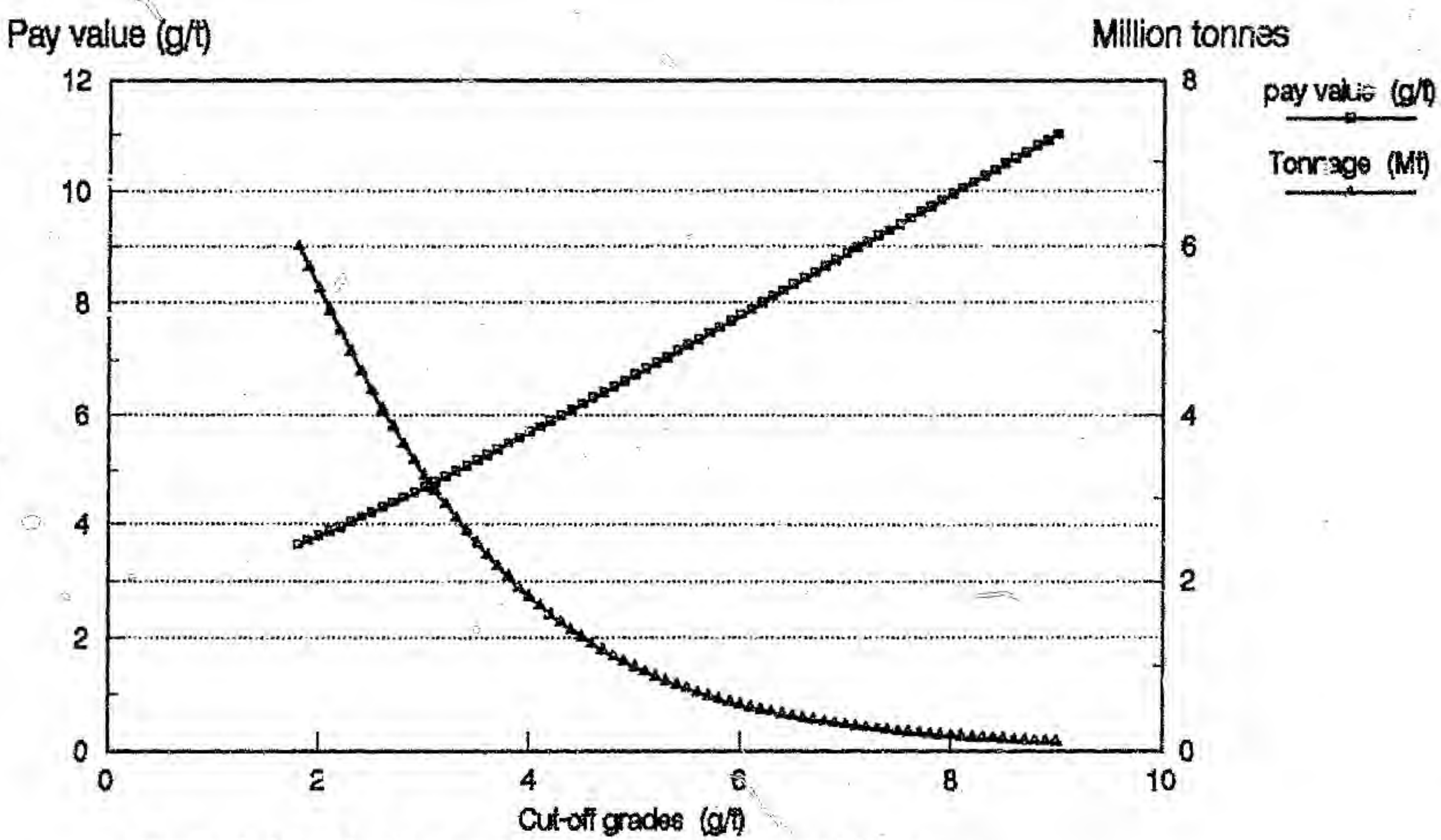

Figure 2.4.4 Grade-tonnage curve for corrected blocks.

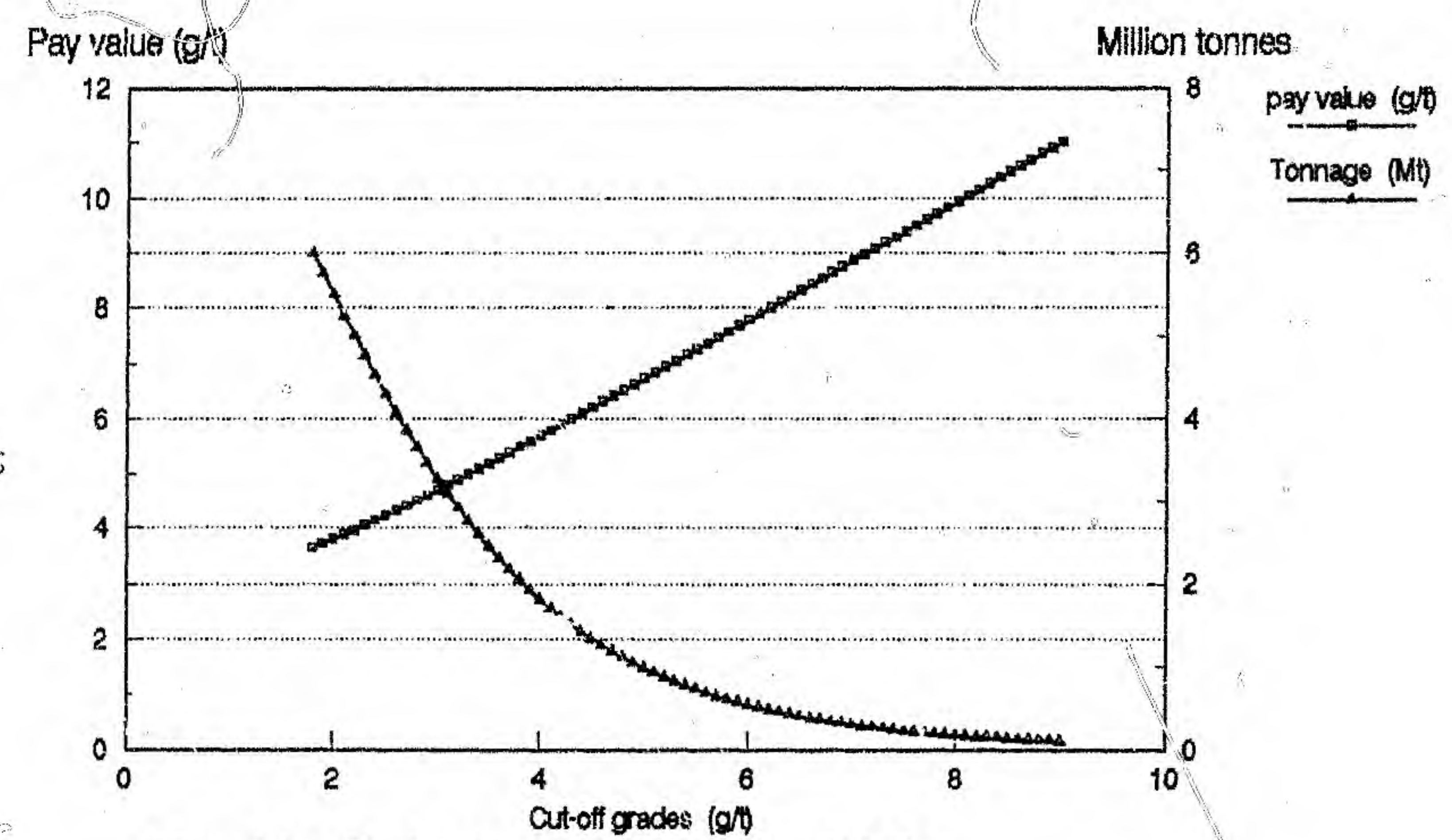

Figure 2.4.4 Grade-tonnage curve for corrected blocks.

A comparison between the "corrected" and "uncorrected" block grades showed that because of the correction the lower values were adjusted upwards while the high values decreased. The overall average was barely changed (uncorrected = 3.035 g/t versus corrected = 3.034 g/t). The log. variance increased (0.2721 versus 0 2777)

The cut-off grade policy analysis, which follows in the subsequent chapters, will use the corrected block grades.
Features of the grade distribution are also conveniently expressed by an ogival plot of cumulative tonnage accompanied by its resulting average grade against cut-off grade (Figure 2.4.5). Also informative is the corresponding plot of cumulative valuable metal content (Figure 2.4.6). These plots are a valuable analytical tool and the starting point of any serious cut-off grade study. Since the grades are in terms of blocks that are likely to be separable by the chosen mining method, these graphs indicate what can be obtained from the deposit, together with the range of flexibility of cut-off choice and the parameters affected thereby. In terms of a policy on cut-off grades, it aids in the choice of various ranges of values that can be reasonably achieved in that particular grade distribution, defining the limits of choice and what is ultimately achievable.

But, despite its usefulness as a basic tool, the plot is limited in use for production scheduling because it ignores spatial relation between the block grades. The higher the cut-off, the smaller the proportion taken and the more spatially spread the

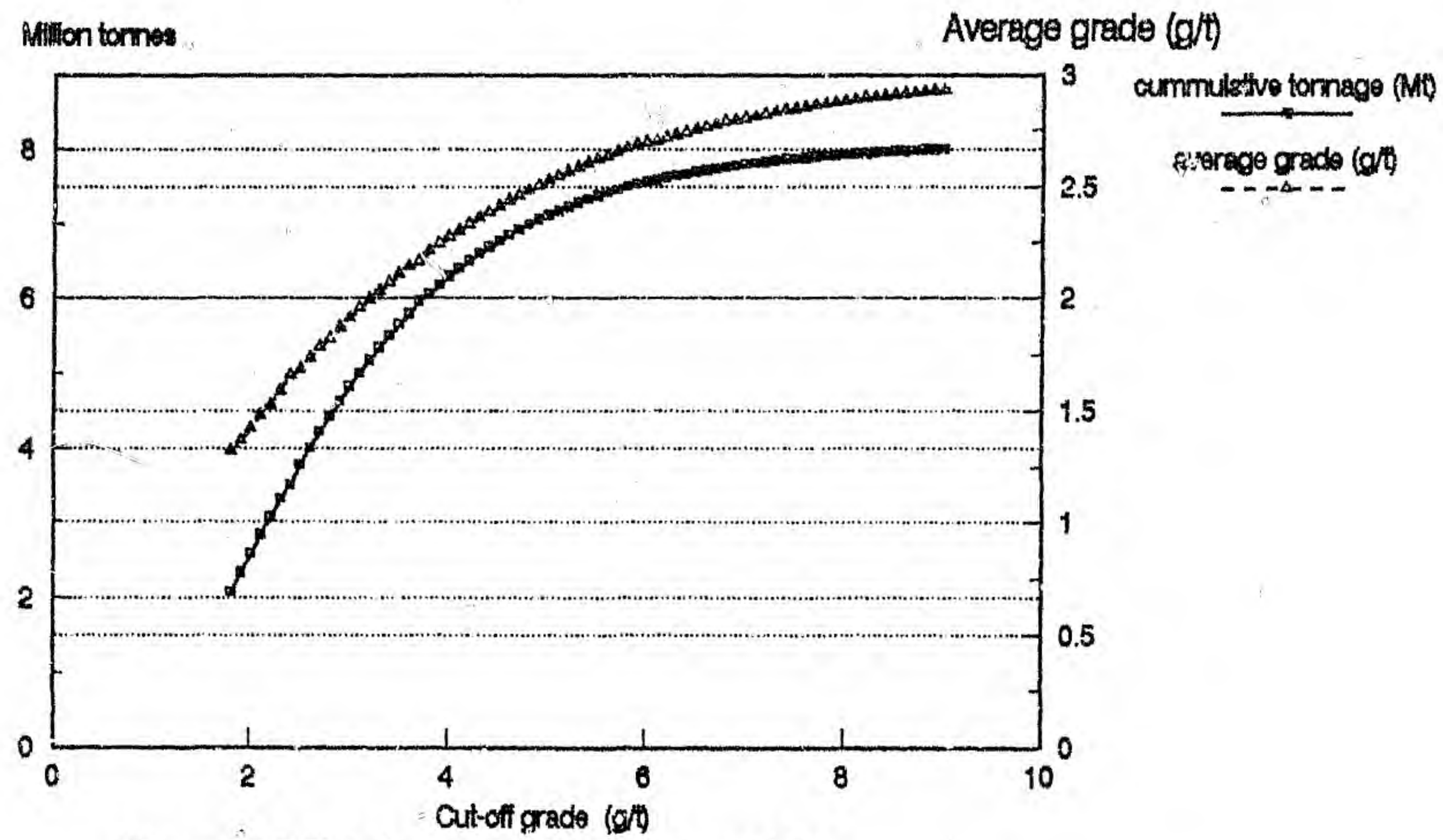

Figure 2.4.5 Plot of cummulative tonnage and the corresponding average grade against cut-off

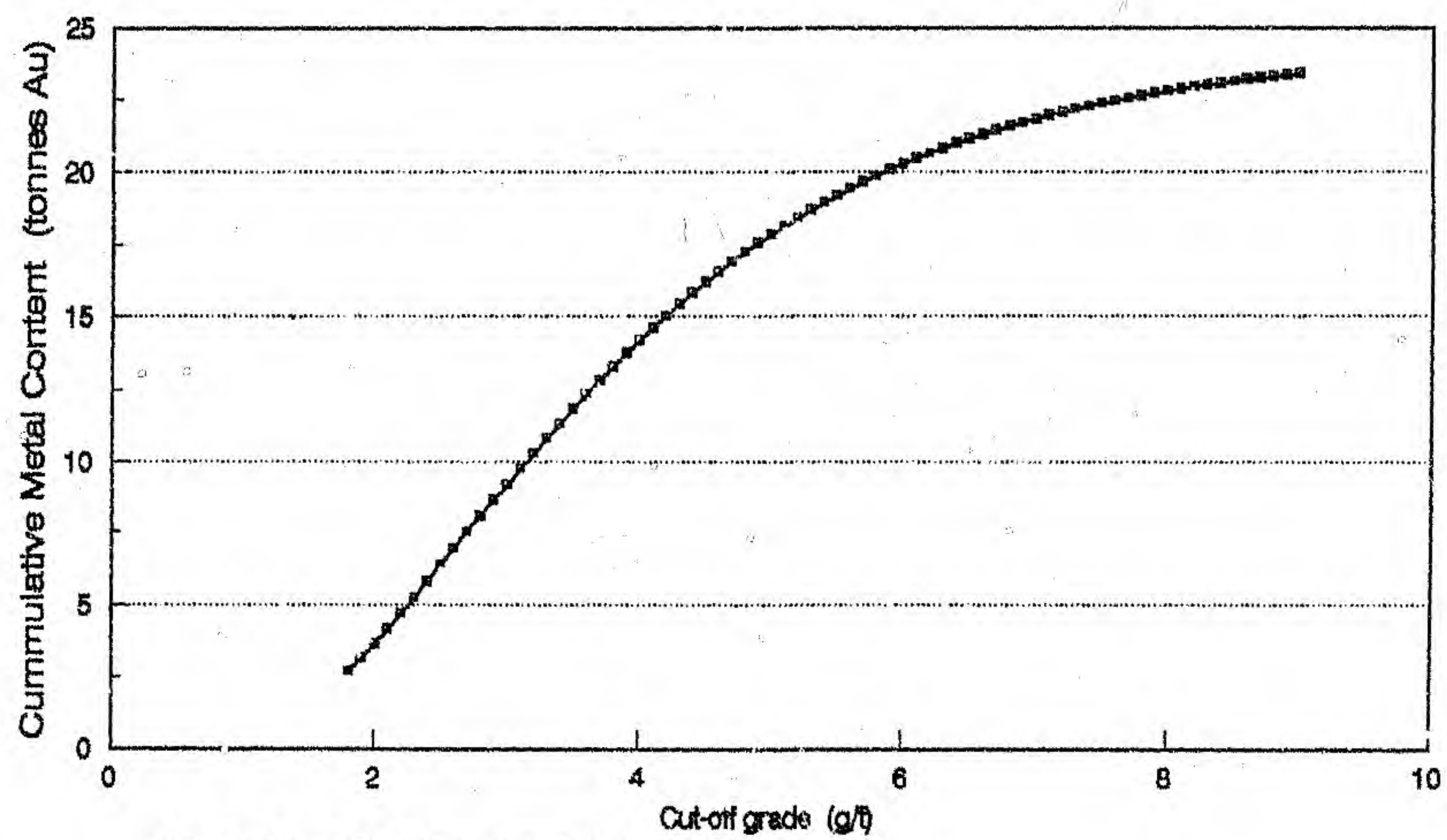

Figure 2.4.6 Plot of cummulative metal content against cut-off

blocks may be, to the point that the generation of economic mining zones may be impaired unless evidence exists of pronounced and continuous grade zoning.

3.2 Model Outline

Two dimension working area: 1 740 m on dip and
1 880 m on strike

Raise spacing: 180 m (10 raise lines on strike)

Level spacing: 240 m (7 levels on dip)

Stoping width: 1.3 m

Selective mining unit: 60 m by 45 m

Mining rate: 300 000 tonnes in first year
400 000 tonnes subsequently

Pre-production development: 2 years

There is no stockpiling of ore and all ore broken within any year will be processed within that year.

3.3 Mining Costs and related parameters

Expansion of the existing mine is to augment the depleted operating unit by increased development to define ore, and provide increased tonnage. This is going to cost R 30 million in the first year and R 20 million in each of the next two years. Financing is by equity so there will be no interest paid. Also being part of an existing unit tax ring-fencing is not applicable and all capital expenditure will be tax deductible. Use is going to be made of available facilities such as the existing shaft system. The capital expenditure will mainly be used on new equipment and the expansion related development in the new area in order to deliver ore to the existing mill facility which has

Taxation: $y = 61 - 305/x$

where y is the tax rate

x is the ratio of taxable to gross income

Lease: $y = 15 - 90/x$ plus 1.25% surcharge.

where y is the percentage of profits payable

x is ratio of profit less capital redemption allowance to mining revenue. (It must noted that lease payments are no longer appplicable after March, 1992).

Cost of capital: Defines the minimum annual profit requirement and depends on the mine's capital structure; is 7 percent real after tax.

Inflation: (Over previous year)

Year (end)	1	2	3	4	5 to end
Rate (% p.a.)	15	14	12	10	8

Cost Escalation: Escalation of capital cost is to be 5 percent over and above the ongoing inflation rate while the escalation of the working cost is 2 percent till it levels with inflation after year 4.

3.3.1 Cost Related Parameters

Dilution: Is related directly to he ore-mining conditions and mining method. The average grade of the in situ blocks should reflect the waste dilution factor associated with the selected mining method and estimated condition. For the chosen mining

CHAPTER 3

THE MINING MODEL

The mining model is simplified by avoiding the complexities involved in a detailed model, so that the overall objective of the project can receive full attention.

3.1 Mining Method

The method of mining is Scattered Mining Open Stoping with stope support because of the low grade, flat, thin and low dipping nature of the reef. Ore handling is essentially by conventional scrapers, drawing the broken ore into cars through a series of gulleys and boxholes.

Generally any ore left unmined may be mined in future, but this can be very difficult when the drives and other underground installations are destroyed because of blasting. More so if a caving method is used. Also in deep underground mines, common in the Witwatersrand, any ore blocks left behind are lost for all practical purposes. It is assumed, for the purpose of this project, that any ore left behind is not retrievable when the grade of the block are higher than the prevailing cut-off.

excess capacity.

Apart from the capital expenditure, other mining related costs to be incurred are outlined as follows:

<u>Variable Mining (Operating) Costs:</u> These costs include all direct and indirect costs that are incurred proportionately per tonne of ore or of concentrate selected, handled and treated.

mining	:	50 R/t
milling	:	35 R/t
Total	:	85 R/t

This figure (85 R/t) is much lower than the industry's average of 136 R/t, but compares well with that of the low working cost producers.

<u>Fixed Costs:</u> Are incurred per unit time irrespective of the level of production. Estimation of fixed cost requires great caution because large increases of output inevitably increase some elements of the supposed fixed cost. For the purpose of the project, increased output after the first year of the project will be assumed not to have an effect on the initial fixed cost. Fixed costs to be incurred are as follows:

ventilation, pumping, amenities, services, administration, maintenance, sundries = 4 000 000 R/year,

by a rough comparison with an operation of similar scale.

system, 10% dilution is acceptable and mining grade, g, will be:

$$g = g_h / (1.10)$$

where g_h = in situ grade.

Mine Closure: On closing down a mine after its life certain costs, like retrenchment allowances which can be very substantial, are made. In this project it is assumed that no costs will be incurred upon the closure of the mine.

3.4 Mining Revenue and related parameters

Metal Price: Within the world's current high economic and political uncertainties, it is very difficult to predict metal prices reliably in the long term. But instead of doing nothing about it, some idea of the trend of the metal price could be formed. Different approaches to forecasting exist and must reflect the available technology, business climate and political intelligence of demand and supply related parameters. In this project the present price of gold is taken as R 35/g and the trend is for the price to grow annually at the same rate as the escalation rate of capital cost, and no price cyclicity is built into the model.

By-products: In some cases the main metal of a mineralisation is associated with one or more additional metals. For example, gold is known to be associated with silver, copper, cobalt and even uranium. If it exists, then allowance for revenue generated by any by-product must be made in the main metal cut-off grade calculations. The average by-product price as a percentage of the main metal price, based mostly on the assumption of constant price relationship, is simply the price of the ratio of the by-product to main product price. No by- product is obtained in the deposit under study.

Salvage Value: As a mine comes to the end of its life, some of the capital equipment may not be necessarily old and useless but can be sold. The revenue from such sale, which can attract taxation if it has been capitalised, has a considerable effect on the project economics and must be addressed in a cut-off grade study. For simplicity this will not be considered , i.e, each equipment comes to the end of its useful life at the same time as the mine.

3.4.1 Revenue Related Parameters

Metallurgical Recovery: It is well known in practical experience that a varying head grade has a significant impact on mill recovery of some deposits such as copper. Cut-off grade calculations, usually dealing with a wide spectrum of ore grades, should therefore reflect this. However, in the case of gold this variation is not very significant (96-97 %) and will thus be taken as 96 %.

CHAPTER 4

CUT-OFF GRADE POLICIES

4.1 Types of cut-off grades

Cut-off grade is determined for the purpose of classi ying mineral reserves as economic or uneconomic in a specified period. It is usually based on unit costs, constant metal prices and fixed capacity. Many definitions exist for various purposes, some of which are discussed below.

__Geological cut-off grade__

Is used to truncate a frequency distribution of grades into graded fractions. Its main function is to separate **mineralised material** from **waste** and to assess mineral reserves as a starting basis for determining ore reserves. From the grade and tonnage relationships derived from the geological cut-off grade, economic and technical criteria are imposed to determine the planning cut-off grade and, ultimately, the mineable reserves.

__Planning cut-off grade__

Is the lowest grade of ore for which the mining operation's net present value is equal to zero. The cut-off grade is calculated on the premise that each tonne of ore with this grade must generate revenue equal on a cash basis to all allocated capital, operating and financial cost, including all taxes and royalty payments during the mine's life.

The purpose of planning cut-off grade is two-fold:

- to define the minimum mineable grade used for estimation of the reserves, which in turn serves as one criterion for selection of an appropriate mine/plant capacity;
- to quantify the operation's ore grade safety margin or spread between the estimated average ore reserve grade and the calculated planning cut-off grade.

Once established, the overall ore reserves and planning cut-off grades are supposed to be reviewed annually in order to reflect any substantial technological and/or economic changes.

__Budgeted cut-off grade__

Is the required annual head grade for a mine at which it should operate throughout the year in order to generate a budgeted net cash flow for a specified capacity. The purpose of this cut-off grade is to convey the operation's corporate net cash flow objectives for the budgeted year.

__Accounting cut-off grade__

Is the annual head grade which is sufficient, for a specified capacity, to generate revenue equal to all operating costs plus both annual depreciation allocated to fixed asset and minimum profit requirements. The purpose of this grade is to satisfy the accounting requirements for profitability as net of revenue and operating costs, including taxes and depreciation. This application of cut-off grade is limited in scope. Its proper use is to monitor the economic utilisation of a company from a purely accounting point of view.

Break-even cut-off grade

Is a grade that will generate an annual revenue equal to all out-of-pocket cash expenses, including fixed and variable operating costs, corporate and mining taxes, and all allocated capital expenditure throughout the operating year. If the mine operates at this head grade, the annual cash flow will be equal to zero. The purpose of this grade is to:

- separate ore from waste in underground mining operations; and
- delineate orebody limits in the case of assay walls.

Minimum (Marginal) Break-even cut-off grade

Is the same as the abovementioned grade except that allocated administrative and other fixed operating and capital costs are not included in the cash flow calculation, i.e. this cut-off grade meets only the variable operating costs. This grade is used to define the lowest grade which could be mined without losses, provided there is no mineralised material available which can generate a positive cash flow. It is the absolute lowest limit and is used as an operating cut-off if the mill is starving due to, say, lack of development or stoping capacity.

Operational cut-off grade

Is used to define, on a short range basis, what parcels may contribute to unmined ore reserves, needed when production starts.

Lane[25] describes other cut-off grades according to his definition of various stages in mining as:

Limiting Economic Cut-off grade

Is the cut-off according to the capacity which is limiting the output. The three stages of mining : development, mining and treatment are related to the throughput with which the stage is concerned, giving rise to three cut-off grades known as limiting cut-off.

Balancing Cut-off grade

In a mine, no single stage or component may be necessarily limiting, the balancing cut-off grade is supposed to be the intermediate value at which the capacities are fully utilised in pairs.

Others

Other types of cut-off grades are imposed mainly for quality control. These affect ores of physically high grade e.g. iron, dolomite, magnesite etc. and are set so that the minimum acceptable average grades are achieved. The inverse cut-off grades (maxima instead of minima) used for impurity control, are essentially similar.

4.2 Determination of break-even cut-off grade or pay limit

In the South African context, the average-cost break-even grade or pay limit in its simplest form requires, conceptually, that all ore "shows a profit" by determining a grade from which revenue is equal to total working cost. Thereafter, all superior grades are described as payable and those inferior as unpayable.

The four basic tenets of Witwatersrand valuation, as often set out in examinations for certificates of competency, are:

(i) that all ore mined must be able to bear its proportionate share of working costs, including overhead and (sometimes) development expenses;

(ii) the average break-even or pay limit grade so determined is the only legitimate cut-off grade, whereby any better ore automatically becomes part of the official ore reserve;

(iii) overall stoping must be almost exactly at the average grade of the whole ore reserve; and

(iv) in these matters time and capital expenditure are not wholly relevant.

It may be clearly recognised here, and must be emphasised that cut-off grades and break-even grades are inherently different quantities that may coincide only as a reflection of some particular operative condition.

A generally accepted principle underlying the objective of any mining investment is that over the life of the mine it will be necessary to recover the capital cost of providing the mine facilities, the working cost of sustaining the operations and sufficient surplus to make the venture worthwhile. The average grade of ore mined over the life of the mine must, therefore, at least provide sufficient revenue to satisfy these requirements. However, assuming that these minimum requirements can be satisfied, any additional tonne of ore that can be mined at an additional (marginal) cost, less than the revenue accruing from it will make a positive contribution towards the gross mining profit. The lowest grade that will just cover the direct cost of mining and treating it is the pay limit, and the gross profit over the life of the mine will be maximised by all ore above this grade.

At the pay limit, g_p, the revenue from one tonne of ore must equal the direct cost of mining, transporting, treating it etc., therefore simply;

unit variable cost = g_p x recovery x metal price[26]

$$V_c = g_p x \varepsilon x P_m$$

where V_c = unit variable costs (mining + treatment)

= R 85/t

ε = metallurgical recovery = 0.96

P_m = metal price = R 35/g

$g_p = (V_c) / \{(\varepsilon)(P_m)\}$

= 85/{(0.96)(35)} g/t

= 2.53 g/t

Reference to Figure 2.5.4 indicates that the following tonnage and grade will correspond to the pay limit of 2.5 g/t

Payable tonnage = 4,300,000 tonnes

Average grade of payable ore = 4.32 g/t

It has been determined by Taylor[27] that if the distribution is slightly skewed and the cut-off grade is well down the lower tail, very little tonnage or grade is affected, operations are insensitive to cut-off variations and precise optimisation of the cut-off grade is neither possible nor critical. But more often (not necessarily in this project), pay limit (2.53 g/t) is at or above the median (2.64 g/t) where the tonnage affected is proportionally higher, (i.e., the pay reserve is less than the unpay), thus affecting selectivity. A noted feature of the lognormal distribution is an almost perfectly linear relation between average grade above pay limit and the pay limit for all cut-offs above the geometric mean or median.

4.3 The cut-off grade policies

An investment in a mining project aims to increase the worth of all involved. Governments seek to benefit from increased foreign exchange earnings, guarantee employment for citizens, etc., while a private investor seeks to recoup all his investment plus a return for his time and risks taken. There is an apparent conflict here between the interests of the contributing parties, but from a strictly economic point of view it is not an inevitable conflict. If the mine is planned in such a way that its net present value is maximised, then, in theory, there is more wealth to share between the participants. Everyone will be better off.

An essential preliminary to an analysis of cut-off grade strategy is, therefore, an examination of present value maximisation for the operation.

Denoting the present value of the operation and the future cash flows year by year as CF_1, CF_2, ...,CF_n and r the cost of capital, then

$$NPV = CF_1/(1 + r) + CF_2/(1 + r)^2 + ...+ CF_n/(1 + r)^n$$

The cash flows ,CF_1, CF_2,....CF_n are dependent on the prices and costs prevailing at the time.

V and, for that matter, CF_1, CF_2,....,CF_n must depend on many more factors which describe the way in which the operation is to be conducted. Denote this by Ω. In the case of cut-off strategies for a mine, Ω will consist of the variable cut-off, g, which can take on different values g_1, g_2, ... ,g_n for the remaining years of the mine's life. Such a sequence of values can be called a policy and therefore g_1, g_2, ... ,g_n define a cut-off policy. The purpose of this study is to investigate three suggested cut-off grade policies:

- constant (conventional or traditional)
- decreasing and
- increasing

and determine which provides the maximum net present value under the same base economic conditions.

4.3.1 Constant cut-off policy

The conventionally accepted approach, in preparation of a production schedule for feasibility study purposes, is to plan extraction at a constant average grade over the life of the mine. The average is determined from a consideration of actual values within the orebody and a pay limit or economic break-even and the actual production average grade hardly differs from this.

A study of yield grades (directly proportional to the average mine grade determined after pay limit) for some 30 gold mines in South Africa over a period of 10 years, is shown in Table 4.3. It indicates that a third of the mines have had an increase in their yield grades over the period under study, and of these only half of them showed any strong trend in the rate of increase of more than 5 percent per year. The remaining two-thirds, which showed a declining yield, had only 20 percent showing any consistent and strong rates of decline. Overall there is no definite pattern of a cut-off policy. The decline in yield grades in the majority of the mines could be due more to the depletion of richer grades than a deliberate policy. The absence of a definite increasing or declining trend of the yield grades (the yearly cut-off) is suggestive of the traditional constant policy under dynamic economic (e.g. rising working costs, unstable revenues, etc) and technical (e.g. erratic grades) influences.

By applying a constant cut-off policy to our data, the results are shown in Table 4.3.1. The policy is shown in Figure 4.3.1.

	YEAR											
NAME OF MINE	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991
Barberton	10.03	7.31	7.42	8.23	8.10	7.44	6.50	6.75	5.88	5.58	6.02	6.20
Blyvooruitzicht	8.73	8.85	8.85	7.79	6.78	6.32	5.94	5.25	4.56	4.24	3.96	3.92
Bracken	3.96	3.50	3.37	3.65	3.49	3.80	3.26	2.91	2.95	3.41	4.12	5.35
Buffelsfontein	8.04	8.41	8.57	9.09	8.59	8.38	7.87	6.18	5.98	6.25	5.90	5.82
Deelkraal	3.81	4.41	4.99	4.47	5.05	4.79	5.06	5.45	6.08	6.07	5.82	5.88
Doornfontein	8.47	8.22	7.40	6.79	6.69	6.51	5.92	5.51	5.16	5.08	5.19	4.94
Durban Deep	3.40	3.67	3.56	3.31	3.18	3.08	3.14	3.14	3.09	2.45	2.25	2.46
East Driefontein	15.11	13.17	12.51	12.05	12.25	10.08	9.80	11.70	11.37	8.97	8.22	8.01
E.R.P.M.	4.95	4.43	4.03	3.95	3.79	3.64	3.40	3.61	3.65	2.58	2.46	2.80
Elandsrand	5.33	4.28	5.32	5.74	5.59	5.87	6.25	6.54	6.12	6.67	6.41	6.54
Free State Geduld	10.68	9.27	7.96	6.79	6.09	5.86	5.02	4.85	4.59	4.53	4.87	4.68
Free State Saaiplaas	2.59	–	–	–	–	–	4.53	3.90	4.03	3.96	3.95	3.89
Grootvlei	3.67	3.75	4.10	3.81	3.85	3.47	3.22	3.11	3.29	3.63	4.27	5.10
Hartebeesfontein	10.87	10.16	10.20	9.86	9.99	9.30	9.80	9.45	6.91	6.48	6.29	8.91
Kinross	5.70	5.85	5.88	6.25	6.30	6.58	6.06	5.96	5.70	5.70	6.16	6.30
Kloof	14.51	14.63	14.95	15.20	15.52	14.42	13.50	13.96	13.45	12.24	11.83	13.24
Leslie	3.00	3.23	3.42	3.42	3.73	2.77	2.49	2.38	2.39	3.30	4.71	5.14
Libanon	6.43	6.03	6.35	6.08	5.28	5.11	5.13	4.89	4.23	4.19	4.45	4.51
Loraine	3.26	4.02	5.05	5.34	5.38	5.56	5.59	5.33	5.36	4.99	5.06	5.05
Marievale	1.43	1.49	2.32	3.34	3.35	3.20	2.88	2.58	2.38	2.19	3.93	3.57
President Brand	8.93	8.04	7.25	6.79	6.42	6.16	–	–	–	–	–	–
President Steyn	6.78	6.46	6.30	6.58	6.52	5.95	–	–	–	–	–	–
Randfontein	5.10	5.23	5.00	5.00	5.42	5.28	4.12	3.17	3.34	3.06	3.14	3.34
St. Helena	7.74	7.28	6.80	6.09	5.37	5.17	4.41	4.00	4.82	5.23	6.18	5.97
Stilfontein	8.76	7.79	7.28	6.89	6.03	5.61	5.05	4.64	3.60	2.54	2.26	1.76
Unisel	6.23	6.84	7.20	7.15	6.81	6.73	6.88	6.15	5.41	5.99	6.05	6.09
Vaal Reefs (North & South	8.27	8.67	8.56	8.45	8.60	7.74	7.40	6.90	7.08	6.91	6.53	6.52
Vaal Reefs (Afrikander)	0.44	0.26	1.83	2.05	2.47	1.71	1.67	1.36	–	–	–	–
Venterspost	4.42	3.99	4.35	4.06	4.29	3.77	3.78	4.03	3.72	3.69	3.67	3.98
Welkom	4.77	–	–	–	–	–	–	–	–	–	–	–
West Driefontein	16.22	14.24	14.40	13.72	13.38	12.50	11.16	11.38	9.90	10.12	10.19	11.38
Western Areas	4.53	4.13	4.49	4.80	4.68	4.35	4.10	3.48	3.40	3.55	4.74	5.25
Western Deep Levels	13.70	12.36	11.28	11.20	10.58	9.19	6.73	5.89	5.96	6.16	5.83	6.21
Western Holdings	8.51	5.31	4.83	4.38	4.28	4.39	–	–	–	–	–	–
Western Rand Cons.	–	–	1.80	1.79	2.05	2.05	1.90	2.02	2.02	2.22	2.58	2.43
Winkelhaak	6.53	6.53	6.30	6.24	6.04	5.72	5.81	5.48	5.61	5.66	6.11	5.94
Wit Nigel	3.49	3.65	3.23	3.02	3.27	–	–	–	–	–	–	–
AVERAGE	7.73	7.32	7.00	6.76	6.40	6.28	5.81	5.49	5.33	4.99	5.05	5.20

Table 4.3 Trend of yearly production yield grade* of some major South African gold mines from 1980 to 1991 (Source: Minerals Bureau)

* (yield grade = 1 000 tonnes of ore milled / Kilogrammes fine gold)

	YEAR											
NAME OF MINE	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991
Barberton	10.03	7.31	7.42	8.23	8.10	7.44	6.50	6.75	5.88	5.58	6.02	6.20
Blyvooruitzicht	8.73	8.85	8.85	7.79	6.78	6.32	5.94	5.25	4.56	4.24	3.96	3.92
Bracken	3.96	3.50	3.37	3.65	3.49	3.80	3.26	2.91	2.95	3.41	4.12	5.35
Buffelsfontein	8.04	8.41	8.57	9.09	8.59	8.38	7.87	6.18	5.98	6.25	5.90	5.82
Deelkraal	3.81	4.41	4.39	4.47	5.05	4.79	5.06	5.45	6.08	6.07	5.82	5.88
Doornfontein	8.47	8.22	7.40	6.79	6.69	6.51	5.92	5.51	5.18	5.08	5.19	4.94
Durban Deep	3.40	3.67	3.56	3.31	3.18	3.08	3.14	3.14	3.09	2.45	2.25	2.46
East Driefontein	15.11	13.17	12.51	12.05	12.25	10.08	9.80	11.70	11.37	8.97	8.22	8.01
E.R.P.M.	4.95	4.43	4.03	3.95	3.79	3.64	3.40	3.61	3.65	2.58	2.46	2.80
Elandsrand	5.33	4.28	5.32	5.74	5.59	5.87	6.25	6.54	6.12	6.67	6.41	6.54
Free State Geduld	10.68	9.27	7.96	6.79	6.09	5.86	5.02	4.85	4.59	4.53	4.87	4.68
Free State Saaiplaas	2.59	–	–	–	–	–	4.53	3.90	4.03	3.96	3.95	3.89
Grootvlei	3.67	3.75	4.10	3.81	3.85	3.47	3.22	3.11	3.29	3.63	4.27	5.10
Hartebeesfontein	10.87	10.16	10.20	9.86	9.99	9.30	9.80	9.45	6.91	6.48	6.29	8.91
Kinross	5.70	5.85	5.88	6.25	6.30	6.58	6.06	5.96	5.70	5.70	6.16	6.30
Kloof	14.51	14.63	14.95	15.20	15.52	14.42	13.50	13.36	13.45	12.24	11.83	13.24
Leslie	3.00	3.23	3.42	3.42	3.73	2.77	2.49	2.38	2.39	3.30	4.71	5.14
Libanon	6.43	6.03	6.35	6.08	5.28	5.11	5.13	4.89	4.23	4.19	4.45	4.51
Loraine	3.26	4.02	5.05	5.34	5.38	5.56	5.59	5.33	5.36	4.99	5.06	5.05
Marievale	1.43	1.49	2.32	3.34	3.35	3.20	2.88	2.58	2.38	2.19	3.93	3.57
President Brand	8.93	8.04	7.25	6.79	6.42	6.16	–	–	–	–	–	–
President Steyn	6.78	6.46	6.30	6.58	6.52	5.95	–	–	–	–	–	–
Randfontein	5.10	5.23	5.00	5.00	5.42	5.28	4.12	3.17	3.34	3.06	3.14	3.34
St. Helena	7.74	7.28	6.80	6.09	5.37	5.17	4.41	4.00	4.82	5.23	6.18	5.97
Stilfontein	8.76	7.79	7.28	6.89	6.03	5.61	5.05	4.64	3.60	2.54	2.26	1.76
Unisel	6.23	6.84	7.20	7.15	6.81	6.73	6.88	6.15	5.41	5.99	6.05	6.09
Vaal Reefs (North & South	8.27	8.67	8.56	8.45	8.60	7.74	7.40	6.90	7.08	6.91	6.53	6.52
Vaal Reefs (Afri[illegible])	0.44	0.26	1.83	2.05	2.47	1.71	1.67	1.26	–	–	–	–
Venterspost	4.42	3.99	4.35	4.06	4.29	3.77	3.78	4.03	3.72	3.69	3.67	3.98
Welkom	4.77	–	–	–	–	–	–	–	–	–	–	–
West Driefontein	16.22	14.24	14.40	13.72	13.38	12.50	11.16	11.38	9.90	10.12	10.19	11.38
Western Areas	4.53	4.13	4.49	4.80	4.68	4.35	4.10	3.48	3.40	3.55	4.74	5.25
Western Deep Levels	13.7[illegible]	12.36	11.28	11.20	10.08	9.19	6.73	5.89	5.96	6.16	5.63	6.21
Western Holdings	8.51	5.31	4.83	4.38	4.28	4.39	–	–	–	–	–	–
Western Rand Cons.	–	–	1.80	1.79	2.05	2.05	1.90	2.02	2.02	2.22	2.58	2.43
Winkelhaak	6.53	6.53	6.30	6.24	6.04	5.72	5.81	5.48	5.61	5.66	6.11	5.94
Wit Nigel	3.49	3.65	3.23	3.02	3.27	–	–	–	–	–	–	–
AVERAGE	7.7[illegible]	7.32	7.00	6.76	6.40	6.28	5.81	5.49	5.33	4.99	5.05	5.20

Table 4.3 Trend of yearly production yield grade* of some major South African gold mines from 1980 to 1991 (Source: Minerals Bureau)

* (yield grade = 1 000 tonnes of ore milled / Kilogrammes fine gold)

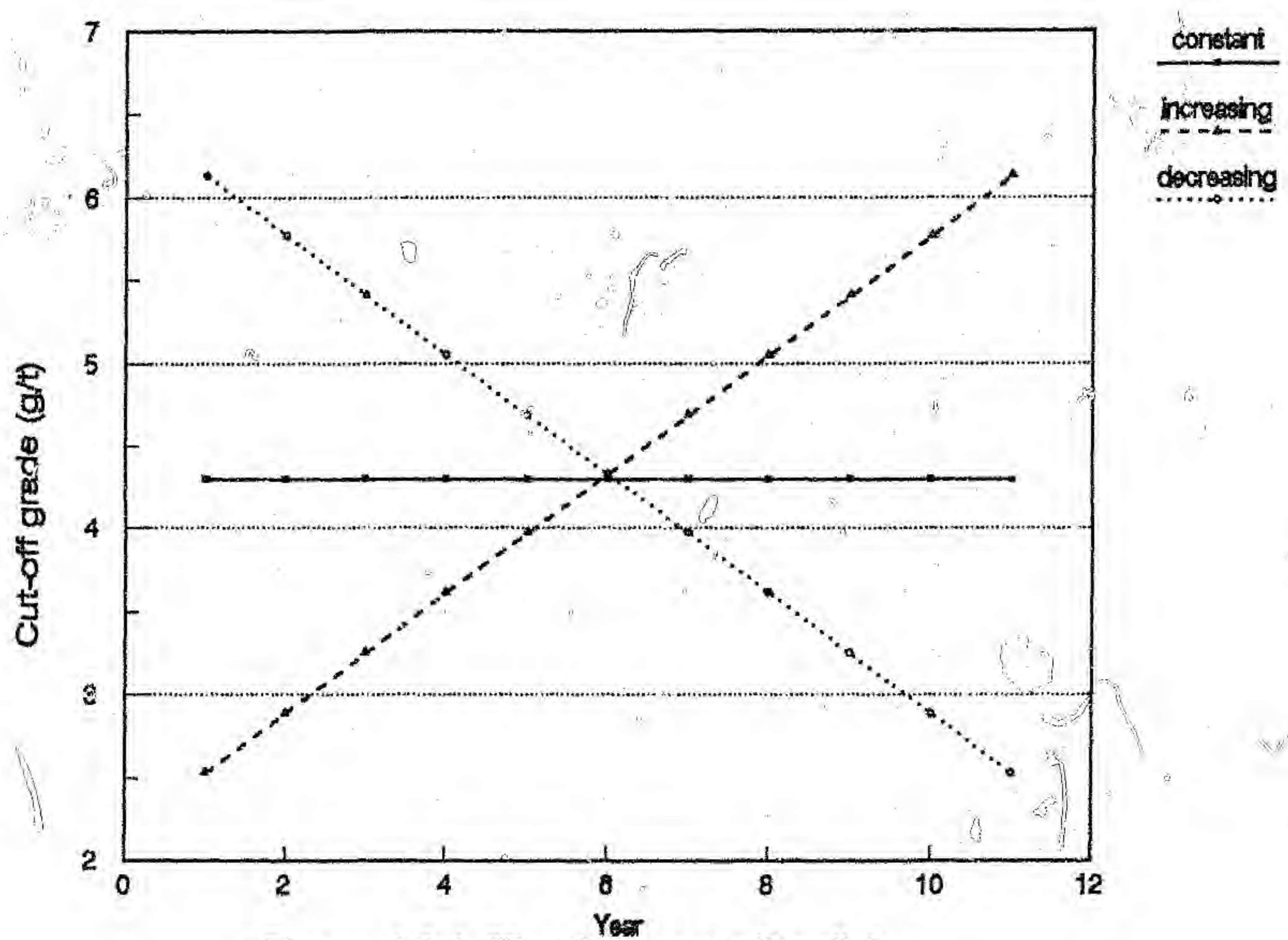

Figure 4.3.1 The three cut-off policies

Year	Tonnage (tonnes)	Grade (gmAu/t)	Capital Expenditure (R Million)	Working Cost (R Million)	Revenue (R Million)	Net Profit (R Million)	Taxable Profit (R Million)	Tax + Lease (R Million)	Real Net Cash Flow (R Million)
1	–	–	30	0	0	(30)	0	0	(30)
2	–	–	24	0	0	(24)	0	0	(20.87)
3	300 000	4.3	28.32	40.04	61.40	6.99	21.33	12.17	(14.62)
4	400 000	4.3	0	58.79	94.9	36.13	36.13	20.92	10.36
5	400 000	4.3	0	64.67	106.00	41.64	41.64	24.22	10.79
6	400 000	4.3	0	69.85	115.00	44.97	44.97	26.16	10.79
7	400 000	4.3	0	75.3	124.00	48.57	48.57	28.25	10.79
8	400 000	4.3	0	81.47	134.00	52.46	52.46	30.51	10.79
9	400 000	4.3	0	87.99	145.00	56.65	56.65	32.95	10.79
10	400 000	4.3	0	95.03	156.00	61.18	61.18	35.59	10.79
11	400 000	4.3	0	102.60	169.00	66.08	66.08	38.44	10.79
12	400 000	4.3	0	110.80	182.00	71.37	71.37	41.51	10.79
13	400 000	4.3	0	119.70	197.00	77.07	77.07	44.83	10.79

Net Present Value @ 7 percent = R 3.31 Million
DCFROR = 7.95801 % p.a.
(Tax + Lease = 0.6685 x profit – 0.03401 x revenue)
(All estimates are as at the end of each year)

Table 4.3.1 Results of economic analysis based on constant cut-off policy

4.3.2 Increasing cut-off grade policy

This policy was suggested quite long ago by Berry[28], and it applies the present 8value theory to justify permanent raising of cut-off grades well above the break-even point. This approach has been supported by Hotchlass and Douglass[29] and Halls et al.[30]

The approach to be applied in this analysis is to choose the cut-off grade lower than the average during the first half of the mine's life and greater than the average during the latter half, the grade increasing in progressively relatively small steps from start to finish.

Results of calculations on the declining cut-off grade policy are presented in Table 4.3.2.

4.3.3 Decreasing cut-off policy

Lane[31], and Taylor[32], among many others, propose this policy to be the best if net present value is to be maximised by optimising the cut-off grade.

The approach is simply reversing the increasing cut-off strategy. During the first part of the mine's life, ore is mined above the average grade. There is a progressive deterioration in the cut-off to the end, Figure 4.3.1. Results are in Table 4.3.3.

Year	Tonnage (tonnes)	Grade (gmAu/t)	Capital Expenditure (R Million)	Working Cost (R Million)	Revenue (R Million)	Net Profit (R Million)	Taxable Profit (R Million)	Tax + Lease (R Million)	Real Net Cash Flow (R Million)
1	–	–	30	0	0	(30)	0	0	(30)
2	–	–	24	0	0	(24)	0	0	(20.87)
3	400 000	2.50	28.32	40.04	35.70	(32.67)	0	0	(24.92)
4	400 000	2.86	0	58.79	63.10	4.35	0	0	2.96
5	400 000	3.22	0	64.67	79.60	14.95	14.95	7.28	4.75
6	400 000	3.58	0	69.85	95.60	25.76	25.76	13.97	5.76
7	400 000	3.94	0	75.43	114.00	38.18	38.18	21.86	8.77
8	400 000	4.30	0	81.47	134.00	52.46	52.46	30.16	10.79
9	400 000	4.66	0	87.99	157.00	68.77	68.77	40.64	12.80
10	400 000	5.02	0	95.03	182.00	87.35	87.35	52.19	14.62
11	400 000	5.38	0	102.60	211.00	108.50	108.50	65.33	16.63
12	400 000	5.74	0	110.60	243.00	132.40	132.40	80.24	18.85
13	400 000	6.10	0	119.70	279.00	159.40	159.40	97.09	20.80

Net Present Value @ 7 percent = – R 5.21 Million
DCFROR = 5.8574 percent p.a.
(Tax + Lease = 0.6685 x profit – 0.03401 x revenue)
(All estimates are as at the end of each year)

Table 4.3.2 Results of economic analysis based on increasing cut–off policy

Year	Tonnage (tonnes)	Grade (gmAu/t)	Capital Expenditure (R Million)	Working Cost (R Million)	Revenue (R Million)	Net Profit (R Million)	Taxable Profit (R Million)	Tax + Lease (R Million)	Real Net Cash Flow (R Million)
1	–	–	30	0	0	(30)	0	0	(30)
2	–	–	24	0	0	(24)	0	0	(20.87)
3	300 000	6.10	28.32	40.04	87.10	18.71	47 03	28.48	(7.45)
4	400 000	5.74	0	58.79	127.00	67.93	67.93	41.10	18.27
5	400 000	5.38	0	64.67	133.00	68.35	68.35	41.17	16.83
6	400 000	5.02	0	69.67	134.00	64.20	64.20	38.36	14.82
7	400 000	4.66	0	75.43	134.00	58.96	58.96	34.84	12.86
8	400 000	4.30	0	81.47	134.00	52.46	52.46	30.51	10.79
9	400 000	3.94	0	87.99	133.00	44.54	44.54	25.27	8.77
10	400 000	3.58	0	95.03	130.00	35.05	35.05	19.00	6.76
11	400 000	3.22	0	102.60	126.00	23.72	23.72	11.56	4.74
12	400 000	2.86	0	110.80	121.00	10.36	10.36	2.80	2.73
13	400 000	2.50	0	119.70	114.00	(5.30)	0	0	(1.77)

Net Present Value @ 7 percent = R 8.65 Million
DCFROR (Real) = 10.5215 percent
(Tax + Lease = 0.6685 x profit – 0.03401 x revenue)
(All estimates are as at the end of each year)

Table 4.3.3 Results of economic analysis based on decreasing cut-off policy

4.4 Discussion of Results

A summary of the results of the three policies investigated is shown in Table 4.4. From the table, the decreasing cut-off policy was rated the best, even though alone it registered a loss in the last year of operation, with all the criteria showing it to be the superior strategy. This outcome underpins two fundamental ways of achieving an improvement in the net present value of an investment opportunity:

- decrease or defer expenditure
- increase revenue or bring it closer in time.

In the decreasing cut-off policy, revenue has been increased at the expense of reducing the cash flow in later years by progressively deteriorating the average grade of the reserve over the life of the mine. The decreasing cut-off policy has been referred to as "highgrading" by Thomas[33], and "overmining" by Wells[34], but this fact is only true if an artificially high discount rate is used in the calculation (Border[35]), so that only the higher grades can be mined economically.

From this analysis, it is evidently convincing that under the same economic conditions a declining cut-off over the life of the operation will be the most attractive policy over the others investigated. It is noted, however, that the declining policy will necessitate higher initial development costs to define high grade ores than the other policies. But the effect will be overshadowed by the resulting higher initial revenues. Variations which can be made in the policy to achieve the optimum (which

	CUT-OFF POLICY		
	Constant	Increasing	Decreasing
Tonnage (tonnes)	4 300 000	4 300 000	4 300 000
Ave. grade (g/t)	4.3	4.3	4.3
Life (years)	11	11	11
NPV (R Million) @ 7% real	3.31	(5.21)	8.65
Real DCFROR (% pa)	7.9580	5.8574	10.5215
Discounted Payback (years)	2	4	2
Rating	2	3	1

Table 4.4 Summary of analysis of cut-off policies.

considers reasonable cutting of grades) to give the maximum net present value, is the subject of Chapter Six.

CHAPTER 5

FACTORS AFFECTING POLICIES

Cut-off grades are dependent upon many factors including prices, costs, grade distribution and, to a lesser extent, the mine planning which is the cumulative of the other three factors. Whilst all these factors affect cut-off grades differently, there is one element which is common to them: uncertainty related directly to the unknown or future.

5.1 Costs

The effect of increasing costs of equipment, labour, energy and capital needed to undertake a mining operation, means higher expenses incurred and thus the need to mine higher grades before any return on investment can be obtained.

The general trend in costs seems to point in only one direction; a continual increase due to inflation and escalation effects. However, advances in technology have countered this general trend in some areas, where costs have risen to unsustainable levels and have thus attracted research efforts to bring about cheaper substitutes or methods.

To incorporate the effect of the trend in cost in the policy analysis inflation and escalation were built into the model. Since the general trend points to growing costs, the inflation and escalation effects, accurately and reasonably estimated, may be adequate for cut-off grade policy analysis.

5.2 Grade distribution

In this analysis a simplistic view has been taken of the grade distribution of the mining blocks. The spatial relationship between the block grades has been ignored as far as production scheduling is concerned. Because of this, all the blocks have been assumed to bear the same development cost, which may not necessarily be the case. The high grades may not be mined without mining the lower grades first; or they may be spaced such that mining of the high grades alone can not support the necessary development. Nonetheless, if the spatial relationship is taken into account the effect on the comparison of policies may cancel out, because ultimately all those blocks marked above a particular cut-off will be mined, whatever the policy.

Lane[36] acknowledges that cut-off grade is calculated with reference solely to costs, prices and capacities regardless of the way grades actually vary within the mineralised body to be mined. He notes that this characteristic is intuitively unlikely even though it is well accepted in industry. However, he is of the opinion that in the particular situation where only one component of the mining system limits the throughput, the actual grade distribution is not directly relevant.

There has been a widespread belief that most mineral assays fit lognormal distributions. However, each deposit is unique and the lognormal fit may be an adequate approximation for orebodies other than those which are rich.

5.3 Prices

It is the wish of every supplier of a commodity that the price of that commodity be high enough to enable him to cover his costs and make some profit, at the same time not being too high to attract uncomfortable competition. But for metal producers, the past decades have seen very depressed prices. The situation for metal prices has been exacerbated by rising costs. Historical studies have indicated that metal prices have actually dropped in real terms over the past decades. Nordhaus[37], in a study on resource scarcity of several minerals covering the period of 1900-1970, found a general marked decline in price in all cases except copper. In recent times many operations have been mothballed to await better prices. Recent events in the South African gold industry on cutting down on some operations (e.g. Harmony) provide instances. All mining houses have indicated that jobs are under serious threat if the gold price remains at current levels. However, it is being envisaged in many circles of opinion that the downturn in prices have actually bottomed and that it is about to pick up again in the near future.[d]

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[d] In a study commissioned by stockbroker Malcolm Stewart of Kaplan & Stewart, the scenario for gold is that if the global investment scene that prevailed through the 1980s - high positive

In the analysis in Chapter Four, the trend in gold price was taken to be at par with the cost escalation. This trend can be taken as constant neither rising nor falling, i.e. not differential to cost. The three cut-off policies will now be subjected to two gold price scenarios to determine which will be favourable under the various price regimes.

5.3.1 Scenario 1 - Rising Gold Price

> " .....It seems very likely indeed that over the next couple of years a recovery in industrial demand for gold and a flat supply profile at best, could see a fairly steady rise in the bullion price. .....This healthy trend will be maintained throughout the next economic upturn..."[38]

Comments such as the above have become very prevalent in articles seeking to forecast the trend of gold price. Jewellery has been identified as a potential growth area with increasing demand in the Far East.

With this optimistic outlook in mind, the policies have been analysed for the effect of various price rises and the result is shown in Table 5.3.1. The results indicate that over a small rise in price of between 2 to 8 percent over the life of the mine,

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returns from shares, property and cash - cannot be sustained in this decade, the "glitter" might return to gold. (Sunday Times, Business, May 3, 1992)

		CUT-OFF POLICY		
Percent Rise in Metal Price	Criterion	Constant	Increasing	Decreasing
0	NPV	3.31	(5.21)	8.65
	IRRR	7.9580	5.8740	10.5215
2	NPV	21.92	16.24	26.63
	IRRR	12.5625	10.1846	16.0430
5	NPV	54.60	52.43	55.89
	IRRR	18.6602	15.7422	22.7246
8	NPV	90.43	97.03	87.06
	IRRR	23.621	20.9082	28.0371
10	NPV	124.87	132.49	117.17
	IRRR	27.6465	24.2676	32.9326
15	NPV	221.19	243.73	198.70
	IRRR	35.8469	31.9626	41.1328
20	NPV	353.17	399.05	307.34
	IRRR	43.6035	39.2481	49.3652

Table 5.3.1 Effect of Rising Metal Price Trend on Cut-off Policies.

(The NPV is dicounted at 7 percent real per annum.
Figures in million Rands)

the declining cut-off policy gives the best outcome. The increasing cut-off policy becomes the best strategy over higher rates of increase of greater than 10 percent. The constant cut-off policy is better than the increasing policy but worse than the declining cut-off policy for the lower price increases. At the higher rates of increase, it switches to become better than the declining cut-off policy but worse than the increasing cut-off policy. The important question is which price trend is realistic and achievable.

The outcome of this price trend study can be highly influenced by the cut-off policy itself, especially by the two sliding policies (increasing or declining). A policy in which the growth, or otherwise, of the cut-off grades is steep with respect to time, will be affected differently by the same price rise or fall than a flatter cut-off grade regime. For modest growth in price, which is a realistic assumption for any forecast, the declining policy surpasses the other policies. It is able to take advantage of high initial revenues because of the mining of higher grades and also reaps still high, or at least constant, levels of revenue because of the opposite movement of the grades and prices. A case may be made for the increasing cut-off policy, that by deferring mining of higher grades higher revenues can be received in the latter years when the price is high. While this is true it must also be appreciated that the further the positive cash flow is from the base year the smaller it becomes on discounting, and thus the smaller the net present value. Actually the increasing policy is very attractive in the light of of fantastically increasing price trends.

5.3.2 Scenario 2 - Declining Gold Price

Much as it has been anticipated that the price of gold would rise, the performance in the last few years has been very disappointing. The expected trend for 1992 was confidently forecast to be US$480 per ounce [39]. Meanwhile for most part of the first half of this year (1992) the price has actually fallen. The price has not seen the US$[illegible]30 mark very often and at times threatened to fall below US$330. In short there is the possibility for the gold price to fall, even in the medium term, with the subdued economic recovery and decline in price generating additional sales by holders of gold, thus continuing the period of market weakness for some time to come[40]. Gold is also perceived to be increasingly losing its monetary value[e].

The cut-off grade policies have been briefly subjected to the declining gold price scenario in Table 5.3.2. The results show

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[e] In a feature on gold; "The magic is gone", the Financial Mail (April 24, 1992) quotes a Bank of Lisbon Economic Focus on the limited role of gold as a medium of settlement between central banks because of the presence of such international agencies such as the I.M.F. and Bank of International Settlements (B.I.S.). This means that countries facing external financial difficulties can apply for short- term assistance from them and need not fall on their gold reserves. Also the metal has lost much of its investment attraction due to the emergence of sophisticated hedging instruments on the financial markets in the eighties.

		CUT–OFF POLICY		
Percent Fall in Metal Price	Criterion	Constant	Increasing	Decreasing
0	NPV	3.31	(5.21)	8.65
	IRRR	7.9580	5.8740	10.5215
2	NPV	(13.11)	(23.13)	(10.60)
	IRRR	2.5752	(1.2627)	0.0439
3	NPV	(20.59)	(32.12)	(20.49)
	IRRR	(0.6209)	(1.5195)	22.7246
5	NPV	(36.29)	(48.25)	(40.44)
	IRRR	< (4.0000)	< (4.0000)	< (4.0000)
6	NPV	(46.50)	(56.23)	(50.56)
	IRRR	< (4.0000)	< (4.0000)	< (4.0000)

Table 5.3.2 Effect of Falling Metal Price Trend on Cut–off Policies

(The NPV is discounted at 7 percent real per annum.
Figures in million Rands)

that at declining trends comparable to present day short-term figures (2 to 5 percent), the declining cut-off policy is the best strategy. With gold set to pull out from what has proved to be a long 11-year bear market[41] the reflected decline in the price trend this scenario does not seem to be a threatening issue.

From the foregoing, the declining cut-off policy has proved a versatile strategy as far as meeting the objective of a mining enterprise is concerned, under the tested scenarios and bound by reasonably conservative limits. Figure 5.3.1 is a summary plot of the tested scenarios indicating the assumed limits (-4 to 8 percent change). This result, however, does not play down the perception of future gold price as, if there is going to be a fantastic increase, all operators will definitely resort to the increasing cut-off policy.

In the next section the project will address the issue c implementing the policy by optimising the cut-off grades in the model.

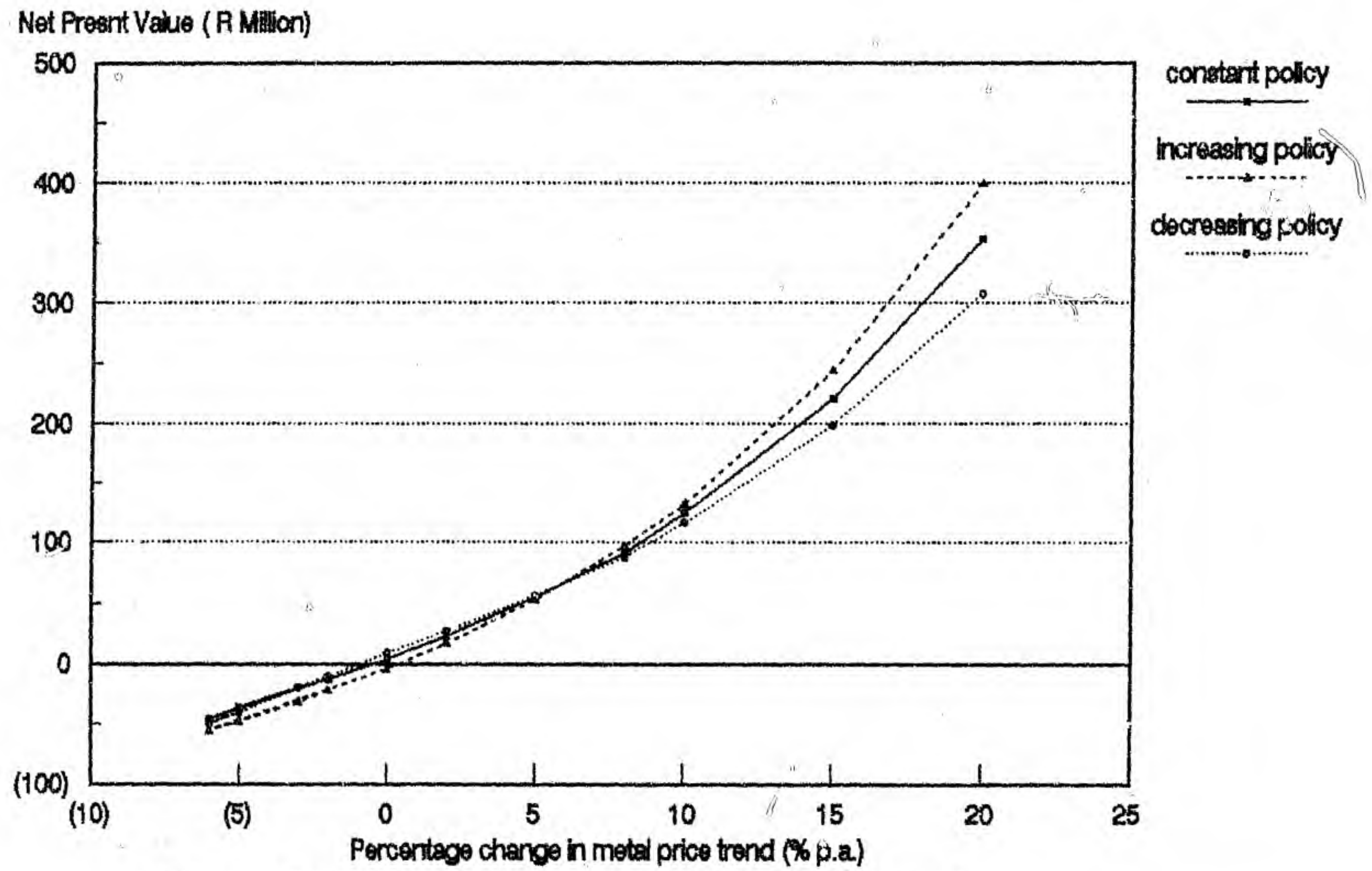

Figure 5.3.1 Summary of Effect of Price Trend on Policies.

CHAPTER 6

IMPLEMENTATION OF OPTIMAL CUT-OFF GRADE POLICY

This is essentially the determination of a sequence of optimum cut-off grades over an extended period, over the life of the mine. It constitutes what has been defined already as a cut-off policy. Every cut-off grade in the sequence must be an optimum consistently calculated to give the maximum present value throughout the whole period of the mine's life. So an optimum cut-off policy is a complete exploitational strategy for the mineralised body. The whole issue is to find a track of cut-off grades in the periods (mostly years) along which a consistent sequence of present values of a given terminal value, (zero if the mineral is depleted at the end of its life, otherwise some value if there is still some mineable reserves left), can be achieved. On this optimal track the present values at every stage along it are maxima.

Perfect application of an optimum cut-off grade policy should ensure that only ore generating more than a minimum return is mined, and that as much ore capable of generating that return as possible is mined. The following is a generalised procedure on how to integrate optimisation of cut-off grades with conventional project evaluation (e.g. feasibility or mine design) (Border[42]). It is an iterative process using the theory of dynamic programming and is applicable to the majority of mines where cut-off grades have a significant effect on reserves.

6.1 Outline of the procedure

Step 1: Geological Model and Ore Reserves

Cross sections, long sections and contour plans should be constructed. The orebody outline is shown using a feasible cut-off grade C_i and this will be used to estimate the initial reserves. This initial cut-off may be chosen from experience with mines using the same mining system. Then, in most cases, a geostatistical study will be performed, and this may include a grade-tonnage curve which is corrected to a block size approximating that of the smallest anticipated stope.

Step 2: Choose probable mining method(s)

Orebody geometry and rock strength generally limit the choice of mining method. Massive orebodies will call for bulk mining methods whilst veins may be mined more selectively. The grade-tonnage curve will indicate whether the geometry is likely to change significantly at different cut-offs. If so geological sections must be constructed for both high and low cut-off cases.

Step 3: Choose probable mining rate

A number of empirical formulae have been proposed for the estimation of optimum mine output, one of the most commonly used being the Taylor's rule[43], stated as;

$$\text{Annual capacity, } Q = 5R^{0.75}$$

where R is the expected mineable reserve.

Taylor states that the error associated with the use of this formula should not exceed 20 percent. However, caution may be needed where a mine's reserves are high, as the formula may predict a rate which will not be achievable in the short term due to geometric limitations on the number of operating stope faces.

Step 4: Mining costs

Inaccuracies associated with estimating mining costs are very easy to make. Over-estimation leads to the project being too costly to be not profitable whilst under-estimation gives a false signal which is always detected too late. Updating of costs figures used in the pre-feasibility or feasibility stage will make the final mine design cost more accurate. Mining costs are directly related to the planned mining method and mining rate. Estimates of mining recovery and dilution must be made. Assessment of mining equipment requirements will lead to working estimates of capital and operating costs for the mine.

Step 5: Plant and mill cost

Metallurgical and testwork will have indicated the most suitable treatment methods and will have determined the expected recovery. Preliminary mill design will allow reasonable estimates of both capital and operating costs. Any other non-mine factors ffecting capital or operating costs, such as commitment with environmental regulations or administration, costs should also be assessed.

Step 6: Find the initial optimum cut-off grade

Lane[44] has shown that where milling or hoisting capacity is limiting a mine's output, the optimum reserve cut-off grade C_o is given by:

$$C_o = \frac{(1+d)\,(V+[f+F]/A)}{NSRx\varepsilon}$$

where f = annual fixed costs

F = opportunity cost

A = annual production

V = variable cost per tonne

d = mining dilution

NSR = sale price per tonne of product after all transport, insurance, marketing and smelter costs have been deducted.

ϵ = mill recovery.

F, the opportunity cost, is the change in the value of the mine over the time period being considered (usually one year). If the economic conditions (prices and costs) over this period are assumed to be constant, then F = discount rate x mine value (NPV). Obviously F will decline over the mine's life. The net present value, as estimated at the pre-feasibility stage, is normally used to give an initial estimate of the opportunity cost, F. The underlying philosophy about the opportunity cost is that every deposit has a given present value associated with it at any given point in time, and that every tonne of material mined and processed during a given year should pay for the cost not receiving that future cash-flow.

Step 7: Estimate cut-off grade at the end of mine's life C_e

At the end of the mine's life will approach zero, so the equation becomes:

$$C_e = \frac{(1+d)\ (V+f/A)}{NSRx\varepsilon}$$

Unless there is a major price increase forecast, C_e will be lower than C_o.

Step 8: Estimate mineable reserves using C_o and C_e

Sections and plans showing the distribution of reserves at these cut-offs should be prepared and used in conjunction with the grade tonnage plots

Step 9: Estimate mine life

The mine life is the reserve tonnage divided by the annual production. The most accurate estimate of mine life will be produced by preparing a mine planning schedule for the whole reserve. However, this requires a complete schedule of planned cut-off grades to be used each year and these in turn depend on the estimated life. An estimate may be made by taking the average of the lives determined from the reserves at cut-off C_o and C_e. Border[45] describes a reasonably accurate estimate using a prescribed procedure.

Step 10: Check mining parameters

An examination must be made to check whether the cha... ...n cut-off grades makes any significant difference to the development required, and hence the mining costs. Any major change in expected mine life may necessitate the revision of the planned mining rate. Where changes are made, the optimisation must start from step 4.

Step 11: Estimate the net present value

A complete cut-off policy requires product price forecasts over the life of the mine. As estimates of likely cut-off grades must be made before future cash flows are made, a linear decrease of F over the life of the mine is assumed in order to derive an optimum cut-off grade for each year of the expected mine life.

Step 12: Recalculate F for computed net present value in 11

If F has changed significantly, the optimisation must be repeated from step 7. Because the initial levels of the present values are unknown an assumption is made, the policy calculated and the present value on termination is compared with the assumed opportunity cost. Depending on the difference, the initial levels are modified and a new policy calculated. When a point is reached where there is no significant difference, the optimum policy has been reached.

Occasionally the optimisation procedure described will not end, successive iterations providing very different estimates of the NPV and C_0. This can be caused by a deposit in which mining costs are very dependent of the cut-off grade, due to different requirements at different cut-off grades. An elaborate technique is therefore required for predicting how development costs will vary with cut-off grade by use of a graph of development metres (or costs) plotted against cut-off grade.

In the next section the procedure will be applied to the model.

6.2 Application of the procedure to the model

For the model, we assume that all the necessary steps from 1 to 5 have been taken already. What is left to be done, is to calculate the complete cut-off policy for the deposit under study under the defined economic conditions and forecast costs and prices.

To find C_0 the following terms are defined:

d, dilution is not considered because grade is diluted

V, variable costs = R 85 per tonne of ore

A, annual production = 400 000 tonnes

f, the annual fixed costs = R 4 Million

r, the discount rate = 7 percent p.a.

NPV, the present value of the mine, from say pre-feasibility study, in this model the NPV from the constant policy analysis

= R 3.11 Million

F, opportunity cost = NPV x r = 0.07 x 3.11

= R 217 700

NSR, the received price of gold = Gold price x Percentage net smelter return for mine (0.98 in gold mines)

= R 35 000 x 0.98

= R 34 300 /t (R 34.3 /gm)

ϵ, mill recovery = 0.96

Because we are using diluted grades the initial optimum cut-off grade C_o becomes:

$$C_o = \frac{(85+[4+0.2177]\,x10^6/400\,000)}{34.3x0.96}$$

$$= 2.90 \ g/t.$$

By the same argument, the estimated cut-off grade at the end of the mine' life, C_e, is given by:

$$C_e = \frac{85+4\,000\,000/400\,000}{34.3x0.96}$$

$$= 2.89 \ g/t$$

At these optimum cut-off grades the following is the outcome:

Cut-off	Pay Value	Payability	Tonnage	Mine Life
2.89 g/t	4.70 g/t	42.70 %	3.458 M	8.6 years

Following from Step 10 these results show that the mine will come to its end some 3 years before its designed life of 11. This will necessitate the revision of the designed or planned mining rate, so in keeping with the procedure one must start at step 4 or 5 by revising the costs associated with this different mining requirement. This will also affect the net present value of the mine. The main reason behind this state of affairs is the fact that the costing and the design was done on the marginal break-even cut-off, which made no provision for any other costs apart from the tonnage dependent variable cost. The lesson here is the danger in assuming that once the pay limit has been defined all other parcels of ore of higher grade could be mined to cover all the other costs. This may not necessarily be so, especially in a low grade deposit such as the one in the model.

In this case, since both the initial and end of life cut-off grades are almost the same, it points to the use of the constant policy if the chosen present value of the mine is to be achieved under the existing economic conditions. A calculation summarised in Table 6.2.1 gave a net present value of only R 0.043 million. But from the previous analysis, if there is a possibility of applying a declining policy it may give a better net present value. But in this case it is not possible. Also because the C_t, being the lowest cut-off grade to be mined in the declining policy, gave mine life of nearly 9 years, and the life given by the C_0 will definitely give some lower, value it means that the mine may have to be re-designed to exploit the ore at a reduced life if an improvement is to made in the net present value. Of

Year	Tonnage (tonnes)	Grade (gmAu/t)	Capital Expenditure (R Million)	Working Cost (R Million)	Revenue (R Million)	Net Profit (R Million)	Taxable Profit (R Million)	Tax + Lease (R Million)	Real Net Cash Flow (R Million)
1	–	–	30	0	0	(30)	0	0	(30)
2	–	–	24	0	0	(24)	0	0	(20.87)
3	300 000	4.70	28.32	40.04	67.1	(1.27)	21.33	15.80	(13.02)
4	400 000	4.70	0	58.79	104	44.96	36.13	26.53	12.55
5	400 000	4.70	0	64.67	116	51.53	41.64	30.50	13.02
6	400 000	4.70	0	69.35	126	55.66	44.97	32.94	13.02
7	400 000	4.70	0	75.3	136	60.11	48.57	35.57	13.02
8	400 000	4.70	0	81.47	146	64.92	52.46	38.42	13.02
9	400 000	4.70	0	87.99	158	70.11	56.65	41.49	13.02
10	400 000	4.70	0	95.03	171	75.72	61.18	44.81	13.02

Net Present Value @ 7 percent = R 0.043 Million
DCFROR = 7.01563 % p.a.
(Tax + Lease = 0.6685 x profit – 0.03401 x revenue)
(All estimates are as at the end of each year)

Table 6.2.1 Results of economic analysis on implementation of procedure with the same costs but mine life 8 years.

Year	Tonnage (tonnes)	Grade (gmAu/t)	Capital Expenditure (R Million)	Working Cost (R Million)	Revenue (R Million)	Net Profit (R Million)	Taxable Profit (R Million)	Tax + Lease (R Million)	Real Net Cash Flow (R Million)
1	–	–	30	0	0	(30)	0	0	(30)
2	–	–	24	0	0	(24)	0	0	(20.87)
3	300 000	4.70	28.32	40.04	67.1	(1.27)	21.33	15.80	(13.02)
4	400 000	4.70	0	58.79	104	44.96	36.13	26.53	12.55
5	400 000	4.70	0	64.67	116	51.53	41.64	30.50	13.02
6	400 000	4.70	0	69.85	126	55.66	44.97	32.94	13.02
7	400 000	4.70	0	75.3	136	60.11	48.57	35.57	13.02
8	400 000	4.70	0	81.47	146	64.92	52.46	38.42	13.02
9	400 000	4.70	0	87.99	158	70.11	56.65	41.49	13.02
10	400 000	4.70	0	95.03	171	75.72	61.18	44.81	13.02

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(Tax + Lease = 0.6685 x profit − 0.03401 x revenue)
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1	–	–	25	0	0	(25)	0	0	(25)
2	–	–	21	0	0	(21.6)	0	0	(18.78)
3	300 000	4.99	25.49	40.04	71.2	84.12	21.33	20.24	(9.02)
4	400 000	4.94	0	58.79	109	53.36	36.13	31.96	14.57
5	400 000	4.89	0	64.67	121	59.64	41.64	35.76	14.79
6	400 000	4.85	0	69.85	129	63.05	44.97	37.76	14.50
7	400 000	4.80	0	75.3	138	66.97	48.57	40.06	14.28
8	400 000	4.75	0	81.47	148	70.76	52.46	42.27	14.00
9	400 000	4.70	0	87.99	158	74.74	56.65	44.59	13.72

Net Present Value @ 7 percent = R 8.70 Million
DCFROR = 11.04688 % p.a.
(Tax + Lease = 0.6685 x profit – 0.03401 x revenue)
(All estimates are as at the end of each year)

Table 6.2.2 Results of economic analysis on implementation of procedure with modified costs and mine life 7 years.

course, the shorter the life of the mine the higher will the net present value be, and this brings in the subject of high-grading the mine.

The model needs to be modified in order to mine the deposit in a shorter time. So the costs will have to be re-defined to cater for the shorter life. Assume that the capital expenditures will now be R 25 million in year 1 and R 18 million the next two and the required fixed costs to support the logistics to be R 2 million a year. All inflation and escalation levels remain the same. Say, a particular value of R 120 million has been placed on the mine giving it an opportunity cost of R 8.4 million. This gives a C_o of 3.84 g/t, mineable tonnage of 2.04 million at 5.5 g/t pay grade. At the selected mining rate the life of the mine is 6.6 years. On averaging with the life give by C_e, the combined average is 7 years. A summary of the analysis is given in Table 6.2.2 in which the net present value of the project is R 8.7 million, comparing very well with the opportunity cost of R 8.4 million. Because of the closeness of the two figures, it can be reasonably assumed that the cut-off policy as calculated in Table 6.2.2 is the optimum. One thing that proponents of the declining policy do not clarify is the question of how significantly different the two values must be for them to be rejected. According to Lane[46], the question of the most suitable convergence formula is a complex one, depending on the behaviour of the underlying model and the relative importanceof speed and robustness (insensitivity to assumptions and small errors).

CHAPTER 7

CONCLUSION

The foregoing project has been on the important subject of cut-off grades. Since the main objective of a mining enterprise is to deplete a mineral reserve of its good grades in order to recover costs and reap a return, the question of optimising the cut-off grades by a well founded policy cannot be under-estimated. Yet this vital aspect of the mining operation does not receive due attention because of the use of traditional or conventional approaches which, without a doubt, has worked quite well. Some workers in the area of mine design (mining engineers, geologists, financial analysts) have proposed other cut-off schemes which have been analysed in this report.

With the constant cut-off policy or the traditional approach, the mine's annual head grade is usually estimated on the basis of a calculated break-even, and is constant unless the metal price and costs change significantly during the life of the mine. It has been known also that cut-off grades have been set by comparison with existing mines of similar type, without regard to differences in geology and cost structure.[47]

The increasing cut-off policy will be preferable if the mine is to be opened up slowly, with low initial development increasing with time. The low initial revenue will be matched by the lower costs. It will also be the best policy, in this analysis, if the metal price is anticipated to increase at a rate greater than 8 percent each year, differential to costs, over the life of the mine.

The declining cut-off policy was shown to be the best of the three policies under the same economic conditions. It is the outstanding policy as far as optimisation of cut-off grades are concerned. High rates of positive cash-flows are desirable in the early years of a mine's life, for such objectives as liquidation of debts, early recovery of capital and consequent reduction of residual risks, early dividends to stockholders, or other purposes. The discounting rate must reflect these legitimate objectives, as well as more broader social ones, if depletion is to be optimal in total.

In keeping with the first condition of optimal depletion of an exhaustible resource, that the extraction and consumption must involve an opportunity cost, the declining cut-off policy is able to incorporate the opportunity cost to give a continuous maximum present value. The opportunity cost is simply the discount rate multiplied by the present value of the mine remaining at the start of each new year. This is added to the annual fixed cost. Because the present value of the mine decreases as the mine ages, so must the cut-off grade.

One apparent drawback of the declining cut-off grade policy is in the fact that the lower g de material left during the early years. In open casts this material can be stockpiled and milled later, but in deep underground mines it is practically lost. This is seen by some as waste of resources. However, the overall objective in optimising cut-off grades - maximum economic benefit - may not be achieved by mining every part of the mineralised body. Thus the declining cut-off policy may appear as "high-grading", but if cut-off grades are to be optimised to achieve a maximum present value, logic and mathematics can lead to no other recommendation.

The various cut-off policies were examined under certain important factors namely; grade distribution, costs and prices. Because of the varied nature of characteristics of deposits such as location, size, chemistry, disposition, genesis and depth of burial, different deposits have different grade distribution of the mineralisation. Grades may be uniformly distributed, with respect to value and position (e.g porphyry copper and alluvial gold), may be graded, or may be localised, with high grades occupying areas different from the lower grades (most hydrothermal gold deposits). Different grade distributions will affect cut-off grades differently; uniformly distributed grades will be more amendable to the constant cut-off grade policy, whilst graded values will be easily handled by either the increasing or declining cut-off policy. Localised mineralisation may do with a combination of any of the policies. To determine how different grade distributions affect cut-off grade policy will be an area for further study.

As richer and more accessible deposits are depleted, and as mines become deeper, the remaining leaner and less accessible ones become more expensive to deplete. The effect of increasing costs associated with mining are not necessarily reflected by the price of the commodity, because mine operators are essentially price-takers. In the analysis, no study was done on the effect of cost trends on the cut-off grade policies. The effect will be the reverse of the effect of prices. Costs are generally on the increase, and a reasonable estimate of inflation and cost escalation built into the model captures the effect of increasing cost. Rapidly increasing costs, seen by some as a sign of scarcity, may reduce depletion with time, when future high costs make it prohibitive to deplete. Uncertainty about future costs will also bring depletion near in time. A slow increase in cost, differential to commodity price, will favour depletion in later years. Shifting depletion to the present is comparable to the declining policy, and to the future, the increasing policy. So in the event of high inflation and escalation or uncertainty about future costs, the declining cut-off policy will provide the optimal depletion, and the reverse is true for the increasing policy. All other things remaining the same, the constant cut-off grade policy will be the favourable strategy if there is no difference in cost and price, but the fact of discounting must be borne in mind.

Under the various price scenarios studied, the declining cut-off policy was the best if the metal price is to change between -4 and 8 percent per year. But the increasing cut-off policy crosses over quickly to be the best policy when the future metal price increase is greater than 8 percent, followed by the constant cut-off policy. It stands to reason, that if higher future prices are anticipated then the policy that will be able to postpone depletion to reap higher future revenue will be the optimum, with the discount rate playing an important part. So cut-off grade policies will be affected by perception of future prices.

In conclusion, the declining cut-off policy seems to have greatest potential as far as optimising cut-off grades is concerned, even though it involves a lot more work and time. However, with the advances in the handling of massive amounts of data in a relatively short time by means of computers, it is possible to do many iterations to arrive at the best choice. Though the analyses in this study have not involved all the fine details involved in an actual mine design, the model has proved the potential of the declining cut-off policy in achieving a better result than the conventional approach.

It is therefore recommended that mining concerns acquaint themselves with this policy and make further research into the underlying theory to determine its suitability to particular mining models (e.g. mining methods, geological environment, grade distribution, modes of financing, etc), in these times when the best outcome should be sought from any investment.

APPENDICES

APPENDIX H: INCREASING POLICY ANALYSIS; 5% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	25	17	20
3	14	23	16	18
4	12	21	14	16
5	10	17	10	12
6	8	13	8	8
7	8	13	8	8
8	8	13	8	8
9	8	13	8	8
10	8	13	8	8
11	8	13	8	8
12	8	13	8	8
13	8	13	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	38.70	40.04	28.32	(29.61)
4	71.50	58.79	0.00	12.72
5	94.20	64.67	0.00	29.53
6	118.00	69.85	0.00	48.51
7	147.00	75.43	0.00	71.73
8	181.00	81.47	0.00	100.00
9	222.00	87.99	0.00	134.30
10	271.00	95.03	0.00	175.50
11	328.00	102.60	0.00	225.10
12	395.00	110.80	0.00	284.20
13	474.00	119.70	0.00	354.70

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	0.00	0.00	(22.59)
4	11.43	5.21	5.12
5	29.53	16.54	8.05
6	48.51	28.40	11.53
7	71.73	42.95	15.89
8	100.00	60.70	19.33
9	134.30	82.21	23.70
10	175.50	108.20	28.40
11	225.10	139.30	33.46
12	284.20	176.60	38.89
13	354.70	221.00	44.72

Internal Real Rate of Return: 15.74219 p.a.
Net Present Value @ 7 percent: R 52.4316 million.

APPENDIX J: CONSTANT POLICY ANALYSIS; 8% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	28	17	20
3	14	26	16	18
4	12	22	14	16
5	10	20	10	12
6	8	16	8	8
7	8	16	8	8
8	8	16	8	8
9	8	16	8	8
10	8	16	8	8
11	8	16	8	8
12	8	16	8	8
13	8	16	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	69.90	40.04	28.32	(1.54)
4	114.00	58.79	0.00	54.91
5	136.00	64.67	0.00	71.78
6	158.00	69.85	0.00	88.44
7	184.00	75.43	0.00	108.20
8	213.00	81.47	0.00	131.50
9	247.00	87.99	0.00	159.10
10	287.00	95.03	0.00	191.60
11	332.00	102.60	0.00	229.80
12	386.00	110.80	0.00	274.80
13	447.00	119.70	0.00	327.60

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	29.86	17.59	(12.24)
4	54.91	32.84	15.03
5	71.78	43.34	17.60
6	88.44	53.74	19.89
7	108.20	66.07	22.35
8	131.50	80.67	24.99
9	159.10	97.94	27.82
10	191.60	118.30	30.87
11	229.80	142.30	34.14
12	274.80	170.60	34.14
13	327.60	203.80	41.42

Internal Real Rate of Return: 23.66211 p.a.
Net Present Value @ 7 percent: R 90.4275 million

APPENDIX L: DECLINING POLICY ANALYSIS; 8% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	28	17	20
3	14	26	16	18
4	12	22	14	16
5	10	20	10	12
6	8	16	8	8
7	8	16	8	8
8	8	16	8	8
9	8	16	8	8
10	8	16	8	8
11	8	16	8	8
12	8	16	8	8
13	8	16	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	99.20	40.04	28.32	30.81
4	152.00	58.79	0.00	93.01
5	171.00	64.67	0.00	106.10
6	185.00	69.85	0.00	114.90
7	199.00	75.43	0.00	123.50
8	213.00	81.47	0.00	131.50
9	226.00	87.99	0.00	138.40
10	239.00	95.03	0.00	143.60
11	249.00	102.60	0.00	146.30
12	257.00	110.80	0.00	145.70
13	260.00	119.70	0.00	140.40

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	59.13	36.16	(40.77)
4	93.01	57.01	24.51
5	106.10	65.09	25.36
6	114.90	70.56	25.45
7	123.50	75.83	25.33
8	131.50	80.67	24.99
9	138.40	84.81	24.38
10	143.60	87.89	23.48
11	146.30	89.37	22.23
12	145.70	88.66	20.60
13	140.40	85.00	18.53

Internal Real Rate of Return: 28.03711 p.a.
Net Present Value @ 7 percent: R 87.0606 million.

APPENDIX A: CONSTANT POLICY ANALYSIS

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	20	17	20
3	14	18	16	18
4	12	16	14	16
5	10	12	10	12
6	8	8	8	8
7	8	8	8	8
8	8	8	8	8
9	8	8	8	8
10	8	8	8	8
11	8	8	8	8
12	8	8	8	8
13	8	8	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	61.40	40.04	28.32	(6.99)
4	94.90	58.79	0.00	36.13
5	106.00	64.67	0.00	41.64
6	115.00	69.85	0.00	44.97
7	124.00	75.43	0.00	48.57
8	134.00	81.47	0.00	52.46
9	145.00	87.99	0.00	56.65
10	156.00	95.03	0.00	61.18
11	169.00	102.60	0.00	66.08
12	182.00	110.80	0.00	71.37
13	197.00	119.70	0.00	77.07

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	21.33	12.17	(14.62)
4	36.13	20.92	10.36
5	41.64	24.22	10.79
6	44.97	26.16	10.79
7	48.57	28.25	10.79
8	52.46	30.51	10.79
9	56.65	32.95	10.79
10	61.18	35.59	10.79
11	66.08	38.44	10.79
12	71.37	41.51	10.79
13	77.07	44.83	10.79

Internal Real Rate of Return: 7.958008 p.a.
Net Present Value @ 7 percent: R 3.31102 million

APPENDIX B: INCREASING POLICY ANALYSIS

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	20	17	20
3	14	18	16	18
4	12	16	14	16
5	10	12	10	12
6	8	8	8	8
7	8	8	8	8
8	8	8	8	8
9	8	8	8	8
10	8	8	8	8
11	8	8	8	8
12	8	8	8	8
13	8	8	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	35.70	40.04	28.32	(32.67)
4	63.10	58.79	0.00	4.35
5	79.60	64.67	0.00	14.95
6	95.60	69.85	0.00	25.76
7	114.00	75.43	0.00	38.18
8	134.00	81.47	0.00	52.46
9	157.00	87.99	0.00	68.77
10	182.00	95.03	0.00	87.35
11	211.00	102.60	0.00	108.50
12	243.00	110.80	0.00	132.40
13	279.00	119.70	0.00	159.40

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	0.00	0.00	(24.92)
4	0.00	0.00	2.96
5	14.49	7.28	4.75
6	25.76	13.97	6.76
7	38.18	21.39	8.77
8	52.46	30.51	10.79
9	68.77	40.64	12.80
10	87.35	52.19	14.82
11	108.50	65.33	16.83
12	132.40	80.24	18.85
13	159.40	97.09	20.86

Internal Real Rate of Return: 5.857422 p.a.
Net Present Value @ 7 percent: - R 5.21273 million

APPENDIX C: DECLINING POLICY ANALYSIS

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year· % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	20	17	20
3	14	18	16	18
4	12	16	14	16
5	10	12	10	12
6	8	8	8	8
7	8	8	8	8
8	8	8	8	8
9	8	8	8	8
10	8	8	8	8
11	8	8	8	8
12	8	8	8	8
13	8	8	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	87.10	40.04	28.32	(18.71)
4	127.00	58.79	0.00	67.93
5	133.00	64.67	0.00	68.35
6	134.00	69.85	0.00	64.20
7	134.00	75.43	0.00	58.96
8	134.00	81.47	0.00	52.46
9	133.00	87.99	0.00	44.54
10	130.00	95.03	0.00	35.05
11	126.00	102.60	0.00	23.72
12	121.00	110.80	0.00	10.36
13	114.00	119.70	0.00	(5.30)

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	47.03	28.43	(7.45)
4	67.93	41.10	18.27
5	68.35	41.17	16.83
6	64.20	38.36	14.82
7	58.96	34.84	12.80
8	52.46	30.51	10.79
9	44.54	25.27	8.77
10	35.05	19.00	6.76
11	23.72	11.56	4.74
12	10.36	2.80	2.73
13	0.00	0.00	(1.77)

Internal Real Rate of Return: 10.52149 p.a.
Net Present Value @ 7 percent: R 8.64955 million.

APPENDIX D: CONSTANT POLICY ANALYSIS; 2% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	C	0	0
2	15	22	17	20
3	14	20	16	18
4	12	18	14	16
5	10	14	10	12
6	8	10	8	8
7	8	10	8	8
8	8	10	8	8
9	8	10	8	8
10	8	10	8	8
11	8	10	8	8
12	8	10	8	8
13	8	10	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	63.40	40.04	28.32	(4.91)
4	99.80	58.79	0.00	41.04
5	114.00	64.67	0.00	49.14
6	125.00	69.85	0.00	55.34
7	138.00	75.43	0.00	62.28
8	151.00	81.47	0.00	70.01
9	167.00	87.99	0.00	78.64
10	183.00	95.03	0.00	88.27
11	202.00	102.60	0.00	98.99
12	222.00	110.80	0.00	110.90
13	244.00	119.70	0.00	124.30

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	23.41	13.49	(14.04)
4	41.04	24.04	11.58
5	49.14	28.98	12.48
6	55.34	32.74	12.96
7	62.28	36.95	13.44
8	70.01	41.65	13.94
9	78.64	46.91	14.44
10	88.27	52.77	14.96
11	98.99	59.32	15.48
12	110.97	66.62	16.01
13	124.30	74.77	16.55

Internal Real Rate of Return: 12.5624 p.a.
Net Present Value @ 7 percent: R 21.9200 million.

APPENDIX E: INCREASING POLICY ANALYSIS; 2% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	22	17	20
3	14	20	16	18
4	12	18	14	16
5	10	14	10	12
6	8	10	8	8
7	8	10	8	8
8	8	10	8	8
9	8	10	8	8
10	8	10	8	8
11	8	10	8	8
12	8	10	8	8
13	8	10	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	36.90	40.04	28.32	(31.46)
4	66.40	58.79	0.00	7.61
5	85.20	64.67	0.00	20.56
6	104.00	69.85	0.00	34.40
7	126.00	75.43	0.00	50.74
8	151.00	81.47	0.00	70.01
9	181.00	87.99	0.00	92.60
10	214.00	95.03	0.00	119.00
11	252.00	102.60	0.00	149.70
12	296.00	110.80	0.00	185.20
13	346.00	119.70	0.00	226.40

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	0.00	0.00	(24.00)
4	4.47	0.73	4.69
5	20.56	10.85	6.02
6	34.40	19.45	8.57
7	50.74	19.45	11.21
8	70.01	29.63	13.94
9	92.60	41.65	16.76
10	119.00	55.76	19.68
11	149.78	72.25	22.70
12	185.27	91.46	25.82
13	226.40	113.80	29.04

Internal Real Rate of Return: 10.18164 p.a.
Net Present Value @ 7 percent: R 16.2365 million.

APPENDIX F: DECLINING POLICY ANALYSIS; 2% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	22	17	20
3	14	20	16	18
4	12	18	14	16
5	10	14	10	12
6	8	10	8	8
7	8	10	8	8
8	8	10	8	8
9	8	10	8	8
10	8	10	8	8
11	8	10	8	8
12	8	10	8	8
13	8	10	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	90.00	40.04	28.32	21.66
4	133.00	58.75	0.00	74.48
5	142.00	64.67	0.00	77.73
6	146.00	69.85	0.00	76.31
7	149.00	75.43	0.00	73.81
8	151.00	81.47	0.00	70.01
9	153.00	[illegible].99	0.00	64.69
10	153.00	9[illegible].03	0.00	57.60
11	151.00	102.60	0.00	48.37
12	148.00	110.80	0.00	36.68
13	142.00	119.70	0.00	22.14

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	49.98	30.35	(6.63)
4	74.48	45.26	19.90
5	77.73	47.12	18.95
6	76.31	46.04	17.35
7	73.81	44.27	15.68
8	70.01	41.65	13.94
9	64.69	38.05	12.12
10	57.60	33.31	10.23
11	48.37	27.20	8.26
12	36.68	19.51	6.21
13	22.14	9.98	4.07

Internal Real Rate of Return: 16.04297 p.a.
Net Present Value @ 7 percent: R 26.6254 million

APPENDIX G: CONSTANT POLICY ANALYSIS; 5% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	25	17	20
3	14	23	16	18
4	12	21	14	16
5	10	17	10	12
6	8	13	8	8
7	8	13	8	8
8	8	13	8	8
9	8	13	8	8
10	8	13	8	8
11	8	13	8	8
12	8	13	8	8
13	8	13	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	66.60	40.04	28.32	(1.72)
4	108.00	58.79	0.00	48.72
5	126.00	64.67	0.00	61.11
6	142.00	69.85	0.00	72.29
7	161.00	75.43	0.00	85.18
8	181.00	81.47	0.00	100.00
9	205.00	87.99	0.00	171.10
10	232.00	95.03	0.00	136.70
11	262.00	102.60	0.00	159.30
12	296.00	110.80	0.00	185.10
13	334.00	119.70	0.00	214.70

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	26.60	15.51	(13.15)
4	48.72	23.91	13.49
5	61.11	36.58	15.19
6	72.29	43.49	16.51
7	85.18	51.48	17.89
8	100.00	60.70	19.33
9	171.10	71.31	20.84
10	136.70	83.52	22.42
11	159.30	97.56	24.07
12	185.10	113.70	25.80
13	214.70	132.20	27.61

Internal Real Rate of Return: 18.66016 p.a.
Net Present Value @ 7 percent: R 54.6015 million.

APPENDIX I: DECLINING POLICY ANALYSIS; 5% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	25	17	20
3	14	23	16	18
4	12	21	14	16
5	10	17	10	12
6	8	13	8	8
7	8	13	8	8
8	8	13	8	8
9	8	13	8	8
10	8	13	8	8
11	8	13	8	8
12	8	13	8	8
13	8	13	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	94.50	40.04	28.32	26.18
4	144.00	58.79	0.00	84.73
5	157.00	64.67	0.00	92.72
6	166.00	69.85	0.00	96.10
7	174.00	75.43	0.00	98.64
8	181.00	81.47	0.00	100.00
9	188.00	87.99	0.00	99.93
10	193.00	95.03	0.00	97.95
11	196.00	102.60	0.00	93.50
12	197.00	110.80	0.00	86.00
13	194.00	119.70	0.00	74.72

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	54.50	33.22	(5.37)
4	84.73	51.76	22.45
5	92.72	56.63	22.34
6	96.10	58.60	21.50
7	98.64	60.02	20.50
8	100.00	60.70	19.33
9	99.93	60.41	17.98
10	97.95	58.92	16.45
11	93.50	55.84	14.70
12	86.00	50.80	12.72
13	74.72	43.34	10.50

Internal Real Rate of Return: 7.958008 p.a.
Net Present Value @ 7 percent: R 3.31102 million

APPENDIX K: INCREASING POLICY ANALYSIS; 8% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	28	17	20
3	14	26	16	18
4	12	22	14	16
5	10	20	10	12
6	8	16	8	8
7	8	16	8	8
8	8	16	8	8
9	8	16	8	8
10	8	16	8	8
11	8	16	8	8
12	8	16	8	8
13	8	16	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	40.60	40.04	28.32	(27.71)
4	76.90	58.79	0.00	18.08
5	104.00	64.67	0.00	39.19
6	134.00	69.85	0.00	64.11
7	171.00	75.43	0.00	95.55
8	216.00	81.47	0.00	135.00
9	272.00	87.99	0.00	184.20
10	340.00	95.03	0.00	245.10
11	423.00	102.60	0.00	320.20
12	523.00	110.80	0.00	412.40
13	645.00	119.70	0.00	525.30

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	0.61	0.00	(21.14)
4	18.08	9.47	5.86
5	39.19	22.67	10.23
6	64.11	38.30	14.80
7	95.55	58.06	19.90
8	135.00	82.89	25.62
9	184.20	113.90	31.99
10	245.10	15.309	39.10
11	320.20	199.70	47.02
12	412.40	257.90	55.82
13	525.30	329.20	65.59

Internal Real Rate of Return: 20.9082 p.a.
Net Present Value @ 7 percent: R 97.0257 million.

APPENDIX M: CONSTANT POLICY ANALYSIS 10% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	30	17	20
3	14	28	16	18
4	12	26	14	16
5	10	22	10	12
6	9	18	8	8
7	8	18	8	8
8	8	18	8	8
9	8	18	8	8
10	8	18	8	8
11	8	18	8	8
12	8	18	8	8
13	8	18	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	72.10	40.04	28.32	3.76
4	121.00	58.79	0.00	62.37
5	148.00	64.67	0.00	83.15
6	174.00	69.85	0.00	104.60
7	206.00	75.43	0.00	130.40
8	243.00	81.47	0.00	161.40
9	287.00	87.99	0.00	198.60
10	338.00	95.03	0.00	243.20
11	399.00	102.60	0.00	296.40
12	471.00	110.80	0.00	360.00
13	556.00	119.70	0.00	435.90

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	32.18	18.99	(11.62)
4	62.37	37.75	16.89
5	83.15	50.56	20.18
6	104.60	63.98	23.28
7	130.40	80.17	26.66
8	161.40	99.64	30.36
9	198.60	123.00	34.40
10	243.20	151.00	38.81
11	296.40	184.60	43.63
12	360.00	224.70	48.90
13	435.90	272.50	54.66

Internal Real Rate of Return: 27.64649 p.a.
Net Present Value @ 7 percent: R 124.866 million

APPENDIX N: INCREASING POLICY ANALYSIS; 10% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	30	17	20
3	14	28	16	18
4	12	26	14	16
5	10	22	10	12
6	8	18	8	8
7	8	18	8	8
8	8	18	8	8
9	8	18	8	8
10	8	18	8	8
11	8	18	8	8
12	8	18	8	8
13	8	18	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	41.90	40.04	28.32	(26.42)
4	80.60	58.79	0.00	21.80
5	111.00	64.67	0.00	46.03
6	1[illegible]9.00	69.85	0.00	75.39
7	243.00	75.43	0.00	113.20
8	311.00	81.47	0.00	161.40
9	395.00	87.99	0.00	222.60
10	499.00	95.03	0.00	299.80
11	629.00	102.60	0.00	396.70
12	788.00	110.80	0.00	517.80
13	197.00	119.70	0.00	668.50

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	1.90	0.00	(20.16)
4	21.80	11.83	6.79
5	46.03	27.01	11.78
6	75.39	45.46	17.16
7	113.20	69.23	23.32
8	161.40	99.64	30.36
9	222.60	138.20	38.39
10	299.80	187.00	47.54
11	396.70	248.20	57.93
12	517.80	324.80	69.73
13	668.50	420.10	83.10

Internal Real Rate of Return: 24.2675% p.a.
Net Present Value @ 7 percent: R 132.494 million

APPENDIX O: DECLINING POLICY ANALYSIS, 10% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	30	17	20
3	14	28	16	18
4	12	26	14	16
5	10	22	10	12
6	8	18	8	8
7	8	18	8	8
8	8	18	8	8
9	8	18	8	8
10	8	18	8	8
11	8	18	8	8
12	8	18	8	8
13	8	18	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	102.00	40.04	28 32	33.96)
4	162.00	58.79	0.00	103.00
5	185.00	64.67	0.00	120.30
6	204.00	69.85	0.00	133.80
7	223.00	75.43	0.00	147.60
8	243.00	81.47	0.00	161.40
9	263.00	87.99	0.00	174.60
10	282.00	95.03	0.00	186.60
11	299.00	102.60	0.00	196.20
12	313.00	110.80	0.00	202.40
13	323.00	119.70	0.00	203.40

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	62.28	38.16	(3.20)
4	103.00	63.33	26.99
5	120.30	74.12	28.58
6	133.80	82.52	29.40
7	147.60	91.10	30.00
8	161.40	99.64	30.30
9	174.60	107.80	30.40
10	186.60	115.10	30.10
11	196.20	121.00	29.35
12	202.40	124.60	28.08
13	203.40	125.00	26.22

Internal Real Rate of Return: 32.39258% p.a.
Net Present Value @ 7 percent: R 117.168 million.

APPENDIX P: CONSTANT POLICY ANALYSIS; 15% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	35	17	20
3	14	33	16	18
4	12	31	14	16
5	10	27	10	12
6	8	23	8	8
7	8	23	8	8
8	8	23	8	8
9	8	23	8	8
10	8	23	8	8
11	8	23	8	8
12	8	23	8	8
13	8	23	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	77.80	40.04	28.32	9.46
4	136.00	58.79	0.00	77.13
5	173.00	64.67	0.00	108.00
6	212.00	69.85	0.00	142.50
7	261.00	75.43	0.00	185.70
8	321.00	81.47	0.00	239.80
9	395.00	87.99	0.00	307.10
10	486.00	95.03	0.00	391.00
11	598.00	102.60	0.00	495.20
12	735.00	110.80	0.00	624.40
13	904.00	119.70	0.00	784.70

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	37.78	22.61	(10.03)
4	77.13	46.94	20.56
5	108.00	66.30	25.79
6	142.50	88.03	31.22
7	185.70	115.30	37.40
8	239.80	149.40	44.44
9	307.10	191.90	52.45
10	391.00	244.80	61.58
11	495.20	310.70	71.97
12	624.40	392.40	83.81
13	784.70	493.60	97.30

Internal Real Rate of Return: 35.84961 p.a.
Net Present Value @ 7 percent: R 221.194 million.

APPENDIX Q: INCREASING POLICY ANALYSIS; 15% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	35	17	20
3	14	33	16	18
4	12	31	14	16
5	10	27	10	12
6	8	23	8	8
7	8	23	8	8
8	8	23	8	8
9	8	23	8	8
10	8	23	8	8
11	8	23	8	8
12	8	23	8	8
13	8	23	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	45.20	40.04	28.32	(23.11)
4	90.40	58.79	0.00	31.62
5	129.00	64.67	0.00	64.61
6	177.00	69.85	0.00	107.00
7	239.00	75.43	0.00	163.90
8	321.00	81.47	0.00	239.80
9	428.00	87.99	0.00	340.20
10	567.00	95.03	0.00	472.40
11	748.00	102.60	0.00	645.40
12	982.00	110.80	0.00	870.80
13	1280.00	119.70	0.00	1163.00

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	5.21	1.94	(19.11)
4	31.62	18.06	9.23
5	64.61	38.80	15.98
6	107.00	65.49	23.77
7	163.90	101.40	33.15
8	239.80	149.40	44.44
9	340.20	212.90	57.95
10	472.40	296.50	74.12
11	645.40	406.00	93.39
12	870.80	548.70	116.30
13	1163.00	734.00	143.60

Internal Real Rate of Return: 31.96289 p.a.
Net Present Value @ 7 percent: R 243.733 million

APPENDIX R: DECLINING POLICY ANALYSIS; 15% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	35	17	20
3	14	33	16	18
4	12	31	14	16
5	10	27	10	12
6	8	23	8	8
7	8	23	8	8
8	8	23	8	8
9	8	23	8	8
10	8	23	8	8
11	8	23	8	8
12	8	23	8	8
13	8	23	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	110.00	40.04	28.32	42.05
4	181.00	58.79	0.00	122.70
5	216.00	64.67	0.00	151.30
6	248.00	69.85	0.00	178.10
7	283.00	75.43	0.00	207.60
8	321.00	81.47	0.00	239.80
9	362.00	87.99	0.00	274.00
10	405.00	95.03	0.00	309.70
11	448.00	102.60	0.00	345.10
12	489.00	110.80	0.00	378.20
13	526.00	119.70	0.00	406.10

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	70.37	43.29	(00.94)
4	122.70	75.83	31.90
5	151.30	93.82	35.61
6	178.10	110.60	38.67
7	207.60	129.20	41.64
8	239.80	149.40	44.44
9	274.00	170.90	46.95
10	309.70	193.20	49.05
11	345.10	215.40	50.57
12	378.20	236.20	51.31
13	406.10	253.60	51.02

Internal Real Rate of Return: 41.13281 p.a.
Net Present Value @ 7 percent: R 198.695 million

APPENDIX 8: CONSTANT POLICY ANALYSIS; 20% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	40	17	20
3	14	38	16	18
4	12	36	14	16
5	10	32	10	12
6	8	28	8	8
7	8	28	8	8
8	8	28	8	8
9	8	28	8	8
10	8	28	8	8
11	8	28	8	8
12	8	28	8	8
13	8	28	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	83.70	40.04	28.32	15.38
4	152.00	58.79	0.00	93.05
5	200.00	64.67	0.00	135.80
6	257.00	69.85	0.00	186.70
7	328.00	75.43	0.00	253.00
8	420.00	81.47	0.00	338.90
9	538.00	87.99	0.00	450.10
10	689.00	95.03	0.00	593.70
11	882.00	102.60	0.00	778.90
12	1130.00	110.80	0.00	1018.00
13	1440.00	119.70	0.00	1325.00

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	43.70	26.36	(8.38)
4	93.05	57.04	24.52
5	135.80	83.94	32.08
6	186.70	116.10	40.48
7	253.00	157.90	50.44
8	338.90	212.20	62.24
9	450.10	282.60	76.22
10	593.70	373.40	92.80
11	778.90	490.70	112.40
12	1018.00	641.80	135.70
13	1325.00	836.40	163.30

Internal Real Rate of Return: 43.60352 p.a.
Net Present Value @ 7 percent: R 353.168 million.

APPENDIX T: INCREASING POLICY ANALYSIS; 20% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	30	17	20
3	14	38	16	18
4	12	36	14	16
5	10	32	10	12
6	8	28	8	8
7	8	28	8	8
8	8	28	8	8
9	8	28	8	8
10	8	28	8	8
11	8	28	8	8
12	8	28	8	8
13	8	28	8	8

Tax + Lease = 0.5685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	48.70	40.04	28.32	(19.67)
4	101.00	58.79	0.00	42.21
5	150.00	64.67	0.00	85.44
6	214.00	69.85	0.00	143.80
7	301.00	75.43	0.00	225.50
8	420.00	81.47	0.00	338.90
9	583.00	87.99	0.00	495.10
10	804.00	95.03	0.00	709.00
11	1100.00	102.60	0.00	1000.00
12	1510.00	110.80	0.00	1396.00
13	2050.00	119.70	0.00	1929.00

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	8.65	4.13	(18.15)
4	42.21	24.78	11.87
5	85.44	52.01	20.70
6	143.80	88.85	31.49
7	225.50	140.50	45.10
8	338.90	212.20	62.24
9	495.10	311.20	83.72
10	709.00	446.60	110.60
11	1000.00	631.20	144.00
12	1396.00	881.70	185.60
13	1929.00	1220.00	237.20

Internal Real Rate of Return: 39.24805 p.a.
Net Present Value @ 7 percent: R 339.049 million.

APPENDIX U: DECLINING POLICY ANALYSIS; 20% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	30	17	20
3	14	38	16	18
4	12	36	14	16
5	10	32	10	12
6	8	28	8	8
7	8	28	8	8
8	8	28	8	8
9	8	28	8	8
10	8	28	8	8
11	8	28	8	8
12	8	28	8	8
13	8	28	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	119.00	40.04	28.32	50.44
4	203.00	58.79	0.00	143.90
5	251.00	64.67	0.00	185.10
6	300.00	69.85	0.00	229.70
7	356.00	75.43	0.00	280.50
8	420.00	81.47	0.00	338.90
9	493.00	87.99	0.00	405.00
10	573.00	95.03	0.00	478.40
11	660.00	102.60	0.00	557.60
12	751.00	110.80	0.00	639.70
13	840.00	119.70	0.00	720.00

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	78.76	48.61	1.40)
4	143.90	89.32	37.19
5	186.10	115.90	43.48
6	229.70	143.40	49.49
7	280.50	175.40	55.78
8	338.90	212.20	62.24
9	405.00	254.00	68.73
10	473.40	300.30	75.05
11	557.60	350.30	80.87
12	639.70	402.10	85.83
13	720.00	452.80	89.39

Internal Real Rate of Return: 49.3652% p.a.
Net Present Value @ 7 percent: R 307.340 million

APPENDIX V: CONSTANT POLICY ANALYSIS; 2% METAL PRICE FALL

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	18	17	20
3	14	16	16	18
4	12	14	14	16
5	10	10	10	12
6	8	6	8	8
7	8	6	8	8
8	8	6	8	8
9	8	6	8	8
10	8	6	8	8
11	8	6	8	8
12	8	6	8	8
13	8	6	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	59.30	40.04	28.32	(9.03)
4	90.20	58.79	0.00	31.38
5	99.20	64.67	0.00	34.52
6	105.00	69.85	0.00	35.30
7	111.00	75.43	0.00	36.02
8	118.00	81.47	0.00	36.67
9	125.00	87.99	0.00	37.24
10	133.00	95.03	0.00	37.72
11	141.00	102.60	0.00	38.08
12	149.00	110.80	0.00	38.31
13	158.00	119.70	0.00	38.40

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	19.29	10.88	(15.19)
4	31.38	17.91	9.17
5	34.52	19.70	9.17
6	35.30	20.02	8.76
7	36.02	20.29	8.35
8	36.67	20.50	7.95
9	37.24	20.64	7.56
10	37.72	20.70	7.17
11	38.08	20.67	6.79
12	38.31	20.54	6.42
13	38.40	20.29	6.06

Internal Real Rate of Return: 2.575195 p.a.
Net Present Value @ 7 percent: -R 13.1117 million

APPENDIX W: INCREASING POLICY ANALYSIS; 2% METAL PRICE FALL

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	18	17	20
3	14	16	16	18
4	12	14	14	16
5	10	10	10	12
6	8	6	8	8
7	8	5	8	8
8	8	6	8	8
9	8	6	8	8
10	8	6	8	8
11	8	6	8	8
12	8	6	8	8
13	8	6	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	34.50	40.04	38.32	(33.86)
4	60.00	58.79	0.00	1.19
5	74.30	64.67	0.00	9.62
6	87.60	69.85	0.00	17.71
7	102.00	75.43	0.00	26.69
8	118.00	81.47	0.00	36.67
9	136.00	87.99	0.00	47.73
10	155.00	95.03	0.00	59.95
11	176.00	102.60	0.00	73.44
12	199.00	110.80	0.00	88.28
13	224.00	119.7[illegible]	0.00	104.60

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	0.00	0.00	(25.83)
4	0.00	0.00	0.81
5	5.26	0.91	5.34
6	17.71	8.86	5.07
7	26.69	14.37	7.95
8	36.67	20.50	7.95
9	47.73	27.29	9.30
10	59.95	34.18	10.60
11	73.44	43.10	11.83
12	88.28	52.24	13.02
13	104.60	62.28	14.15

Internal Real Rate of Return: 1.262695 p.a.
Net Present Value @ 7 percent: -R 23.1274 million.

APPENDIX X: INING POLICY ANALYSIS; 2% METAL PRICE FALL

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	18	17	20
3	14	16	16	18
4	12	14	14	16
5	10	10	10	12
6	8	6	8	8
7	8	6	8	8
8	8	6	8	8
9	8	6	8	8
10	8	6	8	8
11	8	6	8	8
12	8	6	8	8
13	8	6	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue
8

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	84.20	40.04	28.32	15.81
4	120.00	58.79	0.00	61.59
5	124.00	64.67	0.00	59.45
6	123.00	69.85	0.00	52.91
7	121.00	75.43	0.00	45.36
8	118.00	81.47	0.00	36.67
9	115.00	87.99	0.00	26.76
10	111.00	95.03	0.00	15.51
11	105.00	102.60	0.00	2.75
12	99.20	110.80	0.00	(11.63)
13	91.90	119.70	0.00	(27.78)

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	44.13	26.64	(8.26)
4	61.59	37.08	16.69
5	59.45	35.52	14.81
6	52.91	31.20	12.45
7	45.36	26.21	10.16
8	36.67	20.50	7.95
9	26.76	13.98	5.81
10	15.51	6.61	3.75
11	27.52	0.00	1.07
12	0.00	0.00	(4.20)
13	0.00	0.00	(9.29)

Internal Real Rate of Return: 4.394532 p.a.
Net Present Value @ 7 percent: -R 10.6028 million.

APPENDIX Y: CONSTANT POLICY ANALYSIS; 3% METAL PRICE RISE

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	17	17	20
3	14	15	16	18
4	12	13	14	16
5	10	9	10	12
6	8	5	8	8
7	8	5	8	8
8	8	5	8	8
9	8	5	8	8
10	8	5	8	8
11	8	5	8	8
12	8	5	8	8
13	8	5	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	58.30	40.04	28.32	(10.04)
4	87.90	58.79	0.00	29.07
5	95.80	64.67	0.00	31.10
6	101.00	69.85	0.00	30.71
7	106.00	75.43	0.00	30.15
8	111.00	81.47	0.00	29.40
9	116.00	87.99	0.00	28.42
10	122.00	95.03	0.00	27.21
11	128.00	102.60	0.00	25.72
12	135.00	110.80	0.00	23.92
13	141.00	119.70	0.00	21.79

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	18.28	10.23	(15.47)
4	29.07	16.45	8.60
5	31.10	17.53	8.40
6	30.71	17.11	7.80
7	30.15	16.57	7.21
8	29.40	15.88	6.64
9	28.42	15.04	6.09
10	27.21	14.03	5.55
11	25.72	12.83	5.03
12	23.92	11.41	4.52
13	21.79	9.76	4.03

Internal Real Rate of Return: -0.6210938 p.a.
Net Present Value @ 7 percent: -R 20.5874 million.

APPENDIX 2: INCREASING POLICY ANALYSIS; 3% METAL PRICE FALL

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	17	17	20
3	14	15	16	18
4	12	13	14	16
5	10	9	10	12
6	8	5	8	8
7	8	5	8	8
8	8	5	8	8
9	8	5	8	8
10	8	5	8	8
11	8	5	8	8
12	8	5	8	8
13	8	5	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	33.90	40.04	28.32	(34.45)
4	58.40	58.79	0.00	(34.92)
5	71.70	64.67	0.00	7.05
6	83.70	69.85	0.00	13.89
7	96.70	75.43	0.00	21.31
8	111.00	81.47	0.00	29.40
9	126.00	87.99	0.00	38.17
10	143.00	95.03	0.00	47.68
11	161.00	102.60	0.00	57.96
12	180.00	110.80	0.00	69.07
13	201.00	119.70	0.00	81.02

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	0.00	0.00	(26.28)
4	0.00	0.00	(23.78)
5	0.57	0.00	4.37
6	13.89	6.44	4.27
7	21.31	10.96	5.50
8	29.40	15.88	6.64
9	38.17	21.23	7.71
10	47.68	27.02	8.71
11	57.96	33.29	9.63
12	69.07	40.05	10.48
13	81.02	47.34	11.27

Internal Real Rate of Return: -1.519531 p.a.
Net Present Value @ 7 percent: -R 32.1249 million

APPENDIX AA: DECLINING POLICY ANALYSIS; 5% METAL PRICE FALL

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	17	17	20
3	14	15	16	18
4	12	13	14	16
5	10	9	10	12
6	8	5	8	8
7	8	5	8	8
8	8	5	8	8
9	8	5	8	8
10	8	5	8	8
11	8	5	8	8
12	8	5	8	8
13	8	5	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	82.70	40.04	28.32	14.38
4	117.00	58.79	0.00	58.51
5	120.00	64.67	0.00	55.16
6	117.00	69.85	0.00	47.56
7	114.00	75.43	0.00	39.00
8	111.00	81.47	0.00	29.40
9	107.00	87.99	0.00	18.68
10	102.00	95.03	0.00	6.75
11	96.10	102.60	0.00	(6.51)
12	89.60	110.80	0.00	(21.20)
13	82.30	119.70	0.00	(37.44)

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	42.70	25.73	(429.40)
4	58.51	35.12	(141.00)
5	55.16	32.80	3.38
6	47.56	27.80	20.62
7	39.00	22.18	19.45
8	29.40	15.88	18.32
9	18.68	8.86	17.23
10	6.75	1.05	16.12
11	0.00	0.00	12.18
12	0.00	0.00	7.95
13	0.00	0.00	1.87

Internal Real Rate of Return: < -4.00 p.a.
Net Present Value @ 7 percent: -R 20.490010 million

APPENDIX A [illegible]NSTANT POLICY ANALYSIS; 5% METAL PRICE FALL

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	15	17	20
3	14	13	16	18
4	12	11	14	16
5	10	7	10	12
6	8	3	8	8
7	8	3	8	8
8	8	3	8	8
9	8	3	8	8
10	8	3	8	8
11	8	3	8	8
12	8	3	8	8
13	8	3	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	56.32	40.04	28.32	(12.04)
4	83.40	58.79	0.00	24.57
5	89.20	64.67	0.00	24.52
6	91.90	69.85	0.00	22.02
7	94.60	75.43	0.00	19.19
8	97.50	81.47	0.00	15.99
9	100.00	87.99	0.00	12.40
10	103.00	95.03	0.00	8.37
11	107.00	102.60	0.00	3.87
12	110.00	110.80	0.00	(1.41)
13	113.00	119.70	0.00	(6.72)

Year	Taxable Profit	Tax + Lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	16.28	8.97	(16.02)
4	24.57	13.59	7.48
5	24.52	13.36	6.91
6	22.02	11.60	5.98
7	19.19	9.61	5.09
8	15.99	7.38	4.24
9	12.40	4.88	3.43
10	8.37	2.08	2.65
11	3.87	0.00	1.51
12	0.00	0.00	(0.41)
13	0.00	0.00	(2.25)

Internal Real Rate of Return: < -4.00 p.a.
Net Present Value @ 7 percent: -R 36.2869 million.

APPENDIX AC: INCREASING POLICY ANALYSIS; 5% METAL PRICE FALL

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	15	17	20
3	14	13	16	18
4	12	11	14	16
5	10	7	10	12
6	8	3	8	8
7	8	3	8	8
8	8	3	8	8
9	8	3	8	8
10	8	3	8	8
11	8	3	8	8
12	8	3	8	8
13	8	3	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	32.[illegible]0	40.04	28.32	(35.61)
4	55.[illegible]0	58.79	0.00	(33.46)
5	66.80	64.67	0.00	2.13
6	76.50	69.85	0.00	6.65
7	86.70	75.43	0.00	11.27
8	97.50	81.47	0.00	15.99
9	109.00	87.99	0.00	20.81
10	121.00	95.03	0.00	25.69
11	133.00	102.60	0.00	30.63
12	146.00	110.80	0.00	35.61
13	160.00	119.70	0.00	40.58

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	0.00	0.00	(27.16)
4	0.00	0.00	(22.79)
5	0.00	0.00	1.32
6	0.00	0.00	3.81
7	9.41	3.34	4.21
8	15.99	7.38	4.24
9	20.81	10.21	4.82
10	25.69	13.07	5.32
11	30.63	15.95	5.73
12	35.61	18.82	6.06
13	40.58	21.67	6.32

Internal Real Rate of Return: < - 4.00 p.a.
Net Present Value @ 7 percent: - R 48.2470 million.

APPENDIX AD: DECLINING POLICY ANALYSIS; 5% METAL PRICE FALL

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	15	17	20
3	14	13	16	18
4	12	11	14	16
5	10	7	10	12
6	8	3	8	8
7	8	3	8	8
8	8	3	8	8
9	8	3	8	8
10	8	3	8	8
11	8	3	8	8
12	8	3	8	8
13	8	3	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	79.90	40.04	28.32	11.55
4	111.00	58.79	0.00	52.49
5	112.00	64.67	0.00	46.93
6	107.00	69.85	0.00	37.41
7	103.00	75.43	0.00	27.12
8	97.50	81.47	0.00	15.99
9	92.00	87.99	0.00	3.99
10	86.10	95.03	0.00	(8.93)
11	79.80	102.60	0.00	(2.29)
12	73.00	110.80	0.00	(3.79)
13	65.70	119.70	0.00	(5.40)

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	39.87	23.93	(9.45)
4	52.49	31.31	14.43
5	46.93	27.58	11.98
6	37.41	21.36	9.20
7	27.12	14.64	6.62
8	15.99	7.38	4.24
9	3.99	0.00	1.82
10	0.00	0.00	(3.76)
11	0.00	0.00	(8.92)
12	0.00	0.00	(13.68)
13	0.00	0.00	(18.07)

Internal Real Rate of Return: < - 4.00 p.a.
Net Present Value @ 7 percent: - R 40.4415 million

APPENDIX AE: CONSTANT POLICY ANALYSIS; 6% METAL PRICE FALL

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	14	17	20
3	14	12	16	18
4	12	10	14	16
5	10	6	10	12
6	8	2	8	8
7	8	2	8	8
8	8	2	8	8
9	8	2	8	8
10	8	2	8	8
11	8	2	8	8
12	8	2	8	8
13	8	2	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	55.30	40.04	28.32	(13.02)
4	81.20	58.79	0.00	22.37
5	86.00	64.67	0.00	21.36
6	87.80	69.85	0.00	17.91
7	89.50	75.43	0.00	14.08
8	91.30	81.47	0.00	9.83
9	93.10	87.99	0.00	5.14
10	95. 0	95.03	0.00	(0.04)
11	96.90	102.60	0.00	(5.74)
12	98.80	110.80	0.00	(12.01)
13	101.00	119.70	0.00	(18.90)

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	15.30	8.35	(16.30)
4	22.37	12.19	6.93
5	21.36	11.35	6.20
6	17.91	8.99	5.11
7	14.08	6.37	4.09
8	9.83	3.47	3.13
9	5.14	0.27	2.22
10	0.00	0.00	(0.02)
11	0.00	0.00	(2.24)
12	0.00	0.00	(4.34)
13	0.00	0.00	(6.32)

Internal Real Rate of Return: < - 4.00 p.a.
Net Present Value @ 7 percent: - R 46.5039 million

APPENDIX AF: INCREASING POLICY ANALYSIS; 6% METAL PRICE FALL

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	14	17	20
3	14	12	16	18
4	12	10	14	16
5	10	5	10	12
6	8	2	8	8
7	8	2	8	8
8	8	2	8	8
9	8	2	8	8
10	8	2	8	8
11	8	2	8	8
12	8	2	8	8
13	8	2	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	32.20	40.04	28.32	(36.18)
4	54.00	58.79	0.00	(4.81)
5	64.4[illegible]	64.67	0.00	(0.24)
6	73.1	69.85	0.00	3.22
7	82.00	75.43	0.00	6.58
8	91.30	81.47	0.00	9.83
9	101.00	87.99	0.00	12.94
10	111.00	95.03	0.00	15.87
11	121.00	102.60	0.00	18.60
12	132.00	110.80	0.00	21.10
13	143.00	119.70	0.00	23.29

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	0.00	0.00	(27.60)
4	0.00	0.00	(3.27)
5	0.00	0.00	(0.15)
6	0.00	0.00	1.85
7	0.00	0.00	3.49
8	6.73	1.39	4.15
9	12.94	5.22	3.51
10	15.87	6.84	3.81
11	18.60	8.31	4.02
12	21.10	9.32	4.15
13	23.29	10.71	4.21

Internal Real Rate of Return: < - 4.00 p.a.
Net Present Value @ 7 percent: - R 56.2345 million.

APPENDIX AG: DECLINING POLICY ANALYSIS; 6% METAL PRICE FALL
Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	14	17	20
3	14	12	16	18
4	12	10	14	16
5	10	6	10	12
6	8	2	8	8
7	8	2	8	8
8	8	2	8	8
9	8	2	8	8
10	8	2	8	8
11	8	2	8	8
12	8	2	8	8
13	8	2	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	78.50	40.04	28.32	10.15
4	108.00	58.79	0.00	49.56
5	108.00	64.67	0.00	42.98
6	102.00	69.85	0.00	32.61
7	97.00	75.43	0.00	21.57
8	91.30	81.47	0.00	9.83
9	85.30	87.99	0.00	(2.66)
10	79.10	95.03	0.00	(15.93)
11	72.60	102.60	0.00	(30.07)
12	65.70	110.80	0.00	(45.10)
13	58.60	119.70	0.00	(61.10)

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	38.47	23.05	(9.84)
4	49.56	29.45	13.70
5	42.98	25.07	11.09
6	32.61	18.31	8.19
7	21.57	11.12	5.55
8	9.83	3.47	3.13
9	0.00	0.00	(1.21)
10	0.00	0.00	(6.71)
11	0.00	0.00	(11.73)
12	0.00	0.00	(16.29)
13	0.00	0.00	(20.44)

Internal Real Rate of Return: < - 4.00 p.a.
Net Present Value @ 7 percent: - R 50.5631 million.

APPENDIX AH: DECLINING POLICY IMPLEMENTATION, MINE LIFE - 8 YEARS, SAME COSTS.

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	20	17	20
3	14	18	16	18
4	12	16	14	16
5	10	12	10	12
6	8	8	8	8
7	8	8	8	8
8	8	8	8	8
9	8	8	8	8
10	8	8	8	8

Tax + Lease = 0.6686 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	30.00	(30.00)
2	0.00	0.00	24.00	(24.00)
3	67.10	40.04	28.32	(1.27)
4	104.00	58.79	0.00	44.96
5	116.00	64.67	0.00	51.53
6	126.00	69.85	0.00	55.66
7	136.00	75.43	0.00	60.11
8	146.00	81.47	0.00	64.92
9	158.00	87.99	0.00	70.11
10	171.00	95.03	0.00	75.72

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	27.05	15.80	(13.02)
4	44.96	26.53	12.55
5	51.53	30.50	13.02
6	55.66	32.94	13.02
7	60.11	35.57	13.02
8	6 .92	38.42	13.02
9	70.11	41.49	13.02
10	75.72	44.81	13.02

Internal Real Rate of Return: 7.015625 p.a.
Net Present Value @ 7 percent: R 0.427017 million

APPENDIX AI: DECLINING POLICY IMPLEMENTATION, MINE LIFE - 7 YEARS, MODIFIED COSTS.

Period of capital redemption: 1 year
Capital Allowance % rates for tax & lease: 0 & 0

Inflation and Escalation rates over previous year: % p.a.

Year	Inflation	Revenue	Working Costs	Capital Expend.
1	0	0	0	0
2	15	20	17	20
3	14	18	16	18
4	12	16	14	16
5	10	12	10	12
6	8	8	8	8
7	8	8	8	8
8	8	8	8	8
9	8	8	8	8

Tax + Lease = 0.6685 x profit - 0.3401 x revenue

Year	Revenue	Working Costs	Capital Expend.	Net Profit
1	0.00	0.00	25.00	(25.00)
2	0.00	0.00	21.60	(21.60)
3	71.20	37.32	25.49	8.41
4	109.00	55.70	0.00	53.36
5	121.00	61.27	0.00	59.64
6	129.00	66.17	0.00	63.05
7	138.00	71.46	0.00	66.97
8	148.00	77.18	0.00	70.76
9	158.00	83.36	0.00	74.74

Year	Taxable Profit	Tax + lease	Net cash flow real
1	0.00	0.00	(30.00)
2	0.00	0.00	(20.87)
3	33.90	20.24	(9.02)
4	53.36	31.96	14.57
5	59.64	35.76	14.79
6	63.05	37.76	14.50
7	66.97	40.06	14.28
8	70.76	42.27	14.00
9	74.74	44.59	13.72

Internal Real Rate of Return: 11.04688 p.a.
Net Present Value @ 7 percent: R 8.69725 million

REFERENCES

1. **Berry, E.S.** Present value in its relation to ore reserves, plant capacity, and grade of ore. Min. Metall., July 1922, p11-16.

2. **Henning, U.** Calculation of cut-off grade, Can. Min. J., vol. 84, March 1963, 54-7.

3. **Lane, K.F.** Choosing the optimum cut-off grade, Colo. Sch. of Mines Quarterly .., 59, 1964, p811-29.

4. **Hotelling, H.** Economics of exhaustible resource, Journal of Political Economy, vol. 39, no. 2, 1931, p137-175.

5. **Fischer, A.C.** Resources and environmental economics, Cambridge Surveys of Economic Literatures, 1981, p10-126.

6. **Hotelling, H.** op. cit., p139.

7. **Lipsey, R.G and Steiner, P.O.** Economics, Harper and Row Inc., New York, 1969, p397.

8. **Fischer, A.C.** op. cit., p13.

9. **Herfindahl, O.C.** Depletion and economic theory, in: M. M. Gaffney, ed. Extractive Resources and Taxation (University of Wisconsin Press, Madison, Wisconsin), 1967, pp63-9.

10. **Kemp, M.C. and Long, N.V.** Exhaustion Resources, Optimality and Trade, North-Holland Publishing Company, New York, 1980, p35.

11. **Taylor, H.K.** General background theory of cut-off grades, Trans. Inst. Min. Metall., Section A vol. 81, July 1972, p160-179

12. Ibid., p172.

13. **Dowd, P.** Application of dynamic and stochastic programming to optimise cutoff grades and production rates, Trans. Inst. Min. Metall., section A, vol. 85, Jan. 1976 p22-31.

14. **Pasieka, A.R. and Sotirow, G.V.** Planning and operational cutoff grades based on computerized net present value and net cash flow, CIM Bull., vol 78, June 1985, p47-53.

15. **Berry, E.S.,** op. cit., p11-16.

16. **Carlisle, D.** The economics of a fund resource with particular reference to mining, American Econ. Rev., vol. 44, Sept. 1954, p595-616.

17. **Mortimer, G.J.** Grade control, Trans. Inst. Min. Metall., vol. 59, Feb. 1950, p357-399.

18. **Krige, D.G.** Effective pay limits for selective mining, J. S. Afr. Inst. Min. Metall., vol. 62, Jan, 1962, p345-363.

19. **Henning, V.,** op. cit., p54-7.

20. **Lane, K.F.,** op. cit., p811-29.

21. **Wells, H.M.** Optimising of mining engineering design in mineral evaluation, Min. Eng., Dec. 1978, p1676-1684.

22. **Border, S.N.** Optimisation of cut-off grades during design of underground mines, Aus. Inst. Min. Metall, no. 5, Sept. 1991, p14-20.

23. **Gomez-Hernadez, J.J. and Sirvastava, R.M.** Sequential Indicator Simulation, ISIM3D Program, Computers in Geoscience, vol. 16, 1990, p395-440.

24. **Clark, I.** Regression Revisited, Mathematical Geology, vol. 15, No. 4, 1983, p517-536.

25. **Lane, K.F.** The Economic Definition of Ore - cut-off grades in theory and practice, Mining Journal Books Limited, London. 1988, 149pp.

26. **Well, H.M.,** op. cit. p1681.

27. **Taylor, H.K.** Cut-off grades - some further reflections, Trans. Inst. Min. Metall., sect A, vol. 94, October 1985, p.A204-216.

28. **Berry, E.S.**, op. cit., p11-16.

29. **Douglass, E.J.** How to make the most of a mining investment. Mining Engineering, vol.23, Oct. 1971, p64-7.

30. **Halls, J.L., et al** Determination of optimum ore reserves and plant size by incremental financial analysis. Trans. Inst. Min. Metall., sect A vol. 78, 1969, p.A20-26.

31. **Lane, K.F.**, 1964, op. cit. p811-29.

32. **Taylor, H.K.** 1972, op. cit. p160-179.

33. **Thomas, E.G.** Justification of the concept of high-grading metalliferous orebodies, Mining Magazine. May 1976, vol. 134 No. 5 p393-397.

34. **Wells, H.M.**, op. cit. p1682.

35. **Border, S.N.**, op. cit. p14.

36. **Lane, K.F.**, 1988, op. cit. p32.

37. **Nordhaus, W.D.** Resources as a Constraint to growth, American Economic Review., vol. 64, 1974, p22-6.

38. **Coulson, M** Gold in the Nineties, International Gold Mining Newsletter, vol. 18, no 7, July 1991, pp 107-109.

39. Ibid., p109.

40. **Bird, D.** Studying the Detail, International Gold Mining Newsletter, vol. 19, no 6, June 1992, pp 107-109.

41. **Coulson, M**, op. cit. p109.

42. **Border, S.N.** op. cit. p14.

43. Ibid., p18.

44. Ibid., p17.

45. Ibid., p19.

46. Lane, K.F. 1988, op. cit. p59.

47. Border, S.N. op. cit. p14.