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Hungarian-Murty Soft Decision Decoding Scheme for a MIMO-STBC-SPM Communication System

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Declaration

I declare that, for the Masters Degree at the University of Witwatersrand in Johannesburg, unless I have specifically acknowledged sources in this thesis submission, I verify that it is original and not presented for any degrees or tests at different universities.

KGAUGELO MARILYN BOPAPE

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Abstract

The integration of Multiple-Input-Multiple-Output (MIMO), Space-Time Block Coding (STBC), and Spatial Permutation Modulation (SPM) has led to significant advancements in spectral efficiency and system performance. However, the decoding process in such systems can be computationally demanding. To address this challenge, a soft-decision decoding algorithm based on the Hungarian-Murty algorithm is proposed.

The study aims to evaluate the impact and effectiveness of the Hungarian-Murty algorithm on the system's performance, particularly in terms of Bit Error Rate (BER). The proposed algorithm seeks to enhance decoding efficiency while maintaining reliable error correction performance. The research reviews various low-complexity decoding algorithms and methods used to assess Space-Time Block Code performance, which have guided the development of the proposed solution.

The study employs a MIMO-STBC-SPM transceiver system utilizing Quadrature Amplitude Modulation (QAM) and permutation in low-complexity schemes. The BER performance of these schemes is simulated in a Rayleigh fading multi-path channel with additional additive white Gaussian noise to support the evaluation of the proposed soft-decision decoding algorithm.

The Hungarian-Murty Algorithm, a combinatorial optimization algorithm, is used to solve assignment problems within MIMO systems with STBC-generalized spatial permutation modulation. This algorithm optimizes the signal-to-noise ratio (SNR) at the receiver by finding the optimal permutation matrix for data bit encoding. The soft-decision decoder iteratively ranks the costs of the input signal matrix until the assignment produces a valid code word, thereby improving decoding efficiency and error correction.

STBC-SM (Space-Time Block Code with Spatial Modulation) and STBC-SPM are discussed, highlighting their enhancements in error rate performance through the integration of STBC techniques and the use of permutation vectors. Spatial Permutation Modulation (SPM) and Generalized Spatial Modulation (GSM) are also explored, demonstrating their contributions to improving diversity and reducing error rates.

The findings of this study are expected to provide valuable insights into the Hungarian-Murty algorithm's effectiveness in reducing computational complexity

while maintaining reliable performance in MIMO-STBC-SPM systems, thereby advancing the field of wireless communications.

An analysis of complexity shows that the Hungarian Murty Algorithm (HMA) considerably lowers computational requirements. While traditional Maximum Likelihood (ML) Hard Decision Decoding operates with a factorial time complexity of $O(K! \cdot N_t)$, the HMA reduces this to a more manageable $O(K^4 \cdot N_t)$, where K is the size of the permutation vector, and N_t denotes the number of transmit antennas. This simplification makes the HMA particularly advantageous for large MIMO systems.

Regarding BER performance, the HMA-based decoder shows significant improvements, especially in systems utilizing 16-QAM modulation. Simulation results indicate that at higher SNR, the HMA can achieve up to a 2 dB improvement in BER compared to traditional ML Hard Decision Decoding. In contrast, at lower SNRs, 4-QAM systems perform better due to their resilience to noise, while the higher-order 16-QAM system benefits more from the HMA at increased SNRs, leading to better data throughput and reduced errors. Additionally, systems equipped with multiple receiving antennas ($N_r = 2$) experience up to a 30% decrease in BER at critical SNR levels.

The results from this research underscore the potential of the Hungarian-Murty Algorithm to improve both computational efficiency and error performance in MIMO-STBC-SPM systems. By effectively balancing complexity and performance, the HMA offers a promising solution for enhancing the reliability and efficiency of wireless communication systems, particularly in challenging or high-noise conditions.

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This work is a tribute, to my parents who have passed away and whose love and wisdom continue to inspire me every day. I also dedicate it to my family and friends for their support that means the world to me.

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List of Abbreviations

4G - Forth Generation

5G - Fifth-Generation

AGWN - Additive White Gaussian Noise

BER - Bit Error Rate

BPSK - Binary Phase-shift keying

CDMA - Code Division Multiple Access

ES-C - Exhaustive Search-Correlation

Eb/N0 - Energy per bit to noise power spectral density ratio

FBE-SM - Fractional Bit Encoded Spatial Modulation

FDMA - Frequency Division Multiple Access

FSK - Frequency Shift Key

FFT - Fast Fourier Transform

GSM - Generalized Spatial Modulation

GSPM - Generalized Spatial Permutation Modulation

GSSK - Generalized Space Shift Keying Modulation

HD - Hard Decision

HM - Hungarian-Murty

HMA - Hungarian-Murty Algorithm

i.i.d. - Independent Identically Distributed

ICI - Inter-Channel Interference

IFFT - Inverse Fast Fourier Transform

LED - Light-Emitting Diode

LTE - Long-Term Evolution

MFSK - Multiple Frequency-Shift Keying

MIMO - Multiple Input Multiple Output

MISO - Multiple Input Single Output

ML - Maximum Likelihood

MMSE - Minimum Mean Square Error

MRC - Maximal Ratio Combining

MRRC - Maximal Ratio Receiver Combining

OFDM - Orthogonal Frequency Division Multiplexing

OPEX - Operational Expenditure

OSTBC - Orthogonal Space-Time Block Code

QAM - Quadrature Amplitude Modulation

QR - Quick Response

QSPM - Quadrature Spatial Permutation Modulation

QSTBC - Quasi Space-Time Block Code

RF - Radio-Frequency

SD - Soft Decision

SER - Symbol Error Ratio

SM - Spatial Modulation

SNR - Signal-To-Noise Ratio

SPM - Spatial Permutation Modulation

SSK - Space Shift Keying

STBC - Space Time Block Code

STTC - Space Time Trellis Code

TCM - Trellis-Coded Modulation

TDMA - Time Division Multiple Access

V-BLAST–OFDM - Vertical Bell Labs Layered Space-Time OFDM

WiFi - Wireless Fidelity

List of Symbols

N_t - Number of transmit antennas

N_r - Number of receive antennas

T - Time instance

K - Size of permutation set

$d_{\min}$ - Minimum Hamming distance

s_{ij} - STBC encoded symbols

p_{ij} - Spatially permuted symbols

y_{ij} - Received signal at receive antenna i from transmit antenna j

h_{ij} - Fading coefficient

n_{ij} - Additive white Gaussian noise

c_{ij} - Cost associated with assigning the received symbol y_{ij} to the estimated symbol s_{ij}

$A(s_1, s_2, s_3, s_4)$ - Transmitted code-word for STBC-SPM

e - Encoding or modulation done to the QAM symbol

Y - Received signal matrix, dimensions $N_r \times T$

p_k - Code-word index in permutation set

P - Permutation matrix

$\mathbf{r}_t$ - Received signal vector

$\mathbf{H}_t$ - Channel matrix

$\mathbf{s}_t$ - Transmit signal vector

$\mathbf{n}_t$ - Noise vector

p_i - Groups of permutation vectors

θ_i - Rotation angle for group i

H_{sc} - Spatially correlated channel matrix

$R_t^{1/2}$ and $R_r^{1/2}$ - Correlation matrices at the transmitter and receiver ends respectively

$\hat{s}_{ij}$ - Estimated symbols

Z - Cost in the linear programming formulation

$a(r)$ - Assignment in ranking (r^{th})

K - Dimensionality of the vectors

n - Number of elements/nodes

O - Big O notation

$\in$ - An element of a set

$\notin$ - Not an element of the set

$\sum$ - The sum of the terms

$\forall$ - For all

Publications

Under Review

Kgaugelo Marilyn Bopape, Mulundumina Shimaponda-Nawa and Ling Cheng. Soft-Decision Decoding Scheme for a MIMO-STBC-SPM Communication System. Submitting to *IEEE Transactions on Communications*.

CHAPTER 1

Introduction

Multiple-Input Multiple-Output, Space Time Block Coding, Spatial Permutation Modulation (MIMO-STBC-SPM) represents a cutting-edge solution that leverages advanced techniques to optimize data transmission in modern wireless communication environments [27]. Combining the strength of MIMO, STBC, and SPM this integrated approach improves the system performance, dependability and spectral efficiency in wireless communication systems. However, the MIMO-STBC-SPM also results in substantial computational complexity, which is one of the primary drawbacks of incorporating these advanced techniques. This is because of several factors which contribute to this computational complexity. The computational requirements are mainly due to the signal processing algorithms for encoding, modulating, and decoding signals across multiple antennas and spatial streams.

The computational complexity is further increased due to the matrix operations required for spatial processing and diversity gain, together with other transformations represented by SPM. Complexity is increased by accurate channel state estimation across multiple antennas and spatial dimensions, as well as the intricate encoding and decoding processes inherent in SPM, STBC, and MIMO systems [27]. Furthermore, managing interference patterns and optimizing spatial resources necessitates sophisticated interference mitigation techniques, which further increase the computational overhead. However, the integration of these advanced techniques requires applying complex algorithms and computations to derive their benefits without being overly penalized by the inherent complexities and interactions between them.

This thesis proposes the Hungarian-Murty Algorithm (HMA) soft decision decoding algorithm for MIMO-STBC-SPM system to reduce the computational complexity in MIMO-STBC-SPM wireless communication. The HMA is a combinatorial optimization algorithm employed to solve assignment problems and is classified as a soft decision decoding algorithm. The task is to determine the best way to allocate tasks to resources to minimize or maximize overall cost or benefit. The HMA is famous for solving big assignment problems efficiently, because it is able to find the optimum solution in a short time [35]. The primary characteristic of the HMA is that it is able to find the optimal solution in polynomial time and therefore is very efficient for solving large scale assignment problems. The algorithm operates by repeatedly improving a set of dual variables and potentials to determine the optimum assignment that yields the minimum total cost, or the maximum total benefit [41].

For the MIMO-STBC-SPM wireless communication system, the Soft Decision (SD) was applied on the receiver side. Our study employed the HM as the SD decoding algorithm. First, we see the complexity reduction with the introduction of HM SD for its effectiveness of its implementation [26]. This research investigates the effect of the algorithm on the performance of the system and the potential benefits in terms of computational complexity reduction through simulation. To compare their performance, the simulation of MIMO-STBC-SPM was carried out with and without using the Hungarian-Murty algorithm. Other parameters such as modulation order and transceiver diversity were also analyzed to compare how they affect the performance of the system.

1.1 Research Questions

- Does the Hungarian-Murty Algorithm reduce computational complexity in a MIMO-STBC-SPM System?
- How does the HMA impact the performance of the MIMO-STBC-SPM system?

1.2 Aim and Objectives

The aim is to improve the performance in terms of the BER and computational complexity of a MIMO-STBC-SPM system by considering using an soft decision decoding algorithm.

The objectives are:

- To analyze how the Hungarian-Murty Algorithm performs in order to improve the computational complexity of MIMO-STBC-SPM.
- To describe how the Soft Decision Hungarian-Murty decoding algorithm works for enhancing computation of the MIMO-STBC-SPM system.
- To investigate the BER performance of the MIMO-STBC-SPM system with the Hungarian-Murty Algorithm embedded in it.

1.3 Research Significance and Author's Contribution

This study proposes the use of a soft decision decoding algorithm to reduce the computational complexity of wireless communication systems, especially MIMO-STBC-SPM. The soft decision decoding algorithm will reduce the system computational complexity without compromising on the reliability, capacity, error correction capability and spectral efficiency.

The implementation of the Hungarian-Murty Soft Decision algorithm in the MIMO-STBC-SPM system in order to reduce the computational complexity. The Hungarian-Murty Algorithm has not been applied in a MIMO-STBC-SPM system before. This research investigates potential gains in applying the Hungarian-Murty Algorithm to reduce the computational complexity as well as to improve the BER of the MIMO-STBC-SPM system.

1.4 Thesis Outline

The thesis has been presented as a six-chapter document and explores the integration of the Hungarian Murty algorithm into MIMO-STBC-SPM systems and its performance in terms of decoding accuracy and computational complexity.

Chapter 1 describes all key considered techniques applied to the MIMO wireless communication system and provides an overview. The research questions, aim, and objectives, as well as the research significance and author's contribution are also outlined under this chapter. Chapter 2 provides a comprehensive review of the existing literature on MIMO, STBC, and SPM techniques. It includes an in-depth analysis of the system models, parameter definitions, and equation notations relevant to each component. Additionally, it examines decoding algorithms, including their complexity analysis and practical examples, setting the stage for the subsequent discussion on Hungarian Murty integration. Chapter 3 provides the background and preliminaries that contain the baseline system (MIMO-STBC-SPM) for which the soft decision decoding algorithm will be integrated.

In Chapter 4 we learn about the Hungarian Murty algorithm, which can be used for soft decision decoding in MIMO-STBC-SPM systems. It includes the mathematical equations that describe the algorithm's operation and a flowchart for its implementation. In addition, a complete system model for MIMO, STBC, SPM, and Hungarian Murty is presented with a detailed description of transmitter, channel, and receiver aspects, together with complete parameter definitions and equation notations. The computational complexity of the proposed decoding algorithm is also thoroughly analyzed.

Chapter 5 performs analysis of the MIMO-STBC-SPM system with Hungarian Murty decoding. Simulation results are presented to compare the proposed method with previous methods based on decoding accuracy and computational complexity Based on the analysis insights, implications for real-world deployment and further research endeavors are made. The research contributions, limitations, and implications are outlined in Chapter 6. This chapter further presents potential research directions, such as applying and optimizing the Hungarian Murty parameters for real life communication systems to stimulate further growth in this area.

CHAPTER 2

Background and Preliminaries

This chapter is based on a paper that is currently under review and all our proposed contributions are built on top of this paper. The discussion starts with a detailed presentation of the MIMO-STBC-SPM system model that explains the basic ideas, the main characteristics and the main equations of the system. In this way, we create the foundation and the theoretical framework that enable the more sophisticated approaches and improvements discussed in the later chapters. This chapter is very important as it helps to connect the basic concepts with the new findings of this research.

2.1 Multiple-Input Multiple-Output, Space Time Block Code, Spatial Permutation Modulation

To improve the error rate performance of SM, STBC techniques were introduced to STBC-SM system, specifically Alamouti code was used. For a MIMO system with $N_t = 4$, four transmitted STBC-SM code-words, $\tilde{A}_i$, simultaneously activate two transmit antennas over two time instants. Each code-word comprises a specific structure involving QAM symbols (s_1, s_2) and additional bits represented by the selected code-word indices. The rotation angle θ_1, dependent on the modulation alphabet, is applied to maximize coding gain and diversity in the system [27].

For STBC-SPM, an extension of this concept, the transmission period is expanded T times larger than that of STBC-SM. Initially, data bits are utilized to select a permutation vector from the set $\mathcal{C}_{N_t,T}(K, d_{\min})$, with the vector's entries serving as code-word indices to concatenate specific code-words from the STBC-SM set as an example, for $T = 2$, an STBC-SPM code-word combines two STBC-SM code-words $\tilde{A}_i$ and $\tilde{A}_j$ which carry different QAM symbols. Further, for STBC-SM with $N_t = 4$, two other code-words $\tilde{A}_5$ and $\tilde{A}_6$ are also brought into the system, each having a different rotation angle θ_2 for enhancing the performance. These code-words, when multiplied by the same rotation angles, exhibit orthogonality to each other, enhancing system efficiency. Typically, in the STBC-SM system, only a subset of code-words ($\tilde{A}_1$ to $\tilde{A}_4$) are utilized for transmitting two bits effectively, rendering $\tilde{A}_5$ and $\tilde{A}_6$ redundant in practice [27].

Here, STBC-SPM uses the best properties of these two code words. For example, when two STBC-SM code words are combined, STBC-SPM can employ only 12 out of the 24 combinations of the four transmit antennas, which is sufficient to transmit three bits. With the help of 6 code-words the code-word combinations are increased to 30 from 15 and hence 16 combinations can be chosen to effectively transmit 4 bits. Thus, STBC-SPM offers a superior solution in terms of BER performance compared to other hybrid systems. This is because STBC-SPM virtually eliminates the ML problem by transferring the spatial domain's channel estimation complexity to the temporal domain, where it is more manageable. The BER performance of STBC-SPM improves as the number of spatial streams increases up to a point, beyond which additional streams become less beneficial due to increased ISI and the consequential need for more powerful signal processing techniques. In this scenario, both STBC-SM and STBC-SPM exhibit identical throughput for spatial symbols at 1 bps/Hz, while STBC-SPM achieves a lower error rate due to the permutation strategy that enhances diversity [27].

Similar to STBC-SM, which optimizes coding gain and diversity by applying rotation angles to code-words, we categorize permutation vectors into groups and introduce distinct rotation angles to transmitted code-words when different groups of permutation vectors are employed. This approach is designed to optimize the minimum Hamming distance of the permutation set via rotation as in [29] [27]. For example, when two STBC-SM code-words are cascaded, the permutation vectors can be arranged as follows:

$$
\begin{aligned}
P_1 &= \left\{ \begin{bmatrix} 6 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix}, \right\}, \\
P_2 &= \left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \end{bmatrix}, \right\}, \\
P_3 &= \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \end{bmatrix}, \right\}, \\
P_4 &= \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \end{bmatrix} \right\},
\end{aligned}
\tag{2.1}
$$

In which the Hamming distance is defined as; $d(\mathbf{p}_a, \mathbf{p}_b) = 2$ for permutation vectors within the same group, $\mathbf{p}_a, \mathbf{p}_b \in P_i$ and $d(\mathbf{p}_a, \mathbf{p}_b) \leq 2$ for vectors in different groups, $\mathbf{p}_a \in P_i$ and $\mathbf{p}_b \in P_j$ and $i \neq j$, where, P_i and P_j are the i^{th} and j^{th} group of permutation vectors, respectively and each group of P_i or P_j contains a subset of permutation vectors $\mathbf{p}_k$. Additionally, depending on the group index i, an extra rotation angle

$$
\phi_i = \frac{(i-1)\pi}{8} \tag{2.2}
$$

is applied to the code-words using QPSK and QAM modulations. The transmitted code-word for STBC-SPM is then expressed as:

$$
\mathbf{A}(s_1, s_2, s_3, s_4) = \left[\tilde{\mathbf{A}} p_{k,1}(s_1, s_2) e^{j\phi_i} \quad \tilde{\mathbf{A}} p_{k,2}(s_3, s_4) e^{j\phi_i} \right] \tag{2.3}
$$

with $\mathbf{p}_k \in P_i$ and $p_{k,j} = 1, 2, ..., 6$. Where j is the index of the antennas, i is the index of permutation subset, and k index of code in the vector.[1]

The extension from STBC-SM to STBC-SPM can be applied to other advanced SM techniques such as Differential Transmission (DT-SM) [55], [27]. Particularly, the transmit signal of DT-SM includes the double space-time transmit diversity (DSTTD) codeword [56] and the spatial constellation code-word [50], [6]. Since up to $16\binom{N_t}{N_t-4}$ [27] spatial constellation code-words can be generated and 4 QAM

[1] i and j are locally defined

symbols are conveyed by the DSTTD code-word, the DT-SM system achieves higher spectral efficiency compared to other SM systems. By utilizing permutation vectors to cascade the DT-SM code-words and conducting in-depth performance analysis, one can design the DT-SPM that further enhances transmission reliability [50].

2.1.1 Spatial Permutation Modulation

The design of the permutation set and the subsequent mapping strategy between the permutation vector and the information bits are crucial to the decoding process. The minimum Hamming distance ($d_{\min}$) and spatial correlations are critical in designing permutation sets for SPM [14][27].

The Hamming distance in simple terms, this is the number of positions where any two code-words differ. The Hamming distance is used to characterize a permutation set and is given by matrix $\mathbf{D}$, whose $(i;j)^{th}$ entry d_{ij} represents the Hamming distance between the i^{th} and j^{th} permutation vectors [27][29]. Suppose $\tilde{C}_{N_t,T}$ denotes the set of N_t permutations of a T-element vector, where N_t is the number of transmit antennas as earlier mentioned, T is the number of antennas transmitting at a time; we use a similar approach from [27] to illustrate the characterisation of a $\tilde{C}_{3,3}$ permutation subset $\{[1,3,2]^\top\ [2,1,3]^\top\ [2,3,1]^\top, [3,1,2]^\top\}$. The matrix $\mathbf{D}$ of such a permutation set is given by:

$$\mathbf{D} = \begin{bmatrix} 0 & 3 & 2 & 2 \\ 3 & 0 & 2 & 2 \\ 2 & 2 & 0 & 3 \\ 2 & 2 & 3 & 0 \end{bmatrix} \tag{2.4}$$

The distance between nodes i and j is given by $d_{(i,j)} = D_{(j,i)}$. For instance, the distance between nodes 1 and 3 is $d_{(1,3)} = \mathbf{D}_{(3,1)} = 2$, and the distance between nodes 2 and 4 is $d_{(2,4)} = \mathbf{D}_{(4,2)} = 2$. The shortest path between nodes i and j is the sequence of adjacent nodes that connects them, with a total edge weight no greater than the distance $d_{(i,j)}$. For example, the shortest path from node 1 to node 4 is (1, 2, 3, 4), with a length of 6. The diameter of the graph is the longest shortest path among all pairwise node combinations. In this case, the diameter is 4, corresponding to the shortest path between nodes 3 and 4, which has a length of 4.

The Hamming distance $d_{\min}$ is defined as the smallest off-diagonal component in the matrix $\mathbf{D}$, is derived from this Hamming distance, i.e.,

$$d_{\min} = \min_{i,j;i \neq j} d_{i,j} \tag{2.5}$$

As can be seen from the Hamming distance example above, $d_{\min} = 2$. Therefore, in a scenario where $\tilde{C}_{N_t,T}$ is defined by parameters (N_t, T, K), one approach entails the initial selection of permutation sets that maximizes $d_{\min}$, as outlined in [27]; K denotes the size of the permutation vectors with $d_{\min}$ from $\tilde{C}_{N_t,T}$. Consequently, the subset which selects K permutation vectors with $d_{\min}$ from $\tilde{C}_{N_t,T}$ is denoted as $C_{N_t,T}(K, d_{\min}) \subseteq \tilde{C}_{N_t,T}$.

By selecting permutation pairs with larger minimum Hamming distances, the mapping rule is designed to decrease pair-wise error probabilities to improve the error rate performance. The SPM is designed to adapt its permutation set $C_{N_t,T}(K, d_{\min})$ to dynamically trade-off between throughput and error rate performance, thus, involving a meticulous design process [29]. The SPM spatial symbol is then produced by selecting a specific permutation vector in $C_{N_t,T}(K, d_{\min})$ based on the data bits.

Increasing the number of antennas equips the transmitter with more options for usable permutation vectors, influencing both error rates and throughput in SPM transmissions.Table 2.1 shows the permutation sets adopted from [27] employed in the MIMO-STBC-SPM HMA system considered in this work.

TABLE 2.1: Permutation sets for different values of T, K, and d_{min}

T	K	d_{min}	$C_{4,T}(K, d_{\mathbf{min}})$
2	4	2	$[1;3]^\top, [2;4]^\top, [3;1]^\top, [4;2]^\top$
3	4	3	$[1;2;3]^\top, [2;3;4]^\top, [3;4;1]^\top, [4;1;2]^\top$
3	16	1	$[1;2;3]^\top, [1;3;2]^\top, [1;4;2]^\top, [1;4;3]^\top,$ $[2;1;3]^\top, [2;1;4]^\top, [2;3;4]^\top, [2;4;1]^\top,$ $[3;1;4]^\top, [3;2;1]^\top, [3;2;4]^\top, [3;4;1]^\top,$ $[4;1;2]^\top, [4;2;3]^\top, [4;3;1]^\top, [4;3;2]^\top$
4	4	4	$[1;2;3;4]^\top, [2;3;4;1]^\top, [3;4;1;2]^\top, [4;1;2;3]^\top$
4	8	3	$[1;2;3;4]^\top, [1;3;4;2]^\top, [1;4;2;3]^\top, [2;3;1;4]^\top,$ $[2;4;3;1]^\top, [2;1;4;3]^\top, [3;2;4;1]^\top, [4;1;3;2]^\top$
4	16	2	$[1;2;3;4]^\top, [1;2;4;3]^\top, [1;3;2;4]^\top, [1;3;4;2]^\top,$ $[1;4;2;3]^\top, [1;4;3;2]^\top, [2;1;3;4]^\top, [2;1;4;3]^\top,$ $[2;3;1;4]^\top, [2;3;4;1]^\top, [3;1;2;4]^\top, [3;1;4;2]^\top,$ $[3;2;1;4]^\top, [3;2;4;1]^\top, [3;4;1;2]^\top, [3;4;2;1]^\top$

2.1.2 Spatial correlation

The ideal scenario in a MIMO setup is that the propagation channels between each pair of transmit and receive antennas is statistically independent and identically distributed. In this way, the multiple independent channels with identical characteristics are created and used to enhance the bit error performance of the system. However, MIMO links are prone to spatial correlation between transmit and receive antennas due to very short antenna spacing and the propagation environment. As such, it is imperative to consider spatial correlation when designing permutation sets. The spatial correlation matrix $\mathbf{H}_{\text{t}}$ can be expressed as [6]:

$$\mathbf{H}_{\text{t}} = \mathbf{R}_t^{1/2} \mathbf{H} \mathbf{R}_r^{1/2} \tag{2.6}$$

where $\mathbf{R}_t^{1/2} \in \mathbb{C}^{N_t \times N_t}$ and $\mathbf{R}_r^{1/2} \in \mathbb{C}^{N_r \times N_r}$ are correlation matrices at the transmitter and receiver, respectively. To model spatial correlation accurately, $\mathbf{R}_t^{1/2}$, $\mathbf{R}_r^{1/2}$ are matrices that capture the correlation between antennas.

2.1.3 MIMO-STBC-SPM System Model

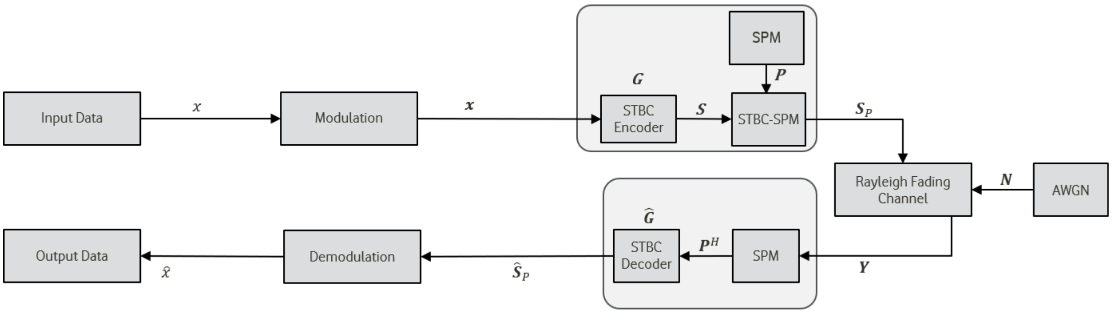

FIGURE 2.1: MIMO-STBC-SPM Wireless Communication System Model 2.7, 2.8

TABLE 2.2: Notation of MIMO-STBC-SPM Transceiver

Symbols	**Description**
N_t	Number of transmit antennas
N_r	Number of receive antennas
$\mathbf{P}$	Permutation Matrix
T	Time instance
K	Size of permutation set
$d_{\min}$	Minimum Hamming distance
QAM	Quadrature Amplitude Modulation
$STBC$	Space-Time Block Coding
$STBC - SPM$	Space-Time Block Coding with Spatial Permutation Modulation

TABLE 2.3: Notation of MIMO-STBC-SPM Transceiver Equations

Equation	**Description**
$\tilde{C}_{N_t,T}$	The collection of all possible orderings of a T-element subset of an N_t-set.
D	Hamming distance matrix
$d_{i,j}$	The Hamming distance between the i-th and j-th permutation vectors.
$\tilde{C}_{N_t,T}(K, d_{\min})$	The subset of K permutation vectors with the minimum Hamming distance $d_{\min}$ from $\tilde{C}_{N_t,T}$.
H_{sc}	Spatially correlated channel matrix
$R_t^{1/2}$ and $R_r^{1/2}$	Correlation matrices at the transmitter and receiver, respectively
P_i	The groups of permutation vector
θ_i	Rotation angle for group i

2.1.4 Transmitter

At the transmitter, input data symbols x_i are first processed by an STBC encoder to produce STBC code-words $s_{ij} \in \mathbf{S}$, where, $i = 1, 2, \ldots, N_t$ and $j = 1, 2, \ldots, T$ (number of time instances). The spatially permuted symbols $p_{ij} \in \mathbf{P}$ are derived from the STBC symbols s_{ij} by using a permutation matrix $\mathbf{P}$ as follows [57]:

$$\mathbf{S}_{\mathrm{p}}(i) = \mathbf{P}(i) \cdot \mathbf{S}(i), \mathbf{S}(i) = f_{STBC}(\mathbf{X}(i)) \tag{2.7}$$

where $\mathbf{X}(i)$ is the input data symbol at time index i, $\mathbf{S}(i)$ is the STBC encoded matrix at time i, $\mathbf{P}(i)$ is the permutation matrix at time i(which records the STBC symbols in space time), $\mathbf{S}_p(i)$ is the final permuted space time block, and $f_{STBC}(\mathbf{X}(i))$ is the STBC encoding function.

2.1.5 Channel

The system operates over a channel that is MIMO and the channel is assumed to follow Rayleigh fading. The received signal at each antenna is subjected to fading which is modeled by complex Gaussian random variables. The channel matrix $\mathbf{H}_t$ describes the propagation characteristics of the channel at time t, and is $N_r \times N_t$ in size.

2.1.6 Received Signal

At the receiving antenna j and time instant t, the received signal can be written as:

$$r_{t,j} = \sum_{i=1}^{N_t} h_{t,(j,i)} \cdot p_{t,i} + n_{t,j} \tag{2.8}$$

where $h_{t,(j,i)}$ is the channel coefficient between the i^{th} transmit antenna and j^{th} receive antenna at time t, and $n_{t,j}$ is the AWGN at j^{th} the receive antenna at time

t. The received signal vector at time t is defined as:

$$\mathbf{r}_t = [r_{t,1}, r_{t,2}, \ldots, r_{t,N_r}]^T$$

Decoding is performed by optimizing the permutation matrix $\mathbf{P}$ to maximize the likelihood of the received symbols given the transmitted symbols.

Decoding in the MIMO-STBC-SPM decoder is achieved by the following:

Algorithm 1 MIMO STBC-SPM Decoding

Input: Received signal vector $\mathbf{r}_t$, channel matrix $\mathbf{H}_t$, permutation vectors $\mathbf{P}$, rotation angles θ, code-word indices p_k, constellation set $\mathcal{S}$

Output: Decoded QAM symbols s_1, s_2, s_3, s_4

Split the received signal vector $\mathbf{r}_t$ into individual components corresponding to each receive antenna.
for each receive antenna j **do**
    Find the inner product of $\mathbf{Y}$ with the conjugate transpose of the channel matrix $\mathbf{H}_t$ for the j^{th} antenna: Find the conjugate of $\mathbf{h}_j^H$ and multiply it with $\mathbf{Y}$.
    Perform symbol demodulation on each component of the received signal to estimate the transmitted symbols as $\hat{s}_{1,j}, \hat{s}_{2,j}, \hat{s}_{3,j}, \hat{s}_{4,j}$.
end for
Decide on the length of the shortest d_{min}
for each permutation vectors $\mathbf{p}_i \in \mathbf{P}$ **do**
    Form the hypothesis signal $\mathbf{X}$ based on the permutation vector $\mathbf{p}_i$ and rotation angles.
    Calculate the Euclidean distance $d(\mathbf{Y}, \mathbf{H}, \mathbf{X})$
end for
if $d(\mathbf{Y}, \mathbf{H}, \mathbf{X}) <$ min_distance **then**
    Update d_{min}
    Update $\hat{\mathbf{s}}$ with the current hypothesis signal.
end if
Determine the group index i of the permutation vector $\mathbf{P}_i$ which is used in transmission.
The modulation scheme is 16 QAM then apply the rotation angleθ_i to the decoded symbols.
Select the appropriate code-word indices p_k from the permutation vector group $\mathbf{p}_i$.
if $p_k = 1$ or $p_k = 4$ **then**
    Then decode symbols s_1 and s_2.
end if
if $p_k = 2$ or $p_k = 5$ **then**
    Then decode symbols s_3 and s_4.
end if
if $p_k = 3$ or $p_k = 6$ **then**
    Then decode symbols s_1 and s_4.
end if
Output the decoded QAM symbols s_1, s_2, s_3, s_4.

Computational Complexity:

- $O(K!)$: Permutation generation
- $O(K! \cdot N_t)$: Total computational complexity

2.1.7 Chapter Summary

In this chapter, we have introduced the basic idea of the MIMO-STBC-SPM system and explained what its key elements, namely MIMO, STBC, and SPM, mean. We gave a detailed explanation of how STBC-SPM is better than the conventional STBC-SM system by incorporating a permutation-based approach to increase diversity and reduce error probabilities. When we studied the system's model, and the encoding and decoding processes, we explained how permutation vectors enhance the coding gain and error control. These foundations will help the approaches and ideas presented in the following chapters. Furthermore, in this chapter, through theoretical analysis and performance assessments which include an examination of the computational complexity of the decoding algorithm. The result of the work presented in this chapter forms a basis for further enhancements and optimizations of MIMO communication systems in the remaining chapters.

CHAPTER 3

Literature Survey

This chapter is a detailed review of the main concepts and major advancements in Multiple-Input Multiple-Output (MIMO) communication systems, along with discussions on Space-Time Block Coding (STBC), Spatial Modulation (SM), and Space-Permutation Modulation (SPM). All these techniques are very important to enhance the efficiency of the current wireless communication networks. These techniques are fundamental in finding the solutions to the challenges in the wireless communications systems and the complexity versus error rate trade-off of systems. This thesis has employed the soft decision decoding using the Hungarian Murty Algorithm, and selecting these techniques is beneficial to understand how other decoding techniques can be employed to enhance the existing methods. Additionally, STBC enhances diversity; SM enhances spectral efficiency; and SPM is a new modulation technique for MIMO systems. All these are in line with the objectives of implementing HMA to optimize the MIMO wireless communication system.

3.1 Multiple - Input Multiple - Output, Space Time Block Code

Multiple Input Multiple Output (MIMO) is a technique which is incorporated in the wireless communication systems to increase the data transfer rates, link reliability and spectral efficiency [56][17]. The MIMO system incorporates several antennas at the transmission and reception end that take advantage of spatial diversity and spatial multiplexing to combat interference and fading as well as improve on bandwidth and power utilization [16]. It has been widely adopted in current wireless standards, including LTE, 5G, and Wi-Fi because of its potential to enhance the communication quality [18]. These systems, which incorporate several antennas at each end of the link, use spatial diversity to increase data throughputs under conditions of band and power limitations. Space-time block coding is one of the MIMO transmit methods which focuses on the utilization of transmit diversity [48].

The MIMO channel is a critical part of the communication system and causes various effects that affect how signals are transmitted from the sender to the receiver [10]. It is usually represented by a channel matrix, which describes the complex fading coefficient between transmit and receive antennas. This matrix contains information on multi path propagation, fading, and spatial attributes, which can affect all the quality and dependability of transmitted signals [34]. These effects and their understanding are vital in the establishment of efficient communication systems that can minimize impairments and enhance performance.

Channel fading is a major issue for MIMO systems, but various strategies exist to combat it [18]. This leads to diversity, such that if several transmissions are made, there is a high probability that at least one transmission will not be affected by severe fading, thus reducing the probability of information loss [13]. In MIMO systems, the operation of the transmitter and receiver antennas is similar to one person sending a message and ensuring it has been received by repeating the question [7].

The diversity order of a MIMO system is defined as the number of independent receptions of the same signal. In a system with multiple transmit and receive antennas, the maximum diversity gain achievable is the product of the number of transmit and receive antennas [13]. This is accomplished through the generation of replicas by an encoder, which exploits diversity by encoding a single data stream across all transmit antennas in both spatial and temporal dimensions. This principle forms the basis of space-time coding, which is a technique used to guarantee the reliability of data transmission [55]. However, like any other wireless systems, MIMO channels are likely to experience impairments such as Additive

White Gaussian Noise (AWGN) and multipath fading, which affect the quality of the signal [43]. This corresponds to the complex Gaussian noise present at each receive antenna in the noise vector. This signal has several properties, including its statistical distribution and power spectral density, which define large portions of the system's overall performance and capacity [22].

To counteract the effects of MIMO channels and AWGN, effective signal processing techniques have to be employed at both the transmitter and receiver sides [4]. They include channel coding, adaptive modulation, diversity combining, and equalization with the aim of combating the negative consequences of channel fading and noise. In addition, sophisticated receiver algorithms such as maximum likelihood estimation and iterative detection are employed to accurately reconstruct transmitted symbols from received signals [18]. The primary task of the receiver is to reconstruct the original symbols from the received signal after the signals have been transmitted through the MIMO channel [22]. The received signal vector is a function of the channel and noise. To estimate the transmitted symbols, the receiver has to remove these distortions accurately.

STBCs are a key technology that is now widely used in MIMO systems in modern wireless communication systems. Introduced to utilize the spatial diversity available from multiple antennas, STBCs improve the reliability and robustness of the transmitted signal in the presence of channel impairments such as fading and noise. While encoding data across multiple antennas over time, STBCs achieve huge diversity gains with efficient transmission [26].

Because they achieve a good trade-off of diversity and data rate while keeping reasonable decoding complexity, STBCs are very popular and are central to wireless standards such as LTE, Wi-Fi and 5G. The development of the Alamouti scheme has also further advanced the field, providing a simple and efficient way of implementing transmit diversity in the real world [33]. These coding schemes remain central to solving the problems of high bit rate demand and reliability in wireless communication networks.

Due to the maximum diversity order for a given number of transmit and receive antennas, along with a simple linear decoding algorithm, STBCs have been developed [6]. Because of this, STBCs have found their way into many wireless applications. Training-based methods are employed to assess the performance of channel estimation at the receiver end. Pure training-based proaches are effective for MIMO channel estimation [26], but at the cost of bandwidth efficiency due to the long training sequences that are necessitated for reliable channel estimation [27]. Although blind and semi-blind methods are computationally complex, many wireless systems still use pilot sequences for channel parameter estimation.

Based on the availability of information, high-reliability STBCs can be divided into orthogonal STBCs and non-orthogonal STBCs. The achievement of full diversity comes at the cost of reduced decoding complexity in orthogonal STBCs, however [6]. However, quasi-orthogonal STBCs achieve higher data rates, but at the cost of diversity gain [26]. Under partial channel state information, transmission of Quadrature Space-Time Block Codes (QSTBCs) without channel knowledge at the transmitter has also been explored [48]. The generating matrix of each STBC transforms input symbols into space and time and determines the transmitting symbols from various antennas in each time slot. This matrix is important in specifying the characteristics of the code, such as diversity, code gain, rate, detector complexity, and time delay [22].

The challenge of transmitting diversity across multiple fading channels has been effectively addressed by the development of STBCs, specifically through the Alamouti code, using innovative signal processing and error correction techniques [33]. For instance, space–time trellis coding (STTC) approaches that combine spatial diversity from antenna arrays with channel coding to enhance wireless communication performance. These methodologies are vital for mitigating fading channel effects and improving data transmission reliability in MIMO systems [13]. The transmitter is also involved in the preparation of the information bits to be transmitted through the MIMO channel. First of all, the information bits are converted into symbols by a constellation mapper after undergoing constellation mapping [23]. These symbols are then processed through an STBC encoder to construct a transmission matrix [49]. The most widely used encoding method is the Alamouti scheme, which develops a specific code matrix for multiple antenna systems to improve the robustness of the transmission against fading and interference [6].

For the encoding scheme used by the transmitter, the receiver uses a Space-Time Block Decoder for instance, in STBC systems such as the Alamouti scheme, the decoder correctly decomposes the received signals and reconstructs the original symbols [59]. To combat channel fading and noise decoding, perform the inverse of the encoding operations performed by the transmitter [47]. The receiver also uses the maximum likelihood estimation to accurately estimate the transmitted symbols [54]. This method involves evaluating the received signal against all possible sequences of transmitted symbols and choosing the sequence that best matches the received signal based on certain statistics. Maximum likelihood estimation is a powerful technique for symbol detection and is robust for noise and fading [18]. This whole communication process is enclosed in this comprehensive system model, which considers the transmission, the channel effects, the noise, the reception and the symbol recovery. It gives a complete idea of how MIMO-STBC systems work and their parameters to guarantee almost error-free communication in poor signal reception conditions.

The advantages and disadvantages of a Multiple-Input Multiple-Output Space-Time Block Coding wireless communication system are represented in Table 3.1.

3.1.1 Alamouti

The Alamouti code is a fundamental space-time block code that sends different copies of the same data stream over unique antennas to improve the signal quality. STBCs are a further development of the Alamouti scheme and are used to enhance the reliability of the wireless communication link. These codes are orthogonal and achieve the full transmit diversity proportional to the number of transmit antennas [56]. This particular STBC is more complex than the Alamouti scheme. The encoding and decoding process in STBC is similar to that of the Alamouti code but

Table 3.1: Pros and Cons of a Multiple-Input Multiple-Output Space-Time Block Coding Wireless Communication System

Pros	**Cons**
• The orthogonality of Space-Time Block Coding leads to enhanced diversity gain, which improves the error performance and signal reliability [6, 56]. • Decoding algorithms like maximum likelihood decoding are relatively simple and computationally efficient [56]. • It has strong resistance to multi-path fading and interference and can be easily applied to harsh wireless environments [13]. • It can be easily integrated with existing Multiple-Input Multiple-Output systems and is widely applicable to current wireless technologies [48].	• The capacity is not highly improved as for the advanced Multiple-Input Multiple-Output techniques such as Spatial Modulation or Space-Permutation Modulation [6]. • The optimal performance is required for accurate Channel State Information (CSI) at the receiver end [26]. • It has poor performance in rapidly changing channels, such as those encountered in high-speed mobile scenarios [26]. • Achieving full diversity often results in reduced data rates, particularly in orthogonal Space-Time Block Coding designs [6, 25].

is applied at both the transmitter and receiver and with additional complexity as a result of larger block size or more spatial dimensions [6]. The data is represented in a matrix form where the number of columns represents the number of transmit antennas and the number of rows represents the time slots for data transmission. The signals are combined and processed through a maximum likelihood detector at the receiving end, which uses decision rules.

The Alamouti scheme with two transmit antennas and many receive antennas has a maximum diversity order proportional to the number of receive antennas. It was the first STBC to achieve complete diversity at full rates for two transmit antenna systems. The encoding is straightforward, where information bits are first modulated into symbols using a digital modulation scheme, then two symbols are grouped together and transmitted according to a specific code matrix. This configuration is such that the diversity order is comparable to that of a conventional maximal ratio receiver combining that has one transmit and multiple receive antennas [52]. The efficiency of the system's orthogonal design is such that the encoder and decoder are rather straightforward. For example, for the purpose of explaining the process, suppose we are dealing with two receive antennas. The encoding procedure can be described in the form of a matrix, where the rows represent different time slots and the columns represent transmitted symbols. In one time slot, both antennas transmit symbols to guarantee that full diversity is obtained, even with a single antenna.

The Alamouti scheme has transformed multi-antenna systems over the last few years with a guarantee of full diversity without estimating the channel state at the transmitter and with a less complex maximum likelihood decoding system at the receiver. Maximum likelihood decoders guarantee overall diversity gain at the receiver end, even in the absence of transmitter end channel state information [26]. The critical feature of orthogonality between the sequences transmitted by the two antennas is what achieves this.

From simulation results, it is observed that if the receiver has good knowledge of the channel state and if the fading between the transmit and receive antennas is independent then there are significant improvements in the system performance [6]. In MIMO systems, the Alamouti scheme with two transmit antennas and the receiver diversity with a single transmit antenna require similar overall transmit power. This is because in the Alamouti scheme, the power allocation strategy used is such that the power is equally divided between the two transmit antennas, hence, about the same transmit power as a single antenna in the receiver diversity scheme [44][1][53]. In the Alamouti scheme, power is allocated equally. Without increasing significantly the power requirement of a single antenna system, the Alamouti scheme gets the benefit of spatial diversity by dividing the total transmit power between the two antennas.

There are three kinds of diversity gains from this literature:

- **Coding gain:** The gain is provided by temporal codings such as convolutional coding or block coding.

- **Array gain:** This is defined as the average increase in the signal-to-noise ratio at the receiver side as a result of the coherent combining effect of multiple antennas at the receiver or transmitter or both [56]. This gain is achieved through the optimal weighting of the signals from each antenna based on the knowledge of the channel state.

- **Diversity gain:** This gain is achieved through spatial diversity across channels at the transmitter or receiver or both in order to combat fading. The gaps across the antenna bank should be separated by distances greater than the coherent distance for that channel [26]. This ensures that the channel between each transmit and receive antenna pair fades independently. Alamouti's scheme does not require channel state information to achieve transmit diversity because it is based on orthogonal data streams. The diversity order is equal to the product of the number of transmit and receive antennas.

3.1.2 Orthogonal Space-Time Block Codes

For the pursuit of the full diversity of wireless communication systems, techniques such as Orthogonal Space-Time Block Codes (OSTBC) are developed Because real orthogonal codes are optimal for their linear decoding and are orthogonal by their nature, they are particularly useful in achieving full diversity [25].There are complex orthogonal codes, but they can fully utilize the diversity gain available only up to two transmit antennas [28]. The channel capacity in MIMO systems can be derived under the assumption of a flat-fading channel where the channel coefficients are assumed to remain constant for several symbol transmissions [49]. Flat fading happens when the bandwidth of the transmitted signal is much smaller than the coherence bandwidth of the channel; thus, all the frequency components of the signal suffer roughly the same amount of attenuation and phase shift [40].

This condition makes the channel model simpler and at the same time stresses the need for good diversity techniques, such as OSTBC, to combat the signal degradation. The two main components of the OSTBC encoder are:

- Constellation Mapper: It produces different data sequences based on the chosen modulation scheme.
- Space-Time Block Encoder: It converts the original data into a set of block sequences of complex symbols The output transmission matrix contains data that is to be transmitted simultaneously over multiple transmit antennas, each column of the output transmission matrix .

Before being transmitted through the antennas, the encoded symbols are first pulse shaped and modulated. This process is such that the utilization of spatial and temporal diversity resources is maximized in order for the communication to be reliable even in fading channels. The OSTBC decoder performs the following important functions to reconstruct the original transmitted data with minimal errors in MIMO systems [3].

- Signal reception: Handling of the signals received from different antennas.
- Space-Time Block Decoding: Obtaining the actual information symbols from the encoded data.
- Symbol Detection: Using techniques like maximum likelihood detection to identify the transmitted symbols.
- Error Correction: Correcting errors in the received signal to improve the credibility of the data even in the presence of noise and interference.

Even in noisy and interference prone environments, the decoder is able to provide a good reconstruction of the transmitted data streams under these techniques [47].

To compare, simulations were conducted for transmit and receive diversity using coherent binary phase-shift keying modulation over flat fading rayleigh channels. The setups included:

- Two transmit antennas and one receive antenna referred to as transmit diversity
- One transmit antenna and two receive antennas referred to as receive diversity.

BER is measured to find that the diversity order of two transmit antennas and one receive antenna is the same as that of the maximal ratio combined system with one transmit and two receive antennas. But performance is noticeable poor when the channel estimation at the receiver is imperfect, as against systems with perfect channel knowledge. In comparison to maximal ratio receive diversity, transmit diversity has similar computational complexity but poor performance because both the scenarios have equal total power transmitted. It is also shown that normalizing the total power across all diversity branches, then the theoretical performance of a second order diversity link is very close to the transmit diversity system. The simulation results also show that the combination of maximum likelihood detection with OSTBC yields better performance. The reliability of wireless communication is enhanced by OSTBC as BER is always decreasing with the signal-to-noise ratio.

3.2 Generalised Spatial Modulation

One of the major drawbacks of the SM is that the number of transmit antennas has to be a power of two, which is not always practical. This limitation leads to the development of a Generalized Spatial Modulation (GSM) that can overcome such restriction. GSM, a block of information bits is mapped to a constellation symbol as well as a spatial symbol [60][9].

SM was developed as a transmission technique that uses multiple antennas. The basic idea of SM is to map a block of bits to two information-carrying units, one symbol being of a constellation and the other as a unique transmit antenna number which holds an information-bearing unit that increases the overall spectral efficiency by the base two logarithm of the number of transmit antennas [42][46]. At the receiver, a combination of the maximum receive ratio and an algorithm is used to retrieve the transmitted block of information bits. An application of SM to OFDM transmission and development of an analytical approach for symbol error ratio (SER) analysis of the SM algorithm in independent identically distributed (i.i.d.) Rayleigh channels are conducted in [2][19].

SM is a spatial multiplexing MIMO technique that was proposed in [32] to increase the spectral efficiency and to overcome inter channel interference (ICI). This is followed by activating only a single transmit antenna at each instance to transmit a certain data symbol, where the active antenna index and the data sent depend on the incoming random data bits [45]. This made the spectral efficiency increase by a base two logarithm of the number of transmit antennas. In addition, the number of transmit antennas must be a power of two. A detector that jointly estimates the active antenna index and the sent data symbol is required on the receiver side.

The optimal SM decoder is proposed by authors in [31] and SM combined with trellis-coded modulation (TCM) is recently proposed in [60]. In addition, space shift keying (SSK) with partial channel state information is presented by authors in [31], and a general framework for performance analysis of SSK for multiple input single output (MISO) systems over correlated Nakagami-m fading channels is shown in [60]. It is shown in [39] that ICI avoidance results in better BER performance and a significant reduction in detection complexity as compared to V-BLAST, for instance [2]. However, the logarithmic increase in spectral efficiency and the requirement that the number of antennas must be a power of two would require a large number of antennas. Fractional bit encoded spatial modulation (FBE–SM) is proposed by authors in [32] to overcome the limitation in the number of transmit antennas. FBE–SM is based on the theory of modulus conversion and allows an arbitrary number of transmit antennas. However, the system suffers from error propagation [60].

The complexity of the SM–OFDM technique and its performance is compared with those of the vertical Bell Labs layered space–time (V-BLAST–OFDM) and

Alamouti–OFDM algorithms. The V-BLAST uses the MMSE detection with ordered successive interference cancellation. Simulations were made in [5] on both coded and uncoded systems to illustrate how spectral efficiency can be increased through spatial correlation, mutual antenna coupling, and Rician fading. It is shown that for the same spectral efficiency, around 90 percent reduction on the receiver side of high complexity as compared to V-BLAST and nearly the same receiver complexity as Alamouti can be achieved by SM [2].

In addition, techniques such as MIMO-OFDM-SM have advantages such as achieving better performance in all studied channel conditions. MIMO-OFDM-SM also efficiently works for any configuration of transmit and receive antennas, even in the case of fewer receive antennas than transmit antennas [32].

While MIMO-OFDM-SM has its advantages, within the multiple antenna transmission schemes, there are limitations and challenges, including:

- There is high ICI at the receiver end in BLAST transmission systems because all the antennas transmit simultaneously on the same frequency [2].
- This requires complex receiver algorithm and hence increases the overall system complexity [2].
- The system performance is quite sensitive to the receiver complexity and though the BLAST systems achieve a fairly good performance in ideal channel conditions, their performance is poor in non ideal channel conditions [10].
- This is because MIMO-OFDM-SM with full-diversity STCs has no limitations. As they are orthogonally designed, they are easily decoded at the receiver end. Moreover, STCs are robust in the presence of channel condition imperfections [15]. The drawback is that the spectral efficiency of full-diversity STC systems is one symbol per symbol duration for any number of transmit antennas. In other words, in order for the full-diversity STCs to achieve spectral efficiency that is comparable to that of the BLAST techniques, they must employ higher modulation orders [15].

- The ideal number of transmit antennas for efficient operation of BLAST techniques is less than or equal to the number of receive antennas [2]. In STCs, the situation is different. The STCs can be designed for different number of transmit and receive antennas and can work efficiently even when the number of receive antennas is less than the number of transmit antennas. However, an orthogonal design of full rate code STC is only known for the case of two transmit antennas, and there is no known solution for more than two transmit antennas. Therefore, for STCs with more than two transmit antennas, a portion of the data rate is sacrificed to achieve full orthogonality and hence full diversity [15].

In some cases, high complexities are a result of hard decision decoding from the receiver end in a MIMO-OFDM SM system. SM uses hard decision decoding in which a binary decision is made on each bit based on a pre determined threshold [61]. The threshold can either be set based on the SNR or the probability distribution of the received signal [11].

For the reduction of the errors that come with the hard decision (HD) decoding in a MIMO-OFDM-SM, mechanisms like SD are taken into account. The SD involves the probability distribution of the received signal, where the computation of the likelihood of each bit value is based on the received signal and further makes a decision with it. Soft decision decoding can further provide better performance than hard decision decoding, especially in low-SNR scenarios [39].

The selection between hard decision decoding and soft decision decoding in SM is dependent on use cases The complexity of the decoding system and the processing power required differ significantly between hard decision decoding and soft decision decoding. The hard decision decoding is easier to implement and needs less processing power, whereas the soft decision decoding can offer better performance at the cost of increased complexity. The GSM MIMO technique uses multiple antennas at the transmitter to transmit in the same time slot information bits on both the amplitude of the signal and the index of the active antenna subset. In

GSM, only a subset of the antennas is active at any given time, and the receiver uses the active antenna subset information to decode the transmitted bits [60][38].

The spatial symbol contains both the transmit antennas, which are activated at each instance. The actual combination of active transmit antennas depends on the random incoming data stream. This is unlike SM where only a single transmit antenna is activated at each instance. GSM has the ability to increase the overall spectral efficiency by base two logarithm of the number of antenna combinations. A decrease in the number of transmit antennas, which are needed for the same spectral efficiency [60].

GSM activates more than one transmit antenna at a time to simultaneously transmit a data symbol. In GSM, the transmitted information is conveyed in the activated combination of transmit antennas and the transmitted symbol from a signal constellation. As a result, the number of transmit antennas required to achieve a certain spectral efficiency is reduced by more than half in GSM as compared to SM, and generalized space shift keying modulation (GSSK) proposed by authors in [60] [38].

This omits the ICI at the receiver, but retaining the key advantage of SM, transmitting the same data symbol from more than one antenna at a time. The benefit is that GSM increases the reliability of the wireless channel, and offers spatial diversity gains by providing replicas of the transmitted signal to the receiver [58]. However, the activated transmit antennas must be synchronized to avoid intersymbol interference (ISI).

At the receiver, a ML detection algorithm is vetted to estimate the activated combination of transmit antennas and the transmitted constellation symbol. A thorough analytical upper bound for the BER performance of GSM is derived in this paper and analytical results are validated through Monte Carlo simulation results. Moreover, GSM performance is shown to be very close to the performance of SM but with a major reduction in the required number of transmit antennas [58].

Like the other spatial modulation techniques, the decision process at the receiver in GSM can be either hard decision decoding or soft decision decoding. Based on the SNR of the incoming signal, the receiver can set a threshold to which each bit is evaluated in GSM hard decision decoding. This method classifies the transmitted bit as strictly either 0 or 1, with the likelihood of different values based on the received signal ignored [60]. In GSM, on the other hand, the soft decision decoding takes into account the probability distribution of the received signal. To make its decision, the receiver assesses the signal to compute the likelihood of each bit value using these probabilities. This approach can yield superior performance, particularly in low SNR conditions, than hard decision decoding [31].

Depending on the application and system requirements, choose hard or soft decision decoding in GSM. Hard decision decoding is simpler and needs less processing power, however soft decision decoding, which is more complex, offers better performance. In low-SNR environments, GSM systems usually favour soft decision decoding [31].

Some of the key highlights of GSM are as follows:

- GSM is an extension of the SM that overcomes the restriction that the number of transmit antennas in SM must be a power of two. GSM is a more general system that can be configured to have a block of information bits mapped to both a constellation symbol and a spatial symbol [60].

- SM has a single transmit antenna that is activated at each instance to transmit a data symbol and the active antenna index carries additional information. This technique increases spectral efficiency by the base-two logarithm of the number of transmit antennas [32].

- GSM activates a subset of antennas at each instance and transmits information through both the constellation symbol and the active antenna subset index. This reduces the number of antennas required for the same spectral efficiency as SM and offers spatial diversity gains and improved reliability [60].

- The transmission of data symbols from multiple antennas in GSM reduces ICI, a key limitation in traditional multi-antenna systems. GSM has a better performance with fewer transmit antennas than SM.

- GSM systems require synchronization of the activated antennas to prevent inter-symbol interference (ISI), and the receiver typically employs maximum likelihood (ML) detection to estimate both the active antenna combination and the transmitted symbol [58].

- Like other spatial modulation techniques, GSM can use either hard decision or soft decision decoding. Hard decision decoding is rather simple but may be less effective in low-SNR scenarios, whereas soft decision decoding performs better in such environments by leveraging probability distributions [31, 60].

- GSM reduces the complexity of the receiver design without compromising the performance of SM. In addition, GSM is an attractive solution for systems that need high spectral efficiency but do not have a large number of transmit antennas available [38, 60].

3.3 Spatial Permutation Modulation

From studies conducted earlier, it is known that spatial modulation is beneficial for multiple input multiple output systems. Such capabilities include high energy efficiency and spectral efficiency. In addition, it reduces the complexity and the operational expenditure (OPEX) of multiple antenna systems without compromising on the performance and rather enhancing the poor data rates. In [27], authors have come up with SPM where data bits are modulated into a permutation vector and transmit antennas are activated at time instances. This technique also has a fast increase in high density that enables low rate data to spread over time instances in permutation [27] [21].

SPM can be defined as the general case of SM, the antenna index in spatial modulation is generalized to the permutation vector in SPM[14]. SPM modulates data bits to a permutation vector and activates the transmit antenna at successive time instants accordingly. The SPM achieves higher diversity and thus lower error rate since the permutation vector disperses data along the time coordinate. The theoretical models of diversity and error rate are derived in both the slow-fading channel and fast-fading channel, leading to systematic and fast SPM design exploration [20].

There are very few contributions on this topic, and most of the available works focus on combining SPM with other techniques for further improvements. Some of them are:

- High compatibility: SPM has been presented as a new approach to the design of transmission techniques that is orthogonal to most of the advanced SM/Intensity Modulation techniques; hence, it can be easily combined with these techniques to form a more efficient MIMO system, for instance, STBC-SPM and QSPM techniques [20].
- Systematic and fast design exploration: It is possible to analyze theoretically the performance of SPM, and use the valuable researches on permutations.
- High transmission reliability: Due to the higher transmit diversity, SPM offers better error rate performance than SM, particularly in adverse conditions such as low receive diversity (few receive antennas) or spatially correlated channels [8].
- Low complexity: Due to the fact that SPM uses one transmit antenna at a time it enjoys the low complexity benefit similar to SM for instance, single RF chain and simple decoding algorithm [8].

For both SM and SPM, the decision at the receiver end can be through hard decision decoding or soft decision decoding. In the hard decision process in SPM, the receiver end interprets 0 or 1 for every bit based on a certain threshold that is a function of the SNR of the received signal. This method simply classifies the received signal as corresponding to the sent bit of 0 or 1, ignoring the likelihood of the received signal [27].

On the flip side, the soft decision decoding in SPM takes into account the likelihood distribution of the incoming signal. The recipient forms its conclusion based on the probability of each bit value according to the data received and assesses this probability. This method is often better than hard decision decoding, and it excels particularly in low-SNR situations.

The selection between hard and soft decision decoding in SPM is dependent on the application and system requirements. The easier to implement hard decision decoding has a higher power demand, while the more complex soft decision decoding offers better performance. This generalization improves transmission efficiency and is compatible with advanced spatial modulation diversity techniques, including STBC [27].

In MIMO systems it is necessary to construct sets of permutations for the purpose of effective data transmission. The sets are constructed having regard to the spatial correlation between antennas and with the additional goal of maximizing the minimum euclidean distance between constellation points. Spatial correlation describes how signals from different antennas are connected in time due to their spatial arrangement [30]. By maximizing the smallest distance, the system becomes stronger against noise and interference and so reduces the possibility of error in the received signal. So, the goal of arranging permutation sets is to find a balance between the benefits of spatial correlation for communication and the need for reliable reception.

After the permutation sets have been produced, the next step is to place data bits into the permutation vectors that were developed from these sets. This mapping procedure selects proper permutation vectors to ensure that the received signals can be properly decoded at the receiver end [29]. Mapping is an important process that helps in ensuring that the information to be transmitted is accurate.

The next step involves producing QAM symbols from a predetermined constellation set, after mapping the information bits. This group of stars, or constellation set, selected impacts on what symbolic values may be sent through the channel of communication. The data rate, robustness and spectral efficiency of the system is dependent on the constellation set chosen, as stated in Lee's research [29].

An important part of the system MIMO system is the channel matrix, which describes the relationship between the transmit and receive antennas and embodies crucial spatial information and the fading that is imposed on the transmitted signals as they travel through the medium. MIMO technology is also crucial in improving the data transmission rates of various present day communication systems.

The information transmitted uses QAM symbols chosen from the constellation set to assist with efficient encoding. This process often consists of repeating each QAM symbol to improve signal robustness. These symbols' encoding and modulation techniques are central to Spatial Permutation Modulation (SPM), which further increases the efficiency of data transmission [14].

The signals are organized into a matrix form after being received, which makes them more easily processed and analyzed. Such arrangement is convenient for various signal processing operations which are required for decoding the transmitted information [12]. When the received signals are arranged in this manner, researchers are better placed to develop better algorithms for decoding.

This is an important process in the communication process to decode information bits from received signals to recover the original information encoded through SPM techniques. The SPM decoding algorithms are developed to reconstruct the original information bits from the transmitted permutation vectors such that reliable communication is achieved with minimal errors [14].

To the received QAM symbols, demodulation is involved as the next step. Techniques for demodulation are crucial for properly extracting the transmitted symbols, even in the presence of noise and other channel impairments [30]. This process is often the inverse of operations that occur during modulation, such as phase detection and amplitude estimation [26].

To increase the reliability of data transmission, error correction techniques are employed. Such algorithms work by looking for discrepancies between the received and expected values of the symbols to be transmitted. Using error correcting codes, these methods make the total communication system much more reliable, especially in wireless networks which are prone to noise and interference [14].

It is also important to consider handling of the spatial correlation for the purpose of optimizing the performance in MIMO systems. Techniques that change the channel matrix based on correlation information assist in reducing the impact of spatial correlation between transmit and receive antennas, thus improving the probability of correct detection and decoding of the signal [37].

The effectiveness of SPM depends greatly on the design of the permutation set and the mapping strategy between the permutation vector and the information bits. When developing a permutation set, it is important to consider the characteristics of spatially correlated channels. This consideration often results in the application of a spatial correlation matrix model which describes the inter-relationships of antennas at both the transmitter and receiver end [26].

Considering the minimum Hamming distance and the effect of spatial correlations when setting up the permutation set. For example, for a given system parameters, e.g., the number of transmit antennas, the number of time slots, and the number of permutations, a reasonable approach begins with identifying permutation sets that have their minimum Hamming distance maximized. This initial step is important to guarantee robust performance. As the design develops, one needs to consider the effect of spatial correlations; when nearby antennas are subject to similar fading statistics, certain permutation orders may be detrimental to detection accuracy. Some permutations may thus be excluded. To this end, for reducing the probabilities of pairwise errors, the mapping strategy attempts to select permutation pairs that have larger Hamming distances without considering whether they are near or far in the permutation order.

This almost meticulous design process enables SPM to dynamically adapt its permutation set to achieve a balance between error rate performance and throughput. Increasing the number of antennas provides the transmitter with a broader selection of usable permutation vectors that in turn influence both error rates and throughput for SPM transmissions. Such considerations are vital for optimizing the performance of communication systems utilizing SPM.

CHAPTER 4

Soft-Decision Decoding Scheme for a MIMO-STBC-SPM Communication System

In this chapter, we delve into the Hungarian Murty Algorithm (HMA) and its application as a soft-decision decoding scheme within the framework of the MIMO-STBC SPM communication system model. The Hungarian Murty Algorithm, known for its efficacy in solving assignment problems with multiple criteria, is adapted here to enhance the decoding process of space-time block codes (STBC) in a MIMO setting. By leveraging the algorithm's capabilities, we aim to address the computational complexity and performance challenges associated with soft-decision decoding. This chapter provides a comprehensive introduction to the HMA, explores its integration into the MIMO-STBC-SPM system, and evaluates its impact on system performance, focusing on improvements in decoding accuracy and computational efficiency.

4.1 MIMO-STBC-SPM-HMA System model

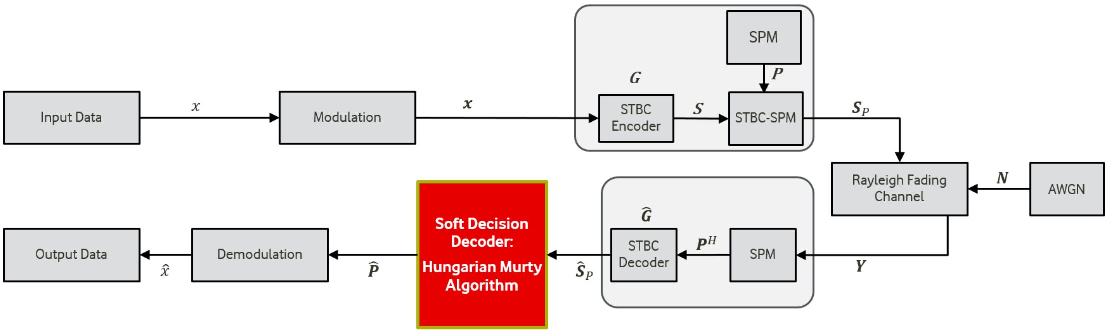

FIGURE 4.1: MIMO-STBC-SPM-HMA Wireless Communication System Model, in red is the Hungarian Murty Algorithm contribution we add in the MIMO-SMIMO-STBC-SPM system model

TABLE 4.1: Notation of MIMO-STBC-SPM-HMA Transceiver

Symbols	**Description**
N_t	Number of transmit antennas
N_r	Number of receive antennas
T	Time instance
K	Size of permutation set
$d_{\min}$	Minimum Hamming distance
QAM	Quadrature Amplitude Modulation
$STBC$	Space-Time Block Coding
$STBC - SPM$	Space-Time Block Coding with Spatial Permutation Modulation
$\mathbf{P}$	Permutation matrix
p_{ij}	Spatially permuted symbols
$\mathbf{S}$	STBC output matrix
s_{ij}	STBC encoded symbols
$\mathbf{G}$	STBC encoding matrix
$\mathbf{Y}$	Received signal
y_{ij}	Received signal symbols at receive antenna i from transmit antenna j
h_{ij}	Fading coefficient
n_{ij}	Additive white Gaussian noise
c_{ij}	The cost associated with assigning the received symbol y_ij to the estimated symbol s_{ij}
H_t	Channel matrix at time t
s_t	Transmit signal vector at time t
n_t	Noise vector at time t
r_t	The received signal vector at time t

Table 4.2: Notation of MIMO-STBC-SPM-HMA Transceiver Equations

Equation	**Description**
$\mathbf{S}_p(i) = \mathbf{P}(i)\mathbf{S}(i)$	STBC-SPM-encoded matrices into spatially permuted matrices
$y_{ij} = h_{ij} \cdot sp_{ij} + n_{ij}$	Symbol received signal equation
$\mathbf{Y} = \mathbf{H}_\mathrm{t}\mathbf{S}_\mathrm{p} + \mathbf{N}$	Matrix received signal equation
$c_{ij} = \lVert y_{ij} - \hat{s}_{ij} \rVert^2$	Cost calculation equation
$r_t = H_t \cdot s_t + n_t$	Relationship between received signal vector, channel matrix, transmit signal vector, and noise vector

4.1.1 MIMO-STBC-SPM-HMA Transmitter

Several important symbols are crucial in the communication process within a MIMO system with N_t transmit antennas. To begin with, the input data symbols, denoted as x_i with i ranging from 1 to N_t, make up the initial information bits for transmission. These symbols are the basic units of data in the system. After using a Space-Time Block Code (STBC) encoder for encoding, the system produces STBC encoded symbols represented as $s_{ij} \in \mathbf{S}$, with i varying from 1 to N_t and j varying from 1 to T. Every s_{ij} symbolizes a symbol sent by the i^{th} antenna at the j^{th} time moment. In addition, spatial diversity is introduced and system reliability is enhanced by performing spatial permutation on the encoded symbols, creating spatially permuted symbols labeled as p_{ij}. Like the STBC encoded symbols, $p_{ij} \in \mathbf{P}$ denotes symbols sent by the i^{th} antenna at the j^{th} time moment, but with a spatial permutation applied. The symbols together create the foundation for signal transmission in the MIMO-STBC-SPM system, guaranteeing effective and strong communication across wireless channels. and results in the following:

$$\mathbf{S}_\mathrm{p}(i) = \mathbf{P}(i) \cdot \mathbf{S}(i), \mathbf{S}(i) = f_{STBC}(\mathbf{X}(i)) \tag{4.1}$$

This equation signifies the transformation of STBC-encoded symbols into spatially permuted symbols through the multiplication with the permutation matrix $\mathbf{P}$. The spatially permuted symbols, p_{ij}, capture the rearranged order of the encoded symbols, incorporating spatial diversity to enhance the reliability and performance of the MIMO system during transmission over wireless channels.

4.1.2 MIMO-STBC-SPM-HMA Channel

In the MIMO system under consideration, the transmission channel undergoes Rayleigh fading, which is defined by fading coefficients h_{ij} for every combination of transmit and receive antennas, with i ranging from 1 to N_r and j ranging from 1 to N_t. These diminishing coefficients represent the unpredictable fluctuations in signal strength and phase caused by multi-path propagation, resulting in dynamic and probabilistic channel conditions. As a result, the fading coefficient h_{ij} affects the received signal y_{ij} at receive antenna i from transmit antenna j. The received signal y_{ij} is obtained by multiplying the symbol p_{ij} with the fading coefficient h_{ij} after permuting it spatially, and then adding AWGN n_{ij}. This depiction of the incoming signal considers both the impact of the channel, represented by the fading coefficients, and the inherent noise in the communication channel, allowing for a thorough model to analyze and enhance the MIMO system's performance.

Samples of Additive White Gaussian Noise (AWGN): n_{ij}, with i ranging from 1 to N_r and j ranging from 1 to T.

4.1.3 MIMO-STBC-SPM-HMA Receiver

During the MIMO communication system's post-demodulation and decoding stage, the estimated symbols $\hat{s}_{ij}$ are essential for reconstructing the initial symbol sequence. The estimated symbols are symbols reconstructed from decoding, with i ranging from 1 to N_t and j ranging from 1 to T. Following the process of demodulation and decoding, the predicted symbols are reorganized according to the encoding and modulation method employed during the transmission stage. The goal of this reorganization process is to rebuild the initial arrangement of symbols sent by the sender. When the receiver correctly arranges the predicted symbols, they are able to accurately retrieve the desired data bits. This process is crucial in guaranteeing the accuracy and dependability of data transfer in the MIMO system by enabling the recovery of the initial message from the symbols that were received.

Implementing the Hungarian Murty Decoding algorithm requires optimizing the permutation matrix $\mathbf{P}$ in order to increase the probability of the transmitted symbols matching the received symbols.

Upon completion of the STBC decoding process, we receive the estimated symbols $\hat{s}_{ij}$ for i ranging from 1 to N_t and j ranging from 1 to T. After the STBC decoding, the estimated symbol matrix $\hat{\mathbf{S}}$ is created.

$$\hat{\mathbf{S}} = [\hat{s}_{ij}], i \in [1, N_t], j \in [1, T] \tag{4.2}$$

A matrix $\mathbf{C}$ is created to illustrate the cost of assigning each received symbol y_{ij} to the estimated symbol $\hat{s}_{ij}$.

$$\mathrm{c}_{ij} = ||y_{ij} - \hat{s}_{ij}||^2, c_{ij} \in \mathbf{C} \tag{4.3}$$

The application of the HMA takes place to find the optimal permutation matrix $\mathbf{P}$ such that the total cost $\sum c_{ij}$ is minimized.

The optimal permutation matrix $\hat{\mathbf{P}}$ is obtained, where:

$$\hat{\mathbf{P}} = \arg\min_{P} \sum_{i,j} \mathbf{P}_{ij} c_{ij} \tag{4.4}$$

Once $\hat{\mathbf{P}}$ is obtained, the final estimated symbols are:

$$\hat{\mathbf{X}} = \hat{\mathbf{P}}\mathbf{P}^{\mathbf{H}}\mathbf{Y}\mathbf{H}_{\mathbf{t}}^{\mathbf{H}} \tag{4.5}$$

Where $\mathbf{P}^{\mathbf{H}}$ is the conjugate transpose of the initial permutation matrix and $\mathbf{H}_{\mathbf{t}}^{\mathbf{H}}$ is the hermitian transpose of the channel matrix.

4.1.4 Additional Model Components

During each time step t, the signal transmitted from the i^{th} antenna is represented as $s_{t,i}$. This notation signifies the distinct signal emitted by each antenna at that specific moment. Collectively, the signal vector at time t can be expressed as

$$\mathbf{s}_t = [s_{t,1}, s_{t,2}, \ldots, s_{t,N_t}]^T,$$

which encompasses all the signals sent by the transmitting antennas at that instant.

The channel matrix $\mathbf{H}_t$ plays a crucial role in this process, as it captures the properties of the channel's propagation at time t. With dimensions $N_r \times N_t$, the matrix $\mathbf{H}_t$ effectively represents the attenuation, delay, and phase shift that the signals experience as they travel from each transmitting antenna to every receiving antenna.

When the signals arrive, the signal received at each antenna j at time t, denoted as $r_{t,j}$, is a combination of the transmitted signals. This combination is influenced by the channel coefficients $h_{t,(j,i)}$ and the additive white Gaussian noise (AWGN) $n_{t,j}$. Consequently, the received signal vector at time t can be represented as

$$\mathbf{r}_t = [r_{t,1}, r_{t,2}, \ldots, r_{t,N_r}]^T,$$

which effectively captures the mixture of transmitted signals and noise at each receiving antenna.

In summary, the matrix representation succinctly illustrates the relationship between the received signal vector $\mathbf{r}_t$, the channel matrix $\mathbf{H}_t$, the transmit signal vector $\mathbf{s}_t$, and the noise vector $\mathbf{n}_t$.

$$\mathbf{r}_t = \mathbf{H}_t \cdot \mathbf{s}_t + \mathbf{n}_t \tag{4.6}$$

Here, $\mathbf{n}_t = [n_{t,1}, n_{t,2}, \ldots, n_{t,N_r}]^T$ embodies the noise vector at time t, while $\mathbf{s}_t$ signifies the transmit signal vector at time t. This matrix representation provides a concise yet comprehensive depiction of the interplay between channel effects, transmitted signals, and noise in the reception process of the MIMO system.

This model captures the essential aspects of MIMO communication, including the spatial diversity and multiplexing gains provided by multiple antennas at both the transmitter and receiver, as well as the effects of channel fading and noise.

4.2 Soft Decision Decoding Scheme Based on Hungarian-Murty Algorithm for MIMO-STBC-SPM

The Hungarian-Murty Algorithm is a combinatorial optimization algorithm used to solve assignment problems. In the context of MIMO systems with STBC-generalized spatial permutation modulation, this algorithm can be used to improve spectral efficiency [55] [27]. The basic idea of STBC-SPM is to use a set of complex symbols to encode data bits, and then permute these symbols in a predefined way to achieve spatial diversity. Multiplying the symbol vector with a preset permutation matrix can convert it into a permutation. Identifying the optimal permutation matrix is essential for maximizing spectral efficiency by improving the signal-to-noise ratio (SNR) at the receiver [30].

The soft-decision decoder's purposes include the iterative rank of the costs of the input signal matrix until the assignment produces a code word in C [24].

The Hungarian process can be illustrated by the following flow chart:

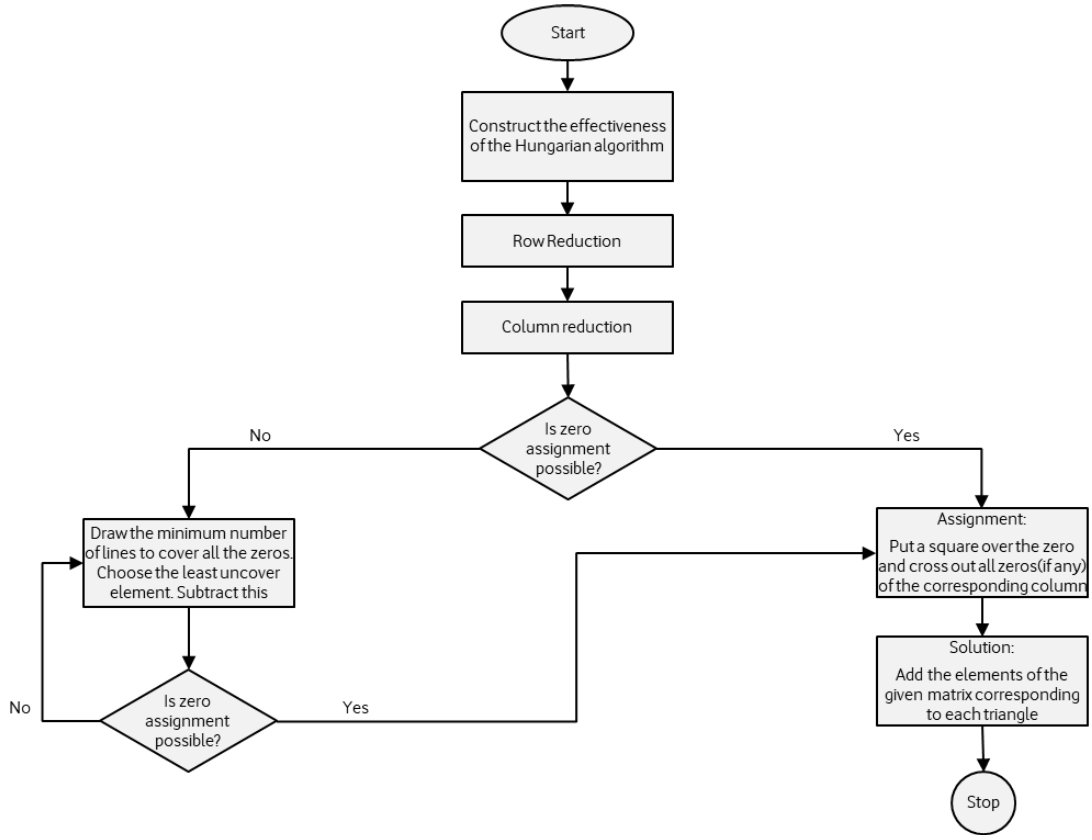

Figure 4.2: Flow Chart illustration of the Hungarian Algorithm

The Hungarian algorithm steps can be conducted as follows:

Algorithm 2 Hungarian Algorithm for Optimal Assignment

```
Input: A cost matrix C of size n × n
Output: Optimal assignment minimizing total cost
Step 1: Row Reduction
for each row i in C do
    Subtract the minimum value in row i from all elements of row i
end for
Step 2: Column Reduction
for each column j in C do
    Subtract the minimum value in column j from all elements of column j
end for
Step 3: Cover all zeros using minimum number of horizontal and vertical lines
repeat
    Find rows with exactly one uncovered zero and assign it
    Cross out other zeros in the corresponding columns
    Find columns with exactly one uncovered zero and assign it
    Cross out other zeros in the corresponding rows
until all assignments are made or no more unique zeros are left
Step 4: Optimality Test
Count the number of lines needed to cover all zeros
if number of lines = n then
    Return the current assignment (Optimal solution found)
else
    Go to Step 5
end if
Step 5: Modify Cost Matrix
Find the smallest uncovered value m
for each element in C do
    if element is uncovered then
        Subtract m from it
    else if element lies at the intersection of lines then
        Add m to it
    end if
end for
Go to Step 3
```

Considering the linear programming technique we obtain the following if Z is the cost:

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}, \tag{4.7}$$

is minimized so that it is subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \forall i = 1, 2 \ldots, n \tag{4.8}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \forall j = 1, 2 \ldots, n \tag{4.9}$$

$$x_{ij} \in (0, 1), \quad \forall (i, j) \tag{4.10}$$

where $\mathbf{C} = \left[c_{ij} \right]$ is the given $n \times n$ cost matrix, n is the is the number of tasks which determine the size of the assignment problem. The assignment problem is represented by the matrix $\mathbf{X} = \left[x_{ij} \right]$ of order $n \times n$, where one of the elements in each row and column is equal to 1 while the remaining elements are 0. This then supports the matrix being a permutation matrix and an assignment. For convenience's sake, we denote an assignment by the set of ordered pairs where each of them represents the unit cells of the permutation matrix, such that

$$\mathbf{a} = \big\{ (1, j_1), (2, j_2), \ldots, (n, j_n) \big\}, \tag{4.11}$$

where $\mathbf{a}$ is the letter that denotes the assignment, and $j_1, j_2, \ldots, j_n$ represent the permutation of the numbers $1, 2, \ldots, n$ in the assignment. In congruence $(i, j) \in \mathbf{a}$ or $(i, j) \notin \mathbf{a}$ can be used to illustrate the matrix $\mathbf{X}$ representing the assignment of $\mathbf{a}$, where (i, j) is the cell entry which is either one or zero respectively.

In order to produce the ranking of all assignments, the increasing order should be defined [24]. This includes the minimum cost assignments with the consideration of the index of the positive integral arguments i.e $\mathbf{a}(r)$ and $\mathbf{a}(s)$ for $r < s$ which means that the cost of $\mathbf{a}(r)$ is smaller than that of $\mathbf{a}(s)$, where $\mathbf{a}$ is used to denote assignments.

We further look at how we can obtain an ascending order of cost in the set of all the assignments, $\mathbf{A}$. We start by defining $\mathbf{a}(r)$ as the minimum cost assignment in $\mathbf{A}$ after excluding all other higher assignments. Such that,

$$\mathbf{A}/\{\mathbf{a}(1), \ldots, \mathbf{a}(r-1)\}, \tag{4.12}$$

where / is the set-theoretic difference then $\mathbf{a}(1), \mathbf{a}(2), \ldots, \mathbf{a}(n)$ is the ranking of all assignments in ascending order of the cost [36].

The Murty algorithm demands that a routine for solving the assignment problem is given and utilizes a set of X, which is characterized by capturing the best possible k. The first step is to obtain the minimum cost assignment by the Hungarian method so, let it be:

$$\mathbf{a}(1) = (i_1, j_1), (i_2, j_2), \ldots, (i_n, j_n), \tag{4.13}$$

which contain a set of nodes that comply with the properties that satisfy the next elements in the sequence of ranked assignments [36]. Let the nodes $M_1, M_2 \ldots, M_{n-1}$ form part of the list in the Hungarian - Murty Algorithm as part of the sequence defined by:

$$M_1, M_2, \ldots, M_{n-1} \tag{4.14}$$

then

$$M_1 = \left\{(\overline{i_1, j_1})\right\}, \tag{4.15}$$

$$M_2 = \left\{(i_1, j_1), (\overline{i_2, j_2})\right\}, \tag{4.16}$$

$$\vdots \tag{4.17}$$

$$M_v = \{(i_1, j_1), (i_2, j_2), \ldots, (i_{v-1}, j_{v-1}), (\overline{i_v, j_v})\} \quad for \quad v = 1, 2, \ldots, n-1, \tag{4.18}$$

where v illustrates the iteration index for ranked assignment sequence. All nodes in M_v contain a non-empty subset of $\mathbf{A}$ and have the properties of being disjoint and having union $\mathbf{A}/\{\mathbf{a}(1)\}$. For the general stage k , with an assumption that $\mathbf{a}(1), \mathbf{a}(2), \ldots, \mathbf{a}(k)$ and a list of nodes $M_1, M_2, \ldots, M_{n-1}$ have been determind , we take the denotation in '**a**' and continue with the sequence $\mathbf{a}(k+1)$ [36]. Thus

$$\mathbf{a}(k+1) = b_{M_d}, \tag{4.19}$$

where b is the letter to denote the chosen assignments and M_d in any of the nodes in the list which additionally can be determined by:

$$Z_{M_d} = min[Z_{M_v}] \qquad \forall i = 1, 2, \ldots, n-1 \tag{4.20}$$

and M_d is the cost, making Z_{M_d} the minimal cost in the node list of M_v. Then the partition of nodes becomes $M_{d1}, M_{d2}, \ldots, M_{df}$, where f illustrates the last component in the partition. The process to determine b_{Md} and the corresponding list for the next stage continues as provide in [36]. If we partition a node we can yield at most $(n-1)$, which requires that for each stage of solving the assignment problems, $(n-1)$ has to be considered for each size $2, 3, \ldots, n$ [36]. By utilizing the murty algorithm a determination of many numbers in a sequence of ranked assignments is desired. Computations can be efficiently organized by capturing the list at each stage, for every node in the first list the storage can be done alongside its minimum cost assignment containing it. The values in Z_M corresponding to each node in the list can be stored in another location like the core memory. At each stage, a node that has the least value for Z_M among all other nodes in the list is further taken out and the remaining sub-generated sub-nodes by its partition are added to the list and so on [36].

4.3 MIMO STBC-SPM-HMA Decoding Algorithm

Algorithm 3 MIMO STBC-SPM-HMA Decoding

Input: Received signal vector $\mathbf{r}_t$, channel matrix $\mathbf{H}_t$, permutation vectors $\mathbf{P}$, rotation angles θ, code-word indices p_k, constellation set $\mathcal{S}$

Output: Decoded QAM symbols s_1, s_2, s_3, s_4

Compute the received signal vector y using the channel output matrix H: $y = Hx + n$, where x is the transmitted signal vector and n is the noise vector.

Perform Hungarian Murty Algorithm to obtain the minimum cost assignment $a(1)$.

Let $M_1, M_2, \ldots, M_{n-1}$ be the nodes in the list from the Hungarian Murty Algorithm.

for $k = 1$ **to** $n - 1$ **do**

    Perform the k^{th} iteration stage of the Murty Algorithm:

    Obtain the next assignment $\mathbf{a}(k+1)$ and the updated list $M_1, M_2, \ldots, M_{n-1}$.

    Compute the expected received signal vector $\hat{y}$ based on the permutation code-word represented by $\mathbf{a}(k + 1)$.

    Compute the Euclidean distance between the received signal vector y and the expected received signal vector $\hat{y}$.

    Store the permutation code-word represented by $\mathbf{a}(k+1)$ with the minimum Euclidean distance as the decoded code-word.

end for

Output the array of decoded permutation code-words.

Computational Complexity:

- Hungarian Murty Algorithm: $O(K^4)$, where K is the size of the permutation vectors.
- Permutation Codewords: Typically $O(K!)$, but optimized heuristics may reduce this

4.4 Chapter Summary

In this chapter, we introduced the Hungarian Murty Algorithm (HMA) as a soft-decision decoding method and examined its application within the MIMO-STBC-SPM communication system framework. By adapting the HMA, which is particularly effective for solving assignment problems, to the MIMO context, we aimed to enhance decoding precision while simultaneously reducing computational complexity, thereby boosting overall system performance.

We outlined the steps involved in the Hungarian Murty Algorithm and demonstrated its relevance in optimizing the decoding process for space-time block-coded systems. Additionally, the chapter covered the mathematical formulations related to the assignment problem, linear programming methods, and the cost functions necessary for implementing the HMA in the MIMO-STBC-SPM framework.

CHAPTER 5

Results and analysis

This chapter presents a comprehensive evaluation of the Hungarian Murty Algorithm (HMA) applied within the MIMO-STBC-SPM communication system, focusing on both performance metrics and computational complexity. The analysis demonstrates significant improvements in decoding accuracy and system reliability, particularly at low SNR levels, achieved through HMA's soft-decision decoding. Additionally, the chapter provides a comparative complexity analysis, highlighting how the HMA offers a more scalable and computationally efficient solution compared to traditional Maximum Likelihood (ML) hard decision approaches. By optimizing both time and space complexities, the HMA proves to be a practical and effective enhancement for modern communication systems.

5.1 Simulation Results

In this section, we look at the SNR Vs BER results of applying the HMA in the MIMO-STBC-SPM decoder.

The performance of three different MIMO systems is compared, highlighting the benefits of the Hungarian-based decoder. The baseline MIMO-STBC-SM system serves as a reference point, achieving a certain BER at different Signal-to-Noise Ratio (SNR) levels. This system highlights the performance gains achieved by integrating more advanced techniques into MIMO systems.

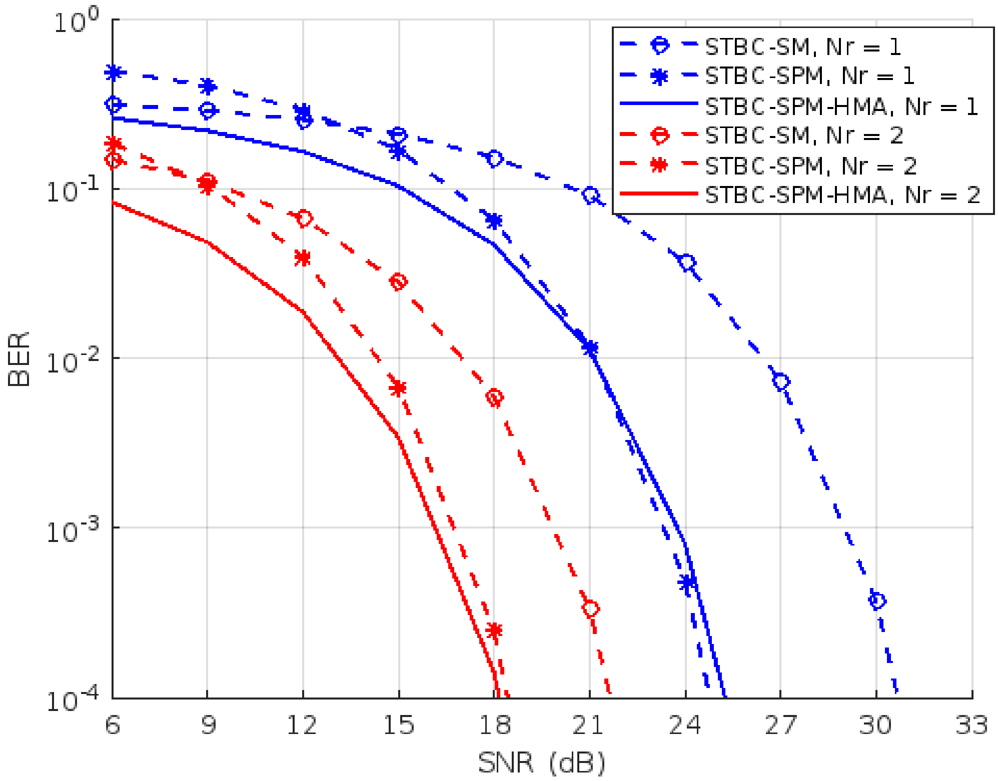

FIGURE 5.1: BER performance comparison of MIMO-STBC-SM, MIMO-STBC-SPM and MIMO-STBC-SPM-HMA, for $N_r = 1$ and $N_r = 2$ - showing up to 10× BER improvement at 18 dB SNR.

The standard MIMO-STBC-SPM system is used as a benchmark for this analysis, achieving a defined BER at different SNR levels. This system allows for a clear evaluation of the enhancements brought about by the Hungarian-based decoder.

When the conventional MIMO-STBC-SPM system is compared with that employing the HMA, a notable improvement in BER performance, particularly at lower SNR values, is observed. The HMA effectively utilizes soft information to enhance decoding accuracy, resulting in better spectral efficiency. At lower SNR values, this system significantly reduces BER compared to the standard configuration. However, at higher SNR values, its performance can fluctuate, sometimes outperforming and at other times underperforming relative to the conventional STBC-SPM decoder particularly for $N_r = 1$. Despite this variability, the Hungarian-based system ultimately achieves a better BER at higher SNR values with increasing number of N_r.

Both ML hard decision decoding and the Hungarian Murty Algorithm (HMA) soft decision decoding rely on Euclidean distance in their decoding processes. This distance measures how close the received signal is to the possible transmitted signals in Euclidean space, which is crucial for identifying the most likely transmitted signal.

In ML hard decision decoding, the decoder selects the transmitted signal that minimizes the Euclidean distance to the received signal. While this method is straightforward, it can be less effective in noisy conditions, where the exact transmitted signal may not be easily distinguishable, leading to suboptimal BER performance.

On the other hand, HMA soft-decision decoding leverages the Euclidean distance more effectively by incorporating soft information. This additional probabilistic data about the transmitted signal enhances the accuracy of the decoding process. At lower SNR values for lower number of N_r, this approach significantly lowers BER because the decoder can more precisely identify the transmitted signal despite the noise. Although performance may vary at higher SNR values, the system generally achieves better BER results compared to hard-decision decoding.

The baseline MIMO-STBC-SM system is included as a reference, showcasing the performance improvements gained through the integration of spatial permutation modulation and the Hungarian-based decoder. While the increased computational complexity of the Hungarian-based system is a factor to consider, its superior BER performance, especially in noisy environments, highlights its potential to enhance the reliability and efficiency of communication systems.

The graph illustrates a comparison of BER against SNR for various Quadrature Amplitude Modulation (QAM) configurations, STBC-SPM system that is enhanced by the Hungarian Murty Algorithm. The findings reveal how BER improves with increasing SNR across different modulation schemes and varying numbers of receive antennas.

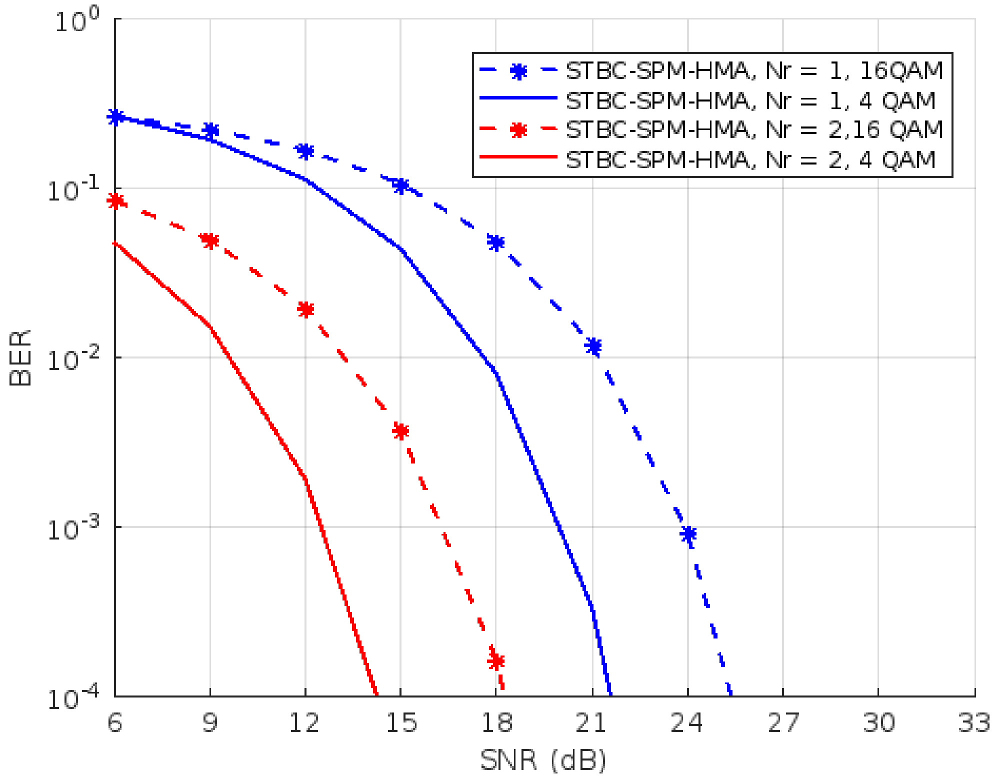

FIGURE 5.2: BER performance of the MIMO-STBC-SPM-HMA for varying QAM constellations

The graph features two modulation schemes: 4-QAM and 16-QAM. It is evident that systems employing 4-QAM whether using a single antenna or dual antennas—exhibit consistently lower BER values compared to those utilizing 16-QAM at any given SNR. This outcome aligns with expectations, as 4-QAM has fewer symbols, making it inherently more robust against noise. In contrast, 16-QAM, which incorporates more symbols, is more susceptible to errors induced by noise.

The introduction of multiple receive antennas ($N_r = 2$) leads to a significant enhancement in BER performance for both 4-QAM and 16-QAM, particularly at elevated SNR levels. For example, at an SNR of 12 dB, the 4-QAM system with $N_r = 2$ demonstrates a markedly lower BER than its counterpart with $N_r = 1$. The diversity achieved through additional antennas bolsters the system's resilience against noise, thereby improving performance in terms of BER.

All observed curves adhere to the anticipated trend, showing a decrease in BER as SNR increases. This reflects the principle that as the signal strength becomes greater relative to the noise, the system experiences fewer errors. At lower SNR values (below 10 dB), the differences in performance between the various modulation schemes and antenna configurations are more pronounced. However, as SNR rises beyond 20 dB, all systems converge toward low BER values, showcasing the STBC-SPM-HMA system's capability to facilitate highly reliable communication when SNR is sufficiently high.

In the mid-SNR range (15 to 20 dB), the configuration utilizing $N_r = 2$ with 4-QAM significantly outperforms the other setups, with its BER decreasing sharply. Conversely, the system with $N_r = 1$ and 16-QAM consistently displays the highest BER across all SNR values, indicating that a single receive antenna faces challenges in effectively decoding the more complex symbol sets under noisy conditions.

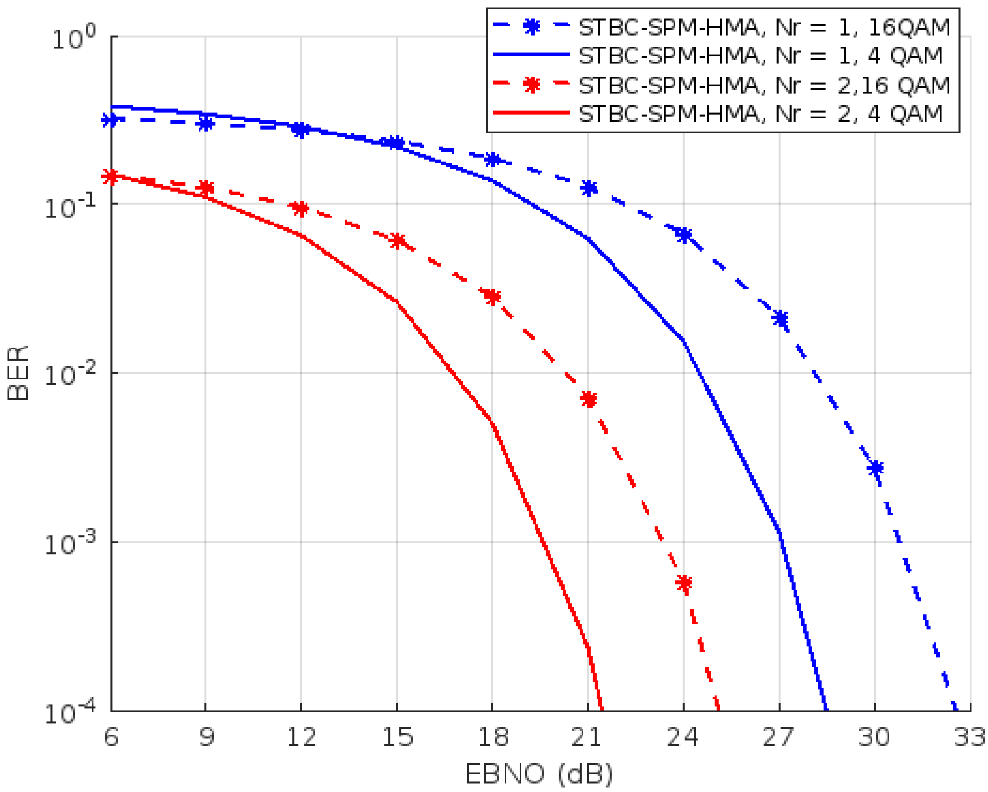

FIGURE 5.3: BER performance of the MIMO-STBC-SPM-HMA for varying QAM constellations with an increased bits per symbol rate 5

From the graph in Figure 5.3, it can be observed that the performance of 4-QAM is better than that of 16-QAM for all increased bits per symbol rate. This result is in line with expectations, as 4-QAM, due to the presence of a smaller number of symbols, is less affected by noise. On the other hand, 16-QAM, due to the presence of a larger number of symbols, is more sensitive to noise, which leads to increased error rates.

Thus, employing two receive antennas ($N_r = 2$) enhances the BER performance of both modulation formats. For E_b/N_0 values less than 15 dB, it can be seen that there is a benefit in using $N_r = 2$, especially for 4-QAM, and the system provides a better BER than the single receive antenna case. This trend demonstrates the effectiveness of spatial diversity in reducing the effects of noise E_b/N_0 and, hence, improving BER.

The BER curve improvement illustrates the standard inverse relationship between the system's capacity in reducing errors as the signal energy dominates over noise. The comparison of the performances of different configurations is most marked at the low E_b/N_0 indices (below 15 dB), which confirms once again the importance of the number of receive antennas and modulation modes in the conditions of interference. In the mid-range E_b/N_0 (15 to 25 dB), the 4-QAM channel model $N_r = 2$ is seen to be the best, as it has the steepest slope of BER.

This configuration is more reliable and efficient than the other setups and can therefore be regarded as the most effective. However, the 16-QAM system with a single receive antenna always has the highest BER for all E_b/N_0 values, which shows the problem associated with using a single receive antenna and high-order modulations in the presence of noise. At even higher E_b/N_0 levels, that is, greater than 25 dB, all the configurations tend to reach very low BERs, which shows that the MIMO-STBC-SPM-HMA results system also supports the reliable system's capability of high signal-providing levels for reliable data transmission even when operating under high constraint rates so long as the SNR is high and the number of antennas is adequate.

In figure 5.4, we can see the BER performance of a MIMO-STBC-SPM communication system with decoding being done using SD HMA. The analysis is done with a set of transmit antennas being fixed at $4 (N_t = 4)$ and the receive antennas are changed to take values of $N_r = 1, 2, 3, 4$ respectively. To observe the performance

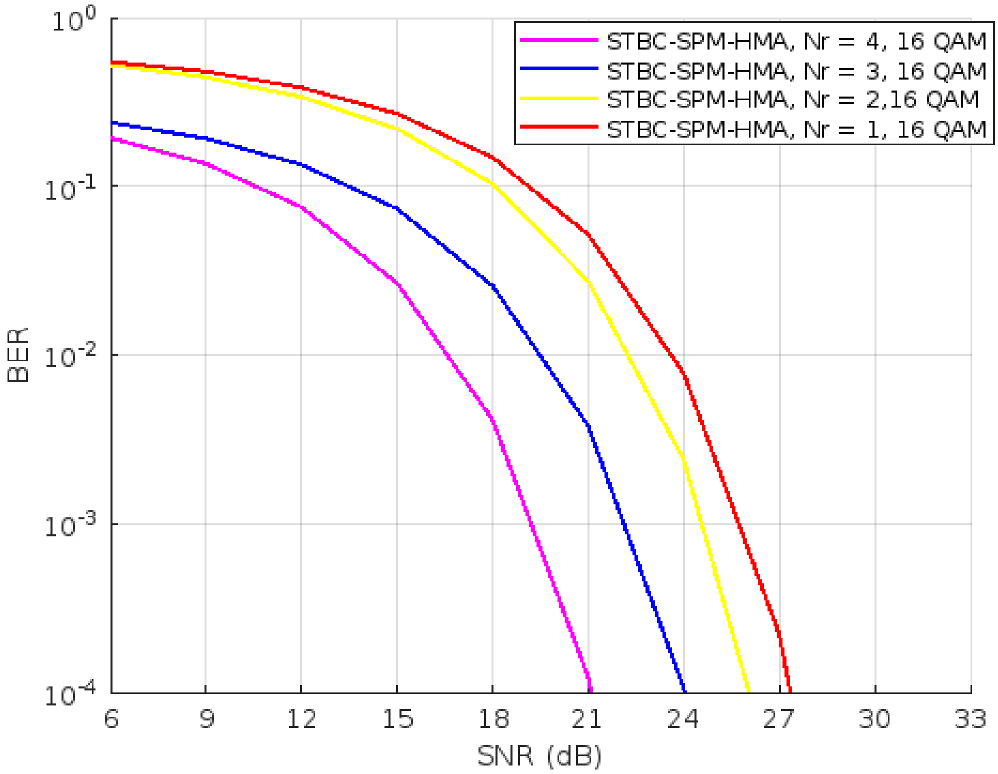

FIGURE 5.4: BER performance of the MIMO-STBC-SPM-HMA 16 QAM for varying recieve antennas

of the system, 16-QAM modulation is used and the graph presents the dependency between BER and SNR to show how receiving antenna diversity enhances the system performance.

With regards to the number of receive antennas, it can be observed that when the antennas are four, the BER performance of the system is much better as compared to when the antennas are one. The $N_r = 4$ shows the best-level result of spatial diversity at SNR the values.On the other hand, the receiver $N_r = 1$ provides the worst performance, with higher BER throughout the entire channel. This difference clearly depicts the role of receive antenna multiplicity in combating signal fading and noise, which are crucial to arbitration.

It is also observable that the behavior of BER versus SNR is similar for all the configurations and BER always decreases with the increase of SNR. At low SNR, for instance, below 15 dB, the system has high BER and there are distinct differences according to the configuration. These gaps start to become less noticeable as SNR increases, but the options that have more antennas at the receiving end provide better results than those with fewer antennas. The mid-SNR range of 15 to 20 dB is quite informative; for instance, the configurations with $N_r = 3$ and $N_r = 4$ exhibit much lower BER as compared to $N_r = 1$ and $N_r = 2$. This is evidence of the benefit of in enhancing system performance where the signal is not weak.

At very high SNR, i.e., greater than 25 dB, the system is highly reliable, and all the configurations have very low BER. However, even in this range, the higher the N_r, the better are the results obtained, which further strengthens the argument for the use of diversity. Although 16-QAM modulation provides better data transmission in a given bandwidth as compared to other modulation schemes, it is more sensitive to bit errors. This effectively counters this weakness through the addition of receive antennas and thus improves the error performance.

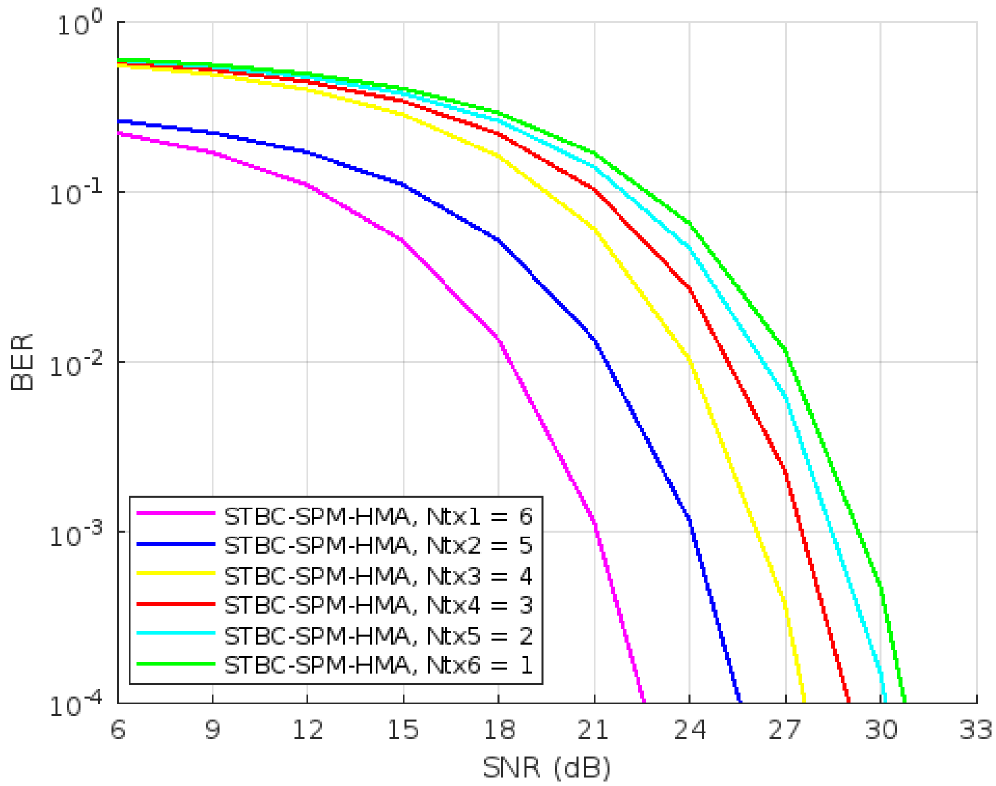

FIGURE 5.5: BER performance of the MIMO-STBC-SPM-HMA 16 QAM for varying trasnmit antennas

The graph on figure 5.5 shows the BER performance of a MIMO-STBC-SPM transmission system with decoding performed using Hungarian Murty Algorithm (HMA). The system applies 16-QAM modulation with a fixed number of receive antenna ($N_r = 6$) while changing the number of transmit antenna (N_t) from 1 to 6. The results show the relationship between BER and SNR to observe the effects of antenna diversity and transmit configuration.

When the number of transmit antennas is increased the system performance also increases significantly. For instance, when the number of transmit antennas is 6, the system performs best with the lowest BER for all the SNR values due to spatial diversity. On the other hand, those with fewer number of transmit antennas for instance 1 dimentional stream as in $N_t = 1$ have high BER because of lack of spatial diversity. This is to highlight the importance of transmit antenna diversity in combating signal fading and enhancing the reliability of the system.

BER decreases with SNR as a general trend which suggests that the number of errors is decreasing as the signal strength is increasing in comparison to the noise. At relatively high SNR levels for example greater than 25 dB all the configurations are quite stable with BER values reheasing to less than 1×10^{-3}. Nevertheless, those with higher number of transmit antenna, for instance, with five antennas or six antennas, provide better performance as compared with the others, especially in the low to medium SNR level.

The employment of $N_r = 6$ receive antennas also increases the spatial diversity and thus enhances the system performance. Both these factors of high transmit and receive diversity enable the system to nullify the effects of fading and hence provide reliable communication even at low SNR. For instance, at mid SNR level (15-20 dB), the configurations with $N_t = 5$, $N_t = 6$ outperform the others in terms of BER especially where the number of transmit antenna is concerned.

Although 16-QAM modulation provides high data rates per unit of bandwidth it is more vulnerable to errors as compared to other modulation schemes with lower order constellations. The results also show that to counteract such susceptibility, it is necessary to apply diversity measures, including the increasing of N_t and

N_r. Both the Hungarian Murty Algorithm and diversity configurations help in reducing the error rate and therefore enhance system reliability.

5.2 Complexity Analysis

Key factors that determine the computational complexity of a decoder include the size of the permutation vectors, the cardinality of the code set, which is influenced by the minimum Hamming distance (d_{min}), and the number of transmitting antennas (N_t). In the context of the hard decision approach, which uses ML to decode the transmitted symbols, the following steps are involved:

1. Generating all permutations of a set, where each permutation vector has K elements as mentioned earlier. Thus, contributing $K!$ to the computational complexity of a system. As mentioned in Table 2.1.

2. Mapping the likely transmitted codeword to one of the codewords in the the generated set of permutation vectors.

3. Computing the Euclidean distance for each permutation vector which requires N_t operations; since the total number of permutations is $K!$ the computational complexity results in $O(K! \cdot N_t)$

Which can also be expressed in terms of the cardinality of the permutation set as $O(|\mathbf{P}| \cdot N_t)$, because $|\mathbf{P}| \approx K!$.

In contrast, the HMA Soft Decision approach offers a more efficient solution because its computational complexity is based on the size of the permutation vector unlike the size of the full set of the permutation vector. To solve the assignment problem, the HMA complexity combines complexities of the Hungarian and Murty's algorithms of $O(K^3)$ and $O(K^4)$, respectively [51]. Hence, considering the worst order of $O(K^4)$ and the Euclidean distance computation which requires $O(N_t)$ operations, the overall computational complexity of the HMA decoder becomes $O(K^4 \cdot N_t)$. Table 5.1 below presents a summary of the complexity analysis factors of the STBC-SPM using both hard and soft decision approches.

TABLE 5.1: Comparison of ML HD vs. HMA SD

Aspect	**ML Hard Decision**	**HMA Soft Decision**
Computational Complexity	$O(\lvert\mathbf{P}\rvert \cdot N_t)$, where $\lvert\mathbf{P}\rvert \approx K!$, thus $O(K! \cdot N_t)$	$O(K^4 \cdot N_t)$
Growth Rate	Factorial, extreamly fast due to $K!$	Polynomial with K, linear with N_t for computational complexity
Feasibility for Large K	Becomes infeasible quickly	More feasible compared to ML Hard Decision
Total Complexity	Impractical for large K	Practical and efficient for larger K

As observed in Figure 5.6, the HMA Soft Decision approach significantly reduces the computational burden by decreasing the number of permutations that need to be generated and examined, resulting in a more efficient process in terms of computational complexity (which is similar to time complexity) compared to the ML Hard Decision approach.

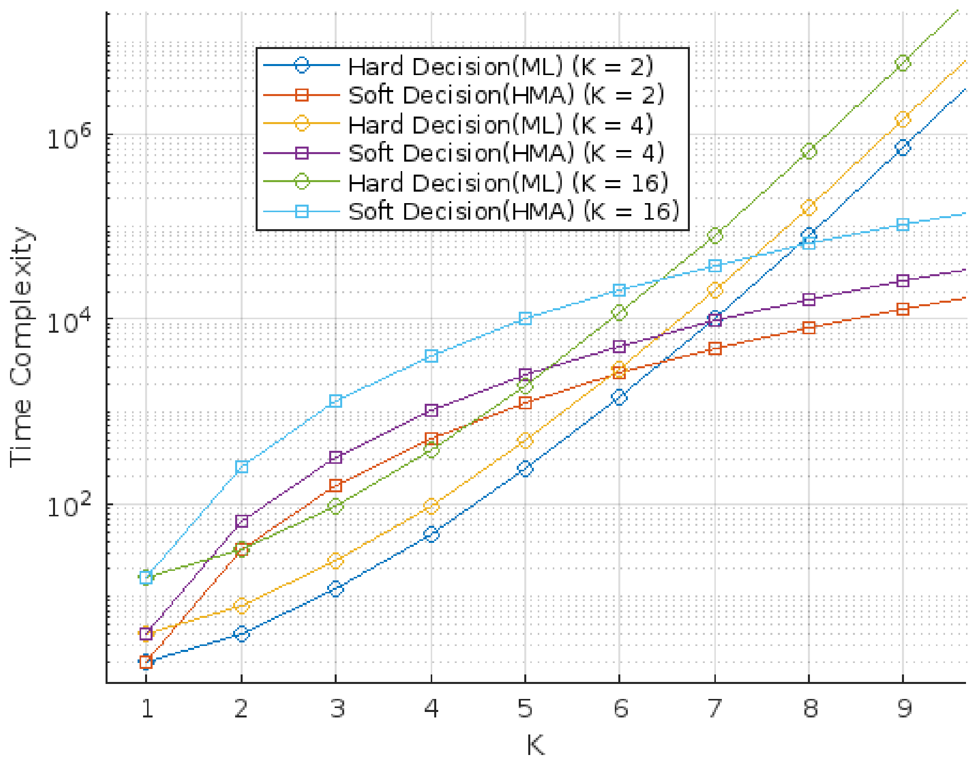

FIGURE 5.6: Time complexity comparison for various K values in hard and soft-decision decoding techniques

The provided figures 5.6 compares the computational complexity of two algorithms: ML hard decision and HMA doft decision algorithm, particularly in the context of a MIMO-STBC-SPM communication system. The ML Hard Decision algorithm, often used in exhaustive search methods for decoding combinatorial problems, exhibits a time complexity of $O(K! \cdot N_t)$. This complexity arises from the factorial growth ($K!$), which signifies a super-exponential increase in the number of operations as K increases, with each operation scaling with N_t, the dimensionality of the problem.

In contrast, the HMA Soft Decision algorithm optimizes the assignment problem by finding the optimal permutation matrix to maximize the SNR. Its time complexity is $O(K^4 \cdot N_t)$, characterized by polynomial growth (K^4), which is more manageable compared to the factorial growth of the ML algorithm. Each operation still scales with N_t.

The time complexity analysis, depicted in Figure 5.6, uses the number of tasks or agents (K) on the X-axis and the time complexity on a logarithmic scale on the Y-axis. The curves show that the time complexity of the ML Hard Decision algorithm grows exponentially due to its factorial nature, making it impractical for larger K. Conversely, the HMA Soft Decision algorithm demonstrates polynomial growth, indicating better scalability and feasibility for larger problems. This illustrates that as K increases, the HMA algorithm remains more practical for implementation compared to the rapidly escalating time complexity of the ML Hard Decision algorithm.

The real world impact of these discoveries is quite important indeed! The ML hard decision algorithm proves to be impractical, as K grows rapidly because of the exponential increase in time complexity. This restricts its use to problems that do not face limitations in resources. On the hand the HMA Soft Decision algorithm is a choice, for larger K as it exhibits polynomial growth in time complexity. The HMA algorithm strikes a balance, between efficiency and accuracy in real world settings particularly useful, for tasks that involve optimization and assignment solutions.

5.3 Chapter Summary

In this chapter, we presented the findings and analysis of the HMA implemented within the MIMO-STBC-SPM communication system. Our comparative study revealed that the HMA-based soft-decision decoding significantly enhances Bit Error Rate (BER) performance, especially in low Signal-to-Noise Ratio (SNR) conditions, when compared to traditional Maximum Likelihood (ML) hard-decision decoding. This improvement underscores the effectiveness of using HMA to boost decoding accuracy.

We also evaluated various QAM configurations and introduced multiple receiving antennas to demonstrate how diversity and modulation complexity influence system performance. Notably, systems employing 4-QAM consistently outperformed those using 16-QAM in terms of BER, particularly with multiple receive antennas, highlighting the resilience of the MIMO-STBC-SPM-HMA system in challenging environments.

The study also emphasizes the significance of diversity techniques, both spatial and transmit, in enhancing the BER performance as the findings reveal that diversity techniques are very effective in improving the BER especially when operating under noisy and interference environment. Although 16-QAM modulation is known to provide higher data transmission rates, the performance of this scheme can be enhanced through the employment of diversity techniques. The conclusions also show that the complex system, which includes MIMO-STBC-SPM-HMA, offers remarkable reliability and dependability with the help of diversity techniques especially in terms of BER, which proves its potential for efficient data transmission even in the conditions of high constraint rates and noise interference. This is therefore useful and can be applied in advanced communication systems.

Furthermore, our complexity analysis contrasted HMA with ML hard-decision decoding, showing that while the latter's complexity increases exponentially with the number of permutations, HMA provides a more scalable and computationally efficient alternative. By utilizing HMA, we can reduce both time and space complexities, making it a more viable option for contemporary communication systems that handle larger problem sizes.

CHAPTER 6

Conclusion

The HMA is a soft decision decoding algorithm that has been integrated into the MIMO-STBC-SPM wireless communication system to improve the computational complexity and error rate performance. For this dissertation, the HMA has shown its effectiveness by enhancing the decoding process by solving the assignment problem efficiently in polynomial time, unlike exponential time needed by the known hard decoding techniques. The bit error rate has been enhanced by the HMA especially at lower SNRs. The utilization of the soft information from the receiver enables the HMA to enhance the decoding decisions while illuminating the likelihood of errors.

6.1 Answers to Research Questions and Corresponding Contributions

1. Does the Hungarian-Murty Algorithm reduce computational complexity in a MIMO-STBC-SPM System? The HMA is better than the ML Hard decision decoding method in terms of computational complexity. The time complexity of ML Hard Decision Decoding is $O(K! \cdot N_t)$ The complexity of this method is $O(K!)$ which increases very steeply with K. As such, the HMA is a soft decision decoding method with time complexity of $O(K^4 \cdot N_t))$. This is a significant reduction in complexity and therefore makes the HMA decoding approach more efficient. Hence, HMA is a significant enhancement to the conventional ML hard decision decoding in terms of computational complexity.

2. How Does the HMA impact the performance of the MIMO-STBC-SPM system? HMA being a soft decision algorithm enhances the performance of the MIMO-STBC-SPM system by an improvement in the BER at lower SNRs. At higher SNRs, there's a difference in performance, the analysis made In the BER metrics, indicate that HMA provides good BER performance. How ever there is a trade-off between the computation complexity and the error rate performance, this makes HMA a good decoding technique required to enhance reliability and spectral efficiency in the MIMO-STBC-SPM wireless communication system.

6.2 Future Research Directions

This thesis demonstrates the integration of HMA into the MIMO-STBC-SPM wireless communication system. Having accomplished this, numerous avenues for future research may further elucidate the application of HMA. The subsequent points may be regarded:

- **Optimization Techniques**: Additional exploration of superior optimization techniques to reduce the computational complexity of HMA is required. It may encompass the examination of parallel processing, hardware acceleration, artificial intelligence, and machine learning techniques, among others. It can either enhance or ensure consistency in the BER performance while diminishing computing requirements. This could result in practical application scenarios.

- **Energy efficiency assessment**: Moreover, it would be intriguing to examine the energy efficiency of HMA in MIMO-STBC-SPM systems comprehensively. This can be examined by comparing the energy consumption during the decoding process with that of conventional hard decision decoding in practical applications.

- **Real-time implementation and testing**: HMA in a real-life scenario trasnciever of a MIMO-STBC-SPM wireless communication system, assessing the practicality of the performance would be a valuable step. First create a prototype and then convert it into a live system; this has the potential to open the way for commercial adoption and utilization.

- **Comparison with emerging algorithms:** To get a detailed comparison of recent decoding methods with integrated HMA in the MIMO-STBC-SPM wireless communication system would be very helpful. This would help to identify gains, trade-offs, and opportunities with respect to different decoding algorithms.

The solutions of these future research directions will enhance the performance and efficiency of the Hungarian-Murty Algorithm and advance MIMO-STBC-SPM communication systems to support their wide use in industries and applications.

References

[1] Abdealrahman Abuqamar, Jehad M. Hamamreh, and Mohamedou Abewa. Stbc-assisted ofdm with subcarrier power modulation. *RS Open Journal on Innovative Communication Technologies*, 2, 2021.

[2] Karam Ahmed, Sherif Abuelenin, Heba Soliman, and Khairy Al-Barbary. Low complexity v-blast mimo-ofdm detector by successive iterations reduction. *Advances in Electrical and Computer Engineering*, 15:77–82, 2015.

[3] Bakir Hadj Ali, Debbat Fatima, and Bendimerad Fethi Tarik. Comparative analysis performance of ostbc/stbc-mimo under rayleigh fading channel. *2014 2nd World Conference on Complex Systems, WCCS 2014*, pages 237–241, 2014.

[4] Anand Arunachalam and Hesham El Gamal. Space-time coding for mimo systems with co-channel interference. *IEEE Transactions on Wireless Communications*, 3:1953–1958, 11 2004.

[5] Dauda Olayinka Ayanda. Uncoded space-time labeling diversity with three transmit antennas : Symbol mapping designs and error performance analysis. 2020.

[6] Ertuğrul Başar, Ümit Aygölü, Erdal Panayirci, and H. Vincent Poor. Space-time block coding for spatial modulation. *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, PIMRC*, pages 803–808, 2010.

[7] Fatma Benkhelifa, Abdoulaye Tall, Zouheir Rezki, and Mohamed Slim Alouini. On the low snr capacity of mimo fading channels with imperfect

channel state information. *IEEE Transactions on Communications*, 62:1921–1930, 2014.

[8] Praveen Sai Bere, Student Member, Mohammed Zafar, Ali Khan, and Senior Member. Diversity preserving , universal hard decision decoder for linear block codes. pages 1–10.

[9] Hanumantharao Bitra and Panlanisamy Ponnusamy. Performance analysis of adaptive generalized spatial modulation. *2020 International Conference on Artificial Intelligence and Signal Processing, AISP 2020*, 2020.

[10] Emil Björnson, Per Zetterberg, Mats Bengtsson, and Bjorn Ottersten. Capacity limits and multiplexing gains of mimo channels with transceiver impairments. *IEEE Communications Letters*, 17:91–94, 2013.

[11] Chienchun Cheng, Chienchun Cheng, and Other Université. Mimo signal design , channel estimation , and symbol detection to cite this version : Hal id : tel-01306917. 2016.

[12] Jung Chun Chi, Yu Cheng Yeh, I. Wei Lai, and Yuan Hao Huang. Sphere decoding for spatial permutation modulation mimo systems. *IEEE International Conference on Communications*, pages 1–8, 2017.

[13] Shakti Raj Chopra, Navjot Kaur, and Himanshu Monga. Space time coding techniques in mimo: A review. *Indian Journal of Science and Technology*, 9, 2016.

[14] Yu An Chu, Po Yu Chen, Yi Hung Chiang, Tzung Sheng Yang, Yu Chieh Liao, and I. Wei Lai. Permutation design for ultra-low latency communication and spatial permutation modulation (spm). *2020 IEEE Globecom Workshops, GC Wkshps 2020 - Proceedings*, 2020.

[15] Mohamed Oussama Damen, Ali Abdi, and Mostafa Kaveh. On the effect of correlated fading on several space-time coding and detection schemes. *IEEE Vehicular Technology Conference*, 1:13–16, 2001.

[16] Wang Dong, Wang Fanggang, L I Xiran, Yuan Pu, and Jiang Dajie. Orthogonal time frequency space modulation in multiple - antenna systems. 19:71–78, 2021.

[17] Gerard J. Foschini. Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas. *Bell Labs Technical Journal*, 1:41–59, 1996.

[18] Hesham El Gamal and Anand Arunachalam. Space-time coding techniques for mimo block fading channels with co-channel interference, 2003.

[19] Hany S. Hussein, Mohamed Elsayed, Usama Sayed Mohamed, Hamada Esmaiel, and Ehab Mahmoud Mohamed. Spectral efficient spatial modulation techniques. *IEEE Access*, 7:1454–1469, 2019.

[20] Naoki Ishikawa. Space-, time-, and frequency-domain permutation modulation designed for microwave and optical wireless communications. 2017.

[21] Naoki Ishikawa, Shinya Sugiura, and Lajos Hanzo. 50 years of permutation, spatial and index modulation: From classic rf to visible light communications and data storage. *IEEE Communications Surveys and Tutorials*, 20:1905–1938, 2018.

[22] Amr Ismail, Jocelyn Fiorina, and Hikmet Sari. A new low-complexity decodable rate-1 full-diversity 4×4 stbc with nonvanishing determinants. *IEEE Transactions on Wireless Communications*, 10:2456–2460, 2011.

[23] Jeyadeepan Jeganathan, Ali Ghrayeb, and Leszek Szczecinski. for mimo channels if. pages 8–11, 2008.

[24] Oluwafemi Kolade, Adnan M. Abu-Mahfouz, and Ling Cheng. A subcarrier permutation scheme for noise mitigation and multi-access in powerline channels. *2021 IEEE International Symposium on Power Line Communications and its Applications, ISPLC 2021*, pages 19–24, 2021.

[25] Ravi Kumar and Rajiv Saxena. Performance analysis of mimo-stbc systems with higher coding rate using adaptive semiblind channel estimation scheme. *The Scientific World Journal*, 2014, 2014.

[26] I. Wei Lai, Jhih Wei Shih, Kwang Cheng Chen, and Ezio Biglieri. Generalized path-permutation clodes for reliable end-to-end networking of opportunistic links. *IEEE Transactions on Wireless Communications*, 19:4579–4593, 2020.

[27] I. Wei Lai, Jhih Wei Shih, Che Wei Lee, Hsu Hsuan Tu, Jung Chun Chi, Jyun Sian Wu, and Yuan Hao Huang. Spatial permutation modulation for multiple-input multiple-output (mimo) systems. *IEEE Access*, 7:68206–68218, 2019.

[28] Christopher M. Lau. Performance of mimo systems using space time block codes (stbc). *Open Journal of Applied Sciences*, 11:273–286, 2021.

[29] Che Wei Lee, Hsu Hsuan Tu, and I. Wei Lai. Mapping and permutation set design for spatial permutation modulation (spm). *Proceedings - 2019 International Symposium on Intelligent Signal Processing and Communication Systems, ISPACS 2019*, pages 6–7, 2019.

[30] Yongho Lee, Yuro Lee, and Sok Kyu Lee. Performance of antenna permutation for multi-rate gigabit mimo-ldpc. *Proceedings - The 2nd International Conference on Mobile Ubiquitous Computing, Systems, Services and Technologies, UBICOMM 2008*, pages 139–143, 2008.

[31] Cai Yan Li, Xue Qin Jiang, Miaowen Wen, Jun Li, Yuyang Peng, and Sijing Chen. Space-time block coded generalized receive spatial modulation. *2019 IEEE 2nd International Conference on Electronics and Communication Engineering, ICECE 2019*, pages 89–93, 2019.

[32] Raed Y. Mesleh, Harald Haas, Sinan Sinanović, Chang Wook Ahn, and Sangboh Yun. Spatial modulation. *IEEE Transactions on Vehicular Technology*, 57:2228–2241, 2008.

[33] Priyanka Mishra, Gurpreet Singh, Rahul Vij, and Gaurav Chandil. Ber analysis of alamouti space time block coded 2×2 mimo systems using rayleigh dent mobile radio channel. *Proceedings of the 2013 3rd IEEE International Advance Computing Conference, IACC 2013*, pages 154–158, 2013.

[34] Mehrbod Mohajer, Seyed Yahya Mortazavi, Essam Elkhouly, and Chen Chen. System simulations for mimo antenna designs. *2021 IEEE International Symposium on Antennas and Propagation and North American Radio Science Meeting, APS/URSI 2021 - Proceedings*, pages 1009–1010, 2021.

[35] Katta G. Murty. Letter to the editor—an algorithm for ranking all the assignments in order of increasing cost. *Operations Research*, 16:682–687, 1968.

[36] Katta G. Murty. Letter to the editor—an algorithm for ranking all the assignments in order of increasing cost. *Operations Research*, 16:682–687, 1968.

[37] Ayman F. Naguib and R. Calderbank. Space - time coding and signal processing for high data rate wireless communications. *Wireless Communications and Mobile Computing*, 1:13–34, 2001.

[38] T. Lakshmi Narasimhan, P. Raviteja, and A. Chockalingam. Generalized spatial modulation in large-scale multiuser mimo systems. *IEEE Transactions on Wireless Communications*, 14:3764–3779, 2015.

[39] Van Duc Nguyen, Matthias Pätzold, Fumiaki Maehara, Harald Haas, and Minh Viet Pham. Channel estimation and interference cancellation for mimo-ofdm systems. *IEICE Transactions on Communications*, E90-B:277–290, 2007.

[40] Purva Nikhate, Atul R. Deshmukh, and Swapna Choudhari. Performance analysis of 2x2 mimo systems for alamouti stbc and zero forcing equalization. *2021 12th International Conference on Computing Communication and Networking Technologies, ICCCNT 2021*, pages 1–6, 2021.

[41] Stephen Kwahu Okrah and Anthony Kwarteng. Application of workforce optimization to the award of projects in a biding tender.(a case study of shame senior high school). *Theijes.Com*, 2020.

[42] Zeyad A.H. Qasem, Hamada Esmaiel, Haixin Sun, Junfeng Wang, Yongchun Miao, and Sheraz Anwar. Enhanced fully generalized spatial modulation for the internet of underwater things. *Sensors (Switzerland)*, 19:1–16, 2019.

[43] Laith Ali Abdul rahaim El-anzy. Design and simulation of stbc- (ofdm and cdma) transceivers based on hybrid transforms laith ali abdul-rahaim el-anzy. 2006.

[44] G. Susinder Rajan and B. Sundar Rajan. Multigroup ml decodable collocated and distributed space-time block codes. *IEEE Transactions on Information Theory*, 56:3221–3247, 2010.

[45] T. Ravibabu and C. Dharma Raj. Ber analysis of spatial multiplexing and stbc mimo-ofdm system. *Proceedings of the 4th International Conference on Devices, Circuits and Systems, ICDCS 2018*, pages 110–116, 2019.

[46] Di Renzo, Miaowen Wen, Senior Member, Beixiong Zheng, Kyeong Jin Kim, Marco Di Renzo, Theodoros A Tsiftsis, Kwang-Cheng Chen, and Naofal Al-Dhahir. A survey on spatial modulation in emerging wireless systems: Research progresses and applications. *IEEE Journal on Selected Areas in Communications*, 2019.

[47] M. Roopa and B. N. Shobha. Performance improvement of mimo system using ostbc scheme and ml detection technique under rayleigh channel. *2019 4th IEEE International Conference on Recent Trends on Electronics, Information, Communication and Technology, RTEICT 2019 - Proceedings*, pages 1431–1435, 2019.

[48] Khodr A. SAAIFAN and Emad K. AL-HUSSAINI. Efficient time/frequency permutation of mimo-ofdm systems through independent and correlated nakagami fading channels. *International Journal of Communications, Network and System Sciences*, 02:903–911, 2009.

[49] Vibhav Kumar Sachan and Syed Akhtar Imam. Orthogonal space-time block coding for wireless sensor networks: Performance results. *2013 International Conference on Signal Processing and Communication, ICSC 2013*, pages 170–175, 2013.

[50] Pingping Shang, Sooyoung Kim, and Xue Qin Jiang. Efficient alamouti-coded spatial modulation for secrecy enhancing. *ICTC 2019 - 10th International Conference on ICT Convergence: ICT Convergence Leading the Autonomous Future*, pages 860–864, 2019.

[51] Mulundumina Shimaponda-Nawa, Oluwafemi Kolade, Wenbo Ding, Dushantha Nalin K. Jayakody, and Ling Cheng. Soft-decision decoding of permutation-based optical codes for a multiple access system. *IEEE Communications Letters*, 25:2824–2828, 2021.

[52] A. Slaney and Y. Sun. Space-time coding for wireless communications: An overview. *IEE Proceedings: Communications*, 153:509–518, 2006.

[53] Emna Ben Slimane, Slaheddine Jarboui, Zouhed Ben Mabrouk, and Ammar Bouallegue. Pilot assisted channel estimation in mimo-stbc systems over time-varying fading channels. *2014 12th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, WiOpt 2014*, pages 119–124, 2014.

[54] Qinfang Sun, Donald C. Cox, Howard C. Huang, and Angel Lozano. Estimation of continuous flat fading mimo channels. *IEEE Wireless Communications and Networking Conference, WCNC*, 1:189–193, 2002.

[55] K. Sundaravadivu and S. Bharathi. Stbc codes for generalized spatial modulation in mimo systems. *2013 IEEE International Conference on Emerging Trends in Computing, Communication and Nanotechnology, ICE-CCN 2013*, pages 486–490, 2013.

[56] Vahid Tarokh, Hamid Jafarkhani, and A. Robert Calderbank. Space-time block coding for wireless communications: Performance results. *IEEE Journal on Selected Areas in Communications*, 17:451–460, 1999.

[57] Vahid Tarokh, Nambi Seshadri, Senior Member, and A R Calderbank. Space-time codes for high data rate wireless communication: Performance criterion and code construction, 1998.

[58] Miaowen Wen, Beixiong Zheng, Kyeong Jin Kim, Marco Di Renzo, Theodoros A. Tsiftsis, Kwang Cheng Chen, and Naofal Al-Dhahir. A survey on spatial modulation in emerging wireless systems: Research progresses and applications. *IEEE Journal on Selected Areas in Communications*, 37:1949–1972, 2019.

[59] Qun Wu, Xiao Zhou, Chengyou Wang, and Hai Cao. Variable pilot assisted channel estimation in mimo-ofdm system with stbc and different modulation modes. *Eurasip Journal on Wireless Communications and Networking*, 2022:183–195, 2022.

[60] Abdelhamid Younis, Nikola Serafimovski, Raed Mesleh, and Harald Haas. Generalised spatial modulation. *Conference Record - Asilomar Conference on Signals, Systems and Computers*, pages 1498–1502, 2010.

[61] Xin Zhong, Chen Chen, Shu Fu, Xin Jian, Min Liu, and Xiong Deng. Ofdm-based generalized spatial modulation for optical wireless communication. In *Proceedings of the 16th IEEE Conference on Industrial Electronics and Applications, ICIEA 2021*, pages 1311–1316. Institute of Electrical and Electronics Engineers Inc., 8 2021.