UNIVERSITY OF THE WITWATERSRAND

Tasks used in mathematics classrooms

By: Phathumusa Mdladla

0217853V

Supervisor: Professor Margot Berger

A research report submitted to the Faculty of Science, University of Witwatersrand, in partial fulfilment for the degree of Masters of Mathematics Education by coursework and research report.

Johannesburg,

March 2017

DECLARATION

I, Phathumusa Mdladla, declare that this research report is my own, unaided work. It is being submitted for the degree of Master of Science Education in the University of the Witwatersrand Johannesburg. It has not been submitted before for any other degree or examination at any other university.

____

Signature

March 2017

ABSTRACT

The current mathematics curriculum in South Africa require that learners are provided with opportunities to develop abilities to be methodical, to generalise, to make conjectures and try to justify and prove their conjectures. These objectives call for the use of teaching strategies and tasks that support learners' participation in the development of mathematical thinking and reasoning. This means that teachers have to be cautious when selecting tasks and deciding on teaching strategies for their classes. Tasks differ in their cognitive and difficulty levels and opportunities they afford for learner to learn mathematics competently. The levels of tasks selected by the teachers; the kinds of questions asked by the teachers during the implementation of the selected tasks and how the questions asked by the teachers and the teachers' actions at implementations affected the levels of the tasks were the focus of this research report.

The study was carried out in one high poverty high school in South Africa. Two teachers were observed teaching and each teacher taught their allocated grades. One teacher was observed teaching Grade 9s while the other taught Grade 11s. Both teacher taught number patterns at the time their lessons were observed. The research was qualitative. Methods of data collection and instruments included lesson observations; collection of tasks used in the observed classes, audio-taping and field notes. Pictures of the teachers' work and copies of learners' workbooks also provided some data.

The analysis of data shows that the teachers not only selected and used lower-level cognitive demand and 'easy' tasks, that did not support mathematical thinking, but also did not lift up the levels and/or maintain the 'difficulty levels' of the task at implementation. Teachers were unable to initiate class discussions. Their teaching focused on 'drill and practice' learning and teaching practices.

ACKNOWLEDGEMENTS

I give all thanks and praises to the Creator of the universe (uSomandlawonke) for His guidance through this MSc journey.

Dedication

To my children – Lungisa and Gugulethu Mdladla, you are my force!

&

To the memory of my late father – Siphosethu Mdladla

Keyword Set

Learning and Practice task; Mathematical thinking; Conceptual understanding; Curriculum; Cognitive demand of task; Difficulty levels; Number patterns; Generalisation & High-poverty schools

Table of Contents

Chapter One

> *"… failure of poor and minority learners were due to lack of opportunity to participate in meaningful and challenging learning experience rather than to lack of ability or potential"*
>
> (Stein, Groven and Henningsen, 1996)

1.1 Introduction

Before democracy, education in South Africa was characterised by racial segregation and separate (according to race) departments operating on different syllabi. Schools in the townships and other black areas were provided with fewer resources and minimal capital expenditure compared to schools in non-black areas (Mda and Mathata, 2000). Unfortunately, the social transformation initiatives embarked on since the dawn of democracy have not upturned the scales or unequivocally redressed these calamities of the past. Currently, the resources per learner in high-poverty schools have improved, but a great challenge is still to improve learners' academic performance. Indeed, the potential of most learners from high-poverty schools remains locked and undetected.

The reports on the Senior Certificate (Matric) examination and Annual National Assessment (ANA) reveal the persisting crisis in South Africa's mathematics education. In 2013, the Grade 9 national average was 14% and only 3% of learners achieved above 50% (Department of Basic Education (DBE), 2013). The debacle is more pronounced in high-poverty schools. The opening citation above, from Stein et al (1996) suggests that high expectations for learners in high-poverty schools may help to improve learners' academic performance. Stein et al (1996) argue that exposing learners to cognitively demanding mathematical classroom experiences can unleash the learners' academic potential. Cognitive demand refers to the kind of thinking required of learners in order to successfully engage with and solve tasks (Stein, Smith, Henningsen and Silver 2000).

Research studies point to the challenges in relation to the quality of education: the availability of appropriately trained mathematics teachers; the quality of the mathematical content and instruction in South African mathematics classrooms (Adler, 2005; Adler, 2000; Adler, Dickson, Mofolo, Sethole, 2001 Ensor 2002; Stoffle, 2007). The Chisholm report (2000) itemised challenges about the quality, the availability, and use of learning support material, these included mathematical tasks. This study is an extension to the efforts by these and other researchers of identifying and understanding the quality of education afforded to South African learners, particularly those in high-poverty schools. In this study, a special attention is given to the use of learning support material. The focus of the study is on the quality of *learning and practice tasks* used in high-poverty mathematics classrooms. The terms learning *and practice tasks* as used in this document are respectively defined as tasks the teacher uses to teach the learners new concepts or skills and tasks used during the lessons to either illuminate the concept or demonstrate the skill further (Kaur, 2010). More discussions on learning and practice tasks will follow in subsequent chapters. However, a distinction is not made in the analysis between learning and practice tasks.

This study is underpinned by three assumptions and/or speculations: firstly, that the quality (level of cognitive demand and difficulty level) of number pattern tasks used in a mathematics classroom can be measured using taxonomies. Secondly, that cognitively demanding mathematical task develops learners' mathematical reasoning (Clarke, 2010) and thinking. Thirdly, that township learners are often deprived of opportunities to work on higher-level cognitive demanding *learning and practice* tasks and that frequently the few high-level tasks that are selected are 'scaffolded' to a point in which the cognitive demand levels of the tasks are lowered significantly. In parts, this can be used to explain the achievement gaps (in mathematics) observed between different social groups (Brodie 2005; Taylor, Muller & Vinjevold, 2003).

Learners gain mathematical experience largely by working on tasks (Sullivan and Clarke, 1991). The selection of tasks for learners to work on is one of the most important functions teachers perform daily. Different tasks require different levels and kinds of thinking. The tasks that learners work on, structure their experience of mathematics and are pivotal in their mathematical development. One of the 'specific aims' of the South African mathematics curriculum (i.e. see discussion in section 1.3.2 below) is "to promote problem solving and cognitive skills" (DBE, 2011, p. 8). Stein et al (2000) argue that task selection should match the goals/aims of learning. They also argue that "if a teacher wants learners to learn how to justify or explain their solution processes, she should select a task that is deep and rich enough to afford such opportunities" (p. 12).

Brodie (2010) argued that "choosing appropriate tasks is necessary but not sufficient to support a learner to develop reasoning" (p. 19) and developing problem solving and cognitive skills. Well thought-out implementation strategies of the tasks are crucial (Brodie, 2010; Stein et al 2000; Stein et al, 1996). At implementation, learners need to be supported by skilled teachers who can systematically raise or maintain the demands of tasks or lead learners into more cognitively demanding tasks (Stein et al 2000). This can be done through a process referred to as 'scaffolding'. In an elaborated form of Vygotsky's theory, 'scaffolding' is where a teacher changes the degree and quality of support provided to the learners when working on unfamiliar problems (Wood, Bruner & Ross, 1976). This support can be provided through communication. Sullivan and Clarke (1991) argue that teachers and learners communicate mainly by methods of questions and answers involving sequential turns of talks and actions. Teachers often ask a series of questions which assist learners to negotiate new meanings.

Being aware of the importance of cognitively demanding tasks and role of teacher questions in developing learners' mathematical skills, I investigated the levels of cognitive demands of *learning and practice* tasks and the kinds of questions asked by teachers during instruction. In this way, I tested the soundness of the 'hypothesis' that township learners are often deprived of opportunities to work on higher-level cognitive demanding *learning and practice* tasks.

1.2 Research Questions

My case study is guided by the following research questions:

1. What are the cognitive levels and difficulty levels of the mathematical tasks used in the case study township classrooms?
2. How do the case study teachers' classroom practice impacts on the cognitive demand levels and/or difficulty levels of mathematical tasks?

Background question

2a. What kinds of questions are asked by the case study teachers to assist learners when working on learning and practice tasks?

1.3 Rationale

This study largely is informed by the work done in South Africa by Brodie, Jina and Modau (2009) and by Kaur (2010) in Singapore, where the quality of mathematical tasks was examined. The work by Brodie et al (2009) was based on two curricula, the interim core syllabus in Grade 11 and National Curriculum Statement (NCS) in grade 10. More about these different South African curricula is presented in section 1.3.2 below. Brodie et al (2009) checked for differences in the teachers' questions and interaction patterns in the different curricula. Kaur (2010) focused mainly on the source and nature of mathematical tasks used by teachers to implement the intended curriculum and the link between the tasks and the primary goals of the curriculum. This case study is an adaption of two studies and focuses on the quality of the mathematical tasks selected by teachers in high-poverty setting and how teachers' classroom practice (focusing on teachers' questions) affected the quality of tasks selected.

1.3.1 A comparable Study

In 2006 the NCS was introduced in Grade 10. In that same year Brodie et al (2009) conducted a case study. In this study, one teacher was observed teaching a Grade 10 and Grade 11 class, each of which were working on two different curricula, NCS and interim core syllabus respectively. The study was conducted at a township school with inadequate classrooms and the majority of learners were from poor families. Jina & Brodie (2008) argued that due to learners' lack of learning aids, for example calculators, the classroom activities took longer than the predictable time to complete. The study explored how the teacher selected and implemented tasks and interacted with learners.

Brodie et al (2009) argued that the NCS (i.e. the curriculum which replaced the interim core syllabus) promoted principles which were underpinned by the thinking shared with Kilpatrick et al (2001), that learning with understanding is more powerful than simply memorising. Through the NCS the Department of Education (2003) envisioned a shift from traditional ways of teaching and learning to more interactive approaches. Brodie et al (2009) acknowledged that what was encouraged in the official curriculum is often very different from what happened on in the classrooms. Using previous research from other studies (i.e. Cuban, 1998; Jansen, 1999; Taylor and Vinjevold, a 1999; Taylor, 1999; Todd & Mason, 2005) Brodie et al (2009) crafted compelling arguments that pointed to the contrasts between classroom practice and what is encouraged in the endorsed curriculums. This led to a clear-cut notion that there are often disjoints between the intended curriculum and how it plays out in the classrooms.

Brodie et al (2009) found that textbooks were the main sources of tasks. The teacher used one traditional textbook in Grade 11 and various NCS attuned textbooks in Grade 10. Using Stein et al.'s framework (a description and discussion of this framework is presented in Section 2.3), Brodie et al (2009) classified the tasks used by the teacher and found that the teacher used more *procedures with connection* (level 3) tasks in the NCS (Grade 10) than in the interim core syllabus (Grade 11). On the other hand, more *procedure without connection* (level 2) tasks was used in the interim core syllabus than in NCS. However, there were neither *memorization* (level 1) nor *doing mathematics* (level 4) tasks in any of the curricula. Brodie et al (2009) attributed the difference in the selection of tasks to the awareness of the teacher to "the supposedly different constraints placed on teachers" (p. 25) by the NCS and interim core curriculum. They also claimed that the teacher's "view was that the NCS allowed for some teacher autonomy in selecting tasks" (p.25). They argued that the teacher's awareness of NCS goals and availability of teaching and learning support materials (i.e. new textbooks) could be used to explain the difference in the selection of tasks used in the two curricula.

Brodie et al (2009) found that the teacher's selected tasks in Grade 10 were in line with the goals of NCS, but that on implementation the cognitive levels of the tasks declined. The findings from their case study reproduced a pattern (i.e. the decline of cognitive levels of tasks on implementation) observed in prior studies and literature. In conclusion they argued that the decline in the cognitive levels was due to 'funnelling' of questions by the teacher. My study seeks to address similar issues to those addressed by Brodie et al.'s (2009) study, but the setting, period and focus are different.

1.3.2 The South African Context

> *"To improve implementation, the National Curriculum Statement was amended, with the amendments coming into effect in January 2012. A single comprehensive Curriculum and*

Assessment Policy document was developed for each subject to replace Subject Statement Learning Programme Guideline and Subject Assessment Guidelines in Grade R – 12" (DBE, 2011, p. 3)

Since the dawn of democracy in 1994 there have been three major amendments to the South African curriculum: Curriculum 2005 (DoE, 1997) followed by (Revised) National Curriculum Statement (DoE, 2002) and now Curriculum and Assessment Policy Statement (DBE, 2011). Reviews of the outcome-based structural design of C2005 resulted in (R)NCS. (R)NCS (also called NCS) was introduced into lower schooling grades (i.e. R – 9) in 2004 and in higher school Grades (10 – 12) in 2006. On-going implementation challenges with NCS resulted in another review in 2009. This resulted in the introduction of CAPS document. Although the Department of Basic Education (DBE) together with the stakeholders keeps on crafting contemporary systematic curricula documents, the challenges of implementation are still manifest in the schooling system. The challenges are often linked to teachers' lack of content knowledge, poor teaching practices, lack of high-quality teaching and learning resources et cetera (see Kazima & Adler, 2006 and Makgato & Mji, 2006). I suggest that these challenges experienced at implementation are more pronounced in high-poverty settings where teachers have a less proactive work ethic and are textbook-bound.

An analysis of the curricula documents reveals similarities and differences between them. The underlying philosophies relating to the purposes of mathematics in schools and societies, communicated by these curricula remain relatively constant. The philosophies are driven by aspirations of social transformation, principles enshrined in the Constitution of the Republic of South Africa (Act 108 of 1996) and skills and values which seek to ensure that learners become global citizens. The documents indicate that these aspirations ought to be achieved regardless of the learners' socio-economic backgrounds and other adverse social factors (DoE, 1997; DoE, 2003 and DBE, 2011).

Amendments have focused on structural design; instructional programmes (i.e. work schedules, pace setters and assessment programmes); mathematical content; context; ways of assessing and teaching and learning resources (i.e. the latter is habitually supplementary since it is outsourced from private organisations, such as textbook writers and publishers). It is stated in CAPS (DBE, 2011) document that “text books and other resources should be consulted for a complete treatment of all the material” (p. 16).

A browse through each curriculum document reveals progressive improvements in the structural designs from C2005 to CAPS. For example, NCS presented four ‘learning outcomes’ as opposed to ten ‘specific outcomes’ in C2005. The NCS had forty-four assessment standards as opposed to two hundred and seventy-four performance indicators in C2005. In NCS clarity was given in terms of the content to be covered in each grade as opposed to over a phase (i.e. a group of grades, for example Foundation Phase is grade 1; 2 and 3), as was a case in C2005. The NCS had shifted from being a “critical curriculum” to being more of a “technocratic curriculum” (Cornbleth, 1990, p. 13), aimed at ensuring that all learners in South African mathematics classes learned content that was relatively the same. The assessment standards talked directly to what mathematical knowledge needed to be taught; thus I infer that it was much easier for teachers, regardless of their socio-economic backgrounds, level of education, to select problems which addressed the targeted mathematical knowledge. In NCS there was still a focus on application of mathematics; however, the use of ‘authentic’ life problems were not made explicit.

The shift from NCS to CAPS saw a further change in the structural design of the documents. Although there are five ‘content areas’ in CAPS and five ‘learning outcomes’ in NCS, the CAPS document outlined the instructional programmes and content more clearly than did the NCS. The CAPS document stipulates what needs to be taught; time frames for when every topic must start and end and how to assess per grade. More specification is given not only to the mathematical knowledge and content, but also detailed work schedules, pace setters, assessment programmes and, in some cases, lesson plans are provided. These instructional programs dictate the content to be covered and the deadlines. The CAPS document also gives “some clarification notes or teaching guidelines” (DBE, p. 32). There is a concerted effort to link prior knowledge to what is to be learned. CAPS is less focused of ‘contextualizing mathematics’ and the use of ‘real-life’ problems as a strategy for teaching and learning mathematics as was the case in C2005. The approach is decontextualized both conceptually and operationally.

The shift from one curriculum to the next often also saw a shift in the 'nature' of mathematics and the topics offered at different grades. The conventional views of mathematics are those of a subject that is isolated from other subjects. However, the Outcome-based views introduced in C2005 viewed mathematics as a human activity that incorporates other learning areas. Thus mathematics as a learning area could not be treated in isolation from other learning areas.

An understanding of a particular curriculum reached through analysis, can be used to explain how a curriculum influences classroom practice and by extension may explain why a set of activities or tasks were selected and how those tasks were used and/or managed in classrooms. As part of my course-work for the Master's degree I analysed the three curricula documents, that is C2005, RNCS and CAPS. It is worth noting that the crafting and introduction of these curricula happened in phases. The reviews of C2005 happened before it could even reach the Further Education and Training (FET) phase (i.e. Grade 10 -12) and where, subsequently, C2005 was phased out. In the upper phases an interim Core Syllabus (DoE, 1995) was used and it ran parallel to C2005. However, the two curricula were different in principles. The analysis gave a fairly detailed trajectory made over the years and I present relevant aspects here.

The analysis focused on content and also looked at the 'domain of mathematical practice and integration' promoted by the curricula using Dowling's (1998) domain of mathematics practice. Parker (2006, p. 68 – p. 70) summarized Dowling's (1998) domain of mathematical practice as:

The *esoteric domain* is most strongly classified (i.e. highly specialized, 'traditionally' abstract mathematical statements or tasks) with respect to other subjects in the curriculum. Both the forms of expression and the content are specialized. Ambiguity is minimized and therefore specialized denotations and connotations are prioritized. It is within this domain that the principles which regulate the practice of the activity can gain their full expression. Highly specialized abstract mathematical statements which might be elaborative either as a set of principles or set of procedures are identified as belonging to this domain. An example of a task in the esoteric domain: *determine the value of P*: $P = \sum_{k=1}^{13} 3^{k-5}$

The *public domain* is where there is relatively weak classification of content and mode of expression. Here the forms of expression and content are generally selected from the public domain context and they are referred, by the mathematical gaze of the esoteric domain, to mathematics contexts. An example of a task in the public domain: *Twenty water tanks are decreasing in size in such a way that the volume of each tank is half the volume of the previous tank. The first tank is empty, but the other nineteen tanks are full of water. Would it be possible for the water tank to hold all the water from the other nineteen tanks?*

The *expressive domain* is produced through a different type of contextualization, where the gaze combines specialized content with non-specialised forms of expressions. Here the classification of content is fairly strong and the classification of expression is weak. Here the non-mathematical element is re-contextualised within mathematical practice in order to give expression to the mathematical content. An example of a task in the expressive domain: *An athlete runs along a straight road. His distance d from a fixed point P on the road is measured at different times, n, and the form* $d(n) = an^2 + bn + c$ *. The distances are recorded in the table below:*

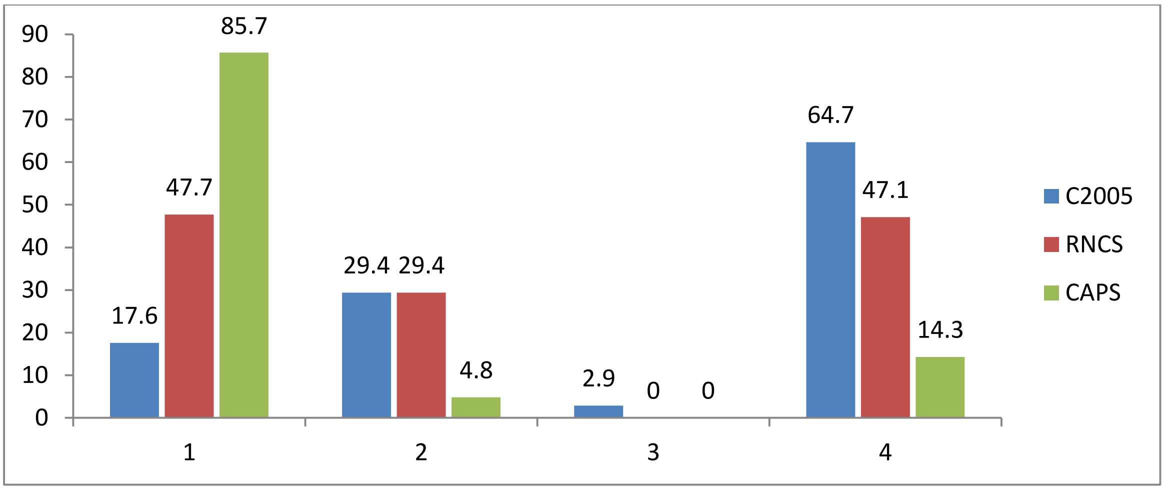
Time(s)	1	2	3	4	5	6
Distance(m)	17	10	5	2	r	S

Determine the values of a, b and c.

The *descriptive domain* arises when specialized mathematical expressions are imposed on non-specialised contents. So here the contents are weakly classified whereas the expression is relatively strongly classified. Here specialized expressions such as formulae are imposed on non-specialised content from the position of the esoteric domain. An example of a task in the descriptive domain: *A tuck-shop owner orders p brown loaves and q white loaves daily for x days. What does the expression* $x(p+q)$ *tell you?*

Crude counts were made across the various 'outcomes' for the three curricula using the kind of coding described above. These are illustrated in the bar graph (Figure 1.1) below. The horizontal axis has the following codes: 1 - esoteric domain; 2 – descriptive domain; 3- expressive domain and 4 - public domain. The graph indicates the change in the 'domains of mathematics' across the three curriculums.

Figure 1.1 Changes in the domain of mathematics



The graph shows extreme shifts in the domains of practice. The C2005 document focused on promoting mathematics as a useful subject for public usage. But the notion of mathematics for the public (4) lost its dominance as amendments were made from one document to the next. The expressive domain (3) form a small percentage in C2005 and it dies out in the RNCS document. Although the descriptive domain (2) was visible in C2005 and RNCS, there was less focus on this domain in the CAPS document. Over the years there was an increase in esoteric domain (1). In the CAPS document the esoteric became more prominent. Accordingly, in this study I anticipated that the tasks used in the observed mathematics classrooms would have striking features of esoteric domain, whereby learners are expected to gain mastery of mathematics with very little reference to integration and/or contextualization. A browse through the LTMS (i.e. textbook and tasks that were said to be CAPS compliant) used by teachers in this case study; gave an unambiguous idea about the dominance of the features of the esoteric domain in CAPS.

1.4 Overview of the report

This report is divided into five chapters. In chapter one, I have provided the background of the study, the South African mathematics education context and the research questions that shaped my analysis. In chapter two, I present a review of related literature. I also discuss the theoretical background and analytic framework, followed by the discussion on data collection and methodology. I present data analysis and findings in chapter four. In the last chapter of this report, chapter five, I conclude the research by discussing the findings from my analysis, implications and limitations.

Chapter Two

Literature Review

2.1 Introduction

As part of the background for this study, in chapter one, I reviewed at length the study by Brodie et al. (2009). In this chapter I review other literature, firstly focusing on Mathematics curricula issues: mathematics curricula; sources of mathematical tasks; mathematical tasks; number patterns. Secondly, I focus on the instruments used to measure features investigated in this study: taxonomies; cognitive levels; difficulty levels and the decline or increase of the mathematical task levels. Thirdly, I focus on the implementation of task in the classroom, with a direct focus on teachers' questions. The reviews assisted in relating previous studies to this study and directed my thinking. Lastly, I discuss the analytical frameworks I used in this study.

2.2 Mathematics Curricula Issues

2.2.1 Mathematics Curricula and classroom practices

In chapter one I allude on the changes made on the South African curriculum over the years since 1994. In this segment, I look at the 'mathematics curriculum' and review studies that examine classroom practices and the forms of learning opportunities some teaching practices afford mathematics learners. The construct 'curriculum' is often used as an all-encompassing term which describes what happens within a schooling system. Stein, Remillard and Smith (2007) argue that the term curriculum has multiple meanings. In the Cambridge Advanced Learner's Dictionary curriculum is defined as "the group of subjects studied in a school". Some authors (see Eisner, 1979; Dossey, 1992) use the term programme instead of the curriculum. The mathematics curriculum can also be talk about to as a 'mathematics programme' or a mathematics teaching programme.

Kelly (2004) defines curriculum as "all the learning which is planned and guided by the school, whether it is carried on in groups or individually, inside or outside the school" (p 10). However, the more elaborated definition of the construct curriculum refers to the entire program provided by an educational department; district; school or the teachers. Stein et al. (2007) outlined three modes of using the term curriculum. Firstly, the term curriculum can be used to refer to the substance or content of teaching and learning. However, it has more to do with “what” to teach, rather than “how” to teach. Secondly, it can also be used to refer to expectations for instructions laid out in policy documents and frameworks. And thirdly, it can refer to the material resource designed to be used by teachers in the classroom” (p. 321). The first and third meanings were found to be particularly significant for this study, since the main focus is on the enacted curriculum or ‘implemented curriculum’ (TIMSS, 1999) that is practiced in the classroom, rather than the ‘intended curriculum’ (TIMSS, 1999) pronounced in curriculum documents.

In South Africa, the curriculum or rather ‘what’ to teach in public schools, is determined at national level by the Department of Basic Education (DBE). Then the information is cascaded downwards to the provincial levels; the district levels and then to schools. Finally, it ends up in the classrooms. Information is carried through curriculum documents and other supporting documents, including textbooks. Although these documents stipulate what content/topics ought to be covered, they also carry some conceptions on the nature of mathematics, which in turn has implications for how mathematic should be taught. Dossey (1992) argues that “the understanding of different conceptions of mathematics is as important to the development and successful implementation of programme in the school mathematics as it is to the conduct and interpretation of research studies” (p.39). In South Africa Mathematics is seen as:

> ***“…*** *a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental process that enhances logical and critical thinking, accuracy and problem solving that will contribute in decision-making. Mathematical problem solving enables us to understand the world (physical, social and economic) around us, and, most of all, to teach us to think creatively.” (DBE, 2011, p. 8)*

The citation above carries a philosophy of mathematics. This philosophy relates to: comprehension of mathematical concepts, operations and relations; capacity for logical thought, reflection, explanation and justification; skill in carrying out procedures flexibly, accurately, efficiently and appropriately; ability to formulate, represent and solve mathematical problems; habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy as theorised by Kilpatrick, Swafford, & Findell (2001).

An intended philosophy of mathematics can be realised through the teachers' classroom practices. Boaler (2002, 2004, 2006) argued that the change in teaching practice of teachers can influence the learning of mathematics. Qualitative research conducted by McLaughlin & Talbert (1993) indicates that there are two contrary classroom practices. On one end is a traditional practice that is teacher controlled classroom marked: by many rules and sanctions; transmission teaching; more worksheets and tests and the content emphasis is on traditional fact-based curriculum. Traditional fact-based curriculum puts emphasis on computation and rigid systemic externally dictated principles such as standards of accuracy, speed and memorising. On the other end is a contemporary practice where teachers facilitate learning; construct group norms; work interactively with learners; encouraging an active learner role and the emphasis is on conceptual understanding. Contemporary teachers allow their learners to "wrestle with problems and puzzles of the subject matter and achieve deeper understanding than is possible with traditional modes of instruction" (McLaughlin & Talbert, 1993, p. 7).

Mathematical education researchers (e.g. Dossey, 1992; Remillard & Bryans, 2004, Stacey, 2003; Stein et al. 2007) acknowledge that classroom practices have over the years shifted away from being purely traditional. In present times more emphasis is placed on developing mathematical thinking, rather than on understanding mathematical procedures, principles and structures (Lakatos, 1976; Kitcher, 1984 and Schoenfeld, 1992). Thus contemporary practices that place more emphasis on learners' active construction of and communication about solutions to challenging problems are promoted. The new goals of learning mathematics have more to do with mathematical thinking, reasoning, problem-solving, connecting, communicating, seeking evidence and constructing arguments to make predictions and support conclusions (Stein et al. 2007, p. 320). These new goals are closely linked with the contemporary classroom practices.

The contemporary classroom practices aim for the co-production of knowledge where there are interactions between the teacher and the learners. It envisages a kind of mathematics that assists in developing mental processes. This cannot be a watered down kind of mathematics, where only memorization and the application of routine procedures are promoted. Rather it is the kind of mathematics that is taxing to the brain, provokes and promotes mathematical thinking (Stein et al, 1996; Ball and Bass, 2003, Kilpatrick, Swafford, & Findell, 2001). That is, it allows learners opportunities to engage in the processes of mathematical thinking, doing what makers and users of mathematics do. This includes framing and solving problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on (Stein et al., 1996, p. 456). Brodie (2008 & 2007) acknowledges the manifestation of contemporary practices in the South African mathematics classrooms. She argued that the NCS encouraged teachers to shift their teaching practices in ways that support mathematical thinking, such as reasoning, problem-solving, connecting, communicating, seeking evidence and constructing arguments to make predictions and support conclusions.

Contemporary classroom practices are supposed to produce learners who are problem solvers. A British researcher, Boaler (1997, 1998, 1999) believe that learners who are taught via contemporary classroom practices learn not only abstract procedures, but also learn a particular set of practices and associated beliefs. In one longitudinal study, she monitored about 300 learners as they attended two schools that used different classroom practices. He study was conducted over a period of three years. At one school learners were taught mathematics using textbooks that asked a series of short, closed questions. Using the domain of mathematical practice discussed in chapter one, we can say the school was operating from the esoteric domain, the mathematics knowledge taught was strongly classified. “Lessons began with a method and techniques being demonstrated by teachers from the front of the room, learners would then practice the methods through their books” (Boaler, 2001, p. 122). The other school used a series of open-ended projects to teach. The school operated from the public domain, the mathematical content was weakly classified. The classroom practice was based on the belief that learners should “encounter situations in which they need to use and apply mathematical methods" (p. 122). However, it is not clear how teachers introduced new methods and content that learners needed in order to complete their projects. From the findings of her study, Boaler (1997, 1998, 1999, 2001) concluded that learners who are taught to reproduce mathematical fact, rules and procedures do not learn as much as learners who are required to explore and understand the nature of mathematical concepts, processes or relations. She concluded "that knowledge and practices are intricately related and that studies of learning need to go beyond knowledge to consider the practices in which learners engage and in which they need to engage in the future" (p. 125).

In the late 1990s and early 2000s, most educational researchers recognised that knowledge is socially shared and that learning may be represented as participation in social practice. Thus, learning to manipulate mathematical procedures was no longer sufficient, but learners had to learn complex and non-algorithmic thinking that would enable them to solve any mathematical problems. I believe that this perspective brought about changes noted in Boalers (1997, 1998 & 1999) study. Qualitative research focusing on classroom practices, appreciate some of the attempts made in mathematics education to shift towards contemporary practices (see Staples, 2007, Lobato; Clarke & Ellis, 2005, Hufferd-Ackles, Fuson, Sherin, 2004, Kazemi & Stipek, 2001, Davis, 1997). Most teachers who were noted to be changing their classroom practices or attempting to shifts from traditional practices belonged to professional communities which encourages and/or enabled them to consider teaching differently (McLaughlin & Talbert, 1993). One example of such teachers is report about in Davi's (1997) work. While conducting an extended collaborative research project with a mathematics teacher, Davids (1997) noted a change in the teacher's classroom practice and attributed the change to the influence of "a graduate course in the mathematics education" (p. 361) the teacher was attending.

Although teacher and researcher recognised the need to shift from traditional ways of teaching, there are still many challenges. Researcher have identified multiple new classroom practices used by teachers to help learners develop conceptual understanding of mathematics. These new classroom practices combination of the traditional and contemporary practices. In one of the studies, Staples (2007) reported on teachers that integrated traditional and contemporary practises. Staples (2007) explored how whole-class collaborative inquiry was used to support the classroom practice that allowed learners to reason and construct their understanding of mathematics. Staples (2007) argues that collaboration "implies a joint production of ideas where learners offer their thoughts, attend and respond to each other's ideas, and generate shared meaning or understanding through their joined efforts" (p. 162). Accordingly, collaboration is one of the features of contemporary teaching practices. Whereas, whole-class teaching is often viewed as a traditional teaching strategy.

Staple's (2007) study focused on a teacher working with a lower-attaining group of learners. Staples (2007) purposefully selected a teacher who had a wealth of teaching experience. The teacher's classroom practice was based on developing learners' abilities to analyse problem situations, generate solution methods and develop conceptual and procedural fluency. The teacher shared a philosophy that views mathematics as a product of human thinking. The teacher applied teaching practices that accommodated learners' needs. Staples collected data by means of ethnographic methods. She observed and analysed all classroom interaction and examined learners' and teacher's understanding of classroom practices and participation structures. In the findings Staples (2007), reflected on the role of the teacher in the observed classroom. She noted that the teacher's roles were to: "supported learners in making contributions; establishing and monitoring a common ground; and guiding the mathematics" (p. 172). The teacher consolidated learners' thinking and systematised their work around mathematical ideas and practices.

In Boaler (1997, 1998, 1999) it is not clear how teachers introduced new methods and content that learners needed in order to complete their projects. One can only assume that the teachers told and demonstrated procedures to their learners. In another study, Lobato, Clarke and Ellis (2005) a made up a compelling argument that suggested that learning theories can change teachers' action and intentions, as a result their teaching practices. They argued that a constructivist theory could assist teachers to:

- differentiate their actions that are aimed at generating understanding from those aimed at the repetition of procedures
- shift away from external; responses towards what can be inferred about learners' mental actions

- gain understanding that knowledge cannot be directly transferred to learners via language
- become more interested in learners' actions as clues to understand what models learners are constructing (p. 102)

Based on their understanding of the constructivist theory Lobato et al. (2005) reformulated a feature of the traditional teaching practice that is 'telling'. Lobato et al (2005) argues that 'telling' traditionally means stating information or demonstrating procedures, thus promote transmission model of teachers that opposes contemporary teaching practices. Traditionally, telling actions require the teacher to stand in front of the class. Lobato et al (2005) continued to argue that among many other drawbacks traditional telling actions limit mathematical exploration, minimises the possibility of cognitive engagement and may communicate to learners that there is only one solution path (p. 103). Lobato et al (2005) using Romagnano's (1994) argument highlighted some of the dilemmas teachers face when having to choose between traditional and contemporary teaching practices, that is to tell or not to tell. They acknowledge that teachers know that telling can "pose the danger of restricting further mathematical exploration, but never telling can result in learners disengaging with the mathematics or engaging at a superficial level" (p. 105). Lobato et al. (2005) argue that there is a need for telling in contemporary classrooms and that the telling actions must not take away the need for learners to reflect on the mathematics and develop their own solution processes. Accordingly, Lobato et al. (2005) claimed that telling can occur in three ways:

1. In terms of the function rather than the form of teachers' communicative acts
2. In terms of the conceptual rather than the procedural content of the new information
3. In terms of its relationship to other actions rather than as an isolated action

Lobato et al. (2005) also considered how teachers can exploit one of the traditional teaching practices feature (i.e. telling) and use it to promote conceptual understanding. They examined the processes of telling empirically and concluded that the definition of what it means to tell is context sensitive.

From the discussion above one can deduce that the teacher's intentions and actions are the main realisms that determine what learners will learner. A teacher whose focus is on conceptual understand rather than procedural understanding is likely to promote conceptual growth regardless of the classroom set-up. Teachers who prefer traditional ways of teaching, can reformulate certain features and align them with their intentions.

2.2.2 Source(s) of mathematical tasks - Textbooks

In many mathematics classrooms learners spend most of the time working on tasks selected from textbooks. Worldwide, textbooks are recognised as the main sources of information for teachers, learners and parents. Over the years, textbooks and other written teaching and learning materials used in South Africa have become subjects of interest for many researchers (see Adler, 2000; Adler, Dickson, Mofolo, Sethole, 2001; Ensor, Dunne, Galant, Gumdze, Jafters, Reeves, Tawodzea, 2002; Stoffels 2007). In South Africa, due to contextual factors, the availability of various textbooks at a school is taken as an indication of an effective education system. Stoffles (2007) found that the government appraisal of textbooks was phony (i.e. not sincere), and recommended that the role of textbooks as resources be taken more seriously by giving more time for evaluation and trialling. The larger variety of textbooks that are habitually published makes the task of selecting textbooks quite daunting for teachers. Hence, many teachers limit themselves to the use of one or two specific textbook(s).

Studies (see Remillard and Bryans, 2004; Lloyd, 1999; Remillard, 2000; Adler, 2000) focusing on the use of mathematics curriculum materials, including textbooks, indicate that textbooks help teachers understand the curriculum and also help develop teachers' knowledge. Textbooks are presumed to save a lot of time when teachers prepare lessons and select tasks. Ensor et al (2002) explored the impact of one textbook which was said to promote a kind of approach to mathematics which was in line with the approach proposed in the curriculum. Ensor et al (2002) argued that curriculum materials promote certain patterns of teaching and learning of mathematics She concluded that textbooks have a role to play in promoting the desired curriculum transformations; furthermore, she argued that teachers need to learn to develop mathematics activities which support transformation. Her argument implies that textbooks can be used to promote certain curriculum objectives. However, Remillard and Bryans (2004) found that the way teachers use and interact with curriculum materials were primarily influenced by the teachers' initial orientations towards a curriculum. There is often a difference between the intended curriculum promoted through textbooks and the enacted curriculum and however the tasks in the textbook are implemented.

2.2. 3 Mathematical Tasks

The role of mathematical tasks in classrooms cannot be downplayed. Shimiza et al (2010) argue that there is theoretical and empirical evidence from studies conducted in different countries which show that tasks play a pivotal role in mathematics classrooms. Mathematical instruction and classroom activities are directed through the use of tasks (Shimiza et al, 2010 and Doyle, 1988). Mathematical tasks are in essence what learners 'do' in the classrooms. Mason & Johnston-Wilder (2006) defined mathematical tasks as "what learners are asked to do, be it computations to be performed, symbols to be manipulated, diagrammatic representations to be made or translations of word problems into mathematical statements or models" (cited in Kaur, 2010, p. 15). Mathematical tasks are more than just objects used to convey information; they also guide learners to a particular mathematical concept (Stein, Grover and Henningsen, 1996). "The task that teachers assign can determine how learners come to understand what is taught. In other words, tasks serve as a context for learners' thinking, during and after instruction" (Shimiza, Kaur, Huang, and Clarke, 2010, p. 1).

In South Africa, the Department of Education (1997) required teachers "to think and prepare interesting and appropriate learning activities …" (p. 11) which promote full understanding of mathematics, not just knowledge of mathematical concepts, principles, and their structure. Stein et al (1996) argue that learning activities should have more than one solution strategy and multiple representations; they should require students to communicate and justify their procedures and understanding.

Researchers around the globe support the notion that the type of mathematical task that ought to be used in classrooms are those that promote mathematical thinking, reasoning and problem solving (Brodie, 2008; Stein and Lane, 1996; Anthony & Walshaw, 2009). Brodie (2008) argues that "a key aspect of reform-orientated practice is choosing tasks that allow for conceptual thinking, reasoning, justifications and communication of mathematical ideas" (p. 34).

Through the use of mathematical tasks learners ought to be encouraged to make links between mathematical ideas; construct mathematical meanings; manage to perform non-procedural and multi-layered thinking (Anthony & Walshaw, 2009). This means that mathematical tasks that are selected or created for teaching and learning ought to promote what Kilpatrick, Swafford and Findell (2001) refer to as mathematical proficiency. Brodie (2000) argues that teachers can promote mathematical proficiency by giving learners tasks which encourages formulation, justification and testing of conjectures.

2.2.4 Number Patterns

During the period of the data collection, number patterns were taught in the classes I observed (refer to appendix D and E, for Annual Teaching Plans). Number patterns are studied across all grades of the South African curriculum. In grades 4 -6 (i.e. Intermedia Phase) number patterns tasks are used in “laying the foundation for study of formal algebra in the Senior Phase while at the same time developing important mathematical thinking skills” (DoE, 2003a, p. 37). The curriculum is designed such that number pattern tasks in the grades 7 – 9 (i.e. Senior Phase) develop on the skills and content imparted in the lower grades. Through the curriculum learners in the grade 8 and 9 are expected to “use algebraic processes in the description of these patterns” (DoE, 2003a, p. 39). From Grade 10 to 12 learners are expected to “solve problems relating to arithmetic, geometric and other sequences and series” in addition “explore real-life and pure mathematical number patterns and problems which develop the ability to generalise, justify and prove” (DoE, 2003b, p. 12).

The South African curriculum highlights the necessity for learner across grades to investigate; justify and generalise a rule/formula in words or algebraically of patterns. Accord to Chua & Hoyles (2012) “pattern generalising tasks typically involve getting learners to examine specific cases to search for a pattern, extend the pattern to predict other cases, and articulate the functional relationship underpinning the pattern using mathematical symbols” (p. 161). Driscoll (1999) argues that ‘generalisation’ is a cognitively demanding thinking process that does not only apply in patterns, but throughout algebra.

Mathematics education researchers acknowledge that learners often find it difficult to generalise or miss the step(s) to generalising/globalising. However, learners can without difficulty describe patterns in recursive terms (Driscoll, 1999; Warren, 2005, Warren and Copper, 2008). Teachers are encouraged to capitalise on learners’ understanding of recursive relationships to build a close form of a rule. From the work of Driscoll (1999); Warren (2005) and Warren and Copper (2008) I deduced that an important undertaking of a teacher is to take learners from the stage(s) of describing patterns in recursive terms to a stage(s) where they are able to generalise. Driscoll (1999) argues for the uses of questions to encourage generalisation and convincing arguments in the classrooms. Some of the question suggest by Driscoll (1999) are:

- How did you get your answer?
- How can you be sure that your answer is correct?
- Why is your formula right?
- Why did you do that?
- What are you trying to find out? (p. 106 – 107)

Driscoll (1999) argues that algebraic thinking involves the ability to "think about *functions* and how they work, and to think about the impact that a system's *structure* has on calculations" (p. 1). He then theorise that algebraic thinking should involve three habits of mind: doing-undoing; building rules to represent functions (i.e. including generalising) and Abstracting from Computation. Habits of the mind can be developed through "guiding questions" (p. 3). Driscoll (1999) gives examples of guiding questions which can the used to develop each of the three habits of the mind. This study is about the use of tasks in the classrooms and teacher questions, thus I found Driscoll's (1999) work pertinent.

Table 2.1 below indicates the grades, topic and content(s), as specified in the CAPS document (DBE, 2011) of the topic 'number patterns' taught in the classes I observed.

Grade	**Topic**	**Content**
11	Number Pattern	• Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic
9	Numeric and geometric patterns	Investigate and extend patterns • Investigate & extend numeric & geometric patterns looking for relationships between numbers; including patterns:

		- Represented in physical or diagram form - Not limited to sequences involving a constant difference or ration - Of learners' own creation - Represented in tables - Represented algebraically • Describe & justify the general rules for observed relationships between numbers in own words or in algebraic language

Table 2.1 Number Pattern in CAPS curriculum

In grade 11 the specified content refers to quadratic number patterns. Samson (2008) through his interaction "with colleagues and learners alike in the exploration of patterns strategies for generalising decontextualized number patterns based on quadratic formulae" (p. 9) came up with eight.

2.3 Instruments and Features gauged

2.3.1 Instruments of analysis: Taxonomies

In mathematics taxonomies are developed and used for different aims (see Stein et al, 2000; Stein et al 1998; Stein et al 1996; Porter, 2000; Berger, Bowie & Nyaumwe, 2010). Generally, taxonomies are used in education to measure the quality of tasks and/or the levels of thinking required to perform a mathematical task. The underlying ideologies and designs of most taxonomies go along with Bloom's Taxonomy (1956), whereby levels/orders range from the lower levels to higher levels.

In South Africa, in the past the national department of education provided a separate Subject Assessment Guidelines for Mathematics (SAGM) that outlined the taxonomy meant to assist teachers in measuring and creating assessment tasks. At present, the department of basic education through CAPS (2011, p. 53) (see appendix D) provides a taxonomy which has four levels: knowledge, routine procedures, complex procedures and problem solving. Each level has descriptors. Both taxonomies proved to be very difficult to work with. However, the difficulty is not unique for DoE or DBE taxonomies. For example, Thompson (2008) found that mathematics teachers have difficulties with using (i.e. decoding and understanding) Bloom's

Taxonomy in order to create higher-order thinking assessment tasks. The difficulties stem from the fact that taxonomies do not separate the 'levels' from 'the kind of thinking'. Berger, Bowie, Nyaumwe (2010) argued that the elements of the SAGM taxonomy made it difficult to apply and remarked that the taxonomy "conflates cognitive levels with the type of mathematical activity" (p. 30). In the CAPS (2011) document there seem to be common characteristics between 'routine procedure' and 'complex procedure', which makes it tricky to use this taxonomy to measure the cognitive level of some tasks. The taxonomy is not clear whether a well-known procedure that contains many steps or tricky algebraic manipulation should be classified as a routine or complex procedure. This implies that much care is needed when measuring the cognitive levels of task, since it is tricky to locate a task in an appropriate level (Stein et al 2000)

Taxonomies can and/or are used to measure the cognitive demands of tasks. Although there are challenges (some were noted above) with using taxonomies, a suitable taxonomy can be very useful. In other cases, a proposed taxonomy can be improved, or a new and more favourable taxonomy can be developed. For example, Berger et al (2010) saw gaps in the SAGM taxonomy and thus used other available taxonomies to close those gaps. In the end they proposed a 'new' taxonomy.

The levels of cognitive demand or levels of thinking required for undertaking a task depend on the familiarity of the learners with the content of the task (Berger et al, 2010; Thompson, 2008). Thus, one should determine whether or not a learner is familiar with the measured task and how the learner will do the task before categorising the task. Berger et al (2010) argues that since the cognitive demand of task depend on familiarity of the learner with the task, it is important to consider the age and grade level of the learner. Studies have shown that teachers tend to categorise tasks at higher levels than they actually are, and to develop task of lower cognitive demands (see Thompson, 2008; Senk, Beckmann, Thompson, 1997). With this in mind, the following questions are worth considering: Do South Africa teachers ever use taxonomies to measure the cognitive levels of tasks they setup for learner? Could it be that teachers select or create learning tasks of lower order thinking without the knowledge that they are doing so? In this study these questions are considered, but are not addressed directly.

Stein and Smith's (1998) taxonomy for which the early version was designed by Stein, Grover and Henningsen (1996), was chosen as a suitable taxonomy for my study for two reasons. One, it was used in several studies successfully (see Kaur, 2010 and Brodie et al, 2009). Two, it was designed for mathematical learning tasks. In this study Stein and Smith's (1998) taxonomy is used to measure the cognitive levels of both the learning and practice tasks. Stein and Smith (1998) explain the multiple roles of mathematical tasks and provide a detailed analysis guide for measuring the cognitive demands of the tasks used by teachers in their classrooms.

2.3.2 Cognitive levels of mathematical tasks

In the preceding section I have reflected on issues concerning use of taxonomies in education. In this session I consider and discuss the taxonomy used in this study.

The *very low level (zero)* is used to classify tasks which focus on the *memorization* of facts, rules or formulae with no explanations required from learners. The *low level (one)* is used to classify tasks which are procedural in nature and are concerned mainly with producing correct solutions; such tasks require no explanations. The focus of 'level one' tasks are on producing correct solutions rather than developing understanding. The *high level (two)* is used to classify tasks where learners are required to make connections and meanings which may enrich their mathematical understanding. In these tasks explanations are required. The *very high level (three)* is assigned to tasks where learners are required to solve problems (i.e. work as true mathematicians); such a problem will be presented without direction or hints. At this level, learners are expected to demonstrate a deeper understanding of mathematical concepts or, sometimes, the application of these concepts to 'real-world' contexts. These tasks often require that several solutions are produced and no methods are suggested for the learners. Learner working with such tasks need to provide solutions in a clear mathematical structure with comprehensive explanations.

The fact that the cognitive demands of tasks depend on who the task is designed for; how it is administered in the classroom and that the teacher and learners can change the initial apparent level of the task (Brodie et al, 2009) makes it very difficult to use the descriptors of the cognitive levels. The teacher can affect the level of the task through the kind and level of assistance s/he provides to the learners. Learners can affect the level of the task by how they engage with the task. This means that there is an initial apparent level of the task and the actual level of the task that is influenced by how the teacher and learners engage with the task.

2.3.3 Difficulty levels of mathematical tasks

At some stage during the write up of this study I had an opportunity to work with a team put together by a Council for Quality Assurance in General and Further Education and Training (uMalusi). The team was tasked to analyse the 2012, 2013, 2014 National Senior Certificate examination papers from both DBE and IEB (Independent Examination Board). Using description of cognitive levels from the CAPS document (DoE, 2011, p. 53), see appendix C.

In my engagements with the team, after each team member (i.e. including myself) had analysed papers independently, we compared our results and had discussion of our findings. Table 2.1 below gives an example of the consolidated group analysis of a question dealing with number patterns.

	CD	DL
QUESTION 3		
3.1 Given the arithmetic sequence: $-3 ; 1 ; 5 ; ... ; 393$		
3.1.1 Determine a formula for the n^{th} term of the sequence. (2)	P	E
3.1.2 Write down the 4^{th}, 5^{th}, 6^{th} and 7^{th} terms of the sequence. (2)	M	E
3.1.3 Write down the remainders when each of the first seven terms of the sequence is divided by 3. (2)	M	E
3.1.4 Calculate the sum of the terms in the arithmetic sequence that are divisible by 3. (5)	P/W	D

Table 2.1: Categorisation by Umalusi team

P- stands for routine procedures or procedures without connections, E stands for easy, M stands for knowledge or memorisation, P/W stands for procedures with connection or complex procedures and D stands for difficult. From the whole process I learned that categorisation of cognitive demands (CD) cannot completely indicate the tasks' difficulty or complexity of the some of the procedural mathematical tasks and solutions. For this reason, I considered looking at the 'difficulty levels' (DL) of the tasks used in the observed lessons. The difficulty of a task is closely linked with the number of knowledge elements and the combination of knowledge elements that are not often combined. The Umalusi team based their measurement of the difficulty of task on the framework by Leong (2006) for thinking about question difficulty. The framework comprised the following four general categories:

- Content (subject/conceptual) difficulty
- Stimulus (question) difficulty
- Task (process) difficulty
- Expected response difficulty

Leong (2006) acknowledged that difficulty framework he proposed was "merely an attempt at creating a conceptual framework to think about item (tasks) difficulty" (p.6), because it cannot explain why certain low-order tasks on specific knowledge can be more difficult than others. He also stated that the item (tasks) difficulty framework "does not state the relationships and interactions among the concepts of the framework" (p.6). Difficulty level as used in this study, was an attempt to compare tasks against each other to check which tasks are harder that the others. No attempt was made to compare the frameworks by Leong (2006) and Stein & Smith (1998) was made.

2.4 Implementation of Tasks

2.4.1 Teachers' Questions

Literature suggests that there is close connection between the teachers' questions and what happens in the classrooms, including how task are implemented (Sullivan & Clarke, 1991; Boaler & Brodie, 2004). The teachers' questions can elicit learners' explanations and justifications, and promote mathematical discourses (Kazemi & Stipet, 2001; Silver and Smith 1996). The manner in which teachers arrange their questions, and by extension their classroom discourses, can assist learners in communicating and thinking about mathematics

(Sullivan & Clarke, 1991). Sullivan & Clarke (1991) argue that the discourse between the teachers and learners happens mainly through questions and instructions. Boaler and Brodie (2004) argue that questions need to be viewed within the context of the kind of classroom practice that they are used and in relation to the tasks. All these arguments shape theories which suppose that good questions can support certain classroom practices, initiate mathematical conversations, provoke learners into constructing their own mathematical meaning, understanding and enrich their experience of mathematics.

Questions need to be viewed within the context of a classroom practice (Hiebert and Wearne, 1993). Traditionally, teachers' questions are aimed at testing learners' knowledge and are usually posed after teachers have demonstrated some mathematical procedures. In such instances the teacher's role is to transmit mathematical knowledge to attentive and silent learners. In contrast, in contemporary teaching practices, learners work independently. Contemporary, teachers support learning by: asking good questions, offering hints, suggesting forms of representation, and asking for clarification and justification (Stein, Engle, Smith, and Hughes, 2008). Stein et al (2008) argue that at some stage during a lesson a teacher must facilitate the discussion of the task. The aim of the discussion of the task is the construction of knowledge through communication and the search for common grounds. Stein et al (2008) understands the teacher's role as that of a regulator. Teachers regulate interaction through questioning, asking for explanations and the underlying reasons as to why learners chose particular strategies when working on a particular task. The arguments presented in this segment point to the importance of focusing on teacher questions in order to understand the relationship between teaching and learning.

Like tasks, questions can also be graded according to their supposed cognitive demand levels or codes. Sullivan and Clarke (1991) differentiate between lower order and higher order questions and reasoned that asking higher order questions enhances learning. In their monograph they found that smaller numbers, but nonzero, of teachers asked higher order questions that "required the pupils to think independently or to give more than one answer" (p. 9). They argued that good questions promote active learning. Good questions were found to be open ended and required more than just recalling facts. They also presented strategy of asking good questions to assist teachers stimulate higher order thinking among learners. Sullivan and Clarke (1991) provided ways in which teachers can move away from perceiving learning mathematics as mastering procedures. Whereas, Boaler and Brodie (2004) noted that different questions shape the nature and flow of classroom discussions and the cognitive opportunities offered to learners.

2.4.2 Decline in the cognitive levels

Classroom researchers noted some of the tasks that teachers select are of high cognitive demands. However, there was a decline in the level of those tasks at implementation (Stein et al., 1996, Stein et al., 1999, Stein et al., 2000). Stein et al. (1999) found that only about one-third of the high level demand tasks that were selected remained that way at implementation. Staple (2007) note that teachers took over the challenging aspects when there was evidence that learners had stopped making progress or when they seem to be struggling or showing signs of frustration

Stein et al. (2000) looked at using cognitively complex tasks in the classroom at length. They conceptualised tasks as “classroom-based activities” (p. 25) rather than mere problems written in textbooks or other learning and teaching support materials. Viewing tasks as classroom-based activities, suggests that tasks can change form and emphasis at implementation, in the interactions of teaching and learning. However, Stein et al. (2000) also acknowledged that tasks can be changed during the setup phase. For example, a teacher who presume that her learners are ‘weak’ may setup tasks in such a manner that the challenging aspects of the tasks are removed. The teachers and their learners contribute to how goals and intentions of the tasks are realised during class interactions. The kind of support given by the teachers to their learners influence the level of tasks (Henningsen & Stein, 1997). Henningsen and Stein (1997) argue that teacher can promote deeper levels of understanding by consistently probing learners’ understanding of tasks in class.

The book by Stein et al (2000) was a culmination of a series of studies on classroom instruction, including task setup and implementation, conducted over a period of 5 years (see Stein et al., 1999; Stein and Smith, 1998; Henningsen & Stein, 1997; Stein et al., 1996,). The findings from these studies suggested factors associated with the decline of high level of cognitive demands of tasks, as the following:

- Problematic aspects of the tasks become routinized
- The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer
- Not enough time provided to wrestle with the demanding aspects of the task or too much time is allowed and students drift into off-task behaviour.
- Classroom management problems prevent sustained engagement in high-level cognitive activities.
- Inappropriateness of task for a given group of learners
- Learners are not held accountable for high-level products or processes.

Stein et al. (2000, p. 27)

Stein et al. (2000) found that tasks that declined during implementation transformed into: procedures without connection to meaning; un-systemic exploration and/or non-mathematical activities. They noted that one of the teachers became disappointed when learners were not making progress and were complaining that the tasks are too difficult. The teacher gave-in and provided learners with a procedure for solving the task. In another case, the teacher allowed learners to continue without her kind of support needed when learners are working on cognitively demanding tasks. As a result, learners spent too much time on the task and they struggled and end-up failing to make progress towards mathematical understanding. They also observe classes were learners played absentmindedly with their manipulatives or talked with their peers about subjects that are not related to the tasks at hand. They concluded that this happens when the task is not matched with the learners' prior learning and other classroom management problems.

2.5 Analytic framework

2.5.1 Cognitive demands of tasks

In line with Kaur's (2010) study, the framework by Stein and Smith (1998) was used to ascertain the cognitive demands of tasks selected by the teachers observed in this study. This framework is similar to the one used by Stein, Grover and Henningsen (1996) and Stein, Smith and Henningsen (2000). It categorises tasks into four levels of cognitive demands. Table 2.2 below shows the description of the features in each category.

Lower-level demand (memorisation) • Involve either previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous. Such task involves the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced. Lower-levels (procedures without connections) • Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers instead of developing mathematical understanding. • Require no explanation or explanations that focus solely on describing the procedure that was used. Higher –level demand (procedures with connection) • Focus students’ attention on the use of procedure for the purpose of developing deeper levels of understanding of mathematical concepts and ideas • Suggest explicitly or implicitly pathways to follow that are broad general procedures that have closed connection to underlying conceptual ideas as

opposed to narrow algorithmic that are opaque with respect to underlying concepts. • Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representation help develop meaning. • Require some degree of cognitive effort. Although general procedure may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlies the procedure to complete the task successfully and that develop understanding. Higher-level demands (doing mathematics) • Require complex and non-algorithmic thinking – a predictable well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example. • Require students to explore and understand the nature of mathematical concepts, processes or relationships. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to analyse the task and actively examine task constraints that may limit possible solutions strategies and solutions • Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

Table 2.2 Cognitive demands (Smith and Stein, 1998, p. 348)

2.5.2 Teacher's Questions

Boaler and Brodie (2004) provided a comprehensive framework for analysing teachers' questions. The framework has nine categories, however, Brodie et al (2009) and Jina (2008) found that only four of the categories were relevant in the South African context. Table 2.3 below shows the description of the four categories used in this study.

Question type	Description

1. Gathering information, leading learners through a method	Require immediate answer Rehears known facts/procedures Enable learners to state facts/procedure
2. Inserting terminology	Once ideas are under discussion, enable correct mathematical language to be used to talk about them
3.Exploring mathematical meanings and/or relationships	Points to underlying mathematical relationships and meaning. Makes links between mathematical ideas and representations
4. Probing, getting learners to explain their reasoning	Ask learners to articulate, elaborate or clarify ideas

Table 2.3: Categories of teacher's question (Boaler and Brodie, 2004)

Boaler and Brodie (2004) analysis schedule was used for coding the observed teachers' questions. As with the tasks, questions were graded according to their supposed cognitive demand level.

2.6 Conclusion

In this chapter, I presented a review of literature about mathematics curricula; sources of mathematical tasks; mathematical tasks; number patterns; the instruments used to measure features investigated in this study and on the implementation of task in the classroom, with a direct focus on teachers' questions. I also discussed the offered the analytic frameworks that were used in chapter four, to analyses data that was collected.

Chapter Three

Data Collection and Methodology

3.1 Introduction

In this section I present the methodology and the strategies followed when collecting information in relation to mathematics tasks and teaching actions in the observed classrooms. I also describe the tools for data collection. Furthermore, I provide details about the sample used for the study; challenges faced during the process of data collection and issues relating to the validity and reliability of the findings.

3.2 Research Design and Methodology

The primary objective of this study was to gain insight into the cognitive levels and difficulty levels of mathematics tasks selected and implemented by the teachers in the study. I also wanted to focus on the teachers' classroom practice, particularly at the types of teachers' questions used during the implementation of the tasks, to determine how the preferred teaching actions affected the cognitive demands of the tasks. Accordingly, an interactive qualitative research design was adopted.

Features measured in this study were foregrounded by the fact that the sampled school was located in a high-poverty community and was not well resourced. In literature there is little information that talks to or about mathematics teaching and learning in high-poverty schools and communities. However, there is a great need to explore what happens in mathematics classrooms in such communities. This is particularly urgent given the dire state of mathematics in such schools. Hence, I supposed it was meaningful to observe mathematics teachers teaching in a school located in high-poverty community.

A case study data collection strategy was selected as the most appropriate method for this research. Creswell (2008) and Opie (2004) share a comparable interpretation of a case study as "an in-depth study of interactions of a single instance in an enclosed system" (Opie, 2004, p. 74). Two teachers and their classes of learners at a single school were 'cases' in this study. Since I looked at more than one case (i.e. two teachers), this is referred to as "multiple case studies" (Merriam, 1998, p. 40). Data from the two teachers and their classes was collected, analysed and findings were presented as two individual case studies.

3.3 Data collection and instruments

This study investigated the tasks used by teachers in their mathematics classrooms and how teachers' classroom practice, more particularly whether their questions lowered or raised the cognitive demand levels of the instructional tasks they selected. To engage in this type of investigation I had to observe teachers in practice. Two methods of data collection were employed: classroom observation and collection of instructional material (i.e. learning and practice tasks used by in the classes). As part of the observation, the teachers and their classes were audio tape-recorded. I would have preferred to observe one teacher teaching two classes at different times during normal schooling time. However, due to my obligation as a full-time teacher I had limited time for collecting data as I preferred. I managed to secure periods for observing the two teachers teaching their classes. It was very difficult to find times which matched well my plans and duties to the lessons I preferred observing. However, I managed to observe a total of ten mathematics lessons for both teachers. The ten mathematics lessons were taught by the two teachers who agreed to participate in the study.

The length of each lesson ranged from thirty minutes to an hour depending on the time allocated on the observed teacher's timetable. I collected and noted the sources of all tasks used during the lessons and those given as 'homework', these made up the total of all tasks that were analysed. The table 4.3 below maps the tool used to research questions addressed.

Research Questions	**Methods of data collection**			
	Lesson observation	Task Collection	Audio-taping	Other artefacts (notes etc.)
1. What are the cognitive levels of the learning and practice tasks use in the case study township classrooms?	**X**	**X**		
2. How do the case study teachers' classroom practice impact on the cognitive demands levels and/or difficulty levels of the mathematical tasks?	**X**		**X**	

2a. What kind of questions are asked by the case study teachers to assist learners when working on learning and practice tasks?	**X**	**X**	**X**	**X**

Table: 3.1: Mapping data collection methods to research questions

In addition, I took photos of the teachers' board work, made brief notes of some of the moments I found attention-grabbing, had informal conversations with the observed teachers and interviewed one of the teachers. The interview was taped and I intended to transcribe the interview. Unfortunately, my bag with the digital audio-tape and other material was stolen at the school. The bag and other materials in it were recovered at the later stage, but the audio-tape was never found and I strongly believe that it was stolen for the hardware rather than for any other reason. Fortunately, most of the other audio data had been saved on my computer. I used the data collected to trace questions asked by teachers, to evaluate the cognitive demand levels of the tasks in the two classrooms, to qualitatively examine the teachers' practice (i.e. the teachers' use of questions) and thus drawing conclusions.

3.3.1 Observations

Observations were a key strategy for data collection in this study. Patton argues that observations gives a researcher "the opportunity to look at what is taking place *in situ* rather than at second hand" (as cited in Cohen, Manion and Morrison, 2002, p. 305). I audio-taped and took notes and never participated in any of the classes activities; thus my role was that of a 'non-participant' observer. Opie (2204) describe non-participant as "where the researcher has no interaction with the subject during data collection" (p. 126). I believe if I had taken part in classes' activities, I would have affected some of decisions and influenced the validity and reliability of the study.

I observed two classes, one grade 11 class and one grade 9 class at all times. Lesson observation in my study was through audio-taping; observation schedule and note taking. During observations attention was paid to the teachers' utterances; what they wrote on the board; the tasks used during the lessons; sources of tasks and the solutions to the tasks and the teachers' general classroom practice. I kept notes of specific features of the two observed classes and some of the concerns I wished to discuss with the teachers. The notes were also useful in capturing particular details about the individual teacher. The table 4.4 below shows which lessons observed were audio-taped; transcribed and where the observation schedule was used.

	Grade 9 – teacher B			Grade 11 – teacher A		
	Audio-taped	Transcribed	Observation Schedule	Audio-taped	Transcribed	Observation schedule
Lesson 1	0	0	1	0	0	1
Lesson 2	0	0	1	0	0	1
Lesson 3	1	1	1	1	1	1
Lesson 4	1	1	1	1	1	1
Lesson 5	1	1	1	0	0	1
Total	3	3	5	2	2	5

Table 3.2: General characteristic of observations

I planned on collecting data on the same topic (i.e. number patterns) in both grades observed. In the week I had arranged with both teachers and the school to collect data by, the grade 11 teacher had started teaching number pattern according to the work schedule. However, the grade 9 teacher was behind schedule, thus the first two lessons I observed were on exponents. For that reason, I did not audio-tape those lessons but I used the observations schedule and wrote some notes focusing more on the teachers' classroom practice. In lesson three teacher B introduced number patterns and all thirty-minute lesson time was spent on writing and the discussion of 'note'. In lesson four, teacher B wrote a very short activity on the board consisting of only seven tasks which required learners to find the next three terms of given sequences. Lesson 5 was on 'finding the general rule/formula'. Teacher B spent eight minutes talking about last lessons work and thereafter wrote on the board a task taken from Gauteng Secondary school support programme ANA revision 2013 (see Appendix B) and then later learners work on three tasks from the 'blue book'. Teacher A used the textbook extensively and I think during my observations the teacher wanted to present the content and skill she thought I would be more interested in observing. Lessons 3 and 4 were audio-taped and transcribed, in these lessons the teacher dealt with number patterns where the second difference was constant. More details and discussions on *learning and practice* tasks and how these tasks were implemented in both classes are provided in the analysis chapter.

3.3.2 Task Collection

In the notes I wrote the sources of tasks that were selected and used by the teachers during the observed lessons. In addition, I photocopied relevant pages from the textbooks and learners' workbooks. The learners' work was not analysed, but was used to confirm that the tasks that were analysed were indeed the ones that were selected by the teachers and implemented in the observed classrooms. Also note that some of the task were not from the textbooks. The learners' workbooks showed to a certain degree how learners engaged with the tasks. A total of 101 grade 11 and 21 grade 9 *learning and practice* tasks on number patterns were collected and analysed.

3.3.3 Audio-taping

McMillan and Schumacher (2010) argue that "tape or digital recording (the interview) ensure completeness of the verbal interaction and provides material for reliability checks" (p. 360). I digitally recorded the teachers while teaching. The digital audio-tape was placed in the classroom in such a way so as to capture all sounds in the classroom. This enabled me to capture all questions and responses and to listen intently afterwards. I was aware of the limitations of using an audio-tape, for instance not capturing actions and gestures. However, I opted against using a video recorder due to contextual concerns and ethical reasons. Preliminary talks with the teachers who consented to participation in the study reveal some nervousness from their side when I declared my intentions of using a video recorder. I also consider the fact that video–taping could fuel disruptions in the lessons observed. I aimed at keeping the class environment as normal as possible, thus I opted for audio-taping.

The *learning and practice* tasks collected in the observed lessons and the audio transcripts were a large part of the data that was analysed. In addition, an observation schedule (see appendix A) was used. An observation schedule that consists of grids that required a tick every time a question is asked could not be used in this research study. The sequence of events played out at a fast pace and the teacher never gave learners enough time to reflect on their questions. The observation schedule was used mainly to track how time was spent in observed lessons. The aim was to use the data collected to connect class activities to a certain classroom practice. The use of the audio-tape came in handy and the transcripts provided all utterances accurately. I transcribed all the data myself; the process was time consuming and very tedious. I replayed the audio several times to ensure that the transcripts remained as close to the data as possible.

3.4 The teachers

In Table 3.2 below, I provide a brief description of qualifications and experience of the 2 teachers, who participated in this study

Teacher	Qualifications	Mathematics teaching experience
Teacher A	Three-year STD (1998) in mathematics and ACE (2009) in mathematics (Senior/FET)	14 years Gr (10 – 12)
Teacher B	Four-year Bachelor Degree in Education, with mathematics and computer applications as teaching subjects	5 years (Gr 8 – 10)

Table 3.3

Teacher A came to Gauteng in 2013 from KwaZulu Natal. In the year of the study she was teaching one Grade 10, two Grade 11 and two Grade 12 pure mathematics classes.

Teacher B had been at the school since its inception in 2010. In the year of the study she was teaching three grade 9 classes and two grade 10 mathematical literacy classes.

3.5 The school

The school is in the Ekurhuleni Municipality which is part of the larger Johannesburg locality. It is one of several 'new' schools that are found in the constantly developing parts of the Kathorus region. The school was established in 2010, in one of the extensions of the old townships. These extensions have resulted from the exponential increase in residents and the influx of people from the rural parts of South Africa and other countries in search for greener pastures in the City of Gold. It is situated among low cost houses; houses provided by the government to the poor, popularly known as RDP houses and shacks.

There are 1503 learners at the school with a staff of 45 teachers. The teacher: learner ratio is 1: 34. In 2014 the school did not have adequate classrooms due to the increasing number of learners enrolling at the school; 35% of the classrooms are mobile. The school's first matric result was an 83% pass rate in 2013. The majority of learners are from underprivileged families; the majority of them cannot afford to buy stationeries, textbooks and other learning aids such as calculators. It is categorised as a 'no fee' school or rather a quintile one school. All government schools are categorised into one of five quintile categories for the purpose of government funding. The quintile system categorise schools based on certain criteria, for example whether parents can afford to pay school fees or not, the rates of income, unemployment and illiteracy of the community around the school. Quintile one schools refers to the poorest schools while quintile five refers to the least poor public schools. Learners in quintile one, two and three schools, now referred to as 'no fee schools' get a much bigger subsidy from the government compare with learners in quintile four and five schools, now referred to as 'fee schools' that supplement their state allocation by charging school fees and fund-raising. The school where the mathematics classrooms were observed benefit from the National School Nutrition Program (NSNP) and learners are served free meals daily. At the beginning of every year learners are issued with the basic stationery and some grades receive textbooks. Due to the changes in the curriculum, an increasing number of learners enrolled each year and poor administration of the Learning and Leaching Support Material (LTSM) there are often LTSM shortages (i.e. particularly textbooks).

The study was conducted in two mathematics classes. There were 43 learners in grade 9 and 37 learners in grade 11. The grade 11 was a 'science' class and was allocated a permanent classroom in the formal structure of the school. The grade 9 class was located in one of the mobile classrooms. The teachers followed learners were they were located and carried all the teaching aids with them and consequently in moving around, time was wasted.

3.6 Data analysis

Data were mostly in the form of transcripts of audio-taped lessons; number pattern tasked used for learning and practice in the observed classes and information gather through the use of the observation schedules. Since the focus of this study was mainly on the cognitive levels of *learning and practice* tasks, the first phase of data analysis focused on classifying the collected task according to their cognitive levels using Stein et al.'s framework. I used Excel spreadsheets to record the codes of the cognitive level of each task. Tables 5.1.2 and 5.1.3 provides tasks analysed and the related classification for each task. The results gave an idea that there were gaps in Stein et al.'s framework. For this reason, difficulty levels of the tasks were explored, details on this analytical approach are provided in the analysis chapter. The tasks from each of the two grades were analysed separately.

The transcripts of the observed lessons served as good sources for teachers' questions. I used these transcripts to identify and code all the questions asked by the teachers using the codes developed by Boaler and Brodie (2004). Boaler and Brodie (2004) identified nine categories of teachers' questions, however Jina (2008) found that only the first four categories were relevant in South African context, thus I limited the categories to those four. The rationale for choosing this analytical framework was directed by findings from literature which suggests a very close link between the use of good questions and the development of mathematical reasoning. In turn mathematical reasoning is closely linked to the cognitive demands of learning tasks, see chapter two and four for more details.

3.7 Validity and reliability

In a broad sense, validity refer to the relationship between an account and something outside of the account (i.e. can include other possible interpretations) and not limited to research instruments used to produce and validate the account (Maxwell, 1992). Maxwell (1992) also acknowledges that "it is possible for there to be different, equally valid accounts from different perspectives" (p. 283). Research requires rigorous efforts to marry data interpretations made through analysis and claims by the researcher, creating a strong relationship between the accounts.

This means that data collection must be accurate and bona fide. Analysis must confirm an adequate account of realism and relate to those matters that the study claims to be about. This can be achieved by selecting research instruments carefully and doing in-depth reviews of the matters they each measure. I had the opportunity to pilot observations and test and adjust the observation instruments. I tried my best to keep records and a chain of evidence to assure validity. Transcripts of all audio-recoding; records of tasks used; field notes and other items were stored. Some of these instruments are included in the report as examples.

Reliability is the extent to which a method gives consistent results over a range of settings (MacMillan and Schumacher, 2001 and Wellington 2000). The reliability of this case study is questionable since there are gaps in the data collected and I analysed the data alone. I strongly believe that data collection and analysis could be done much better by a group of researchers than an individual. Some of the shortfalls of the research methodology were only identified while I was busy writing this report and there was no way of going back to the classrooms to re-observe again. Due to my personal and professional obligation often I was forced to leave issues for a long period and ended up losing my trail of thoughts. The research group would have help in the collection and interpretation of data, thus add to the reliability and validity of this case study.

3.8 Ethical issues

Cohen, Manion and Morrison (2000) argue that ethics in research represents shared codes of conducts founded upon adherence to a set of principles. I applied to the Wits University Human Research Ethics committee (Non-Medical) for ethical clearance to conduct the research. The universities ethics committee approved the study: Protocol 2013ECE143M. In addition, I negotiated access to the observed classes. The Gauteng Department of Education (reference no: D2014/301); the principal and volunteering teachers approved the terms of data collection. Volunteering teachers and parents of learners were requested to sign consent forms. These consent forms informed the participants of the study and assured them that they could withdraw at any point in the research.

3.9 Conclusion

In this chapter, I have described my research methodology, which is mainly a qualitative case study; my data collection methods; the sample; my methods of data analysis; the validity and reliability of the study; ethical issues and limitation. When describing the sample: teachers and schools, I gave more details about the school to show that it is indeed in a high-poverty area and it was intended for lower income citizens. In the next chapter, I present the analysis of data collected during the study.

Chapter Four

Data Analysis: Cognitive levels of tasks and teachers' questions

"*Analysis of qualitative data requires understanding how to make sense of text and images so that one can form answers to one's research questions*" (Creswell, 2012, p. 236)

4.1 Introduction

In this chapter, concepts from the literature discussed in *chapters two* and *three* are used to analyse data. Firstly, I tabulate the analysis of cognitive levels and 'difficulty levels' of tasks used in the observed classrooms prior implementation (i.e. as they appear in the text-books or other curriculum materials). Secondly, I focused on how tasks were implemented in the observed classes. Information from observation schedules and transcripts was used to measure how time was spent in those lessons and for coding the teacher questions. Accordingly, I argue how the teachers' decisions on implementation in their respective classrooms enhanced or caused a decline on the levels of cognitive demand and/or 'difficulty levels' of tasks.

4.2 Task analysis

In CAPS (p. 19) and the Annual Teaching Plans (ATP; 2014), DBE stipulates that grade 11 and grade 9 must spend two weeks (10 days) and a week (5 days) respectively *on Number Patterns*. The grade 11 class I observed worked to a total of 87 tasks in 9 lessons and the grade 9 class worked to a total of only 21 tasks on number patterns in 3 lessons. In conversations with both teachers, they said they were going to do more tasks on number patterns later on as part of 'revision'. However, I could not check whether this was the case. I made copies of all tasks on number patterns that were given to the learners, including some instructional tasks that were used by the grade 11 class in my absence. I wrote solutions of all tasks and thereafter analysed them.

The Grade 11 class used a textbook: Everything Maths by Siyavula, version 1 CAPS, Grade 11 Mathematics, written by volunteers (2013) and 'examination aid books'. The textbooks were supplied by the government and the learners were requested to buy 'mathematics examination aid books or study guides', with the contents that contained past mathematics examination questions and solutions. From the lessons observed, most tasks selected by teacher A were from the prescribed textbook. The class followed the sequence set in the textbook (i.e. from exercise 3.1 to 3.4). Teacher A required the learners to find solutions for almost all tasks in the selected exercises. The teacher used some of the tasks from those exercises as examples or instructional tasks and others were used as practice tasks that were either done in class or done at home. In conversation, the teacher said she uses “a variety of textbooks and manuals to finding more information; challenging and examination related problems and examples to use in class”. She seemed concerned with preparing learners for the end of the year examinations and getting them ready for grade 12.

The grade 9 learners did not have textbooks. However, they all had DBE workbooks (i.e. DBE workbook 1 Numeracy Grade 9, the English version). The grade 9 teacher was observed using one textbook, the DBE workbook and the ANA grade 9 Mathematics Exemplar 1 of 2012 as sources for tasks. She wrote all the work on the board, and that wasted a lot of time. After the lesson, in conversation, I asked her why she did not photocopy work for the learners; she said that the schools photocopying machine was non-functional at the time. I also gathered that the school was often short of printing paper and that the photocopying machine was often out of service.

4.2.1 Differentiating levels of cognitive demand

Initially I worked exclusively with Stein et al (1998) framework and made a judgement on the kind of thinking demands required of learners to do a particular task. The framework consists of four levels: lower level (zero)- *memorization* of fact; *low level (one) – procedures without connection; high level (two)- procedures with connection* and *very high level (three) – doing mathematics.* More discussions about the levels and reasons for using this framework were presented in *chapter two*. However, it is worth emphasising that Stein et al (2000) warns that “when determining the level of cognitive demand provided by a mathematical task, it is important not to become distracted by superficial features of the task and to keep in mind the learners for whom the task is intended” (p. 15). Thus in this report a task intended for grade 11 learners that is at the lower-level was rated at a higher-level for grade 9 learners and contrariwise.

The process of differentiating the cognitive levels of tasks started after I had written solutions of each task and also considered possible alternative methods of solving the tasks. Tables in appendix F presents the tasks used in grade 11 and grade 9 classes I observed. The tables indicate the sources; purpose of the task; the cognitive levels and the difficulty levels of the tasks. The crude counts of cognitive levels categories resulted in the summary presented in Table 4.1. and a figure 4.1. below.

Teacher	**Grade**	**Memorization**	**Procedure without Connection**	**Procedure with connection**	**Doing Math**	**Total**
		Level 0	**Level 1**	**Level 2**	**Level 3**	
Teacher A	11	7	68	11	1	87
Teacher B	9	0	16	1	4	21

Table 4.1.: Task categorisation (by teachers) using Stein et al.'s categories

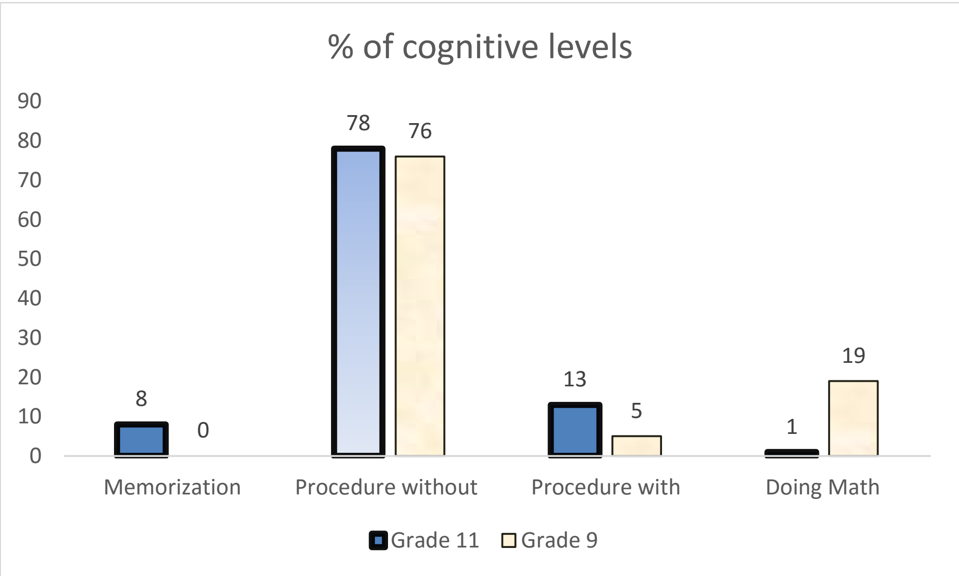


Figure 4.1. % of cognitive levels per grade

The analysis suggested that, 8% of the tasks were memorisation and these tasks were found in grade 11 only. This made sense, since it is not common to find memorisation tasks in mathematics, 'memorisation' is not hugely promoted in mathematics. In this study, elementary number pattern tasks selected for grade 11s' were classified as memorisation. Linear patterns that required learners to write down the next three terms of the sequence were classified as memorisation. For example: “write down the next three terms of the following sequence 45; 29; 13; -3 ...” was classified as *level zero – memorisation.* Considering that learners start doing number patterns in primary school, for me such tasks should be considered the least demanding for grade 11s'. Stein et al.'s categories do not unambiguously factor in all the considerations (i.e. learners' ages, grade, prior knowledge and experience) that are indispensable when assessing tasks.

78% and 76% of the grade 11 and grade 9 respectively were found to be procedure without connections. While 13% and 5% of grade 11 and grade 9 respectively were found to be procedures with connections. Only one task of 87 tasks in grade 11 was categorised as doing mathematics. The four tasks of 21 tasks that were categorised as doing mathematics in grade 9. Three of those tasks were given as homework at the end of the lesson. I felt that these were given as an afterthought by the teacher. I do not think these tasks were discussed in the grade 9 class. I later discovered that the teacher could not provide solutions for these tasks. These sequences are quadratic and thus are covered for the first time in grade 11 according to CAPS and the ATPs. I thought she only gave them to her learners because they were in the textbook and she had to give learners a 'homework'. The implications are that there was lack of fore planning, preparation and innovation from the grade 9 teacher's side. Both grades did not really get any chance of engaging with number patterns tasks which required working at the higher levels of demand. The grade 11s' had a very small number of opportunities (i.e. 13% of the tasks were classified as procedures with connections) of making connections and meaning which could enrich and/or stretch their mathematical understanding and their ability to solve mathematical problems. I noted that although most tasks were found to be 'procedures without connections' the task differed in their 'levels of difficulty'.

4.2.2 Level of difficulty of the tasks

At some stage during the write up of this study I had an opportunity to analyse and work with a team analysing the 2012, 2013, 2014, 2015 National Senior Certificate examination papers from both DBE and IEB (Independent Examination Board). In my engagements with the team, after each team member (i.e. including myself) had analysed papers independently, we compared our results and had discussion of our findings. From the whole process I learned that categorises of cognitive demands cannot completely indicate the tasks' difficulty or complexity of the some of the procedural mathematical tasks and solutions. For this reason, I considered looking at the ‘difficulty levels’ of the tasks used in the observed lessons. In this subdivision, I present some analyses of the tasks looking at the ‘levels of difficulty’ of the tasks. In the literature I have searched there was no analytic framework for measuring the ‘level of difficulty’ of tasks. Thus, for this exercise I used my own experience as a Mathematics teacher, my knowledge of the subject and the knowledge I gained through analysing and working with the uMalusi (Council for Quality Assurance in General and Further Education and Training) team. I then made judgments about whether each task was easy, moderate, difficult or very difficult compared to other tasks at the same cognitive level. Table 4.2 and figure 4.2 below shows summaries of the analysis from tables in appendix F as follows:

Teacher	Grade	Easy	Moderate	Difficult	V. Difficult	Total
Teacher 1	11	74	3	9	1	87
Teacher 2	9	17	1	0	3	21

Table 4.2: Task’s difficulty levels

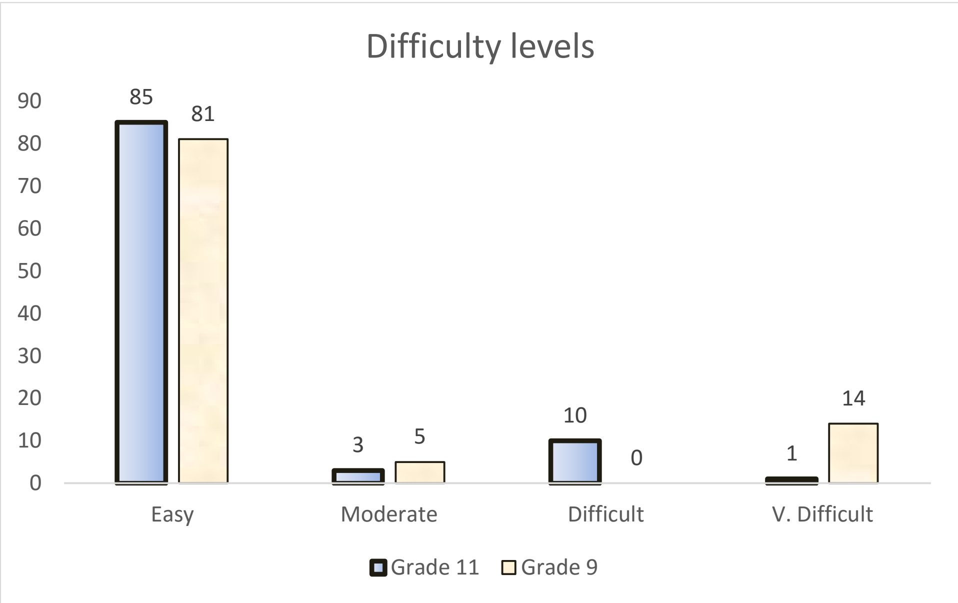


Figure 4.2: % tasks difficulty levels

The analysis suggested that, although most tasks were found to be ‘level 1 – procedure without connections’ following the framework by Stein et al (1998), some of the tasks in grade 11 were difficult. Table 5.2 and figure 5.2 above shows that 85% and 81% of the grade 11 and grade 9 tasks respectively were found to be easy. Only one task (i.e. number 16) of the tasks selected for grade 9s’ required learners to investigate or to determine the general term of a linear sequence. This task was taken from an ANA exemplar question paper and learners were not given enough time to work on this task. Teacher B relied heavily on stating facts and demonstrating procedures. 3% and 5% of the grade 11 and grade 9 tasks respectively were found to be moderately difficult. Only 10% of the grade 11 tasks were found to be difficult; these were tasks of quadratics sequences that required learners to find the general term. Finding the general term of a quadratic sequence is procedural and involves multiple steps and there are several ways of reaching the solution. For example: “Find the general formula for sequence 16; 27; 42; 61 ...”. Learners, particularly the grade 11s’ manage to only master a certain part of the procedure demonstrated to them by the teacher. Most learners end up not finding the correct solution, for this reason such task were supposed ‘difficult’ for grade 11s in comparison to other tasks on number patterns.

4.3 Task at implementation

4.3.1 Time spend on activities

I used the classroom observation schedule (i.e. discussed in detail in chapter four) to track activities in the lessons I observed. A summary is presented in the table below:

	Teacher writing on the board	Teacher Talking	Teacher Questioning	Individual work by learners	Other, for example disciplining class or Admin.
Grade 9	30%	33%	15%	10%	12%
Grade 11	8%	30%	12%	49%	1%

Table: 4.3. Percentage of time spent

Both teachers in the study used traditional “chalk and talk” and whole class teaching methods. The lessons of the two teachers were teacher-centred, but the classroom environments were different. There were far less disciplinary (i.e. dealing with misbehaving learners) incidences in the grade 11 class than in the grade 9 class. The two teachers each spent about 30% of the lesson time talking to learners, ‘motivating learners’, explaining steps to perform when working towards a solution of a task, and/or concepts. Each teacher spent approximately 13.5% of the lessons’ time questioning learners. The grade 11s’ spent more time, 49% of the time, doing individual work, contrary to Grade 9 who spent 10% of the time doing individual work. The Grade 11 also did more tasks than grade 9s. This I attributed to the fact that the district officials put more pressure on grade 10; 11 and 12 teachers to give more tasks. The stipulation is that 5 – 8 tasks must be done in class daily and another 5 -8 tasks for 'homework' daily. The grade 9 learners spend about 10% of the time on individual work. The grade 9 teacher stood in front of the class or wrote on the board. She hardly sat down during lessons.

4.3.2 Teacher questioning

Boaler and Brodie (2004) recognised and valued the use of questions as important tools in developing learner mathematical understanding. They argued that teachers need to know their learners well; have a good understanding of mathematical content and teaching methods and they also need to think deeper in order to be able to ask good questions. Boaler and Brodie (2004) developed an analysis schedule for coding teacher questions, the codes are as follows:

Question type	Description
1. Gathering information, leading learners through a method (fact)	Require immediate answer Rehears known facts/procedures Enable learners to state facts/procedure
2. Inserting terminology (term)	Once ideas are under discussion, enable correct mathematical language to be used to talk about them
3.Exploring mathematical meanings and/or relationships (concept)	Points to underlying mathematical relationships and meaning. Makes links between mathematical ideas and representations
4. Probing, getting learners to explain their reasoning (probe)	Ask learners to articulate, elaborate or clarify ideas

Table: 4.3.1: codes of teachers' questions

As in Boaler and Brodie (2004), I also considered 'teacher questions' as utterances that had both the form and function of questions and which were mathematical. I coded repeated questions as a single question. Table 4.3.2 below shows the results from the coding of the grade 9 and grade 11 lessons I observed.

Question Type	Frequency: Grade 9	Frequency: Grade 11
1. (fact)	98%	97%
2. (term)	1%	1%
3. (concept)	1%	1%
4. (probe)	0	1%

Table: 4.3.2: Grade 9 and Grade 11 coding of questions

Table 4.3.2 suggests that nearly all (> 97%) questions asked by the teachers in both Grade 9 and Grade 11 classes that I observed required immediate answers or required learners to reply with a simple 'yes or no'. In both cases teachers spent most of the lesson time explaining method and concepts than allowing learners to think and do mathematics. Teachers engaged learners in discussions or asked questions infrequently and most of their questions were what I termed pseudo-questions. A pseudo-question is a statement which is turned into a question by adding words like 'ok?' or 'right?'. Pseudo-questions require a respondent to simple say 'yes', often said as a confirmation that the respondent was listening or heard what was said. For example: "so each and every term I will multiply it by 3 to get the next term, right?". Such questions were coded as type 1, and they were more common in the grade 9 lessons.

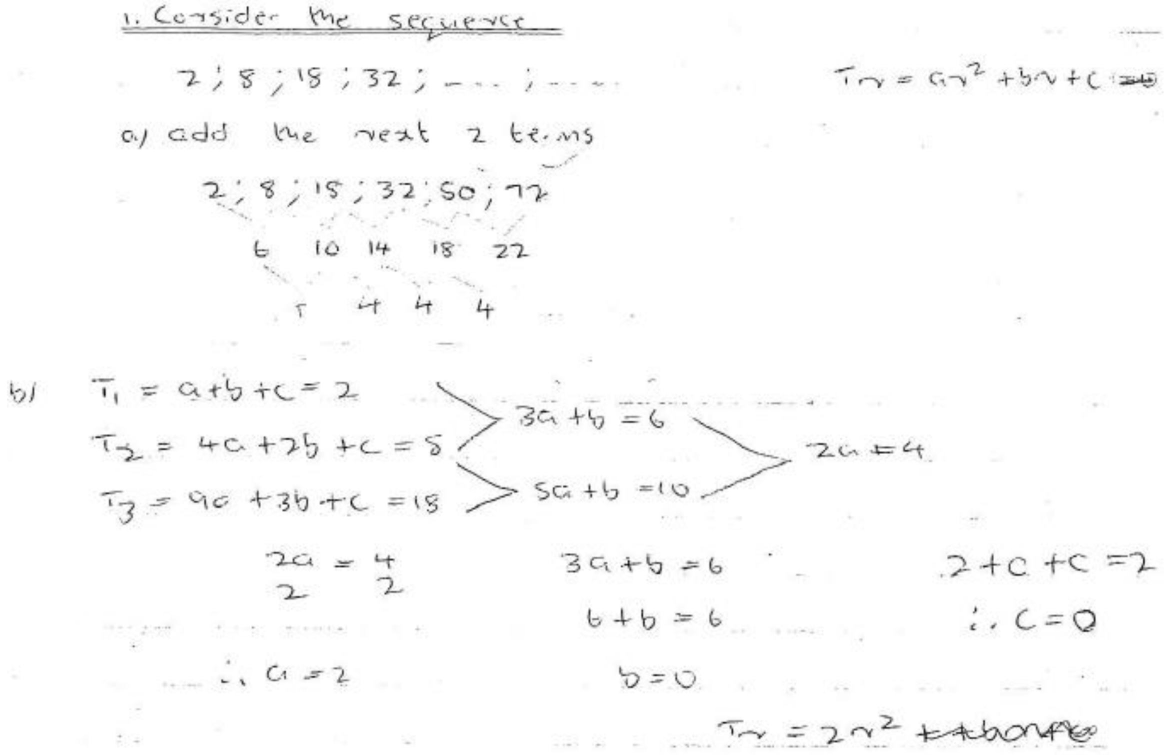
4.4 Decline in the 'levels of difficulty' of tasks

The analysis of the cognitive demand of tasks suggested that a mere 14% of the Grade 11 tasks were of higher-level demands. 78% of the tasks were 'level 1 – procedures without connection', however 10% of these tasks were found to be 'difficult'. The tasks that were classified as 'difficult' required learners to investigate number patterns with the constant second difference. Special attention was paid on how such tasks were implemented. For example, the extract on Table 4.4.1 below comes from a lesson in the grade 11 class. The class had to find the general formula for 2; 8; 18; 32; ... (see appendix F: tasks number 41; 42 & 43). This was a practice task selected, from a textbook, by the teacher A. The teacher had demonstrated the procedure of finding the general formula of a quadratic sequence in previous lessons. She gave learners some time (± 10 minutes) to complete the tasks individually, although some learners discussed the task in pairs and other drifted into off-task behaviour. This extract indicates the talk the class had in relation to task number 42 (see appendix F) and also serves as an example of how discussions took place in the Grade 11 lessons observed.

Turn No.	Speaker	Dialogue	Code	Description
42	Teacher	... we need to find the next two numbers or the next two terms. So we have the first term 2, the second one 8; 18 and 32. Then we need to find out by taking term 2 minus term 1. What is 8 -2?	1	Require an immediate answer
43	Class	6		
44	Teacher	18 – 10?	1	Require an immediate answer
45	Class	10		
46	Teacher	32 – 18?	1	Require an immediate answer
47	Class	14		
48	Teacher	10 – 6?	1	Require an immediate answer
49	Class	4		
50	Teacher	14 -10?	1	Require an immediate answer
51	Class	4		
52	Teacher	so to add the next two we start from 4, so 4 + 14?	1	Require an immediate answer
53	Class	18		
54	Teacher	18 + 32?	1	Require an immediate answer
55	Learner	50		
56	Teacher	Hawu unamanga 46 (you are lying 46)		

57	Class	46!		
59	Learner	50		
60	Teacher	50! And then we add the next term,…		
61	Learner	Kanjani? (how?)		
62	Teacher	kanjani? (how?) 8 +2 = 10, then we carry out 1, then 1 +3 is 4 + this 1 is 5. Ubani lo othi kanjani (who said how?) … 4 + 18?	1	Require an immediate answer
63	Class	22		
64	Teacher	22 + 50?	1	Require an immediate answer
65	Class	72		
66	Teacher	so these are the next two terms. Now we need to calculate the value of a, the value of b and the value of c, because this is the quadratic since we have there constant on the second difference. So it is Tn= an^2 + bn + c, so we start by substituting T1, T1 we substitute 1		

Table 4.4.1: Grade 11 lesson 4



1. Consider the sequence

2 ; 8 ; 18 ; 32 ; ... ; ... $T_n = an^2 + bn + c$

a) add the next 2 terms

2 ; 8 ; 18 ; 32 ; 50 ; 72

6   10   14   18   22

4   4   4   4

b) $T_1 = a + b + c = 2$

$T_2 = 4a + 2b + c = 8$ $3a + b = 6$

$T_3 = 9a + 3b + c = 18$ $5a + b = 10$ $2a = 4$

$\frac{2a}{2} = \frac{4}{2}$ $3a + b = 6$ $2 + c + c = 2$

$6 + b = 6$ $\therefore c = 0$

$\therefore a = 2$ $b = 0$

$T_n = 2n^2$

Figure 4.4.1 Learner's work

In this lesson the teacher spent most of the time asking arithmetic questions, (turns 42, 44, 46, 48, 50, 52, 54, 62, 64) explaining the procedure and writing on the board while explaining. Teacher A provided few, if any at all, opportunities for learners to participate in the process of generalising. This extract show that the teacher's questions did not encourage learners to think nor to familiarise themselves with the procedures involved in finding the general term of a quadratic sequence. Indeed, all the questions were on addition and subtraction of integers. I argue that the grade 11 teacher's practice reduced the 'difficulty' of the tasks. Firstly, the teacher gave learners too much time to complete the task. Secondly the teacher reduced the 'difficulty' of the task by showing learners 'steps' to follow. The teacher's questioning was poor and 'scaffolded' the tasks to the disadvantage of learners' thinking and reasoning. The teacher did neither build on previous lessons nor considered what learners had done. See an example of the learners' work in figure 4.4.1 above. From what can be seen in extract, teacher only did arithmetic. The demonstration of the procedure was evident from what she wrote on the board and the learners' work.

The grade 9 tasks were found to be of lower- level demands and easy. The teachers did not increase the levels at implementation. The lessons often started on the work the teacher assumed learners knew. The teacher did not check the learners' previous knowledge. The demonstrations, explanation, shouting (i.e. perceived as some form of discipline) and writing on the board were permanent fixtures of grade 9 lessons observed. The teacher's movement were limited to the front part of the classroom. The level of learner engagement and mathematical thinking in those lessons was very low and the tasks were inappropriate for Grade 9 learners. At Grade 8 and 9 learners are expected to experiment with find the general term of linear patterns. This was missing in all the lessons I observed. Table 4.3.2 below gives an indication of the kind of engagement between teacher B and her learners. It is presented here in order to show that teacher B spent an huge part of the lesson writing and explaining notes (see turn 1).

Turn No.	**Speaker**	**Dialogue**
1	Teacher	T: Right, patterns, you know patterns from last year. Is it? There are representations of the way in which numbers are placed or objects are placed. Even your clothes some of you have patterns. … So we are going to look at patterns in Maths and that also apply in real life. So let's look at the notes. A pattern or a sequence, sometimes they call them sequences sometimes they call them patterns, it is the same thing. #Starts reading the notes, from p. 88. "A pattern or sequence … We have numeric patterns and geometric patterns. Numeric patterns it is where we deal mostly with numbers and Geometric patterns it where we have shapes. They can give you match sticks, they can give you balls, they can give you dots, they can give you whatever, triangles; rectangles and you have to figure out the pattern there. The sequence of how many match-sticks or how many triangles or whatever, so in numeric patterns, this is a pattern or sequence which is an ordered set of numbers. This is a set of number that can start from zero to something or one or two or three or whatever number that the first term is. It is normally defined by a rule so that a formula can be given, so there is always a rule, either you

		add one each time or you add 7 each time or you divide by 3 or you multiply by 7 or you subtract each time. There is always a rule that tells you how are you going to be able to get the next term and with that rule you are able to define a formula whereby you are able to find term number million without actually counting, 1; 2; 3; 4; 5; … You can move from term number 3 and automatically be able to find term number million just by using that formula. Defined by the rule. For example, we have natural numbers; you remember natural numbers when we were doing number systems, right? Your counting numbers, 1; 2; 3; 4; 5 … right? Natural numbers follow the rule that you add one to the previous number to get the next number that is the rule. If I am one, then we have another, then we are now 2. They have added one person, if they add the third person; then there will be 3 of us. For each time for natural counting you add one; that is the rule. Each number in a sequence is known as a term, so you can have term 1; term 2; term 3 until, until, until the infinite term, which most of the time is represented by n. Other rules are as follows, so we can have other rules it is not always were you add a constant number. You can have other rules these are just some of the examples. The first one we can have a rule where we add constant number to one term in order to get the next term. An example 2; 6; 10; 14; … and those dots means it continues on. So the first term here will be 2, the second term 6, so now if you move from 2 and go to 6, how much do we add? You count 2; 3; 4; 5; 6 so you add 4. To move from 6 to 10, you say 7; 8; 9; 10, you add 4. So each and every time you add 4 to the previous term to get the next term. So in this kind of sequence we add a constant number each time to one term to get the next term. We say in that sequence we have a constant difference. Constant difference is the opposite of addition, right? Difference is the result you get when you do subtraction. So; if you add the same number each time the opposite becomes the difference. So if I want to know the difference between 4 and 6, I will say 6 – 2 then I will get 4. 10 - 6 will give me? 4.

		14 -10 will give me? 4. So we have a constant difference of 4. Then another rule could be that we multiply one term to get the next, so if I'm multiplying let's say I have 1 and I multiply it by 3, the next term how be what?
2	Learner	4
3	Teacher	(repeat) if my first term is 1, I multiply 1 by 3, what will be my next term?
4	Learner 1	4
5	Learner 2	1
6	Learner 3	3 …. (giving varied responses, almost concurrently)
7	Teacher	listen, listen, if my first term in the sequence, the value of my first term is 1 and I multiply this 1 by 3, what will be my next term?
8	Learners	9
9	Teacher	oh my Lord! You don't know the times table of 1?
10	Learners	3!
11	Teacher	it is 3, multiply any number by 1 the result is that number.

Table: 4.4.2 Grade 9 lesson

Teacher B copied notes from a textbook on to the blackboard, thereafter read and explained the most part of those notes. The teacher stated that class was "going to look at patterns in Math and that also apply in real life" (turn 1). In the lessons observed none of the task required learners to work on pattern in real life contexts. The tasks selected by the teacher were of lower-level demands and not challenging, however the teacher lowered the demands ever further on implementation. While observing I got a feeling that most learners learned very little from those lessons, moreover there were few practice tasks given.

4.5 Conclusion

In this chapter I have attempted to make sense of the collected data in relation to my research questions. I analysed the tasks with the aim of understanding the levels of cognitive demand

and the levels of difficulty of the tasks. I also look at how time was spent in the observed classes and the type of questions teachers answered at implementation of tasks. Based on the findings of the analysis of the tasks; how time was spent in the classrooms and the teachers' questions, I attempted to verify whether or not the teachers' classroom practice affected the cognitive demands and/or levels of difficulty of tasks at implementation. In the next chapter, I present discussions of finding and draw conclusions based on literature and findings of this study.

Chapter Five

Conclusion

5.1 Introduction

This chapter draws conclusions from the findings of the study and present possible explanations for the teachers' practices as well as some recommendations.

5.2 Discussion of findings

This study explored and responded to the following questions:

1. What are the cognitive levels and 'difficulty levels' of the mathematical tasks used in the case study township classrooms?
2. How do the case study teachers' classroom practice impact on the cognitive demand and/or difficulty levels of mathematical tasks?

Background question

2a. What kind of questions are asked by the case study teachers to assist learners when working on learning and practice tasks?

According to CAPS (2011) learning mathematics requires learners be provided with opportunities to develop abilities to be methodical, to generalise, make conjectures and try to justify and prove their conjectures. Justifying and generalising are significant practices involved in mathematical thinking. The literature suggests that high-level cognitive demanding tasks that are implemented properly support learners' mathematical reasoning (see Brodie, 2010; Lampert 2001 and Stein et al 1996). This study investigated the levels of tasks selected by the teachers; the kinds of questions asked by the teachers during the implementation of the selected tasks and how the questions asked by the teachers and the teachers' actions at implementations affected the levels of the tasks. In this section I show how the study advanced in order to answer the research questions above.

5.2.1 Level of mathematical tasks

In chapter five, the analysis of tasks was presented. The findings suggested that most of the tasks were selected from prescribed curriculum materials. Using the analytical framework(s) discussed in chapter three, I found that teachers selected and implemented many tasks that were classified as lower-level tasks. Many of the selected tasks were level 1 - procedures without connection and were easy. The grade 11 teacher selected an adequate amount of tasks for her learners. However, a small amount of those tasks afforded the learners opportunities to generalise, make conjectures and try to justify and prove their conjectures. The grade 9 teacher selected an inadequate amount of tasks for her learners. The greater part of those tasks were of the lower level; very easy and there were no opportunities for learners to generalise, make conjectures and try to justify and prove their conjectures.

5.2.2 Kinds of question asked by the teachers

A focus on classroom activities revealed that both teachers spent a considerable amount of lesson time talking with learners being 'inactive'. Whole-class teaching was the single pedagogic approach preferred by both teachers. I argued that the teachers' classroom practice leaned more towards a teacher-centred or traditional rather than a learners-centred approach. Predominantly, the teachers' questions were meant to lead learners through a method and required immediate responses from the learners. Both teachers seemed more interested in demonstrating how to solve a particular task than engaging learners and using question to solicit for learners' ideas and thoughts. The Grade 11 class had a better chance of getting the tasks of the same form correct in the examination, because they have adequate practice tasks. However, it is likely that only a selected few Grade 9s would know what to do in order to find the general term of a linear sequence. The teacher did what can be referred to as 'show and tell' on the board and gave few practice tasks. Considering that the observed teachers used whole-class teaching approach, I expected teachers to ask questions that were going to allow learners to reason and reflect on their own and others thinking, thus building on the class experience. Class discussions were lacking and not promoted by the teachers.

5.2.3 Decline in the level of tasks

This study has shown that both teachers frequently selected lower-level mathematical tasks. A closer look at how the small amount of higher-level tasks were implemented revealed that the teachers' classroom practice reduced the level of the tasks. The cognitive and difficulty levels were lowered further when teachers demonstrated and talked of mathematics as a practice that focuses mainly on procedures and drilling. It was evident that teacher A knew the content rather well and was very confident when demonstrating procedures. However, her teaching practice promote neither conceptual understanding nor procedural fluency. On the other hand, teacher B's content knowledge was questionable. She also selected tasks that were beyond her reach. This I attributed to lack of fore planning and an indication that the teacher did not careful select tasks for her learners. Her selection of task was worryingly depended on the prescribed curriculum materials. She did not evaluate the curriculum material before using it. Both teachers' classroom practice under-estimated the learners' competency by reducing all tasks to be about practicing newly learned skills and procedures.

5.3 Implication for teacher education and development

This study has shown that not only do teachers select tasks that do not promote mathematical thinking and reasoning, but also their content knowledge and pedagogical content knowledge is questionable. The lack of thought provoking mathematical discussions and use of higher-level tasks in the observed lessoned was disturbing. The findings of this study in terms of task selection and tasks implementation suggests some areas of focus in teacher education and development.

In order to meet the aims of CAPS there need to be a shift in how teachers work with their learners. Teacher development programs need to identify the adequacy of the mathematics teachers' knowledge to teach mathematics. This could be done by administering competency tests that test both teachers' content knowledge and instructional knowledge. The current teacher development models by DBE focuses mainly on subject content knowledge and neglect pedagogical content knowledge (DBE, 2011b, p.76). This study highlights the need for teacher development programs to focus on selection of tasks and how tasks can be used to effectively teach mathematics and maximise the learning opportunities for learners from any walks of life.

5.4 Limitation of the study

Small scaled case studies by their nature hold many limitations. The results of a case study are not generalizable in the statistical sense. The participants were not chosen to be representatives of teachers in high-poverty schools and the lessons observed were too few to give a generalized view of all situations.

Considering that during the course of this study, I was a part-time student and was expected to be at my teaching post daily data collection was a huge challenge. I worked with two teachers who consented to being observed, I did not have the opportunity to select the teachers. I late discovered that I had not collected rich data from my classroom observations and there was no way of repeating the process. If time and resource allowed, I would have preferred to video record the teachers' lessons and observe teachers teaching other topics.

REFERENCES

Adler, J. (2000) Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education*, 3, 205-224.

Adler, J. (2005). Mathematics for teaching: what is it and why is it important that we talk about it*? Pythagoras*, 62, 2 – 11

Adler, J., Dickson, M., Mofolo, B. & Sethole, G. (2001). *The use of written texts in mathematics classrooms. A study of grade 7 and 9 classes in selected Gauteng and North West schools in 2000*. Johannesburg: University of Witwatersrand. Unpublished manuscript.

Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of Mathematics: A view from the West. *Journal of Mathematics Education,* 2(2), 147-164.

Ball, D.L., & Bass, H. (2003). Making mathematics reasonable in school. In G. Martin (Ed.), *Research compendium for the Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Berger, M., Bowie, L., & Nyaumwe, L. J. (2010). Taxonomy matters: Cognitive levels and types of mathematical activities in mathematics examinations. *Pythagoras*, (71), 30 - 40.

Boaler, J. & Brodie, K. (2004). The importance, nature and impact of teacher questions. In D. E. McDougall & J. A. Ross (Eds.), *Proceedings of the 26th Annual meeting of the North American International Group for the Psychology of Mathematics Education* (Volume 2, pp. 773–781). Toronto: OISE/UT.

Boaler, J. (1997). *Experiencing school mathematics: Teaching styles, sex and setting.* Buckingham, England: Open University Press.

Boaler, J. (1998). Open and closed mathematics approaches: student experience and understanding. *Journal for Research in Mathematics Education*, 29, 41 -62.

Boaler, J. (1999). Participation, Knowledge and Beliefs: A Community Perspective on Mathematics learning. *Education Studies in Mathematics learning*, 40, 259 -281.

Boaler, J. (Ed.). (2000). *Multiple Perspectives on Mathematics Teaching and Learning*. Westport, CT: Ablex Publishing.

Brodie, K. (2007) Dialogue in mathematics classrooms: beyond question –and-answer methods. *Pythagoras, 66, 3-13*

Brodie, K. (2008). Describing teacher change: interaction between teacher moves and learner contributions. In J.F. Matos, P. Valero & K. Yasukawa (Eds.), *Proceedings o the 5th International Mathematics Education and Society Conference.* Lisbon.

Brodie, K and Berger,M. (2010). Towards a discursive framework for learner errors in mathematics. In V. Mudaly (Ed.) *Proceedings of 18th meeting of SAARMST*, Vol 1, University of KwaZulu Natal, pp 169- 181

Brodie, K. (2010). *Teaching mathematical reasoning in secondary schools*. New York: Springer.

Chisholm, L. (chair) (2000) *A South African Curriculum for the Twenty First Century: Report of the Review Committee on Curriculum 2005*. *Presented to the Minister of Education, Professor Kader Asmal*. http://education.pwv.gov.za/content/documents/44.pdf (visited 24 April 2013).

Cohen, L., Manion, L. and Morrison, K. (2000). *Research Methods in Education*. (5th ed.). London: Routledge Falmer.

Cohen, L., Manon, L., and Morrison, K. (2002). *Research methods in Education 5th edition*. Routledge Falmer. London.

Chua, B. L., & Hoyles, C. (2012). The effect of different pattern formats on secondary two students' ability to generalise. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 155 - 162). Taipei, Taiwan.

Daniel, H. (2007). Pedagogy. In H. Daniels, M Cole and J. Wertsch (Eds). *The Cambridge companion to Vygotsky* (pp. 307 – 331)

DoE. (2011). Curriculum and Assessment Policy Statement (CAPS): Senior Phase Mathematics, Grade 7 -9. Department of Basic Education.

DoE. (2002). Revised National Curriculum Statement Grade R - 12 (Schools): Mathematics. Department of Education.

DoE. (1997). National Department of Education (1997) Curriculum 2005: Lifelong Learning for the 21st century. Pretoria: NDE.

Department of Education (1997). Curriculum 2005. Implementing OBE-1: Classroom practice. Lifelong learning for 21st century. Pretoria: Author.

Department of Education (1998) Norms and Standards for Funding of School Regulations. Pretoria: Government printers

Department of Education (2003). National Curriculum Statement Grade 10-12 (General). Mathematics. Pretoria: Author

Department of Education (2004). Norms and Standards for Funding of School Regulations. Pretoria: Government printers

Department of Basic Education (2011). Curriculum and Assessment Policy Statement. Pretoria: Government printers

Department of Basic Education (2011b). Integrated Strategic Planning Framework for Teacher Education and Development in South Africa 2011-2025 Technical Report. Pretoria: Government printers

Diaz-Maggioli, G. (2004). *Teacher-centered professional development*. Alexandria, VA: Association for Supervision and Curriculum Development.

Dossey, J. [1992]. The nature of mathematics: Its role and its influence. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 39-48). New York: Macmillan

Dowling, P. (1998). The Sociology of Mathematics Education; mathematical Myths/pedagogic Texts. London: Falmer Press, Ch 1, Ch 5, p 115, para 3 onwards, Ch 6, p 130 onwards.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist*, 23 (2), 167-180.

Driscoll, M. (1999). '*Fostering Algebraic Thinking: A Guide for Teachers Grades 6–10* Portsmouth: Heinemann

Ensor, P., Dunne, T., Galant, J., Gumdze, F., Jafters, S., Reeves, C. and Tawodzea, G. (2002). Textbooks, teaching and learning in primary mathematics classrooms. *African Journal of Research in Mathematics, Science and Technology Education*. 6, 21 – 35.

Gordon Calvert, L. (2001) Mathematical conversations within the practice of mathematics. New York: Peter Lang (chapter 2 and 3 – up till page 37)

Hatano, G. (1996). A conception of knowledge acquisition and its implications for mathematics education. In P. Steffe, P. Nesher, P. Cobb, G. Goldin & B. Greer (Eds.), *Theories of mathematical learning* (pp. 197-217). New Jersey: Lawrence Erlbaum.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second grade arithmetic*. American Educational Research Journal*, 30, 393 - 425.

Jina, Z., & Brodie, K. (2008). Teacher questions and interaction patterns in the new and old curriculum: A case study. In M. V. Polaki, T. Mokuku & T. Nyabanyaba (Eds.), Proceedings of the sixteenth annual congress of the Southern African Association for Research in Mathematics, Science and Technology education (SAARMSTE). Maseru, Lesotho

Kaur, B. (2010). A study of Mathematical Tasks from Three Classrooms in Singapore. In Shimiza, Y., Kaur, B., Huang, R., and Clarke D. (Eds): Mathematical Tasks in Classrooms Around the world (pp. 13-33). Rotterdam: Sense Publishers B.V.

Kazima, M., & Adler, J. (2006). Mathematical knowledge for teaching: Adding to the description through a study of probability in practice. *Pythagoras*, *63*, 46-59.

Kelly, A. V., (1999). The Curriculum. Theory and Practice. Sage. London.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Komoski, P.K. (1985) Instructional Materials will not improve until we change the system. *Education Leadership*, 42, 31-37

Lobato, J., Clarke, D., & Ellis, A. B. (2005). *Initiating and eliciting in teaching: A reformulation of telling.* Journal for Research in Mathematics Education, 36(2), 101–136.

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery*. New York: Cambridge University Press.

Lave, J. (1993). "Situating Learning in Communities of Practice" In L B. Resnick, J.M. Levine and S.D, Teasley (Eds) Perspectives on Social Shared Cognition. Washington, DC: American Psychology Association.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge University Press.

Leong, S.C. (2006). '*On varying the difficulty of test items*', Paper presented at the 32nd Annual Conference of the International Association for Educational Assessment, Singapore

Lloyd, G. M. (1999). Two teachers' conceptions of a reform-oriented curriculum: Implications for mathematics teacher development. *Journal of Mathematics Teacher Education,* 2, 227-252.

Lobato, J., Clarke, D., & Ellis, A. B. (2005). *Initiating and eliciting in teaching: A reformulation of telling.* Journal for Research in Mathematics Education, 36(2), 101–136.

Makgato, M & Mji, A. (2006). Factors associated with high school learner's poor performance: a spotlight on mathematics and physical science. *South African Journal of Education.* Vol 26(2) 253-266.

Maxwell, J.A. (1992). Understanding and validity in qualitative research. Harvard Educational Review, 62, 279-300.

Mda, T.V., and Mothata, M.S. 2000. Critical Issues in *South African Education* –after 1994. Juta and Company Limited. South Africa.

Meadows, S. (2004). Models of cognition in childhood: metaphors, achievements and problems. In H.Daniels, H. and A. Edwards. (Eds.) *The Routledge Falmer Reader in Psychology of Education.* (pp. 165 – 183). London: Routledge/Falmer.

Mogari, D. (2001). Geometrical thinking embedded in the construction of a kite. Pythagorus, 60, 36-43.

Mok, I.A.C. & Kaur, B. (2006). Learning Tasks. In Clarke, D., Emanuelsson, J., Jablonka, E., and Mok, I.A.C. (ed.) *Making Connections: Comparing Mathematics Classrooms Around the World* (pp. 147-200). Rotterdam: Sense Publishers B.V.

Molefe N & Brodie K 2010. Teaching mathematics in the context of curriculum change. *Pythagoras*, 71:3-12.

Opie, C. (2004). *Doing Educational Research*. Sage. London.

Parker, D. (2006). Grade 10-12 Mathematics curriculum reform in South Africa: A textual analysis of new national Curriculum Statements. African Journal of Research in SMT Education, Volume 10(2) 2006, pp. 59-73. African Journal of Research in SMT Education, 10(2), pp. 59-73.

Punch, K. (2009). *Introduction to Research Methods in Education*. Sage. London.

Remillard, J. T. (2000). Can curriculum materials support teachers' learning*? Elementary School Journal, 100*(4), 331–350.

Remillard, J., & Bryans, M. (2004). Teachers' orientations toward mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education*, *35*, 352-388.

Resnick, L. B. (1983). Towards a cognitive theory of instruction. In S. G. Paris, G. M. Olson, & W. H. Stevenson (Ed.), *Learning and motivation in the classroom*. Hillsdale, NJ: Erlbaum

Sanni (2008) Mathematics teachers' task practice and teacher knowledge. Unpublished PhD Thesis, University of the Witwatersrand, Johannesburg

Sfard, A. (1998). "On two metaphors for learning and the dangers of choosing just one." Educational Researcher 27 (2): 4–13.

Sfard, A. (2006). Participationist discourse on mathematics learning. In J. Maasz and W. Schloeglmann (Eds.). *New Mathematics Education Research and Practice*. 153 – 170 Rotterdam: Sense Publishers

Sfard, A. (2007). When the rules of discourse change but nobody tells you: making sense of mathematics learning from a commognitive standpoint. *Journal of the Learning Science*, 16(4), 567-613.

Shimiza, Y., Kaur, B., Huang, R., & Clarke D. (2010). The Role of Mathematical Tasks in Different Cultures. In Shimiza, Y., Kaur, B., Huang, R., and Clarke D. (Eds): *Mathematical Tasks in Classrooms Around the world* (pp. 13-33). Rotterdam: Sense Publishers B.V.

Skovsmose, O., & Yasukawa, K. (2009). Formatting power of 'mathematics in a package': A challenge for social theorizing? In P. Ernest, B. Greer & B. Sriraman (Eds.), Critical Issues in Mathematics Education (pp. 255-281). Charlotte, NC: Information Age Publishing.

Stacey, K. (2003). The need to increase attention to mathematical reasoning. In H. Hollingsworth, J. Lokan, & B. McCrae, *Teaching mathematics in Australia: Result from the TIMSS 1999 Video Study* (pp. 119-122). Melbourne: Australian Council for Educational Research.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50-80.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33(2), 455-488.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, 3, 268-275.

Stein, M. K., Remillard, J. T. & Smith, M. S., (2007). How Curriculum Influences Student Learning. In Frank. K. Lester (Ed.). *Second handbook of research on mathematics teaching and learning, Vol. 1-2* (pp. 319-369). Greenwich, CT: Information Age Publishing.

Stoffels, N. (2007). A process-oriented study of the development of science textbooks in South Africa. *African Journal of Research in SMT Education, 11*, 1-13.

Von Glaserfeld, E. (1984). Radical constructivism. In P. Watzlawick (Ed.) *The invented reality,* (pp. 17-40). Cambridge, MA: Harvard University Press

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press. Chapters 4 and 6.

Warren, E., & Cooper, T. (2008). Generalising the pattern rule for visual growth patterns: Actions that support 8 year olds' thinking. *Educational Studies in Mathematics, 67*, 171–185.

Warren, E. (2005). Young children's ability to generalise the pattern rule for growing patterns. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 305 - 312). Melbourne, Australia.

Wells, G. (1999) Dialogic Inquiry. Toward a Sociocultural Practice and Theory of Education. Cambridge: Cambridge University Press.

Woolfolk, A. E. (1993). *Educational psychology,* Bosten: Allyn and Bacon.

Appendix A – Classroom Observation Schedule

University of the Witwatersrand

Classroom Observation Schedule

Research: Tasks used in mathematical classrooms

Researcher: Phathumusa Mdladla

Observed lesson number: ________ Date of observation: ______________

Number of female learners: ________ Number of male learners: __________

Lesson topic: _____________________________

Duration of lesson: __________ Time of lesson: _________

Tape recording of lesson (tick appropriate block):

YES	NO

(*Capture observations by placing key(s) in the appropriate block at each time interval. Each key outlines a category, if the categories provided are insufficient, list the catergory that best captures the observation*.)

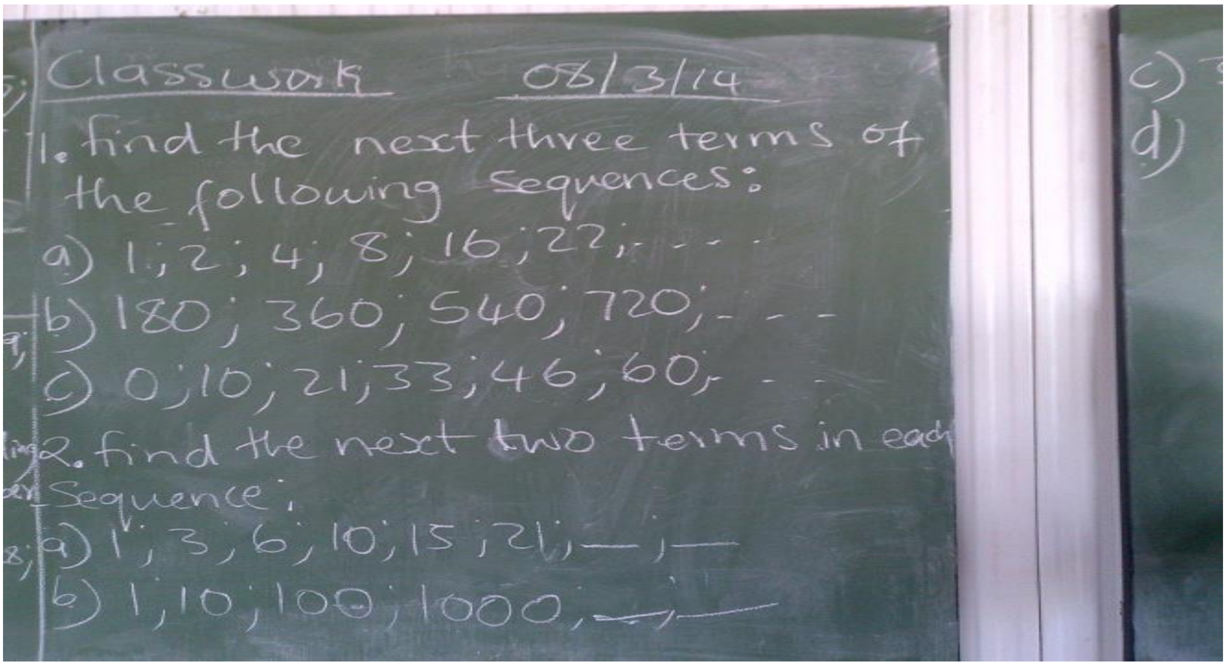
Codes: **W** = writing on the board; **E** = explaining/ demonstrating; **D** = disciplining; **Q**= Questions and Answers **I** = individual work

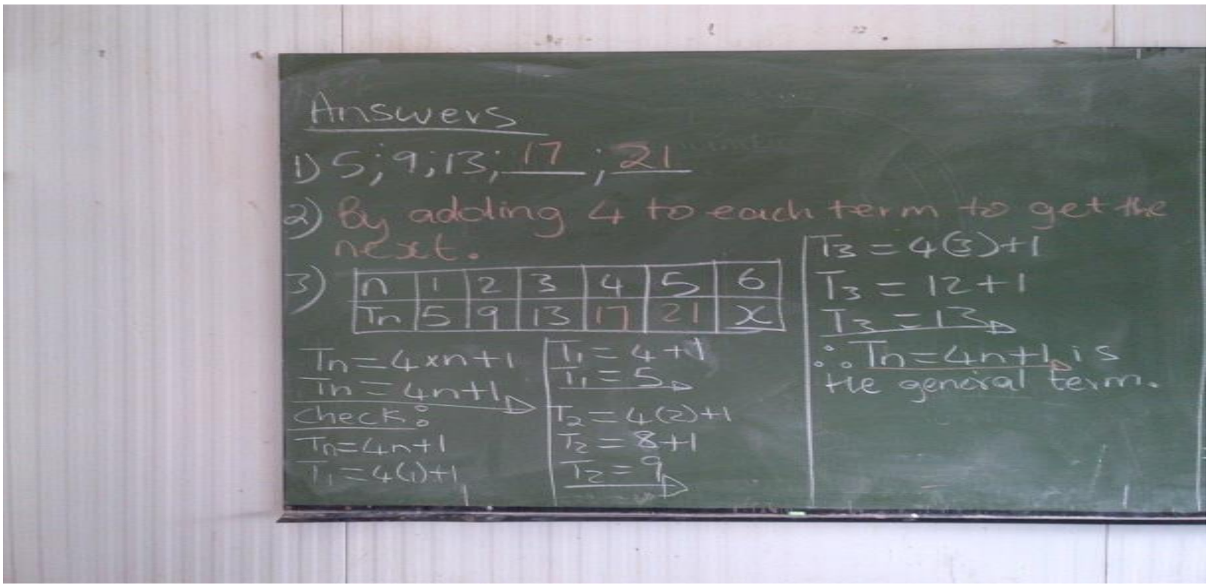
Rough sketch of the learners seating arrangement

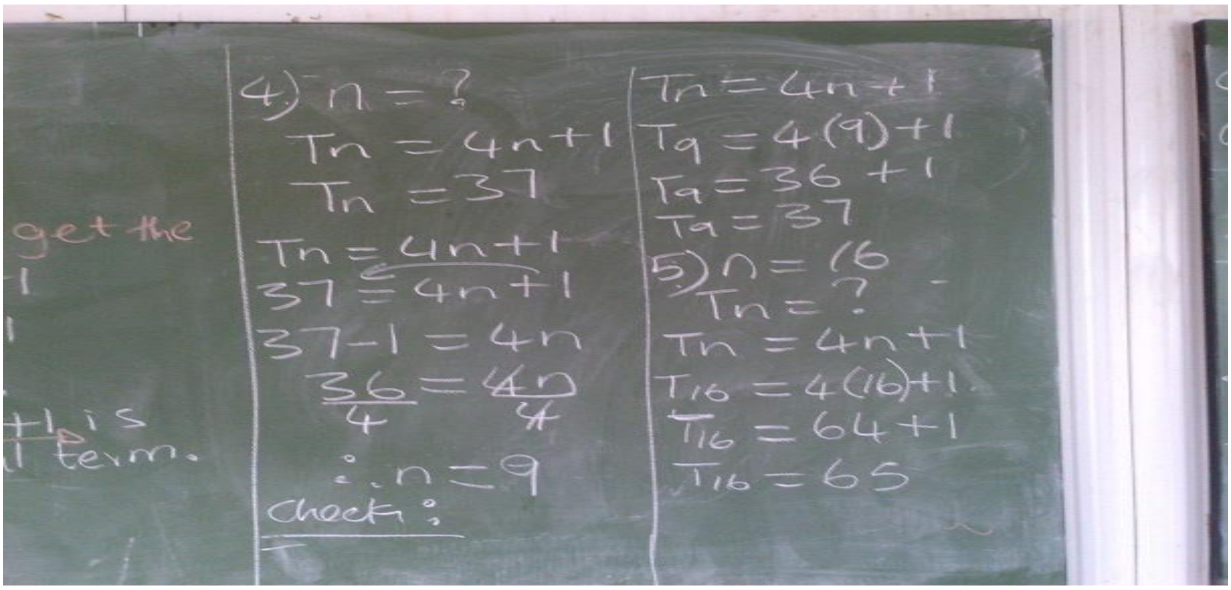
Time	Teacher's Action	Learners' Action
5 min		
10 min		
15 min		

20 min		
25 min		
30 min		
35 min		
40 min		
45 min		
50 min		
55 min		
60 min		

Appendix B – Sample of work done in Grade 9




Answers
1) 5; 9; 13; 17 ; 21
2) By adding 4 to each term to get the next.
3)
n 1 2 3 4 5 6
Tn 5 9 13 17 21 x
Tn = 4xn+1
Tn = 4n+1
Check:
Tn=4n+1
T1=4(1)+1
T1 = 4+1
T1 = 5
T2 = 4(2)+1
T2 = 8+1
T2 = 9
T3 = 4(3)+1
T3 = 12+1
T3 = 13
∴ Tn=4n+1 is the general term.


4) n = ?
Tn = 4n+1
Tn = 37
Tn = 4n+1
37 = 4n+1
37-1 = 4n
36/4 = 4n/4
∴ n = 9
Check:
Tn = 4n+1
T9 = 4(9)+1
T9 = 36+1
T9 = 37
5) n = 16
Tn = ?
Tn = 4n+1
T16 = 4(16)+1
T16 = 64+1
T16 = 65

Need (7) sums

7 Functions and patterns

By the end of this module, you should be able to:

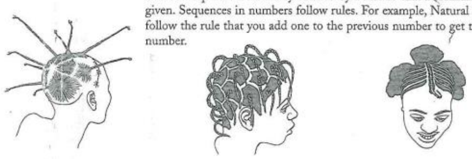
- discover, analyse, extend, and create patterns, relations, or functions to model mathematical ideas in a variety of forms
- use physical materials or shapes to represent numerical patterns
- use patterns to make generalisations and predictions
- describe the rule or general plan of existing patterns
- create new patterns with consistent rules or plans
- classify patterns and functional relationships
- use pattern-based thinking to understand and represent mathematical and other real-world phenomena
- investigate various types of numeric and geometric patterns and functions
- represent numbers and analyse mathematical situations to determine input-values, output-values and the relationship between them using: verbal descriptions, flow diagrams, input output relationships, tables, formulae and equations
- use mathematical models to analyse change in real and abstract context
- improve visual thinking skills.

Introduction

Mathematics is, at heart, a search for patterns. The patterns can occur naturally like seasons, population growth and tidal waves. Patterns are commonly used in hair braiding and weaving in African communities.

NB READ

A **pattern** or **sequence** is an arrangement of objects or numbers in a specified order. Relationships can be established in a sequence. A number sequence is normally defined by a rule so that a **formula** can be given. Sequences in numbers follow rules. For example, Natural numbers follow the rule that you add one to the previous number to get the next number.



1. Number sequences

Mathematics is especially useful when it helps us to predict. In order to find patterns in numbers, you need to predict. Recognising patterns is also a valuable problem-solving skill. When you see a pattern you can use that pattern to generalise in a broader solution.
A number sequence is a set of numbers formed by using a rule to establish each term. A sequence is an ordered set of numbers.
For example, consider the sequence 4; 11; 32; 95.
The next number in this sequence is obtained by: $(n \times 3) - 1$ where n is the previous number.
Each number in a sequence is known as a term. The first term of this sequence is 4, the second term is 11. The rule must be true for every term of the sequence.

Once you know the pattern, the next term in the sequence can be found.

Example:

Make a rule for generating the next three terms in the following sequence: 2; 4; 7; 11; ___; ___; ___.

The first term in the sequence is 2. If you add 2 you will get the second term. Then you add 3 to find the third term.

$$2 \xrightarrow{+2} 4 \xrightarrow{+3} 7 \xrightarrow{+4} 11$$

To find the next three terms you continue the pattern: add three to the second, add 4 to the third term.

$$11 \xrightarrow{+5} 16 \xrightarrow{+6} 22 \xrightarrow{+7} 29$$

So, the next three terms in the sequence will be 16, 22 and 29.

Different kinds of sequences

Numbers can have interesting sequences.
Table showing common number sequences:

Type of sequence	Description	Examples
Arithmetic sequence	Arithmetic sequence is made by adding a constant value each time.	1; 4; 7; 10; 13; 16; 19; 22 ... The pattern is continued by adding 3 to the last number each time. 3; 4; 8; 13; 18; 23; 28; 33; 38... This sequence has a difference of 5 between each number. The pattern is continued by adding 5 to the last number each time.
Geometric sequence	Geometric sequence is made by multiplying by some value each time	9; 27; 81; 243; 729; 2 187 ... The pattern is continued by multiplying the last term by 3 each time.

Appendix C - Description of cognitive levels as provided in CAPS (2011, p. 53)

Cognitive levels	Description of skills to be demonstrated	Examples
Knowledge **20%**	• Straight recall • Identification of correct formula on the information sheet (no changing of the subject) Use of mathematical facts • Appropriate use of mathematical vocabulary	1. Write down the domain of the function $y = f(x) = \frac{3}{x} + 2$ (Grade 10) 2. The angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle ....
Routine Procedures **35%**	• Estimation and appropriate rounding of numbers • Proofs of prescribed theorems and derivation of formulae • Identification and direct use of correct formula on the information sheet (no changing of the subject) • Perform well known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	1. Solve for $x : x^2 - 5x = 14$ (Grade 10) 2. Determine the general solution of the equation $2\sin(x - 30°) + 1 = 0$ (Grade 11) 2. Prove that the angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle is double the size of the angle $A\hat{C}B$ which the same arc subtends at the circle. (Grade 11)
Complex Procedures **30%**	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding	1. What is the average speed covered on a round trip to and from a destination if the average speed going to the destination is $100km/h$ and the average speed for the return journey is $80km/h$? (Grade 11) 2. Differentiate $\frac{(x+2)^2}{\sqrt{x}}$ with respect to x. (Grade 12)
Problem Solving **15%**	• Non-routine problems (which are not necessarily difficult) • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts	Suppose a piece of wire could be tied tightly around the earth at the equator. Imagine that this wire is then lengthened by exactly one metre and held so that it is still around the earth at the equator. Would a mouse be able to crawl between the wire and the earth? Why or why not? (Any grade)

Appendix D – Grade 11 Annual Teaching Plan

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG PROVINCE
MATHEMATICS – WORK SCHEDULE – GRADE 11
2014

DATE	TOPIC	CONTENT	F	ASSESSMENT	DATE Completed	% Completed
TERM 1				2 TASKS FOR TERM 1		
15/1 – 17/1 (3 days)	Exponents and Surds	• Simplify expressions using the laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p}$; $x > 0; q > 0$.				3%
20/1 – 24/1	Exponents and Surds	• Solve equations using the laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p}$; $x > 0; q > 0$. • Add, subtract, multiply and divide simple surds.				6%
27/1 – 31/1	Exponents and Surds	• Add, subtract, multiply and divide simple surds. • Solve simple equations involving surds				9%
03/2 – 07/2	Equations	• Revision of factorisation • Quadratic equations (by factorisation) • Complete the square.				12%
10/2 – 14/2	Equations and Inequalities	• Quadratic equations (by using the quadratic formula). • Quadratic inequalities in one unknown (Interpret solutions graphically).	F	PROJECT/ INVESTIGATION SBA marks: 20		15%
17/2 – 21/2	Simultaneous equations Nature of roots	• Equations in two unknowns, one of which is linear and the other quadratic. • Nature of roots.				18%
24/2 - 28/2	Number patterns	• Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.				21%
03/3 - 07/3	Number patterns	• Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	F	TEST SBA marks: 10		24%
10/3 – 14/3	Analytical Geometry	• The equation of a line through two points • The equation of a line through one point and parallel or perpendicular to a given line				27%
17/3 – 21/3 (4 days)	Analytical Geometry	• The inclination(θ) of a given line • Application				30%
24/3 – 28/3	Analytical Geometry	• Application				33%
TERM 2				2 TASKS FOR TERM 2		
07/4 – 11/4	Functions	• Revise the effect of *a* and *q* and investigate the effect of p on the graphs of the functions defined by: • $y = f(x) = a(x + p) + q$ • $y = f(x) = a(x + p)^2 + q$				36%
14/4 - 18/4 (4 days)	Functions	• Revise the effect of *a* and *q* and investigate the effect of p on the graphs of the functions defined by: • $y = f(x) = a(x + p)^2 + q$ • $y = f(x) = \dfrac{a}{x+p} + q$				49%

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

EKURHULENI SOUTH
MATHEMATICS – WORK SCHEDULE – GRADE 9 2014

DATE	TOPIC	CONTENT	F	ASSESSMENT	DATE Completed	% Com-pleted
		• Common fractions & decimal fraction forms of the same number • Common fraction, decimal fraction & percentage forms of the same number **Decimal fractions** **Calculations using decimal fractions** • Multiple operations with decimal fractions, using a calculator where appropriate • Multiple operations, with or without brackets, with numbers that involved the squares, cubes, square roots & cube roots of decimal fractions **Calculation techniques** • Use knowledge of place value to estimate the number of decimal places in the result before performing calculations • Use rounding off & a calculator to check results where appropiate **Solving problems** Solving problems in context involving decimal fractions **Equivalent forms** • Revise equivalent forms between: o Common fractions & decimal fraction forms of the same number o Common fraction, decimal fraction & percentage forms of the same number.				
17/2 – 21/2	Exponents 4,5 hours Done	**Comparing & representing numbers in exponential form** • Revise: o compare & represent whole numbers in exponential form o Compare & represent integers in exponential form • Extend scientific notation to include negative exponents **Calculations using numbers in exponential form** • Revise the following general laws of exponents: o $a^m \times a^n = a^{m+n}$ o $a^m \div a^n = a^{m-n}$ $if\ m > n$ o $(a^m)^n = a^{mn}$ o $(a \times t)^n = a^n \times t^n$ o $a^0 = 1$ • Extend the general laws of exponents to include: o Integer exponents o $a^{-n} = \frac{1}{a^n}$ • Perform calculations involving all four operations using numbers in exponential form **Solving problems** • Solve problems in contexts involving numbers in exponential form, including scientific notation	F	**TEST** **SBA marks: 50**		13,6
24/2 - 28/2	Numeric and geometric Patterns 4,5 hours Done	**Investigate and extend patterns** • Investigate & extend numeric & geometric patterns looking for relationships between numbers; including patterns: o Represented in physical or diagram form o Not limited to sequences involving a constant difference or ratio o Of learner's own creation o Represented in tables o Represented algebraically • Describe & justify the general rules for observed relationships between numbers in own words or in algebraic language				16,2
03/3 - 07/3	Functions and relationships 4,5 hours Done	**Input & output values** • Determine input values, output values or rules for patterns & relationships using: o Flow diagrams o Tables o Formulae o Equations **Equivalent forms** • Determine, interpret & justify equivalence of different descriptions of the same relationship or rule presented; o Verbally o In flow diagrams o In tables o By formulae o By equations o By graphs on a Cartesian plane				18,9
10/3 – 14/3	Algebraic Expressions 4,5 hours Done	**Algebraic language** • Revise the following done in Grade 8: o Recognize & identify conventions for writing algebraic expressions o Identify & classify like & unlike terms in algebraic expressions o Recognize & identify coefficients & exponents in algebraic expressions • Recognize & differentiate between monomials, binomials & trinomials	F	**ASSIGNMENT** **SBA marks: 50**		21,6

Appendix E – Task analysis

Ref Number	Task	Source	Type of cognitive demand	Difficulty level
	Homework (Linear sequences) Revision (connection between new and old concepts and/or skills)			
1	Write down the next three terms in each of the following sequence 45; 29; 13; -3; ...	siyavula	**memorisation**	**Easy**
	The general term is given for each sequence below. Calculate the missing terms.			
2	-4; -9; -14; ...; -24 $T_n = 1 - 5n$	Siyavula	**p/without**	**Easy**
3	6; ...; 24; ...; 42 $T_n = 9n - 3$	Siyavula	**p/without**	**Easy**
	Find the general formula for the following sequences and then find T_{10} , T_{15} and T_{30}			
4	13; 16; 19; 22; ...	Siyavula	**p/without**	**Easy**
5	18; 24; 30; 36; ...	Siyavula	**p/without**	**Easy**
6	-10; -15; -20; -25;	Siyavula	**p/without**	**Easy**

7	The seating in a classroom is arranged so that the first row has 20 desks, the second row has 22 desks; the third row has 24 desks and so on. Calculate how many desks are in the ninth row.		**p/without**	**Easy**
8	Complete the following: $13+31=$ $24+42=$ $38+83=$	Siyavula	**memorisation**	**Easy**
9	Look at the numbers on the left-hand side, what do you notice about the unit digit and the tens-digits?	Siyavula	**memorisation**	**Easy**
10	Investigate the pattern by trying other examples of 2-digit numbers.	Siyavula	**p/with**	**Easy**
11	Make a conjecture about the pattern that you notice.	Siyavula	**p/with**	**Easy**
12	Prove the conjecture.	Siyavula	**Doing Math**	**Difficult**
	Classwork (Introduction to quadratic sequences) Introducing new concept			
	Determine the second difference between the terms for the following sequences:			
13	5; 20; 45; 80; ...	Siyavula	**p/without**	**Easy**
14	6; 11; 18; 27; ...	Siyavula	**p/without**	**Easy**
15	1; 4; 9; 16; ...	Siyavula	**p/without**	**Easy**

16	3; 0; -5; -12; ...	Siyavula	**p/without**	**Easy**
17	1; 3; 7; 13; ...	Siyavula	**p/without**	**Easy**
18	0; -6; -16; -30; ...	Siyavula	**p/without**	**Easy**
19	-1; 2; 9; 20; ...	Siyavula	**p/without**	**Easy**
20	1; -3; -9; -17; ...	Siyavula	**p/without**	**Easy**
21	3a + 1; 12a + 1; 27a + 1; 48a + 1; ...	Siyavula	**p/without**	**Easy**
22	2; 10; 24; 44; ...	Siyavula	**p/without**	**Easy**
23	t – 2; 4t – 1; 9t; 16t + 1; ...	Siyavula	**p/without**	**Easy**
	Homework			
	Complete the sequence by filling in the missing term:			
24	11; 21; 35; ...; 75	Siyavula	**p/without**	**Easy**
25	20; ...; 42; 56; 72	Siyavula	**p/without**	**Easy**
26	...; 37; 65; 101	Siyavula	**p/without**	**Easy**
27	3; ...; -13; - 27; - 45	Siyavula	**p/without**	**Easy**
28	24; 35; 48; ...; 80	Siyavula	**p/without**	**Easy**
29	....; 11; 26; 47	Siyavula	**p/without**	**Easy**
	Use the general term to generate the first four terms in each sequence:			
30	$T_n = n^2 + 3n - 1$	Siyavula	**p/without**	**Easy**
31	$T_n = -n^2 - 5$	Siyavula	**p/without**	**Easy**

32	$T_n = 3n^2 - 2n$	Siyavula	**p/without**	**Easy**
33	$T_n = -2n^2 + n + 1$	Siyavula	**p/without**	**Easy**
	Classwork			
	Given: 7; 11; 15; ...			
34	Calculate the n^{th} term.		**p/without**	**Easy**
35	Calculate the 15^{th} term.		**p/without**	**Easy**
36	Which term will be 83?		**p/without**	**Easy**
37	Given: 3; 6; 9; 12; ... find the equation of the general term.		**p/without**	**Easy**
	Consider the sequence 8; 18; 30; 44; …			
38	Add the next two terms.		**p/without**	**Easy**
39	Calculate the nth term.		**P/with**	**Difficult**
40	Which term of the sequence is 330?		**p/without**	**Moderate**
	Consider the sequence 2; 8; 18; 32; …			
41	Add the next 2 terms	Teacher's resource	**p/without**	**Easy**
42	Calculate the n^{th} term	Teacher's resource	**p/with**	**Difficult**
43	Which term is 1250?	Teacher's resource	**p/without**	**Moderate**

	Consider the sequence -3; -2; -3; -6; …	Teacher's resource		
44	Add the next 2 terms.	Teacher's resource	**p/without**	**Easy**
45	Determine an expression of the nth term.	Teacher's resource	**p/with**	**Difficult**
46	Calculate the 35^{th} term	Teacher's resource	**p/without**	**Easy**
	Consider the sequence 3; 7; 11; 15; …			
47	Add the next two terms	Teacher's resource	**p/without**	**Easy**
48	Write the general formula	Teacher's resource	**p/with**	**Difficult**
49	Calculate the 15^{th} term	Teacher's resource	**p/without**	**Easy**
	Consider the sequence 6; 11; 16; 21; …			
50	Add the next 2 terms	Teacher's resource	**p/without**	**Easy**
51	Write the general formula	Teacher's resource	**p/with**	**Difficult**

52	Calculate the 15th term	Teacher's resource	**p/without**	**Easy**
	Homework			
	Calculate the common second difference for each of the following quadratic sequences:			
53	3; 6; 10; 15; 21; ...	Siyavula	**p/without**	**Easy**
54	4; 9; 16; 25; 36; ...	Siyavula	**p/without**	**Easy**
55	7; 17; 31; 49; 71; ...	Siyavula	**p/without**	**Easy**
56	2; 10; 26; 50; 82; ...	Siyavula	**p/without**	**Easy**
57	31; 30; 27; 22; 15; ...	Siyavula	**p/without**	**Easy**
58	Find the first five terms of the quadratic sequence defined by: $T_n = 5n^2 + 3n + 4$	Siyavula	**p/without**	**Easy**
59	Given $T_n = 4n^2 + 5n + 10$, find T_9.	Siyavula	**p/without**	**Easy**
60	Given $T_n = 2n^2$, for which value of n does $T_n = 32$?	Siyavula	**p/without**	**Easy**
61	Write down the next two terms of the quadratic sequence: 16; 27; 42; 61;...	Siyavula	**p/without**	**Easy**
62	Find the general formula for the quadratic sequence above.	Siyavula	**p/with**	**Difficult**
	Homework (on Inequalities)			
	Classwork			

	Find the first five terms of the quadratic sequence defined by:			
63	$T_n = n^2 + 2n + 1$	Siyavula	**p/without**	**Easy**
64	Given the pattern: 16; x; 46 ..., determine the value of x if the pattern is linear.	Siyavula	**p/without**	**Easy**
	For each of the following patterns, determine: • The next term in the pattern • And the general term, • The tenth term in the pattern			
65	3; 7; 11; 15; ...	Siyavula	**p/without**	**Easy**
66	17; 12; 7; 2; ...	Siyavula	**p/without**	**Easy**
67	½; 1; 1 ½ ; 2; ...	Siyavula	**p/without**	**Easy**
68	a; a + b; a + 2b; a + 3b; ...	Siyavula	**p/without**	**Easy**
69	1; -1; -3; -5; ...	Siyavula	**p/without**	**Easy**
	Homework			
	Determine whether each of the following sequences is: linear, quadratic or neither			
70	6; 9; 14; 21; 30 ...	Siyavula	**Memorisation**	**Easy**
71	1; 7; 17; 31; 49; ...	Siyavula	**Memorisation**	**Easy**

72	8; 17; 31; 49; ...	Siyavula	**Memorisation**	**Easy**
73	3; 9; 15; 21; 27; ...	Siyavula	**Memorisation**	**Easy**
74	Given $T_n = 2n^2$, for which value of n does $T_n = 242$?	Siyavula	**p/without**	**Easy**
75	Given $T_n = 3n^2$, find T_{11}.	Siyavula	**p/without**	**Easy**
76	Given $T_n = n^2 + 4$, for which value of n does $T_n = 85$?	Siyavula	**p/without**	**Easy**
	Given the following sequence: -15; -11; -7; ...: 173	Siyavula		
77	Determine the equation for the general term.	Siyavula	**p/without**	**Easy**
78	Calculate how many terms there are in the sequence.	Siyavula	**p/without**	**Easy**
	Classwork			
	Given: 2; 7; 14; 23; 34; ...			
79	Add the next two terms.	Teacher's resource	**p/without**	**Easy**
80	Write the general formula.		**p/with**	**Difficult**
81	Which term is 2599?	Teacher's resource	**p/without**	**Moderate**
	Given 18; 13; 9; 6; 4; ...			
82	Add the next two terms.		**p/without**	**Easy**
83	Write the general formula.		**p/with**	**Difficult**
84	Write the 25th term.		**p/without**	**Easy**

	Given 10; 2; - 6; -14; ...			
85	Add the next two terms		**p/without**	**Easy**
86	Write the general formula		**p/with**	**Difficult**
87	Write the 25^{th} term.		**p/without**	**Easy**

Table 5.1.1 Classification of grade 11 tasks

Grade 9 tasks

Ref Number	**Task**	**Source**	**Type of cognitive demand**	**Difficulty level**
	Lesson 1: To introduce concept			
	Find the next three terms of the following sequences:			
1	1; 2; 3; 4; 5; ...		p/without	Easy
2	2; 6; 10; 14; ...		p/without	Easy
3	1; 3; 9; 27; ...		p/without	Easy
4	2; 6; 18; 54; ...		p/without	Easy
	Lesson 2			
	Find the next three terms of the following sequences:	p. 91		
5	1; 2; 4; 8; 16; 32; ...	Spot On	P/without	Easy
6	180; 360; 540; 720; ...	Spot On	P/without	Easy

7	0; 10; 21; 33; 46; 60; ...	Spot On	P/without	Easy
	Find the next two terms in each sequence:			
8	1; 3; 6; 10; 15; 21; ...	Spot On	P/without	Easy
9	1; 10; 100; 1000; ...	Spot On	P/without	Easy
10	32; 30; 26; 20; 12; 2; ...	Spot On	P/without	Easy
11	11; 7; 3; - 1; - 5; - 9; - 13; ...	Spot On	P/without	Easy
12	54; 63; 72; 81; ...	Spot On	P/without	Easy
13	12; 36; 108; 324; 972; ...	Spot On	P/without	Easy
	Classwork			
14	Write down the next two terms in the given sequence: 5; 9; 13; ...	ANA exemplar	p/without	Easy
15	Describe the sequence in question 19 above in your own words.	ANA exemplar		Easy
16	Write down the general term of the given sequence in the form: Tn = ______________	ANA exemplar	Doing math	Moderate
17	Which term in the sequence is equal to 37?	ANA exemplar	p/without	Easy
18	Find the 16^{th} term.	ANA exemplar	p/without	Easy
	Homework (Geometric and numeric pattern) Not clear			

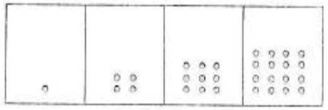
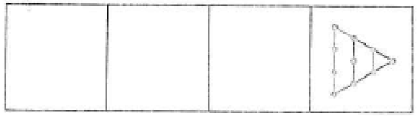
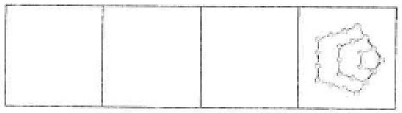
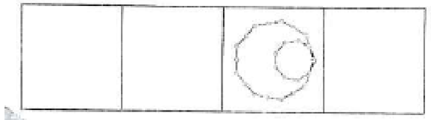
	Read the top row. The position: 1st term, 2nd term, 3rd term, 4th term, 5th term, n^{th} term If the 2nd term position is 2 and its value is 16 the rule is $2 \times 8 = 16$. What is the 1st term? You need to make sure that you know what the geometric figures are.  **1. Create and complete the following geometric patterns.** • Draw the first four terms in each of the following geometric patterns. • Write them in a table determining the 1st, 2nd, 3rd, 4th, 10th and n^{th} terms, where applicable. **Example: Square**  Position 1st 2nd 3rd 4th 10th n^{th} Value 1 4 9 16 100 n^2			
19	a. Triangle  Position 1st 2nd 3rd 4th n^{th} Value 10	DBE workbook (p. 70)	Doing Math	V. difficult
20	b. Pentagon  Position 1st 2nd 3rd 4th n^{th} Value 22	DBE workbook (p. 70)	Doing Math	V. difficult
21	c. Nonagon  Position 1st 2nd 3rd 4th n^{th} Value 24	DBE workbook (p. 70)	Doing Math	V. difficult

Table: Classification of Grade 9 tasks

Appendix F – Ethic Clearance Letter

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa
Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

Student Number:
Protocol Number: 2013ECE143M

Date: 24 October 2013

Dear Emmanuel Mdladla

Application for Ethics Clearance: Master of Education

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

Tasks used in mathematics classrooms

The committee recently met and I am pleased to inform you that clearance was granted.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely

MMabeta

Matsie Mabeta
Wits School of Education

011 717 3416
CC Supervisor: Ms Margot Berger

Appendix G –GDE Research Approval Letter

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

For administrative use:
Reference no: D2014/301

GDE RESEARCH APPROVAL LETTER

Date:	**20 November 2013**
Validity of Research Approval:	**10 February to 3 October 2014**
Name of Researcher:	**Mdladla E.P.**
Address of Researcher:	**17516 Monomane Street**
	Vosloorus Extension 25
	1475
Telephone Number:	**011 865 2526 / 073 755 2400**
Email address:	**phathumusamdladla@yahoo.com**
Research Topic:	**Tasks used in Mathematics classrooms**
Number and type of schools:	**ONE Secondary School**
District/s/HO	**Ekurhuleni South**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*

Making education a societal priority

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

[signature]
....

Dr David Makhado

Director: Education Research and Knowledge Management

DATE: 2013/11/21

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za