7. EXTRACTION OF THE PILLAR AS A REMNANT

Introduction

While a shaft pillar affords the highest degree of protection to a vertical shaft, it also presents the problem of final extraction of the reef in the pillar area. This is usually a slow and expensive process and involves the most difficult mining conditions.

Extraction of the shaft pillar has been carried out a great number of times and a large amount of experience is on record.

It can be claimed that the theoretical and practical tools are available for the careful planning of pillar extraction.

This chapter covers briefly the planning of the extraction process and by looking at a number of cases the mechanisms of damage related to each method of extraction are discussed.

7.1 Planning of the extraction of the pillar

Planning of the extraction of a shaft pillar as a remnant is subject to two considerations. From the economic management point of view it is desireable to mine certain grades and tonnages to suit general economic climates and particular economic needs of the mine. Secondly, the planning of the pillar extraction is subject to engineering considerations. If a part of the pillar is to be left as a remnant it should be located in the lowest grade area. This is an example of how engineering design must take economic conditions into account. Whatever the case might be, it is necessary for the rock mechanics engineer to make the economic mining of the shaft pillar possible and safe.

It follows then that in each case the planning of pillar extraction will be different. It is then necessary to have an engineering tool available that is both versatile and accurate. In the field of rock mechanics the best tool that can be used is a three-dimensional boundary element program such as the MINSIM program that was developed by the Chamber of Mines of South Africa.

With MINSIM it is possible to model different sequences of mining while the influence of pillars and backfill can also be tested. The final aim is to plan the extraction process to be safe and economical.

Since the extraction of a shaft pillar is different in every case, it is difficult to lay down any theoretical guidelines for the pillar extraction process. The writer proposes to go about this study in the following manner:

i)A number of cases where shaft pillars were extracted as remnants will be discussed and an attempt will be made to identify common problem areas during pillar extraction,

ii)finally these problem areas will be examined and discussed.

Table 7.1

LISTING OF ELEVEN CASES WHERE SHAFT PILLARS WERE EXTRACTED AS REMNANTS

CASE	DEPTH	DIP	MINING METHOD	DAMAGE
A	914	21	Inner pillar then outer pillar. Extensive stiff support in inner pillar area and extensive compressible support in outer pillar area. 9 percent of original pillar left intact.	Severe fracturing above reef intersection. Pressure burst damage to buntons.
B	564	33	Inner pillar then outer pillar. Extensive stiff support in inner pillar area.	Hangingwall ride downdip. Shaft sidewall sliped to compensate.
C	661	21	Inner pillar then outer pillar. Stiff support in inner and outer pillar area.	Damage to pipe column. Hanging moves updip. Scaling in ore-bin.
D	778	13	Inner pillar then	Damage to pipe

			outer pillar. Sandfill.	column. Hanging moves updip. Damage to concrete brick lining below reef. Large amount of rock fell down shaft.
E	360	22	Stiff support in inner and outer pillar area. Sandfill in old stopes.	No trouble in vertical hoisting during and long after extraction.
F	409	42	Downdip longwall advancing from one boundary of pillar to other edge. Compressive support and waste packing later.	No sign of parting on bedding planes. Pressure on station below reef after full closure.
G	451	17	Inner pillar then outer pillar. Dense stiff support close to shaft getting less dense further from shaft.	Slight subsidence. No difficulties in shaft.
H	168	17	Inner pillar then outer pillar. Dense stiff support close to shaft getting less dense further from shaft.	Parting on bedding planes above reef horizon. No damage to shaft.
I	122	4	Inner pillar then outer pillar. Dense stiff support close to shaft getting less dense further away from shaft.	Minor cracks in shaft lining. One minor pressure burst when pillar 95 percent mined.

J 487	42	Inner pillar then outer pillar. Dense stiff support close to shaft getting less dense further away from shaft. Shaft brow tied in with cables.	Pressure and scaling during mining of final remnant. Scaling in shaft below reef station.
K 1890	42	Dip longwall advancing along strike from one side of the pillar.	Shaft and station steelwork extensively distorted 5 m above reef intersection. Concrete side-walls damaged. Buntons distorted from 40 m above to 20 m below reef.

7.2 Discussion of Tabulated Information

The extraction of shaft pillars can present all the problems associated with leaving a protection pillar and with the early extraction of reef in the pillar area. This includes:

i) high compressive vertical stress and strain around the vertical shaft due to a small remnant,

ii) high tensile vertical stress and strain around the shaft when the reef in the vicinity of the shaft is extracted and

iii) high induced tilt values due to asymmetric mining around the shaft.

The extent to which the above conditions will apply depends on the mining sequence employed during the mining of the shaft protection pillar.

The most commonly used methods are:

i)inner pillar then outer pillar,

ii)downdip longwall advancing along strike from one edge of the pillar.

The mechanisms of damage related to each of the mining sequences mentioned above will now be discussed.

7.2.1 <u>Inner pillar then outer pillar</u>

The most popular method of extracting the shaft pillar is to first mine the inner pillar area and then the outer pillar. This has the advantage that the extraction process is started in a low stress zone. Experience has shown that this method affords the highest degree of protection to the vertical shaft.

When the inner pillar is mined first the vertical shaft is placed in a tensile zone. Tensile strain causes downward movement of the hangingwall and a rise of the footwall. Bed separations can set in. Damage is usually confined to the region close to the shaft-reef intersection.

- High tensile strains and bed separation can cause cracks and loosening of the shaft lining.
- Shaft steelwork and pipe columns can be damaged by displacement of the surrounding rock mass.
- Excessive inflow of water can result where water bearing rock is encountered.

The major advantage of this sequence of mining the shaft pillar is that the maximum unsupported span remains small while the inner pillar is being mined. This means that the vertical shaft remains in a relatively undisturbed environment. Strain criteria arrived at in Chapter 6 should apply in this case.

7.2.2 Downdip longwall

This method was employed a number of times. Mining is started at the highly stressed pillar edge and the vertical shaft can suffer from high vertical compressive stresses as well as excessive tilt. From a production point of view it is advantageous to have a longwall system.

- High compressive stresses can damage auxiliary excavations such as sumps and settlers and shaft stations. Soft layers such as clay or coal could be squeezed out by excessive compressive stresses. This could damage the shaft lining.

- High strains can lead to distortion of shaft steelwork and pipe columns and damage to the shaft lining.
- Excessively high tilt values could make winding dangerous or impossible if shaft steelwork is severely distorted.

When the pillar is mined in this sequence the vertical shaft is placed in a highly disturbed zone as mining progresses. As mining gets closer to the shaft, compressive stresses and strains increase in the rock surrounding the shaft. Tilt and shear values are also high while the mining face is close to the shaft. When the face has passed the shaft high tensile stress values dominate in the rock surrounding the vertical shaft.

CONCLUSIONS

The mechanisms of damage associated with different sequences of shaft pillar extraction have been outlined. It can be seen that in a number of cases the damage to vertical shaft and auxiliary excavations was severe. The extraction of shaft pillars need to be executed with the greatest care. If a computer simulation indicates that conditions may be unacceptable the use of stabilising pillars, backfill and waste packing could control the extent of damage induced by mining. Monitoring devices should be employed in the vertical shaft and stopes so that excessive ground movement can not occur without the knowledge of mine management.

8. SUMMARY AND CONCLUSIONS

The effects of three configurations of mining around a vertical shaft have been studied. In the first case the effects of creating a shaft pillar are examined, the second part of the dissertation covers the early mining of reef in the shaft pillar area and finally, the extraction of an existing shaft pillar is investigated. Conclusions have been drawn, where appropriate in the main body of the dissertation and are summarised here.

A number of methods, aimed at predicting conditions associated with the creation of a shaft pillar, are discussed. One of these methods is a closed form solution that was derived to estimate stresses and differential movements when a circular shaft pillar is created at infinite depth. These equations are based on elastic theory and have been reduced to a simple form by making a number of assumptions valid at infinite depth.

In addition, a large part of the first section of this dissertation is based on another closed form solution that takes the effects of the surface into account. This is therefore also valid at both shallow and moderate depth. In this solution the shaft pillar is still considered to be circular but these equations are more complex and cannot be integrated with normal integration rules. Numerical integration with Simpson's rule is used.

The possibility of simulating the elastic effects of mining around a shaft pillar using a three dimensional boundary element program is also discussed but this method is not evaluated in great detail.

The definitions of shallow, moderate and great depth are given and it is shown that the closed form solutions mentioned above converge in the moderate depth range. It is, however, advisable to use the complete solution, that takes into account the effects of the surface, whenever time and facilities allow.

Using this equation, it has been shown that mining at a distance greater than 1000 m from the edge of the protective pillar has little or no influence on the vertical shaft. This conclusion was employed later in the study to support the assumption that shaft pillars are subjected to the maximum stress soon after their creation.

An important assumption of the complete closed form solution was investigated. This is the assumption that closure of the mined out area is complete, even at the edge of the pillar. It emerged that an error of approximately 12 percent results at the lower limit of the moderate depth range, while at great depth the error becomes negligible.

The interpretation of stresses and differential movements, calculated with the closed form solution, were also analysed, as were the factors to be considered during the design of a shaft pillar. The influence of variations in these factors, their maximum allowable values and the positions where these occur were all studied.

It was shown that the normal method of designing a shaft pillar according to the calculated strain at the shaft-reef intersection could cause damage, since the maximum strain along the shaft axis can occur some distance above or below the reef. On the other hand the maximum tilt is expected to occur some distance into the hangingwall. The critical strain value of 1 mm/m, normally used in the South African mining industry was found to be acceptable. The critical tilt is of the same order of magnitude, but is of lesser importance since the strain rather than the tilt will be the limiting differential movement criterion.

The influence of high stress, especially on auxiliary excavations, has also been examined. The practice of using 100 MPa as the maximum stress when designing shaft pillars could lead to instability of auxiliary excavations. Shaft pillars are usually designed so that the maximum stress at the shaft-reef intersection is below 100 MPa, however, the stress in the protective pillar increases as the distance from the centre increases. Stresses at distances of 0,5R above and below the reef intersection also tend to be significantly higher than the values at the shaft-reef intersection. Service excavations in these positions might be difficult to maintain. Damage to a vertical shaft protected by a shaft pillar, could also occur if the combined stoping width is not taken into account where multi-reef mining is done.

A case study of ten existing shaft pillars showed that shaft pillars are generally smaller, in practice, than they would be if designed using the 100 MPa and 1 mm/m criteria. In the cases where damage did occur, it was usually in the form of deterioration of auxiliary excavations, as could be anticipated from the conclusions presented in the preceding paragraph. Although some of these old pillars were very small, they did actually remain stable. It was found that solid ground, waste packing, sandfill and compressible support have a significant stabilising influence on shaft pillars.

The section covering the early mining of the reef in the shaft pillar area opened with a case study showing that the early mining of the reef around the Harmony Ventilation Shaft was not without hazard. The water problems experienced made travelling in the shaft very unpleasant and this particularly prevented this exercise from being repeated on other mines. The case study showed that damage caused by this sequence of mining can be predicted fairly accurately. A reasonable correlation was found between the formation of cracks in the shaft lining and a tensile strain of 0,2 mm/m while water inflow corresponded well with tensile strain values of 0,3 mm/m.

It was shown that using an elastic model footwall rise can be estimated accurately. However, strains in the dislodged hangingwall can be much larger than calculated. Local geology determines the extent of difference between calculated and observed strain in this region. Beyond the dislodged hangingwall, approximately 140 m in this case, measured strains agreed well with corresponding values calculated with an elastic model.

Finally, the MINSIM program was used to predict conditions associated with the early mining of reef at great depth. It was assumed that the mining method would be two downdip longwall faces advancing away from the shaft. Calculated strains were found to be large enough to cause some damage along a considerable length of the shaft and the ride component became significant when the dip was as large as 30 degrees. Mining of the reef in the shaft pillar area can only be carried out at great depth if precautions are taken to protect the vertical shaft.

The damaging effects of mining during the extraction of an existing shaft were investigated. Since economic factors often determine the time during the life of the mine and the method whereby a pillar is to be extracted, it is difficult to generalize about this procedure. A number of case studies showed that damage to the vertical shaft is common during pillar extractions. Damage generally takes the form of distortion of shaft sidewalls and scaling or collapse of auxiliary excavations. The mechanisms of damage associated with the two most common methods of extracting a shaft pillar have been discussed. In the first of these methods the inner pillar is first mined and then the outer pillar. More recently shaft pillars have been extracted using a downdip longwall advancing along strike from one edge of the pillar.

When the inner pillar is extracted first, the results showed that the shaft is placed in a tensile zone. The span across this tensile zone is usually kept small so that the rock mass around the shaft is not subjected to an extreme situation. As the pillar is reduced in size, stresses will increase and endanger excavations in the pillar. This is well understood in practice and the generally accepted solution is to move any service excavations before pillar extraction is started.

The downdip longwall method has the advantage that production is continuous and can therefore be maintained at a higher level. However, high stresses can be expected as the pillar becomes smaller and damage to auxiliary excavations can result. Large compressive strains can damage shaft equipment and if this procedure is not carried out symmetrically, a tilt component can endanger the operation of the shaft.

This study has shown that methods based on elastic theory can be used to predict damage caused by mining in the vicinity of a vertical shaft. The mechanisms and extent of damage associated with a number of mining configurations have been quartified against the background of actual case histories. Presently accepted limiting criteria were also examined and where necessary new values have been proposed.

APPENDIX A1

```
Program Stressequation(input,output);

 **   THIS PROGRAM CALCULATES INDUCED AND RESULTANT STRESSES          **
 **   IN AND AROUND A VERTICAL SHAFT WITH A CIRCULAR SHAFT PILLAR.    **
 **   IT IS POSSIBLE TO CALCULATE STRESSES DOWN THE SHAFT OR IN       **
 **   A HORIZONTAL DIRECTION AWAY FROM THE SHAFT.                     **
 **   IT IS ASSUMED THAT THE REEF AROUND THE PILLAR IS EXTENSIVELY    **
 **   MINED OUT.                                                      **
      -----------------------------------------------------------------


 **        DECLARATION OF CONSTANTS AND VARIABLES                     **
           --------------------------------------

Var   iout                                     :text;
      fi,si,rsp,xl,d,zi,zl,z2,p,a,b,rvar,z,
      hi,hfi,al,ae,ao,an,be,bo bn,ji,
      sw,upper,young                           :real;
      m,n,c                                    :integer;
      direction                                :char;


Function Stressfirsthalf(d,p,r,zi,zl,z2:real):real;
         ---------------

 **              STRESS FUNCTION TO BE INTEGRATED (FIRST HALF)      **
                 ---------------------------------------------

Var a,b,cc,dd,e,f,g,h,i,j,l:real;
   Begin
     a:=4*(1-p)/(exp(1.5*ln(r*r+z2*z2)));
     b:=(12*z2*z2*(1-p))/exp(2.5*ln(r*r+z2*z2));
     cc:=(2*(2*z2*z2-r*r))/exp(2.5*ln(r*r+z2*z2));
     dd:=8*z2*(zi+2*d*(1-p))/exp(2.5*ln(r*R+Z2*z2));
     e:=(6*z1*z1*(2-p)+(1-2*p)*r*r)/exp(2.5*ln(r*r+z1*z1));
     f:=(6*z2*z2*(2-p)+(1-2*p)*R*r)/exp(2.5*ln(r*r+z2*z2));
     g:=(5*z1*z1*(2*z1*z1*(2-p)+(1-2*p)*r*r))/exp(3.5*ln(r*r+z1*z1));
     h:=(5*z2*z2*(2*z2*z2*(2-P)+(1-2*p)*r*r))/exp(3.5*ln(r*r+z2*z2));
     i:=(10*z2*(2*z2*z2-r*r)*(zi+2*d*(1-p)))/exp(3.5*ln(r*r+z2*z2));
     j:=(6*D*(z2*(2*z2*z2-3*r*r)+3*zi*(2*z2*z2-r*r)))
                                        /exp(3.5*ln(r*r+z2*z2));
     l:=(42*d*zi*z2*z2*(2*z2*z2-3*r*r))/exp(4.5*ln(r*r+z2*z2));

     stressfirsthalf:=r*(a-b+cc+dd-e-f+g+h-i+j-l);

   End; (*stressfirsthalf*)


Function Stresssecondhalf(d,p,r,zi,zl,z2:real):real;
         ----------------

 **           STRESS FUNCTION TO BE INTEGRATED (SECOND HALF)        **
              ----------------------------------------------
```

```
Var a,b,cc,dd,e,f,g,h,i,l,j,k:real;
Begin
  a:=(2*(1-2*p))/exp(1.5*ln(r*R+z2*z2));
  b:=((1-2*p)*r*r)/exp(2.5*ln(r*r+z1*z1));
  cc:=(4*(1-2*p)*r*r)/exp(2.5*ln(r*r+z2*z2));
  dd:=(6*z2*(z1+2*p*d))/exp(2.5*ln(r*r+z2*z2));
  e:=(2*(1+p)*z1*z1-(1-2*p)*r*r)/exp(2.5*ln(r*r+z1*z1));
  f:=(2*(1+p)*z2*z2-(1-2*p)*r*r)/exp(2.5*ln(r*r+z2*z2));
  g:=(12*d*zi*(2*z2*z2-r*r))/exp(3.5*ln(r*r+z2*z2));
  h:=(15*z2*(z1+2*p*d)*r*r)/exp(3.5*ln(r*r+z2*z2));
  i:=(5*(2*(1+p)*z1*z1-(1-2*p)*r*r)*r*r)/(2*exp(3.5*ln(r*r+z1*z1)));
  j:=(5*(2*(1+p)*z2*z2-(1-2*p)*r*r)*r*r)/(2*exp(3.5*ln(r*r+z2*z2)));
  l:=(21*d*zi*(4*z2*z2-r*r)*r*r)/exp(4.5*ln(r*r+z2*z2));

  stresssecondhalf:=r*(a-b-cc-dd+e+f+g+h-i-j-l);
End; (* stresssecond half *)


Function Lower(fi,rsp,xl:real):real;
         -----
 *  LOWER LIMIT OF INTEGRATION    *
    -------------------------

Begin
  lower:=sqrt(xl*xl*(cos(fi)*cos(fi)-1)+sqr(rsp))-xl*cos(fi);
End;


Procedure Intergratefunctions(a,b:real;m,n,c:integer);
          ------------------

 **          SIMPSONS RULE FOR DOUBLE INTEGRATION                    **
             -----------------------------------

Var i,j,mm,nn                     :integer;
    hi,hr,an,ae,ao,bn,be,bo,z :real;
Begin
  nn:=2*n+1;
  mm:=2*m-1;
  hi:=(b-a)/(2*n);
  an:=0;
  ae:=0;
  ao:=0;
  for i:=1 to nn do
    begin
      fi:=a+(i-1)*hi;
      hfi:=(upper-lower(fi,rsp,xl))/(2*m);
      if c=1 then
        bn:=stressfirsthalf(d,p,lower(fi,rsp,xl),zi,z1,z2)+
            stressfirsthalf(d,p,upper,zi,z1,z2);
      if c=2 then
        bn:=stresssecondhalf(d,p,lower(fi,rsp,xl),zi,z1,z2)+
            stresssecondhalf(d,p,upper,zi,z1,z2);
      be:=0;
      bo:=0;
      for j:=1 to mm do
        begin
```

```
          rvar:=lower(fi,rsp,xl)+j*hfi;
          if c=1 then z:=stressfirsthalf(d,p,rvar,zi z1,z2);
          if c=2 then z:=stresssecondhalf(d,p,rvar,zi,z1,z2);
          if (not(odd(j))) then be:=be+z else bo:=bo+z;
        end;
      a1:=(bn+2*be+4*bo)*hfi/3;
      if ((i=1) or (i=nn)) then an:=an+a1
         else if (not(odd(i))) then ae:=ae+a1
                else ao:=ao+a1;
    end;
  ji:=(an+2*ae+4*ao)*hi/3;
End; (*integrate*)


Procedure Writedownshaft;
          --------------

Begin
 writeln(iout,'INPUT DATA : ');
 writeln(iout,'----------   ');
 writeln(iout,'        DEPTH OF REEF=',d:8:1);
 writeln(iout,'        RADIUS OF SHAFTPILLAR=',rsp:8:1);
 writeln(iout,'        OUTER LIMIT OF MINING/integration=',upper:8:1);
 writeln(iout,'        STOPING WIDTH=',sw:8:2);
 writeln(iout,'        POISSONS RATIO=',p:8:2);
 writeln(iout,'        YOUNG''S MODULUS=',young:8:2);
 writeln(iout);
 writeln(iout,'OUTPUT DATA :');
 writeln(iout,'-----------  ');
 writeln(iout,'  Z     INDUCED STRESS   RESULTANT STRESS     RATIO');
 writeln(iout,'METRES        MPA              MPA           -Z1/RSP');
 writeln(iout,'------   --------------   ----------------    -------');
 writeln;
 writeln('  Z     INDUCED STRESS   RESULTANT STRESS     RATIO');
 writeln('METRES        MPA              MPA           -Z1/RSP');
 writeln('------   --------------   ----------------    -------');
End;


Procedure Writeacross;
          -----------

Begin
 writeln(iout,'INPUT DATA :');
 writeln(iout,'----------  ');
 writeln(iout,'        DEPTH OF REEF=',d:8:1);
 writeln(iout,'        RADIUS OF SHAFTPILLAR=',rsp:8:1);
 writeln(iout,'        OUTER LIMIT OF MINING/integration=',upper:8:1);
 writeln(iout,'        STOPING WIDTH=',sw:8:2);
 writeln(iout,'        POISSONS RATIO=',p:8:2);
 writeln(iout,'        YOUNG''S MODULUS=',young:8:2);
 writeln(iout,'        DEPTH AT WHICH VALUES ARE CALCULATED=',zi:8:1);
 writeln(iout);
 writeln(iout,'OUTPUT DATA :');
 writeln(iout,'-----------  ');
 writeln(iout,'  X     INDUCED STRESS   RESULTANT STRESS     RATIO');
 writeln(iout,'METRES        MPA              MPA            X/RSP');
```

```
 writeln(iout,'------   -------------   ---------------     -----');
 writeln;
 writeln('   X     INDUCED STRESS   RESULTANT STRESS     RATIO');
 writeln('METRES        MPA               MPA            X/RSP');
 writeln('------   -------------   ---------------     -----');
End;


Procedure Downshaft(a,b:real;m,n,c:integer);
          ---------

 **          TO CALCULATE STRESSES DOWN THE SHAFT                     **
             ------------------------------------

Var interval        :real;
    numberpoints,q  :integer;
Begin
  x1:=0;
  write('At what depth do you want your first value(that means ');
  writeln('your shallowest point)?');
  read(zi);
  writeln('At what intervals do you want values calculated?');
  read(interval);
  writeln('How many points do you want to calculate?');
  read(numberpoints);
  writeln('THIS WILL TAKE A FEW MINUTES!BE PATIENT!');
  writedownshaft;
  for q:=1 to numberpoints+1 do
   begin
     z1:=zi-d;
     z2:=zi+d;
     for c:=1 to 2 do
       begin
         intergratefunctions(a,b,m,n,c);
         if c=1 then si:=-ji/(8*3.141593*(1+p)*(1-2*p));
         if c=2 then si:=si+(ji*2*p)/(8*3.141593*(1-sqr(p))*(1-2*p));
       end; *** of for c *******
   write(iout,zi:6:1,si*young*sw:14:4);
   writeln(iout,zi*9.81*2.7E-3+si*young*sw:18:4,-z1/rsp:13:2);
   write(zi:6:1,si*young*sw:14:4);
   writeln(zi*9.81*2.7E-3+si*young*sw:18:4,-z1/rsp:13:2);
    zi:=zi+interval;
   end;
End; * downshaft  *


Procedure Across;
 ------

 **        TO CALCULATE STRESSES IN THE HORIZONTAL DIRECTION         **
           -------------------------------------------------

Var c,q,numberpoints :integer;
    interval         :real;
Begin
```

```
  writeln('At what depth do you want to calculate values?');
  read(zi);
  z1:=zi-d;
  z2:=zi+d;
  writeln('At what intervals do you want values calculated?');
  read(interval);
  writeln( How many points do you want to calculate?');
  read(numberpoints);
  writeln('THIS WILL TAKE A FEW MINUTES!BE PATIENT!');
  writeacross;
  xl:=0;
  for q:=1 to numberpoints+1 do
    begin
      for c:=1 to 2 do
        begin
          intergratefunctions(a,b,m,n,c);
          if c=1 then si:=-ji/(8*3.141593*(1+p)*(1-2*p));
          if c=2 then si:=si+(ji*2*p)/(8*3.141593*(1-sqr(p))*(1-2*p));
        end;
      write(iout,xl:6:1,si*young*sw:14:4);
      writeln(iout,zi*9.81*2.7E-3+si*young*sw:18:4,xl/rsp:13:2);
      write(xl:6:1,si*young*sw:14:4);
      writeln(zi*9.81*2.7E-3+si*72000*sw:18:4,xl/rsp:13:2);
      xl:=xl+interval;
    end;
End; * Across *



BEGIN  * Main Program *
  writeln('ALL DIMENSIONS MUST BE GIVEN IN METRES!');
  a:=0.0;
  b:=6.283185307;
  m:=100;
  n:=100;

  writeln('Depth of reef ?');
  read(d);
  writeln('Radius of shaftpillar ? ');
  read(rsp);
  writeln('Stoping width ?');
  read(sw);
  writeln('Outer limit/radius of mining ?');
  read(upper);
  write('Poissons ratio of host rock(usually between ');
  writeln('.15 and .5) ?');
  read(p);
  writeln('Young''s modulus (Mpa) = ?');
  read(young);
  writeln('Do you want to calculate stresses down the shaft or');
  writeln('in a horizontal direction away from the shaft? (D/H)');
  readln;read(direction);
  if (direction='d') or (direction='D') then downshaft(a,b,m,n,c);
  if (direction='h') or (direction='H') then across;
END.
```

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APPENDIX A2

```
Program Strainequation(input,output);
 ***  THIS PROGRAM CALCULATES INDUCED STRAIN DOWN A VERTICAL      ***
 ***  SHAFT WITH A CIRCULAR SHAFT PILLAR.                         ***
 ***  IT IS ASSUMED THAT THE REEF BEYOND THE SHAFT PILLAR IS      ***
 ***  EXTENSIVELY MINED OUT.                                      ***
 ***  ----------------------------------------------------        ***
var
    iout                                    :text;
    si,d,zi,z1,z2,p,
    a,b,hi,xi0,xi1,xi2,x,xi,interval   :real;
    m,q,numberpoints                       :integer;


 **             FUNCTION TO BE INTEGRATED                       **
                -------------------------

function strain(r,p,d,zi,z1,z2:real):real;
var aa,bb,c,dd,e,f,g,hh,ii,jj,kk     :real;
    begin
     aa:=(4*R*(1-P))/exp(1.5*ln(R*R+Z2*Z2));
     bb:=(12*R*Z2*z2*(1-P))/exp(2.5*ln(R*R+Z2*Z2));
     c:=(6*R*Z1*Z1*(2-P)+R*R*R*(1-2*P))/exp(2.5*ln(R*R+Z1*Z1));
     dd:=(5*R*Z1*Z1*(2*Z1*Z1*(2-P)+R*R*(1-2*P)))/exp(3.5*ln(R*R+Z1*Z1));
     e:=(6*R*Z2*Z2*(2-P)+R*R*R*(1-2*P))/exp(2.5*ln(R*R+Z2*Z2));
     f:=(5*R*Z2*Z2*(2*Z2*Z2*(2-P)+(1-2*P)*R*R))/exp(3.5*ln(R*R+Z2*Z2));
     g:=(8*R*Z2*(ZI+2*D*(1-P)))/exp(2.5*ln(R*R+Z2*Z2));
     hh:=(2*R*(2*Z2*Z2-R*R))/exp(2.5*ln(R*R+Z2*Z2));
     ii:=(10*R*Z2*(2*Z2*Z2-R*R)*(ZI+2*D*(1-P)))/exp(3.5*ln(R*R+Z2*Z2));
     jj:=(6*R*D*(Z2*(2*Z2*Z2-3*R*R)+3*ZI*(2*Z2*Z2-R*R)))
     /exp(3.5*ln(R*R+Z2*Z2));
     kk:=(42*R*D*ZI*Z2*Z2*(2*Z2*Z2-3*R*R))/exp(4.5*ln(R*R+Z2*Z2));
     strain:=aa-bb-c+dd-e+f+g+hh-ii+jj-kk;
    end; (*strain*)


 **             SIMPSONS RULE FOR INTEGRATION                   **
                -----------------------------

Procedure Intergratefunctions(a,b:real;m:integer);
Var    i,mm  :integer;
Begin
  hi:=(b-a)/(2*m);
  xi0:=0;
  xi0:=xi0+strain(a,p,d,zi,z1,z2);
  xi0:=xi0+strain(b,p,d,zi,z1,z2);
  xi1:=0;
  xi2:=0;
  mm:=2*m-1;
   for i:=1 to mm do
   begin
     x:=a+i*hi;
     if (odd(i)) then xi1:=xi1+strain(x,p,d,zi,z1,z2)
     else             xi2:=xi2+strain(x,p,d,zi,z1,z2);
   end;
```

```
  xi:=(xi0+2*xi2+4*xi1)*hi/3;
End; (* procedure integrate *)


Procedure Writeresults;
 Begin
  writeln(iout,'  ');
  writeln(iout,' INPUT DATA:');
  writeln(iout,' -----------');
  writeln(iout,'    DEPTH OF REEF=',d:8:1);
  writeln(iout,'    RADIUS OF SHAFTPILLAR=',a:8:1);
  writeln(iout,'    POISSONS RATIO=',p:8:1);
  writeln(iout,'  ');
  writeln(iout,' OUTPUT DATA:');
  writeln(iout,' ------------');
  writeln(iout,'  DEPTH    -Z1/Rsp    STRAIN/Sm   STRAIN*Rsp/Sm');
  writeln(iout,'  -----    -------    ---------   -------------');
  writeln('  ');
  writeln(' OUTPUT DATA:');
  writeln(' ------------');
  writeln('  DEPTH    -Z1/Rsp    STRAIN/Sm   STRAIN*Rsp/Sm');
  writeln('  -----    -------    ---------   -------------');
 End;


 **               MAIN PROGRAM               **
                  ------------

BEGIN
   writeln('ALL DIMENSIONS MUST BE GIVEN IN METRES!');
   writeln('Radius of shaftpillar?');
   read(a);
   b:=30000;
   m:=100;
   writeln('At what depth does the shaft intersect the reef?');
   read(d);
   writeln('Poissons ratio of host rock(usually between .15 and .5)?');
   read(p);
   writeln('At what depth do you want to calculate your first value?');
   read(zi);
   writeln('At what intervals do you want to calculate values?');
   read(interval);
   writeln('How many values do you want to calculate?');
   read(numberpoints);
   writeresults;
       for q:=1 to numberpoints+1 do
          begin;
            z1:=zi-d;
            z2:=zi+d;
            intergratefunctions(a,b,m);
            si:=-(a/(4*(1-P)))*xi;
            writeln(iout,zi:7:1,-z1/a:10:2,si/a:14:6,si:13:5);
            writeln(zi:7:1,-z1/a:10:2,si/a:14:6,si:13:5);
            zi:=zi+interval;
          end;  of for i
END.
```

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APPENDIX A3

```
Program Tiltequation(input,output);

 **   THIS PROGRAM CALCULATES TILT IN A VERTICAL SHAFT WITH A          **
 **   CIRCULAR PROTECTIVE PILLAR.                                      **
 **   IT IS ASSUMED THAT HALF OF THE PILLAR HAS BEEN MINED OU.         **
 **   AND THAT THE REEF BEYOND THAT HAS BEEN EXTENSIVELY MINED OUT.    **
 **   --------------------------------------------------------------   **


 **          DECLARATION OF CONSTANTS AND VARIABLES                    **
             --------------------------------------

const
     upper=1.570796327;
     lower=-1.570796327;
var
    iout                                 :text;
    fi,fiincrement,si,d,zi,z1,z2,p,
    a,b,radius,z,hfi,hradius,a1,ae,ao,an,be,bo,bn,ji,
    interval                             :real;
    m,n,i,numberpoints                   :integer;

 **             FUNCTION TO BE INTEGRATED                   **
                -------------------------

function tilt(d,fi,p,r,zi,z1,z2:real):real;
var a,b,c,dd,e,f,g          :real;
        begin
         a:=4*(1+p)*z1/exp(2.5*ln(r*r+z1*z1));
         b:=5*z1*(2*(1+p)*z1*z1-(1-2*p)*r*r)/exp(3.5*ln(r*r+z1*z1));
         c:=(8*z2*(1-2*p)+6*(z1+2*p*d))/exp(2.5*ln(r*r+z2*z2));
         dd:=5*z2*(2*(1+p)*z2*z2-(1-2*p)*r*r)/exp(3.5*ln(r*r+z2*z2));
         e:=30*z2*z2*(z1+2*p*d)/exp(3.5*ln(r*r+z2*z2));
         f:=6*d*(8*zi*z2+4*z2*z2-r*r)/exp(3.5*ln(r*r+z2*z2));
         g:=42*zi*z2*d*(4*z2*z2-r*r)/exp(4.5*ln(r*r+z2*z2));
             tilt:=r*r*cos(fi)*(a-b-c-dd+e+f-g);
        end; (*tilt*)


 **             SIMPSONS RULE FOR INTEGRATION               **
                -----------------------------

Procedure Intergratefunctions(a,b:real;m,n:integer);
Var    i,j,mm,nn  :integer;
Begin
  nn:=2*n+1;
  mm:=2*m-1;
  hradius:=(b-a)/(2*n);
  an:=0;
  ae:=0;
  ao:=0;
    for i:=1 to nn do
     begin
```

```
      radius:=a+(i-1)*hradius;
      hfi:=(upper-lower)/(2*m);
      bn:=tilt(d,lower,p,radius,zi,z1,z2)+
                                  tilt(d,upper,p,radius,zi,z1,z2);
      be:=0;
      bo:=0;
         for j:=1 to mm do
           begin
             fiincrement:=lower+j*hfi;
             z:=tilt(d,fiincrement,p,radius,zi,z1,z2);
             if (not(odd(j))) then be:=be+z else bo:=bo+z;
           end;
      a1:=(bn+2*be+4*bo)*hfi/3;
      if ((i=1) or (i=nn)) then an:=an+a1
      else if (not(odd(i))) then ae:=ae+a1
      else ao:=ao+a1;
    end;
 ji:=(an+2*ae+4*ao)*hradius/3;
End; (* procedure integrate *)


Procedure Writeresults;
 Begin
  writeln(iout,'  ');
  writeln(iout,' INPUT DATA:');
  writeln(iout,' -----------');
  writeln(iout,'    DEPTH OF REEF=',d:8:1);
  writeln(iout,'    RADIUS OF SHAFTPILLAR=',a:8:1);
  writeln(iout,'    POISSONS RATIO=',p:8:1);
  writeln(iout,'    UPPER LIMIT OF FI=',upper:15:10);
  writeln(iout,'    LOWER LIMIT OF FI=',lower:15:10);
  writeln(iout,'  ');
  writeln(iout,' OUTPUT DATA:');
  writeln(iout,' ------------');
  writeln(iout,'  DEPTH    -Z1/Rsp    TILT/Sm    Rsp*TILT/Sm');
  writeln(iout,'  -----    -------    -------    -----------');
  writeln('  ');
  writeln(' OUTPUT DATA:');
  writeln(' ------------');
  writeln('  DEPTH    -Z1/Rsp    TILT/Sm    Rsp*TILT/Sm');
  writeln('  -----    -------    -------    -----------');
 End;


 **              MAIN PROGRAM              **
                 ------------


BEGIN
   writeln('ALL DIMENSIONS MUST BE GIVEN IN METRES!');
   writeln('Radius of shaftpillar?');
   read(a);
   b:=30000;
   m:=100;
   n:=100;
   writeln('At what depth does the shaft intersect the reef?');
   read(d);
   writeln('Poissons ratio of host rock(usually between .15 and .5)?');
```

```
   read(p);
   writeln('At what depth do you want to calculate your first value?');
   read(zi);
   writeln('At what intervals do you want to calculate values?');
   read(interval);
   writeln('How many values do you want to calculate?');
   read(numberpoints);
   writeresults;
      for i:=1 to numberpoints+1 do
         begin;
           z1:=zi-d;
           z2:=zi+d;
           intergratefunctions(a,b,m,n);
           si:=-ji/(8*3.141592654*(1-p));
           writeln(iout,zi:7:1,-z1/a:10:2,si:12:5,si*a:13:5);
           writeln(zi:7:1,-z1/a:10:2,si:12:5,si*a:13:5);
           zi:=zi+interval;
         end;  of for i
END.
```

APPENDIX B

```
Program Nopillar(input,output);

 *******************************************************************************
 **  THIS PROGRAM CALCULATES STRESSES AND STRAINS IN THE SITUATION          **
 **  OF EARLY MINING OF THE REEF AROUND A VERTICAL SHAFT,                    **
 **  BEFORE CLOSURE.                                                          **
 *******************************************************************************
Const
 gravityacc=9.81;pi=3.1415927;
Var
   iout                                                              :text;
   halfspan,criticalhalfspan,z,depth,young,density,poisson,
   primitiveratio,stopingwidth,vertinstress,horinstress,
   verttotstress,hortotstress,vertinstrain,horinstrain,
   primstress,G,lzsqr,y,interval,
   horz,a,bb,p,vertz,tolerance,
   vertstrainz,limstrain,
   horzfootwall                                                       :real;
   i,number                                                          :integer;

Function Vertstress(z:real):real;
Begin
    primstress:=density*gravityacc*(depth+z);
    g:=young/(2*(1+poisson));
    criticalhalfspan:=(stopingwidth*g)/(2*(1-poisson)*
    density*gravityacc*depth);

   lzsqr:=sqr(halfspan)+sqr(z);

   vertstress:=primstress*z/depth+
   primstress*abs(z*z*z)/
   exp(1.5*ln(lzsqr));
End;

Function Horstress(z:real):real;
Begin
    primstress:=density*gravityacc*(depth+z);
    g:=young/(2*(1+poisson));
    criticalhalfspan:=(stopingwidth*g)/(2*(1-poisson)*
    density*gravityacc*depth);

   lzsqr:=sqr(halfspan)+sqr(z);

   horstress:=(primitiveratio-1)*primstress+primstress*
   primitiveratio*z/depth+
   primstress*abs(z)*(2*sqr(halfspan)+sqr(z))/
   exp(1.5*ln(lzsqr));
End;


Function vertstrain(z:real):real;
Begin
    primstress:=density*gravityacc*(depth+z);
    g:=young/(2*(1+poisson));
```

```
    criticalhalfspan:=(stopingwidth*g)/(2*(1-poisson)*
    density*gravityacc*depth);

   lzsqr:=sqr(halfspan)+sqr(z);

   vertstrain:=(1+poisson)*primstress/young*
   ((1-poisson)*(abs(z*z*z)/
   exp(1.5*ln(lzsqr))-1)-
   poisson*(abs(z)*(2*sqr(halfspan)+sqr(z))/
   exp(1.5*ln(lzsqr))-1));

End;


Procedure Calculateqlcg;
Begin
    primstress:=density*gravityacc*(depth+z);
    g:=young/(2*(1+poisson));
    criticalhalfspan:=(stopingwidth*g)/(2*(1-poisson)*
    density*gravityacc*depth);
End;

Procedure Beforeclosure;
Begin
    primstress:=density*gravityacc*(depth+z);
    g:=young/(2*(1+poisson));
    criticalhalfspan:=(stopingwidth*g)/(2*(1-poisson)*
    density*gravityacc*depth);

   lzsqr:=sqr(halfspan)+sqr(z);
   horinstrain:=(1+poisson)*primstress/young*
   ((1-poisson)*(abs(z)*(2*sqr(halfspan)+sqr(z))/
   exp(1.5*ln(lzsqr))-1)-
   poisson*(abs(z*z*z)/exp(1.5*ln(lzsqr))-1));
   vertinstrain:=(1+poisson)*primstress/young*
   ((1-poisson)*(abs(z*z*z)/
   exp(1.5*ln(lzsqr))-1)-
   poisson*(abs(z)*(2*sqr(halfspan)+sqr(z))/
   exp(1.5*ln(lzsqr))-1));


   horinstress:=primstress*(abs(z)*(2*sqr(halfspan)+sqr(z))/
   exp(1.5*ln(lzsqr))-1);
   vertinstress:=primstress*(abs(z*z*z)/
   exp(1.5*ln(lzsqr))-1);
   hortotstress:=(primitiveratio-1)*primstress+primstress*
   primitiveratio*z/depth+
   primstress*abs(z)*(2*sqr(halfspan)+sqr(z))/
   exp(1.5*ln(lzsqr));
   verttotstress:=primstress*z/depth+
   primstress*abs(z*z*z)/
   exp(1.5*ln(lzsqr));
End;


Procedure Writeheadings;
Begin
```

```
writeln;
writeln('INPUT:');
writeln;
writeln('Depth of origin =',depth:7:1);
writeln('Halfspan =',halfspan:7:1);
writeln('Stoping width =',stopingwidth:5:2);
writeln('Density in kg/cubic metre =',density:7:1);
writeln('Youngs modulus in Gpa =',y:5:1);
writeln('Poissons ratio =',poisson:5:2);
writeln('Ratio: Horizontal primitive to vertical primitive stress =',
primitiveratio:4:1);
writeln;
writeln('OUTPUT:');
writeln;
writeln('Critical halfspan =',criticalhalfspan:7:1);
writeln;
 if halfspan>=criticalhalfspan then
 writeln('THIS PROGRAM DOES NOT GIVE CORRECT RESULTS ');
 if halfspan>=criticalhalfspan then
 write('BEYOND THE POINT OF FULL CLOSURE!!!!!!!');
writeln;
writeln;
write('Z':5,'INDUCED STRESS':19,'INDUCED STRAIN':19);
writeln('TOTAL  STRESS':18,'PRIM. STRESS':16);
writeln('(METRES)':9,'(MPA)':11,'(MPA)':19,'(MPA)':18,'(MPA)':17);
writeln(' ':11,'Vert.   Hor.       Vert.    Hor.       Vert.    Hor.');
write('------':8,'-----  -----':15,'-----     -----':20);
writeln('-----    -----':19,'----------':14);


writeln(iout,'  ');
writeln(iout,'INPUT:');
writeln(iout,'  ');
writeln(iout,'Depth of origin =',depth:7:1);
writeln(iout,'Halfspan =',halfspan:7:1);
writeln(iout,'Stoping width =',stopingwidth:5:2);
writeln(iout,'Density in kg/cubic metre =',density:7:1);
writeln(iout,'Youngs modulus in Gpa =',y:5:1);
writeln(iout,'Poissons ratio =',poisson:5:2);
writeln(iout,'Ratio: Hor. primitive to vert. primitive stress =',
primitiveratio:4:1);
writeln(iout,'  ');
writeln(iout,'OUTPUT:');
writeln(iout,'  ');
writeln(iout,'Critical halfspan =',criticalhalfspan:7:1);
writeln(iout,'  ');
 if halfspan>=criticalhalfspan then
 writeln(iout,'THIS PROGRAM DOES NOT GIVE CORRECT RESULTS ');
 if halfspan>=criticalhalfspan then
 write(iout,'BEYOND THE POINT OF FULL CLOSURE!!!!!!!');
writeln(iout,'  ');
writeln(iout,'  ');
write(iout,'Z':5,'INDUCED STRESS':19,'INDUCED STRAIN':19);
writeln(iout,'TOTAL  STRESS':18,'PRIM. STRESS':16);
writeln(iout,'(METRES)':9,
'(MPA)':11,'(MPA)':19,'(MPA)':18,'(MPA)':17);
writeln(iout,' ':11,
```

```
 'Vert.   Hor.        Vert.    Hor.       Vert.    Hor.');
  write(iout,'------':8,'-----  -----':15,'-----     -----':20);
  writeln(iout,'-----    -----':19,'----------':14);
End;


Procedure Writeresults;
Begin
   writeln(z:8:1,vertinstress/1E6:8:2,horinstress/1E6:7:2,
   vertinstrain:12:5,horinstrain:9:5,
   verttotstress/1E6:10:2,hortotstress/1E6:8:2,
   primstress/1E6:11:1);


   writeln(iout,z:8:1,vertinstress/1E6:8:2,horinstress/1E6:7:2,
   vertinstrain:12:5,horinstrain:9:5,
   verttotstress/1E6:10:2,hortotstress/1E6:8:2,
   primstress/1E6:11:1);
End;


Procedure zeropoints;
Var
    i,ni    :integer;

  Procedure sub1;
   Begin
   i:=i+1;
   if (vertstress(a)*vertstress(p)>0) then a:=p
   else bb:=p;
   if i=ni then writeln('Iteration procedure failed after ',
   i:5,' iterations!');
   End;

  Procedure sub2;
   Begin
   i:=i+1;
   if (horstress(a)*horstress(p)>0) then a:=p
   else bb:=p;
   if i=ni then writeln('Iteration procedure failed after ',
   i:5,' iterations!');
   End;

  Procedure sub3;
   Begin
   i:=i+1;
   if ((vertstrain(a)-limstrain)*(vertstrain(p)-limstrain)<0)
   then a:=p
   else bb:=p;
   if i=ni then writeln('Iteration procedure failed after ',
   i:5,' iterations!');
   End;

Begin
  i:=1;
  tolerance:=0.001;
  bb:=vertz;
```

```
a:=vertz+interval;
ni:=40;
 while i<ni do
 begin;
   p:=a+(bb-a)/2;

   if (vertstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln('The smallest value of vert.stress (MPA) =',
   vertstress(p)/1E6:7:2);
   if (vertstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln(iout,'The smallest value of vert.stress (MPA) =',
   vertstress(p)/1E6:7:2);
   if (vertstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln('The corresponding Z-value =',p:7:2);
   if (vertstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln(iout,'The corresponding Z-value =',p:7:2);
   if (vertstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln('Number of iterations =',i:5);
   if (vertstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln(iout,'                              ');
   if (vertstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then ni:=1
   else sub1;
 end;


i:=1;
tolerance:=0.001;
bb:=horz;
a:=horz+interval;
ni:=40;
 while i<ni do
 begin;
   p:=a+(bb-a)/2;

   if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln('The smallest value of hor. stress (MPA) =',
   horstress(p)/1E6:7:2);
   if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln(iout,'The smallest value of hor. stress (MPA) =',
   horstress(p)/1E6:7:2);
   if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln('The corresponding Z-value =',p:7:2);
   if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln(iout,'The corresponding Z-value =',p:7:2);
   if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln('Number of iterations =',i:5);
   if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then writeln(iout,'                              ');
   if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
   then ni:=1
   else sub2;
 end;


i:=1;
tolerance:=0.001;
```

```
  bb:=horzfootwall;
  a:=horzfootwall+interval;
  ni:=40;
   while i<ni do
   begin;
     p:=a+(bb-a)/2;

     if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
     then writeln('The smallest value of hor. stress (MPA)',
     'in the footwall =',horstress(p)/1E6:7:2);
     if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
     then writeln(iout,'The smallest value of hor. stress (MPA)',
     'in the footwall =',horstress(p)/1E6:7:2);
     if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
     then writeln('The corresponding Z-value =',p:7:2);
     if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
     then writeln(iout,'The corresponding Z-value =',p:7:2);
     if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
     then writeln('Number of iterations =',i:5);
     if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
     then writeln(iout,'                              ');
     if (horstress(p)=0) or (abs((bb-a)/2)<tolerance)
     then ni:=1
     else sub2;
   end;


  i:=1;
  tolerance:=0.001;
  bb:=vertstrainz;
  a:=vertstrainz+interval;
  ni:=40;
   while i<ni do
   begin;
     p:=a+(bb-a)/2;

     if (vertstrain(p)-limstrain=0) or (abs((bb-a)/2)<tolerance)
     then writeln('The limiting value of vert. strain =',
     limstrain:9:5);
     if (vertstrain(p)-limstrain=0) or (abs((bb-a)/2)<tolerance)
     then writeln(iout,'The limiting value of vert. strain =',
     limstrain:9:5);
     if (vertstrain(p)-limstrain=0) or (abs((bb-a)/2)<tolerance)
     then writeln('The corresponding Z-value =',p:7:2);
     if (vertstrain(p)-limstrain=0) or (abs((bb-a)/2)<tolerance)
     then writeln(iout,'The corresponding Z-value =',p:7:2);
     if (vertstrain(p)-limstrain=0) or (abs((bb-a)/2)<tolerance)
     then writeln('Number of iterations =',i:5);
     if (vertstrain(p)-limstrain=0) or (abs((bb-a)/2)<tolerance)
     then writeln(iout,'                              ');
     if (vertstrain(p)-limstrain=0) or (abs((bb-a)/2)<tolerance)
     then ni:=1
     else sub3;
   end;

End;
```

```
**                          MAIN PROGRAM                          **
                            ------------
BEGIN
  writeln('Depth of shaft-reef intersection?');
  read(depth);
  writeln('Halfspan value?');
  read(halfspan);
  writeln('Stopingwidth?');
  read(stopingwidth);
  writeln('Density of host-rock (kg/cubic metre) ?');
  read(density);
  writeln('Youngs modulus of host-rock(Gpa) ?');
  read(y);
  young:=y*1E9;
  writeln('Poissons ratio?');
  read(poisson);
  writeln('Ratio of primitive hor. to primitive vert. stress?');
  read(primitiveratio);
  writeln('Smallest Z-value, positive below reef, negative above?');
  read(z);
  writeln('Intervals ?');
  read(interval);
  writeln('Number of values to be calculated ?');
  read(number);
  limstrain:=-0.00040;
    for i:=1 to (number+1) do
    begin
    calculateqLcG;
    beforeclosure;

    if ((hortotstress>0.0) and (z<0.0))
    then horz:=z;


    if ((hortotstress<0.0) and ((z>0.0) or (z=0.0)))
    then horzfootwall:=z;


    if ((verttotstress>0.0) and (z<0.0))
    then vertz:=z;


    if ((vertinstrain-limstrain>0.0) and (z<0.0))
    then vertstrainz:=z;


    if i=1 then writeheadings;
    writeresults;
    z:=z+interval;
    end;
  zeropoints;
END.
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