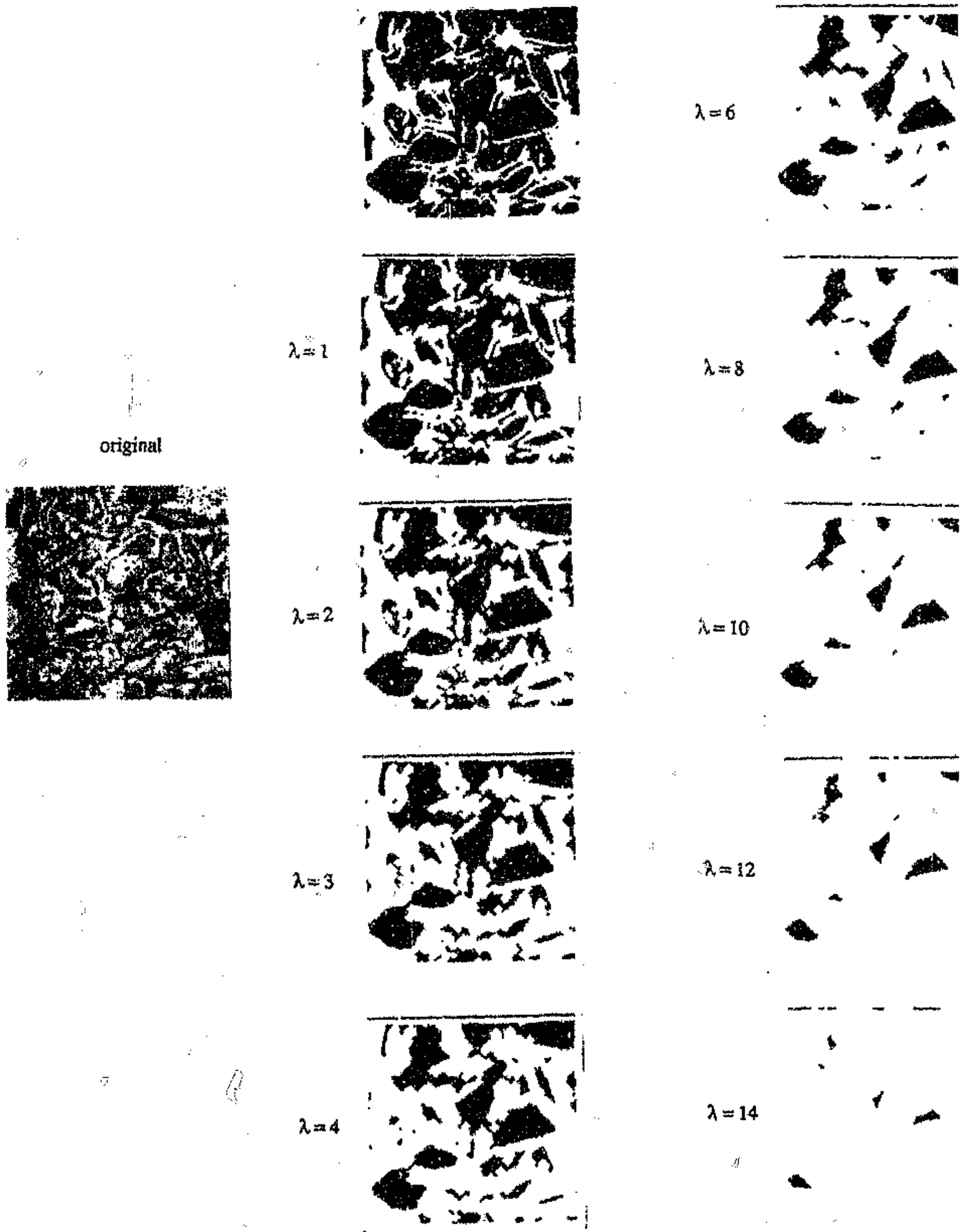
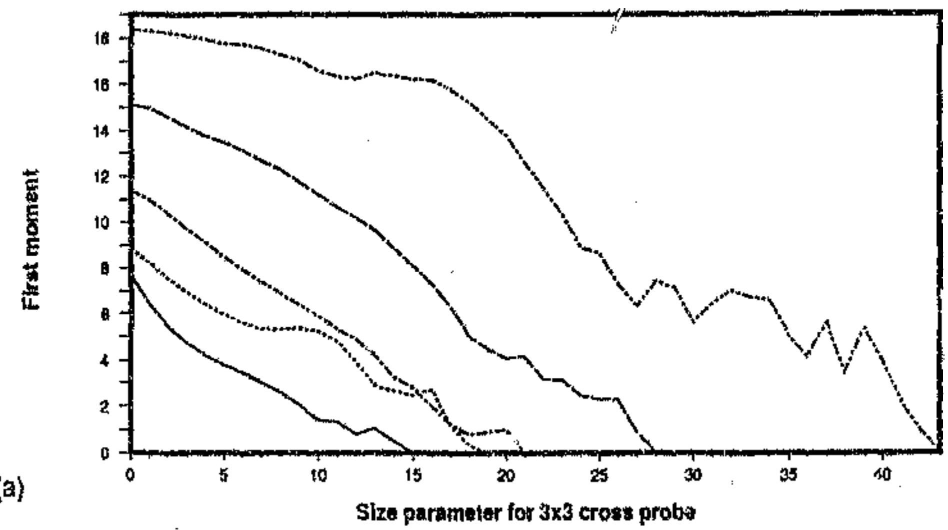
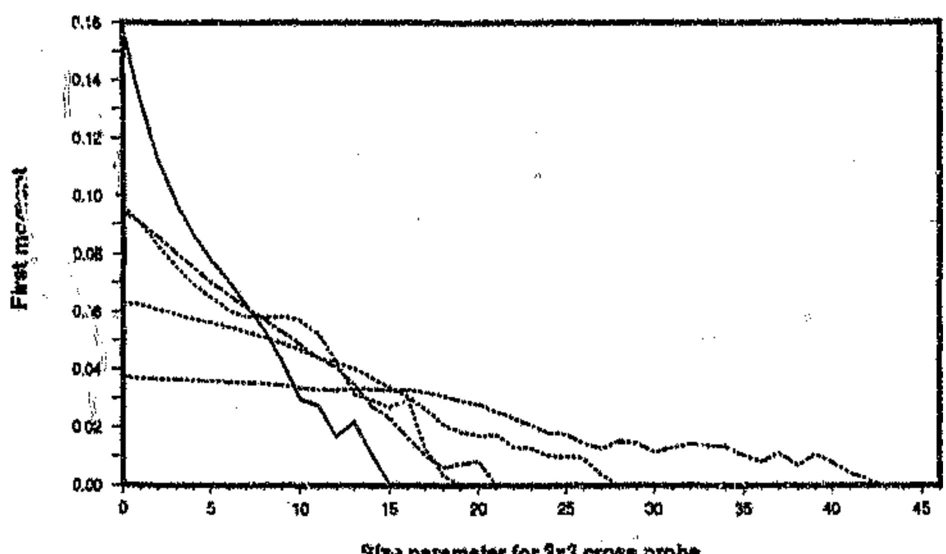


Figure 7.31: Example of successive erosions of an image.



(a)



(b)

Figure 7.32: (a) The first moment function $\nu_{\Theta,\mu_j}(\lambda)$ of the chord-length distributions measured from different rockstreams Ω_j which have been eroded. (b) The same data as in (a) except the distributions have been transformed into density functions.

7.6 INTERPRETATION OF THESE NEW MEASUREMENTS

The question that needs to be answered is : "Are these morphological and moment size measurements valid and of any use to the mining industry, and can they be used for process control ?" The answer to these questions depends on exactly what is required by industry.

For example in autogenous milling it has been suggested that a measure of the *number* or relative frequency *occurrence* of large rocks in proportion to small rocks is required. Weighted distributions, first moments, opening and erosions can provide this information. With opening, for example, a point on the λ axis must be selected which corresponds to the lower point of the range of larger rock that is of interest, and occurrence of larger rock will result in the first moment function extending beyond this point, (see figure 7.27).

With morphological sizing transforms it must be noted that the resultant data reflects the size distribution with respect to the shape of the probe chosen. This is an extremely important concept because the user can then define a probe of a certain shape, for example, a vertical or circular probe, and the size distributions will reflect the sizes of objects similar in shape and with the same relative orientation of the probe. Thus for a morphological measurement insensitive to the orientation of the rocks, a spherical probe should be used.

The *measure* of μ_L^1 could also be linked to the number and size of the rocks above the size point $D = kL$.

Alternatively the first moment of the chord length distribution can be used to estimate the average rocksize. From the cumulative chord-length distribution $P(L)$ the slope near the origin appears to vary with the average size or amount of smaller rock and any other point at size $L = aD$ can be used as some of the "percentage passing as certain size".

There is ultimately an infinite range of different measurements which can be extracted depending on the application. The interpretation and correlation of this measured image data to sieve size has to be established through experimentation. Perhaps the independent variable (usually sieve size) with which this transformed data is to be compared should not necessarily be sieve size, but some other mill control variable, which is in line with the new philosophy developed above. Knowledge of the explicit relationship between the parameter and sieve size is of secondary importance to the fact that the measurement or extracted parameter exists and is *related* to *size*, *is useful*, *repeatable*, and *reliable*.

7.7 SUMMARY

The conversion of measurements on images to sieve size is a difficult and inherently ambiguous task with all transformations introducing bias of some form. In this chapter direct inversion and deconvolution methods were used as a means to transform the chord-length distribution to equivalent sieve or mesh size distributions. The difficulties and inherent ambiguities in this inversion process does not allow for these methods to be used to infer rock size with great accuracy. As a consequence it is proposed to transform the chord-length data to a form that is *understandable* and useful rather than synonymous with sieve size. This philosophy led to the development of moment methods for transforming and analyzing chord-length distributions which intuitively behave reasonably and in accordance with changing rock size.

Established methods such as constrained-linear-inversion and King's method produce distributions which are logically correct although quantitatively inaccurate. The *cumulative* and *weighting* methods developed appear to reflect the size distributions of the rocks with better accuracy quantitatively, although the *meaning* of the quantitative measurements from these transformed chord-length distributions is debatable and still needs to interpreted correctly. The new weighting method produces bell-shaped distributions that correspond, although not accurately, with the sieve-size distributions.

With all these new transformed distributions, there are difficulties in distinguishing between the various quantities of *smaller* rock, and inferring the amount of larger rocks is complicated by the large variations caused by insufficient sampling.

The raw chord-length distributions measured show a sharp and narrow peak in the region of short chords and it is assumed that this peak forms part of the problem of understanding and transforming the chord data. Monte Carlo simulations of random linear intercepts on irregular shaped bodies show that no sharp peak is expected in the lower chord-length regions. Thus the peak has to be attributed to, i) experimental error, ii) extraordinary angular particles tested in this work, iii) the fact that the erroneous segmentation modifies the rock outlines to be more angular than their natural appearance, and iv) the fact that the chord probe lines are adjacent to one another. However with regard to the short chords and small particles, DeHoff and Rhines (1961) in their paper on the detection of the number of particles per unit volume, comment that the measurement of small particle outlines introduces the largest errors because they are the most difficult to measure.

Observation of the results of most of the different transformations show that there is inherent instability and uncertainty in determining the size distribution of the smaller rocks in the size range being analyzed. King's method reveals oscillations in the region of smaller rocks, and with the moment method there appears to be no method of clearly distinguishing between the amount small rocks in each different distribution.

In the light of these problems a new philosophy is proposed which frees the evaluation of rocksize from the limiting objective of aiming for estimating sieve size. Any other sizing parameter, as long as it obeys Matherons sizing criteria and it useful to the application, may be considered a measure of rocksize.

The *first moment* of chord-length distribution has been found to be linearly related to average rock size and thus average rocksize can be measured within certain limitations, this being the variance about the mean which is function of rock size and the number of chord samples.

Morphological *opening* and *erosions* were used as examples of new methods for sizing the rock "projections" in images. The results of these operations show that these operations do produce size measurements which vary logically with sieve size and on average do obey Matherons criteria. These methods thus provide motivation for evaluating rocksize distribution according to a different interpretation of size.

8 CONCLUSION AND RECOMMENDATIONS

8.1 SUMMARY OF WORK UNDERTAKEN

The determination of the size distribution of rocks on a moving conveyor belt is an important measurement for the mineral processing industry, and in particular for the control of comminution and crushing processes. A reliable instrument to measure the size distribution of rocks on conveyor belts has generally not been readily available to the mining industry because of poor performance of commercially available systems and misapprehensions about the underlying principles used, specifically chord-length intercepts. These factors have motivated an investigation of the principles underlying an online rocksize measurement, and the feasibility and limitations of constructing such an instrument based on machine vision.

In this thesis the principles of estimating the *size* distribution of ideal particle system from populations of lower dimensional measurements are presented, and extended to the rocks as found on conveyor-belts. The rockstream which consists of arbitrary shaped rocks of random size packed together, is assumed to be a stochastic process, characterized by its *rocksize distribution*, which is a random variable.

Any method for determining rocksize distribution must rely on remote sensing and the modulation of probing radiation, and the subsequent analysis thereof. Computer vision techniques using the visible spectrum are motivated as the best method for probing the rockstream. The determination of the size distribution of the rockstream from images returned by remote sensing, is a non-trivial problem that cannot be solved analytically. This stems from the complexities of modelling the dynamics of the rockstream, and linking the image data to real physical properties of the rockstream.

Computer vision and image-processing techniques which have been selected as the sensing methodology, can only provide one- or two- dimensional data which ultimately has to be transformed into three-dimensional data such as sieve size. This transformation can be realized through the use of stereological methods. However most stereological size transformations are application specific, or derived for ideal particulate systems where the particles are uniformly and sparsely distributed in space, and are simple in shape. The rocks on the conveyor belt do not conform to this model and thus such models can only be applied to the rocksize problem with extremely limited success.

The failure of existing techniques to effect a reliable rocksize measurement has inspired a heuristic and experimental approach towards solving this rocksize problem, which in turn has led to the development of the *moment* and *morphological* transformations which produce time varying size data indicative of rock size. Although this data is not the full sieve size distribution, it is closely related to sieve size.

The size transformations developed, measure chord-length distributions on transformed areal projections of the segmented images of rockscenes. Image processing techniques, using pre-filtering, edge detection, thresholding, and mathematical morphology have been combined to perform adequate segmentation and rock boundary detection for the purpose of measuring chord-length distributions. Chord-length distributions are selected as the basic parameter measurement because of the ease of computation, and experimental evidence which strongly suggests that the effect of edge and segmentation errors are less severe for chord-length measurements than areal measurements.

The measurement of accurate chord-length or area distributions depends entirely on the precise and error-free determination of the boundaries of the rock projections. However the identification and segmentation of rock images to derive their edges is inherently error prone because of the random nature of the rockscenes which vary with time. The stochastic nature of the rocks, as well as the serious problem of occlusion guarantees that any edge of areal pattern will be noisy, and that any parameter measurement will be inaccurate. The errors in the edge patterns is minimized through the use of edge correction techniques which make use of a-priori knowledge of the most probable shape of the rocks. However the edge detection and correction procedures are limited and cannot be significantly improved because of realtime and cost constraints of a practical instrument.

In addition to segmentation constraints introduced by the natural limitation of the complexity of the edge detection and correction algorithms that can be employed, hardware constraints also impose serious limitations on the ability of the system to sample the rockstream for rocksize estimation. Limited sampling affects the determination of the statistics of the chord-length distributions, their variances, and the level of confidence in the measurements. The chord data from at least a few images should be averaged to give a reliable indication of the trends of the rocksize distribution.

Direct inversion, constrained-linear inversion, as well as King's methods are used to perform the conversion of chord-length distributions to size. These methods are fairly unstable and require iteration in order to force the resultant rocksize data to be meaningful ie no negative distributions. The problems with these methods are attributed to erroneous chord-length data, to which inversion transformations are extremely sensitive.

The utilisation of inversion techniques arises from the goal of deriving *sieve* size distribution. Sieve size is only one of many methods for describing particle size. It is thus hypothesized that alternative size descriptions which preclude the use of ambiguous inversion methods may be more useful in deriving size data for control systems. Supporting this hypothesis is the inescapable fact that rock edge detection will never be perfect and that the measurement process is highly ambiguous because of the dependence of an imaging system on the *surface* characteristics of the rockstream.

This thesis thus describes moments, and weighted moment functions, developed as alternatives to sieve size. In particular the average sieve size of rockstream can be estimated from the first moment of the chord-length distributions with a variance of between 5% to 25% over a size range of 6:1. The variance depends on the relative size of the rocks and the resolution of image system.

The weighted moment functions of the chord-length distributions also provide qualitative and quantitative visual indications of the size distributions of the rocks. However the estimation of the lower size ranges of the rocks in a given distribution is difficult to estimate from such curves. The problem with the lower sizes stems from the fundamental ambiguity of determining three dimensional parameters from sections where information is lost, and which is exacerbated by the segmentation and edge detection errors. Chord-length moment curves also allow for the visual display of the changing proportions of a certain size class of rocks in proportion to another.

Morphological sizing methods also provide a new insight into the size of the rocks on the conveyor belt. Morphological sizing methods especially *opening*, commute very well with the sieving process and again the size data extracted correlates with sieve size.

8.2 CONCLUSIONS

The principles behind the estimation of the size distribution of rocks from visual images has been formally established. A relationship based on the probability theory as developed by King (1982) has been extended to relate parameters measured from images of rock streams to the size distribution of the rockstream. However this relationship cannot be expressed in a final equation because of the intractability of the mathematics for modelling the rockstream, and the uncertainties and ambiguities associated with the relationship between the images and the rockscenes.

Experimental evidence shows that online rocksize measurement can be made, despite the fact that only the top surface of the rockstream is sampled. Fundamental to this measurement process is the fact that chord-length distributions do allow for the measurement of rocksize. The full size distribution may only be estimated to a limited extent and is restricted by the nature of the rockstream. For example, a very muddy rockstream which hides most of the rock features from the camera, cannot be instrumented.

Imaging techniques have been developed through the marriage of many diverse techniques into a single useful system and which advances the universal theory that all imaging techniques should be combined where appropriate to solve a particular problem.

A concept has been proposed, that any repeatable size measurement which is appropriate to the application, although not fully understood, is a viable and useful measurement. This leads to the hypothesis that with todays powerful computers an *empirical* and computer based solution may be more appropriate than the traditional analytical approach usually aspired to in this work.

8.3 FUTURE WORK

The above research work has touched upon a large number of areas in science and engineering which in themselves are vast areas of research. These areas could not be covered fully in this thesis, and it is therefore recommended that the following research be undertaken in order to improve upon the measurement process developed here.

A major area that needs to be researched is that of the fundamental stereological tools required to measure the size distribution of particles in a non-ideal situation which includes occlusion and touching. A more powerful combined probe consisting of both point, linear and areal probes should be investigated together with the objective of quantifying the biases inherent in the measurements displayed here. A multidimensional probe should help to minimize these biases thus making this measurement more robust and reliable. As was shown in chapter 3, modelling the rockstream, the spatial arrangement of rocks on a conveyor belt, and the sampling mechanisms of an imaging systems are complex problems which still need to be resolved.

The image processing and object recognition algorithms as developed in this thesis do form the foundation for the experimental size measurement developed in this thesis. However these imaging techniques could be improved upon. In particular the relationship between the rocks and the intensity pattern as captured by the camera needs to be investigated in order to determine more robust methods of identifying the rocks boundaries. New advances in high-speed computer technology as well as the continuing research and development for better 3-D pattern recognition strategies that work on natural scenes of irregular and complex objects all bode well for improved rock edge detection. Neural computers and parallel processing hardware and strategies may also be part of the answer to the solution to the rock recognition problem.

A few of the image processing areas which can be researched further, for example, are adaptive edge detection algorithm incorporating multi-resolution techniques, hierarchical and pyramidal techniques for unambiguously identifying rocks from a global as well as local point of view. The use of colour or multi-frequency sensing techniques as well stereo imaging could also have potential as far as this measurement is concerned.

The combining of the results from different image processing strategies including the optimization of image knowledge for better understanding could also improve rock identification and the elimination of noise entities and the closing of edge gaps.

Structured light could potentially prove a better indication of the edges of the rock and could be incorporated into the existing algorithms or be used as a new stand alone method for determining rock edges. The use of parallel bands of light can be conceptually linked to chord-length measurements, and may ultimately provide a better and faster method for deriving chord-lengths. In accordance with the philosophy put forward that sieve size and other conventional methods may be restricting the derivation of rocksize data, perhaps the distance between the broken end-points of the structured light lines, or their actual length, could be used as the parameter distributions to be transformed to size.

Mathematical morphology has proved to be a most useful tool for image processing as well as estimating rocksize distribution. Much more research is required in order to derive a better and faster morphological size measurement technique. A theory tying together size distributions derived from kernels of different shape may prove to be a better universal size measurement, which at present is still very diversified.

The moment and morphological methods provide useful data with which an observer can infer trends with regard to the size of the rocks on a conveyor belt. The extraction of quantitative information is at present limited, although a large amount of potential exists as far as the extraction of quantitative rocksize information is concerned. In particular the results from *opening* and the moment methods could be combined to give more reliable size data. It is suggested that in time, these extracted measurements could be developed to a stage where they could form a new size standard for rockstreams or even converge to the ideal sieve distribution.

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Appendix A

Selected Definitions of Particle Size.

The table below presents a number different diameters with which the size of a particle may be defined. For a more comprehensive survey see Allen (1981).

Symbol	**Name**	**Definition**	**Formula**
d_v	Volume diameter	Diameter of a sphere having the same volume as the particle.	$V = \frac{\pi}{6} d_v^3$
d_{Stk}	Stokes diameter	The free falling diameter of a particle in the laminar flow region (Re <0.2)	$d^2 = \frac{d_v^3}{d_d}$
d_a	Projected area diameter	Diameter of circle having the same area as the projected area of the particle resting in a stable position.	
d_p	Projected area diameter.	Diameter of circle having the same area as the projected area of the particle resting in a random orientation.	
d_A	Sieve diameter	The width of the minimum square aperture through which the particle will pass.	
d_M	Martin's diameter	The mean chord length of the projected outline of the particle.	
d_F	Ferret's diameter	The mean value of the distance between pairs of parallel tangents to the projected outline of the particle.	
d_c	Perimeter diameter	Diameter of a circle having the same perimeter of the projected outline of the particle.	

Appendix B

Sample calculatio[n o]f weighted chord-length distributions and moments.

This section presents samples of measured chord-length distributions $g(L_i)$, together with the calculated density distributions $p(L_i)$, cumulative density distributions $P(L_i)$, amplified density distributions $p^*(L_i)$ as well as the n'th moments μ_L^n .

The chord-length distributions measured are from a single image of the following size class

- (i) -75+50mm
- (ii) -12.7+9.53mm
- (iii) -38+12.7mm

The rock material is quartzite, and all data is captured from the closed loop conveyor belt system described in chapter 5. Belt [sp]eed for this data was approximately 0.5m/s. and the rocks were lightly sprayed with water. The clock frequency of the controller was 1.4MHz. The chord length distributions measured here were made from adjacent linear probes spaced 1 pixel apart. The image processed is the centre subimage of size 200 by 200 pixels of the full 512 by 512 image. The im[age] processing algorithm consisted of morphological edge detection together with morphological edge correction as described in chapter 5. Threshold parameters are $k_{high} = 0.625$ $k_{low} = 0.250$.

The calculations were made as follows

Chord length density distribution $$p(L_i) = \frac{g(L_i)}{\sum g(L_i)}$$

Cumulative density function $$P(L_i) = \sum_{0}^{i=139} p(L_i)$$

Weighted distributions $$p_\alpha^*(L_i) = L_i^\alpha \cdot p(L_i)$$

n'th moments $$V_L^n(L_i) = \sum_{i=0}^{139} p_\alpha^*(L_i)$$

Rock : quartzite. Size :-75+50mm.
Total number of chords measured : 920

$v_L^1 = 27.784$

$\mu_L^2 = 1733.971$

L	$g(L)$	$p(L)$	$P(L)$	p^*_{n+1}	p^*_{n+2}
0	0	0.000	0.000	0.000	0
1	86	0.093	0.093	0.093	[illegible]
2	51	0.055	0.149	0.111	0.222
3	48	0.052	0.201	0.157	0.470
4	42	0.046	0.247	0.183	0.730
5	32	0.035	0.282	0.174	0.870
6	42	0.046	0.327	0.274	1.643
7	31	0.034	0.361	0.236	1.651
8	19	0.021	0.382	0.165	1.322
9	24	0.026	0.408	0.235	2.113
10	20	0.022	0.429	0.217	2.174
11	17	0.018	0.448	0.203	2.236
12	13	0.014	0.462	0.170	2.035
13	19	0.021	0.483	0.268	3.490
14	19	0.021	0.503	0.289	4.048
15	19	0.021	0.524	0.310	4.647
16	16	0.017	0.541	0.278	4.452
17	10	0.011	0.552	0.185	3.141
18	5	0.005	0.558	0.098	1.761
19	3	0.003	0.561	0.062	1.177
20	16	0.017	0.578	0.348	6.957
21	4	0.004	0.583	0.091	1.917
22	9	0.010	0.592	0.21	4.735
23	10	0.011	0.603	0.250	5.750
24	5	0.005	0.609	0.130	3.130
25	13	0.014	0.623	0.353	8.832
26	5	0.005	0.628	0.141	3.674
27	8	0.009	0.637	0.235	6.339
28	4	0.004	0.641	0.122	3.409
29	3	0.003	0.643	0.095	2.742
30	3	0.003	0.646	0.098	2.935
31	12	0.013	0.661	0.404	12.535
32	13	0.014	0.675	0.452	14.470
33	5	0.005	0.680	0.179	5.918
34	9	0.010	0.690	0.333	11.309
35	7	0.008	0.698	0.266	9.321
36	4	0.004	0.702	0.157	5.635
37	4	0.004	0.707	0.161	5.952
38	0	0.000	0.707	0.000	0.000
39	7	0.008	0.714	0.297	11.673
40	5	0.005	0.720	0.217	8.696
41	5	0.005	0.725	0.223	9.136
42	8	0.009	0.734	0.365	15.339
43	6	0.007	0.740	0.280	12.059
44	7	0.008	0.748	0.335	14.730
45	6	0.007	0.754	0.293	13.207
46	[illegible]	0.006	0.760	0.250	11.500
47	5	0.006	0.765	0.255	12.005
48	13	0.014	0.779	0.678	32.557
49	8	0.009	0.788	0.426	20.878
50	10	0.011	0.799	0.543	27.174

Rock : quartzite. Size :-75+50mm.
Total number of chords measured : 920
$\nu_L^1 = 27.784$ $\mu_L^2 = 1733.971$

L	$g(L)$	$p(L)$	$P(L)$	$p^*_{\alpha=1}$	$p^*_{\alpha=2}$
51	10	0.011	0.810	0.554	28.272
52	8	0.009	0.818	0.452	23.513
53	10	0.011	0.829	0.576	30.533
54	5	0.005	0.835	0.293	15.848
55	6	0.007	0.841	0.359	19.728
56	2	0.002	0.843	0.122	6.817
57	7	0.008	0.851	0.434	24.721
58	7	0.008	0.859	0.441	25.596
59	8	0.009	0.867	0.513	30.270
60	8	0.009	0.876	0.522	31.304
61	4	0.004	0.880	0.265	16.178
62	1	0.001	0.882	0.067	4.176
63	2	0.002	0.884	0.137	8.628
64	3	0.003	0.887	0.209	13.357
65	2	0.002	0.889	0.141	9.185
66	3	0.003	0.892	0.215	14.204
67	2	0.002	0.895	0.146	9.759
68	1	0.001	0.896	0.074	5.026
69	2	0.002	0.898	0.150	10.350
70	1	0.001	0.899	0.076	5.326
71	4	0.004	0.903	0.309	21.917
72	4	0.004	0.908	0.313	22.539
73	1	0.001	0.909	0.079	5.792
74	4	0.004	0.913	0.322	23.809
75	2	0.002	0.915	0.163	12.223
76	6	0.007	0.922	0.496	37.670
77	1	0.001	0.923	0.084	6.445
78	2	0.002	0.925	0.170	13.226
79	2	0.002	0.927	0.172	13.567
80	2	0.002	0.929	0.171	13.[illegible]
81	1	0.001	0.930	0.088	7.1[illegible]2
82	0	0.000	0.930	0.000	0.000
83	2	0.002	0.933	0.180	14.976
84	1	0.001	0.934	0.091	7.670
85	1	0.001	0.935	0.092	7.853
86	1	0.001	0.936	0.093	8.039
87	0	0.000	0.936	0.000	0.000
88	0	0.000	0.936	0.000	0.000
89	0	0.000	0.936	0.000	0.000
90	0	0.000	0.936	0.000	0.000
91	0	0.000	0.936	0.000	0.000
92	0	0.000	0.936	0.000	0.000
93	0	0.000	0.936	0.000	0.000
94	1	0.001	0.937	0.102	9.604
95	2	0.002	0.939	0.207	19.620
96	0	0.000	0.939	0.000	0.000
97	2	0.002	0.941	0.211	20.464
98	2	0.002	0.943	0.213	20.875
99	1	0.001	0.945	0.108	10.653
100	1	0.001	0.946	0.109	10.870

Rock : quartzite, Size -75+50mm.
Total number of chords measured : 920
$v_L^1 = 27.734$ $\mu_L^2 = 1733.971$

L	$g(L)$	$p(L)$	$P(L)$	$p^*_{n=1}$	$p^*_{n=2}$
101	2	0.002	0.948	0.220	22.176
102	1	0.001	0.949	0.111	11.309
103	1	0.001	0.950	0.112	11.532
104	1	0.001	0.951	0.113	11.757
105	1	0.001	0.952	0.114	11.984
106	2	0.002	0.954	0.230	24.426
107	1	0.001	0.955	0.116	12.445
108	1	0.001	0.957	0.117	12.678
109	0	0.000	0.957	0.000	0.000
110	4	0.004	0.961	0.478	52.609
111	1	0.001	0.962	0.121	13.392
112	2	0.002	0.964	0.243	27.270
113	4	0.004	0.968	0.491	55.517
114	2	0.002	0.971	0.248	28.252
115	2	0.002	0.973	0.250	28.750
116	2	0.002	0.975	0.252	29.252
117	3	0.003	0.978	0.382	44.638
118	3	0.003	0.982	0.385	45.404
119	2	0.002	0.984	0.259	30.785
120	2	0.002	0.986	0.261	31.304
121	0	0.000	0.986	0.000	0.000
122	1	0.001	0.987	0.133	16.178
123	0	0.000	0.987	0.000	0.000
124	0	0.000	0.987	0.000	0.000
125	0	0.000	0.987	0.000	0.000
126	0	0.000	0.987	0.000	0.000
127	0	0.000	0.987	0.000	0.000
128	0	0.000	0.987	0.000	0.000
129	2	0.002	0.989	0.280	36.176
130	3	0.003	0.992	0.424	55.109
131	3	0.003	0.996	0.427	55.960
132	0	0.000	0.996	0.000	0.000
133	1	0.001	0.997	0.146	19.227
134	0	0.000	0.997	0.000	0.000
135	0	0.000	0.997	0.000	0.000
136	1	0.001	0.998	0.148	20.104
137	1	0.001	0.999	0.149	20.401
138	0	0.000	0.999	0.000	0.000
139	1	0.001	1.000	0.151	21.001

Rock : quarizite. Size :-12.7+9.53mm.
Total number of chords measured : 7509

$v_L^1 = 6.115$

$\mu_L^1 = 68.686$

L	$g(L)$	$p(L)$	$P(L)$	$p^*_{n=1}$	$p^*_{n=2}$
0	0	0.000	0.000	0.000	0.000
1	1311	0.175	0.175	0.175	0.175
2	933	0.124	0.299	0.249	0.497
3	872	0.116	0.415	0.348	1.045
4	682	0.091	0.506	0.363	1.453
5	613	0.082	0.587	0.408	2.041
6	524	0.070	0.657	0.419	2.512
7	451	0.060	0.717	0.420	2.943
8	327	0.044	0.761	0.348	2.787
9	301	0.040	0.801	0.361	3.247
10	223	0.030	0.831	0.297	2.970
11	210	0.028	0.859	0.308	3.384
12	144	0.019	0.878	0.230	2.761
13	168	0.022	0.900	0.291	3.781
14	120	0.016	0.916	0.224	3.132
15	103	0.014	0.930	0.206	3.086
16	93	0.013	0.943	0.205	3.273
17	75	0.010	0.953	0.170	2.887
18	72	0.010	0.962	0.173	3.107
19	39	0.005	0.967	0.099	1.875
20	49	0.007	0.974	0.131	2.610
21	25	0.003	0.977	0.070	1.468
22	22	0.003	0.980	0.064	1.418
23	15	0.002	0.982	0.046	1.057
24	19	0.003	0.985	0.061	1.457
25	21	0.003	0.987	0.070	1.748
26	16	0.002	0.990	0.055	1.440
27	16	0.002	0.992	0.058	1.553
28	9	0.001	0.993	0.034	0.940
29	8	0.001	0.994	0.031	0.896
30	9	0.001	0.995	0.036	1.079
31	10	0.001	0.997	0.041	1.280
32	4	0.001	0.997	0.017	0.545
33	5	0.001	0.998	0.022	0.725
34	5	0.001	0.998	0.023	0.770
35	1	0.000	0.999	0.005	0.163
36	4	0.001	0.999	0.019	0.690
37	2	0.000	0.999	0.010	0.365
38	0	0.000	0.999	0.000	0.000
39	2	0.000	1.000	0.010	0.405
40	0	0.000	1.000	0.000	0.000
41	1	0.000	1.000	0.005	0.224
42	0	0.000	1.000	0.000	0.000
43	0	0.000	1.000	0.000	0.000
44	0	0.000	1.000	0.000	0.000
45	0	0.000	1.000	0.000	0.000
46	0	0.000	1.000	0.000	0.000
47	0	0.000	1.000	0.000	0.000
48	0	0.000	1.000	0.000	0.000
49	0	0.000	1.000	0.000	0.000
50	0	0.000	1.000	0.000	0.000

Rock : quartzite. Size :-12.7+9.53mm. **Total number of chords measured : 7509** $v_L^1 = 6.115$ $\mu_L^2 = 68.686$					
L	$g(L)$	$p(L)$	$P(L)$	$p^*_{n=1}$	$p^*_{n=2}$
51	0	0.000	1.000	0.000	0.000
52	0	0.000	1.000	0.000	0.000
53	0	0.000	1.000	0.000	0.000
54	0	0.000	1.000	0.000	0.000
55	0	0.000	1.000	0.000	0.000
56	0	0.000	1.000	0.000	0.000
57	1	0.000	1.000	0.008	0.433
58	0	0.000	1.000	0.000	0.000
59	1	0.000	1.000	0.008	0.464
60	0	0.000	1.000	0.000	0.000
61	0	0.000	1.000	0.000	0.000
62	0	0.000	1.000	0.000	0.000
63	0	0.000	1.000	0.000	0.000
64	0	0.000	1.000	0.000	0.000
65	0	0.000	1.000	0.000	0.000
66	0	0.000	1.000	0.000	0.000
67	0	0.000	1.000	0.000	0.000
68	0	0.000	1.000	0.000	0.000
69	0	0.000	1.000	0.000	0.000
70	0	0.000	1.000	0.000	0.000
71	0	0.000	1.000	0.000	0.000
72	0	0.000	1.000	0.000	0.000
73	0	0.000	1.000	0.000	0.000
74	0	0.000	1.000	0.000	0.000
75	0	0.000	1.000	0.000	0.000
76	0	0.000	1.000	0.000	0.000
77	0	0.000	1.000	0.000	0.000
78	0	0.000	1.000	0.000	0.000
79	0	0.000	1.000	0.000	0.000
80	0	0.000	1.000	0.000	0.000
81	0	0.000	1.000	0.000	0.000
82	0	0.000	1.000	0.000	0.000
83	0	0.000	1.000	0.000	0.000
84	0	0.000	1.000	0.000	0.000
85	0	0.000	1.000	0.000	0.000
86	0	0.000	1.000	0.000	0.000
87	0	0.000	1.000	0.000	0.000
88	0	0.000	1.000	0.000	0.000
89	0	0.000	1.000	0.000	0.000
90	0	0.000	1.000	0.000	0.000
91	0	0.000	1.000	0.000	0.000
92	0	0.000	1.000	0.000	0.000
93	0	0.000	1.000	0.000	0.000
94	0	0.000	1.000	0.000	0.000
95	0	0.000	1.000	0.000	0.000
96	0	0.000	1.000	0.000	0.000
97	0	0.000	1.000	0.000	0.000
98	0	0.000	1.000	0.000	0.000
99	0	0.000	1.000	0.000	0.000
100	0	0.000	1.000	0.000	0.000

Rock : quartzite. Size :-12.7+9.53mm.
Total number of chords measured : 7509

$\hat{v}_L^2 = 6.115$

$\hat{\mu}_L^2 = 68.686$

L	$g(L)$	$p(L)$	$P(L)$	$p^*_{\alpha=1}$	$p^*_{\alpha=2}$
101	0	0.000	1.000	0.000	0.000
102	0	0.000	1.000	0.000	0.000
103	0	0.000	1.000	0.000	0.000
104	0	0.000	1.000	0.000	0.000
105	0	0.000	1.000	0.000	0.000
106	0	0.000	1.000	0.000	0.000
107	0	0.000	1.000	0.000	0.000
108	0	0.000	1.000	0.000	0.000
109	0	0.000	1.000	0.000	0.000
110	0	0.000	1.000	0.000	0.000
111	0	0.000	1.000	0.000	0.000
112	0	0.000	1.000	0.000	0.000
113	0	0.000	1.000	0.000	0.000
114	0	0.000	1.000	0.000	0.000
115	0	0.000	1.000	0.000	0.000
116	0	0.000	1.000	0.000	0.000
117	0	0.000	1.000	0.000	0.000
118	0	0.000	1.000	0.000	0.000
119	0	0.000	1.000	0.000	0.000
120	0	0.000	1.000	0.000	0.000
121	0	0.000	1.000	0.000	0.000
122	0	0.000	1.000	0.000	0.000
123	0	0.000	1.000	0.000	0.000
124	0	0.000	1.000	0.000	0.000
125	0	0.000	1.000	0.000	0.000
126	0	0.000	1.000	0.000	0.000
127	0	0.000	1.000	0.000	0.000
128	0	0.000	1.000	0.000	0.000
129	0	0.000	1.000	0.000	0.000
130	0	0.000	1.000	0.000	0.000
131	0	0.000	1.000	0.000	0.000
132	0	0.000	1.000	0.000	0.000
133	0	0.000	1.000	0.000	0.000
134	0	0.000	1.000	0.000	0.000
135	0	0.000	1.000	0.000	0.000
136	0	0.000	1.000	0.000	0.000
137	0	0.000	1.000	0.000	0.000
138	0	0.000	1.000	0.000	0.000
139	0	0.000	1.000	0.000	0.000

Rock : quartzite. Size :-38+12.7mm.
Total number of chords measured : 1551

$\nu_L^1 = 13.172$

$\mu_L^2 = 331.667$

L	$g(L)$	$p(L)$	$P(L)$	$p^*_{\alpha=1}$	$p^*_{\alpha=2}$
0	0	0.000	0.000	0.000	0.000
1	196	0.126	0.126	0.126	0.126
2	90	0.058	0.184	0.116	0.232
3	116	0.075	0.259	0.224	0.673
4	84	0.054	0.313	0.217	0.867
5	77	0.050	0.363	0.248	1.241
6	69	0.044	0.407	0.267	1.602
7	66	0.043	0.450	0.298	2.085
8	49	0.032	0.482	0.253	2.022
9	51	0.033	0.515	0.296	2.663
10	38	0.025	0.539	0.245	2.450
11	46	0.030	0.569	0.326	3.589
12	35	0.023	0.591	0.271	3.250
13	47	0.030	0.622	0.394	5.121
14	34	0.022	0.643	0.307	4.297
15	41	0.026	0.670	0.397	5.948
16	32	0.021	0.691	0.330	5.282
17	37	0.024	0.714	0.406	6.894
18	33	0.021	0.736	0.383	6.894
19	22	0.014	0.750	0.270	5.121
20	28	0.018	0.768	0.361	7.221
21	24	0.015	0.783	0.325	6.824
22	31	0.020	0.803	0.440	9.674
23	24	0.015	0.819	0.356	8.186
24	18	0.012	0.830	0.279	6.685
25	19	0.012	0.843	0.306	7.656
26	17	0.011	0.854	0.285	7.409
27	13	0.008	0.862	0.226	6.110
28	16	0.010	0.872	0.289	8.088
29	22	0.014	0.887	0.411	11.929
30	12	0.008	0.894	0.232	6.963
31	16	0.010	0.906	0.320	9.914
32	13	0.008	0.913	0.268	8.583
33	15	0.010	0.923	0.319	10.532
34	9	0.006	0.928	0.197	6.708
35	7	0.005	0.933	0.158	5.529
36	9	0.006	0.939	0.209	7.520
37	7	0.005	0.943	0.167	6.179
38	5	0.003	0.946	0.123	4.655
39	9	0.006	0.95[illegible]	0.226	8.826
40	7	0.005	0.957	0.181	7.221
41	4	0.003	0.959	0.106	4.335
42	7	0.005	0.964	0.190	7.961
43	9	0.006	0.970	0.250	10.729
44	4	0.003	0.972	0.113	4.993
45	1	0.001	0.973	0.029	1.306
46	6	0.004	0.977	0.178	8.186
47	10	0.006	0.983	0.303	14.242
48	3	0.002	0.985	0.093	4.456
49	6	0.004	0.989	0.190	9.288
50	5	0.003	0.992	0.161	8.069

Rock : quartzite. Size :-38+12.7mm.
Total number of chords measured : 1551
$v_L^1 = 13.172$
$\mu_L^2 = 331.667$

L	$g(L)$	$p(L)$	$P(L)$	$p^*_{\alpha+1}$	$p^*_{\alpha+2}$
51	0	0.000	0.992	0.000	0.000
52	2	0.001	0.994	0.067	3.487
53	0	0.000	0.994	0.000	0.000
54	1	0.001	0.994	0.035	1.880
55	1	0.001	0.995	0.035	1.950
56	0	0.000	0.995	0.000	0.000
57	0	0.000	0.995	0.000	0.000
58	1	0.001	0.995	0.037	2.169
59	1	0.001	0.996	0.038	2.244
60	1	0.001	0.997	0.039	2.321
61	0	0.000	0.997	0.000	0.000
62	0	0.000	0.997	0.000	0.000
63	1	0.001	0.997	0.041	2.559
64	0	0.000	0.997	0.000	0.000
65	0	0.000	0.997	0.000	0.000
66	0	0.000	0.997	0.000	0.000
67	0	0.000	0.997	0.000	0.000
68	0	0.000	0.997	0.000	0.000
69	0	0.000	0.997	0.000	0.000
70	1	0.001	0.998	0.045	3.159
71	0	0.000	0.998	0.000	0.000
72	0	0.000	0.998	0.000	0.000
73	0	0.000	0.998	0.000	0.000
74	0	0.000	0.998	0.000	0.000
75	0	0.000	0.998	0.000	0.000
76	0	0.000	0.998	0.000	0.000
77	0	0.000	0.998	0.000	0.000
78	0	0.000	0.998	0.000	0.000
79	1	0.001	0.999	0.051	4.024
80	0	0.000	0.999	0.000	0.000
81	0	0.000	0.999	0.000	0.000
82	0	0.000	0.999	0.000	0.000
83	1	0.001	0.999	0.054	4.442
84	0	0.000	0.999	0.000	0.000
85	0	0.000	0.999	0.000	0.000
86	0	0.000	0.999	0.000	0.000
87	0	0.000	0.999	0.000	0.000
88	0	0.000	0.999	0.000	0.000
89	0	0.000	0.999	0.000	0.000
90	0	0.000	0.999	0.000	0.000
91	0	0.000	0.999	0.000	0.000
92	0	0.000	0.999	0.000	0.000
93	0	0.000	0.999	0.000	0.000
94	0	0.000	0.999	0.000	0.000
95	0	0.000	0.999	0.000	0.000
96	0	0.000	0.999	0.000	0.000
97	0	0.000	0.999	0.000	0.000
98	0	0.000	0.999	0.000	0.000
99	0	0.000	0.999	0.000	0.000
100	0	0.000	0.999	0.000	0.000

Rock : quartzite. Size :-38+12.7mm.
Total number of chords measured : 1551
$v_L^1 = 13.172$
$\mu_L^2 = 331.667$

L	$g(L)$	$p(L)$	$P(L)$	$p^*_{\alpha=1}$	$p^*_{\alpha=3}$
101	0	0.000	0.999	0.000	0.000
102	0	0.000	0.999	0.000	0.000
103	0	0.000	0.999	0.000	0.000
104	0	0.000	0.999	0.000	0.000
105	1	0.001	1.000	0.068	7.108
106	0	0.000	1.000	0.000	0.000
107	0	0.000	1.000	0.000	0.000
108	0	0.000	1.000	0.000	0.000
109	0	0.000	1.000	0.000	0.000
110	0	0.000	1.000	0.000	0.000
111	0	0.000	1.000	0.000	0.000
112	0	0.000	1.000	0.000	0.000
113	0	0.000	1.000	0.000	0.000
114	0	0.000	1.000	0.000	0.000
115	0	0.000	1.000	0.000	0.000
116	0	0.000	1.000	0.000	0.000
117	0	0.000	1.000	0.000	0.000
118	0	0.000	1.000	0.000	0.000
119	0	0.000	1.000	0.000	0.000
120	0	0.000	1.000	0.000	0.000
121	0	0.000	1.000	0.000	0.000
122	0	0.000	1.000	0.000	0.000
123	0	0.000	1.000	0.000	0.000
124	0	0.000	1.000	0.000	0.000
125	0	0.000	1.000	0.000	0.000
126	0	0.000	1.000	0.000	0.000
127	0	0.000	1.000	0.000	0.000
128	0	0.000	1.000	0.000	0.000
129	0	0.000	1.000	0.000	0.000
130	0	0.000	1.000	0.000	0.000
131	0	0.000	1.000	0.000	0.000
132	0	0.000	1.000	0.000	0.000
133	0	0.000	1.000	0.000	0.000
134	0	0.000	1.000	0.000	0.000
135	0	0.000	1.000	0.000	0.000
136	0	0.000	1.000	0.000	0.000
137	0	0.000	1.000	0.000	0.000
138	0	0.000	1.000	0.000	0.000
139	0	0.000	1.000	0.000	0.000

Appendix C

Sample calculation using a simplified version of King's solution.

The formulae used is the solution derived for spheres as given in equation [7.31]. Note that $C(M,\beta)$ is assumed to be zero, and that μ_1 can be factorized out and for convenience is assumed to be 1.

In the first step the raw chord length distribution $g(L_i)$ is low-pass filtered with a 3 element moving avergae filter to give $g_f(L_i)$ where $g_f(L_i) = \{g(L_{i-1}) + g(L_i) + g(L_{i+1})\}/3$. The filtered distribution is then reduced to a density distribution $p_f(L_i)$.

The estimation of the cumulative sieve-size distribution $F(D)$ is then calculated with the following equation taken from equation [7.31]. $F(D_i) = \frac{1}{\mu_1}\left\{P_1(L_i) - \frac{L_i^2}{\beta}p(L)\right\}$ where $P_1(L_i) = \sum_{j=0}^{i} L\,p_f(L_i)$.

Rock : quartzite. Size : -38+25mm
Total number of chords measured : 320
$\beta = 1.28$

L_i	L_i(mm)	$g(L_i)$	$g_f(L_i)$filtered	$p_f(L_i)$	$P_1(L_i)$	$\frac{L}{\beta}p(L_i)$	$F(D)$
0	0.0	0.001	0.002	0.000	0.000	0.000	0.000
1	0.5	0.002	0.002	0.002	0.002	0.000	0.000
2	1.0	0.003	0.003	0.008	0.010	-0.002	0.000
3	1.5	0.004	0.004	0.020	0.029	-0.009	0.000
4	2.0	0.005	0.005	0.040	0.064	-0.024	-0.001
5	2.5	0.006	0.006	0.070	0.120	-0.050	-0.002
6	3.0	0.007	0.007	0.112	0.202	-0.090	-0.003
7	3.5	0.008	0.008	0.168	0.314	-0.146	-0.006
8	4.0	0.009	0.009	0.241	0.462	-0.221	-0.008
9	4.5	0.010	0.010	0.331	0.650	-0.319	-0.012
10	5.0	0.011	0.023	0.557	1.808	-1.251	-0.047
11	5.5	0.047	0.034	0.926	3.249	-2.323	-0.088
12	6.0	0.043	0.043	1.447	5.003	-3.555	-0.134
13	6.5	0.041	0.040	1.966	5.396	-3.430	-0.129
14	7.0	0.036	0.038	2.492	5.888	-3.397	-0.128
15	7.5	0.036	0.035	3.015	6.274	-3.260	-0.123
16	8.0	0.033	0.034	3.553	6.886	-3.333	-0.126
17	8.5	0.033	0.032	4.096	7.373	-3.278	-0.124
18	9.0	0.031	0.031	4.648	7.967	-3.319	-0.125
19	9.5	0.029	0.031	5.236	8.933	-3.608	-0.139
20	10.0	0.033	0.030	5.839	9.649	-3.810	-0.144
21	10.5	0.028	0.029	6.458	10.399	-3.941	-0.149
22	11.0	0.027	0.027	7.061	10.623	-3.562	-0.134
23	11.5	0.027	0.027	7.684	11.450	-3.766	-0.142
24	12.0	0.027	0.027	8.321	12.236	-3.915	-0.148
25	12.5	0.026	0.025	8.939	12.353	-3.414	-0.129
26	13.0	0.021	0.023	9.547	12.646	-3.099	-0.117
27	13.5	0.023	0.022	10.154	13.113	-2.960	-0.112
28	14.0	0.023	0.022	10.775	13.908	-3.134	-0.118
29	14.5	0.021	0.021	11.393	14.345	-2.952	-0.111
30	15.0	0.021	0.020	12.006	14.716	-2.710	-0.102
31	15.5	0.020	0.020	12.642	15.760	-3.118	-0.118
32	16.0	0.021	0.020	13.271	16.109	-2.838	-0.107
33	16.5	0.018	0.019	13.899	16.594	-2.623	-0.102
34	17.0	0.019	0.018	14.503	16.415	-1.912	-0.072
35	17.5	0.017	0.017	15.107	16.901	-1.794	-0.068
36	18.0	0.017	0.016	15.673	16.315	-0.642	-0.024
37	18.5	0.014	0.015	16.229	16.467	-0.237	-0.009
38	19.0	0.015	0.014	16.743	15.621	1.122	0.042
39	19.5	0.012	0.013	17.239	15.456	1.783	0.067
40	20.0	0.011	0.011	17.695	14.608	3.088	0.116

L_i	$L_i$mm	$g(L_i)$	$g_f(L_i)$filtered	$p_f(L_i)$	$P_i(L_i)$	$\frac{i}{n}p(L_i)$	$F(D)$
41	20.5	0.011	0.011	18.145	14.769	3.377	0.127
42	21.0	0.011	0.011	18.599	15.250	3.349	0.126
43	21.5	0.011	0.011	19.060	15.856	3.204	0.121
44	22.0	0.011	0.010	19.516	16.057	3.460	0.131
45	22.5	0.010	0.010	19.945	15.444	4.501	0.170
46	23.0	0.008	0.009	20.345	14.726	5.620	0.212
47	23.5	0.008	0.008	20.729	14.417	6.312	0.238
48	24.0	0.008	0.008	21.113	14.765	6.348	0.239
49	24.5	0.008	0.008	21.487	14.662	6.826	0.258
50	25.0	0.007	0.007	21.817	13.190	8.628	0.325
51	25.5	0.005	0.006	22.107	11.828	10.275	0.388
52	26.0	0.005	0.005	22.384	11.5[illegible]4	10.870	0.410
53	26.5	0.006	0.005	22.661	11.738	10.923	0.412
54	27.0	0.005	0.005	22.928	11.550	11.378	0.429
55	27.5	0.004	0.004	23.173	10.798	12.375	0.467
56	28.0	0.004	0.004	23.409	10.553	12.856	0.485
57	28.5	0.004	0.004	23.653	11.117	12.535	0.473
58	29.0	0.005	0.004	23.898	11.372	12.526	0.473
59	29.5	0.004	0.004	24.135	11.170	12.965	0.489
60	30.0	0.003	0.003	24.328	9.279	15.049	0.568
61	30.5	0.002	0.003	24.481	7.456	17.025	0.642
62	31.0	0.002	0.002	24.611	6.448	18.163	0.685
63	31.5	0.002	0.002	24.735	6.255	18.480	0.697
64	32.0	0.002	0.002	24.872	7.006	17.866	0.674
65	32.5	0.003	0.002	25.003	6.821	18.182	0.686
66	33.0	0.002	0.002	25.121	6.262	18.859	0.711
67	33.5	0.001	0.002	25.226	5.595	19.631	0.741
68	34.0	0.002	0.002	25.329	5.592	19.736	0.745
69	34.5	0.002	0.002	25.434	5.819	19.615	0.740
70	35.0	0.001	0.001	25.520	4.799	20.721	0.782
71	35.5	0.001	0.001	25.584	3.669	21.916	0.827
72	36.0	0.001	0.001	25.640	3.232	22.408	0.845
73	36.5	0.001	0.001	25.696	3.265	22.431	0.846
74	37.0	0.001	0.001	25.756	3.543	22.214	0.838
75	37.5	0.001	0.001	25.812	3.332	22.479	0.848
76	38.0	0.001	0.001	25.856	2.712	23.144	0.873
77	38.5	0.000	0.001	25.895	2.381	23.513	0.887
78	39.0	0.000	0.001	25.943	3.019	22.924	0.865
79	39.5	0.001	0.001	25.996	3.335	22.662	0.855
80	40.0	0.001	0.001	26.064	4.326	21.738	0.820
81	40.5	0.001	0.001	26.122	3.786	22.336	0.843
82	41.0	0.001	0.001	26.175	3.435	22.739	0.858
83	41.5	0.000	0.000	26.208	2.219	23.989	0.905
84	42.0	0.000	0.000	26.246	2.529	23.717	0.895
85	42.5	0.001	0.000	26.279	2.297	23.982	0.905
86	43.0	0.000	0.000	26.306	1.860	24.446	0.922
87	43.5	0.000	0.000	26.315	0.579	25.735	0.971
88	44.0	0.000	0.000	26.321	0.410	25.910	0.977
89	44.5	0.000	0.000	26.329	0.640	25.[illegible]90	0.969
90	45.0	0.000	0.000	26.341	0.860	25.482	0.961
91	45.5	0.000	0.000	26.354	0.879	25.475	0.961
92	46.0	0.000	0.000	26.367	1.006	25.362	0.957
93	46.5	0.000	0.000	26.383	1.160	25.223	0.952
94	47.0	0.000	0.000	26.397	1.073	25.324	0.955
95	47.5	0.000	0.000	26.406	0.714	25.692	0.969
96	48.0	0.000	0.000	26.410	0.247	26.162	0.987
97	48.5	0.000	0.000	26.411	0.112	26.299	0.992
98	49.0	0.000	0.000	26.411	0.000	26.411	0.996
99	49.5	0.000	0.000	26.411	0.000	26.411	0.996
100	50.0	0.000	0.000	26.411	0.000	26.411	0.996
101	50.5	0.000	0.000	26.413	0.134	26.279	0.991
102	51.0	0.000	0.000	26.419	0.532	25.887	0.977
103	51.5	0.000	0.000	26.427	0.675	25.752	0.972
104	52.0	0.000	0.000	26.434	0.546	25.888	0.977
105	52.5	0.000	0.000	26.436	0.137	26.298	0.992
106	53.0	0.000	0.000	26.437	0.132	26.306	0.992
107	53.5	0.000	0.000	26.454	1.405	25.049	0.945
108	54.0	0.000	0.000	26.474	1.738	24.736	0.933
109	54.5	0.000	0.000	26.492	1.631	24.861	0.938
110	55.0	0.000	0.000	26.496	0.318	26.178	0.988

L_i	$L_i$mm	$g(L_i)$	$g_f(L_i)$filtered	$p_f(L_i)$	$P_1(L_i)$	$\frac{L}{\beta}p(L_i)$	$F(D)$
111	55.5	0.000	0.000	26.496	0.000	26.496	1.000
112	56.0	0.000	0.000	26.496	0.000	26.496	1.000
113	56.5	0.000	0.000	26.496	0.000	26.496	1.000
114	57.0	0.000	0.000	26.496	0.000	26.496	1.000
115	57.5	0.000	0.000	26.496	0.000	26.496	1.000
116	58.0	0.000	0.000	26.496	0.000	26.496	1.000
117	58.5	0.000	0.000	26.496	0.000	26.496	1.000
118	59.0	0.000	0.000	26.496	0.000	26.496	1.000
119	59.5	0.000	0.000	26.496	0.000	26.496	1.000
120	60.0	0.000	0.000	26.496	0.000	26.496	1.000
121	60.5	0.000	0.000	26.496	0.000	26.496	1.000
122	61.0	0.000	0.000	26.496	0.000	26.496	1.000
123	61.5	0.000	0.000	26.496	0.000	26.496	1.000
124	62.0	0.000	0.000	26.498	0.180	26.318	0.993
125	62.5	0.000	0.000	26.502	0.366	26.136	0.986
126	63.0	0.000	0.000	26.505	0.372	26.134	0.986
127	63.5	0.000	0.000	26.507	0.189	26.318	0.993
128	64.0	0.000	0.000	26.507	0.000	26.507	1.000
129	64.5	0.000	0.000	26.507	0.000	26.507	1.000
130	65.0	0.000	0.000	26.507	0.000	26.507	1.000
131	65.5	0.000	0.000	26.507	0.000	26.507	1.000
132	66.0	0.000	0.000	26.507	0.000	26.507	1.000
133	66.5	0.000	0.000	26.507	0.000	26.507	1.000
134	67.0	0.000	0.000	26.507	0.000	26.507	1.000
135	67.5	0.000	0.000	26.507	0.000	26.507	1.000
136	68.0	0.000	0.000	26.507	0.000	26.507	1.000
137	68.5	0.000	0.000	26.507	0.000	26.507	1.000
138	69.0	0.000	0.000	26.507	0.000	26.507	1.000
139	69.5	0.000	0.000	26.507	0.000	26.507	1.000
140	70.0	0.000	0.000	26.507	0.000	26.507	1.000
141	70.5	0.000	0.000	26.507	0.000	26.507	1.000
142	71.0	0.000	0.000	26.507	0.000	26.507	1.000
143	71.5	0.000	0.000	26.507	0.000	26.507	1.000
144	72.0	0.000	0.000	26.507	0.000	26.507	1.000
145	72.5	0.000	0.000	26.507	0.000	26.507	1.000
146	73.0	0.000	0.000	26.507	0.000	26.507	1.000
147	73.5	0.000	0.000	26.507	0.000	26.507	1.000
148	74.0	0.000	0.000	26.507	0.000	26.507	1.000
149	74.5	0.000	0.000	26.507	0.000	26.507	1.000
150	75.0	0.000	0.000	26.507	0.000	26.507	1.000
151	75.5	0.000	0.000	26.507	0.000	26.507	1.000
152	76.0	0.000	0.000	26.507	0.000	26.507	1.000
153	76.5	0.000	0.000	26.507	0.000	26.507	1.000
154	77.0	0.000	0.000	26.507	0.000	26.507	1.000
155	77.5	[illegible]	0.000	26.507	0.000	26.507	1.000
156	78.0	[illegible]	0.000	26.507	0.000	26.507	1.000
157	78.5	0.000	0.000	26.507	0.000	26.507	1.000
158	79.0	0.000	0.000	26.507	0.000	26.507	1.000
159	79.5	0.000	0.000	26.507	0.000	26.507	1.000
160	80.0	0.000	0.000	26.507	0.000	26.507	1.000

Appendix D

Sample Calculations of Direct and Constrained Linear Inversion

In this appendix *direct* and *constrained linear* inversion technqiues are applied to two typical chord-length distribution $g(L_i)$ in order to estimate size distributions $f^*(D_i)$ as described in chapter 7.2.2 and 7.2.3.

Two classes of chord-lengths distributions $g(L_i)$ are to be transformed to size distributions. The chord-length distributions were measured from circulating rockstreams of -50+38mm and -19+12.7mm quartzite. The chord-length distributions were derived using morphological edge detection and correction, and the distributions were measured using adjacent linear probes. Belt speed was approximately 0.5ms^{-1}. The kernel matrix presented in these calculations is derived from 5 size classes of quartzite namely, (i) -75+50mm, (ii) -38mm, (iii) -38+25mm, (iv) -25+19mm, and (v) -19+12.7mm.

Direct and constrained linear inversion use equations [7.19] and [7.25] repsectively. The chord-length vectors for the kernel matrice for these calculations were derived by averaging 30 chord-length distributions for each size class, and then compressing the averaged chord-length vector from 140 to 5 elements by averaging blocks of elements according to the mapping scheme given in figure table D1.

The raw chord-length distributions to be transformed are given in table D2, and table D3 presents the compressed chord-length distributions. Note that no data other than zero occurs after $i = 80$ and thus the full distribution to $i = 200$ is not presented in table D1.

As an example the -50+38mm chord-length data, the second term of the compressed distribution g_{comp} is calculated as follows:

$$g_{comp}(L_2) = \frac{g(L_3)+g(L_4)+g(L_5)+g(L_6)+g(L_7)+g(L_8)}{6}$$

$$= \frac{87+65+62+56+54+53}{6}$$

$$= 62.83$$

Each term is calculated in a similar manner, and then nomlaized to give $p_{comp}(L_i)$ where $p_{comp}(L_i) = \frac{g_{comp}(L_i)}{max[g_{comp}(L_i)]}$

and $max[g_{comp}]$ *is the element with the maximum value in* $g_{comp}(L_i)$. *For the chord-length distribution measured* for -50+38mm the first compressed term is 51.67. The maximum value of the conmpressed value $g_{comp}(L_1) = 51.67$, thus the first and second terms are $\frac{51.67}{62.83} = 0.822$ and 1.00 respectively, see table D3.

Table D1 : Table showing the particular mapping scheme for mapping elements from the chord-length distribution to the compressed distribution for this particular set of calculations.

elements i in $g(L_i)$	maps to element i in g_{comp}
0 - 2	1
3 - 8	2
9 - 20	3
21 - 52	4
53 -	5

Table D2 : First 80 points of two chord-length distributions measured from tests on the conveyor belt rig.

L_i	$g(L_i)$ -50+28	$g(L_i)$ -19+12.7	L_i	$g(L_i)$ -50+28	$g(L_i)$ -19+12.7	L_i	$g(L_i)$ -50+28	$g(L_i)$ -19+12.7	L_i	$g(L_i)$ -50+28	$g(L_i)$ -19+12.7
0	0	0	20	10	58	40	4	5	60	7	0
1	108	946	21	12	78	41	8	0	61	1	1
2	47	820	22	16	25	42	6	5	62	1	0
3	87	648	23	11	31	43	8	0	63	0	0
4	65	[illegible]44	24	17	17	44	9	1	64	2	0
5	62	486	25	12	17	45	22	0	65	0	0
6	56	395	26	18	15	46	20	1	66	2	0
7	54	349	27	14	22	47	17	0	67	0	0
8	63	302	28	10	14	48	9	0	68	0	0
9	58	271	29	18	18	49	3	0	69	1	0
10	42	252	30	8	16	50	6	0	70	0	0
11	23	246	31	8	12	51	9	0	71	0	0
12	28	202	32	9	5	5[illegible]	9	0	72	0	0
13	26	221	33	10	9	53	10	0	73	0	0
14	32	177	34	11	5	54	9	0	74	0	0
15	24	159	35	5	3	55	13	1	75	0	0
16	19	142	36	10	4	56	9	0	76	1	0
17	25	137	37	6	5	57	10	1	77	0	0
18	21	96	38	4	6	58	6	0	78	0	0
19	11	87	39	6	4	59	3	0	79	0	0

Table D3 : **Results of compressing the chord-length distribution data given in table D2 using the mapping given in table D1.**

g_{comp} -50+38mm	g_{comp} -19+12.7mm
0.8222812	1.0000000
1.0000000	0.7712344
0.4230769	0.2899207
0.1666114	0.0167752
0.0160981	0.0000586

The following steps progress through the calculation of the estimated size distribution $f^*(D_i)$ using the data given in table D3 as input. Two methods are used, *direct* and *constrained* linear inversion where $f^*(D_i) = \mathrm{A}^{-1}p(L_i)$ and $f^*_{CLI}(D_i) = (\mathrm{A}^T\mathrm{A} + \lambda H)^{-1} \cdot \mathrm{A}^T \cdot p_s(L_i)$ respectively. Note that the the smoothing function **H** used in this example is the quadratic.

A =

1.0000	1.0000	1.0000	1.0000	1.0000
0.8938	0.9069	0.8596	0.7896	0.8287
0.3981	0.4195	0.3706	0.3472	0.3453
0.1313	0.1508	0.1510	0.0951	0.0567
0.0289	0.0160	0.0073	0.0017	0.0007

A^T =

1.0000	0.8938	0.3981	0.1316	0.0289
1.0000	0.9069	0.4195	0.1508	0.0160
1.0000	0.8596	0.3706	0.1510	0.0073
1.0000	0.7896	0.3472	0.0951	0.0017
1.0000	0.8287	0.3453	0.0567	0.0007

$\mathrm{A}^T \cdot \mathrm{A}$ =

1.9756	1.9980	1.9360	1.8566	1.8857
1.9980	2.0216	1.9580	1.8761	1.9050
1.9360	1.9580	1.8992	1.8218	1.8489
1.8566	1.8761	1.8218	1.7531	1.7796
1.8857	1.9050	1.8489	1.7796	1.8092

H =

1	-1			
-1	2	-1		
	-1	2	1-	
		-1	2	-1
			-1	1

λH = (λ = 0.127)

0.1270	-0.1270	0.0000	0.0000	0.0000
-0.1270	0.2540	-0.1270	0.0000	0.0000
0.0000	-0.1270	0.2540	-0.1270	0.0000
0.0000	0.0000	-0.1270	0.2540	-0.1270
0.0000	0.0000	0.0000	-0.1270	0.1270

λH = (λ = 0.066)

0.0660	-0.0660	0.0000	0.0000	0.0000
-0.0660	0.1320	-0.0660	0.0000	0.0000
0.0000	-0.0660	0.1320	-0.0660	0.0000
0.0000	0.0000	-0.0660	0.1320	-0.0660
0.0000	0.0000	0.0000	-0.0660	0.0660

$A^T \cdot A + \lambda H =$, $\lambda = 0.127$

2.1026	1.8710	1.9360	1.8566	1.8857
1.8710	2.2765	1.8310	1.8761	1.9050
1.9360	1.8310	2.1532	1.6948	1.8439
1.8566	1.8761	1.6948	2.0071	1.6526
1.8857	1.9050	1.8489	1.6526	1.9362

$H =$, $\lambda = 0.066$

2.0416	1.9320	1.9360	1.8566	1.8857
1.9320	2.1535	1.8920	1.8761	1.9050
1.9360	1.8920	2.01312	1.7558	1.8489
1.8566	1.8761	1.7558	1.8851	1.7136
1.8857	1.9050	1.8489	1.7136	1.8752

$(A^T \cdot A + \lambda H)^{-1} =$

8.22	2.22	-1.73	-4.00	-5.12
2.22	4.02	-0.12	-2.58	-3.80
-1.73	-0.12	3.14	0.15	-1.32
-4.00	-2.58	0.15	4.38	2.54
-5.12	-3.80	-1.32	2.54	8.34

$(A^T \cdot A + \lambda H)^{-1} =$

14.52	3.32	-3.42	-6.77	-8.42
3.32	7.02	-0.32	-4.30	-6.23
-3.42	-0.32	5.98	0.32	-2.43
-6.77	-4.30	0.32	7.72	3.8
-8.42	-6.23	-2.43	3.80	14.25

$(A^T \cdot A + \lambda H)^{-1} \cdot A^T =$

-0.613	0.469	0.405	0.485	0.250
-0.261	0.340	0.317	0.419	0.120
0.117	0.066	0.020	0.167	-0.030
0.503	-0.212	-0.217	-0.331	-0.146
0.634	-0.246	-0.362	-0.733	-0.208

$(A^T \cdot A + \lambda H)^{-1} \cdot A^T =$

-0.765	0.730	0.649	0.766	0.430
-0.505	0.505	0.505	0.686	0.194
0.136	.037	-0.005	0.298	-0.061
0.775	-0.426	-0.335	-0.541	-0.246
0.973	-0.453	-0.625	-1.246	-0.344

$A^{-1} =$

3.707	-0.193	-10.170	-1.315	58.422
-7.152	-3.831	30.698	-4.388	-36.928
-1.310	13.340	-30.740	15.408	-14.506
12.032	-23.029	20.185	1.294	11.973
-6.278	13.713	-9.973	-11.090	-18.962

Results of Direct (D) and Constrained Linear Inversion (CLI)					
-50+38mm			-19+12.7mm		
[illegible]	$f_{CLI}(D_i)\lambda=0.127$	$f_D(D_i)$	[illegible]	$f_{CLI}(D_i)\lambda=0.066$	$f_D(D_i)$
0.8223	0.386	-0.726	1.0000	0.3973	-0.0682
1.0000	0.332	1.950	0.7712	0.3419	0.1832
0.4231	0.198	1.807	0.2899	0.2038	0.1509
0.1686	0.053	-4.187	0.0168	0.0649	-0.3932
0.0161	-0.003	2.178	0.0001	-0.0026	0.2048