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Diversifying and hedging REIT portfolios with cryptocurrencies: Evidence from global and regional REIT indices

Babatunde O. Odusami [a,*], Omokolade Akinsomi [b]

[a] School of Business Administration, Widener University, PA, United States

[b] School of Construction Economics and Management, University of Witwatersrand, Johannesburg, South Africa

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ABSTRACT

Hedging the financial risk of portfolios of securitized real estate assets is daunting because of the unique nature of the underlying assets and because no direct market exists to trade on property-related derivatives. In this paper, we conduct a global study of the diversification and risk mitigation benefits of cryptocurrencies for REIT stocks. Specifically, we examine the dynamic relationship between Bitcoin and REIT indices obtained from multiple countries and regions of the world. We also explore whether portfolios of REIT stocks that are hedged with Bitcoin futures obtain significant improvement in portfolio risk mitigation and return performance. Furthermore, given that Bitcoin and REIT stocks are prone to large volatility swings, we examine the nature of the tail dependences in the multivariate distributions of our REIT indices universe and Bitcoin using Archimedean copula functions. Lastly, we compare the market risk of Bitcoin-hedged REIT portfolios to the unhedged portfolios of REIT stocks using value-at-risk. We find region-specific benefits from using Bitcoin to hedge REIT exposure. Furthermore, we show that using Bitcoin to dynamically hedged REIT portfolios had minimal impact on the bivariate and multivariate joint distributions of REIT portfolios. We also find evidence of region-specific reductions in the market risk exposure of dynamically hedged REIT portfolios relative to unhedged and naively hedged REIT portfolios. We posit that Bitcoin could improve the mean-variance frontiers of REIT investors in countries with higher market friction and lower global integration.

1. Introduction

Since the spectacular collapse in the global prices of Real Estate Investment Trusts (REITs) stocks in the periods that ensued the advent of the COVID-19 pandemic[1] there has been a renewed interest in their portfolio risk and how to hedge them. This is because there are many financial benefits to investors in REIT stocks that exposures to other sectors of the market do not provide. REITs provide a low-entry cost, high liquidity, and tax-efficient exposure to diversified portfolios of real estate assets. REITs also provide a steady income stream, statutory protection, and transparency in asset holdings that other types of equity claims do not provide. Despite these attractive properties, interest in holding REIT stocks amongst institutional and retail investors, in general, has remained stubbornly lower than their holdings of stocks in other sectors of the market that have underperformed REITs over the past decade. Over a ten-year period which began on January 1st, 2012, and ended on December 31st, 2021, the returns on the US REIT sector index outperformed the consumer staples, energy, financial services, and communication services sectors, yet the sector allocations to REITs in the S&P 500 index remained for the most part, lower than the allocations to all of these sectors.[2] We theorize that this puzzle is a byproduct of the challenges investors face in managing the risk of REIT portfolios. Odusami (2021) shows that REITs are particularly susceptible to interest rate fluctuations and crashes during turbulent market periods, however without an effective mechanism to actively manage the risk of REIT exposures, investors rationally choose to not hold REIT stocks to avoid catastrophic losses in the portfolio value.

Indeed, unlike other classes of securities that possess a liquid futures market, REIT stocks do not enjoy such access, due to the specialty nature of the securities. Liang, Seiler, and Chatrath (1998) examined whether

* Corresponding author at: School of Business Administration, Widener University, PA 19013, United States.
E-mail addresses: boodusami@widener.edu (B.O. Odusami), kola.akinsomi@wits.ac.za (O. Akinsomi).

[1] See Akinsomi (2021) for the summary of the performances of REIT stocks during the pandemic.

[2] Between January 1st, 2012, and December 31st, 2021, the total return on the REIT sector of the S&P 500 Index was 149.52%, compared to 139.79%, 106.04%, 98.77%, and − 18.83%, for the consumer staples, communications services, utilities, and energy sectors, respectively.

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the volatility of REIT index returns can be hedged using index futures contracts on equity, bonds, commodities, and precious metals. They found that futures contracts perform poorly as hedges for REIT stocks. They conclude that due to the unique nature of REIT stocks, special types of futures contracts may need to be created for REITs. However, more than two decades later, no such contract has been introduced.[3] Furthermore, the market for other financial products that can be employed to manage the risk of REIT investments such as options are extremely thin relative to similar markets for the stocks of other sectors.[4] Thus, motivating the need to continue to explore new opportunities for cross-market hedging and diversification for REIT investors and portfolio managers.

Although they are widely regarded as speculative assets, cryptocurrencies were one of the asset classes that performed especially well during the period of the pandemic. For example, between March 2020 and December 2020, the prices of cryptocurrencies such as Bitcoin rose by over 224% (from $8945.53 to $28,996.28) while the global REIT index fell by 5.32% (from 3040.11 to 2878.44). Spurred by the recent innovations in blockchain technology and their remarkable performance during the pandemic period, the number of cryptocurrencies in circulation has grown tremendously in recent years. Indeed, there are close to ten thousand cryptocurrencies in circulation around the globe.[5] However, in terms of market capitalization and global adoption, Bitcoin (XBT) dominates. With its current market capitalization of $456 billion, Bitcoin accounts for almost half of the approximately $1 trillion worth of cryptocurrencies in circulation. In terms of trading volume, Bitcoin is the second-highest traded cryptocurrency in the world, after Tether. It is a well-known and globally traded digital asset. Indeed, Foley, Frijns, Garel, and Roh (2022) document evidence of Bitcoin usage in 80 countries as of 2018. As of the time of writing this paper, Bitcoin is owned in over 148 countries by over 420 million investors.[6] The diversification benefits of Bitcoin have also been examined in a number of literature. These include Mohamad and Stavroyiannis (2022) which examined and found the herding effect between Bitcoin and major foreign exchange rates, Haffar and Le Fur (2022) which employ copula-GARCH to examine the time-varying dependence between Bitcoin and six stock market indices, and Nekhili and Sultan (2022), which examined whether Bitcoin risk can be hedged using the equities, commodities, bonds, currencies, and volatility index futures, to name a few. Altogether, the above suggests that Bitcoin could potentially offer some hedging and diversification benefits to REIT investors. Indeed, cryptocurrencies and Bitcoin futures are arguably the only seminal financial innovation since the publication of Liang et al. (1998) that has not been explored in terms of its global cross-market diversification and hedging characteristics for REITs.

Motivated by the above, in this paper, we examine the diversification and risk mitigation benefits of cryptocurrencies for REIT portfolios. Specifically, we examine the dynamics of the conditional covariances between a set of global and regional REIT index portfolios and Bitcoin using a Vector Autoregression with multivariate GARCH (MGARCH) framework. We test whether Bitcoin can mitigate the riskiness and enhance the returns performance of REIT portfolios in a dynamic investment scenario by estimating conditional hedge ratios for REIT portfolios that are dynamically hedged with Bitcoin futures. We assess the effectiveness of the hedged portfolios by backtesting their performances against the unhedged, constant hedge ratio, and naïvely hedged portfolios using in-sample portfolio volatility and return measures. Furthermore, we conduct a robustness assessment of our results by dividing our dataset into sub-periods and reestimating our VARX-MGARCH specification, along with the test of the performance of our dynamic Bitcoin-hedging strategy. We also extend the robustness of our results by analyzing the joint multivariate distribution of Bitcoin and REIT portfolios using copula functions. In doing so, we examined tail dependences between Bitcoin and REITs and compared the performance of the hedged REITs portfolios against the benchmark portfolios by simulating the tails of their distribution and calculating their value-at-risks (VaR).

This paper provides three distinct contributions to literature. First, this paper is the first to explore the hedging potentials of Bitcoin for REIT stocks, using a comprehensive set of global and regional REIT measures. This approach allows us to explore the heterogeneity that may exist amongst the regional real estate markets and the securitized claims on the assets that they own. It also allows us to observe potential heterogeneity in their relationships with Bitcoin using both time-varying and cross-sectional methods. From an investor point of view, our findings highlight the variation in region-specific investment opportunities in REITs. We find that the investment performances of REITs and the dynamics of REIT returns are unique to each region. For example, although it is documented in the literature that REITs in general behave more like stocks than bonds.[7] In reality, this finding is region-specific. The pay-off profiles of REITs in some regions of the world, such as the REITs in Asia, appear more in line with bonds than with stocks. This finding implies that investors can potentially benefit from diversifying their REIT holdings across countries or regions. From a public policy perspective, our study sheds light on the novel roles that cryptocurrencies are performing in the portfolios of investors in economies where the financial markets are not robust enough to support the creation of specialized markets for the trading of contingent claims such as the derivative markets.

Second, this paper is also the first to examine the dependences between the marginal distribution of multinational REIT indices and Bitcoin using copula functions. By employing copula functions, the paper side-steps the constraint that Bitcoin and REITs must share the same univariate marginal distributions before their dependence structure can be examined. It is broadly documented in the literature that the magnitude of the correlations between financial markets tends to change significantly during periods of high market turmoil.[8] This so-called contagion has significant ramifications for the hedging effectiveness given the role that correlations play in computing the hedge ratio. Indeed, the main contribution of the copula approach in this study is that we can examine dependences across the quantiles of the respective univariate distributions of REIT stocks and bitcoin using rank correlations as well as whether the nature of the dependences at the tails of the distribution is statistically different such that it can potentially alter the hedging effectiveness of Bitcoin for REIT portfolios.

Lastly, we also extend the copula approach we employed in this paper to calculate the value-at-risk (VaR) for REIT portfolios after taking into consideration the dependences in their joint distributions. Although effective hedging improves the mean-variance frontier of the combined portfolio, the literature is unclear on whether hedging can improve the reward-to-risk ratio at the tails of the distributions of the returns of the hedged portfolio, particularly if the hedged returns are not normally distributed at the tails of the distribution. This manuscript contributes to the literature by using the copula-VaR to examine whether there are

[3] Property-related derivatives that currently exist are in the form of swap contracts. Fabozzi et al. (2010), present some of the recent implementation of property-related derivatives in Europe. It also discusses some of the implementation challenges for the US.

[4] For example, at the time of writing this manuscript, the average 20-day total volume of options trades on the stock of American Towers Corporation (the largest US REITs by market capitalization) was 5937, compared to 759,387 for Apple Inc., 761,018 for Amazon Inc., and 7012 for ULTA Inc. Although the market capitalization of American Towers Corporation is approximately five times greater than the market capitalization of ULTA Inc.

[5] Statistics obtained from Statista.com at De Best (2023)

[6] See this and other Bitcoin statistics at Triple (2023)

[7] See Gilberto and Shulman (2017)

[8] For example, see Bekaert, Harvey, and Ng (2005),

improvements in the tail risk of Bitcoin-hedged REIT portfolios relative to unhedged REIT portfolios.

The rest of this paper is organized as follows. In Section II, we present a brief review of the literature that examines various topics relating to diversifying and hedging mixed-asset portfolios using REITs and Bitcoin and highlight gaps in the literature. The multivariate GARCH framework and the conditional hedge ratios employed in this research are briefly discussed in Section III. In Section IV, the global and regional REITs and Bitcoin series employed in this paper are described. Also discussed are the statistical properties of the exogenous variables employed in the paper. Section V examines the bidirectional causal relationships between REITs, Bitcoin, and the exogenous variables employed in this paper. In Section VI we present the comparative performance of the dynamically-hedged REIT portfolios to the three benchmark portfolios and highlight the results from our robustness test. Section VII presents the model specification of the copulas employed to analyze tail dependences between the REIT indices and Bitcoin. We also present the results from the VaR specification drawn from the tails of the joint distribution of global and regional REITs. Section VIII provides some concluding remarks.

2. Review of related literature

The diversification benefits of REIT returns have long been examined in real estate literature. Past works that have examined the impact of REITs in a mixed asset portfolio include the works of Chandrashekaran (1999), which examines the time series of REIT returns and finds that REIT stocks may have an important role to play in dynamic asset allocation strategies. Furthermore, the paper shows that conditioning on lagged REIT return offers investors an improved method to predict volatilities and correlations of REITs with other asset classes. Yang, Zhou, and Leung (2012) investigates the index returns of the S&P 500, US corporate bonds, REITs, and commercial mortgage-backed Securities (CMBS) from 1999 to 2008 and found evidence of asymmetric volatilities and correlations in CMBS and REITs. In particular, they found that REIT returns exhibit stronger asymmetric volatilities than other asset classes. Furthermore, the authors show that corporate bonds and CMBS may provide diversification benefits for stocks and REITs. Lee and Stevenson (2005) examine the role of REITs in a mixed asset portfolio. The results of their study show that REITs offer different benefits to different asset classes and mixed-asset portfolios and that these benefits have changed over time. They conclude that the benefits of REITs in a mixed asset portfolio depend on the relative performance of the REITs as well as the alternative assets within the mixed asset portfolio. Glascock, Lu, and So (2000) examined the integration of REIT, bond, and stock returns. The findings from their paper show that REITs behave more like stocks and less like bonds after structural changes that occurred in 1990. Their findings are also buttressed by the findings of Lee and Chiang (2010). They also found that the diversification benefits of including REITs in a mixed asset portfolio diminished significantly after 1992. Further analysis of the diversification benefits of REITs includes the work of Niskanen and Falkenbach (2010) which examined the diversification benefits of REITs from a European context and found a significant positive correlation between REITs and equities. Other papers, such as Lee and Stevenson (2005) which investigate the consistency of the role REITs play in a mixed asset portfolio over different investment horizons show that the benefit of REITs as portfolio diversifiers increases as the holding period increases.

Recently, Boudry, deRoos, and Ukhov (2020) examined the diversification benefits of preferred and common stock of REITs using a utility-based framework in which investors are segmented based on their risk tolerance. The findings in their paper reveal that the common stocks of REITs help low-risk aversion investors attain portfolios with higher returns while preferred REIT stocks assist high-risk aversion investors by providing an avenue for reducing risk. Thereby suggesting that the diversification benefits of REITs are based on the risk tolerance of investors.

Although cryptocurrency is a relatively new asset class, several papers have explored the performance of cryptocurrencies relative to other asset classes. Kurka (2019) examined the asymmetric transmission of shocks between cryptocurrencies (Bitcoin in particular) and other traditional asset classes such as commodities, foreign exchange, stocks, and financials. It finds in general that unconditional connectedness between cryptocurrencies and traditional assets is minimal, while the conditional connectedness analysis shows a significant degree of shock transmission in some periods. Gil-Alana, Abakah, and Rojo (2020) report further evidence in support of these findings in their paper, which tests for cointegration between cryptocurrencies and stock market indices and finds no evidence of cointegration. Charfeddine, Benlagha, and Maouchi (2020) compare the financial properties of cryptocurrencies with major financial securities and commodities and find weak time-varying correlations between cryptocurrencies and conventional assets. Their results support the notion that while cryptocurrencies could be suitable for portfolio diversification, they may be unsuitable as hedging instruments for many asset classes. In contrast, Borri (2019) shows that cryptocurrencies are not exposed to tail risk with respect to other global assets and finds that portfolios of cryptocurrencies offer better risk-adjusted and conditional returns than individual cryptocurrencies. Lastly, a recent paper by Huang et al. (2022) examined the diversification benefits of cryptocurrency assets and finds that the lower the risk aversion of the investor, the more beneficial cryptocurrencies are as portfolio diversifiers.

Together, these results imply that there is little to no connectedness between cryptocurrencies and the stock market, hence positioning cryptocurrencies as a possible diversifier in mixed-asset portfolios. Past literature has also shown that although REITs, in general, have return-enhancing potential, they also possess the ability to offer different benefits to different asset classes at different points over time. Due to the novel nature of cryptocurrencies, there are still gaps in current real estate literature in terms of the study of the potential benefits of cryptocurrency for REIT investors. Indeed, no study to the best of our knowledge at the time of writing this paper has examined this question from both national, regional, and global points of view. This is particularly important given that many REIT companies hold multi-country assets, and that the statutory and business structures of REITs vary slightly amongst countries and regions.[9] Furthermore, while the number of investors who hold cryptocurrencies in their portfolio has grown significantly over the years, the degree of adoption is not the same across countries and regions of the world. This raises the potential for heterogeneity in the diversification and hedging characteristics of mixed asset portfolios of REITs and Bitcoin.[10] Motivated by these points, this research aims to fill the gap by conducting multi-country scrutiny of the diversification and risk mitigation benefits of mixed-asset portfolios that are comprised of REIT stocks and cryptocurrencies.

3. Estimating multivariate GARCH model

To extract the conditional hedging benefits of Bitcoin for our universe of REIT portfolios, we model the dynamic relationship between REITs and Bitcoin returns, while controlling for the effects of other exogenous variables on the dynamics of their return by proposing a multivariate vector autoregression with exogenous variables and GARCH errors (VARX-MGARCH) framework. There are many appealing features of the VARX-MGARCH approach relative to the univariate

[9] [3] Interested reader can review the structure of REITs in different countries by reviewing the Worldwide Real Estate Investment Trust (REIT) regimes (Compare and contrast), published by PWC at PricewaterhouseCoopers (PWC) (2019)

[10] Data on the global cryptocurrency adoption index can be found at ChainalysisTeam (2023)

framework for our modeling objective. First, the VARX framework allows for dynamic feedback across markets from Bitcoin to REITs and vice versa. This feature is particularly important given the differences in the microstructure of each market. Bitcoin trades in a continuous 24-h global market, while REITs trade in country/regional markets during the official trading hours. Second, the VARX framework allows for other exogenous variables to influence the dynamics of the returns of both indices, such that we can control for their effects, as well as test whether they contain meaningful information about future realizations of the returns of both asset classes. Lastly, the multivariate GARCH specification provides a flexible framework to implement various types of covariance relationships that may exist between Bitcoin and REITs. These include conditional covariance, constant and conditional correlations, and asymmetric covariance, to name a few. Given the differences in the microstructure of the Bitcoin and the equity markets from which our universe is drawn, our VARX-MGARCH specification is identified by assuming no contemporaneous transmission of shocks between the Bitcoin and REIT markets. Furthermore, we assume that shock transmission between both markets is short-term and heteroscedastic by nature. Lastly, we assume that shocks to endogenous variables from the exogenous variables are unidirectional such that the direction of causality in the system is from the exogenous to the endogenous variables only.

Due to the lack of a direct futures contract for many of the REIT indices we examine in our empirical model, we assume investors in global and regional REIT portfolios can hedge their long position by taking an offsetting position in Bitcoin futures. Bitcoin is a highly liquid and globally traded asset. In contrast to assets that are traded on centralized exchanges that are located within national boundaries, cryptocurrencies, including Bitcoin are traded on decentralized exchanges using market microstructures that are built on the blockchain algorithm. However, cryptocurrency futures are traded on organized exchanges such as the Chicago Mercantile Exchange (CME), which offers investors futures and options products on Bitcoin and Ether.[11]

In general, the optimal dynamic hedge ratios are the ratios of the conditional covariances of the returns of the reference and hedging assets to the conditional variance of the hedging asset. Since the conditional covariance and variances are conditioned on the stochastic properties of the residuals of the regression, we control for the likelihood of serial correlations in the residuals of the VAR model by specifying a vector of exogenous variables.[12] It is well established in the literature that REIT stocks comove with the broader equity market, although the payoff profiles of REIT companies often resemble those of fixed-income investments.[13] Furthermore, it is common for REIT companies to own assets in multiple countries as a means to diversify their portfolio. This implies that REIT investors may be exposed to global risk factors and the returns of the REIT stock may move in tandem with the global equity market. Chaudhry, Christie-David, and Webb (2010) find that the returns of REIT stock are highly cointegrated with energy prices and suggest hedging potentials using energy futures may exist for REIT investors. More recently, Raza, Ali, Shahzad, and Raza (2018) used the asymmetric DCC-GARCH specification to examine hedging effectiveness for real estate stock returns using futures returns on a commodity index, gold, oil, and bonds. They found that oil and commodities provide the best hedges to real estate stock returns over the short run. Odusami (2021) applied heterogeneous autoregressive volatility and conditional hazard models on the realized jumps derived from six US REIT indices and found that volatility jumps in the returns of REIT stocks could arise from changes in term spread, default spread, VIX, equity market returns, commodity returns, and the U.S. dollar exchange rate. Thus, the exogenous variables for our econometric model include the Morgan Stanley International (MSCI) global equity index, which is a free float market value-weighted index that is comprised of stocks from 23 countries. The index includes countries in the Americas, Europe, the Middle East, and the Pacific regions.[14] Other exogenous variables include the Bloomberg Commodity total return index, which is a diversified hypothetical portfolio that is comprised of six types of commodities (namely: energy, livestock, soft, grains, precious metals, and industrial metals),[15] and the Bloomberg Dollar Spot index, which compares the performance of the U. S. dollar against top 20 currencies in terms of bilateral trading activity.[16]

Our multivariate GARCH augmented vector autoregressive (VARX-MGARCH) model of Bitcoin and REIT index returns with exogenous variables is specified as follows.

$$Y_t = \delta + \sum_{i=1}^{P=1} \phi_i Y_{t-i} + \sum_{m=1}^{S=3} \varphi_m X_{m,t} + \varepsilon_t$$
$$Y_t = \begin{bmatrix} RREIT_t \\ RBXT_t \end{bmatrix}, X_{m,t} = \left[RMSCI_{1,t}, RCOM_{2,t}, REXCH_{3,t}\right] \tag{1}$$

where $RREIT_t$ is the daily return on the REITs index portfolio, $RBXT_t$ represents the daily returns on Bitcoin, and $\left[RMSCI_{1,t}, RCOM_{2,t}, REXCH_{3,t}\right]$ represents the vectors of daily returns on the Morgan Stanley International World Equity index, the Bloomberg Commodity index, and the Bloomberg Dollar Spot index, respectively. ε_t which is the vector of error terms is assumed to be heteroscedastic and follows a conditional variance process defined by H_t, [i.e., $\varepsilon_t = H_t^{1/2}(\theta) z_t$, $z_t \sim N(0, I_N)$]. H_t, which is a positive definite conditional covariance matrix of Y_t can be estimated using any of the numerous MGARCH specifications that have been proposed. In this paper, we employ the well-known BEKK specification of Baba, Engle, Kraft, and Kroner (1990) and Engle and Kroner (1995), which in its simplest form can be written as:

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'H_{t-1}B \tag{2}$$

where C is a lower triangular matrix of constants, A and B are $K \times K$ square matrices of parameters, which together with C are stacked into the vector θ.

There are several benefits of the BEKK specification over alternative multivariate GARCH models. First, the BEKK specification allows the conditional variance and covariance processes for both markets to be directly impacted by intra-market and spill-over shocks from other markets in a dynamic manner. Second, the parameterization of the BEKK specification guarantees the conditional covariance matrix is positive definite. Lastly, the parameters of the mean and variance components of the VARX-MGARCH specifications are all estimated in a single optimization, which guarantees that the dynamic relationships between the mean and covariance process are accounted for simultaneously. Indeed,

[11] Bitcoin futures was first introduced by the CME in December 2017. Currently the CME offers three types of cryptocurrencies derivatives product on Bitcoin and Ether. They include Bitcoin and Ether futures and options, as well as Micro Bitcoin and Micro Ether futures. More information on the cryptocurrency futures traded on the CME can be found at: https://www.cmegroup.com/markets/cryptocurrencies/bitcoin/bitcoin.html

[12] We do not rule the possibility that the dynamics of REIT and Bitcoin returns can be explained by other factors, which if omitted from the VAR specification may result in the violation of the White noise assumptions.

[13] See for example, Myer and Webb (1993).

[14] More information on the index methodology can be found at: https://www.msci.com/World

[15] The weightings for the portfolio components are set to embody four main attributes: economic significance, which reflects the global production and investment holdings data, diversification, continuity, and liquidity. More information on the index methodology can be found at: https://data.bloomberglp.com/professional/sites/10/BCOM-Methodology-December-2019.pdf

[16] More information on the index methodology can be found at: https://data.bloomberglp.com/indices/sites/2/2015/12/Bloomberg_Currency_Indices_Methodology_-FINAL_1.04.2017.pdf

Table 1
Index factsheet table.

Symbol	Description	Top 10% Holding	P/E	P/B	DY	Members	Average Market Cap ($ Billions)	Total Market Cap ($ Billion)
REIT Indices								
ENXG	The FTSE EPRA/NAREIT Global REITs Index	22%	33.10	2.16	3.05	353	$4.29	$1512.68
TERASU	FTSE EPRA Nareit Asia REITs Index	34%	18.40	1.05	4.39	136	$2.78	$267.96
TENEXAPU	FTSE EPRA Nareit Emerging Asia Pacific REITs Index	33%	18.32	0.96	4.90	124	$1.99	$246.28
TERNAU	FTSE EPRA Nareit Developed North America REITs Index	43%	28.94	1.95	4.21	132	$8.22	$1084.24
TENEXEMU	FTSE EPRA Nareit Emerging EMEA REITs Index	71%	7.15	0.75	5.64	39	$0.60	$14.31
TEREEU	FTSE EPRA Nareit Developed Europe REITs Index	39%	13.37	0.73	4.82	67	$1.93	$137.22
TENEXMEU	FTSE EPRA Nareit Emerging Middle East and Africa REITs Index	75%	10.42	0.72	5.79	34	$0.69	$11.16
TENEXU	FTSE EPRA Nareit Emerging REITs Index	44%	10.93	0.80	7.34	47	$0.85	$42.26
Average			**17.58**	**1.14**	**5.02**	**116.50**	**$2.67**	**$414.51**
Equity Indices								
MSCI	MSCI World Index		19.60	2.79	2.19	1508	$38.24	$57,668.60
S&P 500	S&P 500 Index		22.86	3.92	1.71	503	$68.70	$34,556.19
MXMS	MSCI Asia Index		13.15	1.43	2.66	1130	$13.07	$14,770.58
MXAP	MSCI Asia Pacific Index		13.89	1.46	2.93	1485	$14.30	$21,230.40
MXNA	MSCI North America Index		22.46	3.77	1.77	714	$54.17	$38,676.04
MXEU	MSCI Europe Index		14.65	1.93	3.24	424	$27.75	$11,766.89
MXEF	MSCI Emerging Market Index		11.67	1.48	3.19	1373	$14.05	$19,293.92
MXEE	MSCI Emerging Europe Index		9.82	1.69	3.36	154	$22.62	$3483.67
MXEA	MSCI EAFE Index		14.87	1.69	3.24	794	$23.92	$18,992.55
Average			**15.89**	**2.24**	**2.70**	**898.33**	**$31.61**	**$25,180.78**
Other Explanatory Variables								
MXWO	The MSCI World Index		17.12	2.80	2.17	1508	$38.67	$58,315.65
BCOM	Bloomberg Commodity Index					24		

The number 24 represents the number of commodities that are included in the Bloomberg commodity index.

Table 2
Summary statistics.

	Labels	Mean	Min	Median	Max	STDEV	Skewness	Kurtosis	KS-Stats	*P*-Value
Panel A: Regional And Global REIT Indices										
RENXG	Global REIT Index	0.009	−17.009	0.067	8.926	1.182	−2.451	37.330	0.11	0.01
RTERASU	Nareit Asia REIT Index	0.008	−16.181	0.045	9.599	1.032	−2.263	49.866	0.10	0.01
RTENEXAPU	Emerging Asia Pacific REIT Index	0.001	−9.988	0.023	5.808	0.814	−2.319	29.954	0.11	0.01
RTERNAU	Developed North America	0.014	−21.177	0.041	8.617	1.436	−2.383	35.866	0.10	0.01
RTENEXEMU	Emerging EMEA REITs Index	−0.014	−17.422	0.014	10.600	1.866	−0.913	11.786	0.07	0.01
RTEREEU	Europe REIT Index	−0.022	−15.482	0.018	10.496	1.511	−1.195	17.556	0.10	0.01
RTENEXMEU	Emerging Middle East and African REIT Index	−0.011	−18.608	0.007	11.824	1.972	−0.859	11.949	0.07	0.01
RTENEXU	Emerging Market REIT Index	−0.006	−12.813	0.009	10.292	1.297	−1.139	15.229	0.08	0.01
Panel B: Cryptocurrencies and Other Explanatory Variables										
RXBT	Bitcoin	0.219	−31.728	0.173	29.261	4.437	−0.217	5.703	0.11	0.01
RMSCI	MSCI World Index	0.025	−10.442	0.059	8.406	1.018	−1.252	18.519	0.06	0.01
REXCH	Bloomberg Dollar Exchange Rate	0.004	−1.922	0.000	1.793	0.363	0.022	1.914	0.12	0.01
RCOM	Bloomberg Commodity Index	0.020	−5.259	0.048	4.057	0.936	−0.539	3.371	0.04	0.01

Descriptive statistics were obtained from the daily returns of the eight REIT indices, and four exogenous variables. KS-Stats are the test statistics obtained from the Kolmogrov-Smirnov test of normality. The p-value column reports the associated *p*-values of the test.

other statistics of interest, such as the dynamic correlations between asset classes and the dynamic hedge ratios for asset classes can be easily computed from the elements of the conditional covariance matrix of the BEKK-MGARCH specification.

Given T number of observations, and a vector θ of unknown parameters of the mean and BEKK specifications, the conditional density function of the VAR-MGARCH model is:

$$f(Y_t|I_{t-1};\theta) = (2\pi)^{-1}|H_t|^{\frac{1}{2}}exp\left(-\frac{\varepsilon_t^{'}(H_t^{-1})\varepsilon_t}{2}\right) \tag{3}$$

where I_{t-1} denotes the information set that is available at the time $t-1$. From the above, the multivariate log-likelihood function of the VAR-MGARCH model can be specified as:

$$L = \sum_{t=1}^{T} log(f(\varepsilon_t|I_{t-1};\theta)) \tag{4}$$

We derive the parameter estimates of the model by maximizing the log-likelihood function over the parameter space θ using the BFGS algorithm.

Drawing on the specification of H_t, the conditional hedge ratio is calculated as:

$$\pi_t^* = \frac{H_{i,j,t}}{H_{j,t}} \tag{5}$$

Table 3
Correlation coefficients.

	RXBT	RMSCI	REXCH	RCOM	RTERASU	RTENEXAPU	RTERNAU	RTENEXEMU	RTEREEU	RTENEXMEU	RTENEXU
RENXG	0.175	0.761	−0.361	0.277	0.499	0.399	0.972	0.507	0.646	0.497	0.562
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
RTERASU	0.062	0.338	−0.388	0.223	1.000	0.608	0.318	0.427	0.466	0.423	0.533
	(0.01)	(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
RTENEXAPU	0.044	0.343	−0.310	0.224	0.608	1.000	0.274	0.486	0.429	0.480	0.650
	(0.06)	(0.00)	(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
RTERNAU	0.172	0.730	−0.273	0.236	0.318	0.274	1.000	0.406	0.506	0.397	0.442
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)	(0.00)
RTENEXEMU	0.085	0.478	−0.398	0.259	0.427	0.486	0.406	1.000	0.552	0.994	0.926
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)
RTEREEU	0.132	0.572	−0.427	0.271	0.466	0.429	0.506	0.552	1.000	0.537	0.595
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)		(0.00)	(0.00)
RTENEXMEU	0.077	0.459	−0.392	0.251	0.423	0.480	0.397	0.994	0.537	1.000	0.918
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)		(0.00)
RTENEXU	0.100	0.541	−0.459	0.296	0.533	0.650	0.442	0.926	0.595	0.918	1.000
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	

Reported above are the Pearson correlation estimates for the daily returns of the eight REIT indices with exogenous variables and with the other regional and global indices. p-values are shown in parentheses.

Table 4
Granger causality table.

	Panel A: REIT Index					Panel B: RBXT				
	RXBT	REXCH	RMSCI	RCOM	(Joint)	RREIT	REXCH	RMSCI	RCOM	(Joint)
RENXG	**10.28**	**9.78**	11.62	**3.79**	**28.51**	**8.47**	0.38	**5.23**	0.15	7.35
	(0.01)	**(0.01)**	(0.00)	**(0.15)**	**(0.00)**	**(0.01)**	(0.83)	**(0.07)**	(0.93)	(0.29)
RTERASU	**23.19**	**48.78**	**184.68**	**26.98**	**210.41**	0.75	0.38	**5.23**	0.15	7.35
	(0.00)	**(0.00)**	**(0.00)**	**(0.00)**	**(0.00)**	(0.69)	(0.83)	**(0.07)**	(0.93)	(0.29)
RTENEXAPU	**35.17**	**100.22**	**291.83**	**75.76**	**348.96**	4.30	0.38	**5.17**	0.15	7.29
	(0.00)	**(0.00)**	**(0.00)**	**(0.00)**	**(0.00)**	(0.12)	(0.83)	**(0.08)**	(0.93)	(0.29)
RTERNAU	**11.78**	**7.58**	**26.82**	**6.19**	**43.86**	**9.13**	0.38	**5.23**	0.15	7.35
	(0.00)	**(0.02)**	**(0.00)**	**(0.05)**	**(0.00)**	**(0.01)**	(0.83)	**(0.07)**	(0.93)	(0.29)
RTENEXEMU	**8.05**	**54.85**	**52.68**	**16.78**	**90.05**	2.02	0.38	**5.17**	0.15	7.29
	(0.02)	**(0.00)**	**(0.00)**	**(0.00)**	**(0.00)**	(0.36)	(0.83)	**(0.08)**	(0.93)	(0.29)
RTEREEU	**10.96**	**12.04**	**33.83**	**5.82**	**42.05**	4.92	0.38	**5.23**	0.15	7.35
	(0.00)	**(0.00)**	**(0.00)**	**(0.05)**	**(0.00)**	(0.09)	(0.83)	**(0.07)**	(0.93)	(0.29)
RTENEXMEU	**7.60**	**53.18**	**53.22**	**16.98**	**88.19**	2.13	0.38	**5.17**	0.15	7.29
	(0.02)	**(0.00)**	**(0.00)**	**(0.00)**	**(0.00)**	(0.34)	(0.83)	**(0.08)**	(0.93)	(0.29)
RTENEXU	**9.54**	**60.67**	**76.70**	**19.98**	**126.25**	0.14	0.38	**5.17**	0.15	7.29
	(0.01)	**(0.00)**	**(0.00)**	**(0.00)**	**(0.00)**	(0.93)	(0.83)	**(0.08)**	(0.93)	(0.29)

The table above reports the results of the Granger causality test for the null hypothesis that the column variables do not Granger cause the row variables. The p-values of the test statistics are reported in parentheses.

Table 5
Correlations and Hedge Ratios Statistics.

	Panel A: Conditional Correlation				Panel B: Conditional Hedge Ratios				
Mean	Min	Median	Max	STDEV	Mean	Min	Median	Max	STDEV
−0.002	−0.544	−0.003	0.704	0.080	−0.002	−0.362	0.000	0.340	0.024
−0.003	−0.773	0.009	0.459	0.074	−0.004	−0.579	0.001	0.215	0.030
−0.020	−0.577	−0.012	0.388	0.068	−0.006	−0.382	−0.002	0.047	0.027
0.005	−0.503	0.006	0.437	0.135	0.001	−0.147	0.001	0.195	0.031
−0.035	−0.571	−0.030	0.399	0.057	−0.014	−0.502	−0.010	0.140	0.028
−0.010	−0.559	0.000	0.202	0.044	−0.004	−0.438	0.000	0.142	0.019
−0.035	−0.597	−0.028	0.369	0.061	−0.017	−0.568	−0.010	0.134	0.038
−0.036	−0.656	−0.015	0.295	0.076	−0.009	−0.466	−0.003	0.042	0.026

Reported in the table above are the sample averages of the conditional correlations and conditional hedge ratios that were derived from the BEKK GARCH specification. Conditional correlations are derived using the specification shown in Eq. (8), while the conditional hedge ratios were calculated using the specification shown in Eq. (5).

where i and $j$represents REITs and Bitcoin, respectively. Thus, $H_{i,j,t}$ represents the daily conditional covariances between REITs and Bitcoin, and $H_{j,t}$ represents the daily conditional variance of Bitcoin. $H_{i,j,t}$ and $H_{j,t}$ are diagonal elements of the conditional covariance matrix H_t. From the above, the hedge ratio in its simple form is the ratio of the covariance of REITs and Bitcoin returns, and the variance of Bitcoin. The hedged REITs portfolio is obtained by holding a unit position in REIT stocks and an offsetting π_t^* unit position in Bitcoin futures as shown below.

$$HREIT = RREIT - \pi_t^* RXBT \tag{6}$$

where $HREIT$is the daily return on the hedged REIT position.

Following (Geppert, 1995), we measure the degree of the hedging effectiveness of Bitcoin for REIT stock by estimating the percentage

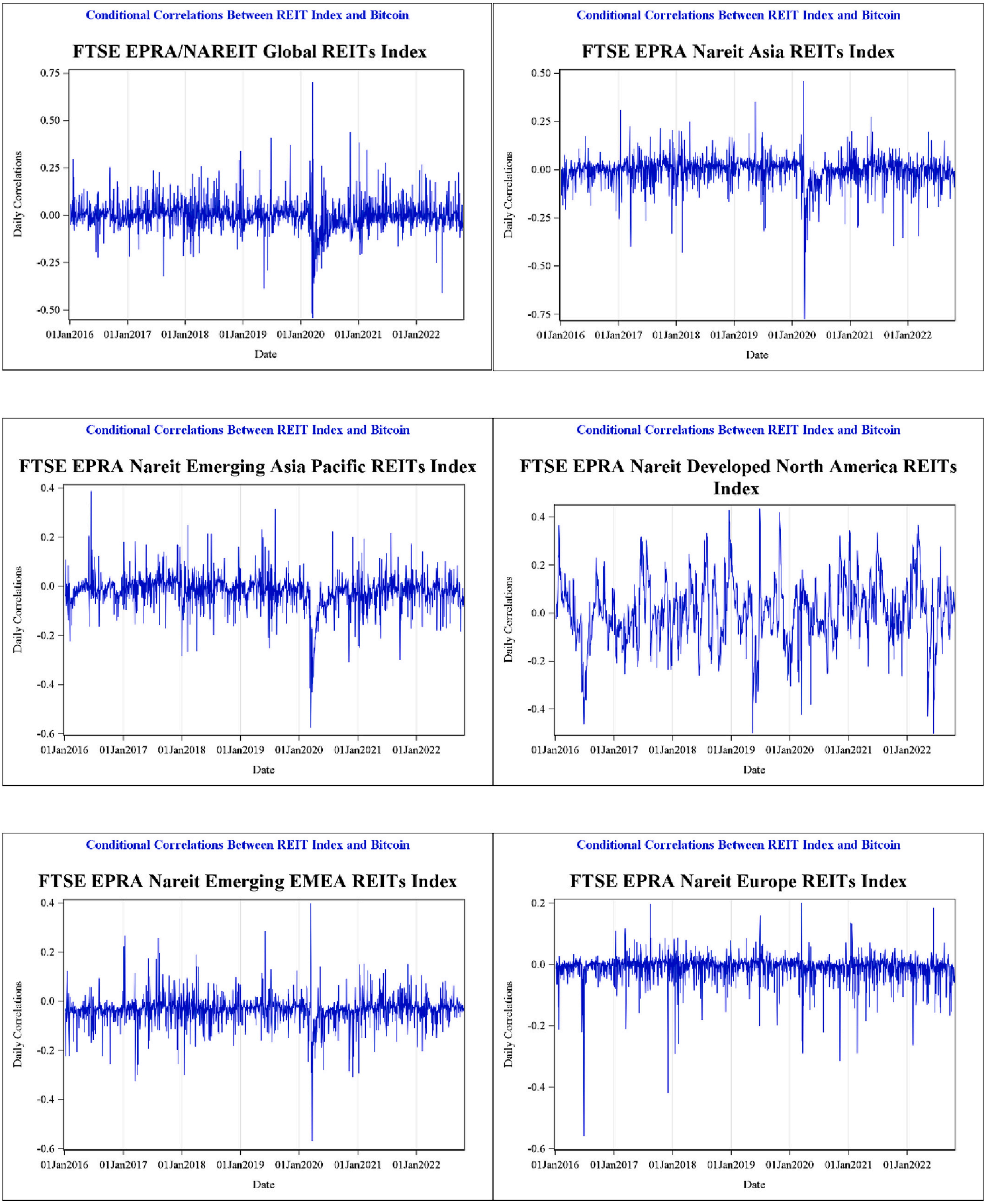
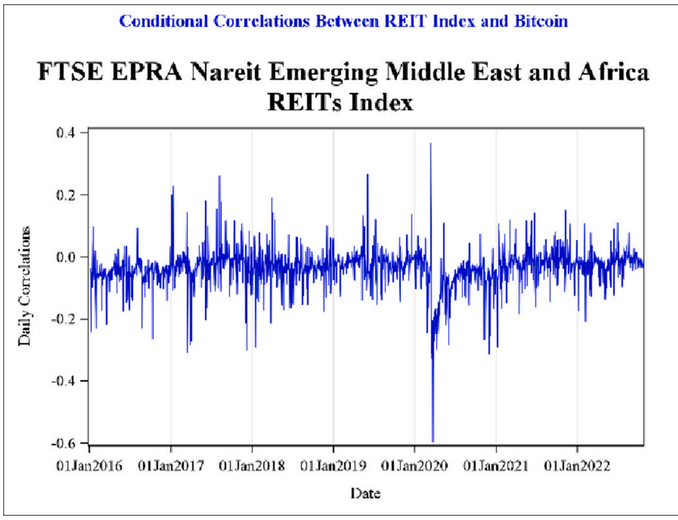


Fig. 1. Conditional correlations between REITs indices and bitcoin.



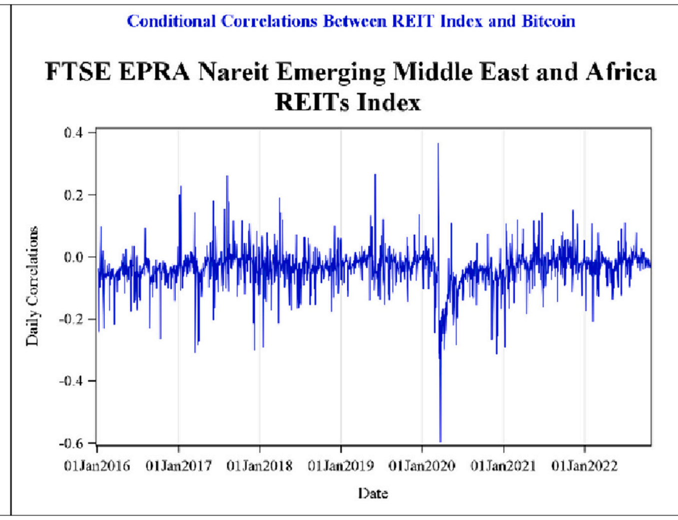


Fig. 1. (*continued*).

reduction in the volatility of the REIT returns in the hedged portfolio relative to an unhedged position. The degree of hedging effectiveness, which is called variance reduction is calculated as.

$$EH = 1 - \frac{\sigma_{HREIT}}{\sigma_{RREIT}} \tag{7}$$

4. Data

In this study, we utilize two sets of data. The first set of data is comprised of the daily returns of eight REIT indices (one global REIT and seven regional REIT indices). The regional REIT indices include those from Asia, Europe, North America, the Middle East and Africa, Emerging Markets, Asia Pacific, and Developed Markets. All indices are published through a partnership between the Financial Times (FTSE), the European Public Real Estate Association (EPRA), and the National Association of Real Estate Investment Trust (NAREITs) and hence carry branding prefix (FTSE EPRA NAREITs). All indices are US dollar-denominated, market value-weighted, and represent the combined value of the largest publicly traded equity REITs in the countries covered. We also obtained daily values of Bitcoin which were sampled at the daily close of the US market, as well as the daily values of the MSCI index, Bloomberg commodity, and Bloomberg US dollar foreign exchange index. The data, which was obtained from Bloomberg Professional Services, spans the sample period, beginning on January 1st, 2016, and ending on October 31st, 2022. In total, the REIT data set is comprised of 1776 daily observations of REIT indices from seven regions and one global REIT index. For each REIT index and explanatory variable, we calculated daily returns by computing the daily log difference in the prices of the variable.

Table 1 presents a factsheet of the global and regional REIT indices in our sample dataset (Panel A). We also include in Panel B, data on the broader equity market in each of these regions to highlight the unique attributes of REITs relative to other sectors of the equity market. Going by the data in Table 1, REITs in general appear to have higher price-to-earnings (PE) ratios and lower price-to-book (PB) ratios than the broader equity market. In line with their unique tax treatment, REITs on average also have higher dividend yield (DY) than the broader equity market. Within the global REIT sectors, significant dispersion in their characteristics also exists. The global REIT index (RENXG), which is comprised of 353 REIT stocks from Australia, the United Kingdom, the US, and Japan, has an average market capitalization of \$4.29 billion. However, that value is about half of the average market capitalization of the North American REITs (\$8.22 billion). The global REIT index (which is dominated by US REITs) and REITs in North America in general, have higher valuation multiples. The price-to-earnings and price-to-book ratios for the global and the North American REIT indices are 33.1 and 28.94, and 2.16 and 1.95, respectively. These values are significantly greater than the total sample average price-to-earnings and price-to-book ratios of 17.58 and 1.14, respectively. Furthermore, the North American REIT index also has the lowest dividend yield and highest market valuations. Together, these stylized facts suggest that there is significant heterogeneity in the financial and investment performance of REITs across the different regions of the world. From a hedging point of view, higher valuation ratios imply higher economic gains and consequently costs from hedging. Indeed, if all things remain the same, we expect to see higher economic gains from hedging in regions of the world in which REITs receive higher valuation ratios such as the US and Europe. Nevertheless, the cost of hedging expensive stocks is generally much higher than the cost of hedging cheaper stocks.[17]

Panel A of Table 2 reports the summary statistics for the REITs and the exogenous variables. The average daily returns on REITs in the US and Asia are positive, while the same statistics for the rest of our sample are negative. REITs in emerging markets, North America, and Europe appear to have the highest volatility, going by the range of the returns and the standard deviations of returns obtained during the sample period. In contrast, North American and Asian REITs appear to have the highest tail risk, going by the kurtosis estimate for the REIT series. All REIT series display negative skewness, which implies that more than half of the daily returns are above the daily average. The KS-stat and *p*-values of the Kolmogorov-Smirnov statistics suggest that the returns series for REITs are not normally distributed. When taken together with the results in Panel A of Table 1, the results suggest that there are elements of uniqueness in the characteristics of REITs amongst the regions. This may lead to varying degrees of hedging effectiveness of Bitcoin for REIT portfolios. REITs in North America enjoy higher valuations but generate lower distributions, they also have higher tail risk than REITs in other regions of the world. REITs in Asia have lower valuations, and volatilities, and generate lower-than-average distributions. Giliberto and Shulman (2017) posit that the dividend yields could explain some of the variations in the interest rate sensitivity of REIT stocks. This is because, during periods of low bond yields, some investors may consider yields on REIT stocks as an alternative to bonds. Together the results suggest that investors in Asia could be perceiving REITs as asset classes that share bond-like attributes whereas investors in the US may be

[17] If we go by the cost of put option on expensive stocks or the margin cost of holding the offsetting position in the futures market.

Table 6
Portfolio performance statistics for cryptocurrency hedged REIT portfolios.

	Panel A: Dynamically Hedged REITs Index					Panel B: Unconditionally Hedged REITs Index					Panel C: Naïvely Hedged REITs Index				
	Mean	Min	Max	Variance Reduction	Economic Gains (Loss)	Mean	Min	Max	Variance Reduction	Economic Gains (Loss)	Mean	Min	Max	Variance Reduction	Economic Gains (Loss)
RENXG	0.01	−16.87	9.48	0.61	−0.002	0.01	−17.03	8.94	0.16	0.001	−0.21	−29.26	26.17	271.31	−0.217
RTERASU	0.01	−11.10	10.49	−3.33	0.004	0.01	−16.07	9.64	0.24	0.002	−0.21	−32.02	25.64	335.16	−0.217
RTENEXAPU	0.00	−10.32	6.35	−1.05	0.003	0.00	−9.84	5.79	0.34	0.002	−0.22	−29.26	26.36	450.07	−0.219
RTERNAU	0.01	−21.65	8.36	−0.40	−0.004	0.01	−21.18	8.62	0.00	0.000	−0.20	−28.97	26.16	207.94	−0.217
RTENEXEMU	0.00	−15.90	10.23	−0.37	0.011	−0.01	−17.30	10.56	0.42	0.004	−0.23	−30.71	28.46	149.98	−0.219
RTEREEU	−0.02	−15.44	11.05	0.22	0.004	−0.02	−15.50	10.52	0.18	0.001	−0.24	−30.23	26.69	197.43	−0.217
RTENEXMEU	0.00	−16.71	11.22	−0.28	0.013	−0.01	−18.47	11.78	0.43	0.004	−0.23	−30.69	28.54	139.05	−0.219
RTENEXU	0.00	−11.36	9.94	−0.23	0.005	0.00	−12.73	10.26	0.51	0.003	−0.22	−30.08	27.37	246.74	−0.219

Average, minimum, and maximum returns (in percent) from dynamically hedged, unconditionally hedged, and naively hedged REIT indices are reported. Variance reduction and economic gains (loss) are reported (in percent) relative to an unhedged REIT index portfolio. Concerning variance reduction, negative values imply lower variance (i.e., variance reduction). Concerning economic gains (loss), positive values imply economic gains from hedging, while negative values imply economic loss from hedging relative to an unhedged REIT position.

perceiving US REITs as equity-type securities, which they truly are. We note that hedging cost is invariably linked to the volatility of the underlying assets. It is generally more expensive to hedge when volatility is higher due to the increased vega risk that the option holder bears if options are used as the vehicle for hedging or the higher cash flow risk from marking-to-market that the futures investor faces from rapid changes in the prices of the underlying. Altogether, hedging REITs should be relatively more beneficial and expensive in regions of the world with higher volatility of REIT returns.

Turning our attention to Bitcoin and the other explanatory variables. The daily average returns for all four variables are positive during the sample period, with Bitcoin displaying the highest average daily return of 0.219%. Bitcoin also displays the highest daily return dispersion with a range of over 60% between the lowest and the highest recorded daily returns. Given this result, it is surprising to see Bitcoin display a kurtosis value that is relatively lower than the same for the commodity index. This may be an artifact of the stylized distribution of Bitcoin returns. Bitcoin is generally perceived as a high-risk investment due to the predominance of large swings in its prices relative to the other asset classes. Indeed, the standard deviation of Bitcoin is approximately four times greater than the standard deviation of the other asset classes in the sample. The average daily return on the MSCI index (RMSCI) is positive. Although the MSCI is the most diversified portfolio in the sample, the emerging Asia, and Pacific REIT index (RTENEXAPU) surpassed the MSCI in terms of total risk, while the Developed Asia REIT index (RTERASU) slightly underperformed it, using the standard deviations of the indices as our measure of risk (0.814%, 1.018%, and 1.032%, for RTENEXAPU, RTERASU, and RMSCI, respectively).

5. Correlation and causality

In this section, we explore linear dependence and causality between REIT returns, Bitcoin (RXBT), and the three covariates (RMSCI, REXCH, and RCOM). In Table 3, we report the unconditional correlation coefficients of the variables. Table 4 reports the bi-directional causality between them. Going by the results in Table 3, it is evident that global and regional REIT returns are correlated with the performances of Bitcoin, as well as those of the global equity, exchange rate, and commodity markets. Notable in the table is the variation in the degree of linear association between these variables and the REIT indices, and amongst the exogenous variables. All REIT indices are shown to be positively correlated with Bitcoin; however, the strength of the linear relationship varies. It is stronger for REITs in developed markets such as North America and Europe than REITs in developing markets and Asia. The same findings hold for the relationship between REITs and MSCI. In contrast, REITs in emerging markets and Asia have stronger negative correlations with exchange rates than REITs in North America and Europe. In general REIT returns are positively correlated with commodity returns. However, the degree of the correlation is stronger for REITs in high commodity-related economies such as those with high commodity production (emerging markets) and consumption (North America and Europe), than REITs in lower commodity-related economies.

Moving on to correlations amongst the REITs. In general, we find evidence of correlation intensity that is related to economic similarities. REITs in developed markets display higher correlations amongst themselves than with REITs in emerging economies. Similarly, REITs in Asia and emerging economies have higher correlations amongst themselves than with the global REIT index and REIT indices from developed countries.

Given the existence of correlations amongst the variables, it is also expedient to explore if causality exists and the direction of the causality. This is because futures markets are generally viewed as mechanisms for transmitting information about prices and transferring risk. From a hedging point of view, understanding the existence and direction of causality can assist investors in determining the optimal diversification

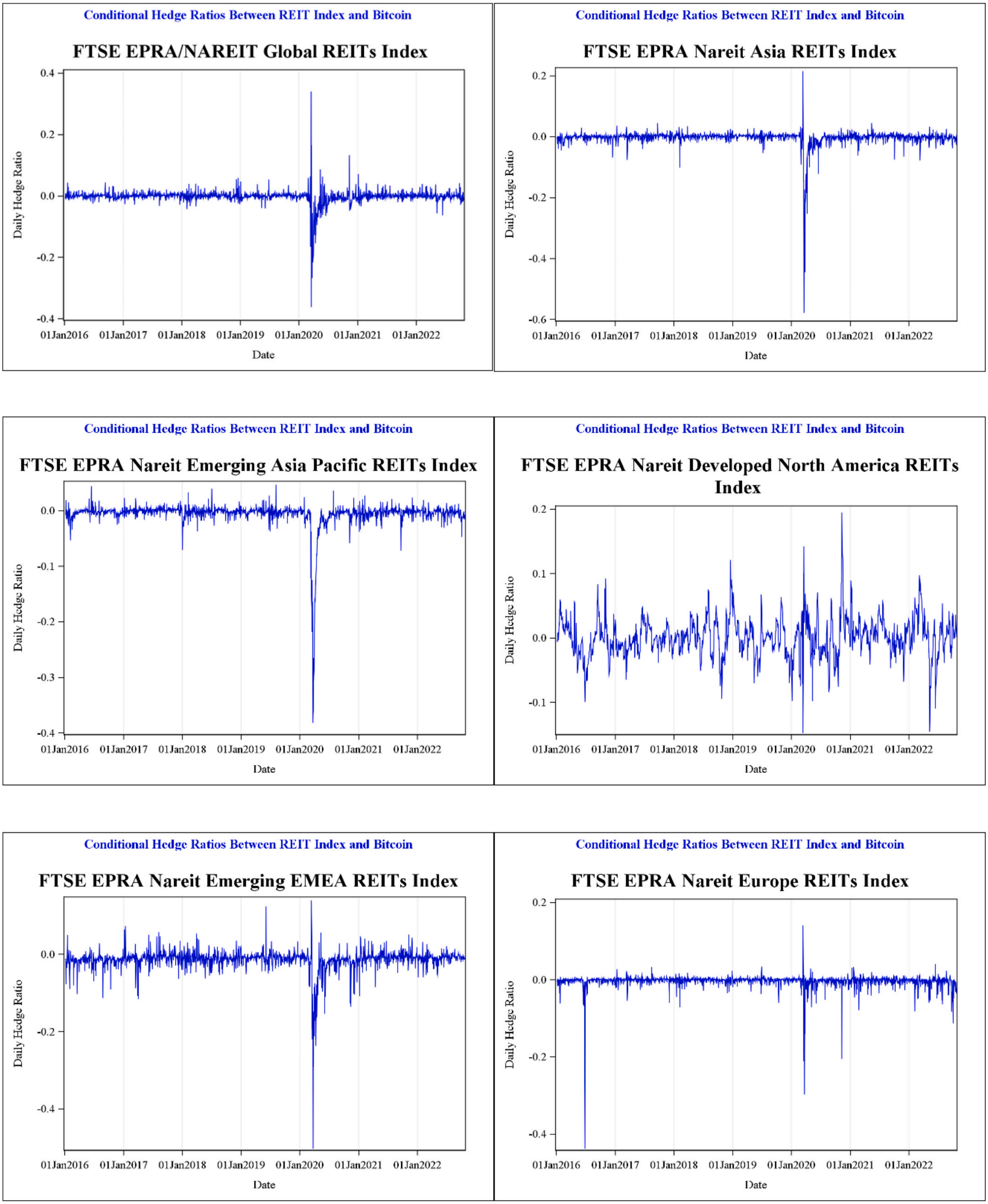


Fig. 2. Conditional hedge ratios for REITs portfolios using bitcoin.

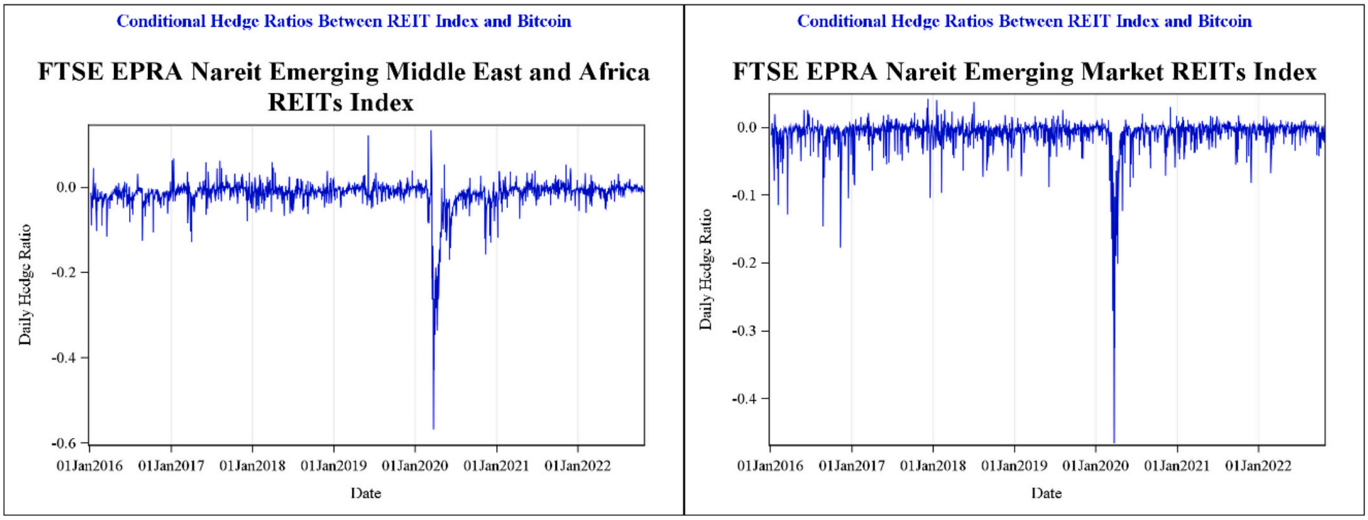


Fig. 2. (*continued*).

and hedging strategies. For example, if a bidirectional causal relationship exists between Bitcoin and REITs, then Bitcoin could serve as an effective hedge for portfolios in REITs and vice versa. If only unidirectional causality exists between both assets, then hedging benefits may also be directional, however, diversification benefits would still exist if the portfolio holdings are properly calibrated.

In Table 4, we report the Granger causality tests for the series. In Panel A, we report the results of the test of the null hypotheses that the column variables (RXBT, RMSCI, REXCH, RCOM) do not Granger cause the row variables (REIT indices). Panel B of the table reports the same statistics for Bitcoin, with the values in the first column showing the statistics for the corresponding REIT series. Overall, the results suggest that all four variables (RXBT, RMSCI, REXCH, RCOM) Granger causes changes in REIT returns. However, the findings appear to be unidirectional relative to Bitcoin. We do not find statistical evidence that REITs in general Granger cause Bitcoin at the 5% level of significance (except for the global REIT index). Furthermore, we also do not find statistically significant evidence that the regression covariates (exchange rates and commodity return) Granger cause Bitcoin. A notable exception to this finding is the MSCI index, which displays statistically significant causality with Bitcoin.

The overall implication of the results reported in this section is the existence of joint dependence amongst the variables examined, thereby motivating the need to further explore the impact of the phenomenon in subsequent sections of the paper. We also establish that the potential hedging benefits between Bitcoin and REITs may be unidirectional. Furthermore, we find that variations in Bitcoin prices appear to have implications for REIT returns, which implies that elements of price discovery for REIT portfolios may exist in the dynamics of Bitcoin prices.

6. Hedging global and regional REIT Portfolios

In Table 5, we report some descriptive statistics for the conditional correlations (Panel A) between the REIT indices returns and Bitcoin, and the conditional hedge ratios (Panel B) that were obtained from the BEKK GARCH specification. Overall, the averages of the conditional correlations are negative (except for the North American REIT index), which is remarkably different from the unconditional correlation coefficients reported in Table 3. The discrepancies arise from the differences in the methodologies employed to calculate the values. Table 3 reports the linear estimates of the correlations, while Table 5 reports the correlation statistics that pertain to the conditional covariance process between Bitcoin and REIT index returns. They are estimated using the following specifications.

$$\rho_{i,j,t} = \frac{H_{i,j,t}}{\sqrt{H_{i,t}} \bullet \sqrt{H_{j,t}}} \tag{8}$$

where i and $j$represents REITs and Bitcoin, respectively. The flexible form of the correlation structure presented in Eq. (8) allows it to evolve as the dynamics between the REIT index and Bitcoin change over time. The low standard deviation of the correlation estimate suggests that correlations are relatively stable over time, however, the values of the minimum and maximum correlation statistics indicate that spikes in correlations occur periodically in the data. In Fig. 1, we highlight the periodic spikes in the daily correlations on the time series plots of the same statistics for the eight REIT indices. The graphs denote that the relationships between Bitcoin and REITs are dynamic and that the directions of the relationship could change depending on the prevailing conditions in either market. These patterns imply that static hedges for REIT portfolios using Bitcoin may lead to suboptimal results, particularly in periods of high volatility in either market.

Panel B of Table 5 reports similar descriptive statistics for the conditional hedge ratios for REIT portfolios using Bitcoin as the hedge. In line with the correlation estimates, the averages of the conditional hedge ratio are also negative (except for the North American REIT index), which implies that on average, long positions in REITs should be offset by short positions in Bitcoin. We note some interesting variations in the hedge ratios across the regions. First, the average hedge ratios appear to be pronouncedly more negative for emerging market REITs than for Asian REITs and REITs in developed markets. The findings also hold for the minimum conditional hedge ratios. Second, the average maximum hedge ratios for the emerging market REITs are lower than the values of the statistics for the REITs in Asia and developed markets. Altogether these results suggest that each region requires unique hedging strategies for optimal risk mitigation for REIT portfolios when using Bitcoin as hedges.

In Table 6, we report the hedging performances of Bitcoin for the REIT indices. Specifically, we compare the variance reduction achievable from the hedge REIT portfolio relative to the unhedged, constant hedge, and naively hedged REIT index portfolios. Also reported are the average economic gains(loss) from the hedged portfolios, relative to the competing portfolios. Except for the hedged global REIT index and the European REIT index, the dynamically hedged REIT index portfolio generated improvements by way of variance reduction, and economic gains (except for North America). Specifically, the dynamically hedged portfolio outperformed the unhedged, unconditionally hedged, and naively hedged portfolios in terms of variance reduction and economic

Table 7
Sub-period portfolio performance statistics for cryptocurrency hedged REIT portfolios.

	Panel A: Dynamically Hedged REITs Index						Panel B: Unconditionally Hedged REITs Index						Panel C: Naïvely Hedged REITs Index					
January 1st, 2016, to December 31st, 2018																		
Series	Mean	Min	Median	Max	Variance Reduction	Economic Gains (Loss)	Mean	Min	Median	Max	Variance Reduction	Economic Gains (Loss)	Mean	Min	Median	Max	Variance Reduction	Economic Gains (Loss)
RENXG	0.01	−3.48	0.06	2.33	−41.8%	−0.001	0.01	−3.49	0.06	2.50	0.00	0.000	−0.26	−29.26	−0.25	26.17	524.51	−0.27
RTERASU	0.03	−3.73	0.07	2.98	−41.1%	0.000	0.03	−3.73	0.06	2.98	0.00	0.000	−0.24	−29.53	−0.24	25.64	549.30	−0.27
RTENEXAPU	0.03	−2.05	0.04	3.23	17.6%	0.003	0.03	−2.08	0.04	3.25	0.01	0.001	−0.24	−29.26	−0.18	26.36	719.72	−0.27
RTERNAU	0.01	−4.09	0.01	3.23	−23.2%	−0.003	0.01	−4.09	0.01	3.23	−0.03	−0.001	−0.26	−28.97	−0.24	26.16	405.93	−0.27
RTENEXEMU	0.02	−7.87	0.07	7.51	−64.3%	0.004	0.02	−7.87	0.06	7.50	0.01	0.001	−0.26	−30.71	−0.19	28.46	182.78	−0.27
RTEREEU	−0.01	−14.98	0.02	5.37	−105.0%	0.003	−0.02	−15.42	0.03	5.41	0.02	0.002	−0.29	−30.23	−0.34	26.69	297.14	−0.27
RTENEXMEU	0.04	−8.65	0.07	7.92	−61.0%	0.004	0.03	−8.65	0.09	7.91	0.01	0.001	−0.24	−30.69	−0.17	28.54	167.92	−0.27
RTENEXU	0.01	−5.67	0.04	5.75	0.8%	0.001	0.01	−5.68	0.03	5.75	0.02	0.001	−0.27	−30.08	−0.23	27.37	295.46	−0.27
January 1st, 2019, to October 31st, 2022																		
RENXG	0.00	−17.22	0.06	9.73	111.3%	−0.002	0.01	−17.05	0.06	8.95	0.29	0.001	−0.20	−29.26	−0.16	26.17	275.27	−0.20
RTERASU	0.01	−10.64	0.04	11.60	−232.2%	0.006	0.00	−16.05	0.04	9.64	0.32	0.002	−0.20	−32.02	−0.20	25.64	347.97	−0.20
RTENEXAPU	0.00	−10.37	0.03	6.00	−82.6%	0.001	0.00	−9.86	0.03	5.79	0.36	0.002	−0.21	−29.26	−0.14	26.36	462.40	−0.20
RTERNAU	0.01	−20.91	0.05	9.67	42.2%	−0.001	0.01	−21.21	0.04	8.64	0.21	0.001	−0.19	−28.97	−0.18	26.16	209.82	−0.20
RTENEXEMU	−0.02	−15.66	0.00	10.14	−19.2%	0.012	−0.03	−17.32	0.00	10.57	0.44	0.003	−0.24	−30.71	−0.18	28.46	165.37	−0.20
RTEREEU	−0.01	−14.65	0.02	10.82	−43.4%	0.004	−0.02	−15.24	0.01	10.48	−0.20	−0.001	−0.22	−30.23	−0.18	26.69	215.06	−0.20
RTENEXMEU	−0.02	−16.19	0.01	10.69	−52.4%	0.014	−0.03	−18.48	0.00	11.78	0.47	0.004	−0.24	−30.69	−0.19	28.54	154.50	−0.20
RTENEXU	−0.01	−10.95	0.00	9.66	−38.0%	0.005	−0.01	−12.74	0.00	10.27	0.52	0.002	−0.22	−30.08	−0.17	27.37	272.24	−0.20

gains. The variance reduction ranges from 0.61% for the global REIT index to −3.33% for the Asian REITs, while the economic gains fall within the range of −0.004 for North American REITs to 0.013 for the Middle East and African REITs. In terms of variance reduction, Asian REITs benefited the most from Bitcoin hedging, followed by North American REITs, and then REITs in emerging markets. The unconditionally and naively hedged portfolios recorded higher variances than the unhedged portfolios. Fig. 2 displays a time series plot of the conditional hedge ratios for the REIT indices. Evident on the graphs are notable spikes in the magnitudes and directions of the hedges around certain periods. From a portfolio management point of view, dynamic hedging provides efficiencies, because significant portfolio resources are deployed for rebalancing only when the market condition warrants them. Furthermore, given that market conditions can change in favor or against certain assets, dynamic hedging provides managers the flexibility to align the portfolio with the prevailing opportunities or risks in the market. Indeed, the findings reported above suggest that REIT portfolios that are dynamically hedged with Bitcoin can achieve higher Sharpe ratios and alphas than unhedged REIT portfolios. Furthermore, the results are robust to REITs in different regions of the world.

6.1. Robustness

We assess whether our findings are statistical artifacts of the sample period we employed by splitting our dataset into two sub-periods. The first sub-period is from January 1st, 2016, to December 31st, 2018, while the second sub-period is from January 1st, 2019, to October 31st' 2022. We then re-estimated our VARX-MGARCH model and computed the dynamic hedge ratios for the two sub-periods. We report the results in Table 7. Overall, we continue to observe meaningful improvement in variance reduction in both sub-periods for the REIT indices in all regions, except for Asia Pacific and emerging REITs in the first sub-period and North American and Global REITs in the second period. Furthermore, the dynamically hedged REIT indices continue to outperform the unconditional and naively hedged REIT indices in all regions and in both sub-periods. Together, these results reinforce that our findings are robust to sample period selection bias.

7. Joint dependences between bitcoin and REITs

In the previous section, we documented the existence of linear correlations, as well as the notable spikes in conditional correlations between REITs and Bitcoin. We also document evidence of spikes in the conditional hedge ratios of dynamically Bitcoin-hedged REIT portfolios. Indeed, the findings that correlation patterns often change during turbulent market periods are well established in the literature.[18] From a hedging perspective, variations in correlation patterns increases the risk that the hedged portfolio may not achieve the desired outcome in extremely volatile periods. Furthermore, common correlation measures such as the Pearson correlations are unreliable for statistical inference because they are susceptible to outliers. Motivated by these factors, we examine dependences across the quantiles of the respective univariate distributions of REIT stocks and Bitcoin using rank correlations as well as whether the magnitude of the dependences at the tails of their respective distribution are materially significant to such an extent that they can potentially alter the hedging effectiveness of Bitcoin for REIT portfolios.

We also employ the copula approach to model the joint distribution of Bitcoin and REITs, and the joint distribution amongst the regional REIT indices. In this approach, we are particularly interested in whether the properties of the joint distributions of REIT indices can explain the heterogeneity observed in the hedging effectiveness of Bitcoin for the

[18] Scott (2020) provides a comprehensive review of the literature that have examined connectedness, contagion, and correlations in financial markets.

Table 8
Copula estimates and tail dependences.

Panel A: Unhedged Portfolio					Panel B: Dynamically Hedged Portfolio			
	Theta		Kendall's Tau	Lambda	Theta		Kendall's Tau	Lambda
RENXG	0.18	(0.00)	0.08	0.02	0.18	(0.00)	0.08	0.02
RTERASU	0.09	(0.00)	0.04	0.00	0.10	(0.00)	0.05	0.00
RTENEXAPU	0.07	(0.01)	0.04	0.00	0.10	(0.00)	0.05	0.00
RTERNAU	0.17	(0.00)	0.08	0.02	0.16	(0.00)	0.07	0.01
RTENEXEMU	0.10	(0.00)	0.05	0.00	0.13	(0.00)	0.06	0.01
RTEREEU	0.14	(0.00)	0.07	0.01	0.15	(0.00)	0.07	0.01
RTENEXMEU	0.09	(0.00)	0.04	0.00	0.13	(0.00)	0.06	0.00
RTENEXU	0.13	(0.00)	0.06	0.00	0.16	(0.00)	0.07	0.01
Joint	0.49	(0.00)	0.20	0.24	0.49	(0.00)	0.20	0.24

Coefficients of the bivariate and multivariate joint dependence between Bitcoin and REITs are reported. *P*-values are in parentheses. The copula parameters were calculated using the specifications shown in Eqs. (11–13).

regional indices.

In normal times, effective hedging reduces the standard deviations of portfolios. However, in turbulent times, which could cause the structure of the dynamic relationship between the hedging and hedged asset to change, hedging may actually lead to catastrophic portfolio losses if the changes in the multivariate extreme value distributions of the assets in the portfolio are not accounted for in constructing the hedge.[19] Furthermore, Bitcoin returns (as shown in Table 1) are generally more volatile than REIT returns, and REIT returns generally have higher tail risks than Bitcoin returns. Motivated by these concerns, we also examined the tail risk performance of the Bitcoin-hedged REIT portfolios by drawing on our copula model. Specifically, we estimated the parameters of the copula functions of the multivariate joint distribution of Bitcoin and the Bitcoin-hedged REIT portfolios which we obtained from the BEKK GARCH specification. We then simulated 10,000 observations of portfolio returns for the Bitcoin-hedged and unhedged REIT portfolios and calculated their 90%, 95%, and 99% value-at-risk (VaR) quantiles.

Following Sklar (1973) theorem, the joint distribution function for a random vector $Y = (Y_1, \ldots\ldots, Y_d)$ with continuous marginal cumulative distribution functions $F_1, \ldots\ldots, F_d$ can be expressed as:

$$G(Y) = C(F_1(y_1), \ldots\ldots, F_d(y_d)) \tag{9}$$

where $C : [0,1]^d \rightarrow [0,1]$ denotes the d-dimensional copula that is the multivariate distribution of the vector of random variables with the uniformly distributed marginal distribution. Such that:

$$C(u_1, \ldots\ldots, u_n) = G\left(F_1^{-1}(u_1), \ldots\ldots, F_1^{-1}(u_1)\right) \tag{10}$$

where $u_1 = F_1(y_1)$ and F_1^{-1} are the quantile functions of the marginals. Conceptually, copulas couple the marginal CDF distribution u_1 to the joint distribution G. Therefore, they provide an efficient means to extract the dependence structures between univariate marginal distributions by modeling their joint distributions using copula functions. In recent years, various copula functions have been proposed. They include the elliptical copulas such as the Gaussian and Student-t copulas, as well as the Archimedean copula functions such as the Clayton, Gumbel, and Frank copulas. Elliptical copulas are radially symmetric because they are copulas of elliptical marginal distributions such as the normal and the Student-*t*, although the Student-t has fatter tails than the normal copula. Given that the objective of this paper is to scrutinize the hedging potentials of cryptocurrency for REITs, we are especially interested in copulas that can account for dependences and asymmetry at the lower tail of the distributions. Thus, we select the Clayton copula, which has asymmetric lower tail dependence as our choice copula. The Clayton copula with the generator function $\phi(u) = \theta^{-1}\left(u^{-1} - 1\right)$ is generalized as

$$C_\theta(u_1, \ldots\ldots, u_d) = max\left(\left[\sum_{i=1}^{d} u_i^{-\theta} - d + 1\right]^{-1/\theta}, 0\right) \tag{11}$$

where θ is the copula parameter. The generator function $\phi(u)$ is the CDF of the distribution with a mean of 0 and variance of 1. The relationship between its shape parameter θ and its corresponding ranked correlation Kendall's tau (τ) which measures dependence is given by.

$$\tau = \frac{\theta}{\theta + 2} \tag{12}$$

The coefficient of tail dependence (lambda) of the Clayton copula is zero$[\lambda_u(Y_1, Y_2) = 0]$ in the upper tails, while the lower tail dependence coefficient is expressed as:

$$\lambda_l(Y_1, Y_2) = limP\left(Y_1 < F_2^{-1}(u_2) \,|\, Y_2 < F_1^{-1}(u_1),\right) = 2^{-\frac{1}{\theta}}, \quad (\theta > 0) \tag{13}$$

In Table 8, we report the estimates of the copula parameters, Kendall's Tau, and the lower tail dependence coefficients for the bivariate joint distributions of Bitcoin and unhedged REIT indices returns (in Panel A), and the bivariate joint distribution of Bitcoin and hedged REIT indices returns (in Panel B). Given that we found statistically significant evidence of linear correlations amongst the REIT indices, we also report the copula parameter, Kendall's Tau, and lower tail dependence coefficient for the multivariate joint distribution that is comprised of Bitcoin and all the REIT indices examined in this study. Going by the estimates of the copula parameter θ, the results support the existence of a bivariate joint distribution between Bitcoin and the REIT indices, as well as a multivariate joint distribution amongst the REIT indices. A notable aspect of the result is the minimal impact of Bitcoin in the dynamically hedged portfolio on the bivariate and multivariate joint distribution copula parameters. The average increase in θ and Kendall's tau for all REIT indices were 0.02 and 0.01, respectively. We also found that on average, the lower tail dependence between Bitcoin and REITs is very low (0.01). Therefore, we surmise that dynamically hedging REIT portfolios with Bitcoin would not increase the likelihood of lower tail dependences between Bitcoin and REIT indices. In Fig. 3, we present scatter plots that graph the daily returns of Bitcoin against the corresponding dynamically hedged and unhedged REIT indices, and Tail plots, which graph the marginal densities of Bitcoin against their corresponding values for the dynamically hedged and unhedged REIT indices. In general, the scatter plots show clustering of return performances in the regions of their respective means of the series and a systematic association in the performance at the upper tail of the

[19] In May 2012, JPMorgan Chase announced that it had lost over $2 billion on a failed hedging strategy. By the end of the year, the total loss from the failed strategy had grown to over $6 billion. Indeed, a large proportion of the losses were incurred in unwinding the failed hedge in subsequent months, after the bank realized that the hedged position would actually result in increased risk for the bank due to improving economic conditions. See Zeissler, Ikeda, and Metrick (2019) for a detailed discussion of the JPMorgan Chase London Whale crisis.

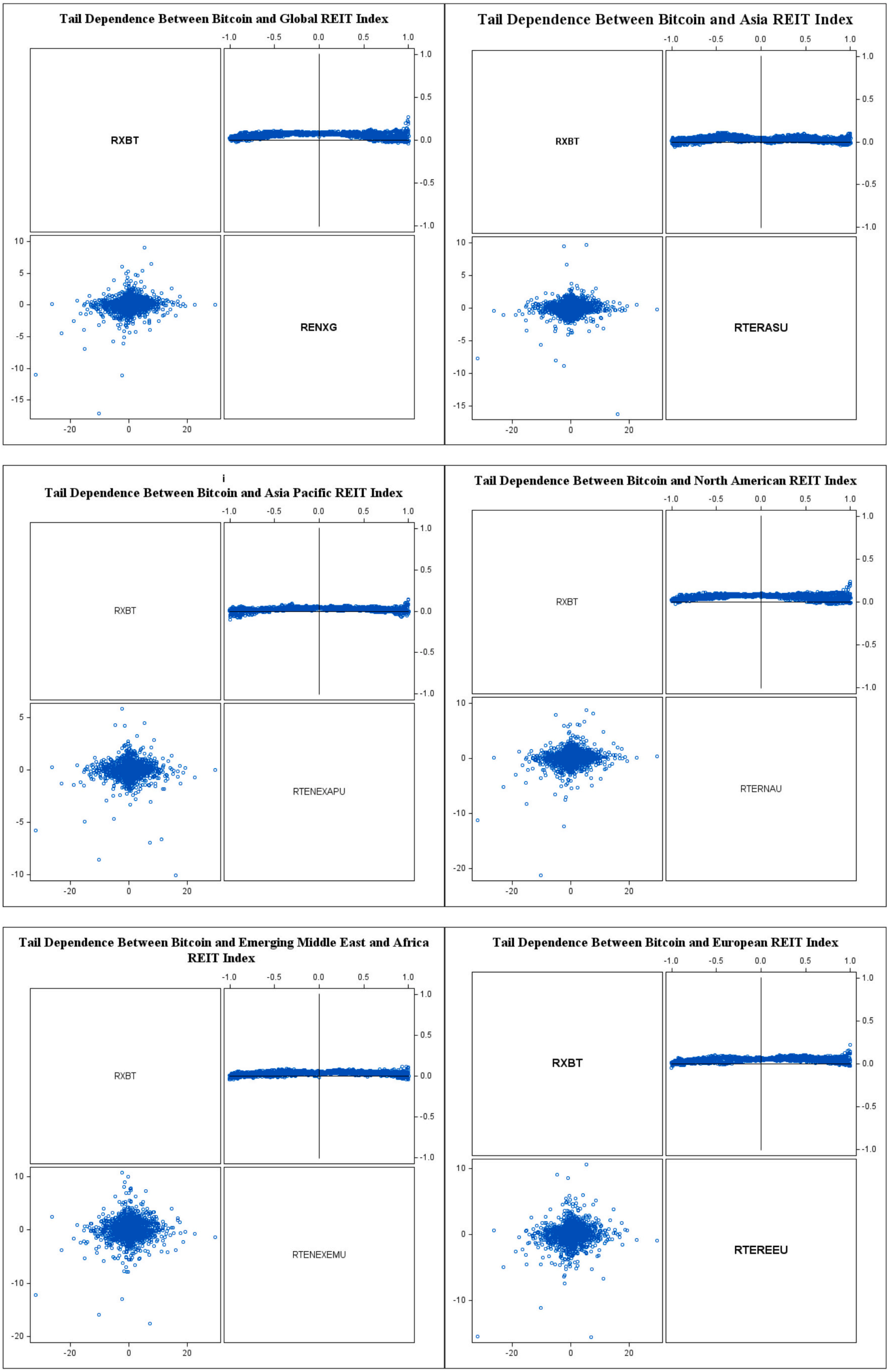


Fig. 3. Tail dependence between bitcoin and REITs indices.

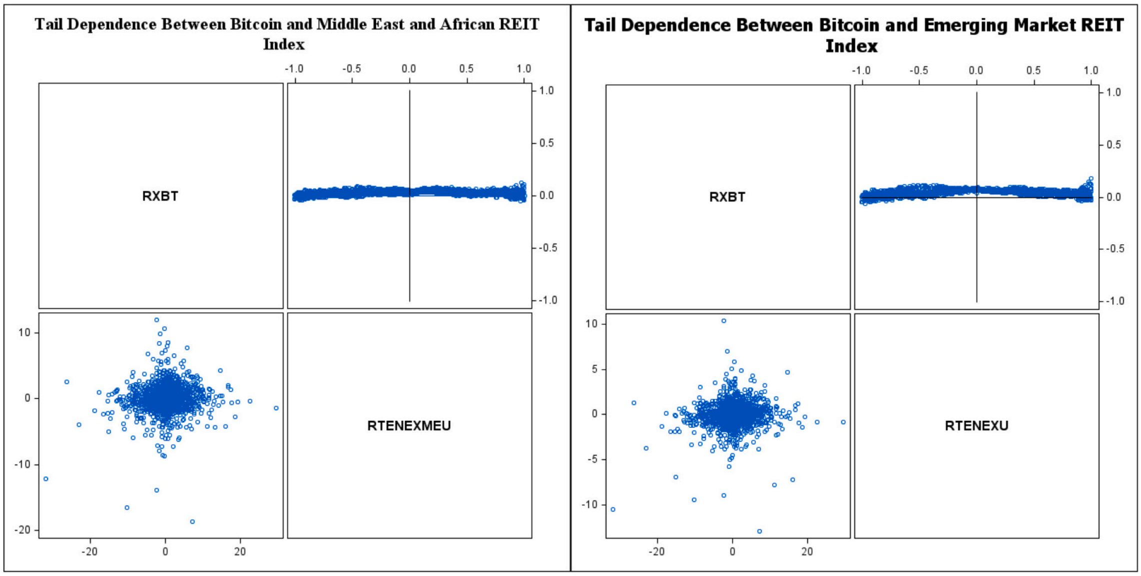


Fig. 3. (*continued*).

Table 9
Value-at-risk for REIT portfolios.

	Panel A: Unhedged			Panel B: Dynamically Hedged		
Variable	VaR (99%)	VaR (95%)	VaR (90%)	VaR (99%)	VaR (95%)	VaR (90%)
RENXG	−3.216	−1.594	−1.103	−3.370	−1.615	−1.096
RTERASU	−2.622	−1.331	−0.894	−2.364	−1.371	−0.900
RTENEXAPU	−1.881	−1.128	−0.687	−1.978	−1.137	−0.715
RTERNAU	−4.280	−1.910	−1.355	−3.983	−1.944	−1.363
RTENEXEMU	−5.336	−2.687	−1.863	−5.437	−2.660	−1.878
RTEREEU	−4.390	−2.165	−1.490	−4.442	−2.280	−1.520
RTENEXMEU	−5.643	−2.753	−1.926	−5.577	−2.685	−1.924
RTENEXU	−3.600	−1.799	−1.233	−3.752	−1.769	−1.245

Value at Risk estimates are derived from simulating 10,000 observations of the daily returns of each REIT index from their joint bivariate distribution with Bitcoin using the Clayton Copula. The expression for deriving the Clayton copula is shown in Eq. (11).

distributions. This pattern is further buttressed by the Tail plots, which show weak evidence of tail dependences at the lower tails, but more pronounced dependences at the upper tail of the distributions. Altogether, the results suggest that Bitcoin may have potential benefits for hedging the risks of REIT portfolios, particularly during periods of high volatility in the REIT markets.

We further examine the hedging potentials of Bitcoin for REIT portfolios by simulating 10,000 daily returns for Bitcoin and the respective REIT index. These returns are drawn from the estimated bivariate and multivariate joint distribution of Bitcoin and the REITs indices using the Clayton copula. In Table 9, we report the 90%, 95%, and 99% value-at-risk (VaR) thresholds for the unhedged (Panel A) and dynamically hedged REITs (Panel B) portfolios. In terms of the VaR estimates, we find in general, varying levels of exposure to market risk across the regions. REITs in North America and in the Emerging Middle East and Africa have the highest market exposure, while REITs in Asia and the Asia Pacific regions have the lowest. When we compare the VaR estimates for the dynamically hedged portfolio to the VaRs for the unhedged REIT portfolios, we obtain mixed results for the VaR estimates for the REIT indices. At the 99% VaR, dynamically hedged Asian, North American, and Emerging EMEA REIT portfolios have lower market risk than unhedged REIT portfolios. For the other regions, unhedged REIT portfolios have lower market risk than the dynamically hedged portfolios.[20]

8. Conclusion

The exponential growth in securitized real estate investments has renewed interest in exploring the unique challenges of REIT investment, such as using other asset classes for cross-hedging REIT exposures. Over the past decade, the sector has evolved noticeably from two broad categories of REITs to over ten distinct categories, which cover a broad range of investment options and exposures to real assets.[21] In this paper, we conduct a multinational examination of the diversification and risk mitigation benefits of cryptocurrencies for REIT portfolios. Specifically, we model the conditional covariances between a set of global and regional REIT index portfolios and Bitcoin using a multivariate GARCH (MGARCH) framework. Using the covariance matrix that we obtained from the MGARCH specification, we construct dynamically hedged REIT portfolios that are cross-hedged with Bitcoin. We backtest the hedged portfolios, against the unhedged, constant hedge ratio, naïvely hedged, and naively diversified portfolios using in-sample portfolio volatility and return measures. Furthermore, we check the robustness of our results by examining tail-dependence between the global and regional REIT portfolios and Bitcoin in a joint multivariate distribution framework using the Clayton copula. We compare the performance of the hedged portfolios against the benchmarks by simulating the tails of the distribution and calculating their value-at-risk.

We find that except for North American REITs, the dynamically hedged REIT index portfolio outperforms the unhedged, unconditionally hedged, and naively hedged REIT portfolios based on variance reduction and economic gains. The variance reduction ranges from 0.61% for the

[20] At the 95% VaR, dynamically hedged REITs in Emerging markets outperformed the unhedged portfolios. At the 90% VaR, only the dynamically hedged Emerging Middle East and Africa index outperformed it unhedged equivalent.

[21] For example, we now have specialty REITs such as Timber, Cannabis, Communication Tower, Hospital, and Trailer Parks REIT to name a few.

global REIT index to −3.33% for the Asian REITs. Asian REITs benefited the most from Bitcoin hedging, followed by REITs in emerging markets.

From our copula estimation, we find statistically significant evidence to support the existence of a bivariate joint distribution between Bitcoin and the REIT indices, as well as multivariate joint dependences amongst the REIT indices. We show that using Bitcoin to dynamically hedged REIT portfolios had minimal impact on the bivariate and multivariate joint distribution. We also found that on average, low left-tail dependence between Bitcoin and REITs. Altogether, these results provide further evidence of the utility of Bitcoin for hedging REIT portfolios.

Concerning the market risk of dynamically hedged REIT portfolios, we find mixed evidence of the impact of Bitcoin on the daily value-at-risk forecast for our REIT sample. In particular, we obtained mixed results for the VaR estimate for the REIT indices. At the 99% VaR, the dynamically hedged Asian, North American, and Emerging EMEA REIT portfolios outperformed the unhedged portfolios.

Our findings suggest that using Bitcoin can significantly improve the mean-variance frontier for REIT investors in markets that are not fully integrated into the global capital market. Indeed, in many of the regions where we find significant economic gains for using Bitcoin to dynamically hedge REIT portfolios, REITs are relatively novel investment products[22] in financial markets that have relatively higher degrees of market friction. Bitcoin appears to be the channel by which REIT investors in these markets can expand their investment opportunity set to include a global asset class such that their mean-variance frontier can be optimized over a global portfolio rather than a purely domestic portfolio. A limited amount of such benefit appears to be evident for investors in frictionless markets with higher levels of global integration, such as the US and developed Europe.

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Data availability

The authors do not have permission to share data.

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Further reading

Lee, S. (2010). The changing benefit of REITs to the mixed-asset portfolio. *Journal of Real Estate Portfolio Management, 16*(3), 201–215.

[22] For example, REITs was launched in South Africa in 2013, in 2016 in Saudi Arabia, in 2021 in China, and 2019 in India, to name a few.