Risk-optimisation strategies and co-movement between the BRICS countries
Master of Commerce (50% Research) in Finance

Taariq Davids **[1388262]**

Supervisor: Mr. D. Mcclelland

School of Economics and Finance

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Abstract

This study aimed to implement various risk-optimisation portfolio strategies to determine if there are greater risk-adjusted returns than a market-capitalisation weighted portfolio from 1999 until 2019 at both the domestic level (South Africa) and international level (BRICS) using a 1-year estimation and holding period.

To implement the international portfolio, all stocks listed on the BRICS countries' stock exchange are considered and will be combined to create an investable universe of stocks. However, this investible universe of stocks will first be reduced yearly to the stocks that make up the top 90% of each stock market and then combined to create a single investible universe. Resulting in two investable universes that are the BRICS markets combined and the South African market that includes the same market-capitalisation filter. Each risk-optimisation strategy will be compared to the market-capitalisation weighted portfolio.

Furthermore, the study looks at the co-movement between the BRICS markets to determine if there is a change in financial integration among the BRICS indices using the DCC-GARCH and Pearson's correlation to examine if the risk-optimisation portfolios at an international stage can produce alphas due to international diversification benefits.

Based on the results from a domestic and global level when comparing the risk optimisation strategies to the market-capitalisation weighted portfolio. The market-capitalisation weighted portfolio performed poorly on a risk-adjusted basis during the full sample period and the four sub-sample periods it also performed poorly in terms of the VaR and maximum drawdown values. In contrast to this, the minimum variance portfolio and the maximum diversification portfolio was able to perform well in most of the periods and during the periods in which it performed poorly, it was at least able to maintain one of the lowest standard deviations between the portfolios which the market-capitalisation portfolio was not able to do.

Based on the DCC-GARCH results and the Pearson correlation results between 2000 and 2010 the correlation between the BRICS countries increased but they have decreased slightly between 2011 and 2019. Indicating that there were still diversification benefits during this period.

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1. Introduction

The market-capitalisation weighted portfolio strategy is a passive investing strategy that is extremely popular due to it having limited transaction costs and low implementation costs (van Vliet, 2018). Passive investing strategies assumes that stock markets are efficient, however, the performance of the market capitalisation weighted portfolio has question if markets are efficient as alternative portfolio strategies were able to produce a greater return for the same level of risk or the same return for lower risk.

Hsu (2004) stated that when prices are more volatile than is expected by its fundamentals, the market-capitalisation weighted portfolio will not compensate one adequately for their risk resulting in an inefficient portfolio. This has increased desire to examine alternative strategies such as the various risk-optimisation strategies which include the global minimum variance portfolio, equal risk contribution portfolio, maximum diversification portfolio, maximum decorrelation portfolio, and inverse volatility portfolio (Choueifaty, 2006; Choueifaty & Coignard, 2008; Clarke, Da Silva & Thorley, 2006; Maillard, Roncalli & Teiletche, 2010; Clarke, Da Silva & Thorley, 2011). The market-capitalisation weighted portfolio will contain minimal turnover and transaction cost due to it automatically rebalancing as stock prices fluctuate. Whereas, the risk-optimisation portfolios often contain significantly more transaction costs.

Implementing risk-optimisation portfolios in the real world often results in the estimation risk outweighing the greater risk-adjusted returns, causing ex-ante efficient portfolios to become inefficient ex-post (Choueifaty & Coignard, 2008). This means that when an investor or practitioner is investigating alternative portfolios to the market-capitalisation weighted portfolio they will need to consider the potential decrease in performance resulting from the estimation error. Therefore, when comparing the performance of the market-capitalisation weighted portfolio and the risk-optimisation weighted portfolio it is imperative to consider the transaction cost and estimation error that will be present in these portfolios.

The risk-optimisation strategies will be tested at both a domestic level and an international level to examine if the increase in transaction cost and increase in the estimation errors that will occur with the implementation of a one year estimation period to prevent a look-ahead bias and have methodologies that can be implemented in the real world. The risk-adjusted performance of these risk-optimisation portfolios needs to be compared to that of the market-capitalisation weighted portfolio in the hope of obtaining portfolio strategies that can provide greater risk-adjusted returns than the market-capitalisation weighted portfolio.

At the international level, the rapid increase of globalisation among developed, emerging, and frontier economies of the world have brought attention to the subject of co-movement between stock markets around the globe since the early 2000s (Billio, Donadelli, Paradiso & Riedel, 2017). The existence of robust economic and trading links, the increase of liberalisation activities of country leaders, the expansion of international trade and finance, the drastic increase in telecommunication and trading systems, and the formation of trading are some of the contributing factors to the increase of financial integration (Billio et al., 2017; Boamah, Akotey & Aaawaar, 2020). An increase in financial integration induces strong positive cross-country stock return correlations which is a major disadvantage. This will result in a decrease in the benefits of international diversification (Billio et al., 2017). This might result in a persistent decrease in the risk-adjusted performance of internationally diversified portfolios.

Wilson and Purushothaman (2003) examined the growth projections of Brazil, Russia, India, and China. They stated that these BRICs economies could be a large force in the world economy. Based on the demographic projections and the Solow growth model, they estimated GDP growth, income per capita, and currency movements in these economies up to 2050. They found that if policy implementation is successful it is safe to assume that in less than 40 years these BRICs economies might be larger than the G6 in dollar terms.

In 2011 South Africa was included in the BRICs group which was seen as a major foreign policy achievement. South Africa was not included due to its market size or growth rate but rather South Africa demonstrated that its foreign policies aligned with the group. South Africa altered the nature of the group as it gave it a more global structure. This increased the legitimacy of BRICS to represent the emerging world. The inclusion of South Africa did not reduce the capacity of cooperation between BRICS but instead, it increased their ability to develop joint positions (Oliver, 2013).

With the continual increasing role that the BRICS countries have played in the world economy with its combined purchasing power equal to that of the G7 countries in 2015. The makes the BRICS countries a good proxy of measuring performance of emerging markets. Furthermore, BRICS accounted for 56% of the growth of the global gross national product during 2017-2018 and the expectations of it accounting for more than half of the global economic growth is expected to persist up to 2030. This contribution would be higher if the investment rates increase within the BRICS countries (Reddy, 2018).

The primary objective of the study is to investigate whether risk-optimisation strategies can be implemented on a domestic and an international scale such that the risk-adjusted returns outperform the market-capitalisation weighted portfolio. The unique contribution of this study will be implementing a variety of risk-optimisation strategies while implementing an estimation period and accounting for transaction costs at both a domestic and international scale.

The reason for implementing an estimation period as well as accounting for transaction costs is to provide a realistic representation of how these risk-optimisation strategies can be implemented in practice and if this realistic approach can outperform the market-capitalisation weighted portfolio at either a domestic or international level. This results in the following hypothesis:

H1: The risk-optimisation portfolios and market-capitalisation weighted portfolios for South Africa and BRICS produce insignificant performance differences on a risk-adjusted basis.

H1a: There is a significant performance difference between the risk-optimisation portfolio and the market-capitalisation weighted portfolio for South Africa and BRICS.

The secondary objective is to investigate and evaluate the cross-country correlations of the BRICS countries using the dynamic standard (Pearson's) correlation and the dynamic conditional correlation generalised autoregressive conditional heteroscedasticity (DCC-GARCH) model of Engle (2002), to determine if the international diversification benefits between the BRICS countries evolve. This could indirectly appear in the performance of the international diversified portfolio. If there are persistent decreases in the performance of the international portfolios while the performance of the domestic portfolio remains the same or increases over the same period. It will indicate a decrease in the benefits of international diversification.

The use of these two models was inspired by the findings of Billio et al. (2017) which found that most of the methodologies used in estimating co-movement provided the same result irrespective of their level of complexity. Based on that, this study will implement the simplest methodology (dynamic Pearson’s correlation) and a more complex heteroscedastic measure (DCC-GARCH). This will provide robustness to the results. For the Pearson correlation to be dynamic, it will be calculated at various time intervals which will make it vary between periods but be constant within a time interval. This results in the following hypothesis:

H2: There is no change in the correlation relationships between the BRICS indices over the entire period.

H2a: There is a change in the correlation relationships between the BRICS indices over the entire period.

The sample period will be from 1999 up to and including 2019. This will provide an analysis of the performance of the South African equity market and BRICS equity market over an extended period which includes both recessionary and expansion periods. This sample period will also provide an analysis of the trend of the co-movement between the BRICS equity markets to investigate whether there is an effect on the performance of an international diversified portfolio. However, it should be noted that the change in performance might be due to the decrease in the barriers to trade which would inherently increase the co-movement between stock markets.

When looking at the returns of South Africa post-transaction cost, the worst performing portfolio was the market-capitalisation weighted portfolio which had an annualised return of under 1% and an annualised standard deviation of 27.52%. Whereas, the best performing portfolio was the maximum diversification portfolio with an annualised return of greater than 5% and an annualised standard deviation of 24.54%. Looking at the performance (no transaction cost) of the JALSH index it had an annualised return of 4.80% and an annualised standard deviation of 26.56%.

These results are echoed in the performance of the BRICS investment universe. The market-capitalisation weighted portfolio had an annualised return of under 1% and an annualised standard deviation of 14.48% which was the worst-performing portfolio. The best performing portfolio was the low volatility portfolio which had an annualised return of 3.90% and an annualised standard deviation of 10.35%.

When examining the DCC-GARCH and Pearson's correlation matrix between the BRICS indices it was found that the indices are not highly correlated in which South Africa contained the highest correlation with the remaining BRICS countries and China been the least correlated. This means that there were international diversification benefits over this period and this could also be seen in that the equal-weighted BRICS index had a higher return and lower volatility than the BRICS indices individually. However, it should be noted that all of the currencies except for the Yuan depreciated significantly in comparison to the Dollar. Due to this depreciation, it significantly decreased the returns and increased the volatility of the indices and stocks over this period.

The study proceeds as follows. Section 2 looks at various aspects of prior literature relating to portfolio construction and the performance of those portfolios and then looking at the co-movement between international markets. Section 3 outlines the data and methodology that is implemented in the study. Section 4 presents the results of the various portfolios and the co-movement of the BRICS indices, and Section 5 concludes.

2. Literature review

This literature review will look at prior literature on the topics of co-movement, portfolio strategies, and the performance of various portfolio strategies.

2.1 Weighting strategies

2.1.1 Market-capitalisation weighted

Fama (1976) showed that in an environment that has unrestricted short-selling and in the presence of a risk-free asset, the market-capitalisation weighted (value-weighted) market index will be efficient in equilibrium when the weighted averages of all investors' expectations of stock returns and covariance are viewed as efficient. Haugen and Baker (1991) state that market-capitalisation stock indices are likely to be a suboptimal investment strategy when investors disagree on expected return and risk. There will be alternatives to the market-capitalisation portfolio that can deliver similar expected returns at lower volatility, or for the same level of volatility deliver higher expected returns.

Reinganum (1991) stated that a stock market-capitalisation is a vital variable in portfolio management. Over the long-term, small-cap stocks tend to outperform large-cap stocks, however, in the short-term there is a great deal of variability. This means that investors or managers that can dynamically manage their capitalisation exposures over the short-term will be able to reap significant gains in comparison to that of investors or managers that maintain a fixed allocation to small and large-cap stocks, or ignored the capitalisation effects altogether. Therefore, managers and investors should create their portfolio with greater flexibility in altering their capitalisation instead of only using the market capitalisation weighted methodology.

Reinganum (1999) explained that market capitalisation is one of the salient factors of a portfolio's returns. An investor or practitioner's ability to manage their exposure to market capitalisation will have a significant impact on the performance of the portfolio. He found that over the long-term smaller capitalisation stocks outperformed larger capitalisation stocks. However, in the short-term the relationship is much more variable. This means that investors who can dynamically manage their capitalisation exposer in the short-term can achieve significantly greater returns than investors who maintain a fixed allocation to small- and large capitalisation stocks.

Treynor (2005) provided a different approach to index fund investing in which he defined market valuation indifferent, as an index built on any weighting mechanism that can avoid the problem with market-capitalisation. The reason for this is that stocks are priced incorrectly but the distribution between overpriced and underpriced stocks is symmetrical, as long as a weighting mechanism does not assign a large proportion to a single stock. This will result in the avoidance of over-investing in overpriced stocks. As such, by investing randomly in the market pricing errors, one can enjoy the benefits of an index fund without the need to know the true values to avoid the problem with market-capitalisation funds.

Hsu (2004) illustrated that under fairly simple assumptions on the inefficiency of prices, the market-capitalisation weighted portfolios are sub-optimal. When prices are more volatile than what is expected by its fundamentals, then the market-capitalisation portfolios do not compensate adequately for their risk. Thus, sub-optimality occurs because market-capitalisation portfolios tend to underweight stocks whose prices are relatively low to their fundamentals whilst overweighting stocks whose prices are relatively high to their fundamentals. As the noise in the volatility of prices increases, the market-capitalisation underperformance increases in magnitude.

In addition, Hsu (2004) stated four advantages of the market-capitalisation portfolios with the first being that market-capitalisation portfolios are a passive strategy that requires little to no active management resulting in little to no active management fees. Secondly, there is no rebalancing cost associated with the market-capitalisation portfolios as they automatically rebalance as stock prices fluctuate. The only rebalancing costs will be removing or adding a constituent stock in the portfolio. Thirdly, market-capitalisation portfolios assign greater weights to the larger companies, which is highly correlated with liquidity. Since market-capitalisation portfolios ensure that the portfolio mainly consists of highly liquid stocks, it will reduce the expected transaction costs of the portfolio. Lastly, under the capital asset pricing model, a broad-based market-capitalisation portfolio is a mean-variance portfolio as it is the market portfolio.

Bhattacharya and Galpin (2011) investigated the popularity of the market-capitalisation portfolio between 1995 and 2007 from 46 countries which were made up of 23 developed and 23 emerging markets. South Africa, India, Brazil, and China were part of the 23 emerging markets. It was found that the market-capitalisation method is less popular in emerging markets than in developed markets, but the popularity is increasing in both developed and emerging markets. They used US data to explore the reasons why this is occurring. They concluded that the reasons are that large, old, and greater analyst coverage stocks are more likely to be market-capitalisation as stock picking is less likely to occur. Further, they found that institutional shareholder or mutual fund shareholders that use the S&P 500 or the Russell 2000 as market-capitalisation performance benchmarks, tend to use value-weighting as well.

Bradfield and Munro (2017) stated that due to the concentrated portfolios that are typical in South Africa it will require a greater number of stocks to diversify one's portfolio. As such, they compared the market-capitalisation weighted method to that of the equal-weighted method in the construction of portfolios. They used weekly data from 167 stocks from 2002 to 2014, where they divided this period into several three-year sub-periods to establish consistency in various market conditions. They found that for the market-capitalisation portfolio an investor will require is at least 33 stocks to obtain a 90% risk reduction and as many as 60 stocks to obtain a 95% risk reduction. Whereas, for the equal-weighted method an investor required 15 and 29 stocks to gain a 90% and 95% risk reduction respectively.

2.1.2 Equally Weighted

Windcliff and Boyle (2004) used a five-asset universe to illustrate their idea that if the return distribution parameters are assumed to be known, equal-weighting will lead to a portfolio below the efficient frontier. The assets they looked at were large cap, medium cap, small-cap, world index, and long bond index. Assuming that they have the true population parameters for the expected return, standard deviation, and correlation, the equal-weighted portfolio fell below the efficient frontier making it a suboptimal portfolio. However, they showed that equally weighted portfolios are optimal when the assets are uncorrelated and indistinguishable.

In reality, investors do not know the true future return distributions of assets because these are estimated using historical data with the assumption that the parameters are stationary. As such, when the impact of estimation risk is taken into account the mean-variance optimisation no longer dominates the equally-weighted portfolios. These results are in agreement with the naïve investment strategy which states that equally-weighted portfolios are difficult to improve in when there is an estimation period (Windcliff & Boyle, 2004).

DeMiguel, Garlappi and Uppal (2009) compared the performance of the equal-weighted portfolio to that of 14 models of optimal asset allocation. They found that the out-of-sample Sharpe ratio, of the sample-based equal-weighted portfolio, was higher than the mean-variance strategies. This indicated the estimation errors of means and covariance eroded all the gains from the optimal strategy relative to the equal-weighted strategy. Even the models that were proposed to deal with the estimation errors were not able to outperform the equal-weighted portfolio consistently.

Plyakha, Uppal and Volkov (2014) showed that an equal-weighted portfolio with monthly rebalancing outperforms the market-capitalisation portfolio in terms of average returns, Sharpe ratio, and the four-factor alpha model. The reason for this outperformance is due to an equal-weighted portfolio having higher exposure to systematic risk factors. However, 42% of the total outperformance is from the difference in alphas from rebalancing to maintain constant weights in the equal-weighted portfolio. This outperformance of the equal-weighted portfolios occurred despite it having higher volatility and kurtosis compared to the market-capitalisation portfolio.

Malladi and Fabozzi (2017) stated that the equal-weighted method is more robust than the market-capitalisation strategies as it does not rely on average expected returns. They demonstrated that using Monte-Carlo simulations and real-world data from 1926 to 2014 that the equal-weighted method did indeed outperform the market-capitalisation strategy. A significant portion of this outperformance is attributable to annual portfolio rebalancing. Even after factoring in the costs incurred due to higher portfolio turnover the equal-weighted strategy still outperformed the market-capitalisation strategy. They concluded that the reason for this outperformance was due to smaller-cap stocks outperforming the larger-cap stocks, given that smaller-cap stocks are inherently riskier. Therefore, it is natural to expect a higher return from these stocks in the long-term.

2.1.3 Low volatility weighted

Low volatility investing has started to attract considerable interest since the global financial crisis. Haugen and Baker (1991) documented that it is due to the performance advantage of low-beta or low-volatility stocks that has been outperforming the high-beta or high-volatility stocks, which is contrary to that the capital asset pricing model. The outperformance of the low volatility method is directly related to the low-volatility anomaly.

Hsu and Li (2013) state that market participants are viewing low volatility investing as a Smart-Beta strategy because a portion of the outperformance relative to the market-capitalisation benchmarks comes from the decoupling of weight and price. The risk-reducing mechanism tends to involve dropping large-cap, glamour stocks that are growth-orientated, which tends to drastically increase the tracking error. As such, low volatility investing will appeal to investors who are seeking to maximise their portfolio's Sharpe ratio, which reflects total risk and is less concerned with the information ratio that reflects risk relative to the benchmark. A low-volatility portfolio can be seen as an effective method to diversify risk factor exposures. Lowering the exposure to market risk factors allows greater allocation to value and volatility factors that can provide attractive risk premiums. They conclude that adding low volatility strategies can result in greater diversification and a greater efficient frontier.

Van Vliet (2018) stated that market-capitalisation passive investing is extremely popular due to it having limited transaction costs and low implementation costs. However, with passive buy-and-hold strategies, one assumes that the stock markets are efficient. In contrast to this, low-volatility investing is not a passive strategy, as it requires rebalancing because the risk of stocks does not remain constant over time. Based on this the aim of the study was to evaluate the relation between turnover and the reduction in risk. The results indicated that an efficient low-volatility portfolio strategy requires little trading, with low volatility being more liquid and cheaper to trade because the stocks are much larger on average. With the low volatility indices having annualised transactions between 5 and 11 bps and the market-capitalisation index only having 1 bps. As such, he concluded that in order to implement an efficient low volatility strategy the turnover per year should not exceed 30%.

2.1.4 Minimum Variance weighted

The minimum variance portfolio is located at the left-most tip of the efficient frontier. The minimum variance portfolio is unique in the sense that the asset weights are independent of the forecasted or expected returns of individual assets. This is the only portfolio always located on the efficient frontier that does not use expected returns, hence, it is designed to minimise risk for a given return.

Clarke et al. (2006) used covariance matrix methodologies that avoid pre-specified factor models such as the three-factor and four-factor model for covariance matrix estimation. The reason for this is that it should reduce data mining and provide more robust results. Firstly, they reported that large-scale optimisation procedures for volatility reduction provide sound results. Secondly, minimum variance portfolios that do not rely on any expected returns do show signs of adding rewards over the market-capitalisation portfolio. Lastly, the minimum variance portfolios tended to be exposed to both value and small-size beta.

Clarke et al. (2011) examined the structure of minimum variance portfolios with an emphasis on the analytic form and parameter values of individual stock weights. Traditionally the efficient frontier in a mean-variance optimiser is described mathematically on unconstrained portfolios but implemented in a constrained long-only numerical optimiser. They aimed to find an analytic solution for optimal stock security weights under the long-only constrained implementation. According to the analytic solution, optimal stock weights within a portfolio declines with the market beta, and only stocks with betas below a threshold value remained in the long-only solution.

However, the higher idiosyncratic risk decreases the optimal stock weight, but cannot in itself remove a stock out of the solution. The optimal stock weight equation showed that the volatility minimisation component in a general mean-variance objective function is enough to disqualify the majority of investable stocks and is not due to complex exposure constraints, transaction costs, or turnover considerations.

Scherer (2011) stated that investors are disappointed with the performance of the market-capitalisation benchmark yet cautious about the merits of active portfolio management, which has caused investors to turn to alternative weightings for indices. Minimum variance investing is one such method in which he found that the construction process for minimum variance implicitly selects risk-based pricing anomalies. Consequently, it tends to hold low residual risk and low beta stocks. He found that over 80% of the excess returns from minimum variance portfolios relative to the market-capitalisation portfolio can be attributed to the Fama and French three factors, low residual risk and low beta stocks. However, Yanushevsky and Yanushevsky (2015) contradict this finding by proving that low residual stocks and low beta stocks create a positive weight in the minimum variance portfolio and that the minimum variance portfolio is unlikely to select low residual stocks and low beta stocks.

2.1.5 Maximum Diversification weighted

Choueifaty (2006) proposed the diversification ratio to measure a portfolio's diversification, which is defined as the ratio of the portfolio's weighted average volatility to its overall volatility. The volatility of a long-only portfolio of stocks is less than or equal to the weighted sum of volatilities. Therefore, the diversification ratio is greater than or equal to one for a long-only portfolio, and for a single stock is equal to unity.

Choueifaty and Coignard (2008) stated the increasing popularity in the claims that the market-capitalisation indices are not efficient. As such, they investigated the theoretical and empirical properties in the construction of portfolios using the criterion of diversification which they called the maximum diversification portfolio. To achieve the maximum diversification ratio the diversification ratio is used, in which the portfolio that can maximise the ratio is defined as the maximum diversification portfolio. If all stocks in a portfolio have the same volatility, then the maximum diversification portfolio will be the same as the minimum variance portfolio. If the maximum diversification portfolio is continuously rebalanced one should get a significant portion of the benefits from diversification returns when compared to a buy-and-hold strategy of the market-capitalisation benchmark.

Similarly, Choueifaty and Coignard (2008) results confirmed the theoretical framework for diversification. They found that it is difficult to determine if the maximum diversification portfolio is ex-ante on the efficient frontier. However, the evidence indicated that ex-post the maximum diversification portfolio was more efficient than the market-capitalisation benchmark, equal weight portfolio, and the minimum variance portfolio. Therefore, they concluded that this new investment strategy favours diversification and avoids gambles based on the prediction of returns or confidence in the performance of the market-capitalisation benchmarks.

In the real world, estimation risk will outweigh the diversification benefits more often than not for mean-variance optimisation, causing ex-ante efficient portfolios to become inefficient ex-post. However, both the maximum diversification and minimum-variance strategies are concentrated in low-volatility stocks. Lohre, Neugebauer and Zimmer (2012) found that when benchmarked against the risk-based allocation schemes, the maximum diversified portfolio provides the best risk-adjusted performance and contains the highest degree of diversification among the alternatives.

Choueifaty, Froidure and Reynier (2013) proposed a set of rules that an unbiased, agnostic portfolio construction process should follow. These were called the portfolio invariance properties which were made up of three properties. Firstly, the duplication invariance states that an unbiased construction of the portfolio should produce the same portfolio, irrespective if a stock was duplicated. Secondly, the leverage invariance states that if a company decides to deleverage, all else equal, the allocated weights by the portfolio to the company's underlying business should not be affected due to cash exposure being dealt with separately. Thirdly, the positive linear combination invariance states that positively correlated stocks already in the universe should not affect the portfolio weights to the original assets, as they were accounted for in the original universe. They found that the maximum diversification portfolio adhered to these rules. Lastly, they found that when using the MSCI World index as a benchmark the maximum diversification stood out in comparison to the minimum variance, equal risk contribution, and equal weight in terms of relative performance and exposures to the factors of Fama and French.

Lohre, Opfer and Orszag (2014) stated the diversified strategy is a reasonable alternative for risk-based asset allocation. The framework allows it to be compared with strategies such as equal weight, minimum-variance, or equal risk contribution. The minimum variance is known for having concentrated risks, and the equal risk contribution being more balanced. However, when benchmarking the equal risk contribution portfolio against the maximum diversification portfolio one observes a degeneration in the diversification characteristics over time, causing the equal risk contribution portfolio to be a concentrated gamble in the current environment.

2.1.6 Equal Risk Contribution weighted

Maillard et al. (2010) stated that the equal risk contribution is a heuristic measure that falls in between the minimum variance and equal-weighted portfolios. The equal risk contribution portfolio aims to equalise the total contribution of risk. Risk contribution has become synonymous with risk budgeting which is the analysis of a portfolio in terms of contribution to risk rather than in terms of weight to a portfolio.

Maillard et al. (2010) investigated the out-of-sample risk-adjusted properties of the equal risk contribution portfolio. They found that the equal risk contribution portfolio mimics the diversification effects of the equal-weighted portfolios while taking into account both single and joint volatility contributions of the stocks. This means that no stock contributed more volatility to the overall portfolio than its peers. The minimum variance portfolio is similar to the equal risk contribution portfolio in that it also equalises the contribution to risk, but only on a marginal basis. This means that for a minimum variance portfolio, an increase in any stock will lead to the same increase in the total volatility of a portfolio on an ex-ante basis.

Optimising the trade-off between risk and reward causes significant computational and modelling challenges for institutional investors. This is due to small errors in the estimated returns for stocks that can lead to optimised portfolios to incur a much higher risk for the returns they receive. Attributable to this adverse effect, portfolio construction techniques based solely on risk have become popular, for example, the equal risk contribution portfolio which is fully diversified from a risk perspective (Mausser & Romanko, 2014).

2.1.7 Maximum Decorrelation weighted

The maximum decorrelation strategy aims to minimise the volatility of a portfolio based on the assumption that volatility is identical across all stocks. Hence, this strategy aimed at exploiting the differences in the stock correlations instead of exploiting differences in the volatility of the stock. Maximum decorrelation optimisation is a method in exploiting the low correlations across stocks in the reduction of portfolio volatility rather than concentrating on low volatility stocks, which is a limitation of the minimum variance approach. Therefore, The maximum decorrelation weighting method aims at reducing the volatility of a portfolio by exploiting the interaction effects between stocks (Christoffersen, Errunza, Jacobs & Xisong. 2010).

The maximum decorrelation strategy is an additional alternative to using the constraints within the framework of volatility minimisation. The main idea is to exploit the risk reduction originating from low correlations rather than by the concentration of low volatility (Al , Goltz & Lodh, 2012). The maximum decorrelation portfolio only uses information provided by the correlation matrix. This approach does not imply any explicit control on the risk of the total volatility of the portfolio that results from the volatility of each stock. However, the risk reduction provided from the minimum variance strategy can be attained by the maximum decorrelation strategy, while avoiding the concentration by ignoring the individual volatility differences (Kremer, Talmaciu & Paterlini, 2018)

Maximum decorrelation is closely related to the maximum diversification and minimum variance strategies, but the maximum decorrelation strategy applies to the case when an investor has confidence in that all stocks will have similar volatility and returns, but heterogeneous correlations, or that the correlation estimates are reliable, however, the volatility or return estimates are not. Therefore, the maximum decorrelation strategy will be optimised using the correlation matrix instead of the covariance matrix that the minimum variance strategy uses when optimizing a portfolio. The portfolio weights of the maximum decorrelation will maximise the diversification ratio when all stocks have the same volatility and maximise the Sharpe ratio when all stocks have equal volatility and returns (Butler, 2018).

2.1.8 Inverse Volatility weighted

Maillard et al. (2010) showed that when assuming uniform cross-correlations between each stock, the equal-risk contribution portfolio weights are proportional to the individual stocks' inverse volatilities. This means, that if investors trust that the volatility estimates, assume constant correlations that disregard any estimation errors of the correlation matrix, an alternative method to obtain risk parity is to weigh stocks by their inverse volatility.

The equal-risk budget strategy is also known as the inverse volatility strategy. The equal-risk budget strategy invests in a method in which each stock in a portfolio will have the same risk budget. This is defined as the product of a stock's volatility and its weight. Since volatility is equally distributed among the stocks. This is an extension of the equal-weight portfolio if the volatility forecasts are accurate. While this method is different from the equal-risk contribution method as it does not consider the correlation between each stock. Lastly, the equal-risk budget portfolio is tilted away from high-risk stocks and in favour of low-risk, low-beta stocks (De Carvalho, Lu & Moulin, 2012).

A method that demonstrates this is the heuristic inverse volatility method, which assigns greater weight to stocks with lower volatility. Chow, Hsu, Kuo, and Li (2014) state that low-volatility portfolios will earn a duration premium due to their greater stability in their cash flows, which makes them share bond-like characteristics. In contrast to the market-capitalisation strategy, which has market beta near unity, the low volatility portfolio will have a lower exposure market risk factor and will have a greater diversified allocation to different sources of returns such as value, duration, market, and volatility.

Chow et al. (2014) found that the low volatility portfolio provided greater returns at lower volatility than the traditional market-capitalisation method. This anomaly was persistent over time and in different countries. They found that the inverse volatility method was able to deliver higher returns than the minimum variance, but the minimum variance tended to have lower volatility. The drawdowns of the low volatility investing are the underperformance in upward-trending market environments in the short-term and the significant tracking error against the market-capitalisation benchmarks, which is undesirable for some investors. The heuristic low volatility method also tended to have limited investment capacity, lower liquidity, and higher turnover. The high turnover may make this strategy inappropriate for larger institutional investors. Lastly, the heuristic low volatility tends to have a high concentration in countries or sectors, which could result in small allocations to BRICS countries for US investors. Therefore, these outcomes could create asset allocation challenges.

Butler (2018) stated that when portfolios have similar Sharpe ratios, and an investor is unable to reliably estimate the correlations or they share homogenous correlations, the optimal portfolio which is mean-variance efficient would be weighted proportionally to the inverse of the stocks volatilities. The equal-risk budget portfolio will be consistent with the maximum diversification portfolio, and approximately the minimum variance portfolio if all stocks' pairwise correlations are consistent.

2.2 Summary of prior literature related to the performance of portfolio strategies

DeMiguel et al. (2009) evaluated the out-of-sample performance of the sample-based mean-variance strategy and 13 other extensions of the model that were designed to reduce the out-of-sample estimation error, relative to the simple equal-weighted portfolio. They found that across various sample periods none of the 14 mean-variance portfolios were able to consistently outperform the equal-weighted portfolio in terms of certainty-equivalent return, turnover, or the Sharpe ratio. This indicated that in an out-of-sample scenario, the offset due to estimation errors more than offsets any gains that were obtained through optimal diversification strategies.

Amenc et al. (2012) compared the performance of risk-optimisation strategies such as efficient maximum Sharpe ratio, minimum volatility with long-only constraints, maximum decorrelation, fundamentally weighed, and equal-weighted portfolios which they referred to as diversification based weighting strategies. They compared the diversification-based strategies to a variety of benchmarks that used stock selection strategies such as high and low volatility, market cap, and dividend yield.

This provided a wide range of construction possibilities for assessments of commercial indices that implement similar strategies. The sample included all stocks listed on the S&P 500 from 1959 to 2010. Firstly, their results indicated focusing solely on stock selection allowed an investor for improving a prior objective at the expense of having a higher concentration within one's portfolio. For example, selecting low volatility stocks and then equal weighting them to achieve similar volatility to the minimum variance benchmark will lead the investor to own a drastically higher concentration.

Secondly, Amenc et al. (2012) found that diversification-based weighting strategies were more effective in attaining a specific diversification objective than using only a stock selection strategy. The results further showed that investors who maintain their desire for stock selection, ignoring the diversification benefits can cause one to incur higher opportunity costs because the diversification-based strategies consistently allowed improvement of the risk-adjusted returns. The results recommended that stock selection can be used as a tool in the correction of risk factor exposures that are undesirable by excluding these stocks before the implementation of diversification-based weighting strategies. Lastly, they reported that the risk-optimisation benchmark approach found that different strategies have different objectives. As such, the construction of these strategies can be compared to that of commercial index strategies that implement a similar risk-optimisation approach. These strategies must also answer the question of being able to outperform the market-capitalisation index in which they found that each of the risk-optimisation strategies reported a higher Sharpe ratio for each of the stock selection strategies.

De Carvalho et al. (2012) investigated equal weight, inverse volatility, equal risk contribution, minimum variance, and maximum diversification in comparison to the market-capitalisation benchmark. They look at the global universe of stocks from the developed market, and also the US, Europe, and Japan in isolation from 1997 to 2010 with the use of quarterly rebalancing. They found that all five-risk-based strategies outperformed the market-capitalisation portfolio.

De Carvalho et al. (2012) found that equal weight, inverse volatility, and equal risk contribution are similar to each other. These three strategies invest in all stocks from the investment universe and produced a similar turnover. The equal-weight strategy had greater exposure to small-cap stocks than the inverse volatility strategy which in turn has greater exposure than equal risk contribution. Equal risk contribution is considered a defensive strategy due to their exposure to low beta stocks.

De Carvalho et al. (2012) reported that the minimum variance and maximum diversification are different from the other three as they tended to have a large short position. As such, they compared both the long-only constraint and the unconstraint version of each. The long-only portfolios of minimum variance and maximum diversification invest in a small number of stocks, with an average of 120 stocks from the investment universe. Their unconstrained versions, on the other hand, invests 60% long and 40% short. Both the long-only constraint and unconstrained versions of the minimum variance and maximum diversification strategies are exposed to low beta stocks and are considered defensive. The difference between the two strategies arises from minimum variance having a higher exposure to low residual volatility stocks. The minimum variance portfolio reaches similar levels of diversification in which the maximum diversification maximises. The diversification ratio from the minimum variance and maximum diversification portfolios were much larger than the equal weight, inverse volatility, and equal risk contribution.

Clarke, De Silva, and Thorley (2013) compared and contrasted risk-based portfolio strategies using long-only constraints. The risk-optimisation strategies they looked at were minimum variance, maximum diversification, and equal risk contribution from 1968 to 2012. The methodology they employed was a back-test using standard OLS risk estimates on a single-index model with no maximum position limit or any other portfolio constraints. The results of the single-factor solution on the 1000 largest US stocks confirmed the superiority of the ex-post minimum variance portfolio in the objective of minimizing volatility, which increases the Sharpe ratio. The equal risk contribution portfolios produced the highest ex-post Sharpe ratio.

There were four implications from Clarke et al. (2013). Firstly, objective function-based portfolios with long-only constraints exclude a large portion of the investable universe, with the proportion of exclusion becoming greater as the universe set increases. Secondly, all three risk-optimisation portfolios have stock weights that decrease with both higher systematic and idiosyncratic risk, with systematic risk been the dominant factor, especially for the equal risk contribution portfolio. Thirdly, long-only constraints on stock volatility parameters varied with the ratio of systematic volatility to idiosyncratic volatility over time. The risk-optimisation portfolios became more concentrated with lower idiosyncratic volatility and higher systematic volatility. However, the higher concentration does not equate to higher ex-ante or ex-post volatility. Lastly, stocks that have negative betas receive large weights in all three risk-optimisation strategies.

Choueifaty et al. (2013) compared the long-only portfolios for the maximum diversification, equal risk contribution, and minimum variance strategy to that of the MSCI world index by using the invariance properties and unbiased portfolio construction process. Based on the invariance results they found that the maximum diversification portfolio is the only method to meet all three criteria with equal risk contribution only meeting the leverage invariance rule, the minimum variance only meeting the duplication invariance rule, and the equal weight meeting none of them.

Choueifaty et al. (2013) reported the performance results of the portfolios found that all risk-optimisation portfolios outperformed the market-capitalisation index even at unrealistically high trading costs of 2% to account for the higher turnover. The equal risk contribution, minimum variance, and maximum diversification delivered significantly greater returns and lower volatility, whereas the equal weight provided significantly higher returns at the same volatility. The equal risk contribution portfolio functioned as a risk-weighted version of the equal weight as it provided similar returns at significantly lower volatility. Lastly, the market-capitalisation portfolio had the lowest diversification ratio due to its high concentration in large capitalisation stocks and risk factors. The equal-weight portfolio has a slightly greater diversification in comparison to the market-capitalisation, but less than the other portfolios that implement stock covariance information. The maximum diversification portfolio had both the highest diversification and Sharpe ratio. As such, they are the closest portfolio to being a tangency portfolio. The minimum variance portfolio did deliver in its objective by having the lowest ex-post volatility.

Amenc and Goltz (2013) stated that the majority of the risk-optimisation indices have a high probability of outperforming the market-capitalisation indices over the long-term due to high concentration levels present in the market-capitalisation index. However, the risk-optimisation strategies have exposures to other forms of risk in which the market-capitalisation index is not exposed, which can cause the underperformance of these risk-optimisation strategies relative to the market-capitalisation index. Therefore, their paper aimed to examine various kinds of risk that the minimum-variance, maximum Sharpe ratio, and maximum decorrelation strategies performed relative to the market-capitalisation index.

Amenc and Goltz (2013) firstly examined the performance based on the objective of the risk-optimisation strategy. They found that after controlling for stock selection through the specific risk factor, the risk-return objectives of the risk-optimisation strategies still provided significant improvements over the market-capitalisation portfolio. Next, they examined if the performance of the risk-optimisation strategies will be compromised by controlling liquidity risk by reducing the US investment universe to the top 50% of stocks. The results indicated the performance is insignificantly affected by the high-liquidity stock selection as the annualised returns and Sharpe ratios decreased insignificantly.

Therefore, this implies that by avoiding the liquidity problem by investing in the most liquid stocks, an investor can main the potential for greater performance relative to the market-capitalisation portfolio (Amenc & Goltz, 2013). Lastly, Amenc and Goltz (2013) for the minimum variance portfolio they examined the performance maximum and minimum sector exposure constrained optimisation. The results showed that the imposition of sector-neutrality constraints did not affect the performance relative to the market-capitalisation index as it was still able to obtain a significant risk reduction. However, the risk reduction was greater for the unconstraint minimum variance portfolio.

Watson and Seetharam (2014) aimed at quantifying the investment prospects of the BRICS emerging countries from a global investors' perspective. They wanted to identify whether the addition of BRICS equity to a developed market portfolio will increase the risk-adjusted returns of the portfolio. They focused on using the minimum variance strategy for the BRICS portfolio from 1995 to 2013 using monthly data for the MSCI country indices which accounted for approximately 85% of the market-capitalisation of the country. Since there is no natural index for the BRICS countries they created a pseudo index in which they equally weighted each index into a portfolio.

Watson and Seetharam (2014) reported that the minimum variance portfolio omitted both Russia and Brazil, with China, India, and South Africa receiving 14.4%, 35%, and 50.6%, respectively. The reason why Russia was excluded could be due to it having the highest standard deviation, whereas Brazil was omitted due to it having an extremely high correlation with the other BRICS countries and the second-highest standard deviation. China's inclusion must be caused by its low correlation with South Africa and India since it had an average return of zero. South Africa's inclusion must be due to it having the lowest standard deviation because it had a high correlation with the remaining BRICS countries.

Kremer et al. (2018) investigated eight risk minimisation strategies which were; equal-weighted portfolio, minimum variance, equal risk contribution, inverse volatility, maximum diversification, and maximum decorrelation, and minimum linear torsion in a conditional performance analysis compared risk factor performance. The countries that were included in the sample were the US, UK, Developed Europe excluding the UK, Japan, and Asia excluding Japan from 2004 to 2015. The results showed that none of the portfolio strategies were statistically different from each other concerning the Sharpe ratio, and none of the strategies dominating another one. The results did, however, find that the minimum variance portfolio is the best for those seeking minimal risk, shorter drawdown periods, and a higher certainty equivalent value. The minimum variance portfolio does have high turnover as such, this strategy will perform well when there are low transaction costs.

Butler (2018) explored the fundamentals of the market-capitalisation portfolio and equal-weighted portfolio and several risk-based optimisations which were minimum variance, inverse volatility, equal risk contribution, and maximum diversification methods. They examined the returns of various asset classes, including stocks, bonds, and commodities. The different asset classes had a positive relationship between risk and return. The results found that the minimum variance method produced the highest Sharpe ratios, with equal risk contribution and inverse volatility been the best from a statistical point. The reason for the equal risk contribution and inverse volatility outperforming the equal weight is due to the smaller amount of active risk relative to the equal-weighted portfolio. However, the equal risk contribution and inverse volatility underperformed relative to the minimum variance and maximum diversification methods. Lastly, both the equal weight and risk-based optimisation outperformed the market-capitalisation portfolio.

Raza and Ashraf (2019) investigated the effects of fundamental weighting, equal-weighting, and low volatility weighting strategies on a constrained portfolio, the constraint they implemented was on Shariah-compliant equity portfolios to investigate whether outperformance relative to market-capitalisation portfolios will persist on a reduced equity universe. The sample was from 2003 to 2016 from equity markets in Indonesia, Malaysia, Middle East, Australia, Europe, the US, and Canada. The results found that the risk-optimisation strategies of the Shariah-compliant constraints outperformed both the conventional market-capitalisation portfolio as well as the Shariah-compliant market-capitalisation portfolio. Therefore, they suggest that passive investors must consider the importance of risk-optimisation strategies as they provided both higher risk-adjusted returns as well as lower drawdown.

Raza, L'Huillier and Ashraf (2019) investigated the effects of Shariah-compliant stock performance using market-capitalisation, fundamental weighted, equal-weighted, and low-volatility strategies of the S&P 500 from 1986 to 2016 during bullish and bearish markets. They found that there was a significant difference in the performance during bull and bear markets. During bull markets, the market-capitalisation and fundamentally weighted portfolios performed the best. However, during the bear market, the low volatility strategy was the best. Hence, they recommended that the low-volatility strategy be used as a hedge since it contains significantly lower value-at-risk and expected shortfalls relative to the other strategies.

Bender, Blackburn and Sun (2019) compared the mean-variance framework and market-capitalisation portfolio against a vast amount of risk-optimisation strategies which included minimum variance, maximum decorrelation, maximum diversification, and equal risk contribution. The sample period was from 1995 to 2016. The results found that based on the information ratio, the mean-variance portfolio was the best performing portfolio, while the maximum diversification and maximum decorrelation were the only two strategies that underperformed the market-capitalisation benchmark. However, based on the Sharpe ratio the maximum decorrelation was the lowest and the only strategy to underperform the market-capitalisation benchmark. The best performing portfolio based on the Sharpe ratio was the mean-variance followed by the minimum variance.

2.3 The co-movement of market indices

Markowitz (1952) stated that investors should not concern themselves with the expected returns of an individual asset, but instead, the weighted average of the expected returns of the portfolio's component asset as well as how it moves in conjunction with others. This is known as the co-movement quantified by covariance.

Grubel (1968) extended the analyses of Markowitz to include international assets in which he found that international diversification of portfolios can be used as an extra source to gain additional diversification as it will allow investors to achieve higher levels of returns and lower volatility relative to a purely country-specific diversification of assets. The method that Grubel (1968) employed was to compute a set of efficient portfolios from eleven different equity markets. He found that the US portfolio based on Moody's industrial index was not efficient. Given the ability of a US investor to invest globally, the investor can move to a higher efficient frontier by taking advantage of international diversification.

Hilliard (1979) examined the structure of the international equity market indices during a financial crisis that spreads worldwide. This caused him to examine the daily return from 7 July 1972 to 30 April 1974 for 10 major industrial indices across the world. This period included the OPEC Embargo being announced. The methodology employed was the Cramer Decomposition Theorem as this model will not incur the risk of model misspecification as regression analysis.

Hilliard (1979) results indicated that there is no apparent world-wide financial market factor that is common because if there were markets as such Tokyo and Milan would have had greater significant coupling (co-movement), especially around the OPEC Embargo. These types of relationships were seen with most market pairs except for New York and Amsterdam. This means that lead and lag relationships are not relevant. Therefore, most international markets had low correlations and international diversification benefits were prominent during this OPEC Embargo crisis.

Bekaert and Harvey (1995) measured market integration using a conditional regime-switching model. This model allowed them to use expected returns that were segmented in the part of the sample period that became integrated in a different part of the sample period. They found that a number of emerging markets displayed time-varying integration in the later part of the sample period. They found that some markets were more integrated than expected despite the investment restrictions. While other markets appeared to be segmented despite having relatively free access to their financial market. Lastly, they found that all financial markets have not become more integrated, suggesting that international diversification is still a viable strategy.

Brooks and Del Negro (2004) used data monthly returns denominated in US dollars and market-capitalisation from 1985 to 2002 which consisted of 42 developed and emerging countries. They found that global industry effects have become more important than country effects in 1999-2000. Next, they found that beyond the technology sector, which was at the heart of the 2000 stock market bubble, there is no evidence that industry effects have become significantly more important than country effects. Lastly, they found that the evidence suggests that the rise in sector effects after the mid-1990s was a temporary phenomenon associated with the technology bubble. Therefore, they conclude that for portfolio managers that in the aftermath of the technology bubble, the strategy of diversifying internationally rather than by industry may still have merit in reducing portfolio risk.

Samitas, Kenourgios and Paltalidis (2007) analysed the correlation dependency among the BRIC emerging countries and the developed markets of the US and UK when at least of the markets is under financial crisis. They used the asymmetric generalised dynamic conditional correlation model (AG-DCC) with the use of daily returns for the respective countries from 1995 to 2006. They found that the conditional volatilities of equity indices contain prevalent evidence of asymmetry. The AG-DCC results found higher joint dependence during stock market crises as the equity correlation among BRICs and developed markets dramatically increase. As such, when international diversification is sought out most by investing in multiple markets is when diversification is at its worse because markets become increasingly correlated.

Asness, Israelov and Liew (2011) investigated the difference between short and long-term international diversification benefits. The hypothesis investigated was that short-term market downturns are due to panics and selling frenzies. However, long-term results tend to be due to economic performance. This caused them to split returns into a component arising from multiple contractions or expansions and a component arising from economic performance. They analysed international diversification benefits from a local investor's perspective in 22 countries from 1950 to 2008.

Asness et al. (2011) found that over the short-term, international diversification can disappoint as markets tend to crash at the same time. Thus, international diversification fails investors the most at crucial times. Short-term crashes are painful, but long-term returns are more important to the creation or destruction of wealth. In which they reported that markets do not crash together as economic performance plays a bigger role in the performance of a portfolio in the long-term. They concluded that International diversification protects investors against any adverse effects of having a concentrated position in a country with poor long-term economic prospects.

Lehkonen and Heimonen (2014) investigated the difference in asset return co-movement of the BRIC countries as the emerging economy group, Canada, Hong Kong, and Australia as the developed economies, and finally the UK, Germany, and Japan as the major industrialised economies. The methodology employed on the daily return indices from the respective countries between 1994 and 2010 was a decomposed timescale wavelet analysis which was used as inputs for the DCC framework and is used to measure co-movement. The results for the smaller timescales find that markets can be divided into the three following clusters: Asia, Europe, and America. The correlation structures between these three clusters vary significantly during this period and differ significantly between markets.

Mensi, Hammoudeh, Nguyen and Kang (2016) aimed to examine the dynamic spill-overs between the BRICS emerging countries, as these were the five fast-growing economies, and the US, which is considered the most important and developed market, with an emphasis on the financial crisis of 2008- 2009. They used the multivariate dynamic conditional correlation fractionally integrated asymmetric power ARCH (DCC-FIAPARCH) model to explore the spill-over effect of the daily spot indices of the BRICS and US stock markets from 1997 to 2013. The DCC modelling provides an efficient method to capture the conditional correlations between the sample markets which change through time concerning market conditions.

Mensi et al. (2016) results of the DCC-FIAPARCH model showed significant time-varying correlations between the BRICS and the US stock markets. They found several structural changes to the unconditional volatility of the individual BRICS and US stock markets which correspond to the collapse of Lehman Brothers on September 15, 2008. The results of the co-movement before and after this date support the strengthening re-coupling hypothesis between the US and Brazil, India, China, and South Africa. This is a sign of increasing contagion for these countries after the financial crisis. In contrast, they found the support of the decoupling hypothesis between the US and the Russian stock market during the financial crisis. Therefore, from a US investor perspective, the greatest international diversification benefits can be obtained by investing in the Russian stock market due to the decoupling between the countries' stock markets. While US investors' international diversification benefits would have decreased after the financial crisis.

Mohanasundaram and Karthikeyan (2015) explored the possibility of a short-term and long-term relationship between South Africa, India, and the US's stock market indices as these three countries are prominently related to each other. They used monthly return data from the stock indices which were JALSH, NIFTY, and NASDAQ for South Africa, India, and the US respectively from 2004 to 2014. They employed granger causality to test for causality, while the vector autoregression model and Johansen and Juselius multivariate co-integration approach test for short and long-term relationships.

Mohanasundaram and Karthikeyan (2015) results indicated that the NASDAQ has no predictive ability on the JALSH or NIFTY. However, the JALSH has a predictive ability on the NIFTY. In the short-term, both the US and South African markets are predicted by their lags, with the Indian market being a function of both its lags and South Africa's lags. Whereas, the long-term relationship was absent for the three markets. This means that in the Short-term there are international diversification benefits only between the US and South Africa. However, in the long-term, there are international diversification benefits between all three markets.

Al Nasser and Hajilee (2016) investigated stock market integration among Brazil, China, Russia, Mexico, and Turkey as the emerging markets and the developed markets being the US, the UK, and Germany. The methodology they employed is the bounds testing approach to co-integration and error-correction modelling to determine both the short and long-term relationship between emerging stock market returns on the developed stock market returns. They used the monthly returns for the country-based MSCI indices from 2001 to 2014 which was expressed in the US dollar to get homogenous data and to account for currency risk.

Al Nasser and Hajilee (2016) reported that their results found evidence of short-term integration between emerging markets and developed markets. However, the long-term coefficient for stock price indices in all emerging markets only had a significant relationship with the German stock market index. Consequently, they concluded that although there is no arbitrage opportunity in the long-term in emerging markets, local investors can still obtain arbitrage profits through international portfolio diversification in the short-term. However, the opportunities to achieve international portfolio diversification gains in the long-term through emerging markets are still there.

Billio et al. (2017) set out to compare the financial integration patterns from a variety of methodologies. The aim was to find an integration measure that was able to balance the trade-off between computational complexity and accuracy in measurement. As such, they examined the degree of information disparity between the measures and quantitatively evaluate the ability of each measure to examine the relationship between financial integration and international diversification. They used monthly return data from 16 developed countries from 1973 to 2016 and 11 emerging countries from 1990 to 2016, which included South Africa and India from the BRICS group. Monthly return data was used as it avoided a daily return of zero, non-synchronicity of data, and excess noise.

Billio et al. (2017) main results are as follows. Firstly, they observe that the standard correlation, 1st PC, and Adjusted R-square results are almost identical in their equity market integration patterns. Secondly, the heteroscedasticity-adjusted measures produce more volatile integration patterns, but over the long-run, they give similar integration trends. As such, they find that the use of different methods does not show any major differences. Lastly, they find that on average, the standard correlation measure explains movements in international diversification better than the more sophisticated measures.

Kanuri, Malhotra and Malm (2018) evaluated the international diversification benefits and performance of emerging market exchange-traded funds (ETFs) from a US investor's perspective from 2003 to 2015. The emerging-market ETFs were equally weighted. The results showed that the emerging market ETFs had higher returns but twice the volatility of the S&P 500 ETF, thus, the US still had a higher risk-adjusted return. Lastly, they found that a US investor adding their desirable percentage of emerging market ETFs based on their risk tolerance to their domestic allocation will increase returns and performance. This provides evidence that international diversification can improve returns.

Boamah et al. (2020) explored the co-movements between 20 emerging markets in which they employed a dynamic conditional correlation method. These emerging markets included South Africa, Brazil, and India from the BRICS group. The sample period was from 1997 to 2015 and included monthly data returns for the countries' indices that were used as this helped control for the potential impact of thin trading. They found that integration increased around the crisis period within emerging markets, which suggests that international diversification gains within emerging markets may be minimal around crisis times. However, some emerging markets were exhibiting a lesser degree of country pair integration even in the financial crisis, and that across emerging markets investing can offer investors significant diversification potential. Lastly, they provided evidence that trade with high-income countries is more important than trade among emerging markets for driving financial market integration within emerging markets. This suggests that emerging markets are less economically engaged with each other.

Based on this extensive literature review this study will look to fill the gap in the literature that looks at the use of a vast array of conventional and Smart Beta strategies on an international scale. The strategies included are market-capitalisation, equal weight, low volatility, minimum variance, maximum diversification, equal risk contribution, maximum decorrelation, and inverse volatility. Further, this study will aim at quantifying the effects of globalisation by investigating whether there is a decrease or eradication of international diversification benefits for investors. This will be done by using DCC-GARCH and standard correlation to quantify the co-movement between the BRICS and how would this affect a South African investor looking at selecting an appropriate investment strategy to maximise risk-adjusted returns.

3. Data and Methodology

This section looks at the various data and methodologies that are implemented in this study. The first part examines the data that is used in this study. The second and third part of the methodology looks at the portfolio construction techniques and the performance analysis tools that are used in the study. The last section looks at the co-movement methodologies that are implemented. It should be noted that time t refers to the time at the beginning of the year for the portfolio construction and performance analysis section, whereas, time t is the beginning of each day for the co-movement section.

3.1 Data

The co-movement between the BRICS countries' daily return data will be calculated from the daily total return price data. GARCH modelling is used to capture any heteroscedasticity present in the daily movements of the indices. The return index from Brazil will be Bolsa de Valores do Estado de Sao Paulo (Ibovespa) index, as it is the main indicator for the Brazilian stock market's average performance. The return index for Russia will be the Moex index as this index captures the largest and most liquid stocks listed on the Russian stock exchange. The return index for India is the Bombay stock exchange SENSEX index. The return index for China is the Shanghai Stock Exchange Composite index which is the largest stock exchange in Mainland China and the second biggest in the world (An & Brown, 2010). TableLastly, the return index for South Africa will be the JSE All Share Index as this is the main index for the South African stock market.

The data for all the indices are retrieved from Bloomberg. The sample period is based on the availability of data provided on Bloomberg and to match the results of the risk-optimised portfolios which was 2000 until the end of 2019. The period in which the DCC-GARCH and correlation measures will be implemented is based on the entire sample period.

To determine the returns of the risk-optimisation portfolios and the market-capitalisation weighted portfolio the daily stock prices and market-capitalisation for all the individual stocks that were listed on the Sao Paulo Stock Exchange, Moscow Stock exchange, Bombay Stock Exchange, Shanghai Stock Exchange, and Johannesburg Stock Exchange were retrieved from Bloomberg. This resulted in a total of 5749 stocks in the BRICS equity universe available from Bloomberg. The sample period was based on the availability of data provided on Bloomberg which was 1999 until the end of 2019. However, 1999 will only be used to calculate the weights of the respective portfolios that will be implemented in 2000 to provide an estimation period.

Thus, the performance of the portfolios will be examined over 20 years (2000-2019) with annual rebalancing in which the estimation period will be one year.

To construct the portfolios the methodology of Table 1 will be used. The weights for the risk-optimisation portfolio will be based on the previous year's returns. Annual rebalancing is used to minimise transaction costs for the portfolio strategies and allow the portfolio strategies to be more comparable.

Table 1: Example of implementing an out-of-sample portfolio return strategy				
Year	1999	2000	2001	2002
Calculate Weights				
Calculate Returns (performance)				

The annual portfolio turnover will be calculated to obtain an estimate of the amount of trading required to implement each of the portfolio strategies with annual rebalancing. Turnover will be calculated by subtracting the opening day volume of a stock from the previous day's closing volume of each stock at the end of each year. The sum of absolute values of the individual stocks turnover at the end of each year will provide an estimate of the amount of trading that has occurred to obtain annual rebalancing of the portfolio. The transaction costs of each portfolio strategy will be 50 basis points for both buying and selling of a stock at a given point in time which will be multiplied by the turnover (DeMiguel et al., 2009).

By accounting for transaction cost it will provide a more realistic portfolio return that is achievable because one will need to pay for each transaction that they undertake when rebalancing the portfolio. However, this method does not account for the liquidity of a stock, which is the ability to buy and sell a stock at any given point in time without impacting the price. This method does not account for price impact which would be a function of portfolio fund size and the market-capitalisation and liquidity of the stock in question. With one being able to buy and sell at the same cost for each stock at every point in time (DeMiguel et al., 2009). The value-weighted portfolio will be based on the market-capitalisation of the stock at the start of the year.

For a stock to be considered as part of the investment universe it will need to be part of the stocks that make up the top 90% of the total market-capitalisation of each country at the end of the previous year. For example, if the portfolio construction occurs on 01 January 2000, the market-capitalisation used to determine the top 90% is from 31 December 1999. This will result in a market-capitalisation filter that removes the small stocks that tend not to be traded regularly as these stocks are difficult to invest into in reality and distort the returns and standard deviation of the portfolios. Therefore, for the South African universe, only the stocks that form part of the top 90% of the overall market will be considered. Whereas, to create the investment universe for the BRICS portfolios each country will undergo the filter separately before been combined to create a single investment universe. This will allow for fair representation for each country when performing the BRICS portfolios because on average each country will have 90% of its market represented before the construction of the risk-optimisation and market-capitalisation weighted portfolios. While still maintaining a realistic portfolio that can be invested in, which is not susceptible to illiquid stocks (Malladi & Fabozzi, 2017).

All data including the market-capitalisation of each stock will be denominated in US Dollars to obtain a comparable result on an international scale. The daily exchange rates were obtained from Metadata. The risk-free rate that will be used is the US T-bill rate which was obtained from the Macrotrends website. The method in which this will be done is that the prices of both the stocks and indices will be converted to the US Dollar daily for the stocks and indices that will be used. This is due to emerging markets being highly susceptible to high inflation, and the risk-free rate differing drastically among countries which will affect the returns as well as providing comparable results (Zaremba, 2015).

3.2 Portfolio weighting methodology

This section of the methodology looks at the various portfolio strategies that will be implemented both at a domestic level and international level. The strategies include the basic market-capitalisation weighted portfolio and the following risk optimisation strategies equal-weighted portfolio, low volatility weighted portfolio, minimum variance portfolio, maximum diversification portfolio, equal risk contribution portfolio, maximum decorrelation portfolio, and inverse volatility portfolio. For the risk optimisation portfolios that are not inherently a long-only portfolio, a long-only constraint will be imposed. It should be noted that all of the portfolio construction occurs after the filter has been implemented.

3.2.1 Market-capitalisation weighted portfolio

The value-weighted portfolio is constructed by taking the product of an individual stock's price and the number of common stocks outstanding divided by the total sum of the product of the number of stocks outstanding and the current stock price at the time for all available stocks in the investment universe. Stocks that have a higher market-capitalisation value will receive a greater weight in the portfolio. This method inherently has a long-only constraint. The market-capitalisation weighted portfolio will be mean-variance efficient if CAPM holds. The following methodology is used which is similar to De Carvalho et al. (2012):

$$w_{i,t}^{vw} = \frac{n_{i,t}p_{i,t}}{\sum n_{i,t}p_{i,t}} \quad (1)$$

Where:

$w_{i,t}^{vw}$ = Weight for a value-weighted portfolio

n_i = Number of outstanding stock at time t

$p_{i,t}$ = Will be the price of the stock at time t

3.2.2 Equally weighted portfolio

The equal-weighted portfolio is the easiest weighting methodology available. This method simply allocates an equal weight to stock in the universe irrespective of the price or market-capitalisation of the stock. Similar, to the market-capitalisation weighted portfolio, this portfolio is long-only (Windcliff & Boyle, 2004).

$$w_{i,t}^{ew} = \frac{1}{N_t} \quad (2)$$

Where:

$w_{i,t}^{ew}$ = Weight of an equal-weighted portfolio

N_t = Total number of stocks at time t

3.2.3 Low-volatility weighted portfolio

The objective of the low-volatility strategy is to reduce the volatility of the overall portfolio by assigning weights to each stock that will reduce the volatility. This will be done using a similar methodology as Boudt, Raza and Wauters (2017) using a two-step process. Firstly, the standard deviations for each stock will be calculated (Equation 3.1) and ranked in ascending order. Due to the stocks been ranked in ascending order, quintile 1 contains 20% of the stocks from the universe that produced the lowest volatility over the period. The second step in the process weighted using the equal-weighted methodology on quintile 1 (Q1) with the remaining stocks receive a weight of zero (Qx) (Equation 3.2). The weighting mechanism of this strategy is an inherently long-only strategy.

$$\sigma_{Si,t} = \sqrt{\frac{\sum\left(S_{i,t} - \mu_{i,t}\right)^2}{N}} \quad (3.1)$$

$$w_{i,t}^{Q1} = \frac{1}{N(Q_1)_t} \;\; ; \;\; w_{i,t}^{QX} = 0 \quad (3.2)$$

$\sigma_{Si,t}$ = Standard deviation of stock i at time t

$w_{i,t}^{Q1}$ = Weight of stocks in quintile 1 at time t

$w_{i,t}^{QX}$ = Weight of stocks not in quintile 1 at time t

3.2.4 Minimum Variance weighted portfolio

The minimum variance portfolio is determined by the covariance matrix. The weight of the portfolio is determined from the universe of risky stocks which is expected to deliver the lowest possible volatility. The minimum variance portfolio is the only portfolio that will always be located on the efficient frontier without using expected returns as inputs for determining the weights of a portfolio. The minimum variance portfolio optimises the weights such that the marginal contribution to portfolio volatility is equal (Kremer et al., 2018). However, this will cause many stocks to receive no weight, while low idiosyncratic and lowly correlated stocks will receive a greater proportion of the weight. This will occur in diminishing amounts based on the size of the investment universe. Lee (2011) stated that this does not suggest that the portfolio is diversified from the view of portfolio weights and contributions as it is still subjected to concentration risk.

The methodology that is implemented for the minimum variance is the daily covariance matrix for the previous year is used to calculate the weight vector that will be used in the current year. There will be two restrictions imposed on the minimum variance portfolio. The first restriction states that the sum of the individual weights must equal one and the second restriction states that the weight of individual stock must either be greater than or equal to zero which means that there is no short-selling. Thus, the minimum variance optimisation with a long-only constraint will identify a vector of weights that minimizes the portfolio volatility. The optimisation formula is implemented similar to that of Kremer et al. (2018).

$$w_t^{mv} = {}^{Min}_{w}(W_t^T COV_t W_t) \quad (4)$$

Where:

w_t^{mv} = The optimised weights at time t for the minimum variance portfolio

W_t = [N x 1] vector of portfolio weights

W_t^T = Transposed version of W_t

COV_t = The daily stock price covariance matrix for year t

Restrictions:

$$\sum_{i=1}^{n} W_{i,t} = 1$$

$$W_{i,t} \geq 0$$

3.2.5 Maximum Diversification weighted portfolio

Choueifaty and Coignard (2008) created the diversification ratio which is the ratio of the weighted average volatilities of the stocks to the volatility of the portfolio containing the same stocks when accounting for correlation. The diversification ratio is proportional to the level of portfolio diversification, in that the greater the ratio the greater the diversification the portfolio achieves. Therefore, the maximum diversification portfolio will be constructed so that the diversification ratio is maximised. This is achieved by maximising the distance between an imaginary portfolio that contains perfect correlation and the volatility of the same portfolio in a real-world scenario using their estimated correlation coefficients and thus allowing diversification benefits. The maximum diversification ratio tends to favour higher idiosyncratic and low correlated stock. Due to the maximum diversification inherently allowing short-positions in stocks there will be a long-only constraint in the optimisation tool similar to that of Bender et al. (2019).

$$DR(W_t) = \frac{\sigma_t W_t}{\sqrt{(W_t^T COV_t W_t)}} \quad (5)$$

$$W_t^{MDP} = \frac{maxDR(W_t)}{W_t} \quad (6)$$

Where:

$DR(W_t)$ = Diversification ratio at time t

W_t = [N x 1] vector of portfolio weights

W_t^T = Transposed version of W_t

COV_t = The daily stock price covariance matrix for year t

W_t^{MDP} = The optimized weights of the maximum diversification portfolio at time t

Restrictions:

$$\sum_{i=1}^{n} W_{i,t} = 1$$

$$W_{i,t} \geq 0$$

3.2.6 Equal Risk Contribution weighted portfolio

The equal risk contribution aims to ensure that each stock gives an equal contribution of volatility to the global volatility of the portfolio. The equal risk contribution is a refinement to that of the equal-weighted portfolio. Instead of each stock producing the same weight, each stock will contribute the same volatility. This method is inherently a long-only strategy that invests in all available stocks while favouring low idiosyncratic and low correlated stocks similar to that of the minimum variance portfolio. This means that this portfolio strategy optimises to have an equal contribution to the total risk instead of the marginal contribution of risk been equal to the minimum variance portfolio. The optimisation function that is used similar to Sharma (2015).

$$W_{it}^{ERC} = W_{i,t} \frac{(COV_t W_t)_i}{\sqrt{(W_t^T COV_t W_t)}} \quad (7)$$

Where:

W_{it}^{ERC} = The risk contribution at time t for stock i

W_t = [N x 1] vector of portfolio weights

W_t^T = Transposed version of W_t

COV_t = The daily stock price covariance matrix for year t

Restriction:

$$\sum_{i=1}^{n} W_{i,t} = 1$$

3.2.7 Maximum Decorrelation weighted portfolio

The maximum decorrelation aims at minimizing portfolio volatility through exploiting the information provided by correlation coefficients. The main idea of the maximum decorrelation is combining stocks into a portfolio that exhibits low correlations, rather than focusing on low volatility stocks to minimise volatility. Due to maximum decorrelation containing short positions in stocks, a long-only constraint will be added. The maximum decorrelation strategy only uses information provided by the correlation matrix which will make this strategy favour stock that has a low correlation similar to that of the maximum diversification portfolio, except this portfolio strategy ignores idiosyncratic risk as it uses the correlation matrix and not the covariance matrix. However, the risk reduction of the minimum variance portfolio can still be attained, with the advantage of avoiding the higher concentration minimum variance portfolios suffer from. This strategy does not have any implicit control on the total portfolio volatility that results from the volatility of each stock (Kremer et al., 2018). The optimisation method implemented is similar to that of Bender et al. (2019) and Butler (2018).

$$w_t^{MDec} = \ {}^{Min}_{w}(W_t^T COR_t W_t) \qquad (8)$$

Where:

w_t^{MDec} = Weight matrix for the maximum decorrelation at time t

W_t = [N x 1] vector of portfolio weights

W_t^T = The transposed version of W_t

COR_t = Daily stock price correlation matrix for year t

Restrictions:

$$\sum_{i=1}^{n} W_{i,t} = 1$$

$$W_{i,t} \geq 0$$

3.2.8 Inverse volatility weighted portfolio

Maillard et al. (2010) proved that under the assumption of constant cross-correlation between the stocks, the equal risk contribution portfolio weights are proportional to the inverse of the stocks' volatilities. The inverse volatility is inherently a long-only portfolio in which it is similar to the market-capitalisation weighted strategy, but instead of using market-capitalisation as a weighting determinant, it uses volatility. The inverse volatility assigns a greater weight to stocks that contain low volatility and stocks that have a high volatility will have a much lower proportion of weight. Following Kremer et al. (2018) the weights of the inverse volatility are calculated as follows:

$$w_i^{IV} = \frac{\frac{1}{\sigma_i}}{\sum_{i=1}^{N} \frac{1}{\sigma_i}} \quad (9)$$

w_i^{IV} = Weight of stock i using the inverse volatility method

σ_i = Standard deviation of stock i

3.3 Portfolio Construction

For each of the portfolio strategies, the turnover and transaction cost will need to be accounted for, to determine the cost associated with these strategies. In addition, the risk optimisation strategies that implement a covariance or correlation matrix will have to undergo shrinkage to minimise the estimation errors inherent in the covariance and correlation matrix.

3.3.1 Shrinkage Estimation

Ledoit and Wolff (2004) stated that the sample covariance matrix that is used for portfolio optimisation should not be used as it contains extreme levels of estimation error. They suggested using the sample covariance matrix and applying a shrinkage transformation. This shrinkage method tends to pull the most extreme coefficients towards more central values. By doing this it will reduce the estimation error where it matters most.

The core of this method is that the estimated coefficients in the sample covariance matrix that are exceptionally high will tend to contain abnormal amounts of positive error with the need to be pulled down to compensate for this. The opposite is also true for exceptionally low estimated coefficients that need to be pulled upwards. This shrinkage method will solve the problem of the estimation errors in the sample covariance matrix (Ledoit and Wolff, 2004). Therefore, shrinkage estimators will be applied to all risk-optimisation portfolios that require a covariance matrix.

$$\widehat{\sum_{shrink}} = \widehat{\delta^*}F + \left(1 - \widehat{\delta^*}\right)S \qquad (10)$$

Using the methodology of Lediot and Wolff (2004) with S been the sample covariance matrix which has the advantages of being easy to calculate and it is unbiased. However, the disadvantage of the sample covariance matrix is that it contains a lot of estimation error when there is the same or fewer number of data points than there are of individual stocks. F is a highly structured covariance matrix that follows Sharpe (1966). These estimators contain relatively few estimation errors but tend to be misspecified while been severely biased at the same time. As such, Lediot and Wolff (2004) finds a compromise between the two by calculating a convex linear combination which is Equation 10. This is the technique known as shrinkage due to the sample covariance matrix been shrunk towards a structured estimator. With δ been a number between 0 and 1, it is the shrinkage constant that measures the weight that is given to the structured estimator.

The optimal shrinkage constant is determined by minimising the expected distance between the shrinkage estimator and the true covariance matrix which is δ*. The optimal shrinkage estimator will be calculated at each rebalance. This is done by applying the shrinkage equation to the covariance matrix of returns from the previous period which is used when determining the optimised weights for the portfolio strategies.

3.3.2 Turnover and transaction costs measures

Turnover will provide a sense of the amount of trading required in the implementation of each portfolio strategy. To compute the portfolio turnover it is the average sum of the absolute value of trades for the N stocks available when t is measured in years (DeMiguel et al., 2009).

$$TO = \sum_{t=2}^{T}\sum_{i=1}^{N} |w_{i,t} - w_{i,t-1}| \qquad (11)$$

Where:

TO = Turnover

$w_{i,t}$ = The weight of stock i at time t

$w_{i,t-1}$ = The weight of stock i at time t – 1

To account for the transaction costs needed to implement the portfolio strategy with yearly rebalancing, a trading cost of 50 basis points (0.50%) will be implemented (DeMiguel et al., 2009; Plyakha et al., 2014). To obtain the performance of the portfolio after cost the methodology of Plyakha et al. (2014) will be followed. This results in the performance both before and after transaction costs be reported.

$$R_{post-trans} = R_{pre-trans} - (TO * TC) \quad (12)$$

Where:

$R_{post-trans}$ = Returns post-transaction cost

$R_{pre-trans}$ = Returns pre-transaction cost

TO = Turnover

TC = Transaction costs

3.3.3 Portfolio construction

The process of constructing each of the portfolio strategies is similar for each holding period and consists of seven steps. The first step is to convert all of the price data and market-capitalisation data for each stock into dollars as the price data that is initially used is denominated in the domestic currency. Using this price data, the continuous returns of each stock calculated. The second step is to remove the stocks that do not make up 90% of the total market-capitalisation for the country in that given year. The two investment universes are created at this step. For the international universe, each of the countries' stocks is combined and for the domestic universe, the remaining stocks after the filter are the completed domestic investment universe.

The third step involves the calculation of the weights for each stock. For the portfolio strategies that do not require any covariance or correlation matrix, the weights are calculated using the formulas above. For the portfolio strategies that do require the covariance or correlation matrix, the covariance or correlation matrix is shrunk using the Ledoit and Wolff shrinkage methodology, and the shrunk matrix is used in the formulas above that calculates the weights of the portfolio strategy. The fourth step uses the weights that were calculated above to calculate the returns of the portfolio strategy. The weights that were calculated are based on the previous year's returns to avoid a look-ahead bias.

The fifth step is to repeat steps 1 to 4 for each year in order to obtain the full sample periods of return data for each portfolio strategy. The sixth step calculates the turnover and the transaction cost of each portfolio. The way this is done is by calculating the absolute difference between the weights of a stock at the beginning of a period minus the lagged end of the period weight of a stock resulting in the turnover of the individual stock. The turnover of each stock is summed together and multiplied by 5bps to obtain the transaction cost for both buying and selling of the individual stocks that make up the portfolio strategy. Lastly, the seventh step is to calculate the performance of each portfolio strategy both before and after taking into account the transaction costs.

3.4 Performance analysis

To analyse the performance of the portfolio strategies, various performance analyses measures are used to account for a variety of different risk-adjusted performances. The performance measures that are implemented are (1) the Treynor ratio which is the risk premium per unit of risk. (2) The Sharpe ratio is the risk premium per unit of total risk. (3) The Sortino ratio measures excess returns of a portfolio's average return to a minimum threshold of acceptable return. (4) Value at risk is the value of the portfolio that might be lost at a given probability level for a given day. (5) The maximum drawdown is the maximum loss from the peak to trough that an investor can suffer over the entire holding period.

3.4.1 Treynor ratio portfolio performance measure

$$T_i = \frac{\bar{R}_i - \overline{RFR}}{\beta_i} \quad (13)$$

Where:

$\bar{R}_i$ = Average rate of return for Portfolio i.

$\overline{RFR}$ = Average daily return for the 90-day T-bill rate at time t.

β_i = Slope of the Portfolio’s characteristic line.

Treynor (1965) developed a measure of portfolio performance that incorporated a risk factor. The Treynor ratio is composed of two components, risk produced by the market and unique risk from a stock. He proved that any rational risk-averse investor would always prefer a portfolio that produced a higher T value. Since a larger T value is an indicator of a better performing portfolio regardless of the individual’s risk preference. The Treynor ratio is known as the risk premium per unit of risk. Lastly, the Beta measures the systematic risk and does not take into account the decrease in volatility due to portfolio diversification.

3.4.2 Sharpe ratio portfolio performance measure

$$S_i = \frac{\bar{R}_i - \overline{RFR}}{\sigma_i} \quad (14)$$

Where:

$\bar{R}_i$ = Average rate of return for Portfolio i.

$\overline{RFR}$ = Average daily return for the 90-day T-bill rate at time t.

σ_i = Standard deviation of the rate of return for Portfolio i.

Sharpe (1966) developed a similar performance measure to that of Treynor (1965). He noted that a higher S value of a portfolio will indicate a superior portfolio irrespective of the individuals' risk preferences. In contrast to the Treynor ratio, the Sharpe ratio measures the total risk of the portfolio using the standard deviation of a stock instead of the systematic risk. Thus, the Sharpe ratio measures the risk premium per unit of total risk. Sharpe (1966) stated that if a portfolio is well-diversified the Treynor and Sharpe ratio will provide similar results. However, for a portfolio that is not diversified the two ratios can provide results that differ significantly.

3.4.3 Sortino ratio portfolio performance measure

$$ST_i = \frac{\bar{R}_i - \tau}{DR_i} \quad (15)$$

Where:

$\bar{R}_i$ = Average rate of return for Portfolio i.

τ = The minimum acceptable average rate of return threshold (US T-bill rate).

DR_i = The downside risk coefficient for Portfolio i.

Another risk-adjusted measure is the Sortino ratio which was developed by Sortino and Price (1994). This risk-adjusted performance measure differs fundamentally from the Sharpe ratio in two ways. Firstly, the Sortino ratio measures excess returns of a portfolio's average return to a self-selected minimum threshold of acceptable returns. The US T-bill rate will be considered as the minimum acceptable threshold. Secondly, the Sortino ratio captures the downside risk of a stock only, as such, the measure does not penalize a stock for been volatile on the upside, which the Sharpe ratio does. Similar to the Treynor and Sharpe ratio, a higher Sortino ratio is an indication of a superior portfolio.

The downside risk is the volatility of returns below a hurdle rate of the investors' choice. The hurdle rate that will be used is 0% performance. This means that all negative returns will be considered as downside risks. To measure downside risk below a minimum threshold, the semi-standard deviation measure developed by Harlow (1991) is implemented.

$$Semi - Deviation = \sqrt{\frac{1}{n}\sum_{R<\bar{R}}(R_{it} - \bar{R}_i)^2} \quad (16)$$

Where:

n = The number of portfolios returns that are negative

R_{it} = Return of portfolio i at time t

$\bar{R}_i$ = The average expected returns for portfolio i

3.4.4 Value at Risk

The VaR is based on the historical returns over the entire holding period and is defined as the maximum loss an investor can suffer in a given period for at the given level of confidence. The alpha value is defined as the level of significance of the normal distribution of returns (Kremer et al., 2018).

$$VaR_\alpha(R) = \inf\{l : Pr\{-R < 1\} \geq \alpha\} \quad (17)$$

3.4.5 Maximum Drawdown

The maximum drawdown is the maximum loss from the peak to trough that a portfolio can suffer over the entire holding period (Kremer et al., 2018).

$$MDD = \frac{(P - L)}{P} \quad (18)$$

3.5 Co-movement

3.5.1 The Dynamic Standard Correlation

Kearney and Lucey (2004) found that Pearson's correlation proxy for measuring co-movement of markets and thus financial integration to be one of the most popular measures. This is due to it being one of the easiest proxies to measure and interpret. Following Billio et al. (2017), this study concentrates on bilateral correlations to avoid the choice of a benchmark index. A window of 5 years is used in the estimation of the bilateral correlation. The formula that is used is Pearson's correlation (Gujarati & Porter, 2009).

$$\rho_{xy} = \frac{n\sum x_t y_t - \sum x_t \sum y_t}{\sqrt{n\sum x_t^2 - (\sum x_t)^2}\sqrt{n\sum y_t^2 - (\sum y_t)^2} \qquad (19)$$

Where:

ρ_{xy} = Pearson's r correlation coefficient between index x and index y

n = number of observations

x_t = value of Index x at time t

y_t = value of Index y at time t

3.5.2 DCC-GARCH

Due to the volatilities of stocks moving together over time, it is a necessity to take into account the co-movement of asset returns by using multivariate GARCH models. This can be done by extending the univariate GARCH model for the estimation of the covariance and correlation matrix. Trying to construct a parsimonious multivariate GARCH model that is still flexible is difficult. One such approach is the decomposition of the conditional covariance matrix into its conational standard deviation and conditional correlation matrix. Bollerslev (1990) introduced the constant conditional correlation GARCH model (CCC-GARCH). This model assumed the conditional correlation is constant over time and the conditional standard deviation is time-varying.

Assuming that the conditional correlation is constant over time is not reasonable. Hence, Engle (2002) proposed a dynamic conditional correlation estimation that will be as flexible as the univariate GARCH model but not as complex as the multivariate GARCH model. This model is known as the DCC-GARCH model because the conditional correlation matrix and the conditional standard deviation will vary over time.

Following Engle (2002) the DCC-GARCH model is given by;

$$\Delta r_t \equiv \Delta r_{1t}, \Delta r_{2t} \qquad (20)$$

$$\Delta r_t | \Omega_{t-1} \sim N(0, D_t R_t D_t) \quad (21)$$

$$D_t^2 = diag\{w_1 w_2\} + diag\{k_1 k_2\}\, o\Delta r_{t-1}\Delta r_{t-1}' + diag\{\lambda_1 \lambda_2\} o D_{t-1}^2 \qquad (22)$$

$$\varepsilon_t = D_t^{-1} \Delta r_t \quad (23)$$

$$Q_t = S(1 - \alpha - \beta) + \alpha(\varepsilon_t \varepsilon_{t-1}')\beta Q_{t-1} \quad (24)$$

$$R_t = \, diag\{Q_t\}^{-1} Q_t diag\{Q_t\}^{-1} \; (25)$$

Δr_t is the bivariate vector of stock index returns. The $D_t R_t D_t$ is a 2x2 diagonal matrix of the conditional variance-covariance matrix of time-varying standard deviation from a univariate GARCH model and a correlation matrix containing the conditional correlation coefficients. The w_1, k_1, λ_1 symbols are the constants and coefficients from the ARCH and GARCH terms, respectively. The standardised residuals are indicated by ε_t. In the unconditional correlation matrix S, Q is the standardised residuals. The symbol o is the Hadamard product of two matrices of the same size (Égert & Kocenda, 2009).

This model is estimated in two steps: in the first step the univariate GARCH models are estimated and in the second step the conditional correlation matrix and the parameters are estimated. The DCC-GARCH model can be estimated using this two-step approach as it will avoid the dimensionality issues prevalent in multivariate GARCH models. Lastly, this DCC-GARCH model is parsimonious and ensures that there are time-varying correlation matrices produced between indices returns that are positive definite (Égert & Kocenda, 2009).

The implementation of the DCC-GARCH model will contain the daily return data from the BRICS indices. The univariate GARCH model for each of the indices will be determined individually which is used to determine the DCC-GARCH model. Once the DCC-GARCH model is estimated the mean from the daily correlation matrix will be calculated with a 5-year window.

4. Results

This section of the study will look at the results of the portfolio strategies and the co-movement between the BRICS countries. This section will be split into four main sections with the first and second sections analysing the performance of the risk optimisation portfolios and market-capitalisation weighted portfolio for the South African investment universe and the BRICS investment universe, respectively. The third section will be analysing the effects of converting the price of the BRICS indices from their domestic currency to the dollar as this had a major impact on the returns of the portfolios. The final section will look at the co-movement between the BRICS indices.

4.1 Performance of the risk optimisation portfolio in South Africa

This section of the results examines the performance of the risk optimisation portfolios and the market-capitalisation weighted portfolio when using the domestic investment universe of South Africa. The results will look at the entire sample period which is from the start of 2000 up to the end of 2019. The results of the sub-sample periods of the domestic investment universe are explored in Appendix 1.

Figure 1: South Africa's Performance Summary Pre Transaction Costs for 2000:2019

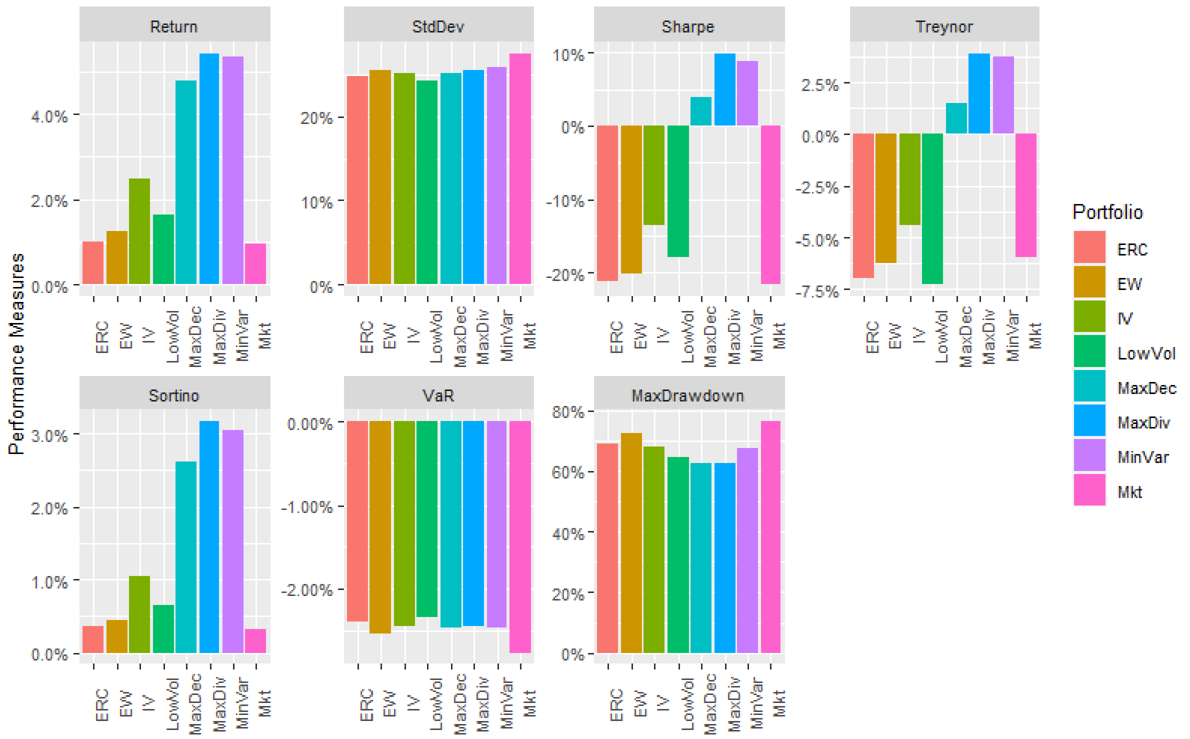


Figure 2: South Africa's Performance Summary Post Transaction Costs for 2000:2019

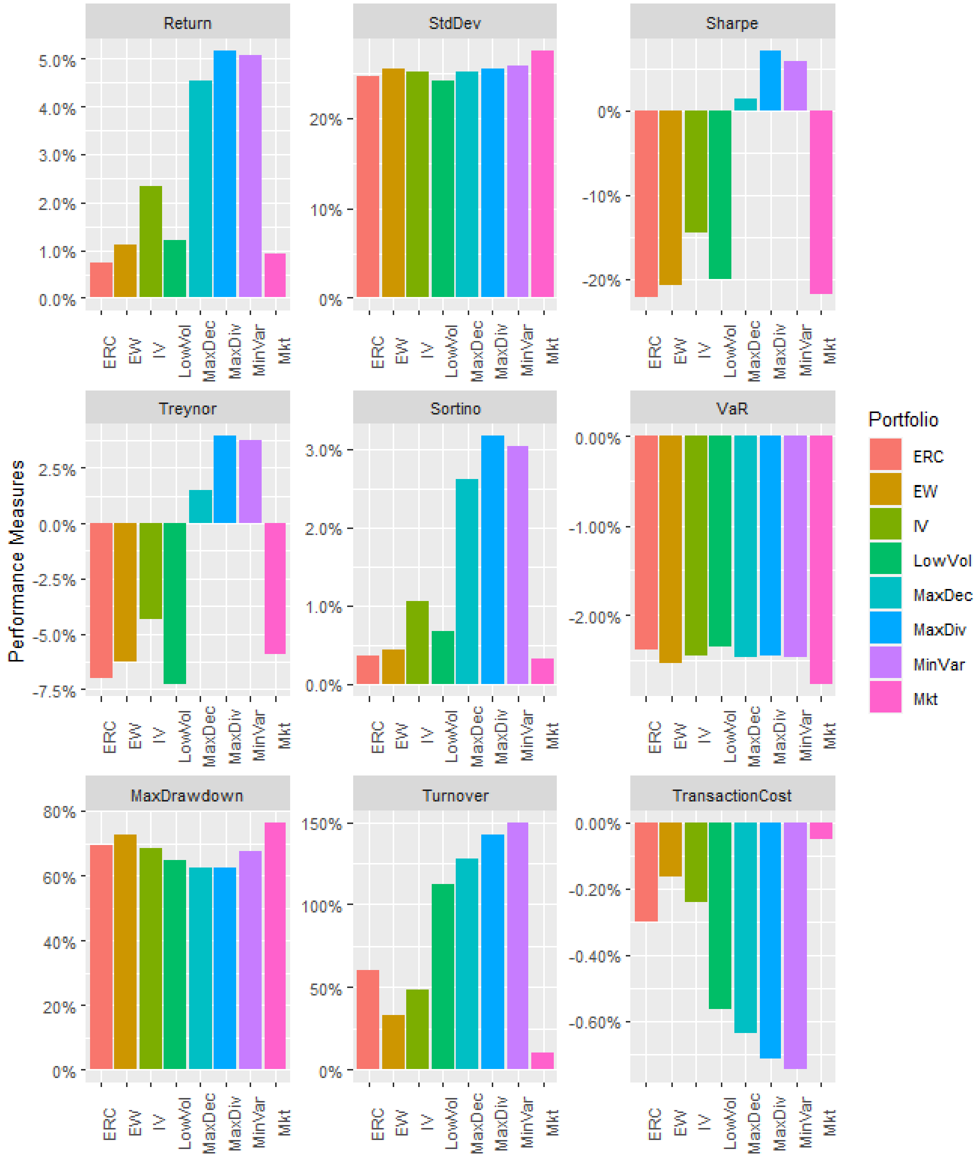


Figure 3: Cumulative Returns for SA between 2000:2020

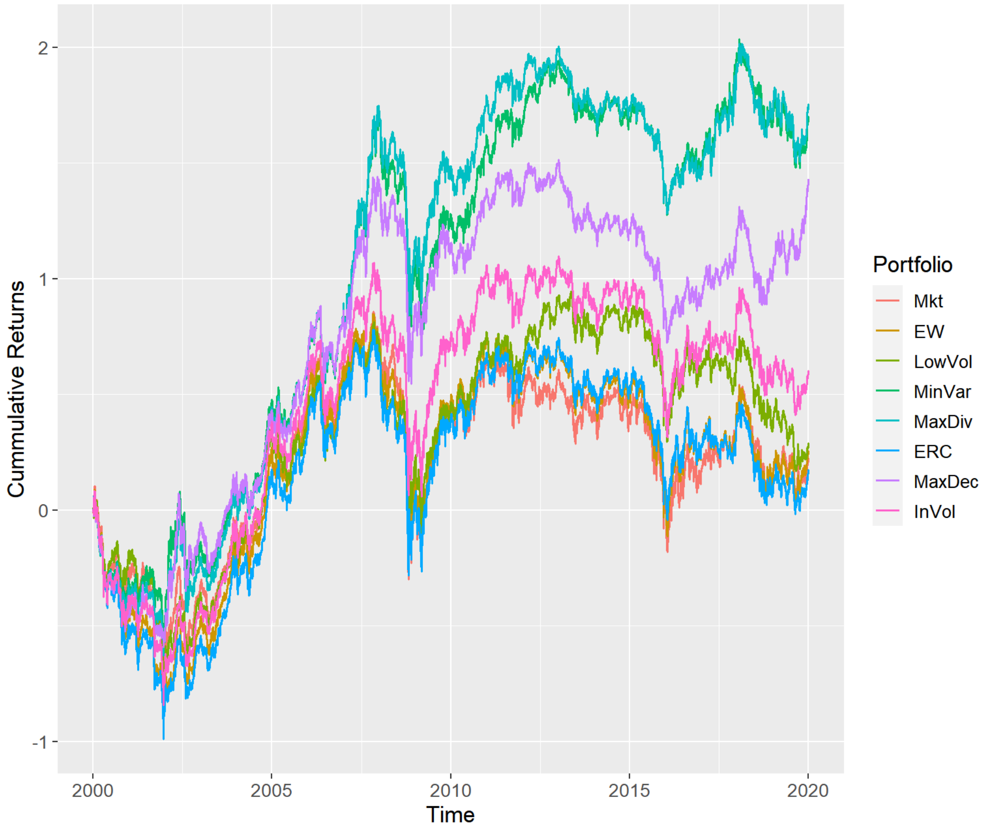


Figure 1 reports the performance of the risk optimisation and market-capitalisation portfolio before taking into account transaction cost. It should be noted that all of the performance measures are based on Dollars and not Rand's. Based on the returns in Figure 1 it can be seen that the all of the portfolios had a positive return over the 20 years with the market-capitalisation weighted and equal risk contribution portfolio having the worst returns. The maximum diversification portfolio had the highest returns over the sample period followed by the minimum variance and maximum decorrelation portfolio.

The portfolio with the lowest standard deviation was the inverse volatility portfolio whereas, the market-capitalisation weighted portfolio had the highest standard deviation. The maximum diversification portfolio had the highest Sharpe, Treynor, and Sortino ratio with the average risk-free rate been 3.35% for the 20 year sample period. The minimum variance portfolio and maximum decorrelation portfolio was the only other portfolio that had a positive Sharpe and Treynor ratio. The market-capitalisation weighted portfolio had the worst Sharpe and Sortino ratio which corresponds to it having the lowest return and highest standard deviation.

Therefore, based on these performance measures, on a risk-adjusted basis, the market-capitalisation weighted portfolio was the worst overall portfolio, and the maximum diversification portfolio been the best portfolio which is in agreement with the literature that the market-capitalisation weighted portfolio is indeed inefficient. While the alternative risk optimisation strategies such as the maximum diversification portfolio and minimum variance portfolio were better able to compensate one for the risk undertaken despite the estimation errors that these portfolios are prone to when the weights of the portfolio are based on the previous year's returns.

Based on the historical VaR it can be seen that at the 95% level for any given day the portfolio that had the lowest possible loss (the largest VaR value) was the low volatility portfolio followed by the equal risk contribution portfolio, whereas, the market-capitalisation weighted had the highest possible loss. Based on the maximum drawdown the portfolio that had the worst peak to trough performance was the market-capitalisation weighted portfolio followed by the equal-weighted portfolio. The maximum diversification portfolio and maximum decorrelation portfolio had the lowest maximum drawdown. Based on these results it can again be concluded that the market-capitalisation weighted portfolio is the worst portfolio that one can invest in since it was the most likely to suffer the largest loss for a given probability on a given day and had the worst maximum drawdown.

When looking at the results post-transaction cost in Figure 2 there is no meaningful change in the performance of the portfolios when using a transaction cost of 50bps with the market-capitalisation weighted portfolio still performing poorly and the maximum diversification, minimum variance, and maximum decorrelation portfolio been the three best performing portfolios. However, when taking into account transaction cost the equal risk contribution portfolio does perform worse than market capitalisation weighted portfolio. This is in agreement with Hsu (2004) that stated the market capitalisation weighted portfolio requires minimal rebalancing in comparison to the alternative portfolios. This can also be seen in that the market-capitalisation weighted portfolio contains the lowest average yearly turnover over the entire sample period. The minimum variance portfolio had the highest turnover followed by the maximum diversification portfolio. It can be seen that the portfolio strategies that inherently invest in all of the available stocks will have lower turnover and transaction costs. This can be seen with the market-capitalisation weighted portfolio, equal-weighted portfolio, ERC portfolio, and inverse volatility portfolio. Whereas, the portfolio strategies that do not invest in every stock had turnovers of greater than 100% per year which was the low volatility portfolio, minimum variance portfolio, maximum diversification portfolio, and maximum decorrelation portfolio.

Based on the results that the maximum diversification portfolio was able to perform the best on a risk-adjusted basis when taking into account the transaction cost that will occur as well as the estimation errors. Concurring with Haugen and Baker (1991) who stated that the market-capitalisation weighted portfolio is a suboptimal investments as alternative strategies will contain lower risk and higher returns. The market-capitalisation weighted portfolio is by far the most efficient in terms of having the lowest turnover and transaction cost as this portfolio does contain automatic rebalancing. However, in terms of risk-adjusted performance, the market-capitalisation weighted portfolio performs the second worst while alternative risk optimisation strategies such as the maximum diversification portfolio and minimum variance portfolio are the most efficient portfolios. This is in agreement to the finding of Clarke et al. (2006), Choueifaty and Coignard (2008), and Lohre, Neugebauer and Zimmer (2012).

Based on the results in the full and subsample periods for the South African investment universe. The market-capitalisation weighted portfolio was always inefficient in terms of risk-adjusted-performance. It was consistently the most efficient portfolio in terms of maintaining the lowest turnover. Whereas, minimum variance portfolio, maximum diversification portfolio, and maximum decorrelation portfolio were constantly able to outperform the market-capitalisation weighted portfolio on a risk-adjusted basis despite having a higher turnover. Appendix 1 contains an in-depth analysis for the subsample periods due to the results been similar to that of the full sample period.

Figure 3 plots the cumulative returns of the market-capitalisation weighted portfolio and the risk optimisation portfolios after taking into account transaction cost. These results echo the findings of the annualised returns that the market-capitalisation weighted and equal risk contribution portfolios are the worst performing portfolios over the sample period. Whereas, the best performing portfolios were the minimum variance and maximum diversification portfolio. Lastly, it can be seen that there is a great deal of correlation between the various portfolios over the sample period which makes sense due to the investment universe been the same between each portfolio strategy.

4.2 Performance of the risk optimisation portfolio in BRICS

This section of the results examines the performance of the risk optimisation portfolios and the market-capitalisation weighted portfolio when using the international investment universe of BRICS. Appendix 2 looks at the performance of the four sub-sample periods of 5 years each for the BRICS investment universe.

Figure 4: BRICS's Performance Summary Pre Transaction Costs for 2000:2019

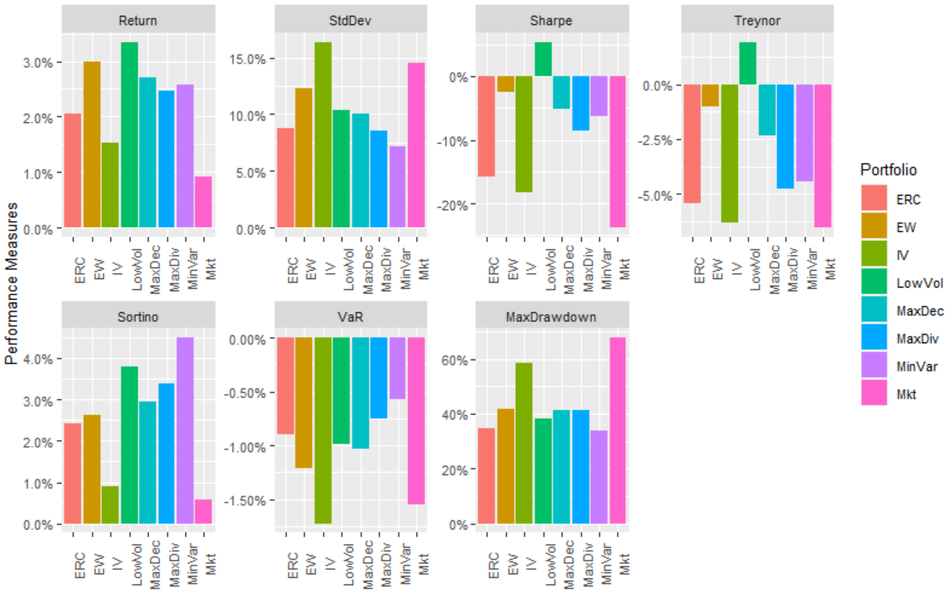


Figure 5: BRICS's Performance Summary Post Transaction Costs for 2000:2019

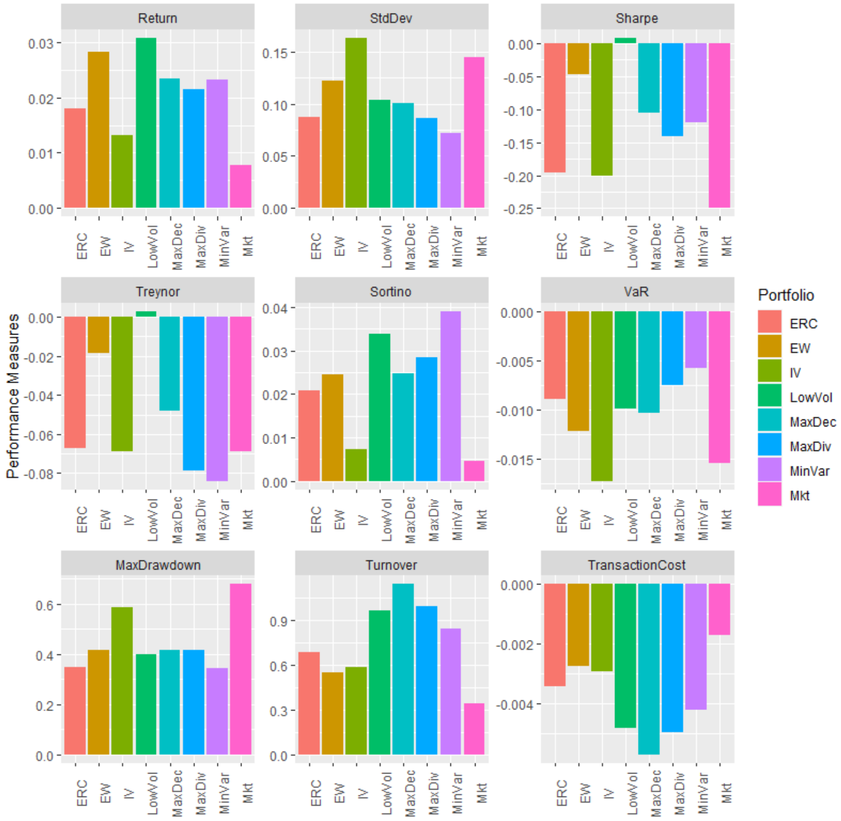


Figure 6: Cumulative Returns for BRICS between 2000:2019

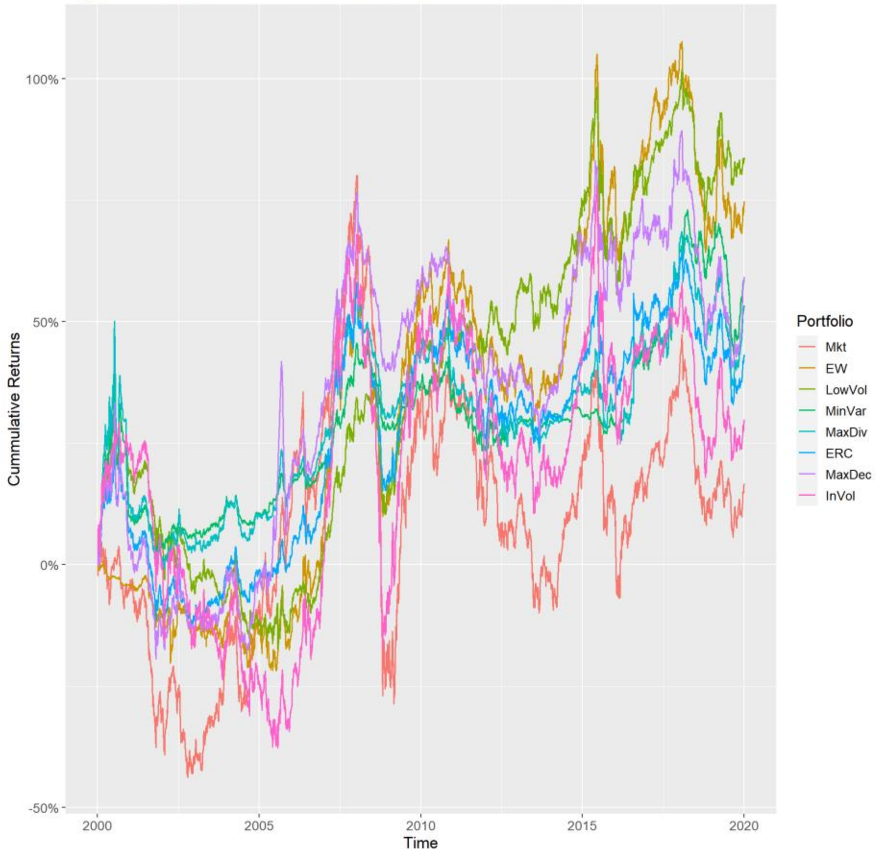


Figure 4 reports the results over the entire sample period for the BRICS investment universe before taking into account transaction cost. Each of the portfolio strategies produced a positive return. With the market-capitalisation weighted portfolio performing the worst. The market-capitalisation weighted portfolio also contained the worst Sharpe, Treynor, and Sortino ratio and maximum drawdown. The low volatility portfolio was the best performing portfolio and the only portfolio with a positive Sharpe and Treynor ratio. The minimum variance portfolio contained the lowest standard deviation followed by the maximum diversification and the equal risk contribution, respectively. Therefore, it can be concluded that even though all of the risk optimisation strategies except for the low volatility portfolio had negative risk-adjusted-performance it was still able to outperform the market-capitalisation weighted portfolio.

Based on the maximum drawdown the portfolio that had the largest peak to trough movement was the inverse volatility portfolio followed by the market-capitalisation weighted portfolio whereas, the maximum diversification portfolio had the lowest maximum drawdown followed by the low volatility portfolio, resulting in the low volatility and maximum diversification portfolio been the best portfolio.

Figure 5 reports the results of post-transaction cost for the BRICS investment universe. In agreement with that of the results in South Africa, the average turnover for the market-capitalisation weighted portfolio is the lowest and the portfolios that inherently invest in all the available stocks have a lower turnover than the portfolios that do not invest in all the of the stocks. These portfolios are the minimum variance, maximum diversification, maximum decorrelation, and low volatility portfolio have much higher turnovers. This means that market-capitalisation weighted portfolio is much more efficient than those risk optimisation portfolios in terms of having a low turnover and transaction cost. However, on a risk-adjusted basis, the market-capitalisation weighted portfolio is the least efficient. Lastly, in conjunction with the findings of South Africa, the relative performance of the portfolios does not change that much after taking into account transaction cost.

These results are in agreement with the finding of Haugen and Bakers (1991), and Hsu (2004) that stated that the market-capitalisation weighted portfolio is a suboptimal portfolio when comparing the returns or standard deviation to alternative portfolio strategies but the most efficient portfolio in terms of minimising transaction cost.

Figure 6 plots the cumulative returns of the full sample period after taking into account transaction cost. The results are in concurrence with the Figure 6 in the worst performing portfolio being the market-capitalisation weighted portfolio and the best performing portfolio being the low volatility portfolio.

The results of the subsample period do not differ significantly as the market-capitalisation weighted portfolio was consistently the worse performing portfolio and the low volatility, maximum diversification, and minimum variance portfolio have been one of the best performing portfolios. Appendix 2 contains an in-depth analysis of the performance of the sub-sample periods for the BRICS investment universe.

4.2 Performance summary of the risk optimisation portfolio in South Africa and BRICS

Based on the results from South Africa and the BRICS countries the market-capitalisation portfolio performed poorly in the full 20 year sample period and the four 5 year sub-sample periods on a risk-adjusted basis and in terms of the VaR and maximum drawdown. In contrast, the minimum variance portfolio and the maximum diversification portfolio was able to perform well in most of the periods and during the periods in which it performed poorly, it was at least able to maintain one of the lowest standard deviations between the portfolios which the market-capitalisation portfolio was not able to do.

However, it should be noted that when the market-capitalisation weighted portfolio was able to obtain the lowest turnover in comparison to the other portfolios. Whereas the risk optimisation portfolios that tend not to invest in all the stocks were more likely to contain the highest turnover. With the exception being the inverse volatility portfolio in the BRICS investment universe that contained a high turnover. This could be due to there been large volatility swings between the countries in the sample period which would result in large changes to the inverse volatility, as this portfolio assigns the least volatile stocks the greatest weight. Despite the minimum variance portfolio and the maximum diversification portfolio having high turnovers, it was still able to perform relatively well with an estimation period.

The variability in correlations between the portfolios are much greater in Figure 6 than that of Figure 3. This is most likely due to the BRICS investment universe being much larger and the amount of stocks in each portfolio having a larger variance than that of the South African portfolios.

4.4 The Dollar effect on the returns of the indices

Figure 7: BRICS Performance Summary

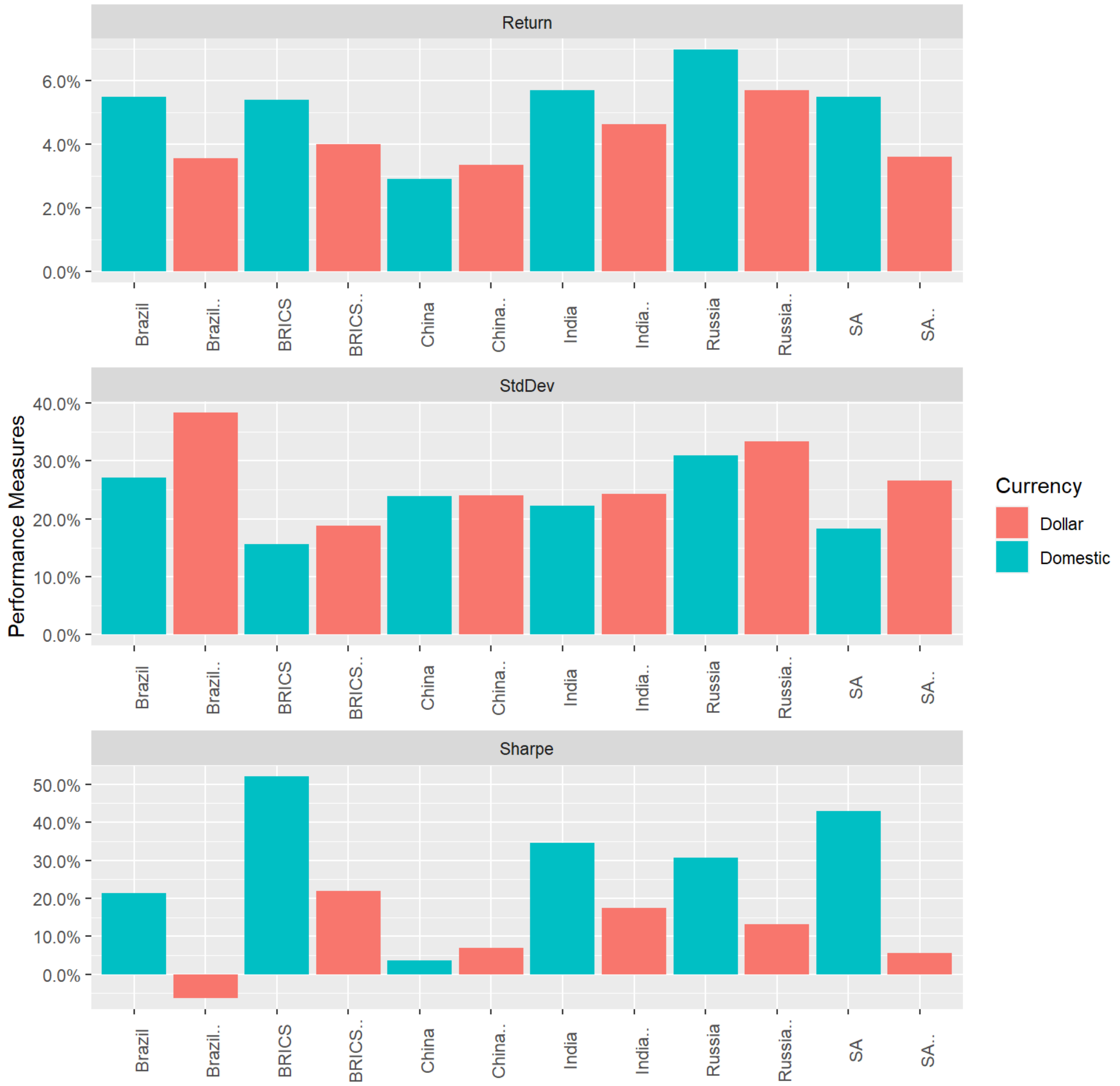


Figure 8: Cumulative Returns for BRICS indices

Due to this study converting all the prices to the dollar. This section will look at the effect of converting the prices of the indices to Dollars to provide a possible reason why the returns of the portfolio strategies might seem to produce underwhelming returns when the prices of each stock are converted from the domestic currency to the US Dollar. Based on the results from Figure 7 it can be seen that the returns of the JALSH (SA) decreased from 5.48% per annum to 3.61% per annum while the volatility of the JALSH increased from 18.34% per annum to 26.56% per annum. Therefore, it can be seen that from 2000 to 2019 the depreciation of the Rand relative to the US Dollar has affected the returns and volatility of the JALSH adversely.

When investigating each of the BRIC indices individually in the home currency vs the price of the indices converted to dollars it can be seen in Figure 7 that the Ibovespa index (Brazil), IMOEX (Russia) and Sensex (India) all decreased over the sample period. The SHCOMP index (China) is the only index whose returns increased over. This means that over the entire sample period the Yuan is the only currency to appreciate relative to the Dollar. The volatility of each of the indices on the other hand all increased when comparing the index in their home currency vs the index price converted to dollars. As such, it can be seen that the Brazilian Real, Russian Ruble, and Indian Rupee all depreciated while the Chinese Yuan appreciated. When comparing the Sharpe ratios between each country assuming a zero percent risk-free rate, it can be seen that both in the domestic currency the JALSH was the best performing risk-adjusted index with SHCOMP been the worst. However, when converting it to the dollar the best performing risk-adjusted index was the Sensex and the worst was the Ibovespa.

The BRICS index portfolio was constructed using the same methodology as Watson and Seetharam (2014) who constructed a BRICS index by creating an equal-weighted portfolio of the 5 country indices as there is currently no available BRICS index available. Similar to the performance of the JALSH the returns of the BRICS indices when converted from their home currency to the Dollar returns decreased while the volatility increased. It contained the highest Sharpe ratio both before and after converting to dollars, which means that there were international diversification benefits from this point of view. The relative effect on the BRICS indices is much smaller than that of the individual indices, which means that the currency depreciation of the BRICS countries combined is less than the depreciation or appreciation of the individual currencies. International diversification will help to reduce the adverse effects of currency depreciation relative to the US Dollar.

4.3 DCC-GARCH and Pearson correlation results

The final section of the results will look at the DCC-GARCH to examine if the correlations between the indices change drastically and examine the general trends in the indices return relationships. Appendix 3 contains the DCC-GARCH and Pearson correlation matrices and the Pearson correlation graphs for the BRICS indices.

Figure 9: Brazil's DCC GARCH with the remaining BRICS

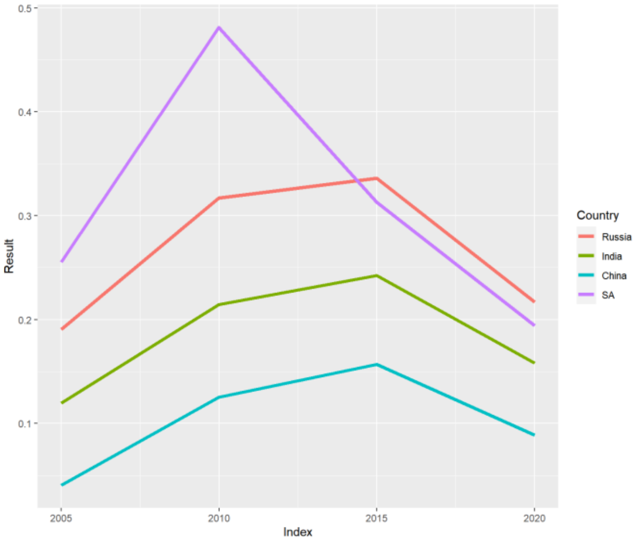


Figure 10: Russia's DCC GARCH with the remaining BRICS

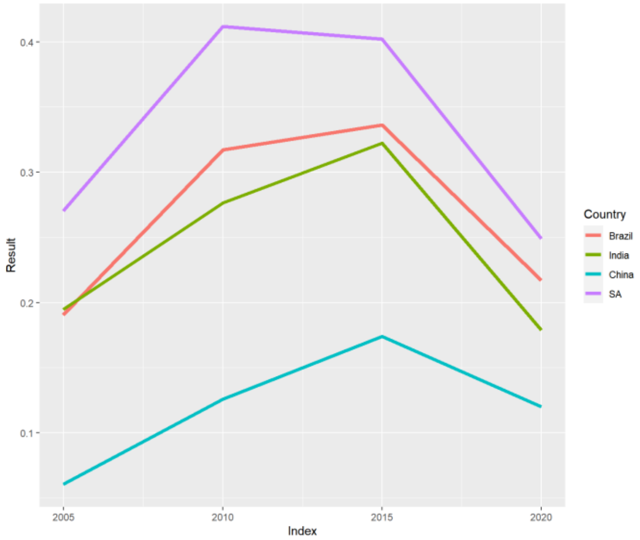


Figure 11: India's DCC GARCH with the remaining BRICS

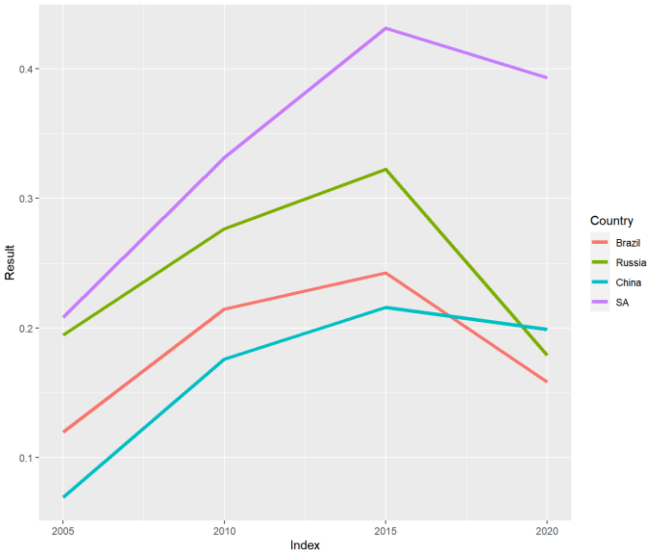


Figure 12: China's DCC GARCH with the remaining BRICS

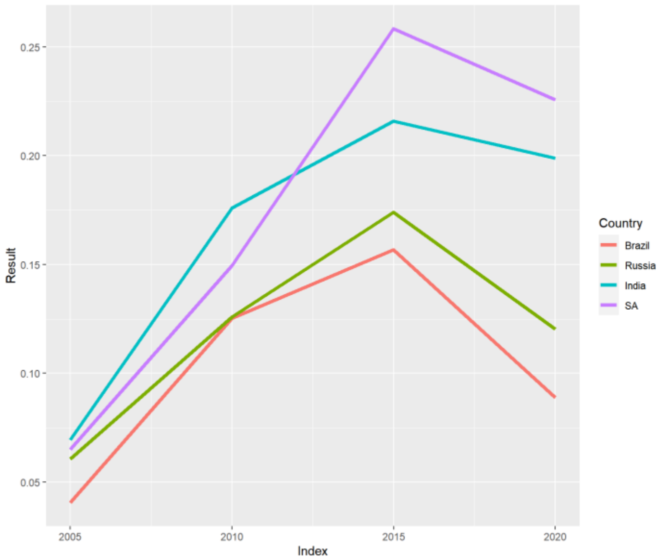
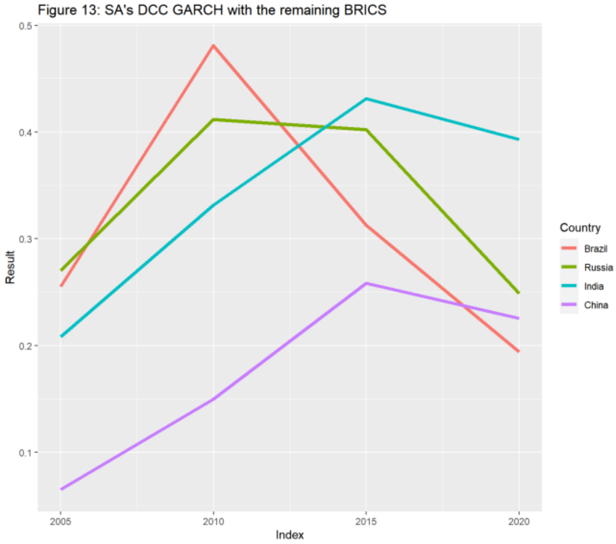


Figure 13: SA's DCC GARCH with the remaining BRICS



Based on the overall trends between the DCC-GARCH (Figure 9:13). Starting with examining the DCC-GARCH correlation of Brazil it can be seen they had the highest correlation with South Africa followed by Russia, India, and China. Brazil's correlation with China never exceeded 0.2 over this period while South Africa's correlation with Brazil peaked at 0.5 between 2005 and 2009 before decreasing to around 0.2 between 2015 and 2019. For all 4 countries' correlation with Brazil increased between 2000 and 2009 before decreasing between 2010 and 2019.

The DCC-GARCH for Russia found that they are most correlated with South Africa peaking at 0.41 and with a minimum of 0.25 between 2005:2009 and 2015:2019 respectively, while they are least correlated with China with a peak and trough of 0.18 and 0.05 between 2010:2014 and 2000:2004 respectively. Lastly, Russia was correlated with Brazil the second most followed by India in third. Based on the DCC-GARCH of India, they are most correlated with South Africa during the entire sample period with a peak of 0.45 between 2010 and 2014 with a trough of 0.20 between 2000 and 2004. India was correlated the second most with Russia between 2000 and 2014 but then they become the least correlated country between 2015 and 2019. India was correlated with Brazil the second least between 2000 and 2014 and they become the least correlated between 2015 and 2019. Lastly, India is correlated with China the least between 2000 and 2014 and they become the second most correlated country for India between 2015 and 2019 starting with a correlation of 0.06 and increasing to 0.2.

The DCC-GARCH for China finds that they have a low correlation with all of the remaining BRICS countries with India been the highest between 2000 and 2004 and then South Africa becomes the highest correlated country for China between 2010 and 2019 with a peak of 0.26. During this time South Africa and India swapped positions from second to first while Russia remains the third most correlated country with China during the entire period and Brazil been the least correlated country.

Lastly, the DCC-GARCH for South Africa found that they are most correlated with Brazil between 2000 and 2010 with a peak of 0.49, however, Brazil becomes the least correlated country between 2015 and 2019 with a minimum of 0.2. Russia was the second most correlated country for South Africa between 2000 and 2009 but then they drop to third place between 2010 and 2019. India on the other hand starts as being the second least correlated country with South Africa between 2000 and 2009 and they become the most correlated country between

2010 and 2019. Lastly, between 2000 and 2014 China was the least correlated country for South Africa and then they become the second least correlated country with South Africa.

Therefore, during this period the highest DCC-GARCH came between South Africa and Brazil which was 0.48 between 2005 and 2009. In contrast to this China and Brazil had the lowest DCC-GARCH of 0.04 and Pearson's correlation of 0.01 between 2000 and 2004. It can also be seen that between 2000 and 2010 the correlation between the BRICS countries increased but they have decreased slightly between 2011 and 2019. This indicates that there were international diversification benefits to be gained between the BRICS countries due to their indices not been highly correlated with each other. The results for Pearson's correlation obtained more a less the same trend pattern for each country in the sample period and each country showed a similar pattern to the other countries. The highest Pearson's correlation was between the 2005 and 2009 periods for South Africa and Brazil with a correlation of 0.62. In contrast to this China and Brazil had the lowest DCC-GARCH of 0.04 and Pearson's correlation of 0.01 between 2000 and 2004.

These results are in agreement with Lehkonen and Heimonen (2014) who stated that the correlation structures between BRIC countries vary significantly between markets. This is in coherence with Mensi et al. (2016) that indicated that investors can gain extensive international diversification benefits.

5. Conclusion

This study looked at various risk-optimisation strategies which included a number of portfolios; the equal-weighted, minimum variance, low volatility, equal risk contribution, maximum diversification, maximum decorrelation, and the inverse volatility portfolio as an alternative portfolio strategy to the market capitalisation weighted portfolio. This is due to the inefficiencies of not compensating one adequately for their risk that is prevalent in the market-capitalisation weighted portfolio. However, the market-capitalisation weighted portfolio has minimal transaction costs and estimation errors that are prevalent in the risk optimisation portfolios. Taking into account that all portfolios had an estimation period and transaction cost was to provide a more realistic approach in the construction of portfolios.

Furthermore, this study wanted to investigate the performance of the risk optimisation portfolios on a risk-adjusted basis for the BRICS countries combined to create a single investment universe and South Africa on its own. To account for inflation and any other issues that might affect the validity of the results, all of the prices for the individual stocks and indices were converted to the Dollar due to it being the vehicle currency of the world.

Based on the results from South Africa and the BRICS countries the market capitalisation portfolio performed poorly in the full 20 year sample period and the four 5 year sub-sample periods on a risk-adjusted basis and in terms of the VaR and maximum drawdown measure. In contrast to this, the minimum variance portfolio and the maximum diversification portfolio were able to perform well in most of the periods and during the periods in which it performed poorly, it was at least able to maintain one of the lowest standard deviations between the portfolios which the market capitalisation portfolio was not able to do.

The sample period for this study was from 1999 to 2019, however, due to this study implementing an estimation period the performance of the portfolios was examined between 2000 and 2019. Based on the results from South Africa and BRICS the market-capitalisation portfolio performed poorly in the full 20 year sample period and the four 5 year sub-sample periods on a risk-adjusted basis and in terms of the VaR and maximum drawdown. In contrast to this, the minimum variance portfolio and the maximum diversification portfolio were able to perform well in most of the periods and during the periods in which it performed poorly, it was at least able to maintain one of the lowest standard deviations between the portfolios which the market capitalisation portfolio was not able to do. These results concurred with the findings of Clarke et al. (2006), Choueifaty and Coignard (2008), and Lohre, Neugebauer and Zimmer (2012) as they indicated that risk optimisation portfolios are more efficient than the market-capitalisation weighted portfolio.

Despite the poor performance from the market capitalisation weighted portfolio it constantly contained the lowest turnover whereas, the low volatility portfolio, minimum variance portfolio, and maximum diversification portfolio contained significantly higher turnover and in some cases contained a turnover of around 100%. Which is comparable to the findings of Haugen and Baker (1991) that stated that the market-capitalisation weighted portfolio is the most efficient portfolio in terms of transaction cost.

When examining the correlation between the indices it was found that the indices are not highly correlated meaning that there were international diversification benefits over this period and this could also be seen in that the equal-weighted BRICS index had a higher return and lower volatility than the BRICS indices individually. However, it should be noted that all currencies except for the Yuan depreciated significantly in comparison to the Dollar. Due to this depreciation, it significantly decreased the returns and increased the volatility of the indices and stocks over this period. This caused many of the portfolios not been able to outperform the US T-bill rate over this period.

Future research can aim to investigate international diversification in Frontier markets and alternative emerging markets such as MIST. Lastly, future research can investigate the effects of different market filters and the effects that these filters will have on the liquidity of the portfolios. This is due to large performance differences between the market capitalisation weighted portfolio and the indices used as a benchmark.

6. References

Al Nasser, O. M., & Hajilee, M. (2016). Integration of emerging stock markets with global stock markets. *Research in International Business and Finance*, *36*, 1-12. DOI: 10.1016/j.ribaf.2015.09.025.

Amenc, N., & Goltz, F. (2013). Smart Beta 2.0. *The Journal of Index Investing*, *4*(3), 15-23. DOI: https://doi.org/10.3905/jii.2013.4.3.015

Amenc, N., Goltz, F., & Lodh, A. (2012). Choose your betas: Benchmarking alternative equity index strategies. *The Journal of Portfolio Management*, *39*(1), 88-111. DOI: https://doi.org/10.3905/jpm.2012.39.1.088

Asness, C. S., Israelov, R., & Liew, J. M. (2011). International diversification works (eventually). *Financial Analysts Journal*, *67*(3), 24-38. DOI: https://doi.org/10.2469/faj.v67.n3.1

Bekaert, G., & Harvey, C. R. (1995). Time‐varying world market integration. *The Journal of Finance*, *50*(2), 403-444. DOI: https://doi.org/10.1111/j.1540-6261.1995.tb04790.x

Bender, J., Blackburn, T., & Sun, X. (2019). Clash of the Titans: Factor Portfolios versus Alternative Weighting Schemes. *The Journal of Portfolio Management*, *45*(3), 38-49. DOI: https://doi.org/10.3905/jpm.2019.45.3.038

Bhattacharya, U., & Galpin, N. (2011). The global rise of the value-weighted portfolio. *Journal of Financial and Quantitative Analysis*, *46*(3), 737-756. DOI: https://doi.org/10.1017/S0022109011000044

Billio, M., Donadelli, M., Paradiso, A., & Riedel, M. (2017). Which market integration measure?. *Journal of Banking & Finance*, *76*, 150-174. DOI: https://doi.org/10.1016/j.jbankfin.2016.12.002

Boamah, N. A., Akotey, J. O., & Aaawaar, G. (2020). Economic engagement and within emerging markets integration. *Research in International Business and Finance*, *52*, DOI: 101106. https://doi.org/10.1016/j.ribaf.2019.101106

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *The review of economics and statistics*, 498-505. DOI: https://doi.org/10.2307/2109358

Boudt, K., Raza, M. W., & Wauters, M. (2017). Evaluating the Shariah-compliance of equity portfolios: The weighting method matters. *International Review of Financial Analysis*. DOI: https://doi.org/10.1016/j.irfa.2017.12.003

Bradfield, D., & Munro, B. (2017). The number of stocks required for effective portfolio diversification: the South African case. *South African Journal of Accounting Research*, *31*(1), 44-59. DOI: https://doi.org/10.1080/10291954.2015.1122284

Brooks, R., & Del Negro, M. (2004). The rise in comovement across national stock markets: market integration or IT bubble?. *Journal of Empirical Finance*, *11*(5), 659-680. DOI: https://doi.org/10.1016/j.jempfin.2003.08.001

Butler, A. (2018). Portfolio Optimization and the Sharpe mulitiplier a case study on managed funds. Retrieved from https://www.advisorperspectives.com/articles/2018/11/12/portfolio-optimization-and-the-sharpe-multiplier-a-case-study-on-managed-futures.pdf

Choueifaty, Y. (2006). Methods and systems for providing an anti-benchmark portfolio. Patent number: USPTO 60/816, 276.

Choueifaty, Y., & Coignard, Y. (2008). Toward maximum diversification. *The Journal of Portfolio Management*, *35*(1), 40-51. DOI: https://doi.org/10.3905/JPM.2008.35.1.40

Choueifaty, Y., Froidure, T., & Reynier, J. (2013). Properties of the most diversified portfolio. *Journal of investment strategies*, *2*(2), 49-70. DOI: https://dx.doi.org/10.2139/ssrn.1895459

Chow, T. M., Hsu, J. C., Kuo, L. L., & Li, F. (2014). A Study of Low-Volatility Portfolio Construction Methods. *The Journal of Portfolio Management*, *40*(4), 89-105. DOI: https://doi.org/10.3905/jpm.2014.40.4.089

Christoffersen, P., Errunza, V., Jacobs, K., & Xisong, J. (2010). *Is the Potential for International Diversification Disappearing?* A Dynamic Copula Approach. *The Review of Financial Studies*, *25* (12), 3711–3751.
DOI: https://doi.org/10.1093/rfs/hhs104

Clarke, R. G., De Silva, H., & Thorley, S. (2006). Minimum-variance portfolios in the US equity market. *The Journal of Portfolio Management*, *33*(1), 10-24. DOI: https://doi.org/10.3905/jpm.2006.661366

Clarke, R., De Silva, H., & Thorley, S. (2011). Minimum-variance portfolio composition. *The Journal of Portfolio Management*, *37*(2), 31-45. DOI: https://doi.org/10.3905/jpm.2011.37.2.031

Clarke, R., De Silva, H., & Thorley, S. (2013). Risk parity, maximum diversification, and minimum variance: An analytic perspective. *The Journal of Portfolio Management*, *39*(3), 39-53. DOI: https://doi.org/10.3905/jpm.2013.39.3.039

De Carvalho, R. L., Lu, X., & Moulin, P. (2012). Demystifying equity risk-based strategies: A simple alpha plus beta description. *The Journal of Portfolio Management*, *38*(3), 56-70. DOI: https://doi.org/10.3905/jpm.2012.38.3.056

DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?. *The review of Financial studies*, *22*(5), 1915-1953. DOI: https://doi.org/10.1093/rfs/hhm075

Égert, B., & Kočenda, E. (2011). Time-varying synchronization of European stock markets. *Empirical Economics*, *40*(2), 393-407. DOI: https://doi.org/10.1007/s00181-010-0341-3

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, *20*(3), 339-350. DOI: https://doi.org/10.1198/073500102288618487

Fama, E. F. (1976). *Foundations of finance: portfolio decisions and securities prices*. Basic Books (AZ).

Grubel, H. G. (1968). Internationally diversified portfolios: welfare gains and capital flows. *The American Economic Review*, *58*(5), 1299-1314. Retrieved from https://www.jstor.org/stable/1814029

Gujarati, D. N., & Porter, D. (2009). Basic Econometrics Mc Graw-Hill International Edition.

Harlow, W. V. (1991). Asset allocation in a downside-risk framework. *Financial analysts' journal*, *47*(5), 28-40. DOI: https://doi.org/10.2469/faj.v47.n5.28

Haugen, R. A., & Baker, N. L. (1991). The efficient market inefficiency of capitalization-weighted stock portfolios. *The Journal of Portfolio Management*, *17*(3), 35-40. Retrieved from http://www.efalken.com/LowVolClassics/HaugenBaker991.pdf

Hsu, J. C. (2004). Cap-weighted portfolios are sub-optimal portfolios. *Journal of Investment Management*, *4*(3). DOI: https://dx.doi.org/10.2139/ssrn.647001

Hsu, J., & Li, F. (2013). Low-volatility investing. *The Journal of Index Investing*, *4*(2), 67-72. DOI: https://doi.org/10.3905/jii.2013.4.2.067

Kanuri, S., Malhotra, D., & Malm, J. (2018). Evaluating the Performance and Diversification Benefits of Emerging-Market Exchange-Traded Funds. *The Journal of Wealth Management*, *20*(4), 85-90. DOI: https://doi.org/10.3905/jwm.2018.20.4.085

Kremer, P. J., Talmaciu, A., & Paterlini, S. (2018). Risk minimization in multi-factor portfolios: What is the best strategy?. *Annals of Operations Research*, *266*(1-2), 255-291. DOI: https://doi.org/10.1007/s10479-017-2467-6

Ledoit, O., & Wolf, M. (2004). Honey, I shrunk the sample covariance matrix. *The Journal of Portfolio Management*, *30*(4), 110-119. Oliver, S. (2013). DOI: https://doi.org/10.3905/jpm.2004.110.

Lee, W. (2011). Risk-Based Asset Allocation: A New Answer to an Old Question?. DOI: https://doi.org/10.3905/jpm.2011.37.4.011

Lehkonen, H., & Heimonen, K. (2014). Timescale-dependent stock market comovement: BRICs vs. developed markets. *Journal of Empirical Finance*, *28*, 90-103. DOI: https://doi.org/10.1016/j.jempfin.2014.06.002

Lohre, H., Neugebauer, U., & Zimmer, C. (2012). Diversified risk parity strategies for equity portfolio selection. *The Journal of Investing*, *21*(3), 111-128. DOI: https://doi.org/10.3905/joi.2012.21.3.111

Lohre, H., Opfer, H., & Orszag, G. (2014). Diversifying risk parity. *Journal of Risk*, *16*(5), 53-79. DOI: https://dx.doi.org/10.2139/ssrn.1974446

Maillard, S., Roncalli, T., & Teiletche, J. (2010). The properties of equally-weighted risk contribution portfolios. *The Journal of Portfolio Management*, *36*(4), 60-70. DOI: https://doi.org/10.3905/jpm.2010.36.4.060

Malladi, R., & Fabozzi, F. J. (2017). Equal-weighted strategy: Why it outperforms value-weighted strategies? Theory and evidence. *Journal of Asset Management*, *18*(3), 188-208. DOI: https://doi.org/10.1057/s41260-016-0033-4

Markowitz, H. (1952). Portfolio selection. The journal of finance, 7(1), 77-91.

Mausser, H., & Romanko, O. (2014). Computing equal risk contribution portfolios. *IBM Journal of Research and Development*, *58*(4), 5-1. DOI: 10.1147/JRD.2014.2325291

Mensi, W., Hammoudeh, S., Nguyen, D. K., & Kang, S. H. (2016). Global financial crisis and spillover effects among the US and BRICS stock markets. *International Review of Economics & Finance*, *42*, 257-276. DOI: https://doi.org/10.1016/j.iref.2015.11.005

Mohanasundaram, T., & Karthikeyan, P. (2015). Cointegration and stock market interdependence: Evidence from South Africa, India and the USA. *South African Journal of Economic and Management Sciences*, *18*(4), 475-485. DOI: https://dx.doi.org/10.17159/2222-3436/2015/V18N4A3

Oliver, S. (2013). South africas brics membership: A win-win situation?. *African Journal of Political Science and International Relations*, *7*(7), 310-319. DOI: https://doi.org/10.5897/AJPSIR2013.0625

Plyakha, Y., Uppal, R., & Vilkov, G. (2014). Equal or value weighting? Implications for asset-pricing tests. DOI: https://dx.doi.org/10.2139/ssrn.1787045

Raza, M. W., & Ashraf, D. (2019). Does the application of smart beta strategies enhance portfolio performance? The case of Islamic equity investments. *International Review of Economics & Finance*, *60*, 46-61. DOI: https://doi.org/10.1016/j.iref.2018.12.001

Raza, M. W., L'Huillier, B., & Ashraf, D. (2019). The Effect of Market Regimes on the Performance of Market Capitalization-Weighted and Smart Beta Shariah-Compliant Equity Portfolios. DOI: https://dx.doi.org/10.2139/ssrn.3338365

Reddy, S. G. (2018). The Role of BRICS in the World Economy and International Development. Retrieved December 14, 2018. Retrieved from https://reddytoread.files.wordpress.com/2017/09/brics-2017.pdf

Reinganum, M. R. (1999). The significance of market capitalization in portfolio management over time. *The Journal of Portfolio Management*, *25*(4), 39-50. DOI: https://doi.org/10.3905/jpm.1999.319750

Reinganum, M. R. (1999). The significance of market capitalization in portfolio management over time. *The Journal of Portfolio Management*, *25*(4), 39-50. DOI: https://doi.org/10.3905/jpm.1999.319750

Samitas, A., Kenourgios, D., & Paltalidis, N. (2007). Financial crises and stock market dependence. In *14th Annual Conference of the Multinational Finance Society, Thessaloniki, Greece*. Retrieved from https://efmaefm.org/0EFMAMEETINGS/EFMA%20ANNUAL%20MEETINGS/2007-Austria/papers/0464.pdf

Scherer, B. (2011). A note on the returns from minimum variance investing. *Journal of Empirical Finance*, *18*(4), 652-660. DOI: https://doi.org/10.1016/j.jempfin.2011.06.001

Sharma, P. (2015). Performance of risk-based portfolios under different market conditions: Evidence from India. *Research in International Business and Finance*, *34*(1), 397-411. DOI: https://doi.org/10.1016/j.ribaf.2015.03.006

Sharpe, W. F. (1966). Mutual fund performance. *The Journal of business*, *39*(1), 119-138. Retrieved from https://www.jstor.org/stable/2351741

Sortino, F. A., & Price, L. N. (1994). Performance measurement in a downside risk framework. *The Journal of Investing*, *3*(3), 59-64. DOI: https://doi.org/10.3905/joi.3.3.59

Treynor, J. (2005). Why market-valuation-indifferent indexing works. *Financial Analysts Journal*, *61*(5), 65-69. DOI: https://doi.org/10.2469/faj.v61.n5.2757

Treynor, J. L. (1965). How to rate mutual fund performance. *Harvard Business Review*, *43*, 63-75. Retrieved from: https://www.jstor.org/stable/2351741

Van Vliet, P. (2018). Low volatility needs little trading. *The Journal of Portfolio Management*, *44*(3), 33-42. DOI: https://doi.org/10.3905/jpm.2018.44.3.033

Watson, J., & Seetharam, Y. (2014). Building a Portfolio with BRICS: The case of the international diversifier. *Investment Risk and Performance Feature Articles*, *1*, 1-4. Retrieved from: https://www.researchgate.net/profile/Yudhvir_Seetharam/publication/260981240_Building_a_portfolio_with_BRICS_The_case_of_the_international_diversifier/links/0a85e5331a0d4b34d6000000/Building-a-portfolio-with-BRICS-The-case-of-the-international-diversifier.pdf

Windcliff, H., & Boyle, P. P. (2004). The 1/n pension investment puzzle. *North American Actuarial Journal*, *8*(3), 32-45. DOI: https://doi.org/10.1080/10920277.2004.10596151

Wilson, D., & Purushothaman, R. (2003). *Dreaming with BRICs: The path to 2050* (Vol. 99, pp. 1-23). New York, NY: Goldman, Sachs & Company. Retrieved from: https://d1wqtxts1xzle7.cloudfront.net/31055102/3

Yanushevsky, R., & Yanushevsky, D. (2015). Comment on “A note on the returns from minimum variance investing”. *Journal of Empirical Finance*, *31*, 109-110. DOI: https://doi.org/10.1016/j.jempfin.2013.11.004

Zaremba, A. (2015). Country selection strategies based on value, size and momentum. *Investment Analysts Journal*, *44*(3), 171-198. DOI: https://doi.org/10.1080/10293523.2015.1060747

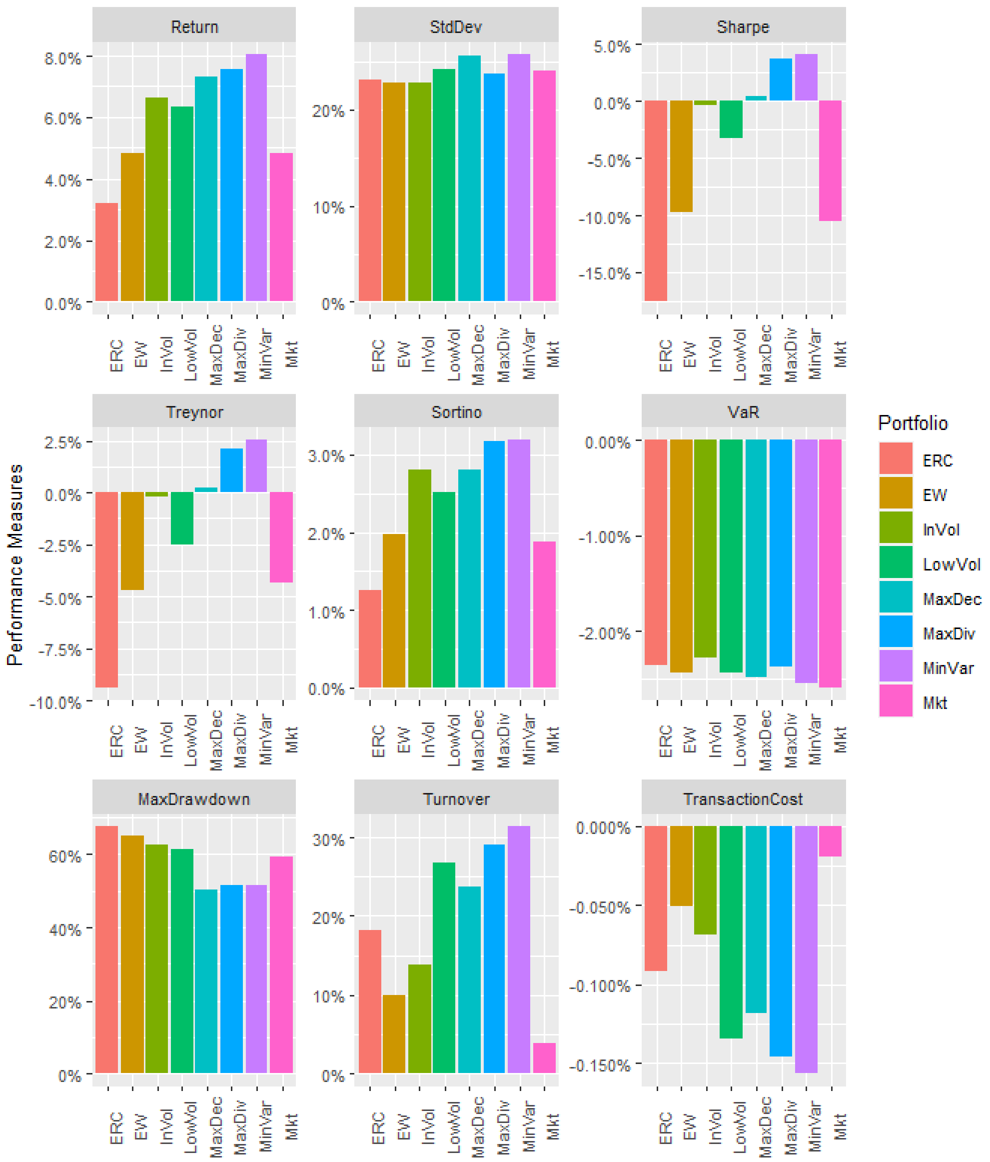
Appendix

The Appendix is split into three sections, the first section looks at the sub-periods of the South African universe, the second section looks at the sub-periods of the BRICS investment universe and the last section looks at the GARCH and Pearson's correlation results.

Appendix 1

Due to the results not showing any significant differences between pre and post-transaction costs that will cause the conclusion to change, for brevity only the results post-transaction cost will be presented.

Figure 14: South Africa's Performance Summary Post Transaction Costs for 2000:2004



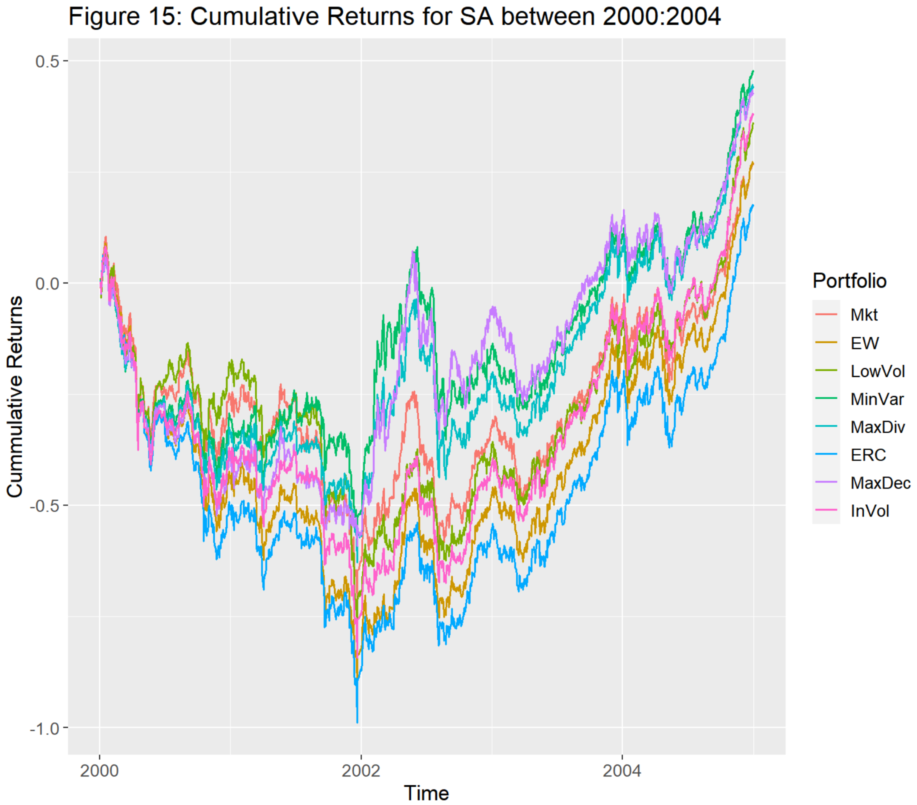


Figure 14 provides the performance summary between 2000 and 2004 post-transaction cost. The equal risk contribution portfolio was the worst-performing portfolio in terms of returns, Sharpe, Treynor, and Sortino ratio followed by the market-capitalisation weighted portfolio. On the other hand, the best performing portfolios were the minimum variance portfolio followed by the maximum diversification portfolio. Based on VaR the worst performing portfolio was the market-capitalisation weighted portfolio and the best been the inverse volatility portfolio. Based on the maximum drawdown, the portfolio that had the worst peak to trough was the equal risk contribution portfolio, while the best was the maximum decorrelation portfolio.

In agreement with prior literature, the market-capitalisation weighted portfolio is inefficient in terms of risk-adjusted performance. The market-capitalisation weighted portfolio had the lowest turnover and the minimum variance portfolio had the highest turnover which is in agreement with prior literature that the market-capitalisation weighted portfolio is efficient in terms of minimising transaction cost. Despite this, it does not make up for the inefficiency due to risk-adjusted performance as the minimum variance portfolio and maximum diversification portfolio were the best performing portfolios when accounting for transaction cost.

Figure 16: South Africa's Performance Summary Post Transaction Costs for 2005:2009

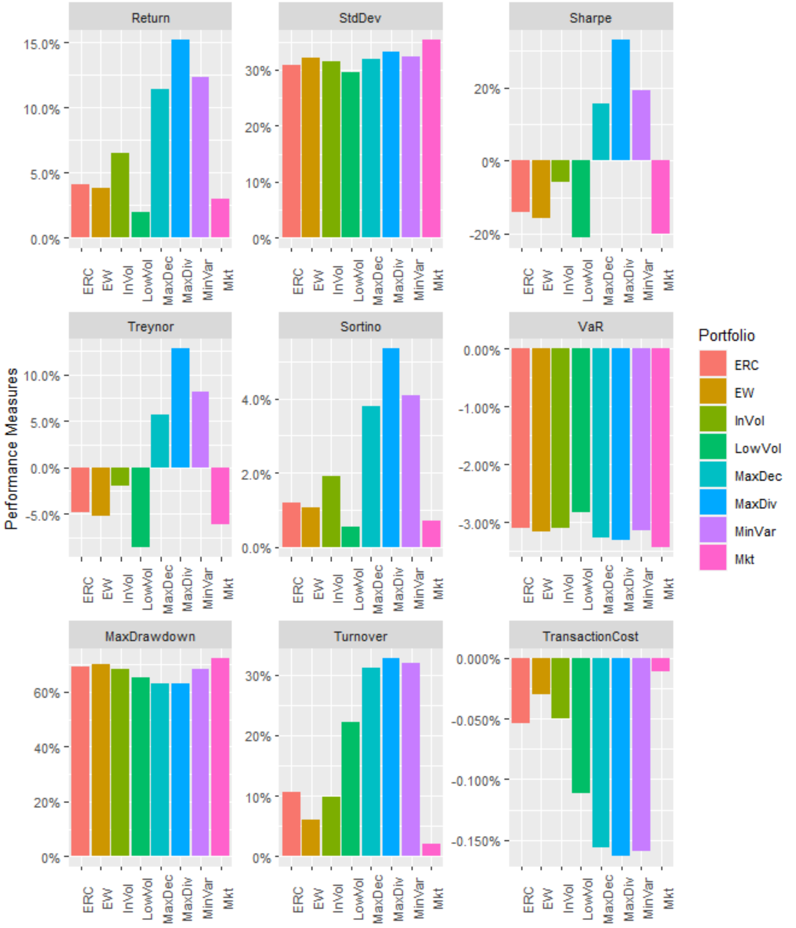


Figure 17: Cumulative Returns for SA between 2005:2009

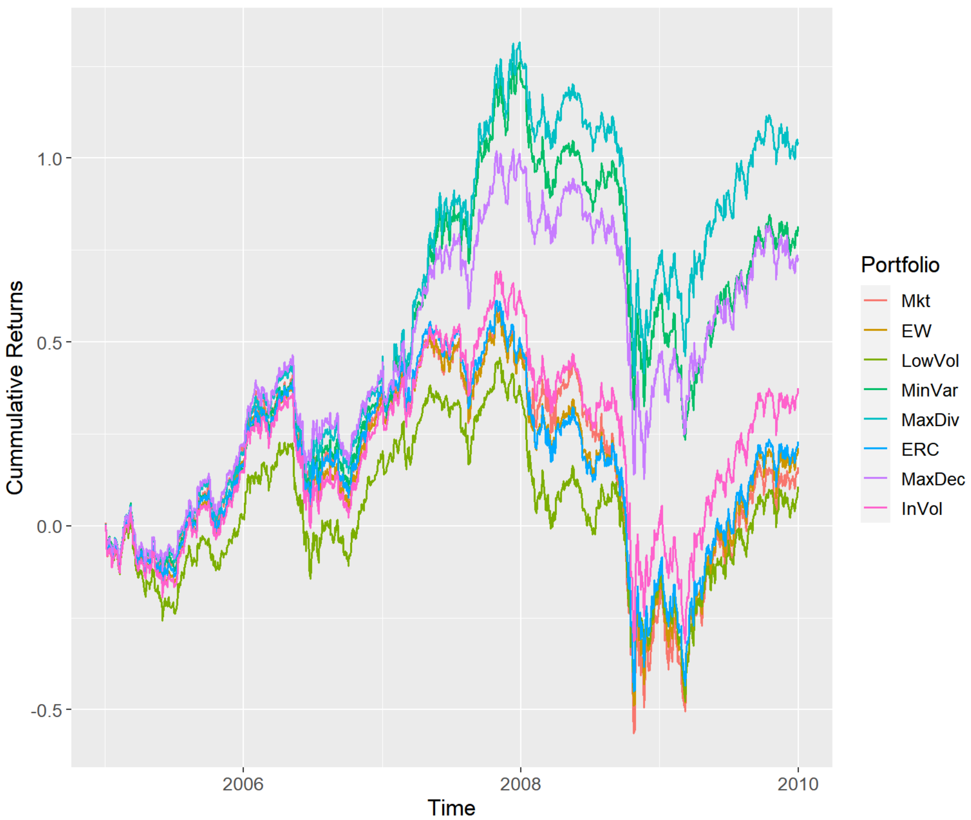


Figure 16 reports the performance results between 2005 and 2009 post-transaction cost. The portfolio had the highest return, Sharpe, Treynor, and Sortino ratio was portfolio the maximum diversification portfolio followed by the minimum variance and maximum decorrelation portfolio. In contrast to this, the low volatility portfolio and market-capitalisation weighted portfolio had the lowest return, Sharpe, Treynor, and Sortino ratio. The market-capitalisation weighted portfolio contained the lowest turnover whereas, the highest turnover is from the maximum diversification portfolio followed by the minimum variance portfolio. In agreement, with prior literature, the market-capitalisation weighted portfolio is inefficient in terms of risk-adjusted performance but efficient in terms of having a low turnover. Therefore, the alternative risk optimisation strategy of the maximum diversification portfolio and the minimum variance portfolio are good alternatives despite having the highest turnover.

Figure 18: South Africa's Performance Summary Post Transaction Costs for 2010:2014

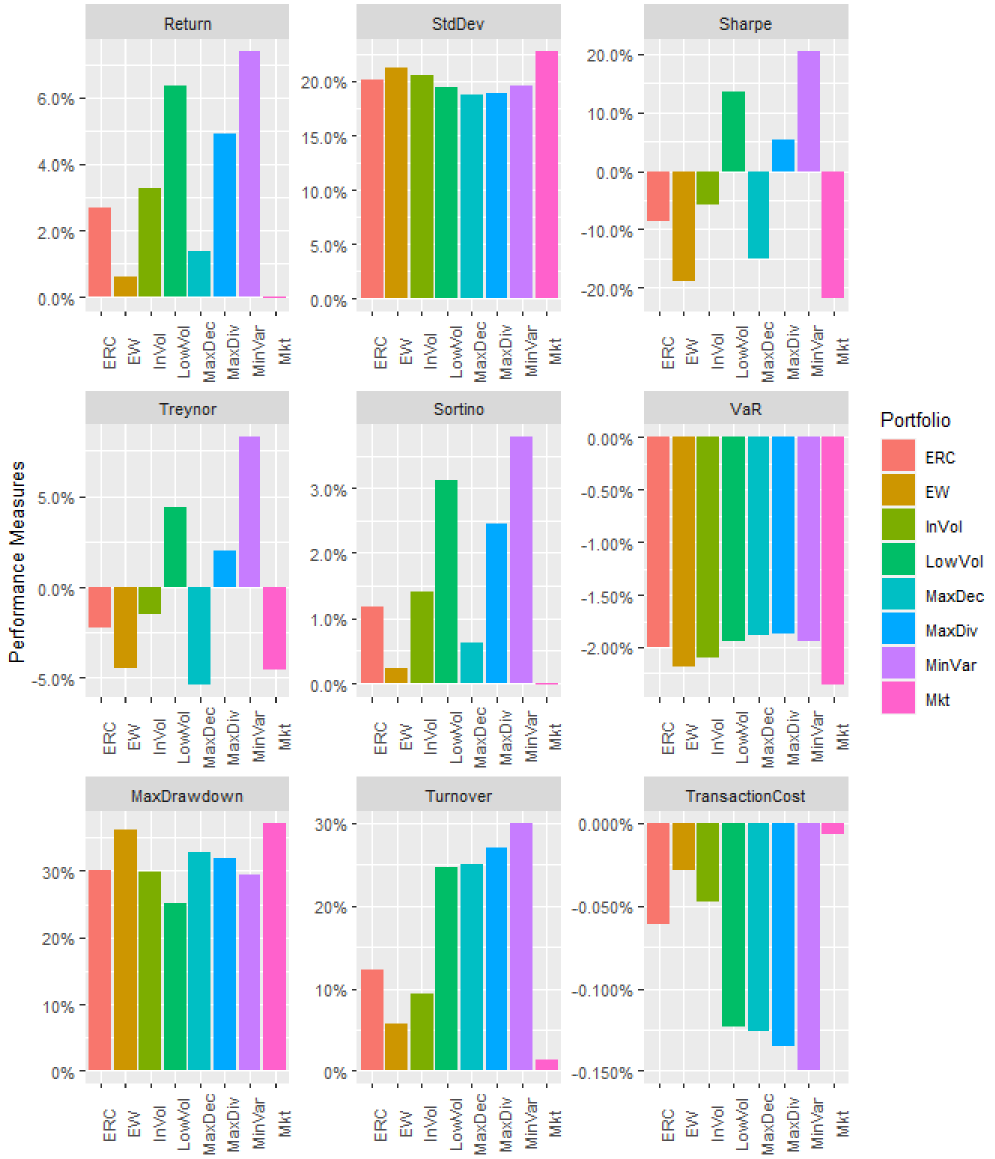


Figure 19: Cumulative Returns for SA between 2010:2014

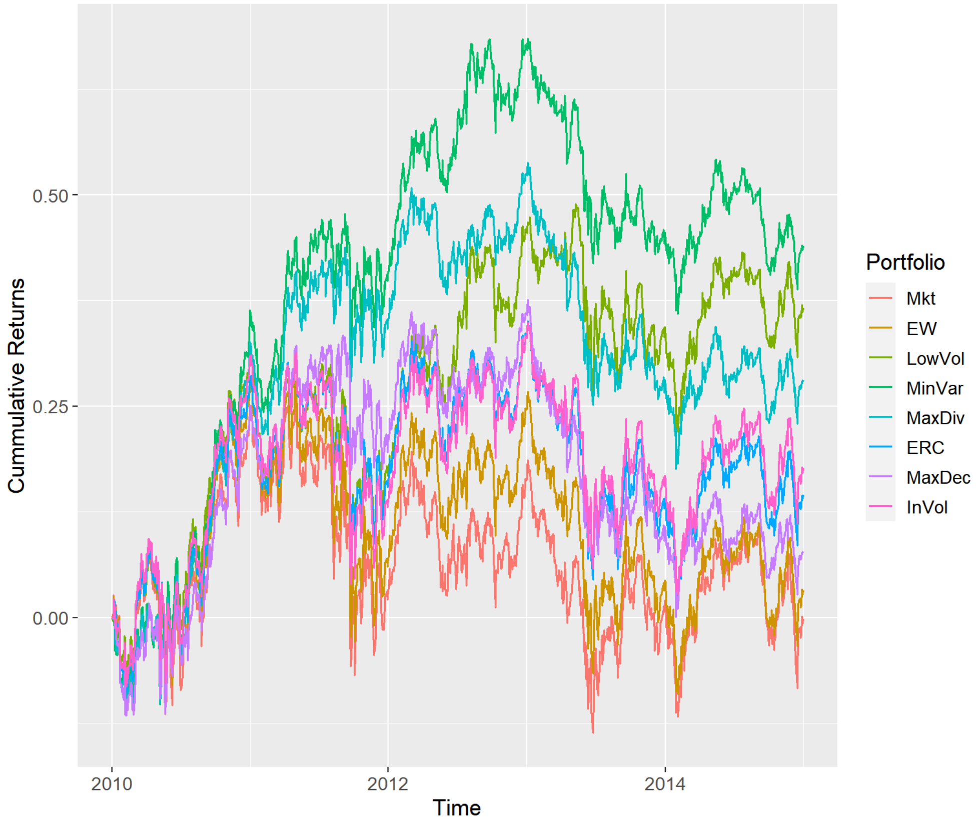


Figure 18 reports the results between 2010 and 2014 post-transaction cost. Based on the returns, Sharpe, Treynor, and Sortino ratio the minimum variance portfolio, followed by low volatility portfolio and the maximum diversification was the best performing portfolio. The worst performing portfolios were the market-capitalisation weighted and maximum decorrelation portfolio. The market-capitalisation weighted portfolio contained the lowest turnover, whereas, the highest turnover is from the minimum variance portfolio followed by the maximum diversification portfolio. Therefore, the alternative risk optimisation strategy of the maximum diversification portfolio and the minimum variance portfolio are good alternatives despite having the highest turnover.

Figure 20: South Africa's Performance Summary Post Transaction Costs for 2015:2019

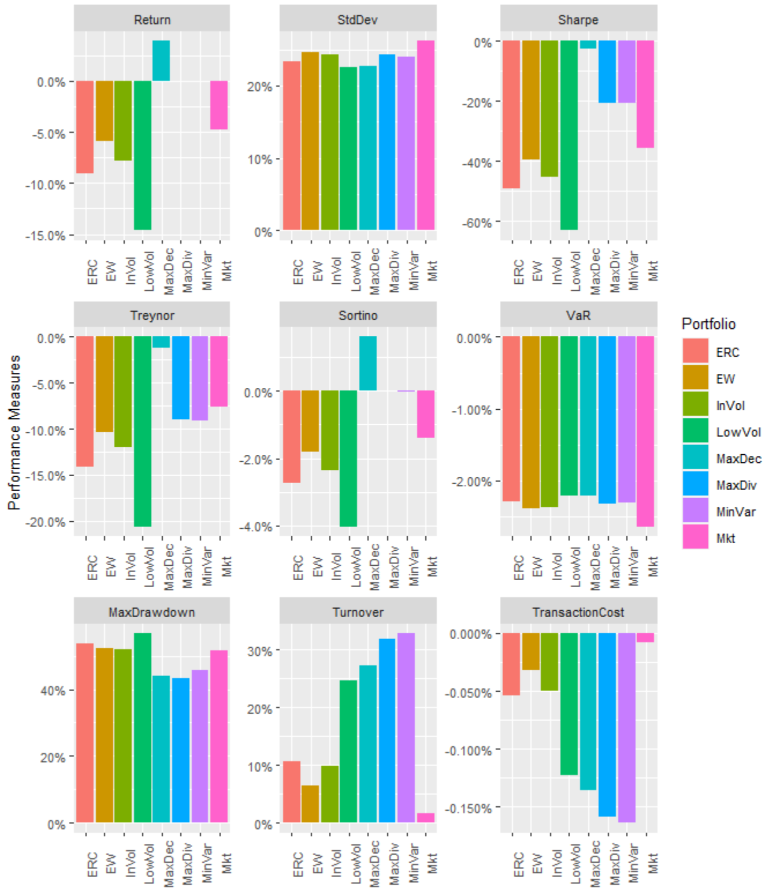


Figure 21: Cumulative Returns for SA between 2015:2019

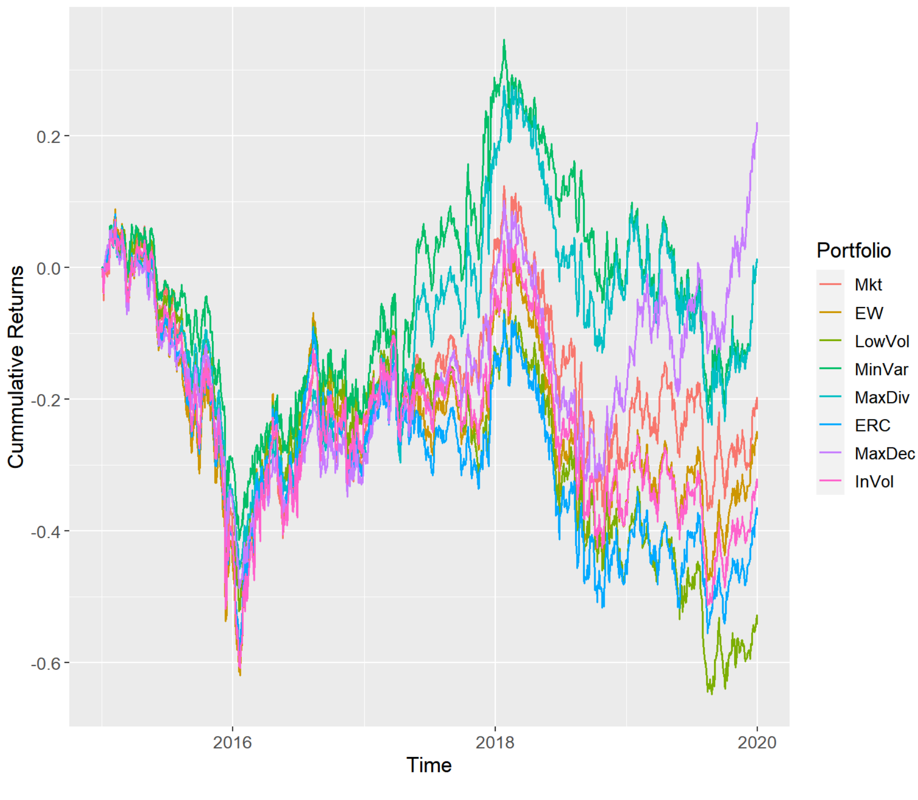


Figure 20 reports the results between 2015 and 2019 post-transaction cost, based on the returns the maximum decorrelation portfolio was the best performing and the only portfolio with a positive return and Sortino ratio. The low volatility portfolio was the worst performer over this period as it had the lowest return, Sharpe, Treynor, and Sortino ratio. It can also be seen that despite having a return close to zero the maximum diversification portfolio and minimum variance portfolio was still able to obtain a higher return, Sharpe and Sortino ratio than the market-capitalisation weighted portfolio.

The low volatility portfolio also contained the lowest standard deviation followed by the minimum variance portfolio with the market-capitalisation weighted portfolio having the highest standard deviation. The maximum decorrelation portfolio was the only portfolio that had a positive Sharpe, Treynor, and Sortino ratio whereas, the low volatility portfolio had the lowest. The minimum variance portfolio had the lowest VaR and maximum drawdown over this period with the market-capitalisation weighted portfolio having the highest VaR and the low volatility portfolio having the highest maximum drawdown. The market-capitalisation weighted portfolio contained the lowest turnover with the highest turnover been from the minimum variance portfolio followed by the maximum diversification portfolio. Therefore, it can be concluded that despite the minimum variance portfolio and maximum diversification portfolio performing poorly it was still able to outperform the market-capitalisation weighted portfolio.

Appendix 2

Figure 22: BRICS's Performance Summary Post Transaction Costs for 2000:2004

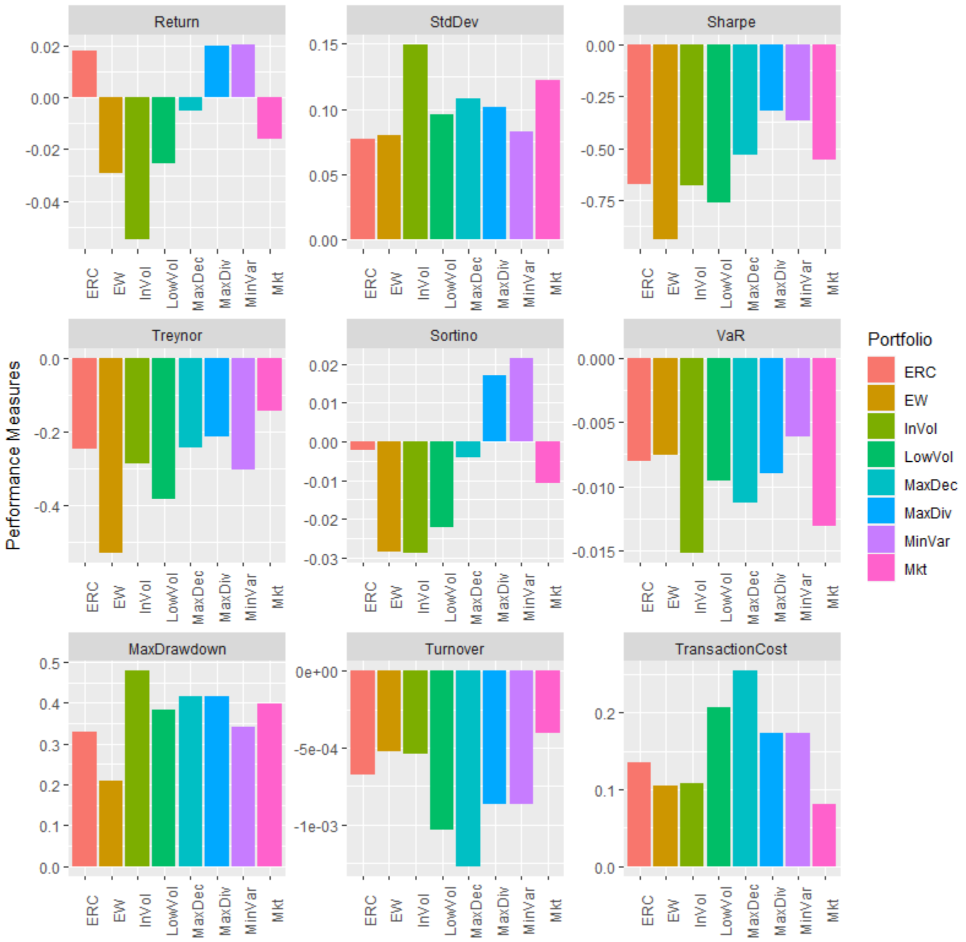
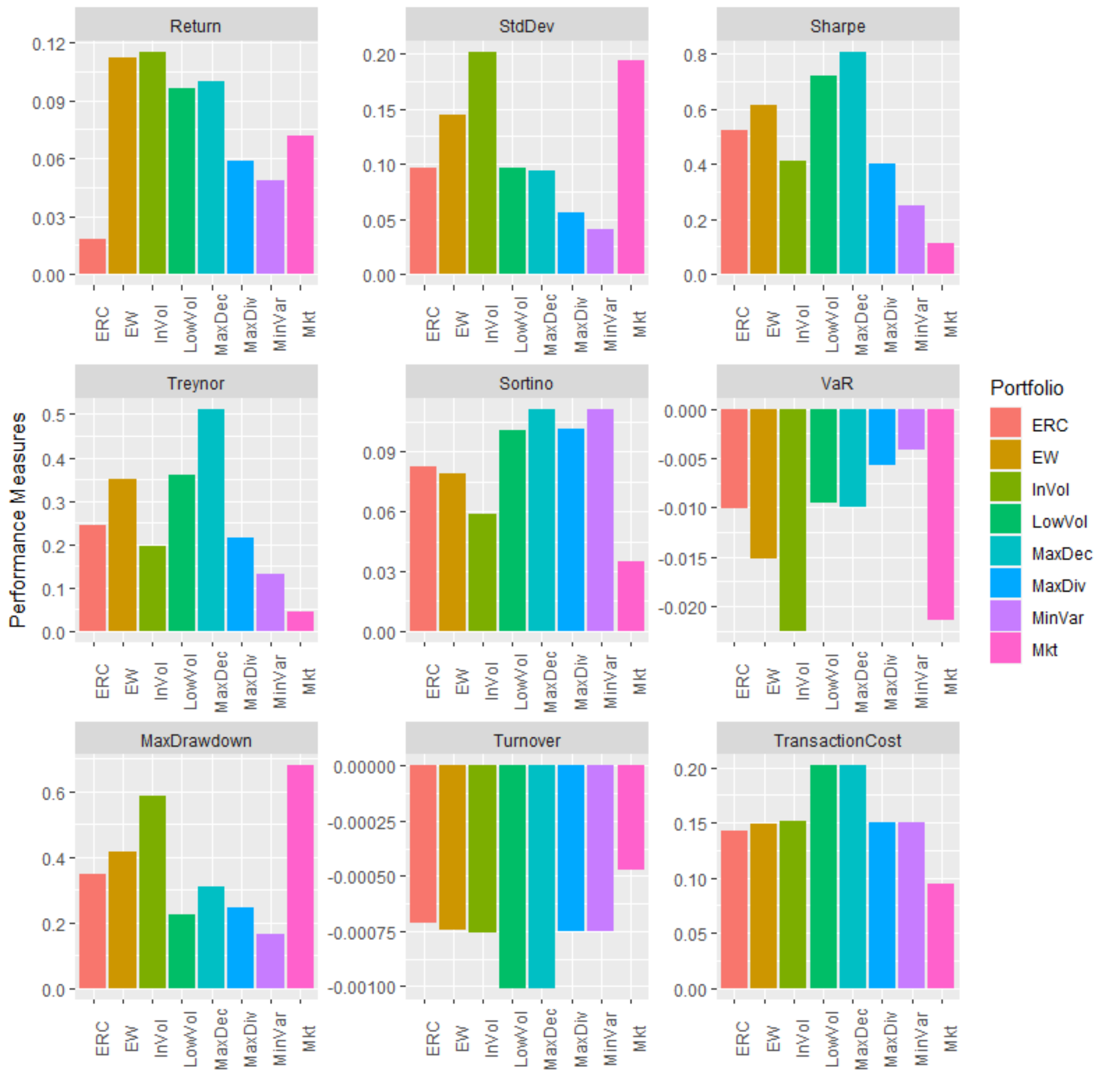


Figure 23: Cumulative Returns for BRICS between 2000:2004

Figure 22 reports the results between 2000 and 2004 for the BRICS investment universe post-transaction cost. The minimum variance portfolio had the highest return followed by the maximum diversification and equal risk contribution portfolio which were the only portfolio to have a positive return and positive Sortino ratio. The inverse volatility portfolio was the worst-performing portfolio followed by the equal-weighted portfolio. Based on the turnover the market-capitalisation weighted portfolio had the lowest turnover followed by the equal-weighted portfolio, whereas, the highest turnover is from the maximum decorrelation portfolio followed by the maximum diversification portfolio and low volatility portfolio. Therefore, despite been the most efficient portfolio in terms of turnover and not been the worst portfolio in terms of risk-adjusted returns, the alternative risk optimisation strategies of minimum variance portfolio and maximum diversification portfolio were still a better alternative portfolio.

Figure 24: BRICS's Performance Summary Post Transaction Costs for 2005:2009



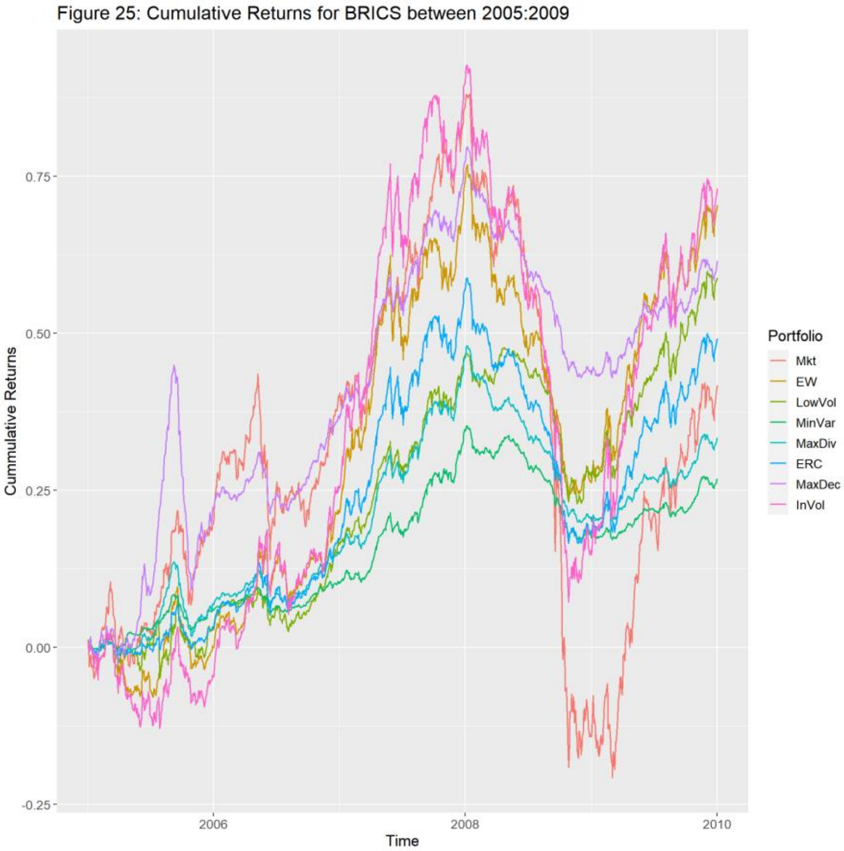


Figure 25 reports the results between 2005 and 2009 post-transaction cost. The equal-weighted portfolio had the highest returns followed by the inverse volatility and maximum decorrelation with the worst-performing portfolio been the minimum variance portfolio followed by the market-capitalisation weighted portfolio. However, the inverse volatility portfolio had the highest standard deviation followed by the market-capitalisation weighted portfolio, whereas, the minimum variance portfolio had the lowest standard deviation followed by the maximum diversification portfolio. This affected the Sharpe, Treynor, and Sortino ratio quite a lot with the maximum decorrelation portfolio having the best risk-adjusted returns followed by the low volatility whereas, the market-capitalisation weighted portfolio the worst ratios.

During this period the minimum variance portfolio had the lowest VaR and maximum drawdown followed by the maximum diversification portfolio. Whereas, the highest VaR and max drawdown followed by the market-capitalisation weighted portfolio. The market-capitalisation weighted portfolio contained the lowest turnover followed by the equal weight portfolio, whereas, the highest turnover is from the maximum decorrelation portfolio followed by the low volatility portfolio. Therefore, despite the market-capitalisation weighted portfolio having a decent return it had such a high standard deviation that it ended up being the worst portfolio in terms of risk-adjusted returns even though it was still the most efficient in terms of turnover.

Figure 26: BRICS's Performance Summary Post Transaction Costs for 2010:2014

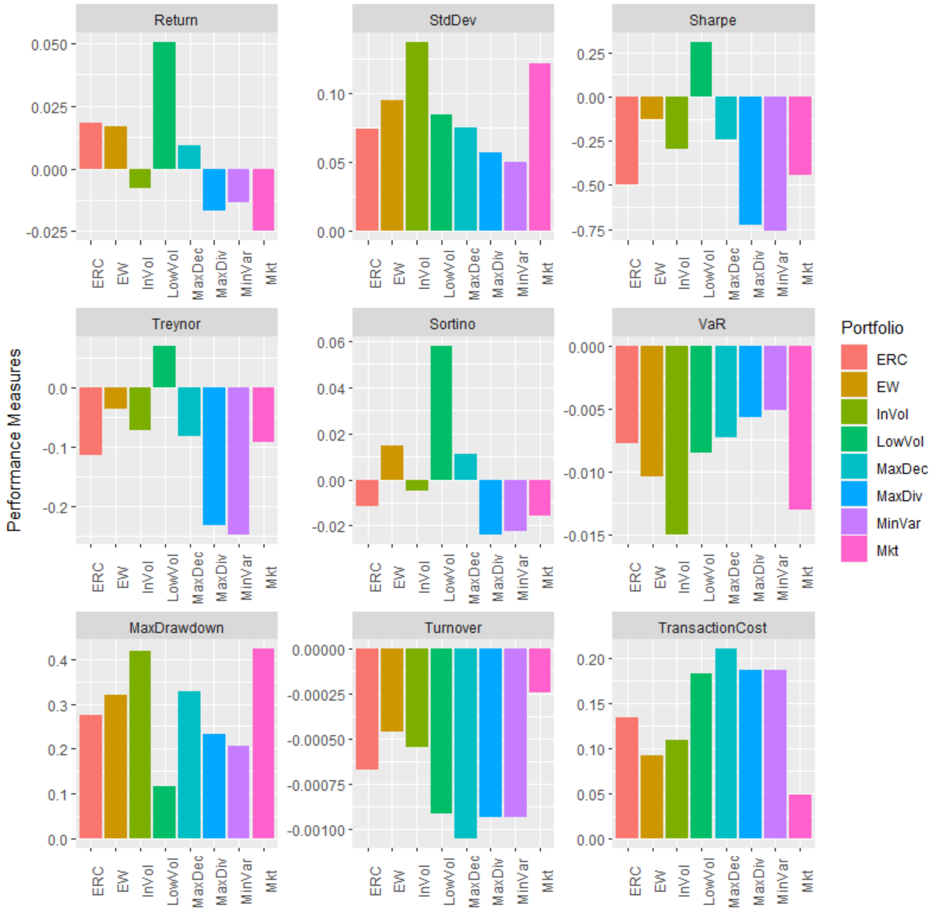


Figure 27: Cumulative Returns for BRICS between 2010:2014

Figure 26 reports the results between 2010 and 2014 for the BRICS investment universe post-transaction cost. The portfolio with the best returns was the low volatility portfolio followed by the equal weight portfolio and equal risk contribution portfolio. Whereas, the portfolio with the worst returns been the market-capitalisation weighted portfolio. The inverse volatility portfolio and the market-capitalisation weighted portfolio had the highest standard deviation. Again the minimum variance portfolio has the lowest standard deviation followed by the maximum diversification portfolio however, they both had negative returns.

On a risk-adjusted basis looking at the Sharpe ratio all the portfolios except for the low volatility were not able to outperform the risk-free rate as they all had a negative Sharpe ratio with the low volatility been the only one with a positive Sharpe ratio. The minimum variance portfolio had the lowest VaR with the BRICS having the highest. However, the low volatility portfolio had the lowest maximum drawdown and the market-capitalisation weighted portfolio had the highest. The market-capitalisation weighted portfolio contained the lowest turnover followed by the equal-weighted portfolio, whereas, the highest turnover is from the inverse volatility portfolio followed by the maximum decorrelation portfolio.

Therefore, during this sample period, the market-capitalisation weighted portfolio able to outperform most of the minimum variance portfolio and maximum diversification portfolio on a risk-adjusted basis but was still not able to be the most efficient portfolio on a risk-adjusted basis which the low volatility portfolio. However, it was the most efficient portfolio in terms of turnover.

Figure 28: BRICS's Performance Summary Post Transaction Costs for 2015:2019

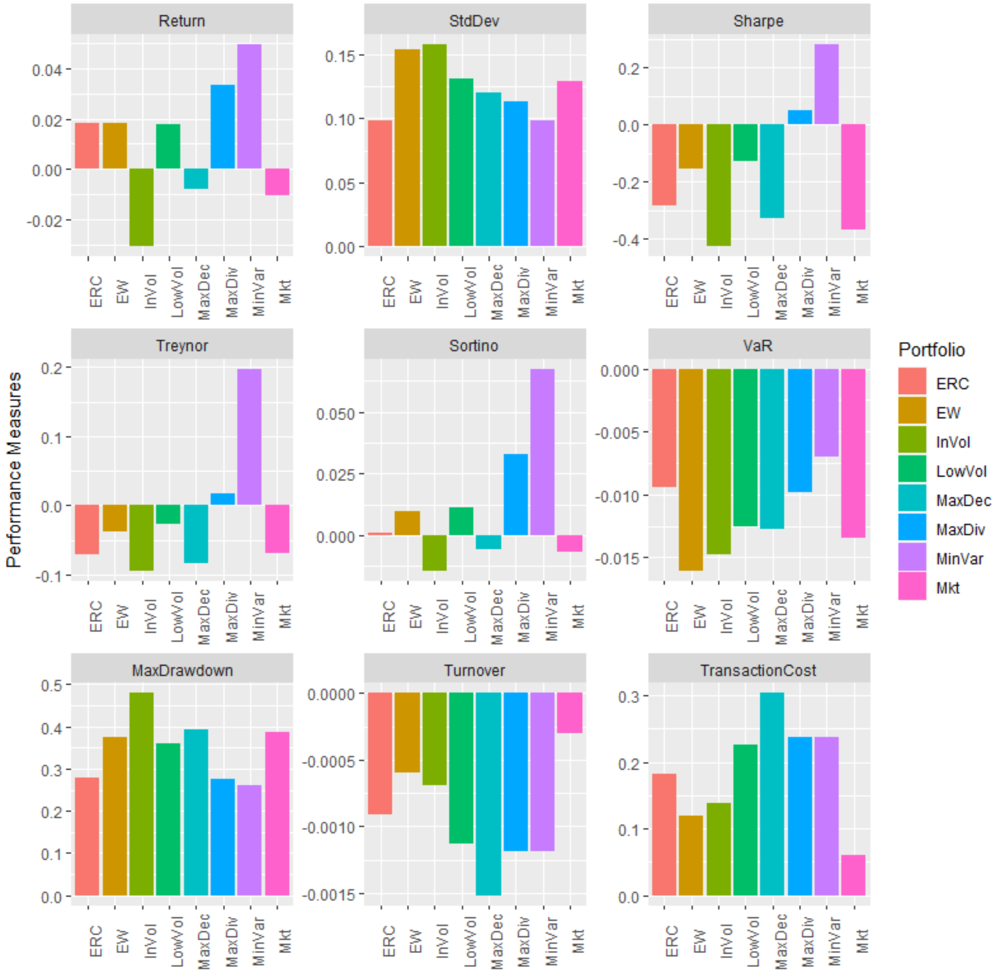


Figure 29: CumulatInVole Returns for BRICS between 2014:2019

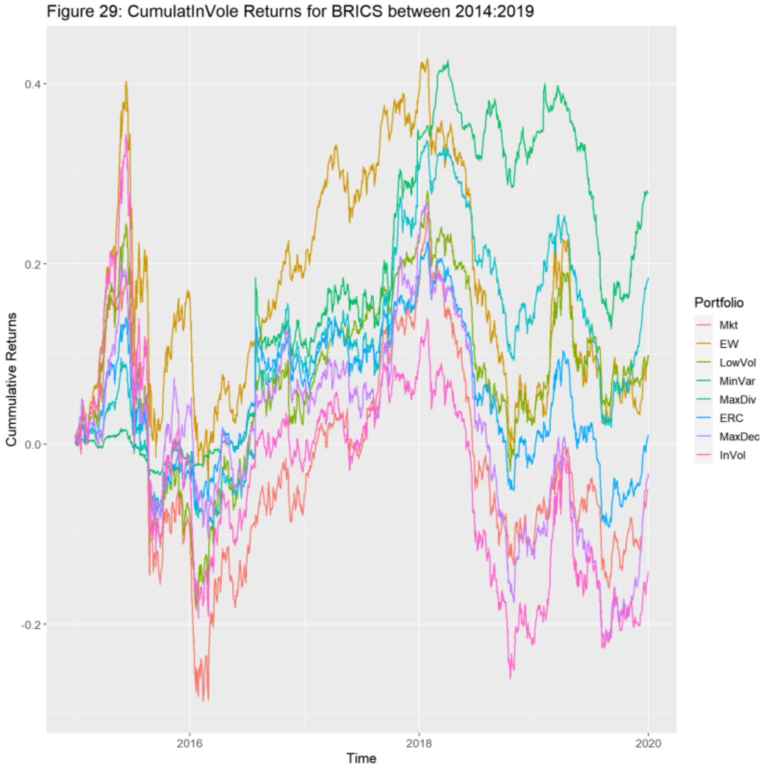


Figure 28 reported the results between 2014 and 2019 for the BRICS investment universe post-transaction cost. The minimum variance portfolio had the highest return followed by the maximum diversification and low volatility portfolio whereas the inverse volatility had the lowest return followed by the market-capitalisation portfolio. The minimum variance portfolio and the maximum diversification portfolio are only portfolios with positive Sharpe and Treynor ratios with the minimum variance portfolio having the highest ratios as such it was the best performing portfolio on a risk-adjusted basis. Whereas, the inverse volatility followed by the market-capitalisation weighted portfolio had the worst ratios during this period.

The minimum variance portfolio followed by the maximum diversification portfolio had the lowest VaR and maximum drawdown with the BRICS portfolio followed by the market-capitalisation weighted portfolio having the highest. The market-capitalisation weighted portfolio containing the lowest turnover followed by the equal weight portfolio, whereas, the highest turnover is from the maximum decorrelation portfolio followed by the maximum diversification portfolio. Therefore, it can be concluded again that the market-capitalisation weighted portfolio is inefficient in terms of risk-adjusted return and the most efficient in terms of turnover. The alternative risk optimisation strategies such as the minimum variance portfolio and maximum diversification portfolio are a much better alternative as they are a lot more efficient in terms of risk-adjusted performance despite having the highest turnover.

Based on the sub-sample periods for the BRICS investment universe it is clear that more often than not the market-capitalisation weighted portfolio is one of the most inefficient portfolios in terms of risk-adjusted performance and the most efficient portfolio in terms of minimising turnover. The minimum variance portfolio, maximum diversification portfolio, and low volatility portfolio were able to outperform the market-capitalisation portfolio on a risk-adjusted basis in every sub-period except for 2010:2014 where the market-capitalisation weighted portfolio was able to outperform the minimum variance portfolio and maximum diversification portfolio but it still had a negative return.

Appendix 3

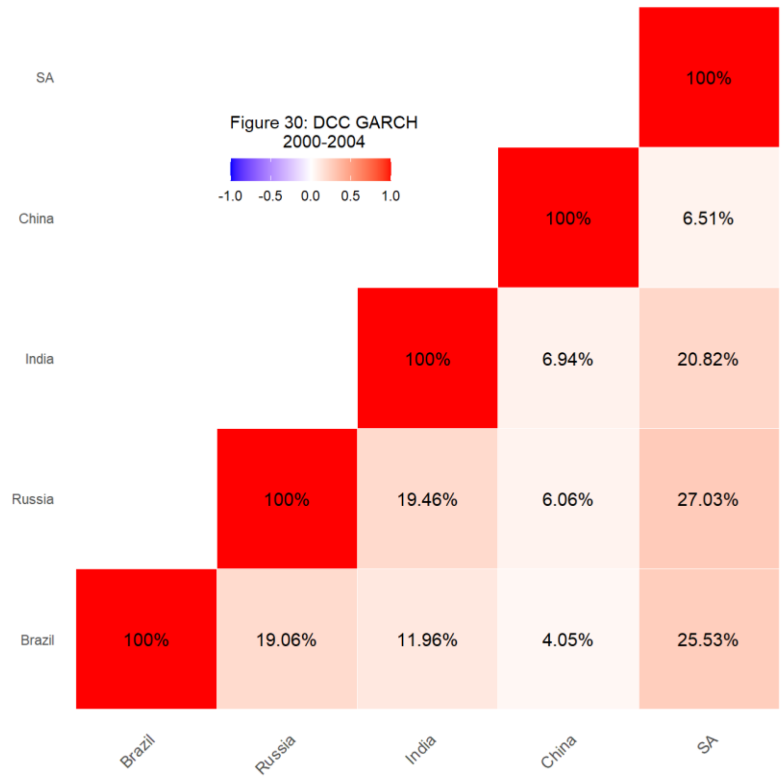


Figure 30: DCC GARCH 2000-2004

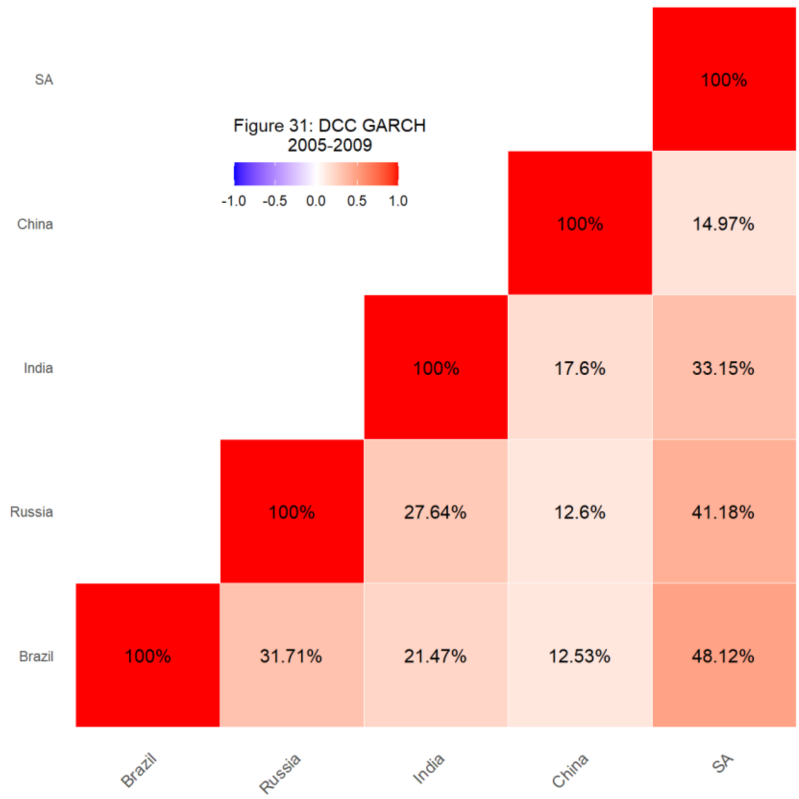


Figure 31: DCC GARCH 2005-2009

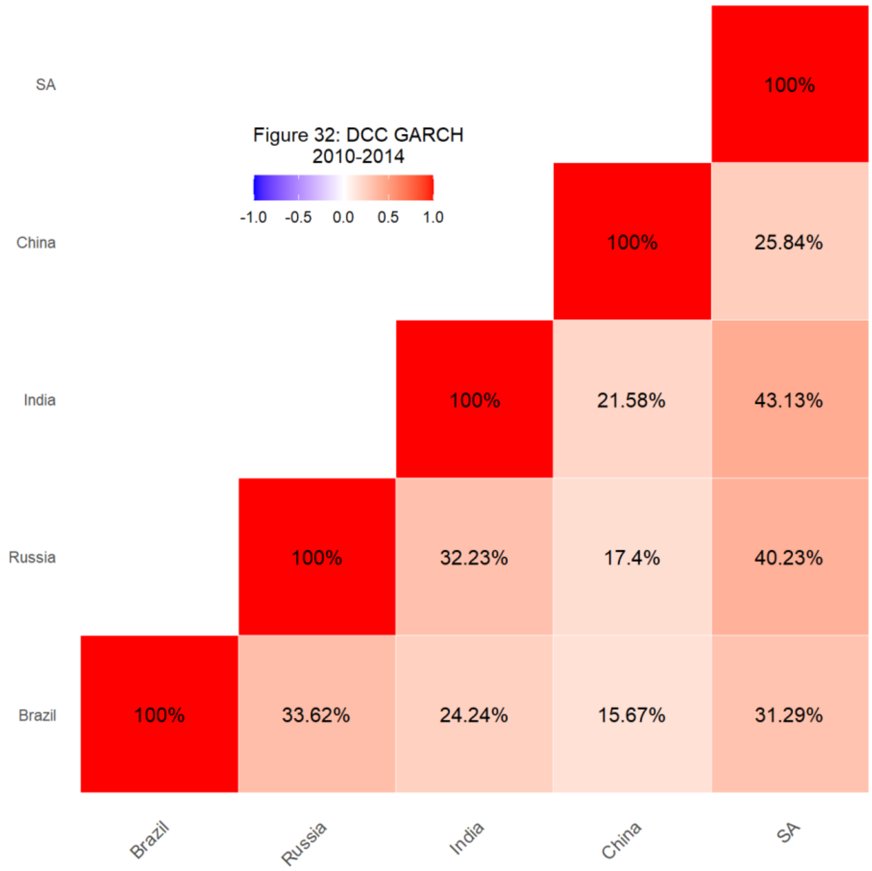


Figure 32: DCC GARCH
2010-2014
-1.0 -0.5 0.0 0.5 1.0
SA
China
India
Russia
Brazil
100%
100% 25.84%
100% 21.58% 43.13%
100% 32.23% 17.4% 40.23%
100% 33.62% 24.24% 15.67% 31.29%
Brazil Russia India China SA

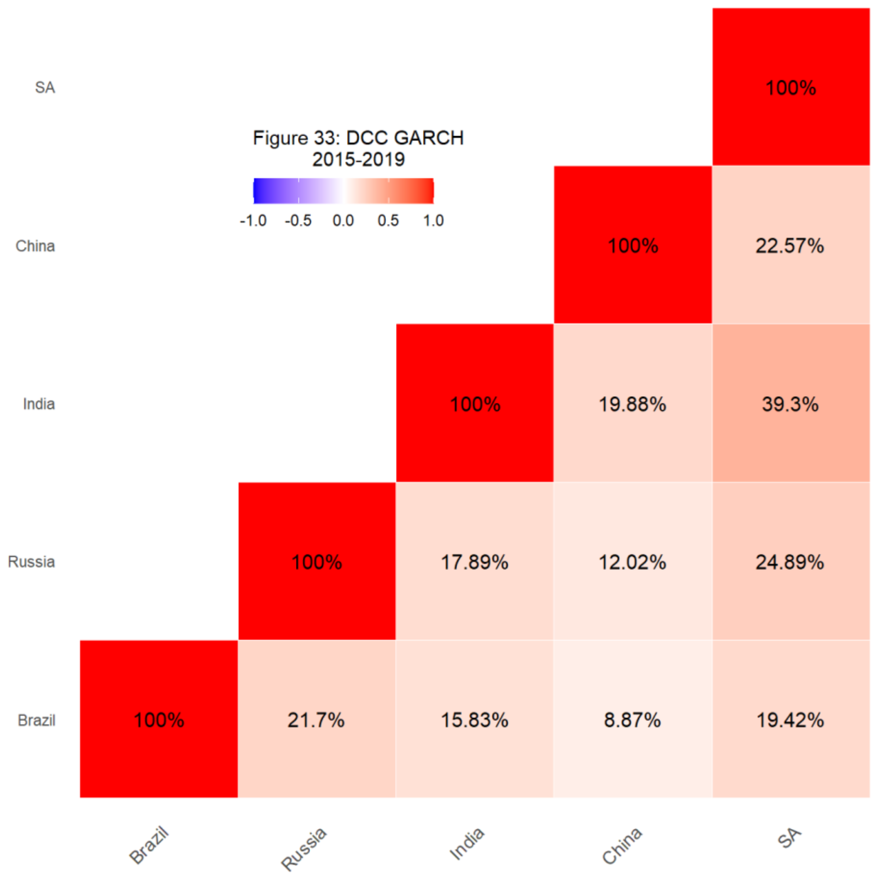


Figure 33: DCC GARCH
2015-2019
-1.0 -0.5 0.0 0.5 1.0
SA
China
India
Russia
Brazil
100%
100% 22.57%
100% 19.88% 39.3%
100% 17.89% 12.02% 24.89%
100% 21.7% 15.83% 8.87% 19.42%
Brazil Russia India China SA

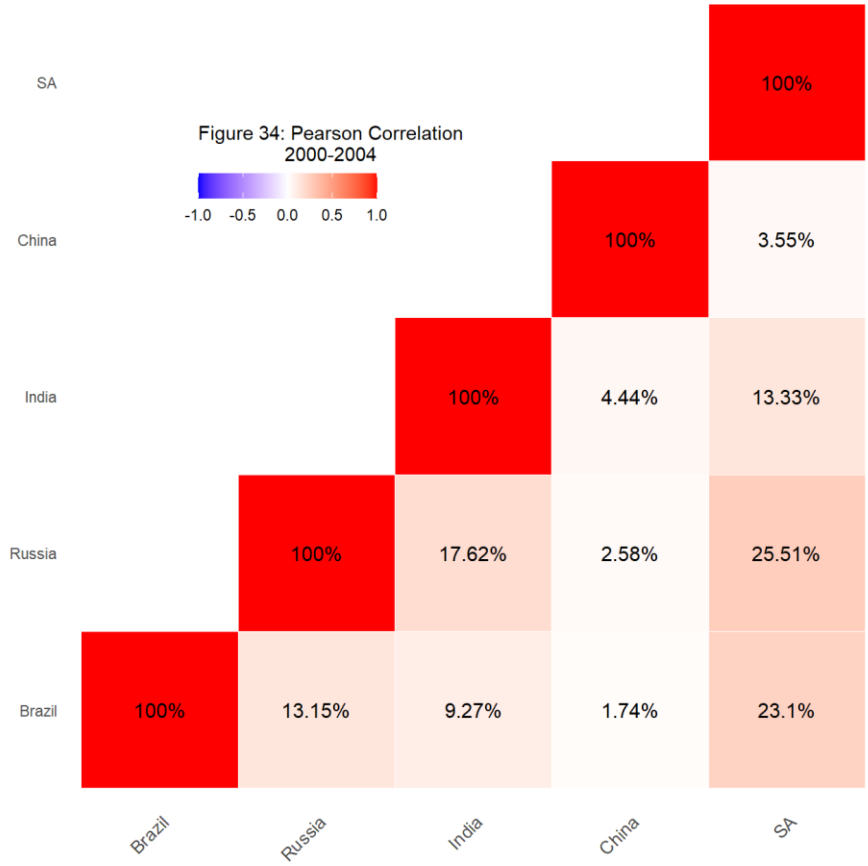


Figure 34: Pearson Correlation 2000-2004

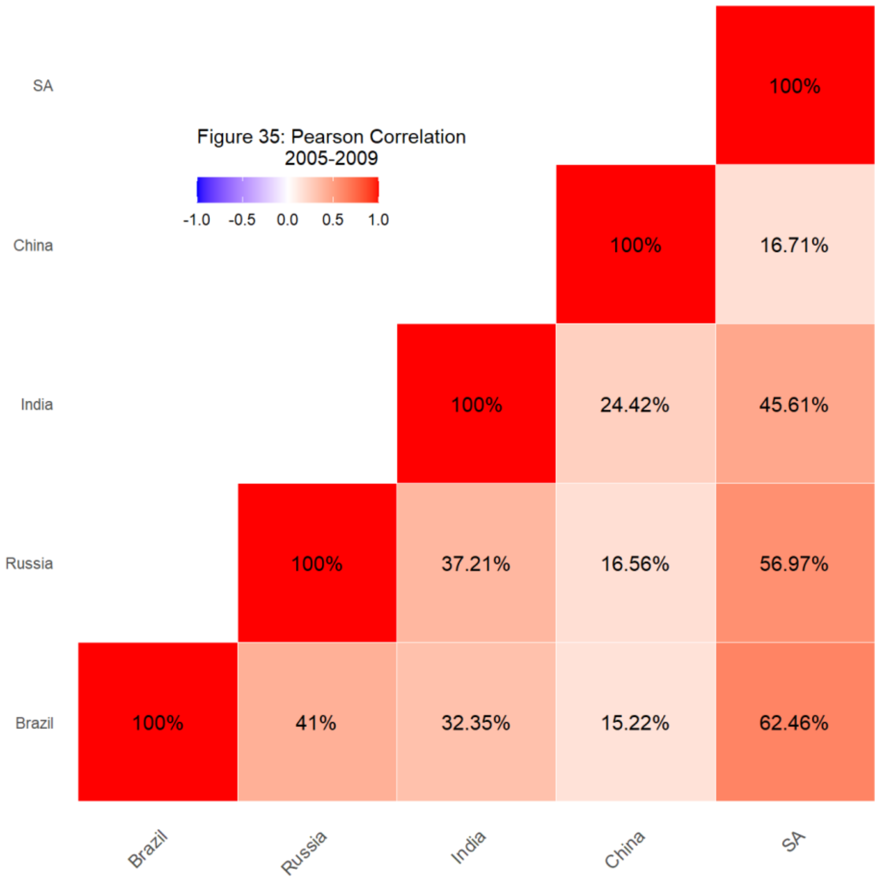


Figure 35: Pearson Correlation 2005-2009

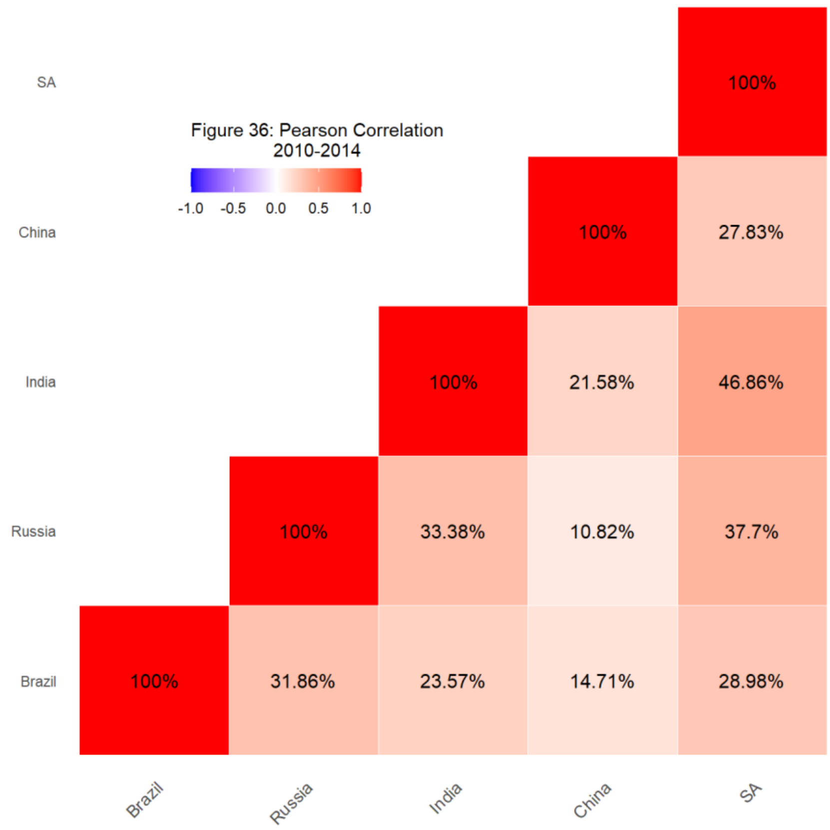


Figure 36: Pearson Correlation
2010-2014
-1.0 -0.5 0.0 0.5 1.0
SA
China
India
Russia
Brazil
100%
27.83%
100%
21.58%
46.86%
100%
33.38%
10.82%
37.7%
100%
31.86%
23.57%
14.71%
28.98%
Brazil
Russia
India
China
SA

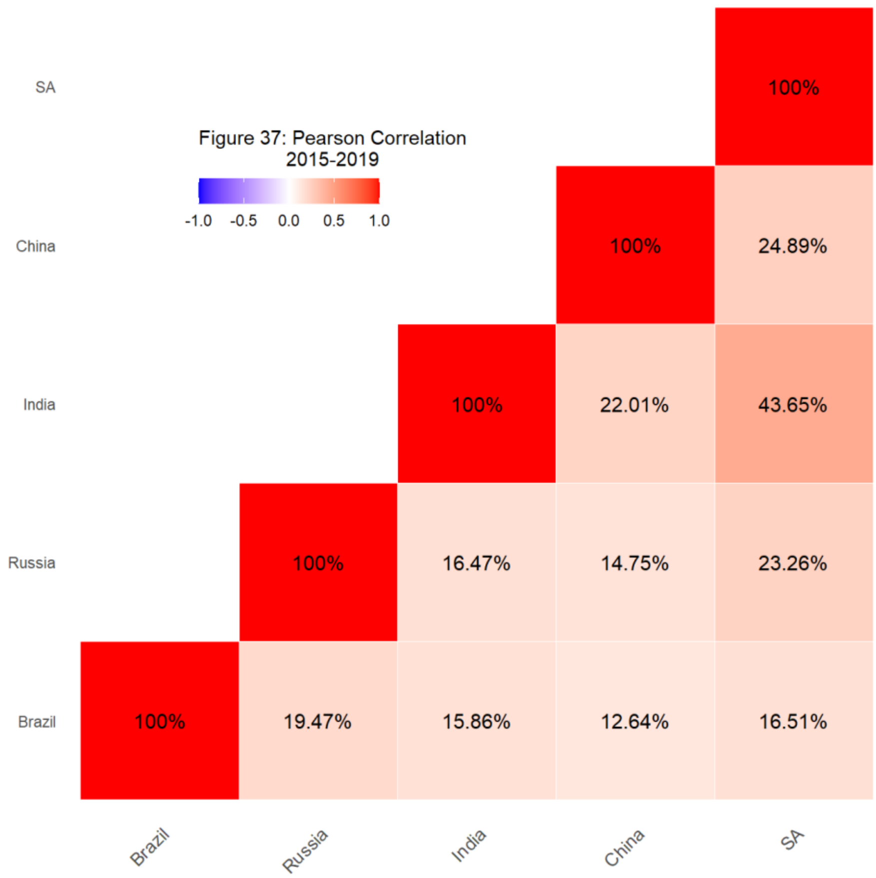


Figure 37: Pearson Correlation
2015-2019
-1.0 -0.5 0.0 0.5 1.0
SA
China
India
Russia
Brazil
100%
24.89%
100%
22.01%
43.65%
100%
16.47%
14.75%
23.26%
100%
19.47%
15.86%
12.64%
16.51%
Brazil
Russia
India
China
SA

Figure 38: Brazil's Pearson Correlation with the remaining BRICS

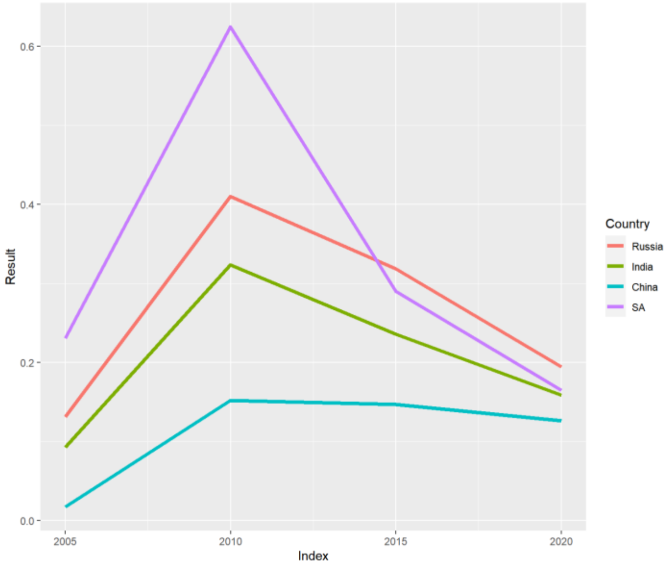


Figure 39: Russia's Pearson Correlation with the remaining BRICS

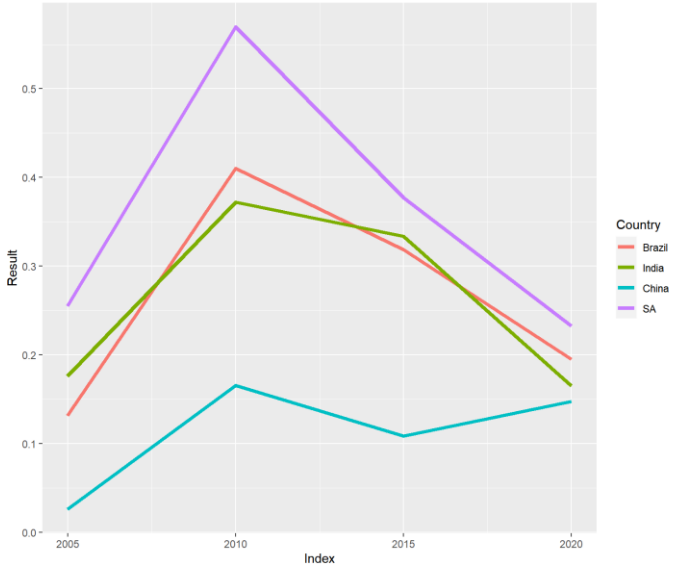


Figure 40: India's Pearson Correlation with the remaining BRICS

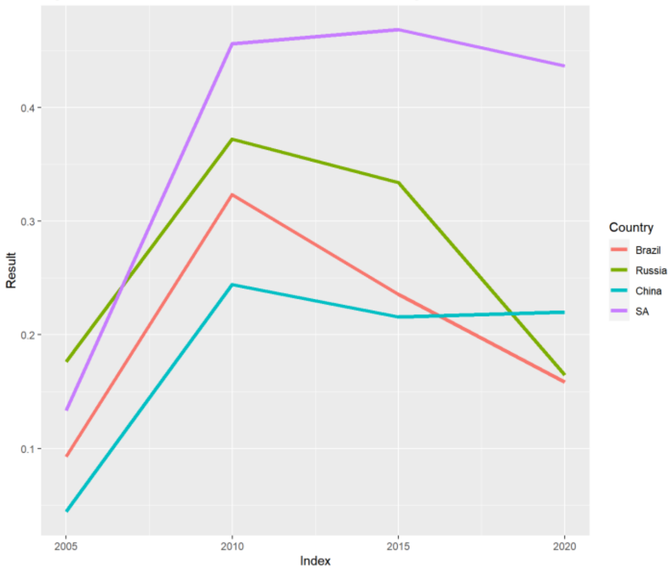


Figure 41: China's Pearson Correlation with the remaining BRICS

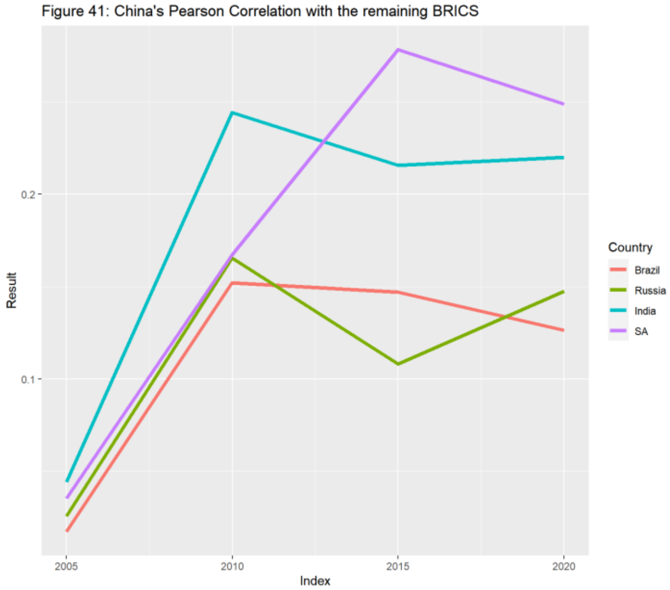


Figure 42: SA's Pearson Correlation with the remaining BRICS