EXPERT AND NOVICE SENIOR PHASE TEACHERS' RESPONSES TO TRIGGERS OF CONTINGENCY IN THE TEACHING OF ALGEBRA

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DECLARATION

I declare that this Research Report is my own, unaided work, except as indicated in the text and the references. It is being submitted for the degree of Master of Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

JM

Signature Date: 12 November 2020

Julian Moodliar

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ABSTRACT

The purpose of this study was to investigate how secondary school novice and expert teachers responded to learners unexpected or unplanned offers in the teaching of algebra. Three teachers; two novices and one expert participated in this study. A total of nine lessons (three from each teacher) were observed and video recorded. The lessons were coded using a modified form of Weston's (2013) coding protocol to analyse the quality of teachers' responses to unexpected learner offers. These were coded as *inappropriate* or *appropriate*. Indicators used to highlight the degree of quality of the response were *minimum*, *middle* and *maximum* if a response were coded as appropriate to a learner's offer. Moreover, Video-Stimulated Recall (VSR) interviews were conducted with each teacher to gain insights into the teachers' thoughts and decision making when they responded to these unexpected or unplanned moments in their classroom. The findings of the study revealed that there was low level press evident in the two novice teachers' responses to unexpected learner offers. Conversely, the expert teacher's responses demonstrated a high level of press that constantly interrogated learners' thinking *in-the-moment*. This suggests that the expert teacher's responses were highly supportive of emergent mathematics learning in the collective classroom space. Another key finding was how the reflection-oriented questions during the VSR interviews proved fruitful in assisting both expert and novice teachers to reflect on how they responded to unexpected learner offers. All the participants in the study were able to articulate a justification for their actions associated with *deliberate reflection*. Two of the teachers were able to envisage alternatives to how they responded to unplanned moments associated with *critical reflection*. This highlights the potential of VSR interviews as noted in other studies (Geiger, Muir & Lamb, 2016) in helping teachers reflect on aspects of their teaching not previously considered and envisage alternative approaches to potential limitations in their practice.

Keywords: Responsive teaching, contingency, secondary mathematics, in-service mathematics teachers, algebra, novice and expert teachers.

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List of Abbreviations

Abbreviation	Meaning
DBE	Department of Basic Education
FET	Further Education and Training
IRE	Initiation-Response-Evaluation
VSR	Video-Stimulated Recall

CHAPTER 1: INTRODUCTION

1.1 Problem statement

In secondary mathematics classrooms in South Africa, Brodie (2007b) has illustrated through empirical evidence that teachers typically ask questions to which they know the answers. In such instances, the teacher is listening evaluatively to the learners since the teacher is not asking questions which are dependent on learner offers (Davis, 1997).

International literature on quality teaching suggests that teaching must be improvisational and contingent on learner offers and thinking in the collective classroom space (Borko & Livingston, 1989; Jacobs & Empson, 2016; Sawyer, 2004). The construct that teaching is creative and improvisational is analogous to theatrical actors working without a script on a stage *in-the-moment* whereby the dialogue is contingent on how interactions unfold between the actors during the improvisation (Sawyer, 2004). By contingency, I mean a situation of unexpected or unplanned events that arose during mathematics teaching. In the same vein, collaboration between the teacher and learners is improvisational since learners' contributions and ideas steer the trajectory of the lesson (Borko & Livingston, 1989; Sawyer, 2004). Ultimately, such an approach ensures that learners engage in meaningful participation since their thinking is respected (Forman & Ansell, 2002).

Researchers have continuously noted that responding constructively to contingent events is a crucial albeit challenging facet of teaching. This knowledge in action revealed during the act of teaching to unanticipated learners' contributions links with the contingency dimension of the Knowledge Quartet (Rowland, Huckstep & Thwaites, 2005). In recent years, three triggers of contingency have been identified and this can be initiated either by: (i) responding to a learner's offer which was unanticipated; (ii) teacher insight whereby the teacher actively reflects *in-the-moment* and senses that something is amiss or (iii) when a pedagogical resource such as technology becomes unexpectedly available or unavailable (Rowland, Thwaites & Jared, 2015).

Currently, I have found limited research in the South African context examining responsive teaching at both primary and secondary school levels to understand teachers' decision making in response to unanticipated events (Abdulhamid, 2016; Abdulhamid, 2017; Abdulhamid & Venkat, 2018). Abdulhamid (2016) recently qualitatively examined responsive teaching in primary mathematics classrooms and has noted that evaluation of learner offers *in-the-moment* are rare in the collective classroom space. At secondary level, I found that there is a paucity of research concerned with teacher's decision making in relation to contingent events that occur in classrooms. Hence, research is needed locally to explore how teachers respond to triggers of contingency in the teaching of a fundamental topic such as algebra. The attention to algebra is associated with the difficulties experienced when working with variables at both teaching and learning levels[1].

Algebra is a difficult area in mathematics for learners to grasp since algebra is abstract (Witzel, Mercer & Miller, 2003). Both locally and internationally, there has been limited research attempting to understand teachers' knowledge concerning algebra (Hallagan, 2006). Doeer (2004, p.268) recognises that "there is a serious need for theory building to describe and explain what it is that teachers need to know to teach algebra, and how it is that this knowledge base is developed by novice teachers and by experienced teachers". Ultimately, research efforts need to be directed to exploring novice and expert teachers' knowledge and decision making during the teaching of algebra.

It was anticipated that understanding how novice and expert teachers responded to the triggers of contingency during the teaching of Grade 8 and Grade 9 algebra would illuminate insight on teachers' decision making *in-the-moment* in response to unexpected events.

1.2 Purpose of the study

The purpose of this study was to investigate how novice and expert Senior Phase (Grade 8 and Grade 9) mathematics teachers respond to triggers of contingency that arose during their lessons. Furthermore, this study intended to understand teachers' thoughts and decision making when they responded to unexpected or unplanned events in their lessons via VSR interviews with each teacher.

1.3 Research questions

1. How do novice and expert Senior Phase teachers respond to unexpected events in the teaching of algebra?
2. What are the novice and expert Senior Phase teachers' thoughts and decision making[2] in response to unexpected events during the teaching of algebra?

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[1] By focusing on algebra, this is not to suggest that algebra has more contingent events than other topics in the mathematics curriculum. Rather, the motivation for pursuing contingency in the learning and teaching of algebra stems from my own difficulty I experienced as a novice teacher in teaching algebra (see section 1.5) and the numerous contingent events I experienced when teaching algebra.

[2] By thoughts and decision making, I am referring specifically to the teachers' degree of reflective awareness to responsive teaching *in-the-moment*.

1.4 Significance of the study

This study will add to the research base on responsive teaching in South Africa as little is known on how secondary novice and expert mathematics teachers respond to unexpected events during their lessons.

1.5 Why focus on contingency?

My interest in contingency emanates from my own experience as a novice teacher in my second year of teaching in 2018 whereby I recall asking my Grade 10 mathematics class: "Will x squared plus one always be positive?" Most learners responded that this will always be the case since the expression will always be positive. However, one learner responded unexpectedly that this is incorrect since x could be an imaginary number (i) where $x = \sqrt{-1} = i$ and $i^2 = -1$. This means that the answer could be zero to the question I asked. The unexpected learner response to my initiated question caught me off guard as I was unsure how to respond *in-the-moment*. Did I need to deviate from my lesson agenda to address complex numbers? Or should I acknowledge the contribution, put it aside and continue with the lesson? This incident served as a catalyst for initiating my research into responsive teaching in order to explore how novice and expert teachers respond *in-the-moment* to unanticipated learner offers.

1.6 Conclusion

This chapter highlighted my interest in contingency and reasons for focusing on responsive teaching. The problem statement illustrated the need for research at secondary level focused on contingency and this chapter also highlighted the purposes and significance of the study. In the next chapter, I situated my study within theory and literature.

CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

The first part of this chapter starts by locating the study in relation to existing literature on responsive teaching and contingency. The first section of the literature review highlights why responsive teaching is critical for quality teaching and characterises the nature of responsive teaching in the South African context. Thereafter, an overview is provided on contingency within the existing mathematics education literature. The subsequent sections provide an overview of how the existing research compares novice and expert teachers in terms of cognitive psychology and improvisational performance. Furthermore, I have highlighted research conducted on the teaching and learning of algebra and have noted a gap in research concerning novice and expert teachers in the teaching of algebra in the South African context. The second part of the chapter outlines the study's theoretical framework in relation to Mason and Spence's (1999) constructs of *knowing*.

2.2 Literature review

2.2.1 Responsive teaching

2.2.1.1 What is responsive teaching?

In the last four decades, classroom interaction patterns such as Initiation-Response-Evaluation (IRE) interactions have been a focal point of investigation for many researchers (Brodie, 2007a; Brodie, 2007b; Mehan, 1979; Sinclair & Coulthard, 1975). IRE interactions are typically characterised by the teacher initiating the interaction whereby the learner responds and the teacher evaluates the learner's contribution. The body of literature regarding responsive teaching can be categorised into two types: *deficit* approaches and *affordance* approaches.

The *deficit* approach is associated with *funnelling* as identified by Bauersfeld (1980). Initially, the teacher initiates the interaction by asking learners a question of high cognitive demand. However, when the teacher notices that learners are unable to respond, this results in the teacher narrowing the questions in the form of a traditional question and answer exchange associated with authoritative interactive discourses (Scott, Mortimer & Aguiar, 2006). Consequently, learners are answering questions far below the cognitive demand of the question initially initiated (Stein, Grover & Henningsen, 1996) and this inhibits moving learners' mathematical thinking forward (Brodie, 2007b). Such response patterns restrict genuine learner contributions and do not provide a platform for expanding learning opportunities since the teacher is not asking questions dependent on learners' contributions and thinking (Brodie, 2007b).

Conversely, the *affordance* approach is characterised by the teacher listening interpretively rather than evaluatively to learner offers (Davis, 1997). In responsive teaching, questions are designed by the teacher *in-the-moment* in response to learner contributions and this engages learners to be meaningful, active participants during the lesson (Jacobs & Empson, 2016). Quality teaching is creative, highly improvisational and the teacher deviates from the scripted instruction since the learners offers and ideas steer the trajectory of the lesson (Borko & Livingston, 1989; Sawyer, 2004). The improvisational nature of responsive teaching is collaborative between the teacher and the learners. Improvisational teaching is analogous to theatrical actors working without a script on a stage whereby the dialogue is contingent on how interactions unfold between the stage actors during the improvisation (Sawyer, 2004). Jacobs, Lamb and Philipp (2010) recognise that this improvisational aspect of responsive teaching associated with professional noticing of learner's mathematical thinking *in-the-moment* is indeed challenging since teachers need to attend to learner's mathematical thinking, make sense of the learner offer without imposing their own thinking upon the learner and decide how to respond to the learner.

2.2.1.2 Characterisation of responsive teaching in South Africa

The nature of responsive teaching in South African primary mathematics classrooms has been recently examined qualitatively by researchers and it has been noted that evaluation of learner offers *in-the-moment* are rare in the collective classroom space (Abdulhamid, 2016; Abdulhamid, 2017; Abdulhamid & Venkat, 2018; Hoadley, 2012). A consequence of not evaluating learner offers is that "learners may well remain unaware of the extent to which their offers and narratives are 'endorsable' from a mathematical perspective" (Abdulhamid, 2017, p.200). In stark contrast to the South African research base, Abdulhamid (2017) notes that the international literature does not focus on whether learner contributions are evaluated but rather the focus is on what opportunities for learning arise as a result of the teacher's evaluation of the learner's contribution.

In secondary mathematics classrooms, Brodie (2007b) has illuminated that there is evidence of evaluation of learner offers which differs from the situation that currently exists at primary school level. However, the nature of teacher engagement with learners' offers is often superficial (Brodie, 2007a) and research highlighting how teachers respond to unexpected learner contributions *in-the-moment* is limited. Ultimately, more research is required to highlight how teachers respond to contingent events as well as investigate teacher's decision making in response to unexpected events.

2.2.2 Contingency in mathematics teaching

During the 1970s, researchers such as Alan Bishop were interested in understanding teachers' decision making and the techniques that teachers employed in buying time when they needed to respond to unanticipated events (Borko, Roberts & Shavelson, 2008). For example, Bishop (2001) has recollected numerous times the fraction in between fraction problem to highlight the importance of *in-the-moment* responsiveness. Bishop asked primary school learners between the ages of 9 and 10 to supply a fraction that lies between $\frac{1}{2}$ and $\frac{3}{4}$. One of the learners stated that the answer was $\frac{2}{3}$ and this answer caught Bishop off guard as it was unexpected and he was unsure how to respond (Borko et al., 2008).

There has been an increase in research output internationally interested in how teachers need to make split-second decisions and respond to unexpected events which were not planned for (Foster, 2015; Mason & Davis, 2013; Mason & Spence, 1999; Rowland & Zazkis, 2013; Rowland, Turner, Thwaites & Huckstep, 2009; Rowland et al., 2015). At the University of Cambridge, Rowland and his colleagues observed classroom data from 12 pre-service participants' classrooms and looked at how subject matter knowledge and pedagogical content knowledge came into play in pre-service teachers' actual teaching practice (Rowland et al., 2005). Four common codes emerged from their data and this led to the formulation of the Knowledge Quartet which has since been utilised to analyse lessons in England and other countries (Rowland et al., 2015) such as South Africa (Abdulhamid & Venkat, 2013). These four different components of the Knowledge Quartet include foundation, transformation, connection and contingency (Rowland et al., 2005). Foundation refers to theoretical knowledge that the teacher possesses such as subject matter knowledge. Transformation refers to how the teacher transforms the subject matter they possess into forms that are pedagogically powerful and this can be demonstrated by the teacher's choice of examples and representations. Connection refers to how the teacher achieves coherence in a lesson or between a series of lessons taught. Contingency refers to how teachers think on their feet (Schön, 1987) in response to unplanned events that arise unexpectedly during the lesson which did not form part of the lesson plan (Rowland et al., 2005). It was this dimension of contingency emanating from the Knowledge Quartet which was the focus of this research project.

A teacher responding to contingency is comparable to a pilot operating an aircraft and responding to what they see on their instrument panel during flight or a doctor needing to quickly diagnose a patient's illness based on readings and symptoms described by the patient (Rowland et al., 2015; Sawyer, 2004). Rowland et al. (2015) recognise that it is impossible for a teacher to be able to anticipate all unexpected classroom situations that could arise in advance and that some kind of improvisation will be needed on the teacher's part. However, "the teacher who anticipates such obstacles to learning is better placed to plan a less 'bumpy', more joined-up learning experience for his or her students" (Rowland et al., 2015, p.76). Research into contingency is focused on analysing how teacher's respond to unanticipated events which did not form part of the lesson agenda (Rowland et al., 2015) as well as the willingness or unwillingness of the teacher to deviate from scripted instruction (Borko & Livingston, 1989).

As stated earlier, Rowland et al. (2015) have identified three triggers of contingency and this can be initiated by either the learner, teacher or a pedagogical resource. This can encompass: (i) responding to a learner's offer which was unanticipated; (ii) teacher insight whereby the teacher actively reflects *in-the-moment* and sense that something is amiss in the lesson or (iii) a pedagogical resource such as technology becoming unexpectedly available or unavailable. In the first trigger, Rowland et al. (2015) propose that the teacher is confronted with the possibility of either (i) ignoring the learner's suggestion; (ii) acknowledging the suggestion but side-lining it or (iii) acknowledging, responding and incorporating the learner's offer into the lesson.

The first trigger of contingency takes into account how the teacher responds to learners' contributions and this is further subdivided into three classifications which include (i) the learner's response to a question initiated by the teacher; (ii) a learner's spontaneous response to an activity or discussion and (iii) the unexpected incorrect offer stated by a learner (Rowland et al., 2015). For example, Rowland (2013) describes how a graduate student teacher asked learners to divide their whiteboards in two using a whiteboard marker. The majority of the learners divided the board by drawing a line through the centre of the oblong, parallel to one of the sides. However, one learner drew a diagonal line to split the board into two parts. The unexpected learner's response to the question resulted in the teacher asking the learners to divide their boards into equal quarters. Rowland (2013) recognises from the observation that the one learner's solution caught the teacher off guard as it took the teacher some time to recognise that splitting the board diagonally into quarters results in four unequal parts not being congruent. This unexpected situation placed particular demands on the teacher's subject matter knowledge and pedagogical content knowledge as he was unsure of how to respond *in-the-moment*.

Moreover, Rowland and his colleagues (2009) recount how a graduate trainee had begun her lesson with oral and mental starters, dividing the class into two groups counting in ones where the two groups would alternate. One of the students spontaneously observed that one group would be counting in odds and the other in evens. In this incident, the teacher acknowledged and incorporated the learner's response since she saw the potential it held for learning and deviated from her lesson agenda to teach about odd and even numbers in an interesting way. Furthermore, in this first trigger of contingency the teacher can be caught off guard by an incorrect learner offer. Abdulhamid and Venkat (2013) highlight an incident whereby a teacher asked a learner to add 19 to 21 effortlessly and the actual response was unanticipated. The learner added 8 to 21 instead of 9 to make a friendly number of 30 and the error was not probed. Errors such as these that transpired should not be ignored since they provide insight into learners' thinking and are a normal part of the learning process which create learnable moments (Brodie 2010; Nesher, 1987).

In the second trigger of contingency, the teacher becomes aware that something is amiss by reflecting in action during the progression of the lesson (Schön, 1987). Rowland et al. (2015) recognise that this trigger is rare amongst pre-service and newly qualified teachers and predict that it is more common amid expert teachers. Corcoran (2008) describes a pre-service teacher in Ireland making use of butter beans to illustrate division to a class of primary school learners. The teacher designed division questions requiring learners to utilise the Quotient structure related with grouping. However, by reflecting *in-the-moment* she realises she unintentionally guided learners to the incorrect division structure of partition. As a result of teacher insight, the teacher re-directed learners to the appropriate division structure needed to answer the division problems.

In the third trigger of contingency, a pedagogical resource can become unexpectedly available or unavailable. For example, when a resource such as technology becomes suddenly unavailable, this can place particular demands on the teacher where the teacher needs to improvise and think on their feet. Rowland et al. (2015) describe an incident of a teacher planning to link completing the square with a graphical representation of a parabola using a computer program, *Autograph*. Unfortunately, the teacher was unable to activate the software and this sudden unavailability of a pedagogical tool required the teacher to deviate from his initially envisaged lesson image. The teacher improvised *in-the-moment* and resorted to using the whiteboard. In other instances, it can be the sudden incorporation of a pedagogical tool that can disrupt the trajectory of the lesson. For example, Rowland et al. (2015) recount an incident of a student teacher spontaneously incorporating a 1-100 square grid she found lying on a table which did not form part of her lesson plan. The teacher intended to teach subtraction symbolically yet learners responded spatially by describing moves up, down, left or right and this undermined the teacher's intentions as she wanted them to describe symbolically what was happening.

2.2.3 Characteristics of novice and expert teachers

2.2.3.1 What is meant by a 'novice' and 'expert' teacher?

There are differing views of what constitutes a *novice* and an *expert* teacher by mathematics education researchers. In Borko and Livingston's (1989) study, they interpreted a novice teacher to be a student teacher with a strong mathematics background. In contrast, Berliner's (1988) conception of a novice is inclusive of both student teachers and teachers currently in their first year of teaching. Furthermore, Berliner's (1988) model recognises that teachers with two to three years teaching experience are considered 'advanced beginner'. I believe that Berliner's classifications are limited and find Huberman's (1993) conception of a novice to be broader. Huberman (1993) recognised that a teacher in their first year of teaching ranging to a teacher having three years of teaching experience can be classified as a novice. Hence, Huberman's (1993) notion of a novice was used in this study.

Yang and Leung (2013) note that there is no universal consensus for defining an expert teacher. The conception of an expert mathematics teacher is highly contingent on the cultural and contextual background of the region that the research is conducted (Berliner, 2001). Palmer, Stough, Burdenski and Gonzales (2005) recognise that limited attention has been devoted to developing a set of common identification criteria to identify and select expert teachers across different teaching domains. Ultimately, Palmer et al. (2005) identified four common characteristics across the literature that researchers used to identify and select expert teachers.

This includes selection criteria such as:

- The number of years a teacher taught the subject at school level (which was typically a minimum of ten years).

- Social recognition and being recommended as an expert teacher by the school principal and colleagues.
- Relevant teaching certifications, qualifications (at least an Honours degree) and belonging to a relevant professional body. This category can also include being a cooperating teacher who acts as a mentor to novice teachers.
- Performance based criteria which were "normative" and "criterion-based".

The notion of an expert teacher that was used in this study encompassed all four criteria delineated above. The expert teacher selected for the study was experienced with a good track record of learner performance, respected by her colleagues and has social recognition as a master teacher, is in possession of at least an Honours degree and has served as mentors to pre-service teachers or teachers in their first few years of teaching.

2.2.3.2 Comparisons between novice and expert teachers

During the 1980s, there was a surge in mathematics education research comparing the characteristics of novice and expert teachers on the basis of cognitive psychology and improvisational performance (Borko & Livingston, 1989; Borko, Livingston & Shavelson, 1990; Leinhardt & Greeno, 1986). Borko and Livingston (1989) noted stark contrasts between novice and expert teachers in the nature of their planning, teaching and reflections after teaching. Novices engaged in time consuming, short-term planning typically at unit level which was written down the day before teaching the lesson. Furthermore, novices engaged in ineffective, rigid planning which was highly scripted word-for-word and rehearsed. Novices' planning did not consider any difficulties that learners may experience or predict unexpected events such as learner questions that could arise during the lesson. Conversely, experts engaged in more flexible planning at chapter level with mental plans for their lessons. Borko and colleagues (1990, p.43) attribute the more effective planning skills of experts to having more advanced, interconnected mental schema which allows them "to determine what information is relevant to their planning tasks and to plan more efficiently". In addition, experts considered unexpected events since they "described plans that explicitly anticipated contingencies that were dependent on student performance" (Borko & Livingston, 1989, p.480).

During teaching, novices experienced difficulties in teaching their lessons due to the nature of their planning, they struggled to improvise and respond to unanticipated learner questions whereby they deferred learner questions to the end of their explanations. Borko and Livingston (1989) attribute the difficulty of novices responding to unexpected events as a result of their inexperience in skilled improvisation, relatively undeveloped schema and limited pedagogical reasoning skills to be able to create suitable explanations *in-the-moment*. Conversely, expert teachers engaged in improvisational teaching where they responded accordingly to learner contributions and were successful at answering questions on the spot (Borko & Livingston, 1989). This finding correlates with research conducted by Mason (2015, p.110) whereby he recognizes that "despite any surprise, an expert teacher has access to a repertoire of pedagogic strategies and didactic tactics informed by a deep appreciation and comprehension of the topic, of pedagogy, of psychology, and of sociology". Post-lesson reflections of novices lacked conciseness and revealed that novices focused more on technical details of the lesson without mentioning specific incidents that arose during the lesson (Borko & Livingston, 1989). However, expert teachers had more focused reflections which succinctly described specific events that occurred during the lesson (Borko & Livingston, 1989).

2.2.4 Teaching and learning of algebra

Algebra is a fundamental branch of mathematics for learners to understand since it acts a precursor to other mathematics topics such as analytical geometry and trigonometry (Watson Jones & Pratt, 2013). Researchers have long recognised that algebra is a difficult area in mathematics for learners to grasp since it is abstract (Witzel et al., 2003). Generally, learners are used to working computationally with numbers in arithmetic at primary school level. However, when learners are exposed to algebra, they sometimes erroneously link arithmetic principles with algebra and experience difficulty in transitioning from arithmetic to algebra which is referred to as a *cognitive gap* (Herscovics & Linchevski, 1994). For instance, when learners are asked to simplify an algebraic expression such as $6x + 7$, some learners will conjoin the terms and state that the solution is $13x$ since their thinking is still rooted in arithmetic (MacGregor & Stacey, 1997).

In South Africa, examiners of high stakes assessments such as the matric examinations conducted by the Department of Basic Education (DBE) have continuously noted in examination reports that the "algebraic skills of the candidates are poor" and that the majority of learners "lacked fundamental and basic mathematical competencies which could have been acquired in the lower grades" (DBE, 2019, p.179). This highlights the urgency that attention needs to be devoted to developing learners' algebraic skills in the Senior Phase.

The dominance of research in algebra is focused on learner errors since it is believed that errors provide a glimpse on how learners can experience algebra (Watson et al., 2013). For instance, researchers in South Africa have focused their attention on understanding learners' errors and misconceptions across different algebra topics (Gardee & Brodie, 2015; Makonye & Khanyile, 2015; Makonye & Luneta, 2014; Pournara, 2020). However, the research base in developed and developing contexts concerning teachers' knowledge of algebra is scarce and more research is needed (Hallagan, 2006). Doerr (2004, p.268) recognises that "there is a serious need for theory building to describe and explain what it is that teachers need to know to teach algebra, and how it is that this knowledge base is developed by novice teachers and by experienced teachers".

Internationally, research highlighting the distinctions between novice and expert teachers' knowledge in teaching algebra is limited and the only known research that I could find has been conducted in Israel over two decades ago (Even, Tirosh & Robinson, 1993; Tirosh, Even & Robinson, 1998). Even et al. (1993) illustrated that novices delivered lessons which lacked connectedness in linking topics and ideas from earlier lessons whereas experts delivered coherent lessons which linked with ideas and topics discussed in previous lessons which provided a platform for meaningful learning and understanding of mathematical content.

Moreover, a study by Tirosh et al. (1998) analysed two novice and two expert teachers teaching simplifying algebraic expressions to Grade 7 learners. Novices taught the lesson focusing on collecting like terms and encouraged a *fruit salad approach* to algebra when learner difficulties arose by using variables to denote objects such as b for banana and coefficients representing adjectives such as $4a$ for four apples. However, researchers have argued that a *fruit salad approach* exacerbates learners' confusion and "detrimentally influenced students' understanding of a variable" (Hallagan, 2006, p. 106) since learners do not understand that a variable does not represent an object but rather a number. Furthermore, novice teachers were unable to think on their feet and anticipate common learner errors such as conjoining unlike terms. On the other hand, the expert teachers were able to anticipate learner errors and tried to assist their learners to avoid conjoining unlike terms. One of the expert teachers engaged in a ritual approach to teaching algebra which involved spending time on identifying like terms which was succeeded by collecting like terms. The other expert teacher in Tirosh et al.'s (1998) study taught more for understanding and utilised substitution to compare the effects of doing operations in different combinations and orders in algebra to engineer conflict in learners' thinking that they would need to resolve. More research is needed both locally and internationally to explore how novice and expert teachers respond to unexpected events when teaching pivotal topics such as algebra.

2.3 Theoretical framework

Mason and Spence (1999) theorised that *knowing* is a more useful idea to consider when we think about teaching and learning compared to *knowledge.* This is attributed to the term *knowledge* creating a barrier in our thinking between the knower and the information they possess when deciding or enacting a decision (Mason & Davis, 2013; Mason & Spence, 1999; Thompson, 2016). *Knowing* in the context of teaching suggests that there is an inseparable link between knowledge and knower which in this case would be the teacher who possesses mathematical knowledge (Mason & Davis, 2013; Mason & Spence, 1999; Thompson, 2016).

Mason and Spence (1999) posit that there are two different forms of knowing referred to as *knowing about* and *knowing to*. The two different forms of knowing are diagrammatically represented in Figure 1. The distinction between *knowing about* and *knowing to* rests in the fact that *knowing about* something refers to planned aspects of teaching whereas *knowing to* involves the teacher dealing with unplanned or unexpected situations as they transpire in the classroom. Mason and Spence (1999) divide *knowing about* into three categories referred to as *knowing that*, *knowing how* and *knowing why* which are outlined below:

- *Knowing that* something is the case means knowing mathematical facts such as being able to state that opposite angles of a cyclic quadrilateral are supplementary.
- *Knowing how* to do something means being able to perform a technique, action or skill such as being able to solve quadratic equations.
- *Knowing why* refers to having a justification for actions performed and "having stories I tell myself to account for something" (Mason & Spence, 1999, p.145).

It is pertinent to note that *knowing about* something is not a precursor for *knowing to* and cannot guarantee that a teacher in practice would be able to respond *in-the-moment* to a contingent incident. This is highlighted by Mason and Spence's (1999, p.138) concern whereby they state that *knowing about* something "gives little indication of whether that knowledge can be used or called upon when required".

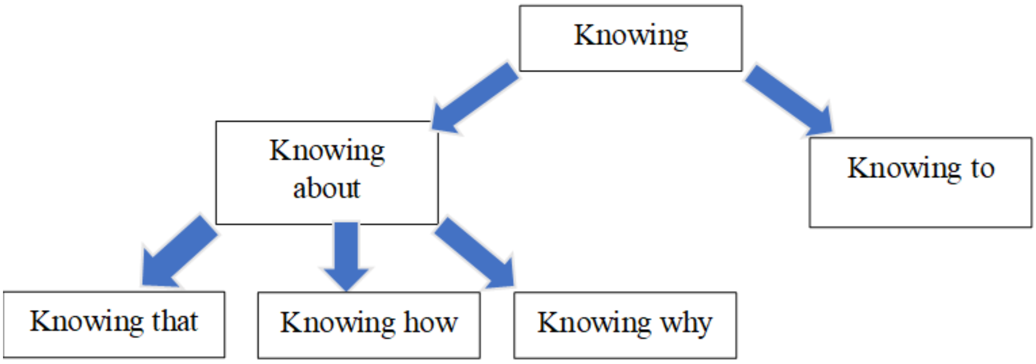

Figure 1: Beyond *knowing about* (Adapted from Mason and Spence, 1999, p.145)

The notion of *knowing to* proved a useful construct to understand knowledge displayed in action and this links with the contingency dimension of the Knowledge Quartet proposed by Rowland et al. (2005). Teachers need awareness to act creatively *in-the-moment* in response to triggers of contingency and they need to swiftly make decisions which can alter the trajectory of their intended lesson (Mason & Davis, 2013; Rowland et al., 2015). As discussed above (see Section 2.2.2), there are three triggers of contingency and this provided a lens to examine interrelated concepts dealing with the phenomenon of contingency in mathematics teaching.

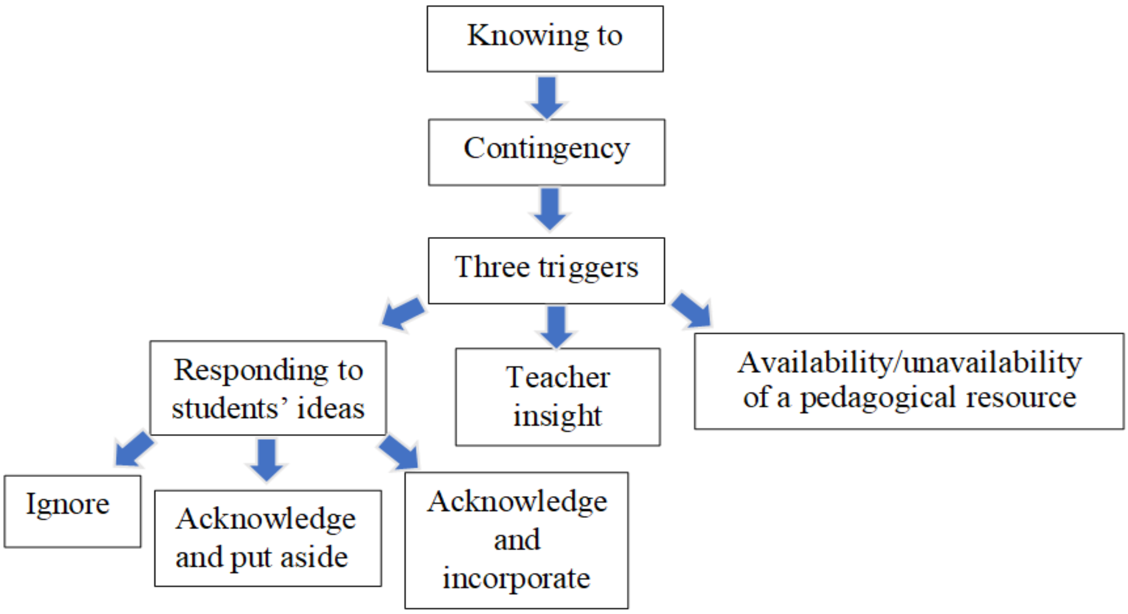

Figure 2: Possible teacher response pattern to triggers of contingency

2.4 Conclusion

This chapter highlighted that a critical element needed for responsive teaching is to embrace teaching as an act of improvisational performance and deviate from scripted instruction. I have provided an overview of contingency and noted that the majority of research regarding responsive teaching has been conducted internationally indicating that there is a gap in the South African context related to how teachers respond *in-the-moment* to contingent events. I have characterised novice and expert teachers in terms of the existing literature and noted that there is a paucity in the algebra research concerning novice and expert teachers. Moreover, this chapter outlined the theoretical framework of this study. The next chapter explores the research design and methodology employed in the study.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter provides information on the research design and data analysis employed in the study. Moreover, I provide information on the participants, limitations and context of the study as well as ethical considerations taken.

3.2 Research design

This study employed a qualitative case study approach since a case study allows for a detailed exploration of a phenomenon within a closed system (Merriam, 1998). Researchers regularly utilise case study in social sciences research (Yazan, 2015) in order to gain "a much richer and more detailed description of human behaviour and experience than can be obtained from the collection of quantitative information" (Dayer, 1995, p.53). Merriam (1998, p.25) recognizes that the most important feature of case study is the case which represents "a thing, a single entity, a unit around which there are boundaries" and the case in this particular study was the triggers of contingency. The bounded system in this study is the investigation of participants' responses to the triggers of contingency that arose within their classrooms in the teaching of algebra.

3.3 Context of the study

Two high schools formed part of the study. School A was an extremely well-resourced private school located in Johannesburg East serving 600 predominantly high socioeconomic status (SES) learners who are all girls. School B was a well-resourced, urban co-educational government school located in Johannesburg East serving approximately 1000 learners from mainly middle SES backgrounds.

3.4 Participants

Participants were identified using purposive sampling and through "information-rich cases" this enabled me to gain insight into the phenomenon "without needing or desiring to generalise to all such cases" (McMillan & Schumacher, 2006, p.319). One novice teacher and one expert teacher from School A and School B were selected. However, due to the emergence of COVID-19 I could not conduct lesson observations for the expert teacher's algebra lessons from School B in April 2020 as envisaged and as a result I excluded the teacher from the study.

The criteria used to select novice teachers was dependent on the number of years the teacher taught mathematics at high school level (Grade 8 to Grade 12) ranging from a teacher in their first year to a teacher who has three years of teaching experience based on Huberman's (1993) conception of a novice. Moreover, the novice teachers needed to be in possession of a relevant academic degree (such as a BEd, PGCE or BSc).

The criteria used to select the expert teacher was dependent on criteria identified by Palmer et al. (2005). These are:

- The number of years a teacher taught the subject at school level (which was typically a minimum of ten years).
- Social recognition and being recommended as an expert teacher by the school principal and colleagues.
- Relevant teaching certifications, qualifications (at least an Honours degree) and belonging to a relevant professional body. This category can also include being a cooperating teacher who acts as a mentor to novice teachers.
- Performance based criteria which were "normative" and "criterion-based".

Based on the criteria listed above for novice and expert teachers, three teachers participated in this research project and their details are summarised in Table 1. Tara[3] is a novice teacher in possession of a BSc (Biochemistry and Microbiology) degree and from School A. She is currently in her first year of teaching and she completed a teacher internship at the same school in 2019. She is finishing her PGCE part-time and is envisaged to complete her PGCE by the end of 2020. Cassandra is an expert teacher from School A and was in her twentieth year of teaching at the time of data collection. It is pertinent to note that Cassandra has been teaching a Grade 9 mathematics class every year since she started teaching. Cassandra is recognised as having a good track record of learner performance, is in possession of a Master's Degree specialising in Education, is widely respected by her colleagues and the Head of Department. She has mentored pre-service and novice teachers and is currently serving as a mentor to Tara. The novice selected from school B was Shandre and she was in her third year of teaching at the time of data collection. Shandre was in possession of a BSc Honours degree and a PGCE.

[3] All the teachers' and learners' names are pseudonyms intended to preserve the anonymity of all participants.

Table 1: Summary of the background characteristics of participating teachers and their schools

Teacher	Expert or novice	Teaching experience	School	Qualifications	Grade and class size of observed class	SES
Tara	Novice	1 year	School A	BSc (Biochemistry and Microbiology, 2018) PGCE (2019-date)	Grade 9, 19 learners	High SES
Cassandra	Expert	20 years	School A	BMus (Ed) (2000) DIPABRSM (2001) MEd (2005)	Grade 9, 18 learners	High SES
Shandre	Novice	3 years	School B	BSc (Mathematics and Ecology, 2008) BSc Honours (Plant systematics, 2009 PGCE (2018)	Grade 8, 24 learners	Mid SES

3.5 Data sources

The data collection process took place in February, March and July 2020. Data sources for the study included lesson plans, video recordings of each lesson and Video-Stimulated Recall interviews (VSR) with each teacher.

3.5.1 Lesson plans

Each participant designed lesson plans for each lesson I observed. The purpose of getting teachers to design lesson plans was to get a sense of the lesson image that each teacher envisaged prior to teaching the lesson and also to see the extent to which teachers deviated from their lesson plan during the lesson.

3.5.2 Lesson observations

At the end of February and beginning of March 2020, I visited each school with a research assistant to observe and video record three sequential lessons for each teacher. Each lesson lasted approximately 40 minutes. I planned my observation schedule around teachers' timetables and work schedules based on when they would be teaching algebra to their Grade 8 and Grade 9 classes. A total of 9 algebra lessons (3 x 3 teachers) were observed and this is summarized in Table 2. Teachers were sent electronically videos of their recorded lessons to watch and to identify points of interest for discussion during the VSR interviews.

Table 2: Summary of the 9 lessons observed in 2020 for the three teachers

Teacher	Date of observation	Duration of lesson	Lesson focus
Tara	24-02-2020	00:25:48	Finding the degree of a polynomial and simplifying algebraic expressions
	25-02-2020	00:37:14	Distinguishing polynomials from non-polynomials
	26-02-2020	00:33:33	Distributive law
Cassandra	02-03-2020	00:37:19	Application of adding and subtracting polynomials
	03-03-2020	00:30:37	Distributive law
	04-03-2020	00:37:20	Distributive law
Shandre	27-02-2020	00:36:51	Subtraction of polynomials
	28-02-2020	00:37:20	Simplifying exponents
	02-03-2020	00:54:17	Simplifying exponents

3.5.3 VSR interviews

Once lessons for each teacher were transcribed and analysed, I prepared semi-structured interview questions (see Appendix A) based on selected incidents from the transcript. Due to COVID-19, this delayed the interview process and in July 2020 Google Meet was utilised as a platform to conduct and record the interviews one-on-one with each teacher. Teachers were sent electronically copies of the recorded lessons prior to the interview and during the interview I chose selected episodes from each lesson to discuss. The purpose for employing VSR was to prompt recollections of teachers' thoughts at the time they taught the lesson and to get insight into teachers' reasons for their responses and decision making *in-the-moment* (Lyle, 2003; Muir, 2010). Teachers were encouraged to stop the selected episodes at any point if they wished to make a comment. Furthermore, I probed teachers on whether there was anything they would change or improve upon when reflecting on the lessons they taught. Each interview lasted for approximately 1 hour and all interviews were subsequently transcribed and analysed.

3.6 Limitations of data collection methods

I note the influence of the "observer effect" as a limitation during lesson observations (Martella, Nelson, Morgan & Marchand-Martella, 2013). Observed teachers and learners may behave unnaturally in the presence of an observer such as myself or the research assistant (Martella et al., 2013). This was noted by Shandre during the VSR interview as she stated that she was "more present" and "I was far more mindful because of your presence".

3.7 Data analysis

3.7.1 Analysis of lesson observations

Once all lessons were transcribed, I parsed the transcripts into episodes based on whether a contingent moment was present in that episode. Since the presence of a contingent event is not indicative of the quality in terms of how a teacher responds to the trigger of contingency, I decided to code the response to the contingent moment as *appropriate* or *inappropriate*. From the analysis of the data, it was seen that the triggers of contingency related to teacher insight and availability or unavailability of a pedagogical tool were absent from the data and coding the data as *appropriate* or *inappropriate* relates only to the first trigger of contingency associated with responsiveness to learners' offers.

An appropriate response is when the teacher acknowledged a learner's contribution and made an attempt to provide mathematical explanations when warranted to ensure that all learners have access to mathematics *in-the-moment* (LMT, 2006; Weston, 2013). However, an inappropriate response was allocated to an episode when a teacher responds by dismissing a learner's contribution or question which is relevant, related to the topic and the response is seen as "significantly limiting learners' access to mathematics in the moment" (LMT, 2006, p.13). I then allocated categories to the appropriate response based on Weston's (2013) coding protocol which were *minimum*, *middle* and *maximum* to analyse the quality of the responses. *Minimum* is when the teacher provides an incorrect or erroneous response to a learner's idea or question which provides no opportunity for learning. *Middle* is when the teacher provides a response which is correct and provides potential for learning but does not engage or press learners in their thinking. *Maximum* was used when the teacher provided correct detailed feedback to a learner's question, engaged in press with the learner about their thinking *in-the-*

moment and the incident provides an ideal learning opportunity for the class. A schematic representation of the data analysis process for analysing the episodes is illustrated in Figure 3.

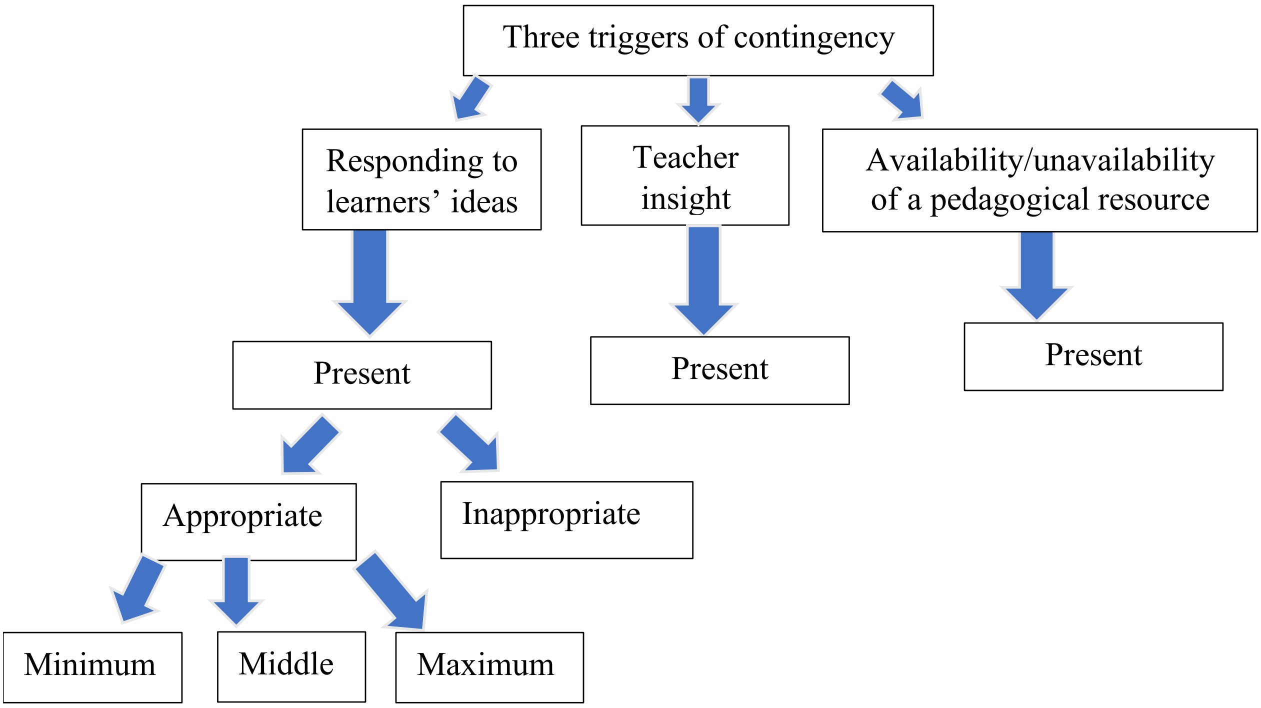

Figure 3: Schematic representation of coding protocol used to analyse episodes for each lesson

3.7.2 Analysis of VSR interviews

During the VSR interviews, my purpose was to gain insight into how each novice and expert teacher reflected on selected episodes I chose for discussion. A two-dimensional framework which incorporated the interviewee's level of reflection (Muir & Beswick, 2007) and the object of their reflection (Geiger et al., 2016) was used. This served as a lens for analysing the VSR interviews.

The first dimension based on the work of Muir and Beswick (2007) recognises three hierarchical levels of the reflective process termed: *technical description*, *deliberate reflection* and *critical reflection*.

- ***Technical description*** is the lowest level of the reflective process whereby the teacher offers a general overview of the lesson but does not offer insight into events or episodes of significance that transpired during the lesson.

- ***Deliberate reflection*** requires the teacher to be able to identify significant events that occurred during the lesson and provide a rationale for the incident or action.
- ***Critical reflection*** is the most advanced level of reflection whereby the teacher transcends the previous level of reflection by now contemplating alternative perspectives and actions which could improve future teaching.

The second dimension of reflection proposed by Geiger et al. (2016) is associated with the object of the reflective process and this can be related to self (the teacher), practice or learners. The two-dimensional framework encompassing the level and object of the reflective process is illustrated by Table 3.

Table 3: Levels of reflection and their associated objects of reflection (Adapted from Geiger et al., 2016, p.463)

Levels of reflection	Object of reflective response		
	Self	**Practice**	**Learners**
Technical	Personal role is described during a teaching event. The description is factual rather than personally insightful	Teaching activity is described in terms of technical aspects. Focus is on consequences or outcomes of their practice	Learners' responses to teaching activity are described in terms of technical aspects. Focus is on consequences or outcomes of teaching practice
Deliberate	Personal role is described during a teaching event. A rationale or explanation for the personal behaviour is provided	Critical incidents are related to teaching practice, and a rationale or explanation for the practice is articulated	Learners' responses to teaching activity are noted, and a rationale or explanation for the response or behaviour is constructed
Critical	Personal role is described during a teaching event. The behaviour is critically analysed and alternative behaviours discussed	The purpose of an activity is clearly articulated, and a judgment is made about the success or otherwise of a teaching practice. When unsuccessful, an alternative practice or activity is suggested	Learners' responses to teaching activity are noted, and a rationale or explanation for the response or behaviour is constructed. Potential improvements to the activity are related to an anticipated learner response

3.8 Rigour

Lincoln and Guba (1985) recognised that there are four important criteria which should be utilised to judge the trustworthiness of qualitative research. This includes criteria such as credibility, transferability, dependability and confirmability.

3.8.1 Credibility

This notion of trustworthiness refers to the researcher presenting plausible interpretations based on the collected data (Lincoln & Guba, 1985). In order to ensure credibility, Scott and Morrison (2005, p.254) stipulate that the researcher must sufficiently interpret participants' perspectives and "constructions of reality". I ensured that I maintained credibility in this study by implementing member checking with each participant to verify that their responses from the transcribed VSR interview were accurately represented which consequently reduced researcher bias (Anney, 2014).

3.8.2 Transferability

Transferability refers to the degree of generalisability of the results of the study to other contexts (Lincoln & Guba, 1985). Denscombe (2010) recognizes that it is difficult to generalise the findings of small-scale qualitative research such as my research project to other studies. However, I have provided thick description to facilitate transferability which will assist other researchers to make a reasonable judgment about whether the findings of this study are applicable to their research contexts (Bitsch, 2005).

3.8.3 Dependability

Dependability refers to the extent to which the findings of the study can be replicated over time to produce the same results (Merriam, 2009). In order to establish dependability within this study, I engaged in regular independent audits with my supervisor to discuss the results and analysis of the findings. Anney (2014, p.729) recognises that engaging in such facets of dependability is important in aiding "the researcher to be honest about his/her study and peers contribute to his or her deeper reflexive analysis".

3.8.4 Confirmability

Confirmability refers to the extent to which the results of the study can be confirmed by other researchers (Lincoln & Guba, 1985). It is vital that researchers of qualitative studies maintain neutrality (Tobin & Begley, 2004) and ensure that the "interpretation should not be based on your own particular preferences and viewpoints but needs to be grounded in the data" (Korstjens & Moser, 2018, p.4). In order to ensure confirmability, a detailed audit trail was undertaken whereby I kept detailed notes of the research process, design, transcribed data and coded data to allow an observer to trace the trajectory of the research study (Holloway & Wheeler, 1996; Shenton, 2004).

3.9 Ethical considerations

Participating teachers were provided an information sheet detailing the purposes of the research project and participants were also required to fill out a consent form (see appendix D). Teachers were informed that participation was voluntary and that their names as well as the name of the schools would be kept anonymous throughout the entire research project. Permission was also obtained from the principals of each high school. Once teachers and the school agreed to participate in the study, information sheets and consent forms were given to learners and their guardians to complete (see appendix D). In instances where learners and/or guardians declined to participate in the research project or did not return the completed consent forms, those learners were placed in a position whereby they were not captured in the frame during video recording. Moreover, permission and ethical clearance was granted by the Gauteng Department of Education (see appendix B) and the University of the Witwatersrand Ethics Committee (see appendix C) before any data collection took place.

3.10 Conclusion

This chapter highlighted the research design and methodology employed for the study in terms of how data was collected and analysed. I provided a rationale for how participants were chosen, provided information on ethical considerations taken and how rigour was used to judge the trustworthiness of the research. In the next chapter, I provide an analysis of the findings from the study.

CHAPTER 4: DATA ANALYSIS AND FINDINGS

4.1 Introduction

The purpose of this chapter is to present findings and discussions related to contingent incidents that emerged within each teacher's lesson and highlight each teacher's degree of reflective awareness to responsive teaching *in-the-moment* conducted during the VSR interviews.

I start the chapter by first providing background information on each teacher, a description of three sequential lessons I observed for each teacher and a qualitative analysis of contingent incidents which transpired in each of the teachers' lessons. For each contingent incident I provide the background to the incident, verbatim evidence in the form of transcript excerpts which are succeeded by interpretation of the contingent incident. Moreover, I present findings from the VSR interview based on teachers' reflective awareness to contingent incidents. Lastly, I provide an overall analysis across the three case study teachers.

4.2 Tara's case

4.2.1 Tara's background and overview of the lessons

Tara is a novice teacher in possession of a BSc degree and is from School A. She is currently in her first year of teaching and completed a teacher internship at the same school in 2019. She is doing her PGCE part-time and is envisaged to complete by the end of 2020.

In Tara's first observed lesson with her Grade 9 class, she started the lesson by asking learners to state the degree of the polynomial $-3x^4y^3 + 4x + 3y - 2x^5y^6 - 7xy + 3$ in terms of x and y. Subsequently, a learner asked Tara to mark the previous lesson's homework as the learner stated that she found the homework difficult and Tara marks three homework questions related to classifying whether an expression is a polynomial or not. Afterwards, Tara moves on to three examples based on simplifying algebraic expressions by looking for like terms as revision of work covered in Grade 8. The lesson lasted 25 minutes and 48 seconds.

In Tara's second lesson, she started the lesson by giving each learner a number line and letting learners know that the objective of the lesson is to go through algebraic and exponents concepts she found they struggled with when marking the homework in the previous lesson. Tara proceeds by writing $x^{-1} = \frac{1}{x}$ and a series of exponents ($2^3 =$; $2^2 =$; $2^1 =$; $2^0 =$) on the whiteboard. Tara asks the class for answers to the exponents she wrote and questions them as to "what do you see happening in the answers?" A learner responds that they are dividing by two. Tara then writes ($2^{-1} =$; $2^{-2} =$) and asks the class for the answer. Subsequently, Tara writes another series of exponents ($6^3 =$; $6^2 =$; $6^1 =$; $6^0 =$; $6^{-1} =$; $6^{-2} =$) and asks the class for answers to each exponent. After this example, Tara erased the whiteboard and moved on to recapping the first law of exponents. She worked out three examples associated with the first exponent law with the class. The last segment of the lesson involved Tara writing five questions related to simplifying algebraic expressions, adding and subtracting polynomials and the class were asked to work individually on these questions before the lesson ends. The lesson lasted 37 minutes and 14 seconds.

In Tara's third lesson, she started the lesson by providing information on their upcoming standardised test and proceeds to state that today they will be looking at distributive law in this lesson. Tara starts working out the example $(x + 1) + (2x + 3)$ with learners by stating that brackets help separate algebraic terms and that the plus sign needs to be distributed into the second bracket. Once Tara simplified the algebraic expression, she proceeded to the second example which involved simplifying $2x - (3 + 2x) - 4$. Subsequently, Tara moved on to the third example of the lesson which involved simplifying $(-2y^2 + 2) - (-4 + 3y^2) + (2y)$. The final example of the lesson involved Tara asking the class to subtract $3z^2 + 4 - 6z$ from $-3z^2 - 7z + 4$. For the remainder of the lesson, learners worked individually on distributive law questions from the textbook. The lesson lasted 33 minutes and 33 seconds.

The analysis as presented in Table 4 highlight that the first trigger associated with responsiveness to learners were present in all of Tara's observed lessons with a notable absence of the second and third trigger of contingency associated with teacher insight and the sudden availability or unavailability of a pedagogical tool respectively. For the observed lessons, I noted four contingent incidents and I provide a qualitative analysis on all four contingent incidents in the next section. For the four contingent incidents which emerged one was coded *inappropriate*, one of her responses were coded as *minimum* and the remaining two responses were coded as *middle*.

Table 4: Overview of triggers of contingency and level of responses provided by Tara

Name of teacher	Trigger of contingency	Level	Lesson 1	Lesson 2	Lesson 3	Total
Tara	Responding to learners' ideas	Maximum	0	0	0	0
		Middle	0	1	1	2
		Minimum	0	1	0	1
		Inappropriate	1	0	0	1
	Teacher insight	Maximum	0	0	0	0
		Middle	0	0	0	0
		Minimum	0	0	0	0
		Inappropriate	0	0	0	0
	Availability/unavailability of a pedagogical resource	Maximum	0	0	0	0
		Middle	0	0	0	0
		Minimum	0	0	0	0
		Inappropriate	0	0	0	0

4.2.2 Qualitative analysis of Tara's responses to contingent incidents

Contingent incident 1: A learner's question - ignore or dismiss

In Tara's first lesson, a learner asks why $\frac{5}{x} - 8x$ can be rewritten as $5(x^{-1}) - 8x$. Tara dismissed the question by stating that she will give an explanation in the next lesson. Tara continues with the lesson on simplifying expressions.

Excerpt 1

Line	Teacher/learner statements
1	Learner: Ma'am can you just explain the "exponentive" one?
2	Tara: "Exponentive" one?
3	Learner: Like why is it negative one?
4	Tara: Let's do that tomorrow, okay? We'll spend time going through all of this stuff, okay.

The learner's question in line 1 was relevant and this knowledge was needed in the lesson in order to classify whether an expression is a polynomial or not. The learner is pressing Tara for a conceptual explanation in line 3. By deferring the explanation in line 4, Tara limited access to mathematics *in-the-moment*. In the VSR interview, Tara admitted she was caught off guard by the question and "jumped through to the answer of what it would be instead of the explanation of how we get there". She further added that assumptions were made "that they already did this stuff in Grade 8" and she did not anticipate for such a question in her planning for the lesson. The findings in this episode are similar to the difficulties experienced by novice teachers in Borko and Livingston's (1989) study, as the authors note that novices had difficulty answering questions on the spot, difficulty anticipating questions and postponing questions they were unsure of answering. I coded her response as *inappropriate* since Tara chose to ignore a grade appropriate question *in-the-moment* and the response does not support learning.

Contingent incident 2: A learner's question – acknowledge but continue

In lesson 2, learners were dealing with negative exponents based on difficulties Tara noted that learners experienced in the previous lesson such as x^{-1} being the same as $\frac{1}{x}$. A learner asked Tara to simplify -2^{-2} in line 2, Tara acknowledges the question but incorrectly states in line 3 that the answer is $\frac{1}{6}$ instead of $\frac{-1}{4}$. Tara refrains from further answering the question and continues with the lesson.

Excerpt 2

Line	Teacher/learner statements
1	Tara: Have fun with your calculator. Try doing all your negative exponents and try see you will still get fractions regardless.
2	Learner: What if it is negative two to the power negative two?
3	Tara: Negative two to the power negative two means that it is one over six. (Tara starts to write $\frac{1}{6}$ and then erases the whiteboard) Can I come back to your question? Can I come back to your question? Otherwise, you're going to get confused there.

Tara's explanation was incorrect and did not support learning *in-the-moment* and she decides to defer answering the learner's question as she thought learners were going to get confused despite showing the learners earlier in the lesson that 2^{-2} is $\frac{1}{4}$. Tara encountered a problem when the learner's question led her to attempt an explanation not prepared in advance and this resulted in an incorrect explanation being constructed on the spot similar to novices' experiences in Borko and Livingston's (1989) study. During the VSR interview, Tara revealed that with this question "I don't want to now go through something else where they're going to look at me so lost and confused again" and was "scared to want to answer their questions" as they were "not really in line with what I actually want them to know". She felt she was "going down a rabbit hole with this question" and decided the best way forward was to defer answering the learner's question. I coded the response as *appropriate* since Tara chose to acknowledge the learner's question but her explanation was incorrect and coded as *minimum* since it did not support learning *in-the-moment*.

Contingent incident 3: A learner's question – acknowledge but continue

Later in lesson 2, a learner asked Tara a question and she affirms that the learner asked a good question about x to the power of a negative fraction before moving on with the lesson.

Excerpt 3

Line	Teacher/learner statements
1	Learner: What if it says, so x to the negative fraction? (unclear because of noise)
2	Tara: Shhh… x to the negative fraction? So, square root of x to the negative fraction?
3	Learner: No, just x to the negative fraction.
4	Tara: x to the negative fraction? You don't really deal with that now. Can we…can we finish off and then I'll answer… cause you had an interesting question because we do deal with it in maths. You do deal with it in maths but it is going to confuse you even further so let's do what we have to do now that this is cemented.

Tara chose to acknowledge the question rather than ignore it. However, the contingent moment is dealt with superficially in line 4 by Tara as the learner's question is unclear and needed to be probed *in-the-moment*. Tara admitted that the learner's question seemed like a difficult question at first when she encountered it in the lesson and was unclear. In the VSR interview, Tara revealed that she decided to spend time with the learner during first break after I recorded the lesson to understand the learner's question as she notes that "I want to ensure that I'm understanding, what she's telling me, I'm actually trying to understand it". Moreover, Tara stated that her thinking at the time "was just trying to keep things as simple as possible" and she was willing to "explore these questions but not right now". Tara's response to this contingent incident is not surprising as Stockero and Van Zoest (2013, p.137) noted similar response patterns amongst novices in their study since novices would affirm that a learner "had posed a good question or comment and giving a brief response before moving on with the lesson" as if the incident did not occur. I coded the response as *appropriate* since Tara acknowledged the question and coded it as *middle* based on Tara's response *in-the-moment* since the learner's thinking could have been probed to clarify exactly what she meant by the question rather than dismissing it as an "interesting question". I want to note that I coded the incident as middle rather than minimum since Tara's response did not introduce any errors and the question is likely to have had neutral impact for the class – neither positive nor negative.

Contingent incident 4: A learner's question - Acknowledge but continue

During lesson 3, Tara is working out an example with the class based on distributive law whereby she simplifies $(x + 1) + (2x + 3)$. She starts working out the question, distributes the invisible one into the first bracket and is asked a question by a learner in line 1.

Excerpt 4

Line	Teacher/learner statements
1	Learner: Madam, isn't there an invisible one before … wouldn't it be four?
2	Tara: No. Remember when one multiplies with a number, what happens to that number? The number stays the same. Right that's what distribution means multiplying. Okay so it's positive one times two x gives me… two x right? The number is still going to stay the same. This positive, and that's three over there. (Tara points to $+3$ on the whiteboard) We are going to have three again okay. Nothing happens there and can I simplify this expression now.

Tara acknowledged the learner's question in line 2 by providing a procedural explanation of distributive law by listening evaluatively rather than interpretively to the learner's question and offers an explanation that distributing the invisible one does not change the number or value of each term (Davis, 1997). I coded the response as *appropriate* and *middle* since Tara chose to acknowledge the question but could have probed or pressed the learner with a follow-up question to understand her thinking as to how she got 4 (Kazemi & Stipek, 2001). In the VSR interview, Tara revealed that she understood the learner's thinking at the time as the learner "wanted to jump in already and say one plus three is four" but Tara thought it would be best to "simplify and go through to how we get the answer" as the learner likes skipping steps.

4.2.3 Analysis of VSR interview with Tara

I started the interview by asking Tara to delineate the objectives she envisaged for each lesson in order to gain insight as to what she wanted learners to grasp at the end of each lesson.

Excerpt 5

Line	Teacher/researcher statements
1	Tara: Okay, so you recorded three lessons from me and the first lesson I think was dealing with … doing a recap on polynomials and then from there we went onto the homework and going through the homework activity and then we did simplifying of expressions. So, that was the objectives for those lessons. Then, I think the second lesson what was occurring was going through more polynomial examples. Then, the third lesson was just looking at the distribution of the signs into those brackets of a polynomial.

As seen in the above excerpt, her reflection was *technical* and factual rather than personally insightful. Her reflections about the objectives of the lesson were related to *practice* content that was covered.

During the interview, I probed Tara's rationale for her actions concerning contingent incidents which transpired during her lessons such as the contingent moment in lesson 2 (excerpt 2) whereby she acknowledged a learner's question about simplifying -2^{-2}, provided an incorrect response and moved on with the lesson.

Excerpt 6

Line	Teacher/researcher statements
1	Researcher: So, Rachel asked you something that was a bit unexpected in that lesson. So, what were you thinking in that moment when she asked you that?
2	Tara: So, it was negative two to the power negative and trying to simplify?
3	Researcher: Mmm …
4	Tara: Umm, I think for me I was already thinking as again I'd go straight to the answer rather instead of trying to explain the process behind it. But also, with that question I know I didn't want to touch that question for a reason. Because I didn't want my other learners to get confused because when my learners ask those higher-level questions and stuff like that because I do have a weaker class so I tend to not want to …I get scared to want to answer their questions that they have that are not really in line with what I actually want them to know. Because what I've seen with learners when you do go on into these questions and stuff like that, they tend to focus on that and they forget the other stuff you want them to learn is more important. I think that's probably why I didn't want to actually answer the question for them. So, yeah there were a whole lot of things going on in my mind thinking about how complicated this thing would be to explain. So, yeah that was kind of the things going through my mind. Also, they tend to struggle with brackets and learning when to put the brackets, so I was thinking already I'm going to go down a rabbit hole with this question.

Tara recognised in her rationale that she had a weaker class and decided not to want to explore the question any further as it could get "complicated". Prior to this incident, the class spent half the second lesson addressing the concept of negative exponents based on difficulties Tara noted that learners experienced in the previous lesson such as x^{-1} being the same as $\frac{1}{x}$ and decided she is "not going to go down a rabbit hole with this question" of negative two to the power negative two. Generally, Tara's insight and reflection on contingent incidents were often *deliberate* as she justified her behaviour for how she responded *in-the-moment* and reflection was associated with *learners* and *self* ("…I didn't want my other learners to get confused because when my learners ask those higher-level questions and stuff like that because I do have a weaker class so I tend to not want to …I get scared to want to answer their questions that they have”). When I probed Tara with a follow-up question as to whether she achieved her objectives for lesson 2, she stated that she did as they did the Grade 8 revision she wanted and felt that the contingent incidents in lesson 2 did not hinder her teaching.

Later in the interview, I probed Tara to reflect on improvements she could have made to her lessons such as lesson 1 and her response is shown in the following excerpt:

Excerpt 7

Line	Teacher/researcher statements
1	Tara: My mannerisms and how I approach topics as well as just ensuring that my classroom management especially when we have disruptions in class to ensure that…because this is a new topic that they are learning, that they are paying attention to the things that I want them to pay attention to.

Her reflection was *technical* as it does not focus on any critical incidents in the lesson, related to classroom management, learners’ behaviour and the object of reflection was associated with *self* and *learners* (“…how I approach topics … this is a new topic that they are learning, that they are paying attention to the things that I want them to pay attention to”). Her concerns were mainly related to her own effectiveness during teaching and this result is unsurprising as Veenman (1984) reported similar patterns across novice teachers in his study.

When I probed Tara on improvements, she could make to latter lessons such as lesson 2 and 3, again there is no reference to how Tara could have improved upon her response to contingent incidents *in-the-moment*. The response is general as shown in Tara’s reflection of lesson 3 in the following excerpt:

Excerpt 8

Line	Teacher/researcher statements
1	Tara: I need to be more aware of my classroom setup, be aware that just because learners are busy doesn’t necessarily mean that they aren’t struggling. Trying to keep them active - maybe even working on the boards with the Exercise to write up their answers and see for myself whether or not they are understanding the work and such. That’s something I would change during that lesson.

In Tara's reflection of lesson 3 similar to her reflection on lesson 1, the reflection is *technical* without any reference to specific incidents such as the contingent incidents that transpired in the lesson. The object of reflection is associated with *self* and *learners* ("I need to be more aware of my classroom setup … trying to keep them active - maybe even working on the boards with the Exercise to write up their answers…").

4.2.4 Linking Tara's practice and reflective awareness

Tara's response to contingent incidents were mainly in the form of acknowledging but putting aside the learner's question to continue with the lesson agenda. This response pattern is similar to how novices responded to learners' questions in Stockero and Van Zoest's (2013) study. Tara's response patterns indicated that she did not maximise the full potential of contingent incidents which transpired and there was a notable absence in all incidents of probing or follow-up questions to understand learners' thinking *in-the-moment*.

Analysis from the VSR interviews indicate that Tara was in a state of *technical reflection* when asked to reflect on objectives and improvements to each lesson as responses were general without referring to specific events which transpired. However, Tara would shift to a state of *deliberate reflection* when I probed her on her thoughts and decision making in response to contingent incidents for each lesson. This was evident as she articulated rationale for decisions as a result of reflection-oriented questions I asked. Tara's reflections concerning improvements that could be made to lessons never indicated a desire to improve specifically her responsiveness to learners' questions *in-the-moment* and she is unaware of changes related to responsive teaching that could be made to her practice.

4.3 Shandre's case

4.3.1 Shandre's background and overview of the lessons

Shandre is a novice teacher from school B. She was in her third year of teaching and is in possession of a BSc Honours degree and PGCE.

In Shandre's first lesson with her Grade 8 class, she starts the lesson by asking the class "I'm taking fifteen apples and from fifteen apples I want to take away six okay, what answer are you going to give me?" and proceeded to ask "if I said to you from x take away three … you're going to give me?" Shandre tells the class that *from* is the key word as "whatever follows *from* goes first". Subsequently, Shandre works the first example from her PowerPoint presentation which states subtract $2x + 3y - 5$ from $5x + 4y - 2$. Shandre moves on to the second example which states from $3x^2 + 2x + 7$ subtract $5x^2 - 4x + 1$ and asks learners in the form of a question-and-answer exchange what needs to be done. For the remainder of the lesson, Shandre asks learners to work individually and do Exercise 6 from the note pack related to subtraction of polynomials and walks around the class to check if learners did the previous day's homework. The lesson lasted 36 minutes and 51 seconds.

In Shandre's second lesson, she starts the lesson by telling the class that they are starting a section called exponents and hands out a note pack to each learner. Afterwards, Shandre uses her PowerPoint slides and asks the class to identify what the 3, x and 2 represent in $3x^2$. Shandre then tells the class that they will be individually investigating law 1 of exponents by filling in a table on the cover of the note pack she handed earlier in the lesson and timed the class 12 minutes to do the investigation. In Shandre's PowerPoint she selected three questions from the investigation (a x a^5; 7^2 x 7 and x^2yz^3 x xyz) and asked the class for the base/s, how to write in expanded notation and for the final simplified answer. Shandre summarises that for "like bases being multiplied, keep the base and add exponents" before proceeding to six examples based on the first law of exponents. Once Shandre finished working out the examples, she asked the class to complete individually Exercise 1 from the note pack related to the first law of exponents. The lesson lasted 37 minutes and 20 seconds.

In Shandre's third lesson, she starts the lesson by looking at more examples of the first law of exponents. Shandre works out six examples (such as simplify: -2 x $-2a^2b^2$ x $-2a^3b$) on the whiteboard and reminds learners to pay attention to the signs, multiply coefficients and add exponents for like bases. Subsequently, Shandre proceeds with two examples of the PowerPoint presentation which require learners to now use knowledge of both BODMAS and law 1 of exponents. The examples on the PowerPoint require learners to simplify $2a^2$ x a + $3a$ x $2a^2$ and $-2x$ x $y^2 + (y)(4xy) - 3y^2$ x $-x$ respectively. Once Shandre worked these examples with the class, learners were asked to do Exercise 2 from the note pack for the remainder of the lesson and Shandre walked around the class individually assisting learners. The lesson lasted 54 minutes and 17 seconds.

The findings from Table 5 highlights that the first trigger associated with responsiveness to learners were present in all of Shandre's observed lessons with a notable absence of the second and third trigger of contingency associated with teacher insight and the sudden availability or unavailability of a pedagogical tool respectively. For Shandre's observed lessons, I noted four contingent incidents and I provide a qualitative analysis on all four contingent incidents in the next section. All four contingent incidents were coded as *appropriate* and *middle*.

Table 5: Overview of triggers of contingency and level of responses provided by Shandre

Name of teacher	Trigger of contingency	Level	Lesson 1	Lesson 2	Lesson 3	Total
Shandre	Responding to learners' ideas	Maximum	0	0	0	0
		Middle	1	2	1	4
		Minimum	0	0	0	0
		Inappropriate	0	0	0	0
	Teacher insight	Maximum	0	0	0	0
		Middle	0	0	0	0
		Minimum	0	0	0	0
		Inappropriate	0	0	0	0
	Availability/unavailability of a pedagogical resource	Maximum	0	0	0	0
		Middle	0	0	0	0
		Minimum	0	0	0	0
		Inappropriate	0	0	0	0

4.3.2 Qualitative analysis of Shandre's responses to contingent incidents

Contingent incident 1: A learner's incorrect answer – Acknowledge but continue

During Shandre's first lesson on subtracting polynomials, she started the lesson with a numerical example by asking the class "I'm taking fifteen apples and from fifteen apples I want to take away six okay, what answer are you going to give me?". She proceeded to ask the class "from x take away three" (line 1) and a learner stated that the answer is x (line 2).

Excerpt 9

Line	Teacher/learner statements
1	Shandre: Okay, from fifteen you took six. So, the from tells you what comes first. Now if I said to you from x take away three … you're going to give me?
2	Learner: x (Shandre shakes her head disapprovingly)
3	Shandre: x minus three.
4	Learners: x minus three.
5	Shandre: Okay, *from* is the key word over here. Okay, whatever follows *from* goes first. Okay, a lot of fs there. You're going to remember that?

The teacher listens evaluatively (Davis, 1997), shook her head in disapproval as she was surprised by the incorrect answer and stated that the answer is $x - 3$ (line 3) since *from* tells you where to start and continues with the lesson. It is unclear how the learner arrived at the answer of x. The teacher could have pursued the learner's thinking to understand the source of his thinking and get more information on his thinking. In the VSR interview, Shandre revealed that quickly correcting the answer was associated with not allowing the wrong thought to linger and to "get them back onto track as quickly as possible". Instead of chastising the error, Nesher (1987) states that errors should be welcomed opportunities for learning since errors are indicative of learners' thinking and are a normal part of the learning process. I coded the response of the teacher as *appropriate* since the learner's response was acknowledged and she gave an explanation for the correct answer which provided potential for learning. I categorised Shandre's response as *middle* since the learner's line of reasoning could have been pressed since his line of reasoning was not obvious to Shandre and the error could have been probed so that the learner could express his thinking (Kazemi & Stipek, 2001).

Contingent incident 2: A learner's question - Acknowledge but continue

In Shandre's second lesson, a learner asks the teacher why the answer cannot be four to an example the teacher did previously on the whiteboard which was 2×2^3. Shandre attempts to offer a different explanation on how to get the correct answer of sixteen by using expanded notation rather than using law one of exponents.

Excerpt 10

Line	Teacher/learner statements
1	Learner: Ma'am, for two times two to the power three, are you not allowed to do like two times two equals four?
2	Shandre: No, only time you're allowed to do that is if you write it in expanded notation. So, if I write it in expanded notation it would be two times two times two times two, right? (Shandre writes 2x2x2x2). So, that's going to be two times two is four times two is eight times two is sixteen. Then, you go straight to sixteen. Okay. So, next one.

There is some potential for learning as a result of the teacher's response to the learner's question. However, the full potential of the learner's question is not exploited as Shandre could have decided *in-the-moment* to probe the learner as to why he thinks the answer could be 4 instead of 16 before proceeding with a procedural explanation of expanded notation. I coded the response as *appropriate* and *middle* as the response does support learning to some extent. When I questioned Shandre as to her explanation, she felt she answered the learner's question successfully but admitted that if she had a chance to reexplain "I might give them the counterexample of look, this is what it does and you agree that two to the power four is two times two times two times two and where the difference is if you multiply the bases".

Contingent incident 3: A learner's question – Acknowledge but continue

Later in the second lesson after Shandre simplified $3a^2b^2$ x $-4ab^3$, a learner asks her if b is also negative in the expression $-12a^3b^5$ (line 1) and Shandre repeats the question to buy herself time to think as the question caught her off guard (line 2). Shandre responded briefly that the whole expression is negative rather than each individual letter and continues with the lesson.

Excerpt 11

Line	Teacher/learner statements
1 2	Learner: Ma'am, is b also a negative? Shandre: Is b also a negative? No, it's not an also. Everything here is negative, okay. Not individually, together it is a negative. Okay, any other questions?

In the VSR interview, Shandre admitted that the "question was very new for me" and a "tricky one to answer definitely". She considered deviating *in-the-moment* but stated that if she tried to show them substitution it would be ineffective as "they haven't been taught substitution yet". Furthermore, she expressed her concern by stating that it would be confusing at Grade 8 level to explain the learner's question using substitution with negative numbers. The learner's question is not straightforward to answer as b is a variable and can represent any number which is positive or negative. I coded Shandre's response as *appropriate* and *middle* since the response to the learner provided some potential for learning. However, Shandre did not capitalise on the contingent moment and engage the learner to understand the thinking behind her question (Stockero & Van Zoest, 2013).

Contingent incident 4: A learner's question – Acknowledge but continue

In the third lesson, Shandre finished simplifying an example 2^a x 2^b and stated that the answer was 2^{a+b}. A learner asks the teacher in line 2 whether she can state it as 2^{ab} or 2^{a+b} and the teacher chooses to acknowledge the question in line 3.

Excerpt 12

Line	Teacher/learner statements
1	Shandre: a plus b. (Shandre writes 2^{a+b} on the whiteboard). Okay, I add my exponents. Same thing here earlier when I did a to the power two times a to the power three which gave me a to the power 5. I added my exponents.

2	Learner: Ma'am is a plus b or ab fine?
3	Shandre: No, you have to write a plus b. ab is multiplication, okay. Be clear about the operations.

Shandre responded in line 3 by explaining that the exponents cannot be multiplied as a plus b is not the same operation as a multiplied with b. Hence, ab cannot be the exponent. Shandre's response was coded as *appropriate* and *middle* since the response holds some potential for supporting learning. However, the possible source of the learner's thinking again is not interrogated by Shandre and remains at the lower level of elaboration (Abdulhamid & Venkat, 2018).

4.3.3 Analysis of VSR interview with Shandre

I started the interview by asking Shandre to reflect on her goals for each lesson as shown in the following excerpt:

Excerpt 13

Line	Teacher/researcher statements
1	Researcher: Okay, Shandre the first question I want to ask you, for the three lessons I watched, if you think back …what was your goal for each lesson? What did you try to achieve for each lesson?
2	Shandre: So, my biggest goal with any given lesson, is for them to have some familiarity with the process and I always hope they come out of it understanding it but understanding it beyond the 'teaching bubble'. Because very often kids are like "Oh yeah it's so easy when ma'am is doing it but I get home and it's so hard and I don't know what's going on". Umm, so a part of it is to teach them the methodical thinking that's required because that's the difference. I have the training and how you need to think and they don't. And if they can remember the steps that I followed, what are the key words, what were the key thoughts for each technique. That's…yeah.

Her reflection was *technical* and generic without offering insight into critical incidents which transpired during her lessons. Her reflection was associated with *practice* and her *learners* ("… for them to have some familiarity with the process … teach them the methodical thinking that's required…").

After Shandre's response, I probed her as to whether she achieved her objectives for her lessons and her response is illustrated in the following excerpt:

Excerpt 14

Line	Teacher/researcher statements
1	Shandre: Did they take it home and remember it? Did they understand what was going on? Usually, there's a good spread of didn't understand at all to had a clear understanding. And the middle group is the largest. They have some understanding but they're still grappling with the finer details or they make mistakes, not misconceptions.

Shandre's reflection in the above excerpt is again *technical* and the object of her reflection is related to *learners* ("…they have some understanding but they're still grappling with the finer details…").

Later in the interview when I asked Shandre why she just corrected but did not probe the learner's incorrect offer of x to a question involving subtraction of polynomials in episode 1 (excerpt 9) where she asked the class from x take away 3, she provided a reason for her decision making:

Excerpt 15

Line	Teacher/researcher statements
1	Shandre: Normally, I'm thinking about don't let the misconceptions stay, so correct it immediately. Umm, yeah it is …don't let the wrong thought linger. Get them back onto track as quickly as possible.

From Shandre's reflection as to how she responded to the unexpected incorrect answer in lesson 1, she articulated a rationale for her actions and was at a level of *deliberate* reflection. The object of her reflection was related to *practice*. Shandre's response also indicates that she wants to quickly fix errors and does not see the value of interrogating learner offers. Literature on quality teaching suggests that this is low level mediation (Abdulhamid & Venkat, 2018).

Shandre again articulated a rationale for her actions when reflecting on lesson 3. For instance, when I probed Shandre about her thinking *in-the-moment* when a learner asked whether 2^a x 2^b can be written as 2^{ab} or 2^{a+b} (excerpt 12), she responded as follows:

Excerpt 16

Line	Teacher/researcher statements
1	Shandre: Learners are very keen to drop signs between variables. So, they think a plus b becomes ab. The misconception is to what operation is represented and the second thing is they do misunderstand when they should add and when they should multiply when it comes to exponents. Umm, particularly in that law. So, my response worked on the first assumption that they are misunderstanding how you represent a plus b – that you can simplify it further to ab.

Shandre engaged in *deliberate reflection* as she articulates that she thinks learners are attempting to conjoin unlike terms and she worked on this "misunderstanding" that "learners are very keen to drop signs between variables". I find this interesting as all the novice teachers in the study by Tirosh et al. (1998) were unaware of the conjoining error yet Shandre was able to anticipate that the learner in lesson 3 possibly made this error. The object of reflection is associated with *learners* ("they think a plus b becomes ab … that they are misunderstanding how you represent a plus b…").

When I asked Shandre if she would make any improvements to her lessons such as lesson 1, she was happy with the overall delivery of the lesson. For instance, she stated generally: "Umm, I don't know. I don't know if I would choose to handle that question differently." Her reflection was *technical*, related to *self* and lacked specificity at this point. However, Shandre shifted to a state of *critical reflection* when I asked her what improvements could be made to the second lesson. For instance, Shandre was envisaging alternatives to how she could have responded when a learner asked Shandre if b is also negative in the expression $-12a^3b^5$ as illustrated by the following excerpt:

Excerpt 17

Line	Teacher/researcher statements
1	Shandre: I could have explained the second example better. I could have shown them a substitution … umm where you have a minus and then you put the values, you substitute for a, b or c. But the problem is they haven't been taught substitution yet so how much do you engage with that? Yeah, and you don't want to substitute a negative number because …so, the second question is a tricky one to answer definitely. Umm, and yeah, I don't know …I might if I were to come across it more regularly. So, I'm used to them having issues with the minus but that specific question was very new for me.

Her reflection is at a level of *critical reflection* as she tried to envisage alternatives to how she could have answered the question. The object of the reflective process is related to *self* and *practice* ("I could have explained the second example better… I'm used to them having issues with the minus…"). I further probed Shandre as to what she meant by "I'm used to them having issues with the minus but that specific question was very new for me" and her response is illuminated below:

Excerpt 18

Line	Teacher/researcher statements
1	Shandre: Umm, so how a minus interacts with a fraction or with an algebraic expression or a term is difficult for the kids to grasp. But nonetheless I can see that I probably could have explained it better, I think … I don't know. Kids struggle with these minuses and I think part of that comes from primary school where they only think of a minus as a subtraction - the action of it, not the value of the term itself. So, when we start integers, they really struggle with the idea of I'm subtracting a negative number which means it becomes a positive so I'm actually adding. Umm, it's very hard for them to grasp. So, I always try to explain it in terms of debt. When you owe someone money and that owing has been taken away, then you have less money that you owe. You have more money because there's less of it that you owe. But still that concept is very abstract for them. So, yes that specific question definitely caught me off guard.

Critical reflection was evident again for lesson 2 when Shandre reflected on improvements that could be made to the second lesson concerning another contingent incident (episode 2 – excerpt 10). For instance, Shandre was envisaging alternatives to how she could have responded to a learner when he asked "are you not allowed to do like two times two equals four?" when Shandre finished simplify 2×2^3. Her response is shown in the following excerpt:

Excerpt 19

Line	Teacher/researcher statements
1	Shandre: I might give them the counterexample of look; this is what it does and you agree that two to the power four is two times two times two times two and where the difference is if you multiply the bases.

The above excerpt is evidence of *critical reflection* as Shandre envisaged alternatives to how she could have improved responding *in-the-moment* and the object of the reflection was associated *practice* and *learners* ("…give them the counterexample … and where the difference is if you multiply the bases"). For lesson 3, Shandre did not suggest any improvements that could be made to the lesson.

4.3.4 Linking Shandre's practice and reflective awareness

Shandre's responses to contingent incidents in all four instances involved acknowledging learners' questions and errors without an attempt to interrogate and probe the learner's offer with follow-up questions *in-the-moment* to understand the learners' thinking. Literature on quality teaching suggests this is associated with low levels of elaboration (Abdulhamid & Venkat, 2018).

Analysis from the VSR interviews indicate that Shandre's reflection on objectives for each lesson were *technical*. Shandre shifted to a state of *deliberate reflection* when probed on her decision making and she articulated a rationale for her actions *in-the-moment* for all three lessons. When I probed Shandre on improvements she could have made to the lessons, her reflection was *technical* for lesson 1 with the exception of her reflection concerning two contingent events in lesson 2. In these two instances, Shandre provided evidence of *critical reflection* envisaging alternatives she could have made concerning how she responded to learners *in-the-moment*. However, Shandre's reflection on improvements that could be made to lesson 3 were again *technical* similar to lesson 1.

4.4 Cassandra's case

4.4.1 Cassandra's background and overview of the lessons

Cassandra is an expert teacher from School A and was in her twentieth year of teaching at the time of data collection. Cassandra is recognised as having a good track record of learner performance, is in possession of a Master's Degree specialising in Education, is widely respected by her colleagues and the Head of Department, has mentored pre-service and novice teachers and is currently serving as a mentor to Tara.

In Cassandra's first lesson with her Grade 9 class, she started the lesson by telling the class that they will be continuing with adding and subtracting polynomials and that the lesson will be focusing on application questions of polynomials. In the first example, she asked the class to simplify $(5x^2 - 2x - 1) - (x^2 - 3x - 5) + (3x^2 - 4)$ and a learner responded that they need to do distributive law and look for like terms. Cassandra proceeded to the second example which required learners to think what needed to be added to $(2x - 3y)$ to get 20 and Cassandra explained to the class how to answer the question. In Cassandra's third example, the question asked: By how much is the sum of $2x^2 + 3x - 2$ greater than the sum of $(-x + 2)$ and $(-5x^2 - 8)$? Cassandra used integers as a tool to substitute in place of the algebraic expression when some learners expressed confusion with the wordiness of the example. Cassandra moved on to the fourth example which required learners to determine the perimeter of an equilateral triangle with sides $5x - 4$ in terms of x and Cassandra asked the class to draw a picture of the equilateral triangle with all sides labelled in order to make the question easier to answer. The final example required learners to individually determine the perimeter of a rectangle in terms of x if the length and breadth with given dimensions are reduced by $x - 2$ and Cassandra walked around the class to check learners' solutions to this example. The lesson ended with Cassandra giving the learners an Exercise from the textbook to do for homework related to application questions of polynomials. The lesson lasted 37 minutes and 19 seconds.

In Cassandra's second lesson, she started the lesson by marking the previous lesson's homework. Subsequently, Cassandra stated that "today we are going to have a look at what you distribute and what you don't always keeping in mind BODMAS" and selected fourteen examples for the class to discuss which had subtle differences such as $2(x + 5)$; $-2(x + 5)$; $(x + 5) - 2$; $(x + 5)2$ et cetera. Before the lesson ended, Cassandra gave the learners homework from the textbook related to distributive law to complete for the next lesson. The lesson lasted 30 minutes and 37 seconds.

In Cassandra's third lesson, she spent the first 10 minutes marking the previous lesson's homework. Cassandra then proceeded with two application examples of distributive law. In the first example, learners were given the dimensions for the length and breadth in terms of x and Cassandra showed the class how to determine the perimeter of the rectangle in terms of x. In the second example, Cassandra asked the class how they would find the missing side of a rectangle if given $6x^2 - 2x$ as the area and $2x$ as a side length of the rectangle. A learner responded that they would take the area and divide it by the missing length and Cassandra worked out this question on the whiteboard. Subsequently, Cassandra gave learners the breakdown of topics for the standardised test before asking the learners to work an Exercise from the textbook in pairs associated with the application of distributive law by using different whiteboards around the classroom. Cassandra walked around the class and checked on learners' work for the remainder of the lesson. The lesson lasted 37 minutes and 20 seconds.

The findings from Table 6 highlight that the first trigger associated with responsiveness to learners were present in all of Cassandra's observed lessons with a notable absence of the second and third trigger of contingency associated with teacher insight and the sudden availability or unavailability of a pedagogical tool respectively. For Cassandra's observed lessons, I noted three contingent incidents and I provide a qualitative analysis on all three contingent incidents in the next section. All three contingent incidents were coded as *appropriate* and *maximum*.

Table 6: Overview of triggers of contingency and level of responses provided by Cassandra

Name of teacher	Trigger of contingency	Level	Lesson 1	Lesson 2	Lesson 3	Total
Cassandra	Responding to learners' ideas	Maximum	1	1	1	3
		Middle	0	0	0	0
		Minimum	0	0	0	0
		Inappropriate	0	0	0	0
	Teacher insight	Maximum	0	0	0	0
		Middle	0	0	0	0
		Minimum	0	0	0	0
		Inappropriate	0	0	0	0
	Availability/unavailability of a pedagogical resource	Maximum	0	0	0	0
		Middle	0	0	0	0

		Minimum Inappropriate	0 0	0 0	0 0	0 0

4.4.2 Qualitative analysis of Cassandra's responses to contingent incidents

Contingent moment 1: A learner's question - Acknowledge and incorporate

In line 1, a learner is confused by the solution to a question Cassandra worked out earlier with the learners whereby the question states: By how much is the sum of $2x^2 + 3x - 2$ greater than the sum of $(-x + 2)$ and $(-5x^2 - 8)$?

Excerpt 20

Line	Teacher/learner statements
1	Learner: I'm so confused.
2	Cassandra: You are so confused. What are you confused about? All I want to know is if you understand this line. (Cassandra points to $2x^2 + 3x - 2$ – [$(-x + 2) + (-5x^2 - 8)$]) Because that's actually the hardest part …. to write the polynomials whether they're added or subtracted. So, this is the hardest part. Did you get this part?
3	Learner: Noooo….I don't understand.
4	Cassandra: So, we need to try understand what's going on here with the words, okay. Answer these questions for me. Now you trying to translate it. Okay, listen to the words. Now you're going to translate the words into Maths. By how much is ten greater than three?
5	Learner: Seven.
6	Cassandra: How did you get the seven?
7	Learner: Ten minus three.
8	Cassandra: Right, by how much is fifteen greater than five?
9	Learner: Fifteen minus five.
10	Cassandra: By how much is twenty greater than seven?
11	Learner: Thirteen

12	Cassandra:	How did you get the thirteen?
13	Learner:	Twenty minus seven.
14	Cassandra:	By how much is twenty greater than x?

Cassandra acknowledges the learner's question and perceives in line 4 that the source of the learner's confusion is struggling to understand and translate the wording of the question. Subsequently, she improvises *in-the-moment* and draws upon her repertoire of pedagogic strategies a series of questions which replace the variables with numbers so that it is easier to interpret. During the VSR interview, Cassandra revealed that over her 20 years of experience learners generally express confusion with such questions and she tries to "replace all this algebra with just some integers" to serve as a "tool that they can use when they sort of get confused with x". Cassandra's skilled improvisation and ability to respond successfully to unexpected questions on the spot was noted similar to how experts in Borko and Livingston's (1989) study were responsive to their learners' questions and willing to deviate from their lesson agenda. I coded Cassandra's response as *appropriate* and *maximum* since her detailed response provides maximum potential to help learners understand the algebra and wording of the question.

Contingent moment 2: A learner's question - Acknowledge and incorporate

In the second lesson dealing with distributive law, Cassandra asked the class to simplify $(x + 5)(+2)$ and writes the solution on the whiteboard. Subsequently, a learner asked Cassandra "if it's x plus five and there is a two outside the bracket?" (line 1). Cassandra chose to acknowledge the "good question" of $(x + 5)2$ and incorporated the learner's example into the lesson.

Excerpt 21

Line	Teacher/learner statements	
1	Learner:	Madam, I have a question. If it's x plus five and there is a two outside the bracket?
2	Cassandra:	I'm getting there. Good question. So, you tell me. Give me that example.
3	Learner:	x plus five in brackets and then there is a two.
	(Cassandra writes $(x + 5)2$)	
4	Cassandra:	What do you think?

5	Learner: Don't you distribute the two.
6	Cassandra: Why?
7	Learner: Because there's a bracket.
8	Cassandra: So, that is two x plus ten. So, what can you tell me about the previous example and this one?

Cassandra was aware in the VSR interview that the learner was a "step ahead and thinking about what if it's a plus two or what if it's just a two" and Cassandra had pre-empted the question in the lesson plan I asked her to design before the lesson. Probing the learner's offer provided a high level of engagement and opportunities for the learner to express her thinking associated with a higher level of elaboration (Abdulhamid & Venkat, 2018). Cassandra recognised that learners could think that the previous example of $(x+5)(+2)$ might yield a different answer to the learner's question of $(x+5)2$. Cassandra wanted learners to realise that both questions produce the same answer so acknowledging and incorporating the example was necessary *in-the-moment*. Her response was coded as *appropriate* and *maximum* since her response of acknowledging and incorporating the example held great potential for learning.

Contingent moment 3: A learner's question - Acknowledge and incorporate

During lesson 3, Cassandra acknowledges and pursues a learner's thinking as a learner is unsure in line 1 whether the answer to $3x(x-5)$ is $3x+3x^2-15x$ or $3x^2-15x$.

Excerpt 22

Line	Teacher/learner statements
1	Learner: I'm saying for the one underneath it after you've distributed it …must I write three x plus three x squared minus fifteen x or must I just write three x squared minus fifteen x.
2	Cassandra: Why do you want to add another three x?
3	Learner: Because it's already there …I'm just asking if I must do that.
4	Cassandra: No. Because you're multiplying. So, if you're going to simplify this … (Cassandra points to $3x(x-5)$ on the whiteboard)

	This means that three x must be multiplied. So, its three x times x. Not three x plus three x squared. You're multiplying. Did you add in extra terms?
5	Learner: No, I didn't add in extra terms. I was just confused.
6	Cassandra: Okay, so these two must be multiplied … (Cassandra circles $3x$ and x) And the answer to that multiplication is three x squared. (Cassandra erases the whiteboard). Alright, let's have a look at this. So, let's have a look at this application. So, this is a rectangle okay.

Cassandra acknowledges the learner's question, spends time trying to diagnose the learner's question and presses the learner with a follow-up question as to why she wants to add an additional $3x$ (line 2). Thereafter, in line 4 she procedurally explains that distributive law yields $3x^2$ for the first term and not $3x$. I coded this as *appropriate* and *maximum* as Cassandra chose to probe and pursue the learner's thinking whilst providing explicit feedback which supported learning (Abdulhamid & Venkat, 2018). During the VSR interview when I probed Cassandra on her response to the learner, she stated that "I was thinking that she probably forgot to multiply" and reflects that "because the last term is $15x$" the learner did not "realise that they also have to multiply by another x which makes it the $3x^2$".

4.4.3 Analysis of VSR interview with Cassandra

I asked Cassandra to reflect on the objectives for each lesson. She provided a justification for what she intended learners to grasp as shown by the following excerpt:

Excerpt 23

Line	Teacher/researcher statements
1	Cassandra: So, the second lesson was distributive law and the goal of that was really for them to look at the subtleties of what distributive law requires. So, it's important: where are the brackets? Umm, where is the sign? Is there a sign in between the brackets and the constant? Is that constant in a bracket? What do you distribute? What do you not distribute? Because that's really important for algebra going forward. So, what do you distribute and what do

	you not? And remembering BODMAS all the time because again they think BODMAS exists in isolation, that now we're on algebra you don't have to do BODMAS. And that's not the case.

Cassandra engaged in *deliberate reflection* when reflecting on her objectives as she provides a rationale for teaching subtle differences of distributive law in lesson 2 since it provides a platform "for algebra going forward". The object of reflection was associated with *learners* ("…for them to look at the subtleties of what distributive law requires …").

When I probed Cassandra on her thoughts and decision making in response to contingent moments such as a contingent incident in lesson 1 (excerpt 20) concerning learners' confusion about a question which asks: By how much is the sum of $2x^2 + 3x - 2$ greater than the sum of $(-x + 2)$ and $(-5x^2 - 8)$? Cassandra provided a clear rationale and justification for her actions in a concise manner as shown in the following excerpt:

Excerpt 24

Line	Teacher/researcher statements
1	Researcher: Okay, thanks Cassandra. Okay umm so now what we're going to do is look at each lesson together specific incidents from each lesson. Umm, so we're going to look at …we're going to talk about lesson 1. There's an episode which is quite interesting, so you ask them a question…Some of the learners were expressing confusion and then one of the girls she asked you why do you need to minus the big bracket. Was that something that was an unexpected question for you in that moment in time?
2	Cassandra: Yes, I would say so. I think I explained that you replace it with normal numbers. So, if I say by how much is the sum ten greater than the sum of two and five? And, then they can do that as soon as you got the numbers but the moment you put the algebra; they don't know what to do anymore. So, I thought I'd explain that.

Her reflection was *deliberate* at this point and related to her *learners* ("…then they can do that as soon as you got the numbers but the moment you put the algebra; they don't know what to do anymore …”). Her thoughts were centred around learners’ understanding of the content. I further probed Cassandra about this contingent incident in lesson 1. Cassandra acknowledged a gap in the sequencing of her example and suggested an alternative by opting to start with lead in examples involving integers before she gets to the algebra. This is illuminated by the following excerpt:

Excerpt 25

Line	Teacher/researcher statements
1	Cassandra: Perhaps, I would have done a few more integer examples before I get to the algebra. So, I wouldn’t have gone straight into that. I would have said “By how much is ten greater than the sum of five and two?” and then they can tell me that. And then I would have said “By how much is twenty greater than the sum of fifteen and three?” Umm, and do more of those and then say okay now “By how much is the sum of two x squared plus three x minus two greater than the sum of negative x plus two and negative five x squared minus eight? and then see if they can figure out.

Cassandra engaged in *critical reflection* in terms of thinking of alternatives to an otherwise successful lesson and the object of reflection was associated with *practice* and *learners* (“…I would have done a few more integer examples before I get to the algebra … then see if they can figure out ...”). Critical reflection was not only limited to Cassandra’s reflection in the episode above but evident in her reflection of other episodes during the VSR interview. For instance, when I asked Cassandra about her reaction and thoughts at the time in response to a learner’s response during a contingent incident in lesson 2 (excerpt 21), she envisaged alternatives to the lesson as illustrated by the following excerpt:

Excerpt 26

Line	Teacher/researcher statements
1	Researcher: Okay. So, Cassandra in that lesson, you were paying attention to the sign and all that, so I remember you did x plus five in the first bracket and the second bracket had a plus two. Then, there’s a girl who asks you if you have x plus five in a bracket with a two outside the bracket. So, she asked you that and you responded that was a good question. Was that something that was an unexpected question for you when you think about that lesson?
2	Cassandra: Umm probably not because I was going to get there but she was already thinking ahead. Yeah.

3	Researcher:	Okay.
4	Cassandra:	Yeah, she was already a step ahead and thinking about what if it's a plus two or what if it's just a two.
5	Researcher:	And yeah, you were going to do that a bit later on.
6	Cassandra:	Another way you could have taught that is you could have said "Okay, here's a two, a minus two and a bracket of x plus five. Now, you come up with different combinations of how we can arrange them with brackets, without brackets". That could be a little bit more …where you involve the pupils a little bit more instead of you coming up with examples, let them come up with the examples.

Cassandra again engaged in *critical reflection* and the object of reflection were her *learners* ("…involve the pupils a little bit more instead of you coming up with examples, let them come up with the examples"). Her reflection envisaged alternate ways she could get the learners more actively involved in their own learning. This resonates with findings found by Borko and Livingston (1989) since they noted that the post lesson reflections of experts were associated with learners and learners' engagement rather than focusing on self.

4.4.4 Linking Cassandra's practice and reflective awareness

In Cassandra's lessons, there were little unexpected incidents for her and she admitted that "you kind of pretty much know what's coming I suppose umm after you've done it for so long" and that "Grade 9 has been the year I've taught every single year for twenty years". Cassandra has learnt to pre-empt most learners' questions as a result of her long engagement with the practice and designs her lessons with these pre-empted questions in mind so that "it allows for better understanding, so there aren't that many questions". Cassandra's response patterns indicated that she maximised the full potential of contingent incidents which transpired by choosing to acknowledge and probe learners with follow-up questions *in-the-moment*. This is indicative of high-quality mediation and a characteristic of effective teaching as Cassandra gave learners an opportunity to express and clarify their thinking by pursuing learners' thinking in all instances (Abdulhamid & Venkat, 2018).

Analysis from the VSR interviews indicate that Cassandra was in a state of *deliberate reflection* when asked to delineate the objectives of her lessons. Cassandra remained at a level of *deliberate reflection* when I probed her on her thoughts and decision making in response to contingent incidents for each lesson since she provided a rationale for her actions. However, Cassandra shifted to a state of *critical reflection* when she envisaged alternatives and improvements after reflecting on contingent incidents in lessons 1, 2 and 3.

4.5 Synthesis across the three case study teachers

Evidence of evaluation at secondary level rather than a lack thereof were prevalent in the observed lessons similar to findings noted by other researchers in South Africa (Brodie, 2007a; Brodie, 2007b). This situation is in contrast to the situation that currently exists at primary school level (Abdulhamid, 2016; Abdulhamid, 2017; Abdulhamid & Venkat, 2018; Hoadley, 2012). Interestingly, the first trigger of contingency associated with responsiveness to learners were present in all of the three case study teachers' observed lessons with a notable absence of the second and third trigger of contingency associated with teacher insight and the sudden availability or unavailability of a pedagogical tool respectively. Moreover, there was a notable difference in the quality of responses provided by the three case study teachers as illustrated by Table 7.

Table 7: Summary of codes used to classify teachers' responses to contingent incidents

	Novice/expert	Inappropriate	Minimum	Middle	Maximum
Tara	Novice	25% (1/4)	25% (1/4)	50% (2/4)	
Shandre	Novice			100% (4/4)	
Cassandra	Expert				100% (3/3)

Overall, the results show that there was a notable absence of press moves when Tara and Shandre (two novice teachers) encountered and responded to unexpected events in their classroom since none of the novice teachers' responses were coded as maximum. This was common especially when novices were confronted with contingent incidents coded *appropriate* and *middle*, they would provide a procedural explanation of how to get the correct answer without trying to understand the learners' thinking. In these instances, responses to learners' offers were evaluative with a notable absence of either "low press" and "high press"[4] follow-up questions to probe learners mathematical thinking *in-the-moment* (Davis, 1997; Kazemi & Stipek, 2001). Such responses characterise the lower level of elaboration since the

[4] Kazemi and Stipek (2001) use the terms "low press" and "high press" to differentiate between the different ways teacher use questions to push their learners' thinking. "Low press" questions require only a procedural justification from the learner whereas "high press" questions require a conceptual explanation which tends to "encourage learners to include mathematical arguments in their explanations" (Kazemi & Stipek, 2001, p.78).

teacher's response did not provide learners with an opportunity to clarify and express their thinking (Abdulhamid & Venkat, 2018). Follow-up questions which require learners to elaborate their thinking can serve as an important resource for teachers' pedagogical decision making *in-the-moment* (Walshaw & Anthony, 2008). It is important to note that the experience of two novice teachers was not the same as Tara was in her first year of teaching (and in essence a pre-service teacher) whereas Shandre was in her third year of teaching. This could possibly explain why Shandre's responses were all coded as *middle* whereas only half of Tara's responses were coded as *middle*. In the case of Cassandra, she pressed learners on their offers with a follow-up question to understand their mathematical thinking when either a learner's thinking was unclear or when she decided to press a learner for a procedural or conceptual explanation. All the expert teacher's responses were coded as *appropriate* and *maximum* indicative of a high level of elaboration and supportive of emergent mathematics learning in the collective classroom space (Abdulhamid & Venkat, 2018).

Another interesting finding across the three case study teachers is that VSR served as a useful catalyst to help the three teachers reflect on their thoughts and decision making in response to contingent events which emerged across their three lessons. This was evident as all three teachers engaged in a *deliberate reflection* at some point during the VSR interview and articulated rationale for their responses to learners' questions *in-the-moment*. Furthermore, the VSR interview allowed Shandre and Cassandra to note potential limitations in how they responded to certain contingent incidents and envisage alternatives to how they could have responded which they might not have previously considered. Cassandra recognised the potential of the VSR interview as it allowed her to consider aspects of her teaching that she did not previously consider and stated that "it's always good to revisit and look at your teaching. We should all be recording our lessons". However, Tara the most inexperienced of the three teachers never reached *critical reflection* when I asked her to reflect on improvements that could be made to her lessons. Tara's reflection in these instances were *technical* and related to her own effectiveness during teaching similar to that of novice teachers' post-lesson reflections in Veenman's (1984) study.

Lastly, it was seen that the novice teachers' desirability to responding to contingent events was influenced by classroom dynamics such as whether they perceived the class to be a strong or weak group. For instance, during the VSR interview Tara revealed that "…I didn't want my other learners to get confused because when my learners ask those higher-level questions and stuff like that because I do have a weaker class so I tend to not want to …I get scared to want to answer their questions that they have". Possibly, Tara's belief about the ability of the learners in her class influenced her lack of desirability to want to answer higher ordered questions which she deemed unexpected since this would confuse the majority of the class. In the case of Shandre, she noted in the VSR interview that I had observed "the smarter of the two (Grade 8) classes" she taught and stated that "I might let them answer even if it's the wrong answer and let them try more than I would a class that struggles more". This could possibly explain why Shandre was more willing to respond to contingent events compared to Tara. Interestingly, Cassandra reflected that she had a "weaker class" yet this did not influence her willingness to respond to contingent incidents.

4.6 Conclusion

In this chapter, I have presented analysis and findings across the three case study teachers in terms of how they responded to contingent incidents in their classrooms. Findings from the analysis indicated that the two novice teachers did not press or probe learners to understand their mathematical thinking i*n-the-moment* when faced with an unexpected question whereas the expert teacher pressed learners in all contingent incidents with follow-up questions to understand their thinking. Also, there was a notable absence of triggers of contingency emanating from teacher insights and (un)availability of resources during teaching across all lessons of the three teachers. Findings from the VSR interview indicated that all three teachers engaged in *deliberate reflection* when reflecting on contingent incidents and a shift to *critical reflection* was noted for Shandre and Cassandra. Lastly, it was noted that the novice teachers' desirability to responding to contingent events was influenced by classroom dynamics such as whether they perceived the class to be a strong or weak group. The next chapter presents the concluding aspect of the research report.

CHAPTER 5: CONCLUSION

5.1 Introduction

Analysing contingent incidents in the three case study teachers' classrooms provide an important insight into how teachers engaged in responsive teaching and illuminated insight into teachers' actions and decision making when faced with contingent incidents, and varied quality of responses in relation to novice and expert teachers.

To conclude this research, I begin this chapter by restating the main foci of the study. Subsequently, I provide a summary of key findings which emerged from the research and implications of the findings in relation to practice and policy. Finally, I discuss limitations of the study and suggest possible directions for future research.

5.2 Foci of the study

This study had two main foci. The first focus was to understand how the three case study teachers responded to contingent incidents which emerged during their lessons. This was guided by the following question:

How do novice and expert Senior Phase teachers respond to unexpected events in the teaching of algebra?

In chapter 2, I noted that in secondary classrooms there is evidence of evaluation of learner offers but the nature of teacher engagement with learners' offers is often superficial (Brodie, 2007a). In order to understand how teachers responded, I employed a modified form of Weston's (2013) coding protocol to analyse the quality of teachers' responses to unexpected learner questions in secondary classrooms. Indicators used to highlight the degree of quality were *minimum*, *middle* and *maximum* if a response were coded as appropriate to a learner's offer.

The second focus was related to understanding teachers' decision making when faced with contingent incidents by conducting VSR interviews with each teacher. This was guided by the following question:

What are the novice and expert Senior Phase teachers' thoughts and decision making in response to unexpected events during the teaching of algebra?

I conducted VSR interviews with the teachers in July 2020 to understand the teachers' actions when faced with an unexpected question by employing a two-dimensional framework which incorporated the interviewee's level of reflection (Muir & Beswick, 2007) and the object of their reflection (Geiger et al., 2016). The key findings which emerged from my research project are summarised in the next section.

5.3 Key findings of the study

1. An absence of press moves or probing amongst novice teachers when faced with a contingent incident

Findings from this study suggests that the response patterns of the novice teachers in this study were all characteristic of low-level mediation since both Tara and Shandre did not probe learners on their mathematical thinking *in-the-moment* (Abdulhamid & Venkat, 2018). In Tara's case, she predominantly acknowledged but put aside answering a learner's question and the potential of the contingent events were not fully exploited. In Shandre's case, she acknowledged learner questions and her responses were coded as *middle* in all instances but wanted to quickly fix errors and did not see the potential of interrogating learner offers. In the case of the expert teacher, her responses were coded *maximum* in all instances as she pressed learners and asked follow-up questions when a learner's thinking was not clear or she probed learners when she wanted a procedural or conceptual explanation.

2. VSR served as a useful catalyst to help teachers reflect on their thoughts and decision making in response to contingent events

The reflection-oriented questions during the VSR interviews proved fruitful in assisting the teachers to reflect on how they responded to contingent incidents during their lessons and enabled me to understand their actions and decision making *in-the-moment*. All three teachers engaged in *deliberate reflection* when reflecting about the contingent incidents and articulated a justification for their actions. Moreover, the VSR interview helped Shandre and Cassandra to envisage alternatives to how they responded to the contingent incidents and reach *critical reflection*. This highlights the potential of VSR interviews as noted in other studies (Geiger et al., 2016) in helping teachers reflect on aspects of their teaching not previously considered, expose limitations in their practice and envisage alternative approaches or strategies to limitations.

3. Absence of triggers of contingency emanating from teacher insights and (un)availability of resources during teaching

The findings from the study highlighted that the first trigger associated with responsiveness to learners were present in all observed lessons with a notable absence of the second and third trigger of contingency associated with teacher insight and the sudden availability or unavailability of a pedagogical tool respectively. However, it is pertinent to note that this is not to say that the second and third triggers of contingency are completely absent in the South African terrain. Rather, these triggers are rare to observe in practice. This was revealed when I probed participants during the VSR interview on their experiences with contingency in their general teaching beyond the lessons I observed and all three teachers were able to describe experiences with the third trigger of contingency. For instance, Shandre succinctly relayed her experiences of having to deviate from her lesson plan when a pedagogical tool such as technology became suddenly unavailable during her lesson. She recounted incidents associated with the third trigger of contingency such as when her "PC decided it had to do a reboot right at the beginning of a lesson" and "the days you have load shedding - you can't use that PowerPoint Presentation you planned on using". It is important to note that the prevalence of the second trigger related to teacher insight during general teaching was only described by Cassandra and this is unsurprising since this trigger is more prevalent amongst expert teachers (Rowland et al., 2015).

5.4 Implications of findings to practice and policy on mathematics teaching

Findings from the study highlighted that novice teachers do not interrogate unexpected learner offers and that responses are of a low level of evaluation. This suggests that novice teachers need long-term professional development opportunities to help them intentionally notice (Mason, 2002) how they respond to unplanned learner offers and understand the nature of their responses *in-the-moment*. This will help novice teachers be better equipped to respond in productive ways to learners offers and will support a shift towards responsive teaching. Research with both pre-service (Lampert, Franke, Kazemi, Ghousseini, Turrou, Beasley & Crowe, 2013; Star & Strickland, 2007) and in-service teachers (Sherin & van Es, 2009) suggests that expertise in noticing improves with support and professional development. Moreover, professional noticing of learners' mathematical thinking associated with attending, interpreting and deciding how to respond *in-the-moment* has been shown to improve amongst participating teachers within the first two years of professional development (Jacobs et al., 2010). The difficulty of noticing and attending to learners' unexpected offers experienced by novice teachers suggests that teacher education programs in South Africa need to do more to equip pre-service teachers to handle unexpected learner offers during responsive teaching. In recent years, research by Lampert et al. (2013) have proposed that pre-service teachers need to be supported to elicit and respond to learner offers through "intellectually ambitious instruction". During such instruction, the pre-service teacher is responsible for teaching an instructional activity whilst the teacher educator "acts as both coach and simulated learner" enabling the pre-service teacher to rehearse how they would respond *in-the-moment* to a potential learner offer and other pre-service teachers in the class are provided an opportunity to "investigate the actions a teacher might make" in response to an unplanned learner offer (Lampert et al., 2013, p.229). Such support as highlighted by Lampert et al.'s (2013) study is necessary for prospective teachers in South Africa to better handle unexpected learner offers.

5.5 Limitations of the study

The number of teachers in this study was a major limitation. This was attributed to the emergence of COVID-19 and I could not conduct lesson observations for the expert teacher's algebra lessons from School B in April 2020 as envisaged. As a result, I excluded the teacher from the study. Consequently, I could not draw any comparisons of Cassandra with another expert teacher.

5.6 Directions for future research

My recommendation for future research is to conduct a professional development program working with a large group of novice and expert teachers over a period of two years to help teachers intentionally notice how they respond to learners' offers *in-the-moment*. This would involve conducting lesson observations for each teacher and working with each teacher during the VSR interview to identify how they responded to contingent incidents in the observed lesson and help teachers reflect on their responsive teaching. Thereafter, a second cycle of lesson observations should be conducted and the researcher could note any shifts in how teachers responded to contingent events and follow-up the lesson with another VSR interview.

5.7 Self-reflection

This study was motivated by my own difficulties I experienced as a novice teacher when responding to unexpected learner offers which caught me off guard and this led me to think how do novice and expert teachers respond to contingent incidents in their own classrooms. This research journey has made significant shifts in my own growth as a teacher as well as a novice researcher. Focusing on responsive teaching has made me more cognisant of how I respond to my own learners when they ask unexpected questions and this had led me to interrogate my own practice. Prior to this research project, I found myself listening evaluatively to learners rather than interpretively (Davis, 1997) and tending to fix incorrect learner offers or dismissing unexpected questions without probing learners to understand their thinking. As a result of this research project, I notice that I tend to try and anticipate unexpected questions before I teach a topic in order “to plan a less ‘bumpy’ more joined-up learning experience” (Rowland et al., 2015, p.76) for my learners. Moreover, I am more attuned to learners’ questions *in-the-moment* and have learnt to probe learners’ thinking when faced with unexpected questions or incorrect offers to exploit the full potential of the contingent moment as it forms a teachable and learnable moment for both myself and the learners.

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APPENDIX A: Sample of VSR interview schedule

Tara interview questions:

Introduction

I would thank Tara for allowing me to have video recorded her lesson and making time available for this interview to discuss about their lesson.

Semi-structured questions

1. Opening conversation

(a) What was your intended goal for each lesson (lesson 1, 2 and 3)? In other words, what were you hoping that learners would be able to do, or understand, at the end of each lesson?
(b) Did you feel that you achieved what you had intended for each lesson? Why or why not?

2. Discussion

Responding to student ideas (Selected episodes from the lesson)

Lesson 1:

Selected episode to discuss:

(a) Considering lesson 1, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?
(b) In your teaching, please can you comment on how you handled a learner's contribution [whereby she asks why there is a x^{-1} when rewriting $\frac{5}{x} - 8x$ as $5(x^{-1}) - 8x$]? What were your thoughts and decision making at that moment in time when responding to the learner's question?
(c) If you were to repeat lesson 1, will you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

Lesson 2:

Selected episodes to discuss:

Episode 1:

(a) Considering lesson 2, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?
(b) In the lesson, a learner asked a question as to simplify -2^{-2}. What was your thinking in-the-moment as to how to respond to the learner and what was your reason for putting aside further answering that question?

Episode 2:

(a) A second contingent event arose in this lesson when a learner asked how you would simplify an expression with a base x which has a negative exponent which is a fraction. What was your thinking in-the-moment as to how to respond to the learner's question and what was your reason for not further exploring the question?

(a) If you were to repeat lesson 2, will you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

<u>Lesson 3</u>:

<u>Selected episode to discuss:</u>

<u>Episode 1:</u>

(a) Considering lesson 3, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?
(b) The class is working on simplifying $(x + 1) + (2x + 3)$ and you were working out the solution with the class. You wrote $x + 1 + 2x$ so far and a learner asks whether the answer would be four. What were you thinking in that specific moment? Please can you tell me why you responded in this way.
(c) If you were to repeat lesson 3, will you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

3. Conclusion

(a) Is there anything else that you would like to add?
(b) What has been your your experience with the three triggers of contingency (I will explain what I mean by the three triggers of contingency to the teacher) beyond the lessons I observed?

<u>Shandre interview questions:</u>

<u>Introduction</u>

I would thank Shandre for allowing me to have video recorded her lesson and making time available for this interview to discuss about their lesson.

<u>Semi-structured questions</u>

1. Opening conversation

(a) What was your intended goal for each lesson (lesson 1, 2 and 3)? In other words, what were you hoping that learners would be able to do, or understand, at the end of each lesson?
(b) Did you feel that you achieved what you had intended for each lesson? Why or why not?

2. Discussion

Responding to student ideas (Selected episodes from the lesson)

<u>Lesson 1:</u>

<u>Selected episode to discuss:</u>

(a) Considering lesson 1, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?
(b) You asked the class a question: "From x take away 3". A learner stated incorrectly x as the answer. Please can you elaborate how you responded. What were you thinking in that specific moment?
(c) If you were to repeat lesson 1, will you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

<u>Lesson 2:</u>

<u>Selected episodes to discuss:</u>

<u>Episode 1:</u>

(a) Considering lesson 2, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?
(b) A learner asked you whether you are allowed to multiply two with two to get four in the following class example: simplify 2 x 2^3. You responded by using expanded notation to explain how to arrive at the answer of sixteen. Please can you elaborate your thought and decision making in-the-moment.

<u>Episode 2:</u>

(a) You finish wrote $-12a^3b^5$ as an answer to an example and a learner asks you if b is also negative in the final answer. What was your thinking in-the-moment as to how to respond to the learner? Please can you tell me why you responded in this way.

(a) If you were to repeat lesson 2, will you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

<u>Lesson 3:</u>

<u>Selected episode to discuss:</u>

(a) Considering lesson 3, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?
(b) You finished simplifying an example 2^a x 2^b and stated the answer was 2^{a+b}. A learner asks you whether she can state it as 2^{ab} or 2^{a+b}. Please can you comment on how you handled the learner's question in the lesson. What were you thinking in that specific moment?
(c) If you were to repeat lesson 3, will you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

3. Conclusion

(a) Is there anything else that you would like to add?
(b) What has been your experience with the three triggers of contingency (I will explain what I mean by the three triggers of contingency to the teacher) beyond the lessons I observed?

Cassandra interview questions:

Introduction

I would thank Cassandra for allowing me to have video recorded her lesson and making time available for this interview to discuss about their lesson.

Semi-structured questions

1. Opening conversation

(a) What was your intended goal for each lesson (lesson 1, 2 and 3)? In other words, what were you hoping that learners would be able to do, or understand, at the end of each lesson?
(b) Did you feel that you achieved what you had intended for each lesson? Why or why not?

2. Discussion

Responding to student ideas (Selected episodes from the lesson)

Lesson 1:

Selected episode to discuss:

(a) Considering lesson 1, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?
(b) You started answering the following question: "By how much is the sum of $2x^2 + 3x - 2$ greater than the sum of $(-x + 2)$ and $(-5x^2 - 8)$?". You wrote $2x^2 + 3x - 2 - [(-x + 2) + (-5x^2 - 8)]$ and a learner asks why do they have to minus the big bracket and expresses confusion. Can you elaborate how you responded and what were you thinking as to how to respond to the learner's question? Was it an expected question?
(c) If you were to repeat this lesson, will you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

Lesson 2:

Selected episode to discuss:

(a) Considering lesson 2, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?
(b) Cassandra, you asked the class a question regarding how to simplify $(x + 5)(+2)$ and you finished answering the question with the class. Subsequently, a learner asks you "if it's $x + 5$ and there is a two outside the bracket?" You stated it was a "good question". Was this an expected question for you? What were you thinking at that moment in time as to how to respond?
(c) If you were to repeat this lesson, will you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

Lesson 3:

Selected episode to discuss:

(a) Considering lesson 3, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?
(b) A learner asks you after simplifying $3x(x-5)$ if you just write $3x$ or $3x+3x^2$ as your answer. What was your thinking as to how to respond to her?
(c) If you were to repeat this lesson, will you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

3. Conclusion

(a) Is there anything else that you would like to add?
(b) What has been your experience with the three triggers of contingency (I will explain what I mean by the three triggers of contingency to the teacher) beyond the lessons I observed?

APPENDIX B: GDE ethics clearance

GAUTENG PROVINCE

Department: Education

REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	28 October 2019
Validity of Research Approval:	10 February 2020 – 30 September 2020 2019/306
Name of Researcher:	Moodliar J
Address of Researcher:	
Telephone Number:	073 784 0841
Email address:	moodliarjulian@gmail.com
Research Topic:	Expert and Novice Senior Phase Teachers' Responses to Triggers of Contingency in the Teaching of Algebra.
Type of qualification	Masters' in Education
Number and type of schools:	Two Secondary Schools
District/s/HO	Johannesburg East and Johannesburg North

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

30/10/2019

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management

7th Floor, 17 Simmonds Street, Johannesburg, 2001

Tel: (011) 355 0488

Email: Faith.Tshabalala@gauteng.gov.za

Website: www.education.gpg.gov.za

1. Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.
2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.
3. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.
4. A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
5. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
6. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
7. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.
8. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
9. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
10. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
11. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
12. On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.
13. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.
14. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 30|10|2019

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX C: University of the Witwatersrand ethics clearance

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE

PROTOCOL NUMBER: 2019ECE050M

PROJECT TITLE: Expert and Novice Senior Phase Teachers' Responses to Triggers of Contingency in the Teaching of Algebra.

INVESTIGATOR: **Julian Moodliar**

SCHOOL/DEPARTMENT OF INVESTIGATOR: WITS SCHOOL OF EDUCATION

DATE CONSIDERED: 10 February 2020

DECISION OF THE COMMITTEE: Approved unconditionally

EXPIRY DATE: Date of submission of the project report

ISSUE DATE OF CERTIFICATE: 17 February 2020

CHAIRPERSON: (Dr. Paul Goldschagg)

cc: Supervisor: Dr. Lawan Abdulhamid

APPENDIX D: Participants information letters and consent forms

Information sheet: Teacher

Dear Teacher

My name is Julian Moodliar and I am a Part-time Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on exploring how novice and expert teachers respond to triggers of contingency that arise during the teaching and learning of algebra in the Senior Phase. This will give insight into teachers' decision making 'in-the-moment' in response to unexpected events that unfold during mathematics lessons.

My research involves using video to observe three sequential lessons to understand how you respond to triggers of contingency that arise during your algebra lessons. I will also be interviewing you as an individual and audiotaping our conversations to gain insight into your reflections and decision making when you taught the videotaped lessons.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data which emanates from this study.

All research data will be destroyed within 3-5 years after completion of this study.

This will be an interesting learning experience for both of us as co-participants in this study. Your participation is voluntary, so you are free to withdraw your participation at any time without any prejudice and/or penalty. Kindly note that there are no financial rewards for your participation in this study.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Thank you very much for your assistance.

Yours sincerely,

JM

Researcher's name: Julian Moodliar
Researcher's email: moodliarjulian@gmail.com or julian.moodliar@students.wits.ac.za
Researcher's contact number: 073 784 0841

Supervisor's name: Dr Lawan Abdulhamid
Supervisor's email: Lawan.Abdulhamid@wits.ac.za
Supervisor's telephone number: 011 717 3468

Teacher's Consent Form:

Kindly complete the consent form and return the reply slip below indicating your willingness to be interviewed for my research project called:

Expert and Novice Senior Phase Teachers' responses to Triggers of Contingency in the Teaching of Algebra

I, ____

Give/do not give (please delete as appropriate)

Permission to be observed in class

I agree to be observed in class. YES/NO

Permission to be videotaped

I agree to be videotaped during the lesson observation YES/NO

I know that the video-recordings will be used for this project only YES/NO

Permission to be interviewed

I agree to be interviewed. YES/NO

Permission to be audiotaped

I agree to be audiotaped during the interview YES/NO

I know that the audio-recordings will be used for this project only YES/NO

[ ] I know that I don't have to answer all the questions and that I may withdraw from the study at any time without prejudice and/or penalty.

[ ] I am aware that the researcher will keep all information confidential in all academic writing.

[ ] I am aware that the audio and video-recordings will be destroyed within 3—5 years after completion of the project.

Teacher's Signature: ______________________ Date: _______________

Information sheet: Learner

Dear learner

My name is Julian Moodliar and I am a Part-time Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on exploring how novice and experienced teachers respond to unexpected events that arise during the teaching and learning of algebra. This will give insight into teachers' decision making 'in-the-moment' in response to unexpected events that unfold during mathematics lessons.

I will observe and record video of your teacher as s/he teaches you mathematics. I want you to know that I will be recording using a video camera and I need you to agree that it is alright to record video of your teacher's teaching of mathematics while you are in the class. I also want you to know that you may appear in the video recording.

Remember, this is not a test. It is not for marks and it is not compulsory, which means that you don't have to appear in the video. Also, if you decide halfway through that you prefer not to appear in my video recording, this is completely your choice and I will position the camera in a way that will not capture you.

I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept secret in all my writing about the research. Also, all video recorded will be stored safely and destroyed after 3-5 years of completion of this research.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you very much for your assistance.

Yours sincerely,

JM

Researcher's name: Julian Moodliar
Researcher's email: moodliarjulian@gmail.com or julian.moodliar@students.wits.ac.za
Researcher's contact number: 073 784 0841

Supervisor's name: Dr Lawan Abdulhamid
Supervisor's email: Lawan.Abdulhamid@wits.ac.za
Supervisor's telephone number: 011 717 3468

Consent form: Learner

Learner
I am happy to be observed and appear in the mathematics lesson video YES/NO
I am happy for the extract from these videos to be viewed by the researchers for research purpose YES/NO
Learner name: ____ Learner signature: ______________________________________ Date: ____

Information sheet: Parent/Guardian

Dear Parent/Guardian

My name is Julian Moodliar and I am a Part-time Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on exploring how novice and expert teachers respond to unexpected events that arise during the teaching and learning of algebra. This will give insight into teachers' decision making 'in-the-moment' in response to unexpected events that unfold during mathematics lessons.

My research involves observing and videotaping mathematics lessons at the school where your child is attending with my focus being on the mathematics teacher. I was wondering whether you would mind if I do the observation and videotaping in your child's class while he/she is present. My study is not focusing on the learners in the classroom however since learners will be interacting with the teacher in the classroom, they will be part of the video-recordings.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed within 3-5 years after completion of the project.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

JM

Researcher's name: Julian Moodliar
Researcher's email: moodliarjulian@gmail.com or julian.moodliar@students.wits.ac.za
Researcher's contact number: 073 784 0841

Supervisor's name: Dr Lawan Abdulhamid
Supervisor's email: Lawan.Abdulhamid@wits.ac.za
Supervisor's telephone number: 011 717 3468

Parent/Guardian consent form

Parent/Guardian
I consent / do not consent* for my child to be observed and appear in the mathematics lesson video
I consent / do not consent* for the extract from these videos to be viewed by the researcher and for research purpose
School name:____ Parent name:____ Child name:_______________________________________ Parent Signature:____________________________________ Date:____

* Please delete as appropriate