

**Exploring the teaching of big ideas
in teaching for mathematical reasoning
while covering content (functions)
meaningfully**

**Exploring the teaching of big ideas in teaching for mathematical reasoning
while covering content (functions) meaningfully**

Kurt Michael Coetzee

Student number: 8806882V

A research report submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg, in partial fulfilment of requirements for the degree of Master of Science by coursework and research report.

Supervisor: Professor Karin Brodie

Johannesburg, September 2011

DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Science by coursework and research report at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university, nor has it been prepared under the aegis or with the assistance of any other body or organisation outside of the University of the Witwatersrand, Johannesburg. All help received with the preparation and/or presentation of this thesis has been clearly acknowledged on the next page.

Kurt Coetzee

Signature: _____



September 2011

ACKNOWLEDGEMENTS

I would like to thank the following people for the part they played in this work:

- My family, for their continuous love, support, patience and encouragement throughout my entire period of study.
- My supervisor, Prof. Karin Brodie, for believing in me and giving me all the necessary support and drive, especially at the time when things were becoming too challenging for me.
- My principal for her support and having allowed me to conduct my research within the school.
- The grade 9 learners without whom this research would not be possible.
- The parents of the learners for allowing them to be part of this research.
- Jade Coetzee for videotaping my lessons.
- Rasheed Sanni, for his tireless efforts in assisting me with the proposal for this research. The sacrifices he endured to complete his Doctoral theses will always remain an inspiration to me.
- Nicholas Molefe and Zaheera Jina whose research reports I used as guidance and inspiration.

ABSTRACT

This study explored the use of big ideas in functions around which mathematical activity is focused in teaching mathematical reasoning to grade 9 learners. I approached the teaching of big ideas in functions through constructivist, socio-cultural and situated perspective on learning. Through analysis of the literature I identified seven big ideas in function. They are: representation, covariance, variable, multiplicative relationships, pattern, slope and additive relationships. In developing tasks to guide mathematical activity, I endeavoured to generate cognitively high level tasks that allowed learners to view the tasks as authentic problems to solve. Authentic problems are problems whose solution paths are not dependent on a set of routine algorithms to be memorised. Through a collaborative approach to implementation of the tasks, I presented four introductory lessons relating to the seven big ideas in functions identified. These lessons were analysed by coding for the seven big ideas in learner responses and by analysing participant interactions using transcripts. Analysis of coding suggests that representation, covariance, variable and multiplicative relationships have strong connections between them. Pattern, slope and additive relationships did not display strong connections with the other four big ideas. Analysis of transcripts revealed startling results in relation to both individual as well as collaborative lines of argumentation. Collaborative lines of argumentation are seen where two or more learners co-construct a solution through mathematical reasoning while engaging the mathematical ideas prevalent in the task. I argue that it is through the implementation of teacher practices, that embraces the learner as a legitimate participant, that learners are able to develop learner practices that promote mathematical reasoning. In teaching big ideas I have found a new way in making mathematical activity more meaningful to my learners.

TABLE OF CONTENTS

	<i>Page no</i>
CHAPTER 1: INTRODUCTION	1
1.1 Background	1
1.2 Problem statement	2
1.3 Critical question	2
1.4 Rationale	3
CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW	6
2.1 Introduction	6
2.2 Theories of learning	6
2.3 Mathematical reasoning	8
2.4 Collaborative learning	12
2.5 Teacher practices	16
2.6 Big ideas	17
2.6.1 What are big ideas?	17
2.6.2 Big ideas in functions	19
CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY	24
3.1 Methodology	24
3.2 Research methods	25
3.2.1 Research context	25
3.2.2 Task design and implementation	26
3.2.3 Data collection	33
3.3 Rigour	34
3.4 Ethical considerations	36
3.5 Analysis	37

CHAPTER 4: DATA ANALYSIS	39
4.1 Introduction	39
4.2 Coding	39
4.3 Big ideas	44
4.4 Reasoning	52
4.4.1 The “Zig Zag” function	52
4.4.2 A collaborative line of argument	58
4.4.3 Big ideas and mathematical reasoning	59
4.5 Practices	61
4.5.1 Learner practices for mathematical reasoning	62
4.5.2 Teacher practices	64
CHAPTER 5: CONCLUSION AND IMPLICATIONS FOR RESEARCH	69
5.1 Summary and discussion	69
5.2 Answering my research question	72
5.3 Limitations	73
5.4 Conclusion	74
REFERENCES	76
APPENDICES	
1. Tasks	80
2. Transcripts	89
3. Coding tables	108
4. Consent forms	150

Chapter 1: Introduction

1.1 Background

With the introduction of Curriculum 2005 into South African schools came the promise of meaningful and equitable learning. Learning that is not based on memorization of twelve years of content but on an interactive process that will provide support to all learners in developing usable skills with which to pursue their aspirations. Curriculum 2005 infused me with excitement to teach in novel, innovative and meaningful ways. Curriculum 2005 was said to embrace the teacher as a professional who guides the learner in developing the essential skills needed for a particular Learning Area. Using an Outcomes Based approach Curriculum 2005 identified specific outcomes that needed to be achieved in order to be deemed competent in a particular Learning Area. The teacher was tasked to develop and administer learning programmes that will allow learners to construct knowledge and develop the necessary skills to achieve these outcomes.

Was this too much to ask of teachers? Enough people with power certainly thought so because in 2002 we saw the implementation of the Revised National Curriculum Statement (RNCS). The RNCS was aimed at narrowing down the scope for content implementation in the curriculum and giving teachers more direction in the form of suggested content. However justified, I believe that with the introduction of the RNCS we've taken one step back in embracing the teacher as a professional. Suggested content in the form of assessment standards runs the risk of becoming prescriptive content that ultimately will need to be learned for exam purposes. Since the outcome of developing a learner that can think critically has remained, the teacher's task has become much more daunting. How does the teacher cope with covering all the content from the old curriculum and a great deal more, while developing the skills in learners necessary to allow them to become the critical learners the RNCS envisage?

At present the government is in the process of again implementing curriculum change. The Curriculum and Assessment Policy Statement (2011) (CAPS), to be introduced at various levels in 2011, introduces more prescriptive content with strong tendencies to the

prescriptive syllabus of apartheid South Africa. Although CAPS still captures some reform notions and principles, much of what was innovative in the RNCS has been removed.

Faced with these challenges posed as a result of a changing curriculum I embarked on an inquiry into my own practice. I started this inquiry with the hope that it would illuminate and give direction to teaching to cover the content as suggested in the curriculum whilst teaching mathematical reasoning. Having embarked on this enquiry I had hoped to get some insights in relation to my current and changing practices to meet the demands of a new and changing curriculum.

1.2 Problem Statement

Since the inception of Curriculum 2005 I have tried to teach mathematics through the creation of a classroom discourse that allows learners to construct mathematics through mathematical reasoning. With the change in curriculum from Curriculum 2005 to the RNCS I found myself faced with the daunting task of having to develop the critical learner while making sure that the suggested content in the curriculum is covered. Being faced with this challenge, I wondered whether connecting and integrating content to a set of big ideas would help. I came across Lampert's (2001) notion of big ideas which refers to mathematical concepts that serve as conceptual links between seemingly different mathematics content domains. Can the teaching of big ideas in mathematics in the context of teaching mathematical reasoning enable me to cover the suggested content meaningfully? In trying to meet these challenges I embarked on an action research project that explored how the notion of teaching big ideas can facilitate teaching functions while teaching mathematical reasoning. In focusing my exploration, I initiated my research with the following question.

1.3 Critical Question

How can teaching the big ideas of functions to grade 9 learners support teaching mathematical reasoning while meeting the content demands of the curriculum?

In an attempt to effectively answer my critical question I have constructed four sub-questions to guide my research and analysis. These four questions will focus analysis on the mechanisms used during the lessons taught.

1. Does the set of activities provide opportunities for learners to engage with the big ideas of functions?
2. How are the big ideas related as students engage in the activities?
3. What mathematical reasoning takes place as students engage with the big ideas of functions in these activities?
4. What teacher and learner practices enable the mathematical reasoning described in (3)?

1.4 Rationale

The RNCS (2002) argues that an important outcome of mathematics is to develop in the learner: “a critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations” (p. 4). This kind of critical application of mathematical knowledge in diverse settings is echoed throughout the document. A critical awareness and application of mathematics as suggested above, suggests high levels of engagement with mathematical knowledge that requires far more than an emphasis on procedural fluency, as in the use of algorithms. Such critical engagement with the mathematics, I argue, is achievable in the context of teaching for the development of *mathematical proficiency* (Kilpatrick et al 2001). Such a critical mathematical orientation requires learners to display proficiency in adaptive reasoning (Kilpatrick et al 2001). Adaptive reasoning according to Kilpatrick et al refers to the capacity to think logically about the relationships among concepts and situations, to reflect, to give explanations and justifications. Adaptive reasoning allows learners to negotiate between meaningful interactions of mathematical ideas and constructs to present a logical whole.

Together with an emphasis on developing critical conceptual knowledge in learners the RNCS proposes content coverage of 5 Learning Outcomes averaging 6 assessment standards per learning outcome. That is about 30 assessment standards with elaborate

descriptions of their content. The content proposed represents the minimum suggested to meet these assessment standards.

The lack of conceptual depth with which mathematics has traditionally been taught can, in part, be argued as being the result of teachers rushing through content to complete a prescriptive syllabus. The need to cover the content is driven by a need to prepare learners for a high stakes national exam (Gipps 1999) at the end of their final year of school. The National Curriculum Statement (2003) (NCS) for the Further Education and Training (FET) phase as well as the new CAPS (2011) requires a high stakes certification exam to be written at the end of grade 12. This will arguably once again result in a content driven curriculum where content coverage is prioritized at the expense of meaningful learning.

A challenge we as teachers and researchers are faced with is in finding ways of addressing the tensions teachers experience between content coverage and meaningful mathematical activity. This challenge is not a recent and isolated South African challenge as the following quotation from Adler (2001) suggests:

In the 1980s and early 1990s, concern with quality, equity and social justice in education in general, and mathematics education in particular, was not confined to obviously unequal contexts like apartheid South Africa. Wider international movements in mathematics education named and explored race, class and gender inequities in mathematical performance. They sought reform of narrow procedural nature of mathematics offered in school and its related pedagogy. They also advocated more interactive and communicative classrooms and hence, meaningful learning. All learners were to be offered more open and authentic mathematical tasks that would engage them in conceptually rich mathematical ideas. Learners were to be given more of a knowledge voice in the classroom—a voice where diverse learner conceptions and ways of knowing were to be taken seriously, and brought into interaction with the set curriculum (p. 5).

The above quotation suggests that the move to more meaningful mathematical activity is a global phenomenon. Reform movements are calling for a shift towards a more interactive and communicative practice, which goes hand in hand with a shift away from an emphasis only on procedural mathematics, toward a comprehensive and conceptually rich meaningful mathematical activity. To avoid the mistakes of the past I believe we

need to emphasize and attend to aspects of the curriculum that speak to the heart of curriculum reform. Teaching towards big ideas has the potential to connect different topics in the curriculum into a meaningful whole which in turn can promote more *flexible ways of knowing and using* mathematics (Boaler 1997).

Functions is a topic in mathematics I thoroughly enjoy studying and teaching. Functions and graphs represent one of the earliest points in mathematics at which a learner uses one symbolic system to expand and understand another, for example algebraic functions and their graphs and data patterns and their graphs (Leinhardt, Zaslavsky, and Stein 1990). This is probably why learners grapple so much with functions as an abstract construct crossing over and organising other topics in mathematics. It is as if all other mathematics at school level was designed and included as tools to be used during the exploration of functions. It is primarily for this reason that I chose to focus on big ideas in functions as a topic to explore in my research.

In the next chapter I explore the paradigm from which I approach this research. While exploring my own paradigm I also endeavour to locate my understanding of how learning and teaching takes place in the literature.

Chapter 2: Theoretical Framework and Literature Review

2.1 Introduction

In an attempt to illuminate the constructs which framed this study I will now attend to broader discussions raised in the literature. I begin this chapter with a discussion of the learning theories which frame my perspective. A second discussion argues for teaching mathematical reasoning. A third discussion focuses on collaborative learning which illuminates my perspective on learning collaboratively in a classroom. A fourth discussion deals with mathematical practices. The fifth and final discussion in this chapter explores the notion of big ideas and identifies seven big ideas in functions.

2.2 Theories of learning

My perspective on how learners learn is largely influenced by constructivist, socio-cultural and situated theories of learning. As argued by Brodie (2010) these theories are often argued as being incommensurable but since they function on different levels, common ground can be found.

As a constructivist, Hatano (1996) argues that knowledge acquisition involves Construction and Restructuring. Construction and Restructuring suggest an active role for the learner to play within the context of learning. Restructuring however also suggests a reorganization of existing knowledge. This active construction and restructuring of knowledge by the learner, I argue, is through a process that can be described as reasoning. Hatano's assertion that knowledge acquisition is situated in context strengthens my conviction about teaching toward big ideas within *conceptual fields* (Lampert 2001). The learners are exposed to a particular concept situated within various contexts and so develop a deeper meaning and *flexibility* about this concept. This in my view can result in what was evident in Boaler's (1997) study where the learners, from Phoenix Park, developed more *flexible mathematical knowledge* compared to their counterparts from Amber Hill.

As a socio-cultural theorist, Vygotsky (1978) defines the Zone of Proximal Development (ZPD) as the difference between actual development and potential development as displayed under guidance by the teacher or a more capable peer. Vygotsky argues that learning is a process of internalization. An external, social activity (interpersonal) is reconstructed internally, inside the individual (intrapersonal). The conversion of interpersonal into intrapersonal activity is a long series of developmental events. Collaboration as a *long conversation* (Mercer 1995) creates a ZPD where learners' interaction with the mathematical concepts is stretched to its potential. The collaboration which takes place here demands from individuals to become accountable and to reach consensus through a process of mathematical argumentation.

Lave (1993) and Lave and Wenger (1991) proposes situated theories of learning that views learning as participation in communities of practice. Learning from this perspective can be seen as increased participation within the community of practice. Learning is seen as inseparable from the position the participant holds within the community. Learning is viewed as legitimate peripheral participation (Lave and Wenger 1991). Newcomers to the community of practice participate legitimately but on the periphery. Viewing learners as legitimate participants gives value to their contributions and a study of their practices promises to provide valuable insights into their development.

The theoretical perspective of situated learning has some commonalities with the socio-cultural perspective. Both provide ways of understanding how learners can learn from the teacher as well as from each other. Situated learning and socio-cultural perspectives relate learning to the social situations in which learning occurs. By focusing on the social interaction between participants, teacher and learner practices within the learning process can be identified.

I conclude this section with what I believe is a powerful assertion based upon the outcomes envisage by the NCS (2002) taken from Ernest (1993, p. 172):

In particular, the metaphor of conversation emphasises the dialogical basis of knowing, learning, and teaching. Since this metaphor also literally provides the basis of democracy, it is not surprising that social constructivism naturally lends support to an emancipatory vision for mathematics education, one which embodies egalitarian values of social justice, and aims at a mathematically literate and empowered critical citizenry.

Ernest's notion of social constructivism includes constructivist, socio-cultural and situated perspectives, which provide the lenses for my study.

2.3 Mathematical reasoning

Brodie (2010) argues that mathematical reasoning is presenting a line of argument that is reasoned. Ball and Bass (2003) capture this notion as they assert that making mathematics reasonable means *making it reasoned and therefore known in useful and usable ways*. Mathematical reasoning is therefore the process whereby a concept, idea or construct is reduced through successive steps from an initially foreign entity to an immediately collectively recognisable mathematical knowledge. Communication is therefore assumed. Validation is not by the authority of a text or person but by the argument presented in an explanatory manner. This explanation must incorporate agreed upon mathematical constructs. The argument is justified within the existing knowledge framework.

Brodie presents a broader view on mathematical reasoning than is traditionally argued as formalistic proofs of theorems. This broader view is in seeing mathematical reasoning inclusive of:

1. Creativity-the use of intuition that generates mathematical ideas.
2. Justification-the need to have one's construct accepted or validated by the community or even judged by the community.
3. Analytical-a more formalistic construct using mathematical rigour in making arguments.

Interactions which involve mathematical reasoning happen at different levels. Not all learners construct the same meaning from mathematical activities and discussions.

Mathematical reasoning is made up of a number of processes. The learner makes observations and then connects these with existing knowledge and in so doing restructures this knowledge (Hatano 1996). Proficiency in *procedural fluency* and *conceptual understanding* (Kilpatrick et al 2001) is needed for such restructuring. The key to enabling restructuring is to explain, communicate and justify assertions made, which are features of *adaptive reasoning* as argued by Kilpatrick et al. Adaptive reasoning allows learners to navigate their way through a series of facts, procedures, concepts, solution paths in fitting them together in so that they make sense. During this communicative process we see the learner evaluating the new knowledge and refining it to a point where the validity of other assertions can be evaluated.

Kilpatrick et al (2001) proposes five strands for the realisation of mathematical proficiency. These five strands are cognitive competencies that must be developed in the learner to be deemed mathematically proficient. The five strands for mathematical proficiency are: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition.

1. *Conceptual understanding* refers to an integrated and functional grasp of mathematical ideas.
2. *Procedural fluency* refers to knowledge of procedures, knowledge of when and how to use them appropriately, skilfully and performing them flexibly, accurately and efficiently.
3. *Strategic competence* refers to the ability to formulate mathematical problems, represent them, and solve them.
4. *Adaptive reasoning* refers to the capacity to think logically about the relationships among concepts and situations, reflection, explanation and justification.
5. *Productive disposition* refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics.

These strands as argued by Kilpatrick et al (2001) are interrelated and should not be viewed in contention with each other but rather complementary to each other. Each strand complements the others, for example learners with better procedural proficiency can focus more readily on the underlying concepts at hand and will be able to reason better. Learners with a lack of procedural fluency will be bogged down by algorithms, leading to these algorithmic procedures becoming the focus of the activity instead of the mathematical concepts and reasoning. In teaching towards mathematical proficiency the teacher should work with these in a balanced way to obtain mathematical proficiency. Teaching all five strands is therefore a holistic approach to teaching for mathematical proficiency. Adaptive reasoning according to Kilpatrick et al, is the glue that holds all the strands together. It is through mathematical reasoning that learners learn to use mathematical *ideas flexibly in diverse cases* (Ball and Bass 2003).

I view the teaching and learning of mathematical reasoning as the vehicle to sense-making of mathematical activity. Here I refer to the sense-making of the mathematics itself as well as making links with everyday life. It is my conviction that only through sense-making of the mathematics can we truly move to mathematics as a worthwhile everyday life activity. Although acknowledging that these five strands work together in creating a competent mathematics learner and user of mathematics, the main thrust of my research has been a focus teaching towards the developing the competencies of conceptual understanding, adaptive reasoning and procedural fluency.

A focus on the three strands mentioned above emanates from the emphasis of developing a critical learner envisioned by the National Curriculum Statement (2003): The NCS expresses the vision of a learner who is able to:

- *demonstrate an ability to think logically and analytically as well as holistically and laterally and*
- *be able to transfer skills from familiar to unfamiliar situations. (p. 5)*

The above vision suggests a thorough understanding of the fundamental concepts using knowledge entrenched in mathematics or *conceptual understanding and procedural fluency*. Secondly the above vision also proposes the development of a capacity by

learners to readily identify situations where their knowledge is of relevance. Adaptive reasoning allows learners to negotiate between meaningful interactions of mathematical ideas and constructs to present a logical whole. It is the glue that holds all other strands together. I argue here that it is adaptive reasoning that allows learners to display a more flexible orientation to knowing and using mathematics.

A flexible conceptual knowledge in relation to context resonates with the research done by Boaler (1997). Learners from two schools, Amber Hill and Phoenix Park, developed different forms of mathematical knowledge as a result of being exposed to different teaching practices. On the one hand was Amber Hill, with a typical textbook approach with lessons consisting of rule-based, procedural activities with much drill and practice. On the other hand was Phoenix Park that adopted an open-ended, problem-solving, real life setting approach to teaching mathematics. Boaler's research concluded that learners gained vastly different experiences of mathematics and developed equally different forms of mathematical knowledge. The majority of learners from Amber Hill were unable to apply their knowledge to new problems and situations. This suggested that they developed knowledge consisting primarily of memory and applying rules that could only be applied within a school setting. Learners at Phoenix Park however developed the kind of knowledge that enabled them to solve new problems they encountered. Learners were therefore able to develop more flexible forms of knowledge. The type of mathematical knowledge developed at Phoenix Park is the type of knowledge the RNCS is calling on our learners to develop.

Boaler's research suggests that the high levels of mathematical activity promoted in the RNCS are not obtainable within a learning environment that holds the more traditional view of an emphasis on procedural fluency only when learning mathematical algorithms. A more *learner-centred* (Brodie et al 2002) approach where learners engage in mathematical reasoning like in Boaler's Phoenix Park School, seems to suggest the ideal. As if in response to Boaler's (1997) findings the Rand Mathematics Study Panel (2002) calls for a study into what people do who learn and use mathematics successfully. In short they call for further study into the practices of mathematics and mathematicians.

They also argue that mathematical reasoning will promote mathematical proficiency and equity. The paper suggests that research into mathematics practices starts with the domains of representational practices, justification and generalization.

I elaborate on representational practices later in this chapter, during the discussion on big ideas. *Justification and generalisation* are processes within a holistic approach to mathematical reasoning (Brodie 2010; Davis 1997). The key to justification and generalisation is the process of communicating mathematical ideas.

It is with the aim of teaching for mathematical reasoning that I strive towards creating a discursive culture within my classroom. In the next section I turn to collaborative learning as creating a discursive culture within a *community of practice* (Lave and Wenger 1991).

2.4 Collaborative learning

Collaborative learning is a communicative process whereby two or more parties gain new knowledge as a result of their interaction. In Staples' (2007) words:

The term collaborative then implies a joint production of ideas, where students offer their thoughts, attend and respond to each other's ideas, and generate shared meaning or understanding through their joint efforts. Collaboration is distinct from cooperation which only implies sharing (p. 4).

Collaborative learning not only refers to an exchange of knowledge between the parties, but the interaction itself serves as a catalyst for the formation of new knowledge by the parties concerned. In my class I think of collaborative learning as a joint venture between learner/s and teacher and among learners themselves. This collaboration is governed by the pursuit of knowledge for the development of learner and teacher. How we reason mathematically or allow our learners to reason mathematically is in part dependent on the nature of collaboration between the parties. The nature of the learning which occurs is a *complex interplay between individual and social construction* (Hatano 1996; Wood et al 1992).

In the classroom the teacher tries to establish an *educational discourse* (Mercer 1995) that he or she would like to see learners emulate in mathematical discussions within the classroom and beyond. The classroom situation consists of learners from different backgrounds with different knowledge and potential knowledge frames. These differences have the potential to facilitate learning in addition to teacher instruction. Direct teacher instruction addresses learner needs from the perspective of the teacher's assessment of learner knowledge. Collaborative learning allows for a greater variety of issues to be addressed simultaneously within the learning environment. Learners are able to raise a variety of content and contexts of learning which relate to their understanding. Alternative concepts are exposed and the teacher or more knowledgeable peer is afforded the opportunity to scaffold new learning.

My concern was always that stronger learners do not benefit from collaborative learning while the weaker learners are moved to reach their potential through the creation of a ZPD (Vygotsky 1978) by their interaction with a more knowledgeable peer. Schifter (2001) provided me with insight into the possibility for stronger learners who engage with weaker learners during mathematical argumentation to develop deeper conceptual understandings. Deeper understandings for the stronger learners unfold as stronger learners have to explain and justify their understandings.

The NCS (2003) places emphasis on collaborative learning firstly as an end in itself and secondly as the means to acquire mathematical knowledge as seen in the following extracts:

- *work effectively with others as members of a team, group, organization and community (p. 2)*
- *Mathematics enables learners to work collaboratively in teams and groups to enhance mathematical understanding (p. 10)*

The first point captures collaborative learning as an end in itself. Learners develop a social conscience based on democratic rule, social justice and human rights through collaborative learning. It would be difficult if not impossible to teach learners to value other people and their opinions, without learners actually learning together from each

other. It is the relevance to the learning of mathematics contained in the second point that tends to be more challenging.

Embedded in the second point is the notion of collaborative learning used as a tool or *means* (Brodie and Pournara 2005) to enhance mathematical learning as opposed to simply being a skill or attitude to be developed within the learner as part of a model citizen with an acute social conscience.

In an attempt to meet the challenge in using collaborative learning as a vehicle for enhancing mathematical understanding, I now turn to the creation of mathematical norms for argumentation. I argue below that it is through the creation of mathematical norms within a discursive and collaborative teaching practice that address the development of the mathematics within a collaborative setting.

A classroom discourse based on mathematical norms for argumentation and mathematical reasoning suggests that learners are able to express their opinion about mathematics in the public domain so that mathematical discussion around their assertions can generate a kind of *intellectual ferment* (Chazan and Ball 1999). With intellectual ferment I understand Chazan and Ball to mean that ideas and arguments presented that feed the thrust and momentum of a progressive mathematical discussion. Learners need to be moved away from a dependence on the teacher as the only mathematical authority in the class to a position where there is what Davis (1997, p. 369) refers to as a *community-established standard: a collective authority*. This requires changes in the way learners perceive their role in class, and also the way in which teachers view their role.

Kazemi and Stipek (2001) argue that learners must move beyond seeing mathematical activity purely in terms of social norms but as *sociomathematical* norms. Social norms refer to the social interactions generally expected in any classroom. Norms such as raise your hand when you want a speaking turn or respect all learners' opinion can therefore be regarded as social norms. Sociomathematical norms are those social norms which are specifically directed at the effective learning of mathematics. Examples of

sociomathematical norms are that statements need to be justified and everyone needs to listen and analyse the arguments made by others. Kazemi and Stipek argue that *high press* moves from teachers enable learners to achieve a deep engagement with the mathematical concepts. By high press moves I understand Kazemi and Stipek referring to teacher's probing for further explanation or justification from learners with regards to statements made. Within this context an explanation consists of a mathematical argument, not simply a procedural description or summary. Mathematical thinking involves understanding relationships among multiple strategies. Errors can provide opportunities to think differently about a problem, explore contradictions in solutions, or pursue alternative strategies.

Collaborative learning calls on teachers to become active participants in the process of seeking understanding from learners' meanings. Teachers are called upon to become what Davis (p. 369) terms *hermeneutic listeners*. This suggests the type of listening that is not evaluative (listening for a preformatted understanding) nor is it interpretive (still listening to learners for that right answer but with a little more respect for learners' own ideas) but a genuine listening (listening with) where the teacher listens as a co-participant in a conversation with learners (Brodie 2010).

Schifter (2001) also expresses the notion of truly listening to the learners in attending to the type of inquiring mathematics suggested by the teaching of mathematical reasoning:

However, unless teachers come to see mathematics as active discovery and assume ownership of their powers of mathematical thought, their own lessons will not yield them comparable insights. They must learn to learn on their feet. (p. 115)

In the above extract Schifter argues for teachers to shift their perspective on mathematics to that of a creative human activity. With this she calls on teachers to become active participants in collaboration with learners for the creation of mathematics in the classroom. Teachers should become flexible doers and users of mathematics in order to mediate between mathematical ideas in conversation. Are teachers not to instruct or teach learners anymore? What happens if learners through class discussion are not able to make progress in a particular topic due to a lack of *intellectual disagreement*, or a presence of

incorrect agreement (Chazan and Ball 1999)? The next section tries to answer these questions raised with regards to teacher instruction through a set of teacher practices, which gives validity to learner contributions and raise the authority of mathematical argument.

2.5 Teacher practices

Chazan and Ball (1999) suggest practical ways in which the teacher can act without giving the answer or direct instruction. These practices, they argue, may affect the intellectual ferment in the discussion or focus and give direction to a particular discussion.

- Attaching conventional terminology to ideas and distinctions learners are already making.
- Appropriating learners' comments by rephrasing them and helping the class to hear them.
- Asking for clarity when they think learners' assertions are not clear.
- Acting in a way to focus learners' attention on a particular aspect of a discussion through instructing learners to perform a certain task such as coming to the board and explaining.

The decision to act and how to act will depend on the context the teacher finds himself/herself with regards to the learning process. The context can be assessed on the following considerations from Chazan and Ball (1999):

- The mathematical value in relation to students' current and future learning. The importance of long term implications for the learner's learning.
- The direction and momentum of the discussion needs to be evaluated continuously to ensure optimal mathematical learning.
- The social and emotional tone of the discussion needs to be continuously monitored to avoid any intellectual debate depleting into a personal grudge match. But also to prevent the not so verbally skilled learner from disappearing into the background.

Aside from orchestrating this complex interplay of the social, the teacher is also charged with imparting mathematical content meaningfully to learners. In seeking to understand ways of doing this I now turn to the notion of big ideas in teaching mathematics. In the next section I argue that the teaching of big ideas holds the potential to allow learners to network mathematical concepts and contexts. In this networking of knowledge lies the creation of the type of flexible knowledge competence within the learner.

2.6 Big ideas

2.6.1 What are big ideas?

With the aid of related literature I will now formulate an argument for what constitutes big ideas in mathematics. I also elaborate on specific features of big ideas that I argue holds the potential for supporting the teaching of mathematical reasoning and functions.

Woodbury (2000) argues that big ideas are those concepts which stretch across different content topics, allowing teachers a holistic approach to teaching seemingly different content topics while emphasizing the concepts which bring these content topics together. Woodbury argues that without the teaching of these big ideas, conceptual understanding is lost and teacher and learners revert to procedures to be memorized. She argues that conceptual understanding creates a knowledge network which gives meaning to mathematical activity. Woodbury reinforces my conviction that the teaching of big ideas can facilitate the teaching for mathematical reasoning. Big ideas are the joints or intersection between mathematical ideas and therefore the key during both conceptual understanding and adaptive reasoning.

Lampert (2001) takes the teaching of big ideas a step further as she elaborates on the teaching of mathematics in Vergnaud's notion of conceptual fields. Conceptual fields, she argues, comprises a set of situations that are needed to master a deeper understanding of concepts. It is in learning a big idea and using different mathematical contexts and problems where the learner is able to expand on what one could call a limited or restricted understanding of a particular concept. What Lampert also points out is that by

teaching within conceptual fields a number of content topics can be covered which would traditionally be taught in isolation. It is in connecting these seemingly different topics under a single big idea where the learner develops a deeper conceptual understanding. I argue here that teaching in conceptual fields holds the potential for developing flexible knowledge schemes. Being exposed to a mathematical concept across situations expands the learners' understanding and usage of this concept and a more comprehensive concept with all its subtle nuances is developed.

Chazan (2000) argues that by *psychologising* the subject matter the teacher is able to mediate sense making and the meaning of algebra to learners. Chazan uses the term *psychologize* to denote the connection the teacher facilitates between the subject matter and the learners' experiences of the subject matter. In doing this Chazan argues that teachers need to know and understand their learners as well as have a thorough conceptual understanding of the subject matter.

Chazan argues that *psychologizing* the subject matter necessitates that the teacher identifies what he terms the objects of study. Chazan, in identifying the object of study while studying algebra chooses functions. He views this as an alternative to studying algebra as constructions of static unknowns. In teaching functions as the object of study Chazan argues that it allows learners to make sense of algebra. Chazan reports that although the object of study might not initially be accessible to learners it must be taught explicitly as learners come to grips with the mathematics at hand. Chazan calls on teachers to mediate between the object of study and learner experiences through making the object of study explicit, having a thorough conceptual understanding of the object under study as well as a thorough knowledge of the learners.

Woodbury (2000), Chazan (2000) and Lampert (2001) illuminates three important aspects of teaching toward mathematical reasoning. Firstly, there is the potential for teaching toward big ideas to develop conceptual understanding through mathematical reasoning in terms of restructuring learners' knowledge frames. Secondly, teachers and learners must be clear on what these ideas are in order for these big ideas to enhance

learner's experiences. Thirdly, teaching within conceptual fields deepens learners' understanding and flexibility in using these concepts across mathematical contexts.

Chazan (2000) uses functions as a big idea in teaching algebra meaningfully. Unlike Chazan, I have chosen to teach functions through a set of big ideas that I will identify and expand on in the next section. I have chosen functions as an object of study firstly because of emphasis placed on functions in the curriculum. Secondly, functions is a topic where many other content topics such as equations and algebraic manipulation feed into. In my experience it is a topic that learners struggle to make sense of. In this research I embarked on searching for big ideas in functions that would allow me to teach functions conceptually and present functions meaningfully to my learners.

2.6.2 Big ideas in functions

In starting this research I hypothesised that if I could teach big ideas I could assist learners in deriving more meaning and sense-making from my classes. Teaching through big ideas meant that I would be aiding learners in acquiring the fundamental or cornerstone ideas which hold the key to better conceptual understanding.

I caught a glimpse of covariance as a first big idea. Silverman (2005) reports on covariance as a key *developmental understanding* in understanding the notion of functions. Key developmental understandings are those concepts on which further development of a variety of other concepts are built. These concepts also have the feature of connecting a range of concepts within the system of mathematics. Silverman describes covariational reasoning as follows:

Covariational reasoning involves the coordination of two variables, each of which can be envisioned as varying independently. Ultimately, this way of thinking allows students to (a) envision a graph as a collection of points; (b) envision the collection of points as being generated by keeping track, simultaneously, of two quantities whose values vary; and (c) envisioning that every point in a graph represents, at once, simultaneous values of two quantities. (p. 21)

Silverman argues that covariance is initially important in shifting understanding from the notion of functions as rules or formulae to functional relationships. This initial understanding of functions allows for a deeper conceptual understanding of calculus and the set theoretic concept of functions.

Variable is the second big idea I have identified. This big idea is implicit in Silverman's notion of covariational reasoning. The notion of covariance relies on the conception of variable representing a set of values. Variable is a letter that is simultaneously representative of a single yet generalized set of values. The distinction between whether a single or set is referred to, would depend on the context it is used in.

The third big idea relating to the notion of functions is that of slope. Stump (2001), in trying to improve on pre-service teachers' pedagogical content knowledge about slope, reports on learners' difficulties in understanding the notions of slope. She also reports on the two fundamental situations in which the concept of slope appears in mathematics. The first situation of slope is found within the physical world, where slope describes the measure of steepness, as in the slope of wheel chair ramps or slope of a road. The second is where slope refers to the measure of rate of change such as the rate at which the water level rises in a dam during seasonal rainfall. Stump argues that it is when the learner is able to make the connections among geometric, algebraic, and real-world representations of slope that deeper conceptual understanding of slope is achieved.

The forth notion of representation as a big idea is prevalent in the literature (Even 1998; Rand Mathematics Study Panel 2002). Representation is important for learners to enrich their conceptual understanding of the mathematical constructs through multiple representations. The ability to represent what is thought about mathematically in itself is a challenge. Moving learners to foster conceptual links between different representations of a mathematical construct is at the heart of reasoning and functions. Even (1998) captures this notion of the importance of multiple representations eloquently:

The ability to identify and represent the same thing in different representations, and flexibility in moving from one representation to another, allow one to see rich relationships, develop a better conceptual understanding, broaden and deepen one's understanding, and strengthen one's ability

to solve problems. Connectedness between different representations develops insights into understandings of the essence as well as the many facets of a concept. (p. 105)

Within the realm of a discursive classroom practice, how one represents what is being thought of is an essential tool and skill that must be developed. Representational skill is therefore crucial in fostering a common understanding about what is being communicated. As an abstract concept the notion of functions cannot stand independently of its representations. It is in understanding these representations where the potential for true understanding of the nature of functions lie.

The fifth notion is that of pattern. I argue that pattern or specifically number patterns is fundamental in understanding functions. It is from the regularity or patterned occurrence of an event where a covariational dependence between input and output is established and the notion of function is conceptualised. A learner's skill in recognizing patterns within mathematical activity holds the key to unlocking generalized mathematical constructs. The Rand Mathematics Panel (2002) calls for research to focus on generalization as they argue that research should focus on *what people do* to learn and use mathematics successfully. The Rand Mathematics Panel argues that mathematics provide tools for abstracting from particulars to see and understand patterns. It is in understanding these patterns that symbolic notations can be used to effectively capture the essence of mathematical ideas at play. The mathematical tools in functions are all symbolic representational constructs derived from generalised patterned occurrences.

These big ideas are also presented in the focus for Learning Outcome 2 in the RNCS (2002) below:

LO 2: PATTERNS, FUNCTIONS AND ALGEBRA

The learner will be able to recognise, describe and represent patterns and relationships, as well as to solve problems using algebraic language and skills.

We know this when the learner:

- 2.1 Investigates, in different ways, a variety of numeric and geometric patterns and relationships by representing and generalising them, and by explaining and justifying the rules that generate them (including patterns found in natural and cultural forms and patterns of the learner's own creation).*

- 2.2 *Represents and uses relationships between variables in order to determine input and/or output values in a variety of ways using: a) verbal descriptions; b) flow diagrams; c) tables; d) formulae and equations.*
- 2.5 *Determines, analyses and interprets the equivalence of different descriptions of the same relationship or rule presented: a) verbally; b) in flow diagrams; c) in tables; d) by equations or expressions; e) by graphs on the Cartesian plane; in order to select the most useful representation for a given situation. (pp. 82-84)*

Assessment standard 2.1 captures the big idea identified above as pattern and the value it holds for abstracting generalizations from mathematical activity. Assessment 2.2 highlights the big ideas of variable, covariance and representation. Assessment standard 2.5 emphasizes the importance of working across representations of a particular function. The fifth big idea of slope identified above is not explicit in the assessment standards on functions however I see it implicitly in assessment standard 2.5 e). I argue here that graphical representation implies a thorough understanding of slope as set out in the assessment standard. Understanding slope as in rate of change is fundamental in selecting an appropriate graphical representation. A curve for example will indicate a continuously changing rate of change whereas a straight line suggests a constant rate of change. It could also be argued that in understanding the equivalence between multiple representations, slope serves as a focal point that connect these various representations. Here I would like to refer to the gradient as a measure of slope of a straight line which corresponds to the coefficient of the independent variable in the equation as an example.

Multiplicative relationships as a sixth big idea in functions I borrowed from Lampert (2001). I argue that understanding of slope and rate of change as ratio is fundamental in seeing ratio in the broader context of multiplicative relationships.

A seventh big idea emanates from the notion of patterns as a study to explore functions. Linear patterns in number patterns, which focus on the regularity of a constant difference, can be argued as impossible to be fully understood without understanding additive relationships. Also the regularity by which the axes of the Cartesian plane are numbered suggests a thorough understanding of additive relationships at a grade 9 level.

At this stage it is important to point out that the seven big ideas are firstly fundamental in developing a conceptual understanding of functions by learners. Secondly these ideas have allowed me to construct mathematical activities that bring together mathematical content and contexts that allow for rich exploration of functions. In the next chapter in attending to issues relating to research design and methodology, I discuss the development and construction of activities geared towards teaching of big ideas in functions while teaching mathematical reasoning.

Chapter 3: Research design and Methodology

3.1 Methodology

There is a kind of credibility afforded in doing research within a familiar methodological frame. I frame my research within the area of qualitative research as described by Miles and Huberman (1994). They argue that good qualitative data allows for the inquiry to be reported with rich descriptions and explanations situated in context. Data and findings can be chronologically presented and this in turn allows the researcher to assign certain consequences as a result of particular events. This kind of explanatory scenario allows for the construction of believable arguments. The power of a descriptive analysis lays within vivid, meaningful and convincing accounts which appeals to the reader.

Defining one's research within known parameters raises the challenge of accommodating for the peculiarities and uniqueness of one's own research setting. This challenge arises as known research methodologies come with a certain amount of accepted methods and understandings. Miles and Huberman (1994) give me confidence that this is not an unusual occurrence:

No study conforms exactly to a standard methodology; each one calls for the researcher to bend the methodology to the peculiarities of the setting (p. 5).

The above assertion gives me the confidence to tap into the strengths which frame known methodologies but at the same time allow for a level of flexibility to accommodate for the peculiarities of my research.

My study is a qualitative action research project (Kemmis and McTaggart 1988).

According to Kemmis and McTaggart action research is best described as a spiral of cyclic events. An intervention is initiated through the development of a plan of critically informed action to improve what is already happening within a particular practice.

Planning is then followed by careful implementation. Observation and reflection is an on-going feature of action research and critical in determining the effects of the informed action in respect to the context in which it occurs. This reflection is the basis on which further planning and subsequent critically informed action is taken, through a succession

of cycles. This project is the first of such cycles. Aside from the outcome of analysis which will determine some of the features of the following cycle, I plan to include a broader spectrum of topics and context to be explored. With the current scope of this project it is not possible to work within conceptual fields (Lampert 2001). It is therefore my intention to work with conceptual fields in the next cycle of action research.

The importance of action research is that it is done by the teacher in his/her own classroom in order to improve the learning environment. This type of research aims to facilitate improvement taking into account the complex nature of doing research while engaging in the day to day teaching environment. In action research the learning process is embedded (Kemmis and McTaggart 1988) within a social context.

I have embarked on the first cycle of research with the aim to create a meaningful interactive learning environment in my own class. In an attempt to create a meaningful and interactive learning environment while at the same time meeting the demands of an overloaded curriculum, I needed to implement changes to my own practice. In seeking to implement changes in my practice I constructed my research questions as outlined in Chapter 1 in order to try and implement sound ideas for teaching mathematical reasoning meaningfully. Action research is therefore an appropriate way to frame my research.

3.2 Research Methods

I will attend to a discussion on the methods I used to address issues of rigour along the following headings: research context, task design and implementation, data collection, rigour, ethical considerations and analysis.

3.2.1 Research context

The research was conducted with one of my grade 9 classes with me teaching. I teach at a school of 1600 learners that is now integrated in terms of historically racial divisions. A teaching staff of 63 puts the teacher : learner ratio at an average of about 1 : 23. Actual class sizes average at 33 learners per class in order to allow for time to perform

administrative duties by teachers. There were 35 learners in the class with which I had conducted the research. Although the school is situated in a middle class suburb, a large number of the learner population travel to school from lower income areas. English is the language of instruction but not the first language of the majority of the learner population.

A degree of differentiation or streaming is done in the school. Learners in junior classes are generally placed in academically stronger or weaker classes in order to facilitate remedial work needed at this level. Even so, classes generally differ with respect to the degree of mathematical proficiency of learners. My classroom is relatively well resourced with desks and chairs for every learner. The building structure in general is well maintained. Aside from the writing board I also have an overhead projector and screen at my disposal.

3.2.2 Task design and implementation

I adapted and constructed a series of activities that served as a catalyst for mathematical activity focusing on the big ideas in the teaching of functions. The use of tasks as a catalyst stems from the value tasks hold for allowing learners to engage with mathematical concepts using high levels of mathematical reasoning and sense making (Stein, Grover, and Henningsen 1996). Stein et al distinguish three phases of task development responsible for student learning:

1. Task as represented in the curriculum or instructional materials
2. Task set up by teacher in the classroom
3. Task as implemented by students in classroom

Stein et al suggest that it is during the two transitional phases from phase 1 to phase 2 and phase 2 to phase 3 where the cognitive demands of the task can be altered. They argue that it is here where the cognitive demands of the task found in the instructional material are often reduced. The challenge of how to discern between the amount of scaffolding learners require during these transitional phases lies with the teacher. Although at some level keeping the cognitive demands high serves as a catalyst for rich mathematical engagement by learners. It could also discourage learners from participating, if mathematical activity is pitched beyond their reach.

I have constructed and adapted a series of tasks that aims to develop the big ideas identified in teaching functions in the Revised National Curriculum Statement. What follows is a discussion on the tasks I used to engage learners with the seven big ideas as discussed in Chapter 2.

Activity 15.1.1 (see Appendix A) consists of 5 short tasks I adapted from a number of texts for the purpose of this research. Each task requires learners to draw a graph of a container being filled with water. Each task depicts a container of a different shape and size. Learners are required to draw graphs of the water level against the time passed.

I've chosen these tasks as my introductory lesson into functions as they deal with four fundamental big ideas in functions: representation; co-variance; variable; and slope as rate of change. These four concepts the learners need to intuitively or explicitly understand and apply in order to successfully complete the tasks. Learners need to understand that as time increases the water level will rise. Deciding whether to use dots or solid lines and straight lines or curves should challenge learners' ability to distinguish between continuous and discrete data as well as the nature of the rate of change depicted in each situation.

The water level tasks appealed to me because it negates the complexities of scales for axes, calculations and plotting points. Appealing more to learners' understanding of the situation of the problem I chose these tasks as an introductory task to explore ground rules for graphical representations of covariance and rate of change. I also chose these tasks because I believe that they encourage a holistic approach to analysing and reading graphs.

Activity 15.1.2 (see Appendix B) I selected as an activity to further practise concepts developed in *activity 15.1.1* as well as to assess how learners' take up of the concepts explored relating specifically to graphical representation of covariance and rate of change. By including vertical and horizontal lines in this task I had hoped to challenge learners' intuitive and deductive reasoning about graphically representing rate of change

and co-variance. Learners did not get an opportunity to explore vertical and horizontal lines in *activity 15.1.1*.

Having to construct a story which justifies the different slopes on the graph through correspondence of events is challenging in that it is a creative task which requires thorough understanding of graphically representing covariance and rate of change.

I have amended *activity 15.2.1* (see Appendix C) *and 15.2.2* (see Appendix D) from the grade 9 text book “Mathematics for All”. I’ve always enjoyed this series as it strives to incorporate the principles imbued in the Revised National Curriculum Statement. This text book promotes conceptual understanding through teaching mathematical reasoning. These tasks situate functions in a real life context, and they encourage a non-algorithmic approach to functions.

Activity 15.2.1 focuses on the linear relationship between altitude and temperature. The table and the graph are given as representative of the real life situation described. Learners have an opportunity to explore the value each representation holds through their attempts to answer the questions set. Although some questions prompt learners to answer the questions based on their understanding of the graph both representations can be used to solve the problems posed. Two questions also prompt learners to extrapolate from the graph. Of the five questions set, four questions are either open ended or have multiple solution paths. These questions therefore warrant justification for answers given and promote the learning of mathematical reasoning.

Activity 15.2.2 focuses on a parabolic function between the cost for tickets and the profit made from selling them. All five questions explore various features of a parabola based on a plausible context for justification. Features such as symmetry, turning point and intersections with axes of the parabola can be explored through the learners’ intuitive understanding of the context. Both global and localised interpretations are needed to fully answer the questions. As in *activity 15.2.1* the task demands that learners discern the mathematics from the context, yet they require the context to justify answers given.

Although the situation depicted in *activity 15.2.1* is not necessary a common everyday context for some of the learners, I believed that most learners should be able to identify with the functional relationship between altitude and temperature. The same can be argued about the functional relationship between cost of tickets and profit made as depicted in *activity 15.2.2*. Both these activities are good examples of modelling with functions. These tasks require learners to be able to discern the mathematics from the context and demand of learners to justify the mathematics using the context.

Activity 15.3.1 to 15.3.3 (see Appendix E, F and G respectively) I developed myself and *activity 15.3.4* (see Appendix H) I adapted from the grade 9 “Classroom Mathematics” text book. The aim behind these tasks was to use measurement as a vehicle within which to teach the three commonly used (common in most texts) representations of functions: tables, graphs and equations.

These four activities were designed to focus on the simplest form of each of five different types of functions (linear, quadratic, cubic, rational and exponential). Because of the nature of measurement these functions were limited to positive real numbers. These tasks also places emphasis on exploring the different types of representations (tables, graphs and equations) of functions.

I envisaged that successful completion of these tasks would raise issues such as scaling, discrete and continuous functions; increasing and decreasing functions and the way we represent these. In relation to co-variance and rate of change and the way we represent these, I expected learners to draw on what they have learnt in the previous activities to successfully complete these tasks. These issues I envisaged would generate the bases for mathematical discussion which in turn provide the fertile ground needed for mathematical reasoning.

I also developed these tasks as an introduction to the various functions dealt with in grade 10 to grade 12. I have tried to keep the contexts of the tasks as simple as possible as all these functions are rarely dealt with at grade 9 level. The Revised National Curriculum

Statement does not restrict the introduction of these functions at this level but traditionally these functions are not taught at grade 9 level. Many new grade 9 text books explore functions beyond the traditional linear function. My motivation for including a wider range of functions to explore is centred on the development of a more holistic concept image of what functions are and to counter the linearity phenomena prevalent in literature (Leinhardt, Zaslavsky, and Stein 1990) where learners tend to treat all functions as linear. An example of this can be seen from the tendency of learners to join all plotted points with straight lines.

In *activity 15.3.1* the length and breadth of a square is used to explore the linear function. The task poses the length to be a function of the breadth. As the breadth is increased so should the length to maintain the construction defined as a square. The function can be defined as $y = x$ where x represents the breadth and y the length.

Activity 15.3.1 also explores the area of the square as a function of its dimensions (length and breadth). This task was used to explore the quadratic function. The function can be defined as $y = x^2$ where x represents the increased dimension and y the area.

Activity 15.3.2 explores the volume of a cube as a function of its dimension. This task was used to introduce and explore the cubic function. The function can be defined by $y = x^3$ where x represents the increased dimension and y the volume of the cube.

Activity 15.3.3 explores the length of a rectangle as a function of its breadth as the area remains fixed. The function can be defined by the equation $y = \frac{36}{x}$ or $yx = 36$ where x represents the breadth, y the length and 36 the constant area.

In *activity 15.3.4* the number of folds and the number of parts a page can be folded into is used to explore the exponential function. The task poses the number of parts as a function

of the number of folds. The function can be defined by $y = 2^x$ where x represents the number of folds and y represents the number of parts.

Activity 15.4.1 (see Appendix I) was taken from the grade 9 “Maths for all” text book. This task challenges learners to match the three common representations dealt with in the previous activities (15.3.1 to 15.3.4). The graphs represented here, unlike the ones in *activity 15.3.1 to 15.3.4*, stretch over all four quadrants of the Cartesian plane. This I saw as a necessary development as the context of measurement limited the concept image of these graphs to the first quadrant only.

In matching the table with equations it was assumed that learners might use substitution therefore calling on their understanding of equations and order of operations. Matching the graphs with the equations I assumed learners will draw on the shape which matches the equations. For example the equations: $y = x^2$, $y = x^3$ and $y = 2^x$ have similar curves if restricted to quadrant 1. Distinguishing between these therefore meant that learners would have to make use of substituting points from the graph into the equation to finally determine the matching equation and graph or match the co-ordinates in the table with those on the graph. The graph for the function $y = \frac{k}{x}$ is a curve different from the others and should be easier to distinguish from the other curves. Matching the table would therefore be a matter of finding co-ordinates from the table which match points on the curve or substituting these into the equation to see if the equation holds true.

The linear function is quite distinct from the curves and I therefore expected learners to have little trouble in matching the straight line with its corresponding equation. Matching the table however would demand knowledge of substitution and equations or matching co-ordinates in the table with points on the graph.

I envisaged that dealing with graphs in all four quadrants could pose a challenge to learners as previous graph sketching had been restricted to the first quadrant only. Would learners recognise the similarities between the equations in this activity and those dealt

with in the activities on measurement? Although some equations were identical others had their parameters changed. Would learners recognise the $y = x$ in $y = -3x$ and be able to associate this with the straight line? Would they recognise the $y = x^2$ in $y = 3x^2 - 3$ and associate it with the parabola? Developing the ability to recognising the correspondence between the graphs and particular features in the equations was a central aim of this activity.

Subsequent to the construction of the tasks they were implemented in a series of four introductory lessons on functions. The table below summarises the order and the manner in which these tasks were implemented across the four introductory lessons.

DAY	ACTIVITY	LESSON	SOCIAL LEARNING CONTEXT					
			INDIVIDUAL WORK	HOMEWORK	GROUP WORK	SPLINTER GROUP	OPEN CLASS DISCUSSION	TECHER LED DISCUSSION
1	15.1.1	1	✓		✓		✓	
2	15.1.1	1			✓		✓	
2	15.2.1	2	✓	✓		✓	✓	
3	15.1.2	1	✓	✓			✓	✓
3	15.2.1	2					✓	
3	15.2.2	2			✓	✓	✓	
4	15.3.2	3			✓		✓	
4	15.3.1	3			✓		✓	
4	15.3.3	3			✓		✓	
5	15.3.3	3			✓		✓	
5	15.3.4	3			✓		✓	
5	15.4.1	4		✓				✓

Table 1: Implementation of tasks

During the lessons I requested learners to engage with the tasks in a number of ways. Individual work was generally used as a time for learners to acquaint themselves with the tasks at hand and to generate some ideas in preparation for discussion in groups that ranged from three to six learners. Group work usually followed an individual session. In some lessons I immediately started engagement with the tasks in small groups. Splinter group describes the use of smaller groups (two or three learners) for initial engagement and then sharing ideas within larger groups (five to six learners). Teacher led discussions were occasions where the teacher dealt with tasks by initiating and heading discussions in open forum. In teacher led discussions the teacher plays the main role in raising topics in open class discussions. I intend to differentiate between open class discussions and whole class discussions as generally used in literature. For me open discussions emphasises the fact that the topic of conversation, from the teachers point of view, is open in that the teacher cannot predict what issues could be raised by learners. Open discussions also places emphasis on the fact that the discussions are open to all to participate. Homework refers to times where the teacher assigned tasks to be completed at home prior to discussing them in groups in class.

As can be seen from Table 1 the construct of a lesson centres on the implementation of particular tasks designed to elicit the learning of particular concepts linked to functions. This implied that a lesson could occur over a number of periods or days.

3.2.3 Data Collection

Four lessons in succession presented on five days were videotaped. I approached my niece, whom at the time was a part time student at UNISA, to video tape the lessons. I provided training in terms of how to use the camera as well as what to focus on. Training consisted of practice on the operational features of the video camera. I instructed her to follow the lessons from the perspective of someone trying to understand the discussions at hand. I instructed her to focus attention on learners who were involved in the discussion and also to capture the visuals written on the board or displayed on the overhead projector that were being referred to during discussions. I also had follow up talks with her after the first and second day of video recording.

Two video recorders were used during the videoing of these lessons. One remained stationary, positioned to focus on learners diagonally across the classroom from the front to the back. The second video recorder was operated by my niece who was positioned diametrically opposite the stationary video. The use of two video recorders was an attempt to maximise the capturing of the activity during every lesson. The videos which came from the recorder operated by my niece served as the primary source of analysis. Videos from the second recorder were used to complement data from the first video recorder.

I kept a reflective journal during the period of presenting these lessons. This journal guided my actions during the implementation of lessons thus conducting an informed action research. Books were issued to learners in which they completed the tasks assigned. I collected these books as an additional source of data. I've used some of the learners' books to corroborate some of the inferences I made during analysis.

3.3 Rigour

Issues of rigour include issues of reliability, validity and bias in reporting on one's inquiry. It is in the way the researcher attends to these matters that builds the reader's trust in what is being reported.

Scaife (2004) reports that reliability attends to consistency of results of particular procedures under certain conditions. Scaife warns that consistency of results is only obtainable where the factors influencing the situation can be controlled. Validity, Scaife asserts, rests upon the extent to which procedures actually measure what they set out or intend to measure. Scaife emphasizes that validity attends to "the relationship between a claim and the data-gathering process" (p. 68) and not in that a particular aspect has validity on its own. Bias is what threatens validity in that it has the potential to direct the outcome of the inquiry. Both Scaife (2004) and Griffiths (1998) emphasize that the presence of bias in itself is not what threatens validity but in the way bias is reported on in the research.

In guarding against bias I have chosen to take an inductive approach to data analysis, which I expand on later in this chapter. The inductive approach to analysis requires that the researcher takes a fresh look at the data collected. Analytical tools are only developed subsequent to thorough scrutiny of the data. Analysis is therefore grounded in the data rather than the researcher's preconceived conceptions. I have also chosen to report on data using the words of the participants. In so doing I allow the reader a window into the data collected. The reader is therefore in a position to verify claims made by the researcher.

Maxwell (1992) writes on the issue of validity in reporting on qualitative inquiry. He characterizes five types of validity attended to in qualitative reports. The first three: descriptive validity, interpretive validity and theoretical validity as discussed by Maxwell I found relevant to me in ensuring that the action research I had embarked on were deemed credible. In the discussion that follows I describe Maxwell's characterizations of validity and how it impacts on my research.

Maxwell (1992) characterizes descriptive validity as attending to the accuracy of the physical observable features of the situation that are part of the inquiry. Interpretive validity attends to reporting on the meanings the participants attribute to physical observables. These meanings are constructed using as far as possible the words of the participants. Although both types of validity attend to the issues of accuracy in reporting, it is threats to descriptive validity that can be effectively countered through verification of the data collected. The use of video recordings used during the analysis can be used to verify the accuracy of the observables in my research. Threats to interpretive validity however are more difficult to counter as reporting rests upon inferences made. In reporting on observables grounded in the words of the participants, I aim to let the reader judge the inferences I make. I have also used transcripts to justify certain claims made in an attempt to allow the reader to verify inferences I make about participant meanings.

Theoretical validity attends to the appropriate use of theoretical constructs in explaining particular phenomena during the inquiry. Threats to theoretical validity are therefore not

eliminated by verifying the facts of the situation reported but through consistency in appropriately applying theoretical constructs as intended by the theorists in ways that do not distort the data. Verifying this appropriateness lies within a particular school of thought. I have surveyed corroborating and conflicting literature in presenting an argument for the appropriate use of any theoretical construct.

Regarding bias, Maxwell (1992), Scaife (2004) and Griffiths (1998) report that transparency during reporting is essential in that it allows the reader to ascertain the basis on which findings are constructed or revealed. Griffiths (1998), in particular, argues that although initial assumptions and intentions guide and inform the research they should not determine its outcome. I have kept a journal throughout the duration of this research to reflect on various aspects during the research but also to keep track of my initial assumptions and intentions and how these have shifted during the course of the research.

3.4 Ethical considerations

In accordance with the ethics committee of the University of the Witwatersrand I have submitted all relevant information for approval and my protocol approval number is 2007ECE19C. In order to uphold ethical principles in my actions, I put the following procedures in place. Letters inviting learners to form part of this research were given to learners and their parents. In this invitation confidentiality and anonymity during reporting of the findings were emphasized. I informed all the learners and their parents that they may withdraw at any point during the research. Consent to participate during the different processes of data collection have been obtained from all but one learner and her parents.

One learner and her parents did not give consent in two of the six criteria stipulated on the consent form. They were: not giving consent for video tapes with her in it to be shown at academic conferences and not giving consent for her written work to be used in this research. This did not pose a problem because I only used a few individual learners' books and no material to date has been used at any conference.

3.5 Analysis

I have formulated my analysis based on Hatch's (2002) frame for interpretive analysis. A number of reasons drew me to this framework for analyzing the data. Firstly the inductive process in which the categories, themes and patterns are developed makes this kind of research grounded within the data collected. As both participant and researcher it is important for me to start my analysis from the ground up with a fresh look at the data collected, no longer as a participant but as a researcher. This is important because it helps to establish an objectivity to eliminate threats of bias.

Secondly Hatch asserts that interpretation is about giving meaning to data: "It's about making sense of the social situation by generating explanations for what's going on within them" (p. 180). The social construct and its impact on how learning and teaching of mathematics takes place within that construct is what I am interested in exploring.

Thirdly Hatch argues that: "Interpretation situates the researcher as an active player in the research process" (p. 180). Being both a participant and researcher as is the nature of action research opens up many advantages such as first hand insight being party to the events reported on. There are challenges as well such as a biased perspective because of being too close to the actual events.

Fourthly Hatch's interpretive analysis framework rests upon the formulation of a story which can be told as the result of the interaction between the reflective impressions about a social context and a systematic analysis informed by an inductive process of the data analysis. Equipped with Hatch's (2002) framework for an interpretive analysis grounded in the data I positioned myself as a researcher to conduct an analysis of the data collected.

In response to the four sub-questions posed in chapter one, analysis of data collected consisted of four phases. The first phase of analysis which addresses sub-questions one and two (p. 3) consisted of coding and analysing relationships between the seven big ideas in teaching functions. The interactions which occurred during open class discussion were written up and then coded. The coded seven big ideas were then subsequently

analysed for any patterns. I expand on how coding was done and the results it yielded in the next chapter.

The second phase of analysis which addresses sub-question three consisted of transcribing the open class discussions in each of the four lessons presented. These transcripts were then used to evaluate the nature of mathematical reasoning that took place. With mathematical reasoning being a communicative process I chose to analyse learner interactions in search of lines of mathematical arguments.

The third phase of analysis which addresses sub-question three was to understand how, if at all, the big ideas were influential in allowing learners to engage in mathematical reasoning. Here again I searched the transcripts for lines of mathematical arguments and the impact the big ideas had on such mathematical reasoning.

The fourth phase of analysis addresses sub-question four. During this phase of analysis I searched the transcripts for the practices learners and teacher engaged in which help facilitate mathematical reasoning. It was in viewing the classroom as a community of practice that I was able to identify these practices.

All the results and inferences made during the four analytical phases are reported on in the next chapter. Due to the limited scope of this project I could not include an analysis of all the transcripts. I therefore focussed my attention on those aspects that were typical of all the transcripts and used a number of extracts to illustrate my findings.

Chapter 4: Data Analysis

4.1 Introduction

Through a descriptive account I hope to give the reader a sense of the complexity involved in teaching big ideas in functions while teaching mathematical reasoning. The descriptive analysis is broken up into five parts. The first is a description of how coding was done in an attempt to illuminate the difficulty in coding for the seven big ideas in learner interactions. The second part seeks to understand the relationship between the seven big ideas. The third analysis deals with the nature of the mathematical reasoning learners are engaged in. A fourth analysis hopes to unravel any connection between the big ideas and learner engagement with mathematical reasoning. The fifth part of the analysis deals with the practices learners and teacher engage in.

4.2 Coding

I identified the big ideas learners engaged in as they grappled to complete the activities through analysis of written accounts of interactions among learners and between learners and teacher. These accounts were written up using as far as possible the learners' own words. I then used these accounts to understand the extent learners engaged in the seven big ideas identified and discussed in chapter 2. Learners' contributions were coded as follow:

- *pattern* – an indication that the learner uses a regular occurrence of an event to abstract a generalization
- *variable* – learner refers to a quantity as a generalized set of values
- *covariance* – learners' use of mathematical constructs that requires coordinating between two variables
- *slope* – learners refers to or uses the incline of line or curve to construct or represent a mathematical construct
- *representation* – learners' verbal, graphical, algebraic and gestures were coded
- *additive relationships* – learners demonstrate their understanding of the task at hand through the use of addition, subtraction or counting

- *multiplicative relationships* – learners discuss their understanding of the tasks at hand through the use of multiplication, division, ratio and proportional reasoning

Table 2 is an example of how these interactions were captured and coded. It must be mentioned that these interactions were coded while watching the videos and reflecting on the interactions described. This particular example comes from the open class discussion led by Zwelile¹ as she reported back a second time on her group's response to activity 15.3.3. My reasons for the coding are discussed after Table 2.

REFERENCE	DESCRIPTION	Pattern	Variable	Covariance	Slope	Representation	Additive relationships	Multiplicative relationships
IA-163	Zwelile comes and presents her corrected graph. She indicates that she used a scale of 2 blocks equal to 1 for both axes. The graph is part of a hyperbolic curve restricted to quadrant 1. The teacher however draws her attention to the fact that the axes does not intersect at (0 ; 0) but (0,5 ; 0). Teacher asks Zwelile to explain her graph. She explains the few points that she could draw relating it to the breadth and length.		✓	✓	✓	✓	✓	✓
IA-164	Teacher then asks her how she thinks it will continue and she indicates verbally and by a hand movement that it will curve down. Teacher then requests that she put the transparency with the table back up.		✓	✓	✓	✓		
IA-165	Teacher enquires about the length and breadth being 0. Zwelile Says: Because at that time I didn't figure it out....inaudible...But what I have thought of is that the length is 0 and the breadth is ..say for example 2 then that means it would be a straight line.... and it wouldn't be area...cause in order to get 36 you need to multiply the length and the breadth to get your area so...and if you multiply 0 by anything else it gives you 0.		✓	✓		✓		✓

Table 2: Coding Zwelile's interaction in open class discussion (see Appendix L for all coded tables)

¹ All learner names are pseudonyms

In IA-163 in Table 2 Zwelile is interacting with the whole class. This interaction I have coded as being working with *variable, covariance, slope, representation, additive representations and multiplicative relationships*. I have coded this interaction as *variable* because Zwelile uses the words length and breadth referring to a collection of different lengths and breadths of a range of rectangles. Zwelile has plotted a number of points correctly and has represented this function correctly by a curve. I coded this interaction as *covariance* as these actions require the coordination between two variables. I have coded this interaction as working with *representation* for the obvious reason that Zwelile graphically and verbally describes her solutions to the task in activity 15.3.3. I've coded Zwelile's correct use of numbering the x and y axes as *additive relationships*. My decision to do this stems from the fact that she previously numbered the axes incorrectly. I've also coded this interaction as *multiplicative relationships* as she now uses a scale for numbering the axes as well as her ability to explain the relationship between the x and y values using multiplication and division. I have also coded this interaction as *slope* because the learner joins the plotted points with a smooth curve indicating a continuously changing rate of change. When working in the domain of constructing graphs the notion of slope is always implicitly present. I have therefore chosen to only code this when learners initially sketch a graph correctly and when particular reference is made to slope in subsequent discussions.

In IA-164 I coded Zwelile's response to my questions as *variable* because Zwelile's hand movement correctly depicts how the x and y values will change in relation to each other. As Zwelile's hand movement coordinates between two varying quantities I coded the interaction as *covariance*. Her hand movement also explicitly depicts a curve indicating a continuously changing rate of change. I therefore coded this interaction as *slope*. Because of Zwelile's use of a verbal description strengthened by her symbolic hand movements I coded the interaction as *representation*.

Although Zwelile explicitly refers to straight line in IA-165 I chose not to code this as slope because her reasoning and explanation around the use of straight line is not sensible. In having a closer look at how I conducted the coding I intend to provide the

reader with a sense of how I coded for the big ideas. I also intend to provide the reader with a sense of the complexity of coding for, in particular, the notions of *slope* and *pattern*.

The next table captures an example of how I typically coded for *pattern* and a not so typical way of coding for *slope*. The example is taken from an open class discussion led initially by Pholani and then Tiego who are both members from the same group. Tiego reports on his thoughts about how to solve question 3.1 from activity 15.2.1 (see Appendix C)

REFERENCE	DESCRIPTION	Pattern	Variable	Covariance	Slope	Representation	Additive relationships	Multiplicative relationships
IA-98	Tiego comes up with a conjecture that if he takes the altitude multiply it by negative 6,5 and then add 15 one would get the temperature. He gives an example of this as he calculates and verifies that the temperature for an altitude of 8km is negative 37 degrees Celsius. On the teacher asking him how he got to that, he answered it just came to him. The teacher then asks him to work out the temperature for 4. He then calculates it and gets to a confirmed negative 11 degrees Celsius. The teacher asks if that works with all of them at which Tiego responds: yes sir. He is then applauded by a large number of learners in the class.	✓	✓	✓	✓	✓	✓	✓
IA-99	Teacher asks Tiego where he got negative 6,5 from. Tiego indicates that the difference in temperature is -13 so if he goes in ones it would be -6,5. And then for 15 I saw for an altitude of 0 the temperature is 15. Teacher says that he hopes everyone got what Tiego was saying. Teacher indicates that Tiego says that for every degree that it increases the difference say for zero and 1 would be -6,5. He then took the difference and multiplied it by the altitude (confirms with Tiego) and then try to see what must be added to or subtracted from that to give the temperature. And he noticed that the value that had to be added or subtracted had to be 15 to give you exactly the temperature that is given here (pointing at the table).	✓	✓	✓	✓	✓	✓	✓

Table 3: Coding Tiego's conjecture

In IA-98 I coded for *pattern* because Tiego constructs an algorithm to determine the temperature for a given altitude using a patterned regularity of a drop of 13 degrees Celsius for every two kilometres rise in altitude. I have coded for *variable* because he uses temperature and altitude as a set of varying quantities. In constructing an algorithm Tiego is constantly coordinating between the variable quantities of temperature and altitude and thus I coded for *covariance*. This construction is embedded in the number patterns he observed. These number patterns are linear and can mathematically be identified through a constant difference and hence I coded for *additive relationships*. I coded for *multiplicative relationships* as Tiego simplifies this ratio to arrive at a drop in temperature of six comma five degrees Celsius for every kilometre rise in altitude. How I coded for *slope* was unconventional in relation to coding for slope in Table 2 which was based on a correct visual representation of the graph. In this instance however, I coded for slope as the learner is working with *rate of change* as representative of slope. Tiego has intuitively stumbled across gradient as a measure of rate of change. Tiego's algorithm in itself is a mathematical representation hence the coding for *representation*, although his arithmetic at this stage can become an equation depicting the functional relationship between altitude and temperature as set out by the task.

In IA-99 I have again coded for all seven big ideas for primarily the same reasons as in IA-98. I've included IA-99 with IA-98 in this Table 2 because of the different modes Tiego is functioning in the two interactions. In IA-98 Tiego gives an explanation of how he arrives at a solution. It is as if he's not sure of his ideas. His explanation serves as a test run. In IA-99 however he justifies his reasoning. It is clear that his explanation is more confident when compared with his assertions in IA-98.

In coding for big ideas in learner responses as they were engaged in the tasks, I started getting a sense that there might be interplay between them. Were some big ideas more fundamental than others? I also wanted to know how each of these ideas present themselves in the various activities. In trying to understand the interplay between the big ideas and between these ideas and the activities I compiled a summary table of the big ideas per activity (see Table 4 in next section).

4.3 Big ideas

Table 4 below displays the frequencies for coding the seven big ideas in learner responses while engaging with the tasks during the activities. The total sum for coding big ideas for a particular big idea across activities is presented in the bottom row. The total sum for coding various big ideas in a particular activity is presented in the last column. A discussion of my interpretation of the findings follows the table.

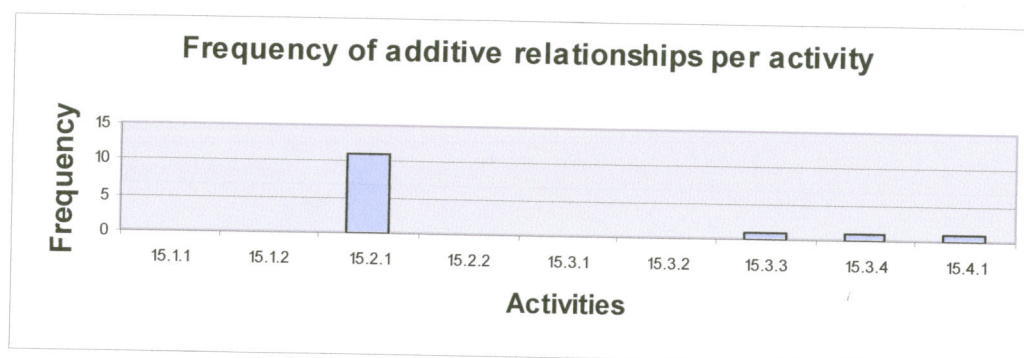
ACTIVITY	Pattern	Variable	Covariance	Slope	Representation	Additive relationships	Multiplicative relationships	TOTAL
15.1.1	0	13	28	18	35	0	10	104
15.1.2	0	5	8	7	9	0	3	32
15.2.1	17	28	29	6	33	11	13	137
15.2.2	1	3	16	5	16	0	1	42
15.3.1	1	9	10	15	10	0	5	50
15.3.2	6	15	14	5	22	0	9	71
15.3.3	2	11	11	3	12	1	3	43
15.3.4	9	9	10	1	11	1	7	48
15.4.1	9	15	17	1	24	1	8	75
TOTAL	45	108	143	61	172	14	59	601

Table 4: Frequency of big ideas in learner responses per activity

The table shows to what extent learners engaged with all seven big ideas across the activities. This is evident from the total number coded for a particular activity. Looking down at the total amount of coding for a particular big idea, it is evident that all seven big ideas are present across activities.

The big idea of *additive relationships* is considerably lower than the other six big ideas. This I expected because *additive relationships* are localised in particular sections of functions such as in the construct of an appropriate scale for the axes of a Cartesian plane

when graphing. The use of additive relationships would be more prevalent in reasoning about linear number patterns, where the identification of a constant difference is essential in formulating a conjecture about the covariance between the dependent and independent variable. Graph 1 below confirms my argument.



Graph1: Frequency of additive relationships across activities

In Graph 1 above it is evident that a substantive number of the *additive relationship* code is only recorded in activity 15.2.1 (see Appendix C). This activity deals with the linear functional relationship between altitude and temperature as depicted in the task.

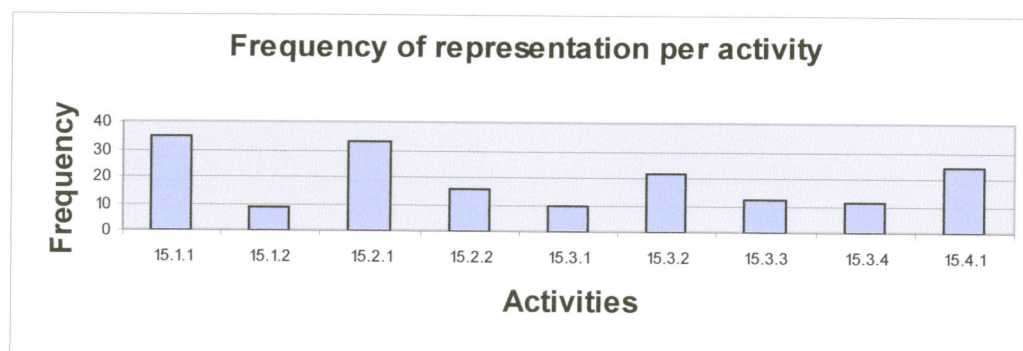
Table 4 indicates that *representation* has the highest frequency. The fact that learners used representation more often than any other big idea in dealing with functions was expected. Representation is fundamental to most mathematical activities. The need to capture mathematical constructs in some form of representation is key in trying to communicate an abstraction. The function concept is one that is difficult to explain without the use of some kind of representation.

Covariance displays the second largest frequency in Table 4. Its total frequency across activities is close to that of *representation*. This is noteworthy because it appears that the need to use mathematical constructs of representation runs parallel to formulating a fundamental concept used in the definition and understanding of functions.

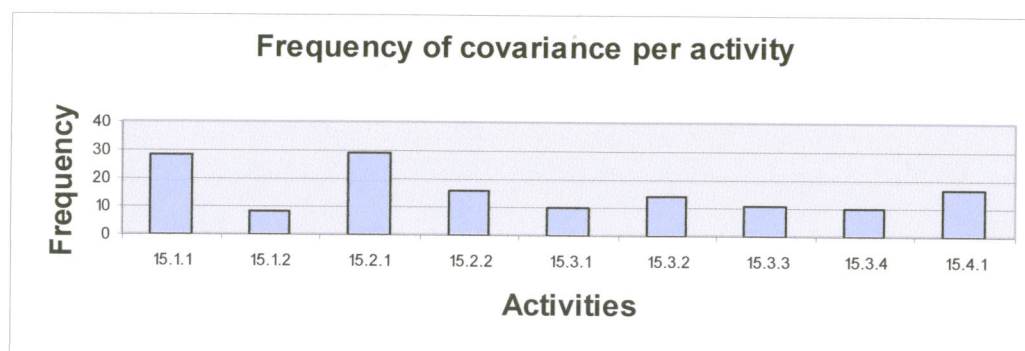
The third highest frequency displayed across activities was that of *variable*. I have argued in Chapter 2 that *variable* precedes *covariance* because covariance is the coordination between a set of varying quantities.

The tracking of patterns across two varying quantities will focus mathematical constructs that use proportional reasoning and therefore we see a more substantive frequency for *multiplicative relationships*. Although the frequency for *multiplicative relationships* displays the fifth largest it is only two counts less than the big idea with the fourth largest frequency, namely *slope*. The big idea of *slope*, I argue, is a particular expression of *covariance* and they are therefore conceptually linked.

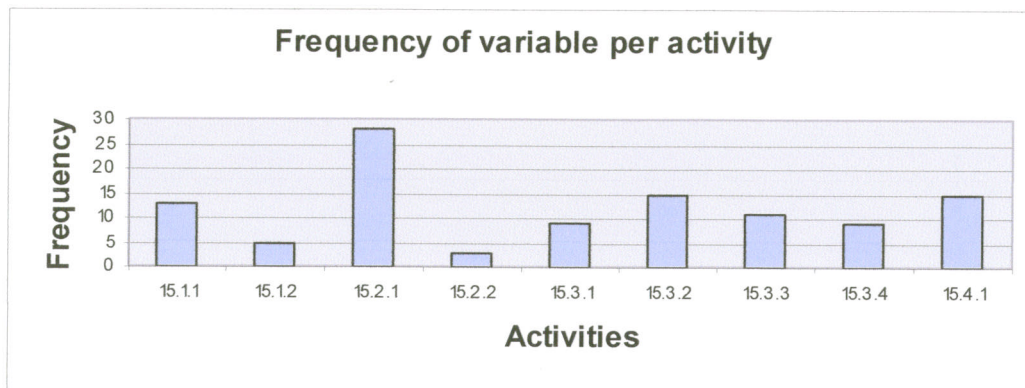
Graphs 2 to 5 below display the frequency of *representation*, *covariance*, *variable* and *multiplicative relationships* across activities respectively. A discussion in relation to these graphs follows below.



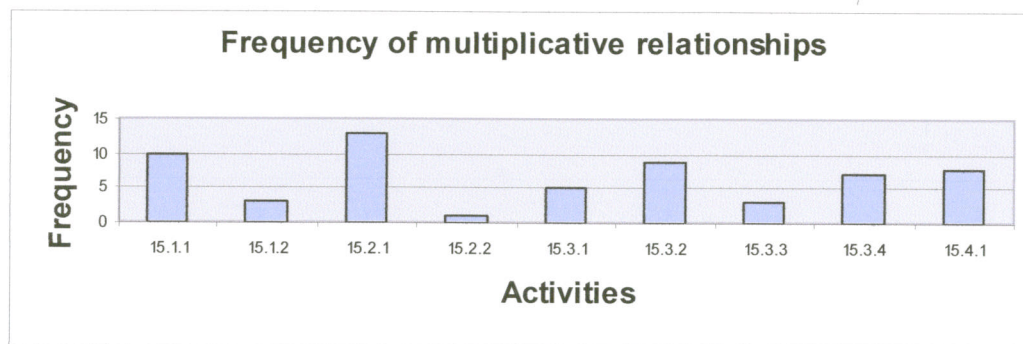
Graph 2: representation



Graph 3: covariance



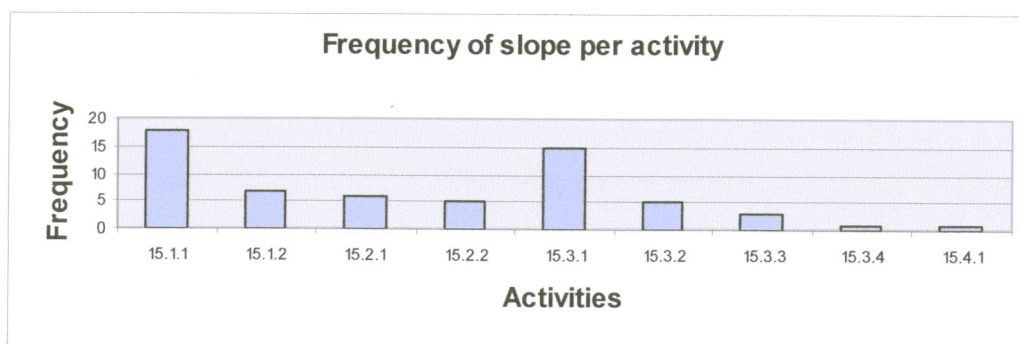
Graph 4: variable



Graph 5: multiplicative relationships

Comparing all four graphs above it is clear that there is a correspondence when looking at the geometric dispersion across activities in all four the big ideas depicted. It is as if these four big ideas are synchronised across activities. The correspondence between *representation* and *covariance* is stronger as well as the correspondence between *variable* and *multiplicative relationships* than the correspondence across all four big ideas. This correspondence confirms my initial assertions relating to strong connections between these four big ideas above.

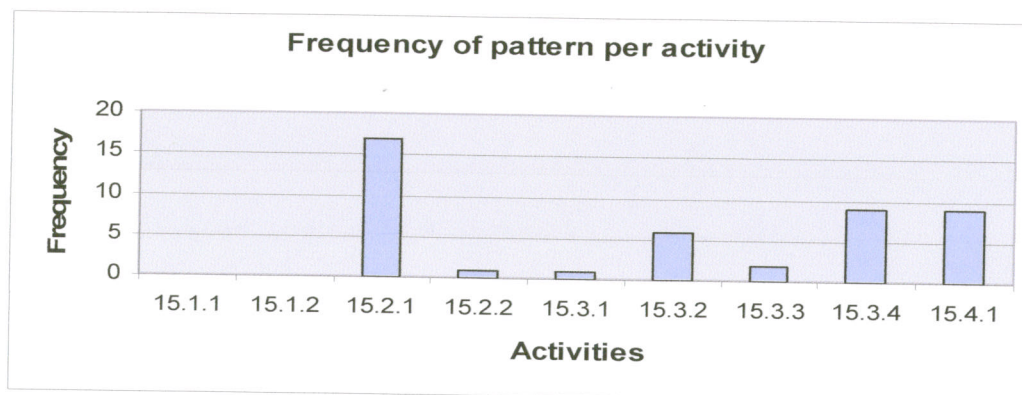
In the light of these connections between the four big ideas mentioned above I expected a similar geometrical dispersion of the frequencies across the activities for the big idea of *slope*. Slope displays a different geometric dispersion pattern in comparison with *representation*, *covariance*, *variable* and *multiplicative relationships* as displayed in Graph 6 below.



Graph 6: slope

Contrary to my expectation the big idea of *slope* followed a pattern corresponding to a more explicit emphasis in teaching for this particular topic. I developed activity 15.1.1 (see Appendix A) with an emphasis on developing the notion of *slope*. Developing a language and the graphical representation for slope as depicted in the task was primary to this activity, hence the substantive frequency of slope counted during this activity. From activity 15.1.2 to 15.4.1, with the exception of 15.3.1, there is firstly rapid then gradual decrease in the number of codes for *slope* across activities. I attribute this decrease in the frequency to a decreased explicit emphasis on *slope* and *rate of change*. The significant increase in frequency in activity 15.3.1 I attribute to the learners take up from the previous group's report back during open class discussion as well as the fact that activity 15.3.1 comprised of two activities in one. Two activities meant more feedback to be given and therefore more learner engagement which could account for the higher frequency of slope coded.

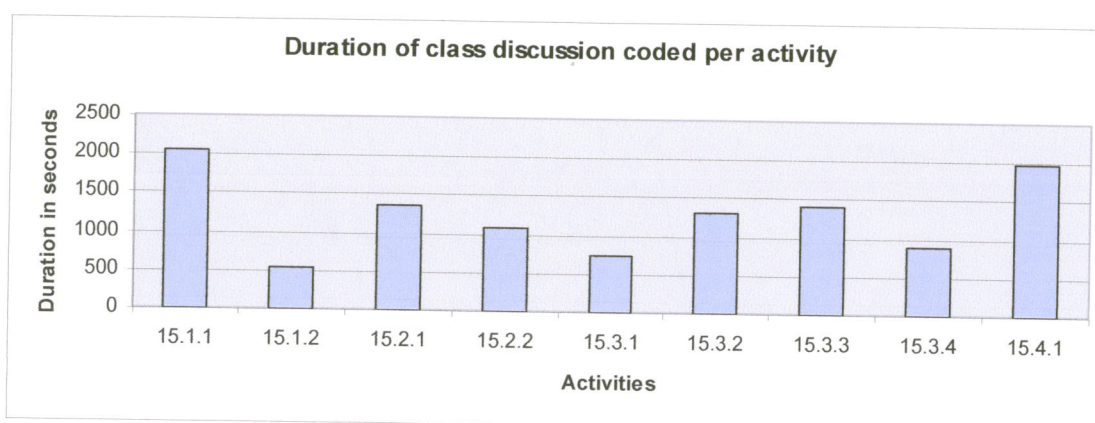
To my surprise *pattern* displayed dispersion patterns across activities distinct from representation, covariance, variable and multiplications. I argue that since I have taken the view of number patterns as opposed to a more general regular occurrence of events, pattern as a big idea was similar to slope and localised to particular tasks which required explicit engagement with this big idea. Graph 7 below displays the dispersion pattern of the frequency at which pattern was coded across activities.



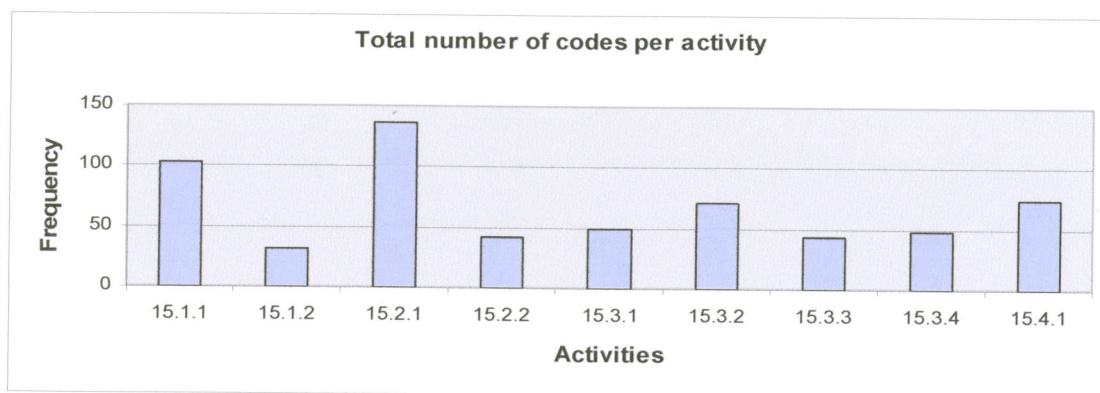
Graph 7: Pattern

In Graph 7 above we see that the highest frequency recorded for pattern occurs in activity 15.2.1. The table values in this activity prompted learners to derive solutions via number patterns identified in the tables. A similar argument can be made for the other activities which recorded a significant number coded for *pattern*.

The indication that slope and pattern were determined by an explicit emphasis in the task design, led me to probe the effectiveness of the designed tasks to elicit learners' engagement with the intended big ideas. If it is assumed that the learners engaged with all the tasks in the same manner, the total number of codes counted across big ideas should show a similar geometrical dispersion across activities as that of the duration of the coded interactions. Graph 8 below displays the duration in seconds of the open discussions across activities. Graph 9 also below displays the total count.



Graph 8: Duration of open discussion across activities



Graph 9: Total number of counted codes across activities

The geometrical dispersion indicating the highs and lows in frequency in Graph 8 displays a strong correspondence with that in Graph 9. There are noticeable differences in the frequencies displayed in activity 15.2.1, 15.2.2 and 15.3.3. These differences displayed in the graphs coincide with events that I infer is not coincidental.

A larger increase proportionally from activity 15.1.2 to 15.2.1 in Graph 9 in relation to the proportional increase in time from 15.1.2 to 15.2.1 in Graph 8 is recorded. I attribute this largely to the level of interaction which activity 15.2.1 elicited from learners. Table 1 in Chapter 3 depicts that activity 15.2.1 was started on day 2 and completed on day 3. The time learners engaged with the activity is significantly longer than what was actually recorded. More time for reflection meant more active engagement the next day.

The total count recorded in Graph 9 for activity 15.2.2 is proportionally lower across activities than the time spent on the same activity as displayed in Graph 8. Very little interaction between learners occurred in relation to the times spent. The questions in activity 15.2.2 were not as open ended as the ones in activity 15.2.1. The fact that this task dealt with the quadratic function, I refrained from pressing too much for deeper engagement with the concepts at play.

Again we see a larger decrease proportionally across activity 15.3.2 to 15.3.3 in Graph 9 when compared with the same activities in Graph 8. Activity 15.3.3 as with activity 15.2.1 was covered over two days. Part of the feedback from the group activity was given

on the 23rd October while the rest was given on the 29th October. The reason for this delay was because I had stopped Zwelile from continuing with her feedback as she displayed too many basic errors relating to the numbering of the axes which completely distorted the shape of the graph. On presenting her corrected feedback on the 29th very little interaction was generated.

During the first part of my analysis I have shown that learners have taken up and worked with the seven big ideas in the four introductory lessons to functions. All seven of the big ideas were evident when coded for in learner contributions across all the activities. I have also shown that a strong correspondence is prevalent in four of the seven big ideas namely representation, covariance, variable and multiplicative relationships.

Additive relationships which reflected the lowest count seemed to have been localised to working with linear patterns, linear functions and scale on number axes only. Does the fact that additive relationships recorded such a low frequency suggest that it is not a big idea? To make such a claim I believe would be premature. Additive relationships as used in the constant difference between consecutive number sequences can be used to identify and explore both linear patterns and quadratic numbers patterns. I believe that a closer analysis of the tasks might shed more light on this matter.

Slope as in rate of change ranked fourth amongst the top five ranking big ideas. Slope however displayed a different dispersion pattern when compared to the dispersion patterns of representation, covariance, variable and multiplicative relationships. Does this indicate to a weaker connection with the other big ideas? The frequency with which slope appears also decreases across activities from start to end. Does this indicate that learners' take up of slope as rate of change is dwindling or can this be attributed to the tasks not directly prompting learners to make use of this concept?

Pattern as in slope displayed distinct dispersion patterns to that of the four connected big ideas. On closer analysis of the tasks, I argue that the presence of tables in tasks prompted

learners to engage with number patterns in order to find solutions to questions set. In the next section I turn to an analysis of the mathematical reasoning learners engaged in.

4.4 Reasoning

During this phase of the analysis I argue that mathematical reasoning frequently occurred throughout the lessons. I selected a section from lesson one which is typical and could have been selected from any lesson. I argue here that the trend depicted here is common to all the lessons.

4.4.1 The “Zig Zag” function

The section below depicts Ruth’s response to activity 15.1.1 number three and the discussion that ensued. Figure 1 is a replication of the graph Ruth drew in response to this task. I present the discussion first and the analysis in the sections that follow.

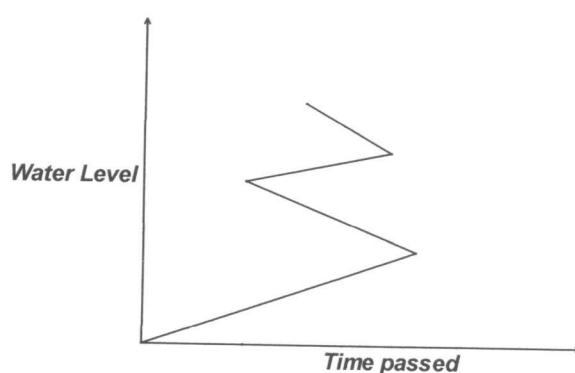


Figure 1

In seeing Ruth’s response to number three a few learners immediately respond to what they feel is an incorrect representation of the situation described. What follows is the discussion that immediately followed Ruth’s solution to the task.

Transcript 1 (See Appendix J for full transcript)

007. Neelan: *I think that the graph...(Inaudible)... shouldn't be going to the left (Pointing with hand movement). Because that is not measuring time.....*

008. Michelle: *.....(Interrupts Neelan) Time's not going back.....*

Neelan tries to explain that time cannot go backwards and that is what Ruth's graph is indicating. Michelle is in support of what Neelan is trying to explain. Ruth tries to defend her construct as follows:

012. Ruth: *Ok sir. I was saying. I never done it like, as it say, ok in one minute this is how much it rose, in two minute this is...I said that, for this, this.. The first section ... It like took this amount, like it took. How can I explain sir? It took longer to like.. It took longer to like fill up that section. And it took shorter. So I'm not saying in one minute it rose ten metres and then in two minutes it went up to twenty like that..*

What Ruth is pointing out is that she is not representing time as an increasing scale on the x axis but rather that the line segments refer to how long it takes for the water to rise in a particular section of the container. Michelle then reaffirms Neelan's previous statement that the graph cannot go back in time.

016. Michelle *Ok, I was just saying that, also what Neelan said, that the lines can't exactly go like "zig zag" because you can't say that like. Because isn't it sir the graph goes like during the time (Across movement of hand) and the level...(Inaudible)...(Upward movement with hand) so it would go that way (Pointing diagonally upward and to the right) not like (Showing "zig zag" movement or "snake like" movements upwards) that way*

017. Teacher: *Ok, so what you say there then, if I'm understanding you correctly, is that the line can't turn back. Because that means time is turning back. You need to consider not just what? Not just the water level but that, that line or that graph that you drawing there represents two things that's occurring at the same time. As the water level is rising time is passing...*

018. Ruth: *...Ah!*

In the next few lines we see Peter coming out in support of the arguments made by Neelan and Michelle that the time cannot go back.

022. Peter: ...*(Inaudible)*...*I was gonna say like at the bottom of this graph it says time passed...*
023. Teacher: ...*Yeah..*
024. Peter: *So basically that's your time, how long it takes you to fill up.*
025. Teacher: *Ok..*
026. Peter: *You can't go back.*

In the light of convincing justification for the convention that the axes assume increasing order from left to right the teachers asks for a volunteer to correct Ruth's graph. Both Miriam and Michelle volunteer to correct Ruth's graph. The teacher affords Michelle the opportunity and she drew the following graph as representative of the situation:

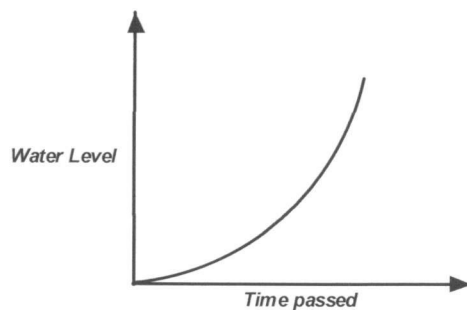


Figure 2

038. Michelle: *(Draws a graph that curves up with increasing rate of change)(Inaudible) Ok what I wrote here...over here it's taking longer (Pointing at start of curve) and as, as the graph gets smaller it will take more quicker to fill up, because it's a narrower (Pointing at top of container) like length to fill. So it will just get quicker and quicker (Running pen along curve from start to finish)*
039. Teacher: *So, you saying because, because the shape is getting narrower you find that it's, it's actually...the, the water level rises faster over time. Ok. We've got hands again it seems. One.. Ester... and then...*

Michelle sees the container on the whole narrowing and therefore represents the situation with a curve that depicts an increasing rate of change. She justifies her graph by stating that the narrower the container the quicker the water level rises. A number of learners who were unhappy with Ruth's representation raised their hands to be afforded an opportunity to respond.

040. Ester: *Ok. Sir. How can it be a..... straight curve? Because sir it's like the blocks (Using both hands to show sides of sections of container) go up and then narrower (Uses both hands to show the shape of container instantly getting narrower) like in a rhythm. And that's just... (Showing curve with hand)*
041. Teacher: *Ok. So, you saying it can't just be a smooth curve (Showing curve with hand movement)...there should be some ...(Inaudible commenting)...Ok.....*
042. Peter: *Because is(Show block movement with hand)*

Both Ester and Peter identify that the graph cannot be represented by a smooth flowing curve. The representation they argue is in contrast with the sudden changes in shape of the container from section to section.

044. Lyle: *Sir, is it wrong if you did a bar graph?*
045. Teacher: *If it's wrong...Ok, Lyle asks: "Is it wrong if you did a bar graph"?*
046. Learners: *....Yes... no....*
047. Teacher: *Ok, let's just come back to that one. We'll come back to: whether it's a bar graph. But before we...Let's just discuss the curve that you (Points in Ester's direction) suggesting. What are you saying (Pointing to Peter)?*

The teacher realises that Lyle has just opened a can of worms and tries to redirect discussion to the curve Michelle has presented. He then affords Peter the opportunity to respond:

048. Peter: *Ok sir. It mustn't be a curve because...(Inaudible)...Each block (Show blocks in a similar way as Ester) is like a sharp thing like this (Show sudden narrowing with hand). So it fills up a lot quicker. Instead of a (Hands [apart] move from bottom up to form vertex [together]. Almost like side view of cone) smooth... Uhm..*
049. Teacher: *Ok. So you say it can't.. So you say it must suddenly change direction (Using same hand movements as Peter). So it can't be a smooth curve (Show curve with hand movement). Yes (Pointing in Craig's direction)*
050. Craig: *...(Can't make out)...But sir. I'd also say it wouldn't be a curve because, the shape is like a constant shape and water is being filled up constantly. It's not like the shape is, like constantly changing and water is filling up at different.....*
051. Teacher: *...Interrupts Craig)...Ok. So you saying the first part. The bottom part. It fills up constant ...at a...*
052. Craig: *...(Interrupts Teacher)...Constant rate...Because of the shape and then...*

053. Teacher: ...*(Interrupts Craig) ...suddenly (teacher uses hand movement to form shape of container) the rate....Changes and it stays constant (Teacher uses hand movements to form shape of container)...*
054. Craig: *Ok, yes*
055. Teacher: *And then suddenly the rate changes and it stays constant (Teacher uses hand movements [shape of container] corresponds to rate of change [spoken words])...*
056. Craig: *Ja.*
057. Teacher: *...So what would you like to see on there (Pointing at OHP). What kind of sha... What kind of things would you like to see? (Pointing in direction of Ester) A curve? What you like to see?*

Here Peter again highlights the fact that the graph needs to represent sudden changes which is evident from the shape of the container. Craig comes in and argues that the different sections should be represented by straight lines as it is a constant rate of change. Craig uses constant shape referring to the width of the sections that stays the same. The constant width giving rise to a constant rate of change had been established in number one of the same activity. It is Ester this time who raises the discussion about the possibility of using a bar graph as representative of the situation described after the teacher tried to shut that door temporarily for another day.

058. Ester: *Sir, Uhm I, I. Would it be wrong to use a histogram sir? Because it could clearly show....the....Like this is like in blocks that are, as they that have...(Inaudible)....So would a histogram be wrong?*

Realising that this argument needs to be laid to rest the teacher engages the learners in conversation with regards to the discrete nature of bar graphs. The teacher makes an argument for the use of a graph that depicts the relationship between two continuous varying quantities. It is Berenice and later Peter who pick up the torch from Ester in arguing for the use of a bar graph.

077. Berenice: *Sir but ...(Inaudible)... time limit? It's like after one minute how much the...*
078. Teacher: *....(Interrupting Berenice)....Ja, but.....*
079. Berenice: *..... water level.....*

080. Teacher: *ti*, time is carrying on. It doesn't say: one minute, stop; two minutes, stop; three minutes, stop. You don't measure after every minute? Did we? No! We just want to represent on a graph what happens when I take the container open the tap and it runs. How it's being filled.
081. Peter: Sir, but you can use a Bar graph because after one minute, say it fills ten millilitres...and after another minute if the shape changes it.....
082. Teacher:(Interrupts Peter)...I'm not saying you can, you cannot do that. What I'm saying is, the situation that's being described is: Draw a graph representing what happens when this container is filled with water. Ok. So as the water level is rising it doesn't stop. So you not measuring after every minute. I'm not saying it can't be done. I'm simply saying that would represent something different. Ok. So you need to choose what you represent. And in this case we want to see as the time is passing what is happening to the water level. So this line that you or curve that you representing should represent what you see happens. Ok. And it ...(Inaudible)....your time and your water level. Ok now, I want to go back here (Refers to number one in activity 15.1.1). Because I can see people are stuck. Why did we have a straight line there? Yes (Points in Craig's direction)
083. Craig: Because the container is being filled up at a constant...(Inaudible)
084. Teacher: Ok, fine. Would you say that for this first part (Referring to bottom section in number three) the water is being filled up at a constant rate?
085. Learners: Yes!
086. Teacher: So why would you want to have a curve? Why can't we represent it with the same thing, a straight line?
087. Learners: Yes!
088. Michelle: Sir because on the second level, if we can call it that, of the drawing it uhm, how do I. It's like smaller. The width is smaller. So it won't take as....
089. Teacher: ...(Interrupts Michelle) Ok. Can i....
090. Michelle:long to fill up
091. Teacher: ...Can I ask you another question? Ok. What Ruth did was: She, she, she had little lines. What mistake did she do?
092. Michelle: She went backwards in time.
093. Ruth: I went backwards in time (Ruth says this and shows with her hand at the same time as Michelle above)
094. Teacher: She went backwards in time. Which she can't do. Can't we use the same concept but still go forward in time?
095. Learners: Yes, Yes we can
096. Miriam: But change direction (Showing zig, zag movement with hand going forward)

097. *Teacher:* So we could just change direction. What would you think that would look like (*Hands transparency pens to Miriam*)? What do you think that would look like?
098. *Learners:* (*Comments [Oh gonna], sounds of frustration as Miriam goes up to OHP to change graph*)
099. *Teacher:* I need your tissue quickly (*Ruth hands tissue to teacher who in turn asks her to give it to Miriam*)

Miriam tries to draw but has great difficulty drawing. It is Jason who then volunteers to assist Miriam in drawing. The teacher gives Jason an opportunity after checking with Miriam. The graph below is a replica of the graph that Jason produced.

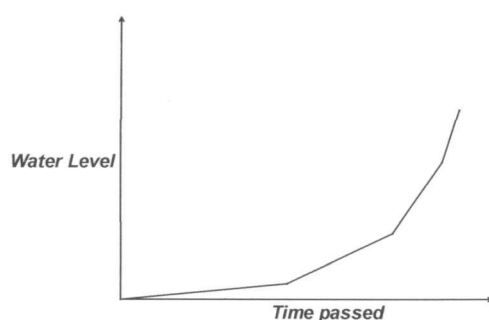


Figure 3

In what follows, I show that the final graph and its justification are collaboratively constructed in the classroom. I also show that the big ideas of functions and mathematical reasoning are developed among the learners.

4.4.2 A Collaborative line of argument

The line of argument is developed from a number of participants in collaboration. Initially we see Ruth identifying that she needed to use four short lines representing the rise of water in the four different sections of the container. Where Ruth makes an error is not taking into account the increase of time from left to right. She also confuses duration and rate of change during her representation.

Neelan and Michelle point out the convention of representing the linearity of the variable of time in an increasing order from left to right. Michelle's representation is consistent with an increase in the rate of change of the water level as seen holistically. She accurately captures the rate of change increasing across the narrowing sections from

bottom to top. She does however not consider the rate of change per section. To her a changing rate of change seems to be represented by a curve, because she uses a curve to represent this changing rate of change.

Lyle, Ester and Peter pick up on the sudden changes in width from one section of the container to the next. They see a smooth curve, as represented by Michelle, not capturing this dynamic. Lyle and later Ester suggest the use of a bar graph to represent these sudden changes. My inference is that their suggestion for using a bar graph is that the bar graph pictorially resembles the sudden changes in width of the container. Craig argues for the graph to represent a constant rate of change as a result of the width of each section remaining constant before it suddenly narrows into the next section.

Miriam suggests a similar zig zag graph as Ruth going forward in time. It is Jason who generates a graph that takes all possible factors into account. He draws a graph using four line segments representing the constant rate of change in each of the four sections of the container. The four segments are drawn with increased slopes representing an increased rate of change. The narrowing of the width from one section to the next is given as justification for this increased rate of change.

Craig takes a step back as he ponders the effect the height of a section has on the rate of change. He suggests that the second section should be represented by a shallower slope because it would take longer to fill. He justifies this argument stating that this section is higher. Craig is confusing two concepts. Firstly there is the duration at which a section of the container is filled and secondly the rate at which the container is filled.

4.4.3 Big ideas and mathematical reasoning

Given the co-construction of the final graph by a number of learners it is useful to ask: What is the impetus for driving the discussion for mathematical reasoning in the above extract?

The need to represent the situation depicted in the task during activity 15.1.1 serves as the catalyst for arguments and counter arguments made by learners. It is through validation by the collaboration of a reasoned argument that progress is made. Trying to find consensus around how to represent a graph depicting the rise of the water level over time, acts as a stimulus for mathematical discussion. An argument made by Ruth as solution to the tasks of graphically representing a graph depicting the situation at hand is quickly countered by both Neelan and Michelle (lines 007, 008 and 016).

Ruth's acceptance of the argument that the x axes must represent time increasing from left to right allows for Michelle to produce a representation that depicts a curve with an increasing rate of change. Michelle justifies her representation by stating that the narrowing of the container from bottom to top causes the water to rise faster (line 038).

Ester followed by Peter, Lyle, Craig and Berenice (lines 039 to 057) present a counter argument. They argue that a smooth curve does not take into account the rapidly changing widths of the container. Craig also argues for the use of line segments because the width of the various sections of the container stays the same and therefore the representation must reflect a constant rate of change.

Covariance is not mentioned as such but is fundamental in that the learners must coordinate between time and water level varying. This coordination between two varying quantities is further complicated by the introduction of the varying width which gives rise to varying rates of change. The water level is a function of time while the rate of change is a function of the width of the container.

Lyle, Ester, Berenice and Peter unknowingly threaten to derail the progression made so far to the current line of collaborative reasoning. Their introduction of the topic of bar graphs (lines 044, 058, 077 and 081) is what threatens to derail the progress made. Seeing this threat I initially try to redirect learners' attention to Michelle's curve. Redirecting the discussion away from bar graphs towards Michelle's curve allows for the discussion to

progress in a positive direction, at least for a while. It is during this time that Craig's assertions about a constant rate of change are made (lines 050 to 057).

It is Ester who forcefully reinitiates the topic of bar graphs (line 058). Her arguments are supported by Berenice (line 077) and Peter (line 081). This compels me to address the topic in greater detail (line 082). It is with an uncomfortable acceptance by the learners of my explanation that I managed to redirect discussion towards refining Michelle's representation. As uncomfortable acceptance as it may be it helps the conversation to progress through careful consideration of the big ideas of representation, covariance and slope as rate of change to generate the correct graph depicted in the situation.

I argue here that the need to understand these concepts and how they interplay, to successfully complete the task, is what contributes to healthy and productive mathematical conversations. The teacher's focus on directing conversation and therefore mathematical activity to focus on these big ideas generates in part the momentum that carries such rich mathematical reasoning. I see the big ideas initially as owned and driven by the teacher. The big ideas then become owned by learners as they become the tools through which they further explore functions.

It is the practices learners engage in, in class, which allows them to progress meaningfully and reach finality in relation to the task set. In the next section that follows I discuss learners' practices which promote mathematical reasoning. During this second section of analysis I have shown that learners engaged each other in rich mathematical conversation. I have shown that their engagement with the big ideas promoted their mathematical reasoning to a point where a collaborative line of reasoning is evident in their conversation. I now turn to a third section of analysis where I look at the nature of the learners' and my own practices.

4.5 Practices

Schifter (2001) defines teacher practices as being skilful, patterned regularities that occur in teachers' classrooms. Molefe (2008) argues that these practices involve the

implementation of social interactive techniques that are employed with regularity that allows learners to develop their mathematics through appropriate interaction. During my discussion on practices I use this definition on both teacher and learner practices. During this discussion I will identify and discuss learner and teacher practices evident in the data.

4.5.1 Learner practices for mathematical reasoning

Learners raise discussion in open class room discussion time.

In line 058 of the above extract we see Ester raising the topic of using bar graphs. This she does aside from the teacher's attempts to redirect the discussion. I argue here that in the light of no justifiable argument presented, Ester feels the need to place this topic back on the table for discussion. Ester is therefore pressing for justification for the exclusion of bar graphs.

Learner prompts for justification from other learners and teacher

Although Ester addresses the teacher in line 039 she is prompting for justification from Michelle as to why she is using what she calls a "straight curve" meaning smooth curve.

Learners justify their arguments presented

With no explicit prompting by teacher or learners, Michelle in line 038 justifies her use of a curve which depicts an increasing rate of change. She argues that as the water level rises the sections are becoming narrower and therefore the rate at which the water level rises is increasing. I argue here that Michelle feels the need to add an explanation for what she has done to present a more complete argument. Answers are not enough. Answers must be justifiable if it is to be accepted within the community of practice.

Learners raise justified counter arguments

Neelan and many others in the extract above, spontaneously counter mathematical constructs by other learners. These learners are not merely disagreeing but also validate or justify their reasons for countering the assertions made by other learners. We see in

The need to represent the situation depicted in the task during activity 15.1.1 serves as the catalyst for arguments and counter arguments made by learners. It is through validation by the collaboration of a reasoned argument that progress is made. Trying to find consensus around how to represent a graph depicting the rise of the water level over time, acts as a stimulus for mathematical discussion. An argument made by Ruth as solution to the tasks of graphically representing a graph depicting the situation at hand is quickly countered by both Neelan and Michelle (lines 007, 008 and 016).

Ruth's acceptance of the argument that the x axes must represent time increasing from left to right allows for Michelle to produce a representation that depicts a curve with an increasing rate of change. Michelle justifies her representation by stating that the narrowing of the container from bottom to top causes the water to rise faster (line 038).

Ester followed by Peter, Lyle, Craig and Berenice (lines 039 to 057) present a counter argument. They argue that a smooth curve does not take into account the rapidly changing widths of the container. Craig also argues for the use of line segments because the width of the various sections of the container stays the same and therefore the representation must reflect a constant rate of change.

Covariance is not mentioned as such but is fundamental in that the learners must coordinate between time and water level varying. This coordination between two varying quantities is further complicated by the introduction of the varying width which gives rise to varying rates of change. The water level is a function of time while the rate of change is a function of the width of the container.

Lyle, Ester, Berenice and Peter unknowingly threaten to derail the progression made so far to the current line of collaborative reasoning. Their introduction of the topic of bar graphs (lines 044, 058, 077 and 081) is what threatens to derail the progress made. Seeing this threat I initially try to redirect learners' attention to Michelle's curve. Redirecting the discussion away from bar graphs towards Michelle's curve allows for the discussion to

progress in a positive direction, at least for a while. It is during this time that Craig's assertions about a constant rate of change are made (lines 050 to 057).

It is Ester who forcefully reinitiates the topic of bar graphs (line 058). Her arguments are supported by Berenice (line 077) and Peter (line 081). This compels me to address the topic in greater detail (line 082). It is with an uncomfortable acceptance by the learners of my explanation that I managed to redirect discussion towards refining Michelle's representation. As uncomfortable acceptance as it may be it helps the conversation to progress through careful consideration of the big ideas of representation, covariance and slope as rate of change to generate the correct graph depicted in the situation.

I argue here that the need to understand these concepts and how they interplay, to successfully complete the task, is what contributes to healthy and productive mathematical conversations. The teacher's focus on directing conversation and therefore mathematical activity to focus on these big ideas generates in part the momentum that carries such rich mathematical reasoning. I see the big ideas initially as owned and driven by the teacher. The big ideas then become owned by learners as they become the tools through which they further explore functions.

It is the practices learners engage in, in class, which allows them to progress meaningfully and reach finality in relation to the task set. In the next section that follows I discuss learners' practices which promote mathematical reasoning. During this second section of analysis I have shown that learners engaged each other in rich mathematical conversation. I have shown that their engagement with the big ideas promoted their mathematical reasoning to a point where a collaborative line of reasoning is evident in their conversation. I now turn to a third section of analysis where I look at the nature of the learners' and my own practices.

4.5 Practices

Schifter (2001) defines teacher practices as being skilful, patterned regularities that occur in teachers' classrooms. Molefe (2008) argues that these practices involve the

implementation of social interactive techniques that are employed with regularity that allows learners to develop their mathematics through appropriate interaction. During my discussion on practices I use this definition on both teacher and learner practices. During this discussion I will identify and discuss learner and teacher practices evident in the data.

4.5.1 Learner practices for mathematical reasoning

Learners raise discussion in open class room discussion time.

In line 058 of the above extract we see Ester raising the topic of using bar graphs. This she does aside from the teacher's attempts to redirect the discussion. I argue here that in the light of no justifiable argument presented, Ester feels the need to place this topic back on the table for discussion. Ester is therefore pressing for justification for the exclusion of bar graphs.

Learner prompts for justification from other learners and teacher

Although Ester addresses the teacher in line 039 she is prompting for justification from Michelle as to why she is using what she calls a "straight curve" meaning smooth curve.

Learners justify their arguments presented

With no explicit prompting by teacher or learners, Michelle in line 038 justifies her use of a curve which depicts an increasing rate of change. She argues that as the water level rises the sections are becoming narrower and therefore the rate at which the water level rises is increasing. I argue here that Michelle feels the need to add an explanation for what she has done to present a more complete argument. Answers are not enough. Answers must be justifiable if it is to be accepted within the community of practice.

Learners raise justified counter arguments

Neelan and many others in the extract above, spontaneously counter mathematical constructs by other learners. These learners are not merely disagreeing but also validate or justify their reasons for countering the assertions made by other learners. We see in

line 007 that Neelan counters Ruth's construct and argues that it is in contrast with the convention of an increased order that is assumed by the horizontal time axes.

Learners support each other's argumentation

Here we see examples of how Michelle supports Neelan's argument for using the horizontal time axes as increasing from left to right. We also see Ester and Peter supporting the argument for a graph that represents the sudden changes in width from one section to the next. Their argument is based on the fact that the smooth curve depicted in Michelle's graph does not capture the dynamic of the container suddenly getting narrower.

Learners shift their initial position presented by themselves in the light of overwhelming evidence or justified argumentation

Ruth's: "Ah!" in line 018 from the extract as well as later during lines 091 – 093 indicate that she has shifted from her initial position and has accepted the overwhelming evidence against her initial position.

Learners challenge arguments that appear to have been swept under the carpet with no justification

Ester raises the discussion of bar graphs as possibly representing the situation of the sudden changes in the shape of the container better. Ester's attempt at reopening the discussion on this topic again I infer as a challenge to a lack of justification on my part for why the use of a bar graph is inappropriate in this situation. She is therefore pressing me for more justification.

As the "old timer" within the classroom practice it is incumbent on the teacher to mediate learning that upholds the integrity of the learner, mathematical discourse and the intended curriculum. Viewed from this perspective I now turn to teacher practices in the next section.

4.5.2 Teacher practices

During my analysis I have identified a number of teacher practices that promote meaningful mathematical activity in the classroom.

Introduces existing mathematical conventions into discourse

The object of this lesson from which the “Zig Zag” extract comes, was amongst others, for learners to work with straight line and curved graphs. The introduction of statistical graphs was both unexpected and at the time seemed to distract from the concepts and content under discussion. The teacher is faced with a dilemma when Lyle introduces the possibility of using bar graphs. Does he allow for Lyle’s thoughts around bar graphs to be the focus of discussion at this time or does he keep the discussion focused on why Michelle used a curve to represent the situation. The teacher chose to redirect discussion towards Michelle’s curve by stating that bar graphs are used in statistics.

Promotes mathematical arguments as authority for decision making

The teacher is the initiator of mathematical activity for investigation. He is therefore also the initiator of the mathematical arguments he hopes to elicit from learners.

As an “old timer” the teacher holds the power to direct and redirect the focus of discussion. The teacher therefore can focus discussion on the mathematical validity of the arguments or the people that makes those arguments. See lines 017, 039, 040, 049, 051, 097 where the teacher revoice learners’ assertions made with an attempt to raise conversation about these arguments made by learners.

Through the teacher pressing for justification he establishes not the originator of the argument but the argument itself as the authority whereby claims are accepted. It is the community that through interrogation of the validity of the claims decides its acceptance.

Assign ownership and validate learner contributions

The teacher refers to the participant from whom the assertions originate in discussion with the aim of assigning ownership of the mathematics. In this way the learners’

contributions are not only validated but the participants become co-producers and legitimate participants of the mathematics at play in the classroom.

The teacher uses learner contributions to connect mathematical objects in order to present an argument. In line 91-97 from the extract, the teacher suggests to learners to connect Ruth, Craig and Michelle's contributions to derive at a possible solution to the task. It is Miriam who initially makes this connection (line 96). Having trouble to sketch the graph as she herself suggested, it is Jason who finally sketched the correct graph. The teacher uses learner contributions to summarise lessons learnt so that learners could progress.

Participate in learning through listening

Listening is key in learning to understand learner contributions made if the learning process is to give legitimacy to learner contributions. Although I have made great effort in giving attention to listening for learners' meanings it is not always possible. During the actual lesson I was oblivious to Ruth's meanings in line 12 of Transcript1. It is now through the eyes of researcher that I hear Ruth's true meanings and can concede that Ruth's only crime was against the convention in working with the axes in an increasing order from left to right.

Active learning, I argue, also implies an openness to accept new learning by the teacher. Schifter (2001) assertion of: "They must learn to learn on their feet" (p. 115) becomes more than a figurative reference to a perspective the teacher needs to adopt but a living reality in the classroom. Through active listening and learning to learners meanings I was able to realise my error in over generalising the notion of direct and indirect proportion. It follows Craig's almost perfect explanation for the graph depicted in activity 15.1.2. Table 5 is an extract from the interaction notes I have made which I used to conduct the coding. This particular section comes from lesson 2 day 3 indicated on Table 2. A discussion on the table is given after the table.

IA-6	Teacher uses this moment to introduce direct and indirect proportion. Stating that with direct proportion both quantities increase or decrease and with indirect proportion one quantity increases while the other decreases.
IA-7	Zwelile asks what would it be called when one is increasing and the other is at a standstill. Teacher is caught off guard and indicates that it is a good question. Zwelile elaborates on her question and refers to F to G. Teacher asks whether one would call that a direct proportion. Pholani responds by saying it would be indirect and Zwelile concurs. Teacher suggests that we should think about what Zwelile is saying and that we might come up with an answer by the end of this period. Teacher suggests that learners use their prior knowledge of proportion in the domain of similarity. Asks class to think back to what a proportion means and recap that it is when one quantity divided by another is equal to something divided by something else. Explains on board that if $1/2 = 4/8$ that it is a proportion. Teacher indicates that $1/2 = 4/8$ is a direct proportion because the top one increases and the bottom increases.

Table 5:

In this extract I prematurely introduce the notion of direct and indirect proportion. During this introduction I state that a direct proportion is when both dependent and independent variables increase or decrease at the same time. I also assert that an indirect proportion is found when the one varying quantity increases as the other is decreasing. In the context of direct and indirect proportion these statements are fine to make but in the context of generalising increase and decrease and constant functions it creates the notion that all increasing functions are direct proportions and all decreasing functions are indirect proportions.

It is in listening and attending to the meanings in Zwelile's assertions that moves me to evaluate my assertions made in relation to direct and indirect proportion. Zwelile's assertion moves me to try and reconstruct my own understanding about these topics from the basis of proportion as it appears in similar shapes. It is being faced with cognitive conflict that I found myself reconstructing my own understanding. I need to state that later that year during a lesson on direct and indirect proportion my understanding of direct and indirect proportion was refined to the tee.

A thorough conceptual and content knowledge is vital if the teacher is to guide mathematical activity through the use of learner contributions. Understanding key elements that give rise to new learning will assist the teacher in orchestrating learning activities that gives rise to learning for his or her learners.

Giving learners time to learn

As teachers driven by the content demands of a loaded curriculum to complete, it is easy to lose sight of allowing learners enough time to learn. Even in light of embracing reform notions and perspectives of learning, being faced with completing the content to be covered for examination and limited time, content coverage becomes the premise for learning. However spectacular fetes are achieved as we've seen in Table 3 where Tiego comes up with a conjecture for the linear function between altitude and temperature through intuitive exploration of the rate of change. Line 154 below from Transcript 2 is taken during an interaction during lesson 2 activity 15.2.1.

Transcript 2 (see Appendix K for full transcript)

146. Teacher: Ja but... What we did was.. Listen carefully. I'm just going to use her book cause I just see it here. What did she do? She saw at seven it's approximately thirty comma five and the graph verifies that. We justified that by his (Pholani) calculation that made sense. Here we weren't too sure because when he (Points to Pholani) drew the graph it showed something different. Because that graph was very inaccurate. So what he then did was ... He actually justified it now. With what? With a calculation. Working out from the pattern. So I'm saying, use the same kind of logic. Try and find a pattern to come to that solution. (long pause) Think quickly. We can't think quickly. (Pause) Let's skip that one and come back. Next. Okay, ...

In this extract I was pressing for an algorithmic verification after learners used estimations and graph reading to find a solution to question 3.2 in activity 15.2.1. After directing learners to use a method, similar to that of Pholani, to try to find a pattern that can be used in solving the problem I pause for quite a while. I then moved on to the next question and left learners with the responsibility to find a solution. It is a while after Pholani gave most of his group's feedback to this activity that Tiego presents his

but the participants become co-producers and
mathematics at play in the classroom.

ons to connect mathematical objects in order to present
extract, the teacher suggests to learners to connect
tions to derive at a possible solution to the task. It is
nection (line 96). Having trouble to sketch the graph
who finally sketched the correct graph. The teacher
arise lessons learnt so that learners could progress.

ening
rstand learner contributions made if the learning
ner contributions. Although I have made great effort
earners' meanings it is not always possible. During the
h's meanings in line 12 of Transcript1. It is now
I hear Ruth's true meanings and can concede that
onvention in working with the axes in an increasing

as an openness to accept new learning by the teacher.
must learn to learn on their feet" (p. 115) becomes
a perspective the teacher needs to adopt but a living
ative listening and learning to learners meanings I was
eralising the notion of direct and indirect proportion. It
anation for the graph depicted in activity 15.1.2. Table
notes I have made which I used to conduct the coding.
lesson 2 day 3 indicated on Table 2. A discussion on

l indirect proportion. Stating that with direct
nd with indirect proportion one quantity

increasing and the other is at a standstill.
s a good question. Zwelile elaborates on her
ier one would call that a direct proportion.
nd Zwelile concurs. Teacher suggests that
that we might come up with an answer by
ers use their prior knowledge of proportion
ck to what a proportion means and recap
equal to something divided by something
proportion. Teacher indicates that $1/2 = 4/8$
s and the bottom increases.

irect and indirect proportion. During
n both dependent and independent
assert that an indirect proportion is
other is decreasing. In the context
fine to make but in the context of
ions it creates the notion that all
reasing functions are indirect

lile's assertions that moves me to
ndirect proportion. Zwelile's
lerstanding about these topics from
It is being faced with cognitive
lerstanding. I need to state that
proportion my understanding of

contributions are not only validated but the participants become co-producers and legitimate participants of the mathematics at play in the classroom.

The teacher uses learner contributions to connect mathematical objects in order to present an argument. In line 91-97 from the extract, the teacher suggests to learners to connect Ruth, Craig and Michelle's contributions to derive at a possible solution to the task. It is Miriam who initially makes this connection (line 96). Having trouble to sketch the graph as she herself suggested, it is Jason who finally sketched the correct graph. The teacher uses learner contributions to summarise lessons learnt so that learners could progress.

Participate in learning through listening

Listening is key in learning to understand learner contributions made if the learning process is to give legitimacy to learner contributions. Although I have made great effort in giving attention to listening for learners' meanings it is not always possible. During the actual lesson I was oblivious to Ruth's meanings in line 12 of Transcript1. It is now through the eyes of researcher that I hear Ruth's true meanings and can concede that Ruth's only crime was against the convention in working with the axes in an increasing order from left to right.

Active learning, I argue, also implies an openness to accept new learning by the teacher. Schifter (2001) assertion of: "They must learn to learn on their feet" (p. 115) becomes more than a figurative reference to a perspective the teacher needs to adopt but a living reality in the classroom. Through active listening and learning to learners meanings I was able to realise my error in over generalising the notion of direct and indirect proportion. It follows Craig's almost perfect explanation for the graph depicted in activity 15.1.2. Table 5 is an extract from the interaction notes I have made which I used to conduct the coding. This particular section comes from lesson 2 day 3 indicated on Table 2. A discussion on the table is given after the table.

IA-6	Teacher uses this moment to introduce direct and indirect proportion. Stating that with direct proportion both quantities increase or decrease and with indirect proportion one quantity increases while the other decreases.
IA-7	Zwelile asks what would it be called when one is increasing and the other is at a standstill. Teacher is caught off guard and indicates that it is a good question. Zwelile elaborates on her question and refers to F to G. Teacher asks whether one would call that a direct proportion. Pholani responds by saying it would be indirect and Zwelile concurs. Teacher suggests that we should think about what Zwelile is saying and that we might come up with an answer by the end of this period. Teacher suggests that learners use their prior knowledge of proportion in the domain of similarity. Asks class to think back to what a proportion means and recap that it is when one quantity divided by another is equal to something divided by something else. Explains on board that if $1/2 = 4/8$ that it is a proportion. Teacher indicates that $1/2 = 4/8$ is a direct proportion because the top one increases and the bottom increases.

Table 5:

In this extract I prematurely introduce the notion of direct and indirect proportion. During this introduction I state that a direct proportion is when both dependent and independent variables increase or decrease at the same time. I also assert that an indirect proportion is found when the one varying quantity increases as the other is decreasing. In the context of direct and indirect proportion these statements are fine to make but in the context of generalising increase and decrease and constant functions it creates the notion that all increasing functions are direct proportions and all decreasing functions are indirect proportions.

It is in listening and attending to the meanings in Zwelile's assertions that moves me to evaluate my assertions made in relation to direct and indirect proportion. Zwelile's assertion moves me to try and reconstruct my own understanding about these topics from the basis of proportion as it appears in similar shapes. It is being faced with cognitive conflict that I found myself reconstructing my own understanding. I need to state that later that year during a lesson on direct and indirect proportion my understanding of direct and indirect proportion was refined to the tee.

conjecture. I argue here that Tiego came up with this conjecture because he was given the time and the responsibility to do this.

During the third and final part of my analysis I have shown that both teacher and learners engaged in practices that promoted learning mathematics through mathematical reasoning. I argue that the practices the teacher engages in promotes the acceptance by learners of the mathematical norms for classroom practices that promote mathematical reasoning.

Chapter 5: Conclusion and implications for research

In this chapter I will address the following questions: What meaningful contributions does this research add to the education community? What are the limitations and challenges highlighted by this research? In attempting to address these questions this chapter is divided into the following headings: Summary and discussion, Answering my research questions, Limitations and Conclusion.

5.1 Summary and discussion

I set out to conduct research with the aim of changing my classroom practices. I set out to create a community of practice where learners engage in genuine mathematical activity. I chose functions as a topic because it is a topic where many mathematical constructs fuse to form an integrated whole.

I chose to work with big ideas in functions for it proposed to be a vehicle around which mathematical content can be focused to achieve meaningful learning. Meaningful learning, as I view it, is reasoned learning. It is learning that connects new and old learning into a meaningful whole. Because knowledge is situated and therefore dependent on the context in which it originated, I chose to work with conceptual fields. Working in conceptual fields held the promise of a more flexible concept construction. In working with concepts in different contexts that illuminate the subtle nuances of that concept and its functions within the broader spectrum of mathematics, I envisioned the learner having better access to the mathematical concepts when called upon.

I chose tasks as an entry point into mathematical activity because learning assumes the learner takes an active role as he/she constructs his/her knowledge while engaged in seeking a solution to the problem in the task. Tasks alone are not sufficient for meaningful learning to occur. The prior knowledge and language competency the learner brings with and with which he or she views the tasks could fundamentally change the task as was intended by the teacher. The teacher's interactions with learners can also change the task. The teacher is therefore charged with staying close to all interactions that occur

in order to ascertain the level of intervention required. Teacher intervention must follow an active engagement with learner interactions such as Schifter (2001) advocates in that the teacher *must learn to learn on their feet*.

The use of *tasks* (Stein, Grover, and Henningsen 1996) to engage learners with the big ideas in functions proved to have been invaluable. Coding revealed that learners engage in all seven of the big ideas in functions during the four introductory lessons on functions. This finding emphasises the value tasks hold for allowing learners to engage with mathematical concepts using high levels of mathematical reasoning and sense making. The varying degree with which learners engaged in the seven big ideas across activities suggests that the tasks both *enabled and constrained* (Stein, Grover, and Henningsen 1996) engagement with the seven big ideas. From a *constructivist perspective* (Hatano 1996) tasks are a vital component for allowing learners to engage in mathematical activity which in turn allows learners to construct and restructure their own knowledge.

Learners adopted practices similar to the way in which mathematicians work. As the mathematical activity in the class progressed so did the learner's take up of certain practices such as justification, challenging for justification, reflection and generalisation. This is in line with a situated perspective on learning as described by Lave (1993) and Lave and Wenger (1991) where learning is viewed as increased participation in a community of practice. The adoption of new learning practices was mediated through deliberate teacher practices. It is in establishing *sociomathematical* norms (Kazemi and Stipek 2001) for mathematical reasoning by the teacher that led learners to become more active participants within the community of practice.

Even though individual learners presented arguments for justification it would be a contradiction to say only individual reasoning took place. Mathematical reasoning is a *communicative process* (Ball and Bass 2003; Brodie 2010) and therefore collaborative in nature. Learners engaged in mathematical reasoning either to convince others or themselves of the truth of mathematical ideas or constructs. From a socio-cultural perspective on learning I argue that the tasks, teacher and other learners served as

instruments in creating a ZPD (Vygotsky 1978) in that it allowed learners to develop their mathematical knowledge beyond what they could accomplish by themselves. Analysis has also revealed that learners engaged more directly in collaborative lines of arguments as argued by Staples (2007). This could be seen from how learners collaboratively constructed the solution during the zig zag problem.

Analysis revealed that the *big ideas* (Lampert 2001) served as an impetus for mathematical reasoning to occur. As learners strived to use the big ideas correctly they engaged in mathematical reasoning. The best example of collaborative argumentation was how the zig zag graph was refined through mathematical reasoning to a graph consisting of four line segments, each constant but increasing rates of change. Here we see how the big ideas improved conceptual understanding (Woodbury 2000) through mathematical reasoning. I argued that it is through the teacher making explicit the object of study (Chazan 2000) which allowed learners to engage with them.

Mathematical reasoning being a line of argument for justification, positions mathematical reasoning as a communicative process. The social norms of the classroom therefore need to make way for socio-mathematical norms. Interaction focused on mathematical constructs between participants need to be forged with explanation and justification.

Collaboration between participants in a discursive environment is essential if any progress is to be seen. Collaboration embraces the integrity of the learner as a legitimate participant. Collaboration also embraces the interdependence of the participants during the learning process. Mathematics is a social construct and its meanings are negotiated and accepted through reasoned argumentation.

5.2 Answering my research questions

Central to any findings is my critical research question which I repeat below.

How can teaching the big ideas of functions to grade 9 learners support teaching mathematical reasoning while meeting the content demands of the curriculum?

As indicated in chapter one I will endeavour to answer this critical question through four sub-questions. The first of these questions is: *Does the set of activities provide opportunities for learners to engage with the big ideas of functions?*

Analysis of the data clearly showed that tasks can be developed and used by the teacher that focuses mathematical activity on big ideas identified in functions. All interactions around the tasks recorded engagement by learners with the seven big ideas identified. The big ideas become the focus around which mathematical constructs were organised in a reasoned manner to give meaning to the mathematical activity.

The second sub-question: *How are the big ideas related as students engage in the activities?* Reflecting on my analyses it is evident that strong connections exist between four of the seven big ideas in functions namely representation, covariance, variable and multiplicative relationships. The big idea of additive relationships recorded the lowest count when coded for in learner contributions during mathematical activity. I argued that this big idea is localised to aspects of functions such as linear functions and scaling of axes for graphing.

My analysis indicated that slope and pattern displayed different dispersion frequency patterns to that of representation, covariance, variable and multiplicative relationships. I have argued that both these two big ideas must be taught explicitly because they are localised in the teaching of specific aspects of functions.

The third sub-question asks: *What mathematical reasoning takes place as students engage with the big ideas of functions in these activities?* Analysis has shown that learners engaged in collaborative lines of argumentation. Learners collaboratively

constructed solutions to problems. Through a series of contested assertions and counter assertions solutions to problems were found within a step by step justification process.

The fourth question: *What teacher and learner practices enable the mathematical reasoning described in the third question?* Both learner and teacher practices afforded and constrained learning. Within the community of practice we see the newcomers adopting the practices of the community as set out by the old timer. Mathematical activity became infused with arguments and counter arguments among participants for justification. If unchecked these arguments can become a reduced to merely asserting ones identity within the community of practice (Chazan and Ball 1999). Arguments solely focused on asserting one's own authority threaten to devalue the process of argumentation for the formation of a collective authority. A collective authority through which mathematical knowledge becomes accepted within the community is therefore established through progressive argumentation.

Through the active promotion of socio-mathematical norms which govern mathematical conversation, threats to progressive lines of reasoning can be overcome. Establishing norms for effective mathematical activity within a classroom context must be driven by the teacher through teacher practices that allows for the establishment of sound mathematical reasoned arguments to become the authority for decision making. From a situated perspective, learners will become full participants as they accept or counter the socio-mathematical norms established by the teacher.

5.3 Limitations

The lessons I had videotaped comprised only the introductory lessons to functions. The lessons therefore aimed to allow learners to explore and construct the tools of functions needed for further exploration and problem solving. This meant that limited follow up on learner take up of the concepts could be analysed. Whether learners developed more flexible knowledge schemes cannot be verified. Due to the limited scope of this project analysis were restricted to the introductory lessons only. Whether learners were able to use the knowledge they acquired during this period in other mathematical contexts and situation could not be verified.

A number of times during the analysis phase of the data I found myself wishing that I could go back and ask learners and check my current understanding of their

A number of times during the analysis phase of the data I found myself wishing that I could go back and ask learners and check my current understanding of their contributions. Follow up interviews would have added tremendous value to inferences I made based on my observations and guided by the literature.

Working over a short term restricted my claim to have worked effectively in conceptual fields. Working in conceptual fields assumed working across content and context where particular concepts can be explored with subtle differences in use and meaning. It is working within a conceptual field where the true nature of a big idea is both deepened and broadened. One of my objectives during the next cycle of action research is to work with big ideas across mathematical content and context domains to understand how conceptual fields can be successfully implemented.

Grade 9 content on functions is traditionally restricted to linear functions. This restriction limited my exploration of teaching big ideas in a broader perspective of functions. This being said, I did make attempts at including other functions in the task design that traditionally goes beyond grade 9 level. Sections which went beyond grade 9 level, although challenging to learners, helped create a broader perspective on functions.

5.4 Conclusion

Aside from the educational benefits brought on by conducting action research it is a humbling experience. A scholar once said: "Become the change you want to see in the world". Action research is driven by this exact sentiment. In my search to render my classroom a hive of meaningful mathematical activity I have encountered both challenges and successes.

The realisation that mathematical discourse is a powerful tool to be used to infuse classroom activity with mathematical reasoning has infused me with new vigour in my teaching. The implementations of goal directed teacher practices are vital in creating a discourse where learners are able to learn with meaning. It is in implementing these

practices that learners have the possibility to become full participants in the community of practice through the actions of a consistently and exemplary old timer.

I have also come to realise that my own mathematical knowledge is not complete but has a number of inconsistencies stemming from my own learning experiences. As a mathematics teacher I have to become an active learner and doer of mathematics in order to create an optimal learning environment for my learners. To teach effectively, one needs to know how to learn effectively.

Active learning also means listening and learning from learner contributions in discussion. This is not an easy task to accomplish as the teacher needs to take other agendas into account in the classroom. Agendas that are not of his/her own creation but stem from the social reality that we live in. In accommodating the social agendas the teacher must become an effective manager of dilemmas (Brodie 2010). However if we have to give legitimacy to learning in our classrooms effective teacher practices should be developed to assist teachers in this task.

References:

- Adler, J. (2001). *Teaching and learning mathematics in multilingual classrooms*. Kluwer Academic Publishers: Dordrecht, Netherlands.
- Ball, D. L. and Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, and D. Schifter (Eds). *A Research Companion to Principals and Standards for School Mathematics* (pp. 27 – 44). Reston, VA: National Council of Teachers of Mathematics.
- Boaler, J. (1997). *Experiencing School Mathematics: Teaching Styles, Sex and Setting*, Buckingham: Open University Press.
- Brodie, K. and Pournara, C. (2005). Towards a framework for developing and researching groupwork in South African classrooms. In: Vital, R., Adler, J., Keitel, C. (Eds). *Mathematics education research in South Africa: possibilities and challenges*. Human Science Research Council, Pretoria.
- Brodie, K., Lelliott, T., and Davis, H. (2002). *Developing learner-centred practices through the FDE programme*. Challenges of teacher development: an investigation of take-up in South Africa. Adler, J. and Reed, Y. Pretoria, Van Schaik.
- Brodie, K. (2010). *Teaching Mathematical reasoning in Secondary School Classrooms*. Springer New York Dordrecht Heidelberg London.
- Chazan, D. (2000). *Beyond Formulas in Mathematics and Teaching: Dynamics of the high school algebra classroom*. P. Wasley, A. Lieberman & J. McDonald (Eds). Teachers College Press. Teachers College. Columbia University. New York and London. (pp. 59 – 111).
- Chazan, D. and Ball, D., (1999). *Beyond being told not to tell*. For the learning of mathematics, 19, 2, (pp. 2 – 10).
- Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, 28 (3), (pp. 355 – 276).
- Maths for all Grade 9 Learner's Activity Book (2001). Macmillan Botswana Publishers (Pty) Ltd.

- National Department of Education (2003). National Curriculum Statement. South Africa.
- Ernest, P. (1993). Constructivism and the problem of the Social. Julie, D. Angelis & Davis, Z. (Eds). *Political Dimensions of Mathematics Education*. Second Conference proceedings. Maskew Miller Longman, Cape Town
- Even, R. (1998). Factors Involved in Linking Representations of Functions. *Journal of Mathematical Behavior*, 17 (1), (pp. 105 – 121).
- Gipps, C. (1999). Socio-cultural aspects of assessment. *Review of research in Education*, 24, (pp. 355 – 392).
- Griffiths, M. (1998). *Educational Research for Social Justice: Getting of the fence*. Open University Press: Buckingham. Philadelphia. Chapter 9: Better knowledge (pp. 29 – 140).
- Hatano, G. (1996). *A concept of knowledge acquisition and its implications for mathematics education*. In Steffe, L., Nesher, P., Cobb, P. and Goldin, G (Eds) *Theories of mathematical learning*. New Jersey: Lawrence Erlbaum, (pp. 197 – 217).
- Hatch, J. A. (2002). Doing qualitative research in educational settings. SUNY. New York. Chapter 4 (pp. 147 – 210).
- Kazemi, E. and Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary school Journal*, 102, (pp. 59 – 80).
- Kemmis, S. and McTaggart. (1988). *The Action Research Planner*. Chapters 1. Deakin University.
- Kilpatrick, J., Swafford, J. and Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Research Council. National Academy press Washington, DC
- Lampert, M. (2001). *Teaching Problems and the Problems of Teaching*. Yale University Press. New Haven & London (pp. 213 – 263).
- Laridon, P., Barnes, H., Jawurek, A., Kitto, A., Pike, M., Sigabi, M., Tebeila, S., Wilson, H. (2001). *Classroom Mathematics Grade 9*. Heineman publishers (Pty) Ltd.
- Lave, J. and Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. New York: Cambridge University Press.

practices that learners have the possibility to become full participants in the community of practice through the actions of a consistently and exemplary old timer.

I have also come to realise that my own mathematical knowledge is not complete but has a number of inconsistencies stemming from my own learning experiences. As a mathematics teacher I have to become an active learner and doer of mathematics in order to create an optimal learning environment for my learners. To teach effectively, one needs to know how to learn effectively.

Active learning also means listening and learning from learner contributions in discussion. This is not an easy task to accomplish as the teacher needs to take other agendas into account in the classroom. Agendas that are not of his/her own creation but stem from the social reality that we live in. In accommodating the social agendas the teacher must become an effective manager of dilemmas (Brodie 2010). However if we have to give legitimacy to learning in our classrooms effective teacher practices should be developed to assist teachers in this task.

References:

- Adler, J. (2001). *Teaching and learning mathematics in multilingual classrooms*. Kluwer Academic Publishers: Dordrecht, Netherlands.
- Ball, D. L. and Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, and D. Schifter (Eds). *A Research Companion to Principals and Standards for School Mathematics* (pp. 27 – 44). Reston, VA: National Council of Teachers of Mathematics.
- Boaler, J. (1997). *Experiencing School Mathematics: Teaching Styles, Sex and Setting*, Buckingham: Open University Press.
- Brodie, K. and Pournara, C. (2005). Towards a framework for developing and researching groupwork in South African classrooms. In: Vital, R., Adler, J., Keitel, C. (Eds). *Mathematics education research in South Africa: possibilities and challenges*. Human Science Research Council, Pretoria.
- Brodie, K., Lelliott, T., and Davis, H. (2002). *Developing learner-centred practices through the FDE programme*. Challenges of teacher development: an investigation of take-up in South Africa. Adler, J. and Reed, Y. Pretoria, Van Schaik.
- Brodie, K. (2010). *Teaching Mathematical reasoning in Secondary School Classrooms*. Springer New York Dordrecht Heidelberg London.
- Chazan, D. (2000). *Beyond Formulas in Mathematics and Teaching: Dynamics of the high school algebra classroom*. P. Wasley, A. Lieberman & j. McDonald (Eds). Teachers Colledge Press. Teachers Colledgez. Columbia University. New York and London. (pp. 59 – 111).
- Chazan, D. and Ball, D., (1999). *Beyond being told not to tell*. For the learning of mathematics, 19, 2, (pp. 2 – 10).
- Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, 28 (3), (pp. 355 – 276).
- Maths for all Grade 9 Learner's Activity Book (2001). Macmillan Boleswa Publishers (Pty) Ltd.

- National Department of Education (2003). National Curriculum Statement. South Africa.
- Ernest, P. (1993). Constructivism and the problem of the Social. Julie, D. Angelis & Davis, Z. (Eds). *Political Dimensions of Mathematics Education*. Second Conference proceedings. Maskew Miller Longman, Cape Town
- Even, R. (1998). Factors Involved in Linking Representations of Functions. *Journal of Mathematical Behavior*, 17 (1), (pp. 105 – 121).
- Gipps, C. (1999). Socio-cultural aspects of assessment. *Review of research in Education*, 24, (pp. 355 – 392).
- Griffiths, M. (1998). *Educational Research for Social Justice: Getting of the fence*. Open University Press: Buckingham. Philadelphia. Chapter 9: Better knowledge (pp. 29 – 140).
- Hatano, G. (1996). *A concept of knowledge acquisition and its implications for mathematics education*. In Steffe, L., Nesher, P., Cobb, P. and Goldin, G (Eds) *Theories of mathematical learning*. New Jersey: Lawrence Erlbaum, (pp. 197 – 217).
- Hatch, J. A. (2002). Doing qualitative research in educational settings. SUNY. New York. Chapter 4 (pp. 147 – 210).
- Kazemi, E. and Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary school Journal*, 102, (pp. 59 – 80).
- Kemmis, S. and McTaggart. (1988). *The Action Research Planner*. Chapters 1. Deakin University.
- Kilpatrick, J., Swafford, J. and Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Research Council. National Academy press Washington, DC
- Lampert, M. (2001). *Teaching Problems and the Problems of Teaching*. Yale University Press. New Haven & London (pp. 213 – 263).
- Laridon, P., Barnes, H., Jawurek, A., Kitto, A., Pike, M., Sigabi, M., Tebeila, S., Wilson, H. (2001). *Classroom Mathematics Grade 9*. Heineman publishers (Pty) Ltd.
- Lave, J. and Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. New York: Cambridge University Press.

- Lave, J. (1993) Situating learning in communities of practice. In: Reanick LB, Levine JM, Teasley SD (Eds) *Perspectives on socially shared cognition*. American Psychological Association, Washington, DC, (pp. 63 – 84).
- Leinhardt, G., Zaslavsky, O., and Stein, M. (1990). Functions, Graphs, and Graphing: Tasks, Learning, and Teaching. *Review of Research in Education*. Vol. 60, No. 1, (pp. 1 – 64)
- Maxwell, J. A. (1992). Understanding and Validity in Qualitative Research in *Harvard Educational Review*, 62 (3) (pp. 279 – 300).
- Mercer, E., (1995). *The Guided construction of Knowledge*. Multilingual Matters. Clevedon. Chapter 5
- Miles, MB. and Huberman, AM. (1994). *An Expanded Sourcebook: Qualitative Data Analysis*. SAGE publications, International Educational and Professional Publisher. Thousand Oaks. London, New Delhi
- Molefe, N. (2008). Unpublished Masters Research Project. University of the Witwatersrand. South Africa.
- Opie, C. (2004). *Doing Educational Research – A Guide to First Time Researchers*. London. Sage Publications.
- Rand Mathematics Study Panel (2002). Teaching and Learning Mathematical Practices Revised National Curriculum Statement Grades R-9 (Schools) (2002). Department of Education. Pretoria
- Scaife, J. (2004). Reliability, Validity and Credibility. In C. Opie (Eds). *Doing Educational Research*. Sage. London. (pp. 58 – 72).
- Schifter, D. (2001). Learning to see the invisible. What skills and knowledge are needed in order to engage with students' mathematics ideas? In Wood, T., Nelson BS. & Warfield, J. (Eds.), *Beyond Classical Pedagogy: Teaching elementary Mathematics* (pp. 109 – 134). Mahwah, NJ: Lawrence Erlbaum Associates.
- Silverman, J. (2005). An Investigation Of Content Knowledge For Teaching: Understanding Its Developments and Its Influence on Pedagogy. Dissertation Submitted to the Faculty of the Graduate School of Vanderbilt University in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Teaching and Learning August, 2005 Nashville, Tennessee

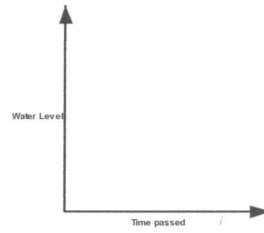
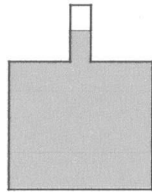
- Staples, M. (2007). Supporting Whole-Class Collaborative Inquiry in a Secondary Mathematics Classroom. Mathematics Education Research Group. *MERG Publications*. Paper 1.
- Stein, MK., Grover, BW and Henningsen, M (1996). Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Task Used in Reform Classrooms. *American educational research journal Summer 1996*, Vol. 33, No. 2 (pp. 455 – 488).
- Stump, S. L. (2001). Developing preservice teachers' pedagogical content knowledge of slope. *Journal of Mathematical Behavior* 20 (2001) (pp. 207 – 227).
- Vygotsky, L. (1978). *Mind in Society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press. Chapters 4 and 6
- Wood, T., Cobb, P., and Yackel, E. (1992). *Investigating Learning Mathematics in School Classrooms: Interweaving Perspectives*. ICME Quebec.
- Woodbury, S. (2000). *Teaching Towards the Big Ideas of Algebra. Mathematics teaching in the middle school*. Vol. 6, No 4. (pp. 226 – 231).

APPENDIX A

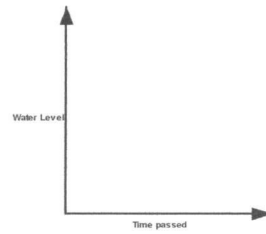
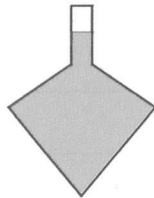
Activity 15.1.1

Water is poured into each of the following containers at constant rate. Draw rough graphs that indicate the manner in which the level of the water rises in each container.

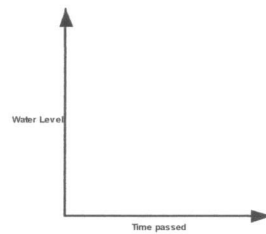
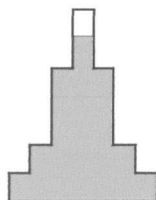
1



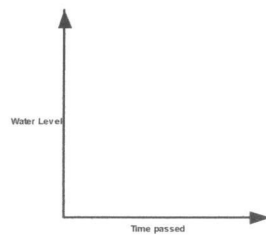
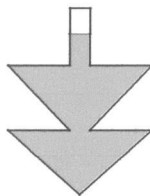
2



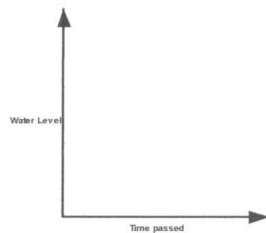
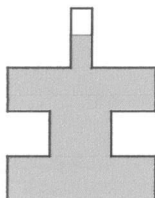
3



4



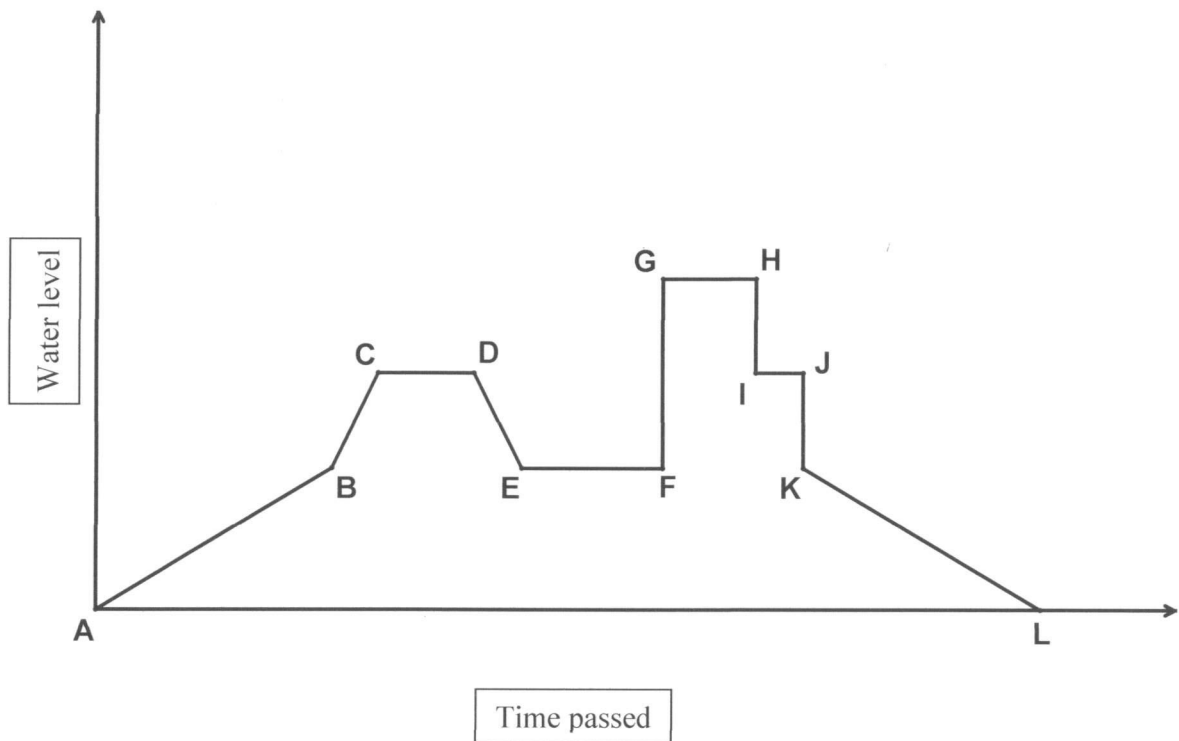
5



APPENDIX B

Activity 15.1.2

Study the graph below and then answer the following questions:



1. Write a story that fits the graph below. The graph records the filling of bath water over time.
2. Comment on what is happening in the graph as it slopes to the left or right, and as it continues in a horizontal line or vertical line.

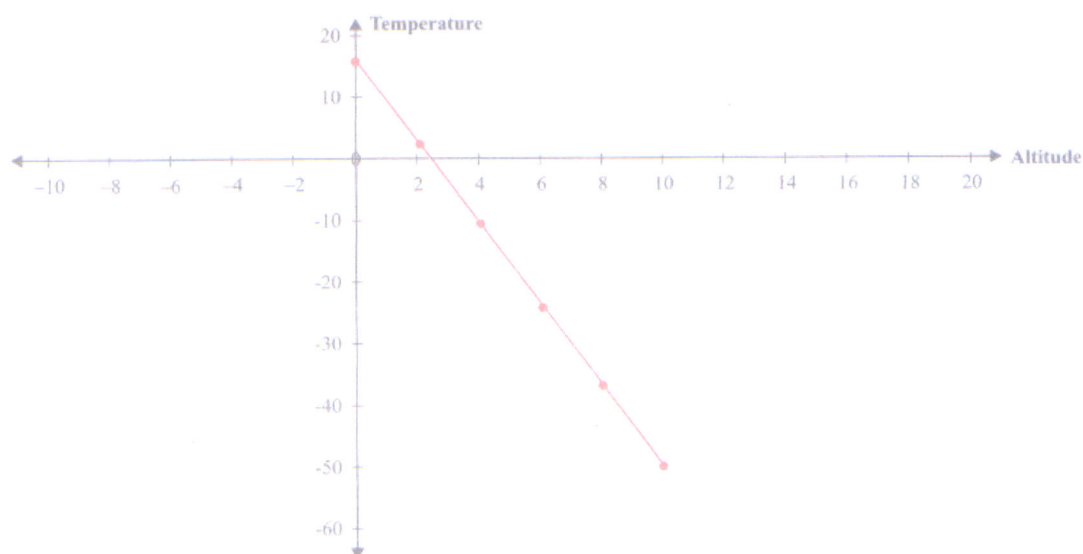
APPENDIX C

Activity 15.2.1

The approximate temperature of the first section of the earth's atmosphere, the troposphere, at different altitudes (heights above sea level) up to 10 km is given in the table:

Altitude (km)	Temperature ($^{\circ}\text{C}$)
0	15
2	2
4	-11
6	-24
8	-37
10	-50

The data in the table is plotted on the graph below:

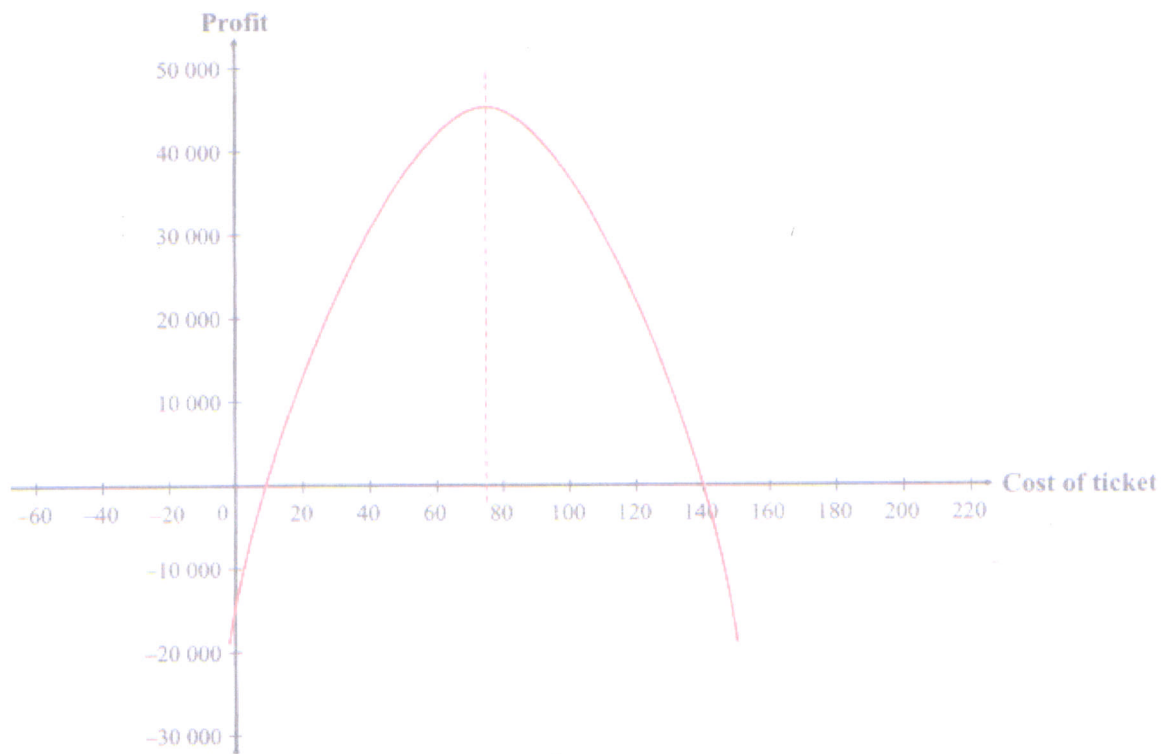


1. What does the graph tell us about the relationship between the temperature and the altitude?
2. Can we continue this graph to the left? Explain.
3. Determine approximately:
 - 3.1 The temperature at an altitude of 7 km.
 - 3.2 The altitude at which the temperature is 0°C .
4. Above 11 km the temperature remains fairly constant at -56°C . At about 13 km, due to the presence of ozone, which absorbs heat from the sun, the temperature starts to rise as the altitude increases. Show this on the graph.

APPENDIX D

Activity 15.2.2

The Skosana High School fundraising project decides to host a rave. The graph below shows us how the profit is affected by the cost of the tickets.

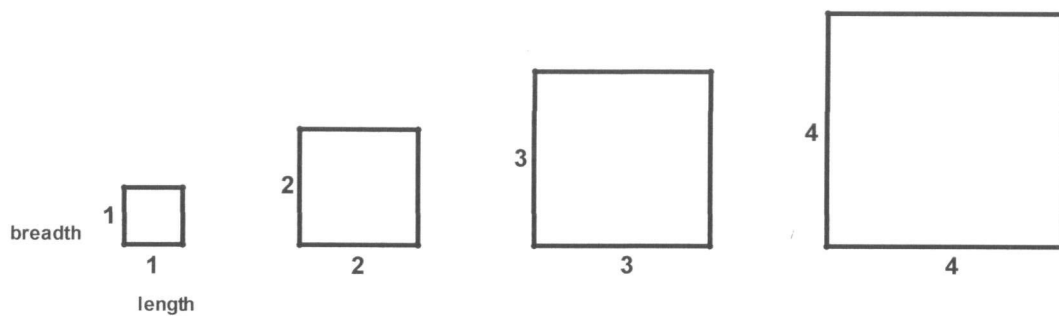


1. What does the graph tell us about the relationship between the profit and the cost of the ticket?
2. The point (0; -15 000) is on the graph. What information does this point give you?
3. What is the cost of a ticket that gives the greatest profit?
4. The points (10; 0) and (140; 0) are on the graph. What information do these points give you?
5. Why do you think the graph goes down after the point (75; 45 268)?

APPENDIX E

Activity 15.3.1

The diagram below shows a sequence of squares indicating the dimensions of the length and breadth of each square as the breadth is increased.



1. Use the above diagram and complete the following table:

Length of square	1	2	3		5		10	15	100
Breadth of square				4		6			

- 1.1 What is the relationship between the length and the breadth of the squares?
- 1.2 If the breadth of the squares is represented by x and the length by y formulate an equation representing the relationship above.
- 1.3 Plot a graph representing the relationship between the length and breadth of the squares as the breadth is increased.

2. Use the above diagram to complete the following table:

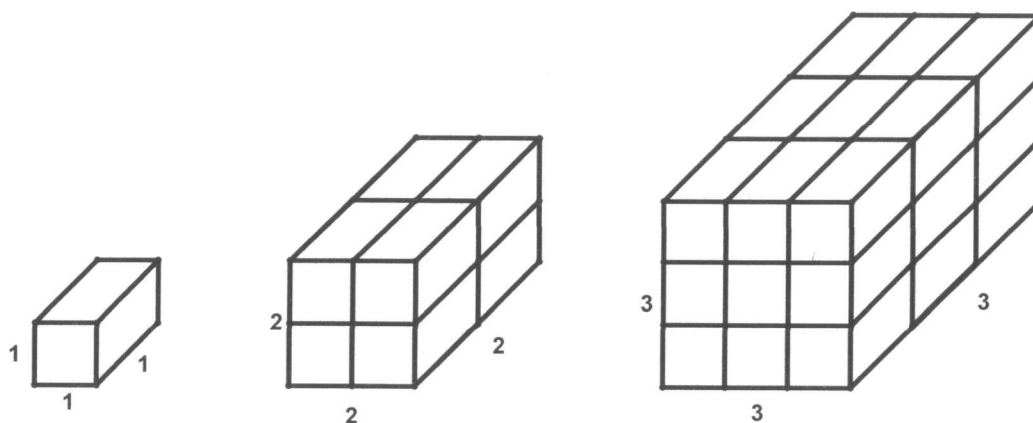
Length of sides	1	2	3		5		10	15	100
Area of square				16		36			

- 2.1 What is the relationship between the length of the sides and the area of the squares as the length is increased.
- 2.2 If the length of the squares are represented by x and the area by y express this relationship as an equation.
- 2.3 Plot a graph representing the relationship between the length and the area of the squares.

APPENDIX F

Activity 15.3.2

The sequence of cubes below show what happens when the dimensions are increased.



1. Use the diagram above to complete the table:

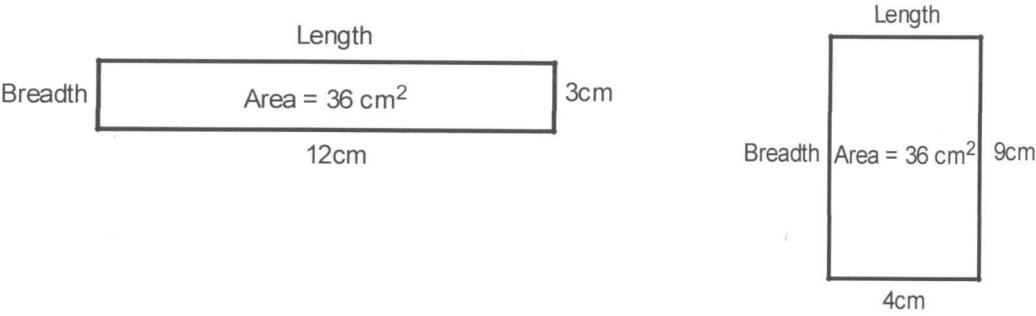
Increase in dimensions (x)	1	2	3	4	5		10	15	100
Volume of cube (y)						64			

2. What is the relationship between the increase in the dimensions of the cube and the volume of the cube?
3. If the increase in dimensions of the cubes is represented by x and its volume by y , represent the above relationship as an equation.
4. Plot a graph representing the relationship between the increase in dimensions and the volume of the cube.

APPENDIX G

Activity 15.3.3

The diagrams below show how the breadth of a rectangle changes when changing the length but at the same time keeping the area at 36 cm^2 .



1. Use the diagrams above to help you complete the table below:

Length (x)	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4	6	9	12	18	36	72	
Breadth (y)														0

2. What is the relationship between the length and the breadth of the rectangle when we keep the area the same (36 cm^2).
3. If the breadth of the rectangle is represented by y and the length by x , write down the equation that represent the relationship in 2 above.
4. Use the table above to plot the graph representing the relationship between the length and the breadth of a rectangle when the area is kept at 36 cm^2 .

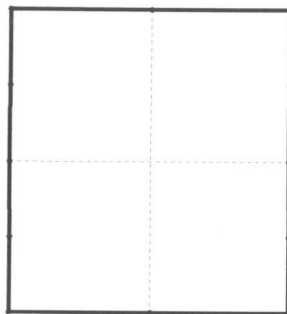
APPENDIX H

Activity 15.3.4

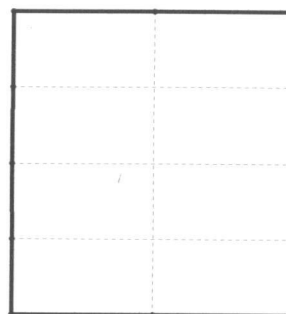
A page can be folded in half several times. The diagrams below show the relationship between the number of folds and the number of parts the page is divided into as a result of the folds.



1st fold



2nd fold



3rd fold

1. Use the diagrams above to complete the table:

Number of folds (x)	1	2	3	4	5	6	7	8	10	55
Number of parts (y)										

- Write down the relationship between the number of folds and the number of parts.
- If the number of folds are represented by x and the number of parts by y complete the following equation: $y =$ _____
- Use the table above to plot the graph representing the relationship between the number of folds and the number of parts.
- How many times can any page realistically be folded?

APPENDIX I

Activity 15.4.1

Match the tables, graphs and rules (equations). Explain how you made your decision.

Tables:

A	
x	y
-1	-1
0	0
1	1
2	8
3	27

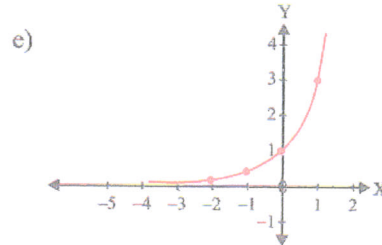
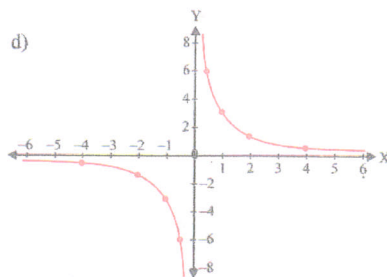
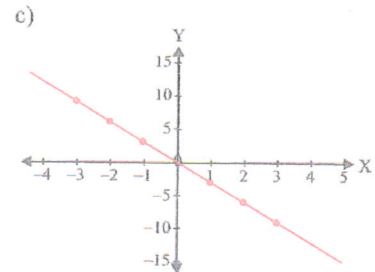
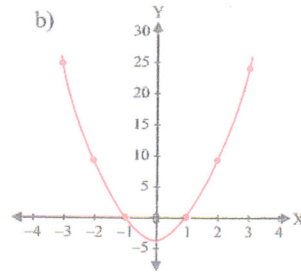
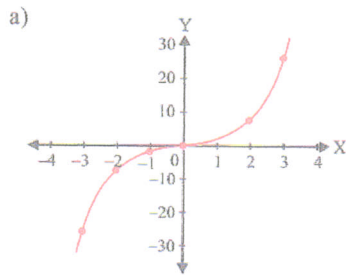
B	
x	y
-1	3
0	0
1	-3
2	-6
3	-9

C	
x	y
-2	$\frac{1}{4}$
1	3
0	1
2	9
3	27

D	
x	y
-0,5	-6
-1	-3
1	3
2	1,5
4	0,75

E	
x	y
-2	9
-1	0
1	0
2	9
3	24

Graphs:



Rules (equations):

(i) $y = x^3$

(ii) $y = 3^x$

(iii) $y = -3x$

(iv) $xy = 3$

(v) $y = 3x^2 - 3$

APPENDIX J

Transcript 1

001. Ruth: *(Ruth draws a graph consisting of four short line segments of varying lengths and slopes) Ok sir. See this bottom piece sir. (Pointing to bottom of container). It's gonna take. Like this right at the bottom. So the water has to travel (Pointing from top of container to bottom of container) right down here and then fill up that bottom piece. So it's gonna take longer (Pointing at first line segment). Then as you get to the top, this piece is slightly wider (Pointing at second section from the bottom) than this (Pointing at bottom section of container). So, it will take less time (Pointing at second line segment), it will take less time, but not like drastically. You understand sir ... (Inaudible)... Ja. Then, this piece here sir (Pointing at third section from the bottom) is like larger than both of these (Pointing at two bottom sections). It's wider. So it will take a longer time (Pointing at third line segment). It won't take as long as this (Pointing at first line segment) but it will take like a longer time. And then this is just a small piece (Pointing at top section of container) so it will take a shorter time (Pointing at top line segment) than all of them*
002. Teacher: *Ok...*
003. Ruth: *That's what I said*
004. Teacher: *We've got lots of hands. Let's hear what they have to say. Let's start with Neelan, then Jason, then..*
005. Michelle: *....Michelle....*
006. Teacher: *....Michelle and then...Neelan!*
007. Neelan: *I think that the graph... (Inaudible)... shouldn't be going to the left (Pointing with hand movement). Because that is not measuring time.....*
008. Michelle: *..... (Interrupts Neelan) Time's not going back.....*
009. Neelan: *..... It's not, it's not Uh...*
010. Ruth: *(Interrupting Neelan) Ja, but I'm not. Sir we not measuring time, like say in one minutes time this is how much it rose.... (Inaudible voices interrupting Ruth) ... Ja but I'm not. Sir we not measuring time, like say in one minutes time this is how much it rose.... (Inaudible voices interrupting Ruth)*
011. Teacher: *Ok... Sssh!... Can we, can we do this? Can we not interrupt each other while they speaking. Because if we gonna do that we going to loose what each other are saying, Ok.*
012. Ruth: *Ok sir. I was saying. I never done it like, as it say, ok in one minute this is how much it rose, in two minutes this is... I said that, for this, this.. the first section. It like took this amount, like it took. How can I explain sir? It took longer to like.. It took longer to like*

- fill up that section. And it took shorter. So I'm not saying in one minute it rose ten metres and then in two minutes it went up to twenty like that..*
013. Teacher: *No, we, we. I think they all agree. So let's just hear what they had to. Just listen to what they saying. Michelle!*
014. Michelle: *I already said what I had to say*
015. Teacher: *Ok, let's all hear what you had to say, because we all couldn't hear.*
016. Michelle: *Ok, I was just saying that, also what Neelan said, that the lines can't exactly go like "zig zag" because you can't say that like. Because isn't it sir the graph goes like during the time (Across movement of hand) and the level...(Inaudible)...(Upward movement with hand) so it would go that way (Pointing diagonally upward and to the right) not like (Showing "zig zag" movement or "snake like" movements upwards) that way*
017. Teacher: *Ok, so what you say there then, if I'm understanding you correctly, is that the line can't turn back. Because that means time is turning back. You need to consider not just what? Not just the water level but that, that line or that graph that you drawing there represents two things that's occurring at the same time. As the water level is rising time is passing...*
018. Ruth: *...Ah!*
019. Teacher: *Ok. So you need to repre...keep both in mind when you drawing that graph. Peter, Peter..*
020. Peter: *(Being disturbed as he is busy working) Yes sir?*
021. Teacher: *You had your hand up?*
022. Peter: *...(Inaudible)...I was gonna say like at the bottom of this graph it says time passed....*
....
023. Teacher: *...Yeah..*
024. Peter: *So basically that's your time, how long it takes you to fill up.*
025. Teacher: *Ok..*
026. Peter: *You can't go back*
027. Teacher: *(As Peter is completing his sentence) And Jason?*
028. Jason: *My question was answered.*
029. Teacher: *Sorry?*
030. Jason: *My question was answered.*
031. Teacher: *Your question was answered. Ok. Can somebody volunteer to fix...Ok (Miriamne's hand is up) I want other peo... (Michelle's hand goes up) Michelle! I mean there was that group over there, that group over there, that group here. One, two, three. You all did the same thing, number three. So I would like some more feedback from you guys. What did you do?*
032. Pholani: *Ja, I'll go after this.*

033. Unknown: *(Laughter and inaudible comments)*
034. Teacher: *Why do you guys do that? If he speaks you guys comment on that. Everybody speaks differently. Let's just respect each other for who they are.*
035. Unknown: *Sorry ... (Inaudible comments and laughter)*
036. Ruth: *Sir, I don't think. Sir I don't think Michelle should do this sir... Sir I don't think Michelle can do this.*
037. Unknown: *... (Inaudible) ... (teacher sort out problem with transparency pens)*
038. Michelle: *(Draws a graph that curves up with increasing rate of change) ... (Inaudible) Ok what I wrote here... over here it's taking longer (Pointing at start of curve) and as, as the graph gets smaller it will take more quicker to fill up, because it's a narrower (Pointing at top of container) like length to fill. So it will just get quicker and quicker (Running pen along curve from start to finish)*
039. Teacher: *So, you saying because, because the shape is getting narrower you find that it's, it's actually... the... the water level rises faster over time. Ok. We've got hands again it seems. One... Ester... and then...*
040. Ester: *Ok. Sir. How can it be a..... straight curve? Because sir it's like the blocks (Using both hands to show sides of sections of container) go up and then narrower (Uses both hands to show the shape of container instantly getting narrower) like in a rhythm. And that's just... (Showing curve with hand)*
041. Teacher: *Ok. So, you saying it can't just be a smooth curve (Showing curve with hand movement)... there should be some ... (Inaudible commenting)... Ok.....*
042. Peter: *Because is (Show block movement with hand)*
043. Teacher: *.... Ok. Ok let's just give.... Lyle a chance.*
044. Lyle: *Sir, is it wrong if you did a Bar graph?*
045. Teacher: *If it's wrong... Ok, Lyle asks: "Is it wrong if you did a Bar graph"*
046. Learners: *... Yes... no...*
047. Teacher: *Ok, let's just come back to that one. We'll come back to: whether it's a bar graph. But before we... Let's just discuss the curve that you (Points in Ester's direction) suggesting. What are you saying (Pointing to Peter)?*
048. Peter: *Ok sir. It mustn't be a curve because... (Can't make out)... Each block (Show blocks in a similar way as Ester) is like a sharp thing like this (Show sudden narrowing with hand). So it fills up a lot quicker. Instead of a (Hands [apart] move from bottom up to form vertex [together]). Almost like side view of cone) smooth... Uhm...*
049. Teacher: *Ok. So you say it can't.. So you say it must suddenly change direction (Using same hand movements as Peter). So it can't be a smooth curve (Show curve with hand movement). Yes (Pointing in Craig's direction)*

050. Craig: ...*(Inaudible)*...But sir. I'd also say it wouldn't be a curve because, the shape is like a constant shape and water is being filled up constantly. It's not like the shape is, like constantly changing and water is filling up at different....
051. Teacher: ...*(Interrupts Craig)*...Ok. So you saying the first part. The bottom part. It fills up constant ...at a...
052. Craig: ...*(Interrupts Teacher)*...Constant rate...Because of the shape and then...
053. Teacher: ...*(Interrupts Craig)*...suddenly *(Teacher uses hand movement to form shape of container)* the rate....changes and it stays constant *(Teacher uses hand movements to form shape of container)*...
054. Craig: Ok, yes
055. Teacher: And then suddenly the rate changes and it stays constant *(Teacher uses hand movements [shape of container] to correspond to rate of change [spoken words])*...
056. Craig: Ja.
057. Teacher: ...So what would you like to see on there *(Pointing at OHP)*. What kind of shape. What kind of things would you like to see? *(Pointing in direction of Ester)*. A curve? What you like to see?
058. Ester: Sir, Uhm I, I. Would it be wrong to use a Histogram sir? Because it could clearly show....the....like this is like in blocks that are, as they that have...*(Inaudible)*...So would a histogram be wrong?
059. Teacher: Would a Histogram be wrong?
060. Learners: No!.....exactly....
061. Craig: I prefer a line!
062. Teacher: Ok. Let's have some other inputs here.
063. Pholani: Sir. I have a question.
064. Teacher: Ja...
065. Pholani: When does like the water fill up more quickly? Is it where the container's wider or when it gets narrower?
066. Learners: Gets Narrower!
067. Pholani: When it gets narrower? That's when it's...the water fills up more faster?
068. Teacher: Obviously when your shape is larger *(Show hands wide apart)* the water level rises slower *(Move hands slowly vertically up)* but when it becomes narrower *(Hands are moved closer together)* it fills up faster *(Hand move vertically up faster than before)*, because the water that is poured in. I open the tap wide and I leave it like that. So I don't change the amount of water that gets put in. It stays the same. That's why they said the water is poured in at a constant rate. So that is not changing. The only thing that can change the level...the rate at which the water level rises is in fact the shape, of the container. Ok. I, I, I, I don't like the Histogram. I also don't like Bar graphs.

Because what we want to do is we want to see the relationship between the time passing, and your what?

069. Learner: *The water level*

070. Teacher: *And your...*

071. Learners: *Water level*

072. Teacher: *And your water level. So how am I going to show that having one bar or another bar or another bar? For bars, that means that it's goes up to there then stops. Up to there then stops. (Teacher draws bars on board and uses it to illustrate) No! It's a continuous thing that happens. It's one event. It is not how many of this do I have. How many of that do I have or different things I have. It is, this is happening. As it is happening, this graph needs to represent as the water is poured in. So it's one event, that is occurring over time. Ok. Bar graphs and Histograms are what? Are like different things at different times, that is measured. Ok. So I can say how many blue cars I have, how many red cars I have. There is no, there's no....how can I say.... link between the two. Here time is passing and the water is rising. Ok. Yes*

073. Berenice: *Sir, would it be correct to use a Bar graph if you had the time span. Like after one minute the water level rises maybe ten millimetres. Would you be able to use a Bar graph then?*

074. Teacher: *What does a bar do? A bar does this. What does a straight line do? What would that mean when you have this?*

075. Miriam: *The water stopped!*

076. Teacher: *The water stopped! Is that possible?*

077. Berenice: *Sir but ...(Inaudible)... time limit? It's like after one minute how much the...*

078. Teacher: *....(Interrupting Berenice)....Ja, but.....*

079. Berenice: *..... water level.....*

080. Teacher: *.... ti, time is carrying on. It doesn't say: one minute, stop; two minutes, stop; three minutes, stop. You don't measure after every minute? Did we? No! We just want to represent on a graph what happens when I take the container, open the tap and it runs. How it's being filled.*

081. Peter: *Sir, but you can use a Bar graph because after one minute, say it fills ten millimetres...and after another minute if the shape changes it.....*

082. Teacher: *....(Interrupts Peter)...I'm not saying you can't, you cannot do that. What I'm saying is, the situation that's being described is: Draw a graph representing what happens when this container is filled with water. Ok. So as the water level is rising it doesn't stop. So you not measuring after every minute. I'm not saying it can't be done. I'm simply saying that would represent something different. Ok. So you need to choose what you represent. And in this case we want to see as the time is passing what is*

- happening to the water level. So this line that you or curve that you representing should represent what you see happens. Ok. And it ...(Inaudible)...your time and your water level. Ok now, I want to go back here (Refers to number one in activity 15.1.1). Because I can see people are stuck. Why did we have a straight line there? Yes (Points in Craig's direction)*
083. Craig: *Because the container is being filled up at a constant...(Inaudible)*
084. Teacher: *Ok, fine. Would you say that for this first part (Referring to bottom section in number three) the water is being filled up at a constant rate?*
085. Learners: *Yes!*
086. Teacher: *So why would you want to have a curve? Why can't we represent it with the same thing, a straight line?*
087. Learners: *Yes!*
088. Michelle: *Sir, because on the second level, if we can call it that, of the drawing it uhm, how do I. It's like smaller. The width is smaller. So it won't take as....*
089. Teacher: *....(Interrupts Michelle) Ok. Can I....*
090. Michelle: *.....long to fill up*
091. Teacher: *.....Can I ask you another question? Ok. What Ruth did was: She, she, she had little lines. What mistake did she do?*
092. Michelle: *She went backwards in time.*
093. Ruth: *I went backwards in time (Ruth says this and show with her hand at the same time as Michelle above)*
094. Teacher: *She went backwards in time. Which she can't do. Can't we use the same concept but still go forward in time?*
095. Learners: *...yes... yes.... we can*
096. Miriam: *But change direction (Showing zig, zag movement with hand going forward)*
097. Teacher: *So we could just change direction. What would you think that would look like (Hands transparency pens to Miriam)? What do you think that would look like?*
098. Learners: *....Oh gonna... (Sounds of frustration as Miriam goes up to OHP to change graph)*
099. Teacher: *I need your tissue quickly (Ruth hands tissue to teacher who in turn asks her to give it to Miriam)*
100. Jason: *(Draws four line segments with each one an increased slope representing an increase in the rate of change as teacher hands out next set of activities)*
101. Teacher: *Ok, let's just have a look at what he's done until we've done this quickly finish. So what he's indicating there (Pointing at first line segment) is that it is increasing at a rate. Then it's increasing (Pointing at second line segment). The rate, it's getting faster at which the water level rises faster (Pointing at second section of container). Water*

- level again rises faster (Pointing at third line segment) and then it's the fastest (Pointing at last line segment).
102. Craig: But sir look you see that curve segment of this shape...
103. Teacher: Ja..
104. Craig: ...It might be more narrower than the second one but look at the length ...(Inaudible, other voices heard)... it will obviously take a longer time than the second part
105. Teacher: So what are you saying?
106. Craig: That the third part of that graph...
107. Teacher: So we should...what are you saying should change on the graph?
108. Learner: The length.
109. Teacher: The length of what?
110. Learners: of the line...(Inaudible)...
111. Teacher: Of the line. The length of that line. So in other words...so what you say is that, that length and that length (Point consecutively at the first two line segments on graph) is about the same (Point at bottom two sections consecutively). So the steepness increases (Pointing at first two line segments) but the length of the line should be about the same...
112. Learner:(Sounds like Craig) No what I'm saying is the steepness increases....
113. Teacher:the distance...
114. Craig: The steepness increases
115. Teacher: ..(Inaudible)...(Craig comes up to OHP to make changes to graph). Ok. While his fixing it, this is what I would like you to do for home work. Please people understand that this work is not just for this research purpose. This is work that we need to understand and be able to do for your CTA test. Ok. This is not other work that I'm doing with you, extra or additional. This is still mathematics that you need to know. So let's just get that straight. Ok. Because I get the idea that people think that this is just like play- play. This is not play- play this is actual work. Ok. The other thing.... that I would like you to do is as follows. This is the old one (Holding up activity 15.1.1 up). Ok. You gonna take that book home. Please paste this sheet (Holding activity 15.1.1 up) in your book, so that it can flap on the first page. And then I would like you to do this for homework (Holding up activity 15.1.2). Ok. I will give you an opportunity to ask just briefly...(Inaudible)....Let's just try and finish off that (Pointing at OHP). Sssh. Let's just see what he does.
116. Craig: (Craig has drawn the graph similar to the previous graph but changed the slope of the third line segment to almost the same as the first line segment) So you see this part here ne (Pointing at bottom section of container) is much wider than that (pointing at first section and second section of container interchangeably) part so it will take a longer

time to fill up (Pointing at graph) then that part. This part (pointing at second section of container) becomes more narrow so it will take (points at second line segment of graph) a shorter time to fill up. This part (Points at third section of container), even though it's more narrow (Points from left to right), it is more longer (Points from bottom to top). Like here, length. So it will take a much longer time (Points at third line segment on graph) to fill up than that part. And this part is much more narrow than any of the ones. So it will take (Points at fourth line segment of graph) extremely quick.

117. Teacher: Ok. Now we asked a question. Is it about how long it takes to fill up or how quickly it fills up.

118. Learners: How quickly!

119. Zwelile: It's the same thing!

APPENDIX K

Transcript 2

001. Teacher: *Okay. End of this. Let's go back to yesterday. We still have kids in front. So please can you come up again. let's just complete that activity. (Teacher walks to the back of the class and Pholani gets up and takes his blazer off.). Okay now. We spoke so much yesterday and I think we got lost. Side tracked a bit. Okay. Let's get back on track. (Pholani gathers his papers and moves to the OHP.) Okay Miriam, you were indicating somebody asked a question...shh... somebody asked a question... I think it was one of you guys here. Who was it? But somebody was saying where do you get six comma five?*
002. Learners: *(Inaudible)*
003. Pholani: *Sir I can explain that.*
004. Teacher: *Okay. Okay let's take it from there.*
005. Pholani: *I got an explanation to that.*
006. Teacher: *Okay so you talk about it there.*
007. Pholani: *Ja*
008. Teacher: *Okay*
009. Pholani: *Okay. Basically we looked at the temperature. And we saw that the difference is (Points to the RHS of the table on transparency) thirteen. Basically it decreases by fifteen minus thirteen is two and two minus thirteen is negative eleven. Then we looked at the altitude (Pointing to the LHS of the table on the transparency)... the level in the altitude which is two. So basically we took the thirteen and divided by two. So if this side (LHS of transparency) the difference was three then we would divide by three. But then since well the difference is two we divide by two. And that's how we get to the odd numbers. Especially in terms of seven which is going to...*
010. Teacher: *(Interrupts) Why can't you just add two? (Pause) You say you divide by two*
011. Pholani: *I mean we minus we don't divide. We minus.*
012. Teacher: *Okay we minus two. (Interruption by learners.)*
013. Michelle: *Haai, you divide by two it's six and a half.*
014. Pholani: *(Points to Michelle) No divide. Divide it's... Divide you get six comma five. (Class interrupts)*
015. Teacher: *Wait. There we go again. Let's. Let's*
016. Zwelile: *(Something beeping) Sir what's that? It's getting on my nerves.*
017. Teacher: *It's been getting on my nerves as well. I don't know where it comes from. Shh... Let's get back. Guys I asked you the last time. Please I know you want to talk but if you don't talk audibly it gets lost, okay.*

018. Pholani: *(Hand raised) Sir the reason why we divide by two because that's the simplest that works accordingly in terms of finding the altitude. (Pause) Because even if I have to make an example for you.*
019. Teacher: *Okay. What do you divide to get to where? That's what I want to know.*
020. Pholani: *Oh to get to the odd numbers. (Looks at paper on OHP) Oh cause you see for three point for three comma five*
021. Teacher: *(Interrupts) So what do you divide by two to get to the odd numbers*
022. Pholani: *No we take thirteen which is the difference*
023. Teacher: *Ja okay*
024. Pholani: *as far as the temperature is concerned*
025. Teacher: *Okay*
026. Pholani: *And then we divide thirteen by two*
027. Teacher: *Okay*
028. Pholani: *to get six comma five*
029. Teacher: *Okay now I get you. Now I get the point.*
030. Pholani: *(looking at his notes on his lap) Sir you get the point? Good. (Class laughs).*
031. Teacher: *So you want to know what would the ... what would the middle one be because it's halfway of that point dividing by two. To get exactly half way between the two. Okay.*
032. Pholani: *Sir and then what we do next is we take fifteen and minus six comma five to get one. (points on OHP) So one is equal to six comma five. And then just to ... moving (points on OHP)*
033. Teacher: *(interrupts) One is equal to six comma five.*
034. Pholani: *Eight comma five (looks at calculator) eight comma five*
035. Teacher: *Eight comma five*
036. Pholani: *Ja. Eight comma five ja.*
037. Teacher: *so for one kilometre up you'll get what? You'll get eight comma five degrees Celsius.*
038. Pholani: *Ja eight comma five. But then moving one block. But the question says...(looking at transparency)...*
039. Jason: *Why? Sir where did the eight come from?*
040. Zwelile: *Yes*
041. Pholani: *(Raises his hand) okay.*
042. Teacher: *He's going to explain it.*
043. Pholani: *Okay look. First and foremost we said fifteen. No I said thirteen divided by two ... you get...*
044. Learner: *Ja six comma five*
045. Pholani: *Okay fine. But then now to get to one you going to say fifteen minus six comma five.*
046. Jason: *Ohhh*

047. Pholani: *Because thirteen divided by two is six comma five. So now we dividing by that six comma five. Fifteen divide by six comma five is eight comma five.*
048. Teacher: *No we...*
049. Learners: *minus*
050. Pholani: *I mean minus (pointing to teacher smiling) minus... minus. Okay now...*
051. Teacher: *(interrupts) We already established that...Can you see the temperature (learners look at transparency on OHP) is decreasing. So he wants to know what is it decreasing by for everyone. Because what they've done there every two kilometres is increasing by thirteen. So what would the decrease be for every one kilometre. That's what he's basically trying to work out.*
052. Pholani: *So do you understand the concept?*
053. Zwelile: *Yes.*
054. Pholani: *Okay fine. It says here (pointing to transparency) determine approximately the temperature of an altitude of seven kilometres. So to determine that we going to take twenty four (looking at transparency), negative twenty four (working on calculator) okay negative twenty four minus six comma five.*
055. Teacher: *Okay which one are you busy with now?*
056. Carol: *Over here (pointing on OHP) we got six and...*
057. Teacher: *(Interrupts) Okay seven.*
058. Pholani: *Ja. I want to show you how we got...*
059. Teacher: *(Interrupts) We working with the seven?*
060. Pholani: *Ja. Okay we have to take negative twenty four because you want the seven as the altitude. So we going to say negative twenty four minus six comma five and you get negative thirty comma five. (looking at OHP) So basically hmm to determine the altitude of seven kilometres right is negative thirty comma five.*
061. Learners: *Ohhh.*
062. Pholani: *Does it make sense*
063. Learner: *Yes*
064. Teacher: *Okay my question is... How do you know that for every one degree you'll see the same kind of pattern?*
065. Pholani: *How do you know?*
066. Teacher: *How do you know? Yes.*
067. Pholani: *Because you know why?....*
068. Learners: *(Laugh)*
069. Teacher: *Okay.*
070. Pholani: *That's a good question.*
071. Teacher: *It's a good question. Yes.*

072. Pholani: *Ja. Umm*
073. Teacher: *Can somebody else... You don't have to answer Pholani.*
074. Pholani: *Okay.*
075. Teacher: *There's thirty five of us in the class. Thirty six, thirty seven. Yes?*
076. Jason: *Sir that's the pattern how the thing worked out.*
077. Teacher: *That's the pattern. (Pholani raises his hand) But how do you know the same thing will happen. It won't be a sudden jump of temperature or radical. Why? Yes?*
078. Pholani: *Can I answer that? Because you know to get to two you going to minus. It's fifteen minus two. It's fifteen minus thirteen I mean. Then it will give us two (pointing to transparency). Because in terms of the altitude ne, it basically increases by two. So obviously now if you want it to increase like in terms of the numbers when we (Inaudible) them we going to divide by two and you going to get six comma five (Pause)*
079. Teacher: *I hear what you saying. Sorry to interrupt you. I probably shouldn't do it. (Teacher moves to the OHP) But I think you not getting my question. I'm saying that how did you know there were sudden changes (replace transparency with activity 15.1.2) there? (Pauses. Looks at learners)*
080. Neelan: *By the direction*
091. Teacher: *By the direction. Or the steepness of the line we saw there was changes. Question is ... How do we know (Change back to previous transparency) there are no sudden changes on that line.*
082. Jason: *Because it's...*
083. Teacher: *Because it's...(Points to Jason)*
084. Jason: *Constant*
085. Teacher: *It's constant. It's a straight line. So the change... the rate at which the things are changing is constant. So we know by that, that the same thing will occur. It won't suddenly rapidly change. Okay. By simply looking at the graph (Pointing to transparency and indicating the downward slope of the graph with finger) that it is a straight constant rate of change. Okay. But you can look deeper in that. That is what he (Pointing at Pholani) is doing. Because it stays constant, he is trying to figure out what would happen then in between your zero and two, (Pointing to transparency) in between your two and four (Making step like actions using hands). Which is the one, the three and the seven and the nine..*
086. Pholani: *Ja. The odd numbers.*
087. Teacher: *The odd numbers. Okay, fine. Another question before you go further. Is that direct or indirect? Is that a direct proportion or indirect proportion?*
088. Learners: *Direct.*
089. Teacher: *Is it direct?*

090. Learners: Yes.
091. Teacher: Because we said that direct is what? As the one is increasing (Using right hand in upward movement) the other one is decreasing (Using left hand in downward movement) right.
092. Learners: No!
093. Teacher: Nooo!
094. Tabethe They both decreasing or increasing at the same time.
095. Teacher: They both... So can that be direct?
096. Zwelile: No. It's indirect.
097. Learners: because the one is decreasing and the...
098. Teacher: Okay. We're speaking at the same time here. Who wants to talk?
099. Tabethe The one is increasing the other one...
100. Teacher: What is increasing?
101. Neelan: The altitude.
102. Tabethe The altitude.
103. Teacher: The altitude. What is decreasing? (Points to w)
104. Tabethe The temperature.
105. Teacher: So can that be direct?
106. Tabethe No.
107. Teacher: What is that?
108. Learners: Indirect.
109. Teacher: Indirect proportion. Thank you. Okay. (Points to Pholani)
110. Pholani: Then moving right along for three point two. That's complex sir. I tried working it out but then I ...
111. Teacher: Okay you weren't the only one working on it. So let's see. He says he needs help. Okay Ruth. (Points to Ruth)
112. Ruth: Sir we said if you look at the temperature (points to transparency). Ja, between two and eleven, zero would fit in somewhere there. Ja. (Pholani points to table on transparency while Ruth speaks). Because it's two and then you'll get like one and zero and negative one, negative two...Between two and negative eleven, zero should fit in somewhere there. So if you look on the altitude side that's between two and four. So we said it's about approximately three.
113. Teacher: Okay. So you say this value here (Pointing on screen at the point where line intersects x-axis) is that the point on... Is it that point there?
114. Ruth: Yes sir.
115. Teacher: Okay. So on the table we looking at where? We looking at (Pointing to table) between two and four and on the graph we looking at exactly there. (Points to the graph)

between two and four) So what do we notice about that point there? It's there where your temperature is zero degrees. Okay. So you can read it off from here (pointing on graph). It's about...

116. Ruth: *Two comma two*
117. Learners: *Two coma five, two comma nine*
118. Teacher: *What goes here? In the centre (Points between two and four)?*
119. Pholani: *Three*
120. Teacher: *Three. So what would that be (Pointing to screen where line intersects the x-axis).*
121. Ruth: *Two comma five.*
122. Teacher: *It's closer to two.*
123. Pholani: *But then how would we know like exactly? How would you prove that to know you correct?*
124. Teacher: *Okay. How would you know exactly that when the altitude is seven it should be thirty comma five.*
125. Pholani: *How did I know?*
126. Teacher: *How did you know that?*
127. Pholani: *Oh must I explain that again?*
128. Teacher: *No. What I'm saying is, use the same concepts.*
129. Pholani: *Okay it's going to be an increase. It's going to be the other way around.*
130. Teacher: *So what would happen? You tell me. Okay. It is not just Pholani that I am asking that question to. I am asking that question to the entire class. They not the only group that did that. Come now. Let's see isn't there a pattern like the one you did?*
131. Neelan: *To find the altitude you said the temperature minus thirteen equals the altitude. So why don't we just use the converse of that which is the altitude plus thirteen equals the temperature.*
132. Teacher: *Is that possible? How do you see that? I don't get that so nicely. You said that for the temperature decreases by thirteen so you subtract the temperature but for the altitude you add thirteen.*
133. Pholani: *I don't think so.*
134. Neelan: *It does.*
135. Teacher: *Look at this (Points to table on screen). To get from zero to two what is happening?*
136. Neelan: *The temperature drops.*
137. Teacher: *But what has happened to the altitude?*
138. Neelan: *Increases by ...*
139. Pholani: *Two*
140. Teacher: *It increases by two. So for a temperature increase of thirteen, sorry a decrease of thirteen degrees we have an increase in altitude of two. So how can that be equated? A*

thirteen drop in temperature and thirteen increase? I mean if I increase by thirteen where do I end up?(pointing to table on screen) Somewhere... zero plus thirteen it's not even here. (Pause) He (Pointing to Pholani) has worked on a pattern. Find a pattern...(Long pause).... We can see clearly from the graph that it is two comma five. How do we verify that? (Long pause).... You can take a while.

141. Jason: *We can take a wild guess?*

142. Teacher: *You can take a wild guess, yes.*

143. Jason: *Isn't you can look by two and four and see where it is?*

144. Teacher: *Say that again.*

145. Jason: *You see by, by, by ...(Looks at sheet in book) by, (Looks at transparency) two, by the temperature there's two and negative eleven and on the altitude it's two and four. So you can guess in between (Uses hands to indicate between) about where it is.*

146. Teacher: *Ja but... What we did was.. Listen carefully. I'm just going to use her book cause I just see it here. What did she do? She saw at seven it's approximately thirty comma five and the graph verifies that. We justified that by his (Pholani) calculation that made sense. Here we weren't too sure because when he (Points to Pholani) drew the graph it showed something different. Because that graph was very inaccurate. So what he then did was ... He actually justified it now. With what? With a calculation. Working out from the pattern. So I'm saying, use the same kind of logic. Try and find a pattern to come to that solution.(long pause) Think quickly. We can't think quickly. (Pause) Let's skip that one and come back. Next. Okay, (Walks to the OHP and reads from it) about eleven kilometres the temperature remains fairly constant at negative fifty six degrees Celsius. Okay. And then it says, at about thirteen kilometres, due to the presence of ozone, which absorbs heat from the sun, the temperature starts to rise as the altitude increases. Show this on the graph. (Hands pen to Pholani) Okay let's see how you indicate that on the graph. (Pholani takes a ruler and places it on the OHP between negative fifty and negative sixty) And here I just need to remind you about what we just discussed. Okay, same concepts. (Pholani extends the line graph and places the ruler again between negative fifty and negative sixty. He draws a horizontal line from between twelve and fourteen to between negative fifty and negative sixty. Bell rings.) When you participate time goes quickly. You'll be amazed. You won't even feel maths. (Pholani continues to complete graph according to question) Okay fine. Let's just see what Pholani is saying about the last one there. Let's see if we agree. Carry on (Indicates to Pholani).*

147. Pholani: *Oh.*

148. Teacher: *Okay. Explain why you did what you did.*

149. Pholani: *Okay. Looking at number four it says (Reads from book) "above eleven kilometres the temperature remains fairly constant at negative fifty six." (Points to transparency between negative fifty and negative sixty) So basically it remains constant at negative fifty six. (Reads from book) And then at about thirteen kilometres, due to the presence of ozone, which absorbs heat from the sun, the temperature starts to rise as the altitude increases.*
150. Teacher: *Okay now remember, what shows that constant temperature?*
151. Pholani: *Ja it's this (Points to the horizontal line which he drew on the transparency) this line show that at thirteen kilometres, negative fifty six remains constant. This line.*
152. Teacher: *Okay, so the temperature...that solid it means there was no change in the temperature.*
153. Pholani: *Ja, there was no change. It remained constant and then this line (Pointing to the line increasing) indicates that the temperature started increasing. Ja, there was no change. It remained constant and then this line (Pointing to the line increasing) indicates that the temperature started increasing.*
154. Teacher: *Okay. Do they say the temperatures increase or is there any specific description of how does it increase.*
155. Pholani: *It says... (Reads from book) the temperature starts to rise as the altitude increases.*
156. Teacher: *Okay.*
157. Pholani: *Okay so that's how I initiated it. Looked at it.*
158. Teacher: *Okay, so would we assume that it is a curve or can it be a straight line?*
159. Michelle: *It looks like a curve. It looks like a curve.*
160. Teacher: *It looks like a curve. Can we assume that? We don't really know. If we say the temperature increases at a constant rate it would be a straight line. However, we don't know because we don't have data to support whatever we saying. So, it could be a curve or it could be a straight line. But what we know... it goes up okay. (Pause) Okay fine. Tiego says that... (Pholani gets up from the chair) You can vacate the hot seat (Speaking to Pholani) yes.*
161. Pholani: *Thank you very much.*
162. Teacher: *That seat needs to be red. Tiego, please, occupy the hot seat.*
163. Tiego: *Why sir?*
164. Teacher: *Because you indicated that you have some idea of how to work out three point two.*
165. Tiego: *Oh sir. I tried but it doesn't work.*
166. Teacher: *It doesn't work. Okay maybe by you sharing what you thought about it might give some ideas to everybody.*
167. Tiego: *Ja,... (inaudible)*
168. Teacher: *(Points to Theresa) Exactly! (Tiego gets up from his seat and moves to the OHP)*
169. Tiego: *(To Pholani)....(Inaudible)...*

170. Pholani: *(Laughs)*
171. Teacher: *Tiego I'm glad you say it's scary now you know what I feel like every day.*
172. Miriam: *Are you scared? But you don't see it.*
173. Learner: *Ahh come on, are you scared? Ahh come on, are you scared?*
174. Teacher: *Okay guys let's focus. Zelda, we'd like to see some participation from your side. Some questions...*
175. Tiego: *(Missing data. Replace disc. Writing on transparency by Tiego. $8 \times (-6.5) = -52$ and $-52 + 15 = -37$) ...*(Inaudible)* with eight kilometres the temperature is going to be negative fifty two.*
176. Teacher: *And you don't know how you got that? So how would you work out the temperature for let's say ...four? If the altitude is four. (Tiego writes on transparency and the teacher repeats what he is writing) You'd say four times negative six comma five equals to (Tiego works on calculator) negative twenty six. Exactly! (Tiego continues to write) Okay, negative twenty six plus fifteen?*
177. Zwelile: *Is negative eleven.*
178. Teacher: *It's negative eleven. Ohhh! So that works with all of them?*
179. Tiego: *Yes sir. (learners applaud)*
178. Teacher: *Now, how did you get the negative six comma five? Shhh. let's hear him class.*
179. Tiego: *Because at first sir I used what ...said (unclear) the difference was going to be six comma five... negative six comma five. So that's why I used negative six comma five. And then the fifteen, I saw with an altitude zero the temperature was fifteen degrees.*
180. Teacher: *Okay I hope you all heard what he said. Who didn't hear that? (Teacher moves to the front of the class. Learners call out that they did not hear) He says that, he noticed that for every degree that it increases (Points to screen to RHS table) the difference between your first, say for zero and one would be negative six comma five (Pause). So he took that negative six comma five because it's the difference, he took that and multiplied it with the altitude... am I right? (Speaks to Tiego) Okay I hope you all heard what he said. Who didn't hear that? (Teacher moves to the front of the class. Learners call out that they did not hear) He says that, he noticed that for every degree that it increases (points to screen to RHS table) the difference between your first, say for zero and one would be negative six comma five (Pause). So he took that negative six comma five because it's the difference, he took that and multiplied it with the altitude... am I right? (Speaks to Tiego)*
181. Tiego: *Yes sir.*
182. Teacher: *(Looking at Tiego) And you tried to see what must be added or subtracted to get that value. And he noticed that the value that must be added or subtracted had to be fifteen to get exactly the temperature (points to screen) that is given there. Okay. (Pause) Do*

- you guys see that? Let me show you something quickly. (Takes pen and writes on transparency in temperature column) What is the difference between that and that? (First two numbers in table)*
183. Learners: *Thirteen*
184. Teacher: *It's negative thirteen. It's negative thirteen. It's not thirteen. What is the difference between this and this (Points to next two numbers in table)*
185. Learners: *Negative thirteen*
186. Teacher: *It's negative thirteen negative thirteen. (Points to altitude column) What is the difference between this and that. It's two.*
187. Learner: *One.... Negative two*
188. Teacher: *No it's positive two because it increases. Okay it's two. Between that and that (Points to next two numbers)?*
189. Learners: *Two*
190. Teacher: *Between that and that (Points to next two numbers)?*
191. Learners: *Two*
192. Teacher: *So, your change in temperature decreases by thirteen each time. The change in your altitude has an increase of two every time. Take negative thirteen (writes on transparency) over two what do we get?*
193. Learners: *(Inaudible)*
194. Teacher: *You get negative six comma five. So what kind of measure is this negative six comma five? It actually measures (Points to writing on board) the rate of change. It tells you exactly how it's changing at what rate is it changing okay. We get that point? (Looks at learners)*
195. Learners: *Yes*
196. Teacher: *Are we happy?*
197. Learners: *Yes.*
198. Teacher: *So here we starting to actually work out with calculations of what we've been talking about from just the pictures that we saw. So we can actually now calculate rate of change. And we notice that (Moves to OHP) it doesn't matter where we go... okay let's say we go from negative twenty four and negative fifty (Points to temperature column) What is the difference between these two negative twenty four and negative fifty is...*
199. Learners: *Negative twenty six*
200. Teacher: *It's negative twenty six. Let's see what's the difference (Points to altitude column) between six and ten?*
201. Learners: *Four*
202. Teacher: *okay it's four. So what would negative twenty six divided by negative four give you?*
203. Learner: *Not negative four. It's positive four.*

204. *Teacher:* Sorry positive four. (Pause) Six comma five again. So it doesn't matter...listen carefully...it doesn't matter where you choose (Points to graph on screen) the values on the line...the points on the line. It keeps on being six comma five ... negative six comma five. So that is the rate of change that is constant...it doesn't change. So now you can actually calculate it, okay. Are we happy?
205. *Learners:* Yes
206. *Teacher:* Okay. Let's move on. Next one.

APPENDIX L

LESSON 1							
DAY 1	17/10/2008						
VIDEO:1 D - S.A	07:56						
REFERENCE	DESCRIPTION	Pattern	Variable	Covariance	Slope	Representation	Additive relationships Multiplicative relationships
IA-1	Learners seek clarity on how to complete tasks: Miriam asks what kind of graph should we do? Teacher responds by saying that learners should draw any graph that they think describes the situation. Teacher indicates that learners should draw a graph indicating what happens to the water level as time passes. Teacher moves hand upward and across respectively when talking about water level and time. Notice learners becoming frustrated because they are not given an example of what to do.					✓	
IA-2	Teacher asks for volunteer to complete no 1. Craig volunteers quite eagerly to give his group's solution to no 1. Teacher instructs how learners should interact with Craig's contributions (see what he does; compare with yours; he will explain why he does what he does; ask questions and comment for clarity or raise different solutions). <i>The teacher uses the word line instead of graph.</i> Craig gives solutions to no1 by drawing increasing straight line segments followed by steeper shorter line segments. He explains that it will take longer to fill bottom part of container and much quicker to fill top part of container. He moves in between container and graph as he relates sections of container to the graph.		✓	✓	✓	✓	✓
IA-3	Ester asks what is meant by water level. Questions whether it means the volume of water or just how high the water goes. Teacher clarifies this question indicating it's about how high the water is from the bottom of the container.		✓				

IA-4	Teacher asks Craig whether it is a straight line and whether he intended for it to be a straight line. The teacher asks this question because Craig has marked off points and then joined them freehand. Teacher merely wanted him to confirm that he intended for it to be a straight line							
IA-4	Teacher asks question to class: Why would you have a straight line why would it not curve? Peter responds by saying that the box is thick and straight so the water goes up evenly			✓	✓	✓		
IA-5	Teacher asks whether anyone else would like to contribute. Calls on Ruth who had her hand up. Ruth confirms what Peter was saying, saying because the shape is the same it will go up at a constant rate and it will be a straight line because it's not going to change.			✓	✓	✓		
IA-6	Craig volunteers to give his group's solution to no 2. Teacher asks Craig why he's using dots and then joined them. Craig responds by saying it is just an indication					✓		
IA-7	Craig draws a decreasing curve followed by an increasing curve. Without being prompted he explains that in the first part of the container the space is narrow so the water fills up fast but as the space gets narrower the water fills up slower. He then points to the point of inflection indicating that at this point the space closes and therefore the water starts to fill quicker again. And then the last section the space is narrow and then fills much quicker.			✓	✓	✓		✓
IA-8	Teacher asks about the top part. Craig indicates that this is represented by the very quick part while making the last part of the graph, the line segment more prominent. Teacher asks Craig whether that is a straight line. Craig replies by saying no it is a curve. Teacher asks why a curve? He responds jokingly that it's because he likes curves. Michelle indicates that it's because it goes from slow, long, slow again so it would show a curve. Teacher prompts for clarity. Michelle then corrects herself by saying it goes from quick low to quick again.			✓	✓	✓		
IA-9	Peter repeats what Michelle is saying using alternate words			✓	✓	✓		
	Teacher asks whether the water level rises evenly at all. Rephrases question by asking whether the water level rises constantly at any point while indicating upward movement with hands. Craig, while using the container to explain, indicates that the water will only rise constantly in the top part. Teacher stresses the distinction between a straight line indicating a constant rate at which the water level rises and a curve indicates that the rate at which the water level rises is constantly changing.			✓	✓	✓		✓
IA-11	Ruth volunteers to give her group's solution to no 3. She draws a graph using four line segments indicating the manner in which the water rises in the four different sections of the container. The line segments move back and forth in time. Ruth confuses the time it takes to fill a particular section and the rate at which it fills the section. Her explanation tries to take the width as well as the height of the different sections into account as she explains how the water level rises.		✓	✓	✓	✓		

IA-12	Teacher comments that there are many hands going up and suggests an order to how learners will respond Neelan, Jason, Michelle, Peter. Allows Neelan to speak.							
IA-13	Neelan Tries to explain that the graph should not go to the left but is interrupted by Ruth indicating that we are not measuring time. She is then also interrupted by unknown voices.		✓	✓		✓		
IA-14	Ruth indicates that we are not measuring time and then indicates how the water is rising say after every minute. But only how long it takes for the water to rise in the different sections in the container.			✓		✓		
IA-15	Michelle, who is in the same group as Ruth, says that she agrees with Neelan that the graph can't go Zig Zag. It can't go back and indicating with hand movement that it can only increase up.		✓	✓		✓		
IA-16	Teacher calls on Peter who is engaged with learner next to him to give his contribution. He indicates that the x axes represents the time that passes. Basically how long it takes you to fill up.		✓			✓		
IA-17	Teacher calls on Jason who indicates that his question was answered.							
IA-18	Teacher asks for volunteers to fix Ruth's graph. Miriam clearly had her hand up but when Michelle's hand goes up teacher calls on her.							
IA-19	Teacher indicates that he would like feedback from other groups. As Pholani indicates that he will go after Michelle, there is some comments to the manner in which he speaks.							
IA-20	Michelle draws an increasing curve upwards. She explains that as the shape of the container is getting narrower the water level rises faster.		✓	✓	✓	✓		✓
IA-21	Ester questions the fact that the graph is a "straight curve" as she gestures with her hands that the containers rapidly changes shape in sections.			✓	✓	✓		
IA-22	Lyle asks whether it is wrong if he uses a bar graph.			✓		✓		

IA-23	Jason draws a graph using four short line segments of increasing steepness. A line segment for every section of the container. Teacher summarises Jason's graph by linking it with the different sections in the container.		✓	✓	✓	✓	✓
IA-24	Craig contributes voluntarily to Jason's representation indicating something about the length of the line and the time to fill the different sections.			✓	✓	✓	
IA-25	Teachers engages with Craig's contribution trying to assert what it is that Craig feels should be different about what Jason has drawn. With the input of another unknown learner the teacher incorrectly concludes that it is the length of the line segments that seems to be the problem Craig is referring to. The height of the different segments of the container is somehow linked to the length of the line segments.			✓		✓	
IA-26	Teacher emphasizes that the work which the learners are doing is not just for his research only but that it is work that they need to know for their upcoming CTA.					✓	
IA-27	Feeling misunderstood he comes up to make the changes to Jason's graph. Craig redraws Jason's graph with the third line segment less steeper than the previous two, explaining that because this section of the container is longer it will take a longer time to fill.		✓	✓	✓	✓	✓
IA-28	Teacher asks whether it is about how quickly it fills up or how long it takes to fill up. Some learners say it is about how quickly it fills up while others indicate that it is the same thing.			✓		✓	

LESSON 1	DAY 2	VIDEO: 2D - S.A	REFERENCE	SIDE A								
	20/10/2008	10:41	DESCRIPTION			Pattern	Variable	Covariance	Slope	Representation	Additive relationships	Multiplicative relationships
					As Naledi comes up to give her group's solution, the teacher instructs learners to focus on what Naledi is doing. Stressing that they must: compare their work; to be able to have meaningful comments and give input.							
IA-29			Naledi draws their solution to no. 4 using 5 line segments with different slopes. She indicates that the water rises fast then slows down then fast again then slows down then fast again. She uses points to indicate where the rate at which the water level rises changes. She then explains that the water absorbs at a constant rate and as it fills up it goes slower and then it goes faster again and then slow again and then very fast at the end. She explains this as she points to the different parts of the graph only. The teacher requests that she explains using the container and the graph to explain how the water rises. She then explains that the water initially rises fast because the container is narrow but then slower, then faster again and slower and at the end very fast because it is so narrow. This time she explains pointing alternately to both the graph and container. Teacher asks for any comments or questions from the other members of Naledi's group and the rest of the class.				✓	✓	✓	✓		✓

IA-30	Craig says that he feels that due to the shape of the container the graph must be drawn using curves not lines. He uses the phrase upside down Christmas tree. The teacher engages Craig in conversation through questioning and extracts from him the fact that because the shape starts out narrow then widens then immediately narrows then widens the shape is constantly changing.		✓	✓	✓	✓		✓
IA-31	Peter indicates that as the shape slowly gets wider therefore it takes longer and longer to fill up. Teacher indicates that because the shape gets wider so the rate at which the water level rises slows down.		✓	✓		✓		
IA-32	Naledi indicates that they decided to do it this way so as not to confuse anybody. She indicates that it curves but she placed the points to show exactly where it curves. Teacher enquires whether what is in between points are not straight lines. She indicates no it's just (showing curve movement with hand). The teacher then asks if she means that it does the following as he changes the first two line segments into a curve and the second two line segments into a curve and retains the last line segment. Although Naledi is a bit hesitant the members of her group clearly say yes, that is what they meant. The teacher then continues by asking why she drew lines in between the dots reiterating that straight lines imply a constant rate and curves a constantly changing rate.			✓	✓	✓		
IA-33	Zwelile responds to teachers comment by saying: so you say we must not use points. Teacher explains that the points are not the problem but the joining of the points with straight lines and not curves is problematic. Teacher reiterates that a straight line would imply that the shape (width) does not change and that the rate would be constant.					✓		
IA-34	Teacher talks about the importance of contributing even if it might be incorrect it is still valuable as it means you thinking about what's going on. This happens after realising that not all the groups completed the work required.							
IA-35	Ester gives her group's solution to no. 5 by drawing the graph using four line segments with different slopes to represent the different rates at which water level rises. The teacher needs to prompt Ester for an explanation. Ester starts her explanation by stating that the first and the third line segments is supposed to be the same length and angle. She justifies this by stating that the sections of the container representing them are the same size (indicating height with fingers on container) and width. She indicates that the water rises at a consistent rate then faster still at a consistent rate but takes longer because it is larger and then it is the same story as the first and then it's short. (Ester brings up an important concept of parallel lines representing same rate of change and same slope. She also indicates that the duration per section of container is the same and represents this with similar lengths of line segments. She also indicates that there is a difference between the rate and how long it takes to fill a particular section). Teacher asks class if they follow. They indicate no. Teacher then asks Ester to elaborate on explanation using both the container and the graph while trying to explain the correspondence. Ester reluctantly concedes to elaborate.		✓	✓	✓	✓		✓

IA-36	Ester points alternately at the various line segments on her graph and the section of the container, relating it to each other (This was as I suggested). Teacher interjects and adds some explanation as Ester points from section to section.					✓		
IA-37	Pholani asks why is it that the time passed is the same in terms of the first level and second level. Teacher responds by saying that we not very accurate about time passed, not actual measurements. Teacher refocuses Pholani's attention on the slope and therefore rate of change. He explains that the first line segment and the third line segment is like this (using hands to illustrate parallel line segments). He then asks Ester to explain why she did this. Ester explains that section 1 and 3 of the container is the same width and in her theory the same rate. Teacher ones again only picks up on the fact that the slope is the same because of the rate of change being the same.			✓	✓	✓		✓
IA-38	Teacher uses Ester's contribution to associate the steepness of the slope of a line with the rate of change, illustrating using Ester's graph. Teacher then enters a question and answer session to recap/ emphasize/ summarise important learning points. What does the slope of the line represent? (How fast, the speed) We call it the rate. The rate at which the water level rises. More mathematically your rate of change. Your rate of change is indicated by the slope of the line. Learners asks to write it down. Sir what did you say. Teacher repeats so that learners can write down. The rate of change can be seen from the slope of a line or the slope of a curve. Teacher points to curve in no. 4, saying that it starts off steep but then it flattens out. That means the rate of change is decreasing. Then from a very high rate of change to decreasing. Then suddenly it is constant but steeper than both these.							
IA-39	Teacher indicates that a curve refers to a changing rate of change. Like no. 4, as it is getting wider the rate of change is slowing down so it curves constantly. Whereas a straight line would represent what Ester said a constant rate of change, the rate of change stays the same.							
IA-40	Teacher asks is there anything we did not get. Zwelile says that she is not understanding, are they not allowed to use dots? Teacher elaborates on the use of the dots. Explains that the dots represents specific measurements.					✓		
IA-41	Ester questions the curves in no. 4. I don't know how to put this, but for no. 4 the curve goes like up but comes down like sort of gently. Isn't it, it goes up fast and then stops... stops where it(not clear). Teacher associates the curves with the sections on the container section by section, then uses hand movements to illustrate the sudden changes in the rate of change from section 1 to section 2 in the container.			✓		✓		
IA-42	Jason asks : Sir what did you say a straight line indicates. Teacher responds by saying that it indicates a constant rate of change. The rate at which something is changing in this case the water level, stays the same. The rate at which the water rises stays the same.			✓		✓		

IA-43	Michelle asks whether we shouldn't have a curve because the rate of change is changing in the container (referring to the width of the container changing at different stages. Teacher starts by saying that that is the difference. Pointing at the diagram for no.4 the teacher states that the change changes all the time. Teacher elaborates that the rate at which the water rises is different at every point on the diagram. Teacher explains that at every point the rate is different. Michelle responds by saying oh! Teacher carries on explaining by comparing this situation with no. 5. Michelle at the end says that she understands now.			✓		✓			
-------	---	--	--	---	--	---	--	--	--

LESSON 2	DAY 2	VIDEO: 2D - S.A&B	REFERENCE	SIDE A	IA-44	SIDE B	IA-45	IA-46	Pattern	Variable	Covariance	Slope	Representation	Additive relationships	Multiplicative relationships
	20/10/2008		DESCRIPTION		Teacher gives instruction to complete Activity 15.2.1 and 15.2.2 in splinter groups of groups. Each splinter group needs to then report back to larger groups and then every larger group to whole class. Teacher interacts with various groups trying to assist with understanding of activities to be completed.		Pholani responds to question 1 by saying: It tells us that as the altitude increases the temperature decreases. Teacher presses Pholani by saying that he should be more specific and use mathematical terms like rate of change. What do you notice does it just decrease rapidly?	Tiego from the same group responds by saying that as the altitude increases by 2 km the temperature decreases by 13. Teacher tries to repeat what Tiego is saying and makes a mistake upon which Tiego rectifies the teacher's mistake.							
							✓	✓	✓	✓	✓				
									✓	✓	✓				✓

IA-47	Teacher probes by asking if the rate of change is constant. A few learners confirm this. At this point the teacher asks how they know this. Neelan from same group responds by saying it is a straight line.			✓		✓		
IA-48	Pholani responds to question 2 by saying: We said no. Because it would contradict itself because the higher the altitude the lower the temperature and it can't be the lower the altitude the lower the temperature.			✓		✓		
IA-49	Tina responds by saying yes because it would represent the temperatures below sea level. Teacher repeats Tina's response to class. Teacher asks what altitude is ultimately. Learners respond by saying above sea level.		✓	✓		✓		
IA-50	Teacher gives Lyle a turn. He responds by saying that he wanted to say the same. Teacher presses Lyle to add or elaborate more. Lyle responds by saying the temperature could increase or decrease I don't know.		✓	✓				
IA-51	Ruth responds by saying they also said no because you can't get negative kilometres. Carol interjects by saying that if you go to the left you get negative kilometres.		✓			✓		
IA-52	Zwelile responds by saying: there is negative altitude looking on the graph. And secondly the lower you go the higher the temperature is. (Referring to extending the graph in the same way).		✓			✓		
	Teacher repeats what Zwelile says and asks can we speak of altitude below sea level.							
IA-53	Jason basically asserts that the worksheet clearly indicates that altitude refers to height above sea level. So what else can it be? Teacher asks Jason to explain to Zwelile why we can't speak of altitude below sea level. He responds by saying it would not be called altitude. Teacher asks Jason why? He simply respond by: I don't know why.		✓			✓		
IA-54	Teacher suggests to Ruth that negative could imply direction. Ruth responds by asking why negative? Why not left or right or north and south. Why you use negative? Ruth brings up scuba diving by saying would you say divers dive negative five km down. Teacher tries to introduce the concept of negative numbers as indicating direction. Ruth responds by saying that in a lift -1 means one level below not one km below. The reason why I am saying no is because they using km if they were using other units where negatives can be used. She concedes that it could be yes if you were using units that can be negative but km can't be negative. Teacher then speaks about unit as a measurement of distance.		✓			✓		

IA-55	Zwelile adds to the discussion by stating that when we up here we count 1, 2, 3.. So does it mean that we don't count when we down there. And when you working as a miner down there it is hotter down there. You've seen it.		✓				✓	✓	
IA-56	Tabethe challenges Ruth's statement about negative km why can't there be any negatives when on the graph it is shown.		✓				✓		
IA-57	Carol reiterates Ruth's assertion about the fact that that there's no need for negative when we can simply say below sea level. Michelle confirms that divers say 2km under water		✓				✓		
IA-58	Teacher responds by saying that what is meant is not -2km below but 2 km below or simply -2km. Teacher tries to move class to next question by reading 3rd question. While reading, teacher observes that the class has not moved on. Sensing this teacher repeats that there are still two contesting views on no. 2. Both views have compelling arguments but we need to move on. Some learners are then asking what should they write as the answer. The teacher responds by saying that they should choose what convinces them until they are otherwise convinced.								
IA-59	Teacher asserts some push to no. 3. Can we get to no.3?								
IA-60	Pholani reads question 3 and starts to answer by saying: temperature of approximately an altitude of 7km- what he did was he took 15 and he minus by 6,5 the altitude of 8,5 k - ok let me just explain that we said 15 minus 6,5 which is = to 8,5 -that's the relationship we are using.		✓	✓			✓	✓	
IA-61	Ruth interrupts and asks a question that is not audible- Pholani responds by saying that because it works to get to 7-Ruth asks where did you get 15 and 6 and 8-Teacher interrupts and asks Ruth to talk louder and clearer so that everyone can hear and follow. Ruth repeats more audibly that she wants to know where Pholani got 15 , 8 and 6. Pholani responds by saying here's 15 here (pointing to worksheet) and the temperature in response to questions of where...		✓				✓		
IA-62	Pholani responds by saying ja this is 15 and that's what we using to determine the altitude in km. Because here they've given it in 0, 2, 4, 6 they have given in two's but we want to find out the numbers the odd numbers 1, 3 & 5 & 7 understand. Because it says here determine approximately the temperature of an altitude of 7km that's what we trying to find, so how we go about finding that: We start from no.1 is 15 minus 6,5 which is 8,5...	✓	✓	✓			✓		✓

IA-63	Carol interrupts by saying 7,5-(discussion inaudible between Carol, Ruth and Miriam ensues. During this discussion it is as if Carol comes to some realisation that "it is 6,5 ") Initially in disagreement with Pholani she (Carol) now seems to agree with his statements.					✓		
IA-64	Pholani raises his hand to be acknowledged. Pholani responds by saying: look we are using 6,5 and it's going to work -teacher interrupts and asks Pholani: why he uses 6,5-			✓		✓		
IA-65	Pholani refers to Tiego's response by saying that it is half of 13. Pholani tries to speak-	✓	✓	✓		✓		✓
IA-66	Carol interjects by saying: Because the difference between 15 and 2 is 13, 2 and -11 is 13-Pholani is agreeing by saying ja, ja it is, it is -	✓	✓				✓	
IA-67	Zwelile interjects and asks: of all the numbers why 15- Pholani responds by: inaudible, we want to find a negative, because this is a negative to the odd inaudible ,so we trying to find the altitude in km which is an odd number, this is the temperature			✓				
IA-68	Many hands go up. Teacher acknowledges Lyle but tries to ascertain if Lyle's contribution will deviate from the discussion at hand. Lyle indicates that he wants to add on to Pholani's contribution. Lyle explains that the solution can be found by joining a vertical line from 7km with the graph and join the point of intersection with a horizontal line with the y-axes. Pholani finds it somewhat difficult to see what Lyle is saying so teacher assists Lyle and Pholani to execute what Lyle tries to explain to class.			✓		✓		
IA-69	Thope responds by saying: (inaudible) ...you just minus 6,5 from ...(inaudible)...-Teacher responds by saying that Pholani is saying that there's a definite relationship between 0 & 15, it's 2 & 2. But what is he saying - how do I calculate - So he wants to say what is the temperature at 7-I don't fully get what you saying yet- Thope responds by saying: we got -30,5 -because -24 minus 13 is -27, so we want to find 6-the number between 6 and 7-So we -24 minus ..inaudible...Teacher then responds by saying that it is exactly what Pholani's group is saying. Teacher then mediates response to class: She is saying this is how they got, you've got -24 and -37, so the one dead centre would be the middle between the two. Am I right? So you would say, what, 37 and minus 6,5 why 6,5-It's half of 13-Half of 13 so what do you get- 30,5-Negative 30,5-	✓	✓	✓		✓	✓	✓
IA-70	So how did you get it pointing Pholani-The process-Ja the process. We did like Thope half it which is 6,5. We started by 15, which is 15 minus 6,5; it's 8,5 that is 1; and then 2 minus 6,5 is -4,5 which is 3 - a lot of commotion follows.	✓	✓	✓		✓	✓	✓

IA-75	Teacher asks learners to explain the difference between slopes going up and slopes going down. Pholani explains that the slopes going up is when water is poured into the bath and slopes going down is when water is drained from the bath. Teacher uses Pholani's contribution to indicate that lines going up are when the water level rises as time passes and lines going down represents a decrease in the water level as time passes.		✓	✓	✓	✓		
IA-76	Teacher uses this moment to introduce direct and indirect proportion. Stating that with direct proportion both quantities increase or decrease and with indirect proportion one quantity increases while the other decreases.							
IA-77	Zwelile asks what would it be called when one is increasing and the other is at a standstill. Teacher is caught off guard and indicates that it is a good question. Zwelile elaborates on her question and refers to F to G. Teacher asks whether one would call that a direct proportion. Pholani responds by saying it would be indirect and Zwelile concurs. Teacher suggests that we should think about what Zwelile is saying and that we might come up with an answer by the end of this period. Teacher suggests that learners use their prior knowledge of proportion in the domain of similarity. Asks class to think back to what a proportion means and recaps that it is when one quantity divided by another is equal to something divided by something else. Explains on board that if $1/2 = 4/8$ that it is a proportion. Teacher indicates that it is a direct proportion because the top one increases and the bottom increases.		✓	✓	✓	✓		✓
IA-78	Teacher explains that direct proportion is when both quantities are increasing or decreasing at the same time. Indirect proportion is when one increases and the other one decreases. Use bath water story and slopes AB and KL to illustrate these concepts.							
IA-79	Tiego queries the slope EF. Teacher indicates that it is constant and that this is what Zwelile is asking. Would this be direct or indirect proportion? Teacher indicates that we need to think about this before we give an answer.			✓		✓		
IA-80	Teacher asks constant rate of change we said was (points to Michelle)? Michelle indicates that it's like from point A to B because it is going up at a constant rate. Teacher then asks is there any line segment that does not represent a constant rate of change. Some learners say no. Teacher states that all of them are constant why? Answer his own question by saying because we discussed earlier that a straight line represents a constant rate of change. That means it is changing at the same "tempo" at the same rate.			✓	✓	✓		

IA-81	Teacher reads changing rate of change from board. Indicates that this sounds funny. Then explains that changing means that the rate of change is constantly changing. And when did we see this again when it was a curve (Learners chorus with teacher). So a curve would indicate that there is a change in the rate of change. That it happens faster as you go on or slower that it either decreases or increases the rate of change.			✓	✓	✓		
IA-82	Pholani is up front and teacher recaps that someone asked where they got 6,5. Pholani indicates that they looked at the temperature and they noticed that the difference is 13. And we looked at the altitude and noticed that it was 2. So we divided the difference in temperature by 2. If the difference in altitude was 3 we would have divided by 3. And since the difference is 2 we divide by 2 and that is how we get to the odd numbers. Especially in terms of 7.	✓	✓	✓		✓	✓	✓
IA-83	Teacher asks: "why can't you just add 2?" Pholani is baffled. Teacher says you say you divide by 2. Pholani responds by saying: I mean we minus we don't divide by 2. Teacher: Ok you minus 2. Some learners reiterate you divide by 2. Pholani says again we divide by to get 6,5. Too many people talking together and teacher restores order.	✓	✓	✓		✓		
IA-84	Pholani reiterates that the reason they divided by 2 is that it is a sequence that works accordingly in terms of finding the altitude. Cause even if I had to make an example for you. Teacher asks: What do you divide to get to where? Pholani: To get to the odd numbers because you see...Teacher interrupts Pholani: So what do you divide by 2 to get to the odd numbers? Pholani: No we take 13 which is the difference as far as the temperature is concerned (Ja, Ok from teacher as Pholani speaks) then we divide 13 by 2 to get 6,5. Teacher responds by saying ok now I get you.. now I get the point. Pholani: So you get the point? Teacher: So you want to know what would the middle one be because it's half way and that's why you divide by 2, to get exactly half way between the two. Pholani: so what we do next is take 15 and minus 6,5 to get 1. Pholani: so 1 is equal to 6,5. Teacher: 1 is equal to 6,5? Pholani: 8,5. Teacher: So for 1 km up you'll get 8,5 degrees Celsius. Many learners asking random questions indicating that they don't understand.	✓	✓	✓		✓	✓	✓
IA-85	Jason asks; Sir where did the 8 come from? Teacher indicates that Pholani will explain. Pholani: I said 13 divide by 2 right...(Zwelile chorus 6,5) ...Ok now to get to 1 we eventually say 15 minus 6,5. Because 13 divided by 2 is 6,5 so now we dividing by that 6,5 is 8,5. Learners and teacher Chorus minus ...Pholani corrects himself saying minus. Teacher explains that what Pholani is trying to calculate is the decrease in temperature for every kilometre.	✓	✓	✓		✓	✓	✓

IA-86	Pholani: It says here determine approximately the temperature of an altitude of 7km. So, to determine that we going to take negative 24. (Calculating -24 minus 6,5 on calculator). Teacher: Which one you busy with now?We going to take -24 because 7 is the altitude in km. So we going to say -24 minus 6,5 and we get -30,5. So basically to determine the temperature at an altitude of 7km, is -30,5.	✓	✓	✓		✓	✓	✓
IA-87	Teacher: My question is however: How do you know that for every 1...unclear...you will see the same kind of pattern. Pholani: How you'll know? Teacher: How you know this? Pholani: Because you know why....(Learners laugh). Pholani: That's a good question. Teacher: somebody else? You don't have to answer Pholani. There's 35 ah 36, 37. Jason answers by saying because that's the pattern that they've worked out.	✓						
IA-88	Teacher probes further by asking: How do you know that the same thing will happen, there won't be sudden jump of temperature? Pholani explains the fact that the altitude changes consistently by 2 while the temperature changes by 13.	✓	✓	✓				✓
IA-89	Teacher interrupts Pholani by saying that he is not understanding what the teacher is asking. Teacher puts on graph of bath story and asks class how did we know that there were sudden changes there. Some learners indicate by the direction. Teacher repeats: by the direction and steepness of the line we saw that there were changes. Teacher: Question is how do we know (placing the graph of current problem on OHP) that there are no sudden changes on that line. Sounds like Jason who says it's a straight line it's constant. Teacher repeats it's constant it's a straight line, so the change, the rate at which things are changing is constant. So we know by that, that the same thing will occur, it won't suddenly rapidly change Ok. By simply looking at that graph that it is a straight constant rate of change Ok. But you can go deeper in that and that is what he (Pholani) is doing because it stays constant he's trying to figure out what is happening in between your 0 & 2 in between your 2 & 4 which is the 1, the 3, the 5 the 7, the odd numbers. Ok.			✓	✓	✓		
IA-90	Teacher: Fine another question before we go further is that direct or indirect proportion. Teacher through questioning leads learners into revising direct and indirect proportion and to see this is an indirect proportion.			✓	✓	✓		
IA-91	Pholani indicates that he couldn't work out 3.2 and asks for help from the class? Ruth indicates that a temperature of 0 degrees lies between 2 degrees and negative 11 degrees and therefore the altitude for 0 degrees must lie between 2 and 4 km. Using the table Ruth indicates that they said approximately 3 km.			✓		✓		

IA-92	The teacher then tries to show Ruth that a temperature of 0 degrees can be read from the graph (where the graph cuts the x axes). The teacher then asks learners to read off the altitude. Some learners respond by saying 2,5 and a stronger learner insists 2,2. Teacher then indicates where 3 would be on the x axes and then asks again to read off the altitude. A number of learners respond by saying 2,5.		✓	✓	✓	✓		
IA-93	Pholani then responds by asking how one would know exactly what this value is how would we prove it. Teacher responds by asking how did he know exactly what the previous questions answer was. In this, Pholani responds by asking the teacher should he re-explain how he got to that answer. The teacher replies no but that he should use the same concepts. Pholani then immediately starts thinking about how to do this as he says but then it's going to be an increase so it's the other way around. At this the teacher asserts that this is not Pholani's problem only but that everybody else should take ownership of the problem and try and solve it. Teacher goes on that other groups were also given the responsibility to complete this task.							
IA-94	After spending some time speaking to learners about their responsibility Neelan raises his hand and suggests a possible path to a solution. He indicates that since we said the temperature minus 13 is the altitude so why can't we just use the converse of that the altitude plus 13 is the temperature. Teacher tries to make sense of what Neelan just said but finds it difficult and admits that he doesn't get it. Teacher evaluates the information in that the temperature decreases by 13 as the altitude increases by 2. With this in the background the teacher then indicates to add thirteen to the altitude to find the altitude at 0 degrees is not sound reasoning.	✓		✓		✓		
IA-95	Teacher encourages learners to find a pattern that as Pholani indicated that can verify our approximation of 2,5. Jason refers back to the table but teacher insists on a more conclusive accurate method such as the calculation Pholani has done.			✓		✓		
IA-96	Teacher moves over to question 4. Reads the question and then give over to Pholani to give his groups response. Pholani draws a short horizontal line between 11 km and 13 km and then a line going up towards the x axes. Teacher engages learners in some discussion whether this should be a straight line or a curve. Teacher concludes that due to a lack of information it could be either.		✓	✓	✓	✓		✓
IA-97	Teacher offers the "hot seat" to Tiego as the latter indicated that he might have some valuable input regarding no 3.2. Tiego very hesitatingly takes up the "hot seat" and indicates that it is scary. Teacher responds by: now he has some idea of what the teacher goes through.							

	SIDE B							
IA-98	Tiego comes up with a conjecture that if he takes the altitude multiply it by negative 6,5 and then add 15 one would get the temperature. He gives an example of this as he calculates and verifies that the temperature for an altitude of 8km is negative 37 degrees Celsius. On the teacher asking him how he got to that he answered it just came to him. The teacher then asks him to work out the temperature for 4. He then calculates it and get to a confirmed negative 11 degrees Celsius. The teacher asks if that works with all of them at which Tiego responds: yes sir. He is then applauded by a large number of learners in the class.	✓	✓	✓	✓	✓	✓	✓
IA-99	Teacher asks Tiego where he got negative 6,5 from? Tiego indicates that the difference in temperature is -13 so if he goes in ones it would be -6,5. And then for 15 I saw for an altitude of 0 the temperature is 15. Teacher says that he hopes everyone got what Tiego was saying. Teacher indicates that Tiego says that for every degree that it increases the difference say for zero and 1 would be -6,5. He then took the difference and multiplied it by the altitude (confirms with Tiego) and then try to see what must be added to or subtracted from that to give the temperature. And he noticed that the value that had to be added or subtracted had to be 15 to give you exactly the temperature that is given here (pointing at the table).	✓	✓	✓	✓	✓	✓	✓
IA-100	Teacher asks learners: What is the difference between that and that pointing at the first two temperatures in the table. Learners respond by saying 13. Teacher asserts that it is -13. Teacher asks the next two and learners respond -13. Teacher asks what the difference is between first 2 altitudes and learners respond by say -2. Teacher asserts that it is 2 because it increases. Teacher then indicates that the change in temperature is -13 and the change in altitude is 2. Teacher then asks learners to divide -13 by 2 and they get -6,5. teacher explains that this measure is a measure for the rate of change. Teacher illustrates that it is irrelevant from which points we measure this rate of change it will remain the same because it is a straight line. The teacher uses -24 and -50 asking learners what the difference is. Learners respond by saying -26. Teacher then enquire's what the difference between the corresponding altitudes are 6 and 10. Learners respond by saying 4. Teacher then asks what -26 divided by 4 would be. Learners respond by saying -6,5. Teacher then reaffirms that this indicates a constant rate of change which is indicated by the straight line.							

IA-101	Michelle and Karin volunteer to give their answers to activity 15.2.2. Michelle indicates that because the graph curves it represents a changing rate of change. Michelle also indicates that as the ticket price goes up so does the profit but that both is changing.		✓	✓	✓	✓		
IA-102	Michelle protests against hands going up for questions. Teacher reassures her that it is a good thing not a bad thing in that what they ask or add helps us all to better understand.							
IA-103	Craig initially indicates that their answer differs from theirs and states that initially the profit rises with the cost for tickets but at a certain point the profits drop because people will not be prepared to spend so much on a ticket. Teacher enters into discussion with Craig trying to make the point that he is not differing with Michelle but is adding to what Michelle is saying to which he concedes.	✓		✓	✓	✓		
IA-104	Teacher asks Michelle whether the rate of change during the first part of the graphs is increasing or decreasing. To which Michelle responds decreasing. To which teacher responds so it's getting slower. Learner inaudible gives contribution. Teacher respond's by saying that although the profit is getting more the rate at which it is getting more is decreasing. Teacher asks class: I don't know if everybody understands what I am saying.			✓	✓	✓		✓
IA-105	Teacher asserts: Do you all agree to the left of the broken line; the profit is increasing so is your (some learners chorus) ticket price. So both increases so that could be seen as a ... (someone responds: a direct proportion) direct proportion. However the rate at which it is increasing (Michelle says: slows down) slows down. Teacher then asks why? Michelle responds by saying, the tickets become more expensive so people don't want to buy so expensive tickets. Teacher elaborates by saying that at some point there is actually no longer an increase in the profit but that the profit is starting to decrease as the tickets are becoming too expensive.		✓	✓	✓	✓		
IA-106	Teacher asks Michelle to continue to next question. As Michelle starts there are some interruptions by learners, at which teacher calls them to order. Michelle continues that the cost of a ticket is 10, so not many people will buy because they will sit right at the back and therefore no profit is made. Realising that there is confusion the teacher refers back to the point 0, -15 000 and asks what does the 0 represent, that the cost of the ticket is (zero by a number of learners). Michelle and Karin look at each other bewildered. Teacher continues: and the profit is negative 15 000.			✓		✓		

IA-107	Michelle quickly comes back and asks the question: How can there be a loss if you spend 0 rand on tickets. Teacher brings class to order in order to hear Michelle's question again. Michelle starts arguing her point at which the teacher responds by asking her to ask a question. Michelle asks how can there be a loss when 0 Rands are spent. Teacher says that many hands are up.			✓		✓		
IA-108	Peter is acknowledged by the teacher after Ester asserts herself as being the first persons who's hand was up. Peter responds by saying that if you buy a ticket for a ride but you pay nothing they waste money on the ride. At this there is overwhelming response. At this point the teacher calls class to order and indicate that they should talk through a show of hands.			✓		✓		
IA-109	Sophie indicates that the 0 means that the tickets are free which means that the organisers spend 15 000 to organise the event. Meaning that the negative 15000 is a loss for these people. Teacher asks Sophie to repeat because people could not hear her and she does.			✓		✓		
IA-110	Pholani tries to say a similar thing as Sophie, but from a different angle or viewpoint- He's talking about it being the starting point. From what I can make out Pholani is trying to say that the 15 000 is the money spent on making the tickets so the zero 15000 is the point at which tickets are starting to be sold and profit is starting to be made. Most of what is said I can't hear because of the interruptions. Teacher calls class to order again and indicates that everybody's opinion must be respected and that he will try and give everyone an opportunity to speak.			✓		✓		
IA-111	Tabethe explains that if you buy say jerseys so you will use your own money to buy the jerseys in order to make a profit, but if you don't sell any jersey you not going to make anything. You've lost all your money within those jerseys. So over here say they've already paid the DJ. And now they relying on that money of the tickets to increase the profits (NOT CLEAR)more than the DJ.			✓		✓		
IA-112	Teacher gives Thope a turn without responding to Tabethe. Thope tries to explain to Pholani that in the example they had already bought the tickets for 15 000 and they giving them away for free so it is not the starting point for selling the tickets because they already gave them away for free.			✓		✓		
IA-113	Teacher does not respond to Thope and allocates Lita a turn. Lita indicates that if they not selling the tickets and they use 15 000 to say like say decorating the place and pay the DJ and all that stuff and they have to regain the money plus profit but they let the people in for free so they not going to make anything.			✓		✓		
IA-114	Teacher does not respond to Lita and gives Zelda a turn. Zelda indicates that she found her answer. Teacher then indicates that Busi can speak she indicates that she is fine.							

IA-115	Teacher then gives Sophie a turn. Sophie indicates that when they speak of a loss. It says here the point 0, -15000 meaning the profit is -15000 meaning that they not going to sell the tickets. They giving it for free, so it's not a starting point because or they gave it for free that's why we say it's a loss.			✓		✓		
IA-116	Teacher elaborates and states that besides that where do they get the 15 000 to speak of. When you start with a company to make money. Teacher asks learners if they start a company where they going to get 15 000. Learner respond by saying borrow. What does that mean when you borrow money? You owe the money so you running a loss already. Teacher then says alright next... indicating to Michelle to carry on.							
IA-117	Michelle reads the next question and indicates that this is more straight forward. Michelle indicates that they looked at where the curve was the highest and the dotted line kinda gave them that. So it was in between 60 and 80 and which we found that the middle part was 70, right. And we looked in the middle of 70 and 80 which was here, round about right. So we looked in between here so we said it's round about 75. Teacher repeats by saying : so the ticket price is about 75, good excellent. Teacher also indicates that one can do what Tiego did: find a a calculation based on a pattern that will verify the estimated answer that was read from the graph. Teacher also indicates that reading from the graph is less accurate and that is why one would search for such a calculation.	✓	✓		✓			
IA-118	Michelle reads question and then gives their answer: We said that it tells us that there is no profit loss neither is there no profit gained. Michelle goes on by justifying: Because the curve does not go lower than 0 is to 140 or higher. So we said there is no profit gained or loss. Teacher asks so what is the price of a ticket for the first one there where it cuts the x axes. Learners says plus minus 10 rand. Teacher indicates that so if your ticket is 10 you will make not profit or loss. Michelle then indicates so also for 140.			✓		✓		
IA-119	Michelle reads next question and gives their group's response: We said it's because nobody wants to buy such expensive tickets after 75, 75 is highest point to buy (NOT SURE). Teacher says: so you will have less people buying tickets because they so expensive, good excellent. Michelle gets up and leaves hot seat to go sit down. Teacher asks whether there are any other comments or contributions. Did anyone do anything differently that you would like to raise.			✓	✓	✓		

LESSON 3	DAY 4	VIDEO: 4D, S:A&B	REFERENCE	SIDE A	IA-120	IA-121							
	23/10/2008	13:20	DESCRIPTION		Lesson starts by teacher handing out transparencies on which learners must give their solutions to the activities previously handed out. Learners have obviously not completed activities and are busy finding solutions in their groups. Teacher assists some learners in completing the tasks.	Ruth presents her group's solutions: Ruth indicates that how they figured out the table is to use volume is length times breadth times height. She then goes through each cube and indicates that the pattern is the length cubed. She then shows how they completed the pattern by cubing each of the numbers. She gets 1, 8, 27, 64, 125, 216, 1000 and so on. She then reads the second question and responds to it: The volume of the cube is the increase in the dimensions cubed. Teacher asks her to repeat and she says: the volume and teacher completes is the increase in the dimensions cubed.	✓	✓	✓	✓	✓	✓	✓
			Pattern										
			Variable										
			Covariance										
			Slope										
			Representation										
			Additive relationships										
			Multiplicative relationships										

IA-122	Teacher responds by saying: So the increase in the dimension from 1 to 2 would be 1. Teacher asks: What is the increase in the dimension from 1 to 2. Learners respond by saying 1. Teacher asks what is the increase in the dimension from 2 to 3? Learners respond by saying 1. Teacher then asks: So is it 1 cubed all the time. Ruth responds by saying: No it's the number cubed, so it's like. I don't know how to word this.		✓	✓		✓		
IA-123	Carol responds by saying: Every time the dimension increases we cube the number it increases to.	✓	✓	✓		✓		✓
IA-124	Ruth carries on by saying: So you don't increase the difference I mean you don't cube 1 you cube this number(pointing to 2). Teacher responds by saying: So you cube the new dimension. Ruth: Yes. Teacher: Ok.	✓	✓	✓		✓		✓
IA-125	Ruth continues by reading out next the question and proceeds to answer: x times x times x equals to y or x cubed equals to y. She then indicates that Carol will do the next one- the graph.	✓	✓	✓		✓		✓
IA-126	While Carol labels axes Jason asks: Sir did you have to draw a graph with the values or just draw a graph to represent the (unclear). Teacher responds by saying: No, I saw that some people also did it ...Teacher distinguishes between what was expected in these activities from the graphs drawn right in the beginning. He says that the previous graphs had no measurements because we wanted to get an idea of how the rate of change changes and here we have actual measurements. We want to draw graphs using those measurements. Teacher goes on explaining that we want to use the values in the table to plot points and then draw the graph. Teacher also indicates that then we can go back and compare and relate it to what we see happening in the graphs.			✓		✓		
IA-127	Carol continues by saying: Ok so we've put the dimensions over here and the volume over here (indicating with pen). Teacher interrupts: Can I just ask you, why did you choose to put the dimensions at the bottom? Carol responds by saying I don't know. Ruth says: Because you put the dimensions at the bottom and the volume on the side I don't know.			✓		✓		

IA-128	Teacher asks learners if they can remember what we talked about in statistics. He asks them if they can remember the concepts dependent and independent variables. Indicates that learners were talking about bar graphs and now it seems they had forgotten statistics. Teacher asks learners: Is the dimensions in Ruth's, is it a variable? Learners respond: Yes. Teacher: Why can we say that it is a variable? Learner: Because it's changing. Teacher picks up from learner: Because it's changing. We can see it becomes 1 then 2 then 3 then 4 then 5. It varies, your number is increasing, so it changes. So it's a variable. So x is a variable.		✓					
IA-129	Is the volume a variable? Learners respond: yes. Teacher picks up from learners: Yes. Your volume is a variable it was first 1 cubed then 8 cubed then 27 cubed. Ok only 27 not cubed. So it also changes.		✓					
IA-130	So then we have 2 variables but we need to determine which one is the dependent and which one is the independent variable. What did you change? Teacher picks up from learners: You changed the dimensions. What changes because of it? The volume changes because of it. Neelan responds by saying that the volume dependent on the increase in dimensions. Teacher picks up: The volume depends on the increase in dimension.		✓	✓				
IA-131	And we said that your dependant variable is normally represented on your horizontal axes (Mistake). Teacher indicates that it does not have to be but it helps with communicating. Teacher repeats (corrected by Ruth) that the independent variable is represented on the horizontal and the dependant on the vertical. Just for uniformity. So that we get the same picture otherwise my picture is going to look different from yours and how we interpret it then becomes a problem.					✓		
IA-132	Carol plots the first 5 points on the Cartesian plane and then join them with straight line segments. Teacher comments on Carol estimating to plot points and indicate that it is fine for her to do that. He also mentions that she could have used a different scale though.		✓	✓		✓		✓
IA-133	As Carol takes a ruler and joins the points she says that they decided to join the points with straight lines. Teacher asks Carol why they decided on lines. Carol responds: If we were to draw a curve... a curve represents something that's continuous and this is like...(inaudible). Teacher responds by saying: I'm a little bit lost.		✓			✓		

IA-134	Ruth explains: If we had to draw a curve we wouldn't want the dots to be there. Now if we had to draw a curve without dots it would show that it is a continuous thing and this with the dimensions it's not something that continues. That changes at a continuous rate or has a pattern. The only pattern is that it is cubed. So it's not a continuous pattern. Whereas with the dots, number 1 there is where it stops. That is for dimension number 1. Dimension 1 does not like continue into dimension number 2, it stops by that dot.	✓	✓			✓		
IA-135	Teacher asks: Although you have separate pictures can you not think of a possibility where I can have a cube who's length is not 1 but 1,1? And another one who's length is 1,2 and another one who's length is 1,3 and 1,4 , 1,5. So what I'm saying is length continuous or is it discrete? Many learners are saying discrete and other continuous.		✓			✓		
IA-136	Carol says length is continuous but in this case...(inaudible). Teacher explains that what is given in the activity is simply to give learners an idea of how the pattern develops. He says that he could have included other diagrams. Makes an example that he could have included a diagram that has a length of 1, 5.....I tried to make things easier for you to be able to work with numbers so I chose the simplest numbers but you need to think about it...Are we dealing with things that are continuous or discrete ...and distance for me is not discrete.		✓			✓		
IA-137	Ruth conceded that length and distance are continuous but goes on saying: Because it's a cube thing and it's and we measuring each cube by itself it makes that a ...Teacher interrupts and says I get perfectly what you saying but why did you join the lines? Why you join the dots? Because all you have is for 1 (points in air) for 2 (points in air) for 3(points in air) for 4(points in air) You don't have anything in between. So your data is only about the ones that you have. So you won't join them.		✓			✓		
IA-138	Carol is saying something inaudible. Teacher responds by: You just going to have dots. However if you think of it a little bit further like I just explained. We can have the others included....What you trying to tell me is that you only have where the dimension is 1 where the dimension is 2 or 3 so then you only have the dots. And I would if I was you only represent the dots and nothing else. To draw lines means there's something in between going on. Carol starts to erase the lines in between.					✓		

IA-139	Jason gives his contribution which is inaudible but is talking about that's why they joined the dots with lines. Teacher responds to him by saying: I don't mind you joining the dots with lines or curves but then the lines or the curves must represent something. Jason says: But it does. Teacher asks: What does it represent? Jason responds saying: It does represent that there are something that is in between that are (inaudible) but then with the dots. The dots represent the information that we have. We don't have the information for 0, whatever we only have the information for 1, 2, 3, 4, 5.		✓			✓		
IA-140	Teacher indicates that Jason has a good argument. Teacher speaks to class about importance to talk to each other and discipline but looking at video it is not the class that is rowdy but the noise comes from outside. Teacher explains: Jason says that he knows that there is something that we can have that is in between the dots. But how you interpret that. What happens in between is important. Teacher explains to learners that if one looks at the dots the pattern that we see is that the dots is not in a straight line but forms a curve. Therefore what happens in between must also then form a curve.	✓				✓		
SIDE B	13:51							
IA-141	Teacher: Explains what a learner is trying to get at. The graphs slowly curves but then it increases rapidly (teacher uses the words drastically).							
IA-142	Teacher then reminds learners about Pholani's work in a previous lesson: He assumed that the pattern in between stayed the same that is why he could work out what happens in between. If we had to work out what happens between 1 and 2. Would we assume something different, would we assume a straight line when the dots clearly indicate that it is curving.							
IA-143	Teacher then asks learners to cube 1,5. Teacher then asks Carol to plot the points 1,5 and 3, 375.		✓			✓		✓

IA-144	Cube 1,2. Jason responds 1, 728. Carol plots this point. Learner comments : It looks like a curve. Teacher responds: The ones that you have indicates to a pattern...and that pattern shows that it is a curve. So what happens in between I can assume that the rate of change remains the same. The way it's increasing remains the same. So you follow that pattern. So if you have to join these two (points between 1 and 2). What would you do? Would you draw a straight line. Learners respond: No. teacher picks up: No, you would actually draw a curve from one point to the other point. And a curve smooth curve joining all the points. Not little lines in between.			✓		✓		✓
IA-145	Jason asks: This is just out of curiosity. If you spread the dimensions apart from each other would it be a straight line? Like if 1,2 was maybe 2,3 blocks away apart from each other. Teacher indicates that Ester should raise her hand if she wants to speak.			✓		✓		✓
IA-146	Ester says: It wouldn't be any different because what you doing is taking the same number, same scale and making it bigger. Teacher says: You saying that it would only change the scale. You would have a bigger graph still going up a curve, good.			✓		✓		✓
IA-147	Ruth confirms this by saying that she put a number further apart but it still goes up a curve. Teacher asks questions: What does a curve indicate? Learners: Changing rate of change. Teacher pick up: Changing rate of change. Is the changing rate of change increasing or decreasing? Learners: Increasing. Teacher pick up: It's increasing.			✓		✓		
IA-148	Rian and Craig comes up to present their solutions: Rian correctly indicates that the length of the square is the same as the breadth of the square and that the equation becomes y equal to x.	✓	✓	✓		✓		✓
IA-149	Craig then shows the graph as points plotted in a straight line. Teacher elaborates on the fact that these dots can be joined by a straight line indicating what happens in between the dots as well. Emphasizing that length is continuous.		✓	✓		✓		
IA-150	Rian answers the second part of the activity by stating that area is obtained by multiplying the length by itself. Rian states that the equation would be x times x equal to y.	✓	✓	✓		✓		✓
IA-151	Craig adds by saying or x squared would be equal to y.		✓	✓		✓		✓

152	Craig places their graph on the OHP. This graph has been drawn with area on the horizontal and length on the vertical. Teacher asks Craig to change it so that the dimension is on the horizontal. Teacher explains via questioning that his asked Craig to do that because the dimension is the independent (Jason answers these) variable.		✓	✓		✓		✓
IA-153	Ester: If we had the 16and the area at the bottom and you had 16 ... And you had the 4 there. Wouldn't it also make sense? Cause to find the 4 you also need 16. Teacher asks Craig to place the graph the way he initially intended. Teacher then illustrates what Ester is saying but adds that he does not say that it is incorrect to do that but that for uniformity and effective communication we put the independent variable on the horizontal and the dependent on the vertical.		✓	✓		✓		
IA-154	Ester then asks: Can't 16 not also be independent? Teacher: how do you mean?....Teacher asks: So you saying, can the area be independent of the dimension. Ester: Let's not call it that let's just call it length. Teacher: Length can it be? why? Ester: If you wanted to find the length of a square with 16 the (can't verbalise the words). Another learner says: area. Ester: Is like what if you wanted to find the square root of 16 you say 4.		✓	✓		✓		

IA-155	Teacher: What are we busy with in this activity. We had the dimensions didn't we? Didn't we have the dimensions? Some learners say yes. Teacher: So we had the dimensions. And what did we want to find? Some learners say area. Teacher picks up: The area for those particular dimensions as we increasing the dimensions. So if the problem was slightly different. If i give you lots of squares with the areas. Is that what you saying (to Ester)? And I would like to find out what are the lengths then I would completely agree with you. Then I would tell you your relationship in an equation would be x is equal to the square root of y. Then I would agree with you, then in that situation your area is the one that's known and you actually trying to find out what the length of your sides should be in other words your dimensions. And in that case your length is dependent on your area. But in the way the problem was posed to you which one is the dependent. Some learners say the area. Teacher: And that is what I am saying, the way we pose the problem, the way the context of the problem is will determine which one is dependent on which one. Thank you for bringing up that Ester. I know lots of people are tired and they don't want to have this important conversation but they are important. I guarantee you. If you want to go further with mathematics and you want to become a Doctor or Engineer, please do not put blinkers on. And don't just want to come with the idea to let's get it quickly over and done with so that we can go out and have fun. Maths can be fun when you participate.	✓	✓		✓		
IA-156	Craig explains that they calculated the area by squaring the lengths and that is how they plotted the points. He also indicated that the graphs goes up like that. Teacher says it curves. Teacher then asks:	✓	✓		✓		✓
IA-157	So what is the relationship ultimately between the dimensions and the area. Is that an increasing...? Learners: yes. Teacher asks: Is that an indirect or direct proportion. Some learner say: Direct. Teacher asks: Why is it direct? Teacher: As your dimension increases your area is increasing.	✓	✓		✓		
IA-158	Teacher: Rate of change, is it changing or is it the same. Some learners say the same and others changing. Teacher: It's changing, it's increasing, it's a curve. So your rate of change is increasing so that means that rate at which the volume or rather the area increases is increasing. And that is why we see those bigger gaps in between points becomes further apart.		✓		✓		✓

IA-159	Zwelile and Naledi goes up to present their group's solutions. Zwelile explains that to find the breadth they took the area and divided it by the length. She gives examples from their table. She indicates that they said that the relationship between the breadth and the length is: the breadth is 36 divided by length or the length is 36 divided by the breadth. She also gives the equation as y equal to 36 divided by x. Teacher: That's, that's what people? Jeremy: Deep. Teacher picks up: That's deep. Everybody is confused. They wondering what are these people really doing up there. There were some people that did that activity and I'm a little disappointed that the people that did that activity can't even recognise what's going on. Michelle says: I recognise it. Some other learners also indicate that they recognise it.	✓	✓	✓		✓		✓
IA-160	Zwelile asks: Then can I try it again? Teacher: Carry on. Zwelile: In order to find that breadth you obviously have to use length. Teacher interrupts Zwelile and says that in order to understand Zwelile he must be listening...Zwelile: Obviously keeping the same area you must Em...you must divide your area by your length to get your breadth. She then gives examples of each eg 36 divided by 1 is 36, 36 divided by 3 is 12. She also indicates that you can check it by multiplying the length and breadth with each other to get the area and give examples of that: 12 times 3 will give you 36.	✓	✓	✓		✓		✓
IA161	Teacher explains that since the area must be the same the length or breadth must be adjusted to ensure this. Teacher says that one can find the breadth by asking what times a quarter will give you 36. Michelle adds: Or you can just say 36 divided by a quarter. Teacher confirms her equation of y equal to 36 divided by x.		✓	✓		✓		✓

IA-162	Teacher as Zwelile gets up: Graph, let's see what the graph looks like. By the way I noticed that you did something there, you left the 0 and 0 just out what happened there. Zwelile: Let's skip that part. Teacher: We have to come to the 0. Zwelile's groups graph is contrary to all conventions: The y axes scale varies. The x axes is labelled in decreasing order. Teacher stops Zwelile from explaining: I'm going to stop you there because we're running out of time. What have you done? Your axes! You have what you done commit a very grave mistake! But I can understand it because I didn't give you any direction and that's okay. Teacher tries to explain that the labelling of the vertical axes is inconsistent. Zwelile acknowledges that it was her mistake but does not fully understand what the mistake is. Teacher asks class: Who is not following? Asks Carol to explain: Carol indicates that Zwelile should label in quarters. I can hear learner say oh no! as Carol comes up to explain then every two blocks should represent a quarter. Zwelile then asks that how would she fit everything in. Teacher explains that not all the coordinates in the table should be represented. He indicates that Zwelile should go as far as the grid allows. Teacher suggests that to help Zwelile she could use every block as a quarter. Teacher also explains that Zwelile has made an even bigger mistake on her horizontal axes in that she labelled it in descending order. He refers to the fact that the horizontal axes is a number line. Revise concepts of a Cartesian plane through question and answer.					✓		
--------	--	--	--	--	--	---	--	--

LESSON 3	DAY 5	VIDEO: 5D, SA	REFERENCE	SIDE A	DESCRIPTION	Pattern	Variable	Covariance	Slope	Representation	Additive relationships	Multiplicative relationships
	29/10/2008	11:17										
IA-163			Zwelile comes and presents her corrected graph. She indicates that she used a scale of 2 blocks equal to 1 for both axes. The graph is part of a Hyperbolic curve. The teacher however draws her attention to the fact that the axes does not intersect at (0;0) but (0,5;0). Teacher asks Zwelile to explain her graph. She explains the few points that she could draw relating it to the breadth and length.				✓	✓	✓	✓	✓	✓
IA-164			Teacher then asks her how she thinks it will continue and she indicates by a hand movement that it will curve down. Teacher then requests that she put the transparency with the table back up.				✓	✓	✓	✓		
IA-165			Teacher enquires about the length and breadth being 0. Zwelile Says: Because at that time I didn't figure it out....inaudible...But what I have thought of is that is the length is 0 and the breadth is...say for example 2 then that means it would be a straight line.... and it wouldn't be area...cause in order to get 36 you need to multiply the length and the breadth to get your area so...and if you multiply 0 by anything else it gives you 0.				✓	✓		✓		✓

IA-166	Teacher: So what are you saying about? Ok let's take the first one there where the length is 0. What would the breadth have to be? Zwelile: I don't know. Teacher: You don't know. Why you don't know? What is the problem? Zwelile: Because I need the length to tell me what the breadth is. Teacher: So you need the length to tell you what the breadth is because it's the two multiplied. And like you said that 0 times anything will be 0 so you can't get 36. Because area is length times breadth. And what times 0 will give you 36? Is there such a number? Zwelile and other learners say no. Teacher: So what is your answer? It is (Learners say:0)...undefined. There is no value for y. There is no value for the breadth. It is not possible to get a number that we can multiply with 0 that will give you 36. So it's undefined.		✓	✓		✓		✓
IA-167	And at the same at the breadth, at the breadth what is it, it's 0.....so can we find the length? Zwelile: No. Teacher: No we can't we can't find, determine the length, why not? Because there's no value times 0 that will give you 36. Fill it in undefined.		✓	✓		✓		✓
IA-168	Teacher: Now let's just go back to the graph.....Zwelile places the graph back up and the teacher asks learner what the x value on the y axes is. Teacher goes through a few points on the y axes and asks learners what each x value is. Teacher then asks again what the x value on the y axes is. Learners respond by saying 0.		✓	✓		✓		
IA-169	Teacher goes through points on the x axes and asks learners to give the coordinates. Teacher says that you first give the x and then the y value seeing that learners are confusing the two. Teacher then asks: For any point on the x axes what is your y value? Learners say: 0.		✓	✓		✓		
IA-170	Teacher puts Activity with table back on OHP and questions learners about the points where x and y are 0. Teacher puts graph also on OHP to try and link the points on the graph with those in the table. By eliciting responses from some learners explain that the graph gets closer to the y but will never intersect it because of the undefined value. Teacher explains the same for the x.		✓	✓		✓		
IA-171	Lyle comes up to present their group's responses to the activity 15.3.4. Lyle indicates that they've obtained the number of parts by counting them and shows using the diagram that 1 fold gives 2 pieces, 2 folds 4 pieces, 3 folds 8 pieces and says: you keep on counting the last number...inaudible... Learners then interact with each other and Lyle (Inaudible).		✓	✓		✓		
IA-172	Teacher acknowledges Zwelile's hand. Zwelile: If you saying that you must.....If you say that you must multiply by the last number then you going to double it. So shouldn't you add 2 onto the last number? A few learners including Miriam says no. Miriam carries on talking (not clear).	✓	✓			✓	✓	✓
IA-173	Teacher asks Miriam to speak louder. Miriam: Ok with the number of parts in y you say 2 times 2 is 4, 4 times 2 is 8, 8 times 2 is 16 and 16 times 2 is 32. So you don't add 2 you times by 2. Teacher checks whether he understands what Miriam is saying by multiplying the previous number of parts by 2 one gets the next diagram's number of parts.	✓	✓	✓		✓		✓
IA-174	Teacher acknowledges Carol who has her hand up. Carol: Sir I don't, ok see it works like this but won't it be difficult eventually. Because say you have like a number like for 55, are you gonna now like go back and move back from number 1 and you go 2 times. Some learners interrupting.	✓		✓		✓		

IA-175	Teacher asks: Ok let's just listen to Carol carefully to what she's saying ok. She has a valid point just listen. Carol:.... Shouldn't you find a rule that works with the number of folds in order for you to get the number of parts.	✓	✓	✓	✓		
IA-176	Teacher indicates that that is what it's about for example: If I ask you what is the number of parts for 200 folds? Lyle: I wouldn't know. Teacher: you wouldn't know. Why wouldn't you know? Because you got a rule that cannot determine that (Not correct). Ok so you need to determine the number of parts from the number of folds.....So like Carol is saying now you have to work out everything right up to 199 first and then you can work out 200. Which is pointless. It's not getting to where I want to go to.						
IA-177	Seeing her hand is up teacher gives Thope a peaking turn. Thope: Sir isn't the number of parts equal to the number of folds times 2? Teacher: She's saying isn't the number of....Thope: parts equal to the number of folds times 2. Teacher: Isn't the number of parts equal to the number of folds times 2.	✓	✓	✓	✓	✓	✓
IA-178	Teacher: Let's verify that and check that: 1 times 2 is 2, 2 times 2 is 4, but 3 times 2 is 6 not 8. So we have a what? A counter example. And when you have a counter example then your rule cannot be taken into account. It can't work.						
IA-179	Teacher gives Tiego a speaking turn. Tiego: As the number of folds increase the number of parts also increase by raising 2 to the power of x.	✓	✓	✓	✓	✓	✓
IA-180	Teacher: Tiego, Lyle are you following what is he saying. Lyle: What? Say again. Tiego: As the number of folds increase the number of parts also increase by raising 2 to the power of x.	✓	✓	✓	✓	✓	
IA-181	Teacher first verifies that all learners agree with the values Lyle's group gave in the table and then asks Tiego to assist Lyle. .. Teacher reads:Double the last number of parts written according to the folds. Now that's fine what you did Lyle but it limits us like Carol says. It is not incorrect but it is limiting.Let me ask you: How did you work out for 55 folds how did you get that? Did you multiply by 2 until you got for 55? Lyle: Lyle can't answer and tries to work it out but fails to explain successfully. Teacher indicates that we will check that and gives Tiego a turn.						
IA-182	Tiego scratches out Lyle's $2x$ and writes 2 to the power x and then writes: As the number of folds increase the number of parts also increase by raising 2 to the power of x. Tiego does not speak but teacher narrates as Tiego writes.	✓	✓	✓	✓	✓	✓

IA-183	Teacher: Does it work (learners say yes) let's see. 2 to the power 1 is 2 , 2 to the power 2 is 4, 3 to the power 2 is 8 (Learners quickly correct teacher) 2 to the power 3 is 8 and 2 to the power 4,...16, 2 to the power 5... 32...2 to the power 6 ..64 and 2 to the power 7...128. you see what we saying. So what is he working with here. He is working with the number of folds to get to the number of parts. Teacher explains that Lyle's initial method is tedious and that Tiego's method is quicker. Teacher asks:.... Take a page as big as you want to and see if this is really possible realistically..... Can you really fold a page up to even 55?.... An A4 page is 7 times..... do you think by enlarging the page you'll be able to fold it more times. Some learners say yes others no. Teacher: Come and show me that you can fold any page more than 7 times.	✓	✓	✓	✓	✓	✓
IA-184	Teacher points out using the graph that for 0 folds the graph indicates that there are 0 parts. Teacher compares this to the diagrams and learners agree that it should be 1. Lyle indicates that it is not 0 and 0 on the graph but 0 and 1 and that it is the scale they have chosen that makes it look like 0 and 0. They estimated. Teacher indicates that if they had made the scale bigger, then they would have gotten a better picture of what happens. Learners are still folding the page. Zwelile indicates that it is exciting. Teacher adjusts Lyle's graph slightly to show the graph cutting the y axes at (0; 1).			✓	✓	✓	✓

IA-185	SIDE A	REFERENCE	VIDEO: 5D, S.A&B	DAY 5	LESSON 4
Teachers places first graph on OHP and asks for volunteer. Teacher asks which table matches with that? Learners say A. Teacher asks which equation? Learners say (i).		DESCRIPTION		29/10/2008	
	Pattern				
	Variable				
	Covariance				
	Slope				
✓	Representation				
	Additive relationships				
	Multiplicative relationships				

IA-175	Teacher asks: Ok let's just listen to Carol carefully to what she's saying ok. She has a valid point just listen. Carol:.... Shouldn't you find a rule that works with the number of folds in order for you to get the number of parts.	✓	✓	✓	✓		
IA-176	Teacher indicates that that is what it's about for example: If I ask you what is the number of parts for 200 folds? Lyle: I wouldn't know. Teacher: you wouldn't know. Why wouldn't you know? Because you got a rule that cannot determine that (Not correct). Ok so you need to determine the number of parts from the number of folds.....So like Carol is saying now you have to work out everything right up to 199 first and then you can work out 200. Which is pointless. It's not getting to where I want to go to.						
IA-177	Seeing her hand is up teacher gives Thope a peaking turn. Thope: Sir isn't the number of parts equal to the number of folds times 2? Teacher: She's saying isn't the number of....Thope: parts equal to the number of folds times 2. Teacher: Isn't the number of parts equal to the number of folds times 2.	✓	✓	✓	✓	✓	✓
IA-178	Teacher: Let's verify that and check that: 1 times 2 is 2, 2 times 2 is 4, but 3 times 2 is 6 not 8. So we have a what? A counter example. And when you have a counter example then your rule cannot be taken into account. It can't work.						
IA-179	Teacher gives Tiego a speaking turn. Tiego: As the number of folds increase the number of parts also increase by raising 2 to the power of x.	✓	✓	✓	✓	✓	✓
IA-180	Teacher: Tiego, Lyle are you following what is he saying. Lyle: What? Say again. Tiego: As the number of folds increase the number of parts also increase by raising 2 to the power of x.	✓	✓	✓	✓		
IA-181	Teacher first verifies that all learners agree with the values Lyle's group gave in the table and then asks Tiego to assist Lyle. .. Teacher reads:Double the last number of parts written according to the folds. Now that's fine what you did Lyle but it limits us like Carol says. It is not incorrect but it is limiting.Let me ask you: How did you work out for 55 folds how did you get that? Did you multiply by 2 until you got for 55? Lyle: Lyle can't answer and tries to work it out but fails to explain successfully. Teacher indicates that we will check that and gives Tiego a turn.						
IA-182	Tiego scratches out Lyle's $2x$ and writes 2 to the power x and then writes: As the number of folds increase the number of parts also increase by raising 2 to the power of x. Tiego does not speak but teacher narrates as Tiego writes.	✓	✓	✓	✓	✓	✓

IA-183	Teacher: Does it work (learners say yes) let's see. 2 to the power 1 is 2 , 2 to the power 2 is 4, 3 to the power 2 is 8 (Learners quickly correct teacher) 2 to the power 3 is 8 and 2 to the power 4,...16, 2 to the power 5... 32...2 to the power 6 ..64 and 2 to the power 7...128. you see what we saying. So what is he working with here. He is working with the number of folds to get to the number of parts. Teacher explains that Lyle's initial method is tedious and that Tiego's method is quicker. Teacher asks:.... Take a page as big as you want to and see if this is really possible realistically..... Can you really fold a page up to even 55?.... An A4 page is 7 times..... do you think by enlarging the page you'll be able to fold it more times. Some learners say yes others no. Teacher: Come and show me that you can fold any page more than 7 times.	✓	✓	✓	✓	✓	✓
IA-184	Teacher points out using the graph that for 0 folds the graph indicates that there are 0 parts. Teacher compares this to the diagrams and learners agree that it should be 1. Lyle indicates that it is not 0 and 0 on the graph but 0 and 1 and that it is the scale they have chosen that makes it look like 0 and 0. They estimated. Teacher indicates that if they had made the scale bigger, then they would have gotten a better picture of what happens. Learners are still folding the page. Zwelile indicates that it is exciting. Teacher adjusts Lyle's graph slightly to show the graph cutting the y axes at (0; 1).			✓	✓	✓	✓

IA-185	SIDE A	REFERENCE	VIDEO: 5D, S.A&B	DAY 5	LESSON 4
Teachers places first graph on OHP and asks for volunteer. Teacher asks which table matches with that? Learners say A. Teacher asks which equation? Learners say (i).		DESCRIPTION		29/10/2008	
	Pattern				
	Variable				
	Covariance				
	Slope				
✓	Representation				
	Additive relationships				
	Multiplicative relationships				

IA-186	Teacher then asks: Why did you choose to do this, anybody? Ruth says that she matched all the others first and then matched the ones that were left. Ruth then justifies further: The reason why I say that sir if you look at a ok and the table ...If you look -1 and -1there's a point there.....there's a point at 0, 0Teacher: Which is the origin.....Ruth: The line goes through 1 and 1 but there's no point there ...Teacher: the line goes through? Ruth: The curve goes through, if look ne take 1 and 1...Teacher: 1 and 1 yes...Ruth: (inaudible)...Teacher: If you look at this, this is 1 and 1...Ruth: But there's no point there! Teacher: there's no point there. Not all the points are indicated. Ruth: So that's it ...Teacher: So they used a couple of points like I suggested you do. I said you can't go so high like the numbers in the table. So choose a few points to at least get an idea of what the graph looks like. So you don't have to plot every single point as long as you can get enough points to give you an indication of what the graph looks like Ok. So what you saying here is you chose this table because it corresponds to this (pointing at graph) Fine.					✓		
IA-187	Teacher: How did you choose this (pointing at equation) equation. Ruth:...3 to the power 3 is 27 ...Teacher: 3 to the power 3 is 27, So your y value there is equal to your 3 Ok. Ruth: And then 2 to the power 3 is also 8. Teacher: So 2 to the power, if I substitute in there (Pointing at transparency), 2 to the power 3 will give me 8 which is the y over there. So there is definitely a correspondence between the graph, the table and that (pointing at equation).	✓				✓		✓
IA-188	Teacher points at Ruth's group and said that they came up with an equation similar to that. Teacher then refreshes their memories with the transparency of the activity where they had to work with the volume of a cube as the dimensions are increased. Teacher shows them that they came up with the equation $x^3 = y$.							
SIDE B	11:49							
IA-189	Teacher:..... So what are we saying if it's x^3. Basically what I'm trying to say is that your graph will have this shape (pointing at graph on OHP). If it's $y = x^3$ (pointing at equation), that kind of rule we will always have that kind of table (pointing at graph).							
IA-190	Teacher puts graph b on the transparency to complete the next matching task but is taken aback by a question from Carol. Teacher: She (Ruth) said that the table points correspond to what's on the line. In other words the x and y values correspond to what's on the line and that if you look at your equation (pointing to equation) and you look at these values (pointing at table). If you make your x 3 then it becomes 3 to the power 3 which is 27. So the equation generates the values that's in the table. And the points in the table will be on the curve that's there. That's what she said.	✓				✓		
IA-191	Teacher asks Neelan to place the various representations on the OHP as the discussion takes place. Teacher asks learners which table they chose for graph b. Learners indicate table E.					✓		

IA-192	Teacher asks why E. Ruth: Because the values correspond. Teacher: Say that again, Ruth: Because the value, whatever they called, the table corresponds to the graph. Teacher: How? In what sense? Is Ruth the only one in this class. Ruth: If you take for example -2, -2 on the x axes ok, and then 9 on the y axes you can see that there is a dot there. Teacher: Ja....Ruth: Then if you take -1 and 0 there is a dot there as well. Teacher: There we go -1 and 0 (pointing at graph). Ruth: And 1 and 0. Teacher: And 1 and 0 (pointing at graph). Ruth: and 2 and 9. Teacher And 2 and 9 (pointing at graph). Ruth: And 3 and 24. Teacher: And then 3 and 24 (Pointing at graph)..... Ok so what are we saying that the values in the table correspond to the coordinates of the points that's on the graph. Exactly on the graph.	✓				✓		
IA-193	And what equation then?Sophie: 5....Teacher: Equation 5 why?...Sophie:Inaudible....Teacher: Speak louder. I can't hear you. Sophie: Substitute the variables with the numbers...(inaudible)...Teacher: If you substitute what? Sophie: For instance Sir ...3 x, x squared minus 3, If you substitute 3x with 3 ...Teacher: If you substitute x...Sophie: with 3... Teacher: with 3 ok, you get 3 squared which is? Sophie: 9... Teacher: 9 times 3?...Sophie: 27...Teacher: 27 Minus 3 Sophie: is 24...Teacher: Is 24. So your y values here will be exactly the same if you substitute your x values in there.....					✓	✓	✓
IA-194	Does it work for all the points on the table, by the way. Sophie: Yes sir.....	✓				✓		
IA-195	Zwelile says: I get it. I also got the same as her. But when I got to -1 everything went wrong. Teacher: Ok let's just check -1. She's saying something went wrong at -1 so let's substitute. Teacher writes 3(-1) squared minus 3 on board and goes through calculation step by step to get an answer of 3. Zwelile says something about squaring first but not very audible.					✓		
IA-196	The teacher draws learners' attention to the x squared in the equation and asks where learners have seen this before. A learner says the area one. Teacher places graph on OHP that was dealt with before. Teacher makes a point of the fact that the previous graphs only dealt with positive values but if the equation was taken as purely as a rule and negative values were included then the graph would be similar to the one they currently dealing with. Learners actually indicate that the graphs are different because of the negative x values.					✓		
IA-197	Teacher; Next, straight line? Lyle: B. Teacher: B, so table B...					✓		

IA-198	Why that table? Lyle: ...middle...naturally.0,0..Teacher: Ok so the 0, 0 corresponds to the origin (Pointing at table then graph). Lyle: and -1 almost by 5 so it's 3....Teacher: and the -1 which is almost by 5 which is about 3 ...Teacher:Any other correspondence between table and the ...Lyle: 2 and -6 ...Teacher: 2 and -6. so if we go to 2 we see that's just passed -5 which is -6 good. So we see that there is a definite correspondence. The values that's here (table) gives you the coordinates that's the points that's on the line (graph) ok good.					✓		
IA-199	Next, equation? Come on there's more people in this classroom. Teacher says: Yes, Karin. Jason answers though. Jason: I said 3. Teacher: You said 3.					✓		
IA-200	Ok 3, why 3?...Jason: ...If you look at the x values andso -3 times 3 gives you -9 and then 2 times -3 will give you -6...Teacher: Can you see 2 (table) if you got x 2 (equation) there. -3 times 2 give you -6. It is the y value there (table). So if you substitute into this equation (equation) Your x here (table) will then generate whatever y is here (table)..... Anybody who doesn't get that?	✓				✓		✓
IA-201	Thope: The numbers of y are all multiples of 3. Teacher: They all? Thope: multiples of 3. Teacher: Ok she's noticed something else. She saw that you have your x values which are there (table). And then you have your 3 times x (equation). And all these values (Table) are multiples of 3. That means you had to multiply each x value with a 3 (equation) to get these values here (table) ok. You see there (pointing at values in table). A negative 3 times -1 is 3. 0 times 3 is 0. 1 times -3 is -3. They are all multiples of negative 3.	✓				✓		✓
IA-202	What does that mean? It means that... if you look at these values (pointing at y values in table) ... What is happening here? What is it changing here? What is the difference between these numbers? From 3 to 0 is? Learners: 3 Teacher: 1...It's minus 3. From 0 to -3 is...minus 3. from -3 to -6 is?...minus 3...From -6 to -9 ...minus 3. What is the difference here from -1 to 0 (pointing to x values in table)plus 1. Ok so what are we saying? The y's have a difference of negative 3 (writes -3/ on board)...the x's have a difference between them of (Learners say plus 1) plus one(Writes +1 below -3/ and =). And if we say negative 3 divided by plus 1 what do we get? Learner: negative 3. Teacher: We getting negative 3 (Writes on board). Remember when Tiego, who is sleeping right now. But Tiego came up with a wonderful thing. Remember that straight line that we had. And then he said 1,6. Or was it 1,65 (at Tiego). And we saw that if we calculate the difference in the table and difference in the x, y values and divide them we got exactly that constant value. Same thing. What does this give me (pointing at answer of -3 on board) What does this value give me? It gives me a measurement for what? Learner says: (inaudible)...change. Teacher: For the rate of change. And what do we notice here (pointing at equation). That your rate of change corresponds to the value there in front of the x value. we also call that value, your rate of change, we call it your gradient. It's known as your gradient for any line.	✓			✓	✓		

IA-203	Teacher: Graph d corresponding to what? Learner: D. Teacher: corresponding to D.					✓		
IA-204	Why? Why? Why? Why? Yes Tiego? Tiego: Inaudible. Teacher: Yes why do you choose table D to correspond to that graph? ...Yes Zwelile? Zwelile: Ok, my -0,5 is -6 . Teacher: If you look at that point there -0,5 is there and -6 would be there. So the point is on the graph. Zwelile: and -1 would be -3. Teacher: What did you say? -1 is -3. Ok can you see that -1, -3 is there. Zwelile: And then the 1 is 3. Teacher: and then the 1 is 3. If x is 1 then y would be 3 which is that point on...So every single x and y value. We have a point for it that lies exactly on that curve. Which means it satisfies the graph, it satisfies the equation. Not the equation the table.	✓				✓		
IA-205	Equation which equation is that? Rian: x, y equal to 3. Teacher: x, y equal to 3. Zwelile: Number 4. Teacher: Number 4.					✓		✓
IA-206	Why did you choose that? Rian: Because eh. You say. I think you timesing here. You say. You times x by y You multiply x by y and you'll get 3. Teacher: You multiply x by y and you will get 3.					✓		✓
IA-207	Teacher: What does this remind you of? We've done something like this just recently. Yeah your one (Pointing at Zwelile who's hand is up). Recap on previous activity where $xy=36$. Expand on the shape of the graph moving closer to axes but never intersecting it because of undefined values. Draw similarities and differences between Zwelile's graph and this one. Indicate that Zwelile's was limited because of axes and a better picture can be seen because of extended x axes.					✓		
IA-208	Zwelile: If we have a test and we could only see the (Part of graph indicated by hand movement) curve. Teacher: What we are learning now is that if we look at an equation like x times y we know that the graph is going to be doing something like that (pointing at graph). Teacher indicates that the fact that we have negative x values here is a further difference from Zwelile's graph.							
IA-209	Teacher: Ok Let's look at graph e...Learner: Obvious. Teacher: Obvious what is obvious, there's only two left so let's put them there. But let's find reason why they belong together even though it's obvious.							

IA-210	Rian: When we look at the table. Teacher: Ok we look at the table. Pholani: Ok negative 2 on the graph....Teacher: -2 on graph....Pholani: correspond with 1 over 9.Teacher: 1 over 9. You see it's a small fraction of 1. ...Pholani: Then 1...Teacher: 1...Pholani: on the graph also corresponds to 3....Teacher: 1 Corresponds to 3. Ok, there you go.Pholani: Then 3 ...Teacher: Then 3? Pholani: no there's no 3. Teacher: What about the 0 and 1? If x is 0 what is your y value?1...Pholani: 1 but there's no indication. Teacher: There's no indication but it is 1. So it will be cutting at 1. Where did we see that? Did we see that before? Learner: No....Teacher: Yes we have. You didn't see that but I know it was there. The one Lyle just did. I'm looking for it.	✓					✓	
IA-211	Teacher places the graph Lyle did on OHP. Teacher draws a comparison between Lyle's and the graphs at hand. He indicates that the graph intersected at 0, 1 just like this one does. He also indicates that in Lyle's graph we only worked with positive values but if we regard the equation strictly as a rule we could substitute negative values for x and then we will have the part of the graph indicated in the second quadrant.							
IA-213	Teacher: And what did Tiego give us as a rule? He said that it was y equal to 2 to the power x. ...And if we look at it nicely (writes on board: $y = 2^x$). What is this value here? If this value is x(circle x in equation on board). x value is? (points at y intercept on graph). Learners: 1.Teacher: Ha ah...Learner: zero....Teacher: What is your x value for that point there (points at y intercept on graph) Learners: zero....Teacher: It's zero. So y is equal to 2 to the power 0(writes $y = 2^0$ on board). And what have we learnt in exponents. 2 to, any value to the power 0 is? ...Learners: 1...Teacher: 1. Always one. Ok. So we have your exponent being x (Points at equation). And if your value there your exponent becomes a 0 the graph will always cut through 1 because then your y becomes equal to 1.							✓
IA-214	Teacher: Equation? And that should be obvious, because it's obvious but now we know a little more. We can see that that equation should have what? Should have to the power something, x. so let's see what it is.We have y equal to 3 to the power x. Pholani (working on calculator): In this regard we get 3 to the power 2. No wait. Oh 3 to the power x which is -2. We get 0,1 which is equivalent to 1 over 9....Teacher: Ok good. So it does correspond. He is saying that when you substitute the value for x(point at equation) here and if you substitute -2 (point at graph) So you say 3 to the power -2 (Pointing at equation). He did it on his calculator. He noticed that he got a value that is equivalent to, zero comma? ..Pholani: I got 0,1 which is equivalent to 1 over 9. ..Teacher: Zero comma 1 which is equivalent to 1 over 9. We know that if we make 0,1 a proper fraction (writes on board 0,1). What do we get? we get 1 over?...Pholani: 9...Teacher: 1 over 9 (writes 1/9 on board). Is that correct? Pholani: Yes. Teacher: it's 1 over 10. But what is your value? Pholani: 0 comma 1...Teacher: Just double check that.					✓		✓

IA-215	Teacher: Ok I've got a better way. Let's use exponents. You guys done exponents. So let's just have a look at this quickly. What is it? It's y equal to 3 to the power negative 2(writes $y = 3^{-2}$ on board). What did we learn about negative exponents? Come now. Dig, dig, dig, dig, dig way back way back when. Negative exponents, is it negative numbers? No, what did we say? Please I'm praying, I'm praying very hard. I know it's late. I know it's very hot. I know you tired. CTA's are killing you. Learner: Yes!...Teacher: But really. Let's go. Negative exponents people are?...fractions, fractions. 2 to the power -2 means y is equal to 1 over 3 to the power 2 (writes $y = 1/3^2$ on board)..Can here Zwelile go: Oh! And therefore y is equal to 1(writes on board $y = 1/$). What is 3 to the power 2? ...Learners Say: nine...Teacher: 9 (writes 9 below $1/$). So y is equal to 1 ninth which corresponds to x being equal to -2 and y being equal to 1 over nine (points at table values). Please people negative exponents, invert to make it positive. Teacher speaks about learning to understand and not just to get it over and done with because it all comes back again.						
IA-216	Pholani: 1 over 9 is equivalent to 0,1....Teacher: It can't be. It cannot be. Let's just do that again. What did we say? 1 divided by 9 as a decimal number. What do you guys get. Just do that quickly...Learner says: 11111....Teacher: It's 1111111 so it's 1 recurring. zero comma 1 recurring. It is not exactly 0,1..Pholani: It's approximately...Teacher: It's approximately. One over 10 (writes $1/10$ on board) is very close to one over 9 but it's not exactly. It's slightly bigger. It's not 0,1 like in 10 but it's 0,1 recurring writes(0,1' on board). And 0,1 recurring is a bigger number then 0,1. Slightly bigger..... 1 over 9 is larger than 1 over 10. So 0,1 recurring is a bigger number then 0,1 exactly. 1 over 10 is 0,1 but 1 over 9 is 0,1 recurring				✓		

APPENDIX M

Kurt Coetzee
School of Education
Wits University
Cell: 084 606 5533

Dear Learner

Information and Consent to participate in a research project

DESCRIPTION

You are invited to participate in a research study on mathematics teaching and learning. The study aims to understand teaching practices in high school mathematics classrooms and how these promote or inhibit mathematical reasoning among learners. This study being conducted also forms part of the requirements needed by me to be awarded a Masters degree in Science education at Wits University. As part of the study I would like to observe and videotape a number of lessons in your mathematics class. I will also need to use your written work during these lessons to allow me insight into how learners reason mathematically. If you choose to participate you need to know that where needed data would be used for research purposes and teacher-education purposes for a maximum period of five years.

CONFIDENTIALITY

Unless you request otherwise, your name will be kept completely confidential at all times and in all academic writing about the study.

RISKS AND BENEFITS/PAYMENT

There are no foreseeable risks in participating in this study. The study will have no effects on your mathematics marks. You will not be paid for participating in the study. Benefits of the project will be a contribution to understandings of mathematics teaching and learning. If you have any concerns about participation, or any questions that you would like to ask, please contact me at any time.

TIME INVOLVEMENT

Classroom observations will take place during class time.

SUBJECT'S RIGHTS

If you have read this form and have decided to participate in this project, please understand your participation is voluntary and you have the right to withdraw your consent or discontinue participation at any time without penalty. You have the right to refuse to answer particular questions. Your individual privacy will be maintained in all published and written data resulting from the study.

VIDEO TAPES

I like to reassure you that if you agree to participate in the study the video tapes will be kept strictly confidential. Once the tapes are no longer needed for research purposes, they will be destroyed.

CONSENT

Please complete, sign and return the form below, indicating whether you agree or do not agree to participate.

Learner's consent form

Please tick the appropriate box and **sign on the flip side**.

Participation in study:

- ☐ I am **willing** to participate in the study
- ☐ I am **not willing** to participate in the study

Video recording:

- ☐ I am **willing** to be videotaped in my maths class.
- ☐ I am **not willing** to be videotaped in my class.

Use of video:

- ☐ I **give consent** for video tapes with me in them to be used for research purposes in this study.
- ☐ I **do not give consent** for video tapes with me in them to be used for research purposes in this study.

Use of video for other educational purposes:

- ☐ I **give consent** for video tapes with me in them resulting from this study to be shown at academic conferences.
- ☐ I **do not give consent** for video tapes with me in them resulting from this study to be shown at academic conferences.
- ☐ I **give consent** for video tapes with me in them resulting from this study to be used for teaching purposes, in teacher education programmes.
- ☐ I **do not give consent** for video tapes with me in them resulting from this study to be used for teaching purposes, in teacher education programmes.

Written work:

☐ I **give consent** for my written work completed during this study to be used for research purposes in this study.

☐ I **do not give consent** for my written work completed during this study to be used for research purposes in this study.

Learner's name: _____

Signature: _____

Date: _____

APPENDIX N

Kurt Coetzee
School of Education
Wits University
Cell: 084 606 55 33

Dear Parent/Guardian

Information and Consent to participate in a research project

DESCRIPTION

Your child has been invited to participate in a research study on mathematics teaching and learning. The study aims to understand teaching practices in high school mathematics classrooms and how these promote or inhibit mathematical reasoning among learners. This study being conducted also forms part of the requirements needed by me to be awarded a Masters degree in Science education at Wits University. As part of the study I would like to observe and videotape a number of lessons in your child's mathematics class. I will also need to use your child's written work during these lessons to allow me insight into how learners reason mathematically and how my practices could have promoted or inhibited their reasoning. It is important to know that where you have given permission, data would be used for research purposes and teacher-education purposes for a maximum period of five years.

CONFIDENTIALITY

Unless you request otherwise, your child's name will be kept completely confidential at all times and in all academic writing about the study.

RISKS AND BENEFITS/PAYMENT

There are no foreseeable risks in participating in this study. The study will have no effects on your child's mathematics marks. Your child will not be paid for participating in the study. Benefits of the project will be a contribution to understandings of mathematics teaching and learning. If you have any concerns about participation, or any questions that you would like to ask, please contact me at any time.

TIME INVOLVEMENT

Classroom observations will take place during class time.

SUBJECT'S RIGHTS

If you have read this form and have decided to allow your child to participate in this project, please understand your child's participation is voluntary and your child has the right to withdraw his/her consent or discontinue participation at any time without penalty. Your child has the right to refuse to answer particular questions. Your child's individual privacy will be maintained in all published and written data resulting from the study.

VIDEO TAPES

I like to reassure you that if you agree to let your child participate in the study the video tapes will be kept strictly confidential. Once the tapes are no longer needed for research purposes, they will be destroyed.

CONSENT

Please complete, sign and return the form below, indicating whether you agree or do not agree to allow your child to participate.

CONSENT (Parent/Guardian)

Please tick the appropriate box and **sign on the flip side**

Participation in study:

- ☐ I **give consent** for my child to participate in the study
- ☐ I **do not give consent** for my child to participate in the study

Video recording:

- ☐ I **give consent** for my child to be videotaped in his/her mathematics class.
- ☐ I **do not give consent** for my child to be videotaped in my class.

Use of video of this research:

- ☐ I **give consent** for video tapes with my child in them to be used for research purposes in this study.
- ☐ I **do not give consent** for video tapes with my child in them to be used for research purposes in this study.

Use of video for other educational purposes:

- ☐ I **give consent** for video tapes with my child in them resulting from this study to be shown at academic conferences.
- ☐ I **do not give consent** for video tapes with my child in them resulting from this study to be shown at academic conferences.
- ☐ I **give consent** for video tapes with my child in them resulting from this study to be used for teaching purposes, in teacher education programmes.
- ☐ I **do not give consent** for video tapes with my child in them resulting from this study to be used for teaching purposes, in teacher education programmes.

Written work:

- ☐ I **give consent** for my child's written work completed during this study to be used for research purposes in this study.
- ☐ I **do not give consent** for my child's written work completed during this study to be used for research purposes in this study.

Parent/Guardian's name: _____

Signature: _____

Date: _____