

# CONTEXTUALIZED RISK MITIGATION BASED ON GEOLOGICAL PROXIES IN ALLUVIAL DIAMOND MINING USING GEOSTATISTICAL TECHNIQUES

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## DECLARATION

I declare that this thesis is my own, unaided work. It is being submitted for the Degree of Doctor of Philosophy to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.

A handwritten signature in blue ink, appearing to read "Jacob", is written over a horizontal black line.

March 2016

$$P(X = l) = q^v \frac{\Gamma(v+l) p^l}{\Gamma(v) l!}$$

$$L(X = l) = d_l$$

$$l = 0, 1, 2, 3, \dots \text{ and } p, q, v \geq 1$$

$$v = 0, 1, 2, 3, \dots \text{ and } p, q, \lambda \geq 1$$



Image adapted from <http://ferdi-rizkiyanto.blogspot.com>

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## ABSTRACT

### Background

Quantifying risk in the absence of hard data presents a significant challenge. Onshore mining of the diamondiferous linear beach deposit along the south western coast of Namibia has been ongoing for more than 80 years. A historical delineated campaign from the 1930s to 1960s used coast perpendicular trenches spaced 500 m apart, comprising a total of 26 000 individual samples, to identify 6 onshore raised beaches. These linear beaches extend offshore and are successfully mined in water depths deeper than 30 m. There is, however, a roughly 4 km wide submerged coast parallel strip adjacent to the mostly mined out onshore beaches for which no real hard data is available at present. The submerged beaches within the 4 km coast parallel strip hold great potential for being highly diamondiferous. To date hard data is not yet available to quantify or validate this potential. The question is how to obtain sufficient hard data within the techno economic constraints to enable a resource with an acceptable level of confidence to be developed. The work presented in this thesis illustrates how virtual orebodies (VOBs) are created based on geological proxies in order to have a basis to assess and rank different sampling and drilling strategies.

### Overview of 4 papers

*Paper I* demonstrates the challenge of obtaining a realistic variogram that can be used in variogram-based geostatistical simulations. Simulated annealing is used to unfold the coastline and improve the detectable variography for a number of the beaches. *Paper II* shows how expert opinion interpretation is used to supplement sparse data that is utilised to create an indicator simulation to study the presence and absence of diamondiferous gravel. When only the sparse data is used the resultant simulation is unsuitable as a VOB upon which drilling strategies can be assessed. *Paper III* outlines how expert opinion hand sketches are used to create a VOB. The composite probability map based on geological proxies is adjusted using a grade profile based on adjacent onshore data before it is seeded with stones and used as a VOB for strategy testing. *Paper IV* illustrates how the Nachman model based on a Negative Binomial Distribution (NBD) is used to predict a minimum background grade by considering only the zero proportions ( $Z_p$ ) of the grade data.

## Conclusions and future work

In the realm of creating spatial simulations that can serve as VOBs it is very difficult to attempt to quantify uncertainty when no hard data is available. In the absence of hard data, geological proxies and expert opinion are the only inputs that can be used to create VOBs. Subsequently these VOBs are used as a base to be analysed in order to evaluate and rank different sampling and drilling strategies based on techno economic constraints. VOBs must be updated and reviewed as hard data becomes available after which sampling strategies should be reassessed. During early stage exploration projects the  $Z_p$  of sample results can be used to predict a minimum background grade and rank different targets for further sampling and valuation. The research highlights the possibility that multi point statistics (MPS) can be used. Higher order MPS should be further investigated as an additional method for creating VOBs upon which sampling strategies can be assessed.

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Jurgen for being so different to me and loving me so much

slice of wind

in the most vibrant moment of a raging sandstorm  
the cutting wind scorching all surrounds  
turns its head and looks at a framed instant  
of calmness within itself

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# 1. INTRODUCTION

In general, an increase in available information leads to better decision making. On the other hand, with less data, poorly informed decisions can have devastating consequences. When decisions have to be made there is a fine line as to how one should or could go about generating information that can inform a decision without derailing a process or introducing undue bias when hard data is not available. As part of the decision making process some quantification of the risk involved is needed. A conventional approach would be to run a series of simulations in an attempt to quantify risk and inform decision making. The question, however, is how to quantify risk in the absence of hard data when constrained by economic considerations.

Diamonds do not have a unit price like US\$/ounce for gold or US\$/metric tonne for aluminum. The value of a parcel of diamonds depends on the colour, cut, clarity and carat weight of individual stones. This makes it particularly difficult to sample and estimate a diamond mineral resource. Not only is sampling for discrete particles required but enough of these discrete particles are required to assess their size frequency distribution and assortment characteristics in order to do diamond valuation and obtain a revenue estimate. Diamonds are therefore not seen as a true commodity. A comparison of concentration and homogeneity (King, *et al.*, 1982, Oosterveld, 2003) shows that alluvial diamonds have by far the lowest concentration and lowest homogeneity of all commodities (Figure 1). Alluvial diamond resource estimation is thus challenging on several fronts.

The delivery of a mineral resource and reserve starts off with sample optimization based on an understanding of the geological emplacement model (Figure 2). During sample optimization cognizance must be taken of the desired confidence level. The purpose may be to obtain initial scout or exploration samples only. Alternatively the objective of the sampling could be to generate a resource estimate at inferred, indicated or measured level of confidence based on the SAMREC code (Rupprecht, 2015) to be mined on a specific support size. Cost effectiveness and the integrity of sample process and results are critical aspects underpinning any resource estimate. The decision of how to sample spatially is based on analyzing representative simulations. Wide spaced exploration sampling gives an opportunity to decide whether to walk away or continue with closer spaced 2<sup>nd</sup> phase sampling. The stringent process of data validation and management is always required when samples are gathered. The geological

understanding, volume representation, grade and density estimation are all parameters used for the classification of model. A reasonability test: the reasonable prospect of eventual economic extraction (RPEEE) is applied to assess what portion of the resource model can be declared as a mineral resource. The resource model is used to inform short and long term strategic decisions forming the “Life of Mine” plan. Once economic and other factors are applied to the resource the mineral resource is converted to a mineral reserve which represents the practical obtainable expected financial return from the mineral resource.

In the mining industry there is always time pressure on delineating and generating mineral resources that can be used to inform the “Life of Mine” plan. The critical time pressure drives assessing sampling strategies in terms of efficiency and the quality of information generated. This in turn will inform decision making affecting the whole business.

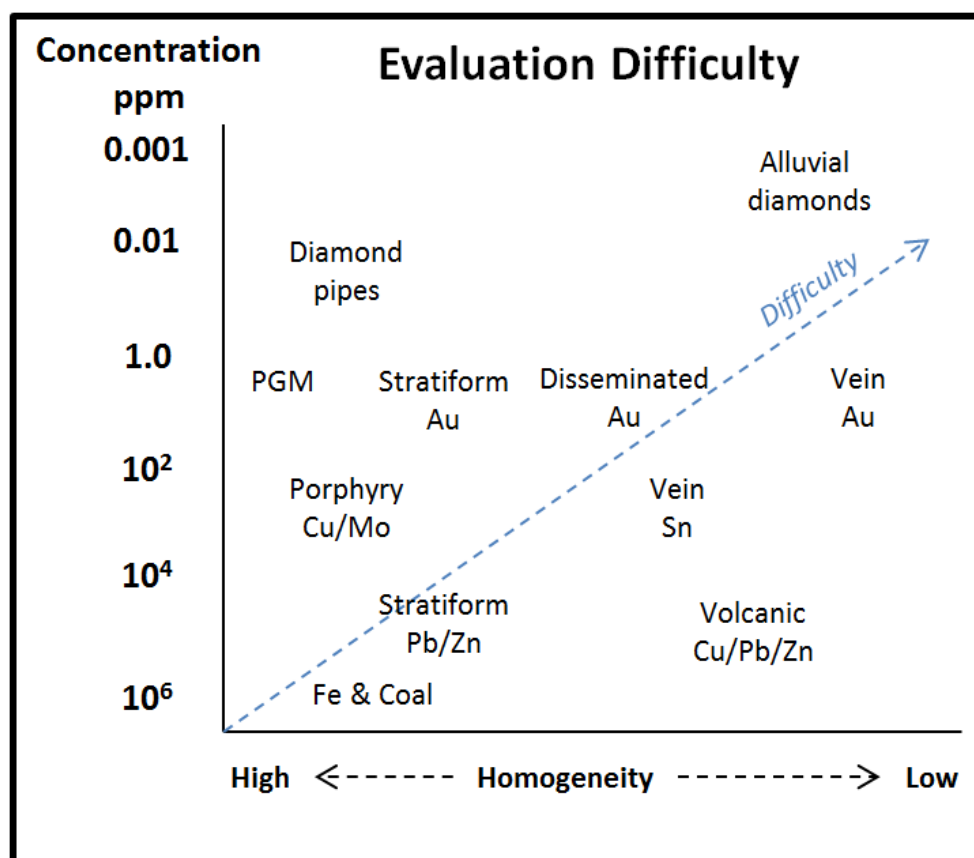


Figure 1 A comparison of evaluation difficulties (after King, et al., 1982 modified by Oosterveld, 2003)

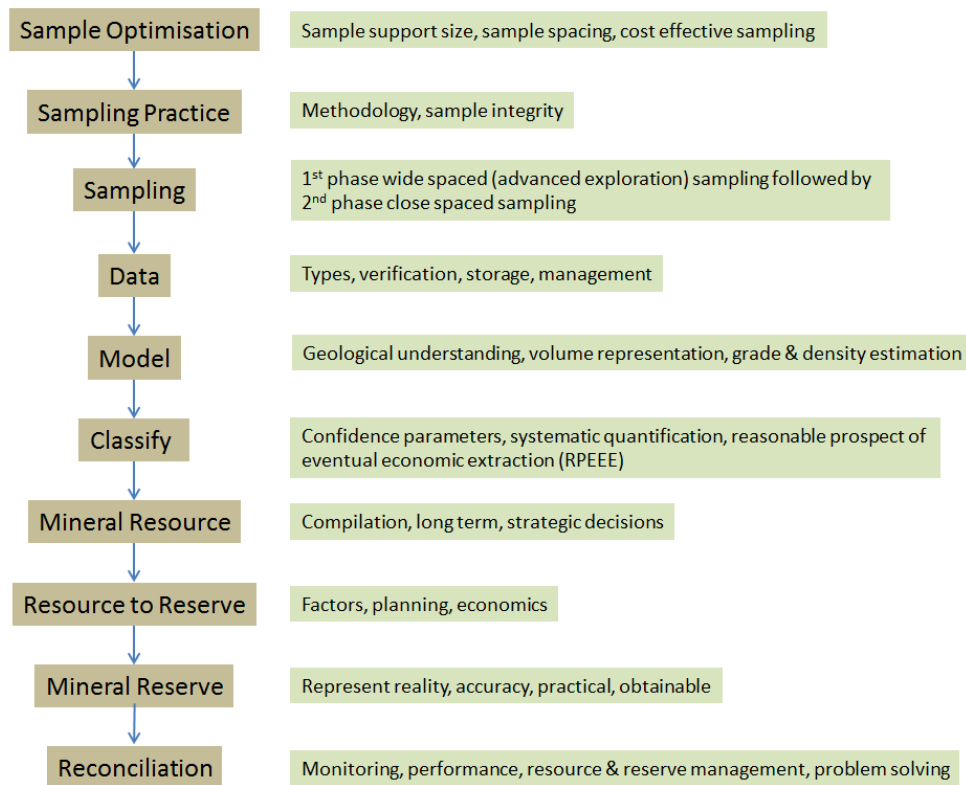


Figure 2 A generalized overview of resource and reserve cycle

In alluvial diamond mining the cost and technical constraints of sampling are very high. These constraints present the unique situation that the initial sampling attempt must be the best possible option to ensure that optimal data is generated upon which further decisions can be based. The decisions taken will only be as good as the samples collected hence the first sampling campaign is the most important. Great emphasis should thus be put on sampling in such a way that adequate and representative information is obtained at minimum cost without compromising sample integrity. In a geospatial context simulations for typical marine deposits are variogram based. The work presented here illustrates how simulation scenarios are handled when one has to overcome the void left when conventional variograms or MPS based simulations do not yield satisfactory results.

Four papers researching various aspects of creating virtual orebodies (VOBs), informing sampling strategies and assessing early stage exploration risk indicators are presented in this thesis. *Papers I, III*

and IV focus on the stones per square meter variable and *Paper II* on the indicator variable for presence or absence of diamondiferous marine gravel.

- The first paper (I) documents how simulated annealing is used to improve detectable spatial variography for the stones per square meter variable by “unfolding” the geology along the coastline where spatial continuity is governed by sea-level stands when considering the stones per square meter variable.
- The second paper (II) illustrates how expert opinion, hand sketch interpretation is used to create an indicator simulation VOB of diamondiferous gravel occurrence. The indicator, gravel present or absent, is the simulated variable presented in this paper.
- The third paper (III) deals with creating a VOB of diamond occurrence based on an expert opinion hand sketch for the stones per square metre variable. .
- The fourth paper (IV) researches the appropriateness of applying the Negative Binomial Distribution (NBD) Nachman model for stones per square metre variable as an early stage exploration risk assessment tool.

An overview mind map (Figure 3) shows of the work presented in this thesis from variogram improvement, geological and grade simulation to determining minimum grades in exploration targets. Chapter 2 provides the context within which research is conducted with associated background information, a description of the problem statement and the knowledge gaps identified. Chapter 3 gives an overview of literature that was considered during the research journey and contextualizes shortcomings of existing techniques in solving problems specific to the study area. Chapters 4 to 7 represent published papers I, II, III and IV, documenting the methodologies, case studies and implementation of the solutions to the overall challenge of risk mitigation in the shallow-water submarine resource development context. Chapter 8 concludes and summarizes the achievements of the research and documents future work planned to continue the journey of discovery and knowledge gathering.

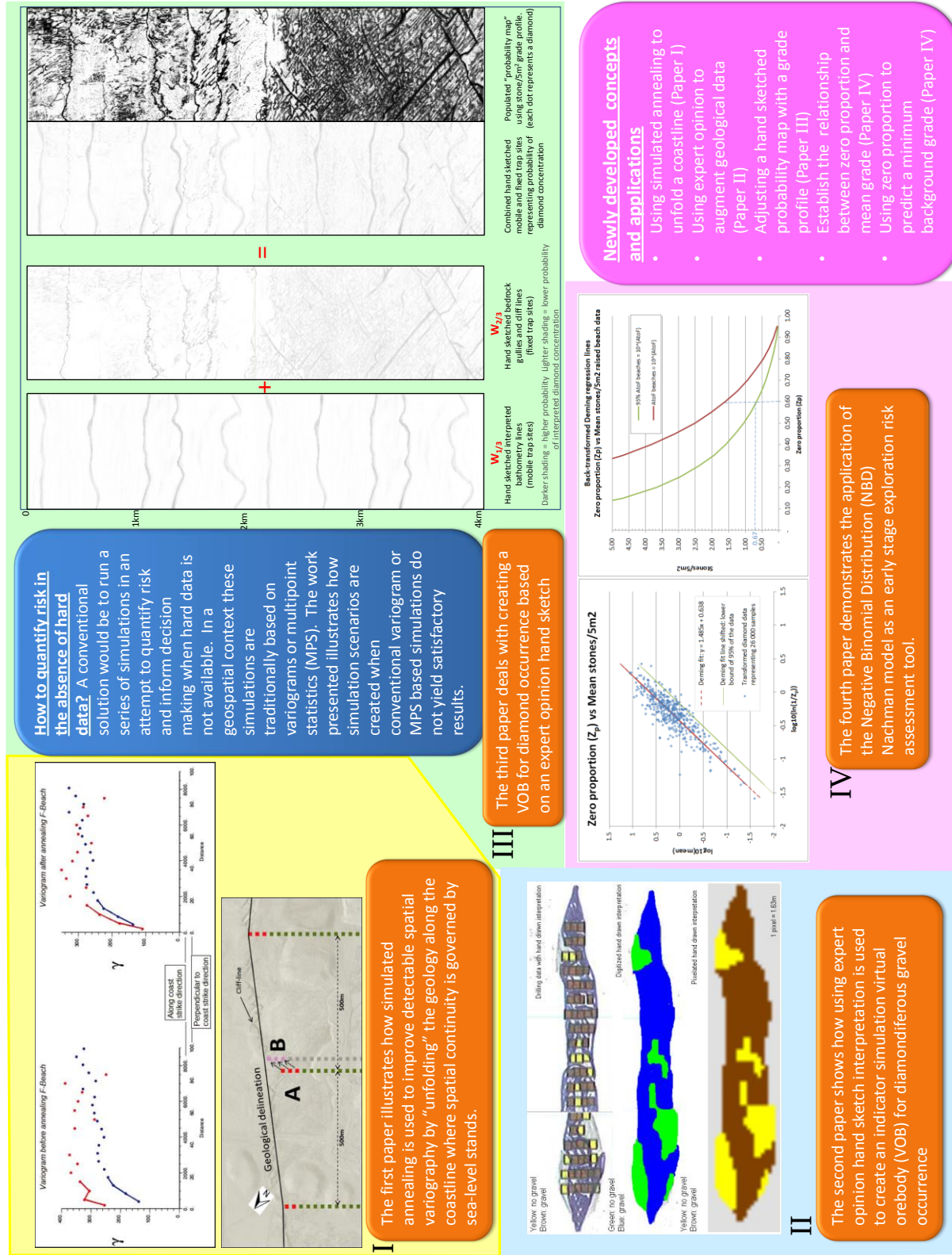


Figure 3 Overview mind map of the research

## 1.1 Overview of published papers

All four papers carry an element of constructing the risk quantification tools when only limited or no hard data is available. Innovative solutions are required to overcome challenges related to this type of problem without losing sight of the objective, i.e. to find a solution with an appropriate expression of the associated risk.

### *Paper I*

#### *Improved Variography using Simulated Annealing to Adjust Sample Locations to Align with Diamondiferous Linear Beach Structures*

Chris Prins and Jana Jacob (2014, SAIMM)

*Paper I* shows how the difficulty of not being able to obtain variography from diamond grade data along the coast when there are strong geological controls, that do indicate structure, can be overcome. Kleingeld (1987) and Caers (1996b) identified the original problem of an absence of variography in diamond sample data along the coast. I provided geological context for *Paper I* and postulated that, by aligning the higher grade samples obtained from beach crests and cliff-lines, useable variograms may materialize. Chris Prins developed a novel way of setting up an optimization system where the minimization is done through a simulated annealing approach with discrete solutions that adjusts the position of the data within a distance constraint and, in so doing, effectively unfolds the cliff line and/or aligns the beach crests. This technique worked well in some areas but in other cases did not result in a useable variogram for geological reasons. I analysed the outcome of the simulated annealing process, assessed whether the resultant variograms made geological sense and tested outlier sensitivity affecting the objective function. The knowledge gained and resultant variogram models from this research assisted in the decision making process regarding grade sample spacing along the coast.

### *Paper II*

#### *Determination of Sampling Configurations for Nearshore Diamondiferous Gravel Occurrence using Geostatistical Methods*

Jana Jacob, Chris Prins and Andre Oelofsen (2013, SAIMM)

*Paper II* details the research on how limited data can be used in conjunction with expert opinion to improve the input parameters required to generate an indicator simulation to model the presence or absence of diamondiferous marine gravel. A simulation based solely on the limited hard data did not yield satisfactory results and was inconsistent with the expected geological texture. Only once expert opinion was incorporated as part of the parameter determination, the resulting simulation was deemed reasonable. I provided the geological framework, interpreted the short scale geological continuity, tested alternative subsets of input data, interpreted resultant variogram parameters and co-developed the simulations with Chris Prins. Andre Oelofsen provided the technical aspects and the constraints of the drilling platform. I validated and analysed the resultant simulation and established the drill configuration that is currently being used at the operation.

### *Paper III*

#### *Construction of Expert Opinion Based Virtual Ore Bodies for a Diamondiferous Linear Beach Deposit*

Jana Jacob and Chris Prins (2013, SAIMM)

*Paper III* discusses the methodology for creating a VOB for diamond occurrence based on expert opinion in the absence of hard data. A composite grey scale probability map, composed by experts, is adjusted with a grade profile from proxy data to form a single realization simulation on which different sample configurations and support sizes can be tested. I hypothesized the construction of the different components required as inputs for the composite sketch, prepared the proxy data and consulted experts with collective experience of more than 50 years on the deposit regarding the acceptability of the sketch as a model of the texture of the spatial diamond distribution on the seabed. Chris Prins statistically aligned the probability map with the grade profile and created the spatial simulation based on the sketch that I prepared. I ran multitudes of sampling and estimation scenarios to test sample size and spacing strategies and validated the simulated and estimated outcomes through statistical comparison with known sample and production data. I assessed the multitudes of solutions (testing 13 500 combinations in total), provided a spectrum of results of which the correlation between actual and estimate, techno-economic assessment of n (number of samples), assessment of outliers and appropriateness of estimation methodology were combined into a final sample strategy based on a risk ranking scenario. I designed the risk ranking method and as a result of this research a sample strategy could be proposed which is currently implemented in practice at the operation.

## *Paper IV*

### *Using the Proportion of Barren Samples as a Proxy for Minimum Grade in a Diamondiferous Linear Beach Deposit – an Application of the Nachman Model*

Jana Jacob (2016, SAIMM)

*Paper IV* documents the application of the NBD Nachman model as an early stage exploration risk assessment tool. Although there has been a lot of focus on the statistical distributions of diamonds in the past (Sichel, 1973, Kleingeld, 1987, Oosterveld *et al.*, 1987, Kleingeld *et al.*, 1996, Caers, 1996a and others), the relationship between the zero proportion ( $Z_p$ ) and mean has not been expressed before. I researched various alternatives and identified the NBD based Nachman model as applied in a botanical context as an appropriate model to use for diamond data analysis and application. Following an initial assessment it became apparent that the Nachman model can provide a framework within which diamond sample results can be analysed. This paper shows a model of  $Z_p$  predict a minimum background grade during the early stages of an exploration project.

## 1.2 “New” concepts and applications developed as solutions

The following new concepts and applications were researched, developed and implemented:

- Using simulated annealing (with a discrete solution) to unfold a coastline (*Paper I*)

The use of a mesh to unfold complex geology (Thibert *et al.*, 2005) has been utilized before but the available data in this study is too sparse to use this technique. A hypothesized minimization problem with a simulated annealing solution is used to align geological features (beach crests and cliff lines - which are known to contain higher diamond concentrations) to define the objective function.

- Using expert opinion to in-fill geological data (*Paper II*)

This paper illustrates how limited hard data, that did not yield a satisfactory simulation, was complimented by expert opinion generated data. The resultant simulation, based on a combination of hard data and expert opinion generated data, yielded far superior results (Appendix 1).

- Adjusting a hand sketched probability map with a grade profile using proxy data (*Paper III*)

Building upon the previously proposed approach of creating probability maps using expert opinion hand sketches (Prins, 2011), this paper introduces the concept of adjusting the hand sketched probability map using a grade profile.

- Expressing a generalized relationship between zero proportion and mean grade for use in the ranking of exploration targets (*Paper IV*)

In depth work has been undertaken on the statistical distributions of the Namibian diamond placer (Sichel, 1973, Kleingeld, 1987, Oosterveld *et al.*, 1987, Kleingeld *et al.*, 1996, Caers, 1996a and others). This paper is the first to express the strong relationship between the  $Z_p$  and mean of these diamond distributions and illustrate how the relationship can be used to rank exploration targets.

- Using  $Z_p$  to predict a minimum background grade (*Paper IV*)

After establishing the existence of a strong relationship between the  $Z_p$  and mean this paper also shows how  $Z_p$  can be used to predict a minimum background grade during the early exploration phase of a project.

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## **2. BACKGROUND AND PROBLEM STATEMENT**

Mining Area No. 1 (MA1) comprising of diamondiferous linear beaches has been the mainstay of Consolidated Diamond Mines and later Namdeb Diamond Corporation's diamond production for more than 80 years (Figure 4). Today, the vast majority of the onshore beaches have been mined out. Thus far more than 62 million carats have been produced from the gravel beaches since their discovery in 1928. MA1 still has significant remaining potential in areas that are currently submerged under water. Beach accretion is a natural process resulting from an above unity ratio of sediment supply/beach erosion, which in effect builds the coastline out into the sea. During 2010, the concept of planned, controlled beach accretion was initiated, where, up until that point, accretion was a consequence of the stripping and dredging process during which overburden sand is removed. The diamond resource that has been made accessible and mined in recent years, due to the non-deliberate beach accretion process, is approximately 2.4 million carats.

The current life of mine plan encompasses the transfer of enough sand and processed gravel into the sea to accrete the coastline by approximately 700m. This 700m wide strip is referred to as the "Ultra Shallow Water Zone" (USW-A&B) (Figure 4). As beach accretion advances a resource extension program utilizing a small diameter probe drill is conducted. This is followed by a phase of large diameter grade sampling. The current assumption is that the deposits are geologically continuous and grades have thus been extrapolated westwards from the immediately adjacent blocks. This approach to express a potential resource remains a significant risk until geological continuity is proven (McCallum, 2011). A sonic drilling test campaign performed during October/November 2011 revealed excellent results, confirming geological continuity in the most recently accreted areas (Jacob, 2011). Furthermore, the deployment of the Probe Drill Platform walking stage, which is able to walk out 300m from the beach, in the 2<sup>nd</sup> quarter of 2012 confirmed gravel in the initial 150m portion that was targeted.

### **2.1 Geological setting of diamondiferous linear beaches**

The linear beach deposit in MA1 is one of the world's greatest diamond-bearing marine deposits. Discovered in 1928, these fossil gravel deposits extend continuously for about 100 km along the coastline. The beaches are developed at different elevations and occur as high as 30 m above sea-level and as low as 25 m below sea-level. The present day gravel beaches along the Black Sea within the

Bolshoi Sochi region are, perhaps, good examples of what the MA1 beaches would have looked like thousands of years ago (Spaggiari, *et al.*, 2006).

The width of the marine platform on which the MA1 diamond deposit rests varies from 3000 m at the Orange River to narrower than 500 m in the north near Chameis Bay (Figure 4). The marine abrasive platform is cut by several major river channels which reflect low stands of sea-level prior to the deposition of the diamondiferous beach sediments. A total of six raised beaches above present sea-level have been grouped into younger (lower) terraces and older (upper) terraces (Figure 5). The beaches of the upper terrace all possess a warm-water fossil fauna *Donax rogersi*. The younger terraces are characterized by modern cold water fauna. Emplacement of the younger beaches occurred subsequent to down-warping which lowered the elevation of the upper terraces from south to north.

The marine shelves (platforms) have been cut into the predominantly siliciclastic rocks of the Oranjemund Formation of the Gariep Belt. The Oranjemund Formation outcrops along a narrow coastal belt from south of the Orange River mouth to Chameis Bay. A fundamental parameter controlling the distribution of alluvial diamonds in the mining area is the marine erosion pattern on the bedrock surface (Jacob, *et al.*, 2006).

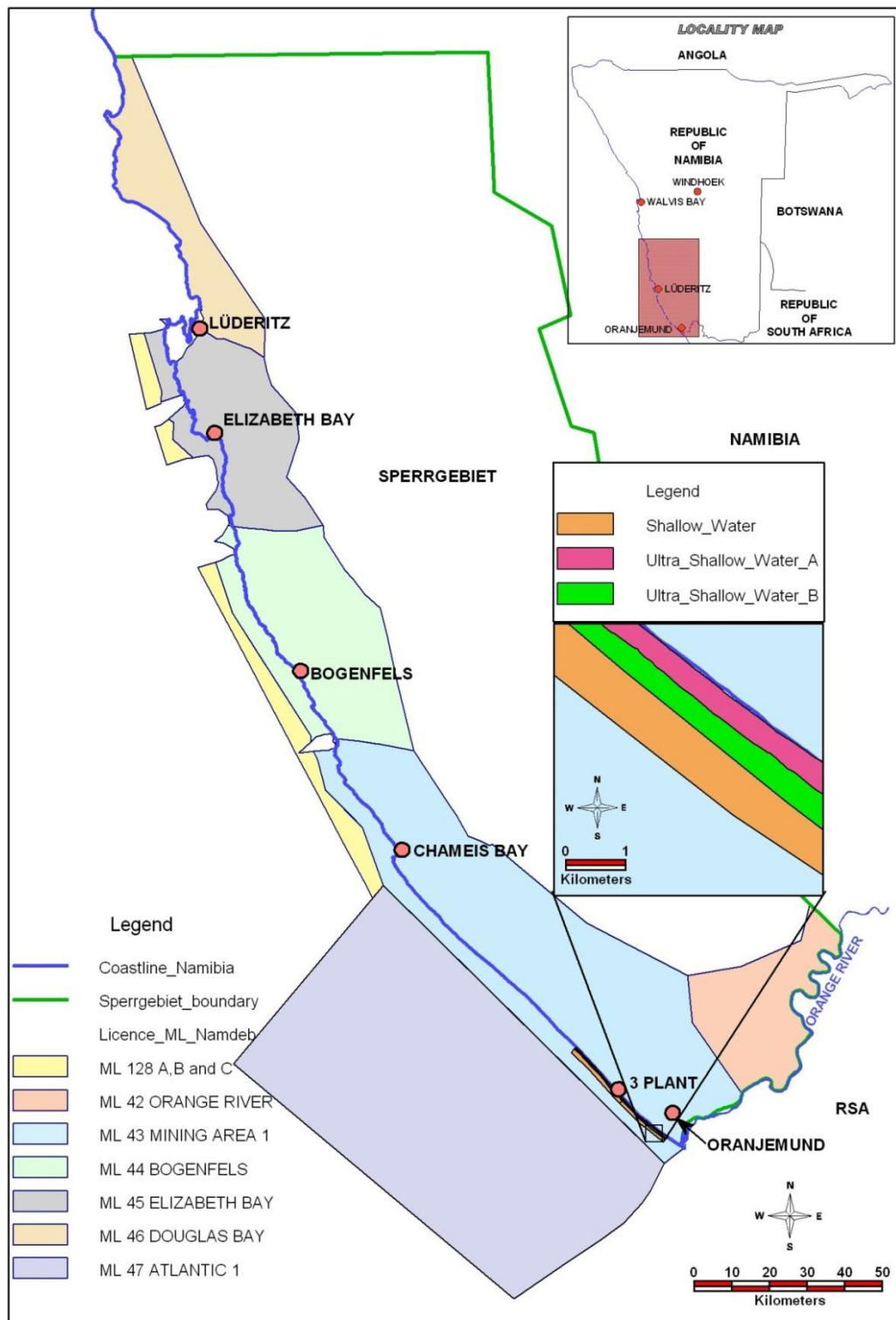


Figure 4 Location map showing the location of ML43 (Mining Area No. 1) that forms the basis of this study

Six major wave-cut shelves all above present day sea-level, backed for most of their length by an impressive sea cliff, have been identified in MA1. The older bedrock shelves have been preserved owing to a cover of diamondiferous conglomerate that has in turn undergone subsequent erosion. The four older shelves are characterized by smooth wave-cut platforms and slope-controlled gullies that have originated through the abrasive effect of boulders agitated in wave surges. On the two younger shelves structural control dominates the orientation of gullies, due to the relative lack of boulders as abrasive agents in the system (Jacob, *et al.*, 2006). The older shelves seem to host higher diamond concentrations than the younger marine shelves, but in both cases diamond concentration remains a function of the amount of reworking.

The geology of the raised beaches, and their counterparts immediately below sea-level, has been well studied over the years. Coast perpendicular trenching has provided mapable cross sections of the beaches and more recently mud and aircore drilling has provided extensive geological logs for interpretation and modelling

The distribution of the diamonds is directly related to the presence or absence of diamond-carrying gravel, the quantity and quality of the trapsites and the degree of reworking. Diamond concentration occurs preferentially below bedrock peaks where maximum turbulence occurs. Thus, in general, grades are higher on the more gullied B and F beaches (Figure 5) and lower along parts of any given beach where the bedrock was less competent and therefore the gullies were less able to form.

While the quality of trap site plays the major role in determining the local grade of the diamond deposit, there are observable trends that may be explained in terms of geological observations or reasoning. Overprinted on local diamond distribution along the entire mega-placer is the trend of decreasing average diamond size from south to north. This is explained in terms of the longshore drift process as smaller diamonds are transported more easily than larger stones that may be preferentially trapped. At any given distance from the river mouth, the oldest F beach contained a larger average size than the other beaches, possibly as a result of that beach having a longer sea-level still-stand that allowed extensive reworking of the diamond supply and fractionation of diamond sizes along the coast. Stone densities are highest towards the north, possibly as a result of the down-warping of the older beaches which reduced the beach width and promoted more reworking of the deposit.

The remaining ore in MA1 is below sea-level. It is known that the sequence of raised beaches extends below the +2 m beach as illustrated in Figure 6 and that the general trends of diamond grade, stone density and average size are reflected in the deposits below sea-level.

The classic complete beach shape is not recognized, however, it is likely that beaches that were emplaced below present sea-level have been truncated, modified or in other ways disturbed by the wave action of a transgressing sea on a rising sea-level following the lowest level reached during the last glacial maximum. Observations of gravel in the vicinity of the 16th century shipwreck discovered in 2008 approximately 15 km north of the Orange River mouth give evidence that the diamondiferous gravel was being moved by wave action as recently as 500 years ago.

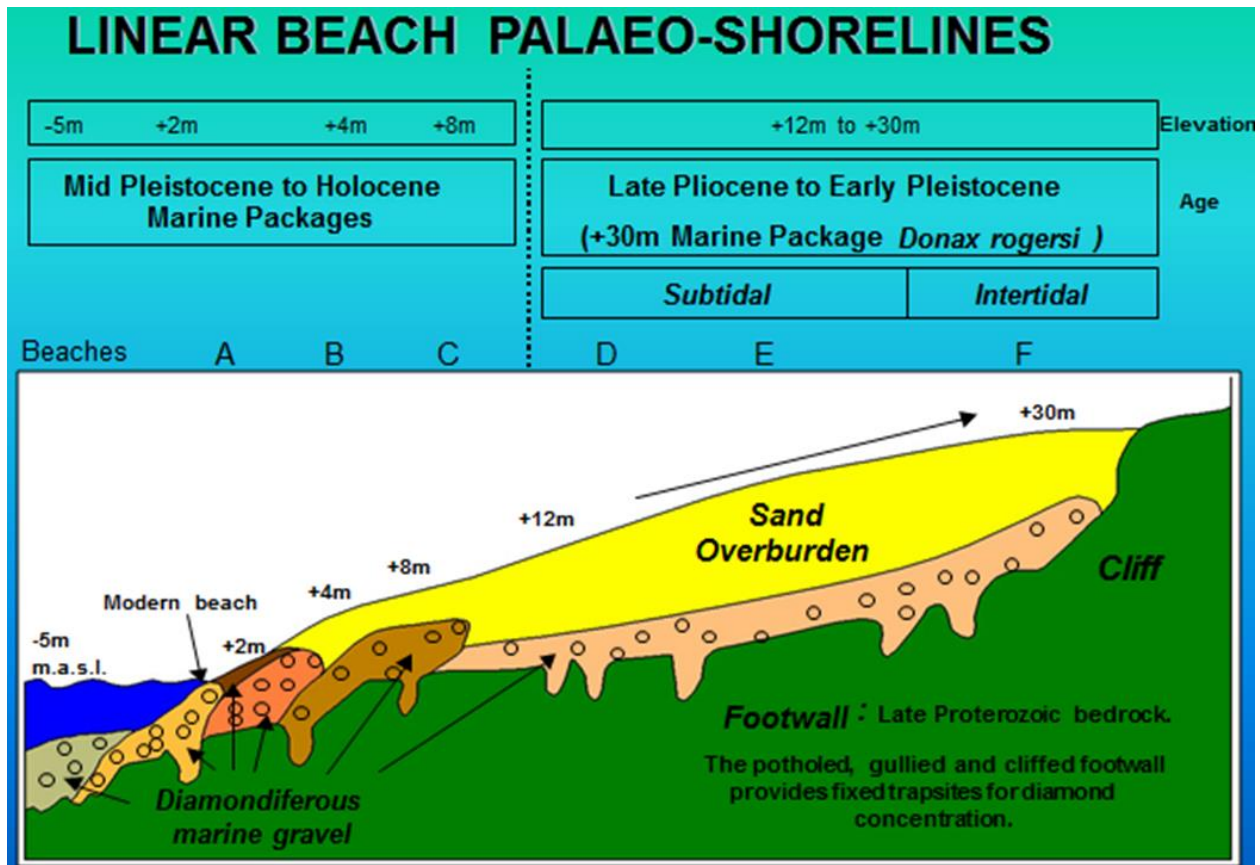


Figure 5 A schematic diagram of raised beaches of MA1

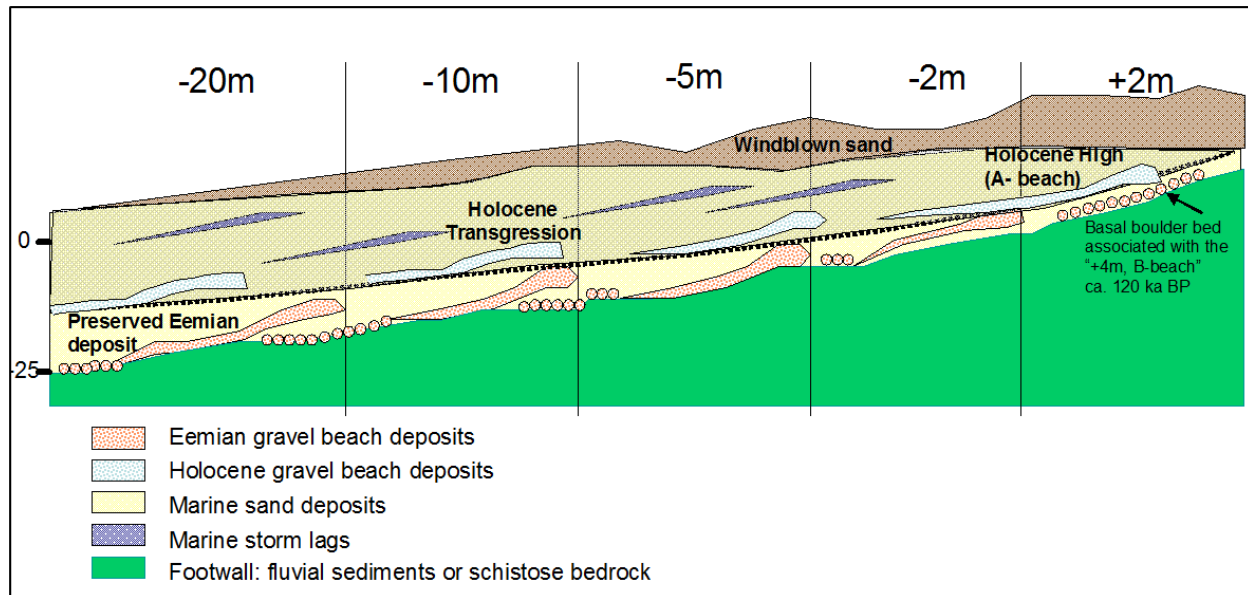


Figure 6 A schematic cross-section of marine beaches extending below current sea-level

Diamondiferous gravel possessing the same sedimentological and underlying bedrock features as the raised beaches have been, and are being, mined to a depth of 25 m below present sea-level. The present mining method of beach accretion, whereby sand overburden is first pushed out to sea and then used to build “sea walls” designed to hold the sea back to allow mining behind them (to below sea-level), does not allow sampling of the ore prior to a decision to mine.

Mega-trenching (10 m wide) and 1 m-wide trenching was used to map the raised beaches on land and to determine the diamond distributions (grade and stone size). However, such trenching was unable to extend into the sea and therefore no direct information on gravel bodies below sea-level is available ahead of stripping, sea-wall construction and accretion of the beach (Figure 7 and 8).

More than 80 years of mining has resulted in a collective knowledge base being built up. Expert production geologists have developed a good “gut-feel” opinion on the quality of diamondiferous gravel and trapsites. These experts were consulted to assess models produced during the course of the research presented in this thesis. In all cases, sampling strategies were only tested once the experts confirmed that the spatial models for diamond distribution were reasonable.

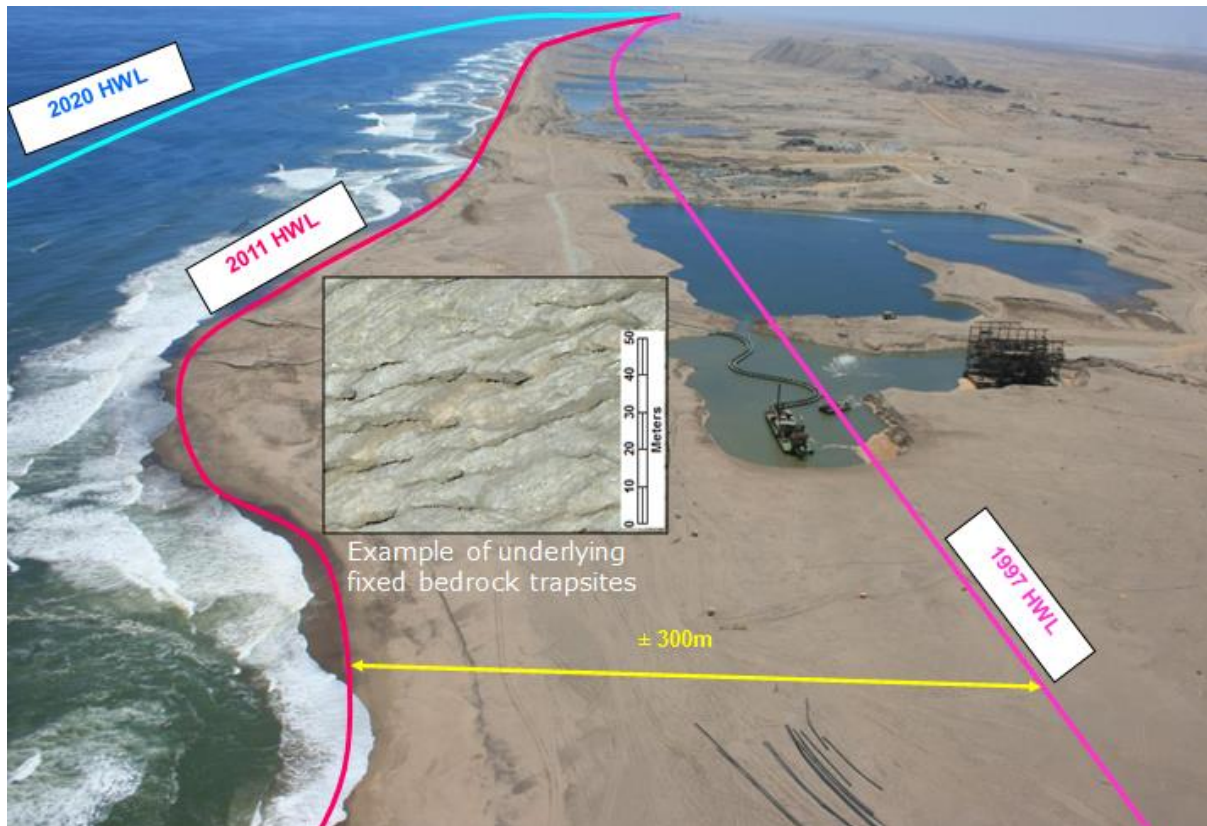


Figure 7 Oblique aerial photograph showing beach accretion and extension of the high water line (HWL)

## 2.1 Problem statement

*“Sampling is defined as the operation of removing a representative part, convenient in size for testing, from a whole of much greater bulk, in such a way that the proportion and distribution of the quality to be measured are, within reasonable limits, the same in both the whole (population) and the part removed (the sample)” (Covacic & Clarke, 1990).*

Bearing the definition of sampling in mind, the technical and economic constraints of delineating and sampling a diamondiferous submerged beach deposit in water depths shallower than 30m are numerous. One of the major challenges is the relentless wave energy since the west coast of Namibia is

deemed to be one of the most vigorous coastlines in the world. The wave energy in the swash zone and aerated water column that overlies up to 25 m of sand cover which in turn overlies a thin diamondiferous gravel layer makes it very difficult to conduct geophysical surveys. In addition, sampling vessels that gather diamond information in deeper water cannot operate in water depths shallower than 30 m. The target area is a 100 km coast parallel strip stretching roughly 4 km into the sea, with roughly half of the target area in water depths shallower than 30 m so all the constraints mentioned above hampers information gathering attempts.



Figure 8 Mining below sea-level in an accreted area

The Stanford Encyclopaedia of Philosophy defines risk as follows:

- 1. risk = an unwanted event which may or may not occur.*
- 2. risk = the cause of an unwanted event which may or may not occur.*
- 3. risk = the probability of an unwanted event which may or may not occur.*
- 4. risk = the statistical expectation value of an unwanted event which may or may not occur.*
- 5. risk = the fact that a decision is made under conditions of known probabilities (“decision under risk” as opposed to “decision under uncertainty”)*

In the context of the research conducted in this thesis the risk of missed opportunities when sampling an orebody can come from non-representative sampling of a specific target. Potentially this can result in erroneously abandoning the target as one is not able to quantify its full potential. At the other end of the scale over sampling results in excessive and unnecessary costs that could have been utilised more appropriately elsewhere.

Risk quantification within the constraints of a marginal mine where profitability margins are very narrow is best attempted by simulating VOBs upon which scenarios can be run to evaluate different delineation and sampling strategy options. In the context of the study area the problem is the lack of hard data which makes it extremely challenging to create VOBs. Analog, geological proxy data and expert opinion are the only additional inputs that can be used to simulate VOBs. Fortunately, there is an awareness of the limitation of this approach based on the collective knowledge of years of production data and the expert geological knowledge related to bedrock topography and the quality of diamondiferous gravel. However, due to the lack of hard data the outcome of this work is only deemed suitable as a ranking tool for risk quantification.

A meticulously recorded dataset comprising 26 000 sample results gathered from the 1930s to the 1960s from the raised beaches adjacent to the target area is available as analog data. The expected observable variography governed by the well understood spatial continuity in the geology, controlled by sea-level stands, is however not evident in this dataset (Chapter 4 – *Paper I*). The creation of variogram

based VOBs for diamond occurrence is thus problematic. A method of determining the expected observable variography was identified as a requirement for which a suitable solution needs to be developed.

To date, the only way in which geological delineation can be conducted successfully within 300m of the shoreline is by using a custom built probe drill platform (PDP). The PDP (Chapter 5 - *Paper II*) is too small to sample for diamond content and is only used to assess the footwall type, footwall depth and nature of the diamondiferous gravel. The strategic sampling question then becomes “How can the PDP be best used for the delineation of gravel occurrence?” The use of analog data from newly accreted beach areas as input for an indicator simulation did not yield a satisfactory output (Appendix 1). A way of creating additional input information is thus required and a topic of research was identified in which available data could be supplemented in order to get scientific alignment between available data, input parameters, the resultant variogram and the indicator simulation texture to represent geological field observations.

To sample and evaluate a 4km wide, 100 km long coast parallel diamondiferous strip for which only very sparse hard data exists presents a unique challenge in terms of collecting sufficient information to create a sampling strategy within economic and practical constraints and thus provide sufficient scope for a risk assessment to be completed. Research that pursues different avenues for constructing a virtual orebody for a highly non-stationary environment is outlined in Chapter 6 (*Paper III*).

One has to consider the high cost of collecting information; the development of a gauge (Chapter 7 - *Paper IV*) that can be used during the early stages of an exploration project would be very useful in giving guidance as to where further exploration effort should go. An aid that facilitates contextualized risk mitigating decisions will greatly reduce costs within a framework of developing a resource at an appropriate level of confidence.

The combined research of the 4 papers has significantly aided in strategic decisions. As data is collected strategies will be revisited and risk mitigating adjustments will be made to ensure that activities remain within an optimal envelope.

### 3. LITERATURE REVIEW

#### 3.1 Virtual Ore Body (VOB) construction methodologies

In general simulation models can be used to represent what natural or man-made systems could possibly be under changing conditions. Manchuk, *et al.* (2015) illustrates how Monte Carlo simulation is used for predicting resource uncertainty during different phases of resource development focusing on exploration for kimberlitic diamond pipe deposits. Simulations pertinent to this research focus on the spatial distribution of variables. Lantuéjoul (2002) gives an overview of practical and theoretical aspects of geostatistical simulations.

Spatial simulations are used to determine the probability distribution of a regionalized variable at a specific location. Two-point variogram based techniques include Turning Bands (Matheron, 1973, Mantoglou, 1987, Tomson, *et al.*, 1989), Sequential Gaussian (Deutsch & Journel, 1992, Daly & Verly, 1994, Goovaerts, 1997, Deutch & Journel, 1998, Journel, 2002 and others) and the Cox Process (Cox, 1955, Kleingeld & Lantuéjoul, 1993, Kleingeld, *et al.*, 1996, Brown, *et al.*, 2004). Multipoint statistics (MPS) include SNESIM (Strebelle, 2002, Remy, *et al.*, 2009 and Liu, 2006), FILTERSIM (Zhang, *et al.*, 2006) (Wu, *et al.*, 2008), IMPALA (Straubhaar, *et al.*, 2011) and HOSIM (Mustapha & Dimitrakopoulos, 2010 & 2011) and a Direct Sampling (DS) algorithm (Mariethoz, *et al.*, 2010, Meerschman, *et al.*, 2013, Mariethoz & Lefebvre, 2014).

For both two-point variogram based and MPS simulations the size of the simulated domain and the number of simulated realizations should be assessed carefully. The integral range (Lantuéjoul, 1991) can be used to assess the required size of the training image required for MPS. The number of realizations should be large enough to stabilize the mean and variance but small enough to allow post processing of the simulations (Remy, *et al.*, 2009). The MPS method is still contentious (Emery & Lantuéjoul, 2014) but considering the visual nature of interpreting MPS it was decided to consider the method in this research.

In cases where limited or no data is available for a specific study the simulation of different scenarios is a good place to start. Journel and Kyriakidis (2004) point out that the mining industry could benefit from the general concept of virtual simulation for decision making, as other fields like engineering have done. The availability of advanced computer hardware and software should make virtual simulation a routine

step in decision making in a range of aspects related to mineral resource quantification and mine planning.

Unfortunately, attempts to create both variogram and MPS based VOBs did not yield satisfactory results during this research study. Although further investigation in using both of these techniques to create useable VOBs is ongoing the only satisfactory VOB created for the purpose of this specific problem was expert opinion based. An overview of variogram and MPS based simulation techniques are given in sections 3.1.1 to 3.1.6.

### **3.1.1 Sequential Gaussian Simulation (SGSim)**

The first step in SGSim is to determine the ccdf as per the conditional cumulative distribution function (Goovaerts, 1997):

$$Prob\{Z(u) \leq z | n(u)\} = Prob\{z(u) \leq z, n(u)\} / Prob\{n(u)\}$$

The SGSim and SISim algorithms (Deutsch & Journel, 1998 and others) recombine the elementary conditional probabilities in a spatially dependent context. The second step involves assessing the possibility of homogeneous subdomains within a large complex domain (Journel, 2002). SGSim (Deutsch & Journel, 1992) involves drawing simulated values one after the other from joint distribution functions. A random simulation path (Daly & Verly, 1994) is used to avoid artifacts. Only the closest data to a previously un-simulated node is retained to inform local conditioning of the unknown node.

The principle of SGSim for unknown nodes is based on conditioning data including both the original data and previously simulated nodes. SGSim uses simple kriging at a node to estimate the posterior mean and variance with random sampling of the posterior distribution (Journel, 1994). When using SGSim the following assumptions are made: firstly that the data (or transformed data) has a normal distribution; secondly, that the data is stationary implying the mean, standard deviation and variogram do not change over the spatial domain; and thirdly, that the function being modelled is a multivariate Gaussian random function (Deutsch & Journel, 1992). The resulting simulations' statistics will correspond to the random function model statistics on a global scale. Sensitivity analysis can be undertaken by, e.g. changing the variogram model (Journel & Kyriakidis, 2004).

Practical implementation and checks during SGSim include: trend analysis, bootstrap grades, checks/adjustment of the normal score values and checks/adjustment of the distribution of simulated grades (Nowak & Verly, 2004).

The elegance of using the Gaussian Random Function (GRF) for probabilistic models for continuous variables is that it is fully characterized by its mean vector and covariance matrix. Furthermore the conditional mean and variance are given by simple kriging (Journel & Huijbregts, 1978, Anderson, 2003). A GRF has the disadvantage that it maximizes disorder (entropy) beyond the input covariance (Journel & Deutch, 1993, Chiles & Delfiner, 1999). This is why Gaussian-based simulation algorithms can't produce an image with definite patterns involving more than two locations at a time. Gaussian simulation that is governed by specific spatial and distribution laws, where median value correlation is maximized and extreme value correlation is less correlated, is known as the destruction effect (Goovaerts, 1997). Gaussian algorithms are less suitable for simulating spatial phenomenon with well correlated extreme values or high and low values that are not correlated in the same way.

The use of a combination of different simulation techniques for different situations gives the best results. A study by de Almeida (2010) utilizes 5 different simulation techniques for categorical variables for lithoclass classification in carbonate reservoirs. Parallelization of classic simulation techniques is a research topic that has started to come to the fore in recent years (Nunes & de Almeida, 2010). A comparison of simulated annealing (discussed in Section 3.1.4) and SGSim of soil Pb content in rice paddy fields (Lin, *et al.*, 2001) revealed that for this specific case SGSim's realizations showed higher local heterogeneity when compared to the simulated annealing realizations.

### **3.1.2 Turning Band Simulations**

The Turning Band approach was originally developed by Matheron to generate realizations for standardized normal distributed data for isotropic models (Matheron, 1973). It has been applied in the mining industry (Journel & Huijbregts, 1978). Mantoglou (1987) developed the Turning Band approach further to accommodate anisotropic models. An advantage of this technique is its computational efficiency, a series of one-dimensional random processes along lines radiating from a specified center coordinate are generated. The subsequent projections are placed as random points in space (Tomson, *et al.*, 1989).

### 3.1.3 Cox Process simulation

The Cox Process (Cox, 1955) based on the doubly stochastic model was introduced in 1955. Kleingeld and Lantuéjoul (1993) applied the Cox Process to diamonds. In their paper the term 'potential' is introduced to describe parts of a deposit that may potentially be richer than others. The Cox Process is a simulation model based on a generalization of the standard Poisson process (Kleingeld, *et al.*, 1996). A numerical value of the potential for diamond occurrence at each point  $x$  is represented by  $z(x)$ . The potential attached to a region  $v$  comprises the sum of all the potentials of all the points within  $v$  (Kleingeld, *et al.*, 1996), given by:

$$Z(v) = \int_v Z(x) dx$$

1

It is assumed that the number of particles is distributed according to a Poisson process with a regionalized intensity  $z$  (Kleingeld, *et al.*, 1996), so that:

- i) The number of particles  $N(v)$  falling in region  $v$  follows a Poisson distribution with a mean  $z(v)$

$$P\{N(v) = n|z\} = e^{-z(v)} \frac{(z(v))^n}{n!}$$

2

- ii) The number of stones falling in disjoint domains are mutually independent

$$P\left\{\prod_{i=1}^k N(v_i) = n_i \middle| z\right\} = \prod_{i=1}^k P\{N(v_i) = n_i | z\}$$

3

Cox Process simulations are particularly useful when dealing with highly skewed discrete distributions (Brown, *et al.*, 2004).

#### **3.1.4 Simulated Annealing**

Simulated Annealing unlike conventional numerical optimization methods explores a function's entire surface and combines both uphill and downhill movement during optimization attempts. It is therefore largely independent of starting values and can escape local optima. In addition, a mathematical model is not a required input for most Simulated Annealing solutions (Goffe, *et al.*, 1994). Simulated Annealing makes use of probabilistic methods and Markov chains providing non-unique practical solutions in cases where the structure of the problem may not be well understood (Bertsimas & Tsitsiklis, 1993). Although Simulated Annealing always provides a solution it may not be optimal, thus it could be used to supply practical solutions for optimization problems that are difficult to solve (Ledesma, *et al.*, 2008).

Caers (1996) used Simulated Annealing to generate conditioning data by extracting the spatial model statistics from a non-conditional simulation model. The extracted spatial model statistics from the non-conditional simulation is then used as a training image for the conditional simulation.

#### **3.1.5 Multipoint Statistics (MPS)**

Entropy beyond the variogram's range cannot be captured by two-point simulation techniques. MPS on the other hand can accommodate this requirement (Journel & Zhang 2006). To move away from an explicit two-point model, a training image (TI) is introduced where MPS are lifted directly from the image (Guardiano & Srivastava, 1993, Strebel, 2002 & Zhang, *et al.*, 2006). As with the two-point approach, results are no better than the multipoint model upon which it is based for both Gaussian based and algorithm models (Remy, *et al.*, 2009). Two-point generated simulation realizations honour both the data and a variogram model whereas multipoint generated simulations honour the data and the structure present in the training image (Remy, *et al.*, 2009).

The original MPS implementation by Guardiano and Srivastava (1993) required that the whole TI had to be rescanned for conditioning data at each simulation node. Single normal equation simulation (SNESIM) (Strebel, 2002) requires that the TI be scanned only once and is, consequently, much faster. All conditional data for a given search template is stored in a hierarchical search tree structure and are therefore available to be retrieved for subsequent simulations.

The construction of a search tree entails a search template (Remy, *et al.*, 2009 and Liu, 2006)  $T_j$  defined by  $J$  vectors  $h_j = 1, \dots, J$  radiating from a central node  $u_0$ . Training patterns:  $pat(u'_0) = \{t(u'_0): t(u'_0 + h_i), i = 1, \dots, J\}$  for any central node  $u'_0$  of the TI are obtained. Firstly, the total number of patterns ( $n$ ) with exactly the same  $J$  data values  $D_j = \{d_j, 1, \dots, D\}$  are delineated. Secondly, for these patterns, the number ( $n_k$ ) which features a specific value  $t'(u'_0) = k, (k = 0, \dots, K - 1)$  at the central location  $t(u'_0)$  is obtained.  $K$  is then the total number of categories; hence SNESIM is used for categorical and not continuous variable simulations. The proportion of the training patterns featuring the central value  $t(u'_0) = K$  are then defined by  $P(t(u'_0) = n_k | D_g) = \frac{n_k}{n}, k = 0, \dots, k - 1$ , (Strebelle, 2002, Remy, *et al.*, 2009).

SNESIM sequentially visits one pixel at a time following a random path to visit all the nodes within the simulated grid. The search tree template is then used to retrieve the conditional data event  $dev(u)$ , defined as  $dev_j(u) = \{z^l(u + h_1), \dots, z^l(u + h_j)\}$ , where  $z^l(u + h_j)$  is an informed nodal value in the  $l$ -th SNESIM realization. The informed node could either be hard data or a previously simulated node.

FILTERSIM (Zhang, *et al.*, 2006, Wu, *et al.*, 2008) differs from SNESIM in that it can handle both categorical and continuous variables. FILTERSIM has a lower random access memory (RAM) demand because it only saves the central location of each training pattern in memory, compared to SNESIM that saves all the training replicates in a search tree. A filter comprises a set of weights associated with a specific data configuration in a specific template size  $J, T_j = \{u_{0i}, h_i, i = 1, \dots, J\}$  (Remy, *et al.*, 2009, Zhang, *et al.*, 2006). Each node  $u_i$  is defined by a relative offset vector  $h_i = (x, y, z)$  from the template centre  $u_0$  associated with a specific filter value of weight  $f_i$ . For a  $J$ -node template its associated filter is  $\{f(h_i) = i = 1, \dots, J\}$  (Zhang, *et al.*, 2006, Wu, *et al.*, 2008).

IMPALA (an improved parallel multipoint algorithm using a list approach) is only suitable for categorical (*not continuous*) variables. The introduction of a list structure (Straubhaar, *et al.*, 2011), replaced the tree structure used in FILTERSIM. A list structure has a number of advantages. Firstly, it allows the use of larger templates because it is less RAM intensive. Secondly, the list structure is parsimonious and hence can include additional information, e.g. non-stationary training images. Thirdly, the list structure allows the part of the algorithm in which the conditional density function is being calculated to be parallelized, leading to improved processing efficiency.

The list structure can deal with non-stationarity by using categorical fields and auxiliary variables (Chugunova & Hu, 2008). It can also make use of the multigrid approach (Tran, 1994, Strebelle, 2002).

Traditional variogram-based geostatistical methods are insufficient when it comes to the reproduction of high order structure as found in geology (Lyster & Deutch, 2008). Pixel-based two-point simulations are good at conditioning data compared to object-based simulations which are good at capturing geological structures (Liu, 2006). MPS combines the strengths of the above mentioned simulation techniques.

Previous pixel based methods that make use of multipoint information rely on conversion considerations which are costly in terms of computer processing time. These methods include Simulated Annealing (Deutsch, 1992), Markov Chain, Monte Carlo (Tjelmeland, 1996) and Gibbs Sampler (Srivastava, 1995).

The concept of indicator simulation further developed with the introduction of higher order MPS (Guardiano & Srivastava, 1993) that make use of training images (TI). This differs from the work described above in that it does not rely on iterative processes. Guardiano and Srivastava's method obtains probabilities from direct scans of the same training image. The method is elegant in its simplicity but still requires extensive computer processing time since a rescan of the full TI is required at each node. Strebelle (2002) built on this work to develop a SNESIM algorithm. The SNESIM algorithm combines the flexibility of easy data conditioning based on pixel-based algorithms and the object based algorithms' ability to reproduce shapes. The efficiency of the SNESIM algorithm stems from the use of multiple grids on different scales and makes use of tree structures that allow conditional probabilities to be retrieved.

The SNESIM algorithm works on the same principal as Guardiano and Srivastava's method in that each simulated node becomes a conditional input data point in the next step of the sequential simulation. The SNESIM algorithm differs in the following 2 ways. Firstly, the dimensions of the TI influences the number of conditional probability density functions (cpdfs) and secondly the cpdf conditional to a specific event can be obtained from a subset of the total TI template.

Steps comprising the SNESIM algorithm (Strebelle, 2002) include:

1. Scanning of the TI(s) in order to construct a search tree.
2. Assigning of the original sample data to the closest grid nodes and defining a random search path to visit all unsampled nodes once only.
3. Retain the conditioning data present within the maximum search template used to construct the search tree at each unsampled location.
4. Obtain a simulated value from the cpdf from the search tree. This value becomes part of the conditioning input data for further simulation.
5. Move to the next node to be simulated on the random path and repeat steps 3 and 4.
6. Continue the process from steps 3 to 5 until all the nodes are simulated.
7. A second realization can be created by repeating steps 2 to 6 using a different random path.

A multiple grid approach can be followed by capturing large scale structures using a relatively small number of grid nodes (Tran, 1994). Nodes simulated using coarse grids that account for large scale structures can then be included as conditioning data for finer grid simulations.

Tree structures are however extremely RAM demanding. The IMPALA list approach (Straubhaar, *et al.*, 2011) reduces the RAM requirements and has a number of advantages. It allows the use of larger templates; the list structure can be extended to include additional information which includes non-stationary training images and finally the list approach allows parallel processing of the conditional probability density function.

Challenges that arise when MPS techniques are used include the choice of statistics that one uses and how to reproduce these complex statistics during simulation (Lyster & Deutch, 2008). Srivastava demonstrated the use of the Gibbs Sampler in geostatistics (Srivastava, 1992). The Gibbs Sampler algorithm utilizes an iterative process using conditional distributions until convergence is reached. A MPS algorithm using a Gibbs Sampler (MPS-GS) as developed by the Centre for Computational Geostatistics (CCG) includes the following steps (Lyster & Deutch, 2008):

1. Use a randomly-populated field on the coarsest grid as a start.
2. At a random location:
  - a. Calculate the cpdf,
  - b. Adjust the cpdf to account for secondary information and global facies proportions,
  - c. Apply noise reduction or cleaning and correct cpdf to sum to 1.0,
  - d. Draw a new facies value from the corrected cpdf, and
  - e. Move to another unsampled location and repeat Step 2.
3. After every location has been visited, check for convergence:
  - a. If there is no convergence yet, repeat Step 2 over all locations, and
  - b. If convergence has been achieved, populate the next-coarsest grid.
4. After the final grid has been simulated write out the results and start the next realization.

The MPS-GS algorithm (Lyster & Deutch, 2008) uses a multipoint event (MPEs) concept for determination of the conditional probabilities as in Step 2a above. MPEs can be expressed as indicators similar to single point facies data.

As mentioned earlier, one of the challenges when dealing with MPS is the selection of an appropriate TI. Theory regarding appropriate checks for the suitability of selected training images is outlined by van den Boogaart (2006).

A direct sampling (DS) algorithm (Mariethoz, *et al.*, 2010 and Meerschman, *et al.*, 2013) is a recently developed technique that scans the TI directly for a given data event. DS uses the distances between given data events and TI patterns which allows it to simulate categorical, continuous and multivariate problems.

The DS algorithm (Meerschman, *et al.*, 2013) entails the following steps:

1. A quantitative sensitivity analysis is conducted on the three parameters balancing simulation quality and CPU time:
  - a. The acceptance threshold ( $t$ ),
  - b. The fraction of the TI to scan ( $f$ ), and
  - c. The number of neighbors ( $n$ ).

2. In conjunction with a visual inspection of the resulting simulations, the speed of the calculation and quality of the pattern reproduction is analyzed.
3. Finally, the impact of post processing on quality improvement of the resulting simulations is assessed.

MPS overlaps with computer graphic texture synthesis techniques in terms of the algorithmic tools used to generate realistic textures (Mariethoz & Lefebvre, 2014). Both MPS and computer graphics make use of TIs. MPS has superior conditioning capabilities compared to texture synthesis technique's superior computational efficiencies. Markov Random Fields (MRF) is the basis for both MPS and example based texture synthesis (Mariethoz & Lefebvre, 2014).

Caers (1996) points out that it is hard to state *a priori* if a random function model will be sufficient for a specific application. In the case of secondary (alluvial) diamond deposits point data is, in general, limited. It is thus best to rely on simple models to describe point processes since they are only inferable *a posteriori* from data that does not capture the full complexity of the deposit.

Validation techniques for geological patterns that have been simulated based on both variogram and MPS techniques include objective measures and higher order spatial cumulants based on third and fourth order spatial templates (De Laco & Maggio, 2011).

### **3.1.6 High-order stochastic simulation**

Following on multipoint simulation approaches, high-order measures of spatial complexity, represented by simulation of high-order spatial cumulants (for continuous variables), are introduced by Mustapha and Dimitrakopoulos (2010, 2011). High-order conditional spatial cumulants are inferred from training images by using Legendre polynomials. One of the advantages of high-order sequential simulation (HOSIM) is that available hard data dominates the simulation process and training images are only used to fill in high-order relations not obtainable from hard data (Mustapha & Dimitrakopoulos, 2010). Furthermore, the HOSIM approach is able to reproduce both lower and higher-order spatial complexity. A technique developed by Perez *et al* verifies high-order consistency of training images by ranking TI based on their relative compatibility to the data and quantifying consistency between the TI and the data in terms of spatial structure (Perez, *et al.*, 2014).

### 3.1.7 Hand Sketching

Expert opinion is used as input “data” in various fields where sparse or no other analog data is available for construction of simulated models. Lele and Das (2000) show how hierarchical modeling is used to combine expert opinion data for inferential purposes. The elicitation of data is much more natural to experts, e.g. in the field of environmental pollutants, compared to the elicitation of prior distributions on the parameters of probability models (Lele & Das, 2000). One method is to incorporate statistical models using stepwise statistical procedures. This application has been applied to fine-scale vegetation mapping (Pearce, *et al.*, 2001). In another example Kuhnert *et al.* (2009) outlines the use of expert opinion for the modeling of ecosystems (Kuhnert, *et al.*, 2009).

The examples referred to above all utilize expert opinions to partially inform statistical distributions or parameter estimation. A technique whereby the expert opinion is captured by hand sketching of the probability of an occurrence being present on a 2 dimensional grey scale has been developed by Prins (2011). Detail regarding this technique is outlined in Chapters 5 (*Paper II*) and 6 (*Paper III*). Enhancements of this technique were required as a result of challenges and shortcoming identified in the research presented in this thesis pertaining specifically to the unique study area.

## 3.2 Overview of statistical distribution work on diamondiferous marine placers

Historical published work on the statistical distribution of the diamonds on the Namibian onshore beaches dates back to the 1970s. A summary of the historical work is presented in *Paper IV* (Chapter 7) in this thesis. This paper primarily revolves around the modelling of stone potential for highly skewed distributions. Sichel (1973) derived a probability function describing the distribution of individual diamonds based on a mixed Poisson distribution. Kleingeld (1987) recognized the importance of regionalized potential and combined the dependent factors that influence diamond concentration. Based on the hypothesis that the number of particles within any region can be described to be the product of the presence of Poison pockets and an intensity of particle distribution within each pocket, distributed randomly, the following function is used:

$$\phi(r) = \int_0^{\infty} P(r|\lambda)f(\lambda)d\lambda$$

with  $P(r|\lambda)$  a Poisson and  $f(\lambda)$  a Gamma distribution function.

Through exhaustive testing Kleingeld (1987) concluded that it is far superior to use the  $Z_p$  and mean to determine the parameters for the Sichel distribution rather than the method of moments. This can only be undertaken with some degree of accuracy when a large number of samples are available.

Matheron (1981a & b) found that the generalized NBD and Sichel distributions did not fit the data of the Namibian coast if the support size is changed and larger areas are considered. Matheron extended the Poisson mixing distribution by developing a four parameter distribution model. Kleingeld researched and concluded that this can only be done when a large number of samples are available. For the research in this thesis a simpler generalized model with fewer parameters would produce more stable results and limit the required number of assumptions since one is dealing with proxy data only.

Oosterveld (1987) attempted to determine the relationship between geology and statistical parameters using the Sichel distribution. It was found that the highly complex geological environment could only be described in very broad terms when using statistical parameters. Large coast parallel blocks up to 5 km long were estimated and subsequently sub-blocked into smaller 500 m blocks with 100 m or 300 m widths. This process is followed in order to overcome the small scale variability characteristic of this deposit.

Sutherland and Dale (1984) used Sichel's mixed Poisson distribution to determine a sample size when an acceptable  $Z_p$  and mean number of stones per sample are selected upfront. Kleingeld (personal communication) points out that geological interpretation is required in designing a spatial sampling campaign and that acceptable  $Z_p$  and stones per sample are unknown upfront. Chapter 7 (*Paper IV*) of this thesis outlines how additional research has led to a novel way of applying  $Z_p$  as a risk mitigation ranking tool when certain distribution assumptions are made.

Caers (1996) uses the Neyman-Scott cluster model where centers of clusters are generated and then stones are spread around the cluster according to a spatial distribution. Caers makes use of the mixed

Poisson process and the Neyman-Scott cluster model to produce different degrees of clustering for diamond deposits. The complex and non-stationary geological entrapment environment of the study area warranted research into the creation of a suitable VOB that can represent diamond distribution on a point support scale. Chapter 6 (*Paper III*) illustrates how such a VOB is created.

Deviating from pure distribution work and looking at sampling theory, Lyman (2011) investigated the relationship between in-situ and particulate heterogeneity. In the case of diamond deposits with a relative uniform stone size, number of stones and thus carats per cubic metre, Lyman used simpler Poisson models to describe this relationship. Research outside the scope of this thesis to assess the wider applicability of nomograph type solutions for assessing sampling strategies is ongoing.

The literature study gave insight into multiple approaches and ideas. Solutions for problems encountered in the study area as described in Section 2.2 could not be found. Research presented in this thesis has been undertaken following the lack of finding study area specific solutions in existing literature.

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## 4. PUBLISHED PAPER I

*The Southern African Institute of Mining and Metallurgy  
The Danie Krige Geostatistical Conference  
C.F. Prins and J. Jacob*

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# Improved variography using simulated annealing to adjust sample locations to align with diamondiferous linear beach structures

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Diamondiferous submerged linear beaches along the western coast of Namibia are earmarked for sampling and future mining and various sampling configurations are tested for optimality through the use of spatial simulations. For the creation of these virtual orebodies, basic statistics and variograms are needed, but in this specific instance no data exists from which the necessary parameters can be determined. The best that can be done is to use proxy data from onshore beaches, with adaptations where needed. The reworking of raised beaches during periods of rising and falling sea-levels destroyed the internal beach structures in some cases, and it is very difficult to determine the variograms. A method is proposed whereby simulated annealing is used to adjust the sample locations to align the data pertaining to beach crests or cliff lines, thus improving the variogram structure along the shoreline in the direction of the highest geological continuity.

**Keywords:** submerged beaches, simulated annealing, variography.

## Introduction and geological framework

The diamondiferous linear beach deposits of Namibia's Sperrgebiet (Figure 1) form part of the world's greatest marine diamond-bearing deposit. The onshore beach components, which are now mostly mined out, extend northwards along the coastline from the Orange River mouth for about 100 km. Six raised beaches, from 30 m above present sea-level, are developed at different elevations and extend to the current day sea-level. Strong geological continuity governed by sea-level stands in a coast-parallel direction is evident for all six beaches. Continuity in the direction normal to the coast line is not so strong. The geological delineation of the six raised beaches is based on 1 m trenches comprising 1x5 m sample paddocks and mega-trenches comprising 10x50 m sample paddocks (Figure 2).

Through a process of beach accretion (overburden sand resulting from the mining process used to extend the beach seawards) the linear beach deposits are currently mined to 25 m below sea-level. With the onshore *in-situ* deposits largely depleted, new resources below sea-level are targeted for exploration, and a sample optimization study is required.

Since there is very little sample data available for the submerged beaches, the onshore data is the only information available that can be used as analogue with which to design optimized sampling programmes to be applied in the exploration of the submerged linear beaches. This is deemed acceptable based on the results obtained from the -25 m mining results and geological understanding of the depositional environment. Over time, as sample information from the submerged beaches becomes available, the sampling programmes will be reviewed.

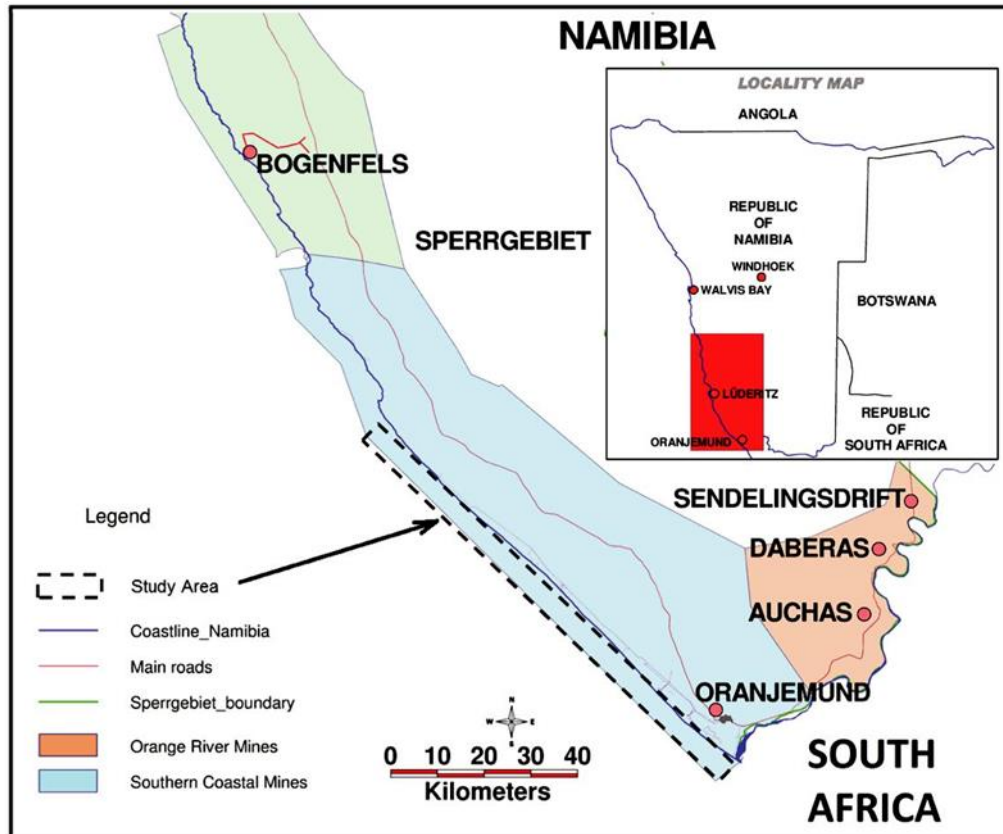


Figure 1 – Location map showing the study area

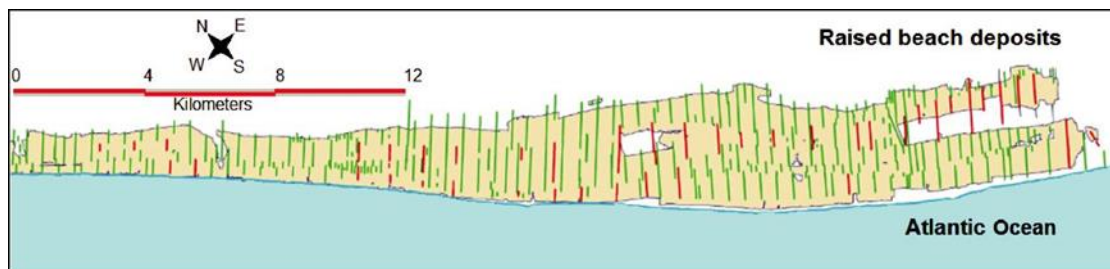


Figure 2 – An extract of the data showing delineation trenches sampled in 1x5 m (green) and 10x50 m (red) paddocks orientated perpendicular to the coastline

## Data

Traditional trench sampling campaigns used to delineate raised beach deposits are not feasible in a submerged beach environment. Current mining at depths of 25 m below sea-level has, however, yielded enough evidence to show that data collected from onshore beach sampling campaigns can be used as proxy information (with some modifications) in the sample optimization studies.

Data from the 1 m wide trenches provides information regarding the nature and elevation of the footwall, diamond content, and gravel occurrence on a 1x5 m scale. Cliff-lines (which are not always visible due to sand cover) and internal beach morphology are governed by sea-level stands. The cliff-lines are not equally well developed for all sea-level stands and internal beach structures are sometimes destroyed during multiple cycles of sea-level transgression and regression.

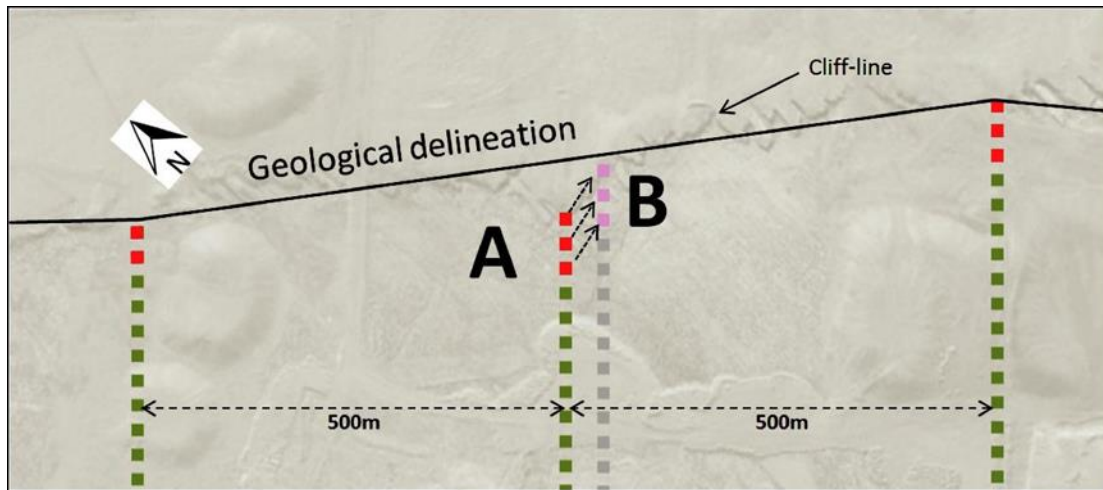
Based on the simultaneous interpretation of a series of adjacent trenches, the six beaches were delineated and split into geological zones, each with its own statistical characteristics which are analysed separately.

The outlines of the geological zonation are straight lines which have, due to the 500 m spacing of the data, inherent inaccuracies. Previous work (Jacob, *et al.*, 2006) showed that cliff lines are generally associated with zones of higher diamond concentration, and this feature of the depositional environment can be used to adjust data to align with the cliff lines. Adjustment of the data locations to align with the cliff lines (or beach crests) would result in the optimum variogram. This is equivalent to calculations along an unfolded structure, aligned to the underlying geological structures of the beach. The complexity of this method is underpinned by adjusting the data in such a way as to obtain the most robust structured variogram (clearly defined variogram type, range, and expected low nugget effect) through alignment of the higher grades in each sample line. To achieve this, a squared difference, weighted by the distance between the sample lines is proposed while sample locations are adjusted through the use of simulated annealing.

### Problem statement

In order to create virtual orebodies for use in sample optimization studies, spatial simulations, based on the characteristics of the different beach deposits with associated variograms, are used. High ratios of anisotropy in the direction parallel to the current shoreline exist, but reworking of raised beaches during periods of rising and falling sea-levels in some cases destroy the internal beach structures. The expected strong spatial correlation is thus not clearly visible in the data. This could also be a function of the data spacing, which is wide relative to the expected ranges for variograms of diamondiferous beach deposits, or else the jagged nature of the coastline. This makes the determination of variogram parameters challenging.

A method is proposed whereby simulated annealing is used to adjust the sample locations to align the data pertaining to beach crests or cliff lines, thus improving the variogram structure along the shoreline in the direction of the highest geological continuity. Figure 3 shows schematic trench positions on a Google Earth backdrop. The middle trench needs to be aligned to the cliff line.



*Figure 3 – A well-developed cliff line with superimposed 1 m wide trench results. Schematically, higher grade samples (red) associated with the cliff line are shown in line A with the pink and grey sample line (B) representing the two-step data shift needed to align the higher grades with the geological delineation*

### Methodology : Adjusting sample locations using simulated annealing

The sample data is migrated onto a 1\_5 m grid with the aim of aligning the highest data values per trench with the adjacent trenches subject to constraints and minimizing an objective function. The adjustment process is illustrated in Figure 4.

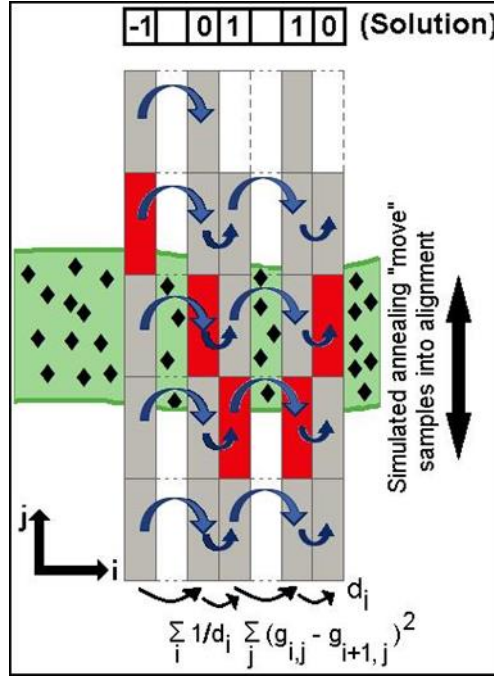


Figure 4 – The high-grade blocks (red) must be aligned to the underlying geological structure (green) through sample location adjustments

By iterating the sample data locations (constrained to a -2 to +2 block movement in a north/south direction), the weighted grade difference between the lines can be minimized, implying alignment of the data to the underlying geological structure. The solution to the problem illustrated in Figure 4 shows how the high-grade (red) blocks need to be adjusted in order for the grade data to honour the underlying geological structure.

Simulated annealing is a perturbing search method, whereby a change in the system causes it to deviate slightly, which is used for finding a best outcome within a solution space. In this study an objective function is minimized using simulated annealing by changing one factor at a time. The deviations are therefore evaluated and solutions accepted that reduces the objective function. This is done repeatedly in a structured manner until a best solution is found.

During the process, over and above accepting the solutions that improve the objective function, rejected solutions are randomly tested against the Metropolis criterion to occasionally accept less favourable solutions. This avoids the solution becoming entrapped in a local, sub-optimal solution. The probability of acceptance of a worse solution is determined as follows:

$$P(\text{accept worse solution}) = e^{-(obj_{new} - obj_{old})/T} > \text{uniform}(0,1)$$

where a temperature value (T) and the new and old objective function values are used. As the number of iterations increases, the temperature is reduced according to an annealing schedule, and combined with a smaller difference between the two objective functions, it becomes less likely that worse solutions will be accepted. The annealing process eventually converges to a 'best' solution.

The simulated annealing algorithm as implemented by Goffe *et al.* (1994) was modified so that each random adjustment can contain only integer values.

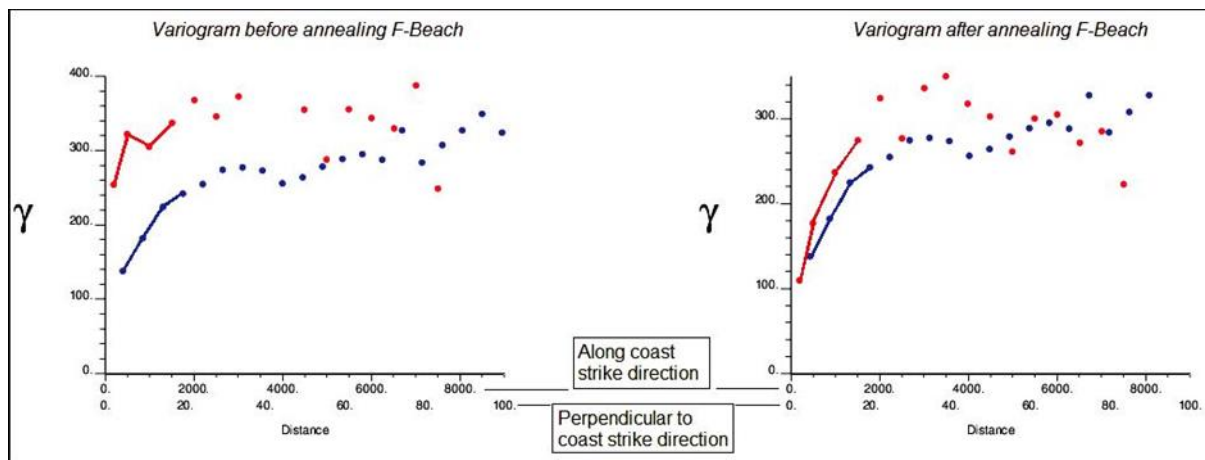
To take cognisance of the distance ( $d_i$ ) between sample lines, the objective function includes a weighting based on the inverse distance between two adjacent sample lines. The objective function to be minimized is:

$$\min \left[ \sum_{i=1}^{\# \text{ samples lines}} 1/d_i \sum_{j=1}^{\# \text{ samples per line}} (\text{stone grade}_{i,j} - \text{stone grade}_{i+1,j})^2 \right]$$

The movement for each sample line is initially set to zero and, after running the annealing process inspection of the results shows that the most logical solutions are obtained when the movements are not allowed to deviate too much from zero. These slight movements of the sample locations were thought to be realistic, as the geological boundaries are not expected to be radically misaligned with the underlying beach crests/cliff lines. The constraints of the annealing process thus allow only relatively small adjustments to the sample locations and were restricted to not more than 20 m in the cross-beach direction.

### Case study

The F beach is the oldest, highest grade, and most eastward of the six raised beaches. The samples covering the beach are spaced at 500 m in the north-south and 5 m in the east-west directions. With the samples closely spaced across the beaches, the east-west direction should be more representative in terms of the modelling of the nugget effect of the variogram. Figure 5 shows the variograms before and after the data locations were adjusted.



*Figure 5 – Variograms before and after simulated annealing, highlighting the modelling of the nugget effect. Strong anisotropy along (red) and across (blue) the coast strike directions is evident*

The variograms are plotted using two different scales on the x-axes, as the anisotropy along and across the beach cannot otherwise be shown in a single graph due to scale. It can clearly be seen that after the sample locations were adjusted, the nugget effect of the along-beach variogram is in agreement with the nugget effect determined from the across-beach data.

Similar results (although slightly less impressive than from beach F) were obtained from some beaches, but not in all cases. The reason for this is thought to be the reworking of beaches during the multiple transgression and regression cycles of sea-level stands, which destroyed so much of the beach structures that it is very difficult to reconstruct the linearity of the beach based on the sample grade only. Examples of before and after data adjustments within sample lines are shown in Figure 6.

|               |    |     |    |     |    |    |    |  |  |
|---------------|----|-----|----|-----|----|----|----|--|--|
| <b>BEFORE</b> |    |     |    |     |    |    |    |  |  |
| 1             | 3  | 2   | 42 | 116 | 22 | 11 | 9  |  |  |
| 68            | 28 | 20  | 7  | 31  | 11 | 17 | 17 |  |  |
| 1             | 2  | 1   | 9  | 6   | 0  | 18 | 16 |  |  |
| <b>AFTER</b>  |    |     |    |     |    |    |    |  |  |
| 1             | 3  | 2   | 42 | 116 | 22 | 11 | 9  |  |  |
| 6             | 50 | 116 | 68 | 28  | 20 | 7  | 31 |  |  |
| 1             | 2  | 1   | 9  | 6   | 0  | 18 | 16 |  |  |

|               |     |    |    |    |     |     |    |    |   |
|---------------|-----|----|----|----|-----|-----|----|----|---|
| <b>BEFORE</b> |     |    |    |    |     |     |    |    |   |
| 7             | 7   | 5  | 12 | 7  | 16  | 25  | 22 | 6  | 5 |
| 0             | 0   | 1  | 16 | 16 | 2   | 1   | 0  | 1  | 0 |
| 34            | 62  | 39 | 97 | 15 | 311 | 166 | 62 | 12 | 8 |
| 22            | 213 | 65 | 21 | 15 | 4   | 6   | 4  | 4  | 3 |
| <b>AFTER</b>  |     |    |    |    |     |     |    |    |   |
| 7             | 7   | 5  | 12 | 7  | 16  | 25  | 22 | 6  | 5 |
| 0             | 0   | 0  | 0  | 1  | 16  | 16  | 2  | 1  | 0 |
| 34            | 62  | 39 | 97 | 15 | 311 | 166 | 62 | 12 | 8 |
| 13            | 8   | 3  | 5  | 22 | 213 | 65  | 21 | 15 | 4 |

Figure 6 – Examples showing the alignment of stone count per sample data

## Results and conclusion

Through inspection of the example in Figure 4, an alternative solution of (-2, -1, 0, 0, -1) can be proposed. Similarly, the results after annealing the sample locations of the beaches must be contextualized within the framework of the more complex data configurations and the possibility that the annealing probably does not converge to a unique solution. The multiple outcomes of annealing runs are therefore analysed and a range of variogram parameters determined, leading to an array of simulations for use in the sample optimization studies. In the inspection of the output to determine if cliff lines or beach crests can be identified from the data, the minimized objective function and variogram parameters are used to establish the acceptability of the results. The outcomes thus far have been promising.

The research is ongoing and is focusing on eliminating outliers from the data, testing the effect of different constraints on sample movements, as well as testing the effect of using multiple random starting points in the annealing.

## Acknowledgements

The authors wish to thank Anglo American and Namdeb for permission to publish the results. The research forms part of a PhD study by Jana Jacob.

We also acknowledge Danie Krige for inspiring an appreciation for geostatistics by conveying his knowledge and sharing his passion for the subject over many years.

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## References

- Goffe, W.L., Ferrier, G.D., and Rogers, J. 1994. Global optimization of statistical functions with simulated annealing. *Journal of Econometrics*, vol. 60. pp. 65-99.
- Jacob, J., Ward, J.D., Bluck, B.J., Scholz, R.A., and Frimmel, H. E. 2006. Some observations on diamondiferous bedrock gully trapsites on late Cainzoic, marine-cut platforms of the Sperrgebiet, Namibia. *Ore Geology Reviews*, vol. 28, no. 4. pp. 493–506.
- Wilde, J. (jochen.wild@dlr.de) 2001. Simann.f - bug in description of judge's function. E-mail to: [bgoffe@whale.st.usm.edu](mailto:bgoffe@whale.st.usm.edu) <http://www.netlib.no/netlib/opt/simann.f> [Accessed 19 November 2013].
- Steve, L., Leuangthong, O., and Deutsch, C.V. 2004. Simulated annealing post processing for multiple point statistical reproduction. *Sixth annual report for CCG members*, Centre for Computational Geostatistics, University of Alberta. pp. 109-1–109-15.

## **5. PUBLISHED PAPER II**

### **DETERMINATION OF SAMPLING CONFIGURATION FOR NEAR SHORE DIAMONDIFEROUS GRAVEL OCCURRENCE USING GEOSTATISTICAL METHODS**

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**\*Namdeb Diamond Corporation, <sup>†</sup>Anglo American plc**

#### **Abstract**

Diamondiferous linear beaches in Mining Area No. 1 have been the mainstay of Namdeb diamond production for more than 80 years. Most of the onshore beaches have been mined out but in recent years mining has been extended offshore into the surf zone through a process of beach accretion. A total of 61.6 million carats has been produced from the gravel beaches since their discovery in 1928 and Mining Area No. 1 is considered to have great remaining potential from areas currently submerged under water.

To date, the surf zone has remained largely unexplored due to the consistently strong winds, currents and large waves which make access to this area extremely difficult. This paper describes the processes used in developing a practical methodology for exploration of the surf zone in a domain extending approximately 22 kilometres (km) long in a north-west southeast direction and 0.3 km wide in a perpendicular direction adjacent to the current Mining Area No 1.

The vigorous surf zone poses multiple technical challenges in terms of obtaining geological and diamond information. In order to access the area for sampling a jack-up walking Probe Drill Platform (PDP), with a 5-inch diameter reverse-circulation drill has been developed to carry out exploration drilling in the dynamic surf zone. The hydraulically driven platform has eight legs each of which is 18m long. Four of the legs are in fixed positions on the fore and aft sides of the platform. The other four moveable legs are fitted to sliding frames attached to the port and starboard sides of the platform. The sets of fixed and moveable legs can be raised and lowered by hydraulically powered jacking stations. By alternatively lifting and lowering the sets of fixed and moveable legs in conjunction with the frames moving back and forth horizontally, a walking action is performed by the PDP. The platform can walk at a speed of 10-15m/hour, depending on weather and footwall conditions.

Optimisation of sampling for diamondiferous gravel occurrence in Mining Area No.1 was undertaken through creation of a virtual ore body on which different sample configurations were tested. The input data for the construction of a virtual ore body comprises a set of drilling data, collected from recently accreted areas directly adjacent to the 22 km x 0.3 km target domain. The input drilling data covers only 34 % of the domain, and for the purpose of this study, a single realization is deemed to be sufficient.

The texture obtained from only using the drilling data to construct a simulation did not make geological sense; hence it was necessary to make use of analog data in order to improve the

simulation. The first analog data used are the gully patterns found in the meta-sedimentary Precambrian bedrock footwall. Gullies are, in general, gravel filled and it is fair to assume that gully patterns form a subset of the total gravel occurrence. Total gravel occurrence includes marine terraces (governed by sea-level stands) occurring above bedrock peaks, together with gravel within gullies below bedrock peaks. The second analog “data” introduced is through the use of “Expert interpretation” as outlined in (Prins, C. F., 2011). The indicator drilling data is interpreted by expert opinion and the 2 dimensional result is hand sketched, digitized and then pixelated. The pixelated data set is then used as input for variogram calculation.

This study cannot provide a single definite optimisation result as the nature of the data does not permit this. The use of different validation approaches (conditioning data, where available; expert interpretation and gully pattern data), however, can give a very good indication of how to balance sampling effort with de-risking aspects related to geological continuity.

Work undertaken by sampling the simulation will give an assessment of the relative probabilities to determine gravel occurrence in the study area. This study showed that a 50m x 400m cross configuration will be a good initial sample spacing for highlighting areas where gravel may be absent, and further infill drilling may be required.

The relative efficiency of a 50m x 400m cross off-set sampling configuration has been demonstrated using a trumpet curve *versus* sampling effort when using kriging as the estimation method.

## **Introduction**

The world class diamond placer along the southern Namibian coast (Figure 1) is characterized by raised and submerged gravel beach terraces and bedrock controlled trapsites. As the diamond resources on land are being depleted, the focus is shifting towards gravel deposits under the sea in the near shore surf zone. To date, the surf zone has remained largely unexplored due to the consistently strong winds, currents and large waves which make access to this area extremely difficult. This paper describes the processes used in developing a practical methodology for exploration of the surf zone in a domain extending approximately 22 kilometres (km) long in a north-west southeast direction and 0.3 km wide in a perpendicular direction adjacent to the current Mining Area No 1.

To optimize sampling for diamondiferous gravel occurrence in the surf zone areas, a virtual ore body is required on which different sample configurations can be tested. The input data for the construction of a virtual ore body comprises a set of drilling data, collected from recently accreted inshore areas directly adjacent to the 22 km x 0.3 km domain. The input drilling data covers only 34 % of the domain, and for the purpose of this study, a single realization is deemed to be sufficient.

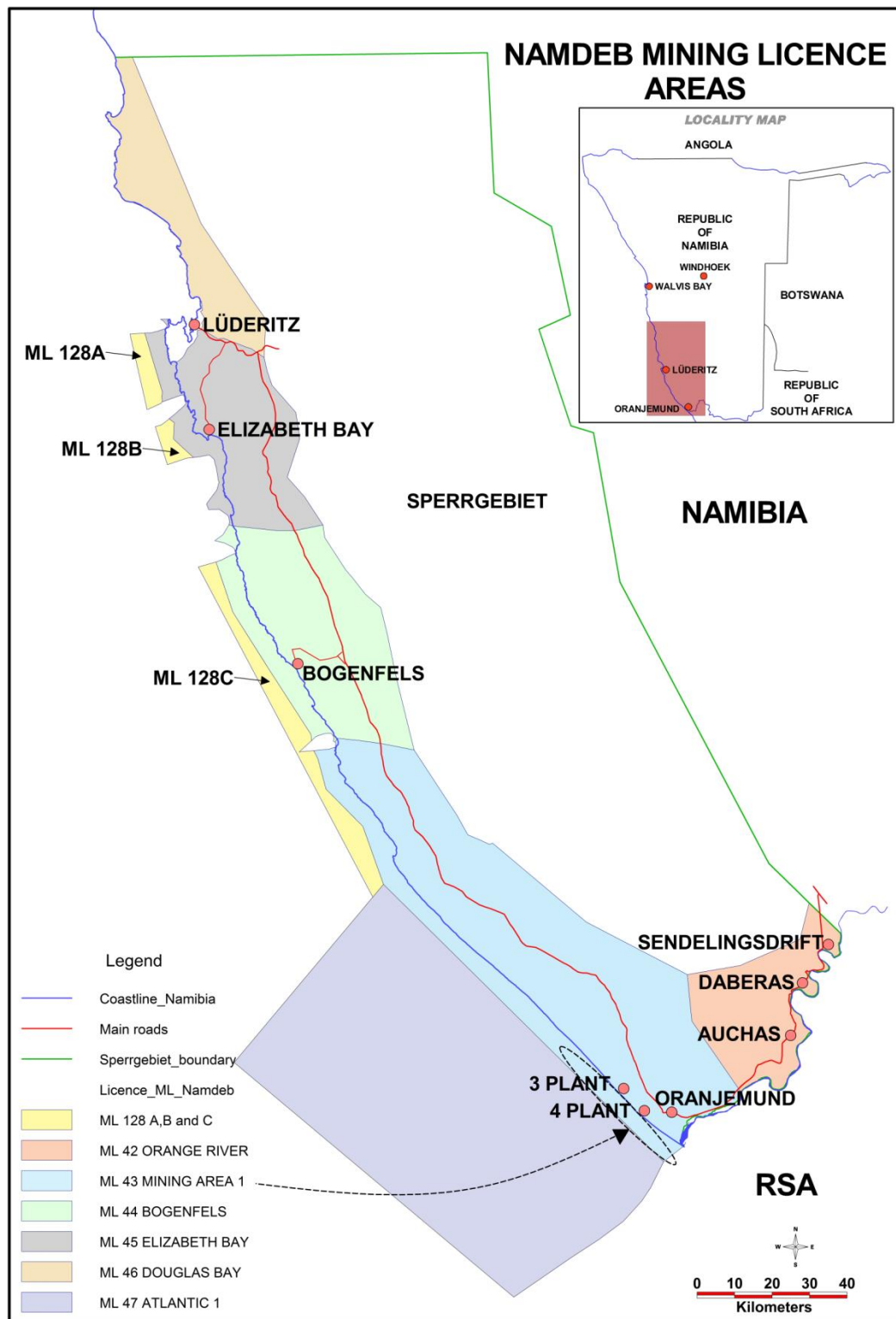


Figure 1 Location map showing the portion of ML43 (Mining Area No. 1) which is the study area

## Methodology

In order to access the area to allow delineation and sampling of diamondiferous gravels covered by the sea, a jack-up walking Probe Drill Platform (PDP), with a 5-inch diameter reverse-circulation drill has been developed to carry out exploration drilling in the dynamic surf zone (Figures 2 and 3). The platform weighs 195 tons, its deck measures 12m x 12m and it is designed to withstand 144km/h winds combined with 5m swell. It can operate in up to 7m water depth in calm seas, and drill through more than 30m of sediment with its 5-inch diameter reverse circulation drill. A 2 m air gap must be maintained between the base of the platform and the top of the waves. The platform is connected to land with an aerial ropeway system which has a maximum extension of 350m which is used to transfer personnel to and from the platform. The hydraulically driven platform has eight legs each of which is 18m long. Four of the legs are in fixed positions on the fore and aft sides of the platform. The other four moveable legs are fitted to sliding frames attached to the port and starboard sides of the platform. The sets of fixed and moveable legs can be raised and lowered by hydraulically powered jacking stations. By alternatively lifting and lowering the sets of fixed and moveable legs in conjunction with the frames moving back and forth horizontally, a walking action is performed by the PDP. The platform can walk at a speed of 10-15m/hour, depending on weather and footwall conditions.



Figure 2 The Probe Drill Platform in the surf zone

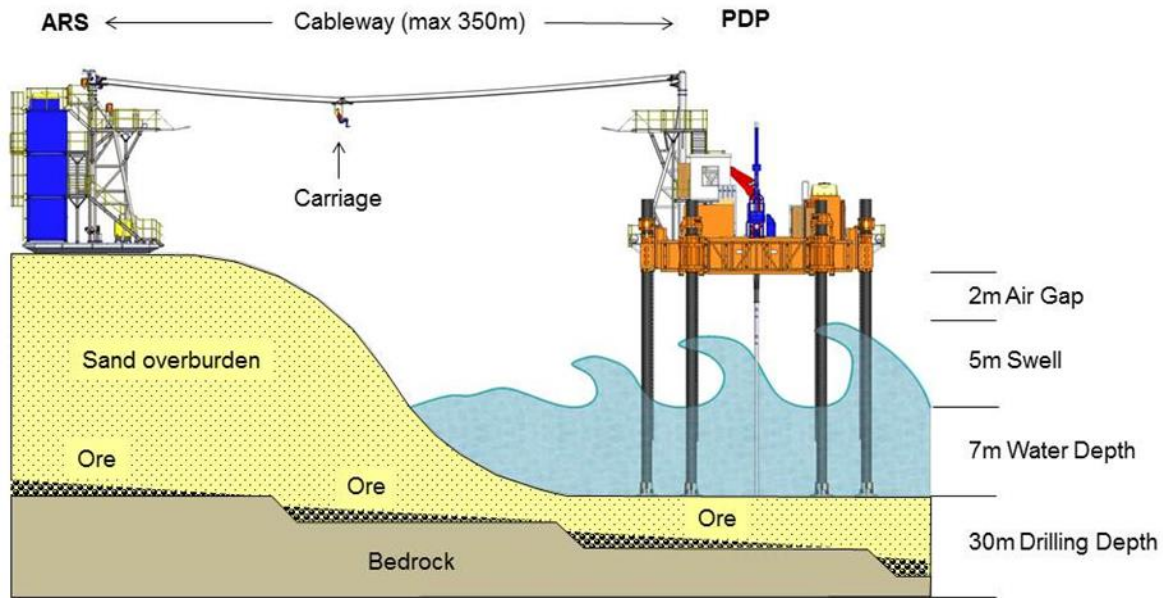


Figure 3 Schematic of the aerial ropeway system (ARS) and probe drill platform (PDP).

Knowledge and experience gained over many years of onshore exploration used to simulate and optimise an exploration plan for the surf zone areas. The following methodology was followed to combine the different sources of input data into the process of creating and analysing the simulation:

- A review of the input data was undertaken to assess the aerial coverage, spatial clustering and location of the data relative to the study area.
- An exploratory data analysis considered the statistics of the data and the percentage split of the gravel/no-gravel indicator data.
- Variography testing was carried out, based on input data and geological interpretation.
- A hand sketch was used to capture the interpretation of the input data and generate “expert interpreted” data and parameters for later use in the process of creating the simulation.
- The data was kriged to get an overall perspective of gravel occurrence and the results used to validate the simulation outcome.
- Parameter determination was undertaken using the input data and “expert interpretation” data to create a conditional indicator simulation.
- The acceptability of the simulation realization was assessed against geological understanding and available data.
- A “measure of success” was determined so that the different sample configurations could be compared against each other.

- A sample optimization study was performed.
- Conclusions were drawn and a decision was made about what would be considered an efficient sample configuration

## **Data**

The gravel occurrence data used in the simulation is sourced from a combination of onshore aircore, GB50 hydraulic grab and BG36 auger drilling from Mining Area No. 1. The drilling covers an elongated roughly north-south coast parallel strip of about 22 km long and 0.30 km wide. For this sample optimization study the data was rotated in order to create a strictly north-south domain and to improve the efficiency of simulation and subsequent analysis. The relative data positions remain unchanged. After the rotation, the input data was simplified to consist of a location co-ordinate and an indicator value representing the presence or absence of diamondiferous gravel.

High water lines (HWL) are shown for 1997 and 2011 in pink and red (Figure 4) and the expected HWL for 2020 is shown in blue (based on Council for Scientific and Industrial Research (CSIR) accretion models). Beach accretion is an on-going outflow of mining activities. The task at hand is to determine a drill configuration to determine diamondiferous gravel occurrence for the areas currently under water for resource generation purposes. The PDP will be applied to drill in the area currently under water (between the 2011 and 2020 HWLs (Figure 4)).

Linear beach deposits show greater continuity in the direction along the coast as opposed the the direction normal to the coast line. Hence the onshore drilling data accumulated on an accreted “sand spit” is ideal for simulation purposes in a long narrow coast parallel strip. The input data used for the simulation comes from areas where most recent accretion occurred (i.e there is dry land for conventional reverse circulation and large diameter BG36 drills to have operated on - illustrated by the black dots (Figure 4)). These patches of beach accretion are limited to and governed by the proximity of recent mining activities. This is the reason why the input data covers only 34% of the simulation domain of 22 km x 0.3 km.

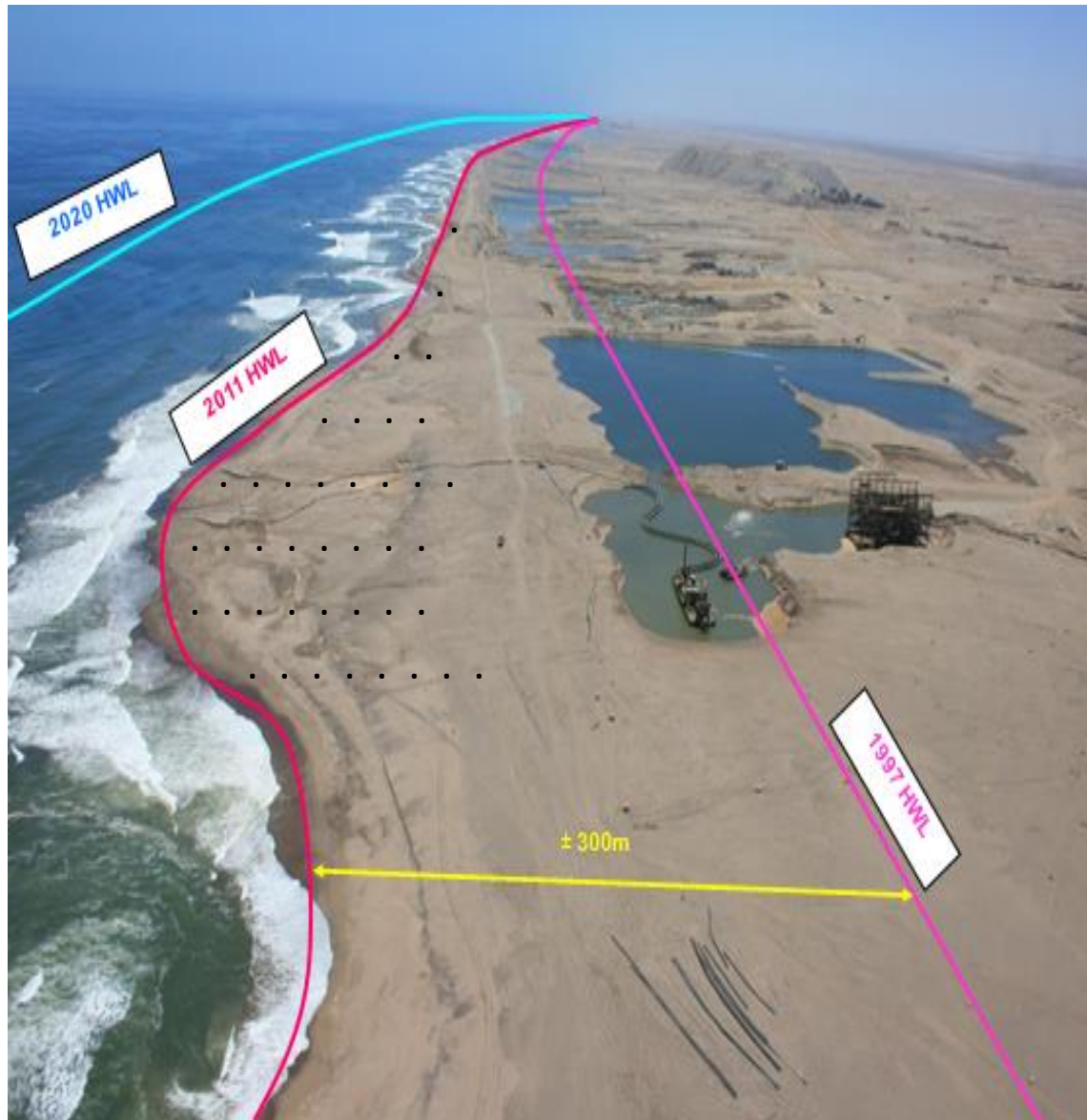


Figure 4. An oblique aerial photograph showing shifting high water lines (HWL) caused by beach accretion. Black dots schematically represent onshore drilling activities on recently accreted areas.

## Variography

The variography obtained using the **drilling data** is shown in Appendix 1 (Figures A1 and A2). The simulated texture based on the variogram structure and conditioning drilling data did not make geological sense, hence additional **analog data** is considered to supplement the variography interpretation.

The footwall underlying the diamondiferous beaches is highly gullied Precambrian meta-sedimentary bedrock. In general, the gullies are filled with gravel and it is fair to assume that gully patterns, as shown in Appendix 2, represent gravel occurrence. Appendix 2 outlines variography obtained from **bedrock gully patterns**. However, gravel found in gullies is a “subset” of the total gravel occurrence (as gravel also occurs above bedrock peaks) and cannot be used in isolation. This interpretation was undertaken to obtain a sense of the short range spatial behaviour and distances over which gravel occurrence is correlated.

In order to further assess the nature of gravel occurrence an “**Expert interpretation**” method is followed (Prins, C. F., 2011). The indicator drilling data (Figure 5a) is interpreted by hand (similar to interpretations involved when building 2 or 3 dimensional geological volume models) by geologists with an intimate knowledge of the deposit. In this case, the 2 dimensional interpretation is digitized (Figure 5b) and then pixilated (Figure 5c). The pixilated data set is then used as input data to the variogram calculation and ultimate modelling process (Figure 6).

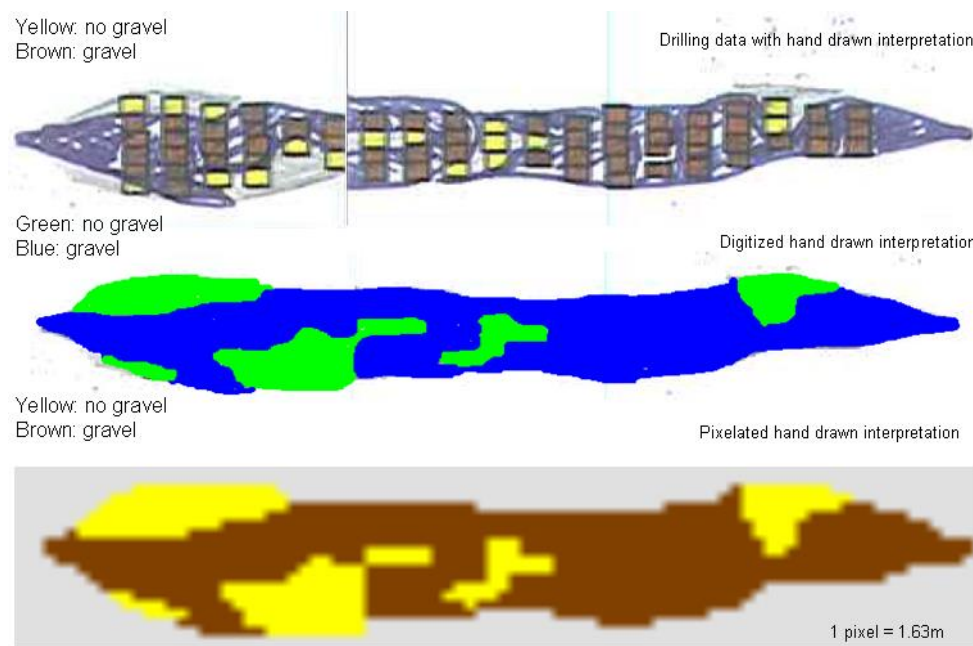


Figure 5. Steps followed in creating “Expert interpretation” data (a-top, b-middle, c-bottom)

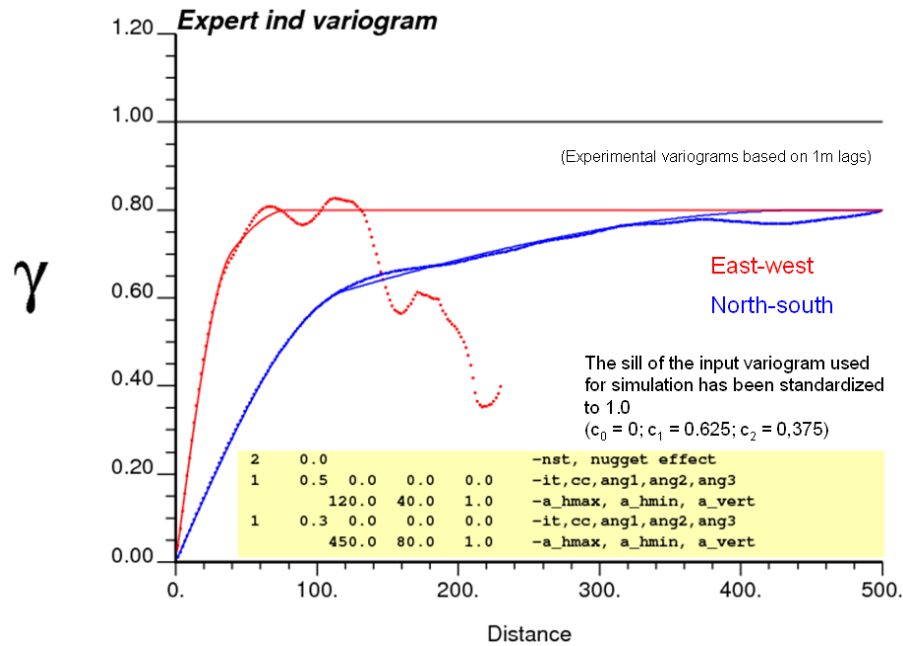


Figure 6. Variogram based on pixilated indicator data

Variogram parameters obtained from different datasets are summarized in Table I. All variogram models are spherical unless indicated differently. The  $C_0$  value for all data sets is 0. The  $C_1$  and  $C_2$  values for the drilling and “Expert interpretation” data is similar at 0.6 vs 0.625 and 0.3 (+ $C_3$  (0.1) = 0.4) and 0.375.

Table I Variography parameters obtained from different data sets.

| Data                         | Direction | Nugget ( $C_0$ ) | $C_1$ | $C_2$ | $C_3$ | Range1 | Range2      | Range3 |
|------------------------------|-----------|------------------|-------|-------|-------|--------|-------------|--------|
| Drilling data                | EW        | 0                | 0.6   | 0.3   | 0.1   | 5      | 160         | 160    |
|                              | NS        | 0                | 0.6   | 0.3   | 0.1   | 90     | 90          | 700    |
| Gully pattern data           | EW        | 0                | 0.9   | 0.1   |       | 60     | exponential | model  |
|                              | NS        | 0                | 0.9   | 0.1   |       | 5      | 60          |        |
| “Expert interpretation” data | EW        | 0                | 0.625 | 0.375 |       | 40 *   | 80          |        |
|                              | NS        | 0                | 0.625 | 0.375 |       | 120    | 450         |        |

\* Similar ballpark range

In general, analysis of data from previously mined areas shows that about 70% of diamonds occur below bedrock peaks of gullied (fixed trap site) Precambrian footwall and diamond occurrence is associated with gravel occurrence. Swash gullies (parallel to the swash direction of the waves – EW direction) are the dominant gully type in the study area. It is thus fair to assume that there should be correlation between the gully pattern in an EW direction and gravel occurrence.

The range of 60m for the “Gully pattern” data in an east-west direction concurs with the geometric ranges of 40m and 80m as modelled on the “Expert interpretation” dataset. The gully pattern in a north-south direction has a very short range as evident from the texture of the image (Appendix 2) of exposed bedrock. In reality there is, however, strong continuity in overall gravel occurrence (marine terraces above bedrock peaks) in a north-south direction governed by sea-level stands. It has been decided, therefore, to acknowledge the variogram parameters observed from the conditioning data and gully patterns, but the variogram obtained from the “Expert interpretation” will be used as a basis for kriging, and to create the simulation, as the parameters are well aligned with the geological interpretation.

## **Kriging**

Kriging of the indicators using the drilled data with zero (gravel absence) and one (gravel presence) is used as a means of visual validation of the simulation outcome as well as to obtain a perspective on the scale and continuity of gravel occurrence. The input to the kriging is the indicator drilling data and the variogram parameters obtained from the “Expert interpretation”.

The output of the indicator kriging was re-proportioned in the ratio of 0.77 : 0.23 to convert the kriging results back to indicators of zeros and ones, and to align the output with the zero:one ratio observed in the data (Soares, 1990). The visual similarity between the pixelated hand-drawn interpretation and the kriged result (Figure 7) is deemed to be acceptable.

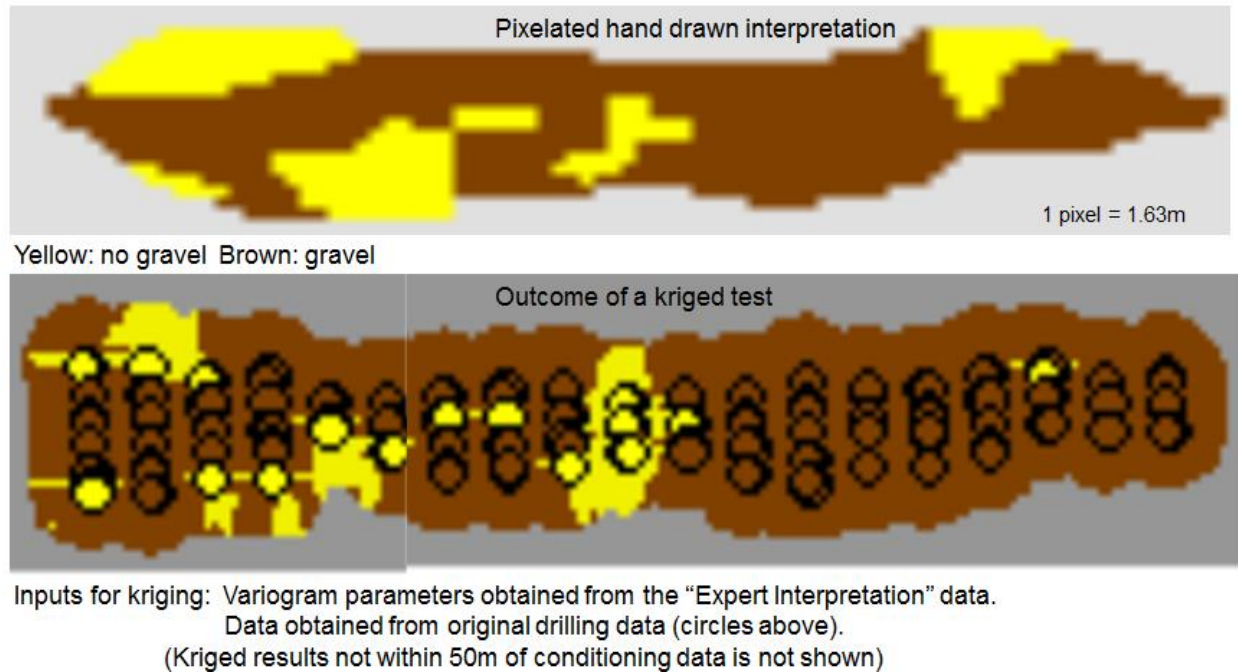


Figure 7. A comparison between kriged results and the pixilated hand-drawn sketch, used for validation

## Simulation

After some experimentation, the most satisfactory simulated texture is obtained when a 5x5m cell spacing is used for simulation. In order to simplify calculations related to the sampling of the simulation, a 1 x 1m cell spacing is preferable. Several attempts at reproducing the input variogram from different realizations (on 1 x 1m spacing) did not give satisfactory results. Hence, both the drilling data and the 5 x 5m simulated data is used in combination as conditioning data (Tran, 1994) to create a simulation on a 1 x 1m cell size (Figure 8). This approach produced a simulation realization with a spatial structure reasonably comparable to the input variogram (Figure 9) albeit not perfect. The deviation from the original input variogram is deemed acceptable in light of the type of study undertaken, the approach followed, paucity of data and inability to obtain an acceptable variogram from the drilling data in creating the simulation.

The zero:one proportion for the simulation is very close to that of the conditioning data and the mean of the simulation is 0.7875. The success of sample campaigns will be measured against this average of 0.7875.

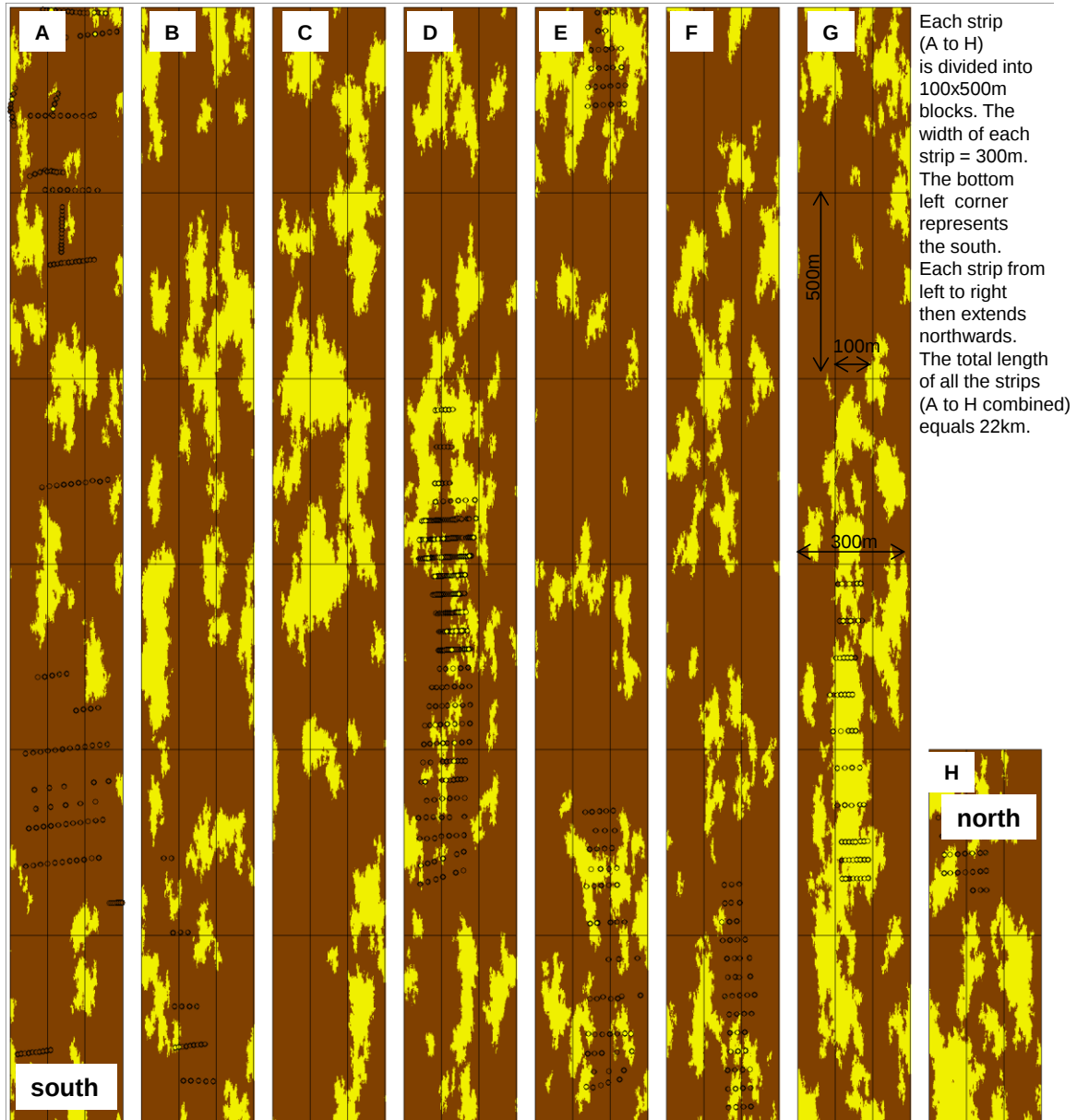


Figure 8. Simulated texture of gravel occurrence with conditioning data

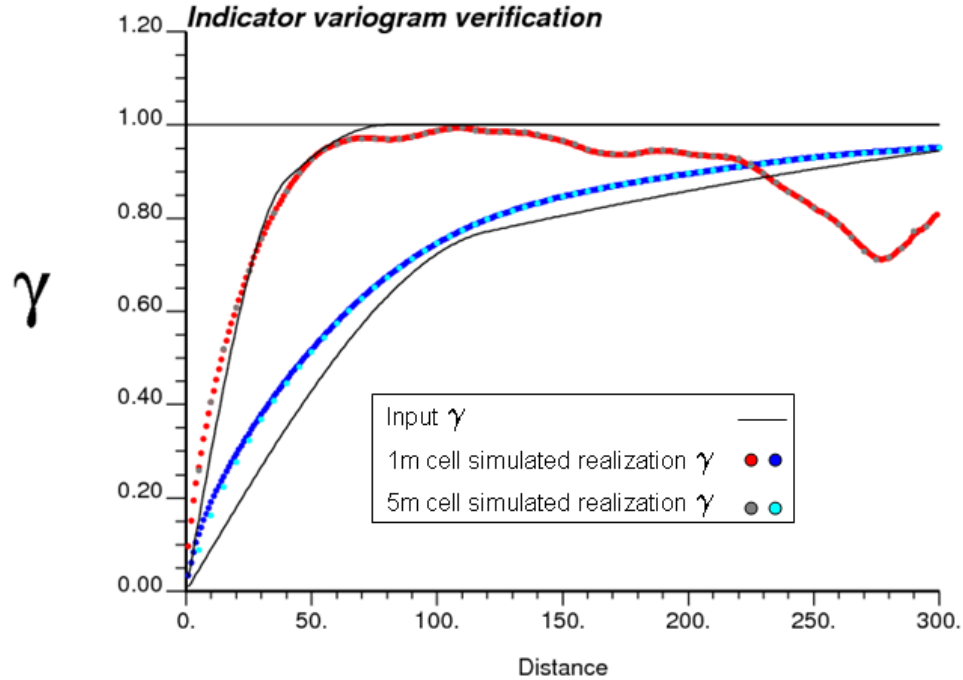


Figure 9. Comparison between input variogram and 1x1m and 5x5m realizations

### Sampling of Simulation and Measure of success

Different sample strategies can be tested using the created simulation to determine which best predicts gravel occurrence. A measure of success of a sampling campaign is required and for this, different scenarios were used:

#### A) Trumpet curve

Consider the outcomes of 100 sampling campaigns, each using a different offset for the origin when sampling the simulation. Different sample configuration for each of the 100 campaigns (Table II) is used and for each realization the estimated percentage of gravel for the domain is plotted.

Table II Different sample configurations used to sample the simulation (C = cross)

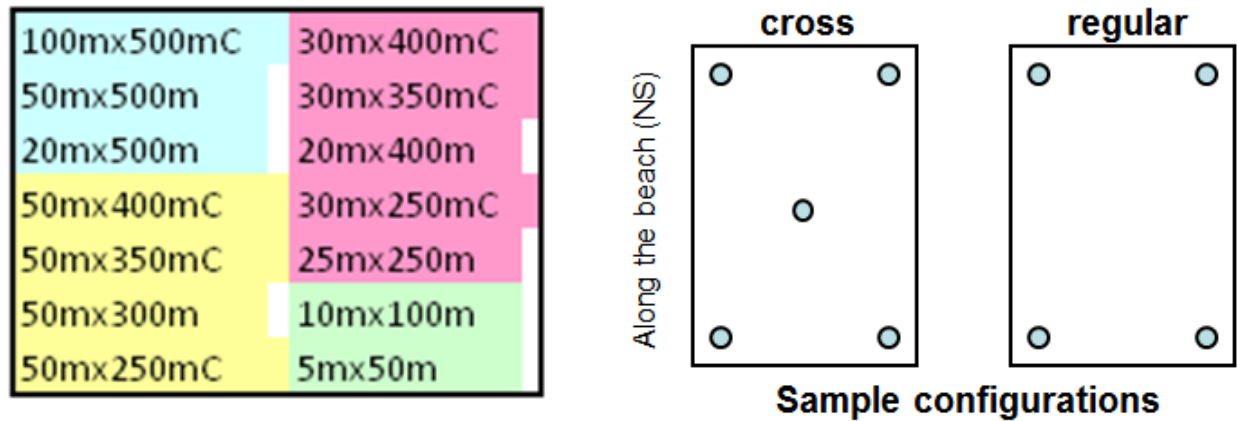


Figure 10 shows a trumpet curve with the different sampling realizations and estimates for the proportion of gravel for the entire simulated domain. The trumpet curve is a relative comparison of one campaign against another and is not meant to give a measure against a 15% accuracy benchmark.

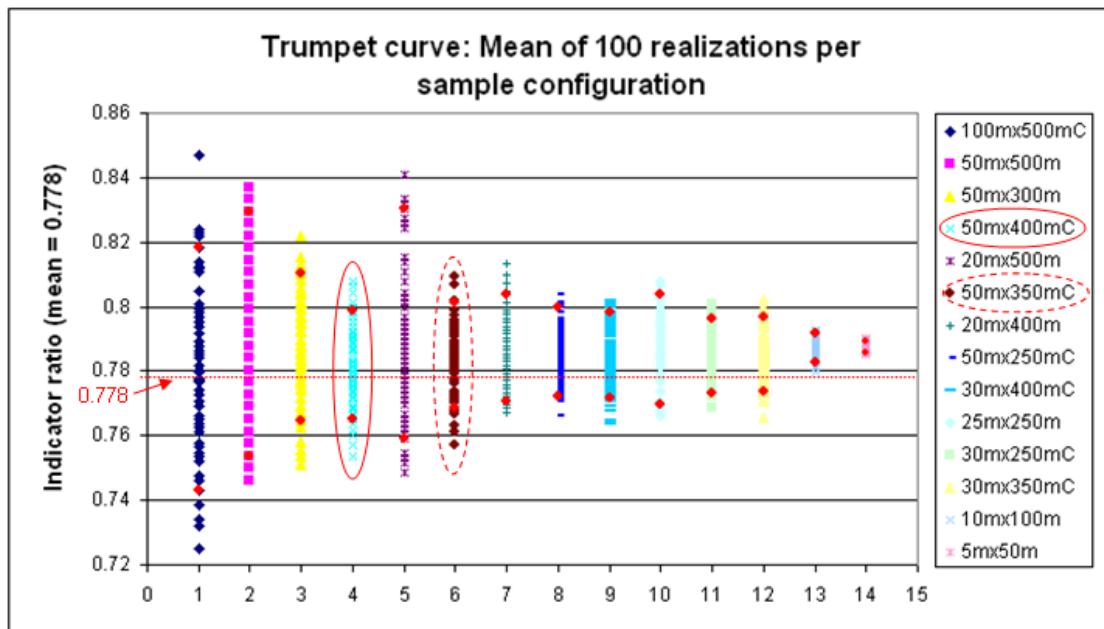


Figure 10. Trumpet curve showing comparison between different sample configurations. Red diamonds represent 90% confidence limits

B) Kriging and correlation coefficients are used to rank the efficiency of different sample configurations.

Two strategies of search criteria were used in estimating the percentage gravel for blocks of 50 x 250m through kriging (based on 100 realizations created by offsetting). A small neighbourhood, not using more than the nearest 4 samples and a second strategy, using up to 24 neighbours, were used in the kriging. The estimates from the kriging exercises were plotted against the “actuals” determined from the simulation and the correlation coefficients calculated.

Figure 11 shows a higher correlation coefficient when comparing the kriged results, using a maximum of 4 samples, than when comparing the kriged result using 24 samples against the simulated reality (except for the highly dense sampling campaigns based on 10 x 100m and 5 x 50m sample configurations). The 50 x 400mC sample configuration is interpreted to be relatively efficient.

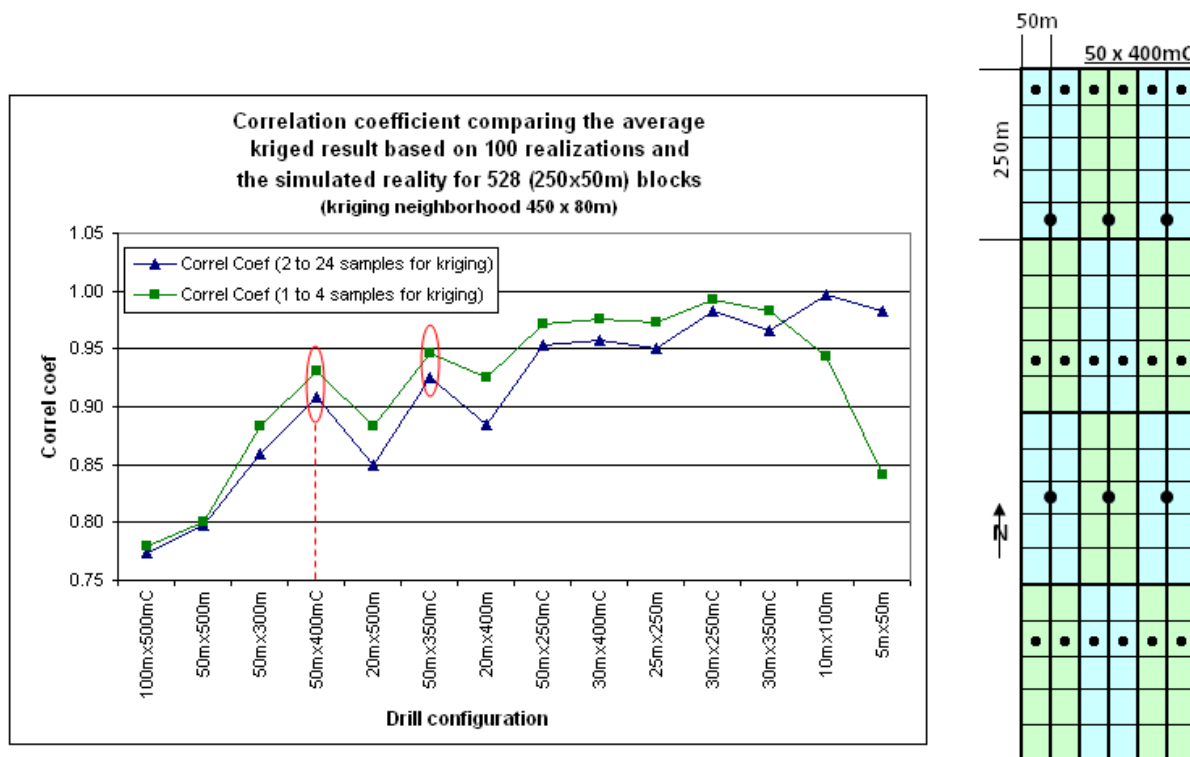


Figure 11. Comparison of correlation coefficients for the simulated reality vs. the average of the kriged result using a maximum of 4 and 24 samples.

Interpreting the results shows that a smaller search neighbourhood has better correlation with “actual” that using a large search neighbourhood. Furthermore, the kriging exercise confirms that reasonable results can be obtained using a sample spacing of 50 x 400mC.

## **Summary and Interpretation of results**

This study does not provide a single optimisation result with defensible confidence limits, as the nature and lack of conditioning data do not permit this. Different validation approaches, however, resulted in the creation of a simulation on which a comparative study could be done. The results gave a very good indication of how to balance sampling effort with de-risking aspects related to different sampling configurations, and the determination of gravel occurrence and geological continuity.

The simulation obtained from using only the drilling data and associated variogram did not make geological sense; hence, it was decided to make use of analog data in order to create a more acceptable simulated reality which is geologically acceptable. The first analog data used are the gully patterns and the second analog data set used is the “Expert interpretation”, as outlined in Figure 5. This was used to parameterize the simulation to produce a geologically acceptable outcome, which was sampled and analysed to determine a relative efficient sample configuration.

The trumpet curve (Figure 10) shows that a sample spacing of 50m x400mC is relatively efficient (compared to other sample configurations) and when used as input to kriging, is also considered as being relatively efficient.

The 50 x 400mC configuration will give a well-balanced indication of sampling effort, and risk aspects related to geological continuity, and will be implemented in future sampling strategies. As more sample data becomes available from PDP campaigns, the “Expert interpretation” will systematically and incrementally be replaced by more conventional simulation studies and the sample strategy revisited.

## **Acknowledgements**

The authors would like to thank the Namdeb Mineral Resource Department for its support and providing access to data. The work presented in this paper forms part of a larger project partially incorporated in a PhD study.

## References

Jacob, R. J. 2011, Namdeb Business Plan Technical Report (BPTR) for 2011 FINAL – Namdeb Internal Docs: #103352

McCallum, K., 2011, MRM CHQ Additional Assurance Note to the Namdeb Business Plan 2012 to 2020 – Namdeb Internal Docs: #104078

Prins, C. F., 2011, Échantillonnage, simulation et estimation des gisements secondaires de diamante, Geostatistics Doctoral Thesis, School of Mines of Paris

Tran, T. T., 1994, Improving variogram reproduction on dense simulation grids, Computers and Geosciences Vol 20, No. 7-8, pp.1161 - 1168

Soares, A. , 1990, Geostatistical estimation of orebody geometry: Morphological kriging, Mathematical Geology, October 1990, Volume 22, Issue 7, pp. 787-802

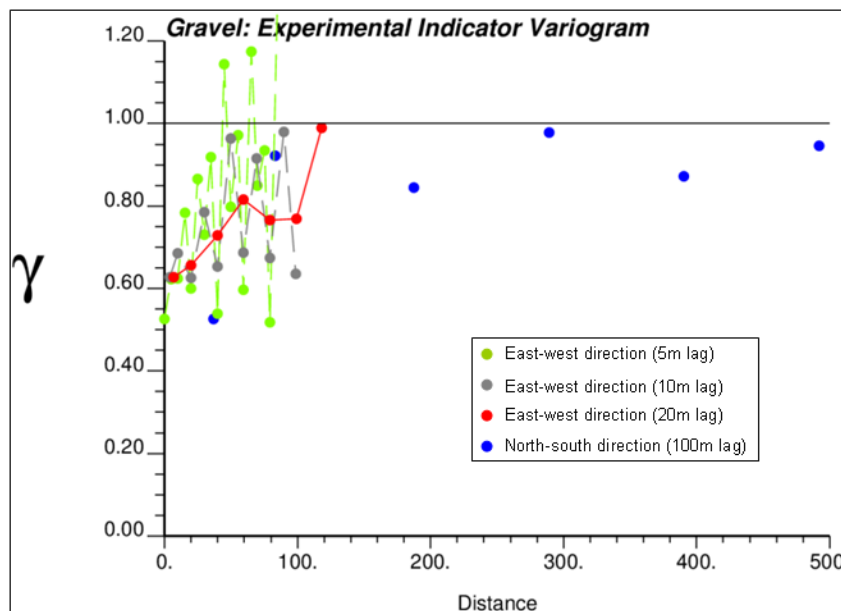
## Appendix 1 – Variography based on drilling data

### Experimental variograms

The input drilling data comprises multiple spatial configurations, but, very roughly, the majority of data is spaced at 50 x 20m.

East-west direction: The spacing of the various drilling campaigns varies in the east-west direction. Multiple lag distances have been tested in the variogram calculations in order to obtain the most satisfying spatial structure in terms of detail and structure stability. Five, ten and twenty metre lags have been used (Figure A1). The result, from using a 20m lag spacing, is deemed to be the most stable structure which also represents the average structural shapes of the 5 and 10m lag spacing. Due to the complexity in modelling the variogram, the 5, 10 and 20m data has been overlaid on each other to help guide the variogram fitting.

North-south direction: The domain size is about 22km in a north-south direction. There are areas of up to 6 km in length which have no data present, with other areas having sufficient data to enable the use of a lag spacing of 100m.



**Figure A1. Experimental variograms in north-south and east-west directions (using different lag distances)**

Modelled variograms

The best fit obtained to the experimental variogram points is shown in Figure A2.

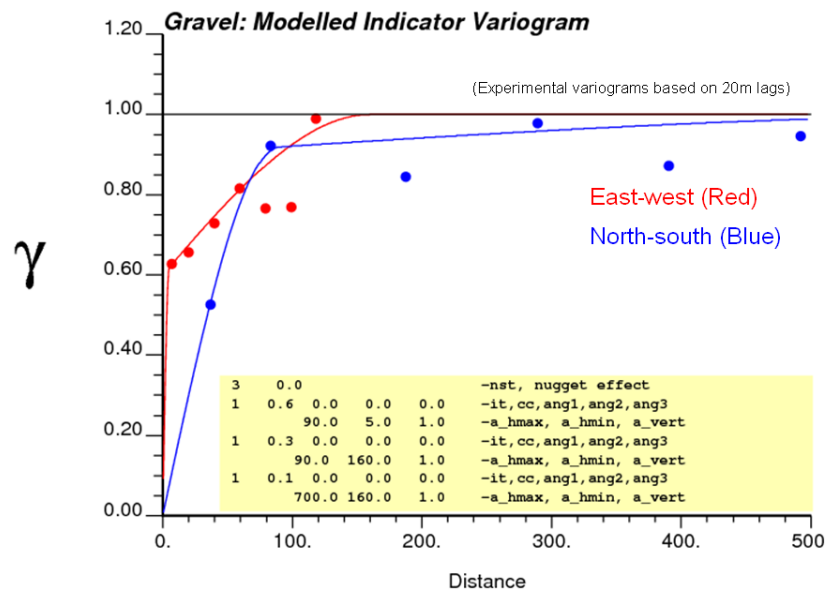


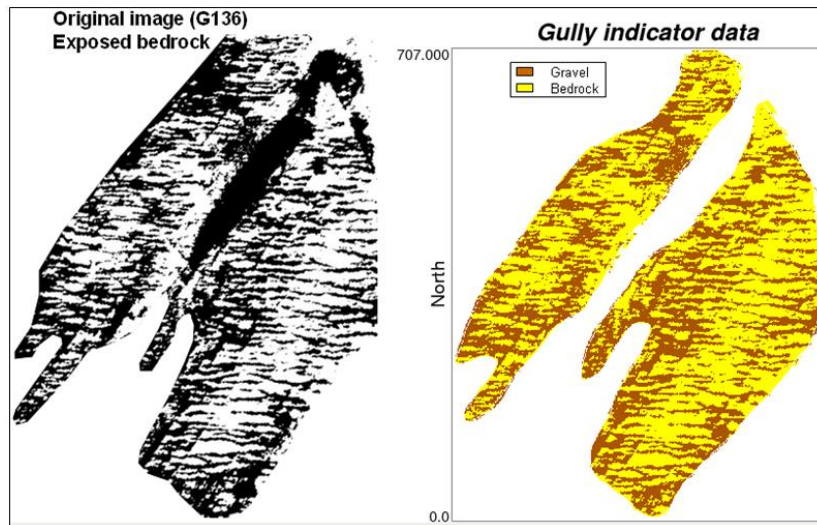
Figure A2. Modelled variograms in north-south and east-west directions

Although a model could be fitted to the experimental variogram points, the parameters to enable a fit through the points near the origin (distance close to 0m), and the geometric anisotropy did not make geological sense. Considering other fits, for example modelling the nugget effect value at 0.6 (plus geometric anisotropic structures), would result in a simulation that will not have continuous gravel occurrences, which would not be comparable to real observations.

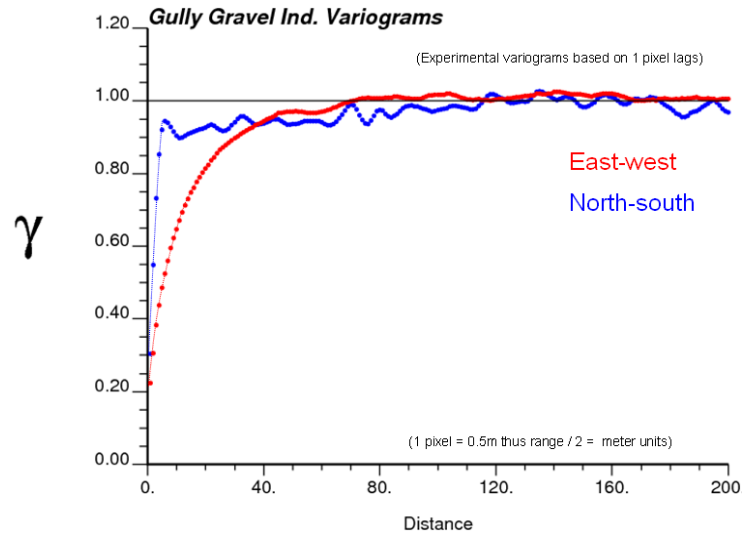
Geological knowledge of the continuity and size of gravel occurrences needs to be introduced into the determination of an appropriate variogram model. This is done through the introduction of “expert knowledge” captured as a hand-drawn sketch.

Appendix 2 – Variography of Gully patterns

An aerial photograph of exposed bedrock was used to create an indicator dataset of gravel/no gravel occurrence, in the form of a pixel map. A variogram was calculated using this interpretation of gullies as proxy to gravel occurrence and the output analyzed.



**Figure B1. Original and pixilated images of exposed bedrock showing gully patterns**



**Figure B2. Experimental variograms in north-south and east-west directions**

The practical range for an Exponential variogram model is interpreted as 3 x modelled range. In this case, the practical range was determined as a 60m range in an east-west direction. (A scaling factor of 0.5 is used to translate the pixels to real space and a range of 40m was modelled).

It should be kept in mind that gully patterns represent a subset of overall gravel occurrence. The range of 60m in an east-west direction broadly concurs with the ranges of 40m and 80m as obtained from the “Expert interpretation dataset” calculated in an EW direction. The gully pattern in a north-south direction has a very short range, but this is due to the texture of the above image of the exposed bedrock. In reality, there is great continuity in overall gravel occurrence in a north-south direction governed by sea-level stands.

Analysis of the occurrence of gravel in bedrock gullies was undertaken to broaden the understanding of variogram behaviour, and to help determine and double check the reasonableness of parameters determined from the “Expert interpretation”.

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## **6. PUBLISHED PAPER III**

### **CONSTRUCTION OF EXPERT OPINION BASED VIRTUAL ORE BODIES FOR A DIAMONDIFEROUS LINEAR BEACH DEPOSIT**

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#### **Abstract**

During early stage diamond exploration projects, hard data underpinning spatial continuity is often very limited. An extreme example of this is a diamondiferous marine placer target area alongside a current onshore mining area in south western Namibia. Although an abundance of geology and grade data exists in the adjacent onshore mining area, the target area itself contains no such information. Notwithstanding this apparent abundance of data, it is extremely difficult to obtain a variogram (Prins & Jacob, 2014) for use in this study area. In addition, the highly gullied footwall, which controls most of the diamond entrapment is non-stationary, further hindering the use of more traditional simulation techniques. An alternate approach for creating a simulated virtual ore body (VOB) is thus required in order to enable the assessment of sampling strategies.

This paper demonstrates how, in the absence of any hard data, expert opinion is used to generate a composite probability map for diamond concentration using a grey scale hand sketching technique. The composite probability map, after statistical manipulation, is then used to create a grade simulation on which different sample scenarios can be tested.

The composite map takes into account the two types of diamond entrapment observed geologically: fixed trap sites (gullies) and mobile trap sites (suspended gravel lenses). For the fixed trap sites, analog data in the form of bedrock gully patterns, as exposed by onshore mining activities, are used as a basis for creating the trapping potential maps. A mosaic of scaled, exposed bedrock gully patterns obtained from aerial photography is placed on a light table. Expert opinion grey scale hand sketching is then applied to shade areas of higher potential for diamond concentration darker while areas of lower potential are progressively shaded lighter. Mobile trap sites in the form of gravel lenses, suspended above gullied bedrock peaks, are associated with sea-level stands. Bathymetry data in the area of interest is then used as a basis for creating the shaded maps and dealt with in the same way as the fixed trap site data to create a trapping potential map.

Proxy diamond grade data is obtained from adjacent historical onshore sampling results. Delineation of the raised beaches was done using 1m wide sampling trenches orientated normal to the coastline comprising 5 m<sup>2</sup> sampling paddocks spaced 500m apart along the coast. About 26 000 detailed sample results (collected from the 1930s to 1960s) are available from these

trenches. These sample results are used to populate the potential map in order to create different grade scenarios.

From the detailed sample results it is evident that on the whole, one third of the diamonds are trapped in the mobile trap site environment suspended above bedrock peaks and two thirds are trapped below bedrock peaks in the fixed trap sites. The final potential map is constructed by taking this information into account through a weighted combination of the pixilated shaded intensities of the fixed and mobile trap site sketches.

A submerged coast parallel strip (roughly 100 km long) wedged between the mostly mined out onshore linear beaches (that have yielded more than 63 million carat gem quality diamonds) and the partially mined deep water resource (which has yielded more than 15 million carats to date) is the target identified for future exploration. This paper shows how a 1 x 4 km probability map is created based on expert opinion as a first step towards determining an appropriate sampling strategy for the target area. The probability map is calibrated and populated using the diamond distribution of different raised beaches obtained from analog data based on sample results adjacent to the target area. The potential map then becomes a VOB, into which individual diamond locations are seeded. This VOB is used to analyse and rank the efficiency of different sampling strategies for grade determination of submerged diamondiferous linear beach exploration targets.

## **1. Introduction**

Expert opinion is being used in various fields (engineering, biological research, economics *etc.* Kuhnert *et al*, 2009, Pearce *et al*, 2001) to assess uncertainty where limited or no hard data is available. Several approaches exist for combining probabilities obtained from expert opinion. The linear opinion pool (Stone, 1961, Winkler, 1968) is a weighted combination of expert opinion probabilities which satisfies the marginalization property. This requires that the combined probability is the same for combining either the marginal distributions or the joint distributions and then calculating the marginal distribution (Clemen & Winkler, 1999). Game-theory is applied in deciding on when to combine which probabilities obtained from different methods that are equally appropriate based on the available data (Bickel, 2012).

A hierarchical modelling framework for combining expert opinion data and actual observed data for inferential purposes in a spatial context demonstrated that even a misleading expert opinion can be useful in cases where hard data refutes the expert opinion data (Lele & Das, 2000). The expert opinion data is influenced by hard data hence the datasets are not independent; for example, probabilities generated from different likelihood functions (Journel, 1986). Truong *et al.*, illustrate how expert opinion is used as input to determine variogram parameters for down scaling from block support observations to point support where no point support observations are available (Truong *et al*, 2014).

Spatially it is often very difficult to obtain or get access to hard data to be considered for a sampling optimisation strategy. This paper demonstrates how expert geological opinion is firstly used to generate a composite probability map for diamond concentration using a grey scale hand sketching technique. Secondly, how the probability map is then calibrated to the correct sample support size and thirdly, populated using the diamond grade distribution histogram obtained from observed analog data. Figure 1 shows the orientation and size of the VOB in relation to Oranjemund and the coast line of Namibia.

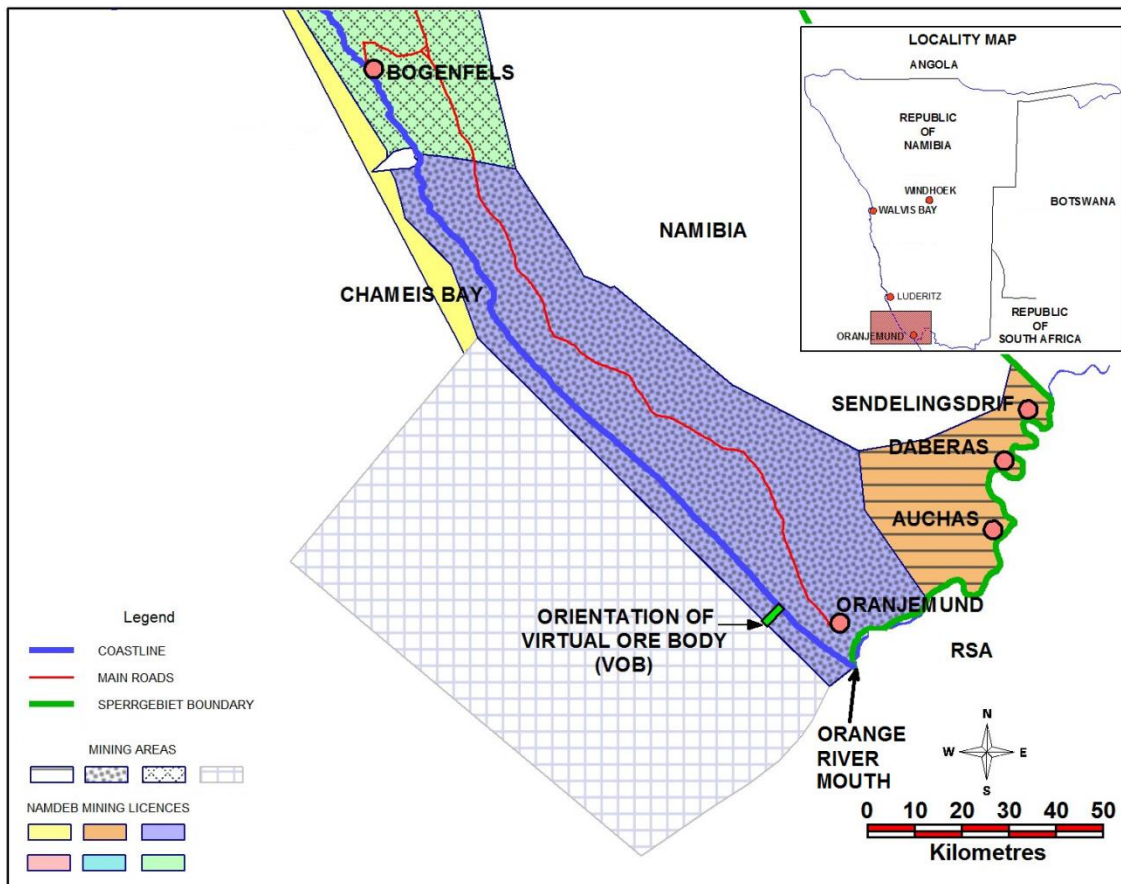


Figure 1 Locality map showing the size and orientation of the virtual ore body (VOB) - South Western Namibia

## 2. Background

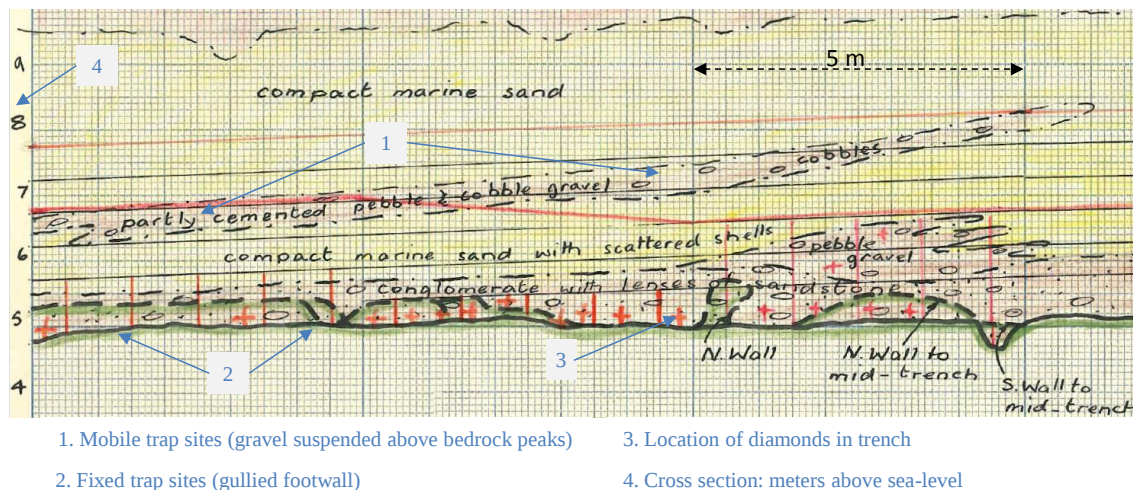
Onshore diamondiferous linear beaches along the Namibian coast (Figure 1) have been the mainstay of Namdeb's diamond production for more than 80 years. These beaches are developed at different elevations, occurring as high as 30 m above sea-level and as low as 30 m below sea-level. A modern day analog of this would be the present day gravel beaches along the Black Sea within the Bolshoi Sochi region and are perhaps good examples of what the study area's beaches would have looked like thousands of years ago (Spaggiari *et al*, 2006). The fossil gravel deposits of the study area extend continuously for approximately 100 km northward from the Orange River mouth along the coastline. Most of the onshore beaches have been mined out

to date, there exists however, great potential in remaining areas that are currently submerged under water. During 2010, the concept of planned, controlled shoreline accretion was initiated, while, up until then accretion was a consequence of the stripping and dredging process. Beach accretion is a natural process resulting from over-abundance in sediment supply, which in effect builds the coastline outwards. The stripping and dredging processes are used to remove overburden sand in order to gain access to diamondiferous basal marine gravel. At present, focused deliberate beach accretion is inherent of the mine plan and is also considered a possible means for making the mining of the submerged beaches reality. Therefore it is imperative to find a way to determine the diamond grade to justify the accretion for future mining areas.

The sea's high energy swash zone makes obtaining upfront data well in advance of mining a challenge. In addition to the high energy swash zone aerated water column, a sand overburden sequence must be penetrated before the diamondiferous basal gravel sequence is reached. At present, a Probe Drill Platform (PDP) is the only implemented technology that successfully withstands these energies in the vigorous swash zone, and it provides geological data only. Furthermore, the sample size is too small for grade determination in the submerged target areas, due to the low grade nature of the deposit (Jacob *et al*, 2013). The PDP is restricted by a land bound base station and can only operate within 300m from the current shoreline. The potential of the offshore linear beaches however extends up to 4 km into the sea and the challenge is thus to generate a VOB which can be used for sampling strategy and risk studies in the absence of any hard data.

### **3. Nature of the analog data**

The initial delineation of the onshore raised beaches was done between the 1930s and 1960s by a comprehensive 1m wide trench campaign. The 1m trenches were spaced 500m apart along the coast covering the 100 km from the Orange River mouth northwards to Chameis Bay (Figure 1). These 1m trenches, orientated normal to the coastline, spanned six distinct onshore raised beaches. Continuous trench paddocks of 5m lengths resulted in more than 26 000 samples at 5 m<sup>2</sup> support. Diamonds are concentrated in both gravel lenses (mobile trap sites) suspended above the bedrock footwall and highly gullied footwall. An example of the very detailed methodical mapping (1930s to 1960s) of the 1m trench sections where the locations of individual diamonds are recorded is shown in Figure 2.



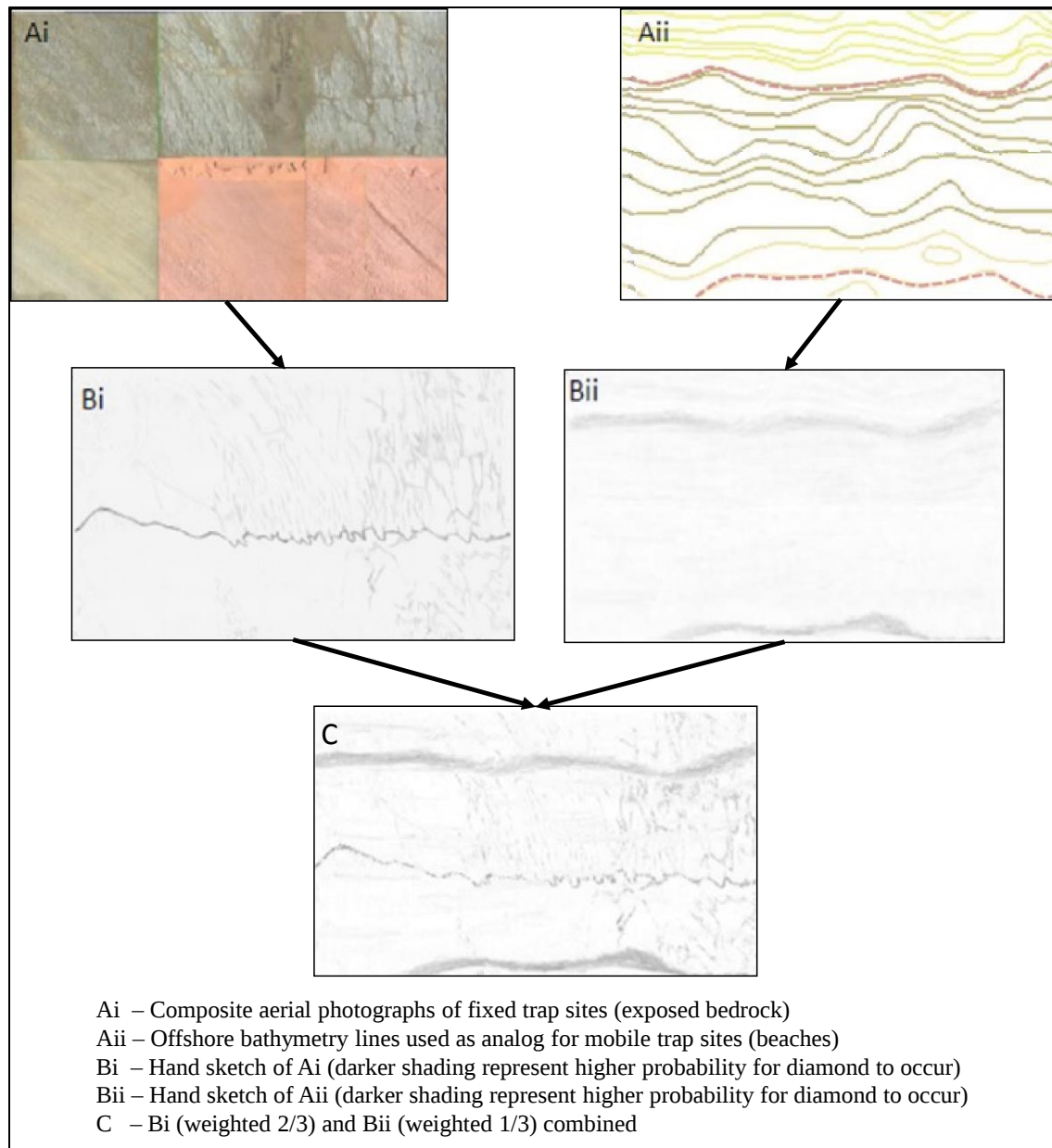
**Figure 2** Vertical cross section of a 1 m sample trench showing the spatial relation of fixed and mobile trap sites. Each sample paddock comprises a 5 x 1 m (5m<sup>2</sup>) support size.

The morphology of the marine erosion pattern on the bedrock surface (fixed trap sites) dominates the distribution of alluvial diamonds (Jacob *et al*, 2006). From detailed sample results it is evident that two thirds of the diamonds occur in the fixed trap sites with one third of the diamonds in the mobile lenses and this observation is incorporated in the construction of the VOB.

#### 4. Methodology

Prins (2011) developed a method using an “expert opinion” based hand sketch to simulate the occurrence of diamonds in a VOB, in cases where no hard data (spatial diamond grade) is available. The hand sketch is constructed on the principle that relative darker areas in the sketch represent areas with a higher probability to contain diamonds. This VOB is designed to form a strip connecting the current onshore area to the edge of the mining license approximately 4km offshore (Figure 1). The methodology of developing the probability map based on expert opinion is shown in Figure 3. The first step is the bedrock morphology sketch by the expert. The bedrock morphology is based on actual bedrock patterns currently exposed by onshore mining activities and exposed bedrock patterns obtained from bathymetry surveys in water depths deeper than 30m. Secondly, sea-floor bathymetry contour lines are used as a proxy for gravel beach location suspended above the bedrock fixed trap sites. The two sketches referred to above are merged in such a way that the shading reflects the  $\frac{2}{3}:\frac{1}{3}$  proportion of diamond potential of the two trap site types, resulting in a single combined probability model that can be used as a diamond potential entrapment map for the 1 x 4 km strip.

Geologists with combined experience in the order of 50 years were asked to assess the resultant sketch after the merging described above. All agreed that the proposed model reflects the current understanding of what could be a reasonable representation of diamond distribution in the study area based on their extensive onshore production experience.



**Figure 3 Steps involved in constructing a probability map for potential diamond entrapment**

This paper introduces an additional aspect to the method proposed by Prins (2011), by applying the grade (stones/m<sup>2</sup>) profile observed across the different beaches resulting in a grade distance profile in the VOB (Figure 4). This profile is based on the average grade of the 6 onshore linear beaches obtained from the 26 000 sample results spanning roughly 100 km from the Orange River mouth to Chameis Bay (Figure 1).

Statistical manipulation to align the hand sketch with the proxy grade data requires four steps. The first step is to regularise the sketch, mapped to scaled coordinates, into pixels representing 5m<sup>2</sup> data so that it is aligned with the support size of the proxy data. Secondly, a normal score

transformation is applied to the potential obtained from the sketch. Thirdly, a back transformation of the regularised sketch data is done using the distribution of the proxy data. Finally, a 5<sup>th</sup> degree polynomial fitted to the grade in stones per 5 m<sup>2</sup> per beach (Figure 4) is used, with the y-axis as independent variable, to adjust the grade prior to seeding stones into the VOB. The grade/distance profile observed is thus matched to the grade/distance of the hand sketched diamond entrapment potential map.

The outcome is finally adjusted by shifting the grey scale potential through histogram transformation, adjustment of individual histogram classes and the random removal of stones (decimation). This is done in order to honour the zero proportions, tail characteristics of the histogram and univariate statistics of the proxy data.

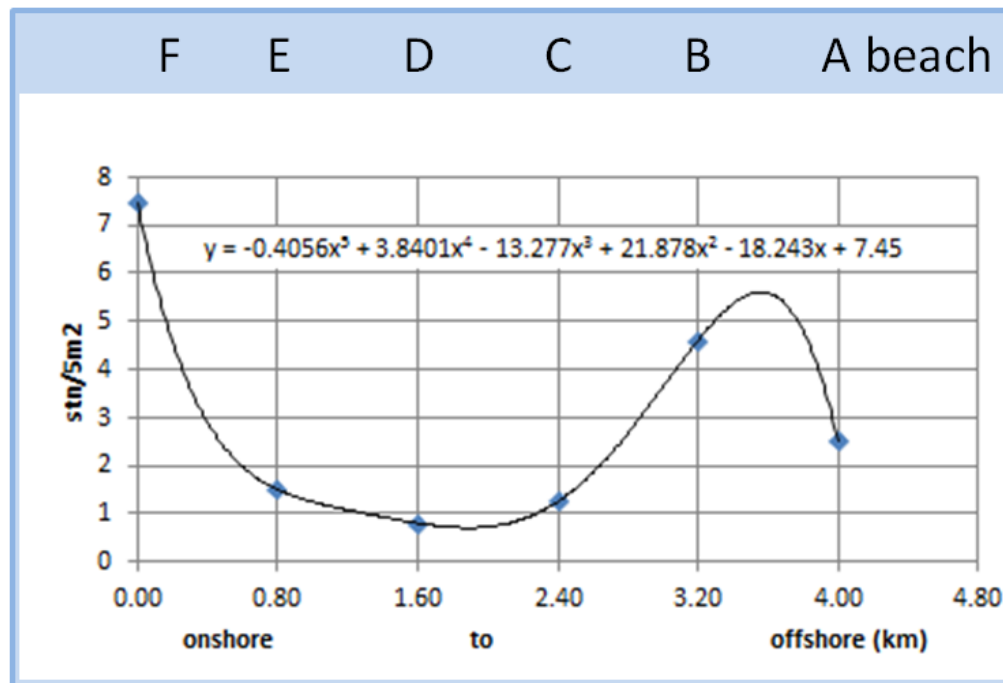


Figure 4 Grade profile used to adjust the diamond potential entrapment map

Once the grade for a particular line of pixels is determined, the total number of stones for that line can be seeded. The individual stone locations simulated into the VOB based on the 5m<sup>2</sup> pixels' grey scale potential is shown in Figure 5. For example, the first pixel will have the potential to randomly receive 5/96<sup>th</sup> (the second pixel 2/96<sup>th</sup>, the third pixel 6/96<sup>th</sup> and so forth) of the number of stones for that line.

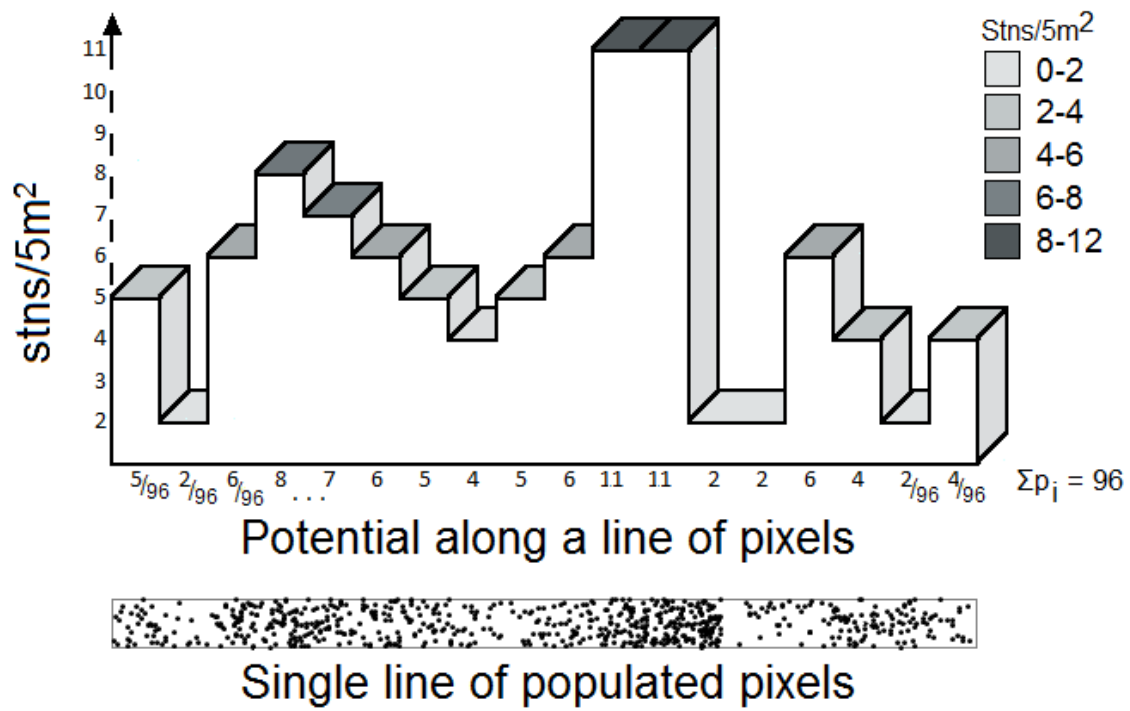
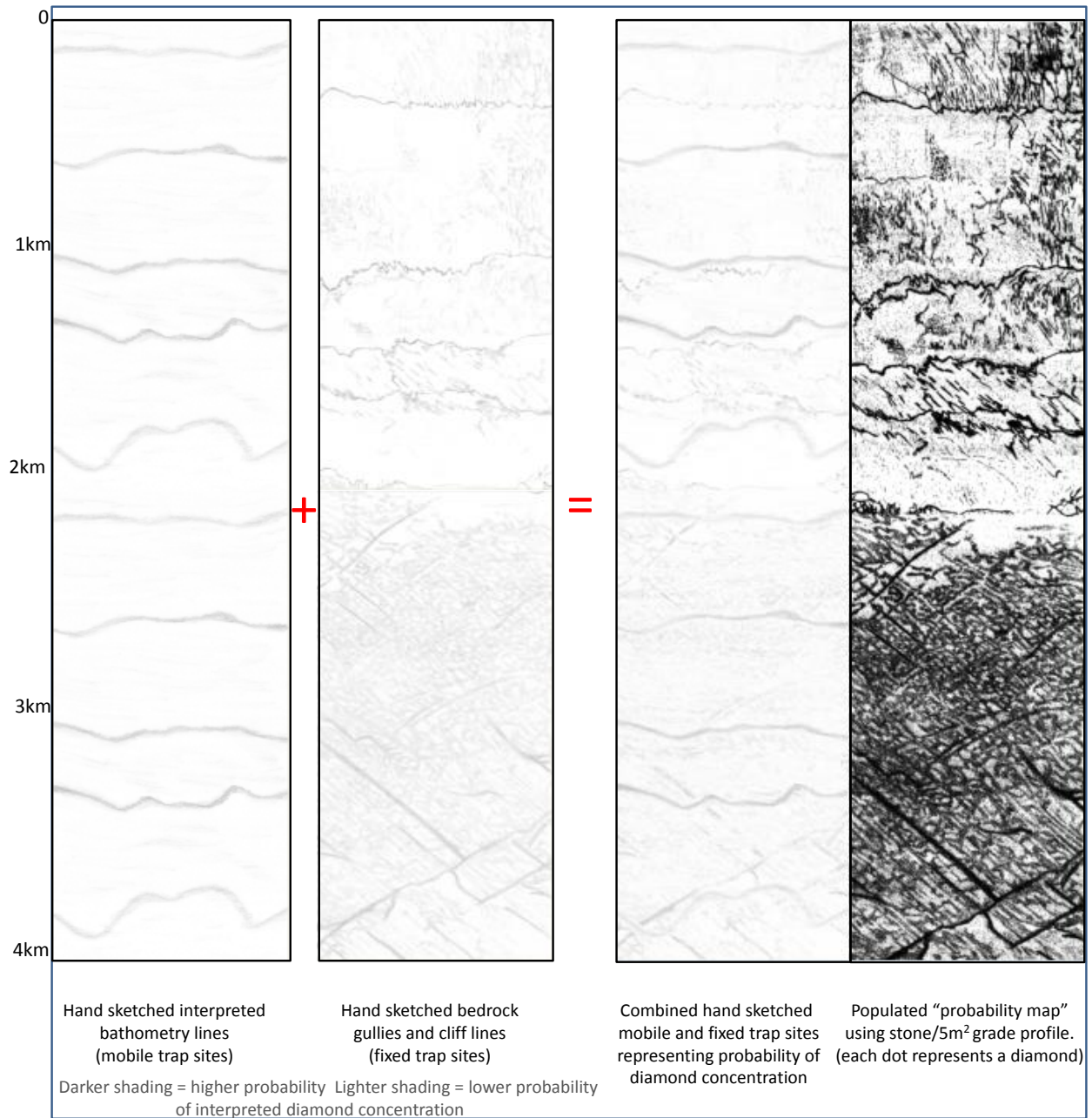


Figure 5 Diamond potential (grey scale), with a line of simulated stone locations

The process is graphically depicted in Figure 6 and it shows how the two hand sketches are merged, and the resulting 1x4km VOB.



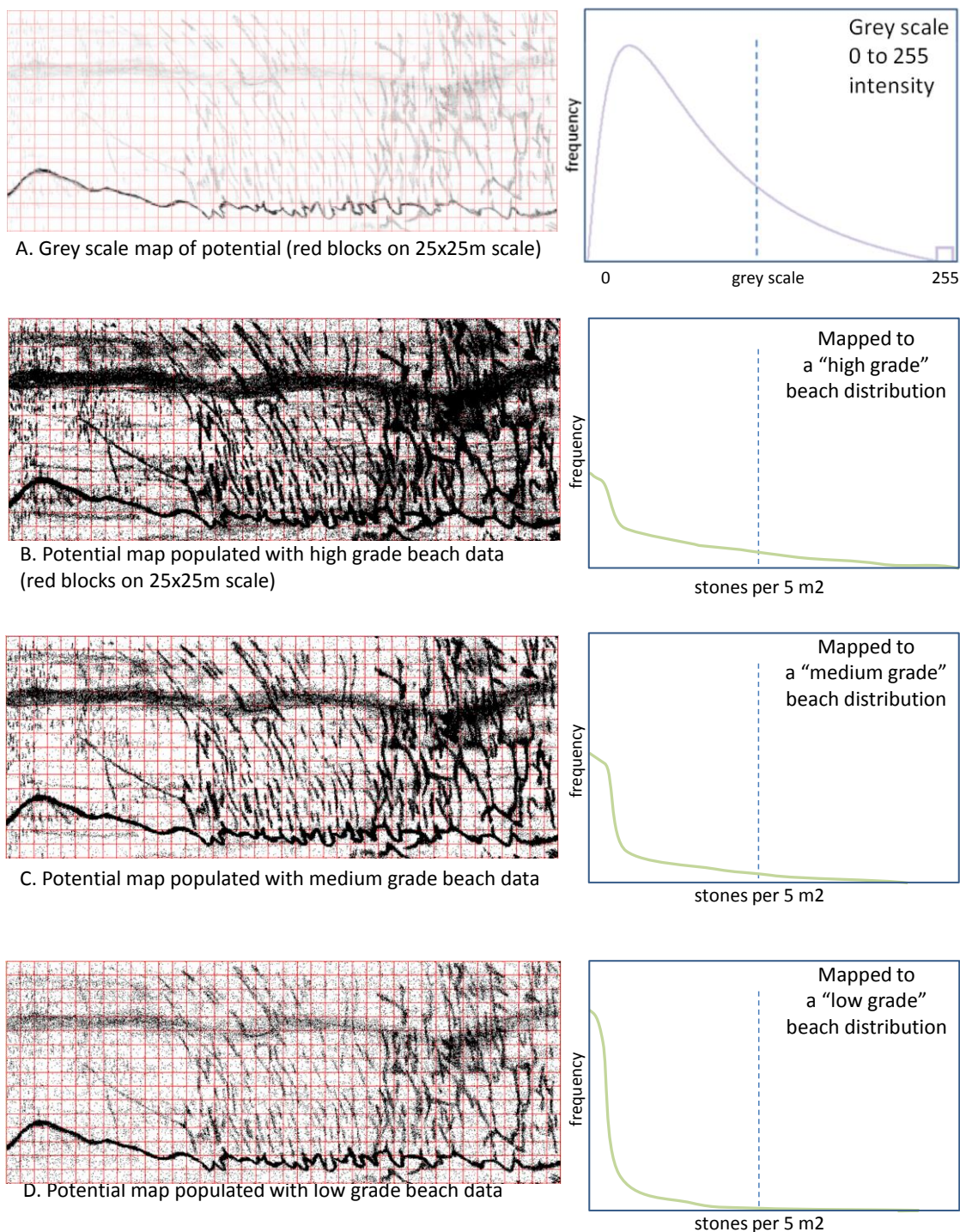
**Figure 6 Process showing the combining of probability maps and grade profile to seed stones.**

This simulation is one realisation that is sampled, evaluated and used to rank different sample spacing/size combinations to determine the optimum sampling strategy.

## **5. Creating different grade scenarios using the VOB**

To create multiple realisations to facilitate decision making for the exploration project the hand sketch is also populated with the grade characteristics of individual beaches.

This is done by regularising the sketch to a sample support size of  $5 \text{ m}^2$  so that it is directly comparable to the support of the proxy sample data. A Gaussian transform of the potential map is back transformed using the transformation table of the proxy data. The underlying potential sketch remains unchanged but different results are obtained when back transforming as the resultant realisations will have low, medium and high grades with different higher order statistics (Figure 7).



**Figure 7** The simulated outcome for a 400 x 1000 m portion of the study area, populated respectively by low, medium and high (schematically presented) beach grades.

In this way a range of possible outcomes are generated as realisations available for analysis. Since grade (the spatial intensity of the stones per area) affects the confidence of the resource estimates for a specific sample campaign, the realisations give an indication of the variability in the outcome of the assessments of the sample campaign effectiveness. The benefit of this work

is that it can be used in risk rankings and assessments. Based on acceptable levels of uncertainty and taking into account the cost to execute a sampling campaign and the confidence obtained for the block estimates, the sampling scenarios can be ranked and a decision made on the appropriate course of action.

## **6. Discussion and Conclusions**

The authors admit to an acute awareness of the challenges of this approach. The quality of the hand sketch, paper texture, grey shading and scan quality all impact on the final result of the VOB. In highly non-stationary environments where kriging is sometimes not possible, sample size and spacing are the most influential factors impacting on the outcome of the final estimate. The VOB presented in this paper provides a first attempt at ranking different sampling scenarios. It also provides a preliminary risk quantification tool until such time when hard data becomes available for incorporation into conditional simulations. As data becomes available a re-assessment of sample size and spacing should be done to confirm the outcome of the sketching technique, using well established geostatistical simulation methods (Kleingeld *et al*, 1996) developed for discrete particles.

## **7. Acknowledgements**

The authors would like to thank the De Beers, Anglo American and Namdeb Mineral Resource Departments for their support, providing access to data and permission to publish this work. The research presented in this paper forms part of a greater resource development project, also incorporated in a PhD study. The authors appreciate the comments and helpful suggestions from the reviewers.

## **8. References**

- Bickel, D. R. (2012). Game-theory probability combination with applications to resolving conflicts between statistical methods. *International Journal of Approximate Reasoning*, 880-891.
- Clemen, R. T., & Winkler, R. L. (1999). Combining probability distributions from experts in risk analysis. *Risk Analysis*, 187-203.
- Cox, D. R. (1955). Some statistical methods connected with series of events (with discussion). *Royal Statistical Society(B17)*, 129-164.
- Jacob, J., Prins, C., & Oelofsen, A. (2013). Determination of sampling configuration for diamondiferous gravel occurrence using geostatistical methods - applied to a probe drill platform. *The Southern African Institute of Mining and Metallurgy and Assay: Best-practise in African mining*.
- Jacob, J., Ward, J. D., Bluck, B. J., Scholz, R. A., & Frimmel, H. E. (2006). Some observations on diamondiferous bedrock gully trapsites on Late Cainzoic, marine-cut platforms of the Sperrgebiet, Namibia. *Ore Geology Reviews*, 493-506.

- Journal, A. G. (1986). Constrained interpolation and qualitative information - the soft kriging approach. *Mathematical Geology*, 269-289.
- Journal, A. G. (1994). Modelling uncertainty: Some conceptual thoughts. In R. Dimitrakopoulos (Ed.), *Geostatistics for the next century* (pp. 30-43). Dordrecht: Kluwer.
- Kleingeld, W. J., Thurston, M. L., Prins, C. F., & Lantuéjoul, C. (1996). The conditional simulation of a Cox process with application to deposits with discrete particles. (E. Y. Baafi, & N. A. Schofield, Eds.) *Geostatistics Wollongong*, 638-694.
- Kuhnert, P. M., Hayes, K., Martin, T. G., & McBride, M. F. (2009). Expert opinion in statistical models. Cairns, Australia: 18th World IMACS/MODSIM Congress.
- Lele, S. R., & Das, A. (2000). Elicited data and incorporation of expert opinion for statistical inference in spatial studies. *Mathematical Geology*, 4, 465-487.
- Pearce, J. L., Cherry, K., Dielsma, M., Ferrier, S., & Whish, G. (2001). Incorporating expert opinion and fine-scale vegetation mapping into statistical models of faunal distribution. *Journal of Applied Ecology*, 38, 412-424.
- Prins, C. F. (2011). *Échantillonnage, simulation et estimation des gisements secondaires de diamante (PhD) Thesis, E.N.S des Mines de Paris.*
- Prins, C. F., & Jacob, J. (2014). Improved variography using simulated annealing to align sample locations within beach structures. *The Journal of the South African Institute of Mining and Metallurgy*, 114.
- Spaggiari, R. I., Bluck, B. J., & Ward, J. D. (2006). Characteristics of diamondiferous Plio-Pleistocene littoral deposits within the palaeo-Orange River mouth, Namibia. *Ore Geology Reviews*, 28, 475-492.
- Stone, M. (1961). The opinion pool. *Annals of Mathematical Statistics*, 1339-1342.
- Truong, P. N., Heuvelink, G. B., & Pebesma, E. (2014). Bayesian area-to-point kriging using expert knowledge as informative priors. *International Journal of Applied Earth Observation and Geoinformation*, 128-138.
- Winkler, R. L. (1968). The consensus of subjective probability distributions. *Management Science*, 361-375.

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## **7. PUBLISHED PAPER IV**

# **Using the Proportion of Barren Samples as a Proxy for Minimum Grade in a Diamondiferous Linear Beach Deposit – An Application of the Nachman Model**

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Over the past 80 years, the Namibian diamondiferous marine placer has been studied extensively to provide solutions for mining and sampling challenges. The types of studies include the statistical modelling of the distributions of the stone counts per sample; investigating the relationship between geology and the grade distribution; assessing the quality potential of the entrapment of the available diamond pulse; using predetermined acceptability of barren samples (zero proportion ( $Z_p$ ) samples) to model distributions; optimal sample sizes, and more. During early-stage project evaluation it is more important to find out if a particular area is likely to be above a specific cut-off grade than to focus on sampling for the purpose of accurate resource estimation. Previous work using mixed Poisson and Sichel distributions to model the abundant onshore diamond data has been very successful in modelling the long-tailed nature of these linear beach deposits. The means of these distributions are, however, sensitive to the occurrence of extreme values. Technical and cost constraints prevent a similar scale of sample collection in an adjacent, geologically equivalent submerged beach environment. A method not sensitive to the occurrence of extreme values is thus required to make early-stage assessments as to the likelihood that the grade of a particular target is above a certain minimum cut-off grade. The Nachman model describes the functional relationship between the mean population density and proportion of unoccupied patches ( $Z_p$ ) in a patchy environment. A prerequisite for using the Nachman model is that the underlying data can be modelled using a negative binomial distribution (NBD). The case study data is from an analogous area adjacent to the exploration target and meets the NBD requirement. It is thus not unreasonable to apply the Nachman model. The Nachman model provides an opportunity to use the observed  $Z_p$  to predict the mean grade for an area at the very early stage of an exploration project. In future, early-stage exploration data from a single geological zone exhibiting characteristics of the Nachman

model assumptions can thus be used to rank and target those areas that show potential to be above the minimum critical grade cut-off for follow-up sampling and inclusion in the mine planning cycle.

Keywords: zero proportion, minimum grade, project evaluation

## INTRODUCTION

Diamond mining of linear beaches along the southwestern coast of Namibia has been carried out since the early 1930s. The onshore raised beaches, stretching 100 km north of the Orange River mouth, have been virtually mined out. Mining of the extension of the raised onshore beaches continues below present-day sea level using a process of beach accretion (Jacob *et al.*, 2013).

Between the 1930s and the 1960s some 26 000 samples were collected during the initial delineation of the diamondiferous raised beach deposit along the 100 km coastal strip north of the Orange River mouth. The delineation took the form of 1 m wide coast-perpendicular trenches sampled at 5 m intervals, resulting in a sample support size of 5 m<sup>2</sup> (1 m × 5 m). These trenches are mostly spaced 500 m apart along the coast (Figure 1). Detailed logging and record keeping of the exact locations where diamonds were found are preserved in hand-drawn trench sections. During this delineation trenching campaign, six raised beaches, A- to F-beach, were identified, with the A-beach closest to the present-day shoreline and the F-beach the furthest inland.

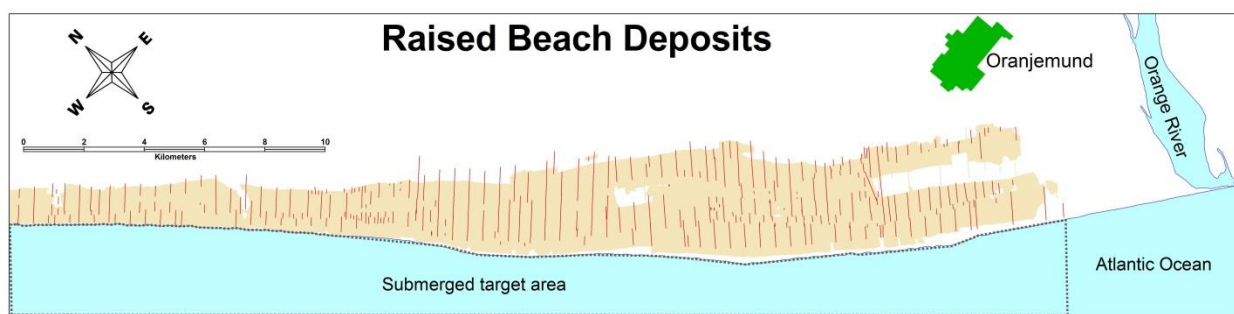


Figure 1. Overview map of the southern portion of diamondiferous raised linear beach deposits north of the Orange River mouth showing 1 m wide delineation trenches and the submerged target area.

## OVERVIEW OF PREVIOUS STATISTICAL WORK ON DIAMONDIFEROUS MARINE PLACERS

The Namibian diamondiferous marine placer has been studied extensively over the past 80 years to provide solutions for mining and sampling challenges. The types of studies include (a) statistical modelling of stone count distributions per sample; (b) using mixed distributions to describe the occurrence of diamonds in trapsites; (c) investigating the relationship between geology and sample grade; (d) determining an optimal sample size using a preselected sample zero proportions ( $Z_p$ ); (e) determining optimal sample sizes, and more. A summary of these studies is given below.

### Statistical Modelling of the Stone Count per Sample

The grade valuation of diamond populations is governed by two aspects: the number of individual stones per sample and the size distribution of the individual stones. Sichel (1973) derived a distribution function that could successfully model the number of stones per sample using a mixed Poisson probability distribution. This distribution is especially useful in modelling populations with a high  $Z_p$  and a long positive tail.

Sichel's distribution function (Sichel, 1973) of the number of stones is given by:

$$\phi(r) = \sqrt{2\alpha/\pi} \exp\left(\alpha\sqrt{1 - 2\beta/\alpha}\right) \frac{\beta^r}{r!} K_{r-\frac{1}{2}}(\alpha) \quad r = 0, 1, 2, 3 \dots, \alpha > 0, \beta > 0$$

with  $K_{\gamma(\cdot)}$  the modified Bessel function of the second kind of order  $\gamma$ , with  $\gamma = r - \frac{1}{2}$  and

$$\hat{\alpha} = [-\ln \hat{\phi}(0)] \left[ 1 - \frac{\ln \hat{\phi}(0)}{2[\bar{r} + \ln \hat{\phi}(0)]} \right] \quad \text{and}$$

$$\hat{\beta} = \frac{\bar{r}}{\hat{\alpha}} \sqrt{\bar{r}^2 + \hat{\alpha}^2} - \bar{r} \quad \text{and}$$

$$\hat{\theta} = \frac{2\hat{\beta}}{\hat{\alpha}}$$

where  $\bar{r}$  is the average number of stones per sample.

The expected frequencies are calculated from the recurrence formula

$$\phi(r) = \frac{\beta}{\alpha} \left( \frac{2r-3}{r} \right) \phi(r-1) + \frac{\beta^2}{r(r-1)} \phi(r-2) \quad r > 1$$

starting with the first two frequencies:

$$\phi(0) = \exp[-\alpha(1 - \sqrt{1 - \theta})]$$

$$\phi(1) = \beta\phi(0) \quad 0 < \theta < 1$$

Sufficient samples are required to define the parameters of the distribution in order to determine the estimated proportion of barren samples,  $\hat{\phi}(0)$ , and the mean  $\bar{r}$  with reasonable confidence. Once such a distribution has been fitted, the mean and associated confidence limits can be determined.

### Using Mixed Distributions to Describe the Occurrence of Diamonds in Trapsites

Diamond (discrete particle) concentration in a marine placer environment is influenced by the quality of diamond pulses available to be concentrated and the quality of the trapsites. A concept of regionalized potential described by Kleingeld (1987) combines the impact of dependant factors influencing diamond concentration. Geological processes at play during diamond deposition are complex. One scenario could be that a diamond pulse containing a relatively high proportion of diamonds is deposited over an area with poor trapsites. A different scenario could be that a high diamond content pulse of gravel is deposited over an area with good trapsites, but that subsequent marine reworking processes could have removed diamonds from these trapsites. The model developed by Kleingeld (1987) makes use of the hypothesis that the number of particles within any region is considered to be a sum of Poisson pockets, with the particles per trap site randomly distributed.

Kleingeld (1987) uses  $\phi(r) = \int_0^\infty P(r|\lambda)f(\lambda)d\lambda$ , with  $P(r|\lambda)$  a Poisson and  $f(\lambda)$  a gamma distribution function, to model the number of particles per sample and the distribution of the trapsites. Kleingeld concluded that when applying Sichel's model,  $\hat{\theta}$  can be estimated with some degree of accuracy only by using large numbers of samples so that a good statistical chance exists for encountering the richer trapsites. Also, through exhaustive testing, Kleingeld determined that it is far superior to use the  $Z_p$  of samples and the mean to determine the Sichel distribution parameters, than using the method of moments.

With  $\bar{r}$  the sample mean and  $\phi(0)$  the sample  $Z_p$ ,  $\hat{\alpha}$  and  $\hat{\theta}$  can be determined from

$$\hat{\alpha} = -\frac{1}{2} \log(\phi(0)) \left( 1 + \frac{\bar{r}}{\bar{r} + \log(\phi(0))} \right)$$

$$\hat{\theta} = 1 - \left[ \frac{-\log(\phi(0))}{2\bar{r} + \log(\phi(0))} \right]^2$$

For early-stage exploration decisions, when only a few data points are available, it will be challenging to model the Sichel distribution with confidence.

### Relationship between Geology and Sample Grade

The complexity of diamondiferous marine placer geological models cannot be overstated. Oosterveld *et al.* (1987) made an attempt to determine the relationship between the geology and statistical parameters obtained from diamond sampling for the diamondiferous marine placer of southwestern Namibia. It was found that those highly complex geological controls influencing the distribution of diamonds could only be described in broad terms by the statistical parameters. For this reason, blocks up to 5 km in length along the coast were estimated (Oosterveld *et al.*, 1987). The large blocks were subsequently sub-blocked into 500 m lengths situated between 1 m trenches and 100 m to 300 m widths perpendicular to the coast. Two smoothed parameters of the Sichel distribution are used as follows: the  $\beta$  contribution is associated with the quantity of trapsites and the  $\theta$  contribution with the abundance of diamonds in trapsites (*i.e.* the quality of trapsites). Oosterveld *et al.* (1987) stabilized the  $\theta$  term by smoothing  $\theta/\sqrt{1-\theta}$  along the coastline with a moving average, based on groups of at least 50 samples. The  $\alpha$  parameter is calculated with  $\theta$  fixed *a priori*. The estimation of Sichel's mean (Sichel, 1973) is log-transformed into

$$\log(D) = \log(\beta) + \log(1/\sqrt{1-\theta})$$

where

$$D = \alpha\theta/2(\sqrt{1-\theta})$$

$$= \beta(\sqrt{1-\theta})$$

and

$$\beta = \alpha\theta/2$$

Oosterveld *et al.* (1987) related the smoothed parameters to broad scale geological observations, diamond pulse, and footwall conditions determining the abundance of trapsites. It was concluded that the highly complex geological controls influencing the spatial distribution of diamonds could only be modelled in broad terms by the statistical parameters (Figure 2).

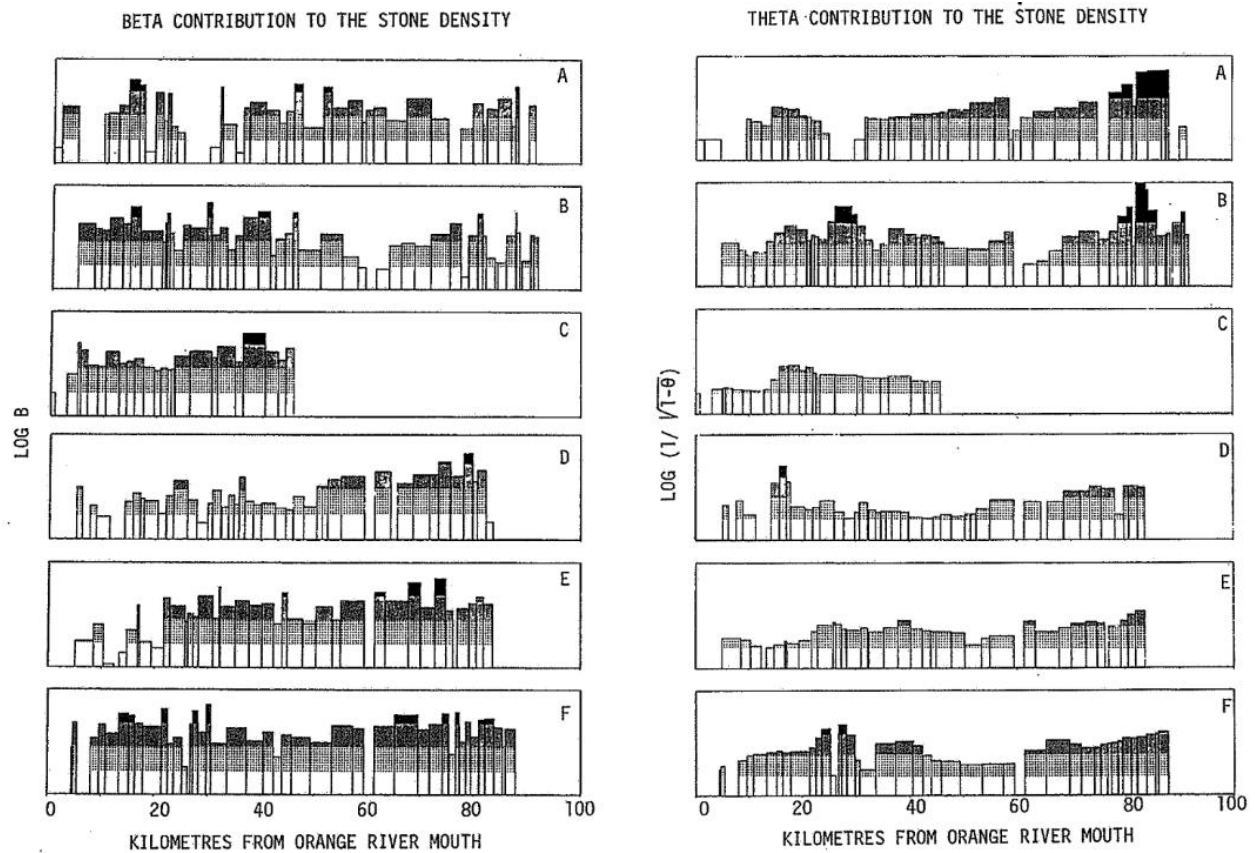


Figure 2. The  $\beta$  and  $\theta$  contributions plotted along the coastline, split into data groups containing at least 50 samples (after Oosterveld *et al.*, 1987).

### Determining an Optimal Sample Size using a Preselected Sample $Z_p$

A challenge that exists prior to the modelling of the individual stone data is the decision about an appropriate sample support size for data collection. In determining a sample size, Sutherland and Dale (1984) developed an approach based on Sichel's mixed Poisson distribution, where an acceptable sample  $Z_p$  is preselected. Based on assumptions about the underlying probability distribution and mean number of stones per sample, a minimum sample size can be determined. In early-stage resource exploration prior to sufficient number of samples being available it is difficult to quantify the mean value  $\bar{r}$  of the target. If, however, a reasonable value for  $\bar{r}$  can be determined early on in a project then Figure 3 (after

Sutherland and Dale, 1984) is very helpful in determining a minimum sample size. This approach does not, however, guide early-stage decision-making regarding continuing or abandoning an exploration target when there is still insufficient data available and the mean and tail of a target's distribution have not yet been well established.

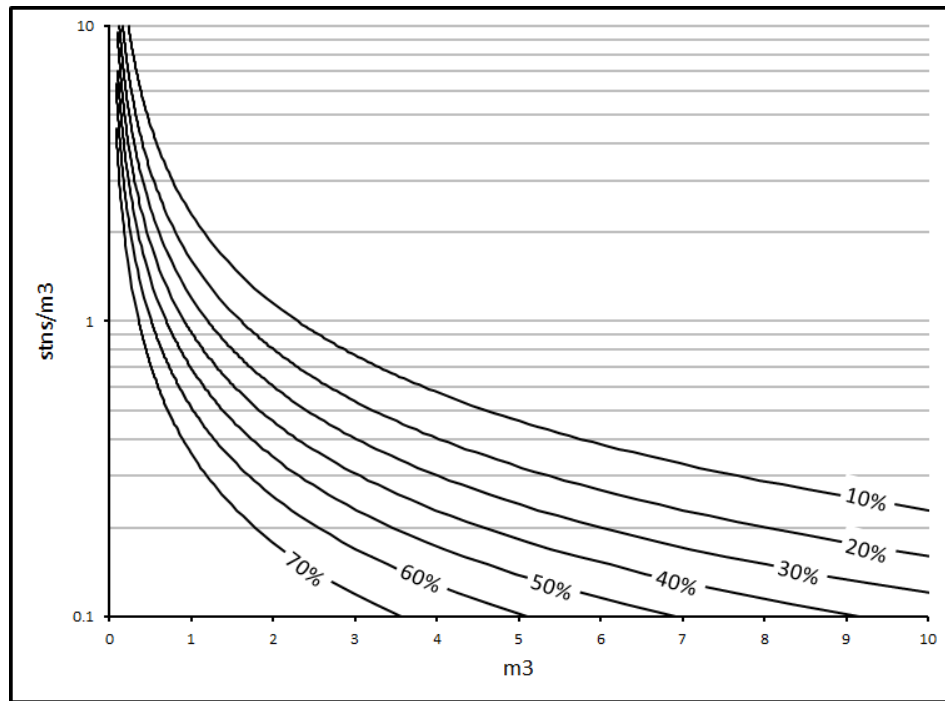


Figure 3. Determining a minimum sample size using a preselected sample  $Z_p$  (lines represent  $Z_p$  as % here) and  $\bar{r}$  (after Sutherland and Dale, 1984).

### Determining an Optimal Sample Size using Pierre Gy's Sample Theory

The application of Pierre Gy's sampling formula to express the sampling error when a volume of material is considered is used by Royle (1986) in an alluvial diamond deposit example. The formula makes use of the physical characteristics of the material being sampled, with the mean squared sampling error expressed as

$$e = \frac{cflg d^3 A^2}{M}$$

with  $c = D_m/a$ ,  $D_m$  the density of the mineral,  $a$  a proportion of the mineral,  $f$  a shape factor,  $l$  expressing liberations,  $g$  a size range factor,  $d$  the mesh size,  $A$  the mean value of the sampled material, and  $M$  the mass of the sample. Royle (1986) outlines detail regarding values to be used for these constants.

From an indication of the expected grade of a deposit, or making use of a minimum economically required cut-off grade, the sample error can be determined. By changing the sample volume and making use of an assumed  $1.96\sqrt{e}$ , the error variance can be used to adjust and express 95% confidence limits for the mean value of the material.

Royle (1986) notes that Gy's formula gives an order of magnitude sample size and that if possible, larger samples would be better. Of utmost importance, however, is that no mathematical or statistical procedure can improve poor quality samples. All effort should be focused on ensuring sample integrity and minimizing the introduction of sample error.

## **PROBLEM STATEMENT**

Technical and cost constraints will always be present in exploration efforts targeting diamondiferous submerged beaches. During early-stage project evaluation it is more important to find out if a particular area is likely to be above a specific cut-off grade than to focus on sampling for the purpose of accurate resource estimation. Previous work on statistical distributions of diamonds was based on abundant onshore data, which has been used very successfully in modelling the long-tailed nature of these linear beach deposits. Technical and cost constraints prevent a similar sampling campaign in a geological equivalent submerged beach target environment. A method not sensitive to the occurrence of extreme values and reliant on less data is thus required in order to make early-stage assessments as to the likelihood that the grade of a particular target is above a certain cut-off grade.

## **PROPOSED METHOD OF APPLYING THE NACHMAN MODEL TO DIAMOND OCCURRENCES**

### **Description of the Nachman Model**

Nachman (1981) describes the functional relationship between the mean population density and the proportion of unoccupied patches ( $Z_p$ ) in a patchy environment, as initially applied to mite counts observed on cucumbers in glasshouses. The Nachman model assumes that the underlying population follows a NBD. Plotting the following transformations against each other shows a linear relationship between the mean discrete counts and the zero proportion.

Zero proportion ( $Z_p$ ) transformation:  $\log\left[\ln\left(\frac{1}{Z_p}\right)\right]$

Mean number of occurrences ( $m$ ) transformation:  $\log(m)$

This linear relationship can then be plotted and used to predict the mean of the data based on the observed  $Zp$ .

### Case Study

As discussed previously, the Sichel distribution fits the long-tailed linear beach data reasonably well. Looking at the 26 000 sample results per trench spaced at about 500 m intervals along the 100 km long coastal strip (discarding any grouping containing fewer than 30 samples), the two-parameter NBD also shows a reasonable good fit to all the data (Figure 4).

The two-parameter NBD depends on parameters  $\nu$  and  $p$  (Kleingeld, 1987). The probability density function is given by:

$$P(X = i) = q^\nu \frac{\Gamma(\nu + i)}{\Gamma(\nu)} \frac{p^i}{i!} \quad i = 0, 1, 2, 3, \dots \quad \text{and} \quad p, q, \nu > 0$$

The mean  $m$  and variance  $\sigma^2$  can be determined from its moments (Kleingeld, 1987):

$$m = \nu p/q \text{ and } \sigma^2 = \nu \frac{p}{q^2}$$

from which

$$p = \frac{\sigma^2 - m}{\sigma^2} \text{ and } \nu = \frac{m^2}{\sigma^2 - m}$$

The assumption required for using a Nachman model, that the data can be described with a NBD, is thus valid. For this reason it is thus reasonable to use the Nachman model to guide exploration effort based on the  $Zp$  of data collected.

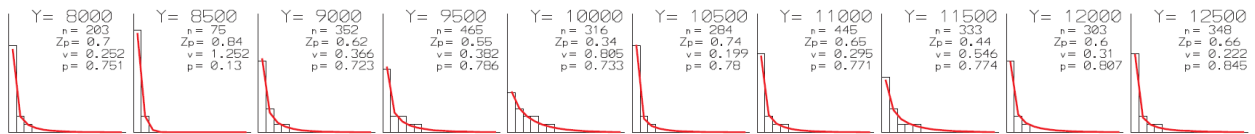


Figure 4. Snapshot of the NBD distribution fitted to diamondiferous linear beach sample data grouped per trench.  $Y$  represents the distance from the Orange River mouth northwards along the coast in metres.

## RESULTS

Recognizing that there would be measurement error in both  $Z_p$  and the mean, a Deming regression (Deming, 1943) is fitted to the Nachman transformed data (Figure 5a). The regression line is subsequently shifted in such a way that 95% of the data falls above it (Figure 5a). The back-transformed Deming regression line (Figure 5b) illustrates that if, for example,  $Z_p$  is equal to 0.60 then 95% of the data will be above a mean of 0.67 stones per 5 m<sup>2</sup>. Based on these results it is perhaps thus not surprising that more than 95% of the onshore diamondiferous raised beach deposit along the 100 km strip north of the Orange River mouth has been mined out to date because it is above the cut-off grade of 0.05 stones per 5 m<sup>2</sup>.

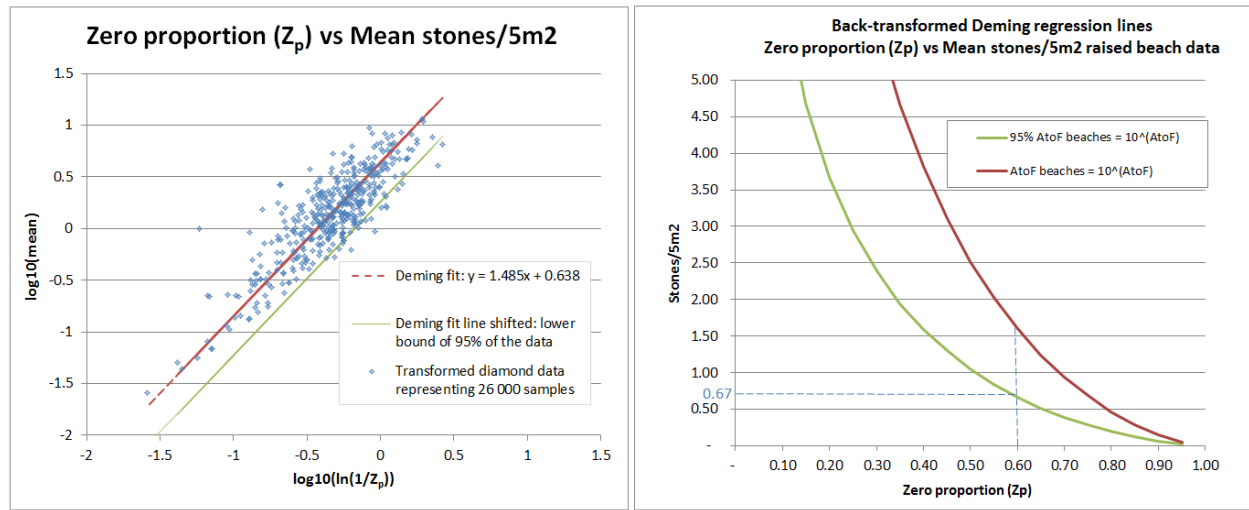


Figure 5. Transformed (a) and back-transformation (b) linear beach diamond data of the Nachman model based on raised onshore beach data.

## DISCUSSION AND CONCLUSION

Previous publications (Sichel, 1966; Sutherland and Dale, 1984; Sichel, 1987; Kleingeld, 1987; Oosterveld *et al.*, 1987) described diamond distributions using variations of the mixed Poisson model. The means of these distributions are, however, sensitive to the occurrence of extreme values and many data points are needed to stabilize the results.

The Nachman model (Nachman, 1981) describes the functional relationship between the mean population density and proportion of unoccupied patches ( $Z_p$ ). In the case study the available analogous data from an area adjacent to the exploration target can be fitted using an NBD. It is thus reasonable to apply the Nachman model. The Nachman model approach provides an opportunity to use the proportion of barren samples to predict the mean grade for an area at the very early stage of an exploration project. In future, early stage exploration data from a single geological zone exhibiting characteristics associated with the Nachman model assumptions can thus be used to rank and target those areas that show potential to be above the minimum critical grade cut-off for follow-up sampling and inclusion in the mine planning cycle.

Comparing Sutherland's approach for determining a minimum sample size using a preselected sample  $Z_p$  and mean (Figure 3) and the outcome of applying the Nachman model (Figure 5) the following is observed:

- A  $Z_p$  of 0.60 (according to the Nachman model) predicts a minimum grade of 0.67 stones per 5 m<sup>2</sup> or 0.134 stones per m<sup>2</sup>.
- Considering a diamondiferous gravel thickness of 1 m, Sutherland's model based on Sichel's distribution at a 5 m<sup>2</sup> support predicts a grade of 0.10 stones per m<sup>2</sup> for  $Z_p$  equal to 0.60.
- The minimum grade predicted by the Nachman model is higher than Sutherland's theoretical model based on the Sichel distribution. Since the Nachman model results are based on actual empirical data, it is deemed more appropriate compared to the theoretical Sichel model results. This affirms the reasonableness of using the Nachman model's  $Z_p$  to predict a potential minimum grade based on samples coming from a single geological zone.

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## REFERENCES

- Deming, W.E. (1943). *Statistical Adjustment of Data*. Wiley, New York.
- Jacob, J., Prins, C., and Oelofsen, A. (2013). Determination of sampling configuration for diamondiferous gravel occurrence using geostatistical methods - applied to a probe drill platform. *Sampling and Analysis: Best Practice in African Mining*, Misty Hills, Mulderdrift, Gauteng, South Africa, 4–6 June 2013. Southern African Institute of Mining, Johannesburg.
- Kleingeld, W.J. (1987). La géostatistique pour des variables discrète. PhD thesis, School of Mines, Paris.
- Nachman, G. (1981). A mathematical model of the functional relationship between density and spatial distribution of a population. *Journal of Animal Ecology*, 50 (2), 453–460.
- Oosterveld, M.M. (1972). Ore reserve estimation and depletion planning for a beach diamond deposit. *Proceedings of the Tenth International Symposium on Application of Computer Methods in Mineral Industry*: South African Institute of Mining and Metallurgy, Johannesburg, South Africa. pp. 65–71).
- Oosterveld, M.M., Campbell, D., and Hazell, K.R. (1987). Geology related to statistical evaluation parameters for a diamondiferous beach deposit. *APCOM 87: Geostatistics. 87. Proceedings of the Twentieth International Symposium on the Application of Computers and Mathematics in the Mineral Industries. Vol. 3*. Lemmer, I.C. (ed.). South African Institute of Mining and Metallurgy in association with the Council for Mineral Technology, Johannesburg. pp. 129–136.
- Royle, A.G. (1986). Alluvial sampling formula and recent advances in alluvial deposit valuation. *Transactions of the Institution of Mining and Metallurgy*, B179–B182.
- Sichel, H.S. (1966). The estimation of means and associated confidence limits for small samples from lognormal populations. *Proceedings of the Symposium on Mathematical Statistics and Computer Applications in Ore Evaluation.*, South African Institute of Mining and Metallurgy, Johannesburg. pp. 106–122.
- Sichel, H.S. (1973). Statistical valuation of diamondiferous deposits. *Journal of the South African Institute of Mining and Metallurgy*, 73, 235–243.
- Sichel, H. S. (1987). Some advances in lognormal theory. *APCOM 87: Geostatistics. 87. Proceedings of the Twentieth International Symposium on the Application of Computers and Mathematics in the Mineral Industries. Vol. 3*. Lemmer, I.C. (ed.). South African Institute of Mining and Metallurgy in association with the Council for Mineral Technology, Johannesburg. pp.3–8.

Sutherland, D.G. and Dale, M.L. (1984). Method of establishing the minimum sample size for sampling alluvial diamond deposits. *Transactions of the Institute of Mining and Metallurgy Section B: Applied Earth Science*, 93, B55–B58.

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## 8. CONCLUSIONS AND FUTURE WORK

In a mining context the characteristics of the sample should represent the characteristics of the whole population. In this case the sample comprises  $n$  individual samples making up the whole sample representing the specific domain for which an estimate is required. Having considered sample integrity and technical and economic constraints the decisions on the location and size of the  $n$  individual samples becomes the most important aspect in terms of generating representative data for mineral resource estimation. Creating simulations is an integral step in the risk mitigation decision making process. The new concepts and applications developed, as outlined in Section 1.2, are highlighted in Section 8. The work presented in this thesis demonstrates how to assess risk mitigating sampling strategy decisions when no or only limited hard data is available for a unique non-stationary geological environment (*Papers II and III*).

The sample configuration for determining gravel occurrence (*Paper II*) was implemented (in 2013) and has proven to be successful in an ongoing delineation campaign. The gravel occurrence detection success is beyond expectation and the expected outcome of the simulation work has been confirmed. A re-assessment of the current sample strategy is thus not required, but ongoing monitoring of the proportion of holes yielding gravel is used to ensure that it remains aligned with the outcome predicted by the simulated model.

The diamond occurrence simulation (*Paper III*) has been the basis upon which a sampling configuration was determined. The shortcomings and limitations of a proxy based simulation are clearly understood. Sample theory and other methods are continuously researched for models and approaches to ensure that no unidentified risk remains until hard data becomes available to inform future decision making. As data become available it will be incorporated into conditional simulations and the sample configuration will be re-assessed.

Once a sampling configuration has been determined an initial indicator of where to focus further exploration effort would be very useful. The Nachman model was researched and tested to ensure that its application would be appropriate for use during early stage exploration projects to assist with ranking the targets based on  $Z_p$  samples as presented in Chapter 7 (*Paper IV*). The generalized relationship between the  $Z_p$  and mean grade has been successfully modelled and will be applied in future.

The scale of spatial structures quantified through variograms is very useful when designing sampling configurations. The usual directional approach with tolerance limits when experimental variogram are calculated did not yield a satisfactory model. Yet, geologically there is an expectation for spatial structure to exist. Inspection of the data showed that directional misalignment (in the along strike direction) with tolerances over-smoothing the data resulted in weak spatial structures. Simulated Annealing (*Paper I*) is used as a minimization technique to improve the detectable variography by aligning / unfolding according to the cliff-lines and beach crests. Results give a clear insight into understanding the large scale structures of the deposit which in turn help to determine appropriate sample line spacing.

In terms of the nonstationary environment of this study area MPS (section 3.1.5) could potentially provide a suitable solution. The MPS training images on point support scale tested for this purpose were too information rich with the resultant simulations proving to be too similar to the initial training image. The HOSIM (section 3.1.6) avenue was abandoned in favour of the hand sketch as the output was judged in terms of the expectation of the experts' opinions. Some scope for future testing using MPS remains and will be pursued to increase the pool of VOBs available for use as the basis for assessing different sampling configurations on exploration target areas.

The use of MPS to simulate different block support sizes in order to assess the risks of mining on various support scales and multiple locations within the orebody should be investigated as this may be beneficial for future mine planning processes in terms of de-risking resource performance.

The number of mining blocks to be mined simultaneously to maintain monthly or quarterly acceptable variability in cash flow requirements can be assessed based on block support size simulations. Historically, resource blocks were up to 5km in length and sub-blocked into 500m lengths with widths of 100 to 300m. When multiple blocks are mined and treated at the same time the resource carat ratio balances out and approximates unity, but the small scale variability will always be high.

When considering the standard Gaussian error =  $\frac{\sigma}{\sqrt{n}}$  where  $n$  is the number of blocks being mined at the same time and  $\sigma$  the standard deviation it is evident that increasing the  $n$  will reduce the standard error.

In highly complex geological environments it is better to rely on simpler robust models limiting the number of required assumptions. The application of the NBD based Nachman model where the  $Z_p$  of samples is used to predict a minimum background grade can have far reaching financial implications for exploration target selection and resource development ranking. In future, once data is collected the NBD assumption will be reassessed.

Solutions found for practical problems related to the sampling of shallow-water marine diamond deposits researched in this thesis could be applied elsewhere with minor modifications to the methodologies followed.

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## 9. REFERENCES

- Anderson, T. W. (2003). *An introduction to multivariate statistical analysis* (3rd ed.). New York: John Wiley and Sons.
- Bertsimas, D., & Tsitsiklis, J. (1993). Simulated Annealing. *Statistical Science*, 8(1), 10-15.
- Bickel, D. R. (2012). Game-theory probability combination with applications to resolving conflicts between statistical methods. *International Journal of Approximate Reasoning*, 880-891.
- Brown, G. J., Lantuejoul, C., & Prins, C. F. (2004). Sample optimization and confidence assessment of marine diamond deposits using Cox simulations. (O. Leuangthong, & C. V. Deutch, Eds.) *Geostatistics Banff*, 315-324.
- Caers, J. (1996a). A General Family of Counting Distributions Suitable for Modeling Cluster Phenomena. *Mathematical Geology*, 601-624.
- Caers, J. (1996b). *PhD Thesis: Statistical and geostatistical valuation of diamond deposits*. Leuven: Katholieke Universiteit te Leuven.
- Chiles, J., & Delfiner, P. (1999). *Geostatistics: Modeling spatial uncertainty*. New York: John Wiley & Sons.
- Chugunova, T., & Hu, L. (2008). Multiple-point statistical simulations constrained by continuous auxiliary data. *Mathematical Geosciences*, 40(2), 133-146.
- Clemen, R. T., & Winkler, R. L. (1999). Combining probability distributions from experts in risk analysis. *Risk Analysis*, 187-203.
- Covacic, D., & Clarke, J. (1990, October). Sampling Techniques. *Mining Magazine*, p. 270.
- Cox, D. R. (1955). Some statistical methods connected with series of events (with discussion). *Royal Statistical Society*(B17), 129-164.
- Daly, C., & Verly, G. W. (1994). Geostatistics and data integration. In R. Dimitrakouloulos (Ed.), *Geostatistics for the next century* (pp. 94-107). Kluwer.
- de Almeida, J. A. (2010). Stochastic simulation methods for characterization of lithoclasses in carbonate reservoirs. *Earth-Science Reviews*, 101, 250-270.
- De Laco, S., & Maggio, S. (2011). Validation techniques for geological patterns simulation based on variogram and multi-point statistics. *Mathematical Geoscience*, 43, 483-500.
- Deming, W. E. (1943). *Statistical adjustment of data*. New York: Wiley.

- Deutsch, C. V., & Journel, A. G. (1998). *GSLIB: Geostatistical software library and user's guide* (2nd ed.). New York: Oxford University Press.
- Deutsch, C. V. (1992). *Annealing techniques applied to reservoir modeling and the integration of geological and engineering (well test) data*. Unpublished Doctoral dissertation, Stanford University.
- Deutsch, C. V., & Journel, A. G. (1992). *Geostatistical software library and user's guide*. New York: Oxford University Press.
- Emery, J., & Lantuéjoul, C. (2014). Can a Training Image be a substitute for a random field model? *Mathematical Geosciences*, 46, 133-147.
- Goffe, W. L., Ferrier, G. D., & Rogers, J. (1994). Global Optimization of Statistical Functions with Simulated Annealing. *Journal of Econometrics*, 60, 65-99.
- Goovaerts, P. (1997). *Geostatistics for natural resources*. New York: Oxford University Press.
- Guardiano, F. S., & Srivastava, R. M. (1993). Multivariate geostatistics: beyond bivariate moments. In A. Soars (Ed.), *Geostatistics-Troia* (pp. 133-144). Dordrecht: Kluwer Academic.
- Jacob, J. (2016). Using the proportion of barren samples as a proxy for minimum grade in a diamondiferous linear beach deposit - an application of the Nachman model. *Diamonds Still Sparkling, Symposium Series S88* (pp. 47-57). Gaborone, Botswana: SAIMM.
- Jacob, J., & Prins, C. F. (2015). Construction of expert opinion based virtual ore bodies for a diamondiferous linear beach deposit. *Geostatistical Geovalue Rewards and Returns of Spatial Modelling* (pp. 155-164). Johannesburg: SAIMM.
- Jacob, J., Prins, C., & Oelofsen, A. (2013). Determination of sampling configuration for diamondiferous gravel occurrence using geostatistical methods - applied to a probe drill platform. *The Southern African Institute of Mining and Metallurgy and Assay: Best-practise in African mining*.
- Jacob, J., Ward, J. D., Bluck, B. J., Scholz, R. A., & Frimmel, H. E. (2006). Some observations on diamondiferous bedrock gully trapsites on Late Cainzoic, marine-cut platforms of the Sperrgebiet, Namibia. *Ore Geology Reviews*, 493-506.
- Jacob, R. J. (2011). *Namdeb Business Plan Technical Report (BPTR) 2011*. Namdeb Diamond Corporation.
- Journel, A. G. (1986). Constrained interpolation and qualitative information - the soft kriging approach. *Mathematical Geology*, 269-289.
- Journel, A. G. (1994). Modeling uncertainty: Some conceptual thoughts. In R. Dimitrakopoulos (Ed.), *Geostatistics for the next century* (pp. 30-43). Dordrecht: Kluwer.
- Journel, A. G. (2002). Combining knowledge from diverse sources: An alternative to traditional data independence hypotheses. *Mathematical Geology*, 34, 573-596.

- Journel, A. G., & Deutch, C. V. (1993). Entropy and spatial disorder. *Mathematical Geology*, 35, 329-355.
- Journel, A. G., & Huijbregts, C. J. (1978). *Mining Geostatistics*. New York: New York Academic Press.
- Journel, A. G., & Kyriakidis, P. C. (2004). *Evaluation of mineral reserves - A simulation approach*. New York: Oxford University Press Inc.
- King, H. F., McHahon, D. W., & Bujtor, G. J. (1982). A Guide to the Understanding of Ore Reserve Estimation. 281. Parkville, Victoria: The Australian Institute of Mining and Metallurgy.
- Kleingeld, W. J. (1987). PhD Thesis "La Geostatistique pour des variables discrete". Paris: School of Mines Paris.
- Kleingeld, W. J., & Lantuejoul, C. H. (1993). Sampling of orebodies with highly dispersed mineralization. In A. Soars (Ed.), *Geostatistics Troia* (pp. 953 - 964). Dordrecht: Kluwer.
- Kleingeld, W. J., Thurston, M. L., Prins, C. F., & Lantuejoul, C. (1996). The conditional simulation of a Cox process with application to deposits with discrete particles. (E. Y. Baafi, & N. A. Schofield, Eds.) *Geostatistics Wollongong*, 638-694.
- Kuhnert, P. M., Hayes, K., Martin, T. G., & McBride, M. F. (2009). Expert opinion in statistical models. Cairns, Australia: 18th World IMACS/MODSIM Congress.
- Lantu  joul, C. (1991). Ergodicity and integral range. *Journal of Microscopy*, 161, 387-403.
- Lantu  joul, C. (2002). *Geostatistical Simulations, models and algorithms*. Springer.
- Lee, Y. M., & Ellis, J. H. (2000). Turning bands method and its application to geostatistical analysis of groundwater contaminant concentration fields. *Journal of Chinese Institute of Environmental Engineering*, 10(1), 1-12.
- Lele, S. R., & Das, A. (2000). Elicited data and incorporation of expert opinion for statistical inference in spatial studies. *Mathematical Geology*, 4, 465-487.
- Liu, Y. (2006). Using the Snesim program for multi-point statistical simulation. *Computers and Geoscience*, 32, 1544-1563.
- Lyman, G. (2011). In situ and particulate material heterogeneity. *5th World Conference on Sampling and Blending*. Santiago, Chile.
- Lyster, S., & Deutch, C. V. (2008). MPS Simulation in a Gibbs sampler algorithm. *Geostatistics, 8th International congress, Chile, Santiago*, 1, 79-88.
- Manchuk, J. G., Stiefenhofer, J., Thurston, M., & Deutch, C. V. (2015). Framework for resource uncertainty prediction and data valuation: An application to diamond deposits. *Canadian Institute of Mining, Metallurgy and Petroleum*, 178-190.

- Mantoglou, A. (1987). Digital simulation of multivariate two- and three-dimensional stochastic processes with a spectral turning bands method. *Mathematical Geology*, 19(2), 129-149.
- Mariethoz, G., & Lefebvre, S. (2014). Bridges between multiple-point geostatistics and texture synthesis: review and guidelines for future research. *Computers and Geosciences*, in press.
- Mariethoz, G., Renard, P., & Straubhaar, J. (2010). The direct sampling method to perform multiple-point geostatistical simulations. *Water Resources Research*, 1-14.
- Matheron, G. (1973). The intrinsic random functions and their applications. *Advanced applied probability*, 439-468.
- Matheron, G. (1981a). Quatre familles discretes. *Centre de Geostatistique, Fontainebleau, Research Report N-703*, 48.
- Matheron, G. (1981b). Deux autres familles, non moins discretes, mais plus nombreuses. *Centre de Geostatistique, Fontainebleau. Research Report N-717*, 63.
- McCallum, K. (2011). MRM CHQ Additional Assurance Note to the Namdeb Business Plant 2012 to 2020.
- Meerschman, E., Pirot, G., Mariethoz, G., Straubhaar, J., van Meirvenne, M., & Renard, P. (2013). A practical guide to performing multiple-point statistical simulations with the Direct Sampling algorithm. *Computers and Geoscience*, 52, 307-324.
- Mustapha, H., & Dimistrakopoulos, R. (2011). HOSIM: A high-order stochastic simulation algorithm for generating three-dimensional complex geological patterns. *Computers and Geosciences*, 37(9), 1242-1253.
- Mustapha, H., & Dimitrakopoulos, R. (2010). High-order stochastic simulation of complex spatially distributed natural phenomena. *Mathematical Geosciences*, 42, 457-485.
- Nachman, G. (1981). A mathematical model of the functional relationship between density and spatial distribution of a population. *Journal of Animal Ecology*, 50(2), 453-460.
- Nowak, M., & Verly, G. (2004). The practice of sequential gaussian simulation. In *Geostatistics Banff* (pp. 387-398). Netherlands: Springer.
- Nunes, R., & de Almeida, J. A. (2010). Parallelization of sequential Gaussian, indicator and direct simulation algorithms. *Computers and Geosciences*, 36, 1042-1052.
- Oosterveld, M. M. (1972). Ore reserve estimation and depletion planning for a beach diamond deposit. *Application of Computer Methods in Mineral Industry: Proceedings of the Tenth International Symposium* (pp. 65-71). Johannesburg, South Africa: SAIMM.
- Oosterveld, M. M. (2003). Evaluation of alluvial diamond deposits. *Geological Society of South Africa: Alluvial diamonds in South Africa workshop*, (pp. 1-7).

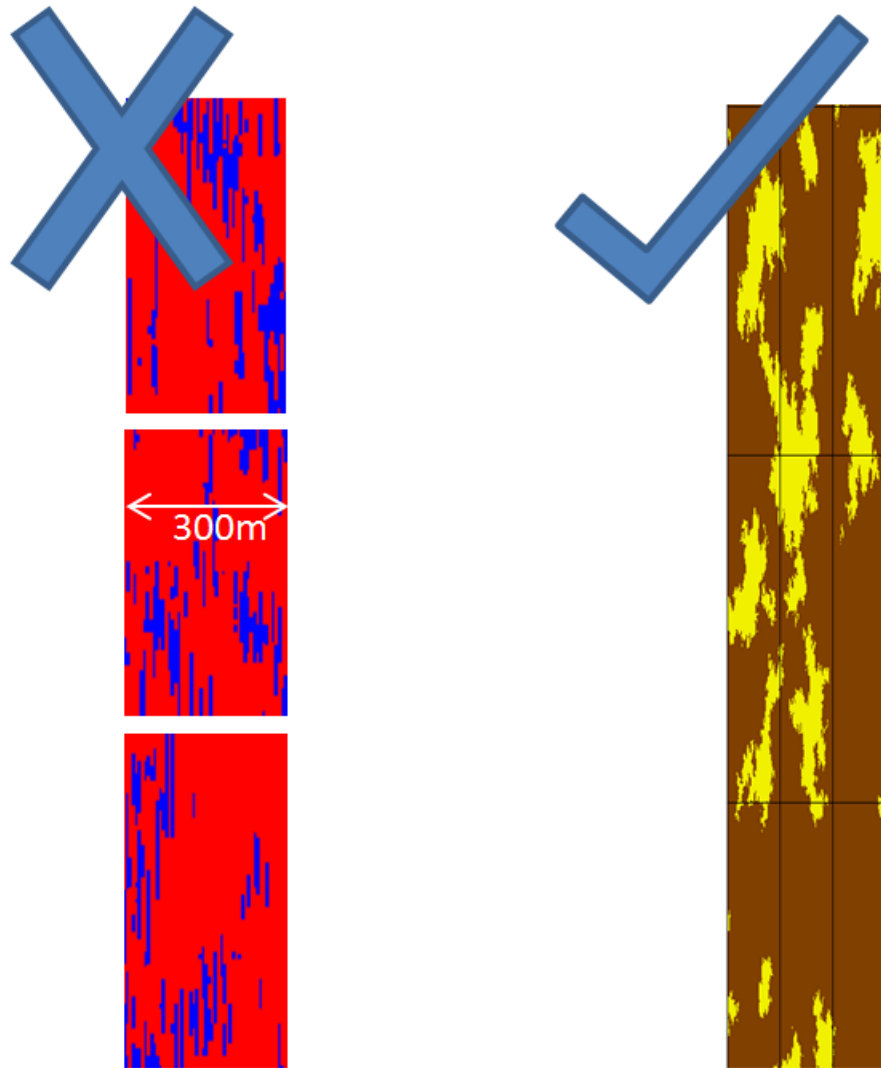
- Oosterveld, M. M., Campbell, D., & Hazell, K. R. (1987). Geology related to statistical evaluation parameters for a diamondiferous beach deposit. *Geostatistics, SAIMM*, 87, 129-136.
- Pearce, J. L., Cherry, K., Dielsma, M., Ferrier, S., & Whish, G. (2001). Incorporating expert opinion and fine-scale vegetation mapping into statistical models of faunal distribution. *Journal of Applied Ecology*, 38, 412-424.
- Perez, C., Mariethoz, G., & Ortiz, J. (2014). Verifying the high-order consistency of training images with data for multiple-point geostatistics. *Computers and Geosciences*, 70, 190-205.
- Prins, C. F. (2011). *Échantillonnage, simulation et estimation des gisements secondaires de diamante (PhD) Thesis, E.N.S des Mines de Paris.*
- Prins, C. F., & Jacob, J. (2014). Improved variography using simulated annealing to align sample locations within beach structures. *The Journal of the South African Institute of Mining and Metallurgy*, 114.
- Remy, N., Boucher, A., & Wu, J. (2009). *Applied Geostatistics with SGems*. New York: Cambridge University Press.
- Royle, A. G. (1986). Alluvial sampling formula and recent advances in alluvial deposit valuation. *Transactions of the Institution of Mining and Metallurgy*, B179-B182.
- Rupprecht, S. (2015). The Proposed 2016 SAMREC Code. *Copper Cobalt, Africa incorporated in the 8th Southern African Base Metals Conference, SAIMM*, 11-20.
- Sichel, H. S. (1966). The estimation of means and associated confidence limits for small samples from lognormal populations. *SAIMM(Symposium on Mathematical Statistics and Computer Applications in Ore Evaluation)*, 106-122.
- Sichel, H. S. (1973). Statistical Valuation of Diamondiferous Deposits. *Journal of the South African Institute of Mining and Metallurgy*, 235-243.
- Sichel, H. S. (1987). Some advances in lognormal theory. *Geostatistics, SAIMM*, 3(APCOM 87), 3-8.
- Soars, A. (1990). Geostatistical estimation of orebody geometry: morphological kriging. *Mathematical Geology*, 22(7), 787-802.
- Spaggiari, R. I., Bluck, B. J., & Ward, J. D. (2006). Characteristics of diamondiferous Plio-Pleistocene littoral deposits within the palaeo-Orange River mouth, Namibia. *Ore Geology Reviews*, 28, 475-492.
- Srivastava, M. (1992). Iterative methods for spatial simulation. *Stanford Center for Reservoir Forecasting*, 5, 24.
- Srivastava, M. (1995). An overview of stochastic methods for reservoir characterization. In J. Yarus, & R. Chambers (Eds.), *Stochastic modeling and geostatistics: principles, methods, and case studies* (pp. 3-16). AAPG Computer Applications in Geology.

- Steve, L., Leuanthong, O., & Deutch, C. V. (2004). Simulated annealing post processing for multiple point statistical reproduction. *Sixth annual report for CCG members, Centre for Computational Geostatistics, University of Alberta*, 109-1 to 109-15.
- Stone, M. (1961). The opinion pool. *Annals of Mathematical Statistics*, 1339-1342.
- Straubhaar, J., Renard, P., Mariethoz, G., Froidevaux, R., & Besson, O. (2011). An improved parallel multi-point algorithm using a list approach. *Mathematical Geoscience*, 43, 305-328.
- Strebelle, S. (2002). Conditional simulation of complex geological structures using multi-points statistics. *Mathematical Geology*, 34, 1-21.
- Sutherland, D. G., & Dale, M. L. (1984). Method of establishing the minimum sample size for sampling alluvial diamond deposits. *Transactions of the Institute of Mining and Metallurgy (Section B: Applied Earth Science)*, 93, B55-B58.
- Thibert, B., Gratier, J. P., & Morvan, J. M. (2005). A direct method for modeling and unfolding developable surfaces and its application to the Ventura Basin (California). *Journal of Structural Geology*, 303-316.
- Tjelmeland, H. (1996). *Stochastic models in reservoir characterization and Markov random fields for compact objects*. Unpublished Doctoral dissertation, Norwegian University of Science and Technology.
- Tomson, A. F., Ababou, R., & Gelhar, L. W. (1989). Implementation of the three-dimensional turning bands random field generator. *Water Resources Research*, 25(10), 2227-2243.
- Tran, T. T. (1994). Improving variogram reproduction on dense simulation grids. *Computers and Geosciences Vol 20*, 1161-1168.
- Truong, P. N., Heuvelink, G. B., & Pebesma, E. (2014). Bayesian area-to-point kriging using expert knowledge as informative priors. *International Journal of Applied Earth Observation and Geoinformation*, 128-138.
- van den Boogaart, K. G. (2006). Some theory for Multiple-point statistics: Fitting, checking and optimally exploiting the training image. *Mathematical Geology*, S12-05.
- Wilde, B. (2009). Minimizing Error Variance in Estimates by Optimum Placement of Samples - A Comparison of Optimization Techniques. *Centre for Computational Geostatistics Annual Report - Paper 406*, 406 1 - 10.
- Winkler, R. L. (1968). The consensus of subjective probability distributions. *Management Science*, 361-375.
- Wu, J., Boucher, A., & Zhang, T. (2008). A SGems code for pattern simulation of continuous and categorical variables: Filtersim. *Computers and Geoscience*, 34(12), 1863-1876.

Zhang, T., Switzer, P., & Journel, A. (2006). Filter-based classification of training image patterns for spatial simulation. *Mathematical Geology*, 38, 63-80.

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## 10. APPENDIX 1 – Texture of gravel occurrence simulation



The linear needle-like texture on the left is an indicator simulation based on a variogram obtained directly from the drilling data. The diffused texture on the right is based on a variogram obtained from interpreted drilling data. The simulation on the right is deemed to be a much better representation of the geological texture of the deposit (See Chapter 6 - *Paper II* for more detail).

# 11. APPENDIX 2 – Generalized NBD model fitted on trench data along the coast

See Chapter 7 (*Paper IV*) for more detail. Each graph represents the NBD fit (red) and parameters for trench data along the coast. Y represents the distance (m) away from the Orange River mouth for each 1m wide sampling trench subdivided into 5m long paddocks.

