



Versatile all-digital transport-of-intensity based wavefront sensor and adaptive optics using a DMD

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Abstract: Measuring and correcting wavefront aberrations is an important process in a wide variety of disciplines, from ophthalmology, laser cutting, and astronomy to free-space communication and microscopy, and always relies on measuring intensities to infer phase. One approach is to use the transport-of-intensity as a means for phase retrieval, exploiting the connection between observed energy flow in optical fields and their wavefronts. Here we present a simple scheme, using a digital micro-mirror device (DMD), to perform angular spectrum propagation and extract the wavefront of optical fields at various wavelengths, dynamically, with high resolution and tuneable sensitivity. We verify the capability of our approach by extracting common Zernike aberrations, turbulent phase screens, and lens phases under static and dynamic conditions at multiple wavelengths and polarizations. We use this setup for adaptive optics, correcting distortion using a second DMD to apply conjugate phase modulation. We observed effective wavefront recovery under a variety of conditions which allowed for convenient real-time adaptive correction in a compact arrangement. Our approach provides an all-digital system that is versatile, cheap, fast, accurate, broadband and polarization invariant.

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1. Introduction

An optical wavefront is defined as the surface which lies (approximately) normal to the Poynting vector field, thus indicating where the field's energy will flow as it propagates [1]. Determining the effect a system has on the wavefront of an optical field passing through it is therefore important in characterizing, correcting and optimizing the system. In microscopy and astronomy the quality of microscope and telescope images is dictated by the point-spread-function (PSF) of the optical system, with deviations from the ideal PSF (dictated by the diffraction limit) due to wavefront aberrations induced by manufacturing limitations [2–4]. However, if the aberrations are known, computational techniques can be used to improve the quality of the images [5,6]. In ophthalmology, wavefront aberrations inherent in the human eye which cause vision impairment are surgically compensated for once measured [7,8]. In free-space optical communication, modal crosstalk due to aberrations caused by atmospheric turbulence result in signal errors, post-/pre-correction of these aberrations can reduce the bit error rate [9–12]. In laser cutting the ability of focusing lenses to concentrate the power of an optical field into a small region is compromised by wavefront aberrations which distort the focal field distribution [13–15], correction of these aberrations can be performed using robust deformable mirrors. In all the above cases the accurate determination of aberrations caused by a given optical system is essential in effectively compensating for them, either through physical or computational means. Currently employed methods include interferometric sensors which provide high accuracy and sensitivity, but involve large systems with reference beams which are susceptible to environmental perturbations [16–18]. The ophthalmology and astronomy communities often utilise Shack-Hartman lenslet arrays to acquire fast, low resolution wavefront measurements which are then interpolated [19]. Certain specialized applications, such as calibration of large interferometers, use modal techniques to

estimate aberrations with extremely high sensitivity [20,21]. The transport-of-intensity equation (TIE) describes how the phase of light is related to its energy flow under propagation, and was proposed for use in wavefront sensing and adaptive compensation decades ago [22,23]. Since, the TIE has found applications most notably in quantitative phase imaging (QPI) [24–26] while also being used to measure lens induced curvature [27], as well as atmospheric turbulence [28]. The equation itself can be derived using Poynting's theorem, which highlights the elegance of the technique: that it is based directly on the conservation of energy [29]. Conventionally the intensity images required to extract phase using the TIE are acquired through translation of a camera along the light's path, single shot measurements at fixed propagation lengths can be achieved using beam displacement setups [30] and tune-able propagation lengths can be implemented using liquid crystal spatial light modulators (SLMs) [31,32]. These methods do, however, suffer from a variety of drawbacks such as bulky arrangements, high costs, alignment instability, wavelength dependent calibration and polarization constraints.

Here we present an all-digital scheme that can accurately extract the global wavefront of an optical field in a fast, cheap and versatile manner. Our approach exploits a digital micro-mirror device (DMD). Initially developed for the digital projection industry [33], DMDs offer relatively low-cost, rapid modulation, polarization insensitivity and broadband operation making them an attractive alternative for manipulating structured light. The essential component consists of an array of individually-addressable micro-mirrors situated behind a glass protective film and attached to a driver unit which facilitates control via an external source. Each micro-mirror can pivot about a hinge in either one of two directions, representing the two possible states for operation, namely: on and off. This allows the DMD to mimic a dynamic amplitude-only grating [34]. The DMD is placed at the focal plane of a $4f$ imaging system to perform angular spectrum propagation of the field, the intensity of which is imaged using a CCD. The captured intensities at 3 (or more) planes are used to determine the wavefront of the field using the TIE. Our approach offers a number of advantages over alternative methods: the polarization insensitivity and broadband nature of DMDs removes the need to calibrate for the specific polarization and wavelength required by SLMs, while still retaining the tune-ability and automation provided by a digital propagation scheme [35]. Additionally, the kHz order frame rates offered by DMDs make them a more powerful tool for adaptive optics applications where rapidly changing wavefronts need to be characterized and corrected. We highlight the resolution and sensitivity of our approach by measuring induced Zernike aberrations in static samples, and demonstrate the real-time ability on dynamically changing turbulence phase screens. We then demonstrate broadband operation in a single optical setup probed at multiple wavelengths and with multiple polarizations. We believe this work will be of relevance to fields where wavefront measurements are ubiquitous.

2. Theory

2.1. Transport-of-intensity equation

The Poynting vector, $\vec{S} = \vec{S}(\vec{r})$, represents the directional energy flux of an electromagnetic field. Conservation of energy provides us with the condition that the total flux over a closed surface s must be zero for a system free of charge and current densities, namely

$$\int_s \vec{S} d\vec{s} = \vec{\nabla} \cdot \vec{S} = 0, \quad (1)$$

where $\vec{\nabla} = (\partial_x, \partial_y, \partial_z)$. For monochromatic light (of frequency ω) we can use a Helmholtz decomposition to separate $\vec{S}$ into a curl-free scalar potential $\varphi = \varphi(\vec{r})$ and divergence-free vector potential $\vec{A} = \vec{A}(\vec{r})$, according to

$$\vec{S} = \frac{\omega}{4\pi} \left(\vec{\nabla} \varphi + \vec{\nabla} \times \vec{A} \right). \quad (2)$$

We can now substitute Eq. (2) into Eq. (1), and use the paraxial approximation (assuming propagation along z) to find

$$-k\partial_z I = \vec{\nabla}_T \cdot (I\vec{\nabla}_T \varphi) + \vec{\nabla}_T I \cdot (\vec{\nabla} \times \vec{A})_T, \quad (3)$$

where I is the field intensity at some plane of interest, say $z = 0$, and the subscript T indicates quantities in the plane transverse to propagation [36]. The term involving $\vec{\nabla} \times \vec{A}$ is only dominant in scalar optical fields that have embedded phase singularities and is commonly disregarded when using the TIE for phase retrieval (we will return to this point later in the discussion). After some manipulation a relaxation/iterative method can be applied to approximate solutions Eq. (3) [37]. We observed the solution converging after only a single iteration and chose to use the following reduced form of the approximation (see [Supplement 1](#))

$$\phi \approx \vec{\nabla}_T^{-2} \left(\frac{-k\partial_z I}{I + C} \right), \quad (4)$$

where $\vec{\nabla}_T^{-2}$ is the inverse of the transverse Laplacian operator, which can be numerically solved using fast Fourier transforms (as described in [Supplement 1](#)). Here C is a constant chosen by the user, introduced to deal with intensity nulls (i.e., regions where $I = 0$). It should be noted that C should be chosen to be small in order to not distort the recovered phase, throughout this work we set C to 1 % of the maximum intensity at the plane of interest. The wavefront (WF) can then be calculated through constant multiplication $WF = \frac{\lambda}{2\pi} \phi$ for homogeneously polarized fields. It should be noted that there are some limitations on the ϕ distributions which can be recovered by Eq. (4) due to the approximations and assumptions made in its derivation. Notably if ϕ has a (non-flat) structure which results in $\partial_z I = 0$, Eq. (4) will (incorrectly) recover a flat profile due to disregarding $\vec{\nabla} \times \vec{A}$ (we later discuss some methods with potential to overcome this). Additionally the relaxation approximation requires the system to behave linearly between iterations, potential errors due to non-linearity will scale with the distance over which $\partial_z I$ is approximated (we discuss this in the following section).

2.2. Determining the intensity gradient

In order to determine the unknown $\partial_z I(x, y, z)$, images of the intensity at multiple planes along the propagation axis z are required. For our purpose we use three planes (i.e., $z \in [-\delta z, 0, \delta z]$). In order to acquire these images we utilise a 4f imaging system with a DMD modulating the far field by a transmission function $P(z) = e^{ik_z(k_x, k_y)z}$, the intensity at the end of the imaging system is described by

$$I_{\text{out}}(x, y) = |\mathcal{F}^{-1}(\mathcal{F}(U(x, y, 0))P(z))|^2, \quad (5)$$

where $U(x, y, 0)$ is the field of interest. Coordinate substitutions applied to the Fresnel diffraction integral, combined with an application of the convolution theorem have revealed that the observed intensity is equal to that of the field at the chosen distance z , therefore

$$I_{\text{out}}(x, y) = |U(x, y, z)|^2 = I(x, y, z). \quad (6)$$

This holographic implementation of the angular spectrum propagation technique has been reliably used to investigate the propagation dynamics for various purposes [35,38]. We approximate our intensity gradient along the z axis as

$$\partial_z I(x, y, z) \approx \frac{I_{\text{out}}(x, y, \delta z) - I_{\text{out}}(x, y, -\delta z)}{2\delta z}. \quad (7)$$

Using Eq. (7), $I(x, y, 0)$ and Eq. (4) we can extract $\phi(x, y, 0)$. To facilitate the required modulation of the field by the DMD we employed a conventional complex amplitude modulation

scheme [34,39,40]. The associated binary holograms took the following form

$$H(\vec{r}_T) = \frac{1}{2} + \frac{1}{2} \text{sign}(\cos(\vec{r}_T \cdot \vec{g}_T + \arg(P(z))) - \cos(\arcsin(|P(z)|))), \quad (8)$$

where $\vec{r}_T = (x, y)$ are the transverse coordinates and $\vec{g}_T = (g_x, g_y)$ are the Cartesian grating frequencies. A benefit presented by the holographic approach to acquire $\partial_z I$ is that the parameter δz can be adjusted based on the desired application. It has been shown in previous investigations that larger δz leads to higher sensitivity measurements, while smaller values are effective for stronger aberrations and measuring phase objects with high spatial frequencies [29]. It should be noted that while the TIE alone cannot resolve certain structures such as: phase vortices (i.e., wavefronts that helically wrap around the propagation axis corresponding to energy swirling around an intensity null) and flat phase discontinuities (i.e., discontinuous wavefronts lying parallel to the propagation axis), phase flattening techniques can be implemented alongside the TIE allowing these features to be probed [41]. Furthermore these phase flattening procedures have been effectively implemented using DMDs and can be readily integrated into our proposed scheme [42].

3. Experiment

To validate our technique we utilized the experimental arrangements shown in Fig. 1(a). A Gaussian beam produced by a laser (both a HeNe operating at $\lambda = 633$ nm and an Ar-ion operating at either $\lambda = 457(488)$ nm were used) was expanded and collimated using lenses L_E and L_C , respectively. The expanded beam was directed onto the screen of DMD₁ (DLP6500). The DMD was addressed with a binary hologram which facilitated modulating the amplitude and phase of the input field following the approach outlined in Ref. [40] (see Supplement 1). It should be noted that only the 1st diffraction order holds the desired modulated beam and an additional $4f$ system (omitted from Fig. 1(a) for clarity) was used to isolate this order at the focal plane with an aperture. We chose to encode a Gaussian amplitude profile as well as two cases of phase distributions: for measuring digitally controlled aberrations an aberrating phase in the form of Zernike aberrations or a numerically generated turbulence phase screen, for measuring physically induced aberrations a flat phase (acting as a probe) was encoded at DMD₁ with an additional $4f$ imaging system allowing phase distributions from a silica turbulence phase plate or NBK7 spherical lens to be measured. The sensor is constructed using lens L_1 which projects the Fourier transform of the aberrated field onto a second DMD₂ (DLP4710 [43]), which was encoded with the transmission function $P(z)$. It should be noted that three holograms corresponding to the different propagation distances can be multiplexed for single shot operation (see Supplement 1) or implemented sequentially if retrieval at the full CCD resolution is desired. Lens L_2 was then used to project the inverse Fourier transform of the field at DMD₂ onto a CCD camera (FLIR Grasshopper). It should be noted that multiple diffraction orders will appear in the plane of the CCD, however only the 1st diffraction order(s) are used in the wavefront estimation. For the exemplar measurements presented in this work $\delta z = 10$ mm was chosen, the Gaussian was given a waist radius of $w_0 = 1$ mm and the focal lengths used were: $f_E = 2$ mm, $f_C = 250$ mm, $f_1 = 1$ m and $f_2 = 300$ mm. Since the TIE is only valid in regions with $I > 0$ the recovered phase screens were truncated below the threshold of $I < 0.1 I_{\max}$, where $I_{\max}$ is the max intensity at the plane of interest (see Supplement 1). Once the wavefront aberrations were recovered using Eq. (4) the conjugate of the recovered phase was added to DMD₁ in order to adaptively correct the aberration as illustrated in Fig. 1(b). The effectiveness of this adaptive correction was determined by measuring the far-field intensities of the aberrated and corrected fields with a second CCD_F (FLIR Chameleon) at the focal plane of L_1 accessed using a 50:50 beam-splitter (BS) before DMD₂.

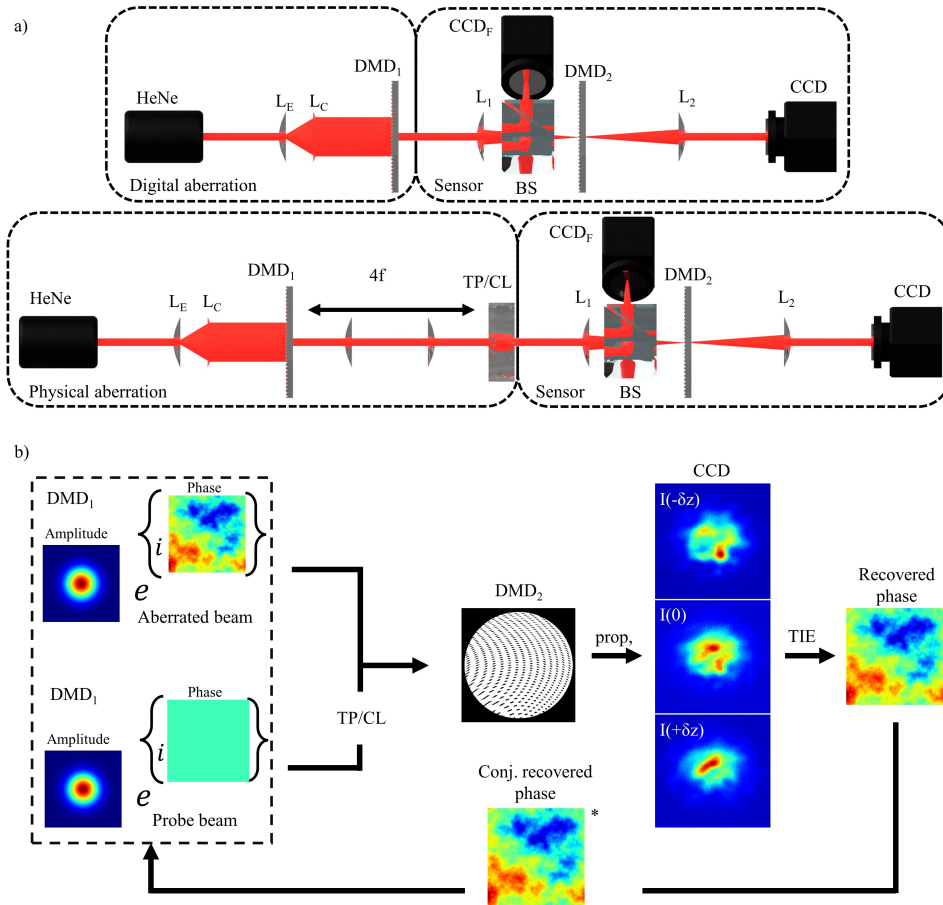


Fig. 1. (a) Experimental arrangements used to demonstrate how the DMD-TIE based wavefront sensor can recover digital (top) and physical (bottom) aberrations (note the sensor is comprised of $L_{1,2}$, DMD₂ and CCD while the BS and CCD_F were only used to assess the effectiveness of adaptive compensation). (b) Diagram showing how the aberrated/probe beams generated by DMD₁ are used to create digital/physical wavefront distortions, how DMD₂ is used to digitally propagate the field at three distances and how phase recovered using the TIE is conjugated to perform adaptive (pre-)correction using DMD₁.

4. Results and discussion

As a demonstration of effective operation, Fig. 2(a) shows the performance of the DMD enabled TIE phase recovery scheme on astigmatic (Z_2^2) and trefoil (Z_3^3) Zernike aberrations of various strengths. The general structure and strength of the aberrations are readily recovered as is evident by comparing the recovered and encoded aberrations, the latter shown in the insets. Displaying how recovered phases can be used for adaptive correction in our arrangement Fig. 2(b) shows encoded and recovered wavefronts from turbulence phase screens (generated using a Fourier-transform technique [44]). The insets display how the conjugate of the recovered phase is able to drastically improve the far-field intensity distribution when used for adaptive correction, evident in the Gaussian like shapes of the corrected intensities. It is expected that any residual aberrations can be removed through a second iteration of the correction procedure. The real-time operation of our adaptive correction procedure is highlighted in Fig. 3, where we can see

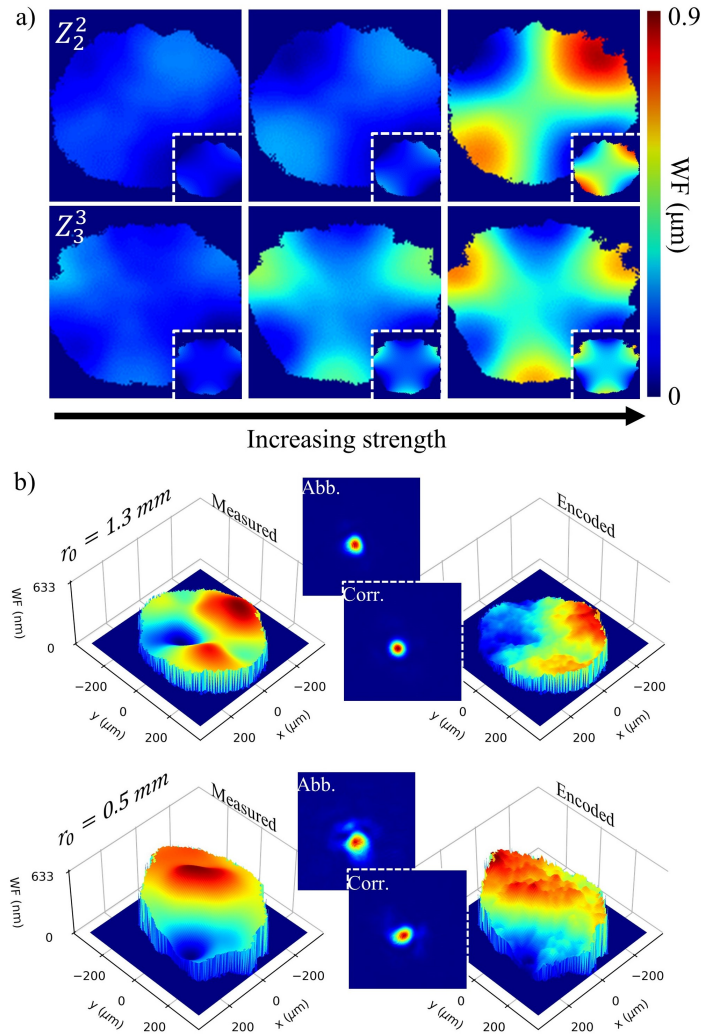


Fig. 2. (a) Measured astigmatic (top) and trefoil (bottom) aberrations at various strengths, insets show the encoded aberrations. (b) Measured (left) and recorded (right) wavefronts of turbulence phase aberrations with Fried parameters of $r_0 = 1.3 \text{ mm}$ (top) and $r_0 = 0.5 \text{ mm}$ (bottom) - insets show the aberrated and adaptively corrected far field intensities.

wavefront distortions produced by a turbulence phase plate being translated across a probe Gaussian beam. The wavefronts were captured at $\approx 5 \text{ Hz}$, a video depicting dynamically acquired frames is given in [Visualization 1](#). The translation of features (such as the two prominent valleys) in the recovered wavefront plots is clear and additionally the dynamically captured aberrated and corrected far-field intensities reveal the accuracy of the recovered wavefronts. It should be noted that only the frame rate of the CCD camera restricted the dynamic acquisition rate, while Eq. (4) is evaluated in $<2 \text{ ms}$ (with an Intel i5-8400 running MatLab) indicating the potential for $>500 \text{ Hz}$ acquisition rates if appropriate high-speed CMOS cameras are implemented. In order to highlight the benefit provided by the broadband nature of DMDs, Fig. 4 shows the effectiveness of the DMD based sensor at multiple wavelengths. The phase profile of a $f = 50 \text{ mm}$, BK7, spherical lens was measured using 488 nm and 457 nm wavelength light the results are shown in in Fig. 4(a). The radially symmetric pattern is clear in both cases with the 488

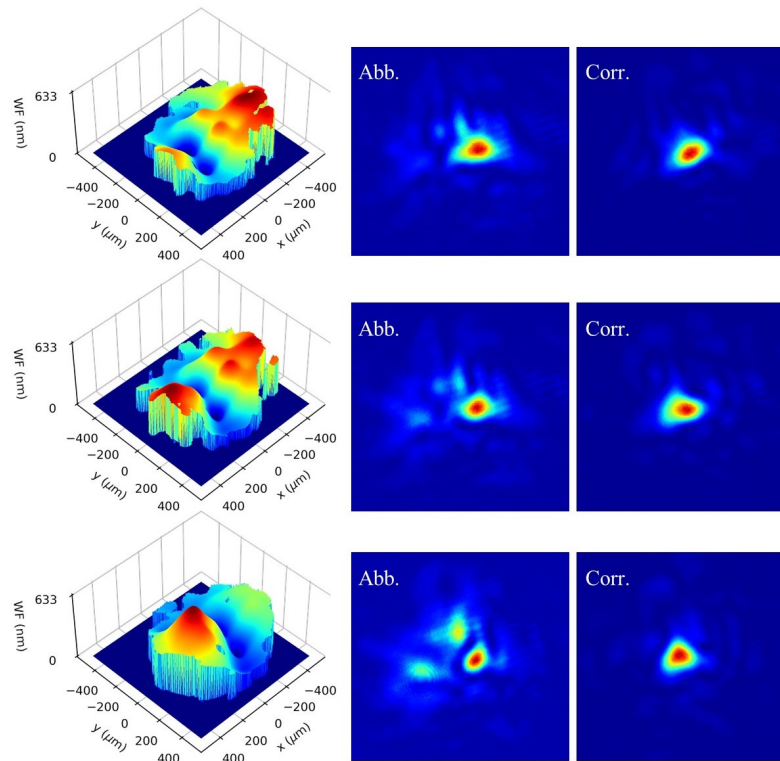


Fig. 3. Recovered wavefront aberrations (left) produced by a turbulence phase plate translated in the x direction, along with the aberrated (middle) and corrected (right) far-field intensities.

nm measurement showing some slight aberration in the region highlighted by the dotted block. Radial sections along the dashed lines are shown in Fig. 4(b) showing a comparison to the theoretical curves predicted by the thin lens approximation $\phi_{Th.} = kr^2/2f$, while there is some deviation from the ideal curves at both wavelengths the general trend and relative relationship between the wavelengths agrees well with expectations. This type of broadband operation is expected to function for the 400 nm to 700 nm operating wavelength range of the DMD without the need for re-calibration, revealing a significant advantage over SLM based approaches. In order to demonstrate the polarization invariance of the DMD based sensor turbulence phase plate results were taken with vertically and horizontally polarized light as shown in Fig. 5(a), the strong agreement between the two cases is clear with the small deviation likely originating due to the half-wave plate (used to rotate the plane of polarization) increasing the effective propagation distance. A verification of the polarizations captured at the CCD is given in Fig. 5(b) where each was sent through a vertical and horizontal linear polarizer, the non-zero intensities through the orthogonal polarizers are likely due to extinction ratio limitations.

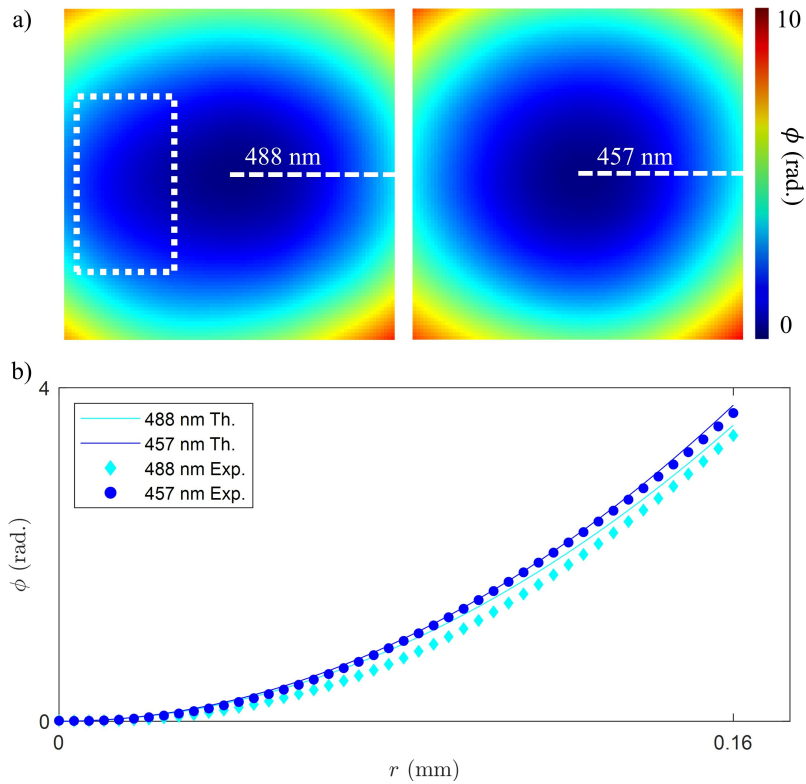


Fig. 4. (a) Recovered transverse phase profiles of a $f = 50$ mm focal length, N-BK7 spherical lens using TIE sensing implemented with a DMD at wavelengths of 457 nm and 488 nm. (b) Radial cross sections of the phase profiles at 457 nm and 488 nm along the dashed lines. Dotted square indicates a cylindrically assymmetric aberration in the 488 nm measurement.

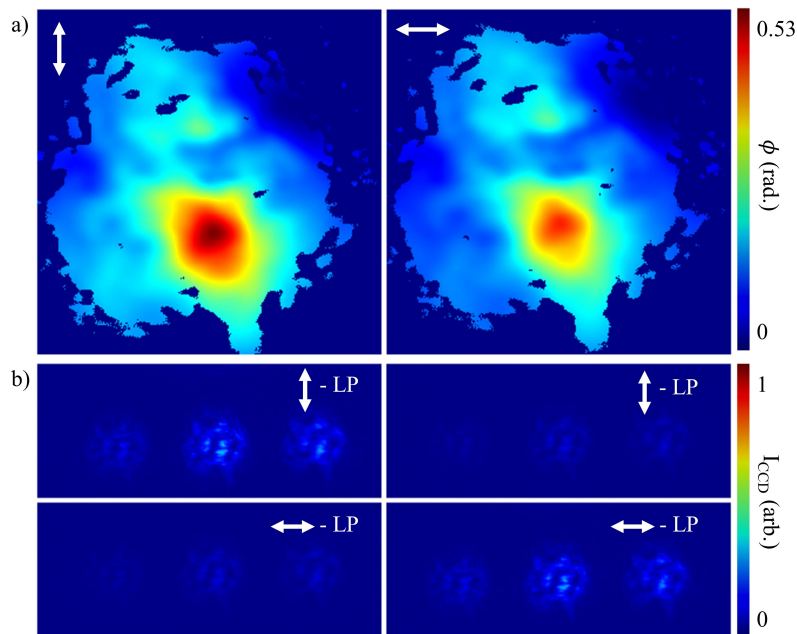


Fig. 5. (a) Phase profiles recovered from a turbulence phase plate using Gaussian probe beams with vertical (left) and horizontal (right) polarization. (b) CCD images containing the intensity profiles (captured in a single shot at each of the three propagation distances) for each polarized beam through vertical (top) and horizontal (bottom) linear polarizers.

5. Conclusion

We have demonstrated a proof-of-principle for a simple, micro-mirror enabled, digital wavefront sensor based on the transport of intensity. We have demonstrated the approach using standard Zernike aberrations, static and dynamic turbulence screens as well as broadband optics and at multiple polarizations. Additionally we have shown how the device can be effectively integrated into an adaptive optics arrangement. It can also be noted that the polarization insensitivity of DMDs indicates the potential for the approach to be used with vectorial light, which has been receiving growing interest in recent years [42]. The low cost and dynamic capabilities of the DMD in conjunction with the broadband operation and versatility of the holographic propagation implementation make this a promising addition to the adaptive optics toolkit.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

Supplemental document. See [Supplement 1](#) for supporting content.

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