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A COMPARISON OF ORDINARY AND SIMPLE KRIGING ON A PGE RESOURCE IN THE EASTERN LIMB OF THE BUSHVELD COMPLEX

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A research report submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

Johannesburg, 2015

DECLARATION

I declare that this research report is my own, unaided work. Where use has been made of the work of others, it has been duly acknowledged. It is being submitted for the Degree of Master of Science in the University of the Witwatersrand, Johannesburg.

Signed:

_____

Mbalenhle Mpanza

This _____ day of _____ 2015

ABSTRACT

The selection of an appropriate estimation method is one of the fundamental decisions in resource estimation. The effects of selecting an inappropriate estimation method can lead to ± 50% error in the estimate (Dominy et al., 2002). In selective mining, for example it is the mining block estimates that determine which of the ore blocks are to be mined and processed and which of the ore blocks are waste. The choice of the estimation method amongst others is based on the geology and complexity of grade distribution within the deposit. For example polygonal estimation methods are suitable for producing a volume weighted global mean grade, and in this estimation method there is one fixed and biased answer. The inverse distance method is unbiased but does not minimise the estimation variance, while kriging is subject to certain conditions, such as providing the best estimate possible by a linear combination of the available weighted data as well as minimising the error variance of the estimate.

This dissertation presents a detailed study of the application of two linear geostatistical estimation techniques; Ordinary and Simple Kriging. Included in this study is a detailed discussion on variography and its necessity in resource estimation. The theory of kriging as well as the kriging equations is discussed in great detail. The differences between Ordinary and Simple Kriging estimation techniques are drawn from this study by the consideration of the kriging variance, kriging efficiency, kriged estimate, kriging neighbourhood as well as the block variance.

The suitability of the application of both Ordinary and Simple Kriging estimation techniques is highlighted by this study. The two techniques are applied on a PGE (4E) deposit from an undisclosed locality due to confidentiality. This dissertation highlights the differences that are not discussed in most literature between Ordinary and Simple Kriging and the way that these techniques influence the outcomes of mineral resource estimation.

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DEDICATION

This thesis is dedicated to my family

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LIST OF SYMBOLS

v Block support volume

V Larger block support volume

x Location in space

h Distance vector

Zv Random variable for support size v

$Zv(x_i)$ Grade for a block of size v at x_i

$v(x)$ Domain of volume v centered at x

$V(x)$ Domain of volume V centered at x

$E\{Z\}$ Expected value or mean of Z

m Mean value

σ^2 Variance

Var {Z} Variance of Z

CoV Coefficient of variation (σ/m)

$r(h)$ Semi-variogram function at lag h

$\bar{r}\ (V, v)$ Average variogram value of the domains V and v

$C(h)$ Covariance function at lag h

Chapter 1: Introduction

1.1 Introduction

Geostatistics is the application of random functions to the description and estimation of natural phenomena (Journel and Huijbregts, 1978). Isaaks and Srivastava (1989) state that geostatistical methods describe spatial continuity of natural phenomena. In its origins, geostatistics was started in the mining industry with the aim of improving the estimation of mineral resources. For example when considering a region or mineral deposit with a particular grade distribution; geostatistics estimates and describes the spatial relationship existent between all locations within that region. A geostatistical approach to mineral resource estimation relies on some form of kriging, in which the weights given to each sample are derived from using the semi-variogram model that expresses the continuity of grades in two or three dimensions. In geostatistics two categories of estimation methods exist, linear and non- linear methods. Linear methods provide an estimate which is a linear combination of data, while non- linear methods use non-linear functions to obtain conditional expectations (Vann and Guibal, 2001). This study discusses two of the linear methods namely Ordinary Kriging (OK) and Simple Kriging (SK). It focuses on the differences in the application of SK and OK, for mineral resource estimation.

A number of studies have been conducted with the aim of comparing SK and OK. These studies were undertaken by Goovaerts (1997), Isaaks and Srivastava (1989), Armstrong (1998), Journel and Huijbregts (1978) as well as Clark (2000) just to mention a few. Some of the work by these authors is discussed in detail and adopted in this study.

SK and OK techniques are generally based on classical statistics, which are affected by the distribution of the grade population underlying the data. Glacken and Snowden (2001) suggest that SK has a much stronger emphasis on the assumption of stationarity of the mean than OK, and that OK can be applied optimally for normal or Gaussian distributions. It is important to note that no single estimation technique is appropriate for all mineral resources (Isaaks and Srivastava, 1989). It is therefore imperative to fully understand the capabilities of each estimation technique before it is applied.

Goovaerts (1997) notes that the significant difference between SK and OK is in the constraints imposed during the variance minimisation. In OK there is a condition that the sum of the weights must be equal to one, which is not the case in SK.OK assumes that the mean is unknown whereas SK assumes that the mean is known and constant throughout the deposit. Armstrong (1998) suggests that OK accounts for the local fluctuations of the mean by limiting the area of stationarity of the mean to the local neighbourhood, which means that the mean may vary in the area and does not remain constant. She further notes that OK better estimates resources, where data sets have large areas with low values and large areas with high values. Local means appear more meaningful in a situation where the global mean is not constant.

1.2 Problem Statement

The growing number of technologically advanced geostatistical software packages, provides practitioners access to powerful algorithms. Two of the mineral resource estimation techniques developed by Krige in the early 1950s include Ordinary and Simple Kriging. The latter has been commonly used in the South African gold mining industry to estimate the local mean of the mineral resources. The *aim of this study is to examine and highlight the differences between Ordinary Kriging and Simple Kriging,* using a shallow dipping portion of the UG2 Reef in the Eastern Limb of the Bushveld Complex. Having examined the differences between the two techniques, the outcomes of this study are compared with the differences as well as similarities obtained by other eminent geostatisticians using the available literature.

1.3 Plan and Layout of this study

This study aims to discuss the theory of SK and OK as estimation techniques, and offers insight into work done in the past using both SK and OK. The study also offers a concise discussion on the theory of semi-variograms and how they affect the estimation process. The study also shows the selection of the most appropriate semi-variogram parameters to be used in the estimation process

Furthermore the study critically analyses whether other differences exist between SK and OK apart from those discussed by Goovaerts (1997). To investigate this, the study uses a PGE (4E) mining data set. This study also investigates the outcomes of applying SK and OK as well as compares the resultant differences between the two techniques. Before estimation, the statistical analysis of the PGE (4E) data is undertaken to investigate the data distribution and thereafter the differences between SK and OK are investigated by means of:

a) A nine point support sample exercise on a 20 m x 20 m block V, applying SK and OK in order to observe the differences between them. In the exercise, kriging variance and the kriged estimate are outputs used to analyse the differences.

b) Estimation of the PGE resource on a 250 m x 250 m x 10 m block size model in order to emphasize and examine the differences that exist between the two techniques.

c) Observation of the behaviour of weights and the nugget effect for both SK and OK techniques.

d) Observation of the mean squared error against the kriging neighbourhood for both SK and OK.

This study will use mining data from, a Platinum Group Element (PGE 4E deposit) which occurs in the UG2 Chromitite Layer of the Eastern Limb, in the Bushveld Complex. The Platinum Group Elements 4E comprises platinum (Pt), palladium (Pd), rhodium (Rh) and gold (Au). All these elements have different uses, with platinum and palladium having the most applications of all the PGEs. Platinum is used in motor vehicles as catalytic converters and it is also used in jewellery, while palladium

is used in electronics, hydrogen purification, chemical applications and ground water purification (Cramer et al., 2004). In the UG2 Reef the platinum and palladium occur in amounts of 46% Pt and 30% Pd. Since 1923 South Africa has been the largest producer of platinum in the World. Until the 1970s most of the platinum came from the Merensky Reef in South Africa (Cawthorn, 1999). Lonmin began mining the UG2 Reef in the 1980s because of its high grade followed by Anglo Platinum Ltd, which now reports that 40% of platinum produced comes from the UG2 Reef (Cawthorn, 1999).

1.4 Justification for this study

The estimation of mineral resources provides the primary inputs for any decision making and financial forecasting of a mining project. Cash flow calculations often fail to incorporate the uncertainty associated with resource and reserve estimates (Morley et al., 1999). Reliable estimates of mineral resource grades and tonnages, with appropriate measures of uncertainty, are essential to mining operations in order to prevent financial losses. This also pertains to feasibility studies on new mining projects where data are sparse and the geological information is often uncertain. Dominy (2002a) reviewed the performance of resources and reserves of small to medium Australian gold operations. He found that most problems were related to grade estimation. A common trend he found on most operations was that more tonnes were produced (up to 15%) and less grade (up to -55%). Establishing accurate estimates of mineral resources provides confidence in mining for the purpose of a mine design. Kriging provides estimates that can be used in mine planning when selecting which mining blocks to be mined and in making future decisions about resource allocations.

Carras (2001) suggests that assumptions governing algorithms of the geostatistical estimation techniques are rarely understood, stated or questioned. The lack of a detailed understanding of such assumptions can result in wrong decisions being made with no profitability in mining. However the full understanding of the geostatistical techniques i.e. SK and OK will enable practitioners to select the most appropriate technique to use, for a particular context. Attention to detail is vital and can lead to recognition of important features. Dominy et al. (2002) suggests that the effects of unsuitable estimation methods could lead to errors of ± 50% in the

estimate. This study will assist mineral resource practitioners to obtain a clear overview of SK and OK, and the context in which they can be applied.

1.5 Description of the Study Area

Data for this study was provided by Anglo Platinum and comes from an exploration project on the Eastern Limb of the Bushveld Complex which hosts the world's largest platinum resources. The study area is approximately 6000 ha, and occurs in the rural area of Steelpoort in the Limpopo Province (see Figure 1.1). Although the Merensky Reef and UG2 Chromitite Layer (UG2 Reef) occur in the area, this study will only consider the UG2 Reef, as the project plans to start mining the UG2 Reef first and at a later stage mine the Merensky Reef. The UG2 Reef is of high priority because of its high grade.

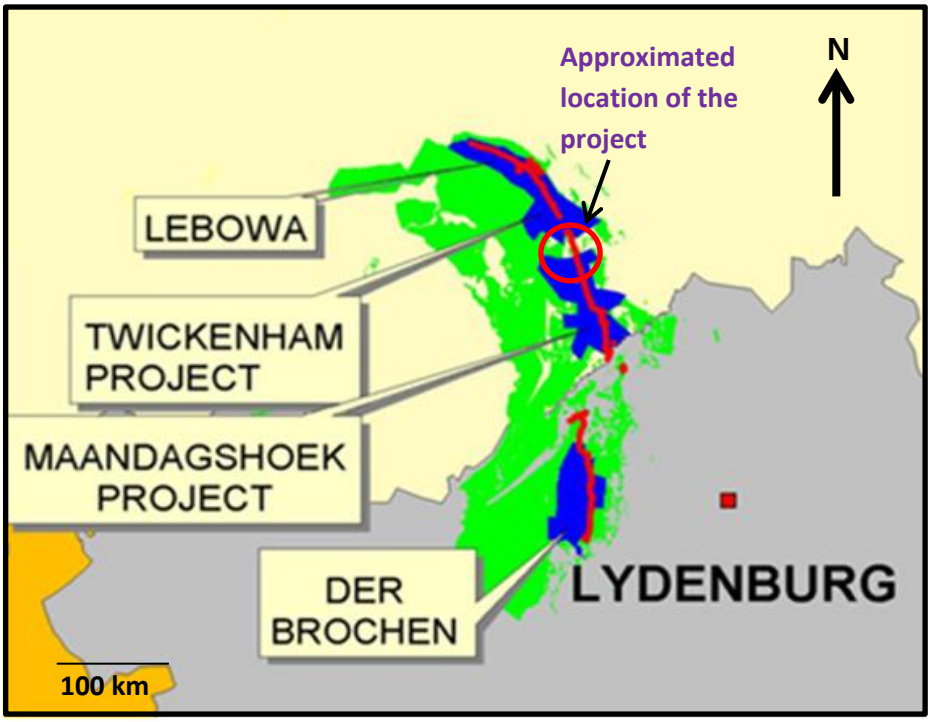


Figure 1.1: Map of the Eastern Limb of the Bushveld Complex and the approximate location of the project area as well as actual and potential mines (Anglo Platinum, 2011)

1.6 Research Overview

In this section the structure of the research report is described. The report is made up of seven chapters with supplementary material located at the end as an appendix. A description of the chapters is as follows:

Chapter 1 is the introduction of the study where the importance of the study is highlighted. A brief project background and the basic concepts of SK and OK are discussed. The issues that the research aims to address are highlighted in this chapter.

Chapter 2 is the presentation of selected information and fundamental concepts related to geostatistics and the mining industry. First the classical statistical theory is discussed in this chapter, followed by the theory of regionalised variables because it is the basis of geostatistics and it assists in fully understanding the geostatistical concepts. The concept of change of support is briefly discussed as one of the crucial concepts in this study as well as the theory of variography.
The theory of kriging and kriging equations are discussed. A comprehensive discussion on the theory of SK and OK is undertaken in this chapter. Included is an example of the application of SK and OK adopted from the Geostatistical Evaluation Assignment Exercise, by C.E Dohm (2011).

Chapter 3 discusses the effects of the nugget effect on SK and OK weights. This chapter also includes case studies by Goovaerts (1997) and Deutsch et al. (2014) on the application of SK and OK. The case studies include the description of the trend estimates and the use of the number of search data for both SK and OK.

Chapter 4 first discusses the geology of the study area and that is followed by the Exploratory Data Analysis (EDA) where a full statistical analysis of the PGE (4E) data is undertaken. Included in the statistical analysis is the investigation of data integrity by data validation. These analyses are carried out to check for spurious data, because errors in the data can significantly affect and influence the estimation process.
The construction of histograms as well as a probability plot to diagrammatically present the PGE (4E) deposit is undertaken in this chapter.

The PG2000 (Clark 2000), Excel 2010 and Surpac 6.2.1 are the software packages used in the statistical analysis.

Chapter 5 is the application of variography to the PGE (4E) data. The two important assumptions that govern the theory of variograms are introduced here.
A brief discussion on domaining and how it affects estimation of mineral resources is undertaken in this chapter. The investigation of distinct domains in the PGE deposit is also undertaken.
A brief section on contour maps is undertaken to investigate whether any trends exist in the PGE (4E) grades.
Variogram fans are constructed to further investigate preferred directions of maximum continuity of PGE (4E) grades.
Towards the end of the chapter the construction of semi-variogram models and the selection of the appropriate semi-variogram parameters for the estimation of the PGE deposit conclude the chapter. Supervisor 8 software from Snowden is used specifically for the purpose of semi-variogram modelling.

Chapter 6 considers the estimation of the PGE (4E) grades.SK and OK techniques are applied on the grade block model of 250 m x 250 m x 10 m block support size. Kriging estimators are used to investigate why SK and OK produce different results. To understand the difference between the two, a detailed analysis of the kriged estimate, kriging variance, block variance, and kriging efficiency is undertaken. The methods applied by Deutsch et al. (2014) and by Goovaerts (1997) are adopted to investigate further the differences between SK and OK using the PGE (4E) data. At the end of this chapter domaining is considered to investigate further OK and SK.

Chapter 7 is the concluding remarks and recommendations of this study.

Chapter 2: Literature Review

This chapter presents selected information and fundamental concepts of geostatistics related to the mining industry. The classical statistical theory is discussed as it is effectively applied in geostatistics. The theory of regionalised variables is the basis of geostatistics and assists in fully understanding geostatistical concepts such as the theory of variography and kriging which are discussed in depth in this chapter.

2.1 Introduction

Geostatistics is suitable to be used in the mining industry, because of the spatial nature of mining sample data. Mining companies sample the mineral deposits they mine, the sample locations and other measurements of interest i.e. grade values are recorded and this constitutes the mining sample data. The sample data is thus used to estimate the quantity and quality of the mineral deposit in unmined areas. It was the problems encountered in the mining industry that led to the pioneer work by H.S Sichel and D.G. Krige and developments by G. Matheron in statistics and geostatistics.

Geostatistics is of benefit to the mining operations as it provides estimates which assist in decision making and maintain profitability in a mine. Geostatistical techniques are advantageous because they provide a measure of accuracy of all its estimates. These techniques are also used to determine the optimal sampling pattern and can estimate contour maps of the mineral deposit.

2.2 Classical statistics theory

This study considers statistical theory applied in the mining industry. Statistics is the science of collecting and analysing numerical data in large quantities. Geostatistics requires extensive use of statistics for organising and interpreting data as well as drawing conclusions and making reasonable decisions. In mining, statistical theory includes the notion that a sample is a representative subset selected from the population. A good representative sample must capture the essential features of the population from which it is drawn. The population is made up of infinite collection of samples that form a mineral deposit. When a sample is considered representative, statistical inference can be undertaken, meaning that conclusions about the

population can be inferred. There is a certain level of uncertainty when inference is considered; therefore probabilities are used when stating conclusions.

In mineral resource estimation statistics is applied for:

a) Improved viewing, validation and understanding of data and the mineral deposit.
b) Ensuring data quality and condense information to make inferences as well as estimations.

Geostatistical studies require a set of sample values taken at various locations within a spatial area. Statistics allows the analysis of samples without considering the location at which that sample was measured. Statistics also assist in understanding the behaviour and properties of samples by using tools such as the histograms, probability plots, coefficient of variation (CoV) as well as the measures of spread and central tendency. These tools are used for analysing the PGE (4E) data used in this study. The histogram provides insight into the possible distribution of the sample population. Once the histogram is constructed the data distribution is defined whether it is normal or lognormally distributed. The probability plot defines the different sample populations that exist in a data set.

The normal distribution is used to model mineral deposits that display symmetric value distribution where the mean and median are the same. A lognormal distribution is commonly used to model mineral deposits that have skew value distributions. Probability plots also assist in checking distribution models, a straight line on a logarithmic scale suggests a lognormal distribution while on arithmetic scale suggests a normal distribution.

This study assumes that the reader has a background in statistics and geostatistics, thus classical statistical theory such as the CoV, measures of central tendency (mean, mode and median) and measures of spread such as (variance, standard deviation and range) have not been extensively discussed here. However if the reader is interested, references such as Lapin (1983), Davis (1986) and Ripley (1987) discuss in depth the classical statistical theory.

2.3 Theory of regionalised variables

Geostatistics is based on the theory of regionalised variables and provides a set of statistical tools for understanding spatial correlation of observations in data processing (Goovaerts, 1997). This section discusses this theory as well as the theory of variograms as they are essential tools required in the application of kriging.

The theory of regionalised variables states that natural phenomena are characterised by a distribution in space of one or more variables. Sample grade, for example is a regionalised variable because it is distributed throughout a space (Journel and Huijbregts, 1978). Sample grade distribution characterises the mineralisation of a mineral deposit which can be quantified and estimated. There are two aspects considered when defining regionalised variables, the first one is local randomness and the second one is the structural pattern. The random aspect considers the variations from one point to another. Structural aspects reflect large-scale tendencies of regionalised variables. The estimation of regionalised variables depends on both these characteristics. For example the error of estimation becomes greater when regionalised variables are irregular and not continuous in their spatial variations (Matheron, 1971).

Let $Z(x)$ be the random variable with its outcome $z(x)$, which is the observed value at each data point x. A random variable is a variable of which the values are randomly distributed in space. A set of random variables that have spatial locations and depend on each other are specified by a probabilistic mechanism called a random function i.e. $Z(x_1), Z(x_2) \dots Z(x_k)$ (Isaaks and Srivastava, 1989). Geostatistics is a method that allows one to estimate $z(x)$ at point x where no data is available.

When random variables are correlated their correlation depends on distance h, separating points i.e. x_i and $x_i + h$, direction and the nature of the variable considered (Journel and Huijbregts, 1978). The actual grade $z(x)$ at any point x , is a realisation of a random variable $Z(x_i)$, while a set of actual grades defining a deposit is a single realisation of the random function $\{Z(x_i), \forall x_i \in D\}$.

The mathematical tool that is used to characterise the spatial variability of a regionalised variable $z(x)$ is known as a variogram. Consider two values $z(x)$ and $z(x+h)$ at point x and $x+h$ separated by a distance h; the variability of the two values can be characterised by a variogram function. The variogram function is given by:

$\gamma(h) = 0.5\, N \sum_{i=1}^{n} [Z(x+h) - Z(x)]^2$

where N is the number of pairs $[z(x_i), z(x_i+h)]$ of data separated by the vector h.

Certain assumptions are considered when characterising the variability of random variables. Stationarity is assumed meaning that the mean of the random variable must be constant in any location. Matheron (1963) developed the "intrinsic hypothesis", which assumes that the mean and variance of increments $Z(x+h) - Z(x)$ exist and are independent of point x. In reality this assumption is true if the mineralisation within a mineral deposit is homogeneous. Once a variogram of the random function is computed, kriging can be undertaken.

2.3.1 The support of a regionalised variable

In most situations a regionalised variable is measured as the average over a certain volume or surface rather than a point (Armstrong, 1998). The basic volume at which a regionalised variable is measured is called its support. The change in support changes the structural characteristics of the regionalised variable under study. For example the grades measured on a 50 mm diameter core have a higher variance than those measured on larger diameter cores or blocks. It is therefore imperative to know the relationship between the variables i.e. the grade of blocks and cores. The dispersion and variograms of both variables should be considered.

Let $Z(x)$ be a random variable of a point support and let $Z_V(x)$ be a block support random function or a block support random variable, with blocks of volume V. A block support random function over the spatial region Ω is defined as a set of random variables:

$\{Z_V(x), x \in \Omega\}$

where the random variable $Z_V(x)$ represents the value of a block V centred at point x.

2.4 Variography

In order to perform SK and OK in any data set, variogram models should be constructed before the actual kriging process. Kriging outcomes can be significantly affected by variability and spatial structure of the data as well as the choice of the variogram model. A variogram according to Clark (2001) is a graph describing the expected difference in value between pairs of samples a distance apart with a relative orientation. Journel and Huijbregts (1978) define a variogram as a function that characterises the variability of samples, and which is an expectation of the random field $[Z(x) - Z(x+h)]^2$.

Variograms characterise spatial continuity, by comparing samples in terms of distance and orientation as well as describing the way in which samples relate to one another in space. This information is used to create an expectation about grades in a deposit based on weighting the surrounding samples according to the variogram. The variogram indicates the difference in sample values as the distance increases in a fixed direction. Half of the variogram $\gamma(h)$ is referred to as the semi-variogram.

Semi-variograms summarise all the information pertaining to the spatial distribution of a variable considered. The variogram $2\gamma(h)$ represents a vector $|h|$ which by definition starts at zero because it is impossible to take two samples closer than no distance apart (Clark, 2001) therefore $\gamma(0) = 0$. In general, but not always, the variability between two samples at different positions increases as h between them increases (Journel and Huijbregts, 1978). The manner in which the variogram increases over a distance (h) characterises the spatial continuity of the variable.

It is unlikely that the variability in mineralisation will be the same in every direction. For example in some deposits a variogram in the North-South direction may display stronger variability than in the East-West direction, which could suggest that there is maximum continuity in the East-West direction as opposed to the North-South direction. Variograms that display variability in different directions are known as anisotropic variograms, whereas the variograms that display the same variability in different directions are called isotropic.

2.4.1 Characteristics of variograms

Variograms are characterised by a random component, a structured component and a variance component. The random component is called the nugget effect, and the structured component is located between the nugget effect and the sill (the variance component) (see Figure 2.1).

The nugget effect is a vertical jump (on the y axis) from the origin to a variance at very small separation distances where $\gamma(h) = 0$. For example the nugget effect is observed when two halves of drilled core are analysed and different results in grade are obtained. This shows that no matter how close samples are, there will be differences in values between them (Clark, 2000). The nugget effect is a result of the error in the measurements and microvariability in mineralisation (Journel and Huijbregts, 1978). At zero separation distance which is the origin by definition, where $\gamma(0) = 0$, sample values have no variability.

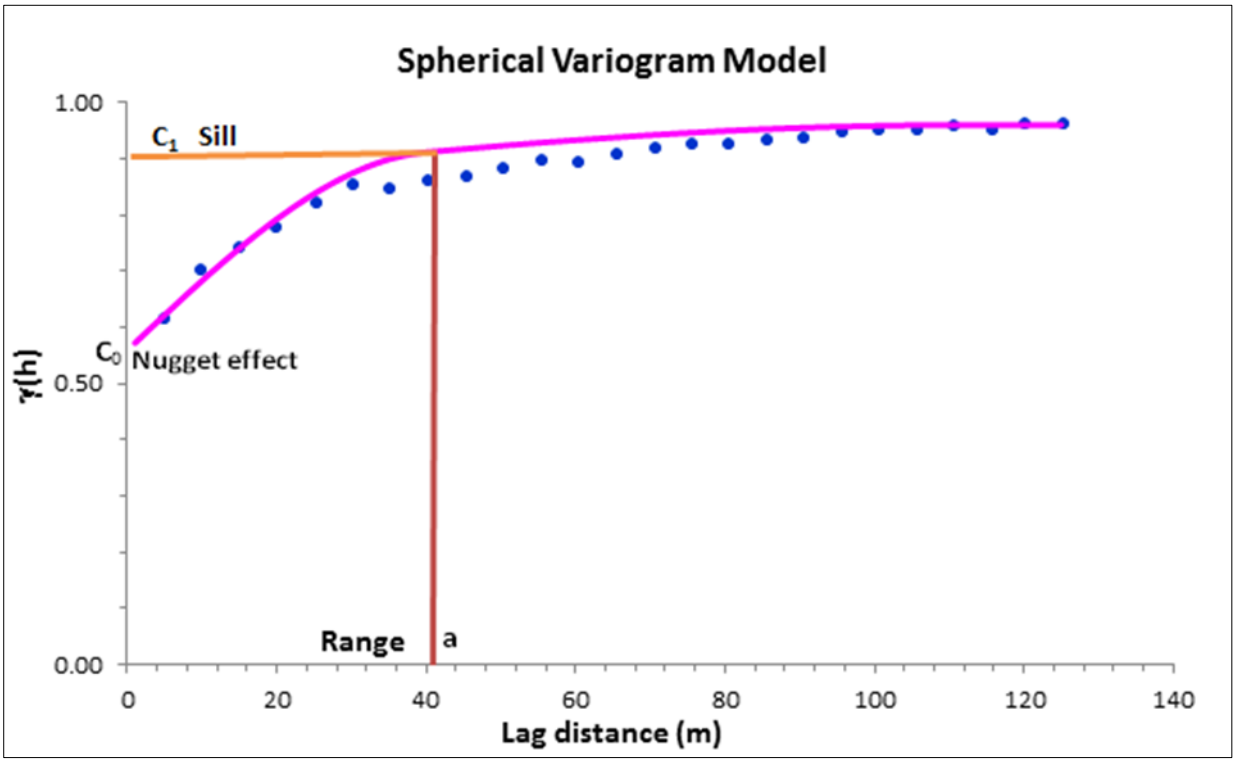


Figure 2.1: A generic variogram model showing the sill, nugget effect and a range, for the commonly used spherical model (Geostatistical Class Exercise C.E Dohm, 2011)

The behaviour of semi-variograms near the origin reveals the continuity and spatial uniformity of a random function $Z(x)$ (Armstrong, 1998). Journel and Huijbregts (1978) researched the common behaviours of semi- variograms near the origin (see Figure 2.2). They found that quadratic behaviours exist near the origin, which indicates highly continuous spatial data (see Figure 2.2 a)). Linear behaviours near the origin occur when the regionalised variable is continuous but not differentiable (see Figure 2.2 b)). Discontinuity at the origin occurs when $\gamma(h)$ does not tend towards zero when h tends towards zero, the regionalised variable is not continuous in this case. The discontinuity at the origin is called the nugget effect (see Figure 2.2 c) and d)) and most deposits have discontinuity at the origin.

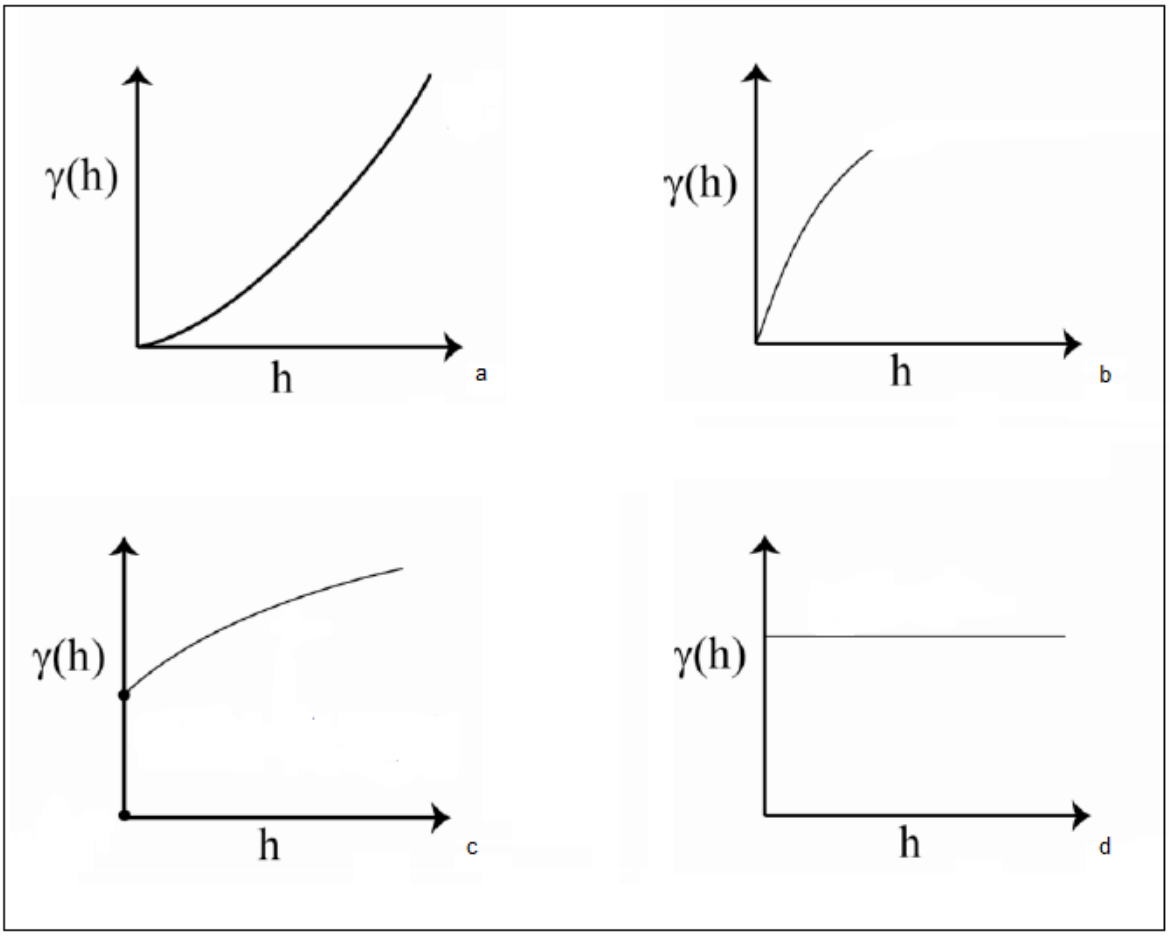


Figure 2.2: The behaviour of variograms near the origin. Quadratic shape a), linear b), nugget effect c) and pure nugget effect d) (Armstrong, 1998)

A typical variogram reaches a limit which is known as the sill (C_1) at a distance called the range (a), (see Figure 2.1). Once a variogram reaches the sill the samples $z(x)$ and $z(x+h)$ no longer depend on the vector h between them and are no longer correlated. The sill represents the variance of the random field where:

$$\gamma(\infty) = Var\{Z(x)\} = C_1.$$

The range (a) corresponds to the "zone of influence", which refers to the influence of one sample value on another sample value. When sample value $z(x)$ is correlated with any other sample value its influence on the other sample will decrease as the distance between the two samples increase (Journel and Huijbregts, 1978).

2.4.2 Variogram models

There are different types of variogram models such as the Spherical, Linear, Exponential, Gaussian and Power model (see Figure 2.3.). This study only discusses the Spherical variogram model as it is used to characterise spatial continuity of the PGE (4E) deposit in Chapter 5. The Spherical variogram model is one of the more commonly used models (see Figure 2.1). Its shape appropriately matches natural observations; first a linear growth up to a distance then stabilisation. Spherical variogram models reach a sill at a certain distance (the range), it therefore models minimal correlation at large distances beyond the range.

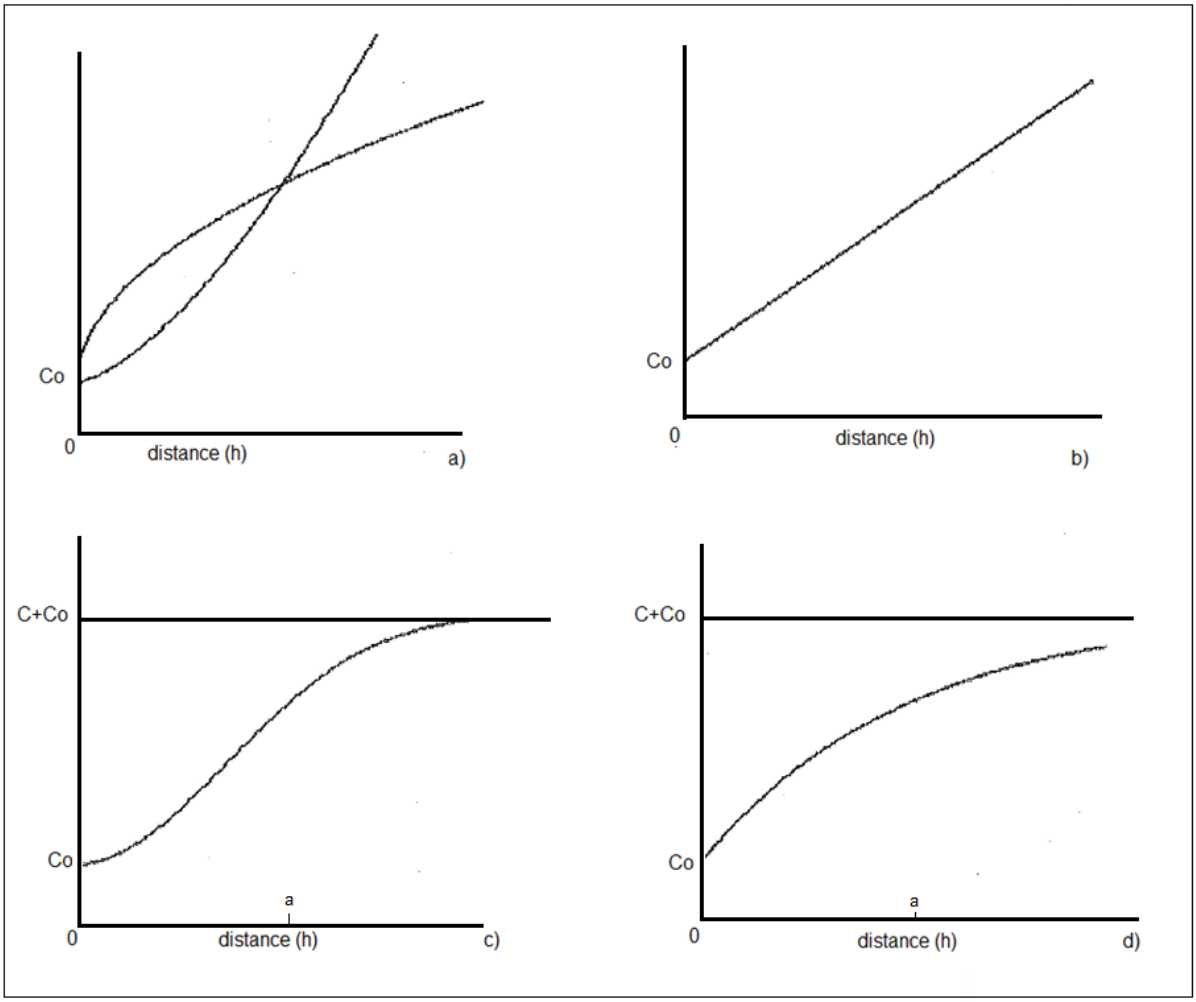


Figure 2.3: Variogram models, Power model a), Linear model b), Gaussian model c) and Exponential model d) (Clark, 2000).

2.4.3 Analysing spatial continuity

The nature and distribution of the mineral deposit determines which variogram type will be used to characterise its spatial continuity. Omnidirectional (isotropic) variograms are used for analysing data with the same degree of continuity in all directions such as some coal deposits. On the contrary when a deposit does not display the same degree of continuity in all directions, its spatial continuity can be characterised by anisotropic variograms. It is standard practise to investigate different directions when calculating variograms in order to identify the possible existence of anisotropy.

2.4.4 Construction of the Experimental Variogram

The construction of variograms requires consideration of the azimuth, angle of tolerance; lag distance and band width (see Figure 2.4). Deposits are unique, therefore appropriate directions and angles for semi-variograms need to be investigated for each deposit. The azimuth, angle of tolerance, lag distance and band width are search parameters used to find the reasonable number of pairs to calculate semi-variograms. The lag distance defines the distances at which the experimental variogram pairs are calculated. The angle of tolerance assists in establishing distance bins for lag increments in order to accommodate unevenly spaced observations (Leuangthong et al., 2008).

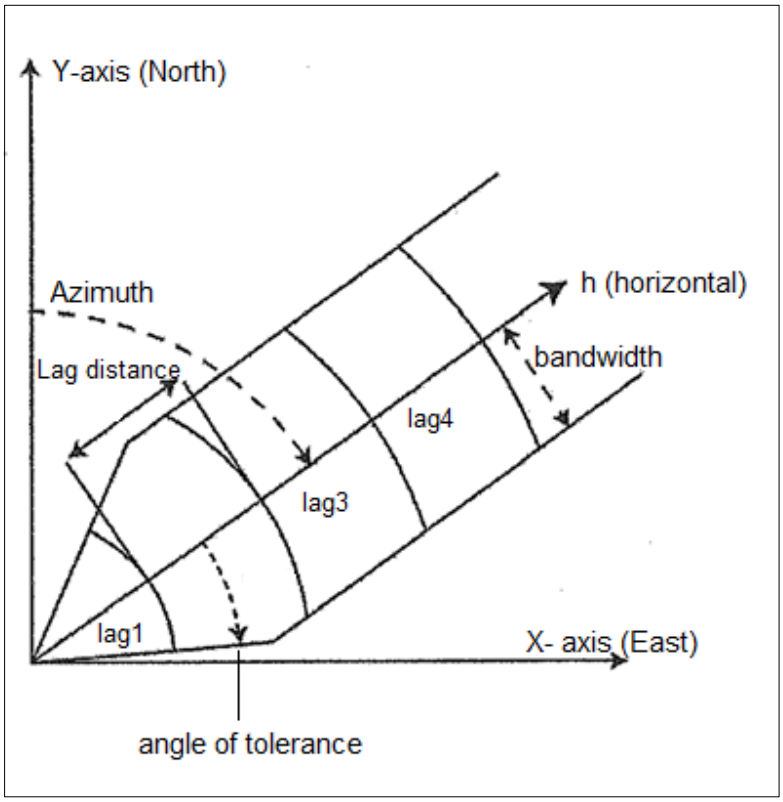


Figure 2.4: Schematic explanations of tolerance parameters (Leuangthong et al., 2008).

The tolerance parameters are significant when calculating semi-variograms, for instance if they are too small the variogram becomes too noisy (Leuangthong et al., 2008). This occurs due to a lack of information, and having too few data pairs in a lag (see Figure 2.5 (a)). If the tolerance parameters are too large, the data pairs might look similar in all directions because the information will have been averaged out (see Figure 2.5 (b)).

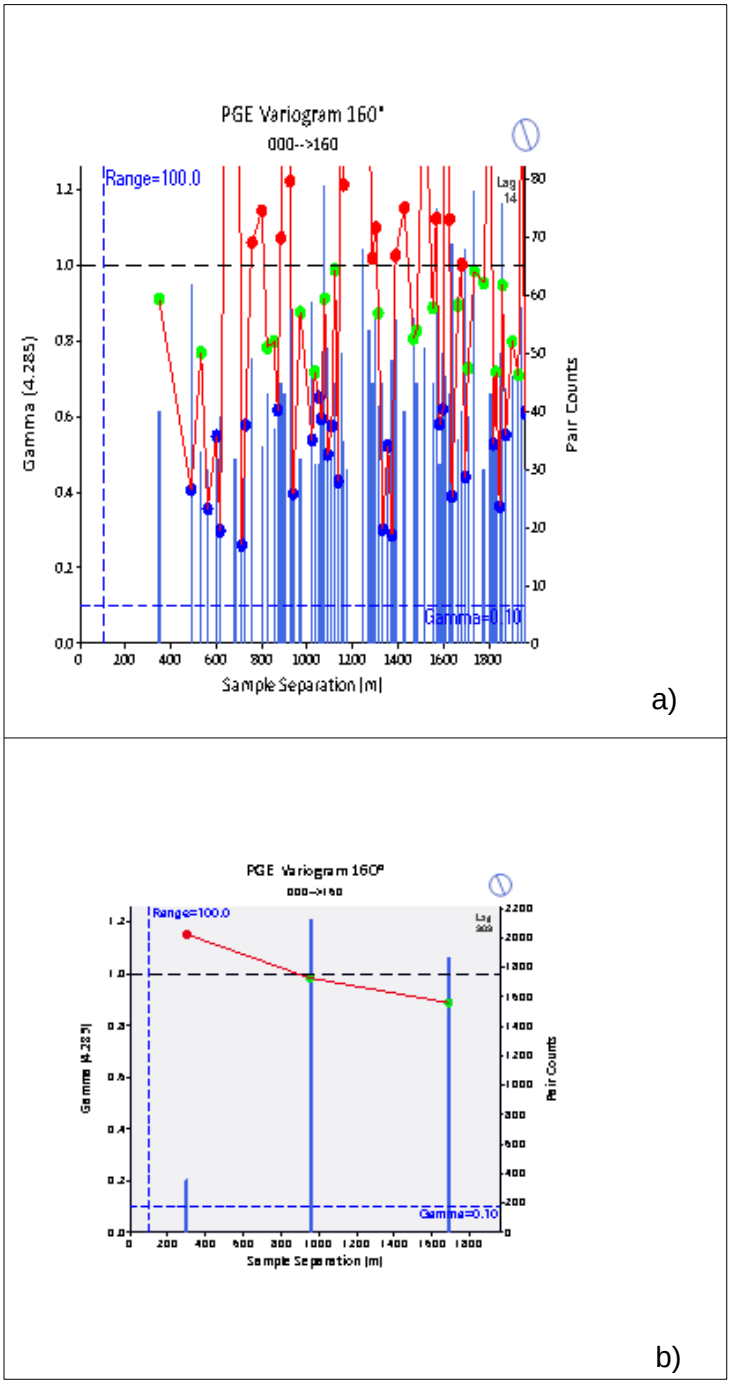


Figure 2.5: (a) Example of a noisy variogram with a small lag of 14 m and (b) a variogram with a large lag parameter of 903 m (Supervisor 8, Snowden)

2.5 Theory of Kriging

This section mainly discusses the theory of SK and OK and includes examples of the application of the two techniques. Furthermore this chapter emphasises the differences between SK and OK.

2.5.1 Kriging

Kriging is a method of obtaining the best (or minimum variance) linear unbiased estimates (B.L.U.E) of point values or of block averages (Armstrong, 1998). Kriging is an interpolation technique that considers both the distance and the degree of variation between known data points when estimating values in unknown areas.

In its original formulation a kriged estimate at a locality is simply a linear sum or weighted average of the data in its neighbourhood. The weights are allocated to the sample data within the neighbourhood of the point or block support to be estimated in such a way to minimise the estimation variance, and the estimates are unbiased.

2.5.2 The theory of Kriging

Kriging estimates are the linear function of the random variable $Z\ (x)$, at one or more unsampled points or over large blocks, where there is N data values available i.e. $Z(x_1)$ … … … … $Z(x_N)$. The data may be distributed in one, two or three dimensions, though applications in geology are usually in two or three-dimensions.

Kriging is easy to apply; it is designed to give the minimum variance linear estimate (Armstrong, 1998). According to Armstrong (1998) the accuracy of the estimate depends on the following:

a) The number of samples and quality of the data at each point
b) The position of samples within a deposit
c) The distance between samples and the point or block to be estimated
d) The spatial continuity of the variable under consideration. It is easier to estimate the value of a fairly regular variable than an irregular one.

Kriging has an advantage in that it is more reliable than other interpolation methods such as the inverse distance estimator and polygonal method. Kriging involves a selection of weights which depends on how the variable of interest varies in space

(Samui and Sitharam, 2011). The weights are based on the variogram model unlike the polygonal method where the same weights are used regardless of the variability.

2.5.3 Equations of Kriging

If $Z(x)$ is the random function and is stationary at a point support level, with the expectation $E\{Z(x)\} = m$, then $Z_V(x)$ is a random function at a block support level (see section 2.3.1). In a similar way to the point support under the hypothesis of stationarity, the expectation of $Z_V(x)$ is: $E\{Z_V\} = m$ for block support.

A kriged estimate is a weighted linear combination of the surrounding data values given by equation 1.

$$\left(\mathrm{Z}_{\mathrm{V}}^{*}\right) = \sum \lambda_{\mathrm{i}} \,.\, \mathrm{Z}\,(\mathrm{x}_{\mathrm{i}}) \quad \ldots\ldots\ldots\

study SK and OK are explored because in actual fact all the different kriging types use the same principle of minimising the error variance.

2.5.4 Simple Kriging

The assumption that governs SK is the theory of stationarity. The theory states that the mean and variance remain constant and are known in all locations (Goovaerts, 1997). SK is an estimation method where the condition that $\sum\lambda_i = 1$ does not apply.

Consider a random variable $Z(x_i)$ where Z is at some location x within a domain A

$Z(x_i)\ x \in A$

The assumption of stationarity in SK allows random functions to be defined as residuals by $Y(x) = Z(x) - m$ with a zero mean.

The estimation of the random variable is thus given by:

$$Y_V^* = \sum\lambda_i Y(x_i) \quad \text{.....(4)}$$

where: Y_V^* is the weighted linear estimate at a point being estimated, λ_i are the weights at sample locations and $Y(x_i)$ is the regionalised variable.

SK must be unbiased and must have a minimum variance. The estimation error must have an expected value of zero to avoid bias:

$$E\,[Y_V^* - Y_V] = E\,[\sum\lambda_i . Y(x_i) - Y_V] = 0 \quad \text{.....(5)}$$

The mean of the estimation error is zero therefore the estimator is unbiased, and there is no constraint stated on the sum of weights. The variance of the estimation error is given by:

$$Var\,[Y_V^* - Y_V] = E\,[\sum\lambda_i . Y(x_i) - Y_V]^2 \quad \text{.....(6)}$$

When the estimation variance is minimised it becomes the kriging variance which can be written in terms of the semi-variogram:

$$\sigma^2 = \sum\sum\lambda_i . \lambda_j . \gamma\,(x_i, x_j) + \bar{\gamma}\,(V, V) - 2\sum\lambda_i . \bar{\gamma}(x_i, V) \quad \text{.....(7)}$$

There is no need for a Lagrange multiplier since there is no constraint that the sum of weights must be equal to one. After partially differentiating equation 7, the SK system therefore becomes:

$$\sum \lambda_i . \bar{\gamma}\,(x_i, x_j) = \bar{\gamma}(x_i, V) \quad \text{.....(8)}$$

This equation indicates that kriging weights are based on the spacing of samples relative to one another and to the point being estimated. The weights do not depend in anyway on the grade of samples at points used in the estimation.

The SK variance is given by:

$$\sigma_{sk}^2 = \sum \lambda_i . \bar{\gamma}(x_i, V) - \bar{\gamma}(V, V) \quad \text{.....(9)}$$

The SK estimator can also be written in terms of the weight of the mean by replacing $Y(x)$ with the $Z(x) - m$ expression:

$$Z_{sk}^*\,(x_i) = Y_V^* + m = \sum \lambda_i [Z\,(x_i) - m] + m \quad \text{.....(10)}$$

$$= \sum \lambda_i\; Z\,(x_i) + m[1 - \sum \lambda_i]$$

$$= \sum \lambda_i\, Z\,(x_i) + \lambda_m\, m$$

where the weight λ_m is the weight of the mean in SK.

This weight is equal to $\lambda_m = 1 - \sum \lambda_i$ .....(11)

The system of equations in SK can also be expressed and summarised by a matrix as indicated in equation 12:

$$K_{sk} . \lambda_{sk} = M_{sk}$$

$$\begin{bmatrix} \gamma(x_1, x_1)\,\gamma(x_1, x_2) & \cdots & \gamma(x_1, x_n)1 \\ \gamma(x_2, x_1)\gamma(x_2, x_2) & \ddots & \gamma(x_2, x_n)1 \\ \gamma(x_n, x_1)\gamma(x_n, x_2) & \cdots & \gamma(x_n, x_n)1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_n \end{bmatrix} = \begin{bmatrix} \bar{\gamma}(x_1, V) \\ \bar{\gamma}(x_2, V) \\ \bar{\gamma}(x_n, V) \end{bmatrix} \quad \text{.....(12)}$$

To illustrate the practical application of SK, an example adopted from the Geostatistical Evaluation Assignment Exercise, by C.E Dohm (2011) is used. The mean of the samples is assumed to be known, and is equal to 16.67.

Suppose a 20 m x 20 m block V to be estimated by SK using a 9 point support sample, located on a regular 30 m grid (see Figure 2.6).

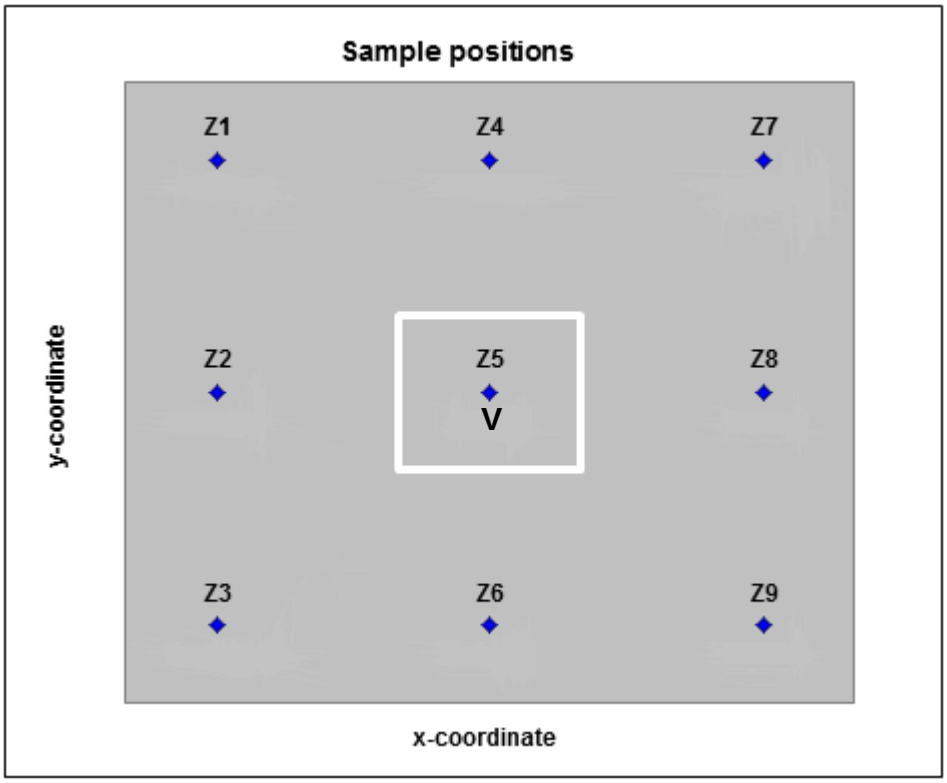


Figure 2.6: Data location of the 9 samples and the 20 m x 20 m block to be estimated (Geostatistics Assignment C.E Dohm, 2011)

The variogram of this block is a one-structure isotropic spherical semi-variogram with a sill of 1 and a range of 120 m (see Appendix A). The gamma values are $\overline{\gamma}(V,V) = 0.1063$, $\bar{\gamma}(Zi,Zj) = 0.503$, $\bar{\gamma}(z,V) = 0.402$

The Z values: $Z_1 = 19,\ Z_2 = 25,\ Z_3 = 17,\ Z_4 = 13,\ Z_5 = 21,\ Z_6 = 8,\ Z_7 = 12,\ Z_8 = 15,\ Z_9 = 20$

The SK system of equations shown in equation 12 when applied to the layout of points in Figure 2.6 produces a matrix form indicated below in Table 2.1. The reader is referred to (Appendix A) for the full calculation of this matrix.

Table 2.1: SK matrix of the 9 point support sample

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	λ			RHS
Z_1	0.000	0.367	0.688	0.367	0.508	0.751	0.688	0.751	0.884		w_1	=	0.5105
Z_2	0.367	0.000	0.367	0.508	0.367	0.508	0.751	0.688	0.751		w_2	=	0.3715
Z_3	0.688	0.367	0.000	0.751	0.508	0.367	0.884	0.751	0.688		w_3	=	0.5105
Z_4	0.367	0.508	0.751	0.000	0.367	0.688	0.367	0.508	0.751		w_4	=	0.3715
Z_5	0.508	0.367	0.508	0.367	0.000	0.367	0.508	0.367	0.508		w_5	=	0.0883
Z_6	0.751	0.508	0.367	0.688	0.367	0.000	0.751	0.508	0.367		w_6	=	0.3715
Z_7	0.688	0.751	0.884	0.367	0.508	0.751	0.000	0.367	0.688		w_7	=	0.5105
Z_8	0.751	0.688	0.751	0.508	0.367	0.508	0.367	0.000	0.367		w_8	=	0.3715
Z_9	0.884	0.751	0.688	0.751	0.508	0.367	0.688	0.367	0.000		w_9	=	0.5105
weights													

Solving for the weights gave:

$w_1 = w_3 = w_7 = w_9 = -0.005,\ w_2 = w_4 = w_6 = w_8 = 0.068, w_5 = 0.731$

The sum of weights was therefore: $\sum w_i = 4(-0.005) + 4(0.068) + (0.731)$

$$= 0.98$$

Therefore the SK estimate is:

$$Z_{sk}^{*}v = \sum w * z_i + (1 - \sum w_i) * m$$

$$Z_{sk}^{*}v = -0.005\,(19 + 17 + 12 + 20) + 0.731(21) + 0.068(25 + 13 + 8 + 15) + (1 - 0.98) * 16.67$$

$$= 19.49$$

The variance given by: $\sigma_{sk}^2 = \sum w_i . \bar{\gamma}(x_i, V) - \bar{\gamma}(V, V)$

$$= (0.1700) - (0.1063)$$

.

$$= 0.064$$

2.5.5 Ordinary Kriging

OK is a linear geostatistical method which provides local estimation by interpolation. D Krige and G. Matheron introduced this linear estimation technique with the aim to reduce the volume variance effect. They decided on a linear technique because it is believed to provide the least amount of difference between the actual and estimated mine grades. OK assumes that regionalised variables are stationary where the mean (m) is unknown (Armstrong, 1998).

In OK, all the points with no sample values are assigned a value using a weighted linear combination of known neighbouring sample values.

The estimated value can be presented by the following formula:

$$Z^*_{Vok} = \sum \lambda_i Z(x_i) \quad \text{.....(13)}$$

To ensure that there is no bias, the OK error $E\,[Z_V^* - Z_V] = 0$ and is estimated in terms of weights by substituting the estimate Z_V^* with the $\sum \lambda_i . V_i$, therefore the error can be expressed as

$$r_i = \sum \lambda_i . V_i - V_i \quad \text{.....(14)}$$

with $Z(x_i)$ being represented by V_i

The error made when estimating unknown values is an outcome of a random variable (Isaaks and Srivastava, 1989). The expected value of the error at any particular location is zero and that is verified by substituting the equation of the expected value on the estimation error equation. The expected value equation is

$$E(r) = E\,\{\sum \lambda_i . V_i - V_i\} \quad \text{.....(15)}$$

This can be expressed as:

$$Er = \sum \lambda_i . EV_i - EV_i \quad \text{.....(16)}$$

Isaaks and Srivastava (1989) state that the expected error is referred to as the bias. The expected value equation is set to zero and the resulting equation satisfies the condition of un-biasedness and is given by:

$$E(r) = 0 = \sum \lambda_i . EV_i - EV_i \quad \text{.....(17)}$$

$E\sum\lambda_i.V_i = EV_i$……..……..(18)

meaning that:

$\sum\lambda_i = 1$……..……..…….... (19)

and thus $E\{Z_V^*\} = m\sum\lambda_i = m = E\{Z_V\}$……..……..……..……..…(20)

OK also ensures minimum estimation variance $E\{[Z_V^* - Z_V]^2\}$ which can be expressed by first obtaining the variance of the error: Isaaks and Srivastava (1989) suggest that this error is a random variable which can be expressed as:

$Var\{\sum\lambda_i.V\} = \sum\sum\lambda_i\,\lambda_j.\gamma\{V_i, V_j\}$.....(21)

Using $[Z_V^* - Z_V]$ and equation 21, the variance of the error can be expressed as:

$$Var\{E(r)\} = \gamma\{V^*(x_0)V^*(x_0)\} - \gamma\{V^*(x_0)V(x_0)\} - \gamma\{V(x_0)V^*(x_0)\} + \gamma\{V(x_0)V(x_0)\}$$

$$= \gamma\{V^*(x_0)V^*(x_0)\} - 2\gamma\{V^*(x_0)V(x_0)\} + \gamma\{V(x_0)V(x_0)$$.....(22)

The first term $\gamma\{V^*(x_0)V^*(x_0)\}$ is the variogram of $V^*(x_0)$ with itself, which is equal to the variance of $V^*(x_0)$:

$Var\{V^*(x_0)V^*(x_0)\} = Var\{\sum\lambda_i.V\} = \sum\sum\lambda_i\,\lambda_j\bar{\gamma}_{ij}$.....(23)

The third term in equation 22,$\gamma\{V(x_0)V(x_0)\}$, is the variogram of random variable $V(x_0)$ with itself and is equal to the variance of $V(x_0)$. If the assumption that random variables have the same variance σ^2, then the third term can be expressed as:

$\gamma\{V(x_0)V(x_0)\} = \sigma^2$.....(24)

The second term in equation 22, can be expressed as:

$$2\gamma\{V^*(x_0)V^*(x_0)\} = 2\{(\sum\lambda_i.V)V_0\} = 2E\{\sum\lambda_i\,V.V_0\} - 2E\{\sum\lambda_i\,V\}.E\{V_0\}$$

$$= 2\sum\lambda_i\,\gamma\{V, V_0\}$$.....(25)

Combining the three terms we have the following expression:

$Var\{E(r)\} = \sigma^2 + \sum\sum\lambda_i\,\lambda_j\bar{\gamma}_{ij} - 2\sum\lambda_i\,\gamma_{ij}$.....(26)

Equation 26 can also be expressed as:

$$\sigma_{\varepsilon}^{2} = 2\sum \lambda_i \overline{\gamma}\ (V, x_i) - \sum\sum \lambda_i\, \lambda_j\, \gamma(x_i\, x_j) - \bar{\gamma}(V, V) \quad \text{.....(27)}$$

2.5.5.1 Introducing the Lagrange multiplier

The Lagrange multiplier uses the equation for the error variance which is constrained by the requirement that the weights must add up to one namely, $\sum \lambda_i = 1$ to minimise the estimation variance. Lagrange requires that the constraint be set equal to zero and multiplied by the Lagrange multiplier, μ to give:

$$\mu(\sum \lambda_i - 1) = 0$$

This constraint is added to equation 27, but it does not change its value, giving:

$$\sigma_{\varepsilon}^{2} = 2\sum \lambda_i \overline{\gamma}\ (V, x_i) - \sum\sum \lambda_i\, \lambda_j\, \gamma(x_i\, x_j) - \bar{\gamma}(V, V) + \mu(\sum \lambda_i - 1) \quad \text{.....(28)}$$

This equation can be expanded as follows

$$\sigma_{\varepsilon}^{2} =$$
$$2\,\lambda_1\gamma(V, x_1) + 2\lambda_2\gamma(V, x_2) + 2\lambda_3\gamma(V, x_3) -$$
$$\begin{Bmatrix} \gamma\,(x_1, x_1)\lambda_1\lambda_1 + \gamma(x_1, x_2)\lambda_1\lambda_2 + \gamma(x_1, x_3)\lambda_1\lambda_3 \\ \gamma(x_2, x_1)\lambda_2\lambda_1 + \gamma(x_2, x_2)\lambda_2\lambda_2 + \gamma(x_2, x_3)\lambda_2\lambda_3 \\ \gamma(x_3, x_1)\lambda_3\lambda_1 + \gamma(x_3, x_2)\lambda_3\lambda_2 + \gamma(x_3, x_3)\lambda_3\lambda_3 \end{Bmatrix} \text{-}[\gamma(V, V) + \mu(\lambda_1, \lambda_2, \lambda_3 - 1)]$$

In order to minimise the error variance Equation 28 is partially differentiated with respect to the weights (λ_i) and the Lagrange multiplier (μ). These 4 equations with 4 unknowns are set to zero and solved:

$$\frac{\partial \sigma_{\varepsilon}^{2}}{\partial \lambda_1} = \gamma(V, x_1) - (\lambda_1\gamma(x_1, x_1) + \lambda_2\gamma(x_1, x_2) + \lambda_3\gamma(x_1, x_3) + \mu = 0$$

$$\frac{\partial \sigma_{\varepsilon}^{2}}{\partial \lambda_2} = \gamma(V, x_2) - (\lambda_1\gamma(x_2, x_1) + \lambda_2\gamma(x_2, x_2) + \lambda_3\gamma(x_2, x_3) + \mu = 0$$

$$\frac{\partial \sigma_{\varepsilon}^{2}}{\partial \lambda_3} = \gamma(V, x_3) - (\lambda_1\gamma(x_3, x_1) + \lambda_2\gamma(x_3, x_2) + \lambda_3\gamma(x_3, x_3) + \mu = 0$$

$$\frac{\partial \sigma_{\varepsilon}^{2}}{\partial \mu} = \lambda_1 + \lambda_2 + \lambda_3 - 1 = 0$$

Differentiating with respect to the Lagrange multiplier gives:

$\lambda_1 + \lambda_2 + \lambda_3 = 1$

The kriging system is then:

$$\sum \lambda_i \gamma(x_i x_j) + \mu = \bar{\gamma}(x_i, V)$$

The kriging variance in OK is given by:

$\sigma_{ok}^2 = \sum \lambda_i \bar{\gamma}(x_i, V) - \bar{\gamma}(V, V) + \mu$ ……..……..…. (29)

where lambda μ the Lagrangian multiplier measures the bias. The Lagrange multiplier is the balancing factor that ensures the optimisation of weights calculated for the OK system of equations. Equation 29 states that kriging variance equals the sum of variogram for point to block distance multiplied by kriging weights ($\sum \lambda_i \bar{\gamma}(x_i, V)$ minus average variogram between each and every discretisation point in a block ($\bar{\gamma}(V, V)$ plus the Lagrange multiplier μ.

The Lagrange multiplier is a reflection of the balances between the samples and the point being estimated and the relationship between the samples themselves.

The relationships between the samples and the point to be estimated are $\sum \lambda_i \gamma(V, x_i)$ and the relationships amongst the samples themselves are $\sum\sum \lambda_i \lambda_j \gamma(x_i x_j)$

The Lagrangian multiplier is therefore:

$\mu = 2\sum \lambda_i \bar{\gamma}(V, x_i) - \sum\sum \lambda_i \lambda_j \gamma(x_i x_j)$……(30)

The first term of equation 30 increases the error variance and the second term decreases the error variance, but at half the rate of the first term. There is a balance between these two functions where the Lagrange multiplier is zero such that:

$2\sum \lambda_i \bar{\gamma}(V, x_i) = \sum\sum \lambda_i \lambda_j \gamma(x_i x_j)$……..……(31)

The kriging variance measures the quality of the estimation. It depends on the structural models i.e. semi-variogram $\gamma(h)$ as well as the exact data configuration. However the kriging variance does not depend on the actual values of the samples

used in the estimation. Equation 29 indicates that, the kriging variance takes into account the geometry of the domain V to be estimated, expressed in the term $\bar{\gamma}(V,V)$. It also takes into account the distance between V and x expressed by $\bar{\gamma}(x_i,V)$.

The system of equations in OK can be expressed and summarised by a matrix as indicated in equation 32:

$$K_{ok}.\lambda_{ok} = M_{ok}$$

$$\begin{bmatrix} \gamma(x_1,x_1) & \gamma(x_1,x_2) & \cdots & \gamma(x_1,x_n) & 1 \\ \gamma(x_2,x_1) & \gamma(x_2,x_2) & \ddots & \gamma(x_2,x_n) & 1 \\ \gamma(x_n,x_1) & \gamma(x_n,x_2) & \cdots & \gamma(x_n,x_n) & 1 \\ 1 & 1 \ldots\ldots\ldots\ldots & & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_n \\ \mu \end{bmatrix} = \begin{bmatrix} \bar{\gamma}(x_1,V) \\ \bar{\gamma}(x_2,V) \\ \bar{\gamma}(x_n,V) \\ 1 \end{bmatrix} \quad \text{.....(32)}$$

In Equation 32, the first matrix represents variogram values between each sample and all other samples. The symbol (γ) gamma represents the corresponding variogram between the points. The weights are represented by symbol (λ) are calculated and (μ) is the Langrage multiplier; the weights are multiplied with sample grades to produce an Ordinary kriged estimate. Armstrong (1998) suggests that the matrix K_{ok} will always be non-singular, provided that the point variogram model $\gamma\,(h)$ is valid and none of the available data points are situated at the exact same location. Non- singular matrix means that there is an existing inverse of that particular matrix. This will ensure existence and uniqueness of the solution to the OK system of equations and will also ensure that the OK variance is always positive (Journel & Huijbregts, 1978).

In the same way that SK was applied to the layout of points in Figure 2.6 OK is now applied to the same layout.

The variogram of this deposit is a one-structure isotropic spherical semi-variogram with a sill of 1 and a range of 120 m. The gamma values are $\bar{\gamma}(V,V) = 0.1063$, $\bar{\gamma}(Z_i,Z_j) = 0.503$, $\bar{\gamma}(z,V) = 0.402$ (see Appendix A).

The z values are: $Z_1 = 19,\ Z_2 = 25,\ Z_3 = 17,\ Z_4 = 13,\ \ Z_5 = 21,\ Z_6 = 8,\ Z_7 = 12,\ Z_8 = 15,\ Z_9 = 20$

The OK system of equations applied to the layout of points in Figure 2.6 produces a matrix form indicated in Table 2.2. The reader is referred to (Appendix A) for the full calculation of this matrix.

Table 2.2: OK matrix of the 9 point support sample

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	λ			RHS
Z_1	0.0000	0.3672	0.6875	0.3672	0.5082	0.7512	0.6875	0.7512	0.8839	1.00	w_1	=	**0.5105**
Z_2	0.3672	0.0000	0.3672	0.5082	0.3672	0.5082	0.7512	0.6875	0.7512	1.00	w_2	=	**0.3715**
Z_3	0.6875	0.3672	0.0000	0.7512	0.5082	0.3672	0.8839	0.7512	0.6875	1.00	w_3	=	**0.5105**
Z_4	0.3672	0.5082	0.7512	0.0000	0.3672	0.6875	0.3672	0.5082	0.7512	1.00	w_4	=	**0.3715**
Z_5	0.5082	0.3672	0.5082	0.3672	0.0000	0.3672	0.5082	0.3672	0.5082	1.00	w_5	=	**0.0883**
Z_6	0.7512	0.5082	0.3672	0.6875	0.3672	0.0000	0.7512	0.5082	0.3672	1.00	w_6	=	**0.3715**
Z_7	0.6875	0.7512	0.8839	0.3672	0.5082	0.7512	0.0000	0.3672	0.6875	1.00	w_7	=	**0.5105**
Z_8	0.7512	0.6875	0.7512	0.5082	0.3672	0.5082	0.3672	0.0000	0.3672	1.00	w_8	=	**0.3715**
Z_9	0.8839	0.7512	0.6875	0.7512	0.5082	0.3672	0.6875	0.3672	0.0000	1.00	w_9	=	**0.5064**
weights	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	λ	=	1.0000

Solving for the weights gave:

$w_1 = w_3 = w_7 = w_9 = -0.0006$,$w_5 = 0.7280$, w_2=w_4=$w_6 = w_8$ =0.0687

The sum of weights is $\sum w_i = 4(-0.006) + (0.7280) + 4(0.0687)$

$$\sum w_i = 1.00$$

and the Lagrange multiplier: $\lambda = -0.0127$

Therefore the OK estimate is:

$$Z_{ok}^{*}v = \sum w_i . z(v)$$

=-$0.0006(19 + 17 + 12 + 20) + 0.0687(25 + 13 + 8 + 15) + 0.7280(21)$

$= 19.51$

The variance given by: $\sigma_{ok}^2 = \sum w_i \bar{\gamma}(z, V) - \bar{\gamma}(V, V) + \mu$

$$= 0.17 - 0.1063 + (-0.0127)$$

$$= 0.051$$

Table 2.3 below summarises the results obtained from the 9 point support samples estimated using SK and OK.

Table 2.3: 9 point support sample kriging results

Output	SK	OK
Kriged estimate	19.49	19.51
Kriging variance	0.064	0.051

When comparing the results of SK and OK for the 9 point support sample example; the first difference is that the mean is known in SK and unknown in OK. When comparing the arithmetic mean value of 19.67 with the SK kriged estimate of 19.49, there is a significant difference between the two. The arithmetic mean of the data influences the SK estimate (see SK example section 2.5.4). The SK value of the kriged estimate is less than the OK kriged estimate of 19.51. The OK estimate is not influenced by the arithmetic mean of the data. The OK variance of 0.051 is smaller than the SK variance of 0.064; meaning that for this particular estimation OK minimises the variance better than SK.

2.5.6 Differences between SK and OK

The difference between the two kriging types are the constraints imposed during the variance minimisation. OK involves the condition that the sum of the weights must be equal to one while in SK that condition does not apply. This condition of having weights summing up to one has a Langrage factor μ accompanying it and SK does not have that parameter (see Table 2.4).

Table 2.4: A table of comparison between SK and OK

Ordinary Kriging (OK)	Simple Kriging (SK)
Sum of weights is equal to one $\sum\lambda_i = 1$	Sum of the weights is not equal to one,
Assumes that the mean is unknown and can fluctuate over the deposit.	Assumes that the mean is known and remains constant throughout the deposit
OK estimator is : $Z^*_{ok}(v) = \sum\lambda_i Z(x)$	SK estimator is : $Z^*_{sk}(v) = \sum\lambda_i Z(x_i) + [1 - \sum\lambda_i]m$
Stationary OK adapts well to trends since the mean does not remain constant	Stationary SK does not adapt well to trends since the mean is assumed to be constant
OK has a Lagrange parameter associated with the condition that $\sum\lambda_i = 1$ $\mu = 2\sum\lambda_i\bar{\gamma}\ (V,x_i) - \sum\sum\lambda_i\lambda_j\gamma\ (x_i x_j)$	Does not have the Lagrange parameter associated with the weights and therefore has no condition on the sum of weights.
$\sigma^2_{ok} = \sum\lambda_i\bar{\gamma}\ (x_i,V) - \bar{\gamma}(V,V) + \mu$ Kriging variance for OK	$\sigma^2_{sk} = \sum\lambda_i.\bar{\gamma}(x_i,V) - \bar{\gamma}(V,V)$ Kriging variance for SK
Block variance for OK: $BV = \sigma^2 - \bar{\gamma}\ (V,V)$	Block variance for SK: $BV = \sigma^2 - \bar{\gamma}\ (V,V)$
$KE = \left(\frac{\sigma^2_B - \sigma^2_{ok}}{\sigma^2_B}\right)$ Kriging efficiency for OK	$KE = \left(\frac{\sigma^2_B - \sigma^2_{sk}}{\sigma^2_B}\right)$ Kriging efficiency for SK

OK assumes that the mean is unknown whereas SK assumes that the mean is known and constant throughout the deposit (Goovaerts, 1997). OK accounts for the local fluctuations of the mean by limiting the area of stationarity of the mean to the local neighbourhood (Goovaerts, 1997), which means that the mean may vary in the study area and does not remain constant. The local mean in OK is not the same as the global mean; therefore in low grade areas in a deposit the OK estimate will be lower than the SK estimate since the local mean is smaller than the global mean. In high grade areas the OK estimate is larger than the SK estimate because the local mean is larger than the global mean (see Table 2.4). SK emphasises strong stationarity, where the mean value remains constant throughout the deposit.

Chapter 3: Research Methods

In this chapter the statistical approach used in this study for analysis of PGE (4E) data is briefly described. The assumptions made regarding the data are stated, and the key statistical tools applied are described. Thereafter, the methods employed by Dohm (2011), Goovaerts (1997) and Deutsch et al. (2014) are discussed in this chapter as they are adopted by this study. These methods include the application of SK and OK in mining of various mineral deposits.

3.1 Statistical approach

In this study statistics is applied to describe the PGE (4E) data, giving all essential population parameters as well as relevant and meaningful diagrammatic presentations of the data.

The PGE (4E) sample data is used to draw conclusions about the underlying population. The geology of the study area is first understood before the attempt of the statistical study. The geological study is undertaken to understand the geological controls, and making decisions of how to group the data see Chapter 4.

To view, analyse and understand the PGE data, the descriptive statistics is undertaken. In descriptive statistics a few concepts are considered i.e. the measures of central tendency, measures of variability and the measures of symmetry. To further describe the PGE (4E) data, diagrammatic presentations are also used i.e. histograms, probability plot and grade sample location plots.

3.1.1 Descriptive statistics

The measures of central tendency include the following:

a) Mean which is the arithmetic average of the data set
b) Median which is the middle or central value of the whole data set
c) Mode is the most frequently occurring value or common value in a data set

The measures of central variability include the following:

a) Range measures spread, the difference between the smallest and largest value in a data set.
b) Variance which is the spread of data values around the mean. This is an important measure of deviance.
c) Coefficient of variation provides an estimate of the variability of the data i.e. grade variability of the orebody.

The measures of symmetry considered in this study are skewness and kurtosis. Most natural data distributions are skewed; this skew measures the extent to which a distribution departs from symmetry. Symmetrical distributions are mirror images of one another i.e. normal distribution (bell shape). Kurtosis refers to the shape of the distribution, how peaked a distribution is.

The diagrammatic presentations i.e. the histogram and probability plot were discussed in Chapter 2 and will not be discussed here. There are certain assumptions made regarding the sample data in statistics. These assumptions are applied to the sample data to be analysed, and they state the following:

a) Data values are precise
b) Data values are accurate
c) Data values are random and independent
d) Samples are very small proportion of the population.

These assumptions are applied also for the PGE (4E) data set used in this study.

3.1.2 Spatial data analysis

This section describes the tools used for spatial analysis. The spatial analysis is undertaken to confirm and validate the information supplied in the statistical analysis. In this study as a tool of spatial analysis the colour coded sample location plots are produced as well as the two dimensional grade contour maps. The grade contour maps are constructed at different grade intervals see Chapter 5 and are used for understanding grade trends. The colour coded sample locations plots provide assessment of the continuity of high and low grade see Chapter 4. To characterise the continuity of the PGE (4E) data the application of variography is undertaken in

Chapter 5. The variogram maps are produced as well the semi-variogram models, in order to further analyse the spatial chacteristics of the PGE (4E) data.

3.2 The Kriging weights for OK and SK

This section is the continuation of the example discussed in Chapter 2 of the 9 point support sample values and has been slightly modified from work done by C.E Dohm (2011). This section discusses in detail what occurs to the kriging weights as the nugget effect increases for both OK and SK. This section is undertaken as a view of what technologically advanced software packages do, such as Surpac 6.2.1 used in this study see Chapter 6.

For both SK and OK the kriging weights can be calculated by means of matrix algebra as shown earlier in Chapter 2 (equation 12 and 32) respectively. According to Goovaerts (1997), the kriging weighting system accounts for:
a). the relationship of the data to the location being estimated through the semi-variogram.
b). data redundancy through the data semi-variogram matrix.

Isaaks and Srivastava (1989) mention that the kriging weights depend on the following:

a) The spatial correlation of the available samples with respect to each other.
b) The spatial location of the available samples with respect to the block being estimated
c) The spatial continuity and structure of the deposit under study, this is presented by the semi-variogram function (nugget effect, anisotropy and range).

To observe the behaviour of kriging weights for both SK and OK, the data outline shown in Figure 2.6 is used. In each case to be discussed an isotropic semi-variogram model is assumed at a various number of nugget values. The nugget effect is set at 0.1 increments until 1 for each range of influence and a sill of 1. The ranges used are 30 m, 90 m and 120 m respectively.

Considering the data configuration in Figure 2.6 and assuming an isotropic semi-variogram model at a range of 30 m, the OK and SK weights compared with the nugget effect appear as illustrated in Figure 3.1

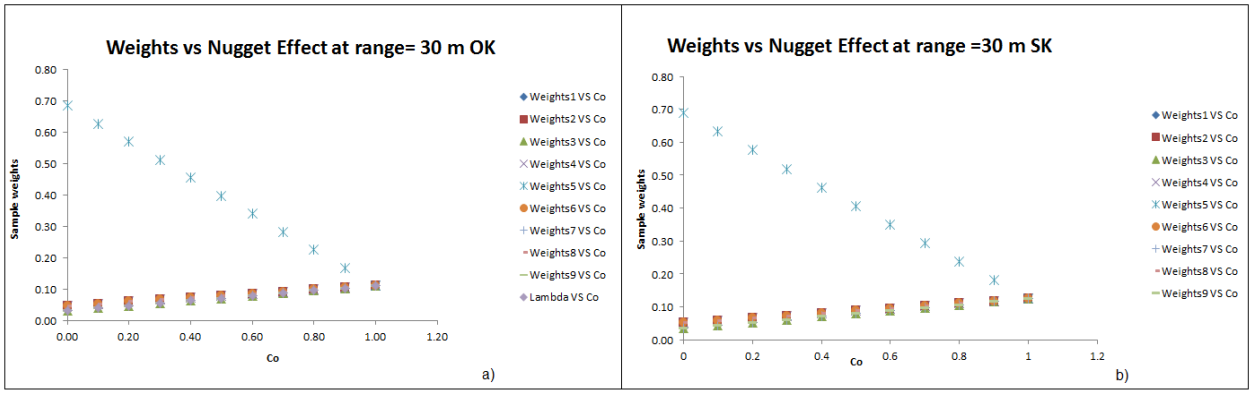


Figure 3.1: Impact of the nugget effect on the OK and SK weights at a 30 m range

For OK, what is observed is that the weights decrease as the nugget effect increases at sample point Z5, where block V being estimated is situated. The same is observed for SK weights, sample point Z5 has the highest weight because it is located close to the block V being estimated. The sample at points Z1, Z3, Z7 and Z9 have equal weights and behave the same both in OK and SK. These sample points are assigned the same weights because they are located at the same distance from the point being estimated and an isotropic semi-variogram is assumed which only considers the sample distances regardless of the direction. Similarly the sample points Z2, Z4, Z6 and Z8 have equal weights and behave the same. The samples located around the block being estimated have their weights increasing with the nugget effect. What is observed at this range is that the SK weights are slightly larger than the OK weights (see Appendix B). This is due to the difference in the OK matrix and SK matrix, the OK matrix includes the Lagrange multiplier which ensures that the weights sum up to 1 and SK matrix does not.

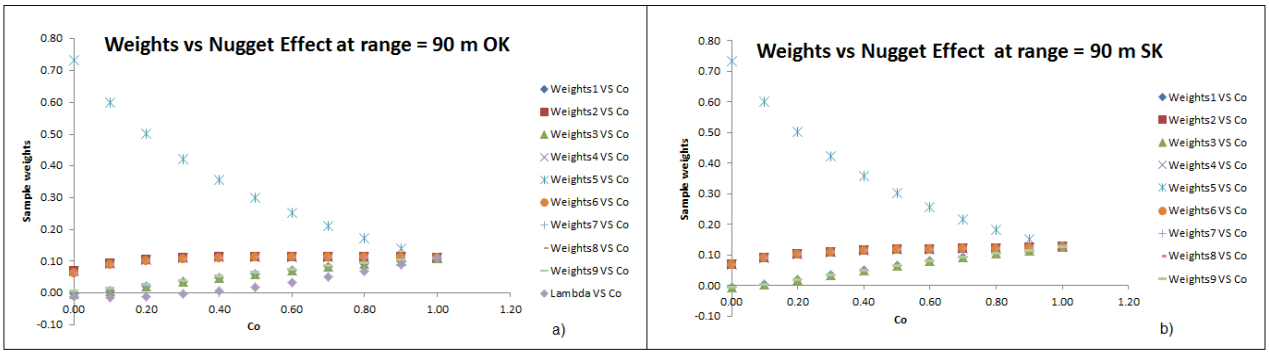


Figure 3.2: Impact of the nugget effect on the OK and SK weights at a 90 m range

At a range of 90 m, the weights at sample point Z5 decreases with the increase in the nugget effect which is similar to what is observed in the range of 30 m. The weights of the points surrounding the block V estimated increase as nugget effect increases (see Figure 3.2). According to Goovaerts (1997) the increase in the nugget effect reduces the impact of distance of the data locations to the point or block being estimated.

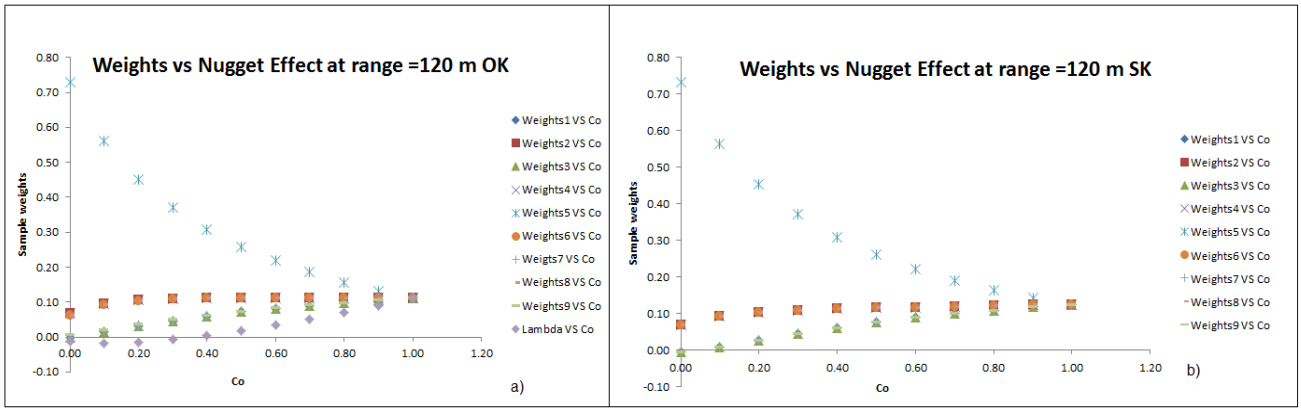


Figure 3.3: Impact of the nugget effect on the OK and SK weights at a 120 m range

At a 120 m range an introdcution of negative weights is observed at sample point Z1, Z3, Z7 and Z9 at zero nugget effect. These sample points are screened by the closer samples Z2, Z4, Z6 and Z8, hence they obtain negative weights.The furthest samples at zero nugget effect have negative weights in SK, which is not observed in OK . Nagative weights are undesirable as they can result in negative kriged estimates. Goovaerts (1997) notes that the increase in the nugget effect reduces the

screening effect. Hence what is observed in all ranges is that at pure nugget effect (C_o = 1) all samples are assigned equal weights and are positive (see Appendix B).

Nugget effects indicate the variability of samples over short distances. In Chapter 2 it is mentioned that the behaviour of the semi-variogram near the origin has implications on the kriging results and their stability. A common feature of the semi-variogram is the discontinuity at the origin, given by the nugget effect. The increase in the nugget effect allows for the kriging weights of points far away from the block being estimated, to be assigned similar weights as the points closer to the block (see Appendix B). This causes great averaging of the kriging process and a smooth appearance of the kriged grades. In mining this means that waste can be mistaken for ore if a very high nugget effect is used and this can lead to misinterpretation and financial loss.

3.2.1 Differences in the application of OK and SK weights

As the range increased, negative weights were obtained in SK and the Lagrange multiplier in OK also started being negative. At all ranges some of the SK weights are larger than the OK weights and all weights converge as the nugget effect increases. For all ranges in both OK and SK the weights at sample point Z5 decreased as the nugget effect increased. The change of the range seems to have minor effects on the weights for OK. The sample points surrounding block V being estimated has weights increasing as the nugget effect increases for both OK and SK, hence the convergence of weights at pure nugget effect. It can be concluded that the higher the nugget effect the higher the degree of smoothing. When the nugget effect is high samples are more evenly weighted and the block estimate is derived from all available sample data. Conversely when the nugget effect is low the block estimate is derived from the closest samples within the range of influence. The real major difference observed here between OK and SK is in the computation of the matrices of the two techniques. For OK there is a Lagrange multiplier ensuring that the weights sum up to 1 and there is no Lagrange multiplier in SK. Overall similar behaviours on the weights are observed for both OK and SK, since the same isotropic variogram is assumed for both.

3.3 Kriging performance measures

A number of decisions are required to make an appropriate kriging estimate, such as the kriging type .i.e. SK or OK, search parameters and data selection. This section discusses some of the performance measures used to assess a kriging estimate. A case study from Deutsch et al. (2014) is discussed and the methods used in his case study are adopted for this study.

3.3.1 Number of search data

The number of search data is part of the decision made to make an appropriate kriging estimate. A number of authors have studied in detail the impacts that the number of search data has on the kriged estimate, and found that OK performs better than SK when a large number of search data is used in most cases.

According to Deutsch et al. (2014), Rivoirard (1987) and Boyle (2010) a restricted search is considered when kriging is used as an estimation method. This restricted search is considered to reduce the reliance on the hypothesis of a stationary mean for OK and to reduce the presence of negative weights for SK and therefore reducing the weight assigned to the mean (Rivoirard, 1987). The search in kriging refers to the process of finding the sufficient samples to represent a local distribution function and to minimise conditional bias. The search parameters include:

a) a maximum range around the location being estimated to search for local data
b) maximum number of local samples to consider
c) maximum number of data to be used from each borehole and the maximum number of data to use from each octant or quadrant searches. Together all these parameters are referred to as the kriging neighbourhood (Deutsch and Journel, 1998). A detailed explanation about the Kriging neighbourhood is found in Rivoirard (1987), Boyle (2010) and Vann et al. (2003).According to Boyle (2010) the testing of the number of search data in kriging can be referred to as the kriging neighbourhood analysis (KNA).

According to Vann et al. (2003) the fact that kriging is a minimum variance estimator is true when the neighbourhood is properly defined. He also suggests that kriging neighbourhood can assist with block size selection, choice of discretisation and mineral resource classification decisions.

Rivoirard (1987) suggests two parameters to assess, when investigating the appropriate number of search data. The two parameters are the weight of the mean, which shows how kriging depends on the number of search data as well as the slope of regression, which shows if the number of search data used is sufficient or not. The mathematics behind the kriging weight of the mean as well as the slope of regression is covered in detail in Rivoirard (1987). This study assumes the reader has an understanding of linear geostatistics.

According to Boyle (2010) in SK where the mean is known, the weight of the mean shows the dependency of kriging on local samples rather than the whole deposit and samples further away. He explains that, if the weight assigned to the mean is low then mainly local samples are used to estimate the grade and the assumption of stationarity is relaxed. Conversely if the weight assigned to the mean is large then that suggests that the local sample information is limited, therefore the global mean and stationarity are more important.

In OK the mean is not known, and the weights are assigned to the local samples and to the local mean, kriged from local close samples. Rivoirard (1987) suggests that if the weight of the mean in SK is greater than 20% of the original mean, the estimate of the local mean becomes more important for OK. This estimation of the local mean involves samples that are further away if the samples close by are insufficient to estimate the data. (Boyle, 2010).

Rivoirard (1987) suggests that if the slope of regression is less than 1 it means that true grades estimated to have high grade values are most likely lower than estimated. The variance of estimated grades is normally greater than the variance of true/original grades; this suggests a highly restricted neighbourhood search. Conversely if the slope of regression is greater than 1, then over smoothing of estimated grades exists.

Krige (1996) suggests that if there is insufficient number of data there is no way to avoid smoothing and conditional bias. The number of search data should be increased to obtain a slope of regression close to 1, improving estimation accuracy as well minimising conditional bias.

3.3.2 Number of search data analysis (KNA)

Deutsch et al. (2014), adopted case studies from different authors which made use of the kriging neighbourhood to assess the kriging estimate. The case studies considered both SK and OK which is what is also considered with this PGE (4E) data. The studies investigated the effects of the number of search data against the mean squared error between the estimates and true values $(Z_V^* - Z_V)^2$.

Three case studies were considered where between 5 and 100 local data was used to produce each estimate. The first case study was of a low grade porphyry copper deposit with 134 drill holes (see Figure 3.4 a)). The second case study was of bitumen data (oil sands) with 280 drill holes (see Figure 3.4 b)). The final case study was of a zinc deposit with 367 drill holes (see Figure 3.4 c)). The results from the three studies are shown in Figure 3.4.

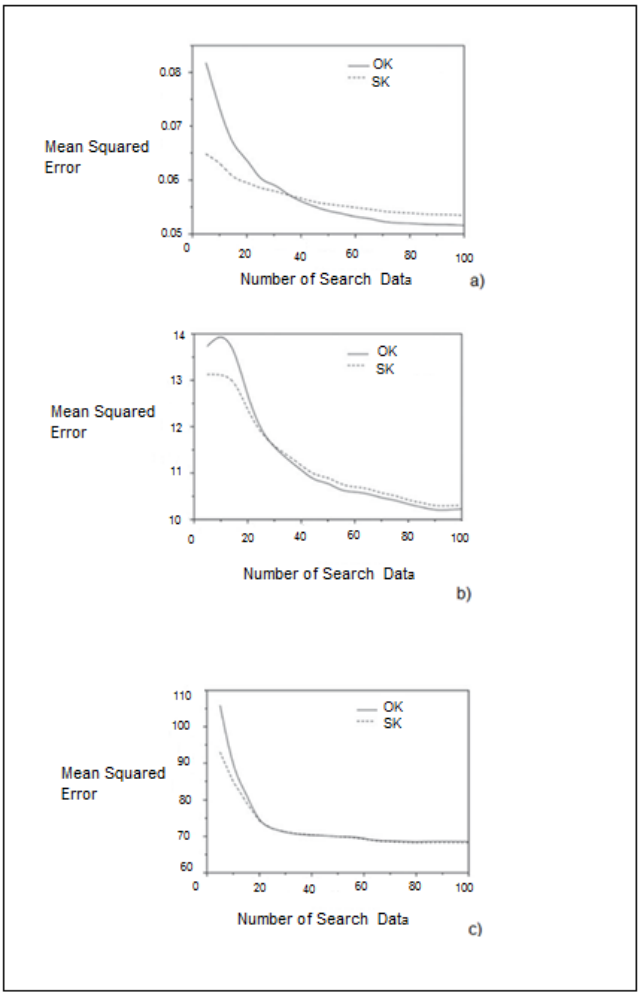


Figure 3.4: The effect of the number of search data on OK and SK (Deutsch et al., 2014).

In the first case study of the low grade porphyry copper with 134 drill holes, the average drill hole spacing is 100 m. This porphyry copper deposit was composited into 3 m sections. The porphyry copper assays are moderately skewed with a mean of 0.25% and standard deviation of 0.27%. The variogram model for this deposit is isotropic and has a nugget effect of 20% of the total sill (Deutsch et al., 2014).

In this low grade porphyry copper deposit the mean squared error decreases as the number of search data increases, improving the kriging estimate. Figure 3.4 a) shows the effects of the number of search data on SK and OK using the low grade porphyry copper .For a low number of search data SK performed better than OK. Conversely for a large number of search data OK was the better estimator.

In the second case study of the bitumen data with 280 drill holes, the data was composited into 3 m sections. The bitumen deposit is stratified and the deposit displays strong vertical to horizontal anisotropy at ratio 150:1.For this deposit the histogram displays a normal distribution with a mean of 7.7%.A similar case to the porphyry copper was observed, where OK performed better when a large number of search data was used (see Figure 3.4 b)).

The third case study of the zinc deposit with 367 drill holes, where the zinc assays are skewed and have a moderate anisotropy between horizontal and vertical directions. For this deposit, the mean squared error for both OK and SK with a large number of search data performed in the same manner.

Increasing the number of search data decreases the mean squared error for both OK and SK. For OK increasing the number of search data increases the accuracy of the estimate of the local mean since the mean squared error was decreased drastically when the search data increased. The conclusions that were made in these case studies was that, SK will always result in a lower mean squared error compared to OK when few number of data are used, provided the mean in that deposit is not globally stationary. When more number of data is used OK performs better than SK.

3.4 Trend estimates

The analysis of trend estimates in a mineral deposit assist in the evaluation of local mean departures from the overall mean value, thus providing an overview picture of global trends of that deposit. This section discusses work done by Goovaerts (1997) on the application of SK and OK in trend estimates. The methods employed by Goovaerts (1997) are adopted and used by this study in order to evaluate the differences in the application of SK and OK.

3.4.1 Cadmium trend estimates

An example of a study undertaken by Goovaerts (1997) is discussed to further show the differences in the application of SK and OK. Cadmium (Cd) local mean was estimated using SK and OK along a NE-SW direction or orientation of the data. Figure 3.5 a) shows ten Cd concentrations at locations u_1 to u_{10}.The local mean was estimated every 50 m using the 5 closest data values; Figure 3.5 b) and c) show the results from the estimation. The OK estimate of the mean is different from one segment to another depending on the neighbouring data retained. It is however identical at locations where the same neighbouring data are involved in the estimation. The OK estimate therefore results in a trend estimate that follows the general increase of Cd values which increases with an increase in distance. The mean of the 10 data values of Cd is 1.49 ppm presented by a horizontal dashed line in the third graph (see Figure 3.5 c)). The horizontal line overestimates the lower left Cd local mean and underestimates the local mean on the right.

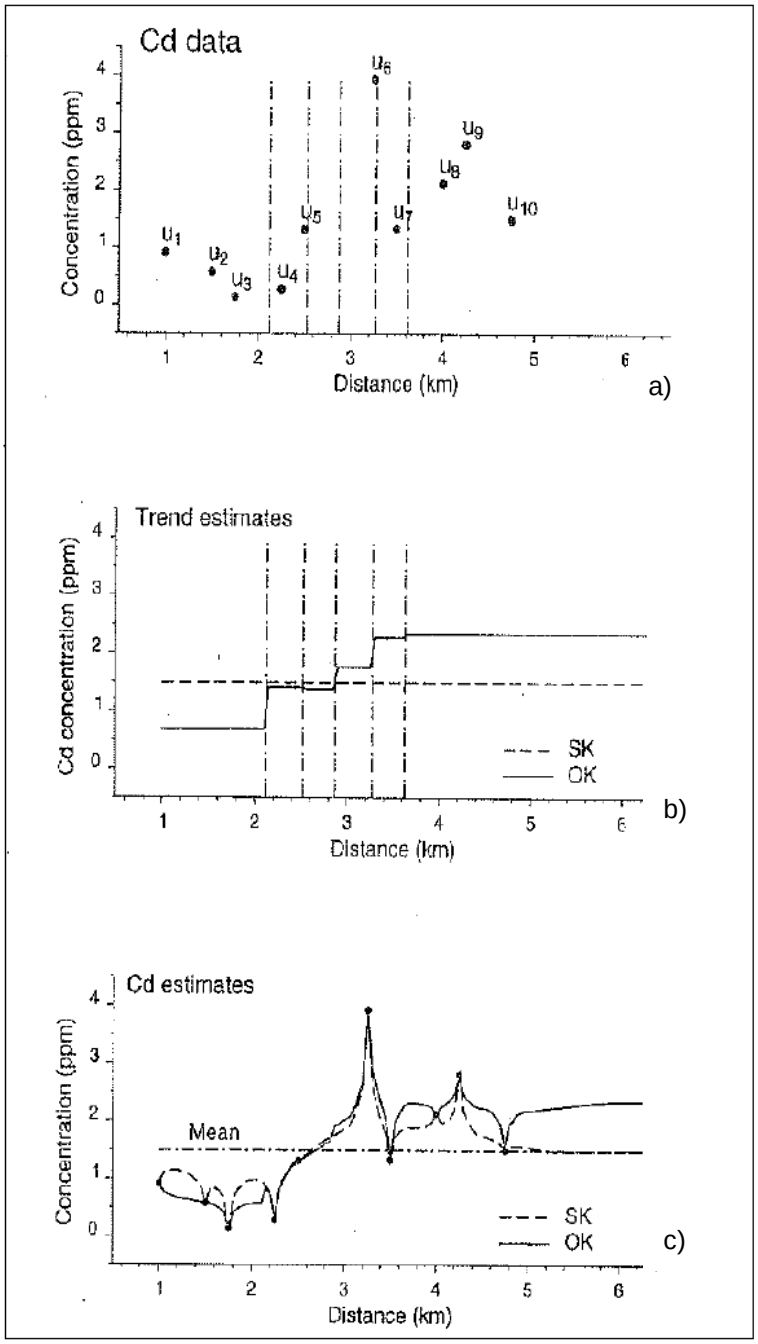


Figure 3.5: SK and OK trend estimates of Cadmium (Goovaerts, 1997)

The third graph shows the results obtained from both SK and OK. The estimates from OK are smaller than the estimates from SK in the left part of the graph (see third graph Figure 3.5 c)), where the local mean is smaller than the global mean of 1.49 ppm.

The OK estimate is larger than the SK estimate in the right part of the graph (see Figure 3.5 b)), where the local mean is larger than the global mean of 1.49 ppm. The OK estimates better follow the data fluctuations with smaller values in the left part and larger values in the right part of the graph. OK better estimates the Cd data since it follows the Cd trend better than the SK estimate.

Goovaerts (1997) notes that the use of stationary mean yields SK estimates that are close to that mean value (1.49 ppm) away from the data values (see the right edge of Figure 3.5 c)). In contrast, local estimation of the mean within search neighbourhoods yields OK estimates that better follow the data fluctuations as seen in Figure 3.5 c); small values in the left part and large values in the right part of the graph. Table 3.1 summarises the differences between SK and OK obtained by Goovaerts (1997).

Table 3.1: Comparison between OK and SK (Goovaerts, 1997)

Ordinary Kriging (OK)	Simple Kriging (SK)
Sum of weights is equal to one $\sum\lambda_i = 1$	Sum of the weights does not have to be equal to one
Does not require knowledge or stationarity of the mean over the entire deposit The mean can fluctuate over the deposit.	Assumes that the mean is known and remains constant throughout the deposit. Emphasises strong stationarity.
OK estimator is : $Z^*_{ok}(v) = \sum\lambda_i Z(x)$ It estimates the local mean at each location with data specific to the neighbourhood	SK estimator is : $Z^*_{sk}(v) = \sum\lambda_i Z(x_i) + [1 - \sum\lambda_i]m$ Assumes a stationary mean
OK adapts well to trends since the mean does not remain constant	Stationary SK does not adapt well to trends since the mean is assumed to be constant

Chapter 4: Geological Setting and Exploratory Data Analysis

Without the knowledge of geology of the orebody, the grade estimates obtained would be poor; therefore a brief overview of the geology of the study area is discussed in this chapter. To fully understand the main characteristics of the PGE (4E) data, Exploratory Data Analysis (EDA) is also undertaken in this chapter. EDA involves classical statistics, which provides an idea about the distribution of grades in a mineral deposit.

4.1 Project background

This research project is based on a new platinum development owned by Anglo Platinum Plc; the data was supplied by the company. To preserve confidentiality of the site location, Anglo Platinum has translated and rotated the data (Anglo Platinum, 2011). No mining has occurred in the study area, except in the surrounding mines. The project only considers the UG2 Reef as was mentioned in Chapter 1.

4.2 Geological Setting

4.2.1 Regional Geology

As mentioned in Chapter 1 the project area is located in the Eastern Limb of the Bushveld Complex, which extends from the north in Lebowagomu and to the south in Roossenekal. It is divided into northern, central and southern sectors and hosts the Rustenburg Layered Suite (RLS). The RLS contains Ni-Cu-PGE mineralisation and is subdivided into the following zones:

-Marginal Zone

-Lower Zone

-Critical Zone

-Main Zone and

-Upper Zone

The Critical Zone (CZ) is the most important as it contains the world's largest reserves of PGEs and chrome hosted in the Merensky, UG2, MG and LG6 Reefs (see Figure 4.1).

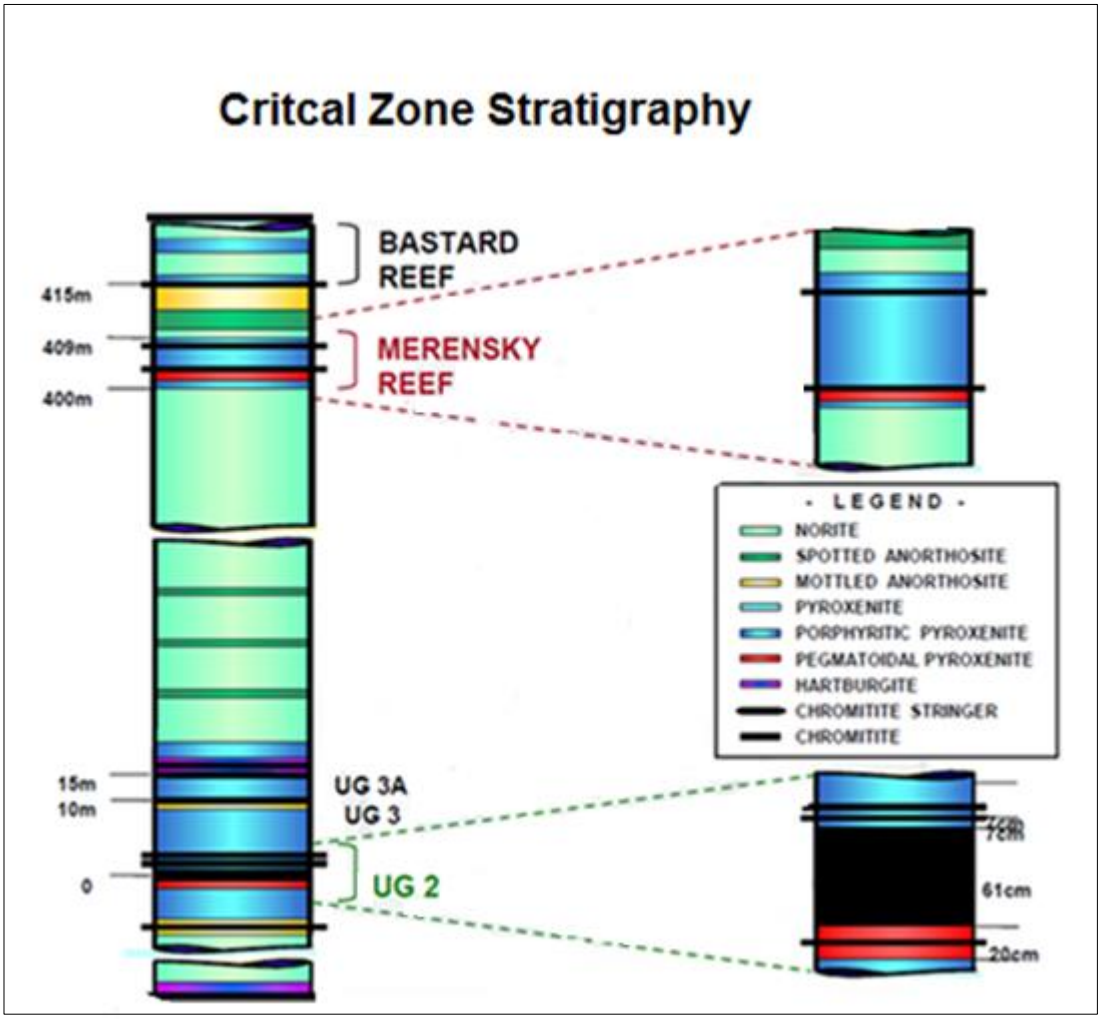


Figure 4.1: Stratigraphic column of the Merensky, Bastard and UG2 Reef (Anglo Platinum, 2011)

4.2.2 Local Geology

UG2 refers to the Upper Group 2 Chromitite Layer in the upper Critical Zone of the RLS. The UG2 occurs at 15 – 400 m below the Merensky Reef. Based on the borehole data analysed the layer is 0.5-1 m thick with a feldspathic pyroxenite base or footwall and feldspathic pyroxenite hanging wall. There are usually two to three chromitite stringers 10 to 15 cm above UG2 Reef stratigraphy (see Figure 4.2).

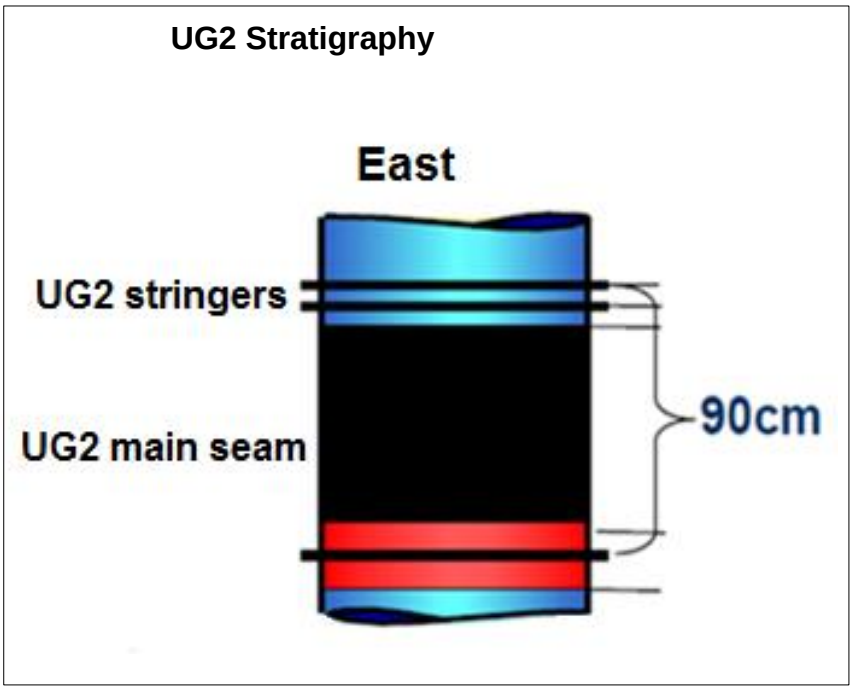


Figure 4.2: Stratigraphic column of the UG2 Reef (Anglo Platinum, 2011)

There is about 60-90% chromite with an average Cr: Fe ratio of 1.26 to 1.4 and 43% Cr_2O_3. The PGEs occur between the chromite cubic grains (interstitial). Lee, (1996) measured the concentration of the PGEs and gold within the UG2 up to 10 ppm with a platinum content of 3.6 ppm, 3.81 ppm palladium, 0.3 ppm rhodium with copper and nickel being low at 0.05%. The Pt: Pd ratio varies with geographical location. Mineralisation is from the top stringer through the chromitite layer down to the feldspathic pyroxenite. The formation of PGE mineralisation is a result of magmatic pulses that have been subjected to later remobilisation.

The prominent structural features associated with the UG2 Reef are potholes, dykes, faults and Iron Rich Ultramafic Pegmatites (IRUPs) (see Figure 4.3).

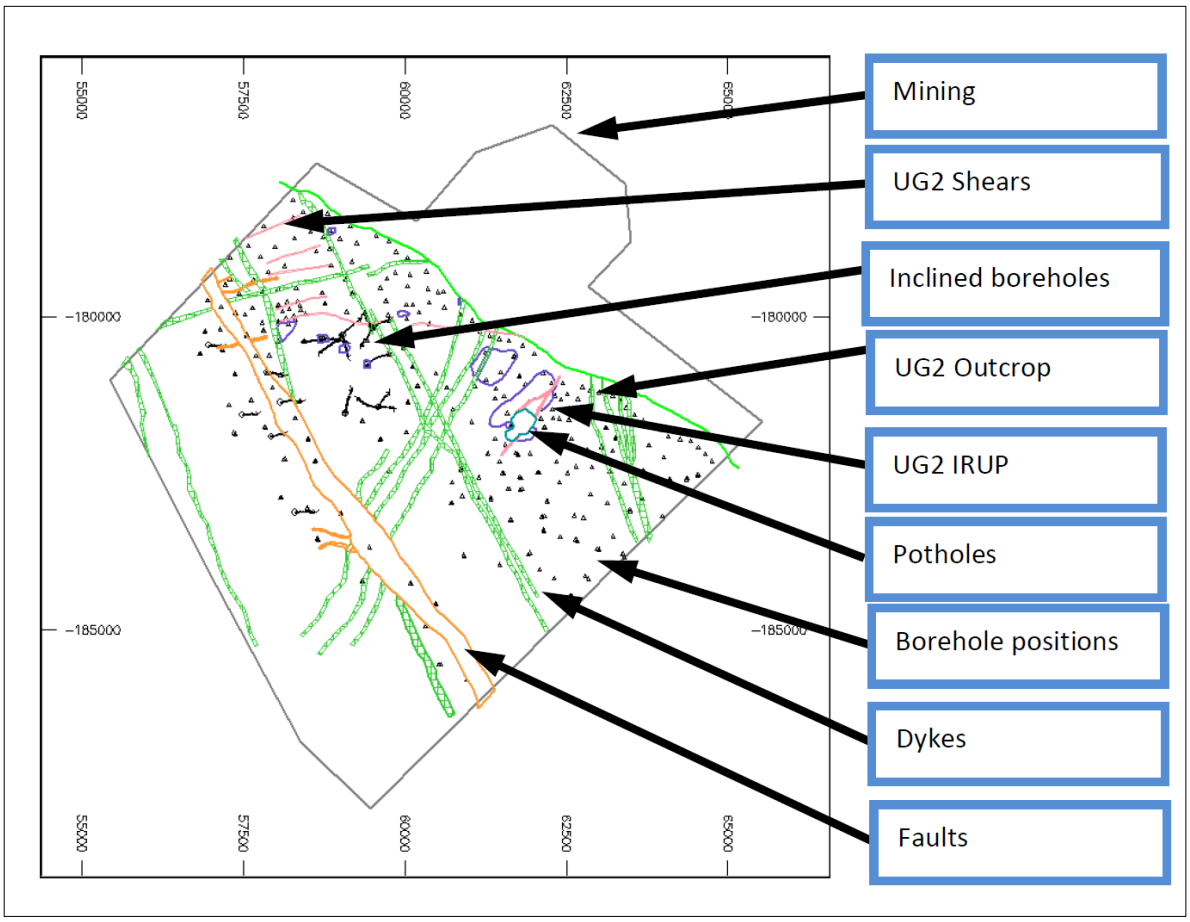


Figure 4.3: Geological structural features through the UG2 Reef (Anglo Platinum, 2011)

A major fault separates the north of the deposit from the south and it throws the South block down by 35 m, (see Figure 4.3) orange coloured fault (Anglo platinum, 2011). Several shear zones were also inferred from the drill holes as well as prominent dykes. Dykes vary in thickness from 1 m to 10 m. Both dolerite and lamprophyre dykes occur in the area.

4.3 Exploratory Data Analysis

In statistics EDA is used to analyse data sets with the aim of summarising their main characteristics, often with visual methods.

4.3.1 The Sample Data Set

This project has a total of 570 drill holes with X, Y and Z coordinates and the variable considered, is the PGE grade measured in (g/t). The deposit strikes NW-SE and has an average dip of 9° to the south west.

4.3.2 Data locations

The boreholes in this data set are located as shown in Figure 4.4. The boreholes appear to be evenly distributed across the project area except towards the southern part, which is characterised by severe faulting.

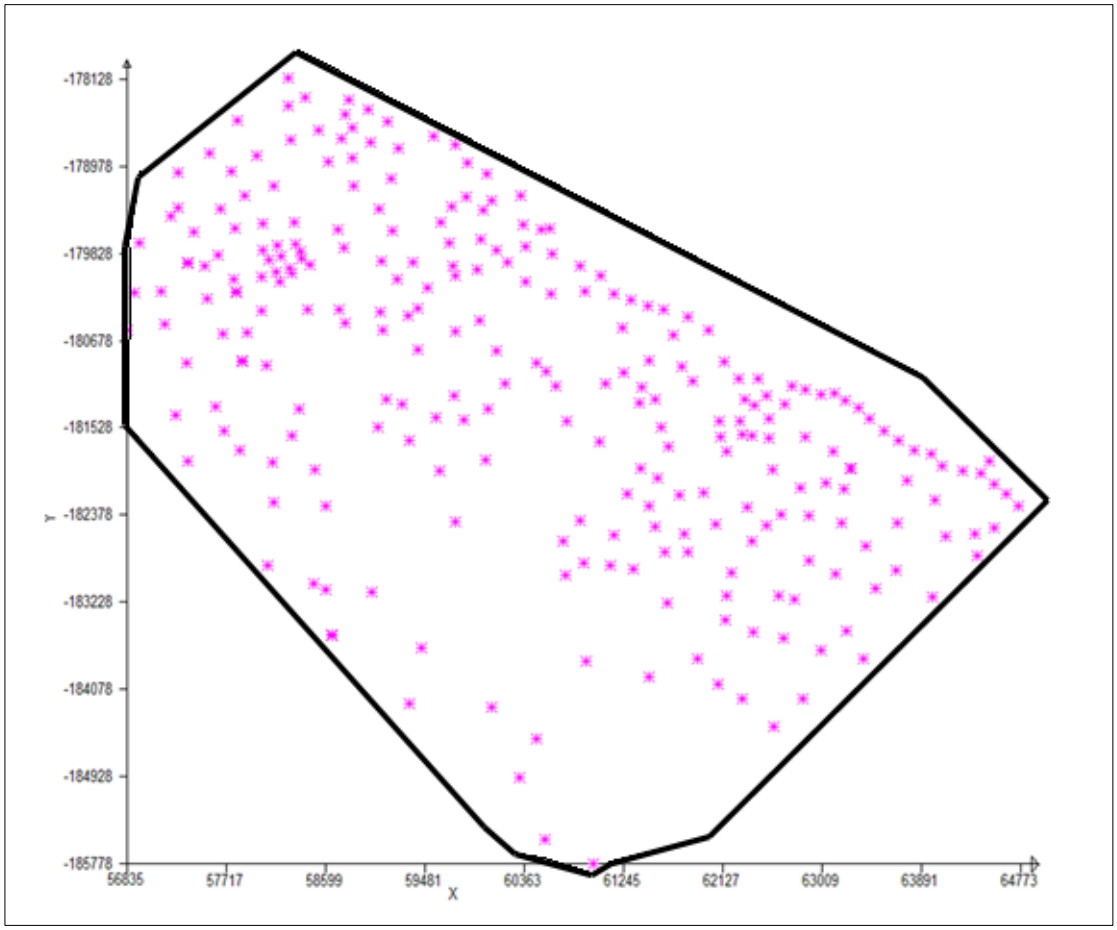
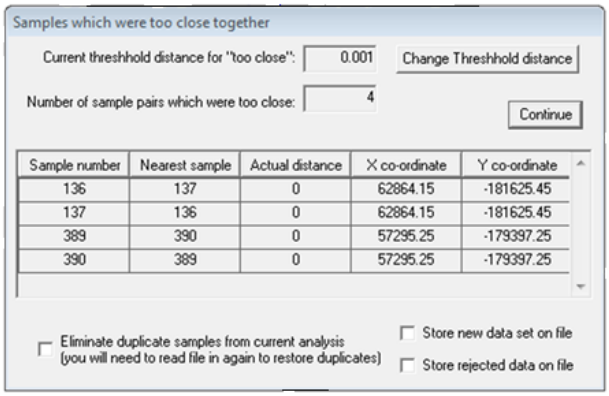


Figure 4.4: Location of the 570 borehole intersections of the UG2 reef

4.3.3 Data validation

Initially when the data was received and validated 4 pairs of data had the same x and y coordinates with different grade PGE (g/t) and thickness (m) values (see Table 4.1).

Table 4.1: Showing duplicate boreholes from the data set



PG 2000 software was used to assess data duplication, Figure 4.5 shows the pairs duplicated in red, and pairs not duplicated, in blue.

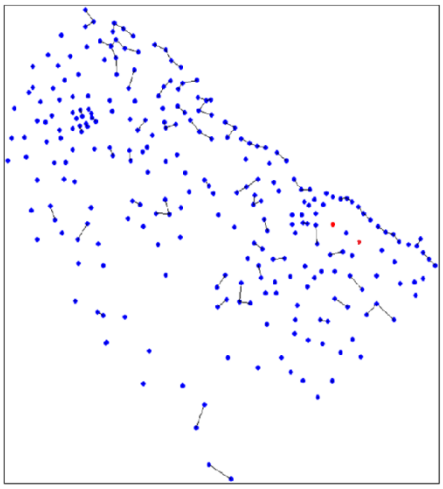


Figure 4.5: Location of pairs and the duplicate pairs presented by a red dot

The data custodian from Anglo Platinum advised that the coordinates be increased by 0.001 to remove the effect of duplication. Therefore all sample values were used for analysis.

4.3.4 PGE (4E) Sample statistics

The sample statistics for the PGE (UG2) grades are summarised in Table 4.2.

Table 4.2: Descriptive statistics of PGE grades

Statistic	Value
Mean	5.76 g/t
Median	5.91 g/t
Mode	4.42 g/t
Standard Deviation	2.20 g/t
Sample Variance	4.84 $(g/t)^2$
Kurtosis	3.44
Skewness	0.20
Range	13.75 g/t
Minimum	1.12 g/t
Maximum	14.87 g/t
CoV	0.38
N	570

4.3.5 The PGE (4E) grades

The histogram has an underlying distribution of the data that appears bimodal (see Figure 4.6). The bimodality of this data was further investigated by plotting a probability plot (see Figure 4.8).The histogram shows that the distribution of PGEs is non-normal. In mineral deposits several geological factors and processes contribute to the final sample values, such as the intrusion of magma and remobilisation. There is no obvious reason for this bimodality, but it could be explained by the intrusion of magma pulses and remobilisation known to have occurred in the Bushveld Complex.

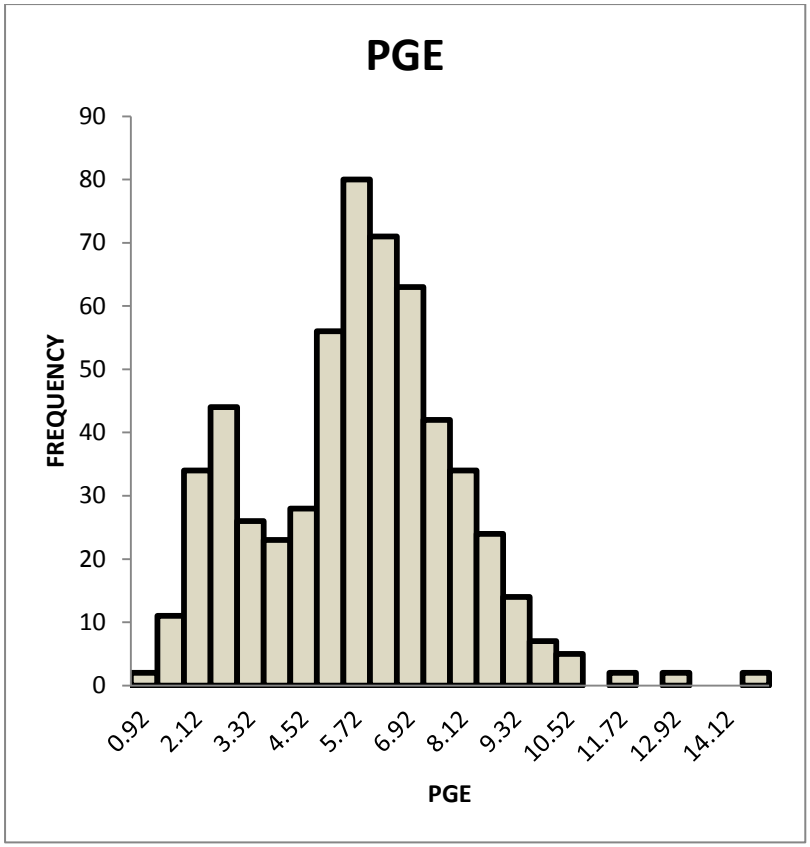


Figure 4.6: Original PGE grades histogram with a class width of 0.6

4.3.6 Colour coding the PGE grades

In Figure 4.7 the PGE grades are colour coded according to grade location to try and observe if there is a clear distinction between areas of low and high grades.

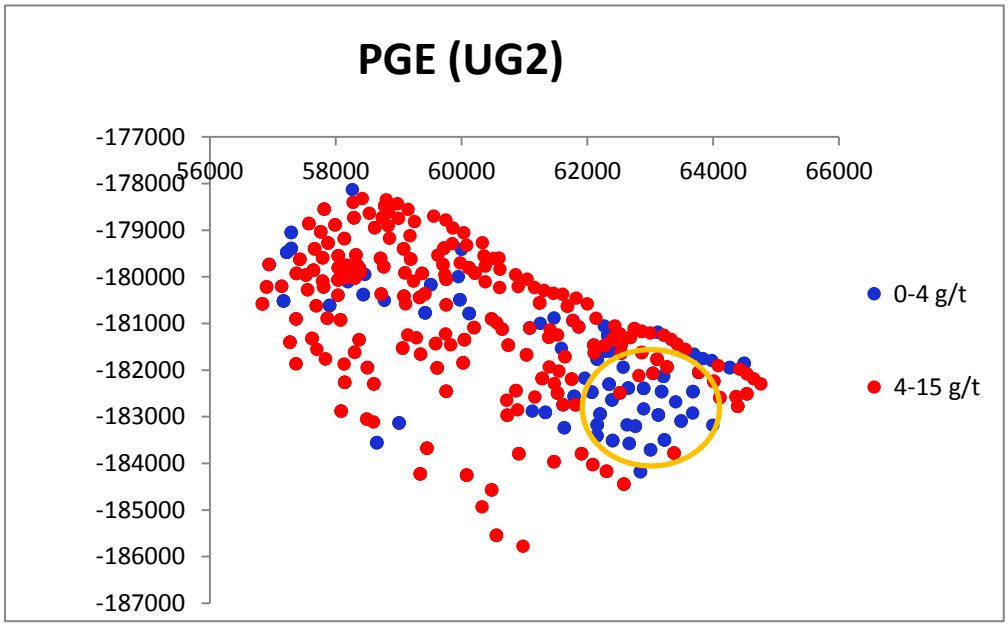


Figure 4.7: Location of low and high grade areas

The criterion to try and separate grades was derived from the histogram in Figure 4.6. In Figure 4.7 the grades from 0 g/t to 4g/t represent the lower grade portion of the histogram and grades from 4g/t to 15g/t represent the higher grade portion of the histogram. It appears that there is a mixture of low and high grades throughout the deposit. There is however a grouping of low grades that stands out on the SE corner of the deposit (see Figure 4.7 circle).

4.3.7 The Probability Plot

The probability plot in Figure 4.8 suggests that the data has 3 population distributions instead of 2. The first distribution varies with grades from 0.0 g/t to 3.0 g/t, while the second distribution of medium grade varies from 3.0 g/t to 7.0 g/t and the third distribution varies from 7.0 g/t to 15 g/t.

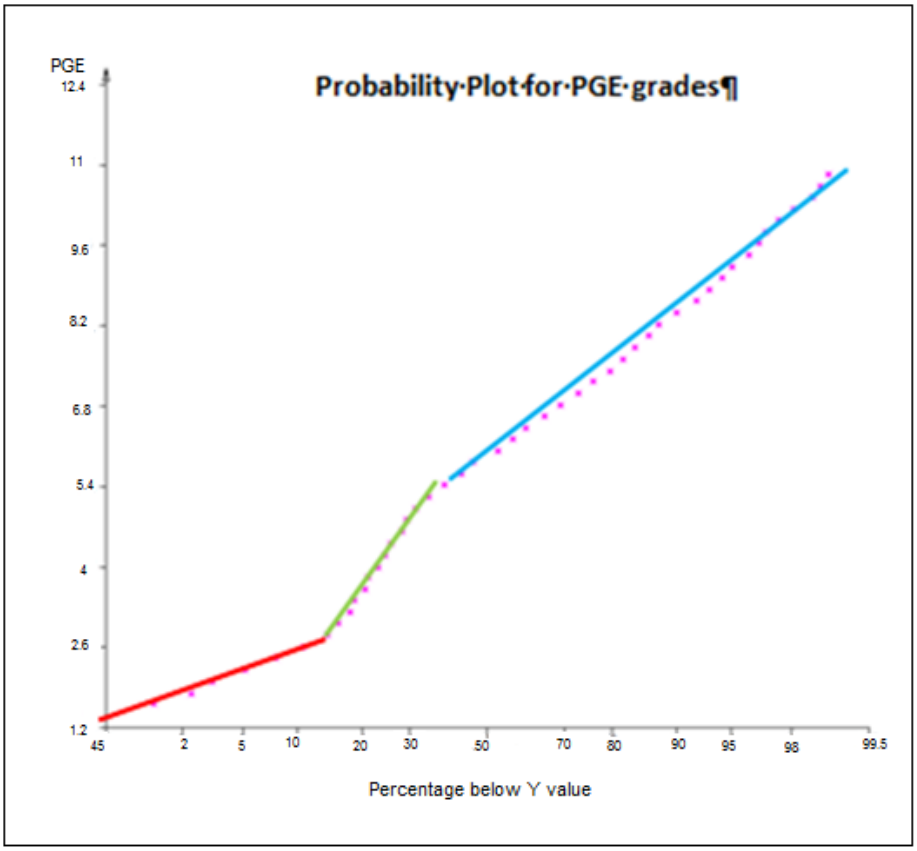


Figure 4.8: Probability plot of the PGE (UG2) data

The different populations displayed by the probability plot could be explained by the knowledge that the Bushveld Complex resulted from the intrusions of more than one phase of magma pulses and was subjected to later remobilisation. It is not certain

whether the hydrothermal effect played a major role in creating these different populations.

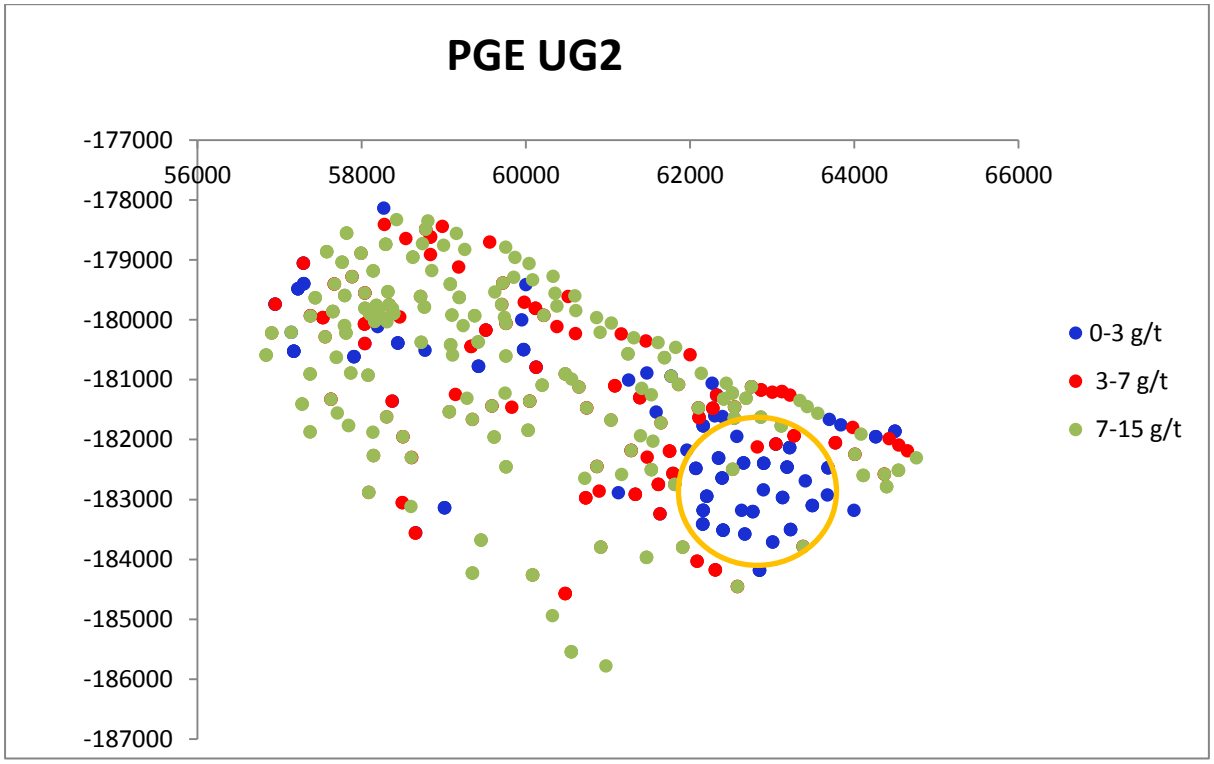


Figure 4.9: Location of low and high grade areas

In Figure 4.9 the PGE grades were also colour coded according to grade location to try and observe if there are any distinct populations as indicated by the probability plot. Similarly to Figure 4.7 there is no distinction of low and high grade areas, due to the mixture of grades observed also in Figure 4.9. What is also noted in this figure is the SE corner grouping of low grade PGE values. The SE portion of low grade PGE values is associated with cross cutting dykes as shown in Figure 4.3

The coefficient of variation (CoV) is given by:

$$CoV = \frac{\sigma}{\bar{Z}}$$

where σ is the standard deviation and $\bar{Z}$ is the mean value of samples

Coefficient of variation is a normalised measure of variation after the influence of the arithmetic mean has been removed (Isaaks and Srivastava, 1989). The larger the coefficient of variation the wider the dispersion of the data set.

According to Wellmer (1989) when a data set has a coefficient of variation less than 0.33, that data set is symmetrical and has a normal distribution. The coefficient of variation for this PGE deposit is 0.38 which is evidence that this data is non-normal.

To verify the observations made in Figure 4.7 and 4.9, PG2000 software is used to produce the overall grade sample location plot of PGE grades shown in Figure 4.10.

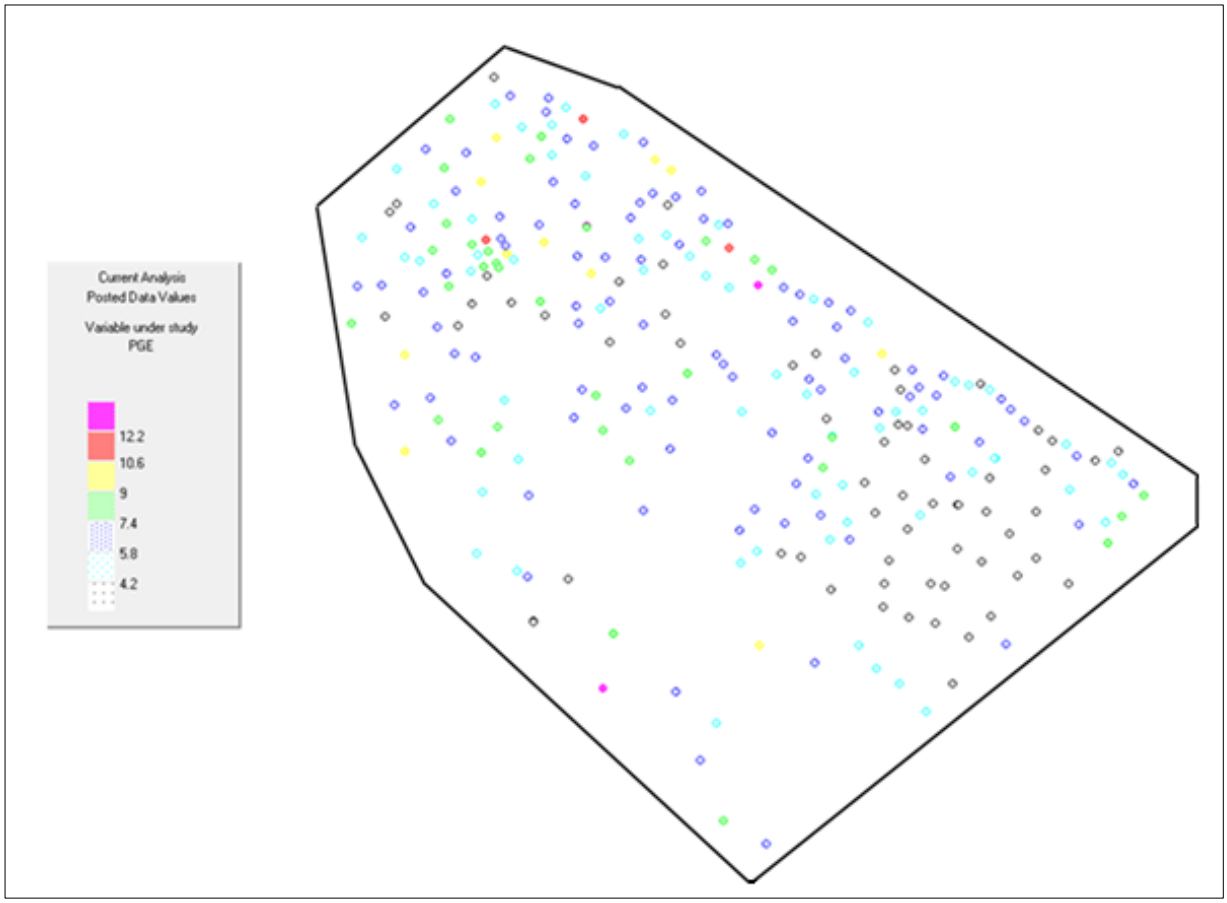


Figure 4.10: Grade sample location plot

In summary, there is an even distribution of low grade areas and high grade areas; there are no distinct areas of only low or high grade PGE values except for the small grouping of low grades in the SE corner. A mixture of both low and high grade PGE values is observed in the grade sample location plot in Figure 4.10 throughout the mineral deposit.

4.3.8 PGE composition

To further investigate the bimodality of this 4E PGE deposit, the histograms and statistics of platinum (Pt.), palladium (Pd), rhodium (Rh) and gold (Au) were produced. The proportions of these elements also indicate which element influences the distribution of this deposit the most. The Pt contributes 56% of the total PGE (see Figure 4.11 c)) appears to have a normal distribution with the mean of 4.23 g/t and a median of 3.64 g/t. The Pd is slightly skewed with the mean of 2.65 ppm and a median of 2.25 ppm. The Pd contributes 46% of the total PGE (see Figure 4.11 b)).

Similarly to the platinum the rhodium appears normal with a mean of 0.59 and median of 0.51. The Rh contributes 0.07% of the total PGE (see Figure 4.11 a)).The Au is positively skewed which is expected of the gold. It has a long tail to the right, with a mean of 0.02 and median of 0.01. The gold contributes 0.003% of the total PGE (see Figure 4.11 d)).

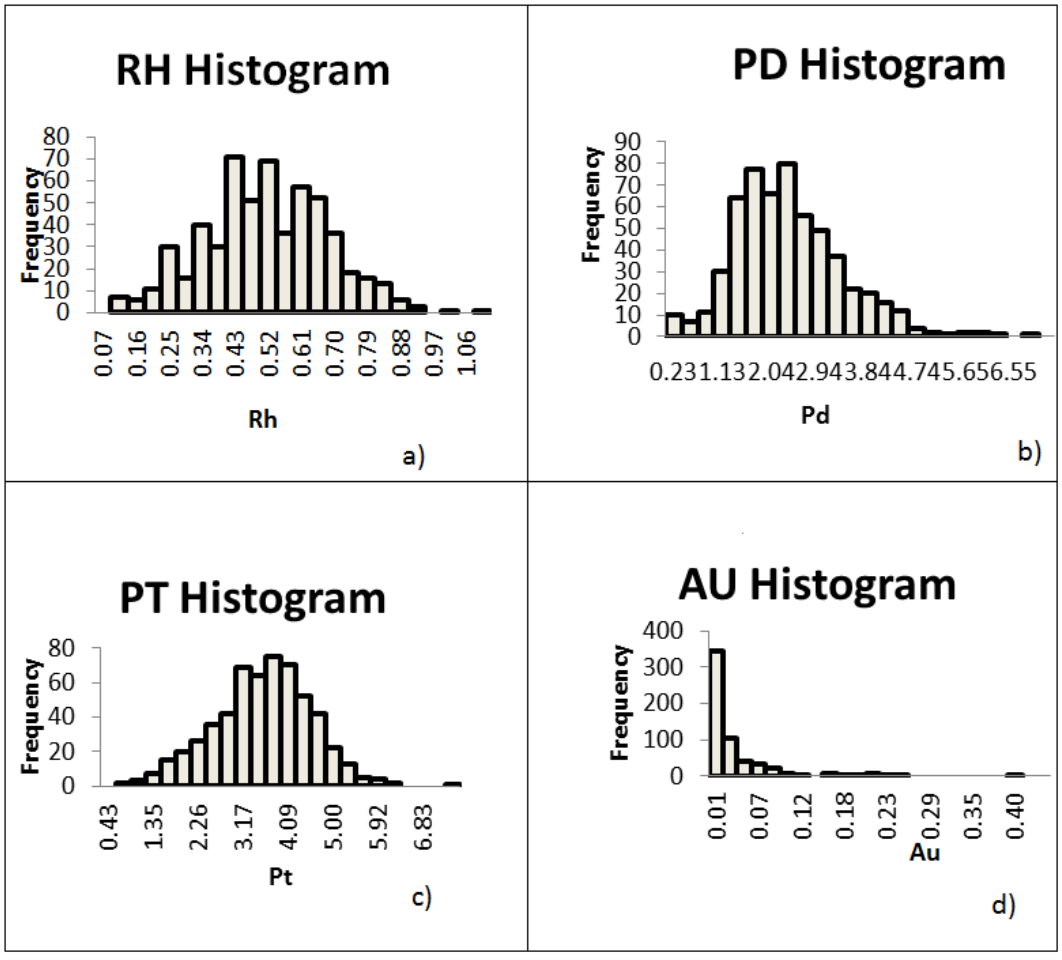


Figure 4.11: Histograms of the 570 analyses of Rh, Pd, Pt and Au

It is quite clear that platinum contributes the most to this PGE deposit followed by palladium. The two elements have the most influence on this PGE deposit and are possibly responsible for the bimodality of this deposit since both elements have slightly different distributions (see Figure 4.11).

Table 4.3: Descriptive statistical table for Pt, Pd, Rh and Au

Statistic	Pt	Pd	Rh	Au	PGE
mean	4.23	2.65	0.59	0.02	5.76
mode	3.59	1.72	0.44	0.05	4.42
median	3.64	2.255	0.51	0.01	5.91
min	0.78	0.13	0.1	0.007	1.12
max	7.46	6.73	1.08	0.41	14.87
St dev	0.98	1.06	0.16	0.04	2.2
Cov	0.23	0.4	0.3	0.5	0.38
Distribution	Normal	Lognormal	Normal	Lognormal	Bi-modal

The PGE deposit has a mixture of the distributions with Pt, Rh being normal and Pd and Au being lognormal with CoVs greater than 0.33. The mixture of normal and lognormal distribution could explain why the combination of these elements (PGE) has a bimodal distribution.

4.4 Conclusion

The grade distribution of the PGEs is bimodal, it is non-normal. This suggests that there is more than one population that exists in this data set. However no concrete conclusions can be made about the formation of these populations at this stage. The probability plot also suggests that more than a single population exists in this data; it is possible that a trimodal distribution exists. The existence of a trimodal distribution is investigated further in Chapter 5. The probability plot is not a straight line, which supports the idea that this data is non-normal and that the parent population could be lognormally distributed. The PGE grade sample plots (see Figure 4.10, 4.9 and 4.7) suggest that an even distribution of low and high grades exists, and that there is an overall mixture of low and high grade values throughout the deposit. A small distinct pocket of low grade PGE values is observed in the SE corner of these grade sample location plots. This pocket of low grade PGE values is small and seems not to affect the overall observation of the mixture of low and high grade values in this deposit, this will however be investigated further in the following chapter.

Chapter 5: Application of Variography

5.1 Introduction

In preparation for variography the data statistics should be understood in order to identify the distribution of the underlying data and the existing populations. Not all data sets will have a single population distribution; this will depend on the style of mineralisation and the geological structural controls. In Chapter 4 first, the PGE data appeared to have at least two populations because the data has a bimodal distribution. Secondly, the log probability plot of the PGE data in Figure 4.8 suggested that there could be at least three populations in this data and this confirms that the distribution of this data is non-normal.

To further investigate the spatial characteristics of the data before variography, it must be determined whether it is possible to divide the data into domains or not. Domaining is a process that involves separating data according to common characteristics until a single population of the data exists (Coombes, 2008). A deposit can have more than one domain if the data has a number of preferred orientations for continuity and complex structural controls. Domains need to be defined concisely so that there is a good understanding and handle of a given data set to be used for estimation.

5.2 Domaining

Domaining should always be considered when preparing data for estimation. Glacken and Snowden (2001) define domains as areas or volumes with similar geological and mineralisation characteristics. Glacken and Snowden (2001) suggest that domains can be defined by cut-off grades, or by global and local statistical means. Duke and Hanna (2001) suggest that not all deposits contain mineralisation which has clearly defined domain boundaries.

5.2.1 Domaining of PGE data

The spatial distribution of PGE grades seems to show no strong trend in any particular direction or orientation, even though high and low grades are displayed, but the sections to follow investigate this idea further. The low and high grades appear to be evenly distributed which is evident in the colour-coded plots of the PGE grades in Figure 4.7, 4.9 and 4.10. There is a small cluster of low grades in the

South-Eastern corner of the mineral deposit but it does not have a significant effect on the overall distribution of grades in this deposit. There appears to be a mixture of low and high grade values and there are no clearly defined geological features to allow domaining.

5.3 Contouring

To further investigate whether there are any trends displayed by the data, grade contour maps were computed at different intervals on Surfer 7. The different colours indicate the PGE grades with their preferred direction of mineralisation. The contour maps a),b),c) and d) are drawn at 0.5 g/t, 0.7g/t, 1g/t and 2 g/t intervals respectively (see Figure 5.1). The different intervals are investigated so that there can be more than one view of the spatial distribution of the PGE grades and not to miss any trends that might exist in this mineral deposit.

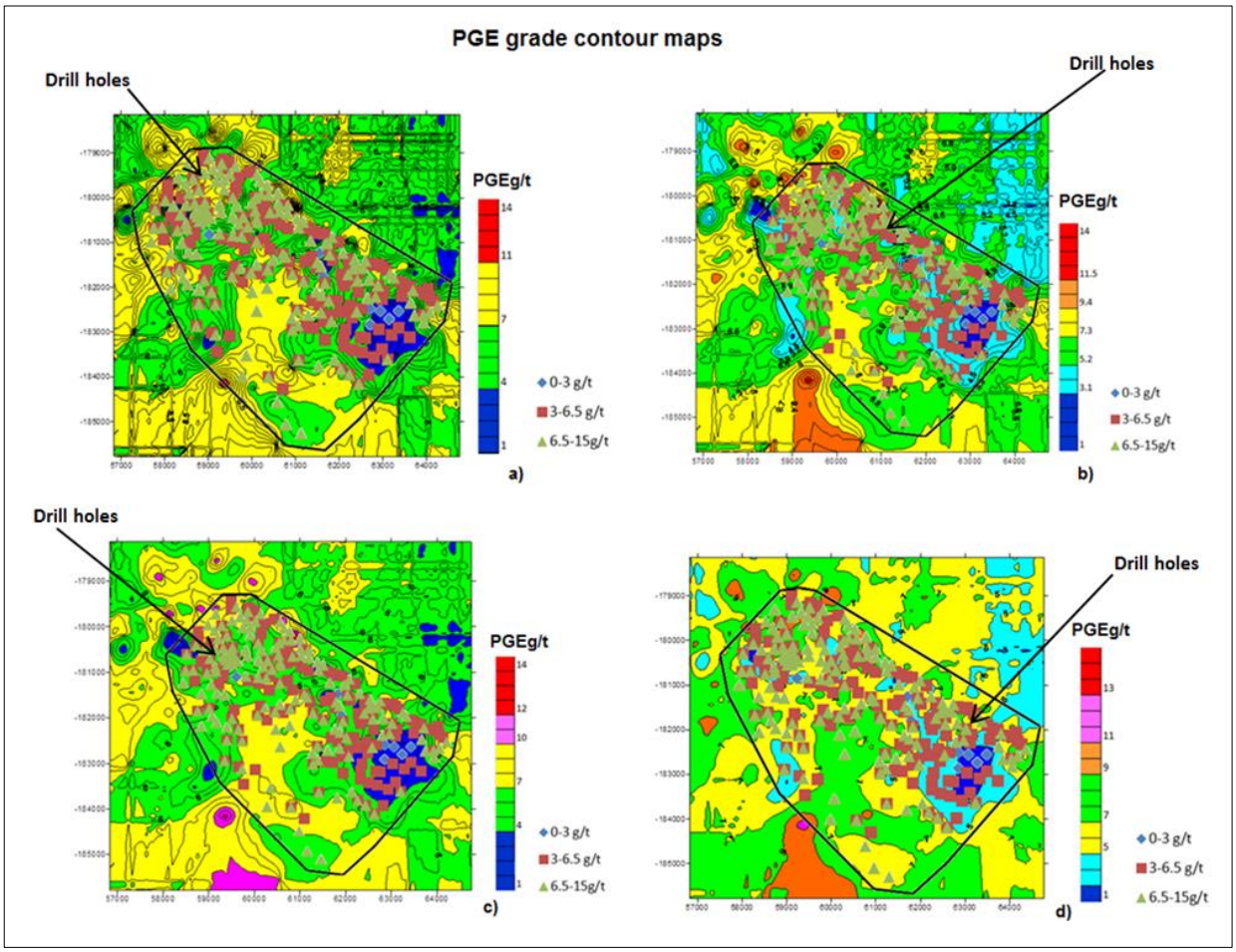


Figure 5.1: Contour maps of the PGE grades a) at 0.5 g/t interval b) at 0.7 g/t interval c) at 1 g/t interval and d) at 2 g/t interval

In Figure 5.1 no strong trend is visible in the contour maps; however the section on variograms will further investigate this. Low grades and high grades are for the most part, evenly distributed throughout the deposit. There is however a significant concentration of low grades in the eastern corner of the deposit. There seems to be no clear preferred orientation of the PGE grades in all the contour maps.

The PGE grades ranging from 4 g/t to 10 g/t prove to be dominant and are evenly distributed in the contour maps (see Figure 5.1 b)) at a 0.7 g/t interval. There seems to be a weakly developed trend in the NW-SE direction in these contour maps. There is, however not enough evidence to support the idea that a trend exists in the NW-SE direction.

In all the contour maps, medium to high grade areas stand out. Some weak trends are developed and continue for a short distance in some areas. The mineral continuity in the NS direction seems to be equivalent to the mineral continuity in the EW direction. It is anticipated that the variogram nugget will be low and the range will extend to large lag distances as the variability of grades seems to be low. Low nugget is an indication of low variability between samples next to each other, so the probability of change is low, and the mineralisation is continuous. The long range shows strong spatial dependency or relationship between sample values over a long distance.

5.4 Variograms

In Chapter 1 it is mentioned that Supervisor 8 will be used for variogram analysis. The PGE data is imported to Supervisor 8 and a further investigation of what is indicated in section 5.2.1 of domaining is undertaken. The following assumptions are essential when computing semi- variograms:

1) The sample grades are sourced from a single grade population i.e. only one domain and
2) The difference in grade between pairs depends on their relative separation (Coombes, 2008).

In order to assess the existence of the direction of maximum continuity, the semi-variogram fans and semi-variogram models are constructed. The semi-variograms are constructed in different directions and Figure 5.2 a) and b) show the horizontal continuity semi-variogram fans at lag 175 m and 500 m respectively.

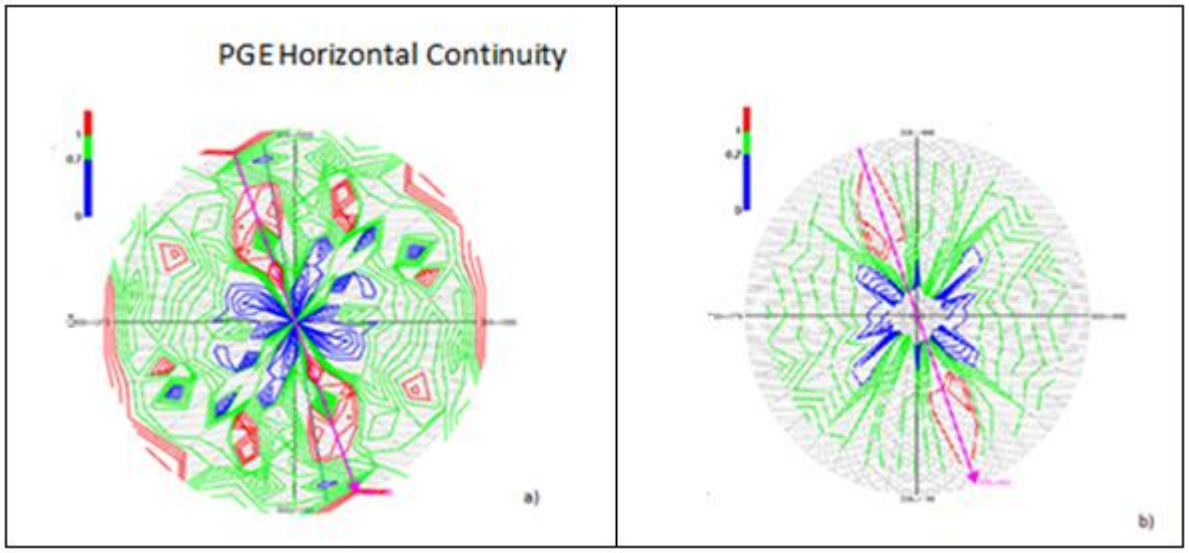


Figure 5.2: Horizontal continuity variogram fans of the PGE grades a) at lag 175 m and b) at lag 500 m

A lag distance of 175 m is chosen, which equates to half the number of the average drill hole spacing of the deposit. It is chosen to capture a clear continuity of the deposit and indicate if there are any preferred directions of the PGE grades at this lag distance. A larger lag of 500 m is also investigated to observe the same (see Figure 5.2 b)). On both the variogram fans there is no clear preferred direction of maximum continuity.

Figure 5.3 shows variograms for both 175 m and 500 m lags in a 160^0 direction. The variograms obtained are not clear and for the 175 m lag the variogram is noisy (see Figure 5.3 a)). The variogram at 500 m lag does not give a good presentation of the mineral deposit; it has a very high nugget effect (see Figure 5.3 c)). A better variogram is obtained at 130^0 (see Figure 5.3 b)) at lag 175 m.

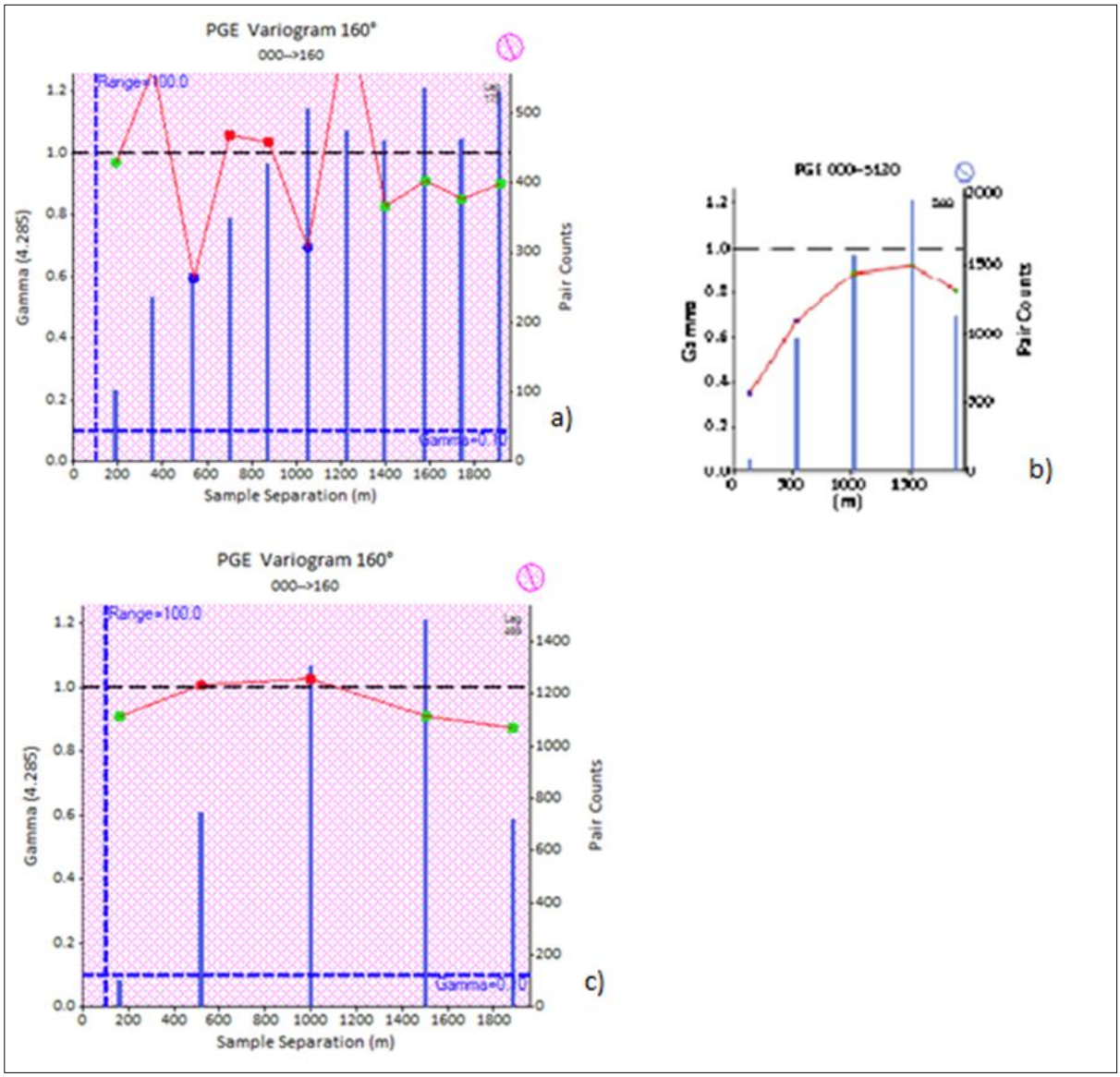


Figure 5.3: a) Variogram at 160^0 with a lag of 175 m and b) at 130^0 which looks better than the a) and c) Variogram at 160^0 with a lag of 500 m

Further investigations on the across strike and dip plane variogram fans are analysed. The across strike variogram fan shows some continuity at 70^0 which does not provide sufficient information about the continuity of the overall PGE grades (see Figure 5.4 a)). The dip plane variogram fan shows some unclear continuity at 9° (see Figure 5.4 b)).

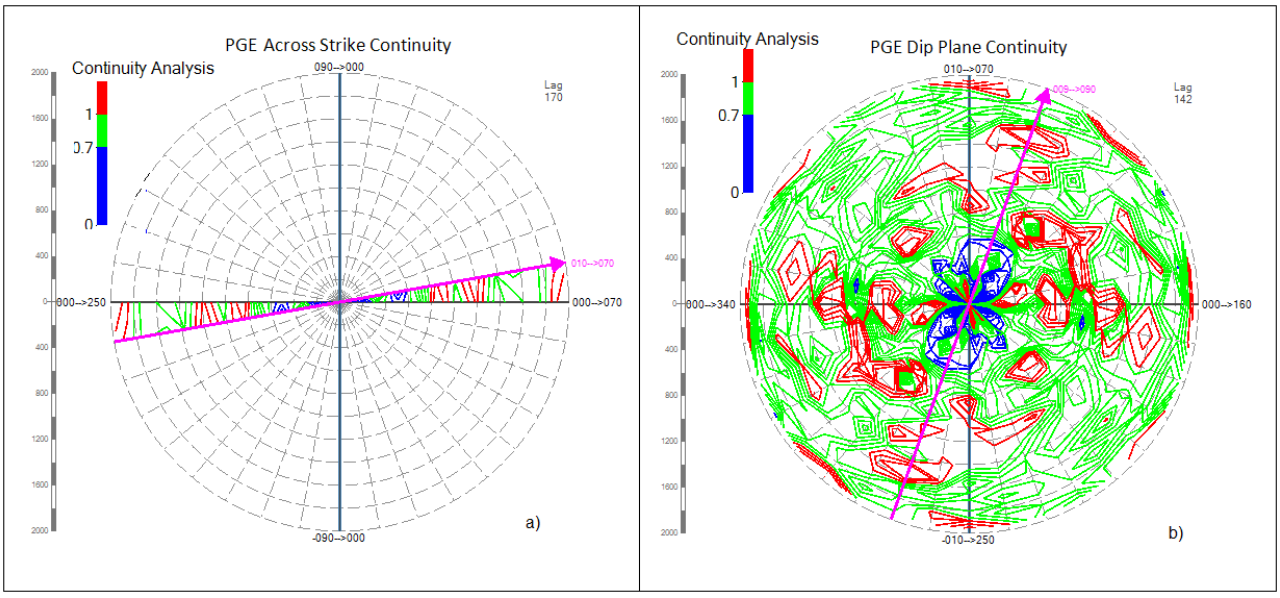


Figure 5.4: a) Across Strike variogram fan and b) Dip plane variogram fan

It can be concluded that, there is no clear preferred direction of maximum continuity in this PGE deposit. This conclusion is justified by the lack of anisotropy,displayed by the semi-variogram fans, the colour-coded plots in Figure 4.7,4.9, 4.10 and the contour grade maps in Figure 5.1 .

5.5 Experimental Variograms

In the absence of a clear or strong trend displayed by the PGE data, the omnidirectional semi-variogram was selected as a semi-variogram that best represents this data.

A series of omnidirectional semi-variograms have been modelled (see section 5.5.1). In each omnidirectional semi-variogram, a spherical model was fitted and different lags were chosen to model these semi-variograms.

5.5.1 Variogram modelling

All the omnidirectional semi-variograms modelled are two structured with sill components C_1 and C_2. The first omnidirectional semi-variogram is modelled at lag 70 m which is the smallest lag at which this PGE data is modelled (see Figure 5.5 a)). The omnidirectional semi-variogram shows significant variability and when this semi-variogram was modelled, fitting a spherical semi-variogram was a challenge due to the erratic behaviour of this semi-variogram. The omnidirectional semi-

variogram at a lag of 175 m is clearer than the omnidirectional semi-variogram at a lag of 70 m (see Figure 5.5 b)).

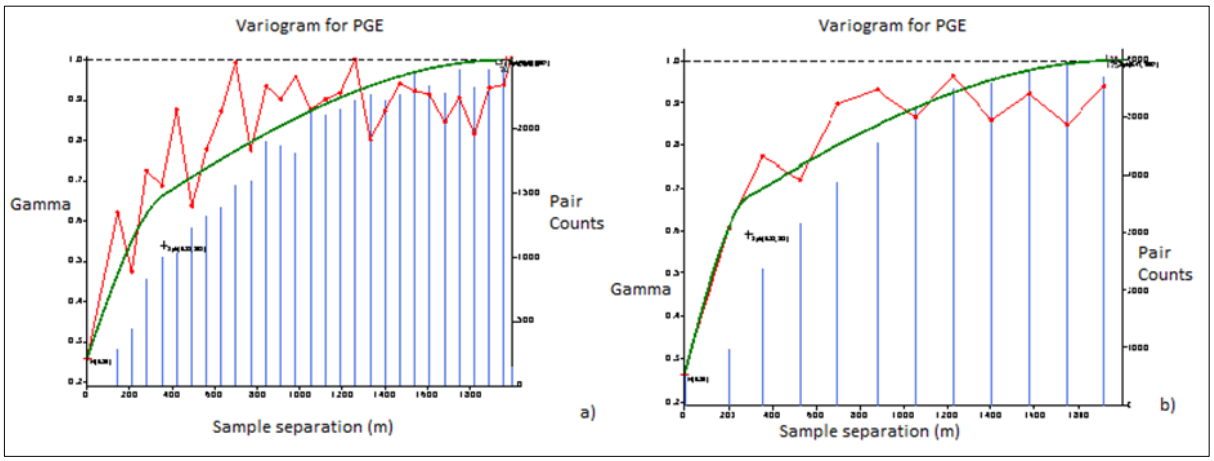


Figure 5.5: a) Omnidirectional semi-variogram at lag 70 m b) and at lag 175 m

Table 5.1: The parameters of the PGE semi-variograms at lag 70 m and 175 m

Parameter	**PGE(4E) Lag of 70 m**	**PGE(4E) Lag of 175 m**
Nugget Effect	0.26	0.26
Type of variogram	Spherical	Spherical
No. of structures	2	2
Sill of component 1 C_1	0.72	0.67
Sill of component 2 C_2	0.02	0.08
First range of influence	392 m	511 m
Second range of influence	1543 m	2304 m
Lag	70 m	175 m
No. of Pairs	2466	5924

The semi-variogram parameters are listed in Table 5.1 for the omnidirectional semi-variograms at a lag of 70 m and 175 m respectively. Both the semi-variograms have a nugget effect of 0.26 but their sill components differ. The semi-variogram at a 70 m lag has fewer data pairs than the semi-variogram at a175 m lag.

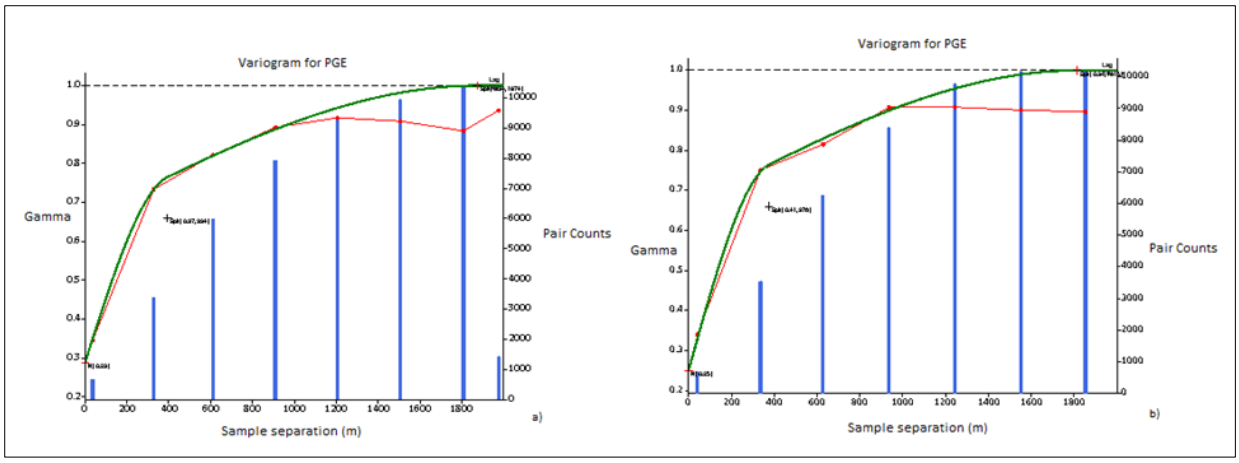


Figure 5.6: a) Omnidirectional semivariogram at lag 300 m and b) at lag 310 m

Figure 5.6 a) and b) shows omnidirectional semi-variograms at lag 300 m and 310 m respectively. Both semi-variograms appear smooth and show a lack of variability in the data, this does not appear to be representative of the behaviour of the original data.

Table 5.2: The parameters of the PGE semivariogram at lag 300 m and 310 m

Parameter	**PGE(4E) Lag of 300 m**	**PGE(4E)Lag of 310 m**
Nugget Effect	0.29	0.25
Type of variogram	Spherical	Spherical
No. of structures	2	2
Sill of component 1 C_1	0.37	0.41
Sill of component 2 C_2	0.34	0.34
First range of influence	394 m	376 m
Second range of influence	1874 m	1815 m
Lag	300	310
No. of Pairs	10404	10178

Table 5.2 shows the semi-variogram parameters of these omnidirectional variograms. It is noted that the semi-variograms start to smooth out as the lag size increases. In Section 5.3 it is mentioned that a low nugget value is expected from this deposit as it appears to have low variability, therefore the semi-variogram with a nugget value of 0.25 may be more favorable than a semi-variogram with a nugget value of 0.29.

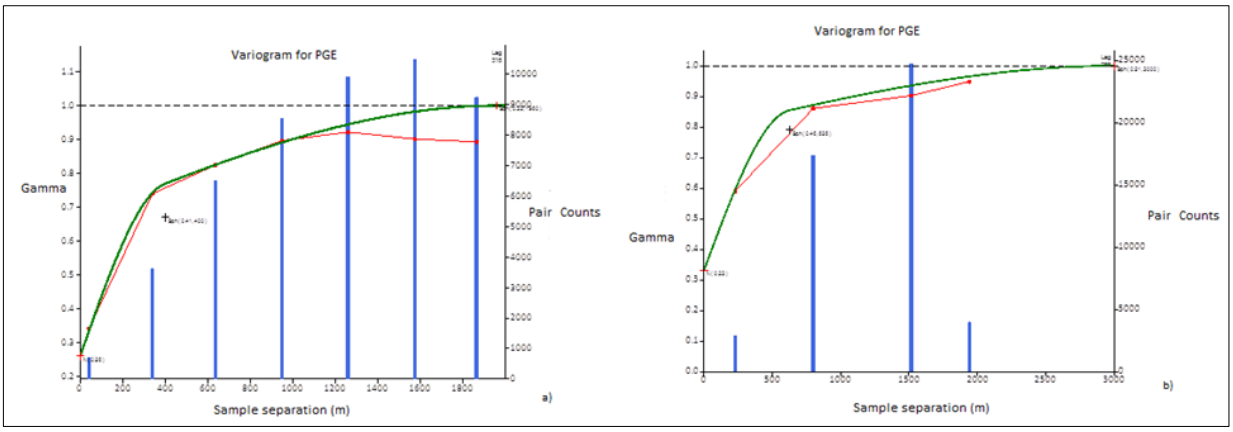


Figure 5.7: a) Omnidirectional semivariogram at lag 315 m and b) at lag 755 m

Figure 5.7 a) and b) shows omnidirectional semi-variograms at lag 315 m and 755 m respectively. While the semi-variogram at lag 315 m displays the best results, it seems to not necessarily be representative of the deposit. The omnidirectional semi-variogram at lag 755 m (the largest lag chosen to model these omnidirectional semi-variograms) is too smooth and does not accurately represent the deposit.

Table 5.3: The parameters of the PGE variogram at lag 315 m and 755 m

Parameter/Variable	PGE(4E) Lag of 315 m	PGE(4E) Lag of 755 m
Nugget Effect	0.27	0.33
Type of variogram	Spherical	Spherical
No. of structures	2	2
Sill of component 1 C_1	0.46	0.46
Sill of component 2 C_2	0.27	0.21
First range of influence	472 m	626 m
Second range of influence	1995m	3000m
Lag	315	755
No. of Pairs	10494	24764

Table 5.3 shows the semi-variogram parameters of the omnidirectional semi-variograms at lag 315 m and 755 m. The omnidirectional semi-variogram at lag 755 m has a relatively higher nugget effect of 0.33 (see Table 5.3) which further justifies the notion that this semi-variogram model is not representative of this PGE deposit.

5.6 Conclusion

Larger lag tolerances accommodate many numbers of pairs for estimation; however some detail is lost in the semi-variogram (see Figure 5.7 b)). The semi-variogram in Figure 5.7 b) has 24764 pairs but appears smooth, it seems that some detail is averaged and lost; this is due to the fact that the semi-variogram is modelled at a large lag distance of 755 m and at a relatively high nugget value. The first range of influence in this semi-variogram is at 626 m which is larger than the average borehole spacing and there is minimal correlation of samples beyond this distance. The nugget effect of this semi-variogram is 0.33 (see Table 5.3) which is relatively high therefore it is concluded that the parameters of this semi-variogram are inappropriate to be used for the purpose of this estimation.

High nugget values have a smoothing effect on the kriging results. At high nugget values sample points far away from the block estimated are assigned equal weights to the points closer to the block estimated.

At a lag of 300 m the semi-variogram has a relatively high nugget effect of 0.29 (see Table 5.2).Even though the semi-variogram appears to be representative; its parameters cannot be used for the estimation process. At a lag of 315 m the nugget effect is high and the semi-variogram does not appear to be representative. The selection of the appropriate semi-variogram remains with the three semi-variograms from Figure 5.5 a), b) and 5.6 b). The three semi-variograms have low nugget effects; Figure 5.5 a) and 5.5 b) have semi-variograms with nugget effects of 0.26 and which are at different lags. Figure 5.6 at a lag of 310 m has a semi-variogram with the smallest nugget effect of 0.25 (see Table 5.2). The semi-variogram model is however smooth.

It has been a challenge to decide which semi-variogram to use for estimation between the semi-variogram at a lag of 70 m and 175 m. The reason being that the lag of 175 m clearly captures the behaviour of this PGE deposit, the semi-variogram shows some variability and seems to be representative (see Figure 5.5 b)).

The semi-variogram at a lag of 70 m includes the smallest data pairs (see Table 5.1) but shows variability at a reasonable range of influence and has a small nugget effect. The semi-variogram at a lag of 70 m was chosen to be used for estimation.

Chapter 6: Application of OK and SK

This chapter focuses on the application of SK and OK to the PGE (4E) deposit. The grade block model, where both SK and OK are applied is created and the key differences between the two techniques are summarised.

6.1 The PGE Model

The PGE (4E) deposit was modelled using Surpac version 6.2.1 and Figure 6.1 summarises the procedure followed to create the PGE (4E) block model.

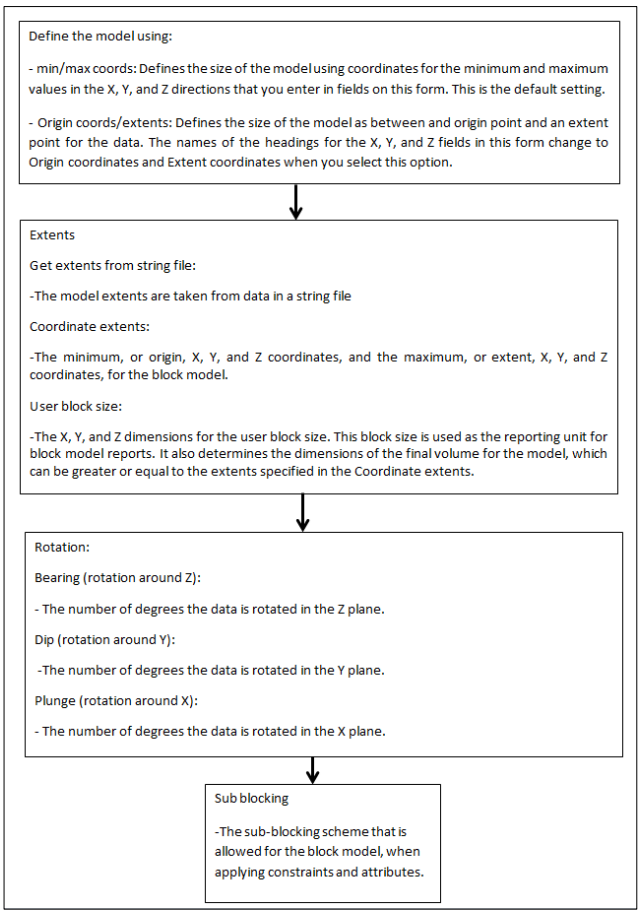


Figure 6.1: Block model creation procedure (Gemcom, 2012)

A 250 m x 250 m x 10 m block size was chosen to model this PGE resource. This size was chosen because industry standards state that the block sizes must not be smaller than half the drill hole spacing when classifying measured resources (SAMREC, 2007). The average distance between drill holes was calculated to be 350 m hence the 250 m x 250 m x 10 m block size was used to model this PGE resource.

The choice of an appropriate block size improves the reliability of the estimates such as large blocks with dimensions close to that of the average sample spacing. Very small blocks, lower than the calculated average drill hole spacing normally have high estimation variances and that is undesirable in mineral resource estimation. A high estimation variance is associated with smoothing of estimated values which can lead to overestimation of the mineral resource (Dominy et al., 2002).

In Surpac 6.2.1 software the block model origin is defined using minimum X, Y and minimum Z (see Table 6.1). The 250 m x 250 m x 10 m block model generated about 6103 blocks.

Table 6.1: PGE block model parameters

Description	250 m x 250 m x 10 m block model
Origin	X= 52000 m; Y= -18700 m, Z = 870 m
Block size	250 m x 250 m x 10 m
Number of blocks	6103

SK and OK estimation techniques were applied on the model created and the estimation covers the whole deposit which is 35381009 m^2 in extent (see Figure 6.2). Figure 6.2 shows the 250 m x 250 m x 10 m block model generated from Surpac version 6.2.1.

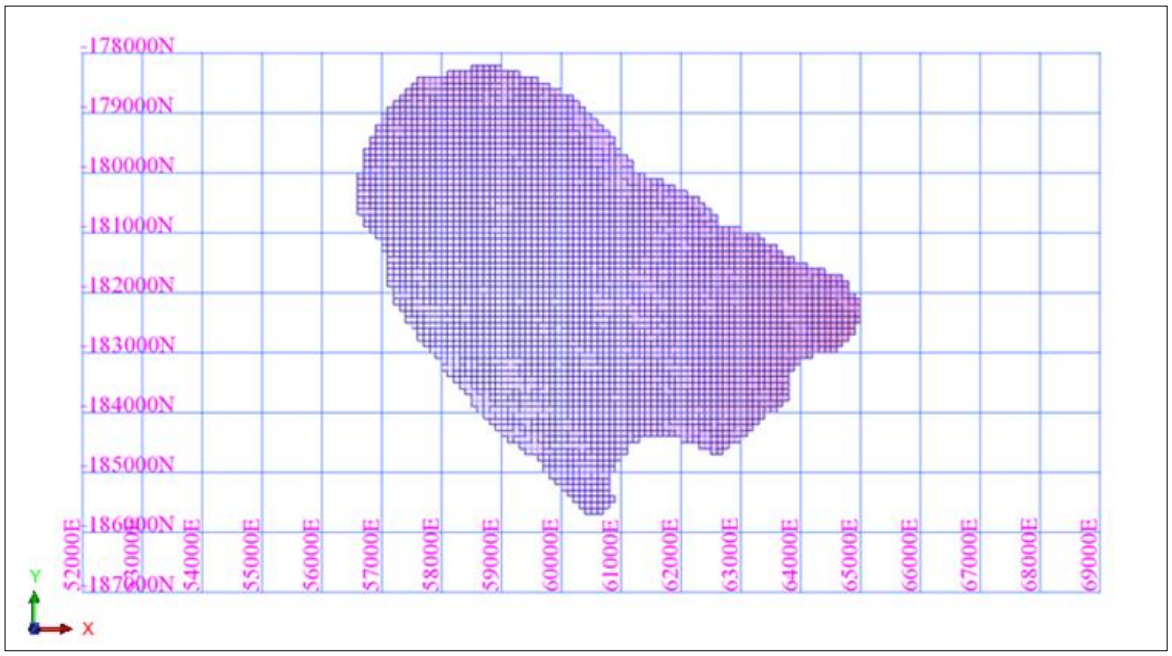


Figure 6.2: PGE (4E) block model generated using Surpac Version 6.2.1

6.2 Kriging

In Chapter 2, Kriging equations are discussed and those equations are used in Surpac 6.2.1 software. First, OK is applied to the data set after which SK is applied; for both OK and SK the same semi-variogram parameters are used (see Table 5.1). The outputs which Surpac produces are the kriging variance, block variance, kriging efficiency, and number of samples used; the kriged estimate as well as the Lagrange multiplier (see Table 6.2).

According to Snowden (2001) a perfect estimation would give values of kriging variance = 0, kriging efficiency = 100% and a slope of regression =1

Kriging variance equations for SK and OK are provided by equation 9 and 29 respectively in Chapter 2. According to Snowden (2001), kriging variance highlights the relative confidence from block to block and also exposes areas which require more drilling.

Kriging efficiency (KE) is defined as shown in equation 1:

$$KE = \frac{(BV - KV)}{BV}$$

$$KE = \frac{({\sigma_B}^2 - \sigma_K^2)}{\sigma_B^2}$$

$$KE = \frac{\bar{\gamma}(V,V) - \sigma_K^2}{\bar{\gamma}(V,V)} \quad \text{..... (1)}$$

BV is the block variance (variance of actual block values)

KV is the kriging variance.

According to Coombes (2008), kriging efficiency estimates the percentage overlap expected between the estimated grades and the true grades. A 100 % kriging efficiency indicates a perfect match between the estimated and true grade distributions. Krige (1996) defines kriging efficiency as a measure of the efficiency of the estimation procedure. Negative kriging efficiency indicates sparse data or an extrapolation more than interpolation of data (Coombes, 2008).

Krige (1996) states that when a global estimate of blocks is practical, all blocks get assigned a global mean, the global estimate of all blocks is the only estimate made and KV = BV, therefore KE is:

$$KE = \frac{(BV-BV)}{BV} = 0\%$$

He further suggests that this results to imperfect estimation. Deutsch et al. (2006) suggests that negative kriging efficiency results when KV> BV. This negative efficiency is normally observed when there is inadequate data per block (Deutsch et al., 2006).

BV is the block variance, the error of block values, defined by:

$$\sigma_B^2 = \sigma^2 - \bar{\gamma}(V,V) \quad \text{..... (2)}$$

Block variance is defined as the sample variance less the within block variance (the average variogram value inside the block) (Clark, 2000).

6.2.1 Results and analysis

Table 6.2 summarises the results from the OK and SK on the 250 m x 250 m x 10 m block model.

Table 6.2: Summary results of the estimation using OK and SK of the 250 m x 250 x m 10 m block model

Attribute	OK	SK
Kriging variance	1.31 $(g/t)^2$	0.56 $(g/t)^2$
Std.dev	2.40 g/t	1.50 g/t
Estimated grade	7.41 g/t	5.76 g/t
Block variance	0.56 $(g/t)^2$	0.56 $(g/t)^2$
Kriging efficiency	-1.32	0.00
CoV	0.15	0.13
Lagrange Multiplier	-0.91	-

The OK variance is greater than the SK variance (see Table 6.2). When recalling the kriging variance equations for both OK and SK, the OK variance has the Lagrange factor added to it, which could explain why it is bigger than the SK variance which does not have the Lagrange factor added to it. This can also be explained by the

idea that the SK mean used in the estimation provides significant, useful and additional information (Assibey-Bonsu, 2014 personal communication).

The OK average estimated PGE grade is greater than the SK estimated grade (see Table 6.2), this means the local mean is greater than the global mean. According to Boyle (2010) in SK, the weights are assigned both to local samples and to the global mean. So if more weights are assigned to the local samples, the global mean can be small.

The block variance for both SK and OK is equal to 0.56 $(g/t)^2$ (see Table 6.2), this could be explained by the fact that both SK and OK block variance equations are the same as well as the block support used for both is the same. The block variance is not helpful, when it comes to differentiating between SK and OK.

The Lagrange multiplier is obtained from OK, but not from SK. Isobel Clark (personal communication, 2012), stated that when the Lagrange multiplier value is large and positive it means that the samples are too far from the point or block being estimated. On the contrary when the Lagrange multiplier is large and negative it means that samples are close to the point or block being estimated. The latter is observed in Table 6.2 where the Lagrange multiplier is negative meaning that the data values are spaced appropriately.

There are insufficient interpretations to be made from just the tabulated results. To further interpret and compare SK and OK, the effects of the number of search data is considered for this PGE data for both techniques.

6.3 Number of search data for the PGE deposit

In the same way that Deutsch et al., (2014) used the three case studies to discuss the effects of the number of search data as shown in Chapter 3, this study also adopts the method of analysis to this PGE data.

Three cases are considered:

a) The first case considered between 3 and 20 local data.
b) The second case considered between 3 and 200 local data
c) The third case considered between 3 and 500 local data to be used to produce each estimate.

For all the three cases the same number of drill holes is used which are 570 and also the same semi-variogram model is used (see Figure 5.5). Strong stationarity of the mean is assumed for this deposit therefore the average global mean of 5.76 g/t is used as the known SK mean.

6.3.1 Case 1

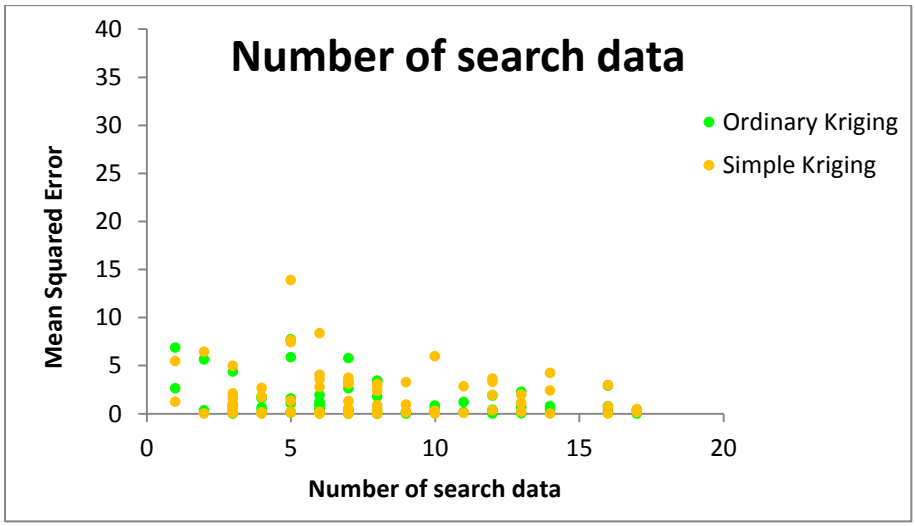


Figure 6.3: Number of search data influence on OK and SK

When local data between 3 and 20 samples is considered for this PGE deposit; as the number of search data increases the mean squared error decreases in the same manner for both OK and SK (see Figure 6.3).

6.3.2 Case 2

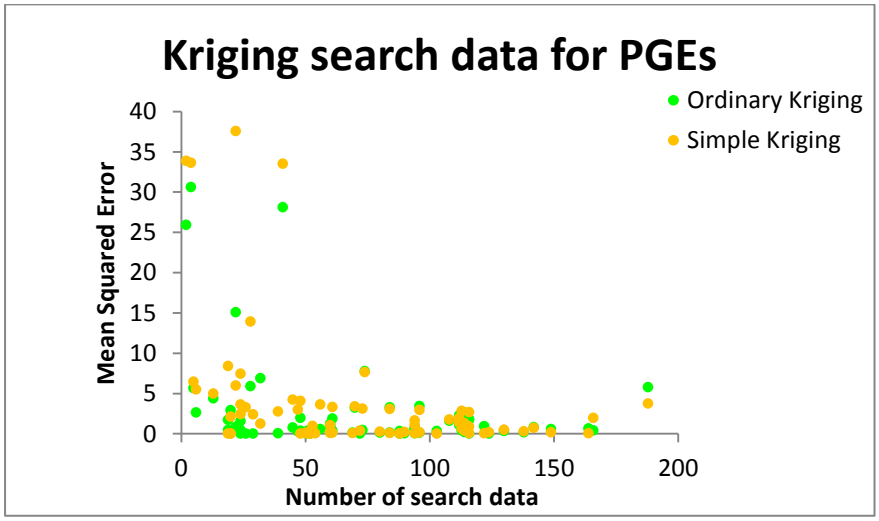


Figure 6.4: Number of search data influence on OK and SK

When local data between 3 and 200 is considered, a similar case is observed wherein as the number search data increases the mean squared error decreases for both SK and OK (see Figure 6.4).

6.3.3 Case 3

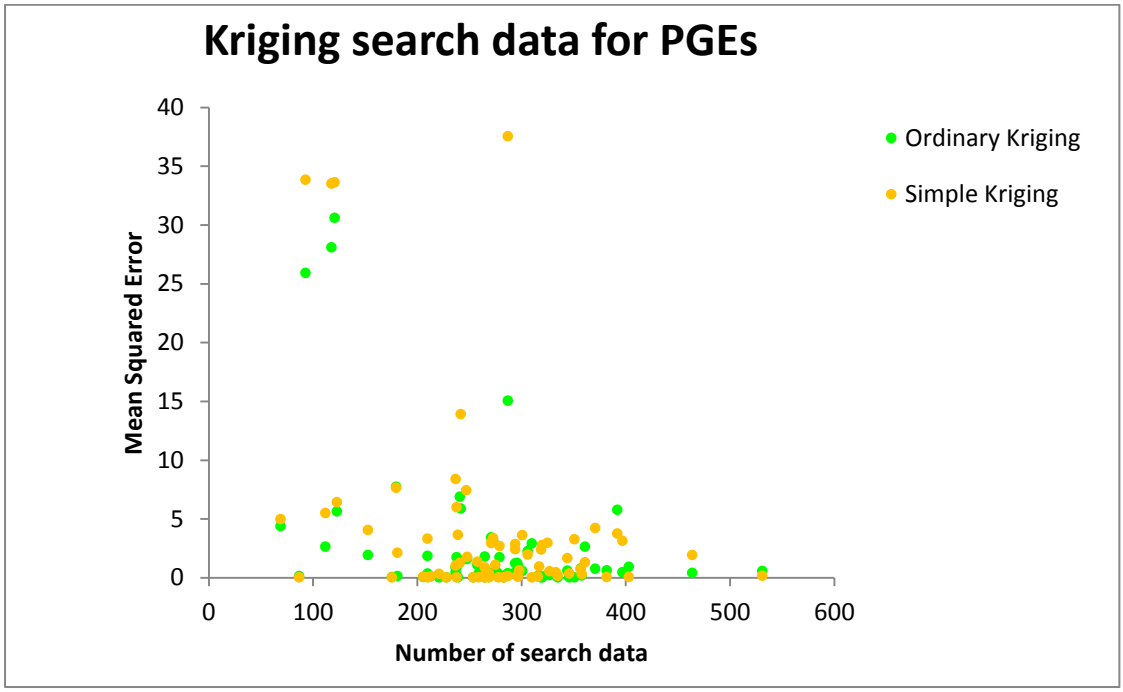


Figure 6.5: Number of search data influence on OK and SK

When local data between 3 and 500 is considered, increasing the number of search data decreased the mean squared error for both OK and SK. For a low number of search data OK performed slightly better than SK, the OK has a lower mean squared error than SK (see Figure 6.5). This could be explained by the suggestion made by Rivoirard (1987), when he said if the weight applied to the mean in SK is small, then the local neighbourhood has a strong influence hence OK performs better.

6.3.4 Analysis

Increasing the number of search data decreased the mean squared error for both SK and OK. Increasing the number of search data for OK increases the accuracy in the estimate of the local mean. What can be noticed is that case1 where the local data is between 3 and 20, a very low mean squared error is observed which suggests that this search is more accurate and would result in more accurate estimates only if the assumption of a stationary mean is emphasised. OK and SK in this particular PGE deposit is little affected by the number of search data, hence in all three cases

investigated there is similar estimation accuracy. In OK the mean squared error average is 3 g/t and for SK the average is 4 g/t this is more evidence showing that there is a slight difference in the performance of OK and SK for this PGE deposit.

To further investigate the differences in the performance of OK and SK, trend analysis is undertaken in section 6.4. This section is undertaken to confirm whether the observation made in section 6.3 of the similar performance in OK and SK is valid or not.

6.4 The PGE data trend estimates

In the same way that Goovaerts (1997) analysed the 10 cadmium samples as shown in Chapter 3, the PGE data of this study is analysed.

Figure 6.2 shows the PGE block model produced from Surpac version 6.2.1. In the figure it is shown that the direction chosen for the analysis is E-W starting from 56000 m to 65500 m displayed by the grid, this covers the whole deposit in this particular direction. Any direction could have been chosen for this analysis, since earlier in the study it was concluded that the grades are evenly distributed and the same level of spatial continuity is observed in all directions. For this analysis the grades were considered every 100 m (see Appendix C the data results).

First the original data of the PGE grades was analysed over the distance between 56000 m and 65500 m. The results obtained are shown in Figure 6.6, where low grade values appear to occupy the right most part of the graph as indicated with a circle. This corresponds with what has been observed of this PGE deposit where a small grouping of low grade PGEs occupies the SE corner of the deposit (see Figure 5.1).

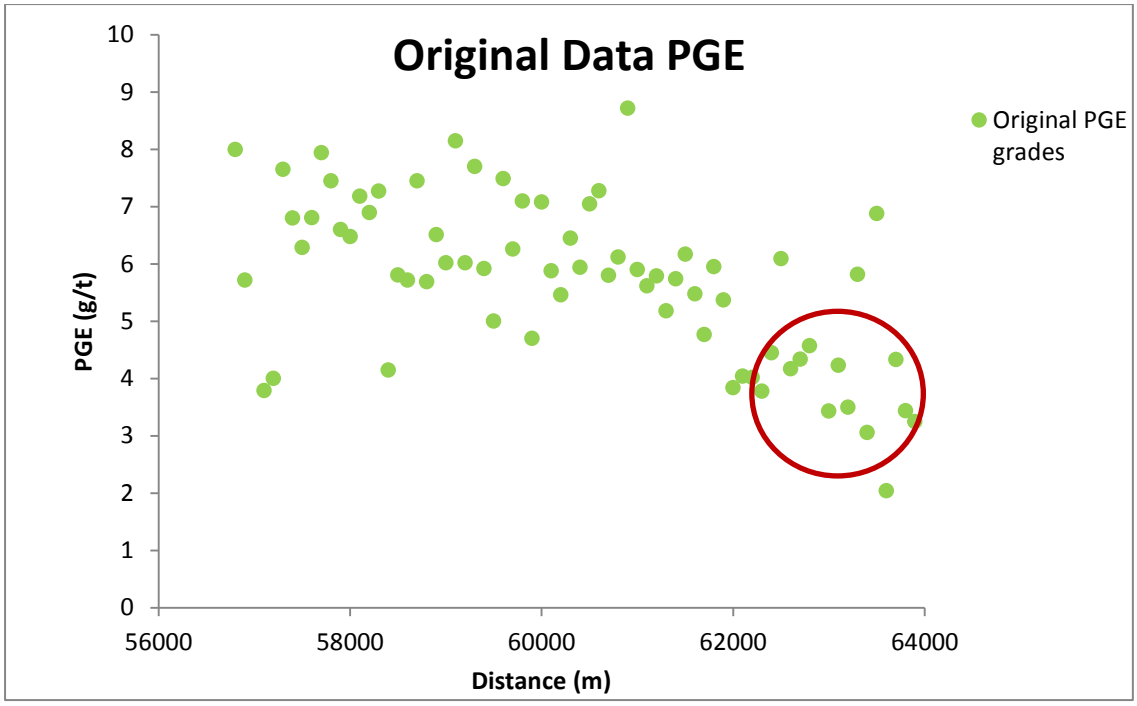


Figure 6.6: Original PGE grades trends

Most of the high grade PGE values appear to occupy the left edge of the graph (see Figure 6.6), but in between at 58000 m and 62000 m is the general trend of a mixture of low and high grades.

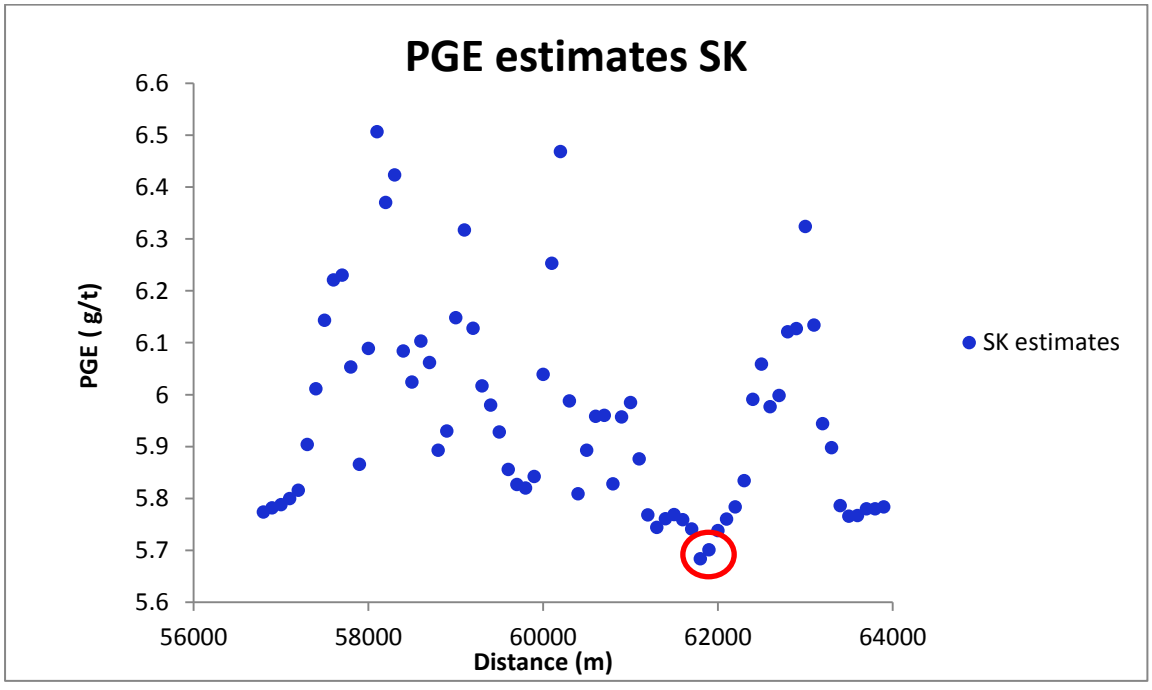


Figure 6.7: SK PGE estimates

The PGE SK estimates were also generated over the same distance of 56000 m and 65500 m. The results obtained are shown in Figure 6.7, where a slightly different behaviour of the PGE grades is observed from that of the original data. Only a mixture of grades is observed throughout the investigated distance. What can be noted though is a small portion of grades less than 5.76 g/t between 61000 m and 62000 m (see circle Figure 6.7).

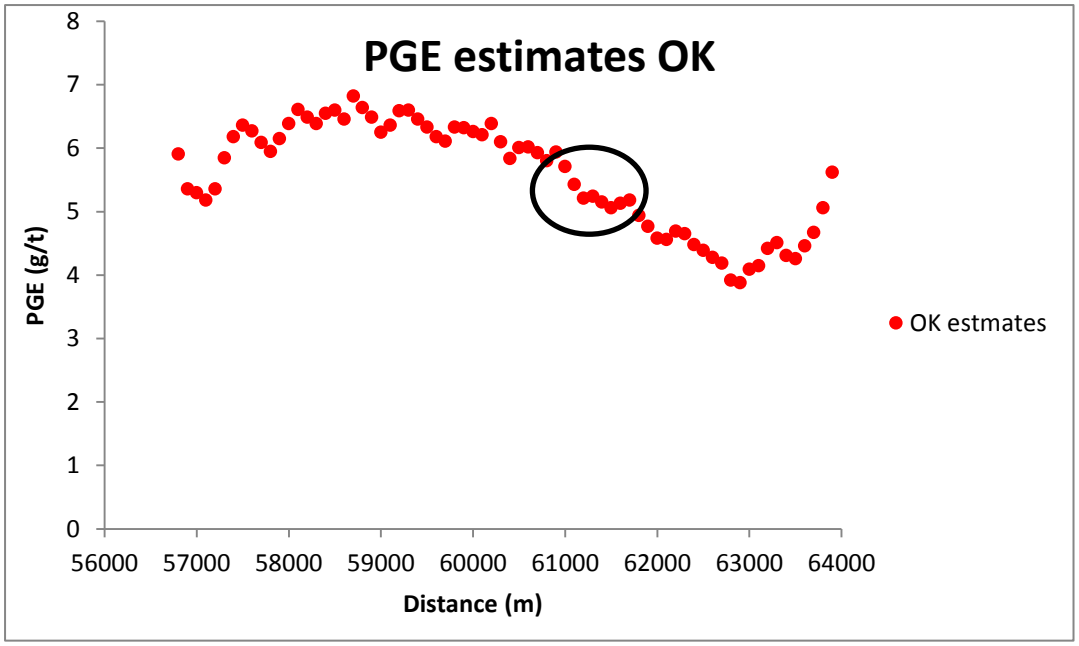


Figure 6.8: OK PGE estimates

The results of the PGE OK estimates are shown in Figure 6.8. What can be noted from this graph is that the low grades appear to occupy the right most edge of the graph which is similar to the behaviour of the original PGE grades. The left most edge of the data appears to be occupied by high grades .What should be noted in this graph is that grade values are not scattered widely as seen in SK, and there is a clearly defined point of only high to low grades at 61000 m (see the circle in Figure 6.8).

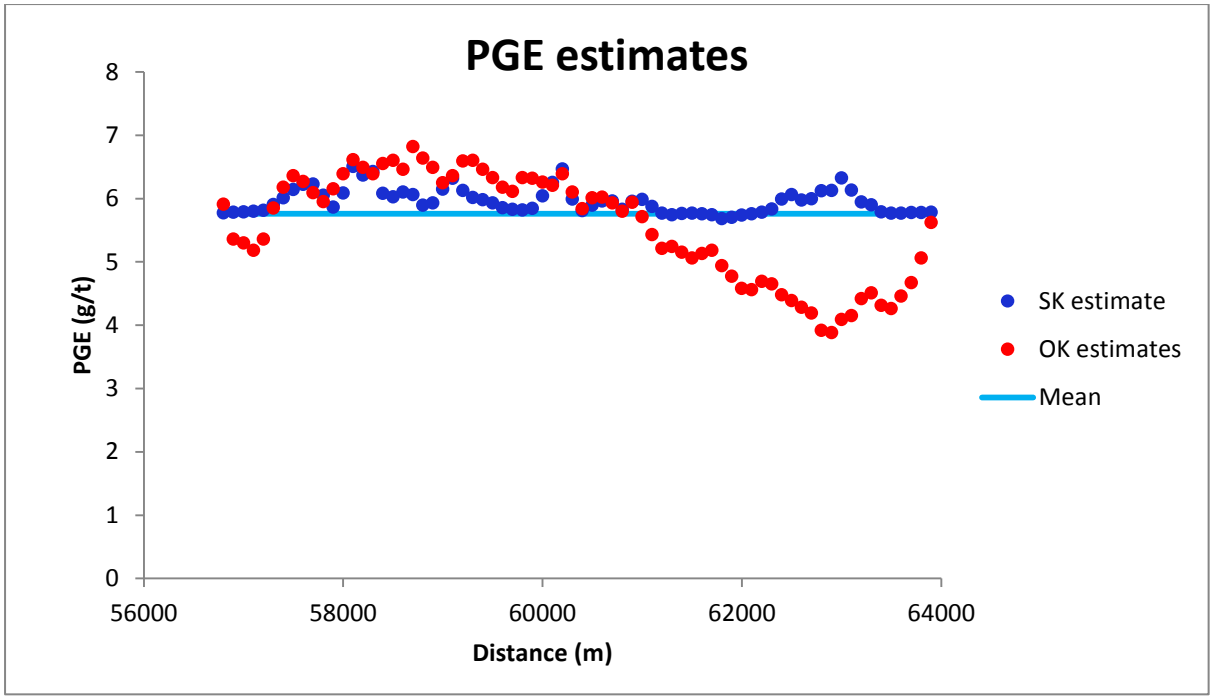


Figure 6.9: SK, OK estimates and the global mean

The global mean of the PGE data is compared with the OK and SK estimates (see Figure 6.9). The overall SK estimates appear closer to the global mean. The OK estimates are larger than the global mean between 57000 m and 61000 m. This is explained by the fact that the local mean in this area is larger than the global mean (see Appendix C results). On the right edge of this graph the OK estimates are smaller than the global mean. This can also be explained by the fact that the local mean in this area is smaller than the global mean (see Appendix C results).

In summary it appears that the OK estimate is smaller than the SK estimate in low-valued areas where the local mean is smaller than the global mean. In contrast, the OK estimate is larger than the SK estimate in high valued areas where the local mean is larger than the global mean. The discrepancy between the two estimates increases as the weight of the mean increases for example when the location being estimated moves further away from the data locations (the farthest edges in the graph) (Goovaerts, 1997). This means that in OK the weights are assigned to local samples and to locally varying mean whereas in SK weights are assigned to local samples and the global mean (Boyle, 2010).

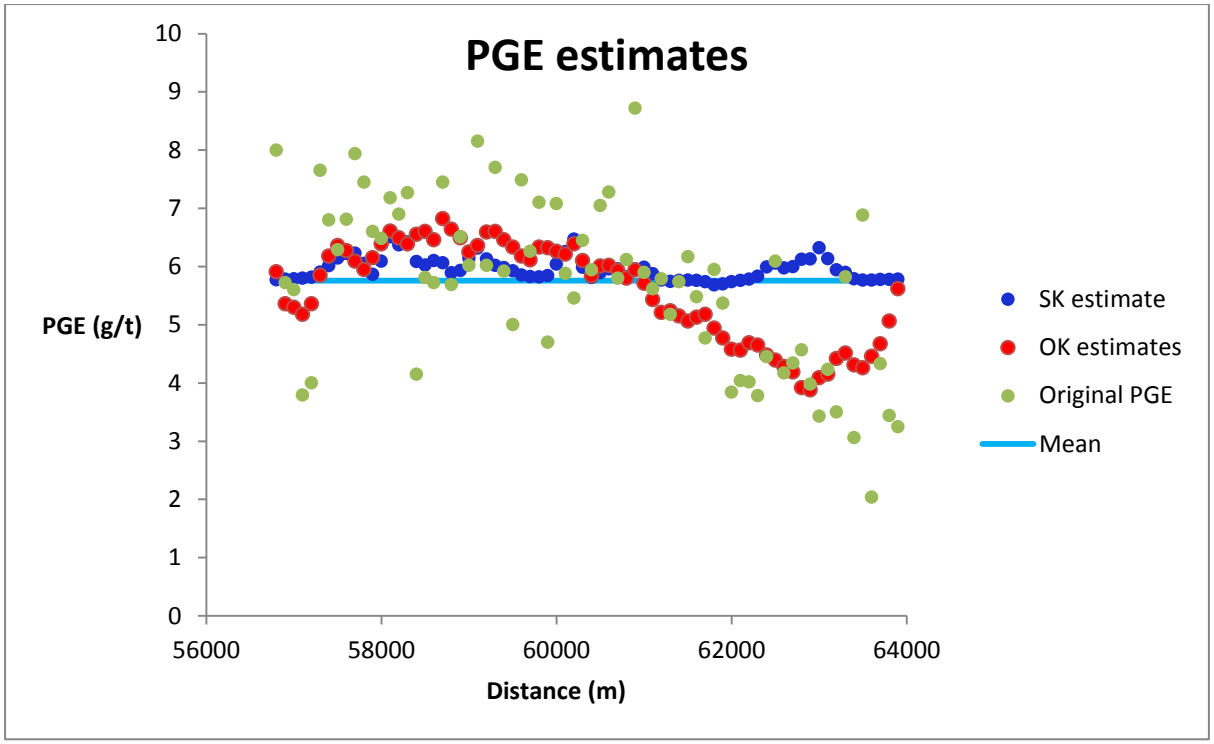


Figure 6.10: SK, OK estimates, global mean and original PGE data

The original PGE data is compared with the OK and SK estimates (see Figure 6.10). Most of the original data appears to be greater than the global mean in the left part of the graph between 57000 m and 61000 m and it appears to be less than the global mean in the right part of the graph (see Figure 6.10). The same is observed for the OK estimates meaning that the OK method better estimates this PGE data. The low grade values are observed between 61000 m and 64000 m for both OK estimates and the original data (see Appendix C the data results). The SK estimate almost remains constant between 61000 m and 65000 m, but follows the original data slightly better on left side of the graph (high grade area).

In summary the SK estimates are closer to the global mean of 5.76 g/t whereas OK estimates are closer to the local mean which fluctuates. The SK estimate overestimates the PGE values between 61000 m and 64000 m (right part of the graph) where there are actually low grade PGE values. The SK estimates slightly underestimates the PGE grades between 57000 m and 61000 m the left part of the graph. In contrast OK estimates better follows the original PGE data with large values on the left part and small values on the right part of the graph. It can be concluded that, if the weight assigned to the mean is low (i.e. low grade areas), then

local samples are mostly used for estimating the grade and stationarity is ignored. However if the weight assigned to the mean is large (i.e. high grade areas), then local sample information is not relied upon and therefore the stationarity and global mean are considered.

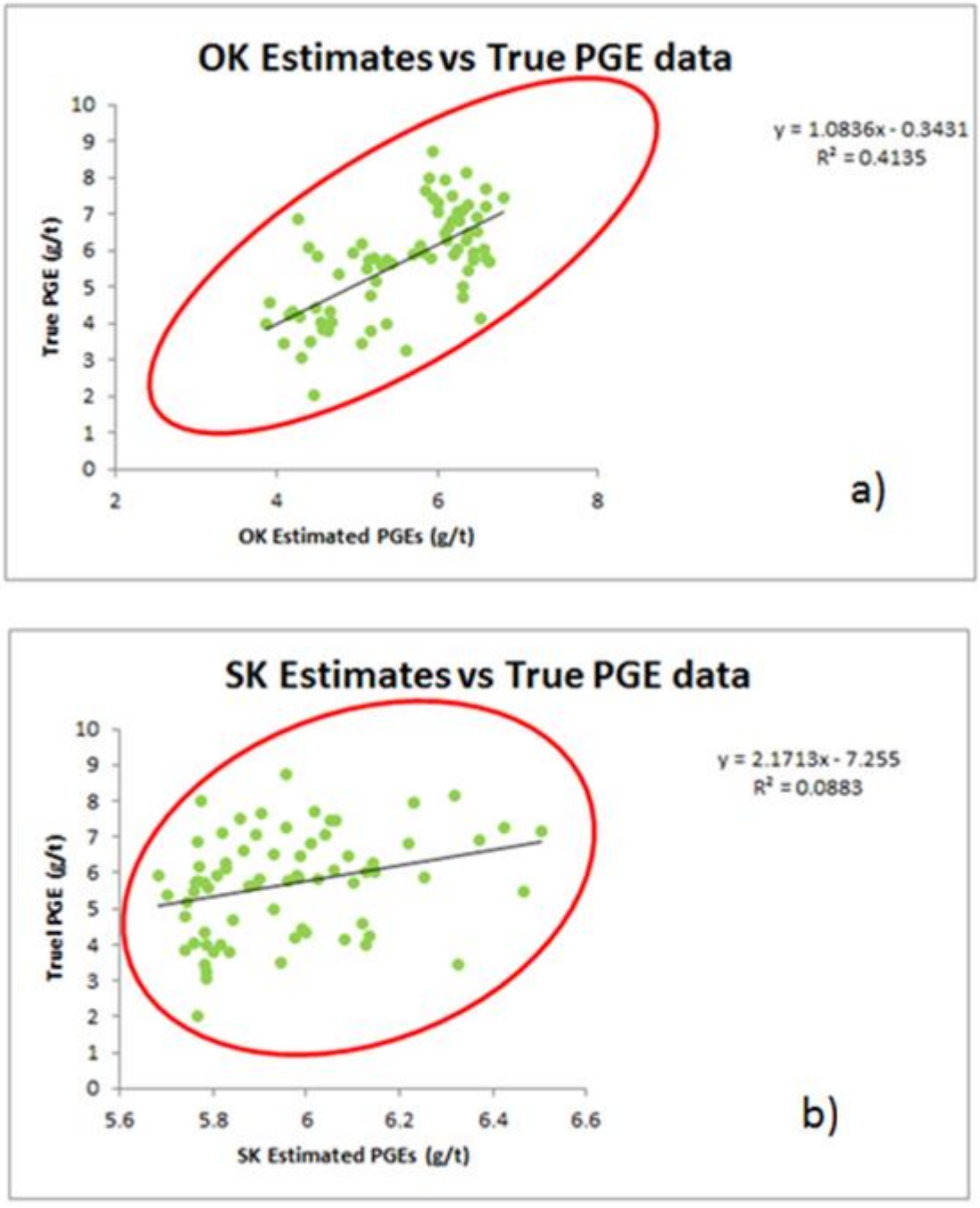


Figure 6.11: Actual and estimated PGE grades using OK and SK

Figure 6.11 a) and b) compares the original PGE data with the OK and SK estimates respectively. A difference is observed from the two scatter plots, there is a wider

scatter of points in SK than in OK, more points lie in the 45° line in OK estimated values. OK has a higher correlation coefficient of 0.64 than SK. This means that there is less accuracy in the estimates from SK than in OK. The SK estimated high grades appear lower than the true high grades. Conversely, the estimated low grades are higher than the true low grade values. When comparing the OK estimates and the true PGE grades, the true low grades appear to be similar to the estimated low grade values. In the same way, the true high grade values appear to be similar to the estimated high grade values. This results in further support of the idea that the OK estimation method better estimates this PGE data.

6.5 Comparing estimated grade and the true grade

As a method of cross validation of the resource model, the comparison of original and estimated grade was undertaken. Table 6.3 shows a comparison of original and estimated grade values of both OK and SK for the 250 m x 250 m x 10 m block model.

Table 6.3: Summary statistics of the estimated versus original PGE (UG2) data

Attribute	OK (250 m x 250 m)	SK (250 m x 250 m)	Original (PGE)
Kriging variance	1.31 $(g/t)^2$	0.56 $(g/t)^2$	-
Sample variance	-	-	4.84 $(g/t)^2$
Std.dev	2.4 g/t	1.5 g/t	2.2 g/t
Estimated grade	7.41 g/t	5.76 g/t	5.76 g/t
Block variance	0.56 $(g/t)^2$	0.56 $(g/t)^2$	-
Kriging efficiency	-1.32	0	-
CoV	0.15	0.13	0.38
Lagrange Multiplier	-0.91	-	-

The difference between the original standard deviation and the OK estimated standard deviation is 0.2 g/t on the 250 m x 250 m x 10 m block model. The difference in grade between original and OK estimated grade is 1.65 g/t on the same block model. There is a significant difference in the OK estimated grade and the original grade values; this could be explained by the idea that OK uses the local neighbourhood to estimate its mean value as opposed to the global mean used for SK in this study.

The difference between the original standard deviation and the SK estimated standard deviation is 0.7g/t on the 250 m x 250 m x 10 m block model. The

difference in grade between the original and estimated grade is 0 from SK on the 250 m x 250 m x 10 m block model. This could be explained by the fact that in SK the mean is assumed to be known, even in this case the mean was assumed to be equal to 5.76 g/t the global mean hence the difference is zero. The difference between OK standard deviation and the original values is less than that of SK, meaning there will be lesser smoothing from OK estimated values. The block variance for OK and SK is smaller than the original sample variance; this can be explained by the fact that the original sample variance is computed from a point sample support and the OK and SK block variance is computed from a block support, 250 m x 250 m x 10 m. Table 6.4 summarises the overall observed differences between OK and SK.

Table 6.4: The overall comparison of the OK and SK techniques

Output	OK	SK
Kriged estimate (z*k)	7.41 g/t OK estimate is > SK estimate in high grade areas Condition, weights should sum up to 1 Equation does not include $m\ (1-\sum\lambda)$	5.76 g/t SK estimate is > OK estimate in low grade areas Weights do not necessarily have to sum up to 1 Hence equation includes $m(1-\sum\lambda)$
Kriging variance (Sk^2)	1.31 $(g/t)^2$ The OK variance > SK kriging variance There is a Lagrange factor added to the OK variance equation	0.56 $(g/t)^2$ The SK kriging variance <OK variance There is no Lagrange factor added to the SK variance equation
Block variance (sB)	0.56 $(g/t)^2$ Block variance of OK = SK Verification is observed in the equation The same block size support is used for both OK and SK	0.56 $(g/t)^2$ Block variance of SK = OK See block variance equation The same block size support is used for OK and SK
Kriging efficiency (KE)	-1.32 Negative Kriging efficiency Block variance < kriging variance	0 Is equal to zero Block variance and kriging variance are equal
Lagrange multiplier (μ)	-0.91 Indicates that samples are relatively close to the blocks being estimated	No Lagrange factor

6.6 The identified major differences between OK and SK

The kriging variance for OK is higher than the kriging variance for SK. The OK variance equation (see Chapter 2 equation 29), has the Lagrange multiplier added to it which is the factor that ensures that the OK weights are optimal and add up to 1. On the contrary, SK variance does not have the Lagrange multiplier added and there is no condition on the weights.

The value of the kriging efficiency is zero for SK since both the block variance and kriging variance are equal. A kriging efficiency of zero indicates that most blocks are assigned the value of the global mean, and that is actually the case in SK with the strong assumption of stationarity and known mean. The OK variance is not equal to the block variance in the 250 m x 250 m x 10 m block model. The block variance is less than the kriging variance, resulting in a negative average kriging efficiency of -1.32 for OK (see Table 6.2). The negative kriging efficiency could be explained by the idea that in some blocks there is no sample data, which could increase the kriging variance. The block variance is the same for both OK and SK which is equal to 0.56 $(g/t)^2$. When the same neighbourhood and block size support are used for OK and SK, the block variance obtained is equal, because their block variance equations are the same. The mean in SK is assumed to be known and was used in Surpac version 6.2.1 and in OK it is unknown. The mean of OK is local since it is estimated from the neighbouring data values in each block whereas the SK mean is a global mean. The OK mean estimated is larger than the global mean in the case of this PGE deposit, while the estimated SK mean is equal to the global mean of 5.76 g/t due to strong assumption of stationarity in this mineral deposit.

6.7 Domaining

Upon the realisation from trend estimates that the PGE grades clearly display two distinct populations, domaining was taken into consideration. The PGE grade scatter plot in Figure 4.9 is used as a guide to separate the PGE grades into two populations thus this domaining is grade based. Domaining this PGE deposit should improve the estimation results and prevent over smoothing of estimated block grades across the different zones of mineralisation. There are two distinct zones of mineralisation identified as shown in Figure 6.12, the SE corner circled in green is the low grade zone called domain 1 and the rest of the deposit is named domain 2 which is high grade.

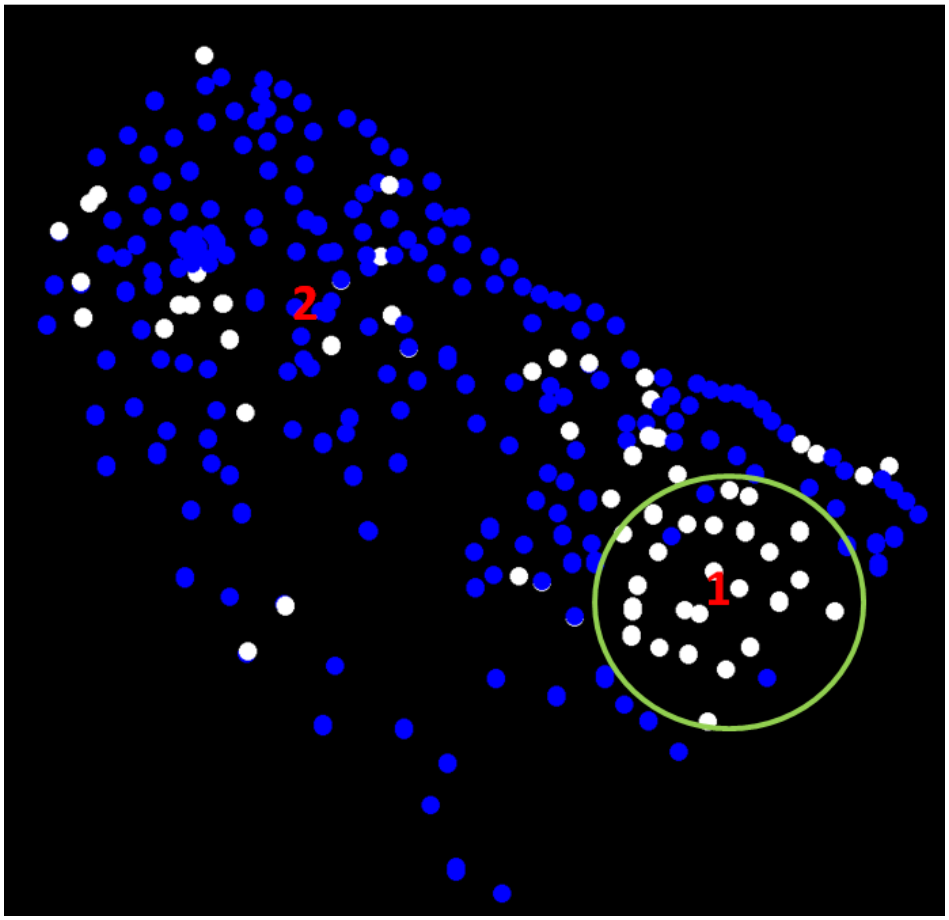


Figure 6.12: PGE grades digitised into two domains 1 and 2

6.7.1 Domain1

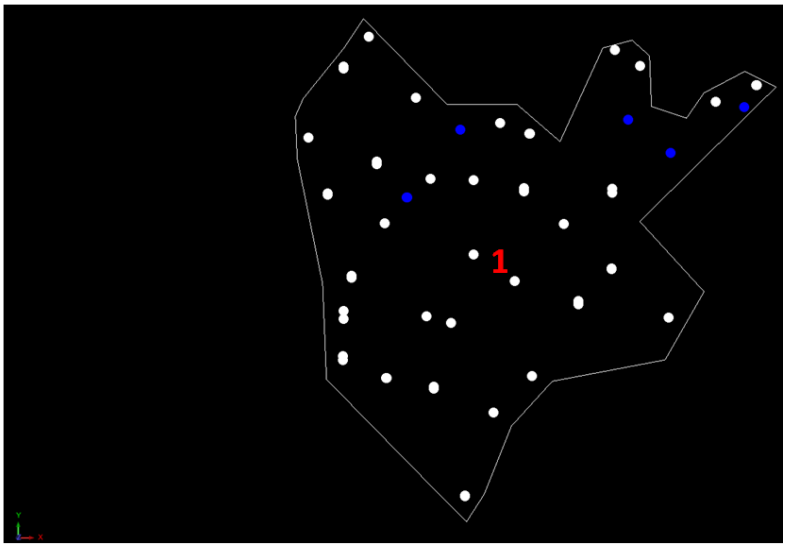


Figure 6.13: PGE low grades domain 1

Domain 1 has PGE grades ranging from 0 to 3g/t and is classified as the low grade area (see Figure 6.13).

6.7.2 Domain 2

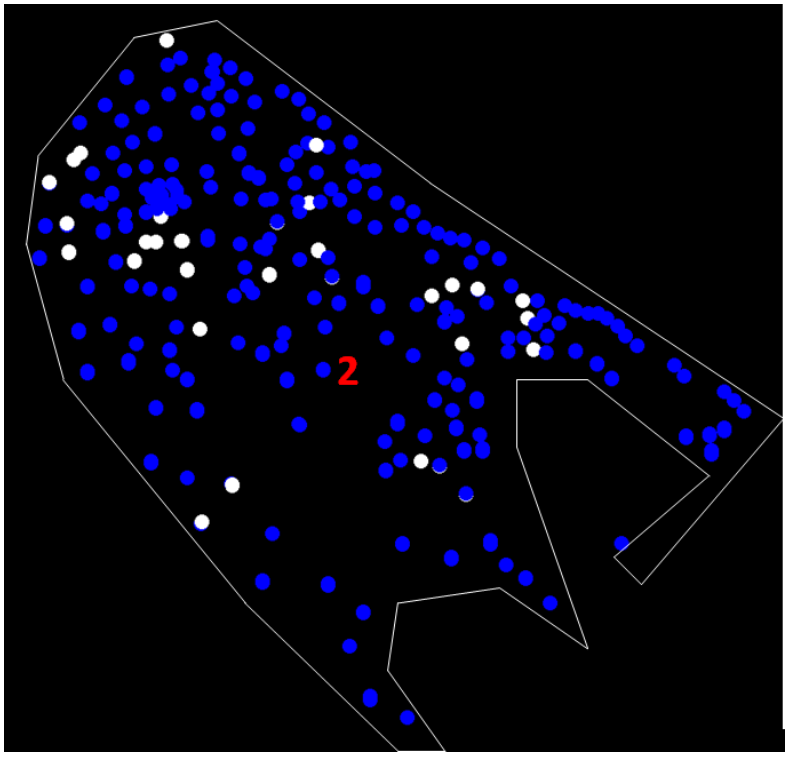


Figure 6.14: PGE high grades domain 2

Domain 2 has PGE grades ranging from 3 g/t to 15 g/t and is classified as the high grade area (see Figure 6.14).

Ordinary and Simple Kriging were performed on both the domains and the results were compared (see Table 6.5).

Table 6.5: Summary statistics of the domains; estimated versus original PGE data

Attribute	Original_Domain 1	Original _Domain 2	OK_Domain1	OK_Domain2	SK_Domain1	SK_Domain2
Estimated grade	2.88(g/t)	6.29 (g/t)	2.73 (g/t)	11.31 (g/t)	2.88 (g/t)	6.32(g/t)
Kriging variance	-	-	0.87 $(g/t)^2$	1.21 $(g/t)^2$	0.421 $(g/t)^2$	0.42$(g/t)^2$
Sample variance	1.49$(g/t)^2$	3.66 $(g/t)^2$	-	-	-	-
Std.dev	1.22 g/t	1.91 g/t	0.93 g/t	1.1 g/t	0.64 g/t	0.65 g/t
Block variance	-	-	0.42 $(g/t)^2$	0.42 $(g/t)^2$	0.42 $(g/t)^2$	0.42 $(g/t)^2$
Kriging efficiency	-	-	1.06	-1.88	0.00	0.00
CoV	0.42	0.30	0.34	0.09	0.23	0.10
Lagrange Multiplier	-	-	0.52	-0.79	-	-

A number of interpretations are drawn from the estimation results. SK average estimated grade for domain 1 is the same as the original mean grade however for domain 2 it is slightly larger. For OK the average estimated grade for domain 1 shows a slight difference from the original mean grade. In domain 2 there is a significant difference between the OK estimated average grade and the original mean grade. A possible explanation for this is that there are only few areas within domain 2 which consist of low grade PGEs; and this might be the cause of the notable increase in the estimated grade.

When comparing Table 6.3 and Table 6.5 there is a considerable improvement in the various measures of kriging, for instance the kriging variance for both OK and SK decreased by 0.44 and 0.14 respectively in domain 1. In domain 2 the kriging variance for both OK and SK decreased by 0.1 and 0.14 respectively. Of note is that in domain 1 the low grade area, the OK and SK performance is similar.

The block variance also followed the same pattern as the kriging variance; in the two domains the block variance for both OK and SK decreased by 0.14 in both domains.

It can be drawn from the estimation results that domaining does improve the estimation of a mineral resource since the variance is minimised. In domain 1 the grade ranges from 0 to 3 g/t and a total of 177 drill holes formed part of the estimation while the grades in domain 2 ranges from 3 to 15 g/t and a total of 393 drill holes formed part of the estimation process. Hence it is expected that the

estimated PGE grade values be not the same as the initial results produced in Table 6.3 where all 570 drill holes were used in the estimation process.

To conclude the domaining section it is quite clear that when there are a few data involved in the estimation process OK and SK behave almost the same however when more data is used there are significant differences observed between OK and SK estimated grades.

Chapter 7: Concluding Remarks

7.1 Conclusion

In real geological sites there are large scale variations in structures and spatial continuity of grades. Studying the geology of the research area assisted in the understanding of the PGE grades distribution and spatial continuity. In the statistical analysis of the PGEs, grades were found to have a bimodal distribution. The reason for the bimodality could not be verified; but a possible reason is the injection of different magma pulses at different times and remobilisation that occurred in the Bushveld Complex. The mixture of the different PGE elements could also be another possible reason for the bimodality. In the grade sample location plots and contour maps (in Chapter 4 and Chapter 5 respectively); initially the low and high PGE grades appeared to be evenly distributed throughout the deposit, hence no domains were defined. The variogram fans further verified that there is no preferred direction of maximum continuity and therefore omnidirectional semi-variograms were modelled.

The application of the estimation techniques SK and OK was undertaken on the PGE grades and the mean in SK was assumed to be known due to the strong assumption of stationarity while in OK it was unknown. OK does not strongly emphasise stationarity and it depends only on the local neighbourhood to estimate its value of the mean.

SK would be misleading in a non-homogenous deposit, when estimating grades since it assumes a constant mean and variance across the deposit (theory of stationarity). In SK the PGE grades are averaged out, which is not representative of the true PGE grade values. The manner in which SK was applied for this PGE deposit can be misleading and that can cause great financial loss in a mining project because waste can be sometimes estimated as ore; due to the strong assumption of stationarity.

In Figure 6.10 it is clearly shown that SK overestimates the PGE grades in the SW edge of this PGE deposit. SK can however be suitable in estimating deposits with few data because it does not heavily rely on the local neighbourhood for estimating values like OK does. Thus it can be concluded that if the mean is not globally

stationary in the mineral deposit, then using a local stationary mean with OK will result in better estimates. OK proves to be suitable for estimating deposits with fluctuating mean and variance (see Figure 6.10) which is what is common in reality.

For this study the SK mean is global whereas the OK mean is local. In the case where the OK mean is greater than the SK mean; whether the support is increased or remains the same the kriging variance of OK is always greater than the kriging variance of SK, provided strong stationarity is assumed. This occurs because in the computation of OK variance there is a Lagrange multiplier which is the factor that ensures that there are optimum weights, whereas SK does not have that factor. This can also be explained by the idea that the SK mean used in the estimation provides significant, useful and additional information (Assibey-Bonsu, 2014 personal communication). When taking the kriged estimate results into consideration, it shows that OK better follows the original data than an SK estimate (this is evident in Figure 6.9 and Figure 6.10).

Towards the end of the study after doing the trend analysis it was clear that there are distinct low grade and high grade areas and therefore domaining was considered. The estimation results were significantly different as shown in Table 6.5. It was concluded that when there are a few data involved in the estimation process OK and SK behave almost the same however when more data is used there are significant differences observed between OK and SK estimated grades. This idea will be developed further in the future studies to be done.

The mining industry is a high risk business. In some mining projects risks are escalated by a lack of data availability as the cost of acquiring data is sometimes high, resulting in high uncertainty. In other mining projects the risks can be exacerbated by the errors associated with the available data. Therefore appropriate use of mineral resource estimation techniques are needed to quantify the risk. The understanding of the application and suitability of these estimation techniques is of vital importance to accurately quantify, mitigate and minimise this risk.

7.2 Recommendations

To improve the estimation results and the manner in which the estimation techniques were applied for this PGE resource the following are strongly recommended:

- The domaining section should be developed further to properly investigate more differences of OK and SK as domaining has indicated improvements in the estimation results.
- The PGE histogram, probability plot and trend analysis indicated mixed populations, therefore a method of separating the mixed populations should be employed in this data.
- Instead of applying the average global mean as the known SK mean, the local SK mean should be applied as there are evident fluctuations in this PGE data.
- Trend analysis should be done earlier in any study in fact it should be included immediately after the statistical analysis, to clearly identify trends that may exist in any dataset.
- For this particular PGE data, the four PGE elements should be estimated separately and not mixed to avoid obtaining a bi-modal distribution, to also obtain a better view of the behaviour of SK and OK for each element.
- The assumption of stationarity should also not be heavily relied upon specifically for this particular PGE data as it resulted in the poor performance of the SK technique; improvements in SK were only observed when domaining was considered.
- Declustering should be considered for such data and unfortunately the software that was used in this study does not have a declustering function.

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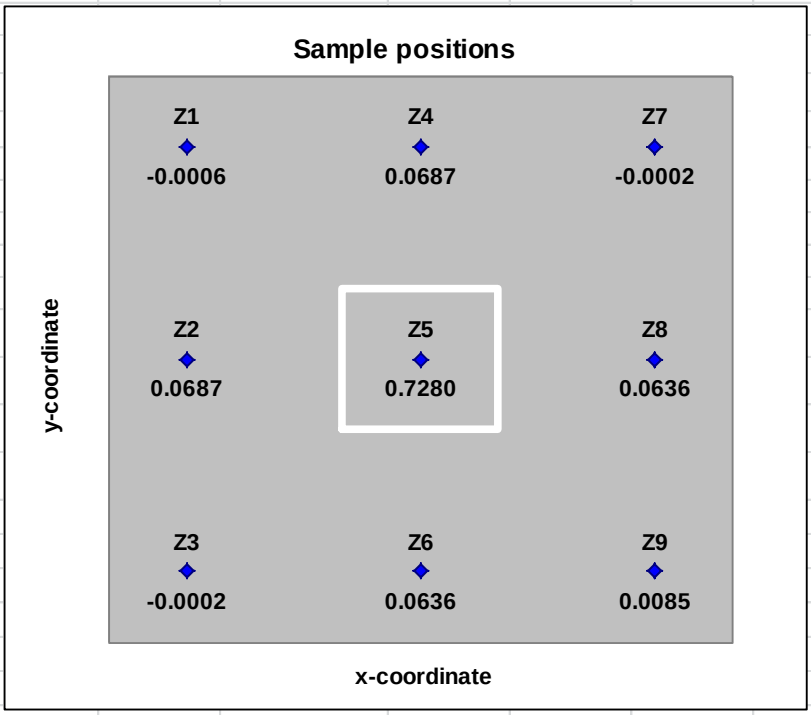
APPENDICES

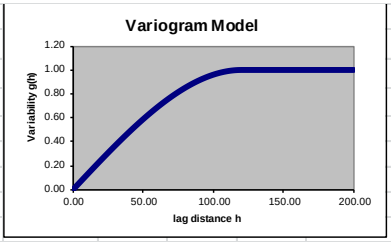
APPENDIX A

Ordinary Kriging (OK)

x	y
20.00	20.00
40.00	20.00
40.00	40.00
20.00	40.00
20.00	20.00
Σ wi	**1.00**
λ	**-0.0127**
Z^*_K	**19.5123**
σ_K^2	**0.0473**
σ_ε^2	**0.8937**
σ_B^2	**0.1933**
slope	**1.0154**
w_1	-0.0006
w_2	0.0687
w_3	-0.0002
w_4	0.0687
w_5	0.7280
w_6	0.0636
w_7	-0.0002
w_8	0.0636
w_9	0.0085
λ	-0.0127



Variogram Model														
Co =	0.0													
C1 =	1.0	a 1 =	120											
C2 =	0.0	a 2 =	120											
C =	1.0													



Sample	Sample	coordinates		Distance Sample i to Sample j = h_{ij}					= SQRT($(X_i-X_j)^2+(Y_i-Y_j)^2$)				
Value	No	x	y	Distance	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9
19	Z_1	0	60	Z_1	0	30.00	60.00	30.00	42.43	67.08	60.00	67.08	84.85
25	Z_2	0	30	Z_2	30	0	30.00	42.43	30.00	42.43	67.08	60.00	67.08
17	Z_3	0	0	Z_3	60	30	0	67.08	42.43	30.00	84.85	67.08	60.00
13	Z_4	30	60	Z_4	30	42	67	0	30.00	60.00	30.00	42.43	67.08
21	Z_5	30	30	Z_5	42	30	42	30	0	30.00	42.43	30.00	42.43
8	Z_6	30	0	Z_6	67	42	30	60	30	0	67.08	42.43	30.00
12	Z_7	60	60	Z_7	60	67	85	30	42	67	0	30.00	60.00
15	Z_8	60	30	Z_8	67	60	67	42	30	42	30	0	30.00
20	Z_9	60	0	Z_9	85	67	60	67	42	30	60	30	0
16.67													

				Average distance from sample to block for all blocks								
	x	y		Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9
a	25.0	35.0	Aa	35	25	43	25	7	35	43	35	49.5
b	25.0	25.0	Ab	43	25	35	35	7	25	49	35	35.4
c	35.0	35.0	Ac	43	35	49	25	7	35	35	25	49.5
d	35.0	25.0	Ad	49	35	43	35	7	25	43	25	35.4
Average distance from sample to block				42.72	30.43	42.72	30.43	7.07	30.43	42.72	30.43	43.0

Calculation of weights using matrix algebra

Calculation of $\gamma(h_{ij})$

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	λ			RHS	
Z_1	0.0000	0.3672	0.6875	0.3672	0.5082	0.7512	0.6875	0.7512	0.8839	1.00	w_1	=	**0.5105**	X w = y
Z_2	0.3672	0.0000	0.3672	0.5082	0.3672	0.5082	0.7512	0.6875	0.7512	1.00	w_2	=	**0.3715**	
Z_3	0.6875	0.3672	0.0000	0.7512	0.5082	0.3672	0.8839	0.7512	0.6875	1.00	w_3	=	**0.5105**	$X^{-1}X$ w =X^{-1} y
Z_4	0.3672	0.5082	0.7512	0.0000	0.3672	0.6875	0.3672	0.5082	0.7512	1.00	w_4	=	**0.3715**	
Z_5	0.5082	0.3672	0.5082	0.3672	0.0000	0.3672	0.5082	0.3672	0.5082	1.00	w_5	=	**0.0883**	I w =X^{-1} y
Z_6	0.7512	0.5082	0.3672	0.6875	0.3672	0.0000	0.7512	0.5082	0.3672	1.00	w_6	=	**0.3715**	
Z_7	0.6875	0.7512	0.8839	0.3672	0.5082	0.7512	0.0000	0.3672	0.6875	1.00	w_7	=	**0.5105**	w =X^{-1} y
Z_8	0.7512	0.6875	0.7512	0.5082	0.3672	0.5082	0.3672	0.0000	0.3672	1.00	w_8	=	**0.3715**	
Z_9	0.8839	0.7512	0.6875	0.7512	0.5082	0.3672	0.6875	0.3672	0.0000	1.00	w_9	=	**0.5064**	
weights	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	λ	=	1.0000	

inverse matrix	X^{-1}											y		
-2.098	1.117	0.019	1.117	-0.050	-0.111	0.019	-0.111	0.100	0.258		0.5105		$X^{-1}X$ w =X^{-1} y
1.117	-2.891	1.117	0.042	0.862	0.042	-0.111	-0.065	-0.111	0.033		0.3715		
0.019	1.117	-2.098	-0.111	-0.050	1.117	0.100	-0.111	0.019	0.258		0.5105		
1.117	0.042	-0.111	-2.891	0.862	-0.065	1.117	0.042	-0.111	0.033		0.3715		
-0.050	0.862	-0.050	0.862	-3.248	0.862	-0.050	0.862	-0.050	-0.164		0.0883		
-0.111	0.042	1.117	-0.065	0.862	-2.891	-0.111	0.042	1.117	0.033		0.3715		
0.019	-0.111	0.100	1.117	-0.050	-0.111	-2.098	1.117	0.019	0.258		0.5105		
-0.111	-0.065	-0.111	0.042	0.862	0.042	1.117	-2.891	1.117	0.033		0.3715		
0.100	-0.111	0.019	-0.111	-0.050	1.117	0.019	1.117	-2.098	0.258		0.5064		
0.258	0.033	0.258	0.033	-0.164	0.033	0.258	0.033	0.258	-0.573		1.0000		
									w_1	=	-0.0006		w =X^{-1} y
									w_2	=	0.0687		
									w_3	=	-0.0002		
									w_4	=	0.0687		
									w_5	=	0.7280		
									w_6	=	0.0636		
									w_7	=	-0.0002		
									w_8	=	0.0636		
									w_9	=	0.0085		
			Σwi must =1	1.00					λ	=	-0.0127		

Calculation of the γ value for each discretisation point									
	Z1	**Z2**	**Z3**	**Z4**	**Z5**	**Z6**	**Z7**	**Z8**	**Z9**
Aa	0.4292	0.3139	0.5146	0.3139	0.0883	0.4292	0.5146	0.4292	0.5836
Ab	0.5146	0.3139	0.4292	0.4292	0.0883	0.3139	0.5836	0.4292	0.4292
Ac	0.5146	0.4292	0.5836	0.3139	0.0883	0.4292	0.4292	0.3139	0.5836
Ad	0.5836	0.4292	0.5146	0.4292	0.0883	0.3139	0.5146	0.3139	0.4292
RHS	**0.5105**	**0.3715**	**0.5105**	**0.3715**	**0.0883**	**0.3715**	**0.5105**	**0.3715**	**0.5064**

Distance between discretisation points				
	a	b	c	d
a	0.00	10	10.00	14.14
b	10	0	14.14	10.00
c	10	14	0	10.00
d	14	10	10	0.00

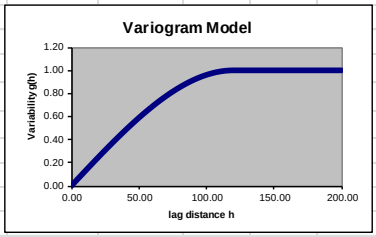
Variogram for discretisation points

	a	b	c	d
a	0.000	0.125	0.125	0.176
b	0.125	0.000	0.176	0.125
c	0.125	0.176	0.000	0.125
d	0.176	0.125	0.125	0.000

γ bar(A,A) =	0.106
γ bar(z_i,z_j) =	0.503
γ bar (z,A) =	0.401

Simple Kriging (SK)

Variogram Model			
Co =	0.00		
C1 =	1.00	a 1 =	120
C2 =	0.00	a 2 =	120
C =	1.00		



Sample	Sample	coordinates		Distance Sample i to Sample j = h_{ij}					= SQRT($(X_i-X_j)^2+(Y_i-Y_j)^2$)				
Value	No	x	y	Distance	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9
19	Z_1	0	60	Z_1	0	30.00	60.00	30.00	42.43	67.08	60.00	67.08	84.85
25	Z_2	0	30	Z_2	30	0	30.00	42.43	30.00	42.43	67.08	60.00	67.08
17	Z_3	0	0	Z_3	60	30	0	67.08	42.43	30.00	84.85	67.08	60.00
13	Z_4	30	60	Z_4	30	42	67	0	30.00	60.00	30.00	42.43	67.08
21	Z_5	30	30	Z_5	42	30	42	30	0	30.00	42.43	30.00	42.43
8	Z_6	30	0	Z_6	67	42	30	60	30	0	67.08	42.43	30.00
12	Z_7	60	60	Z_7	60	67	85	30	42	67	0	30.00	60.00
15	Z_8	60	30	Z_8	67	60	67	42	30	42	30	0	30.00
20	Z_9	60	0	Z_9	85	67	60	67	42	30	60	30	0
Average	16.67												

Average distance from sample to block for all blocks

	x	y		Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9
a	25.0	35.0	Aa	35.36	25.50	43.01	25.50	7.07	35.36	43.01	35.36	49.50
b	25.0	25.0	Ab	43.01	25.50	35.36	35.36	7.07	25.50	49.50	35.36	43.01
c	35.0	35.0	Ac	43.01	35.36	49.50	25.50	7.07	35.36	35.36	25.50	43.01
d	35.0	25.0	Ad	49.50	35.36	43.01	35.36	7.07	25.50	43.01	25.50	35.36

Calculation of weights using matrix algebra

Calculation of $\gamma(h_{ij})$

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	λ			RHS		
Z_1	0.000	0.367	0.688	0.367	0.508	0.751	0.688	0.751	0.884		w_1	=	0.5105		X w = y
Z_2	0.367	0.000	0.367	0.508	0.367	0.508	0.751	0.688	0.751		w_2	=	0.3715		
Z_3	0.688	0.367	0.000	0.751	0.508	0.367	0.884	0.751	0.688		w_3	=	0.5105		$X^{-1}X$ w = X^{-1} y
Z_4	0.367	0.508	0.751	0.000	0.367	0.688	0.367	0.508	0.751		w_4	=	0.3715		
Z_5	0.508	0.367	0.508	0.367	0.000	0.367	0.508	0.367	0.508		w_5	=	0.0883		I w = X^{-1} y
Z_6	0.751	0.508	0.367	0.688	0.367	0.000	0.751	0.508	0.367		w_6	=	0.3715		
Z_7	0.688	0.751	0.884	0.367	0.508	0.751	0.000	0.367	0.688		w_7	=	0.5105		w = X^{-1} y
Z_8	0.751	0.688	0.751	0.508	0.367	0.508	0.367	0.000	0.367		w_8	=	0.3715		
Z_9	0.884	0.751	0.688	0.751	0.508	0.367	0.688	0.367	0.000		w_9	=	0.5105		
weights															

inverse matrix		X^{-1}										y			
	-1.982	1.131	0.135	1.131	-0.124	-0.096	0.135	-0.096	0.216			0.5105			$X^{-1}X$ w = X^{-1} y
	1.131	-2.889	1.131	0.043	0.853	0.043	-0.096	-0.063	-0.096			0.3715			
	0.135	1.131	-1.982	-0.096	-0.124	1.131	0.216	-0.096	0.135			0.5105			
	1.131	0.043	-0.096	-2.889	0.853	-0.063	1.131	0.043	-0.096			0.3715			
	-0.124	0.853	-0.124	0.853	-3.201	0.853	-0.124	0.853	-0.124			0.0883			
	-0.096	0.043	1.131	-0.063	0.853	-2.889	-0.096	0.043	1.131			0.3715			
	0.135	-0.096	0.216	1.131	-0.124	-0.096	-1.982	1.131	0.135			0.5105			
	-0.096	-0.063	-0.096	0.043	0.853	0.043	1.131	-2.889	1.131			0.3715			
	0.216	-0.096	0.135	-0.096	-0.124	1.131	0.135	1.131	-1.982			0.5105			
										w_1	=	-0.0054		w = X^{-1} y	
										w_2	=	0.0675			
										w_3	=	-0.0054			
										w_4	=	0.0675			
										w_5	=	0.7311			
										w_6	=	0.0675			
										w_7	=	-0.0054			
										w_8	=	0.0675			
										w_9	=	-0.0054			
										λ	=				

Calculation of the γ value for each discretisation point									
	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9
Aa	0.429	0.314	0.515	0.314	0.088	0.429	0.515	0.429	0.584
Ab	0.515	0.314	0.429	0.429	0.088	0.314	0.584	0.429	0.515
Ac	0.515	0.429	0.584	0.314	0.088	0.429	0.429	0.314	0.515
Ad	0.584	0.429	0.515	0.429	0.088	0.314	0.515	0.314	0.429
RHS	**0.5105**	**0.3715**	**0.5105**	**0.3715**	**0.0883**	**0.3715**	**0.5105**	**0.3715**	**0.5105**

Distance between discretisation points				
	a	b	c	d
a	0.00	10.00	10.00	14.14
b	10.00	0.00	14.14	10.00
c	10.00	14.14	0.00	10.00
d	14.14	10.00	10.00	0.00

Variogram for discretisation points				
	a	b	c	d
a	0.00	0.12	0.12	0.18
b	0.12	0.00	0.18	0.12
c	0.12	0.18	0.00	0.12
d	0.18	0.12	0.12	0.00

γ bar(A,A) =	0.1063
γ bar(z_i,z_j) =	0.5031
γ bar (z,A) =	0.4018

Z^*_K	**19.4448**
σ_K^2	**0.0678**
σ_B^2	**0.8937**
σ_ε^2	**0.1942**
slope	**1.0000**

APPENDIX B

OK weights 30 m range

Co	w1	w2	w3	w4	w5	w6	w7	w8	w9	λ
0.00	0.03	0.05	0.03	0.05	0.68	0.05	0.03	0.05	0.03	0.03
0.10	0.04	0.05	0.04	0.05	0.63	0.05	0.04	0.05	0.04	0.04
0.20	0.05	0.06	0.05	0.06	0.57	0.06	0.05	0.06	0.05	0.05
0.30	0.06	0.07	0.06	0.07	0.51	0.07	0.06	0.07	0.06	0.06
0.40	0.06	0.07	0.06	0.07	0.46	0.07	0.06	0.07	0.06	0.06
0.50	0.07	0.08	0.07	0.08	0.40	0.08	0.07	0.08	0.07	0.07
0.60	0.08	0.09	0.08	0.09	0.34	0.09	0.08	0.09	0.08	0.08
0.70	0.09	0.09	0.09	0.09	0.28	0.09	0.09	0.09	0.09	0.09
0.80	0.10	0.10	0.10	0.10	0.23	0.10	0.10	0.10	0.10	0.10
0.90	0.10	0.10	0.10	0.10	0.17	0.10	0.10	0.10	0.10	0.10
1.00	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11

OK weights 90 m range

Co	w1	w2	w3	w4	w5	w6	w7	w8	w9	λ
0.00	0.00	0.07	0.00	0.07	0.73	0.06	0.00	0.06	0.01	-0.01
0.10	0.01	0.09	0.01	0.09	0.60	0.09	0.01	0.09	0.02	-0.01
0.20	0.02	0.10	0.02	0.10	0.50	0.10	0.02	0.10	0.03	-0.01
0.30	0.03	0.11	0.03	0.11	0.42	0.11	0.03	0.11	0.04	0.00
0.40	0.05	0.11	0.05	0.11	0.36	0.11	0.05	0.11	0.05	0.01
0.50	0.06	0.11	0.06	0.11	0.30	0.11	0.06	0.11	0.06	0.02
0.60	0.07	0.11	0.07	0.11	0.25	0.11	0.07	0.11	0.08	0.03
0.70	0.08	0.11	0.08	0.11	0.21	0.11	0.08	0.11	0.09	0.05
0.80	0.09	0.11	0.09	0.11	0.17	0.11	0.09	0.11	0.09	0.07
0.90	0.10	0.11	0.10	0.11	0.14	0.11	0.10	0.11	0.10	0.09
1.00	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11

OK weights at 120 m

Co	w1	w2	w3	w4	w5	w6	w7	w8	w9	λ
0.00	0.00	0.07	0.00	0.07	0.73	0.06	0.00	0.06	0.01	-0.01
0.10	0.01	0.10	0.01	0.10	0.56	0.09	0.01	0.09	0.02	-0.02
0.20	0.03	0.11	0.03	0.11	0.45	0.10	0.03	0.10	0.04	-0.02
0.30	0.05	0.11	0.05	0.11	0.37	0.11	0.05	0.11	0.05	-0.01
0.40	0.06	0.11	0.06	0.11	0.31	0.11	0.06	0.11	0.06	0.00
0.50	0.07	0.11	0.07	0.11	0.26	0.11	0.07	0.11	0.07	0.02
0.60	0.08	0.11	0.08	0.11	0.22	0.11	0.08	0.11	0.08	0.03
0.70	0.09	0.11	0.09	0.11	0.19	0.11	0.09	0.11	0.09	0.05
0.80	0.10	0.11	0.10	0.11	0.16	0.11	0.10	0.11	0.10	0.07
0.90	0.11	0.11	0.11	0.11	0.13	0.11	0.11	0.11	0.11	0.09
1.00	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11

SK weights 30 m range

Co	w1	w2	w3	w4	w5	w6	w7	w8	w9	Co
0.00	0.04	0.05	0.04	0.05	0.69	0.05	0.04	0.05	0.04	0.00
0.10	0.04	0.06	0.04	0.06	0.63	0.06	0.04	0.06	0.04	0.10
0.20	0.05	0.07	0.05	0.07	0.58	0.07	0.05	0.07	0.05	0.20
0.30	0.06	0.07	0.06	0.07	0.52	0.07	0.06	0.07	0.06	0.30
0.40	0.07	0.08	0.07	0.08	0.46	0.08	0.07	0.08	0.07	0.40
0.50	0.08	0.09	0.08	0.09	0.41	0.09	0.08	0.09	0.08	0.50
0.60	0.09	0.10	0.09	0.10	0.35	0.10	0.09	0.10	0.09	0.60
0.70	0.10	0.10	0.10	0.10	0.29	0.10	0.10	0.10	0.10	0.70
0.80	0.11	0.11	0.11	0.11	0.24	0.11	0.11	0.11	0.11	0.80
0.90	0.12	0.12	0.12	0.12	0.18	0.12	0.12	0.12	0.12	0.90
1.00	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	1.00

SK weights 90 m range

Co	w1	w2	w3	w4	w5	w6	w7	w8	w9
0.00	-0.01	0.07	-0.01	0.07	0.73	0.07	-0.01	0.07	-0.01
0.10	0.00	0.09	0.00	0.09	0.60	0.09	0.00	0.09	0.00
0.20	0.02	0.10	0.02	0.10	0.50	0.10	0.02	0.10	0.02
0.30	0.03	0.11	0.03	0.11	0.42	0.11	0.03	0.11	0.03
0.40	0.05	0.11	0.05	0.11	0.36	0.11	0.05	0.11	0.05
0.50	0.06	0.12	0.06	0.12	0.30	0.12	0.06	0.12	0.06
0.60	0.08	0.12	0.08	0.12	0.25	0.12	0.08	0.12	0.08
0.70	0.09	0.12	0.09	0.12	0.22	0.12	0.09	0.12	0.09
0.80	0.10	0.12	0.10	0.12	0.18	0.12	0.10	0.12	0.10
0.90	0.11	0.12	0.11	0.12	0.15	0.12	0.11	0.12	0.11
1.00	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13

SK weights 120 m range

Co	w1	w2	w3	w4	w5	w6	w7	w8	w9
0.00	-0.01	0.07	-0.01	0.07	0.73	0.07	-0.01	0.07	-0.01
0.10	0.01	0.09	0.01	0.09	0.56	0.09	0.01	0.09	0.01
0.20	0.03	0.10	0.03	0.10	0.45	0.10	0.03	0.10	0.03
0.30	0.05	0.11	0.05	0.11	0.37	0.11	0.05	0.11	0.05
0.40	0.06	0.11	0.06	0.11	0.31	0.11	0.06	0.11	0.06
0.50	0.08	0.12	0.08	0.12	0.26	0.12	0.08	0.12	0.08
0.60	0.09	0.12	0.09	0.12	0.22	0.12	0.09	0.12	0.09
0.70	0.10	0.12	0.10	0.12	0.19	0.12	0.10	0.12	0.10
0.80	0.11	0.12	0.11	0.12	0.16	0.12	0.11	0.12	0.11
0.90	0.12	0.12	0.12	0.12	0.14	0.12	0.12	0.12	0.12
1.00	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13

Appendix C

Trend Estimate Data

X (m)	SK (g/t)	OK (g/t)	Original pge (g/t)
56800	5.774	5.91	8
56900	5.782	5.36	5.72
57000	5.788	5.3	5.6
57100	5.8	5.18	3.79
57200	5.816	5.36	4
57300	5.904	5.85	7.65
57400	6.011	6.18	6.8
57500	6.143	6.36	6.29
57600	6.221	6.27	6.81
57700	6.23	6.09	7.94
57800	6.053	5.95	7.45
57900	5.866	6.15	6.6
58000	6.089	6.39	6.48
58100	6.506	6.61	7.18
58200	6.37	6.49	6.9
58300	6.423	6.39	7.27
58400	6.084	6.55	4.15
58500	6.024	6.6	5.81
58600	6.103	6.46	5.72
58700	6.062	6.82	7.45
58800	5.893	6.64	5.69
58900	5.93	6.49	6.51
59000	6.148	6.25	6.02
59100	6.317	6.36	8.15
59200	6.128	6.59	6.02
59300	6.017	6.6	7.7
59400	5.98	6.46	5.92
59500	5.928	6.33	5
59600	5.856	6.18	7.49
59700	5.827	6.11	6.26
59800	5.82	6.33	7.1
59900	5.842	6.32	4.7
60000	6.039	6.26	7.08
60100	6.253	6.21	5.88
60200	6.468	6.39	5.46
60300	5.988	6.1	6.45
60400	5.809	5.84	5.94
60500	5.893	6.01	7.05
60600	5.958	6.02	7.28
60700	5.96	5.93	5.8

60800	5.828	5.8	6.12
60900	5.957	5.94	8.72
61000	5.985	5.71	5.9
61100	5.876	5.43	5.62
61200	5.768	5.21	5.79
61300	5.744	5.24	5.18
61400	5.761	5.15	5.74
61500	5.769	5.06	6.17
61600	5.759	5.13	5.48
61700	5.741	5.18	4.77
61800	5.684	4.94	5.95
61900	5.701	4.77	5.37
62000	5.738	4.58	3.84
62100	5.76	4.56	4.04
62200	5.784	4.69	4.02
62300	5.834	4.65	3.78
62400	5.991	4.48	4.45
62500	6.059	4.39	6.09
62600	5.977	4.28	4.17
62700	5.998	4.19	4.34
62800	6.121	3.92	4.57
62900	6.127	3.88	3.98
63000	6.324	4.09	3.43
63100	6.134	4.15	4.23
63200	5.944	4.42	3.5
63300	5.898	4.51	5.82
63400	5.786	4.31	3.06
63500	5.766	4.26	6.88
63600	5.767	4.46	2.04
63700	5.78	4.67	4.33
63800	5.78	5.06	3.44
63900	5.784	5.62	3.25

Appendix D

ORIGINAL PGE DATA										
BHID	X	Y	Z	LENGTH	PGE	DENSITY	PT	PD	RH	AU
WA100D0	62103.3	-181471	872.157	1.07001	5.42736	4.3036	3.2152	1.76685	0.40244	0.04288
WA100D1	62103.2	-181471	872.111	0.82001	6.20238	4.41098	3.48398	2.18424	0.47929	0.05488
WA100D2	62103.1	-181471	871.756	1.21002	5.46125	4.30308	3.09438	1.87955	0.44312	0.04419
WA101D0	62070.4	-182482	762.898	1.54999	4.2269	3.82775	2.0755	1.7564	0.33445	0.06055
WA101D1	62070.3	-182482	763.012	1.45001	3.76998	3.81978	2.04822	1.34622	0.33561	0.03993
WA101D2	62070.2	-182482	762.855	1.69	4.56153	3.74976	2.21188	1.89381	0.39624	0.0596
WA102D0	63379	-183784	721.1	0.73999	4.76142	3.76998	2.14803	2.18322	0.35358	0.07659
WA102D1	63379.1	-183784	721.068	0.66998	6.55069	3.95161	2.91302	3.07747	0.48979	0.07041
WA102D2	63379.2	-183784	720.993	0.72998	6.79802	3.88902	2.87308	3.3738	0.46638	0.08476
WA103D0	61474.3	-180890	968.288	1.25	1.23198	3.50747	0.88216	0.20064	0.13391	0.01527
WA105D0	63112.2	-181770	923.243	0.91	7.95526	4.1322	4.06359	3.0287	0.77833	0.08463
WA105D1	63112.1	-181770	923.656	0.89999	7.82938	4.08523	4.08004	3.01148	0.65746	0.0804
WA105D2	63111.9	-181770	922.908	0.89002	8.79552	4.01742	4.3936	3.58624	0.73052	0.08517
WA105D3	63111.8	-181770	922.531	0.90002	6.4595	3.92591	3.65643	2.1609	0.58632	0.05585
WA106D0	63267.3	-181935	900.931	1.05002	5.30627	3.9023	2.81832	2.03349	0.3947	0.05976
WA106D2	63267.1	-181935	900.805	1.11002	5.52041	3.89037	2.73037	2.30675	0.42435	0.05894
WA106D5	63266.8	-181935	900.768	1.06	5.32531	3.99584	2.97323	1.88851	0.40366	0.05992
WA107D1	63770.2	-182055	898.133	1.34	5.39516	3.91523	2.87675	2.06925	0.41071	0.03845
WA107D2	63770.2	-182055	897.926	1.31	5.45567	3.98962	2.9085	2.09776	0.41415	0.03525
WA107D3	63770.2	-182055	897.812	1.54001	4.83071	3.89877	2.54112	1.9015	0.35817	0.02992
WA108D0	64010.3	-182244	893.587	1.98002	7.7707	3.87949	4.12411	2.97875	0.61112	0.05671
WA108D1	64010.3	-182244	893.438	2.01999	9.78407	3.94119	5.15824	3.67442	0.88206	0.06935
WA108D2	64010.4	-182244	893.683	1.61002	6.74532	3.93374	3.43276	2.7402	0.5125	0.05986
WA109D0	64367.1	-182575	901.13	0.84	7.06456	4.18071	4.03167	2.40023	0.59263	0.04003
WA109D1	64367.1	-182575	901.145	0.82001	6.31786	4.13744	3.71484	2.00917	0.56189	0.03198
WA109D2	64367	-182575	900.628	0.87997	5.54327	4.15172	3.47021	1.54745	0.49456	0.03105
WA113D0	61389.6	-181299	917.238	1.44998	6.18765	4.01483	3.44896	2.21872	0.48401	0.03596
WA113D1	61389.7	-181300	917.535	1.19	5.9647	4.12723	3.24308	2.20591	0.46817	0.04753
WA113D2	61389.8	-181300	917.366	1.35999	5.67228	4.15434	3.12524	2.00443	0.50422	0.03839
WA114D0	61084.8	-181103	935.105	0.91	4.89902	4.10681	3.38393	0.96526	0.54219	0.00763
WA114D1	61084.7	-181103	935.001	0.88	4.43463	4.12602	3.13055	0.7909	0.50495	0.00824
WA114D2	61084.6	-181103	934.803	0.98	4.44377	4.03163	3.1226	0.81544	0.49222	0.01351
WA115D0	62846.3	-184181	635.1	2.09497	3.35115	3.45256	1.65136	1.41006	0.24038	0.04936
WA115D1	62846.3	-184181	635.048	2.14001	2.74987	3.54878	1.49904	0.99304	0.21705	0.04074
WA115D2	62846.2	-184181	635.01	2.23999	4.56433	3.60227	2.27614	1.93478	0.30385	0.04955
WA116D0	63006.6	-183711	698.305	2.02997	3.78968	3.62561	1.76267	1.66779	0.31316	0.04606
WA116D2	63006.4	-183711	698.208	1.95001	3.37279	3.59262	1.82708	1.22769	0.28136	0.03666
WA117D0	62629.9	-183179	711.497	1.58002	5.19505	3.94115	2.70354	2.026	0.40586	0.05964
WA118D0	62402.2	-183515	664.09	1.94	3.52031	3.65636	1.93534	1.26731	0.28149	0.03616
WA118D1	62402.1	-183515	664.126	2.16998	4.39457	3.7608	2.42688	1.56127	0.37603	0.03037
WA118D2	62402	-183514	664.158	1.91004	4.27696	3.73285	2.1622	1.7552	0.31529	0.04426
WA118D3	62401.9	-183514	664.155	1.96601	4.45975	3.73265	2.27617	1.79712	0.33821	0.04826
WA11D0	62325.6	-181256	959.475	0.57001	5.82587	3.9986	3.6261	1.76318	0.38153	0.05507
WA11D1	62325.5	-181256	959.337	0.59	6.05455	4.00644	3.79165	1.80944	0.39476	0.0587
WA11D2	62325.4	-181256	959.728	0.47	5.91901	3.95362	3.78582	1.73601	0.33722	0.05996
WA120D0	61861.9	-181081	970.055	0.92999	6.37521	3.96581	3.98354	1.71379	0.66494	0.01293
WA120D2	61861.6	-181081	969.752	0.815	4.72898	3.82522	3.0026	1.24141	0.47523	0.00974
WA121D0	62897.1	-182397	811.702	3.78503	2.89724	3.4615	1.41041	1.21895	0.23564	0.03225
WA121D2	62896.8	-182397	811.734	3.65997	2.29095	3.62057	1.26672	0.8087	0.19116	0.02437
WA121D4	62897	-182397	812.222	3.62	2.54058	3.56875	1.32422	0.96221	0.22806	0.02608
WA122D0	62086.3	-184031	603.545	1.33002	7.54325	3.91607	4.03807	2.78692	0.6664	0.05186
WA122D3	62085.9	-184031	602.341	1.04999	6.15926	3.86339	3.40464	2.15577	0.54382	0.05503
WA123D0	62579.6	-184451	593.797	1.64502	8.61041	4.21191	4.33137	3.46355	0.71786	0.09763
WA123D3	62579.1	-184451	594.223	1.15002	6.73526	3.86239	3.57876	2.60157	0.50807	0.04686
WA123D4	62579	-184451	594.158	1.03003	5.81584	3.99931	3.45149	1.80548	0.523	0.03587
WA124D0	62518.5	-182496	787.19	0.72	8.39902	3.86626	3.89829	3.75758	0.68425	0.0589
WA124D3	62518.1	-182496	786.986	0.72	8.53073	3.98807	3.96033	3.76797	0.74597	0.05646
WA124D5	62518.3	-182496	786.724	0.51999	7.10666	4.09769	3.52237	2.94868	0.57073	0.06489
WA127D0	64393.6	-182784	875.873	0.815	8.35428	4.07202	4.45396	3.19416	0.64901	0.05716

WA127D1	64393.4	-182784	875.772	0.88	7.78305	4.05296	4.65989	2.40025	0.66944	0.05348
WA127D2	64393.2	-182784	875.412	0.97	8.23847	4.15051	4.47988	3.05258	0.63325	0.07276
WA128D0	61614.1	-182747	713.332	1.28998	5.80021	3.79609	3.13411	2.11358	0.49949	0.05302
WA128D1	61613.8	-182747	713.446	1.25	6.03766	3.78101	3.06944	2.49366	0.40582	0.06874
WA128D2	61613.7	-182747	713.299	1.29004	5.77404	3.80139	3.02836	2.20719	0.46841	0.07009
WA129D0	61815.6	-182748	720.873	1.18994	6.50567	3.92429	3.5231	2.37349	0.55156	0.05751
WA129D1	61815.3	-182748	721.228	0.95996	7.01045	4.0049	3.83452	2.53278	0.59444	0.04871
WA129D2	61815.2	-182748	721.127	1.04999	7.82748	3.93378	4.07449	3.04533	0.63506	0.0726
WA12D0	62865.1	-181173	1031.34	1.81	4.78655	3.64033	2.93497	1.48654	0.31037	0.05467
WA12D1	62865	-181173	1031.3	1.8	6.1686	3.66128	3.40346	2.33786	0.36124	0.06603
WA12D2	62864.9	-181173	1030.77	1.905	5.14898	3.65244	3.01944	1.75933	0.32505	0.04515
WA130D0	61792.5	-182569	737.994	1.375	5.57125	3.86945	2.75807	2.30631	0.45281	0.05405
WA130D1	61792.4	-182569	738.017	1.43005	5.08817	3.73117	2.76843	1.86221	0.4031	0.05445
WA130D2	61792.2	-182568	737.987	1.32996	5.16769	3.75251	2.9325	1.77034	0.40914	0.05571
WA131D0	62161.1	-183181	688.52	2	3.98801	3.48871	2.15003	1.48044	0.31296	0.04458
WA131D1	62160.7	-183180	688.233	1.86005	4.41629	3.58479	2.35722	1.64492	0.36259	0.05157
WA131D2	62160.6	-183180	688.365	2.03998	4.00932	3.66185	2.14673	1.51316	0.30162	0.0478
WA132D0	62766.4	-183205	718.698	3.58496	2.22998	3.34614	1.19137	0.82774	0.18673	0.02414
WA132D1	62766.1	-183205	713.567	3.77503	2.75687	3.4169	1.31893	1.20711	0.19808	0.03274
WA132D2	62766.2	-183205	718.294	4.27997	3.60162	3.47386	1.78555	1.49765	0.28644	0.03199
WA133D0	62392.1	-182641	758.147	1.95502	3.81722	3.6642	2.11964	1.35348	0.30236	0.04174
WA133D1	62391.8	-182641	758.23	1.98999	4.39597	3.68579	2.29273	1.74373	0.31485	0.04467
WA133D2	62391.7	-182641	758.821	2.01001	4.07747	3.61615	2.04495	1.69678	0.28793	0.04782
WA134D0	62652.3	-182390	807.478	2.86502	3.17677	3.53607	1.63524	1.26969	0.24403	0.02781
WA134D1	62652.2	-182390	807.038	2.97	3.0964	3.52775	1.53055	1.291	0.24419	0.03066
WA134D2	62652	-182389	807.063	2.82501	3.76804	3.5883	1.77065	1.67689	0.2789	0.0416
WA135D0	63212.7	-182135	861.44	2.57001	5.57677	3.6786	2.65794	2.44734	0.41615	0.05535
WA135D1	63212.4	-182135	861.326	2.40003	4.75342	3.73971	2.32245	2.04106	0.34528	0.04462
WA135D2	63212.2	-182134	861.994	2.09	4.61381	3.69024	2.39268	1.81305	0.3705	0.03758
WA138D0	63226.4	-183505	731.758	2.46496	3.16645	3.59185	1.68808	1.20691	0.23385	0.03762
WA138D1	63226.3	-183505	731.764	2.435	3.21937	3.64085	1.6912	1.28251	0.21326	0.0324
WA138D2	63226.1	-183504	731.648	2.46503	2.74096	3.63643	1.40258	1.12288	0.19179	0.02372
WA139D0	63045.2	-182077	871.396	1.67499	5.45141	3.87685	3.07283	1.86345	0.46727	0.04785
WA139D1	63044.9	-182076	871.322	1.79501	5.20946	3.72142	2.56215	2.23686	0.35951	0.05093
WA139D2	63044.8	-182076	871.566	1.63999	6.9101	3.83773	3.13377	3.27207	0.4094	0.09486
WA13D1	63695.4	-181661	980.072	2.3	3.15745	3.77608	1.867	1.03733	0.21749	0.03563
WA142D0	63129.7	-182968	763.37	1.06	1.92502	3.65361	1.4838	0.23365	0.19849	0.00907
WA142D0	63129.7	-182968	759.63	1.56	4.1738	3.84624	2.15088	1.61941	0.36508	0.03843
WA142D1	63129.5	-182968	763.588	1.04999	2.91138	3.8509	1.46303	1.21898	0.20719	0.02218
WA142D1	63129.4	-182968	759.647	1.59497	3.72069	3.9239	1.75368	1.61684	0.31031	0.03986
WA142D2	63129.4	-182968	763.545	0.94	1.56658	3.41873	1.14095	0.25339	0.16225	0.00999
WA142D2	63129.3	-182968	759.681	1.52997	3.93465	3.67882	2.01363	1.54781	0.34457	0.02864
WA15D0	64495.7	-181861	1007.71	0.29	3.49619	4.295	2.77522	0.57369	0.11865	0.02863
WA15D1	64495.6	-181861	1008.22	0.235	3.10802	4.26	2.46578	0.37565	0.24456	0.02203
WA15D2	64495.5	-181861	1007.77	0.305	3.65017	4.2241	2.98748	0.39931	0.23329	0.03009
WA16D0	63997.2	-183183	777.834	3.44998	3.41551	3.81333	1.88302	1.24913	0.24991	0.03345
WA19D0	64264.2	-181954	950.964	2.86	2.39905	3.58542	1.39516	0.80314	0.16208	0.03868
WA19D1	64264.3	-181954	950.76	2.81002	2.80113	3.60726	1.38989	1.20816	0.15767	0.04542
WA19D2	64264.3	-181954	951.056	2.40001	3.08827	3.61876	1.674	1.19191	0.16648	0.05588
WA20D0	63121.3	-181198	1038.29	0.96	4.73868	4.12375	2.96415	1.47714	0.26031	0.03707
WA20D1	63121.2	-181198	1038.15	1.05	4.48803	3.94809	2.97639	1.17398	0.30214	0.03551
WA25D0	62748.6	-181117	1038.57	1.05	6.62142	3.69629	4.1486	1.79128	0.66034	0.02121
WA25D1	62748.5	-181117	1037.74	0.95	6.52803	3.57821	3.71189	2.22192	0.56011	0.03411
WA25D2	62748.4	-181117	1038.27	1.07	7.01205	3.71477	4.2551	2.10709	0.61946	0.03041
WA27D0	63003.8	-181211	1031.95	1.26501	5.11426	4.03166	3.32646	1.28603	0.47176	0.03001
WA28D0	61529.4	-181253	929.188	1	6.93764	4.07849	4.03211	2.29291	0.57574	0.03689
WA30D0	62669.6	-183576	677.644	2.52997	3.16786	3.52681	1.65648	1.27922	0.19519	0.03697
WA30D2	62669.5	-183576	677.598	2.52002	3.42745	3.56954	1.82309	1.34855	0.2086	0.04721
WA31D0	61910.1	-183794	619.057	1.29999	8.11859	3.9097	3.50804	4.11525	0.38164	0.11366
WA31D1	61910.1	-183794	618.797	1.23004	6.8642	3.93489	3.59869	2.7948	0.38701	0.0837
WA31D2	61910.2	-183794	618.985	1.23999	7.44596	3.8959	3.26259	3.72699	0.37803	0.07834

WA32D0	61770	-180940	1005	0.95001	3.85954	3.852	2.93979	0.59604	0.3106	0.0131
WA32D1	61770	-180940	1005.01	0.91001	3.57258	3.93032	2.65756	0.63304	0.27263	0.00935
WA32D2	61770.1	-180940	1005.2	1.04001	6.71791	3.97519	4.33587	1.85721	0.50156	0.02326
WA33D0	61687.5	-180631	1053.95	1.16	7.85429	4.14116	4.71072	2.61652	0.48275	0.04429
WA33D1	61687.4	-180631	1053.94	1.11	7.30873	4.18874	4.60745	2.20397	0.46041	0.0369
WA33D2	61687.3	-180631	1053.9	1.06001	7.52118	4.24367	4.55071	2.43908	0.47693	0.05446
WA34D0	62204.6	-182949	712.4	2.11005	3.57246	3.65855	1.94185	1.31548	0.26381	0.05132
WA34D1	62204.5	-182949	712.649	2.13001	4.37242	3.64813	2.04488	1.98225	0.29594	0.04935
WA34D2	62204.4	-182949	712.441	2.04004	3.31946	3.64921	1.7983	1.22427	0.2523	0.04459
WA36D0	62864.1	-181625	936.971	0.89999	7.92269	4.23157	4.57099	2.66234	0.63373	0.05563
WA36D1	62864.1	-181625	937.071	0.84001	7.33307	4.23071	4.30341	2.39271	0.58861	0.04834
WA36D2	62864.1	-181625	937.036	0.87	8.15066	4.26207	4.61631	2.82611	0.66138	0.04687
WA39D0	62540.4	-181644	910.134	1.12	6.89472	3.85715	3.82981	2.62345	0.38815	0.0533
WA39D1	62540.4	-181644	910.048	1.19	6.72297	3.87733	3.65406	2.60535	0.40829	0.05527
WA39D2	62540.3	-181644	910.028	1.15003	7.1045	3.93096	3.76677	2.83724	0.40804	0.09245
WA40D0	61635.3	-183241	661.347	1.48999	5.19177	3.78657	2.74869	1.99585	0.40225	0.04497
WA40D1	61635.3	-183241	661.236	1.54999	5.49823	3.72241	2.99794	2.01298	0.43657	0.05075
WA41D0	61243.4	-180567	1035.08	1.17499	9.79502	4.15502	5.12457	3.79197	0.76356	0.11491
WA41D1	61243.3	-180567	1035.05	1.13	9.60046	4.12159	5.33599	3.36441	0.82047	0.07958
WA41D2	61243.2	-180567	1035.22	1.09	8.18988	4.03496	4.47997	3.03893	0.63246	0.03853
WA42D0	63182	-182462	821.081	3.94	1.83245	3.34262	0.95034	0.71337	0.14525	0.02348
WA42D1	63181.9	-182462	820.942	3.85003	2.46487	3.38516	1.12352	1.13293	0.17471	0.03371
WA42D2	63181.8	-182462	821.055	3.97998	2.18589	3.38377	1.13881	0.84339	0.16919	0.03451
WA43D0	63681.3	-182469	833.942	2.07999	2.93625	3.53784	1.58768	1.08424	0.23091	0.03343
WA43D1	63681.4	-182469	833.99	2.06	3.02771	3.62223	1.79893	0.93385	0.26634	0.02859
WA43D2	63681.4	-182469	832.932	2.16999	2.72205	3.56318	1.42999	1.04457	0.21581	0.03168
WA44D0	61750.2	-182195	789.597	1.17999	6.67765	3.92999	3.36825	2.7923	0.43166	0.08543
WA44D2	61749.6	-182194	789.448	1.19	5.36628	3.91206	2.80251	2.10228	0.40379	0.0577
WA44D3	61750.2	-182195	789.455	1.20996	6.40765	3.93756	3.22377	2.64973	0.46763	0.06652
WA45D0	62309.1	-184178	600.674	1.19	6.90694	3.98461	3.37272	2.92459	0.52582	0.08382
WA45D1	62309.2	-184178	601.19	1.24005	6.20297	3.85512	3.25354	2.40763	0.47678	0.06502
WA45D2	62309.1	-184177	601.172	1.53998	7.12217	4.05553	3.69491	2.83461	0.54009	0.05256
WA49D0	63490.1	-183100	775.149	2.51001	3.19362	3.66454	1.68892	1.2359	0.23149	0.03732
WA49D1	63490.1	-183100	775.034	2.59003	3.1008	3.67792	1.75393	1.07624	0.23641	0.03422
WA49D2	63490.1	-183100	775.139	2.56995	3.17493	3.644	1.68007	1.22163	0.23405	0.03918
WA50D0	62346.7	-182308	804.678	1.84	4.1108	3.73135	2.21695	1.6003	0.24693	0.04661
WA50D1	62346.7	-182308	804.3	2.22	5.53049	3.75473	2.733	2.31409	0.43048	0.05292
WA50D3	62346.8	-182307	804.306	2.35001	5.2019	3.76188	2.49176	2.27012	0.3721	0.06791
WA51D0	62895	-182837	757.793	4.48999	2.07948	3.35584	1.0173	0.86828	0.16634	0.02756
WA52D0	62161.6	-181769	841.416	1.63001	3.70249	3.80101	1.9278	1.44246	0.28332	0.04892
WA52D1	62161.6	-181769	841.538	1.69998	4.41937	3.88304	2.07613	1.9752	0.31939	0.04865
WA52D2	62161.7	-181769	841.595	1.54001	3.68291	3.81069	1.99406	1.35214	0.29681	0.0399
WA53ID0	61253.1	-181004	937.83	0.44	3.74399	4.11	3.01483	0.34115	0.37275	0.01527
WA54ID0	61411.7	-181143	945.49	0.98	6.58155	4.19648	3.77558	2.1701	0.56261	0.07326
WA57ID0	62301.8	-181604	880.674	2.14999	3.67223	3.55944	1.86852	1.48244	0.28143	0.03984
WA58ID0	62569.7	-181945	857.749	1.62	4.68301	3.42068	2.43265	1.82359	0.36994	0.05683
WA59ID0	62820.4	-182121	851.649	0.63001	7.50476	3.50937	3.20552	3.67752	0.55108	0.07064
WA60ID0	63406.4	-182687	797.659	3.64001	2.0033	3.53202	1.01865	0.78073	0.17173	0.03219
WA61ID0	61161.7	-180235	1075.02	0.98	6.08497	4.18944	3.82838	1.63761	0.60068	0.01831
WA62ID0	61312.8	-180300	1083	1.03	6.09913	4.04087	3.85165	1.67089	0.55998	0.0166
WA63ID0	61462.1	-180355	1089.82	0.97	5.86791	4.00165	3.61148	1.71345	0.52771	0.01527
WA64ID0	61610	-180380	1100.87	1.27	6.11332	4.12339	3.68482	1.87048	0.51739	0.04063
WA65ID0	61824.6	-180462	1101.92	0.96501	6.79306	4.27513	4.33748	1.82896	0.60123	0.02539
WA66ID0	62002.8	-180584	1089.66	1.14	5.71445	3.96351	3.67956	1.43022	0.5875	0.01718
WA67ID0	62138.7	-180896	1037.82	1.095	8.00355	3.96644	3.96328	3.33264	0.6284	0.07923
WA68ID0	62271.2	-181058	1005.24	0.59	3.20149	3.58102	1.75145	1.12367	0.30864	0.01773
WA69ID0	62441	-181059	1024.76	1.215	8.22137	3.95881	4.19603	3.31439	0.64607	0.06487
WA70ID0	63217	-181257	1033.24	1.585	4.44058	3.85981	2.61697	1.40751	0.38136	0.03475
WA71ID0	63336.4	-181348	1029.59	0.89999	6.17544	4.27489	4.20612	1.33562	0.61157	0.02214
WA72ID0	63428.9	-181451	1010.99	0.73	7.1776	3.94699	3.65825	2.91313	0.53406	0.07215

WA73ID0	63558.4	-181560	993.622	1.09	7.58683	3.97761	4.01517	3.07228	0.11588	0.3835
WA74ID0	63836.6	-181756	961.434	2.09999	4.47392	3.84695	2.43675	1.59338	0.40543	0.03837
WA75ID0	63980.7	-181798	971.163	1.58	5.29308	3.70405	2.35432	2.45944	0.4105	0.06882
WA76ID0	64084.6	-181911	955.102	0.7	8.05219	4.05115	3.98609	3.33588	0.6408	0.08942
WA77ID0	64424.9	-181986	983.694	0.95	5.58326	3.73368	3.0574	1.97288	0.5081	0.04488
WA78ID0	64541.6	-182091	997.752	1.27001	6.12619	3.70795	3.75434	1.73102	0.58565	0.05518
WA79ID0	63675.5	-182926	791.955	1.91998	3.77395	3.63056	1.942	1.51281	0.28333	0.03582
WA80D0	62155	-183413	661.309	1.56	4.00378	3.70156	2.05996	1.61325	0.28397	0.04659
WA80D1	62154.9	-183413	661.102	1.83002	5.43226	3.73787	2.83076	2.11575	0.42965	0.0561
WA80D2	62154.8	-183413	661.153	1.79999	4.58318	3.65477	2.25695	1.92613	0.34132	0.05878
WA84D0	61644.7	-181720	817.049	1.44998	4.38227	3.53234	2.5227	1.44226	0.37404	0.04327
WA84D1	61644.5	-181720	817.08	1.60001	6.80606	3.65319	3.23939	3.02441	0.48038	0.06188
WA86D0	62515	-181228	990.295	0.93001	7.6858	4.24774	4.60854	2.34812	0.67742	0.05171
WA86D1	62514.9	-181227	990.212	0.92	7.31705	4.24908	4.4737	2.12311	0.67457	0.04568
WA86D2	62514.8	-181227	990.309	1.03	6.94504	4.25281	4.28927	1.94074	0.67975	0.03528
WA87D0	62411.9	-181324	958.42	0.81999	6.44989	4.56646	3.94409	1.8864	0.57686	0.04254
WA87D1	62411.7	-181323	958.343	0.84	6.33591	4.53404	3.93018	1.7392	0.61368	0.05284
WA87D2	62411.6	-181323	958.036	0.81999	7.17104	4.44207	4.41838	2.02241	0.69869	0.03156
WA88D0	62279	-181475	913.5	1.18002	6.16745	4.20942	3.5583	2.03009	0.53297	0.04609
WA88D1	62278.9	-181475	913.461	1.22	6.86857	4.22413	3.97818	2.14223	0.70045	0.04771
WA88D2	62278.8	-181475	913.593	1.22	6.42289	4.22099	3.72574	2.10418	0.54794	0.04502
WA89D0	62111.4	-181632	840.169	1.23001	5.04111	4.0461	2.77404	1.78761	0.4416	0.03786
WA89D1	62111.3	-181632	840.296	1.31	6.15095	4.02344	3.21042	2.41076	0.49193	0.03785
WA89D2	62111.2	-181632	840.301	1.19	6.12267	4.08539	3.14108	2.40969	0.509	0.0629
WA91D0	62682.9	-181309	987.77	0.88001	6.95159	4.19023	4.03142	2.27494	0.60446	0.04077
WA91D1	62682.7	-181308	987.787	0.97	7.32733	4.09923	4.15028	2.49878	0.63368	0.04458
WA91D2	62682.6	-181308	987.809	0.93001	7.13042	4.0843	4.32806	2.09893	0.66449	0.03895
WA92D0	62542.2	-181455	945.023	1.21001	5.40538	3.96785	3.09298	1.77235	0.48482	0.05523
WA92D1	62542	-181455	944.916	1.31	6.24084	3.97901	3.29156	2.40581	0.46124	0.08222
WA92D2	62541.9	-181455	944.698	1.40999	5.76628	4.11262	3.45496	1.73691	0.52084	0.05357
WA93D0	62393.6	-181615	895.747	1.58002	5.76217	3.70196	2.91384	2.34374	0.43497	0.06961
WA94D0	61963.1	-182176	788.927	1.35999	5.0477	3.85598	2.50211	2.10233	0.38155	0.06171
WA95D0	61549.7	-182028	807.783	0.89001	7.81388	4.25899	4.13762	2.91366	0.68483	0.07777
WA96D0	61396.6	-181938	821.939	0.94	6.83038	4.00182	3.74004	2.50055	0.52052	0.06927
WA97D0	61477.2	-182296	773.125	1.39001	6.03782	3.86434	3.71941	1.76947	0.5211	0.02784
WA98D0	61528.2	-182501	743.57	1.10004	7.56203	4.32388	3.70871	3.18219	0.56678	0.10436
WA98D1	61528.2	-182501	743.449	1.01996	9.10347	4.19088	4.44622	3.86405	0.67824	0.11495
WA98D2	61528.1	-182501	743.406	1.02002	7.73258	4.18024	4.12194	2.87881	0.64795	0.08388
WA99D0	61587.1	-181539	796.457	5.94998	1.1329	3.79334	0.72283	0.28121	0.11965	0.00922
WC10D2	60325.4	-184938	370.945	0.79999	7.07088	4.15248	3.97011	2.6025	0.44387	0.05439
WC12D0	59454.6	-183677	456.662	0.79001	7.24304	4.2057	4.07795	2.62416	0.4688	0.07213
WC12D1	59454.6	-183677	456.513	0.79001	9.04087	4.26708	4.79185	3.49524	0.64606	0.10771
WC12D2	59454.6	-183677	456.658	0.75998	8.57502	4.35737	4.46245	3.44425	0.56129	0.10703
WC13D0	60555.7	-185547	296.24	1.19	7.12248	4.1122	3.90115	2.69553	0.45427	0.07153
WC13D1	60555.6	-185547	296.268	1.22003	9.04035	4.1173	4.65601	3.74075	0.53365	0.10993
WC13D2	60555.5	-185547	296.2	1.24005	10.7221	4.08694	4.40046	5.69321	0.49099	0.13742
WC14D0	59347.6	-184227	388.719	0.91003	10.3412	4.27395	5.48953	3.99454	0.7594	0.09774
WC14D1	59347.6	-184227	388.645	0.96997	10.0495	4.26639	4.95004	4.27302	0.6975	0.12898
WC14D2	59347.7	-184226	388.721	1.10999	13.5157	4.26948	6.2638	6.20911	0.85087	0.19194
WC15D0	60082.6	-184259	438.501	0.85999	5.49753	4.03145	3.93057	1.18994	0.35362	0.0234
WC15D1	60082.7	-184259	438.59	0.89997	7.01537	4.04145	4.32067	2.21636	0.45379	0.02455
WC15D2	60082.9	-184259	438.373	0.96997	8.81123	4.03639	4.86824	3.3371	0.50592	0.09997
WC15D3	60083	-184259	438.626	1.04004	7.70542	4.48114	4.69943	2.49588	0.47074	0.03937
WC16D0	60977.3	-185779	288.698	0.83997	7.49651	3.94869	4.29025	2.52468	0.61033	0.07125
WC17D0	61036.8	-181676	842.51	1.07999	7.7757	4.0388	4.5363	2.4992	0.6857	0.0545
WC17D1	61036.7	-181676	842.547	0.97	5.9264	4.03412	3.76626	1.55048	0.56916	0.0405
WC17D2	61036.6	-181676	842.473	1.16	8.93804	4.35947	4.84242	3.32938	0.69331	0.07293
WC18D0	60743.1	-181473	869.705	0.78999	7.05429	4.38785	4.22554	2.16751	0.64409	0.01715
WC18D1	60743.1	-181473	869.641	0.82001	6.51861	4.355	4.31738	1.52664	0.65933	0.01527

WC18D2	60742.9	-181473	869.703	0.79999	5.74465	4.40863	3.80089	1.35264	0.57586	0.01527
WC19D0	60027.9	-181844	761.143	1.11499	7.10156	4.42	4.15834	2.24929	0.63844	0.05549
WC19D1	60027.8	-181844	761.034	1.17999	8.54391	4.53916	4.56914	3.16319	0.73557	0.07601
WC19D2	60027.7	-181844	761.262	1.17999	7.7521	4.31771	4.48031	2.46061	0.70568	0.10549
WC20D0	59615	-181956	700.609	0.94	6.91794	4.43298	4.06396	2.23276	0.57725	0.04397
WC20D1	59615	-181956	700.622	0.94998	7.83277	4.35926	4.3995	2.71388	0.65452	0.06487
WC20D2	59615.1	-181956	700.629	0.96997	8.08512	4.20227	4.74327	2.59389	0.68648	0.06148
WC21D0	60864.8	-182448	726.338	1.23999	6.92466	4.06052	4.05984	2.28752	0.54317	0.03414
WC21D1	60864.9	-182448	726.387	1.34998	5.86175	3.95977	3.61433	1.73215	0.4935	0.02177
WC21D2	60864.9	-182448	726.291	1.13001	6.897	4.03761	3.51591	2.89309	0.44194	0.04607
WC22D0	61334	-182912	674.116	1.28003	4.89706	3.95046	2.6027	1.9304	0.0867	0.27726
WC22D1	61333.9	-182912	673.889	1.34003	5.35072	3.94909	3.07041	1.84777	0.07912	0.35342
WC22D2	61333.9	-182912	673.951	1.51001	5.98253	3.89471	2.82121	2.76334	0.09808	0.2999
WC23D0	60729.3	-182971	635.305	1.12994	6.53591	3.97485	3.50818	2.47827	0.4962	0.05327
WC23D1	60729.2	-182971	635.156	1.18	6.02129	3.97832	3.22657	2.29598	0.4467	0.05204
WC23D2	60729.2	-182971	635.052	1.22003	7.48978	3.99025	4.03288	2.79885	0.59279	0.06526
WC24D0	61283.8	-182183	786.329	1.09998	5.25185	3.98691	3.3471	1.34271	0.53306	0.02898
WC24D1	61283.7	-182183	786.252	1.07001	5.53869	3.9929	3.5169	1.44591	0.53942	0.03646
WC24D2	61283.5	-182183	785.314	1.11999	6.40936	3.97643	3.54205	2.2278	0.59466	0.04485
WC24D3	61283.3	-182183	786.291	1.22003	6.43309	4.01377	3.67928	2.14883	0.56056	0.04443
WC26SD0	61128.3	-182886	663.923	1.19	5.3156	3.80229	2.71091	2.11164	0.43054	0.06252
WC27D0	61471.7	-183966	571.589	0.92999	7.34482	3.85817	3.94559	2.66546	0.64561	0.08816
WC27D1	61471.7	-183966	571.67	0.98004	8.02273	3.83897	4.24625	3.00207	0.68584	0.08857
WC27D2	61471.8	-183966	571.932	0.91003	6.67234	3.86341	3.82817	2.20819	0.58154	0.05444
WC28D0	60914.3	-183797	546.575	0.85999	6.26462	3.91628	3.43462	2.27435	0.502	0.05366
WC28D1	60914.3	-183797	546.739	0.81	5.86991	3.91383	3.35764	1.96094	0.50101	0.05032
WC28D2	60914.2	-183797	546.473	0.90002	9.97584	3.92111	4.82859	4.34014	0.70625	0.10085
WC29D0	59757.8	-182457	632.281	1.16	7.65162	3.93724	3.92872	3.10281	0.57678	0.0433
WC29D1	59757.9	-182457	632.129	1.28	6.77405	3.89054	3.41931	2.76665	0.5196	0.0685
WC29D2	59758.1	-182456	632.061	1.12997	7.22786	3.95045	3.91897	2.67104	0.5754	0.06246
WC30D0	58087.2	-182881	395.371	1.16004	9.06789	4.2717	4.94668	3.23502	0.84536	0.04083
WC30D1	58087.1	-182881	395.51	1.20001	7.24172	4.38625	4.08799	2.49764	0.60844	0.04765
WC30D2	58087	-182881	395.384	1.25	6.0296	4.29312	4.13751	1.2374	0.63815	0.01655
WC31D0	61166.2	-182580	715.701	1.23999	8.56918	4.02589	4.61807	3.15505	0.73529	0.06076
WC32D0	60891.8	-182859	654.872	1.51001	7.06382	3.95304	3.49325	2.93571	0.56654	0.06832
WC33ID0	60717.4	-182648	683.883	1.08002	7.04406	4.06018	3.79951	2.62558	0.56433	0.05464
WC36D0	58602.3	-183119	447.973	0.87006	8.16057	4.12183	3.99841	3.49665	0.59808	0.06743
WC5D1	58495.1	-183053	441.492	0.76001	6.01591	3.95697	3.4684	2.0119	0.48596	0.04965
WC5D2	58495.2	-183053	442.119	0.83002	6.17655	3.99372	3.8481	1.76349	0.51438	0.05059
WC6D0	59008.6	-183138	497.333	1.85999	2.25631	3.47338	1.4938	0.55834	0.17838	0.02579
WC6D1	59008.7	-183138	497.101	2.04001	3.27277	3.53651	1.98969	0.97684	0.2787	0.02754
WC6D2	59008.7	-183138	497.209	2.14002	2.98506	3.5443	1.62674	1.10207	0.23045	0.0258
WC7D1	58658.9	-183554	407.047	1.53003	3.47493	3.67309	1.9715	1.16074	0.29695	0.04574
WC7D2	58655.1	-183561	407.634	1.60004	4.77744	3.65239	2.61453	1.76695	0.32178	0.07418
WC7D3	58655.1	-183561	407.382	1.54999	4.68529	3.65522	2.46604	1.84442	0.29643	0.07841
WC9D0	60479.7	-184570	394.029	0.82001	5.78798	3.97707	3.33798	1.93573	0.45395	0.06032
WC9D1	60479.8	-184570	395.185	0.80005	5.71741	3.99725	3.3407	1.93115	0.38317	0.06239
WC9D2	60479.8	-184571	395.253	0.76001	5.61076	4.04368	3.46356	1.63702	0.46759	0.04259
WQ005D0	60332.3	-179274	1085.56	1.03	6.67247	3.68	3.64726	2.44221	0.5248	0.0582
WQ006D0	60597.6	-179601	1081.87	1.1	6.55652	3.88	3.59247	2.3902	0.51673	0.05712
WQ007D0	59753.2	-178789	1086.92	1.08	6.95152	3.93241	3.72298	2.63005	0.53697	0.06152
WQ008D0	60038.9	-179063	1079.22	1.24	10.213	3.78371	5.28687	4.0675	0.76692	0.09167
WQ011D0	61043	-180055	1088.1	1.36	7.91313	3.94	4.23346	2.99875	0.61112	0.0698
WQ012D0	60903.8	-180211	1054.36	1.03	14.7541	3.71	7.46565	6.06765	1.08705	0.13377
WQ014D0	58984.2	-178439	1049.65	0.85	6.05794	3.94	3.35689	2.16655	0.48204	0.05245
WQ015D0	59153.2	-178557	1057.8	0.82	11.7163	3.74	6.03036	4.70483	0.87571	0.10537
WQ016D0	59561.3	-178703	1078.32	1.08	4.96802	3.66	2.84187	1.67767	0.40621	0.04226
WQ017D0	59866.7	-178961	1076.42	1.36	10.2553	3.94	5.34009	4.04944	0.77407	0.0917
WQ018D0	58805.9	-178352	1047.63	1.09	6.28984	3.73	3.46646	2.27058	0.49818	0.05462
WQ019D0	60863.2	-179962	1073.08	1.36	7.65914	3.69559	4.1134	2.88488	0.59344	0.06743

WQ022D0	60606.5	-180232	1020.44	1.21001	6.24149	4.13661	3.7525	1.87902	0.58306	0.02691
WQ023D0	60610.6	-179846	1054.28	0.93	11.8115	4.30613	5.42042	5.38344	0.79416	0.21345
WQ024D0	60379	-180114	1005.65	1.10001	6.29407	4.07418	4.11302	1.53522	0.62709	0.01874
WQ025D0	60377.2	-179772	1041.15	1.13	8.22135	4.16089	4.09561	3.44784	0.60055	0.07735
WQ027D0	59738.9	-179961	947.884	1.09998	8.01882	4.02673	4.23395	3.12985	0.55803	0.09699
WQ028D0	59852.7	-179291	1034.73	1.21001	8.53369	4.32025	4.6648	3.10761	0.67066	0.09063
WQ029D0	59624.1	-179538	983.839	1.35999	8.81905	4.09969	4.28093	3.81943	0.5897	0.12899
WQ031D0	59181.2	-179119	985.408	1.05	6.4939	4.02876	4.02266	1.81667	0.6304	0.02417
WQ032D0	59254.2	-178823	1026.98	0.98	6.19621	4.00939	3.959	1.65267	0.55082	0.03373
WQ033D0	58837	-178913	972.759	1.17	6.51377	4.16009	3.64552	2.25941	0.54793	0.06091
WQ034D0	58538.3	-178641	978.452	1.14	7.218	3.97855	4.30924	2.20628	0.67827	0.02421
WQ035D0	58275.4	-178409	981.034	1.10999	6.30899	3.9827	3.75452	1.96795	0.56317	0.02335
WQ036D0	58268.2	-178135	1015.41	1.57999	4.43262	3.80437	2.88546	1.08169	0.43185	0.03363
WQ039D0	58739.8	-178729	988.662	0.9	7.43463	4.05755	4.07345	2.70307	0.60061	0.05751
WQ040D0	60356.7	-179557	1059.36	1.06	7.07756	4.09434	4.11628	2.3266	0.61598	0.01869
WQ041D0	60082.4	-179330	1053.01	1.13	8.55125	4.38027	4.5509	3.26922	0.65141	0.07971
WQ042D0	58998.3	-178755	1011.73	1.09	7.25064	4.03018	3.71112	2.92031	0.52134	0.09787
WQ052D0	58425.2	-178329	1007.72	1.32001	6.85533	3.82954	3.59008	2.66042	0.53266	0.07217
WQ502D0	58779.1	-178488	1023.77	1.20001	6.85715	3.89941	4.14636	2.05797	0.6269	0.02592
WQ502D1	58779	-178488	1023.8	1.12999	6.33434	3.8308	4.06019	1.60995	0.64244	0.02175
WQ502D2	58778.9	-178488	1023.94	1.14	6.72453	3.8368	3.73039	2.36111	0.5798	0.05323
WQ503D0	58840.3	-178617	1015.82	1.28	5.81287	3.84046	3.32242	1.87729	0.55495	0.0582
WQ503D1	58840.2	-178617	1015.65	1.25	5.32292	3.86081	3.19425	1.61859	0.47084	0.03924
WQ503D2	58840.1	-178617	1015.68	1.28	5.02205	3.80179	3.00054	1.4645	0.51753	0.03948
WQ506D0	60513.7	-179609	1069.1	1.23	5.90914	3.98081	3.67942	1.78323	0.43887	0.00763
WQ508D0	59705	-179744	968.301	1.14999	6.98982	4.03183	4.24407	2.20956	0.47761	0.05857
WQ508D1	59706.8	-179742	968.854	1.19	5.75013	3.96488	4.02279	1.23406	0.47175	0.02153
WQ509D0	59721	-179387	1010.46	1.32001	6.18781	4.03417	4.03242	1.67821	0.43282	0.04437
WQ509D1	59721.1	-179387	1009.99	1.13	5.98229	4.0315	3.72201	1.8478	0.38227	0.03022
WQ509D2	59721.1	-179387	1010.26	1.31003	7.90285	4.03405	4.08006	3.33656	0.41946	0.06677
WQ510D0	60118.8	-179806	1007.4	1.16	6.85097	4.03198	4.27495	2.14176	0.42613	0.00813
WQ510D1	60118.7	-179806	1007.66	1.08	5.84783	3.99824	4.02055	1.41211	0.40561	0.00956
WQ510D2	60118.6	-179806	1007.56	1.05	6.19586	3.97676	3.88992	1.88766	0.398	0.02028
WQ513D0	60217.6	-179924	1005.41	1.14	5.58939	3.98158	3.81287	1.35262	0.41328	0.01062
WQ513D1	60217.5	-179924	1005.61	1.12	6.35419	4.03036	4.13844	1.7733	0.42755	0.01491
WQ513D2	60217.4	-179924	1005.59	1.16	6.25604	3.91733	4.28654	1.50216	0.44765	0.01969
WQ527D0	59982.9	-179709	1005.6	1.46001	6.12081	4.02603	3.69916	1.79895	0.60587	0.01684
WQ529D0	59951.2	-179999	966.333	2.48999	3.8775	3.52414	2.18996	1.30812	0.36293	0.01649
WQ530D0	59999.5	-179415	1038.76	1.24001	4.69684	4.0096	3.01757	1.20653	0.45747	0.01527
WV124D0	64538	-182515	940.567	0.95999	9.25294	4.15729	3.92374	4.68998	0.53248	0.10674
WV124D1	64537.9	-182516	940.671	0.87	6.76644	4.30483	3.88344	2.27017	0.59747	0.01535
WV124D2	64537.8	-182516	940.597	1.06	8.36199	4.03111	4.24301	3.39967	0.6191	0.10021
WV93ID0	64758.2	-182304	1014.72	1.27001	7.38863	4.15858	4.30444	2.33051	0.69363	0.06005
WV94ID0	64647.3	-182189	1010.77	1.06001	6.10389	4.36038	3.67947	1.84011	0.56418	0.02013
WW100LUD	59144.6	-181249	739.031	1.20001	6.26203	4.11975	4.14687	1.51017	0.5968	0.00821
WW101LSD	59109.1	-180587	809.262	2.12994	6.30295	3.98066	3.48349	2.30656	0.466	0.04689
WW102LDD	58773.8	-180511	783.112	2.06	3.84345	3.71806	2.05038	1.43917	0.32062	0.03327
WW103D0	58855.4	-179179	945.092	0.76	7.74469	4.22448	4.59776	2.41577	0.67145	0.05971
WW104D0	60648.7	-181124	911.786	1.27	6.18839	3.92976	3.60178	2.01278	0.53897	0.03486
WW104D2	60648.4	-181124	910.961	1.33	5.86936	3.83673	2.88382	2.42907	0.50329	0.05318
WW105LND	60559.6	-180992	919.733	1.07001	7.0978	4.01903	3.61729	2.90518	0.50905	0.06628
WW106D0	58624.9	-178951	947.362	1	10.2487	4.1251	4.83901	4.57411	0.74402	0.09156
WW106D1	58624.9	-178951	947.38	1	6.88184	4.1245	4.16158	2.03821	0.63189	0.05015
WW106D2	58624.8	-178951	947.363	0.90999	8.10212	4.24561	4.3136	3.03812	0.68803	0.06237
WW107D0	57819	-178552	917.686	0.98	8.90792	4.33759	4.81636	3.23171	0.75656	0.10329
WW107D1	57818.9	-178552	917.512	0.89001	8.26305	4.47387	4.74552	2.69826	0.74521	0.07406
WW107D2	57818.8	-178552	917.537	1.02001	8.66781	4.32882	4.61462	3.27931	0.70263	0.07125
WW108D0	57991.1	-178887	890.416	1.25	12.3008	4.0576	5.90711	5.49733	0.76151	0.13487
WW108D1	57991.2	-178887	890.447	1.17	6.12364	4.07299	3.85372	1.61823	0.61914	0.03256
WW108D2	57991.2	-178887	890.667	0.98999	7.02817	4.06899	3.88362	2.53688	0.56387	0.0438

WW109D0	59078.7	-179405	940.19	1	6.27692	3.8943	3.7852	1.86196	0.59083	0.03893
WW109D1	59078.7	-179405	940.268	0.88001	6.97969	3.89875	4.35485	1.94143	0.66294	0.02047
WW109D2	59078.8	-179405	940.57	1.14999	8.92937	3.90592	4.59729	3.47346	0.78465	0.07398
WW110D0	58302.9	-181619	571.204	1.32999	6.25754	3.86053	3.45207	2.16214	0.5919	0.05143
WW110D2	58303.2	-181619	571.199	1.19	10.3709	3.91399	4.73297	4.85794	0.64315	0.1368
WW112D0	57905.3	-180618	656.854	1.48001	5.55425	3.83939	2.99836	2.05366	0.45201	0.05021
WW112D1	57905.2	-180618	656.776	1.37	3.11693	3.72182	2.04419	0.75168	0.29172	0.02934
WW112D2	57905.1	-180618	656.809	1.35999	4.87059	3.7108	2.92772	1.49416	0.40075	0.04796
WW114D0	57142.5	-180210	648.141	1.25	4.74169	3.92057	2.94635	1.3404	0.4408	0.01414
WW114D2	57142.5	-180209	647.957	1.14001	8.62582	4.02158	4.28615	3.69339	0.59408	0.0522
WW116D0	57434.3	-179633	727.857	1.18001	7.63635	4.06203	4.42194	2.58261	0.56309	0.0687
WW116D1	57434.4	-179633	727.628	1.21001	7.4099	4.00372	4.43507	2.3058	0.632	0.03703
WW116D2	57434.4	-179633	727.344	1.41	7.28487	4.00234	4.75593	1.81077	0.68383	0.03435
WW119D0	56836.9	-180587	565.829	0.87	6.54111	4.22276	4.27685	1.62362	0.59422	0.04642
WW119D1	56836.8	-180587	565.702	0.91998	9.26899	4.04641	4.6489	3.69552	0.85699	0.06758
WW119D3	56836.5	-180587	565.516	0.84	8.58947	4.15333	4.96828	2.8643	0.68474	0.07215
WW121D1	59098.6	-179921	880.675	1.07001	8.45898	4.1915	4.07408	3.68251	0.62392	0.07848
WW121D3	59098.5	-179920	880.574	1.19998	8.38153	4.16979	4.11183	3.56396	0.62323	0.0825
WW122D0	59418.8	-180370	873.79	1.38001	5.09282	3.62671	3.11548	1.49277	0.44648	0.03809
WW122D1	59418.7	-180370	873.812	1.60001	6.06332	3.62669	3.41236	2.15406	0.46047	0.03642
WW122D2	59418.6	-180370	874.149	1.31	6.08045	3.69168	3.5655	1.99313	0.46603	0.05579
WW125D1	57292.6	-179057	793.722	1.36	4.26562	3.805	2.43095	1.42887	0.37431	0.03149
WW125D2	57292.5	-179057	793.455	1.52	3.92861	3.80796	2.50446	1.02004	0.38994	0.01416
WW125D4	57292.3	-179056	793.684	1.54999	4.72441	3.80406	2.88136	1.36829	0.43905	0.0357
WW126D0	57703.1	-181557	528.983	0.82999	8.8557	4.21445	4.70384	3.37913	0.70798	0.06475
WW128D0	58140.4	-179185	865.531	0.89	8.68834	4.11697	4.63255	3.31893	0.68433	0.05253
WW128D1	58140.3	-179185	865.275	0.86	6.25013	4.08291	3.84856	1.82307	0.53629	0.04221
WW128D2	58140.3	-179185	865.324	1	9.57434	4.0433	4.9341	3.84132	0.70308	0.09584
WW129D0	58135.5	-181875	558.357	1.09	8.74594	4.11895	4.67165	3.27582	0.72272	0.07574
WW132D0	57223.6	-179480	728.718	2.09	6.55952	3.77098	3.57519	2.35065	0.57932	0.05437
WW132D2	57223.6	-179480	728.624	2.16	4.21494	3.74722	2.4988	1.31421	0.36573	0.03621
WW132D3	57223.4	-179480	728.74	2.165	5.00167	4.05723	2.79074	1.7626	0.40495	0.04339
WW133D0	60124	-180792	903.62	2.51001	4.42959	3.65271	2.32085	1.74518	0.33342	0.03014
WW133D1	60123.9	-180793	903.513	2.63001	3.93351	3.64795	2.26073	1.31553	0.32927	0.02798
WW133D2	60123.8	-180793	903.909	2.53	2.93736	3.64087	1.70495	0.97658	0.23944	0.0164
WW134D0	57807.5	-180218	682.094	0.82999	7.1663	4.05072	4.23792	2.29347	0.59503	0.03987
WW134D1	57807.6	-180218	681.529	0.92502	10.0461	4.21135	4.77482	4.47962	0.68884	0.10281
WW134D3	57807.7	-180218	681.475	0.98001	9.89593	4.17346	4.46644	4.68866	0.65913	0.0817
WW135D0	57527.2	-179965	691.201	1.28	5.20455	3.81937	3.22709	1.48437	0.46676	0.02633
WW135D1	57527.1	-179965	691.18	1.22	4.93067	3.97651	2.98543	1.50633	0.39903	0.03989
WW135D2	57527.1	-179965	691.35	1.16001	5.6258	4.10631	3.47471	1.62646	0.49019	0.03445
WW138D0	60480.2	-180906	923.612	1.22501	6.07927	3.86967	3.96039	1.51143	0.58101	0.02644
WW138D2	60480	-180905	923.464	1.29999	6.09247	3.78223	3.5594	1.91473	0.57645	0.0419
WW138D5	60480.1	-180906	923.679	1.23502	7.13791	3.96518	3.94568	2.54059	0.59369	0.05795
WW15D0	57172.1	-180524	620.772	1.12	2.34913	3.455	1.7416	0.33122	0.25136	0.02494
WW15D1	57172.2	-180524	620.487	1.60998	5.89008	3.45981	3.26446	2.05667	0.52975	0.0392
WW15D2	57172.3	-180524	620.64	1.52002	2.98938	3.42271	1.75356	0.94836	0.24863	0.03883
WW15D3	57172.4	-180524	620.654	1.23001	4.12149	3.21098	2.17393	1.6411	0.26646	0.04
WW16D0	57627.7	-181329	561.964	0.77002	9.42329	3.94169	4.65228	4.18015	0.48591	0.10496
WW16D1	57627.6	-181329	562.039	0.75998	6.00866	3.96304	3.48929	2.00374	0.45539	0.06024
WW16D2	57627.6	-181329	562.044	0.79001	5.93348	3.96137	3.29151	2.18379	0.40374	0.05444
WW17D0	58142.9	-182268	478.905	0.94998	7.90791	3.63558	3.30527	3.98318	0.52435	0.09512
WW17D1	58142.8	-182268	478.785	0.96002	7.2979	3.73124	3.98326	2.76231	0.49773	0.05459
WW17D2	58142.7	-182268	479.082	0.92999	5.18944	3.77547	3.13912	1.59097	0.42734	0.032
WW18D0	58041	-179553	804	1.06	8.69885	3.68227	4.29597	3.69244	0.60679	0.10365
WW18D1	58040.9	-179553	804.087	1.05	6.56212	3.7	3.63793	2.3853	0.47056	0.06834
WW18D2	58040.8	-179553	804.064	1.04001	5.50283	3.63615	3.24468	1.82123	0.38795	0.04897
WW18D3	58040.7	-179553	804.085	1.06	5.28466	3.72	3.17818	1.67108	0.39726	0.03814
WW19D0	57377.6	-179934	677.07	0.86002	6.33238	3.7372	2.91005	3.03091	0.35165	0.03977
WW19D1	57377.5	-179934	677.702	0.85999	6.12084	3.81244	3.45692	2.21802	0.38949	0.05641
WW19D2	57377.4	-179934	677.164	0.87998	5.56334	3.7292	3.20754	1.96155	0.34997	0.04428

WW20D0	57693.1	-180625	627.536	0.73001	7.1545	3.83095	3.72552	2.78679	0.5512	0.09099
WW20D1	57693.1	-180625	627.516	0.66998	8.68964	3.87508	4.32094	3.693	0.57754	0.09817
WW21D0	58503.6	-181949	554.327	0.84	7.59487	3.95333	3.88911	3.03731	0.57932	0.08913
WW21D1	58503.5	-181949	554.261	0.81	6.09647	3.8579	3.46755	1.98955	0.58736	0.05201
WW21D2	58503.4	-181950	554.685	0.73999	4.70209	3.77001	2.68829	1.56225	0.40406	0.0475
WW22D0	57648.3	-179861	719.762	0.98001	9.67251	4.09428	4.51351	4.33286	0.68023	0.14592
WW22D1	57648.2	-179861	719.644	1.00999	10.1227	3.96139	4.71148	4.621	0.65606	0.13417
WW22D2	57648.1	-179861	719.711	0.98999	5.4901	4.00253	3.14081	1.80552	0.49344	0.05033
WW22D3	57647.9	-179861	719.813	0.92	7.72371	4.03022	3.78048	3.37593	0.45197	0.11533
WW23D0	57667.3	-179404	788.189	0.875	5.96123	3.9816	3.53257	1.8326	0.50891	0.08715
WW23D1	57667.1	-179404	787.621	1.03999	5.94328	3.75181	3.47424	1.93392	0.46146	0.07366
WW23D2	57666.9	-179404	788.113	0.95	5.61006	4.00263	3.06775	1.97725	0.49197	0.07308
WW24D0	56945.7	-179738	697.436	0.5	5.88097	3.62202	3.95816	1.34202	0.5395	0.04129
WW24D1	56945.8	-179738	697.163	0.5	5.65242	3.85639	4.22452	0.78804	0.59238	0.04748
WW24D2	56945.9	-179738	697.126	0.47	8.69359	3.94128	3.98662	4.02779	0.57722	0.10196
WW29D0	58716.2	-179611	871.624	0.73	7.65884	4.30425	4.24524	2.7337	0.59765	0.08224
WW29D1	58716.2	-179611	871.419	1.06	11.4584	4.22963	6.11602	4.35833	0.88632	0.09768
WW29D2	58716.2	-179611	871.654	0.73	6.88564	4.26397	4.53658	1.71565	0.58276	0.05065
WW30D0	58292.2	-178741	939.518	0.85501	6.77834	4.3048	4.28661	1.84214	0.60438	0.04521
WW30D1	58292.1	-178741	939.462	0.8	6.7454	4.40675	4.1679	1.97399	0.54585	0.05766
WW30D2	58292	-178741	939.688	1.05	8.32504	4.29286	4.56842	3.04972	0.60557	0.10134
WW30D3	58291.9	-178741	939.265	0.84001	9.23494	4.42845	4.89944	3.62611	0.63456	0.07484
WW31D0	60199	-181092	875.317	1.14999	7.55254	4.02991	4.40727	2.47928	0.64654	0.01945
WW31D1	60199	-181092	875.147	1.28	11.6898	4.03367	6.16356	4.5778	0.90134	0.04714
WW31D2	60199	-181092	875.077	1.34998	8.58805	4.03533	4.4496	3.38813	0.69644	0.05387
WW33D0	59191.1	-179626	927.365	0.88998	12.57	4.43595	6.15522	5.53494	0.76949	0.11035
WW33D1	59191.2	-179626	927.417	0.85999	8.78836	4.24489	5.05436	2.92871	0.7288	0.07649
WW33D2	59191.3	-179627	927.244	0.92002	8.2244	4.36445	4.66094	2.84328	0.65048	0.0697
WW34D0	59512.8	-180174	902.003	1.70001	7.05291	3.96865	3.60261	2.90409	0.48257	0.06363
WW34D1	59512.6	-180174	902.165	1.63001	6.71373	3.9916	3.48047	2.65822	0.50061	0.07443
WW34D2	59512.5	-180174	902.178	1.70001	5.64986	3.92817	3.25387	1.86891	0.47865	0.04844
WW35D0	58725.9	-180374	788.36	1.75	8.45361	4.0756	4.56167	3.14943	0.69271	0.04979
WW35D1	58726	-180374	788.417	1.65002	7.49496	3.99233	4.19828	2.59088	0.66131	0.04449
WW36D0	59587.2	-181437	768.44	1.15	6.06152	4.04174	3.53562	1.93578	0.52998	0.06015
WW36D1	59587.1	-181437	768.512	1.20001	7.47775	4.0225	4.033	2.77899	0.59943	0.06632
WW36D2	59587.1	-181437	768.559	1.16	8.36101	4.02681	4.51418	3.12665	0.63298	0.0872
WW37D0	60045.9	-181356	827.241	1.155	6.24121	4.22117	3.93511	1.75265	0.51564	0.03782
WW37D1	60045.9	-181356	827.073	1.22	7.70473	4.29148	3.94574	3.13136	0.57557	0.05206
WW37D2	60046	-181356	827.093	1.17999	7.01275	4.22085	4.73395	1.54675	0.71678	0.01527
WW38D0	59348.5	-181661	707.203	2.57498	3.81898	3.76054	2.41211	1.02628	0.35978	0.0208
WW38D2	59348.2	-181661	707.436	2.35001	4.72518	3.75999	2.14405	2.24914	0.28569	0.04631
WW40D0	59974.8	-180496	921.484	1.46997	4.66699	3.92707	3.03052	1.22976	0.39017	0.01655
WW40D1	59974.9	-180496	921.235	1.51001	4.69465	3.86723	3.07732	1.20394	0.39684	0.01656
WW40D2	59975	-180496	921.586	1.51001	4.84311	3.89702	3.18966	1.1963	0.4398	0.01735
WW41D0	59067.3	-181535	689.887	0.88001	6.93769	4.09529	4.18791	2.07947	0.58991	0.08039
WW41D1	59067.2	-181535	689.681	0.9	5.97186	4.11445	3.95625	1.42343	0.53888	0.05331
WW41D2	59067.1	-181535	689.955	0.95001	6.92714	4.15389	4.26803	2.0144	0.58862	0.05608
WW42D1	58039.2	-180396	702.839	1.57999	4.12599	3.54234	2.62518	1.14053	0.33013	0.03015
WW42D2	58039.2	-180396	702.942	1.48999	2.73433	3.53242	1.89971	0.56705	0.25143	0.01614
WW44D0	59424.3	-180778	825.975	0.36002	4.48647	4.04999	2.88078	1.22961	0.3449	0.03117
WW44D1	59424	-180778	825.971	0.36002	3.71445	3.86835	2.77571	0.54912	0.37275	0.01686
WW44D2	59423.9	-180778	825.868	0.31998	3.46782	3.89842	2.4962	0.63791	0.31844	0.01527
WW45D0	57555.2	-180284	649.073	1.17001	5.85543	3.95939	2.98104	2.33632	0.45899	0.07908
WW45D1	57555.1	-180284	648.989	1.29001	7.15688	3.96604	3.69456	2.85445	0.51414	0.09373
WW46D0	57371.7	-180907	593.991	0.91998	8.02211	4.13348	4.16924	3.01806	0.76569	0.06912
WW46D1	57371.7	-180907	593.795	0.85999	8.34937	4.14558	4.4388	3.10264	0.71744	0.0905
WW46D2	57371.7	-180907	593.87	0.89999	9.90971	4.16445	5.07933	3.95243	0.78873	0.08923

WW47D0	57841.4	-181763	523.589	0.80499	6.93887	3.86137	4.14924	2.1465	0.59674	0.04639
WW47D1	57841.3	-181763	523.399	0.82001	7.23242	3.80464	4.15218	2.47315	0.55464	0.05246
WW48D0	57373.3	-181870	426.557	0.78003	9.19818	4.31429	4.8195	3.66632	0.68338	0.02899
WW48D2	57373.1	-181870	426.456	0.76996	10.5382	4.42077	5.7048	3.95638	0.84467	0.03232
WW49D0	57274.3	-181405	504.579	0.85004	8.6978	4.0373	4.77586	3.12143	0.7222	0.07831
WW49D1	57274.2	-181405	504.224	0.83997	8.09867	4.0444	4.46552	2.89132	0.67158	0.07025
WW49D2	57274.3	-181405	504.227	0.82996	6.24072	4.01185	4.09825	1.52651	0.58796	0.028
WW50D0	57792	-180096	694.715	0.95001	8.72015	4.161	4.7482	3.22909	0.67564	0.06722
WW50D2	57791.7	-180096	694.644	1.07999	8.75817	4.14195	4.57106	3.44267	0.6586	0.08584
WW51D0	58321.2	-179533	838.923	1.03	6.74078	3.88388	3.71015	2.43668	0.53948	0.05447
WW51D1	58321.1	-179533	838.957	1.03999	7.13407	3.8949	3.96574	2.52097	0.58779	0.05956
WW51D2	58321.1	-179533	838.746	1.08	6.13211	3.8788	3.40941	2.17795	0.47988	0.06488
WW52D0	58763.5	-179786	856.709	0.84	9.01199	4.29262	4.95434	3.24661	0.72213	0.08892
WW52D1	58763.4	-179786	856.404	0.84999	8.37121	4.22524	4.87348	2.73208	0.69633	0.06933
WW52D2	58763.4	-179786	856.53	0.87999	9.22297	4.29216	4.84454	3.61803	0.66746	0.09295
WW53D0	58391.9	-179896	799.638	0.9	8.75183	4.16934	4.6768	3.29337	0.65923	0.12243
WW53D1	58392	-179896	799.644	0.95001	9.78085	4.19905	4.80684	4.14414	0.7281	0.10177
WW53D2	58392	-179896	799.674	0.93	9.57002	4.24129	5.08332	3.61207	0.77107	0.10355
WW54D0	57881.8	-179279	822.385	1.20999	7.42028	4.03496	3.93407	2.8073	0.60244	0.07646
WW54D1	57881.8	-179279	822.501	1.14	6.75186	3.94404	3.40535	2.74161	0.53181	0.07309
WW54D2	57881.6	-179279	822.439	1.20999	8.4453	4.05198	4.29865	3.38081	0.69171	0.07412
WW55D0	57763.3	-179037	843.045	0.98999	7.46688	4.4296	4.32283	2.39473	0.68294	0.06639
WW55D1	57763.2	-179037	843.017	0.95001	8.43608	4.49705	4.53886	3.12501	0.69861	0.0736
WW55D2	57763.1	-179037	843.103	0.84	9.04884	4.59714	5.11176	3.12699	0.73088	0.0792
WW56D0	57577	-178862	850.135	1.10999	8.87732	4.11356	4.63147	3.46634	0.70173	0.07778
WW56D1	57576.9	-178862	849.377	1.01	8.85392	4.14089	5.29175	2.74393	0.75804	0.0602
WW56D2	57576.8	-178862	850.263	1.04	7.58095	4.07087	4.02815	2.87332	0.60557	0.07391
WW58D0	57793.9	-179594	774.266	0.80499	8.10052	3.89416	5.22235	2.06547	0.75404	0.05865
WW58D1	57794	-179594	774.07	0.87	8.41657	3.90069	5.07121	2.50433	0.74495	0.09608
WW58D2	57794	-179594	773.991	0.87	7.74443	3.85863	4.52447	2.50048	0.66723	0.05225
WW59D0	57295.2	-179397	745.946	2.15001	3.8992	3.3413	2.06907	1.47873	0.32562	0.02578
WW59D1	57295.3	-179397	745.691	2.16	3.74551	3.32213	2.11806	1.26823	0.34078	0.01845
WW59D2	57295.3	-179397	745.981	2.08	3.30827	3.32933	2.00279	0.96575	0.32263	0.0171
WW66D0	56903.2	-180220	619.124	1.13	9.27871	4.05319	4.4976	4.05484	0.62185	0.10443
WW66D1	56903.1	-180220	618.847	1.01999	6.09375	4.06981	3.9024	1.57511	0.59858	0.01766
WW66D2	56903.1	-180221	618.975	1.13999	7.59484	4.00088	3.60184	3.46769	0.4942	0.03111
WW67D0	58036.4	-180071	729.272	1.01	5.17653	3.77772	2.96886	1.73279	0.42334	0.05155
WW67D1	58036.4	-180071	729.079	1.05	6.07265	3.84371	3.37885	2.08211	0.56534	0.04635
WW67D3	58036.5	-180071	729.283	1.05	5.75541	3.90658	3.49481	1.73267	0.475	0.05293
WW68D0	58606.5	-182299	491.075	2.41	6.59924	3.72112	3.41402	2.62111	0.47751	0.08659
WW68D1	58606.8	-182299	491.121	2.50998	4.97512	3.82252	2.69246	1.82659	0.41003	0.04604
WW68D2	58606.9	-182299	490.944	2.72	4.14371	3.62832	2.26544	1.49175	0.34329	0.04323
WW69D0	57866.8	-180890	615.275	0.83499	8.97621	4.1821	4.65805	3.49202	0.78234	0.04379
WW69D3	57867.1	-180891	615.028	0.88	6.70132	4.1684	4.09127	1.88107	0.70013	0.02884
WW70D0	58077.5	-180927	635.288	1.12	7.7953	4.35241	4.27974	2.80315	0.64299	0.06942
WW70D1	58077.5	-180927	635.021	1.10001	8.75295	4.33464	4.57394	3.41911	0.67097	0.08893
WW70D2	58077.2	-180926	635.162	1.06	7.62334	4.34642	4.41879	2.47543	0.66581	0.0633
WW71LUD0	58373.5	-181357	623.755	2.73999	4.89249	3.78332	2.85346	1.57852	0.43423	0.02629
WW71LUD1	58372.8	-181357	623.914	2.63998	6.38293	3.80069	3.0391	2.81545	0.47844	0.04994
WW73LND0	59377.7	-179930	909.633	1.42999	7.6293	4.11031	4.44079	2.41996	0.71767	0.05087
WW73LND1	59377.7	-179930	909.471	1.39999	8.09633	4.10603	4.33988	2.99185	0.69797	0.06663
WW73LND2	59377.8	-179930	909.311	1.42999	6.46218	4.0628	3.79883	1.94806	0.66735	0.04794
WW74LUD0	59759.6	-180057	943.009	0.98999	7.02038	3.93111	3.91363	2.42783	0.62587	0.05305
WW74LUD1	59759.6	-180057	942.719	1.32501	5.50007	3.96968	3.16651	1.75598	0.56084	0.01674
WW74LUD2	59759.4	-180057	942.665	1.04001	5.82711	3.94382	3.67859	1.56836	0.56489	0.01527
WW75LDD0	59333.5	-180447	850.881	1.88995	6.1281	4.1038	3.38705	2.1447	0.55228	0.04407
WW75LDD1	59333.5	-180447	850.536	2.02002	7.37074	4.09481	4.09597	2.56819	0.65747	0.04911
WW75LDD2	59333.8	-180447	850.634	1.83002	8.0549	4.01075	3.85504	3.551	0.58454	0.06432
WW76LDD0	58440.8	-180388	749.345	2.16998	1.35578	3.43283	1.04165	0.16342	0.13469	0.01602
WW76LDD1	58440.5	-180388	748.726	2.52002	1.09115	3.53794	0.81044	0.13271	0.13227	0.01572

WW76LDD2	58440.7	-180389	748.896	2.66998	1.2991	3.46551	0.79249	0.37141	0.11844	0.01677
WW77LUD0	59086.1	-180418	831.818	2.06	7.45642	4.0199	4.11724	2.61674	0.67901	0.04343
WW77LUD1	59085.8	-180418	831.834	1.83002	6.98835	3.99983	3.81384	2.49824	0.64218	0.03409
WW77LUD2	59086	-180418	831.588	1.94	7.14984	3.94842	3.9663	2.49017	0.64855	0.04482
WW78LUD0	59755.9	-180603	885.473	2.21002	7.0686	4.15004	4.30436	2.14024	0.57689	0.0471
WW78LUD2	59758.6	-180603	887.186	2.20996	6.50242	4.20239	3.58893	2.31737	0.53891	0.05722
WW82LUD0	59237.7	-180099	886.185	1.03	9.88094	4.4102	5.6117	3.35832	0.84907	0.06185
WW83D0	58041.4	-179810	769.652	1.01	10.0472	4.14713	4.88296	4.40186	0.68308	0.07932
WW83D1	58041.2	-179810	770.111	1.01001	8.38577	4.19381	4.40621	3.27533	0.62631	0.07792
WW83D2	58041.2	-179811	769.839	0.97	8.06419	4.27623	4.44184	2.88139	0.66254	0.07842
WW84D0	58179.3	-179763	794.05	1.14001	9.79521	4.26149	4.63525	4.33691	0.73426	0.08879
WW84D1	58179.2	-179763	793.952	1.15999	8.00383	4.30034	4.3457	2.94662	0.63556	0.07594
WW84D2	58179.1	-179762	794.064	1.03999	13.3233	4.14096	5.59184	6.70092	0.93706	0.09351
WW85D0	58335.7	-179750	811.525	1.14999	7.44503	4.09948	4.12661	2.61013	0.65811	0.05018
WW85D1	58335.6	-179750	811.782	0.86	6.99927	4.06674	4.13052	2.24268	0.5756	0.05046
WW86D0	58376.1	-179817	806.675	1.45	6.13388	4.00462	3.37197	2.16269	0.55347	0.04575
WW86D1	58376	-179817	806.722	1.31999	5.40327	4.02159	2.92915	1.99258	0.41949	0.06205
WW86D2	58375.9	-179816	807.114	0.92	7.15461	4.15489	4.05402	2.42923	0.61477	0.05659
WW87BD0	58210.9	-179871	781.804	0.90799	7.9423	4.25529	4.17306	3.01737	0.67421	0.07766
WW88D0	58103.9	-179908	762.955	0.99001	7.77427	4.0998	3.84327	3.29464	0.59256	0.0438
WW88D1	58103.8	-179908	762.188	1.09001	8.48322	4.16605	4.68931	3.00717	0.70567	0.08106
WW88D2	58103.7	-179908	763.159	0.90999	5.72686	4.08518	3.83789	1.32683	0.54032	0.02181
WW89D0	58161.2	-180025	753.865	1.05	7.26096	4.24019	4.17535	2.38182	0.6417	0.06209
WW89D1	58161.1	-180025	753.555	1.03499	8.01247	4.22362	4.54935	2.74318	0.65232	0.06761
WW89D2	58161	-180025	753.659	1.14999	10.2932	4.29139	5.2433	4.15008	0.80129	0.09851
WW90D0	58196.6	-180114	744.86	1.78	1.83814	3.39085	1.33181	0.30776	0.1833	0.01527
WW91D0	58286.8	-179987	773.278	0.735	6.92775	4.4019	3.80031	2.54457	0.53621	0.04666
WW91D2	58286.4	-179987	773.138	0.85501	8.77747	4.21579	5.0257	2.89872	0.7894	0.06366
WW92D0	58306.7	-180034	770.239	0.89999	8.70041	4.27742	4.60034	3.38386	0.6414	0.07481
WW93D0	58465.1	-179953	798.337	0.49501	7.18488	3.98899	4.59816	1.94013	0.62894	0.01766
WW94LSD0	59827.3	-181461	795.494	1.51001	5.43014	4.12887	3.63127	1.28646	0.49785	0.01456
WW96LND0	59284.8	-181312	744.941	1.23999	7.56483	3.98975	4.23684	2.67907	0.57647	0.07245
WW97LUD0	59747.4	-181227	812.146	1.36999	7.77208	4.10139	4.36501	2.67411	0.67226	0.0607