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Large mass hierarchies from strongly-coupled dynamics

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Abstract: Besides the Higgs particle discovered in 2012, with mass 125 GeV, recent LHC data show tentative signals for new resonances in diboson as well as diphoton searches at high center-of-mass energies (2 TeV and 750 GeV, respectively). If these signals are confirmed (or other new resonances are discovered at the TeV scale), the large hierarchies between masses of new bosons require a dynamical explanation. Motivated by these tentative signals of new physics, we investigate the theoretical possibility that large hierarchies in the masses of glueballs could arise dynamically in new strongly-coupled gauge theories extending the standard model of particle physics. We study lattice data on non-Abelian gauge theories in the (near-)conformal regime as well as a simple toy model in the context of gauge/gravity dualities. We focus our attention on the ratio R between the mass of the lightest spin-2 and spin-0 resonances, that for technical reasons is a particularly convenient and clean observable to study. For models in which (non-perturbative) large anomalous dimensions arise dynamically, we show indications that this mass ratio can be large, with $R > 5$. Moreover, our results suggest that R might be related to universal properties of the IR fixed point. Our findings provide an interesting step towards understanding large mass ratios in the non-perturbative regime of quantum field theories with (near) IR conformal behaviour.

Keywords: Technicolor and Composite Models, Gauge-gravity correspondence, Lattice Quantum Field Theory

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1 Introduction

The Higgs particle [1, 2] is the first example in nature of a boson the mass of which is not protected by symmetry arguments. Its low-energy Effective Field Theory (EFT) description in terms of a weakly-coupled scalar field is fine-tuned, as additive renormalisation makes it sensitive to unknown physics up to high scales. This is the hierarchy problem, one of the main motivations to investigate theoretical extensions of the Standard Model (SM).

Searches at the LHC for resonant production of particles decaying into diphoton or diboson final states show excesses at center-of-mass energies around 750 GeV [3, 4] and 2 TeV [5–9], respectively. Such tentative signals bring into further question the possibility of writing EFTs with hierarchical scales without invoking fine-tuning.

If a new strongly-coupled theory is responsible for electroweak symmetry breaking and all the physical phenomena connected with it [10–20],[1] it would provide an elegant and conclusive solution to the hierarchy problem(s) of the electroweak theory, in a context in which new composite states appear at energies accessible to the LHC.

A simple rescaled version of QCD cannot explain the phenomenology we see at the electroweak scale and above. Precision electroweak studies have already ruled out this possibility — a conclusion further supported by the discovery of the Higgs particle. A realistic model must exhibit dynamical properties that are radically different from QCD, yielding large hierarchies in the masses of particles.

[1] For reviews see also ref. [21].

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Non-perturbative methods are needed to test this broadly defined scenario. Over the past decades, great progress has been made by using lattice gauge theories, as well as gauge/gravity dualities. The two approaches can be considered to be complementary to one another, as we will discuss later in the paper.

We want to identify models that dynamically produce a large *mass hierarchy* between composite states, that cannot be explained in simple terms by symmetry arguments in a low energy EFT context (and without fine-tuning). We are aiming at something more than what in QCD is captured by the chiral Lagrangian, or Heavy Meson Chiral Perturbation Theory (χ-PT), that explain the masses and properties of pions and of heavy-light mesons, respectively. We want to find appropriate physical observables that allow such an identification to be assessed in a clean, unambiguous way distinctive from simple arguments formulated at weak coupling on the basis of internal symmetries. This should be based on the recent developments in non-perturbative techniques, both from lattice studies and in the context of gauge/gravity duality.

In this paper, we provide one interesting step in this direction, and suggest possible ways to further develop this challenging research program in the future. Our starting point is the observation that, irrespectively of the microscopic details, all Lorentz-invariant four-dimensional field theories admit a stress-energy tensor $T_{\mu\nu}$. Correlation functions involving $T_{\mu\nu}$ can be analysed in terms of their scalar (trace) part and tensor (transverse and traceless) part. A well-defined observable is the ratio

$$R \equiv \frac{M_T}{M_0}\,, \tag{1.1}$$

where M_T is the mass of the lightest spin-2 composite state, while M_0 is the mass of the lightest spin-0 state. This quantity is defined universally, it can be computed explicitly in a wide variety of models, it is scheme-independent, and it is not directly controlled by internal global symmetries of the theory. It is legitimate to compare R computed in theories with completely different internal symmetries and symmetry-breaking patterns. This is a particularly welcome feature in the context of gauge theories with fermionic field content, where the physics of chiral symmetry and its breaking introduces non-trivial model-dependent features.

In recent years a large number of different theories with non-QCD-like dynamics have been studied on the lattice (see the reviews [22–26]). Much emphasis has gone into the discussion of their mesonic properties, while technical difficulties have so far hindered the progress in understanding the glueballs of the theories, with the exception of some particularly neat cases. Among the latter, pure Yang-Mills SU(N) theories are comparatively well understood, and the spectra of glueballs of various quantum numbers are known. The ratio R has been computed to be $1.4 \lesssim R_{\rm YM} \lesssim 1.7$ with the lower bound reached for small N and the upper bound by extrapolating to large N [27–32]. The question we want to address is whether there exist models in which $R \gg R_{\rm YM}$.

We focus on SU(2) theories with adjoint matter, for which there are indications that the dynamics is IR conformal [33–54]. As explained below, the investigation of a conformal theory with numerical simulations[2] requires an extrapolation to the zero residual mass,

[2]Needless to say, on a finite lattice and at finite fermion mass, the spectrum will be different from that

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infinite volume, and continuum limit. In this conformal limit all masses are zero. It is, nevertheless, possible to extrapolate a finite value of R in an unambiguous way. We will show, also by comparison with our toy model, that this procedure depends on the precise way the different limits are taken. We are also interested to see whether the ratio R obtained in this way contains non-trivial information about the anomalous dimensions, generalising [35].

By exploiting arguments originating from scale invariance, the authors of [35] derived scaling relations for spectral masses in a mass-deformed IR conformal gauge theory as a function of the mass deformation m. In particular, all spectral scales follow the same power law in m when approaching the massless limit, with the exponent being determined solely by the anomalous dimension of the deforming operator. In general, the coefficients in front of such a power law depend on the state considered and on details of the theory. However, scale invariance in the IR might provide extra constraints on them.

Consequences of scale invariance are well studied in statistical systems. One of the main results is universality, which states that at a second order phase transition (where the theory is scale invariant) certain properties of a Statistical Mechanics system, including the critical exponents (anomalous dimensions), depend only on the symmetries of the Hamiltonian and on the dimensionality, and not on microscopic details such as the specific elementary degrees of freedom.

Universality applies also to less familiar examples. The power law approach (as a function of deformation parameters such as temperature or external magnetic field) to the critical point is governed by model-dependent coefficients. But there are particular combinations that only depend on symmetries and dimensionality, and are hence universal. For a detailed discussion pertaining to the example of the two-dimensional Ising model, we refer the reader to [55].

Here, we would like to stress that, while explicit calculations can be carried out mostly in two dimensions, the concept of *universal amplitude ratios* is more general and descends from the concept of universality (see e.g. [56]). It is hence a legitimate question to ask whether there are universal ratios in IR conformal gauge theories, and in particular whether R (which, we stress, can be defined in many local QFTs) is one of them.

Gauge/gravity dualities allow the use of weakly-coupled classical field theory coupled to gravity in higher dimensions to explore the dynamics of strongly-coupled four-dimensional theories [57–60]. There exists a fully algorithmic procedure for computing glueball spectra, at leading order in the large-N and large 't Hooft coupling limits. It uses the action of a sigma-model coupled to gravity in five dimensions, that in the top-down approach may be obtained as a consistent truncation of the dimensional reduction of a more fundamental gravity theory in higher dimensions [61–65].

These techniques have been applied successfully to the dual of confining gauge theories, such as the Witten model [66–68], the Klebanov-Strassler model [61–63, 69], the Maldacena-Nunez model [61–63, 70, 71] and several generalisations, including cases in which the mass

of the continuum massless theory. In this paper we collectively call *lattice artefacts* all effects that are not present in the continuum massless field theory, inclusive of finite volume, finite spacing and finite fermion mass effects.

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spectra include an anomalously light state [72–75]. For instance, the Witten model has many properties that make it resemble the gravity dual of a non-supersymmetric Yang-Mills theory, and the gravity calculation yields $R \simeq 1.7$ [67].

Less sophisticated models exist for which the asymptotic high-energy behaviour of the theory is simpler to interpret, but in which the geometry is not smooth, indicating that these models provide incomplete descriptions of long-distance physics. An example is the GPPZ model [76] (see also [77]), in which the dual field theory is $\mathcal{N} = 4$ super-Yang-Mills, deformed by a particular symmetry-breaking mass term. The (gravity) calculation of the glueball spectrum yields $R = \sqrt{2} \simeq 1.4$ [78].

We are interested in modelling with gravity a physical situation that is similar to the one found on the lattice: a CFT is deformed by a relevant coupling, thus inducing a departure from AdS of the gravity background, ultimately leading to the geometry ending along the extra dimension. The spectrum consists of masses that all scale in a universal way with the deformation scale [35], and hence R can be given a physical meaning. While the individual coefficients controlling the masses are model-dependent, we want to investigate whether the ratio R shows universal properties. In the absence of a known class of supergravity backgrounds that describe the deformation of a CFT with tuneable anomalous dimension, we resort to a toy model, in the spirit of the bottom-up approach to holography.

The main idea of this paper is the following. We consider a set of conformal theories that admit a relevant deformation, and compute the spectra of scalar and tensor bound states in the presence of this deformation. Inspired by the idea of universality, in particular of universal amplitude ratios borrowed from statistical field theory, we question whether it is possible that the ratio R defined earlier might be a quantity exhibiting such universal properties.

We compare the results obtained from lattice SU(2) theories with two different field contents, and hence two different anomalous dimensions $\gamma^* \equiv \Delta - 1$ for the $\bar{\psi}\psi$ operator, to results obtained in a completely different class of models, built within the bottom-up approach to holography. In the latter class of theories, Δ is a tuneable parameter: we compute the value of R for generic Δ, and compare to the results of the SU(2) lattice theories for which the same values of Δ are available. As we will see, we find a surprisingly good level of agreement, though subject to numerous caveats that we discuss in detail.

Only with further future work on the subject it will be possible to ascertain whether this is an indication of a strong form of universality, manifesting itself in the fact that R is just a function of Δ (and the space-time dimensionality), but not of the details of the theory, or whether the agreement we uncover is a more modest result of generic similarities specific to the theories we analysed. We will comment on further steps we suggest for future work in this direction at the end of the paper.

The paper is organised as follows. In section 2 we describe the lattice results for R in two models, both based on SU(2) gauge group, in which the field content consists of either one or two Dirac fermions transforming in the adjoint representation. In section 3 we illustrate the results for R as a function of the dimension of the operator deforming a CFT for a toy model built as a generalisation of the five-dimensional consistent truncation leading to the GPPZ background. In section 4 we analyse the physical implications of

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our results, and extract from them some useful lessons of general relevance. We critically discuss the limitations intrinsic in our work and we outline future avenues for research that might overcome these limitations in section 5.

2 Lattice SU(2) gauge theories with adjoint matter

2.1 Lattice formulation and setup

The SU(2) gauge theory with two Dirac fermions in the adjoint representation of the gauge group is the first that has been shown to be infrared (IR) conformal in the context of lattice studies of the conformal window. A vast body of literature exists on its spectrum [33–42], on the running of its coupling [43–47] and on the anomalous dimension of its chiral condensate [48–52]. From a phenomenological point of view, this theory is likely to be of limited relevance, since the anomalous dimension of the condensate is small. Yet the study of this gauge theory has been a crucial milestone in numerical explorations of strongly interacting dynamics beyond the Standard Model, fostering the development of specific investigation techniques for nearly conformal gauge theories discretised on a spacetime lattice.

It has been shown in [54] that the SU(2) gauge theory with a single adjoint Dirac flavour (or, equivalently, two Majorana flavours) is near the onset of the conformal window and has an anomalous dimension of order one. This makes it an ideal lattice playground for non-perturbative tests of near-conformal gauge theories with large anomalous dimensions.

In Minkowski space, the Lagrangian of an SU(2) gauge theory coupled to N_f flavours of adjoint Dirac fermions of mass m is

$$\mathcal{L} = \sum_{i=1}^{N_f} \overline{\psi}_i \left(i \not{D} - m \right) \psi_i - \frac{1}{2} \mathrm{Tr} \left[G_{\mu\nu} G^{\mu\nu} \right] , \tag{2.1}$$

where $\not{D} \equiv (\partial_\mu + i g A_\mu) \gamma^\mu$, γ^μ are the Dirac matrices, $A_\mu \equiv \sum_a T^a A^a_\mu$ with $a = 1, 2, 3$, and T^a are the 3×3 generators of SU(2) in the adjoint representation. The field strength tensor is $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i g [A_\mu, A_\nu]$, with g the coupling. The trace is over the gauge indexes, with the generators T^a normalised such that $\mathrm{Tr}(T^a T^b) = \delta_{ab}/2$. We shall consider the cases $N_f = 1$ and $N_f = 2$.

The action on the Euclidean spacetime grid is

$$S = S_\mathrm{G} + S_\mathrm{F} , \tag{2.2}$$

where

$$S_\mathrm{G} = \beta \sum_p \mathrm{Tr} \left[1 - U(p) \right] , \tag{2.3}$$

with $U(p)$ the lattice plaquette, is the pure gauge part (referred to as the Wilson plaquette action), and

$$S_\mathrm{F} = \sum_{x,y} \sum_{i=1}^{N_f} \overline{\psi}_i(x) D(x,y) \psi_i(y) \tag{2.4}$$

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is the fermionic contribution. The massive Dirac operator $D(x,y)$ in the Wilson fermion discretisation used throughout this work is

$$D(x,y) = \delta_{x,y} - \kappa \left[(1-\gamma_\mu)\, U_\mu(x) \delta_{y,x+\mu} \, (1+\gamma_\mu)\, U_\mu^\dagger(x-\mu) \delta_{y,x-\mu} \right] , \tag{2.5}$$

with γ^μ the Euclidean Dirac matrices. x and y are points on the lattice. In the previous equation, the lattice links $U_\mu(x)$ are written in the adjoint representation. $\kappa = 1/(8+2am)$ is the hopping parameter, with a the lattice spacing and m the bare fermion mass. For computational reasons, in numerical simulations one has to consider the theory in the presence of a non-zero mass m. The behaviour of the model as the mass is taken to zero allows to disentangle between the near-conformal and the chiral-symmetry broken cases.

The path integral of the theory is

$$Z = \int \mathcal{D}U \mathcal{D}\overline{\psi} \mathcal{D}\psi e^{-S} , \tag{2.6}$$

and the vacuum expectation value of any operator $O(U,\psi,\overline{\psi})$ is given by

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\overline{\psi} \mathcal{D}\psi O(U,\psi,\overline{\psi}) e^{-S} . \tag{2.7}$$

If O carries specific J^{PC} quantum numbers, at large distance $r = |x-y|$ the correlator

$$\mathcal{C}(r) = \langle O^\dagger(x) O(y) \rangle , \tag{2.8}$$

decays as

$$\mathcal{C}(r) = k_1 e^{-M_1 r} + k_2 e^{-M_2 r} + \cdots , \tag{2.9}$$

with M_i the lowest spectral masses in the given J^{PC} channel. For large enough r, only the contribution from the state with lowest mass M_1 survives, provided the coefficient k_1 (related to the decay constant) is not suppressed. Note that in the previous expression the invariant mass M_1 is not associated necessarily to a stable state or a resonance, but could correspond to a scattering state. For instance, in QCD, for which the mass of the vector is above the threshold of the two pion system, unless the corresponding weighting coefficient k_1 is suppressed for dynamical reasons or by an appropriate choice of the probe operators used in the calculation, in general with this technique one would extract the invariant mass associated to the scattering of two pions with $J = 1$, and the mass of the resonance will only manifest as an excitation [79]. The latter discussion about this somewhat technical aspect of the calculation will be relevant when analysing the numerical results obtained for the investigated lattice theories.

We shall focus on glueball masses,[3] that following consolidated procedures are extracted using a variational calculation including spatial Wilson loops of various sizes and shapes transforming in the irreducible representation of the symmetry group of the cube. Continuum spins are then reconstructed by looking at the embedding of this group into the continuum rotation group. Further technical details on how glueball masses are extracted using this method are provided for instance in [27].

[3]Due to the choice of the probe, in the following we will refer to the corresponding spectral states as glueballs, although one has to keep in mind that these states will mix with meson-like states with the same quantum numbers.

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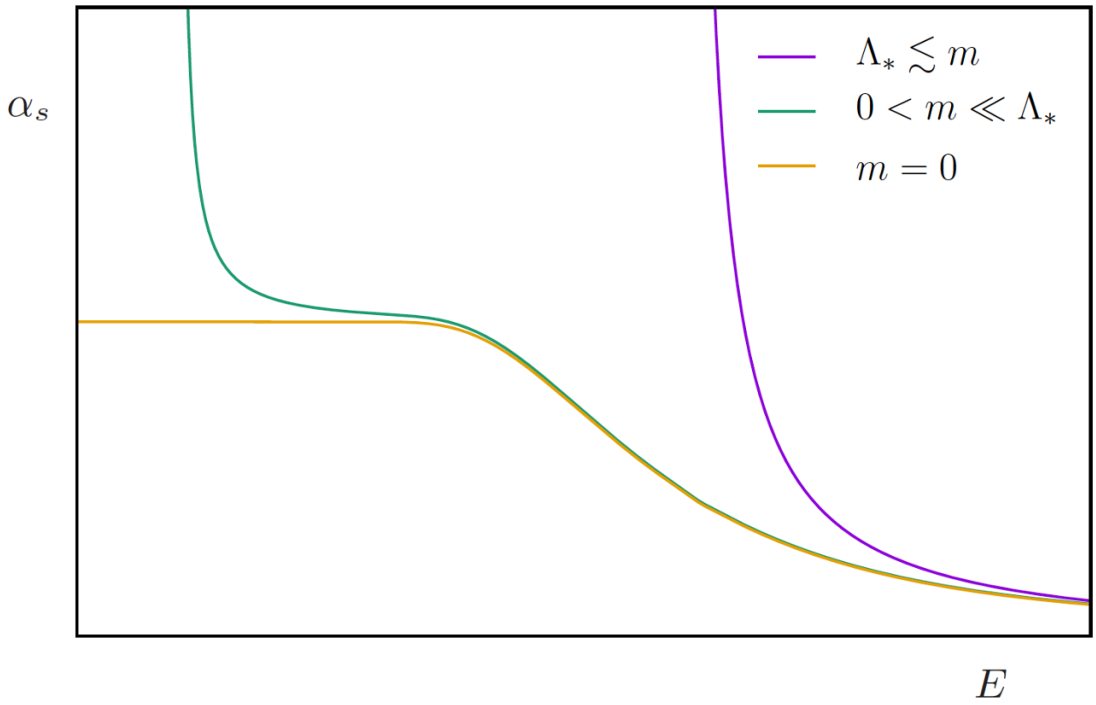


Figure 1. Schematic representation of $\alpha_s = g^2/(4\pi)$ as a function of E for three representative choices of fermion mass m. In yellow the massless $m = 0$ case, in which the only scale is Λ_*. In purple a case in which the mass m is very large. In green a case in which $m \ll \Lambda_*$, and hence $\Lambda_0 \ll \Lambda_*$.

2.2 Mass deformation and infrared behaviour

The claimed IR-conformality of the two theories of interest is true only in the chiral limit. In the presence of a mass deformation, we must understand how the would-be IR-conformal theory reacts to the explicit breaking of scale invariance introduced via $m \neq 0$.[4]

We show in figure 1 a cartoon of the running coupling $\alpha_s \equiv \frac{g^2}{4\pi}$ as a function of the energy scale E, for various choices of m, in order to illustrate the various possible regimes of m that yield physically distinctive features. Firstly, in the $m = 0$ case the IR-conformal behaviour features a dynamical scale Λ_* that separates the perturbative regime from the long-distance behaviour. The latter is characterised by the asymptotic approach of the coupling to a finite value g^* at low E. The scale Λ_* may be estimated on the basis of perturbation theory, and is the lowest scale E at which the dynamics is well captured by the physics of the trivial UV fixed point.

If we deform the theory with a large mass m, chosen to be much larger than the dynamical scale Λ_* of the massless theory, we completely destroy infrared conformality, giving rise to a confining behaviour similar to that of a Yang-Mills theory with heavy quarks. The scale at which the theory confines is $\Lambda_0 \neq \Lambda_*$, and at this scale the fermions are effectively decoupled.

[4]With abuse of notation, we denote with m both the Lagrangian bare mass and the renormalised fermion mass, with the latter being the one that is relevant for the present discussion. A prescription to introduce the renormalised mass is given in [35, 80].

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For small mass $m \ll \Lambda_*$, the breaking term acts as a soft deformation. The theory behaves as IR-conformal down to some energy of the order of a scale Λ_0, which is determined in a dynamically non-trivial way by the mass deformation m. For $E \sim \Lambda_0$ the confining behaviour reappears. An explicit calculation is possible when the fixed point is weakly-coupled [80]. In this case, one finds $\Lambda_0 \simeq m e^{-\frac{1}{b_0 \alpha_s^*}}$ with $b_0 = (11/6\pi)N_c$ (N_c being the number of colours) and $\alpha_s^* = g^{*2}/(4\pi)$ the value of the coupling at the fixed point. The mass spectrum is determined by Λ_0 and the meson masses are enhanced by a factor $e^{\frac{1}{b_0 \alpha_s^*}}$ [33, 80].

Regardless of the strength of g^*, in the energy region

$$\Lambda_0 \ll E \ll \Lambda_* \,, \tag{2.10}$$

in which the mass-deformed theory behaves as if it were IR conformal, all spectral masses M_i scale as

$$M_i \propto m^{\frac{1}{\Delta}} \,, \tag{2.11}$$

where $\Delta = 1 + \gamma^*$ and γ^* is the anomalous dimension of the chiral condensate [81].

When the theory is formulated on a spacetime lattice, an ultraviolet cutoff $a \equiv \frac{1}{\Lambda_{\rm UV}}$ is introduced. The value of a can be tuned by changing the coupling β. The continuum limit itself is realised at $\beta = \infty$. Computing observables numerically causes the introduction of an infrared cutoff related to the lattice size $L = na \equiv \frac{1}{\Lambda_{\rm IR}}$, with n the number of lattice points in the considered direction. Physical observables must not be affected by cutoff effects. In the case of a particle of mass M, this requires

$$a \ll \frac{1}{M} \ll L \,. \tag{2.12}$$

Conversely, if $1/M \lesssim a$ the state is affected by discretisation artefacts, while if $1/M \gtrsim L$ finite volume effects dominate the calculation.

When analysing the approach of lattice data to the chiral point, in addition to the mass deformation, the infrared cutoff scale L needs to be taken explicitly into account. Borrowing a consolidated analysis technique from the investigation of critical phenomena, the finite size can be seen as a renormalisation group relevant direction with mass dimension -1. The dimensionless variable describing scaling with m and L is then $x = Lm^{1/\Delta}$. From the Widom form of the effective action, one can derive the scaling law [34, 35, 48, 82]

$$LM_i = f_i(x) \,, \tag{2.13}$$

i.e. spectral masses in units of the lattice size are universal functions of the scaling variable. One can use the lowest-order relation for a particle of mass M_0

$$LM_0 \propto x \tag{2.14}$$

to rewrite the scaling in eq. (2.13) as [48]

$$LM_i = f_i(LM_0) \,, \tag{2.15}$$

with ratios of spectral masses assuming a universal form:

$$M_i/M_j = f_i(LM_0)/f_j(LM_0) \,. \tag{2.16}$$

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This scaling relation is particularly useful for an unbiased and direct comparison of lattice data obtained at different values of m and L, as it accounts for the finite size without making explicit reference to the anomalous dimension. At large L, the leading behaviour with M_0 of the previous equation predicts constant mass ratios as m is varied [33–35].

Equipped with these considerations, in subsections 2.4 and 2.5 we present lattice results for the ratio $R \equiv M_T/M_0$ with M_T and M_0 respectively the mass of the tensor and scalar, determined by probing the theory with glueball-like operators.

2.3 The femto-universe

Since we will comment also on predictions in the small-volume limit, it is convenient to discuss some generic features of the glueball spectrum obtained in this limit for Yang-Mills theories. Here we are referring to the scenario of the *femto-universe* [83], which is realised when the size of the system is smaller than the shortest intrinsic (dynamically generated) length scale in the theory [84]. In the case of QCD, this implies

$$1/L > \Lambda_{\rm QCD}. \tag{2.17}$$

In this limit, the spectrum of an asymptotically-free gauge theory can be extracted with perturbation theory.

In this section, we restrict our discussion to the case of a (3+1)-dimensional hypercubic volume $L^3 \times T$, with L satisfying the corresponding condition in eq. (2.17). Perturbation theory in such small cubic boxes can be complicated by the global toroidal structure ("torons") of the periodic lattice [85, 86]. This complication can be removed by employing the colour-twisted boundary conditions (TBC) [87, 88]. To keep our discussion simple, we concentrate on results obtained using TBC in refs. [89–91].

The glueball spectrum of the pure Yang-Mills theories in the femto-universe with TBC takes the generic form

$$M_G = \frac{X_0}{L} + \frac{g^2 X_1}{L} + \dots, \tag{2.18}$$

where g is the coupling, and X_0 is a constant that is completely determined by the geometry and the boundary condition of the finite volume, as well as the cubic-group representation of the glueball state. Eq. (2.18) is the result of perturbation theory. With a specific choice of the twist that preserves the cubic symmetry, X_0 takes the same value for all the states associated with the irreducible representations of the cubic group [90]. In particular, this means that the scalar and the tensor glueballs are degenerate, $R = M_T/M_0 = 1$, in the $g^2 \to 0$ limit. We stress that this value of the mass ratio, R, is the consequence of the box geometry, the boundary condition and group theory. It does not result from the underlying dynamics.

The gauge-field dynamics begins to set in at the first non-trivial order in the expansion of eq. (2.18). In general, the coefficient X_1 depends on the spin of the glueball. For the case of TBC considered in ref. [90], it increases R.[5] This coefficient depends on the boundary conditions as well. In fact, in the computation employing periodic boundary conditions, the

[5]This statement is also found to be valid when lattice artefacts are accounted for in the perturbative calculations [91].

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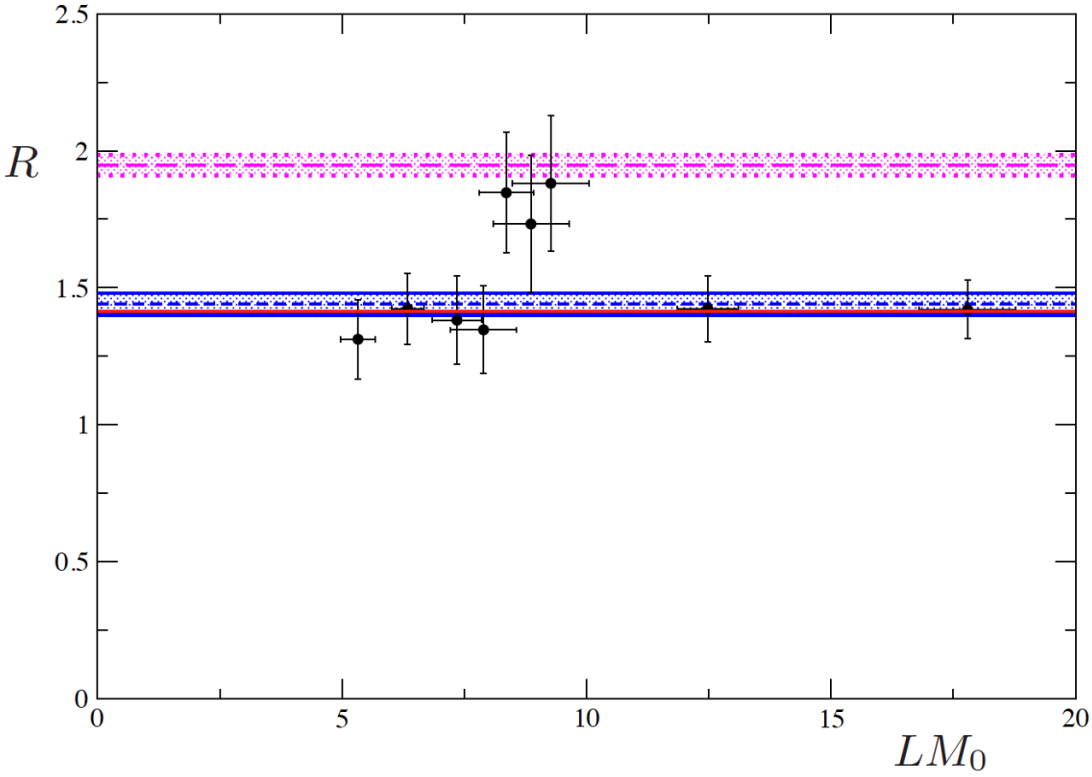


Figure 2. The ratio R as a function of LM_0 for the SU(2) gauge theory with two adjoint Dirac fermions. The horizontal blue band indicates the SU(2) Yang-Mills value $R = 1.44(4)$. The pink band $R = 1.95(4)$, and the red line at $R = \sqrt{2}$ are discussed in section 3. The interpretation of the data is explained and discussed in section 2.4.

authors of refs. [92, 93] find that the tensor glueball is lighter than the scalar state at the first non-trivial order of perturbation theory, although the mass ratio, R, is also close to unity.

In the femto-universe, the light-fermion masses are well below the scale $1/L$. Therefore, practically they can be considered as massless. Since the fermions couple to the pure gauge degrees of freedom only perturbatively, they will not have any significant (non-perturbative) effects on the glueball spectrum discussed above. The authors of ref. [91] computed the fermionic contribution in continuum perturbation theory to $O(g^2)$, and found it to be small.

2.4 Numerical results for the two flavour theory

The construction of the interpolating operators for the measurements of the bound state spectrum in adjoint QCD is similar to QCD with fermions in the fundamental representation. The physics of these bound states is, however, much different. One important difference is the fact that, due to the real representation, the massless theory has SU($2N_f$) chiral symmetry that is broken to SO($2N_f$) by the fermion condensate.

The spectrum of the theory has been widely studied, with an accurate analysis of possible systematic effects related to the choice of interpolating operators and to finite size effects given in [36] and a careful large volume extrapolation presented in [37]. It is worth noting that M_0 is the lightest state of the theory and $M_T < 2M_0$. Hence, the

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mass extracted in the tensor channel is associated with a stable particle. For small fermion masses, the spectrum has spectral mass ratios that are constant as a function of m. This signals (near-)conformal behaviour. Studies of the running of the coupling have exposed the presence of an infrared fixed point. The anomalous dimension of the condensate has been found to be $\gamma^* = 0.371(20)$ [37, 52] (see also [33, 35, 80]).

The most accurate studies for the mass spectrum have been done at only one value of the lattice coupling $\beta = 2.25$. Taking the data from the simulations available in the literature (mostly [33, 35] supplemented with the two largest volumes at $am = -1.05$ given in [37]), we plot the ratio R as a function of LM_0 in figure 2. We show also the pure SU(2) Yang-Mills value $R \simeq 1.44(4)$ as determined by lattice calculations [28, 29] with a horizontal line. A manifest feature of the data is the appearance of three distinct plateaus in R. For the largest values of LM_0 we find good agreement with the Yang-Mills value. At intermediate values of LM_0 there is evidence of an enhancement of $R > 1.4$. At the smallest available values of LM_0, R is systematically below the pure Yang-Mills value, although still compatible with $R = 1.44(4)$ within the large errors.

We conclude this subsection by noting that for the smallest volumes at $am = -1.15$ studied in [37] R is compatible with the expected femto-universe results. The explicit observation of the deviation from the femto-universe regime in the results discussed here is a good indication of absence of severe small volume effects for the data presented in figure 2.

2.5 Numerical results for the one flavour theory

In the $N_f = 1$ adjoint flavour theory, upon decomposition of the Dirac fermion into its two Majorana constituents, the SU(2) chiral symmetry is manifest in the action. It is broken to SO(2) if a non-zero fermion condensate forms. Hence, chiral symmetry breaking would result in the production of two Goldstone bosons. A detailed analysis of the symmetries and numerical results for various spectral states are provided in [54]. In particular two independent degenerate signals for the scalar channel, the 0^{++} glueball and the mesonic isoscalar, have been considered.

The simulations (performed for the single value of $\beta = 2.05$) show again a spectrum with a markedly different signature than one would expect in the chiral symmetry broken case. Ratios of the lowest-lying spectral masses appear to be constant as a function of the mass deformation m, with the lightest state being a 0^{++} scalar. The would-be Goldstone bosons expected from the anticipated symmetry breaking pattern SU(2) $\to$ SO(2) look like other ordinary massive states. The condensate anomalous dimension has been found to be $\gamma^* = 0.925(25)$.

We report in figure 3 numerical results for the ratio R as a function of LM_0, using the data for the 0^{++} glueball obtained in [54] supplemented by additional calculations that will be discussed elsewhere. While at the largest available values of LM_0 the ratio R is compatible with the pure SU(2) Yang-Mills theory, the ratio is significantly different from it for lower $M_0 L$. In this latter regime, R is considerably enhanced, even with respect to the peak value for the $N_f = 2$ case.

In the light of the discussion of potential contributions of scattering states to the two-point function (see section 2.1), the existence of a region where $M_T > 2M_0$ deserves further

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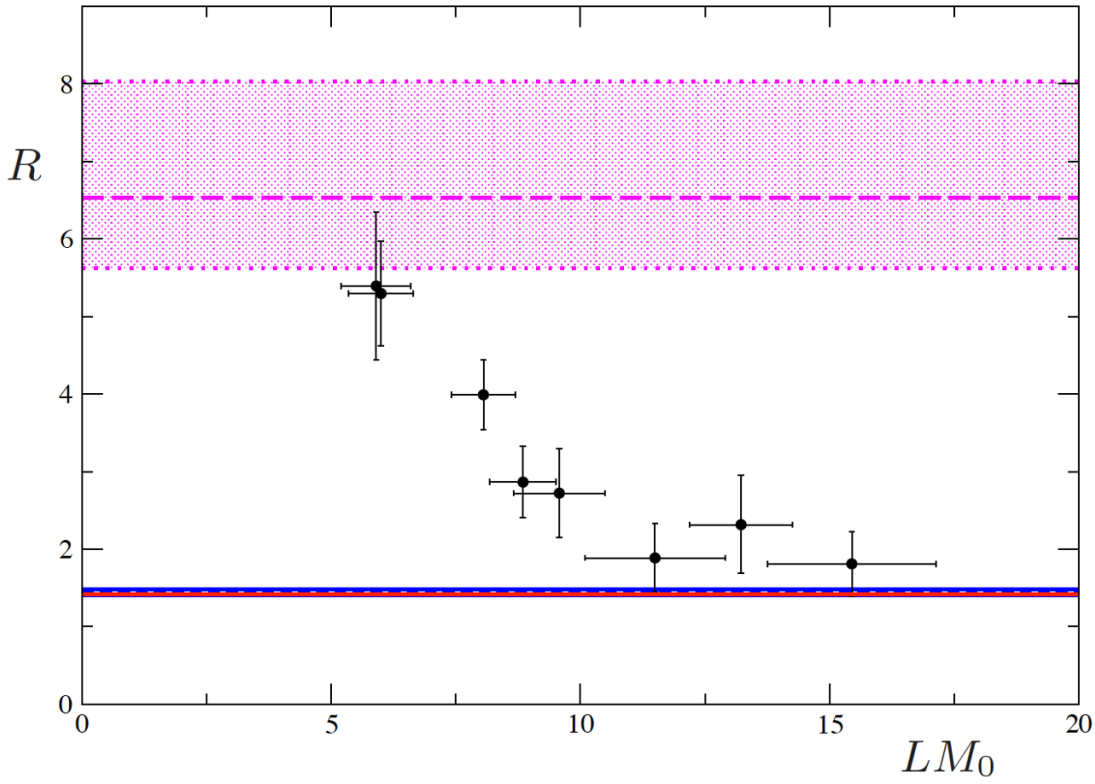


Figure 3. The ratio R as a function of LM_0 for SU(2) gauge theory with one adjoint Dirac fermion. The horizontal blue band indicates the SU(2) Yang-Mills value $R = 1.44(4)$. The pink band at $R = 6.53^{+1.50}_{-0.91}$ and the red line at $R = \sqrt{2}$ are discussed in section 3. The interpretation of the data is explained and discussed in section 2.5.

comments. In fact, one should expect that if the resonance in the tensor channel has a mass that is much larger than that of the scalar, the tensor correlator will be asymptotically dominated by scattering states [79]. Hence, one might wonder whether we have identified the wanted resonance or we are observing some spurious object.

In order to provide a convincing answer to this question, calculations with extended statistics and purpose designed analysis methods [94, 95] are needed, which would go well beyond our current aims. Our preliminary investigation, based on the scaling of the mass with the lattice volume and on the expected degeneracy of the continuum tensor state in two representations of the rotational group of the cube in the case of a single particle, suggests that the extracted mass identifies a resonance. The apparent absence of scattering state contributions in the correlator might be due to the particular construction of the trial variational operators, which are optimised for single particles.

3 A string-inspired toy model and the dual mass spectrum

We want to model the dynamics of a conformal gauge theory in four dimensions, in which the insertion of a relevant deformation via the coupling of an operator $\mathcal{O}$ of dimension $4 - \Delta$ introduces a scale in the theory that discretises the spectrum and introduces a mass

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gap. In the spirit of bottom-up holography, we consider a five-dimensional sigma-model consisting of one scalar Φ coupled to gravity. The 5-dimensional action is

$$\int \mathrm{d}^4x\mathrm{d}r\sqrt{-g}\left[\frac{R_5}{4}-\frac{1}{2}g^{MN}\partial_M\Phi\partial_N\Phi-V(\Phi)\right], \tag{3.1}$$

where R_5 is the Ricci scalar in five dimensions, $V(\Phi)$ is the scalar potential and g_{MN} the five-dimensional metric. The dynamics of the (canonically normalised) bulk scalar field descends from a superpotential for which, taking inspiration from GPPZ [76, 77], we reconsider the toy model proposed in [65]:

$$W=-\frac{3}{4}\left(1+\cosh 2\sqrt{\frac{\Delta}{3}}\Phi\right), \tag{3.2}$$

where Δ is the scaling dimension of the parameter controlling the deformation. The scalar potential is

$$V=\frac{1}{2}\left(W_\Phi\right)^2-\frac{4}{3}W^2, \tag{3.3}$$

where $W_\Phi\equiv\frac{\partial W}{\partial\Phi}$. We write the metric as

$$\mathrm{d}s_5^2=\mathrm{d}r^2+e^{2A}\mathrm{d}x_{1,3}^2, \tag{3.4}$$

and search for classical solutions of the form $A=A(r)$ and $\Phi=\bar{\Phi}(r)$, manifestly preserving Lorentz invariance.

Any solutions to the first-order equations

$$A'=-\frac{2}{3}W, \qquad \bar{\Phi}'=W_\Phi, \tag{3.5}$$

where $'$ denotes derivatives in respect to r, solve also the full set of coupled second-order differential equations of the five-dimensional system. The solution of the first-order equations is

$$\bar{\Phi}(r)=\sqrt{\frac{3}{\Delta}}\mathrm{arctanh}\left(e^{-\Delta(r-c_1)}\right), \tag{3.6}$$

$$A(r)=A_0+\frac{1}{2\Delta}\ln\left(-1+e^{2\Delta(r-c_1)}\right). \tag{3.7}$$

We will be interested in ratios of masses, and hence we set the two integration constants to $c_1=0$ and $A_0=0$. In the UV (for large r) the background is asymptotically AdS_5 with $A\simeq r$, and $\bar{\Phi}\sim\sqrt{\frac{3}{\Delta}}e^{-\Delta r}$, while the space ends at $r=0$.

From the UV expansion, one sees that $m\equiv\sqrt{\frac{3}{\Delta}}e^{\Delta c_1}$ is the dimensionful parameter that introduces a scale, analogous to the mass deformation m in section 2. This deformation makes the space end at $r\to c_1$, and yields a mass gap in the spectrum, controlled by a scale Λ_0, in the dual theory. Setting m to zero ($c_1\to-\infty$) would yield the exact AdS_5 background that is the gravity dual of a CFT. We are modelling a scenario that is qualitatively the same as in the lattice calculations in section 2, except for the fact that the gravity dual cannot be used to describe a weakly-coupled fixed point, and hence $\Lambda_*\to+\infty$.

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We compute the spectrum of scalar and tensor fluctuations, that we interpret in terms of the glueballs of the (putative) dual field theory, by deriving the linearised equations of motions around the background solution, and by constructing explicitly gauge-invariant combinations of the fluctuations [61–64].

We introduce two cutoffs, by restricting the radial direction to the segment $0 < r_{\rm IR} < r < r_{\rm UV}$. $r_{\rm IR}$ plays the role of $\Lambda_{\rm IR}$ (the lattice volume L), while $r_{\rm UV}$ corresponds to $\Lambda_{\rm UV}$ (the lattice spacing a) in section 2.

Notice how the function $A(r)$ is monotonic and takes values from $-\infty$ for $r \to 0$ to $+\infty$ for $r \to +\infty$. We indicatively identify the cutoff scales as

$$\frac{\Lambda_0}{\Lambda_{\rm IR,UV}} \equiv e^{-A(r_{\rm IR,UV})}\,. \tag{3.8}$$

After Fourier transforming, the bulk equation for the gauge-invariant scalar $\mathfrak{a}(r, q_\mu) \equiv \varphi - \frac{\bar\Phi'}{6A'}h$ is [64]

$$\left[\partial_r^2 + 4A'\partial_r + e^{-2A}M^2\right]\mathfrak{a} - \left[V_{\Phi\Phi} + \frac{8\bar\Phi' V_\Phi}{3A'} + \frac{16V\bar\Phi'^2}{9A'^2}\right]\mathfrak{a} = 0\,. \tag{3.9}$$

We define $M^2 \equiv -\eta^{\mu\nu}q_\mu q_\nu$, in terms of the 4-momentum q^μ. We impose boundary conditions according to [65],[6] and repeat the calculation to extrapolate the results to the (physical) case $r_{\rm IR} \to 0$ and $r_{\rm UV} \to +\infty$. The physical results do not depend on the spurious regulators r_i. The boundary conditions are [65]

$$\left[\partial_r + \frac{M^2}{e^{2A}}\frac{3A'}{2\bar\Phi'^2} - \left(\frac{4V\bar\Phi'}{3A'} + V_\Phi\right)\right]\mathfrak{a}\,\Big|_{r_i} = 0\,. \tag{3.10}$$

The traceless transverse components $\mathfrak{e}^\mu{}_\nu$ of the fluctuations of the metric obey the same equations as a scalar field with canonical kinetic term and no potential [78]:

$$\left[\partial_r^2 + 4A'\partial_r + e^{-2A}M^2\right]\mathfrak{e}^\mu{}_\nu = 0\,. \tag{3.11}$$

We impose Neumann boundary conditions at the boundaries:

$$\partial_r\mathfrak{e}^\mu{}_\nu\,|_{r_i} = 0\,. \tag{3.12}$$

There is only one physical scale, fixed by the deformation itself — equivalently, by the end of space $c_1 = 0$, or by the scale Λ_0. The spectrum of scalar and tensor modes is a function of the one free parameter Δ. We focus on the range $1 < \Delta < 2$. The results are shown in figure 4. In particular, for $\Delta = 1$ we find $R \equiv M_T/M_0 = \sqrt{2}$, which reproduces the GPPZ case [78, 96].

From the numerical study, we find that for $\Delta = 1.371(20)$ we have $R \simeq 1.95(4)$, obtained with $r_{\rm IR} = 10^{-6}$ and $r_{\rm UV} = 20$. For $\Delta = 1.925(25)$ the calculation requires to use higher values of the UV cutoff, because of the proximity to $\Delta \to 2$. We find $R \simeq 6.53^{+1.50}_{-0.91}$ with $r_{\rm IR} = 10^{-6}$ and $r_{\rm UV} = 50$. These results are the pink shaded regions in figures 2 and 3, which can be thought of as indicative predictions from the gravity calculations.

[6] We take to $+\infty$ two boundary mass terms that are allowed by the symmetries of the model [65]. This procedure is equivalent, in the present context, to requiring regularity and normalisability, according to the standard prescription of gauge/gravity dualities.

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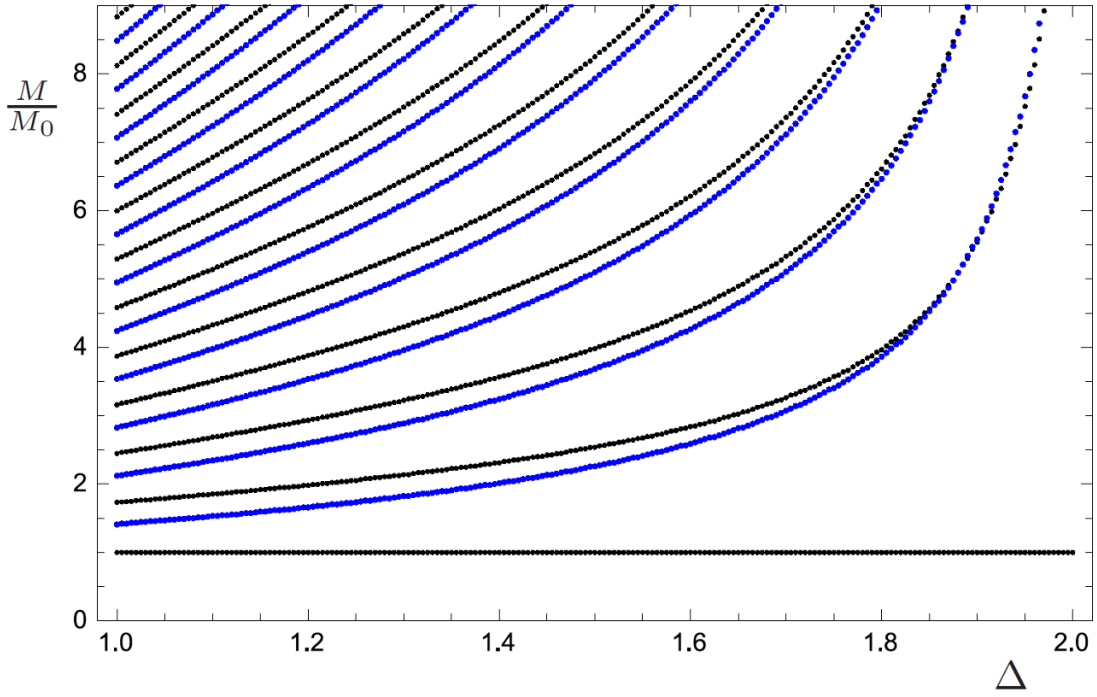


Figure 4. The mass M of the composite spin-0 (black) and spin-2 (blue) states, as well as their excitations, computed for $c_1 = 0 = A_0$, as a function of Δ, obtained for $r_{\rm UV} = 25$ and $r_{\rm IR} = 10^{-6}$, normalised to the mass M_0 of the lightest scalar. The lowest blue line is also the ratio R.

In proximity of $\Delta = 2$, the mass of the lightest scalar state approaches zero. The $\Delta = 2$ case is special because it corresponds to saturating the Breitenlohner-Freedman bound [97], in proximity of which non-trivial phenomena appear (see for instance [98–100]).

In addition, when $\Delta \simeq 2$ the mass of the tensor modes becomes approximately degenerate with the masses of the excited scalars. This is probably accidental, yet it might be relevant phenomenologically. In some of the analysis in [3, 4] the hypothesis that the diphoton resonance has a comparatively large width seems to improve the fit to the data. If the diphoton signal were to be interpreted in terms of two new narrow resonances, one with spin 2 and the other with spin 0, with masses close to one another, the large visible width would be easier to explain.

3.1 Cutoff effects

We perform here an exercise aimed at illustrating some subtleties related to the role of the cutoffs used in the calculation of the spectrum. This discussion is intended to be read in parallel with section 2.

There are three scales in the gravity calculation of the mass spectrum. One is the physical scale Λ_0 induced non-trivially by the deformation m. The other two are spurious scales, due to the finite values of $r_{\rm IR}$ and $r_{\rm UV}$ — corresponding to $\Lambda_{\rm IR}$ and $\Lambda_{\rm UV}$, respectively. One must check that by repeating the calculations with larger and larger $r_{\rm UV}$ (smaller and smaller $r_{\rm IR}$), eventually the results become insensitive to $r_{\rm UV}$ ($r_{\rm IR}$). Further-

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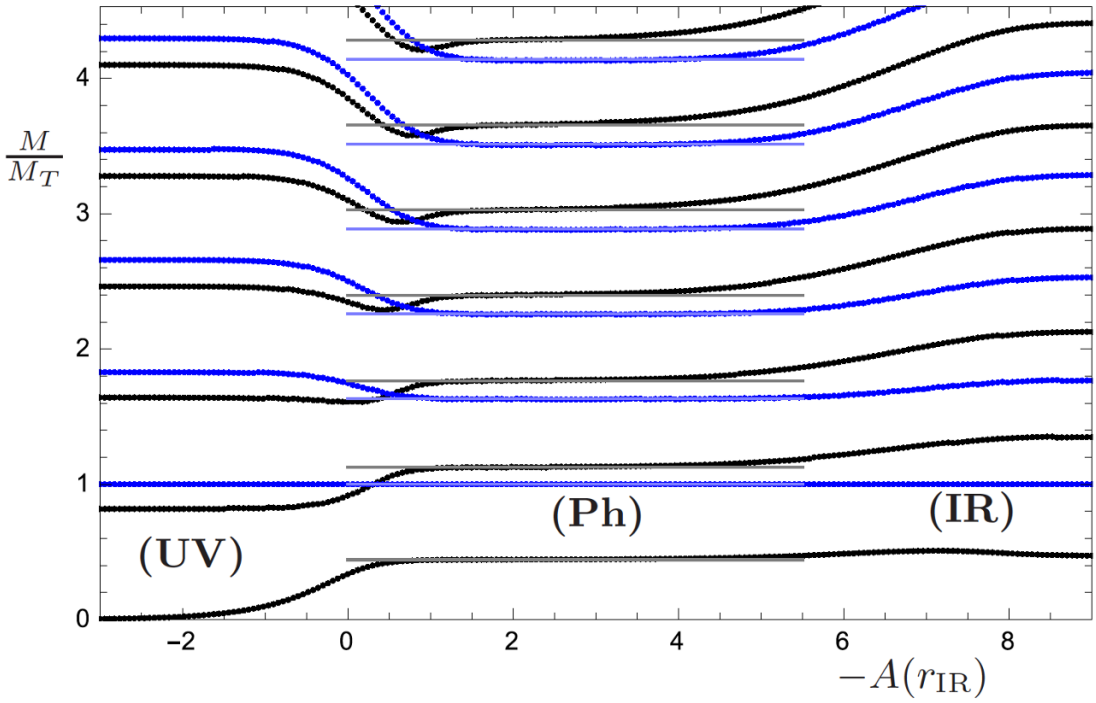


Figure 5. The mass M of the composite spin-0 (black) and spin-2 (blue) states, computed for $c_1 = 0 = A_0$, as a function of $-A(r_{\rm IR}) = \log(\Lambda_0/\Lambda_{\rm IR})$, for $\Delta = 1.5$. We also vary the UV cutoff, by keeping fixed $A(r_{\rm UV}) - A(r_{\rm IR}) = 8$. M_T is the mass of the lightest tensor. The straight lines are the result from figure 4, showing agreement between the two calculations in the (Ph) region, as explained in the text.

more, this must be true while varying independently the three mass scales. This is how figure 4 has been obtained.

Figure 5 illustrates the artificial distortions of the spectrum in the presence of correlated finite cutoffs. We fix $c_1 = 0 = A_0$ and compute the spectra for $\Delta = 1.5$, by varying both the IR and UV cutoffs, but imposing the constraint that $A(r_{\rm UV}) - A(r_{\rm IR}) = 8$.

The ratio $\Lambda_0/\Lambda_{\rm IR}$ defines the IR cutoff. The spectrum is normalised to the mass M_T of the lightest tensor excitation. The horizontal axis of the plots has the same meaning as in figures 2 and 3, as $\Lambda_0/\Lambda_{\rm IR} \propto LM_0$.

There are in general three regions of $\Lambda_0/\Lambda_{\rm IR}$, when the separation between the cutoffs is kept finite.

(UV) For small $\Lambda_0/\Lambda_{\rm IR}$ the gravity calculations probe only a region where the background is close to AdS.

(IR) For large $\Lambda_0/\Lambda_{\rm IR}$, because the ratio of cutoff scales is kept fixed, the calculation probes only the region close to the end-of-space, far from AdS.

(Ph) For intermediate values of $\Lambda_0/\Lambda_{\rm IR}$, and provided the ratio of cutoffs is large, the numerical study is probing both asymptotic regions of the geometry.

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The physical region (Ph) is the one corresponding to figure 4. If we take $\Lambda_{\rm IR}$ small enough and $\Lambda_{\rm UV}$ large enough, the gravity calculation is sensitive to both the physics of the (dual) fixed point — in particular to Δ — as well as to confinement. In figure 5 this region is reached in the middle of the plot: the ratio of the lowest blue line and lowest black line agrees with $R \simeq 2.2$ from figure 4 for $\Delta = 1.5$. In this region, universality is expected to play a role, and hence gravity, lattice and field theory to yield the same results. The width of this intermediate plateau depends on how far separated the cutoffs are. In particular, this plateau disappears if one takes the cutoffs too close to one another.

In the (IR) region the calculation is performed with cutoffs that are both near enough the end of space to be insensitive to the AdS region — see the right part of figure 5. The results are completely unphysical, affected by the artificial modelling of the mass gap. The comparison to the right-end part of figures 2 and 3 has to be done carefully. In this region the lattice calculation agrees with the $\Delta = 1$ gravity calculation, in which $R = \sqrt{2}$ (see the discussion in section 3.2). This is not the result shown in figure 5, which is performed with $\Delta = 1.5$, and entirely contained in a region of the geometry in which the gravity calculation cannot be trusted.

In the (UV) region the spectrum results from a finite-volume artefact: an approximately scale-invariant theory is forced inside a small box. The most striking feature is the presence of a parametrically light scalar state. Finite volume introduces a VEV, spontaneously breaking scale invariance, and hence yields a Goldstone boson (dilaton). In the limit of exact CFT, this state would be exactly massless, and decoupled from physical correlation functions. For finite $\Lambda_0/\Lambda_{\rm IR}$, the deformation provides both mass and couplings to the dilaton.

The lightness and small coupling of the first scalar imply that either one gets arbitrarily large $R = M_T/M_0$, or one computes R from the second scalar excitation. In the latter case, R is smaller than in the (Ph) region, eventually approaching $R \simeq 1$, along the lines of the femto-universe [84, 93]. The specific details are model-dependent (not universal).

It is interesting to notice that some semi-quantitative cutoff features in gravity reproduce those of lattice calculations even in the (UV) region. First of all, $R < \sqrt{2}$ (which is obtained ignoring the lightest scalar) is close to 1 [101], as can be seen from figure 5, as well as in the region with $LM_0 < 8$ of figure 2. Secondly, at least in some regions of parameter space, $M_T < M_0$, as seen in figure 5 as well as in the perturbative calculation in [93]. Thirdly, at large $\Lambda_{\rm UV}/\Lambda_{\rm IR}$ one finds $R \simeq 1.2$, similar to the lattice strong-coupling expansion [102]. It would require a dedicated study to establish how many of these observations are more than just accidental results.

3.2 Comments about pure Yang-Mills theory

We notice the striking coincidence that three different theories agree on $R \simeq \sqrt{2}$: the SU(2) Yang-Mills theory [28, 29], the SU(2) theory with adjoint fermions [33, 35] in the regime in which LM_0 is large, and the GPPZ gravity model [76, 78]. These independent results are not original to this work, yet the level of agreement is so good that it deserves some further independent discussion. It is especially surprising that the agreement extends to

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the GPPZ model. We devote this digressive subsection to further suggest that this might be the consequence of possible underlying universality properties.

Among the many attempts to describe within gravity a confining, large-N Yang-Mills-like theory, we would like to highlight three special ones. In GPPZ [76, 78], the asymptotic (in the UV) geometry is $\mathrm{AdS}_5 \times S^5$ and $R = \sqrt{2}$. A less known example exists [103] (see also [68]), in which the local geometry is asymptotically $\mathrm{Ad}S_6 \times S^4$ (one dimension is compactified on a shrinking circle), and for which $R \simeq 1.6$. In the Witten model [66], the local asymptotic geometry is $\mathrm{Ad}S_7 \times S^4$ (two dimensions are compactified on shrinking circles), and $R \simeq 1.7$ [67]. $\mathrm{Ad}S_{d+1}$ gravity backgrounds provide the dual of d-dimensional CFTs, and hence the three examples we reported are related (in the far UV) to CFTs living in 4, 5 and 6 dimensions, respectively. Coincidentally, notice that in the three cases $R \simeq \sqrt{d/2}$.

Large-N theories should not agree with SU(2) in four dimensions. But if the ratio R is universal, this special observable should depend only on universal quantities, such as the dimensionality of the space-time, or the dimensions of the relevant operators defined by the CFT, and not on microscopic details.

The extrapolation of lattice SU(N) data is compatible with the prediction from the Witten model. Yet the comparison with GPPZ — the dual of which is a four-dimensional CFT, and in which the anomalous dimension $\gamma^* = 0$ — suggests that at least in the proximity of the fixed point (which seems to be crucial in computing R, as our study shows) the dynamics of Yang-Mills theory is better captured by GPPZ. Even some field theory arguments in [104, 105] — in which the author also draws a comparison with large-N studies, but without relying on infinite-N extrapolation — yield $R = \sqrt{2}$.

The remarkable coincidence on $R = \sqrt{2}$ between several different lattice, field theory and gravity calculations, at large-N as well as at small-N, for theories in which anomalous dimensions are trivial, might be the result of deep universality properties of the ratio R, although a firm conclusion is premature.

4 Physics lessons

In this paper, we are looking for an example of a strongly-coupled field theory yielding the ratio of masses $R \gg 1$. Besides intrinsic theoretical reasons, we are interested to find evidence of this because the recent signals of a new particle with mass $M \simeq 750 \gg 125\,\mathrm{GeV}$ might admit an explanation in terms of a heavy glueball in a strongly-coupled extension of the Standard Model.

The working hypotheses under which we discuss lattice data on SU(2) with adjoint matter are the following.

- The SU(2) theory with one or two adjoint matter fields is asymptotically free and close to conformal in the IR.
- In it, the masses of composite states shows a universal scaling with m, with scaling exponent determined by the anomalous dimension of the $\bar{\psi}\psi$ operator.
- While the coefficients in front of these scaling laws are model-dependent, certain ratios, in particular R, exhibit universal characteristics.

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In this section, we critically discuss whether the numerical studies support these hypotheses. We want to disentangle physical results from lattice artefacts and assess whether there are indications that $R \gg 1$.

We are aided in our task by the intuition gained in section 3. The toy model in the gauge/gravity context suggests that the ratio R is a monotonic function of $\Delta = 1 + \gamma^*$, with a minimum $R = \sqrt{2}$ for $\Delta = 1$. R diverges at $\Delta \to 2$. This fact indicates that for gauge theories with large anomalous dimensions one should find $R \gg 1$.

We want to check whether the behaviour indicated by the toy model is confirmed by the more rigorous and concrete lattice calculations. We start from the case of $N_f = 2$ in figure 2, that can be summarised by three possible values of R. For the largest values of LM_0 the results agree with the Yang-Mills theory value $R = 1.44(4)$, as well as the gravity result for $\Delta = 1$ — in GPPZ, $R = \sqrt{2}$. For intermediate values of LM_0, R is close (within errors) to the prediction of the gravity toy model $R \simeq 1.95(4)$. For smaller values of LM_0, R is visibly lower, with the decrease likely to be a sign of the approach to the femto-universe regime.

The large LM_0 region is interpreted in terms of artefacts that make the system lose memory of the presence of the IR fixed point. We interpret the intermediate one as the physical region, in which the lattice results can be extrapolated to the continuum limit. The low LM_0 region is interpreted as a lattice artefact: given that the value of β is fixed, and that the maximum number of lattice sites n is finite, for small values of LM_0 we are exploring the region of (lattice) parameter space in which m is so small that the discretisation of the spectrum is due to the finite volume effects, with $\Lambda_0 \ll \Lambda_{\rm IR}$, as in the femto-universe.

This interpretation is in line with what is shown in figure 5, obtained from gravity, in spite of the fact that cutoff effects are not universal. A non-trivial test of our interpretation would require a more detailed and precise measurement of the spectrum of scalars at small values of m, with higher statistics. Finite volume effects should be associated with the appearance of a spurious, weakly-coupled, light spin-0 state, that may have escaped detection.

The study of SU(2) gauge theory with two adjoint fermions does not contradict the results from gravity. In the physical region (Ph) we do see numerical evidence of an enhancement of the ratio R in respect to pure Yang-Mills theory, by an amount compatible with gravity results, though modest — the anomalous dimension γ^* is small.

In the case of one Dirac adjoint fermion, the anomalous dimension is large, and $\Delta = 1.925(25)$ is close to its natural upper bound. The gravity calculation yields large $R = 6.53^{+1.50}_{-0.91}$. In figure 3, for large LM_0 the ratio R is small, broadly speaking compatible with the result of pure Yang-Mills $R = 1.44(4)$ (within the large errors). Going to smaller LM_0, R is growing monotonically until a maximum of $R \simeq 5.4$.

We interpret this behaviour in the following terms. For large LM_0 the numerical results are dominated by lattice artefacts that hide the effects of the IR fixed point. Going to smaller LM_0, the calculations are approaching the physical plateau at large R, but do not quite reach it. The physically relevant value of R is realised in a region of (lattice) parameter space to the left of the points available to this study.

In order to assess whether this interpretation is correct, it would be necessary to perform additional lattice simulations, for larger lattices (bigger n), and smaller values of the mass m. Such a study should show the appearance of a plateau at large R. The reader

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should exercise caution in using the quantitative comparison to the gravity toy model. For Δ close to its natural maximum value $\Delta \sim 2$, the ratio R is predicted to diverge. This means that the result is very sensitive to the exact value of Δ itself, or equivalently of the anomalous dimension γ^*, the precise determination of which is non-trivial.

There is an alternative logical possibility in interpreting figure 3, in which the raise in R is attributed to a lattice artefact — the large volume effects that introduce a spurious light scalar in the spectrum. This is the same scalar we claimed to be hard to identify in the $N_f = 2$ case. For this reason, and because we do not see a plateau at a large value of R, we discard this possibility.

In summary, the lattice results are consistent with our working hypothesis. We find evidence of an enhancement of R for the theory in which the IR fixed point has large anomalous dimensions. The enhancement we see is compatible with the prediction from gravity. Accidentally, R comes close to what would be needed to interpret the diphoton signal ($R \sim 6$).

We conclude with a cautionary remark about phenomenological applications. The themes of this paper are to enquire on whether R exhibits a universal character in diverse theories and whether there exist theories for which $R \gg 1$. We focus on R because this quantity is well defined in a broad class of field theories, and not directly affected by model-dependent details. In particular, we exhibited explicitly the chiral symmetry and chiral-symmetry breaking pattern of the two SU(2) theories, showing that they are very different. And we compared to a gravity model in which there is no chiral symmetry at all. The fact that we find large values of R for the $N_f = 1$ theory is encouraging for phenomenological purposes, in particular in reference to the diphoton anomaly at the LHC, as a step towards a proof of principle that large mass hierarchies can arise in strongly-coupled theories.

Conversely, figure 5 of [54] shows that the spectrum of mesons includes several particles that are lighter than the tensor glueball, and do not correspond to states observed at the LHC. The phenomenological viability of any model requires studying carefully many other model-dependent details that go beyond our present purposes, including the task of finding a strongly-coupled model that has large anomalous dimensions without introducing in the spectrum a plethora of light mesons.

5 Conclusions and outlook

We have exhibited what is, to the best of our knowledge, the first example of a lattice study of a strongly-coupled theory in which the ratio of tensor to scalar glueball mass is large ($R > 5$). This is encouraging, in the light of the recently observed LHC diphoton excess, which would require $R \sim 6$.

The model in which these indications arise is the mass deformation of a gauge theory with SU(2) gauge group and $N_f = 1$ generations of Dirac fermions in the adjoint representation. In the massless limit, this theory is believed to have an IR fixed point with large anomalous dimensions. As a consistency check, we compared the numerical results to the lattice results for the case $N_f = 2$, in which the IR fixed point has small anomalous dimensions. We found $R \sim \mathcal{O}(1)$, which is compatible with expectations.

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We also compared to a toy model built in the context of gauge/gravity dualities. We drew a parallel between the cutoff effects on the lattice and in gravity models, finding qualitative agreement. In the physically relevant region of parameter space, we found remarkably good numerical agreement with the lattice for the relation between R and the dimension Δ of the deforming parameter.

These are preliminary results, and require further investigation with dedicated studies. We summarise some of the directions this research program could encompass.

On the lattice side, a dedicated program for the study of glueball spectra, both in SU(2) with adjoint matter, as well as in other candidate theories with IR conformal dynamics, is needed. Particular attention should be devoted to the SU(2) theory with $N_f = 1$, the only known case to date for which R is found to be large. New studies with larger lattices might ascertain whether large values of R are truly physical or affected by lattice artefacts.

On the gravity side, a more general exploration of models that describe the dynamics of deformations of IR-conformal theories is needed. The results presented here are based on a simplified model built within the bottom-up approach to holography. It would be useful to identify and study a full model derived from string theory in the top-down context. Yet, it should be possible to carry out similar studies in more general contexts, within the bottom-up approach.

In the field theory context, it would be interesting to study the ratios of masses, aided by statistical field theory arguments. The claim of universality that underpins the comparison we make of lattice and gravity results might be taken literally (in the sense that R is only a function of Δ), or just in a broad sense: the behaviours emerging in different models share qualitative features. In this paper, we took the latter view, in the absence of a rigorous proof of a more robust relation, though our numerical results show such a good level of agreement with gravity calculations that it might be an indication of a more fundamental physical principle.

On the phenomenological side, several additional questions arise. Under the assumption that new strong dynamics can explain a large mass hierarchy $R \gtrsim 5-6$, the construction of a realistic model of electroweak symmetry breaking requires to introduce many additional ingredients. It is worth bringing to the attention of the reader for example the papers in [106–108], where some relevant considerations pertaining to model-building and LHC phenomenology are discussed. This paper is a first step towards a future proof of principle that comparatively heavy spin-2 resonances can emerge as composite states of new strong dynamics, leaving all model-building and phenomenological issues aside.

We close by repeating the main results of this study. We collected a significant body of empirical evidence, both from lattice studies of SU(2) theories with adjoint matter and from simplified gravity dual models, that points in two intertwined directions. First of all, we find agreement (within errors) in the results for R computed in completely different theories that share the same dimension Δ for the relevant deformation, suggesting that the quantity R might be a manifestation of some form of universality. Secondly, when the deforming operator has large anomalous dimension, we find a parametric enhancement of R, the ratio of masses of the lightest tensor and scalar glueballs. Both results need to be further tested with more extensive, specific future studies.

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References

[1] ATLAS collaboration, *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, *Phys. Lett.* **B 716** (2012) 1 [arXiv:1207.7214] [INSPIRE].

[2] CMS collaboration, *Observation of a new boson at a mass of* 125 *GeV with the CMS experiment at the LHC*, *Phys. Lett.* **B 716** (2012) 30 [arXiv:1207.7235] [INSPIRE].

[3] ATLAS collaboration, *Search for resonances decaying to photon pairs in* 3.2 fb^{-1} *of pp collisions at* $\sqrt{s} = 13$ *TeV with the ATLAS detector*, ATLAS-CONF-2015-081, CERN, Geneva Switzerland (2015).

[4] CMS collaboration, *Search for new physics in high mass diphoton events in proton-proton collisions at* $\sqrt{s} = 13$ *TeV*, CMS-PAS-EXO-15-004, CERN, Geneva Switzerland (2015).

JHEP06(2016)114

[5] CMS collaboration, *Search for massive resonances decaying into pairs of boosted bosons in semi-leptonic final states at* $\sqrt{s} = 8$ *TeV*, *JHEP* **08** (2014) 174 [arXiv:1405.3447] [INSPIRE].

[6] CMS collaboration, *Search for heavy neutrinos and W bosons with right-handed couplings in proton-proton collisions at* $\sqrt{s} = 8$ *TeV*, *Eur. Phys. J.* **C 74** (2014) 3149 [arXiv:1407.3683] [INSPIRE].

[7] CMS collaboration, *Search for resonances and quantum black holes using dijet mass spectra in proton-proton collisions at* $\sqrt{s} = 8$ *TeV*, *Phys. Rev.* **D 91** (2015) 052009 [arXiv:1501.04198] [INSPIRE].

[8] ATLAS collaboration, *Search for high-mass diboson resonances with boson-tagged jets in proton-proton collisions at* $\sqrt{s} = 8$ *TeV with the ATLAS detector*, *JHEP* **12** (2015) 055 [arXiv:1506.00962] [INSPIRE].

[9] CMS collaboration, *Search for massive WH resonances decaying into the* $\ell\nu\mathrm{b\bar{b}}$ *final state at* $\sqrt{s} = 8$ *TeV*, *Eur. Phys. J.* **C 76** (2016) 237 [arXiv:1601.06431] [INSPIRE].

[10] S. Weinberg, *Implications of dynamical symmetry breaking: an addendum*, *Phys. Rev.* **D 19** (1979) 1277 [INSPIRE].

[11] L. Susskind, *Dynamics of spontaneous symmetry breaking in the Weinberg-Salam theory*, *Phys. Rev.* **D 20** (1979) 2619 [INSPIRE].

[12] S. Weinberg, *Implications of dynamical symmetry breaking*, *Phys. Rev.* **D 13** (1976) 974 [INSPIRE].

[13] B. Holdom, *Techniodor*, *Phys. Lett.* **B 150** (1985) 301 [INSPIRE].

[14] K. Yamawaki, M. Bando and K.-I. Matumoto, *Scale invariant technicolor model and a technidilaton*, *Phys. Rev. Lett.* **56** (1986) 1335 [INSPIRE].

[15] T.W. Appelquist, D. Karabali and L.C.R. Wijewardhana, *Chiral hierarchies and the flavor changing neutral current problem in technicolor*, *Phys. Rev. Lett.* **57** (1986) 957 [INSPIRE].

[16] K. Lane, *Two lectures on technicolor*, hep-ph/0202255 [INSPIRE].

[17] C.T. Hill and E.H. Simmons, *Strong dynamics and electroweak symmetry breaking*, *Phys. Rept.* **381** (2003) 235 [*Erratum ibid.* **390** (2004) 553] [hep-ph/0203079] [INSPIRE].

[18] A. Martin, *Predicted signals at the LHC from technicolor: Erice lecture*, *Subnucl. Ser.* **46** (2011) 135 [arXiv:0812.1841] [INSPIRE].

[19] F. Sannino, *Conformal dynamics for TeV physics and cosmology*, *Acta Phys. Polon.* **B 40** (2009) 3533 [arXiv:0911.0931] [INSPIRE].

[20] M. Piai, *Lectures on walking technicolor, holography and gauge/gravity dualities*, *Adv. High Energy Phys.* **2010** (2010) 464302 [arXiv:1004.0176] [INSPIRE].

[21] R.S. Chivukula, *Lectures on technicolor and compositeness*, hep-ph/0011264 [INSPIRE].

[22] L. Del Debbio, *IR fixed points in lattice field theories*, *Int. J. Mod. Phys.* **A 29** (2014) 1445006 [INSPIRE].

[23] B. Lucini, *Numerical results for gauge theories near the conformal window*, *J. Phys. Conf. Ser.* **631** (2015) 012065 [arXiv:1503.00371] [INSPIRE].

[24] T. DeGrand, *Lattice tests of beyond Standard Model dynamics*, *Rev. Mod. Phys.* **88** (2016) 015001 [arXiv:1510.05018] [INSPIRE].

JHEP06(2016)114

[25] J. Giedt, *Anomalous dimensions on the lattice*, *Int. J. Mod. Phys.* **A 31** (2016) 1630011 [arXiv:1512.09330] [INSPIRE].

[26] D. Nogradi and A. Patella, *Strong dynamics, composite Higgs and the conformal window*, in *Lattice gauge theories beyond QCD*, B. Lucini and M. Panero eds., to be published in a special issue of *Int. J. Mod. Phys.* **A**.

[27] C.J. Morningstar and M.J. Peardon, *The glueball spectrum from an anisotropic lattice study*, *Phys. Rev.* **D 60** (1999) 034509 [hep-lat/9901004] [INSPIRE].

[28] B. Lucini and M. Teper, SU(N) *gauge theories in four-dimensions: exploring the approach to* $N = \infty$, *JHEP* **06** (2001) 050 [hep-lat/0103027] [INSPIRE].

[29] B. Lucini, M. Teper and U. Wenger, *Glueballs and k-strings in* SU(N) *gauge theories: calculations with improved operators*, *JHEP* **06** (2004) 012 [hep-lat/0404008] [INSPIRE].

[30] Y. Chen et al., *Glueball spectrum and matrix elements on anisotropic lattices*, *Phys. Rev.* **D 73** (2006) 014516 [hep-lat/0510074] [INSPIRE].

[31] E. Gregory et al., *Towards the glueball spectrum from unquenched lattice QCD*, *JHEP* **10** (2012) 170 [arXiv:1208.1858] [INSPIRE].

[32] B. Lucini, A. Rago and E. Rinaldi, *Glueball masses in the large-N limit*, *JHEP* **08** (2010) 119 [arXiv:1007.3879] [INSPIRE].

[33] L. Del Debbio, B. Lucini, A. Patella, C. Pica and A. Rago, *Conformal versus confining scenario in* SU(2) *with adjoint fermions*, *Phys. Rev.* **D 80** (2009) 074507 [arXiv:0907.3896] [INSPIRE].

[34] L. Del Debbio, B. Lucini, A. Patella, C. Pica and A. Rago, *Mesonic spectroscopy of minimal walking technicolor*, *Phys. Rev.* **D 82** (2010) 014509 [arXiv:1004.3197] [INSPIRE].

[35] L. Del Debbio, B. Lucini, A. Patella, C. Pica and A. Rago, *The infrared dynamics of minimal walking technicolor*, *Phys. Rev.* **D 82** (2010) 014510 [arXiv:1004.3206] [INSPIRE].

[36] F. Bursa et al., *Improved lattice spectroscopy of minimal walking technicolor*, *Phys. Rev.* **D 84** (2011) 034506 [arXiv:1104.4301] [INSPIRE].

[37] L. Del Debbio, B. Lucini, A. Patella, C. Pica and A. Rago, *Large volumes and spectroscopy of walking theories*, *Phys. Rev.* **D 93** (2016) 054505 [arXiv:1512.08242] [INSPIRE].

[38] S. Catterall and F. Sannino, *Minimal walking on the lattice*, *Phys. Rev.* **D 76** (2007) 034504 [arXiv:0705.1664] [INSPIRE].

[39] L. Del Debbio, A. Patella and C. Pica, *Higher representations on the lattice: numerical simulations.* SU(2) *with adjoint fermions*, *Phys. Rev.* **D 81** (2010) 094503 [arXiv:0805.2058] [INSPIRE].

[40] S. Catterall, J. Giedt, F. Sannino and J. Schneible, *Phase diagram of* SU(2) *with* 2 *flavors of dynamical adjoint quarks*, *JHEP* **11** (2008) 009 [arXiv:0807.0792] [INSPIRE].

[41] A. Hietanen, J. Rantaharju, K. Rummukainen and K. Tuominen, *Minimal technicolor on the lattice*, *Nucl. Phys.* **A 820** (2009) 191c [INSPIRE].

[42] S. Catterall, J. Giedt, F. Sannino and J. Schneible, *Probes of nearly conformal behavior in lattice simulations of minimal walking technicolor*, arXiv:0910.4387 [INSPIRE].

[43] A.J. Hietanen, K. Rummukainen and K. Tuominen, *Evolution of the coupling constant in* SU(2) *lattice gauge theory with two adjoint fermions*, *Phys. Rev.* **D 80** (2009) 094504 [arXiv:0904.0864] [INSPIRE].

JHEP06(2016)114

[44] T. DeGrand, Y. Shamir and B. Svetitsky, *Infrared fixed point in* SU(2) *gauge theory with adjoint fermions*, *Phys. Rev.* **D 83** (2011) 074507 [arXiv:1102.2843] [INSPIRE].

[45] J. Rantaharju, K. Rummukainen and K. Tuominen, *Running coupling in* SU(2) *with adjoint fermions*, arXiv:1301.2373 [INSPIRE].

[46] J. Rantaharju, T. Rantalaiho, K. Rummukainen and K. Tuominen, *Running coupling in* SU(2) *gauge theory with two adjoint fermions*, *Phys. Rev.* **D 93** (2016) 094509 [arXiv:1510.03335] [INSPIRE].

[47] J. Rantaharju, *Gradient flow coupling in the* SU(2) *gauge theory with two adjoint fermions*, *Phys. Rev.* **D 93** (2016) 094516 [arXiv:1512.02793] [INSPIRE].

[48] B. Lucini, *Strongly interacting dynamics beyond the Standard Model on a spacetime lattice*, *Phil. Trans. Roy. Soc. Lond.* **A 368** (2010) 3657 [arXiv:0911.0020] [INSPIRE].

[49] F. Bursa, L. Del Debbio, L. Keegan, C. Pica and T. Pickup, *Mass anomalous dimension in* SU(2) *with two adjoint fermions*, *Phys. Rev.* **D 81** (2010) 014505 [arXiv:0910.4535] [INSPIRE].

[50] S. Catterall, L. Del Debbio, J. Giedt and L. Keegan, *MCRG minimal walking technicolor*, *Phys. Rev.* **D 85** (2012) 094501 [arXiv:1108.3794] [INSPIRE].

[51] J. Giedt and E. Weinberg, *Finite size scaling in minimal walking technicolor*, *Phys. Rev.* **D 85** (2012) 097503 [arXiv:1201.6262] [INSPIRE].

[52] A. Patella, *A precise determination of the* $\bar{\psi}\psi$ *anomalous dimension in conformal gauge theories*, *Phys. Rev.* **D 86** (2012) 025006 [arXiv:1204.4432] [INSPIRE].

[53] B. Lucini, A. Patella, A. Rago and E. Rinaldi, *Infrared conformality and bulk critical points:* SU(2) *with heavy adjoint quarks*, *JHEP* **11** (2013) 106 [arXiv:1309.1614] [INSPIRE].

[54] A. Athenodorou, E. Bennett, G. Bergner and B. Lucini, *Infrared regime of* SU(2) *with one adjoint Dirac flavor*, *Phys. Rev.* **D 91** (2015) 114508 [arXiv:1412.5994] [INSPIRE].

[55] G. Delfino, *Integrable field theory and critical phenomena: the Ising model in a magnetic field*, *J. Phys.* **A 37** (2004) R45 [hep-th/0312119] [INSPIRE].

[56] J.L. Cardy, *Scaling and renormalization in statistical physics*, Cambridge lecture notes in physics **3**, Cambridge Univ. Pr., Cambridge U.K. (1996) [INSPIRE].

[57] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [hep-th/9711200] [INSPIRE].

[58] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [INSPIRE].

[59] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [INSPIRE].

[60] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111] [INSPIRE].

[61] M. Bianchi, M. Prisco and W. Mueck, *New results on holographic three point functions*, *JHEP* **11** (2003) 052 [hep-th/0310129] [INSPIRE].

[62] M. Berg, M. Haack and W. Mueck, *Bulk dynamics in confining gauge theories*, *Nucl. Phys.* **B 736** (2006) 82 [hep-th/0507285] [INSPIRE].

JHEP06(2016)114

[63] M. Berg, M. Haack and W. Mueck, *Glueballs vs. gluinoballs: fluctuation spectra in non-AdS/non-CFT*, *Nucl. Phys.* **B 789** (2008) 1 [hep-th/0612224] [INSPIRE].

[64] D. Elander, *Glueball spectra of SQCD-like theories*, *JHEP* **03** (2010) 114 [arXiv:0912.1600] [INSPIRE].

[65] D. Elander and M. Piai, *Light scalars from a compact fifth dimension*, *JHEP* **01** (2011) 026 [arXiv:1010.1964] [INSPIRE].

[66] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131] [INSPIRE].

[67] R.C. Brower, S.D. Mathur and C.-I. Tan, *Glueball spectrum for QCD from AdS supergravity duality*, *Nucl. Phys.* **B 587** (2000) 249 [hep-th/0003115] [INSPIRE].

[68] D. Elander, A.F. Faedo, C. Hoyos, D. Mateos and M. Piai, *Multiscale confining dynamics from holographic RG flows*, *JHEP* **05** (2014) 003 [arXiv:1312.7160] [INSPIRE].

[69] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χSB resolution of naked singularities*, *JHEP* **08** (2000) 052 [hep-th/0007191] [INSPIRE].

[70] J.M. Maldacena and C. Núñez, *Towards the large-N limit of pure $N = 1$ super Yang-Mills*, *Phys. Rev. Lett.* **86** (2001) 588 [hep-th/0008001] [INSPIRE].

[71] A.H. Chamseddine and M.S. Volkov, *Non-Abelian BPS monopoles in $N = 4$ gauged supergravity*, *Phys. Rev. Lett.* **79** (1997) 3343 [hep-th/9707176] [INSPIRE].

[72] C. Núñez, I. Papadimitriou and M. Piai, *Walking dynamics from string duals*, *Int. J. Mod. Phys.* **A 25** (2010) 2837 [arXiv:0812.3655] [INSPIRE].

[73] D. Elander, C. Núñez and M. Piai, *A light scalar from walking solutions in gauge-string duality*, *Phys. Lett.* **B 686** (2010) 64 [arXiv:0908.2808] [INSPIRE].

[74] D. Elander and M. Piai, *On the glueball spectrum of walking backgrounds from wrapped-D5 gravity duals*, *Nucl. Phys.* **B 871** (2013) 164 [arXiv:1212.2600] [INSPIRE].

[75] D. Elander, *Light scalar from deformations of the Klebanov-Strassler background*, *Phys. Rev.* **D 91** (2015) 126012 [arXiv:1401.3412] [INSPIRE].

[76] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *The supergravity dual of $N = 1$ super Yang-Mills theory*, *Nucl. Phys.* **B 569** (2000) 451 [hep-th/9909047] [INSPIRE].

[77] K. Pilch and N.P. Warner, *$N = 1$ supersymmetric renormalization group flows from IIB supergravity*, *Adv. Theor. Math. Phys.* **4** (2002) 627 [hep-th/0006066] [INSPIRE].

[78] W. Mueck and M. Prisco, *Glueball scattering amplitudes from holography*, *JHEP* **04** (2004) 037 [hep-th/0402068] [INSPIRE].

[79] L. Maiani and M. Testa, *Final state interactions from Euclidean correlation functions*, *Phys. Lett.* **B 245** (1990) 585 [INSPIRE].

[80] V.A. Miransky, *Dynamics in the conformal window in QCD like theories*, *Phys. Rev.* **D 59** (1999) 105003 [hep-ph/9812350] [INSPIRE].

[81] L. Del Debbio and R. Zwicky, *Scaling relations for the entire spectrum in mass-deformed conformal gauge theories*, *Phys. Lett.* **B 700** (2011) 217 [arXiv:1009.2894] [INSPIRE].

[82] L. Del Debbio and R. Zwicky, *Hyperscaling relations in mass-deformed conformal gauge theories*, *Phys. Rev.* **D 82** (2010) 014502 [arXiv:1005.2371] [INSPIRE].

JHEP06(2016)114

[83] J.D. Bjorken, *Elements of quantum chromodynamics*, in *Proc. Summer Institute on Particle Physics*, A. Mosher ed., SLAC report no 224, U.S.A. (1980) [INSPIRE].

[84] M. Lüscher, *Advanced lattice QCD*, hep-lat/9802029 [INSPIRE].

[85] A. González-Arroyo, J. Jurkiewicz and C.P. Korthals-Altes, *Ground state metamorphosis for Yang-Mills fields on a finite periodic lattice*, CPT-81-P-1336, Freiburg Germany (1981) [INSPIRE].

[86] A. Coste, A. González-Arroyo, J. Jurkiewicz and C.P. Korthals Altes, *Zero momentum contribution to Wilson loops in periodic boxes*, *Nucl. Phys.* **B 262** (1985) 67 [INSPIRE].

[87] G. 't Hooft, *A property of electric and magnetic flux in non-Abelian gauge theories*, *Nucl. Phys.* **B 153** (1979) 141 [INSPIRE].

[88] A. Coste, A. González-Arroyo, C.P. Korthals Altes, B. Soderberg and A. Tarancon, *Finite size effects and twisted boundary conditions*, *Nucl. Phys.* **B 287** (1987) 569 [INSPIRE].

[89] A. González Arroyo and C.P. Korthals Altes, *The spectrum of Yang-Mills theory in a small twisted box*, *Nucl. Phys.* **B 311** (1988) 433 [INSPIRE].

[90] D. Daniel, A. González-Arroyo, C.P. Korthals Altes and B. Soderberg, *Energy spectrum of* SU(2) *Yang-Mills fields with space-like symmetric twist*, *Phys. Lett.* **B 221** (1989) 136 [INSPIRE].

[91] D. Daniel, A. González-Arroyo and C.P. Korthals Altes, *The energy levels of lattice gauge theory in a small twisted box*, *Phys. Lett.* **B 251** (1990) 559 [INSPIRE].

[92] M. Lüscher, *Some analytic results concerning the mass spectrum of Yang-Mills gauge theories on a torus*, *Nucl. Phys.* **B 219** (1983) 233 [INSPIRE].

[93] M. Lüscher and G. Münster, *Weak coupling expansion of the low lying energy values in the* SU(2) *gauge theory on a torus*, *Nucl. Phys.* **B 232** (1984) 445 [INSPIRE].

[94] M. Lüscher, *Volume dependence of the energy spectrum in massive quantum field theories. 1. Stable particle states*, *Commun. Math. Phys.* **104** (1986) 177 [INSPIRE].

[95] M. Lüscher, *Volume dependence of the energy spectrum in massive quantum field theories. 2. Scattering states*, *Commun. Math. Phys.* **105** (1986) 153 [INSPIRE].

[96] R. Apreda, D.E. Crooks, N.J. Evans and M. Petrini, *Confinement, glueballs and strings from deformed AdS*, *JHEP* **05** (2004) 065 [hep-th/0308006] [INSPIRE].

[97] P. Breitenlohner and D.Z. Freedman, *Stability in gauged extended supergravity*, *Annals Phys.* **144** (1982) 249 [INSPIRE].

[98] D.B. Kaplan, J.-W. Lee, D.T. Son and M.A. Stephanov, *Conformality lost*, *Phys. Rev.* **D 80** (2009) 125005 [arXiv:0905.4752] [INSPIRE].

[99] L. Vecchi, *Multitrace deformations, Gamow states and stability of AdS/CFT*, *JHEP* **04** (2011) 056 [arXiv:1005.4921] [INSPIRE].

[100] L. Vecchi, *Technicolor at criticality*, *JHEP* **04** (2011) 127 [arXiv:1007.4573] [INSPIRE].

[101] M. Falcioni et al., *On the masses of the glueballs in pure* SU(2) *lattice gauge theory*, *Phys. Lett.* **B 110** (1982) 295 [INSPIRE].

[102] G. Münster, *Strong coupling expansions for the mass gap in lattice gauge theories*, *Nucl. Phys.* **B 190** (1981) 439 [*Erratum ibid.* **B 200** (1982) 536] [*Erratum ibid.* **B 205** (1982) 648] [INSPIRE].

JHEP06(2016)114

[103] C.-K. Wen and H.-X. Yang, *QCD_4 glueball masses from AdS_6 black hole description*, *Mod. Phys. Lett.* **A 20** (2005) 997 [hep-th/0404152] [INSPIRE].

[104] M. Bochicchio, *Glueball and meson propagators of any spin in large-N QCD*, *Nucl. Phys.* **B 875** (2013) 621 [arXiv:1305.0273] [INSPIRE].

[105] M. Bochicchio, *Glueball and meson spectrum in large-N massless QCD*, arXiv:1308.2925 [INSPIRE].

[106] R. Franceschini et al., *What is the $\gamma\gamma$ resonance at 750 GeV?*, *JHEP* **03** (2016) 144 [arXiv:1512.04933] [INSPIRE].

[107] C. Han, H.M. Lee, M. Park and V. Sanz, *The diphoton resonance as a gravity mediator of dark matter*, *Phys. Lett.* **B 755** (2016) 371 [arXiv:1512.06376] [INSPIRE].

[108] V. Sanz, *Theoretical interpretation of a spin-two diphoton excess*, arXiv:1603.05574 [INSPIRE].

JHEP06(2016)114