



HUNTING DARK MATTER WITH FAINT RADIO HALOS

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Thesis

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Abstract

The nature of Dark Matter (DM), the elusive substance that constitutes a significant amount of the total matter in the universe, remains an unsolved problem in modern physics despite a decades-long search effort. One approach to this problem has been to search for faint emission signatures that are produced indirectly from the DM present in large astrophysical structures, and thus infer properties about theoretical DM models from observational data. In recent years, the results from studies that use this type of indirect search have produced stringent constraints on the most popular DM particle candidate parameter spaces, ruling out swathes of viable DM models. These compelling results have been enabled by the arrival of sophisticated interferometric radio telescopes, which are excellent DM hunters due to their high sensitivity and resolution. In this thesis, we focus on the use of the latest data from the MeerKAT radio interferometry telescope, through the first public release of the MeerKAT Galaxy Cluster Legacy Survey, to search for DM emissions in a set of nearby galaxy clusters. Each step of this process, from the creation of theoretical DM emission models to the statistical analysis of the observational data, has been described in detail in this thesis. With this data, we find an almost universal improvement to results found with corresponding modelling scenarios in the literature. Since this work is among the first to use MeerKAT data in astrophysical DM searches, these results present a strong argument for continued work in this field. Another central focus of this thesis is the accurate modelling of the physical processes involved in the production of the DM-induced radio emissions, as the quality of current radio data requires theoretical models that are sufficiently accurate to describe the emission at such high resolutions. One aspect of the modelling that has lacked this accuracy has been the solution to the diffusion-loss equation, which is a crucial factor in determining indirect DM emissions. A new algorithm for solving this equation, which provides higher accuracy and computational efficiency than previous public methods, has thus been developed and presented in this thesis. These aspects of DM indirect detection study will become ever more important as we approach the inauguration of the Square Kilometre Array (SKA), which will provide data with unprecedented potential with which to continue the hunt for DM.

Declaration

I declare that this thesis is my own, unaided work. It is being submitted for the Degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

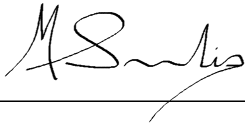


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Declaration - Publications

The following are details of publications that include research presented in this thesis. The candidate undersigned declares that following data is correct and accurate.

- Beck, G. and Sarkis, M. (2023a). “Galaxy Clusters in High Definition: A Dark Matter Search”. In: *Physical Review D* 107.2, p. 023006. DOI: [10.1103/PhysRevD.107.023006](https://doi.org/10.1103/PhysRevD.107.023006). arXiv: [2210.00796](https://arxiv.org/abs/2210.00796)
- Sarkis, M., Beck, G., and Lavis, N. (2023b). “A Radio-Frequency Search for WIMPs in RXC J0225.1-2928”. In: *Letters in High Energy Physics* 2023. DOI: [10.31526/LHEP.2023.361](https://doi.org/10.31526/LHEP.2023.361). arXiv: [2303.00684](https://arxiv.org/abs/2303.00684)
- Sarkis, M. and Beck, G. (2023a). “Simulating the Radio Emissions of Dark Matter for New High-Resolution Observations with MeerKAT”. in: *The Proceedings of SAIP2022*. The 66th Annual Conference of the South African Institute of Physics. UJ, ed. by Prof A. Prinsloo, pp. 408–413. DOI: [10.48550/ARXIV.2301.03326](https://doi.org/10.48550/ARXIV.2301.03326)
- Sarkis, M. and Beck, G. (2022). “Diffusing Assumptions in Astroparticle Physics”. In: *The Proceedings of SAIP2021*. The 65th Annual Conference of the South African Institute of Physics. UJ, ed. by Prof A. Prinsloo, pp. 316–321. URL: <http://saip.org.za/Proceedings/Track%20D/51.pdf>



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This work makes use of MGCLS data products, which were provided by the South African Radio Astronomy Observatory and the MGCLS team and were derived from observations with the MeerKAT radio telescope. The MeerKAT telescope is operated by the South African Radio Astronomy Observatory, which is a facility of the National Research Foundation, an agency of the Department of Science and Innovation.

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Glossary

Common abbreviations

Abbreviation	Description
ASKAP	Australian SKA Pathfinder (radio interferometry telescope)
CLT	Central Limit Theorem (statistical theorem)
DM	Dark Matter
DMID	Dark Matter Indirect Detection
dSph	dwarf Spheroidal galaxy
FITS	Flexible Image Transport System (data file format)
gNFW	Generalised NFW (DM halo density profile)
IC	Inverse Compton scattering (particle interaction)
LHC	Large Hadron Collider (particle collider)
MCXC	Meta-Catalogue of X-ray-detected Clusters (observational catalogue)
MGCLS	MeerKAT Galaxy Cluster Legacy Survey (observational programme)
MW	Milky Way
NFW	Navarro-Frenk-White (DM halo density profile)
OS	Operator Splitting (numerical technique)
RMS	Root Mean Square (typically as an error estimate)
RoI	Region of Interest (subset of a data image)
SKA	Square Kilometre Array (radio interferometry telescope)
SM	Standard Model (of particle physics)
WIMP	Weakly Interacting Massive Particle (DM candidate)
2HDM+S	Two-Higgs Doublet Model (SM extension, DM candidate)
Λ CDM	Λ + Cold DM (standard model of cosmology)

Prominent software packages or tools

Package	URL
PyBDSF	https://github.com/lofar-astron/PyBDSF
PPPC4DMID	http://www.marcocirelli.net/PPPC4DMID.html
RX-DMFIT	https://github.com/alex-mcdaniel/RX-DMFIT
astropy.reproject	https://github.com/astropy/reproject
SAOImageDS9	https://sites.google.com/cfa.harvard.edu/saoimageds9
WebPlotDigitizer	https://automeris.io/WebPlotDigitizer/
L ^A T _E X-Cookbook	https://github.com/alexpovel/latex-cookbook

Introduction

In the current standard model of cosmology, over a quarter of the energy density in the universe exists in the form of some type of Dark Matter (DM) (Planck Collaboration et al. 2020). Although there is strong evidence for its existence, including numerous observations of galaxy cluster dynamics, galactic rotation curves, anisotropies in the Cosmic Microwave Background (CMB) and gravitational lensing, as well as implications from the current layout of large-scale structure in the universe, its physical nature remains unknown (see for instance the excellent reviews in Bergstrom (2000), Bertone et al. (2005, sec. 2), and Bertone et al. (2018)). This evidence has come primarily from the gravitational effects of DM (Rubin 2004; Bertone et al. 2005; Clowe et al. 2006; Springel et al. 2006; Dietrich et al. 2012; Planck Collaboration et al. 2020), as it does not seem to directly interact with our usual astrophysical messengers. The inherently weak interactions of DM have thus made any kind of detection difficult, with more sophisticated and precise detection schemes being developed at an increasing pace, always to miss out on an unambiguous detection signal. Further suggestions of the existence of DM come from deficiencies within the Standard Model (SM) of particle physics, which is currently the most well-tested physical theory in the history of science. Several extensions to the SM that account for these deficiencies also predict the existence of particles which could constitute DM, and so when the much-anticipated solution to the DM problem does arrive, it is likely to bring new physics with it.

Given the significant implications of this DM problem, there has been a global search effort dedicated to its solution for several decades, with the first studies on this topic dating back to the early 1930s (Bertone et al. 2018). Modern searches for DM are taking place in large international collaborations, like in the Large Hadron Collider (LHC) (Kahlhoefer 2017), in enormous underground detectors like XENONnT (XENON Collaboration et al. 2023), or with sophisticated astronomical observatories like the Fermi-LAT space telescope (Ackermann et al. 2017). These examples highlight the feats of modern technology and engineering that have been utilised in the search for DM, and give some indication of the true scale of this problem. As the nature of these detection schemes are also complementary to each other, with each probing different parameter spaces of potential DM candidates, we have cast a very wide net into the dark side of the universe.

The most common candidate for DM considered in the literature is the generic Weakly Interacting Massive Particle (WIMP), which conforms to many of the most stringent requirements of a DM candidate set by observation, while also appearing in several models that extend the SM. These particles are neutral and weakly-interacting, and it is usually assumed that they formed a thermal relic population in the early universe (Steigman et al. 2012), which has resulted in the current abundance we observe today. They are often considered to be collisionless and cold, i.e. non-relativistic, which is a scenario implied by the evolution of large-scale structure in the universe (Springel et al. 2006; Kuhlen et al. 2012) and forms a core component of the Λ CDM model of cosmology. In many studies, WIMPs are treated independently of specific underlying particle physics models, possessing the bare qualities needed to satisfy DM candidature. While we do consider model-independent WIMPs in this thesis, we also consider a particular particle model that was recently proposed in von Buddenbrock et al. (2015) and von Buddenbrock et al. (2016), which is a variation of a two-Higgs-Double model, and is referred to as the 2HDM+S. This is an extension to the SM that was proposed to explain anomalous lepton production in the data gathered from Run 1 and Run 2 of the CMS and ATLAS experiments in the LHC (The CMS Collaboration et al. 2008; Aad et al. 2008), and the model posits the existence of a DM candidate particle, also outside the SM.

The existence of WIMPs can be probed through several independent physical mechanisms, which are possible in all the example detection schemes referred to above. The first method relies on the production of WIMPs via particle collisions in large particle accelerators, and is referred to as collider detection. The second method attempts to detect the signatures of collisions between

WIMPs and SM nuclei in large underground detectors, and is referred to as direct detection. The final method, relevant to this thesis, searches for emissions produced by WIMPs that reside in astrophysical structures through the use of telescope observations, and is referred to as indirect detection (ID/DMID). This detection scheme is enabled by the expectation that WIMPs have the ability to self-annihilate, which would lead to the production of a set of kinematically-accessible annihilation products within the SM, either directly or through some mediator that couples to the SM. There is the additional possibility that WIMPs have a finite lifetime and decay into SM particles, again producing a set of SM products. These are crucial mechanisms, as they could lead to a method for observing DM signatures indirectly, through the SM products of WIMP-WIMP interactions. As such, a primary goal of ID studies is to probe the annihilation cross-section parameter, which describes the amount of annihilation that occurs and is thus a direct factor in the emission produced by a region of WIMPs.

The field of DMID has matured substantially over the past couple of decades, and now includes comprehensive studies of multi-wavelength and multi-messenger emissions from DM that encompass the range of cosmic-ray antimatter to radio frequencies (for reviews, see Bertone (2010, part V), Cirelli (2013), and Gaskins (2016)). The most common form of DMID study in the literature focuses on the prompt gamma-ray emission from WIMP annihilations. Study of this messenger has been driven in popularity by several factors, including a set of observations with the Fermi-LAT gamma-ray telescope (Fermi/LAT Collaboration et al. 2009) and the fact that the calculation of the expected signal on Earth is relatively simple, since the gamma rays are not deflected by ambient environments like magnetic fields. Other messengers, like neutrinos or anti-protons, have also gained popularity with the operation of new, sensitive detectors like IceCube (Abbasi et al. 2012; Abbasi et al. 2023) or the Alpha Magnetic Spectrometer (AMS-02) on the International Space Station (Barao 2004; Calore et al. 2022). The particular focus of this thesis is however on radio frequency ID, which has a long history originating in the 1990s (Berezinsky et al. 1992) and in its modern form has produced some of the most advanced ID results, consisting of relatively strong constraints on the WIMP annihilation cross-section (Regis et al. 2021; Egorov 2022; Beck et al. 2023a). The stringent constraints from these studies are a consequence of the latest generation of radio interferometry observatories (Thompson et al. 2017), which are comprised of arrays of single radio dishes that correlate incoming signals, allowing for baselines on the order of tens to hundreds of kilometres. This sophisticated method of determining the incoming flux of radio emission can result in extremely high sensitivity and spatial resolution, which are both crucial aspects of DM hunting.

The Square Kilometre Array (SKA), a multinational radio observatory with a planned observing area of a square kilometre, will be the largest radio interferometry device built to date, and will be the flagship radio telescope of its generation (Dewdney et al. 2009). While we await its inauguration, precursor instruments like the South African MeerKAT telescope (Jonas et al. 2018) and the Australian SKA Pathfinder (ASKAP) (Johnston et al. 2008) have already had their first-light observations. There is now consensus in the literature that the SKA and its precursors have excellent potential to further advance the DMID search effort (Colafrancesco et al. 2015a; Colafrancesco et al. 2015b; Beck et al. 2017; Storm et al. 2017; Beck 2019b; Chen et al. 2021). In particular, the high resolution of these instruments is important for distinguishing the emission from different regions of DM density and for disentangling any compact source emissions from diffuse fluxes, and the high sensitivity is necessary to observe the relatively faint DM signal; factors which both lend to a better overall search potential compared to previous or single dish instruments. Indications of this potential have already begun to be realised with initial studies that use data from precursor observation programmes, through the ASKAP Evolutionary Map of the Universe (EMU) survey (Norris et al. 2021) and the MeerKAT Galaxy Cluster Legacy Survey (MGCLS) (Knowles et al. 2022), respectively (Regis et al. 2021; Sarkis et al. 2023b).

In this thesis we focus on the MGCLS in particular, which had its first public data release (DR1) on the 11th November 2021. This survey consists of 115 galaxy clusters, positioned between 0° and −85° declination and at a median redshift of 0.14, with a total observing time of ~ 1000 hours. The use of galaxy clusters as test-beds for DM study has a rich history, with some of the very first evidence for DM coming from observations of the Coma galaxy cluster (Zwicky 1933; Andernach et al. 2017), and more recent studies of the Coma cluster being seminal works in the field of radio-based DMID (Colafrancesco et al. 2001; Colafrancesco et al. 2006). Since then,

there have also been a number of searches for specific models of DM, or studies of the general morphology of the radio emissions from galaxy clusters (Colafrancesco et al. 2006; Colafrancesco et al. 2011; Marchegiani et al. 2018; Marchegiani et al. 2019), and studies searching for constraints on the annihilation cross-section (Storm et al. 2013; Storm et al. 2017; Kiew et al. 2017; Chan et al. 2020; Chan et al. 2022; Beck et al. 2023a).

Despite the above-mentioned sources, the use of galaxy clusters in radio DMID is still generally a less popular choice in the literature compared to small satellite galaxies of the MW (particularly those referred to as dwarf spheroidal galaxies (dSphs), see the review in Beck (2019b) for instance). This is primarily because of the presence of large background emissions from non-DM sources, often in the form of large diffuse radio halos, that make isolating the DM signal from other sources difficult. Since dSphs are nearby objects with masses dominated by DM, and contain low amounts of stars and gas (implying relatively low background emission from baryonic sources and thus easier isolation of pure DM emission), they are attractive targets for DM hunting. However, dSphs possess several sources of uncertainty, such as their internal magnetic fields and WIMP density distributions, which as we will see later in this thesis, play an important role in the determination of the radio emissions from WIMP annihilation products. These physical environments are significantly more constrained in galaxy clusters, while still being DM-dominated. Further, the selection of galaxy clusters that lack bright radio halos as search targets can mitigate (to some degree) the afore-mentioned drawback of high background baryonic emissions. These points then generally allow for search results that are based on less uncertainty than those found using dSphs, and suggest that they are deserving of further detailed investigation.

A crucial factor in the reliability and accuracy of DMID studies is the modelling of the predicted emissions. In radio studies, this modelling process is a complicated one. The scenario we consider in this thesis is the creation of electrons and positrons (referred to as just electrons) as final-state products of WIMP annihilations, which due to the large masses of allowed WIMP models, are highly-relativistic. Being charged, these electrons are also subject to many interactions with the ambient astrophysical environment, and the interaction with magnetic fields is specifically what leads to the emission of radio waves, through the process of synchrotron radiation. This synchrotron radiation is the ultimate target of DMID searches, and its precise determination relies on accurate models of the spatial number density and energy distribution of the produced electrons. The evolution of these properties is usually described by a general cosmic-ray propagation equation (for example in Strong et al. (1998, app. eq. 1) and Bertone (2010, eq. 27.2)), though in the astrophysical environments considered here, it is sufficient to consider a truncated form of this equation which encapsulates the dominant effects of spatial diffusion and energy losses – a form generally referred to as the diffusion-loss equation.

A recurring theme of this thesis is a focus on the accuracy of the above modelling scenario. The diffusion-loss equation is not trivial to solve, which has led to the development of several mathematical techniques that have various degrees of accuracy and ease of use. The most common technique employed in the literature uses a semi-analytical approach, popularised by Colafrancesco et al. (2006) and based on the method shown in Baltz et al. (1998) (see also Vollmann (2021) for a variation on this method), and while directly producing a solution to the equation, it requires the approximation that all involved physical quantities are spatially independent. This requirement is typically satisfied by substituting the spatially-dependant quantities, like magnetic fields and thermal gas densities, with scalar averages. This loss of spatial information is the primary motivation for the use of numerical techniques, which are based on finite-differencing schemes and used extensively in the field of galactic cosmic-ray physics (Strong et al. 1998; Porter et al. 2022). Solutions found with the numerical method are more accurate and computationally efficient than their semi-analytical counterparts, however these benefits come at the cost of a significantly more complicated implementation. This has caused a relatively low adoption of this technique in the literature, with the only uses of it by the authors Regis et al. (2015b), Regis et al. (2017), and Regis et al. (2021) with a proprietary implementation, and Egorov (2022) with a modified version¹ of the popular GALPROP package (Strong et al. 1998).

The practical and theoretical aspects of each solution method discussed above are investigated in detail in this thesis. As the accuracy of the electron distribution calculated with the diffusion-

¹Online repository  https://github.com/a-e-egorov/GALPROP_DM

loss equation directly impacts the expected DM-induced synchrotron radiation, it is a salient factor for DMID studies performed with high-resolution observational data. The anticipation of radio interferometry data has motivated the development of a numerical solution method that is accurate and computationally efficient, while also being accessible and easy to integrate into a general radio-based DMID framework. While the two instances of the use of numerical methods given above do exist, the current implementations are either completely inaccessible or based on code that was designed specifically for galactic cosmic-ray propagation, which is not fully aligned with general DMID studies (particularly with galaxy cluster search targets). The numerical solution method developed and presented in this work is expected to be released, together with all necessary computational tools for modelling DM emission, in a public release sometime in the near future.

The overall structure of the rest of the thesis is as follows. In Chapter 2 we provide a brief outline of the DM problem. The main focus of this Chapter is a review of the current status of the radio-based DMID field, though it also contains a summary of ID schemes using other astrophysical messengers. We also give a cursory description of some historical aspects to the DM problem, briefly summarising the most prominent pieces of evidence and prevailing DM candidate models.

Chapter 3 then provides a detailed account of the physical modelling scenarios that are used to predict the radio emissions from WIMP DM. This Chapter contains the bulk of the theoretical aspects involved in the work, which includes the modelling of WIMP annihilations and DM halos, the relevant physical effects involved in the propagation of electrons (particularly the various aspects of diffusion and energy loss), and the calculation of synchrotron emissions from a population of high energy electrons.

Chapter 4 is dedicated to the analysis of the diffusion-loss equation. It contains a review of solution methods that are commonly used in the literature, which is followed by a rigorous presentation of a computational algorithm, developed in this project, to solve this equation. It also includes a set of comparisons between results found with this algorithm and semi-analytical methods, with a comprehensive discussion around the use and merits of each method. Given the rigorous nature of this Chapter, a brief summary of all pertinent points has also been provided.

In Chapter 5 we provide a showcase of the observational data obtained by the MGCLS programme, and discuss in detail the relevant data processing techniques that were used to prepare the radio images for analysis. We also discuss the preparation of DM emission models, found by the methods in the previous Chapters, for comparisons to the data.

Chapter 6 presents the main results of this work, in terms of the constraints found for the WIMP annihilation cross-section based on the data from the MGCLS prepared in the previous chapter. It also contains a detailed explanation of the statistical analysis that was employed to find the constraints. A comprehensive discussion on these results is provided at the end of the Chapter, which also contains a comparison to comparable work in the literature. The thesis is then finally concluded with a brief summary of the main outcomes of this work in Chapter 7.

The Dark Matter Problem

The problem of DM has a long and rich history of research, which has allowed it to grow into a global collaborative and interdisciplinary field. Finding the solution to the DM problem is now one of the most pressing priorities in modern physics, and it is likely to have far-reaching implications in fields from particle physics to cosmology. The aim of this Chapter is to review the overall status of the field, with a particular focus on the DM paradigm that is used in this thesis. As such, the main focus will be on reviewing WIMPs and the various indirect detection schemes that are used in the attempt to detect them. As the field of DM research is vast, for other DM models and phenomenology not used here we have provided sources to excellent works in the literature for further reading. The precise physical modelling of the various aspects of DM detection will be described in Chapters 3, 4 and 5.

2.1 Background review

Before the modern understanding of the DM problem, there were a number of astronomers and mathematicians in the 19th and 20th centuries that observed behaviour which appeared anomalous to their understanding of the celestial bodies (Bertone et al. 2018, cha. II). Indeed, if there was an indication of a massive object that was undetectable in space, the object was often referred to as ‘dark matter’. However, these astronomical phenomena were often attributed to known objects that were just too dark to be observed, like dark stars or planets. In the early 1930s, the Swiss astronomer Fritz Zwicky produced a now-famous piece of work (Zwicky 1933; Andernach et al. 2017), which showed a large scatter in the line-of-sight velocities of galaxies within the Coma galaxy cluster, as measured by their redshift. When the virial theorem was applied to this cluster, it was found that the mass inferred from the mass-to-luminosity ratio was only 2% of the total calculated mass, signifying a significant amount of unaccounted-for dark matter. This is typically considered as the beginning of the modern DM problem in the literature, though it should be noted that there was a significant contribution to the field at this time from a host of other scientists, including Sinclair Smith, Jan Oort and Edwin Hubble, for example (Bertone et al. 2018).

Another prominent piece of historical literature is based on the measurement of the stellar rotation curves of galaxies. A particularly famous piece of work was authored by Vera Rubin and Kent Ford Jr in 1970 (Rubin et al. 1970) for the rotation curve of the Andromeda Galaxy, with further notable works that studied more galaxies coming in the following years (Rubin et al. 1978; Rubin 1983). The conclusion from these studies was that the orbital velocity of stars far from the central (and luminous) bulge of the galaxy were independent of radius, resulting in a ‘flat’ curve at higher radius. Since Newtonian gravity predicts a decrease in the stellar orbital velocity at large radii, there was again implication for some unseen contribution to the total mass of the galaxies. By this time, the consensus in the astronomical community was that there had to be some explicit component of missing matter, which was now based on a large body of work from analyses of astronomical observations (Bertone et al. 2018).

At roughly the same time as the above works, the theoretical effects of gravitational lensing were being discovered. The ability for matter to act as a lens for background photons, within the framework of the theory of General Relativity, provided another probe for finding potentially non-luminous matter. Since then, there have been a number of modern studies that, through observations of gravitational lensing, suggest a significant amount of DM in galaxies and galaxy clusters (Bergmann et al. 1990; Becker et al. 2004; collaboration et al. 2000; Tisserand et al. 2007). A result of observations of weak-lensing in the merging of two galaxy clusters 1E0657-558 published in 2006 (Clowe et al. 2006), known colloquially as the ‘Bullet Cluster’, is often cited as more strong evidence for the existence of DM in large amounts. In this cluster merger, the X-ray emitting

intracluster gas – which traces the dominant baryonic component of the cluster – has become separated from the centre of mass of each cluster determined through gravitational lensing, due to ram pressure of the intracluster gas (see Figure 2.1). This spatial separation was measured at a significance level of 8σ , and suggests that the centre of mass is following a large, unseen contributor to the gravitational potential. These observations are explained well by the presence of collisionless DM in the cluster, and the DM interpretation of this phenomenon is widely accepted in the literature.

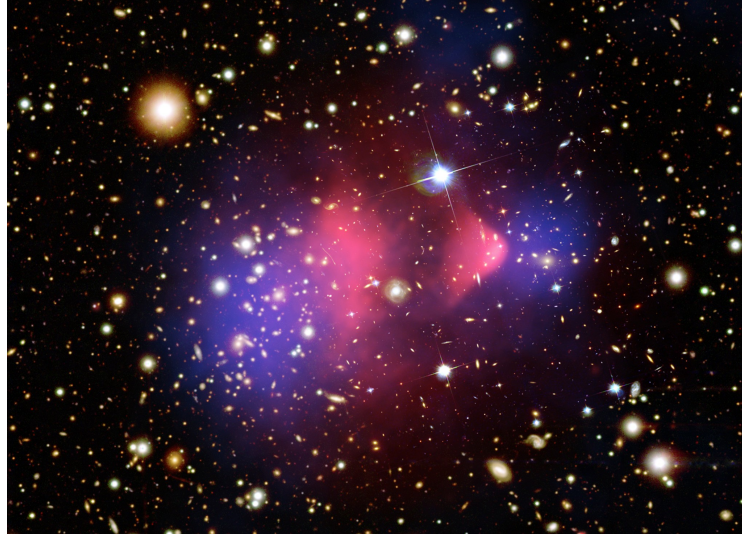


Figure 2.1: The famous composite image of the ‘Bullet Cluster’ (1E0657-558) galaxy cluster merger, taken from the Chandra photo album¹. This composite shows the diffuse central X-ray emission (pink) overlaid on the optical image which shows the distinct member galaxies (white/orange) of the merging clusters, and an estimate of the mass densities of the clusters found from lensing measurements (blue). As the X-ray emission traces the baryonic content of the region, the separation between the pink and blue regions in this image indicates the presence of a large, unseen gravitational component, which is typically attributed to DM.

¹<https://chandra.harvard.edu/photo/2006/1e0657/>

Some of the strongest evidence for DM has come from observations on a cosmological scale, which possess the additional feature of being able to constrain the total amount of DM the universe, compared to observations of localised targets. These observations have been enabled by the extraordinary precision of modern instruments, in particular the Planck space telescope (Planck Collaboration et al. 2020), which superseded the Cosmic Background Explorer (COBE) (Bennett et al. 1993), Wilkinson Microwave Anisotropy Probe (WMAP) (Spergel et al. 2003). Measurements of the temperature anisotropies present in the Cosmic Microwave Background (CMB) performed with the above instruments have shown that the amplitude of primordial fluctuations, which relate to the early state of the universe, are at the scale of a factor of 10^{-5} . By analysing the power spectrum obtained from the CMB anisotropies, a set of constraints on the cosmological parameters that govern the evolution of the universe can be obtained. From the Planck 2018 results (Planck Collaboration et al. 2020) for example, the total matter density parameter was determined to be $\Omega_m h^2 = 0.1432 \pm 0.0013$ (where h is the usual Hubble parameter and the factor Ω represents the fraction of total matter in the universe). However, the baryonic density parameter was determined to be $\Omega_b h^2 = 0.02236 \pm 0.00015$. This discrepancy is a clear indication for the existence of a matter component that is non-baryonic. Further, the ‘missing’ component needs to account for $\sim 84\%$ of all matter in the universe. As the results from these experiments have found no significant evidence for cosmological parameters that vary from the Λ CDM model of cosmology, the existence of cold DM in our universe is therefore well-established.

Another aspect of DM research, the theoretical simulation of the effects of DM through large N -body simulations, has helped to define our understanding of DM on a huge variety of physical scales. These simulations are ever-improving alongside the development of computer technology,

and modern simulations have been performed with trillions of particles in simulation volumes that span cubic Gpc scales. In particular, simulations of the large-scale evolution of structure in the universe have shown that the inclusion of cold DM models more accurately represents the observations from large galaxy surveys, such as the Sloan Digital Sky Survey (SDSS) (York et al. 2000) (or more recently those performed with the Baryon Oscillation Spectroscopic Survey (BOSS) (Tröster et al. 2020)), compared to simulations with the absence of DM (Springel et al. 2006; Garrett et al. 2011; Kuhlen et al. 2012). These results provide extremely useful information when modelling DM in various astrophysical scenarios, and help drive our understanding of DM in a complementary way to the other astrophysical pieces of evidence we have discussed.

When combining all pieces of modern observational evidence, the existence of some kind of non-baryonic DM in large quantities in the universe is strongly supported. There have thus been a number of proposed particle models that have the necessary physical traits to account for these effects, each with various theoretical motivations. For a comprehensive review of the proposed models, we refer the reader to the studies by Bertone et al. (2005, cha. 3), Bertone (2010, part II), Feng (2010), Garrett et al. (2011), and Bertone et al. (2018, cha. V). As the focus of the work in this thesis is on a particular candidate particle known generally as a Weakly Interacting Particle (WIMP), we continue by focusing our discussion on this candidate.

2.2 Are WIMPs the solution?

By far the most popular candidate particle model for solving the DM problem in the literature is the WIMP. This term could technically refer to any number of specific particle models, as it only defines a set of phenomenological aspects necessary to account for the observational evidence of DM. Of these, it must (i) be massive enough to explain observations of gravitational effects, (ii) interact weakly with regular matter (and without electromagnetic interactions), to explain the lack of detected photons from DM, and (iii) have been created in a sufficient number so that after thermal freeze-out their abundance can account for the current observed abundance of DM. These points are usually supplemented by a further requirement on the average velocity of the particles, with non-relativistic particles being referred to as ‘cold’, relativistic particles as ‘hot’, and those with intermediate velocities as ‘warm’. The current layout of large-scale structure in the universe (Springel et al. 2006) suggests that massive structures formed through a bottom-up process, which entails small objects collapsing first and then gradually merging with other collapsed objects to create complex structures. This scenario is only possible with cold WIMPs, as highly-relativistic particles would be able to stream further than the size of small-scale structures, prohibiting their growth.

There are a variety of specific particle models that conform to the above WIMP requirements. The most common models considered in the literature are usually members of extensions to the SM, particularly in the supersymmetric extensions like the Minimal Supersymmetric Standard Model (MSSM) (such as neutralinos, gravitinos, sneutrinos, axinos) (Bertone et al. 2018). However, more exotic models like Kaluza-Klein particles that are predicted in models that contain extra spatial dimensions, are also viable. (Feng 2010). In many studies in the literature, and in this thesis, WIMPs are considered as model-independent generic particles that have the required properties. For example, the DM cookbook (Cirelli et al. 2011) which is popular in the literature has computed a number of particle characteristics, like annihilation and decay pathways, for completely model-independent WIMPs.

The reason that generic WIMPs are considered favourites in the pool of candidate particles is due to theoretical motivations, and in particular a deficiency in the Standard Model (SM) of particle physics known as the Gauge Hierarchy Problem (Feng 2010). This problem is sometimes framed in terms of the discrepancy between the mass of the Higgs Boson, at 125 GeV, and the expected mass of dimensionful parameters in the case of electroweak symmetry breaking, which is the Planck mass of $\sim 1 \times 10^{19}$ GeV. A solution to this problem is the existence of particles at the weak scale, i.e. 10 GeV – 10 TeV, which also happen to fit the characteristics of the described WIMPs from astrophysical evidence. In the simple case of WIMPs being created thermally after the Big Bang, the number density (n) can be described by the Boltzmann equation (Feng 2010,

eqn. 5)

$$\frac{dn}{dt} = -3Hn - \langle\sigma v\rangle(n^2 - n_{\text{eq}}^2), \quad (2.1)$$

where H is the Hubble constant, n_{eq} is the thermal equilibrium number density of WIMPs, and $\langle\sigma v\rangle$ represents the thermally-averaged cross-section for WIMP annihilations. In the early primordial environment, the production of WIMPs through interactions of SM particles would be equalled by their own annihilations, so that WIMPs would be in thermal equilibrium with all other particle species. However, the expansion of the universe would cause the average temperature in this environment to drop below the WIMPs mass, which would suppress the creation of new WIMPs through SM interactions. At this stage, the number density of WIMPs in the expanded universe is low enough that their annihilations effectively cease, creating the constant density that has persisted until the current epoch. The annihilation cross-section is an important parameter in this process, and as it determines the rate of WIMP annihilations, it is the primary parameter of interest in indirect detection studies (discussed in the following Section).

2.3 Current status of the hunt

The attempt to detect a WIMP has been a huge scientific endeavour, and the current experimental landscape can be divided into three distinct classes. The first of these, and the most direct, attempts to search for indications of the scattering of a WIMP with a target nucleus. The primary parameter of interest in these studies is therefore the WIMP-nucleon interaction cross-section. The detectors that are sensitive enough to detect the low energy signatures of these collisions are usually embedded in large quantities of material that maximise any potential signals, such as liquid Xenon. They are also commonly built in underground facilities to reduce the background signals from non-DM scattering events. Some examples of state-of-the-art direct detection experiments are the PandaX-4T (Meng et al. 2021), LUX-ZEPLIN (LZ) (Aalbers et al. 2022), XENON1T (Aprile et al. 2018) or the recently expanded version XENONnT (XENON Collaboration et al. 2023), and a set of upper limits on the WIMP-nucleon cross-section determined in XENON Collaboration et al. (2023) have been displayed in Figure 2.2a. The results from the latest set of experiments are all relatively close, with the strongest limits coming from WIMP masses $\gtrsim 10$ GeV, with a steep increase in the limits below this value.

The second of these search techniques is known as collider detection, and relies on the creation of WIMPs in particle collider experiments through the scattering of highly energetic particles. Even though the detectors within these experiments would be unable to detect the created WIMPs, the creation process would result in missing energy in the detectors, which could be analysed to implicate a WIMP signature. There have been a number of WIMP searches at the LHC (see Kahlhoefer (2017) for a review), which also attempt to determine the WIMP-nucleon cross-section. In Figure 2.2b we show a set of recent upper limits on the WIMP-nucleon cross-section determined by ATLAS Collaboration (2022), using proton-proton collisions with the LHC and detected by the ATLAS experiment. The value of complementary detection schemes can be seen directly here, with the collider experiment providing strong limits, almost competitive with direct detection results, in the WIMP mass range that is only weakly probed by direct detection (i.e. $\lesssim 10$ GeV).

2.3.1 Indirect Detection

We now shift our focus to the detection scheme that is relevant for our work – ID. For reviews that contain excellent accounts of the various aspects this field, see Bertone (2010, part V), Cirelli (2013), and Gaskins (2016). The common thread between all DMID schemes is the search for DM that resides in astrophysical structures, and produces some kind of observable emission. In the case of WIMP DM, this emission is generated either as a result of WIMP-WIMP annihilations or decay of individual WIMPs that produce a set of SM particles. The detection, or so far non-detection, of these products then allows us to infer properties about the underlying DM models, which can act as useful complementary datasets to the alternative detection schemes and particle parameters already discussed. As the emission from these processes is stronger in regions where there is a

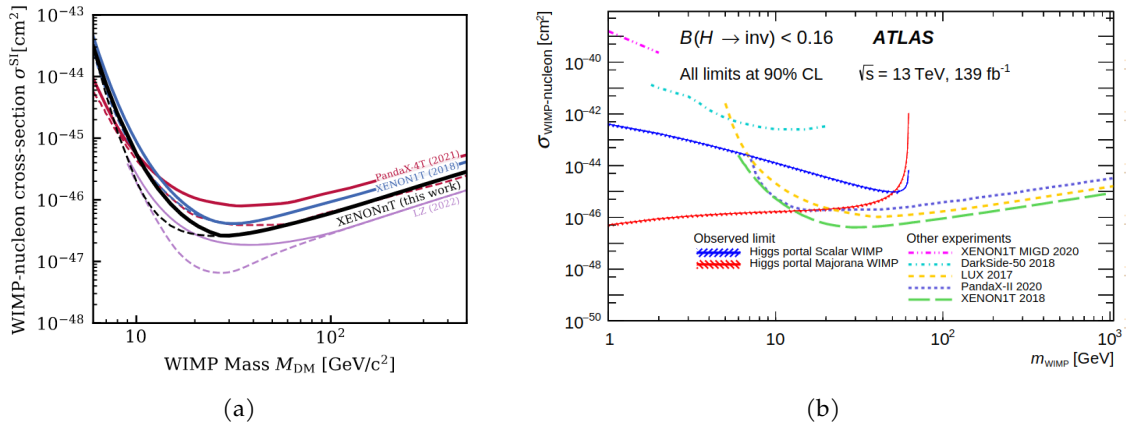


Figure 2.2: Examples of state-of-the-art results from direct and collider detection experiments, showing sets of upper limits (at 90% confidence level) on viable spin-independent WIMP-nucleon cross-sections. (a): Upper limits from studies of various direct detection experiments (see text for details). Figure taken from latest (2023) XENONnT results (XENON Collaboration et al. 2023, fig. A.4). (b): Upper limits found with proton-proton collisions at the LHC, detected by the ATLAS experiment. Also shown are some recent limits from direct detection searches, for comparison. Figure taken from latest (2022) ATLAS results (ATLAS Collaboration 2022, fig. 7).

larger abundance of WIMPs, astrophysical structures that are dominated by DM or contain dense regions of DM are generally favourable as search targets.

Another commonality in all ID schemes is that the defining parameter of the searches, the annihilation cross-section, is currently constrained by the thermal relic WIMP population that is required to reproduce the current observed cosmological abundance of DM (Steigman et al. 2012). The precise calculation of this cross-section in Steigman et al. (2012) results in a value of $\langle\sigma v\rangle \approx 2.2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ in the regions of WIMP mass that are generally considered in the literature, which is slightly – but importantly – different from the previously popular trend of using a canonical value of $\langle\sigma v\rangle \sim 3.0 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. This small value of the cross-section further motivates the use of search targets that contain high densities of DM, as low densities simply interact too infrequently to produce the necessary SM products for an observable signal on Earth. Since the expected emissions from WIMP annihilations or decay are directly proportional to this cross-section, it is often treated as a free parameter in ID studies. This treatment results in a set of curves within the $\langle\sigma v\rangle - M_\chi$ parameter space (where M_χ is the mass of the WIMP), which define the upper limit on $\langle\sigma v\rangle$ which is required to produce emission compatible with observational data. These curves can then be compared to the precise value of the thermal relic $\langle\sigma v\rangle$ value calculated in Steigman et al. (2012), and models that overpredict this value (based on some statistical confidence level) would be considered unable to produce the total current cosmological abundance of DM (this process will be described in further detail in Chapter 6).

As alluded to in Chapter 1, the field of DMID study has matured into a strong global search effort, spanning multiple wavelengths and astrophysical messengers. The most popular of these schemes in recent years searches for the prompt gamma-ray photons originating from WIMP annihilations. This popularity was driven by a wealth of gamma-ray data from observatories like the Fermi-LAT space telescope (Fermi/LAT Collaboration et al. 2009), and Imaging Atmospheric Cherenkov Telescopes (IACTs) such as H.E.S.S., MAGIC and VERITAS (H. E. S. S. collaboration et al. 2006; Sitarek et al. 2013; Park 2016), and the water Cherenkov array HAWC (Abeysekara et al. 2017). However, the use of gamma rays also has a major benefit, in that the expected emission from WIMP annihilations can be determined relatively simply compared to other wavelengths. The simplicity of gamma-ray emission is a result of the neutral high-energy photons being able to propagate directly from the source of the emission to the detectors, without complicated intermediary physical interactions with environments like magnetic fields. This allows the calculation of the expected emission to be neatly split into two factors that contain independent information

relevant to the WIMP particle models and the astrophysical environment. The separation of these two factors has allowed for simple estimates of the expected emission from various astrophysical targets in what are commonly referred to as J -factors, without the need for specifying information about the particle model, which can make gamma-ray ID studies easily accessible for analysis and comparison.

In gamma-ray ID studies, there are two principle astrophysical search targets. The first is the Galactic Centre (GC), which has had arguably the most attention of any DMID study in the literature. In particular, studies showed that observations of the region with the first two years of the Fermi space telescope were underspecified by models of astrophysical gamma-ray backgrounds, but could be compatible with an extra component of annihilating DM (Goodenough et al. 2009; Hooper et al. 2011). This led to a significant effort to characterise and analyse these observations in a DM context, represented by the following list of noteworthy papers (Calore et al. 2015b; Gordon et al. 2013; Calore et al. 2015a; Daylan et al. 2016; Ackermann et al. 2017; Karwin et al. 2017; Cholis et al. 2022). As shown in the work by Di Mauro (2021) and Cholis et al. (2022) for example, interest in this region is still strong. The main setback and cause for debate in this discovery has been the astrophysical background associated with the GC, and the accurate modelling that is needed to match the characteristics of the excess signal. The study by Di Mauro (2021) has placed a set of new constraints on various physical parameters and DM modelling scenarios for the DM interpretation of the excess using 11 years of Fermi-LAT data, although conclude that even more accurate modelling of the GC region is needed to improve the results. In Cholis et al. (2022), some new modelling techniques for this GC background emission were employed, using cosmic-ray data from the AMS-02 (Barao 2004) on board the International Space Station and the *Voyager 1* probe (Stone et al. 1977). The results of this detailed analysis confirm previous indications that the known abundance of millisecond pulsars around the GC are unable to independently explain the signal detected by the Fermi telescope (Zhong et al. 2020), and provide a set of best-fit DM parameters that constitute a DM interpretation of this signal. This result is shown in Figure 2.3a, which contains the best-fit contours for 1,2 and 3 σ limits of the DM interpretation, corresponding to a model of ($M_\chi \approx 40$ GeV, $\langle\sigma v\rangle \approx 1.4 \times 10^{-26}$ cm³ s⁻²) and annihilation into gamma-ray photons through the bottom quark channel.

The other prominent astrophysical search target in gamma-ray ID studies is of the satellite dSph galaxies of the MW (see Hooper et al. (2015) or Fermi-LAT Collaboration (2015) for an overview). The motivation for these sources are that they are extremely DM-dominated and contain little quantities of baryonic matter, which makes disentangling DM-induced gamma-ray signals from background sources much easier. There have been a multitude of studies that investigated gamma-ray emissions from these dSphs with instruments like Fermi-LAT (see Hoof et al. (2020) for a comprehensive list of references). In this thesis we draw particular attention to two recent studies: Hoof et al. (2020) and Armand et al. (2021), which are both noteworthy for their completeness compared to previous studies – they combine datasets for a number of different dSphs (27 and 20, respectively), and in the case of Armand et al. (2021), 5 different gamma-ray observatories (with the caveat that the published results are preliminary). The combination of datasets used in these works provide a robust analysis, which also leads to strong limits on the annihilation cross-section. However, it should be noted that recent work by Ando et al. (2020) has shown the use of updated priors on the DM halo density in dSphs, motivated by physical models of structure formation, leads to annihilation cross-section constraints that are up to a factor of 7 times weaker than when using the priors previously considered in the literature. An example set of results for DM annihilations through the bottom quark channel from Hoof et al. (2020) (the more detailed of the two publications) is displayed in Figure 2.3b, which also contain some comparisons to previous limits in the literature.

In general, the trend that emerges when considering the full set of gamma-ray ID result is that lower mass WIMPs, with the exact values dependant on the annihilation channel, are incompatible with the value of the thermal relic cross-section. The preferred parameter space from analysis of the GC excess is in tension with $\langle\sigma v\rangle$ upper limits from dSphs, though some uncertainty regarding the modelling of the DM density in these objects could resolve this tension. Due to the points mentioned before regarding the benefits of gamma-ray searches, and the sheer amount of analysis available in the literature, the constraints on the WIMP parameter space from these studies are strong, especially compared to other messengers like neutrinos or antiprotons. This can be seen in

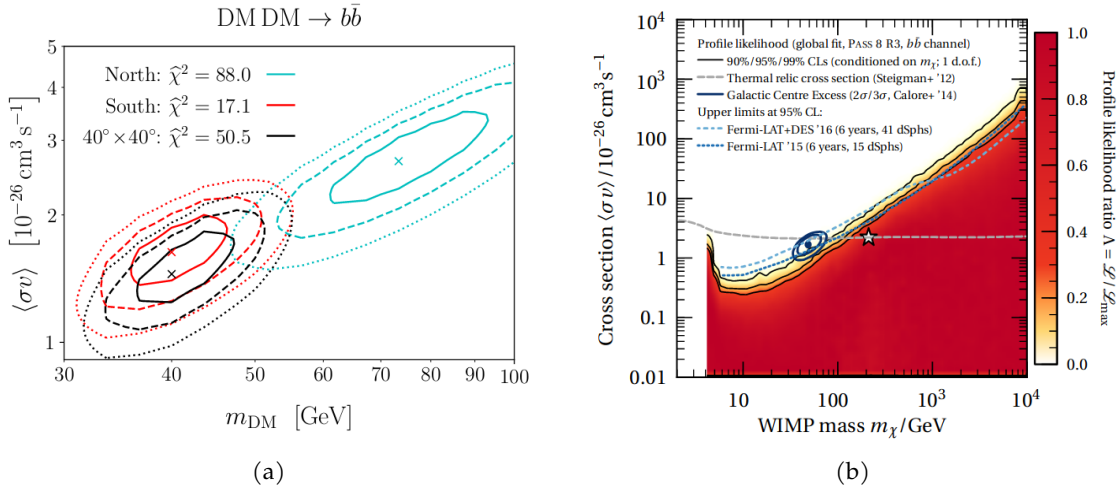


Figure 2.3: Examples of current state-of-the-art results from gamma-ray ID studies. (a): The preferred parameter space for WIMP annihilations into bottom quarks, given a DM interpretation of the excess GC gamma-ray signal. The solid, dashed and dotted lines correspond to 1, 2 and 3 σ contours, and the three colours correspond to different regions of interest considered in the data. The considered regions were a full $40^\circ \times 40^\circ$ area around the GC, and then that area confined to only the northern and southern hemispheres centred on the GC. Although the excess signal was found to be present in all regions, the energy spectrum of the excess signal differs between the north and south, which leads to the tension in the best fit values seen in the Figure. Figure taken from Cholis et al. (2022, fig. 18). (b): A set of upper limits, calculated using a profile likelihood technique, for WIMP annihilations into bottom quarks. The star represents the best-fit parameters, and several curves representing results from the literature have been included (shown on the figure). Figure taken from Hoof et al. (2020, fig. 9).

Figure 2.4, in which a few noteworthy results have been collated and compared to other messengers (see below). In this plot the results from the combined analysis of dSphs (Hoof et al. 2020; Armand et al. 2021) seem to produce the strongest constraints on the cross-section when compared to the thermal relic value, ruling-out WIMP masses below $\lesssim 100 - 200$ GeV. Note that for the sake of clarity, most of the uncertainty bounds in these results have not been displayed (with the more constraining cases selected here), and since there are many results for varying parameters (like analysis techniques, confidence levels and annihilation channels), we have chosen only compatible results as far as possible. Figure 2.4 is thus for comparative purposes and the observation of general trends only.

The use of other astrophysical messengers in ID studies, like neutrinos and cosmic rays, has seen significant advancement in the past several years, primarily due to the improving capabilities of detectors like Super-Kamiokande (SK) (Walter 2008), IceCube (Abbasi et al. 2012) and ANTARES (Maggiotta 2009), or the AMS-02 instrument (Barao 2004). In the case of neutrino-based ID, the neutrinos produced during the WIMP annihilation process (similarly to gamma-rays) propagate directly towards the Earth without significant interactions due to the intermediate environment. A popular source target in the literature for these searches is the DM halo within the MW itself, with a particular focus on the GC due to the increased density of WIMPs (Mijakowski 2020). However, there is also interest in the detection of neutrinos via the process of WIMP capture by astrophysical bodies such as the Sun or the Earth. In this process, WIMPs become bound in the gravitational potential wells of large bodies when they lose energy after scattering with resident nuclei within these objects, accumulating until their density is high enough to annihilate in sufficient numbers. Unlike other SM annihilation products, neutrinos would be able to escape these environments due to their own weakly-interacting nature, which would allow them to be detected on Earth (Mijakowski 2020). There have been several neutrino-based searches for WIMPs, most prominently at the SK, IceCube and ANTARES detectors (some large and noteworthy studies include Aartsen et al. (2017), Albert et al. (2020), and Abe et al. (2020)). Another interesting

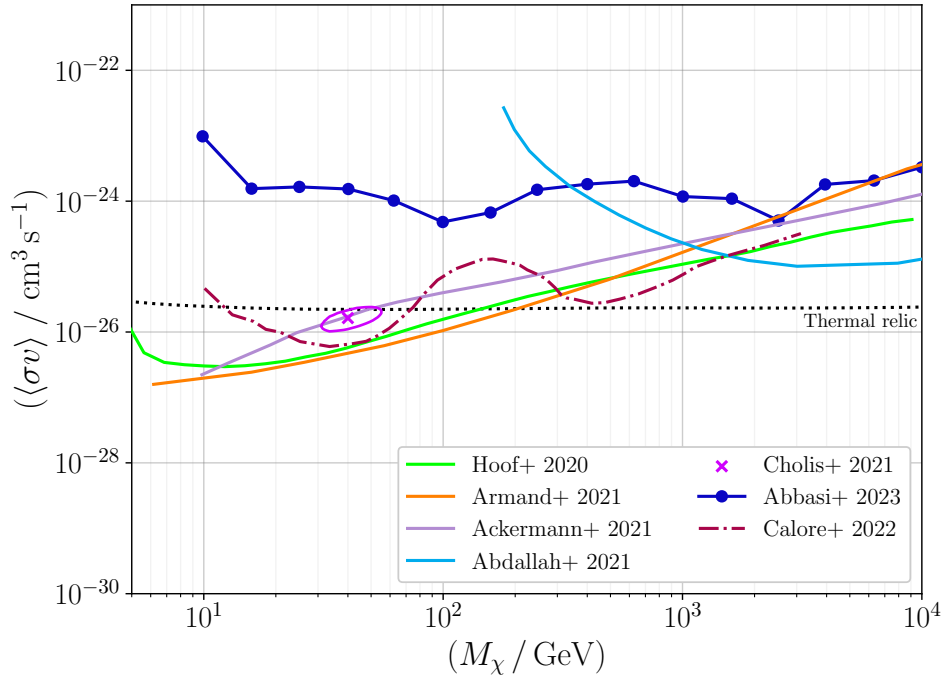


Figure 2.4: A non-exhaustive list of several noteworthy ID studies, and the constraints on the annihilation cross-section they have produced. The results shown here are limited to non-radio-based studies (particularly gamma-ray shown by the solid lines, neutrino by the connected scatter plot and antiproton by the dot-dashed line) that investigate the bottom quark annihilation channel, with the exception of those from Abbasi et al. (2023) which show a neutrino channel. The value of the thermal relic cross-section, calculated in Steigman et al. (2012), is shown as a dotted black line. For a corresponding summary of constraints from radio studies, see Figure 2.6. For information about each of the publications shown here, see Table 2.1 or the text.

detection scenario is possible with neutrinos, in that a monochromatic flux is expected from certain annihilation processes from cold WIMPs (see Abbasi et al. (2023) for details). As this monochromatic flux is not expected from any other known astrophysical processes, its detection would be a so-called ‘smoking gun’ detection, unambiguously implying a DM interpretation. With all detection scenarios considered however, the limits from neutrino-based ID are significantly weaker than other messengers. This can be seen in Figure 2.4, where we have plotted the current strongest limits from Abbasi et al. (2023, fig. 9) (note that we have used the results for the $\nu_e \bar{\nu}_e$ channel, as the results from the $b\bar{b}$ channel, shown in all other results, are roughly two orders of magnitude weaker). A summary of the limits obtained from a variety of neutrino experiments can be found in Figure 2.5a.

The last of the major astrophysical messengers discussed in this thesis outside radio ID studies is of cosmic rays, with the bulk of studies in the field typically investigating electrons/positrons and protons/antiprotons. Cosmic-ray ID is usually performed with space-bound particle detectors, like the afore-mentioned AMS-02 instrument and the specialised Dark Matter Particle Explorer (DAMPE) instrument (Chang 2014). The use of cosmic rays as astrophysical messengers is made more challenging than gamma rays or neutrinos by the fact that the charged particles are highly susceptible to interactions with ambient media, which makes the accurate modelling of their propagation difficult (see Chapter 4 for a discussion of relevant effects). Additionally, the modelling of background or primary sources of cosmic rays can have significant uncertainty, which makes the disentanglement of a pure dark matter signal from the background challenging, especially when using instruments with poor resolution or high systematic uncertainty (Heisig 2021). Nonetheless, there has been keen interest in this field in the literature, with the recent history of cosmic-ray ID searches being dominated by claims of excess detections which were accompanied

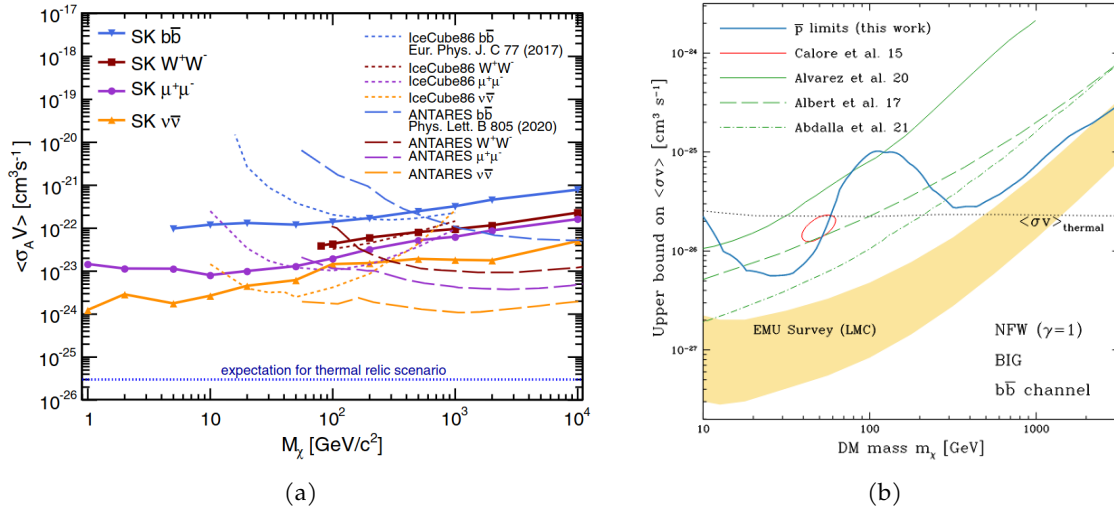


Figure 2.5: Current state-of-the-art results from multi-messenger ID studies, showing results from neutrinos, antiproton, gamma-ray and radio observations. (a): Upper limits on $\langle\sigma v\rangle$ at a 90% confidence level for a number of neutrino-based ID studies, using data from the SK, IceCube and ANTARES detectors. Figure taken from Abe et al. (2020, fig. 8). (b): Upper limits on $\langle\sigma v\rangle$ at a 95% confidence level using antiproton data from the AMS-02 instrument. Also shown are results from gamma-ray searches of the GC and a radio search of the LMC. Figure taken from Calore et al. (2022, fig. 3).

by DM interpretations. Two notable examples are of the excess antiproton ($\bar{p}$) signal discovered by the AMS-02 instrument (Aguilar et al. 2016) and the ~ 1.4 TeV excess flux of electrons/positrons discovered with the DAMPE instrument Ambrosi et al. (2017), with examples of the posited DM interpretations coming from Cholis et al. (2019) and Yuan et al. (2017), Yang et al. (2017), and Fan et al. (2018), respectively (also see Beck (2019a) for an analysis of these claims, specifically in the context of multi-wavelength DMID, or Cirelli (2013) for phenomenological aspects). In the case of the $\bar{p}$ excess, recent work in Calore et al. (2022) (based originally on earlier work in Giesen et al. (2015)) has shown that with updated modelling techniques and data analysis, the DM interpretation of this excess is unnecessary. This work has also produced a set of limits that are competitive with other ID messengers, seen in Figures 2.4 and 2.5b.

Radio searches

We now focus on the DMID scheme relevant to this work, which is based on the radio emission from WIMP annihilations. Unlike the messengers described above, the observable radio waves are not a result of prompt emission from WIMP annihilations, instead being radiated from physical interactions of the charged annihilation products. In ID studies the products are typically considered to be electrons and positrons, as these are produced more abundantly than hadronic final states in WIMP annihilations (Colafrancesco et al. 2006; Cirelli et al. 2011). In this case, the emission from the high-energy electrons is dominated by synchrotron radiation (Bertone 2010, cha. 27) which is produced when the electrons interact with a magnetic field. We have discussed all relevant physical interactions and characteristics of this process, from the creation of electrons by WIMP annihilations to the emission of synchrotron radiation, in detail in Chapters 3 and 4.

Some notable early works in the field of radio ID include Baltz et al. (1998) and Baltz et al. (2004), which were the first studies to provide a detailed account of the diffusion of the electrons produced by WIMP annihilations in the MW and its satellite galaxies, and their resulting emissions. Later, the studies by Colafrancesco et al. (2001) and Colafrancesco et al. (2006) expanded upon this work by creating a detailed picture of the potential multi-frequency nature of emissions from DM annihilations in the context of galaxy clusters. In particular, Colafrancesco et al. (2006) became a standard reference in radio-based ID studies, given the detailed mathematical description of

the radio emissions produced by annihilating WIMPs. An important factor in this work was the prescription for a semi-analytical solution to the electron propagation equation, commonly known as the diffusion-loss equation, which was necessary to model the spatial and energy distribution of the electrons and ultimately determine the synchrotron flux emitted from them. The presented solution method became the basis for a vast array of radio ID studies, and will be discussed in more detail later in the thesis. Alongside this solution method was an analysis of the primary physical effects involved in the equation, namely diffusion and energy loss, which showed the general behaviour and characteristics of each in various astrophysical scenarios.

Since then, the interest in radio-based DMID has grown substantially, with numerous studies being performed on different targets and with data from different observatories. Some recent review articles like Beck (2019b) and Chan (2021) contain comprehensive analyses of the literature, and in this section we focus on a few of the most recent and noteworthy results from various targets and instruments. Because of the significant gain in spatial resolution and sensitivity when using an array of radio dishes through interferometry (Thompson et al. 2017; Beck 2019b), the strongest results on the WIMP annihilation parameter space have come from array telescopes. The prominent observatories used in radio ID studies are the LOw Frequency ARray (LOFAR) (van Haarlem et al. 2013), the Australia Compact Telescope Array (ATCA) (Wilson et al. 2011), the Karl G. Jansky Very Large Array (JVLA) (Napier et al. 1983; Lane et al. 2014), and most recently the SKA precursor instruments MeerKAT (Jonas et al. 2018) and the Australia SKA Pathfinder (ASKAP) (Johnston et al. 2008); although observations from single dish instruments like the Green Bank Telescope (GBT) (Prestage et al. 2009), Giant Meterwave Radio Telescope (GMRT) (Swarup et al. 1991) or the Effelsberg telescope (Hachenberg et al. 1973) are also sometimes used.

In terms of observational targets, the most commonly studied objects have been the satellite dSphs of the MW, which benefit from the same characteristics discussed above in the context of gamma rays. The first noteworthy set of results to be obtained from dSphs were presented in Colafrancesco et al. (2007) for the Draco dSph (using VLA data and projections for LOFAR, but also considered through a multi-wavelength approach). The constraints found in this work were relatively weak, with even the most optimistic projections failing to probe the thermal relic cross-section. Further analysis of dSphs came from Spekkens et al. (2013) and Natarajan et al. (2013) using the GBT and VLA instruments. Although these results suffer from significant uncertainties in the modelling of the magnetic field, diffusive environment and DM halo within the considered targets (a common feature in searches of dSph targets), the thermal relic value was probed in some favourable modelling scenarios, for example in the case of annihilation through the e^-e^+ channel and strong magnetic fields. Further searches in six dSphs: Carina, Fornax, Sculptor, Hercules Segue 2 and Bootes II, were then carried out in the series of papers Regis et al. (2014), Regis et al. (2015a), and Regis et al. (2015b), with the limits on $\langle\sigma v\rangle$ presented in Regis et al. (2014, figs. 4-5). The shown limits again vary drastically between modelling scenarios, with the optimistic scenario strongly ruling out the entire WIMP parameter space, while the pessimistic scenario not getting close to probing the thermal relic value of $\langle\sigma v\rangle$. The average modelling scenario is able to reach the thermal relic value for a small range of WIMP masses ($\sim 20 - 40$ GeV) when annihilating through the electron channel, but not at all for the $b\bar{b}$ channel. These limits, in the case of an average modelling scenario, were improved by roughly an order of magnitude when the Reticulum II dSph was used as a search target, again with the use of ATCA data in (Regis et al. 2017). The results found in Regis et al. (2014) and Regis et al. (2017) are of particular relevance to this thesis, since they were the first to employ a numerical approach in the solution to the diffusion-loss equation, as opposed to the semi-analytical method popularised by Colafrancesco et al. (2006). This approach offers an improvement to the accuracy of the solution in terms of the spatial dependence of the electron population, and all aspects surrounding this choice of solution method will be discussed in detail in Chapter 4.

The recent study on the Canes Venatici I dSph in Vollmann et al. (2020) is noteworthy, as, like in Regis et al. (2014) and Regis et al. (2017), it forewent the usual solution method of the diffusion-loss equation. Instead, it employed a solution technique based on approximations of the diffusion environment in various ‘regimes’ that ultimately result in a universal profile for the expected synchrotron emissions in dSphs. The mathematical description of this method was presented in detail in a follow-up paper (Vollmann 2021), and is summarised in this thesis in Chapter 4 (it should also be noted that in Vollmann (2021) a correction was made to the commonly-used

semi-analytical method in Colafrancesco et al. (2006)). Again, uncertainties in the modelling of diffusion parameters lead the results in Vollmann et al. (2020) to vary significantly, with the average case displayed in Figure 2.6. The only other published set of results in the literature that use the ‘universal profile’ solution method are from Gajović et al. (2023), which used data from the LOFAR Two-metre Sky Survey (LoTSS) (Shimwell et al. 2022) to obtain upper limits on six dSphs, namely: Canes Venatici I, Ursa Major I, Ursa Major II, Ursa Minor, Willman I, and Canes Venatici II. Although the average modelling scenario presented therein results in limits that probe the $\langle\sigma v\rangle$ at low masses (with similar characteristics to the results in Vollmann et al. (2020)), the modelling of the DM halos in this work was done with a cuspy DM density profile. Cuspy density profiles are currently disfavoured for the modelling of dSphs, based on the analysis of simulations which instead suggest these halos have flatter central densities (see Chapter 3 or de Blok (2010)).

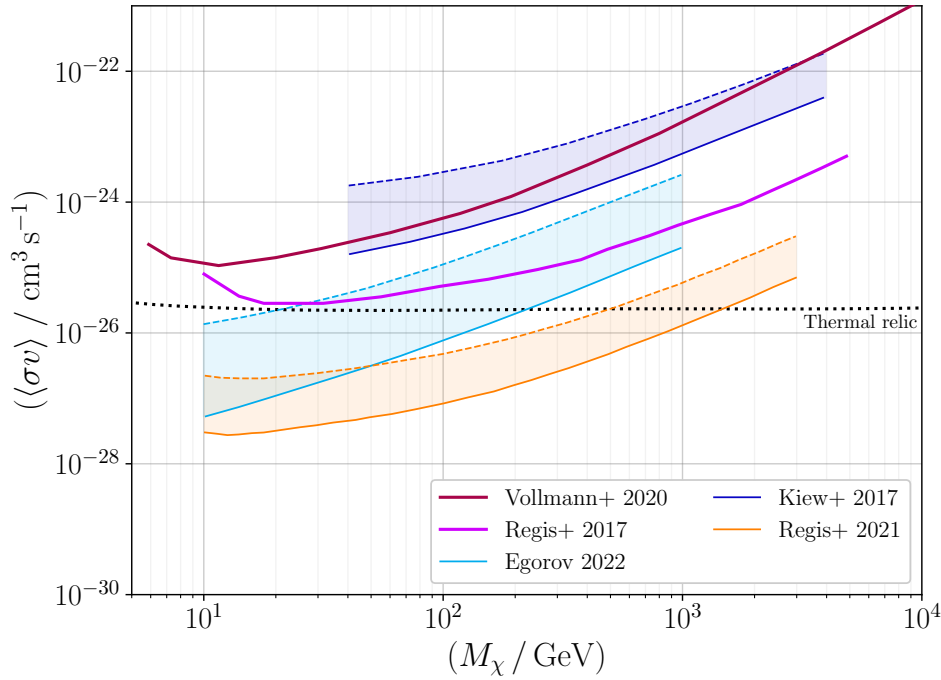


Figure 2.6: A non-exhaustive list of several noteworthy ID studies, and the constraints on the annihilation cross-section they have produced. The results shown here are limited to radio-based studies that investigate the bottom quark annihilation channel. The value of the thermal relic cross-section, calculated in Steigman et al. (2012), is shown as a dotted black line. Similar to Figure 2.4. For information about each of the publications shown here, see Table 2.1 or the text.

Although less common, notable results have come from search targets other than dSphs. The first of these is Egorov (2022), which presented updated limits on $\langle\sigma v\rangle$ from the M31 galaxy from an earlier work (Egorov et al. 2013). In this work, a variety of radio datasets were used, including survey data from the LoTSS and the VLA Low-frequency Sky Survey Redux (Lane et al. 2014), as well as data from the Effelsberg telescope and the VLA. This work was also the first to solve the diffusion-loss equation in two spatial dimensions, which required a detail modelling of the magnetic field in the spiral galaxy, and was carried out with a numerical implementation of the solution to the diffusion-loss equation. This implementation involves the adaptation of the GALPROP galactic cosmic-ray propagation solver code package, which is discussed in detail in Chapter 4. We also note that an error in the calculation of the energy loss rate due to bremsstrahlung radiation, common in many previous studies, was corrected in this paper (discussed further in Section 3.2.2). The resulting limits from this analysis produce strong constraints on the $\langle\sigma v\rangle$, which are plotted in Figures 2.7a and 2.7 and rule out the thermal relic cross-section for WIMP masses $\lesssim 70$ GeV for annihilation through the $b\bar{b}$ channel and the median modelling scenario.

Another notable result has been presented in Regis et al. (2021), which was the first study

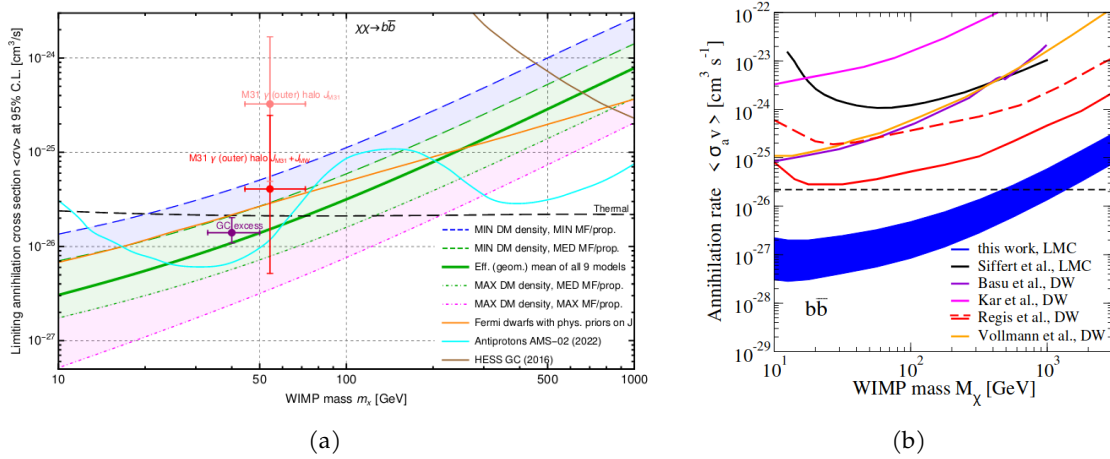


Figure 2.7: Current state-of-the-art results from radio-frequency ID studies, showing 95% confidence levels on the upper limits of $\langle\sigma v\rangle$ when annihilation occurs through the bottom quark channel. (a): Upper limits found from various radio observations of the M31 galaxy. The uncertainty bands represent variations in the considered modelling scenario. Also shown are select results from other studies. Figure taken from Egorov (2022, fig. 10). (b): Upper limits found from observations of the LMC performed by the EMU survey, with the ASKAP instrument. Uncertainty bands represent results found using different DM halo density profiles. Also shown are select results from other studies. Figure taken from Regis et al. (2021, fig. 4).

to use data from an SKA precursor instrument, ASKAP, in an ID search of the Large Magellanic Cloud (LMC). This data was obtained through the Evolutionary Map of the Universe (EMU) survey programme (Norris et al. 2021), and has resulted in the strongest constraints on the thermal relic cross-section yet obtained, for any DMID study. The upper limits on $\langle\sigma v\rangle$ from this work, which even in the least constraining case rule out the thermal relic cross-section for WIMP masses $\lesssim 480$ GeV, are shown in Figures 2.7b and 2.7. The large improvement in the constraining power of these results were attributed to the more sensitive radio observations from ASKAP compared to ATCA, as well as an improved statistical analysis. The statistical analysis adopted in Regis et al. (2021), which entails a detailed comparison of two-dimensional image data to the model, is similar to the one used in this thesis, which will be discussed in detail in Chapter 6.

The final observational target used in radio-based ID is of galaxy clusters, which have been comparatively less common in recent literature (outside our own work) (Storm et al. 2013; Storm et al. 2017; Kiew et al. 2017; Chan et al. 2020). Of these studies, those in Storm et al. (2013) and Kiew et al. (2017) have considered a set of galaxy cluster targets (11 and 6, respectively) and used a variety of archival radio datasets and upper limits on the fluxes observed in radio halos to find upper limits on the annihilation cross-section. The resulting upper limits on $\langle\sigma v\rangle$ obtained in these studies are noticeably weaker than other observation targets, with the slightly stronger limits from Kiew et al. (2017, fig. 3) shown in Figure 2.6. In Chan et al. (2020), a search using radio data from the GMRT of the Abell 4038 cluster (Kale et al. 2012) is made, wherein they find a best-fit value of $M_\chi = 40$ GeV for annihilation through bottom quarks. A feature of the results in Storm et al. (2013) and Chan et al. (2020) is that the effect of substructure in the DM halo (Zavala et al. 2019) significantly improves the limits; however with the simplified modelling scenarios used for the DM halo density (especially in Chan et al. (2020)) and the lack of observational evidence for the substructure models considered, this could be considered as an extra source of uncertainty in the results. Finally, the study performed in Storm et al. (2017) produces a set of predicted upper limits on $\langle\sigma v\rangle$ based on the projected instrument capabilities of several telescope arrays, namely: ASKAP (through the EMU programme), LOFAR and the APERTIF array (Oosterloo et al. 2009), for a set of generic galaxy cluster models. The upper limit results probe the thermal relic annihilation cross-section at some WIMP masses for most of the considered annihilation channels, but are not as constraining as the results from M31 or the LMC (Egorov 2022; Regis et al. 2021). We have discussed the results from the above sources, in the context of the constraints found on

the cross-section in this work, in Chapter 6.

The overall picture of radio-based ID studies is therefore a promising one, given the stringent constraints on the annihilation cross-section obtained with several recent studies. However, there is also a prevailing trend of large uncertainty in these results, due either to underspecified modelling scenarios of the diffusive environment or a lack of sophistication in the modelling of DM halos. With the availability of high-quality interferometric radio data from SKA precursors like MeerKAT and ASKAP, and eventually from the SKA itself, accurate solutions to the diffusion-loss equation that are supplemented by accurate models of the diffusive environment will be especially important factors to consider for reliable results.

Reference	Observational target	Instruments	Messenger
Kiew et al. (2017)	Galaxy Clusters	Misc.	Radio
Regis et al. (2017)	Reticulum II dSph	ATCA	Radio
Vollmann et al. (2020)	Canes Venatici I dSph	LOFAR (LoTSS DR2)	Radio
Egorov (2022)	M31	LOFAR, VLA, Effelsberg	Radio
Regis et al. (2021)	LMC	ASKAP (EMU)	Radio
Abdallah et al. (2016)	GC	H.E.S.S.	Gamma-ray
Ackermann et al. (2017)	GC	Fermi-LAT	Gamma-ray
Hoof et al. (2020)	27 dSphs	Fermi-LAT	Gamma-ray
Armand et al. (2021)	20 dSphs	Misc. (see text)	Gamma-ray
Cholis et al. (2022)	GC (best fit)	Fermi-LAT	Gamma-ray
Abbasi et al. (2023)	MW	IceCube (DeepCore)	Neutrinos
Calore et al. (2022)	MW	AMS-02	Antiprotons

Table 2.1: Description of the highlighted DMID studies used in Figures 2.4 and 2.6.

Physical Modelling

A core component of indirect detection studies is the correct prediction of the emissions that are produced by DM interactions. This typically involves choosing a particular DM candidate particle model, which determines the interaction rate and output spectra of individual particles, and a halo model, which describes the overall structure and distribution of virialised clumps of DM. For some indirect detection studies (those that investigate only the prompt emission of gamma rays, for example), the above factors can be sufficient to determine the expected emission, if the products of the DM interactions do not interact with ambient astrophysical environments. The modelling of radio emission, however, involves various physical processes that affect the DM products and can entail additional calculations of magnetic fields, thermal gas populations, particle diffusion and energy losses. In the following Chapter, we present the complete radio emission modelling process used in this project, from the DM particle candidates to the expected observable emission here on Earth. However, the solution of the diffusion-loss equation, which is also a prominent part of the modelling process, requires a more comprehensive description and is the sole topic of Chapter 4.

3.1 Dark matter

3.1.1 Particle models

In the vast sea of viable, theoretically-motivated DM particle models, many of the most popular candidates have been studied extensively by large interdisciplinary teams of particle and astrophysicists. The phenomenology of these models has thus been captured in software that is available to the community, making the modelling of otherwise complicated particle interactions and processes relatively easy to perform. Notable software packages in this respect (among others) are the `micrOMEGAS` (Belanger et al. 2002) and `DarkSUSY` (Gondolo et al. 2004) packages (the latter of which is also used by the `RX-DMFIT` package, which is relevant to this work and will be discussed in Detail in the next Chapter). An alternative to these computational solutions, which has become popular for DM indirect detection studies in recent years, is the Poor Particle Physicist Cookbook for Dark Matter Indirect Detection (PPPC4DMID) (Cirelli et al. 2011). This cookbook provides pre-computed numerical tables of particle spectra (ingredients), functions for computing the propagation of charged annihilation products through the galaxy (recipes), and even results of flux calculations that are ready for consumption. All of the relevant tables and results are hosted in an online repository¹, which is where the relevant material for this project has been obtained.

The aspect of the PPC4DMID we are most interested in here is the energy spectrum of particles produced during DM self-annihilations (which are denoted hereafter as DM-DM or $\chi\chi$). The form of this spectrum is usually taken to be dN_i / dE , where N_i is the multiplicity, or number of produced particles, and E is their corresponding energy. The subscript i represents the stable SM particle that is the *final state* of the annihilation, i.e. $i \in \{e^+, e^-, \gamma, p, \bar{p}, \nu, \bar{\nu}, \bar{d}\}$. The final state can be reached through various intermediate *channels*, denoted by f , through the general reaction

¹Website homepage: <http://www.marcocirelli.net/PPPC4DMID.html>, Release 6.0

pathways of $\chi\chi \rightarrow f \rightarrow i$.² A large set of channels, given by

$$\begin{aligned} f = \{ & e_L^+ e_L^-, e_R^+ e_R^-, \mu_L^+ \mu_L^-, \mu_R^+ \mu_R^-, \tau_L^+ \tau_L^-, \tau_R^+ \tau_R^-, q\bar{q}, c\bar{c}, b\bar{b}, t\bar{t}, \gamma\gamma, gg, \\ & W_L^+ W_L^-, W_T^+ W_T^-, Z_L Z_L, Z_T Z_T, hh, \nu_e \bar{\nu}_e, \nu_\mu \bar{\nu}_\mu, \nu_\tau \bar{\nu}_\tau, \\ & VV \rightarrow 4e, VV \rightarrow 4\mu, VV \rightarrow 4\tau \} \end{aligned} \quad (3.1)$$

has been used in the PPPC4DMID, where $g = u, s, d$ denotes light quarks, the subscripts L, R and L, T refer to the polarisation of each lepton (*Left, Right*) or vector boson (*Longitudinal, Transverse*), and V represents a new light boson in some proposed DM models (Pospelov et al. 2009; Arkani-Hamed et al. 2009). Although the channels shown in Equation (3.1) also undergo a range of interactions before reaching the final state, they are simply used and referred to in the literature by the labels of their primary products, as in Equation (3.1). It has also become common practice for indirect detection studies to only consider a subset of the channels shown above, usually selecting one channel from a group with similar particle characteristics, whose spectra are then taken as representative of the wider set. Further, since these channels represent the particle pairs that are produced directly from the DM-DM collision, only channels with a total mass less than 2 times the individual DM particle mass are viable within a certain DM model.

The energy spectra in PPPC4DMID were calculated using two Monte Carlo event-generator programs, PYTHIA (Sjöstrand et al. 2008) and HERWIG (Corcella et al. 2001), which simulate the parton showers and hadronisation processes that are necessary for the production of SM particles in a DM-DM collision event. While both programs perform the same function, they do so using independent techniques, which allows Cirelli et al. (2011) to compare the results of each and obtain uncertainty estimates related to the Monte Carlo method itself. For further details regarding the difference between these programs, we refer the reader to Cirelli et al. (2011, sec. 3.2), and note that the final spectra in the cookbook are found using the PYTHIA code. We also explicitly make note of the presence of electroweak radiative corrections in the cookbook results, applied by the authors following the prescription in Ciafaloni et al. (2011). These corrections can have a large impact on the final energy spectrum, changing both the multiplicity and the set of SM particles available in the final state. This effect is especially significant when the energy of the particles is much larger than the typical electroweak scale (relevant for DM particle models with large masses), and is due to the radiation of electroweak gauge bosons from the highly energetic products of the DM-DM annihilations (for further details about these electroweak interactions, we refer the reader to Ciafaloni et al. (2011)).

An example set of results from the PPPC4DMID, showing the final energy spectrum, is shown in Figure 3.1. We download these quantities directly from the online repository, and import them into our modelling pipeline after some minor unit conversions. A large emphasis has been placed on the model independent nature of these results, through both the event generators and the electroweak corrections in Ciafaloni et al. (2011), which allows us to very conveniently use the full set of results from the cookbook. The models considered in this work, still of a generic WIMP nature, will then be parameterised by the annihilation cross-section ($\langle\sigma v\rangle$) and the WIMP mass (M_χ). The energy spectrum, in its most general form, will be a combination of the spectra from all relevant intermediate channels f , i.e.

$$\frac{dN_i}{dE} = \sum_f B_f \frac{dN_{f \rightarrow i}}{dE}, \quad (3.2)$$

where B_f represents a branching ratio that determines the overall contribution from each channel. In indirect detection studies, it is standard to consider results for individual channels, which implies setting $B_f = 1$ and selecting a single channel from the set in Equation (3.1). Following the points made above, we consider 3 channels in this work: bottom quarks ($b\bar{b}$), muons ($\mu^- \mu^+$) and tau leptons ($\tau^- \tau^+$). In terms of the final state i , we consider both electrons (e^-) and positrons (e^+), which have effectively identical behaviour for the processes relevant in our modelling. Henceforth,

²The notation of these quantities in the literature has been defined in a number of different ways, so we choose to generally follow the notation of Cirelli et al. (2011) in this Section.

we may refer to the final state as just ‘electrons’, but account for both products in the calculations. The relevance of this choice of final state, in the context of radio emissions, will be made clear in Section 3.2. Then, with a given DM model ($M_\chi, \langle\sigma v\rangle$) and intermediate channel, we define a source function, $Q(\mathbf{x}, E)$, which describes the continual production of final state electrons in a region of annihilating WIMPs, as follows:

$$Q(\mathbf{x}, E) = \mathcal{N}_\chi(\mathbf{x}) \langle\sigma v\rangle \frac{dN_{e^\pm}}{dE}. \quad (3.3)$$

In this equation the factor $\mathcal{N}_\chi(\mathbf{x})$ represents the number density of WIMPs which can annihilate in the region of interest. So far we have discussed the case of annihilating WIMPs, which is the relevant phenomenon for our work. However, there also exists the possibility that individual WIMPs decay with a finite lifetime, with the decay chain $\chi \rightarrow f \rightarrow e^\pm$. In this case, the annihilation cross-section in Equation (3.3) should be replaced by the decay rate Γ for the respective channel (as we will see in Chapter 6, these quantities are usually treated as the free parameters in the model to be constrained through observation). The relevant WIMP number density is then simply found by

$$\mathcal{N}_\chi(\mathbf{x}) = \frac{1}{2} \left(\frac{\rho_\chi(\mathbf{x})}{M_\chi} \right)^2 \quad (\text{annihilation}), \quad (3.4)$$

$$\mathcal{N}_\chi(\mathbf{x}) = \left(\frac{\rho_\chi(\mathbf{x})}{M_\chi} \right) \quad (\text{decay}), \quad (3.5)$$

where ρ_χ is the DM density, discussed further in Section 3.1.2. The factor of 1/2 in the annihilation term assumes that the WIMPs are Majorana fermions, allowing all particles to annihilate with each other. If they are instead treated as Dirac fermions, which would only permit particle-antiparticle annihilations, this factor would change to 1/4.

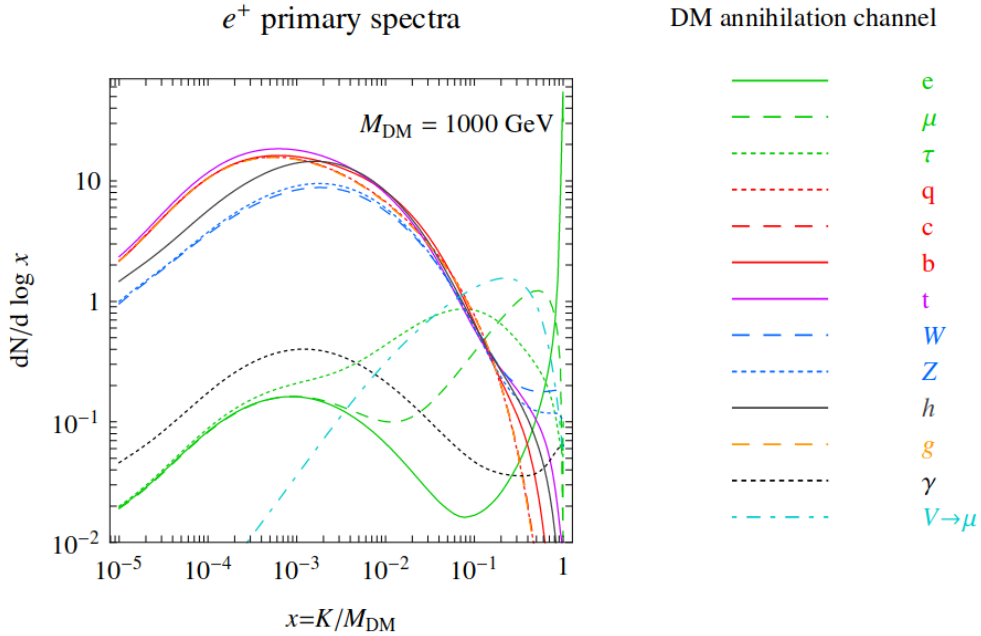


Figure 3.1: Example energy spectrum of positrons produced during a single DM-DM annihilation, via different intermediate channels. In Cirelli et al. (2011), quantities are expressed in terms of the dimensionless energy fraction $x = K/M_\chi$, where K is the kinetic energy of a single DM particle of mass M_χ , so the expression $dN_f/d \log x$ represents the energy spectrum of the produced final state particles. This plot has been normalised to the average multiplicity in the sample. Figure from Cirelli et al. (2011, fig. 3).

2HDM+S and the Madala hypothesis

In the above section we have described the particle modelling scenarios that are usually considered in indirect detection literature. Here, we briefly outline the phenomenology of a relatively new model, which also provides a DM candidate particle. The model is based on a generic extension to the SM, known as the two-Higgs Doublet Model (2HDM) (proposed in Lee (1973), see also Branco et al. (2012)), which simply introduces the possibility of a second doublet of the Higgs field. This type of model is well-motivated by some theoretical deficiencies of the SM, and an extra Higgs doublet is necessary in some models of supersymmetry or models that contain axions. A specific type of 2HDM model, which is known as a Type-II 2HDM (the most similar to supersymmetric models, and popular in the literature) that also permits the existence of a new Higgs-like particle, S , and a DM candidate particle, χ , was originally proposed in von Buddenbrock et al. (2015), and then later expanded upon in von Buddenbrock et al. (2016). The addition of S to the generic 2HDM has motivated the naming convention of ‘2HDM+S’ for this model, which we adopt for this thesis. The model has also been referred to as the *Madala* hypothesis in the literature, which is a word in the isiZulu language that translates to ‘old person’. This name is a reference to the extra Higgs scalar (denoted H , the *Madala boson*) that is introduced by the second Higgs doublet, since H is conventionally heavier (due to being ‘older’) than the SM Higgs (von Buddenbrock 2017).

An observable signature of the 2HDM+S model is the production of a number of leptons in particle colliders that would be considered anomalous, or in excess, of SM theories. In this regard, the authors of von Buddenbrock et al. (2016) found that the relevant parameters of the 2HDM+S model were compatible with data gathered from Run 1 of the CMS and ATLAS experiments in the Large Hadron Collider (LHC) (The CMS Collaboration et al. 2008; Aad et al. 2008). Then, after analysis of further Run 2 data, the model parameters were constrained, but still viable (von Buddenbrock et al. 2019) (see also von Buddenbrock et al. (2017), von Buddenbrock (2017), and von Buddenbrock et al. (2018)). Of these constraints, we only focus on a few that are most relevant to this work. In particular, we follow the general analyses of Beck et al. (2023b) and Beck et al. (2017), which were the first to investigate the possibility of using this new model in indirect DM searches. In these works, a fixed mass of the Madala boson of $m_H = 270$ GeV and the scalar S of $m_S = 150$ GeV are used. Since the χ particle is still unconstrained by LHC data, results were found for a range of possible masses. However, as the particle χ is assumed to couple directly to S , there are still some theoretical limits on its mass which are determined by the associated interaction chains. In the cases of the χ particle having spin of 0, 1/2, or 1, the Lagrangians that define these interactions are given by (Beck et al. 2023b, eqs. 7-9):

$$\begin{aligned}\mathcal{L}_0 &= \frac{1}{2}M_\chi g_\chi^S \chi\chi S, \\ \mathcal{L}_{1/2} &= \bar{\chi}(g_\chi^S + ig_\chi^P \gamma_5)\chi S, \\ \mathcal{L}_1 &= g_\chi^S \chi^\mu \chi_\mu S.\end{aligned}\tag{3.6}$$

In these expressions the g_χ^S factor represents the strength of the scalar coupling between S and χ , g_χ^P is the strength of the pseudo-scalar coupling, and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, where i is the complex unit and the γ 's are the standard gamma (or Dirac) matrices. We note that in the hypothesis that the DM candidate is a vector boson, it would need to interact with the Higgs/Scalar field to acquire its mass, in a similar manner to other SM vector bosons. Without these interaction terms in the Lagrangian, a vector DM candidate could break the symmetry that gives mass to vector bosons. Any potential interactions with the Higgs field in different particle hypotheses would be specified by the underlying particle physics framework, and are outside the scope of this thesis (for further details on vector DM, see for example Arcadi et al. (2020)). For this work, we consider the following processes (similar to the annihilation channels described in the Section above) defined in Beck et al. (2023b):

$$\chi\chi \rightarrow S \rightarrow e^{+,-}\tag{3.7}$$

and

$$\chi\chi \rightarrow S \rightarrow HS/h \rightarrow e^{+,-}.\tag{3.8}$$

Clearly, for the above reactions to be possible, the mass of χ needs to obey $M_\chi > 1/2 m_S = 75 \text{ GeV}$ in Equation (3.7) and $M_\chi > 1/2 (m_H + m_h) \sim 200 \text{ GeV}$ for Equation (3.8). The energy spectra from these channels, in a similar manner to the PPPC4DMID, were then found using Monte Carlo event generators and presented in Beck et al. (2023b, figs. 1, 2). Specifically, the MG5_@MC (Alwall et al. 2011) tool was interfaced with the PYTHIA package to produce results for a range of χ masses and final states. An example spectrum for positrons, most relevant to this work, is shown in Figure 3.2. These results use the reaction in Equation (3.7), with a spin-0 χ particle. In the full set of results, the difference in the spectra for different spin parameters of χ are not significant, but the process in Equation (3.7) leads to spectra that are several orders of magnitude larger than those found from Equation (3.8). Thus, the use of the 2HDM+S model in this work is restricted to the case of a spin-0 χ particle annihilating through the process in Equation (3.7).

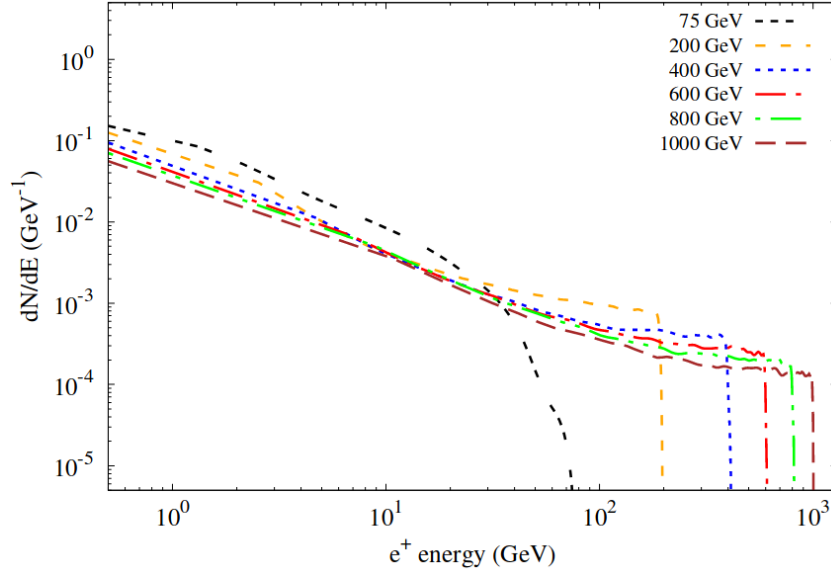


Figure 3.2: Example energy spectra of positrons produced during a single DM-DM annihilation, with varying χ mass. These spectra are found through the annihilation process described in Equation (3.7), for a spin-0 χ particle. Figure from Beck et al. (2023b, fig. 1).

3.1.2 Halo models

With the characteristics and behaviour of the individual DM particles set, we then need to model the distribution of these particles in the regions we are interested in. There is substantial evidence to suggest that DM particles have a tendency to clump together and form gravitationally-bound *halos*, coming from both analytical arguments (see for instance Silk et al. (1993)) and from a host of large N -body simulations (see Kuhlen et al. (2012) or Vogelsberger et al. (2020) for excellent reviews). Since the properties of these halos – including their formation histories, inner structures and mass distributions – have been the topic of several dedicated studies in recent years, the current paradigm of DM halo modelling has become quite robust. In this Section we will provide a brief outline of some pertinent aspects of DM halo modelling, and present the specific equations that are used in this work. For more comprehensive summaries or reviews on this topic, we refer the reader to the following works: Navarro et al. (2010), Bertone (2010, ch. 2), Kuhlen et al. (2012), Zavala et al. (2019), and Vogelsberger et al. (2020).

In this work, only the formation of Cold, non-relativistic DM (CDM) halos is considered, as the particle models used here all behave like CDM. We note that although other DM scenarios, like Warm DM (WDM) or Self-Interacting DM (SIDM), can also lead to the formation of halos, there can be significant departures from CDM halo characteristics (especially in the case of SIDM) (Kuhlen et al. 2012; Zavala et al. 2019). The majority of these halo characteristics have been sourced from analyses of modern N -body simulations, which are run on high-performance supercomputers

utilising thousands of CPU cores and taking millions of CPU hours. These simulations usually track an ensemble of N particles (consisting of dark or baryonic matter, or both) from the very beginnings of structure formation, seeded by tiny density perturbations that grow with inflation and produce large self-gravitating halos of DM, up to the current epoch. Treating the evolution of these particles in a numerical fashion also allows the non-linear regimes of standard structure formation theories to be more accurately probed, avoiding many of the assumptions and limitations of analytical methods. Many such simulations have been performed over the past three decades, and a complete review of them could be the subject of an entire thesis. Since we are only focused on a few very particular outcomes from these simulations, we provide just a brief list of some of the more notable projects, which can serve as a starting point for further reading in this subject.

The most long-standing of these simulation projects is the Millennium project, which, in its various forms, has been running since 2005. The inaugural simulation, now known as Millennium I (Springel et al. 2005), tracked the evolution of 10 billion DM particles in a $(685 \text{ Mpc})^3$ volume. Since then, the Millennium II, XXL and most recent MillenniumTNG simulations (Boylan-Kolchin et al. 2009; Angulo et al. 2012; Hernández-Aguayo et al. 2023) have all improved either the number of simulated particles (up to over a trillion with the inclusion of neutrinos) or the simulated volume (up to $(3000 \text{ Mpc})^3$). The DEUS FUR (Dark Energy Universe Simulation Full Universe Run) (Alimi et al. 2012) is another notable project, with 550 billion simulated particles and three different dark energy models in an enormous $(29 \text{ Gpc})^3$ volume that spans the entire observable universe. Finally, the PKDGRAV3 code (Potter et al. 2017) has performed a 2 trillion particle cosmological simulation that makes use of extremely fast and efficient computational techniques that should scale with improvements to particle number, ensuring the viability of larger and more detailed simulations in the future.

The results of these CDM simulations on cosmological scales also agree well with observational surveys of large-scale structure in the universe (Springel et al. 2006), which reinforces the notion that the large-scale dynamics of structure in the universe behaves in a manner consistent with our standard Λ CDM cosmology. However, a major limitation of older, large-scale N -body simulations is that they lacked the spatial resolution to probe the small-scale structure of individual DM halos. Though, this problem has diminished in recent years with larger simulations and more dedicated studies. In this work, the structure of individual halos is vital to the accurate modelling of expected DM-related emissions, and the DM halo density (which appears in Equation (3.4)) plays a particularly important role. Throughout the rest of the thesis a large emphasis is placed on the very inner regions of DM halos, since these regions contain the largest density of DM and thus produce the strongest observable emissions.

Before describing the modelling of halo density profiles, we note that descriptions of the physical properties of DM halos – their spatial boundaries and points of interest, enclosed mass, etc. – depend entirely on the mathematical definitions of these properties. Real halos would be clumps of virialised particles with no distinct border and potentially complex three-dimensional geometry. In this work, we make use of very common definitions in the literature, the first of which is an assumption that the halos are spherically symmetric and can be described by a single radial parameter. Evidence from some N -body simulations suggest that halos would very likely have triaxial shapes (Dubinski et al. 1991; Bett et al. 2007), with prolate central regions and oblate outer regions. However, since spherical density profiles still approximate the simulations quite well (discussed more below), a spherical approximation should not affect the final results significantly (although, adaptations of the numerical modelling schemes used in this work could be a potential area for future work). The second definition we make use of is that of the *virial* radius, R_{vir} , and mass, M_{vir} . These are defined in terms of the spherical collapse model, whereby the virial radius is chosen at the point where the enclosed (collapsed) halo has a mean density that matches some overdensity parameter, Δ_{vir} , times the cosmological critical density ρ_c . As such, these definitions will necessarily depend on the chosen cosmological model. In general, we can calculate the critical density as a function of redshift by

$$\rho_c(z) = \frac{3H(z)^2}{8\pi G}, \quad (3.9)$$

where H and G are the usual Hubble parameter and gravitational constant. Then, the overdensity

parameter Δ_{vir} can be estimated as (Bryan et al. 1998, eq. 6):

$$\Delta_{\text{vir}} \approx 18\pi^2 - 82x - 39x^2 \quad (3.10)$$

for flat cosmological models (used in this work), or

$$\Delta_{\text{vir}} \approx 18\pi^2 - 60x - 32x^2 \quad (3.11)$$

for an Einstein/de-Sitter (flat, matter-only) universe. Note that in these expressions, x has been parameterised as in Colafrancesco et al. (2015a, eq. 2.6) by $x = 1 - \Omega_m(z)$, where $\Omega_m(z)$ is the matter density parameter and is calculated by

$$\Omega(z) = \frac{1}{1 + \frac{\Omega_\Lambda(0)}{\Omega_m(0)}(1+z)^3}, \quad (3.12)$$

with Ω_Λ as the usual cosmological constant density parameter. The overdensity parameter Δ is also commonly taken as a flat value of 200, which is a simple approximation of the boundary between regions of the halo that have fully collapsed and those that are still undergoing collapse, and is also close to the case of an Einstein/de-Sitter universe (which has $\Delta \approx 178$) (Zavala et al. 2019). Once a value of Δ_{vir} has been calculated, the virial mass of the halo can be found by

$$M_{\text{vir}} = \frac{4}{3}\pi\Delta_{\text{vir}}\rho_c R_{\text{vir}}^3. \quad (3.13)$$

The other definition we use is of the *concentration* parameter, which roughly describes the overall shape of the DM density distribution within the halo. This parameter can be calculated as a simple ratio of the virial radius (defined with any of the above methods) and a characteristic inner radius, as

$$c_{\text{vir}} = \frac{R_{\text{vir}}}{r_{-2}}. \quad (3.14)$$

The inner radius r_{-2} used above is most commonly defined as the radius within the halo at which the logarithmic slope of the radial density function is equal to -2. In many popular density profiles, this parameter is also termed the *scale-radius*, denoted by r_s . There is some evidence of correlation between the virial concentration and mass of halos (lower mass halos generally have higher concentrations), found in studies of large N -body simulations (Navarro et al. 1997; Prada et al. 2012). This makes the concentration a powerful parameter when describing the density of DM halos, where in some cases the entire density profile of the halo can be fully specified by a single parameter.

The inner structure of these density profiles has been the topic of a huge amount of work in the literature, with particular focus on the central regions within the scale radius. A compelling result of N -body simulations in the late 1990s was the concept of a ‘universal’ density profile, which accurately described a large portion of the halo density and was independent of halo mass (Navarro et al. 1996; Navarro et al. 1997). This profile has since become known as the Navarro-Frenk-White (NFW) profile after the authors of the work, and has become a standard modelling tool in the literature for CDM halo modelling. The profile is given by

$$\rho_{\text{nfw}}(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1 + \frac{r}{r_s}\right)^2}, \quad (3.15)$$

where $\rho_s = \delta_c \rho_c$, and δ_c is a dimensionless characteristic density. Equation (3.15) exhibits a central *cusp*, resulting from the asymptotic density of the regions where $r \rightarrow 0$. The nature of this cusp – and the non-physical values at $r = 0$ – are a consequence of the lack of sufficient spatial resolution in the simulations used at the time, and still represents the general difficulty in accurately modelling this region. However, the excellent overall fits to the simulations in the rest of the halo, and its universal nature, ensured its use as a standard halo density profile even to this day. In modern studies, it is becoming common to consider a generalised version of the

NFW profile (termed gNFW) by including an extra shape parameter α (sometimes denoted γ) as follows:

$$\rho_{\text{gnfw}}(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)^\alpha \left(1 + \frac{r}{r_s}\right)^{3-\alpha}}. \quad (3.16)$$

Here α can be varied to adjust the shape of the profile in the inner region of the halo, with $\alpha = 1$ reproducing the standard NFW profile. Although successful in fitting many simulation results, the need for an alternative profile quickly became apparent with more detailed simulations that probed further into the centre of the halo (Navarro et al. 2004; Navarro et al. 2010). These simulations suggest that not only do halos exhibit less self-similarity (universality) as previously thought, but that they are also less well fit by the standard NFW profile at small radii. This is put succinctly in Navarro et al. (2010, sec. 3.4): “*extrapolating [...] the NFW [...] fits further inwards, to regions less well resolved numerically, is almost guaranteed to incur substantial error*”. The remedy to this problem is the introduction of an alternative profile which includes its own extra shape parameter – the Einasto profile, named after the author who first discussed the profile in the context of modelling the dynamics of the Milky Way (Einasto 1965; Einasto 2010). In its initial form, the profile is of a simple, generalised exponential function, though in modern studies it is usually expressed as:

$$\rho_{\text{ein}}(r) = \rho_s \exp \left\{ -\frac{2}{\alpha} \left[\left(\frac{r}{r_s} \right)^\alpha - 1 \right] \right\}, \quad (3.17)$$

where α plays the part of an adjustable shape parameter. This Einasto profile, with α treated as a free parameter, has provided overall better fits to the simulations in Navarro et al. (2004) and Navarro et al. (2010) than the NFW profile in Equation (3.15) (although the NFW profile still performs well with only $\sim 6\%$ RMS deviations over a large radius range). The best profile to use in any given scenario should, however, be informed by observations wherever possible, and the choice of parameters should always be well motivated. In this work, we primarily consider the gNFW profile with α varying between 1 (standard NFW) and 0.5 (shallower inner cusp). The choice of halo parameters relating to the specific observational targets used in this work is discussed further in Chapter 5. A visual representation of these profiles, for illustrative purposes, is provided in Figure 3.3.

For the sake of completeness, we also outline another important halo density profile, the final one considered in this thesis. The Burkert profile, named after the author (following the density profile naming trend) in Burkert (1995), was proposed because the rotation curves of several dwarf galaxies that were observed at the time conflicted with cuspy models of DM halos. Since then, significant evidence has been gathered that dwarf spheroidal galaxies (dSphs) or low surface brightness galaxies should be hosted by DM halos that possess a characteristic central *core* (i.e., a central region with nearly constant density), rather than a cusp like in the NFW profile (see for example (de Blok et al. 2002; Bertone 2010, ch. 5.2; de Blok 2010; Walker et al. 2011)); a discrepancy that has come to be called the ‘core-cusp problem’ in the literature (de Blok 2010). The Burkert profile is thus given by

$$\rho_{\text{bur}}(r) = \frac{\rho_s}{\left(1 + \frac{r}{r_s}\right) \left(1 + \left(\frac{r}{r_s}\right)^2\right)}, \quad (3.18)$$

and its behaviour can be compared to the cuspy profiles of the NFW and Einasto profiles in Figure 3.3. We also note that since the logarithmic slope of this profile only reaches -2 at $r \sim 1.52 r_s$ (Bullock et al. 2001, eq. 8), the definition of r_s above is slightly different to the one for Equations (3.15), (3.16) and (3.17). However, as the above discussion is only relevant for DM halos in relatively small structures like dSphs, and halos that host large clusters of galaxies are the focus of this work, we leave the reader with the definition of this profile and concentrate our modelling scenarios on halos with cuspy centres.

3.2 Electron propagation

The SM particles produced in the WIMP annihilations described in Equation (3.3) will undergo various physical interactions as they travel around their host environment. These include in-

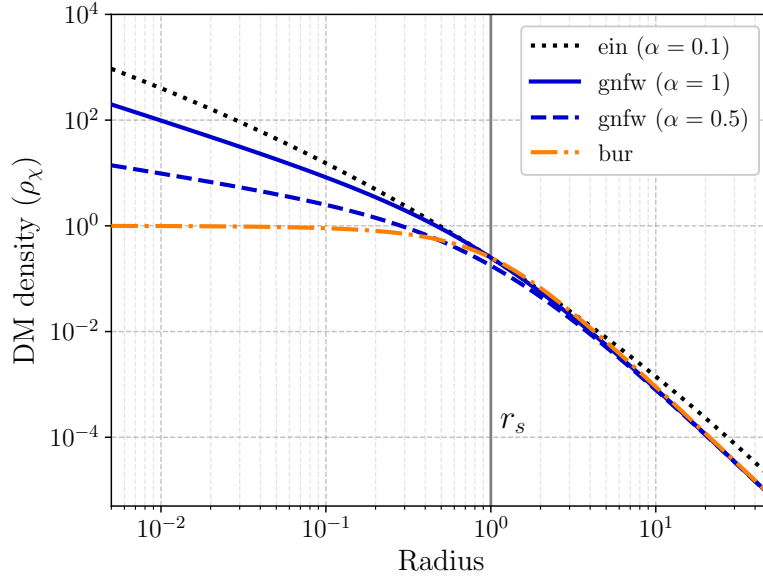


Figure 3.3: Representation of halo density profile behaviour, with arbitrary scaling values. Note that the α parameter of the Einasto (ein) profile can be adjusted to produce significantly different behaviour; for example, a value of $\alpha = 0.17$ closely mimics that of the standard NFW profile, and larger values like $\alpha \sim 0.5$ lead to profiles with cores that are more similar to the Burkert (bur) profile.

interactions with the magnetic fields and thermal gas populations that are ubiquitous in large astrophysical structures, both of which play a role in determining how the annihilation products are distributed in terms of their position and total energy. Here we summarise these interactions, following the standard theories used in the literature. For more comprehensive details we refer the reader to textbooks on the subject, like Longair (2011) or Günter (2016).

In a general form, a transport equation for charged particles in a magnetic field may resemble the one given in Strong et al. (1998, Appendix eq. 1) or Bertone (2010, eq. 27.2). This general form encapsulates all possible effects, including spatial diffusion, energy gains/losses, convection, diffusive re-acceleration and particle injection. In practice though, the different environments within astrophysical structures cause some of these effects to dominate the evolution of the particles. Thus, the effects that are sub-dominant can often safely be ignored. These simplifications – where appropriate – are usually a great help in finding results, as the transport equation can be difficult to solve (as will be discussed in detail in Chapter 4).

These equations are usually written in terms of an electron distribution, here represented by $\psi(\mathbf{x}, E)$ ³. This factor contains all the positional ($\mathbf{x}$) and energy (E) dependence of the electrons in the host environment, and is what gets influenced over time by the above-mentioned interactions. When the effects of spatial diffusion and energy losses are dominant, the full transport equation reduces to the standard diffusion-loss equation, which is written as

$$\frac{\partial \psi(\mathbf{x}, E)}{\partial t} = \nabla \cdot (D(\mathbf{x}, E) \nabla \psi(\mathbf{x}, E)) + \frac{\partial}{\partial E} (b(\mathbf{x}, E) \psi(\mathbf{x}, E)) + Q(\mathbf{x}, E). \quad (3.19)$$

In this equation we see the time evolution of the electron distribution influenced by spatial diffusion effects (first term on the RHS), energy losses (second term on the RHS) and injection of new DM annihilation products (final term on the RHS). The diffusion and energy loss effects are governed by the diffusion coefficient $D(\mathbf{x}, E)$ and the energy loss coefficient $b(\mathbf{x}, E)$ respectively, and will be discussed in more detail in the following Sections. The various mathematical techniques used to solve this equation are presented in Chapter 4.

³We note that the electron distribution is often denoted by n_e or dn_e/dE in the literature. Since we use the variable n_e to refer to the gas density within a halo in this work, we instead use ψ for the electron distribution (as in GALPROP).

3.2.1 Diffusion

Finding an exact form of the diffusion coefficient $D(\mathbf{x}, E)$ can often be challenging, due largely to the uncertainties in the structures of the host magnetic fields. Due to the Lorentz force, we know that the charged, high-energy electrons should spiral around the magnetic field lines, however the particles can scatter or deviate from these field lines if there are irregularities in the magnetic field. Since we expect real magnetic fields in large structures to have some level of irregularity, and if we assume the scatterings are random, the resulting movement of electrons can be approximated as isotropic diffusion away from areas of high density (the approximation of isotropic diffusion also helps in reducing the dimensionality of the problem, as the diffusion coefficient should be considered as a tensor otherwise). Since the vast majority of studies in the literature make use of this approximation, we likewise adopt it in this work.

The magnitude of the aforementioned scattering effect is dependent on the scale of the irregularities when compared to the gyro-radius of the particle's orbit; the scattering is only appreciable when these two scales are similar, whereas relatively small irregularities are ignored by the gyrating electron. However, irregularities can also be generated by the streaming of the high energy particles in the field through the production of Alfvén and hydromagnetic waves (Longair 2011), which adds an extra layer of complexity to their analytical description.

Due to the theoretical complexity and the observational uncertainties when describing diffusion in these scenarios, it is common for studies to use simple expressions for the diffusion coefficient that use various approximations. The simplest of these would be the case of Bohm diffusion (see, for example, Günter 2016), which is considered as a lower limit for the diffusion coefficient as it is just related to the gyro-radius r_g of the particle in the magnetic field, i.e.

$$D \gtrsim \frac{1}{3} r_g \propto \frac{E}{B_0}. \quad (3.20)$$

In a more general way, the diffusion coefficient can be represented by a single power law of the energy of the particles

$$D(E) = D_0 \left(\frac{E}{1 \text{ GeV}} \right)^\delta, \quad (3.21)$$

where the normalisation constant D_0 and the index δ depend on the particular magnetic field environment under study. In terms of the normalisation D_0 , chosen values usually lie in the range $10^{27} - 10^{30} \text{ cm}^2 \text{ s}^{-1}$. Measurements of cosmic rays in the MW with the AMS-02 experiment quote values of $D_0 \sim 10^{28} \text{ cm}^2 \text{ s}^{-1}$ at 1 GeV energies, for instance (Korsmeier et al. 2016). The index δ describes the scale of the turbulence in the magnetic field, and takes on the value of $\delta = 1/3$ for Kolmogorov turbulence or $\delta = 1/2$ for Kraichnan turbulence (or back to $\delta = 1$ in the case of Bohm-like diffusion). In any case, the choice of these two parameters should be informed as closely as possible by observations of the magnetic field in the source.

A still more sophisticated expression for the diffusion coefficient, that includes some information of the magnetic field's spatial profile, is given by

$$D(r, E) = D_0 \left(\frac{E}{1 \text{ GeV}} \right)^\delta \left(\frac{B(r)}{B(0)} \right)^{-\delta}. \quad (3.22)$$

This expression is only useful for studies that make use of numerical techniques to solve Equation (3.19), and a form of it is used in Regis et al. (2017) and Beck et al. (2023a). A comparison of two extreme diffusion scenarios, Bohm diffusion and Kolmogorov turbulence, was carried out in Beck et al. (2023a) in the context of galaxy clusters. The results of this study show that the outside of the inner regions of the halo, there is negligible difference in the output surface brightnesses for each case. Thus, in practice most results should be robust to the different choices of the diffusion coefficients outlined here (although, as hinted at throughout this manuscript, the differences in these inner regions are likely to be important when finding accurate results with high-resolution data and surface-brightness calculations).

3.2.2 Energy Loss

The other dominant effect in the propagation of electrons is the loss of energy, due to various physical phenomena. Although technically possible, the environments we are considering make any gains in energy rare, so we will proceed under the definition of an explicit energy *loss* coefficient $b(\mathbf{x}, E) \equiv -dE/dt$, as is standard. The most relevant physical phenomena that contribute to b are Inverse Compton (IC) scattering, synchrotron emissions, Coulomb scattering and bremsstrahlung. A brief outline of each of these phenomena will be described below, after which we will present the full version of the energy loss coefficient. For a detailed collection of relevant results, see also Strong et al. (1998, Appendix C.2).

Inverse Compton (IC) When the relativistic electrons scatter with ambient photons, either in the CMB or in an interstellar radiation field (ISRF), they will transfer part of their kinetic energy to the photons. Since the energies of the electrons will be much larger than those of the photons, the process can be described by Thomson scattering (Longair 2011), with the total energy loss rate given by

$$b_{\text{IC}} = \frac{4}{3} \sigma_T c u_{ph} (\gamma^2 - 1) \approx \frac{4}{3} \sigma_T c u_{ph} \gamma^2, \quad (3.23)$$

where σ_T is the Thomson scattering cross-section, u_{ph} is the energy density of the photon field through which the electron is travelling, and γ is the usual Lorentz factor ($\gamma = E/m_e c^2$, where m_e is the rest mass of an electron). The photon field is usually chosen to be the CMB, in which case $u_{ph} = 0.25 \text{ eV cm}^{-3}$.

Synchrotron In the presence of a background magnetic field, the charged electrons gyrate around the magnetic field lines and constantly emit radiation due to their acceleration (towards the centre of their orbits) (Longair 2011). This effect is precisely the source of the radio emissions we use to search for DM, and we will explore the emission in more detail in Section 3.3.1. The relevant energy loss rate from this process, for the contribution to $b(\mathbf{x}, E)$, is given by

$$b_{\text{synch}} = \frac{4}{3} \sigma_T c u_{mag} \gamma^2, \quad (3.24)$$

where u_{mag} is the energy density of the magnetic field, given by $u_{mag} = B^2/2\mu_0$ where μ_0 is the permeability constant. Here, as in the case of IC scattering, we have made the approximation $\gamma^2 - 1 \approx \gamma^2$ in the case of highly-relativistic electrons.

Coulomb At lower energies, the interactions between electrons and ambient nuclei are dominated by Coulombic scattering events. In the case of a cold plasma, the energy losses from this process can be written as (Strong et al. 1998, eq. C7)

$$b_{\text{Coul}} = -2\pi r_e^2 m_e c^3 n_e \frac{1}{\beta} \left[\ln \left(\frac{Em_e c^2}{4\pi r_e \hbar^2 c^2 n_e} \right) - 0.75 \right], \quad (3.25)$$

where r_e , m_e , $\beta = v/c$ are the usual quantities of the electron's classical radius, mass and velocity in the particle physics nomenclature, and n_e is the number density of the ambient ions. We note that this expression is related to the energy loss experienced due to the ionisation of a neutral gas, which is given by the Bethe-Bloch equation,

$$b_{\text{ion}} = -2\pi r_e^2 m_e c^3 n_e \frac{1}{\beta} \left[\ln \left(\frac{(\gamma - 1)\beta^2 E^2}{2I^2} \right) + 0.125 \right], \quad (3.26)$$

where I is the ionisation potential of the ambient atoms. While an important process in general cosmic ray physics, energy loss due to ionisation is usually neglected in DM studies. This is due to the lack of a large density of neutral gas to which the electrons can lose energy, as the centre of DM halos are usually in regions where many other ionising processes of astrophysical origin exist, like in the centres of galaxies or galaxy clusters.

Bremsstrahlung Bremsstrahlung, or *braking radiation*, sometimes known as free-free emission, is the emission from a high-energy electron that accelerates within the electrostatic field of a nucleus. This is a complicated process to describe completely – there are various derivations of the energy loss rate which are applicable to different regimes and ambient particle fields. For relativistic electrons interacting with neutral atoms we quote the full result from Longair (2011, eq. 6.74) as

$$b_{\text{brem},0} = \frac{Z(Z+1.3)e^6}{16\pi^2\epsilon_0^3 m_e^2 c^4 \hbar} n_e E \left[\ln \left(\frac{183}{Z^{1/3}} \right) + \frac{1}{8} \right] \propto n_e E, \quad (3.27)$$

where Z is the atomic number of the ambient gas. The important relation has been highlighted in Equation (3.27); with a particular species of nuclei (say, a neutral hydrogen gas) the energy loss will scale proportionally with the electron's energy and the number density of the gas. On the other hand, an approximate form of bremsstrahlung energy loss of an electron in a fully ionised gas is given in Ginzburg (1979) and Strong et al. (1998, eq. C10) as

$$b_{\text{brem},I} = \frac{4Z(Z+1)e^6}{m_e^2 c^4 \hbar} n_e E \left[\ln \left(\frac{2E}{m_e c^2} \right) - \frac{1}{3} \right]. \quad (3.28)$$

Thus, in the ionised case, there is an extra dependance on the electron energy within the logarithm. The use of either form, for the case of neutral or ionised gas, is likely the cause of a discrepancy in the expression used for bremsstrahlung energy loss present in a number of radio DMID studies. This was noted in Egorov (2022), and seems to have stemmed from an earlier study by Egorov et al. (2013) which deviated from the expressions used in the oft-cited paper by Colafrancesco et al. (2006). The form given in Colafrancesco et al. (2006, eq. A.16) is

$$b_{\text{brem}} = 1.51 \times 10^{-16} n_e \left(\ln \left(\frac{\gamma}{n_e} \right) + 0.36 \right) \text{ GeV s}^{-1}. \quad (3.29)$$

As far as we are aware, there is no obvious source for the form of b_{brem} given above. We thus consider the equations given by the textbooks (which contain detailed derivations of the formulae with motivations) as more reliable, in either the case of a neutral or ionised gas.

To elucidate this discrepancy in versions of the bremsstrahlung equation, we have performed a simple analysis of the three common forms, as found in the reference papers Colafrancesco et al. (2006), Egorov et al. (2013), and Egorov (2022), and present a representative sample of the results in Figure 3.4. The notable points from this analysis are that at high energies, IC effects (and synchrotron in the case of mid-to-high strength magnetic fields) dominate the energy loss coefficient. At low energies, Coulomb scattering should dominate. The contribution of bremsstrahlung is highly dependent on the gas density: in the case we consider the most extreme, when the gas density is very high (and the magnetic field is weak to reduce synchrotron contributions), bremsstrahlung dominates the energy losses in the mid-energy range (typically around 1-100 GeV). In most of the astrophysical scenarios we are considering however – galaxy clusters, dSphs and galaxies – gas densities are far lower than this extreme case, which results in bremsstrahlung effects not significantly impacting the total energy loss at any energy. Thus, the use of either Equation (3.27) or (3.28) should not have a noticeable impact on the accuracy of results in the vast majority of DMID studies.

The expressions we have given for each of the above energy loss mechanisms are rooted in theoretical considerations, and are cumbersome to work with in practical settings. As such, we now provide an equation that encapsulates all of the above effects in a single function that is easy to implement in a more computational setting. The full form of the energy loss coefficient that is

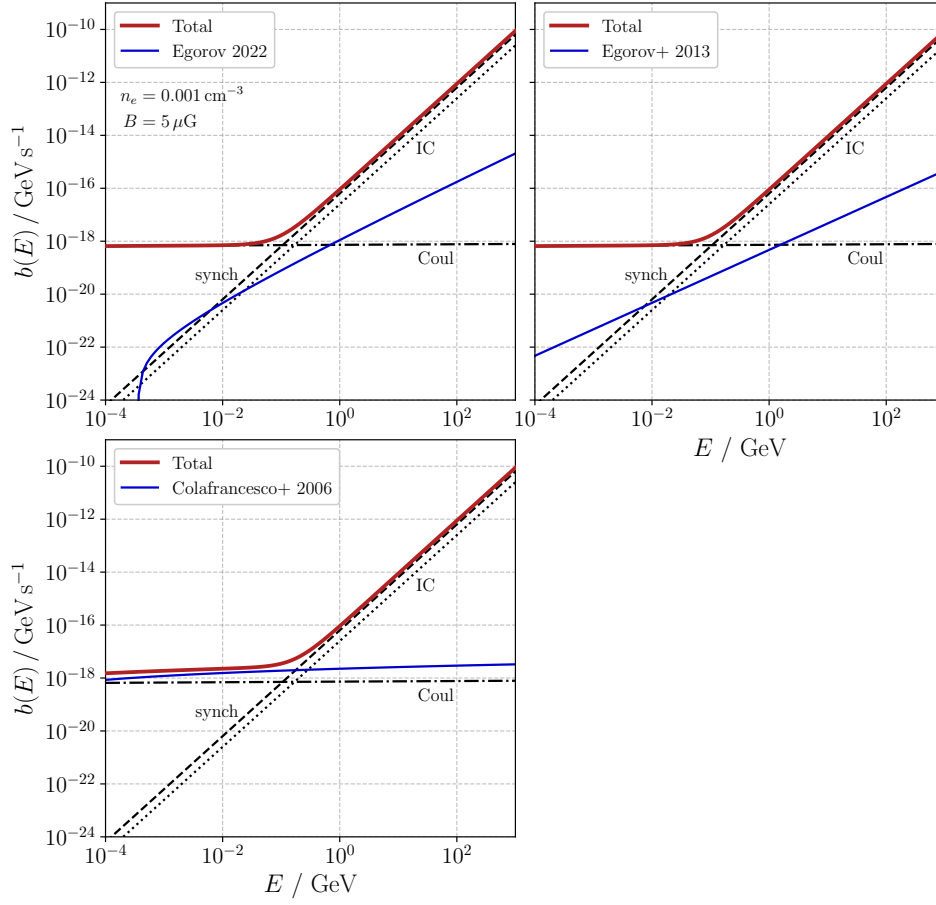


Figure 3.4: The bremsstrahlung problem: here we show the contribution of each energy loss mechanism (explained in Section 3.2.2) to the total energy loss coefficient $b(E)$. We highlight the use of different forms of the bremsstrahlung energy loss equation in reference papers Colafrancesco et al. (2006), Egorov et al. (2013), and Egorov (2022) by the blue lines in each panel. For many parameter configurations, the bremsstrahlung contribution to the total energy loss coefficient is sub-dominant to the other relevant energy loss mechanisms.

used in the diffusion-loss equation is then

$$\begin{aligned}
 b(r, E) = & b_{\text{IC}} \left(\frac{E}{1 \text{ GeV}} \right)^2 \\
 & + b_{\text{synch}} \left(\frac{E}{1 \text{ GeV}} \right)^2 \left(\frac{B(r)}{1 \mu\text{G}} \right)^2 \\
 & + b_{\text{Coul}} \left(\frac{n_e(r)}{1 \text{ cm}^{-3}} \right) \left(\log \left(\frac{\gamma}{n_e(r)/1 \text{ cm}^{-3}} \right) + 73 \right) \\
 & + b_{\text{brem}} \left(\frac{n_e(r)}{1 \text{ cm}^{-3}} \right) \gamma (\log(\gamma) + 0.36), \quad (3.30)
 \end{aligned}$$

where we have considered a fully ionised plasma in the bremsstrahlung formula, as in Egorov (2022, eq. 12). This equation is only dependant on observable quantities like the magnetic field and gas density (these will be described in the following Section). Each of the coefficient prefactors in Equation (3.30) are the result of evaluating the physical constants present in the original equations,

and their exact values are given as in Egorov (2022),

$$b_{\text{IC}} = 0.25 \times 10^{-16} \text{ GeV s}^{-1}, \quad (3.31)$$

$$b_{\text{synch}} = 0.025 \times 10^{-16} \text{ GeV s}^{-1}, \quad (3.32)$$

$$b_{\text{Coul}} = 7.6 \times 10^{-18} \text{ GeV s}^{-1}, \quad (3.33)$$

$$b_{\text{brem}} = 7.1 \times 10^{-20} \text{ GeV s}^{-1}. \quad (3.34)$$

3.2.3 Magnetic fields and gas densities

As discussed in the preceding Section, the structure of the host magnetic field plays a central role in determining the propagation of electrons through their environment once they have been produced. In particular, the turbulence and irregularities inevitably present in these fields are the source of electrons diffusing away from the centre of the halo where their density is highest. While the description of the turbulence (in a very generic way) is incorporated into the diffusion coefficient, the smooth or regular component of the magnetic field still needs to be quantified to appear as a factor in both the diffusion and energy loss coefficients.

The structure of the magnetic field can thus be described by various profiles, similarly to the way we defined various profiles for the density of the DM halo in Section 3.1.2. Although higher dimensional profiles do exist (to describe spiral galaxies for example, see Egorov (2022)), consider spherically symmetric profiles of the radius r as the objects we are primarily interested in this work have approximately spherical halos. Thus, the profiles we define here are the flat, power-law (pl), beta, double-beta (d-beta) and exponential (exp) profiles, defined respectively as:

$$B_{\text{flat}}(r) = B_0, \quad (3.35)$$

$$B_{\text{pl}}(r) = B_0 \left(\frac{r}{r_b} \right)^{q_b}, \quad (3.36)$$

$$B_{\text{beta}}(r) = B_0 \left(1 + \left(\frac{r}{r_b} \right)^2 \right)^{\frac{3\beta_b}{2}}, \quad (3.37)$$

$$B_{\text{d-beta}}(r) = B_0 \left(1 + \left(\frac{r}{r_b} \right)^2 \right)^{\frac{3\beta_b}{2}} + B_{0,2} \left(1 + \left(\frac{r}{r_{b,2}} \right)^2 \right)^{\frac{3\beta_{b,2}}{2}}, \quad (3.38)$$

$$B_{\text{exp}}(r) = B_0 \exp \left(-\frac{r}{r_b} \right). \quad (3.39)$$

In each of the above, B_0 is the magnetic field strength normalisation, r_b is the scaling radius of the profile and q_b, β_b are the indices for the power-law and beta profiles respectively. Note that the double-beta profile uses a second set of parameters which are each denoted by a 2 in the subscript. A visual representation of these profiles, showing their general behaviour, is provided in Figure 3.5.

We also provide profiles for the spatial number density of the gas (plasma), which is necessary for the calculation of energy loss effects like bremsstrahlung and Coulombic interactions, as follows:

$$n_{e,\text{flat}}(r) = n_0, \quad (3.40)$$

$$n_{e,\text{pl}}(r) = n_0 \left(\frac{r}{r_e} \right)^{q_e}, \quad (3.41)$$

$$n_{e,\text{beta}}(r) = n_0 \left(1 + \left(\frac{r}{r_e} \right)^2 \right)^{\frac{3\beta_e}{2}}, \quad (3.42)$$

$$n_{e,\text{beta}}(r) = n_0 \left(1 + \left(\frac{r}{r_e} \right)^2 \right)^{\frac{3\beta_e}{2}} + n_{0,2} \left(1 + \left(\frac{r}{r_{e,2}} \right)^2 \right)^{\frac{3\beta_{e,2}}{2}}, \quad (3.43)$$

$$n_{e,\text{exp}}(r) = n_0 \exp \left(-\frac{r}{r_e} \right), \quad (3.44)$$

where the symbols n_0 , r_e , q_e and β_e have the equivalent meanings for these profiles as those for the magnetic field profiles.

Many of the profiles defined above have been fit from observations of the gas density in astrophysical structures (see for example Chen et al. (2007) and Zhu et al. (2021)). There is also some evidence from cosmological magneto-hydrodynamic simulations and correlations between radio and X-ray observations that suggests the general relationship between the profiles of the magnetic field and gas density can be described by $B(r) \propto (n_e(r))^\eta$, where η is usually between 0.5 - 1 (Dolag et al. 2001; Govoni et al. 2004; Murgia et al. 2004; Dolag et al. 2008; Feretti et al. 2012; Osinga et al. 2022). Therefore, the magnetic field profiles used in the literature are typically chosen to trace the gas density profiles using this relation. More specific details about the choice of magnetic field/gas density parameters for the analysis performed in this work will be discussed in Chapter 5.

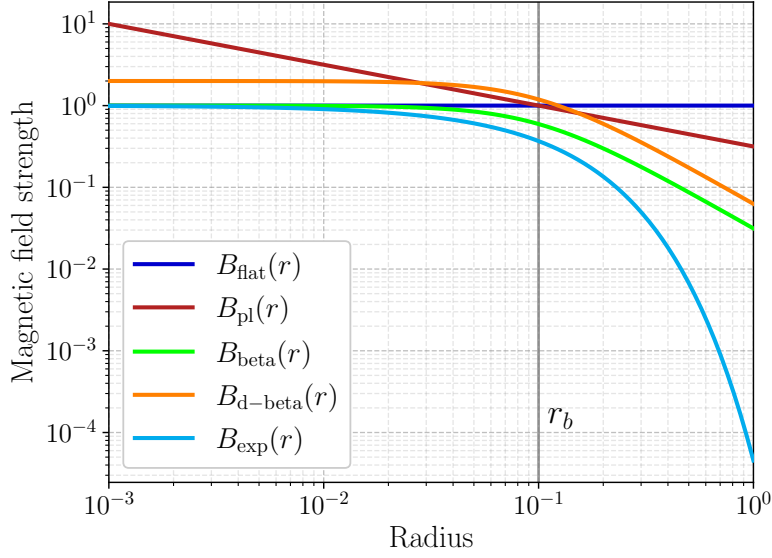


Figure 3.5: Representation of magnetic field profile behaviour. The axes values are arbitrary – we have chosen a normalisation constant of 1, and magnetic indices of $q_b = \beta_b = -0.5$.

3.3 Observable emissions from DM

The emissions from the electrons that are produced by DM annihilation are dominated by synchrotron radiation, which is detectable primarily in the radio frequency band. The accurate modelling of this radiation is one of the integral components of a good indirect DM search, and is a well-studied area of radiative astrophysics. In the following Sections we describe (briefly) some features of synchrotron radiation emitted by ultra-relativistic electrons, drawing mainly from the analysis in Longair (2011), which has become a standard source in the literature.

3.3.1 Synchrotron emission

When considering the case of a single electron in a simple, static magnetic field, its movement is helical with a constant pitch angle (α), representing the angle between the electron velocity ($\mathbf{v}$) and magnetic field ($\mathbf{B}$) vectors. This helical motion is caused by the Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ of a charge q in a magnetic field, which results in a constant velocity that is parallel to the magnetic field lines (the ‘guiding centre’ of the motion) and an acceleration that is exactly perpendicular to this guiding centre. The accelerated motion of the electron is what, through standard electromagnetic theory, generates the synchrotron radiation.

The characteristic frequency spectrum of synchrotron radiation from a single electron is shown in Figure 3.6. This spectrum is determined by the synchrotron kernel function, which is usually

denoted by $F(x)$, and will be defined later in this Section. As the kernel function contains all the frequency dependance in the expression for the emissivity, this spectral behaviour is also a characteristic of the emitted radiation. The critical frequency ν_c , which corresponds roughly to the peak of this spectrum, is given by

$$\nu_c = \frac{3}{2}\gamma^2\nu_g \sin(\alpha), \quad (3.45)$$

where $\nu_g = eB/(2\pi m_e c)$ is the non-relativistic gyro-frequency of the electron.

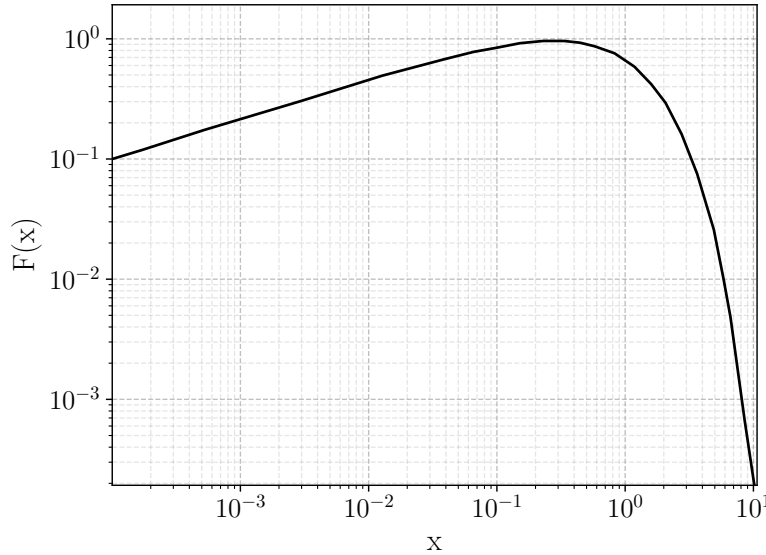


Figure 3.6: Representation of the spectrum of synchrotron radiation from a single ultra-relativistic electron. Here the variable $x = \nu/\nu_c$, where ν_c is the critical frequency, and $F(x)$ – the synchrotron kernel function – represents the factor in the emissivity that contains all the frequency dependance. The peak occurs at $\nu = 0.29\nu_c$. Figure reproduced from Longair (2011, fig. 8.8).

In the case of highly-relativistic electrons, the apparent direction of the emitted radiation is beamed in the direction of the electron's instantaneous velocity. Since electrons produced in DM annihilations can be highly relativistic, they would certainly experience this relativistic beaming effect. However, as we consider large populations of electrons in large astrophysical structures that contain complex magnetic field structures, the pitch angle of each electron can be considered to be random. This should result in an isotropic distribution of pitch angles throughout the electron population, and because of the diffusion effects described in Section 3.2.1, a diffuse region of radio frequency emissions should thus be observable within the DM halo.

The method used to calculate the emissivity of the synchrotron emissions usually proceeds as follows. Firstly, the isotropically-averaged power of the synchrotron emission from a population of electrons is expressed as in Colafrancesco et al. (2006, eqn. 37),

$$P(\nu, E, r) = \int_0^\pi d\theta \frac{\sin(\theta)}{2} 2\pi\sqrt{3} r_e m_e c \nu_g \sin(\theta) F(x/\sin(\theta)). \quad (3.46)$$

Here r_e and m_e are the electron's classical radius and mass respectively. The function F and the variable x are then defined as follows:

$$F(t) \equiv t \int_t^\infty dz K_{5/3}(z) \simeq 1.25 t^{1/3} \exp(-t) [648 + t^2]^{1/12} \quad (3.47)$$

and

$$x \equiv \frac{2\nu}{3\nu_0\gamma^2} \left[1 + \left(\frac{\gamma\nu_p}{\nu} \right)^2 \right]^{3/2}, \quad (3.48)$$

where $K_{5/3}$ is a modified Bessel function of order 5/3 and $\nu_p = 8980(n_e(r)/1 \text{ cm}^{-3})^{1/2} \text{ Hz}$ is the plasma frequency of the gas. With the synchrotron power in hand, the emissivity can then be calculated by

$$j(\nu, r) = \int_{m_e}^{M_x} dE (\psi_-(r, E) + \psi_+(r, E)) P(\nu, E, r), \quad (3.49)$$

where $\psi_{\pm}$ are the equilibrium electron distributions found in the solution to the diffusion-loss equation in Equation (3.19) (the solution methods for this are the subject of Chapter 4). We note that if the DM halo has a redshift $z \gtrsim 0$, instances of the frequency ν should be replaced by $\nu(1+z)$. Equation (3.49) is then used to calculate various observable quantities, such as the radio surface brightness or the flux density spectrum. In this work we use the following forms of these quantities, which are also presented in Beck et al. (2023a, eq. 30 and 31). Firstly, we define the surface brightness as

$$I_{\text{synch}}(\nu, r) = \int dl \frac{j(\nu, \sqrt{r^2 + l^2})}{4\pi}, \quad (3.50)$$

where r is the radial distance from the centre of the halo and l is the line-of-sight coordinate to the position at r . Because $I_{\text{synch}}(\nu, r)$ is spatially dependant (in the form of the radius r), it is a very useful quantity when spatially-resolved data is available (i.e. via the observations of high-resolution interferometers); this fact should allow for high precision comparisons of DM models to the data, especially when compared to (previous) observations that contain few data points. This point will be referred to again throughout the thesis.

Secondly, we define the integrated flux as

$$S_{\text{synch}}(\nu, R) = \int_0^R d^3r' \frac{j(\nu, r')}{4\pi (d_L^2 + (r')^2)}, \quad (3.51)$$

where R represents the maximum radius of the halo within which the emission is integrated, and d_L is the luminosity distance to the halo. With Equations (3.50) and (3.51) now specified, and using the results of the previous Sections, the radio emission from DM can be fully calculated. However, the observational data from the MGCLS has 2, rather than 1, spatial dimension. To account for this, we take the radial surface brightness calculation above and map the results to a 2-dimensional grid. This allows us to create a FITS image of the DM emission model that matches the data, simplifying the statistical analysis that we perform. The result of this is a spherically symmetric emission profile in 2 spatial dimensions, which will be discussed further in Chapter 5.

3.3.2 Monochromatic approximation

In the cases where accurate modelling of the synchrotron emission spectrum can be overlooked in favour of practicality (perhaps for the sake of calculations that are quick and easy to implement), the assumption of a monochromatic spectrum can be made. This involves approximating the synchrotron kernel function to a form in which the only relevant frequency is the one at the peak of the kernel function, i.e.

$$F(\nu) \approx \delta(\nu - 0.29\nu_c). \quad (3.52)$$

The meaning of this approximation can easily be seen in Figure 3.6, as there is a single peak in the function $F(x)$ that occurs at a value $x = 0.29$. When this approximation is made, the energy of the electron which corresponds to the peak frequency output is given by

$$E_{\text{peak}} \simeq 0.463 \left(\frac{\nu}{1 \text{ MHz}} \right)^{1/2} \left(\frac{B}{1 \mu\text{G}} \right)^{1/2}. \quad (3.53)$$

The benefit of this approximation is that the synchrotron luminosity L_ν (and thus other observables) can be calculated rather simply, as in Bertone (2010, eq. 27.9),

$$\nu L_\nu = \frac{9\sqrt{3}}{4} \frac{\langle \sigma v \rangle}{M_\chi^2} \int d^3\mathbf{x} \rho(\mathbf{x})^2 E_{\text{peak}} Y_e(E_{\text{peak}}), \quad (3.54)$$

where $Y_e(E) = \int_E^{M_\chi} dE' \frac{dN_e}{dE'}$. This approximation has been used in Chan (2021) and Chan et al. (2019), and is a useful calculation when rough estimates or behavioural characteristics of a particular effect is desired (as in say, the appendix of Colafrancesco et al. (2006, eqn. A.17)), but it does not have the required accuracy for use in DM exclusion studies with high-resolution data. The most important benefit in using this approximation is that there is no longer a need to solve the diffusion-loss equation, since the electron's energy distribution collapses to a single value. As will be discussed at length in the following chapter, solving the diffusion-loss equation in an accurate way can be a challenging and time-consuming endeavour. An example of this approximation, in comparison to more complete modelling processes, can be found in Sarkis et al. (2022, fig. 1).

Solving the Diffusion-Loss Equation

The nature of Equation (3.19), that of being a second-order partial differential equation, means it cannot trivially be solved by analytical methods. However, since the problem of cosmic ray propagation is a rather well-studied one, there have been a variety of techniques used to find solutions to this equation in the literature. Each method is more or less computationally performant depending on the user's desired level of accuracy in the result, and in the amount of time and effort given to setting up the algorithms. In the following Chapter we will outline each of the variations found in the literature, and then focus in particular on the method we have developed and used in this project. A visual comparison of the results of each method can be found at the end of this Chapter, in Section 4.4.

4.1 Overview of solution methods

Semi-analytical solutions

The most commonly cited technique for solving Equation (3.19) in the literature was introduced by Colafrancesco et al. (2006), although a similar technique, tailored specifically for the MW halo, was used before that in Baltz et al. (1998). This method has since appeared in a multitude of radio indirect detection studies and is the method of choice in the software package RX-DMFIT (McDaniel et al. 2017), which has in turn been used in a variety of studies (this package will be discussed in detail at the end of this Chapter). Here we outline the fundamental parts of the method and comment on its applicability to different astrophysical modelling scenarios; for a detailed analysis and derivation of the formulae we refer the reader to the appendix in Colafrancesco et al. (2006).

To work with Equation (3.19) in a way that provides an analytical solution, there are several assumptions about the physical environment – which lead to simplifying approximations – that are required. The most significant of these approximations is that the diffusion function, D , and energy-loss function, b , depend only on the energy of the electrons and not on any spatial coordinate, i.e. $D(\mathbf{x}, E) \approx D(E)$ and $b(\mathbf{x}, E) \approx b(E)$. Because of the noted uncertainty in the magnetic field and gas density profiles in certain astrophysical objects, this approximation may be unimportant in certain cases. However, as will be further discussed in later Sections, high-resolution radio data that is able to resolve the inner regions of these objects makes this approximation a significant hindrance to finding reliable results when that information is available. The other approximation necessary for this method is that the relevant physical quantities obey spherical symmetry, such that spatial effects are described only in terms of the radius r from the centre of the halo. The validity of this assumption depends entirely on the geometry of the relevant astrophysical target; cool-core galaxy clusters or dSphs should easily qualify, whereas highly elliptical or spiral galaxies would benefit from a more detailed spatial analysis (for an example of a solution that incorporates more than one spatial dimension, see Egorov (2022)).

Now, after defining the variable

$$u = \int_E^{E_{\max}} \frac{dE'}{b(E')}, \quad (4.1)$$

Equation (3.19) can be written as

$$\left[-\frac{\partial}{\partial t} + D(E)\nabla^2 - \frac{\partial}{\partial u} \right] \frac{dn_e}{du} = b(E)Q(E, r), \quad (4.2)$$

where n_e represents the number density of electrons used in the distribution $\psi \equiv dn_e/dE$. After transforming this equation into Fourier space, it has an analytical solution. Transforming the solution back from Fourier space, and then applying boundary conditions through a ‘method of images’ technique, the final equilibrium distribution can be written explicitly as

$$\psi = \frac{1}{b(E)} \int_E^{M_\chi} dE' \frac{1}{\sqrt{4\pi\Delta v}} \sum_{n=-\infty}^{\infty} (-1)^n \int_0^{r_h} dr' \frac{r'}{r_n} \left[\exp\left(-\frac{(r' - r_n)^2}{4\Delta v}\right) - \exp\left(-\frac{(r' + r_n)^2}{4\Delta v}\right) \right] Q(r', E', t'). \quad (4.3)$$

Here M_χ is the mass of the WIMP, $r_n = (-1)^n r + 2nr_h$ are the locations of the n^{th} image charge, $t' = t - (u' - u)$ and the variable v was introduced where $\Delta v = v - v'$ and $v = \int_{u_{\min}}^u d\tilde{u} D(\tilde{u})$. In the method of images technique the diffusion radius r_h (i.e. the radius at which electrons can freely stream outwards from the halo) is used as a boundary condition at which the electron distribution vanishes. Since its derivation however, the analytic validity of this solution method has been revisited by the author of Vollmann (2021). In that study, a corrective factor has been applied to the solution (this factor is given below, see Equation (4.10) and discussion thereafter for details).

The final approximation of note in this method is that of stationarity, i.e. that the equilibrium distribution of electrons does not change with time. This requires that the relevant physical timescales, that of diffusion and energy losses, are much smaller than the age of the host astrophysical object, so that the electron distribution has had sufficient time to reach equilibrium. In this case we can neglect any rate-of-change derivatives, i.e. $\frac{\partial}{\partial t} \rightarrow 0$ in Equation (3.19). Since the typical timescales of these effects (including for each of the energy-loss mechanisms described in Section 3.2.2) will depend on the physical environment, as well as the intrinsic energy of the electrons, they can take a large range of possible values. These will generally scale inversely with energy, so that the assumption of stationarity will in general be more acceptable for higher energy electron populations in older astrophysical structures. In DM indirect detection studies though, this assumption has become quite ubiquitous, often used regardless of the target environment. Under the stationarity approximation then, we can write Equation (4.3) in a form commonly seen in the literature, as

$$\psi = \frac{1}{b(E)} \int_E^{M_\chi} dE' G(r, \Delta v) Q(r, E'). \quad (4.4)$$

where the corresponding time-independent Green’s function is given as

$$G(r, \Delta v) = \frac{1}{\sqrt{4\pi\Delta v}} \sum_{n=-\infty}^{\infty} (-1)^n \int_0^{r_h} dr' \frac{r'}{r_n} \left[\exp\left(-\frac{(r' - r_n)^2}{4\Delta v}\right) - \exp\left(-\frac{(r' + r_n)^2}{4\Delta v}\right) \right] \frac{Q(r')}{Q(r)}. \quad (4.5)$$

By analysing the behaviour of Equation (4.5) for various halo types, Colafrancesco et al. (2006) provided some insight into the relevant effects of diffusion and energy loss. It turns out that in the inner regions of a halo, Equation (4.5) tends to unity and can thus be ignored in calculations of the equilibrium distribution within large halos. This scenario matches the specific case of a loss-only transport equation (i.e. where the diffusion term is neglected), where the analytical solution is given as

$$\psi = \frac{1}{b(E)} \int_E^{M_\chi} dE' Q(r, E'). \quad (4.6)$$

Since Equations (4.5) and (4.6) are equal except for a factor of the Green’s function, we can then infer that the Green’s function determines the spatial diffusion effects in the problem. This insight, together with a brief discussion of the relevant timescales involved in the problem found in the appendix of Colafrancesco et al. (2006), are the reasons why modern studies on this topic often neglect the effects of diffusion in the solution of the diffusion-loss equation.

An alternative semi-analytical method to the one described above has recently been proposed in Vollmann (2021), and has so far been used by Vollmann et al. (2020) and Gajović et al. (2023). This method employs a set of approximations in various regimes (characterisations of the parameter space) that simplify the problem, leading to very fast and efficient computations. Since this method isn't used in this project specifically, we again refer the reader to Vollmann (2021) for more specific details involving the derivation of equations and discussions about accuracy of the results when input parameters are varied.

As in the previous method, this technique also assumes spherical symmetry, diffusion and energy-loss coefficients that are spatially independent and stationarity. It then defines the following variable

$$\lambda^2(E) = \int_E^\infty dE' \frac{D(E')}{b(E')} \quad (4.7)$$

(which is an analogue for $\sqrt{v}$ above), and using the fact that the resulting form of Equation (3.19) becomes like a one-dimensional heat equation, the final form of the electron distribution is then written

$$\psi(r, E) = \frac{1}{b(E)} \int dE' \int dr' \frac{r'}{r} G_{r_h}(\Delta\lambda^2, r, r') Q(r', E'), \quad (4.8)$$

where

$$\Delta\lambda^2 = \lambda^2(E) - \lambda^2(E'). \quad (4.9)$$

The functions G_{r_h} are the one-dimensional Green's functions, whose explicit forms are then obtained in two ways: the method of images technique and a Fourier-type analysis. The resulting expressions for these methods, respectively, are given by the following:

$$G_{r_h}(\Delta\lambda^2, r, r') = \sum_{k=-\infty}^{\infty} \frac{\Theta(\Delta\lambda^2)}{\sqrt{4\pi\Delta\lambda^2}} \left(\exp\left(-\frac{(r-r'-2kr_h)^2}{4\Delta\lambda^2}\right) - \exp\left(-\frac{(r+r'-2kr_h)^2}{4\Delta\lambda^2}\right) \right), \quad (4.10)$$

and

$$G_{r_h}(\Delta\lambda^2, r, r') = \frac{2\Theta(\Delta\lambda^2)}{r_h} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi r'}{r_h}\right) \sin\left(\frac{n\pi r}{r_h}\right) \exp\left(-\frac{n^2\pi^2\Delta\lambda^2}{r_h^2}\right). \quad (4.11)$$

We note here, as in Vollmann (2021), that the form of Equations (4.5) and (4.10) differ slightly, by a quoted factor of $(-1)^k \frac{r}{r-2kr_h}$. After an analysis of mock DM halos of various types in the appendix of Vollmann (2021), this difference seems to be negligible for cuspy halos, while resulting in a factor of at most 3 difference for constant density halos. The two solutions also tend to equality when diffusion effects are neglected (i.e. in the case of the no-diffusion approximation regime defined below).

With these general solutions in hand, the ratio of diffusion to energy-loss timescales (τ_{diff} and τ_{loss} respectively) are used to define three separate regimes in which to solve Equation (4.8), using either Equation (4.10) or (4.11) with relevant approximations. We outline these regimes as follows, and refer the reader to Vollmann (2021) for a more in-depth analysis.

Regime A - No-diffusion approximation When $\tau_{loss} \ll \tau_{diff}$, the electrons lose energy – through the mechanisms described in Section 3.2.2 – quickly enough that their diffusion in the halo can be neglected completely. This regime is defined by the limit $\Delta\lambda^2 \rightarrow 0$ (although it should be noted that this limit is technically only valid in non-cuspy halo profiles).

Regime B - Leading-mode approximation When $\tau_{loss} \approx \tau_{diff}$, the value of $\Delta\lambda$ approaches the diffusion radius. In this case, the author uses Equation (4.11) with the leading-mode (i.e. the $n = 1$ term of the series) as the form of the Green's function. Clearly this solution leads to reduced accuracy compared to using the full series, but it does lead to an analytical form of the final electron distribution.

Regime C - Rapid-diffusion approximation When $\tau_{loss} \gg \tau_{diff}$, the electrons are able to diffuse to the edge of the diffusion boundary without losing a significant amount of energy. The defining limit in this regime is $\lambda(E) \gg r_h$.

Now, the emissivity of a particular DM halo configuration can be written in the following general way,

$$j_\nu(r) = \frac{Q}{4\pi} \times H_{\mathcal{R}}(r) \times X(\nu). \quad (4.12)$$

In this equation, $H_{\mathcal{R}}(r)$ represents the ‘halo part’, containing all of the spatial dependence, and $X(\nu)$ represents the ‘spectral part’, containing all of the energy dependence. Each of these have well-defined and analytical (though not necessarily simple) expressions for each of the regimes A, B and C defined above, which for the sake of brevity won’t be reproduced here. We see in Equation (4.12) the central point of Vollmann (2021)’s study - to have a simple, approximate formula for calculating emissivities from DM halos. This method has been implemented in Vollmann et al. (2020) and Gajović et al. (2023), and seems to be an interesting way to find results, albeit ones that are approximate in nature and specific to cored profiles (dSphs). Notably, the authors of Gajović et al. (2023) adopt a standard (cuspy) NFW profile to describe the halos in the dSphs they are studying while using the approximations described here.

Numerical solutions

The majority of the use cases for numerical solutions to the diffusion-loss equation are in the field of galactic cosmic-ray physics. There are two prominent pieces of software that perform this function: the seminal GALPROP package¹ (Strong et al. 1998; Porter et al. 2022) (also used with extensions in Egorov (2022)) and the DRAGON(2) package² (Evoli et al. 2017). The other notable use of a numerical solution is by the authors of the series Regis et al. (2014), Regis et al. (2015b), Regis et al. (2015a), Regis et al. (2017), and Regis et al. (2021). Their use case is relevant here, because the method presented within those papers is motivated by the same thing as this project – to solve the propagation of charged DM annihilation products in astrophysical objects (and not necessarily within the MW).

In each of the above-mentioned sources, a similar technique has been used to solve Equation (3.19), originally inspired by GALPROP. This method is based on a finite-differencing method known as the Crank-Nicolson scheme, which discretises Equation (3.19) in the spatial and energy (momentum) dimensions. Once it is discretised, the operators of each dimension are solved alternatively through a technique known generally as ‘operator splitting’, a technique which drastically simplifies the computational complexity of the problem, as will be discussed further in detail below. While there are a significant number of numerical effects that need to be considered and potentially tweaked for different use-cases, the outcome of this method is a noticeable computational speed-up over the semi-analytical method described in Section 4.1.

The other important benefit to using numerical solutions is the ability to consider the full spatial dependence of the relevant magnetic field and gas density profiles throughout the modelling process. In various cases, such as for dSphs, the spatial profiles of these properties are still highly uncertain. Thus, in the past they have often been estimated or had their spatial dependence completely averaged out (with the exceptions of Storm et al. (2013) and Beck (2019b), which use a more physically motivated approach to the averaging of these quantities, see also Sarkis et al. (2022) for details of this approach). As the data from astronomical observatories become more precise in the future and we have higher resolution images to hunt for DM in though, our understanding of the physical properties of these objects will also improve. This makes having a solution to Equation (3.19) that contains an accurate spatial treatment of these properties vital in any future searches for DM that use these new data sets.

In the following Sections, the numerical technique that we have developed to solve the diffusion-loss equation is described in detail. This technique addresses some of the issues present in other solution methods, and will hopefully occupy a niche that exists in the DM indirect-detection hunting space: as a computational tool that is efficient, general and accurate.

¹Package website: <https://galprop.stanford.edu/>

²Online repository : <https://github.com/cosmicrays>

4.2 The Crank-Nicolson scheme

The foundation of our numerical solution to Equation (3.19) is a basic finite-differencing method known as the *Crank-Nicolson* (hereafter CN) scheme. In setting up the necessary framework for this solution, we closely follow the textbook Press et al. (2007, ch. 20) which contains a comprehensive list of details for the numerical treatment of partial differential equations, and draw other aspects of the solution from implementations of the scheme in Strong et al. (1998) and Regis et al. (2015b). We also note that the majority of the notation in this Section follows Strong et al. (1998), and any differences are noted where appropriate in the text. Before proceeding to exhaustively detail the method, we first provide a brief outline on some of the concepts and terms used to describe relevant finite-differencing methods by working through a simple, but instructive, example.

Simple Example

Consider a simple diffusion equation with a constant diffusion function D , i.e.

$$\frac{\partial \psi}{\partial t} = D \frac{\partial^2 \psi}{\partial x^2}. \quad (4.13)$$

To work with this equation numerically, we can discretise the temporal (t) and spatial (x) variables into a set number of points that are regularly spaced. These points, collectively referred to here as a grid, are constructed as follows:

$$\begin{aligned} x_i &= x_0 + i\Delta x, & i &= 0, 1, \dots, I \\ t_n &= t_0 + n\Delta t, & n &= 0, 1, \dots, N \end{aligned} \quad (4.14)$$

where the 0 subscript refers to the initial point in both grids, Δx , Δt the spacings between the grid points in each dimension (sometimes referred to as a *step*) and the grid variables i and n are non-negative integers. Each point in the grid, and the function itself, can then be labelled by i and n . This is usually written in a shorthand way, and in this work it is taken to be $\psi_i^n \equiv \psi(x_i, t_n)$. With these grids, a finite-differencing scheme can utilise known values of the function at certain points to iteratively calculate values at further points. A relevant scheme for performing these calculations is the Forward-Time and Centered-Space (FTCS) method, which discretises the derivatives in the equation (just using the definition of a derivate) in the following manner (Press et al. 2007, eq. 20.1.9-10):

$$\left. \frac{\partial \psi}{\partial t} \right|_{i,n} \rightarrow \frac{\psi_i^{n+1} - \psi_i^n}{\Delta t} + \mathcal{O}(\Delta t) \quad (4.15)$$

and

$$\begin{aligned} \left. \frac{\partial \psi}{\partial x} \right|_{i,n} &\rightarrow \frac{\psi_{i+1}^n - \psi_{i-1}^n}{2\Delta x} + \mathcal{O}(\Delta x^2), \\ \left. \frac{\partial^2 \psi}{\partial x^2} \right|_{i,n} &\rightarrow \frac{\psi_{i+1}^n - 2\psi_i^n + \psi_{i-1}^n}{\Delta x^2} + \mathcal{O}(\Delta x^2). \end{aligned} \quad (4.16)$$

In these cases, the temporal derivative has had a *forward* differencing scheme applied to it (i.e. points forward in the time grid are used) and the spatial derivative has had a *central* scheme applied to it (i.e. points both forward and backward in the grid are used). Ignoring higher order terms, the full FTCS discretisation of Equation (4.13) is therefore (Press et al. 2007, eq. 20.2.4)

$$\frac{\psi_i^{n+1} - \psi_i^n}{\Delta t} = D \left[\frac{\psi_{i+1}^n - 2\psi_i^n + \psi_{i-1}^n}{\Delta x^2} \right]. \quad (4.17)$$

Since this equation evolves in time and space, a computational solution method requires some prior knowledge about the function at certain grid points. This information is usually given in the form of initial conditions (a description of the function ψ at time t_0) and boundary conditions

(a description of the behaviour of ψ on its boundary) – these conditions will be discussed in the broader context of the work in Section 4.3.5. For example, with known function values at all current time points n , the above equation could be re-arranged in terms of $n + 1$ and then solved iteratively until the equilibrium distribution is reached. This characteristic is referred to as *single-level explicit* finite-differencing, since there is a single point in the grid needed for the calculations (at, say, point n) and subsequent function values can be calculated explicitly from the known points.

However, while instructive, the explicit FTCS method presented above has a drawback for practical use with general diffusion equations. An application of the standard von Neumann stability analysis (Press et al. 2007, ch. 20.1.1) shows that for the iterative solution to be stable, the criterion

$$\frac{2D\Delta t}{\Delta x^2} \leq 1 \quad (4.18)$$

must be satisfied (for further general details about the numerical stability of finite-differenced equations, see Press et al. (2007, ch. 20), and for details regarding the stability of the solution method developed in this work see Section 4.3.7). In general, the characteristic diffusion length-scale (λ_D) will be larger than the grid spacing Δx . Then, since the corresponding diffusion timescale of $\tau_D \sim \lambda_D^2/D$ will be much larger than the grid spacing Δt required by the stability condition, the number of iterations required to find the equilibrium solution also becomes large. As computational resources are valuable (especially with high-resolution grids) a high iteration count could hinder studies using this method in practice.

The above consideration is the motivation for defining another finite-differencing technique, known as *implicit* differencing, which replaces the explicit nature of the above technique. Implicit methods simply use advanced grid points (i.e. $n + 1$ here) in the calculation, instead of relying solely on known information at current points during each iteration. Such a scheme used on Equation (4.13) leads to the discrete equation

$$\frac{\psi_i^{n+1} - \psi_i^n}{\Delta t} = D \left[\frac{\psi_{i+1}^{n+1} - 2\psi_i^{n+1} + \psi_{i-1}^{n+1}}{\Delta x^2} \right], \quad (4.19)$$

which could be solved for the $n + 1$ grid points through a set of simultaneous linear equations. Now, the von Neumann stability criterion is

$$\left| \frac{1}{1 + 4(D\Delta t/\Delta x^2) \sin^2(k\Delta x/2)} \right| \leq 1, \quad (4.20)$$

where k (a wave-number used in the stability analysis) is arbitrary. Unlike in the explicit case, this stability condition is always satisfied, so that choices of the grid spacing Δt can be made without fear of the solution diverging from the equilibrium solution. However, if the choice of Δt is too large, all effects that occur on smaller characteristic timescales will be completely lost in the function ψ during each iteration. The lack of this small-scale accuracy could be another hindrance to studies that use this method, especially when using modern observational data that have high sensitivities and can resolve a variety of spatial scales.

The above-mentioned issues of stability and accuracy can both be neatly solved by the CN scheme, which uses a combination of explicit and implicit terms in the finite-differenced equation. Specifically, it considers the average of explicit and implicit FTCS terms (Equations (4.17) and (4.19)), which results in the following expression:

$$\frac{\psi_i^{n+1} - \psi_i^n}{\Delta t} = \frac{D}{2} \left(\frac{(\psi_{i+1}^{n+1} - 2\psi_i^{n+1} + \psi_{i-1}^{n+1}) + (\psi_{i+1}^n - 2\psi_i^n + \psi_{i-1}^n)}{\Delta x^2} \right). \quad (4.21)$$

A von Neumann stability analysis reveals that this scheme retains the unconditional stability of the implicit method, and since it relies on both n and $n + 1$ grid points at each iteration, it also has better (second-order) accuracy for small-scale effects. These characteristics are beneficial for a computational study, and are the reason this scheme is chosen as the foundation for all of the popular numerical cosmic-ray propagation codes. The method presented above has, however, been based on a simple example of constant diffusion in one spatial dimension. Since this is not the case in our work (consider Equation (3.19)), we proceed by presenting a generalisation of the scheme that allows for a more efficient description in the case of more complex diffusion problems.

Generalised Scheme (1-D)

In realistic physical scenarios, the diffusion function is not described only by some scalar value. In these cases, and for a one-dimensional grid, it is common to write the CN scheme in terms of generalised α -coefficients which contain some combination of the diffusion function and grid spacings, as follows:

$$\frac{\psi_i^{n+1} - \psi_i^n}{\Delta t} = \underbrace{\frac{\alpha_1 \psi_{i-1}^{n+1} - \alpha_2 \psi_i^{n+1} + \alpha_3 \psi_{i+1}^{n+1}}{2\Delta t}}_{\text{implicit}} + \underbrace{\frac{\alpha_1 \psi_{i-1}^n - \alpha_2 \psi_i^n + \alpha_3 \psi_{i+1}^n}{2\Delta t}}_{\text{explicit}} + Q_i, \quad (4.22)$$

Here we have just formed the average of implicit and explicit FTCS schemes as before, while generalising the coefficients of each term (the benefit of this will become clear when we begin to deal with multi-variate diffusion functions). We have also introduced a source term Q_i , which accounts for the continuous injection of new electrons into the halo by WIMP annihilation. This general expression can now be re-arranged to isolate the implicit and explicit terms:

$$-\frac{\alpha_1}{2} \psi_{i-1}^{n+1} + \left(1 + \frac{\alpha_2}{2}\right) \psi_i^{n+1} - \frac{\alpha_3}{2} \psi_{i+1}^{n+1} = Q_i \Delta t + \frac{\alpha_1}{2} \psi_{i-1}^n + \left(1 - \frac{\alpha_2}{2}\right) \psi_i^n + \frac{\alpha_3}{2} \psi_{i+1}^n. \quad (4.23)$$

Equation (4.23) is the form of the CN scheme used in GALPROP (eq. 30 in the GALPROP Explanatory Supplement found on the project website³) and by Regis et al. (2015b, eq. A3 – although we note a missing negative sign in this equation). It is a representation of a matrix equation, with the α -coefficients constituting the components of tridiagonal matrices in the overall updating equation

$$A\psi^{n+1} = B\psi^n + Q, \quad (4.24)$$

or in a more explicit form:

$$\begin{pmatrix} \left(1 + \frac{\alpha_2}{2}\right) & -\frac{\alpha_3}{2} & & 0 \\ -\frac{\alpha_1}{2} & \left(1 + \frac{\alpha_2}{2}\right) & & \\ & \ddots & \ddots & \\ 0 & & -\frac{\alpha_1}{2} & \left(1 + \frac{\alpha_2}{2}\right) \end{pmatrix} \begin{pmatrix} \psi_0^{n+1} \\ \psi_1^{n+1} \\ \vdots \\ \psi_{I-1}^{n+1} \\ \psi_I^{n+1} \end{pmatrix} = \begin{pmatrix} \left(1 - \frac{\alpha_2}{2}\right) & \frac{\alpha_3}{2} & & 0 \\ \frac{\alpha_1}{2} & \left(1 - \frac{\alpha_2}{2}\right) & & \\ & \ddots & \ddots & \\ 0 & & \frac{\alpha_1}{2} & \left(1 - \frac{\alpha_2}{2}\right) \end{pmatrix} \begin{pmatrix} \psi_0^n \\ \psi_1^n \\ \vdots \\ \psi_{I-1}^n \\ \psi_I^n \end{pmatrix} + Q_i \Delta t. \quad (4.25)$$

Since the α coefficients all contain known quantities (see Section 4.3.4 for their explicit forms), the above equation could now naively be solved for ψ^{n+1} iteratively using some linear algebra. A significant bonus to this scheme is that the matrices are tridiagonal (and also extremely sparse when using high-resolution grids – we will discuss this point further below), which allows for the use of specialised solution algorithms on the matrix equation. Since matrix algebra can be computationally taxing for large grids and many iterations, the tridiagonal nature of these matrices can be exploited for computational solutions that are relatively efficient. The equation above will also be unconditionally stable for any choice of Δt , as already stated. Despite these advantages, finding an accurate equilibrium solution in both an optimal and practical way requires some extra consideration. These will be discussed in detail in the following Section.

³<https://galprop.stanford.edu/code.php?option=manual>

4.3 Algorithm details

As stated, the generalised CN scheme in Equation (4.23) is sufficient to solve simple diffusion problems in one dimension. In the case of the diffusion-loss equation considered in this work however, the electron distribution and diffusion function are not only multivariate, but the equation also has a term associated with energy losses. To effectively make use of the scheme and benefit from its stability, we have thus developed various extensions to the general version presented above. These extensions include a generalisation of the scheme to more than one dimension, techniques that reduce the computational cost of the algorithm, and a method to ensure that the solution is accurate on various relevant physical scales.

4.3.1 Symmetries and Transformations

Coordinate frame Since we assume spherical symmetry at all stages of the method, the spatial derivatives in Equation (3.19) are written as follows:

$$\begin{aligned}\nabla \cdot (D\nabla\psi) &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial\psi}{\partial r} \right), \\ &= \frac{1}{r^2} \left(2rD \frac{\partial\psi}{\partial r} + r^2 \frac{\partial D}{\partial r} \frac{\partial\psi}{\partial r} + r^2 D \frac{\partial^2\psi}{\partial r^2} \right), \\ &= \left(\frac{2D}{r} \frac{\partial\psi}{\partial r} + \frac{\partial D}{\partial r} \frac{\partial\psi}{\partial r} + D \frac{\partial^2\psi}{\partial r^2} \right).\end{aligned}\quad (4.26)$$

Equation (3.19) can now be expanded to:

$$\frac{\partial\psi(r, E)}{\partial t} = \frac{2D}{r} \frac{\partial\psi}{\partial r} + \frac{\partial D}{\partial r} \frac{\partial\psi}{\partial r} + D \frac{\partial^2\psi}{\partial r^2} + \frac{\partial b}{\partial E} \psi + b \frac{\partial\psi}{\partial E} + Q. \quad (4.27)$$

Logarithmic scaling Because of the large range of length and energy scales involved in the processes considered here, we make the variable transformations

$$\tilde{r} = \log_{10} \left(\frac{r}{r_0} \right), \quad \frac{\partial}{\partial r} = \frac{10^{-\tilde{r}}}{\ln(10)r_0} \frac{\partial}{\partial \tilde{r}}, \quad (4.28)$$

and

$$\tilde{E} = \log_{10} \left(\frac{E}{E_0} \right), \quad \frac{\partial}{\partial E} = \frac{10^{-\tilde{E}}}{\ln(10)E_0} \frac{\partial}{\partial \tilde{E}}, \quad (4.29)$$

where r_0, E_0 are constant scale values. The scale values can be chosen somewhat arbitrarily (although for convenience should be chosen to represent the relevant physical scales), and in all cases of interest in this work we use the values of $r_0 = r_s$ (where r_s is the scale radius of the DM halo) and $E_0 = 1$ GeV. Substituting the above definitions into Equation (4.27) yields

$$\frac{\partial\psi}{\partial t} = \frac{10^{-2\tilde{r}}}{(\ln(10)r_0)^2} \left(\ln(10)2D \frac{\partial\psi}{\partial \tilde{r}} + \frac{\partial D}{\partial \tilde{r}} \frac{\partial\psi}{\partial \tilde{r}} + D \frac{\partial^2\psi}{\partial \tilde{r}^2} \right) + \frac{10^{-\tilde{E}}}{\ln(10)E_0} \left(\frac{\partial b}{\partial \tilde{E}} \psi + b \frac{\partial\psi}{\partial \tilde{E}} \right) + Q, \quad (4.30)$$

or grouping by derivatives of ψ (useful for finding the explicit forms of the α -coefficients later),

$$\frac{\partial\psi}{\partial t} = \frac{10^{-2\tilde{r}}}{(\ln(10)r_0)^2} \left(\left[\ln(10)2D + \frac{\partial D}{\partial \tilde{r}} \right] \frac{\partial\psi}{\partial \tilde{r}} + D \frac{\partial^2\psi}{\partial \tilde{r}^2} \right) + \frac{10^{-\tilde{E}}}{\ln(10)E_0} \left(\frac{\partial b}{\partial \tilde{E}} \psi + b \frac{\partial\psi}{\partial \tilde{E}} \right) + Q. \quad (4.31)$$

To not clutter these equations we have dropped the variable dependance of each function, but we note that the functions ψ, Q, D and b all now depend on the variables $\tilde{r}$ and $\tilde{E}$. This variable transformation results in function evaluations and discrete grid points that are more convenient to work with for a wide range of possible values. A similar transformation is made in Regis et al. (2015b), wherein they set $r = \ln(r/r_0)$. Our choice of using base 10

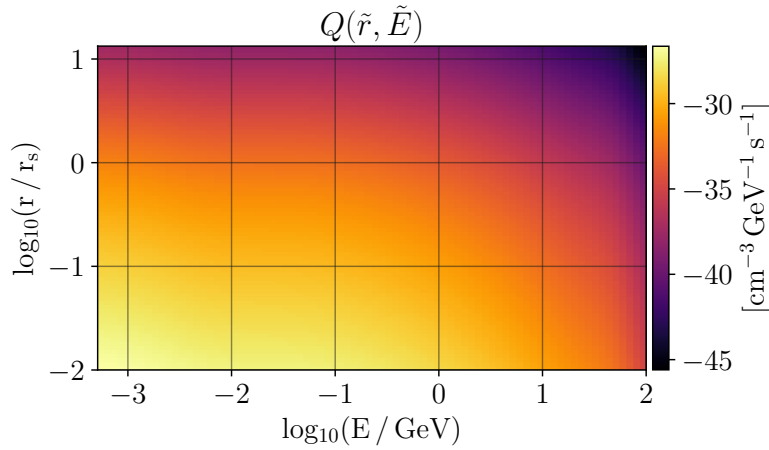


Figure 4.1: The source function $Q(\tilde{r}, \tilde{E})$, from Equation (3.3), represented visually on the $\tilde{r}$ and $\tilde{E}$ grids. This example function has been generated with 100 GeV WIMPs annihilating through the $b\bar{b}$ channel, with a set of halo parameters describing the Abell 4038 galaxy cluster (discussed in more detail in Section 5.3). Note that the injection of electrons tends to occur at larger densities in central regions of the halo and at lower relative energies.

instead of natural logarithms is just for convenience, the purpose of both transformations is identical.

Visual representations of the functions Q (Equation (3.3)), D (Equation (3.22)) and b (Equation (3.30)) occupying the log-transformed radial-energy grid are given in Figures 4.1, 4.2 and 4.3 respectively. These figures have been made with an example computation involving the Abell 4038 cluster, and visual ‘snapshots’ of functions like these during the algorithm run-time often provide insight into the accuracy and stability of the numerical method (these aspects will be discussed further in Section 4.3.7).

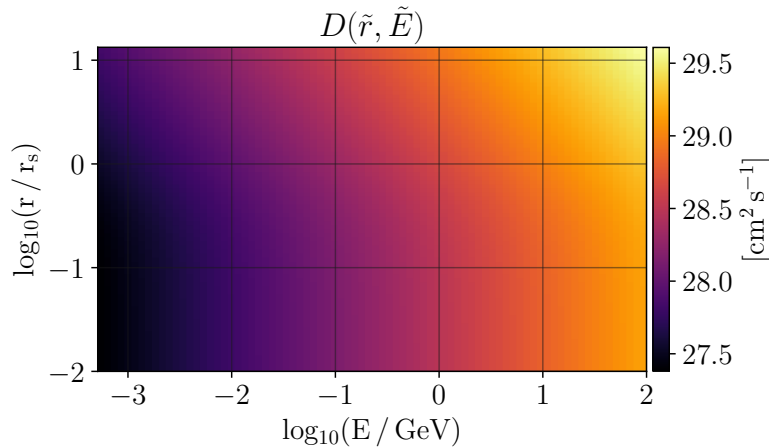


Figure 4.2: The diffusion coefficient function $D(\tilde{r}, \tilde{E})$, from Equation (3.22). Similar to Figure 4.1.

Operator splitting and the ADI method

A vital extension to the general scheme is a method to handle the multiple dimensions in the problem – in our case the radius of the halo (now represented by $\tilde{r}$) and the energy spectrum of the electrons ($\tilde{E}$). One of the major computational benefits of the CN scheme was the tridiagonal and sparse nature of the coefficient matrices A and B . However, if the full form of the scheme is used on a multivariate diffusion equation, the matrices are no longer tridiagonal. Since it

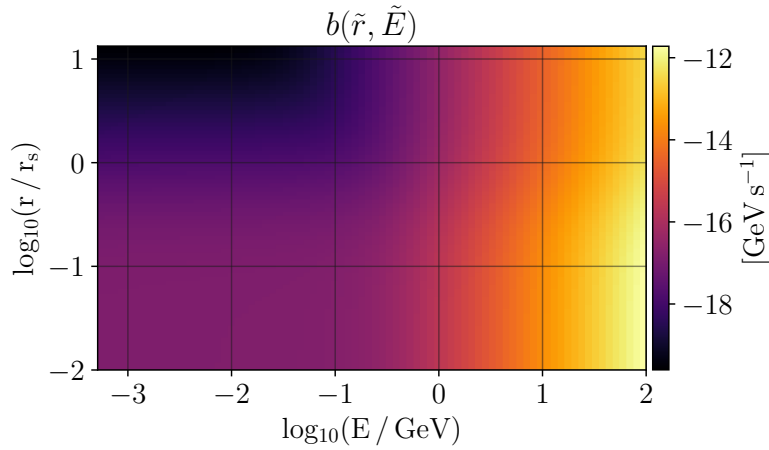


Figure 4.3: The energy-loss coefficient function $b(\tilde{r}, \tilde{E})$, from Equation (3.30). Similar to Figure 4.1.

would be unfortunate to lose this benefit, we instead opt for an alternative approach known as *operator splitting* (OS). This method is explained succinctly in Press et al. (2007, ch. 20.3.3), and is summarised as follows: when an equation can be written with a linear combination of independent operators, i.e. if the operator $\mathcal{L}$ in the equation

$$\frac{\partial \psi}{\partial t} = \mathcal{L} \psi \quad (4.32)$$

can be split up into the operators

$$\mathcal{L} \psi = \mathcal{L}_1 \psi + \mathcal{L}_2 \psi, \quad (4.33)$$

then each of $\mathcal{L}_1$ and $\mathcal{L}_2$ can be independently applied in-turn during an iterative solution algorithm. A requirement of this method is that each of the operators have valid finite differencing schemes when in the absence of all others, which is satisfied in our case. Using this method, we can then split the operators in Equation (4.31) as follows:

$$\mathcal{L}_r = \frac{10^{-2\tilde{r}}}{(10 \ln(10) r_0)^2} \left(2D \frac{\partial}{\partial \tilde{r}} + \frac{\partial D}{\partial \tilde{r}} \frac{\partial}{\partial \tilde{r}} + D \frac{\partial^2}{\partial \tilde{r}^2} \right) \quad (4.34)$$

and

$$\mathcal{L}_E = \frac{\partial b}{\partial \tilde{E}} + b \frac{\partial}{\partial \tilde{E}}. \quad (4.35)$$

If we represent the finite differencing schemes of these operators symbolically as Ψ_r and Ψ_E , then, using the standard Centered-Space scheme for the spatial dimension:

$$\mathcal{L}_r \rightarrow \Psi_r = \frac{10^{-2\tilde{r}_i}}{(10 \ln(10) r_0)^2} \left[\frac{\psi_{i+1} - \psi_{i-1}}{2\Delta\tilde{r}} \left(\ln(10) 2D + \frac{\partial D}{\partial \tilde{r}} \right) + \frac{\psi_{i+1} - 2\psi_i + \psi_{i-1}}{\Delta\tilde{r}^2} D \right]. \quad (4.36)$$

For the energy dimension, as in GALPROP and Regis et al. (2015b), we use an *upstream* differencing scheme. This is a scheme that only uses advanced grid points (i.e. $j+1$), under the assumption that energy is only lost, and not gained. Then,

$$\mathcal{L}_E \rightarrow \Psi_E = \frac{10^{-\tilde{E}_j}}{(E_0 \ln(10))} \left[\frac{b\psi_{j+1} - b\psi_j}{\Delta\tilde{E}} \right]. \quad (4.37)$$

In the above schemes the Δ quantities have their usual meanings and the subscripts i and j have been reserved for the spacial and energy grids, respectively. The overall updating scheme for the points n to $n+1$ can then be expressed symbolically as

$$\begin{aligned} \psi^{n+1/2} &= \Psi_r(\psi^n, \Delta t/2) \\ \psi^{n+1} &= \Psi_E(\psi^{n+1/2}, \Delta t/2). \end{aligned} \quad (4.38)$$

Thus, at each iteration of the solution algorithm, the individual operators (which in our case also encapsulate each dimension) are evaluated sequentially. This ensures that the coefficient matrices A and B retain their tridiagonal forms at each step of the method.

Before proceeding, we make note of a point of ambiguity that has existed in the literature around the naming of this solution method. In Regis et al. (2015b, app. A) and all versions of the GALPROP Explanatory Supplement (sec. 5.3.3) available at the time of writing, there is a description of a solution method that is referred to as the Alternating-Direction-Implicit (ADI) method. The details of this method are similar to those of the operator splitting technique presented above, except for one subtle, but distinct, technical difference. In the ADI method, during each ‘half’-step in Equation (4.38), the implicit part of the CN method is applied to each operator, while the other is treated explicitly (ensuring that the full operator is used during every half-step). Since the outline of the method in Regis et al. (2015b) specifies “the implicit updating scheme is alternately applied to the r - and E -operators in turn, keeping the other coordinate fixed”, it seems (albeit without full knowledge of the algorithms actually used by these authors) as though the use of the term ADI is in error, since this scheme would not use the full operator at every half-step. The most recent publication associated with the development of the GALPROP package (Porter et al. 2022, sec. 3.3.1) refers to the method used only as operator splitting, and makes no mention of the ADI method. We thus refer to the solution shown above likewise, and note that results appearing in previous publications of this work might erroneously refer to it as the ADI method. In all circumstances, the underlying method is identical, and the difference is in name only.

4.3.2 Block matrices

The computation of (4.38) involves the solution of large matrix equations at each step of the algorithm. Below, we describe a method for making use of the fast and efficient vector calculations in the `scipy` package, which reduces the number of iterations and method calls necessary in the algorithm and significantly speeds up the run-time of the code. Now, in our case the electron distribution $\psi_{i,j}^n(\tilde{r}, \tilde{E})$ (and similarly other functions like Q , D and b) can be written in the form:

$$\psi_{i,j} = \begin{pmatrix} \psi_{0,0} & \psi_{1,0} & \cdots & \psi_{I,0} \\ \psi_{0,1} & \psi_{1,1} & \cdots & \psi_{I,1} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{0,J} & \psi_{1,J} & \cdots & \psi_{I,J} \end{pmatrix}, \quad (4.39)$$

where we have dropped the index n for readability, and the subscripts i and j denote the value of ψ at each dimensional grid point, as before. To use the operator splitting method to update ψ in each dimension independently, we restructure the overall matrix equation and the shapes of the matrices and vectors as follows. For each dimension, flatten ψ to the form

$$\tilde{r} : \psi_{i,j} = \left\{ \begin{pmatrix} \psi_{0,0} \\ \psi_{1,0} \\ \psi_{2,0} \\ \vdots \\ \psi_{I,0} \\ \psi_{0,1} \\ \psi_{1,1} \\ \vdots \\ \psi_{I,J} \end{pmatrix} \right\} \text{ const } \tilde{E} \quad \text{and} \quad \tilde{E} : \psi_{i,j} = \left\{ \begin{pmatrix} \psi_{0,0} \\ \psi_{0,1} \\ \psi_{0,2} \\ \vdots \\ \psi_{0,J} \\ \psi_{1,0} \\ \psi_{1,1} \\ \vdots \\ \psi_{I,J} \end{pmatrix} \right\} \text{ const } \tilde{r} \quad (4.40)$$

Now, in each representation of the function ψ there are ‘blocks’ of sub-matrices that are composed solely of function values that have one variable kept constant. Consequently, each of the matrices A and B in the general updating system should then be composed of an equal number of matrices that act on the ‘one-dimensional’ blocks in ψ . We implement this by creating block

matrices, represented here by $\mathcal{M}$, which have the following form:

$$\mathcal{M}(\tilde{r}, E_j) = \begin{pmatrix} \left(1 + \frac{\alpha_2(\tilde{r}_0, \tilde{E}_j)}{2}\right) & -\frac{\alpha_3(\tilde{r}_0, \tilde{E}_j)}{2} & & & \\ -\frac{\alpha_1(\tilde{r}_1, \tilde{E}_j)}{2} & \left(1 + \frac{\alpha_2(\tilde{r}_1, \tilde{E}_j)}{2}\right) & & & \\ & & \ddots & & \\ & & & \left(1 + \frac{\alpha_2(\tilde{r}_{I-1}, \tilde{E}_j)}{2}\right) & -\frac{\alpha_3(\tilde{r}_{I-1}, \tilde{E}_j)}{2} \\ & & & -\frac{\alpha_1(\tilde{r}_I, \tilde{E}_j)}{2} & \left(1 + \frac{\alpha_2(\tilde{r}_I, \tilde{E}_j)}{2}\right) \end{pmatrix} \quad (4.41)$$

and similarly for $\mathcal{M}(\tilde{r}_i, \tilde{E})$. We note that these matrices are constructed similarly to those in the one-dimensional case in Equation (4.25). With this definition, the full A matrices in each step of the algorithm should then resemble:

$$A_{\tilde{r}} = \begin{pmatrix} \mathcal{M}(\tilde{r}, \tilde{E}_0) & & & & 0 \\ & \mathcal{M}(\tilde{r}, \tilde{E}_1) & & & \\ & & \ddots & & \\ & & & \mathcal{M}(\tilde{r}, \tilde{E}_{J-1}) & \\ 0 & & & & \mathcal{M}(\tilde{r}, \tilde{E}_J) \end{pmatrix} \quad (4.42)$$

for the spatial dimension and:

$$A_{\tilde{E}} = \begin{pmatrix} \mathcal{M}(\tilde{r}_0, \tilde{E}) & & & & 0 \\ & \mathcal{M}(\tilde{r}_1, \tilde{E}) & & & \\ & & \ddots & & \\ & & & \mathcal{M}(\tilde{r}_{I-1}, \tilde{E}) & \\ 0 & & & & \mathcal{M}(\tilde{r}_I, \tilde{E}) \end{pmatrix} \quad (4.43)$$

for the energy dimension. For the sake of brevity the B matrices are not written out here, but they are constructed in a corresponding manner. The total size of each matrix will then be the total size of each block matrix times the number of block matrices, for example $\mathcal{M}(\tilde{r}, E_j) \in \mathbb{R}^{I \times I}$ and $A_{\tilde{r}} \in \mathbb{R}^{IJ \times IJ}$. With these quantities, and the general updating scheme from Equation (4.24), each half-step of the operator splitting solution method can now be solved with a single matrix equation function call, instead of I or J times. This results in a significant improvement in the overall speed of the algorithm.

4.3.3 Sparse matrices

The sparsity index, which represents the percentage of zero-valued elements in a matrix, can be calculated by

$$\mathcal{S} = \frac{N - N_{nz}}{N} \times 100\%, \quad (4.44)$$

where N and N_{nz} are the total number and number of non-zero elements in the matrix. From the forms of Equations (4.42) and (4.43), together with the form of each block matrix $\mathcal{M}$, it is quite clear that these matrices are highly sparse – each having only a very small fraction of non-zero entries. For example, when $I = 61$ and $J = 71$, the sparsity indices are $\mathcal{S}(A_{\tilde{E}}) = 99.954\%$ and $\mathcal{S}(A_{\tilde{r}}) = 99.931\%$. In such cases, the use of the full-sized ($IJ \times IJ$) matrices for computations would be extremely resource-inefficient. Thus, when working with these matrices in the code we make use of the `scipy.sparse` Python module, which allows for the representation of sparse matrices in various memory-efficient storage configurations.

Once the A and B matrices are constructed using the methods shown above, they are subsequently stored in the code as sparse matrix objects. We have chosen the ‘csr’ format, which specifies the particular non-zero element storage method: *compressed sparse row*. The inner algorithms used by this format are outside the scope of this work, but it should be noted that this format was chosen for its fast array vector operations, as stated in the `scipy` documentation. Since we are performing vector operations on these objects in the solution of the matrix equation, this

format seems to be the ideal choice.⁴ The solution of the matrix equation is then performed with the `sparse.linalg.spsolve` method. After profiling the algorithm, the bottleneck in runtime performance comes primarily from this method, as it is called during every iteration (each half-step). In contrast, constructing the A and B matrices only happens once for each value of Δt that is used, which is typically only a few dozen at most. Since there could be hundreds or thousands of iterations in a typical solution, the efficiency of the `spsolve` method almost completely determines the computational performance of the algorithm.

4.3.4 α -Coefficients

The explicit forms of the α -coefficients in Equation (4.23) can be found by equating coefficients with the expressions of $\Psi_{\tilde{r}}$ and $\Psi_{\tilde{E}}$. This results in the following:

$$\Psi_{\tilde{r}} : \begin{cases} \frac{\alpha_1}{\Delta t} = C_{\tilde{r}} \left(-\frac{\ln(10)D(\tilde{r}_i, E_j) + \frac{\partial D}{\partial \tilde{r}}|_{\tilde{r}_i, E_j}}{2\Delta \tilde{r}} + \frac{D(\tilde{r}_i, E_j)}{\Delta \tilde{r}^2} \right) \\ \frac{\alpha_2}{\Delta t} = C_{\tilde{r}} \left(\frac{2D(\tilde{r}_i, E_j)}{\Delta \tilde{r}^2} \right) \\ \frac{\alpha_3}{\Delta t} = C_{\tilde{r}} \left(\frac{\ln(10)D(\tilde{r}_i, E_j) + \frac{\partial D}{\partial \tilde{r}}|_{\tilde{r}_i, E_j}}{2\Delta \tilde{r}} + \frac{D(\tilde{r}_i, E_j)}{\Delta \tilde{r}^2} \right) \end{cases} \quad (4.45)$$

for the spatial dimension and

$$\Psi_E : \begin{cases} \frac{\alpha_1}{\Delta t} = 0 \\ \frac{\alpha_2}{\Delta t} = C_{\tilde{E}} \frac{b_j}{\Delta E} \\ \frac{\alpha_3}{\Delta t} = C_{\tilde{E}} \frac{b_{j+1}}{\Delta E} \end{cases} \quad (4.46)$$

for the energy. Here $C_{\tilde{r}} = \frac{10^{-2\tilde{r}_i}}{(\ln(10)r_0)^2}$ and $C_{\tilde{E}} = \frac{10^{-\tilde{E}_j}}{\ln(10)E_0}$.

4.3.5 Initial and boundary conditions

For the initial condition ($n = 0$) of ψ , we simply use the DM electron source function $Q(\tilde{r}, \tilde{E})$, i.e.

$$\psi_{i,j}^0 = Q_{i,j}. \quad (4.47)$$

We also use the following Dirichlet and Neumann boundary conditions:

$$\begin{aligned} \psi(\tilde{r} = \tilde{r}_{\max}) &= 0, \\ \frac{\partial \psi}{\partial \tilde{r}}(\tilde{r} = \tilde{r}_{\min}) &= 0. \end{aligned} \quad (4.48)$$

The Dirichlet condition (which corresponds to the free-streaming of the electrons at the diffusion radius) is simply enforced by setting all values of ψ at $\tilde{r} = \tilde{r}_{\max}$ to 0 during each iteration of the OS method. The value of $\tilde{r}_{\max}$ can be set manually in the code, and suitable values should be chosen based on the relevant environments. For the results of the galaxy clusters shown in this work we have used a value of $\tilde{r}_{\max} = 2R_{\text{vir}}$, where R_{vir} is the virial radius of the DM halo. In smaller structures, like dSphs, such a large value of $\tilde{r}_{\max}$ will not always be suitable. This is due to the confinement of electrons in more compact, exponential magnetic fields that leave the outer regions

⁴There is also a 'dia' (diagonal) storage option listed in the `scipy` documentation. However, this format seems to clash with the algorithm used in the `scipy.spsolve()` method, which we utilise when solving the matrix equation. This may be fixed in the future and (potentially) provide some improvement in memory utilisation, but until then the 'csr' format seems to work well enough for our purposes.

of the DM halo void of radiating electrons. In these cases, values of $\tilde{r}_{\max}$ closer to the scale radius of the halo would be appropriate, although this should be determined by analysing the diffusive environment of individual targets.

The Neumann condition is enforced by using alternate α -coefficients at the points $\tilde{r} = \tilde{r}_{\min}$ in the coefficient matrices. This condition leads to a new differencing scheme, with α -coefficients given by:

$$\Psi_{\tilde{r}=\tilde{r}_{\min}} : \begin{cases} \frac{\alpha_1}{\Delta t} &= 0 \\ \frac{\alpha_2}{\Delta t} &= C_{\tilde{r}} \left(\frac{4D(\tilde{r}_i, \tilde{E}_j)}{\Delta \tilde{r}^2} \right) \\ \frac{\alpha_3}{\Delta t} &= C_{\tilde{r}} \left(\frac{4D(\tilde{r}_i, \tilde{E}_j)}{\Delta \tilde{r}^2} \right) \end{cases} \quad (4.49)$$

The condition $\tilde{r} = \tilde{r}_{\min}$ is a representation of the smallest sampled radius in the spatial grid. This value can also be set manually in the code, and for the results shown in this work we use $\tilde{r}_{\min} = 0.01r_s$, where r_s is the scale radius of the DM halo.

4.3.6 Physical Scales

Characteristic timescales

As discussed in Section 4.1, the characteristic timescales associated with the physical processes in the diffusion-loss equation play an important role in the behaviour of the solution in different regimes. These factors are also important when determining the stability and convergence of the numerical method used. We define the timescales as follows, and discuss their impact on the method in Section 4.3.7.

Energy Losses:

$$\tau_E = \frac{\tilde{E}}{b(\tilde{E})} \quad (4.50)$$

Diffusion:

$$\tau_D = \frac{\Delta \tilde{r}^2}{D(\tilde{r}, \tilde{E})} \quad (4.51)$$

Electron Distribution: To estimate the timescale on which changes to ψ occur, we simply use a Forward-Time differencing scheme as follows:

$$\tau_\psi = \frac{\psi}{\left| \left(\frac{\partial \psi}{\partial t} \right) \right|} \quad (4.52)$$

$$\Rightarrow \tau_\psi = \psi \left| \left(\frac{\psi^{n+1} - \psi^n}{\Delta t} \right)^{-1} \right| \quad (4.53)$$

Diffusion velocity

The diffusion function (3.22) should not be allowed to take arbitrarily large values. We limit this function by requiring that the virial diffusion velocity, $v_D = D(\tilde{r}, \tilde{E})/R_{\text{vir}}$, be smaller than the speed of light. This is then implemented as an upper bound for D ,

$$v_D < c, \quad (4.54)$$

$$\Rightarrow D(\tilde{r}, \tilde{E}) < R_{\text{vir}} c. \quad (4.55)$$

We similarly enforce this condition on the function $\frac{\partial D}{\partial \tilde{r}}$, which appears in the α -coefficients of Equation (4.45).

4.3.7 Stability and convergence

The continuation or termination of the algorithm is managed by several conditions relating to how the electron distribution ψ is changing between iterations. These conditions are split into two categories: stability and convergence, as described below.

Stability

At each step of the operator splitting method (Equation (4.38)), the ψ distribution is updated according to the CN scheme. The changes to ψ during the initial few iterations are large, and then gradually decrease until it nears the equilibrium solution, which should then be independent of time. The condition of stability we require in this method is that the relative change in the distribution from time step $n - 1$ to n is small – i.e. the changes in ψ have begun to reduce and the function approaches a constant value over all grid points in $\tilde{r}$ and $\tilde{E}$. The precise condition enforced in the code depends on the method used in the algorithm, either the ‘accelerated’ method (ACC) or the ‘constant-step-size’ (CSS) method. Both of these methods are inspired by the GALPROP package (the relevant methods are described in the Explanatory Supplement, sec. 5.3.3 or in Strong et al. (1998, app. B)).

ACC method In the ACC method, the initial timestep (Δt_i) is chosen to be large compared to the timescales of all relevant physical effects. This value can be chosen manually in the code, and defaults to 1×10^9 years, but should always follow the condition

$$\Delta t_i > \max\{\tau_E, \tau_D\}. \quad (4.56)$$

The timestep Δt is then, from this initial value, reduced by a certain factor (the default value in the code is 0.5) after a minimum number of iterations ($n_{\min}$) have passed. The value of $n_{\min}$ can be chosen manually, and would usually benefit from some insights about the performance of the algorithm that are gained after running benchmark tests. In the ACC method, iterations above the cap of $n_{\min}$ are considered to be stable. The reduction in the value of Δt continues until some minimum value $\Delta t_{\min}$, which defaults to 100 years in the code.

CSS method In this method there are no hard caps on the number of iterations. Instead, we calculate the relative difference between the ψ distributions at subsequent iterations, and set a certain tolerance threshold for this value (say, 0.1%). If the relative difference, calculated as follows, is less than this tolerance then we consider the algorithm to be stable:

$$\left| 1 - \frac{\psi^{n-1}}{\psi^n} \right| < \text{tolerance}. \quad (4.57)$$

The ACC method is generally preferred for the majority of calculations. The primary reason for this is that in most relevant astrophysical scenarios, there are a wide range of timescales which affect the electron distribution. In the CSS method, using step sizes that are low enough to actually capture the effects of shorter timescales leads to the algorithm needing a huge number of iterations to converge on the equilibrium solution. Conversely, while needing only relatively few iterations, large timesteps can lead to inaccurate final solutions as the impact from short-timescale processes are lost during each iteration. The ACC method addresses both of these drawbacks, since the timestep is changed dynamically and covers all relevant timescales (provided that Δt_i and $\Delta t_{\min}$ are chosen appropriately). When dealing with various astrophysical scenarios, it could be of benefit to tweak the parameters discussed above to find a set that leads to a convergent and accurate final solution in the most computationally efficient way.

Convergence

Together with the conditions of stability mentioned above, the following convergence conditions dictate the exact iteration at which the final equilibrium distribution is found, and thus on which

iteration the algorithm should be terminated. A visual representation of the equilibrium distribution is shown in Figure 4.4. Again these conditions depend on the method, either ACC or CSS, being used. There is one condition we check for universally, regardless of the method used, namely:

$$\tau_\psi > \max\{\tau_E, \tau_D\}. \quad (4.58)$$

This condition ensures that the timescale of changes to the electron distribution is larger than the timescale of any physical effects involved, which should mean that there will be negligible changes to the distribution once this condition is true.

For the ACC method, we add an alternative condition to (4.58). After Δt reaches its lowest value, set by $\Delta t_{\min}$, we consider the algorithm to have converged if the relative difference between distributions snapshots at time $n - 1$ and time n is less than some tolerance (as in (4.57)). The reason for this is based in practicality. For some halo configurations, particularly the dSphs we have tested, the condition (4.58) requires a larger number of iterations to reach. After several hundred iterations, the relative difference between snapshots at time $n - 1$ and time n can be miniscule – on the order of 10^{-7} , but condition (4.58) is still not reached because one of the effect timescales (usually that of energy losses) is very large. This can occur when the energy scale, which relates to the WIMP mass considered, reaches very low values. In these cases, we allow condition (4.58) or (4.57) to signal convergence.

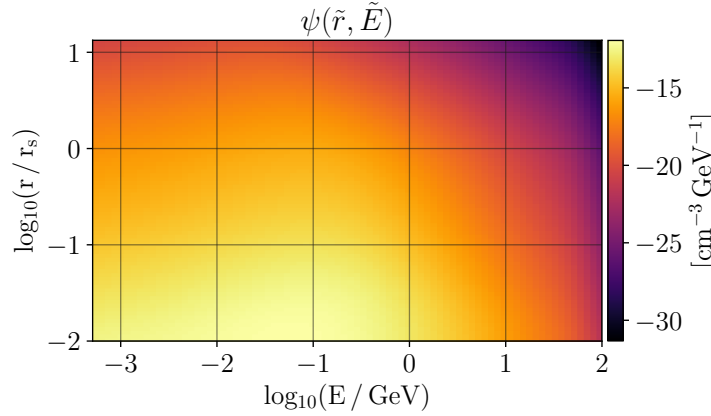


Figure 4.4: The equilibrium distribution of electrons, $\psi(\tilde{r}, \tilde{E})$. This is the final result of the solution method presented in this Chapter, and is used in Equation (3.49) to calculate the radio emissivity of the DM halo. This function can be compared to the initial condition in the algorithm, the WIMP annihilation source function $Q(\tilde{r}, \tilde{E})$, shown in Figure 4.1. Note the change in scale, and the slight shift in values across the $\tilde{r}, \tilde{E}$ grid. Note also the higher densities of electrons around mid-range energies in the inner regions of the halo.

4.4 Method comparisons and testing

There are two main considerations when describing the accuracy of the OS method. The first is the accuracy of the final equilibrium distribution of electrons on all relevant scales, i.e., have all of the physical effects had the correct impact on the distribution throughout the algorithm? The second is, how closely do the results of this numerical method compare to other solution methods, particularly the widely-used semi-analytical methods described in Section 4.1? While aspects of these two points have already been mentioned throughout this Chapter, we discuss them in more detail below.

4.4.1 Accuracy and benchmarking

The accuracy of the final solution on different physical scales is directly related to the size of the timestep Δt and how it compares to the timescales of the diffusion and energy loss processes,

in a similar manner to that mentioned in Section 4.3.7. To quantitatively measure the accuracy of the solution in this regard, we can run a set of ‘benchmark’ tests with the CSS method that have very strict convergence criteria. These benchmarks are based on the ones outlined in the GALPROP Explanatory Supplement (ch. 11), and consist mainly of using the CSS method with a very small timestep value ($\Delta t < \min\{\tau_E, \tau_D\}$) and letting the algorithm run without a cap on the number of iterations. With a suitable value of Δt , this results in a final solution that converges on the equilibrium distribution within the bounds of the precision attainable by the code. We enforce this, like in GALPROP, by changing the convergence condition of Equation (4.58) to be

$$\begin{aligned} \frac{\partial \psi}{\partial t} &= 0 \\ \Rightarrow \frac{\psi^{n-1} - \psi^n}{\Delta t} &= 0. \end{aligned} \quad (4.59)$$

This condition must hold for *all* radial and energy grid points in the electron distribution in order for the benchmark test to successfully converge.

In practice, the benchmarking test described above requires a considerable number of iterations to achieve, which takes a lot of computing time on a personal computer. While it does ensure the most accurate solution possible when using the OS method, it is inefficient to use when there are many computations needed (in the case of survey data being compared to many DM models, for instance). Further, the relative difference in the electron distribution between iterations tends to rapidly diminish at higher iteration numbers. This suggests that cutting out the final iterations that seek the above computer-precision convergence condition can, in general, result in significantly better computation times while having little risk of inaccuracies, which can be valuable in practical settings. To test the accuracy of solutions found with the much more efficient ACC method, we thus use the CSS method with the convergence condition Equation (4.59) as a benchmark test case for a set of results. We then compare the results to those found using the ACC method, and if they are compatible within a certain tolerance, we proceed to use the ACC method for testing various parameter variations. The choices of either method (CSS or ACC) and whether the convergence is based on the benchmark case are all controlled by flags in the code which allow for conveniently switching between methods.

4.4.2 Comparisons to semi-analytical method

The second measure of accuracy we consider is how closely the solution compares to other, independent methods. We have thus performed some basic tests of our algorithm using identical parameters to calculations with the semi-analytical method described in Section 4.1 (referred to from here as the Green’s Function method or GF). We first perform these calculations by using the same GF used in Colafrancesco et al. (2015a), Beck (2019b), and Sarkis et al. (2022), and then use a dedicated software package RX-DMFIT. The pertinent parameters used during the tests are shown in Table 4.1, and any relevant parameters not shown therein or mentioned explicitly are taken to be the standard values referred to at their descriptions in this thesis.

Calculations of the surface brightness for a set of targets and parameters were carried out in Sarkis et al. (2023a), using both the OS and GF methods. We have reproduced a subset of those results here, highlighting the most relevant comparison between methods in a galaxy cluster scenario. In this case we observe very close agreement between the two methods, with a large overlap in the surface brightness values for the full range of WIMP masses. The most notable trend in these results is that for lower mass WIMPs (which produce stronger emissions, due to the inverse mass relationship in the electron source function), there is a universal discrepancy in the surface brightness in the inner regions of the halo. In the case of low mass WIMPs the OS method produces slightly higher surface brightness values than the GF method in these regions, but in all cases the OS method seems to produce relatively brighter values compared to the outer regions of the halo. In the outer regions, there is negligible difference between the methods for low mass WIMPs. These results seem to indicate that the most significant differences between the two solution methods emerge from the centre of the halo. This is expected behaviour, as the spatial dependance of the diffusive environment is retained in the OS method, but not in the GF method.

We note that the GF calculations performed here used the solution method originally presented in Colafrancesco et al. (2006) (i.e. using Equation (4.5)), and not with the corrected form of this method (i.e. Equation (4.10)). Given that the difference between the outputs of these methods is estimated to be negligible for cuspy halos (like those used for the NFW profile of the Coma galaxy cluster), we do not expect significant deviation in the general behaviour of these results. However, we do expect that use of the corrected form of the GF method would yield tighter agreement with the numerical method, if spatially flat magnetic field and gas density profiles are used. We provide a detailed analysis of the discrepancies between the results of these methods in the following Section, which describes a similar set of calculations using an alternative code package.

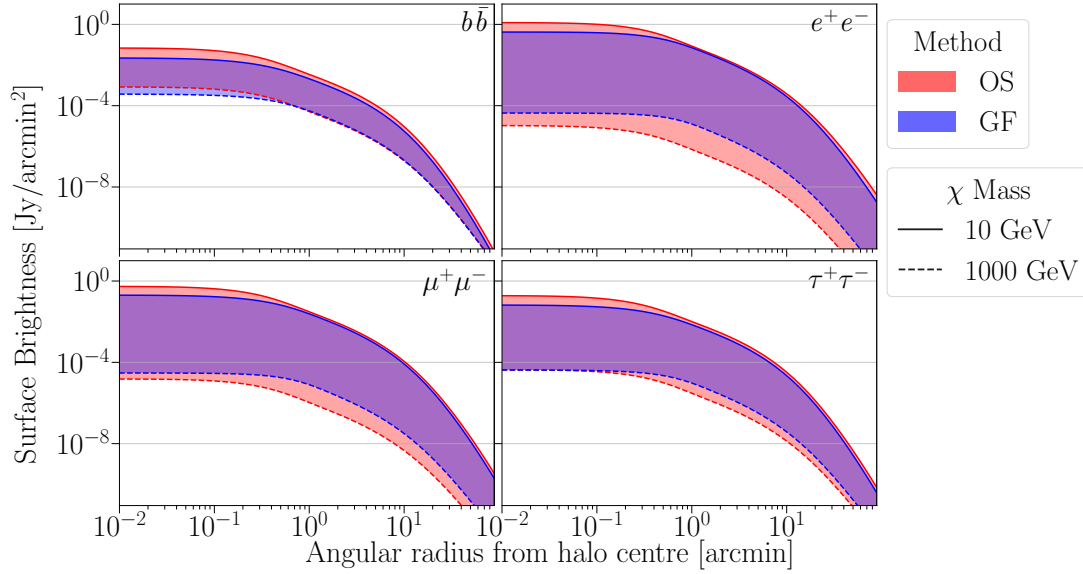


Figure 4.5: Surface brightness simulations for the Coma galaxy cluster, for a range of WIMP masses and four annihilation channels. The shaded regions between the maximum and minimum WIMP masses encapsulate the results for intermediate values. Note the extensive overlap in the regions for the two methods. These results were calculated for a frequency of 500 MHz. Reproduced from Sarkis et al. 2022.

4.4.3 RX-DMFIT

We have performed a set of simple calculations with the Radio and X-ray DMFIT (RX-DMFIT) package, for the purpose of comparing the results to the ones obtained with the method developed in this project. The code was originally written to extend the DMFIT package (Jeltema et al. 2008) with radio and X-ray calculations, particularly the emissivity and flux outputs from synchrotron and Inverse Compton (IC) processes. The code, written in C++, was obtained through an online GitHub repository⁵, which also hosts the package documentation. The information presented in this section has been taken from either the documentation or the publication associated with the package release (McDaniel et al. 2017). RX-DMFIT also necessarily makes use of two main external libraries: the DarkSUSY package (Gondolo et al. 2004; Bringmann et al. 2018) and the GNU Scientific Library (GSL) (Galassi 2009). These external tools provide the particle spectra for the annihilation of WIMPs (necessary for the source term Q) and the numerical integration routines used for various calculations in the solution method, respectively.

⁵Online repository : <https://github.com/alex-mcdaniel/RX-DMFIT> (commit #7e9a616 used in this project)

Description of the code

Because of the nature of the calculations we have performed with this software, (i.e., for comparative purposes) we provide a brief overview of the methods used in the code and some of its primary capabilities. Many of the concepts and methods used have already been described in this thesis, so we will often refer to previous equations and solution techniques. The defining feature of RX-DMFIT is that it uses the GF solution method to the diffusion-loss equation, and the method described in McDaniel et al. (2017, sec. 2) directly follows the one introduced in Colafrancesco et al. (2006) (also discussed above in Section 4.1). There are roughly 15 physical parameters exposed in the code (although other hard-coded parameters could be adjusted if necessary), and either the emissivity or the flux of synchrotron (radio) or IC (X-ray) can be computed from these parameters at any given time. At the time of writing, calculations of the surface brightness are not supported, so we instead use the emissivity as the primary output calculation for our comparisons.

As the code uses the GF method in the solution of the diffusion loss equation, all physical quantities that appear in the diffusion-loss equation must be treated as spatially independent within the Green's functions. This is achieved as follows: the magnetic field is averaged over a chosen radially dependant profile (in a simple non-weighted manner, see Equation (4.61) below), and the gas density is given as a scalar value (equivalent to the flat profile given in Equation (3.40)). The two options provided for the magnetic field profile, equivalent to our definitions in Equation (3.35), are the exponential and beta profiles. There are also two supported forms of the diffusion function D : a simple power law (given in Equation (3.21)) and a form that incorporates the degree of uniformity in the magnetic field (Colafrancesco et al. 2006, eq. A15), as

$$D(E) = D_0 \left(\frac{d_B^{2/3}}{B_{\text{avg}}^{1/3}} \right) E^\gamma, \quad (4.60)$$

where d_B is the coherence length of the magnetic field (in kpc), and the average magnetic field strength is measured in μG . The energy loss formula used is similar to the one described in Equation (3.30), except that it does not contain the correction for the Bremsstrahlung term given in Egorov (2022). In terms of DM parameters, there are two halo density profiles to choose from: a generalised NFW profile and an Einasto profile (see Section 3.1.2). The user may then specify the WIMP mass, annihilation cross-section and a branching ratio consisting of several standard annihilation channels (which will then be passed to DarkSUSY). A representative sample of results from McDaniel et al. (2017) has been provided in Figure 4.6. These results show calculations performed with RX-DMFIT for the synchrotron emissivity emitted by typical astrophysical structures, for a set of diffusion coefficients D_0 .

Treatment of modelling parameters

As in the examples shown in Figure 4.6, we have chosen to focus our comparisons on two cases: a galaxy cluster (Coma) and a dSph (Reticulum II). These two environments should provide two extreme ends of the range of astrophysical environments usually considered in DM ID studies. During all calculations performed in this section we have attempted (as far as possible) to keep a set of control parameters between the two codes to highlight any differences in the solution method itself, rather than any possible differences in preferred parameter choices. The most pertinent of these control parameters are displayed in Table 4.1.

Although the solution methods are different in each code, which necessitates some variation in the modelling process, we also attempt to keep parity between the physical modelling scenarios. The most notable aspect of this is the form of the diffusion function. In the case of the OS method, we use the form given in Equation (3.22), which allows for spatial information about the magnetic field to enter into the solution (we reiterate that this is one of the principle benefits of using a numerical technique). As mentioned above, RX-DMFIT allows for two alternate forms of the diffusion function. All of these forms contain a constant diffusion coefficient and a power law in energy, but there is a factor that varies between each, namely $(B(r)/B_0)^{-\delta}$ for the OS method (usually with $\delta = 1/3$) and $(d_B^{2/3}/B_{\text{avg}}^{1/3})$ for the GF method in RX-DMFIT. These both depend inversely on the magnetic field strength, but the overall effect on the diffusion function in each

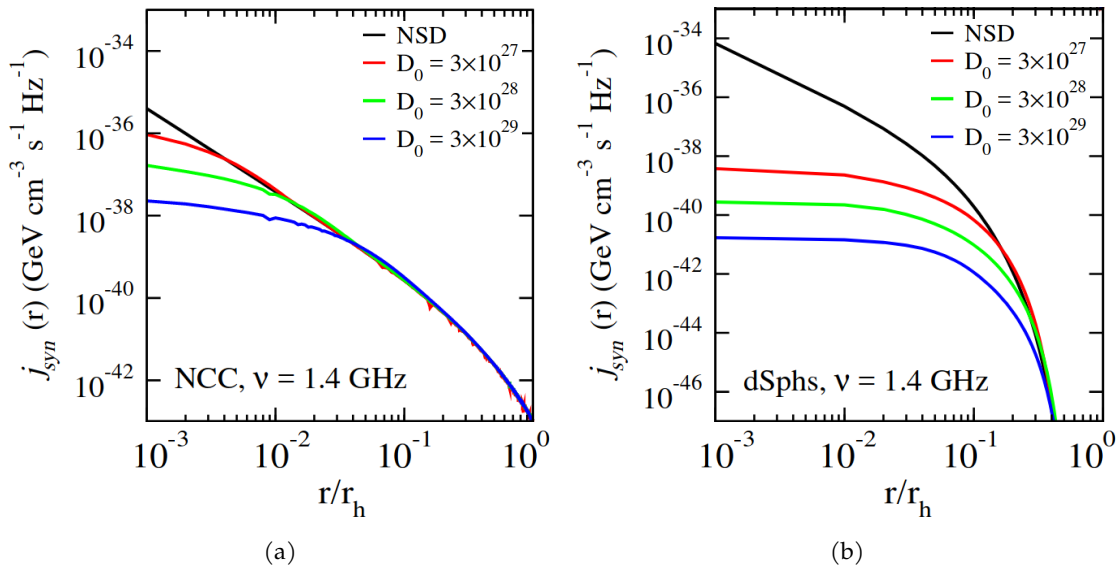


Figure 4.6: For reference, figures (a) and (b) have been taken directly from McDaniel et al. (2017, fig. 6). They show example output synchrotron emissivities using the RX-DMFIT package for typical NCC galaxy cluster (a) and dSph (b) targets for a range of diffusion coefficients ('NSD' refers to the case of No Spatial Diffusion, or $D_0 = 0$).

case can vary depending on how they are treated. In the following paragraphs we briefly discuss the behaviour of these factors and their effective impact on the final results in different scenarios.

Since the magnetic field profiles that are typically found in large astrophysical structures have a roughly constant inner field which decays in the outer regions, the factor in the OS method should be near unity at the centre of the halo. Then, at larger radii the factor will increase (note the negative exponent), which leads to a slight increase in the diffusion strength (an example of this behaviour is shown in Figure 4.7a). The larger overall diffusion function will allow electrons to stream more easily in these regions, leading to an effective reduction in the total emission. This spatial information is built into the algorithmic solution to the diffusion-loss equation (see Section 4.3.4), and produces a solution that possesses more spatial accuracy than one without this information.

In the case of Equation (4.60), the relevant factor reduces to a scalar value calculated based on average properties of the magnetic field. The average magnetic field strength B_{avg} is calculated in RX-DMFIT by an average that corresponds to

$$B_{\text{avg}} \equiv \left\langle \frac{r_h^2}{3r^2} B(r) \right\rangle \equiv \frac{1}{4\pi r_h^3/3} \int dV \frac{r_h^2}{3r^2} B(r) \quad (4.61)$$

$$\Rightarrow B_{\text{avg}} = \frac{1}{r_h} \int_0^{r_h} dr B(r), \quad (4.62)$$

where r_h is an analogue for the parameter $\tilde{r}_{\text{max}}$ in the above sections, i.e. it defines the maximum considered radius of the diffusive environment. Other, more sophisticated methods to calculate this average have been proposed (see Sarkis et al. (2022) for a review), but since this is the one used by RX-DMFIT we limit the discussion to only this method. A sample visualisation of this calculation is provided in Figure 4.7b. From the equation above, it is clear that the value of B_{avg} depends on the value of r_h . Generally, values of r_h that are smaller than the scale radius of the magnetic field r_b will lead to averages that are close to the normalisation constant B_0 , and values of $r_h > r_b$ will lead to averages that are much smaller than B_0 . Later we will see that the r_h parameter also has a large impact on the accuracy of the GF method, but since there is no obvious physical prescription for choosing a value for it here, using it in the modelling of diffusion processes can immediately introduce a source of uncertainty into the final emissivity results. This uncertainty

Parameter	Unit	Coma	Reticulum II
<i>halo parameters</i>			
Profile	-	NFW	Ein
α	-	1.0	0.4
M_{vir}	[$M_{\odot}$]	1.2×10^{15}	-
R_{vir}	[Mpc]	2.7	-
ρ_s	[$M_{\odot}/\text{Mpc}^3$]	-	7.0×10^{16}
r_s	[kpc]	287.0	0.2
D_0	[cm^2/s]	3.1×10^{28}	3.1×10^{28}
z	-	0.0231	-
d^*	[kpc]	-	30.0
<i>WIMP parameters</i>			
M_{χ}	GeV	100	100
Channel	-	$b\bar{b}$	$b\bar{b}$
<i>gas parameters</i>			
Profile	-	flat	flat
n_0	[$1/\text{cm}^3$]	3.49×10^{-3}	1×10^{-6}
<i>magnetic field parameters</i>			
Profile	-	beta	exp
B_0	[μG]	4.7	1.0
r_b	[kpc]	253.0	0.035
q_b	-	-0.327	-

Table 4.1: List of parameters used in the simulations for comparison between methods. Each of these parameters are kept constant when comparing the models between RX-DMFIT and the OS solution method developed in this project.

* d is the distance to the target.

would be compounded by the second parameter required in the factor, the coherence length of the magnetic field d_B . Since there is a general lack of useful magnetic field structural information in many astrophysical objects (especially in targets like dSphs), the use of precise, empirical values of d_B is unlikely in the majority of modelling scenarios.

To provide an instructive example of this scenario, we have calculated the factor ($d_B^{2/3}/B_{\text{avg}}^{1/3}$) for the Coma cluster, using the same general modelling parameters as those in Table 4.1 and using a value of $d_B = 20$ kpc, as in Colafrancesco et al. (2006) (we note here that the Coma cluster has a relatively well-studied magnetic field, so that the impact of parameter uncertainties on these results would be less than in most other targets). When the maximum sampled radius $r_h = 415$ kpc, which is the value given in the RX-DMFIT example model for Coma, the average magnetic field is $B_{\text{avg}} = 3.7 \mu\text{G}$ (very close to the normalisation constant $B_0 = 4.7 \mu\text{G}$) and the total value of the factor is ≈ 4.78 . When the value of r_h changes to the virial radius or twice the virial radius (as in Figure 4.7b), these values change to $B_{\text{avg}} = 1.4 \mu\text{G}$ and $B_{\text{avg}} = 0.85 \mu\text{G}$, while the total factor becomes ≈ 6.62 and ≈ 7.79 , respectively. These scalar values should be compared with the factor plotted in Figure 4.7a, as they perform similar functions in the overall diffusion modelling scenario. The dependance of this factor on the maximum sampled radius r_h is exacerbated in smaller targets like dSphs, where the exponential magnetic field profile leads to average magnetic field values that are much smaller than B_0 , and hence much larger total diffusion functions. Finally, by treating these factors as an effective change in the value of the diffusion coefficient D_0 , their effect on the emissivity can be judged from Figure 4.6, which shows emissivities calculated with RX-DMFIT for a range of D_0 values. Since an order of magnitude increase in D_0 leads roughly to an order of magnitude decrease in the emissivity from these results, the choices of both d_B and r_h clearly have a direct impact on the observable emission from the target. Thus, we have used the simple power law implementation of the diffusion function in all calculations with RX-DMFIT. This should allow

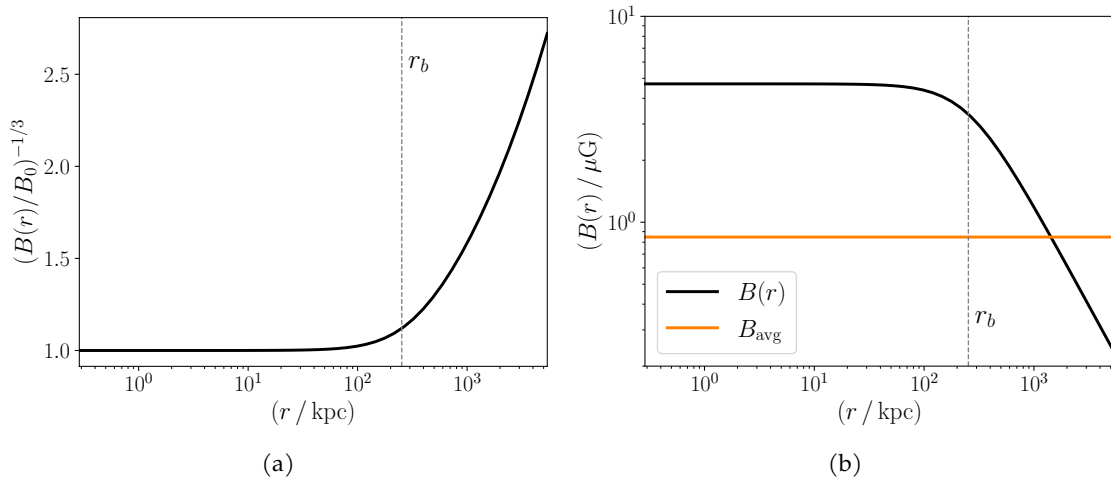


Figure 4.7: Visualisation of the behaviour associated with the magnetic field profile, its average and the coefficient factor which appears in the diffusion function for the OS method. Note the reduction in magnetic field strength of the inner region of the halo between the full spatially-dependant profile and the average value.

for fair comparisons between the methods, particularly in the centre of halos where the factor in Figure 4.7a has no impact on the overall diffusion (and where the emission is most important for DMID studies).

The other major difference in the methods arising from the need for spatially independent physical quantities is the form of the energy-loss function. For the sake of comparison, we have reverted to the old expression for the Bremsstrahlung energy-loss in the OS method to match the one used in RX-DMFIT (although we note that this change is unlikely to have a significant effect on the results considered here, see Section 3.2.2 for details). Since the only spatial dependance in the energy-loss formula comes from synchrotron emission, substituting the magnetic field profile with its average results in the following approximation for the synchrotron term (McDaniel et al. 2017, eq. 2.8):

$$b(r, E) \approx b(E) = b_{\text{synch}} \left(\frac{E}{1 \text{ GeV}} \right)^2 \left(\frac{B_{\text{avg}}}{1 \mu\text{G}} \right)^2. \quad (4.63)$$

As we have seen in the analysis of the diffusion function above, the average magnetic field strength is typically lower than the central strength B_0 (particularly in the inner regions of the halo). Therefore, this approximation leads to lower energy-loss values (in general) compared to the fully spatially dependant version. In RX-DMFIT, Equation (4.63) is used during the evaluation of the Green's function (where the approximation is necessary), whereas the full profile is used for instances of the function that appear outside the Green's function.

Method comparisons

In the following figures we display the results from our set of comparisons. Each of the calculations have been performed in a manner following the discussions given in the preceding sections, and should highlight any differences between the two solution methods. The results are framed in terms of single outputs (either emissivities or fluxes) from each method for the Coma galaxy cluster (Figures 4.8 and 4.9) and the Reticulum II dSph (Figures 4.10 and 4.11).

The first general trend we see in these results is that the calculations performed with RX-DMFIT depend on the maximum sampled radius r_h , whereas the OS method seems robust to changes in this parameter. The values of $r_h = 2 \times R_{\text{vir}}$ for the Coma cluster and $r_h = 0.2 \text{ kpc}$ for Reticulum II in particular seem to produce less reliable results. In Figure 4.8b we see significant numerical artefacts in the outer regions of the halo (this will be discussed further below), and the outputs from Figures 4.10a and 4.11a seem to be underestimated. These r_h values are clearly not applicable

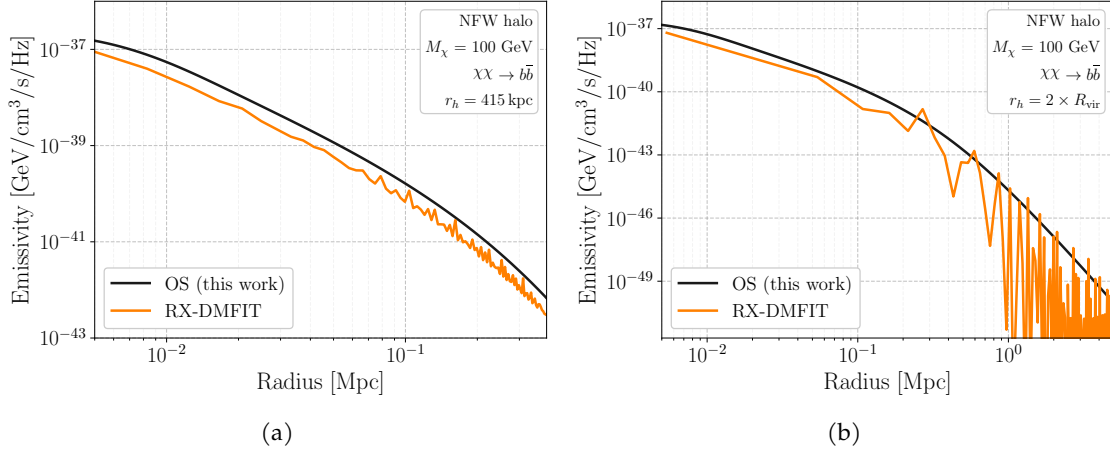


Figure 4.8: Radio emissivities from the Coma galaxy cluster, calculated using Equation (3.49). The equilibrium electron distributions have been determined by the OS method developed in this work and with the code package RX-DMFIT (McDaniel et al. 2017). Each panel shows results found using a different value of the parameter r_h , which represents the maximum sampled radius in the diffusive environment: (a) 415 kpc and (b) $2 \times R_{\text{vir}}$. All relevant modelling details can be found in the text.

to the relevant physical scenarios, but we include their results here to show the potential impact of a misinformed parameter choice in this regard. We thus focus our discussion on the results for the more reliable cases of $r_h = 415$ kpc for the Coma cluster and $r_h = 1$ kpc for Reticulum II.

When comparing each of the methods, we use a simple relative difference to the larger value (typically the OS method) to quantify the difference at important points, calculated with $(|y_1 - y_2|/y_1) \times 100\%$, where $y_1 > y_2$. In the Coma cluster, there is a $\sim 30\%$ relative difference in the emissivity values throughout the halo (excluding the points with numerical artefacts in the outer regions, although these points still retain the general behaviour of the function). This trend is echoed in the flux output, which varies between $\sim 30\%$ at low frequencies to $\sim 15\%$ at high frequencies. At the MeerKAT reference frequency of 1280 MHz (relevant for results found in this work), there is a $\sim 27.5\%$ relative difference. There is no significant change in the behaviour of these functions, and the two methods seem to be generally compatible. In the case of Reticulum II, the emissivity calculations result in a relative difference between the methods of $\sim 74\%$ near the centre of the halo, and the two methods then converge as $r \rightarrow r_h$. The flux varies more significantly than in the case of Coma, with a maximum relative difference of $\sim 81\%$ at 10 MHz and $\sim 45\%$ difference at 1280 MHz.

The differences in the results highlighted above can be attributed to the treatment of spatially dependant functions during the solution method. As mentioned before, we have accounted for the radial dependance of the diffusion function in the centre of the halo with each method having equivalent diffusion strengths. However, the approximation of the energy-loss function (in Equation (4.61)) can lead to an observable difference in the results. With the value of B_{avg} being generally lower than B_0 , and synchrotron being the dominant energy-loss mechanism at higher energies (see Figure 3.4), the overall energy-loss function that depends on B_{avg}^2 can be significantly reduced in the centre of the halo (this effect is also stronger in Reticulum II than in Coma because of the steeper exponential magnetic field profile). In the GF solution method, the Green's functions are defined with the variable v (see Equation (4.3) or McDaniel et al. (2017, eq. 2.7)). This variable, or more specifically $\sqrt{v}$, represents the mean distance an electron travels while losing a specified amount energy, and can be interpreted as a type of diffusive length scale. Since this variable depends inversely on the energy-loss function, lower energy-loss leads to larger values of v , and thus electrons that travel further in the halo while losing the equivalent amount of energy. This effect can be seen in Figure 4.10b: the GF method produces less emission than the OS method in the inner regions of the halo, while slightly more in the outer regions as $r \rightarrow r_h$. This can be attributed to the above-mentioned effect, that electrons are moving outwards from the centre

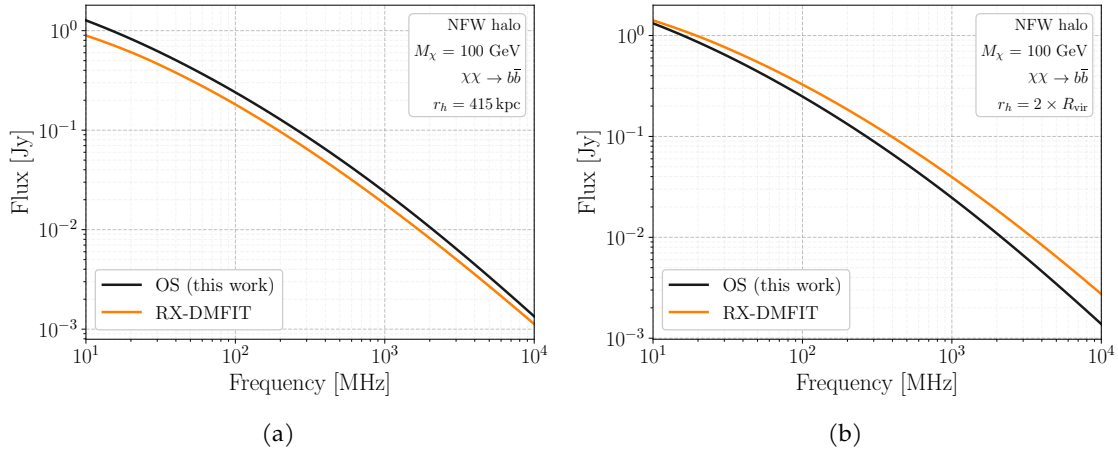


Figure 4.9: Similar to Figure 4.8. The output shown here is the integrated radio flux (Equation (3.51)), for the same values of the parameter r_h .

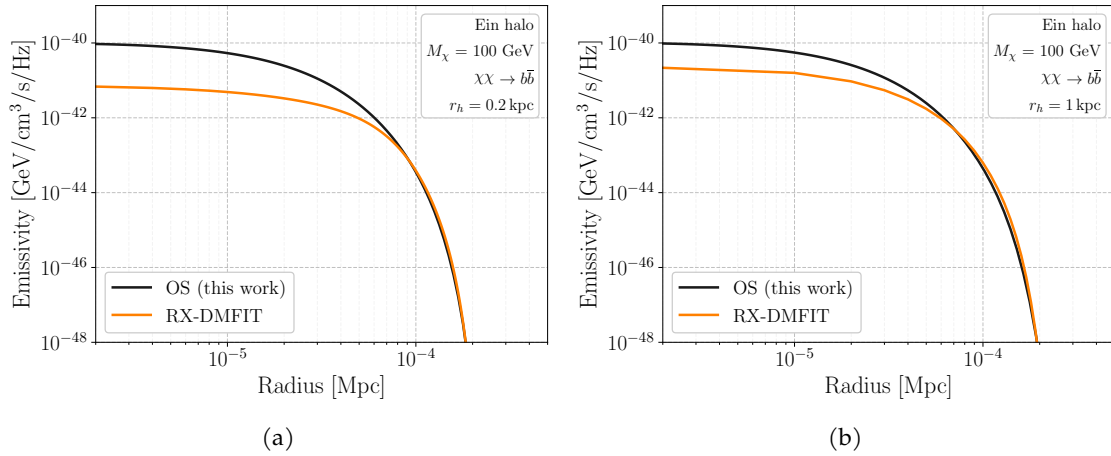


Figure 4.10: Similar to Figure 4.8. These emissivities have been calculated for the Reticulum II dSph, with values of r_h equal to: (a) 0.2 kpc and (b) 1 kpc.

of the halo while radiating the same amount of energy as those that are confined to the centre in the OS method. Since the behaviour of the two methods in Figure 4.10b generally resembles that of different diffusion coefficients in Figure 4.6b, the use of the GF over the OS method could be considered equivalent to using a larger diffusion coefficient, in the context of emissivity output.

The lower emissivities from the GF in the Coma cluster and in the centre of the Reticulum II halo seem to lead directly to lower flux output at all frequencies. The only set of results that do not follow this trend are the ones for $r_h = 2 \times R_{\text{vir}}$ in the Coma cluster, which contain significant numerical artefacts. In these cases (we also note the presence of smaller artefacts in other calculations for Coma), the technique used to numerically integrate the emissivity should be able to handle these artefacts correctly. This has been accounted for in RX-DMFIT, which allows for the user to select an alternative integration method (both of which are provided by the GSL (Galassi 2009)). The default method, a quadrature non-adaptive Gauss-Kronrod (QNG) technique, is only suitable for smooth functions. The alternative method, quadrature adaptive integration with singularities (QAGS), should be able to handle functions that contain some singularities. This is done by adaptively defining small sub-intervals near the singularity, which eventually leads to a convergent approximate of the overall integral. As mentioned in the RX-DMFIT documentation, this method is not recommended for all cases, since it takes significantly longer than the QNG option ($\sim 10\times$ total runtime for these calculations). We have used the QAGS routine on the noisy

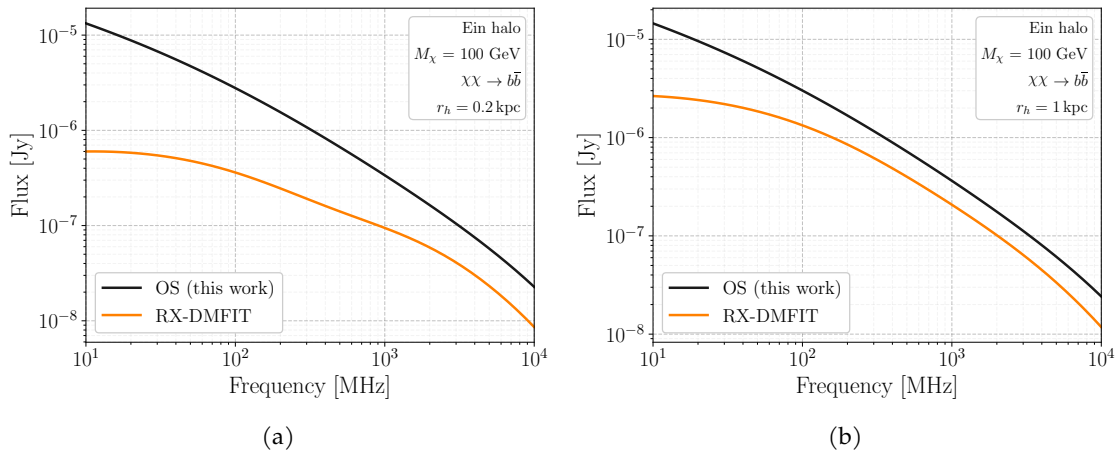


Figure 4.11: Similar to Figure 4.10. The output shown here is the integrated radio flux (Equation (3.51)), for the same values of the parameter r_h .

emissivity values in Figure 4.8b, but found no significant difference to the behaviour of the flux found when using the default QNG routine. A more detailed investigation into the use of different integration techniques for noisy emissivity functions (or into the formation of these artefacts in the first place) could reveal a solution to this problem, but is out of the scope of this work. We instead just note that the alternative integration method is provided in the code, and could be of benefit in some cases of noisy results.

The final aspects of these calculations worth discussing are of a practical nature, and primarily how quickly the equilibrium electron distributions are found in each case. On the author's personal computer, the average duration of the calculations performed in this Section with OS method was ~ 24 s, using the ACC technique. The vast majority of this time is made up of calls to the linear algebra solution method in the `scipy` library to solve the matrix equation described in Equation (4.24). The full calculations, including the emissivity or flux outputs, took an average duration of ~ 28 s. In comparison, the full calculations performed with RX-DMFIT, on the same hardware, took an average of 295 s. This value was found with the recommended options for computation speed,⁶ i.e., using the QNG integration routine and using a lookup table for GF calculations (by setting `SD=1`, see RX-DMFIT documentation for details). The factor mentioned above is over $10\times$ slower than the average OS method for the set of calculations presented in this Section, a result which corresponds to the claim made in Regis et al. (2015b, app. A) that the numerical method is faster than the semi-analytical one. The advanced techniques developed in this work (like the use of sparse, block matrices in a completely vectorised algorithm) and those inspired by GALPROP (like the use of an accelerated timestep-switching technique) have also helped to make the numerical solution method extremely efficient. The use of `scipy` libraries, which are designed for fast numerical computations, also noticeably improve the performance of otherwise sluggish numerical Python code (compared to the C/C++ implementations in RX-DMFIT and GALPROP).

Finally, we note that although the results found in this Section have used a non-exhaustive list of parameters and modelling scenarios, they provide sufficient evidence that the solution method developed in this work can produce reliable results. The emissivity and flux outputs from each method could be compatible, as long as various differences discussed above have been accounted for. A more comprehensive study of the differences between these code packages and solution methods, for a wider range of modelling scenarios, could be the focus of future work.

⁶We do not compare benchmarking runs or those with options not recommended for general use.

4.5 Summary

In this Chapter we began by providing a high-level overview of some of the methods used to solve the diffusion-loss equation found in the literature. The most common technique in DM indirect detection studies involves the use of Green's function approximations that are solved semi-analytically to yield the equilibrium distribution of electrons produced during WIMP annihilations. One requirement of these approximations is the removal of any spatial dependance in functions that describe physical quantities, like the magnetic field and thermal gas density profiles that appear in the diffusion and energy-loss functions. This is usually done by averaging the relevant profiles, and substituting the spatially-dependant functions for scalar averages in the diffusion-loss equation.

An alternative approach to these semi-analytical methods makes use of the Crank-Nicolson finite-difference scheme to discretise the quantities involved in the diffusion-loss equation, and then solve it iteratively and numerically. The primary benefits of this approach are that it is significantly faster computationally, and that the full spatial dependance of the relevant quantities can be kept during the entire calculation. Outside a single other research team that have implemented a numerical technique for radio DM studies, these methods are typically used in the field of galactic cosmic-ray physics. This field has been shaped around the popular and publicly available GALPROP code package, which has pioneered many aspects of the accurate modelling of cosmic-ray propagation. However, the central focus of this package is on galactic cosmic-ray propagation, and not on extragalactic DM radio studies, which is the central focus of this work. We have therefore developed an algorithm that implements and extends the core numerical method that is used in GALPROP, and embedded it into a general code base that contains a comprehensive suite of DM-related calculations and models.

In this Chapter we have discussed, in detail, all relevant aspects of the developed algorithm, including: the variable transformations used, the multidimensional extensions to the Crank-Nicolson scheme, the specific linear algebraic techniques used to solve the related matrix equations, the initial and boundary conditions and several important considerations relating to stability, computational performance, convergence and accuracy. We have also produced a set of benchmarks and tests of the algorithm, to ensure that the results it produces are reliable and accurate. As part of these tests, we have carried out a simple set of comparisons to the publicly available code package RX-DMFIT, which utilises the semi-analytical Green's function solution method. The overall trend in these comparisons is that there is a general agreement in the results, with certain notable exceptions. In the centre of small structures like dSphs there can be a significant difference in the radio emissivity output of both methods, which can in turn lead to an equally significant difference in the total flux from these objects. As an example, we calculated the expected radio emissivity from the Reticulum II dSph, and found a relative difference of nearly 75% between the two methods near the centre of the halo, which led to a difference of 45% in the integrated flux at the MeerKAT reference frequency of 1280 MHz. In the much larger Coma galaxy cluster, the relative difference in output from the methods never went above 30%.

The differences in the predicted emissions from each method can be attributed to the mathematical treatment of various modelling parameters. In particular, the substitution of the radial magnetic field profile with a scalar average results in electrons losing less energy to synchrotron radiation, which allows the electrons modelled by the Green's function method to diffuse outwards from the centre of the halo a relatively further distance than equivalent electrons modelled with the numerical method. Since this effect leads to a lower electron number density, the total radio emissivity output from these regions is subsequently reduced. In the case of Reticulum II, the output emissivity from the Green's function method behaves similarly to the effect of using a larger diffusion constant with the numerical method.

We place a large emphasis on the accuracy of the predicted emissions in the centre of DM halos, as these are the regions where the strongest radio emissions are expected. With the recent availability of observational data from sophisticated radio interferometers that boast extremely high resolution and sensitivity (which is the topic of the next Chapter), accurate modelling of radio emissions from DM on small spatial scales is essential to maximise the scientific potential of the data. Since we also make use of new survey data from the MeerKAT telescope in this work, a solution method that can produce these results in a reliable and efficient way is

important in a practical sense. The solution method we have developed here outperforms the only other comparable public software package by 10× in terms of code runtime, retains the full spatial dependence of relevant physical quantities in the solution, and is generally more robust to different spatial grid parameters. While not directly developed in this project, the code that integrates with the solution method (relevant to DM modelling and the calculation of radio emissions) also provides a range of benefits over packages that use numerical solution methods, like GALPROP. Principle among these benefits are a suite of DM-related calculations and models, with a significant number of physical and numerical parameters to choose from. Additionally, options to calculate radio surface brightnesses or output results in a FITS file format help for the direct analysis of interferometric radio data, and the explicit use of only standard Python libraries helps reduce the dependence on external code packages. These factors place the developed code in a radio DM indirect detection niche, which we anticipate will help advance the field as the DM parameter space becomes increasingly constrained. The code is expected to be released publicly, with an associated guide and documentation, in the near future.

Observational Data

A focus point of the theoretical aspects mentioned in the above Chapters is the need for robust modelling of the DM halo and its emissions, particularly in the dense central regions of the halo. Correspondingly, the need for high resolution and high sensitivity radio data with which to compare the models to is just as important. Because of the large wavelengths of the radiation in radio studies, the spatial resolution achievable with a single radio dish is severely limited by its physical size (which leads to giant radio dishes that are hundreds of metres wide, like the 305 m Arecibo¹ or 500 m FAST² instruments). In contrast, the recent advent of radio interferometry observatories has allowed for much higher resolutions and sensitivities to length scales not feasible with single radio dishes. The capability of these telescopes to image the sky in such detail presents us with a fortunate opportunity to hunt for signs of DM emission using the modelling tools already described (a detailed study of radio instruments and their design principles is outside the scope of this work, so we refer the reader to reference textbooks like Thompson et al. (2017) and Condon et al. (2016) for more information regarding instrumentation). As such, we make use of recent observations performed with the MeerKAT interferometer (Jonas et al. 2018) in this project, which makes it the first analysis of MeerKAT data in the context of DM searches.

The MeerKAT instrument is currently composed of 64 radio dishes arranged in an array with a dense 1 km-diameter core and a maximum baseline between individual dishes of 7.7 km. It is located in the Northern Cape in South Africa, and released first-light images on the 16th July 2016 with the full array being completed in 2018. The pertinent observations to this project were released as part of the first public Data Release (DR1) of the MeerKAT Galaxy Cluster Legacy Survey (MGCLS, (Knowles et al. 2022)), released on 11th November 2021. The following Chapter provides an overview of the data from the MGCLS, how it is accessed and processed, and how the models are concurrently prepared for a robust and statistical search within the data.

5.1 Data from MGCLS

The MGCLS is a radio survey observation programme performed between June 24th, 2018 and June 16th, 2019 with MeerKAT's L-band (900-1670 MHz) receiver. It consists of 115 galaxy clusters, positioned between 0° and −85° Declination (DEC) and at a median redshift of 0.14. Each target was observed for 5.5 - 9.5 hours with a minimum of 59 of MeerKAT's 64 total radio dishes, for a total observing time of ~ 1000 hours. A comprehensive list of details for these observations can be found in Knowles et al. (2022); in this work we focus on the enhanced data products for a small selection of targets from this survey, which will be described as necessary later in this Chapter.

The full sample of clusters observed with MGCLS is made up of two general groups, the so-called *radio-selected* and *X-ray-selected* clusters. The radio-selected group consists of 41 clusters that have been observed in previous radio searches, and are likely to contain large diffuse radio emissions in the form of radio halos or relics. Because of this selection criterion, the radio-selected clusters also have relatively large masses. Since the total mass of these clusters is dominated by DM, a large mass leads (in general) to a higher central DM density and thus stronger indirect emissions. The remaining X-ray selected sample on the other hand is based only on clusters found in the Meta-Catalogue of X-ray-detected Clusters (MCXC, Piffaretti et al. (2011)). This selection has no bias to any radio emission properties.

The selection of targets to use for our analysis is based on several factors, foremost of which is the lack of bright radio halos in the images. Since the expected synchrotron emission from the DM halo (see Section 3.3.1) is diffuse and relatively faint, a bright source of non-thermal

¹Website homepage: <https://www.naic.edu/ao/>

²Website homepage: <https://fast.bao.ac.cn/>

diffuse emission from other astrophysical processes – such as a radio halo – would taint the DM signal or hide it completely. Since the modelling of these structures can be highly uncertain (and complex), including an estimate of this component into the analysis would introduce significant uncertainty to our final result. Based on the modelling procedures laid out in Chapters 3 and 4, the elimination of uncertainty in the modelling process is clearly an important factor in creating reliable and robust DM searches. We thus choose targets for our sample that, as far as possible, lack this type of diffuse emission.

Another salient factor to consider is the availability of DM halo data in the literature, as targets that lack critical information about the halo environment will again create uncertainty in the results. It is however (unfortunately) quite difficult to find reliable DM halo data on a large portion of the MGCLS targets, since many of them lack dedicated studies and only have associated data in surveys or catalogues. For targets in our selection that are missing needed modelling parameters, we have utilised simple methods to estimate these based on trends in galaxy cluster morphologies, and will elucidate these when necessary.

The last factor considered in target selection is the overall quality of the radio image (represented by the Data Quality Factor (DQF) column in Table 5.1). Any non-physical artefacts in the image (typically found around very bright compact sources) or bright, extended emission near the dynamical centre of a cluster will pollute the statistical comparison we make to the fainter DM models, making them less effective. We reiterate here that the principle benefit of using the high-resolution and high-sensitivity data from MeerKAT is to compare our models with the faintest possible sources, in order to get the most constraining results. We therefore attempt to select targets that have a higher DQF (in general). Where extended emissions or compact sources are present, we eliminate them using a source-finding software package to subtract them out of the final image (see Section 5.2.2 for details). All the sources selected in this work have also had astrometric corrections applied to their images, so we omit any details about this subject and refer the reader to Knowles et al. (2022, sec. 4.4.4) for details.

Finally, we note that the sample of clusters chosen here is not exhaustive. A more comprehensive DM search of the MGCLS data is possible, and potentially earmarked for future work – especially if any new modelling parameters for these sources come to light. With the above considerations in mind, the selection of clusters used in this work is presented in Table 5.1. Thereafter, each of the targets are referred to by a simpler, truncated version of their full designation, i.e. RXCJ0225.1–2928 will be referred to as RXCJ0225 (or just J0225). Since the designations in our selection are unique, this naming scheme should be unambiguous.

Cluster name	RA _{J2000} [deg]	Dec _{J2000} [deg]	z	RMS [μ Jy/beam]	DQF	D.E.	Alternate name
<i>Radio selected:</i>							
Abell 133	15.6879	−21.8800	0.057	6.7	1	–	MCXCJ0102.7–2152
Abell 4038	356.8796	−28.2028	0.028	3.0	0	✓	MCXCJ2347.7–2808
RXCJ0225.1–2928	36.3750	−29.5000	0.060	5.1	2	–	MCXCJ0225.1–2928
<i>X-ray selected:</i>							
RXCJ0600.8–5835	90.2013	−58.5872	0.037	2.5	0	–	Abell S560
RXCJ0757.7–5315	119.4438	−53.2636	0.043	3.2	0	–	Abell S606

Table 5.1: Table of selected cluster properties. Reproduced with relevant data from Knowles et al. (2022, tab. 1). RMS values represent the statistical noise component of the images (see Section 5.3.2), the DQF (Data Quality Factor) values indicate both the dynamic range and the presence of artefacts in the image and the D.E. column indicates the detection of some form of cluster diffuse emission (see Knowles et al. (2022) for details).

5.2 Data processing

All the data used in this project is accessed from an online repository through a DOI³ in the form of archived FITS files for each source, Stokes parameter and resolution. We consider only the enhanced data products, which include primary beam corrected images of the central $1.2^\circ \times 1.2^\circ$ map. The relevant files for our targets have been downloaded from this repository and extracted. Each file contains multiple planes (datasets), consisting of: the intensity of the surface brightness at the reference frequency (1280 MHz), the spectral index, the uncertainty associated with each of these quantities, and the χ^2 values of a combined least-squares fit of the data. We note that the uncertainty planes are measures of the statistical noise in the image, and not of the systematic effects involved in the processing of the interferometric data. Then, the relevant planes for this work are the intensity and its associated uncertainty, which are extracted from the cubes for further processing. The full resolution data for the unpolarised intensity (Stokes-I) images are used in this work. These all have synthesised beam sizes (represented by the FWHM - Full Width at Half Maximum) of $\sim 7.5 - 8''$ and RMS noise levels of $\sim 3 - 5 \mu\text{Jy beam}^{-1}$.

Figure 5.1 is a visual representation of the first plane of each FITS data file, as downloaded from the repository. These show the image intensities before any regions of interest are extracted and the compact sources are subtracted. We note in these images the presence of various artefacts (particularly in RXCJ0225, and around various bright compact emissions). These artefacts result from the processing of the interferometric data, through various stages of the highly complex calibration and imaging phases. We make no attempt to categorise these here, and refer the reader to Knowles et al. (2022) for further details.

5.2.1 Selection of RoI

The enhanced product images are of an area of only $1.2^\circ \times 1.2^\circ$ around the pointing coordinates, as this is the region within which the applied primary beam corrections are reliable. However, the good spatial resolution of the images (of the order of the beamwidth FWHM, $7.5 - 8''$) means that they still contain many pixels. This is an important factor for the statistical analysis performed in this work, which will be discussed further below and in Chapter 6. To illustrate the effects of the size of the region on the analysis, we consider the source RXCJ0757 as an example. The image of this target contains $\sim 12 \times 10^6$ pixels in the $1.2^\circ \times 1.2^\circ$ region, which is a huge amount of image data. However, since DM searches find the most benefit from observations in the high-density inner regions of halos (where the strongest emission is expected), the pixels in the outer edges of the image are less useful than those in the centre. Therefore, we crop each target's image to a smaller region that is focused on the centre of the DM halo, usually within a square of size $2.5' \times 2.5'$. For RXCJ0757, this results in a 120×120 pixel image, for a total number of 14.4×10^3 pixels. Whether this size is reasonable or even optimal for each source depends on several factors, described below.

Firstly, there is an issue of correlation between individual pixels in any radio flux density image. This is due to the nature of radio telescope observations, which have non-negligible beamwidths within which the radiation is detected. This correlation can be complex to account for in general, involving calculations of matrices that describe the covariance between each pixel. In this project, following Regis et al. (2015b) and Regis et al. (2021), we make the approximation that all the pixels within a single beamwidth (of the synthesised beam) are fully correlated with each other, while those outside of the beamwidth are completely uncorrelated. This approximation will be described quantitatively in Chapter 6. The effect of correlated pixels, and the above approximation, on the size of the image is that it reduces the effective number of pixels (resolution elements) considered statistically independent, which is a requirement of the statistical analysis we adopt. Therefore, in all cases we consider the number of resolution elements in an image to be the total number of distinct beamwidths, i.e. the total number of pixels divided by the number of pixels in a single beamwidth. This calculation is then performed for each image. For the reduced image size of $2.5' \times 2.5'$ for RXCJ0757 mentioned above, the approximation leads to ~ 377 resolution elements, with 38 pixels per beamwidth.

While the size of the cropped region – which we refer to from here as the Region of Interest

³DOI: <https://doi.org/10.48479/7epd-w356>

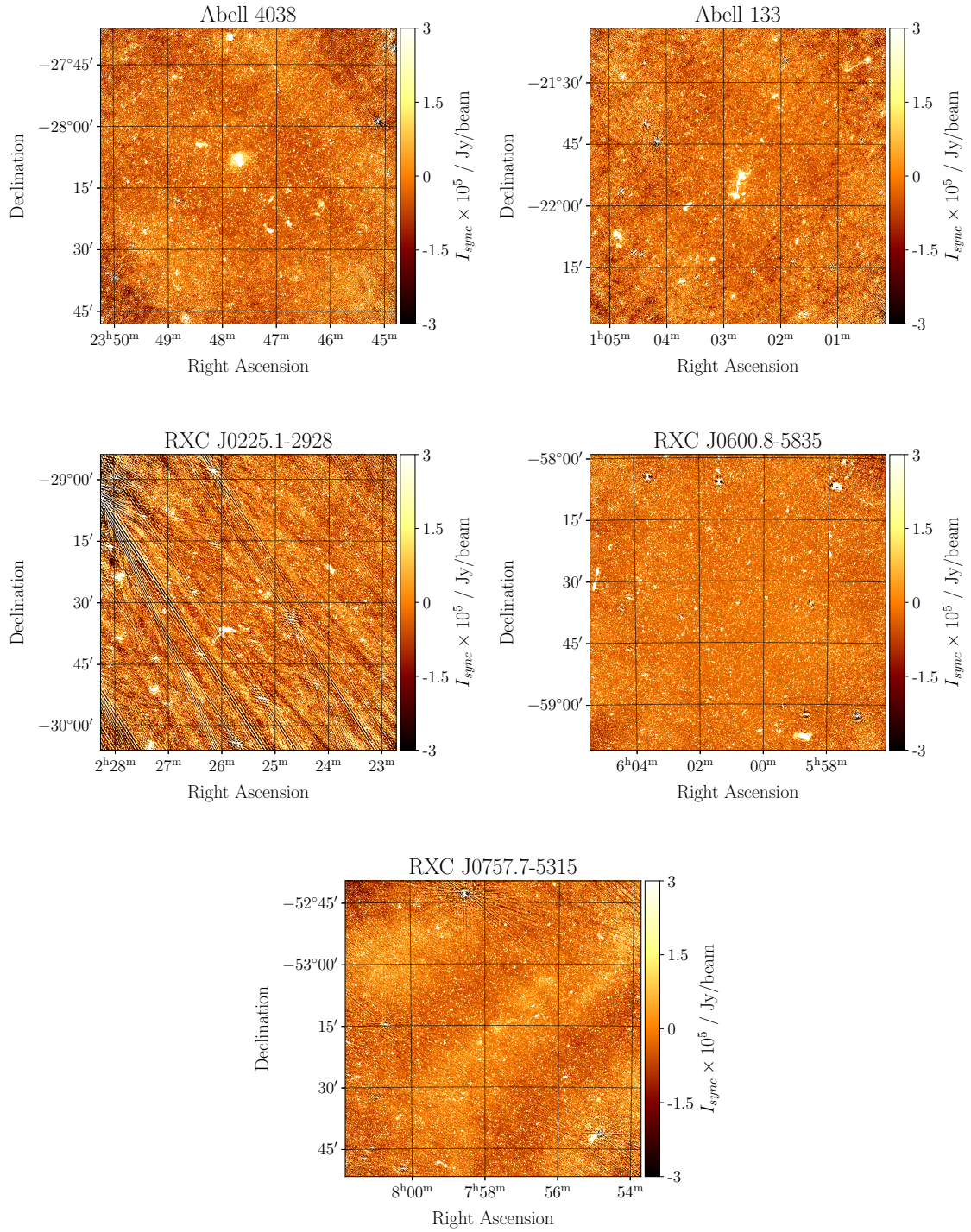


Figure 5.1: Full resolution radio maps for the selected sources in Table 5.1. The scaling and contrast of the colour bars are arbitrary, and chosen to highlight the main features in the images. We note the lack of bright diffuse radio halos, while drawing attention to other extended emissions, compact sources and artefacts in each image (see text for details).

(RoI) – can be chosen somewhat arbitrarily, there are some limiting considerations which are important for the statistical analysis performed on each image. On one hand, smaller regions will lead to better constraints on DM parameters, as they will more directly probe the inner, high-density region of the DM halo. However, smaller regions also contain fewer data points, too few of which results in unreliable statistics. We implement this consideration as a soft lower-limit on the size of the image – keeping the number of resolution elements large enough for us to reliably use the Central Limit Theorem (CLT) and Wilks’ Theorem (WT) (collectively referred to as the Statistical Tests or ST) in the analysis. On the other hand, larger regions have enough data points with which to reliably do any statistical analysis. They, however, produce less stringent constraints on the DM halo emissions, as the faint outer regions of the halo are lost in the data and do not contribute significantly to the overall search.

When finding a RoI that satisfies the above considerations, one also needs to account for any pixels that are deemed unusable. After the source-subtraction procedure (described in the following Section), there may be some pixels that have negative values that are not a feature of the interferometric data itself, but rather as artefacts resulting from extremely bright point sources. Following the analysis performed in Regis et al. (2021), we mask any pixels that have an absolute value that is larger than 3 times the total uncertainty of the data. In the majority of the cases investigated here, we find that this masking criterion has no impact on the results, with most RoI’s containing no pixels that meet the condition. In some clusters there are a non-zero number of pixels that are masked, however the impact on the results is negligible.

An alternative method to choose the RoI’s of the images is to dynamically scale the size of the RoI according to some physical property associated with the target cluster, instead of setting a constant angular size of say, $2.5' \times 2.5'$. In Table 5.2 we show one example of this, with RoI’s found using the scale radius of the DM halo as the characteristic angular size. In these cases, the physical size of the cluster would determine the amount of data that enters into the analysis. Since the resolution of the telescope is finite however, this could cause the RoI’s of some clusters to be either too large or too small to be optimal in regard to the point made about the number of available resolution elements made above. We have therefore investigated two strategies for choosing a RoI in this thesis: one using a constant angular size of $2.5' \times 2.5'$, and one using a dynamic RoI based on half the value of the scale radius of the DM halo (equivalent to half the values in Table 5.2). This choice should represent a reasonable physical scale in most modelling scenarios, although choices based on other physical characteristics may be made. In each case however, we stress that the RoI should necessarily have more than 50 resolution elements in order for the results to be considered valid. The choice of RoI, and its impact on the final results of the analysis, will be discussed further in Chapter 6.

Cluster name	z	Luminosity Distance [Mpc]	Halo scale radius [Mpc]	RoI size [arcmin]
A133	0.057	263.08	0.224	2.92'
A4038	0.028	126.55	0.300	8.15'
RXCJ0225	0.060	277.51	0.108	1.34'
RXCJ0600	0.037	167.87	0.145	2.97'
RXCJ0757	0.039	177.70	0.212	4.10'

Table 5.2: Distance and size information used to calculate dynamic RoI’s based on the physical properties of the DM halo in each cluster. The RoI size represents one side of a square angular region.

5.2.2 Source-subtraction

The surface brightness images obtained through the MGCLS data repository still contain compact point sources and extended emissions, resulting from bright radio galaxies or extended structures like jets. As mentioned in the beginning of this Chapter, these elements hamper the search for DM signatures in the image, as the expected synchrotron emissions from DM are faint and diffuse. We

attempt to remedy this issue by using the Python Blob Detector and Source Finder (PyBDSF)⁴ software package to both identify and remove any of these non-diffuse elements. Indeed, the PyBDSF package has been built specifically for this purpose, as it was first developed as a source-extraction tool for images produced by the Low Frequency Array (LOFAR) telescope. Since LOFAR is a relatively advanced interferometer, producing high sensitivity images with a wide range of emission scales, PyBDSF can successfully identify regions of extended emission (in the form of wavelet and shapelet decomposition), as well as regular Gaussian point sources. The software, written in Python and open-sourced⁵, works with any FITS images as input and is not limited to just LOFAR data. These attributes make it an ideal tool for use with the MGCLS images.

An example use case of PyBDSF is presented in Figure 5.2, which is a direct output from the program when run on the image data of Abell 4038. By inspection, the program seems to have successfully identified the region containing the (significant) bright extended emissions, indicated by the demarcation of islands and Gaussian peaks in the upper central panel. However, the pertinent output for our analysis is the residual image (bottom left panel), which contains the remaining surface brightness pixels after the modelled emissions (bottom central panel) are subtracted. This image contains a large number of negative pixels, identifiable by the darker regions in the lower right area of the image. These are caused by the over-subtraction of surface brightness values in the models (this conclusion is made clear from a comparison of negative surface brightness pixels between this residual image and the original). The issue is visualised more clearly in Figure 5.3b, which shows the same residual image with a diverging colour scale and the negative pixels clearly marked (we show this as an instructive case, as Abell 4038 has the most egregious over-subtraction in our sample). Since some of these pixels are unusable (and subsequently masked according to the criterion mentioned in the above Section) they could represent a loss in data with which to do the analysis.

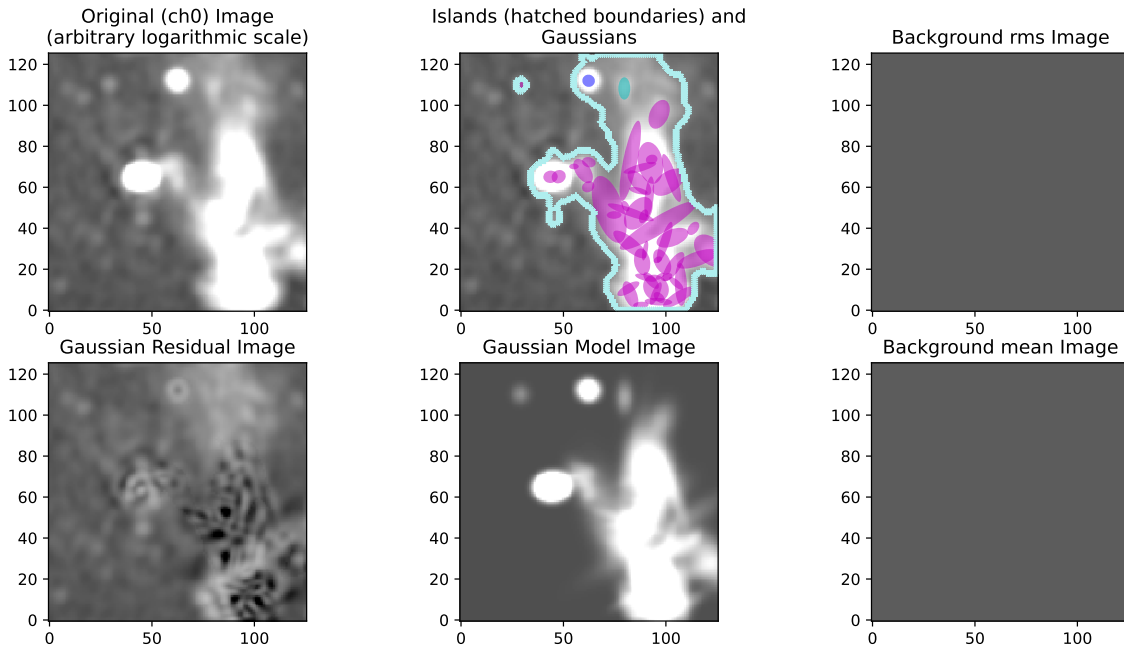


Figure 5.2: The results from running the PyBDSF software package on the Abell 4038 cluster surface brightness image. Note the comparison between the original image (upper left) and the residual image (bottom left), which is the original image but with the model emissions (bottom centre) subtracted out. Also note the presence of over-subtracted pixels in the lower right region, which contained a significant amount of bright extended emission in the original image.

This topic has been discussed briefly in Regis et al. (2015b), wherein they compare various methods for source-subtraction on interferometric data (in their case generated with the Australia

⁴Package documentation: <https://pybdsf.readthedocs.io/>

⁵Online repository  <https://github.com/lofar-astron/PyBDSF> (v1.10.3 used in this project)

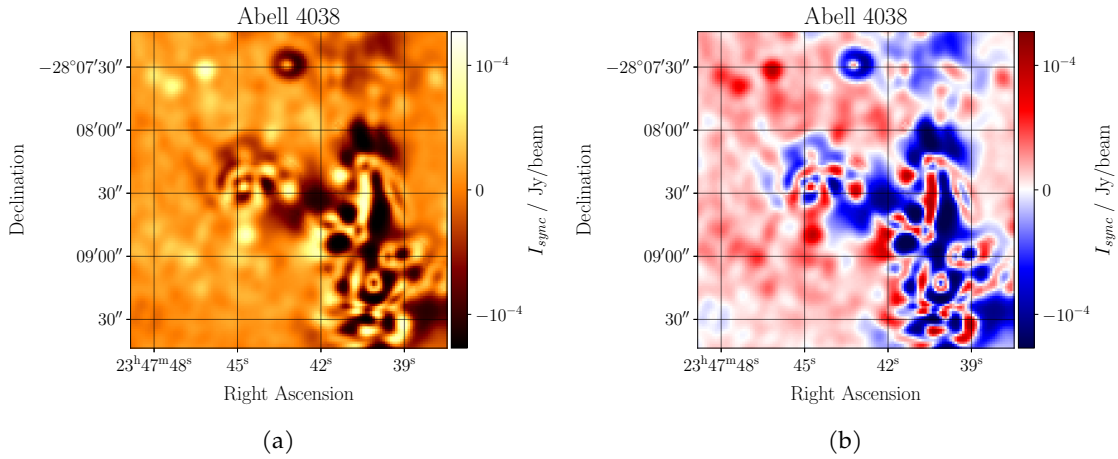


Figure 5.3: The residual surface brightness image of Abell 4038, as computed by PyBDSF. Note the presence of negative pixels, resulting from over-subtraction during the source-subtraction procedure. The two images show the same data, with a diverging colour map in pane (b) that specifically highlights regions with positive and negative pixel values.

Telescope Compact Array (ATCA)). It is argued by the authors that sources should be subtracted in the u, v plane, also known as the visibility plane, which is related to the image plane via a Fourier transformation. While this technique may alleviate the over-subtraction of surface brightness, it is not currently available in this project while using the distributed enhanced data products of DR1. Another improvement to the implementation used here would be the optimal selection of algorithm parameters used in the PyBDSF software. All the residual images in this project have been obtained with the default parameter set, for consistency and ease of use, though there are numerous options which can be tweaked. Some factors which could be of benefit involve intelligently setting the threshold values used to identify peaks, and use of an iterative wavelet scale decomposition (referred to as the *Á trous* module in the documentation). Both of the above considerations constitute possible areas of future investigation.

5.3 Model Images

Once the data images from the MGCLS have been processed according to the above methods, the next step of the analysis is to prepare the predicted model emissions for the statistical comparison. These model emissions will have been generated by the computational procedure outlined in Chapters 3 and 4 and saved in their final form as FITS files. The parameters that define each DM halo environment (corresponding to each target in Table 5.1) are shown in Table 5.3. Additionally, the following modelling configurations, which have been discussed in detail in previous Chapters, are used in each simulation:

- WIMP masses in the range 10 - 1000 GeV
- Annihilation channels in the set $(b\bar{b}, \mu^-\mu^+, \tau^-\tau^+, 2\text{HDM}+\text{S})$
- Two DM halo density profiles, corresponding to the gNFW profile with $\alpha = 1$ and $\alpha = 0.5$
- Use of the full form of Equation (3.19) (i.e. both diffusion and energy loss effects are always taken into account)
- Kolmogorov-type turbulence with a diffusion coefficient of $D_0 = 3.1 \times 10^{28} \text{ cm}^2 \text{ s}$
- Use of the OS solution method
- Synchrotron radiation frequency equal to the MeerKAT reference frequency (1280 MHz)

In some cases (notably for the X-ray-selected sources) there is little available information in the literature about the halo or magnetic field environments in the cluster. Thus, we make a simple estimation of the necessary parameters by taking the values found in the Coma cluster environment and scaling them with the mass of the cluster in our sample. This procedure is laid out by the following equations, which yield the required thermal electron density normalisation ($n_0 \equiv n_e(0)$) and scale-radius for the gas (r_e) and magnetic field (r_b) from the corresponding values in the Coma cluster, respectively:

$$n_0 = n_{0,\text{Coma}} \left(\frac{M_{\text{vir}}}{M_{\text{vir,Coma}}} \right)^{1/3}, \quad (5.1)$$

and

$$r_{e/b} = r_{e/b,\text{Coma}} \left(\frac{R_{\text{vir}}}{R_{\text{vir,Coma}}} \right), \quad (5.2)$$

where M_{vir} and R_{vir} are the virial mass and radius of the sample cluster. Then, we compute the magnetic field by assuming that its profile mimics that of the thermal gas density, i.e. the magnetic field is given by the general relation $B(r) \propto (n_e(r))^\eta$, where η is a parameter usually between 0.5 - 1 and is found by correlating Faraday Rotation Measure (RM) and X-ray flux observations (Dolag et al. 2001). Such a relation between the magnetic field and thermal gas density is common in studies of galaxy cluster magnetic environments (Dolag et al. 2001; Govoni et al. 2004; Murgia et al. 2004; Dolag et al. 2008; Feretti et al. 2012; Osinga et al. 2022), and stems from supporting evidence found in both cosmological magneto-hydrodynamic simulations and in correlations between radio and X-ray observations. More specifically, in this work we calculate the magnetic field profile by

$$B(r) = B_0 \left(\frac{n_e(r)}{n_0} \right)^\eta, \quad (5.3)$$

where we set the index $\eta = 0.5$ and the magnetic field normalisation factor $B_0 = 5 \mu\text{G}$. The former choice corresponds to the case in which the energy density of the magnetic field is linearly proportional to that of the gas density. The latter choice defines the strength of the magnetic field at the centre of the cluster, and is a conservative estimate based on the results shown in Osinga et al. (2022). The results therein show that B_0 values of $5 - 10 \mu\text{G}$ (for $\eta = 0.5$) are needed to fit observations of radio source depolarisation made with the Karl G. Jansky Very Large Array (VLA) telescope. The quoted values are however also consistent with other, previous estimates in the literature. Since the central magnetic field strength directly impacts the propagation of electrons, and hence the synchrotron output of the model, larger values of B_0 will lead to stronger constraints on the annihilation cross-section. This is shown in Chapter 6, and motivates our choice of the conservative value $5 \mu\text{G}$. An example of the use of Equations (5.1), (5.2) and (5.3) for the estimation of modelling parameters is shown in Figure 5.4.

The virial mass of each cluster has been calculated from the estimates of M_{500} that have been provided in the MCXC (Piffaretti et al. 2011). These values are estimated based on the observed luminosity of the cluster, and found with the following L-M relation:

$$h(z)^{-7/3} \left(\frac{L_{500}}{10^{44} \text{ erg s}^{-1}} \right) = C \left(\frac{M_{500}}{3 \times 10^{14} M_\odot} \right), \quad (5.4)$$

where $\log(C) = 0.274 \pm 0.032$ and $\alpha = 1.64 \pm 0.12$. The relation in Equation (5.4) with the quoted parameters was originally described in Arnaud et al. (2010, tab. 1). Since the flux from the DM halo will depend on the square of the total mass, the uncertainty in the final upper limits on the WIMP annihilation cross-section would thus scale approximately with twice the uncertainty in the total mass.

After the model images have been generated with the parameters outlined above, they require two further processing steps before they are ready for the statistical comparison. These steps are: ensuring that the DM halo is centred on the correct sky coordinates, and remapping the simulated emissions onto the same coordinate grid as the data image. These processes will be discussed in detail in the following Sections.

Parameter	Unit	A133	A4038	J0225	J0600	J0757	Coma
M_{vir}	$[10^{14} M_{\odot}]$	3.89	4.60	-	0.701	1.829	12.4
R_{vir}	[Mpc]	1.47	-	-	1.063	1.462	2.7
r_s	[Mpc]	-	-	0.108	0.145	0.212	0.287
ρ_s	$[10^{15} M_{\odot} \text{Mpc}^{-3}]$	-	-	5.277	-	-	-
c_{vir}	-	-	-	-	7.312	6.886	-
z	-	0.057	0.028	0.060	0.0369	0.039	0.0231
n_0	$[10^{-3} \text{cm}^{-3}]$	6.77 (0.988)	22.0	3.955	1.340	1.844	3.49
r_e	[kpc]	81.18 (391.05)	75.87	111.715	99.650	136.991	253.0
q_e	-	-0.745 (-1.081)	-0.54	-0.805	-0.654	-0.654	-0.654
B_0	[μG]	5.79 (2.21)	8.0	5.0	5.0	5.0	4.7
r_b	[kpc]	81.18 (391.05)	75.87	111.715	99.650	136.991	253.0
q_b	-	-0.372 (-0.541)	-0.27	-0.4025	-0.327	-0.327	-0.327

Table 5.3: List of modelling parameters for each cluster in the sample, and the Coma cluster used in various parameter scaling relations in the case of a lack of literature data. For the A133 cluster, which is modelled with a double-beta profile, the secondary parameters for the profile are given in parentheses. The references used for each target are given as follows. A133: Zhu et al. (2021) and Piffaretti et al. (2011). A4038: Wojtak et al. (2007), Piffaretti et al. (2011), and Zhu et al. (2021). J0225: Shakouri et al. (2016), Foëx (2019), and Piffaretti et al. (2011). J0600: Piffaretti et al. (2011). J0757: Piffaretti et al. (2011). Coma: Bonafede et al. (2010), Łokas et al. (2003), and Chen et al. (2007).

5.3.1 Position of DM halo centre

The pointing coordinates used in the MeerKAT observations of the cluster sample have been given in Table 5.1. These values do not necessarily point to the centre of mass of the cluster though, where the centre of the DM halo should be located. The exact location of this central coordinate can be determined in several ways: via lensing measurements, chosen as the peak brightness value in X-ray observations of the intra-cluster medium (ICM) gas, or as the central coordinates of the Brightest Cluster Galaxy (BCG) in optical observations. However, as discussed at length in Seppi et al. (2023), these independent measures of the cluster centre may be offset from each other, and are expected to be offset for clusters that are not dynamically relaxed (due to late-time formation or recent merger activity). The results of Seppi et al. (2023) show that the offset between the BCG and the peak of the X-ray emissions, in a sample of galaxy clusters observed by the eROSITA telescope, can be tens or hundreds of kpc. Since this offset can also be affected by mergers and AGN feedback, the determination of the true centre of a cluster is a complicated endeavour that requires precise observations and dedicated multi-wavelength studies, complemented by large-scale simulations of the DM halo.

In the case of cool-core clusters – or more specifically, those which have dynamically relaxed cores – we can be rather certain that either commonly-used measure, the X-ray brightness peak or the BCG, is a good estimate for the centre of the DM halo. Unfortunately, when there is an offset in these measures as described above, the high-resolution nature of the MeerKAT images causes the RoI around these points to be entirely different (we note that in the lucky event that neither region has artefacts or large extended emissions, there could be a negligible effect on the final results, however this would have to be investigated on a case-by-case basis). Further, the RoI's around the MGCLS pointing coordinates are often completely distinct from these central coordinate estimates, an example of which is shown for the Abell 4038 cluster in Figure 5.5. In this case, the images display extremely different behaviour in the two regions, with one (at the location of the peak X-ray brightness) showing bright extended emissions throughout the region, while the other displays much a much fainter overall surface brightness. Such a large discrepancy in the RoI's would quite clearly lead to large differences in the final results (especially given the imperfect nature of the current implementation of our source-subtraction procedure). This example illustrates the general difficulty in selecting a location for the centre of the DM halo in high-resolution images of galaxy clusters.

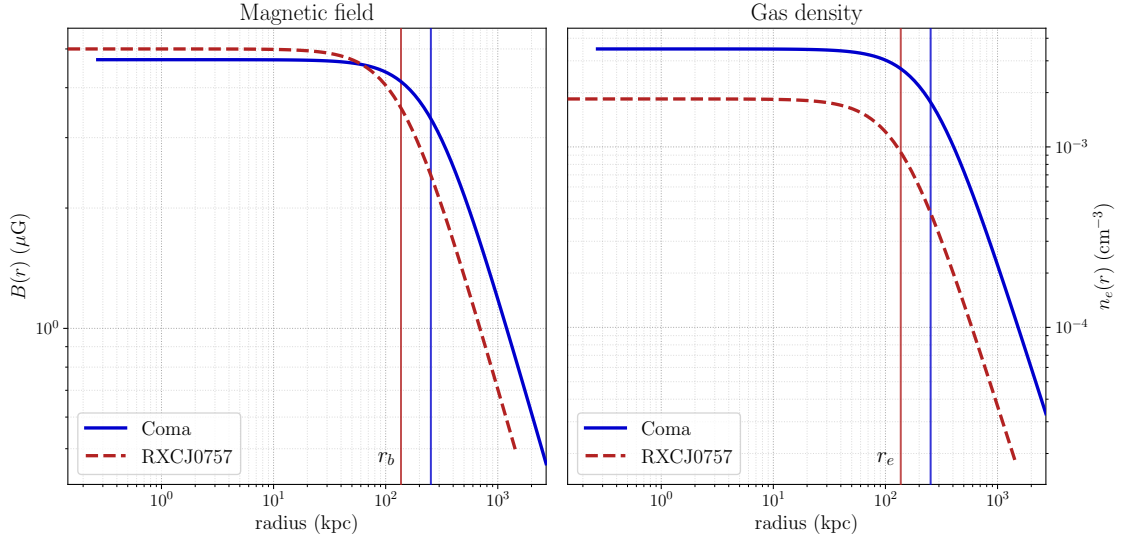


Figure 5.4: The estimation of modelling parameters from the scaling procedure explained in the text (using Equations (5.1), (5.2) and (5.3)). These example profiles are for the RXCJ0757 cluster, whose magnetic field (left panel) and gas density (right panel) have been determined by scaling Coma cluster properties with the virial mass of RXCJ0757. The vertical lines represent the scale-radius of each profile, respectively.

For this work, we choose to use the coordinates provided by the MCXC, which are also used in the NASA/IPAC Extragalactic Database (NED)⁶. These coordinates are the observed centroid X-ray brightness values, and are shown (in comparison to the MGCLS pointing coordinates) in Table 5.4. As discussed above, these coordinates should provide an excellent estimate of the centre of the DM halo for clusters with dynamically relaxed cores. For other cases, this estimate will contain larger uncertainties (from the elements discussed in Seppi et al. (2023)). While still being physically motivated by the above-mentioned arguments, this data set is also simple to access and use, whereas clear information about the BCG in each cluster of our sample has been more difficult to gather. Notably however, most of the clusters in our sample display significant extended emissions in the RoI's around these coordinates, which could be tracers of astrophysical activity in the BCGs (which would reinforce our choice of these coordinates as the centres of the DM halos). This also makes the choice of coordinates a conservative one, as different locations void of these emissions would in general lead to stronger DM constraints. We note that these aspects may be improved upon with future studies that make a more in-depth investigation of each individual cluster.

Cluster name	MGCLS coords		NED coords	
	RA (deg)	DEC (deg)	RA (deg)	DEC (deg)
A133	15.6879	-21.8800	15.675417	-21.873611
A4038	356.8796	-28.2028	356.930000	-28.141389
RXCJ0225	36.3750	-29.5000	36.293750	-29.473889
RXCJ0600	90.2013	-58.5872	90.201250	-58.587222
RXCJ0757	119.4438	-53.2636	119.443750	-53.263611

Table 5.4: List of coordinates given by the MGCLS pointing coordinates (Knowles et al. 2022) and the NED/MCXC cluster centre positions determined by X-ray observations (Piffaretti et al. 2011). Note the high level of precision in these numbers (measured in degrees) that is necessary for arcsecond-scale resolutions.

⁶Website homepage: <https://ned.ipac.caltech.edu/>

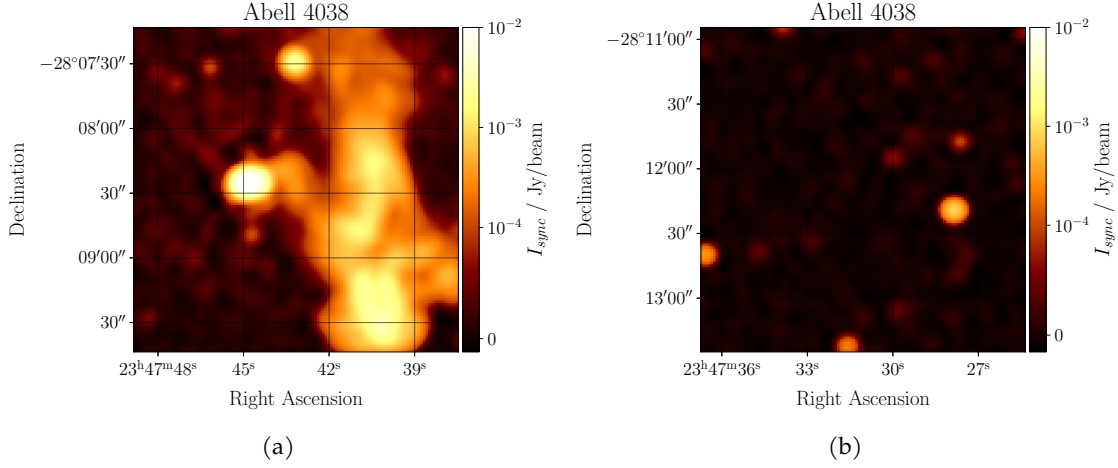


Figure 5.5: Comparison between the RoI's of Abell 4038 surface brightness images that are centred on different coordinates. (a): Using the NED/MCX coordinates. (b): Using the MGCLS pointing coordinates. Note that the colour maps have an equal scale so that the attributes of each image can be directly compared.

5.3.2 Model reprojection

The generated DM emission model, with proper central sky coordinates, is now mapped onto a FITS image. This mapping procedure transforms the radial surface brightness values (calculated with Equation (3.50)) into a two-dimensional space under the assumption of spherical symmetry. The target pixel density of the output image can be set manually, and is chosen to be at least better than the one found in the MGCLS images, i.e. $\sim 1''$ per pixel. Due to the high resolution of these images, the full model (which is calculated up to the diffusion radius of the halo) contains a huge amount of data which is mostly made up of the outer regions of the halo. Since these regions are useless in the analysis, we discard them and keep only pixels within a 'cut-off' radius from the halo centre that is chosen to encapsulate each RoI. An example of an image generated in this way is shown in Figure 5.6a, which shows a model calculated for 100 GeV mass WIMPs in an NFW-like halo that is centred on the coordinates for the Abell 4038 cluster.

The form of the model in Figure 5.6a, while being a complete model of the DM emission, is not yet ready for a full statistical comparison to the MGCLS data. The final step in this process is to ensure that every pixel in the model image corresponds exactly to its counterpart in the MGCLS image, including the pixel sizes and locations (in sky coordinates) and physical units. This ensures that the surface brightness value of each individual pixel, in both model and data images, represents the same physical quantity. To accomplish this task, we use the `reproject`⁷ module, made available under the `astropy`⁸ package (Astropy Collaboration et al. 2022). This tool allows us to perform the described reprojection procedure, completely mapping the model image onto the coordinate system of the data image (further information about this tool and its capabilities can be found in the online documentation). During this procedure, we also manually make a unit conversion, as the model image initially has units of Jy/pixel whereas the data image has units of Jy/beam. This conversion is done quite simply, proceeding as follows. Firstly, we calculate the solid angle of the beam on the sky by

$$\Omega_{\text{beam}} = (\pi/4 \ln 2) \theta_{\text{beam}}^2 = 1.133 \theta_{\text{beam}}^2, \quad (5.5)$$

where the factor $\theta_{\text{beam}}^2 = \theta_{\text{min}} \times \theta_{\text{max}}$, and $\theta_{\text{min}}, \theta_{\text{max}}$ refer to the minor and major axes of the (elliptical) beam. The prefactor in this expression is due to the Gaussian nature of the beam. If we then define the number of pixels per beam as N_{px} and the solid angle of each pixel on the sky as

⁷Package documentation: <https://reproject.readthedocs.io>

⁸Website homepage: <https://www.astropy.org/>

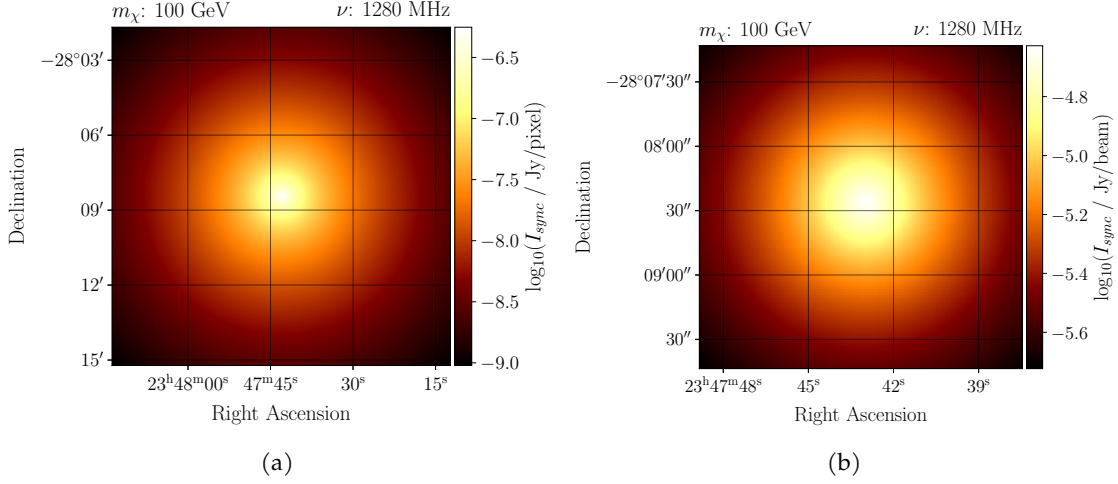


Figure 5.6: Effect of reprojection and unit conversions on the model image. (a): Example of the DM model emission, calculated with 100 GeV WIMPs, at a frequency of 1280 MHz, and with a set of other modelling parameters matching the A4038 cluster. This image represents the model emission before it is reprojected onto the data image’s coordinate system and any pixel unit conversions are applied. (b): The same model from (a) after reprojection onto the data image’s coordinate system and conversion of pixel units. Note the change in both axes coordinates and colour scale between each image.

Ω_{px} , we obtain

$$N_{\text{px}} = \lfloor \Omega_{\text{beam}} / \Omega_{\text{px}} \rfloor, \quad (5.6)$$

where we have used the floor function to ensure we only consider whole pixels in the analysis. As an example, in the case of the Abell 133 cluster we have $\theta_{\text{min}} \approx 7.29''$ and $\theta_{\text{max}} \approx 7.59''$, which results in $N_{\text{px}} \approx \lfloor 44.27 \rfloor \text{ Px/beam} = 44 \text{ Px/beam}$. This value is then used to convert our model image values from Jy/pixel to Jy/beam. We note here, as before, that the u, v map of the data from MeerKAT DR1 products was unavailable at the time of this project. While we expect that the angular scales of the model emission would not be resolved out by MeerKAT observations, the models do not take into account the various angular scales that exist within the observational data. A more sophisticated treatment of the model emission that takes into account the above points would allow for a more rigorous comparison to the data, and this could be the subject of future work in this topic.

Finally, we briefly describe the different forms of noise and uncertainty measures used in the analysis (a quantitative treatment of these factors will be provided in Chapter 6). In the original MGCLS observation data, i.e. in the FITS files obtained from the online repository, there are measures of the statistical noise present in each image, stored as individual planes in the FITS data cubes. An example of such data is shown in Figure 5.7a for the Abell 4038 cluster with a RoI of $2.5' \times 2.5'$. For all clusters in our sample, this noise is relatively uniform except for regions that contain bright point sources or extended emission in the surface brightness images, which consequently have higher noise estimates. The second relevant measure is the root-mean-square (RMS) uncertainty values calculated by PyBDSF during the source subtraction procedure. An example of this, again for the Abell 4038 image, is shown in Figure 5.7b. The calculation of this value follows a non-trivial algorithm that is described in detail in the PyBDSF documentation. As the use of this procedure with default parameters tends to produce reasonable results (when compared to the statistical noise measures of the MGCLS), we continue by using these results in the analysis and refer the reader to the PyBDSF documentation for further details.

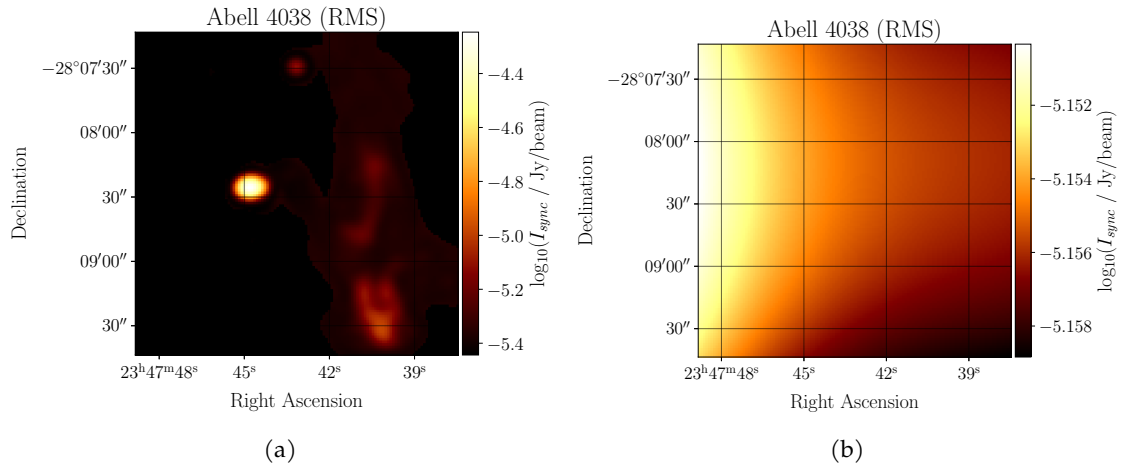


Figure 5.7: Representations of the uncertainty measures in the images. (a): Statistical noise estimates from the MGCLS data. (b): RMS map calculated by PyBDSF during the source-subtraction procedure.

Dark Matter Hunting

In the previous Chapters, we described both the theoretical modelling of the expected DM signal, and the processing of the observational data from various astronomical targets. In this Chapter, we combine the results of each of these (generally referred to from here as just the *model* and *data* respectively) in a robust statistical analysis that searches for any DM emissions in each data image. The primary outcome of this analysis is a set of annihilation cross-section magnitudes, calculated over a range of DM particle masses, that are excluded by non-detection of emission in the data. These $\langle\sigma v\rangle - M_\chi$ curves are the standard way of constraining the DM parameter space in DM indirect detection studies, and were discussed in terms of the current state of the literature in Chapter 2 (seen in Figures 2.4 and 2.6). The following Sections provide an outline of the relevant theoretical aspects involved in the statistical analysis, demonstrate how it is performed, and then present the main results of the analysis with a concluding discussion.

6.1 Statistical framework

To directly compare the data to the models, we adopt a standard framework for hypothesis testing that includes the use of χ^2 statistics and likelihood ratio tests. The methodology roughly follows the one used in Regis et al. (2015b, sec. 5) and Regis et al. (2021, sec. 4) for the same purpose, which is also discussed in a general context in Wall et al. (2003, ch. 6). The details of the specific implementation used here will be described as follows.

6.1.1 Likelihood ratio tests

Firstly, we define the likelihood function $\mathcal{L}_i$, which defines the probability of the obtained data given a certain set of model parameters (represented by the subscript i). In our analysis, since the annihilation cross-section is treated as a free parameter, we use a simple range of $\langle\sigma v\rangle$ values to define each model while keeping all other modelling parameters constant. By comparing the likelihoods of these models, we then find a value of $\langle\sigma v\rangle$ that either best explains the data, or conforms to some statistical constraint. One standard way to perform this comparison is through a *likelihood-ratio-test* (LRT), which produces a test statistic λ as follows:

$$\lambda = -2 \log [\mathcal{L}_i / \mathcal{L}_{\max}] , \quad (6.1)$$

where the likelihood function $\mathcal{L}_{\max}$ is the maximum over any model likelihood $\mathcal{L}_i$. The form of the ratio test in Equation (6.1) (i.e. -2 times the logarithm of the likelihood ratio) is written so as to conform to Wilks' Theorem (Wilks 1938; Silvey 1991, ch. 7), which states that:

Theorem 6.1 (Wilks' Theorem) *For a sample size n that uniformly approaches ∞ , and a parameter space Θ which has a subset Θ_0 associated with the null hypothesis H_0 , then, if H_0 is satisfied, the distribution of $\lambda = -2 \log(\Lambda)$, where Λ is the ratio of likelihoods defined over Θ and Θ_0 , is, up to terms of order $1/\sqrt{n}$, χ^2 -distributed with degrees of freedom equal to the difference in dimensionality of Θ and Θ_0 .*

One of the requirements of Wilks' Theorem is that the null hypothesis be true. In our analysis, the null hypothesis represents the case where no DM emission at all is present in the data images, which is set by the model $\langle\sigma v\rangle = 0$ (with all alternative models having some non-zero value of $\langle\sigma v\rangle$). In Regis et al. (2015b), a test for the detection of non-zero DM signals in six Local Group dSphs is performed by calculating a LRT with the null hypothesis, similarly to Equation (6.1), i.e.

$$\lambda = -2 \log [\mathcal{L}_0 / \mathcal{L}_{\max}] . \quad (6.2)$$

They then find, using Wilks' theorem to produce a χ^2 -distributed λ statistic, the significance with which the null hypothesis could be accepted or rejected (and hence, whether a non-zero DM signal was detected). While there were some spurious detection significances, which were attributed to faulty source-subtraction in the data images, the conclusion was that no statistically significant emission had been detected. This conclusion matches the one made in Regis et al. (2021), where a similar search for DM signals was performed on data from the Large Magellanic Cloud.

As the primary results in both of the referred-to studies are the observed limits on the DM-induced emission (either directly in terms of the flux or as $\langle\sigma v\rangle - M_\chi$ plots), Equation (6.1) is then used as a test to find the statistical upper bounds on the emission from each data set. Though, for both the null hypothesis testing and the finding of upper bounds, a technique of 'profiling out' nuisance parameters, based on Rolke et al. (2005), is used in addition to the standard LRT analysis presented here. While this extra methodology could be the subject of future work, we limit the analysis in this thesis to just using the standard LRT. Additionally, since we are primarily interested in the exclusion of models based on non-detections, we always assume the null hypothesis and proceed by using Equation (6.1) to test for these parameter exclusions (covered in the next Section).

We do follow Regis et al. (2015b, eq. 9) and Regis et al. (2021, eq. 4.1) in the choice of the likelihood function itself, chosen to have a Gaussian-like form expressed as

$$\mathcal{L} = e^{-\chi^2/2}, \quad (6.3)$$

where

$$\chi^2 = \frac{1}{N_{\text{px}}^{\text{beam}}} \sum_{n=1}^{N_{\text{px}}} \left(\frac{I_{\text{model}}^n - I_{\text{data}}^n}{\sigma_{\text{rms}}^n} \right)^2. \quad (6.4)$$

Here the quantities $N_{\text{px}}^{\text{beam}}$ and N_{px} represent the number of pixels in a single beamwidth and the total number of pixels in the data image, respectively. The surface brightness values determined by the model and data are given by I_{model}^n and I_{data}^n , and σ_{rms}^n represents the total uncertainty measure in the data. Chapter 5 contains general details about how all these quantities are found, and the prefactor of $(1/N_{\text{px}}^{\text{beam}})$ unusual to a standard χ^2 formula will be explained in the following Section. In the case of the uncertainty measure, we use the combination of the statistical error estimate provided by the MGCLS (Knowles et al. 2022) and the RMS estimate calculated by the PyBDSF package during the source subtraction procedure (an example visualisation of each uncertainty estimate is given in Figure 5.7). Thus, the final uncertainty estimate is given by

$$\sigma_{\text{rms}} = \sqrt{\sigma_{\text{MGCLS}}^2 + \sigma_{\text{PyBDSF}}^2}. \quad (6.5)$$

With the form of the likelihood given in Equation (6.3), the LRT defined in Equation (6.1) then simplifies to

$$\begin{aligned} \lambda &= -2 \log [\mathcal{L}_i / \mathcal{L}_{\text{max}}], \\ \Rightarrow \lambda &= \chi_i^2 - \chi_{\text{min}}^2, \end{aligned} \quad (6.6)$$

where the χ^2 values are calculated through Equation (6.4) for each model $\langle\sigma v\rangle$ value.

6.1.2 Model exclusion

Now that the LRT defined in the above section has an explicit form, the next step in our analysis involves finding models that are excluded by the data within some standard significance measure. Before we continue with the analysis of the LRT however, we present the definitions of some quantities necessary to this part of the analysis (note that the terminology and notation used throughout this Section mostly follows from Wall et al. (2003, ch. 2, 3)).

Firstly, we define the probability density function of the well-known Gaussian (normal) distribution for some random variable X as:

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right), \quad (6.7)$$

where μ and σ represent the mean and standard deviation of the distribution. Note that the term *random variable* used here refers to the standard definition in probability theory, i.e. a function that maps the measurable outcomes from a certain process acting on some sample space. Then, the cumulative distribution function, which is defined as the probability that the random variable X will have a value less than x , can be expressed as:

$$\begin{aligned} F(x) &= \int_{-\infty}^x dy f(y), \\ \Rightarrow F(x) &= \int_{-\infty}^x dy \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right). \end{aligned} \quad (6.8)$$

The normal distribution is closely linked to the Central Limit Theorem (CLT), which is widely-used in many types of statistical analyses. In a general form, it can be stated as:

Theorem 6.2 (Central Limit Theorem) *Given a sample of n random, independent and identically-distributed variables, represented by X_i with a mean μ and finite standard deviation σ , then as $n \rightarrow \infty$ the distribution of $\sqrt{n}(\bar{X}_i - \mu)$ tends to a normal distribution with mean 0 and standard deviation σ , where $\bar{X}_i$ is the sample mean of the X_i .*

The usefulness of the CLT in our analysis is that the distribution of $\sqrt{n}(\bar{X}_i - \mu)$ tends to a normal distribution regardless of the original distribution of the X_i 's. We use this result by assuming that the deviations in our model predictions from the true values are only due to random statistical fluctuations, which, given by the χ^2 statistic in Equation (6.4), will then tend to be normally distributed (with enough independent data points). As discussed at length in Section 5.2.1, the number of independent data points in each image is affected by the correlations between pixels inside the primary radio beam. This leads to the approximation mentioned therein, which treats all pixels within a single beamwidth as being completely correlated, and those outside a beamwidth as completely uncorrelated, resulting in resolution elements that can be treated as independent data points. This approximation is quantified by the prefactor in the χ^2 statistic in Equation (6.4) (i.e. $1/N_{\text{px}}^{\text{beam}}$). Given the condition of many data points needed for the CLT to be valid, we have limited our analysis to RoI's that contain a minimum of 50 resolution elements (although they typically contain far more than this).

We now return to our goal of finding the model with the lowest value of $\langle\sigma v\rangle$ that is excluded by the data, given some confidence level. As is standard in the literature, we use a confidence level of 95%, which corresponds to a significance level of $\alpha = 0.05$ or about 2σ . These values define the regions within the probability density function which cause a model to be rejected, as they too poorly reproduce the data. The normal probability density function, which has the characteristic "bell-curve" shape, has two tails within which these regions typically exist, representing the case of either underpredicting or overpredicting the expected value. In our analysis, we are interested in models that overpredict the observed data. To illustrate this point (and confirm that the analysis behaves as expected), a set of sample calculations have been performed using the RXCJ0757 galaxy cluster target and considering DM-DM annihilations through the $b\bar{b}$ channel (we will refer to this sample throughout the analysis process).

The χ^2 values for a DM particle mass of 10 GeV are shown in Figure 6.1, and the calculation of Equation (6.6) is shown in Figure 6.1b. From Figure 6.1b, it is clear that there are two tails that diverge from the maximum likelihood (given by the χ_{min}^2 value). Each of the regions split by this value represent $\langle\sigma v\rangle$ models that either underpredict or overpredict the data. Underpredicting models – in our case representing those that would lead to a weaker synchrotron signal than what is seen in the data – provide little empirically useful information, so we discard these and instead focus on models with larger $\langle\sigma v\rangle$ values than the one that results in χ_{min}^2 . The use of only one side of this function clearly motivates the use of a one-sided (also known as a one-tailed) significance test, which we perform by evaluating the cumulative distribution function at the given significance level, in line with the definition given in Equation (6.8). Given that we are looking for overpredicting models however, the cumulative distribution function is then defined as

$$F(\chi) = \int_{\chi_c}^{\infty} d\chi \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\chi^2}{2}\right), \quad (6.9)$$

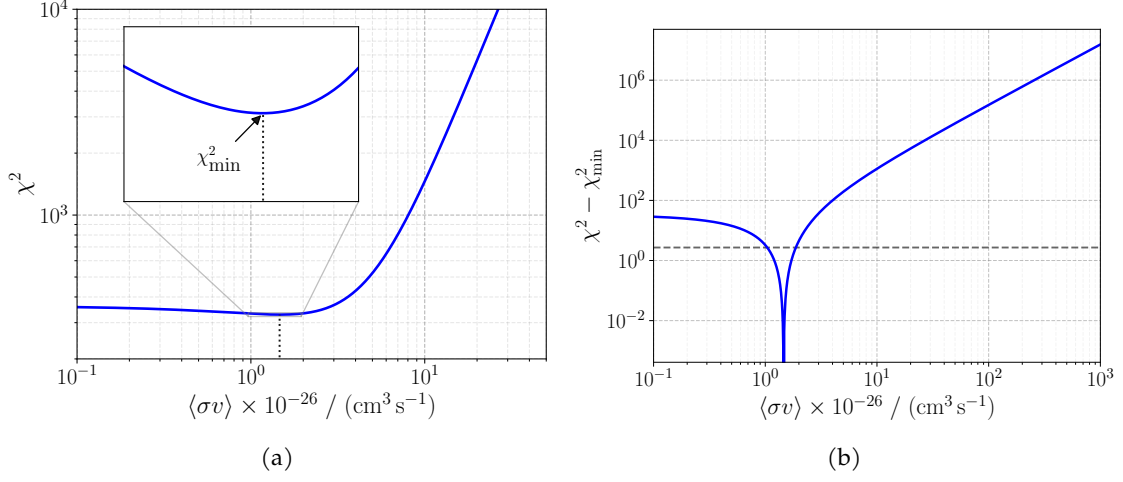


Figure 6.1: Sample calculations of Equation (6.4) (a) and Equation (6.6) (b). Results are shown for the RXCJ0757 galaxy cluster target, with WIMP DM particles of 10 GeV annihilating through the $b\bar{b}$ channel. In (b), the dotted horizontal line at ≈ 2.71 represents a significance level of 0.05 (i.e. a confidence level of 95%) at which models should be rejected. Regions left or right of the point given by $\chi^2_{\min}$ represent $\langle\sigma v\rangle$ models that underpredict or overpredict the data, respectively.

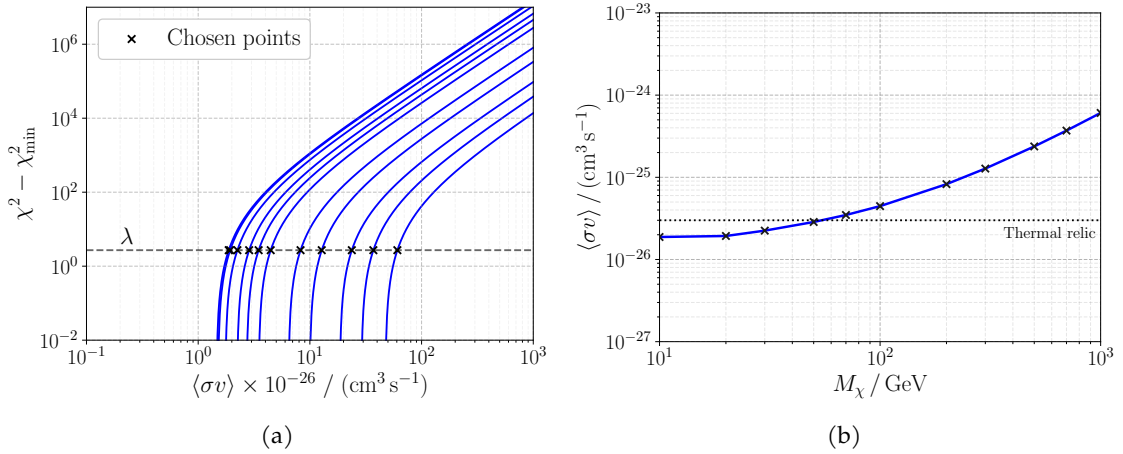


Figure 6.2: Selection of $\langle\sigma v\rangle$ values for exclusion limits. In (a) we show the result of the LRT calculation for a set of DM particle masses (each curve corresponds to a different mass), with each marker showing the value of $\langle\sigma v\rangle$ chosen according to the significance level of $\alpha = 0.05$. These chosen points directly lead to the exclusion curve in (b), which is compared to the canonical thermal relic model of $\langle\sigma v\rangle = 3.0 \times 10^{-26} \text{ cm}^3 \text{s}^{-1}$ plotted by the horizontal dotted black line.

where the integration limit χ_c represents the region defined by the significance level. After evaluating this equation numerically for the specific confidence level of 95% (i.e. $F(\chi) = 0.05$), the limit is found to be $\chi_c = 1.6449$, or $\chi_c^2 = 2.7055$. This value now represents the limit of the LRT statistic from which models should be rejected, so we define $\lambda_c = \chi_c^2$, and Equation (6.6) becomes

$$\lambda_c = \chi_i^2 - \chi_{\min}^2. \quad (6.10)$$

Whichever model i that satisfies the above criterion is then selected as the lower limit of rejected models (or, upper limit of viable models). This selection procedure is shown clearly in Figure 6.2a, which displays only the overpredicting tails of the models calculated using a range of DM particle masses and the intersection points of Equation (6.10). These selected points then fully specify the usual $\langle\sigma v\rangle - M_\chi$ exclusion curve, which is shown in Figure 6.2b. In the visualisation of all exclusion curves in this work we also display the values of $\langle\sigma v\rangle$ that would lead to the thermal relic density of DM necessary to explain the cosmological mass density of DM, as any model with values greater than these points are usually considered to be excluded in the literature. In the sample calculation we display the simplified canonical value of $\langle\sigma v\rangle = 3.0 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$, but in all results from the observational data of the MGCLS shown in the following Section we use the refined (numerical) calculations of $\langle\sigma v\rangle$ from Steigman et al. (2012, fig. 5).

6.2 Cross-section exclusion limits

This Section contains all final results of the modelling processes and analysis described in the previous Chapters. The galaxy cluster targets shown here are found in Table 5.1, and all aspects of their observational image data from the MGCLS can be found in Chapter 5. We consider a range of modelling scenarios for each target, characterised primarily by a variety of particle annihilation channels and halo density models, which are detailed in Chapter 3. The specific parameter configuration of each target can be found in Tables 5.3 and 5.4. All calculations of electron propagation are performed with the operator-splitting numerical method developed in this project, which is described in Chapter 4. Any calculation parameters or procedures not present in the above-mentioned tables can be found at the relevant Sections in the text.

Figures 6.3 to 6.6 show the cross-section exclusion curves of individual annihilation channels, calculated for all targets. The left-hand panels in each figure show results for a standard NFW halo density profile (index parameter $\alpha = 1$), with the right-hand panels showing results for a shallow gNFW halo ($\alpha = 0.5$). Note that we have restricted the mass range of the 2HDM+S models, as lower masses are unable to participate in the chosen annihilation channel of $\chi\chi \rightarrow S \rightarrow e^+e^-$. Each of the images used here have a constant RoI of $2.5' \times 2.5'$.

Figure 6.7 shows the same results, now framed in terms of the individual targets (which are displayed in each panel). These figures show two sets of results, with lower and upper cross-section values connected by a filled-in area. These two limits are defined by the two density profiles, i.e. the NFW and gNFW profiles, and the filled-in area should encapsulate any uncertainty in the density profile between these two choices.

Figure 6.8 shows the impact on the results when varying the chosen RoI. In Figure 6.8a, we have used a constant angular size of $2.5' \times 2.5'$ for all clusters. In Figure 6.8b, we have used a dynamic RoI based on half the scale radius of the DM halo in each cluster, which can be found in Table 5.2. These results are meant to be illustrative of the overall effects, and have been calculated for WIMPs annihilating through the $b\bar{b}$ channel in an NFW halo configuration. Note that this is the only figure in which the RoI is varied, in all other cases it is fixed at $2.5' \times 2.5'$.

Figure 6.9 shows the impact on the results for different magnitudes of the magnetic field normalisation, B_0 . The results were found for the Abell 4038 cluster with an NFW halo profile, and with 100 GeV WIMPs annihilating through the $b\bar{b}$ channel. This result is also meant as an instructive example, and we note that similar variation would be expected in other targets or annihilation channels.

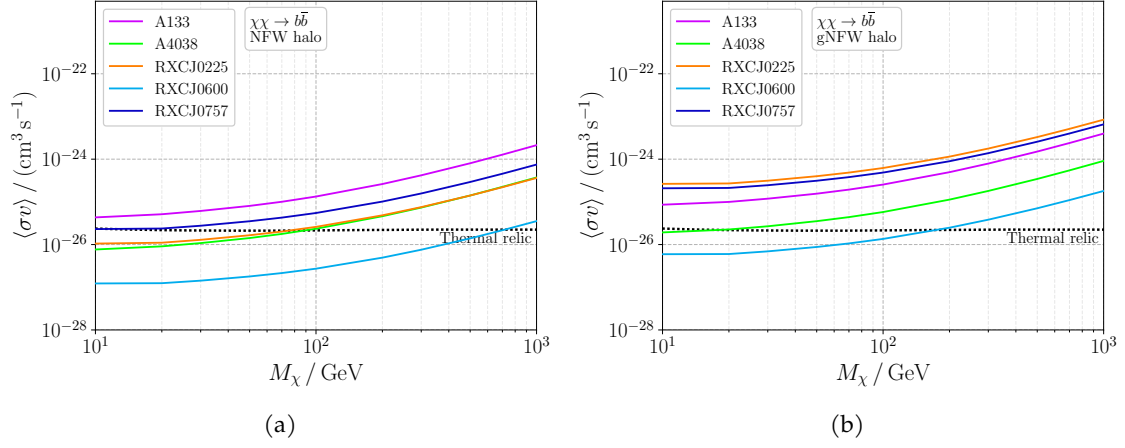


Figure 6.3: Annihilation cross-section exclusion curves calculated for all targets, with annihilations occurring exclusively through the $\chi\chi \rightarrow b\bar{b}$ channel. Panels represent halo density profiles of the (a) standard NFW profile and (b) a more shallow gNFW profile with shape parameter $\alpha = 0.5$. The cross-sections required to explain the thermal relic density of DM are also shown as a dotted horizontal line, with values taken from the numerical calculations in Steigman et al. (2012, fig. 5).

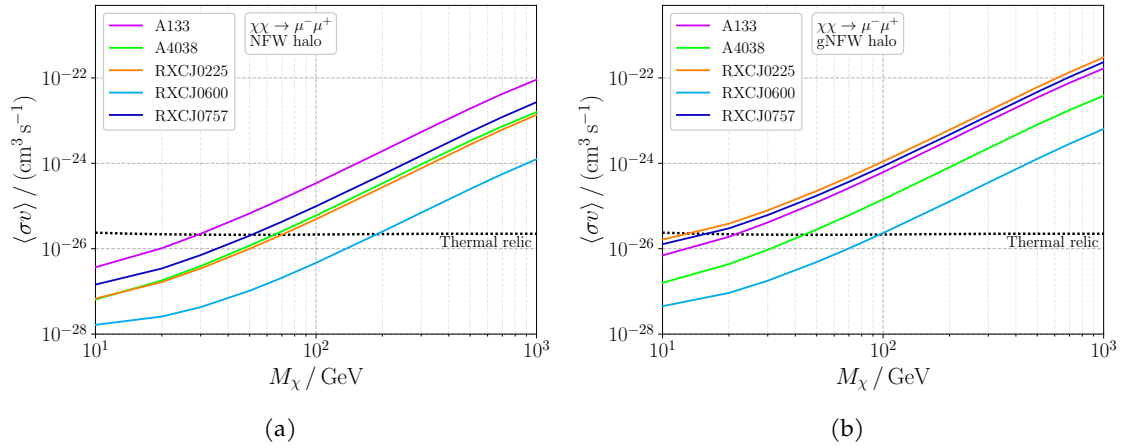


Figure 6.4: Same as Figure 6.3, except with annihilations occurring exclusively through the $\chi\chi \rightarrow \mu^-\mu^+$ channel.

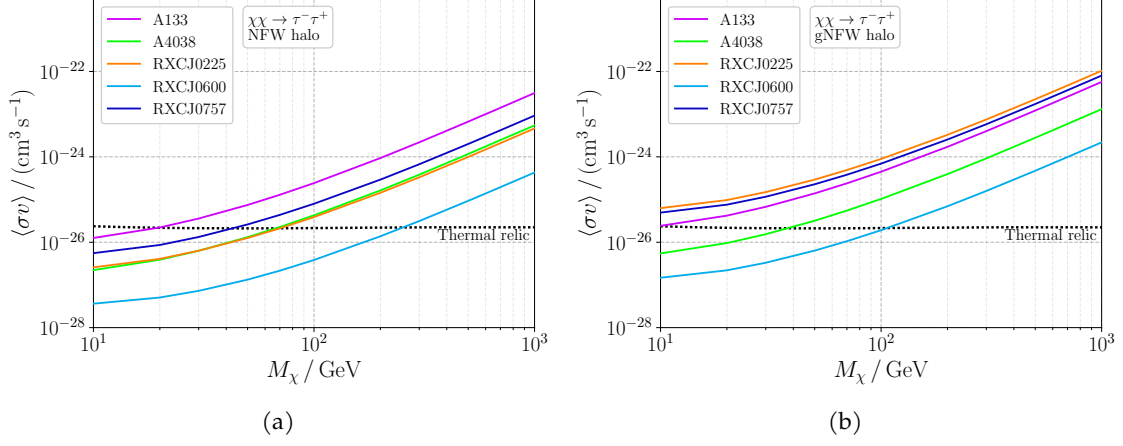


Figure 6.5: Same as Figure 6.3, except with annihilations occurring exclusively through the $\chi\chi \rightarrow \tau^-\tau^+$ channel.

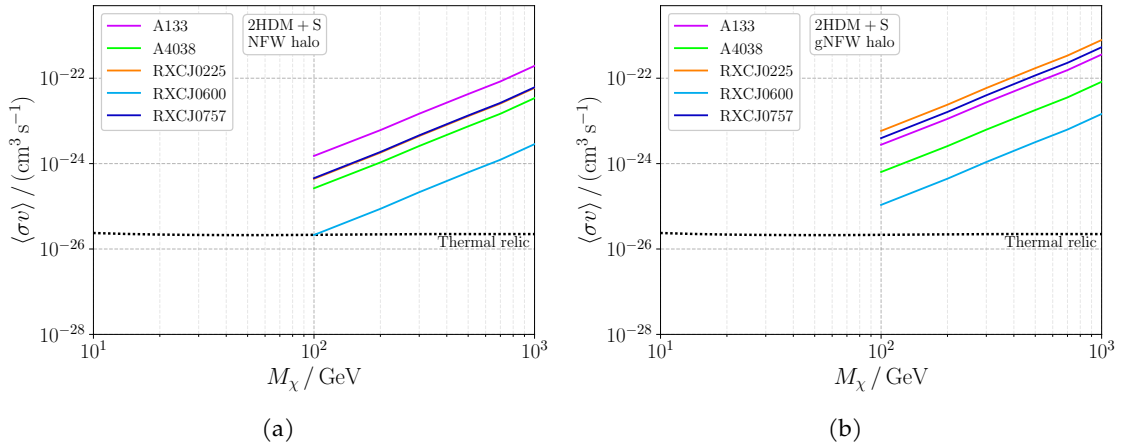


Figure 6.6: Same as Figure 6.3, except with annihilations occurring exclusively through the $\chi\chi \rightarrow 2\text{HDM} + \text{S}$ channel (specifically using spin-0 χ 's and the channel given by Equation (3.7)).

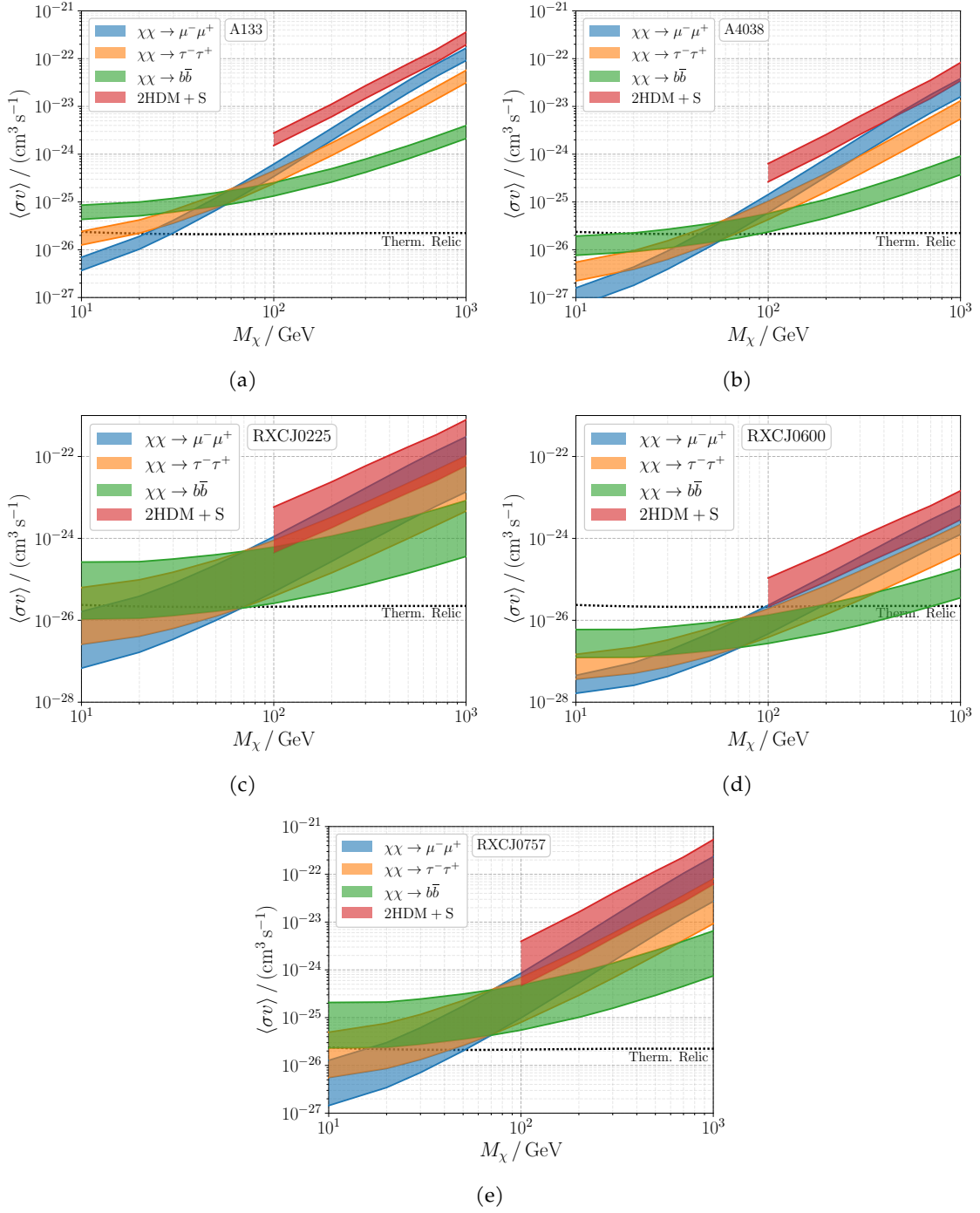


Figure 6.7: Annihilation cross-section exclusion curves calculated for each target (represented by a label in the upper centre of each plot). The lower limit of each band is the result of a standard NFW profile, and the upper limit is the result of a shallow gNFW profile (see text for details). The range between these limits has been filled in with the appropriate colours that represent each annihilation channel, and should encapsulate the results of using halo profiles with indices that vary between the limiting cases.

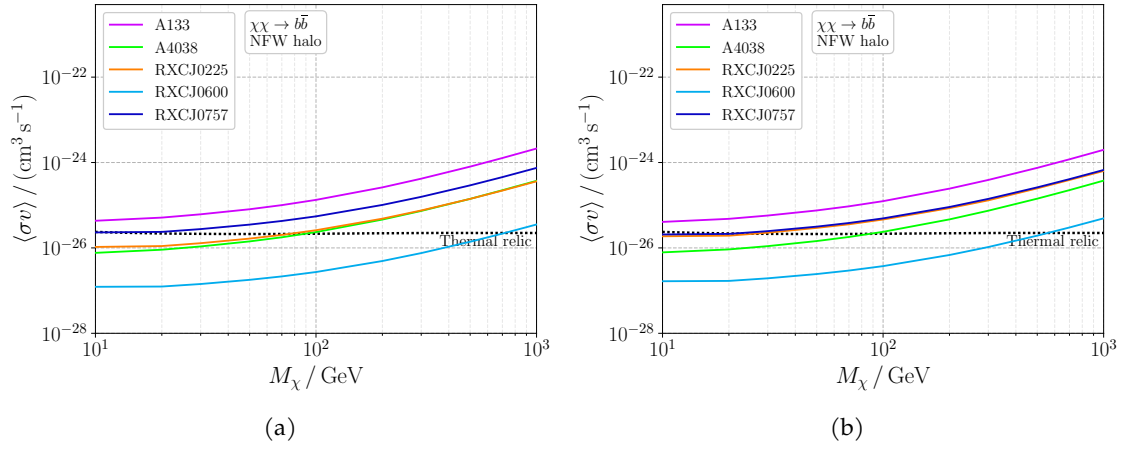


Figure 6.8: Effect of changing the RoI on the limits. (a): Limits found with a constant RoI size of $2.5' \times 2.5'$ for all clusters. (b): Limits found by dynamically adjusting the RoI size according to the physical size of the cluster, with each RoI selected to represent half of the DM halo scale radius. Note that in the cluster RXCJ0225, the RoI selected based on this criterion results in an image that has too few resolution elements to reliably perform the statistical analysis, and the results have been included for comparative purposes only. Further details may be found in the text.

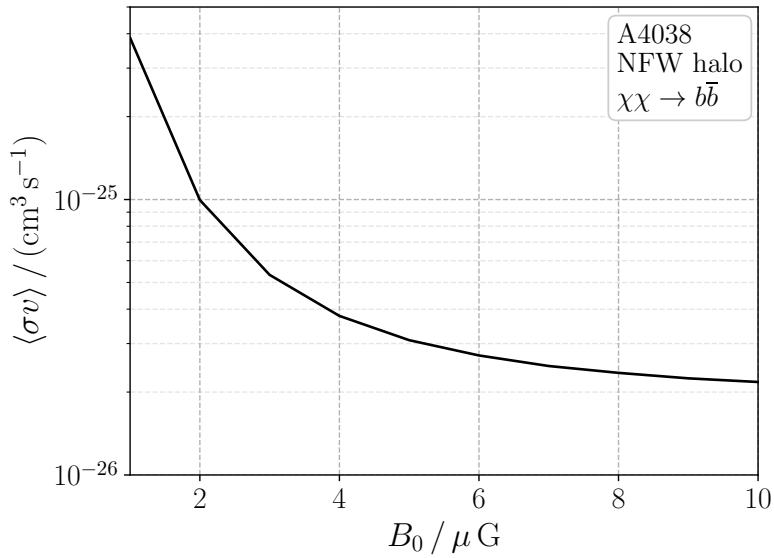


Figure 6.9: Effect of different magnetic field normalisations (B_0) on the limits. These results are for an illustrative case of the A4038 cluster, with 100 GeV WIMPs annihilating through the $b\bar{b}$ channel and with an NFW DM halo profile. The value of B_0 used in all other results shown here corresponds to $B_0 = 8$ G. There is a factor of ≈ 18 difference between the maximum and minimum values shown here.

6.3 Discussion

In this Section we outline some of the general trends that are observed in the exclusion curves presented above, and then compare the results to relevant works in the literature. Firstly, we note that the strength of $\langle\sigma v\rangle$ limits, typically measured by how much of the parameter space below the expected thermal relic value is probed by the results, can often be explained qualitatively in terms of the DM modelling parameters and morphology of the image data. For example, we see significant changes in the $\langle\sigma v\rangle$ limits when different DM halo density profiles are used. This is expected behaviour, as higher DM densities leads to more annihilations in the same volume and thus a larger number of electrons which are able to radiate synchrotron emission. Since the two profiles examined here, the standard NFW profile and the gNFW profile with an index of $\alpha = 0.5$, have contrasting DM density at the centre of the halo (see Figure 3.3), the higher relative central density of the NFW profile results in universally stronger limits on $\langle\sigma v\rangle$. This effect is made clear in Figure 6.7, which compares the results from using both profiles in terms of individual targets and annihilation channels. We see that RXCJ0225, RXCJ0600 and RXCJ0757 are significantly affected, whereas A133 and A4038 are more robust to this change. While A133 and A4038 both have slightly more massive halos than the other targets, they also noticeably have bright extended emissions in the centre of their data images (caused by bright radio galaxies or extended structures like jets, see for example Figure 5.5), which are missing in the other targets. These bright non-DM-related emissions (and the resulting source-subtraction procedure) could be ‘covering’ the central regions where the difference between the profiles is most stark, thus leaving A133 and A4038 less impacted by changes to the central density profile.

Although the results produced from all clusters in the sample are relatively clustered, the two that produce the most stringent upper limits on the annihilation cross-section are A4038 and RXCJ0600, especially in the case of more shallowly-cusped halo profiles. Both of these clusters probe a significant portion of the parameter space below the thermal relic cross-section, for any halo profile index. In the case of RXCJ0600, almost the entire mass range would be ruled out for annihilation in the $b\bar{b}$ channel and an NFW halo, and up to ~ 200 GeV for a gNFW halo. Given that the modelling parameters of the DM halo in RXCJ0225 are relatively modest compared to the rest of the sample, we attribute these stringent results to the observational data, which is both the faintest and most devoid of bright compact sources of the MGCLS images sampled here. In contrast, the RoI of A4038’s data image has a relatively large amount of bright source emission, however it also has the most massive halo, strongest magnetic field normalisation, and largest thermal gas population density of the cluster sample, factors which all contribute to stronger model emissions. Specifically, the synchrotron energy loss rate of the propagating electrons (which is the dominant effect at relevant energies, see Figure 3.4) depends on the square of the magnetic field strength, and the emitted synchrotron power depends directly on the magnetic field strength. While the explicit effect of the thermal gas population on the output synchrotron emission is not trivial, it seems that the higher magnetic field strength and mass of A4038 are sufficient to enable such strong $\langle\sigma v\rangle$ limits.

On the other hand, the prize for the weakest limits goes to A133 with an NFW halo, and is shared between RXCJ0225 and RXCJ0757 with gNFW halos. This could again be explained by the discussion given above, as RXCJ0225 actually produces comparatively strong limits with an NFW halo, but its substantial dependance on the shape of the density profile almost encompasses the differences observed in all other targets. We also note that the RXCJ0225 data has the most egregious presence of non-physical artefacts due to the interferometric image processing (see Figure 5.1), although the worst regions in the full map were ultimately avoided in the final $2.5' \times 2.5'$ RoI.

The characteristics of each annihilation channel in the results shown here agree well with the literature. These channels are sometimes separated into so-called *hard* or *soft* channels (Gaskins 2016), corresponding to the $\mu^-\mu^+$, $\tau^-\tau^+$ (hard) and $b\bar{b}$ (soft) channels shown here. This distinction can clearly be seen in Figure 3.1, with the generally similar spectral behaviour of the quarks and bosons being quite different to that of the leptons. This is precisely the reason only a small subset of possible channels are used in the calculations and then considered as representative of other channels. In the results presented above the leptonic channels (i.e. $\tau^-\tau^+$ and $\mu^-\mu^+$)

result in much steeper $\langle\sigma v\rangle - M_\chi$ curves than the softer quark channel, with regions below $\sim 60 - 70$ GeV being dominated by the $\mu^-\mu^+$ channel, and above by the $b\bar{b}$ channel. This particular dividing value depends on the particle interactions and decay pathways that produce the final state electrons in each channel, which is likely similar for the channels studied here. Compared to these three channels, the limits from the 2HDM+S model, with spin 0 χ 's annihilating through the channel $\chi\chi \rightarrow S \rightarrow e^{+,-}$, are relatively weak (although they can be reconciled with predictions for observations with MeerKAT, see following Section). As low mass WIMPs are being increasingly constrained with modern observations (see Regis et al. (2021) for example), the most promising avenue for constraining large WIMP masses thus seems to be through the quark channels.

In Figure 6.8, we draw comparison between results that are found using different RoI's. In Figure 6.8a we have used a constant RoI size of $2.5' \times 2.5'$ in all clusters, whereas a dynamic RoI calculated based on half of the scale radius of the DM halo in each cluster was used in Figure 6.8b, using values presented in Table 5.2. The motivations for using either of these two schemes were discussed in Chapter 5, and we discuss briefly how the results are affected by these choices here. Between the results, we observe marginal changes in the example case of annihilation through the $b\bar{b}$ channel and NFW profile. Because of the nature of this comparison, the limits produced for each cluster target might increase or decrease based on the physical size and distance of the cluster. Further, each data image contains a unique morphology that would manifest in different ways when the RoI is increased or decreased from the constant value of $2.5' \times 2.5'$, especially in the target images that have bright extended emission associated with the BCG. The total average relative differences (normalised relative to the larger values) between the results using each RoI, are: 6.4%, 1.4%, 43.3%, 27.2% and 10.4% for the A133, A4038, RXCJ0225, RXCJ0600 and RXCJ0757 clusters, respectively. However, we note that the number of resolution elements available in the dynamic RoI of the RXCJ0225 cluster was only 21, which is well short of the 50 elements that we consider to be required for the statistical analysis to be valid. All other clusters had ≥ 100 resolution elements, and can thus be considered statistically valid. We also note that although this dataset was chosen to be displayed, it represents an instructive example for other results; as the selection of the RoI only changes the region of the data image that is analysed, changes in modelling parameters (such as halo profile or annihilation channel) produce results that exhibit identical behaviour. The conclusion we draw from this comparison is that the use of either scheme, a static or dynamic determination of the RoI, is less impactful to the results when compared to some other sources of uncertainty (like halo profile index or magnetic field normalisation). It could thus be chosen optimally for each target, with the only requirement being that the number of resolution elements remains statistically valid in all cases. Since the resolution of the model images can be adjusted freely (with some inevitable uncertainty in the description of the innermost radii of the halo density profile, see Section 3.1.2), higher resolution data in the future could further improve the viability of smaller RoI's, providing stronger $\langle\sigma v\rangle$ limits.

We observe in Figure 6.9 that the upper limits on the annihilation cross-section are more sensitive to changes in the magnetic field normalisation strength at lower values of B_0 , with less impact between larger values. As discussed in Chapters 3 and 5, a precise determination of this value from empirical sources is difficult, which is the reason we have chosen the conservative estimates of $5 \mu\text{G}$ and $8 \mu\text{G}$ for dynamically disturbed (NCC) and dynamically relaxed (CC) clusters, respectively. Studies of magnetic fields in galaxy clusters (Govoni et al. 2006; Guidetti et al. 2008; Govoni et al. 2017; Stuardi et al. 2021; Osinga et al. 2022) indicate central magnetic field strengths (corresponding to the normalisation value of our magnetic field profiles) of larger than $\sim 2 \mu\text{G}$, but in extreme cases this rises to $30 \mu\text{G}$ (Govoni et al. 2004). As with the comparison between RoI's, the general behaviour of the upper limits on the cross-section shown in Figure 6.9 translates to other modelling configurations and targets, and the range in limits shown therein should thus provide a reasonable estimate of the sensitivity of the results to various possible magnetic field normalisations.

Comparisons to related work

With observations from the full SKA project just on the horizon, a number of studies have predicted strong prospects for using the SKA or its pathfinders for radio-based DM indirect detection (Colafrancesco et al. 2015a; Colafrancesco et al. 2015b; Beck et al. 2017; Storm et al. 2017; Beck 2019b;

Chen et al. (2021), and indeed the use of the ASKAP instrument has already produced some of the strongest limits on $\langle\sigma v\rangle$ of any wavelength in the literature (Regis et al. 2021). However, as discussed before in this thesis, there has been relatively little modern interest in the use of galaxy cluster search targets, with much attention going to the satellite dSph galaxies of the Milky Way. Some studies have been performed that test specific models of DM, or investigate the general morphology of the radio emissions from galaxy clusters (for example in Colafrancesco et al. (2006), Colafrancesco et al. (2011), Marchegiani et al. (2018), Marchegiani et al. (2019), and Chan et al. (2020)), but the most direct comparisons to the literature for this work come from studies that produce $\langle\sigma v\rangle - M_\chi$ exclusion curves, as we have above. Of these works, we find that the results presented here are almost always more constraining.

In Kiew et al. (2017), the authors studied the predicted DM-induced radio emission from six galaxy cluster targets, for a very similar range of modelling parameters as those considered here. The halos were modelled with NFW density profiles and $b\bar{b}$ and $\mu^-\mu^+$ channels (the most extreme channels in our results as well, excluding the 2HDM+S model), with a very similar set of magnetic field and gas density models. However, they use the Green's function solution method for the diffusion-loss equation (as described in Section 4.1), a somewhat less sophisticated exclusion framework, and a set of galaxy cluster targets (using a variety of radio flux observations mostly at 1.4 GHz) that has no overlap with the one used here. The limits on $\langle\sigma v\rangle$ can be seen in figure 3 of Kiew et al. (2017). These limits are improved by all of our NFW results, and in the case of our best results (RXCJ0600) the limits are strengthened by more than an order of magnitude. This improvement can be generally attributed to the quality and general faintness of the radio data, as the higher resolution of MeerKAT allows the crucial inner regions of the halo to be more accurately probed with surface brightness measurements.

The above comparison is very similar for the work of Storm et al. (2013), which also notably used the DMFIT software package for solutions to the diffusion-loss equation, and considered magnetic field profiles that are weighted by the halo density profile. Again, the results presented in this thesis significantly strengthen the $\langle\sigma v\rangle$ limits for compatible cases (i.e. excluding the estimated effects of substructure) (Storm et al. 2013, figs. 3-6), which can be explained with similar reasoning as before. A more recent study by the same authors, Storm et al. (2017), investigated the potential indirect detection capabilities of the (at the time of publication) planned radio surveys of LOFAR (van Haarlem et al. 2013), ASKAP (Johnston et al. 2008) and APERTIF (Oosterloo et al. 2009) projects for generic galaxy cluster models at 1.4 GHz. The main projections for these observations are seen in figure 3 of Storm et al. (2017). Compared to the results shown here, there is a small but distinct strengthening of the limits in all cases, with larger differences for the ASKAP and APERTIF projections.

The final set of direct comparisons can be drawn from our own work in Beck et al. (2023a), which presents a set of $\langle\sigma v\rangle$ limits obtained with radio data from the Coma and Ophiuchus galaxy clusters. This study uses a wide range of modelling parameters, and also employs the operator splitting method developed in this thesis (Chapter 4). When compared to the unaltered radio data results from both clusters, the limits presented in this thesis are significantly stronger (especially in the case of Ophiuchus, due to the less concentrated halo). However, the limits found using radio data that was extrapolated to smaller inner radii are much stronger, and probe similar WIMP masses and cross-sections as the best results shown here. The extrapolation of the data was motivated by the enhanced ability of high-resolution interferometers to probe these regions (which was previously unavailable to single-dish radio instruments), and the lack of evidence to suggest that there is any significant change in behaviour of the diffuse radio emission at smaller radii in these targets. The most limiting results in Beck et al. (2023a) are also comparable to those found in Regis et al. (2021), which uses radio data of the Large Magellanic Cloud from the other SKA pathfinder, ASKAP. While the search target used in that study was not a galaxy cluster, it is still a notable set of results given how strong the constraints are (visible in figures 4 and 6) and since the analysis of the data is performed in a very similar way to the one used in this thesis. These comparisons, coupled with the ones discussed above, suggest that limits on $\langle\sigma v\rangle$ found using a combination of MeerKAT data and accurate modelling techniques can live up to the high expectations created by early work in this field.

We also make a brief comparison to work done for the 2HDM+S DM model, as all the previous points apply only to the usual SM channels. As this particular model is relatively new in the

literature (proposed in von Buddenbrock et al. (2015) and von Buddenbrock et al. (2016)), analysis of the model in the context of indirect detection is currently limited to the work done in Lavis et al. (2022), Temo et al. (2022), and Beck et al. (2023b) (although, new studies utilising MeerKAT data are expected to join these in the near future). The limits on $\langle\sigma v\rangle$ found for the 2HDM+S model found in this work are generally in agreement with those found in Temo et al. (2022, fig. 1) for the Coma cluster, while being significantly stronger at lower masses for the Ophiuchus cluster. When compared to the limits found with M31, or to the projected limits in Beck et al. (2023b, fig. 6) for potential 100-hour MeerKAT observations of the Reticulum II dSph however, the limits shown in this work are typically weaker, by over an order of magnitude in some cases. Compared to Lavis et al. (2022), the clusters used here provide significantly stronger constraints in most cases.

Finally, we comment briefly on the status of these results compared to other multi-messenger indirect detection studies in the literature. Our most constraining target case, that of RXCJ0600 with an NFW halo, rules out $\lesssim 800$ GeV WIMPs when annihilating through the $b\bar{b}$ channel. This limit falls within the uncertainty region shown for the same channel in Regis et al. (2021, fig. 4), though is slightly above the most optimistic bound. Given that the limits presented therein are the most constraining indirect detection limits on WIMPs to date, the results shown here suggest that the approach taken in this work contains significant value to the field of radio-based indirect detection. The results for RXCJ0600 are also more constraining than any results found with gamma-rays, neutrinos or anti-protons thus far (see Figure 2.4 and cited references), and notably rule out the proposed best-fit WIMP model associated with the Galactic Centre excess measurements (Cholis et al. 2022). Outside our best case, the clusters A4038 and RXCJ0225 produce limits that are comparable with the strongest results from gamma-ray observations (Hoof et al. 2020; Armand et al. 2021), with all other cases presented here (which correspond to more shallow DM halo profiles or weaker magnetic field normalisations) being less stringent than the current leading-edge results. Notably, many of our results rule out WIMPs with masses below GeV, and almost all exclude those below 10 GeV. Since this is the mass range that is most suited for collider-based detection schemes among the alternative search techniques (see ATLAS Collaboration (2022) and Figure 2.2), improved results from future collider experiments should provide a complementary view on the results presented here.

Conclusion

This thesis provides a comprehensive study of state-of-the-art radio-based indirect DM detection searches. It includes detailed descriptions of the necessary physical modelling scenarios, a rigorous presentation of a newly-developed algorithm to solve the diffusion-loss equation, a showcase of observational data from the MeerKAT telescope, and a statistical search for DM signals within radio images. The motivation for this work was based on one of the most pressing open problems in modern physics, the nature of DM, and the recent emergence of sophisticated radio telescopes that are able to image the sky in unprecedented detail and thus effectively hunt for faint signatures of DM. The beginning of this thesis outlined some of the background and historical aspects of astrophysical DM, the existence of which is suggested from strong observational evidence and motivated by theoretical extensions to the SM. A resolution to both of these points could come from generic WIMPs, weakly-interacting particles that have the potential to create observable signals through self-annihilation processes. The particular process we have focused on in this work is the emission of synchrotron radiation in the radio band, which occurs when electrons and positrons that are produced by WIMP annihilations gyrate around magnetic fields. When there is a sufficient amount of DM, usually in the form of giant halos that encompass compact astrophysical structures like galaxies or galaxy clusters, this synchrotron emission should be visible to us as a region of diffuse radio emission. The premise of this work is thus to search for these emissions by modelling all the necessary physical processes and comparing the predicted emission to observational data of the region.

A large emphasis has been placed on the accuracy of the calculations of the expected DM-induced synchrotron emission, as accurate models are needed when using modern, high-quality radio data. For the modelling of DM parameters, which includes the energy spectra of annihilation channels and the density profile of the halo, we have used the most up-to-date modelling scenarios that are available in the literature. We obtained the spectra for all SM annihilation channels from the most recent version of the PPC4DMID results (Cirelli et al. 2011) (which also contain electroweak corrections (Ciafaloni et al. 2011)), and used the spectra in Beck et al. (2023b) for the annihilation channel in the 2HDM+S model. We have then used two halo density profiles, the standard NFW profile and a generalised version of the NFW with a more shallow central cusp, in all sets of results. Although the NFW profile provides excellent overall fits to the results of large N -body simulations, the small-scale regions around the centres of halos are still not well resolved by these simulations. Given the uncertainty in the central regions and the lack of precise halo modelling data in the literature for many galaxy cluster targets, the use of the two considered profiles should encompass any variation in form of the density profile in these objects. We reiterate that the density in these central regions is a salient factor in modern DM searches, as the ability for high-resolution data to spatially resolve and isolate the strong synchrotron emissions they produce can have a significant impact on the final constraints found, particularly in the case of faint radio data.

The other modelling aspect we focused on in this thesis is the solution to the diffusion-loss equation, which is a truncated form of the general cosmic-ray propagation equation that encapsulates the relevant effects of diffusion and energy-loss. The equilibrium solution to this equation is necessary for the calculation of the expected synchrotron emission from a population of electrons, however the nature of being a second-order partial differential equation means it requires a set of non-trivial mathematical techniques to solve. A common approach to finding these solutions in the literature has been a semi-analytical method that makes use of Green's functions to find the equilibrium solution directly. This approach has two significant drawbacks for general use: it can be computationally expensive, and it requires approximations about the spatial distribution of any involved physical quantities (like the magnetic field or thermal gas density). In this thesis we have therefore presented, in rigorous detail, a numerical solution method that has been developed with

the primary purpose of addressing these two drawbacks, and solving the diffusion-loss equation in a computationally performant and accurate way. The method is based on the Crank-Nicolson finite-differencing scheme (generalised to multiple dimensions with an operator-splitting technique) that calculates the equilibrium solution iteratively. The implementation of the algorithm used in the method draws inspiration from the GALPROP package (Strong et al. 1998), which is a well-developed publicly-available software package used extensively in the field of galactic cosmic-ray physics.

To test the solution method developed in this project, we have provided a set of comparisons between the results of our code and RX-DMFIT (McDaniel et al. 2017), which is another publicly-available software package that uses a semi-analytical method to solve the diffusion-loss equation. The package has been used in the literature to compute the expected synchrotron emissions from DM halos, and in the results presented here we compare the output DM-induced radio emissivity and integrated flux for a model galaxy cluster and dSph galaxy. We found general agreement between the output of each code, with some notable exceptions. Principle among these was a significant difference in the emissivity function at small radii within the dSph halo, with our code leading to a maximum of 75% more emission in this region. The difference was attributed to the mathematical treatment of the physical quantities involved, and in particular the substitution of the full magnetic field profile with a scalar average. This approximation leads to electrons at the centre of a DM halo losing relatively less energy to synchrotron radiation, which allows them to diffuse further through the halo. This effect reduces the electron number density in the inner regions of the halo, and thus also reduces the total output emission from this region. Since the high-density central region of a halo is of such importance in searches for DM-related emission, this lack of spatial accuracy can represent a major hindrance when using these models in conjunction with high-resolution observational data.

Several studies have shown that the SKA and its precursor instruments, with their high sensitivity and spatial resolution, are well-suited to produce novel DM search results, and the first studies to utilise ASKAP and MeerKAT data have already placed stringent constraints on the DM parameter space. In this work, we selected five nearby galaxy clusters, observed as part of the MeerKAT Galaxy Cluster Legacy Survey programme (Knowles et al. 2022), to act as a set of observation targets within which to hunt for radio signals of DM. These galaxy clusters are all at a low redshift ($z \leq 0.06$) and were chosen based on several criteria related to their potential effectiveness in DM searches, such as the overall quality of the data, the amount of bright compact emission present in the image, and the availability of modelling parameters in the literature. The data images for the chosen targets were subject to a number of processing techniques to prepare them for a statistical DM signal search, which included the selection of an optimal region around the centre of the cluster (to leverage the strong emissions in these regions) and the removal of any compact emission in a source-subtraction process (as these sources hide the relatively faint diffuse synchrotron emission from DM). These processed images were compared to the computed model emissions from the DM halos in each target, which were mapped onto two-dimensional FITS images and reprojected onto the same coordinate system as the data, ensuring that each pixel in the model image corresponded to the exact sky location of the data image.

The comparison between model and data images then followed a standard statistical framework, using a likelihood ratio test, that resulted in a set of $\langle\sigma v\rangle - M_\chi$ exclusion curves calculated at a confidence level of 95%. Of the targets considered here, the galaxy cluster RXCJ0600.8-535 produced the strongest constraints on the cross-section $\langle\sigma v\rangle$, regardless of the density profile used to model the DM halo. This was primarily due to the relative faintness of the region in the radio data provided by the MGCLS, which allows for more penetrative testing of the DM model emissions. When using an NFW density profile, there is at least partial coverage of the thermal relic cross-section limit from all targets, with especially strong constraints on lower mass WIMPs. When the gNFW profile is used the resulting constraints are (expectedly) weaker, particularly for targets with faint radio data in the region of interest, although in some cases this modelling scenario still produces constraints on low-mass WIMP models.

Our results also included a variety of representative annihilation channels that are well-studied in the literature, like quark or leptonic channels, as well as the comparatively unexplored 2HDM+S model. The leptonic channels ($\mu^-\mu^+$ especially) produce steep $\langle\sigma v\rangle - M_\chi$ curves, providing strong constraints on $\langle\sigma v\rangle$ at low masses, whereas the quark channel ($b\bar{b}$) has a flatter curve, but

probes the thermal relic cross-section at larger masses. The limits on $\langle\sigma v\rangle$ presented here are almost always more constraining than current results (that use corresponding source targets and modelling scenarios) in the literature. The results from the 2HDM+S channel produce relatively weaker constraints than the SM channels, but are in general comparable to the limits found for this model in the literature. Our exclusion curves agree with the trend in modern radio-based DM indirect studies that rule out low-mass WIMPs, requiring remaining viable models to have relatively higher masses, of hundreds of GeV.

Given that most of the galaxy clusters used as search targets in this work have not been used in indirect detection studies before, these results represent a strong opening move for DM searches in galaxy clusters with MeerKAT data. Future work in this field, which could include directed observations from MeerKAT (rather than the survey observations used here), improved source subtraction procedures, or more advanced analysis techniques (like interferometric data-stacking or baseline tapering), promises to deliver even better results in the ongoing search for the nature of DM. With observations from the full SKA on the horizon, we can also look forward to a future with completely ruled out parameter spaces, or even the detection of a so far elusive DM signal.

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