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Secondary school teachers' knowledge of the dynamics of teaching and learning mathematics in multilingual classrooms

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List of abbreviations

ANC - African National Congress
AECCC - Australian Education Council Curriculum Corporation
COTEP - Committee on Teacher Education Policy
DET - Department of Education and Training
DoE - Department of Education
ESL - English Second Language
INSET - in-service education and training
NCTM - National Council of Teachers of Mathematics
NEPI - National Education Policy Investigation
NESB - non English - speaking backgrounds
NQF - National Qualifications Framework
PRESET - pre-service education and training

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To Taffy, Joshua and Michelle

for their patience, love and support

Declaration

I declare that this thesis is my own unaided work. It is submitted for the degree of Doctor of Philosophy in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any other degree or examination in any other university.

Jillian Beryl Adler

10th day of June, 1996.

Three key dilemmas emerge in this study: the dilemma of code-switching, the dilemma of mediation, and the dilemma of transparency. Evidenced in this study is that while dilemmas suggest the need for a choice between opposites, they are never either/ors in the complex life-blood of classrooms. Instead, they are a potential source of praxis. Teachers manage their dilemmas, sometimes fully aware of the choices they make, choices that are at once personal, practical and contextual. At other times, elements of their practice are obscured.

The theoretical contribution of this thesis lies in broadening the conception of teaching dilemmas and in extending the language of dilemmas to capture the specificity of the multilingual mathematics classroom.

The implications of the study for mathematics teacher education are profound yet simple. A language of dilemmas can assist teachers to recognise, talk about and act on the tensions in their practice, and so empower them to make informed and contextually appropriate pedagogical decisions.

Keywords: mathematics education; teacher education; teachers' knowledge; multilingual; secondary; teaching dilemmas, code-switching, mediation, transparency.

Abstract

This is a study of secondary mathematics teachers' knowledge of the dynamics of learning and teaching mathematics in multilingual classrooms in South Africa. It probes teachers' articulated and tacit knowledge through a qualitative methodology that includes in-depth interviews, classroom observations, and reflective workshops. The sample is purposive and theoretical, comprising six teachers drawn from three different multilingual school contexts. Categories of description and analytic narrative vignettes enable a qualitative, layered analysis of what the teachers said and how they acted.

A sociocultural theoretical framework is developed to explain both teachers' knowledge and the learning and teaching of school mathematics. In a multilingual classroom, it is the teacher's role to harness language as a resource, and to assist learners' movement back and forth between their main language and the language of instruction on the one hand, and between everyday, educational and mathematical discourses on the other. The complex communicative demands on teachers and learners are best understood within a social theory of mind. Here, teaching and learning are dialectical processes, deeply interrelated.

The notion of a 'teaching dilemma' is the key analytic mechanism proposed to open up teachers' knowledge of the complex and dialectical nature of teaching and learning mathematics in multilingual classrooms. This study confirms that teaching dilemmas are at once explanatory tools and analytic devices for teaching. They make explicit the tensions inherent in teaching. Once located in a multilingual mathematics classroom, a language of dilemmas becomes generative of dilemmas that highlight tensions specific to the language and mathematical challenges in such classrooms.

SECONDARY SCHOOL TEACHERS' KNOWLEDGE OF THE DYNAMICS OF TEACHING AND LEARNING MATHEMATICS IN MULTILINGUAL CLASSROOMS

Jillian Beryl Adler

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decontextualisation of learners, and hence silence socio-cultural forces at play in mathematics classroom communication.[11] These studies do not and cannot provide an adequate account of the communication of mathematics in classrooms, many of which are constituted by linguistic and cultural diversity. From a socio-cultural perspective, it is not only what mathematics is taught and learnt, but how and where, and if the how entails a language-rich classroom, then the communication of mathematics cannot be taken for granted (Wertsch, 1985). Communication in the classroom is, therefore, not simply a cognitive resource. This is central to Lerman's argument for the incompatibility of Piagetian and Vygotskian perspectives. Language as a mediational means is inherently social. Meaning never exists outside sets of social practices and sets of discursive relations. There is thus cause for concern with the dominant influence of radical constructivism in research activity in mathematics education curriculum initiatives. As Hicks (1995) argues:

> The work of educational reformers such as those embracing the NCTM standards for mathematics teaching has led to important changes in the form of classroom communication ... (However) educational researchers and teacher-researchers may need to move one step further than altering the participant frameworks that constitute more traditional instructional practices. They may also need to begin serious enquiry into how the heterogeneous voices of students and teachers situationally constitute classroom discourses and what counts as academic knowledge (p. 86).

Secada (1991) also criticises cognitive research in mathematics education in the USA for taking as given or normalising the monolingual child. He posits that this is increasingly not the case[12] and a whole set of different questions related to cognition and curriculum arise if you turn the equation around and make the bilingual child the norm.

There is a similar silencing of language differences amongst research subjects (that is, school learners) in research linked to the Cockcroft Report (Cockcroft, 1982) where, too, Piagetian underpinnings can be discerned. For example, Hoyles

individual constructions, but within a radical constructivist paradigm, Cobb et al (1992), Wood et al (1992), Yackel (1992), Murray (1992), and Murray et al (1993) all report classroom based research on the manner and extent to which mathematical knowledge construction is intersubjective. Their collective research evidences how mathematical knowledge (and not simply rote execution of set procedures) can be, and is, effectively built through interactive classroom processes. And shifting focus from learners to teachers, Jaworski has investigated the tensions that arise for teachers working with a 'constructivist philosophy' (Jaworski, 1989; 1991; 1995), that is, with a philosophy that respects and values learners' conceptions and entails interactive processes in the classroom.

All this research reveals important changes in forms of classroom communication. Teachers co-construct with learners new forms of discourse consonant with the goals of enabling learners to engage in more challenging and authentic mathematical practices. Communication of mathematics either by a learner, or amongst learners, or between learners and teachers respectively constitutes the data in the research cited above. Excerpts from classroom transcripts are used to focus on and explain the *communication* that *does* happen and how this shapes the mathematical knowledge made available. Cobb et al's extensive work, for example, typically provides detailed analysis of the development of intersubjective meanings between a teacher and some pupils, or between pupils themselves. However, the constructivist perspective of this work leads to a silencing of crucial dimensions of communication. The question: 'Who is communicating?' if asked, is mostly posited only in relation to pupils versus the teacher, and not in relation to pupil diversity and difference. One does not know if these classrooms are multilingual. The language learners bring to class is not problematised.

While these studies have gone some way to explaining and illuminating learner conceptions on the one hand and the negotiation and co-construction of mathematical meaning on the other, they simultaneously produce a

mathematical meanings are negotiated and intersubjective (for example, Cobb et al, 1992).

Debate on the primacy of the individual or the social in meaning making and the development of consciousness is often referred to as the Piaget-Vygotsky debate. Building on attempts to resolve this debate theoretically (Confrey, 1994, 1995a, 1995b) and pragmatically (Cobb, 1994a), Lerman (1996) cogently argues that Piagetian radical constructivism and Vygotskian socio cultural constructivism are irreconcilable. Tacking 'intersubjectivity' on to radical constructivist presuppositions, as Cobb endeavours to do, simply creates incoherence. Lerman argues further that it is unhelpful to claim that either Piaget or Vygotsky ignore or downplay the individual or social life in their theories:

> ... both views place the individual and social life at the centre of their theories... the difference is encapsulated in their identification of the source of meaning, the one identifying the cognizing individual and the other cultural and discursive practices.
>
> (Lerman, 1996, p. 147)

Thus dichotomising individual and social forces in learning as either/ors is spurious, as is attaching these opposites to Piagetian and Vygotskian theory respectively.

Lerman situates his critique of the Piaget Vygotsky debate in the current mathematics education community where radical constructivism is 'a major, if not dominant theoretical orientation ... in relation to children's learning' (p. 133). This orientation has shaped a great deal of recent and influential research in mathematics education. Confrey (1991; 1995c), for example, has investigated an individual learner's active and unique construction of mathematical knowledge, as well as what we might learn if we listen to children working with mathematics, that is, if we enable their mathematical voices. Confrey's research is unsettling for teachers who believe that mathematical knowledge is certain and that their pupils cannot generate different and worthy mathematisations. Extending beyond

Neither question is peculiar to South African nor to multilingual classrooms. Even in monolingual settings, pupils bring a range of communicative competences to classroom communication. Situating this research in urban, multilingual classrooms in a changing South Africa serves to highlight these issues as well as reflect their historical and situational specificity.

3.1.2 Related research and development in mathematics education

The past decade has witnessed a range of research linked, directly or indirectly, with the curriculum initiatives cited above.

A proliferation of predominantly North American research has arisen out of von Glasersfeld's radical constructivism (1990)[7]. Radical constructivism derives from Piaget (Piaget, 1970) and combines two main elements: the Piagetian notion of the subject as actively building his or her knowledge; and an ontological position which asserts that cognition serves the subject's organisation of the world, not the discovery of an independent ontological reality. Radical constructivism has been challenged on both epistemological and pedagogical grounds. Epistemological criticisms focus on the implication of solipsistic knowledge in the theory (Lerman, 1989; Ernest, 1991), and the charge that it is no more than 'new wine' in the 'old bottle' of empiricism (Suchting, 1992; Matthews, 1992). Pedagogical challenges concern the interpretations of radical constructivism for the learning and teaching of mathematics. On the one hand, there is sociologically informed critique that constructivist theory denies social forces in learning and the negotiated, social and political dimensions of knowledge and meaning (for example, Gordon, 1993). On the other hand, and from a psychological perspective, debate has focused on whether teacher talk is an intervention or interference in individual learners' constructions or whether it is a crucial dimension in the mediation of cultural knowledge in school (Ellerton and Clements, 1992), and on evidence that

classrooms shape meaning making and hence possibilities for mathematical learning; an advocacy of more open, learner centred and communicative classrooms, that is, of pedagogical (teaching and learning) practices that *require much more language-rich mathematics classrooms than is current practice*; and (d) concerns with improving the quality of mathematics teaching and learning, and success rates for *all* learners at school.

These initiatives are clearly reflected in the new aims of mathematics syllabi recently released in South Africa. To quote but a few:

> **2.2** General Teaching and Learning aims
>
> This syllabus is aimed at fostering the following teaching and learning aims:
>
> ...
>
> **2.2.4** to develop fluency in communicative and linguistic skills e.g. reading, writing, listening and speaking
>
> **2.2.5** to encourage a cooperative learning environment
>
> **2.3** Specific aims of mathematics education
>
> ....
>
> **2.3.8** to develop the ability to understand, interpret, read, speak and write mathematical language
>
> (Department of Education (DoE), 1995b)

The South African example raises two interrelated questions that are addressed in this study:

(i) how might a teacher manage the teaching and learning of mathematics in a more communicative classroom when pupils bring a range of main and additional languages to class?

(ii) how might a teacher manage the development in learners of both the language of learning (in this case English) and the language of mathematics at the same time?

and creativity and flexible and critical thinkers, problem posers and solvers (Keitel 1994; Young, 1994). This is reflected in a number of major national mathematics curriculum proposals, each of which places a new and detailed emphasis on communication in the learning and teaching of mathematics (Cockcroft, 1982; Australian Education Council Curriculum Corporation (AECCC), 1991; National Council of Teachers of Mathematics (NCTM), 1989, 1991). At one level this has to do with *mathematics for communication*, that is, the power of mathematical language in particular forms of communication and hence the importance of access to, and the development of, communicative competence[i] in all mathematics learners: they should be able to use mathematical language, to communicate mathematically. Mathematical competence is thus more than the ability to execute set procedures."

At another level the emphasis on communication has to do with what I call *communicating for mathematics*. 'Students should learn to use language as a tool for reflecting on their mathematical experiences hence for their own mathematical learning', states the AECCC document (p. 19). Similarly, Cockcroft's famous six elements of mathematics classroom practice include pupil-pupil and teacher-pupil discussions to facilitate the development of mathematical ideas and concepts (p.71). In verbalising their ideas, pupils' speech for self (private) and for others (public) function to enhance mathematical learning through meaning making activity. Focusing more broadly on classroom discourse, the authors of NCTM's Professional Standards for Teaching Mathematics (NCTM, 1991) argue that 'discourse ... is central to both *what* students learn about mathematics as well as *how* they learn it' and devote two standards to teachers' and learners' discursive roles (pp. 52-54).

While there are differences between these national proposals, they share: (a) a commitment to communicative competence in mathematics, and hence a broader view of what constitutes mathematical knowledge and competence; (b) an understanding that styles, modes and means of communication in mathematics

Moreover, as a teacher educator, I am interested in teachers' knowledge of their practice.

Secondly, the focus on language (and specifically teachers' knowledge of this) in this study does not imply that language is the only factor in mathematics learning in school classrooms. Teachers and learners harness a range of resources, both material and symbolic in their mathematics classrooms. Verbal communication is a key resource in classrooms, but it obviously works together with visualisation and action (Vygotsky, 1978). The foregrounding of language here is both appropriate for, and necessary to, a focused study.

3 WHY THIS AND WHY NOW?

Having set the scene for the study, I now turn to its rationale. In the rest of this chapter I will discuss why this study should be done now, why it focuses on teachers rather than learners, why *South Africa* is an appropriate and generative setting for the study and why a focus on mathematics is appropriate. In addressing these questions, I also provide a critical review of the literature pertaining to my field of study. I conclude the chapter with a discussion on the limitations of the study.

3.1 Why now?

Recent developments in mathematics education, with a particular focus on literature and research in the field of mathematics education and language, reveal the pertinence of my work at this juncture.

3.1.1 Current curriculum initiatives in mathematics education

Demands of a rapidly changing, increasingly technologised world and global economy have led to advocacy for educational systems that will produce initiative

amongst teachers and learners) is centre stage, involves both appropriation[4] of cultural knowledge and development and change in human consciousness. With language as a medium of communication in school, the nature and quality of communication in the classroom will significantly shape the development of mathematical knowledge in learners.

The underlying assumptions in this study are that (a) teachers are knowledgeable about their practice; (b) teachers build theories just as much as theories build teachers (Levino, 1993); and (c) from a perspective of a social theory of mind, teachers in multilingual school contexts are confronted by situations that might otherwise be taken for granted and manage their teaching and their multilingual environments as part of the complexity of teaching and learning. There is therefore a lot to learn from teachers' experience and understandings of *their* practices in *their* multilingual settings.

An underlying longer term motive is that an increased awareness of the dynamics of language and mathematics in school can contribute to teacher education. What teachers know, both consciously and tacitly, is a crucial element in any INSET (in-service education) practice. If INSET is to be effective it needs to be informed not only by policy but by knowledge and practices on the ground. My hope is that this thesis will usefully inform mathematics teacher education in general, and in a democratising and explicitly multilingual South Africa in particular.

There are two necessary boundaries to this study that require explicit mention. Firstly, while this study is ultimately concerned with learner access to mathematical knowledge, learners are only indirectly part of the study. It is beyond the scope of one thesis to examine the dynamics of multilingual classrooms and mathematics learning in all its facets. The focus here on teachers' knowledge is *one* way into the issue. This route is appropriate because, as I shall argue, teachers' knowledge in the field represents a gap in current literature.

The point of this elaboration of my own interests and growth is to show some of the inspiration for the study of teachers' knowledge of their practices in multilingual mathematics classrooms reported here. The multilingual classroom emerged as a site demanding further study within a wider context where mathematics, and its learning and teaching, were being reinterpreted as social and cultural processes, and where teachers were regarded as the reflective practitioners and curriculum and knowledge producers that they are. Together these constituted the starting points of my study, and have both shaped and been shaped by its processes.

2 AIMS, THEORETICAL ORIENTATION, ASSUMPTIONS, MOTIVES AND BOUNDARIES

The aim of this research is to investigate, that is, to describe, analyse and explain, some secondary mathematics teachers' practical and theoretical knowledge of those dimensions of their practice shaped by particular multilingual school contexts.

The specific research question that guides the study is: **What is secondary teachers' knowledge (both conscious and tacit) of their practices in multilingual classrooms?**

In grappling with this question, the study generates a theoretical framework and language that enables description, analysis and explanation of teachers' knowledge. While theory generation is part of the research process, my study is underpinned by a social theory of mind (Vygotsky, 1978; 1986). This theoretical orientation is elaborated in Chapter 2. Very briefly, knowledge is understood as situated (Lave and Wenger, 1991) and socially and culturally mediated. Language, or more specifically, speech, is a key mediational means (Wertsch, 1985; 1991a). Intersubjectivity and interpersonal communication are the 'driving belts' of the development and structure of human consciousness (Kozulin, in Lerman, 1996, p.148). Schooling, a specific teaching and learning context where speech (talk

Not surprisingly, teachers in my courses shared my growing interest in communicating mathematics. Working from the assumption that knowledge is situated, made and not given, my seminars included critical engagement with mathematics education literature related to the communicative and cognitive function of 'talk' in mathematical meaning-making; the specificity of the language of mathematics; and studies of bilingualism and mathematics learning.

Each year, with each new group of students, the most interesting session would be the one which grappled with the challenge and effects of having to communicate mathematics in English when the main language of most learners and teachers in South Africa was not English. Each teacher had a story to tell - either from teaching in multilingual classrooms or from his or her own learning of mathematics. In a socio-political context where English remains the primary language of government and commerce and hence the language of power, the teachers' stories revealed sometimes contradictory assumptions. For example: learning mathematics in English is necessary, especially at a secondary level; learning mathematics in school in a language that is not the teachers' nor pupils' main language places additional and complex demands on teachers and learners; mathematics is difficult for everyone, irrespective of spoken language since problems of understanding have more to do with the mathematics itself and not with English as the language of instruction; language is learnt through use, thus learners need to be made to use English in the mathematics class.

Yet, despite the universality of the stories and the assertions that learning mathematics in English while learning to speak English was a significant challenge, the dynamics of this challenge were elusive, either too embedded in teachers' tacit knowledge or less of a problem than they articulated. This elusiveness was reflected not only in teachers being unable to specify the challenges, but also in the absence of a multilingual focus in the action-research activities they chose to carry out as part of their course requirements.

These shifts in my own practices and interests occurred within the turbulent and contradictory macro political climate in South Africa at that time, that is, the five years prior to the change of government in 1994. National curriculum initiatives developing in South Africa reflected international trends. For example, there were attempts to generate, particularly in primary mathematics, a more learner-centred and problem-based curriculum. But national syllabus development remained within racially segregated apartheid structures and effectively controlled by 'white' National Government. Draft mathematics syllabi for all race and language groups developed between 1988 and 1992 (see, for example, DEC, 1990) were based on research in predominantly white, resourced classrooms where the language of instruction was also the main language of both learners and teachers. Yet the majority of South African classrooms are characterised by a range of main and additional languages that learners bring, and by a situation where learners' and teachers' main language is not the language of instruction. The lack of attention to language and diversity in these draft syllabi was not only a function of statutory apartheid, but also reflected an underlying psychological perspective that celebrated a self-regulating, . . . ogenised (and hence decontextualised) active and creative learner. Moreover, teachers had no formal representation on syllabus committees and thus little say in syllabus development, confirming the status of South African teachers as implementers rather than developers of curriculum.

While I welcomed the encouragement of more active, meaningful and learner-centred mathematics in school that was evident in these national curriculum initiatives, I was frustrated by their underlying theoretical orientation that, in my view, limited understanding of the complexities of teaching. I shared with many others a growing anger with continuing illegitimate political processes and their explicit disregard for the diversity of classroom contexts that comprise schooling in South Africa. And I was exasperated by a system that continued to devalue its teachers.

CHAPTER 1

THE PROBLEM: WORKING IN MULTILINGUAL SECONDARY MATHEMATICS CLASSROOMS IN SOUTH AFRICA

1 THE PROBLEM DEVELOPS

My engagement with many mathematics teachers from diverse backgrounds over the past momentous decade in South Africa has highlighted the challenge of working in multilingual[1] classrooms. Yet, while teachers talk about this challenge, its nature has remained elusive, begging further study.

The precursors to penetrating the conundrum of the multilingual mathematics classroom in a politically charged South Africa were two parallel processes in my own teaching that reflected development and debate in mathematics education internationally. In my seminars, I worked with mathematics teachers to develop a critical understanding of school mathematics as a social, cultural and political process. I wanted to challenge conceptions of learning and teaching that placed success and failure in school mathematics solely within the minds and abilities of either individual learners or teachers. At a more explicit pedagogical level, I also worked in these seminars to interpret and apply notions of learner-centredness and empowerment in the mathematics classroom.

Learner-centredness and mathematics as a cultural process together expose the limits of teacher-centred approaches to mathematical learning and of treating mathematical knowledge as procedural - both of which still dominate mathematics classroom practices. A shift to learner-centred approaches entails more communicative- and language-rich mathematics classrooms. As a result, I became more and more interested in language, culture and meaning in the learning of mathematics.

that focus on multilingual classrooms. A sociocultural informed study of the three dimensional dynamic of mathematics learning in multilingual classrooms thus makes an important contribution to the important field of mathematics education and language.

4 WHY TEACHERS?

Teachers and teacher educators are the audience to whom most of the language and mathematics education literature is addressed. These texts unite in a call for greater awareness of language issues by mathematics teachers, many going as far as to say that mathematics teachers should also be language teachers (for example, Zepp, 1989; Stephens et al, 1993). What is remarkable is that few appear to be concerned directly with how teachers themselves make sense of the relationship between mathematics teaching and language.

Pimm (1987), for example, carefully examines the language used by teachers in mathematics classes, but not how teachers understand this use. The teachers' voice is but a whisper in the texts reviewed here." As more and more classrooms become multilingual sites, as arguments on the benefits of a rich communicative classroom culture multiply, and as language and education policy demonstrates a shift in regarding multilingual classrooms as resourceful and not problematic, teachers' knowledge of the interrelatedness of mathematics learning and language becomes a pertinent area of study and of particular interest to the mathematics teacher educator. What do teachers understand this relationship to be? How have they come to this understanding? And what might be the effects of their understanding on the practices and the mathematical learning made possible?

Every day, teachers in South Africa (and in many other countries), manage their mathematics teaching in multilingual settings. Starting with what they have learned and what they prioritise makes good sense. This is not to suggest that I

mathematics or alienates them. Teacher 1 'generally presents information and develops ideas in English and then provides a more basic and less detailed translation in Spanish'. Students work quietly and independently. A group of Spanish speaking students is separated out and the teacher's interactions with them are restricted to counting activity in the form of chanting. In contrast, in Teacher 2's classroom, there is a great deal of talk between pupils and the teacher, and it is predominantly in Spanish. The teacher seizes opportunities to 'engage her students in mathematical talk and thinking' and Khisty argues that these different communication processes give out two very different messages to Hispanic students. In the first class students are being told implicitly that they cannot really participate in mathematics. In the second the implied message is just the opposite.

This account of research into the relationship between mathematics and language reflects Ellerton and Clements' (1991) review of the field and resultant claim that the field of language and mathematical education is fragmented, dispersed and infused with diverse emphases and assumptions. More pertinently, however, this review suggests that the dynamics of teaching and learning mathematics in multilingual school classrooms is three dimensional. It is not simply about access to the language of learning. Nor is it simply about access to the (English) mathematics register. Rather, it appears to be about how these intersect with the cultural processes that constitute school mathematics teaching and learning.

To sum up: In motivating why this study should be done now I have argued that recent curriculum development initiatives advocate more communicative classrooms. A great deal of influential research that supports more communicative classrooms has, however, taken communication in classrooms for granted. Research that does problematise the communication of mathematics has begun, but it appears that only socioculturally informed studies connect culturally embedded communicative patterns and functions with access to the mathematics register in an explicit bilingual context. Moreover there appear to be no studies

Two recent mathematics education studies in bilingual settings, both located in a sociocultural framework, do address the complexity of access to English and to mathematical discourse in classroom practices. In addition to Moschkovich (1996), Crawford (1990) and Khisty (1995) are among the few studies that effectively engage the complex interaction between bilingualism and mathematical learning in classrooms. Crawford examines the potential of new educational software to change the social organisation in classrooms with Aboriginal students in a positive way 'so that language functions to inform students about mathematical contexts'. Using a Vygotskian framework, and thus that 'language is, par excellence, the medium for the social construction of both social and physical realities', she shows that learners who speak a different language from the language of learning are not simply disadvantaged by a lack of proficiency in the language of learning. Rather access to mathematics learning is how such proficiency intersects with access to the English mathematical register as well as to cultural processes in the classroom. That is, cultural processes play a fundamental role as pupils work to access mathematical English.

Khisty's research confirms the effects of cultural processes on mathematical learning. Her research focuses on teaching processes in primary (second and fifth grade) classrooms in the USA with sizeable numbers of Hispanic pupils and fully bilingual teachers. It too draws on socio historical psychology and the critical role of social interaction in learning, as well as on Cummins' work and the importance of English development being integrated with academic skill development (CALP) and on Halliday, where she interprets the issue surrounding the mathematics register as not lying in the register itself but in mathematical learners identifying mathematics with its language.

Khisty identifies and contrasts two different teaching processes, which illustrate how talk or absence of talk either draws second grade pupils closer to mathematics or alienates them. Teacher 1 'generally presents information and develops ideas in English and then provides a more basic and less detailed

He shows how teachers themselves use French, despite their own language difficulties, to ensure pupils acquire it. At the same time, to meet the demand of imparting knowledge, they often resort to other linguistic practices like code switching in order to support the concept development and culture of learners. He concludes by arguing that code switching is not something to be 'tolerated'. It should rather be official policy. This is reflected in South Africa's current multilingual education policy.

Herein lies the rub. What does code-switching mean in multilingual mathematics contexts where pupils are not only learning in a language that is not their main language, but within a classroom setting where pupils bring a range of main languages? Despite a growing focus in language education on multilingualism'[4], no literature in mathematics education and language appears to address the more complex situation of multilingual classrooms.

Moreover, the complexities of how mathematical learning in multilingual classrooms is researched, and by whom, are thrown into relief by Campbell's study of a grade six mathematics class in Papua New Guinea (1986). Campbell attempted to understand from a sociolinguistic point of view how the teacher works to develop mathematical discourse with her pupils in English. He shows how the teacher has her pupils say in words that 'number sentences express complete ideas about numbers' and 'set sentences express complete ideas about sets'. Through an analysis of the classroom discourse Campbell shows how the teacher works simultaneously with mathematical and English language development. To a mathematics educator, however, this study is frustrating. It is hard to see whether and how pupils attach any mathematical meaning to what they are saying in English, whether what they say connects in any way with being able to distinguish actual mathematical and set sentences. Thus while Campbell researches what teachers might do to deal with both ESL learners and their acquisition of mathematical discourse, his sociolinguistic focus seems to prevent any critical analysis of the mathematical content in the lesson itself.

bilingualism and bilingual education, Baker (1993) argues that decisions on language choice and use (in terms of both medium and message) involve complex purposeful activity that is simultaneously social, political and psychological (p. 15).

> ... [c]ode switching has more than linguistic properties ... (there) are important social and power aspects of switching between language, as there are between switching between dialects and registers.
> (p. 77)

Code-switching was a salient feature of the Burundi classrooms in Ndayipfukamiye's (1994) study. Here, classroom interaction involved the use of both Kirundi and French. This is not surprising given that Ndayipfukamiye's focus is on Grade 5 classes, when the switch from Kirundi to French as medium of instruction occurs. This situation is similar to the South African context where, under apartheid education policy, the majority were confronted with a more prestigious minority language and a switch over in year five of schooling to that language as language of instruction. Ndayipfukamiye asked how teachers and pupils meet the communicative demands of classroom life. He found that mathematics lessons that mainly involved the completion of exercises were carried out mainly in French. In contrast, in lessons where new concepts were introduced and a great deal of demonstration and explanation were involved, he found that 'it was in practice impossible for teachers to handle all activities involved without resorting to Kirundi'.

Switching to Kirundi thus facilitates learning and he goes on to argue that code switching is a powerful reflection of the ways the teacher and the learners mediate the communicative demands they face in the implementation of the curriculum.

> (Code switching) plays a crucial role in reconciling two contradictory demands in classroom interactions: using Kirundi in domains where French is expected, to ensure understanding; abiding with the official policy that requires the teaching of French (p. 91).

possibilities for learner confusion of the everyday and mathematics registers on the one hand, and between the English and Spanish mathematical registers on the other. 'Students learning mathematics in these two languages would need to sort out not only the differences between the two registers, but the correspondences between the math registers in the two languages as well' (p. 28). However, she goes on to contend that a discontinuity model, in its focus on the conflict between registers or languages as an obstacle in learning mathematics, fails to consider situational resources and thus 'easily turns into a model of students as deficient' (p. 30). Moschkovich argues instead for a 'situated' model which she elaborates through two secondary classroom episodes where the learners' first language (Spanish) is harnessed as a resource for 'participating in mathematical discourse practices' (p. 32).

While I agree that some of the research cited earlier does work from a deficit model, I have also argued that a discontinuity model is not necessarily a deficit model at all. Instead, it is a continuity model that might well have dire consequences for equity and access in mathematics classrooms. However, Moschkovich's argument that any account of learning must take situational resources into account resonates with socio-cultural theory and the theoretical framework developed further in Chapter 2. I agree with Moschkovich that explanations of mathematical learning in bilingual classrooms must extend beyond access to the language of instruction and access to mathematical discourse by being situated in mathematical discourse practices *in classrooms.*

Moschkovich's study resonates with the few sociolinguistic studies of code switching in bilingual classrooms that include a specific focus on mathematics (for example, Ndayipfukamiye, 1994). Code-switching refers to the movement between two languages. Code switching includes using different languages in different domains, for example, English in formal interactions in the classroom, but not anywhere else. It also includes a person using different languages within one domain. In his accessible and broad account of research and debate on

settings and thus limit our understanding of bilingual education and mathematics learning[1].

Dale and Cuevas (1987) provide a useful analysis of mathematical language and suggestions for mathematics teachers in bilingual classrooms. They illustrate how mathematics teachers can attend to the language development of ESL learners, and conversely how ESL teachers can include mathematical language into their learning support programmes. Dale and Cuevas focus on access to mathematical discourse and this resonates with recent arguments that challenge popular views on curriculum reform favouring relevance and the everyday as the route to more meaningful science and mathematics learning. Such reform, it is argued, is premised on a continuity model that wishes away boundaries between everyday and school knowledge. It is therefore potentially disempowering and can inadvertently prevent access to mathematical discourse and science curricula (Dowling, 1995; Brookes and Brookes, 1995) by not providing explicit opportunities for distinguishing and acquiring school mathematical discourse. This argument can be linked with Cummins' distinction (in Baker, 1993, pp. 138-142; in Dale and Cuevas, 1987) between basic interpersonal communication skills (BICS) and cognitive academic language proficiency (CALP). BICS are language skills used in interpersonal relations or in informal situations whose extra-linguistic and linguistic context provide relatively easy access to meaning (the everyday). CALP is the kind of language proficiency required to make sense of and use academic language in less contextually rich situations. As Cummings suggests, BICS are relatively easy to acquire. CALP is more difficult, takes longer and requires explicit teaching of the language in the academic context[1].

Moschkovich (1996) argues against the 'discontinuity' model reflected in Cummins' distinction between BICS and CALP, and in the distinctive elements of a mathematics register. From both these perspectives, she argues, learning is seen as mapping meanings across register or language discontinuities. Drawing on the Spanish English context of many USA classrooms, she argues that there are

bold in placing this question back into mathematics education and cautioning us against rigid polarisation.

3.2.2 Bilingual education and mathematics learning

Zepp (1989), Durkin and Shire (1991), Clarkson (1991) and Stephens et al (1993) take up the issue of bilingualism and mathematics learning, arguing that bilingualism per se does not impede mathematical learning. This field of research has drawn extensively on Cummins' research (in Baker, 1993, pp. 135-146) which distinguished different levels and kinds of bilingualism, and showed a relationship between learning, levels of proficiency in both languages, and the additive or subtractive model of bilingual education used in a school. Bilingualism per se need not impede learning, as is shown by Fafunwa's (1975) interesting study of the success of Nigeria's experiment with mother tongue instruction through the primary school and the simultaneous development of a second language prior to switching language of instruction. Secada's (1992) overview of research on bilingual education and mathematics achievement supports this claim. The studies reviewed by Secada suggest a significant relationship between the development of language and achievement in mathematics. Oral proficiency in English in the absence of mother tongue instruction was negatively related to achievement in mathematics (p. 644). Hence there appear to be significant benefits to mother tongue instruction.

The focus of the research reported by Zepp and others is on cognition. The studies draw primarily on *psycholinguistic* theory, and focus on concepts, conceptual structures and comparative performance studies. 'Problems' are located in the learner, reflecting a deficit model where it is learners who need to change to fix the problem. I have already argued that much of what we accomplish in schools is framed more by cultural and social practices than cognitive structures. The studies by Zepp and others are circumscribed by a lack of attention to the dynamics of bilingualism and mathematics learning *in classroom*

Pimm examines the dynamics of classroom communication, but acknowledges that he has not addressed the additional demands of multilingual classrooms, and the complex social and intellectual problems arising for learners 'attempting to learn mathematics through a language that is not one's mother tongue' (1987, p. 204). Thus, while his metaphor is that learning mathematics is like learning a (second) language, he does not address the related issue of learning *in* a second language. As has been mentioned, MacGregor's (1993) study shows some particular difficulties ESL students experience with mathematical expression. Similarly, Cocking and Mestre (1988) and Spanos, Rhodes, Dale and Crandall (1988) have described Spanish-speaking students' problems in understanding mathematical vocabulary and in translating from English to mathematical symbols. Thus, while access to the mathematical register is an issue for all learners, research suggests that there is a specificity to the challenge it presents in bi- and multilingual classrooms.

Pimm's most recent work shifts focus onto symbols and meanings in school mathematics (Pimm, 1995). He confronts current polarisation in mathematics education where fluency as an outcome is associated with rote learning practices, and contrasted with understanding, an outcome linked to interactive, meaning-making practices. Again, he has not drawn explicitly from multilingual contexts, but does refer to learning situations where through imitation and chanting, a great deal of successful cultural transmission occurs. In South Africa, rote learning, and often chanting, is part of the culture of many mathematics classrooms, and particularly in situations of overcrowding common in many black township and rural schools. There is good reason to be concerned with the quality of learning that transpires in such classrooms. But we perhaps need to understand better the limits and potentials of such practices. Inasmuch as the new movement has challenged the limitations of rote learning, it errs in the opposite direction by not addressing the development of fluency in mathematics, an important element in mathematical competence, particularly in algebra. Pimm, to his credit, has been

3.2.1 Learning mathematics is like learning a language

The most extensive work on mathematics and language has been done by Pimm (1981, 1987, 1991, 1992, 1994, 1995). In *Speaking Mathematically*, Pimm (1987) explores what can be learned about mathematics and its teaching and learning in school if we consider mathematics as a language (in a metaphorical sense). Learning mathematics is thus like learning a language, and more specifically, like learning a second language. One of the useful effects of seeing mathematics teaching as language teaching is that this brings a shift in focus from form to meaning, 'from the study of a rule-governed abstract system, with an emphasis on written forms, to one of the acquisition of communicative competence about certain objects, situations and phenomena, with a concomitant oral emphasis' (p. 203). Pimm's work reflects the shift to more communicative mathematics classrooms. It informs and adds considerably to the research on communicating mathematics cited above.

Pimm draws on a wide range of language and learning literature, including Halliday's sociolinguistic theory (Halliday, 1978). Through an analysis of selected transcripts of classroom talk and conversations, Pimm identifies, describes and explains the complexities and specificity of learning to 'mean' mathematically (acquire mathematical communicative competence) in school. One of his foci relates to Halliday's notion of the mathematics register which is 'a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings' (Halliday, in Pimm, 1987, p. 75). The mathematics register is thus the set of meanings, words and structures appropriate to the practice of mathematics. Pimm illustrates how words like 'difference', phrases like 'let's assume that', and sets of meanings are specific in the mathematics register. If learners are to acquire mathematical communicative competence, they need to learn to work within the mathematics register and understand its specificity.

Australia and the attention paid to the issue of language and diversity in the Australian national curriculum document (AECCC, 1991).

In this section I have highlighted the dominance of radical constructivist theory in the mathematics education community and argued that because communication and social interaction are not the driving force in its explanation of learning, research from this perspective inevitably takes communication in the classroom for granted. This is a serious and growing inadequacy, and particularly so in a multilingual context. In answer to my question 'Why now?', the current curriculum initiatives and related research discussed above suggest that the communication of mathematics in multilingual classrooms from a socio-cultural perspective requires further study.

3.2 Language and mathematics

Those unfamiliar with the field of mathematics education might be misled into thinking that mathematics educationists outside Australia have not been concerned with the relationship between mathematics education and language in general and with bilingual learners in particular. Not so. 'Mathematics is a language', 'mathematics is like a language', 'mathematics in language', 'language in mathematics', 'mathematical language', 'the language of mathematics', 'mathematical discourses', represent but some of the attempts by mathematics educators to create an expression that will reflect appropriately the inter-relationship of mathematics and language. Moreover, since Austin and Howson's seminal article (1979), the complex inter-relatedness of language, mathematics and mathematics education has been the object of a great deal of both empirical and theoretical research[11].

(1985) argues that peer interaction and verbalisation in class facilitate mathematical learning; and Pirie's extensive study (Pirie & Schwarzenberger, 1988; Pirie, 1988; 1989) on the relationship between discussion and mathematical understanding has contributed to an understanding of learning, but is not able to establish conclusively any neat causal link between mathematical talk and improved understanding. Both take communication for granted (that is they do not problematise language as a socio cultural force in the construction of knowledge) and thus imply the homogenised learner with no differential access to the language of learning. An effect of this has been the production of numerous texts for teachers on 'talking and learning mathematics' (for example, Ball, 1990; Backhouse et al, 1992), where multilingual classrooms and diversity in learners is silenced, or, even worse, simply ignored. This is surprising in the increasingly multicultural urban context of the United Kingdom.

Recent studies in Australia do problematise communication of mathematics (Stephens et al, 1993). Some, while not focused on multilingual settings, show from a sociological perspective that peer interaction does not function equitably in a classroom (Gooding and Stacey, 1993; Cooper et al, 1993)[10]; that teachers communicate different messages about mathematics to socially different groups of learners (Cooper et al, 1993). Others engage directly with ESL (English second language) students and show that students whose ability to communicate in the language of learning does not match their ability to reason mathematically are disadvantaged by classroom processes that demand oral and written (verbal as opposed to symbolic) communication of mathematical ideas (MacGregor, 1993). While an elaborated review of bilingualism and mathematics education follows below, it is of note here that Australian teacher resources do acknowledge and address perceived needs of what are termed NESB learners (learners with non-English-speaking backgrounds) (see Bickmore-Brand, 1993; Bell, 1993; Clarkson and Thomas, 1993, all in Stephens et al, 1993). Perhaps this reflects less dominance of radical constructivism in mathematics education research trends in

CHAPTER 2

DEVELOPING A LANGUAGE OF DESCRIPTION AND EXPLANATION

1 INTRODUCTION AND BROAD THEORETICAL BRUSH

The purpose of this chapter is to develop a language to describe and explain teachers' knowledge of their practices in multilingual mathematics classrooms. What is entailed is an analysis of the origins of teachers' knowledge and its effects in day to day classroom practices. The language developed here shapes and is shaped by a social theory of mind and draws on the sociocultural and sociohistorical[1] theory of Lev Vygotsky (1978, 1986), the social practice theory of Lave and Wenger (1991) and the language of dilemmas developed by Lampert (1985) and Berlak and Berlak (1981). Together, these provide theoretical and conceptual tools to examine teachers' knowledge of mathematics teaching and learning in multilingual contexts.

Central to a social theory of mind is the notion that consciousness forms in and through culturally[2], socially and historically mediated activity, and within particular social practices[3]. Within this notion, language, and particularly speech, is a key mediational mean (Vygotsky, 1978; 1986; Wertsch, 1985; 1991a). For Vygotsky,

> To understand another's speech, it is not sufficient to understand his words - we must also understand his thought. But even that is not enough - we must also know its motivation. (1986, p. 253)

Thus, language, consciousness and social context are inextricably linked. Hence classroom communication entails more than proficiency in the language of instruction: it requires understanding thought, intentions and cultural processes.

In addition, speech, for Vygotsky, has a communicative and intellectual function (1978, p. 7). It is both tool, functioning externally, and sign, turned inward - a key mediator in the development of higher psychological systems (p. xxiv). What

15. This is not to suggest that teachers themselves have not written about language and mathematics. A number of authors in Stephens et al (1993) are teachers: see also Brodie (1991) who writes from a South African perspective.

3 I use 'main' language in place of what is often referred to as 'home' language or 'mother tongue'. By main language I mean the language of greatest day to day use and facility for the speaker. I use 'additional' language to mean language spoken in addition to the speaker's main language or languages. It thus replaces 'second' language. In doing so I follow the practices in the Applied English Language Studies department at the University of the Witwatersrand. In our complex multilingual society, many people speak more than two languages, where more than one might be a main language and where it is not appropriate to signal one as the second language; moreover, mother tongue is not necessarily synonymous with main language

4. I use 'appropriation' here in preference to cultural transmission so as to avoid any suggestion that knowledge is ever simply transmitted. However, I am wary of current simplistic dichotomous and dogmatic notions that 'transmission teaching' (exposition, telling by teacher) is necessarily teacher centred and bad practice, and any and all activity by learners, in contrast, is necessarily learner centred and good practice. Teaching and learning is a situated activity. Different kinds of knowledge and diverse contexts require a range of teaching strategies appropriate to the context and the desired knowledge

5. I use 'competence' in its broad sense, that is, to include knowledge, skills and values Communicative competence is thus both knowledge and skill in language use in context

6. What is signalled here is debate on the nature of mathematical knowledge, and selection into the school curriculum. Extensive discussion is beyond the scope of the thesis, but briefly, current curriculum initiatives acknowledge both uncertainty in mathematical knowledge (in contrast to its absolutist history (Ernest, 1991)) as well as mathematical knowledge as a socio cultural (as opposed to a neutral) process (Ernest, 1994).

7. The Proceedings of the past years of the annual international conference of the Psychology of Mathematics Education provide sufficient evidence for this claim See, for example, Geeslin and Graham (1992).

8. I specify language, as this is a concern of this study Equally silenced are race, class and gender relations that must intersect with classroom communication

9. Khisty (1995) presents forecasts that by the year 2000, more than 50% of Americans will have Hispanic origins While this does not necessarily mean their main language will be Spanish, many are likely to live in Spanish speaking communities and speak Spanish at home.

10. Cobb has more recently examined diverse peer interaction and its effects (see Cobb, 1994b).

11. My own research on mathematics for adult distance learners some ten years ago paid particular attention to the construction of readable text material for NESB learners (Adler, 1985, 1988 1989)

12. Similar arguments as to the limitations of psycholinguistic studies of bilingual learning mathematics can be found in Mousley and Marks (1991), Ellerton and Clements (1991), Clarkson and Thomas (1993), Bishop (1988) and Crawford (1990).

13. Krashen's acquisition learning distinction in relation to language learning (in Baker, 1993, p 102) is similar to the distinction between everyday and school mathematical knowledge Some language is acquired in informal situations where language is a means and not a focus or end in itself Formal language learning is learning 'about' the language, and both kinds of learning are part of developing both fluency and accuracy in a language

14. See, for example, the recent series of books under the general publishing title of 'Multilingual Matters'. The books by Baker (1993) and Rakagumya (1994) and Mercer (1995) are all within this series

data collected. Chapters 5 - 8 are the heart of this thesis. Through categories of description and analytic narrative vignettes, they describe, analyse and explain dilemmas teachers face in their complex multilingual mathematics classrooms. It is in Chapter 5 that three central dilemmas emerge as teachers talk about their practice. Each key dilemma is the focus of in-depth study in Chapters 6, 7 and 8. The dilemma of code-switching is elaborated in Chapter 6; dilemmas of mediating mathematical knowledge are the focus of Chapter 7; and dilemmas of implicit and explicit practices are the focus of Chapter 8. In all three chapters, as teachers' articulated and tacit knowledge are interrelated, an extended language of dilemmas emerges.

Moreover, what is clearly evidenced in this study is that while dilemmas are expressed as binary oppositions, they are never either/ors in the complex life blood of classrooms. Instead, they are sources of praxis, of transcending inherent tensions in the dialectical teaching-learning process. In short, teachers manage their dilemmas. Sometimes teachers are fully aware of the choices they make, choices that are at once personal, practical, social and political and specific to mathematics teaching. At other times, in managing the complex three dimensional dynamic of access to the language of instruction (English), access to mathematical discourse and access to classroom discourse, elements of their practice are obscured. The final chapter discusses the implications of this study for further research and for mathematics teacher education.

NOTES

1 I use 'multilingual' as the appropriate term to describe the range of main and additional languages that are brought into school classrooms, particularly urban ones, in South Africa by a diversity of pupils and teachers. However, this does not necessarily imply any deliberate multilingual teaching.

2. For clarity of description through the thesis it becomes necessary at times to refer to apartheid defined racial groups. Here 'white' implies South Africans who are historically European. 'Black' is used generically to refer to so called 'coloured' South Africans (mixed race), as well as Indians (historically Asian) and Africans (native South Africans). All black South Africans were disenfranchised during the apartheid era. Through the thesis I use 'black' in its generic sense and African when I wish to signal South Africans of African origin.

Rubagumya and Garcia reinforce, from the perspectives of Africa and the Americas, two motivations for this study that have already been alluded to. In his introduction to a collection of papers originally presented at a colloquium on teaching and researching language in African classrooms, Rubagumya identifies a need for research in multilingual classrooms that attends to how teachers and learners actually 'get things done'. Garcia strikes a chord with her claim that contemporary teacher education has done little to enable teachers to make 'informed decisions' in their complex multilingual classrooms. Garcia inadvertently intimates that teachers have limited or no 'informed' understanding of the linguistic complexities of their classrooms. In contrast, Rubagumya implies that, in getting things done, that is, through their practice, teachers have a great deal of both conscious and tacit knowledge about their teaching in multilingual settings. But what do we, the wider educational community, know of this knowledge? Specifically, how do mathematics teachers get things done in multilingual settings? What do they know? Should their knowledge be elaborated, and if so, how and why?

The central concern of this thesis is thus mathematics teachers' knowledge of the dynamics of teaching and learning mathematics in multilingual classrooms. Chapter 2 builds on the notion of teaching dilemmas in order to capture at once, the personal, the social and the political in teachers' knowledge. It argues that a description of teaching dilemmas must embrace the specificity of mathematics and language in school if it is to illuminate fully, teachers' knowledge of their practices in multilingual mathematics classrooms. The notion of dilemmas itself is located in a theoretical framework that understands knowledge as constituted in and constitutive of social practice where culture and context create each other.

In Chapter 3, a qualitative methodology that includes in-depth interviews and classroom observation and so accesses teachers' articulated and tacit knowledge (what they say and what they do) is elaborated. Qualitative data analysis entails increasing levels of analysis. Chapter 4 therefore is a first level of analysis of all

Secondly, access to mathematics with its symbol system and ways of meaning is a function of formal schooling. How this impacts in multilingual settings is an important question.

Secada's (1992) review and analysis of research related to race, ethnicity, social class, language and achievement in mathematics, which has already been noted, reveals its 'marginal nature and relative status in mainstream mathematics education research' (p. 654). Typically, such research is not found in maths education journals, nor is it the product of the work of mathematics educators. Such a state of affairs is both unconscionable and untenable' (p. 654).

7 A SKETCH MAP OF THE THESIS

South Africa is probably unique in the dimensions and speed of social and political transition at this juncture in the mid-1990's. However, the issues of flux and of teaching and learning in increasingly multilingual contexts are global ones. Few societies are so sealed off that there is neither flux, nor linguistic diversity in their school classrooms. The challenge posed for research and teacher education elsewhere is reflected in current language education literature:

> ... more attention needs to be given to the ways in which teachers and learners actually get things done with two or more languages in day to day classroom practices.
>
> (Rubagumya, 1994, p. 1)

> The greatest failure of contemporary education has been precisely its inability to help teachers understand the ethnolinguistic complexity of children, classrooms, specific communities, and society, in such a way as to enable them to make informed decisions about language and culture in the classroom.
>
> (Garcia, in Baker, 1993, p. vii)

more likely to be a result of her specific pre- and in-service training, rather than her interaction with the ESL teacher in the school. Implications of the study included the importance of ongoing professional development specifically in relation to ESL learners in the mainstream (pp. 11-12).

Thirdly, in a situation of change, taken-for-granted assumptions are often fundamentally challenged. Since 1990, teachers in South Africa have been coping with changes in a democratising and deracialising society. In their schools and classrooms, they are coping with interpreting multilingualism as policy and with new ideas about quality mathematical learning. This fertile ground of a society in transition makes for a constructive link between curriculum reform in other parts of the world and the search for a democratic educational dispensation in South Africa.

6 WHY MATHEMATICS?

Quite obviously, language is fundamental to thinking and learning in general, and hence the area of study here is of value not only to mathematics. It is important to investigate how the learning of all school subjects is framed by multilingual settings. There are, however, two reasons for a specific responsibility in doing this study in mathematics.

Firstly, at this juncture in South African history, educational access is of paramount importance. It is well known that success rates in mathematics in South Africa are alarmingly poor, and that mathematics is the school subject par excellence that functions here and elsewhere to filter out the vast majority of learners (Adler, 1991a; 1991b; 1993b). There are a growing number of studies that take the production and explanation of difference in mathematics educational practice as their focus (Walkerdine, 1988; 1989; Dowling, 1993; Brown, 1993; Ensor, 1993). But while they have dealt with class, race and gender issues, they have not explicitly examined any of the effects of multilingual classrooms.

Current teacher preparation courses do little to prepare teachers for their task of teaching subjects like mathematics in English to learners who are not English-speaking (NEPI, 1993, 181; Hofmeyr et al, 1995, p.10). If mathematics teachers in South Africa need to regard themselves as language teachers, we must ask what specific language-related teacher education should mathematics teachers have? The answer to this question lies in unravelling the dynamics of the learning and teaching of mathematics in multilingual classrooms.

Relevant to this is the fact that South Africa suffers a profound shortage of appropriately qualified mathematics teachers (Arnott and Bot, 1993; Bot, 1994; Schindler, 1995; The Star, 19 March 1996, p. 2). There is likely to be serious pressure for short and long term strategies for the provision of the mathematics teaching corps. If multilingualism is to be a feature of teaching (as opposed to a characteristic of learners), then teacher preparation and development will need to build language related courses into their curricula. This is particularly important in the light of a study of teachers' views on language of learning (Bot, 1993) which reveals that teachers hold quite contradictory positions. For example, they want learners to use whatever language they can to understand their studies, but they must answer tests only in English. These seemingly opposing views (understanding vs. developing English) are reflected in the teachers in this study. This study could thus usefully inform the formulation of teacher preparation and development courses.

The value of specific language-related courses in pre- and in service teacher education is reinforced by recent research in Australia, where ESL and NESB programmes have been in schools for some time, often with specific staff employed for this role. A survey of provisions for teaching mathematics and science to NESB secondary students in selected Directorate of School Education (DSE) schools was undertaken by Larkin et al (1993). The survey revealed that a subject teacher's ability to deal with a range of NESB students in her class is

in November 1995 by the national Department of Education, *Towards a Language Policy in Education*, is based on the principles of multilingualism and the promotion of all eleven official languages (Mokgalane and Vally, 1996, p. 4). Such policy resonates with recent research on bilingualism and bilingual education (NEPI, 1993; Baker, 1993, pp. 247-254; Heugh, 1995). Models of bilingual education which are built on conceptions of bilingualism as a resource in the classroom are now argued to be both personally and socially more effective than those that treat bilingualism as a problem and work from assumptions of deficits in learners (Baker, 1993, pp. 94-105).

One goal of the proposed language policy in South Africa is to ensure that all learners from all language communities have equitable access to education (Mokgalane and Vally, 1996). But there is a long and arduous road from policy to implementation and an enormous challenge in bridging intent and practice in a complex social and political world. This study takes one of many varied and possible routes into researching multilingual mathematics classrooms and thus hopes to contribute to the challenges we face. It shares the understanding in all recent teacher education policy documents here that teachers are 'key' in any implementation of education change in schools (NEPI. 1993, p.235; ANC, 1994a, p. 67; DoE, 1995a, p. 39; Bengu, 1996).

Secondly, new language education policy encourages multilingual teaching practice (see also ANC, 1994b, p. 62; NEPI, 1993, p. 189). That the use of more than one language in classrooms will be taken seriously in a new dispensation is reflected in new policy for teacher education (see COTEP, 1996, pp. 63-64). All teachers need to become aware of language issues, or actually become language teachers. Multilingual teaching policy reflects a commitment to democracy. In addition, a flexible and developmental language policy in the early years of schooling is clearly crucial for young children. It is likely, however, that with its global, commercial and political currency, English will remain the language of instruction in many, if not most, secondary schools.

many discontinuities, inconsistencies and contradictions, she argues. Drawing further on Secada's argument that work on teaching 'needs to include teacher beliefs, knowledge and behaviours as a function of the sorts of students who are in their classrooms' (1991, pp. 46-7), she proposes 'beliefs as situated dialectical constructions, products of activity, context and culture' (1992, p. 280). Hence the value or inquiry into the complexities of teachers' knowledge of the dynamics of mathematics teaching and learning in multilingual contexts.

The South African curriculum context is further impetus for focusing on teachers' knowledge. Curriculum practices in South Africa cast teachers as mere implementers of the conceptualisations of others (Adler, 1991a; 1992b; 1994). Curriculum practices in South Africa will thus benefit if they are informed by teachers' knowledge of their practices.

In this section I have argued that teachers' knowledge of the dynamics between mathematics and language has not been studied, and that because teacher knowledge is situated and emerges in the context, culture and activity of classroom life, it is a crucial dimension of a socioculturally informed account of the dynamics of teaching and learning mathematics in multilingual school classrooms. Hence the focus of my research.

5 WHY SOUTH AFRICA?

There are three central reasons why South Africa was an appropriate context at the time the research was undertaken, the first half of the 1990's.

Firstly, South Africa is a multilingual society. This fundamental characteristic of our society is recognised in the new constitution adopted by parliament (8 May 1996). There are now eleven official languages, and policy statements at both social and individual levels take multi and bilingualism as a resource rather than simply as a right, and certainly not as a problem. A discussion document issued

education research in Grouws reviews research on the impact of mathematics teachers' knowledge on pupils' learning. In their critical analysis, Fennema and Franke (1992) review studies that range from those focused on teachers' content knowledge as a contained and static entity, to those that understand knowledge as situated. The challenge for research on teachers' knowledge and its impact, they argue, is to develop research methodologies that provide insight into how teachers' knowledge is formed and transformed in action in particular contexts.

These reviews of mathematics teacher education research reflect a growing understanding of teaching as complex and deeply contextualised, and that researching teaching means taking this complexity into account. Thus, while it is possible analytically to separate aspects or elements of the teaching-learning relationship, in practice, beliefs, for example, never operate outside of the context and logic of practice.

Teacher education research typically seeks to contribute to enabling and enhancing mathematics teacher education, in order, obviously, to impact on the quality of learning opportunities for pupils. My research is similarly motived, but it takes a different route to the studies reviewed in Grouws. My concern is not to relate teacher beliefs and/or knowledge to impact on learning or pupil performance. Working from the understanding that teaching as a practice is deeply textured, my concern is rather to learn from mathematics teachers' practices, beliefs and knowledge in action and in context, and to relate these to a particular field in order to further inform that field.

The value of researching teachers' knowledge in order to learn from it is supported by Lampert's (1986) illumination of how differently primary mathematics teachers and researchers viewed, acted on and talked about the same educational research problems. Indeed, Hoyles (1992), like Thompson, questions research that assumes a continuity between teacher beliefs (say about mathematical knowledge) and their practices (see, for example, Lerman, 1983; Ernest, 1989). There are

see no horizons to practical knowledge (Richardson, 1994), nor that I conflate espoused theories with theories in use (Argyris and Schön, 1974), nor that I would hold to notions that teachers have some generalised decontextualised belief system (Hoyles, 1992, Argyris and Schön, 1974). As Levine (1993) argues: 'teachers build theories just as much as theories build teachers' (p. 203). Worthwhile activities in mathematics teacher education are thus those that enable teachers to integrate theoretical, practical and research knowledge and situate this in the contexts of their work (Adler, 1992a; 1992c; 1993a) and as well as research-like activities that enable teachers to initiate and respond to change in their classrooms (Crawford and Adler, 1996).

The considerable range of mathematics teacher education research is usefully surveyed in Grouws (1992). Three chapters reveal the extent and diversity of research concerned with improving mathematics teacher education. Studies interested in enhancing understanding of the relationship between mathematics teaching practices and their effects reflect growing awareness of the complexities in such a field (Koehler and Grouws, 1992). Research here includes large-scale product-process studies, and small-scale, more focused, expert-novice studies. Grouws discusses how the positivist assumptions that underlie much of this research are challenged by recent research informed by constructivist perspectives, cognitively guided instruction and sociological and epistemological perspectives.

Thompson (1992) reviewed research that examined the relationship between mathematics teachers' beliefs and their practices. She challenges two underlying assumptions in these studies: that belief systems are 'static entities to be uncovered' and that 'the relationship between beliefs and practice is a simple linear causal one' (p. 138). Her research suggests that there is, instead, a dynamic and dialectical relationship between beliefs and practice. She nevertheless remains silent, together with the studies reviewed, on the role of context in shaping both beliefs and practices. The third chapter on teacher

While their dilemma language of schooling is an effort to present the thought and action of teachers as an ongoing dynamic of behaviour and consciousness within particular institutional contexts of schools for the young, they remind us that dilemmas are not [illegible]es that may be physically located in a person's head or in society. Rather they are 'linguistic constructions that, like lenses, may be used to focus upon the continuous process of persons acting in the social world' (p. 111).

> Persons' activities cannot be understood apart from their biographies and the histories of the groups with whom they identify, which live on in the consciousness; or apart from the time and place in which they act (a particular school, local education authority, nation or state at a particular juncture of human history) (p. 111).

Thus no matter how effective a description might be, it can never fully capture the intensities of feelings, pain and joy, angers and frustrations, that are part of any school's daily life (p. 164)

The Berlaks' project is about change in schooling. They ask: will teachers change their patterns if they are more aware of trade off in and between dilemmas? This is an important question and of particular relevance in transitional South Africa. What is suggested by dilemma language is that with a language that assists teachers to understand, talk about and act in relation to the tensions in their teaching, they might be better positioned for deliberative and transformatory action.

3.4 Dilemmas in South African multilingual mathematics classrooms

The studies by the Berlaks and Lampert and their arguments for dilemmas in teaching were developed in contexts very different in time and place from multilingual classrooms in South Africa in the 1990s. Crucially, both studies were in primary classrooms with a far more informal approach to teaching and learning than is dominant practice here. Moreover, in primary schools, teacher identity and

dynamic form and need to be managed. Dilemma management and/or resolution will be important to explore in this study.

It is not possible in this chapter to present the richness of the Berlaks' study. The selected sketch does, however, illustrate the potential power of their dilemma language for 'ordering the flux of classroom life, and placing the mundane events of daily school life in the p spectives of culture and time' (p. 37). Thus, just as Lampert's description is co. pelling and resonates with my own experience, so the Berlaks' language could facilitate talking about mutually constitutive macro and micro, social and personal, issues in teaching.

There is a saying about sociology, however, that it tells us all we need to know in words we don't understand. That teaching entails dilemmas is so obvious it is, after all, a social practice. Does a language of dilemmas, and specifically the sixteen dilemmas constructed by the Berlaks, offer a powerful descriptive and explanatory tool for my study? Who will benefit? Will it be in words that don't connect with teachers' points of view, thus confirming Lampert's critique of theories of teaching? I will argue with the Berlaks that, as with any language, the language of dilemmas brings into focus qualities of experience hidden by our familiar languages. Theories thus serve purposes, but, in Bourdieu's (1990) terms, theories can never *be* the practical. There are different logics of practice in play. Dilemma language is a tool in my study that brings into focus the taken for granteds in teaching mathematics in multilingual classrooms.

3.3 Some limitations

Whatever the power of language, there are also limits. As Berlak and Berlak point out, just as language can illuminate, so it can distort and fragment social activity. Language categorises and separates consciousness from observable behaviour and social context, making it difficult to talk and think about schooling as a communal process, wherein context and consciousness are 'joint' in the acting moment.

From their empirical base the Berlaks reveal that patterns of teachers' actions may be viewed through one, several, or all of the dilemmas simultaneously (p. 136). Teaching acts can be viewed as simultaneous resolution to multiple dilemmas (p. 165). Certainly, Lampert's dilemma can be viewed through both control (freedom vs order) and societal (gender) dilemmas. In the Berlaks' dilemma language these would be *whole child vs. child as student* i.e. concern with freedom for the boys to be themselves, vs. order so that all could learn, and *equal vs. differential allocation of resources* i.e. how to manage gender diversity.

The Berlaks distinguish dominant, exceptional and transformational patterns of dilemma resolution. Through dominant or exceptional patterns of resolution to one or more dilemmas, they were able to represent within and across teacher similarities and differences and use their sixteen dilemmas to describe the variations, regularities and apparent contradictions they observed in classrooms. Exceptional or dominant patterns of resolution are not exhaustive of possibilities. There are also solutions or resolutions where the pulls of opposite alternatives are joined, 'when contending presses of the culture at least for the moment are synthesised and thus overcome' (p. 133). These are transformative solutions which in practice are quite rare.

As is explained in Chapter 3, my study is not a full ethnography. Rather, it works with snapshots of teaching. I am nevertheless concerned to understand similarities and differences in teachers' accounts of their practices. Dilemma language becomes a means for doing this.

In contrast to Lampert's argument that effective practical action in classrooms is about the management of tensions, the Berlaks' emphasis and orientation is on resolution, i.e. overcoming contradictions, or competing alternatives. But surely this is not an either/or? Both are correct. There must be instances where tensions can be resolved, just as there are many tensions in teaching that are ever present in

Teachers behaved in complex and often contradictory ways, and their understandings of their actions and teaching exhibited similar complexities and contradictions. Possible descriptions and explanations did not lie within teachers, nor their classrooms, nor the wider context, but rather in their inter-relations. The language of dilemmas became a means for capturing the complex relationship between the context of schooling and the dilemmas of teaching.

3.2 The dilemmas

In their study of primary schooling the Berlaks developed three distinct sets of dilemmas - sixteen in all - that facilitated a description and explanation of what they observed in and across schools and teachers studied (pp. 135 175).

Control dilemmas capture tensions over the locus and extent of control over students. Four control dilemmas are described as: whole child vs. child as student, teacher vs. child control - time, teacher vs. child control - operations, and teacher vs. child control - standards.

Eight *curriculum dilemmas* capture contradictions in how teachers, 'through their schooling acts transmit knowledge and ways of knowing and learning' (p. 144) and are described as: personal vs. public knowledge, knowledge as content vs. knowledge as process, knowledge as given vs. knowledge as problematical, learning is holistic vs. learning is molecular, intrinsic vs. extrinsic motivation, each child unique vs. children have shared characteristics, learning is individual vs. learning is social and child as person vs. child as client.

Societal dilemmas capture contradictions related to equity and social relations and are described as: childhood continuous vs. childhood unique, equal vs. differential allocation of resources, equal justice under the law vs. ad hoc application of rules, common culture vs. sub group consciousness.

> Each dilemma captures not only the dialectic between alternative views, values, beliefs in persons in society, but also the dialectic of subject (the acting 'I') and object (the society and culture that are in us and upon us) ... both the forces which shape teachers' actions (those forces that press towards particular resolutions to a dilemma)... and the capacity of teachers not only to select from alternatives, but to create alternatives. (p. 125).

The Berlaks' project is to understand possibilities for changes in schools and teaching. They share Lampert's concern to recognise and acknowledge the complexity and wisdom in teaching and hold that teachers are able both to inquire into their practices and to change them. Their project is, further, to develop a language that accounts for teachers' actions and that can be used to inquire into, and change teaching.

Berlak and Berlak develop a dialectical account of teachers' action that challenges a subject-object dichotomy. They distinguish between habitual and reflective activity in teachers (and humans in general). Through reflective activity, teachers do not simply adapt to a given world but shape and change that world. Language, and other communicative gestures, are some of the most crucial means humans have for self consciousness: that is, to view themselves through others, though neither uncritically nor unchangingly. Language or communication enables persons to shape the conditions of their own adaptations.

Where Lampert works against a theory practice dichotomy, the Berlaks want to address the sociological structure agency dichotomy, and hence move to incorporate critical Marxist theory. They develop their dilemma language through an ethnographic study of a selection of primary schools in the United Kingdom in the late 1970s, and particularly those primary schools that typified the English informal/open classroom at that time. As they struggled to organise their observations, it became clearer and clearer that, both within and across particular teachers and schools, there were no simple dichotomous ways of capturing the teaching they observed nor how teachers themselves understood their work.

is not accidental nor idiosyncratic that in her teaching in the early 1980s, issues of control, equity and creative classroom organisation were part of her personal goals. They were deeply enmeshed in educational debates in the USA at the time. Managing dilemmas is as much a function of the wider social context of the teacher's work as it is of her or his personal biography.

What begins to emerge here is the contested terrain of knowledge about teaching. There is contestation not only on what counts as knowledge about teaching, but also on who constructs knowledge about teaching and how, and crucially who benefits. This raises methodological questions that are taken up in the next chapter. For Lampert, generalised theories about teaching, while important, are not entirely helpful to teachers. They do not speak to the logic of practice as do detailed descriptions from the teacher's point of view. Her detailed descriptions of two instances of dilemmas are easy to identify with as a teacher and thus confirm her argument. But her focus on the personal and practical limits her and her readers from seeing how her management is as much a function of who she is as where she is - in her school and society. The question emerges as to whether a more generalised but focused language of dilemmas might enable description and explanation of the complexities of teaching in any context that is at once theoretical and meaningful for practitioners.

3 DILEMMAS IN TEACHING: A DIALECTIC ACCOUNT OF TEACHERS' ACTIONS

3.1 The individual and the social

Zeichner and Berlak (1981) make an explicit attempt to develop a language of dilemmas that captures 'contradictions that are simultaneously in consciousness and in society' (p. 124). A language of dilemmas, they argue, can simultaneously represent contradictions that reside in the situation, in the individual and in the larger society, as they are played out in the form of institutional life, and particularly schooling:

additional forms of assessment, making explicit assessment practices and their effects (though this was ethically and practically difficult because it required being open about the students' own performance), and working carefully in other ways on historical racial inequities.

2.2 The personal and the practical in teaching situations

Lampert's project is to argue for the 'practical work involved in managing dilemmas', from the teacher's point of view. In this way, she argues, we will understand how teachers manage their complex professional activity. She criticises Berlak and Berlak (1981), whose dilemma language focuses on possibilities and constraints in change in teaching, for not paying sufficient attention to the practical in dilemmas of teaching and demonstrates her commitment to locate teaching in the arena of wise professional practice. Her position develops out of Schwab's language of the practical in relation to knowledge about teaching (Schwab, 1978): the theory-practice dichotomy is dissolved in the practical. Effective teacher action is neither ad hoc nor theoretically driven. It is practical - in the teaching situation. It is also personal ted to biography. Knowledge about teaching is thus not adequately portrayed in theoretical accounts that are divorced from the practical knowledge of teachers from their point of view. Such accounts only exacerbate the theory practice dichotomy.

Lampert's account of teacher knowledge and action is thus both *situational* and *personal*. However, in her focus on the personal and the practical, she does not pay sufficient attention to the social, political and cultural structuring and constraints in situations within which teachers work. While she does pose questions for further research in relation to strategies teachers adopt in the face of dilemmas, and in relation to the resources teachers have in dealing with dilemmas, through her focus on personal intervention and coping strategies she does not take sufficient account of the contextual dimensions of any dilemma. It

teacher's practice, how the management of the dilemmas they faced was a function of their personal biographies and their identities as teachers. A different teacher, for example, might not be concerned, as was Lampert, with gender equity and with levels of classroom control that enabled both freedom and order. Such a teacher would probably not experience the situation in the same way, and thus not face a dilemma'.

Lampert describes a pedagogical dilemma as an 'argument with oneself' that ultimately involves 'personal coping strategies'.

> My capacity to bring disparate aspects of myself together in the person that I am in the classroom is one of the tools that I used to construct an approach to manage my dilemma (1985, p. 184).

Lampert's argument for the *management and use of conflict* as opposed to attempting to resolve conflict in the face of competing alternative dilemmas is compelling. Working as she does with both her own and another teacher's practice in school, she offers a serious insider/outsider engagement with some of the very real complexities of teaching and learning and the practical work involved.

Understanding teacher decision-making as managing dilemmas resonates with my own experiences in teaching in schools and the university. I can recall, particularly in my pre-service primary mathematics teacher education courses, the dilemmas faced and managed in relation, for example, to my assessment practices and my concern with racial equity. Black students were a minority in the large class, with a very different school history from the majority of white students. I felt the necessity for assessment practices that demonstrated explicitly what was required for success in the institution of teacher education, so enabling black students to recognise clearly what was required, and the simultaneous awareness that a number of such assessment practices reinforced racially defined apartheid outcomes, in that they privileged the kind of cultural capital many white students had acquired in their schooling years. Managing the tension involved seeking

that teaching involves managing this tension. There is no one 'right' solution to this tension in this context of practice, whatever analysis or theorising might suggest. If she stays near the boys, thus satisfying her pedagogical need to have sufficient control in the classroom for teaching and learning to occur, she will betray her desire not only for gender equity but for encouraging girls in a field like mathematics. If she moves away from the boys to satisfy the importance of equity in pedagogical practice, she will undermine classroom order which she believes is a prerequisite for teaching and learning. Thus it is not a matter of making a right choice. Resolution does not lie in one (control) or the other (equity). Rather, what is entailed is finding/constructing strategies that do not ignore but address the tension by diminishing conflict. Lampert managed her tension (with fortuitous assistance - two boys being absent) by carefully reorganising the pupils' seating.

Lampert strengthens her case for managing dilemmas by describing the dilemma faced by another primary mathematics teacher. Two pupils produced two different but meaningful responses to a problem, one of which matched the response in the text book. The pupils wanted the teacher to say which was correct. She did not want to undermine the pupils' confidence in the text book, nor did she want to mislead them into thinking there was only one meaningful and correct response. The dilemma for the teacher was that the text book was her tool, but in providing only one answer and one method, it had limitations in relation to her other mathematical goals. This teacher managed her dilemma by working with the pupils on why both responses were correct

The notion of teaching as *managing dilemmas* runs counter to many images of teachers, for example, as cognitive information processors, as technicians carrying out researchers' knowledge of what produces learning, or as members of opposing camps making choices among dichotomous alternatives". All these images portray the conflict in teaching as resolvable through choosing one or the other competing alternative. Lampert illustrates from her own and the other

faced dilemmas in their teaching in multilingual classrooms. Dilemmas also became an illuminating way in which to organise, that is describe and explain, my observations. It was thus from the empirical work in the study itself that I turned my attention to unde-standing and developing dilemma theory, and a language with which to describe and explain mathematics teachers' knowledge of their practice in a range of multilingual mathematics classrooms in South Africa.

The Berlaks (1981) and Lampert (1985) aim to describe and explain teaching acts in such a way that not only the complexity of teaching is captured, but also the deliberative and multifaceted way in which teachers think and act. In doing this, they both work with a language of dilemmas, though from different perspectives. Lampert's is teacher research, and action research, insider/outsider accounts[b] of the complexities of teaching through detailed illustrative examples from practice. It is based in a theory of the practical. The Berlaks' is a much more extensive ethnographic study grappling with an individual-society dialectic and develops a more elaborate language. Both provide analytic tools for my study: just as both found in dilemma language a way to analyse teaching, so has it been a key analytic tool in this study. I thus begin with a detailed account of how their understanding of teaching and knowledge about teaching can be joined effectively for the purposes of my study in social practice and sociocultural theory.

2 DILEMMAS IN TEACH'NG: A PRACTICAL ACCOUNT OF TEACHERS' ACTIONS

2.1 Managing dilemmas in a teaching situation

Lampert (1985) argues that 'from the teacher's point of view, trying to solve many common pedagogical problems leads to practical dilemmas'. Working from an example from her own primary mathematics classroom, she describes facing the pedagogical need to position herself near the rather difficult boys in the class so as to keep them under her eye (a control problem) but in so doing finds herself distanced as teacher from the girls in the class (an equity problem). She argues

of this study, I then asked: how do teachers understand their practices in this complex multidimensional dynamic?

While it is analytically possible to separate elements of the practice of teaching and learning in multilingual mathematics classroom (as in the three dimensions described above) they are deeply intertwined and enmeshed in the lived practice of teaching. Any practice is deeply contextual and hence complex and often contradictory. It is a function of both actors - their beliefs, values, desires and capabilities - and the social, historical and cultural possibilities and constraints in their context of action. In Bourdieu's (1990) terms, action in lived practice is a function of the 'logic of practice'.

Within the rich textures of the practice of teaching, a useful way of describing and analysing its complexity is through a language of dilemmas - a framework that has developed in the context of this study. As a qualitative and grounded study, and as is elaborated in the next chapter, there is a dialectic between the empirical data of the study and the sociocultural theoretical lens that is brought to bear on its interpretation. They shape each other in a dynamic way in the process of the study.

The notion of teaching dilemmas forms a part of existing literature on teaching (e.g. Berlak and Berlak, 1981; Lampert, 1985). For the Berlaks, a language of dilemmas captures:

> contradictions that are simultaneously in consciousness and society ... (dilemmas) capture not only the dialectic between alternative views, values beliefs in persons and in society, but also in the dialectic of subject (the acting I) and object (the society and culture that are in us and upon us). (pp. 124 125)

'Dilemmas' were not part of my original focus or thinking. However, as I analysed the initial interviews with the teachers in the study, it became apparent that they

this suggests is that speech in school must impact on mathematical learning, and in particular ways in multilingual settings.

Similarly, in working with teachers' knowledge (that is, what they have come to know about teaching) it is not enough to work only with what they say. We need to understand their meanings and intentions, as well as what they do - their actions in context. There is thus a need to reveal and examine bot - their tacit and their articulated knowledge (Polanyi, 1967). In Polanyi's terms, we know more than that which we can self-consciously articulate . Knowledge is both embodied and discursive.

Within a social theory of mind, teachers' knowledge is understood as socially mediated. What they are able to say or self consciously articulate (account and reflect on) and what they do (their actions) form and are formed by their activities and practices which are social (located in institutions of society), cultural (located in language, symbols and ideas) and have a history. This means that teachers in and from different contexts and settings are likely to have different accounts and act in different ways. Simultaneously, since they share the practice of teaching secondary mathematics, their accounts and actions are likely to contain commonalities.

My review of pertinent literature in the field of 'mathematics education and language' reflected the complexity of the field and led to the argument (Chapter 1, p. 22) that teaching and learning mathematics in multilingual school classrooms can be understood as dynamic and at least three-dimensional. The teaching and learning of mathematics in multilingual classrooms is not simply about access to the language of learning. Nor is it simply about access to the (English) mathematics register. Rather, it appears to be about how these intersect with one another, as well as with the social, cultural and discursive processes that constitute school mathematics teaching and learning. In keeping with the focus

Difficulties in interpretation can be located in their privileging the structure of the practice in such a way as to exclude the structure of pedagogy (the mediation of knowledge in the relationship between teaching and learning) as the source of learning. Motivation, identity, conflict, power relations, all reside in the community of practice and will work in different ways to enable centripetal movement to full participation or constrain it. This is why for them, learning is only understood in relation to a learning rather than a teaching curriculum or intentional instruction (p. 40; p. 97). But in so doing, and despite their own commitment to move away from dichotomies, they insert a new and equally problematic dichotomy between teaching and learning.

It is useful to ponder for a moment that in Russian, for example, there is only one word - *obuchenie* - that describes teaching/learning. In other words there is no learning without teaching and vice versa. The teaching/learning relation is a hugely complex one. It is as fundamental a problem in teacher education as it is in school learning. Dominant teacher education practices are structured in both the academy and in the school itself - a combination of a formal and an apprenticeship context. The success of this combination and the relative merits, weightings, contents and processes of the two parts remain the focus of ongoing research and debate. Lave and Wenger's theory of social practice shifts the problematic away from theory/practice dichotomies and questions of transfer and encourages us rather to examine the resources made available in different contexts of teacher education and their possible effects. However, in shifting attention onto a learning curriculum and thus correctly questioning any direct relationship between intentional teaching and learning, they nevertheless move to deny any relationship between learning and intentional teaching. This is problematic in general and particularly so in the context of schooling.

I have argued that while Lave and Wenger provide a framework for understanding teachers' knowledge and identity, their social practice theory is not unproblematically transferable to school learning and teaching. They have,

transparency in the practice and access to resources for newcomers becoming knowledgeable and fashioning a successful identity. I have argued that this conceptualisation of learning within social practice assists the theorising of knowledge about teaching - how teachers learn about teaching. How does Lave and Wenger's conceptualising transfer to theorising learning mathematics (for example) in school? In Lave and Wenger's own terms this question is important: school is a specific social context, involving different communities of practice from those in contexts of apprenticeship.

A shift into school learning raises a number of questions: What/who is the community of practice in school mathematics? What is the community that teachers are old-timers in? mathematicians? mathematics teachers? Or are older students, or mathematically schooled adults the old-timers here? and where are they in relation to the teachers? and pupils? What are pupils new-comers into? Centrally, what might constitute legitimate peripheral participation in the mathematics classroom and towards what is the centripetal process of participation? becoming a mathematician? a mathematically schooled adult?

Lave and Wenger offer a general theory of social practice in which learning is always a part. However, there are clear difficulties in moving into the context of schooling. In school, students remain students until they leave. No matter how much mastery they might have achieved, only a few, after school, might become mathematics teachers and fewer mathematicians. Moreover, their teachers - however mathematical - are not, in the context of schooling, practising mathematicians. There is also a labour intensity in an apprenticeship model that does not transfer easily to mass schooling conditions. Thus, while Lave and Wenger's intentions are for a general theorising, and they attend at moments (for example, pp. 39-41; p. 100) to the specificity of schooling, they in fact sidestep difficulties in using their conceptualisation to interpret and explain teaching and learning in school.

This conception of teacher knowledgeability frames teaching dilemmas. It supports the motivation for my study – teachers have knowledge to share about teaching mathematics in multilingual mathematics classrooms. Moreover, in a study that wishes to access teachers' knowledge, data collection should then include teachers talking about *and* within their practice. In short, Lave and Wenger's social practice theory provides a theoretical framework with design implications for a study entailing teachers' knowledge.

However, and as argued earlier, the dilemma language of Lampert and the Berlaks needs to be elaborated to include the specific knowledge teachers can or might hold in relation to the teaching and learning of mathematics in multilingual classrooms. A study of teachers' knowledge of the teaching and learning of mathematics in school thus needs also to theorise knowledge of subject matter (school mathematics) and knowledge of language in use in classrooms. Does Lave and Wenger's social practice theory transfer from apprenticeships, and other communities of practice like Alcoholics Anonymous and teaching, into school mathematics learning? In other words, can an elaboration of dilemma language be provided by conceptual tools in Lave and Wenger's theory of social practice?

4.5 Shifting into school learning

In order to develop an understanding of learning as part of social practice, Lave and Wenger turn to contexts of successful learning – apprenticeships. They explicitly turn away from the school because learning as intended in schools has been unsuccessful for so many and, moreover, in socially distributed ways. In addition, the formal school has been the dominant and determining domain of learning theory, yet it is not the only context of learning.

Instead of teachers and learners we have *old-timers* – knowledgeable others in a community of practice – and *new comers* whose knowledge and identity evolve through centripetal participation in the practice. They elaborate the importance of

to talk about teaching from outside the practice. For Lave and Wenger this is achieved through a didactic use of language, not itself the discourse of teaching practice, and thus creates a new linguistic practice all of its own.

Talking within and talking about a practice thus need redefinition (p. 109). Talking within a practice itself includes both talking within (for example, exchanging information necessary to the progress of ongoing activities) and talking about (for example, stories, community lore). Inside the shared practice, both forms of talk fulfil specific functions: engaging, focusing and shifting attentions, bringing about co-ordination on the one hand; supporting communal forms of memory and reflection as well as signalling memberships on the other. Stories by full participants, say by experienced teachers about their practice, inform other teachers about teaching and demonstrate or model how to tell stories about teaching from within the practice.

Talking *about* a practice also usually involves talking both within and about. In Lave and Wenger's terms, the effect of this talk is not full membership of the practice, because it is happening from the outside. It is rather what they call 'sequestration' and an alienation from, or prevention of access to, the practice. We know only too well from teacher education courses that a prospective teacher's ability to write a good essay on what is good teaching - where 'good essay' is signalled in the practices of the academy - often bears little relation to good teaching in practice.

Knowledge about teaching is thus not simply in individual teachers' heads: it is tied to their identities and evolves in and through co-participation in the practices of the teaching community. Teachers, particularly if they have been in practice for some time, are more or less knowledgeable about the practice of teaching[5], depending on the community, their access to its resources - particularly to activities related to talking within and about the practice, and to the transparency in the practice.

Extending the concept of transparency furt' r into the classroom and the focus of this study, the example of group discussion of a mathematical task is illuminative. Pupil-pupil discussion of a task should enable the mathematical learning in the task and so be invisible[11]. However, the rules for constructive functioning of a learning group are often left implicit. It is possible that the discussion itself becomes the focus of attention for the group, rather than a means to the mathematics. Here it obscures access to mathematics, by becoming too visible itself. This possibility might well be exaggerated in the workings of a group which has a number of main languages.

In short, practices that are more or less transparent can enable, obstruct or even deny peripheral participation and hence access to the practice. Through transparency, members can exercise control and selection into the practice. Thus, the explanatory burden for learning, and here learning about teaching, is placed in cultural practice. It is placed in the community of teaching, and not on one kind of learning or another. Increasing participation and hence knowledgeability is not about connecting theory and practice, or experience and abstraction, but rather entails the organisation of activities that makes their meaning visible to teachers and all other participants in the practice.

4.4 Learning *to* talk

In addition to transparency, legitimate peripheral participation also involves learning how to talk (and be silent) in the manner of full participants. For newcomers then, the purpose is not to learn *from* talk as a substitute for legitimate peripheral participation, it is to learn *to* talk as a key to LPP. Unpacking these concepts related to talk, Lave and Wenger distinguish between *talking within* and *talking about* a practice. Full participation in a community of practice means learning *to* talk, and this entails talking about and within the practice (p. 109). Talking about the practice from the outside is what often constitutes formal learning (for example, theory of education in teacher education) where student teachers learn

Becoming a full participant means engaging with the technologies of everyday practices in the community, as well as participating in its social relations. Thus, access to artifacts in the community through their use and understanding of their significance is crucial. Often material tools, artifacts - technologies are treated as given. Yet, they embody inner workings tied with the history and development of the practice and which are hidden - these need to be made available. For example, in mathematics teaching, the textbook is a resource. It is used widely and often exclusively to teach school mathematics. Its inner workings, however, are undoubtedly tied with the history and development of school mathematics as the acquisition of dominantly procedural knowledge.

Lave and Wenger elaborate 'transparency' as involving the dual characteristics of *invisibility* and *visibility*[10]:

> ... invisibility in the form of unproblematic interpretation and integration (of the artifact) into activity, and visibility in the form of extended access to information. This is not a simple dichotomous distinction, since these two crucial characteristics are in a complex interplay (p. 102).

Access to a practice relates to the dual visibility and invisibility of its resources. In other words, the invisibility of mediating technologies in a practice is necessary for focus on and supporting the visibility of the subject matter in the practice. Continuing with the example of the textbook as a resource in teaching: the textbook is highly visible but also invisible in that it makes mathematics (the subject matter) visible. However, the dilemma of the teacher in Lampert's study illustrates effectively how the inner workings of the textbook (revealing mathematics as single methods and answers to problems) can obstruct access to participation in wider conceptions of mathematical practice. Effective teaching (becoming a full participant) then depends not only on the availability and use of a textbook, but also knowledge of and insight into its history and inner workings, its possibilities and limits.

participation thus requires a 'decentering' of mastery and pedagogy away from the individual master or learner and into the structuring of resources in the community of practice (p. 94). Learning and mastery are functions of how resources are made available. For Lave and Wenger understanding participation and learning requires a focus on the learning curriculum, and not the teaching curriculum. It is neither teaching intentions, nor planned pedagogy that can enable and explain learning (p. 97). Rather, the social structure of the practice and conditions for legitimacy define the practice and possibilities for learning.

Peripheral and full participation provide a model for considering the positions of a teacher in relation to learning. They also provide a means for distinguishing new and older teachers, as well as for distinguishing within newer or older teachers in such a way that those who remain more peripheral are not so simply because they are 'inadequate'. This might well be the case, but must be seen in relation to a teacher's access to resources in the social structure of teaching. The concept of transparency elaborates this point.

4.3 Transparency

For Lave and Wenger, learning occurs through centripetal participation in the learning curriculum of the community. Becoming a full member, that is, becoming more knowledgeable, entails having access to a wide range of ongoing activity in the practice - access to old timers, other members, to information, resources and opportunities for participation. Such access hinges on the concept of *transparency*.

> The significance of artifacts in the full complexity of their relations with the practice can be more or less *transparent* to learners. Transparency in its simplest form may imply that the inner workings of an artifact are available for the learner's inspection ... transparency refers to the way in which using artifacts and understanding their significance interact to become one learning process (pp. 102-3).

For Lave and Wenger, social practice, and not learning, is the starting point. Learning is rather a dimension of any social practice. It is at once subjective and objective through a focus on whole person-in-the world. Learning is increasing participation in communities of practices and concerns the whole person acting in the world. This is in sharp contrast to dominant learning theory which is concerned with internalisation of knowledge forms and their transfer to and application in a range of contexts. Knowing is thus an activity by specific people in specific circumstances. Identity, knowing and social membership entail one another. Thus 'learning is not a condition for membership, but is itself an evolving form of membership' (p. 53). Knowing about teaching and becoming a teacher evolve, and are deeply interwoven in ongoing activity in the practice of teaching. Knowledge about teaching is not acquired in courses about teaching, but in ongoing participation in the teaching community in which such courses might be a part.

This view of knowledgeability opens another way of understanding teachers' roles in developing knowledge about teaching. Debates on the 'teacher-as-researcher' often polarise researchers and teacher researchers, with arguments about what constitutes research, and, moreover, what knowledge about teaching in fact affects practice[9]. Lave and Wenger's social practice theory clearly identifies teachers as a crucial source of knowledge about teaching. I am not suggesting here that only teachers can know about teaching. Rather, just as carpenters are not the only people who know about carpentry, they certainly are a key source of any understanding of its practice. Simply, we can and must learn about teaching from teachers themselves.

Lave and Wenger distinguish between *peripheral* and *full* participation where both are legitimate but different forms of participation in the practice and both are constantly changing. Full participation signals mastery in the form of full membership in the practice rather than an endpoint in learning/knowing all there is to know about the practice. The process of moving from peripheral to full

For Lave and Wenger, becoming knowledgeable is thus a simultaneous and ongoing fashioning of personal and professional identity within a community of social practice. As a set of relations among persons, activity and world, a community of practice 'is an intrinsic condition for the existence of knowledge, not least because it provides the interpretive support necessary for making sense of its heritage' (p. 98). Learning is thus located in the process of co-participation, and not in the heads of individuals. This is thus a social theory of mind where meaning production is located in social arenas that are at once situationally specific and in the broader society. In Lave and Wenger's terms, knowledge about teaching is thus fundamentally tied to the context of teaching, and cannot be abstracted from it. Knowledge about teaching is also dynamic and simultaneously personal and social.

Reflecting back to Lampert's understanding of teaching acts as personal, these must not be understood as simply located in an individual teacher's head but in her co-participation in the community of teaching. Hence my earlier point that Lampert's concerns were not accidental or idiosyncratic but connected in and with talk in her community of practice.

4.2 Legitimate peripheral participation

'Legitimate peripheral participation' (LPP) is the conceptual bridge between the person and the community of practice. As people participate in communities of practice so they become more knowledgeable in the practice. They move from a position of 'newcomers' to becoming 'old-timers' with greater mastery of the practice and with all the conflicts, contradictions, changes and stability that entails. LPP is a means of explaining both the developing identity of persons in the world, and the production and reproduction of the community of practice. Here is a conceptual framework for integrating the concerns of Lampert and the Berlak's in describing and explaining teaching.

How then does one capture the knowledge about teaching and teacher action as both personal and social, where meaning is constituted in and constitutive of activity in social and cultural settings, that is, in social practice? As an answer to this I turn to social practice theory.

4 A SOCIAL PRACTICE THEORY OF TEACHERS' KNOWLEDGE

4.1 Situating learning in communities of social practice

Lave (1991) and Lave and Wenger (1991) situate learning in communities of social practice. Building on Lave's earlier work on situated cognition (1985; 1988), they develop a theory of social practice - what they call 'legitimate peripheral participation in communities of practice' (LPP). LPP can illuminate how teachers learn about teaching. It can also be used to throw light on teachers' knowledge about teaching. As I shall show, it provides a theoretical framework that incorporates the personal, practical and social dimensions of teachers' knowledge.

Lave and Wenger write:

> Briefly, a theory of social practice emphasizes the relational interdependency of agent and world, activity, meaning, cognition, learning and knowing. It emphasises the inherently socially negotiated character of meaning and the interested, concerned character of the thought and actions of persons-in-activity. This view also claims that learning, thinking, and knowing are relations among people in activity, in, with and arising from the socially and culturally structured world. This world is socially constituted; objective forms and systems of activity, on the one hand, agents' subjective and intersubjective understandings of them, on the other, mutually constitute both the world and its experienced forms. Knowledge of the social world is socially mediated and open ended. Its meaning to given actors, its furnishings, and the relations of humans with/in it, are produced, reproduced, and changed in the course of activity (which includes speech and thought, but cannot be reduced to one or the other). In a theory of practice, cognition and communication in, and with, the social world are situated in the historical development of ongoing activity (pp. 50-51).

understand the forces, internal and external, that prevail in sets of circumstances might well provide our diversity of teachers with some resources and tools to think and act appropriately in the complexity of their classroom life.

> It is our hope that the dilemma language will be useful in clarifying for professionals and the public some of the debates of schooling practices and their relationship to the major political and economic questions of the day, and for helping to identify alternative possibilities for making schooling a richer, more engaging and challenging intellectual, cultural and social experience for all students.
> (Berlak and Berlak, 1981, p. 9)

The Berlaks express this hope in the context of the conflicts over educational reform at the end of the 1970s in the USA and the UK. But their words could have been taken from the mouth of a researcher in South African education in the 1990s as we confront the realities of trying to carve more equitable, more meaningful and higher quality intellectual experiences in schooling for all our children. And we do so in an even more conflictual and contradictory context.

However, while dilemma language in general is appropriate, I have argued above that, for this study, it requires extension. The eight curriculum dilemmas categorised by the Berlaks might not adequately capture dilemmas specific to teaching and learning secondary mathematics, especially in multilingual classrooms. One of the contributions of this study will thus be to extend dilemma language.

My extension of dilemma language (see Chapters 5 - 8) shares the Berlaks' underlying theoretical assumption about the need for a dialectical account of teachers' actions and knowledge. Fundamental to such an account is the view that meaning is not a process that goes on in the mind. Rather, it is constituted in and through social activity. My extension of dilemma language also shares Lampert's insight into the practical and personal (a teacher's identity) in teaching. Here the management of dilemmas is important.

classroom practice is not as clearly bound with the subject taught as is the case with secondary teachers. What then do these studies and the Berlaks' dilemma language offer to a study of teachers' knowledge of their practices in multilingual secondary mathematics classrooms in South Africa?

I am concerned here with problematising teaching dilemmas in relation to the specificity of mathematical subject knowledge in school. As indicated above, this is directly linked to the issue of identity, for one's identity as a secondary teacher is as a subject teacher. The Berlaks' typology, being sociological, is concerned largely with the social organisation of the curriculum and does not deal with the specificity of subject knowledge. While their curriculum dilemmas will apply in mathematics classrooms, there are, perhaps, particular dilemmas teachers face in their attempts to teach mathematical discourse. While Lampert's dilemma examples are drawn from primary mathematics classes, they are control and curriculum dilemmas related to mathematics teaching but with little engagement with the specificity of mathematical content. Moreover, mathematics at the secondary level is more complex - a more intricate web of concepts is at play and thus could lead to more mathematically specific dilemmas.

In short, the forms of dilemmas, particularly at the secondary level, need to relate to what is being taught as well as to how and where. Dilemma language thus needs extension to subject specific teaching. In addition, neither the Berlaks nor Lampert explore the issue of language in the classroom and dilemmas that might emerge. Thus dilemma language needs to be developed here in relation to both language and mathematics[8].

More generally, how does one recontextualise a study into a different context, into a different time and place? At this transitional juncture of South Africa's social and political history, where choices have to be made in new educational policy formulation and its implementation in the life blood of classrooms, dilemma language offers a great deal. Having an accessible language with which to

then is conscious awareness, effective capacity, the ability to think about thinking - metacognition.

Scientific and spontaneous concepts, while distinct, influence each other:

> In the scientific concepts that the child acquires in school, the relation to an object is mediated from the start by some other concept. Thus, the very notion of scientific concept implies a certain position in relation to other concepts, i.e., a place within a system of concepts. It is our contention that the rudiments of systematisation first enter the child's mind by way of his contact with scientific concepts and are then transferred to everyday concepts, changing their psychological structure from the top down.
>
> (Vygotsky, 1986, pp. 172-173)

> One might say that the development of the child's spontaneous concepts proceeds upwards, and the development of his scientific concepts downwards, to a more elementary and concrete level. This is a consequence of the way in which the two kinds of concepts emerge. The inception of a spontaneous concept can usually be traced to a face-to-face meeting with a concrete situation, while a scientific concept involves from the first, a "mediated" attitude towards its object. (Vygotsky, 1986, pp. 193-194)

There are two implications here, particularly for teaching and learning mathematics. First, without the specific experience of schooling, particular forms of intramental functioning (abstract concept formation, metacognition) will not develop. The value and benefits of schooling, however, provide the content of much research and debate. Since Vygotsky's time, schooling has shown itself to be useful in socially distributed ways[19]. An additional question here is whether explicit pedagogy is absent in everyday life. Is decontextualised thinking the privileged realm of formal instruction? The in- and out of school debates are beyond the immediate focus of this thesis, but continue to be the subject of ongoing research activity, particularly in relation to meaningful mathematics

as a distinct context, with particular kinds of learning activities that will shape consciousness and lead to particular kinds of knowledge, and that while these will not simply transfer into other activity, they will nevertheless impact on psychological functioning.

Vygotsky identified three leading activities that shape consciousness formation: play, school and work (Davidov, 1990). A distinct feature of schooling is that it involves formalised instruction, what one might call explicit pedagogy or, in Lave's terms, a teaching curriculum. Vygotsky thus recognised the school as a distinct context entailing distinct kinds of activities leading to qualitatively different kinds of knowledge than those acquired in everyday life, in play or in work.

Schooling is a particular social setting and takes particular forms in different historical conditions. Therefore the kinds of social interaction that children have in school will have intrapersonal effects (Damon, 1990). The particularity of schooling in contrast to everyday life is that schooling is abstracted and structured out of everyday life, and organised with particular time-space relations. Everyday knowledge and school knowledge are thus distinctly different kinds of knowledge.

For Vygotsky, schooling and formalised instruction lead specifically to the development of metacognitive awareness on the one hand and to the development of scientific concepts on the other. The learning of new word meanings in school is not through direct experience with things or through phenomena. Rather it is through a system of concepts. What distinguishes scientific from spontaneous concepts is that they are part of a system of concepts, and they are deliberate and self conscious. In contrast, spontaneous concepts, the concepts that are formed in our everyday activity, are unsystematised and saturated with experience.

Because scientific concepts are tied to other concepts they entail abstract thinking, and generalisations of generalisations. What develops during the school period

context there is no simple or direct use. Rather there is a recontextualisation into the logic of the new context, a recontextualisation that inevitably shifts, changes and affects the knowledge use. Drawing together the language of Lave and Bernstein, knowledge acquired in one set of social practices and used in another undergoes transformation through a delocation from its context of origin and a relocation in another set of social practices[17].

In contrast, in Vygotsky's theory, the higher psychological functions, which includes the formation of concepts, form through socially and culturally mediated activity where cultural tools first used interpersonally, become internalised and used to regulate action. This can be read as the formation of structures and systems that come to operate independently from future experiences and activity and evolve with development. However, word meaning for Vygotsky was never outside of systems of relations.

Perhaps it is possible to combine social practice theory - which does not deal with developmental issues - with sociocultural theory. While social practice theory looks at becoming knowledgeable as the fashioning of identity in sets of social practices, it does not deal with how these might function and change as a result of time and development. For example, it does not engage with how different sets of social practices function in relation to children in contrast with adults. Vygotsky provides a way of understanding psychological functioning as undergoing qualitative transformations *over time*[18] and through sets of experiences (activities in social practices). Thus, when placed in a new situation, previous learning and resultant psychological functioning factors in, though not in any decontextualised way.

6.2 The specificity of schooling and scientific concepts

From a conception that particular activities and learning are tied to their context of production, but not in any static way, comes the understanding that schooling,

of acting on the world and signs operate internally in mediating thinking, in controlling thought. The use of tool and sign brings transformations tool on the object of activity and signs on intramental functioning.

Language, and speech, can function as both tool and sign through interpersonal communication (acting on and with others), and then intrapersonally as a thinking device. Language (speech) thus has both a communicative and an intellectual function. As Wertsch argues (Wertsch, 1985; 1991a), for Vygotsky, human social and mental activity is mediated by technological tools and signs (psychological tools), in other words, through semiotic mediation and hence shaped by sign systems such as natural language. As mentioned in the introduction to this chapter, this understanding of human functioning and the role of language must impact on mathematics learning in general and in multilingual classrooms in particular.

An understanding of human consciousness as culturally and socially mediated and historically contingent implies that the kinds of activities and social interactions that we have, the kinds of tools (both material and symbolic, and particularly language) that we use, and the social and political contexts within which these are lived, will have specific intramental consequences. Thus in Vygotskian theory we see again shared conceptions of situated knowledge.

However, there are fundamental differences in the theoretical orientations of Lave and Vygotsky, particularly in relation to internalisation on the one hand and their different emphases on activity and practice. Lave argues against any conception of internalised knowledge that is independent of the context of its production and its use. For Lave, knowledge acquired is tied to its context of acquisition, and not decontextualised into some abstract form that is then transferable to other different contexts. Bernstein's (1990) notion of recontextualisation (and Brousseau's notion of transposition (in Crawford and Adler, 1996)) are powerful here in describing what happens as knowledge acquired is used in a different

higher psychological functioning. Higher psychological functions (conscious awareness, voluntary attention, logical memory and the formation of concepts), he argues, originate in human social relations (1978, p. 57), through the use of material and symbolic cultural tools, including language, and through interpersonal communication (Vygotsky, 1978, p. 56; 1991). The higher psychological functions are thus specifically human, a function of the internalisation of the tools of culture - including language and thus, too, speech. Whereas language/speech is initially used for communication with others, that is, used to act externally as a tool, it becomes internalised as a sign that guides intellectual functioning. First it is a means of influencing others, and only then a means of influencing oneself.

This is formulated in Vygotsky's genetic law of cultural development:

> Any function in the child's cultural development appears on the stage twice, on two planes, first on the social plane and then on the psychological, first among people as an interpsychological category and then within the child as an intrapsychological category. This equally applies to voluntary attention and logical memory, formation of concepts and development of volition.... Behind all higher functions and their relations genetically stand social relations.
>
> (1991, p. 40)

In a social theory of mind, language, context and subject matter can be woven together.

Throughout his writings, Vygotsky draws important conceptual distinctions e.g. between the biological and the cultural, language and thought, immediate and mediated memory, that in other psychological theorising are often collapsed into each other, or too severely dichotomised. For Vygotsky, there is no thought without word, and vice versa, but language and thought are not the same. Similarly, cultural development cannot be understood divorced from biological development (in humans) but, again, these are distinct forms. Another key distinction Vygotsky draws and is pertinent to this study is that between tool and sign. Both are mediators of activity, yet distinct. Tools operate externally as ways

6 SOCIOCULTURAL THEORY OF LANGUAGE AND THE TEACHING AND LEARNING OF MATHEMATICS IN SCHOOL

6.1 Vygotsky's sociocultural and historical theory of psychological development

Vygotsky posits a theory of human consciousness as formed through purposeful and mediated activity in a social and historically contingent world (Vygotsky, 1986; 1978; 1991; Rieber and Carton, 1987). In other words, Vygotsky's theory of human consciousness is both a sociocultural theory and an historical-cultural theory. What we know and understand (intramental functioning) is a function both of our activity (actions and interactions) in a social world and their location in a particular historical context (intermental functioning). Thus, Vygotsky's sociocultural developmental theory and Lave's social practice theory share an historically contingent and sociocentric conception of human consciousness.

This central thrust in Vygotsky's theory is captured in his law of cultural development:

> From the very first days of the child's development his activities acquire a meaning of their own in a system of social behaviour, and being directed towards a definite purpose, are refracted through the prism of the child's environment. The path from object to child and from child to object passes through another person. This complex human structure is the product of a developmental process deeply rooted in the links between the individual and social history. (1978, p. 30)

For Vygotsky, the linkage between the individual and social history lies in two distinct but interweaving lines of development - a cultural line and a biological/maturational line. Maturation and cultural activity together account for human development. This links with a key concern of Vygotsky: to understand that which distinguishes human consciousness from animal behaviour. Humans make use of auxiliary tools and signs in their ongoing activity and problem solving. This specific human use of tools led to Vygotsky distinguishing elementary from

in a community of practice is problematic. Their continuity argument denies the crossing of any bridges. What do talking within and talking about then mean in classroom practices? How do learners learn to talk about mathematics? That there is a bridge to cross between everyday and educated discourses is at the heart of Walkerdine's (1988) argument for 'good teaching' entailing chains of signification in the classroom where everyday notions have to be prised out of their discursive practice and situated in a new and different discursive practice.

Current debate in mathematics education, stimulated by more fallibilist conceptions of mathematical knowledge, reflects attempts to change the quality of experiences learners have away from the procedural application of rules to more principled, deliberate thinking, problem solving and problem posing. The goal, on the one hand, is to make school mathematical experiences more authentic, more like the mathematics of mathematicians, and on the other, to bring in the real world of problems and applications. But solving a mathematical problem in school is not simply continuous with solving mathematical problems in other real world contexts. Nor is school mathematical practice the practice of mathematicians. It can and should include both. But it will always be through a recontextualisation. And hence the need for a focus on pedagogy to enable the crossings between registers, and between languages and social situations. And crossings in the classroom can create dilemmas for teachers. As Muller and Taylor (1995) argue, such crossings can be dangerous and alienating in school and more for some learners than others. I thus turn now to sociocultural theory to expand dilemma language to incorporate the specificity of mathematics teaching and learning in multilingual classrooms.

Mercer provides a language with which to understand the special nature of classroom education and knowledge produced in the context of schooling. He distinguishes between *educational discourse* - the discourse of teaching and learning in the classroom - and *educated discourse* - new ways of using language, 'ways with words' which will enable pupils to become active members of wider communities of educated discourse (Mercer, 1995, p. 82). In Mercer's terms, educated discourse in school mathematics will include the mathematics register.

Learners can develop familiarity and confidence using new educated and educational discourses only by using them. While pupils all engage in educational discourse, they need opportunities to practise being users of educated discourses. Often there is a mismatch between the educational discourse in play (ways with words in the classroom) and the educated discourse they are meant to be entering. The teacher's role is to translate what is being said into academic discourse, to help frame discussion, pose questions, suggest real life connections, probe arguments and ask for evidence. This does not mean that the teacher's role is simply to explain, but more to be the person who brings the language and the frames of reference of the 'expert' discourse into the 'collective consciousness' of the group (Mercer, p. 81). The language practices of the classroom (educational discourse) must 'scaffold students' entry into educated discourse' (p. 82). This is not a negation of student creativity. Even for creativity, students still need to know the discourse.

> Teachers are expected to help their students develop ways of talking, writing and thinking which will enable them to travel on wider intellectual journeys, understand and being understood by other members of wider communities of educational discourse: but they have to start from where learners are, to use what they already know, and help them go back and forth across the bridge from 'everyday' discourse into 'educated discourse' (Mercer, 1995, p. 83).

It is in this understanding of the aims of school education that Lave and Wenger's seamless web of practices entailed in moving from peripheral to full participation

specific learning. First, a brief elaboration of the nature of school mathematical knowledge.

5 SCHOOL MATHEMATICS AS A DISCOURSE

There are two understandings of school mathematics embedded in this theoretical discussion. The first is that the school is a specific context. There is a distinctiveness to activity in school. The learning and teaching of mathematics in school is thus a very specific social practice. Chapter 1 drew on Pimm's metaphor that learning mathematics is like learning a language to begin a review of the literature on mathematics education and language. From this perspective, learning mathematics entails acquiring new language registers which include both technical skills and ways of meaning. In other words, learning mathematics entails acquiring, recognising and developing specific ways of using language, or, in Lave and Wenger's terms, learning *to* talk.

The second is that school mathematics then needs to be understood as a discursive subject[15] or as a discourse or set of discourses, where 'discourse' means 'language as it is used to carry out the social and intellectual life of a community' (Mercer, 1995, p. 79) where the mathematical register is part of the discourse. School mathematics then is learnt through discourse, through language in use in the classroom. An important question arises in relation to school knowledge. What is the discourse of school mathematics? It is not the discourse of mathematicians - they are in a different community of practice from that of a classroom. Similarly, the practice of school mathematics is not the practice of mathematicians. School mathematics is also not the discourse of apprenticeships. School mathematics is a social practice with specific time-space relations, activities and discursive practices. School mathematics is a distinct practice (Muller and Taylor, 1995; Dowling, 1993; 1995)[16] where there are recontextualisations from the discipline of mathematics into the curriculum.

dilemma of personal vs. public knowledge. In short, what Lave and Wenger's theorising of learning does not explain, is the specific demands of apprenticeship into school mathematics, and its necessary focus on the structure of pedagogy of mediating school mathematics[11].

Within a social theory of mind, that is, sharing some basic assumptions with Lave and Wenger, there has been a great deal of research, theorising and debate on the mediation of mathematical knowledge in school. It is beyond the scope of this thesis to elaborate fully here. Briefly, however, more sociological arguments draw on the work of Paul Dowling (see, for example, Coombe and Davis, 1995; Dowling, 1993; 1995) and the importance of the discursive elaboration of mathematical knowledge in the classroom for access or apprenticeship into mathematics as opposed to widespread alienation. Here, mediation of mathematical knowledge via the everyday and the emphasis on procedural knowledge in the curriculum come under scrutiny. More psychologically oriented research has focused on the question of meaning where both children's meanings and socially constructed mathematical knowledge are important in the pedagogical situation. Alienation is a function of the suppressing or ignoring of learner meanings. Informed by both neo-Piagetian and sociocultural theory, quality and effective mathematics learning and teaching in school involve a blending of both self- and other regulated activity, between scaffolding a task and providing for creative responses to a task, between teaching and learning (see, for example, Cobb, 1994a; Confrey, 1994; 1995a; 1995b).

Explaining access to or sequestration/alienation from school mathematics requires an understanding of the structure of pedagogy, that is, its mediation through the teaching-learning relationship. Lave and Wenger's social practice theory falls short here. I thus turn now to sociocultural theory for a full and effective elaboration of knowing, learning and teaching mathematics in school and particularly in multilingual classrooms. Sociocultural theory deals with both school and subject

and Wenger's terms, educational ground rules are cultural resources - they need to be transparent, with the dual characteristics of visibility and invisibility.

What Lave and Wenger powerfully illuminate is that resources for learning, like language, can enable or exclude. Depending on how they are used, resources can enable access to the practice or sequester participants. This links directly with the Berlaks' societal dilemma of equity vs. differential resource allocation and illuminates what it might mean to treat more than one language in a classroom as a resource rather than as a problem.

For Lave and Wenger, becoming knowledgeable in a practice entails learning to talk within and about the practice, and not learning from talk. Yet, as I argued in Chapter 1, curriculum initiatives in mathematics education reflect that it is about both learning to talk and learning from talk[14]. For Pimm (1987) many activities in mathematics classrooms flowing from an interpretation of the Cockcroft report depend on learning from talk, on pupils' verbal expression being seen as an important part of teaching and learning mathematics (p. 48). But Lave and Wenger's distinction between talking within and about becomes useful. First, it links with distinctions between talk as exploratory vs. talk for displaying knowledge. In mathematics classrooms where there is a move to more exploratory problem-solving mathematical practices, students often work together on tasks, and then report on their working to others in the class and to the teachers. While on tasks, pupils could be said to have opportunity for talking *within* their mathematical practice.

Then, and either to the teacher, or other pupils or both, they talk *about* their mathematical ideas. Thus they are being provided opportunity to learn *to* talk but a question unanswered is: given the distinct practice that is school mathematics, that classroom talk has its own form and function (Mercer, 1995), how are pupils apprenticed into this talking? And what happens in classes where children have a range of spoken languages? This can be linked to the Berlaks' curriculum

nevertheless, constructed concepts that could provoke interesting insights into learning and teaching mathematics in school. Specifically, *access and sequestration*, the *availability of learning resources*, *transparency*, and their distinction between *talking within and about* a practice are easily read into the pedagogical relation in maths teaching in school, and are thus useful to explore further.

In relation to transparency and the focus of this study on mathematics learning in multilingual settings, language - and specifically speech - functions as a tool in the classroom. A great deal of classroom communication occurs through speech. Speech is thus a resource where, in Lave and Wenger's terms, invisibility and visibility are in constant interplay: speech should be invisible so that the subject of inquiry - a mathematical problem, say - can be engaged, i.e. become visible. But language is a cultural tool and never unproblematic. In and of itself, it can mediate the activity in the course of action. For example, a group of learners working on a problem communicate through speech, gestures and so on, about the problem. This communication is supposed to make the problem more visible, more accessible. But the social relations in the discussion and the discussion itself can mediate the problem, particularly if it occurs in a mix of languages. That language itself can mediate activity and obscure the task rather than make it visible seems fairly obvious in a multilingual class. Are teachers aware of whether, when and how language used in their classrooms should be transparent, needing to be invisible, yet make visible? and if so, what is this understanding?

The concept of transparency with its interwoven visibility and invisibility also links, though in a different way, with Edwards and Mercer's (1987) study of classroom talk. They identify implicit rules of educational talk and practice evident in all classrooms. These 'educational ground-rules' are neither arbitrary nor simply imposed by a teacher. They are aspects of culture (p. 59). Successful participation in school is linked to access to these ground rules and they rightfully ask: if these are rules for successful participation why are they implicit? In Lave

but in ongoing participation in the teaching community in which such courses are a part. As argued earlier, this conception of teacher knowledgeability is a motivation for this study – teachers have knowledge to share about teaching mathematics in multilingual mathematics classrooms. It is also a significant determinant of the research design and specifically of the establishment of the sample and the data collection methods. How does one capture teachers' knowledge about teaching and learning mathematics in their multilingual classrooms when (i) knowledge is both tacit and embodied on the one hand, and explicit and discursive on the other, and (ii) knowledge is both personal and social, where meaning is constituted in and constitutive of activity in social and cultural settings?

Crucially, attention had to be paid to talking to teachers about, and observing teachers in, their teaching practices, in their classrooms and in discussions with other teachers. In Lave and Wenger's terms, data collection needed to include teachers talking about and within their practices – what they say about their teaching and what they talk about with other teachers, as well as observation of teachers in practice – what they do. The study thus required direct interaction between the researcher and the teachers, using the qualitative methods of in depth interviewing and participant observation and fits what is called a field study, or field research (Rose, 1982; Erickson, 1986; Smaling, 1990; 1992; Silverman, 1993).

The sample of teachers was to be drawn from the field of secondary mathematics teachers in multilingual classrooms where English was the language of instruction. In South Africa, this field includes a range of different kinds of multilingual contexts, suggesting a number of possible methodologies (for example, an ethnographic study, or a case study). In the elaboration of the field, the selection of a small sample from three distinctive multilingual contexts and thus a design that falls somewhere between an ethnography and a single case study is

language and conceptions in the curriculum that the teacher wishes them to acquire in the context of classroom communication and activity?

In other words, what do teachers say and do in relation to mediating the development of scientific concepts? And, what do teachers say and do in relation to managing to work with and create appropriate zones of proximal development in large and linguistically diverse classes?

* What do teachers say and do in relation to teaching in a more communicative mathematics classroom when pupils bring a range of main languages to class? (learning *from* talk - *about language as a resource).*

This question can be elaborated as: What do teachers say and do in relation to whether, when and how language used in their classrooms (a) is a resource or a problem and (b) should be transparent, needing to be invisible, yet made visible?

* And, at a meta level, what is the relationship between what teachers say and what they do?

It is important to note here that the research questions are not expressed in a language of teaching dilemmas. My coming to dilemmas is empirically grounded in the research itself - it was from an analysis of the initial interviews described in this chapter that I could discern in teachers in different contexts, different dilemmas that framed their views on language related issues in their teaching. Thus the claim in this thesis that teachers face dilemmas, and articulation of what these dilemmas are, are outcomes of the research process. Dilemma language thus arises and develops in the process of the study in response to the questions above.

1.2 Addressing the research questions

From an understanding of knowledge as situated and socially mediated, we know that knowledge about teaching is not simply acquired in courses about teaching,

particular, learning *to* talk mathematics and learning mathematics *from* talk are both crucial aspects of mathematics learning in school[1]. There is thus at least a three-dimensional dynamic at play between the language of instruction, the language *of mathematics* (educated discourse) and the language of classroom cultural processes (educational discourse). The focused question for this thesis has thus become:

What is teachers' knowledge of their practices in this complex multidimensional dynamic?

Less formally, what can we learn from teachers' tacit and conscious knowledge about how they manage to teach secondary school mathematics in complex, dynamic and demanding multilingual settings?

This central question can be elaborated by a number of more specific subsidiary questions that have been posed in the previous two chapters:

- What do teachers in multilingual mathematics classrooms say and do in relation to simultaneously developing the language of instruction, the language of mathematics (educated discourse) and the language of the classroom (educational discourse)?

- What do teachers say and do about teaching and learning in the language of instruction and using the other main language(s) of the pupils? (about learning *from* talk and *to* talk - *about language as a resource in the classroom)*. In other words, what do teachers say and do about code-switching?

- What do teachers say and do in relation to *mediating* (working with both) the informal, expressive language and mathematical conceptions that learners bring to and construct in class, and the formal mathematical

CHAPTER 3

EXPLORING TEACHERS' KNOWLEDGE: METHODOLOGY AND RESEARCH DESIGN

1 INTRODUCTION

1.1 Elaborating the research question

This study is framed by a sociocultural theory of mind. Knowledge is thus understood as at once personally and situationally meaningful and socially and culturally mediated. Knowledge is both tacit and conscious, embodied and discursive. In other words, what we know lies in what we say and what we do in specific situations. Together then, what we are able self consciously to articulate, as well as how we act, form and are formed by our developing identity and our activities and practices in a social, cultural and historically contingent world.

I have also explained the emergence of my interest in the seemingly elusive dynamics of teaching and learning mathematics in a multilingual secondary school classroom. In the course of a review of the literature in the field of language and mathematics I have argued that while there has been a great deal of research into bilinguals' mathematical learning, much of this has been located in a narrow cognitive framework, and more significantly, that teachers' knowledge of the dynamics between mathematics and language in multilingual classrooms has not been studied. Since teachers' knowledge, like all knowledge, is situated and thus shapes and is shaped by the context, culture and activity of classroom life, it is a crucial dimension of a socioculturally informed account of the dynamics of teaching and learning mathematics in multilingual school classrooms.

Together, the theoretical frame and literature review have also revealed the complexity of teaching and learning mathematics in multilingual settings. In

Obviously what counts as a dilemma is also a function of one's identity as teacher

Time is not raised here as an area of teaching dilemmas. I will refer to this at later points in the thesis

See, for example, Crawford and Adler (1996), Cochran-Smith and Lytle (1993) and Richardson (1994)

In the World Book dictionary, adjectival use of transparency refers to the transmission of light so that bodies beyond or behind can be distinctly seen, e.g. a window is transparent. This is the sense in which Lave and Wenger use the term. A window's invisibility is what makes it a window. It is an object through which the outside world becomes visible. That we can see through it, however, is what makes it highly visible. This use of 'transparency' is not to be confused with its figurative meaning as easily seen through or detected, e.g. a transparent lie

Mena's (1995) analysis of tool use resources in mathematics classrooms, distinguishes 'fields of invisibility' which enable smooth entry into a practice, and 'fields of visibility' which extend information by making the world visible

Of course, all teachers are knowledgeable about their own experience. It is the wider practice of teaching that is referred here

This is also reflected in a great deal of literature of language and learning. See, for example, Barnes (1976) and Britton (1970)

In language learning terms, the distinction that is missing in Lave and Wenger's work is that between acquisition and learning. Some language has to be learnt (and thus needs a focus on the structure of pedagogy - on its mediation), some language is acquired (in the structure of practice)

See Pimm (1987, p. 47) for discussion of how, typically, teachers do not conceive of mathematics as a discursive subject - it is not something that can be discussed, that learners could have opinions about - it is a matter, rather, of right and wrong answers to given problems.

Dowling, and Muller and Taylor provide a sociological argument as to the distinctiveness between school knowledge and everyday or 'relevant' knowledge. For Dowling, school mathematics has both discursive saturation (it is embedded in discursive relations), everyday knowledge, in contrast, has low discursive saturation. Everyday and school mathematical knowledge are thus incommensurate and attempts to teach mathematics through everyday meaning and relevance are likely to exclude learners from mathematical knowledge

This is similar to Walkerdine's (1988) analysis of children's mathematical knowledge in school and at home - two different sets of practices (what she refers to as discourses) wherein concepts take on different meanings.

As noted in note 8 above, time is a significant dimension of complex teaching practice (Hargreaves, 1994). In Vygotskian theory, what happens over time is important. However 'time' was not an object of focus in this study - but emerges in the concluding chapter for discussion of further work

See, for example, Bowles and Gintis (1976), Apple (1982) and Aronowitz and Giroux (1986) for arguments on cultural and economic reproduction in education. The distribution of knowledge and power in school is not a function of 'ability', but rather of social relations of power, both cultural and economic

Chapter 1 argued that the teaching and learning of mathematics in multilingual classrooms was three-dimensional: it entails access to the language of instruction, the language of mathematics (educated discourse) and classroom culture (educational discourse). Because of this dynamic, I argued further that only socioculturally informed studies actually illuminated the dynamic. I then asked what is teachers' knowledge of this dynamic, and whether and how this knowledge could and should be elaborated. The purpose of this chapter has been to develop a sociocultural theoretical framework for my study of teachers' knowledge of their practice in multilingual mathematics classrooms.

NOTES

1. Vygotsky's theory is described both as a sociocultural theory (for example, Wertsch, 1985; 1991) and as a sociohistorical theory (Ivic, 1989). As is elaborated later, it is clearly both. It is, however, cumbersome to refer continually to both. So, like Wertsch, I will from now on refer to sociocultural theory, implying also its historical dimension.

2. Vygotsky's use of 'cultural' is consistent with Geertz's notion of culture as 'plans, recipes, rules, instructions for the governing of behaviour'. 'Culture' is understood here as systems of making meaning, or what Luria (1976, p. 164) terms 'social practice'. See Miller (1984).

3. 'Activity' works at the level of the person acting; 'practices' work at the level of social groups.

4. The distinction between, and importance of both what we say and what we do, relates to what Argyris and Schön (1974) describe as 'espoused theory' (theory to which we give allegiance) and 'theory-in-use' (theory which governs actions) (pp. 6-10). Argyris and Schön are particularly interested in whether and how what we say (about what we do or wish to do) might differ from what we actually do. This distinction between espoused and enacted theories can be read as a disjuncture - for example, in repressive terms, we do not like to look at what we do not want to see - and therefore study the disjuncture. In contrast, the relation between espoused and enacted theories could be viewed as a complete relation - there is no disjuncture. Much research on teachers' beliefs and actions works on this assumption and its implication that changing actions thus means changing beliefs. Polanyi's sense, and the way in which this study views the relation, is that there is some relation between our espoused theories and our theories in-use - there is overlap, but we cannot reduce the one to the other. Our knowledge is both embodied and discursive - and these are intertwined. We cannot always say what it is we do. And vice versa, we cannot always do what we say. Some of what we know is implicit, tacit. This study does not dichotomise these two windows on teachers' knowledge about teaching. What teachers say and what they do (or how they act) are both rather seen as crucial, if sometimes conflicting or contradictory, parts of teachers' knowledge.

5. See Cochran-Smith and Lytle (1993) for an interesting account of inside and outside (emic and etic) knowledge of teaching where "outside" and "inside" are not related simply as opposites but as voices that engage one another in dialogue".

6. However, managing teaching is consistent with accounts of teaching as practical wisdom (for example, Fenstermacher, 1994; Richardson, 1994; Pendlebury, 1990).

some of the specific dynamics of teaching and learning mathematics in multilingual classrooms. These ground and expand the language of dilemmas developed. Dilemma language captures the complexities of teachers' knowledge of their practice with its shared assumptions about persons, society and knowing and so becomes a means to describe and explain teachers' knowledge.

I have argued in this chapter that teaching is deeply textured and complex. This is specifically so in multilingual mathematics classrooms. Thus being able to describe and explain teachers' knowledge of their practice in such contexts (and so learn from teachers) requires a language of description that embraces this complexity. A language of dilemmas serves this purpose. Sources of dilemma language can be found in social practice theory, and extensions to dilemma language found in sociocultural theory.

Specifically, the sociocultural theoretical framework developed here is that becoming knowledgeable (both of mathematics in school and of teaching) entails learning to talk (in and as part of a community of practice) where learning to talk includes both talking within and talking about a practice. However, the shift from talking within to talking about mathematics in school is not a seamless web, but one that requires mediation and lies in the dialectical process of teaching and learning. In a multilingual classroom, it is the teacher's role to enable learners to move back and forth between talking within and about mathematics, between educational and educated discourses in the classroom, and between everyday and scientific concepts. But there are tensions in this that can be analysed as dilemmas that teachers face.

Furthermore, becoming knowledgeable is bound up with access to resources in the practice through their transparency their dual characteristics of visibility and invisibility. Language is a resource in the classroom. Teachers in multilingual mathematics classrooms thus need to work with the languages learners bring to the class and the language of instruction. Herein lies another tension.

Thus it is not only learning *to* talk that must be problematised but also learning *from* talk. The use of tools in classrooms and particularly the language resources made available for learning must come under scrutiny. In multilingual classrooms this becomes a particularly interesting question: how language is and is not made use of and why. How do teachers manage the tensions in use of formal mathematical language and informal language on the one hand, and in the language of instruction and the main language of the pupils on the other? Moreover, how are these tensions thought and talked about by teachers?

Thus, specific to this study are tensions around the mediation of school mathematical knowledge; around spontaneous mathematical ideas and those of the mathematical school community; and around language-in-use in the classroom (including *formal* and *informal* mathematical language use) and language as resource for or display of mathematical meanings and learning. Chapter 1 described how current curriculum initiatives in mathematics are attempting to shift to more investigation and communication of mathematical ideas, and to seeing mathematics as a discursive practice. They thus bring with them a new set of challenges for teaching and learning mathematics. Crucially, we must problematise the situation where learners' main language is not the language of instruction and particularly the situation in multilingual classrooms where there are more than two main languages present, thus complicating opportunities and possibilities for code-switching.

It is in relation to these tensions in teaching that dilemma language becomes a useful analytic tool.

7 CONCLUSION

In this chapter I have drawn on sociocultural and social practice theory, first to theorise teachers' knowledge as situated in communities of practice - thus expecting commonalities and differences across teachers and, second, to highlight

Accepting schooling as a specific context, but not the privileged realm of scientific thought, and acknowledging that schooling has functioned to exclude and alienate in socially distributed ways, what then is the role of schooling at this historical juncture? This question takes on added significance in the light of arguments that schooling needs radical change if it is to prepare people for the global changes we face in this knowledge and information era (Young, 1994). In South Africa, where the majority have been denied access to schooling first through apartheid policies and more recently through the breakdown of schooling in the struggle against apartheid, school continues to have status and a perceived important role. Vygotsky is correct in working to understand the specificity of schooling and the kind of learning and development it offers. He is incorrect, however, in making this a privileged and unproblematic site for higher order thinking. It is *a* site for higher order thinking, but certainly not in any decontextualised, ahistorical and unproblematic way. The quality and nature of activity, and the social relations in which it is embedded in school, crucially shape the quality and nature of the knowledge appropriated in school.

Sociocultural developmental theory thus provides significant insight into issues that confront the complexity of teaching and learning mathematics in multilingual school classrooms. Firstly, the school mathematics curriculum needs to be mediated. Many questions then arise around how this is effectively done, and particularly in relation to the concept of the ZPD. Most ZPD studies have been done on pairs of learners - or on learner and teacher or parent and child. How does the ZPD function in the context of large and diverse groups of learners? How does a teacher manage to work with and create appropriate zones in large diverse classes? Secondly, mediation is also between everyday and scientific concepts - between previously acquired mathematics and new mathematics. How do teachers manage the tension between the conceptions learners bring and those the teacher wishes them to acquire (the personal vs. public knowledge) in the context of classroom communication and activity, for it is her responsibility that these are acquired? Further, how is this tension thought of and talked about by teachers?

she s how the situation definitions differed both within the group and between the group and the teacher. Importantly, and in addition to specific motives and goals, their orientation to a geoboard which had them focus on the pegs rather than the spaces or distances between pegs, resulted in interesting but problematic attempts by the group to generate effective scientific concepts in relation to area. This was compounded by the interactions in the group itself and their limited interaction with the teacher. In her attempt to listen to and understand the pupils' orientation, and the limited time she had for this given her management of the whole class, the teacher did not manage to mediate their spontaneous approach to the task in a way that would have facilitated their understanding of area.

The zone of proximal development thus brings the issues of challenge and yet supporting student conceptions and orientations. In more general terms, the ZPD brings issues of effective mediation and/or scaffolding.

Vygotsky did not problematise schooling sufficiently (Bernstein, 1993; Levine, 1993; Daniels, 1993; Ivic, 1989). One of the functions of schooling is to provide access to bodies of knowledge and ways of knowing. Hence school mathematics is about providing pupils with access to the body of knowledge of mathematics, and of developing in learners ways of thinking mathematically. Ways of thinking mathematically in school are a distinctly different practice from learning maths in everyday life. The issue in Vygotskian theory is the privileging of schooling for the development of scientific and higher order thinking. Here the development of complex thinking resides only in the context of schooling. As discussed earlier, this has serious implications for the unschooled. While it is correct to recognise and identify the specificity of schooling, it is incorrect to locate the development of scientific thinking in the formal school only. What about, for instance, the apprenticeship context? And similarly, in Vygotsky we get no exploration of what is spontaneously learned in school. Schooling itself is a social context, where there is both intentional and unintentional learning.

> as determined through problem-solving under adult guidance or in collaboration with more capable peers (1978, p. 86).

Tied to the ZPD is Vygotsky's understanding of the relation between learning and development: there is no learning that is not in advance of development (1978). Schooling (that is, formal instruction) 'brings forth the zone of proximal development' (p. 89).

Vygotsky did little to elaborate the concept of the ZPD himself. This has, however, been done by Wertsch. In Wertsch's (984; 1991b) discussion of activity, he distinguishes three components to functioning in the ZPD: situation definition; intersubjectivity and semiotic mediation. As learners begin a task constructed by their teacher, they adopt an orientation to the task that requires, in the first instance, what can be called the situation definition of the task - how the task is situated and defined by the learners and teacher. What motives, goals, needs and values are read into the task by the learners and the teacher? How is the task understood in relation to its specific classroom context? When the situation definition of a task is shared, then intersubjectivity (between the teacher and learners) in relation to the task is easily established. It is when situation definition is not shared (either within a group of learners or between a learner and the teacher) that mediation is required for intersubjectivity to be established. The issue in adult-child interactions is the changing of the child's situation definition and the kind of mediation that is required to establish intersubjectivity in relation to the task at hand. Issues of power and control and whose knowledge enter here, but pertinent to this study is that if situation definition is shared then intersubjectivity is easier t. blish and mediation easier too. If situation definition is not shared, then establishing intersubjectivity is a more complex process.

Brodie (1995a), in an interesting and detailed study of a group of Standard Seven pupils in a South African school working on a task related to area, effectively

learning (see, for example, Gerdes, 1985, 1988; Carraher et al, 1985; Bishop, 1988; Taylor, 1991; Dowling, 1993).

The second implication is that the inter-relatedness (two-way process) of spontaneous and scientific concepts in school challenges commonsense notions that we always learn best when we move from the familiar to the unfamiliar. This commonsense pervades mathematics education, and assumes that the starting point of any activity and mathematics learning must be either concrete or known. This opens up, again, pedagogical issues of how and when new knowledge is made available in school. What kinds of scaffolding and challenges are appropriate when?

What is thus important for the focus of my study is that, in school, language is a key mediational means (Wertsch, 1985; 1991a). In line with sociocultural theory and social practice theory, if particular kinds of knowledgeability can be accessed in school, or particular kinds of intellectual functioning can develop through school, the question then is: how might the kinds of mathematical knowledgeability that schooling teaches be facilitated *and/or* obstructed by different pedagogies and their use of language? More specifically, how do teachers think and talk about this?

6.3 Pedagogic mediation and the ZPD

In order to deal with the interaction of scientific and spontaneous concepts, and to further elaborate his cultural law of development, Vygotsky posited the notion of the Zone of Proximal Development (ZPD).

The ZPD is:

> the distance between the actual development level as determined by independent problem-solving and the level of potential development

The interview schedule was piloted on Helen, Teacher 1, and found to be effective. The intention was that if the interview questions were inadequate, a follow-up interview with Teacher 1 could have been arranged. This proved to be unnecessary. The remaining initial interviews were conducted at the teachers' schools or in my office (at the teacher's choice) s... after the interview with Helen, th... is, during April and May 1992 as follows:

TEACHER 1	MC	HELEN	9 APRIL 1992
TEACHER 2	MC	SARA	15 APRIL 1992
TEACHER 3	ex-DET	JABU	8 MAY 1992
TEACHER 4	ex-DET	THANDI	12 MAY 1992
TEACHER 5	PR	CLIVE	30 APRIL 1992
TEACHER 6	PR	SUE	6 MAY 1992

No two interviews proceeded in the same way because each teacher raised different issues and challenges and these were followed up in the context of the interview. The schedule served as the crucial reminder of areas that needed to be covered if they were not spontaneously brought up by the teacher. The approach in the interview was informal and conversational in the sense that the researcher followed on from points raised in the attempt to probe, have teachers exemplify points they made, point out things they had or had not mentioned, and commented on issues to provoke further discussion. The underlying objective, however, was to elicit as much discussion from the teacher on the issues as possible within an hour and a half.

5.3 Observations and impressions of the diverse schools

Diverse schooling scenarios that I have described elsewhere (Adler, 1991a) were exemplified in these schools. The interviews in the two ex DET schools were conducted in small, sparsely furnished offices for use by Heads of Departments in the school, reflecting limited resources in each school. During each interview,

(see Appendix A) was designed to elicit discussion from teachers on the context of their work (Questions 7 and 8, 17 and 20), their backgrounds and views about mathematics and teaching it in school (Questions 1-6 and 9-12, 19), as well as specific language related questions (Questions 13-18).

The three open ended key questions were:

1. Describe the context and ethos of your school.
2. What for you are the general tasks and challenges in teaching junior secondary mathematics in your school?
3. More specifically, what language issues do you face in your mathematics teaching at this level?'

Some of the specific prompts that accompanied the second question related to strategies they used to address challenges in class. In addition teachers were asked to describe a typical lesson and to share what for them were their rewards in teaching.

The purpose and function of the first two key questions was to be able to relate knowledge of the dynamics of teaching and learning in multilingual classrooms to conceptions of mathematics and teaching and learning in general - thus reflecting a view of curriculum as relational and in the interaction between task (subject matter), learner, teacher and context (see 6.1 below).

Thus the initial interview can be understood as deliberative:

> ... the central issue of method is to bring the research questions and data collection into a consistent relationship, albeit an evolving one . . Framing research questions explicitly and seeking relevant data deliberately, enable and empower intuition rather than stifle it.
> (Erickson, 1986, p. 140)

5.2 The initial interview

The initial interviews in this study were interactive and semi structured (Hitchcock and Hughes, 1989, p. 83; Cohen and Manion, 1989, p. 309). This means that the interview assumes the appearance of a natural interesting conversation, but is nevertheless controlled. The interviewer 'guides and bends to the service of his research interest' (Hitchcock and Hughes, 1989, p. 79).

What is meant by semi structured is shown in the Interview Schedule in Appendix A. There were key questions to be explored with each of the six teachers, as well as a series of prompts or sub-question within each key question. In other words, the initial key questions posed were open-ended, with possibilities for further prompting if this became necessary. This semi structured form was to ensure that all teachers were asked the same set of key questions - that with each there was a conversation on key aspects of their work in multilingual mathematics classrooms. At the same time, it was possible to follow the flow of what each teacher offered and so enable their differences to emerge and shape their interview.

These initial interviews thus fit Hitchcock and Hughes' description of an interview as 'talk to some purpose' (1989, p. 79). Interviews are situations and thus the accounts obtained are situated accounts. The interviewer is always implicated in the interview, no matter how unstructured it is. Anyway, unstructured does not imply a lack of structure or planning (Hitchcock and Hughes, 1989, p. 87; Burgess, 1982, pp. 107-9). What is implied in an understanding of the interview as a 'conversation with a purpose' is that the interview is a speech event and thus must be seen in sociolinguistic terms - as co-constructive, and in a context.

Because context so obviously shapes what is possible in South African schools, and because views and beliefs of knowledge are situated, the interview schedule

hermeneutic (Smaling, 1992). It is thus crucial that the research process is fully described and itself becomes an object of study.

5 THE RESEARCH PROCESS

One of the tasks in reporting field research is to describe the purposes of the research and how it developed over time: to 'include details of field-work, data collection and methods for keeping field notes, but also an account of the process of data analysis' (Rose, 1982, p. 125; Erickson, 1986). This and the next section fulfil this purpose.

5.1 Setting up access

As discussed, there was no straight-forward access to schools for research purposes in the early 1990s. In March 1992 I approached seven teachers, two or three from each of the three different kinds of classroom contexts described earlier. All wished to participate in the study and were happy to be interviewed and videoed.

I then communicated with each school principal and obtained permission to (a) interview teachers, and if teachers were willing, to (b) observe and video them teaching one class later in the year. Early on in this process, difficulties in one of the schools forced one of the teachers to withdraw. Fortunately, this did not upset the spread across the different schools.

While I designed the study to minimise time spent observing in schools, given the continuing turbulent political and educational context, I did not naively expect an empirical project with teachers that included classroom-based research activity to simply proceed smoothly to plan, however committed the teachers. Surprisingly, other than one teacher withdrawing from the workshops, all other data collection activities proceeded to plan.

1. an initial (tape recorded) semi structured, in depth, interactive interview;
2. a report-back to teachers interviewed to discuss and partially validate (Silverman, 1993, p. 159) my initial analysis and interpretation of the interviews;
3. up to three hours of observation (videotaped) of at least two consecutive lessons in one or two of each teacher's classes;
4. a reflective interview with each teacher on the video of their classroom/s, and
5. a series of workshops (three in all) on issues and aspects of the data that the teachers themselves wanted to discuss with each other and to pursue.

Data were thus gathered in different contexts and through different activities, related to both what teachers said and what they did. These different forms of data we. to enable rich description, diverse perspectives and voices, and the crucial aspects of teachers' knowledge.

With the exception of (2) above, all interviews and workshops were tape-recorded. Field notes were written immediately after the meeting that was not recorded. Videotext of classroom practice was supported by observational field notes.

These data collection processes were mutually developmental. The initial analysis and interpretation of the first set of interviews framed aspects of the video reflections. The video reflections raised issues of mutual and independent interest for teachers to share with each other. The follow-up workshops became opportunities to further explore issues of interest to the teachers.

Interwoven with these data collection methods was what is often regarded as the most important research tool in qualitative research, the researcher herself (Smaling in Meulenberg-Buskens, 1993). It is expected that both researcher and researched will change in and through their interaction over time; this is the double

(d) pragmatically - teachers I knew, and who, given the politics of the day, were able and willing to negotiate and/or enable access to their schools and classrooms.

In short, all six teachers were fully qualified and experienced secondary mathematics teachers with a personal and professional interest in the study, as well as a willingness to participate in the process and facilitate access to their schools and classrooms. Thus, in addition to a theoretical and purposive sample, this sample of six teachers was also an opportunity sample (Cohen and Manion, 1989, p. 103; Rose, 1982, p. 121).

As Silverman (1993) states, field research studies are usually based on one or more cases which are unlikely to have been randomly selected and more likely chosen because they allow access. This raises questions about the size and representivity of the sample. As regards representivity, the issue is rather one of the generalisability of cases to theoretical propositions than to populations or universes. That is, the issue is tied to theory-building which can then be transferred to other cases and contexts (p. 160).

4 THE RESEARCH METHODS

As argued earlier, in order to illuminate teachers' knowledge, two sources of data were needed: it was necessary to talk to teachers, to have them talk about their practice; and data of actual classroom practice was also needed, as well as how teachers justified and explained what they did.

Hence, interviews with teachers had to be supplemented with observations of their classroom practice, and their reflections on these. The methods used to collect data were:

were poorly resourced, and in the previous decade, a learning culture had all but broken down[6]. The black state school teachers in this study, Jabu and Thandi, were both African and Tswana speaking. As mentioned earlier, many, but not all their pupils were Tswana speaking. Jabu was Teacher 3, and Thandi Teacher 4.

Two private (PR) schools which had predominantly black pupils who did not have English as their main language and brought a range of main languages to class, as well as a range of proficiencies in English.

Pupils here included African, Indian and coloured children and youth, and thus some (though few) might have English or Afrikaans as their main language. These schools were well-resourced and most teachers were English-speaking. The two private school teachers in this study, Clive and Sue, were both white and English-speaking. Clive was Teacher 5, and Sue was Teacher 6. Clive's school was predominantly African.

3.2 More about the teachers

I further selected these six teachers according to the following criteria:

(a) **academically** - each teacher had tertiary mathematics qualifications, and thus there could be no question as to the soundness of their mathematical knowledge;

(b) **professionally** - each teacher had at least 3 years of teaching experience, some recent exposure to language issues either in in-service courses or further study, some participative involvement in mathematics education beyond the school, and some were trying new language-rich pedagogical approaches in their classes;

(c) **motivationally** - teachers who were interested in the study and willing to participate; and

where access to the school was possible. In addition, the design needed to provide access to what teachers said and to what they did in their teaching. The design would need to draw on ethnographic methods of participant observation in schools, and on case study - where particular cases are examined in context and in multiple ways.

3 THE SAMPLE

3.1 The teachers and their schools[b]

As with all field research and qualitative methods, the sample in this study was small, purposive and theoretical (Rose, 1982, p. 121; Cohen and Manion, 1989, p. 103). Six secondary mathematics teachers, from the three different multilingual contexts described above were selected, that is, two teachers from each of:

(a) Two recently desegregated historically white state schools (hereinafter Model C (MC) schools).

In both selected MC schools, English was the dominant language in and around the school; teaching staff remains white and English-speaking; there were increasing numbers of pupils with other main languages - hence classes in these schools were multilingual. Both schools were adequately-resourced. The two participating teachers, one from each school, were both white and English-speaking. For reference purposes, Helen was Teacher 1 in this study and Sara, Teacher 2.

(b) Two township-based black state Department of Education and Training (ex-DET) schools.

Here, neither teachers nor pupils had English as their main language. In addition, they did not all share the same main language. These schools

mathematics, their accounts and actions are likely to contain commonalities. Such similarities and differences could be very illuminating.

Moves in the country to a more unified and less racially fragmented education system arguably made it more appropriate to set up a study so that teachers from each of the three language contexts described above could contribute, as opposed to focusing on one of the three contexts. While working across three contexts would inevitably mean less depth in each, the complexity of teaching mathematics in multilingual settings could be more widely explored. Hence the decision to design the study to work across the three language settings described above.

There were additional factors militating against either a long-term ethnographic study or a detailed single case study. Black state schools, despite moves towards a new order, were still politically very turbulent during the period of the research. A long term presence by a white researcher in such schools was not only unlikely but also potentially unsafe. Townships near Johannesburg were volatile and unpredictable and not always safe to travel in, particularly for a white woman. It was thus ill-advised to set up a study that relied on stability in schools and the attendance of the researcher over time. Even the limited travelling that was required brought unexpected dangers. These are elaborated below.

More generally, the tightly controlled apartheid system of education was viewed by many as illegitimate. Access to any state or state-aided school by researchers at that time was, for various reasons, often blocked. In the transitional and turbulent moment of this study, I felt restricted to working with teachers I already knew - teachers whom I could approach where there were sufficient levels of trust in both directions and teachers who would be able to gain access for me into their schools.

Hence a design and a sample were required that did not depend on extended time in any one school, that captured at least three different language contexts, and

language of all pupils. Moreover, pupils bring to the class a range of home languages.

In contrast, the numerous state schools that were historically African schools remain African in student body and teaching staff. Most such schools are still under-resourced as new non-racial government departments of education struggle with limited budgets to effect wide-ranging changes. Large classes prevail, many teachers are under-qualified, with less than three years post-matric teacher education, and material resources are few and far between. These schools have also been significantly affected by political activity and instability over the past two decades. In urban black state schools, many teachers speak English fluently, but English is not their main language. Despite apartheid's attempt to keep ethnic groups separate, teachers, particularly at the secondary level, will share their main language with some, but not necessarily all of their students. For example, in the two black schools that are part of this study, the teachers involved and a number of their pupils are Tswana-speaking. However, some pupils had Zulu, Xhosa or Sotho as their main language. In this sense then, these classes are racially homogeneous but multilingual.

Given this linguistic diversity, a choice had to be made as to how best to study teachers' knowledge of their practices in multilingual classrooms. A full ethnography or case study, for example, would mean selecting one of the linguistic contexts as a focus. This would provide rich and deep insight into the teaching and learning in one particular setting, but could not access the different kinds of knowledge teachers might have as a result of teaching in varied contexts. With an understanding of knowledge as situated, teachers from each of three different contexts could be expected to have similarities and differences in their knowledge. Teachers in and from different contexts are likely to have different accounts of their practices, and are also likely to act in different ways. Simultaneously, however, since they share the practice of teaching secondary

areas. It is really only in the cities and historically white towns that racially mixed schools now exist and will exist in the future.

The deracialising of state schools began during the process of negotiation to a new order in South Africa (1991 - 1994). Within its racially fragmented educational departments, the apartheid government of the day created state-aided, or 'Model C' schools. Most historically white, coloured and Indian schools became, and remain, state-aided schools. In financing and governance terms, these schools are quite distinct from private schools on the one hand, and state schools on the other. While state schools are fully funded by government, state-aided schools are only 75% funded. State-aided schools were given greater autonomy and control over their properties, admission criteria and governance.

The creation of Model C schools resulted in at least three distinct multilingual secondary schooling contexts in South Africa where English is the language of instruction[4]: (1) schools where the teacher and some pupils are English-speaking, but a number of pupils in each class would not have English as their main language (Model C schools, and some private schools); (2) schools where the teachers are English-speaking but none of the pupils have English as their main language (some private schools); and (3) schools where neither the teachers nor the pupils are English-speaking (state African schools).

These diverse multilingual contexts are likely to remain on the South African educational canvas for some time, despite proposals for a two-tiered system comprising only public and independent schools (Bot, 1996). In addition, it is important to acknowledge that within the existing three tiers, there is great variation across schools. By and large, Model C and private schools are adequately resourced and well-functioning. Appropriate financing and governance structures and processes exist in the schools. Here, while the language of instruction (English) is usually the main language of the teacher, it is not the main

motivated and described below. Moreover, schools needed to be accessible to the researcher by car, and the research agreed to by the school and teachers.

2 THE FIELD OR THE WORKING UNIVERSE

In his elaboration of field research, Rose (1982) distinguishes between the 'general' universe which is 'the universe of phenomena to which the theory applies', and the 'working' universe, 'the set of all empirical units which the researcher defines as the basis of the study, and from which a sample is selected'. The general universe in this study is 'teachers' (p. 56). The working universe is secondary mathematics teachers in urban multilingual schools in Johannesburg and neighbouring black townships[2] where English is the language of instruction.

Schools in the Johannesburg area vary enormously, reflecting the race and class legacy of apartheid with its overt racial and more covert economic segregation. Until 1994 there were only two official languages in South Africa, English and Afrikaans. Language policy in African schools was mother tongue instruction for the first five years of primary school. From Std 3 or grade 5, the language of instruction was officially either English or Afrikaans[3]. In 1994, language policy changed. It is now more flexible and accommodating of the eleven official languages. English remains the primary language of government and the language of commerce and is thus likely to continue to be the language of instruction in secondary and tertiary education.

In addition to changes in language policy is the changing racial composition of pupils in some schools. Historically racially restricted white, coloured and Indian only schools are now increasingly multiracial with increasing numbers of African pupils. Private schools have long been racially desegregated, some since the late 1970's. However, given the demography of the country, many African schools will remain with a total African student body, particularly in township and rural

code once again against the new category of 'dilemmas'. Thus the claims in this thesis that teachers face dilemmas, and the articulation of what these dilemmas are, are the outcome of systematic examination of evidence.

As Marton (1988) argues, categories of description are research findings. They are the primary outcome, the most important result. In this study, categories of description are the means to interpreting and presenting the research findings related to the initial interviews and teachers' articulated knowledge.

In this interview data analysis, what can be seen is that:

> In field work, induction and deduction are in constant dialogue ... It is true that specific categories of observation are not determined in advance of entering the field setting as a participant observer. It is also true that the researcher identifies conceptual issues of research interest before entering the field setting.
>
> (Erickson, 1986, p. 121)

The analysis is neither one of unspoilt or unframed grounded theory. Nor is it an attempt to fit data to pre-existing categories. It is rather a dialectical process that involves both induction and deduction, theoretically informed theory generation[8].

Chapter 5 describes, interprets and interrogates the language-related commonalities, divergences, presences and silences in the collective account of the teachers' initial interviews. From my perspective as researcher, these reveal how different contexts and conditions give rise to different language-related dilemmas for teachers as they go about their work[9].

6.2 Analysis of actions and reflections on actions - the analytic narrative vignettes

Analysing and reporting qualitative field research is a complex process. Much has been written in relation to the theoretical presuppositions and research skills

or 'interaction', or the need for 'explicit language teaching'. Language was talked about in relation to working with 'English as additional language' (or more commonly English as second language - ESL), or dealing with the specifics of 'mathematical discourse', or (and overlapping with pedagogy) as 'communication'. In this second level of analysis, categories were grounded in the data itself.

With the above two 'levelled category framework it became possible to generate a map (as above) of each interviewee's response categories. With these maps, similarities and differences between what the teachers emphasised became apparent. In other words, it became possible to attend to presences and silences within and across the six interviews. Interviewee maps are represented in Appendix C. This process of mapping and comparing the interviews shifts over into a form of quantitative analysis - where a form of counting facilitates analysis. In fact in this study this 'counting' was a key to identifying dilemmas, and thus supports Silverman's argument that such quantitative techniques are often important tools in overall qualitative research (1993, p. 162-5). It goes without saying that this quantitative analysis was greatly facilitated by computer coding of the data interview data. Category summaries for each teacher were generated at the touch of a button.

As intimated, the notion of dilemmas emerged in this comparative and quantitative exercise. This is then a third level of categorisation and abstraction. I noticed that teachers in situations of change had more dichotomous accounts, more dilemma type expressions. Teachers who had tried to change their pedagogical approach had their taken-for-granted assumptions about good classroom communication shaken; and teachers whose classes had recently and rapidly deracialised had become particularly sensitive to and aware of their own and their pupils' use of language in their multilingual classrooms. Moreover, within a social theory of mind, it makes sense that particular kinds of practices lead to and shape particular kinds of consciousness - these sets of teaching experiences would thus be more explicit in some teachers than in others. I then returned to all six interviews to

It was possible to assign every utterance in all six interviews either to a view of mathematical knowledge, teaching or teacher, learning or learner, pedagogy (the relationship between teaching and learning), or to language. (Or of course, to more than one of these). In addition, within each broad category, utterances could be further coded to a second level of category analysis. Different ways that mathematical knowledge was talked about could be described, for example, as 'the basics', or as 'reasoning', or 'content' or 'process'. Different ways that pedagogy was talked about could be described as emphasising 'meaning making',

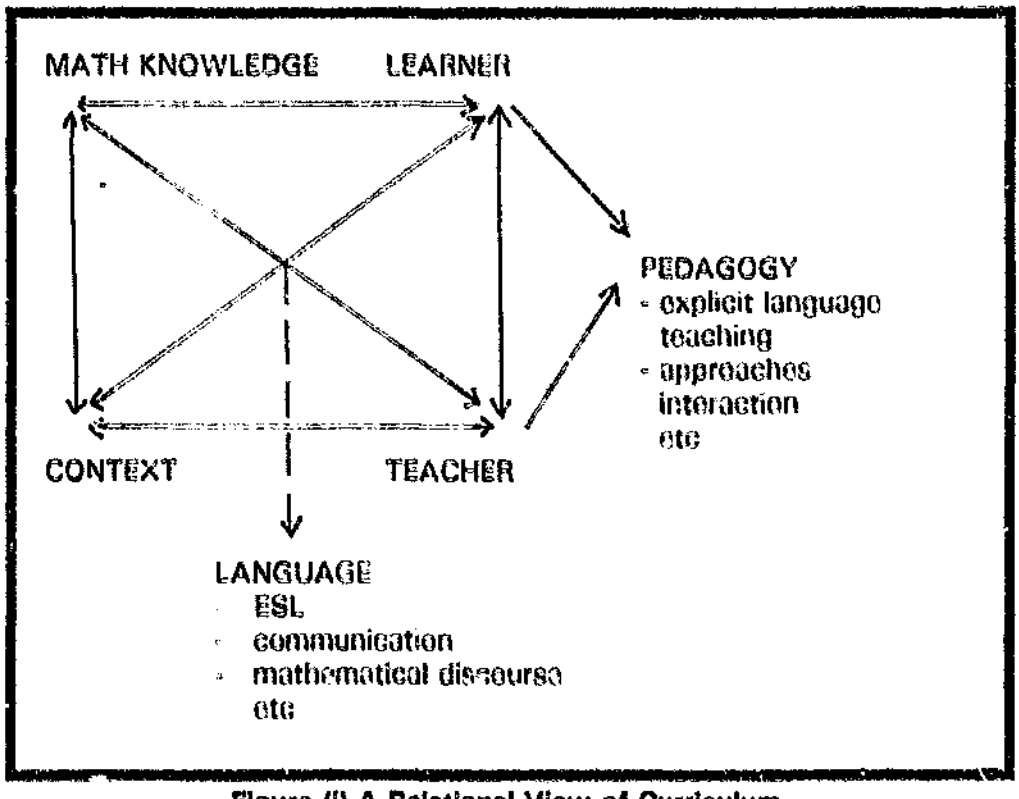


Figure (I) A Relational View of Curriculum

Here I have turned to the ethnographic use of *analytic narrative vignettes* (Erickson, 1986). These vignettes serve the dual function of providing a way of working with the mass of data generated as well as elaborating the dilemma language developed through the initial interviews.

Both strands of analysis were greatly facilitated by the transcripts of tape recorded interviews and well as the video text. As technological resources that could be reviewed over and over again, they enabled travel back and forth in time and space and thus a capacity for completeness, reducing dependence on observation and field notes. Of course, machine recording also brings limitations: events can be viewed over and over again, but contextual information can be lost.

6.1 Analysis of the initial interviews

Each interview generated a great deal of both structured and unstructured data. As is required of qualitative data, systematic analysis of the interview data occurred at increasing levels of abstraction (Woods, 1985; Erickson, 1986; Marton, 1988; Hutchinson, 1988). Notwithstanding the initial analysis that had accompanied the data collection process, my starting point, or first level of more systematic and rigorous analysis and organisation of the initial interview data (see figure (i) below) was according to:

(i) five inter-related broad curriculum categories, *mathematical knowledge, teacher, learner, context, pedagogy*, adapted from a relational conception of curriculum (Christiansen and Walther, 1986); and

(ii) Vygotsky's notion of *language* as mediator.

Careful attention or 'listening to the data', although inevitably shaped by my experience with the field of language and mathematics education, led to the construction of sub-categories illustrated in Figure (i) below.

At the end of 1993, a number of things were apparent to all the teachers and myself: (1) the data collection process had been a participative one and generative for the teachers. Three of the six had started further study in the university and had developed their workshop discussions into action research projects; (2) the workshops and meetings had been a 'safe' place (outside of particular school and staffroom politics) to discuss issues of interest and concern that were often otherwise difficult; (3) the process developed its own direction and momentum that was teacher-led, and while within the project and with a focus on language, the specific interest in multilingual classrooms often slipped out of view. The overlay of learning in an additional language had an elusiveness as teachers confronted the picture of their teaching. Many of the issues discussed, while on language in general, were not necessarily specific to the context being multilingual.

Thus, a first level of analysis was part and parcel of the data collection process. This first level of analysis is a somewhat superficial content analysis and is reported in Chapter 4. However, what was now required was a much more detailed and in-depth cumulative study of all the data gathered, and an attempt to grapple with the research questions about what teachers said and did about their practices in multilingual classrooms. The process of this more extensive data analysis is described below.

6 DATA ANALYSIS AND INTERPRETATION

There are two key strands to the data analysis in this study. First is the analysis of the initial interviews where attention is focused on the development of *categories of description* of what the teachers said about their practices in multilingual classrooms. It was during this process of developing categories out of the data, and then systematically categorising all the interviews, that the notion of dilemmas emerged. The second is the more complicated task of making sense of the video texts, reflective interviews and follow up workshops, that is, making sense of what teachers did and how they justified and reflected on their actions.

5.8 Three reflective workshops

Three reflective workshops, one in each of the first three school terms of 1993, took place on Saturday afternoons (20 February, 15 May and 7 August). These are referred in Chapters 4-7 as W1, W2, W3.

At the February workshop each teacher presented an excerpt from his or her video that they wished to discuss with the other five teachers. At the May and August workshops, teachers presented and discussed related work they had followed up in their classrooms. Each workshop was tape-recorded and transcribed.

As can happen in any long term research process, one of the teachers, Jabu, did not attend any of the workshops. He held two teaching jobs, one that involved him all of Saturday mornings and while he said that he would be able to attend in the afternoon, he did not arrive. I met with him after the February workshop to share our discussions, to ascertain whether there was any problem and to encourage his important contribution to our deliberations. Despite his assurances that he could and would come, and that he was interested and wished to continue participating and even with reminders from me on the day itself, he never arrived. This of course alters the balance of teachers in the workshops, but as these are only one part of the data, and are used primarily to support other data, Jabu's absence from the workshop did not detract in any significant way from the study as a whole.

In the whole data collection process related to teachers' justifications and reflections, what was interesting is that the teachers' choices from their videos for workshop discussions were quite different from aspects of their videos that I had found interesting (in relation to the study). Given work pressures, however, I had time only to record each of the workshops, transcribe and think about them in preparation for the next meeting and attempt to raise questions and keep alive the focus of the study - working in multilingual classrooms.

The video interview with Helen (T1), for example, is referred to as VI1, and each of the six interviews took place as follows:

VI1 - Mon 16 November
VI2 - Thurs 26 November
VI3 - Thurs 19 November
VI4 - Mon 23 November
VI5 - Wed 18 November
VI6 - Tues 17 November

The follow-up interviews with teachers were more complex than I had anticipated. I advised that we would let the tape run until there was something the teacher wanted to talk about, and that I would also stop the tape at times and ask questions. I was interested to hear teachers' spontaneous interests and selections and so initially held back on some of my questions. The main reason for this was that I soon became aware that there was not necessarily good overlap between what I and they focused on for reflective discussion. In addition, there were three hours of video for each teacher and not more than two hours of interview time. I did, however, make sure that incidents that I interpreted as having particular relevance to the study (that is, language issues) were discussed.

After the reflective interviews, the teachers themselves were interested to share and discuss what they had learnt from their videos with the other teachers. During 1993, three workshops were held (February, May and August 1993) where each of the teachers discussed aspects and excerpts from their videos with the other teachers.

classroom walls usually contained APLA posters, I was told, but pupils had removed these for the videoing.

Videoing on the second day in the second black school saw a repeat of what happens later in the day in many schools, and what I observed during the initial interview: while the pupils in the class being videoed were present after the second break (they enjoyed being filmed, I was told), much of the rest of the school was dysfunctional. Classrooms on either side of us had no teachers. Moreover, many pupils had left for home, or wherever else they might go. The teacher told me that there had been discussion with the staff as to whether I should observe and video in the school at that time of day. They agreed that 'the truth will out', anyway, so why try to hide the reality of their context.

The day-to-day functioning of the remaining four schools was 'normal' (pupils and teachers arrived and left on time). However, those with relatively large numbers of black pupils nevertheless faced pupils whose lives outside of school were often traumatised by increasing levels of violence.

5.7 Reflective video interviews - stimulated recall: teachers' justifications and reflections

Reflective video interviews were held with each teacher during November 1992. Each teacher had been given a copy of the video to review and think about before the reflective interview. The reflective interviews were tape-recorded and each was around two hours long. They were conducted either at the teacher's school or in my office, according to each teacher's wish. In the two hours, the teachers were first asked if there were any general comments they wished to make or issues they wished to discuss before we looked at the video text. Thereafter, the videotape was allowed to run until the teacher, or I, wished to discuss an episode. It was then frozen and sometimes reviewed in order to facilitate discussion.

observe each lesson while it was filmed by a professional video photographer. There was to be no extraordinary planning for the lessons. However, we did try to co ordinate planning in such a way that (i) teachers would be teaching topics which they thought would reveal interesting language issues, and (ii) teachers teaching the same level were videoed teaching the same content.

Sue (T6) and Sara (T2) were videoed teaching Std 6 geometry (the triangle); Clive (T5) and Jabu (T3) were teaching Std 7 ratio; Helen (T1) was teaching Std 8 trigonometry and Thandi (T4) was teaching Std 9 linear programming.

Videos and observations of each teacher occurred over two or three consecutive teaching periods (some were double) during September and October 1992. The photographer was instructed to focus on the teacher and her or his interactions with the class and/or groups of pupils. The object was to capture the teachers' actions. While these are never separate from the whole class activity, and quite difficult to capture in a pupil-centred classroom, it remained important to have a consistent camera focus across the six teachers and their classes. (A list of abbreviated references to the video texts and observation notes can be found in Appendix D.)

Stark differences across schooling contexts were apparent. At both black schools, the harsh realities of teaching and working in such schools were exemplified. The second day of videoing in the one school saw only half the class, with the remainder at a funeral of a young male from another school killed in a political clash. On my way out of the township I in fact drove into the crowds returning from the funeral and was sharply reminded about the danger of township life. On the instruction of the photographer with me, I took the car up and over a pavement to get out of the crowd which we both knew could turn ugly at any point. This school was in a Pan-Africanist Congress (PAC)-dominated area and the teacher told me many of the pupils support its armed wing (APLA). In fact,

document and is rererred to as RBS (see Appendix B). On the other hand, this meeting served to include the teachers in the research process.

Because the meeting was initially conceived of as a validation and participation exercise, and not as contributing to the data for the study, it was not recorded. Respondent validation has, in fact, come under attack:

> ... there is no reason to assume that members have privileged status as commentators on their actions ... such processes of so-called 'validation' should be treated as yet another source of data and insight.
>
> (Fielding and Fielding, in Silverman, 1993, p. 159)

The meeting was an occasion for teacher participation in the research. My mistaken conception of the validating function of the report back became clear to me during the meeting. As teachers commented on and discussed my summary I quickly realised that while this meeting did serve to clarify some aspects of my interpretation of what teachers had articulated in their interviews, it was much less about validation of my interpretations, and more a context where teachers talked with each other about their work. It thus added to the data-base of the study as a whole. As a result, notes were taken during the meeting and elaborated as field notes directly after the meeting. (These are referred in Chapters 4-7 as RBFN.)

At this report back meeting, teachers expressed their commitment to continuing with the project and agreed to be observed and videoed for at least two consecutive lessons with one or more of their classes during September and October 1992 and to give time to follow-up interviews and workshops.

5.6 Classroom observation and videos in schools

Each of the six teachers was videoed for at least three hours. It was agreed that videoed lessons were to be part of the normal programme of teaching. I was to

in mathematics teacher associations and through this activity and my teaching, I have come to know a number of secondary mathematics teachers in the Johannesburg region. This professional location means I am familiar with the area of study (language and mathematics) and have a professional relationship with its working universe. I was thus able to approach teachers to participate in the study, teachers whom I knew had some knowledge and interest in the area. Some had participated in teacher in-service courses that I had run and others had done some post graduate study in my department and taken courses I offered.

This familiarity brought both strengths and limitations to the research. Strengths lay in the already established relationship of trust between the teachers and myself, as well as a shared and acknowledged interest in the project. This nevertheless constituted a specific sample and raised questions about generalising from the study. Familiarity at the same time imposes limitations: because teachers know me and hence know some of my professional interests, and, moreover, because in some instances I had been their 'lecturer', they could have wanted to please me in their interviews by expressing views they thought I approved. My participant observation could also have been experienced as evaluative. I attempted to minimise these effects by assuring them at the outset that they were in no way the objects of evaluation in this study, but rather sources of knowledge in a difficult area where it was important to confront the realities they faced in their complex classroom activity.

5.5 Report back on the initial interviews

The report back on the initial set of interviews took place in June 1992. This meeting had a dual purpose. On the one hand it was to clarify and validate my initial analysis and interpretations of the interviews. What I had managed for the June meeting was a first level content analysis of the transcripts of each interview. This was presented to teachers for discussion in the form of a summary

there were a number of interruptions as other teachers entered the room. At one school, the interview took place at the beginning of a break as the teacher was free in the periods following. Despite a bell signalling the end of break and a return to class, the teacher drew my attention to the continuing noise and the fact that the bell had by and large been ignored. At that school most pupils chose not to return after the second break or to come back in their own time. As I left, pupils were moving in and out the main gate; three class rooms I passed had no teachers in them. Here was first hand evidence of the breakdown of a learning culture in black schools that both teachers talked about in their interviews. Moreover, this school is opposite a migrant worker hostel that had recently been the scene of violent confrontation between African National Congress (ANC) and Inkatha Freedom Party (IFP) supporters. The teacher explained how that conflict inevitably spilled over into the school. Some Mondays, at the start of a new week and the end of a violent weekend of clashes, approaching the school gate involved passing dead bodies.

In contrast, the private and Model C schools were distanced physically, though not entirely emotionally, from the conflict in the townships. Some pupils in these schools lived in conflict ridden townships. Nevertheless, these four schools were calm, ordered, adequately or even well resourced and functioning as one would expect schools to function.

After the initial interviews, the research process depended on what had gone before, with a key instrument being myself as researcher and interpreter of data collected.

5.4 The researcher

I am a white, English-speaking South African and a qualified secondary mathematics teacher. Since 1986 I have been employed in mathematics teacher education, working with both primary and secondary teachers. I am also involved

phenomena are divorced from our descriptions. Erickson (1986) captures the dialectic interaction between phenomena and our descriptions.

9. Of course, the data collected for the study was much more than only language-related. But the focus of the thesis required that other data, however interesting, was important for contextualising the language-related data, and not in and of itself a focus of attention.

10. 'Field research' and 'qualitative research' are used in the literature on qualitative research. I too use them both, though initially I specified field research, which involved qualitative methods.

of interests and values. It, too, constructs a language of description where theoretical concepts are grounded in practice - a language that, at once, illuminates teachers' knowledge and offers possibilities for assisting teachers' knowing and thinking about their practice.

Nevertheless, Lather's cautionary note is pertinent:

> ... language is a delimitation, a strategic limitation of possible meanings. It frames; it brings into focus by that which goes unremarked.
> (Lather, 1991, p. xix)

NOTES

1. Obviously, these are not the only aspects of learning and teaching mathematics - they nevertheless can and should be the focus of specific study.

2. Racially segregated residential areas were created in the apartheid era. While areas of all cities have since deracialised, the demography of the country is such that black townships will remain black for the foreseeable future.

3. During the period of negotiation in the early 1990s, African primary schools were allowed greater option as to how and when to switch language of instruction.

4. There are secondary schools where the language of instruction is Afrikaans. They are not included in the working field of this study.

5. All names (teachers, pupils and schools) have been changed to retain confidentiality. However, in Chapters 6-8 (see, for example, p. 192), I draw on some of the teachers' writings of their action research in the project. Each preferred to be properly referred and gave permission for their actual surnames to be used.

6. See Adler (1991a) for a detailed description of different mathematics classroom contexts in South Africa and their implications for exam policy.

7. It was the original intention in this study to focus on the junior secondary level. The initial interviews did have this focus, but also extended into the senior secondary level. By the time the video material was being collected, not all teachers were still teaching at the junior secondary level.

8. The argument that categories of description are dialectically constructed fits between the argument, on the one hand, that in describing the world, a language of description is actually arbitrary (Dowling, 1992). Language is not determined by reality, but rather creates it. This position has language operating somehow independently from the material world, or determining of the reality of that world. Marton (1988), on the other hand, and from a phenomenographic perspective, argues that categories generated are by no means arbitrary but limited by the phenomena themselves, that is, linked to the content of the study. Here

where the research comes at the theory building process empty and blind. This is impossible. The only way we can see is with some orientation.

Nevertheless, it is important to attend carefully to the data and ground interpretation and analysis in the data, to remain focused on finding out from participants, from their point of view, what it is they know, but declaring the perspective which is brought to this listening.

The issue then becomes: how is teachers' knowledge described? What is gained and lost in using participant or theoretical concepts or both in developing a language of description? This study reinforces Rose's contention that a mixture is in fact what occurs. This is because the purpose of the study is to find out what teachers know - to let the data speak - and at the same time develop and provide a useful and meaningful language of description.

For Rose, theory-building, that is, developing concepts and relationships between concepts, should be useful, practical and understandable by laypeople (1982, p. 127). This point is re-iterated by Mercer (1991), in his analysis of the research processes that culminated in the well-known book *Common Knowledge* that he co-authored with Derek Edwards. In discussing the importance of the dissemination of research, Mercer argues that the language that you use in the research report is important. The development of language and specialist terms (like 'ground rules') are helpful. They provide a means for talking about practice for both practitioners and laypeople.

In Rose's terms, the Berlaks' dilemma language - their language of description - is a typology. It is their language of description, abstracted from their participant observation of teachers and schools. The dilemmas they construct are theoretical concepts, but grounded in practice. My research, while grounded in the voices of teacher participants, in the end is presented in my voice - a voice that values and respects teachers' knowledge, but is still a voice of a researcher with her own sets

relation to reliability, generalisability is better viewed as in the relationship between cases and theoretical assertions than from samples to populations. Generalisability in this study thus lies in the language of description that is developed, and whether and how it is generative (Bernstein, 1993) of further research on the one hand, and facilitates talk about the practice of teaching through forming an explanatory model.

As I will argue in the concluding chapter, the dilemma language developed in this thesis, and its elaboration in the vignettes, point to further research, to hypotheses testing. The test of its generative quality is whether this language can be and is used in research and teacher education to develop the field.

And the generative potential of the language of description raises a final but important methodological question.

9 IN WHOSE VOICE?

Rose makes an interesting distinction between participant concepts grounded in the data analysis and expressed in the words of the participants and theoretical concepts grounded in the data but abstracted out through the theoretical language of the researcher (Rose, 1982, p. 119). In practice, Rose contends, we usually find a mixture of theoretical and participant concepts in any field research study.

It is tempting to assign authenticity to participant concepts, to view the words of the teachers themselves as more accurate and meaningful representations of practice. However, in a theory of practice, words are always an abstraction out of the practice, no matter who expresses them. Furthermore, the assumption that participants speak a truth unfettered by their language and context does not hold sway in a social theory of mind. Both Rose and Silverman critique notions of pure theory-building, pure grounded theory which assumes that there is a starting point

8.2 Validity

Validity means the extent to which data and subsequent findings present accurate pictures of the events they claim to be describing (Hitchcock and Hughes, 1989, p. 45; Silverman, 1993, pp. 149 159; Maxwell, 1992). In field research using qualitative techniques, validity lies in the relationship between interpretation and evidence; interpretation must be based on systematic, comprehensive and rigorous use of evidence.

For Maxwell, validity in qualitative research lies in what he calls 'critical realism': an account involves description, inference and explanation, i.e. both accuracy and appropriateness. Qualitative research depends on different kinds of validity. In the first instance, it depends on descriptive validity, and then also on interpretive validity and both of these involve accuracy. There needs be a clear link between interpretations and descriptions and the evidence that is supplied. This is achieved through careful transcriptions, recognisable categories about which there can be consensus and which remains close to experience - to the data itself (1992). Secondly, validity in qualitative research also depends on theoretical and explanatory validity. Theoretical concepts and relations between them are at a more abstracted level and thus must be systematically linked and clearly argued within the realms of the study and its theoretical frame. These kinds of validity correlate with Erickson's list of data analysis and reporting techniques discussed earlier. Descriptive, interpretive and theoretical validity are attended to in the reporting of the research in Chapters 4 - 8, where systematic data analysis is demonstrated in the use of evidence (from interviews and video text) in constructing categories of description and vignettes.

8.3 Generalisability

Generalising from qualitative research[iii] with a purposive and opportunity sample is always problematic (Rose, 1982, p. 123). As I have already discussed in

8.1 Reliability

Reliability refers to replaceability - how does one ensure that another researcher would construct the same categories of description and vignettes out of the data? In relation to categories of description, and from his phenomenographic perspective, Marton (1986) asks instead: Would a category that has been described already be recognised by others? He argues that this second question is a reasonable question. The first is not! This is because categories are a discovery. They need not be replicable, but they need to be recognisable. Inter rater reliability is thus not a matter of generating the same categories but rather that others can systematically and with intersubjective agreement apply the researcher's categories similarly to the data at hand. Inter-rater reliability, as well as using standard methods to write field notes and prepare transcripts, is a well established form for seeking reliability in interview and textual studies (Silverman, 1993, p. 165), and it is crucial if there are others involved in coding the data.

While I was the only researcher, it was nevertheless also important here to improve reliability of the categories generated by checking whether these could be applied effectively by others. Improved reliability on the interview categories was thus sought through a meeting where three of the participating teachers and two additional teachers were asked to code part of one of the interviews using my categories. While they offered other categories, thus re-enforcing Marton's distinction, they nevertheless concurred with the assignments I had made of utterances to my set of categories.

In addition to 'others' recognising the categories constructed, reliability of coding was also enhanced by the successive levels of coding in this study. Here, the same researcher reviews the data again and again at different times. This revisiting enabled a refining of both the coding and the categories constructed.

Walford's claim occurs in the foreword to his edited collection of accounts of the research processes in numerous well known studies. I do not, however, see a dividing line between Walford's claim against simple notions of objectivity, and Smaling's concern that qualitative studies nevertheless do justice to their objects.

This point is reiterated by Erickson:

> ... For any reader, the report must (a) be intelligible in the relations drawn between concrete detail and the more abstract level of assertions and arguments made by linked assertions; (b) display a range of evidence that warrants assertions the author makes; and (c) make explicit the author's own interpretive stance and the grounds of that stance in substantive theory and personal commitments. Presenting all this enables the reader to act as co analyst with the author
>
> (1986, p. 153)

A range of reporting techniques assist in producing an intelligible and rigorous report. Empirical assertions need to be generated and tested. Key linkages need to be established to convince readers of the validity of assertions made. Analytic narrative vignettes, quotes from field notes and interviews, synoptic data reports (frequencies, charts) combine to form what is called evidentiary warranty, that is, adequate evidence. Interpretive commentary that frames both particular and general description, as well as theoretical discussion, lead to theory generation. Finally, report on the natural history of inquiry in the study enables readers to locate the researcher, her assumptions and processes in relation to the report. These reporting techniques are employed in the data analysis in Chapters 4 - 8.

The importance of reliability and validity in field research with its qualitative methods is recognised by other qualitative researchers (Silverman, 1993; Maxwell, 1992), and requires further, more specific elaboration in relation to the categories of description and analytic narrative vignettes in this study.

mathematics education no clear methodologies for studying the complexity of classroom life and teachers within it.

Erickson identifies dangers that lie in field research. Care must be taken to prevent the collection of an inadequate amount of evidence and an inadequate variety of kinds of evidence. These potential pitfalls have been discussed in the sections dealing with sampling and methods. There is further potential for faulty interpretive status of evidence through inadequate engagement with disconfirming evidence and inadequate discrepant case analysis. It is often through discrepant case analysis that the researcher is able to refine and adjust major assertions and their theoretical presuppositions. What is intimated here, is that attention needs to be paid to what is otherwise termed reliability and validity.

8 RELIABILITY, VALIDITY AND GENERALISABILITY

Smaling (1990) has defined what he calls 'Munchhausen Objectivity' in an attempt to bring a concept of objectivity - reliability and validity - into qualitative research, that this can and should be striven for even if never actually attainable. What it means for the researcher is 'doing justice to the object of the study'.

The counter argument, of course, is that there is no such thing as objectivity - all research, even in a positivist paradigm, is infused with values, positions, choices and power relations.

> In practice ... it is now widely recognised that the careful, objective, step-by-step model of the research process is actually a fraud and that, within natural science as well as social science, the standard way in which research methods are taught and real research is often written up for publication perpetuates what is in fact a myth of objectivity (Medawar, 1963). The reality is very different... (and) often centres around compromises, short-cuts, hunches, and serendipitous occurrences.
>
> (Walford, 1991, p. 1)

across six cases. It is a particular kind of field research that cannot be neatly labelled by current types of field research.

For Rose (1982, p. 108), field research involves some or all of the following elements:

> The subject of the study is a social organisation or a more loosely knit social group; there is direct social interaction between the individual researcher (or a small research team) and the organisation or group; the field-work 'evolves' over a period of time; a variety of techniques may be used for data collection ...; the data do not lend themselves easily to quantification, and are collected, analysed and presented in the research report as qualitative data ...

He further distinguishes field research in an idealised typology: field research is comprised of qualitative (not quantitative) data; its theory-evidence links are in theory-building rather than testing; its concepts are constructed from the data (discovered) and not known in advance; sampling is theoretically driven rather than representative; and data analysis and presentation is in author's summaries and illustrations rather than tables (p. 117). Field research involves analytic description and theory-building that are grounded in the data.

The design I have described is undoubtedly field research, grounded in the object of the study, dialectically related to its theoretical frame and has evolved with the study. It is consistent with other field research and qualitative studies where it is assumed that the researcher, the researched and the research are not sealed off from each other - the process is dialogical (Smaling, 1990; 1992). It is also consistent with studies attempting to ask new, different and difficult questions (Hollway, 1989). Hopefully, the detailed description of the research process will offer methodological insight for mathematics educationists studying the diversity and complexity of classroom life. It adds to the growing field of qualitative research on/in mathematics teaching (Lerman,1993). As Hoyles claimed through her review of researching mathematics teaching (1992): there are as yet in

transcripts was not carried out. While such an analysis might add insights into teachers' tacit knowledge, selected excerpts from classroom transcripts function illustratively within each vignette in relation to a dilemma and to the teacher's reflections.

7 A PARTICULAR KIND OF FIELD RESEARCH

I have described the sample, methods and research process in this study. The description reflects that, methodologically, the study is field research, using qualitative methods (Rose, 1982; Erickson, 1986; Silverman, 1993). As field research, it is in an interpretive paradigm, working in-depth and concerned with the meanings in, and complexity of, teaching in context with a critical stance towards human meaning (Erickson, 1986, p. 122).

As Berlak and Berlak (1982) argue, undertaking an interpretive study means taking as a starting point 'a view of persons as beings with the capacity to create culture and transform the conditions of their own living, and a reflexive conception of human consciousness and social context' (p. 14). That is, human consciousness and actions shape and are shaped by culture and history.

They appropriately criticise Lortie's famous study of teachers in that Lortie castigates teachers for their aversion to theory. Lortie's study presents teachers as having no technical vocabulary for talking about their teaching - no systematic language. Their language instead reflects conceptual simplicity. As the Berlaks point out, Lortie only studies what teachers say about teaching - not what teachers actually do nor how they construct and justify their activities (Berlak and Berlak, 1982, p. 235). Hence the Berlaks argue for their ethnographic study which investigated teachers accounts, their actions and their justifications. These same assumptions underpin my study. However, the design I have described is neither a full ethnography, nor a single case study, but employs ethnographic methods

One of the key difficulties in working with analytic narrative vignettes is being able to justify their selection out of the whole data corpus. Do any and all instances count? Would it be enough that I, the researcher, find an incident interesting and assert its significance for the study? Erickson's answer is a clear no! As an ethnographer, he argues that the significance of an instance must be established. It cannot simply be asserted. Vignettes are 'vivid' and 'persuasive' portrayals of an aspect of social life with functions that are rhetorical, analytic and evidentiary. 'Evidentiary warranty' is achieved by 'citing and reporting analogous instances and by showing in summary fashion the overall distribution of instances in the data corpus' (p. 151). In other words, significance is not internal to the incident, but rather in its instantiation and/or illumination of assertions in relation to the study as a whole. In short the particular must be located in the general.

Erickson distinguishes typical and rare events, both of which are significant. And in an ethnography, typical and rare incidents and linkages between them can be established from the data corpus. This study is not a full ethnography. Its classroom data is limited to three hours of teaching by each teacher. It is thus inappropriate to argue the typicality of one classroom incident in Erickson's terms. However, I agree with Erickson that the significance of a selected incident needs to be established in relation to the study. The significance of the three vignettes will be argued and elaborated in relation to the study in Chapters 6-8.

Briefly, the three vignettes serve as illustrations of the three-dimensional dynamic at work in different contexts and with different teachers. The vignettes are not generalisations, nor typical, nor rare, but rather instances that illustrate the dynamics of working in multilingual classrooms. They illustrate a nest of problems that are crucial to the research. They function, furthermore, to elaborate the dilemma language.

It is important to add here that, in weaving together data from the videos, reflective interviews and workshops, detailed discourse analysis of classroom

involved (Erickson, 1985; Rose, 1982; Hitchcock and Hughes, 1989; Burgess, 1985), to issues of validity (Maxwell, 1992) and to the intellectual and emotional demands of the creative processing required (Woods, 1985).

Erickson recommends a 'leap to narration' (p. 151) as a way to stimulate analysis of the mass of data that collects in field work research, particularly ethnographic studies. After an initial reading of the whole corpus of data sources, the researcher should make an assertion, choose an excerpt from the field notes that instantiates the assertion, and write up a narrative vignette reporting the key event chosen. The purpose behind Erickson's 'jump into storytelling' is similar to Woods' view of the necessity of the 'writing-up' process as 'an important inducement to the production of ideas'.

The purpose of a vignette, however, is not simply one of stimulant. For Erickson, the 'analytic narrative vignette' is the foundation of an effective report of fieldwork research (1986, p. 149). It is the means to identify and substantiate assertions, concepts and ideas that emerge through the study. As Brown (1981) claims:

> One incident with one child, seen in all its richness, frequently has more to convey to us than a thousand replications of an experiment conducted with hundreds of children. (p. 11)

And so too for one incident in one classroom. The analytic narrative vignette emerged as the most effective means of reporting on what teachers did, and how they justified their actions, and selected from, and talked about, their work. The vignette was also a means to elaborating the key dilemmas that emerged through the interviews, a means of elaborating dilemma language and exploring how teachers confront and work with dilemmas in their complex, dynamic three-dimensional multilingual contexts. Chapters 6, 7 and 8 develop three key dilemmas that also take particular forms in the three distinct multilingual contexts in this study.

content. For example, Jabu found he had to revise how to simplify the fraction 12/18.

As mentioned in Chapter 3, Jabu did not attend any of the workshops, although he continually said he would be there, and then never turned up. Perhaps this was a way of not facing his loss of dignity and belief in teaching. But this is mere speculation. Jabu's story will hopefully be able to be told one day soon, when we can look at the past and accept the range of experiences that people faced during the struggle for liberation from apartheid.

4.2.1 Code-switching

While a number of teachers notice and comment on main language use during pupil pupil interaction in their classes (for example, T1, T6), Thandi notices *for the first time in her teaching* that when pupils work together, they talk mostly in their main language, which for most pupils is Tswana. Despite her surprise, and accepting that 'maybe it is important so they can understand', she would like them to use English:

> *I only noticed it (that pupils spoke in Tswana) for the first time when I watched the video. Not even in the lesson - because when you get to them then they speak English ... I think they need to understand, so their own language is useful, but I think it is important for them to practise English ...*
>
> (VI4, 59-69)

Referring to her classroom and her practice, she articulates three different justifications for her position: that pupils need to practise English so more of them will be comfortable using English publicly in class; that not all pupils are Tswana speaking; and, as a language, Tswana has constraints in terms of a mathematical register:

> *... I would like them to speak English to each other so that maybe when you speak to them - like you will notice when I call them to the*

As discussed in Chapter 3, there was no direct overlap between what teachers focused on in the reflective interviews and what I had marked to raise for discussion. Nevertheless, across the six teachers, attention had been paid to the issues raised in the initial report back meeting. Code-switching, verbalisation by pupils and pupil-pupil discussion, mathematical discourse and explicit language teaching all emerged quite clearly in the reflective interviews.

During the reflective interviews, there were opportunities for discussing both the teachers' and my interests and thus for eliciting teachers' reflections and justifications for their teaching. The teachers' diverse multilingual classroom contexts and their different pedagogical approaches bring out different emphases in their interviews.

Before describing these interviews, it is important to discuss Jabu's case briefly. Most of Jabu's reflective interview, at his request, was unrecorded. He told the story of his teaching in Soweto since the mid-1980s, how he felt he had been forced into a position that effectively meant a loss of dignity as a teacher, and that he was a 'broken' and a 'damaged' teacher. It is not ethically possible to retell his specific stories. However, in general terms, he, and many other teachers like him, found themselves caught up in political turmoil that brought with it corruption in some schools, particularly around the final matriculation examinations. Events that occurred in the name of the 'struggle for liberation from an illegitimate system' in his school profoundly disturbed Jabu's identity as a teacher. For Jabu, when you teach mathematics you also teach 'sincerity and honesty'. He found himself in a position where he could not support practices that were being condoned in the politics of liberation.

When we did get to focus on his video, we discussed what was most apparent: how pupils struggled to express themselves publicly in English, and how their poor background knowledge continually slowed the pace of learning the actual lesson

4.2 The reflective interviews

In the reflective interviews that followed, teachers asserted that the lessons observed were by no means atypical. Some teachers were nervous to begin with, and in some classes pupils were quieter than usual.

Seeing themselves on video was both interesting and difficult for the teachers. Some expressed how they had learnt a lot by looking at themselves, that it was good to see that some of their intentions were being actualised. It was also good to recognise areas where they felt they could improve. However, whether at the start of the interview, or at points through it, each teacher made negative personal comments that ranged from not liking their voices ('too leading') or the appearance of their hair to irritating habits (fiddling with a pen, saying 'ok') and talking too much. Four of the six teachers also commented that their lessons were less 'exciting' than they had remembered or that they themselves were not so 'dynamic'.

Sue's and Sara's Std 6 classes were working on the triangle. Sue had pupils in pairs discussing, drawing and writing explanations of whether you can have a triangle with two acute angles, with two obtuse angles, with a reflex angle, and so on. Sara had pupils drawing and then tearing the angles of different kinds of triangles to discover the angle sum and exterior angle theorems. Clive and Jabu introduced their Std 7 classes to ratio, working from examples of the concept to exercises on simplifying ratios, or sharing in a given ratio. Jabu's first lesson was on division with exponents. Helen introduced her Std 8 class to the six trigonometric ratios through group activity where all constructed different size right-angled triangles with a 40 degree angle, and then measured lengths of sides and compared ratios between them. Thandi was working in Std 9. She revised inequality graphs in the first lesson and introduced the language 'at most' and 'at least' in the second, both as precursors to linear algebra.

4 INSIDE THE CLASSROOMS, REFLECTIVE INTERVIEWS AND WORKSHOPS

4.1 Inside the classrooms

As discussed in Chapter 3, p. 96, differences across schooling contexts were stark. Pertinent to the language focus of this study were two language-related observations (VFN2A,3A,5A,6A, see Appendix D). First was the considerable difference in what can be called the 'educational ground rules' (Edwards and Mercer, 1987) across classes and schools. 'Chanting' (for example, unison cries of 'yes sir' in response to 'do you understand?') was part of the culture of the township classrooms. The ground rules in most the other classes were such that individual pupils responded to recall or 'why?' questions initiated by the teacher. In contrast, the ground rules in Sue's class were clearly more pupil-centred and interactive and are described in detail in Chapter 7.

Secondly, there were clear differences in pupils' spoken English across schools and across levels within schools. African pupils in township schools struggled far more to express themselves in English than African pupils of a similar age and level in Model C or private schools in town. The latter were both more fluent and had acquired a different pronunciation and greater confidence in (spoken) public use of English. Also, older African pupils (Std 9s) in township schools were more fluent in English than those in Std 7. While these differences are obviously all a function of time, surround and the culture of the schools, both the dominant ground rules and levels of English proficiency are dimensions of the dynamic of teaching and learning mathematics in multilingual classrooms and, as is elaborated in Chapters 6 - 8, they impact on classroom practices and mathematical learning possibilities.

(5) specific aspects of mathematical discourse were problematic, for example, word problems, giving clear instructions, some mathematical terms and ways of meaning.

As will be seen, these s[illegible]ised issues correspond with the 'communication', 'mathematical discourse' and 'ESL' categories developed in Chapter 5.

Each element of the summary (Appendix B) was discussed with and by the teachers. We focused on the discussion of language issues to build a shared sense of what would be attended to and explored in the videoing and subsequent reflective interviews. These were:

* w[illegible]k class interaction, how it works, or breaks down;
* questioning by teachers and pupils, listening;
* small group interaction, quality of discussion;
* when translation is used, when pupils have difficulty saying what they think, moving between two languages;
* making language explicit;
* specific mathematics language issues, including both terminology and meanings.

These foci relate to the study's research questions on what teachers say and do in relation to the dynamics of teaching in multilingual classrooms: code-switching, communicating and mediating mathematical concepts, that is, learning to talk mathematics within the cultural processes of the school.

pupils in their despairing conditions (T3). Some celebrated improved discussions and interactions (T6), while others strove for a relaxed, enjoyable atmosphere with sparks of light, good reasoning and understanding (T2, T4, T5).

The range in teachers' conceptions of problems, challenges, and rewards linked with their school contexts and ethos as well as their beliefs about mathematics and learning. Those who stressed foundations were confronted with learners who had been passed through a system in turmoil. They faced situations where learners had significant gaps in expected pre-knowledge. Those who stressed inquiry and interaction were in schools where this was part of the school ethos. Those who stressed more utilitarian approaches faced the reality that many of their students would not continue with mathematics at senior secondary level.

These conceptions of mathematics, challenges and rewards clearly influenced what teachers perceived to be language issues in their teaching. Notwithstanding this diversity of conceptions, it was important to focus on and summarise the language issues the teachers discussed in their interviews. The interviews showed that:

(1) despite differences, all six teachers expressed the view that teacher-pupil and pupil-pupil communication was a good thing and should be developed;
(2) all found that speaking in English was difficult for ESL pupils in the first years of secondary school. Most felt that translation, and talking in a familiar language, where possible, was necessary at times;
(3) where teachers and pupils did not share a main language there were instances of communication breakdown;
(4) in order to develop mathematical English and general spoken English at the same time, some felt they needed to be more explicit about the language they used and to get pupils to verbalise more, and that this practice in fact benefits everybody; and

background knowledge and 'ability'. Some (T3, T4 and T5) expressed the lack of 'time' to deal adequately with the poor foundations of many pupils, to build these foundations and at the same time cover new material. Most teachers expressed the important role of motivation and attitudes to learning. In ex-DET schools poor motivation, interrelated with gaps in background knowledge, produced serious hindrances to teaching and learning in these schools. In contrast, teachers who were attempting to work more investigatively (T1 and T6) were concerned with diversity in relation to managing group work and promoting effective mathematical discussion and related assessment issues.

The teachers described a range of *pedagogical strategies* they use to deal with these challenges, for example, drilling and revising, having pupils write solutions to exercises on the blackboard and explain what they were doing, organising pupils to work at tasks in pairs or groups. Despite differences in approaches, all teachers talked, though in different ways, about wanting to increase verbalisation and pupil-pupil and/or teacher-pupil communication as a means of dealing with the diverse difficulties and challenges.

Interestingly, there were similarities in most teachers' descriptions of a typical lesson, particularly if they were dealing with a new section. There would be the teacher explaining, but encouraging interaction with pupils, and then pupils actively working on tasks on their own or in groups. Differences lay in how whole-class interaction is structured and p received - whether it precedes or follows pupil tasks - and in the kinds of tasks set. For some, tasks were exploratory and conceptual (T1, T2,T6); for others, tasks might be done in pairs but were essentially exercises on the section discussed (T3, T4, T5).

For each teacher, the *rewards* of teaching related to their conception of mathematics and learning progress, and to their desire for mathematics to be enjoyed and understood. For some, satisfaction came when pupils were able to speak mathematically; for others, rewards lay in the progress of at least some

This analysis drew out the following:

The *context* of teachers' work differed enormously. In ex-DET black schools, teachers coped with poor material conditions as well as the more demoralising breakdowns of relations of authority and management in the school and the absence of a teaching and learning culture. Political differences in teacher organisations also affected their working conditions. These conditions are reflected in how teachers described the *ethos* of their schools: as 'lacking discipline' (T4), 'there is disharmony' but 'still we try for high standards and a friendly environment' (T3). In contrast, teachers from the four other schools which all functioned normally emphasised developing 'dignity' (T1), 'academic excellence' (T2), 'learner-centredness' and so on, as their school ethos.

The *tasks and challenges* in teaching mathematics raised by teachers, particularly at the junior secondary level, illuminate the wide range of curriculum dilemmas the teachers face. For some the challenge was pupils' lack of 'foundations', and providing this while moving on with the curriculum (T3, T4 and T5); for others it was providing skills for school leavers at the end of Std 7 and, at the same time, preparing and selecting those who would continue with mathematics in Std 8 (T1, T5); for others it was enabling enjoyment of mathematical learning in the face of time pressures that 'make you rush' through material (T2); and for T6 it was enabling pupils to develop a questioning, thinking, investigative attitude towards mathematics as well as a grasp of mathematical conventions. Aside from the last which could fit the *knowledge is given vs. knowledge is problematical* dilemma, these dilemmas do not neatly correspond with Berlak and Berlak's (1982) curriculum dilemmas. Clearly, teachers face many dilemmas in their teaching. Those that relate to mathematics and language in school are the focus of following chapters.

Societal and affective issues also informed the challenges teachers believed they faced. All teachers spoke of difficulties working with diversity in pupils'

Before these views are described, it is important to point out that the diversity across schools and teachers that is apparent in Tables 1 and 2, and the range of teachers' views about mathematics and teaching, is necessary contextual information. At the same time, however, this introduces a great deal of 'noise' into the study - noise that is interesting, but, for the purposes of this study, must be backgrounded. For example, there appeared to be a correlation between the teachers' length of experience and interesting differences in the views of mathematics and teaching they emphasised in their interviews. Each teacher's identity, what he/she emphasised about him or herself, is also interesting, as is a range of gender issues. All are important aspects of teaching and learning mathematics in school that need to be examined but fall outside the focus of this study.

3 SUMMARISING THE INITIAL INTERVIEWS

The following abbreviations are used here, and in Chapter 5 - 8, to refer to extracts from the initial interviews, the reflective video interviews and the subsequent workshops:

II1, II2 II3, II4, II5, II6 refer to the six initial interviews respectively. Reference to an utterance in lines 23-25 from the initial interview with Helen, for example, will be (II1, 23-25).

VI1, VI2, VI3, VI4, VI5, VI6 refer to the video interviews; and W1, W2 and W3 refer to the workshops.

As mentioned in Chapter 3, a descriptive summary, or first level content analysis, of the interviews was presented to the teachers at a report-back meeting (see Appendix B).

TABLE 2: TEACHERS' SCHOOLS AND CLASS TAUGHT

TEACHER	SCHOOL TYPE	SCHOOL CONTEXT AND ETHOS	CLASS/ LEVEL	PUPIL NO.S	TEACHERS' MAIN LANG.	PUPILS MAIN LANG
1. HELEN	MODEL C 50% black girls only	*lower-middle class; good resources #good citizens #dignity	STD 8	30	English (French & some Zulu)	English, Portugese, Tswana, Sotho, Taiwanese
2. SARA	MODEL C 25% black co-ed	*middle class; good resources #academic excellence #all round education	STD 6 (two)	32 33	English	English* Portugese, Zulu Tswana, Sotho
3. JABU	EX-DET African co-ed	*lower class poor, poor resources *broken learning culture, disrupted #high standards, friendly	STD 7	42	Tswana (Eng. & other African languages)	Tswana* Pedi
4. THANDI	EX-DET African co-ed	*mixed class, many very poor *large 1700, poor resources #lack of discipline	STD 9	50	Tswana (Eng. & other African languages)	Tswana* Sotho, Xhosa Zulu
5. CLIVE	PRIVATE mainly African co-ed	*mixed class, adequate resources *racially mixed staff #equal, good education	STD 7	30	English	Zulu Tswana, Sotho English
6. SUE	PRIVATE 99% black co-ed	*mixed class, bursaries *racially mixed staff *good resources #liberal values #involvement, inquiry	STD 6 (two)	17 17	English	Zulu, Tswana Sotho, English Afrikaans

TABLE 1: TEACHERS' BIOGRAPHICAL PROFILES

TEACHER	QUALIFICATIONS		EXPERIENCE		INSET
	1992	1995	1992	1995	
1. HELEN	B.A - (MATHS 3) H.DIP.ED B.ED	M.ED (in process)	6 years in Stds 6-10	8 years now 1 year in adult education	yes regular
2. SARA	B.DEGREE- (MATHS 3) H.DIP.ED MATHS HONOURS (Teaching)		20 years in Stds 6-10	23 years	yes regular
3. JABU	SSTC B.SC (MATHS 2) 3rd year in process	B.SC MATHS 3	11 years in Stds 6-10	13 years now 1 year in NGEO	yes regular
4. THANDI	B.A.ED (MATHS 3) MATHS HONOURS (incomplete)	B.ED M.ED (in process)	4 years in Stds 6-10	6 years no 1 year in NGEO (primary level)	yes regular
5. CLIVE	B.A (MATHS 3) U.E.D	B.ED	10 years in Stds 6-10	12 years now 1 year in NGEO (college level)	yes occasionally
6. SUE	B.SC (MATHS 3) MATHS HONOURS H.DIP.ED	M.ED	3 years in Stds 6-10	4 years 1 year P/T student 1 year lecturer	yes regular

CHAPTER 4

THE DATA: A FIRST LEVEL OF ANALYSIS

1 INTRODUCTION

As discussed in Chapter 3, validity depends, in part, on description and interpretation, where both are 'close' to the data (Maxwell, 1992). The descriptions and interpretations in this chapter are all close to the data. My purpose is to provide a first level analysis of all the data collected: the initial interviews, videos, reflective interviews and workshops.

2 THE TEACHERS AND THEIR SCHOOLS

Table 1 below summarises biographical data on the six teachers in the sample. It reflects the purposive sample of the teachers: all have tertiary mathematics qualifications; all are professionally qualified; and all have at least three years of classroom experience as well as ongoing involvement in the profession.

Table 2 below summarises teachers' descriptions of their schools and the particular classes that were to be videoed. It shows that, according to the teachers, their pupils are drawn from diverse socio-economic backgrounds (see *s in Table 2). School ethos (see #s in the table) and conditions of teaching vary considerably across the six schools. Each class videoed is multilingual. In two of the six classes, neither the teacher nor the pupils have their main language as the language of instruction.

In addition to collecting biographical data and information on schools and classes, the initial interview also probed each teacher's view of mathematical knowledge and his or her own mathematics teaching. Such qualitative data is difficult to tabulate for comparison.

Mathematical discourse and how it is learnt is the issue here. That is, mathematics and language cannot be separated here - they are deeply intertwined. Clive's decision to be explicit is a function of his preparation and of what pupils ask him. The effect is that being explicit, which he argues is important, particularly in a multilingual setting, can also be problematic. The complexity of mathematical discourse and situational specificity come into play. This issue of being explicit is explored in depth in Chapter 8.

4.3.2 Pupil verbalisation

Sara argued:

> *Sa: I wanted them to see that no matter what triangle they drew, the sum of the angles is 180, whether they used a big or small, acute or obtuse. So a lot of what I was doing, as opposed to straight drill, was trying to reinforce words ... The main thing I want to point out is that I was trying to encourage them to use correct language and correct terms and also to encourage them to say it aloud. I really believe that if they have used the words, their recall is better. I like letting them discuss it amongst themselves so that they will remember it better.*
>
> (W1, 1050-1056, 1142-1151)

Helen raised the important issue of whether articulation equals 'possession'. On the first day in her class a pupil (Debbie) summarised the meaning of sin 40 degrees, that the ratio of the two sides will be the same even if the lengths of the sides in the triangle change. The next day Helen was trying to work with sin theta and the ratio opposite over hypotenuse:

> *H: Debbie, who did that very nice summary at the end of the last lesson has got absolutely no idea at this stage. For me it seemed that if she had done this great summary the day before, that she should have been able to do that - and she had no idea and back to the 90 degree angle... how could she*

The teachers agreed that ratio is a difficult concept in any class and then debated the benefits of being explicit about 'difference' and 'ratio'. Some suggested that relating ratios to fractions helps, others disagreed. Sue argued that making the difference between 'difference' and 'ratio' explicit can *create* problems - some pupils do get confused and then she deals with them individually. But she does not deal with the difference explicitly unless it is taken up by a pupil. In response Clive justifies his actions:

> *C:* *... There were two reasons why I did that is that it come up right on the first day from the first example when I said 'He has one and she has 5', or whatever, and I said 'how can we compare them?' and somebody said 'Oh well, S is 4 more than F', and then we looked at the two types of comparison because difference is a comparison but also because their language is so weak. It is very very weak and I just felt it was important to go over these words because (that was a) constant - they kept making that mistake. You say "alright, what is so and so's number compared to" or the ratio and they would give the answer as the difference ...*
>
> (W1, 157-173)

> *C:* *... Well I tell you something which I found teaching science, is that you have got be very very careful, not just in terms of language - that comes up often - but if you use a word very casually then it can be totally misunderstood.*
>
> (W1, 215-220)

He goes on to describe how pupils often misinterpret diagrams. For example, he had drawn arrows from crest to crest to show wavelength yet found that pupils *'had drawn it (the wavelength) just anywhere. And that happens often... and its the language ...'*

Sue questions this interpretation:

> *S:* *Ratio and rate - first language speakers, good speakers struggle as well, so it is not just a language thing.*
>
> (W1, 225-260)

mathematical activity which typically includes verbal explanations by pupils. The reflective interviews thus further illuminated teachers' knowledge of coping with the dynamic of learning in English, learning mathematical discourse and learning within classroom processes.

4.3 The first workshop

The workshops that followed the reflective interviews were useful contexts for teachers to talk further *about* their practices. Constituting a site for this small community of practice, the workshops were simultaneously a context for teachers talking *within* their practice. Moreover, it was here that they selected for discussion aspects of their teaching that they wished to pursue. And over and again, they talked about code-switching, pupil discussion and explicit mathematics language teaching.

In the first workshop in February 1993, each teacher selected ten minutes of excerpt(s) from their video to discuss with the other teachers.

4.3.1 Mathematical discourse - the importance of being explicit

Clive selected his ratio incident. He showed his lesson introduction and then the point where communication in the lesson broke down.

> *C: I think one thing was I lost them around the total number of apples and the meaning of the word 'share'. I was looking and thinking that the word 'share' is quite difficult. Also I was using the word 'difference' quite casually and the day before I had stressed the meaning of the word 'difference' as not pertaining to ratio ... What I was really trying to do was to actually get them to understand words like ratio, difference, share. That share is actually a comparison ... it is the number of apples compared to someone else's number of apples or to the whole number of apples ...*
>
> (W1, 13-31)

S: *Another thing is when I do understand them ... and know the rest of the class doesn't. What do I do? Do I make them re-explain it, go up to the board? Or do I just take it up myself and deal with it? ...*

I think you can validate, like 'I understand, I think you have a good idea but I don't think the others understand. Can you explain again?'

J: *What happens then if they get up and explain and it is NOT a good idea?*

S: *People who volunteer usually don't - I do try to choose them when I know they will have something to say ... and (then) you are also saying something to the kids who don't go up. I also choose kids who I know will make a good attempt of it - but then I am also saying something to the kids who don't get to go up. I mean there is one kid who I don't get up because he will just confuse everyone!*

(VI6, 1190-1196, 1202-1212)

Despite Thandi's lament that often pupils 'don't have the language to say what they think' (and this is clearly more prevalent when public speaking in class is not in the learners' main language), behind all reflections here is a belief in a strong relationship between thought and language. Clive and Helen reiterate:

C: *... often I can see that they know what to do and they do it - and they can't say it - they just say what they do, like there with Busi, she just says her steps, they don't explain what they do but I think they do need to be able to say it.*

(VI5, 55-60)

H: *It is still ... cause if they start to describe something to me in accurate mathematical language it does seem to reflect some kind of mastery.*

(VI1, 604-607)

The reflective interviews provided a context for teachers to justify their attempts in class to encourage mathematical explanation and discussion. The teachers grappled with whether learner explanations are necessary and with how learners use discussion. These concerns are a function of the nature of school

S: *With maths there is an added thing - is that discussion is hard and so different. Like if you have a discussion in English say around whether Chapel should be compulsory and everyone has got their own ideas and they accept it and learn from others. But to talk about whether a square is a rectangle - there is more a right and a wrong.*

(V16, 312-320)

Helen has similar concerns with pupil-pupil discussion:

H: *The whole issue of discussion is very complicated for me. And I am struggling with it all the time. It is not something that only comes up occasionally ... they are not talking to each other properly. They are not listening to each other. They only listen for the right word or the right connection that helps them get to the next phase of the problem ... Now look at this group (referring to video) they are not even deciding - I think what happens is that one starts and others look 'what are you doing?' and then copy. There is no deciding together.*

(V11, 595-598, 630 634, 714-717)

In contrast to Sara who 'feels' the benefits of verbalisation for learners, Sue questions the benefits of insisting on pupils expressing their thinking:

S: *And then my other big issue is whether insisting on language use is detrimental or not ... Like that kid who can see, understand, has got the concept, can see how the shapes work - and then can't say it. He at least can see that that is the problem, that he can't say it. You can deal with it. But there may be kids who feel that they can't say it and therefore they don't understand it ... or to what extent insisting on the language and they are not comfortable with it and then it inhibits their understanding - they withdraw. ... So that is my question: to what extent forcing them to explain it creates insecurities where in fact there is understanding and a few years later they will be able to explain it. That's what worries me...*

(V16, 165-178, 217-222)

4.2.4 Pupil-pupil discussion and explanation

The benefits of informal language use as a means to more formal and correct use and conceptualisation are often linked with the benefits of pupil pupil discussion on mathematical tasks. Of the six teachers, Sara and Helen made the greatest progress in developing participative discourses in their class with more group work, pupil-pupil discussion, and investigative tasks. In their reflective interviews they both forcefully expressed the questions they faced in their attempts to teach more meaningful and effective mathematics through different pedagogical approaches.

Sue opens her interview:

> *S:* *... the thing that worries me the most is that I am not sure whether, I am not sure to what extent it helps them learn. I think that talking to each other is not unproblematic. I think a lot of the kids don't listen. Maybe they are too young. I think. You can see it with the questions ... (and) ... I suppose not being able to see afterwards - they can't articulate the concept or how they struggle in tests later. I mean some of them are getting the concept, but some aren't. I think MY problem in teaching is that it is so unmeasurable - it is actually difficult to know.*
>
> (VI6, 7-14, 33-39)

> *S:* *With the Std 6's, because their language is much weaker, and they work in partners, I encourage them to first try it and then they talk about it - but I do question the way they talk to each other - to what extent they really challenge each other.*
>
> (VI6, 234-239)

> *S:* *Then there is also the case when discussion breaks down ... there is confusion about the task though that didn't come up - and then not understanding each other and being so polite about it. If you are not sure of your own ideas then it is hard to challenge and if they are not great at explaining, they don't understand each other. I encourage them to do it but they often don't ...*
>
> (VI6, 276-289)

when they try to do it on their own they can, they've got the words. But I don't know if it actually makes a difference ... but I don't know whether it necessarily benefits them.

(VI1, 517-529)

Clive is also ambivalent about the effects of formal mathematical language and being explicit about its use:

C: *... my way of dealing with the language is to go over it. I remember all the time I would say: 'what did you do?', 'difference', I would go over and over that. I often find I do that whatever class I am in - go over and over the words and the synonym and what it means. And that is what I noticed ... how I would keep going over certain things - like I would keep going back over 'what is the difference between difference and ratio?' - that sort of thing*

J: *Did you think that was a good thing?*

C: *Yes, ... I think I didn't get enough of the pupils involved and maybe I ... um ... could have tried something like group work to involve more students. Like I notice also that there are some who always get chosen and it is not intentional. It sort of happens.*

(VI5, 36-51)

So while teachers believe that the correct use of mathematical language is important, and that it is important that pupils gain access to mathematical English and use it themselves, how this occurs remains problematic. At one level, what pupils themselves say can be undermined; at another it is hard to focus on all pupils simultaneously; and most crucially, it is not absolutely clear whether in fact it really helps. Obviously, access to the register is important for all learners, so an interesting question is whether teachers' concern with using 'correct' language is exaggerated in a multilingual setting.

The issue of explicit language teaching was taken up in the workshops and is discussed below. It is further developed in the vignette in Chapter 8 which focuses on Helen's trigonometry class and her workshop questions.

So: I think it must, because when you look at it again you say angles on the opposite side - opposite side of what? you know ... but there again it is just my feeling that if they can say it properly it is an aid ...

(VI2, 275-285)

Helen and I discuss her frequent rephrasing of what pupils say.

J: Here you rephrase for them. Did you notice that?

First she says:

H: MMmm a lot! ... I think I am doing it 'cause I am not confident that they are actually saying the correct things, so that when they say something the child who is saying it knows what she means but the rest of the class is not necessarily going to get the correct meaning out of it. So I rephrase. But it is a problem for me that I do that ... I think it is undermining of the person who puts forward the idea to have what they say put into another language. But often I get the child to repeat what she has just said and the class still doesn't understand and then I rephrase and say NOW do you understand and of course they say yes, but ... you know ... you can't necessarily have the lie detectors out ...

And then

... like there she says 'the shadow of time' and I could maybe have assumed that she knew what she was talking about and just carried on .. but that is something that I picked up quite often. ... I didn't allow that kind of inaccuracy of language to go through.

J: And do you think that is a problem?

H: No I think it is necessary.

(VI1, 103-138)

And later

H: ... the one thing that strikes me here with this kind of thing is that I am saying specific words, very specific teaching in a way and I don't know that it actually makes a difference. I am hoping that the specific choice of language sinks in so that

and she goes on to argue that this is why it is so important to get pupils to speak in class:

> *Sa: So with that sort of thing you don't realise unless they are actually saying it. I mean otherwise you can be totally confused. There is an acute angle that is less than 90 and now suddenly it adds up to 180. So. um ... that is the sort of thing that you pick up when you are talking to them on a semi-casual basis like that.*
> (VI2, 147-158)

Clive reflects on how a word like 'simplify' has multiple meanings in maths and he tried to work with that, and also how he struggled to work with the ratio P:Q = 2:4 and P's 'share' was 1/3 not 1/2. (VI5, 50-54)

As Pimm states, awareness of different registers and of how the same terms and uses vary within and across them needs to be fostered or cultivated. The question is: how? (1987, p. 109). The teachers' experience and knowledge in this study suggests that explicit language teaching is important. They grapple with explicit language teaching in both their reflective interviews and the workshops.

4.2.3 Explicit language teaching through using 'correct' mathematical language

I noticed, as did some teachers, that in addition to explicitly teaching mathematical language, they were concerned that pupils expressed themselves correctly.

Sara believes that pupils must learn to say what they mean in clear mathematical statements:

> *Sa: Listen to what they are saying: they are equal to the angle on the opposite side ...*
>
> *J: Do you think they know what they mean? How much does it help if they say it right?*

H: *... it is just the process of them starting to understand. It's just like a feeling that you get ... because they were quite hyped ... it's the difference in the words 'opposite' and 'adjacent'. Like who lives opposite to you. And the difference between angles and sides 'cause you come across it in geometry. They talk of angles and sides as if they were the same thing. They'll be talking to you about a side and they will call it an angle.*

J: *Why?*

H: *I am not sure - that is why I go on about it here (referring to excerpt on video) - because sometimes I think they don't know that they are talking about a side and not an angle ... and in trig now we have a side in relation to an angle rather than an angle in relation to another angle.*

(VI1,224-245)

What is interesting here is the multiple meanings of both 'opposite' and 'adjacent'. Their previous use in geometry would have been in relation to opposite angles or opposite sides (for example, vertically opposite angles or opposite sides of a parallelogram) and to adjacent angles. Now opposite and adj must refer to a relationship between a side and an angle. Helen sees the importance of explicitly working on these multiple meanings.

Sara reflects on how having pupils say aloud what kinds of triangles they have drawn, and doing this repeatedly, assists the mathematical thinking of all pupils, but particularly those less fluent in English. It also helps the teacher see difficulties pupils might be having. For example, one pupil confuses 'acute angle' and 'acute-angled triangle', illustrating Pimm's illumination of joining words together to form different meanings in a new register:

Sa: *... like when I asked about an acute-angled triangle and he (a pupil) says 'no an acute angle is less than 90' that is what he was saying there. So it is to put the **acute angle** together with the **acute-angled triangle**.*

We know that mastery of the mathematics register, with its borrowed words from English and their changed meanings; its creation of new words; its use of metaphor; with the way words are put together to form specific meanings (Pimm, 1987), is both necessary and difficult for all learners. Register confusion is possible in any mathematics class and, as Pimm argues, 'unexplained extension of concepts can too often result in the destruction rather than the expansion of meaning' (p. 110). The question then is whether it is any more complex in a multilingual context, and if so, how?

In Pimm's terms there is a double layer of mathematical meaning required in reading and expressing '2-5': there is the symbolic meaning and reading of '-' here which includes the shift from natural to whole numbers, that larger numbers can be subtracted from smaller numbers; and there is the order of the symbols where here 5 must be taken from 2, yet the symbolic order is the reverse. The preposition 'from' is borrowed from English, but takes on a particular meaning in the mathematics here as part of the mathematical register. Sue's experience also suggests preposition use causes specific difficulties for ESL learners. She comments in her initial interview that some pupils who are less fluent in English have difficulties with division and subtraction that link with their verbal expressions. Many say 'we subtract this *with* that' (where 'with' replaces 'from'). This use of 'with' in addition and multiplication where 'adding this with that' or 'multiplying this with that' is unproblematic in practice as the order of 'this' and 'that' does not matter. But it does matter in subtraction. Ensuing misreadings are then likely in 'word problems' and in communication.

These mathematical register difficulties are added to by Helen, Sara and Clive and relate to borrowed words and terms and their often multiple meanings (Pimm, 1987, p. 91). Helen discusses confusions with the terms 'opposite' and 'adjacent' that have to do with pupils not being clear when and whether they are referring to angles or sides.

board I never s: hey you ... because I tried that ... and when I forced them to talk then they never wanted to come up front. So then I felt ... then only the some few will come up - those that feel free to talk and others are not ...

(VI4, 74-83)

It is a good thing to talk to each other but I think in Tswana it becomes a problem because- um, like if he explains in Tswana ... (inaudible) our language is unique and when you come to 'at most' and 'at least' then what are you n an to say? For in our language 'greater and equals to' and 'gr : there is a little difference. I have to use a long sentence for - than and equals to' ...

(VI4, 115 120)

I think if I was to explain in Tswana I would run out of words ... like you can't say 'minus four' - if you try to say 'minus four' it won't really make sense. And for my mixed class it would also be a problem because not everybody speaks Tswana. So must I do it again in Xhosa and then Zulu? I would definitely run out of words and go back to English ... But you see my problem, there (pointing to the video) in the middle of that group is Nomathemba and she is Zulu and I don't know how much Tswana she understands.

(VI4, 140-146, 394-397)

The benefits and effects of code-switching *in practice,* be it in the form of translation or in informal discussion, are by no means clear for Thandi. The issue of code-switching was taken up in the workshops and is discussed further in 4.4.2. The teaching of inequality language in Thandi's Std 9 class is the focus of the vignette in Chapter 6.

4.2.2 Mathematical discourse

Jabu's pupils also talk to each other in Tswana. He is not troubled by this. It is also clear that pupils in Jabu's class struggle to express themselves in English. Working on the value of 2-5, one pupil argues, very haltingly, that the answer is 3 because 'you can minus 2 *into* 5' (my emphasis). Interestingly, most the class agreed with him!

English and Motsoari said 'obviously English'. And two said mixed - that is they could go to Tswana if they needed to. And the four boys who wanted Tswana are all from one group, from Eddie's group ...

Sa: *And when they have their discussions, do th. : also have it in Tswana?*

T: *On the video, they were doing it in Tswana, though I thought they were talking English before I saw the video. It is only Grace who was doing it in English - and in Grace's group they were not talking much - it was mainly her talking like a teacher*

(W3, 212 231)

She explained why she thought they made their different choices for the language of the interview:

T: *I think it is because they wanted to be free - they are weak some of them, always quiet. When I asked them that I wanted to interview them, one of them asked: 'Mam is it going to be in English?' So it seems more wanting to be free. And also in their reasons - when I asked them about learning in English - nobody said they wanted to be taught in Tswana. Even the boys who wanted to be interviewed in Tswana said they wanted to be taught in English because when you look for jobs ... no that's the next one. Andrew said: 'I think it is good because in the discussion I have in the mother tongue I find that others are Xhosa and others are Tswana and we cannot understand each other and also when we speak Tswana we have freedom of speech and we end up hurting others (laughter). 'So English is best', said Andrew, 'because even our final exam will be in English'. And he was not the only one concerned about that. Eddie talked about looking for jobs and one of them said when you go overseas you speak English (laughter). So although they know even if they can do it in Tswana, they know that the way our future is in South Africa means they end up having to do it in English at some stage ...*

... And the next was interesting when I said: 'But I noticed that you were working in Tswana'. Andrew looked surprised and he said: 'in our group?' and when I said 'yes', he said 'but sometimes it is difficult to interpret the questions in English and in that case we use Tswana.' The others said

S: *... the kids are part of that as well. Just thinking about my transcript of my video here - where I don't talk a lot. I talk very little - there are very few points where I come in and yet I redirect the discussion totally when I do - I can take them away from what they are trying to do - I have misunderstood it, bringing my own interpretation and off it goes. And that's the kids and they take that on. They see me as powerful and if I am redirecting it then that is where it must go. And it is hard because you do need to be powerful, you do need to redirect if they are going off.*

(W2, 618 628)

Thandi's interest in discussion was more on the groups of pupils themselves and how they functioned. She interviewed nine pupils from different groups in her class on how they perceived group learning. She asked them about how their groups formed and about gender relations in this process, who became leaders and why, about how they argued with each other and the language they used in their discussions. She found strong differences in pupils' choices about who they work with, some going for stronger peers who will help them with their maths, others going for equals and yet others choosing to work with pupils they can help. These raise interesting questions about pupil preferences and possibilities for functioning within created ZPDs. Thandi's interviews are very interesting and form part of her own research (Setati, 1994).

Most interesting in relation to *this* study was how Thandi's interviewees interacted with her on why they spoke in Tswana in their groups.

4.4.2 Code-switching revisited

Thandi analysed what her interviewees said:

T: *Then about language ... there was something amazing about language. Four of the nine said they want to take the interview in Tswana; three of them said that they want it in*

Helen reflects:

H: *Just after the sentence is written on the board and I ask 'What do you understand by this statement?' - the one child puts forward a perfect explanation: she talks about the angle being the same in both triangles and then she talks about the depth of the triangles or whatever and I pick up on that ... and then this child now does it absolutely perfectly. So, that is two very good expressions of what is going on. And yet when you ask the class: 'Is this sentence correct?', there is this complete silence. So the question for me is: even in the minds of those two children who put forward such consistent explanations, what's going on with them? that they cannot um pick up incorrectness in the sentence.*

... I think that that sentence came out of something that the group was working with ... if you actually take a sentence like that which is supposed to be concise, and it carries a whole lot of meaning there is difficulty ... They can talk to you about it and they can give you a long explanation of what to do ... so its seems to me to be also a problem of expressing a lot of maths in one clear sentence. For me that is also linked to the issue of how we transmit maths to each other. If you make a mathematical statement you are involved in getting it down to a simple, short hand language that we can all share ...

(W2, 60-121)

H: *... in retrospect, when I look at that lesson, I went on but MUCH TOO LONG (laughter) on and on and on and I keep saying the same thing and I repeat myself, on and on ... and I watch the video and I think I wonder why they are still sitting in their seats and I am falling on the floor falling asleep. But the thing is then if you have sense that there is a shared meaning amongst the group can you go with it? um when the sentence is completely wrong? ... Can you let it go? Can a teacher use a sense of shared meaning to move on? I think this is a central question in terms of the verbalisation and discussion.*

(W2, 151-185)

Some of the discussion amongst the teachers here focused on teacher intervention and mathematical language development and Sue added the important insight that teachers' behaviours here are not simply of their own making:

For Sue, Thandi and Helen, there are clear dilemmas in *mediating* mathematics in a more participative and enquiring approach in schools where classes are also multilingual. This issue is taken further in the vignette in Chapter 7, and focuses on Sue's Std 6 class.

Helen also explored discussion and verbalisation by videoing the introductory lesson with Std 9's who were the Std 8 class in trigonometry the year before. She asked pupils to work in groups to summarise what they thought trigonometry was and what they felt about group discussion in their mathematics class, and whether language was an issue for them at all. Very interesting issues emerged for discussion. Helen's action research highlighted the specific problems that can arise in whole class interaction with pronunciation on the one hand, and with pupils' articulation of their mathematical thinking, or more generally, how pronunciation and articulation interact with mathematical meaning-making.

> *H: One of the issues was linguistic ... the sound issue between sides with an s and sizze. A lot were hearing size when I was saying sides and we picked up on that issue and then (on what trig is) on e ratio being the same for a given angle in different sized triangles. When Jill and I discussed this we talked about the one part where a child put forward what she thinks is going on in relation to that issue and it is a question of even though her language is not clear is there understanding amongst the rest of the students and it seems like the rest of the students do understand even though she is using incorrect language. So we can watch and think around that.*
>
> (W2, 19-33)

(On the video, the student says: *the ratio of the two sides is independent to the size of the angles in the two triangles* and Helen writes this word for word on the board for the class to think about.)

She struck a chord with the other teachers in relation to a key dilemma in teacher-pupil classroom interaction:

> *T:* *What, like, maybe sometimes if a teacher doesn't get in and tell and give direction - then some of the other kids might in the end not know or appreciate things that are not said or are not right or whatever, during the discussion. So if you come in I think it might help in giving direction ...*
>
> (W2, 1515-1520)

> *H:* *um ... and it is such a difficult thing to know when to give that direction, (lots of agreement). I think of trying to teach y=tan x to the Std 9's, and say to them investigate and then just leaving them for five minutes and they have got to 90 degrees nd (..) things happening there and they kept on, the graph ould just suddenly go through 90 so they didn't know where it should go and then I struggled with myself for a long time as to whether I should tell them to look at 89 and then 89,99 and then withdraw. Um, um and is it selling out on the process to already tell them to do that? So then I sold out and told them to look at that and then left them in the hope that they would look at 90,1 and 90,2 and they didn't. And so then I didn't know in myself whether I had spoon fed them to look at 89,9 and then they were waiting for me to now say look at 90,1 ...*
>
> (W2, 1524 - 1545)

> *H:* *... Ja, you see I think that is interesting because the one thing that there is always a tension for me between telling them where we are going and letting it be a surprise. (lots of agreement). Like if I go to a workshop I sometimes don't want to know what is happening at the end of the workshop I want it to unfold - beautiful spiral stuff, um whereas then the next week you do a reading that says it is important to tell the children where you are going and so you spend the next week going: we are going to do this and this and then this - so that they know wh ere you are going and there is this constant tension um and what you are saying kind of fits in with that ...*
>
> (W2, 1601-1617)

C: *I think that is going on all the time - particularly second language classes ... I think the command of language is very different and their use of words is different - I think - because the way they are talking here - just coming in to that discussion sounds like gibberish to me.*

(W2, 1186-1191)

But Sue was not so sure:

S: *I originally thought the same as Clive ... but it also might be, thinking about it now, we talk about 'you draw an angle' that is quite a common phrase, and what does that mean? Does it mean you draw that or that or does it mean you indicate, which is what he is meaning here ...?*

(W2, 1192-1200)

The crucial point for Sue was how, as teacher, you assess the value of such discussion in the mathematics class:

S: *Um, you see I don't know how to assess it ... 'cause in the end (..) I mean my way of assessing it is with the other class. And in the end, I think this class got the same as the other class got - so I don't think it was negative. But I think I could possibly have made it more positive. . if I had heard what was happening. I could have come in in a better way - and also drawn in more other kids, you know, although that is also a difficult issue. You know 'cause there were some who don't feature here at all - and I don't know how they perceived it, and what they got from it.*

(W2, 15. 3-1375)

Sue also observed in the transcript that her interaction with the pupils was often telling pupils things rather than questioning them in the typical I-R-F (initiation, response, feedback) mode. And she then argued:

S: *What I am trying to say is that I think that is how we structure the lesson - that in fact it is not so much I have to ask questions. I think I am trying to say that I think tell is also OK. That it is more honest ...*

(W2, 1501-1506)

seemed to be quite peripheral and then ask them first of all how they felt about the discussion, whether it helped or confused - I thought confusion was quite an important thing to focus on - and then I also wanted to try to get at their understanding so I asked them the question again to see if they had changed their mind and to be able to ask them why ...

(W2, 816-831)

Through this process she made interesting observations:

- that whole class interaction in this instance does not help all pupils in the same way, and some not at all;
- that whole class discussion here is side-tracked by the teacher's misunderstanding of the pupils' debate, and this is not necessarily a function of different languages in the class;
- that telling the class about mathematics can serve a positive teaching function.

After interviewing pupils:

S: *... the assumption is that the discussion is helpful to everybody in some way. In one sense it was not helpful to her (the pupil interviewed) at all - she did not change at all ...*

(W2, 1060-1072)

After analysing the transcript:

S: *... another interesting thing that I found was what in fact the discussion was about ... it seems here (the transcript) that the meanings of the words 'choose' and 'draw' were quite central which I didn't pick up at all. ... And then I come in and change it totally (by asking) is there one or is there three - (Laughs) - and they are choosing and drawing - (lots of laughing).*

(W2, 1163-1183)

Clive interpreted the misunderstanding as related to different languages in the class:

> ... in the solitude of this most private act, I write, in my public language, in order to update what might have been my other self. The diary is about me and not about me at all. (Hoffman, 1989, p. 121)

Though writing and speaking are different, Hoffman forces us to think about what it means to explain ourselves in a language other than the language of the self. Writing is not a focus in this study. This extract nevertheless raises questions about journal writing as part of continuous assessment in mathematics classrooms.

In this teacher-led workshop, there was discussion of what teachers said and justification of how they acted in relation to: discussion and verbalisation and whether and how they help mathematics learning and teaching; explicit language teaching which can both help and hinder; and, though most obscure, the particular layer of multilingualism. These relate directly to my research questions and are interwoven into the three vignettes developed in Chapters 6-8.

4.4 Follow-up action research: the two remaining workshops

While all five teachers participated in the two follow-up workshops in May and August 1993, three actually engaged in some action research. Sue transcribed the lesson in the class where there was lengthy discussion on the reflex angle. She wanted to see exactly how she and the pupils interacted. She also interviewed a few pupils from the class to see what their understandings of the lessons were.

4.4.1 Discussion and pupil learning

Sue's overall question for the workshops was:

> S: ... *does discussion help with ... kids mathematical understanding? I don't know how to get at that 'cause I don't know how you measure understanding. I first thought I would take two kids who had participated a lot and then two who*

latched on to it to try to get to this concept of inclusion 'cause I knew it was coming up later on, and I couldn't use it. They were more confused about the inclusion at the end of it. I don't think they were confused about the reflex angle and the triangle. I couldn't use that (the inclusion) in the way that I had tried to and I wouldn't have realised that if I hadn't seen the video afterwards. I probably would have done the same thing again. Now I know I won't.

... I wouldn't let a discussion like that carry on. What worries me is that in a different situation another thing will come up, which I will go with, not having the advantage of hindsight afterwards. That is what worries me. I think it is good in general. I prefer a more open debating approach. It feels better but I don't know if it actually is better.

(W1, 1380-1436)

Aside from the specific issue of language-use in Thandi's class, the discussion and issues in the first workshop were not specific to pupils who were learning maths and English at same time and in different contexts. It is interesting here to reflect on a comment from Clive:

C: What is interesting is that in guidance they have been asked to write about their interests in their own language and in English and the two are completely different. And in fact, you develop your personality in your first language - you don't have the same personality in your second language.

This affirms Eva Hoffman's discussion of her adolescent diary writing. Her main language is Polish, but she is now a resident of Canada. English is her present language but it is not the language of her emotions, so in what language does she write her diary?

> If I am to write in the present, I have to write in the language of the present, even if it is not the language of the self. As a result, the diary becomes surely one of the most impersonal exercises of the sort produced by an adolescent girl. There are no sentimental effusions of rejected love, eruptions of familial anger, or consoling broodings about death. English is not the language of such emotions

others just listening and at the end of it just writing it down. I also spotted one group where the group leader was talking and she was asking them questions - sort of forcing them to think - while others felt the good one leads the others - must do it for them. She was different - she asked them questions, then the answer then she writes down and they go along with her ... also most were talking Tswana. It was I think only one group who were talking English - all the others were all talking Tswana.

(W1, 750-764)

For Sue, the major question was: 'does discussion help or does it confuse?' Sue shows two discussions from two classes on the same problem: *if you have a reflex angle and you try to make a triangle* it is impossible, you will get a quadrilateral. In both classes, some pupils' interpretation was that there could be a triangle with a reflex angle - it was outside the triangle. Sue had 'just assumed that wouldn't be a problem'. The issue was dealt with quickly in the first class where one pupil says 'the angle must be inside', but in the second class a whole-class debate occurred with pupils arguing not about inside or outside angles but whether there were one or three reflex angles, and whether you can draw three but then choose one.

S: *I do wonder with the second class - the other girl was saying 'there are three reflex angles but she chose one'. It is interesting. I think he saw there were three but he was only referring to the one - he chose one - and to me that discussion in the second class was a total diversion and it went on for a long time - and I don't know if it helped ...*

(W1, 1358-1364)

S: *I don't feel it was a problem, because I think in both classes, the discussions achieved the same thing. The second class, it was I who said the interior angle and in the first class it came up from them. Maybe it was when I was going round - I can't remember maybe I said something then. So in fact the only difference in them getting what I wanted to teach them was that it took longer in the second class - but what they got was quite an interesting discussion where they were beginning to challenge each other. But I feel uncomfortable because I*

setting, where a language of instruction is adhered to, talking to learn means having language to talk with, and displaying learning means having acquired the language to do so. Sue and Thandi both bring out their difficulties in their different multilingual contexts - the issue of not having the English to display knowledge - and the issue of how this might be developed.

Helen went on to question who actually benefits from pupils' verbalisation or articulating their thinking and thoughts:

> *H: One of the things that struck me seeing the range of our different work is that there is a lot of teacher talk and how much of what we call language, verbalisation and all that is in fact ... we have this thing and we land up doing much more verbalising than they do and then we leave our lessons feeling OK and in fact there hasn't been much pupil talk ...*
> (W1, 1450-1465)

Others agreed that there is less pupil verbalisation than they had thought there was and they would have liked. The teachers then also discussed whether talk benefits learning. It does benefit teaching because 'hearing what it is pupils think and articulate can help you see what they understand'. Of course, within the framework of a social theory of mind, the dichotomy between teaching and learning is questioned: if a teacher has access to pupils' thinking, this could aid the mediation process and hence pupils' learning.

4.3.3 Group discussion and classroom interaction

Thandi showed an excerpt pointing out how differently the groups in her class interact with each other, and how this is predominantly in their main language.

> *T: What I was interested in is how my pupils worked in groups. I checked my students where they worked in groups - how did they run their discussion. In other groups the group leader was dominating. He or she was the only person talking, the*

do it the one day and then not the next? ... Like Debbie had put it into words but hadn't come to full possession of it. ...
(W1, 345-365)

Explanations from the other teachers ranged from pupils not knowing how to say what they are thinking - 'they don't have the language' (Thandi) - to the conceptual jump from sin 40 to sin theta being underestimated, and suggested strategies for enabling pupils to express what they think in pairs or writing down their thoughts.

Helen held to her question:

> *H: I still think it is not answering the central question 'cause those strategies when I have used them, they still mask that central question. Whether you write it down, say it to your partner - all those things that I use a hell of a lot - is still not answering the question of whether it is bringing them into full possession of it.*
> (W1, 546-552)

For Sue, who was constantly concerned with pupils' meanings:

> *S: My question is a bit different I think. I also get the feeling that just talking about it is not enough and either the kid has the concept and they can't say it or else saying it doesn't reflect the concept. So my question is shouldn't I be getting them to talk about it in a different way, or intervening in some way so that they learn to use language ... No, that is not what I am trying to say - um ... so that their act of verbalising gets more sophisticated so that it does help them ... like with Debbie if the kid says something then we accept it and we could probe more - then we could start getting it to be more worthwhile - but I don't know how to do that.*
> (W1, 680-697)

We see teachers struggling here with the formative and knowledge display functions of talk. *Talking to learn* and simultaneously *demonstrating what has been learnt by talking*, bring new sets of pedagogical questions. In a multilingual

Linked to the reality that code switching is not straightforward, Thandi said:

> *T: ... there are Xhosa speakers in the class - so if I am speaking Tswana then they complain I am favouring them ...*
> (II4, 538 591)

On the other hand, Jabu acknowledged difficulties teaching in English and explained how some ideas (those easily linked to everyday life) are conveyed better if explained in Tswana.

> *Ja: ... at times (teaching in English) is a problem. You feel bad that you don't succeed to reach them, and then this is bad. But it only happens in the lower classes ... with the tens it is English all the time.*
> (II3, 460-464)

Talking about the 2a - a problem above:

> *Ja: I say, 'my girl, bring me that book'. Then I say 'I have two books, and she brought one book'. And that, problems like that. I go on in English. But there are similar cases like say half plus half, and they have serious difficulties, and then I say, in Tswana: 'I have a half a loaf, and a half a loaf, how many ...'?*
> (II3, 490-500)

He acknowledged that 'unfortunately' at a Std 7 level pupils interact with each other 'in their own language', but that 'they need to talk to each other in a language that they understand' (II3, 560-62), and further, that if he did need to explain something, he would do it in both Zulu and Tswana as these were the languages in the school.

It is clear from these two accounts that while these teachers hold different views in relation to English use in class it is possible to argue that Jabu treats home language more as a resource than does Thandi. both spoke about using

4.1 Developing English vs. developing meaning: or, to code-switch or not?

Both teachers in the ex-DET schools were fully bilingual (English/Tswana). Jabu, when he needs to, also speaks Zulu in class. This asset was not shared by the four English-speaking teachers, though one did understand and speak some Zulu. With this bi-, tri- or multilingualism in teachers comes the possibility of code-switching, of communicating in both English and the learners' main language. As discussed in Chapter 1, there are tensions in code-switching as teachers work to enable understanding and follow language of instruction policy (Rubagumya, 1994) and these create dilemmas for teachers:

> *T: ... in Std 7, where I asked a question and one answered in Tswana. And I said: 'Can you please try answer that in English - I don't understand that?' and he said, crossly 'No, mam, but you are Tswana - you are not white!' He was angry. But it is not like that in Std 9 ... they like to work in English.*
>
> (II4, 426 - 434)

And later in the interview:

> *T: Sometimes you find that you get stuck because students cannot communicate - then, though not much, you resort to Tswana. You are careful because if you do that then they want you to do it all the time, and they turn the situation to a Tswana class. Then they will never improve.*
>
> (II4, 546-554)

Earlier, this teacher was also quite adamant that she 'never translate(s)...'. This could be related to the strong policy enacted in her school by a grouping of teachers which she explained as follows:

> *T: We have sort of formed a group. We have said that if there is someone who doesn't come to school we confront them, and if you don't appear in class we confront them. If we pass your class and you are teaching in vernac, we confront you. So that is the thing the group adopted.*
>
> (II4, 52 60)

so there is a confusion over commutativity ... (they) also say 'this subtracted with that'.

(I16, 859 884)

The teachers in this study were thus aware that the *mathematics register* (mathematical English) presents difficulties in general, that it can clash with everyday language, and that there are specific instances of clashes and confusions for ESL speakers.

Difficulties with mathematical discourse and algebraic language are well known and other examples from the data have been discussed in Chapter 4. Such difficulties are extensively discussed by Pimm (1987) and experienced by many learners, irrespective of their spoken language. MacGregor (1993) pointed to ESL difficulties with mathematical expression in English. Teachers' experiences and knowledge that ways of speaking mathematically in African languages are different are borne out by the common example of people whose main language is an African one saying in English: 'there are 4 or 3 people needed'. In English and mathematical English the typical ordering would be ascending. This language in-use could well impact on mathematical communication. Nevertheless, as I argued earlier, the issue here is not simply one of mathematical discourse, the register and symbolic form. It is also about how mathematical discourse (educated discourse) is learned (in English), *within school mathematics culture* (educational discourse).

4 DIVERGENCES AND THE EMERGENCE OF DILEMMAS

More interesting than that which was common across what all six teachers said are dimensions of language that emerge for *some* of the teachers in their particular multilingual settings. These language dimensions are interesting because they are linked to the context of the teacher's work and to situations of change, and because they reveal dilemmas that teachers face in their practice.

problems', all six teachers described difficulties with mathematical English and with algebraic language. For example:

> *Ja:* *... what did I have recently? Oh yes! 2a - a. To me they said: (pointing to the a) "There is nothing here, so the answer is 2a". They see 2a and 0a and so get 2a.*
> (II3, 470-475)

> *Sa:* *... "simplify" for Std 6 is a real problem - sometimes it means add the terms and sometimes it means other things...*
> (II2, 605-607)

Some grappled with how and whether mathematical discourse, particularly the use of ordinary English in mathematics, presents greater difficulties for ESL learners:

> *C:* *... one is that they (pupils) just don't understand the use of words. They don't understand the links between certain words, words which have similar meanings. They don't get the subtleties. Like they don't understand the difference between factorisation and division, that kind of thing, products and multiplication, units, for example millilitre and millimetre ... and certain words which have rigorous meaning, like 'at least', 'at most', things like that, which are used in everyday language. I think black learners use them differently. They use 'at least' in particular ways ... Logical things and negations are difficult for second language learners, and we have those things in mathematics.*
> (II5, 649 - 685)

> *S:* *Sometimes it is ordinary English that I am using to explain things. The technical words are a problem but I think they are for all kids. The mathematical English is also a problem for all kids, but maybe, ... there is this one issue of 'divide by' which comes first? and 'subtract'. Now that may be a problem for English-speaking kids but it may also be cultural ... At our school, kids say "this subtracted with that" and I have always interpreted to mean "this subtracted from that", but it is not clear because "this added with that" can be the other way ...*

Without wishing to trivialise any of the above, it makes sense that where English is learners' additional language, they will, at times, resort to their main language in their learning, and that they will also struggle when forced to speak English. Furthermore, teachers who can speak the main language of the pupils will do so at times. What these excerpts reveal, however, even if somewhat superficially, is that the experience of these learners in school is such that at times they are blocked from using verbal speech. And in terms of a social theory of mind, this must have some effects.

> T: *... even though they can hear like I said, it would be much better if they could also talk about it in English - they would understand more - they would raise their views on it and it would make it easier to understand. I think communication is very important in learning maths, and if you can't communicate then it makes it difficult.*
>
> (II4, 529-540)

Teachers in this study thus know that in their multilingual settings, teaching and learning mathematics in English inevitably means using languages other than English in the class; and that at times, they and their students struggle to communicate with each other in English. These accounts are from multilingual classrooms. Nevertheless, with Sue's comment that some children are just not so good at explaining themselves, these accounts support my criticism in Chapter 1 of recent research in classrooms where *all* pupils are supposedly communicating their mathematical ideas and thinking, yet the research is silent on communicative competence. It takes communicative competence across all pupils for granted.

3.2 Difficulty with mathematical discourse, specifically the mathematics register and mathematics symbolic form

The teachers were aware that mathematical discourse, and specifically the mathematics register and mathematical symbolic form create problems for all learners at least some of the time. Aside from well-known difficulties with 'word-

towards the group, listen, and then hope that they don't stop. If they do stop I say continue and once they have finished and if they agree or establish something then I say OK now explain to me in English or if there is a child in the group who doesn't understand ve . and there sometimes is, then I say 'OK! We don't understand, so can you explain to us?" So they are getting the chance to explain in English after they have understood what they are trying to do in the vernac.

(II6, 930-947)

3.1.2 Communication in English is difficult at times

Each teacher mentioned some occasion when either they had difficulty in explaining through English, or pupils had difficulty in expressing themselves in English.

C: I find if a child wants to ask a question, they very often can't ask the question. They might know in their heads what they want to ask, but what comes out is a jumble ... and then sometimes I think I know what they have said but then I answer the wrong question. I can see from their faces... there is a lot of difficulty with black children communicating what they want to ask.

(II5, 697-718)

I: I don't think they have a problem with hearing. They do hear what you are saying. But maybe answering back, asking you a question in English, its a problem ... when they have to say it - saying in English that is the problem.

(II4, 430-440)

Ja: ... at times ... you feel that you don't succeed to reach them ... At times they struggle to explain in English. And I say: "use any language to explain the concept", 'cause I realise that he is battling with two things. First is the language and then the second thing is the math concept. Let him rather struggle only with the concept and pick up with the language later.

(II3, 460, 565-572)

3 COMMONALITIES

3.1 Communicating in English, the language of instruction

3.1.1 Policy and practice

English was the official language of instruction in all six schools and each teacher spoke of learners and/or themselves, using their main language at times. For example:

Ja: *Eh, okay you see maybe some work is not done, I scold them in English, and if I feel that it has not got the message home, then I do it either in Tswana or Zulu, these are the languages in the school. Then ... I feel that I reach them.*

J: *And if you are trying to explain something new, some maths ...*

Ja: *Okay ... In that case I think of a problem in their day to day situation. Then, I feel, if it will impact to use the language they use at home then I resort to it.*

(and a little later)

... when they come to talk to me on anything, I insist on them speaking to me in English most of the time ...
(II3, 466-480, 517)

C: *... there is a rule in the school that they are not allowed to communicate other than in English ... I don't really stick to that too strictly, unless I have reprimanded someone and they break into vernac ... so quite often I think they do communicate in vernac. But very often, I mean, they are learning maths in English. So they will talk about it in English.*
(II5, 726-739)

S: *(Talk in class is) mainly in English, but when it gets very difficult or when they start fighting or arguing they will go into the vernac ... and I have my own policy, not a classroom policy. But, when I hear that happening, I will kind of walk*

The tallies in each map against the broad curriculum categories described above reflect the number of times the teacher spoke about her practice in relation to that category. For example, in Helen's (T1) transcript there were 23 instances of her talking about pedagogy. Through the six maps and with the tallies in each map, presences and silences within and across the interviews became evident. As the frequency tables below show, *context* featured prominently in both ex-DET teachers' interviews, a reflection of their difficult working conditions. *Pedagogy* (which includes *approaches to teaching and learning*, and *classroom interaction*) was noticeably more dominant for teachers who are trying to shift their classrooms to be more communicative and inquiry-based. All teachers talked about *mathematical discourse (and specifically the mathematics register and symbolic form)* and *ESL* issues. This was a function of the semi-structured nature of the interview: I probed these issues if the teachers themselves did not raise them. However, of significance was that far more was said in relation to ESL by teachers 3 - 6, all of whom had fully, or almost fully, African classes. The silence in the two ex-DET teachers' interviews in relation to what I have termed *communication and culture*, was also interesting. Jabu and Thandi did not discuss communication breakdowns as attributable to a relationship between communication and culture.

TABLE 3: UTTERANCE FREQUENCY ACROSS SELECTED CATEGORIES

	CONTEXT	PEDAGOGY	ESL	COMMUN & CULTURE	MATHEMATICAL DISCOURSE
T1:HELEN	4	23	3	3	9
T2:SARA	5	21	4	2	9
T3:JABU	18	6	9	0	12
T4:THANDI	10	5	12	0	8
T5:CLIVE	3	12	9	1	7
T6:SUE	2	31	10	4	1

curriculum as relational and as an interaction between task (mathematical knowledge), learner, teacher and context.

The broad curriculum categories of *mathematical knowledge, context, teacher, learner, pedagogy (the relationship between teaching and learning)* and *language*, as well as sub-categories within each of these (for example, *ESL, mathematical discourse* and *communication)* are those categories of interpretation that arose both from the form of, and assumptions underlying, the interview, and the teacher responses and discussion. Careful 'listening' to the data in the light of the field of language and mathematics education assisted the construction of the sub-categories and the generation of a 'map' (as in Figure (i) presented again below) of each teacher's interview. The six teacher maps are presented in Appendix C.

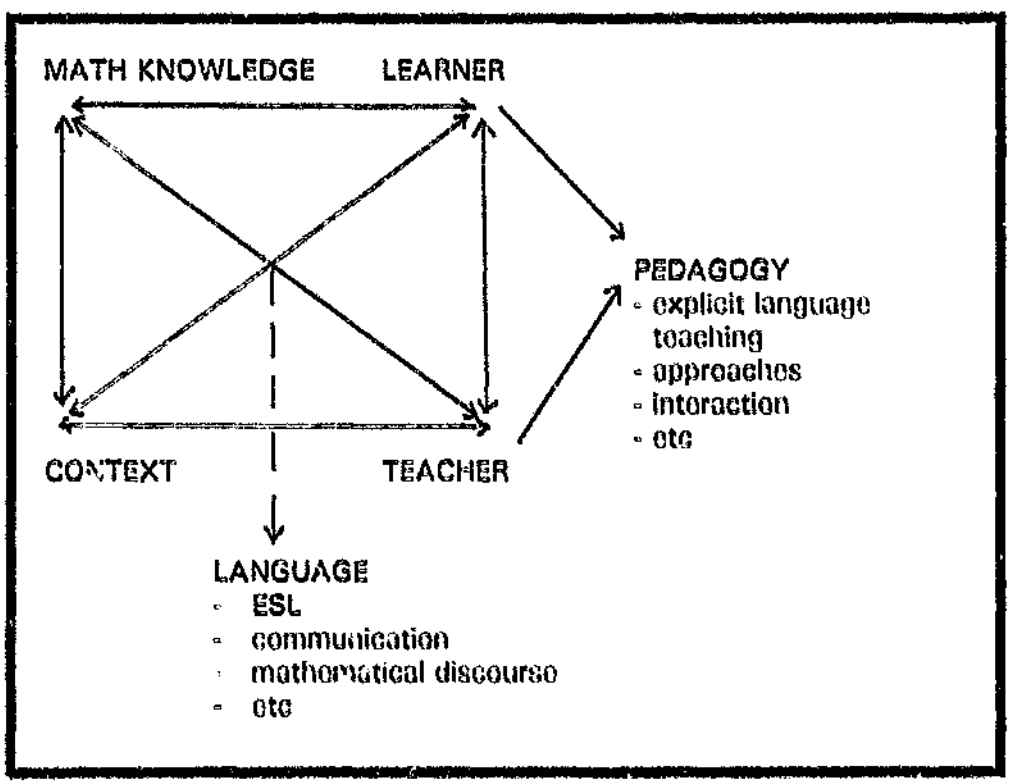


Figure (i) A Relational View of Curriculum

CHAPTER 5:

TALKING *ABOUT* TEACHING: THE INITIAL INTERVIEWS AND THE EMERGENCE OF DILEMMAS

> ... like there are some kids who are really not good at explaining themselves, and I don't do anything to address that ...
> (I16, 1165-1167)

1 INTRODUCTION

The focus of this chapter is the initial interview and what the teachers consciously articulated *about* their practice. The initial interviews were thus a window into what teachers *know about* their practices in multilingual classrooms. While they talked *about* their practice in their reflective interviews and in the follow-up workshops, this talk was more *within* their practice since it was in relation to instances in their videos. Their talk was more within what they did, and with each other.

This chapter analyses, interprets and explains the language-related commonalities, divergences, presences and silences in the six teachers' initial interviews. From my perspective as researcher, these revealed how different contexts and conditions give rise to different dilemmas for teachers as they go about their work.

2 THE INITIAL INTERVIEW AND DATA INTERPRETATION.

As described in Chapter 3, the initial interview was semi-structured, comprising three central questions related to the context and ethos of the teacher's school, general tasks and challenges in teaching (junior) secondary mathematics, and then the specific language issues each teacher thought he or she faced. The function of the first two questions was to provide a context for interpreting accounts related to the third and main focus of the study. This structure reflects a view of

As described in Chapter 3, the August workshop was the last formal meeting with the teachers. While the overall research process was experienced as a participative and informative one for all, the overlay of learning in an additional or second language remained elusive as teachers confronted the whole picture of their teaching. Many of the issues discussed, while on language in general, were not necessarily specific to the context being multilingual. There nevertheless remained a strong view that the area I was trying to unravel was an important one.

5 CONCLUSION

The purpose of this chapter has been to describe the entire data corpus and in so doing provide a first level of descriptive and interpretive validity by remaining close to the data. Three issues emerge over and again: explicit language teaching; code-switching and pupil-pupil discussion are key language issues for teachers in secondary mathematics classrooms. As a qualitative study, however, descriptive and interpretive validity are necessary but not sufficient to support arguments and claims made. There needs to also be theoretical validity (Maxwell, 1992). It is in the detailed analysis of the initial interviews in Chapter 5 and the vignettes that follow in Chapters 6 - 8 that dilemma language emerges and is developed to capture more effectively the three issues in teachers' knowledge of language in their classrooms described in this chapter. Dilemma language becomes an effective means by which to draw more explicitly on the sociocultural theoretical frame developed in Chapter 2 and so engage more sharply with the research question and subsidiary questions in the study. The unravelling of the research question and its subsidiary questions through in depth analysis follows in Chapters 5 - 8.

NOTES

1. Clive enrolled in post graduate study in 1994. Perhaps their involvement in the research project provided a context and some incentive for the teachers to want to study further. However, they were an interested and professionally active group before this study. They thus might well have come to further study anyway. Sara, with 23 years of teaching behind her and an Honours degree, had no desire for further formal study.

different things: like Pogiso said: 'Yes, mam, we were speaking in Tswana. Eddie started (..) ... (laughter)'. So I wonder what would help him get in and change - would it be like he was trying to be better talking in English and so on - and in fact Eddie was talking in Tsotsitaal and so it is not like it was in Tswana. The other boys from Eddie's group said Eddie started in Tswana and because he is the leader they had to follow.

(W3, 241-289)

Thus, just as Sue pointed out, pupils are implicated in and part of establishing classroom culture and interaction processes. Both Thandi and her pupils have strong beliefs about the value of English as language of instruction for mathematics, that it *should* be used. These beliefs are implicated in, and therefore must be considered in any work on code-switching, especially at senior secondary level. Thandi's action research thus provides a valuable source of data for the study which was not set up to include talk with learners about their views of the language of instruction in mathematics learning.

Thandi, Sue and Helen have had most of the floor in the workshop descriptions. This is because Clive and Sara did less explicit research-like activity in preparation for the workshops. They nevertheless were full participants in the discussions and their views have been included in the areas discussed. It is worth pointing out that Sue, Thandi and Helen all enrolled at the university part time for further post graduate study in 1993 and thus had a context that supported their writing about their work - they could build this reflective work into their course work. This may account for their investing more time in preparation for the workshops[1].

The selections the teachers brought to the May and August workshops again brought out discussion on the benefits of and constraints on: explicit language teaching; working in and across languages; and pupil-pupil talk in the classroom in the learning of mathematics.

Linked to this is the argument that in mathematics classes peer discussion and reporting are two different tasks, and that reporting skills should not be left to chance, but explicitly taught. Too much emphasis on process and pupils' meanings can inadvertently perpetuate social difference by not making explicit to all learners exactly what is expected of them (Mousley and Marks, 1991). Pimm also argues the specificity to the communication skill of reporting mathematical thinking (1992, 1994), but raises the tension that if such skills are made explicit, then there is the danger that reporting becomes focused on the form of the report rather than its substance.

The dilemmas of mediation are clearly not new, nor unique to multilingual classrooms. They relate quite clearly to curriculum dilemmas described by the Berlaks. Personal knowledge vs. public knowledge, learning as individual vs. learning as social and each child unique vs. children have shared characteristics are all at play in mediating the school mathematics curriculum and particularly so in classrooms where teachers have tried to shift their pedagogy to value and work with personal meanings, social interaction and diversity. Multilingual contexts bring some dilemmas of mediation inescapably to light. The crucial question now for this study is how and whether the dilemmas that have been inferred from what teachers said relate to what they do. How do they *act* in the face of such dilemmas? This question is taken up in general terms in the concluding section of this chapter and is the focus of the vignette in Chapter 7.

Linked to dilemmas of mediation is what I have termed *explicit mathematics language teaching*. This is a strategy consciously developed by teachers whose classrooms recently changed from being homogeneously English-speaking to being multilingual, and one which they claimed benefits all learners. They were forced to confront what they otherwise took-for-granted - that the pupils, irrespective of spoken language, were not equally competent in English. This diversity was previously hidden in a seemingly homogeneous English-speaking class. Here again is 'good practice' that needs to be further developed and shared.

to each other about their mathematics and to report back publicly on their thinking, different expressive competence is noticed. While this would be the case in any mathematics class, in multilingual settings it is often difficult to work out how both proficiency in English and understanding of mathematics are implicated. In more analytical terms it is difficult to distinguish whether difficulties are linguistic or epistemic and how these are related. There is thus less of a tendency to reduce difference in expression to the 'innate' mathematical ability of the learner and more openness to see it as linked to language and thus possible and necessary to act on in a specific way.

The multilingual context, while having its own layer of complexity (proficiency in the language of learning), thus alerts us to the issue of differential communicative competence across learners. This must be present in any and all classrooms, and sufficient cognisance has not been taken of it in research related to school mathematics reform initiatives.

Chapter 2 argued that from the perspective of social practice theory, mastery entails talking within and about a practice. We need to communicate to learn, and learning is about becoming communicatively competent. If communicative competence, or talking within a practice is a means to learning, then it becomes crucial for teachers to facilitate learners in this. The dilemma is how and when to act! Premature actions could silence the learner and prevent the teacher from really listening to the meanings the learner is trying to convey (Jaworsky, 1991). On the other hand, delayed mediation could be destructive for the learner and create confusion in the class. What are appropriate and timeous mediational actions aimed at learners improving their expression of mathematical thinking (their mathematical language or educated discourse)? In Mercer's terms (1995) how does the teacher assist the learner in moving back and forth between the pupils' informal expression of mathematical ideas and the educated discourse of mathematics?

Ex-DET teachers face what I called the dilemma of *developing English vs. developing meaning*, particularly at the junior secondary level. Because they are bi- or in some cases even multilingual, they can switch in and out of English and other main languages, and the manner and extent to which they do this seems to relate to the context of their school and to their own theories. The importance of English, particularly in the secondary school where there is greater impact of the mathematical (English) register, creates a pressure to teach in English and hence the dilemma for teachers, which the literature appears not to have really discussed. This dilemma relates to, but is not sufficiently specified in, the Berlaks' personal knowledge vs. public knowledge dilemma. This is the dilemma over what knowledge is the most worthwhile: that which connects with the knower or that which is more public and deemed needed to be known. The expression and display of mathematical knowledge in a learner's main language (but not English) is probably more connected to the knower than to mathematical English, but it is the latter that is the route to scholastic success and further political and economic access.

The problem of code-switching seems to disappear in the senior secondary school. How does this happen? What is it that teachers do? The dilemma of code-switching (of developing English vs. developing meaning) is the focus of the vignette in Chapter 6.

Obviously, code-switching and/or the extent of English spoken by the teacher is not an issue for English-speaking teachers who do not speak an African language. These teachers, however, face other dilemmas as they confront the otherwise taken-for-granted in their particular contexts.

Teachers who have attempted to change their pedagogy face dilemmas that I called *developing mathematical communicative competence vs. negotiating and developing meaning*. In multilingual classrooms where pupils are expected to talk

Whether it was about writing a test on paper, or as a group, or doing a mathematical investigation, these pedagogical practices carried with them cultural assumptions about being in a particular school and in the mathematics class. They were clearly not simply about English proficiency nor mathematical skill, but about these within classroom cultural processes. They were situations where all three dimensions of mathematics teaching and learning in multilingual classrooms are in interaction, and the most difficult to interpret. They created what seem to b dilemmas of explanation (in contrast to dilemmas of action): *is it the language and/or the mathematics vs. is it the culture?*

5 DISCUSSION

The teachers' initial interviews (what they said about their practices) reveal that to a greater or lesser extent, mathematics teachers in multilingual settings in South Africa are already multilingual teachers. By this I mean they have developed and are aware of some practices and strategies to deal with and enable the multilingual learners in their mathematics classrooms. In relation to literature in the field, some practices, for example explicit mathematics language teaching and encouraging public reporting in class in English, constitute 'good practice' for all learners. It makes sense then that these should be captured, shared and developed with other teachers.

But practices always have multiple effects. It was possible to infer from these mathematics teachers' accounts that in their multilingual contexts they confront situations that would otherwise be taken-for-granted, and some of these situations produce dilemmas.

The dilemmas are expressed in language that is different from the Berlaks' dilemma language. I have used language that focuses on particularities of language and mathematics. It is important, nevertheless, to examine whether and how the dilemmas described here relate to those developed by the Berlaks.

So: *There are three who really battled at the beginning of the year. I mean the first test I did, one didn't realise this was supposed to be a test and she just stared at it not knowing what to do. She hadn't realised this was a test and that she must write the answers down. This horrified me. I was totally astounded. Surely her understanding couldn't have been that bad that she could write nothing? And when I asked her she said she didn't know she was meant to write the answers 'down here' - she handed this page with 1 - 20 written on and the penny dropped only at the end and then she was too scared to tell me about it... I felt terrible about that. And since then, I have been a lot more specific to make sure everybody is clear what they need to do.*

(II5, 551-571)

These are instances of communication breakdown. Why did they happen? The teachers' struggled to interpret and explain them and this points to the complexities of culture and communication in mathematics classrooms. Sue elaborated at other points in her interview:

S: *I think the issue of second language learning is a real one. I mean the fact that kids are not understanding all the language that is in the classroom, that their language is not the medium of instruction ... I mean sometimes it may be previous educational experience. A child might look blank because they are not used to what you are doing in class and you interpret that as them not understanding (the maths).*

(II6, 809 - 813)

S: *Again it is past experience ... because they are used to filling in words or doing exercises ... with the investigations it is difficult because often they are structured, like in the points of departure ones where there are like five questions. Each question is a real question and they think each question is like a one word answer (laughs). So five minutes later they are finished.*

(II6, 1113-1124)

Furthermore, and as mentioned earlier, what was also interesting was the absence of both miscommunication and the need for more explicit teaching in the interviews with the ex DET teachers. Accounts of communication difficulties were ascribed either to English as language of instruction or to mathematical language but not to what could be called 'culture and communication' issues.

4.3 Culture and Communication: having the language vs. having the culture

Three of the four English speaking teachers in private and Model C schools told stories of communication breakdown.

> *S: It's language as well. We tried (when setting classes off on investigative tasks) to explain it to the Std 6's in terms of exploring, like what do you do when you are exploring? You go looking for something but you maybe find other things along the way, and you try to... we couldn't talk about explorers in history because they hadn't done that we talked about exploring an area like town or a new place for the first time like (school name) when you first get there how you explore it. But it didn't really help as an analogy ... I don't think, it may later ... but a lot of them maybe just didn't understand what we were saying...I mean the actual English I think may have been a problem with the Std 6's.*
>
> (II6, 241 - 253)

> *H: I am not sure if I can explain them but they happen ... for example, we did a test at the end of algebra ... we developed a test where they had to develop a pattern... the little squares that form a T ... we set that as a group work test so part of the assessment was working as a group etc, and then apart from solving and handing in something written as a group, they had to explain what they did as a presentation to the class. The other 3 classes managed. My bridging class didn't begin to handle the task and we had done three weeks of investigation, whereas the others had only done two weeks, we had done 7 - 9 investigations and discussed, groups, explain on the board etc. nothing, nothing, nothing, nothing, handed in pages of nothing, nothing it was weird.*
>
> (II1, 617-632)

J: *that is interesting - they get hidden?*

Sa: *Yes, especially it it is a big class and everybody seems to be carrying on and working away and it is only when you come and look underneath that little hand to see what is going on that you realise nothing is going on.*

and later

Sa: *As I have said it has made one more aware of being careful about how you present things because you know there will be kids who don't understand everything you say. Whereas before you just assumed that because kids spoke English at home they could understand everything you said, but they don't, and having the other children there makes you aware that they don't understand the more adult words - more aware of language.*

(II2, 575-88, 724-751)

These teachers' experience was that explicit language teaching did help, and it helped everybody. Pimm (1987) and Mousoley and Marks (1991) also advocate explicit teaching of aspects of the mathematics register, and different genres within mathematical discourse.

But the value of explicit teaching can lead to a deep pedagogical tension, and hence acute dilemmas for teachers: learner-centred approaches to teaching and learning mathematics ask for far more opportunity in classrooms for learners to investigate and build mathematical meanings and for teachers to listen to, build on, and interact with the meanings learners bring to and make in class. This is in tension with explicit mathematical language and genre teaching and both, in different ways, are about access to mathematical processes and its products for all learners (Adler, 1995).

There are also strong interpretations of constructivism that conflate or reduce all teaching to *intervention in learner's meaning-making*. For teachers who hold this view, explicit teaching will, in fact, present a paradox.

opened up two questions for teachers. One (the last quote above) relates back to the opening quote in this chapter, and that is recognition that language help is needed, but mathematics teachers are not language teachers. So, where and how do such skills develop? The second is when and how to help with ways of speaking mathematically so that as teacher you still listen carefully to what pupils are trying to convey (their exploring mathematical knowledge), that you assist the negotiation and development of meaning with students and do not block student meaning by prematurely working on how it is expressed (their display of mathematical knowledge).

This leads to the issue of *explicit mathematics language* teaching, or what could be phrased as the dilemma of *when to tell, model mathematical language vs. when to listen?*

The need for explicit mathematical language teaching was discussed by both Model C teachers in their interviews:

> *H:* *As you were talking, something struck me ...is the assumption that everyone understands English ... you'll say ... I can't think of an example ... at points when I give an instruction, I write up the word on the board so that no-one is unclear of what it is and I have realised how many kids from English-speaking homes never knew that word, the one I wrote up was the one I was saying and that was interfering with their maths a lot ... I can't think of an example, but it has happened to me several times. Where I would have assumed a few years ago in an all white class I would have just gone ahead and talked aways and now because there were black children in my class and I was writing up in a conscious effort to explain the English that I suddenly realised it was benefitting the English-speakers as well.*
>
> (II1, 404-420)

> *So:* *I think one notices it more with the black kids 'cause it is just so obvious. But there are some of the other kids who I only realised afterwards weren't quite sure what I was on about and they had been too scared to ask ...*

H: *The problem (in group work - with discussion in any language, report back in English) is ... if all your discussion is in Zulu you get to the concept then you can't report back in English so you can't talk about it in English 'cause you never developed it in English - they don't develop the English to speak with. This is a difficult question - it can be dealing with the problem or making it worse - and now I want to be able to explore it further. I am not sure which is better. If you start to try to develop the English while they are reporting then you can be putting words in their mouths instead of hearing what they have construed.*

(II1, 691-706)

S: *... like there are some kids who are really not good at explaining themselves, and I don't do anything to address that except to try to get them to explain it again because the class hasn't understood. And they still do it badly, and then I say can someone help him? And by listening maybe he will get the chance to develop. I haven't spoken to the English teachers about that but - want to because I am sure they have got strategies of actually developing a more exact way of communicating.*

J: *And that would be an important thing to do?*

S: *I am wondering. I don't know - these questions are starting to arise....*

(II6, 1165-1176)

The experience of both these teachers was that learners clearly have difficulties in expressing their thinking - but not just any thinking. It is specifically their mathematical thinking. As is well known, mathematics is hard to speak (Pimm, 1987).

These accounts were from contexts of shifting pedagogy, from teachers whose pedagogical practice had greater emphasis on learners' meaning-making and included learners exploring mathematical ideas and then sharing these with the whole class. It seemed that some learners need help to express their thinking both in English and mathematically (or, in Mercer's terms, in educated discourse). This

and other learners) (see above II6, 930-947), (b) encouraging learners to use whatever language is comfortable as they discuss and develop mathematical ideas with each other (the private space), but that when pupils report on their thinking to the rest of the class (the public space) this must be done in English (II1), and c) repeating ideas in English in different ways (II5) and asking learners to repeat words and phrases in English (II2).

Strategies (a) and (b) are common in ESL teaching and acknowledged as 'good practice'[3]. They are bound into pedagogical situations where learners are seen as active meaning-makers, and hence given an opportunity to discuss their ideas. This also accords with Levine's (1993) research on second language teaching developments (in ESL) that emerged as 'good practice' for all language teaching. The latter strategies discussed above are more linked to traditional teaching modes. Each strategy, of course, brings its own dilemmas and these are dealt with in the next section.

The question might be asked: surely all this holds for *all* teachers, not only mathematics teachers? It does, but in the words of one of the teachers:

> *T: It is complicated - the problem is the maths. I don't think they have a problem with hearing, they do hear what you are saying ... Because as I said, when you are teaching they don't say 'I don't understand' the English you are speaking. It is the maths you did that they don't understand. But when they have to say it - saying in English, that is the problem.*
> (II4, 452-470)

4.2 Developing mathematical communicative competence vs. negotiating and developing meaning.

As argued earlier, and recognised by all teachers, it is not simply the English that is the problem:

viewed as inevitable and necessary though 'unfortunate' thus reinforcing Rubagumya's (1994) claim that in contexts similar to Burundi, code-switching is tolerated rather than encouraged.

There is very little in the literature that alerts us to the dilemma of code-switching for bi- or multilingual teachers in multilingual contexts, and particularly so at a secondary level. As discussed in Chapter 1, research on code-switching has been in bilingual contexts, and mostly at a primary level. Code-switching and treating language as a resource and not a problem makes both political and pedagogical sense. Yet, from what the teachers in this study said, code-switching in secondary multilingual mathematics classrooms in practice is not straightforward.

An obvious question that arises is: does what teachers *say* actually happen in class? Do switches in and out of English occur as they have been talked about here? What are the purposes and the effects of switching? Perhaps, more importantly, detailed research with teachers into their switching in and out of English could illuminate whether and how such practice facilitates and/or obstructs mathematical learning.

The dilemma of whether the teacher should use a main language besides English in the mathematics class (whether the teacher should code-switch) was obviously absent in contexts where the teacher was English-speaking, and did not speak or understand any African language. While the dominant strategy for the ex-DET teachers above was to use and have learners use English as much as possible, this practice was taken-for-granted in settings where the teacher is English-speaking. What speaking strategies emerged as these teachers talked about *their* multilingual settings, that is, where the teacher is not bilingual[2], and some or earners bring to class main languages different from the language of instruction?

Strategies included (a) being alert to pupil-pupil discussion that occurs in another main language and asking that this then be explained in English (for the teacher

multilingual teaching, that is, they engaged in forms of code-switching. Their strategy was to use English themselves most of the time. But in the lower classes it was simply not possible to do so all of the time. Somehow, in their view, this 'problem' disappears in the senior secondary school. The dilemma apparent here is how to develop spoken English without jeopardising mathematical understanding in the lower classes and this dilemma emerges because of the particular multilingual context: teachers and many learners share a main language and this is not the language of learning.

This dilemma is not only a function of educational policy of English as language of instruction. There is a political dimension. For both Jabu and Thandi, pupil facility with English is also about access: access to tertiary education, 'to college or technikon and there is English on campus' (II4, 495-6); and access more generally where without English 'then the child will land up the victim'. (II4, 638)

The dilemma also reflects the common sense assumption noted in Chapter 1, and one that is reinforced by the teachers' own learning experiences, that language is acquired in use. Thus, if English is to be acquired, it must be used as much as possible.[1]

It is interesting that Jabu chose to re-explain the algebraic language of 2a - a in English, and further that the explanation invoked what Pimm has described as the symbol-object confusion so prevalent in teacher explanations of parts of algebra. 2a as '2 books' suggests that 'a' stands for 'books'. But algebraic language is such that 'a' stands for 'a number of books', not 'books' .

That aside, policy advocating multilingual teaching is in fact already in practice here. Bi- or multilingual teachers can and do switch in and out of different languages but face a constant dilemma, particularly in the junior secondary school, over how to develop both mathematical meaning and spoken English; over how, when and for what purposes there should be code-switching. Code-switching was

it is better to have discussions in English because there are other languages in the class besides Tswana.

Thandi discusses their reactions and responses:

> None of the students agreed that they used Tswana deliberately in their discussions. This may be because they know that it is policy of the class that during the maths period all communications must be in English. (Setati, 1994).

As discussed in Chapter 4, Thandi and her pupils are mutually implicated in the view that English must be spoken.

As a result of her action research with her pupils, she goes on to express her dilemma in relation to her pupils' code-switching during their group work.

> This is a dilemma because as a maths teacher I would like to have my students to understand the mathematical concepts and at the same time to have them master English as a language, especially that they learn mathematics in English. Grasping the concepts might mean allowing the students to use the language they understand better; in which case they will be free to communicate in their groups although the usage of English will not improve. On the other hand if they are forced to have their discussion in English they may either not do as required or they may withdraw and not communicate enough in their groups. (Setati, 1994, p. 189)

Her recommendation, for her own and others' teaching, is that pupil-pupil discussion should take place in any language - but 'reporting back', the sharing of *group or paired work in the public domain of the class, must be in English.* Moreover, she reflects on the kind of tasks that are appropriate for group discussion, as well as how to organise groups for productive and constructive functioning. As discussed in Chapter 5, these practices are now well established 'good practice' in language classes themselves.

T: *I think so .. I would like them to speak English to each other so that maybe when you speak to them - like you will notice when I ...l them to the board I never say hey you ... because I tried that ... and when I forced them to talk then they never wanted to come up front. So then I felt then only the same few will come up. Those that feel free to talk and others are not. Also it wastes time. Like I thought of trying and then I thought someone will come up here, write down his solution and not see anything, and then do you wait? Do you interrupt? So I didn't want to try. That is something I would love to improve when they go up ...*

(VI4, 49-80)

This articulated view of the importance of English and its consistent use in class in Thandi's reflective interview supports what she said in her initial interview (Chapter 5). But the reality of her lesson, that is, confronting what she saw in her video, interests her and she selects this as an area of action research to undertake and share with us in the workshops. She writes about her action research:

> ... during the maths period, students are expected to work in English, this has been policy in my class since I started teaching them; I always thought that they practised it, or at least I should say they gave me the impression that they do. The video, however, revealed that to me during that particular period. Ten groups out of twelve had their discussions in Tswana, Zulu or street language (Tsotsitaal). This raised a lot of questions in me: can't they understand English or does communication in English make it difficult to have a discussion?
>
> (Setati, 1994)

In her interviews with her pupils (see Chapter 4 for more detailed description) she asks about their Tswana and/or Zulu discussions, and the pupils, rather defensively, blame each other (for example, 'he started ...'), or suggest it was a 'slip'. They say that English use is better, giving the usual rationalisations: we need it for work; we are examined in the final examination in English. They also show a concern for fairness in a multilingual classroom, pointing out to Thandi that

Thandi's own action research focused on pupils' code-switching and has been described in Chapter 4. It is interesting and illuminating to elaborate here as this locates and contextualises the episode in Thandi's practice over time.

In her initial interview Thandi states that she experiences difficulties teaching in English 'mostly in the lower classes'. By Std 9 pupils have come to value English, understand the access it provides and even talk to each other in English.

> *J:* *And in Std 9 you get them to talk to each other in groups? yes? and how do they talk?*
>
> *T:* *They talk in English! yes, in English. I think some even speak English when they are alone.*
>
> (II4, 561-565)

However, the video of her Std 9 lessons reveals the contrary. After commenting that she talks too much, and that this is partly because she was nervous the first day, she opens her reflective interview with her noticing her pupils are not using English when they are working together.

> *T* *... even though the tape is not clear I can hear, but some others are talking in Zulu or in Xhosa and I can't hear what they are saying ...*
>
> *J:* *And is that usual?*
>
> *T:* *I only noticed it for the first time when I watched the video, not even in the lesson. Because when you get to them then they speak English.*
>
> *J:* *Is that a problem?*
>
> *T:* *I think they need to understand, so their own language is useful. But I think it is important for them to practice English ...*
>
> *J:* *So would you want to change what they do?*

as it is. This is reinforced, though with less frequency, by having pupils themselves explain 'why?' in their public responses to her questions. Her interest in reasoning is also evident in the episode below where she challenges pupils to resolve for themselves their confusing suggestions for the meaning of 'at most'.

In two double (one-hour) consecutive lessons observed over two days, Thandi first provides a number of related linear inequalities to be drawn and shaded. Typically, a number of pupils had not completed these as homework preparation and so time is taken in the hour for them to do so. Here Thandi encourages pupils to work in pairs or groups and moves around the class assisting and prodding where necessary. She also manages in this time, to have whole class reflection on the solutions they had drawn. Here a pupil is encouraged to the board to share his or her work, and others are invited to comment. Within the constraints of her large class, Thandi manages to insert elements of interactive processes into her teaching. Graphing inequalities occupies the time in both periods until the last ten minutes of the second period when the episode (p. 194) takes place.

3.3 Language

Thandi and many (but not all) pupils in her class are Tswana-speaking. As described in Chapter 5, there is a strong political commitment amongst a number of teachers on the staff in this school, to conducting all teaching in English. Thandi is part of this group of teachers. This commitment to using only English in class creates difficulties and challenges - dilemmas - for Thandi.

3.4 Code-switching by pupils

The episode that will be examined in this chapter is focused on the issue of code switching *by the teacher*. As Ndayipfukamiyo (1994) has argued, 'code-switching should be a resource for both pupils and teachers'.

multilingual classrooms. The episode is neither typical of Thandi's practice, nor rare (Erickson, 1986). As discussed in Chapter 3, classroom data in this study is in the form of snapshots of practice. The episode in this chapter (and those in Chapters 7 and 8) are selected because of what they exemplify and illuminate.

3 THE CONTEXT

3.1 The school and class

The episode that is described here occurs in Thandi's (T4) Standard (Std) 9 (Grade 11) mathematics class in a large black state school. Students do not have their own text books. They do have note books. There is a blackboard, chalk and enough desks and chairs. There is no other equipment and the walls of the classroom are somewhat bare. As mentioned in Chapter 4, after the class Thandi tells me that the day before, students removed political posters from the walls.

The class has 56 pupils, but about one third are not present on the day of the episode. The absentees were most likely at the funeral of a student from a neighbouring school. Disruptive absences from school were not uncommon in Thandi's school. In her initial interview, she talked at length about the difficulties of teaching in a highly politicised schooling context where teacher demoralisation had set in, discipline in the younger classes was difficult, and motivation of pupils often seriously lacking.

3.2 Thandi's approach

In her initial interview, Thandi expressed her concern that her pupils learn that mathematics is something 'you reason about' - mathematics is 'not rules, it's reasons'. This view of mathematical knowledge is reflected in her teaching practice. For example, as she works with her pupils to plot points to work out the shading of linear inequality graphs, she continually explains 'why' the shading is

learners, fo their access, through mathematics and English, to further education and the workplace. Thirdly, decision-making on code-switching inter-relates in complex ways with the mathematics register on the one hand and its insertion in school mathematical discourses on the other.

These assertions will be instantiated and illuminated through an analytic narrative vignette (Erickson, 1986) built from one episode in one multilingual classroom with one teacher, and her reflections on her teaching.

The chapter commences with a brief methodological comment. The context in which the episode occurs is then described. The episode, observations, reflections and discussion follow. This structure is to enable and enhance the reading of episode in the context in which it occurs.

2 BRIEF METHODOLOGICAL COMMENT

As discussed in Chapter 3, much has been written about the complexities of analysing, validating and reporting qualitative field research (Erickson, 1986; Rose, 1982; Hitchcock and Hughes, 1989; Maxwell, 1992, Woods, 1985). Erickson recommends a 'leap to narration' as a way of stimulating analysis of the mass of data that collects, with the 'analytic narrative vignette' being the foundation of an effective report of such research.

Erickson also argues that the significance of the selected vignette needs to be established in relation to the data corpus and the study as a whole. Significance is not internal to an incident or episode, but rather in its instantiation and/or illumination of assertions in relation to the study as a whole.

Of the teachers interviewed it is Thandi who most explicitly takes up the issue of code-switching, and the episode selected here exemplifies and illuminates the dilemma of code-switching as part of the three-dimensional dynamic at play in

code-switching even at senior secondary level. The episode used here illustrates a teacher's actions and reflections in relation to switching in and out of the language of instruction (English) and the main language (Tswana) of most learners in her class. The chapter discusses the teacher's actions and those of her pupils. There is no actual code-switching in the particular episode examined here. However, the teacher's reflections on this particular episode illuminate how and why she works with mathematical English the way that she does, and the effects of her actions on learners' interpretations of her mathematical messages.

I have also argued that teaching and learning mathematics in a multilingual classroom needs to be understood as a three-dimensional dynamic: access to English, to mathematical discourse and to classroom culture (educational discourse). This can be understood as managing code-switching (in the sense of changing languages), register changes and discourse (in the sense of working with both mathematical and ordinary English, and with both everyday and school mathematical discourses) and broader discourse changes (in the sense of specific classroom or educational and hence cultural processes).

In this chapter I will argue the following: firstly, while code-switching by both teacher and learners is arguably a means to improving the quality of pupils' mathematical learning, code-switching in a multilingual classroom is a complex activity. Nevertheless, treating the multilingual classroom as a resource and not a problem offers possibilities for improved learning. The dilemma of code-switching is thus not a matter of whether or not to switch, but rather when to switch and for what purposes. Secondly, in the South African context English is the primary language of government and commerce and remains the most prestigious of the now eleven official languages. Mathematics teachers' decision-making and practices are heavily constrained by the politics of access to mathematical English. While they see and express the value of code-switching, that is, also using languages other than English in mathematics classes to assist meaning-making, this pedagogical understanding interacts with strong political goals for their

CHAPTER 6:

TO SWITCH OR NOT TO SWITCH - THAT IS (OR IS THAT?) THE QUESTION

1 INTRODUCTION

In the following chapters, I use the language of dilemmas developed thus far to probe what teachers do and so elaborate their knowledge of their practices. Each chapter shows how context and culture create each other through a particular teacher, the particular context and the dynamics of working in complex classrooms. Each chapter simultaneously serves to develop dilemma language in relation to mathematics and language.

Chapter 5 argued that, like all teachers, mathematics teachers in multilingual classrooms face tensions that can be experienced as dilemmas as they go about their work. Moreover, different contexts give rise to different dilemmas. In particular, mathematics teachers who share a main language with many learners which is different from the language of instruction (this is the situation of most teachers in ex-DET schools in South Africa) face the dilemma of 'developing spoken mathematical English vs. ensuring mathematical meaning; or, whether or not to code-switch (use both English and the learners' main language).

In closing discussion of the dilemmas that emerged through the teachers' initial interviews in Chapter 5, the following questions were posed: what happens in practice? How do teachers manage their dilemmas in practice? What have teachers learnt? What might be the effects of their actions on the mathematical knowledge made available? And then, how and in what ways are teachers aware of their actions and the effects of their actions?

This chapter explores and elaborates the dilemma of code-switching in a secondary mathematics classroom and examines the dilemma teachers face in relation to

NOTES

1. This takes us into the debate in language teaching on the distinction between acquisition and learning - how much, or what parts/kind of language are acquired and what is learned, i.e. a function of instruction (Baker, 1993). It is beyond the scope of this thesis to enter into this debate.

2. It is important to add here that English-speaking white South African teachers all had to study Afrikaans (the other official language in the apartheid era) and in this sense they are bilingual. However, few, if any, learnt a vernacular (indigenous African) language.

3. In discussion with Dr H Janks, English methodology lecturer at the University of the Witwatersrand.

multiracial argue the benefits of explicit language teaching for all their pupils. In relation to learner-centredness, however, tensions reside here on how both to tell (be explicit) and to create opportunities for pupil creativity.

But as has been argued throughout this study, teachers' knowledge of their practices is more than they can self-consciously articulate. Clearly these accounts need to be related to actual classroom practices, to what the teachers do. How do teachers act in the face of the dilemmas described here? How do they manage the dilemmas they face? In Lampert's (1985) terms, how are their actions shaped by the practical and the personal, about who the teachers are at particular moments in their particular practice?

And the Berlaks ask:

> Will teachers change their patterns - co-ordinate and rationalise them more carefully if they become aware of the trade-offs implicit in their classroom behaviour (sic); both the trade-offs represented by each dilemma, and the trade-offs among dilemmas?
>
> (1982, p.165)

Are teachers' actions in the face of dilemmas defaults to dominant practice, resolutions or transformations? How are such actions understood by teachers? Do teachers use a range of strategies as they make choices in the face of dilemmas. Are these choices tacit or informed and conscious? Or are they fall-backs to the familiar?

The next three chapters each focus on an incident in a classroom and attempt to answer these questions while simultaneously developing dilemma language.

explain their occurrence: attempts to communicate are through language, and in the classroom, through a great deal of, but not only, verbal speech. In Vygotskian terms, language, as a bearer of meaning and motivation, is imbued with culture and history and hence not easily interpreted by learners with different cultural and historical experiences to those that underpin classroom mathematical processes.

Practically, however, these incidents were difficult for teachers to explain and understand. They point to a dilemma of explanation for teachers that I described earlier as: Is it the language vs. is it the culture? And as a dilemma of explanation it does not fit with the Berlaks' dilemmas. Again incidents of 'communication breakdown' are absent in the ex-DET teachers' interviews and this suggests either that they are somehow hidden, or that in such contexts, this kind of communication breakdown is simply absent. Video data, reflective interviews and workshops, unfortunately, did not provide sufficient scope for further exploration of this dilemma through a focused vignette. It clearly is an important aspect of working in multilingual mathematics classrooms and requires further study.

6 CONCLUSION

The complexity of teaching and the inherent tensions in the teaching-learning dialectic are such that all teachers face dilemmas. Detailed and systematic analysis of teacher accounts of their mathematics teaching in diverse multilingual settings revealed that teachers in different multilingual contexts face different dilemmas, thus supporting the notion of teaching as a social practice. Specifically teachers in ex-DET schools face the dilemma of code-switching. They need to ensure that pupils understand the mathematics they are trying to teach and that they develop their English. Teachers who have tried to make their classes more participative, communicative and inquiry-based face complex dilemmas of mediation - of moving effectively between learners' more informal expression of their mathematical thinking and more formalised school mathematical discourse. And teachers in schools that have suddenly and rapidly become multilingual and

Two interesting and difficult questions arise from this 'good practice'. The first is whether the silences in the ex DET teacher accounts suggest that aspects of diversity are hidden in the seemingly homogeneous black classroom? This is something to explore. It relates to the question the Berlaks ask about whether and how teachers treat pupils as unique or with shared characteristics, that is, whether teachers act differently towards different groups of children. What is interesting is that the Berlaks ask this within a multicultural context.

The second is the deep tension and paradox mentioned briefly above. Learner-centred curriculum initiatives with their more democratic ideals of listening to learners, of creating space for their creativity and meaning, of less authoritarian classroom practice, require teachers to stand back more. At the same time, such classroom processes rely on communicative competence in learners as means of sharing their ideas with others. The tension is that if communicative competence is to develop in mathematics, and this is equally important from the perspective of democracy, access and equity, then it requires mediation in general and explicit teaching of mathematical discourse in particular. The dilemmas inferred from the teachers' interviews are in support of a sociocultural perspective where the teacher's role is understood as enabling learners to travel across boundaries between knowledge areas. What arises here is the more specific mediational dilemma of *when to model mathematical language vs. when to listen*, and this is the focus of the vignette in Chapter 8. Here again is the personal knowledge vs. public knowledge dilemma in a form that focuses on both language and mathematics simultaneously.

Finally, some of the teachers' accounts categorised as 'culture and communication' revealed situations that point to the interaction of English proficiency, mathematical understanding and classroom culture. None was attributable on their own to any one of the three dimensions of the dynamics of mathematics learning and teaching in multilingual classrooms. These were clearly incidents of communication breakdown. Within a social theory of mind, we can

T: *(still talking about greater than etc at the end of the lesson) I think at the end they were just guessing, because one of them said 'greater than or less than' and then I said 'OK, how would we write that?' ... The next day we still had problems. They still felt that 'at most' was 'greater than', even when I started to try to connect it to 'not more than' ... I think it is something about 'at most', and they are looking at the word 'most' and 'most' means 'more' and so they want to write 'more than'. () It is not like 'at least', because they use at least more ...*

J: *Do you mean in their everyday language?*

T: *Yes. The next day I tried to explain the connections between 'at least' and 'not less than' and 'at most' and 'not more than' and then 'less than and equals to', et cetera, but Jeremiah was still not convinced. They don't just accept - they want to understand ...*

(VI4, 522-561)

Thandi and I do not explore the dilemma of whether or not to model mathematical English, that is, her intentions in relation to modelling through repetition. As already noted, the language of dilemmas only emerged after those reflective interviews. Nor do we explore her shifting into the everyday. Perhaps this is as much a function of my own commonsense at the time of the reflective interview - that the shift into the everyday could only have helped contextualise the concepts.

Thandi is adamant that her shifting in and out of English and Tswana - the teacher code-shifting - would not have helped. The mathematics register in the Tswana language had limitations ('I would run out of words'); moreover, resonating with what she later finds her pupils saying in their interviews with her, this was a multilingual, not a bilingual class, so her switching into one other language is problematic.

Her use of the table, and the manner in which she did so, was planned - it had its purpose - it was not a result of spontaneous action in the classroom nor lack of

a little difference. I have to use a long sentence for 'greater than an equals to'.

J: *And for 'at most'? and 'at least'*

T: *That is going to be problem to say it in Tswana, 'at most' and 'at least'. That is why I talked of 'not more than' and 'not less than'. I feel if they resort to Tswana, then, when they come to those terms what are they going to do?*

J: *... even English speakers battle with those terms. () Would it help to explain the idea in Tswana and then shift to English?*

T: *I think if I was to explain in Tswana I would run out of words. And for my mixed class it would also be a problem because not everyone speaks Tswana. So must I do it again in Xhosa and then Zulu? I would definitely run out of words and go back to English. For example, I can explain it in Tswana but if I am trying to say 'at most' I would say something like 'the limit is this' ... I would explain what it means but trying to find words to say 'at least'? ...*

... we still had a problem with 'at most'. 'At least' wasn't a problem because I said you can say 'at least' or 'not less than' then I () I didn't say 'not more than'. I didn't say 'at most' or 'not more than'. I just did 'at most' ...

J: *I wondered about that, why you didn't link 'not more than'?*

T: *I started with 'not more than' but purposefully felt that if I did 'at most' or 'not more than' together, then perhaps they wouldn't remember 'at most' and they would choose to stick to 'not more than'*

... but you see my problem? There () in the middle of that group is Nomsa, and she is Zulu and I don't know how much Tswana she understands ... like I can understand Xhosa but there are some words ... so there is a problem if the others are all fluent Tswana ... most the school is Tswana speaking ...
(VI4, 115-160)

Thandi thus has a clear justification for setting out the table as she does as well as the order she puts into the task. She also attributes confusion to previous meanings pupils have for 'most' as linked to 'more:

chains or scaffolding that Thandi builds are only partially or initially helpful. As they intermix (the everyday, the table) confusion sets in.

It was abundantly clear that, as the sequence of teacher-led questions developed, there was more and more guessing and confusion, perhaps illustrating the difficulties of managing the complex dynamic between access to English, to mathematics and to classroom discourse. Interestingly, Thandi leaves them with their uncertainty and confusion. Reflecting her concern for 'reasons not rules' she asks her pupils to take some responsibility themselves for thinking about and working out meanings for the next day, and reminding them that she has provided a context of meaning to assist them.

6 THE REFLECTIVE INTERVIEW

As mentioned, the reflective interviews were opportunities for teachers to reflect on and justify their actions. They were opportunities for them to talk more about their practices, as well as opportunities for moving between tacit and articulated knowledge.

While Thandi's opening comment is that she talks too much, her immediate focus of attention is the use of the learners' main languages when they are working in their groups. Her reflection moves quickly to include the difficulties of not developing English in the mathematics class in the long run. As described in Chapter 4, her arguments include the limitations of the Tswana language conveying mathematical meanings, the multilingual nature of her class, and hence equity issues in switching into one other language.

> *T: ... in Tswana it becomes a problem because () um () like if he explains in Tswana, then when it comes to the (terms? unclear) our language is unique; and when you come to 'at most' and 'at least' then what are you going to say? For in our language, 'greater and equals to' and 'greater' () there is*

In relation to the content of the interaction, Thandi appealed to everyday situations to provide a context of meaning for the concepts, and at no point switched into Tswana. She also uses a table to summarise the relationships between the contextual meaning, the concept in words, and the symbolic form. I remained curious as to why she joined 'at least' and 'not less than' in the table, but split 'at most' and 'not more than'. I thought this might have contributed to the pupils' confusion - that because 'at most' was split off it needed to have a different symbolic form. This impression was reinforced when I noticed that pupils initially volunteered the answer 'equals'. This response made little sense in relation to her introduction to the ideas. It did, however, make some sense in relation to the table itself, and typical mathematical classroom practice. Often in demonstrating the meanings of different symbols, and particularly $\geq$ and $\leq$, exercises ask pupils to fill in $\geq, \leq$ or $=$ as three different possibilities for relating numbers, and provide examples of each of these three. Having had the symbols $>$ and $<$ already in the table, and as typical educational discourse in the mathematics classroom would suggest, the other remaining possibility that is different is just $=$.

Thandi switches in and out of different discourses in her attempt to 'scaffold' (Mercer, 1995) these concepts, creating chains of signification from one set of discourses to another (Walkerdine, 1988). She switches or shifts in and out of mathematical classroom discourses (using a table and mathematical terminology and symbols) and everyday discourses (you have at most R50 to spend, lines 80 84). And she explicitly points out the shifts in the opening and closing utterances in the episode ('mathematical symbols and their verbal statements' ...'I gave you examples to think about. You can spend at most R50' lines 11-13; 130-131).

In sum, I observed that in this instance Thandi's tacit knowledge entails modelling (repeating and reformulating) mathematical English as well as shifting (moving back and forth) between discourses as she attempts to communicate the meanings of 'at most' and 'at least'. A point to be discussed later is that the signifying

multilingual classroom together have produced the teaching techniques of repetition and reformulating - modelling strategies - which, in her reflective interview (below), Thandi described as 'talking too much'.

Herein lies a tension and another possible dilemma for teachers in a context like Thandi's: *to model or not to model appropriate mathematical language.* This tension and possible dilemma is interesting. It arises in the current context of both 'progressive' and more 'critical' teaching approaches which call for less teacher talk and more pupil engagement in the classroom. In Thandi's context, how can appropriate mathematical language be modelled? By whom? Does repetition and reformulation by the teacher need to occur differently or more often than in homogeneous English-speaking classrooms or classrooms where there are more English-speaking pupils? In language learning terms, what mathematical language can be acquired (in use) and what needs to be learnt? Does this modelling in effect help or does it remove more opportunity for learners to use mathematical language themselves? These questions point to a further interesting question of whether research into and advocacy of progressive practice elsewhere, specifically in different language contexts, is appropriately recontextualised here, and for whose benefit? The issue is again different in the Botswana context Arthur (1994) has described which is similar to many rural contexts in South Africa. Here the teachers themselves are not fully bilingual - and their use of English is reliant on that learnt in schools and other formal institutional settings. Arthur calls this a 'final draft' style of English, which is often very limiting if that is the only form through which mathematics is communicated in a classroom. She thus challenges the policy of English as language of instruction in the primary school and advocates the importance of using the teacher's and learners' main language in class in the Botswana context.

What this brief discussion suggests is that any action in relation to code switching and mathematical language teaching must be a situated one (Lave and Wenger, 1991), appropriate to the specificity of the circumstances

5 MY OBSERVATIONS

It was abundantly clear that there was confusion (lines 80-112). Also, difficulties with this kind of mathematical language are quite common in monolingual English-speaking classes. But I did wonder whether and how an 'English-only' strategy was implicated, and whether and how the table was causing some of the confusion. That is, whether some code-switching might have helped, and whether access to these kinds of meanings would be more successful through a different kind of task. Pupils had had little opportunity to express their own meanings outside of answering or making sense of Thandi's questions. I obviously marked this episode as one of problematic communication and thus for specific discussion with Thandi in her reflective interview (VFN4B). As is discussed below, Thandi herself raises this episode for reflection and discussion as she talks about code switching difficulties in her reflective interview.

While watching the video of Thandi's class in preparation for the reflective interview I was able to elaborate my observations. In relation to the *form* of her interaction with the class on the concepts 'at most' and 'at least', I noticed that Thandi repeated herself, sometimes saying the same thing over and over again (lines 4-10; 14-17; 26-27; 56-61), and sometimes saying it in a different ways in English (lines 27-28; 64-72). She talks in a way that serves to repeatedly *model (mathematical) English*[1]. She also loudly and clearly re-expresses, sometimes with rephrasing, the responses pupils offer (lines 33-39; 64-72). This seemed to me to be one of her tacit practices learnt in the context of her multilingual setting. As Mercer (1995, p.34) describes, repetitions and reformulations are common techniques used by teachers. However, in Lave and Wenger's terms, while Thandi has learnt to teach she has become a relative old-timer - in a multilingual classroom context, she has simultaneously been immersed in a social practice where, on the one hand, clear exposition by the teacher is highly valued, and on the other where it is understood that it is important for pupils to learn (English) mathematical language. Thus, her identity as mathematics teacher and her

91	T:	More than R50 or less than R50. Is that what
92		you are saying?
93	Sabie:	(trying, mumbling) ... greater ... (then puts head in
94		hands and laughs shyly)
95	T:	How can we write that with a mathematical
96		symbol?
97	Sabie:	x greater than R50, less than R50
98	T:	Huh? x greater then R50, less than R50. Huh?
99		and what is that? If I say you go to the shops
100		and you can spend at most R50 how much would
101		I () would I be happy if you spend seventy
102		rands? (inaudible..).
103	S's:	no, no (mumbling)
104	T:	So what do we say? More than or less than? ...
105	S's:	(mumbling)
106	T:	Peter says it must be more than R50 and Mogapi
107		says more than R50 and Siza says equals to R50.
108		huh?
109	S's:	(some mumbling)
110	S5:	Less than R50?
111	S6:	Equals to R50?
112		(some interchange that is inaudible)
113	S7:	I think, um, the one must get more
114	T:	More? Which means I am saying it means the
115		same as not less then? because (interrupted)
116	S7:	No ... you spend more.
117	T:	How different is it from not more than or not less
118		than? () How are you going to write it in
119		symbols?
120	S7:	You are going to use greater than
121	T:	Which means you are saying 'at most' is the
122		same as 'at least'? huh?
123	S6:	No. It is just greater than.
124	T:	Oh. You mean this one is just greater than () not
125		also equals to?
126	S's:	(some together) yes
127	T:	And others are saying no. OK. I want you to go
128		home and check the meanings of 'at most' and
129		'at least' () and what is going to be your symbol
130		for each. And I gave you examples to think
131		about. You can spend at most R50. OK?

(VT4B)

verbal/word	mathematical symbol
not more than	less than or equals to
at least/ not less than	greater than or equals to

73 So, if I say or the problem says 'not less than',
74 then it is 'greater than or equals to'... OK we can
75 say also 'at most' () At most. () What do I
76 mean if I say 'at most'? (writes it into table).

verbal/word	mathematical symbol
not more than	less than or equals to
at least/ not less than	greater than or equals to
at most	

77 If I say you can spend 'at most R50', what do I
78 mean? huh?
79 S4: (inaudible)
80 T: Mogapi says 'equals to'. You can spend at most
81 R50. Does that mean I can spend R50 exactly?
82 ... Who agrees with him? ... Let me write: (and
83 writes 'at most R50' on the board). What do I
84 mean 'at most R50'? ... Peter disagrees.
85 Peter: You must get more than R50?
86 T: You think more than R50? ... Sabie's hand is
87 also up.
88 Sabie: plus minus
89 T: What do you mean plus minus?
90 Sabie: Not much more than R50 or less than R50.

00:53:21 to 00:55:05

T: And if I say 'at least'? () What do I mean if I say 'at least'? And if I want to say 'at least' I can say 'not less than' (and writes these both into the table).

verbal/word	*mathematical symbol*
not more than	*≤ less than or equals to*
at least/ not less than	

Then I can give an example for that. () If I say 'at least 10 people must attend the meeting' () 'at least 10 people must attend the meeting', how many people am I expecting at the meeting? I am saying at least 10 people must attend the meeting .... at least ten people must attend the meeting.... hm?

S3: It means greater or equals to.

T: I mean greater or equals to. So it can mean anything greater than 10 or less than 10 (slip here - she probably meant equals to 10). If I say at least, which means 10 is the smallest number of people I am expecting at the meeting. OK () So which means I am expecting either 10 people or more than 10 people. So the mathematical symbol for that is 'greater than or equals to' (writes symbol ≥ into the table).

18	T:	Let's look at 'not more than'. If I get into the shop and
19		I () I say you can use not more than 50 cents. What do
20		I mean? How much should I use? What do I mean if I
21		say 'not more than 50 cents' ?
22	S's:	(Murmuring from class in unison)
23	T:	yo, yo, yo (teacher's `way of signalling all are calling
24		out) ... If I say not more than 50 how much can I use?
25	Temba:	You can spend less than 50 cents.
26	T:	You can spend less than 50 cents. Temba says that I
27		can spend less than 50 cents. Not more than 50 cents
28		means I can spend anything less than 50 cents. Is that
29		right?
30	S's:	yes
31	Lindiwe:	(amongst a lot of noise) you can also spend 50 cents.
32	T:	So if I say 'not more than 50 cents', Lindiwe says we
33		can also use 50 cents () or less. () If they say not more
34		than 50 cents, Joe, am I including 50 cents? () Joe is
35		having a problem here. He does not want to include 50
36		cents, which means 50 cents downwards. So if I say
37		the phrase 'not more than 50 cents', what do I mean
38		mathematically?
39	S's:	(all start to call out)
40	T:	yo, yo, yo ... Sable?
41	Sable:	less than or equals to
42	T:	I mean less or equals to () which means the
43		mathematical symbol for that means ≤ (and fills
44		it in table). So if I get a problem in the form of a
45		sentence, and it has got a 'not more than' in it -
46		it means (pointing to the symbol ≤) less than or
47		equal to (and she writes: 'less than or equals to'
48		into the table).

verbal/word	mathematical symbol
not more than	≤ less than or equals to

You can use anything from 50 cents downwards.

KEY: (brackets within a data extract - researcher commentary)
T - teacher
S1, S2 - students whose names are not used
S's - all students
() very short pause
... - longer pause
bold - when speaker places particular emphasis

00:50:00 to 00:53:16 (Time of day: 13h24)

*T: And note that inequalities can be given () sometimes inequalities are given () inequalities may not always be given in **mathematical symbols**. They can be given in **verbal symbols** and you should be able to recognise if they say 'not more than' what will it be? () OK? () And I want us to look at that because sometimes I can use the words 'not more than' () I can use the words 'not more than' () So you need to check as to whether if I use the words 'not more than' do I mean greater than, or less than, or greater than and equals to, or less than and equals to. () OK () So I made a table there and I am going to compare my **verbal statements** () whatever statements I make verbally and then the **mathematical symbol** we use for that statement. So (she draws a table)*

verbal/word	*mathematical symbol*

For instance, if I say 'not more than', usually we use the words 'not more than'. But what does it mean in maths? And we are going to meet such problems when we get into linear programming (writes : 'not more than' into table).

verbal/word	*mathematical symbol*
not more than	

Through her action research, Thandi finds a way to manage her dilemma in a way that could be considered transformational. She is able to shift to seeing the languages in her classroom as a resource rather than a problem, and so move her ideas beyond the commonsense notion that mathematical English is best learnt if the main language of the pupils is not used.

Thandi's action research does not engage with her own use of Tswana (or another language besides English) in her interactions with the class - this is [illegible] focus. As mentioned earlier, advocates of code-switching see it as important [illegible] both the teacher and her pupils. Whether or not a teacher code-switches is crucially also part of the dilemma that emerged through the analysis of the initial interviews. However, at the time of the videos, switching by the teacher was not the practice in either Thandi's or Jabu's classrooms. Given the politics of the day, their location in typically politicised urban township secondary schools, and that their own mathematical and teaching qualifications were obtained in English, this practice makes sense. It is also likely to be mirrored in other urban township school settings by mathematically qualified bi- or multilingual teachers.

The episode that follows is a teacher-led introduction and whole class interaction on the concepts 'at most' and 'at least' in the last five minutes of Thandi's second videoed lesson with her Std 9 class. As the transcript shows, she teaches only in English. This episode is also an object of focus for Thandi in her reflective interview.

4 A VIGNETTE: AN EPISODE WITH 'AT MOST' AND 'AT LEAST'

In the previous lesson, and for most of this lesson (each one hour lessons), Thandi has been revising linear inequality graphs, building up to introducing linear programming. The class has spent the time drawing and shading intersecting inequality graphs.

* What do teachers say and do about mediating the development of scientific concepts?

* What do teachers say and do about working with the zone of proximal development in linguistically diverse classes?

We know that the way teachers cope with the complexities of teaching is deeply textured. As Chapter 6 dealt with code-switching, this chapter explores how one teacher copes with dilemmas of mediation in practice. The focus here is on Sue (T6) who has gone a long way to establish what can be called a participatory-inquiry approach[1] in her classroom. This chapter examines what Sue says and her reflections on how she and her pupils get things done in her multilingual classroom (what she does). It asks: what do we learn from Sue, from both her tacit and articulated knowledge of her practices?

I will argue that in a multilingual classroom, a participatory-inquiry approach to teaching and learning mathematics creates particular dilemmas of mediation for teachers. Managing these dilemmas (Lampert, 1985) can entail trade-offs (Berlak and Berlak, 1982) some of which are deliberate, others tacit. Trade-offs are tied up with worthwhile goals that sometimes conflict in moments of practice. Working to meet the dual goals of negotiating meaning (which entails validating diverse pupil perspectives and working with informal expressive language and learners' conceptions) together with developing mathematical communicative competence (which in turn entails access to formal mathematical language and to specific scientific concepts) is extraordinarily complex within the time-space relations in a school classroom. Trade-offs are inevitable. Moreover, teachers are acutely aware of dilemmas in managing informal, expressive and sometimes incomplete and confusing language, and the abstract and formal language of mathematics. That is, they are aware of dilemmas in managing both talking within and about a practice. What is obscured, however, is that a participatory-inquiry approach, and the possibilities it offers for learner activity and negotiation of

CHAPTER 7

A PARTICIPATORY-INQUIRY APPROACH AND THE DILEMMAS OF MEDIATION

1 INTRODUCTION

Chapter 5 described and explained how mathematics teachers in multilingual classrooms talk about dilemmas they face as they go about their work. These dilemmas are shaped by different contexts and conditions. Chapter 6 elaborated the dilemma of code-switching, of developing spoken English vs. developing mathematical meaning faced by teachers in ex DET schools. In other words, it responded to the research question: what do teachers say and do about using language(s) as a resource in the classroom and about simultaneously developing the language of instruction and mathematical language in their classes? (See Chapter 3, p.81)

This chapter focuses on dilemmas of mediation and particularly the dilemma described in Chapter 5 as developing mathematical communicative competence (educated discourse) vs. negotiating and developing meaning. This dilemma is found in the talk of teachers whose pedagogical approaches are more interactive and participatory. This chapter will thus explore the following research questions (see pp.81-82):

- What do teachers say and do in relation to working with both the informal, expressive language and mathematical conceptions that learners bring to and construct in class, and the formal mathematical language and conceptions in the curriculum that the teacher wishes them to acquire in the context of classroom communication and activity?

sequences. They might very well improve the quality of learning and knowledge constructed, but they also might not.

The question is not 'to switch or not' or whether 'to model or not' but 'when and how to switch and model?', for what purposes and with what effects. Grappling with situations and episodes of practice that highlight the dilemmas of code switching and modelling mathematical English could be a means, as it was for Thandi, to teachers' informed decision making in their multilingual mathematics classrooms.

NOTES

1. It is, of course, possible that Thandi did not intend to 'model' appropriate mathematical language but that with English as one of her additional languages, her repetition and reformulation is rather a function of her own command of the language. Whatever her purposes, the effect was a modelling over and again of mathematical English. The question then is also how this is 'read' or 'heard' by her pupils - what significance they attach to her actions. Do they see this as patronising in any way? or as reinforcing English expression? or as the teacher's insecurity? Also there is the possibility that *reformulating or rephrasing* a point in a bi- or multilingual class might be heard as a different point rather than as a rephrasing of the same point. And further, while 'modelling' by the teacher might serve a function, how does it relate to *learners' use* of mathematical language? These issues were recently discussed with Thandi and are commented on later in the chapter.

2. While there is not sufficient evidence for a clear claim here, Cummins' distinction between basic interpersonal communication skills (BICS) and cognitive academic language proficiency (CALP) discussed in Chapter 1 might also throw light on these occurrences. Thandi does not conflate BICS and CALP, but uses one to access the other, and breaks down the task of understanding inequality language. Cummins argues that a tendency in making a task accessible in English in bilingual settings is to break the task down. This in fact decontextualises the task into small segmented pieces and inadvertently then denies access to the task. Perhaps Thandi's breaking down of the overall task in a table produced an overall decontextualisation that obscured meaning.

It is also too simple to say: get the scaffolding into mathematical discourse or the chain of signification 'right'. Unlike Walkerdine's examples which are at elementary levels in mathematical discourse, here the discourse of the everyday is complicated by the earlier learnt mathematical meaning of most or more. It might, in fact, be more effective to move from the mathematical meaning of 'more than' (which is a concept they know) to its negation 'not more than' i.e. within the mathematical register, to teach the meaning of 'at most' as another way of saying 'not more than', and then embed both in 'at most R50' or 'not more than R50'. That is, move from the scientific to the everyday.

Moreover, the table brought in a whole set of signifiers, part of the educational discourse (ground rules) in the class, that in fact obscure mathematical communication - the table as medium becomes the message itself and the mathematical idea becomes confused. If we understand the table in Lave and Wenger's terms as a symbolic tool or resource in the practice, then we know that it needs to be transparent to the newcomers if it is in fact to be an accessible resource or a resource that enhances participation in the practice. We see from Thandi's experience that the harnessing of this mathematical practice as a resource in her classroom is a complicated affair with unintended effects as it is inserted in school mathematical practice.

Most significantly, we learn from Thandi that it is also too simple to say all would be well if both pupils and teacher were to code-switch. Both the pragmatics and the politics of simultaneously developing spoken English and mathematical meaning are complex. Intertwined with this is the additional dilemma of modelling that is tacitly inserted in Thandi's practice.

Each of these 'interventions' (a different kind of task, classroom organisation and talk, code switching) would make a difference. The activity and the practice would change. The important thing is that these practices too will have their

We also see from Thandi's lessons how the dilemma of code-switching is simultaneously interrelated with the dynamics of access to mathematical classroom discourse, wherein the mathematical register presents its own challenges[4].

As we have seen, there is a mathematical classroom communicative pattern at play here that significantly complicates matters. While Thandi is aware of code-switching and mathematical register issues, the signifying role of the table is more obscure. The listing of 'less than and equals to', and then 'greater than and equals to' brings an anticipation of something different needing to follow. This anticipation is brought in from previous mathematical classroom processes. At this point in the lesson, the table becomes the most visible cue for the pupils. They are inadvertently led to the kind of guessing that ensues and do not entertain the possibility of repeating either of the two symbols already in the table. They seek something different ('equals to') or a peculiar and mathematically meaningless combination of the two ('it is greater than or less than').

8 CONCLUSION

The point in this chapter was neither to applaud nor castigate Thandi's practices and her justifications, but rather to learn from her and understand how it is she goes about enabling access to mathematics for her pupils in her particular context. In particular, the purpose of this chapter has been to elaborate the dilemma of code-switching.

We learn that it is too simple to say: change the activity and all will be well since, as will be argued in Chapter 7, more participative task-based learning brings its own set of dilemmas to be managed. Rather, it is understanding that the nature of the activity does shape the quality of knowledge that is made available.

instruction and the students' L1/indigenous/home/mother tongue or main language. Code-switching is used in 'subtle and skilful ways by both teachers and pupils to manage difficult teaching and learning situations' (Rubagumya, 1994, p. 2). Code-switching in bilingual classrooms is an important classroom resource for both pupils and teachers.

However, from social practice theory, we know that knowledge is situated - tied to the context and activity of its production. As Lave and Wenger (1991) argue:

> ... learning is an integral part of generative practice in the lived in world ... (p. 35)

We learn from Thandi and her pupils that code-switching in a multilingual classroom brings new questions. It is no straightforward matter, both in terms of which language is used if the teacher is to switch, and then how to find appropriate language in Tswana for example.

In addition, dilemma language was extended through Thandi's tacit practices to include the dilemma of modelling mathematical English in a multilingual class. Modelling serves purposes in her class not least of which was contextual (the only thing you could ensure in a dysfunctional system was that you had 'done your job') and thus tied with her identity as a teacher at that time. On reflection, however, she sees this tacit practice as less an intention to repeat and reformulate mathematical English for the benefit of the pupils, and more as the need for teachers to show that they know their mathematics. It thus becomes 'talking too much'. Here is a dilemma created by a disjuncture between her embodied practice and what she now believes. The questions posed earlier in relation to the specific location of teaching techniques like repetition and reformulation in multilingual classrooms suggest the need for further research. These questions are picked up tangentially in Chapter 8 through the elaboration of explicit language teaching.

Pimm's (1987) analysis of the mathematics register and its implications for teaching and learning mathematics.

> Registers have to do with the social usage of particular words and expressions, ways of talking but also ways of meaning ... pupils at all levels must become aware that there are different registers and that the *grammar*, the *meanings* and the *uses* of the same terms and expressions all vary within them and across them ... (pp. 108-9)

What Pimm argues is that even within the mathematics register, meanings shift. Mathematical meanings are not forever fixed but shift in relation to mathematical use. Pupils need to become aware of such shifts.

Furthermore, from the analysis in Chapter 2 we know that mathematical meaning-making is not simply a matter of awareness. What we learn from sociocultural theory is that, in Vygotskian terms, 'at most' is a scientific concept, linked with and emergent from other concepts. It is bound in with meanings of related concepts and their use and thus shifting into the everyday might well not be sufficient to attach the appropriate new conceptual meaning.

For Thandi, all this is complicated by the fact of her working solely in English, and her dilemma of code-switching. To repeat:

> Code-switching is when an individual (more or less deliberately) alternates between two or more languages ... code-switches have purposes ... (and there) are important social and power aspects of switching between languages, as there are between switching between dialects and registers'
> (Bennett, 1994, p. 77)

and

> code-switching facility enhances flexibility ... greater adaptability to changing environments (Bennett, 1994, p. 127)

As has been discussed, research emphasis in bilingual education has been on code-switching which means switching between the official language of

Walkerdine argues:

> ... the object world cannot be known outside the relations of signification in which objects are inscribed. In this sense, then, the shift into mathematical discourse becomes a shift or transposition from one practice and system of signification to another.
>
> (1988, p. 121)

> ... non-mathematical practices become school mathematical practices, by a series of transformations, which retain links between the two practices. This is achieved, not by the same action on objects, but rather by the formation of complex signifying chains, which facilitate the move into new relations of signification which operate with written symbols in which the referential content of the discourse is suppressed ... (p. 128)

Walkerdine comes to this claim through her detailed examination of the teaching and learning of mathematics in the early school years. She shows a teacher skilfully building chains between: a context where three and four are joined, three and four objects joined together, and finally 3 + 4 = 7. She also shows how when chains across discourses are not carefully built, but assumed, confusion can ensue. 'More' for her attaches to the painful memory of the opposite of more being 'no more', not less. And that 'bigger than' relations in the family were more than just physical size. As teachers move into home relations in the attempt to contextualise and make more sense of the mathematics, they bring in other signifiers that could, in fact, cause confusion.

However, Thandi's difficulty here is not chaining across different discourses (mathematical and everyday) but rather within mathematical discourse. She needs to try to dislocate the meaning of 'most' from 'more than' and relocate it as 'at most' and the negation of 'more than'.

Walkerdine's analysis thus does not go far enough in interpreting more complicated mathematical teaching and learning. We need to turn, instead to

in some ways all a teacher could ensure was that 'she has done her job', that she 'has explained it as best as she can, and over again'. So if pupils do not learn then 'she has done all she can' - she is not to blame.

Thandi now stands firmly behind the recommendation she made after her action research, that a key to improved mathematics learning lies in the kinds of tasks learners are given and possibilities for pupils to work effectively in groups. Her view now resonates with a sociocultural theory of practice (Mercer, 1995) where the nature of the activity and talk in a classroom is what fundamentally shapes what is learnt. More task-based learning activity (learners working together and with the teacher on tasks) provides opportunities for language to become a social mode of thinking, for pupils to have more opportunity to be immersed in educational and educated discourse and so appropriate different kinds of knowledge.

However, activity is always situated. Scaffolding and modelling are both appropriate at different times and for different purposes.

7 DISCUSSION

The episode in Thandi's class is the three-dimensional dynamic (*access to English, to mathematical and to classroom discourse*) at play. We see the criss-crossing of discourses that Thandi and her pupils have to manage. Accessing 'at most' and 'at least' can, for example, be through the ordinary English use of those terms, that is, through contextualisation in the everyday.

meaning. However, the mathematical practice of tabulating symbols and differentiating them connects or resonates with similar practices related to the learning of greater than and less than. Thandi does not see how the table itself too might have conveyed an obscuring message rather than being a resource, how it might have been implicated in what she sees as 'just guessing' by the pupils towards the end of the lesson.

It is also possible to argue that her teacher-led interactions in themselves led to students guessing at what she wanted, irrespective of the table. But none of the pupils' guesses were either > or <, that is, the two symbolic forms already in the table. This suggests that the table had a visibility and signifying role.

In a recent and informal discussion with Thandi on my analysis in this chapter, she shared her current thinking and further learning about mathematics teaching. She was clear that in the episode discussed here, 'modelling' was not an intentional strategy for her. She was rather trying to scaffold learners into the meaning of 'at most' by linking and building on what they said, referring them back to what they previously said, and by posing questions that would provoke some conflict. Since the videoing in her classroom Thandi has become more and more aware that she 'talks more' in some circumstances than in others. She is much more comfortable with listening to and talking less when she works with 'young children'. But she noticed herself 'talking a lot in a workshop with teachers the other day'. She explained that while repetition and reformulation by the teacher might play a modelling role for learners, their use is perhaps more a function of needing to demonstrate one's knowledge - of needing to show older learners and adult teachers that, in fact, as teacher, one does know. There is no similar pressure with primary level learners.

She went on to explain that at the time of the video, the breakdown of a learning culture in school created political pressures in the classroom in addition to those of access to English. Conditions were so difficult, discipline such a problem, that

Thandi is currently registered for an M Ed and her research focus is teachers' and pupils' code-switching in a Std 3 (Grade 5) class on the West Rand in Gauteng.

In relation to what we learn from what she knows, Thandi's knowledge at the time of her video and her reflective interview reveals quite clearly that code-switching, and its bilingual assumptions, is no simple matter in a multilingual class. Both she as teacher, and her pupils are conscious that the use of the dominant main language in a class for meaning-making will exclude some, even if it is a minority, of the pupils. Thus research on code-switching and treating bilingualism in a class as a resource needs to be extended to its management in a multilingual classroom.

We learn further that there are practical difficulties in drawing in a language like Tswana to work with the English mathematical register and hence the dilemmas of code-switching for teaching in multilingual settings.

Through Thandi's tacit knowledge we learn of another possible teaching dilemma in the current context of what is considered good (be it critical or progressive) educational practice, and that is the dilemma of whether to model or not to model mathematical English.

We learn too that the everyday is invoked and with some success to access the meanings of the concepts in the lesson. However, previous verbal and symbolic mathematical meanings attached to English words like 'greater than' and 'most' carry over into this new situation. They interweave with and complicate what at elementary mathematical levels are more accessible signifying chains.

Thandi is aware, from her teaching and her own observations of her teaching, of difficulties of access to and through English and to and through mathematical English. What is obscured from her awareness is the signifying role played by the table in this episode. Using the table in the way that she did was a conscious choice. Thandi separates out 'at most' and focuses on it specifically to ensure

forethought and planning. On the basis of her past experience, it seems, Thandi wanted to single out 'at most' for special attention. She expected this to be the most difficult mathematical phrase for her pupils because of the attachment of 'more than' and 'most' to the symbol and meaning 'greater than'.

Thandi struggles to teach the difficult mathematical language of 'at most'. She also struggles with the resources she harnesses to help her (the everyday, and the table). In Lave and Wenger's terms we can see how she attempts to provide access to resources of school mathematical practice - links between symbolic and verbal forms; tabulating; everyday contextualisations.

During 1994, after our workshops and her own research into her pupils' code-switching, I gave Thandi the transcript of this episode to check for accuracy against the video itself. Because I could not understand Tswana, I was concerned there might be instances that I thought were inaudible, but instead, were simply not intelligible to me. This turned out not to be the case and Thandi only made one minor correction to the transcript. However, on returning it to me she commented that it was not just that she could see she 'talked too much', she could have arranged the task differently, and that would have helped - but code-switching, especially by the pupils in her interactions with them, would also help.

Over time in the research project and most importantly, through her action research, Thandi's adamant view of 'Tswana as limited' (that is, a problem in the mathematics class) has shifted to seeing it more as a resource to be harnessed. She has found ways to manage her dilemma of code-switching by her pupils, and has moved to talking about how to manage her own code-switching.

What has Thandi learnt and what do we learn from Thandi? At one level, we see what Thandi has learnt from her own research into her practice. That she has learnt so much from research activity testifies to the value of teacher research.

and its effects on mathematical knowledge in classroom interaction have been shaped in complex and interesting ways.

Such shaping is, of course, made technically possible by electronic recording of classroom life and teacher reflections: any and all events can be observed again and again, going back and forth in time. However, once electronically captured, a reflection, a lesson or any part of it, is taken out of its context (Brown, 1990; Erickson; 1986). Like any other record of the event, it is still a record and not the event. Interpretation requires a context. Joe's stretching, Sue's actions and her reflections needs a fuller context.

5 CONTEXTUALISING THE VIGNETTE

5.1 Thickening the description: the incident and my observations

5.1.1 The school

Sue's school is a well-resourced private school. Classes are small (around 20 pupils). The vast majority of pupils are black. English, the language of instruction is not their main language. Many pupils are on bursaries, and thus not necessarily from economically affluent families. Most teachers (including, Sue) are white and English speaking. All are well qualified and a culture of professionalism and inquiry permeates school and staffroom. Sue's notions of teaching and learning are thus supported in her school. This is important because the difficulties she might face occur despite this support[1]. Of course, Sue's school is located within a broader schooling system, and she must work with the canonical school mathematics curriculum

S: *With the Std 5's, because their language is much weaker, and they work in partners, I encourage them to first try it and then they talk about it - but I do question the way they talk to each other ... to what extent they really challenge each other // If you are not sure of your own ideas then it is hard to challenge and if they are not great at explaining, they don't understand each other. I encourage them to do it, but often they don't.*

(VI6, 234-259)

She particularises her concerns later in the interview when we view Joe's explanation of 'stretching' angles:

He doesn't really answer her question. They are not communicating - and that happens a lot! He can't hear her question and she can't hear his explanation.

(VI6, 641-644)

As the tape reaches the point where she teaches angle labelling, she comments critically on her actions:

Now I am deflecting more.

(VI6, 651-652)

Sue's opening general comment above pertains to the incident and mirrors many of my observations - we share concerns that communication is problematic and that her actions are indeed 'deflecting'. It is pertinent to point out, however, that we are not similarly interested by this incident. In the unstructured reflective interview it is I, the researcher, who brings Joe into focus. The incident thus could stimulate discussion on both what she says and what she does, with the study being enhanced by differences between teacher and research interests and orientations.

Now, some months later, having sat with the data (with a transcript and repeated viewing of the incident and its antecedents during the lesson, with copies of pupils' written work, with Sue's reflective interview and data from the other teachers), my initial thoughts related to mathematical communicative competence

Sue's question about starting with 125 degrees does not resurface.

From my perspective as researcher, this incident promised to provide insight into diverse communicative competence within and across learners, and how teacher actions are shaped by problematic communication and have effects on mathematical knowledge, that is, into a nest of problems pertinent to the research project as a whole.

4.3 Some of Sue's reflections

As mentioned earlier, there are commonsense assumptions about the benefits of interactive learning: that it will democratise; that it will make communication less problematic. More specifically, the Cockcroft Report (Cockcroft, 1982) stimulated what has become a new commonsense notion (though not necessarily common practice) that mathematical learning is enhanced by teacher-pupil and pupil-pupil discussion. After some years of promoting interaction and discussion in her mathematics class, Sue has pedagogical concerns that could be ascribed to managing the democracy-development tension, and they are writ large:

Sue's opening point in her reflective interview is:

> S: *... the thing that worries me the most is that I am not sure whether, I am not sure to what extent it helps them learn. I think that talking to each other is not unproblematic. I think a lot of the kids don't listen. Maybe they are too young. I think. You can see it with the questions [[they'll ask a question and say I don't understand and then the one who is up will try to explain and it doesn't really help but they are being polite and they are not quite sure and they say 'OK fi. e'. I am not sure they understand.*
>
> (V16, 7 28)

As we observe the video of her lesson, she elaborates:

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As we observe the video of her lesson, she elaborates:

4.2 My initial observations

At the outset, while observing the lesson in process, this incident caught my interest. Firstly, I was impressed by Joe's dynamic, relational conception of the angles of a triangle - how angles change in relation to each other[1]. Yet he struggles to explain himself clearly, to find the words and illustrations to express his ideas publicly. Rose's question and Joe's response suggest that they did not understand each other and Sue's mediation focuses on clarifying Rose's question, and then on how to label angle size clearly, on estimated angle values in the diagram, and away from the actual mathematical content of the task - away from 'stretching' to form an obtuse angle to labelling angles.

In my field notes (VFN6A1) I marked this as an instance of problematic communicative competence - of a difficulty with mathematical English - so as to ensure I discussed it in the reflective interview with Sue. Specifically, I noted that the angle labelled 37 degrees was ignored, and that this contradictory 'estimate' might have been confusing. I also recorded my sense that the teacher did not share my enthusiasm for Joe's conception.

After studying the video on my own, prior to and in preparation for the reflective interview with Sue, a number of things interested me that are important to point out now - but will only be evidenced later in the chapter. While Joe battled to explain himself to the class, earlier he had managed to convey his reasoning to both Sue and his partner, albeit with lots of particularist language and pointing. Sue is not entirely happy with his explanation, whether 'it covers all possibilities' and suggests he tries to 'start with an obtuse angle like 125 degrees' (see p. 229). In the recap at the beginning of the following lesson the next day, Joe's partner volunteers and summarises his reasoning quite clearly to the class.

> *... He said miss, um, you stretch B miss, then A and C will get smaller.*

degrees, uh, that 10 degrees, let's say B had ... uh uh if if if one angle stretches then, uh, the the two angles, the two other angles have to contract.

Sue: *Okay what do other [illegible] think? Any questions? Rose?*

Rose: *Isn't that triangle the same as the other one if you measure [(]*

Joe: *I was just doing an example, I forgot what angles I was using in my book [] but [] they are supposed to add up somewhere near to 180 degrees [].*

After some teacher-mediated interaction between Rose and Joe during which Rose is able to clarify that her question is whether the one triangle is the 'same as the other turned upside down', Sue says:

Sue: *I think Joe maybe the first problem is that you haven't shown these angles on the picture and lots of people do this - they write the angles outside the picture. OK Now you know what you mean and I know what you mean and maybe some people know what you mean. But to be clear (she writes the angle sizes inside the triangle), do that. Put it inside [] Now, are these two triangles the same just turned upside down?*

She continues interacting with the class to ensure they understand that while the triangles 'look the same', they are not. So, Joe in not 'wrong'. The bell rings but she continues:

Sue: *[] it does not really matter what they really measure - we still get what he is trying to tell us because he has shown us an example of what he has done [] we will come back to this tomorrow.*

(VT6A1)

4 A VIGNETTE: AN INCIDENT AND REFLECTIONS

4 1 From stretching to labelling angles: an incident

KEY: (brackets within a data extract - research commentary)

(0) - inaudible utterance

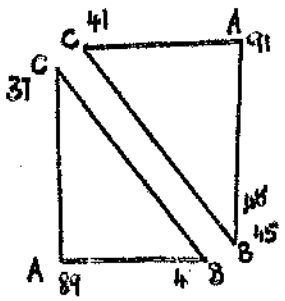
[] - unnecessary utterances edited out

() - short pause

... - longer pause

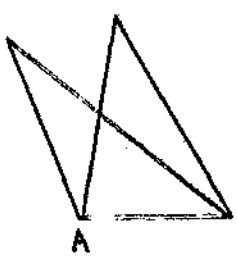
Joe, Std 6, is 'reporting' to the class, his explanation for a worksheet question 'Is it possible to draw a triangle with two obtuse angles?'

While talking, he draws the following two triangles on the board:



Joe: I said all the A's must be like more than () they must, uh, be the biggest in the triangle, um, so that if, uh, if this A here, say, is like 89, () and then these are say 37 and (mumbling to himself, ya, ya) 44, ya. And then in this one, number two, () it will be an obtuse angle. I said 91 and this is 44, () and this here is 46, no (crosses it out and puts 45 - all 'labels' are outside the triangle). And I said like if A, if A is going to stretch, () if A is going to stretch (pointing to 91) then these two angles here ... if it has to stretch then these two, like these two they are going to contract.

He draws another 90 degree angle below, and re explains:



If this here, if this is A, if A is here now miss and if it has to stretch, like these two we gonna have to (()) them both ... if this is 90, and you if you, if you, if it is gonna (()), turn to be lets say 110 or something, () (drawing the obtuse angle) then this one here (pointing to top angle) will be smaller than it was before, it was before, so, so if it was, say, 40 here then it is going to be 30 here, uh () then A is going to be taking that 90

mathematics is a deductive system where you must 'think more mathematically ... reason more logically', not just rote recall (II4, 107-109); for Helen, the challenge is 'to make explicit the mathematical language involved' (II1, 6-7); mathematics (among other things) is a language, and her greatest thrill is when pupils 'use maths language to explain something to their friends... they start to use the correct language' (II1, 345-347). In their reflective interviews, both these teachers comment on the difficulties some pupils have articulating their thinking (VI4, 27-28; VI1, 210-211) and note that when their pupils are interacting, often they do not really engage each other. Indeed, mathematics is hard to speak (Pimm, 1987).

Thandi says that, in their groups, they don't explain, they 'just say what is to happen' (VI4, 46-47). Helen elaborates that while she tried to explicitly teach pupils to ask each other why, to demand an explanation, they don't. Their attention is on getting out a solution, even in open-ended investigations. She is finding

> *... increasing difficulty with the word 'discuss', they can't ... I don't know what it is // They are not talking to each other properly. They are not listening to each other // The whole issue of discussion is very complicated for me and I am struggling with it all the time. It is not something that only comes up occasionally.*
>
> (VI1, 299-320)

In addition, the selected incident occurs during 'report back' on work pupils have been doing. As described earlier, reporting back is common practice in classrooms where pupils work on a task in groups or pairs. Even in traditional classrooms, where pupils will work on an exercise to practice a skill or concept just taught, public articulation of answers to a problem and methods of arriving at them is often sought. For example, public sharing of homework is used as a resource for whole class teaching and learning. The significance of the incident selected in this chapter extends to possibilities for illuminating practices in a wide range of mathematics classrooms.

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embrace democratic and developmental ideals and that are both personal and pragmatic. Insight, that is, into the very real and concrete challenge in education of working the democracy-development tension and its interrelated dilemmas.

3 METHODOLOGICAL COMMENT

In her initial interview, Sue clearly articulates the value she places on mathematics being 'something you can talk about' and 'have your own ideas about' and not something you 'just do'. Further, it is not simply about getting answers; it is also about 'asking questions' (II6, 13-15, 25, 75-76). Her greatest thrill is 'Good questions, when pupils come up with good questions' (II6, 726-727). Sue clearly values mathematical knowledge as social and personal, diverse and problematic.

'Typical' activities in her class are 'pupils working in pairs' on a task, an investigation for example, or comparing homework, and her 'in front with whole class discussion'. However, her range of strategies 'depends on the class'. In classes where there are a lot of weak pupils, there is 'more individual teacher supervision' (II6, 737-754). As will be shown, Sue goes a long way to establishing the participatory-inquiry approach that she values.

According to Sue, Joe and Rose (two pupils prominent in the incident below) 'are not communicating' and she adds that this 'happens a lot'. This is said as she observed the incident. Earlier in the reflective interview she says: 'if they are not great at explaining, they don't understand each other' and this happens 'often'. The incident selected instantiates problematic communicative competence and is thus illustrative of common occurrences in Sue's class.

Two of the five other teachers organise part of their lessons for pupil-pupil discussion, though in different ways. They also have different conceptions of mathematics and value different kinds of pupil behaviour. For Thandi, the challenge of teaching mathematics is getting pupils to 'reason'. For her,

through having pupils explain their ideas to the rest of the class. The task-based, interactive mathematical activity that is provided in such a class offers learners a qualitatively different mathematical experience, and hence possibilities for mathematical learning and knowledge development that extend beyond traditional 'telling and drilling' of procedures (Adler, 1993b).

A participatory-inquiry approach places particular demands on both teachers and learners. As the incident in Sue's lesson will show, this approach can, at times, turn in on itself, constraining possibilities for the development of scientific concepts and so mathematical knowledge development. This highlights what in contemporary South Africa is well known as the 'democracy-development' tension[2], a tension that is expressed at all levels and in all classrooms. In school mathematics, we can understand the tension as between developing democratic practices and developing mathematical knowledge, or in Sue's terms, between mathematics as 'something you can talk about ... have your own ideas about', and access to mathematical knowledge curtonised in the mathematics curriculum[3].

Contained here are a number of the Berlaks' (1981) curriculum dilemmas: of knowledge as given vs. knowledge as problematic, of learning as social vs. learning as individual, and centrally, of knowledge as personal vs. knowledge as public. More obliquely, the societal dilemma of equal vs. differential allocation of resources can be discerned. Each dilemma invokes questions about mediation, about communication and a teacher's and pupils' roles in the construction and appropriation of mathematical knowledge in the classroom. Each dilemma is explored in this chapter and specifically focused on language and mathematical knowledge.

As has been argued, the personal and the practical are deeply intertwined in the management of dilemmas. Through an in-depth analysis of the actions and reflections of one skilled teacher in a particular context, we gain insight into the complexities of teaching mathematics in multilingual classrooms in ways that

meaning, can inadvertedly constrain mediation of mathematical activity and access to scientific concepts.

These assertions will be instantiated and illuminated through an analytic narrative vignette based on an incident in Sue's multilingual classroom and her reflections on it. This chapter commences with a discussion of a participatory-inquiry approach and a brief methodological comment. Unlike Chapter 6, the vignette is provided first and followed by a detailed description of Sue's class and her reflections. The detail is necessary precisely because Sue's approach is distinctly different from dominant approaches to teaching and learning mathematics. It is a rich context for exploring the nest of research questions above.

2 A PARTICIPATORY-INQUIRY APPROACH AND RELATED DILEMMAS

Before attending to Sue, some general comments on what I have called a participatory-inquiry approach are necessary, particularly in relation to dilemma language.

A participatory-inquiry approach to teaching and learning is often driven by democratising intent, with dual goals of moving away from authoritarian approaches to teaching, learning and knowledge, and improving the socially unequal distribution of access and success rates. The underlying working assumptions in this case are that knowledge and learning are social and that a participatory approach to learning and teaching mathematics is more appropriate, meaningful and effective than a traditional teacher-centred approach

In a participatory-inquiry approach, pupils are expected to take responsibility for their learning. Typically, they are provided opportunity to engage with challenging mathematical tasks, either alone, but more likely in pairs or groups. The knowledge pupils bring to class is recognised and valued. Diverse and creative responses are encouraged, and justifications for mathematical ideas sought, often

5.2 Thickening Sue's reflections

As discussed earlier, Sue is aware of difficulties pupils have. On Joe's difficulties articulating his thinking, she says Joe 'does okay' but she doesn't know 'how to move them on. I don't know how to develop the language'.

5.2.1 A good explanation is a generalised one

On the content of Joe's thinking, she says:

> *S: I think what I was saying to him is you started with one triangle and you explained it - so now start with a different triangle // I am definitely pushing Joe more 'cause I felt the others are more generalised explanations. He was starting off with a specific triangle.*
>
> (VI6, 502-521)

After some discussion with her on why I thought his conception was sophisticated and dynamic and had a generality to it, she adds:

> *S: ... Ja, that IS a VERY good explanation, except I did want him to generalise more.*
>
> (VI6, 584-585)

It is difficult for teachers to always see pupil perspectives:

> *J: What is so fascinating about this is how do you see everything*
>
> *S: you can't ... especially when he is not very good at explaining himself //*
>
> (VI6, 587-591

This affects what gets affirmed:

> *S: They must, they have to. I mean in terms of affirmation: how do I know that something is good. how do I say that is a good question. Its because it is*

to the content of the task. Why is it that Joe and Rose battle to engage each others' ideas?

When Joe reports, Rose, who perhaps cannot read his representation on the board, also asks a seemingly off-task question. The two transcripts support my sense, while observing the class, that in this report back session constraints are operating on pupil-pupil negotiation of meaning in relation to the content of the mathematical tasks - most discussion and mediation is related to questions of clarification of elements of the report given. Such questions are, of course, part of learning mathematics', The point here is that if questions are all of this type, deeper mathematical thinking could be constrained.

Sue intervenes when she thinks there is confusion. If we recall her individual interactions with Joe and Rose while they were on task, she listens to what they are saying, supporting their thinking, but also pushing them both to think in more generalised terms (to Joe: try 125; to Rose: are you sure it is always the case?). She thus seems to have a notion in mind of what a more generalised answer could entail. This individual mediation of a scientific concept does not, however, enter the public space of whole class mediation.

With hindsight, it is interesting to me that on observation, I never considered Rose's presentation as at all problematic. While it is more proficient/fluent than Joe's, on closer examination, she presents her conclusions more than the approach to reach these. It thus l[illegible] an accessible sequencing. It too is problematic, though for very differen.. reasons. In Sue's second class that I observed and recorded, the two pupils publicly reporting on this same task also display difficulties in formally articulating their thinking, in talking about mathematics. There are similar elements of problems with fluency and sequencing.

24 *Sue:* *Joe are you talking about acute angles in a triangle or acute*
25 *angles in that diagram (points to the board)? (mumbling) In a*
26 *triangle? Well, what do you think? Can you use other angles*
27 *in a triangle?*
28 *Joe:* *no*
29 *Sue:* *Has Mzi's explanation helped or are you still not sure?*
30 *Joe:* *yes*
31 *Sue:* *Sure? OK ||*

Joe and Rose clearly 'are not communicating' (lines 13 - 17). Despite Mzi's and Sue's participation, and although Joe says the explanation has helped, the benefits of this discussion are far from clear. Joe's 'yes' is unconvincing and it would have been interesting and perhaps beneficial if Sue had asked 'How?' instead of 'Sure?'.

32 *Thanu:* *Uh miss, he said miss to get two obtuse angles you get*
33 *a quadrilateral miss... does it necessarily have to be a*
34 *quadrilateral?*
35 *Rose:* *It does.*
36 *Sue:* *What do you think Thami?*
37 *Thami:* *We got a kite miss ..||*
38 *Many:* *(exclaiming - including Rose). It is a quadrilateral ...*
39 *(laughter some mumbling).*
40 *Rose:* *If you are trying to draw a triangle with two obtuse that*
41 *(pointing to her diagram) is ...what you get.*
42 *Sue:* *Are you happy Thami or is there still something*
43 *worrying you?*
44 *Thami:* *OK...*

Joe's 'stretching angles' follows this.

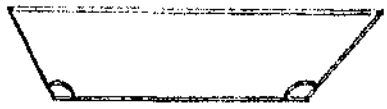
In this report back time, Sue invites pupil interaction and the class, or at least some pupils, respond willingly. Interactions follow, predominantly between Joe and Rose. What is interesting is that Joe and Rose have difficulty communicating on Rose's presentation in much the same way as they had on Joe's. In contrast, the interaction between Rose, Thami and Sue is more successful (lines 32-44). Thanu's question provokes meaningful discussion on the task whereas the questions Joe and Rose ask each other are difficult to interpret and seem unrelated

As mentioned earlier, Joe and Rose provide the two reports given in the 11 minutes remaining for report back during this lesson. They engage each other on their reports. Joe is second. To complete the context of the incident it is important to see what happened before with Rose. As will be shown, Joe and Rose have similar difficulties communicating about her report.

00:25.04 to 00:29:36

Sue: What I want us to do now - I am going to ask one person from each group to go up to the board to tell us what they have done and to explain it. [I OK Rose (Rose goes to the board with her book.]... OK people, listen to her and see if it makes sense to you, and ask her questions if it doesn't.

Rose: (To the class - Sue is standing on the side of the class). First I am going to show you the drawing. (She draws this, silently)



(Then, pointing to the base angles, she says:) This one is obtuse and this one. Then, (pointing to the top angles) these ones are the acute. OK? eh . We said, we said, uh, that it is impossible, uh, because when you draw a triangle with, uh, two obtuse angles and one acute you can't do it, uh, cos when you draw a triangle you have to get two, uh, two or three acute angles. So, we said we get this drawing which is a qua-dri-lateral drawing to get two obtuse angles (then very quietly - the class) Do you understand?

(some murmurings of 'yes')

Joe: Why, why do you need two acute angles?
Rose : Two or three acute angles ...
Joe: Why do you need acute? Why can't you use any type of angle?
Rose: ... so what we did we get two acute angles, or three (shrugs).
Mzi: Can I answer (addressed to Rose)? um Joe, you are talking about a triangle no? already um OK um um can you use a reflex angle to draw a triangle? (Joe answers 'no') So obviously she can't use reflexes so she use acutes and obtuses (not easy to hear, said haltingly) to find the angle, vertical, triangle ...

As described in Chapter 4, during the workshops two other teachers talk of difficulties with pupil expressions that confuse other pupils, and their tendency not to call on these pupils publicly.

Clearly, in a multilingual communicative classroom, the linguistic demands on pupils increase. Teachers in this study are aware of pupil difficulties but they are not sure how to improve language skills and so the issue is not explicitly addressed. This vignette and its larger study suggest that problematic communicative competence in general, and within a participatory inquiry pedagogical approach in particular, leads to selective teacher pupil interactions and these are potentially differentiating. This points to the dilemma of equal vs. differential allocation of resources and Lave and Wenger's argument about participation being a function of access to resources in the practice. Here language (through communication and particularly verbalisation) is a resource for learning that inadvertently becomes differentially distributed. Those not called on have less opportunity to talk about their thinking.

In the time while pupils work in pairs on the worksheet, and Sue circulates, we see her managing the complexities of her participatory inquiry approach in her multilingual classroom - complexities that lie in the interplay between pupils' communicative competence and the deepening of their mathematical knowledge, between democratising the classroom and simultaneously developing mathematical knowledge.

5.1.6 The lesson: from talking within to talking about

The last part of the lesson is report back time which demands public articulation of work done. Sue manages to continue her encouragement of participation and inquiry, but here through whole-class interaction. Mediation and negotiation of mathematical meaning come to take on a different form with different possibilities and constraints.

5.1.5 The lesson: demands on pupils' communicative competence

As is partly illustrated by the five pupils who had difficulty in giving clear explanations, there is differential communicative competence in this class in terms of demonstrating knowledge through written language. These pupils do not volunteer to report on their work publicly. Nor do they engage with those who do report back.

In both her initial and her reflective interviews, Sue discusses problems she faces in encouraging all to participate publicly. She wonders to what extent forcing them to explain ('and they are not comfortable' doing this) 'inhibits their understanding' and 'creates insecurities'. Also, some pupils are poor communicators and confuse the class. She realises that this affects her own responses and interactions which, in turn, may have negative effects on pupils. Pupils who volunteer usually have something to say, and moreover:

> *S: I do try to choose them when I know they will have something to say... and then you are also saying something to the kids who don't get to go up ... there is one kid who I don't get up because he will just confuse everyone*
>
> (VI6, 1202-1212)

Without wishing to detract from the focus of this chapter, it is interesting and useful to the evidentiary nature of this vignette to add here that Thandi is similarly aware of her own exclusionary actions because of poor communicative competence. In her reflections, she says:

> *T: I choose the willing ones, they listen, always willing to do things. Then it seemed to me the other pupils don't get to be seen like those who are not talking on the video.*
>
> (W3, 55-59)

since an obtuse angle is more than 90 degrees, two will give more than 180 degrees and thus will not make a triangle; or that if you start with one obtuse angle and then try to draw a second one, you will land up with a quadrilateral and not a triangle. This latter is Rose's approach.

One pupil, Mzi, wrote:

> It is impossible because if you have an obtuse angle, you will have a line that will have to bind the two points together, so you will get acute angles.

Of the 32 students across both classes, 15 responses were similar to Rose's, 8 were focuses on the angle sum in a triangle, three were like Mzi's and 5 were limited or unclear explanations.

> Impossible because then you will not have a triangle
>
> Impossible _ (drawing _ to support answer)
>
> Impossible (no support for answer)
>
> It is impossible because obtuse is bigger than 90 because all the angles have to add up to 180
>
> You cannot draw it because it is bigger than 90 and a triangle is 180

There are certainly different ways of approaching the task and answering the question. Specifically, we see a more deductive, potentially algebraic argument, on the one hand, and visual, more intuitive conceptualisations on the other. Again we see that what Sue values, she accomplishes. However, diverse responses are not only in orientation but also in quality.

spontaneously asks questions, for example: How many triangles are there in the world?

In addition, she fosters pupil-pupil interaction. Pupils interact with each other while on task, and then during report back. Pupils have learnt that they are expected to ask themselves why, to explain and ask why of others and to interact verbally with each other.

These interactions, while controlled by Sue, also reflect her skill in managing the constant interplay of democracy and development, in listening to and pushing pupils in her interactions with them and the task in hand. What Sue values (mathematics as something you talk about, have your own ideas about, ask questions about; learning as social; knowledge as personal and problematic; and learners as diverse - see p. 219), she accomplishes.

This participatory-inquiry approach stands in sharp contrast to many mathematics classrooms where teacher-initiated recall-type questions and I-R-F interactions (initiation-response- feedback) predominate and where pupils 'go for an answer' (Campbell, 1986). Sue's lessons are better described by pupils' 'going for a question'.

I recall my excitement and admiration in watching this lesson - so different from most others where pupils all do the same thing in the same way. Here, despite curricula constraints and in contrast to dominant practices, a culture was being created that provided pupils with confidence to inquire, interact and develop their mathematical intuitions.

5.1.4 The lesson: mathematical knowledge and diverse pupil perspectives

From the videotape and copies of their written answers, pupil reasoning for why Q3 is 'impossible' includes numerical and visual justifications. Most are either that

More explicit in Sue's interactions is a validation of pupil perspectives on the task provided they are meaningful, that is, they 'make sense'. In addition, in her encouraging of Joe and Rose, we can see possibilities for bringing forth the Zone of Proximal Development (Vygotsky, 1978, p. 90), for deepening their mathematical thinking through developing the concept of generalisation, and her tacit knowledge at work. Working with Wertsch's elaboration of activity and the ZPD (Wertsch, 1984, 1991b), she supports Joe's thinking by saying his explanation is 'OK' but also challenges his situation definition, suggesting while admitting that she is not sure, that his orientation to the task evidenced in his actions is possibly limited, and that he should also start with an obtuse angle.

Of course, at issue here is that situation definition is not simply related to the actions and operations, but also to motives[ii]. It is perhaps Sue's motive for a more generalised response that is not shared by Joe. I will return to this point later.

In Lave and Wenger's (1991) terms, there is ample opportunity for talking within the mathematical practices in this class. As discussed in Chapter 2, Lave and Wenger distinguish between talking within and talking about a practice. They argue that full participation in a community of practice means *learning to* talk, which entails talking about and within the practice (p 109). And both talking about and within a practice themselves entail talking within and about the practice. In class, pupils are within school mathematical practice. While on tasks, Sue's pupils are talking within their mathematical practice. Then, and less so in this part of the lesson, they also talk about their mathematical ideas, either to the teacher, or their partner.

An inquiry approach is evident in Sue's actions and in utterances like Joe's 'But why'. I observed four of Sue's lessons, during which pupils asked her a wide range of questions, for example: Can we have a curved angle? If we have a right angle is there also a left angle? In a triangle, why don't we include straight angles (referring to any point and a 180 degree angle on one of the sides). Sue also

00:13:05 to 00:14:36

Joe: I am confused miss .... It is impossible miss because when you have this, miss, no, it is 90, ne? ((let's make it 89)) but (pointing on his book) then these are both acute ne?

Sue: yes

Joe: You make this one an obtuse angle, ne? So you like stretch this a bit and then while you are stretching this, these two angles (teacher saying right right as he explains with pointing) will get smaller - so that is why.

Sue: That is an okay explanation - but I am not sure if it covers all the possibilities, because what if you start off with an obtuse angle? You started off with 89 and it became 91 - and I think you should write that explanation. But when you have finished writing that think about what happens if you start off with an obtuse angle, like 125 degrees. Could you then have the triangle with another obtuse angle?

(Sue then turns to Joe's partner to see if she shares Joe's view and then encourages them to 'together think about the 125 degree starting angle').

Sue encourages both Rose and Joe to consider the generalisability of their answers. She is not entirely happy with Joe's explanation - but she is not sure - her tacit knowledge is to push him to consider whether his explanation holds if he starts with an obtuse angle. We thus see here that implicitly developing the concept of generalisation is included in Sue's lesson goals.

In this part of the lesson, Sue gets round to the whole class. Her interaction, her mediation and scaffolding, as mentioned above, are predominantly in the form of questions that encourage learners to articulate their thinking, present their reasoning and to question each other. We can see opportunity for conscious reflection, and in Vygotskian terms, possibilities for developing of higher psychological functioning, and specifically though implicitly the scientific concept of generalisation.

This temporal analysis illuminates the pedagogy in operation and Sue's values: she allocates a considerable amount of time to the paired work on the tasks.

Joe's stretching incident occurs in report back time and concerns the possibility of drawing a triangle with two obtuse angles (Q3b above). It is focused on two pupils, Joe and Rose, who work separately, each with his or her own partner. Both offer their work in public report back time and both engage each other on their respective reports.

5.1.3 The lesson: Sue's participatory-inquiry approach

While pupils are working in pairs on the worksheet, Sue circulates. Her interaction with learners is largely through questions of clarification and justifications of their ideas, and encouraging of their (pupil-pupil) interaction. So questions like 'Why?', 'How do you know?', 'Do you understand what (your partner) says? Ask him a question?' predominate. In addition, she also reminds learners to write their explanations in words and/or to draw a picture - thus reinforcing different representations built into the worksheet.

In addition to *descriptions, questions, explanations and justifications*, Sue also encourages some pupils to consider and extend the *generalisability* of their answers. In Vygotskian terms, generalisation is a scientific concept, linked to other concepts and acquired through mediated systematic instruction. Sue's interaction with Rose and Joe recorded below while they are working on their tasks with their partners reflects how at this individual level she works to scaffold this scientific concept and manages the democracy-development tension. She interacts with Rose on her answer to 3b (Rose has drawn a shape like this: _/). Sue validates the response and then asks: 'Will this always be the case? that they don't join?'. And when she gets to Joe and his partner, he says:

5.1.2 The lesson: a temporal analysis

The lesson which produces Joe's stretching is repeated with a different Std 6 class on the same day. Where illuminative I will include occurrences from both classes.

For most of the 40 minute lesson, the 16 pupils in each of the two classes work in pairs on part of a worksheet (Figure (ii) below) where the questions are designed to elaborate the concept of the angles of a triangle.

FORM I: GEOMETRY
If any of these is impossible, explain why, otherwise draw it.
1. Draw a triangle with 3 acute angles.
2. Draw a triangle with 1 obtuse angle.
3. Draw a triangle with 2 obtuse angles.
4. Draw a triangle with 1 reflex angle.
5. Draw a triangle with 1 right angle.

Figure (ii): Extract from worksheet in Sue's Std 6 class

The lesson begins with brief instructions related to the worksheet. Pupils are to discuss their ideas with their partners, and illustrate (draw a diagram) and write (in words) an explanation of their answers in their notebooks. They are also told that later in the lesson, one from each pair will be called on to 'explain' what they have done to the class

There are thus three distinct time-frames in the lesson: the first few minutes orient the whole class to the worksheet. The next 25 minutes, pupils work in pairs on the worksheet. The last 11 minutes are for whole-class report back.

I have described how Sue's idea of what constitutes a 'good explanation', one that is more general, is not made explicit. This implicit intention is evident in individual interactions and absent in Sue's interaction with the whole class in public report back time. However, it is her responsibility to provide appropriate instruction of a more generalised response. But doing this produces a profound dilemma for Sue and others as creative and reflective as she is. She would need to highlight attention and provide a scaffolding process not only on how Joe and Rose differ, but why in her view Rose's response is more generalised and therefore a better explanation. This could be done in a way that continues to encourage pupil participation and interaction but such activity on Sue's part might well undermine her goal for her pupils to take responsibility for their learning. Joe might feel that his thinking is not good enough because it is not like Rose or Sue's. This could inhibit his willingness to participate in future or negatively shape his goals. Yet if Sue does not mediate publicly how and why Rose's response is a general answer, but Joe's a specific case, then her intention to develop 'good explanations' and have pupils negotiate meaning will be thwarted.

In this we can see that pupils' difficulties with engaging each other are more than metacognitive on the one hand, and on the other, more than their ability to express their mathematical thinking. Their difficulties are bound up with the teaching and learning approach in the class. Again, we see the three dimensional dynamic of learning and teaching mathematics in multilingual classrooms.

What we learn so forcefully from Sue is that she knows *that* expression and engagement are problems, but not *how* to deal with these. I am thus arguing from a Vygotskian perspective that Sue does not see how her approach, embedded as it is in her actions, is implicated in neither facilitating the creation of a construction zone for Joe and Rose, nor in mediating the scientific concept of generalisation.

Pragmatically, given time constraints, the question of whether, at this level, the concept of generalisation can only be emergent and individually mediated must be

social. So, she wants them to ask each other more effective questions, perhaps like those she asks Jr and Rose as she interacts with them individually while they are on task. I have already posited one reason why this doesn't happen. There is no effective Zone between Joe and Rose.

The irony here is that Sue's desire for pupils to engage each other is perhaps simultaneously part of and undermined by her participatory inquiry approach, and this appears to be obscured from her. What are her purposes behind not refocusing? It is arguable that Sue's concern to encourage participation and inquiry interacts with the difficulties pupils have in explaining their thinking and engaging each other, and her repeated point that 'she doesn't know how to help them'. Together these mitigate against her mediating across differences and evaluating the differing substance and content of what the pupils offered. Rich mathematical possibilities are thus simultaneously created and partially lost", the trade-off for sustaining what she has worked hard to build - a culture of meaningful inquiry where pupil perspectives are valued and knowledge is treated as personal and problematic.

A second explanation as to why Sue does not refocus relates to her implicit intention in the lesson - the development of the scientific concept of generalisation. As Bartolini Bussi (1995) argues:

> ... in a Vygotskian perspective, a scientific concept is neither a natural development of an everyday concept nor a matter of negotiation, but is acquired through instruction (p. 96).

Bartolini Bussi argues this in her analysis of classroom activity where a teaching intention was the scientific concept of 'patterning' but teacher mediation of this concept was nowhere evident in teacher-pupil interactions on the task set. This then accounts for difficulties pupils had in generating and working with the patterns in the task.

Here we are concerned with learning and development possibilities through pupil-pupil interactions. Joe and Rose do not share the same situation definition. Rose starts by drawing obtuse angles and then cannot form a triangle, only a quadrilateral. Joe starts with a triangle and sees that if he stretches one angle into an obtuse angle, the others will contract. Their orientations to the task, their objects of attention, are different and they struggle to see past their own orientations to engage with each other. There is no effective zone here for either of them in their interaction. That is perhaps why we cannot fathom their questions and why Sue's insights are pertinent. This also explains why Rose and Thami's interactions are more fruitful - they share the same orientation.

There is much potential for quality learning in this situation, for both Rose and Joe reflecting on their own conceptions in relation to others' that are different. But they cannot do it on their own. It requires the teacher to create a construction zone (Newman, Griffin & Cole, 1989, p. 153), perhaps by mediating their differences publicly, bringing to attention different orientations and starting points, their connections and relative mathematical strengths. For here, discussion amongst the pupils is not about who is right or wrong - both their approaches make sense and answer the question - it is about how they are similar and different, how they are related, and then too, which, if either, is the better mathematically.

The role Sue plays in these interactions is to ensure that their questions are clarified and that the two explanations of why a triangle cannot have two obtuse angles are understood by the class. In Joe's case, she deflects to teach labelling and does not refocus back onto the mathematical substance of the task. Sue sees her deflection as an opportunity for teaching in context.

A key issue, however, and here we return to the democracy -development tension and its related dilemmas, is that Sue would like them to negotiate meaning and engage each other. She wants to treat knowledge as problematic and learning as

and how these are inter-related. Mind-language interrelatedness is at issue as we grapple with pupil-pupil interaction difficulties.

6.2 Negotiating meaning

Why are Joe and Rose unable to negotiate meaning effectively? Why can she not hear his explanation and he her question?

Sue's insights from her reflections on her practice are very instructive. Pupils in this class struggle to hear and engage each other. What she understands is how hard it is for them to step out of their own ideas and frames to engage others' mathematical thinking. Indeed, it was hard for her too. The question is: how might pupils have the vantage point that one expects of the teacher - a vantage point from which to interpret and engage a range of ideas different from your own.

As discussed, Wertsch's elaboration of activity and Vygotsky's Zone of Proximal Development (ZPD) are ways of elaborating and naming Sue's insights. Activity in the ZPD involves: situation definition; intersubjectivity and semiotic mediation (Wertsch; 1984, 1991). It is when situation definition (motivation and/or orientation to the task) is not shared that mediation is required for intersubjectivity to be established. The issue in adult-child and particularly teacher-pupil interactions, where motivation and goals are often not shared, is the changing of the child's situation definition and the kind of mediation that is required to establish intersubjectivity in relation to the task at hand. Issues of power and control and ownership of knowledge enter here, but the important one here is that if situation definition is shared then intersubjectivity is easier to establish and mediation easier too. If situation definition is not shared, and it could well be if change in thinking is to occur, then establishing intersubjectivity becomes a complex process.

The teachers in this study raise serious questions about the benefits of verbalisation in their reflective interviews and workshops and these concerns have been described in Chapter 4. In particular, they wonder whether it is not, in fact, that verbalisation benefits the teacher who can hear and then work with pupil thinking, rather than the pupil [illegible] benefitting from the verbalisation. This can be described as a dilemma of *verbalisation as a learning resource vs. verbalisation as a teaching resource*. Pimm points out the tension in purpose between hearing what pupils have done (their perspectives - knowledge as personal) and using this for more general teaching (knowledge as public). However, in relation to the sociocultural theory developed in this thesis, there needs to be some explicitness of what is required in report backs, in managing both the personal and the public, or, in Mercer's (1995) terms, crossing back and forth between educational and educated discourse.

What we thus learn from Sue is that teachers could be usefully informed about aspects and issues of reporting as discussed by Pimm, to become aware that the shift from talking within to talking about is not necessarily spontaneous. Here is empirical ground for the argument in Chapter 2 (p. 59) that Lave and Wenger's theory of social practice needs elaboration if it is to include school contexts. For access to school mathematical practice, and not sequestration, the boundary between talking within and talking about mathematical ideas and thinking must be recognised. Scaffolding into educated discourse, that is more explicit mediation, is required. This is not without tension as Chapter 8, with its focus on explicit language teaching, will illuminate. Nevertheless, practices that assume a seamless web of activity in learning to talk mathematics, a wishing away of the boundary, can result in some pupils being denied access to educated discourse. They are thus unlikely to become full participants in the practice.

Reporting skills are important, but not the key issue here. A deeper question is whether the underlying problem for Joe is a communicative or an epistemic one

to labelling, which, as she herself commented recently, has a real purpose. Sue's labelling thus focuses, to some extent, on this issue as she highlights to Joe and others, how you need to label effectively if others are to understand you. Being explicit about what is required is necessary. But this requires language teaching and Sue 'does not know what to do'. Moreover, Pimm (1992, 1994) raises the pedagogical tension (which relates in part to both the knowledge is personal vs. knowledge is public dilemma, and the learning is social vs. learning is individual dilemma) that arises as teachers attend to being more explicit about what is required in reporting. The more explicit, the more pupils will take form for substance. The less explicit, the less pupils are likely to notice what is going on, what is intended.

We learn further from Sue and her astute awareness, that public articulation of mathematical thinking is not equally pleasurable or beneficial for all learners, and that her own actions at times favour some students through an inadvertent unequal distribution of learning resources.

Pimm asks us to consider 'who benefits' from reporting back, and in what ways. Those reporting have an opportunity for serious, formal conscious reflection. Other students benefit by hearing a range of ideas and orientations and different perspectives can be validated and valued. Teachers can hear what their pupils think and attend to pupils who are presenting. They can mediate through some 'contextually based meta remarks'. But these benefits need to be consciously attended to if they are to be achieved. For Pimm, speaking mathematically assists in acquiring the competence of writing mathematically and it is written mathematics that is highly valued.

It is not clear to what extent Joe or Rose benefited from their reporting. Pupils heard two of the four kinds of responses to the question. And Sue uses the opportunity to do some teaching in context. Whether Joe and Sue, or any of the others, will report any more effectively next time is debatable.

loose selection of angles is confusing. Rose has less sense of audience in that she does not convey the process of her thinking.

To some extent, Sue is aware of the issues of selection and audience. In her reflective interview she described a situation where pupils in another class had worked on an investigative task for a few days, where exciting mathematics had been evident.

> **S:** *You see what happened in another lesson: I gave them an investigation to do - and it was a two to three lesson investigation: given a fixed perimeter which shape has the greatest area? And they all did wonderful things and in different ways, and after three lessons the time was to present it and they presented appallingly - no-one could understand what the other group had done on the board. I knew and was able to draw it out but they just weren't able to present and partly 'cause I have never told them how to present, never told them you can use diagrams and structure it in this way. And also I have never structured explaining to them.*

And

> *they don't know what it is others need to know about their thinking ...*
>
> (V16, 947-963)

We also see here, that *talking within* and *talking about* mathematics within the classroom and its mathematical practices, while deeply related, do not place the same communicative demands on the speaker. Lave and Wenger's seamless web of becoming a full participant through learning in a community of practice is problematic. In this community of practice, Sue's mathematics classroom, the move between talking within and about is not spontaneous or tacitly learnt. It requires some mediation.

Sue knows that there is problem, that there needs be some instruction in relation to reporting. But we learn again from Sue that knowing *that* does not mean knowing *how* to act. We can see Sue's tacit knowledge through her 'deflection'

competence vs. negotiating and developing meaning, and to the knowledge is personal vs. knowledge is social dilemma, the dilemma of validating pupils perspectives vs. developing scientific concepts? What can we learn and what do we learn from Sue, from both her tacit and articulated knowledge? What is she not aware of and why?

How does the democracy-development tension wend its way through both the issues and Sue's knowledge of them? What are the mediation dilemmas at play and how does Sue understand and explain her management of them and the trade-offs she might make?

6.1 From talking within to talking about mathematical practice

Joe has difficulty articulating his thinking in his report back to the class. His language is littered with 'ums', repetitive phrases and hesitancies. An obvious explanation is that Joe, and others who display similar behaviour, are not mother tongue English-speakers. Their task is thus one of double attention - to their new mathematical ideas and to a language they are still learning. Sue is aware of this but 'doesn't know how to move him on - to develop the language'.

However, Pimm's (1992, 1994) work suggests that this issue is not simply about proficiency in or access to English. Reporting mathematical thinking, even for mother-tongue English-speakers, is not a simple process because of the linguistic and communicative demands entailed. 'Skills of reflection and selection' and a 'sense of audience' are important to successful report back. This could explain why Joe could convey his meaning to the teacher and his partner but struggles with his report; and why Rose does not mount the appropriate sequencing of her ideas. Interacting individually with Sue and his partner, Joe is able to point to his work in his book and thus does not have to select in the same way as when he works on the board in front of the whole class. While he displays a sense of audience by trying to recount the process of his thinking, his loose selection of

different purpose in teaching and gives rise to tensions and contradictions in different ways. Listening, chairing discussion, and negotiating meaning serve to maintain pupils' activity and interest, and generate many ideas, not all of which can be taken up or even heard. On the other hand, lecturing and eliciting serve to make available the knowledge of the curriculum. A form hardly used, but one which I think deserves more exploration is answering pupils questions. (Brodie, 1995, p. 246)

6 DISCUSSION

In the complexity of teaching mathematics in Sue's multilingual classroom her participatory-inquiry approach makes particular demands on learners' communicative competence. And while Sue has developed a culture of meaningful mathematical inquiry in her class, we see that:

1. Pupils sometimes struggle to explain their mathematical thinking. They have difficulty with formal public expression or mathematical communication, with moving from talking within to talking about mathematics.

2. Pupils sometimes have difficulty understanding each other. Pupil-pupil interaction (verbal communication) is thus not a taken-for-granted given. Often, they are unable to communicate and negotiate mathematical meaning effectively with each other, and their questions appear restricted to points of clarification.

3. These communicative difficulties with public articulation and negotiation of meaning shape Sue's actions and interact with what she makes explicit and what she leaves implicit, and, in turn, with the mathematical knowledge made available to pupils.

Returning to the object of study: what is Sue's knowledge of these difficulties? How do they relate to the dilemma of developing mathematical communicative

is it a pragmatic and appropriate way of mediating in diversity, and of creating a culture where personal and diverse knowledges are valued and encouraged?

5.2.4 Sometimes, 'telling is more honest'

While not directly related to the incident, it is interesting to note that Sue's interest in her videos is captured in her whole class interactions – both classes in relation to Q4, and whether a triangle can have a reflex angle. In each class, a response from some pupils is that it can, but the angle is outside the triangle. In the first class, this publicly presented response is challenged by other pupils and quickly resolved by all agreeing that the angles must be inside. In the second class, however, whole class discussion occurs not about inside and outside angles but rather whether there are one or three such reflex angles. This discussion goes on for some time and Sue reflects with the research group on whether and how it was beneficial and her role in it. She observes that while she does not talk a lot in the lesson, when she does she is very powerful in shifting the direction of the discussion. Also, when questions are put to her, she often answers them rather than responding with a question for the learner to think about. She suggests to the group that while this might be read as teacher telling, at times it is not only appropriate, but more honest (W3, 1500-1506).

In her action research she carefully analyses mediation throughout the lesson and notices that she interacts with pupils in a number of different ways. She argues that:

> ... it cannot be our aim as teachers to remove ourselves and our power from our pupils' attempts to construct (mathematical) knowledge. Rather, we need to attempt to relate their developing ideas to broader social and cultural knowledge. In attempting to do so, we will always work with tensions and contradictions.
>
> There is no best method of teaching, rather, a number of different strategies that we can use. I have identified some of those that I use in the analysis of a single lesson. Each kind of interaction serves a

It is not only Joe and Rose who have difficulty understanding each other. In this research there are three layers of interaction and communication, each with difficulties, about Joe's 'stretching'. There is Joe and Rose (two pupils), Joe and Sue (pupil and teacher), and Sue and myself (teacher and researcher). While Sue p[nly sees the difficulties between Joe and Rose, it is only in discussion with me that her differences with Joe become apparent to her. And, not surprisingly, it is only in discussion with a colleague on this text, that I can acknowledge my differences with Sue as to what is and is not a general or good explanation and justification of the task[11].

5.2.3 Deflection or teaching in context

In the reflective interview, we did not specifically discuss Sue's 'deflection' to labelling angles. This came up recently, however, when I sought both confirmation for description and interpretation of the data in this paper as well as agreement (from an ethical point of view) to talk about her work. Bearing in mind the time-lag, she said that she had been thinking about 'deflection' and noticed more and more that she used it frequently to 'teach in context'. This was especially the case now as a materials-writer. She often 'deflected' to teach in context.

The issue then is one of refocusing - getting back to the task, or what we could call a *dilemma of deflection*. In Joe's 'stretching' the bell rings. Sue gets back to the task the next day, and Joe's partner expresses his idea clearly. Hence the 'mathematical properties of a triangle' are explicitly dealt with. But the relationship between Joe and Rose's responses, and the mathematical limitations of Joe's response from Sue's perspective, are not publicly explored. Sue's notion of what would qualify as a more generalised, and therefore a better, explanation remains implicit and at the level of individual mediation. It does not enter public discussion - whole class mediation - in report back time. Does this matter and to whom? Is this a case of unintentionally enabling only some (Joe and Rose)? Or

a question I would have asked. So it is bringing up what I think is mathematical thinking and that is my own view, so it definitely does. And often you don't hear what a child is saying because it doesn't match [I]

(VI6, 1215-1225)

Sue certainly does have a notion of a good explanation - one that is general. We can thus read two intentions for her in this task: the development of the mathematical properties of a triangle - that it cannot be made of two obtuse angles; and the development of the scientific concept of generalisation through justifications that are general. The interesting observation here is that the first of her intentions is clearly evident in the construction of the task and the instructions pupils are given. The second intention is, however, more implicit and observable only through her separate interactions with Joe and Rose.

5.2.2 Communication difficulties

We then get to Joe and Rose's interaction and she says:

S: He doesn't understand her - his argument is totally different so he doesn't understand where she is coming from. She is saying you have to have two or three acute and he is saying why acute - why are you doing that? he doesn't understand where she is coming from, 'cause she is not doing what he was doing ...

J: Then you are saying that they find it quite hard to let go of their own understanding ...

S: to get into somebody else's understanding. ... I mean his own is so tenuous that [(]) [] because they are not sure of their own ideas they lose track in their arguments. One kid starts talking about one thing and the other points to something else and then they lose their argument. This happens in class a lot []...

(VI6, 869-884)

Helen, Sara and Clive's classes are, to a greater or lesser extent, interactive. Interaction in Clive's class remains at the level of whole class teacher pupil interaction where concepts are introduced through teacher questioning, examples are demonstrated on the board inviting pupil participation, pupils work individually on exercises and these are then discussed again in whole class teacher-led interactions. Sara shifts between paired pupil activity on tasks where she interacts with and questions individual pupils, checking their understanding through their expression of their mathematical thinking, and then whole class, and typically I-R-F, teacher-pupil interactions as she works to consolidate the mathematics in the task. Helen's classes include organised group activity. Here there is considerable pupil-pupil interaction (talking within), followed by whole class teacher-pupil interactions around the reporting and reformulating of tasks completed (talking about). Of the three classes, Helen's approach is most task-based and provides most opportunity for talking within and about mathematics, for language as a social mode of thinking. In all three classes time is given to whole class teacher-pupil interaction.

The valuing of explicit language teaching thus arises most prominently in multiracial and hence evidently multilingual classrooms but across a continuum of more or less interactive teaching and learning approaches.

Nevertheless, in Chapters 6 and 7 we see other teachers, either tacitly or deliberately, incorporate 'teachers telling learners about mathematics' (Sue), and 'teachers modelling mathematical language' (Thandi). As mentioned in Chapter 7 (p. 240), through her own research, Sue comes to see this as sometimes a 'good thing', appropriate and important in the teaching-learning relationship. Helen's views thus resonate with Sue and Thandi's experiences that 'teacher telling' is sometimes appropriate and important in mathematics teaching and learning.

Difficulties with explicit language teaching are also part of other classroom contexts. We have seen that, in practice, pupils' expression of their mathematical

Now listen to how clearly Rosie verbalises that ... and she is a successful student. There must be a relationship.
(VI1, 823-825)

Helen appears to share Lave and Wenger's notion that becoming knowledgeable means learning *to* talk, learning mathematical discourse. As was argued in Chapter 2, from a sociocultural perspective, learning *to* talk in school mathematics is deeply intertwined with learning *from* talk, with the interpersonal activity - the educational discourse - in a classroom. Helen also provides opportunity in her classes for pupil-pupil interaction and group work mediated by the teacher. Thus, Helen's view of learning mathematics has elements that resonate with both sociocultural and social practice theory.

Helen regards pupils' verbalisation in the mathematics classroom as a resource - a cultural tool[3]. Encouraging pupil verbalisation of mathematical ideas is using language as a resource for thinking and learning. Pupil verbalisation for Helen is also a display of knowledgeability. This acknowledges the fact that all a teacher has access to is forms of language (Pimm, 1996). Explicit language teaching and verbalisation by the teacher further enables access to the practice by making explicit what membership and mastery entails. However, using language as a resource for both displaying knowledge and as a tool for thinking creates tensions in the practice of teaching.

2.2 Linking Helen to the entire data corpus

In his reflective interview, Clive joins Sara and Helen in valuing explicit language teaching. This issue emerges again and again in these three teachers' reflective interviews. Clive was teaching in a private school where the majority of pupils were not English-speaking. His previous teaching experience, nevertheless, included a recently deracialised private school. Helen, Sara and Clive thus share the experience of working in schools where there was a rapid change in the racial composition of the class.

practices in multilingual classrooms - an element quite apparent in newly deracialised schools. However, the importance of the selected episodes must be established, both in relation to Helen herself, and in relation to the other teachers in the study.

2.1 Helen

The description of the workshops in Chapter 4 reflects the teachers' interactions with Helen as she struggles over whether or not explicit language teaching helps, over whether and how working on pupils' mathematical talk - their ability 'to' talk mathematics - is a good thing. How does one manage language, in particularly talk (and verbalisation) by both teacher and learners, as a transparent resource for exploring and displaying mathematical knowledge?

The dilemma of transparency is particularly strong for Helen and not surprising considering her view of mathematics as language, and of language as a crucial resource in the practices in her classroom. In her initial interview, Helen articulated a view of 'mathematics as language' (see Chapters 4 and 5 and Appendix C). This entails the understanding that there is a strong relationship between what her pupils can say and what they understand. In her initial interview she said that her greatest thrill (her rewards) is when pupils can express themselves, their thinking, in mathematical language. She repeats this view in her reflective interview:

> *... 'cause if they start to describe something to me in accurate mathematical language it does seem to reflect some kind of mastery ...*
>
> (VI1, 604-607)

This is reinforced later in the interview:

both what Helen says and what she does about language as a resource in her classroom and thus answers the research question specified above. Furthermore, this focus enables the elaboration of a language-related dilemma.

I will argue that Lave and Wenger's concept of transparency, while not usually applied to language as a resource, is useful and illuminating here. In elaborating the dilemma of transparency, I will argue that explicit language teaching, where teachers attend to pupil language expression as a public resource for class teaching, offers possibilities for enhancing access to mathematics, especially in multilingual classrooms. However, such practices easily slip into possibilities for alienation through a shift of attention off the mathematical problem and onto language per se. Teachers' decision-making at critical moments, while always a reflection of both their personal identity and their teaching context, requires the ability to shift focus off and then back onto the mathematical problem. The challenge, of course, is when and how such shifts are best for whom and for what. Thus, as with code-switching and inquiry learning, explicit language teaching creates dilemmas for teachers that at times will entail trade-offs, and at times will be managed. Whether and how the management of explicit language teaching can be transformatory remains speculative, and beyond the scope of the data in the study.

This chapter proceeds with further discussion on explicit language teaching. It then contextualises Helen's teaching in the study and in education in general. Together these enable the reader to locate the episodes and reflections that follow which form the kernel of this chapter.

2 LOCATING HELEN'S FOCUS ON EXPLICIT LANGUAGE TEACHING

As in the previous two chapters, the episodes from Helen's classroom selected here are neither typical nor rare (Erickson, 1986). Rather they are instances that illustrate and illuminate an important element of teachers' knowledge of their

For Helen, as will be shown later, successful mathematics learning is related to pupils saying what they think concisely and precisely in mathematical language. She has tried to develop language teaching as part of her mathematics classroom. As she reflects on her teaching she begins to question what this means in practice and whether and how explicit language teaching helps. And we are alerted to a dilemma: There is always the problem in explicit language teaching of 'going on too long', of focusing too much on what is said and how it is said. Yet explicit mathematics language teaching appears to be a primary condition for access to mathematics, particularly for those pupils whose main language is not English or for those pupils less familiar with educated discourse.

In other words, how is speech used by teachers and by pupils? How does one pay attention to appropriate ways of speaking mathematically without conflating medium and message? For if this occurs, attention is placed on the form of speech, and the mathematical meaning and concept at play can be lost. This can produce sequestration from, rather than access to, the practice of school mathematics. And how does one work with pupils' mathematical language, scaffolding educated discourse, without inadvertently producing in pupils abbreviated or empty concepts, and in Sue's terms, without devaluing and blocking their thinking, and what they are trying to communicate? How does one work on educated discourse without diminishing opportunities for using language in the classroom as a social mode of thinking, for pupils' exploration of knowledge through educational discourse.

These dilemmas fall under the curriculum dilemma of knowledge as personal vs. knowledge as public. But the generality of this dilemma cannot capture its specific language dimensions.

This chapter focuses on episodes in Helen's Std 9 (Grade 11) trigonometry class and her contributions to the workshops where we have her reflections on her video and her own follow-up research. In other words, the chapter focuses on

As Helen questions whether pupils actually benefit from talk in the classroom, the workshop discussion articulates a fourth element in explicit language teaching:

(d) *Pupil verbalisation as a tool for teaching.* If pupils express what they are thinking, this assists the teacher to know and respond appropriately to what learners are construing.

Thus, while the practice of explicit language teaching entails being explicit about both mathematical and classroom discourse, it is bound up with a strong relationship between language and learning, encompassing a myriad of interweaving but analytically distinct features of talk and learning. In (b) and (c) we see talk as important for exploring and displaying mathematical knowledge (Barnes, 1976) and, while not directly linked, a valuing of talking within and about mathematics in school (Lave and Wenger, 1991). We have language functioning as psychological tool in (b) and as a cultural tool for the sharing and joint construction of knowledge in (d) (Vygotsky, 1979; Mercer, 1995). The complexities of managing these analytically distinct, but practically interwoven, features of classroom talk are highlighted in this chapter, and captured in dilemma language as *the dilemma of transparency.*

In Lave and Wenger's terms, participation and access to full participation in a community of practice are bound up with access to resources. Helen regards language, particularly verbalisation, as a resource for learning in her classroom. Lave and Wenger elaborate access to resources - cultural tools and artifacts - in a practice through the concept of transparency. As discussed in Chapter 2, transparency (like a window) involves both visibility and invisibility. A resource must be visible if it is to be used. We must be aware of it. But, in use, the resource must enable smooth entry or access into the practice (here school mathematics) and so itself become invisible. It cannot constantly be the focus of our attention. It must be transparent. What does this mean in Helen's classroom where talk is a resource, and explicit mathematics language teaching a feature?

or a problem and should be transparent, needing to be both invisible (the means to mathematics) and visible (language itself is the object of attention)? (Chapter 3, p. 82)

From the discussion in Chapter 7, explicit language teaching might be part of the scaffolding process argued as necessary for learners shifting back and forth between talking within and about mathematics in the classroom. In the first two workshops it is Helen who specifically takes up and problematises the issue of explicit language teaching. As described in Chapter 4, in the workshops and reflective interviews, *explicit mathematics language teaching* comes to mean more than the teacher making mathematical and classroom discourse explicit. It includes teachers encouraging and working on pupils' verbalisations in the mathematics classroom. For Helen in particular, explicit language teaching in her mathematics classroom includes:

(a) *Attention to pronunciation and clarity of instructions.* The pronunciation of particular words by either or both pupils and teacher can be a problem in a multilingual mathematics classroom. Teachers' instructions can be misunderstood. Thus clear speech and clear instructions are important and improve clarity for all pupils, not just learners whose main language is not English.

(b) *Pupil verbalisation (putting things into words) as a tool for thinking*[1]. If pupils say what they are thinking, this will help them know the mathematics they are working with.

(c) *Verbalisation of mathematical thinking as a display of mathematical knowledge.* If pupils can clearly say what they are thinking, then they know the mathematics they are working with.

CHAPTER 8

THE DILEMMA OF TRANSPARENCY: LANGUAGE VISIBILITY IN THE MATHEMATICS CLASSROOM

1 INTRODUCTION

This chapter explores the benefits and constraints of explicit mathematics language teaching, or what can be described as a dilemma of transparency for teachers in multilingual secondary mathematics classrooms.

Chapters 6 and 7 explored and elaborated the dilemmas of code-switching and mediation. These emerged in the particular contexts of ex-DET schools and of changing pedagogy respectively. In Chapter 5, an additional area of knowledge about teaching mathematics in multilingual classrooms emerged from contexts of changing pupil-bodies. In their initial interviews Helen (T1) and Sara (T2), English-speaking teachers in recently deracialised Model C schools, both talked about the value and benefit to all learners of what I called explicit mathematics language teaching. Now that their classes included pupils whose main language was not English, it became obvious to Helen and Sara that they needed to be explicit about instructions for tasks, as well as mathematical terms and ideas. This can be understood as the need to be explicit about aspects of both educational discourse (the discourse of teaching and learning) and educated discourse (new ways of using language) (Mercer, 1995). Sara and Helen found, to their surprise, that explicit mathematics language teaching benefited all pupils in their mathematics classes, irrespective of their language background.

Explicit language teaching implies that language itself becomes the object of attention in the mathematics class and a resource in the teaching-learning process. This, in part, answers the research question: what do teachers say and do in relation to whether, when and how language used in their classrooms is a resource

7. See Love and Mason (1992) for a good discussion on questioning in the mathematics class.

8. I am grateful to Lyn Slonimsky for sharpening my awareness here. Moreover, in a recent informal discussion with Sue on this paper, she said that she is still not convinced that Joe's answer is in fact a good one.

9. It is important to remember that this argument has emerged long after the reflective interview with Sue, and that her views and interpretations in relation to this particular claim were thus not sought.

communication about mathematics. That is, both are required, and managing the tension is the challenge!

In the description and discussion of Sue's lesson and her knowledge of her practice, I have identified and then tried to explain issues related to communicative competence and the development of mathematical knowledge. I have illuminated what Sue knows and what we learn from her, and particularly areas where she knows what the problem is, but not how to deal with it. There are also areas in the logic of practice that remain obscure to Sue. In all these lie significant challenges for mathematics teacher education which clearly needs to include opportunities for teachers to engage explicitly with classroom communication and language development in the mathematics class.

NOTES

1. There are many labels for more open classrooms. 'Constructivist', 'investigative' are two that have current currency in mathematics education. I am more concerned here with accurate description of this classroom and hence have specifically avoided any label that might attach other meanings.

2. At a micro level, democracy implies participation and is linked to equity and redress; development implies growth and improvement. Local and individual participation within civil society and national economic and social development are both necessary in building a thriving democratic society. However, in practical activity, these can pull in opposite directions.

3. Of course, there is a prior debate here on the function of schooling as an institution. As I discussed in Chapter 2, while schooling does have a developmental function, it has fulfilled this function in socially distributed ways. This is the development-democracy tension in schooling per se. My focus in this chapter is on the development-democracy tension as it is manifest in mathematics classroom processes.

4. Some time after closely observing the video and the reflective interview with the teacher, I learnt in discussion with a mathematics colleague that Joe's answer closely resembles an attempt to prove the sum of the angles of a triangle in the early part of this century. (See Bates et al, 1993)

5. Sue's teaching context is very different from the over-crowded, under-resourced reality of many South African schools. That she works in optimum conditions is why her experience and struggles are pertinent and illuminating.

6. I am drawing here on Leontiev's elaboration of the concept of activity as including activity correspondent to a motive, action correspondent to a goal, and operation dependent on conditions (Kozulin, 1986).

Furthermore, this vignette has illustrated that it is precisely when perspectives are not shared (and this will occur in any classroom, not only multilingual classrooms) that 'talking about' mathematics in public or whole-class pupil-pupil communication is problematic (questions are restrictive and confusing). An effective construction zone is not spontaneously and necessarily present. The effect it has on Sue's actions is to deflect her from the substance of the task to dealing with confusions. Refocusing is constrained by a number of factors, including her participatory-inquiry approach. The unintended consequence of this is to constrain possibilities for deepening engagement with the mathematical task at hand and hence the development of mathematical knowledge.

The dilemma of validating pupil meanings vs. developing mathematical communicative competence is a profound one for Sue who has worked hard to develop a classroom culture that values and supports personal meanings and knowledge in mathematics as problematic. The dilemma entails recognising and working with the boundary between talking within and about mathematics, as well as working with the dilemma of verbalisation as a teaching resource vs. verbalisation as a learning resource. It also entails recognising teaching intentions in relation to the development of scientific concepts and the instructional role this implies. An instructional or scaffolding role (facilitating the creation of appropriate ZPDs) is also entailed when there is no zone for effective pupil-pupil interaction. Instructional roles are in tension with a desire to elicit, encourage and validate pupils' conceptions. Sue's *trade-off* (deflecting to teach in context) is as much a function of her personal identity as mathematics teacher, as it is of the contextual and knowledge/politics forces at play.

Here, at the level of the classroom, is the democracy-development tension. The implications for teaching are not new: while the *withdrawal* of the teacher as continual intermediary and reference point for pupils enables Sue's participatory classroom culture, her *mediation* is essential to improving the substance of

asked. Perhaps it is sufficient simply for the different answers to the task to be shared. To what extent could and should Sue have pushed mathematical meanings further through this task with the whole class?

However, without some sharing and contrasting of ideas and looking at how they are similar and different, Sue's desire for pupils to engage the substance of their classmates' contributions might well continue to be thwarted. Her pupils' public interactions on the task do not include moving across different views and judging them. Such judgement needs to be gauged from the emphases Sue implicitly gives in her interactions with the class. In this set of interactions Sue's implicit preference is for Rose's explanation.

7 CONCLUSION

A culture of meaningful inquiry, pupil-pupil interaction and multiple perspectives on mathematics is encouraged and achieved in this class both while they are working on a mathematical task and when they publicly report on their work. This is no mean accomplishment in school mathematics. The practice includes talking within and about mathematics. But talking about mathematics is hard, and particularly so in a multilingual classroom where many pupils are learning in a language that is not their spoken language. What we learn from Sue, from what she says and does, is that she knows this is a problem but she does not know what to do about it. From my sociocultural perspective on teaching and learning, and on the basis of the empirical evidence in Sue's classroom, I have argued that the shift from talking within to talking about mathematics requires explicit mediation. While there are tensions in managing this, teachers would be better placed if their decision-making was informed by an understanding of the boundary between such practices and of their role in assisting learners to move back and forth across it.

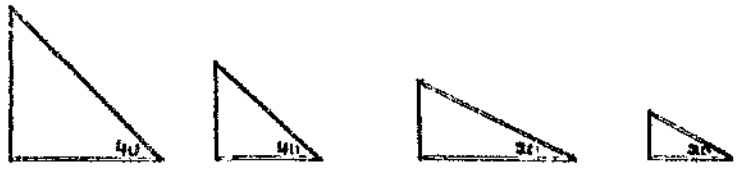
ratios between their sides, as will two different right-angled triangles each with 20 degree angles. But the two sets of ratios will be different precisely because the angles across the triangle pairs are different. And then she asks the pupil who first articulated the sentence to tell her what she understands ... in her own words.)

5 MY OBSERVATIONS[3]

My observational notes taken during this lesson reflect my strong sense that despite problematic verbalisation by some pupils, there was a sense of shared meaning that ratios were constant in similar right-angled triangles. The diagrams of three groups reflected this clearly. I was interested that this was the dominant trigonometric message for the pupils, but that some extended their notion of trigonometry beyond measuring and finding sides and angles of right-angled triangles to include its application to heights and distances problems.

I was also confronted, for the first time, with the issue of pronunciation. I was struck by Helen's insight into possible confusion between 'size' and 'side' in Episode 2. In the literature on language and mathematics there is little focus on difficulties that might arise through different pronunciations. I suppose this reflects the assumption that pronunciation is attended to and further, that in use, sense is made of differing pronunciations and hence that it is not in and of itself a problem. Helen's practice challenged my taken-for-granted assumptions. I became aware that different mathematical terms or different words used in mathematical discourse that nevertheless sound similar could cause confusion particularly in a classroom where pupil talk was encouraged. Moreover, the particular pronunciation of some English words by pupils whose main language is a vernacular one could cause confusion. For example, observing a student teacher a few months later, I noticed some pupils had written 'the sights of the triangle'. This is perhaps an effect of the stress in some African languages being on the first or middle syllable of a word, and a dropping of emphasis at

60 *of the angles is equal, then the ratio of the, of the sides won't*
61 *change.*
62 H: *Now listen to what you're saying. You're saying you've got (),*
63 *you said to me (and H links the bold words below to related*
64 *words on the board as she speaks) you've got the* ***size*** *of two*
65 ***triangles*** *and then you said that the* ***angle inside them*** *is the*
66 *same,* ***OK.*** *So if we want to, is what she said different to what*
67 *is on the board at the moment.*
68 S's: *No/yes ()*
69 H: *she said to me the ratio of the two sides is independent of the*
70 ***SIZE*** *of the triangle,* ***WHEN*** *you've got the same angle in all*
71 *of them. So is* ***NOT*** *true to say that the ratios are independent*
72 *of the size of the* ***ANGLE.*** *The size of the angle is* ***EXACTLY***
73 *what makes the* ***FUNDAMENTAL DIFFERENCE.*** *Because if I've*
74 *got two triangles, these two beautiful triangles over here, 40,*
75 *40 (and she fills in 40 degrees into two similar triangles on the*
76 *board),*



76 *and these two over here, 20, 20 (and again fills in these angle*
77 *sizes onto another set of similar triangles on the board). ()*
78 *Would I get if I say spoke about () sin here and sin here? OK?*
79 *Will I get the same answer?*
80 S's: *No*
81 H: *No! I'll get two different answers. So it is not true to say to me*
82 *it is indepen nt of the size of the angle - because the angle*
83 *if it is 40, makes the difference to 20, right. It's the size of the*
84 ***TRIANGLE*** *that makes the difference. () Does that make sense*
85 *to you?*
86 S's: *No*
87 H: *What doesn't make sense?*
88 S2: *Mam?*
89 H: *Ja*
90 S2: *It makes a difference to what?*
91 H: *It makes a difference ...to ...*
92 S's: *(laugh)*
H: *Where was I starting off? ... um, let me start again...*

(VT1C)

(Helen then recaps by drawing attention to diagrams on the board, to how two different right angled triangles each with 40 degree angles will have the same

22 *H:* *Triangle is a shape.*
23 *S's:* *(Mumblings) The length of the sides.*
24 *H:* *The length of the sides of the triangle. OK. You know. Let's*
25 *just look at this word "independent". OK. Now I know when*
26 *I teach this, I use the word independent and then you think,*
27 *well that's a nice fancy word to use. If I just repeat it nicely in*
28 *the right sentence then she'll be very impressed. But, when*
29 *you use the word independent you've got to know what it*
30 *means. What does it mean? Phindiwe?*
31 *Phin:* *(some mumbling) It stands on its own.*

After distinguishing 'length of sides' from 'sides of triangle' Helen pulls the word independent out on its own, and attends to its meaning. She then returns to focus on the sentence in which it is placed.

32 *H:* *OK. All right. Is that statement true?*
33 *S's:* *No//Yes. ()*
34 *H:* *Must I put a true or a false at the end of it?*
35 *S's:* *True//false*
36 *H:* *OK. Who says it's true?*
37 *S6:* *(Puts her hand up)*
38 *H:* *S6 says its true 'cause she said it.*
39 *S's:* *(laugh)*
40 *H:* *OK, who says its false?*
41 *S's:* *(laugh)*
42 *H:* *What do you think?*
43 *Phin:* *I don't know, I don't understand the sentence.*
44 *H:* *OK, let's try and sort out the sentence. The ratios of two*
45 *sides, that's a true part of the line, uh, of the sentence . Does*
46 *that make sense?*
47 *S's:* *Yes*
48 *H:* *OK. Ratios of two sides, we know we always talk about*
49 *opposite to hypotenuse, or adjacent to opposite or something*
50 *() we are talking about a ratio and we are talking about two*
51 *sides. () is independent. OK. Wait. The most important word*
52 *in the sentence is independent? Right. So one thing is*
53 *independent of another. So maybe if I just change this to of ()*
54 *we can start.*
55 *So the ratio is independent from what? Size of the angle in the*
56 *two triangles? () It's true, who says it's true? Why?*
57 *S7:* *Because, mam, um, I think it means that, no, uh, if it you, it*
58 *you have, uh, one big triangle and you have one small triangle*
59 *and you have the same angle in both of them, uh, the the size*

26	*H:*	*Have I made my point? OK, when I say this (points to*
27		*side) to you, some of you hear it as (circles size)*
28	*S's:*	*Size.*

(VT1C)

Helen then repeats the importance of being clear in what is being said and referred to, and towards the end of the lesson actually assists here by suggesting to pupils that it is more accurate to refer to the 'length' of sides, rather than the 'size' of sides, as sides could have thickness.

Helen then raises the next expression that she is concerned about, those expressions from Groups 1 and 4 in bold above. She states that it is still not clear to her whether they 'understand what trigonometry is saying'. She repeats that most reports talked of two triangles with the same angle in it. But, she asks, where do we go from there? Offerings are made repeating the expressions that she is concerned about:

EPISODE 3

1	*H:*	*Say that to me slowly, the*
2	*S6:*	*(H writes as pil talks) The ratios of the two sides (I is*
3		*independent to the size of the angles (I in the two triangles ...*
4	*H:*	*Is independent to ...?*
5	*S6:*	*The two tri.., is independent, no, the two sides is independent*
6		*...*
7	*H:*	*The ratio of the two sides is independent to?*
8	*S6:*	*The size of the angles in the two triangles (and H finishes*
9		*writing).*
10	*H:*	*Let's look at that statement carefully I need some distance.*
11		*(And she moves back from the board, and then says slowly)*
12		*The ratios of the two sides is independent (I to the size of the*
13		*angle (I in the two triangles. What does that statement mean*
14		*to, uh, to anyone?*
15	*S6:*	*It means that, uh, whether the anges (I when you've got two*
16		*triangles, and the angles come up to the same degree, you,*
17		*uh, it doesn't matter how long or short the triangle is, your*
18		*angles, as long as your angles are equal (inaudible)*
19	*H:*	*Now listen to what you said: how long or short the triangles*
20		*are?*
21	*S6:*	*The length, the length of the triangle.*

and you're asked to find or say what sin, cot and tan is, you can use by using the hypotenuse which is the angle opposite, which is, um, the side opposite your right angle, and your adjacent and your opposite. And there will be your theta which is the angle you must find. So trigonometry is actually the measurement of angles, it can also be the measurement of right angle triangles.

(VT1C)

At the end of all the presentations, some 14 minutes into the lesson, Helen, who has said little other than to direct groups up to speak in turn, moves to the front of the class and says:

EPISODE 2

1	*H:*	*I want to pick up on () some words that you've been*
2		*using () um, that concerned me. ... Right (writes size,*
3		*side on board). What is the difference between those*
4		*two words? () Just look at the words.*
5	*S's:*	*Size // side (mumblings in unison of both words).*
6	*H:*	*One can be measured? ... Penny say those two words*
7		*for me.*
8	*Penny:*	*(reads) Size, Side.*
9	*H:*	*(slowly) Size, side. When you speak, are you clear of*
10		*the difference between the two?*
11	*S's:*	*Yes*
12	*H:*	*Are you sure?*
13	*S's:*	*Yes // No (both said in unison)*
14	*H:*	*Then how come you say the side, the size of the, the*
15		*sizes of the triangles, you said to me the side of the*
16		*angles and you said all sorts of funny things?*
17	*S's:*	*Sides.*
18	*H:*	*Who can distinguish for me very carefully, the*
19		*difference between sizzzzo (she emphasises 'zze') and*
20		*siddde (emphasises 'dde')? 'Cause, when I say sides,*
21		*which word do you hear, the first or the second?*
22	*S's:*	*Size// sides// first// second*
23	*H:*	*Are you hearing the first?*
24	*S1:*	*I am hearing the second.*
25	*S's:*	*Second, (laugh)*

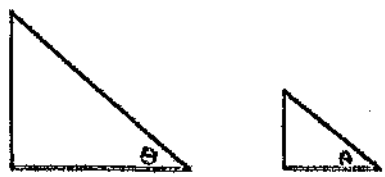


it doesn't matter of the size of the of the two triangles, whether its small or big. But then you find that the two angles are the same. So they work hand in hand.

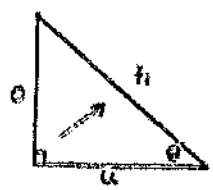
Groups 2, 3 and 5's explanations of trigonometry also focused on the six ratios, and how these, 'with Pythagoras' theorem', can be used to determine the sides and angles of triangles, especially right-angled triangles. Specifically, trigonometric ratios are used to solve problems related to heights and distances.

Group 4's articulation of the meaning of 'trigonometry' is in similar language to Group 1's:

Grp 4: *Okay uh () what you were talking about is actually is what is actually true. You said that trigonometry is the measurement of triangles. It can also be the measurement of right angled triangles. Say, for example, you got uh() two triangles - right angle triangles (and she draws them) of different sizes.*



As long as your theta, which is the angle, is equal () the size, the side uh, uh, the ratio of each angle will be equal. Therefore, we came to the same thing that the ratio of two sides is independent to the size of the tri, of the angle in two - langles. So this angle will equal to that angle because of this theta there, and ... uh, if you got a triang[illegible] like this (draws)



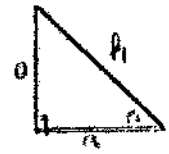
are part of Helen's action research and what she bring to the workshops. In the lesson Helen asks pupils in groups of four first to discuss what 'trigonometry' means to them, and then to report back their meanings to the rest of the class in a 'maximum of two minutes per group ... using key words and putting across your main ideas'. After that she has them discuss, again in their groups, their views of the benefits of group discussion in their mathematics class. My focus here, as in the workshop, is on the first task.

Seven minutes into the lesson, the first group is reporting, first explaining that 't gonor etry is used to determine the size and sides of the angles', and that there are six ratios.

The transcription key follows that used in Chapters 6 and 7, but I have emphasised, through bold type, utterances that will be referred in the discussion following. Emphasised utterances by Helen are in bold caps.

EPISODE 1

GRP 1: *OK, um, ... our group said that algebra is the mixture of geometry ... um No, it said trigonometry is the mixture of algebra and geometry. And () trigonometry is used to determine the **size and sides of the angles.** Therefore it also has six ratios in trigonometry. For instance, you have your triangle (she draws triangle below), the ratios, like you have your hypotenuse, if you have an angle, this is the adjacent, this is the opposite.*



*So six ratios are sin, which has a reciprocal of cosec, and your cos, which has a reciprocal of sec, and your tan, which has a reciprocal cot. So therefore () uh **we said the ratio of two angles is independent to the size of the angle in the other two triangles.** That means if you have two triangles, there you have a small triangle and a bigger triangle (and she draws)*

quite interesting. It helped in that it made me more critical of some of things I do, and one of the times when I was watching it more carefully I, picked up inaccuracies in my questions, or not finishing a question. ...

... It was interesting, very interesting to watch it, to see it all happening - like a sociological interest - this little hour in the middle in the world of this difficult class and this person who thought too much about her teaching - it's just interesting.

(VI1, 1003-1027)

In the first workshop (that is, after expressing her firm commitment in her initial interview to explicit language teaching, and after observing and reflecting on her video) she asks the other teachers to grapple with her about whether 'saying it' actually is indicative of understanding, of knowing. She asks: How come Debbie (a pupil) can say what sin 40 means one day, and then the next day seems unable to shift from sin 40 to sin theta with any kind of meaning? The teachers' discussion on this has been described in Chapter 4. Helen follows this up in her own research and then in the next workshop where she asks: how come some can say clearly and correctly what they think but then cannot seem to detect anything problematic in what their peers say? They can say it 'right' but do not seem to hear when peers are saying it 'wrong'. This and the dilemma of transparency are illustrated by what Helen brings to the second workshop as a result of her research.

The vignette below provides insight first into how Helen copes in practice with pupils' meanings and their mathematical expression, and then into reflections on her practice, and the dilemma she faces.

4 A VIGNETTE - CLASSROOM EPISODES

The episodes below take place in the first trigonometry lesson of Standard 9, the year following video-taping Helen's teaching trigonometry to her Std 8 class. They

J: *But why did you want to do it?*

H: *I think it is a general ideological framework that I come from. Um ... I am very authoritarian and strict in the general school life. But I like, I am comfortable with the idea of respect for the individual and that kind of thing. And the other kind of teaching doesn't do that. I have taught with teachers who teach in other ways and I am horrified at how they talk of pupils - so it is very much an ideological thing.*

J: *Which then is difficult to bring into successful maths teaching?*

H: *Like this year in my matrics - I never punished. But they got something wrong in the prelims and five hundred times they had to write it out and I felt nothing. It is an external exam. It doesn't fit with my ideas of what maths is. They are going to have to learn if they want to pass. I don't feel comfortable with that because I don't see it as respectful. On the other hand you treat a bunch like this with full respect and you just get trampled on. It works at a lot of different levels. But my teaching is always changing and hopefully always improving.*

(VI1, 880 - 939)

Through this extract from her reflective interview we see Helen struggling to take on what she believes is more politically and educationally progressive pedagogical approaches, approaches that bring more language and exploratory activity into her classroom. She does this within her view of mathematics as language and within the constraints of a content-driven examination system forcibly at play in the senior secondary school. Her struggle is captured in her closing comments in the interview:

H: *It was strange to watch. Some things I liked what I was seeing because I knew that I was explicitly trying to do those things but didn't know that I was actually managing to get them done ... and also seeing the extent to which they worked in groups (I like seeing the whole class doing that with me as part of it was also*

She introduces trigonometry to her Std 8 with an outdoor activity that investigates shadow length caused by the sun at different times of the day, and then (and this is when the videos were made) with pupils measuring and comparing the ratios of sides of a right-angled triangle with one angle of 40 degrees. Working on their reports of what they find out, she builds their understanding of constant ratios and relates these to the programming of trigonometric ratios into a calculator. For the workshops and her own research the following year, Helen asks the class to discuss and then report what 'trigonometry' means to them.

While Helen argues for the value of explicit language teaching (Chapter 5), like Sue she is concerned with the actual benefits of pupil-pupil discussion. For Helen this includes worries and dilemmas around exploratory learning and both the performance and mathematical knowledge-growth of her pupils. In her reflective interview she says:

H: *I am not hopeful. I had some of this group - many - last year, and we spent ages measuring circle circumference etc and then this year I asked "what's pi - give me some idea of where it comes from? ... meaning?" and not a clue. How about area of a triangle? Nothing ... and this was a group that I did extensive work with, and for nothing! So what I am saying is that why didn't I just get them to learn the formula for the area of a circle in Std 7 and then you expect them to have forgotten it instead of having spent three double periods on measuring the circle etc. That is the dilemma. They don't remember the string either. You are exhausted at the end of a day of this. And "What for?" you sometimes ask?*

J: *Could you teach in another way?*

H: *Nnnooo ... I have made a lot of the changes academically for myself, and then it has resulted in this kind of thing. Then I become more critical of what is going on and it helped me make decisions I wanted to make but didn't have the skills to do so, or awareness or whatever to do it.*

> For most teachers in schools, the crucial issue is maintaining a suitable balance between offering children opportunities for open-ended exploration and discussion and, on the other, fulfilling a responsibility for achieving established curriculum goals. It is generally accepted that this is a difficult balance to achieve. (1995, p. 29)

The difficulty of accomplishing this balance is illuminated in the episodes selected here and Helen's reflections on them.

3 THE PARTICULAR CONTEXT

3.1 The school and class

Helen teaches in a girls-only Model C (historically white) state school. This school deracialised faster than most other Model C schools and at the time of the research, fewer than 50% of the pupils were white. The school is well-resourced. There is a library and computer centre. At an individual classroom level, for example the classroom where observation and videoing were carried out, there was an OHP and screen, a good blackboard and a whiteboard. Helen was thus able to make use of three different teaching resources. She had prepared overhead transparencies with task instructions; she wrote important notes during class discussion on the whiteboard, and used the blackboard for recording her and the pupils' activity in the lesson.

The class was a 'mixed ability' Standard 6 (Grade 10) of 30 pupils. It included immigrant pupils, one of whom had arrived in the country recently from Taiwan and 'did not speak a word of English'.

3.2 Helen's approach

Helen's classes are interactive and task-based. They reflect her concern that mathematics should be contextualised and learning meaningful and lasting.

thinking is often partial. For example, sometimes pupils say what they did but not why they did it (Clive talks of this - Chapter 4). That is, their articulation of their thinking is only procedural, rather than also principled (Mercer, 1995, p. 41). Sometimes, a pupil's expression of his or her thinking is confusing for others (for example, Joe in Chapter 7). As will be seen in this chapter, sometimes a pupil's expression is actually wrong. Yet, whether limited or wrong, pupils' mathematical expressions often convey some level of understanding or insight. In practice, the issue for teachers, then, is how to work with what pupils bring, how to get at and work with what it is they are trying to say. Clive is not sure how to act when pupils seem unable (or unwilling?) to explain their thinking. And Sue is unsure of how to help Joe and other pupils to 'develop their language'.

2.3 Relating explicit language teaching to the wider educational context

Sue and Thandi 'tell' and 'model' as part of managing their dilemmas in their multilingual classrooms and these practices resonate with Mercer's argument that 'teacher talk' can be an effective strategy for 'guiding the construction of knowledge' - though a strategy to be used sparingly (1995, p. 19).

Mercer theorises a sociocultural theory of classroom talk and argues that 'teacher talk', and question and answer routines are one of many effective strategies for guiding the construction of knowledge. There is good reason to use it, and good reason to use it sparingly, as one of a wide repertoire of communicative activities. Question and answer routines provide little opportunity for language as a social mode of thinking - for pupils to use talk in any but a limiting way (1995, p. 6). However, there is no good reason why the teacher should make a once-and-for all choice between using 'traditional', whole class, didactic methods or the more open-ended discussions and group activities associated with 'progressive' or 'learner-centred' education.

Three key dilemmas emerged in this study. The *dilemma of code-switching*, and its extension to include the dilemma of whether or not to model mathematical English, simultaneously captures, categorises and allows for explanation of the teacher's actions and reflections in a situation where she and most, but not all, her learners share a main language that is not the language of instruction. Here decisions in the classroom often revolve around the tension between developing pupils' English vs. ensuring pupils understand the mathematics; and around whether the tacit practice of modelling mathematical English is in effect, the teacher 'talking too much'.

Over time, the dilemma of code-switching was a powerful source of praxis for Thandi. Thandi interviewed her pupils to find out their views of group work and their use of English in class. Through her action research and her use of dilemma language, Thandi was able to identify and then transcend her dilemma. She also embraced the need not only for code-switching but for change in her mathematical activities. Together with her reflections on her video, Thandi's action research is most convincing that dilemmas are analytic devices and sources of praxis.

The *dilemma of mediation* enables analysis of action and reflection in learner-centred classrooms where a participatory-inquiry approach has effectively been developed. Here, emphases and decisions revolve around validating pupils' meanings vs. developing mathematical communicative competence; around using verbalisation as a learning resource vs. verbalisation as a teaching resource; and around the shifts between talking within vs. talking about mathematics. The dilemma of mediation also entails recognising teaching intentions in relation to the development of scientific concepts and the instructional role this implies. An instructional or scaffolding role is further required when there is no effective construction zone for pupil-pupil interaction. These instructional roles are in tension with a desire to elicit, encourage and validate pupils' conceptions. Such dilemmas emerge for Sue precisely because here is a context of changing pedagogy.

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS FOR TEACHER EDUCATION AND FURTHER RESEARCH

The notion of a 'teaching dilemma' constitutes the key mechanism that captured and opened up teachers knowledge of the elusive, complex and dialectical nature of teaching and learning mathematics in multilingual classrooms. This study has confirmed that teaching dilemmas are at once explanatory tools and analytic devices for teaching. They make explicit the tensions inherent in teaching. At the same time a language of dilemmas can function as a source of praxis. Teachers can use a language of dilemmas to reflect on and transform their practices so as to meet the mathematical needs of their linguistically diverse learners.

This notion is not original. However, for this study, there were two significant limitations to available dilemma language. Firstly, it did not adequately capture the specificity of *mathematics* and *language* learning in school. Once located in a multilingual mathematics classroom, a language of dilemmas becomes generative of dilemmas that highlight tensions specific to the mathematical and language challenges in such classrooms.

Secondly, the conventional notion of a teaching dilemma was polarised between Lampert's emphasis on the personal and the practical in managing dilemmas, and the Berlaks' contextual emphasis in their dialectical account of dilemmas in schooling. *Teaching dilemmas are at once personal, practical and contextual.* This empirically derived observation can be explained by sociocultural theory which not only effectively combines the teacher's context and his or her biography, but also recognises the specificity of school subject knowledge.

The theoretical contribution of this thesis thus lies in broadening the conception of teaching dilemmas and in extending the language of dilemmas to capture the specificity of the multilingual mathematics classroom.

she tries to embrace new practices and make mathematical knowledge available in her particular multilingual classroom.

An important question for mathematics teacher education if it is to address the dynamics of a multilingual classroom is: what significance do teachers attach to pupils use of a 'wrong' word or expression? Opportunities to grapple with implicit and explicit practices on language issues like those highlighted by Helen will enable mathematics teachers' informed decision-making in moments of their practice in their multilingual classrooms.

NOTES

1. In sociocultural terms, this is the dialectic between language and thought, where paraphrasing is associated with personal appropriation of cultural (that is, within a community of practice) concepts and ideas. (Leontiev and Luria, 1968).

2. It is important to repeat here, as Bernstein (1993) so powerfully points out in his introduction to *Charting the Agenda: Educational Activity After Vygotsky* (Daniels, 1993), that language is a cultural tool, and so a tool for learning. But language itself is a producer of relations of power. This point is also made by Ivie (1989). While language is a resource in the classroom, it does not function simply in resourceful and unproblematic ways.

3. While follow-up activity did not necessarily entail my direct participation, Helen was happy that I should assist with arranging the videoing and observe the class. A further reminder here that analysis of classroom episodes has not included detailed discourse nor conversational analysis. Such analysis will no doubt reveal a great deal. However, the focus here is on what Helen says and does. Classroom episodes are illustrations of Helen's knowledge of her practice.

Finally, Lave and Wenger's notion of transparency illuminates classroom processes. Transparency involves both visibility and invisibility, just as with a window. Both visibility and invisibility are part of transparency in the practice. Resources need to be seen to be used. They also need to be invisible to illuminate aspects of practice. So too with language as a resource for mathematics learning. For this resource to be transparent, effectively usable by learners, they must be able to see it and then use it. They must harness language as a resource, focus on it when necessary, but then render it invisible and as a means for building mathematical knowledge.

Teachers like Helen, Sara and Clive, as shown in this study, are concerned about their verbalising and having pupils verbalise 'correct' mathematical language, about using language as a shared public resource in the mathematics classroom. And while access to educated mathematical discourse is important, Helen's classroom illustrates how it can bring a dilemma of transparency where on the one hand making the language visible and itself the focus of attention, obscures the mathematical idea, and on the other of 'going on too long' and diminishing pupils' use of educational and educated discourse. And this is particularly so in a dominant culture of traditional teaching.

Together with Chapter 7, this chapter reveals the fundamental tension between implicit and explicit practices with respect to language issues in multilingual mathematics classrooms. As was argued in Chapter 7, these issues are present in all classrooms, but are present in particularly heightened form in multilingual classrooms. Helen attends to pupils' mathematical language expression as a shared public resource for class teaching. She also sees it as providing a display of mathematical knowledge. As Pimm (1996) points out, the latter attention acknowledges that all I, as teacher, have access to is the forms. And Helen poses these questions: if they can say it 'right', do they know it? And if they don't say it 'right', can I let it go? As in Chapters 6 and 7, there is no simple answer here, rather, an instance of a teacher grappling with the issue as

visibility and invisibility is intertwined with using language both to explore mathematics and as a display of knowledge, with developing both educational and educated discourse.

8 CONCLUSION

Just as from Thandi and Sue we have seen that using main languages in the classroom as a resource is a complicated matter, and that a more participatory inquiry approach creates dilemmas of mediation, so from Helen, we see that explicit mathematics language teaching, while beneficial, is not a straight forward 'good thing'. It brings a language-related dilemma of transparency with its dual characteristics of visibility and invisibility. From Helen we can see Lave and Wenger's argument that talking about a practice from outside the practice can lead to sequestration from, rather than access to, the practice. And we must remember that Helen's practice is not restricted to explicit language teaching and thus in constant and perpetual danger of moving outside the practice. She works on her pupils' expression, on what they bring as a result of their talking within their practice - their mathematics classroom.

Helen's particular questions, reflections and the discussion they provoke in the workshops reflect the real worries of teachers as they try to mount something new and different in their circumstances which, particularly at the more senior levels, include passing external examinations in a wider context where traditional teaching practice remains dominant. From Helen we learn the value of explicit mathematics language teaching some of the time. This is in relation to both pronunciation and to the dilemma of transparency in practice where attempts to make mathematical language visible can actually obscure. It is not simply a matter of 'going on too long' but of managing the shift of focus between mathematical language and the mathematical problem (and of course these are intertwined).

there are both political and educational dimensions to this dilemma for Helen. If she 'goes on too long', she diminishes pupils' opportunities to use educational discourse and inadvertently obscures the mathematics at play. If she leaves too much implicit then she runs the risk of losing or alienating those who most need opportunity for access to educated discourse. She wonders about the possible effects of leaving in play a shared sense of trigonometric ratios but a public display of incorrect mathematical language: 'if they don't say it right, can I let it go?'.

Of course, there is a world of difference between 'what they are saying is wrong' and 'I can't get at what they are trying to say to me' (Pimm, 1996). This is the difference Sue tries to get at in the third workshop when the teachers are discussing how a pupil can talk about sin 40 on one day but not sin theta the next. Sue says she would ask herself what it is she (the teacher) is not understanding and try to ask the pupil another question, something that would help Sue understand what the pupil is trying to say. Helen, more explicitly, is working on the form of what they are saying, and that this form is wrong.

In sur ary, through Helen we gain further insight into a teacher with a strong and pa..ular set of educational and political beliefs about language and learning struggling to understand the potential benefits for pupils of more interactive practices and hence more language in the mathematics classroom. Underpinned by her view of mathematics as a language, and deeply concerned with the politics of access in South Africa, Helen moves between taking 'size' and 'side', and 'dependent' and 'independent' out for scrutiny - making them visible. She then re-inserts them in their particular mathematical use in the lesson. But this is a struggle. It is deeply bound with using language (attending to pupils' language expression) as a shared public resource where, if the mathematical problem is to be addressed, the language itself must also be invisible. It cannot remain the focus of conscious attention. This struggle over

where she questions, bringing into focus the incorrect use of the concept and term 'independent', and finally reformulates and recaps emphasising what she sees as most significant in the description of trigonometry that has emerged from the pupils. But this explicit language teaching is a struggle here.

Helen's knowledge helps us identify a fundamental pedagogic tension between implicit and explicit practices with respect to language issues in her multilingual mathematics class. She harnesses language as a resource in her classroom. As a resource in the practice, its transparency, i.e. its enabling use by learners, is related to both its visibility and invisibility. Specifically, Helen attends to pupils' expression as a shared public resource for class teaching. This is a characteristic of classrooms that is not shared by many other speech settings (Pimm, 1996). The language itself becomes visible and the explicit focus of attention. It is no longer the medium of expression, but the message itself - that to which the pupils now attend.

Episode 3 shows Helen struggling to mediate the scientific concepts of constant ratios and dependence and independence as they arise in school trigonometry. She does this in her multilingual classroom where the complex three dimensional dynamic intersects with her educational and political beliefs as well as her view of mathematics as language. Helen focuses on pronunciation and correct ways of speaking mathematically, thus attempting to provide access to English and to mathematical discourse. However, these attempts occur within her classroom culture where language is used simultaneously to explore and display mathematical knowledge. And problems emerge.

On reflection, Helen feels that her attempt to enable access to mathematical (educated) discourse brings the problem of 'going on too long'. In explicitly making mathematical language visible, it becomes opaque, obscuring the mathematical problem. The dilemma of transparency arises, of whether (and when) to make mathematical language explicit or leave more implicit. Again

Helen's working assumptions of a strong relationship between language and thought are seriously challenged as she experiences and observes pupils expressing their thinking on one day then not the next; of pupils expressing clear and correct mathematical thinking but not being able to discern problematic expression in/of others; and of pupils saying things 'wrong' but creating a sense that they have some grasp of the mathematics in play. She also sees how in her focus on language teaching, in her attention to their use of dependent and independent, the pupils lost their focus on the mathematical and trigonometric problem from which this use arose.

7 DISCUSSION

Helen's action research reveals the tensions in whole class interaction as attention is focused both on pronunciation and on pupils' mathematical verbalisations, and highlights the dilemmas this explicit mathematics language coaching creates for teachers. Through episodes in Helen's class and her reflections we learn to attend to pronunciation in a multilingual mathematics class and the role it might play in meaning-making. And we see what we know only too well, that some mathematics is difficult for pupils to say precisely and with meaning.

The specific dilemma for Helen lies in moving back and forth between educational and educated discourse, between language used for thinking and language used as a display of knowledge, between talking within and about mathematics. I have argued that in sociocultural terms, teaching and learning mathematics entails this moving back and forth, and that Sue, for example, needed to be more explicit in this regard. In contrast to Sue, Helen does work explicitly on pupils talking about mathematics. She provides opportunity for pupils, amongst themselves, to elaborate and then share their meanings of 'trigonometry'. This elicitation of pupils' thinking suggests to her that there is confusion and she moves to clarify this through a particular scaffolding process

And she revisits her question in the first workshop: if they can say it, do they know it? and moreover the recognition that mathematical expression is often very difficult.

> *... I think that that sentence came out of something that the group was working with ... if you actually take a sentence like that which is supposed to be concise, and it carries a whole lot of meaning there is difficulty ... They can talk to you about it and they can give you a long explanation of what to do ... so its seems to me to be also a problem of expressing a lot of maths in one clear sentence. For me that is also linked to the issue of how we transmit maths to each other. If you make a mathematical statement you are involved in getting it down to a simple, short-hand language that we can all share ...*
>
> (W2, 90-120)

Finally, she poses a central question on verbalisation and the dilemma of transparency:

> *... In retrospect, when I look at that lesson, I went on but much too long (laughter) on and on and on and I keep saying the same thing and I repeat myself, on and on .. and I watch the video and I think I wonder why they are still sitting in their seats and I am falling on the floor falling asleep. But the thing is then if you have a sense that there is a shared meaning amongst the group can you go with it? um ... when the sentence is completely wrong? ... Can you let it go? Can a teacher use a sense of shared meaning to move on? I think this is a central question in terms of the verbalisation and discussion.*
>
> (W2, 151-165)

Helen also remarks on and remembers that in her attempt to teach mathematical language explicitly, the mathematical focus of the lesson is lost. She remembers being thrown by a pupil's interjection: 'It makes a difference to what?' (see Episode 3, lines 86-93).

triangles. When Jill and I discussed this we talked about the one part where a child put forward what she thinks is going on in relation to that issue and it is a question of even though her language is not clear is there understanding amongst the rest of the students and it seems like the rest of the students do understand even though she is using incorrect language. So we can watch and think around that.

(W_, 19-33)

She then plays the video from the point where the student says: *the ratio of the two sides is independent to the size of the angles in the two triangles* and she is writing what is being said word for word on the board for the class to think about.

After the tape has run, she also comments that this is not a good student. Thus her concerns about the student's expression interact with her knowledge of the student - and that poor expression might indeed reflect a lack of understanding.

She continues her reflection:

Just after the sentence is written on the board and I ask: 'What do you understand by this statement?' The one child puts forward a perfect explanation. She talks about the angle being the same in both triangles and then she talks about the depth of the triangles or whatever and I pick up on that ... and then this child (getting to the place on the video where a second pupil is responding) now does it absolutely perfectly. So, that is two very good expressions of what is going on. And yet when you ask the class: 'Is this sentence correct?' (Pointing to the sentence she has written verbatim from the first student on the board), there is this complete silence. So the question for me is: even in the minds of those two children who put forward such consistent explanations, what's going on with them? () that they cannot ... um ... pick up incorrectness in the sentence?

(W2, 60 80)

reduce her authoritarianism and control, to allow and encourage on the one hand pupils' voices and meanings, and on the other provide opportunity for language as a social mode of thinking. And she does this without leaving what counts as mathematical language, for example, being precise, at the level of the implicit.

In Episode 3 in particular. Helen asks what the statement with 'independent to the size of the angles' means, inviting rethinking, and further elaboration (lines 10-12). She tries to engage pupils in making sense of the statement (line 44). When S7 expresses a clear explanation (lines 57-60) she focuses on this, reformulates (lines 66-67), and asks the class to compare the two versions (line 64). She assists by recapping and stressing that the 'angle makes the fundamental difference' (line 73) only to find that the focus of the mathematical discussion is lost on the pupils. And so she reformulates and recaps again, and now, in her view, she has gone on too long.

Helen's practice has come to include periodic focusing of her and her pupils' attention onto how we speak mathematics - educated discourse (Mercer, 1995) - and on possible language/pronunciation confusions. This creates what I have called the dilemma of transparency, of language as a resource in the classroom carrying the dual characteristics of visibility and invisibility, and, as a result, Helen faces a new set of challenges.

6 HELEN'S REFLECTIONS

Opening the second workshop, and before showing some of the extract above to the other teachers, she says:

> *One of the issues was linguistic ... the sound issue between sides with an s and size. A lot were hearing size when I was saying sides and we picked up on that issue and then (on what trig is) on the ratio being the same for a given angle in different sized*

the end of the word. While sight, size and side are all monosyllabic, with emphasis at the beginning of a word, they all come to sound roughly the same. Similarly, white and wide, forty and fourteen, multiple and multiply all create possibilities for confusion. In this context, that Helen explicitly distinguishes the spoken words 'dependent' and 'independent' is appropriate. And Sara's insistence that she monitors pupils' written records of their mathematical ideas in their notebooks makes good practical sense.

I noted further that Helen worked explicitly in this lesson with both pronunciation and correct mathematical expression of meanings of trigonometry. I was aware too that as part of her own research, her attention to language issues was probably heightened, bringing more emphasis on explicit language teaching in her practice here than might have been otherwise.

Both in this lesson, and those videoed and observed the year previously, Helen is notably more directive than Sue during report-back time. While Sue invites discussion from the class after each report, Helen has all groups report and then she directs whole class teacher-pupil interaction on what has been presented, focusing attention on problems and reformulating and recapping where necessary. It is in this part of the lesson that explicit language teaching is evident.

Now, on reflection on my own observations and with the hindsight brought by the study as a whole, I am not sure in this instance that 'sides' and 'size' actually caused any confusion of meaning in pupils. Clearly, strictly speaking in mathematics, we do talk of the side opposite or adjacent to an angle, but we do not refer to 'sides of angles'.

What became more and more interesting for me in the context of the study and Helen's views and concerns was her intentional shifting of focus in the classroom onto the mathematical language at play. She is working hard to